THE UNIVERSITY OF MANITOBA

FREE CONVECTION
FROM A VERTICAL PLATE TO
WATER NEAR $4^{o}C$

by

M. P. Hasinoff

A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfilment of the Requirements for the Degree

of Master of Science

Department of Mechanical Engineering

Winnipeg, Manitoba

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ABSTRACT

This thesis is an experimental investigation of the free convection heat transfer process from a vertical isothermal plate to water at temperatures below $4^{\circ}C$, the temperature of the maximum density of water.

Earlier researchers have defined the boundaries of the different flow regions in terms of plate and bulk temperatures. The regions in which unidirectional flow exists have been throughly investigated analytically and experimentally. However, in the region in which bidirectional flow exists an exact theoretical analysis was not available. A number of researchers have made experimental measurements in this region and experimental correlations for the local heat flux are available. The nature of the flow has been investigated and researchers have postulated that certain flow patterns exist.

Initial tests in the present investigation were conducted in a region in which purely downward flow was known to exist. Experimental measurements were recorded and the mean experimental heat transfer coefficients were compared with theoretical predictions obtained using equations developed in an earlier investigation. The results gave an indication of the accuracy

of the experimental apparatus.

The balance of the tests were conducted in the bidirectional flow region. Mean experimental heat transfer coefficients were evaluated and compared with predictions from an earlier experimental correlation. The results of the comparison suggest that a further refinement of the correlation is necessary.

The bidirectional flow patterns were examined using the "thymol blue" flow visualization technique. Neutrally bouyant dye is produced in the region of interest and the dye traces out the flow patterns. The flow visualization studies confirmed the existence of two types of bidirectional flow. In "separated" bidirectional flow the inner upward flowing boundary layer separates from the plate and reverses its direction at the top of the heated section to flow into the outer downward flowing boundary layer. In "non-separated" bidirectional flow the inner upward flowing boundary layer continues flowing up past the top of the heated section and does not separate from the plate.

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B	Coefficients defined by Equations (1-10), (1-11), and (1-15)
C	Coefficient defined by Equations (1-8), (1-11), and (1-16)
C_I	Coefficient defined by Equation (1-10)
C_{III}	Coefficient defined by Equation (1-11)
C_{II-N}	Coefficient defined by Equation (1-14)
C_i	Coefficient defined by Equation (1-15)
C_o	Coefficient defined by Equation (1-16)
D	Coefficient defined by Equations (1-10), (1-11), and (1-16)
E	Coefficient in Equations (1-17) and (1-18)
F	Coefficient in Equation (1-17) and (1-18)
G	Coefficient in Equation (1-10)
Gr	Grashof number
Gr_x	Conventional local Grashof number
Gr'_x	Local Grashof number defined by Equation (1-9)
Gr''_x	Local Grashof number defined by Equation (1-1)
Gr_i	Local inner Grashof number defined by Equation (1-13)

Gr_o	Local outer Grashof number
g	Gravitational acceleration
h	Heat transfer coefficient
k	Thermal conductivity
L	Plate height
m	Exponent defined by Equation (1-18)
Nu	Nusselt number
n	Exponent defined by Equation (1-17)
P	Density function in Equation (1-2)
Pr	Prandtl number
Q	Density function in Equation (1-2)
Ra	Rayleigh number
R_x	Local thermal resistance of the boundary layer
R_T	Electrical resistance of heater at temperature, T
R_5	Electrical resistance of heater at 5^oC
S	Slope of heater resistance versus temperature curve, temperature gradient from last heated to first unheated plate
T	Temperature (^oC)
U	Density function in Equation (1-5)
x	Vertical space coordinate
Z	Coefficient in Equation (1-10)
α	Bouyancy function defined by Equation (1-2)

β	Thermal expansion coefficient
γ	Angle around the point (4^oC, 4^oC) on a graph of plate temperature versus fluid temperature as indicated on Figure 1
θ	$T - T_\infty$
ν	Kinematic viscosity
ρ	Density
ϕ	Temperature function defined on page 17

SUBSCRIPTS AND SUPERSCRIPTS

(See also the variables to which they are attached in the above list)

b	in the bulk fluid
f	at the film temperature, $(T_p + T_\infty)/2$
i	of the inner boundary layer
o	of the outer boundary layer
p	at the plate
T	theoretical
x	the local value at position x
I	in Region I
II-N	in Region II-N
III	in Region III
—	average value
'	a non-standard definition
∞	in the bulk fluid

CHAPTER 1

INTRODUCTION

1.1 The Problem

For most fluids density varies linearly with temperature. However, water, antimony, gallium, and bismuth all possess density maxima above their freezing points. For water the maximum density occurs at $4^{o}C$. The significance of this temperature becomes apparent when one considers the enormous amount of water on the earth's surface that seasonally passes through the point of maximum density.

The density maximum of water is not a factor in forced convection unless the velocity is very low as in mixed free and forced convection.

In the analysis of free convection, with density difference the driving force of fluid movement, one must take into account the peculiar behavior of water at $4^{o}C$ and consider the effects it may have on the process.

A great many natural bodies of water pass through the density maximum in the spring and fall. Discharges of heated condenser water from industrial plants, power plants and chemical plants into ice-covered or partially ice-covered bodies of water can disrupt the normal formation of ice. A thermal plume can influence the physical and biological makeup of a body of water. A knowledge of the behavior of water near $4^{\circ}C$ is important in furthering the study of the physical processes that take place.

1.2 The Phenomenon

Vanier (2) mapped the free convection process into four regions as shown in Figure 1; a map of plate temperature versus bulk temperature. Yuill (1) has provided a complete description of the processes involved in free convection from a vertical flat plate to water at $0^{o}C$ to $26.8^{o}C$. The process is divided into zones of purely downward flow and purely upward flow and a zone in which there is bidirectional flow.

With the bulk temperature at $0^{o}C$ and the plate temperature between $0^{o}C$ and $4^{o}C$ water adjacent to the plate is heated. Its density exceeds that of the bulk fluid and a net downward flow of fluid results. For plate temperatures between $4^{o}C$ and $8^{o}C$, water within the thermal boundary layer is still denser than the bulk fluid and sinks. At a plate temperature just above $8^{o}C$ the water immediately adjacent to the plate is lighter than the bulk fluid but does not rise; it is pulled downward due to the drag of the outer downward moving thermal boundary layer. The water adjacent to the plate surface has sufficient bouyancy to balance the downward viscous shear force at a plate temperature of about $12.4^{o}C$. This results in a zero velocity gradient at the plate. Between $12.4^{o}C$ and $26.8^{o}C$ the inner upward flowing boundary layer

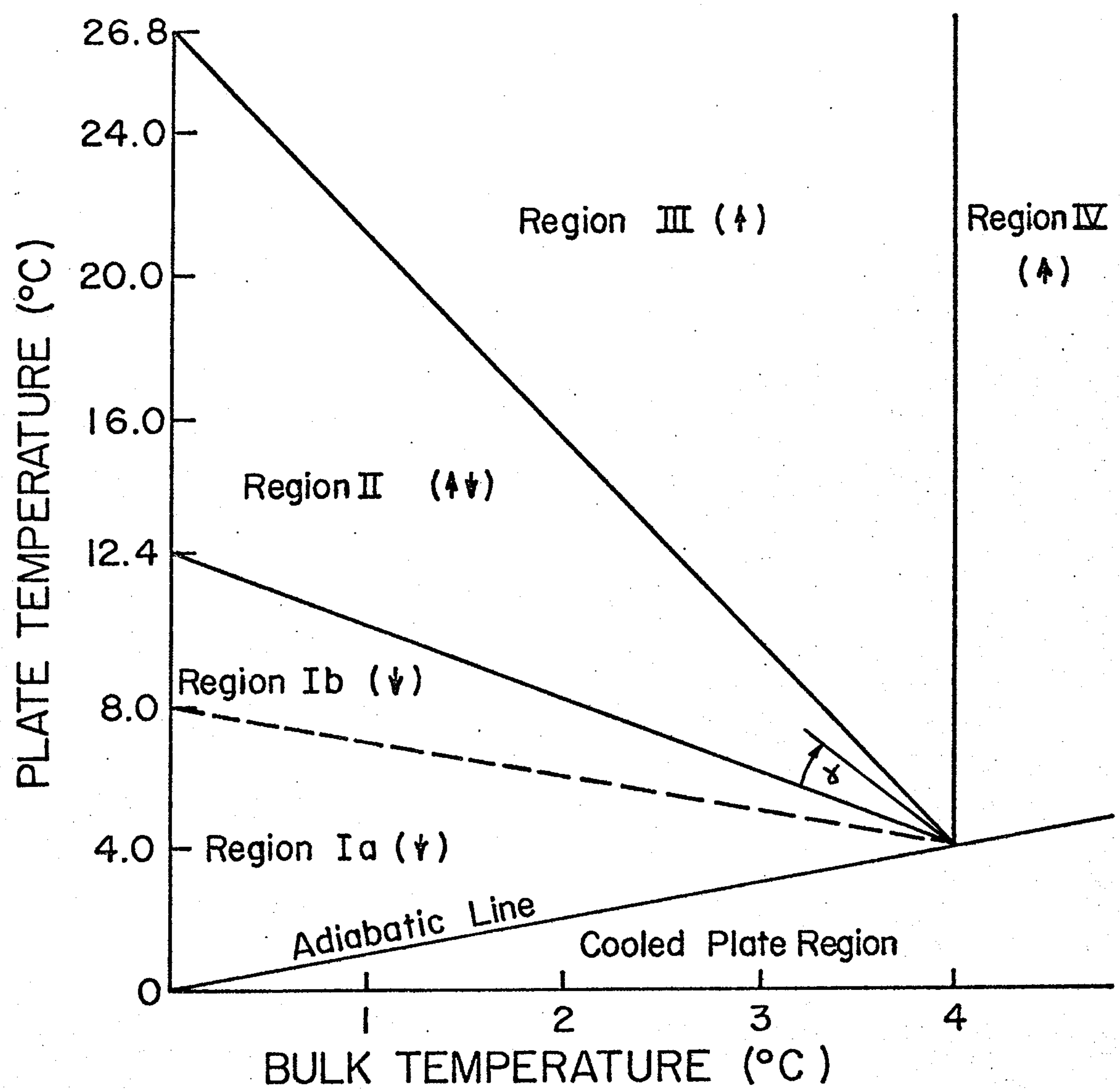

FIGURE 1 MAP OF FREE CONVECTION ZONES IN LOW TEMPERATURE WATER

increases in thickness and bidirectional flow persists. Above 26.8°C the downward motion of the outer boundary layer is stopped by the viscous shear force of the inner layer and unidirectional flow exists. The flow pattern predicted for any combination of plate and bulk temperature is indicated in Figure 1.

1.3 Previous Work

1.3.1 Free Convection

Considerable research has been done in the field of free convective heat transfer since Lorenz (3) first published his analysis of the problem. In 1930 Schmidt and Beckmann applied laminar boundary layer theory to the free convection problem using a series expansion technique (4). Polhausen (5) introduced the similarity transformation technique whereby he transformed the partial differential equations of free convection into ordinary differential equations. Several approximate methods have been proposed since then.

1.3.2 Free Convection in Water Near $4^{o}C$

The first researcher to report the unusual nature of free convection in water near $4^{o}C$ was Codegone (6) in 1939. He found that the rate of heating or cooling a flask of water was at a minimum when the water was at $4^{o}C$.

Ede (7) compiled a study of heat transfer from a vertical plate to several liquids. For all fluids except water the heat transfer rate correlated to the difference between the plate and fluid temperatures. He excluded tests in which the water temperature was below $4^{o}C$ and the plate temperature was above $4^{o}C$ attributing the water results to the density maximum at $4^{o}C$.

Several authors (8, 9) have reported studies on the melting of ice spheres and horizontal and vertical ice cylinders by the process of free convection in water. They found that at temperatures near $4^{o}C$ a minimum value of the Nusselt number occurred. Tkachev (8) and Dumore, Merk, and Prins (9) performed their studies in 1953. Dumore et al could predict the effect by replacing the conventional thermal expansion coefficient in the empirical free convection equation,

$$Nu = 0.6 \ (Pr \ Gr)^{\frac{1}{4}}$$

with a new coefficient that was a linear function of the bulk fluid temperature.

In 1954 Merk (10) applied an integral method to the prediction of melting rates of ice spheres in water. He used a third order polynomial for density as a function of temperature in the boundary layer momentum equation and solved for an infinite Prandtl number. The results were later extrapolated to a Prandtl number of 10. A minimum Nusselt number was predicted to occur at $5.3^{o}C$.

The following year Ede (22) compared Merk's method with the results he had obtained for the heat transfer from an electrically heated vertical flat plate. For Regions I, III, and IV agreement was good. However, in Region II the predicted Nusselt numbers were as much as 50% low.

In 1956 Schechter and Isbin (12,13) studied the heat transfer from a vertical flat plate to water at a temperature of 4°C. They applied the similarity transformation introducing Merk's expression for the thermal expansion coefficient into the boundary layer momentum equation. The equations were then transformed into ordinary differential equations using the method of Ostrach (14). The equations were solved, using an analog computer, for the case of plate temperatures from 1°C to 14°C in a bulk fluid at 0°C.

They found that the Nusselt number decreased up to a plate temperature of 14°C. Schechter and Isbin referred to this point as the transition point to bidirectional flow. At these conditions the computed boundary layer velocity profile showed a zero gradient at the plate. Solutions above 14°C were not possible as the assumptions used in the similarity transformations are not valid in the bidirection flow region.

They applied the Squire-Eckert integral method to the problem to obtain the equations for the local Nusselt number;

$$Nu_x = 0.669\ Pr^{\frac{1}{2}}\ (.952 + Pr)^{-\frac{1}{4}}\ (Gr_x"\alpha)^{\frac{1}{4}} \qquad \text{(1-1)}$$

where $Gr_x" = g\beta_\infty\,\theta_p\ x^3/\nu^2$

Schechter and Isbin evaluated the thermal expansion coefficient for $Gr_x"$ at the bulk temperature instead of the film temperature. The density function α is a dimensionless mean boundary layer density deficit and is defined as:

$$\alpha = 1/3 + P/5 + Q/7 \qquad \text{(1-2)}$$

where P and Q are density functions defined below:

$$P = (D_2 + 3D_3\ T_\infty)\ /\theta_p\ /\ (D_1 + 2D_2\ T_\infty + 3D_3\ T_\infty^2) \qquad \text{(1-2a)}$$

and

$$Q = D_3\theta_p^{\ 2}\ /\ (D_1 + 2D_2\ T_\infty + 3D_3\ T_\infty^2). \qquad \text{(1-2b)}$$

The constants D_1, D_2, and D_3 are density coefficients and their values were given by Vanier (2) as:

$$D_1 = - 0.6669167 \times 10^{-4} \ C^{-1}$$

$$D_2 = 0.871689 \times 10^{-5} \ C^{-2}$$

$$D_3 = - 0.647664 \times 10^{-7} \ C^{-3}.$$

Schecter (13) also performed experimental measurements of free convective heat transfer from a one foot square vertical heated plate to water below $4^{\circ}C$. He attempted to measure the velocity and temperature profiles in the boundary layer.

In three of his tests the flow moved up the plate while for seventeen tests downward flow was observed. In ten tests Schechter observed an upward convective flow near the plate and a downward flow further away from the plate surface. Three of the runs were in Region III but exhibited bidirectional flow. Schechter claimed this was caused either by the effect of limited tank size or temperature conditions that induced a downward flow in the central region of the tank.

Schechter also applied the Squire-Eckert integral method to the bidirectional flow region. He assumed that thermal and momentum boundary layers were equal. He also assumed that all the heat transferred to the inner upward moving boundary layer from the plate returned in the downward flowing boundary layer. By making some further assumptions he obtained the following relations for the local and mean Nusselt number in Region II:

$$Nu_x = 0.489 \, [PrGr_x'' \propto (x/L \, (U-1) + x)]^{1/4} \quad \text{..........(1-3)}$$

and $$\overline{Nu} = 0.652 \, [PrGr_x'']^{1/4} \, (U^{3/4}-1)/(U-1)^{3/4} \quad \text{..............(1-4)}$$

where $$U = [30(1/10 + P/14 + Q/18)/\propto]^{4/3} \quad \text{..................(1-5)}$$

In their paper in 1958 Schechter and Isbin (23) reported a slightly different approximate analysis which yielded the equations:

$$Nu_x = 0.489 \, (Gr_x'' \, Pr_f \propto)^{1/4} \quad \text{........................(1-6)}$$

$$\overline{Nu} = 0.652\ (\overline{Gr}\ "\ Pr_f \propto)^{\frac{1}{4}} \quad \text{.........................(1-7)}$$

In Schechter's analysis the bulk fluid thermal expansion coefficient approaches zero at 4°C. Goren (15), in 1966, developed a method of solution that is valid at 4°C. He introduced into the boundary layer equation a parabolic expression for the density change of water as a function of its temperature deviation from 4°C. He applied similarity and obtained numerical solutions for free convective heat transfer rates for four different temperature differences from 1°C to 0.001°C.

In 1967 Oborin (16) studied free convective heat transfer from a heated sphere and heated cylinder to water in the temperature range from 1°C to 15°C. The sphere was maintained 7°C above the water temperature and the cylinder 2.1°C above the water temperature. For the sphere the minimum Nusselt number occurred at a water temperature of 2.4°C and for the cylinder the minimum occurred at 3.3°C. He assumed that the minimum heat transfer rate would occur when the mean boundary layer temperature was the same as that of the bulk fluid.

Bidirectional convection was observed by Schenk and

Schenkels (17) in their 1968 study of the melting rates of ice spheres in water. They used a high intensity light to make dust particles in water visible and observed upward flow near the melting surface and downward flow further out when the bulk temperature was between $4^{o}C$ and $6^{o}C$. With the bulk temperature below $4^{o}C$ they observed pure downward convection and when the bulk temperature was above $6^{o}C$ they observed pure upward flow.

They observed a minimum heat transfer rate during melting at $5.3^{o}C$. This was also the temperature predicted by Merk (10) to give the lowest Nusselt number. Above $6^{o}C$ Merk's theory was compatible with Schenk and Schenkels observations but between $4^{o}C$ and $6^{o}C$ great discrepancies were observed. Merk's polynomial could not describe the bidirectional flow present in this temperature range. Below $4^{o}C$ they observed a 20% higher melting rate than was predicted.

Vanier and Tien (2, 18, 19) also studied the melting of ice spheres in water. They found a minimum Nusselt number to occur at $5.35^{o}C$ and they correlated their experimental data as

$$Nu = 2 + C(Ra)^{1/4}.$$

They obtained three values of C, each valid for different bulk temperature ranges.

Vanier and Tien (2, 20, 21) performed a theoretical study of convective heat transfer from a vertical flat plate to cold water. The ordinary differential equations of the boundary layer, obtained by a similarity transformation, were numerically integrated. They considered variable properties by using a Prandtl number that was evaluated at the mean boundary layer temperature. A comparison was made using a Prandtl number appropriate to the local temperature at each step of the numerical integration. They did not find large differences and thus used the mean Prandtl number to save computer time. Solutions in Region II were not possible again as similarity does not apply in the bidirectional flow region. The method described above was used, with modifications, to examine the melting of a vertical flat plate of ice.

The most recent investigation of free convective heat transfer from a vertical flat plate to water near $4^{\circ}C$ was carried out by Yuill (1). He performed both experimental measurements and theoretical calculations. Theoretical (analytical) solutions were obtained for the regions of unidirectional flow; namely Regions I, III, and IV. A similarity solution of the boundary layer equations governing free convective heat transfer was used. He included the effect of variable viscosity in the solution. All fluid properties were fitted to polynomial equations. He solved the ordinary differential equations, obtained by a similarity transformation of the partial differential boundary layer equations, using Runge-Kutta forward integration, on a digital computer.

He obtained about 150 solutions in Region I, 260 in Region III, and 120 in Region IV. No solutions were possible for bulk temperatures between 3.3^{o}C and 5.25^{o}C due to a singularity at a bulk temperature of 4^{o}C.

Yuill presented his results in the form traditionally used for the presentation of free convection heat transfer correlations, namely:

$$Nu_x = C(Gr'_x \; Pr)^{\frac{1}{4}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1-8)}$$

$$\text{where } Gr'_x = 3\alpha\beta_o\theta_p \; g \; x^3 / \nu^2, \quad \ldots\ldots\ldots\ldots\ldots \text{(1-9)}$$

and α is defined by Equation (1-2).

The coefficient C was found to correlate well as a function of α in Region I. The relationship is illustrated in Figure 2 with the curve representing the equation:

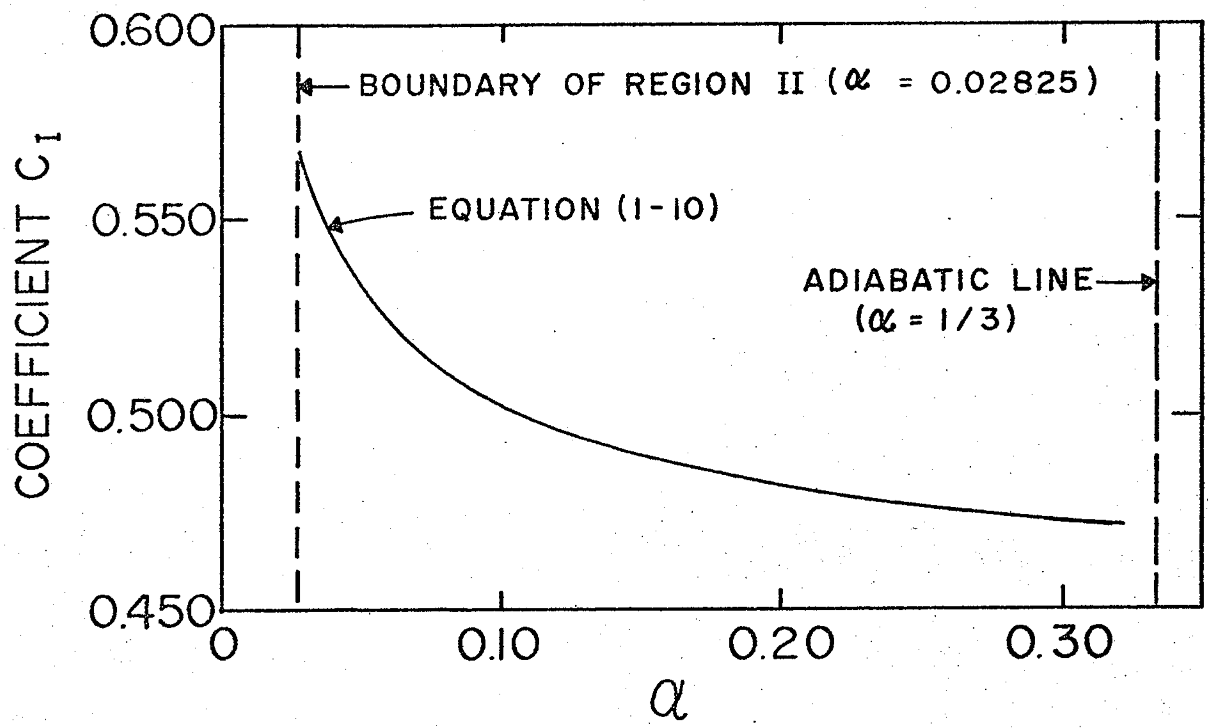

FIGURE 2 COEFFICIENT C_I vs α IN REGION I

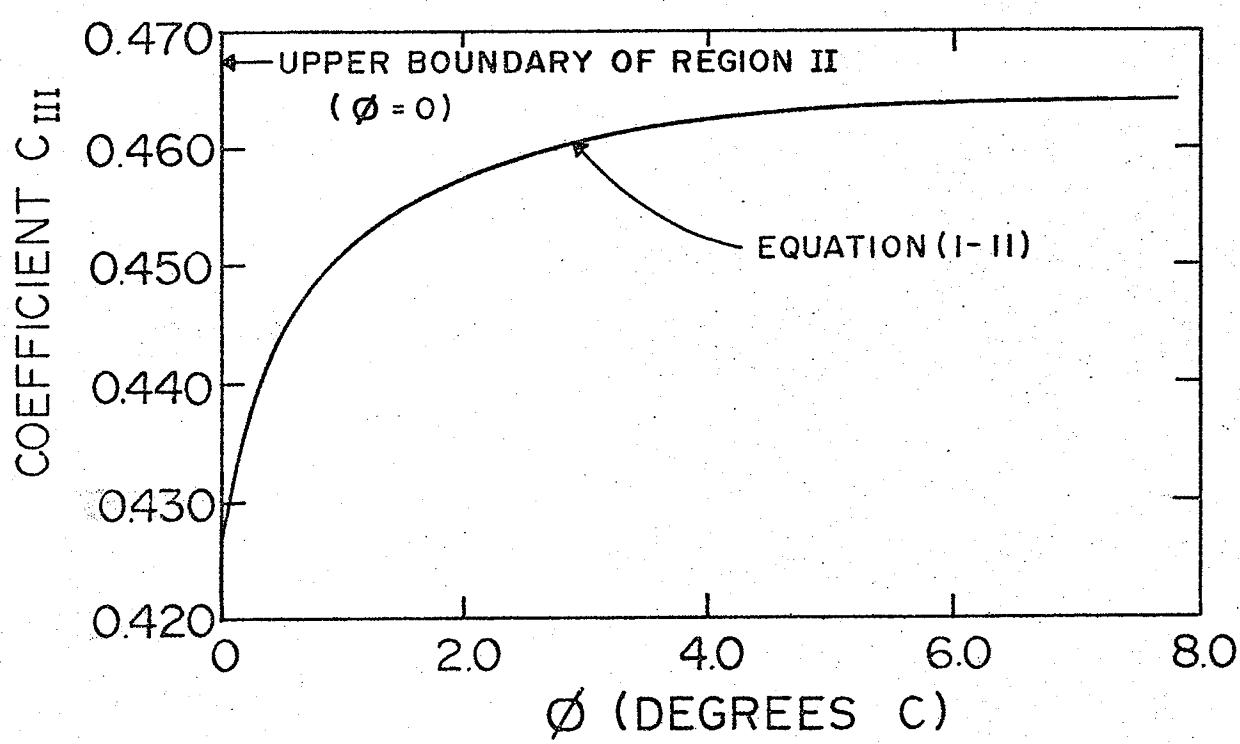

FIGURE 3 COEFFICIENT C_{III} vs ϕ IN REGIONS III AND IV