MEASUREMENTS ON THE RANDOM NATURE OF COSMIC RADIATION

by

Melih OGMEN

A thesis
presented to the University of Manitoba
in partial fulfillment of the
requirements for the degree of
Master of Science

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ABSTRACT

A search for a non-random component in the cosmic radiation has been conducted using a ground based air shower array sensitive to cosmic ray primaries of energy $\geqslant 9 \times 10^{14}$ eV. The time interval between the detection of successive air showers has been measured for more than 149,000 air showers recorded over a period of three years. The data are consistent with the conclusion that these primary particles are incident upon the upper atmosphere randomly in time and uniformly from all portions of the sky swept by the array.

ACKNOWLEDGEMENTS

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CONTENTS

Chapter I

DESCRIPTION OF COSMIC RAY AIR SHOWERS

1.1 DEVELOPMENT OF COSMIC RAY (C.R.) AIR SHOWERS

An incoming primary particle collides at a great height (typically 20-30 Km) with an oxygen or nitrogen nucleus, giving rise to a shower of mesons and nucleons which continue towards the earth approximately along the projected direction of the incoming particle. The central region around this direction is usually called the 'core' of the shower.Further nuclear disintegrations are produced by these mesons and nucleons,giving rise to more mesons and nucleons, constituting the nucleon cascade.In the initial and subsequent collisions, neutral pi-mesons produced,decay into high energy gamma rays which then initiate an electron-photon cascades.Some of the charged pi-mesons decay into muons which form a very penetrating component.The summation of all the individual cascades constitutes the extensive air shower (EAS) at ground level. We see then that an air shower consists of a core of high energy particles,some of which are nuclear interacting(N-Component) i.e. nucleons and mesons and some high energy muons and electrons. Around the core are distributed the electron-photon component and muons.

1.2 LONGITUDINAL DEVELOPMENT OF THE ELECTRON-PHOTON COMPONENT

The electron photon component of the shower is derived almost entirely from the gamma rays produced in the decay of π^0 mesons, which have themselves been produced in nuclear interactions. The electron photon component is thus a secondary product of a nucleon cascade.

The longitudinal development of the cascade has been dealt with in varying degrees of detail in several publications. We will first give a simplified model[48] which shows the qualitative behaviour, and then quote the results of more detailed treatments.[1,2]

Assume interactions of photons that produce electron-positron pairs, or of electrons that produce gamma rays by bremsstrahlung occur at distances 0,1,2,3... units (radiation length)[1] along the path of the shower from the origin. At each interaction the energy is equally divided between two new "particles" (including photons as particles). Then after a distance of t radiation lengths, the number of particles is

$$N = 2^t$$

and the energy of each is

$$E = E_0 / 2^t$$

----

[1] see page 5 for the definition.

where E_0 is the energy of the primary particle. This cascade multiplication continues until E is reduced to some value E_c, at which the electron loses energy rapidly by ionization rather than the above mentioned processes. Beyond this stage, cascade multiplication ceases, and the particles are then absorbed. The depth of the maximum development, T, is given by

$$2^T = E_0 / E_c$$

$$T \propto \log E_0/E_c$$

Thus the depth of the maximum of development increases logarithmically with energy and the number of particles present at the maximum is proportional to the primary energy.

The usual approach, due to Carlson and Oppenheimer[3] to a more satisfactory solution of the cascade problem is through differential equations analogous to diffusion equations which describe the changes occuring in a depth dt at t. We write $N_e(E,t)$ for the spectrum of electrons at the depth t and $\gamma(W,t)$ for the spectrum of photons. Then the number of electrons with energy between E and E+dE (the interval E,dE) changes through the following effects.

(1) photons with energy greater than E can produce electrons in the interval E,dE.

(2) Electrons with energy E′, greater than E, enter the interval by radiating away part of their energy.

(3) Some electrons in the interval leave it by radiation loss.

(4) electrons lose energy ϵ dt by ionization in the depth dt. Thus the number entering the interval (E,dE) is N_e (E+dE) ϵ dt and the number leaving the interval is N_e (E)ϵ dt. The change of number is thus $\epsilon (\partial N_e / \partial E) dE\, dt$.

(5) electrons with energy E ,greater than W radiate photons in the energy interval (W,dW) by bremsstrahlung.

(6) some photons are removed from the interval by pair production.

(7) Photons are removed from the interval by the Compton effect. At the energies we are considering ($\sim 10^{14}$ eV) this loss is small, and is neglected.

The cross-sections for the production of pairs by high energy gamma rays and of gamma rays by bremsstrahlung in the coulomb field of the nucleus have been obtained by use of quantum theory[4]. For the energies with which we are concerned,the important values of the impact parameter are large enough that the approximation of "complete screening" of the nuclear charge by the atomic electrons may be assumed.The probability that a photon of energy E produces an electron pair in one g/cm^2 of matter is

$$4Z^2 \frac{e^2}{hc} \left(\frac{e^2}{mc^2} \right) \frac{N}{A} E \left\{ \frac{7}{9} \ln (183 Z^{-1/3}) - \frac{1}{54} \right\}$$

Where z,A and N are the atomic number,mass number and the number of neutrons (respectively) of the target atom. The loss of energy of an electron of energy E by bremsstrahlung in the same distance is

$$\Delta E = 4Z^2 \frac{e^2}{hc} \left(\frac{e^2}{mc^2} \right) \frac{N}{A} E \left\{ \ln (193 Z^{-1/3}) + \frac{1}{18} \right\}$$

These expressions are similar and nearly proportional to

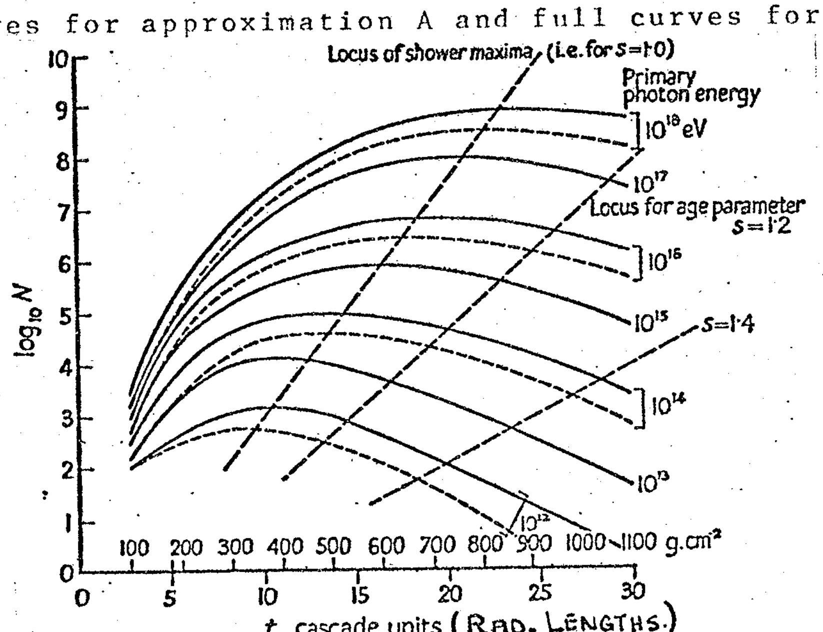
$$\mathcal{E} = 4Z^2 \frac{e^2}{hc} \left(\frac{e^2}{mc^2}\right)^2 \ln\left(183\, Z^{-1/3}\right)$$

It is convenient therefore to introduce a unit X_0 defined by $X_0 = (A/N\mathcal{E})$. This unit is called a RADIATION LENGTH and in it the energy of an electron is reduced to e^{-1} of its value by radiation.

Solutions of the appropriate differential equations[50] have than been obtained in two different approximations. In approximation A ,the ionization loss is neglected,and the asymptotic cross-sections for high energy and complete screening are used. A solution can then be obtained for the number of electrons or gamma rays with energy greater than a given value.

In approximation B ,the ionization loss is included and a partial solution can be obtained for the total number of electrons. Curves which show these results are given in FIG.1.(dotted curves for approximation A and full curves for B)

FIG 1



Photon-electron cascade curves. Full line, Snyder (1949); dotted line, Janossy and Messel (1951).

If $N(t)$ is the number of particles at depth t, the integral $N(t)dt$ is called the track length integral. Since all the energy of the primary particle is ultimately lost in ionizing the air and the energy loss per radiation length by ionization is E_c, it is clear that $\int_0^\infty N(t)dt = E_p/E_c$ where E_p is the primary energy. Greisen[5] has derived an approximate expression for the number of particles.

$$N(t, E_0) = \frac{0.31}{\ln(E_0/E_c)} e^{t(1-3/2 \ln s)}$$

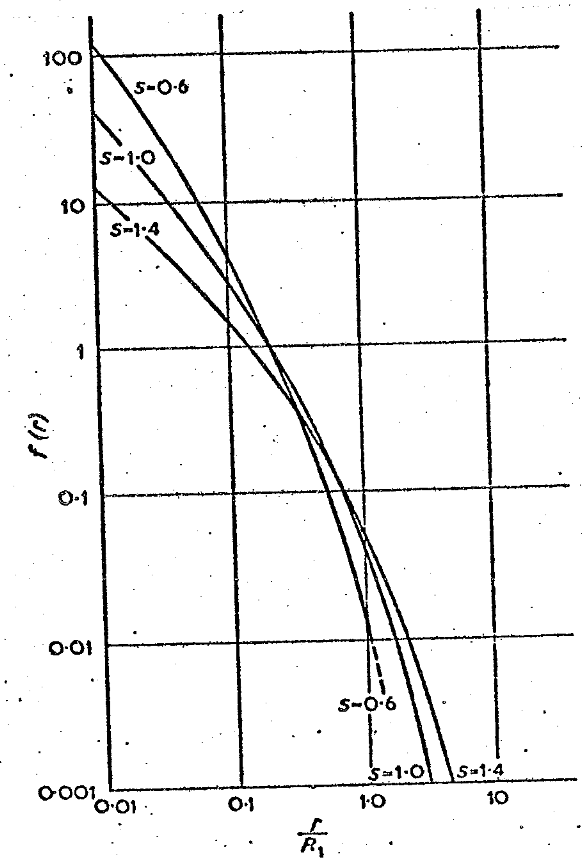
where s is given by $s = 3t/(t + 2\ln E_0/E_c)$.

1.3 LATERAL SPREAD OF COSMIC RAY SHOWERS

So far we have described only the longitudinal development of the electron photon cascade. The lateral spread of the particles is produced mainly by multiple scattering of electrons. The root mean square scattering angle of particles of energy E in one radiation length is given by E_s/E (Rossi and Greisen)[1] where $E_s = 21\times10^6$ eV whereas the mean angle of emission of photons in bremsstrahlung or electrons in pair production is of the order m_e/E, where m_e is the mass of the electron, which is about 5×10^5 eV.

The lateral distribution has been calculated using diffusion equations similar to those for the longitudinal development, but including terms for the lateral displacement and angular displacement of the particles. The resulting equations have been solved by Moliere[6], Eyges and Fernbach[7] and by Nishimura and Kamata[8]. The distributions obtained by Nishimura and Kamata are shown in FIG.2.

FIG 2



THE LATERAL DISTRIBUTION OF ELECTRONS ABOUT THE AXIS OF AN AIR SHOWER ON THE NISHIMURA-KAMATA THEORY (THE STRUCTURE FUNCTION). s is the age parameter and is equal to unity for showers at maximum development. R_1=79 metres at sea level.
(After Galbraith, Extensive Air Showers, Butterworths Scientific Publications, 1958).

1.4 THE DENSITY SPECTRUM

The actual densities of the showers detected depend not only on the area and the number of counters involved, but on the so called density spectrum of showers. By density spectrum is meant the frequency of showers $\gamma(\Delta)d\Delta$, which give densities within the range Δ to $\Delta+d\Delta$ particles per unit area at a particular point. The measurement of the form of the density spectrum $\gamma(\Delta)$ has been the subject of experiments carried out under a variety of conditions both as regards to the location of the experiments (altitude etc.) and the range of densities studied.

The results of all the experiments[9,10] is to show that the density spectrum can be represented by a simple power law of the form

$$\nu(\Delta)d\Delta = C\Delta^{-\gamma-1}d\Delta$$

i.e.an integral spectrum $\nu(>\Delta) = C'\Delta^{-\gamma}$,where C and C′ are constants.The work of Cocconi and Tongiorgi[11] produced the result that

$$\nu(>\Delta) = 720\ \Delta^{-1.48}\ /\mathrm{hr}$$

for showers in the range Δ =10 to 1000 per square meter at the sea level.As more and more data accumulated it has become apparent that there is a slow increase of the exponent with mean density.Greisen[12] concluded that the variation at sea level can be expressed with $\gamma = 1.33 + 0.039\ \ln\Delta$ for $1<\Delta<10^4$

1.5 THE NUMBER SPECTRUM

A quantity of great importance that can be derived from the shower data is the primary spectrum at high energies.To make this derivation it is necessery to know the number spectrum of showers and the relationship between the number of particles in a shower and the primary energy.

By the number spectrum is meant the number of showers, F(N)dN containing a total number of particles between N and

N+dN whose axes are incident upon unit area in unit time.If it is assumed that all showers have the same shape,i.e.the same structure function,than a relation can be found between the density spectrum and the number spectrum.

In order to contribute to a density of Δ, $\Delta+d\Delta$ at a particular point,the axis of a shower of size N ,N+dN must fall at a distance r,dr .The contribution to $\vartheta(\Delta)d\Delta$ is

$$\delta(\vartheta(\Delta)d\Delta) = 2\pi dr\, F(N)\, dN$$

and the density spectrum is then

$$\vartheta(\Delta)d\Delta = 2\pi \int_0^\infty r\, F(N)\, dN\, dr$$

the relation between N,r and Δ is

$$\Delta = f_1\ (r)N$$

$$\Delta(r) = (N/R_1\ )f(r)$$

where R_1 is the product of one radiation length and the r.m.s. deflection of a particle and f(r) is the lateral distribution function or the structure function

$$f_1\ (r) = f(r)R^{-2}$$

Greisen[5] has summarized the data for the integral number spectrum near sea level and gives the relation

$$F(>N) = 5.5 \times 10^{-12}\left(\frac{10^{-6}}{N}\right)^{-\gamma'-1} \quad cm^{-2}\ sec^{-1}\ sterad^{-1}$$

with $\quad \gamma = 1.53 + 0.02 \ln \frac{N}{10^6}$

1.6 THE PRIMARY ENERGY SPECTRUM

The relation between the energy of a cosmic ray primary, E_0 ,and the number of particles in the subsequent shower present at sea level,N,can be written as $N=DE_0^{\delta}$,where D is a constant and δ is an exponent having a value,close to 1, which depends on the details of the nucleon cascade model[5,14].The integral primary energy spectrum can therefore be written as

$$F(>E_0) = D' E^{-\delta\gamma}$$

where the best value of δ is probably 1.14 as given by Olbert's theory[13,14] . For $N=10^5$,Olbert's result gives $E_0 = 10^{15}$ eV and Greisen's relation gives $\gamma=1.48$ .The primary spectrum has thus an exponent

$$\delta\gamma=1.14\times1.48=1.69$$

Chapter II

POISSON STATISTICS AND THE DETECTION OF COSMIC RAYS

2.1 EXPECTED FREQUENCY DISTRIBUTION

In any series of measurements,the frequency of occurrence of particular values is expected to follow some "probability distribution law".In the case of cosmic ray showers this frequency distribution is the Poisson distribution[15] .

The Poisson distribution relates to the number of events that occur per given segment of time or space when the events occur randomly in time or space at a certain average rate.

The necessary and sufficient conditions to choose the particular probability distribution as the one controlling cosmic ray showers are

(1) The chance of a primary cosmic ray particle interacting with the atmospheric atoms or molecules is the same for all primary particles travelling in the same direction.

(2) The fact that a primary particle has interacted in a given time interval does not effect the chance that other primaries may interact in the same time interval (all primary particles are independent).

(3) The chance of a primary to interact during a given time interval is the same for all time intervals of equal size .

(4) The total number of showers and the total number of equal time intervals are large (hence statistical averages are significant).

If K represents a random variable on the sample space then,we can define the Poisson distribution as a distribution in which the probability that K=k is given[16] by $P(K=k)=P(k)=m^{k}\, e^{-m}/k!$, $k=0,1,2,3,\ldots$ We shall refer to such a situation by writing $K \sim Pn(m)$,i.e.K is distributed according to the Poisson distribution with parameter m,where P(k) gives the probability that k events occur in the chosen segment of time or space.

The recurrence relation for the calculation of the P(k)'s is

$$P(k)=P(k-1)\, m/k$$
$$P(0)=e^{-m}$$

2.1.1 The Mean and Variance

The distribution of K is represented by μ or E(K) and defined as[17]

$$\mu=E(K)=\sum_{k=0}^{\infty} k m^{k} e^{-m}/k!$$

$$\mu = \sum_{k=0}^{1} m^k e^{-m}/(k-1)!$$

$$= me^{-m} \sum_{k-1=0}^{\infty} m^{k-1}/(k-1)!$$

$$= me^{-m} e^{m} = m \quad \ldots\ldots\ldots\ldots\ldots\ldots(1).$$

The parameter m is the mean number of events which occur per given segment of time or space.The distribution variance of K is represented by V(K) and defined as;

$$V(K)=E\{(K-\mu)^2\}$$
$$=E(K^2-2\mu K+\mu^2)$$
$$=E(K^2)-\mu^2$$
$$=E(K^2)-\{E(K)\}^2$$
$$=E\{K(K-1)\}+E(K)-\{E(K)\}^2 \quad \ldots\ldots\ldots\ldots(2)$$

$$E(K(K-1))=\sum_{k=0}^{\infty} k(k-1)m^k e^{-m}/k!$$

$$=\sum_{k=2}^{\infty} m^k e^{-m}/(k-2)!$$

$$= m^2 e^{-m} \sum_{k-2=0}^{\infty} m^{k-2}/(k-2)! = m^2$$

by using equation 1 and this result we can write equation 2 as;

$$V(K)=m^2+m-m^2$$

$$V(K)= m$$

Thus the mean and variance of a Poisson distribution are the same.

2.2 THE POISSON PROCESS[15,16,17]

We assume that the probability of the occurance of an event in the time interval (t,t+dt) is $\lambda\delta t+0(\delta t)$ where λ is a constant characteristic of the process.Here δt is small and $0(\delta t)$ means "small compared to δt".We also assume that the probability of occurance of more than one event in the interval in the question is $0(\delta t)$.

Suppose we consider the probability of the occurance of n events in the interval $(0,t+\delta t)$ where $n\geqslant 1$.Under the above assumptions, we need only consider the probabilities of two alternative ways of reaching this situation.

A: n events occur in the interval (0,t) and none in the next δt

B: n-1 events occur in the interval (0,t) and none in the next δt since we do not need to consider other probabilities of much smaller probability.If we let P(n,t) designate the probability that n events have occured in the interval (0,t).We have

$$P(A)=P(n,t)\{(1-\lambda\delta t)+0(\delta t)\}$$

$$P(B)=P(n-1,t)\{(\lambda\delta t)+0(\delta t)\}$$

in the above equations the term $0(\delta t)$ can be ignored since it is very small and the term $(1-\lambda\delta t)$ represents the probability of having no events in the interval (t,t+dt).Also

$$P(n,t+\delta t)=P(A)+P(B)+0(\delta t) \quad \text{Hence}$$

$$P(n,t+\delta t)=P(n,t)(1-\lambda\delta t)+P(n-1,t)(\lambda\delta t)+0(\delta t)$$

, which gives

$$\{P(n,t+\delta t)-P(n,t)\}/\delta t=\lambda\{P(n-1,t)-P(n,t)\}+0(\delta t)/\delta t$$

in the limit, as t goes to 0

$$\dot{P}(n,t) = \lambda\{ P(n-1,t) - P(n,t)\} \quad\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Equation 3 may be recast,thus

$e^{-\lambda t}\ d/dt\{ e^{\lambda t} P(n,t)\} = \lambda P(n-1,t)$ which on integration between the limits 0 (where P(0.0)=1) and t yields

$$e^{\lambda t} P(n,t) = \lambda \int_0^t e^{\lambda t} P(n-1,t)\, dt \quad\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Equation 4 is a recurrence formula by which we may obtain successively $P(1,t), P(2,t), P(3,t)\ldots\ldots P(n,t)$ given $P(0,t)$.

For the case n=0,we have

$$P(0,t+\delta t) = P(0,t)\{(1-\lambda\delta t)+0(\delta t)\} \text{ hence}$$

$$-\lambda = \dot{P}(0,t)/P(0,t) = d/dt\ \{\ln P(0,t)\} \text{ therefore}$$

$$P(0,t) = e^{-\lambda t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Using equations 4 and 5 it can be shown by induction that

$$P(n,t) = \{ (\lambda t)^n \ e^{-\lambda t} \}/n!$$

Hence the number of occurrences in the time interval(0,t) is distributed as $Pn(\lambda t)$.

We now consider the distribution of the time to the first occurrence of an event,by using equation 5 .The probability that the first event will occur in the interval $(t,t+\delta t)$ is the probability that none have occurred up to time t multiplied by the probability one event occurs in the interval (again,neglecting alternatives of very small probability) that is

$$P(0,t)\{(\lambda\delta t)+0(\delta t)\}$$

Substituting for P(0,t) from the equation 5 gives for the probability in question

$$\lambda e^{-\lambda t}\delta t+0(\delta t)$$

Hence, the density of the distribution of the time to the first occurrence of an event, which we shall represent by f_1 (t) is given by

$$f_1(t)=\lambda e^{-\lambda t}$$

Here, the starting time may be just after an event has occurred or any other time.

Therefore the numbers of events per given time have a Poisson distribution while the intervals between consecutive events have an exponential distribution.

2.3 CHI-SQUARE TEST OF GOODNESS OF FIT

This procedure allows a comparison between the actual and expected number of observations (expected under the "assumption ") for various values of the variate. The expected numbers are calculated by using the assumed distribution with the parameters set equal to their sample estimates. We define the quantity

$$\chi^2 = \sum_i \frac{[(\text{observed Value})_i-(\text{expected Value})_i]^2}{(\text{expected Value})_i}$$

where the summation is over the total number of independent classifications i in which the data have been grouped. We determine the number of degrees of freedom F , which is the number of independent classifications.

$$F=k-p-1$$

where p represents the number of parameters estimated by sample statistics.For example if the normality assumption were under test, μ and σ^2 would be estimated and the number of degrees of freedom would be k-3 ,where k represents the number of class intervals used in fitting the distribution.If the assumption of a Poisson distribution were being tested λ would be estimated by X,and the number of degrees of freedom would be k-2.

By using FIG 3[15] and from the values of chi square and F we determine P, which is the probability that chi square would exceed its observed value.If that probability is 0.7 then seven times out of ten one would expect to observe a value of chi square at least as large as the one calculated from the experimental data. In this paper we will use the notation (reduced chi-square value)/degrees of freedom to identify the values of chi square.

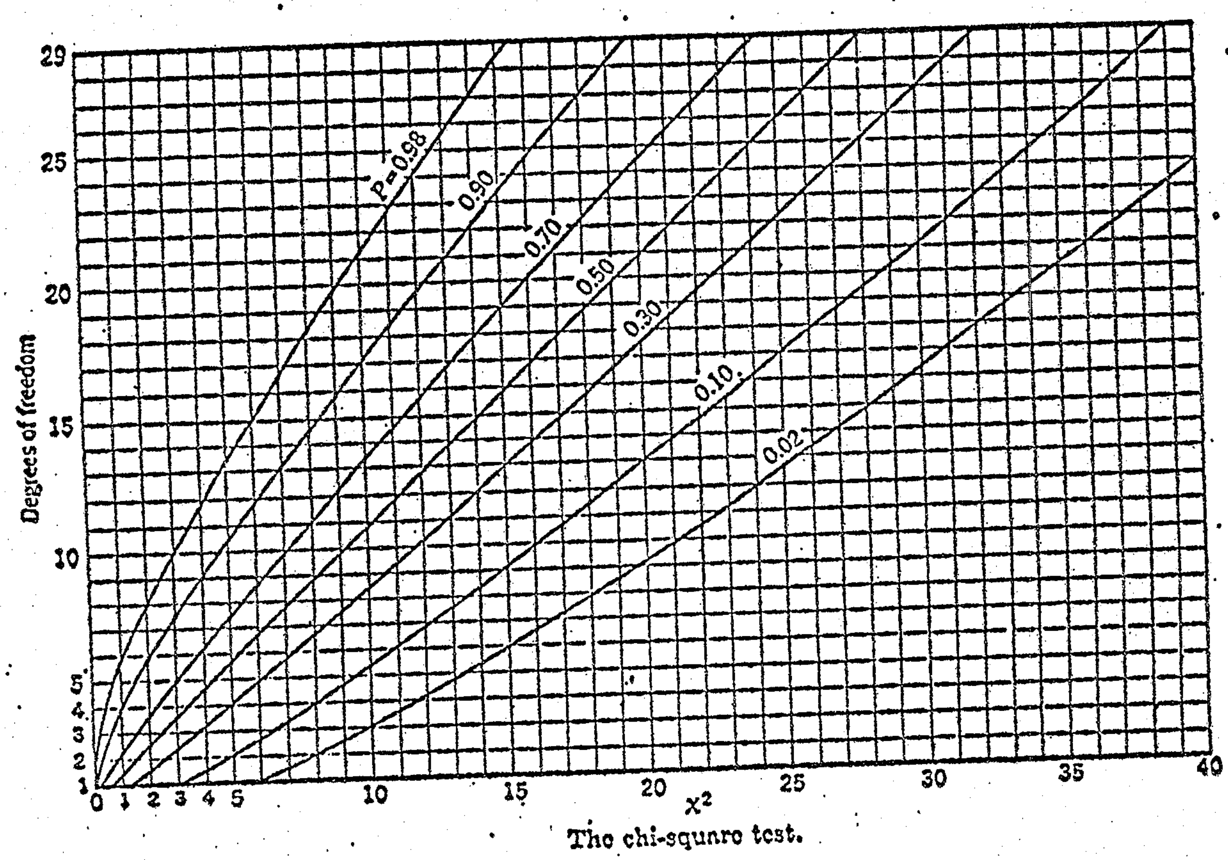

Degrees of freedom
29
25
20
15
10
5
4
3
2
1
0 1 2 3 4 5 10 15 20 25 30 35 40
χ^2
P=0.98
0.90
0.70
0.50
0.30
0.10
0.02
The chi-square test.

FIG3

Chapter III

TIME VARIATIONS OF C.R.SHOWERS

The extensive air shower (E A S) has been a widely studied phenomenon of cosmic ray physics since its discovery in 1938.The first suggestion that the extensive air showers are electron-photon cascades in the atmosphere was made independently by CLAY[18] and experiments in France[19] and Germany[20] .The search for time variations in the rate of arrival of extensive air showers has been a field of great interest for some years,since these events indicate the arrival into the earth's atmosphere of the highest energy particles in the cosmic radiation.Two basic methods have been used to study time variations of cosmic rays and their possible anisotropies in space and time.

1. The counting rate of a given arrangement of apparatus may be recorded as a function of time.Researchers MARTELLI and FORNACA[21] ,J. and A.DAUDIN[22] ,FARLEY and STOREY[23] ,CRANSHAW and GALBRAITH[24] , CRANSHAW and ELLIOT[25] ,McCUSKER[26,27,28] ,have reported results using this method.
2. The directional method is used by a group of researchers working in M.I.T.[29].They determined the direction of the incoming particle from the

tracks, produced by the secondary particles, in the spark or a cloud chamber. These directions are then plotted as a "point" on the celestial sphere. Another version of this method is employed by CLARK[30] and GOODINGS[31]. They deduced the directions from the timing measurements on the arrival of a shower front. The results of all these early experiments indicate that the primary cosmic radiation is isotropic in space and also isotropic in time (i.e. random). There are however very small diurnal ($\sim$0.3 %) and sidereal ($\sim$0.03 %) time variations[49].

The apparent isotropy of primary particles is explained in a slightly different manner in different theories of the origin of cosmic rays. The solar origin theory, suggested by DAUVILLIER[32] and further developed by RICHTMEYER and TELLER[33] and ALFVEN[34], supposes that particles emitted by the sun are trapped in a magnetic field which extends throughout the solar system. The particles circulate in orbits under the action of this extended field of strength about 10^{-5} gauss, and after a period of time 10^{3} - 10^{8} years they ultimately become essentially isotropic.

In a galactic origin theory[35,36,37] the motion of the particle within the spiral arm of the galaxy (where the mean strength of the magnetic field directed along the spiral arms of the galaxy is on the order of 6×10^{-6} gauss) is considered. It is supposed that superimposed on this mean field hydromagnetic waves of amplitude 10^{-6} gauss are travelling along the spiral arms and that between scattering collisions with gas clouds the particles are accelerated by a mechanism similar to that in a betatron accelerator.E. BURBRIDGE and G. BURBRIDGE[38] considered in some detail the consequences of such a model on the observed isotropy of the primary cosmic ray particles.

COCCONI[39] has presented some interesting arguments which point to the source of the highest energy cosmic rays being in intergalactic space.He points out first of all that it is difficult to accelerate primary particles up to 10^{18} -10^{19} eVand at the same time keep them isotropic if we suppose they are to be confined to the galaxy.He suggests that at least the highest energy particles must be present in intergalactic space and probably are accelerated and made isotropic there.

Recently BHAT et al[40,41] have reported a large non-random component in the arrival times of EAS with $E>10^{14}$ eV.They detected these showers by using night sky cerenkov light pulses at Gulmerg,India between June 1976 and June 1978.Their interval distribution analysis is based upon 180 hours of observation (9879 showers) with an average rate of 55 events/hr. and showed a 5σ excess over the expected random (i.e. exponential) interval distribution. Furthermore their data indicates significant periodicities with peaks in the interval distribution at 4 sec.,8 sec.,12 sec. FEGAN et al[42] presented a preliminary report of a similar experiment using a ground level air shower detector array searching for this anomaly among some 20000 showers of primary energy $\geqslant 10^{15}$ eV.They reported negative results in a search for such non-randomness.

Chapter IV

THE EXPERIMENTAL SET UP

FIG 4 shows the arrangement of the University of Manitoba air shower array detectors. The detectors A,B,C were made from sheets of NE102 scintillator (90cm x 90cm x 2.54cm) each viewed by a five inch photomultiplier tube (RCA 8085) located above and to the side of the scintillator.

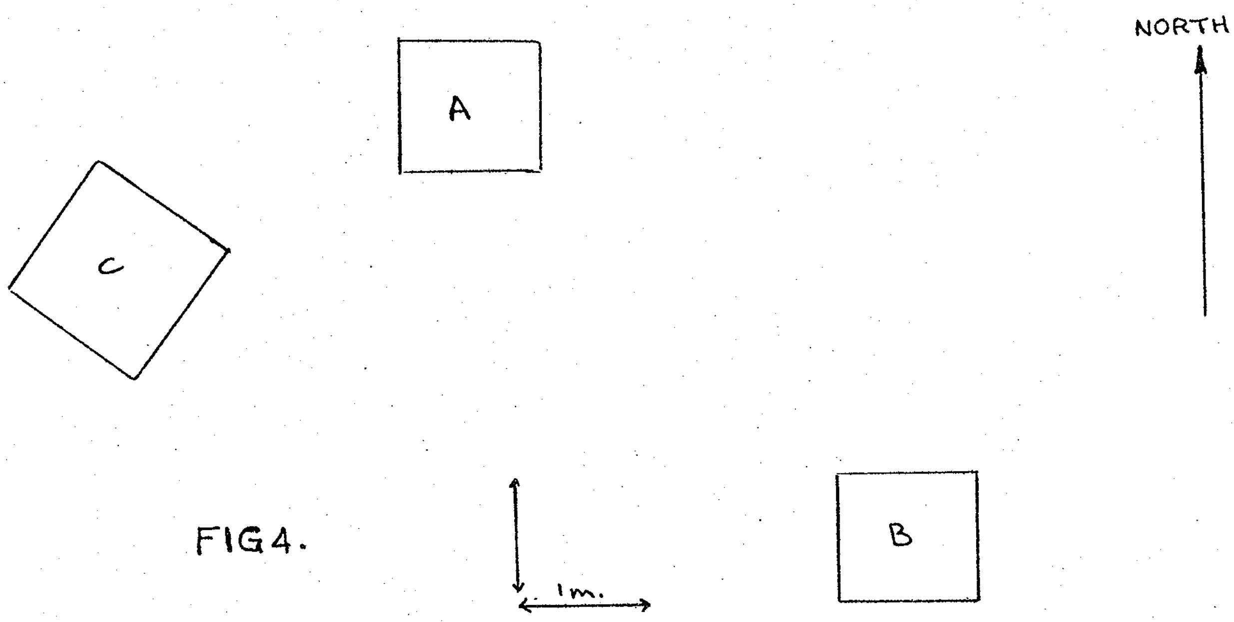


FIG4.

DETECTOR ARRAY FLOOR PLAN.

Each scintillator was enclosed in a white interior, light tight box. An air shower is indicated by an A ,B ,C coincidence (τ =1 μsec). Discrimination levels equivalent to 10 particles/m^2 were set for detectors A ,B ,C .The array was sensitive to showers of energy greater then $\sim 10^{15}$ eV (see appendix C).The block diagram for the electronic set up is shown in FIG 5 .

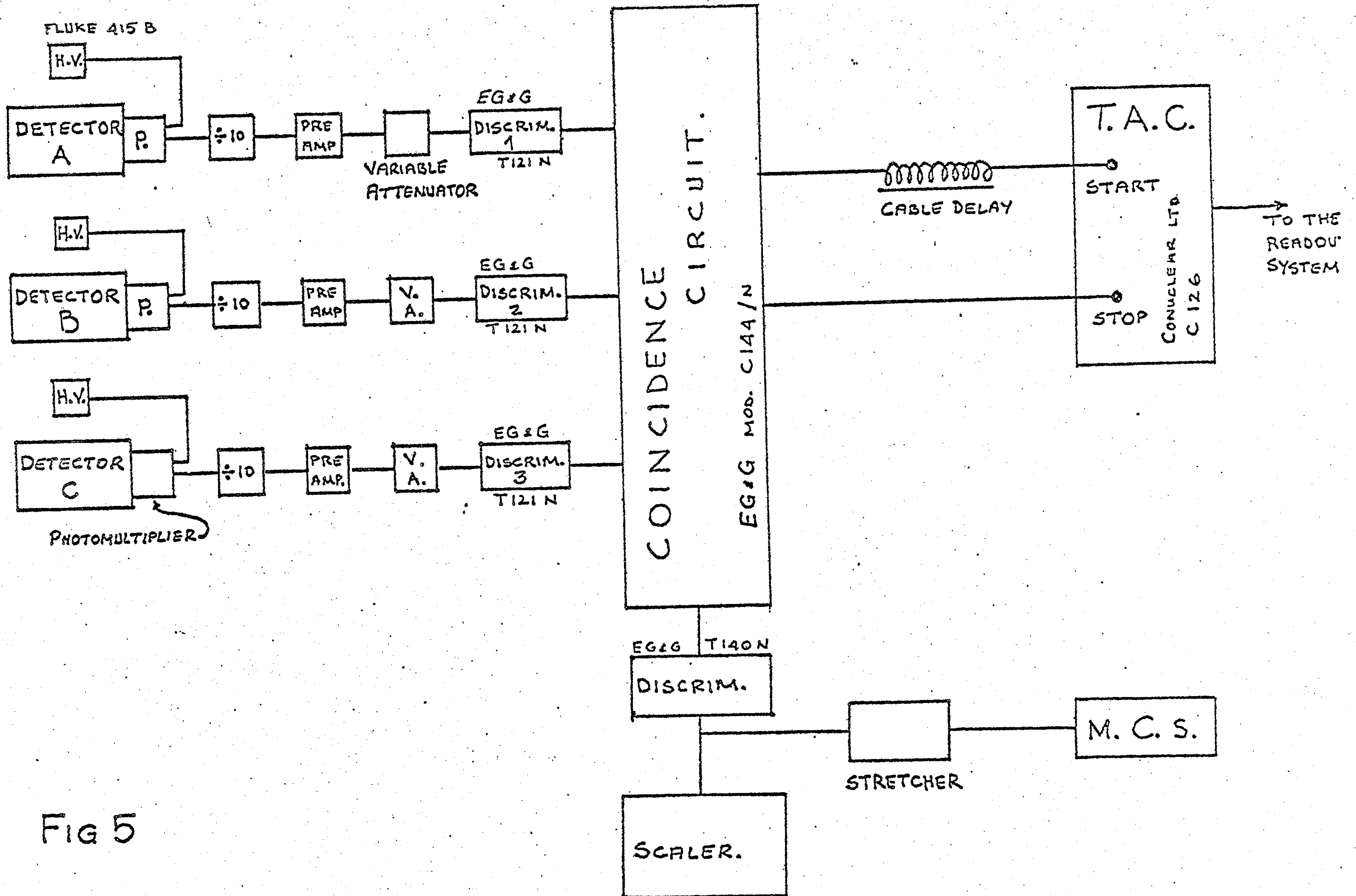

FLUKE 415 B
H.V.
DETECTOR A
P.
÷10
PRE AMP
VARIABLE ATTENUATOR
EG&G
DISCRIM. 1
T121 N
H.V.
DETECTOR B
P.
÷10
PRE AMP
V. A.
EG&G
DISCRIM. 2
T121 N
H.V.
DETECTOR C
÷10
PRE AMP.
V. A.
EG&G
DISCRIM. 3
T121 N
PHOTOMULTIPLIER
COINCIDENCE CIRCUIT.
EG&G MOD. C144/N
CABLE DELAY
T.A.C.
START
STOP
CONUCLEAR LTD C 126
TO THE READOUT SYSTEM
EG&G
T140 N
DISCRIM.
STRETCHER
M. C. S.
SCALER.

Fig 5

The shower detectors were calibrated by observing the muon "through peak"in each of the detectors.Each photomultiplier signal was fed into a preamp and the phototubes were balanced to give the same pulse height for the "through peak" for each of the three detectors.Uniformity between detectors was obtained by setting the discriminator levels to an appropriate value (-100mV) and approximately adjusting the variable attenuator before each discriminator. Outputs from the above mentioned discriminators were then fed into a coincidence circuit. The time interval between succesive showers was monitored in an 800 channel pulse height analyzer (Victoreen Model ST-800M) operating in the multiscaling (MCS) mode.The MCS counted an internally generated 60Hz signal derived from the AC line frequency,so that time intervals could be measured in units of 16.7 μsec.A pulse created in the coincidence circuit by the detection of an air shower,caused the MCS to advance one channel.The MCS took approximately 125 microseconds to switch channels allowing a maximum channel advance rate of 8 khz. Since the time interval between air showers on average was about 4 to 5 minutes, the MCS switching time contributed negligible uncertainty to the measured time interval between air showers.Because the MCS had a maximum channel content of 10^5 counts,an upper limit was imposed on the measurable time interval between successive air showers of 27.8 minutes. Time intervals longer than 27.8 minutes, which are called rollovers, caused the multiscaler to start again at zero counts in the same channel and were undetectable in the experiment.

A data run consisted of recording the time intervals between 800 consecutive showers with the MCS and lasted almost three days. The calibration of the three shower detectors was checked using cosmic ray muons.A detection rate was determined for each detector of the array for events depositing more than 10 times the energy of a minimum ionizing particle. These rates were roughly comparable for the three identical detectors and phototube noise did not adversely affect the shower registration rate.This single event rate was checked for each of the detectors every two weeks and relative attenuation levels were adjusted to maintain these singles rates.By recording start and the stop times for the runs,the solar and sidereal times of detection of each shower could be inferred to an accuracy of 1/2 hour.This limit is set by the fact that we expected 0.8 rollover events/ run,resulting in the loss of 28 minutes of recorded run time.

The resolving time of the coincidence circuit was directly related to the length (τ_w) of a signal produced by a discriminator These were set so that τ_w= 500 nsec.Therefore the resolving time of the coincidence circuit (2 τ_w) was 1 μsec. and the chance coincidence rate for this set up can be worked out approximately as $N_A\, N_B\, N_C (2\,\tau_\omega)^2$ where N_A, N_B and N_C stand for the singles rates for each detector. Substituting the observed values for these rates and τ_ω the chance rate was 2.7×10^{-13} s^{-1} and thus quite negligible.

This experiment was set up not only to record the shower frequency but also to detect possible cosmic ray bursts defined as air showers following each other with a very small time interval between them (up to 120 msec. in this experiment). This interval corresponds to 7 cycles in our MCS. As a check of possible bursts and to be able to record the interval reliably the 120 msec. after each air shower was also monitored by a time to amplitude converter (TAC) (FIG.5). An input "start" pulse to the TAC starts the internal converter circuit (if it is not already busy) via the "start" tunnel diode discriminator and enables the "stop" circuit. An input "stop" pulse can then stop the converter circuit which will then have a voltage stored on it proportional to the time between the "start" and "stop" pulses. The "start" pulses not followed within the conversion time (120 msec.) by a "stop" pulse result in the triggering of the over conversion detector which immediately resets the complete system with no additional dead time. The range setting on the TAC unit selects the time difference between "start" and "stop" input pulses necessary to produce a 10 V output pulse e.g. with range set at 4 μsec the output amplitude will be equal to

10 V for "starts" followed by "stops" in 4 μsec. Both the "start" and the "stop" pulses for the TAC were taken from the coincidence unit.The start signal was delayed via a delay line (cable) for 200 nsec.As a result of a shower,two identical signals are produced from the coincidence unit.One of these pulses activates the "stop" input of the TAC and the other activates the "start" input. But since the start signal is delayed for 200 nsec,the TAC unit starts monitoring the interval between showers 200 nsec later than the actual arrival time.If another shower arrives within the time range set in the TAC it stops the TAC immediately and a corresponding output pulse is produced.200 nsec later the delayed "start" signal from the second shower arrives and activates the TAC again.The pulse sequence for the system is shown in FIG 6.

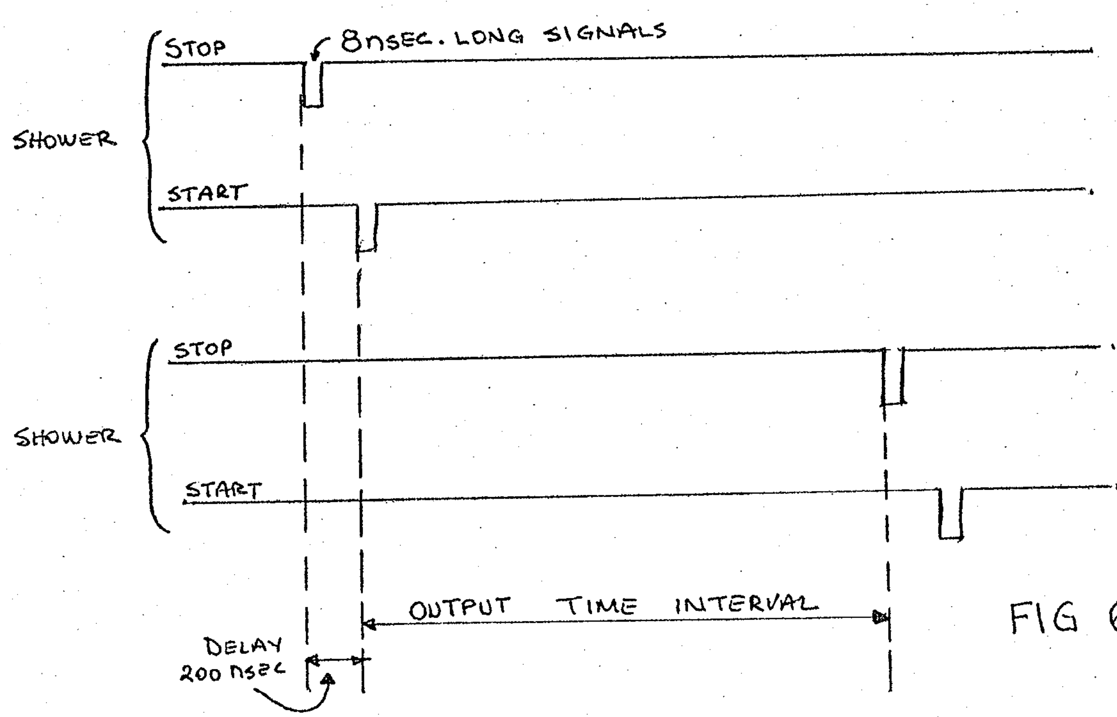


FIG 6.

As can be seen from the pulse sequence diagram the output of the TAC,which indicates the elapsed time between showers,was 200 nsec shorter than the actual interval length but since this interval is on the order of 120 msec the lost time was negligible.

Chapter V

RESULTS AND DISCUSSION

5.1 RANDOMNESS OF PRIMARY PARTICLES

The main objective of this experiment was to look for the large non-random component and possible periodicities in the interval distribution of EAS with $E \geqslant 10^{14}$ eV reported by Bhat et al[40,41]. The non-random component they reported presented itself as an excess for short time intervals (<40 sec) in the interval distribution.Since the intervals between consecutive random events theoretically have an exponential distribution,the plot of such a distribution on a logarithmic scale produces a straight line; the meantime of this line depends on the mean time between events (an event being the detection of an EAS).If there really is an excess as claimed by Bhat et al this would require at least two straight lines with different meantimes for the regions between $0 \leqslant t \leqslant 40$ seconds and $40 \leqslant t \leqslant \infty$ seconds.Therefore we divided our data into two groups and fitted a straight line to events with intervals $t \leqslant 50$ seconds and another one to all of the data.If the resultant meantime values for these two lines agree with each other within the error then that would force us to conclude that one exponential was sufficient for all the data and that primary cosmic rays were truly random events.

Our data was recorded in the University of Manitoba cosmic ray laboratory between September 1980 and June 1982 and consisted of 210 individual runs (a total of 149500 showers).The analysis of any one individual run exhibited a common problem inherent to every single one of them, namely,because of the limited statistics the error values produced for the meantimes were large.A typical example of this can be illustrated for the run recorded between 6^{th} and 8^{th} of July 1981.This particular run included 772 events and 146 of the recorded intervals were in the first fifty seconds.The meantime for the line fitted to the first fifty seconds was 9.04 ± 22.7 mins. and the meantime for the line, fitted to all of the data was 4.52 ± 0.16 mins..The computer program we employed to fit these lines also calculated a chi square value, which is an indication of how good a fit is to a given function. According to statistical tables[15],reduced chi square values 1 ± 0.8 could be accepted as good fits.The chi square values for this particular run (1.69/46 , 1.01/20 where 46 and 20 represents the degerees of freedom in each case) were "good".The rather high error values which make conclusions impossible can be reduced by including several runs in the analysis.This can be illustrated by the following; using the data accumulated between Jan.1^{st}- Aug.6^{th} 1981, which had 92 runs and a total of 72013 events with 14608 of these in the first fifty seconds,the meantime values were 2.85 ± 0.27 mins. for the first fifty seconds and

3.93 ± 0.01 mins. for all of the data.The chi square values were respectively 1.36/48 and 7.34/26 .Even though these "mean time" results seem to indicate that we should fit two exponentials to the interval distribution of EAS(analogous to the Bhat et al results) this is probably not the case because the corresponding chi square values indicate "very poor" fits.

Reduction of chi square values was achieved by the following method. For each individual run we calculated a mean rate.These rates varied from 0.19 showers min^{-1} to 0.35 showers min^{-1} with the majority of runs having data registration rates between 0.19 and 0.27 showers min^{-1}.All the runs with rates between 0.19 - 0.27 showers min^{-1} were put together and analyzed.This grouping of data included 69497 showers which were recorded between September 1980 and December 1981.The first fifty seconds interval had 12329 showers and the fit produced from these selected events had a meantime value 3.94 ± 0.56 mins. and a chi square value of 1.94/48.The meantime resulting from the fit to all of the data was 4.30 ± 0.01 mins.with a chi square value 1.60/26.Such results indicate a single exponential fit to the whole interval distribution with comparatively better values of chi square .Later on with the addition of the data recorded during Jan.-June 1982 the number of showers in this particular rate range was increased up to 125803.To study the effect on the chi square value we divided the above range into two parts and analyzed each part separately. The range $0.19 \leqslant R \leqslant 0.22$ events min^{-1} had 55159 events of which 8532 were in the first fifty seconds interval. Resultant values for the meantimes were 4.86 ± 1.02 mins. for the first fifty seconds and 4.92 ± 0.2 mins. for all of the data with chi square values of 1.18/48 and 1.19/26 respectively. The other range $0.22 \leqslant R \leqslant 0.27$ events min^{-1} had 13121 showers in the first fifty seconds interval and the total number of showers in this group was 70644. The meantime and the corresponding chi square values were 4.50 ± 0.71 mins. , 1.51/48 for the first fifty seconds interval and 4.09 ± 0.02 mins. , 1.11/26 for all of the data. For both groups, meantimes of the first fifty seconds and all of the data agree within the error and the chi square values indicate "good" fits to a single exponential function. It is also clear that the narrower the range of rates which are considered the better is the fit to the single exponential assumed. In Fig.7 we plot the distribution of time intervals between the detection of the 70644 successive air showers in the range $0.22 \leqslant R \leqslant 0.27$ events min^{-1}; errors are not indicated where they are of a size comparable to or less than the plotted points.

It is therefore concluded that for extensive air showers of primary energy $\geqslant 9 \times 10^{14}$ eV, recorded over a period of two years, the time interval between the detection of successive air showers depends exponentially on that time. This is consistent with the conclusion that detection of such showers are truly random event in contradiction to the conclusions of Bhat et al.A possible explanation of the Bhat et al results may well lie in their limited statistics.They observed less than 10000 showers and reported no mesurement of "rates" during their experiment. We have seen that if there is a significant spread in the rates one might well find the need of more than one exponential to fit the data which would then lead to the conclusion of "non-randomness".

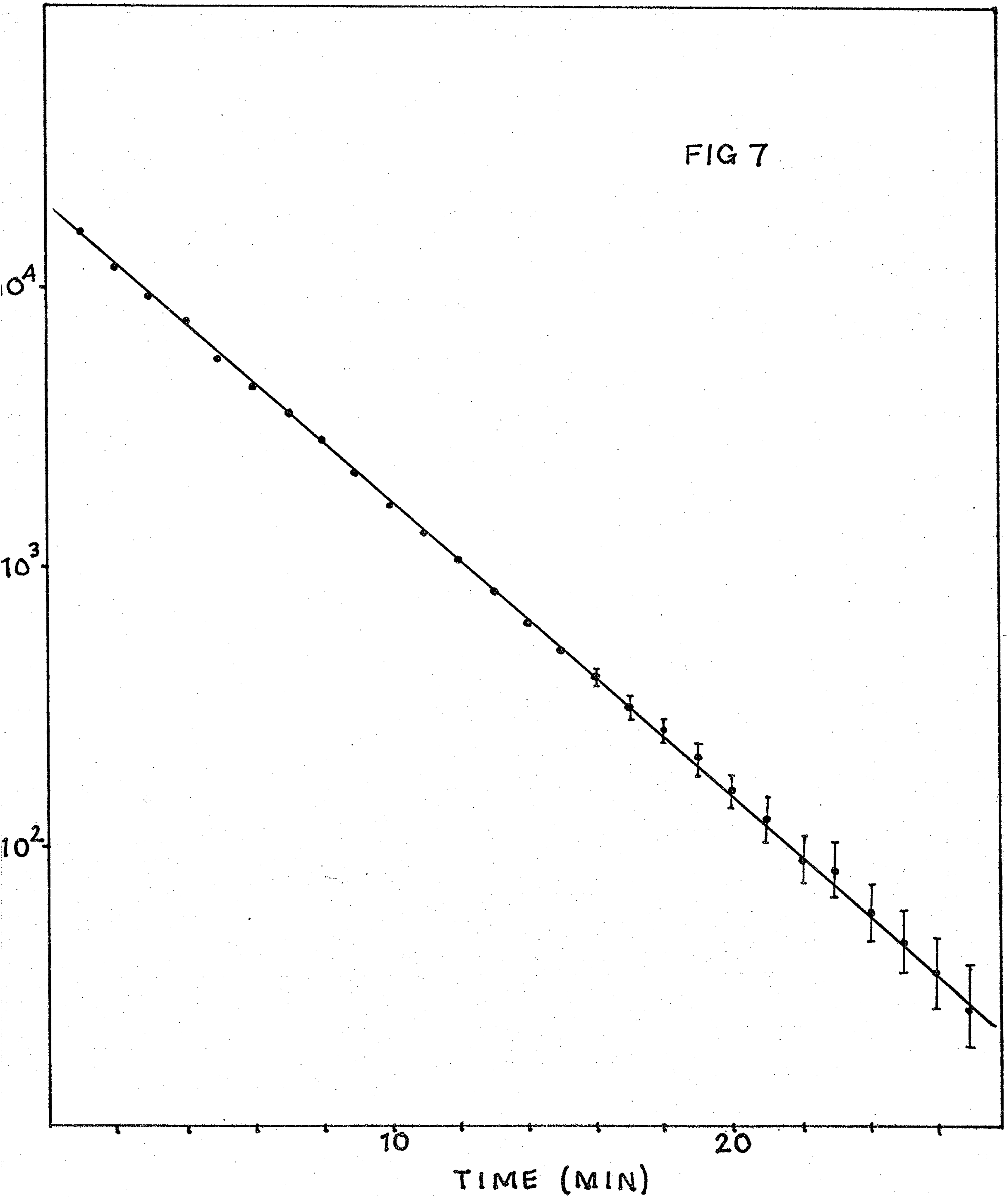


FIG 7
10^4
10^3
10^2
10
20
TIME (MIN)

5.2 SEARCH FOR A POSSIBLE STRUCTURE (PERIODICITY) IN THE FREQUENCY DISTRIBUTION

Since we have to restrict the range of rates to improve the exponential fit in the discussion below we will use the data recorded in runs with rates between .22 and .27 showers/min.While this restriction reduces our sample size from 156000 to 70644 showers the subset is typical of the data recorded in the other rate intervals.Since rejected runs are distributed throughout the data recording period,or occur for deliberately set experimental conditions,this reduction is not unreasonable.

In FIG 8 we plot the distribution of time intervals between successive air showers for time intervals less than 100 seconds,which includes 23815 events out of 70644. To be able to identify any possible structure in this distribution that might be significant,a curve smoothing technique (see appendix B) was applied to the data resulting in the solid curve shown in this figure.The dashed line represents the "best fit" to an exponential. To find out if any of the observed "peaks" represents real structure, artificial time interval data were generated by using an appropriate program (see appendix A) and the results smoothed by our smoothing algorithm.FIG.9 shows the interval distribution of 51030 artificially generated shower data for time intervals less than 100 seconds. There, we see the same general structure i.e. peaks ,which are followed by similar size valleys.In an attempt to eliminate these fluctuations we further smoothed the artificial data by using 575 QHT , 797 QHT and 595 QHT (see appendix B).We were testing to see whether a smooth exponential without the little "waves" superimposed would result from the application of one of these smoothing techniques so that the same technique could then be applied to the real data leaving behind only the marginally significant real structure. The results of smoothing the artificial data with these same techniques showed that even when the 595 QHT algorithm was used these little "waves" persisted.

The effects of the addition of more data to the analysis and smoothing technique was checked by comparing the interval distribution plots of 50000 and 150000 artificially generated events.The results showed no essential change; with "waves",due to statistical fluctuations,in both cases. The conclusion therefore is that the present experiment finds no supporting evidence for the short term periodicities (i.e. structures at 4 sec.,8 sec.,12 sec.) which was reported by Bhat et al[40,41].

5.3 POSSIBLE BURST OF EAS

During the recording of the run,which started at 18th of January and finished at 21st we detected a possible EAS burst.At the time,the TAC system was not operational and the intervals between showers were recorded with the MCS.The "burst" started at the 20th of January 9:55 A.M. The MCS recorded 31 events in 4.4 minutes where the number of expected events was approximately 1.This is the only such event we have observed during our 21 months of operation time. In table A we present the data recorded during this run.The burst starts at the channel 690.

CHANNEL 1				CHANNEL 5				CHANNEL 9		
6468	10600	1879	15284	29573	12103	2351	5175	6898	1995	
5369	20659	759	11968	4721	26219	20299	12991	6026	35304	CHANNEL 20
12792	1675	5257	22614	30809	15986	16729	1508	19440	902	
30926	6101	4349	59	265	4876	4991	9807	2374	16828	
24428	65925	14631	24059	26980	20888	7756	2434	8092	1978	
40690	4986	18883	7937	1027	23311	8432	8779	9044	5450	
66083	23543	40110	28522	42154	6027	8382	26317	9538	4524	
2816	10936	49225	5641	22000	7547	37217	4133	54562	9873	CHANNEL 80
3927	13102	52289	4637	5930	31156	1636	376	10978	1166	
3872	3411	8426	55210	53	29007	11837	6044	14812	9855	
3057	19493	3523	415	5404	1657	28826	2676	724	5022	
6948	8220	26152	28293	4355	11676	16756	2571	33526	17902	
20853	1297	6201	5132	895	27128	10177	2858	8603	1658	
21474	6188	17735	18198	46189	9825	18856	5352	2470	1958	
18171	9933	12364	8336	9781	9640	10095	13906	33176	11707	
1467	1922	27076	8762	20410	38329	1428	22947	951	36356	
1343	1244	29657	7100	31568	7507	5553	19017	4684	10216	
35801	27513	13422	13692	6834	62139	3967	1605	9236	85	
1021	32476	6335	10352	13783	14946	22095	1053	17796	22552	
11212	19741	6800	7759	4713	8666	1063	10895	23492	2544	CHANNEL 200
24277	425	35235	11699	6369	8393	870	6395	15931	36618	
6712	7721	40071	7205	20659	10841	7772	3027	3247	60966	
7092	5807	3255	13241	28838	501	1501	24242	50595	5347	
863	6256	34199	12918	33245	22440	2386	9385	33391	3942	
3385	2528	17175	27505	18307	9944	2973	7337	7066	10994	
17460	20349	18161	2906	14692	7815	7526	38041	920	19758	
14815	11970	22529	6528	2201	2696	2925	47793	23720	1746	
32093	19494	10587	5901	578	12120	2672	26735	10666	33187	
11868	7769	15042	18052	7306	22553	3122	7991	44149	6063	
4524	10526	7418	12877	4304	3043	9824	4921	16816	5144	
40307	8974	5081	2083	2776	17487	13629	16337	15004	12861	
3315	702	3828	12495	2641	38672	15696	7766	16927	841	
8903	12049	5596	16205	8066	28440	28615	20569	25005	1489	
24418	31511	16	7994	9875	7343	1572	17098	9951	32659	
9316	341	1285	4256	73447	22211	16664	13925	7558	2973	
1942	2851	21836	8358	12559	3929	4505	14833	13715	2885	
13558	1750	3463	25302	15593	5440	2849	38296	1525	2321	
997	55850	3906	8272	9029	25664	1143	1124	12709	12140	
8061	3244	7003	18594	1403	5467	5628	3441	1153	23329	
2469	21451	1456	36596	6097	21405	750	12321	2188	11044	
2432	2968	11	7139	280	14081	1998	15454	381	24421	
2822	3091	20099	5989	18245	9594	1567	23032	17603	669	
8015	6596	3383	103	4570	5232	49180	12913	1257	8264	
5708	10122	952	9895	6048	226	11869	33838	34650	34536	
4518	19191	6223	58665	19348	11263	8066	1851	2494	2669	
9044	6208	19906	40510	21431	12744	202	12211	14969	6687	
5452	5852	6475	33697	10427	29468	19974	27194	17499	5553	
1465	20288	221	479	34511	903	2051	3633	52142	10166	
1971	2049	15235	1204	26448	18451	321	13962	11592	52597	
1497	43669	48242	18193	6993	3507	1079	3716	8724	31656	
5301	13385	4150	7134	7372	11029	18447	1391	868	46812	
599	4079	6446	482	8345	925	24013	6839	13130	13917	
2095	882	5231	12888	1561	11100	9773	17536	26724	3539	
24946	6365	38119	2196	8806	24867	4616	3741	7059	15045	
590	1287	49369	9930	3474	17349	4619	588	21791	1962	
73921	1687	2858	1534	2946	38735	7496	1057	10318	14120	
3839	73650	11736	3515	53460	15756	10104	3645	204	2214	
5443	27227	5026	8293	74	2969	4695	11784	8228	44224	
11453	5218	596	16868	25240	7382	16570	5249	10378	17488	
20560	2652	26110	13737	16494	27635	360	21940	145	2134	
3323	10895	14593	21766	47334	8334	8842	24480	7982	16292	
19810	15768	112	66079	9234	7110	2650	24333	13740	6555	
8838	12507	8112	7923	534	934	1282	36495	6738	23697	
184	17424	11177	2506	5730	84760	4610	10056	35241	13004	
32377	10925	11098	10969	3789	14499	2584	5805	2836	16178	
17920	7024	1809	6177	34496	8388	5358	2234	7422	25184	
707	35780	15781	4753	2790	4281	140	27061	29841	3088	
18215	48911	6083	11339	29619	34459	57230	1037	13617	56134	CHANNEL 690
5312	8393	2558	18929	24437	12228	9141	26334	2427	22	BURST DATA
72	60	18	1648	12	12	34	8371	3017	79	
432	6	5	4	2	208	11	213	1386	5	
3	3	3	212	9	6	5	16	525	69	
8966	4356	4134	3602	36183	55033	8964	26275	21154	29065	
4649	5217	420	139	9622	15035	12197	352	2713	13534	
587	44952	3484	54059	17759	25043	21069	31883	9452	1378	
42595	34084	17947	3133	23531	3147	605	10271	26338	36039	
15067	14572	13127	40644	4975	21395					

TABLE A

Fig. 8

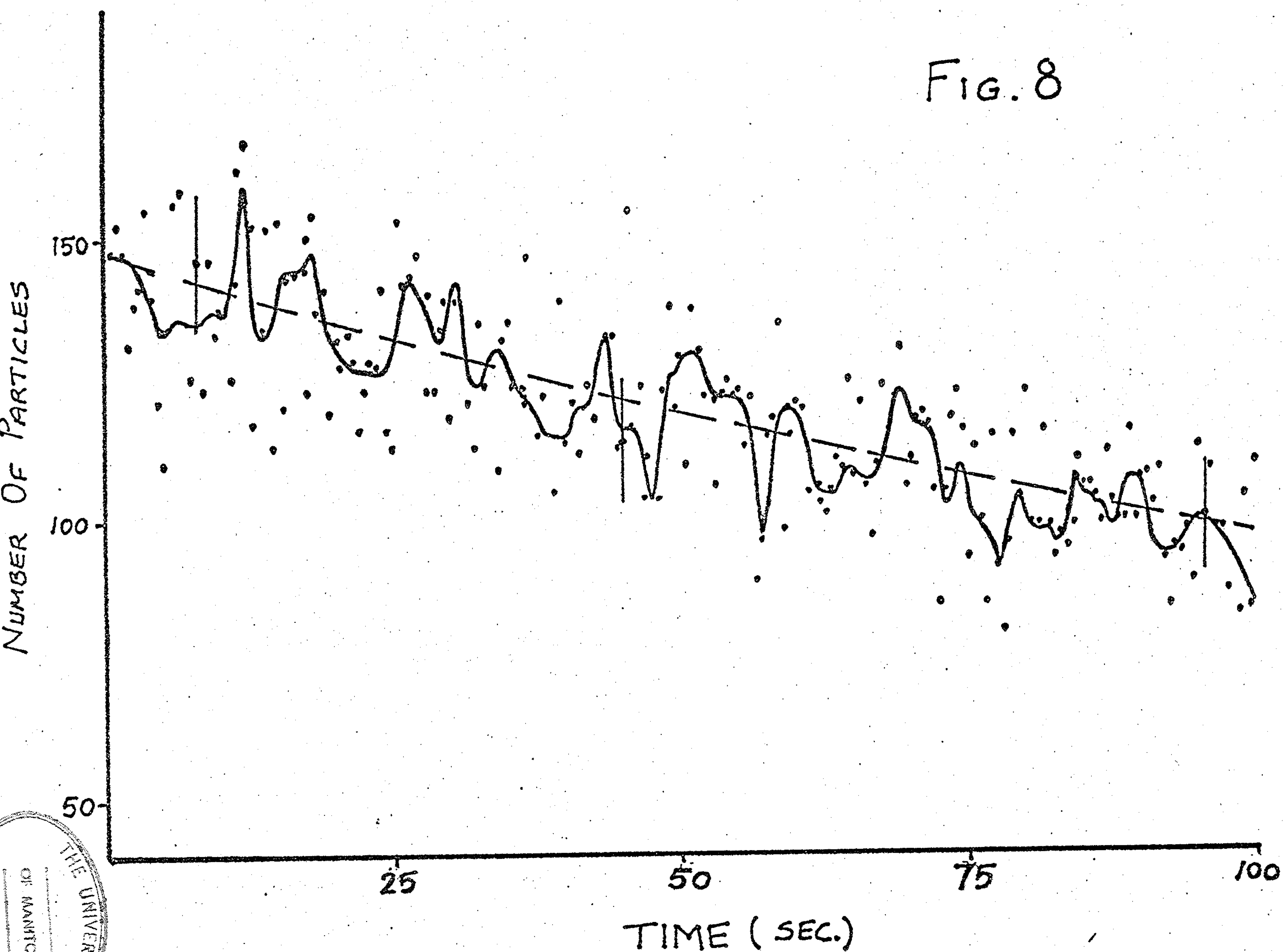
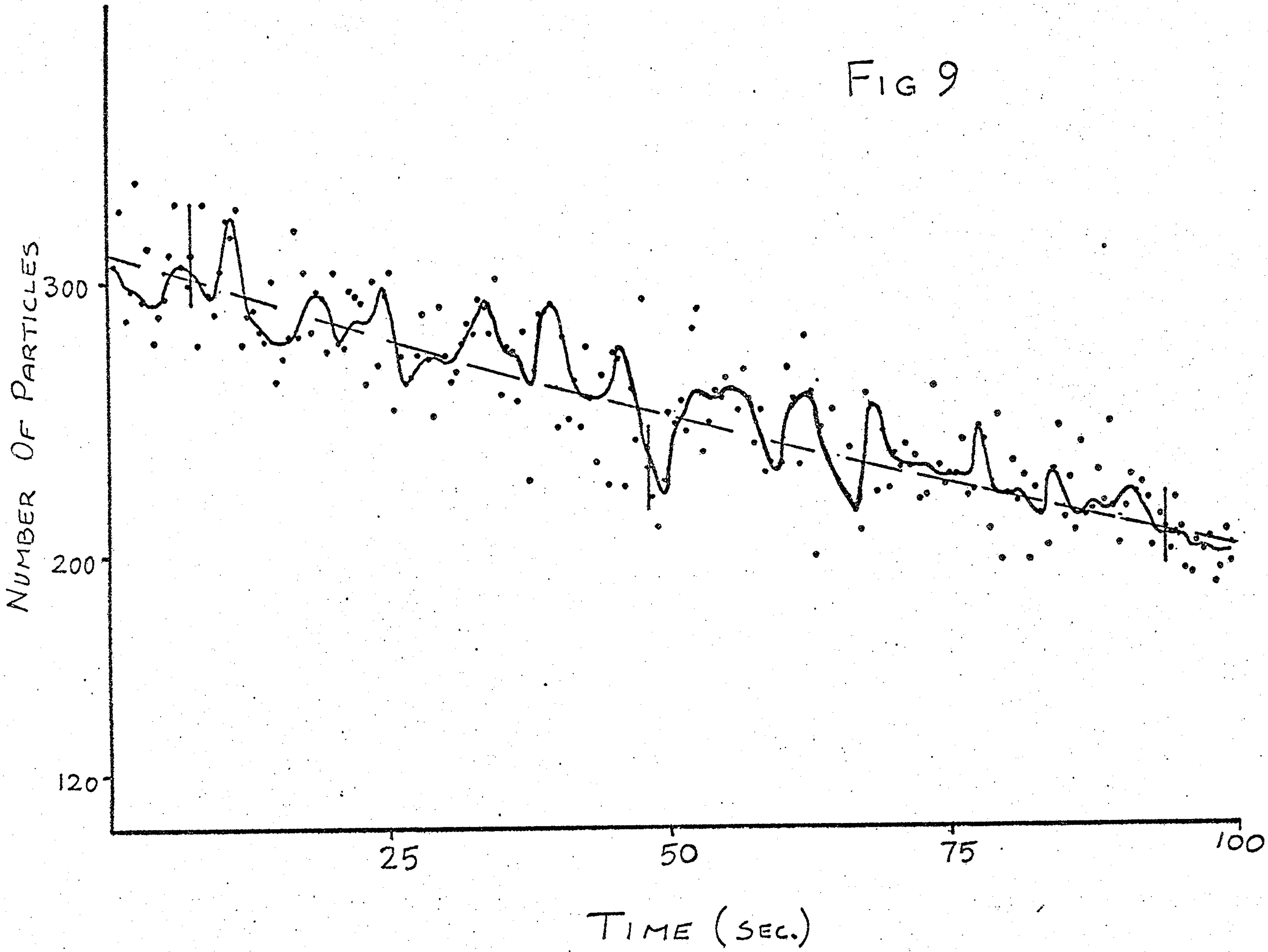


Fig 9



Appendix A

THE GENERATION OF RANDOM NUMBERS

The random number sequence should have the following properties:

1. Uniform distribution over a given interval: usually 0 to 1
2. Independance from each other: ideally,there should be no correlation between the numbers produced,that is,between the i^{th} and the $(i+k)^{th}$ were $k=1,2,3 \ldots\ldots\ldots n$
3. As long a cycle as possible: the cycle is the sequence of numbers produced before repetition.
4. reproducibility: we should be able to reproduce the same sequence.

There are many mathematical processes for generating digits that yield sequences satisfying many of the statistical properties of a truly random process.For example if you examine a long sequence of digits produced by these deterministic formulas,each digit will occur with the same frequency,odd numbers will be followed by even numbers about as often as by odd numbers,different pairs of numbers occur with nearly the same frequency,etc.Since such a process is not really random,it is called a pseudo-random number generator.

A.1 GENERATION OF RANDOM VARIATES

Suppose that we need to generate sample values of a random variable X defined by its probability density function f(x). We compute its cumulative distribution[43]

$$F(x) = \int_{-\infty}^{x} f(t)\, dt$$

which is by definition is equal to $Pr(X \leqslant x)$. The inverse distribution method selects a random number, r, uniformly distributed between 0 and 1, sets $F(x)=r$ and solves for x. For a particular value r_0 of r we get a value x_0, which is a particular sample value of X, and which can be expressed as $x_0 = F^{-1}(r_0)$. This construction is used to generate values x of a random variable X.

The time intervals between cosmic ray showers are described by exponentially distributed random variables. To generate samples of x of a random variable X which is exponentially distributed with average interval time $\tau(X)$

$f(x) = \lambda e^{-\lambda x}$ where $\lambda = 1/\tau(X)$ is the arrival rate

$F(x) = 1 - e^{-\lambda x} = r$ solving for x

$x = (-1/\lambda)\ \ln(r) = -\tau(X)\ \ln(r)$

Therefore, when we require exponentially distributed values with mean $\tau(X)$ we generate a random number r and transform it with the above formula.

Appendix B

A CURVE SMOOTHING TECHNIQUE

The interval distribution plots for different ranges are smoothed to be able to pull out the real structures from the statistical fluctuations.The smoothing algorithm we have employed had four components[44] .

1. Running medians
   a) Running medians of three ("3")
   b) Running medians of five ("5")
   c) Running medians of three ("3")
2. Quadratic interpolation ("Q")
3. Running means ("H")
4. We represent the original unsmoothed sequence by $\{Y_i\}$ and the resultant sequence after the application of steps 1-3 by $\{Z_i\}$. We than form a new sequence r_i where $r_i = Y_i - Z_i$ and apply steps 1 to 3 to this sequence.As a final step we add the resultant r_i to the previously smoothed sequence.This procedure is called "Twicing"

The running medians of three is evaluated by

$$Z_i = \text{median}(Y_{i-1}, Y_i, Y_{i+1})$$

for the end points

$$Z_1 = \text{median}(3Z_{n-1} - 2Z_3, Y_1, Z_2)$$

$$Z_n = \text{median}(Z_{n-1}, Y_n, 3Z_{n-1} - 2Z_{n-2})$$

To this result running medians of five is applied.The next to the end points are calculated as running medians of three.The end points are simply coppied.As a final step runnig medians of three is again applied to the resulting sequence.

The final sequence produced by the above mentioned 353 technique has monotonic discontinuties and discontinuous derivatives.The latter is corrected by the quadratic interpolation.The problem of monotonic discontinuty is dealt by the running means.

$$Z_i = (1/4)Z_{i-1} + (1/2)Z_i + (1/4)Z_{i+1} \quad \text{and}$$

$$Z_i = Z_i \quad , \quad Z_n = Z_n$$

The procedure explained so far is called 353QHT.Other versions of this algorithm are also possible i.e. 575QHT,595QHT,393QHT ...etc.

Appendix C

SENSITIVITY OF SHOWER ARRAY

Presume that we can write the count rate of our experiment as;

$$CR(>N_0) = \int_{\phi=0}^{2\pi}\int_{N_0}^{\infty}\int_{\theta=0}^{\theta_0} R(N,\theta,\phi)\ \mathrm{Aperture}(N,\theta,\phi)\sin\theta\, d\theta\, d\phi\, dN.$$

where $R(N,\theta,\phi)$ is the differantial rate of observation of air showers coming from the direction (θ,ϕ), containing N to N+dN and in a solid angle $d\theta\, d\phi$ at θ,ϕ .We assume that $R(N,\theta,\phi)$ can be written as

$$R(N,\theta,\phi) = - R(N,0,0)\ \ \mathrm{ang}(\theta,\phi)$$

Where $R(N,0,0)$ is the differantial rate of observation of air showers coming perpendicular to our array and $\mathrm{ang}(\theta,\phi)$ is some angular function. Experimentally, we know that $\mathrm{ang}(\theta,\phi)$ only depends on the angle θ but not ϕ . Aperture (N,θ,ϕ) can be written as;

$$\mathrm{Aperture}\ (N,\theta,\phi) = \mathrm{Area}\ (N,\theta) = \pi r^2 \cos\theta$$

$$CR(>N_0) = -\int_{N_0}^{\infty} R(N,0)\ dN \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\theta_0} \mathrm{ang}(\theta)\ \mathrm{Area}(N,\theta)\ \mathrm{Sin}\theta\, d\theta\, d\phi\ .$$

$$= -\int_{N_0}^{\infty} R(N,0) \times \pi\, [r(N)]^2\ dN \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\theta_0} \mathrm{ang}(\theta)\ \cos\theta\ \sin\theta\ d\theta\ d\phi\ .$$

An appropriate expression for ang(Θ) can be found by using the zenith angle distribution curve[45]. After the evaluation of the corresponding integrals the count rate expression can be written as;

$$CR(>N_0) = -1.99 \int_{N_0}^{\infty} R(N,0)\,[r(N)]^2\, dN. \quad \ldots\ldots\ldots\ldots\ldots \quad 3$$

An empirical expression relating the radius of the shower disk to the size of the shower is:

$$r(N) = BN^{\beta} = \begin{cases} 8.593\ 10^{-2}\ N^{0.5268} & 10^3 < N < 9.4\ 10^5 \\ 1.367\ \ N^{0.2108} & 9.4\ 10^5 < N < 10^8 \end{cases}$$

and the differential size spectrum is then,

$$R(N,0) = AN^{-\alpha} = -2.489\ 10^5\ N^{-2.588}.$$

If we insert these values back in equation 3

$$CR(\geqslant N_0) = -1.99 \int_{N_0}^{\infty} -2.489{\times}10^5\, N^{-2.588} \begin{Bmatrix} 8.593\ 10^{-2}\ N^{0.5268} \\ 1.3679\ N^{0.2108} \end{Bmatrix}^2 dN \qquad \begin{matrix} N_0 \leqslant N \leqslant 9.4\,10^5 \\ N > 9.4{\times}10^5 \end{matrix}$$

$$= \quad 3.675\ 10^3 \int_{N_0}^{9.4\times10^5} N^{-1.5344}\ dN + \quad 9.314 \int_{9.4\,10^5}^{\infty} 10^5\ N^{-2.166}\ dN$$

our count rate for the set up was approximately 12 hr^{-1} .If we replace CR ($>N_0$) by 12 and evaluate the above integrals, for N_0 we get

$$N_0 = 8.9{\times}10^4$$

The formula relating the number of particles to the primary energy is given by Cranshaw[48]

$$N_0 = E_p / 10^{10}$$

therefore our energy threshold is approximately $E_t \geqslant 9\ 10^{14}$ eV.

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