GOAL PROGRAMMING FOR PENSION FUND PORTFOLIO MODELING

By

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A Thesis
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THE UNIVERSITY OF MANITOBA

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INTRODUCTION

In this thesis a goal programming approach for the University of Manitoba Pension Plans is presented. The thesis has two purposes: to appreciate a goal programming technique applicable to pension fund portfolio modelling, and, to develop a working model for the University of Manitoba Pension Plans in order to to develop an in-year investment strategy. Unlike linear programming, a goal programming approach attempts to minimize deviations from established goals based on the priority assigned to a goal. Thus, this thesis is based on finding a well-diversified investment portfolio addressing the goals of the plans.

Apart from developing a diversified investment portfolio, this thesis also deals with a demand goal, such that demand (benefits) must be met by the cash and short-term notes held by the pension fund.

The goal programming formulation was based on available data supplied by the University of Manitoba's Staff Benefits Office. The initial model was created with this data, and a forecasting model was subsequently developed. In dealing with the initial model, two different versions were developed:

1. In the first version, the goals and their priorities are set according to the pension plans' mandate. The asset allocation of the fund is done according to the investment policy statement (see Appendix A).

2. The second version deals with the risk associated with the investment strategy. The most interesting feature of this model is that it is formulated to address a worse case scenario. In other words we want to maximize the total return in a bad year.

While dealing with both the initial and forecasted formulations, the future rate of returns for various investments classes were obtained using the "Wilkie Model" (see Appendix B). The simulations using the initial conditions of the Wilkie Model were run using @Risk, a simulation software package, which came as a recent attachment to Microsoft Excel. While dealing with the forecasted formulations, demand was estimated using Holt's method. Normally in future year, demand would be estimated by the actuary and the administrator. However, in absence of this information I adopted Holt's method as a proxy.

The goal programming models are then solved for their optimal solutions by determining the percentage of investment in each asset class. These models are solved using QSB+ software. To validate the formulated goal programming models, the obtained optimal results for the first version model were compared to the actual investment strategy. The thesis is concluded by a discussion of the forecasted models.

CHAPTER ONE

UNIVERSITY OF MANITOBA PENSION PLANS

The University of Manitoba has three pension plans: the University of Manitoba Pension Plan (1970), the University of Manitoba GFT Pension Plan (1986), and the University of Manitoba Pension Plan (1993). Assets of all three plans are held in a Master Trust. The plans' Pension Committee has general authority over the assets. The Committee maintains an important role in designing the investment strategy of the asset mix and retains the services of investment managers to implement this strategy. It is at the discretion of this Committee to formulate an investment policy statement and investment manager mandates.

Net investment income is divided pro-rata to all accounts of the three plans. It is the responsibility of the University of Manitoba Pension Plan trustees to safeguard the assets of the plan, which includes maintaining written agreements with the custodian who is Canada Trust. The plans' trustees review quarterly financial reports and are also responsible for the audited financial statements.

The three plans are administered in accordance with the Pension Benefits Act of the Province of Manitoba and with the provisions of the Income Tax Act (Canada).

The University's responsibilities as administrator include the integrity, objectivity, and preparation of the financial statements and notes. The University

maintains a system of internal control to provide assurances that the records, from which the financial statements are derived, must be complete, accurate, and should properly reflect all transactions. Independent custodians are retained to prepare records of all investment transactions.

The University of Manitoba Pension Plan (1993) was established effective January 1, 1993 by a transfer of assets and liabilities from the 1970 Plan. For the same reason, members of the prior plan automatically became members of the revised plan on the effective date. The transfer of assets did not involve a physical transfer. The assets associated with the 1993 Plan were recorded separately from those with the 1970 Plan. Effective January 1, 1994, most of the balance of the membership in the 1970 Plan also transferred to the 1993 Plan. The major differences between the 1993 Plan and the 1970 Plan are that the new plan requires an additional 1% contribution from both the members and the University and that there has been the introduction of the concept of a "Plan Annuity" whereby the monthly pension after retirement is paid out of the 1993 Plan rather than having an annuity purchased. The 1993 Plan provides for an increase of the Plan Annuity after retirement based on the performance of the pension fund.

Plan Provisions

Pension Benefit: The retirement income is made up of,

> a) A basic pension provided by the provision of the Plan Annuity with the total member required, and the university contributions accumulated with interest to the retirement date, and

b) A supplementary pension being the difference, if positive between a) above on a life guaranteed five year basis and a pension equal to 2% of the average salary in the years when the member's salary is highest multiplied by years of service in the plan less 0.7% of the YMPE in the year of retirement multiplied by service after January 1,1966.

Where a supplementary pension is payable, the sum of the basic pension and the supplementary pension is limited to the lessor of,

a. 2% of the average salary in the five years when salary was highest multiplied by years of service in the plan.

b. $1,722.22 per annum multiplied by years of service in the plan.

For this calculation, service prior to January 1, 1992 is limited to 35 years.

For retirements prior to the normal retirement date of age 65, the formula pension is reduced by 1/4% for each month between actual retirement and the normal retirement date.

Termination Benefit: The plans provide for full and immediate vesting on termination of employment subject to the funds being used to provide a lifetime income, otherwise, provisions of the Pension Benefits Act of the Province of Manitoba are applied.

Pre-retirement Death Benefit: The benefit on death before retirement is the accumulated value of a Member's Contribution Account and the Member's University Contribution Account except that amount represented by contributions made by the University between April 1, 1979 and December 31, 1984.

Contributions:

The University of Manitoba Pension Plan (1970)

The Plan members contribute at the rate of 6% of salary less an adjustment for the Canada Pension Plan. The respective employer matches these contributions. If an actuarial valuation reveals a deficiency in the fund, the Pension Benefits Act of the Province of Manitoba requires that the University makes additional contributions to fund the deficiency.

The University of Manitoba GFT Pension Plan (1996)

The University contributes for each member an amount equal to the lesser of i) 6% of the base salary of full professors at the University minus an adjustment for the Canada Pension Plan and ii) the maximum contribution permitted by the Income Tax Act (Canada). There are no member contributions.

The University of Manitoba Pension Plan (1993)

The Plan members contribute at the rate of 7% of salary less an adjustment for the Canada Pension Plan. The University matches these contributions. If an actuarial valuation reveals a deficiency in the fund, the Pension Benefits Act of the Province of Manitoba requires that the University makes additional contributions to fund the deficiency.

Investment Alternatives

The following investment options were identified for the University of Manitoba Pension Plan. The broad areas were, (1) Fixed income (2) Canadian Equity (3) US-Equity (4) Offshore equity, and (5) Money markets.

As of December 31, 1997, the University of Manitoba Pension Plans' total fund portfolio is $661 Million with 42.7% invested in Fixed Income Securities, 0.5% in Money Market Funds, 11.3% in US Equity, 10.3% in Offshore Equity and 35.2% in Canadian Equity. The total fund had produced returns of 12.9% over 1997.

CHAPTER TWO

PENSION PORTFOLIO AND PENSION RISK

The following chapter provides an introduction to the general pension risks, which are involved in pension portfolio management. A few of these general pension risks apply to the University of Manitoba's Pension Plans' portfolio. In "The Many Dimensions of Risk", Wagner discusses the various sources of risk to a pension fund, managing the risk, and the primary goals in the management of a pension fund. Wagner states that there are three goals of a pension fund: (1) to maximize the probability that the fund meets its benefit obligations, (2) to minimize the cost of the funding source, and (3) to enhance the predictability in the sequence of obligations of the funding source.

Wagner summarizes pension risk as comprising of specific risk, mismatching risk, and business risk. He elaborates by noting that the real risk to a plan member is that the company will not be able to fulfil its pension promise; that is, to provide an employee with a decent standard of living after retirement. Alternatively, the pension risk can be defined as the risk in which the returns will fall below some predetermined acceptable level. Wagner defines his pension risks as follows:

Specific risks:

Overemphasizing sensitivity to specific economic scenarios, by design or inadvertently. Also, failing to take advantage of diversification and ignoring

risks generated within the funding source. The diversification risk will be addressed in the models.

Mismatching risks:

Creating asset portfolios whose time horizons differ from those of the liability stream. As well, investing in securities whose economic sensitivities differ from those of the pension obligations. This is addressed in the models with respect to immediate obligations only.

Business risks:

Creating excessive demands or instabilities on the funding source that exhaust the patience of the managers and the shareholders/ taxpayers/contributors who bear the pension obligations. This cannot be addressed in the models.

The ability of a defined benefit pension plan to fulfil its pension promise depends upon two major factors:

1. Investment performance of the fund being able to at least meet the assumed return by the actuary.

2. Continuing contributions by the plan sponsor to fund (a) past and future service credits, (b) new benefits that are granted, (c) any shortfalls in investment returns, and (d) additional funding in case retirement expenses rise.

Portfolio management has traditionally viewed risk as the variability of returns. This is particularly evident for investors with a short investment time horizon who are mostly concerned with short term fluctuations of return.

However, if the time horizon is long, the portfolio manager may be less concerned with the short-term fluctuations and his/her objective might instead be to maximize the expected rate of return. Also, the portfolio manager may wish to ensure that there is a very low probability that the rate of return will be below a certain allowable minimum. This concept is known as the safety-first theory of portfolio choice. This link suggests that pension fund investment managers may well operate within a lower expected return constraint. The safety-first model then addresses investors' concerns regarding long-term performance, while placing a high emphasis on disaster-avoidance.

To reduce and to control pension portfolio risk, one needs to look at the vulnerabilities that could cause real strains and then consider the assets that could be held in a portfolio to alleviate those vulnerabilities.

This is done by first examining the short-term obligations year-by-year. The present value of these obligations determines the amount that should be placed in equities to meet these obligations. The next step is to match the intermediate-term obligations with bonds of similar duration, and lay aside cash and short-term notes to pay for the known payments that are immediately due. This combination of investments is the best match to the duration and nature of the liability stream, and is defined as the *reserve asset* that minimizes the risk of the pension plan.

Therefore, the best defence against the pension risk obligation is through diversification and matching. However, where this thesis discusses widely about diversification, it only deals with the matching of immediate obligations.

Pension Fund Portfolio Management:

In "An Expected Gain-Confidence Limit Criterion for Portfolio Selection", W.J. Baumol proposes a new efficiency criterion for the Markowitz portfolio selection. Baumol points out that an investment with a relatively high standard deviation of returns will be relatively safe if its expected value is sufficiently high. This contrasts with the Markowitz argument that a portfolio is efficient only if it is impossible to obtain a greater average return without incurring a greater standard deviation. This implies that given an expected return, any increase in standard deviation is undesirable. Baumol points out that with a high enough expected return, an increase in, or high, standard deviation may still provide a sufficiently high lower limit. In other words, Baumol says that the investor is not interested in strictly variability and shows that with normally distributed returns, there is about 2% chance that the returns will be less than two standard deviations below the expected return. Baumol ties all of this together by demonstrating that an increase in expected return may counter-balance an increase in return variability. This may provide an acceptable, safe lower bound on expected returns.

CHAPTER THREE

GOAL PROGRAMMING

Increasing complexity, competition, and social responsibility in the business environment have inspired businesses to consider multiple goals in their decision making process. In classical economic theory the usual objective is to maximize profit, but it is not the only objective. In fact, business firms frequently place higher priorities on non-economic objectives such as public relations, labour relations, and environment protection. In other words, organizations often have multiple objectives and the modern decision making problem becomes one of solving for an optimal mix of goals, which are often incompatible and incommensurable. That is, one needs to solve for an optimal mix of competing objectives.

Goal programming is a modification and extension of linear programming, although both of these programming techniques are linear mathematical models that attempt to provide optimal solutions for constrained objectives. The basic idea is to establish a specific numeric goal for each of the objectives, formulate an objective function for each objective, and then seek a solution that minimizes the sum of deviations of these objective functions from their respective goals. The significance of goal programming lies in its perspective of sharing goals with their priorities and providing an optimal solution, keeping in line the goals and their priorities. Where linear programming usually deals with a one-dimensional

objective such as profit maximization, goal programming solves multiple and frequently conflicting objectives, such as profitability, liquidity, and solvency.

An investor usually specifies the goals in the objective function formulated on the basis of the three following concepts:

Deviational Variables

Unlike linear programming, goal programming attempts to minimize deviations from established goals based on the priority assigned to the goal. We denote d^- and d^+ as deviational variables, where the - (minus) represents a negative deviation and the + (plus) represents a positive deviation. If a specific goal deals with minimizing a negative deviation, such as low returns, then d^- forms a part of the objective function. Similarly, when concerned with minimizing a positive deviation, such as high risk, d^+ forms a part of the objective function.

Pre-emptive Priority Factors

To optimize goals in the order of their importance, a method of ranking the goals must be introduced. This ranking is accomplished by assigning pre-emptive priority factors to the deviational variables for each goal. A pre-emptive priority factor of subscript 1 means that the goal assigned to this rank will be solved first prior to considering the other specified goals. Priority factors of equal rank can also be weighted to make a priority order within the same rank.

Weighting of Deviational Variables

When dealing with portfolio selections it is usually a concern of the investor to assign weights according to the asset mix. For the same reason, it is

necessary to weigh deviational variables that have the same priority level. The effect of these deviational weights is to reflect the relative importance of deviational variables with identical pre-emptive priority factors.

Non Pre-emptive Goal Programming

Unlike standard goal programming, pre-emptive goal programming is where the model assumes that a higher ranked goal will always be satisfied as completely as possible before the next lowered ranked goal is even addressed. However, there can be times where a manager wants several goals to be of nearly the same value.

One way to accomplish the possibility of goal trade-offs is to ignore pre-emptive goal programming (GP) models. More specifically, there are several non pre-emptive GP models that allow the managers to remove the pre-emptive priority coefficients and to replace them with additive weights of importance. Two such models are *archimedean* goal programming and *multigoal* programming. These variations of the standard goal programming model differ only in the way in which the objective function is formulated. The goal and resource constraints continue to have identical form.

Unfortunately, *multigoal* programming does not identify goal weights and does not provide aggregate results that give the manager a measure of worth (being able to identify goal weights). Due to this limitation, as well as several other complications, *multigoal* programming will no longer be discussed.

On the other hand, *archimedean* goal programming has considerable simplicity and is easy to understand. These attractive qualities make a brief discussion of the *archimedean* model worthwhile.

The objective function of the *archimedean* GP model is as follows:

$$\text{Minimize } Z = \sum_{i=1}^{m} (w_i^+ d_i^+ + w_i^- d_i^-)^{\Psi}$$

Where,

Ψ = Power to which the weighted deviations are raised; (can take on any values, but usually $\Psi = 1, 2,$ or ∞)

$w_i^{+,-}$ = relative weight of importance (interval scale) of the ith goal constraint.

The investment manager will need to establish the relative weights of importance of the different "per unit" goal deviation values. The final product of this evaluation will be a summed product value of the weights times the deviation values. It provides a mean for allowing an interplay and/or trade off between goal constraints.

Unlike linear programming, a pre-emptive GP problem does not build a set of parameters that are automatically "ranged", does not have a dual form which provides shadow prices (the shadow price measures the marginal value of the resource, that is, the rate at which Z could be increased by increasing the amount of this resource, b_i), and does not have associated opportunity costs (opportunity cost is the cost involved in obtaining a better solution, but within a

certain range). For these very reasons, some primary issues must be addressed anew for each GP problem. For example,

1. What is the impact of the various goal constraint hierarchical arrangements on the optimal solution? The variety of different arrangements that are possible in ordering the pre-emptive goal constraints makes the priority structure the foremost feature.

2. What is the RHS (a common usage in linear/goal programming pertaining to the changes in the restricted availability of resources) ranging sensitivity of the goal constraint's target values b_i? At what point does a change in this value cause a change in the present optimal solution?

3. How much can the relative weights of a specific priority level change before the present optimal solution is shifted to a pre-emptive GP problem?

4. What are the possible trade-offs among archimedean goal constraints if we relax the priority structure?

5. What is the trade-off among the different deviational variable values of the competing goals?

Goal Programming and Portfolio Management:

In "Goal Programming for Portfolio Selection", S. Lee and D. Chesser applied the concepts of goal programming to the twin objectives of maximum return and minimum risk. The authors present the general goal programming model as one inherently suited to assist investors in selecting efficient portfolios that satisfy a range of objectives. They discuss considerations in selecting efficient portfolios and describe the underlying concepts of goal programming.

Finally, they formulate an illustrative goal programming problem for investment in stocks. It was demonstrated that the goal programming technique either can identify the one portfolio that best satisfies the investors' goals, or it can specify the required trade-offs between conflicting goals in order to achieve a particular goal.

Conclusion

Goal programming is a powerful and useful tool when multiple, conflicting, and incommensurate objectives describe the resource allocation setting. If however, the pre-emptive attributes and incommensurate results of the standard GP model are unacceptable, then a non pre-emptive GP model may be a preferable technique to select. However, in this discussion I feel that pre-emptive GP will be the ideal approach as this plan has a mandate that clearly states its goals while funding for its pension fund liabilities.

CHAPTER FOUR

GOALS AND CONSTRAINTS

In this chapter, I identify the goals and constraints needed for the Goal Programming formulation of the University of Manitoba Pension Plans. In order to assist in formulating this model with realistic goals and constraints, I contacted the Pension Accountant at the University of Manitoba's Staff Benefits office regarding the goals and objectives that the University of Manitoba Pension Plans have mandated for its plan members.

Most of the data with respect to the asset mix, total assets and liabilities, and other income statements were collected from the University of Manitoba Pension Plans Annual Reports and Quarterly Review and are summarized in Exhibit I. The last Annual Report available from the Staff Benefits Office is for the year 1996.

Investment Alternatives

The following investment options were identified for the University of Manitoba Pension Plan. The broad areas of investment seem to be (1) Fixed Income, (2) Canadian Equity, (3) US-Equity, (4) Off-shore Equity, and (5) Money Markets. Based on this information, I have defined the following as my decision variables,

Variables

X_1 = The percentage of money invested in Cash and Short Term Notes.

X_2 = The percentage of money invested in Bonds.

X_3 = The percentage of money invested in Canadian Stocks.

X_4 = The percentage of money invested in Foreign Stocks.

The following are the fund objectives of the University of Manitoba Pension Plans.

a) The basic objective of the fund is to provide retirement benefits at a reasonable and predictable cost to both members and the Plan sponsor. This objective depends on whether the retirement benefits require a supplementary pension. Recent experience has shown that the supplementary pension has not been needed.

b) The overall objective is to maximize investment returns while assuming a level of risk deemed appropriate by the pension committee.

c) The minimum objective for the fund is to achieve the long-term total rate of return including capital gains, dividends and interest, but net of all investment management expenses, equal to the annual change in the Consumer Price Index for Canada, plus at least 4% per annum. Over a four year period, the managers of the fund are expected to achieve an above – median position relative to other similar funds measured by a recognized performance measurement service.

The goal programming model addresses the above objectives as follows:

Goal 1: The rates of return for each asset class should exceed some lower bound. There are many different ways to go about in fixing possible lower bounds, three such approaches are stated below, a) the average investment performance of the fund for the past 10 years. b) Baumol's theory as described in chapter two. c) The funds minimum objective, so that the rates of return should exceed the annual change in CPI plus 4% per annum. However I have decided to go with option three because the average investment performance over the past 10 years was 12.6%, which is too high to be our lower bound as the fund has performed exceptionally over the last few years and this may not be the case all the time. Also Baumol's theory produces a lower bound of negative 3.35%, which seems too pessimistic. Therefore the goal is to minimize the underacheivement (d_1^-) which is listed in the first constraint.

Constraints:

1) The rate of returns obtained by "Wilkie's Model" should exceed the annual change in CPI plus 4% per annum. Therefore, the sum of the rates of return in each class multiplied with the percentages of money to be invested in that particular class should exceed 8.3% ((CPI value = 4.3%) + 4% = 8.3%)). The CPI value of 4.3% is the average nominal annual percentage rates of change from 1948 to 1997 inclusive. This value was extracted from the Canadian Institute of Actuaries Economic Statistics, Report on Canadian Economic Statistics, 1924-1997. A description of the "Wilkie Model" is explained in Appendix B.

2) The amount of cash and short-term notes held by the portfolio is used to pay for the benefits. The idea behind this goal is the widespread belief that in

portfolio management, a long-term investment will yield a better return than short-term investments. A second implicit assumption is that there is a reduction in uncertainty with a long-term investment; the investor can depend upon a known amount of money at some point in the future. On the other hand, investment managers want some cash on hand in order to take advantage of new and better opportunities. Cash is liquid asset. In formulating the demand constraint, it is assumed that the demand for the following year is estimated by the actuary and the administrator beforehand and that this estimation would not deviate significantly from what actually transpires during the year. Hence actual in year demand is used.

According to the investment policy statement of the University of Manitoba Pension Plans:

3) The Minimum percentage of money invested in bonds is 40% of the total portfolio.

4) The Maximum percentage of money invested in bonds is 50% of the total portfolio.

5) The Minimum percentage of money invested in Canadian stocks is 30% of the total portfolio.

6) The Maximum percentage of money invested in Canadian stocks is 40% of the total portfolio.

7) Although a minimum and a maximum is set for foreign investment, it was an observation that the plan is attempting to invest to the allowable maximum of 20% (Revenue Canada restriction).

8) The last constraint is a system constraint, so that the total investment percentage should sum to 1.

In addition to the above goals and constraints, the following goal has been introduced.

Goal 2: An expected rate of return is obtained, which happens to be the worst case rate of return simulated over a 5 year period for each asset class. This second goal thus maximizes these expected rates of return so as to handle the risk for a terrible year. This worst rate scenario is obtained by simulating the rate of returns using @RISK software. The worst case is then a value, which occurs at the tenth percentile of the @RISK output.

While incorporating this goal, two other measures of introducing risk in the model were considered (1) A 60% level of volatility was considered, where a constraint was formulated which took into account the fact that the total volatility of the fund should be no more than 60% times the volatility of the equities invested. (2) the ratio of the variance of the fund's rate of return over the fund's mean return squared should be minimized. It was observed that with the constraint developed in the worst case scenario, both the goals were satisfied. This indicates that the model with its range of investment strategies seems to take into account various risks that are involved while dealing with the University of Manitoba Pension Plans' pension fund.

CHAPTER FIVE

THE GOAL PROGRAMMING MODEL

This chapter primarily deals with the development of the goal programming model using the original data from the University of Manitoba Pension Plans Annual Reports (this data is summarised in EXHIBIT I). The later part of the chapter deals with comparing the actual versus the model results. The initial model was set up for the year 1996. It was observed that the QSB+ program in its very nature is trying to reach the fixed percentage values defined by the constraints and thus it would be appropriate to assume that the results generated by QSB+ could be compared with the actual investment strategies for the years 1994 and 1995. On the same grounds, we could extend the initial model to develop into a forecasting model.

The initial model was created with this data and a forecasting model was subsequently developed. While dealing with the initial model, two different versions of the model were developed.

1. In the first version, the goals and their priorities are set according to the pension plans mandate. The constraints on the asset allocation of the fund are according to the plan's investment policy statement (see Appendix A).

2. The second version deals with the risk associated with the investment strategy. The most interesting feature of this model is that the model is formulated to exceed the expected results in a "terrible year". In other

words, we want to maximize the total return such that it exceeds the return that could be experienced in a bad year (i.e.) the tenth percentile of returns. The input and the output data for each of the following models are included in Appendix C.

MODEL 1(96)

Variables

X_1 = The percentage of money invested in Cash and Short Term Notes.

X_2 = The percentage of money invested in Bonds.

X_3 = The percentage of money invested in Canadian Stocks.

X_4 = The percentage of money invested in Foreign Stocks.

Objective Function

Min Z = $P_1 d_1^-$

Such that,

1. *Yield rates:* obtained using Wilkie's model should exceed the annual change in the CPI plus 4% per annum

$$0.0426 X_1 + 0.0542 X_2 + 0.0754 X_3 + 0.1113 X_4 + d_1^- - d_1^+ = 0.083$$

2. *Demand:*

The percentage of cash and short-term notes available should at least be greater than the ratio of the 1996 demand over the total portfolio amount as at December 31,1995.

$$X_1 \geq \$ 30{,}292{,}187 / \$ 519{,}790{,}137 = 0.058$$

3. *Government and Corporate Bonds (Asset allocation as per the Investment Policy Statement):*

The percentage of bonds invested should be a maximum of 50%

$$X_2 \leq 50\%$$

4. *Government and Corporate Bonds (Asset allocation as per the Investment Policy Statement):*

The percentage of bonds invested should be a minimum of 40%

$$X_2 \geq 40\%$$

5. *Canadian Stocks (Asset allocation as per the Investment Policy Statement):*

The percentage of Canadian stocks invested should be a maximum of 40%

$$X_3 \leq 40\%$$

6. *Canadian Stocks (Asset allocation as per the Investment Policy Statement):*

The percentage of Canadian stocks invested should be a minimum of 30%

$$X_3 \geq 30\%$$

7. *Foreign Stocks (Federal tax legislation):*

The amount of foreign stocks invested should be no more than 20%

$$X_4 \leq 20\%$$

8. *System Constraint:*

$$X_1 + X_2 + X_3 + X_4 = 1$$

Conclusions:

Comparing the Actual results to Model results shows that the investment strategy recommended is very similar to that which the plan used. Thus, it

appears that the model is satisfactory and can be used for future asset allocation strategies. The results are shown in Table 1.

TABLE 1

Asset Allocation as at January 1, 1996

TYPE OF SECURITY	ACTUAL %	QSB+ %
Cash & ST Notes	6.2%	5.8%
Bonds	40.3%	40.0%
Stock Canadian	32.9%	34.2%
Stock Foreign	19.3%	20.0%
Real Estate	0.5%	0.0%
Accrued Interest	0.8%	0.0%
TOTAL	100.0%	100.0%

The above table indicates that our Goal Programming Model does a reasonable job of modelling the plans' investment strategy. It should be noted that the small deviations from the actual results is because of our constraints made during the formulation of the model, noting the demand constraint and not recognizing any investment in Real Estate or Accrued Interest held. The summarized solution for **Model 1(96)** generated by QSB+ shows that the optimal solution is available. It also indicates that there has been a positive underachievement $(d_1^-) = 1.08\%$. Recall, that the objective is to minimize the underachievement, and that the smaller this value, the better.

MODEL 2(96) (Risk adjusted Mandate)

Variables

X_1 = The percentage of money invested in Cash and Short Term Notes.

X_2 = The percentage of money invested in Bonds.

X_3 = The percentage of money invested in Canadian Stocks.

X_4 = The percentage of money invested in Foreign Stocks.

Objective Function

Min $Z = P_1 d_1^-$

Max $Z = 0.0280\, X_1 + 0.0415\, X_2 + 0.0631\, X_3 + 0.0954\, X_4$

Such that,

1. *Yield rates:* obtained using Wilkie's model should exceed the annual change in the CPI plus 4% per annum

$0.0426\, X_1 + 0.0542\, X_2 + 0.0754\, X_3 + 0.1113\, X_4 + d_1^- - d_1^+ = 0.083$

2. *Demand:*

The percentage of cash and short-term notes available should at least greater than the ratio of the 1996 demand over the total portfolio amount as at December 31,1995

$X_1 \geq \$\,30,292,187 / \$\,519,790,137 = 0.058$

3. *Government and Corporate Bonds (Asset allocation as per the Investment Policy Statement):*

The percentage of bonds invested should be a maximum of 50%

$X_2 \leq 50\%$

4. *Government and Corporate Bonds (Asset allocation as per the Investment Policy Statement):*

The percentage of bonds invested should be a minimum of 40%

$X_2 \geq 40\%$

5.*Canadian Stocks (Asset allocation as per the Investment Policy Statement):*

The percentage of Canadian stocks invested should be a maximum of 40%

$X_3 \leq 40\%$

6.*Canadian Stocks (Asset allocation as per the Investment Policy Statement):*

The percentage of Canadian stocks invested should be a minimum of 30%

$X_3 \geq 30\%$

7. *Foreign Stocks (Federal tax legislation):*

The amount of foreign stocks invested should be no more than 20%

$X_4 \leq 20\%$

8. *System Constraint:*

$X_1 + X_2 + X_3 + X_4 = 1$

Conclusions:

Model 2(96)

It is worth noting that even after introducing a minimum rate of return goal, that the optimal solution is the same as the solution of **Model 1(96)**. This suggests that the current goal programming model formulation with the recommended investment strategy can actually handle a worst case scenario of poor rate of returns. The summarized solution again indicates a possible optimal solution, and there is still an underachievement of 1.08% while dealing with goal

1. Similarly, the second goal of 5.89% is achieved. This means that the recommended investment strategy would provide a rate of return, which is expected to be exceeded by the fund 90% of the time.

Comparisons of Fund Balance as at January 1, 1997 (see EXHIBIT A)

Had the funds been invested according to the model's recommendation, then the fund balance as at January 1, 1997 would have been estimated at $607,972,314. This balance is arrived at by taking the actual 1996 invested income of each asset class and pro-rating it according to that class' recommended investment percentage over the actual investment percentage. Actual contributions, payments and other income items are assumed to remain unchanged. Here the actual realized and the unrealized gains were assumed to remain unchanged, as 1) there is no much difference in the actual and estimated Asset Allocation and 2) the same securities have been sold.

Note that this projected balance is 100.3% of the total fund balance of $606,184,388. This is not surprising since the model recommended high percentages to be invested in asset classes which performed well above average during 1996; namely Canadian and Foreign equity.

CHAPTER SIX

The Forecasted Goal Programming Model

In this chapter, a forecasted goal-programming model is developed for the University of Manitoba Pension Plans' fund portfolio for the years 1997, 1998 and 1999. We will look at future investment strategies for a specific year in question based on predictions of future investment and demand for that year.

Demand Estimation

In this thesis, I have incorporated a demand goal such that the demand (benefits to be paid out) must be met by the cash and short-term notes held by the pension portfolio. Normally, demand for the coming year would be reasonably estimated by the actuary and the administrators. However, given the absence of this information, I have developed the following method as a proxy. This method would also be used to estimate demand for any future year which may be beyond the scope of the actuary and administrators.

Holt's Method will be used for obtaining estimates future demand. This method is often an effective forecasting tool for time series data, which exhibits a linear trend. In the present study, this method is appropriate as the smoothing parameters α and β are solved, and are observed to lie between 0 and 1 (see EXHIBIT IIA).

The forecasting function in Holt's method is represented by,

$$Y_{t+k} = E_t + k\, T_t \qquad (1.0)$$

Where,

$$E_t = \alpha Y_t + (1 - \alpha)(E_{t-1} + T_{t-1}) \qquad (1.1)$$

$$T_t = \beta(E_t - E_{t-1}) + (1 - \beta) T_{t-1} \qquad (1.2)$$

Here:

Y_t = The observed value of the time series at time t

E_t = The expected level (base) of time series.

T_t = The expected rate of increase or decrease (trend) per period.

The smoothing parameters α and β can assume any value between 0 and 1. If there is an upward trend in the data, E_t tends to be larger than E_{t-1}, making the quantity $(E_t - E_{t-1})$ in equation (1.2) positive. This tends to increase the value of the trend adjustment factor T_t. Alternatively, if there is a downward trend in the data, E_t tends to be smaller than E_{t-1}, making the quantity $(E_t - E_{t-1})$ in the equation (1.2) negative. This tends to decrease the value of the trend adjustment factor T_t. We can use Excel Solver parameters to identify the values of α and β that minimizes the non-linear MSE objective. An illustration of how to solve for these smoothing parameters is given under Illustration.

The historical data for developing the above trend models consists of years from 1974 to 1996. This was the only information that was made available to the study.

The outflows presented in the financial statements of the University of Manitoba Pension Plans for the period 1974 to 1996 (EXHIBIT II & IIA) were used to estimate demand.

Investment Income

Predicting the rate of return for each asset group in a future year is the next step. In order to determine future expected returns, I have used the "Wilkie Model" for stochastic purposes. These expected rates of return as a goal are to be achieved for the year and the under-achievement is to be minimized.

Forecasted Goals

Goal 1: The expected rate of returns estimated by the "Wilkie Model" should exceed the annual change in CPI plus 4% per annum. Therefore, the sum of the rates of return in each class multiplied with the percentages of money to be invested in that particular class should exceed 8.3% ((CPI value = 4.3%) + 4% = 8.3%)). The goal is to minimize the underachievement (d_1^-) which is listed in the first constraint.

Goal 2: An expected minimum rate of return should be exceeded, which happens to be a worst case rate of return simulated over a 5 year period for each asset class. This second goal thus maximizes these expected rates of return so as to handle the risk for a terrible year. This worst rate scenario is obtained by simulating the rate of returns using @RISK software. The worst case is then a value, which occurred at the tenth percentile of the @RISK output.

Constraints:

1) The rate of returns estimated by the "Wilkie Model" should exceed the annual change in CPI plus 4%. A description of the model is explained in Appendix B.

2) The amount of cash and short-term notes held by the portfolio is used to pay for benefits.

3) The Minimum percentage of money invested in bonds is 40% of the total portfolio.

4) The Maximum percentage of money invested in bonds is 50% of the total Portfolio.

5) The Minimum percentage of money invested in Canadian stocks is 30% of the total portfolio.

6) The Maximum percentage of money invested in Canadian stocks is 40% of the total portfolio.

7) The Maximum percentage of money invested in Foreign stocks is 20% of the total portfolio.

8) The last constraint is a system constraint, so that the total investment percentage should sum to 1.

MODEL 97 (Risk adjusted Mandate)

Variables

X_1 = The percentage of money invested in Cash and Short Term Notes.

X_2 = The percentage of money invested in Bonds.

X_3 = The percentage of money invested in Canadian Stocks.

X_4 = The percentage of money invested in Foreign Stocks.

Objective Function

$$\text{Min } Z = P_1 d_1^-$$

$$\text{Max } Z = 0.0280\, X_1 + 0.0415\, X_2 + 0.0631\, X_3 + 0.0954\, X_4$$

Such that,

1. *Yield rates:* obtained using the "Wilkie Model" should exceed the annual change in CPI plus 4% per annum

$$0.0432 \, X_1 + 0.0547 \, X_2 + 0.0976 \, X_3 + 0.08610 \, X_4 + d_1^- - d_1^+ = 0.083$$

2. *Demand:*

The percentage of cash and short-term notes available should at least be greater than the ratio of the demand over the total portfolio amount.

$$X_1 \geq \$30,564,204 / \$606,184,388 = 0.050$$

3. *Government and Corporate Bonds (Asset allocation as per the Investment Policy Statement):*

The percentage of bonds invested should be a maximum of 50%

$$X_2 \leq 50\%$$

4. *Government and Corporate Bonds (Asset allocation as per the Investment Policy Statement):*

The percentage of bonds invested should be a minimum of 40%

$$X_2 \geq 40\%$$

5. *Canadian Stocks (Asset allocation as per the Investment Policy Statement):*

The percentage of Canadian stock invested should be a maximum of 40%

$$X_3 \leq 40\%$$

6. *Canadian Stocks (Asset allocation as per the Investment Policy Statement):*

The percentage of Canadian stock invested should be a minimum of 30%

$$X_3 \geq 30\%$$

7. *Foreign Stocks (Federal tax legislation):*

The amount of foreign stock invested should be no more than 20%.

$X_4 \leq 20\%$

8. *System Constraint:*

$X_1+X_2+X_3+X_4 = 1$

Conclusions:

Comparing the Actual results to Model results shows that the investment strategy recommended is again similar to that which the plan used. The results are shown in Table 2.

TABLE 2

Asset Allocation as at January 1,1997

TYPE OF SECURITY	ACTUAL %	QSB+ %
Cash & ST Notes	8.5%	5.0%
Bonds	37.1%	40.0%
Stock Canadian	34.6%	40.0%
Stock Foreign	19.5%	15.0%
Real Estate	0.0%	0.0%
Accrued Interest	(0.2%)	0.0%
TOTAL	100.0%	100.0%

The above table indicates that our Goal Programming Model is still doing a reasonable job of modelling the plans' investment strategy. It should be noted that the deviations from the actual results is because of our constraints made during the formulation of the model, noting the estimated demand via Holt's

constraint and not recognizing any investment in Accrued Interest held. The summarized solution for **Model 97** generated by QSB+ shows that the optimal solution is available. It also indicates that there has been a underachievement (d_1^-) = 0.7%. However, the asset strategy has been formulated to produce an overall return which is expected to exceed 5.76% ninety percent of the time. The fund's actual return for 1997 was 12.9%.

Comparisons of Fund Balance as at January 1, 1998 (see EXHIBIT B)

In order to obtain the fund balance for this model, the following approach is developed:

1) For cash asset class,

Closing balance = Starting Balance $(1+i)$ + (Contributions-Benefits)$(1+i)^{1/2}$

2) For non-cash asset class,

Closing balance = Starting balance $(1+i)$

Thus, as of January 1,1998 the outstanding balance is estimated to be $715,379,856.

The actual net assets available for year-end 1997, according to the University of Manitoba Pension Plans Quarterly Review, is equal to $661,000,000. At this point it is hard to explain any difference, as the data for 1997 was not available at this point in time. It should be noted that the model results suggest a good deal of money to be held in Canadian equity. Also, it should be noted that the performance of the current model is based on estimated demand, estimated contributions and as an approximation the realized and unrealized gains of 1997.

MODEL 98

Variables

X_1 = The percentage of money invested in Cash and Short Term Notes.

X_2 = The percentage of money invested in Bonds.

X_3 = The percentage of money invested in Canadian Stocks.

X_4 = The percentage of money invested in Foreign Stocks.

Objective Function

$$\text{Min } Z = P_1 d_1^-$$

$$\text{Max } Z = 0.0280\, X_1 + 0.0415\, X_2 + 0.0631\, X_3 + 0.0954\, X_4$$

Such that,

1. *Yield rates:* obtained using the "Wilkie Model" should exceed the annual change in CPI plus 4% per annum.

$$0.0437\, X_1 + 0.0551\, X_2 + 0.1403\, X_3 + 0.2669\, X_4 + d_1^- - d_1^+ = 0.083$$

2. *Demand:*

The percentage of Cash and Short-term Notes available should be at least greater than the ratio of the demand over the total portfolio amount.

$$X_1 \geq \$32{,}064{,}548 / \$661{,}000{,}000 = 0.049$$

3. *Government and Corporate Bonds (Asset allocation as per the Investment Policy Statement):*

The percentage of bonds invested should be a maximum of 50%

$$X_2 \leq 50\%$$

4. *Government and Corporate Bonds (Asset allocation as per the Investment Policy Statement):*

The percentage of bonds invested should be a minimum of 40%

$$X_2 \geq 40\%$$

5.*Canadian Stocks (Asset allocation as per the Investment Policy Statement):*

The percentage of Canadian stock invested should be a maximum of 40%

$$X_3 \leq 40\%$$

6.*Canadian Stocks (Asset allocation as per the Investment Policy Statement):*

The percentage of Canadian stock invested should be a minimum of 30%

$$X_3 \geq 30\%$$

7. *Foreign Stocks (Federal tax legislation):*

The amount of foreign stock invested should be no more than 20%.

$$X_4 \leq 20\%$$

8. *System Constraint:*

$$X_1+X_2+X_3+X_4 = 1$$

Conclusions:

Comparing the Actual results to Model results shows that the investment strategy recommended is still similar to that which the plan used. The results are shown in Table 3.

TABLE 3

Asset Allocation as at January 1,1998

TYPE OF SECURITY	ACTUAL %	QSB+ %
Cash & ST Notes	5.0%	4.9%
Bonds	39.8%	40.0%
Stock Canadian	35.2%	35.1%
Stock Foreign	20.0%	20.0%
Real Estate	0.0%	0.0%
Accrued Interest	0.0%	0.0%
TOTAL	100.0%	100.0%

The above table indicates that our Goal Programming Model continues to do a reasonable job of modelling the plans' investment strategy. It should be noted that the deviations from the actual results is because of our constraints made during the formulation of the model, noting the estimated demand constraint via Holt's method. The summarized solution for **Model 98** generated by QSB+ shows that the optimal solution is available. It also indicates that the goal has been achieved.

Fund Balance as at January1, 1999 (see EXHIBIT B)

In order to obtain the fund balance for this model, the following approach is developed:

1) For cash asset class,

Closing balance = Starting Balance $(1+i)$ + (Contributions-Benefits)$(1+i)$ $^{1/2}$

2) For non-cash asset class,

Closing balance = Starting balance (1+i)

Employing the same techniques as for 1998 the January 1, 1999 outstanding balance equals to $809,704,410. The actual net assets available as of March 31,1998, according to the University of Manitoba Pension Plans Quarterly Review, is equal to $712,000,000. At this point it is hard to explain, as the data is insufficient to make any conclusions. It should be noted that the model results suggest a good deal of money to be held in Canadian equity and Canadian bonds. Also, it should be noted that the performance of the current model is based on estimated demand, estimated contributions and as an approximation the realized and unrealized gains of 1996.

The outstanding balance as of 1.1.98 becomes the system constraint in the Forecasted Model 99.

Future Research

A next step would be to simulate the investment percentages to see a bigger picture. By running a number of simulations for each asset class return, one could then set up a number of expected fund return constraints and examine the range of asset allocation strategies that the model would recommend. The challenge here for the most part would be to integrate the QSB+ program with the Excel @RISK software so that a investment strategy would be generated with each simulation. This way one could examine a fund's current long-term investment strategy for soundness. During this thesis, a great amount of time was spent to actually see if we could use a range of values for diversification.

Unfortunately, the QSB+ software version 2.1 does not seem to actually work with ranges, even though the ranges could be incorporated in the formulation, but the program most of the time seems to pick only the higher side of the value. A goal programming model to program for some of Wagner's other techniques to reduce or manage risk could be another possible project. One of the ways of looking at Wagner's techniques would be to incorporate deviation matching of assets and liabilities. Also, a goal programming model which address risk by taking into account correlated returns among the asset classes is another possibility.

APPENDIX A

THE UNIVERSITY OF MANITOBA
INVESTMENT POLICY STATEMENT

INVESTMENT POLICY STATEMENT

THE UNIVERSITY OF MANITOBA

PENSION PLANS

EFFECTIVE JUNE 27, 1994

CONTENTS

OVERVIEW

1.1   This investment policy statement ("the Statement") applies to the assets held in trust with respect to The University of Manitoba Pension Plan (1993), The University of Manitoba GFT Pension Plan (1986), and The University of Manitoba Pension Plan (1970), collectively referred to as the "University Pension Plans" (copies of the respective Pension Plan documents are attached as Attachment "A", "B" and "C"). The Statement contains investment objectives, investment guidelines, and monitoring procedures.

1.2   The assets of the University Pension Plans are combined and form the Master Trust Fund ("the Fund") in accordance with a Trust Agreement dated 31st day of December 1973, as amended from time to time, between The University of Manitoba (the "University") and certain individuals as Trustees of the assets of the University Pension Plans (the "Trust Agreement"). The Fund shall be established and maintained pursuant to the provisions of the University Pension Plans for the purpose of providing retirement, death and termination benefits, for the respective plan members and their beneficiaries.

1.3   The Fund will be managed in accordance with all applicable legal requirements notwithstanding any indication to the contrary which might be construed from the Statement. With respect to any portion of the Fund invested in pooled funds, provisions of the investment policy statement of such pooled funds shall prevail over those of the Statement to the extent that they are in conflict.

RESPONSIBILITIES

2.1   The respective pension plan documents of The University of Manitoba (1993), The University of Manitoba GFT (1986), and The University of Manitoba (1970) provide for the establishment of the Pension Committee ("the Committee"). The Committee shall be accountable to the Board of Governors through the Vice-President (Administration) of The University of Manitoba. The Committee has general authority over the investments of the assets in the Fund.

2.2   The Committee may delegate, to the extent it sees fit, authority with respect to services provided in connection with the investment of the Fund to agents and advisors, and for this purpose may retain the services of an investment manager(s) ("the Manager"). The Committee maintains an active role with respect to the following:

        i)   formulation of the Statement and manager mandates;
        ii)   appointment and monitoring of agents and advisors;
        iii)   evaluation of performance.

2.3   Any person to whom the Committee delegates authority with respect to the investment of the Fund must adhere to the provisions of the Statement.

2.4   The responsibilities of the Trustees under the Trust Agreement are to hold and safeguard the assets, to enter into, amend or terminate contracts with competent companies for the holding of the assets in the participating Trusts and to monitor the integrity of said company(ies). Safeguard, as used in the Trust Agreement, does not include investment decisions to the assets of the Participating Trusts.

CONFLICT OF INTEREST GUIDELINES

3.1   Any employee of the University directly or indirectly responsible for the investment activities of the Fund, any employee of the external managers of the Fund and any member of the Pension Committee shall immediately disclose to the Pension Committee any actual or perceived conflict of interest that could be reasonably expected to impair, or could be reasonably interpreted as impairing , his/her ability to render unbiased and objective advice or to fulfil her/his fiduciary responsibilities to act in the best interests of the beneficiaries of the University Pension Plans. A member of the Pension Committee required to make such a disclosure shall not participate in the discussion or vote on any resolution to recommend a transaction in relation to which the disclosure is required.

Examples of a conflict of interest or a perceived conflict of interest would include: (i) the purchase or retention of securities of a company or a fund in which an external investment manager, member(s) of his/her immediate family or his/her firm held a significant financial interest and (ii) the selection of an external investment manager with whom a member of the Pension Committee has an independent business relationship such that the selection of the manager would lead to financial gain for the member.

PLAN CHARACTERISTICS

4.1   The University of Manitoba Pension Plan (1970), The University of Manitoba GFT Pension Plan (1986), and The University of Manitoba Pension Plan (1993), referred to as the University Pension Plans, are trusteed plans registered with the Pension Commission of the Province of Manitoba. The combined assets of the University Pension Plans are held in the name of The Trustees of The University of Manitoba Pension Plans.

The following is a summary of significant provisions of the University Pension Plans[1]:

a.   **The University Pension Plans (all)**

(i)   Termination Benefit

The University Pension Plans provide for full and immediate vesting on termination of employment subject to the funds being used to provide lifetime income, otherwise, provisions of the Pension Benefits Act of the Province of Manitoba are applied.

(ii)   Pre-retirement Death Benefit

The benefit on death prior to retirement is the accumulated values of a Member's Contribution Account and the Member's University Contribution Account except that amount represented by contributions made by the University between April 1, 1979 and December 31, 1984.

b. The University of Manitoba Pension Plan (1970)

   (i)   Retirement Benefits

        This Plan provides that, at retirement, the Member's Contribution
        Account and University Contribution Account are applied to purchase
        a life annuity from a licensed insurance company. The Plan provides
        that if the defined benefit pension based on a formula involving the
        member's years of service and highest average earning, exceeds the
        annuity from the Insurance Company, the difference (known as a
        supplementary pension) is paid from the Plan.

   (ii) Funding

        The Plan members contribute at the rate of 6% of salary less an
        adjustment for the Canada Pension Plan. The University matches these
        contributions. If an actuarial valuation reveals a deficiency in the fund,
        the Pension Benefits Act of the Province of Manitoba requires that the
        University makes additional contributions to fund the deficiency.

c. The University of Manitoba GFT Pension Plan (1986)

   (i)   Retirement Benefits

        This Plan, a defined contribution plan, provides that, at retirement, a
        life annuity is purchased from a licensed insurance company based on
        the accumulated value of the Member's Contribution Account.

   (ii) Funding

        The University contributes for each member an amount equal to the
        lesser of i) 6% of the floor salary of full professor rank at the University
        minus an adjustment for the Canada Pension Plan and ii) the maximum
        contribution permitted by the Income Tax Act (Canada). There are no
        member contributions.

d.   The University of Manitoba Pension Plan (1993)

   (i)   Retirement Benefits

This Plan provides that, at retirement, the Member's Contribution Account and University Contribution Account are applied to establish retirement income known as a plan annuity. The annuity is determined using a pension factor established by the University's Actuary and is paid from the Plan. The Plan provides that if the defined benefit pension based on a formula involving the member's years of service and highest average earnings exceeds the plan annuity, the difference (known as a supplementary pension) is paid from the Plan. The Plan provides for retirement benefits paid from the Plan to be increased using an excess earnings approach.

   (ii)   Funding

The Plan members contribute at the rate of 7% of salary less an adjustment for the Canada Pension Plan. The University matches these contributions. If an actuarial valuation reveals a deficiency in the fund, the Pension Benefits Act of the Province of Manitoba requires that the University makes additional contributions to fund the deficiency.

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[1] The provisions in the University Pension Plans documents may be amended by the University from time to time, and such changes shall prevail over those of the Investment Policy Statement to the extent that they are in conflict.

FUND OBJECTIVES

5.1   The basic objective of the Fund is to provide retirement benefits at a reasonable and predictable cost to both members and the Plan sponsor.

5.2   The overall objective is to maximize investment returns while assuming a level of risk deemed appropriate by the Pension Committee.

5.3   The minimum objective for the Fund is to achieve the long term total rate of return including capital gains, dividends and interest, but net of all investment management expenses, equal to the annual change in the Consumer Price Index for Canada, plus at least 4% per annum. Over a four year-period, the managers of the Fund are expected to achieve an above-median position relative to other similar funds measured by a recognized performance measurement service.

FUND INVESTMENTS

6.1   The Fund may only be invested in the following asset categories:

   i)   cash;
   ii)   demand or term deposits;
   iii)   short term notes;
   iv)   treasury bills;
   v)   bankers acceptances;
   vi)   commercial paper;
   vii)   investment certificates issued by banks, insurance companies or trust
           companies;
   viii)   bonds and non convertible debentures (including strips and residuals,
           retractable and extendable bonds, foreign pay bonds of Canadian issues or
           Supernationals);
   ix)   asset-backed securities (including mortgage backed securities);
   x)   convertible or exchangeable debentures;
   xi)   common and preferred stocks (including warrants and instalment receipts);
   xii)   pooled funds, closed-end investment companies and other structured vehicles
           invested in any or all of the above asset categories;
   xiii). mortgages.

6.2   The Fund may hold derivative financial, commodity or currency related instruments
        such as forward contracts, options, futures or swaps, provided that such participation
        is not for speculative purposes. Any derivative strategy must be approved in writing
        by the Pension Committee.

6.3   The Fund shall not engage in the following:

   i)   purchase of securities on margin;
   ii)   loans to individuals other than to arm's length parties guaranteed by a
          mortgage;
   iii)   short sales.

6.4   The Manager shall not borrow money, pledge or otherwise encumber any of the Fund's assets, except to the extent that temporary overdrafts occur in the normal course of day-to-day portfolio management. However, the Manager may borrow on behalf of the Fund in order to pay out benefits with the written approval of the Committee.

6.5   The Committee may enter into a written agreement with the Custodian for securities lending provided that readily marketable securities having a market value of at least 105% of the market value of the securities lent are maintained on a daily basis.

ASSET ALLOCATION

7.1   The Fund's target asset allocation is the following:

ASSET CLASS (GROUP)   MARKET VALUE OF FUND

	Min.	Target	Max.
Fixed Income	40%	45%	50%
Canadian Equities	30%	35%	40%
Non-Canadian Equities	8%	20%	20%

   (subject to Revenue Canada limits)

Short term investments will be included as a component of the asset classes listed above.

7.2   The target asset allocation has been determined in order to meet Fund objectives. It reflects a risk/return tradeoff which was assessed by the Committee on the basis of long term prospects in the capital markets taking into account the University Pension Plans' benefits, liabilities and financial situation with consideration given to all factors that may affect the funding, solvency and the ability of the Plan to meet its financial obligations.

7.3 The limits within which the asset allocation is to be maintained have been determined with the objective of restricting moves away from the target in order to control the level of risk assumed by the Fund without incurring undue transaction costs.

Should the asset allocation move beyond these limits, the Committee will be advised and action will be taken to correct the situation as soon as possible, taking into account the best interest of the Fund.

7.4 Short term investments include cash and fixed income investments having a maturity of less than one year when purchased.

7.5 Fixed income securities having a maturity of one year and more when issued include convertible debentures.

7.6 Canadian equities include common and preferred stocks of Canadian issuers.

7.7 U.S. stocks include common and preferred stocks of non-Canadian issuers.

International stocks include common and preferred stocks of non-Canadian and non-U.S. issuers.

7.8 Securities held in a pooled fund are classified on the basis of the assets comprising the major portion of such pooled funds.

7.9 Derivative instruments.along with any collateral held therein are included in the asset class comprising the securities whose return or price serves as the basis for the pricing of such derivative instruments.

INVESTMENT RISK

8.1   Diversification among asset classes is provided through the asset allocation guidelines set forth in the Statement.

8.2   Diversification within each asset class is provided by limiting to 10% or less the percentage of the market value of Fund assets invested in a single fixed income security not guaranteed by the Government of Canada or of a Canadian province and by restricting investments in a group of equities whose returns are expected to be highly correlated.

8.3   Liquidity is provided by restricting the use of private placements, by limiting to 10% or less the percentage of a single public issue to be held by the Fund, by limiting to 25% or less the percentage of the Fund to be invested in mortgages or other asset-backed securities, and by requiring that all stocks trade on a recognized exchange unless permission is obtained from the Committee.

8.4   Quality is provided by requiring that 90% of debt securities purchased by the Fund shall have a minimum credit rating of A by the Dominion Bond Rating Service (DBRS) or the equivalent, and that 90% of short-term paper have a minimum DBRS credit rating of R-1 or the equivalent.

8.5   Currency risk is controlled by limiting investments in non-Canadian equities and bonds and by limiting to 20% fixed income investments in foreign-pay bonds of Canadian debtors and Supranational Agencies where debt is considered Canadian content and where such bonds are not 100% hedged into Canadian dollars.

VOTING RIGHTS

9.1   The responsibility for exercising voting rights on Fund securities is delegated to the Manager(s), who shall at all times act prudently and in the best interests of the Fund and its beneficiaries.

9.2   The Manager(s) shall maintain a record of how voting rights have been exercised and report to the Committee.

9.3   In case of doubt as to the best interests of the Fund, the Manager is expected to request instructions from the Committee and act in accordance with such instructions.

MONITORING

10.1 The Manager shall immediately inform the Committee of any violation and shall annually supply a letter indicating compliance with the provisions of the Statement.

10.2 As of the end of each quarter, the market value of each Fund investment is calculated. Investments which are not regularly traded are valued by the Custodian according to a methodology acceptable to the Committee. Any such investment which may represent more than 1% of the market value of the Fund is valued by a qualified independent appraiser or by the Committee through a unanimous resolution at least every three years.

10.3 The Committee shall monitor the performance of each manager. At their discretion, the Committee will deal with unacceptable performance as deemed appropriate in the circumstances.

10.4 The manager shall report to the Committee to:

i) provide information concerning new developments, including personnel changes affecting the firm;

ii) review the transactions in the latest period and the assets held at the end of the period and explain how they relate to the strategy advocated;

iii) explain the latest performance;

iv) provide an economic outlook along with a strategy under such circumstances.

REVIEW

11.1   This Policy shall be reviewed by the Committee at least annually. In addition, a review will be conducted whenever a major change occurs such as the following:

(a)   a fundamental change in the benefit design of the University Pension Plans;

(b)   significant revisions to the expected long-term trade-off between risk and reward on key asset classes, normally dependent upon basic economic/political/social factors;

(c)   a major change in the actuarial calculation basis, the membership/liability distribution, or the contribution/expense expectation;

(d)   a significant shift in the financial risk tolerance of the University;

(e)   shortcomings of the Policy that emerge in its practical application, or substantive modifications that are recommended to the University by a Manager; and

(f)   applicable changes in legislation.

11.2   An in-depth review of this Policy shall be undertaken at the end of each four-year period.

APPENDIX B

WILKIE MODEL FOR ACTUARIAL USE

APPENDIX B

WILKIE'S MODEL FOR ACTUARIAL USE

The purpose of the Wilkie model is to present a stochastic investment model, which can be used for simulating future investment returns and system constraints. This idea was first developed by A. D. Wilkie for the Maturity Guarantees Working Party (MGWP), and later improved in the years 1980, 1981, and 1994. Wilkie refers to his most recent model as a minimum model, which may be used to describe the total investments of a life office or a pension fund.

Since, the actuary's usual projection is many years into the future; he/she is usually content to progress there by annual steps. Therefore, it is desirable to have a stochastic model that describes how appropriate investment variables will move over the long-term, without being too concerned with short-term fluctuations. For this very reason, a wide range of investment areas were identified and accordingly, parameters were defined for future years.

General Features of the Model

A great deal of actuarial thought was developed when the main investments of insurance companies were in fixed interest loans and fixed income securities. Both of these investments provided low yields at a time when long-term inflation was virtually nil. However, the middle 1950 's saw a great deal of investments in ordinary shares, increases in fixed interest rates, and thus, inflation became an important issue to consider while dealing with long-term investments.

The actuary should not only be interested in the average return that may be achieved on investments, but in the range of possible returns. The present Goal programming model was not able to address this approach, as the QSB+ software would not deal with ranges with great efficiency.

The classic models to describe the stochastic movement of ordinary share prices have been that of a random walk. The Wilkie model shows that, over a long term period, a model based on dividends and dividend yields is more appropriate. Thus, the Wilkie model presented in 1994 has functions of dividends and dividend yields within its price variability.

For many purposes, one wishes to forecast both inflation and company dividends and share prices in a consistent way. It is, therefore, appropriate to relate company dividends directly in some way to the index being used as a measure of general prices, which in Canada is the Consumer Price Index (CPI).

Wilkie restricted himself to yields on long-term Government securities. In reality, a complete structure of interest rates varies by term-to-maturity, by the level of coupon payments and, by the characteristics of the borrower.

While it is not easy to measure what "the market" expects inflation to be, one can expect the influence to be historic. It is plausible to assume that the market's estimate of inflation over a long time period does not change rapidly in response to short term price changes. Therefore, in Wilkie's model it is hypothesised that the yields on Consols (bonds that pay inperpetuity) respond with a considerable time lag to changes in the rate of inflation.

A fully comprehensive model should also include overseas shares, which would require a study of exchange rates.

I have used @Risk to run simulations for the next five years with 100,000 iterations. I used the initial conditions as suggested by A. D. Wilkie in his model in a Canadian investment scenario. The idea behind running the simulation is to observe how the rate of returns might look over a short-term period of 5 years. The output is attached at the end.

Wilkie's model is a cascade model and as can be seen below, where the arrows indicate the direction of influence.
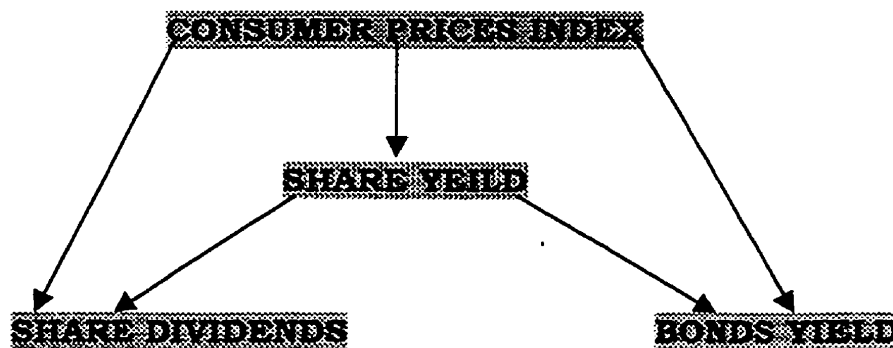
Thus, the Consumer Prices Index series, Q(t), is described first, entirely in terms of its own previous values, and the values of a random "white noise" series. White noise is the name given by electrical engineers to a sequence of independent identically distributed random variables, which thus have no single dominant frequency, and so bear the same relation to sound as white light does to light.

1. The model for Q(t) is

$$Q(t) = Q(t-1) \times \exp\{ I(t) \}$$

So that I(t) = ln Q(t) - ln Q(t - 1) is the rate (strictly force) of inflation over the year (t - 1, t):

$$I(t) = QMU + QA \times [I(t-1) - QMU] + QE(t)$$

$$QE(t) = QSD \times QZ(t)$$

$$QZ(t) \sim iid\ N(0,1)$$

This model says that the annual rate of inflation follows a first order auto-regressive process, with a fixed mean QMU, and a parameter QA such that the expected rate of inflation each year is equal to the mean plus QA times last year's deviation from the mean.

Appropriate values for the parameters:

QMU = 0.0165(adjusted to 1996 CPI), QA = 0.64, QSD = 0.032

2. Y(t) is the dividend yield on ordinary shares. The dividend yield depends on the current level of inflation, on previous values of itself, and white noise series.

$$Y(t) = \exp\{YW \times I(t) + \ln YMU + YN(t)\}$$

Or $\qquad$ ln Y(t) = YW $\times$ I(t) + in YMU + YN(t)

With:

$$YN(t) = YA \times YN(t-1) + YE(t)$$

$$YE(t) = YSD \times YZ(t)$$

$$YZ(t) \sim iid\ N(0,1).$$

The suggested parameter values are:

YW = 1.17, YA = 0.7, YMU% = 3.75%, YSD = 0.19.

3. The original model for share dividends, where D(t) is the value of the dividend index on ordinary shares at time t, is:

$$D(t) = D(t-1) \times \exp\{DW \times DM(t) + DX.I(t) + DMU +$$

$$DY \times YE(t-1) + DB \times DE(t-1) + DE(t)\}$$

$$DM(t) = DD \times I(t) + (1 - DD) \times DM(t-1)$$

$$DE(t) = DSD \times DZ(t)$$

$$DZ(t) \sim \text{iid } N(0,1).$$

The suggested parameters are:

$$DW = 0.19, DD = 0.26, DMU = 0.001, DY = -0.11, DB = 0.58, DSD = 0.07$$

4. The original model for the yield on "bonds", i.e. a perpetual fixed interest stock where $C(t)$ is the yield on bonds at time t, is:

$$C(t) = CW.CM(t) + CMU \times \exp\{CN(t)\}$$

$$CM(t) = CD \times I(t) + [1 - CD] \times CM(t-1)$$

$$CN(t) = CA1 \times CN(t-1) + CA2 \times CN(t-2) + CA3 \times CN(t-3) + CY \times YE(t) + CE(t)$$

$$CE(t) = CSD \times CZ(t)$$

$$CZ(t) \sim \text{iid } N(0,1)$$

The suggested parameters are:

$$CW = 1.0, CD = 0.04, CA = 0.95, CMU\% = 3.7\%, CY = 0.1, CSD = 0.185.$$

5. Short-term interest rates are clearly connected with long-term ones. One approach would be to model the spread:

$$C(t) - B(t)$$

Where $B(t)$ is the value of bank rate at time t; another would be to model the difference between the logarithms:

$$\ln C(t) - \ln B(t) = -\ln [B(t)/C(t)]$$

Thus, we define the short-term rate of interest at time t as $B(t)$ and define:

$$B(t) = C(t) \times \exp\{-BD(t)\}$$

Where:

$$BD(t) = BMU + BA \times [BD(t-1) - BMU] + BE(t)$$

$$BE(t) = BSD \times BZ(t)$$

$$BZ(t) \sim \text{iid } N(0,1)$$

The suggested parameters are:

$$BMU = 0.26, \; BA = 0.38, \; BC * = 0.73, \; BSD = 0.21$$

* for Canada the value of BC is strongly significant so it is included in the model.

The extra component is added to the BD formula:

$$BD = BC \times CSD \times \text{long-term white noise.}$$

In order to obtain the return on equities the following formula was used:

$$LPR(t) = \exp\{K(t)\} \times \exp[YW \times QMU] \times YMU \times [1 + Y(t)]$$

In order to obtain return on bonds the following formula is used:

$$LCR(t) = [1 + C(t)/C(t)] \times [QMU + CMU] - 1$$

In order to obtain cash the following formula is used:

$$LBR(t) = \exp\{-BMU\} \times (QMU + CMU)$$

WILKIE MODEL FOR CANADIAN PARAMETERS

	1996	1997	1998	1999	2000	
Cash & Short-term Notes						
Minimum	1.69%	1.51%	1.43%	1.30%	1.02%	
Maximum	10.10%	14.64%	16.11%	14.21%	18.11%	
Mean	4.26%	4.32%	4.37%	4.43%	4.46%	*4.37%*
Standard deviation	1.07%	1.27%	1.42%	1.54%	1.88%	
Variance	0.01%	0.02%	0.02%	0.02%	0.03%	
5%	2.75%	2.59%	2.48%	2.40%	2.33%	
10%	3.02%	2.87%	2.78%	2.70%	2.64%	*2.80%*
Bonds						
Minimum	3.58%	3.03%	2.58%	2.16%	2.00%	
Maximum	9.25%	11.32%	12.11%	14.87%	18.37%	
Mean	5.42%	5.47%	5.51%	5.56%	5.60%	*5.51%*
Standard deviation	0.71%	1.00%	1.22%	1.42%	1.58%	
Variance	0.01%	0.01%	0.01%	0.02%	0.03%	
5%	4.37%	4.06%	3.82%	3.65%	3.51%	
10%	4.56%	4.29%	4.11%	3.97%	3.84%	*4.15%*
Canadian Stocks						
Minimum	2.99%	2.70%	5.26%	2.70%	2.99%	
Maximum	18.27%	25.75%	36.14%	25.75%	18.27%	
Mean	7.54%	9.76%	14.03%	9.76%	7.54%	*9.73%*
Standard deviation	2.32%	2.99%	4.21%	2.99%	2.32%	
Variance	0.05%	0.09%	0.18%	0.09%	0.05%	
5%	4.19%	5.87%	8.18%	5.87%	4.19%	
10%	4.84%	6.39%	9.07%	6.39%	4.84%	*6.31%*
Foreign Stocks - US						
Minimum	2.95%	3.34%	11.19%	5.97%	2.95%	
Maximum	25.96%	20.83%	63.66%	42.11%	25.96%	
Mean	11.13%	8.61%	26.69%	15.88%	11.13%	*14.69%*
Standard deviation	3.52%	2.75%	8.11%	4.87%	3.52%	
Variance	0.12%	0.08%	0.66%	0.24%	0.12%	
5%	6.61%	4.75%	15.71%	9.15%	6.61%	
10%	7.25%	5.41%	17.58%	10.20%	7.25%	*9.54%*

APPENDIX C

QSB+ INPUT AND OUTPUT RESULTS

```
MIN  +0        X1   +0        X2   +0        X3   +0        X4   +1.00000D1-
     +0        D1+
Subject to
(1)  +1.00000X1  +1.00000X2  +1.00000X3  +1.00000X4  +0        D1-
     +0        D1+  = +1.00000
(2)  +0        X1   +0        X2   +0        X3   +1.00000X4  +0        D1-
     +0        D1+  ≤ +.200000
(3)  +1.00000X1  +0        X2   +0        X3   +0        X4   +0        D1-
     +0        D1+  ≥ +.058300
(4)  +0        X1   +1.00000X2  +0        X3   +0        X4   +0        D1-
     +0        D1+  ≤ +.500000
(5)  +0        X1   +1.00000X2  +0        X3   +0        X4   +0        D1-
     +0        D1+  ≥ +.400000
(6)  +0        X1   +0        X2   +1.00000X3  +0        X4   +0        D1-
     +0        D1+  ≤ +.400000
(7)  +0        X1   +0        X2   +1.00000X3  +0        X4   +0        D1-
     +0        D1+  ≥ +.300000
(8)  +.042600X1  +.054200X2  +.075400X3  +.111300X4  +1.00000D1-
     -1.00000D1+  = +.083000
```

	Summarized Solution for MODEL 96			Page : 1		
Number	Variable	Solution	Opportunity Cost-Obj. 1	Opportunity Cost-Obj. 2	Opportunity Cost-Obj. 3	Opportunity Cost-Obj. 4
1	X1	+.05830000	0			
2	X2	+.40000001	0			
3	X3	+.34169999	0			
4	X4	+.20000000	0			
5	D1-	+.01081224	0			
6	D1+	0	+10.000000			

Priority Level 1: Minimized Objective Function (Goal) = +.01081224
Iteration = 17   Elapsed CPU second = .1601563

```
MIN  +0        X1   +0        X2   +0        X3   +0        X4   +1.00000D1-
     +0        D1+
MAX  +.028000X1  +.041500X2  +.063100X3  +.095400X4  +0        D1-
     +0        D1+
Subject to
(1)  +1.00000X1  +1.00000X2  +1.00000X3  +1.00000X4  +0        D1-
     +0        D1+ = +1.00000
(2)  +0        X1   +0        X2   +0        X3   +1.00000X4  +0        D1-
     +0        D1+ ≤ +.200000
(3)  +1.00000X1  +0        X2   +0        X3   +0        X4   +0        D1-
     +0        D1+ ≥ +.058300
(4)  +0        X1   +1.00000X2  +0        X3   +0        X4   +0        D1-
     +0        D1+ ≤ +.500000
(5)  +0        X1   +1.00000X2  +0        X3   +0        X4   +0        D1-
     +0        D1+ ≥ +.400000
(6)  +0        X1   +0        X2   +1.00000X3  +0        X4   +0        D1-
     +0        D1+ ≤ +.400000
(7)  +0        X1   +0        X2   +1.00000X3  +0        X4   +0        D1-
     +0        D1+ ≥ +.300000
(8)  +.042600X1  +.054200X2  +.075400X3  +.111300X4  +1.00000D1-
     -1.00000D1+ = +.083000
```

			Summarized Solution for MODEL 2(96)			Page : 1

Number	Variable	Solution	Opportunity Cost-Obj. 1	Opportunity Cost-Obj. 2	Opportunity Cost-Obj. 3	Opportunity Cost-Obj. 4
1	X1	+.05830000	0	0		
2	X2	+.40000001	0	0		
3	X3	+.34169999	0	0		
4	X4	+.20000000	0	0		
5	D1-	+.01081224	0	0		
6	D1+	0	+10.000000	0		

```
   Priority Level 1:  Minimized Objective Function (Goal) = +.01081224
   Priority Level 2:  Maximized Objective Function (Goal) = +.05887367
             Iteration = 18   Elapsed CPU second = .171875
```

```
MIN  +0       X1  +0      X2  +0       X3  +0      X4  +1.00000D1-
     +0       D1+
MAX  +.028000X1  +.041500X2  +.063100X3  +.095400X4  +0       D1-
     +0       D1+
Subject to
(1)  +1.00000X1  +1.00000X2  +1.00000X3  +1.00000X4  +0       D1-
     +0       D1+ = +1.00000
(2)  +0       X1  +0      X2  +0       X3  +1.00000X4  +0       D1-
     +0       D1+ ≤ +.200000
(3)  +1.00000X1  +0      X2  +0       X3  +0      X4  +0       D1-
     +0       D1+ ≥ +.050000
(4)  +0       X1  +1.00000X2  +0       X3  +0      X4  +0       D1-
     +0       D1+ ≤ +.500000
(5)  +0       X1  +1.00000X2  +0       X3  +0      X4  +0       D1-
     +0       D1+ ≥ +.400000
(6)  +0       X1  +0      X2  +1.00000X3  +0      X4  +0       D1-
     +0       D1+ ≤ +.400000
(7)  +0       X1  +0      X2  +1.00000X3  +0      X4  +0       D1-
     +0       D1+ ≥ +.300000
(8)  +.043200X1  +.054700X2  +.097600X3  +.086100X4  +1.00000D1-
     -1.00000D1+ = +.083000
```

			Summarized Solution for MODEL 97		Page : 1	
Number	Variable	Solution	Opportunity Cost-Obj. 1	Opportunity Cost-Obj. 2	Opportunity Cost-Obj. 3	Opportunity Cost-Obj. 4
1	X1	+.05000000	0	0		
2	X2	+.40000001	0	0		
3	X3	+.40000001	0	0		
4	X4	+.14999999	0	0		
5	D1-	+.00700500	0	0		
6	D1+	0	+10.000000	0		

Priority Level 1: Minimized Objective Function (Goal) = +.00700500
Priority Level 2: Maximized Objective Function (Goal) = +.05755000
Iteration = 19   Elapsed CPU second = .1601563

```
MIN   +0        X1   +0       X2   +0       X3   +0       X4   +1.00000D1-
      +0        D1+
MAX   +.028000X1 +.041500X2   +.063100X3   +.095400X4   +0       D1-
      +0        D1+
Subject to
(1)   +1.00000X1 +1.00000X2   +1.00000X3   +1.00000X4   +0       D1-
      +0        D1+ = +1.00000
(2)   +0        X1   +0       X2   +0       X3   +1.00000X4   +0       D1-
      +0        D1+ ≤ +.200000
(3)   +1.00000X1 +0       X2   +0       X3   +0       X4   +0       D1-
      +0        D1+ ≥ +.049000
(4)   +0        X1   +1.00000X2   +0       X3   +0       X4   +0       D1-
      +0        D1+ ≤ +.500000
(5)   +0        X1   +1.00000X2   +0       X3   +0       X4   +0       D1-
      +0        D1+ ≥ +.400000
(6)   +0        X1   +0       X2   +1.00000X3   +0       X4   +0       D1-
      +0        D1+ ≤ +.400000
(7)   +0        X1   +0       X2   +1.00000X3   +0       X4   +0       D1-
      +0        D1+ ≥ +.300000
(8)   +.043700X1 +.055100X2   +.140300X3   +.266900X4   +1.00000D1-
      -1.00000D1+ = +.083000
```

Number	Variable	Solution	Opportunity Cost-Obj. 1	Opportunity Cost-Obj. 2	Opportunity Cost-Obj. 3	Opportunity Cost-Obj. 4
		Summarized Solution for MODEL 98 Page : 1				
1	X1	+.04900000	0	0		
2	X2	+.40000001	0	0		
3	X3	+.35099998	0	0		
4	X4	+.20000000	0	0		
5	D1-	0	+1.0000000	0		
6	D1+	+.04380661	0	0		

```
Priority Level 1:  Minimized Objective Function (Goal) =           0
Priority Level 2:  Maximized Objective Function (Goal) = +.05920010
           Iteration = 13   Elapsed CPU second = 5.078125E-02
```

EXHIBITS

EXHIBIT I

## The University Of Manitoba Pension Plans
Investment Income Statements

Investment Asset Mix	Year 1992	Year 1993	Year 1994	Year 1995	Year 1996
Cash & ST Notes	$30,173,489	$25,204,876	$18,331,519	$32,956,342	$51,432,144
Bonds (G & C)	$149,051,372	$184,747,521	$188,114,299	$209,301,573	$224,512,667
Stock - CDN	$117,277,214	$136,362,677	$146,909,652	$171,096,201	$209,579,609
Stock - Foreign	$60,044,426	$81,798,941	$84,963,035	$100,127,118	$118,352,839
Real Estate fund	$5,959,765	$5,311,869	$4,336,015	$2,608,094	$0
Mortgage Fund	$1,947,344	$2,211,814	$0	$0	$0
TOTAL	$364,453,610	$435,637,698	$442,654,520	$516,089,328	$603,877,259

Inv Income Asset Mix	Year 1992	Year 1993	Year 1994	Year 1995	Year 1996
Cash & ST Notes	$1,690,682	$1,592,174	$1,823,785	$1,992,651	$2,453,580
Bonds (G & C)	$13,329,130	$13,997,376	$13,742,616	$13,543,933	$13,058,812
Stock - CDN	$3,941,373	$3,287,447	$3,623,509	$2,931,646	$3,028,335
Stock - Foreign	$1,398,636	$1,759,564	$2,561,517	$1,529,699	$2,799,541
Real Estate Fund	$189,283	$251,436	$161,716	$268,688	$0
Mortgage Fund	$196,526	$279,445	$78,308	$0	$0
TOTAL	$20,745,630	$21,167,442	$21,991,451	$20,266,617	$21,340,268

Asset Mix at MV	1992	1993	1994	1995	1996
Cash & ST Notes	8.20%	5.60%	4.10%	6.20%	8.50%
Bonds (G & C)	40.40%	42.10%	42.30%	40.30%	37.10%
Stock - CDN	31.80%	31.00%	33.00%	32.90%	34.60%
Stock - Foreign	16.30%	18.60%	19.10%	19.30%	19.50%
Real Estate Fund	1.60%	1.00%	1.00%	0.50%	0.00%
Mortgage Fund	0.50%	0.50%	0.00%	0.00%	0.00%
Actual Contributions	$18,171,370	$20,397,829	$21,452,852	$21,587,577	$20,989,969
Wtd Contributions					$20,975,049

DEMAND ESTIMATION USING HOLT'S METHOD
EXHIBIT II

Year	Actual outflows	Base Level	Trend	Predicted outflows
1974	1,317,838	1,317,838	0	0
1975	1,601,341	1,459,590	70,876	1,317,838
1976	2,319,368	1,924,917	268,101	1,530,465
1977	2,156,964	2,174,991	259,088	2,193,018
1978	2,982,384	2,708,231	396,164	2,434,079
1979	3,591,205	3,347,800	517,867	3,104,396
1980	4,081,867	3,973,767	571,917	3,865,667
1981	4,244,602	4,395,143	496,646	4,545,683
1982	3,761,076	4,326,432	213,968	4,891,789
1983	5,440,254	4,990,327	438,931	4,540,400
1984	6,300,195	5,864,727	656,665	5,429,259
1985	9,488,598	8,004,995	1,398,467	6,521,392
1986	14,229,631	11,816,547	2,605,009	9,403,462
1987	17,354,793	15,888,174	3,338,318	14,421,556
1988	19,027,255	19,126,874	3,288,509	19,226,493
1989	17,700,023	20,057,703	2,109,669	22,415,383
1990	17,142,055	19,654,714	853,340	22,167,372
1991	18,122,136	19,315,095	256,860	20,508,053
1992	15,038,947	17,305,451	-876,392	19,571,955
1993	19,484,064	17,956,562	-112,640	16,429,059
1994	12,246,467	15,045,194	-1,512,004	17,843,921
1995	25,174,506	19,353,848	1,398,325	13,533,190
1996	30,292,182	25,522,178	3,783,327	20,752,173
1997				29,305,505
1998				33,088,832
1999				36,872,159
2000				40,655,486

	0.500		MSE	$17,540,517,094,938
	0.500			

DEMAND
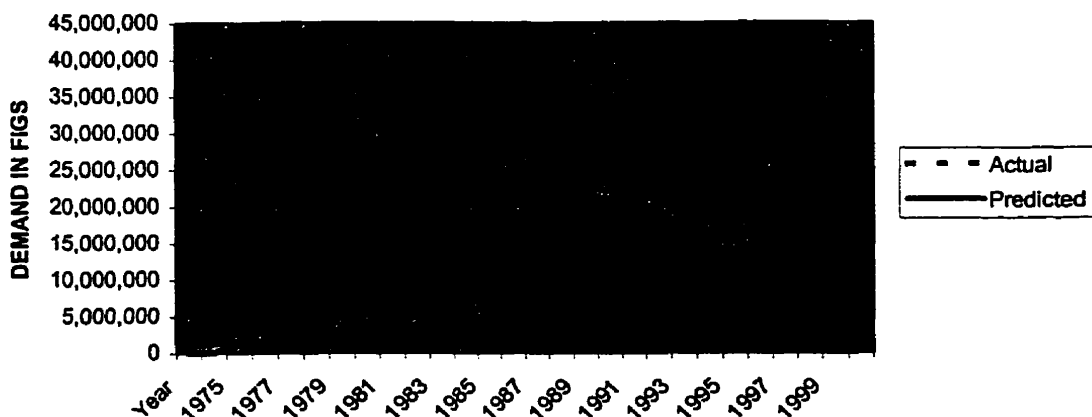
DEMAND ESTIMATION USING HOLT'S METHOD
EXHIBIT IIA

Year	Out Flows	Base Level	Trend	Predicted outflows
1974	1,317,838	1,317,838	0	0
1975	1,601,341	1,545,697	20,019	1,317,838
1976	2,319,368	2,171,447	73,237	1,565,716
1977	2,156,964	2,174,181	67,043	2,244,684
1978	2,982,384	2,836,915	119,379	2,241,224
1979	3,591,205	3,466,589	164,212	2,956,293
1980	4,081,867	3,993,335	196,064	3,630,801
1981	4,244,602	4,233,767	199,962	4,189,399
1982	3,761,076	3,893,099	152,463	4,433,729
1983	5,440,254	5,166,514	250,947	4,045,563
1984	6,300,195	6,126,939	313,280	5,417,462
1985	9,488,598	8,890,284	528,537	6,440,219
1986	14,229,631	13,285,401	868,245	9,418,822
1987	17,354,793	16,726,495	1,094,290	14,153,646
1988	19,027,255	18,790,458	1,179,483	17,820,785
1989	17,700,023	18,145,546	1,019,196	19,969,941
1990	17,142,055	17,539,053	876,367	19,164,742
1991	18,122,136	18,179,700	855,657	18,415,420
1992	15,038,947	15,823,333	573,456	19,035,357
1993	19,484,064	18,878,116	791,460	16,396,789
1994	12,246,467	13,703,421	267,288	19,669,576
1995	25,174,506	22,975,507	1,058,428	13,970,708
1996	30,292,182	29,063,859	1,500,345	24,033,934
1997				30,564,204
1998				32,064,548
1999				33,564,893
2000				35,065,237

	0.804		MSE	$13,807,132,408,564
	0.088			

DEMAND
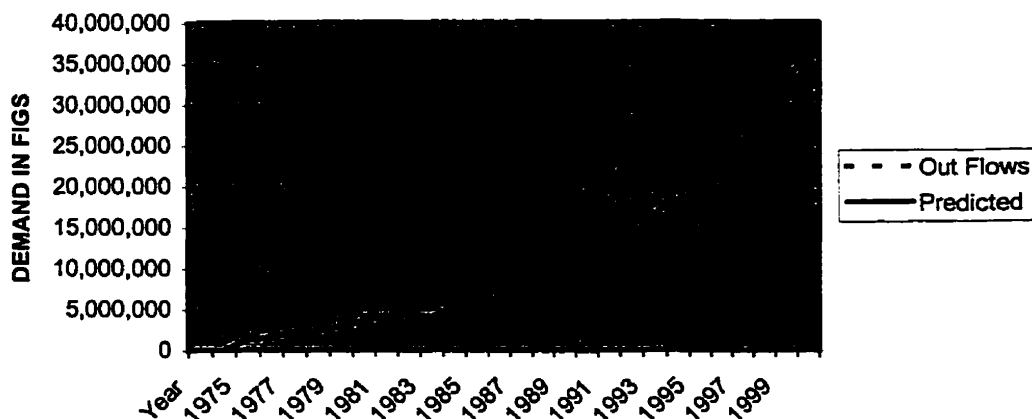
COMPARISION OF FUND BALANCE

EXHIBIT A

MODEL 96

	QSB⁺Results	Actual Holdings	Contrib 96	Demand 96
Cash & ST Notes =	30,147,828	32,956,342		
Bonds =	207,916,055	209,301,573		
Stock Canadian =	177,768,227	171,096,201		
Stock Foreign =	103,958,027	100,127,118		
TOTAL =	519,790,137	513,481,234	20,989,969	30,292,187

	Actual I.I	Net I.I	Realized Gains	Unrealized Gains	o/s as of 1.1.97
Cash & ST Notes =	2,453,580	2,244,488			
Bonds =	13,058,812	12,972,366			
Stock Canadian =	3,028,335	3,146,427			
Stock Foreign =	2,799,541	2,906,653			
TOTAL =	21,269,934	34,194,383	42,020,078		607,972,314

Actual = 606,184,388

Difference -1,787,926

FORECASTED FUND BALANCES

EXHIBIT B

MODEL 97

	QSB⁺Results	End of year	Wtd Contributions	Estimated Demand	o/s as of 1.1.98
Cash & ST Notes =	30,309,219	21,824,487			
Bonds =	242,473,755	255,737,070			
Stock Canadian =	242,473,755	266,139,194			
Stock Foreign =	87,896,736	95,464,645			
TOTAL =	606,184,388	639,165,395	20,975,049	30,564,204	639,165,395
				Gains	76,214,461
				Total	715,379,856
				Actual =	661,000,000
				Difference	-54,379,856

MODEL 98

	QSB⁺Results	End of year	Wtd Contributions	Estimated Demand	o/s as of 1.1.99
Cash & ST Notes =	32,389,000	22,475,186			
Bonds =	264,400,000	278,968,440			
Stock Canadian =	232,011,000	264,562,143			
Stock Foreign =	132,200,000	167,484,180			
TOTAL =	661,000,000	733,489,949	20,975,049	32,064,548	733,489,949
				Gains	76,214,461
				Total	809,704,410

ILLUSTRATION

Define and solve a problem by using Solver

1 On the **Tools** menu, click **Solver**.

   If the **Solver** command is not on the **Tools** menu, you need to install the Solver add-in.

   ≫ How?

2 In the **Set Target Cell** box, enter a cell reference or name for the target cell. The target cell must contain a formula.

3 To have the value of the target cell be as large as possible, click **Max**.

   To have the value of the target cell be as small as possible, click **Min**.

   To have the target cell be a certain value, click **Value of**, and then type the value in the box.

4 In the **By Changing Cells** box, enter a name or reference for each adjustable cell, separating nonadjacent references with commas. The adjustable cells must be related directly or indirectly to the target cell. You can specify up to 200 adjustable cells.

   To have Solver automatically propose the adjustable cells based on the target cell, click **Guess**.

5 In the **Subject to the Constraints** box, enter any constraints you want to apply.

6 Click **Solve**.

7 To keep the solution values on the worksheet, click **Keep Solver Solution** in the **Solver Results** dialog box.

   To restore the original data, click **Restore Original Values**.

Tips

● You can interrupt the solution process by pressing ESC. Microsoft Excel recalculates the worksheet with the last values found for the adjustable cells.

● For more information about options in the **Solver Parameters** dialog box, click ≫

● For more information about options in the **Solver Results** dialog box, click ≫

● For information about the algorithms used by Solver, click ≫

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IMAGE EVALUATION
TEST TARGET (QA-3)
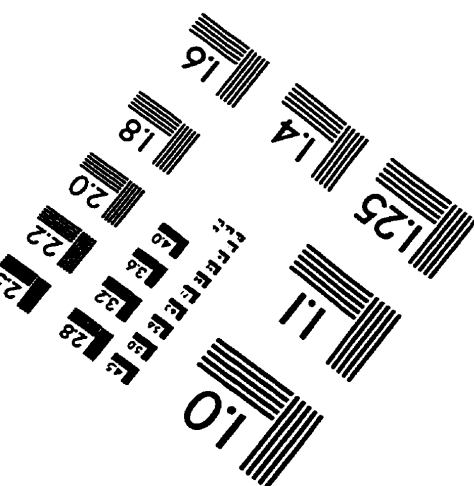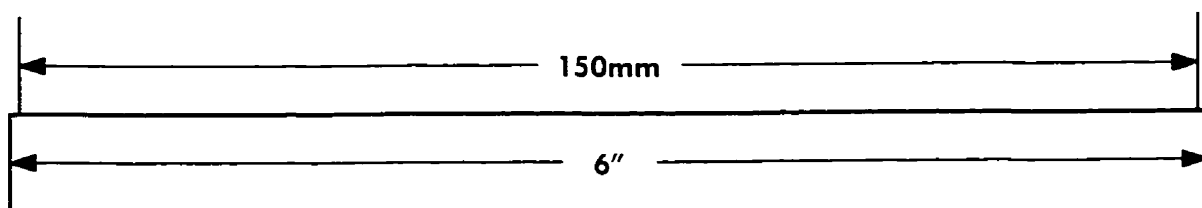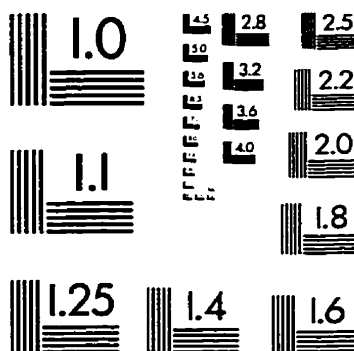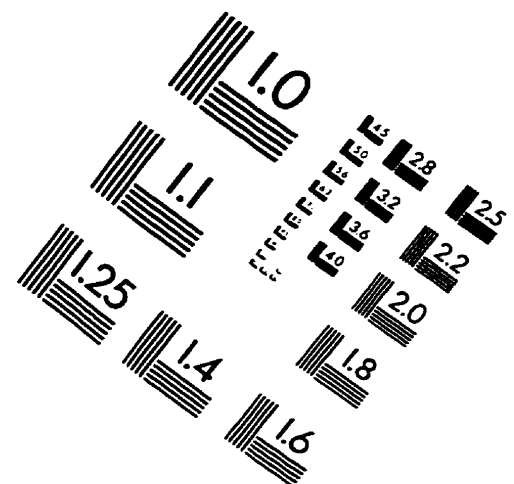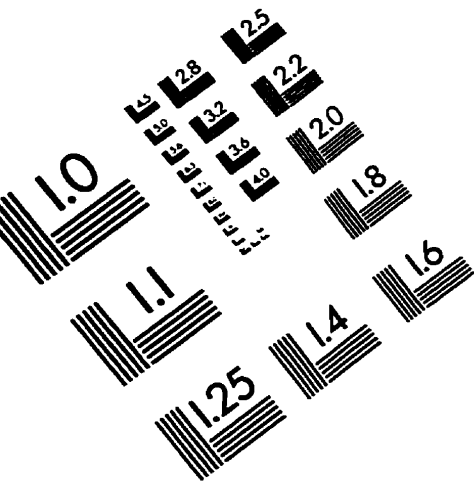
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