MATHEMATICS AND TECHNOLOGY:

A CASE STUDY OF THE TEACHING OF FUNCTIONS

USING MULTIPLE REPRESENTATION SOFTWARE

BY

IAN R. DONNELLY

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A Thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

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Abstract

The purpose of this case study was to determine if a small group of students can use multiple representation software to solve problems involving functions before they have mastered advanced procedural algebraic manipulation skills and to determine if, as students use the software, their mathematical experience is meaningful. After completing a pre-test on algebraic skills, the students worked on five computer activities designed to offer different types of mathematical experiences. The student data from the activities, video-tape transcripts and observation notes were used to answer six study questions.

The results show that students with a higher level of basic algebra skill development were more successful at using and applying software algorithms to solve problems which are algebraically more complex than that which is expected in current mathematics curricula. Students with a lower level of skills achieved moderate success on the problems. The basic algebra skill level was not a factor in the success of students using computer software for exploration.

The type of activity was a factor in how mathematically meaningful the experience was for the students. During an open-ended activity, in contrast to structured activities, more mathematical discussion was evident and students were less likely to rely on the teacher as the only authority.

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Chapter One

INTRODUCTION

Technology in the mathematics classroom offers the potential for major alterations to curriculum and methodology. Software is available at all levels of mathematics education to be used as a tutor for remediation, or as a tool for exploration. Researchers hold great promise of modifications to mathematics curricula to make mathematics more meaningful and more accessible to students.

The National Council of Teachers of Mathematics Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) [hereinafter referred to as the Standards] proposes that the current mathematics curriculum be reformed. With the use of existing technology the algebra curriculum could move "away from a tight focus on manipulative facility to include a greater emphasis on conceptual understanding, on algebra as a means of representation, and on algebraic methods as a problem-solving tool.... Available and projected technology forces a rethinking of the level of skill expectations" (p. 150). Schoenfeld (1988) also suggests that the goals of mathematics curriculum could be transformed:

> That mathematics is a verb (something you *do*) as opposed to a noun (something you *master*) -- causes a radical reconceptualization of the goals of mathematics instruction. If you hold the "mastery" point of view, your goal as a mathematics instructor is to have your students learn and be able to employ the techniques determined by the curriculum....The teacher demonstrates the technique, trains students to use it, and tests them on closely related problems (p. 69).

Students will need to continue to master a wide array of skills some of which will be skills required to use the technology. Additionally, technology can be used as a tool for students to explore patterns and relationships in algebra and geometry. Technology is not a panacea and is not useful for every topic in mathematics education. It is, however, particularly suited to bringing to light the relationship between the different representations of functions: graph, equation and table.

The focus of this study was on students in a technologically rich environment using multiple representation software to solve problems involving functions. A technologically rich environment is one in which graphing calculators and/or computer hardware and software are available to pairs of students. "Multiple representation software" (sometimes called "multiple linked representation software") is software which displays two or more representations simultaneously. The representations applicable to the study of functions are graphs, equations and tables of data. Researchers suggest that technology may have the potential to open up a world of mathematics currently closed to students without a good basis in abstract algebraic manipulations (Demana & Waits, 1990; Leinhardt, Zaslavsky & Stein, 1990; Thorpe, 1989). Student exploration of functions using technology may provide them with an opportunity to discover relationships and patterns for themselves. Recognizing that mathematics is not an experimental science, teachers need to convey to students that relationships and patterns they discover would be accepted as 'true' in the mathematics community when they have been verified deductively using algebra or geometry and not by trying more examples. However, it may be an appropriate sequence in the learning process for students to first discover potential theorems and learn to solve problems using non-algebraic techniques with the help of technology before they learn to verify theorems and solve problems using algebraic techniques. Furthermore, technology may be a tool which will help students to see mathematics as something "you do" rather than as something "you master".

Purpose

This study tested the hypothesis that a technologically rich environment offers students the opportunity to solve challenging problems with a minimal knowledge of algebraic procedures. In this project, students used computer hardware and software to study functions and their representations. The topic of functions is one area for which researchers hold considerable promise for major revisions to mathematics curriculum. These two general questions were addressed: (1) can students use multiple representation software to solve problems involving functions before they have mastered advanced procedural algebraic manipulation skills; and, (2) as students use multiple representation software, are the students' mathematical experiences meaningful? The term 'meaningful' is defined at the beginning of Chapter 3.

Rationale for the Study

Researchers (Dugdale, 1993; Heid, 1988; Tall & West, 1992) say that students might benefit from a change in the instructional sequence of problem solving techniques: students could learn to solve problems using graphing techniques before learning algebraic techniques. To date, there is a limited amount of research which has explored this idea in the classroom setting.

This study was a beginning, to see if students are capable of using computer graphing software and spreadsheet software to solve problems traditionally requiring a mastery of algebraic techniques. As curriculum writers make decisions about the future of mathematics education, they must take the influence of technology into consideration. The new curriculum should not be limited to revamping old ideas - "the past is a very poor guide to the future of this medium in algebra" (Kaput, 1989). With the use of technology, the priorities of the curriculum may shift and the nature of the mathematics taught to students may be altered. Students' experience may range from solving problems by performing algorithmic steps using computer software to exploring and doing their own mathematics.

Some aspects of mathematics are currently not within reach of students because of the demands of algebra. Technology may allow students to study more interesting topics earlier and may help make some mathematics topics accessible to more students. The level of algebraic skill development required of students at various stages of their mathematics education needs to be determined (Kaput, 1989). Results of research looking at technology in mathematics classrooms could assist curriculum writers as they determine the sequence of instruction for mathematics students in the future.

Furthermore, with technology as a tool, the content of mathematics curriculum may change. For example, Fey (1989a) suggests that computer-based successive approximation is a skill worthy of inclusion in new mathematics curricula. Fey goes on to say that students need to acquire a global perspective of the structures of mathematics in order to know how to effectively use the computational power which is available to them. Over and above specific changes in content, as well as mastering some of the mathematics of others, the nature of the mathematics learned may be altered to allow students to do more of their own mathematics by discovering mathematical truths for themselves. Technology gives students the opportunity to do meaningful mathematical exploration with a minimum of skills. Using geometry software, students have been observed discovering theorems for themselves which may help to make the learning more meaningful. That same kind of exploration may be possible in algebra using multiple representation software. It is not clear exactly what shape mathematics curriculum reform must take but it is clear that technological tools will play a part.

The mathematics curriculum is currently being taught by teachers, many of whom do not make regular use of technology during their instruction. Not surprisingly, the teachers who are most likely to use computers regularly as tools for instruction and not just for enrichment are those who are most knowledgeable about computers (Senk, 1989). If technology can benefit students as researchers are suggesting, then computers must be made available to students of mathematics and extensive in-service training is required for teachers of mathematics.

Limitations

This study was a case-study of a small group of students and thus the results of the study may not apply to other classrooms in general. Several grade 11 students missed an excessive number of classes and some of the grade 10 students were absent on one day due to a school ski trip. It is difficult to know how the absenteeism affected the results of the study. Furthermore, this study involved students solving problems related to the roots of functions and the local maxima or minima of functions using specific software. The nature of the students' mathematical experience might be different when other computer software, other graphing tools (that is, graphing calculators) or other activities are used. The research cannot directly imply vast changes to all areas of the curriculum, but can give insight to curricular decisions related to the topics of roots, maxima and minima of functions. Finally, this study did not seek to determine the best sequence of instruction. Rather, the study sought to determine if it is possible for students to have a meaningful mathematical experience when the sequence of instruction is altered.

Chapter Two

REVIEW OF THE LITERATURE

The literature related to mathematics education and technology discussed here concerns four areas: the influence of technology on curriculum, the implications for teacher and student roles, the influence of multiple representation software on the learning of functions, and the scope of graphing utility use.

Mathematics Curriculum

Many researchers (Burrill, 1992; Demana, Schoen & Waits, 1993; Dunham & Osborne, 1991; Fey, 1989a; Markovits, Eylon & Bruckheimer, 1986; Tall & Thomas, 1989; Weigand, 1991) believe that mathematics curricula related to functions should be modified in classrooms where technology is available. In general, this modification can be characterized in three ways. First of all, the current emphasis on symbolic manipulation of the algebraic representation of functions should be accompanied by an emphasis on conceptual understanding of the graphical representation of functions. Secondly, the sequence of instruction may be altered so that mastery of algebraic manipulation will come after and be aided by a mastery of graphic interpretation. Finally, new content related to functions and their graphical interpretation can be added to reflect the new problems accessible by students learning in a technologically rich environment.

Modification of current content. Symbolic manipulation can play a less important role and problem solving processes a more important role when technological tools are available to students. It is widely recognized that technological tools have the potential to help students to focus on the problem solving process rather than on algebraic manipulation (Demana, Schoen & Waits, 1993; Fey, 1989a; Heid, 1988; Tall & Thomas, 1989; Schoenfeld, 1988; Lesh, 1987). Fey (1989b) parallels the use of graphing technology to the experience of students using common calculators:

> In much the same way that numerical computation tools give an opportunity to emphasize planning and interpretation of arithmetic operations for problem solving the existence of computer graphic tools can be used to revise the balance between conceptual and procedural knowledge in mathematics. (p. 250)

Furthermore, when problems are solved by algebraic manipulation, students must work with the most basic of functions. With technology as a tool, the degree of complexity of functions is not a factor - when problems involving functions are solved using a geometric representation they do not necessarily increase in difficulty when more complicated functions are studied (Demana & Waits, 1990; Dunham & Osborne, 1991). Interesting and significant problems which are beyond the reach of students without the use of technology due to the algebraic skills required may be within the reach of computer-using students when representing the problem geometrically (Leinhardt, Zaslavsky, & Stein, 1990; Thorpe, 1989). Furthermore, Tall & Thomas (1989) state that this emphasis on conceptualization may give students long-term conceptual benefits by providing students with a geometric image which can be "a gestalt for a whole concept at an intuitive level" (p. 118). This gestalt may benefit students who will later learn more abstract algebraic techniques for solving problems. At early stages in students' mathematical learning, when technology is available, the curriculum may be modified to focus less on algebraic manipulation and more on problem solving processes.

Sequence of instruction. The instructional sequence of the mathematics curriculum may be altered in a technologically rich environment. Students may first develop a visual notion about the concept of function by studying the graphic representation and later develop the algebraic manipulation skills. The Standards (1989) suggest that in a classroom using technology as a tool, "the formal analysis of polynomial algebra is the culmination of student activity not the beginning" (p. 153). A handful of studies have researched the effect of learning concepts using graphing technology before learning procedures. Heid (1988) found that calculus students, who spent most of their course learning calculus concepts and a small portion learning procedures, performed almost as well on a procedures test as students who spent their whole course learning procedures. Additionally, the concepts-first students "showed more evidence of conceptual understanding than the students in the comparison class" (p. 15). Similarly, Dugdale (1993) reported on a study of two groups of students: one group experienced a traditional treatment of trigonometric identities, a second group was engaged in graphical reasoning tasks as a foundation for trigonometric identities. "The Graphical Foundations Treatment was intended to involve students in building a qualitative perspective before formalizing procedures" (p. 118). The graphical foundations treatment group showed superior post-test performance and was more creative in their approaches to proving identities. Tall & West (1992) state that students could be exposed to "a new kind of learning experience [queue which has students] investigate patterns, conjecture theorems, and test theories experimentally before going on to prove them in a more formal context" (p. 122). The results of these studies looking into teaching concepts before procedures are promising but more research will be needed before reliable conclusions can be drawn.

Several researchers (Burrill, 1992; Cieply, 1993; Demana & Waits, 1990; Phillip, Martin & Richgels, 1993) question the current sequence of instruction which has students learn linear equations before quadratic equations and quadratic equations before cubic and exponential equations. The rationale for this current sequence is attributed to the fact that students are expected to solve problems using algebra and the algebra skills required to solve problems increases as students move from linear to cubic and other equations. Considering the potential of graphing technology as a problem solving tool, the current sequence of instruction may not be the best sequence.

Traditionally, students have been taught to produce graphs from a linear or quadratic equation before being taught to produce an equation from the graph of a linear or quadratic function. Technology could help students to be more comfortable working from a graphic representation to an algebraic representation. In fact, some research suggests that partly due to the equation-to-graph instructional sequence, students have more difficulty converting functions from graphs to equations than from equations to graphs (Leinhardt, Zaslavsky, & Stein, 1990; Markovits, Eylon & Bruckheimer, 1986). Although greater mathematical sophistication may be required for all but the simplest functions, to alter the instructional sequence may be of benefit to students since graph-to-equation is not usually the direction of instruction but is often the direction of use. The sequence of instruction may be altered by teaching concepts before procedures, by not stressing linear equations exclusively as a first algebra experience and by using a graph as a primary representation of an equation rather than a secondary representation. This altered sequence may give students the power to solve more interesting problems earlier through the use of geometric rather than algebraic means.

New content. The content of mathematics curricula may need to be altered and researchers describe several new content areas. It will become important for students to know what kind of information is required as input to the software being used, how to put that information into the computer and how to interpret the results of the mathematical representations generated by the computer (Weigand, 1991; Kaput, 1986). Students will need a comfortable working knowledge of basic families of elementary functions and the use of various computer representations (Fey, 1989a). The curriculum may be able to devote more time to exploration and pattern recognition by using technology to produce graphs of functions (Leinhardt, Zaslavsky & Stein, 1990). Rather than focusing on exact answers, students can be taught to use numerical approximation techniques on problems represented geometrically along with the associated error analysis (Demana & Waits, 1990). More attention to scale changes will need to be given in a technologically enriched curriculum than is usually given in a traditional curriculum (Dunham & Osborne, 1991; Hector, 1992; Leinhardt, Zaslavsky, & Stein, 1990). Rather than only graphing points of a function which are near the origin, students will be expected to generate a picture of the "complete graph" which displays zeros, turning points, y-intercept(s) and an indication of end behaviour (Hector, 1992).

In summary, the availability of technology will require modifications to curricula: symbolic manipulation may be de-emphasized at early stages in students' mathematics experience, the sequence of instruction may be altered, and new skills may need to be taught. As well as a change in curriculum, researchers describe a new role for teachers and students in a technologically rich environment.

Teacher and Student Roles

In a traditional mathematics classroom, mathematical knowledge is passed from the teacher (or text) to the student. In a technologically rich environment, however, the traditional roles of teacher and student may be altered. Rather than the teacher being the only authority in the classroom, technology gives authority to students. The students and computers assume roles often exercised exclusively by the teacher (Burrill, 1992; Heid & Baylor, 1993). With this new authority, students spend more time in problem solving mode and higher order 'thinking about thinking' becomes possible. Technology can give students the power to make and modify their own guesses without teacher intervention. The classroom becomes "an environment where the students are doing their own mathematics -- not memorizing someone else's" (Schoenfeld, 1988, p. 84). This modified student role will aid each student's construction of mathematical knowledge - each will be able to monitor and evaluate his/her own mathematical ideas.

Likewise, researchers have found the teacher's role is transformed. With technology as a tool, the teacher can become a consultant, technical assistant, collaborator and facilitator rather than the purveyor of correct answers

(Barnes, 1994; Heid & Baylor, 1993; Heid, Sheets, & Matras, 1990; Kieren, 1993). With this new role comes new demands. Teachers will not always know the answers to students' problems - they will need to work *with* students to solve problems. Individual students may see a variety of patterns or be led to a variety of conclusions and the teacher will be asked to confirm the validity of each of the conclusions. The demands upon the teacher, both intellectually as well as managerially, can be substantial (Kaput, 1986). The tasks teachers are asked to develop for students may also be altered. Heid, Sheets, & Matras (1990) found that in a technologically rich environment multi-day goals replace single day lessons more often than in a traditional classroom. In a mathematics classroom with technology, traditional student and teacher roles are challenged.

Multiple Representations

Students often have difficulty connecting the graphical, tabular and algebraic representations of a function. Kerslake (1981) when speaking of children aged eleven to sixteen years states: "... while many children will be able to read information from a graph or to plot given data, it seems that only a few will be able to understand the connection between an equation and a graph"

(p. 135). Speaking of the difficulty of teaching and learning the relationship between function equations and graphs, Leinhardt, Zaslavsky and Stein (1990) say:

> ... although much of the prior mathematical work in the student's life may have dealt with concrete representations as the basis for learning more abstract concepts, functions and graphs is a topic in which two symbolic systems are used to illuminate each other....It means that in this topic we have a case in which two symbol systems both contribute to and confound the development of understanding. (p. 3)

The multiple representations offered by computer graphing software, however, may help students to build cognitive links relating different representations. The effects of actions taken in one representation are immediately apparent in a second or third representation (Dunham & Osborne, 1991; Kaput, 1986). Students who develop an understanding of the relationship between the structures of graphs and the symbols of equations have a cognitive skill useful in a variety of topics in mathematics. Lampert (1989) stated, "Multiple representations are at the intersection of mathematical and cognitive ideals; the creation of mathematics itself depends on capturing structures with symbols and using those symbols to move among structures of different types" (p. 256). Recognizing that students have difficulty linking multiple representations of functions, these researchers suggest that technology has the potential to make it less difficult for students.

There are two factors which make technology a powerful tool for students as they use multiple representation software. First, with paper and pencil, students can perform algebraic transformations on the equation of a function. Using technology, students have the added advantage of performing geometric transformations on the graph of a function. Researchers have described the power of the graphical representation using a computer compared to using paper and pencil as dynamic rather than static (Kaput, 1992; Kaput, 1989; Schwarz & Bruckheimer, 1990). Several researchers (Demana & Waits, 1990; Dugdale, 1993; Slavit, 1994; Yerushalmy, 1991) have found that working with dynamic linked representations enhances the understanding of the representations of functions. Second, technology makes it possible for students to work simultaneously with at least two representations. The computer can translate instantaneously across the representations to provide immediate feedback to students. Comparing the results of transformations on two or more representations simultaneously makes the relationship between representations more salient (Kaput, 1989; Leinhardt, Zaslavsky & Stein, 1990; Phillip, Martin & Richgels, 1993).

The immediate feedback and the dynamic nature of computer graphical representations make technology a powerful tool in the classroom.

<u>The Use of Graphing Technology</u>

It is this writer's perception that there has been public concern over calculator use in mathematics classrooms. Likewise, there may be concern that other technological tools (computer graphing software and graphing calculators) will replace meaningful thought processes of students. A large number of researchers, however, do not seem to share that concern. Schwarz and Bruckheimer (1990) describe a meaningful learning sequence using technology:

> When a student defines a function algebraically, turns to the tabular representation in order to locate a set of images, and turns to the graphical representation and chooses a viewing rectangle in the light of the table, the student's actions carry the conviction that the skills are being used meaningfully. (p. 613)

Another study (Barnes, 1994) found that students immersed within a computer environment did not focus on the computer software but rather their mathematical exploration. "The patterns they noticed, the questions they asked, and the general statement they made were about the mathematical content of this setting" (p. 108). Heid (1988) illustrates the real benefit of technology when she quotes a student who suggests that, when working a problem by hand, she thinks until

> ... all of a sudden you get your formula, and you plug in your variables, and then it's like you go into high gear or something. Blinders go on, and your head goes down, and your numbers get put in your mind's calculator, and that's it--you don't think. I only start thinking again when I get my answer. (p. 23)

In contrast, the same student went on to say there was no "blinder" time when using a graphing calculator.

Despite having looked for research with an opposing view, the literature related to the use of graphing technology in mathematics classes indicates that researchers are optimistic about its use. Many of the researchers, however, list concerns about the use of graphing technology along with potential benefits. Dion (1990) compares graphing calculators to traditional calculators: "Students should no more rely on a calculator to graph $y=x^2$ than to compute 5+7" (p. 564). Ruthven (1992) found that symbolic manipulation on a graphing calculator is conceptually more difficult than clearing brackets and combining terms using paper-and-pencil. Dugdale (1993) reported that students establish the connection between the algebraic and graphical representation of functions when they do some graphing by hand rather than computer graphing only. Slavit (1994) found that student use of the graphing calculator "did cause misconceptions to form, including incorrect assumptions about continuity and restricted domains" (p. 25). Goldenberg (1988) pointed out that multiple representation software may clarify functions for some students but the added representations may also add complication for others. The thoughtful use of graphing technology becomes the key. There is some concern among educators that the use of technology will be reserved for those school divisions, schools and individuals who can afford it. The one thing that distinguishes computer graphing tools from graphing calculators is the potential access of students to graphing calculators both at home and at school. Future research will need to determine the appropriate use of graphing technology. Fey (1989a) speculated that in the future, the effective use of technological tools will be of greater importance than paper-and-pencil processing of algorithms.

Summary

Researchers have stated that mathematics curricula will be reformed with the use of technology but the exact nature of the reform is not clear. Manipulation of algebraic symbols as the first problem solving tool may be de-emphasized, the instructional sequence of some topics may be altered, and new content related to information processing may be added. Additionally, researchers have found that a technologically rich environment challenges traditional teacher and student roles. The teacher may become a facilitator and the student an authority who can test his/her own conjectures by experimentation. Also, researchers have recognized the potential of dynamic multiple representation software to lessen some of the difficulties students have linking the representations of functions. Finally, graphing tools in mathematics classrooms open new doors for students, although the appropriate limits on the use of the tools is yet to be determined.

Chapter Three

METHODOLOGY

As stated earlier, the purpose of this study was to determine whether students can use technology to solve problems involving functions before they have acquired advanced algebraic skills. Secondly, this study sought to determine if, as students use multiple representation software, their experience was mathematically meaningful.

Advanced algebraic skills include transformational skills such as polynomial factoring or completing-the-square, and equation solving skills such as solving systems of equations or solving single-variable equations by factoring or by using the quadratic formula. None of the students in this case study have mastered, and in most cases been exposed to, these advanced algebra skills. Furthermore, the term *students* in this study refers to learners whose basic algebra skill development (combining like terms, simplifying expressions, solving equations in one variable) is across the spectrum of ability levels. Operationally, the Pre-test was designed to determine the level of basic algebra skill development of each student.

A mathematically meaningful experience is one in which students can relate what they are learning to concrete experiences in real-life or to concepts they already understand (Novak & Gowin, 1984; Resnick & Ford, 1981). Operationally, a student's experience was mathematically meaningful if he/she did at least one of the following: demonstrated, on the pre-test, his/her understanding of prerequisite calculations; participated in mathematical, as opposed to procedural, discussions about activities he/she were doing; described the relationship between a function's graph and table in terms of local maxima, minima or domain of real-world problems; demonstrated that he/she were making and testing their own conjectures using the software.

To answer the two general questions of the study, more specific questions were considered. With regard to the first question, determining whether students can use technology to solve problems before acquiring advanced algebraic skills, three more specific questions were addressed:

1) Are students able to successfully use an algorithmic software procedure to:
   a) find the roots of equations which have one side equal to zero;
   b) find the maximum volume of an open box formed from a cardboard of fixed dimensions; and
   c) find the minimum surface area of a cylinder with fixed volume.

2) Can students apply or adapt an algorithm to solve these related problems:

   a) find roots of equations in which neither side is equal to zero;
   b) find the relationship between the dimensions of a cardboard and the height of a box with maximum volume formed from the cardboard; and
   c) find the relationship between the radius and the height of a cylinder with minimum surface area and fixed volume?

3) Can students use the software as a tool for exploration to determine the relationship between the characteristics of a polynomial function and the characteristics of its graph?

Similarly, three specific questions were addressed with regard to the second general question of the study involving the meaningfulness of the students' mathematical experience as they solve problems using technology:

4) Are students communicating mathematical ideas with their peers and/or their teacher?

5) Can students describe how a function's table of values relates to its graph (specifically the maximum, minimum and domain)?

6) Are students making and testing their own conjectures?

The next section outlines the data collection procedure used to answer these questions.

Instruction and Data Collection

This research project is a case-study of twenty students from a classroom of Senior 2 (grade 10) students enroled in the specialized mathematics program [hereinafter referred to as 20S] and nineteen students from a classroom of Senior 3 (grade 11) students enroled in the general mathematics program [hereinafter referred to as 30G]. The specialized and general programs are the two options available to Senior 2, 3 and 4 students in Manitoba in 1994-1995. The topics in the curriculum of the specialized program are more abstract than the topics of the general mathematics program. The study was performed with the cooperation of one classroom teacher and two classes of students over a period of 6 (20S group) or 7 (30G group) classes; each class was sixty-five minutes in length. Due to unforeseen circumstances with the staff at the school, the classroom teacher of these groups of students was a long-term substitute. As a substitute teacher, the pressures of curriculum requirements and lack of time particularly with the 20S group were high. As a result, every attempt was made to keep the study within the time frame outlined at the outset. The researcher was the primary teacher during the activities of the study; the classroom teacher and the researcher both offered guidance to the students while they were working on the activities.

During the first session, the Pre-test (see Appendix A) was administered to the students and instruction was given to the students about the basic operation of software to be used. The Pre-test was composed of two parts. Part A consisted of twenty-nine multiple choice questions concerning basic algebra skills such as simplifying expressions with exponents, variables or brackets; evaluating expressions; solving equations in one variable and solving problems involving perimeter or area of triangles and rectangles. On the basis of the results of the twenty-nine questions of Part A of the Pre-test, students were placed in categories rating their basic algebraic skill level as low, medium or high. The students clustered in three reasonably distinct groups. They were placed in the low algebra skill level if they correctly answered fifteen questions or less, in the medium algebra ability group if they correctly answered from sixteen to twenty-one questions, and in the high algebra ability group if they correctly answered twenty-two questions or more. The results of Part A of the Pre-test were used as part of the basis for pairing students. Students with low ability were paired with compatible students of medium ability, high ability students were paired with compatible students of medium ability or of high ability. Also, the results of Part A of the Pre-test were used to determine the relationship between the students' success on the first three questions of the study and their level of basic algebra skills.

The second part of the Pre-test, Part B, consisted of four questions involving the volume of an open box, the circumference and area of a circle and the surface area of a cylinder. The students wrote detailed solutions on this part of the Pre-test and the results were used in two ways: first, to get an indication of the students' level of understanding of volume, and surface area calculations so that an appropriate introduction to each Activity could be developed; second, to relate the success of the students on the spreadsheet activities to their ability to do similar calculations on paper. The students' ability to use these formulas will affect the meaningfulness of the activities. They are, however, going beyond these calculations on the Activities to solve problems involving maxima and minima.

During subsequent instructional sessions the students worked in pairs using computer software. The students were paired according to their ability and compatibility as determined by their teacher's evaluation and, as previously described, by their level of algebra skill development as shown on the Pre-test. The instructional phase of the study was in two parts: in the first part students were observed using the <u>Mathematics Exploration Toolkit</u> (WICAT/IBM, 1988) [hereinafter referred to as the Toolkit] to find the roots to polynomial equations of varying degree; the second part involved students using the Microsoft Works (various dates) spreadsheet program to solve problems involving the volume and surface area of solids.

During the first part of the instructional experience, an algorithm for finding roots of equations in which one side of the equation is equal to zero by 'zooming in' on the x-intercepts of the graph of the equations was demonstrated to the students. Following the demonstration, the students worked on Activity A (see Appendix B) during which they used the algorithm to find the roots of equations. After practising that skill, they did Activity B (see Appendix B) where they generalized the procedure of the first activity to solve equations in which neither side of the equation was equal to zero. In the following session, students worked on a more open-ended activity, Activity C (see Appendix B), to determine the relationship between the degree of a polynomial equation (the algebraic representation) and the general appearance of the graph of the equation (graphic representation). To record their trials and observations, the students were given an Activity C data sheet (see Appendix B) modelled after the records of Barnes (1994).

The second part of the instructional experience involved work with the tabular and graphic representations of functions using Microsoft Works spreadsheet software. The following problem adapted from the Standards was the focus of this session:

> To make an open box out of a 10 cm by 10 cm rectangular piece of cardboard, cut squares of equal sizes out of each of the four corners and fold up the sides. Determine the exact size of the squares which should be cut out to make a box with the largest possible volume. (p. 151)

The spreadsheet already had some formulas entered into the cells to lessen the degree of proficiency required to use the software. A sample spreadsheet screen print-out of the open box calculations and corresponding graph is shown in Appendix C. The ability of each student to do the required calculations of length, width, height and volume of a box was determined in Part B of the Pre-test. As a result of a lower than expected rate of success of the students on Part B of the Pre-test and to make the subsequent activity more meaningful for these students, a demonstration was given to show how an open box could be formed, and the volume calculated, using a rectangle with square corners cut out.

The details of the student work were recorded on the data sheet of Activity D (see Appendix B). The initial square size chosen, the size-increment chosen, the maximum volume for the attempt and a sketch of the graph was recorded for each trial on the spreadsheet. The solutions were to be refined to obtain the most accurate square size and volume values possible with the software. As an extension of the problem (Problem 2), the students were asked to find the size of the square to be cut out of rectangular pieces of cardboard which have length and width values of their own choosing. For each new rectangle, a new data sheet was used. The students were asked to determine the relationship between the size of the original rectangle and the size of the cut-out square.

In the final session, students explored the surface area and volume of a cylinder. When studying a cylinder they answered the following questions:

> Are Cola companies using the best shape of can to hold their 355 ml drink? What should the radius and height of a Cola can be to have 355 ml of volume and use the least amount of metal?

As with the previous session, the students used a spreadsheet program in which much of the structure of the spreadsheet was created for them. A sample spreadsheet screen print-out of the surface area calculations and corresponding graph is shown in Appendix C. Again, each student's ability to calculate the area of a circle and the surface area of a cylinder was determined in the Pre-test. Since many of the students did not successfully calculate the surface area of a cylinder and to make the subsequent activity more meaningful for these students, they were given a demonstration of how a cylinder can be unfolded to form two circles and a rectangle. The details of the student work were recorded on the data sheet of Activity E (see Appendix B). The radius and increment values, the minimum area for the attempt and a sketch of the graph were recorded for each attempt on the spreadsheet. Finally, the students investigated cylinders with volumes other than 355 ml to determine the general relationship between the radius and the height of a cylinder with minimum surface area and a fixed volume (Problem 2). For each cylinder tried a new data sheet was used to record the attempts.

In addition to the student records described above, the interaction of one pair of students (at a time) and their monitor was recorded on video tape. A total of eight pairs of students were video taped - one pair of students and their monitor from each of the two groups during Activities A, B, C, and D. The video-tape recorder malfunctioned during Activity E so that only the first few minutes of the 20S students were recorded. The video tape was used to record the interaction between peers with the computer software and was transcribed for analysis. Furthermore, observation notes were kept by the researcher to record some of the interaction of other pairs of students. The observation notes were recorded while the students worked on the activities or immediately after the sessions. The data for the study consists of the student records of Activities A to E, transcripts of the video-tape recordings, and researcher observation notes.

Data Analysis and Interpretation

The data are used to answer the specific questions of the study. There are two aspects to the questions; namely, success with the software and meaningfulness of the activities. Operationally, decisions on success and meaningfulness are as described below.

Question 1

The student records of Activity A and problem 1 of Activities D and E were analyzed to determine if students can follow a series of software steps to solve a problem. The students were considered to be successful if they wrote the roots of the equations of Activity A within two decimal places of accuracy. After analysing the data, it was decided that the students used the procedure successfully if they found ten of the fifteen possible roots to the desired accuracy. When doing problem 1 of Activity D, successful students will have written that the size of the cut-out square is 1.667 cm and the maximum volume is 74.074 cm^3. Students who successfully completed problem 1 of Activity E will have written that the minimum surface area of the can is 277.545 cm^2 when the radius is 3.837 cm and the height is 7.674 cm. The answers to these problems are well defined; to solve them only requires that the students use the demonstrated algorithmic steps on the software. The number of successful students for each of the Activities was tallied.

The meaningfulness of the activities related to the first question is not a major issue since the students are being asked to use an algorithm. The software algorithm for solving equations may or may not be as meaningful as an algebraic algorithm for solving equations. The meaningfulness of Activities D and E will be partially determined using the Pre-test results. Each student's ability to do the calculations required for Activities D and E as determined in Part B of the Pre-test will be compared with his/her level of success on the Activities. The number of students who were successful on the Pre-test and/or successful with each of Activities D and E is tallied in 2 by 2 matrices. Since formulas were entered into the spreadsheet for the students, operationally, the spreadsheet activity was most meaningful to those students who were capable of doing the calculations on the Pre-test. The determination of the meaningfulness of the other activities will be described later (Questions 4, 5 and 6)

Furthermore, the progression of size of square and increment (Activity D) and of radius and increment (Activity E) recorded on the sheets was analyzed to determine if the students understood how to use the successive approximation procedure. Operationally, they understood how to use the procedure if the progression was such that the calculated maximum volume or minimum surface area value gets closer to the actual value with each trial.

<u>Question 2</u>

The results of students' work on Activity B and problem 2 of Activities D and E gives an indication of the students' ability to adapt or apply a procedure to solve problems. The students experienced a hardware failure for approximately twenty minutes at the beginning of Activity B. Due to the limited time available, the researcher decided to modify the requirements of the activity rather than extend the length of the study. Instead of asking the students to find all roots for each equation, they were asked to find at least one root for each equation. Students successfully adapted the procedure if they wrote at least one root of the equations on the Activity B data sheet within two decimal places of accuracy.

The students were also asked to give a description of how they modified the procedure of Activity A to find the roots of equations which have neither side equal to zero. The descriptions of the modified procedure are organized into similar response types. The response types are reported with some examples.

Records from Activity D (problem 2) were analyzed to determine if students can apply the spreadsheet procedure. Students were considered to be successful if they could state that the relationship between the side of the original square cardboard and the side of the cut-out square is 6:1 (in fact, none of them did). The records from Activity E (problem 2) were analyzed to determine if students could apply the procedure to find a general relationship between the radius and height of a cylinder with minimum surface area and fixed volume. Successful students found the radius to height ratio is 1:2.

Question 3

A measure of whether the students perceived the relationship between an equation and its graph was derived from the results of Activity C. These records gave an indication of the students' understanding of the link between graphic and algebraic representations of functions. On the Activity C data sheet students were expected to record that the degree of an equation partly determines its shape. More specifically, the following two types of responses, which were determined after analysing the results, were sufficiently complete to be accepted as successful responses. Students successfully determined the relationship between the characteristics of the equation of a polynomial function and its graph if they wrote a general description such as, "even exponents make a U shaped graph and odd exponents make a zig-zag graph". Alternately they were successful if they wrote the three specific descriptions: "no exponent makes a line", "squared exponent makes a U shaped graph", and "cubed exponent makes a zig-zag graph". They were considered to be partly successful if they wrote 'true' statements but did not give a complete description of the relationship between equations and graphs.

The descriptions of the relationship between algebraic and graphic representations were organized into response types which demonstrate understandings or misconceptions. The response types are reported with examples. Finally, the students were asked to relate the volume graph of Activity D to a possible equation. Operationally, students have a good understanding of the relationship between the function's graph and equation if they described that the equation to produce the graph of Activity D is cubic. The number of students who have a good understanding of the relationship between a function's equation and graph was tallied.

To get an idea of the relationship between students' basic algebra level and their success using multiple representation software to solve problems, the success of each student on the first three questions of the study was compared with his/her basic algebra ability as determined by the Pre-test. The results are summarized in a 3 by 3 matrix listing each student's Pre-test algebra skill level (low, medium or high) and his/her degree of success on each of the first three questions.

Question 4

The remaining three questions concern the meaningfulness of the students' experience doing mathematics using technological tools. Researcher observation records and video-taped recordings of pairs of students and their computer monitor were analyzed to find evidence of students communicating mathematical ideas with their peers. Operationally, occurrences of discussions about the shape of a graph, the number of roots of an equation or the accuracy of a result were viewed as discussions which were mathematical in nature. Discussions about the use of the software or the instructions of the activities were viewed as discussions which were non-mathematical in nature. The discussions were grouped into types and a tally was kept to get an impression of the nature of the students' experience. The response types are reported with examples.

Question 5

On Activity D and E data sheets, students were asked to describe the relationship between the characteristics of the graph and the values of the table. Operationally, they have a meaningful understanding of the relationship if they described that the maximum volume occurs at the peak of the crest of the graph of size versus volume on the Activity D sheet and that the minimum surface area is found at the bottom of the trough of the graph of radius versus area on the Activity E sheet. Additionally, students have a meaningful understanding of the relationship if they described the real-world connection of the domain of the functions of Activities D and E to the shape of the graph. The descriptions of the relationship between a table and a graph were organized into types of responses and are reported with examples.

Question 6

The video tape and observation records were also searched for incidence of students making and testing their own conjectures. Additionally, general observations recorded on student Activity sheets B, C, D and E provided evidence of students making and testing conjectures. Operationally, students who took on an authoritative role by making and testing conjectures were considered to be using the software meaningfully. Examples of students making and testing conjectures are reported.

In summary, the analysis of the student records, video tape and observation records resulted in an answer to the general questions of the study. Whether students can use technology to solve problems involving functions before they are skilled at algebraic techniques was determined from the first three specific questions. The results of the first three questions are summarized in a 3 by 3 matrix. Whether the students' mathematical experience with the technology is meaningful was determined from the last three specific questions. The number of students who were video taped is limited and the observation records of the other students is also limited so that the reporting of the second general question is descriptive in nature.

Chapter Four

ANALYSIS OF THE RESULTS

In this chapter the results of the analysis of the data will be discussed as they relate to the six questions of the study. The discussion begins with an analysis of the Pre-test results. The study questions are discussed under the following headings: Question 1, using an algorithm to solve problems; Question 2, applying or adapting an algorithm; Question 3, using software for exploration. Next, a comparison of the Pre-test results and success on the first three questions is written. The final three questions related to the meaningfulness of the experience are discussed under the headings: Question 4, mathematical communication; Question 5, tabular and graphic representations; Question 6, making and testing conjectures.

Pre-test Results

The results of the Pre-test (see Appendix A) are given in Table 1. In Part A, the students answered twenty-nine multiple choice questions and were categorized as having a low, medium or high level of basic algebraic skills depending on the number of questions answered correctly.

The number of students in each category is listed. In Part B of the Pre-test the students were asked to calculate the volume of a box, the area and circumference of a circle, and the surface area of a cylinder. The students were categorized as successful on Part B of the Pre-test if their work showed computational success and understanding of how to do the calculations. As expected, the level of basic

Table 1

<u>Pre-test Result Summary</u>

		20S	30G
Part A Score out of 29	High (21-26) Medium (16-20) Low (8-15)	8 9 3	3 4 6
Part B Volume Calculations	successful not successful	11 9	2 11
Cylinder Calculations	successful not successful	9 11	2 11
	absent	0	6

algebra skills in both groups ranges from low to high. Furthermore, it is reasonable that the specialized mathematics class (20S) has a higher proportion of students with a high level of basic algebra skills. Each student's Pre-test result is compared with his/her success on research Questions 1, 2 and 3 to get an indication of how the level of basic algebra skill is related to success with the questions.

Question 1: Using an Algorithm to Solve Problems

Three Activities were analyzed with regard to the first question: Are students able to successfully use an algorithmic software procedure to solve the problems given? In Activity A (see Appendix B) students were asked to find roots of equations in which one side of the equation was zero (Question 1(a)). The number of students who successfully found ten or more (of the fifteen possible) roots as accurately as the software would allow is given in Table 2. The meaningfulness of this activity is not a major issue since the students are only required to use an algorithm. As a result of the striking difference between the 20S group and the 30G group, they will be discussed separately.

Table 2

Student Success on Activity A

	Math 20S	Math 30G
number successful:	17	0
number unsuccessful:	1	16
number absent:	2	3
average # of roots found:	14.1	5.5
average # of roots found accurately:	12.5	3.3

Seventeen of eighteen 20S students were able to successfully use an algorithmic software procedure to find the roots of equations in which one side is equal to zero. On average, the 20S group found 14.1 of 15 roots although only 12.5 were found to the desired accuracy. For many of the students in the 20S class the software algorithm seemed trivial and they quickly moved from zooming in on one root to the next.

None of the 30G students successfully found ten or more of the fifteen possible roots. Nevertheless, six 30G students were able to find one of the roots for each equation. On average, each student in the 30G group found 5.5 of the fifteen roots with only 3.3 to the desired accuracy. The 30G students were, generally, not able to use the software algorithm to find the roots to equations.

The 20S students achieved almost 100% success and the 30G students almost no success on Activity A. This is in contrast to the Pre-test results where the students from both groups covered the range of basic algebra skill level from low to high. The lack of success on the part of the 30G students using the software may have been a result of their anxiety level related to the use of computers. Based on questions they asked and comments they made during the activity, it is the perception of the researcher that the

30G students were considerably more anxious than the 20S students. Considering the Pre-test results, the difference in success between the two groups may also be related to the weaker algebra skills of the 30G group - although some of the 30G students with a high level of basic algebra skills were expected to be successful. The results of subsequent activities do not show a distinct difference between the 30G students and the 20S students other than that which can be explained by the difference in their algebra skill level as measured by the Pre-test. It may be that the 30G group was particularly anxious with the first Activity and became less anxious and more comfortable with the software as the study progressed.

During the first part of Activity D (see Appendix B), students were to use a spreadsheet program to find the maximum volume of an open box formed from a 10 cm by 10 cm square cardboard (Question 1(b)). Based on the number of procedural questions they asked, it is the researcher's perception that the students did not find the spreadsheet algorithm as easy to use as the graphing software. Nevertheless, approximately half of the students were able to successfully use the software algorithm to solve the problems. The number of students who were successfully able to use the spreadsheet procedure to find the size of the cut-out squares and the maximum volume was tallied (see Table 3). Also, on the Pre-test, the students were asked to calculate the volume of an open box given specific rectangle dimensions and cut-out square corner dimensions. The number of students who successfully did the Pre-test calculation

Table 3

Comparison of Pre-test and Activity D Success

		Pre-test successful	Pre-test not successful
Spreadsheet Activity D	successful	7	5
	not successful	3	9

was tallied. Table 3 shows the success of each student on the activity using the spreadsheet to find the maximum volume and size of cut-out square corners, and their success on the volume calculation of Part B of the Pre-test. Eleven of the 30G students and four of the 20S students are not included in this table because they were absent for either the activity or the Pre-test. As a whole, the students were as successful on the Pre-test calculation as they were on the activity using the computer. As can be seen in Table 3, however, the students who were successful on the Pre-test are not necessarily the ones who were successful on the activity. It appears that it was not necessary for some of the students to be able to calculate the length, width, height and volume of a box from a square of given size and cut-out corner sizes for them to be able to use the spreadsheet program to find the maximum volume of a box and the size of cut-out corners. Furthermore, it appears that students who can calculate the volume of a box with specific dimensions are not necessarily going to be able to successfully use spreadsheet software to determine the maximum volume of a box and the size of the cut-out corners. As is sometimes the case with mathematics, students can use a procedure successfully without fully understanding the mathematics of the procedure. The spreadsheet activity would be most meaningful, however, to the seven students who were both capable of doing the calculations on the Pre-test and were able to successfully use the spreadsheet to find the maximum volume of the open box. Additionally, as a result of the demonstration of the method of forming an open box from a rectangle with corners cut out, the activity may have been meaningful for some of the students who were not successful with the Pre-test calculation.

The students who successfully found the required dimensions and maximum volume either used the procedure in an apparently random fashion or by using logical steps. The degree to which the successive approximation steps the students used were logical is shown in Table 4. It is assumed that students used the successive approximation procedure with understanding only if the steps they used were logical. Of the eighteen students who successfully found the required dimensions and maximum volume, six did not appear to use logical steps but rather found the required measures by apparently random guesses of what the initial square size and increment parameters should be. Figure 1 gives two examples of work done by students who successfully found the maximum volume and dimensions required - in one case the progression of size and

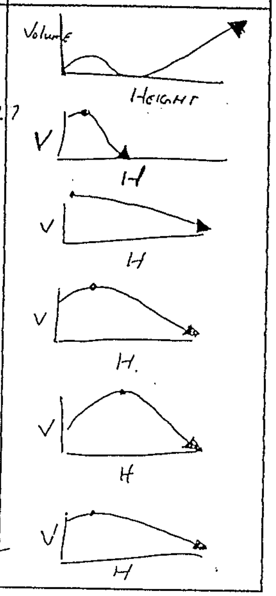
Table 4

Student Success on Activity D

	Math 20S	Math 30G	Totals
successful, logical steps:	6	6	12
successful, random steps:	4	2	6
not able to find square size:	6	4	10
number absent:	4	7	11

increment values appears to be logical, in the other case, however, the progression appears to be random. It is apparent from the work of these two students that the graphical representation of the function gives information which is useful for determining if the maximum is listed in the domain of the spreadsheet table (that is, when the

Original Cardboard Dimensions 10 x 10

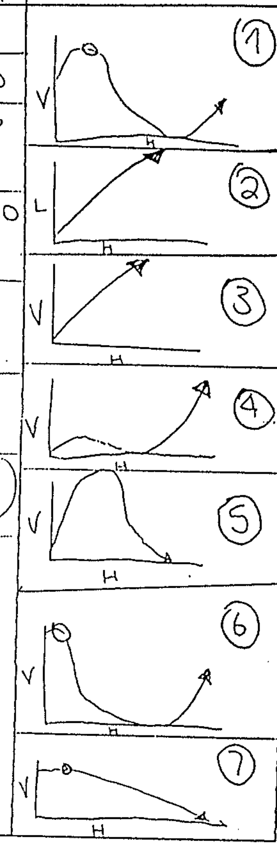
size/increment	Why Chosen?	Max. Volume	Graph
.1/.5		73.9840000 HEIGHT - 1.6	
1.5/.1		74.052 Height - 1.7	
2/0.05		72 HEIGHT - 2.0	
4 1.65/0.005		74.0740185 HEIGHT - 1.665	
1.66/0.001		74.0740719 HEIGHT - 1.667	
1.~~666~~/0.0005		74.0740735 HEIGHT - 1.6665	

Size of cut-out square: 1.666/0.0005

Maximum Volume: 74.0740735

Logical Procedure

Original Cardboard Dimensions 20x20

	size/increment	Why Chosen?	Max. Volume volume	Height	Graph
1	1.5/0.5		591.5	3.5	
2	1/0.05		512.0	2.0	
3	0.5/0.005		212.064	0.6	
4	0.66/1		9390.8159810	H- 20.66	
5	0.05/0.5		590.7555	H 3.55	
6	3/0.5		591.5	3.5	
7	3.2/0.1		592.548	3.3	

Size of cut-out square: 3.2/0.1

Maximum Volume: 592.548

Random Procedure

Figure 1. Examples of logical and random use of the successive approximation procedure for Activity D

turning point is visible) and if the result is near the maximum (the change in slope is small).

During the first part of Activity E (see Appendix B) students used a spreadsheet program to find the minimum surface area of a can with a volume of 355 ml (Question 1(c)). The number of students who were able to successfully find the minimum surface area and the dimensions of the can was tallied (see Table 5). Also, each student was asked, on

Table 5

Comparison of Pre-test and Activity E Success

		Pre-test successful	Pre-test not successful
Spreadsheet Activity E	successful	8	11
	not successful	1	9

the Pre-test, to do calculations involving the circumference and area of a circle and the height, surface area and volume of a cylinder. The success of students on the Pre-test calculations was tallied. A tally of the success of each student on the Pre-test calculations (Part B, questions 2, 3 and 4) and the spreadsheet activity is shown in Table 5. One 20S student and nine 30G students are not included in this table because they were absent for either Activity E or the Pre-test. As with the previous activity, success of students on the Pre-test calculations appears to be independent of students' success finding the minimum surface area and dimensions of the cylinder using the spreadsheet procedure. A majority of students were successful using the spreadsheet software to find the minimum surface area, and much less than half of the students successfully calculated the area and circumference of a circle and surface area of a cylinder on Part B of the Pre-test. The activity would have been most meaningful to the eight students who both did the calculations correctly and used the spreadsheet algorithm successfully. Additionally, as a result of the demonstration of how a cylinder can be unfolded to form a rectangle and two circles, the activity may have been meaningful to some of the students who were not successful on the Pre-test surface area calculations.

Many of the students were able to use the successive approximation procedure with understanding. The number of students who successfully found the required radius, height and surface area and the degree to which the successive approximation steps were logical is shown in Table 6. The number of students who used the procedure logically increased in this activity for 20S students probably because they had the previous experience of Activity D. The number of 30G students who used the procedure logically did not improve. There were seven students absent from the 30G

Table 6

Success of Students on Activity E

	Math 20S	Math 30G	Totals
successful logical steps:	9	2	11
successful random steps:	4	5	9
not able to find radius & ht.:	6	7	13
number absent:	1	5	6

group for Activity D, five of whom returned to class the next day for Activity E. Thus, the 30G students did not show a marked improvement partly because Activity E was the first time some of them had used the spreadsheet procedure. Examples of the steps taken by students are shown in Figure 2. Both of these examples illustrate a logical progression of radius and increment values. The students' inaccurate and unscaled drawings of the graphs are an indication that the graphic representation of the function was less useful for helping them to find the minimum than in the previous activity.

To summarize the results of Question 1, the students were somewhat successful at finding roots of equations using multiple representation software. The students appear to be no more or less successful at finding roots of equations

Original Volume: 355 mL

radius/increment	Why Chosen?	Min. Area	Graph
.5/.1		315.55126	
5/1		279.82616 Radius 3.5	
3.4/.1		277.57130 Radius 3.8	
3.5/0.01		277.85705 Radius 3.71	
3.6/0.01		277.5556 Radius 3.81	
3.6/0.1		277.57130 (3.8)	
3.8/.01		277.54517 (3.84)	
3.83/.001		277.5402 3.837	

Size of radius: 3.83 Size of height: 7.68

Minimum Surface Area: 277.5402

Original Volume: 355 ml

radius/increment	Why Chosen?	Min. Area	Graph
0.5/1		279.82616 RADIUS (3.5)	
3/0.1		277.57130 R (3.8)	
3.5/0.05		277.54810 R (3.85)	
3.771/0.005		277.54505 R (3.836)	
3.82/0.005		277.5451 R (3.835)	
3.837/0.0005		277.54502 R (3.837)	

Size of radius: 3.837 Size of height: 7.67529

Minimum Surface Area: 277.54502

Figure 2. Examples of the steps taken in the procedure of Activity E

using graphing software than would be expected based on the Pre-test results. However, the level of difficulty of the equations solved using the graphing software was considerably higher than would be possible for these students using algebraic methods. None of the students at this level of mathematics would have been able to solve the maximum and minimum problems algebraically. Approximately one-third of the students appeared to have a good understanding of the successive approximation procedure since they demonstrated a logical progression of steps as they found the required measures. Thus, for a majority of 20S students and for a smaller proportion of 30G students, the computer software offers students the opportunity to solve more complex and more interesting problems than they would be capable of with limited algebra skills. As will be discussed later, the experience of using the software algorithms was not particularly meaningful mathematically.

Question 2: Applying or Adapting an Algorithm

Activities B, D and E were analyzed with regard to the second question: Can students apply or adapt an algorithm to solve problems? On Activity B (see Appendix B), less than half of the students were able to modify the algorithm used in the first activity to find the roots of equations which have neither side equal to zero (Question 2(a)). The success of the students is listed in Table 7. Some students

Table 7

Student Success on Activity B

	Math 20S	Math 30G	Totals
number successful:	8	3	11
number unsuccessful:	12	15	27
number absent:	0	1	1

may have had difficulty applying the software algorithm since they were not zooming in at the intersection of two lines as they did on the first activity (that is, the x-axis and a function curve).

Students who successfully applied the software procedure to these equations zoomed in on the part of the graph where the function was equal to some constant (either 10 or 6). They moved the cursor to the part of the graph where the value of the function was near 10 or 6 and zoomed in on that area (see Figure 3). One pair of students was proud to inform the teacher that they had found the values of x which make $6x^2 - 5x$ equal to 6 by graphing $6x^2 - 5x = 6$ and finding the x-intercepts of the resulting two vertical lines. When trying the next questions, they were disappointed to find that the software was unable to graph

Describe how you modified the method from Activity A

In activity a we found for y at 0. This time we guessed around the area where 6 and 10 were and zoomed in there.

The way I modified the method is instead of going to 0 to zoom in you go to the ~~an~~ answer 6 or 10

Figure 3. Descriptions of the modified procedure for Activity A, questions 1 and 2.

cubic equations in one variable. It was expected that some of the students might try other solution methods but none did.

Even though the students have the algebra ability necessary, none of the students transformed the equation algebraically to make one side of the equation zero so that they could proceed with the algorithm as in Activity A. It is possible that the students did not realize that the solution to the transformed equation would be the same as the solution to the original equation since the graph is somewhat different. The students would need only a little more experience with the graphing software to confirm a fact learned in algebra: the solution to an equation is not affected by an algebraic transformation. Also, none of the students graphed the equations $y = 10$ or $y = 6$ in order to produce a horizontal line. Had they thought to graph the horizontal line, then more students might have been able to successfully find the solution at the intersection of the two function curves. The fact that students had limited success and did not transform the equations algebraically may be an indication that they did not have a meaningful understanding of the relationship between a function equation and its graph.

With the last two equations of Activity B, where both sides of each equation had variable terms, none of the students transformed the equations to equivalent equations in which one side was zero. All of the students graphed the two sides of the equations separately and then recognized that they should zoom in on their points of intersection (see Figure 4). Some students were confused by the y-coordinate at the intersection point of the functions. Since it was not equal to zero or some other whole number, some students confused the y-coordinate value with solutions to the equation (x-coordinates).

Describe how you modified the method to solve these equations.

We also graphed in the equation the first equation was to equal. for example $6x^3 - 5x = 7x^2$. We graphed the $7x^2$ also

we made both the variables equal "y" and graphed them both

The way to modify it is to put in the equation then put in the answer and were ever it crosses (wherever) is were you zoom in at

Figure 4. Descriptions of the modified procedure for Activity A, questions 3 and 4.

On Activity D, none of the students successfully determined the relationship between the dimensions of a cardboard square and the size of the square corners to be cut out to make an open box of maximum volume (Question 2(b)). This lack of success may be due to at least two factors. First, students were able to find the required size of cut-out square for only two or, for a few students, three cases of original cardboard dimensions in the time available to them. This limited number of cases may not have been enough for students to make a conjecture about the relationship between the size of the squares. Second, the relationship is 6:1. In the first case tried by all students, the original square was 10 cm on a side and the cut-out square was 1.6667 cm on a side. It may be that it was difficult for students to recognize that the original square was 6 times longer than the rational number generated for the cut-out square. In a second case tried by many students the square was 50 cm or 100 cm on a side - this dimension also resulted in a cut-out square which was not integral. Relationships between rational numbers may be more difficult for students to recognize than relationships between whole numbers particularly when using technology since they must be represented as decimal approximations rather than as exact answers.

On Activity E, a little less than half of the students were able to determine the relationship between the radius and height of a cylinder of fixed volume and minimum surface area (Question 2(c)). Thirteen (out of twenty) students in the 20S group and two students (out of twelve) in the 30G group were able to determine the required relationship. Possibly as a result of limited experience with this type of activity, some of the students had a hard time understanding what was meant by "relationship" between the radius and height of a cylinder.

To summarize the results of Question 2, approximately one-third of the students were able to apply a software algorithm to find roots of equations in which neither side is equal to zero. The level of difficulty of the equations they solved is usually reserved for students using algebraic methods in more advanced mathematics classes. All students had difficulty discovering the 6:1 relationship between the length of the open box and the length of the cut-out square corner to make a box of maximum volume out of a square sheet. None of the students from either group were able to successfully determine the relationship. Approximately half of the students were able to determine the 2:1 relationship between the radius and height of cans with fixed volume and minimum surface area of Activity E. The second relationship may have been easier to determine because a 2:1 relationship may be easier to recognize than a 6:1 relationship. The students may have had difficulty applying the software algorithms since their level of understanding of the algorithms may have been limited. The meaningfulness of the experience of applying the algorithms will be discussed in detail under the headings of Questions 4 to 6.

Question 3: Using Software for Exploration

There were sixteen 20S students and eleven 30G students who participated in Activity C (see Appendix B) in which they used graphing software as a tool for exploration to determine the relationship between the characteristics of the equation of a polynomial function and the characteristics of its graph (Question 3). Each student made as many observations as they could about the relationship between a function equation and the appearance of the graph. Table 8 shows the response types. Each response listed comes from the student records, the students

Table 8

Descriptions of Polynomial Equation and Graph Relationship

Response Types	Description of Response
A	- Even exponents make a U; odd exponents make a zig zag.
B	- no exponent makes a line - squared exponent makes a U shape - cubed exponent makes a zig zag line
C	- negative numbers do the opposite (mirror image) - constant value is the value of the y-intercept - higher exponents makes more curves in the graph

stated a variety of combinations of the following response types. Students are considered to have successfullydetermined the relationship between the characteristics of the equation of a polynomial function and its graph if they wrote either response type A or each of the three statements in response type B in Table 8 since they are reasonably complete descriptions. Students are considered to be partly successful if they wrote a 'true' but less than complete description of the relationship by writing one of the statements from type B or type C in Table 8. A summary of the students' success and the degree to which they were successful is shown in Table 9. A larger proportion of students were successful with this exploration activity than with the previous activities involving software algorithms. The students made independent observations without direction from the teacher. As will be

Table 9

Success of Students on Activity C

	Math 20S	Math 30G	Totals
number successful:	7	4	11
number partly successful:	7	4	11
number unsuccessful:	2	3	5
number absent:	4	8	12

discussed in the next section, the students' success on this activity seems to be independent of their basic algebra skill level.

Some students showed logical reasoning as they worked to confirm conjectures. After a thoughtful selection of equations were graphed, one student stated,

> "We thought that if the exponent was cubed it would have 3 roots. We tried different equations and didn't come to that conclusion."

Other students seemed to rely on a seemingly random assortment of graphs to make conjectures, their conjectures were generally not tested extensively. Examples of both a logical progression and a random assortment of equations and graphs are shown in Figure 5.

One pair of students stumbled upon the opportunity to use algebra in their reasoning of why a function's graph looked the way it did. They graphed a fourth degree equation which did not produce the graph they had come to expect. The students graphed the equation:

$$9x^4 - 6x + 3x^4 + 4x - 12x^4 = y$$

and were somewhat surprised to discover that, unlike other fourth degree equations they had graphed, this one produced a linear graph. Upon reflection, the students realized using algebra, that the equation simplified to a linear equation. This apparent conflict with their general

	Equation	Why Chosen?	Observations
1	$6x^2+3x^3=y$	—	2 roots
2	$5x^4-5x^2=y$	—	3 roots
3	$8x^3-5x^2-2x^4=y$	—	Scale = 20 2 roots
4	$9x^5-2x^3+9=y$	—	1 root Scale = 20
5	$2x^2-7x^3=y$	The x^3 & x^2 are the same in equation 1 & 5 therefore there shape is basically the same.	1 root
6	$4x^2-4x^4=y$	we choose it to find one like #(2) and they are somewhat similar equation	3 roots
7	$5x+3x=y$	we choose this for a difference in equations didn't want to ~~do~~ exponents	1 root

Random assortment of equations and graphs

Equation	Why Chosen?	Observations
$2x^3-15x-5=y$	GIVEN	3 ROOTS
$2x^3-15=y$	TO SEE IF # OF VARIABLES MATCH # OF ROOTS	1 ROOT
$6x^2-20x+3=y$	TO TRY TO MAKE A DIFFERENT GRAPH THAN PREVIOUS ONES.	2 ROOTS
$-6x^2+20x-3=y$	CHANGE ALL SIGNS FROM PREVIOUS QUESTION TO OPPOSITE	2 ROOTS
$5x^3+8x+8=y$	TO CHANGE + SEE IF 1 VARIABLE HAS ONE ROOT:	ONE ROOT...
$10+5x-8x^3=y$		1 ROOT;
$2x^3+19x-10=y$	TO SEE WHERE POSITIVE VARIABLE CUBED HAS AN OPPOSITE PATTERN THAN NEGATIVE CUBED	1 ROOT — NEGATIVE + POSITIVE VARIABLES APPEAR ON OPPOSITE SIDES ON GRAPH
$-2x^3-19x+10=y$		1 ROOT
$2x^3-15x-5=y$	TO SEE IF POWERS AFFECT LINE ON GRAPH	3 ROOTS — NUMBER OF POWERS AFFECT LINE ON GRAPH
$2x^2-15x-5=y$		2 ROOTS,
$2x-15x-5=y$		1 ROOT (SAME # OF MONOMIAL)
$5x-123=y$		1 root — one variable is or line.

Logical collection of equations and graphs

Figure 5. A random assortment and a logical collection of equations and graphs for Activity C.

observations served to solidify their conjecture about the relationship between a function's equation and graph. As will be discussed later,this exploration activity was mathematically meaningful for other students as well.

Many students were able to classify the families of polynomials as linear, quadratic or cubic. On the other hand, when looking at the graph of the volume function of Activity D only two students recognized that the graph could be represented by a cubic equation. Neither of those two students were able to articulate why a cubic equation is reasonable for a function representing volume. The students would require considerably more experience working with real-world functions to be able to make conjectures about the expected shapes of associated graphs.

In summary, a large portion of students were able to describe some of the relationships between a function's equation and its graph. Only two students were able to apply what they learned to determine that the volume function of Activity D is cubic. The knowledge that a relationship does exist between a family of equations and their graphs seemed surprising to some of the students. The power of graphing software as a tool for students with limited algebra skills was particularly apparent with this exploration activity. As will be discussed later, this exploration activity seemed more mathematically meaningful to the students than the more structured activities in which algorithms were used or applied.

Comparison of Pre-test and Study Question Success

The results for the first three questions of the study are summarized in Table 10. As mentioned previously, none

Table 10

Comparison of Pre-test Success and Success on Study Questions

PRETEST SCORE		Question 1	Question 2	Question 3
	High (22-26)	three- 2 (33%) two - 3 (50%) one - 1 (17%) none - 0 (0%) abs - 5	three- 0 (0%) two - 3 (50%) one - 2 (33%) none - 1 (17%) abs - 5	succ. - 4 (40%) partly- 5 (50%) not - 1 (10%) abs - 1
	Med (16-21)	three- 3 (27%) two - 4 (36%) one - 3 (27%) none - 1 (9%) abs - 2	three- 0 (0%) two - 4 (36%) one - 4 (36%) none - 3 (27%) abs - 2	succ. - 4 (40%) partly- 4 (40%) not - 2 (20%) abs - 3
	Low (8-15)	three- 0 (0 %) two - 2 (67%) one - 1 (33%) none - 0 (0%) abs - 4	three- 0 (0%) two - 0 (0%) one - 2 (33%) none - 4 (67%) abs - 1	succ. - 3 (75%) partly- 1 (25%) not - 0 (0%) abs - 3
	Totals for all Students	three- 5 (25%) two - 9 (45%) one - 5 (25%) none - 1 (5%) abs - 11	three- 0 (0%) two - 7 (30%) one - 8 (35%) none - 8 (35%) abs - 8	succ.- 11 (46%) partly-10 (42%) not - 3 (13%) abs - 7

of the students have advanced algebraic skills, the algebra Pre-test levels indicate the level of basic algebra skill development. This summary is a comparison of the success of the students on each of the first three questions of the study and the level of basic algebraic skill development of each student as determined by the Pre-test. Eight students were absent for the Pre-test or for all of the activities and are not included in Table 10. There are three parts to study Questions 1 and 2 and students were successful with all three parts, two parts, one part or none of the parts of the questions. The number of parts of Questions 1 and 2 with which each student was successful and their Pre-test level is tallied in Table 10 (the number of students absent was also recorded). For Question 3, students were to describe the relationship between a function's equation and graph. The summary grid shows a tally of the number of students in each of the three Pre-test ability levels who successfully described the relationship, the number of students who partly described the relationship and the number of students who did not describe the relationship. To ensure that the students' Pre-test ability levels are sufficiently distinct, only the results of the students who achieved a high level of success on the Pre-test and a low level of success on the Pre-test are compared in the following discussion of the results.

Question 1. A majority of students (70%), disregarding their basic algebra ability, successfully used a software algorithm on at least 2 of 3 parts. Of the students who had a high level of achievement on the Pre-test, 83% successfully used the software on at least 2 of 3 parts compared with 67% of students with a low level of achievement on the Pre-test. As might be expected, since the Pre-test results are an indication of students aptitude, the students who achieved a higher level of algebra ability on the Pre-test were the same students who were more successful using the software as a tool. Regardless of basic algebraic skill development, a majority of students were able to use technology to find roots of equations or find the dimensions associated with a box of maximum volume or a cylinder of minimum surface area.

Question 2. A majority of students were not able to apply or adapt an algorithm to solve related problems - only 30% were successful on 2 or more parts of the question. The success of students appears to be related to their ability to do algebra as measured by the Pre-test. Of the students who had a high level of achievement on the Pre-test, 50% successfully applied or adapted the software algorithm on 2 or more parts of the question while none (0%) of the students who had a low level of achievement on the Pre-test were successful on 2 or more parts. Students who had a high level of achievement on the Pre-test successfully solved equations using graphing software and successfully found the relationship between the radius and height (1:2) of a can with minimum surface area. However, as discussed previously, all students had difficulty finding the relationship between the size of cut-out square corners and the size of the original square cardboard to form a box of maximum volume.

Question 3. A large percentage of students, regardless of basic algebra ability were successfully able to use technology to determine the relationship between an equation and its graph. 46% of the students were able to make a relatively complete description of the relationship, another 42% were able to partly describe the relationship. The success of students was independent of their algebra ability level: 40% of students scoring high on the Pre-test were successful and 75% of students scoring low on the Pre-test were successful.

Question 4: Mathematical Communication

The second general question of this study looks into how meaningful the students' mathematical experience is as they solve problems using technology. Three specific questions are addressed: Are students communicating mathematical ideas with their peers and/or their teacher; Can students describe how a function's table of values relates to its graph; Are students making and testing their own conjectures. The analysis of the transcribed video tape records and the researcher notes will be presented as they relate to each of the three specific questions in turn.

Over the course of these activities, a significant portion of the on-task discussion between partners was not mathematical in nature but was about the use of the software or about the procedure involved with each activity. The amount of off-task discussion was not different than expected in a classroom setting. As the students gained experience using the computer hardware and software throughout the week and throughout each activity, the portion of their conversation involving the use of the software decreased. However, even those students who were very proficient with the software continued to discuss the software procedure throughout the activities of the study. The amount of mathematical discussion between partners varied with the type of activity. The video-tape transcripts and the observation notes were used to get a feel for the frequency of mathematical and procedural communication (see Table 11). It was sometimes difficult to distinguish between the type of communication or to know when one incident stopped and another began. As a result, the number of occurrences listed in Table 11 gives a rough indication of the frequency of the types of

Table 11

Occurrences of Mathematical and Procedural Communication

Activity	Mathematical Discussions	Procedural Discussions
A	3	10
B	4	4
C	12	5
D	3	5

communication for each Activity. Results from Activity E are not included in the Table since the video-tape recorder malfunctioned during that activity.

During Activity A, there was a considerable amount of procedural discussion. It is worth noting that most of the procedural discussion was from the 30G students who were feeling rather anxious and who were having difficulty with the software algorithm. The proportion of procedural discussion diminished in subsequent activities presumably because the students were somewhat more familiar with the software. Some of the procedural questions the students asked of each other were:

> "Where's the little zoom box?";
> "How do we find the other *x*?";
> "Do you have to find the *x* and the *y* or two *x*'s?";
> "What do you do when you get to this point [the equation was entered but the GRAPH command was not given]? How do you get the thing up here [pointing to the coordinate axes on the monitor]? What do we press?";

One student asked, "Do we need to zoom in on one point or both of them?" His partner replied, "I think both of them." All of these examples are questions about the use of the software or about the procedure of the activity. Directions given by one partner to another were also often about the procedure: "Move over because that's the axis."; "Make it [the zoom box] bigger, just go to the left more."; "Type in SCALE 20." Although a considerable portion of the conversation was not mathematical in nature, there were mathematical discussions. There was some discussion about where the roots for the linear equation were. In the first few questions of Activity A, the students had found two or more roots. As a result, several pairs of students were looking for more than one root and considered writing the *x*-intercept and the *y*-intercept as roots of the linear equation. After easily zooming in on the *x*-intercept several times, two students had the following discussion:

S1: "Is that it?"

S2: "No, just a second. Zoom in here [pointing to the *y*-intercept]."
S1: "Ok, you do it."
S2: "Is this right [after zooming in on the *y*-intercept]?"
S1: "No, it is supposed to be zero [pointing to the *y* coordinate]. Maybe we should put *y* is equal to 4.99 [the *y*-intercept value]."

The result on the computer did not quite fit with the result they expected. They were looking for the values of *x* when *y* was zero. They recognized that something was wrong but they were not sure enough to eliminate the *y*-intercept as a root of the equation. This discussion demonstrates that these students have limited understanding of the graphic representation of functions but it also demonstrates how the students can use the software to identify their errors in thinking.

During the second part of Activity B, the students were to find the value(s) of *x* which would make two expressions equal. Some comments made, which were both procedural and mathematical in nature, were: "Do I graph these two equations at the same time?"; "The part I need to zoom in on is where the two lines meet, I need to find where they connect." One partner asked of the other, "Do we zoom in here [pointing to the *x*-axis]?" The other partner replied, "I think the answer is in here [indicating the points of intersection of the graphs]." Since they did not have experience with systems of equations, there was some discussion by most pairs of students about the location on the graph which held the solution to both equations. Based on their understanding of the graphs of the functions they determined that they should zoom in on the points of intersection. After making that decision the task of using the procedure was less difficult.

During the first part of Activity B the students were to find the value of *x* for a particular value of the function. One pair of students, who was particularly good at using the software, communicated very little verbally. As one partner zoomed in on the appropriate parts of each graph and pointed to the solution written on the monitor, the other partner wrote the solution down. However, there was some communication when this pair of students was doing the second part of the activity in which they were to find the value(s) of *x* to make two expressions equal. After having successfully zoomed in on the intersection point one student asked of the teacher,

S1: "How do you find the answer? Where does $6x^3 - 5x$ equal $7x^2$?"

T: [Pointing to the monitor and the *x*-coordinate of the ordered pair] "What is this number, what does it represent?"

S1: "*x*, the first number is *x* and the second number is *y*. We want to find *x*."

T: "Do you have any idea what the *y*-value of 1.73 might represent? What is the significance of that value?"

S1: "I don't know."

The pair of students then proceeded to find the appropriate *x*-values but they did not discuss, nor determine the significance of the *y*-coordinates at the intersection points. Their lack of discussion about the *y*-coordinate may indicate that they were satisfied with using the algorithm proficiently and were not interested in spending energy on fully understanding the procedure.

At the end of Activity B, the students were to make up their own polynomial. One student asked, "What if I make up a polynomial that has no solution?" The teacher responded, "Then write that down." Surprised, the student asked, "You mean it's O.K. to have an equation with no solutions?" The student's question suggests that he has not been exposed to equations with no solutions in his previous algebra experience. These students observed how an equation with no solution is represented on a graph. This observation may help them to better understand algebraic transformations of equations which yield no solution.

During Activity C there was considerably more mathematical discussion than during the previous two activities. This may be partially due to the exploratory nature of the activity and also due to the increasing degree of familiarity the students had with the software. A pair of students had a discussion about equations with no roots:

S1: "It's supposed to be on this line, right [pointing to *x*-axis]?"
S2: "I don't know."
S1: "Does there have to be roots?"
S2: "No, that's good."

They drew another parabola with a vertex closer to the *x*-axis but the equation still had no roots. Despite confirming with her partner that equations with no roots are valid, the student insisted on being frustrated by the graphs of equations with no roots.

S1: "Oh man, I don't want that." [The graph was cleared and a cubic function was drawn] "OK, that's good."
S2: "Yah, crosses the *x*-axis." [Another cubic equation was drawn with slight modifications] "That's the same thing, eh?"
S1: "Put down that the graph looks the same." [The student entered another cubic equation] "Oh, its always the same thing. Would you say that's on the line or just below it [referring to the part of the graph near the origin which was flat]?"
S2: "Just below it."

To verify that the line was just below the axis, the student used the zoom feature to view the part of the graph near the *x*-axis at a smaller scale. They saw that there were two roots of the cubic equation since the curve was tangent to the *x*-axis. This exchange between the two students is clearly mathematically meaningful. They have a good understanding of how roots of an equation are represented on a graph, and they have a good command of the software used to manipulate the graph. It did not occur to the students, however, that the line may not actually be tangent but may only appear tangent due to the limited display capability of the graphing technology.

During Activity D, new software was introduced so the frequency of procedural discussions increased as might be expected. One pair of students had a discussion about how to use the successive approximation procedure, the discussion was as much mathematical in nature as it was procedural:

> S1: "Are we just supposed to get the maximum volume higher than 74? How high are we supposed to get it?"
> T: "As large as you can get it."
> S2: "So that means the initial size may be like 10 something."
> S1: "No, it is getting smaller isn't it?"
> S2: "So, we are making the size bigger to try to make the maximum volume bigger."

One student made the observation, "as we continued to decrease the size and increment, the volume increased in small amounts, ... the slope lost its curve as the volume increased." Observations about the slope of a curve are certainly mathematical in nature and this knowledge will be valuable in future calculus classes.

Another group discussed the maximum of the function using the graphic representation. There was some confusion about whether they should be looking at a local maximum on the graph or at the part of the graph which continued to rise to infinity.

S1: "The maximum could be there [indicating local maximum]."
S2: "But the volume keeps rising over there [indicating the increasing part of the cubic graph which is in an invalid domain]."

The students then changed the starting value and increment parameters in the spreadsheet table and produced a graph which showed only the local maximum within a valid domain. The partners were then satisfied that they had found the maximum of the function. They were no longer concerned about the function increasing to infinity presumably because it was not happening on the monitor's display of the graph. They did not discuss the fact that part of the graph (to the far right) was in an invalid domain. It was clear from their lack of discussion that they did not have a good understanding of the real-world implications of the domain of the function representing the volume of the box.

In summary, there was a smaller proportion of mathematical discussion compared to procedural discussion during the activities which used an algorithm as taught by the teacher. During the more open-ended Activity C there was more discussion which was mathematical in nature. Additionally, the procedural discussions decreased as the students became more familiar with the software and the type of tasks they were working on. The proportion of discussion on the various activities indicates that the exploration activity was more meaningful than the activities involving maxima and minima and the activities involving maxima and minima were more meaningful than the activities using graphing software to find roots.

Question 5: Tabular and Graphic Representations

To help to determine if the experience with the software is mathematically meaningful to students the video-tape transcripts and the observation records were searched for students' descriptions of how a function's table of values relates to its graph (specifically the maximum, minimum and domain). The maximum value of Activity D and the minimum value of Activity E were reasonably easy for students to recognize both in the table and on the graph. The following response from one student was typical of most others: "This value is the maximum [indicating the largest value in the spreadsheet table]." Then, after drawing the graph of the data: "The maximum is there [pointing to the top of the graph and reading the numbers on the scale]." Many students used the table to read an accurate value of the maximum and used the graph primarily to isolate the location of the maximum. All students quickly recognized that the maximum value in the table is at the top of the graph.

The significance of details of the domain of the volume and surface area functions were less obvious to many of the students than the maximum and minimum values. Nevertheless, some of the students did have some insights into the domain of the function as it relates to the table and the graph. During Activity E, a pair of students observed that if they were too far to the left of the minimum the graph sloped down from left to right and the values in the table decreased. If they were too far to the right of the minimum the graph sloped up from left to right and the values in the table increased. This observation helped them to locate the appropriate domain to find the maximum or the minimum value.

On the other hand, one pair of students demonstrated their lack of understanding of the domain or of the successive approximation procedure. They had the maximum value within the range of data in the spreadsheet table and on their graph and tried to get a more accurate answer by making the increment smaller. The result was that the maximum value was off the graph to the right. The volume values continued to get larger to the bottom of the table and the graph of the function continued to rise to the far right of the computer monitor. These students did not understand that the initial size should be increased as the increment gets smaller because a smaller part of the domain was visible on the graph. They were successful only after choosing a variety of increments and initial size values apparently in a random fashion.

In summary, the students communicated their understanding of the relationship between the table of a function and its graph related to a local maximum or minimum and were less successful at communicating their understanding of the domain of a function.

<u>Question 6: Making and Testing Conjectures</u>

The experience with the different representations of functions will have been meaningful to students if they have made conjectures about the relationship between the graphic, tabular and algebraic representations and then used the software to test their conjectures. The video-tape transcripts and the observation records were searched for incidence of students making and testing conjectures.

Students used both the teacher and their peers as authorities during several of the activities of the study. Many of the students lacked the confidence to proceed with the activities without reassurance from the teacher. The 30G students, particularly at the beginning of the study, required the teacher to be the authority. They directed these questions to the teacher:

"Do you write this down?";
"O.K., now do we go to the next question?";
"Do you have to find two [roots]?";
"What should your other answer be? We got 3 something."

Some students used each other to verify that they were proceeding correctly. This exchange between two students during Activity A, is an example of one student seeking the advice of another student who is not her partner,

S1: "Do we go onto the next one when we are done?"
S2: "No, is that your first answer?" [S1 nods]. "You have to figure out another answer."
S1: "The teacher said to write that one down."
S2: "Yah, and you have to figure out another, he told us that. You may have two answers and this one's wrong."
S1: "Do you have to find the *x* and the *y* or two *x*'s?"
S2: "Two *x*'s."

At another point, one student was teaching another about the zooming procedure of Activity A. The far left root had been zoomed in on correctly,

"Unzoom and SCALE 10. No. You should go up more because you see, you are right on the line. You should go up more so you can draw the box around it [indicating around the *x*-intercept]."

The partner proceeded to zoom in two more levels successfully. In these cases the students with some doubts were not able to make decisions on their own nor use the software as an aid in decision making - they simply replaced the teacher authority with one of their peers.

Some students, on the other hand, were able to resolve their own conflicts without outside assistance. During

Activity A, one pair of students used the computer software to help them to act as authorities. After finding the values of the *x*-intercepts, they discussed whether they also needed to find the *y*-intercept value as an example of where the function was equal to zero. As they looked at the coordinates of the *y*-intercept they concluded that they had found where *x* was zero rather than where *y* was zero. The students used the computer to help them to resolve this conflict.

The most evidence of students acting as their own authority was found during Activity C. This activity was more exploratory in nature and the students were, by then, confident using the software. Following are four examples of pairs of students making and testing their own conjectures using the software. First, partners were testing the conjecture that the *y*-intercept of the graph of the function was related to the equation of the function. They had noticed that the graph of the function shifted up or down depending on the constant value added to the function. To investigate the relationship, they used the zoom feature of the software to find the values of the *y*-intercepts of a variety of functions. They were somewhat surprised to find that the constant value added to the function was equal to the *y*-intercept. They were able to use the computer as a tool to verify their conjecture about the constant in the equation and the position of the graph of the function.

Second, a pair of students graphed $y=5x$ then $y=-5x$ on the same set of axes. They explored further by graphing $y=5x-5x$ and then finally $x=5y-5y$. This symmetrical pattern of equations resulted in a symmetrical pattern of graphs looking like an asterisk. They were excited with their discovery and called the teacher over, "Look at this, look at this!" They had learned that there is clearly a relationship between a function's equation and its graph although they were not able to articulate the precise nature of the relationship.

A third pair of students used the software features to help them make a decision. After graphing a cubic equation, the students had a discussion about the number of roots of the equation. Their graph did not clearly reveal if the function crossed the x-axis twice, once, or not at all.

> S1: "Would you say that's on the line or just below it?"
> S2: "Just below it."

The student at the computer proceeded to use the zoom feature to get a clearer picture of the graph near the x-axis. The students concluded that there were only two roots for the cubic equation since the graph appeared to be tangent to the x-axis.

A fourth pair of students was able to test their conjecture using the software:

S1: "Let's try to get a straight line."
S2: "Try not putting any exponents in it."
S1: "There [after successfully graphing a line]."

They were able to confirm their idea within seconds of having the idea. They did not try another equation with an x-term with no exponent. Presumably, they were confident with the result after testing only one case.

As in traditional mathematics classes, the students continued to use the teacher as the authority for many of the activities. More students were observed making and testing their own conjectures during the open ended activity (Activity C) than during the more structured activities.

Summary

The student activity records, the video tape transcripts and the observation records were analyzed to answer the questions of the study. The experience for the students seems to be somewhat related to their algebra skill level as determined by the Pre-test. Students with a high basic algebra ability level had greater success both using (Question 1) and applying (Question 2) a software algorithm to solve problems. Students in the low algebra level achieved moderate success on the problems requiring the use of an algorithm (Activities A, D and E) but very limited success on the problems requiring application of the algorithm (Activities B, D and E). The success of students using the software for exploration (Activity C) of the relationship between the graphic and algebraic representations of polynomial functions (Question 3) seemed to be independent of their basic algebra skill level. Students with a wide range of algebra skills were able to use the software for exploration. The success rate of the students on the problems is similar to the success rate of the students on the Pre-test - success in both cases may be due, in part, to the students' innate ability. Although the students were no more successful than on traditional mathematics activities, the complexity of the problems the students worked on goes well beyond that which they are exposed to in current mathematics curricula.

The meaningfulness of the students' experience with the technology was varied. A large proportion of the on-task communication (Question 4) between partners was not mathematical in nature. However, several examples of mathematical communication were evident and the incidence of procedural discussions decreased as the students gained experience with the software. The amount of mathematical communication seemed to vary with the type of activity. There was considerably more mathematical discussion and thus more meaning during the exploration activity than the other activities. The activities using the graphing software to find roots had the least amount of mathematical communication which indicates that they were less meaningful. A large number of students were able to describe the relationship between a function's table and its graph (Question 5) with respect to local maxima or minima and to a lesser degree, to the domain of the function indicating that these activities were somewhat meaningful. Examples of students making and testing conjectures were given (Question 6). The most evidence of students making and testing their own conjectures was found during the exploration activity. This evidence is confirmation that the exploration activity was a meaningful activity. The records suggest that the students' experience was particularly meaningful when doing the specific problems involving maxima and minima of functions and when doing the exploration activity involving the relationship between representations of functions.

Chapter Five

SUMMARY AND CONCLUSIONS

Summary of the Study

Questions and Procedures

The purpose of this case-study was to determine if students can use multiple representation software to solve problems involving functions before they have mastered advanced procedural algebraic manipulation skills and to determine if, as students use the software, their mathematical experience is meaningful. The following six specific questions were investigated and the results analyzed:

1) Are students able to successfully use an algorithmic software procedure to find roots, maxima and minima of functions?

2) Can students apply or adapt a software algorithm to solve related but different problems?

3) Can students use software as a tool for exploration?

4) Are students communicating mathematical ideas with their peers and/or their teacher?

5) Can students describe how a function's table of values relates to its graph?

6) Are students making and testing their own conjectures?

In order to answer these questions, the students worked through five activities using multiple representation software. As they worked through the activities, students recorded information on data sheets, researcher observation notes were kept and the interaction of one pair of students (at a time) and their monitor was video taped. The video-tape records were transcribed and analyzed along with the student records and observation notes.

Results

The data were analyzed in detail in Chapter 4 as they relate to each of the six specific questions and are summarized here. The Pre-test results demonstrated that there was a wide range of algebra ability in both the grade 10 specialized mathematics class (20S) and the grade 11 general mathematics class (30G). Generally, however, the 20S students had better basic algebra skills than the 30G students. The study addresses both the degree of success of the students using multiple representation software and the meaningfulness of the experience.

The results of the study suggest that students have reasonable success using a software algorithm (Question 1) to solve more algebraically complex problems than would be possible with limited algebra skills since 70% of them used a software algorithm to successfully solve two of three problems. The results support researchers' (Demana & Waits, 1990; Dunham & Osborne, 1991) claim that students can do more algebraically complex problems when they are represented geometrically.

The results also suggest, however, that success of students with a low level of basic algebra skill was <u>not</u> enhanced by using the software tools. The students with a higher level of basic algebra skills achieved an 83% success rate on two of three problems compared to only a 67% success rate for the lower level students. Contrary to the speculation of some researchers, the evidence did not indicate that lower ability students would achieve greater success using technological tools than they were accustomed to when solving problems without the tools. Regardless of ability level, the students achieved as much or as little success using the graphing software to solve more algebraically complex equations as they did on the algebraically simpler Pre-test questions.

The students were not particularly successful at applying a software algorithm (Question 2) to related problems since only 30% of the students were successful on two of three parts. Students with a higher level of algebraic skill as indicated by the Pre-test were somewhat successful (50% of them solved two of three problems) and the students with a low level of algebraic skill had very limited success (none of them solved two of three problems). Since the levels as determined by the Pre-test are an indication of the students facility for understanding mathematics, it may be that the high level students had more success applying the software algorithms because some of them were using the algorithms meaningfully.

All students, independent of algebra ability level, were somewhat successful using the graphing software for exploration (Question 3). On average, 46% of the students gave a complete description of the relationships between the equation and the graph of polynomial functions. Another 42% gave a partial description of the relationships. Additionally, the analysis of the nature of the communication of students (Question 4) and the conjectures made and tested (Question 6) during this activity revealed that the exploration activity was the most meaningful activity for the students. This is consistent with other researchers (Demana & Waits, 1990; Dugdale, 1993) who observed students using multiple representation software to enhance their understanding of the link between representations.

Based on the nature of the communication between partners (Question 4), the meaningfulness of the activity depended on the type of activity on which the students were working. The least meaningful activities, based on the nature of the communication between partners, were the activities using the graphing software to find roots. A large proportion of the communication between students was procedural rather than mathematical in nature but, as recorded in Table 11, the proportion of procedural communication decreased as the students became more familiar with the software. As with other procedural algorithms, the students can use these software algorithms without fully understanding the mathematics behind the procedures.

The meaningfulness of the activities involving maximum volume and minimum surface area is less clear. Most students described the relationship between a function's table and its graph (Question 5) with respect to the local minima and maxima. On the other hand, only a small percentage of the students were able to describe details of the domain of the functions. Also, the nature of the communication (Question 4) between partners during the maximum and minimum activities included a similar portion of procedural and mathematical discussion.

The results of this study were affected by the students' limited experience with the software. Contrary to what might occur in a classroom over the course of a year, the students spent a large proportion of their time in the study learning how to use the software. In a complete course, the students would require the same amount of time (a small fraction of the whole course) learning the software procedures used in the study. As a result, the proportion of procedural rather than mathematical discussion and the proportion of time the students were anxious about the software may have been much higher in the study than would be expected if this approach were implemented over time as a part of the normal classroom.

An unknown factor in the implications of this study's results is the attitude of the students. The reason for the excessive absenteeism of the 30G students in this study is not clear. It may be partly due to the nature of the relationship between students and a long-term substitute teacher. It may also be due in part to the lack of desire of the 30G students to do mathematics on the computer. As discussed previously, the 30G group was particularly anxious about using the computer software. Some of the students vocalized their lack of enthusiasm for using computers. The attitudes of the students will need to be considered when teachers determine to what extent they will integrate activities using technology into their mathematics classes.

Implications for Practise

Not unexpectedly, the students' ability to find the roots of algebraically complex equations using graphing software was consistent with their ability doing algebra as measured by the Pre-test. Students who found equation solving difficult using algebra found equation solving with the aid of graphing tools no less difficult. For students with limited graphing experience, the graphic representation of functions may be as abstract as the algebraic representation. Nevertheless, in a technologically rich environment students could be taught a graphic method of equation solving. The advantage appears not to be that more students would have success but rather that the algebraic complexity of problems would not be a factor in problem selection. More problems which may be of interest to students with basic algebra skills could be accessible to them. To make the graphic representation more concrete for students, the students could be exposed to problems involving the graphic representation of functions using real-world data at earlier levels in mathematics.

A large proportion of the students successfully solved problems involving local maxima and minima. The descriptions students gave about the relationship between the table of a function and its graph related to maxima and minima suggest the problems can be solved meaningfully. In a technologically rich environment, both specialized and general mathematics curricula could include problems involving the maxima, minima and roots of equations earlier than is currently considered due to the algebra skills required. Prerequisite knowledge (for example, of volume or surface area calculations) should be considered when doing the problems to ensure their meaningfulness. Problems involving local maxima and minima are potentially more interesting since they can be less contrived and closer to real-world problems than traditional 'type' problems.

Based on the communication of students and the conjectures made and tested, the exploration activity, in which students were to describe the relationship between the algebraic and graphic representations of functions, was the most mathematically meaningful activity for students. Also, the level of basic algebra skill indicated on the Pre-test was independent of success. Exploration activities using multiple representation software could be used to help students develop an understanding of the various representations of functions. Mathematics curricula could include exploration activities using graphing software before students have honed their algebraic manipulation skills. The graphic representation could be used as an alternative representation for algebraic expressions useful for verifying the logic of algebraic transformations.

There is some concern among mathematicians that exploration activities as discussed above may give students the wrong impression of what mathematics is. Students may incorrectly conclude that mathematics is a science in which theories are proved based on empirical data rather than by deduction. The students did not prove the exact nature of the relationship between equations and graphs, they observed patterns of behaviour of families of functions. Exploration activities should not be done to the exclusion of deduction but may be a valuable precursor to more theoretical mathematics. It may also be that technology will influence the aspects of mathematics which receive the most attention and influence the field of mathematics so that the nature of mathematics itself may change with time.

As observed in this study and in support of other researchers (Heid & Baylor, 1993), the roles of teacher and students in a technologically rich environment can be different than traditional roles to which teachers and students have become accustomed. The roles of students and teachers are affected by at least two factors; the use of technology may not be as big a factor as the nature of the activity on which students are engaged. During the open-ended activity more than during the structured activities, the students were seen to use the software to help them to make decisions rather than seek answers from the teacher. Teachers may not be entirely comfortable with their new role as collaborator and facilitator when doing more open-ended activities with technology. Furthermore, teachers need to be aware that students may also be uncomfortable with their new role since the students would be expected to participate more fully than in traditional mathematics classes. Teacher and student attitudes about new role expectations will affect the success or failure of curriculum revisions.

Changes to curriculum are not going to be successful unless teachers are prepared for the change. Teachers would need to become comfortable with software, they may need to adjust the style of lessons they are accustomed to planning, and they would need to consider forms of evaluation other than paper-and-pencil tests. A considerable amount of professional development training to pre-service and in-service teachers would be required to implement the changes suggested here.

Future Research

This study did not determine which sequence of instruction was best, only that it is possible for students to solve problems using technology with limited algebra skills. The best sequence of instruction to balance the teaching of algebra with the use of graphing technology will need to be determined by further research as technology becomes more available to mathematics students. The details of the relationship between the use of graphing technology to clarify and verify algebraic procedures and the use of algebra to aid in the understanding and interpretation of graphs will need to be worked out by careful research and teacher experience.

Depending on the activities, students' experience in a mathematics class in which technology is regularly used may be different from their experience in a traditional mathematics class. The mathematics can be more exploratory but may also be more empirical. Research should be done to determine if students' beliefs about the nature of mathematics are altered after being taught in a technologically rich environment for a prolonged period of time. Students' beliefs about mathematics may affect their future course selections and career choices.

Students currently have some problems understanding the significance of scale on the axes of a graph and they tend to rely on linear relationships to the exclusion of other relationships. To what extent are the difficulties that students experience related to the current focus of the mathematics curriculum on linear equations and simple scales with limits from negative ten to positive ten on each axis? After new curricula are in place and technology is available, research could be done to determine the effect of the new curricula on current student difficulties.

If technology becomes readily available to students, the goals of instruction may change. To make curriculum change effective, testing techniques and methods of assessment may need to be altered accordingly. Educators and researchers will need to determine what evaluation techniques are most appropriate.

Conclusions

In a technologically rich environment, mathematics curricula can be modified. The results of this study indicate that the sequence of instruction could change to include solving equations and problems using the graphic representation of a function before using algebraic techniques. Further research is necessary to determine if the change in sequence is desirable particularly since it is not clear that the students always used the technological tools to solve the problems meaningfully. The different levels of success achieved by students of low and high basic algebra ability may indicate that graphing technology is not as useful to low ability students as some researchers are predicting. The experience of students using computer software can be meaningful but the experience may be more or less meaningful depending on the type of activity in which the students are engaged. Since the use of technological tools to solve problems should not be an 'end' itself, curriculum writers and educators will need to plan student activities carefully so that the tools are being used to solve problems meaningfully.

Mathematics curriculum can change to reflect the power of technological tools but along with a modification to curriculum must come support from governments, administrators, and teachers. Currently, there is a small percentage of mathematics classes with access to the required technology - governments and administrators have limited financial resources and so will need to decide if the availability of technology to mathematics students is a priority. Due to the nature of technological developments, tools purchased today may be obsolete in a few years. It will be important for mathematics educators to learn how general tools such as spreadsheet and graphing software, which do not depend on particular hardware or particular software, can be used to benefit mathematics students. The knowledge of how to make good use of technology will not become obsolete. Administrators need to provide the time and teachers the energy for professional development related to the use of technology in mathematics classrooms. With the required support, technology can enrich the mathematical experience of students.

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Appendix A

Pre-Test

Part A. This test will be used to determine the level of algebra skills you have acquired. Do not write on this test paper. Answer the following multiple choice questions on the sheet provided. Write the letter of the most suitable answer in the space next to each question number.

1. 64 is equivalent to:

A) 4^4 B) 8^8 C) 2^5

D) 2^6 E) 32^2

2. Allan has 54 jawbreakers some of which are red, the others black. He has five times as many red as black. How many of each kind does he have?

A) 6 red, 48 black B) 9 black, 45 red

C) 6 black, 48 red D) 8 black, 40 red

E) 9 red, 45 black

3. When simplified, $(a^2b^4)(ab^3)$ is

A) a^2b^{12} B) ab^6ab^3 C) a^3b^7

D) ab E) a^2b^7

4. A simpler form of $2a(3 - 4b)$ is

A) $6a - 4b$ B) $5a - 4b$ C) $5a - 8b$

D) $6a - 6b$ E) $6a - 8ab$

5. A simpler equivalent expression for $-2x + 3y - 5x - 7y$ is

A) $7x - 10y$ B) $-7x - 10y$ C) $7x - 4y$

D) $-7x - 4y$ E) $-3x - 4y$

6. $-4(x - 7) - 5$ is equivalent to

A) $4x - 33$ B) $4x + 23$ C) $-4x - 33$

D) $-4x + 23$ E) $-4x - 12$

7. Simplify $(7a^4)(-6b^3)$

A) $-42a^4b^3$ B) $-a^4b^3$ C) $42a^4b^3$

D) $-42ab^7$ E) $-42(ab)^{12}$

8. If $x = 4$, $y = 2$, and $z = 0.5$, the value of $2xy^2z$ is

A) 16 B) 32 C) 128

D) 32 E) 64

9. Evaluate $-4a(a - 3b)$ when $a = 2$ and $b = -1$.

A) 8 B) 13 C) -40

D) -8 E) 40

10. If $a = 4$ and $b = 2$, the value of $5a - b$ is

A) 7 B) 18 C) 22

B) 117 E) -6

11. If $a = 4$, $b = 2$, and $c = \frac{1}{2}$, the value of $2ab^2c$ is

A) 16 B) 32 C) 64

D) 128 E) 256

12. The area of the shaded region of the diagram is

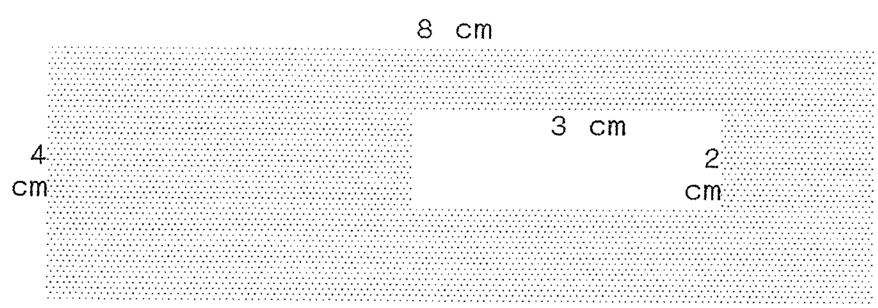
A) 10 cm^2

B) 26 cm^2

C) 16 cm^2

D) 38 cm^2

E) 32 cm^2



13. If the bottom of a 13 metre ramp is 12 metres from the loading platform, how high is the platform?

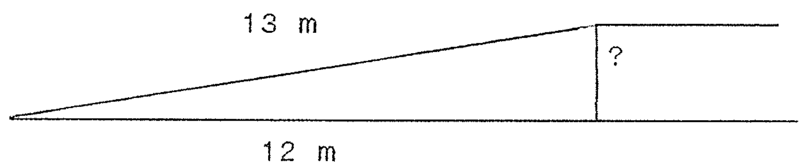
A) 1 m

B) 5 m

C) 8 m

D) 12.5 m

E) 25 m



14. The perimeter of the rectangle ABCD is 100 metres. The width is 20 metres. The length is

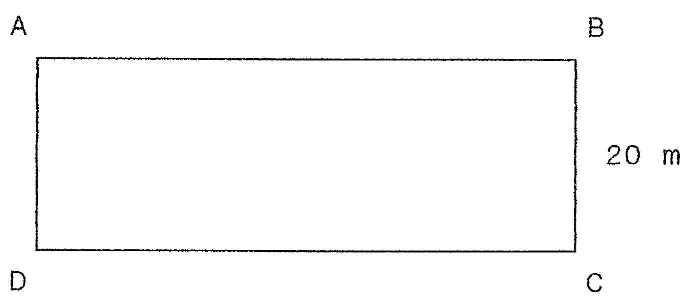
A) 80 m

B) 40 m

C) 30 m

D) 25 m

E) 60 m



15. $12x + 16y =$

A) $12(x + 16y)$ B) $4(3x + 4y)$ C) $4(3x + 6y)$

D) $2(6x + 16y)$ E) $12(x + 4y)$

16. A simpler form of $-6x(2y + 2x - 2w)$ is

A) $-12x - 12xy - 6xw$ B) $-12x^2 + 12xy + 12xw$

C) $-12x^2 - 12xy + 12xw$ D) $-12x^2 - 12xy - 12xw$

17. If $7x = 63$, then $x =$

A) 8   B) -8   C) 9

D) -9   E) 56

18. If $2.5x = 15$, then $x =$

A) 7.5 B) 6 C) 8

D) -6 E) 12.5

19. If $6 + 5n = 41$, then $n =$

A) 6 B) $\frac{47}{5}$ C) $\frac{41}{11}$

D) 9 E) 7

20. If $12 + 7x = 11 - 2x$, then $x =$

A) $\frac{-1}{9}$ B) $\frac{-1}{5}$ C) $\frac{23}{9}$

E) -1 F) 1

21. If $4(2m - 3) = -12$, then $m =$

A) 3 B) $\frac{-9}{8}$ C) $\frac{15}{8}$

D) -1 E) 0

22. If $8(3x - 5) - 6(x + 5) = 20$, then $x=$

A) $\frac{25}{9}$ B) 5 C) $\frac{5}{3}$

D) -5 E) $\frac{10}{9}$

23. $\frac{2x - 1}{3} = \frac{2x + 1}{5}$, then $x=$

A) $\frac{1}{2}$ B) $\frac{-1}{2}$ C) 2

D) -2 E) no solution

24. $x^2 + 15 = 64$, a value of x is

A) 79 B) 49 C) 8

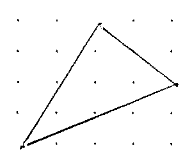
D) 7 E) 6

25. The perimeter of an isosceles triangle is 34 cm. If the length of each equal side is 1 cm less than 4 times the length of the base, what is the length of each side?

A) 6 cm, 14 cm, 14 cm B) 2 cm, 16 cm, 16 cm

C) 4 cm, 15 cm, 15 cm D) 5 cm, 19 cm, 19 cm

26. What is the area of the enclosed region?



A) 6 square units

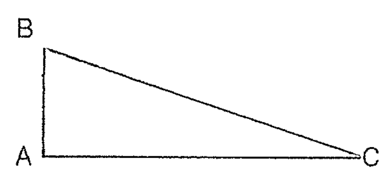
B) 6.5 square units

C) 7 square units

D) 7.5 square units

E) 8 square units

27. The length of AB is twice as long as the length of AC. The area of the triangle is 49 cm^2. The length of AC is



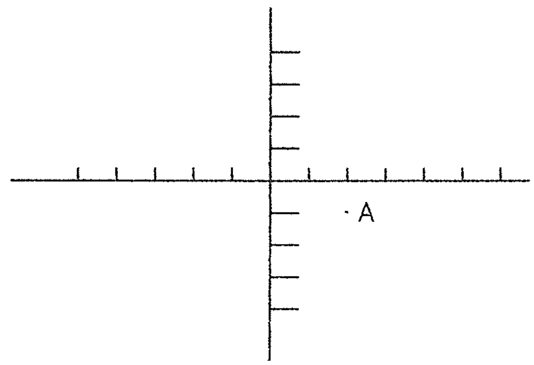
A) 7 cm

B) 14 cm

C) 21 cm

D) 28 cm

28. Point A has the coordinates



A) (2,1)

B) (1,2)

C) (-1,2)

D) (-2,-1)

E) (2,-1)

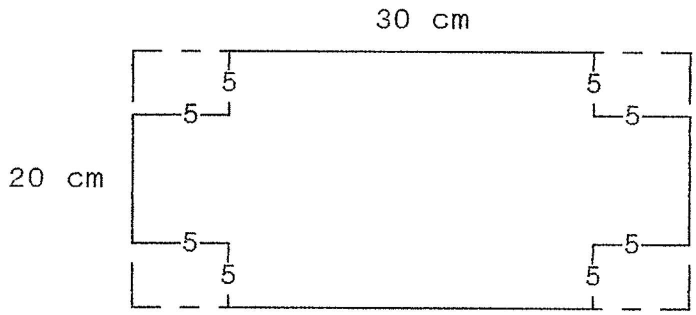
29. Which of the following ordered pairs (x,y) would <u>not</u> be a solution to the equation $3x - 2y = 12$

A) (4,0) B) (6,3) C) (2,-3)

D) (-2,-9) E) (0,6)

Part B. Write the solutions to the following problems on this paper. Make sure the method of your solution is clear. If you are not sure of a solution, do your best to work toward a solution.

1. Squares (5 cm by 5 cm) have been cut out of the corners of the rectangle (originally 30 cm by 20 cm) as shown. What would be the volume of the open box formed by folding up the sides?



2. A pizza has a diameter of 20 cm. Write the radius, circumference ($C=2\pi r$) and area ($A=\pi r^2$) of the pizza.

3. A cylinder has a radius of 10 cm and a volume of 628 cm^3. What is the height of the cylinder? ($Vol=\pi r^2 h$)

4. A cylinder has a radius of 20 cm and a height of 10 cm. What is the total area of the two circular ends and the curved rectangular side?

Appendix B

Student Activities

Activity A
Solving Equations Using Technology

Name: ____________________ Date: ______________________

To find the roots of $2x^2 - x - 15 = 0$ graph the function $y = 2x^2 - x - 15$ then zoom in on the part of the graph where the function is zero (ie. $y = 0$).

Find the roots of the following equations (your answers should be as accurate as the software will allow). Write your solutions on this paper.

ROOTS

1. $2x^2 - 4x - 8 = 0$

2. $x^4 + 2x^3 - 5x^2 + 1 = 0$

3. $3x^3 - 3x^2 - 6x = 0$

4. $x^5 + 5x^4 - 13x^3 - 65x^2 + 36x + 176 = 0$

5. $3x + 5 = 0$

Write patterns discovered or observations made (if any) as a result of the work you have done on this activity.

____

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Activity B
Solving Equations using Technology

Name: ____________________ Date: ______________________

How can the procedure from Activity A be generalized to solve equations in which neither side of the equation is zero.

1. For what values of x is $6x^2 - 5x$ equal to 6

2. For what values of x is $6x^2 - 5x$ equal to 10

Describe how you modified the method from Activity A

____

____

____

3. For what values of x will $6x^3 - 5x$ be the same as $7x^2$.

4. For what values of x will $x^4 + 2x^3$ be the same as $5x^2 - 1$

Describe how you modified the method to solve these equations

____

____

____

Make up your own polynomial equation which could be solved using this method. Write the solution(s) of the equation.

____

____

____

Activity C
Exploring Roots and Equations

Problem:

What are the features of an equation which determine its shape when graphed and the number of roots.

Procedure:

Graph several equations of varying degree. For each equation you try, make a record on the Activity C - Data Sheet listing:

1) the equation,
2) why you chose the equation,
3) any observations you make based on the graph of the equation
4) a sketch of the graph (drawn in the observations column).

Activity C - Data Sheet

Name: ____________________ Date: __________________

Equation	Why Chosen?	Observations

What is the relationship between the characteristics of the equations and the number of roots or the shape of the graph? Describe what led you to your conclusion (use the back too!).

____

____

Activity D
Exploring Volume

Problem 1:

To make an open box out of a rectangular piece of cardboard, cut squares of equal sizes out of each of the four corners and fold up the sides. You are to determine the size of the squares which should be cut out of a 10 cm by 10 cm cardboard rectangle to make a box with the largest **possible volume.**

Procedure:

Load the MS-Works spreadsheet file called "boxvol.wks". Put in the formula to calculate the volume of the box then copy the formula to all cells in the column (fill down).

Use the Activity D data sheet to record, for each attempt, your choice of initial box size and increment and why they were chosen. Also list the maximum volume for the attempt and draw a rough sketch of the shape of the graph produced from the data and indicate on it which part of the graph represents the maximum volume for the attempt. Stop when you have determined the largest possible volume to the greatest possible accuracy using the computer.

Finally, create a spreadsheet table with an initial box size of -20 cm and an increment of 5 cm. View the graph of the data. At the bottom of the data sheet in the general observations area, describe what an equation might look like to produce the same graph as this table.

Problem 2:

Determine the relationship between the size of the original cardboard and the size of the cut-out square needed to form a box with the largest possible volume.

Procedure:

Continue using the "boxvol.wks" file. Replace the dimensions of the Original Cardboard with dimensions other than 10 cm by 10 cm and find the size of the cut-out square. Record your attempts on the Activity D data sheet in the same manner as problem 1. Try at least two other sizes of cardboard and record each on a separate data sheet.

Under general observations, describe how the shape of the graph relates to this problem about maximum volume. Also write any patterns or other observations you may notice.

Activity D - Data Sheet

Name: ______________________ Date: _______________

Original Cardboard Dimensions ________________

size/increment	Why Chosen?	Max. Volume	Graph

Size of cut-out square: ___________

Maximum Volume: ____________________

General Observations (continue on the back if necessary):

____

____

____

Activity E
Exploring Surface Area

Problem1:

Are Cola companies using the best shape of can to hold their 355 ml drink? What should the radius and height of a Cola can be to have 355 ml of volume and use the least amount of metal?

Procedure:

Load the MS-Works spreadsheet file called "surfarea.wks". Put in the formula to calculate the area of the circular top and copy it (fill down). The formulas for height and surface area are entered for you.

On the Activity E Data Sheet record, for each attempt, your choice of initial radius and increment and why they were chosen. Also list the minimum surface area for the attempt and draw a rough sketch of the shape of the graph produced from the data and indicate on it which part of the graph represents the minimum surface area for the attempt. Stop when you have determined the smallest possible area to the greatest possible accuracy using the computer.

Problem 2:

Determine the relationship between the radius and height of a can which has the smallest surface area possible for any given volume.

Procedure:

Continue using the "surfarea.wks" file. Replace the original volume with a value other than 355 ml and find the radius and height of the cylinder with minimum surface area. Record your attempts on the Activity E Data Sheet in the same manner as problem 1. Try at least two other can volumes and record each on a separate data sheet.

Under general observations write why the shape of this graph relates to this problem about minimum surface area. Also write any patterns or other observations you may notice.

Activity E - Data Sheet

Name: ____________________ Date: __________

Original Volume: ____________

radius/increment	Why Chosen?	Min. Area	Graph

Size of radius: ____________ Size of height: ____________

Minimum Surface Area: ____________________

General Observations (continue on the back if necessary):

____

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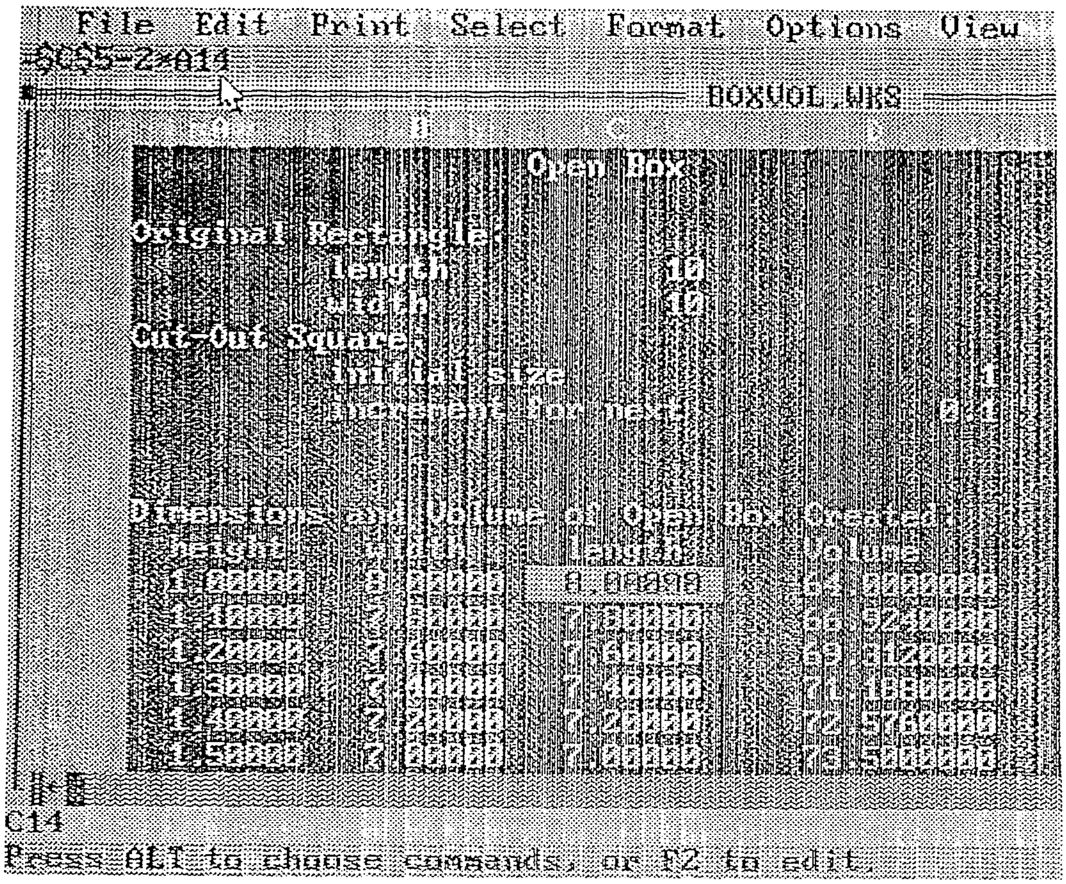
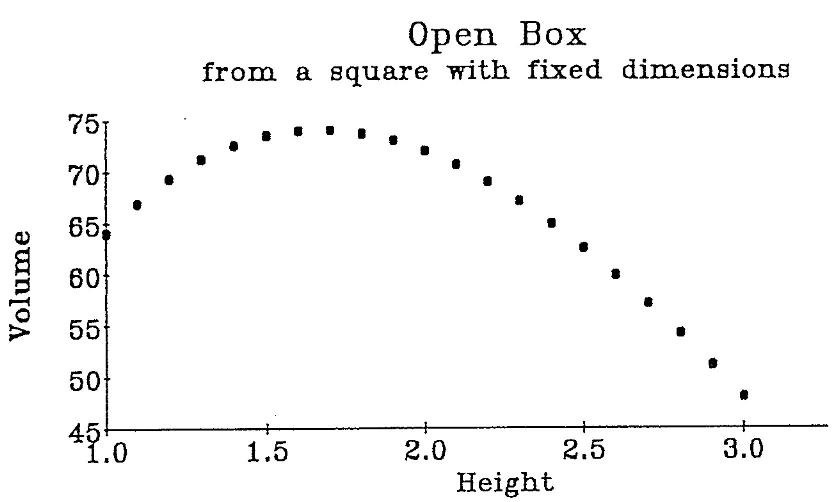
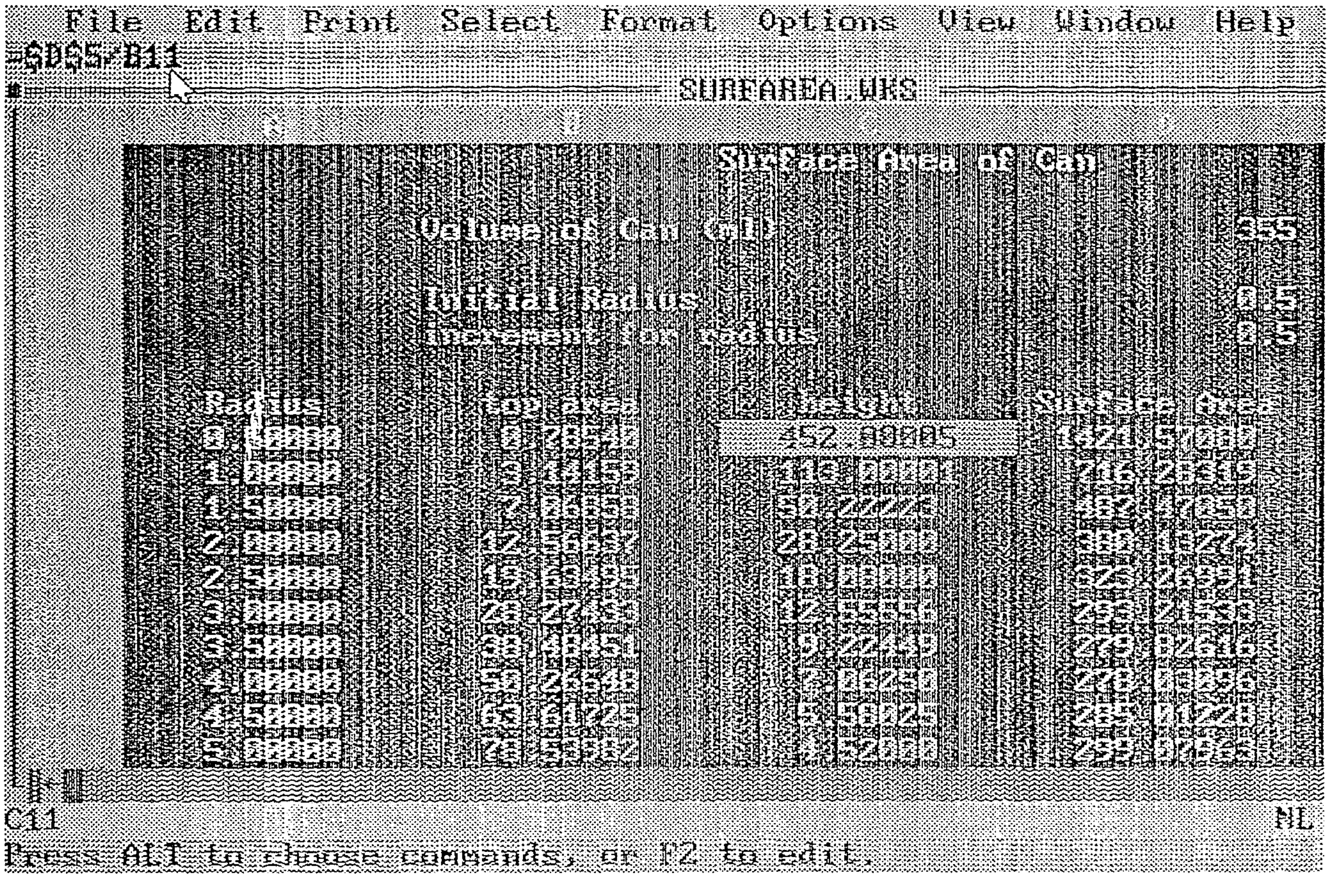
Appendix C

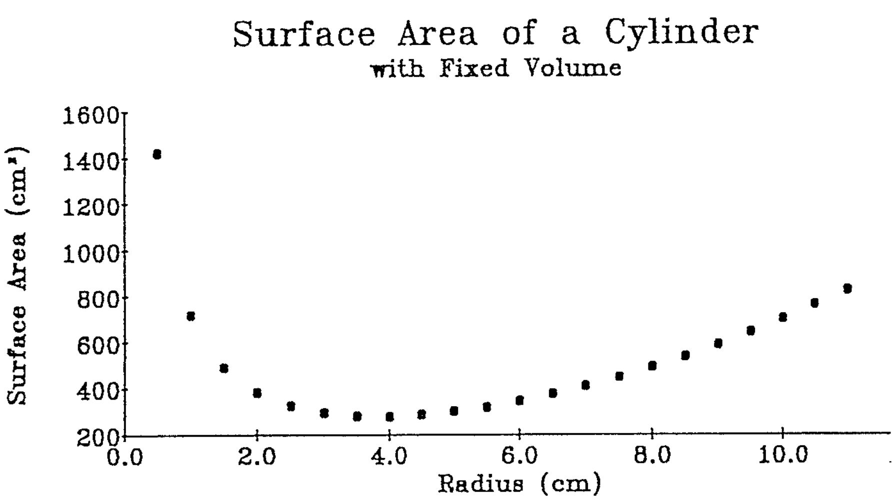
Spread-sheet Screen Print-outs





This is the Micro-Soft Works spreadsheet data for Activity D and the corresponding graph. The formula for cell C14 is in the formula bar in the upper left corner as indicated by the mouse pointer. After entering the formula for the volume calculations, the students adjusted the 'initial size' and 'increment for next' parameters to find the maximum volume given the original rectangle dimensions.





This is the <u>Micro-Soft Works</u> spreadsheet data for Activity E and the corresponding graph. The formula for cell C11 is in the formula bar in the upper left corner as indicated by the mouse pointer. After entering the formula for the top area calculations, the students adjusted the 'initial radius' and 'increment for radius' parameters to find the minimum surface area given the original volume of the can.