

F.E.L.A.R.M
FINITE ELEMENT LAYERED ANALYSIS
OF REINFORCED MASONRY
AN EVALUATION OF THE POTENTIAL
FOR CONVERSION OF THE
PROGRAM F.E.L.A.R.C.

by

Trevor Small

A Thesis
Presented to the University of Manitoba
In Partial Fulfillment
of the Requirements for the Degree of
Master of Science in Civil Engineering

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TREVOR SMALL

A thesis submitted to the Faculty of Graduate Studies of
the University of Manitoba in partial fulfillment of the requirements
of the degree of

MASTER OF SCIENCE

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ABSTRACT

FELARM is the acronym for Finite Element Layered Analysis of Reinforced Masonry. FELARM was developed by modifying the computer program FELARC.

FELARC was originally developed at the University of Calgary by Ghoneim A.M. Ghoneim. The program uses a nonlinear finite element approach that permits the analysis of reinforced and/or prestressed concrete structures.

In the present investigation, the program FELARM associates each layer within each element with a different concrete material. This approach enables the modelling of masonry structures with relative ease.

This thesis also includes a description of the programs, their scope and limitations, input instructions, sample input and output, and an evaluation of the programs in their present state.

CHAPTER 1

INTRODUCTION

1.1 HISTORY

FELARC is the acronym for Finite Element Layered Analysis of Reinforced Concrete. It was originally developed in 1979 by Ghoneim A.M. Ghoneim as part of his Ph.D. thesis, under Dr. Amin Ghali at the University of Calgary. The program was coded in FORTRAN and designed to be utilized on the CDC CYBER 172 computer system at the University of Calgary.

By 1982 FELARC had been brought to the University of Manitoba by Professor John Glanville to aid in the analysis of thermal effects in masonry walls. It was at this time that the author was given the opportunity to adapt FELARC to the AMDAHL computer system at the University of Manitoba. With the aid of Dr. Bruce Pinkney, the program had achieved executable status by 1985.

However, upon executing the program it was discovered that the program had severe limitations. The program has been claimed to be able to perform creep and shrinkage analysis of concrete structures. However the author has not been able to get FELARC to perform creep or shrinkage analysis on the simplest of examples. Whenever instructed to do so it was found that divergence of the solution always occurred, causing the program to terminate the analysis.

Upon consultation with Dr. Amin Ghali in 1986, it was discovered that this was not just a local problem. In fact the supposition that FELARC can perform creep and shrinkage analysis is false. FELARC is able to perform a nonlinear analysis of a loaded structure, but the analysis cannot include the effects of creep or shrinkage. As the original purpose of bringing FELARC to the University of Manitoba was to apply the program to masonry structures, the author decided to convert FELARC to handle this type of analysis more easily and exclude effects of creep and shrinkage.

FELARC uses the layered stiffness approach, wherein a structure is modelled as a group of elements. Each element is in turn divided into layers. Therefore the model of a structure consists of an assemblage of thin plates. In FELARC each element may be composed of one concrete material type only. This feature makes FELARC quite acceptable for concrete structures only. Since grouted masonry walls consist of three different concrete materials, namely the block, mortar and grout, the modelling of such a structure is a difficult operation.

Therefore, the author decided to convert FELARC to FELARM such that each layer of each element of a model may be identified as a separate concrete material if required. This unique approach enabled modelling of a masonry wall or any masonry structure to be done with relative ease.

1.2 LITERATURE REVIEW

The last twenty years has produced a large variety of finite element programs for all fields of engineering. The earliest programs were based on linear models. Today, programs exist that perform a nonlinear material analysis as well as a nonlinear geometric analysis. This literature review will focus on the application of finite element analysis to concrete structures.

In 1967, Ngo and Scordelis [1] published the first paper on the application of finite element analysis to reinforced concrete beams. They performed a linear elastic analysis using constant strain triangular elements for concrete and steel. They incorporated special link elements to simulate bond between the steel and concrete elements. To incorporate the effects of cracking, when cracking was indicated by the separation of nodal points, the topology of the structure was redefined using predefined crack patterns.

Nilson [2] (1967, 1968) introduced nonlinear material properties to finite element analysis. He applied the method to concrete members under concentric and eccentric tension. Load was applied directly to the reinforcing steel in increments. After each increment cracking was checked and if cracking occurred the analysis was stopped and repeated with predefined cracks in the new topology.

In 1968, Bresler and Bertero [3] developed a finite element analysis using a "boundary layer" between the steel reinforcement and concrete. The "boundary layer" had modified elastic modulus to account for bond deterioration under cyclic loading.

In 1968, Rashid [4] proposed modelling cracked concrete as an orthotropic material. This method was employed by Franklin [5], in 1970, who introduced nonlinear material properties to analyze plane stress systems for the entire range of loading. Franklin applied the load in increments and after each increment the material properties were modified to account for cracking, and the unbalanced stresses determined were redistributed.

Zienkiewicz et al. (1969) [6], Cervenku and Gerstle (1971) [7], eliminated the need for modelling the steel as separate elements by superimposing the stiffness of the steel into that of the concrete in the constitutive matrix.

Yuzugullu [8] and Schnobrich [9] (1972, 1973) in their study of shear-wall--frame systems used concrete tensile strain as a measure of crack widths. They allowed cracks to open and close during loading. In 1976 Darwin and Pecknold [10] proposed a solution for shear-wall--frame systems under cyclic loading.

In the 1970's numerous researchers investigated the plane stress behavior of concrete structures using various in-plane elements, constitutive relationships and failure criteria. No attempt is made here to review each of these in the literature. The following will present some of the techniques used in finite element analysis from a macroscopic viewpoint.

In 1971 Jofriet and McNiece [11] used a bilinear moment-curvature relationship in the study of reinforced concrete slabs. Each finite element was divided into layers across the thickness. Cracking was accounted for by modifying the stiffness as the cracks propagated layer by layer. This approach was very popular. Berg (1973) [12] and Rajagobul [13] (1976) included nonlinear geometry effects in their analysis.

Hand et al. [14] (1972, 1973) used the layering approach to study slabs and shells. They coupled in-plane and bending stiffnesses. Lin and Scordelis [15] (1975) used a similar approach by using a triangular shell element composed of a constant strain in-plane element and a linearly constrained triangular plate bending element.

Bell and Elms [16] (1974) used a "modified EI" approach. They reduced the flexural and membrane stiffnesses as functions of the stress level.

Lin [17] (1975) assumed a biaxial state of stress for both concrete and steel. Hand et al. (1973) used an approach similar to that of Lin for the analysis of plates and shells. They used a layered, shallow-shell, rectangular element.

Kabir [18] (1976) extended Lin's work to include the time-dependent effects of creep, shrinkage and load history.

Arnesen [19] (1979) used a triangular shell element with numerical integration through the thickness. He used endochronic theory for the concrete and a trilinear stress-strain relationship for steel. Arnesen also included the effects of nonlinear geometry by using an updated Lagrangian approach.

Floegl [20] (1981) used curved triangular shell elements. He included both nonlinear material and nonlinear geometry. The effect of tension stiffening was based on bond slip between reinforcing steel and concrete.

There has not been much work done in modelling the nonlinear behavior of three-dimensional concrete structures. This is due mainly to the lack of knowledge of the triaxial state of stress of concrete and the expensive computational effort required for such models.

More recently there have been advances made in shell analysis. Curved shell finite elements are being used to give better geometric representation. The trend is toward using more nodes per element to obtain as many as 40 degrees per element, and thus making the applications more versatile. This has allowed for a relatively coarse mesh to be used in the analysis and thus reduce the amount of storage required for the solution.

1.3 OBJECTIVE AND SCOPE

The objective of the present study was to develop a computer program for the nonlinear analysis of masonry structures including time-dependent effects.

Masonry structures are, from a macroscopic viewpoint, concrete structures. It follows that the theories involved in the analysis of concrete structures should be applicable to masonry structures. Therefore it was the author's intention to use an available nonlinear finite element program for concrete structures and use it to model masonry structures. The method employed can be broken down into three phases.

The first phase was to obtain a suitable nonlinear finite element program and adapt it to the computer system at the University of Manitoba. For this purpose the program FELARC (Finite Element Layered Analysis of Reinforced Concrete) was chosen because the range of non-linear analysis and time-dependent effects were reportedly operational. The reported scope of FELARC will be presented later.

The second phase was to make any modifications necessary to the program to facilitate its use in analyzing masonry structures. This was done and the program FELARM was created. FELARM is the acronym for Finite Element Layered Analysis of Reinforced Masonry. The theory and modifications to FELARC will be presented later.

The third phase of the study was to evaluate FELARC and FELARM as to their usefulness by comparing the program results with existing experimental research.

As FELARC has been verified, under certain applications, by experimental research, FELARM was verified by comparisons with FELARC. This was achieved by comparing output data of FELARM with output data from FELARC supplied by the University of Calgary.

1.4 SCOPE OF FELARC

FELARC uses a finite element method in the analysis of concrete structures that may be considered as an assemblage of thin plates subjected to bending and in-plane forces. The method employed should predict the behaviour of concrete structures over the full range of loading, up to failure, while accounting for the following effects:

- a) material nonlinearity of concrete, including yielding, strain-softening and crushing of concrete in compression.
- b) bilinear material effects of steels, including yielding and strain-hardening of reinforcing and prestressing steels.
- c) cracking and tension stiffening of concrete.
- d) cyclic loading.
- e) baushinger effect on steel under cyclic stress.
- f) time-dependent effects of creep and shrinkage including differential shrinkage.
- g) changes in temperature of concrete surfaces or changes of temperature within the thickness of the element with respect to age of concrete.
- h) prestress loss due to elastic shortening and time-dependent effects.

It should be noted that the program does not take into account the effect of bond slip between reinforcing steels and concrete. Perfect bond is assumed. Nonlinear geometric effects and fatigue are not considered. The program uses the following finite elements in the analysis:

- a) A quadrilateral shell element composed of:
 - i) A quadrilateral in-plane element with 12 degrees of freedom, developed by Sisodiya et al. [27] (1972).
 - ii) A quadrilateral plate bending element with 12 degrees of freedom. The element uses natural coordinates to allow for elements other than rectangles.

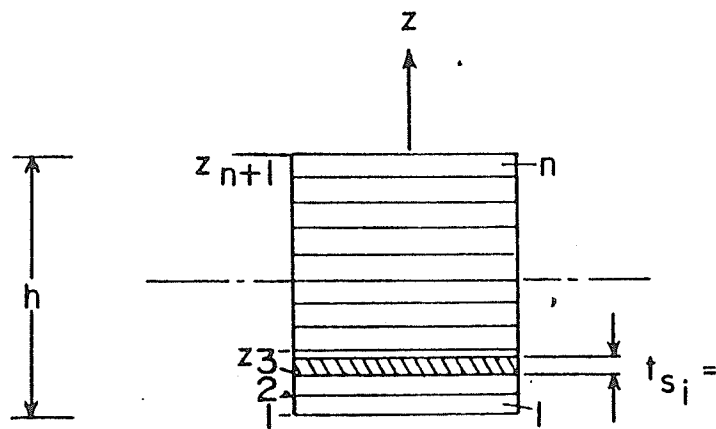
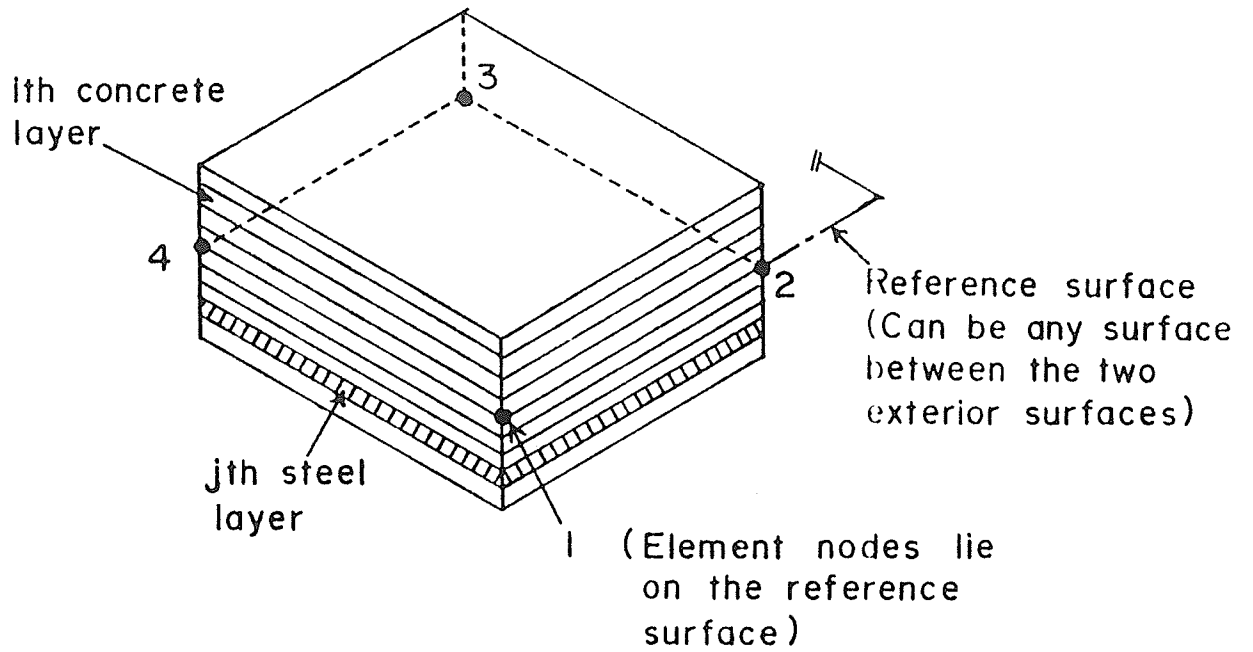
- b) One-dimensional truss element, capable of axial load only.
- c) A boundary element developed by Wilson [21] (1972), used to model prescribed displacements, elastic supports and skewed boundary conditions.

The stiffness matrix of the quadrilateral shell element is determined by dividing the thickness of the element into prescribed layers of concrete and "smeared" steel layers, as shown in Figure 1. The layer stiffnesses are determined by numerical integration at each integration point in the Z-direction. The layer stiffnesses are then summed to create the stiffness matrix of the element.

The "smeared" steel layers occupy the same space as the concrete and therefore have no volume. This method of modelling the steel is appropriate only when the steel is distributed evenly over the area of the element such as in slabs.

When the steel is concentrated in heavy reinforcing bars or prestressing tendons it is advisable to model the steel as separate elements. In this method the stiffness of the steel is calculated by numerical integration and superimposed directly on the stiffness matrix of the host element. Perfect bond is assumed. Internal forces due to initial stress and initial strain are calculated as equivalent nodal forces and added to the element nodal force vector.

The program models unbonded prestressing cables, steel ties, and concrete axial members with the one-dimensional truss element. For prestressing cables, the end anchorages should coincide with nodal points.



Volume occupied by steel layer occupies the same volume as part of a concrete layer

smeared thickness of steel layer

$$= \frac{A_s}{s} = \frac{\text{Area of one bar}}{\text{Spacing between bars}}$$

Figure 1. Quadrilateral Shell Element

Nonlinear material analysis is performed by an incremental iterative tangent stiffness method. The model developed by Kabir (1976) [18] is adapted to account for time-dependent effects. The time domain of loading with respect to the age of the concrete is divided into prescribed time steps. The nonlinear response of the structure is determined during each time step using the aforementioned technique.

1.5 SCOPE OF FELARM

FELARM has exactly the same scope as FELARC except for the following modifications.

FELARC identifies only one concrete material type with each quadrilateral shell element. FELARM identifies one concrete material type with each layer of the quadrilateral shell element. With this method, as each layer stiffness is calculated and then summed to get the stiffness matrix of the element, the true stiffness of the element is obtained with relative ease. The changes required in the input and programming are presented later in this report.

CHAPTER 2

THEORETICAL BACKGROUND OF PROGRAM FELARC

A complete theoretical background was given in the Ph.D. dissertation of Ghoneim A.M. Ghoneim. In this report only the pertinent facts directly related to the program will be presented in brief form.

2.1 CONCRETE STRESS-STRAIN RELATIONSHIP

The stress-strain envelope used is composed of two relationships. The Saenz relationship is used up to concrete's ultimate compressive strength. The Smith-Young curve is used from ultimate compressive strength to the ultimate strain of the concrete. Algebraically the two curves are as follows:

Saenz [23] (1969)

$$\sigma = \frac{E_0 \epsilon}{1 + \left[\frac{E_0}{E_s} - 2 \right] \frac{\epsilon}{\epsilon_{cu}} + \left[\frac{\epsilon}{\epsilon_{cu}} \right]^2} \quad (1)$$

Smith and Young (1955) [24]

$$\sigma = \sigma_0 \left[\frac{\epsilon}{\epsilon_{cu}} \right] e^{\left[1 - \frac{\epsilon}{\epsilon_{cu}} \right]} \quad (2)$$

Where σ = stress

ϵ = strain corresponding to stress

E_0 = initial tangent modulus of elasticity

ϵ_{cu} = strain at maximum compressive strength

E_s = secant modulus = $\frac{\sigma_0}{\epsilon_{cu}}$

f'_0 = maximum compressive strength

2.2 TIME-DEPENDENT PROPERTIES

The age of concrete determines the amount of hydration that has taken place and has a direct effect on the concrete strength.

The program allows the user to input the concrete properties obtained from experiments, or use equations developed by ACI Committee 209. The equations used are:

$$f'_c(t) = \frac{t}{4.0 + 0.86t} f'_c(28) \quad (3)$$

$$f'_t(t) = \frac{1}{3} \sqrt{w f'_c(t)} \quad (4)$$

$$E_0(t) = 33.0 w^{1.5} \sqrt{f'_c(t)} \quad (5)$$

where t = age in days after casting

$f'_c(t)$ = uniaxial compressive strength in psi at time t

$f'_t(t)$ = uniaxial tensile strength in psi at time t

$f'_c(28)$ = 28 day compressive strength in psi

$E_0(t)$ = initial tangent modulus of elasticity in psi at time t

w = unit weight of concrete in lb/in.³

The strain corresponding to maximum compressive strength, proposed by Hognestad [25] (1951), is given by:

$$\epsilon_w(t) = w f_c(t) / E_o(t) \quad (6)$$

2.2.1 Shrinkage

Shrinkage strains obtained from experimental data may be input in the program as initial strains or they may be calculated from ACI Committee 209 formulae. The formulae is given as:

$$\epsilon_{sh}(t) = K_S K_H K_h \left[\frac{(t - \zeta_o)^c}{f + (t - \zeta_o)^c} \right] \epsilon_{sh}(t_\infty) \quad (7)$$

Where $\epsilon_{sh}(t)$ = shrinkage strain at time t

$\epsilon_{sh}(t_\infty)$ = ultimate shrinkage strain

= $415 (10)^{-6}$ to $1070 (10)^{-6}$

t = time in days

ζ_o = age of curing in days

f, c = constants determined from experiments for standard conditions

c = 0.90 to 1.10

f = 20 to 130

K_S = slump correction factor 0.8 to 1.0

K_H = humidity correction factor

= $1.40 - 0.01H$ for $40 \leq H \leq 80$

= $3.0 - 3.03 H$ for $80 \leq H \leq 100$

H = relative humidity

K_h = size correction factor

The program selects the suitable correction factors and constants according to the slump, relative humidity and volume-to-surface ratio input as data.

2.2.2 Creep

Creep may be defined as the part of deformation in excess of instantaneous elastic deformation which is due to sustained levels of stress. The most important factors affecting creep in concrete are age of concrete loading, duration of loading, and temperature variations during the creep period.

For a uniaxial state of stress and an aging material, creep strain may be expressed as:

$$\epsilon^c(t) = \int^t C(\tau, t-\tau) \frac{\partial \sigma(\tau)}{\partial \tau} d\tau \quad (8)$$

where

- $\epsilon^c(t)$ = creep strain at time t
- $C(\tau, t-\tau)$ = creep compliance function at time t for age of loading τ
- $\nu(\tau, t-\tau) = \frac{1}{E(\tau)}$
- $\sigma(\tau)$ = stress at time τ
- $E(\tau)$ = tangent modulus on concrete at age τ
- $\nu(\tau, t-\tau)$ = creep coefficient defined as the ratio between creep strain occurring in the interval $t - \tau$, and the instantaneous elastic strain at age τ
- τ = age at which stress is applied

The program uses Kabir's (1976) [18] formulation for determining creep as an incremental method. The creep compliance function used is given as:

$$C(\tau, t-\tau, T) = \sum_{i=1}^m a_i(\tau) [1 - e^{-\lambda_i \Phi(T)(t-\tau)}] \quad (9)$$

where $a_i(\tau)$ = scale factor at age of loading τ
 λ_i = exponential constant determining shape of logarithmically decaying creep curve
 $\Phi(T)$ = temperature shift function for temperature T
 m = number of terms in the summation

Thus the creep strain increment, $\Delta \epsilon^c$, at any time step n , going from t_n to t_{n+1} , may be represented as:

$$\{\Delta \epsilon_n^c\} = [D_0]^{-1} \sum_{i=1}^m \{A_{in}\} [1 - e^{-\lambda_i \Phi(T_n) \Delta t_n}] \quad (10)$$

Where $\{A_{in}\} = \{A_{in-1}\} [e^{-\lambda_i \Phi(T_{n-1}) \Delta t_{n-1}}] +$ (11)

$$\{\Delta \sigma_n\} a_i(t_n) \quad (12)$$

$$\{A_{i1}\} = \{\Delta \sigma_1\} a_i(t_1)$$

$$[D_0] = \begin{vmatrix} 1 & -\nu_c & 0 \\ -\nu_c & 1 & 0 \\ 0 & 0 & 2(1+\nu_c) \end{vmatrix} \quad (13)$$

ν_c = Poisson's ratio

The advantage of this method is that for the computation of each new creep strain increment, only the stress of the last time step is required, rather than the total stress history of the structure. In

order to use this method, the parameters m , $a_i(\tau)$, λ_i and $\phi(T)$ have to be calculated. The procedures for determining these parameters are discussed later in the user manual for the program.

It has been shown that creep is proportional to stress until the onset of internal microcracking. The microcracking occurs at stress levels of about 35% of the compressive strength of the material. To account for creep after microcracking, Bresler et al. [28] (1973) proposed using an "effective stress" in calculating creep strain. "Effective stress" may be calculated as

$$\sigma_{\text{eff}} = \sigma \quad \text{for } \frac{\sigma}{f'_c} \leq 0.35 \quad (14)$$

$$\sigma_{\text{eff}} = 2.33\sigma - 0.465 f'_c \quad \text{for } 1.0 \geq \frac{\sigma}{f'_c} > 0.35 \quad (15)$$

2.2.3 Time Shift Parameters

The time shift principle can be explained as follows. Let curve C represent the specific creep curve at temperature T (Figure 2). Let curve C_{T_0} represent the specific creep curve at temperature T_0 less than T . For a thermorheologically simple material the curve C_T can be shifted a distance $\psi(T)$, parallel to the logarithmic time-scale to coincide with C_{T_0} . Mathematically,

$$C_T(\ln t) = C_{T_0}(\ln t + \psi(T)) \quad (16)$$

letting $\phi(T) = e^{\psi(T)}$ = shift function for temperature T

$$\begin{aligned} C_T(\ln t) &= C_{T_0}(\ln t + \ln \phi(T)) & (17) \\ &= C_{T_0}(\ln [t\phi(T)]) \\ &= C_{T_0}(\ln t^*) \end{aligned}$$

where $t^* = t \phi(T)$.

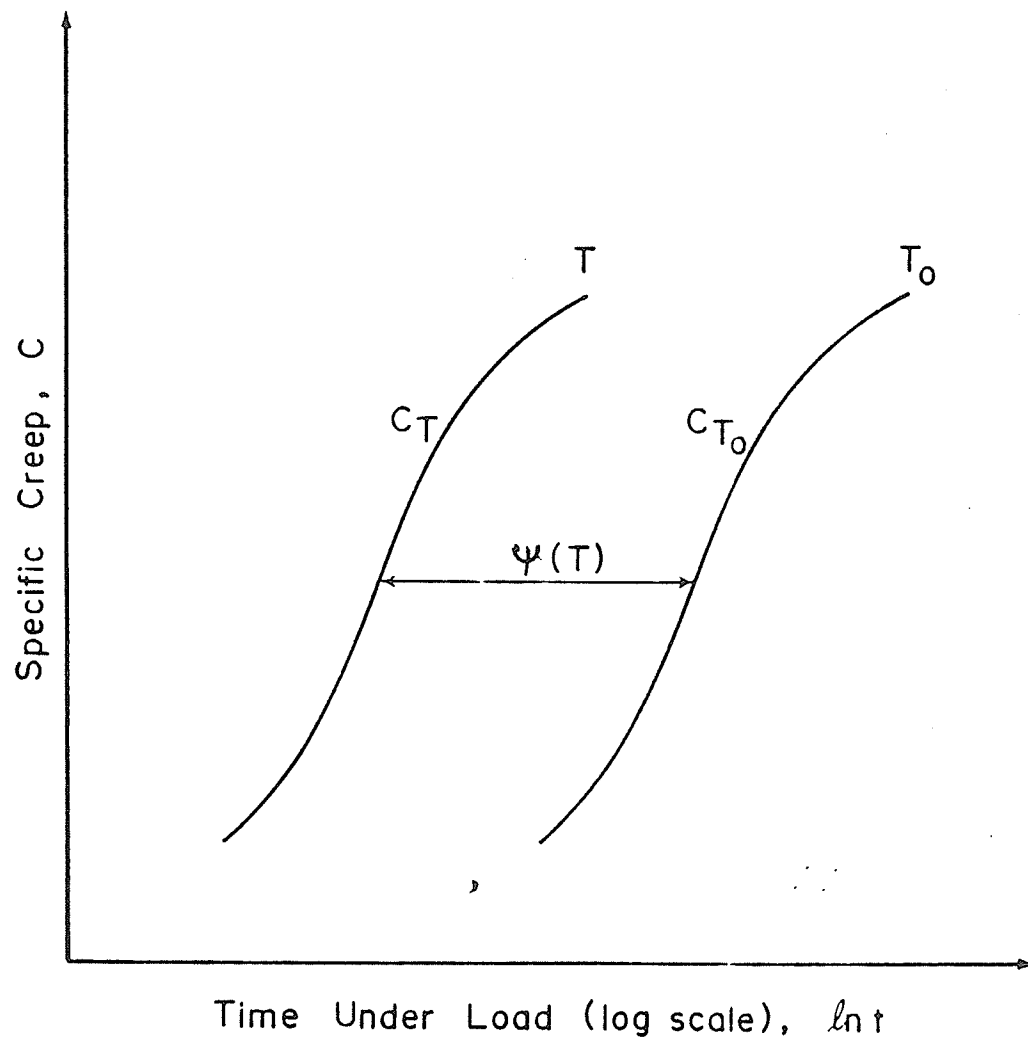


Figure 2. Specific Creep Curve