Internat. J. Math. & Math. Sci.
Vol.7 No.4 (1984) 755-764

LONGITUDINAL DISPERSION OF TRACER PARTICLES IN A CHANNEL BOUNDED BY POROUS MEDIA USING SLIP CONDITION

DULAL PAL, R. VEERABHADRAIAH, P.N. SHIVAKUMAR* AND N. RUDRAIAH

UGC-DSA Centre in Fluid Mechanics
Department of Mathematics
Bangalore University
Bangalore-56001, India

*Permanent Address: Department of Applied Mathematics
University of Manitoba
Winnipeg, Manitoba, Canada
R3T 2N2

(Received April 5, 1983)

ABSTRACT. Longitudinal dispersion of solute in a channel bounded by porous layers is studied using the analysis of Taylor [4] with BJ slip condition. The results of the present analysis are compared with those of Fung and Tang [2] obtained from using the no-slip condition. It is found that the effect of slip is significant only in the case when the membrane is permeable to solvent but not to the tracer. However, in the case when the membrane is permeable to both the tracer and the solvent, we find that our results coincide with those of Fung and Tang [2].

KEY WORDS AND PHRASES. Dispersion, boundary layers, porous media, blood flows.
1980 MATHEMATICS SUBJECT CLASSIFICATION CODES. 76D10, 76Z05

1. INTRODUCTION

Flow through and past porous media has attracted considerable interest in recent years because of its several important applications [1], particularly in biomechanics [2]. The study of mass transfer in the lung, in particular in the capillary blood vessels of the lung, is one of the important problems in biomechanics. These small blood vessels, enclosed in a thin layer of tissues which separate the blood from the air, may be described as sheets bounded by porous layers [2]. It is common to assume that blood vessels are circular cylindrical tubes, but in the lung this assumption may not apply at the alveolar level [2].

This problem was investigated by Fung and Tang [2] using the no-slip boundary condition and a proper matching condition at the interface separating the flow in the channel and the porous media. Now it is well known that this no-slip condition is no longer valid at the mermeable surface [3]. Beavers and Joseph [3] (hereafter called BJ) have postulated the existence of slip at the interface and they have confirmed it experimentally. Therefore, the chief aim of the present paper is to extend the work of [2] to include the BJ condition [3] at the interface. In other words, we study the longitudinal dispersion of a tracer in a configuration shown in Fig. 1 using the BJ slip condition. This gives a technique to measure the blood volume and the volume of interstitial water in the lung tissue. For this we first calculate the velocity distribution and then the concentration distribution taking into account both convection and diffusion. This leads to the calculation of the effective dispersion coefficient which is useful in determining the flow characteristics under abnormality conditions.

2. FORMULATION OF THE PROBLEM

The physical configuration and the co-ordinate system chosen are shown in Fig. 1. The problem considered here is restricted to two-dimensional flow so that the physical quantities are independent of y. The flow in the channel and in the external porous space is assumed to be homogeneous, incompressible and Newtonian and coupled through the boundary conditions.

The flow in porous space is assumed to be governed by Darcy's law

$$u_1 = -\frac{k}{\rho}\frac{\partial p_1}{\partial x}, \quad w_1 = -\frac{k}{\rho}\frac{\partial p_1}{\partial z} \tag{2.1}$$

where the subscript 1 refers to the porous layer, u_1 and w_1 are the velocities in x and z directions respectively, p_1 is the hydrostatic pressure, $k = \frac{K\rho}{\mu}$ is Darcy's constant, K is the permeability, μ is the viscosity and ρ is the density of the fluid. The boundary conditions are [2]:

$$u_1 = 0 \qquad \text{at } x = 0, L \tag{2.2a}$$

$$w_1 = 0 \qquad \text{at } z = \pm\left(\frac{h}{2} + \delta\right) \tag{2.2b}$$

$$w_1 = \frac{K}{\rho}\left[(p_2 - p_1) - \sigma(\pi_2 - \pi_1)\right] \quad \text{at } z = \pm\frac{h}{2} \tag{2.3}$$

where p_1, p_2 are the hydrostatic pressures in the porous space and the channel respectively, π_1 and π_2 are the corresponding osmotic pressures, σ is the reflection coefficient of the wall, h is the thickness of the channel, and δ is the thickness of the porous layer. Equation (2.3) is the well-known Starling's hypothesis commonly accepted in physiology. If the porosity of the porous layer is assumed to be the same at all times, the equation of continuity is of the form

$$\frac{\partial u_1}{\partial x} + \frac{\partial w_1}{\partial z} = 0. \tag{2.4}$$

The flow in the channel is governed by

$$\frac{\partial p_2}{\partial x} = \mu\,\nabla^2 u_2, \quad \frac{\partial p_2}{\partial z} = \mu\,\nabla^2 w_2 \tag{2.5}$$

with the boundary conditions

$$u_2 = \frac{3}{2}U\left(1 - 4\frac{z^2}{h^2}\right), \quad w_2 = 0 \quad \text{at } x = 0, L \tag{2.6}$$

$$\frac{\partial u_2}{\partial z} = \frac{\alpha}{\sqrt{K}}(u_{B1} - Q_1), \quad w_2 = w_1 \quad \text{at } z = -\frac{h}{2} \tag{2.7a}$$

$$\frac{\partial u_2}{\partial z} = -\frac{\alpha}{\sqrt{K}}(u_{B2} - Q_2), \quad w_2 = w_1 \quad \text{at } z = +\frac{h}{2}. \tag{2.7b}$$

Here the subscript 2 refers to physical quantites in the channel, U is the mean velocity of flow in the channel as given by [2], u_{B1} and u_{B2} are the slip velocities at $z = -\frac{h}{2}, \frac{h}{2}$, Q_1 and Q_2 are $-\frac{k}{\rho}\frac{\partial p}{\partial x}$, α is the slip parameter. The equation of continuity is the same as (2.4) with u_1 replaced by u_2.

3. SOLUTIONS OF FLOW EQUATIONS

(a) Solutions in porous space

Equations (2.1)-(2.4), using the stream functions defined by

$$u_1 = -\frac{\partial\psi_1}{\partial z}, \quad w_1 = \frac{\partial\psi_1}{\partial x} \tag{3.1}$$

take the form

$$\nabla^2 p_1 = 0 \; ; \; \nabla^2\psi_1 = 0 . \tag{3.2}$$

The solutions of (3.2) for $z \geq 0$ using the conditions (2.2a) and (2.2b) are

$$\psi_1(x,z) = \sum_{n=1}^{\infty} a_n \sin(\lambda_n x)\sinh\lambda_n\,[(\tfrac{h}{2}+\delta)-z]/\sinh\lambda_n\delta \tag{3.3}$$

$$p_1(x,z) = \sum_{n=1}^{\infty} \frac{\rho a_n}{k} \cos\lambda_n x\,\cosh\lambda_n[(\tfrac{h}{2}+\delta)-z]/\sinh\lambda_n\delta + b_1 \tag{3.4}$$

where $\lambda_n = \frac{n\pi}{L}$ and b_1 is an integration constant which is obtained later using the matching condition (2.3) at $z = \pm\frac{h}{2}$. For $z \leq 0$, ψ_1 and p_1 are mirror images of (3.3) and (3.4).

(b) Solutions in the channel

The continuity equation is satisfied if we introduce the stream function ψ_2 as in (3.1) with the subscript changed. Then (3.2) becomes

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)p_2 = 0 , \quad \left(\frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2\partial z^2}+\frac{\partial^4}{\partial z^4}\right)\psi_2 = 0 . \tag{3.5}$$

To solve these equations, we let

$$u_2 = u_2^{(0)} + u_2' , \quad w_2 = w_2' \tag{3.6}$$

where $u_2^{(0)} = \frac{3}{2}U[1-4\frac{z^2}{h^2}]$; u_2' and w_2' are perturbations due to permeability of the wall. Then the solution of the perturbed stream function ψ_2', with the condition

$$u_2' = 0 , \quad w_2' = 0 \qquad \text{at } x = 0, L \tag{3.7}$$

with (2.7a) and (2.7b), replacing u_2 by u_2' and w_2 by w_2' is

$$\psi_2 = \sum_{n=1}^{\infty}(A_n \sin\lambda_n x \sinh\lambda_n z + C_n \frac{2z}{h}\sinh\lambda_n x\cosh\lambda_n z) \tag{3.8}$$

where

$$C_n = -[A_n\lambda_n + \frac{\lambda_n^2 h}{\alpha\sigma}A_n\tanh\lambda_n\frac{h}{2} + a_n\lambda_n\frac{\coth\lambda_n\delta}{\cosh\lambda_n\frac{h}{2}}]$$
$$\times\,[\frac{2}{h}+\lambda_n\tanh\lambda_n\frac{h}{2}+\frac{\lambda_n h}{\alpha\sigma}(\lambda_n+\frac{4}{h}\tanh\lambda_n\frac{h}{2})]^{-1} , \tag{3.9}$$

$$A_n = \frac{a_n}{\cosh(\lambda_n\frac{h}{2})}\,[1+\lambda_n h\left\{2+\lambda_n h\tanh\lambda_n\frac{h}{2}+\frac{\lambda_n h}{\alpha\sigma}(\lambda_n h+4\tanh\lambda_n\frac{h}{2})\right\}^{-1}\coth\lambda_n\delta]$$
$$\times\,[\tanh\lambda_n\frac{h}{2}-\left\{2+\lambda_n h\tanh(\lambda_n\frac{h}{2})+\frac{\lambda_n h}{\alpha\sigma}(\lambda_n h+4\tanh\lambda_n\frac{h}{2})\right\}^{-1}$$
$$\times\left\{\lambda_n h+\frac{\lambda_n^2 h^2}{\alpha\sigma}\tanh(\lambda_n\frac{h}{2})\right\}]^{-1} , \tag{3.10}$$

$$\delta = \frac{h}{L}\left(\frac{L^2\rho}{\mu K}\right)^{-1/2}$$

and $\lambda_n = \frac{n\pi}{L}$. Here we have relaxed the condition $w_2 = w_2' = 0$ at $x = 0$, L except for symmetry with respect to z.

From (3.5) using (3.8), we obtain the pressure distribution

$$p_2 = \mu\left\{-\frac{12U}{h^2}x + \sum_{n=1}^{\infty}\frac{4}{h}C_n\lambda_n \cos\lambda_n x \cosh\lambda_n z\right\} + b_2 \tag{3.11}$$

where b_2 is again an integration constant to be determined using the matching condition (2.3).

A substitution into equation (2.3) yields,

$$\sum_{n=1}^{\infty} a_n\lambda_n \cos\lambda_n x = \sum_{n=1}^{\infty}\frac{K}{\rho}\cos\lambda_n x\left[\mu\frac{4}{h}C_n\lambda_n \cosh\lambda_n\frac{h}{2} - \frac{a_n\rho}{k}\coth\lambda_n\delta\right]$$

$$+\frac{K}{\rho}\left(b_2 - b_1 - \mu\frac{12U}{h^2}x\right) - \frac{K}{\rho}\sigma(\pi_2 - \pi_1). \tag{3.12}$$

On representing x as a Fourier series in $\cos\lambda_n x$ and assuming that the variation of the osmotic pressures π_2 and π_1 due to concentration distribution is very small (to be discussed later), we can solve (3.12) for a_n $(n = 1,2...)$ to obtain

$$a_n = [1-(-1)^n]\frac{24}{n^2}\frac{\mu K}{\rho}\frac{U}{L}\frac{1}{\lambda_n^2}\Big[4\frac{\mu K\lambda_n}{h\rho}\left\{\lambda_n h + \frac{\lambda_n^2 h^2}{\alpha\sigma}\tanh\lambda_n\frac{h}{2} + \lambda_n h\tanh\lambda_n\frac{h}{2}\coth\lambda_n\delta\right\}$$

$$\times\left\{\tan\lambda_n\frac{h}{2}\left(2 + \lambda_n h\tanh\lambda_n\frac{h}{2} + \frac{\lambda_n h}{\alpha\sigma}\left\{\lambda_n h + 4\tanh\lambda_n\frac{h}{2}\right\}\right) - \left(\lambda_n h + \frac{\lambda_n^2 h^2}{\alpha\sigma}\right.\right.$$

$$\left.+\frac{K}{k}\coth\lambda_n\delta + \lambda_n\right]^{-1}. \tag{3.13}$$

Note that $a_n = 0$ if n is an even number this condition together with the expressions for p_1 and p_2 lead to the following interpretations:

$$b_1 = p_1(x,z) \qquad \text{evaluated at } x = \frac{L}{2} \tag{3.14a}$$

$$b_2 = p_2(x,z) + \mu\frac{6LU}{h^2} \qquad \text{evaluated at } x = \frac{L}{2} \tag{3.14b}$$

$$\sigma(\pi_2 - \pi_1) = (p_2 - p_1) \qquad \text{evaluated at } x = \frac{L}{2}. \tag{3.14c}$$

Since the filtration flow in the porous walls and the perturbation flow in the channel are determined by the coefficients a_n, it would be interesting to see how a_n depends on the geometrical and physical parameters of our model.

4. DISPERSION OF TRACER PARTICLES

We are now concerned with the Taylor's [4] dispersion of tracer particles in the capillary blood vessels of the lung. These vessels may be described as sheets bounded by porous layers. Here we are interested in the case when the channel walls are semipermeable and the assumptions employed by [2] are also true in the present paper.

When the tracer cloud reaches the alveolar sheet, its dispersion is subjected to further disturbance by the porous wall. We are interested in the overall effect of this disturbance, because we can measure the dispersion only in the vein, after the tracer cloud leaves the alveolar sheet. The required concentration distribution is obtained from

$$\frac{\partial c}{\partial t} + u\,\frac{\partial c}{\partial x} + w\,\frac{\partial c}{\partial z} = D\,\nabla^2 c \tag{4.1}$$

with

$$u = (3/2)U[1 - r(z/h)^2]\ ,\ w = 0\ , \tag{4.2}$$

where U is the mean velocity of flow.

To solve this equation we shall introduce the following approximation. Since the solute flow in the alveolar sheet is quasi-steady, it seems reasonable that the overall effect of convection can be obtained by replacing the velocities in the convective term in (4.1) by the average of $u(x,z,t)$ and $w(x,z,t)$ over x from 0 to L. This transforms (4.1) to

$$\frac{\partial c}{\partial t} + \bar{u}\,\frac{\partial c}{\partial x} + s\,\frac{\partial c}{\partial z} = D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial z^2}\right)\ . \tag{4.3}$$

Following Taylor's analysis, we obtain

$$\frac{\partial}{\partial t}\,(\bar{c} + c') + u'\,\frac{\partial}{\partial \xi}\,(\bar{c} + c') = D\,\frac{\partial^2 c}{\partial_z{}^2} \tag{4.4}$$

in which $\xi = x - \bar{\bar{u}}t$, $\bar{c}$ is the value of $c(x,z,t)$ on the centre line $z = 0$ (hence independent of z), c' is the deviation from $\bar{c}$ and u' is the deviation of $\bar{u}$ from mean velocity $\bar{\bar{u}}$. The boundary conditions are $c' = 0$ when $z = 0$ and $\partial c'/\partial z = 0$ at the impermeable boundary. Following Taylor [4], we assume that $\bar{c}$ is a function only of ξ and c' a function only of z, and then integrating (4.4), we obtain

$$c' = f(z)\,\frac{d\bar{c}}{d\xi} \tag{4.5}$$

where

$$f(z) = \int_0^z \left(\int_{z_0}^z \frac{1}{D}\,u'dz\right)dz\ . \tag{4.6}$$

Here, $z_0 = \frac{h}{2} + \delta$ if the membrane is permeable to both the tracer and the solvent only, whereas $z_0 = \frac{h}{2}$ if the membrane is permeable to solvent only. $\bar{\bar{u}}$ is the average of $\bar{u}(z,t)$ over $0 \le z \le z_0$, $\frac{\partial c'}{\partial z} = 0$ at $z = z_0$.

The rate at which mass is transported through a cross section moving at the mean velocity is $M = A\overline{u'c'}$, where A is the cross-sectional area and the overbar indicates a cross-sectional mean. Equation (4.5) yields $M = A\overline{u'f}\,\frac{d\bar{c}}{d\xi}$, which shows that the dispersion follows Fick's law, with a coefficient of apparent difussivity D^* given by,

$$D^* = -\overline{u'f}\ , \tag{4.7}$$

The mean concentration is then governed by the equation

$$\frac{\partial \bar{c}}{\partial t} = D^*\,\frac{\partial^2 \bar{c}}{\partial \xi^2}\ . \tag{4.8}$$

We can now evaluate the details according to the nature of the tracer relative to the membrane separating the channel and the porous space.

CASE 1: <u>The membrane is permeable to both the tracer and the solvent</u>

In this case the velocity distribution can be obtained from (3.8) as

$$u_2' = -\sum_{n=1}^{\infty} \Big(\lambda_n A_n \sin\lambda_n x \cosh\lambda_n z + \lambda_n C_n \frac{2z}{h} \sin\lambda_n x \sinh\lambda_n z + \frac{2}{h}\,C_h \sin\lambda_n x \cosh\lambda_n z\Big). \tag{4.9}$$

Now integrating (4.9) w.r.t. x and dividing by L using (3.6), we get

$$\bar{u}_2 = \frac{3}{2} U(1 - 4\frac{z^2}{h^2}) + \frac{1}{L}\left\{\sum_{n=1}^{\infty} [(-1)^n - 1](A_n \cos\lambda_n z + \frac{2C_n}{h\lambda_n} \cosh\lambda_n z + C_n \frac{2z}{h} \sinh\lambda_n z)\right\} \quad \text{for } 0 \le z \le \frac{h}{2}. \tag{4.10}$$

Equation (4.10) gives the average velocity distribution in the channel space.

Similarly we obtain the average velocity distribution in the porous space from (3.1) and (3.3) as

$$\bar{u}_1 = \frac{1}{L}\left\{- \sum_{n=1}^{\infty} [(-1)^n - 1]\, a_n \cosh\lambda_n[(\frac{h}{2}+\delta)-z]/\sinh\lambda_n\delta\right\} \quad \text{for } \frac{h}{2} \le z \le \frac{h}{2} + \delta. \tag{4.11}$$

By further integration of $\bar{u}_2$ from 0 to $\frac{h}{2}$ and $\bar{u}_1$ from $\frac{h}{2}$ to $\frac{h}{2}+\delta$, summing and dividing by $\frac{h}{2}+\delta$, we obtain

$$\bar{\bar{u}} = \frac{1}{(\frac{h}{2}+\delta)}\left\{\frac{1}{2} hU + \frac{1}{L}\sum_{n=1}^{\infty} [(-1)^n - 1][-a_n/\lambda_n + (A_n/\lambda_n)\sinh \frac{\lambda_n h}{2} + (C_n/\lambda_n) \cosh(\lambda_n \frac{h}{2})]\right\}. \tag{4.12}$$

Since $\delta \ll h$, the deviation u' is approximately

$$u' = \frac{1}{2} \bar{\bar{u}}[1 - 12z^2(h+2\delta)^{-2}]. \tag{4.13}$$

Now the coefficient of apparent diffusivity, from (4.6), (4.7) and (4.13) is

$$D^* = \frac{\bar{\bar{u}}^2(h+2\delta)^2}{210\ D}. \tag{4.14}$$

The effect of the slip is reflected in $\bar{\bar{u}}$.

From (4.12), we have

$$\bar{\bar{u}}(h+2\delta) - Uh = \frac{2}{L}\sum_{n=1}^{\infty} [1 - (-1)^n][\frac{a_n}{\lambda_n} - \frac{A_n}{\lambda_n}\sinh(\lambda_n \frac{h}{2}) - \frac{C_n}{\lambda_n}\cosh(\lambda_n \frac{h}{2})]. \tag{4.15}$$

The values of (4.15) were also evaluated for different physiological parameters. They all turned out to be nearly zero for higher values of $L^2\rho/\mu k$ and $L\rho/\mu K$. In other words $\bar{\bar{u}}(h+2\delta) - Uh = 0$ for $L^2\rho/\mu k = 5 \times 10^9$ and $L\rho/\mu K = 5 \times 10^6$. This indicates that the apparent diffusivity for a channel with impermeable wall is about the same as that of a channel with walls permeable to both the tracer and the solvent.

CASE 2: The membrane is permeable to the solvent but not to the tracer

In this case, only flow in the channel needs to be considered. $\bar{u} = \bar{u}_2$ is given by

$$\bar{\bar{u}} = U + \frac{2}{Lh}\sum_{n=1}^{\infty} [(-1)^n - 1][\frac{A_n}{\lambda_n}\sinh(\lambda_n \frac{h}{2}) + \frac{C_n}{\lambda_n}\cosh(\lambda_n \frac{h}{2})].$$

If we assume the approximation

$$u' - \bar{u} - \bar{\bar{u}} \approx \frac{1}{2}\bar{\bar{u}}[1 - 12z^2/h^2],$$

then the coefficient of apparent diffusivity is

$$D^* = \frac{\bar{\bar{u}}^2 h^2}{210\ D}.$$

The values of $D^*/(h^2U^2/D)$ are tabulated in Table 1 for different values of $L^2\rho/\mu k$ and $L\rho/\mu K$. It is seen from the Table 1, that there is significant effect of slip on co-efficient of apparent diffusivity for small values of $L^2\rho/\mu k$, $L\rho/\mu K$ and for large values of δ/L.

5. DISCUSSION

The results of $\bar{w}_1 = \frac{w_1}{U}$ and $\bar{p}_1 = \frac{Lp_1}{\mu U}$ at $z = \frac{h}{2}$ are plotted in figs. (2) and (3) for several specific values of the dimensionless parameters. From these figures it is observed that the effect of slip is significant only when the parameters $L^2\rho/\mu k$ and $L\rho/\mu K$ are small and its effect is negligible for their large values.

The analytical solution of the co-efficient of apparent diffusivity of longitudinal dispersion of a tracer in a channel bounded by porous layers is obtained for the two cases and the results are given Table 1. This table pertains to the case when the membrane is permeable to the solvent but not to the tracer, where the co-efficient of apparent diffusivity is $D^* = \bar{\bar{u}}^2h^2/210D$. From this it is clear that $D^*/(h^2U^2/D)$ decreases with increase in δ/L and increases with an increase in $L\rho/\mu K$. In the case when the membrane is permeable to both the tracer and the solvent, we find the effect of slip is not so significant and hence they are not reported here. This may be due to the fact that the BJ condition [3] is applicable to a situation where the thickness of the membrane is large compared to the width of flow in the channel. In many bio-mechanical problems, including the one we have discussed here, this condition may not be valid. In that case the BJ condition has to be altered and the work in this direction is under progress.

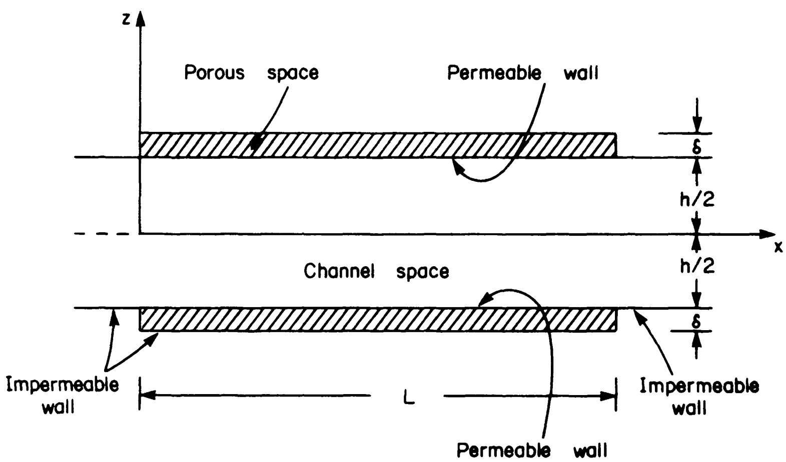

Figure 1 Schematic drawing of the idealized system Fluid flows in the channel between z=h/2 and z=h/2 in the region $0 \leq X \leq L$

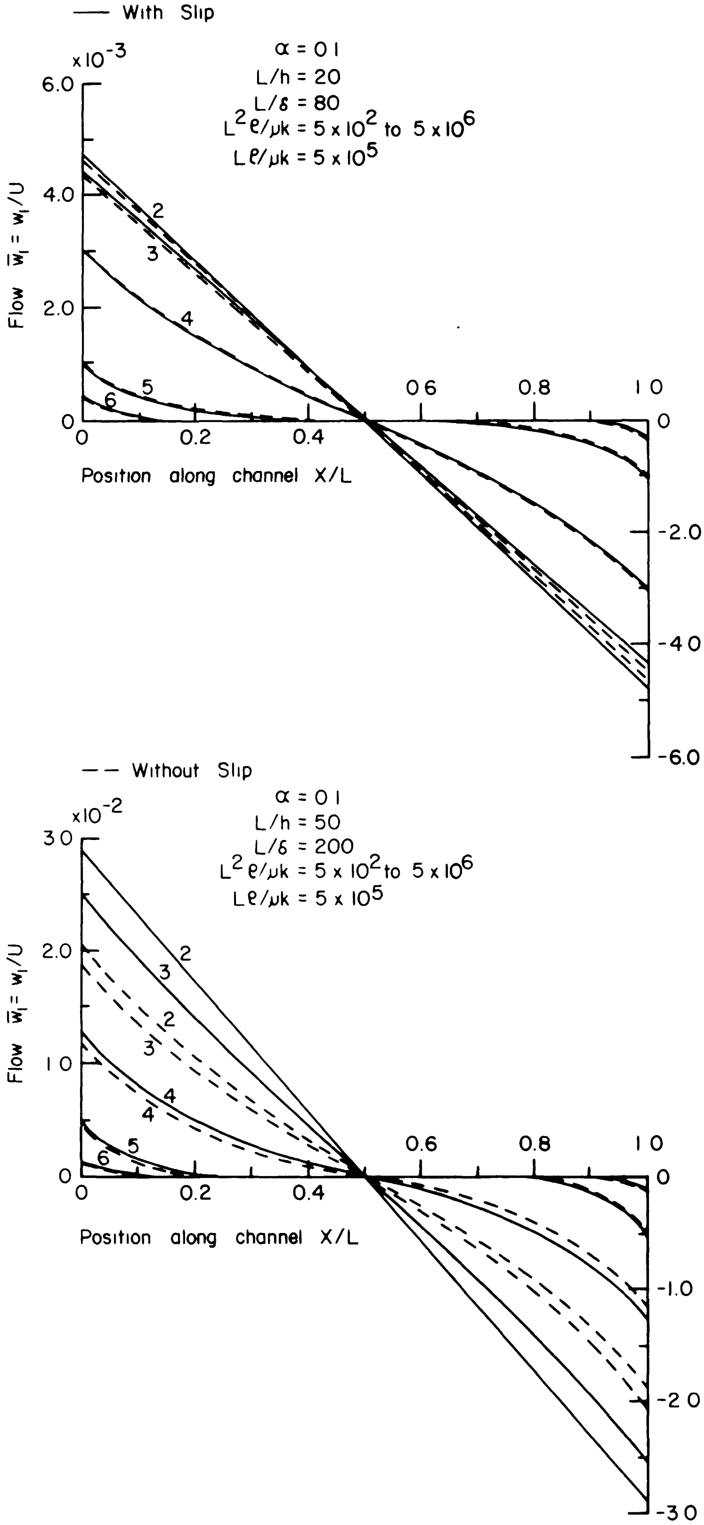

Figure 2. The vertical velocity component of fluid moving into the porous space at the interface $z = h/2$, given in nondimensional form $\bar{w}_l = w_l/U$ for various values of the parameter $L^2 \rho/\mu k$

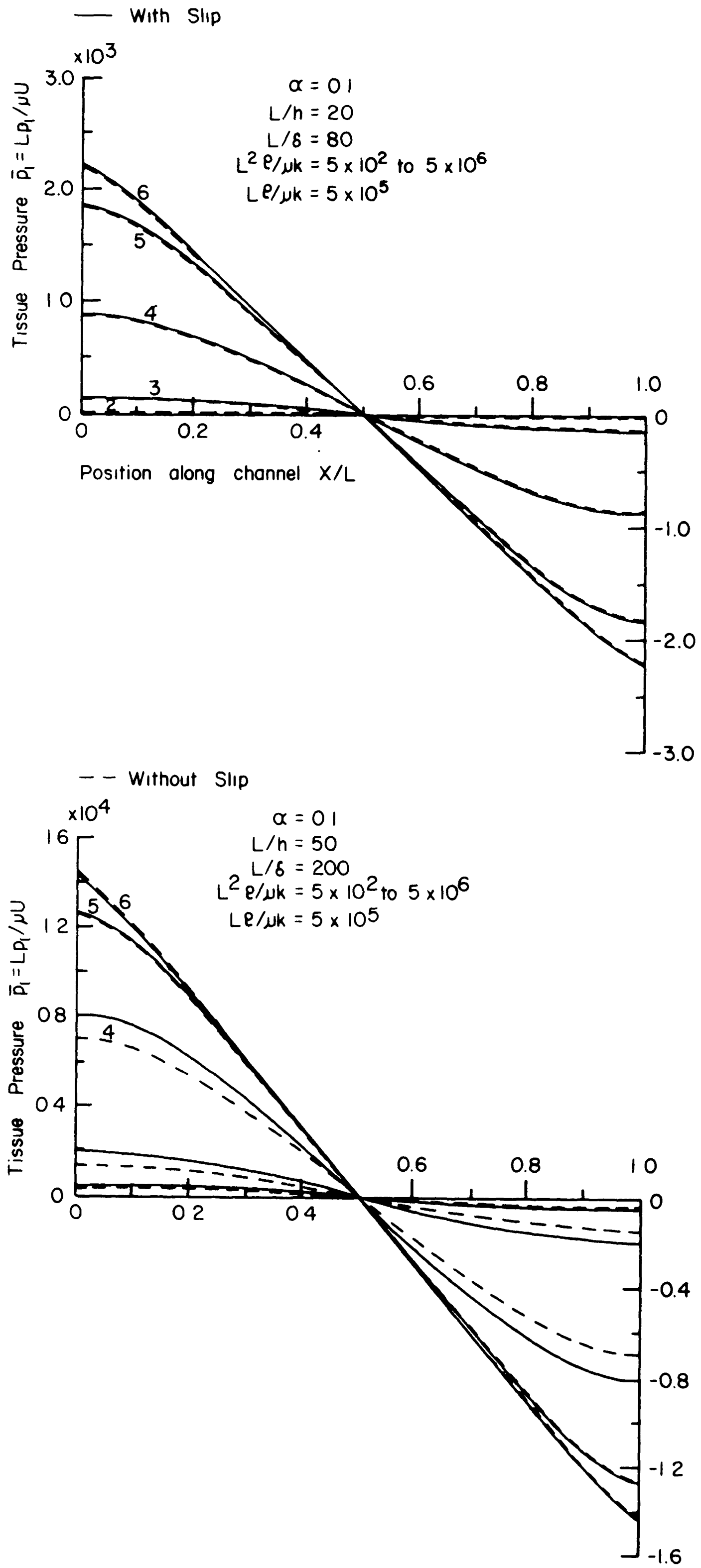

Figure 3 The hydrostatic pressure in the porous space at the interface $z = h/2$. Given in nondimensional form $\bar{p}_l = Lp_l/\mu U$ for various values of the parameter $L^2 e/\mu k$

TABLE 1: Values of $D^*/(h^2U^2/D)$, when the membrane is permeable to the solvent but not to the tracer.

$h/L = 0.02$, $\alpha = 0.1$
$L^2\rho/\mu k = 5\text{x}10^4$, $L\rho/\mu K = 5\text{x}10^3$

δ/L	values with slip condition	values without slip condition
0.001	$0.4241695\text{x}10^{-2}$	$0.4244318\text{x}10^{-2}$
0.002	$0.3763006\text{x}10^{-2}$	$0.3814161\text{x}10^{-2}$
0.003	$0.3321234\text{x}10^{-2}$	$0.3450856\text{x}10^{-2}$
0.004	$0.2813788\text{x}10^{-2}$	$0.3140601\text{x}10^{-2}$
0.005	$0.2538809\text{x}10^{-2}$	$0.2873280\text{x}10^{-2}$
0.006	$0.2184673\text{x}10^{-2}$	$0.2641097\text{x}10^{-2}$
0.007	$0.1879983\text{x}10^{-2}$	$0.2437985\text{x}10^{-2}$
0.008	$0.1593492\text{x}10^{-2}$	$0.2259212\text{x}10^{-2}$
0.009	$0.1334070\text{x}10^{-2}$	$0.2100999\text{x}10^{-2}$
0.010	$0.1100651\text{x}10^{-2}$	$0.1960175\text{x}10^{-2}$

$h/L = 0.02$, $\alpha = 0.1$
$L^2\rho/\mu k = 5\text{x}10^4$, $L\rho/\mu K = 5\text{x}10^6$

δ/L	values with slip condition	values without slip condition
0.001	$0.4530775\text{x}10^{-2}$	$0.4530103\text{x}10^{-2}$
0.002	$0.4454217\text{x}10^{-2}$	$0.4458083\text{x}10^{-2}$
0.003	$0.4415753\text{x}10^{-2}$	$0.4422741\text{x}10^{-2}$
0.004	$0.4392608\text{x}10^{-2}$	$0.4401736\text{x}10^{-2}$
0.005	$0.4377157\text{x}10^{-2}$	$0.4387822\text{x}10^{-2}$
0.006	$0.4366102\text{x}10^{-2}$	$0.4377915\text{x}10^{-2}$
0.007	$0.4357804\text{x}10^{-2}$	$0.4370514\text{x}10^{-2}$
0.008	$0.4351347\text{x}10^{-2}$	$0.4364766\text{x}10^{-2}$
0.009	$0.4346178\text{x}10^{-2}$	$0.4360187\text{x}10^{-2}$
0.010	$0.4341946\text{x}10^{-2}$	$0.4356429\text{x}10^{-2}$

ACKNOWLEDGEMENT. This work was partly supported by the UGC under DSA programme and by the National Research Council of Canada under Grant NRC-A-8783.

REFERENCES

1. RUDRAIAH, N., CHANDRASEKHARA, B.C., VEERABHADRAIAH, R. and NAGARAJ, S.T. "Some flow problems in porous media", PGSAM, vol. 2, Bangalore, 1979.

2. FUNG, Y.C. and TANG, H.T. "Longitudinal dispersion of tracer particles in the blood flowing in a pulmonary alveolar sheet", vol. 92, No. 3, TRANS. ASME, vol. 97, Series E, September 1975, pp. 536-540.

3. BEAVERS, G.S. and JOSEPH, D.D. "Boundary conditions at a naturally permeable wall", Journal of Fluid Mechanics, vol. 30, 1975, pp. 197-207.

4. TAYLOR, G.I. "Dispersion of soluble matter in solvent flowing slowly through a tube", Proceedings of the Royal Society, London, Series A, vol. 219, 1953, pp. 186-203.