OPTIMAL CONTROL OF A SYNCHRONOUS GENERATOR

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KAI K. CHENG
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ABSTRACT

This thesis studies the optimal control of a synchronous generator. The control minimizes the excursion of the squared values of the system parameters during disturbances and faults. The effect of different system models of varying complexity is investigated. Then using a fifth order model with practical data supplied by the Manitoba Hydro, a linear feedback control is obtained by solving the corresponding matrix Ricatti equation. The solution is by a new numerical method called the "Matrix Derivative Method." This method is compared with various existing numerical techniques for solving the matrix Ricatti equation. This control is then applied to the system. For comparison purposes, the optimal control is also derived for a similar system with much lighter damping. The possibilities of suboptimal control and implementation problems are also considered.

I. INTRODUCTION

With the rapid development of modern control theory, applications of these new concepts to power systems become a popular topic. Some try to minimize the time needed to restore the system to normal operating conditions after a fault or a disturbance using the 'bang bang' principle. Others try to minimize the excursions of the variables such as frequency, power, etc., or in other words, to increase stability. An actual power system is a highly complicated thing, but an understanding can be gained by representing it by a simplified system that contains the main features and studying the various aspects in deriving an optimal control for it.

In this thesis, we try to represent a simplified generator by a 5th order differential equation and derive an optimal control that will minimize the excursion of the variables after a fault or disturbance. The generator has both voltage regulator and speed governor controls, though both of these are simplified. The damper windings of the machine are neglected. Emphasis is not on the actual "optimal control" derived, but in studying the problems, difficulties and insights in going through the process.

II. MODELING OF THE SYSTEM

The simplest form of a generating unit is a single synchronous generator connected to an infinite bus by a transmission line. The synchronous machine's prime mover is driven by hydraulic power. The transmission line is assumed to be so short that it has negligible resistance and reactance. This is the system that we want to study.

This system can be modelled to different degrees of accuracy by different numbers of differential equations. One may take into account all aspects of the synchronous machine as well as the regulating devices and end up with a 10th or 12th order system, but for the purpose of this paper, a model that includes only the main features of the system is adequate.

There are various forms of representation of a machine by a set of equations. The classical books and papers represent the machine in the form of a voltage current relationship[*3,4,5] or the circuit form

$$\underline{V} = Z\,\underline{I} \ .$$

But the modern trend is to represent the system in state variable form

$$\dot{\underline{x}} = A\,\underline{x} \ .$$

This form has the advantage of being easily programmed and computed by modern high speed computers. Also, since much research has been done in this direction, the mathematical basis for differential equations of this form is very well founded and well known. For example, the effect of the eigenvalues of A on the system stability is well studied and well known. Also, a large number of theories have been established or conjectured for systems of this form. So in this paper, we shall represent our system by a set of state equations.

In order to determine the complexity of our model that is needed to achieve appropriate accuracy and also to study the relative weights and interactions of various parameters, we shall study systems of various orders and with different state variables. A set of data is obtained from one of the stations of Manitoba Hydro and applied to various models under study.

TABLE I

Nomenclature:

δ	--	power angle
ω	--	frequency
ψ_f	--	field flux linkage
V_f	--	excitation voltage
Pm	--	input power of prime mover
x_d	--	synchronous reactance, d-axis
x_d'	--	transient reactance, d-axis
x_q	--	synchronous reactance, q-axis
τ_{do}	--	open circuit time constant of field
τ_e	--	exciter time constant
τ_a	--	equivalent time constant of turbine unit
τ_w	--	water time constant
τ_{ga}	--	governor actuator time constant
τ_g	--	governor time constant
D	--	damping coefficient
H	--	inertia constant
V_o	--	infinite bus voltage
ω_o	--	infinite bus frequency
Pe	--	electrical power
Pa	--	accelerating power of turbine
u_e, u_g	--	control signals
δ_o	--	power angle at operating point
g	--	gate position of governor-turbine unit

TABLE II

Data

Rated power = 115 MVA

Rated voltage = 13.8 KV

D =	0.23 for 25% load
	0.53 for 50% load
	0.84 for 80% load
	1.05 for 100% load

$H = 634.42$ MW - sec^2

$x_d = 69.6\%$

$x_q = 29.8\%$

$x_d' = 20.1\%$

$\tau_{do}' = 6.21$ sec

$\tau_e = 0.05$ sec

$\tau_g = 5.5$ sec

$\tau_{ga} = 12.2$ sec

$\tau_w = 2.43$ sec

$\delta_o = 23°$

$v_o = 1$ p.u. (13.8 KV)

The simplest form is the 2nd order system (also known as the inertial model)[*1]

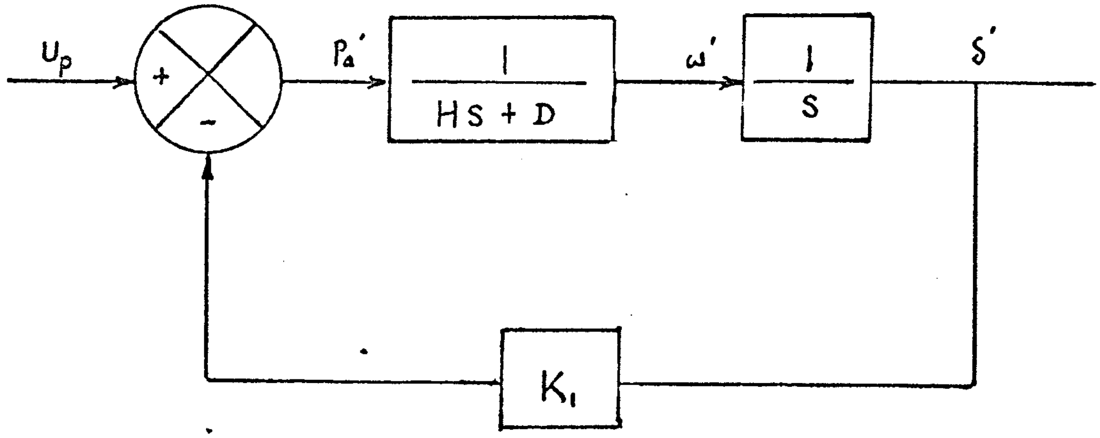


Fig. 2-1 Inertial Model

$$K_1 = \frac{V_o^2 \cos \delta_o}{x_d'} + \frac{V_o^2 \cos 2\,\delta_o (x_d' - x_q)}{x_d' \; x_q} = 118.0431 \text{ p.u.}$$

The primed variables indicate deviations from operating point; thus the differential equations represent the relations between the incremental changes of the variables. This notation will be used throughout the thesis.

Suppose the system is disturbed from normal operating conditions so that $\delta'(0) = 0.8$ pu, $\omega'(0) = 0.9$ pu, the subsequent behaviour of the system, calculated and plotted by a digital computer using an incremental time of 0.0002 sec., is shown in Fig. 2.2 .

The system behaviour with the application of an optimal control u_p^* which minimizes the performance index

$$\int_0^\infty (\,10\delta'^2 + 10\omega'^2 + u_p^2\,)\, dt$$

*) u_p = shaft input power

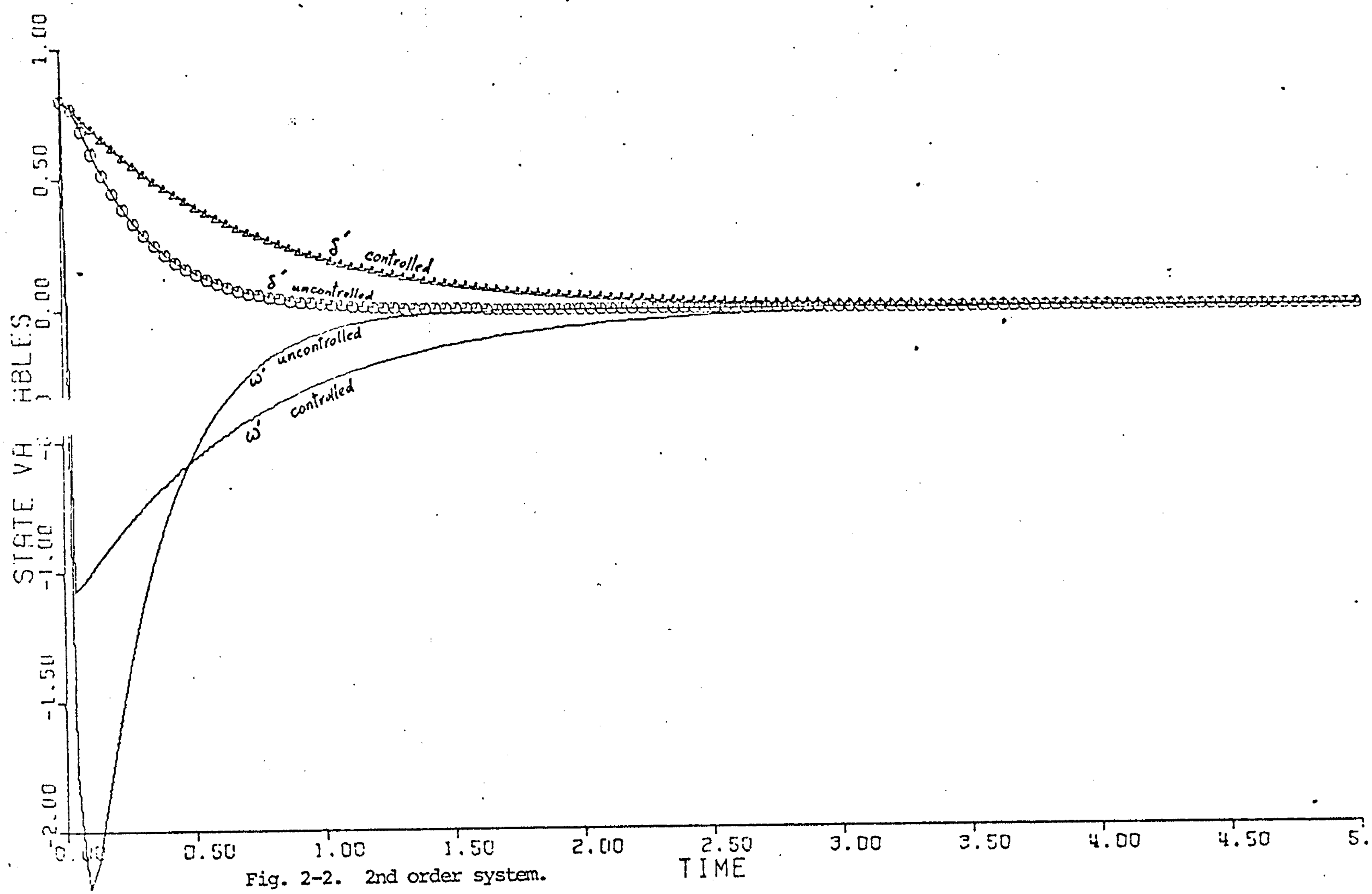


Fig. 2-2. 2nd order system.

is also shown in Fig. 2.2. Details of obtaining the optimal control will be discussed later.

There is a significant improvement, but this model is impracticable as it is impossible to make the shaft power change instantaneously with change of states and therefore Pm' cannot be made an optimal control signal.

Then a 3rd order system with Pm' (input shaft power) as the third variable is introduced. With control signal u_g fed to the governor of the hydraulic turbine, there are several steps between the control signal and the shaft power.

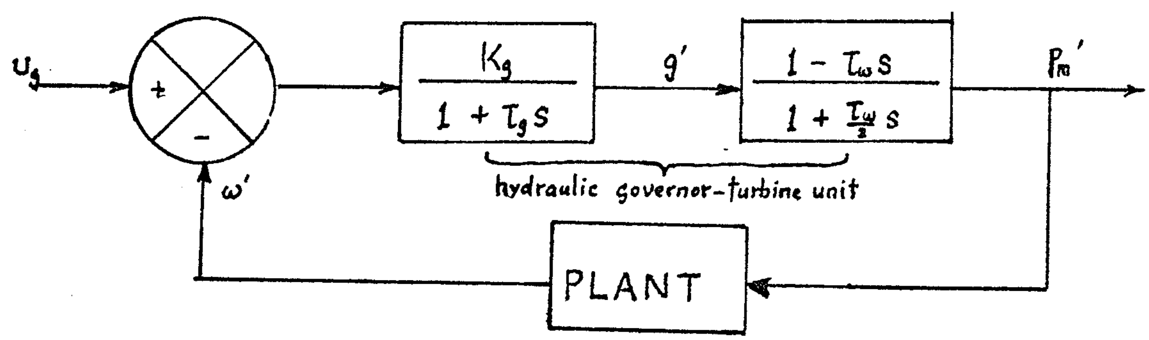


Fig. 2-3

The step response of the hydraulic governor-turbine unit is as shown:

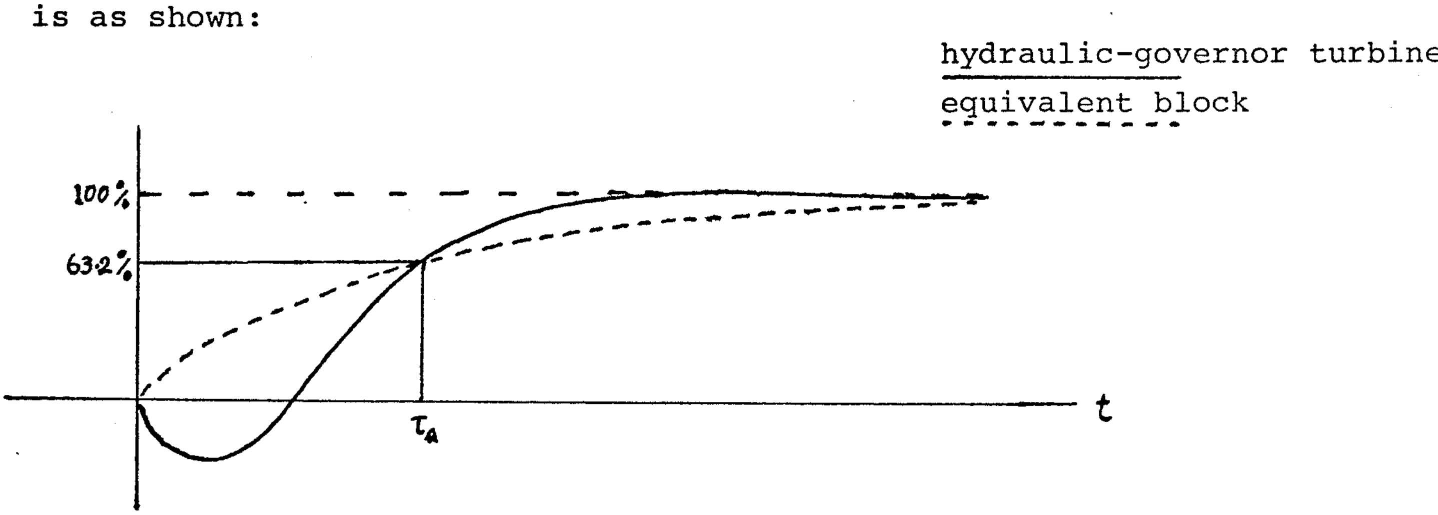


Fig. 2-4 Step response of hydraulic governor-turbine unit

To reduce the order of the system, the governor-turbine unit is approximated by a transfer function[*2] $\dfrac{K_g}{1 + \tau_a s}$

τ_a being the time for the governor-turbine unit to reach 63.2% of its step response.

Furthermore, since u_g (optimal control) is a linear combination of state variables, the quantities K_g and $-\omega'$ can be eliminated from the block diagram as they are included in u_g. The simplified system is:

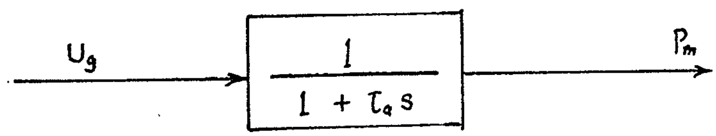


Fig. 2- 5 Approximate governor-turbine unit.

With $\tau_g = 5.5$ sec., $\tau_\omega = 2.43$ sec., the equivalent

$$\tau_a = 9.0 \text{ sec.}$$

The block diagram and the uncontrolled response of the whole system are shown in Fig. 2-6 and Fig. 2-7 respectively.

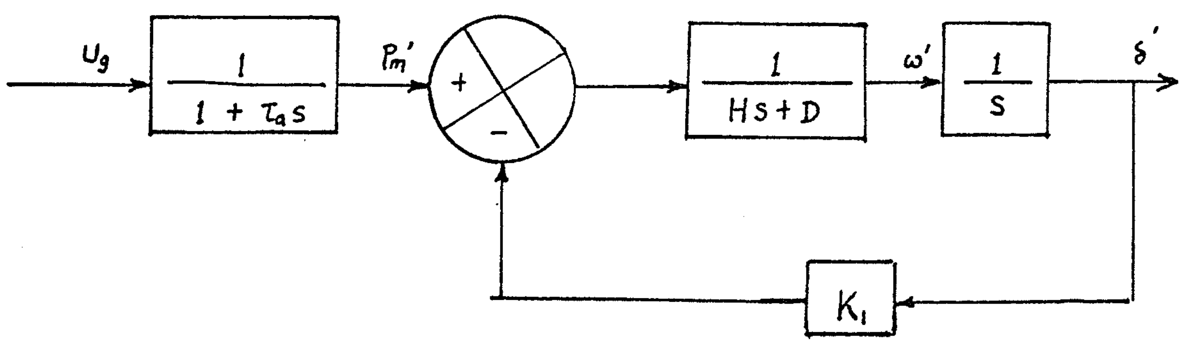


Fig. 2-6 , Third order system.

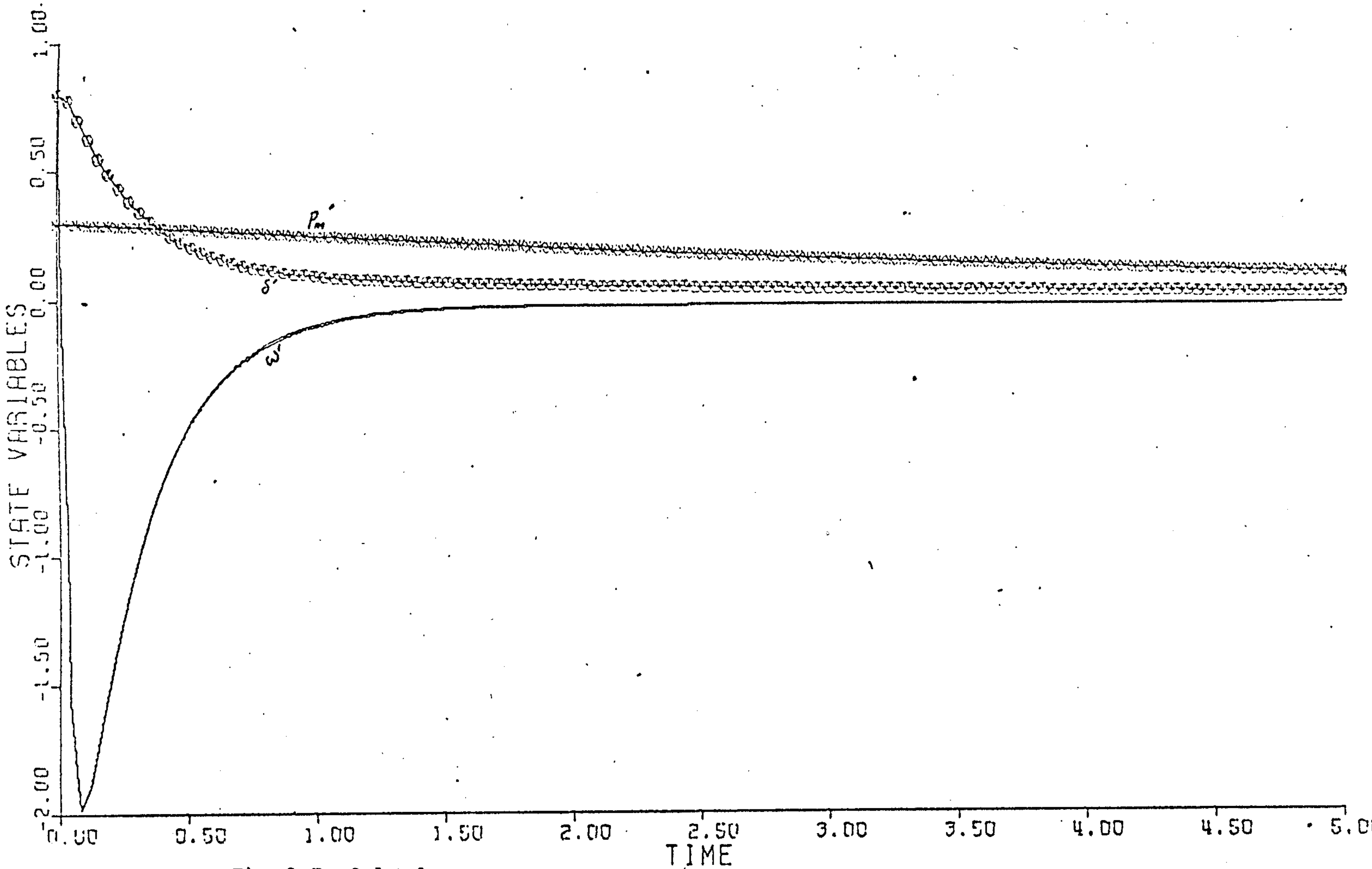


Fig. 2-7. 3rd order system.

We can also take into account the transient electromagnetic effects of the machine by including the field flux linkage ψ_f. The model becomes

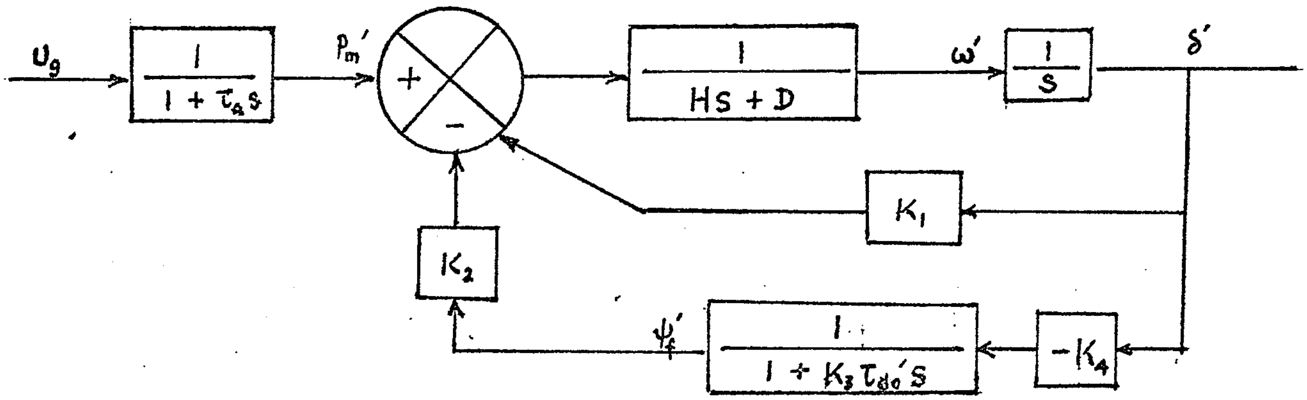


Fig. 2-8 4th order system.

K_1 is as before

$$K_3 = \frac{x_d'}{x_d}$$

$$K_4 = + V_o \sin\delta_o \frac{(x_d - x_d')\tau_{do}'}{x_d}$$

$$K_2 = \frac{V_o \sin\delta_o}{x_d' \tau_{do}'}$$

$$\tau_{do}' = 6.21 \text{ sec.}$$

This model and the constants are modifications of the Heffron-Phillips models.*3

With an initial disturbance of $\delta'(0) = 0.8$, $\omega'(0) = 0.9$, $\psi_f'(0) = 0.5$, $P_m'(0) = +0.3$, the system response is shown in Fig. 2-9.

Finally, to represent the system more accurately, the effect of the field voltage is included. This produces a 5th order system:

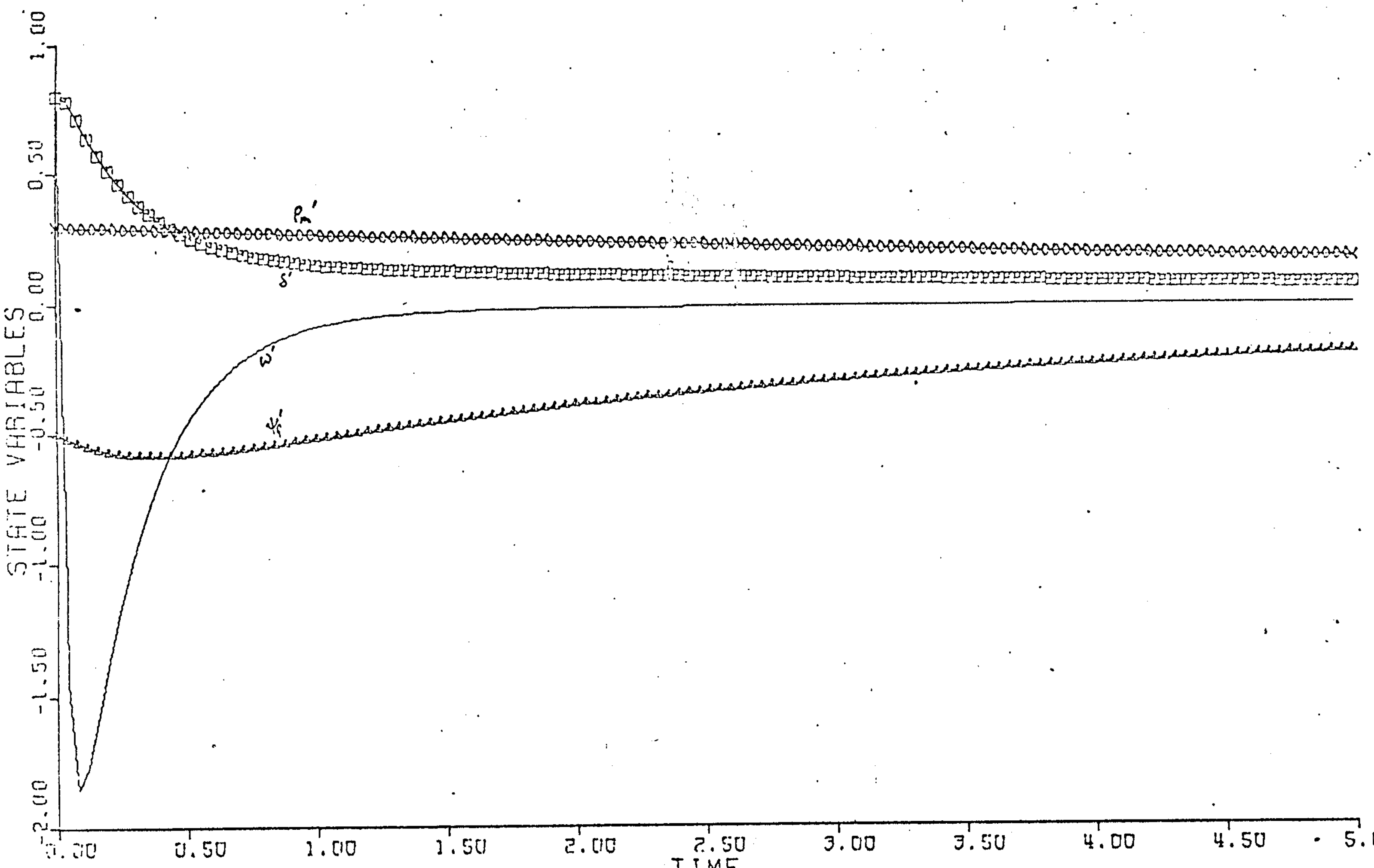


Fig. 2-9. 4th order system (uncontrolled)

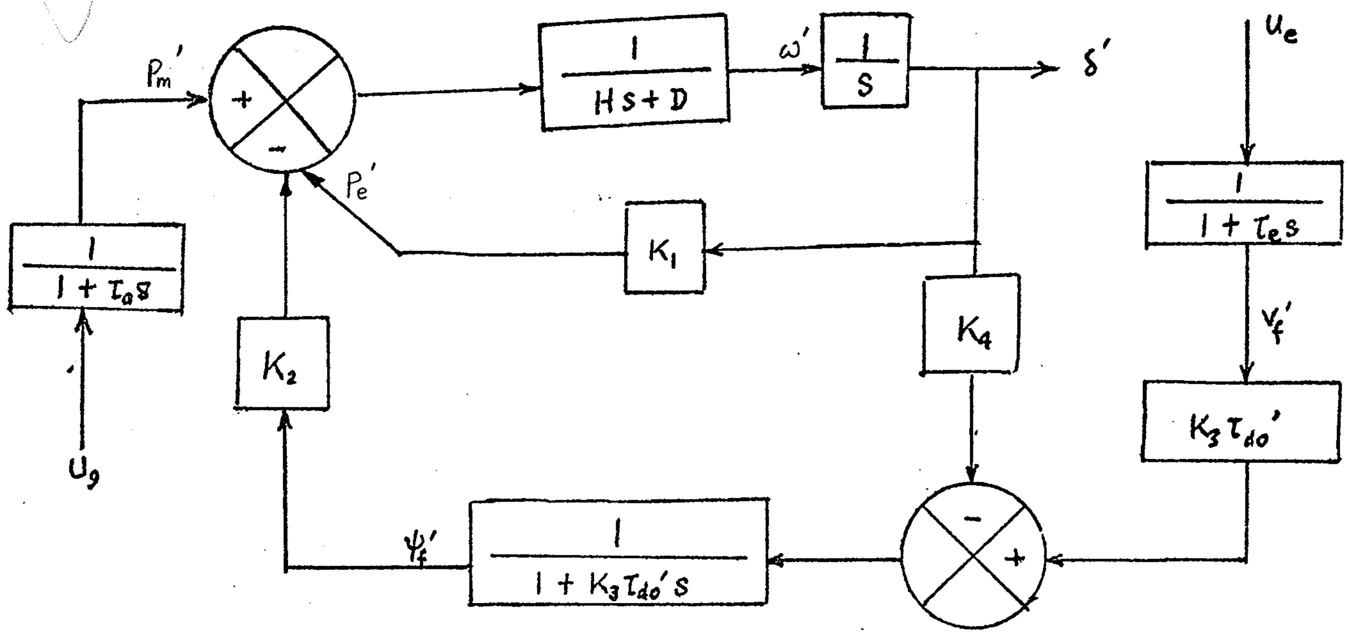


Fig. 2-10 5th order system.

Again, as u_e is a linear combination of the state variables there is no need to feed any other signal to the voltage regulator.

$\tau_e = 0.05$ sec.

With initial disturbances $\delta'(0) = 0.8$ pu, $\omega'(0) = 0.9$ pu, $\psi_f'(0) = -0.5$ pu, $V'_f(0) = -0.8$ pu, $P_m'(0) = 0.3$ pu, the system response is shown in Fig. 2.11 .

This model, which includes all the basic dynamics of the synchronous generator, will be used in the following text of this thesis. However, it is worth noting that the simpler models give a fairly good picture of the dynamics of the two most important parameters, namely δ' (power angle deviation which is associated with the power generated) and ω' (frequency deviation).

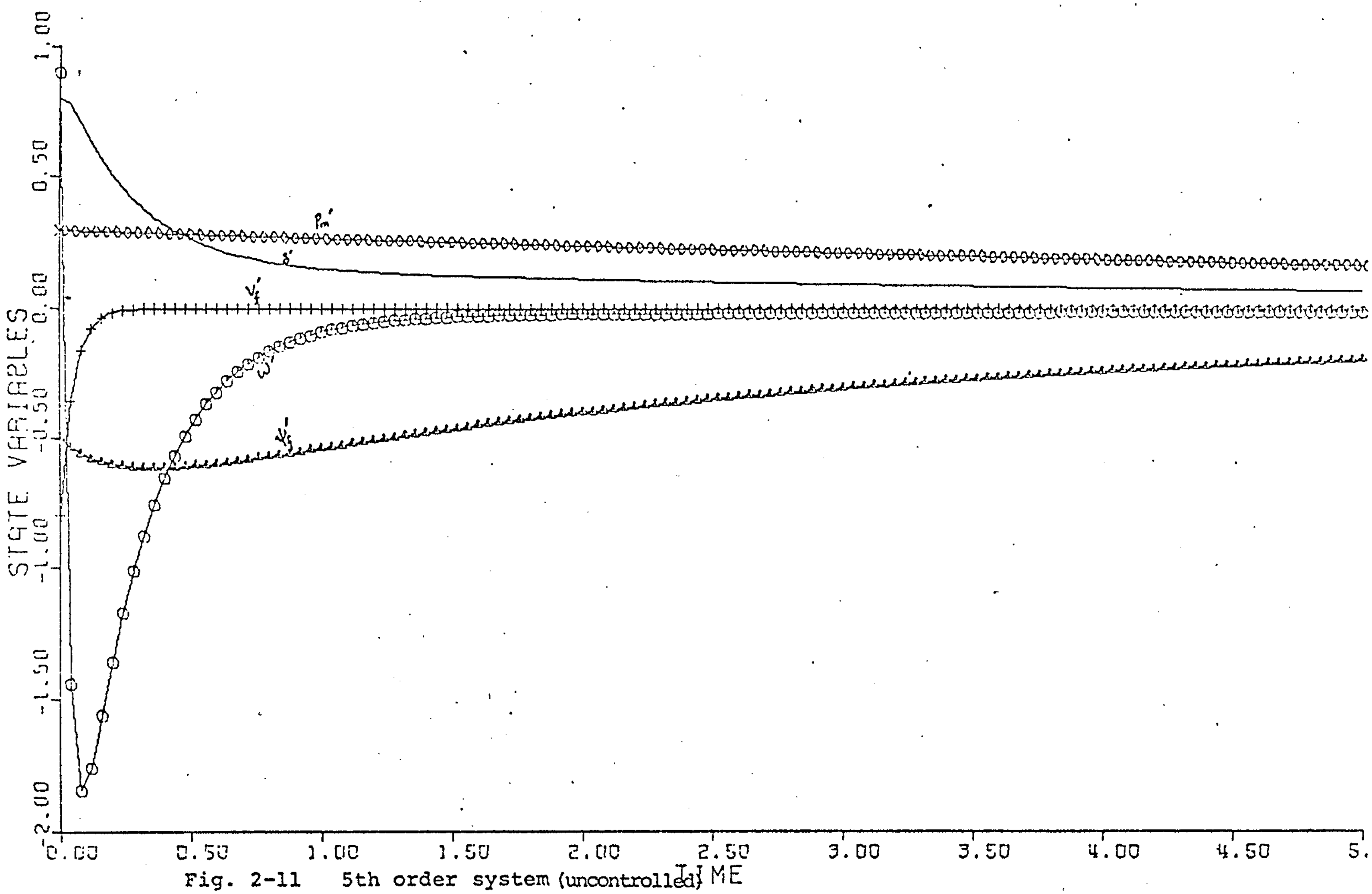


Fig. 2-11 5th order system (uncontrolled)

III. THE LINEAR REGULATOR PROBLEM

In optimal control of a system, a performance index must be defined. The optimal control is then defined as the control function that minimizes the performance index. If a system is in the state-variable form

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

and a performance index is defined as

$$J = \tfrac{1}{2} \int_{o}^{T} (\underline{x}^{T} Q\underline{x} + \underline{u}^{T}R\underline{u})\ dt$$

which is known as the quadratic-form cost functional, then the optimal control will be a linear combination of the states. This is known as the feedback form

$$\underline{u}^{*} = R^{-1} B^{T} K \underline{x} \qquad \text{where } K \text{ is some matrix.}$$

The problem of finding $\underline{u}^{*}$, the optimal control, is called the linear regulator problem. The derivation of these equations can be found in standard texts on optimal control theory.*4

The power system considered in this paper is represented by the 5th order differential equation

$$\begin{bmatrix} \dot{\delta}' \\ \dot{\omega}' \\ \dot{\psi}'_f \\ \dot{V}'_f \\ \dot{Pm}' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & \frac{1}{h} \\ a_{31} & 0 & a_{33} & 1 & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_e} & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{\tau_a} \end{bmatrix} \begin{bmatrix} \delta' \\ \omega' \\ \psi' \\ V'_f \\ Pm' \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\tau_e} & 0 \\ 0 & \frac{1}{\tau_g} \end{bmatrix} \begin{bmatrix} u_e \\ u_g \end{bmatrix}$$

where

$$a_{21} = \frac{-1}{H}\left(\frac{V_o^2 \cos \delta_o}{x_d'} + \frac{V_o^2 \cos 2\delta_o \; (x_d' - x_q)}{x_d' \; x_q'}\right)$$

$$a_{22} = \frac{-D}{H}$$

$$a_{23} = \frac{-V_o \sin \delta_o}{H \; x_d' \; \tau_{do}'}$$

$$a_{31} = \frac{-V_o \sin \delta_o \; (x_d - x_d')}{x_d'}$$

$$a_{33} = \frac{-x_d}{x_d' \; \tau_{do}'}$$

The state variables, all measured in per unit quantities, are deviations of the parameters from normal operating point after a disturbance. The performance index is chosen to be

$$\int_o^\infty \left(\underline{x}^T Q\underline{x} + \underline{u}^T R \, \underline{u}\right) dt$$

where

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

This performance index penalizes the excursions of the state variables after a disturbance with emphasis on δ' and ω'. The penalty on the control vector is required to make the control system practical because otherwise, the control vector will have an infinite magnitude.

The choice of the relative weights of the penalties on the state variables is arbitrary. In fact, some research has been done in how to choose the "best" or optimal Q so that an optimal "optimal control" can be found.[*5] With the upper time limit $T = \infty$ the problem is significantly simplified. An important equation associated with optimal control theory, The Ricatti Equation, is

$$KA + A^TK + KBR^{-1}B^TK = Q$$

The feedback control $\underline{u}$ is given by

$\underline{u} = R^{-1}B^TK\underline{x}$ where K is the solution of the Ricatti Equation.

With $\underline{u}$ decided, the closed loop system equations become

$$\dot{\underline{x}} = G\underline{x}$$

where $G = A + BR^{-1}B^TK.$

Thus the eigenvalues of the closed loop system G depend upon the selection of Q.

To stabilize the system, the dominant eigenvalue of G is shifted as far left on the complex plane as practically possible. For the eigenvalue shift of an n-th order system, it is found that adjusting the diagonal elements in Q will be enough. The shift is restricted to the real part and to the left.

The actual theory is very involved and details can be found in the reference papers. The algorithm can be summarized as follows:

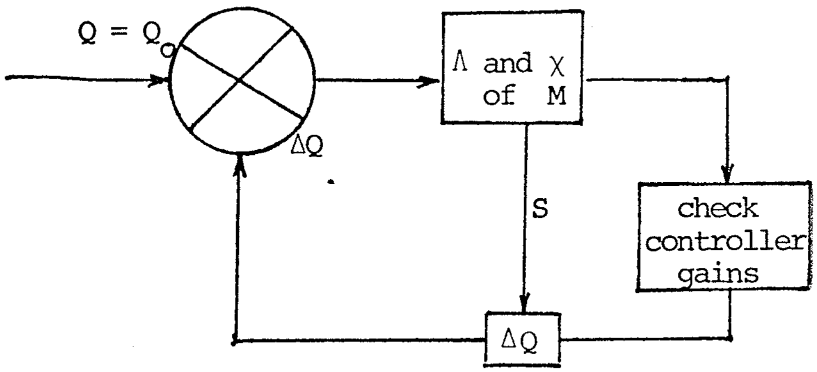
1. Start with a small arbitrary Q.
2. Find the eigenvalues Λ and eigenvectors χ of the matrix

$$M = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}$$

3. Calculate K from the stable eigenvectors of χ[*20] to check the controller gains at each shift.

4. Find ΔQ from the sensitivity coefficient $\frac{\Delta\lambda}{\Delta Q}$.
5. Update Q and repeat the process until a satisfactory eigenvalue shift is made or until the practical controller's limit is reached.

Schematic of operation:



where $S = \text{Real}\ (\lambda, q)$ (sensitivity matrix)

Fig. 3-1. Finding Optimal Q.

The optimal control for a linear regulator problem is

$$\underline{u}^* = R^{-1} B^T K \underline{x}$$

The only unknown is K which is a matrix of the same order as the system. It can be shown that K satisfies the Riccatti Equation*4

$$-\dot{K} = KA + A^T K + KBR^{-1} B^T K - Q\ .$$

If $T = \infty$, K will be a constant for all finite time ($\dot{K} = 0$ when $T \to \infty$) and the Ricatti Equation is simplified to

$$F(K) = KA + A^T K + KBR^{-1} B^T K - Q = 0\ .$$

The solution of this Ricatti Equation is not easy. In fact, for high order systems, it is impossible to solve analytically. With the help of high speed digital computers, several schemes of solving this equation are available:

1) Iterative method based on Kleinman's iteration scheme[*7]. Given an initial estimate K_o, a sequence K_i, $i = 1, 2, 3, \ldots$ is determined by solving the Lyapunov's Equation

$$K_i\ (A - BB^T K_{i-1}) + (A - BB^T K_{i-1})^T K_i + K_{i-1} BB^T K_{i-1} + Q = 0 \quad .$$

The solution requires a number of multiplications and additions which grow as n^3 or n^4 depending on the method used to solve the Lyapunov's Equation.

2) Iterative method based on finite difference approximation

$$K_i = K_{i-1} - \alpha F\ (K_{i-1})$$

The parameter α must be judiciously chosen to avoid instability while yielding K in a reasonable time. Computations grow as n^3.

3) Direct method based on spectral factorization approach.[*8] A matrix

$$W = \begin{bmatrix} A & -BB^T \\ -Q & -A^T \end{bmatrix}$$

is formed and the solutions of the Ricatti Equation can be constructed from the eigenvectors of W. This method suffers from the need to determine eigenvectors of a $2n \times 2n$ matrix which can be tedious when n is large.

4) Direct analog simulation:

The Ricatti Equation can be written out as $\frac{n^2 + n}{2}$ simultaneous first order quadratic differential equations which can be solved on the analog computer. This solution can also be done on the digital computer numerically using the C S M P program.[*9]

The solution is obtained directly using reverse time as the elements of K are constants except near the final time.

The last method is a powerful one. But in this thesis another numerical method is tried.

It is known that for a real analytic function F(x) the solution of the equation $F(x) = 0$ can be obtained from the algorithm

$$x_i = x_{i-1} - \frac{F_{i-1}}{F'_{i-1}}$$

where x_o is chosen arbitrarily. This is illustrated in Fig. 3.2

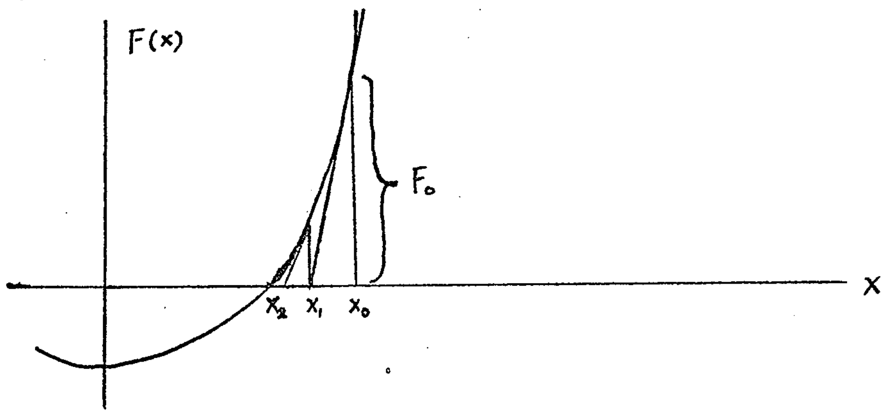


Fig. 3-2. Iterative algorithm.

This iteration scheme is used in solving our Ricatti Equation here.

The method uses the algorithm

$$x_i = x_{i-1} - \frac{f_{i-1}}{f'_{i-1}} \quad .$$

In our case, the variable is a matrix and F is also a matrix

$$K_i = K_{i-1} - \frac{F(K_{i-1})}{F'(K_{i-1})} \qquad .$$

The expressions in this equation are not strictly mathematically valid, and need to be defined.

First $F'(K) = \frac{dF}{dK}$:

ordinary definition of a derivative is:

$$\frac{df}{dx} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon) - f(x)}{\varepsilon}$$

K consists of $n \times n$ elements the change of each of which affects K and hence $F(K)$. Recall the chain rule for single-valued functions

$$df(u,v) = \frac{\partial f}{\partial u}\,du + \frac{\partial f}{\partial v}\,dv \quad .$$

But K is not exactly a function of $n \times n$ variables. The only reasonable way to represent this relationship of $\frac{dF}{dK}$ where K consists of $n \times n$ elements is:

$$n \updownarrow \underbrace{\begin{bmatrix} \frac{\partial F}{\partial k_{11}} & \frac{\partial F}{\partial k_{12}} & \frac{\partial F}{\partial k_{13}} & \cdots & \\ \frac{\partial F}{\partial k_{21}} & \frac{\partial F}{\partial k_{22}} & & & \\ \frac{\partial F}{\partial k_{31}} & & & & \\ \vdots & & & & \\ & & & & \frac{\partial F}{\partial k_{nn}} \end{bmatrix}}_{\longleftarrow\; n \;\longrightarrow} = \frac{\partial F}{\partial K}$$

Each element $\frac{\partial F}{\partial k_{ij}}$ is an $n \times n$ matrix defined as

$$\lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \left[F(K)_{ij} - F(K) \right]$$

where $F(K)_{ij}$ means the i-j th element of K is increased by ε while the rest of the elements are unchanged and F is computed of this new K.

Hence we have defined $\frac{\partial F}{\partial K} = F'(K)$.

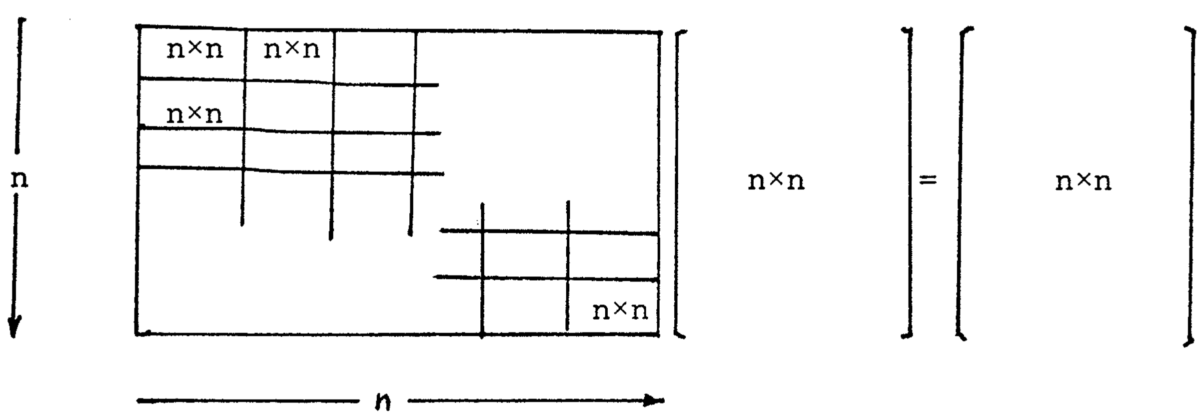
The next step is to define and calculate $\frac{F(K)}{F'(K)}$. Treat $F'(K)$ as an $n \times n$ matrix (each element of which is an $n \times n$ matrix).

$$\frac{F(K)}{F'(K)} = F(K) \ (F'(K))^{-1} = D(K)\,(n \times n),$$

$$F'(K) \quad D(K) \quad = \quad F(K) .$$

In the computer storage, an $n \times n$ matrix A is stored as a list of numbers in the order a_{11} , a_{21} , a_{31} , ..., a_{n1}, a_{12} , a_{22} , ..., a_{nn} .* Therefore the mathematical operation of the solution of the equation

$F'(K)\ D(K) = F(K)$ which in matrix form is



* In PL-1, the order is different, but the principle is the same.

is equivalent to

$$n^2 \downarrow \underbrace{\begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}}_{n^2 \longrightarrow} \begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \\ \vdots \\ d_{n1} \\ d_{12} \\ \vdots \\ d_{nn} \end{bmatrix} = \begin{bmatrix} f_{11} \\ \vdots \\ \\ \vdots \\ f_{nn} \end{bmatrix} \downarrow n^2$$

Care has to be taken in "stretching" this out, but if the proper terms are multiplied together, the two operations are identical. The second operation is of the standard form

$$A\underline{x} = \underline{b}$$

and the n^2 vector $\underline{d}$ can be solved readily using standard matrix routines. Then it is converted back to an $n \times n$ matrix $D(K)$.

The iteration scheme

$$K_i = K_{i-1} - \frac{F(K_{i-1})}{F'(K_{i-1})}$$

$$= K_{i-1} - D(K_{i-1})$$

can now be performed.

Using this scheme, namely,

$$K_i = K_{i-1} - \frac{F(K_{i-1})}{F'(K_{i-1})}$$

the solution of the Ricatti Equation is obtained very rapidly. The solution is checked with those obtained using the finite difference method (method 2) and CSMP program (method 4) and this method is much faster with virtually no error. (The exit criterion is set to be 0.001 for each element of $F(K)$).

For a 5th order system,

CSMP -- 9.36 unit CPU

Matrix Derivative -- 1.93 unit.

For a 3rd order system, the matrix iteration finite difference method takes 124 iterations to arrive at the solution which the matrix derivative method takes 8 iterations to get to with the same accuracy.

However, powerful as this method is, the theoretical grounds are not well founded. The method may theoretically give an answer that is not the optimal control, although with a K_o chosen to be $-I$, the negative identity matrix, this has not happened. It is not the intention of this paper to investigate into details of this method, but it is certainly a point of interest both from the control and numerical analysis point of view.

It is of interest, therefore, to see what effect the starting value K_o will have on this scheme. In Chapter IV we solve for the optimal control of the 5th order system with $K_o = -I$. The same data are used using $K_o = \Phi$ (null matrix) and $K_o = I$ (identity matrix). The results are of interest. With $K_o = \Phi$, the result which is identical with that using $K_o = -I$ is obtained in 4 iterations as compared to 8 using $K_o = -I$. But with $K_o = I$, the algorithm does not converge even though corrective measures are applied in the program and the algorithm ends up in an oscillation. This is intuitively reasonable as $\underline{u} = R^{-1} BK\underline{x}$, and a positive K implies a positive feedback which is physically unstable. On the other hand, $K_o = \Phi$ and $-I$ do not lead to this situation.

IV. OPTIMAL CONTROL OF THE SYNCHRONOUS GENERATOR

A set of data was obtained from the Grand Rapids Station* of the Manitoba Hydro and used in our 5th order model. The following state equation was obtained.

$$\begin{bmatrix} \dot{\delta}' \\ \dot{\omega}' \\ \dot{\psi}_f' \\ \dot{V}_f' \\ \dot{P}_m' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -118.0431 & -35.8774 & -10.696 & 0 & 34.169 \\ -0.9622 & 0 & -0.5576 & 1 & 0 \\ 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & -0.1111 \end{bmatrix} \begin{bmatrix} \delta' \\ \omega' \\ \psi_f' \\ V_f' \\ P_m' \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 0.1111 \end{bmatrix} \begin{bmatrix} u_e \\ u_g \end{bmatrix}$$

The algebraic Ricatti Equation

$$F(K) = KA + A^T K + KBR^{-1}B^T K - Q = 0$$

was solved using the matrix derivative method up to an accuracy of less than 0.00001 error in each entry of the matrix $F(K)$.

The resultant K, obtained in 8 iterations is:

* Table II.

$$K = \begin{bmatrix} -17.7345 & -0.0332 & -0.8615 & -0.0283 & 3.9004 \\ -0.0332 & -0.1403 & 0.0176 & -0.0002 & -0.0248 \\ -0.8615 & 0.0176 & -0.8946 & -0.0305 & 0.7326 \\ -0.0283 & -0.0002 & -0.0305 & -0.0218 & 0.0251 \\ 3.9004 & -0.0248 & 0.7326 & 0.0251 & -7.6068 \end{bmatrix}$$

The optimal control is $\underline{u}^* = -B^T R^{-1} K\underline{x}$

$$= -\begin{bmatrix} 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0.1111 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} K\underline{x} = \begin{bmatrix} u_e \\ u_g \end{bmatrix}$$

$$u_e = 20 * (-0.0283\delta' - 0.0002\omega' - 0.0305\psi_f' - 0.0218V_f' + 0.0251P_m')$$

$$u_g = 0.1111 * (3.9004\delta' - 0.0248\omega' + 0.7326\psi_f' + 0.0251V_f' - 7.6968P_m')$$

The system is disturbed to give an initial disturbance

$$\underline{x}'(0) = [0.8 \quad 0.9 \quad -0.5 \quad -0.8 \quad 0.3]^T .$$

The uncontrolled response was shown in Fig. 2-11. If the optimal control is applied, the response will be as shown in Fig. 4-2, 4-3, 4-4, 4-5, 4-6.

It can be seen by comparing Fig. 4-1 and Fig. 2-11 and from Fig. 4-2 through Fig. 4-6 that the improvement in stability by applying the optimal control is not too significant. Indeed, in the first 1.9 sec., the performance index of the uncontrolled system is lower than the performance index of the optimally controlled system. After $\tau = 1.9$ sec., the controlled system has a smaller performance index, but the state variables are so close to zero in both systems that the deviation in performance indices never becomes very large as both curves are rather horizontal (small slope).

This fact is not surprising because the uncontrolled system (which actually has other forms of control, but not an optimal linear feedback control) is one that is in service and therefore must have a high degree of stability. The reason that the optimal control does not show a great effect on the system is due to its large damping coefficient, large time constants of the hydraulic governor-turbine unit and the open circuit time constant of the field.

The control signals u_e and u_g are fed into the governor and voltage regulator. The field voltage v_f responds quickly to u_e but it has no direct effect on other parameters because its effect is felt through ψ_f, the field flux linkage. Due to the large time constants of both the governor-turbine unit (equivalent time constant $\tau_a = 9$ sec.) and the machine (open-circuit field constant $\tau_{do} = 6.21$ sec.) the effect of the control vector $\underline{u}^* = [\, u_e \;\; u_g]^T$ is not felt until a certain length of time has elapsed. But long before this, in approximately 2 seconds, the oscillations are damped out by the large damping coefficient of the system, as shown in Fig. 2-11, and by the time the control signals become effective, the system has no more oscillations (practically) so the control seems redundant.

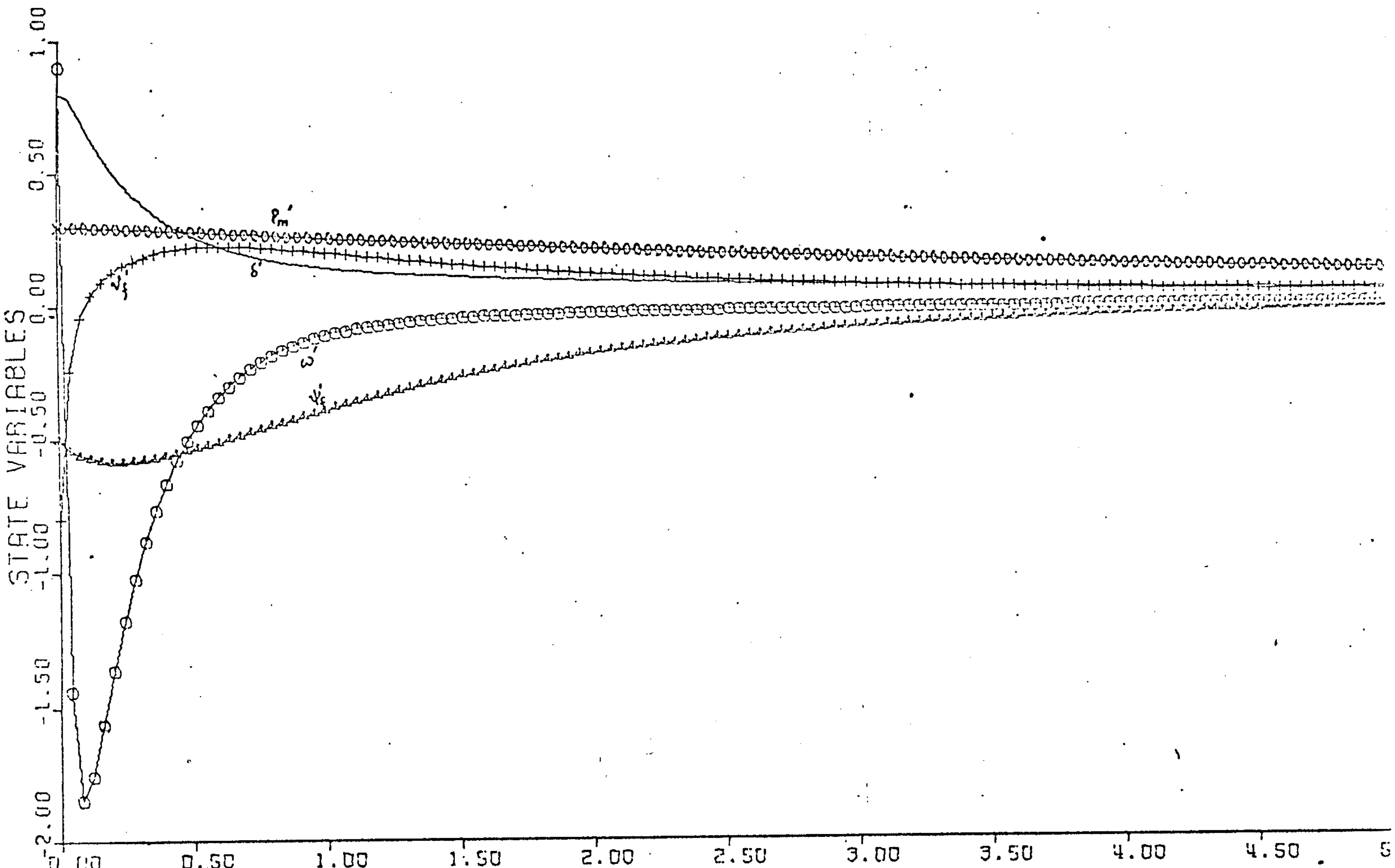


Fig. 4-1 Optimally controlled system

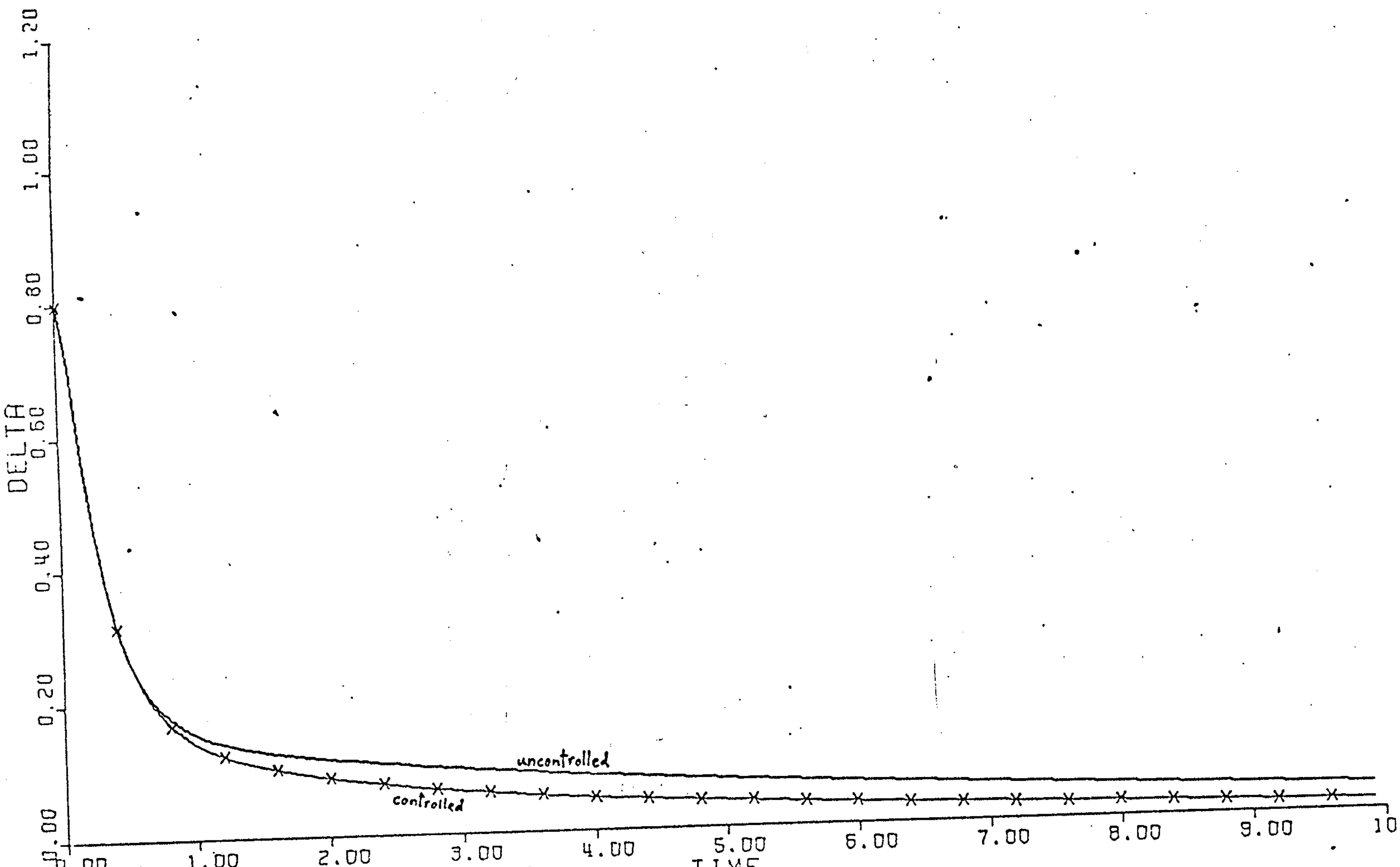


Fig. 4-2. Variation of δ'

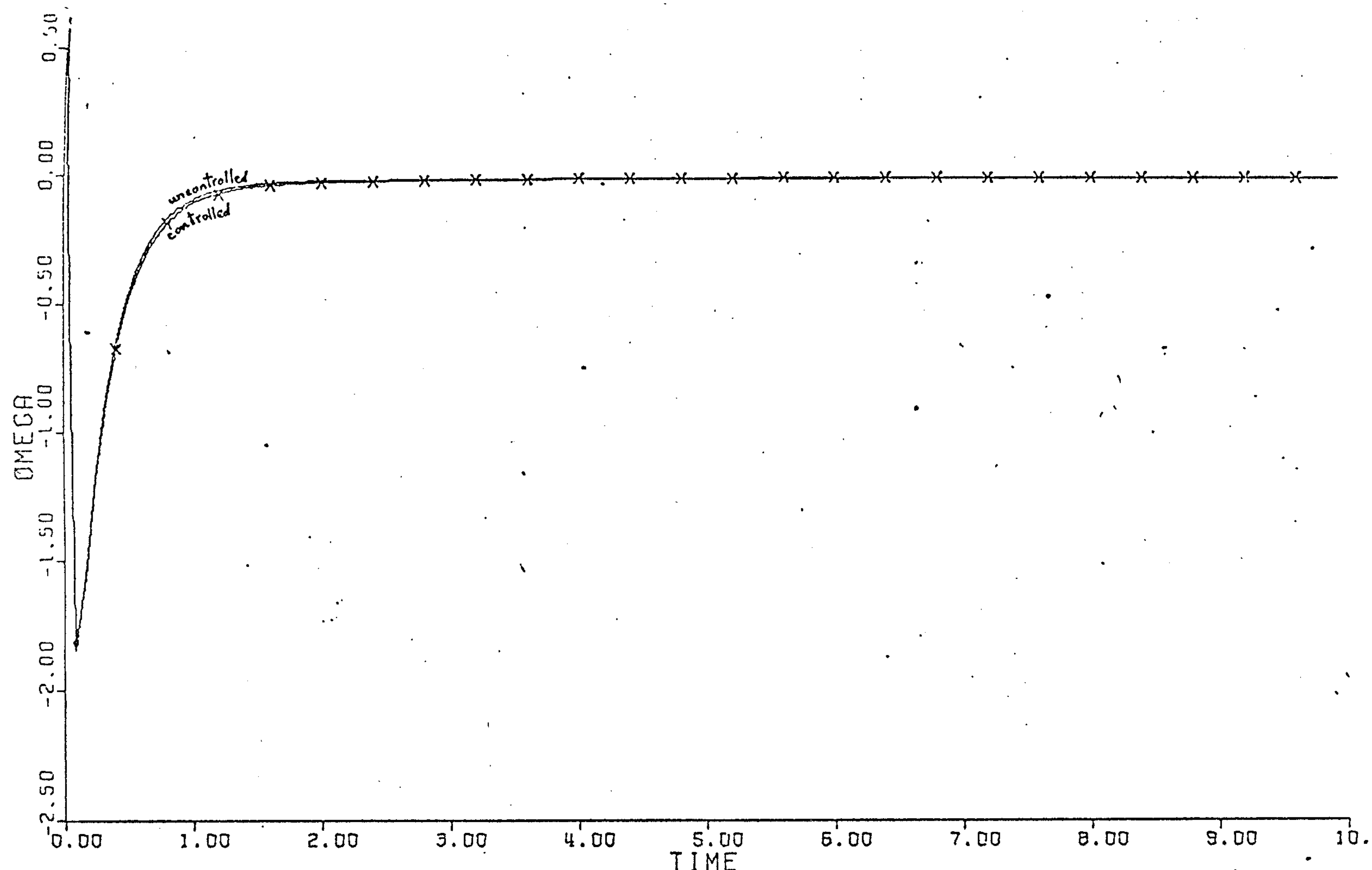


Fig. 4-3. Variation of ω'

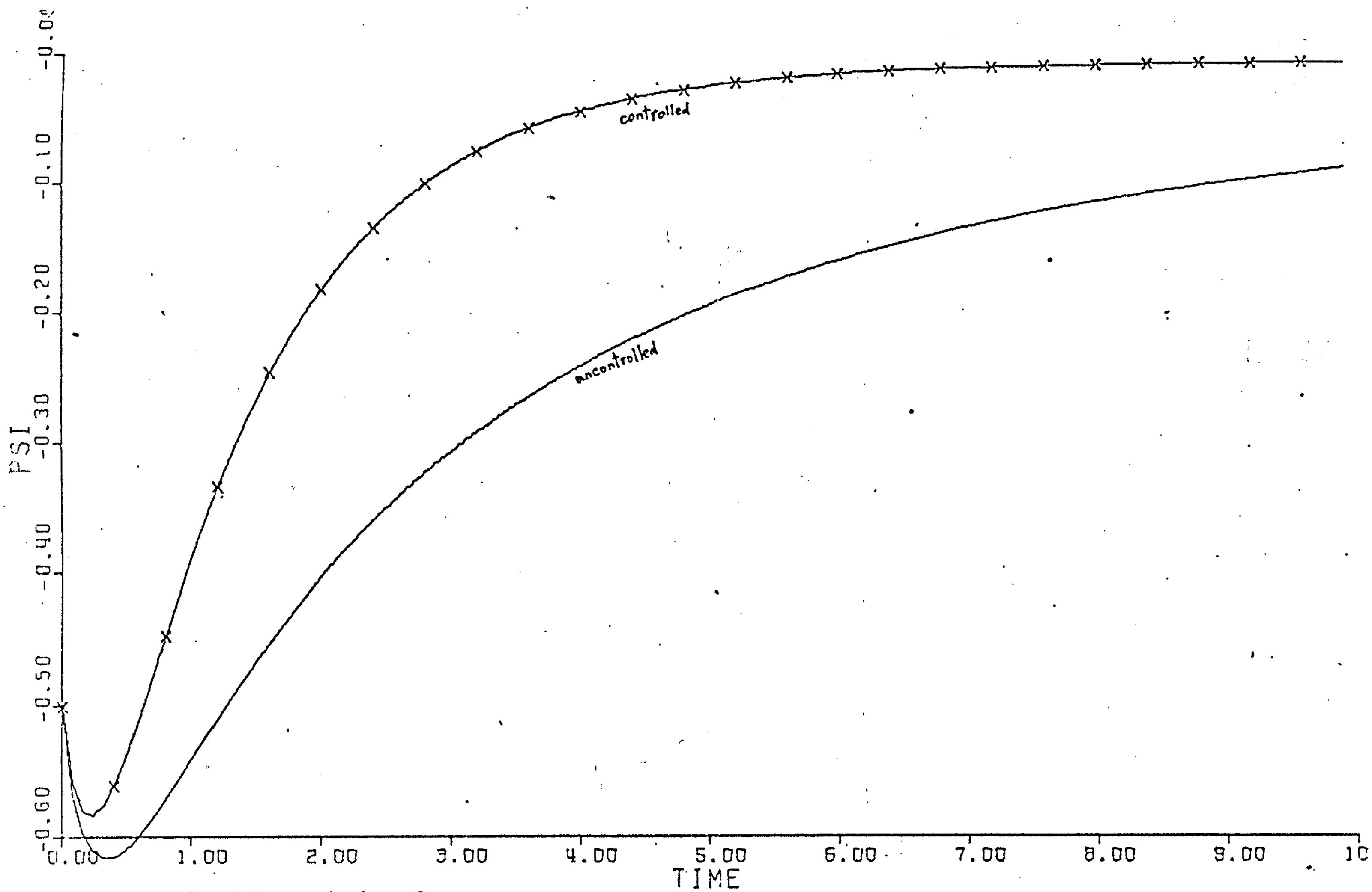


Fig. 4-4. Variation of ψ_f .

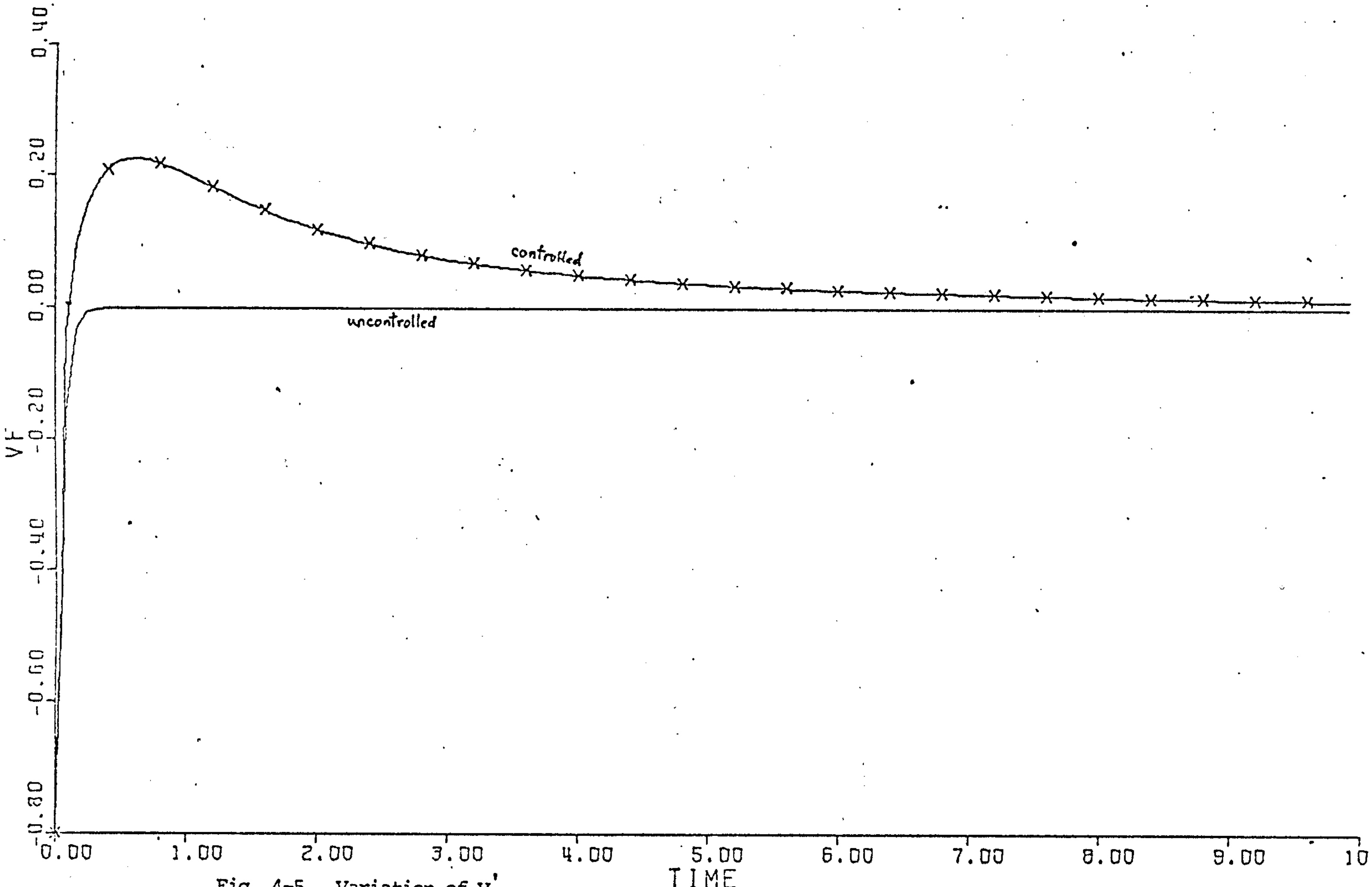


Fig. 4-5. Variation of V_f' .

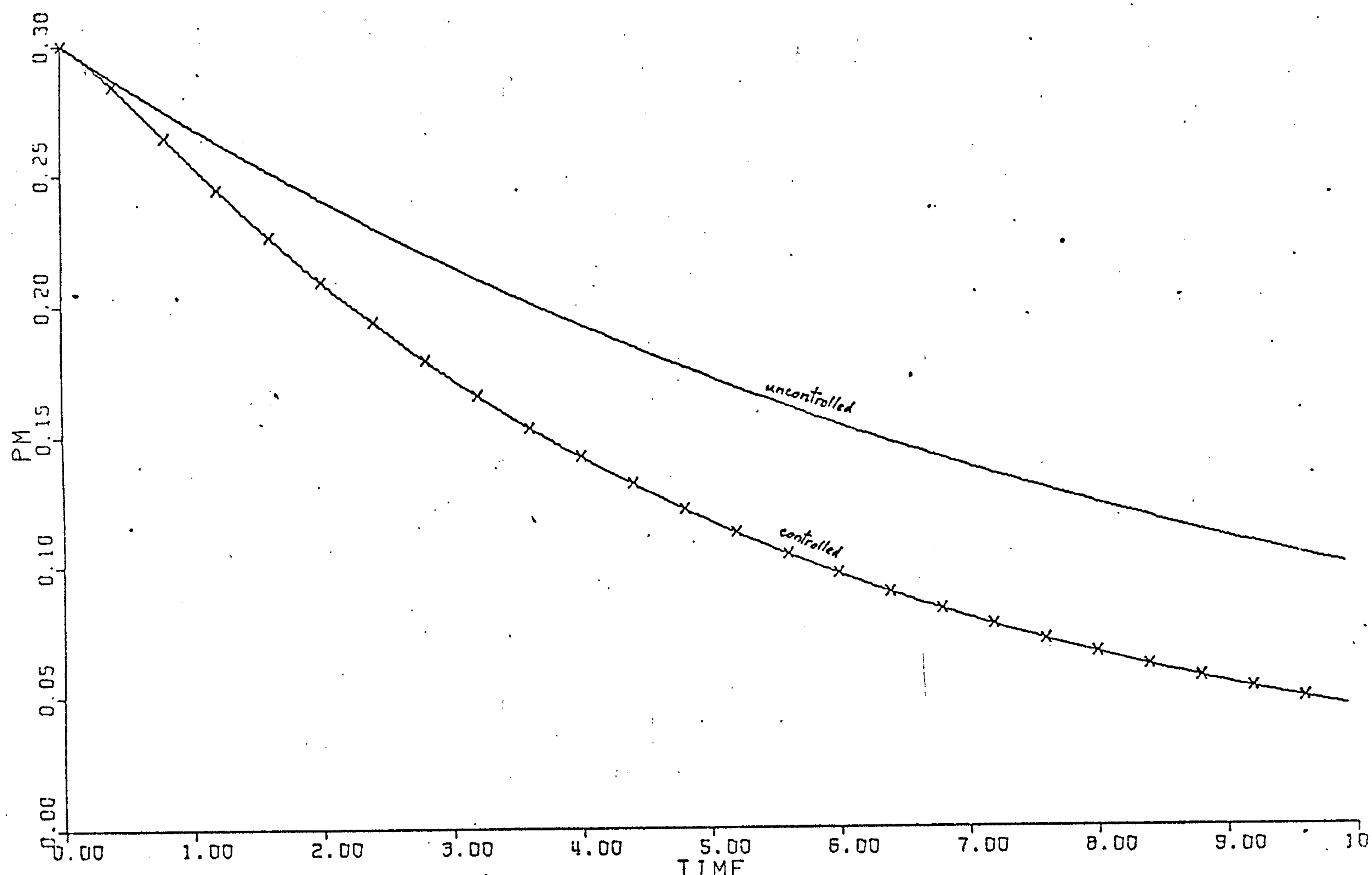


Fig. 4-6. Variation of P_m'

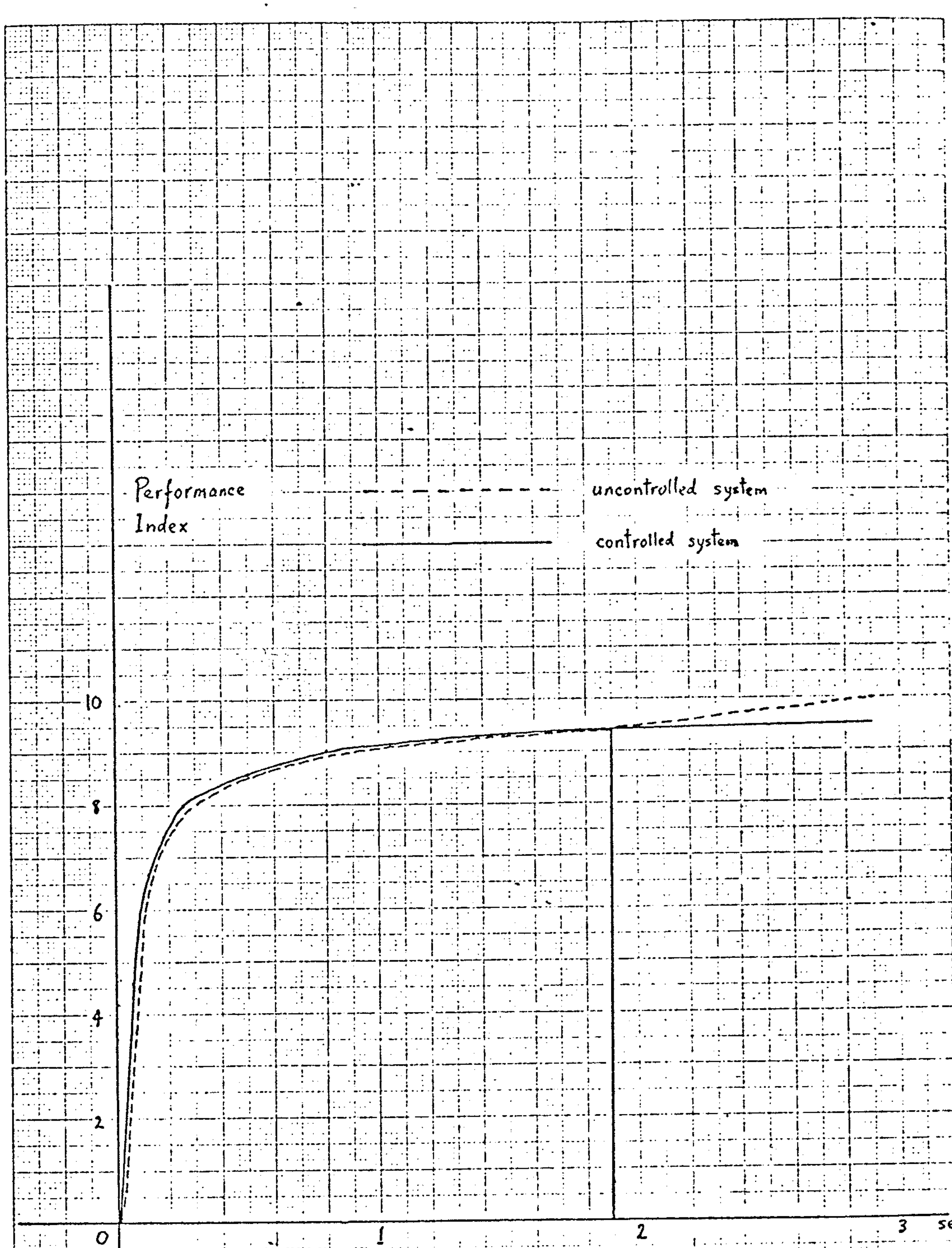


Fig. 4-7 Performance Index

To show that this is actually the case, and not that the "optimal control" is wrong, we repeat the process of calculating the optimal control for a system with $D = \frac{1.05}{115.00}$, instead of $D = 1.05$ and everything else identical, and again plot the re-spnse of the uncontrolled and optimally controlled systems subject to the same initial conditions. The performance indices are also plotted. The great improvement can be noted in Fig. 4-9 to Fig. 4-13. This model will hence be referred to as model A. The optimal control is:

$$u_e = 20\ (-1.2344\ \delta' - 0.02\omega' - 0.138\psi_f' - 0.0254V_f' + 0.3734Pm')$$

$$u_g = 0.1111\ (129.4\delta' - 1.2961\omega' + 12.1897\psi_f' + 0.3734V_f' - 44.1049Pm') \quad .$$

This optimal control which minimizes a cost functional is in continuous linear state feedback form. To be able to put it into practice, one must be able to continuously measure the state variables and transduce them into electrical signals in the case of u_e (u_e, the input to the voltage regulator, is a voltage) or mechanical signals in the case of u_g (u_g, the input to the governor-turbine unit, is a frequency signal).

The power angle deviation δ' can be measured by measuring the power output and the power factor. In fact, there are different kinds of instruments for sensing change in power factor in the power industry being used.

The measurement of frequency is also a routine job in the power system operations. However the accuracy of these measurements still has ample room for improvement.

The direct measurement of the field flux linkage ψ_f is not an easy task. However, according to the Heffron-Phillips model[*3], $V_t' = K_6\psi_f'+K_5\delta'$ where K_5, K_6 are constants $\psi_f' = \frac{V_t'}{K_6} - \frac{K_5}{K_6}\delta'$.

V_t the terminal voltage can be easily determined using a voltmeter, and K_5, K_6 for the system can be determined from experiment.

Measurement of V_f can be done with a voltmeter.

The measurement of the shaft power Pm is probably the hardest. It is possible to derive the mechanical input power signal from the nominal (operating point) value of the electrical power output signal[*11]; or one may try some direct measurement methods . . .

Some research has been done in "observers" which are networks attached to the outputs of the system to transduce the state variables into electrical signals. Since outputs are measurable quantities, the problem of measuring state variables is solved. However, the research is still in development stage, and is beyond the scope of this thesis.

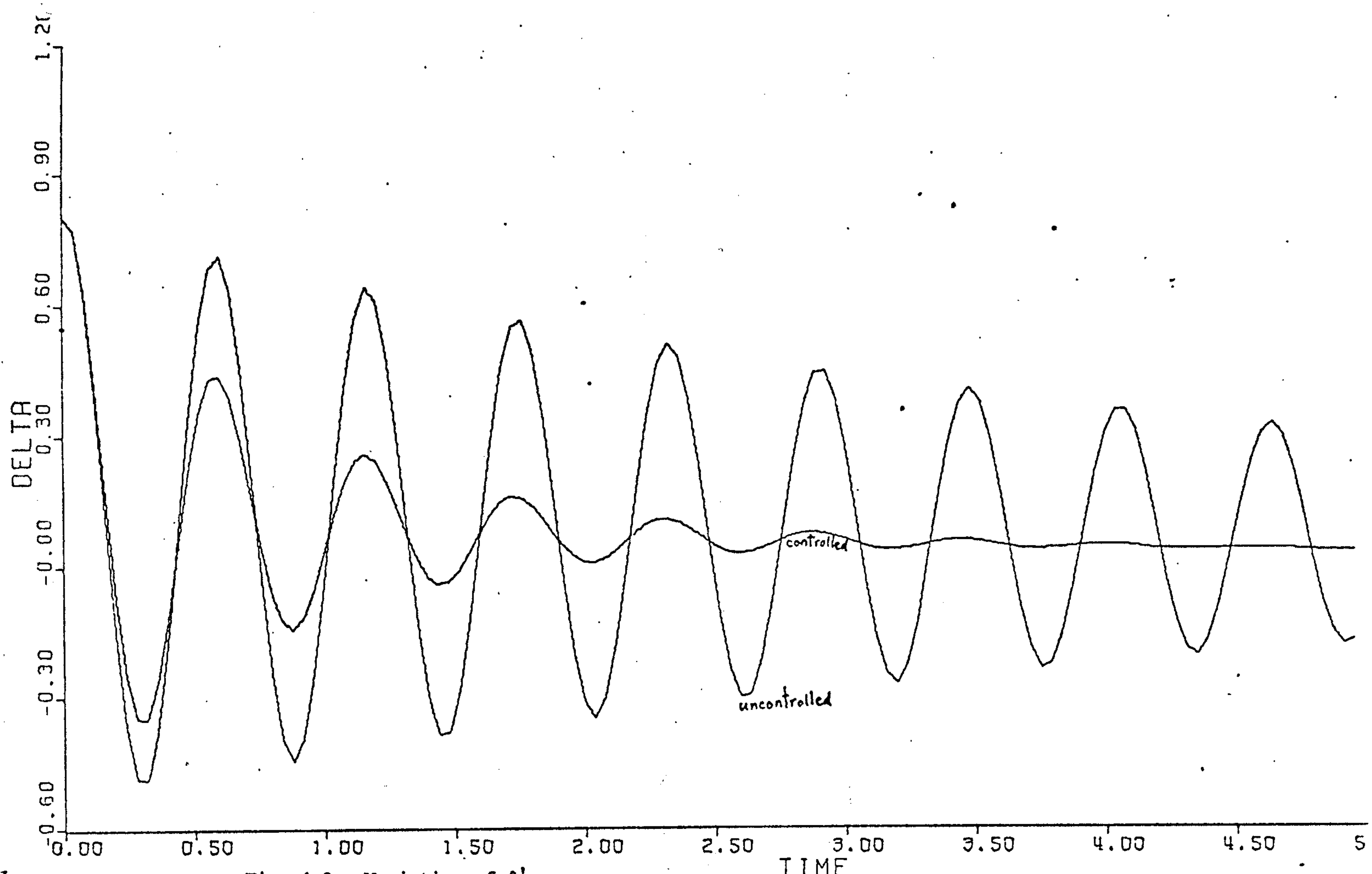


Fig. 4-9. Variation of δ'

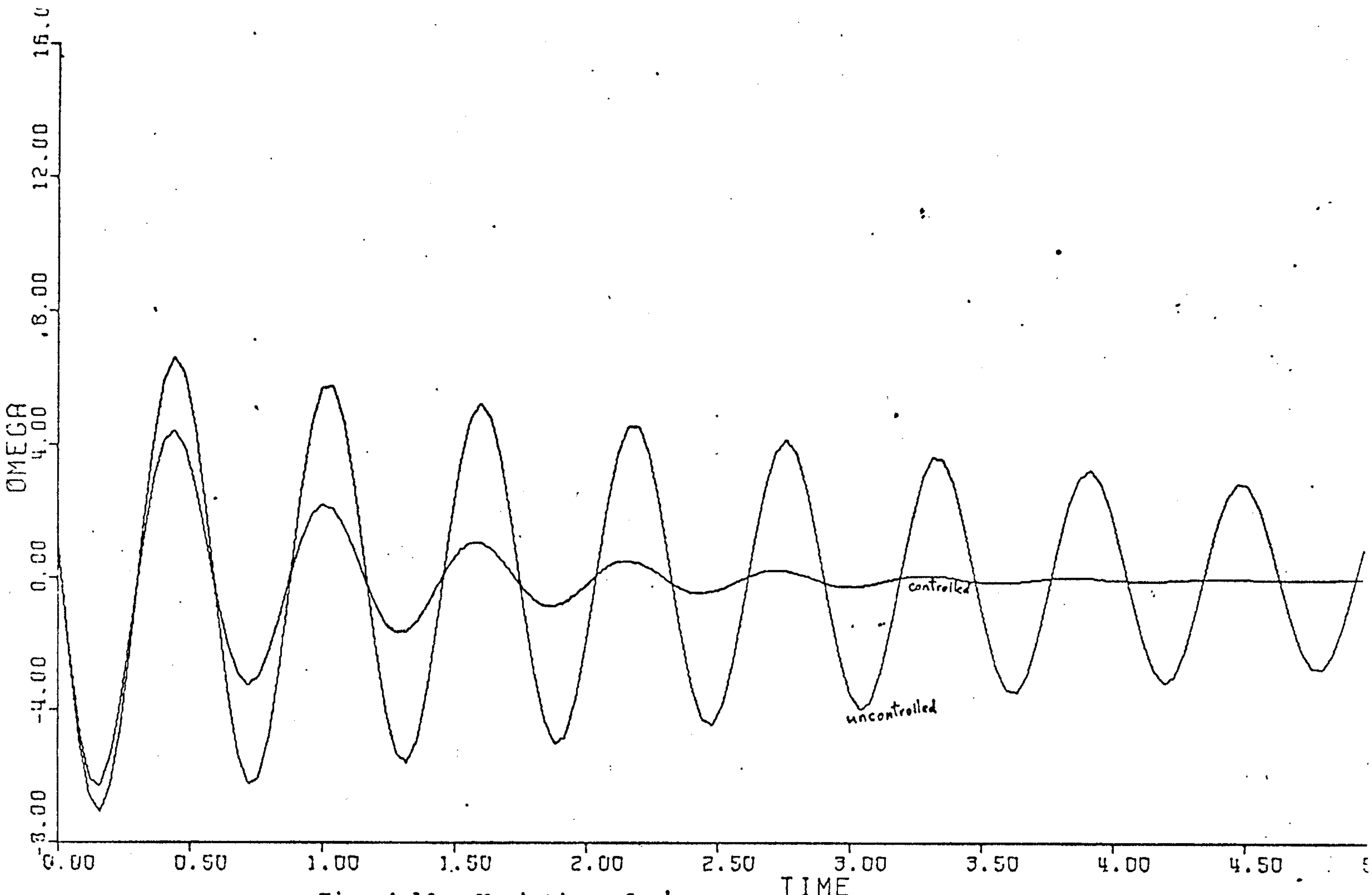


Fig. 4-10. Variation of ω' .

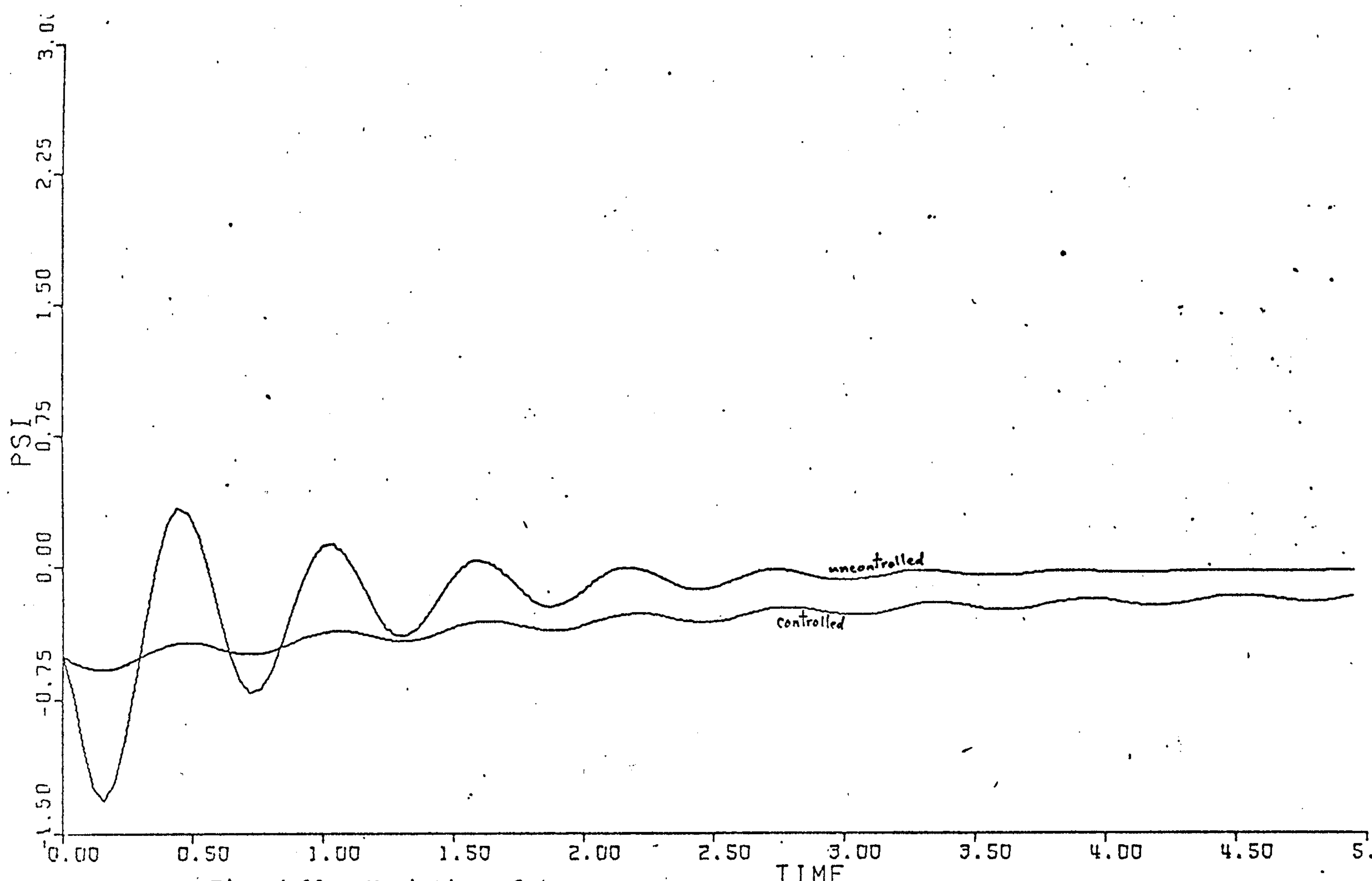


Fig. 4-11. Variation of ψ_f

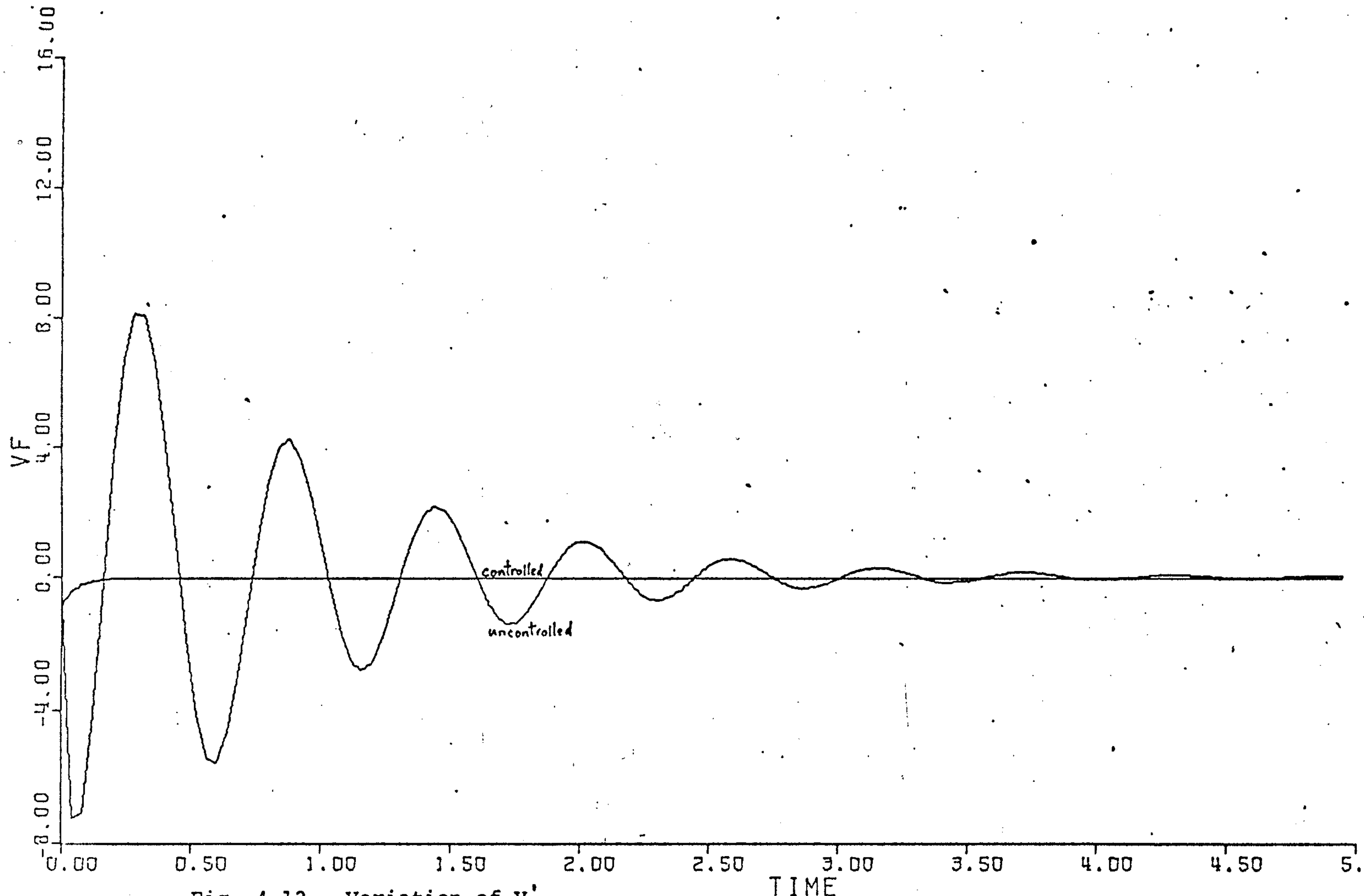


Fig. 4-12. Variation of V_f'.

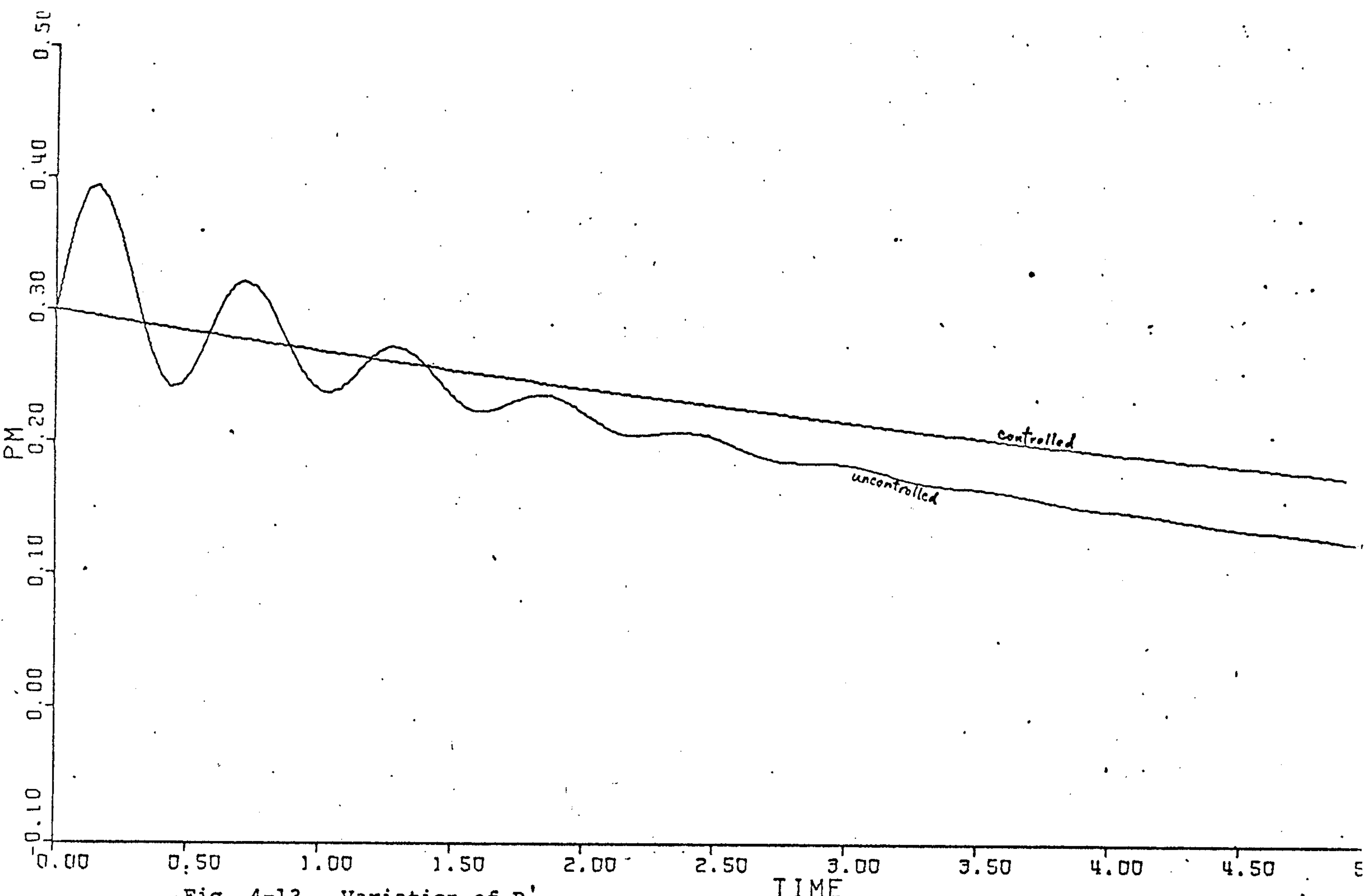


Fig. 4-13. Variation of P_m'

V. SUBOPTIMAL AND ADAPTIVE CONTROL

The difficulty and inaccuracy in the measurement of some state variables make the continuous state feedback control not directly physically realizable. It is more practical to feed back only those states which can readily be measured or some output quantities (such as terminal voltage) which may be a linear combination of some states or nonlinear functions of the states. In the case of feedback output of quantities which are nonlinear functions of states, linearization and approximations have to be made or we would get into involved nonlinear problems beyond the scope of "linear regulator problem" which we are investigating. In the simpler case of feedback of output quantities that are linear combinations of states and the case of partial feedback of states, we are looking into the problem of suboptimal control, because the control we obtained is not optimal according to our original definition.

There are many forms of suboptimal control possible.[*12,10,6] Using the "minimum norm" suboptimal control, it can be shown that[*13] the suboptimal control can be obtained from the optimal control by deleting the terms involving the state variables whose feedback is not desired.

The stability of the suboptimal control is not guaranteed and has to be checked. The performance of this "minimum norm" suboptimal control depends greatly on the proper choice of Q, the weighting matrix in the cost functional. No analytical relationship is known to exist between the weighting coefficients and the behaviour of the machine and the choice of Q is therefore empirical. It is, however, intuitively reasonable to assume that a large weighting coefficient will cause a large feedback of that particular state variable, therefore when the feedback of a state variable is not needed, the weighting coefficient of it should be relatively small.

With our model, we tried the suboptimal control obtained from the optimal control by deleting the feedback of ψ_f, V_f and Pm'. Then we tried suboptimal control on the model A (with small damping coefficient and everything else identical) where the effect of the control is more markedly shown. The responses of the two systems are shown in Fig. 5-1 and Fig. 5-2. The Q matrix in all cases remained the same as previously set.

With Q changed to $Q = \text{diag}\ [10 \quad 10 \quad 0 \quad 0 \quad 0]$ the process was repeated and the responses plotted in Fig. 5-3 through 5-4.

From the figures it can be seen that the application of suboptimal control, whether with the original Q matrix (as in Fig. 5-1 and Fig. 5-2) or with a reduced Q matrix (Fig. 5-3 and Fig 5-4) leads to less stability. In fact, in the case of the lightly damped model A, the system becomes unstable. In this case, the suboptimal control derived from the original Q (penalty on every state variable) is more stable than the suboptimally controlled system with a reduced Q matrix, but this is not conclusive as in these suboptimal-control systems, the choice of Q based on eigenvalue-shifting techniques*5 is of paramount importance.

By reducing the weighting coefficient on the control vectors in the cost functional (i.e. R), a slightly better system is obtained, but the difference is very small as seen in Fig. 5-5.

Another aspect of the optimal control of the synchronous generator and other systems as well is the variation of the operating point and variation of parameters. The operating point will not be varied continuously all the time. It is usually fixed on a few ratings, e.g. 115 MVA, 13-8KV, 0.8 p.f. lagging etc., so it is possible to derive a few sets of control to suit the various standard ratings.

But another problem is the variation of parameters with respect to the change of load, temperature, etc.*14 Assuming some tests have been done to obtain the characteristic curve of a certain parameter with respect to another quantity, e.g. load,*15 it now remains to design a control device that can constantly update the feedback matrix in the light of the parameter values. It is impossible to perform repeated solutions of the Ricatti Equation; (it takes too much time even for a 5th-order system like this, and is highly uneconomical). So a formulation that can constantly update the feedback matrix upon a nominal pre-computed feedback matrix without repeated solutions of the Ricatti Equation, given this characteristic curve of the parameters, is needed to maintain near optimality.

If the parameter dependence is not known then the system will be even harder to control, but it is still possible, to a certain extent, to constantly update the model and find a control for it. These techniques require an application of the model-reference adaptive control techniques and will not be dealt with here.

To illustrate the relation between the K matrix and the operating conditions, we take the most common situation of change of operating point, namely, the change of electrical power, or load. The two optimal controls u_e and u_g corresponding to different values of P_e, the electrical power output, were plotted in Fig. 5-6 and Fig. 5-7.

The power is limited by the power angle δ which cannot exceed 30° according to Manitoba Hydro data. The variation of the K matrix is rather linear. So it should not be difficult to develop a function generator in the feedback mechanism that varies linearly with the electrical power. In this way, loading should not affect the stability very much.

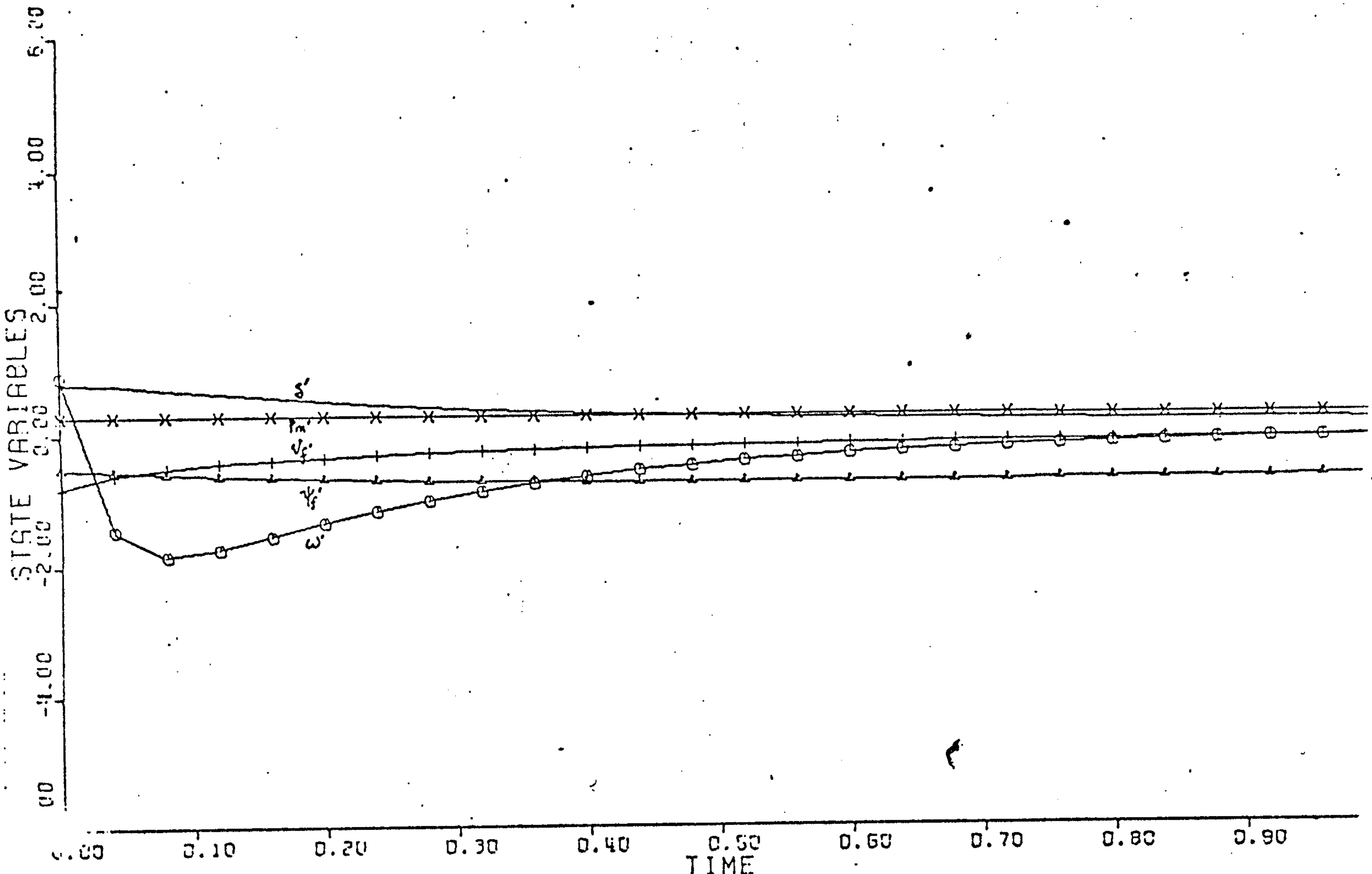


Fig. 5-1. Suboptimal control

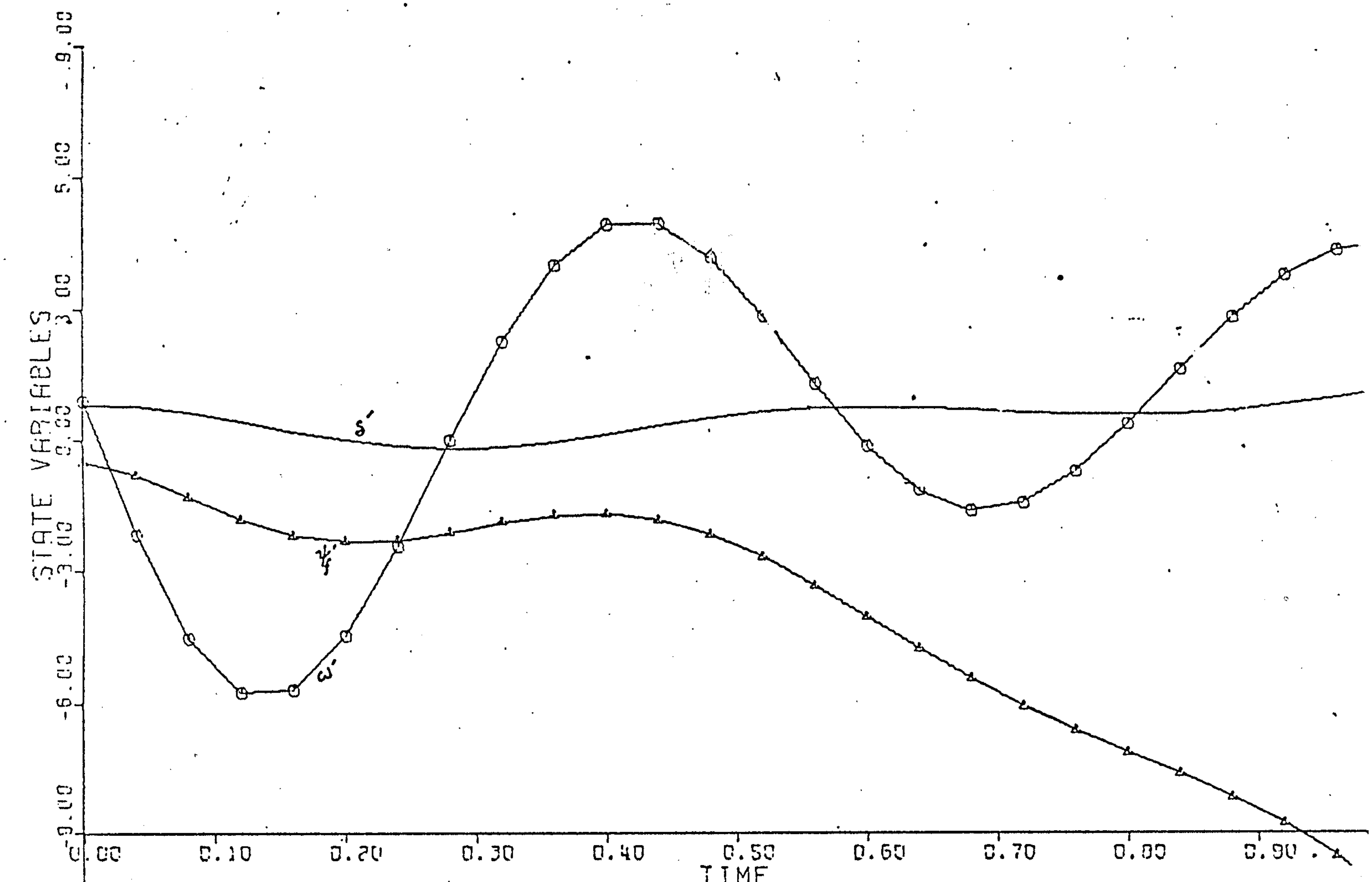


Fig. 5-2. Suboptimal Control of Model A.

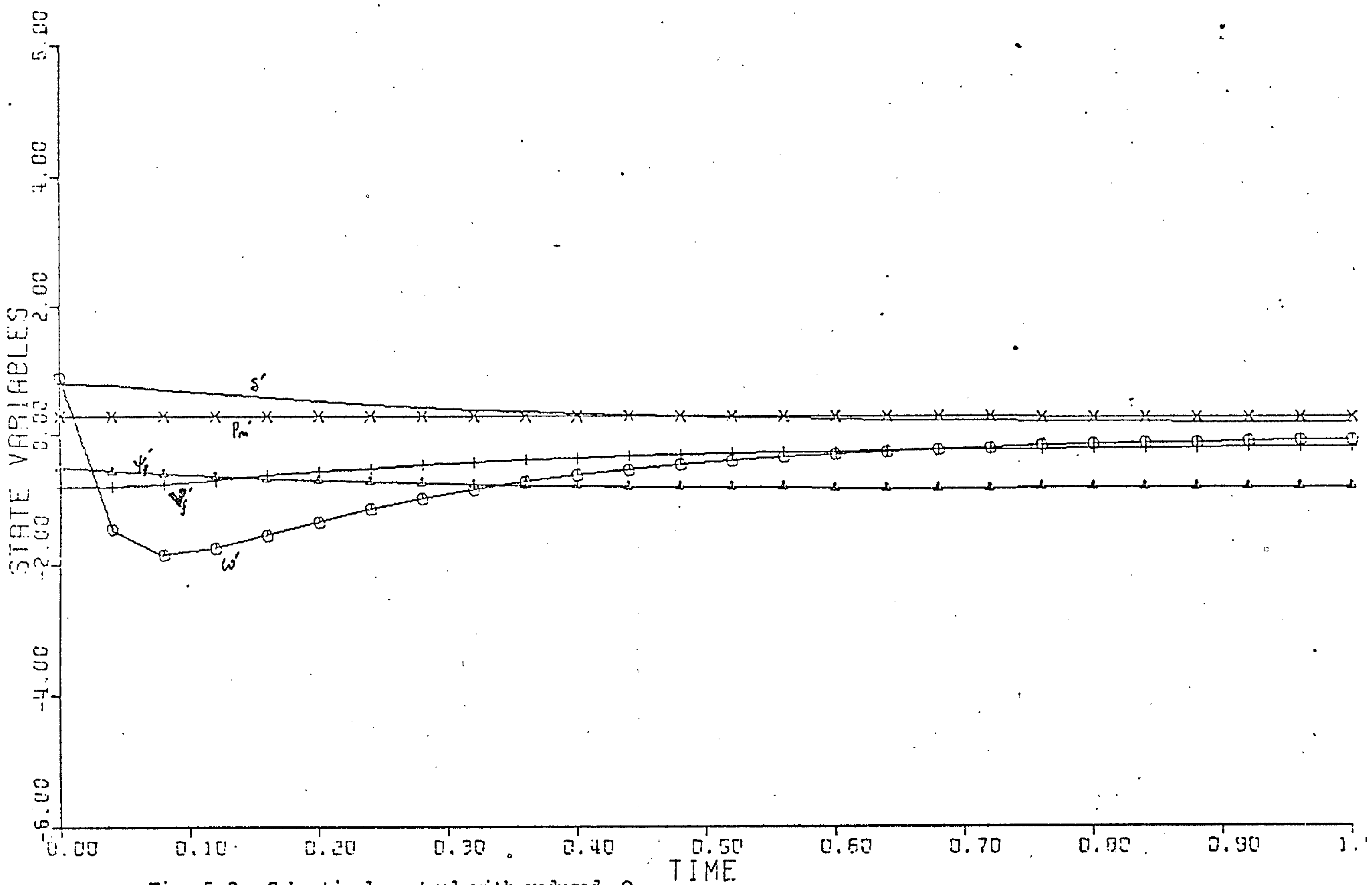


Fig. 5-3. Suboptimal control with reduced Q.

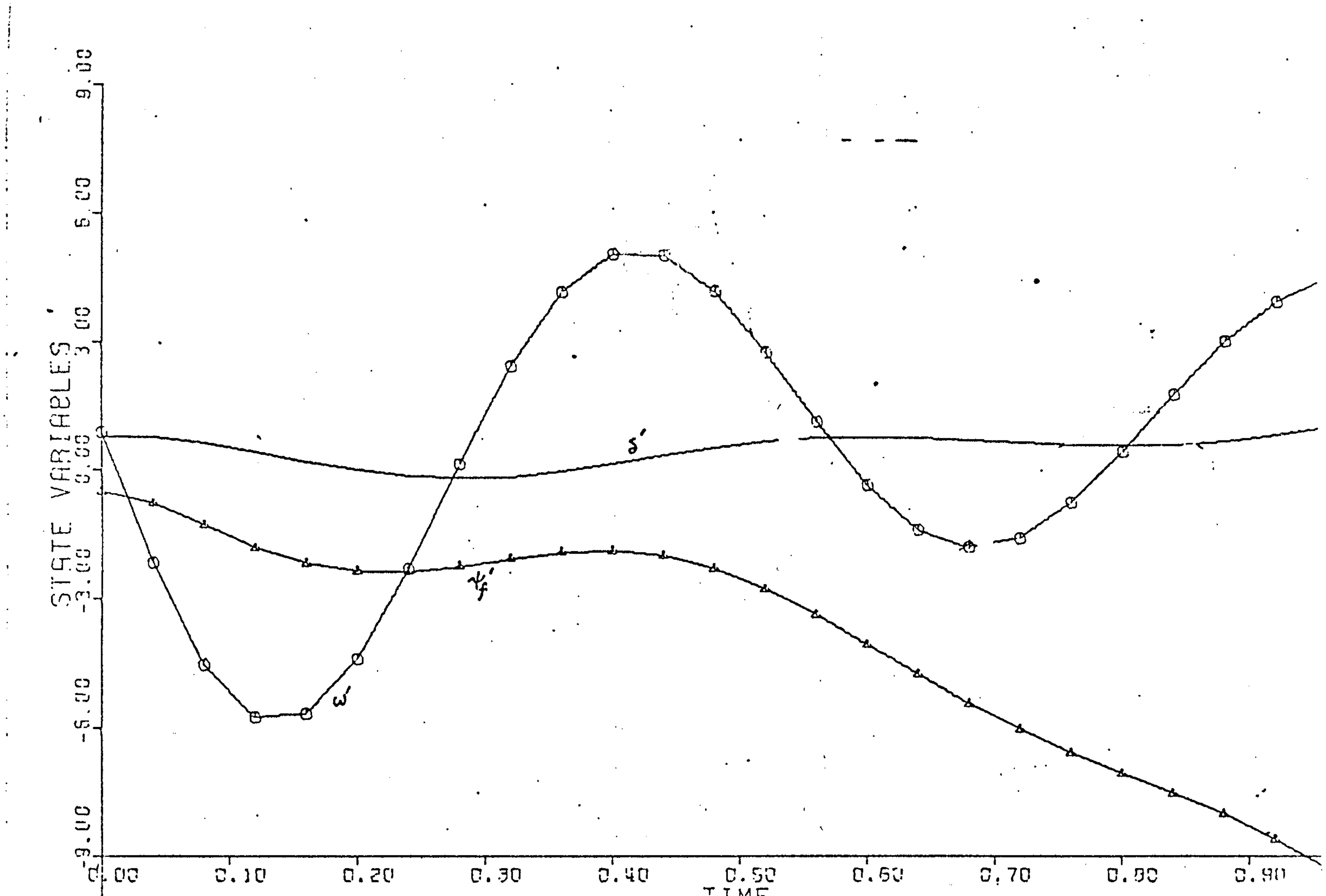


Fig. 5-4. Suboptimal control with reduced Q of model A.

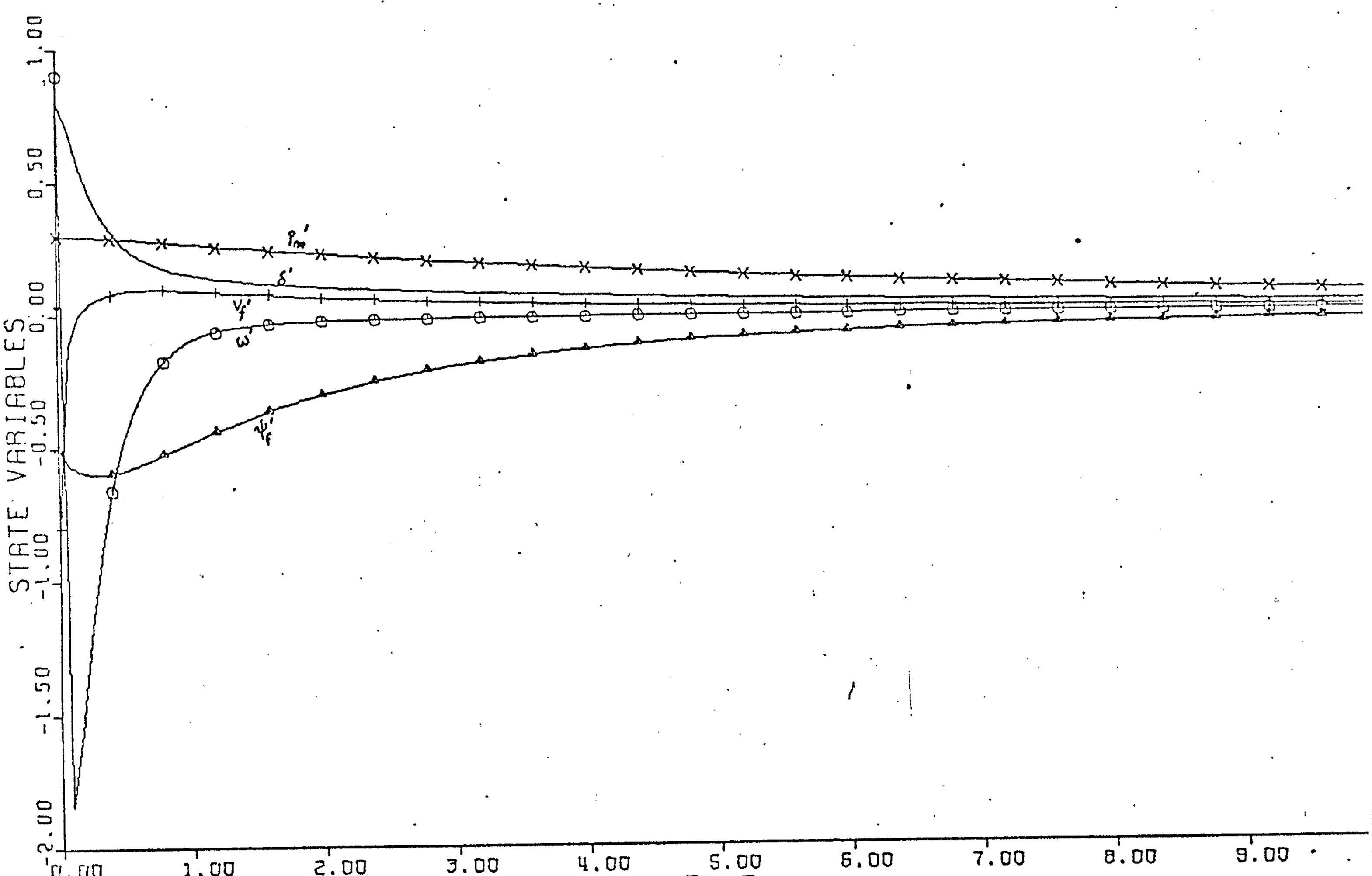


Fig. 5-5. System performance with reduced R.

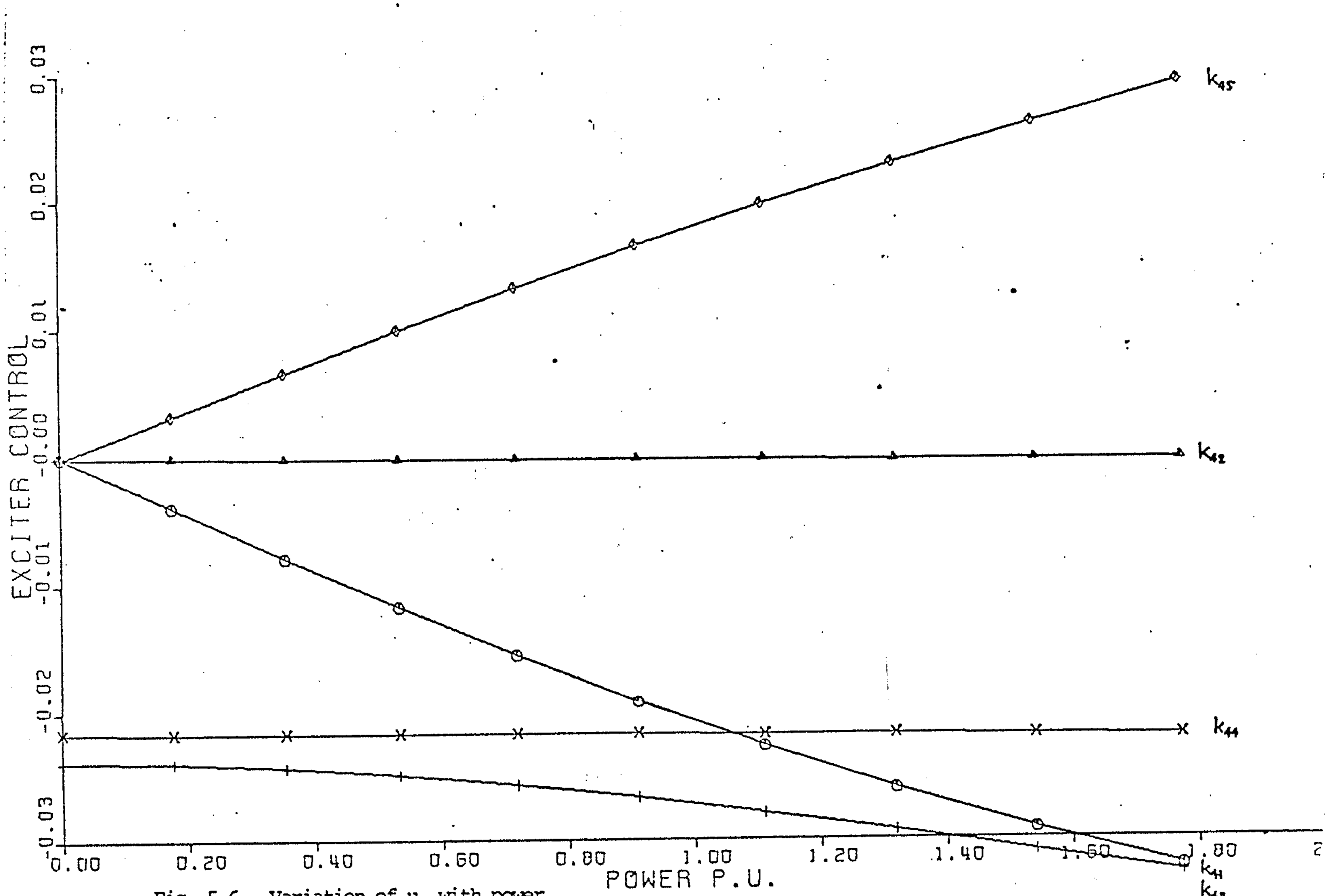


Fig. 5-6. Variation of u_e with power.

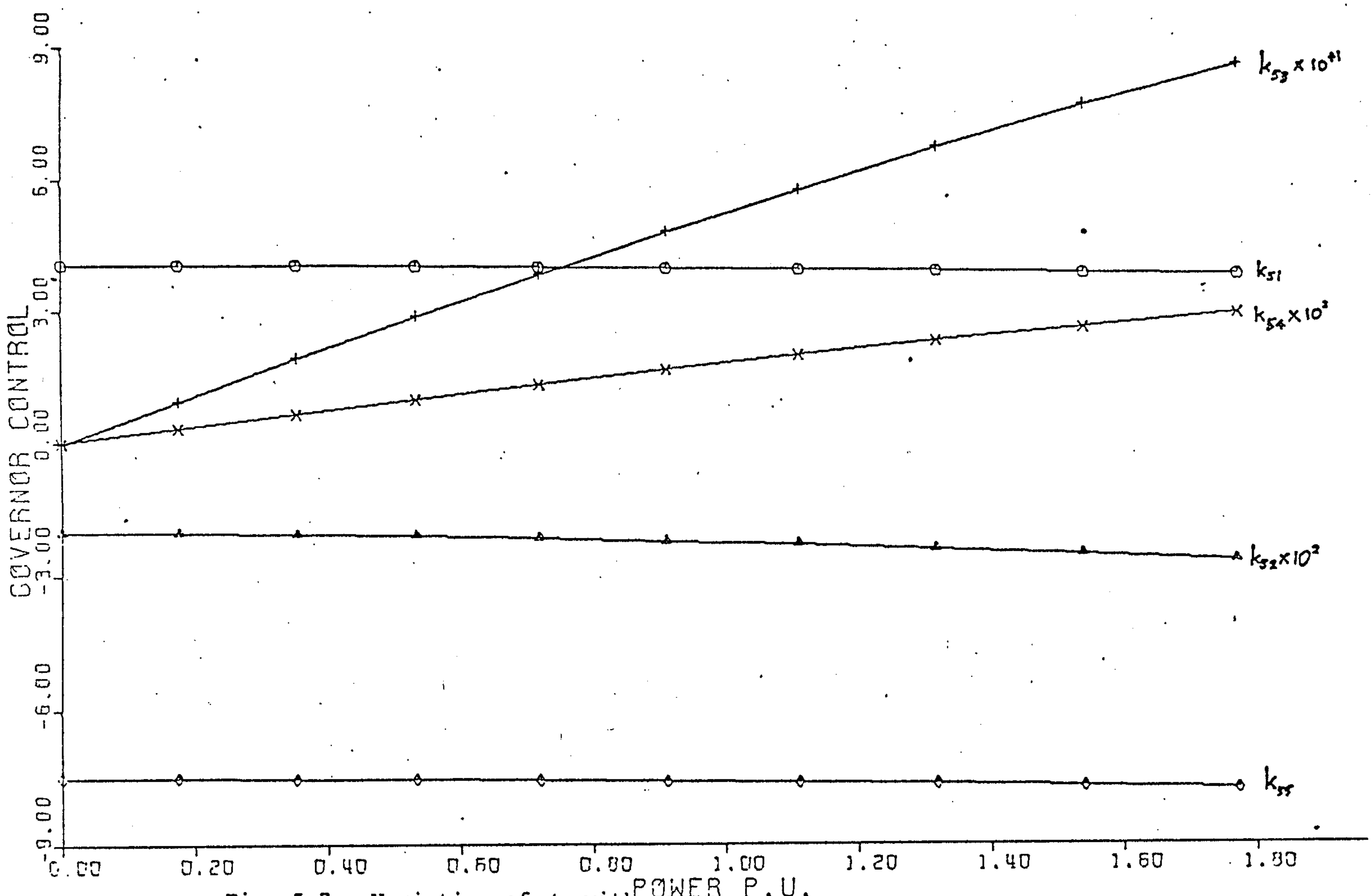


Fig. 5-7. Variation of u_g with power.

CONCLUSION

The application of optimal control to power systems has been a popular topic for researchers in the past few years, but has not reached the point where it can be widely and economically implemented yet. This thesis, therefore, should be considered as a contribution to the development in the application of optimal control to a power system, and not the derivation of a practical controller that can be implemented immediately. As it is now, the optimal control does not seem superior to the conventional methods. But once other problems, such as measurement of parameter, choice of optimal Q matrix etc., have been overcome, the optimal control method allows the designer to emphasize on whichever aspect he desires whether it be time, fuel, or stability and is therefore a very versatile tool.

The new matrix derivative method used in this thesis is very powerful in solving the problem in this thesis. It also makes sense intuitively. The drawback is that its mathematical background is not well established. It may theoretically lead to a different solution from the optimal one if there exist more than one solution. And in the case of a 5th order quadratic matrix equation, there are certainly more than one solution. However, other iterative methods have the same problem but some of them have methods to guide the iterative scheme towards the optimal solution (along eigenvalue lines or slope of steepest descent). The same may be applied to this method. In all, this method is worth studying.

The linear relationship of elements of K and the power is most encouraging. It means a feedback mechanism can cope with nonlinear plants at varying load conditions in a rather simple way. It makes implementation of this type of control more practicable.

APPENDIX I.

Machine Equations:

Power Conversion:

$$P_m = P_a + P_{loss} + P_e$$

$$P_a = H\dot{\omega}$$

$$P_{loss} = D\omega$$

$$P_e = \frac{V_o \sin \delta \psi_f}{x_d' \tau_{do}'} + \frac{(x_d - x_q)}{2x_d x_q} \sin 2\delta$$

Linearize about an operating point; the incremental equation is:

$$P_m' = H\dot{\omega}' + D\omega' + \frac{V_o \sin \delta_o}{x_d' \tau_{do}'} \psi_f' + \frac{V_o \cos \delta_o \psi_{fo}}{x_d' \tau_{do}'} \delta' + \frac{(x_d - x_q)}{x_d x_q} \cos 2\delta_o \delta' \quad \text{--- (1)}$$

Simplified governor-turbine unit:

$$\dot{P}_m' = \frac{1}{\tau_a} [-P_m' + u_g] \quad \text{-- (2)}$$

Exciter unit:

$$\dot{V}_f' = \frac{-V_f'}{\tau_e} + \frac{u_e}{\tau_e} \quad \text{-- (3)}$$

Transient property of machine:

$$\frac{d\psi_f}{dt} = V_f - \frac{x_d \psi_f}{x_d' \tau_{do}'} + \frac{(x_d - x_d')}{x_d'} V_o \cos \delta$$

Linearizing, it becomes

$$\dot{\psi}_f' = V_f' - \frac{x_d}{x_d' \tau_{do}'} \psi_f' - \frac{(x_d - x_d')}{x_d'} V_o \sin \delta_o \delta' \quad \text{-- (4)}$$

By definition:

$$\omega = \dot{\delta}$$
$$\dot{\delta}' = \omega' \qquad \text{-- (5)}$$

Equations (1) to (5) form the state equations of the system.

Assumptions and approximations are:

1. All damper windings excluded;
2. Subtransients excluded;
3. Property of the governor-turbine unit approximated by a simpler transfer function;
4. Transmission line between generator and infinite bus has negligible impedence;
5. Saturation of machine excluded.

APPENDIX II.

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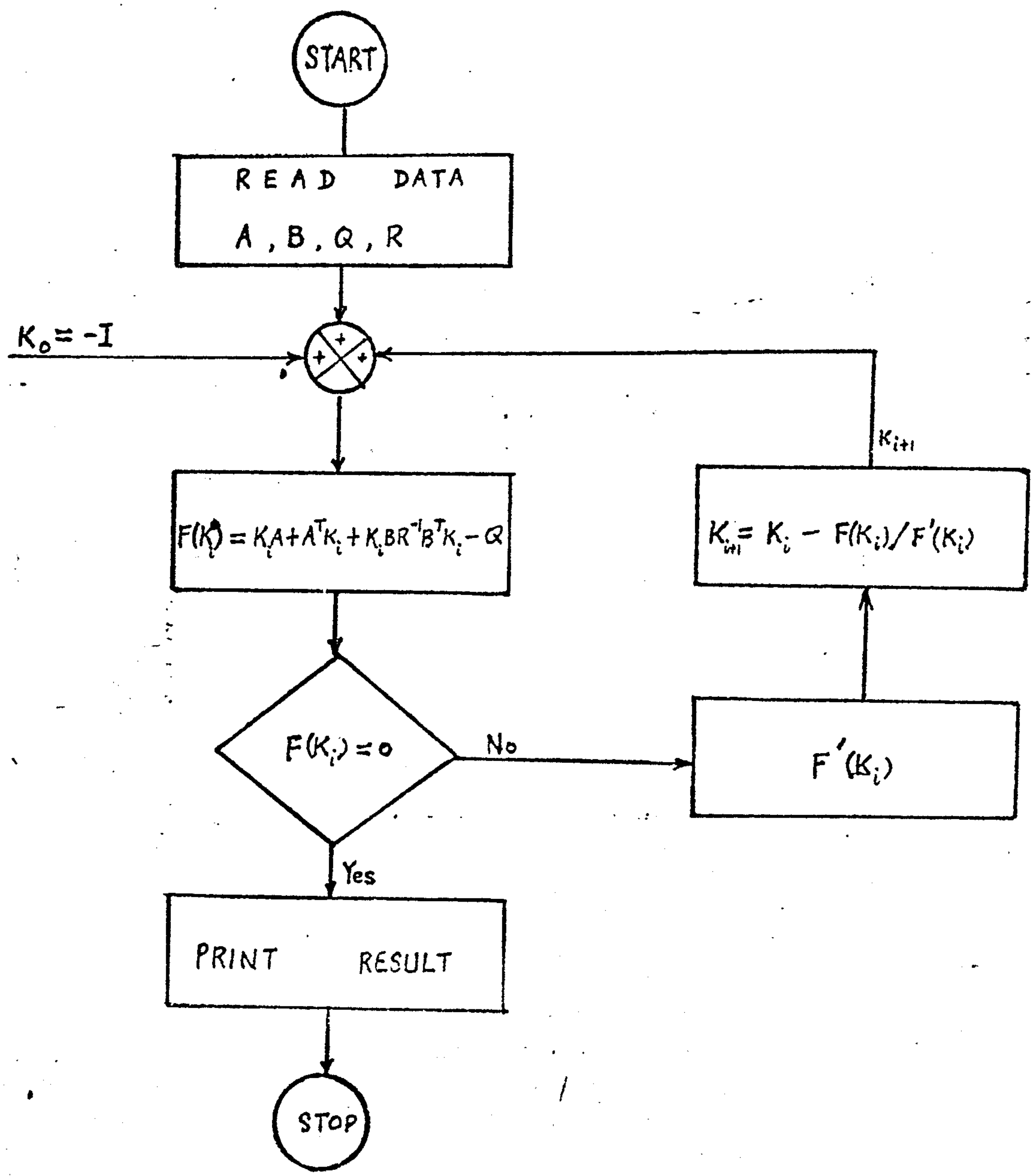
APPENDIX III.

The computer program for the numerical solution of the matrix Ricatti Equation using the "Matrix Derivative Method" is quite complicated and sectionalized. A schematic of the method is as follows:



```
      [illegible]
      DIMENSION A(5,5),B(5,5),C(5,5),FK(5,5)
      EQUIVALENCE (VF,F),(X,FF)
      COMMON F,  ,B,C,N
6     FORMAT(1H0,'OVERSHOOT'/' S1=',G10.4,5X,'S2=',G10.4,I3/)
7     FORMAT(/////30X,'K',57X,'F(K)'/3X,2(50('-'),5X),5(/10(F10.4,2X)),5
     1X,'M=',I3//' FMAX=',F10.4)
10    FORMAT(1H1,'SOLUTION OF RICATTI EQUATION BY MATRIX DERIVATIVE ',
     1'METHOD'//24X,'A',34X,'B',34X,'C'/3X,47('-'),3X,17('-'),3X,47('-')
     2,5(/125F10.4)///' ALGORITHM: K(I+1)=K(I)-CONST*F',1H','(K(I))'//)
17    FORMAT(F4.0)
19    FORMAT(I3)
20    FORMAT('+',5(/3X,10(G10.4,2X)),5X,2HM=,I3/)
      READ(5,19) N
      NN=N**2
      EPS=0.004
      DO 3 I=1,N
      DO 4 J=1,N
      C(I,J)=0.0
      K(I,J)=-1.0
4     CONTINUE
      READ(5,17) C(I,I)
3     CONTINUE
      WO=377.
      VO=1.0
      H=2.*634.42/115./WO
      D=1.05
      XD=0.696
      XQ=0.208
      XDD=0.201
      TDO=6.21
      TE=0.05
      TA=0.0
      DEL=23./360.*2.*3.1416
      C1=VO**2*COS(DEL)/XDD+VO**2*COS(2.*DEL)*(XDD-XQ)/XDD/XQ
      A(1,1)=0.
      A(1,2)=1.0
      A(1,3)=0.
      A(1,4)=0.
      A(1,5)=0.
      A(2,1)=-C1/H
      A(2,2)=-D/H
      A(2,3)=-VO*SIN(DEL)/H/XDD/TDO
      A(2,4)=0.
      A(2,5)=1./H
      A(3,1)=-VO*SIN(DEL)*(XD-XDD)/XDD
      A(3,2)=0.
      A(3,3)=-XD/XDD/TDO
      A(3.4)=1.
      A(3,5)=0
      A(4,1)=0.
      A(4,2)=0.
      A(4,3)=0.
      A(4,4)=-1./TE
      A(4,5)=0.
      A(5,1)=0.
      A(5,2)=0.
      A(5,3)=0.
```

```
      A(5,4)=0.
      A(5,5)=-1./TA
      VB(1,1)=0.
      VB(1,2)=0.
      VB(2,1)=0.
      VB(2,2)=0.
      VB(3,1)=0.
      VB(3,2)=0.
      VB(4,1)=1./TF
      VB(4,2)=0.
      VB(5,1)=0.
      VB(5,2)=1./TA
      WRITE(6,10)((A(I,J),J=1,N),(VB(I,J),J=1,2),(Q(I,J),J=1,N),I=1,N)
      CALL BBT(VB,B,N)
      CALL FK(F)
      S2=ANORM(F,N,N)
      M=1
100   S1=S2
      WRITE(6,63)((K(I,J),J=1,N),(F(I,J),J=1,N),I=1,N),M
      LP=1
      DO 1 J=1,N
      DO 1 I=1,N
      K(I,J)=K(I,J)+EPS
      CALL FK(FF)
      K(I,J)=K(I,J)-EPS
      CALL GMSUB(FF,F,FF,N,N)
      CALL SMPY(FF,1./EPS,DERF(1,LP),N,N,0)
      LP=LP+1
1     CONTINUE
      DO 2 L=1,NN
      X(L)=VF(L)
2     CONTINUE
      CALL GELG(X,DERF,NN,1,EPS,IER)
      CALL GMSUB(K,FF,K,N,N)
      CALL FK(F)
      S2=ANORM(F,N,N)
      M=M+1
      IF(S2.LT.S1) GO TO 200
      CALL MCPY(FF,DK,N,N,0)
      IN=0
300   IN=IN+1
      IF(IN.GT.10) CALL EXIT
      WRITE(6,5) S1,S2,IER
      CALL GMADD(K,DK,K,N,N)
      CALL SMPY(FF,S1/(S2*IN),DK,N,N,0)
      CALL GMSUB(K,DK,K,N,N)
      CALL FK(F)
      S2=ANORM(F,N,N)
      IF(S2.LT.S1) GO TO 200
      GO TO 300
200   IF(S2.GT.EPS) GO TO 100
      WRITE(6,7)((K(I,J),J=1,N),(F(I,J),J=1,N),I=1,N),M,S2
      CALL EXIT
      END
```

```
      FUNCTION ANORM(K,NROWS,NCOLS)
      REAL K(NROWS,NCOLS)
      VALUE=0.0
      DO 22 J=1,NCOLS
      DO 22 I=1,NROWS
      VALUE=VALUE+K(I,J)**2
22    CONTINUE
      ANORM=SQRT(VALUE)
      RETURN
      END
```

```
      SUBROUTINE FK(F)
      REAL F(5,5),K(5,5),A(5,5),B(5,5),C(5,5),Z(5,5)
      COMMON K,A,B,C,N
      CALL GMPRD(K,B,Z,N,N,N)
      CALL GMPRD(Z,K,F,N,N,N)
      CALL GMPRD(K,A,Z,N,N,N)
      CALL GMADD(F,Z,F,N,N)
      CALL GTPRD(A,K,Z,N,N,N)
      CALL GMADD(F,Z,F,N,N)
      CALL GMSUB(F,C,F,N,N)
      RETURN
      END
```

```
      SUBROUTINE BBT(VB,B,N)
      DIMENSION VB(N,2),F(N,N),B(2,2),Z(5,2),VBT(2,5),IL(2),IM(2)
2     FORMAT(4F4.0)
4     FORMAT(/13X,'B'/3X,20('-'),2(/2(F10.4,3X))///)
1     FORMAT(30X,'-1 T'/28X,'BF  B'/3X,55('-'),5(/5(F10.4,3X)))////30X,
     1'B',57X,'F(K)'/3X,2(55('-'),5X))
      READ(5,12) B
      WRITE(6,49) B
      CALL MINV(B,2,D,IL,IM)
      CALL GMPRD(VB,F,Z,N,2,2)
      CALL GMTRA(VB,VBT,N,2)
      CALL GMPRD(Z,VBT,B,N,2,N)
      PRINT 71, B
      RETURN
      END
```

SOLUTION OF RICATTI EQUATION BY MATRIX DERIVATIVE METHOD

A					B		Q				
0.0	1.0000	0.0	0.0	0.0	0.0	0.0	10.0000	0.0	0.0	0.0	0.0
-118.0431	-35.8774	-10.6960	0.0	34.1690	0.0	0.0	0.0	10.0000	0.0	0.0	0.0
-0.[illegible]	0.0	-0.5576	-1.0000	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0
0.0	0.0	0.0	-20.0000	0.0	20.0000	0.0	0.0	0.0	0.0	1.0000	0.0
0.0	0.0	0.0	0.0	-0.1111	0.0	0.1111	0.0	0.0	0.0	0.0	1.0000

ALGORITHM: K(I+1)=K(I)-CONST*F'(K(I))

R	
1.0000	0.0
0.0	1.0000

$BR^{-1}B^{T}$				
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	400.0000	0.0
0.0	0.0	0.0	0.0	0.0123

K					F(K)					
-1.000	-1.000	-1.000	-1.000	-1.000	628.0	553.9	530.3	538.0	485.0	
-1.000	-1.000	-1.000	-1.000	-1.000	553.9	459.8	446.1	453.9	400.8	
-1.000	-1.000	-1.000	-1.000	-1.000	530.3	446.1	421.5	430.3	377.2	
-1.000	-1.000	-1.000	-1.000	-1.000	538.0	453.9	430.3	437.0	385.0	
-1.000	-1.000	-1.000	-1.000	-1.000	485.0	400.8	377.2	385.0	330.9	M= 1
-19.16	-.4747	-1.104	-.3698	5.430	159.2	136.8	129.0	131.7	114.6	
-.4747	-.5970	-.4224	-.4601	-.4605	136.8	116.6	110.5	112.8	100.3	
-1.104	-.4224	-1.271	-.4982	-.1656	129.0	110.5	104.8	107.0	95.18	
-.3698	-.4601	-.4982	-.4776	-.5352	131.7	112.8	107.0	108.4	97.36	
5.430	-.4605	-.1656	-.5352	-34.35	114.6	100.3	95.18	97.35	104.3	M= 2
-19.89	-.1718	-.4392	-.7539E-01	5.834	34.83	31.32	29.90	30.53	28.49	
-.1729	-.2308	-.1654	-.1940	-.2321	31.40	29.45	27.10	27.59	25.78	
-.4876	-.1668	-1.123	-.2345	-.9068E-01	30.09	27.13	25.84	26.21	24.47	
-.7541E-01	-.1940	-.2342	-.2190	-.2943	30.54	27.60	26.34	26.47	25.05	
5.869	-.2332	-.9553E-01	-.2944	-18.84	28.54	25.86	24.58	25.05	26.22	M= 3
-19.57	-.5476E-01	-.5970	0.3491E-01	5.968	4.925	5.569	5.428	5.541	6.024	
-.5472E-01	-.2041	-.4316E-01	-.6787E-01	-.9937E-01	5.564	6.373	6.232	6.333	6.889	
-.5590	-.4316E-01	-1.038	-.1116	-.7482E-01	5.479	6.236	6.074	6.173	6.698	
0.3494E-01	-.6773E-01	-.1118	-.9380E-01	-.1584	5.521	6.320	6.181	6.059	6.784	
5.963	-.9975E-01	-.1651E-01	-.1587	-13.86	6.021	6.895	6.699	6.841	7.701	M= 4
-19.[illegible]	-.[illegible]E-01	-.[illegible]	0.3557E-01	[illegible].646	0.1420E-02	0.6058E-02	0.1426E-01	0.1626E-01	0.2088E-01	
-.[illegible]E-01	-.1566	0.[illegible]E-02	-.1526E-01	-.3144E-01	0.7016E-02	1.107	1.167	1.144	1.919	
-.[illegible]	0.[illegible]E-02	-1.[illegible]	-.5619E-01	0.[illegible]E-01	0.1165E-01	1.164	1.232	1.206	2.025	
0.[illegible]E-01	-.[illegible]E-01	-.[illegible]E-01	-.3[illegible]E-01	-.[illegible]E-01	0.1464E-01	1.142	1.209	1.097	1.985	
5.647	-.313[illegible]E-01	0.[illegible]E-01	-.6737E-01	-12.46	0.1551E-01	1.916	2.030	1.986	2.353	M= 5
-1[illegible]	-.[illegible]E-01	-.1239	-.9666E-02	4.700	0.7930	-.2544	-.3657	-.2882	-1.165	

-.3[illegible]E-01	-.1[illegible]	[illegible]E-01	-.1[illegible]E-02	-.12[illegible]E-01	-.[illegible]	0.[illegible]211E-01	0.1161	0.9166E-01	0.3641	
-.7247	0.1[illegible]E-01	-.[illegible]	-.3679E-01	0.7904	-.3042	0.1154	0.1682	0.1327	0.9340	
-.[illegible]708E-02	-.1748E-02	-.[illegible]E-01	-.2323E-01	-.2800E-02	-.2300	0.9142E-01	0.1333	0.7954E-01	0.4209	
4.700	-.1384E-01	0.[illegible]769	-.2478E-02	-10.47	-1.165	0.3673	0.5361	0.4214	1.715	M= 6
-17.78	-.3401E-01	-.2473	-.2733E-01	3.983	0.1400	-.1413E-01	-.3940E-01	-.1198E-01	-.2091	
-.3393E-01	-.1403	0.1792E-01	-.1579E-03	-.2295E-01	-.1600E-01	0.2376E-02	0.2559E-02	0.6799E-03	0.1056E-01	
-.8475	0.1792E-01	-.[illegible]028	-.3039E-01	0.7749	-.3918E-01	0.2586E-02	0.1031E-01	0.3133E-02	0.6759E-01	
-.2739E-01	-.1572E-03	-.3039E-01	-.2177E-01	0.2255E-01	-.1194E-01	0.6786E-03	0.3141E-02	-.1518E-02	0.1661E-01	
3.983	-.2283E-01	0.7746	0.2256E-01	-8.251	-.2091	0.1045E-01	0.5799E-01	0.1664E-01	0.3169	M= 7
-17.74	-.3315E-01	-.2614	-.2426E-01	3.901	0.3760E-03	-.1923E-02	-.1936E-03	-.6221E-05	-.1542E-02	
-.3319E-01	-.1403	0.1759E-01	-.1653E-03	-.2473E-01	-.2226E-02	0.5341E-03	0.1287E-03	-.3725E-06	-.1774E-03	
-.2614	0.1759E-01	-.7947	-.3250E-01	0.7316	-.1869E-03	0.1316E-03	0.1020E-03	-.8259E-05	0.7715E-03	
-.2826E-01	-.1653E-03	-.7950E-01	-.2178E-01	0.2511E-01	-.4056E-05	-.7451E-08	-.9000E-05	0.2098E-04	0.1001E-04	
3.901	-.2478E-01	0.7318	0.2511E-01	-7.711	-.1398E-02	-.1841E-03	0.7753E-03	0.9630E-05	0.6110E-02	M= 8

K					F(K)					
-17.7345	-0.0332	-0.2615	-0.0283	3.9004	0.0	-0.0000	-0.0000	-0.0000	-0.0000	
-0.0332	-0.1403	0.0176	-0.0002	-0.0248	-0.0000	0.0000	0.0000	0.0000	-0.0000	
-0.2615	0.0176	-0.8746	-0.0305	0.7326	-0.0000	0.0000	0.0	-0.0000	0.0000	
-0.0283	-0.0002	-0.0305	-0.1218	0.0251	-0.0000	0.0000	-0.0000	-0.0000	0.0000	
3.9004	-0.0248	0.7326	0.0251	-7.6968	-0.0000	-0.0000	-0.0000	0.0000	0.0000	M= 9

FMAX= 0.0001