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WIDTH:DEPTH REGIME ANALYSIS FOR ALLUVIAL RIVERS

Submitted by

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in Civil Engineering

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BY

PHILLIP BRUCE SHEWFELT

A thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

MASTER OF SCIENCE

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ABSTRACT

This thesis is concerned with the hydraulic geometry of wide straight alluvial rivers, not including braided reaches. In particular, Tamburi's (1979) simple non-linear equation relating width and depth, is investigated to determine if it can be used to represent equilibrium channel geometry (i.e., regime). The width:depth equation attempts to account for deviations, from the traditional regime methods, resulting from varying bank conditions. The engineering applications claimed for the width:depth equation are dependent on the veracity of Tamburi's claim that the equation describes equilibrium conditions.

Analytical methods are employed herein to understand the theoretical underpinnings of the width:depth equation, as well as to investigate the potential sensitivity of the equation to the reference flow condition (e.g., bed sediment concentration), and to the transition between flow regimes. In addition, the relationship of the width:depth equation to a commonly used sediment transport equation is defined.

The independent variables that control the exponent and coefficient of the width:depth equation are defined explicitly herein. A two-dimensional analytical model is used to quantify these underlying relationships. The results of the analysis reveal that width:depth equations can be used to represent equilibrium channel geometries, inclusive of the transition between the flow regimes. The conditions required to maintain this equilibrium are quantified. The sensitivity of the width:depth equations to the reference flow condition, and to sediment size, is presented in graphical format. Lastly, equations are presented which relate the width:depth equation to a simple non-linear bed material load equation.

Empirical investigations are undertaken, using data from the Middle Loup, Niobrara, and Rio Grande Rivers, and provide results which generally verify the analytical model.

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CHAPTER 1

INTRODUCTION

1.1 Hydraulic Geometry: Traditional Regime Methods versus Width:Depth Analysis

This thesis is concerned with the hydraulic geometry (i.e., width, depth and slope) of wide, straight alluvial rivers. For the purpose of this thesis *alluvial rivers* are defined as:

> "... being comprised of alluvial bed material, meaning material in the silt to gravel size range, of sufficient depth to represent a free surface"[1]

The ability to predict the hydraulic geometry of alluvial rivers is a basic requirement of river engineering.

Two approaches[2] for establishing relationships for the hydraulic geometry of alluvial rivers are (Paris, 1984a):

1. Regime - methods based on using observed data to derive relationships between the variables.
2. Analytical - methods based on equations describing the dominant processes (e.g., sediment transport, flow resistance and bank stability).

WCHL (1978a) further defined *regime analysis*, and Blench (1986) defined the condition of being in *regime*, as follows:

> "*regime analysis* consists of the development of empirical equations from data collected on canals and rivers which have obtained *equilibrium geometry* for their spectra of water and sediment discharges." (WCHL, 1978a)

> "*regime*, applied to a channel will imply having acquired *long-term average steady state* in boundaries of its transported sediment (i.e., equilibrium geometry)." (Blench, 1986)

From the above definitions it is concluded that a channel in *regime* will have attained an *equilibrium hydraulic geometry*. Equilibrium geometry in turn requires *equilibrium sediment transport*, as per ASCE's (1975) definition of the behavior of natural streams over an extended period of time:

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1 This definition does not qualify the bank material size or freedom to erode.

2 A third approach is the use of physical hydraulic models (e.g., WCHL, 1977).

> "The stream (i.e., hydraulic geometry) must evolve so that ... each reach (and each section) transmits as much water and sediment as delivered to its upstream end"

It is important to recognize that equilibrium does not require that the hydraulic geometry at a particular river section is never changing but that the reach under study is maintaining an overall balance with its sediment transport. Any changes that do occur in the hydraulic geometry must maintain this sediment equilibrium. This thesis is concerned with short-term equilibrium, which is a valid base for many engineering applications (ASCE, 1975), rather then long-term or cyclic time (i.e., graded stream). As will be shown in Chapter 4, sediment and fluid properties (e.g., sediment size, gradation) can be considered to be independent variables for the short-term. Over the longer-term (i.e., cyclic time) these variables become dependent on the sediment supply.

Two *equilibrium states* were defined by Komura (1963), as follows:

a. *dynamic equilibrium*

> - the state of full sediment transport where shear stress is greater than critical and neither aggradation or degradation is occuring, requiring that sediment load passing through the upstream of the reach be equal to that of all points along the reach. Henderson (1963) relates this condition to being in *regime*. Komura (1963) suggests that dynamic equilibrium applies to *live-bed* channels in fine-grained material.

b. *static equilibrium*

> - that state associated with no sediment transport, as the shear stress along the reach is everywhere just equal to the critical or tractive shear stress for that sediment. Henderson (1963) refers to this as the tractive force condition. Komura (1963) suggests that this condition is most applicable to channels formed in coarse-grained material. It may also result from a cut-off of sediment supply to the upstream end of a reach.

Therefore, *static equilibrium* is associated with a flow condition described by a bed sediment concentration, X_{bm}, equal to 0 PPM. *Dynamic equilibrium* requires that bed sediment concentration, X_{bm}, equal a value greater than 0 PPM. These definitions presume a two-dimensional approach (i.e., vertical/downstream) where the banks are assumed to be stable, and the transport of bed material is concerned only with the interaction of the bed and the flow (Stevens, 1989). Several mathematical models used to study alluvial rivers employ such an

assumption (e.g., Lopez-Garcia, 1977; Krishnappan, 1981). This is a simplification of a complex three-dimensional flow phenomena[3], where equilibrium may involve deposition and erosion of bank materials.

Traditional regime analyses (i.e., Lacey, see Blench, 1969) predict the equilibrium depth, d, width, W, and slope, S, as a function of discharge, Q [i.e., d, W, S = f (Q)]. These relations apply to conditions along a river rather than at a station. Typically, as one goes downstream, encountering larger discharges, the relations predict wider, deeper and more gently sloped river reaches. Inherent to the development of such relations is the choice of the *reference flow condition* (e.g., discharge) for each reach. The traditional regime methods have successfully related channel geometry (i.e., d, W, S) to the so-called dominant flow conditions[4] (e.g., dominant discharge, Q_{DOM}), despite the influence of flow variability (Wolman and Yu, 1987).

Consider next a *discharge defined reach,* which is described as follows:

> "A reach of river which has insignificant local inflow of water or sediment in relation to that entering its upstream boundary."

The traditional regime methods predict that a *discharge defined reach,* having constant bed and bank material, will have an optimal width, depth and slope. However, field conditions may dictate that the dependent variables are constrained from meeting the optimal situation. Width, in particular, is often constrained by such factors as vegetation, cohesive bank materials and/or river training works (Stevens, 1989; White et al. 1982). Modified regime equations, such as those of Thorne and Hey (1986) and Blench (1969), attempt to account for constraints on river width.

In a separate and independent analysis, Tamburi (1976, 1979; WCHL, 1977, 1978a) proposed a method to establish depth (and area) for a single discharge defined reach, employing width as an independent variable. This method was referred to as *width:depth analysis*. Width:depth analysis is not a replacement for the traditional methods

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3 For example, Parker (1978a) studied the impact of lateral diffusion of suspended sediment on the formation of equilibrium channel geometry.
4 For example, Morisawa (1985) stated that the channel forming flow is the frequent bankfull discharge (i.e., 1.5 yr. recurrence interval).

since it is not predictive of width (or slope). It might be considered to be an addendum to the traditional methods, in which one considers the constraints to attaining the optimal width, and where the results are presented in a fashion that is useful for certain engineering applications.

Width:depth analysis involves the application of statistical methods to a data base in order to derive equations of the form of equations (1.1) and (1.2) (Tamburi et al., 1983)[5].

$$d = K\,W^{(-n)} \qquad \text{(1.1)}$$

$$A = K\,W^{(1-n)} \qquad \text{(1.2)}$$

where d = mean flow depth
W = top width
A = area of flow (= W d)
K = width:depth coefficient
n = width:depth exponent

Tamburi (1976, 1979, et al. 1983) claimed that the depths predicted by the width:depth equation (1.1) represented regime conditions (i.e., equilibrium channel geometries). Tamburi also claimed that the state of equilibrium (i.e., static or dynamic) of a particular river reach is implied by the value of n in equation (1.1)[6]. Other engineering applications were claimed for equations (1.1) and (1.2), as will be detailed in Chapter 2. In general the traditional regime methods are not directly suited to these applications, nor are the modified forms that pertain to the influence of bank stability. For example traditional regime methods do not distinguish between static and dynamic equilibrium as defined above. Ultimately the validity of the engineering applications is dependent on verifying Tamburi's claim that width:depth equations represent equilibrium channel geometry. It is the potential value of the engineering applications that makes the verification necessary.

If equation (1.1) is to be used to describe equilibrium channel geometry, it is important to define how n and K vary with selected

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5 The width and depth measurements are taken at a single reference flow condition (e.g., discharge) that is considered to be channel forming
6 Tamburi ignored the value of K, for the purpose of defining the state of channel equilibrium (i.e., dynamic or static), although he recognized that it too varied as a function of the sediment equilibrium (Public Works, 1978).

independent variables. As with the traditional regime methods one can expect a need to define the reference flow condition. Since the equilibrium geometry is related to equilibrium sediment transport, bed sediment concentration, X_{bm}, is a particularily important flow variable. Tamburi did not examine the sensitivity of the width:depth equation to the reference flow condition, or for that matter to the variation in flow regimes or channel pattern.

This thesis attempts to test the veracity of Tamburi's claims for width:depth analysis, as well as to rectify several outstanding issues related to the application of width:depth analysis alluded to above.

1.2 Statement of Problem

Width:depth analysis, as developed by Tamburi, has not found general application, outside of the originators' applications[7]. Indeed it has been criticized in the technical community; Andres (discussion of Tamburi, 1983b) felt that width:depth analysis is contradictory to traditional regime analysis. As alluded to in Section 1.1, the veracity of Tamburi's claims for width:depth analysis is in question.

This thesis attempts to:

1. Critically evaluate the theoretical underpinnings for width:depth analysis, and thereby confirm that width:depth equations represent equilibrium channel geometries (i.e., regime conditions).

2. Examine the sensitivity of width:depth equations (e.g., n, K) to the choice of reference flow condition (e.g., X_{bm}), and to the transition between flow regimes (i.e., lower, transitional and upper regimes) and channel pattern states.

3. Provide verification of the relationships developed in points 1 and 2 with field data.

4. Examine the theoretical basis for a relationship between equation (1.1) and a commonly used bedload transportation

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7 See Section 2.3 for further details on potential applications.

equation, given by equation (1.3).

$$q_b = C_b \, p \, V^m \quad \text{.................................. (1.3)}$$

where q_b = bed load per unit width
V = mean velocity
p = density of water
C_b = bedload coefficient
m = bedload exponent

One of Tamburi's engineering applications involves the development of a bedload equation solely from width and depth data.

1.3 Outline of Chapters

The Chapters that follow attempt to address the problem as stated in Section 1.2. A brief outline of the Chapters is presented below.

Chapter 2 presents a description of width:depth analysis as developed by its originators and claims made for this method. Included in this Chapter are: the origin of equation (1.1), the methodology for developing site specific equations and examples of the same, and a list of engineering applications claimed for width:depth analysis.

Chapter 3 presents a literature review, which attempts to relate width:depth analysis to other regime and analytical methods pertaining to hydraulic geometry. The review permits a qualitative assessment of Tamburi's claim that width:depth equations represent equilibrium conditions. Moreover, as a by-product of the review, two alternate forms of equation (1.1) are proposed. The review also allows for a qualitative assessment of the sensitivity of width:depth analysis to the reference flow condition, the flow regime and the channel pattern state. Lastly, the review provides a foundation for the required field verification of width:depth analysis by presenting several rivers on which width:depth analysis appears to be an appropriate tool.

Chapter 4 employs analytical methods[8] to address the theoretical underpinnings of width:depth analysis. Generalized rational[9] regime equations are derived from a review of the variables and process

8 The work of Henderson (1963) and Gill (1968) exemplifies this approach. These authors employed analytical methods in an attempt to understand the underlying theory to Lacey's regime equations.
9 The term *rational* implies that their development is based on a structured consideration of the variables and process equations, as opposed to being a purely empirical correlation of variables (after Parker, 1977, 1979)

equations governing alluvial rivers. Analytical methods are employed to provide solutions to the rational regime equations, for the lower flow regime only. The solution data are used to test whether simple non-linear equations (e.g., equation (1.1)) can be used to represent equilibrium conditions. Moreover, the analytical results are used to quantify the sensitivity of the width:depth equations (e.g., n, K in equation (1.1)) to the reference flow condition and sediment size. Bed material concentration, X_{bm}, is employed for the purpose of defining the reference flow condition. Field data from the Middle Loup River are used to test the analytical results.

Chapter 5 quantifies the sensitivity of width:depth equations to the transition between flow regimes. Analytical methods are again employed to provide solutions to the rational regime equations, this time covering all flow regimes. The solution data are used to test whether simple non-linear equations (e.g., equation (1.1)) can be used to represent equilibrium conditions, inclusive of the transition between flow regimes. Field data from the Niobrara and Rio Grande Rivers are employed to verify the analytical results. The analytical solutions of this chapter quantify the conditions (e.g., slope variations) that allow equilibrium sediment transport to be maintained.

Chapter 6 examines the relationship between the bedload equation of the form of equation (1.3) and the width:depth equation (1.1), through consideration of the governing variables and process equations. Included is verification of Tamburi's et al. (1983) claim that n in equation (1.1) and m in equation (1.3) are related by equation (1.4).

$$m = \frac{1}{1-n} \qquad \text{.....(1.4)}$$

Moreover, a rational method for calculating C_b[10] in equation (1.3) is proposed, in order to compliment equation (1.4) and to allow for development of a bed material load formula from a width and depth data base.

Chapter 7 presents the conclusions of this thesis, as well as recommendations for future research.

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10 Bed material load is proposed to replace bedload, and C_{bm} is proposed to replace C_b, as detailed in Chapter 6.

CHAPTER 2

PREVIOUS APPLICATIONS OF WIDTH:DEPTH ANALYSIS

2.1 Origin of the Width:Depth Equation

Western Canada Hydraulics Laboratory used multiple approaches (i.e., hydraulic models, 1977; analytical models, 1980; and regime analysis, 1976, 1978a) to predict navigational depths on the Fraser River. Width:depth analysis evolved out of the regime studies.

Available regime relations could not account for the observed variations in hydraulic geometry on the Fraser River. For example, Schumn's M-factor (Schumn, 1960; Shen, 1971), which is used to account for bank material variations, was inadequate in predicting the influence of the Fraser River geology (i.e., non-regime banks) on channel width (WCHL, 1976). Consequently, site specific regime relations were developed.

The first step in developing the site specific regime relations was to define the pertinent variables. The variables identified were: width, W; discharge as a percentage of the discharge at the river mouth, %Q; non-regime[11] bed, NRB; non-regime banks, NRS; bend curvature, I; salt wedge influence, T; and depth, dmax (maximum depth) and d (average depth). Conspicuous by their absence are the variables slope, S and sediment concentration, X. The general relationship between the variables was defined by equation (2.1) (Tamburi, 1976; Tamburi and Demlow, 1979; WCHL, 1978a):

$$dmax,d = f(W,\%Q,NRB,NRS,I,T) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.1)$$

The multivariate equation (2.1) was reduced to a single variate equation, at least for the dependent variable d, by noting the following points (Tamburi, 1976):

- NRS and W had a high covariance and therefore NRS was a redundant variable
- I affected dmax but not d significantly

11 Non-regime refers to materials not transported and deposited by the present day river (e.g., bedrock, glacial till, etc.) (Tamburi, 1976)

- salt wedge location could be used qualitatively (e.g., separate the data base at the influence limit) to explain discrepencies
- known non-regime bed conditions were limited and generally appeared as explainable outliers to the observed regime relations
- discharge defined reaches were identified from maps and the data for each reach (i.e., %Q constant) plotted separately

Therefore, single variate equations of the general form in equation (2.2) were investigated:

$$d = f(W) \qquad (2.2)$$

For the Fraser River the specific form of equation (2.2) given by equation (1.1) [$d = K W^{-n}$] provided a good fit to the observed data (WCHL, 1978).

2.2 Methodology for Data Analysis and Examples

A brief outline of the methodology employed in compiling and analyzing the data on the Fraser and Mississippi Rivers is given below (Tamburi et al., 1983):

1. *obtain a data base;* A series of surveyed cross-sections of the reach being studied are required (e.g., sounding maps for a specific year). The number of data points (i.e., survey lines) will vary depending on the length of the reach and the required accuracy. Tamburi et al. (1983) suggested 35-50 points as desirable, but felt that 8-10 points would provide a cursory estimate.

2. *choose a reference flow condition;* A reference flow condition must be chosen on which to base the top width and mean depth calculations. In the case of the Fraser and Mississippi Rivers, a navigational step datum called local low water (LLW) was used. Tamburi et al. (1983) suggested that backwater analysis could be used to establish datums for each cross section for low flows in the absence of LLW datums.

3. *calculate top width and mean flow depth relative to the datum;* for each cross section, giving a series of paired data points.

4.*plot the data;* Plots of mean depth versus top width on normal graph paper allow outliers to be quickly identified. For the case of the Fraser and Mississippi Rivers outliers were dropped at this point if any of the following reasons were deemed to be the cause of the anomaly:

-scour or deposition caused by local man-made or natural anomalies (i.e., local turbulence induced effects)

-non-regime bed material

-divided flow; and entrance and exit to the same

The data are split to account for variation in discharge and salt wedge shoaling effects.

5.*statistical analysis of width:depth data pairs;* Equation (1.1) can be represented by a straight line by employing logarithms as in equation (2.3).

$$\log_{10} d = (-n) \log_{10} W + \log_{10} K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.3)$$

Least square regression techniques are then used to find the best fit straight line with slope n and intercept $\log_{10} K$. Other output to the regression analysis can include coefficient of determination, r^2, standard error of estimate, Se, etc. The same technique can be used to find the relationships between area and width, where area is calculated as the product of top width and mean depth, and area replaces depth and (1-n) replaces (-n) in equation (2.3).

Several examples of width:depth analysis [Tamburi (1976, 1979, 1983), Shewfelt and Pomalaza (1981) and WCHL (1976, 1977, 1978a)] are presented below in order to illustrate the above discussion.

Figure 2.1 (WCHL, 1978a) shows the Lower Fraser River. The sub-reaches (e.g., Range 1-52) were chosen to separate reaches of substantially different flow (i.e., %Q), and to separate out the salt wedge influence. Figures 2.2, 2.3 and 2.4 (WCHL, 1978a) illustrate the inverse non-linear relationships that were observed to exist between width and mean depth, in each of the sub-reaches. Table 2-1 (WCHL, 1978a) summarizes the results of fitting equation (1.1) to the data.

TABLE 2-1: WIDTH:DEPTH EQUATIONS LOWER FRASER RIVER (WCHL, 1978a)

RANGE	EQUATION	CORRELATION COEFFICIENT SQUARED r^2
1-52	$d = 5,087\ W^{-0.71}$	.88
53-77	$d = 13,846\ W^{-0.81}$	.94
78-107	$d = 1,699\ W^{-0.54}$	.64
108-139	$d = 5,338\ W^{-0.68}$	.74
140-242	$d = 353,677\ W^{-1.27}$	.94
244-294	$d = 12,971\ W^{-0.85}$	.70

Figure 2.5 shows the location of a reach of the Mississippi River on which width:depth analysis was performed (Shewfelt and Pomalaza, 1981; Tamburi et al., 1983). Figure 2.6 presents the width:depth plots and best fit equations equations (2.4) and (2.5) (Shewfelt and Pomalaza, 1981).

$$d = 89,866\ W^{-1.02} \quad \text{(for 1968)} \quad r^2 = 0.8 \quad \ldots\ldots\ldots (2.4)$$

$$d = 20,237\ W^{-0.83} \quad \text{(for 1981)} \quad r^2 = 0.8 \quad \ldots\ldots\ldots (2.5)$$

Illustration of the equation (1.2) [$A = K\ W^{(1-n)}$], which relates area to width, is given by Figure 2.7, for the Mississippi River, and Figures 2.8 and 2.9 for the Fraser River. The equation for the 1981 Mississippi data is given by equation (2.6).

$$A = 20,237\ W^{0.17} \quad \text{(for 1981)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.6)$$

The 1968 Mississippi data on the other hand did not strictly follow the non-linear increasing trend of area with width, and equation (1.2) did not explain the data satisfactorily. Figures 2.8 and 2.9 show that the trends in the Fraser data are explained reasonably well by equation (1.2).

2.3 Engineering Applications

As stated in Chapter 1, the significance of width:depth analysis is revealed in its application to the solution of river engineering problems. Some previous and/or potential engineering applications claimed for width:depth analysis are outlined below (Tamburi et al., 1983). These applications remain dependent on the veracity of Tamburi's claim that width:depth equations represent equilibrium conditions.

1. *determination of channel top width required for a given navigational depth ;* Width:depth curves allow the engineer to estimate the required trained river width. An example of a potential application for this method is Nakato et al. (1979). Nakato's (1979) analysis of the Mississippi River included a discussion of training works that would be required to reduce channel area and increase mean velocities by 10%. Equation (1.2) could be used to solve this problem.

2. *analysis of channel equilibrium;* Tamburi (1979) concluded, by comparison with Straub's (1935) theory on long constrictions, that a value of n of approximately 9/14 (.64) indicates that a reach is in dynamic equilibrium. Conversely, a value of n of approximately 6/7 (.86) was said to be indicative of a reach in static equilibrium. Furthermore, Tamburi (1979) suggested that n values in excess of 6/7 indicate a negative sediment wave (e.g., dredging effects) and n values lower than 9/14 could indicate positive sediment waves (e.g., aggradation). This type of information can help river engineers to verify their qualitative or quantitative assessments of changes due to stress on a riverine environment. An example of a potential application for this method is given in Walters (1976). Walters (1976) used specific gauge readings to assess the equilibrium state of the Mississippi River. Stevens et al. (1976) provide another example of a geomorphic study which could have employed width:depth analysis.

3. *determination of flow augmentation requirements for a hydraulic model;* WCHL (1977) model tests of the Lower Fraser River included scour tests using the prototype sand size. Model scale factors dictated that the discharge in the model be augmented in order that the bed material transport was properly scaled. The proper flow augmentation was determined iteratively by matching the model and the prototype width:depth curves.

4. *verification of flow distribution trends in delta channels;* In Public Works (1978), observed changes in the proportionate discharge carried by separate channels in the Fraser River delta were verified by demonstrating coincident changes in the width:depth curves for those channels.

5. *development of bedload equations of the form of equation (1.3) from width:depth data;* Work towards this goal has progressed (Tamburi et al., 1983) but to date is incomplete. However, one can appreciate the advantages of a custom fit sediment transport curve, especially in light of the relatively poor performance of various accepted sediment transport curves, as illustrated by Figure 2.10 (ASCE, 1975).

Further details on the above applications can be found in the cited references.

CHAPTER 3

LITERATURE REVIEW AND QUALITATIVE ASSESSMENT OF WIDTH:DEPTH ANALYSIS

3.1 Introduction

A literature review is the first step towards a better understanding of the width:depth equation (1.1). The review presented herein revolves around a comparison and contrasting of width:depth analysis with other theories pertaining to the hydraulic geometry of rivers. The review allows for a qualitative assessment of whether the width:depth equation (1.1) can be used to represent equilibrium channel geometries (i.e., regime), as well as specifying the conditions that are required to maintain this equilibrium. In addition the potential sensitivity of the width:depth equation (i.e., n, K in equation (1.1)) to the choice of the reference flow condition, and to the transition between flow regimes and channel pattern states is discussed. Discussion of the relationship between sediment transport and width:depth analysis is left to Chapter 6.

The review has been subdivided as follows:

3.2 *Previously proposed analytical bases*, used to understand the theoretical underpinnings of width:depth analysis, are presented and the inadequacies of the previous work are discussed.

3.3 Contrasting width:depth analysis to *other regime methods* that have gained acceptance in the scientific community, provides insight into the relationship of the width:depth equation to regime channel geometries.

3.4 Comparison of width:depth analysis to *analytical methods* for determining hydraulic geometry, provides a foundation for overcoming the inadequacies of past attempts (i.e., Section 3.2). Chapter 4 formalizes the analytical approach to understanding the width:depth equation, by building on this review.

3.5 *Observed river behavior*, outside of the Fraser and Mississippi Rivers, will be required in Chapters 4 and 5 to verify the analytically derived results to the width:depth equation. The literature review provides several rivers on which width:depth analysis appears to be an appropriate tool.

3.6 The *sensitivity of width:depth analysis* to the reference flow condition, to flow regime and to channel pattern is examined in the light of pertinent references.

3.2 Previously Proposed Analytical Bases

In addition to the empirical basis for equation (1.1) presented in Chapter 2, Tamburi (1979, 1983a; WCHL 1974, 1976, 1977) claimed that an analytical basis for equation (1.1) could also be developed.

WCHL (1976) derived a relationship between width and depth through the simultaneous solution of the bed load equation (1.3) [i.e., $q_b = C_b\ p\ V^m$], and the equations defining discharge and total bed load, equation (3.1) and equation (3.2), respectively. The method assumes continuity of discharge and total bed load between two sections of varying width and depth. Appendix A presents the details of this analytical derivation, for the Fraser River.

$$Q = V\ W\ d \text{ (also } = V\ P\ r) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

where Q = fluid discharge
P = wetted perimeter
r = hydraulic radius
V = velocity

$$Q_b = q_b\ W \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

where Q_b = total bed load

The result of the analytical methodology is the non-dimensional equation (3.3).

$$d_2/d_1 = (W_1/W_2)^n \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

where the subscripts 1 and 2 refer to adjacent sections having varying widths and depths

WCHL (1976) made the jump to the dimensional equation (1.1) [$d = K\ W^{-n}$] from the non-dimensional equation (3.3) without formal proof.

Tamburi (1979, 1983a; WCHL, 1977) indicated that the studies of scour in long constrictions (i.e., Straub ,1935; Komura 1963, 1966, 1971; see also ASCE, 1975) used a similar solution methodology to arrive at equations of the form of equation (3.3). Straub (1935) solved the DuBoys sediment transport formula (ASCE, 1975), the equations for continuity, equation (3.1), Manning's resistance equation (3.4) and

assumed equal sediment and water discarge through the two sections.

$$Q = [\text{constant}/N]\ W\ d^{5/3}\ S_f^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

where N = Manning's constant (equal between 1 and 2)
S_f = slope of the energy grade line

The resultant values of n in equation (3.3) were found to vary between 0.642 for $\tau_c/\tau_1 \Rightarrow 0$ and 0.857 for $\tau_c/\tau_1 = 1$, where τ_c is the critical bed shear stress and τ_1 is the bed shear stress at the normal section. The variance of n between these two end points is reported in Figure 3.1 (ASCE ,1975). As stated in Section 2.3, Tamburi (1976) interpreted Straub's results as implying that n = 0.857 for static equilibrium and n = 0.642 for dynamic equilibrium. The definition of static equilibrium (i.e., $\tau_c/\tau_1 = 1$) agrees with Tamburi's interpretation. However, from Figure 3.1 it appears that the case of dynamic equilibrium (i.e., $\tau_c/\tau_1 < 1$) should be characterized by $0.64 < n < 0.86$, depending on the value of τ_1.

Komura (1963, 1966, 1971) determined n = 0.686 for dynamic equilibrium and n = 0.857 for static equilibrium in equation (3.3). Komura's approach involved simultaneous solution of the Kalinske-Brown sediment transport formuala (ASCE, 1975), a flow resistance equation, the equation for continuity (equation 3.1) and assuming equilibrium water and sediment transport between the normal and constricted sections. Komura assumed that bed forms were the same in the normal and contracted sections, and steady uniform flow or steady non-uniform flow persisted throughout the reach. Komura (1966, 1971) also investigated the effect of sediment sizes (D_2 and D_1) on the width:depth relationships and found that while the magnitude of the relative scour (d_2/d_1) was affected, the slope n of the relations were not.

Tamburi (1979) claimed that equation (3.3) was the non-dimensional equivalent to equation (1.1) [$d = K\ W^{-n}$], and that the n values for long constrictions should be equally applicable to a discharge defined reach of a natural river which exhibits width variations. The above presentation shows that equation (3.3) was developed assuming equilibrium sediment transport conditions. Thus if equation (1.1) is a form of equation (3.3), then it too should represent equilibrium conditions.

While the above analysis gives promise that the width:depth equation (1.1) defines equilibrium geometry, it is neither generalized or rigourous enough to fully explain the limitations and assumptions of the width:depth equation (1.1). The solution method lacks the specification of all the variables and process equations that govern alluvial rivers (e.g., ASCE, 1975; Hey, 1978; Parker, 1977,1979). As a result it is unclear whether n depends on other variables in addition to those discussed above. Moreover, the relationship of K in equation (1.1) to the controlling variables is not even developed as a part of the analysis. In addition, it is not apparent from the solutions provided how n and K vary with each other, or with different process equations (e.g., sediment transport; recall Figure 2.8) from those chosen above. All of these points bring into question the use of n to define the state of equilibrium. Furthermore, this lack of understanding of the theoretical underpinnings leaves some doubt as to whether the width:depth equation (1.1) represents equilibrium conditions. The work of Chapter 4 attempts to rectify the concerns with the above analyses.

3.3 Other Regime Methods

This Section contrasts the width:depth equations to the traditional regime methods alluded to in Chapter 1. The objective is a qualitative assessment of whether the width:depth equation (1.1) can be used to represent equilibrium conditions (i.e., regime).

In order to properly discuss regime analysis one needs to review the origins of the regime theory (Blench 1969, 1980, 1986). Lacey's work on irrigation canals in India, allowed him to formulate the following equations for canals in regime (Henderson, 1963):

$$V = 1.17\ (f_1 r)^{1/2} \quad \text{.....(3.5a)}$$

$$V = 16\ r^{2/3}\ S_f^{1/2} \quad \text{.....(3.5b)}$$

$$P = 2.67\ (Q)^{1/2} \quad \text{.....(3.5c)}$$

$$S = f_1^{5/3}/(1750\ Q^{1/6}) \quad \text{.....(3.5d)}$$

$$\text{where } f_1 = 8(D_g)^{1/2} \quad \text{.....(3.5e)}$$

D_g = representative grain size of bed sediment

Blench (1980) states that these equations can be manipulated to

the following convenient forms:

$$W \propto Q^{1/2} \quad \text{.....(3.6a)}$$

$$d \propto Q^{1/3} \quad \text{.....(3.6b)}$$

$$S \propto Q^{-1/6} \quad \text{.....(3.6c)}$$

$$V \propto Q^{1/6} \quad \text{.....(3.6d)}$$

Blench (1986) cites Neill (1973) as showing that several sand bed rivers in Alberta conform to the canal regime equations (3.6). Figure 3.2 (Neill, 1973) illustrates that the equations (3.6 a,b) for width and depth represent the data well, while equation (3.6c) for slope does not. Only straight reaches of river at bankfull discharge were used in Neill's (1973) analysis. Figure 3.3 compares Lacey's equations (i.e., equations 3.5a,c) to data from several New Zealand rivers (Henderson, 1963).

Blench (1986) cites Leopold and Maddock (1953) for confirmation that equations (3.6a,b) generally apply to gravel bed rivers in regime. However, the exponent for the regime equation (3.6b) varied slightly for the case of gravel rivers, as in equation (3.7b).

$$W \propto Q^{1/2} \quad \text{.....(3.7a)}$$

$$d \propto Q^{2/5} \quad \text{.....(3.7b)}$$

Kellerhals (1967) studied gravel bed rivers in regime. Considering only rivers of small bed material load, and adding a variable to account for the size of the bed material he found that gravel rivers in regime conformed to equations (3.8). Kellerhal's depth exponent coincides with that of Leopold and Maddock (1953).

$$W \propto Q^{1/2} \quad \text{.....(3.8a)}$$

$$d \propto Q^{2/5} D_g^{-0.12} \quad \text{.....(3.8b)}$$

$$S \propto Q^{-2/5} D_g^{0.92} \quad \text{.....(3.8c)}$$

Initial consideration of equations (3.6), (3.7) and (3.8) might suggest that a self-formed discharge defined reach in regime conforms to a singular regime width. This appears to be in direct contradiction with the suggestion that width:depth equations can be used to represent equilibrium channel geometries (i.e., regime conditions). Andrews (1982) concluded from observations on the East Fork River in Wyoming, that channel sections initially narrower and wider than the observed mean channel width, scour and deposit at their banks in a manner which moves them towards the mean width. Furthermore, these mean widths were

in line with the regime theories of Leopold and Maddock (1953). One might conclude at this point, as did Andres (Tamburi, 1983b), that width:depth analysis is not a regime analysis. Such a judgement is considered premature, since bank competence has yet to be considered.

Application of the canal regime concepts to natural rivers requires consideration of bank stability. Bank material often varies in composition from the bed material, even in an alluvial river. It seems logical to suggest that the width of a river will vary as a function of bank conditions (e.g., cohesiveness, vegetation). Thus, a river may be constrained in reaching the *regime* width that would apply to a homogeneous reach, where bed and banks are of the same material.

Schumn (Schen, 1971) recognized that the resistance of river banks to erosion could strongly influence width. Schumn's proposed modifications to equation (3.7) (Leopold and Maddock, 1954) involved an adjustment for silt-clay percentage, M_f, which is alternately defined by equation (3.9a) (Schen, 1971) or (3.9b) (Morisawa, 1985).

$$M_f = 55/Q_x \qquad \text{.....(3.9a)}$$

where $Q_x = Q_{BM}/Q_T$ = bed material load/total sediment load

or

$$M_f = \frac{W\ Sc + 2d\ Sb}{W + 2d} \qquad \text{.....(3.9b)}$$

where S_c = percent silt/clay in banks
S_b = percent silt/clay in beds

The alternate forms of equations (3.7) are given by equations (3.10).

$$W = 2.3\ Q^{0.38} / M_f{}^{0.39} \qquad \text{.....(3.10a)}$$

$$d = 0.6\ Q^{0.29}\ M_f{}^{0.39} \qquad \text{.....(3.10a)}$$

Schumn's adjustment factors were found to be inadequate in predicting the observed variation in river widths on the Fraser River (WCHL, 1976). Furthermore, Morisawa (1985) suggests that Schumn's relations are spurious due to the method in which they were derived. Equations (3.10) do, however, show that regime width can vary along a reach of constant discharge, if the bank materials vary. Moreover, Schumn's study of 36 stable sand-bed alluvial reaches revealed that only 40 % of the variability of channel dimensions could be accounted for by discharge alone. It is presumed that since Schumn was concerned with regime conditions, that the reaches studied were considered to be in regime.

Andrews (1982) suggested, in regards to sandy-gravel to gravel bed rivers, that much of the observed longitudinal variations in channel characterictics can be attributed to the sequence of riffles and pools. Figure 3.4 (Chang, 1985d) demonstrates the riffle/pool sequence. Richards (Morisawa, 1985) further stated that riffle/pool bank erosion sequences, not variation in bank competence, explain the majority of observed variations in channel width. Thorne and Hey (1986) investigated width variation in gravel rivers (D_{50} > 2 mm) taking into account both the width variations due to riffle/pool sequences and those attributable to vegetation type or incision into floodplain material (i.e., change in bank competence). Thorne and Hey's (1986) regime equations (3.11) reveal that width changes of up to 185% (of the minimum) can be attributed to vegetation and bank cohesion, whereas their equation (3.12) relating riffle and pool width suggests variations of only 107% (of the pool width, which is the minimum of the two). This data seems to strongly contradict the conclusions of Morisawa (1985) and Andrews (1982). Clearly, significant variations in regime width along a discharge defined reach can result from bank conditions.

$W = 4.33\ Q_{bf}^{0.5}$, vegetation type I .....(3.11a)

$W = 3.33\ Q_{bf}^{0.5}$, vegetation type II .....(3.11b)

$W = 2.73\ Q_{bf}^{0.5}$, vegetation type III .....(3.11c)

$W = 2.34\ Q_{bf}^{0.5}$, vegetation type IV .....(3.11d)

$RW = 1.070\ PW$ .....(3.12)

where RW = mean waterline width, riffles, m
PW = mean waterline width, pools,m
Q_{bf}= bankfull discharge; defined as the stage at which W/d ratio associated with the active flood plain is a minimum

and where information on bank vegetation and density were used to define the vegetation type:

vegetation type I = grassy banks, no trees or bushes

vegetation type II = 1-5% tree/shrub cover

vegetation type III= 5-50% tree/shrub cover

vegetation type IV = > 50% tree/shrub or incised into flood plain

Blench (1969, 1980, 1986) proposed modifications to Lacey's original regime equations (3.5) in order to account for bed and side

(i.e., bank) factors as shown in equations (3.13) (ASCE, 1975).

$$W = (F_b/F_s)^{1/2}\ Q^{1/2} \quad \text{.....(3.13a)}$$

$$d = (F_s/F_b)^{2/3}\ Q^{1/3} \quad \text{.....(3.13b)}$$

$$S = \frac{F_b^{7/8}}{K_b\ W^{1/4}\ d^{1/8}\ (1+X/2330)} \quad \text{.....(3.13c)}$$

where F_s = side factor = V^3/W
F_b = bed factor = V^2/d
X = concentration of sediment (PPM)
K_b = 3.63 g / $k_v^{1/4}$
k_v = kinematic viscosity

ASCE (1975) suggested, in the absence of better information, that the following values could be used:

$$F_b = 1.9\ (D_g)^{1/2} \quad \text{.....(3.14)}$$

F_s = 0.1 for friable banks .....(3.15)

F_s = 0.2 for silty, clay, loam banks .....(3.16)

F_s = 0.3 for tough clay banks .....(3.17)

Employing the extremes of F_s (i.e., 0.1 and 0.3) in equations (3.13a,b) gives a range of width and depth variations of 173% and 144% of the minimums, respectively. Henderson (1963) reveals that the coefficient of Lacey's equation (3.5c) actually ranges from 2.0 to 3.4, implying a similar variation in width of approximately 170% for a singular discharge.

The relationship of the width:depth equation to the traditional regime methods can be illustrated with Blench's equations. Equations (3.13a,b) can be related to width:depth analysis by computing relative data points assuming a constant discharge. For example, assuming arbitrarily values of discharge equal to 10 cfs, and bed sediment size, D_g, equal to 0.30 mm, results in the three depth versus width data points plotted in Figure 3.5. Figure 3.5 shows that the data can be represented by a straight line on logarithmic paper. Therefore, equation (1.1) [$d = K\ W^{-n}$] can be used to represent these regime data for the chosen reference flow, Q, and sediment size. Fitting equation (1.1) to Blench's data in Figure 3.5 results in a value of n = 0.67. Further quantitative analysis would be required to prove how n (and K) varies with the flow condition and sediment size. However, from a qualitative point of view, equation (1.1) appears to model the regime channel geometry adequately. Since, most of the regime formulations are

simply a variation of Blench's equations one would expect that equation (1.1) could be used to model the regime conditions predicted by them, also.

Some more recent and slightly variant approaches to deriving *rational regime relations*, namely Parker et al. (1977), Parker (1979) and Peterson (1975), merit consideration. Parker et al. (1977) and Parker (1979) employ a logically structured approach containing the following steps:

1. definition of the system to be studied (e.g., alluvial rivers, straight reaches, uniform flow, infinite width);
2. definition of the variables that describe the system;
3. dimensional analysis to reduce the number of total variables (but not the number of dependent variables);
4. identification of formulated process equations (e.g., continuity, equation (3.1), and momentum, equation (3.18)):

$$\tau = p\ g\ r\ S \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.18);$$

where p = fluid density

5. identification of unformulated process equations (e.g., sediment transport, flow resistance, and self-formed width);
6. partial reduction of the unformulated equations using the formulated equations, to arrive at a set of resultant *rational regime relations*.

The resultant *rational regime relations* are given by equations (3.19) (Parker et al., 1977).

$$S = f(q\sim, R;\ R_p, R_b, p_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.19a)$$

$$X_{bm} = f(q\sim, R;\ R_p, R_b, p_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.19b)$$

where S = slope

$$R = d/D_g = \text{relative roughness} \quad \ldots\ldots\ldots\ldots\ldots (3.20a)$$

$$R_p = (R_b\ g\ D_g)^{0.5}\ D_g/k_v = \text{particle Reynolds no.} \quad (3.20b)$$

$$R_b = (p_s/p - 1) = \text{sediment bouyancy parameter} \quad (3.20c)$$

p_s = sediment density

p_i = dimensionless sediment parameters (e.g., σ_g, gradation)

$$q\sim = q/(R_b\ g\ D_g)^{0.5}\ D_g = \text{dimensionless q} \quad \ldots\ldots (3.20d)$$

$$q = Q/W = \text{unit discharge} \quad \ldots\ldots\ldots\ldots\ldots (3.20e)$$

$$X_{bm} = (1-X/10^6)[(R_b+1)q^*/q\sim]10^6 \quad \ldots\ldots\ldots (3.20f)$$

$$= \text{approximately } [(R_b+1)q^*/q\sim]10^6$$

=concentration of bed material PPM

$$q^* = q_{bm} / [(R_b\ g\ D_g)^{0.5}\ D_g] \quad \text{.....(3.20g)}$$

q_{bm}= unit bed sediment load

The method of Parker et al. (1977) is classified here as regime analysis, since the Peterson and Howell (see Brownlie, 1981b) compendium of field and flume data were used to quantify his rational regime equations (3.19a,b). On the other hand his approach is analytical in nature, and therefore his assumptions are laid out in a clear concise manner.

Typical results of Parker's analysis are presented in Figure 3.6a [i.e., $S = f(q\sim, R;\ R_p, R_b, p_i)$ ..(3.19a)], and Figure 3.6b [i.e., $X_{bm} = f(q\sim, R;\ R_p, R_b, p_i)$ ..(3.19b); where $c = X_{bm}$]. Segments of the equi-concentration lines of Figure 3.6b can be represented by straight lines, and therefore by equation (3.21).

$$R = K\sim\ q\sim^{n} \quad \text{.....(3.21)}$$

where n = the equivalent exponent to equation (1.1)
K~ = coefficient (not equal to K in equation (1.1) since R and q~ are non-dimensional

For a discharge defined reach (Q constant), with uniform bed sediment, q is inversely proportional to W (i.e., q = Q/W = Constant/W), and therefore so is dimensionless unit discharge, q~. The variable R on the other hand is simply non-dimensional depth (d/D_g). Therefore equation (3.21) can be considered the non-dimensional equivalent to the width:depth equation (1.1), where n is equivalent and some relationship exists between K and K~. The formal relationship between equations (1.1) and (3.21) is presented in Chapter 4. From herein equation (3.21) will be considered the non-dimensional alternative to the width:depth equation (1.1). The same questions raised in Chapter 1 with respect to equation (1.1), must be answered with respect to equation (3.21). Since portions of the equi-concentration lines in Figure 3.6b conform to equation (3.21), then by definition (i.e., constant X_{bm} = sediment equilibrium) equation (3.21) represents equilibrium geometry for these portions. The width:depth equation (3.21), of course does not account for the curvilinear portions of Figure 3.6b, but this is left to the discussion of the transition between flow regimes.

Peterson (1975) developed a list of dimensionless parameters for straight rectangular channels with granular bed material. Peterson's resultant equations (3.22) were arrived at in a somewhat more subjective

manner than equations (3.19), but contain essentially the same variables.

$$(q_f,\ d/D_{50},\ Vig,\ X_{bm})=0 \quad \text{.....(3.22a)}$$

$$(q_f/S^{1/2},\ d/D_{50},\ Vig,\ X_{bm})=0 \quad \text{.....(3.22b)}$$

where q_f = Q-factor = q~ (Parker et al., 1977)

$$Vig = (k_v\ g)^{1/3}\ D_{50}/\ k_v \quad \text{.....(3.22c)}$$

Parker et al. (1977) presented one of Peterson's (1975) graphed results, reproduced here as Figure 3.7, for direct comparison with Figures 3.6. Even more than with Figure 3.6 the resultant lines in Figure 3.7, for equal concentrations (i.e., $c = X_{bm}$), plot as straight lines, thus conforming to equation (3.21). Therefore, the alternate form of the width:depth equation given by equation (3.21) appears to adequately model equilibrium (i.e., X_{bm} = constant) channel geometry.

Peterson (1975) used equation (3.23) to represent the results of Figure 3.7.

$$q_f = q\sim = K_1\ R^{m1}\ (X/10)^{n1} \quad \text{.....(3.23a)}$$

$$q_f = q\sim = S^{1/2}\ K_2\ R^{m2}\ (X/10)^{n2} \quad \text{.....(3.23b)}$$

The values of K_1, m_1, n_1, K_2, m_2, and n_2 were related only to sediment size ranges expressed by the vig number, and as detailed in Table 3-1.

TABLE 3-1 CONSTANTS FOR PETERSON'S(1975) EQUATIONS (3.23)

D_{50}(mm)	K_1	m_1	n_1	K_2	m_2	n_2
< 0.2	0.532	1.298	0.281	4.511	1.577	0.044
0.2-0.3	1.177	1.228	0.171	4.939	1.569	0.057
0.3-0.4	0.971	1.246	0.181	8.065	1.509	0.031
0.4-0.5	0.907	1.250	0.178	10.591	1.467	0.017
0.5-0.6	0.914	1.237	0.174	10.117	1.481	0.0073
0.6-0.8	0.877	1.231	0.181	11.059	1.469	-0.0008
0.8-1.3	0.974	1.222	0.149	11.840	1.464	-0.024
1.3-2.3	1.352	1.179	0.097	12.750	1.441	-0.026
2.3-5.5	1.473	1.172	0.068	10.760	1.462	-0.024
> 5.5	1.532	1.173	0.045	9.160	1.484	-0.017

Equations (3.21) and (3.23a) can be equated, implying that that $n = 1/m_1$, and $K\sim = [1/(K_1\ (X/10)^{n1})]^{1/m1}$. Therefore, one can compute that the value of n ranges from 0.77 to 0.85, as a function of sediment

size. This is in direct conflict with the use of n to define the state of equilibrium. The value of K~ is, however, related to the sediment concentration and therefore might be used to define the equilibrium state.

Blench (1975) criticized Peterson's (1975) graphs of q~ versus d/D_{50} as being spurious. Blench (1975) claimed that graphs such as Figure 3.7, were really plots of d/D_{50} versus $(V^2/gd)(1/(p_s/p-1))(d/D_{50})^3$, containing the term d/D_{50} on both sides of the equation. An interesting parallel appeared in the discussion of Parker et al. (1977) by Thomas (1978). Thomas (1978) suggested plotting V_* rather then q~ versus R, where

$$V_* = V/ (g R_b D_{50})^{0.5} \quad \text{.....(3.24)}$$

In reply, Parker et al. (1978) stated that in order to confirm Thomas's assertion, one would need to determine which of V_* or q~ was independent. To this author's knowledge, Thomas's claim that the Parker et al. (1977) relations are spurious has not been verified.

In the case of Blench's (1975) criticism of Peterson (1975), Blench appears to have accepted the validity of Peterson's plots, as they appear in both Blench (1980) and (1986) as regime graphs. In fact, in Blench and Peterson (1980), equation (3.25) is proposed as an alternative to equations (3.23).

$$q_f = q\sim = f\ [(d/D_{50})^{1.25},\ X_{bm}{}^{1/5}] \quad \text{.....(3.25a)}$$

for gravel bed streams and small flumes

$$q_f = q\sim = f\ [(d/D_{50})^{1.50},\ X_{bm}{}^{1/x}] \quad \text{.....(3.25b)}$$

for sand bed streams

Peterson and Blench (1980) felt that the varying exponent was a function of sediment size. However, recalling that q~ is inversely proportional to width for a constant Q, then equations (3.25) imply n values of 0.8 (i.e., 1/1.25) for gravel beds which typically exhibit near threshold conditions (i.e., static equilibrium), and 0.66 (i.e., 1/1.5) for sand bed streams which typically exhibit live bed conditions (i.e., dynamic equilibrium). Thus Blench's exponents could easily be related to the sediment transport condition (i.e., X_{bm}) rather than sediment size. This same argument might account for the observed variation in the exponent of the regime depth equations between sand-bed rivers (equation (3.6b) and gravel bed rivers (equation 3.7b). In summary, this regime

approach has provided results that boil down to n values very close to the range originally expounded by Straub (1935) for long constrictions, suggesting that the use of n to define equilibrium state may yet be valid.

This section began with skepticism that the width:depth equation (1.1) [$d = K W^{-n}$] can be used to represent equilibrium channel geometry, as a result of considering Lacey's regime equations and Andrews (1982) observations on width adjustment. Consideration of bank stability provided reasons for and observations of substantial variations in channel width within a discharge defined reach. Moreover, calculations using Blench's side factors provided regime width and depth values that conformed to equation (1.1) with a value of $n = 0.67$. This fact suggests that width:depth equations can be used to represent regime conditions, although the variation of n with flow and sediment size was not investigated.

Of a more recent nature, Parker (1977) and Peterson (1975) attempted to provide *rational regime relations* for alluvial rivers, arriving at resultant equations relating q~ and R. Consideration of their solution data has suggested an non-dimensional alternative to equation (1.1) given by equation (3.21) [$R = K\sim q^{n}$]. Peterson's data in particular, confirm that equation (3.21) can be used to represent regime geometry, but suggest that n varies with sediment size only, while K~ varies with sediment size and the reference flow condition, X_{bm}. Parker provided a sound theoretical basis for his equations, suggesting that width:depth equations (i.e., equations (3.21) and (1.1)) could also be investigated using such an approach.

3.4 Analytical Approaches

Analytical methods are based on the equations describing the dominant flow processes. However, strict theoretical derivation of the process equations (e.g., sediment transport) appears to be the exception rather then the rule. Many sediment transport and flow resistance equations involve varying degrees of empirically determined constants and relationships between variables (e.g., Ackers-White, 1978; White et al., 1987; Brownlie, 1981a; Van Rijn, 1984). Parker's (1977) rational regime method straddles the border between regime and analytical

methods; since his generalized resultant equations are derived from the governing process equations, but are fitted empirically. Despite the somewhat misleading distinction between the two methods, it does appear that analytical methods are often given more weight (Paris, 1984). There seems to be a presumption that less experience and judgement are required to use analytical methods (ASCE, 1975) and a suspicion that instantaneous measurements of field conditions are inadequate for use in deriving equilibrium hydraulic relationships (Chang, 1979). However, as revealed in Figure 2.8 (e.g., sediment transport equations of Einstein-Brown), the use of analytical methods does not gaurantee good results. Having thus qualified analytical methods, it is instructive to review width:depth analysis in comparison to some analytical methods for hydraulic geometry.

Paris (1984) derived hydraulic relations for alluvial rivers by solving the equations of continuity (equation 3.1), the Ackers-White (1973) sediment transport function and the White-Paris-Bettess (1980) resistance function. Figures 3.8 and 3.9 present Paris's (1984) analytical results for D_{35} equal to 0.5 mm and 20 mm, respectively. Review of these Figures suggests that the resultant relations conform to equations (3.26). Paris's results (i.e., equations (3.26)) apply to straight alluvial channels with non-cohesive bed and banks, subject to steady uniform subcritical flow and exhibiting a sediment equilibrium.

$$S = f_1 (q, X_{bm}, D_{35}) \quad \text{.....(3.26a)}$$

$$d = f_2 (q, X_{bm}, D_{35}) \quad \text{.....(3.26b)}$$

where D_{35} = bed sediment size for which 35% of the sample is finer
X_{bm} = bed sediment concentration (= X in Figures 3.8 and 3.9).
S = slope
d = depth
q = unit discharge

Using the continuity equation (3.1), the momentum equation (3.18) , and the definition of Chezy's friction factor, equation (3.27), Paris also backsolved for Chezy's C and velocity, V as functions of unit discharge, q, concentration, X_{bm}, and sediment size, D_{35}.

$$C = V/U_* \quad \text{.....(3.27)}$$

where U_* = shear velocity = $(g\ r\ S)^{0.5}$

Paris's (1984) analysis is two-dimensional (i.e., downstream/vertical) in nature. Yet, Paris (1984) used his results to study the impact of artificially narrowing or widening of a channel. While this is a simplification of a complex three-dimensional problem, others have used a similar two dimensional approach to study the stable width of alluvial channels. Stevens (1989) noted that for the banks of a river to be stable that:

> "Some deposition and erosion (of the banks) might occur, but it should remain on the average insignificant. "

Therefore, precedents exist for extending the two-dimensional case to explain three-dimensional conditions.

The straight line log-log plot of d versus q for a given X_{bm} in Figures 3.8 and 3.9, implies that an equation of the form of equation (3.28) is applicable.

$$d = K' q^{n} \quad \text{.....(3.28)}$$

where n = the equivalent exponent to equation (1.1)
K' = coefficient (not equal to K in equation (1.1) since q not W is used

Employing the simple two dimensional model, the definition of unit discharge ($q=Q/W$) relates equation (3.28) to equation (1.1) [$d = K W^{-n}$], so that n is equivalent in both equations, and some relationship exists between K' and K. Equation (3.28) is therefore considered herein as an alternative form to equation (1.1). The same questions posed of equation (1.1) in Chapter 1 apply to equation (3.28). In this regard, since the alternative width:depth equation (3.28) fits the equi-concentration lines in Figures 3.8 and 3.9, it appears to adequately model equilibrium (i.e., X_{bm} = constant) channel geometry.

The varying slopes associated with the lines of equal concentration in Figures 3.8 and 3.9 imply that n varies as a function of concentration, X_{bm}. This is in contrast with the regime methods of Blench et al. (1980) and Peterson (1975), which implied that n varied with sediment size. Paris (1984) does not present enough information to allow quantitative evaluation of K' and n in equation (3.28), but it appears that the variation of n with concentration X_{bm}, can be

paralleled to Komura's variation of n with the state of equilibrium.[12] In Chapter 4, Paris's equations are used to develop analytically a continuum of n values[13] (and K' values) corresponding to increasing sediment concentrations, starting at $X_{bm} = 0$.

White et al. (1982) extended Paris's (1984) analysis by determining the *optimum* width for an alluvial river. The constraints were defined by the equations given above for Paris (1984), as well as the maximizing of sediment transport capacity. Specifying the independent variables as the discharge, the sediment size and the slope, allowed the stable hydraulic geometry (i.e., the optimal section width, W, and therefore depth, d) to be determined. Figure 3.10 (White et al., 1982) presents analytical results for a discharge of 500 m^3/s and a sediment size of 40 mm. Figure 3.10 reveals an optimum width of 43 m corresponding to a maximum sediment concentration of 100 PPM, for the given slope of 0.00214. Figure 3.10 also shows that if slope was varied that the minimum slope for the concentration of 100 PPM, is 0.00214. Comparison to the Figures 3.8 and 3.9 Paris (1984) suggests that in some cases (e.g., smaller sediment sizes or lower concentrations) optimal widths may not exist, as evidenced by the lack of a minimum in the S, q, X plots.

Closer inspection of Figure 3.10 shows that local slope changes of less than 120% (of the minimium) are required to maintain equilibrium sediment transport over width variations of up to 300%. In fact, White et al. (1982) state that

> " A channel with banks which are less erodible will have a width smaller than that corresponding to the maximum sediment transport case, while a channel whose banks are more erodible than the bed will have a width correspondingly larger"

Despite their observation White et al. (1982) did not comment on the sediment equilibrium in a reach exhibiting varying widths.

The idea of maximizing sediment transport appears to have been first proposed by Griffith (1939), who was studying long constrictions. Griffith (1939) also developed an equation of the form of equation (3.3), and for conditions corresponding to dynamic equilibrium

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12 Static equilibrium is equivalent to $X_{bm} = 0$ ppm and dynamic equilibrium is equivalent to $X_{bm} > 0$ ppm.
13 Similar to Figure 3.1.

determined that n = 0.637. Griffith's analysis involved the simultaneous solution of the regime equation (3.29) along with the equations of continuity (equation 3.1), while assuming equilibrium sediment and fluid transport.

$$V = f_2 \; c \; (d)^{n1} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.29)$$

where f_2 = shape factor
c = coefficient that varies with concentration, X
n_1 = a constant

Griffith stated that his analysis was valid only for steady non-uniform or uniform flow.

Inglis (discussion contained in Griffith, 1939) stated that equations of the form of equations (3.3) are applicable to non-regime conditions only. Griffith's agreed with Lacey's assessment of the nature of equation (3.3), and attempted to support it as follows. Griffith's developed analytical solutions for a series of widths (at a constant slope and discharge), and concluded that only one width produces the maximum silt carrying capacity. Griffith reasoned that this width is therefore the most stable width. Moreover, by varying the discharge Griffith was able to develop the equation (3.30) relating the optimal width (i.e., at maximum silt capacity) to discharge. Interestingly, Griffith's equation (3.30) closely follows Lacey's equation (3.5c) except that width replaces wetted perimeter.

$$W = 2.60 \; Q^{0.5} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.30)$$

Griffith (1939) concluded that

> "... it appears to establish the law that <u>permanent regime</u> in a channel can be attained only for the sectional dimensions which provide the maximum silt-carrying efficiency (i.e., equation 3.30)"

However, as with White et al. (1982), Griffith's premise of a maximum silt carrying section was based on a constant slope throughout the reach.

Henderson (1963) derived an equation equivalent to Lacey's equation (3.5b) using Lane's tractive force theory and Strickler's (resistance) equation. The equivalent tractive force equation to Lacey's equation (3.5a) took the slightly variant form of equation (3.31).

$$V \propto D_g^{1/3} \; r^{1/6} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.31)$$

Henderson claimed that in order for Lacey's width equation (3.5c) to hold true the regime slope must have been everywhere attained (i.e., equation (3.5d) must apply). Henderson suggested that it may be appropriate in some cases to use the observed river slope rather then the regime slope. If the regime slope equation (3.5d) is not everywhere in force, then simultaneous solution of the equation of continuity, and equation (3.5a) or equation (3.31) yields equations (3.32) and (3.33) relating P, r and Q that are true for varying slopes.

$$P\, r^{3/2} \alpha\, Q\, f_1^{1/2} \quad \text{..(regime conditions}^{14}\text{)} \quad \ldots\ldots(3.32)$$

$$P\, r^{7/6} \alpha\, Q\, D_g^{-1/3} \quad \text{..(threshold conditions}^{14}\text{)} \quad \ldots.(3.33)$$

If the reach under consideration is discharge defined (i.e., constant Q) then equations (3.32) and (3.33) can be reduced to equation (3.34).

$$r \alpha P^{-n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.34)$$

where $n = 1/(3/2) = 0.666$ for regime conditions

$= 1/(7/6) = 0.857$ for threshold conditions

In support of his arguments Henderson (1963) presented the data shown in Figure 3.3 for several New Zealand rivers. Figure 3.3 reveals good correlation of the data with Lacey's equation (3.5b), but poor correlation with Lacey's width equation (3.5c). Equation (3.34) is in agreement with equation (1.1) (for wide rivers) and the limits for n are in line with those for long constrictions; if one equates Henderson's regime to dynamic equilibrium and threshold to static equilibrium. In addition, Henderson's analysis establishes a condition for the application of equation (1.1), namely the variation of slope along the reach.

From the above discussion it appears that a condition for maintaining equilibrium along a discharge defined reach having width variations is the variation of slope along the reach. Therefore, this is also a condition for the application of equation (1.1). This concept may, at first, appear to contradict regime theory. Lacey (discussion contained in Henderson, 1963) claimed that slope must be considered a dependent variable in regime analysis, and therefore felt that Henderson's equation (3.34) did not represent equilibrium conditions. However, one must realize that the slope adjustments required to

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14 Inherent in the assumptions of regime and threshold conditions are equilibrium sediment transport.

maintain equilibrium are small, and may be locally related to coincident changes in depth, while not affecting to a large extent the overall reach slope. The analytical methods of White et al. (1982) confirm that slope adjustments do not have to be large in order to maintain equilibrium. In support of this argument, Bray (1972) analyzing gravel bed streams, found that slope-discharge relations were poor and that;

> "Channels essentially are capable of maintaining orderly cross-section adjustment regardless of the state of the slope adjustment"

The large (i.e., reach) slope adjustments which Lacey refers to are required by long-term equilibrium, and for the purpose of the short-term (i.e., engineering time) Parker (1977) suggests that;

> "Time scales associated with watershed denudation and tectonic lifting are so large that for many purposes, including the analysis of meandering and braiding, it is sufficient to consider the discharge (Q), and river channel slope, S, to be known specific constraints (over the length of the reach)"

As stated in the introductory Chapter, width:depth equations are concerned with this short-term equilibrium.

The analytical methods of Chang (1978, 1979) provide further insight into the conditions that are required to maintain equilibrium in a discharged defined reach with varying top widths. Chang (1979) postulated that channels strive to minimize stream power, thereby controlling width adjustment. White et al. (1982) suggested that the condition of maximum sediment transport, corresponds to Chang's (1979) condition of minimum stream power. For the case of uniform flow, stream power is defined by equation (3.35).

$$\text{stream power} = p\ g\ Q\ S \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.35)$$

For a reach with constant discharge minimum stream power corresponds to minimum slope. Figure 3.10 shows that Chang's (1979) theory of minimum stream power (i.e., minimum slope) results in the same optimal width as the theory of maximum sediment transport (White et al., 1982), for the given assumptions.

Yang (1978) claimed that his minimum <u>unit</u> stream power theory was also equivalent to Chang's (1979) minimum stream power theory. Unit stream power is defined by equation (3.36).

$$\text{unit stream power} = V\ S \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.36)$$

Yang (1976) derived equilibrium channel geometry by solving his own sediment transport formula (Yang, 1972) simultaneously with the criteria of minimum unit stream power along a reach. However, Chang (1978) showed that Yang's (1976) methodology, applied to a discharge defined reach, would only give the same results as minimum stream power if channel area were held constant in the minimization process. Chang (1978) used analytical methods to show that depth and area do vary with width along a channel of given discharge, sediment size and sediment discharge. Figure 3.11 [15] illustrates Chang's (1978) results. The center depth, d_c, in Figure 3.11 is close to the mean depth, d, for straight wide channels. The plot of depth versus width in Figure 3.11 agrees with the inverse non-lineal equation (1.1). Furthermore, the computed non linear increase in channel area with width is in agreement equation (1.2). Also evident in Figure 3.11 is that width and depth changes of 300-400% can be acheived with local slope variations of less than 120% (of the minimum). Since Figure 3.11 applies to a constant discharge, Q, and sediment load, Q_s, then it once again appears that equation (1.1), and also (1.2), can be used to represent equilibrium channel geometries, and that minor local slope variations are required to maintain this equilibrium.

Chang (1979) presented analytical methods pertaining to wide straight alluvial sand-bed rivers having no secondary currents, and subject to lower, transitional, and upper flow regimes. Width, depth and slope were solved for using the continuity equation (equation 3.1), the sediment transport formulae of DuBoys and Einstein-Brown (ASCE,1975), a flow resistance formula (Engelund, 1967) and by minimizing stream power. Figure 3.12 illustrates typical results of this approach (Chang, 1979). In general, two minima for S were found to exist and therefore two stable widths, W, corresponding to the upper and lower flow regimes. Further evidence of the inverse non-lineal relationship (i.e., equation 1.1) between width, W, and depth, d_c is provided by Figure 3.12. Each of the two plots shown in Figure 3.12 corresponds to a single discharge and sediment load (i.e., equilibrium conditions). In addition, since velocity is an indicator of channel

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15 Figure 3.11 applies to stable alluvial trapazoidal canals, lower flow regime only.

area, Figure 3.12 confirms the non-lineal relationship between area and width. The implications of the obvious discontinuity in the width:depth and width:velocity plots are left to Section 3.6.

The conclusions of White et al. (1982) and Chang (1979) present interesting parallels to the traditional regime methods of Lacey (Blench, 1980), Leopold and Maddock (1953) and Kellerhals (1967). In particular the optimal width theories coincide to some degree with the idea of a singular relationship between discharge and width as expounded in the various regime equations (equations (3.5) - (3.8)). This parallel has been quantified for stable canal design (Chang 1980a, 1985b), for gravel bed rivers (Chang, 1980b) and for sand-bed rivers (Chang, 1985c).

Chang's (1980a, 1985b) analytical applications to sand-bed canal design were based on lower flow regime only, and resulted in equations of the following form:

$$W = 4.17\ [(S-S_c)/D^{0.5}]^{0.05}\ Q^{0.5} \quad \text{.....(3.37a)}$$

$$d = 0.055\ [(S-S_c)/D^{0.5}]^{-0.3}\ Q^{0.3} \quad \text{.....(3.37b)}$$

$$S/D^{0.5} = 0.433\ Q_s^{0.736}/Q^{0.789} + S_c/D^{0.5} \quad \text{.....(3.37c)}$$

where S_c = critical channel slope corresponding to the threshold for bed-load sediment transport
Q_s = sediment discharge

Figure 3.13 demonstrates, using equation (3.37a), that slope has only a minor impact on width, as expected from the small exponent on the slope variable (i.e., $S^{0.05}$ in equation (3.37a)). Moreover, the exponent for Q in equation (3.37a) is exactly similar to that of the regime equation (3.6a) (Blench, 1980), implying general concurrance of the two approaches. The discharge exponent in equation (3.37b) for depth (i.e., 0.3) is similar in magnitude to the exponent of the regime equation (3.6b) (i.e., 1/3) but Chang (1980a) also recognized the influence of slope. The exponent for the slope equation (3.37c) is quite different from equation (3.6c) (i.e., -.789 versus -1/6), as is the general form of the equations.

Chang's (1980b) application to gravel rivers used Bray's resistance equation and a form of Parker's (1978) bedload equation, to derive equations similar in form to equation (3.37). As was the case for canals, slope was found to have only a minor influence on width except at relatively steep slopes. Chang (1980b) noted that his

equations were in general agreement with regime gravel bed equations (e.g., equations (3.7) and (3.8)).

The relationship between the width:depth equation (1.1) [$d=K\ W^{-n}$] and the traditional regime methods as expounded in Chapter 1, applies equally well to the analytical methods which determine the optimal hydraulic geometry. That is width:depth analysis is not predictive of width and therefore does not replace these analytical methods. On the other hand, the methods of White et al. (1982) and Chang (1979, 1982) do not directly address the question of equilibrium in a reach where bank factors prevent the channel from attaining the optimal width (Stevens, 1989). Chang (1979) indicated that a river system may not attain self-formed stable geometry (i.e., optimum width) due to hetrogeneity such as vegetation, sediment content, placement of rock, fallen trees, etc. Width:depth analysis can therefore be considered to be an addendum to the optimal width methods, in which one considers the constraints to attaining the optimal width, and where the results are presented in a fashion that is useful for certain engineering applications.

Chang (1985c,d) extended his work on meandering sand-bed equi-width point bar streams (Chang, 1984a), to encompass steep rivers with heavy sediment loads and tendencies to braid. Figure 3.14 (Chang, 1985c) details the results these analytical studies into the hydraulic geometry of alluvial rivers *in regime*. Figure 3.14 is divided into four regions, which are distinguished by obvious inflections in the relationships, and represent transition between differing channel patterns. Figure 3.15 presents the channel classifications with which Chang (1985c) was concerned. At this point what is important to note is that the regime width equation (3.6a) for sand bed rivers conforms to the results for region 1 (Figure 3.14), which are equi-width point bar streams and stable canals. The equations describing the relationship in region 1 are given by equations (3.38) (Chang, 1985c).

$$W = 3.49\ [(S-S_c)/D^{0.5}]^{0.02}\ Q^{0.47} \qquad \text{.....(3.38a)}$$

$$d = 0.51\ \exp\ [-0.38\ (S/S_c-1)^{0.4}]\ Q^{0.47} \qquad \text{.....(3.38b)}$$

The width equation (3.38a) varies slightly from equation (3.37a) for canals but is in general agreement with equation (3.6a), since slope has a very minor influence. The discharge exponent for the depth equation

(3.38b) does not agree with equations (3.37b) or (3.6b), but is closer to the exponent proposed for gravel rivers (e.g., equation 3.9b). Also slope has a significant influence on depth, even in region 1. No equation for slope is available for comparison to equation (3.6c). It is important to note that once the thresholds between regions are exceeded, the regime formulas for width (i.e., equations (3.5) and (3.6)) are quite different from the analytical results of Chang (1985c). The impact of changes in channel pattern states on hydraulic geometry will be further discussed in Section 3.6.

Chang (1983, 1984a,c) included an additional variable, channel curvature, r_c, to model the effects of meandering on the hydraulic geometry of equi-width point bar streams subject to flow in the lower regime. Chang (1985c) schematized point-bar streams as a series of pool and riffle sequences as shown in Figure 3.4. The inclusion of r_c required that another governing process equation be introduced. The condition that minimum stream power over a reach, corresponding to equal slope along a reach, was proposed.

Several additional points regarding width:depth analysis can be gleaned from Chang's (1983, 1984a,c, 1985c, etc.) papers. Figure 3.16 (Chang, 1984a) reinforces Tamburi's (1976) conclusion that curvature has little effect on the mean depth versus width relationship. In particular, comparing velocity (and therefore area) versus width curves shows that the degree of curvature has only minor impact on the relationship of area to width. The relationship of equations (1.2) and (1.1) allows us to conclude that curvature should have little impact on the width versus mean depth relation also. Area is used since Chang's depth, d_c is not representative of the mean depth. However, the inverse non-linear relationship between d_c (i.e., D in Figure 3.16) and W is still evident in Figure 3.16. Also, slope variations of only 107% (of the minimum) are required to accomodate width and depth variations of up to 200%. Figure 3.17 (Chang, 1985c) reveals that pool and riffle sections can be made to have the same stream power by varying the bank slope, z. This supports the possible existence of multiple channel widths under equilibrium conditions, and gives an idea of a further adjustment technique in addition to slope, available to rivers.

Parker (1977) developed, in addition to his own rational regime graphs (i.e., Figure 3.6), analytical solutions to his resultant equations (3.19), by substituting pairs of formulated sediment transport and flow resistance equations into his methodology.[16] Figure 3.18 report Parker's (1977) analytical solutions using a. Engelund's (1967) sediment tranport and flow resistance relations for the lower flow regime, and b. Rottner's (1959) sediment transport equation with the flow resistance relation of Alam et al. (1969). Figure 3.18 reveals that, in both cases, the logarithmic plots of q~ versus R for equal concentration are virtually linear and therefore can be represented by equation (3.21). This confirms that the alternative width:depth equation represents equilibrium conditions. Furthermore, the varying slope of the equi-concentration lines in Figure 3.18, imply that n varies as a function of concentration, X_{bm}, in agreement with Paris (1984).

Parker (1978b,c; 1979) proposed analytical models to describe the stable hydraulic geometry of wide, self-formed, laterally symetrical, and straight sand-silt and gravel rivers with uniform non-cohesive mobile beds and equilibrium banks. In these papers Parker attempts to provide a three-dimensional approach to explaining the paradoxical existence of a mobile sand or gravel bed with a stable channel width.

For the case of the sand-silt river, Parker (1978b) solved the paradox by describing the process of lateral diffusion of suspended sediment from the channel center to the banks, in order to counter the export of bank material due to erosion. One result of his analysis, for sand-silt rivers in regime, is the hydraulic geometry equation (3.39).

$$R_c = 85.1\ R_f^{1/2}\ S^{-1/2} \qquad \text{.....(3.39)}$$

where $R_c = d_c/D_s$ = center relative roughness
$R_f = v_s/(R_b\ g\ D_s)^{1/2}$
v_s = fall velocity
$R_p = [(R_b\ g\ D_s)^{1/2}\ D_s]/k_v$
and Figure 3.19 relates R_f and R_p

In principle at least, equation (3.39) can be equated to the optimal width theories (Chang, 1979, 1985c; White et al., 1982), since for a given discharge defined reach with a constant slope and bed material equation (3.39) defines one depth and, by continuity, therefore one

16 See Section 3.3

width. In practice Parker uses observed variations in slope and sediment size to try to account for observed variations in depth.

Parker (1978b) tested equation (3.39) with data from the Middle Loup (Colby and Hembree, 1955) and Niobrara Rivers (Hubbell and Matejka, 1959). These two rivers were noted to be close field analogues to the theoretical case under consideration. Parker (1978b) chose as his dominant discharge that flow which if it continued constantly for one year would carry the total load for the year. This equated to 11.0 m^3/s for the Middle Loup River and 9.9 m^3/s for the Niobrara River. Table 3.2 (Parker, 1978b) represents the data used to verify equation (3.39). The predicted values of d_c using equation (3.39) and assuming D_{15}[17] as the representative particle size are also given in Table 3-2.

TABLE 3-2 HYDRAULIC GEOMETRY FOR THE MIDDLE LOUP AND NIOBRARA RIVERS: OBSERVED VERSUS COMPUTED (EQUATION 3.39) PARKER (1978B)

RIVER	DATE	SECTION	Q (m^3/s)	S (*1000)	D_{15} (mm)	W (m)	observed d_c (m)	computed d_c (m)
Middle Loup	25/06/51	A	11.1	1.29	0.18	87.8	0.210	0.318
	16/08/51	B1	10.7	1.33	0.20	28.6	0.483	0.313
	04/12/51	C2	11.5	1.34	0.23	25.0	0.472	0.312
	25/05/86	E	11.1	1.19	0.20	44.2	0.324	0.331
Niobrara		C2	9.6	1.23	0.18	33.5	0.396	0.277
		C6	9.9	1.40	0.175	40.5	0.316	0.259

Parker (1978b) concluded from this data (along with some canal data of Simons and Albertson (1963)), that his method predicted grossly correct depths and that the mechanism he proposed for maintaining channel equilibrium is a reasonable one. However, Figure 3.20 shows that, for the Middle Loup River data, Parker's mechanism does not account very well for the observed variations in depth. In fact a single curve could be fitted to the observed Middle Loup data as shown in Figure 3.20, indicating clearly that width is more important then slope or bed sediment size in determining the observed depths. Figure 3.20 gives graphic illustration of the inverse non-linear relationship

17 D_{15} = .20 and .18 mm assumed for the Middle Loup and Niobrara respectively.

between width and depth for a straight alluvial discharge defined reach of river. Since Parker (1978b) was concerned with equilibrium conditions, it is apparent that the width:depth equation (1.1) describes equilibrium, in this case much better than a complex three-dimensional analytical model. Lastly, Table 3-2 illustrates that small changes in local slope can and do occur along a reach of varying widths, perhaps in direct response to changing bank conditions in an attempt to maintain sediment equilibrium.

Parker (1978c) solved the paradox of mobile bed and stable banks for gravel rivers by considering lateral redistribution of perimeter shear stress due to lateral turbulent diffusion of downstream momentum. The method allows for an active bed with low rates of transport of coarse material, with inactive banks. Parker (1979) modified his previous methodology (Parker, 1977) to account for self formed straight gravel rivers, arriving at the resultant rational regime relations given by equation (3.40).

$$R_c = f_1(Q\sim, W_*);\ S = f_2(Q\sim, W_*);\ Q_* = f_3(Q\sim, W_*); \quad \ldots.(3.40)$$

where R_p, R_b and σ_g are implicit to the right side of the equations but are suppressed for brevity.

and

$$R_c = d_c/D_{50} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.41a)$$

$$W_* = W/D_{50} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.41b)$$

$$Q\sim = Q/(R_b g D_{50})^{0.5} D_{50}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.41c)$$

$$Q_* = Q_s/(R_b g D_{50})^{0.5} D_{50}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.41d)$$

Equations (3.42) are specific forms of the resultant equations (3.40) derived analytically (Parker, 1979), by solution to the momentum equation (3.26), a sediment transport equation and a hydraulic resistance equation for gravel rivers[18], and assuming that the central dimensionless shear stress exceeded the threshold value by the nearly constant value of 20%.

$$R_c = 0.866\ Q\sim^{0.830}\ W_*^{-0.830} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.42a)$$

$$S = 6.62\ 10^{-2}\ Q\sim^{-0.819}\ W_*^{0.819} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.42b)$$

$$Q_* = 9.80\ 10^{-6}\ Q\sim^{0.228}\ W_*^{0.772} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.42c)$$

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18 Parker (1979) developed empirical sediment transport and resistance equations using the data of Peterson and Howells (1973)

Equations (3.42) were further reduced by back substituting the empirically determined equation (3.43).

$$W_* = 4.4\ Q_{\sim}^{0.5} \quad \ldots\ldots(3.43)$$

The resultant equations for straight self-formed gravel bed rivers were given by equations (3.44).

$$R_c = 0.253\ Q_{\sim}^{0.415} \quad \ldots\ldots(3.44a)$$

$$S = 0.223\ Q_{\sim}^{-0.410} \quad \ldots\ldots(3.44b)$$

$$V_* = 0.898\ Q_{\sim}^{0.085} \quad \ldots\ldots(3.44c)$$

$$Q_* = 3.08\ 10^{-5}\ Q_{\sim}^{0.614} \quad \ldots\ldots(3.44d)$$

Parker (1978b, 1979) verified equations (3.42) with data from several sources including Neill (1973) (i.e., Figure 3.2) and Kellerhals (1963) (cited Parker, 1979) covering a large range of discharges, Q~. The exponents in equations (3.43) and (3.44a,b) are in close agreement with the regime equations (3.7) and (3.8).

Parker's (1978b, 1979) analysis of gravel bed rivers can be related to width:depth analysis. Firstly, if the reach under consideration is constrained from attaining the width prescribed by equation (3.43)[19], due to bank conditions, then equations (3.42) represent equilibrium conditions. If discharge (Q~) is constant (i.e., a discharge defined reach), then equation (3.42a) reduces to an equivalent form of equation (1.1) with an implied n value of 0.83. This corresponds closely to Komura's (1963) n value for static equilibrium, which could be expected since Parker's (1979) equations are developed for near threshold conditions on gravel bed streams. As the exponent found by Blench (1980) (i.e., equation 3.25a) was also around 0.8, it would seem that near threshold conditions are quite common in gravel bed streams. This fact is also highlighted in the difference in the regime depth equations for sand bed (equation (3.6b)) and gravel bed (equations (3.7b) and (3.8b)) streams. Secondly, the form of equation (3.42a) also indicates the variables that control the coefficient K in equation (1.1), implying that K is equal to $0.866\ Q_{\sim}^{0.830}$. This suggests that K is sensitive to the reference flow condition, Q. Thirdly, equation (3.42b) confirms that the slope will change with the width. Lastly, equation (3.42c) implies that sediment discharge will vary directly with

19 That is W does not equal $4.4\ Q_{\sim}^{0.5}$, but varies say as per equations (3.11).

width (i.e., $Q_* \propto W^{0.722}$). Equation (3.42c), however, implies disequilibrium of sediment transport for a reach with varying widths, which is contrary to the conclusion that these equations represent equilibrium conditions. No reason was found for this discrepency.

This Section has reviewed several analytical techniques for determination of hydraulic geometry of alluvial rivers. In general the review supports the supposition that width:depth equations of the form equation (1.1) [$d = K W^{-n}$], and also equation 1.2 [$A = K W^{1-n}$], can be used to represent equilibrium conditions. Chang (1977) in particular reports results which confirm this qualitatively. Moreover, Chang's (1984a) results for meandering sand-bed equi-width point bar streams suggest that curvature has only a minor influence on the relationship between width and mean depth, in agreement with Tamburi (1976). Parker (1979) uses analytical methods to provide regime relations for gravel rivers that appear to coincide with the width:depth equation (1.1). Another alternate form of the width:depth equation (1.1) has been proposed, as given by equation (3.28) [$d = K' q^{n}$], in addition to that proposed from the review of the regime methods (i.e., equation (3.21) [$R = K\sim q\sim^{n}$]). Equations (3.21) and (3.28) appear to adequately describe portions or all of the equal concentration lines (i.e., equilibrium channel geometry) reported in the analytical approaches of Paris (1984) and Parker (1977), respectively. The review suggests that $n = f(X_{bm})$, with a continuum existing from $X_{bm} = 0$ to $X_{bm} > 0$ corresponding to the continuum from static to dynamic equilibriums. It appears from the review that generalized analytical methods (e.g., Parker, 1977) can be used to quantify the relationship between the width:depth equations (i.e., n, K, K' and K~) and the governing variables, thus overcoming the inadequacies with the analytical bases proposed by Tamburi, in Section 3.2.

Paris's (1984) results, along with the results for the optimal width methods of White et al. (1982) and Chang (1979) suggest that the conditions for maintaining equilibrium along a discharge defined reach with varying widths would include local variations in the slope. Parker (1978) provides data for the Middle Loup River that confirms the strong influence of width on depth, and the potential for local slope

variations, although the data is far from conclusive. Chang (1985c) further illustrates the flexibility of natural rivers, by showing that adjustment of channel side slopes can allow multiple widths to represent stable (i.e., equilibrium) conditions.

The optimal width methods correspond to the traditional regime methods for canals, gravel rivers and sand-bed rivers of the equi-width point bar type with flow in the lower flow regime, and homogeneous bed and bank materials. As such width:depth analysis can be considered to be an addendum to the optimal width methods, in which one considers the constraints to attaining the optimal width (e.g., bank competence). Width:depth analysis does not replace the optimal width methods since it is not predictive of width. The stability condition of minimum and equal slope in a discharge defined reach, as implied by the optimal width methods (White et al., 1982; Chang, 1979, 1980a,b, 1985b,c) is not necessarily contradictory to the conditions required for width:depth analysis to be in regime, since equilibrium sediment transport can occur for non-optimal conditions.

In Chapter 4, analytical methods will be used to understand the theoretical underpinnings of the width:depth equations (1.1), (3.21) and (3.28). The relationship of n, K, K', and K~ to the controlling variables is defined, and then quantified with an analytical approach.

3.5 Observed River Behavior

The transition from the analytical consideration of an idealized straight unconfined alluvial river to the observed behavior of natural rivers can be large (Vanoni, 1978). The attainment of optimal width, as proposed by White et al. (1982), Chang (1979), Parker (1978b,c), and Griffiths (1939) may be constrained by bank resistance, vegetation or other factors. This Section will illustrate this last statement with observations of alluvial rivers. Observed river behavior, outside of the Fraser and Mississippi Rivers, will be required in Chapters 4 and 5 to verify the analytically derived results to the width:depth equations (1.1), (3.21) and (3.28). The Section provides several rivers on which width:depth analysis appears to be an appropriate tool. The question of whether the rivers being discussed are in equilibrium should be kept in mind, even though the nature of this question will make it hard to

answer. Detailed analysis of the observed river data is left to Chapters 4 and 5.

Consider first the Middle Loup River near Dunning, Nebraska (Hubbell and Majteka, 1959). Parker (1978b) notes that sand predominates in the bed, banks, and flood plain of this straight reach of river. Furthermore, it has a relatively uniform discharge maintained by groundwater inflow. Even though this river is a close analogue to the idealized analytical river (Parker, 1978b), and Table 3-2 excludes any data points with explicit indication of locally inerodible banks, width and depth variations of 350% and 230% (of the minimums) respectively are observed in Table 3-2. The width:depth equation (1.1) can be used to represent Parker's (1978b) data, as shown in Figure 3.20 by the regression curve. The width:depth equation (1.1) in Figure 3.20 is described by $n = 0.68$ and $K = 4.4$. The correlation coefficient $r^2 = 0.97$ is significant at the 5% level for this small number of data, implying that a relationship does exist. The fact that Parker (1978b) used discharge to choose his data points is an indication that discharge might be substituted for sediment concentration in the relation proposed in Section 3.4 (i.e., $n = f(X_{bm})$).

Next, consider the work of Williams and Wolman (1984), who studied selected reaches of alluvial rivers before and after construction of major storage works. Their criteria for selection of suitable reaches of rivers included:

1. an alluvial bed at the time the dam was built .
2. surveyed channel cross-sections at various sites downstream of the dam (one of which was done at about the time of dam construction).
3. no significant dredging, channelization, or similar operations in the study reach.

Of interest is the fact that many of the alluvial rivers studied by Williams and Wolman (1984) exhibit significant width variations along discharge defined reaches of river, for any given season of surveys.

Considering the Missouri River below Garrison Dam, North Dakota (dam closure in 1954), Williams and Wolman (1984) made the following

statement:

> "Although the mean width of the Missourri downstream of the Garrison Dam in North Dakota in 1957 was 415 m for a reach 87 km long, the standard deviation of 24 measurements was approximately 129m or 29 percent. The actual widths ranged from a minimum of 255m to a maximum of 845 m."

Figure 3.21 demonstrates that the 87 km reach of the Missouri River has relatively little local inflow compared to its upstream drainage basin, and therefore can be considered discharge defined. Figure 3.22 presents channel width measurements in 1946, 1949 and 1954 on the Missouri R. below Garrison Dam, prior to or just after dam closure, illustrating variablity in excess of that in 1957. Noting that silt content in the banks of this reach varied from 3 to 100 percent, Williams and Wolman (1984) concluded that cohesive banks will limit both channel width and lateral migration. Not enough data on bed material variation are presented to be able to comment on whether non-regime beds have a role in determining the observed channel width variations. However, several locations along this reach are severely constrained on one or both sides by bedrock.

The North Canadian River in Oklahoma, is stated to have nearly constant bed material sediment sizes for long distances downstream (Williams and Wolman, 1984). The reach considered by Williams and Wolman (1984) is below the Canton Dam (closure in 1948) and its general location is illustrated in Figure 3.23. Review of topographic maps reveal that this reach has little local surface inflow, and therefore is likely discharge defined. Figure 3.24 demonstrates observed width variations for the 58 km reach below the Canton Dam for both pre-dam and post-dam conditions.

Williams and Wolman (1984) concluded with regard to the rate of bank erosion observed on rivers, that:

> ".. the rates of change (of width with time) range from a few meters to hundreds of metres per year (on larger rivers such as Yellow River, China); however, most changes are less than 1 per cent (of the width) per year"

This relatively slow rate of erosion is an important consideration in establishing top width measurements as being relatively stable over the short-term and therefore potentially useful as an independent variable in regime analysis. It is suggested here that depth adjustments to

compensate for such small changes in width, could proceed at least as quickly, thus allowing adjustment to maintain continuity and equilibrium.

Kuiper and Galay (1972) indicated that the South Saskatchewan River between the Alberta-Saskatchewan border and Saskatoon is alluvial in nature. Berry et al.(1961) demonstrated that little local inflow occurs along this reach, reporting annual flows at the Alberta boundary and Saskatoon of 7,760,000 ac-ft. and 7,950,000 ac-ft., respectively. Furthermore, this reach has a relatively constant bed slope as revealed in Figure 3.25 (North West Hydraulics, 1980), and an alluvial bed composed of uniform sand with a mean diameter of 0.3 mm (Kuiper and Galay, 1972). Figure 3.26 (Kuiper and Galay, 1972) reveals that bankfull width varies significantly along the South Saskatchewan River, even within the alluvial reaches of river. Kuiper and Galay (1972) note that the river in Alberta flows in a narrow valley which widens markedly in Saskatchewan, allowing the river to freely meander over the flood plain. The river bed reverts to glacial till and boulders at Saskatoon, as shown in Figure 3.25. The width and character of this river is closely controlled by the geology of the oversized glacial valleys in which the present day river flows.

Griffith's (1939) case study on the Hundred Foot River, provides insight into river response to channel widening. Approximately 5/8 of a mile of this river was widened, to which the river responded by reducing the channel depth. Figure 3.27 is a plot of width:depth data for two natural (and their average) and one widened cross-sections on this river. The regression curve fitted to the data of Figure 3.27 is equation (1.1). While the statistical analysis is not very meaningful, due to the lack of data, equation (1.1) does explain the observed variation between the wide and narrow sections.

Nordin (1961) studied the nature of an alluvial reach of the Rio Grande River near Bernalillo, New Mexico. Figure 3.28 is a plan view of the reach, demonstrating the observed width variations, and the lack of local inflow (i.e., no tributaries). The banks of this reach are composed of sand, silt and clay and are partially stabilized by vegetation (Nordin, 1961), while the bed is composed of sand. The flow and bed conditions in this reach have been observed to vary from lower

flow regime over a duned bed, through the transition to upper flow regime over a plane bed. Regarding the equilibrium status of the river Nordin (1977) claimed that within this reach :

> ".. on the average the same water discharge and the same sediment load pass both sections (i.e., A-2 and F, and therefore all sections in between these two)"

Figure 3.29 (Yang, 1978b) illustrate the basis for Nordin's statement. Nordin's statement implies that dynamic equilibrium is being maintained in this reach, suggesting that a width:depth equation developed for this river reach would be for regime conditions.

Parker (1978b) states that reaches of the Niobrara River approach his idealized analytical river. In particular, the Niobrara River near Cody, Nebraska (Colby and Humbree, 1955) has locally straight reaches in a generally weakly meandering river (Parker, 1978b). Figure 3.30 is a plan view of this reach, and serves to indicate the discharge defined nature of the reach. The Niobrara River flows through the Nebraska sand hills and exhibits a very uniform flow, transporting predominantly sand size material (see Figure 3.31, Colby and Hembree, 1955). The flow and bed conditions in this reach have been observed to vary from lower flow regime to transitional and upper flow regimes. The cross-sections C1 to C10, as located in Figure 3.30, were specifically chosen as exhibiting alluvial beds and having little lateral confinement. Yet, despite the uniform nature of this reach, width variation of up to 200% (of the minimum) are observed, as exemplified by Figure 3.32.

Obviously other examples of alluvial rivers exhibiting width variatons along discharge defined reaches exist. The intent of this section is not to document all cases, but simply to illustrate several practical examples of deviations from the theories of optimal river widths. The Niobrara, Middle Loup and Rio Grande Rivers especially lend themselves to further quantatative study due to such factors as: the availability of a large data base, accurate sediment transport measurements, and observations on flow regimes. These three rivers form the basis for verifying the analytical solutions to the width:depth equations that are developed in Chapters 4 and 5. Further data on these rivers in presented in those Chapters.

3.6 Sensitivity of Width:depth Analysis

The potential sensitivity of the width:depth equations (1.1), (3.21) and (3.28) to the reference flow condition (e.g., Q or X_{bm}), to flow regime and to channel pattern will impact on whether they can be used to describe equilibrium channel geometries. In this Section the potential sensitivity of the width:depth equations is examined in the light of several pertinent references. The intent is to provide a foundation for the quantitative analyses presented in Chapters 4 and 5.

Local Low Water (LLW) was employed in both previous applications of width:depth analysis (Tamburi, 1976,1979, et al. 1983, Shewfelt and Pomalaza, 1981) as the basis for developing the width and depth data base. Tamburi et al. (1983) stated that LLW encompased the zone of significant conveyance and that the hydraulic geometry of the LLW channel is in equilibrium with typical flow conditions. However, the terms *significant conveyance* and *typical flow conditions* are not given a quantitative reference (e.g., discharge). As an alternative for future studies, Tamburi et al. (1983) suggested employing backwater analysis using low flow hydrology. Again, however, the descriptive term *low flow* was not quantified.

Inherent to the use of LLW as datum is that equation (1.1) and therefore n and K are defined at some distinct or dominant discharge, Q_{dom}. This conclusion leads naturally to the question of how n and K (and also K', K~) vary with the choice of reference flow condition (i.e., Q_{dom}), and to the postulation that a relationship of the form of equation (3.45) exists.

$$n, K, K', K\sim = f(Q) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.45)$$

Alternately, as discovered in Sections 3.4, n appears to be related to bed material concentration, X_{bm}, and Peterson's (1975) equation (3.23a) suggested that K may be related to the sediment concentration also. It may therefore be appropriate to use X_{bm} as the reference flow conditon, and as per equation (3.46). As shown in Section 3.4, the use of sediment concentration as the reference flow condition corresponds to the concept of equilibrium (i.e., lines of equal concentration in Figure 3.9).

$$n, K, K', K\sim = f(X_{bm}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.46)$$

If Q and X_{bm} are related then equations (3.45) and (3.46) can be interchanged. One must achieve an understanding of the sensitivity to the choice of datum in order to determine whether the computed value of n (and K) reflects the state of equilibrium for that reach or simply the choice of the reference flow condition. As will be shown in Chapter 4, the values of n, K, K' and K~ are related to other independent variables as well.

The link between discharge and channel flow regime is illustrated by Figure 3.33 (Nordin, cited ASCE, 1975). Bed forms change from dunes, to transition to flat bed and antidunes as discharge increases, corresponding to a transition from lower to upper flow regime. On a broader scale, discharge and channel slope are also related to channel pattern, as shown in Figure 3.14 (Chang, 1985c). Therefore, if n and K vary with the reference flow condition, possible changes in channel flow regime or pattern must also be accounted for.

A brief discussion on the potential sensitivity of width:depth analysis to these various factors follows.

3.6.1 Sensitivity to Reference Flow Condition

Maclean (discussion Tamburi, 1983b) noted, for the Fraser River, that the soundings on which the width:depth equations are based were taken during the approximate mean annual discharge. It was further suggested that since degradation (scour) would be observed in the contracted reaches during higher flows, the exponent of equation (1.1) (i.e., n) would vary with the flow, Q. This process of scour (and fill) was noted by Dawdy and Culbertson (1961) to be a logical adjustment of a contracting reach in maintaining a constant slope. Dawdy and Culbertson (1961) concluded, via theoretical consideration of Bernoulli's equation, that during periods of increased discharge wide sections would fill and narrow sections scour, with the inverse occuring during low flows. Chang (1985e) examined the San Diego River during a specific flood event, and documented this same process of scour of narrow sections and filling of wider sections. Chang (1985e) also suggested that these types of adjustments allowed the river to maintain a constant slope (i.e., uniform energy loss). If one relates this scour and fill process directly to corresponding changes in depth (i.e., scour increases depth,

fill decreases depth) then one might initially judge that filling of narrow sections at lower discharges and scour at higher discharges, with the reverse for wide sections, would lead to an increase of n with discharge. That is the slope of the width:depth curve increases with discharge due to the scour/fill process. Figure 3.34 illustrates this concept.

The tendency exemplified by Figure 3.34 is not necessarily valid. Dawdy and Culbertson (1961) concluded that W_1/W_2 is proportional to r_1/r_2 and therefore area along a reach is relatively constant. This is contrary to the observations on the Mississippi and the Fraser Rivers (Figures 2.6 and 2.7), and to Chang's (1978) analytical solutions for stable alluvial canals (Figure 3.11). Furthermore, Dawdy and Culbertson (1961) related scour and fill to bed elevation (i.e., stage) as opposed to the depth, d, or hydraulic radius, r. Simons and Richardson (1961) concluded that the use of depth rather than stage allows for the elimination of discontinuities in the stage-discharge relationship due to scour and fill processes. In other words an increase in the bed stage due to fill is not necessarily coincident with a decrease in depth. Therefore, the initial judgement that n increases with discharge (i.e., Figure 3.34) may be incorrect.

Limited insight into the sensitivity of n to the choice of datum was obtained by reconsidering ranges 53 - 77 (refer to Figure 1.1) of the Fraser River. Width:depth equations (1.1) were developed for LLW, LLW - 5 feet, and LLW - 10 feet. The results of least square regression analysis are reported in Table 3-3 and Figures 3.35 and 3.36.

TABLE 3-3 FRASER RIVER 1977 WIDTH:DEPTH ANALYSIS: SENSITIVITY OF EQUATION (1.1) TO DATUM

Datum	n	K	r^2
LLW	0.795	11,956	0.86
LLW-5'	1.005	48,009	0.86
LLW-10'	1.195	158,938	0.86

The results of this cursory analysis imply the opposite trend to that of Figure 3.34. Specifically, if lowering the datum below LLW is equated with a decrease in dominant discharge, then n appears to increase with decreasing discharge. This comparison is not entirely

valid of course since LLW by defintion is the lowest observed water level. Furthermore, the values of n exceed the milestones suggested by Tamburi (1976) for static equilibrium. Although, Tamburi (1976, 1979) stated that such a condition can be approached for a dredged reach, this would not explain $n = 0.795$ for LLW. Further analysis of the Fraser River in this regard was not undertaken for the several reasons, including: the unknown effects of dredging; the lack of readily available elevations above LLW; the lack of coincident discharge, profile and total concentration measurements; and the complicating effects of split flow and sediment load, non-regime bed materials and existing training works.

The tendency of n to decrease with increasing discharge and/or sediment concentration can be supported by reconsidering Figure 3.1 (ASCE, 1975). Assume for the moment that equations (3.45) and/or (3.46) are valid. Since sediment transport will increase with shear stress, Figure 3.1 implies that n will decrease with increasing sediment concentration. Assuming that sediment concentration increases with discharge implies that n decreases with increasing discharge.

Analytical methods are employed in Chapters 4 and 5 to quantify the relationship of n (and K) to the reference flow condition.

3.6.2 Sensitivity to Flow Regime

To comprehend, at least qualitatively, the implications of changing flow regime on the application of the width:depth equations, consider the data collected for the Rio Grande River near Bernalillo, New Mexico (Dawdy, 1961; Nordin, 1964). Lower, transitional and upper flow regimes have been observed on this reach of river (Figure 3.28). Lower flow regime is typified by dune bed forms, and sometimes ripple bed forms for smaller sediment sizes (Van Rijn, 1982). The transitional flow regime is characterized by washed out dunes and plane beds, while the upper flow regime exhibits both plane bed and antidune configurations (Van Rijn, 1982). The variations in bed roughness as a result of changes in bed forms, account for observed discontinuities in the Rio Grande rating curves, as illustrated by Figure 3.37 (Brownlie, 1981a). Although the influence of this discontinuity on width:depth analysis is unclear, it seems possible that a similar discontinuity may

appear in the width:depth relations. Conversely, Figure 3.38 shows that despite the discontinuities in Figures 3.37 and 3.33, the resulting velocity versus sediment discharge curve is continuous. Since it is apparent that the rating curve in Figure 3.38 could be represented by equation (1.3), and since a relationship has been proposed between equations (1.3) and (1.1) then one might anticipate the width:depth relation to be continuous (i.e., single-valued) over the transition between flow regimes. If this holds true then a simple width:depth equation of the form of equations (1.1), (3.21) or (3.28) may adequately describe the data.

On the other hand, Chang's (1979) analytical approach to hydraulic geometry of sand-bed rivers demonstrated the possibility of discontinuities in the width:depth and width:velocity (i.e., area) relations for constant Q and Q_s, as a result of transition from the lower to upper flow regime. Figure 3.12 illustrates this fact. It is important to realize that Figure 3.12 was developed for a constant discharge and the changes in flow regime are a direct result of changes in channel width. Therefore, different flow regimes might occur simultaneously within a discharge defined reach with varying channel widths, as well as resulting from changes to the reference flow condition. The apparent discontinuities in Figures 3.12 result from analytical assumptions regarding the transition zone, and could be modified by field conditions. The dashed lines on the S versus B curves in Figure 3.12 represent this type of gradual change (Chang, 1979). Figure 3.12 also shows that as Q_s increases at a given Q (i.e., increasing X_{bm}), the magnitude of the discontinuity decreases. Finally, it is worth noting that the results in Figure 3.12 are for a trapezoidal cross-section with constant 3:1 side slopes. Chang (1985a) shows that a side slope adjustment within a reach can create equal minima and perhaps reduce the impact of transition from the lower to upper regime, as in Figure 3.18. The limited amount of results presented in Chang (1979, 1985c) prevent any general conclusions from being drawn.

Figure 3.6 (Parker, 1977) reveals double valued behaviour in the empirical relationships for q~ versus R. Parker attributed this behavior to transitional phenomena associated with high slopes. It is apparent that a simple equation of the form of equation (3.21) does not

account for this transitional phase, and therefore appears to inadequate to describe equilibrium channel geometry under such conditions. Engelund's (1967) and Paris's (1984) relationships as presented in Figure 3.19a, 3.8 and 3.9 respectively, are valid for the lower flow regime only, and therefore do not exhibit this transitional behavior. As stated previously equations (3.21) and (3.28) appear to adequately model these results for the lower flow regime. White et al. (1987) have recently proposed a resistance function for the upper flow regime but have not yet provided solutions equivalent to that in either Paris (1984) or White et al. (1982).

Chang (1980) stated that flow resistance in gravel rivers is mainly due to grain as opposed to bed-form roughness. Therefore, one would not expect the discontinuous relationships such as in Figure 3.37 to exist for gravel bed rivers.

A need exists to determine the sensitivity of the width:depth equation to the transition between flow regimes. Chapter 5 presents quantitative results in this regard.

3.6.3 Sensitivity to Channel Pattern and Thresholds

Channel pattern is often used to distinguish rivers with similar hyraulic characteristics (Morisawa, 1985; Chang, 1979, 1985c; Leopold and Wolman, 1953) or to form part of comprehensive classification systems (Kellerhals et al., 1976). The general channel pattern states are considered to be straight, meandering and braided (Morisawa, 1985).

The threshold line proposed by Leopold and Wolman (1957) separating meandering and braided rivers is given by equation (3.47), and Figure 3.39.

$$S = 0.06\ Q^{-0.44} \qquad \text{.....(3.47)}$$

Chang's (1985c) definition of four distinct river patterns is illustrated by Figure 3.15 and described as follows:

a. *Equi-width point bar streams* - characterizations include: flat slope, low velocity, small bedload, lower flow regime (i.e., ripples and dunes), relatively narrow and deep channel (e.g., meandering), width and depth relatively constant.

b. *Wide-bend point bar streams* - characterizations include: steeper slopes, lower to upper flow regimes, width and depths vary due to the riffle/pool sequences (Figure 3.4).

c. *Braided point bar streams* - characterizations include: as per b., somewhat braided due to increased discharge, slope or both.

d. *Braided streams* - characterizations include: high sediment discharge rates, large width:depth ratios (e.g.,100), pool sections generally non-existant, quite often straight although anabranches may be sinuous.

Of interest here, is the sensitivity of the width:depth relationships to the transition between channel pattern states. Considering the relationship of the traditional regime methods to channel patterns is instructional.

Chang (1985c) investigated the thresholds between the channel patterns states listed above. His analytical solutions (Figure 3.14) illustrated that traditional hydraulic geometry regime relations (i.e., equations (3.7) and (3.8)) tend to coincide with region 1 or equi-width point bar streams and canals, and do not recognize the increasing influence of slope on width as one crosses the thresholds between channel pattern states. ASCE (1975) agreed that regime relations are generally applicable only to the lower flow regime. Since the lower flow regime persists exclusively in equi-width point bar streams and Chang's (1985c) region 1 (Figure 3.14), discontinuities in the width:depth relationships might not be expected for these types of reaches. However, for all other pattern states and regions (Chang, 1985c) any flow regime is possible, as shown in Figure 3.18, depending on the river width and therefore subject to any discontinuities that might be caused by transition between flow regimes.

Neill (1973) contradicts Chang's (1985c) conclusions regarding the application of traditional regime equations. Neill considered 12 reaches of sand-bed rivers classifying them according to his own subjective criteria, as illustrated in Figure 3.2. Figure 3.2a, presents Neill's data for width versus discharge, showing no apparent thresholds association with channel pattern, and indicating that the

regime equations (3.6) are continuous over several channel pattern states.

In lieu of firm conclusions, the following observations offer guidance as to the possible influence of channel pattern states on width:depth analysis. Channel pattern state for any river being studied is obviously important and should be established prior to undertaking width:depth analysis. It is unlikely that a singular reach will exhibit multiple channel pattern states unless it is located near a threshold (e.g., Chang, 1985c). Width:depth analysis may not account for reaches bordering on thresholds between channel pattern states, due to the possibility of multiple pattern states, and the resulting thresholds in hydraulic geometry relationships. The separation of reaches by channel pattern is indicated as a method of accounting for the influence of channel pattern. The following example illustrates this proceedure.

Consider the South Saskatchewan River in Figure 3.25 and the threshold line between meandering and braided streams as proposed by Leopold and Wolman (1957) (i.e., equation (3.47)). NWHC (1980) reports an average river slope of 0.00024 for this reach, with a bed material size (D_{50}) of 0.3 mm, and a bankfull discharge of 4250 m^3/s. Plotting this data on Figure 3.39 indicates that the South Saskatchewan borders the threshold given by equation (3.47). Confinement of the river at Saskatoon causes a meandering pattern to be maintained as opposed to a braided reach upstream. For purpose of analysis one might sub-divide the reach into two sub-reaches, being braided and meandering.

In Chapter 4 the simple case of a straight wide alluvial river is considered, and Chapters 4 and 5 report analytical solutions to the width:depth equations (1.1), (3.21) and (3.28) for such a river, for the lower, transitional and upper flow regimes. Data from the Rio Grande, Niobrara and Middle Loup rivers are used to verify the analytical solutions. These rivers are all considered meandering in accordance with the Leopold and Wolman (1957) criteria, as shown in Figure 3.25, and detailed further in the subsequent Chapters. Furthermore, the reaches of these rivers that are used are locally straight to weakly meandering (e.g., Figure 3.28). The Niobrara and Rio Grande Rivers report flow regimes ranging from the lower to the upper flow regime. In comparison to Chang's (1985c) criteria (i.e., Figure 3.15), these rivers

appear to correspond to either the a. equi-width point bar streams or b. wide-bend point bar streams. For the purpose of this thesis these two channel pattern states are assumed to be equivalent to the meandering state described by Leopold and Wolman (1954). Therefore, this thesis does not address the applicability of width:depth equations to braided reaches of rivers.

3.7 Summary

The literature review presented in this Chapter has provided an understanding of the relationship between width:depth regime analysis and other regime and analytical methods for studying hydraulic geometry. The review suggests, in a qualitative manner, that the width:depth equation (1.1) can be used to represent equilibrium channel geometries (i.e., regime). Furthermore, the review has indicated possible conditions (e.g., local variations in slope) that are required to maintain this equilibrium. In addition the review has suggested two alternative forms of the width:depth equation given by equations (3.21) and (3.28)

$$R = K\sim q\sim^{n} \quad ..(3.21) \qquad\qquad d = K' q^{n} \quad ..(3.28)$$

Lastly, the potential sensitivity of the width:depth equation (e.g., n, K, K', K~ in equations (1.1), (3.21) and (3.28)) to the choice of the reference flow condition, and to the transition between flow regimes and channel pattern states has been discussed.

This Chapter has provided a foundation for the formal understanding of and quantified analytical solutions to the width:depth equations, presented in Chapters 4 and 5.

CHAPTER 4

THEORETICAL UNDERPINNINGS OF WIDTH:DEPTH ANALYSIS

4.1 Introduction

This chapter employs analytical methods[20] to address the theoretical underpinnings of width:depth analysis. Empirical methods are used to test the analytical results. The topics to be covered in their natural progression are:

4.2 A statement of case for alluvial rivers is developed, which includes consideration of the governing variables and process equations, and the resultant relationships between the variables.

4.3 The statement of case provides a semi-theoretical foundation for the development of rational forms of the width:depth equations[21]. The rational forms, developed herein, illustrate the theoretical underpinnings of width:depth analysis by defining the variables which should affect the exponent and coefficients of the width:depth equations (1.1), (3.21) and (3.28)

$d = K \ W^{-n}$ ...(1.1);

$R = K\sim \ q\sim^{n}$ ...(3.21);

$d = K' \ q^{n}$ ...(3.28).

4.4 Analytical solutions to the rational width:depth equations are developed through simultaneous solution of the process equations. The solution data are used to test whether width:depth equations (3.21) and (3.28) can be used to represent equilibrium conditions. Moreover, the solution data are used to quantify the sensitivity of the width:depth equations (i.e., n, K' and K~) to the reference flow condition (i.e., bed sediment concentration, X_{bm}) and the bed sediment size, D_g, for the lower flow regime. In addition, the relationship between the various regime and analytical methods is formalized.

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20 See footnote 8 page 6 for precedents
21 See footnote 9 page 6 for definition.

4.5 Empirical investigations involving data from the Middle Loup River are undertaken to field test the analytical solutions of Section 4.4. In addition, discharge, Q is evaluated as an alternative to concentration, X_{bm}, for defining the reference flow condition.

4.2 Statement of Case for Alluvial Rivers

4.2.1 General

A statement of case is a framework for describing and understanding alluvial rivers (Hey, 1978, 1986; Blench, 1986; ASCE, 1975), involving specification of points A. to D. below.

A. the general characteristics of the river system being considered [e.g., alluvial, cohesive, narrow, wide, meandering, braided, etc.].

B. the variables that sufficiently describe that river [e.g., width, depth, slope, bed material, etc.].

C. the relationships that describe the governing processes [e.g., sediment transport, flow resistance, etc.].

D. the resultant relations describing the dependency of the governing variables [i.e., dependent variables = f (independent variables].

Analytical methods solve the equations proposed in step 3 to arrive at the resultant relations of step 4 , while regime methods ignore step 3 and find the resultant relations directly using empirical means.

An example of the above process is developed from consideration of Hey (1978).

A. Hey considered sand-bed alluvial rivers, without qualifications on the plan form geometry.

B. Hey's variable list was as follows:

dependent : $r, P, V, S, d_{max}, Z_m, bf_d, bf_a, S_m$

independent: $Q_{bm}, Q, D_b, o_b, D_r, o_r, D_l, o_l$

where r = hydraulic radius
P = wetted perimeter
V = mean velocity
S = channel slope
d_{max} = maximum channel depth
Z_m = meander arc length
bf_d = bed form height
bf_a = bed form amplitude
S_m = sinuousity
Q_{bm} = bed sediment load
Q = discharge
D_g, D_r, D_l = sediment size of bed, right and left banks
o_g, o_r, o_l = gradation of bed, right and left bank material

C. Hey (1978) proposed the following nine linking equations, describing the process of channel adjustment to changes in the independent variables. The exact form of the equations is not important to understanding the proceedure.

1. continuity (i.e equation 3.1)
2. flow resistance
3. sediment transport
4. and 5. bank competence equations
6. and 7. bed form equations
8. and 9. river meanders; planform geometry equations

D. Hey concluded that hydraulic geometry is determinate, through the simultaneous solution of the nine process equations; giving the resultant equations of the form of equation (4.1).

$$r, P, d_{max}, V, S, Z_m, bf_d, bf_a, S_m = (Q_{bm}, Q, D_g, o_g, D_r, o_r, D_l, o_l) \ldots\ldots (4.1)$$

4.2.2 <u>Specific statement of case</u>

For the purposes of the work that follows it is necessary to present a concise statement of case. The statement of case is part and parcel to the development of *rational regime relations*, and also provides a basis to illustrate the differences among the methods for determining hydraulic geometry relationships.

Various authors have recognized the fact that studying a simplified case for alluvial river flow can serve as a basis for understanding the more complex case (Paris, 1984; Parker, 1977). Chang (1979, 1985c) for example, developed an increasingly more complex model

from a simplified beginning. For the purpose of this thesis, a relatively simple case is examined, as described below, with the hope that more complex situations can be built upon the results.

A. The river being considered is characterized as follows:

"Straight, alluvial river, comprised of non-cohesive beds, having steady non-uniform flow, sufficiently wide so that d_{max}=d=r and W=P, and channel geometry is described by top width W and average depth, d"[22]

B. Choose the minimum list of variables to describe this river as follows[23]:

$W, d, f, V, S, Q_{bm}, Q, X_{bm}, D_g, o_g, p, p_s, k_v, g$

where

f, d'Arcy-Weishbach friction factor = $V^2/(8grS)$ ...(4.2);

and other variables previously defined.

This list is deemed sufficient, contingent on the following assumptions:

1. D_r, o_r, D_l, o_l mainly influence width and bank shape. Therefore they have been excluded, since W is to be considered as an independent variable[24], and also since the river is wide enough that bank shape does not significantly affect the flow.

2. Wash load concentration, X_w has been excluded to maintain the simplicity of the model. The analytical results obtained and reported herein do not consider wash load. The field data that will be presented have been screened for excessively high wash loads, which might impact fluid viscosity and therefore sediment transport, bed forms and channel shape.

3. Fall velocity, w has been excluded to avoid the complexity associated with the influence of particle shape on sediment transport. Again the data presented are not influenced to any great extent by particle shape.

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22 Hey (1978), considering a more generalized model, stated that wetted perimeter, P, hydraulic radius, r, and maximum depth, d_{max}, are required to totally specify cross-section geometry.

23 This list derives from a review of Hey (1978, 1986), Blench (1986), ASCE (1975), and Nordin and Stevens (1987).

24 If prediction of W was required these variables cannot be ignored.

4. The friction factor includes both grain and bed-form roughness.

Some authors prefer to non-dimensionlize the variables. The degrees of freedom which a river may adjust to its imposed conditions is not reduced. Simply put, the variables g, p, and p_s are internalized within the other variables in a way that makes the variables non-dimensional. The following set of non-dimensional variables is developed from a review of Parker (1977, 1979) and Stevens and Nordin (1987):

$W_* = W/D_g$, dimensionless width;

$R = d/D_g$, relative roughness (depth);

$Q_* = Q/((R_b\ g\ D_g)^{0.5}\ D_g^{\ 2})$, dimensionless discharge;

$Q_{bm*} = Q_{bm}/((R_b\ g\ D_g)^{0.5}\ D_g^{\ 2}$, dimensionless sediment load;

$V_* = V/(R_b\ g\ D_g)^{0.5}$, dimensionless velocity;

S, slope ; o_g, sediment gradation;

X_{bm}, bed material concentration;

f, d'Arcy-Weishbach friction factor;

$R_b = (p_s/p-1)$, sediment bouyancy parameter;

$R_p = (R_b\ g\ D_g)^{0.5}\ D_g/k_v$, particle Reynolds number.

C. The following process equations are chosen for this specific case, based on a review of Hey (1978, 1986), ASCE (1975) and Stevens and Nordin (1987)[25]:

1. continuity of water ; equation 3.1
2. conservation of sediment mass; equation 4.3 (Stevens and Nordin, 1987):

$$X_{bm} = [(R_b+1)\ Q_{bm}/Q]\ 10^6 = [(R_b+1)\ q_{bm}/q]\ 10^6 \quad .....(4.3)$$

3. momentum; definition of f, equation 4.2 (Stevens and Nordin, 1987)[26]
4. sediment transport equation; (unformulated, bed material only)

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25 Hey (1978) and ASCE (1975) are in general agreement as to the number and type of equations required; their equations related to planform geometry are not required for a straight river; Stevens and Nordin (1987) included the conservation of mass equation, to account for the variable X_{bm}.

26 Other valid forms of the momentum equation are the Manning's and Chezy equations (Stevens and Nordin, 1987)

5. flow resistance equation; (unformulated)
6. bank competency equation [27] (unformulated, 1 req'd since d=r, W=P).

D. The 14 dimensional variables can be reduced to 8 independent and 6 dependent variables using the six process equations. Similarily, the 11 dimensionless variables can be reduced to 5 independent and 6 dependent variables. The following division is proposed for this thesis:

Dimensional

dependent d, f, V, S, Q, Q_{bm}

independent $X_{bm}, W, D_g, \sigma_g, g, \rho, \rho_s, k_v$

Non-Dimensional

dependent $R, f, V_*, S, Q_*, Q_{bm*}$

independent $X_{bm}, W_*, R_p, \sigma_g, R_b$

Equations (4.4) describe the resultant relationships.

$$d, Q, f, V, S, Q_{bm} = f(W, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v) \quad \text{.....(4.4a)}$$

$$R, Q_*, f, V_*, S, Q_{bm*} = f(W_*, X_{bm}; R_p, \sigma_g, R_b) \quad \text{.....(4.4b)}$$

Equations (4.4) are one appropriate form for the case of short-term equilibrium (ASCE, 1975). ASCE (1975) summarized several alternative resultant relations in the form of Table 4-1. For the short-term (i.e., engineering time) the sediment and fluid properties (i.e., $D_{50}, \sigma_g, g, \rho, \rho_s, k_v$) are logically considered independent variables. Long-term equilibrium suggests that sediment size becomes dependent, which would imply cyclic (i.e., a graded stream) rather than engineering time. Table 4-1 indicates that, for short-term equilibrium, sediment load, G_s (equivalent to Q_{bm}), and discharge, Q, can be considered to be dependent variables if so desired. In the case of equations (4.4), sediment concentration, X_{bm}, acts as an independent variable in that it is one of the factors determining which of a variety of range of values of possible depths and velocities will occur (ASCE, 1975). This assumes

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27 ASCE (1975) defined this as a width:depth relation.

that the stream adjusts to maintain sediment equilibrium. Width has been chosen, in equations (4.4), as an independent variable to be consistent with the philosophy of width:depth analysis.[28]

TABLE 4-1: RESULTANT RELATIONSHIPS FOR FLOW IN ALLUVIAL STREAMS (ASCE, 1975)

System (1)	Independent Variables[a]		Dependent[a] variables (not all combinations listed) (4)	Functional Relations	
	Properties of fluid, sediment, gravity, etc. (2)	Characteristics of flow systems (not all combinations listed) (3)		Single-valued (5)	Multiple-valued for some ranges (6)
Flumes	$\nu, \rho, \rho_s, d_g, \sigma_g, w, g$	Q, d, b	G_s, V, r, S, f	x	
		Q, G_s, b	d, r, U, S, f	x	
		V, d, b	Q, G_s, r, S, f	x	
		d, S, b	Q, G_s, r, V, f		x
		r, S, b	Q, G_s, d, V, f		x
		Q, S, b	G_s, d, r, V, f		x
Natural streams Short term	$\nu, \rho, \rho_s, d_g, \sigma_g, w, g$	Q, d	G_s, b, r, V, S, f	x	
		d, S	Q, G_s, b, r, V, f		x
		r, S	Q, G_s, b, d, V, f		x
		Q, S	G_s, b, d, r, V, f		x
Long term (graded stream)	ν, ρ, ρ_s, g	Q, G_s	$b, d, r, V, S, f, d_g, \sigma_g, w$	x	
Very long term	ν, ρ, ρ_s, g, geology	climate, man-made works	$Q, G_s, b, d, r, V, S, f, d_g, \sigma_g, w$	x	

[a]Note that plan-form geometry and wash-load concentration are not considered.

Notes for Table 4-1:

$d_g = D_g$, $v = k_v$, $G_s = Q_{bm}$, w = fall velocity, r = hydraulic radius

Development of the rational forms of the width:depth equations involves a closer look at the process and resultant equations identified above.

28 Recall that it was shown in Section 3.5 that width is an appropriate independent variable for regime applications.

4.3 Rational Forms of the Width:Depth Equations

Rational forms of width:depth relations are proposed herein. The analysis borrows logic from Parker (1977, 1979) and ASCE (1975), but the conclusions are unique, to this author's knowledge. The rational width:depth relations are simply one of the possible resultant equations (4.4).

The analytical problem presented in Section 4.4.2 can be simplified by using the formulated process equations to reduce the number of variables (Parker, 1977). In particular, the first three process equations, namely: 1. continuity of water (equation 3.1), 2. conservation of mass (equation 4.3), and 3. momentum (equation 4.2), can be used to eliminate three variables; for instance Q, Q_{bm}, and f. The unformulated sediment transport, flow resistance, and bank competency relations must therefore take the form of equations (4.5) for the dimensional case (after Parker, et al. 1977, 1979).

4. sediment transport; $f_4\ (W,d,S,V,X_{bm},D_g,o_g,g,p,p_s,k_v) = 0$ ...(4.5a)

5. flow resistance; $f_5\ (W,d,S,V,X_{bm},D_g,o_g,g,p,p_s,k_v) = 0$ ...(4.5b)

6. bank resistance; $f_6\ (W,d,S,V,X_{bm},D_g,o_g,g,p,p_s,k_v) = 0$ ...(4.5c)

Simultaneous solution of equations (4.5) allows three dependent variables to be solved for in terms of eight others. Specification of the sediment and fluid properties (D_g,o_g,g,p,p_s,k_v) along with <u>two</u> of the five remaining hydraulic parameters (W,d,S,V,X_{bm}) allows solution for the remaining three hydraulic parameters. Interestingly, this also confirms the general form of Chang's equations (3.37) and (3.38). Parker (1979) notes that the pair d and S are not independent, and therefore cannot be chosen together.

Specifying W and X_{bm} as independent variables results in equations (4.6).

$d = f(W,X_{bm};D_g,o_g,g,p,p_s,k_v)$ ..(width:depth).....(4.6a)

$S = f(W,X_{bm};D_g,o_g,g,p,p_s,k_v)$ ..(flow resistance).....(4.6b)

$V = f(W,X_{bm};D_g,o_g,g,p,p_s,k_v)$ ..(sediment transport)...(4.6c)

Equations (4.6) are herein referred to as *rational regime relations* (after Parker, 1979). Parker referred to these as rational regime relations since they were developed from solutions to the generalized

process equations (i.e., rational), but could be solved empirically (i.e., regime).

The inclusion of q as a variable and equation (4.7) (definition of q) as the required additional equation, allows alternate regime equations to be derived as given by equations (4.8).

$$q = Q/W \; (= V\, d) \quad \ldots\ldots (4.7)$$
$$d = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v) \quad \ldots\ldots (4.8a)$$
$$S = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v) \quad \ldots\ldots (4.8b)$$
$$V = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v) \quad \ldots\ldots (4.8c)$$

In a discharge defined reach, with constant Q, equation (4.7) equates q and 1/W and therefore equates equations (4.6) and (4.8). Paris's (1984) solutions (i.e., Figure 3.8 and 3.9) fit the mold of the rational regime equations (4.8a,b) (i.e., $d, S = f(q, X_{bm}; D_{35}, \sigma_g, g, \rho, \rho_s, k_v)$. Paris (1984) intentionally simplified his analysis by disregarding sediment gradation, and assuming constant sediment and fluid densities, and kinematic viscosity.

Non-dimensional variables could have been used in the above analysis, and in a similar fashion to equations (4.7) and (4.8), produce equations (4.9).

$$R = f(q\sim, X_{bm}; R_p, R_b, \sigma_g) \quad \ldots\ldots (4.9a)$$
$$S = f(q\sim, X_{bm}; R_p, R_b, \sigma_g) \quad \ldots\ldots (4.9b)$$
$$V_* = f(q\sim, X_{bm}; R_p, R_b, \sigma_g) \quad \ldots\ldots (4.9c)$$

Parker's (1977) rational regime equations (3.19) are a variation equations (4.9).

Considering the definition of equilibrium (i.e., constant X_{bm}) and the proposed use of **n** in equation (1.1) to determine equilibrium state, then the *rational* forms of the width:depth equations are given by equations (4.6a), (4.8a), and (4.9a). If single-valued relationships exist between Q and X_{bm}, equation (4.10) could be substituted for equation (4.6a).

$$d = f(W, Q; D_g, \sigma_g, g, \rho, \rho_s, k_v) \quad \ldots\ldots (4.10)$$

The development of the rational regime relations provides insight the variables which are inherent to the various forms of the width:depth equation. Equations (4.6a) [and (4.10)], (4.8a) and (4.9a) are the

generalized forms of equations (1.1), (3.28) and (3.21).

$d = K\ W^{-n}$ ...(1.1); $d = f(W, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$ .....(4.6a)

$d = K'\ q^{n}$ ...(3.28); $d = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$ .....(4.8a)

$R = K\sim\ q\sim^{n}$ ...(3.21); $R = f(q\sim, X_{bm}; R_p, R_b, \sigma_g)$ .....(4.9a)

Equations (1.1), (3.21) and (3.28) mask the underlying variables that, in addition to width or unit discharge, control depth. Therefore, one can expect the constants n, K, K' and K~ to vary with sediment concentration and with sediment and fluid properties, as per equations (4.11).

$$n,\ K,\ K',\ = f(X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v) \qquad \text{.....(4.11a)}$$

or $$n,\ K\sim\ = f(X_{bm}; R_p, R_b, \sigma_g) \qquad \text{.....(4.11b)}$$

The above analysis describes the theoretical underpinnings of the width:depth equations. Analytical methods are next used to quantify this analysis.

4.4 Analytical Solutions to the Rational Width:Depth Equations

This section presents quantitative analytical solutions relating equations (3.28) and (3.21) to equations (4.8a) and (4.9a), and quantifies the relationship of n, K' and K~ to the independent variables of the rational regime relations [i.e., equations (4.11)]. Therein, the objective of demonstrating that equations (3.28) and (3.21) can be used to represent equilibrium sediment transport conditions is realized.

4.4.1 Analytically Methodologies

Prior to quantifying the analytical solutions to the rational width:depth equations, the comparison of analytical and regime methods presented in Chapter 3 is formalized within the framework outlined in Sections 4.2 and 4.3. The methods are first grouped according to their general solution methodology and resultant equations. Then the review provides a basis for the analytical solution methodology to the rational regime equations, presented in Section 4.4.2.

Firstly, the various methods are reviewed. The intent is to formalize how width:depth analysis relates to other methods for analyzing hydraulic geometry. The review is presented here rather than in Chapter 3 because it relies on the logic of Sections 4.2 and 4.3.

Table 4-2 presents 10 separate methodologies for determination of hydraulic geometry. The types of rivers considered range from sand to gravel, straight to meandering to braided, infinite width to flumes, but all are considered alluvial. The table involves a review of the assumptions, variables, process and resultant equations for each methodology. The following conclusions regarding analytical approaches, are based on Table 4-2:

1. Inclusion of the equation for conservation of mass is optional and depends on whether the particular author has included more than one of the variables q_{bm}, Q_{bm}, or X_{bm}. This observation applies equally to the analysis presented in Section 4.3.

2. Sediment and fluid properties ($D_g, p_s, p, g, k_v, o_g, p_i : i=1,2 \ldots N$ sediment parameters) are generally considered independent variables. Considering Table 4-1, this corresponds to ASCE's (1975) short-term condition, which sufficiently describes conditions for many engineering applications. None of the methods have explicitly included o_g in their resultant equations, presumably to maintain simplicity. The variables p_s, p, g, and k_v are implicit to all methods, whether formally stated or not.

3. The momentum equation can take on various forms, as given by equations (3.27), (3.4), or (4.2) to name a few (Stevens and Nordin, 1987).

4. The number of resultant equations equals the number of process equations, albeit implicitly in some cases. The number of independent variables is a function of the number of variables, the number of process equations, and dimensionality, as shown in the comments column of Table 4-2.

TABLE 4-2: ANALYTICAL HYDRAULIC GEOMETRY COMPARISON

SUB-GROUP	AUTHOR	STATED RIVER TYPE	STATED FLOW CONDITIONS	VARIABLES	PROCESS EQUATIONS	RESULTANT EQUATIONS	COMMENTS
A	1.PARKER (1977)	- ALLUVIAL - NONCOHESIVE BED - NONERODIBLE BANK - STRAIGHT	- 1-D FLOW - STEADY, UNIFORM FLOW - INFINITE WIDTH	DIMENSIONAL q,d,S,Xbm,V,qbm t; Ds,ps,p,kv, g,pi:i=1,2,3,.. NON-DIMENSIONAL qf,R,S,Xbm,C,q* t*;Rp,Rb,pi: i=1,2,3,..	1.CONTINUITY [q=Vd] 2.MOMENTUM [t=pgdS] 3.RESISTANCE(see note) 4.SEDIMENT TRANSPORT (see note) 5.MASS CONSERVATION [Xbm=qbm/q]	STATED S=f(qf,R; Rp,Re,pi) Xbm=f(qf,R; Rp,Re,pi) IMPLIED BY EQUATIONS C=f(qf,R; Rp,Re,pi) t*=f(qf,R; Rp,Re,pi) q*=f(qf,R; Rp,Re,pi) ALTERNATES (GRAPHED) R=f(qf,Xbm; Rp,Rb,pi) S=f(qf,Xbm; Rp,Rb,pi)	- SOLVED EMPIRICALLY - ALSO ANALYTICALLY USING ENGELUND-HANSEN, AND ROTTNER AND KENNEDY, FOR EQUATIONS 3 & 4 - COMPARES TO PETERSON ('75) - 13 VARIABLES- 5 EQN.- 3 DIMENSIONS = 5 INDEPENDENT NON-DIMENSIONAL VARIABLES & 5 RESULTANT EQNS.
	2.PARIS (1984)	- ALLUVIAL - DYNAMIC EQUILIBRIUM - STRAIGHT - NONCOHESIVE BED - NONCOHESIVE BANK	- STEADY, UNIFORM FLOW - SUBCRITICAL FLOW - LOWER REGIME IMPLIED	q,d,S,Xbm,V,C; Ds,ps,p,kv	1.CONTINUITY [q=Vd] 2.MOMENTUM [C=V/U*] 3.RESISTANCE (WHITE et al. 1980) 4.SEDIMENT TRANSPORT (ACKERS-WHITE 1973)	IMPLIED (GRAPHICALLY) d=f(q,Xbm;Ds,p,ps,kv) S=f(q,Xbm;Ds,p,ps,kv) C=f(q,Xbm;Ds,p,ps,kv) V=f(q,Xbm;,Ds,p,ps,kv)	- SOLVED ANALYTICALLY - 10 VARIABLES - 4 EQN. = 6 INDEPENDENT VARIABLES & 4 RESULTANTS
	3.PETERSON (1975)	- FLUME AND RIVER - STRAIGHT - RECTANGULAR CHANNEL - SMOOTH SIDES - GRANULAR BED	- STEADY UNIFORM FLOW	DIMENSIONAL q,W,d,S,Xbm,V; D50,pb,p,kv,g, fg NON-DIMENSIONAL qf,W/d,R,S,Xbm; VIG NO.,ps/p,fg	1. CONTINUITY [q=Vd]	STATED Xbm=f(qf,R;VIG NO.) AND S=f(qf,R;VIG NO.) OR Xbm=f(qf/S^.5,R;VIG NO) ALTERNATE (GRAPHED) R=f(qf,Xbm;VIG NO.)	- SOLVED EMPIRICALLY - DELETES THREE NON-DIMMENSIONAL VARIABLES BY OBSERVATION: W/d, ps/p, and fg - 12 VARIABLES-3 DIMENSIONS 1 EQN.-3 DELETIONS = 3 IND. NONDIMEN. VARIABLES
B	4.PARKER (1979)	- ALLUVIAL - LOOSE COARSE GRAVEL BED AND BANKS - STRAIGHT - SELF-FORMED WITH STABLE WIDTH - MOST GRAIN SIZES MOBILE AT DOMINANT DISCHARGE BUT NOT SUSPENDED - D50 > 16mm	- STEADY UNIFORM FLOW - BED SHEAR EXCEEDS THRESHOLD FOR MOTION (I.E. STABLE BANKS/ MOBILE BED)	DIMENSIONAL Q,W,d,S,V,Qbm; t;D50,ps,p,kv, g,pi:i=1,2,3.. NON-DIMENSIONAL Q~,W*,R,S,V*,Q* t*;R*,Rb,pi: i=1,2,3,..	1.CONTINUITY [Q=VWd] 2.MOMENTUM [t=pgRS] 3.RESISTANCE (PARKER,1979) 4.SEDIMENT TRANSPORT (PARKER,1978) 5.SELF-FORMED WIDTH [R=.0553 S^-1.013] (STABLE BANK/MOBILE BED)	STATED R=f(Q~,Q*;R*,Re,pi) S=f(Q~,Q*;R*,Re,pi) W*=f(Q~,Q*;R*,Re,pi) OR R=f(Q~,W*;R*,Re,pi) S=f(Q~,W*;R*,Re,pi) Q*=f(Q~,W*;R*,Re,pi) IMPLIED BY EQUATIONS V*=f(Q~,W*;R*,Re,pi) t*=f(Q~,W*;R*,Re,pi)	- SOLVED ANALYTICALLY - EQUATION 5. ASSUMES SHEAR EXCEEDS THRESHOLD VALUE BY A CONSTANT 20% - 13 VAR. - 3 DIMENSIONS - 5 EQUATIONS = 5 IND. NON-DIM. VAR. & 5 RESULTANTS - EQUATIONS FURTHER REDUCED TO SINGLE HYDRAULIC INDEPENDENT VARIABLE (Q~) USING EQN.[W*=4.4Q^0.5]
	5.BETTESS WHITE, PARIS (1982)	- ALLUVIAL - STRAIGHT - NONCOHESIVE BED AND BANKS - TRAPEZOIDAL SECTION z=0.5; Q<1CMS z=0.5Q^.25; Q>1CMS	- STEADY UNIFORM FLOW	Q,W,d,S,Xbm,V, U*;Ds,ps,p,kv,g	1.CONTINUITY [Q=VWd] 2.MOMENTUM [U*=(grS)^0.5] 3.RESISTANCE (WHITE et al., 1980) 4.SEDIMENT TRANSPORT (ACKERS-WHITE,1973) 5.MAXIMIZE SEDIMENT TRANSPORT	STATED W=f(Q,S;Ds,p,ps,kv,g) Xbm=f(Q,S;Ds,p,ps,kv,g) IMPLIED BY EQUATIONS V=f(Q,S;Ds,p,ps,kv,g) d=f(Q,S;Ds,p,ps,kv,g) U*=f(Q,S;Ds,p,ps,kv,g) W=f(Q,X;Ds,p,ps,kv,g) V=f(Q,X;Ds,p,ps,kv,g)	- SOLVED ANALYTICALLY - 6 STATED VAR. + 6 IMPLIED - 5 EQUATIONS = 2 STATED AND 5 IMPLIED IND. VAR. - LIMITED RESULTS REPORTED - IMPLIED RESULTANT EQN.S NOT REPORTED - TRAPEZOIDAL X-SECTION

TABLE 4-2 (CONT'D) : ANALYTICAL HYDRAULIC GEOMETRY COMPARISON

SUB-GROUP	AUTHOR	STATED RIVER TYPE	STATED FLOW CONDITIONS	VARIABLES	PROCESS EQUATIONS	RESULTANT EQUATIONS	COMMENTS
B	6.CHANG (1979)	- ALLUVIAL - IMPLIES STRAIGHT - TENDENCY TO BRAID CONSIDERED - SAND BED RIVERS	- STEADY UNIFORM FLOW - LOWER AND UPPER FLOW REGIMES	STATED Q,W,d,S,Qbm IMPLIED V,N;Ds,p,ps,kv	IMPLIED 1.CONTINUITY [Q=VWd] 2.MOMENTUM V=r^0.66 S^.5/N STATED 3.RESISTANCE(ENGELUND-HANSEN, 1967) 4.SEDIMENT TRANSPORT [DUBOYS,EINSTEIN-BROWN, (ASCE 1975)] 5.MINIMIZE STREAM POWER [=pgQS]	STATED W=f(Q,Qs;Ds,p,ps,kv) S=f(Q,Qs;Ds,p,ps,kv) d=f(Q,Qs;Ds,p,ps,kv) V=f(Q,Qs;Ds,p,ps,kv) N=f(Q,Qs;Ds,p,ps,kv) AND ALSO REPORTS W=f(Q,S;Ds,p,ps,kv)	- SOLVED ANALYTICALLY - 5 STATED VAR. + 6 IMPLIED - 5 EQUATIONS = 2 STATED AND 4 IMPLIED IND. VAR. - THRESHOLDS RELATED TO CHANGING RIVER PATTERNS
	7.CHANG (1980b)	- ALLUVIAL - IMPLIES STRAIGHT - GRAVEL BED RIVERS	- STEADY UNIFORM FLOW - LOWER AND UPPER FLOW REGIMES	STATED Q,W,d,S,Qbm IMPLIED V,N;Ds,p,ps,kv	IMPLIED 1.CONTINUITY [Q=VWd] 2.MOMENTUM V=r^0.66 S^.5/N STATED 3.RESISTANCE (BRAY,HEY,19??) 4.SEDIMENT TRANSPORT (PARKER, 1978,1979) 5.MINIMIZE STREAM POWER [=pgQS]	STATED W=f(Q,Qs;Ds,p,ps,kv) S=f(Q,Qs;Ds,p,ps,kv) d=f(Q,Qs;Ds,p,ps,kv) V=f(Q,Qs;Ds,p,ps,kv) N=f(Q,Qs;Ds,p,ps,kv) AND ALSO REPORTS W=f(Q,S;Ds,p,ps,kv)	- SOLVED ANALYTICALLY - 5 STATED VAR. + 6 IMPLIED - 5 EQUATIONS = 2 STATED AND 4 IMPLIED IND. VAR.
C	8.CHANG (1985c)	- ALLUVIAL - SAND BED RIVERS - CURVED CHANNELS - BRAIDING CONSIDERED BY NOTING THRESHOLD IN DEVELOPED RELATIONS, AND EXAMINING VALLEY SLOPE	- STEADY UNIFORM FLOW - LOWER AND UPPER FLOW REGIMES - TRANSVERSE FLOW IN CURVED CHANNELS ADDRESSED	STATED Q,W,d,S,V,Qbm, rc,z;Ds IMPLIED p,ps,kv,N,St	STATED 1.CONTINUITY [Q=VWd] 2.RESISTANCE (ENGELUND,1967) 3.SEDIMENT TRANSPORT [DUBOY,(ASCE,'75)] 4.MINIMIZE STREAM POWER [=pgQS] 5.EQUAL STREAM POWER PER UNIT LENGTH IMPLIED 6.MOMENTUM V=r^0.66*S^.5/N 7.ODGAARD'S TRANSVERSE BED SLOPE EQUATION	W=f(Q,Qs,Rc;Ds,p,ps,kv) V=f(Q,Qs,rc;Ds,p,ps,kv) d=f(Q,Qs,rc;Ds,p,ps,kv) S=f(Q,Qs,rc;Ds,p,ps,kv) z=f(Q,Qs,rc;Ds,p,ps,kv) N=f(Q,Qs,rc;Ds,p,ps,kv) St=f(Q,Qs,rc;Ds, p,ps,kv)	- SOLVED ANALYTICALLY - 9 STATED VAR. + 5 IMPLIED - 7 EQUATIONS = 3 STATED AND 4 IMPLIED IND. VAR. - LIMITED RESULTS REPORTED - z CAN BE SPECIFIED TO SATISFY EQN. 5, SUGGESTING MULTIPLE STABLE GEOMETRIES - VALLEY SLOPE, Sv, IS USED TO QUALITATIVELY EXAMINE TENDENCY TO CURVATURE

TABLE 4-2 (CONT'D) : ANALYTICAL HYDRAULIC GEOMETRY COMPARISON

SUB-GROUP	AUTHOR	STATED RIVER TYPE	STATED FLOW CONDITIONS	VARIABLES	PROCESS EQUATIONS	RESULTANT EQUATIONS	COMMENTS
D	9.KOMURA (1963)	- ALLUVIAL - STUDY OF CHANNEL CONSTRICTIONS	- STEADY UNIFORM FLOW OR STEADY NONUNIFORM FLOW	IMPLIED Q,W,d,U*,qbm,V,Qbm,ks;D50,ps,p,g NOTES: 1.d=r ASSUMED 2.ANALYSIS FOR TWO SECTIONS (W1>W2) 3.D50,p,ps,g ARE CONSTANTS	DYNAMIC CASE 1.CONTINUITY [Q=VWd] 2.MASS CONSERVATION [Qbm=qbm W] 3.MOMENTUM V/U*=8(d/ks)^.166 4.RESISTANCE [ks/Ds=CONST(U*^2/(ps/p-1)gDs)^m'] 5.SEDIMENT TRANSPORT [KALINSKE-BROWN] 6.EQUILIBRIUM SEDIMENT [Qbm = CONST.] 7.EQUILIB. Q [Q=CONST] STATIC CASE 1.,2.,3.,4.,7.AS ABOVE 5.CRITICAL TRACTIVE FORCE EQUATION 6.EQUILIBRIUM SEDIMENT [Qbm = 0]	DYNAMIC CASE d=f(W;D50,ps,p,g) U*=f(W ;D50,ps,p,g) STATIC CASE d=f(W;D50,ps,p,g)	- SOLVED ANALYTICALLY - 12 VARIABLES - 7 EQUATIONS = 5 INDEPENDENT VARIABLES - m' VARIES WITH BED FORM m' = 0 PLANE BED m' = 0.769 SMOOTH BED m' = 2 SMOOTH/RIPPLES m' = 3 RIPPLES/DUNES - SOLUTION FOR m'=0 IMPLIES n=0.686 FOR DYNAMIC CASE - SOLUTION FOR STATIC CASE GIVES n=0.857
	10.STRAUB (CITED ASCE, 1975)	- ALLUVIAL - STUDY OF CHANNEL CONSTRICTIONS	- STEADY UNIFORM FLOW	IMPLIED Q,W,d,S,V,Qbm qbm,N,t;f(Ds),tc,p,g NOTES: 1.d=r IMPLIED 2.ANALYSIS FOR TWO SECTIONS (W1>W2) 3.Ds,f(Ds),tc,p g,N ASSUMED CONSTANTS	1.CONTINUITY [Q=VWd] 2.MASS CONSERVATION [Qbm=qbm W] 3.MOMENTUM [t=pgdS] 4.RESISTANCE V=r^0.66 S^.5/N 5.SEDIMENT TRANSPORT [DUBOYS,(ASCE,1975)] 6.EQUILIBRIUM SEDIMENT [Qbm = CONST.] 7.EQUILIB. DISCHARGE [Q=CONST]	d=f(W,t;tc,f(Ds),p,g)	- SOLVED ANALYTICALLY - 13 VARIABLES - 7 EQUATIONS = 6 INDEPENDENT VARIABLES - MANNING'S USED AS A RESISTANCE EQUATION WITH "N" NOT VARYING BETWEEN SECTIONS

The methodologies have been subdivided into four catagories A, B, C and D, as detailed below.

Group A, in Table 4-2, consists of Paris (1984), Parker (1977), and Peterson (1975) which are analytical, semi-analytical, and regime methods, respectively. The analytical approaches define the process equations inherent to the regime analysis. The following observations are made regarding this group:

1. These methods are grouped on the basis of channel type, exclusion of width, W, and the similarity of resultant equations to each other and to the rational forms of the width:depth equation (i.e., equations 4.6a, 4.8a, and 4.9a).
2. Group A methods employ only 4 process equations (mass conservation deleted), as compared to the 5 equations (mass conservation deleted) necessary in the development of the rational forms of the width:depth equation (i.e., equations 4.6, 4.8, 4.9). The missing equation corresponds to bank competency, and is not required since width, W, has been excluded.[29] This corresponds to the simple two-dimensional case (i.e., downstream and vertical), in which the transport of bed material is concerned only with the bed and not the banks (Parker, 1977; Stevens, 1989).
3. For a discharge defined reach (Q=constant), one can introduce the variable width, W, into Group A by adding the equation q=constant/W, and considering W an independent variable (Paris, 1984).

Group B, in Table 4-2, consists of Chang (1979), Chang (1980b), Parker (1979), and White (et al., 1982), all of which are analytical methods, applying to sand and/or gravel bed rivers. The following observations are made regarding this group:

1. These methods are grouped together on the basis that all explicitly include width, W, and all derive an optimum width for a given water and sediment discharge, Q and Q_{bm}, respectively. White et al. (1982) and Chang (1979, 1980b), also investigate solutions assuming discharge, Q and slope, S are independent.

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29 Parker (1977) and Paris (1984) assume infinite widths. Peterson (1975) assumes W/d has no effect on the resultants.

Parker (1979) also examines the case of discharge, Q, and width, W as independent variables.

2. In general, Group B methods employ 5 process equations (since mass conservation is arbitrarly excluded), which is equivalent to the 5 equations (mass conservation deleted) necessary in the development of the rational forms of the width:depth equation (i.e., equations 4.6a, 4.8a, 4.9a).

3. However, all methods substitute freely for the bank competency equation with either:

a. self-formed width (Parker, 1979), or;

b. maximum sediment discharge (White et al., 1982), or;

c. minimum stream power (Chang, 1979; 1980b).

White et al. (1982) showed that methods b. and c. are equivalent. Method a. (Parker, 1979), on the other hand predicts that sediment discharge increases monotonically with width, in direct contradiction to methods b. and c. In either case, these equations proposed as an alternatives to the bank competency relation, have not gained total acceptance as being representative of "real" river conditions (Vanoni, 1984; Carson and Griffiths, 1987; Stevens, 1989).

Group C, in Table 4-2, consists of Chang (1985c) and is included only to illustrate the complexities of rivers. The following observations are made regarding this group:

1. Chang has added variables to describe curvature, r_c, transverse bed slope in river bends, S_t, and bank side slopes, z. This necessitates the inclusion of two more process equations, and one more independent variable.

2. Chang's (1985c) hypothesis of equal stream power corresponds to equal slope along a discharge defined reach, in contradiction to the implied conditions for equilibrium in a reach with varying widths.[30]

30 See Section 3.4; small variations in slope accompany large variations in width

3. Chang's (1985c) manipulation of bank slope, z, to attain equal slope through riffle and pool sections, demonstrates the flexibility that a river has in adjusting its hydraulic geometry to suit the prevailing conditions.

Group D, in Table 4-2, consists of Komura's (1963) and Straub's (1935) methods for scour in long constrictions. These methods are contrasted to the more generalized analytical treatments of alluvial rivers. The following observations are made regarding this group:

1. In comparison to Group A, Komura (1963) introduces the variables width, W, and sediment discharge, Q_{bm}, thereby requiring two additional equations to the 4 equations of Group A. The additional equations consist of equilibrium discharge (i.e., Q=constant) and mass conservation. Furthermore, Komura's (1963) inclusion of equilibrium sediment transport (Q_{bm} = constant), allows the resultant to take the single variate form of equation (2.2). Komura (1963) substituted a critical tractive shear equation for the sediment transport equation for the case of static equilibrium.
2. Straub (1935) is equivalent to Komura, but for the extra variables N, and τ_c. Straub (1935) uses the relative difference in τ and τ_c, to explore his resultant equations in terms of static and dynamic equilibrium.
3. Review of Komura (1963), shows that the value of n in equation (3.3) varies from 0.686 to 0.857 for the case of dynamic equilibrium depending on the assumed value of m', with 0.686 corresponding to m'=0, representing a plane bed. Since Komura (1963) did not tie the sediment transport to his choice of m', it would appear that his results should treated with caution.

The above discussion formalizes the comparison of alternative approaches to defining the hydraulic geometry of alluvial rivers, in a manner which has not been presented previously (to this author's knowledge). In general two approaches exist as summarized below; width:depth analysis belongs to the second type (i.e., equilibrium methods).

a. The *optimal width* approaches (Groups B, C) assume that sediment discharge, Q_{bm} and water discharge, Q (or Q,S) are independent variables and introduce a formula in place of bank competency to solve for optimum channel width. No allowance is made for bank hetrogeniety.

b. The *equilibrium* approach (Groups A, D; rational forms of width:depth equations) assume that unit discharge, q, or width, W, and sediment concentration, X_{bm}, are independent variables, over the short-term, thus accounting for observed width variations under equilibrium conditions, while implying other then optimal slope, depth, and width conditions. These methods cannot predict width.

The difference between the optimal width methods and the equilibrium approach is essentially the choice of the independent variables. The *optimum width methods* are concerned with a homogeneous reach, where width is a dependent variable and the river strives to attain an equal hydraulic geometry everywhere. The *equilibrium* approach (e.g., width:depth analysis) is concerned with the equilibrium conditions in a reach of varying widths. While it appears that along its overall length a river conforms to its sediment and discharge loading to attain mean widths (e.g., Andrews, 1982; Neill, 1973), for discharge defined reaches deviations around the optimum caused by constraints on the system (see Section 3.5) may be the rule rather then the exception. Determination of how channel shape adjusts to maintain sediment equilibrium is the challenge. The next step is to quantify the analytical solutions to the rational regime relations.

A review of Group A reveals that the methodology employed is similar to that for the development of the rational width:depth equations (4.8a) and (4.9a), except for the inclusion of the bank competency equation and width, W. Moreover, comparison of the resultant equations for Group A to equations (4.8) and (4.9), reveals that they are quite similar in composition. Therefore, it is concluded that equations (4.8a) and (4.9a) can be investigated analytically by

considering the variables:

$$q, d, S, X_{bm}, V, f, D_g, p_s, p, k_v, g, o_g$$

while simultaneously solving the equations of:

1. continuity (formulated, $q = V\ d$) ;
2. momentum (formulated, equation 4.2);
3. sediment transport (optional formulations) and;
4. flow resistance (optional formulations).

The momentum and continuity equations are precisely formulated, while any pair of sediment transport and resistance equations can be substituted for equations 3. and 4. Section 4.4.2 employs this methodology to provide solution data.

As stated in point 2 of Group A this approach is two-dimensional (downstream/vertical). While this approach is obviously a simplification of a complex three-dimensional problem, it is consistent with the simple case presented in Section 4.2.2 (i.e., wide straight river) and is a traditional abstract approach to the study of water and sediment interaction (Parker, 1977). As pointed out in Section 3.4 this two-dimensional abstraction has previously been used to establish stable channel widths (Stevens, 1989).

4.4.2 Analytical Solutions

This section presents analytical solutions to the rational regime equations (4.8a) and (4.9a). Equation (4.7a) is not directly suited to the proposed analytical solution method.

$$d = f(q, X_{bm}; D_g, o_g, g, p, p_s, k_v) \quad \text{.....(4.8a)}$$

$$R = f(q\sim, X_{bm}; R_p, R_b, o_g) \quad \text{.....(4.9a)}$$

The methodology employed follows from Section 4.4.1, and involves substitution of formulated sediment transport and resistance equations into the following set of process equations:

1. continuity (formulated);
2. momentum (formulated);
3. sediment transport (optional formulations) and;
4. flow resistance (optional formulations).

A computer code was written to solve the equations simultaneously for given combinations of the independent variables (e.g., $q, X_{bm}; D_g, o_g, g, p, p_s, k_v$) in equations (4.8) and (4.9). The resultant data sets describe

the relationships generalized in equations (4.8) and (4.9), and are presented in graphical format.

Two separate solution sets have been developed using the following two alternative sets of equations:

a. Engelund's (1967) sediment transport (4.) and flow resistance (3.) equations;
b. Ackers-White (1973) sediment transport equation (4.) and White et al. (1980) flow resistance (3.) equation.

These equations were substituted for equations 3. and 4. above. The White et al. (1980) resistance equation is applicable to the lower flow regime, and only the lower regime portion of Engelund's (1967) resistance equation is used in this section. Therefore, the conclusions apply only to the case of the lower flow regime.

The solution data that were developed from the process equations were next fitted to the specific forms of equations (4.8a) and (4.9a) given by equations (3.28) and (3.21), using statistical methods.

$$R = K\sim\ q\sim^{n} \quad ...(3.21);$$

$$d = K'\ q^{n} \quad ...(3.28).$$

By holding the bed sediment concentration, X_{bm}, constant, the objective of testing whether simple width:depth equations [i.e., equations (3.21) and (3.28)] represent equilibrium conditions was realized. Furthermore, the sensitivity of the exponent and coefficients, n, K' and K~, to the independent variables [i.e., equations (4.11)] has been quantified, and is presented herein in graphical format.

$$n,\ K' = f(X_{bm};D_g,o_g,g,p,p_s,k_v) \quad .....(4.11a)$$

$$n,\ K\sim = f(X_{bm};R_p,R_b,o_g) \quad .....(4.11b)$$

The graphical relationships will illustrate the sensitivity of the width:depth equations to the reference flow condition, X_{bm}.

The details of the above manipulations and the results are not presented within the main body of the thesis, but are instead included in the attached Appendices C and D. A short description of Appendices C and D and the main findings follows.

Appendix C: Engelund (1967)

Engelund's sediment transport and flow resistance equations, the solution methodology employed, and detailed results are presented in Appendix C. Solution data (i.e., R) were developed by solving the four process equations outlined above, for a range of dimensionless unit discharge, q~, values for each of the following bed material concentrations, X_{bm}:

X_{bm} = 0, 5, 10, 30, 60, 100, 200, 400, 600, 1000, 3000 ppm.

Since Engelund's equations do not include sediment size (i.e., R_p) as a variable, solutions are a function only of bed sediment concentration, X_{bm}. Solution data are given in Table C-1 [i.e., $R = f(q\sim, X_{bm}; R_p, R_b, o_g)$ (4.9a)].[31] Figure C-3 presents a plot of some of the calculated data points. Table C-2 documents the fitting of equation (3.21) [i.e., $R = K\sim\ q\sim^{n}$] to the solution data [i.e., giving n, $K\sim = f(X_{bm}; R_p, R_b, o_g)$; see footnote 31]. Figure C-1 plots the resulting best-fit equations (3.21) for a number of sediment concentrations, X_{bm}. Figure C-1 also represents the analytical solutions to equation (4.9a), since the equation (3.21) fits the solution data very closely. Figure C-2 is a plot of the relationship between n, K~ and X_{bm}.

Appendix D: Paris (1984) [re: Ackers (1973) and White (1980)]

The Ackers-White (1973) sediment transport and White et al. (1980) flow resistance equations, the solution methodology, the computer coding written to solve the equations, and detailed results are presented in Appendix D. Solution data (i.e., d) were developed by solving the four process equations outlined above, for a range of unit discharge, q, values for each of the following bed material concentrations, X_{bm}:

X_{bm}= 10, 20, 40, 100, 200, 400, 800, 1400, 2000, 4000, 8000 ppm.

Moreover, nine distinct sediment sizes were investigated, namely:

D_{35} = .1, .26, .3, .5, 1, 2.5, 5, 10, 20 mm.

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31 Simultaneous solution of Engelund's equations produces results which conform to equation (4.9a) except for the fact that sediment gradation , o_g, and sediment size, as represented by R_p, do not appear as variables. The variables p, p_s, (and therefore R_b), k_v, and g are presumed to be constants, although not explicitly stated.

Figure D-1 plots the solution data for the bed sediment concentration X_{bm} = 1000 ppm and D_{35} = .3 mm, on a logarithmic scale. As shown in Figure D-1, a straight line regression (i.e., $d = K' q^{n}$) fits the calculated data very closely. The entirety of the solution data are represented by Figures D-2 to D-10, for various sediment sizes [i.e., $d = f(q, X_{bm}; D_{35}, o_g, g, p, p_s, k_v)$ ..(4.8a)].[32] Figures D-2 to D-10 actually represent best fit logarithmic equations of depth, d, versus unit discharge, q, but as shown in Figure D-1 the fit is extremely good. Tables D-1 and D-2 document the results of fitting equation (3.28) [i.e., $d=K' q^{n}$] to the solution data [i.e., giving n, $K\sim = f(X_{bm}; D_{35}, o_g, g, p, p_s, k_v)$; see footnote 32]. Figures D-11 to D-16 illustrate the trends of the data in Tables D-1 and D-2, plotting the relationship between n, K' and X_{bm} , D_{35}.

The following observations/conclusions are made from consideration of Appendices C and D, and the data presented therein:

1. Equation (3.21) [$R = K\sim q\sim^{n}$] closely fits the Engelund solution data to equation (4.9a) [$R = f(q\sim, X_{bm}; R_p, R_b, o_g)$], while equation (3.28) [$d = K' q^{n}$] closely fits the Paris solution data to equation (4.8a) [$d = f(q, X_{bm}; D_{35}, o_g, g, p, p_s, k_v)$] for equi-concentration (X_{bm}) lines. This fact is evident in the high resulting correlation coefficients given in Table C-2 and Appendix D, and illustrated by Figures C-3 and D-1. Therefore, these simple non-linear equations (3.21) and (3.28) represent equilibrium sediment transport conditions (i.e., regime), and therefore equilibrium channel geometry.

2. The exponent, n, and the coefficient, K', from equation (3.28) both vary with sediment size, D_{35}, as well sediment concentration, X_{bm}, for Paris's results. For Engelund's results the exponent, n, and coefficient K~ of equation (3.21) are solely dependent on X_{bm}. In both cases the sensitivity of width:depth analysis to the reference flow condition, X_{bm}, is evident.

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32 Simultaneous solution of the Ackers-White (1973) and White et al. (1980) equations produces results which conform to equation (4.8a), but for the fact that sediment gradation does not appear as a variable. The assumptions for p, p_s, g and K_v are documented in Appendix D.

3. Peterson (1975) claimed that Engelund overlooked sediment size in his analysis. Brownlie (1981a) used flume data to illustrate that Engelund required another variable to explain certain data (e.g., sediment size; see Brownlie 1981a, Fig. 3.7c, pp 35). However, due to the non-dimensional nature of q~ and R, sediment size is inherent to the resultant equations of the form of equation (3.21). This fact is illustrated by converting K~ to K' as detailed in Appendix C and summarized in Table C-3 and Figure C-4. The coefficient K' for Engelund's results varies directly with sediment size in a similar fashion to Paris's results (i.e., Figures C-4 and D-15). Engelund's exponent, n, remains dependent only on concentration, X_{bm}.

4. As static equilibrium is approached (i.e., by definition $X_{bm} = 0$), Engelund's equations predict n = 0.90, while Paris's predict n values of 0.86 to 0.82 depending on sediment size. Solution of Paris's equations for $X_{bm} = 0$ was not possible with the numerical scheme employed, but the results are not expected to vary significantly from those for $X_{bm} = 10$ ppm. These results compare reasonably well with Tamburi's (1976, 1979) presumption that n = 0.86 for static equlibrium.

5. For the case of dynamic equilibrium Tamburi (1976, 1979) assumed n = 0.64. Figure C-2 and Figure D-11 reveal that for both Engelund's and Paris's equations, the exponent, n, approaches an asymtotic limit as X_{bm} increases. However, while the limiting n value for Engelund's equations is about 0.68, the lower limit of n for Paris's equations varies with sediment size; ranging from about 0.82 for $D_{35} = 0.7$ mm to 0.60 for $D_{35} = 1.0$ mm and then increasing back to 0.63 at $D_{35} = 20.0$ mm. In, both cases the relationship of n and X_{bm} is continuous and decreasing between $X_{bm} = 0$ and $X_{bm} \gg 0$, confirming the general conclusions of Chapter 3 (i.e., recall Figure 3.1).

6. Figures D-12 and D-13 clearly illustrate the difference between the solutions using Paris's equations versus Engelund's equations. Whereas Engelund's equations predict horizontal lines straight lines for these graphs, Paris's equations show a marked tendency for n to increase as D_{35} gets smaller, for any given concentration. Figure D-12 further

illustrates that n increases for Dgr > 60 (i.e., D_{35} > 2.5mm) for any given concentration. This transition corresponds to the assumed boundary between fine and coarse sediment, as recommended by Ackers-White (1973).

7. Figure C-2 illustrates the variation of K~ with X_{bm} for Engelund's equations. The double-valued behaviour of this relationship is attributed to the choice of variables since the variation of K' with X_{bm} is generally single valued for any given sediment size as shown in Figure C-4 . The variation of K' with concentration, X_{bm} for Paris's equations in Figures D-14 to D-16, is similar in nature to Engelund's Figure C-4. In both cases there is a marked tendency for K' to increase as sediment size decreases. Thus, for any given q, depth increases and velocity decreases, in order to maintain an equivalent sediment concentration. The magnitude of K' at any given sediment size varies significantly from one method to the other, while the range in K' is significantly greater than for the exponent n.

In summary, equations (3.28) and (3.21) have been shown to adequately represent the analytically derived solutions to the rational width:depth equations (4.8a) and (4.9a), for constant sediment and fluid properties and bed sediment concentration, in the lower flow regime.

$d = K' q^{n}$ ...(3.28); $d = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$ .....(4.8a)

$R = K\sim q\sim^{n}$ ...(3.21); $R = f(q\sim, X_{bm}; R_p, R_b, \sigma_g)$ .....(4.9a)

Since bed sediment concentration, X_{bm}, defines equilibrium, then equations (3.28) and (3.21) define the equilibrium channel shape. As previously stated the sediment and fluid properties are suitable independent variables for the short-term case, and for the purpose of studying a discharge-defined reach.

The sensitivity of width:depth analysis to reference flow condition (X_{bm}) is quantified by the graphical relationships between n, K, K~ and X_{bm}, D_g. Clearly, in using width:depth analysis for the stated engineering applications one must be cognisant of the

relationship between the reference flow condition, X_{bm}, (and D_g) and the value of n, K, and K~.[33]

In particular Tamburi's proposal to use the value of n to define the state of equilibrium (i.e., static or dynamic), is in question. The analytical results suggest that the value of n which represents dynamic equilibrium could vary between the range $0.9 < n < 0.60$, depending on the sediment inflow to the reach and possibly the sediment size. The rapid decrease in the value of n with increasing concentration, X_{bm}, for Engelund's equations indicates that for many rivers with significant sediment loads (e.g., sand-bed rivers), n at dynamic equilibrium will approach Tamburi's value of 0.64. On the other hand for rivers with little sediment load (e.g., gravel rivers) the value of n at dynamic equilibium should approach Tamburi's value of 0.86 for static equilibrium. However, Paris's (1984) results point out that the range of n for some sediment sizes might be quite small (e.g., Figure D-11; $0.82< n < 0.86$ for $D_{35} = .1$ mm). In the next section it will be shown that the value of n derived from regression on field data can have a confidence limit which exceeds the analytical range for n. Therefore, it may not be practical to use n as a means of defining the equilibrium state. On the other hand the analytically predicted range in the coefficient, K', is quite large and it may be a better indicator of the equilibrium state. This hypothesis will be elaborated on in the next section.[34]

The results of this section apply only to the lower flow regime. An alternative set of sediment transport and flow resistance equations (i.e., Brownlie (1981b)) valid for flow in the lower, transition and upper flow regimes is presented in Chapter 5.

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33 The substitution of Q for X_{bm}, as the reference flow condition, is explored in Section 4.5.

34 Realize that K' is an indicator of the relative position of the width:depth curve, whereas n is an indicator of the slope; Section 4.5 will demonstrate that it is easier to define static equilibrium from the relative position of the width:depth curve, then from its slope.

4.5 Empirical Investigations

Analytical results must always be independently verified with field data. In this section, Hubbell and Matejka's (1959) observations for the Middle Loup River near Dunning, Nebraska are employed to verify the analytical solutions to the rational width:depth equations presented in Section 4.4.2. Thereby, the conclusion from the analytical studies that equations (3.28) and (3.21) define the equilibrium channel shape is verified. In addition, the field data are employed to demonstrate the following points:

a. Discharge, Q, is presented as a practical alternative to concentration, X_{bm}, for purposes of defining the reference flow condition.

b. The sensitivity of the width:depth equations to the choice of reference flow condition is illustrated, confirming that K' is a better indicator of channel equilibrium than n. Moreover, the concept that n and K' can be considered independently will be discussed.

The results are applicable to the lower regime only. More complex situations, including sensitivity to changing flow regime are presented in Chapter 5.

Brief descriptions of the data base and the statistical and data analysis methods are presented in Sections 4.5.1 and 4.5.2.

4.5.1 Middle Loup River near Dunning, Nebraska

The study reach is shown in Figure 4.1 (Hubbell and Matejka, 1959). The Middle Loup River at this point has a drainage area of 4500 Km^2. Discharge ranges from 5 to 17 m^3/s and total sediment load from 450 to 3200 tonnes/day. Typical water slopes for this reach are in the order of 0.0013 (Brownlie, 1981a). The reach is classified as meandering, according to Leopold and Wolman's criteria (i.e., Figure 3.39, equation (3.47)). The characterictics of the reach are detailed below.

Sediment and Fluid Properties (D_{50}, σ_g, ρ_s, ρ, k_v, g)

Hubbell and Matejka (1959) presented size distribution curves for each cross-section in Figure 4.1. The curves were based on a minimum of 21 samples and a maximum of 208 samples, with a total of 561 samples

determining the weighted mean size distribution curve. The data shows that bed sediment characteristics, D_{50} and σ_g are relatively constant thoroughout the reach. Based on the weighted mean distribution, the sediment sizes D_{35} and D_{50} equal 0.27 mm and 0.34 mm, respectively. Sediment density, ρ_s, is assumed to be constant within the reach.

The properties of the fluid, ρ and k_v, are indirectly reported in the form of temperature records.

Hydraulic Geometry (d, W, Q, q)

The variables d, W, and V were measured directly at each of the river cross-sections shown on Figure 4.1. Discharge, Q, unit discharge q, and area A are computed indirectly using the continuity equation. Other variables such as water surface gradient, S_w, were also measured .

For the purposes of this investigation only hydraulic geometry data from cross-sections A, B_1, C_2, C_1, and E were included (see Figure 4.1). Section B, D and C were excluded because the turbulence and local scour patterns associated with bridges. Width:depth relations are not applicable to the unsteady non-uniform flow conditions associated with local scour. A total of 137 records were available for analyzing hydraulic geometry.

Sediment Concentration (X_{bm})

A unique feature of this sand-bed reach is the turbulence flume located at section D [see Figure 4.1]. This flume was engineered to suspend virtually all sediment being transported by the river, so that the total sediment load could be measured by suspended-sediment sampling equipment. Total bed material concentration, X_{bm}, was calculated by subtracting the percentage of the total suspended concentration which was finer than 0.062 mm (i.e., the wash load). Sediment sizes smaller than 0.062 mm are not naturally occuring in the bed sediments.

Dominant Flow Condition

The dominant flow conditions are assumed to be $Q_{dom} = 10.8\ m^3/s$ and $X_{bm\ dom} = 1150$ PPM. Table 4-3 illustrates the basis for the choice of dominant flow conditions. This agrees with Parker's [1978b] chosen dominant discharge equal to $11.0\ m^3/s$. The variation of monthly average sediment concentration over the period of record is illustrated by Figure 4.2.

TABLE 4-3 Middle Loup River at Dunning Nebr.; Annual Summary of Water and Total Sediment Discharge, Section D

PERIOD OF RECORD	AVERAGE DISCHARGE (m^3/s)	SUSPENDED SEDIMENT AVERAGE LOAD (TONNES/DY)	SUSPENDED SEDIMENT AVERAGE CONCENTRATION (PPM)	SUSPENDED BED MATERIAL AVERAGE CONCENTRATION[1] (PPM)
03/22/50 30/09/50	10.3	544	1220	1100
10/50 to 09/51	10.8	585	1240	1120
10/51 to 09/52	10.9	635	1340	1200
Weighted Average	10.8	596	1275	1150

[1] Assumes average wash load of 10% [Hubbell and Matejka, 1959, pp. 20].

4.5.2 Data Analysis and Statistical Methods

The methods employed for data selection and analysis are detailed below.

Data Selection

The data base was subdivided on the basis of the reference flow condition, X_{bm}. Alternately, discharge was employed as a reference flow condition, on the presumption of a direct relation between Q and X_{bm}.

Data extraction required the assumption of a confidence range on either side of the chosen flow condition. For the Middle Loup River the ranges assumed were +- 10% for concentration, X_{bm}, and +- 5% for discharge, Q. In the author's experience these values are within the accuracy limits of present measurement techniques. As an example, the resulting selection criteria for the dominant flow condition are as follows:

$$10.5 < Q_{dom} < 11.1 m^3/s \quad \text{or} \quad 1040 < X_{bm\ dom} < 1260 \text{ PPM}$$

The data required for this investigation included the variables d, W, Q, q, X_{bm}, D_{50}, σ_g, ρ_s, ρ, k_v and g. In order to simplify the problem at hand, variations in the fluid properties, ρ_s and k_v, due to temperature were assumed to be of secondary importance in the rational regime relationships. Therefore, the impact of temperature change on sediment transport and flow resistance is <u>not</u> accounted for. In order

to account for the impact of wash load on the fluid viscovity, k_v, the data were screened to exclude any data with unusually high wash load concentrations (i.e., X_{ss} > 10000 PPM). As detailed above the sediment properties of the bed sediment are constants for the reach.

Statistical Methods

Least square regression techniques were used to fit equations of the form of equations (1.1) (3.28) and (1.2) to the data chosen in accordance with the rational equations (4.6a), (4.8a), and variations given by equations (4.10), (4.12), (4.13) and (4.14).

$$d = K W^{-n} \quad \text{.....(1.1)}$$

$$d = K' q^{n} \quad \text{.....(3.28)}$$

$$A = K W^{(1-n)} \quad \text{.....(1.2)}$$

$$d = f(W, X_{bm}; D_{50}, {}^{o}_{g}, g, p, p_s, k_v) \quad \text{.....(4.6a)}$$

$$d = f(q, X_{bm}; D_{50}, {}^{o}_{g}, g, p, p_s, k_v) \quad \text{.....(4.8a)}$$

$$d = f(W, Q; D_{50}, og, g, p, p_s, k_v) \quad \text{.....(4.10)}$$

$$d = f(q, Q; D_{50}, og, g, p, p_s, k_v) \quad \text{.....(4.12)}$$

$$A = f(W, X_{bm}; D_{50}, og, g, p, p_s, k_v) \quad \text{.....(4.13)}$$

$$A = f(W, Q; D_{50}, og, g, p, p_s, k_v) \quad \text{.....(4.14)}$$

Equations (4.10), (4.12) and (4.14) correspond to the assumption of discharge as the reference flow condition.

Logarithms were employed to linearize the data, allowing determination of the slope, n, and the intercepts , K as K', of the equations. Statistics associated with the regression included the correlation coefficient, r^2, the standard error of estimate, Se, the 95% confidence limits for the slope, n and the intercepts K and K', and the confidence interval, CI. McBean et al. (1989) defined the confidence interval as per equation (4.15).

$$CI = +- \; t_{a/2} \; S_e \; [1 + 1/n_d + (X - (\bar{X}))^2 / S_{xx}]^{0.5} \quad \text{.....(4.15)}$$

where CI = confidence interval at (100 - a) %
$t_{a/2}$ = percentage points of the t-distribution for the (1-a) CI
S_e = standard error of estimate
n_d = no. of data points
$\underline{X}$ = Value of the X variable under consideration
X = mean of all the X variables
S_{xx} = measure of dispersion of X_i over all i

The confidence interval was plotted along with the data and regression results to provide an appreciation of the variability of the data.

Results: Statitiscal Methods

The results of the statistical analyses are presented in Table 4-4. This table is divided into three parts, corresponding to regression results for equations of the form of equation (1.1) [$d = K W^{-n}$], (3.28) [$d = K' q^{n}$], and (1.2) [$A = K W^{(1-n)}$], respectively. Results are presented using both discharge, Q, and bed sediment concentration, X_{bm}, for the reference flow condition. For the Middle Loup River, only one bed sediment concentration, X_{bm}, was investigated, while a range of reference discharges, Q, were examined.

As well as reporting the least-square regression value for n, and K, Table 4-4 reports the 95% confidence limits for the slope, n; the correlation coefficient, r^2, and the standard error of estimate, S_e. The coefficient K and K' were related by equation (4.16).

$$K = K' Q^{n} \qquad \text{.....(4.16)}$$

where Q = the reference discharge for the data.

TABLE 4-4: MIDDLE LOUP RIVER AT DUNNING: EMPIRICAL REGRESSION RESULTS

4-4-1

independent variable : log10 (width, W)
dependent variable : log10 (depth, d)
basic equation : $d = K W^{-n}$

DISCHARGE RANGE (m^3/s)	REFERENCE DISCHARGE (m^3/s)	NUMBER OF DATA	n	n +- 95% CONF.LIM.	K	K'	r^2	Se (log_{10})
8.8 TO 9.6	9.2	17	.73	.04	5.13	1.02	.99	.02
9.6 TO 10.5	10	39	.71	.05	5.02	.98	.95	.04
10.5 TO 11.1	10.8	27	.74	.09	5.71	.98	.91	.05
11.1 TO 11.9	11.5	30	.73	.10	5.33	.90	.90	.05
11.9 TO 12.7	12.3	18	.81	.14	7.39	.97	.90	.06
12.7 TO 13.6	13.1	6	.80	.16	7.76	.99	.98	.03
RANGE IN X_{bm} (PPM)	REFERENCE X_{bm} (PPM)							
1040 TO 1260	1150	7	.71	.26	5.13	.94	.91	.05

4-4-2

independent variable : log10 (unit discharge,q)
dependent variable : log10 (depth,d)
basic equation : $d = K' q^{n}$

RANGE IN Q	REFERENCE Q (m^3/s)							
8.8 TO 9.6	9.2	17	.72	.04	5.02	1.01	.99	.02
9.6 TO 10.5	10	39	.71	.06	5.03	.97	.95	.04
10.5 TO 11.1	10.8	27	.74	.10	5.69	.98	.91	.05
11.1 TO 11.9	11.5	30	.72	.09	5.20	.89	.90	.05
11.9 TO 12.7	12.3	18	.80	.14	7.09	.96	.90	.06
12.7 TO 13.6	13.1	6	.81	.16	8.12	1.01	.98	.03
RANGE IN X_{bm} (PPM)	REFERENCE X_{bm} (PPM)							
1040 TO 1260	1150	7	.88	.29	8.14	1.14	.91	.05

4-4-3

independent variable : log10 (width,W)
dependent variable : log10 (area,A)
basic equation : $A = K W^{(1-n)}$

RANGE IN Q	REFERENCE Q (m^3/s)		(1-n)	(1-n) +- 95% CONF.LIM.	K	K'	r^2	Se (log_{10})
8.8 TO 9.6	9.2	17	.27	.04	5.13	1.02	.92	.02
9.6 TO 10.5	10	39	.29	.05	5.02	.98	.76	.04
10.5 TO 11.1	10.8	27	.26	.09	5.71	.98	.57	.05
11.1 TO 11.9	11.5	30	.28	.10	5.33	.90	.55	.05
11.9 TO 12.7	12.3	18	.19	.14	7.39	.97	.34	.06
12.7 TO 13.6	13.1	6	.20	.16	7.76	.99	.76	.03
RANGE IN X_{bm} (PPM)	REFERENCE X_{bm} (PPM)							
1040 TO 1260	1150	7	.29	.26	5.13	.94	.63	.05

Analytical Methods and Results

Analytical solutions were developed for the Middle Loup River to allow comparison to the observed data and to the empirical results of Table 4-4. The analytical methods reported in Appendices C and D for the Engelund (1967) and Paris (1984) equations were used. Values for n, K' and K~, were linearly interpolated from the Tables C-1, D-1 and D-2 for the reference concentration, X_{bm}, and the sediment sizes, D_{35} = 0.27mm and D_{50} = 0.34mm. The relationship developed in Section 4.5.4 is used to relate discharge and concentration, where discharge is employed as the reference flow condition (see note 2 Table 4-5). Where required, k_v = 0.011 cm^2/s, p = 1000 kg/m^3 and p_s = 2650 kg/m^3 have been assumed.

Equations (3.28) and (3.21) were then employed directly to calculate the depth of flow for each given q or q~

$$d = K' q^n \quad \text{.....(3.28)}$$

$$R = K\sim q\sim^n \quad \text{.....(3.21)}$$

Table 4-5 allows direct comparision of the regression results from Table 4-4 [i.e., 4-4-2] to analytically derived exponents and coefficients for equation (3.28) [$d = K' q^n$]. The coefficients K, K' and K~ were related by equations (4.16) and (4.17).

$$K = K' Q^n \quad \text{.....(4.16)}$$

$$K' = K\sim D_g / ((R_b g D_g)^{0.5} D_g)^n \quad \text{.....(4.17)}$$

where Q = the reference discharge for the data.

TABLE 4-5: MIDDLE LOUP RIVER NEAR DUNNING, NEBR.; COMPARISON OF EMPIRICAL AND ANALYTICAL EQUATIONS (3.28)

OBSERVED REFERENCE DISCHARGE (M^3/S)	COMPUTED REFERENCE CONC., X_{bm} (PPM)	EMPIRICAL (TABLE 4-4) n	EMPIRICAL (TABLE 4-4) K	EMPIRICAL (TABLE 4-4) K'	ANALYTICAL: EQUATION (3.28) ENGELUND n	ENGELUND K'	PARIS n	PARIS K'
9.2	655	.72	5.02	1.01	.691	.95	.771	1.07
10	861	.71	5.03	.97	.689	.89	.761	.97
10.8	1150	.74	5.69	.98	.687	.83	.757	.92
11.5	1248	.72	5.20	.89	.686	.82	.756	.91
12.3	1454	.80	7.09.	..96	.685	.79	.754	.88
13.1	1660	.81	8.12	1.01	.685	.77	.752	.86

NOTES: 1. CONCENTRATION FOR Q=10.8 M^3/S IS AVERAGE MAR/50 TO SEPT/52.
2. ALL OTHER CONCENTRATIONS CALCULATED; Xbm(PPM) = 257.7 Q - 1716

4.5.3 Alternatives for Reference Flow Condition

For the Middle Loup River at Dunning the availability of the turbulence flume allows direct measurement of bed sediment concentration, X_{bm}. However, for many situations it is impractical to use sediment concentration as the reference flow condition, due to a lack of direct measurements of the total bed material load. The potential for substitution of discharge for sediment concentration as the reference flow condition has been alluded to previously.[35] In terms of the rational regime relations, this process is typified by the substitution of equation (4.10) for (4.6a).

$$d = f(W, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.6a)$$

$$d = f(W, Q; D_g, \sigma_g, g, \rho, \rho_s, k_v) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.10)$$

Such a substitution requires the development of a relationship between Q and X_{bm}.

Other reasons exist for the substitution of discharge for concentration for defining the reference flow condition. As demonstrated by Brownlie (1981a) significant variations in X_{bm} over a narrow range of hydraulic and bed-material conditions can be expected. For the Middle Loup River, application of the data selection criteria resulted in selection of 7 data points for $X_{bm\ dom}$ = 1150 PPM, and 27 data points for Q_{dom} = 10.8 m^3/s (see Table 4-4). The difference in the number of data points reflects the large variation in sediment concentration for any given discharge. Part of this variance is no doubt due to the fact that the rate of adjustment to bed transients is several orders of magnitude smaller than to water transients (Lopez-Garcia, 1977). Given the fact that instantaneous measurements are being made for the purposes of determining the regime condition, it would appear to be adviseable to ensure that at least the water continuity equation holds. In addition, where possible, data should be taken during steady flow conditions to avoid the influence of transient flow phenomena.

A relationship between X_{bm} and Q was developed for the Middle Loup River using data provided in Brownlie (1981b). The data were input to a statistical model to determine the least square regression fit between discharge, Q, and bed sediment concentration, X_{bm}. Table 4-6

35 See Section 4.3.

summarizes the results of the least square regression analyses and Figure 4.3 displays the results. Two types of equations were employed, linear and logarithmic, as shown in Table 4-6. The data pair (Q_{dom}, X_{bmdom}) = (10.8, 1150) falls reasonably close to the linear regression line (i.e., equation 1.) from Table 4-6, so that it was chosen for relating Q to X_{bm}. This equation was used to related discharge to sediment concentration in Table 4-5.

Table 4-6: Middle Loup River near Dunning, Nebr.: Summary of Computed Relationships between Q and X_{bm}

Regression Formula	Least Square Regression			X_{bm} @ Q=10.8 m^3/s
	A	B	r^2	
1. $X_{bm} = A\ Q + B$	257.7	-1716	0.28	1064
2. $X_{bm} = A\ Q^{B}$	2.04	2.59	0.29	964

The relationships in Table 4-6 are quite poor statistically, although still significant at the 1% level (Table A-11, Neville, 1970). Moreover, the scatter in the sediment concentration data was not unexpected. McBean and Al-Nassri (1988) demonstrated the extreme range in the relationships between Q and X_{bm}, reporting correlation coefficients from 0.2 to 0.9 for various rivers. Relationships for the Niobrara River, also presented in Figure 4.3, show a much better correlation[36].

It is concluded that discharge can be substituted for bed sediment concentration for the purpose of defining the reference flow condition.

4.5.4 Equilibrium

The analytical solutions in Section 4.4.2 indicate that equations (3.21) and (3.28) can be used to represent equilibrium sediment transport conditions (i.e., equilibrium hydraulic geometry). This section attempts to verify those results with the Middle Loup River data. The reference flow condition for this task is chosen to be the

36 Equations for the Niobrara River relations as well as for the Rio Grande River are presented in Chapter 5.

dominant discharge ($Q = 10.8\ m^3/s$) which corresponds to the dominant bed sediment concentration ($X_{bm} = 1150$ ppm).

The analytical solutions and empirical regression results of Tables 4-4 and 4-5, for the dominant flow condition (i.e., either 10.8 m^3/s or 1150 ppm), are plotted against the observed data in Figures 4.4 to 4.7[37]. Figures 4.4 and 4.6 employ the format of equation (3.28) [$d = K' q^n$] whereas Figures 4.5 and 4.7 employ the format of equation (1.1) [$d = K W^{-n}$]. Figures 4.4 to 4.7 show that the analytical solutions for equal sediment concentration (i.e., equilibrium) model the data quite well, thus verifying that equations (1.1) and (3.28) represent equilibrium conditions. An interesting contrast is provided by Figure 3.20; which reveals that optimal or semi-optimal methods (e.g., Parker, 1978a) predict mean depths (and therefore widths) correctly, but do not explain the observed depth or width variation.

The confidence level for the regression results (i.e., n, K, K') for the data chosen on the basis of $X_{dom} = 1150$ ppm is low, even though the correlation coefficient is high . In particular the slope of the regression line in appears to be sensitive to the number of data points, as indicated by the 95% confidence limit of 0.29 and 0.26 (see Table 4-4). Figure 4.4 suggests that the upper data point at $q = 0.48\ m^2/s$ may influence the value of n somewhat disproportionately. Conversely, the confidence level for the regression results for the data chosen on the bases of Q_{dom} is high. The larger number of data points results in a lower 95% confidence limit on n of 0.09.

The width:area relationship helps explain how sediment equilibrium is maintained in a reach with varying top widths. The width:area relationship for the dominant condition (Table 4-4, part 4-4-3) is plotted in Figure 4.8. Figure 4.8 shows the observed data versus the regression results on that data, and the analytical solutions derived using Paris's equations and Engelund's equations. The scatter in the width:area curve, Figure 4.8, indicates the sensitivity of area to small deviations in depth at any given width. Wider differences between the Paris and Engelund solutions than for the width:depth

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37 Figures 4.4 and 4.7 include the 95% confidence interval (upper and lower) boundaries as determined via equation (4.15) for each of the data points.

curves, illustrate this fact. Regardless of this, the reported results demonstrated that area increases with width [i.e., n is not equal to 1][38], confirming the trend of area with width in Figure 3.11 (Chang, 1978). The significance of this statement derives from Chang's (1978) criticisms of Yang's (1976) methodology. Yang, among others claimed that area remains constant along a reach, which requires that $n = 1$ (i.e., $1-n = 0$). The results of the regression, show that this is not true for the Middle Loup River at the dominant discharge. The increase of area with width corresponds to a decrease in velocity, and as will be shown in Chapter 6 a decrease in unit sediment load. Equilibrium sediment transport is maintained by the fact that the total transport is balanced by the increasing width and the decreasing unit sediment transport (i.e., $Q_{bm} = q_{bm} W$).

4.5.5 Evaluation of Sensitivity of Width:Depth Analysis

The evaluation of the sensitivity of width:depth analysis to the choice of the reference flow condition is accomplished through comparison of the analytical solutions of Section 4.4.2 to the full range of observed data.

Figures 4.9 to 4.11 compare the entire data base to the analytical solutions calculated using Paris's (1984) equations (i.e., Table 4-5). Figure 4.9 employs the format of equation (3.28) [$d = K' q^n$], Figure 4.10 that of equation (1.1) [$d = K W^{-n}$], and Figure 4.11 the format of equation (1.2) [$A = K W^{1-n}$]. Analytical solutions are provided for either end of the observed range of bed sediment concentration for the data base (i.e., 655 ppm and 1660 ppm; see Table 4-5). Figures 4.9 and 4.10 reveal that the range in the observed data (i.e., range in d) for any particular width or unit discharge, albeit small, is explained by Paris's analytical solutions using the observed range of sediment concentrations, X_{bm} (i.e., 655 ppm to 1660 ppm). The scatter around the width:area curve in Figure 4.11 is to be expected on the basis of Figure 4.8.

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38 The 95% confidence limits fo n for the reference flow $Q_{dom} = 10.8$ m^3/s imply $0.649 < n < .829$, and therefore $0.16 < (1-n) < 0.35$.

Figures 4.9 to 4.11 also compare the data to the analytical results derived from assuming X_{bm} = 10 ppm [39]. The conclusion that the analytical model fits the data is supported by the plotting of analytical solutions for X_{bm} = 10 ppm on Figures 4.9 and 4.10. Since X_{bm} = 10 ppm is well outside the range of observed concentrations one would not expect any data points to plot close to these analytical solutions. The observed data are significantly different then the analytical solutions at X_{bm} = 10 ppm (i.e., approximate static equilibrium), as one would expect. A similar analysis is given in Figure 4.12, using the analytical results of Engelund (1967). The Middle Loup River data of Table 3-2 Parker (1978a) is plotted against solutions for X_{bm} = 0, 600, 1000 and 3000 ppm. Figure 4.12 shows that X_{bm} = 1000 ppm provides a close fit to the observed data at dominant discharge. Again, the observed data are significantly variant from the static equilibrium case (i.e., X_{bm} = 0 ppm), thereby verifying the analytical model.

Consideration of Tables 4-4 and 4-5 confirms the conclusion drawn in Section 4.4.2, with regard to using n to define the state of equilibrium. Firstly, the computed values of n for the empirical regression results vary from 0.71 to 0.81, covering a significant part of the range suggested by Tamburi as defining the state of equilibrium. Furthermore, the 95 % confidence limits on the empirical values of n, vary from 0.04 to 0.16, thereby covering the whole range of n values proposed by Tamburi as defining the state of equilibrium. Therefore, it is impossible to define the state of equilibrium from n alone. On the other hand, the values of K' for the empirical regressions generally agree with the analytically derived value of K', using Paris's and Engelund's equations (see Table 4-5). Moreover, the empirical range of K' is 0.90 to 1.02, which is significantly different from the analytical solutions for the static equilibrium case, given by K' = 2.17 for Engelund (at X_{bm} = 0 ppm) and K'= 1.77 for Paris (at X_{bm} = 10 ppm). Therefore, based on K', one can conclude that this reach is in dynamic equilibrium. This conclusion is verified by Figure 4.9.

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39 Since X_{bm} = 10 ppm represents a hypothetical case, it was necessary to assume a discharge; Q_{dom} is the assumed discharge required by Figures 4.10 and 4.11.

In summary, K' is a better indicator of the state of equilibrium than is n, for the Middle Loup River. It is important to recognize that K' indicates the relative position of the width:depth curve and that n indicates the slope. Thus the large range in the relative position (i.e., K') is a better indicator of the equilibrium state than the small range in slope (i.e., n), even though the two are not entirely independent. Finally, it is concluded that an even better means of defining the state of equilibrium from an observed width:depth curve is to graphically compare it to appropriate analytical solutions (e.g., Figures 4.9, 4.12). The interelationship between K' and n is accounted for in a graphical display, thereby making it less susceptible to misinterpretation than using the value of K' alone. This does not negate the need for width:depth analysis, since the verified curve has many useful engineering applications.

4.6 Summary

This Chapter has presented a semi-theoretical foundation for width:depth analysis. A statement of case was provided, leading to the development of rational forms of the width:depth equations. The rational regime equations (4.6a) [and (4.10)], (4.8a) and (4.9a) describe the variables which are inherent to the width:depth equations (1.1), (3.28) and (3.21).

$d = K\ W^{-n}$ ...(1.1); $d = f(W, X_{bm}; D_g, o_g, g, p, p_s, k_v)$ .....(4.6a)

$d = K'\ q^{n}$ ...(3.28); $d = f(q, X_{bm}; D_g, o_g, g, p, p_s, k_v)$ .....(4.8a)

$R = K\sim\ q\sim^{n}$ ...(3.21); $R = f(q\sim, X_{bm}; R_p, R_b, o_g)$ .....(4.9a)

The constants n, K, K' and K~ therefore must vary with sediment concentration and with sediment and fluid properties, as per equations (4.11).

$$n,\ K,\ K',\ = f(X_{bm}; D_g, o_g, g, p, p_s, k_v) \qquad \text{.....(4.11a)}$$

or $$n,\ K\sim\ = f(X_{bm}; R_p, R_b, o_g) \qquad \text{.....(4.11b)}$$

A two-dimensional (vertical/downstream) analytical solution to the required process equations has provided solution data conforming to

the rational width:depth equations. Two solution sets have been developed corresponding to the a. Engelund (1967) and b. Paris (1984) sediment transport and flow resistance equations. Equations (3.21) and (3.28) adequately model the solution data for constant bed sediment concentration, and constant sediment and fluid properties, confirming that these width:depth equations represent equilibrium channel geometries. The results of the analytical methods graphically demonstrate the sensitivity of n, K~ and K' to the reference flow condition, X_{bm}, as well as to sediment size, D_g.

Empirical methods, using data from the Middle Loup River, have verified the applicability of the analytical results. Moreover, discharge has been presented as a practical alternative to concentration for the purpose of defining the reference flow condition. Lastly, the sensitivity of width:depth analysis to the reference flow condition has been demonstrated by comparing the data to a range of analytical solutions corresponding to the observed range in sediment concentration. It was concluded, on the basis of the value of K', and via graphical methods, that the Middle Loup River at Dunning, Nebraska is in dynamic equilibrium. Furthermore, the exponent, n, does not appear to be suitable variable for defining the state of equilibrium, from empirical data.

The analytical and empirical results presented in this Chapter are applicable to the lower flow regime only. Chapter 5 explores the impact of the transition between flow regimes on the applicability of width:depth analysis.

CHAPTER 5

INFLUENCE OF FLOW REGIME

5.1 Introduction

This Chapter examines the sensitivity of width:depth analysis to transitions between flow regimes. Analytical and empirical methods are employed as follows:

5.2 Analytical solutions to the rational width:depth equation (4.8a) [$d = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$] are developed, which are valid for all flow regimes. The solution data are used to test whether the width:depth equation (3.28) [$d = K' q^{n}$] adequately models equilibrium channel geometry, in a continuous fashion over the lower, transition and upper flow regimes. In addition, the relationship between the width:depth constants, K' and n, and the reference flow condition, X_{bm}, is quantified, inclusive of the transition between flow regimes. Lastly, the slope adjustments required to maintain equilibrium are quantified and displayed graphically. In the process the results are contrasted to the analytical solutions from Chapter 4 for the lower regime.

5.3 Empirical investigations, involving data from the Niobrara and Rio Grande Rivers, are undertaken to field test the analytical solutions of Section 5.2. The analytical solutions are tested against the complete range of observed data and compared to regression results on the data. The conclusions of Section 4.5 with respect to the use of discharge as a reference flow condition and the preference for using K' as an indicator of channel equilibrium are reinforced.

5.2 Analytical Solutions

In Sections 4.4.1 and 4.4.2 it was revealed that the rational width:depth equation (4.8a) [i.e., $d = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$] could be quantified by substituting any known pair of sediment transport and resistance equations into the following set of process equations, and solving them simultaneously:

1. continuity; 2. momentum; 3. sediment transport; 4. flow resistance.

By substituting a flow resistance equation which models the transition between flow regimes, solution data can be developed for equation (4.8a) which are valid for all flow regimes (i.e., lower, transition, upper). Brownlie's (1981a) equations for flow resistance and sediment transport are valid for the lower, transition and upper regimes.

The sediment transport and flow resistance equations of Brownlie (1981a) are employed in this section to provide solutions to the rational regime equation (4.8a). A computer code was written to solve the above process equations simultaneously for given combinations of the independent variables (i.e., $q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v$) in equation (4.8a). Solution data (i.e., d)[40] were developed for a range of unit discharge, q, values for each of the following bed material concentrations, X_{bm}:

X_{bm} = 1, 10, 20, 40, 100, 200, 400, 800, 1400, 2000, 4000 and 8000 ppm.

Moreover, six distinct sediment sizes were investigated, namely:

D_{50} = .1, .3, .5, 1, 1.5, and 2 mm.

Brownlie's (1981a) equations, the solution methodology, the computer code and detailed results are presented in Appendix E. The solution data conform to equation (4.8a) [i.e., $d = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$]. Sediment gradation has been assumed to be constant and equal to 1.6 in order to simplify the problem. The assumptions for ρ, ρ_s, g, and k_v are documented in Appendix E.

Brownlie (1981a) developed two equations for the flow depth in the lower and upper flow regimes, respectively. The primary criteria for transition between the flow regime states is based on a critical velocity approach (V_{cr}). An equation which models the flow depth in the transition region is proposed in Appendix E. Brownlie (1981a) presents a second criteria which controls the transition between lower and upper flow regimes, based on a critical slope, S_{cr} = 0.006. Figures 3.39 and 3.14 reveal that a slope of 0.006 is associated with a steep braided stream. Such a steep braided stream is considered beyond the scope of this thesis.

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40 The solution methods also predict S, and V.

The results of the simultaneous solution of Brownlie's equations in conjunction with the equations for continuity and momentum are presented in Figures E-12 to E-23. Figures E-12, E-14, E-16, E-18, E-20, and E-22 are logarithmic plots of the depth, d, versus unit discharge, q, solution data for each of the sediment sizes, D_{50}, and bed sediment concentrations, X_{bm}, given above [i.e., $d = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$ ..(4.8a)]. Figures E-13, E-15, E-17, E-19, E-21, and E-23 are logarithmic plots of the slope, S, versus unit discharge, q, solution data for each of the sediment sizes, D_{50}, and bed sediment concentrations, X_{bm}, given above. This case corresponds to the rational equation (4.8b) [i.e., $S = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$]. Data corresponding to equation (4.8c) [i.e., $V = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$] was also produced but is not presented here. The slope graphs quantify the conditions required to maintain equilibrium (i.e., equi-concentration, X_{bm}) for varying unit discharges.

The solution data developed from the process equations (e.g., Figure E-12) were next fit to the specific form of the equation (4.8a) given by equation (3.28) [$d = K' q^n$] via statistical methods. As was the case in Section 4.4.2, holding the bed sediment concentration constant allowed testing of whether the simple width:depth equations can be used to represent equilibrium conditions. Tables E-9 and E-10 document the results of fitting of equation (3.28) [$d = K' q^n$] to the data in Figures E-12, E-14, E-16, E-18, E-20, and E-22 (i.e., giving n, $K' = f(X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$.[41] Figures E-24 and E-25 illustrate the variation of n with concentration, X_{bm}, and sediment size, D_{50}, while Figures E-26 and E-27 show the variation of K' with these same variables. The boundaries between the flow regimes are demarcated on the Tables E-9 and E-10, indicating which concentration and sediment combinations have solution data occuring in the lower regime only; the upper regime only; or the lower, upper and transitional regimes.

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41 Recall that σ_g, ρ, ρ_s, g, and k_v are constants.

Equations of the form of equation (5.1)[42] were fit to the slope versus unit discharge curves in order to illustrate trends in the data of Figures E-13, E-15, E-17, E-19, E-21, and E-23.

$$S = SK' \, q^{Sn} \quad \dots\dots\dots\dots\dots\dots\dots\dots (5.1)$$

As revealed in Figure E-17 equation (5.1) is not representative of all of the data. The majority of the deviation from the line described by (5.1) (i.e., straight line on log-log plot) can be attributed to the transition based on $S > 0.006$, which results in upper regime flow. Tables E-11 and E-12 document the variation of SK' and S_n with concentration and sediment size, and Figures E-28 to E-31 illustrate the same.

The following observations and conclusions are based on consideration of the information described above and in Appendix E.

1. Equation (3.28) [$d = K' \, q^n$] closely fits the analytical solution data for equation (4.8a) [i.e., $d = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$] for all regimes ($S<0.006$), for any given equi-concentration line. This point is illustrated by Figures E-32 to E-39, which compare the solution data to equation (4.8a), for a range of concentration and sediment sizes, to regression analysis using equation (3.28). The calculated flow regime is indicated on Figures E-32 to E-39 by L - lower, T - transition, and U - upper. Figures E-38 and E-39 illustrate all three regimes can exist for a given concentration. Furthermore, Figures E-38 and E-39 show that equation (3.28) is continuous over all three regimes. Therefore, the simple non-linear equation (3.28) can be employed to represent equilibrium sediment transport conditions (i.e. regime), inclusive of the transition between flow regimes.

2. The upper regime data associated with low values of $\log_{10}(q)$ (i.e., -2, -1.3) in Figures E-36 and E-37, are a result of the slope criteria ($S>0.006$). The discontinuity caused by this criteria is exemplified by the slope versus unit discharge plot for the same concentration (i.e., Figure E-23). It is recommended that the equation (3.28) not be applied to situations for $S>0.006$.

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42 Equation (5.1) was chosen simply to emulate the width:depth equation; others forms might fit the data better.

3. For a given reference flow condition the results imply that as q decreases (i.e., width increases) flow regime moves from upper to transitional to lower flow regime. Furthermore, only one regime is feasible at each unit discharge (for a given concentration), implying that the relationship between q and d is single-valued.

4. While the large range of unit discharges considered allowed all regimes to be present at certain concentrations (e.g., Figure E-39), it is anticipated that the lesser range of q associated with field conditions will limit the number of flow regimes to two or only one (e.g., lower and transition, or transition and upper) for any reference flow condition, X_{bm}.

5. The width:depth constants, n and K', vary with both concentration and sediment size, in agreement with Paris's results (i.e., Section 4.4.2; Appendix D). Despite the transition between flow regimes the relationships between n, K' and X_{bm}, D_{50} are relatively continuous as shown in Figures E-24 to E-27. Moreover, K' is strictly decreasing with increasing concentration, X_{bm}, while n generally decreases with increasing X_{bm}, again in agreement with the lower regime results of Paris. The exponent n appears to be more sensitive than K' to the transition between flow regimes, as indicated by the somewhat convoluted shape of Figure E-24.[43]

6. The range of n is even less than for the lower regime methods of Engelund and Paris, having a lower end of 0.73 and an upper limit of 0.85 (i.e., X_{bm} = 1 ppm).[44] Despite the inclusion of transition and upper regime data, the range of K' with concentration and sediment size (Figure E-27) is remarkably similar to those of Paris (Figure D-16) and Engelund (Figure C-4). For example, considering the concentration range 10 ppm to 3000 ppm, and the sediment range .1 mm to 2 mm, the Paris results (Table D-2) predict $0.3 < K' < 1.8$, the Engelund results (Table C-3) predict $0.6 < K' < 3.0$, and the Brownlie results (Table E-10) predict $0.5 < K' < 2.3$. While absolute concurrence cannot be expected,

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43 Another reason to prefer K' over n, for the purpose of defining the equilibrium state.

44 n for Engelund ranges 0.6 to 0.9; and for Paris ranges 0.6 to 0.86 (see Tables C-3 and D-1).

the Brownlie's results for K' for certain sediment sizes (e.g., 0.5 mm) are very close to those of Paris and Engelund. The analytical results for n and K' support the conclusion, from Chapter 4, that K' is a better indicator than n of the state of equilibrium of the channel. The analytically predicted range in K' is again quite large in comparison to that for n. In Section 5.3, it is shown that the value of n derived empirically from regression on field data can have a confidence limit as large as the analytical range for n. The range of K' makes it less susceptible to this problem.

In summary, the analytical solutions indicate the applicability of equation (3.28) to reaches exhibiting varied flow regimes. Equation (3.28) has again been shown adequately describe the analytically derived solution data to equation (4.8a), for constant sediment and fluid properties and bed sediment concentration. Moreover, equation (3.28) is continuous over the full range of flow regimes. Again, since bed sediment concentration, X_{bm}, defines equilibrium, then equation (3.28) defines the equilibrium channel geometry. Certain conditions are required to support these conclusions, including a forward progression from lower to transition to upper flow regime with increasing unit discharge, q. Lastly, the relationship between K' and n, and the reference flow condition, X_{bm}, has been shown to be continuous through the transition, and has been quantified in graphical fashion.

5.3 Empirical Investigations

In this section, observations for the Niobrara River (Colby and Hembree, 1953) and the Rio Grande River (Dawdy, 1961a; Nordin and Culbertson , 1961b; Nordin and Dempster, 1963; Dawdy and Culbertson, 1964; Nordin, 1964; Nordin and Beverage, 1965) are employed to verify the analytical solutions of Section 5.2. Thereby, the conclusion that equation (3.28) defines the equilibrium channel shape is verified, inclusive of the transition between flow regimes.

In addition, the field data are employed to demonstrate the following points:

a. Discharge is demonstrated as an alternative to concentration for defining the reference flow condition, using data encompasing all flow regimes.

b. The conditions required for maintaining equilibrium as presented in Section 5.2 are tested against field data.

c. The conclusion that K' is a better indicator of channel equilibrium than n is reinforced, and expanded to include conditions for all flow regimes.

Brief descriptions of the data bases are presented in Sections 5.3.1 and 5.3.2. The results of regression analysis of the data are summarized in Sections 5.3.3 and 5.3.4.

5.3.1 Niobrara River at Cody, Nebraska

The study reach of the Niobrara River is shown in Figure 3.30 (Colby and Hembree, 1953). Discharge for this reach ranges from 6 to 21 m^3/s and sediment load from 450 to 1800 tonnes/day. Typical water slopes for this reach are in the order of 0.0014 (Brownlie, 1981a). The reach is classified as meandering, according to Leopold and Wolman's criteria (i.e., Figure 3.39, equation (3.47)).

Sediment and Fluid Properties (D_{50}, o_g, p_s, p, k_v, g)

The bed sediment size distribution has been determined for each section, as shown in Figure 3.31. The bed sediment characteristics, D_{50} and o_g are relatively constant thoroughout the reach, with sediment sizes D_{35} and D_{50} equal to 0.23 mm and 0.28 mm, respectively. Sediment density, p_s, is assumed to be constant.

The properties of the fluid, p and k_v, are indirectly reported in the form of temperature records (Brownlie, 1981b).

Hydraulic Geometry (d, W, Q, q)

For the purposes of this investigation only cross-sections C1 to C10 were included (see Figure 3.30). The gauging station, ford section and contracted section were excluded due to the existance of non-regime bed sediments (i.e., clayey siltstone), leaving a total of 118 records for analyzing hydraulic geometry.

The variables d and W were measured at each cross-section. The velocity was measured at the gauging station or at sections C2 or C6, allowing computation of Q, q, and A. The gauging station discharge is applied to all sections. Water surface gradients, S_w, were also measured .

Sediment Concentration (X_{bm})

The contracted section (see Figure 3.30) induces sufficient turbulence to suspend virtually all sediment being transported, allowing suspended-sediment sampling equipment to measure the total sediment load. Total bed material concentration, X_{bm}, was calculated by subtracting the wash load (i.e., sediment sizes < 0.062 mm). Wash load was observed to increase with the total load from a low of 8% to a high of 41%.

Dominant Flow Condition

The dominant flow conditions are assumed to be $Q_{dom} = 9.9\ m^3/s$ (Parker, 1978b) and $X_{bm\ dom} = 1500$ ppm. The dominant discharge is exceeded approximately 50% of the time. Colby and Hembree (1953) indicated that the average total sediment concentration over the period of record was 1800 ppm. The wash load was typically 16% at the dominant condition, leaving a bed material load of 1500 ppm.

Bed Forms

Brownlie (1981b) reported the bed forms at the gauging station. The bed forms for sections C-1 to C-10 were assumed to be the same as for the gauging station on any given date.[45]

5.3.2 Rio Grande River near Bernallilo, and at Belen (Casa Colorada), New Mexico

The study reaches of the Rio Grande River are shown in Figures 3.28 and 5.1 (Culbertson and Dawdy, 1964). Discharge for these reaches range from 3 to 280 m^3/s and sediment load from 20 to 90,000 tonnes/day. Typical water slopes for the Bernallilo and the Belen (Casa Colorada) reaches are 0.00095 and 0.00082, respectively (Culbertson and Dawdy, 1964). Both reaches are classified as meandering, according to Leopold and Wolman's criteria, even using the flow exceeded only 2% of the time (i.e., Figure 3.39, equation (3.47)).

Sediment and Fluid Properties (D_{50}, σ_g, P_s, p, k_v, g)

The bed sediment of the study reaches are uniform as illustrated by Table 5-1 (Nordin and Culbertson, 1961).

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45 Bed forms are not reported for sections C-1 to C-10. This limits the usefulness of the data, as will be shown in Sections 5.3.6 and 5.3.7.

TABLE 5-1: RIO GRANDE RIVER, N.M.: BED SEDIMENT SIZE DISTRIBUTION

Reach	Number of Observations	D_{50} (mm) average	D_{50} standard deviation	σ_g average	σ_g standard deviation
near Bernallilo	139	.29	.02	1.68	.29
near Belen	85	.23	.06	1.55	.16

Sediment density, p_s, is assumed to be constant.

The properties of the fluid, p and k_v, are reported indirectly in the form of temperature records (Brownlie, 1981b).

Hydraulic Geometry (d, W, Q, q)

This investigation is confined to hydraulic geometry data for cross-sections A-2 to F (see Figure 3.30) and 118.15 to 114.77 (see Figure 5.1) from the Bernalillo and Casa Colorada reaches, respectively. Data from the gauging station at Belen are considered in conjunction with the Casa Colorada data. The number of data records available for the Casa Colorada, Belen, and Bernallilo reaches was 60, 17 and 111, respectively.

As indicated in Section 3.5, the Bernallilo reach is alluvial having a sand-bed and banks composed of sand-silt and clay. Section A-2 is partially confined by a bluff of calcareous sandstone on its right bank. On the Casa Colorada reach, Culbertson and Dawdy (1964) report jetty construction, but it appears that the data were taken prior to construction.

The variables d, and W were measured at each cross-section. For the Bernallilo reach, the velocity was measured at sections A-2 and/or F, allowing computation of Q, q, A. For the Casa Colorada reach, the velocity was measured at each cross-section. Water surface gradients, S_w, were also measured .

Sediment Concentration (X_{bm})

The modified Einstein method was used to determine the total bed sediment load from the measured suspended sediment discharge (Nordin and Beverage, 1965). The sediment discharge was calculated at those sections where velocity was measured. A sediment size of 0.062 mm was used to seperate the wash load.

Dominant Flow Condition

The flow exceeded 50% of the time for the Bernallilo and Belen stations are equal to 620 and 440 cfs, while those exceeded 2% of the time are 8100 and 6200 cfs. The large range in flows makes the choice of a dominant condition rather nebulous (Wolman and Yu, 1987), and therefore none is proposed.

Bed Forms

The bed forms were reported indirectly as flow regime (i.e., lower, transition, or upper) for each of the sections where hydraulic geometry measurements were made.

5.3.3 Data Analysis: Niobrara River

Data Selection

The data base was subdivided on the basis of the reference flow condition, X_{bm}, and/or discharge, Q. The range considered for any one regression analysis is summarized in Table 5-2. Typically, one regression represents one to three days recorded data for the entire reach.

TABLE 5-2: NIOBRARA RIVER; DATA EXTRACTION CRITERIA, EMPIRICAL ANALYSES

RUN NO.	DISCHARGE RANGE (m^3/s)	REFERENCE DISCHARGE (WEIGHTED) (m^3/s)	X_{bm} RANGE (PPM)	REFERENCE X_{bm} (WEIGHTED) (PPM)	CHOSEN REFERENCE FLOW CONDITION	FLOW REGIME
1	6.5 TO 7.1	6.7	670	670	Q & X_{bm}	L
2	8.3 TO 8.8	8.4	1104	1100	Q	L
3	9.0 TO 9.1	9.0	1206	1200	Q	T
4	8.3 TO 9.1	8.7	1104 - 1206	1155	Q	L,T
5	9.3 TO 10.2	9.5	1206 - 1454	1430	Q	L
6	11.0 TO 11.9	11.8	1370 - 1430	1425	Q	T
7	18.7 TO 22.7	20.6	2550e - 2803	2700	Q	U
8	8.3 TO 9.4	8.8	1104 - 1206	1155	X_{bm}	L,T
9	9.3 TO 11.9	10.6	1370 - 1454	1440	X_{bm}	T,U
10	22.1 to 22.7	22.1	2803	2800	X_{bm}	U

The data required for this investigation included the variables d, W, Q, q, X_{bm}, D_{50}, σ_g, ρ_s, ρ, k_v and g. In order to simplify the problem at hand, variations in the fluid properties, ρ_s and k_v were assumed to be of secondary importance in the rational regime relationships. The data were screened to exclude any data with unusually high wash load concentrations (i.e. $X_{ss} > 10000$ PPM). As

detailed above the sediment properties of the bed sediment are constants for the reach.

Statistical Methods

Least square regression techniques were used to fit equations of the form of equations (1.1) (3.28) and (1.2) to the data chosen in accordance with the rational equations (4.6a), (4.8a), and variations given by equations (4.10), (4.12), (4.13) and (4.14).

$$d = K\,W^{-n} \quad \ldots\ldots (1.1)$$

$$d = K'\,q^{n} \quad \ldots\ldots (3.28)$$

$$A = K\,W^{(1-n)} \quad \ldots\ldots (1.2)$$

$$d = f(W, X_{bm}; D_{50}, {}^{o}g, g, p, p_s, k_v) \quad \ldots\ldots (4.6a)$$

$$d = f(q, X_{bm}; D_{50}, {}^{o}g, g, p, p_s, k_v) \quad \ldots\ldots (4.8a)$$

$$d = f(W, Q; D_{50}, og, g, p, p_s, k_v) \quad \ldots\ldots (4.10)$$

$$d = f(q, Q; D_{50}, og, g, p, p_s, k_v) \quad \ldots\ldots (4.12)$$

$$A = f(W, X_{bm}; D_{50}, og, g, p, p_s, k_v) \quad \ldots\ldots (4.13)$$

$$A = f(W, Q; D_{50}, og, g, p, p_s, k_v) \quad \ldots\ldots (4.14)$$

Equations (4.10), (4.12) and (4.14) correspond to the assumption of discharge as the reference flow condition.

Logarithms were employed to linearize the data, allowing empirical determination of the slope, n, and the intercepts , K as K'. Statistics associated with the regression are as per Section 4.5.2.

Results: Statitiscal Methods

The results of the statistical analyses are presented in Table 5-3. This table is divided into three parts, corresponding to empirical regression results for equations of the form of equation (1.1) [$d = K\,W^{-n}$], (1.2) [$A = K\,W^{(1-n)}$], and (3.28) [$d = K'\,q^{n}$], respectively. Results are presented using both discharge, Q, and bed sediment concentration, X_{bm} , for the reference flow condition. For the Niobrara River, a range of bed sediment concentrations, X_{bm}, and reference discharges, Q, were examined (i.e., Table 5-2).

As well as reporting the least-square regression value for n, and K', Table 5-3 reports the 95% confidence limits for the slope, n; the correlation coefficient, r^2, and the standard error of estimate, S_e. The coefficient K and K' were related by equation (4.16).

$$K = K'\,Q^{n} \quad \ldots\ldots (4.16)$$

where Q = the reference discharge for the data.

TABLE 5-3: NIOBRARA RIVER; SUMMARY OF EMPIRICAL REGRESSION RESULTS

5-3-1 independent variable : log10 (width, W)
dependent variable : log10 (depth, d)
basic equation : $d = K W^{-n}$

RUN NO.	REFERENCE DISCHARGE (m^3/s)	X_{bm} (PPM)	DATA NUMBER	n	n +- 95% CONF. LIM.	K	K'	r^2	S_e (log_{10})
1	6.7	670	24	.71	.19	4.17	1.08	.74	.04
2	8.4	1100	12	.71	.16	4.52	.99	.91	.02
3	9.0	1200	11	.50	.33	2.11	.70	.57	.05
4	8.7	1155	23	.61	.16	3.10	.83	.74	.03
5	9.5	1430	14	.49	.31	2.06	.68	.50	.05
6	11.8	1425	13	.48	.40	2.42	.75	.39	.06
7	20.6	2700	21	.51	.15	3.02	.65	.74	.04

5-3-2 independent variable : log10 (width,W)
dependent variable : log10 (area,A)
basic equation : $A = K W^{(1-n)}$

RUN NO.	REFERENCE DISCHARGE (m^3/s)	X_{bm} (PPM)	DATA NUMBER	1-n	1-n +- 95% CONF. LIM.	K	K'	r^2	S_e (log_{10})
1	6.7	670	24	.29	.19	4.17	n/a	.32	.04
2	8.4	1100	12	.29	.16	4.52	n/a	.63	.02
3	9.0	1200	11	.50	.33	2.11	n/a	.56	.05
4	8.7	1155	23	.39	.16	3.10	n/a	.54	.03
5	9.5	1430	14	.51	.31	2.06	n/a	.52	.05
6	11.8	1425	13	.52	.40	2.42	n/a	.44	.06
7	20.6	2700	21	.49	.15	3.02	n/a	.73	.04

5-3-3 independent variable : log10 (unit discharge,q)
dependent variable : log10 (depth,d)
basic equation : $d = K' q^{n}$

RUN NO.	REFERENCE DISCHARGE (m^3/s)	X_{bm} (PPM)	DATA NUMBER	n	n +- 95% CONF. LIM.	K	K'	r^2	S_e (log_{10})
1	6.7	670	24	.71	.18	4.09	1.07	.74	.04
2	8.4	1100	12	.72	.15	4.54	.99	.92	.02
3	9.0	1200	11	.50	.33	2.09	.69	.57	.05
4	8.7	1155	23	.60	.17	2.95	.81	.73	.04
5	9.5	1430	14	.47	.32	1.91	.66	.47	.05
6	11.8	1425	13	.52	.39	2.80	.78	.45	.06
7	20.6	2700	21	.51	.13	3.09	.65	.77	.03
8	8.8	1155	24	.60	.17	3.04	.82	.72	.04
9	10.6	1440	26	.61	.22	3.57	.84	.59	.06
10	22.1	2800	12	.59	.18	4.23	.68	.85	.03

Analytical Methods and Results

Analytical solutions were developed for the Niobrara River to allow comparison to the selected data and to the empirical results. The analytical results reported in Appendix E for the Brownlie (1981a) equations were used. Brownlie's (1981a) equations are valid for all flow regimes. Analytical results for the Engelund (1967) and Paris (1984) equations, reported in Appendices C and D, are included for runs which contain lower regime data, for comparison purposes. Values for n, K' and K~, were linearly interpolated from the Tables C-1, D-1, D-2, E-9 and E-10, for the reference concentration, X_{bm} and the reach averaged sediment size. The recorded bed sediment concentration was considered in using the analytical methods, except for one discharge, where curve relating Q and X_{bm} was employed.[46] Where required, $k_v = 0.011$ cm^2/s, $p = 1000$ kg/m^3 and $p_s = 2650$ kg/m^3 have been assumed.

Equations (3.28) and (3.21) were employed to calculate the depth of flow for each given q or q~, and the coefficients K, K' and K~ were related by equations (4.16) and (4.17).

$$d = K' \, q^{n} \qquad \text{.....(3.28)}$$

$$R = K\sim \, q\sim^{n} \qquad \text{.....(3.21)}$$

$$K = K' \, Q^{n} \qquad \text{.....(4.16)}$$

$$K' = K\sim \, D_g \, / \, ((R_b \, g \, D_g)^{0.5} \, D_g)^{n} \qquad \text{.....(4.17)}$$

where Q = the reference discharge for the data.

Table 5-4 allows direct comparision of the regression results from Table 5-3 [i.e., 5-3-3] to analytically derived exponents and coefficients for equation (3.28) [$d = K' \, q^{n}$].

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46 See Section 5.3.5 for equation relating Q and X_{bm}.

TABLE 5-4: NIOBRARA RIVER: SUMMARY OF EMPIRICAL/ANALYTICAL RESULTS

RUN NO.	REFERENCE Q (m^3/s)	REFERENCE Xb (PPM)	EMPIRICAL EQUATION(3.28) [$d = K' q^n$] n	EMPIRICAL EQUATION(3.28) [$d = K' q^n$] K'	ANALYTICAL: EQUATION (3.28)[$d = K' q^n$] BROWNLIE n	BROWNLIE K'	PARIS n	PARIS K'	ENGELUND n	ENGELUND K'
1	6.7	670	.71	1.07	.759	1.06	.778	1.06	.691	.95
2	8.4	1100	.72	.99	.747	.94	.770	.97	.687	.84
3	9.0	1200	.50	.69	.744	.91				
4	8.7	1155	.60	.81	.745	.93	.770	.97	.686	.83
5	9.5	1430	.47	.66	.738	.86	.767	.93	.685	.80
6	11.8	1425	.52	.78	.738	.86				
7	20.6	2700	.51	.65	.735	.73				
8	8.8	1155	.60	.82	.770	.97	.745	.93	.686	.83
9	10.6	1440	.61	.84	.767	.93				
10	22.1	2800	.59	.68	.760	.84				

5.3.4 Data Analysis: Rio Grande River

Data Selection

The data base was subdivided on the basis of the reference flow condition, X_{bm}, and/or discharge, Q, as summarized in Tables 5-5 to 5-8.

TABLE 5-5: RIO GRANDE RIVER NEAR BERNALLILO, DATA EXTRACTION REFERENCED TO DISCHARGE, Q

RUN	DISCHARGE RANGE (m^3/s)	CALCULATED[1] CONCENTRATION (PPM)	REFERENCE Q (m^3/s)	DATA NO.	FLOW REGIME
1. BERNQ1	0 to 28.3	0 to 1450	14.2	25	L
2. BERNQ2	32.8 to 35.1	1550 to 1600	34	12	L
3. BERNQ3	36.0 to 44.7	1620 to 1790	40.3	10	L,T
4. BERNQ2&3	32.8 to 44.7	1550 to 1790	39	22	L,T
5. BERNQ4A	50.4 to 60.9	1890 to 2060	55.6	22	L,T
6. BERNQ4L	50.4 to 60.9	1890 to 2060	55.6	13	L
7. BERNQ4T	50.4 to 60.9	1890 to 2060	55.6	9	T
8. BERNQ5	63.1 to 72.8	2100 to 2240	67.6	6	T
9. BERNQ6	77.3 to 87.2	2300 to 2440	82.3	6	L,T,U
10. BERNQ7	95.1 to 124.0	2540 to 2870	110	6	T,U
11. BERNQ8	129.4 to 171.0	2920 to 3330	150	10	T,U
12. BERNQ9	172.7 to 208.4	3340 to 3640	190	6	U
13. BERNQ10	220.6 to 246.4	3740 to 3940	232	6	U
14. BERNQ11	282.3 to 286.0	4190 to 4220	284	2	U

NOTES: 1. BASED ON $X_{bm} = 309.1\ Q^{0.462}$

TABLE 5-6: RIO GRANDE RIVER NEAR BERNALLILO, DATA EXTRACTION REFERENCED TO CONCENTRATION, X_{bm}

RUN	CALCULATED[1] DISCHARGE (m^3/s)	CONCENTRATION RANGE (PPM)	REFERENCE X_{bm} (PPM)	DATA NO.	FLOW REGIME
1. BERNX1	.3 TO 3.6	183 TO 562	370	8	L
2. BERNX2	5.4 TO 13.3	671 TO 1020	850	13	L,T
3. BERNX2L	5.4 TO 13.3	671 TO 1020	850	11	L
4. BERNX3	14.2 TO 23.3	1054 TO 1324	1180	22	L,T
5. BERNX4	29.7 TO 45.9	1480 TO 1810	1650	10	L,T
6. BERNX5	59.4 TO 102.5	2040 TO 2624	2330	8	T,U
7. BERNX6	111.6 TO 135.9	2730 TO 2990	2860	9	U
8. BERNX7	153.2 TO 207.1	3160 TO 3632	3400	6	T,U
9. BERNX8	247.8 TO 374.7	3946 TO 4777	4360	13	U
10. BERNX9	422.6 +	5050 +	5050	2	U

NOTES: 1. BASED ON $X_{bm} = 309.1\ Q^{0.462}$

TABLE 5-7: RIO GRANDE RIVER, CASA COLORADA REACH, DATA EXTRACTION REFERENCED TO DISCHARGE, Q

RUN	DISCHARGE RANGE (m^3/s)	CALCULATED[1] CONCENTRATION (PPM)	REFERENCE Q (m^3/s)	DATA NO.	FLOW REGIME
1. CASAQ1	27 TO 34	840 TO 1030	30.5	6	L
2. CASAQ2	51.8 TO 78.2	1270 TO 2190	65	6	U,T
3. CASAQ3	82.1 TO 96.6	2290 TO 2650	89.4	9	U,T
4. CASAQ4	100.8 TO 107.6	2750 TO 2920	104.2	9	U,T
5. CASAQ5	111.9 TO 125.7	3030 TO 3360	118.8	6	U,T
6. CASAQ6	149.5 TO 186.3	3940 TO 4800	168	12	U,T
7. CASAQ7	202.8 TO 234.2	5190 TO 5910	223.5	12	U,T

NOTES: 1. BASED ON $X_{bm} = 42.35\ Q^{0.905}$

TABLE 5-8: RIO GRANDE RIVER, CASA COLORADA REACH, DATA EXTRACTION REFERENCED TO CONCENTRATION, X_{bm}

RUN	CALCULATED[2] DISCHARGE (m^3/s)	CONCENTRATION RANGE (PPM)	REFERENCE X_{bm} (PPM)	DATA NO.	FLOW REGIME
1.BELENX1	1.1 TO 15.4	11 TO 558	280	17	L
2.CASAX1	16.7 TO 49.2	540 TO 1440	990	5	L
3.CASAX2	57.6 TO 88.2	1660 TO 2440	2050	7	T,U
4.CASAX3A	95.4 TO 115.3	2620 TO 3110	2865	16	L,T,U
5.CASAX3tu	95.4 TO 115.3	2620 TO 3110	2865	16	T,U
6.CASAX2&3A	57.6 TO 115.3	1660 TO 3110	2385	23	L,T,U
7.CASAX2&3tu	57.6 TO 115.3	1660 TO 3110	2385	23	T,U
8.CASAX4	120.6 TO 144.7	3240 TO 3820	3530	14	T,U
9.CASAX5	153.5 TO 173.8	4030 TO 4510	4270	13	T,U
10.CASAX6	184.5 TO 267.9	4760 TO 6670	5715	5	T,U

NOTES: 1. BASED ON $Xbm = 2.597\ Xssbm^{0.945}$ and $Xssbm = 32.69\ Q^{1.013}$
2. BASED ON $X_{bm} = 42.35\ Q^{0.905}$

The data required for this investigation included the variables d, W, Q, q, X_{bm}, D_{50}, σ_g, ρ_s, ρ, k_v and g. The analysis was simplified by assuming that variations in the fluid properties, were of secondary importance, and by screening the data for high wash load concentrations (i.e., X_{ss} > 10000 PPM). As detailed above the sediment properties of the bed sediment are constants for these reaches.

Statistical Methods

The statistical methods were the same as described for the Niobrara River in Section 5.3.3. Least square regression techniques were used to fit equations of the form of equations (1.1) (3.28) and (1.2) to the data chosen in accordance with the rational equations (4.6a), (4.8a), and variations given by equations (4.10), (4.12), (4.13) and (4.14). Logarithms were employed to linearize the data, allowing determination of the slope, n, and the intercepts , K as K', of the equations. Statistics are as per Section 4.5.2.

Results: Statitiscal Methods

The results of the empirical statistical analyses are presented in Tables 5-9 and 5-10 for the Bernallilo and Casa Colorada reaches, respectively. These two tables report the results of fitting the data to equations (1.1) [$d = K W^{-n}$], (1.2) [$A = K W^{(1-n)}$], and (3.28) [$d = K' q^{n}$]. Results are presented using both discharge, Q, and bed sediment concentration, X_{bm} , for the reference flow condition. For the case of the data referenced to (i.e., chosen on the basis of) sediment concentration, X_{bm}, only equation (3.28) [$d = K' q^{n}$] was fitted to the data. The regression results reported in Table 5-9.A correspond to the data chosen in accordance with Table 5-5, and those of Table 5-9.B correspond to the data chosen according to Table 5-6. Similarily Table 5-10.A corresponds to the data of Table 5-7 and Table 5-10.B corresponds to the data of Table 5-8.

As well as reporting the least-square regression value for n, and K', Tables 5-9 and 5-10 report the 95% confidence limits for the slope, n; the correlation coefficient, r^2, and the standard error of estimate, S_e. The coefficient K and K' were related by equation (4.16).

TABLE 5-9: RIO GRANDE RIVER NEAR BERNALLILO: SUMMARY OF EMPIRICAL REGRESSION RESULTS

5-9.A REFERENCED TO DISCHARGE, Q
independent variable : log10 (width,W)

dependent variable: basic equation :			$\log_{10}$ (depth,d) $d = K\ W^{-n}$				$\log_{10}$(area,A) $A = K\ W^{1-n}$		either	
RUN	RATED Q (m^3/s)	DATA NO.	n	K	K'	r^2	1-n	r^2	n, 1-n +- 95% LIMIT	S_e ($\log_{10}$)
1. BERNQ1	14	25	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
2. BERNQ2	34	12	.74	15.4	1.12	.87	.26	.44	.21	.05
3. BERNQ3	40	10	.48	4.8	.81	.36	.52	.39	.54	.06
4. BERNQ2&3	39	22	.70	12.8	.98	.74	.30	.35	.19	.05
5. BERNQ4A	56	22	.87	36.0	1.09	.78	.13	.07	.22	.06
6. BERNQ4L	56	13	.69	17.0	1.06	.93	.31	.73	.13	.03
7. BERNQ4T	56	9	.91	39.4	1.00	.79	.09	.03	.42	.05
8. BERNQ5	68	6	.75	20.2	.87	.93	.25	.60	.29	.04
9. BERNQ6	82	6	.81	27.4	.78	.84	.19	.23	.49	.06
10. BERNQ7	110	6	.63	14.4	.74	.94	.37	.85	.22	.03
11. BERNQ8	150	10	.65	17.6	.68	.92	.35	.76	.16	.03
12. BERNQ9	190	6	.80	37.6	.58	.99	.20	.84	.12	.02
13. BERNQ10	232	6	.76	38.0	.60	.99	.24	.91	.10	.01
14. BERNQ11	284	2	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A

independent variable : log10 (unit discharge,q)

dependent variable: basic equation :			$\log_{10}$ (depth,d) $d = K\ q^{-n}$				
RUN	RATED Q (m^3/s)	DATA NO.	n	n +- 95% LIMIT	K'	r^2	S_e ($\log_{10}$)
1. BERNQ1	14	25	N/A	N/A	N/A	N/A	N/A
2. BERNQ2	34	12	.73	.22	1.08	.85	.05
3. BERNQ3	40	10	.45	.55	.79	.31	.06
4. BERNQ2&3	39	22	.63	.19	.95	.71	.05
5. BERNQ4A	56	22	.86	.23	1.04	.76	.06
6. BERNQ4L	56	13	.68	.13	1.03	.93	.03
7. BERNQ4T	56	9	.91	.41	.95	.80	.05
8. BERNQ5	68	6	.75	.23	.88	.95	.03
9. BERNQ6	82	6	.79	.59	.78	.78	.07
10. BERNQ7	110	6	.59	.41	.73	.80	.06
11. BERNQ8	150	10	.62	.19	.69	.88	.03
12. BERNQ9	190	6	.78	.15	.60	.98	.02
13. BERNQ10	232	6	.75	.19	.60	.97	.03
14. BERNQ11	284	2	N/A	N/A	N/A	N/A	N/A

5-9.B REFERENCED TO CONCENTRATION, X_{bm}
independent variable : log10 (unit discharge,q)

dependent variable: basic equation :			$\log_{10}$ (depth,d) $d = K\ q^{-n}$				
RUN	RATED Xbm (PPM)	DATA NO.	n	n 95% CONF LIMIT +-	K'	r^2	S_e ($\log_{10}$)
1. BERNX1	370	8	.70	.14	1.16	.96	.04
2. BERNX2	850	13	.57	.15	.93	.87	.05
3. BERNX2L	850	11	.68	.12	1.07	.95	.03
4. BERNX3	1180	22	.66	.12	.96	.87	.05
5. BERNX4	1650	10	.63	.10	.88	.96	.03
6. BERNX5	2330	8	.49	.14	.76	.93	.06
7. BERNX6	2860	9	.56	.12	.70	.94	.05
8. BERNX7	3400	6	.36	.20	.73	.87	.05
9. BERNX8	4360	13	.62	.11	.67	.93	.03
10. BERNX9	5050	2	N/A	N/A	N/A	N/A	N/A

TABLE 5-10: RIO GRANDE RIVER, CASA COLORADA REACH: SUMMARY OF EMPIRICAL REGRESSION RESULTS

5-10.A REFERENCED TO DISCHARGE, Q
independent variable : log10 (width,W)

dependent variable: basic equation :			$\log_{10}$ (depth,d) $d = K\ W^{-n}$				$\log_{10}$(area,A) $A = K\ W^{1-n}$		either	
RUN	RATED Q (m^3/s)	DATA NO.	n	K	K'	r^2	1-n	r^2	n, 1-n +- 95% CONF. LIMIT	S_e ($\log_{10}$)
1.CASAQ1	31	6	.61	8.5	1.04	.89	.39	.76	.30	.03
2.CASAQ2	65	6	.36	3.1	.70	.79	.64	.92	.26	.03
3.CASAQ3	89	9	.69	15.3	.69	.99	.31	.97	.05	.02
4.CASAQ4	104	9	.79	25.8	.66	.96	.21	.61	.15	.05
5.CASAQ5	119	6	.84	38.4	.68	.99	.16	.75	.13	.02
6.CASAQ6	168	12	.81	41.1	.64	.95	.19	.52	.13	.03
7.CASAQ7	224	12	.80	44.8	.58	.98	.20	.74	.08	.02

independent variable : log10 (unit discharge,q)

dependent variable: basic equation :			$\log_{10}$ (depth,d) $d = K\ q^{-n}$				
RUN	RATED Q (m^3/s)	DATA NO.	n	n 95% CONF LIMIT +-	K'	r^2	S_e ($\log_{10}$)
1. CASAQ1	31	6	.82	.41	1.24	.88	.03
2. CASAQ2	65	6	.50	.21	.71	.92	.02
3. CASAQ3	89	9	.75	.08	.68	.99	.02
4. CASAQ4	104	9	.80	.16	.67	.96	.03
5. CASAQ5	119	6	.82	.23	.69	.96	.03
6. CASAQ6	168	12	.76	.11	.64	.96	.03
7. CASAQ7	224	12	.79	.07	.61	.98	.02

5-10.B REFERENCED TO CONCENTRATION, X_{bm}

independent variable : log10 (unit discharge,q)

dependent variable: basic equation :			$\log_{10}$ (depth,d) $d = K\ q^{-n}$				
RUN	RATED X_{bm} (PPM)	DATA NO.	n	n 95% CONF LIMIT +-	K'	r^2	S_e ($\log_{10}$)
1. BELENX1	280	17	.63	.20	1.16	.75	.07
2. CASAX1	990	5	.91	.58	1.34	.89	.03
3. CASAX2	2050	7	.47	.18	.70	.90	.02
4. CASAX3A	2865	16	.47	.18	.69	.70	.05
5. CASAX3tu	2865	16	.84	.20	.67	.87	.03
6. CASAX2&3A	2385	23	.47	.13	.70	.73	.05
7. CASAX2&3tu	2385	23	.65	.15	.69	.80	.04
8. CASAX4	3530	14	.78	.12	.63	.95	.03
9. CASAX5	4270	13	.76	.08	.67	.97	.03
10. CASAX6	5715	5	.76	.17	.65	.99	.02

Analytical Methods and Results

Analytical solutions were developed for the Rio Grande River to allow comparison to the selected data and to the empirical results. Analytical solutions were developed as detailed in Section 5.3.3. The recorded bed sediment concentration was considered in using the analytical methods. Only the runs referenced to sediment concentration (i.e., Tables 5-9.B and 5-10.B) were compared to the analytical solutions.

Equations (3.28) and (3.21) were employed to calculate the depth of flow for each given q or q~, and the coefficients K, K' and K~ were related by equations (4.16) and (4.17).

$$d = K' q^n \quad \text{....} \qquad (3.28)$$

$$R = K\sim q\sim^n \qquad (3.21)$$

Tables 5-11 and 5-12 allow direct comparision of the empirical regression results from Table 5-9.B and 5-10.B to analytically derived exponents and coefficients for equation (3.28) [$d = K' q^n$], for the Bernallilo and Casa Colorada reaches, respectively.

TABLE 5-11: RIO GRANDE R. NEAR BERNALLILO, COMPARISON OF EMPIRICAL AND ANALYTICAL RESULTS

DESCRIPTION		EMPIRICAL (TAB.5-9.B)		ANALYTICAL: [$d = K' q^n$]			
RUN REFERENCE	X_{bm} (PPM)	[$d = K' q^n$]		BROWNLIE		ENGELUND	
		n	K'	n	K'	n	K'
1. BERNX1	370	.70	1.16	.77	1.18	.71	1.10
2. BERNX2	850	.57	.93	.76	1.00	.69	0.90
3. BERNX2L	850	.68	1.07				
4. BERNX3	1180	.66	.96	.75	0.92	.69	0.83
5. BERNX4	1650	.63	.88	.73	0.83	.69	0.78
6. BERNX5	2330	.49	.76	.73	0.75		
7. BERNX6	2860	.56	.70	.74	0.72		
8. BERNX7	3400	.36	.73	.74	0.68		
9. BERNX8	4360	.62	.67	.75	0.64		

TABLE 5-12: RIO GRANDE R., CASA COLORADA REACH, COMPARISON OF EMPIRICAL AND ANALYTICAL RESULTS

DESCRIPTION		EMPIRICAL (TAB.5-10.B)		ANALYTICAL: [$d = K' q^n$]			
RUN	REFERENCE X_{bm} (PPM)	[$d = K' q^n$]		BROWNLIE		ENGELUND	
		n	K'	n	K'	n	K'
1.	BELENX1 280	.63	1.16	.77	1.30	.71	1.22
2.	CASAX1 990	.91	1.34	.75	0.98	.69	0.86
3.	CASAX2 2050	.47	0.70	.73	0.80	.68	0.74
4.	CASAX3A 2865	.47	0.69	.74	0.73	.68	0.65
5.	CASAX3tu 2865	.84	0.67				
6.	CASAX2&3A 2385	.47	0.70	.73	0.77	.68	0.71
7.	CASAX2&3tu 2385	.65	0.69				
8.	CASAX4 3530	.78	0.63	.74	0.69		
9.	CASAX5 4270	.76	0.67	.75	0.65		
10.	CASAX6 5715	.76	0.65	.75	0.62		

5.3.5 Alternatives for Reference Flow Condition

As pointed out in Section 4.5.3, several practical reasons exist for the substitution of discharge, Q, for bed sediment concentration, X_{bm}, for the purpose of defining the reference flow condition. In terms of the rational regime relations, this process is typified by the substitution of equation (4.10) for (4.6a).

$$d = f(W, X_{bm}; D_{50}, \sigma_g, g, \rho, \rho_s, k_v) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.6a)$$

$$d = f(W, Q; D_{50}, \sigma_g, g, \rho, \rho_s, k_v) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.10)$$

Such a substitution requires the development of a relationship between Q and X_{bm}.

From a physical viewpoint, the width:depth equations are defined for equilibrium conditions (i.e., constant sediment transport). However, their development, as stated in Chapter 4, is also contigent on steady flow conditions (i.e., constant discharge). In the absense of unsteady conditions (e.g., transient flood waves) and barring multiple-valued behavior, one might anticipate each sediment concentration to be associated with a certain discharge. The statistical analysis, presented herein tests for a single-valued relationship betweem Q and X_{bm}.

Relationships between X_{bm} and Q were developed for the Niobrara and Rio Grande Rivers using data provided in Brownlie (1981b). The data were input to a statistical model to determine the least square regression fit between discharge, Q, and bed sediment concentration, X_{bm}. Table 5-13 summarizes the results of the least square regression analyses and Figures 4.3 and 5.2 display the results. Two types of equations were employed, linear and logarithmic (i.e., exponential), as shown in Table 5-13.

Table 5-13: Niobrara and Rio Grande Rivers: Summary of Relationships between Q and X_{bm}

River	Regression Formula	Least Square Regression		
		A	B	r^2
Niobrara	1. $X_{tot} = A\ Q + B$	193	-470	0.75
	2. $X_{tot} = A\ Q^{B}$	35.7	1.60	0.75
Rio Grande Bernalillo	3. $X_{bm} = A\ Q^{B}$	309	0.46	0.46
	4. $X_{bm} = A\ Q + B$	12.6	1329	0.36
	5. $X_{bm} = A\ X_{ssbm}^{B}$	2.6	0.94	0.93
Rio Grande Belen	6. $X_{bm} = A\ Q^{B}$	32.7	1.03	0.68
Rio Grande Casa Colorada	7. $X_{bm} = A\ Q^{B}$	42.4	0.91	0.84
[assumes Bernallilo equation 5. applicable]				

notes: 1. X_{ssbm} = suspended bed material concentration (ppm)

The equation for the Niobrara River calculates total concentration since the data of Brownlie (1981b) were not adjusted for wash load. As shown in Figure 4.3 the scatter in the Niobrara data is significantly less than the Middle Loup data. This fact is reflected in the higher correlation coefficients for the Niobrara data. Nordin's (1965) relations for Q versus Q_{BM} can be converted to Q versus X_{bm} and shown to be equivalent to those for Bernallilo (equation 4) and Belen (equation 5). McBean et al. (1988) suggested that relations between Q and Q_T are spurious, and recommended relations between X_{bm} and Q as more statistically valid. While a certain amount of scatter exists in the relationships, the correlation coefficients are significant at the 1% level (Neville, 1970).

The existence of a clear relationship between Q and X_{bm} allows discharge to be substituted for sediment concentration for the purposes of defining the reference flow condition.

5.3.6 Equilibrium

The analytical solutions presented in Section 5.2 indicated that equation (3.28) [$d = K' q^n$] can be used to represent equilibrium sediment transport conditions (i.e., equilibrium hydraulic geometry), even where transition between flow regimes is encountered. This section attempts to verify those results with data from the Niobrara and Rio Grande Rivers.

NIOBRARA RIVER

The analytical solutions and empirical regression results for Runs 1, 8, 9 and 10 in Table 5-4 (i.e., Niobrara River), are plotted against the observed data in Figures 5.3 to 5.6, respectively. Figures 5.3 to 5.6 employ the format of equation (3.28) [$d = K' q^n$].

Figures 5.3 to 5.6 demonstrate that *absolute* concurrance between the analytical and empirical results is not attained for the Niobrara River data. Figure 5.3 compares the observed unit discharge versus depth data to the empirical regression with equation (3.28) (Table 5-3-3; Run No. 1) and to the analytical methods of Brownlie, Engelund, and Paris (Table 5-4; Run No. 1). Even for this lower flow regime case, none of the analytical methods coincide exactly with the empirical regression on the data. However, the regression on the data and the analytical methods parallel each other suggesting *general* concurrance. In other words, since the shape and direction of the empirically derived curve is the same as for the analytical solutions, then the equi-concentration model appears to be valid. The fact that for Figures 5.3, the analytically derived curve is everywhere below the empirical curve suggests that a slightly lower, but still equi-concentration, X_{bm}, would allow the analytical model results to coincide with the observed data. This observation applies to Figures 5.4 and 5.5 also, for data encompasing the lower, transition and upper flow regimes. Finally, for the upper regime data of Figure 5.6 the regression on the data and Brownlie's analytical techniques match each other fairly well. It

appears from the above data that the general premise of the analytical model works, but the absolute results are not 100% accurate.

Next consider the scatter of the observed unit discharge versus depth data from the empirically and analytically derived curves. Regime methods must rely on instantaneous measurements for prediction of equilibrium channel geometry. Instantaneous variations in slope, water temperature, and water and sediment discharge have the potential to account for many of the deviations of the observed data from the regression curve. In each of Figures 5.3 to 5.6 the scatter in the observed data, as indicated by the 95% confidence interval, is as great as the variation between the analytically and empirically derived results. The scatter in the observed data has the potential to account for some of the difference between the relative position of the empirical and analytical curves.

The conformance of the Niobrara data to the analytical solutions using Brownlie's equations is best considered in the form of Figures 5.7, 5.9 and 5.11. These Figures compare the entire observed data base to the analytical solutions calculated using Brownlie's (1981b) equations (i.e., Table 5-4; Runs No. 1, 9, and 10). Figure 5.7 employs the format of equation (3.28) [$d = K' q^{n}$], Figure 5.9 that of equation (1.1) [$d = K W^{-n}$], and Figure 5.11 the format of equation (1.2) [$A = K W^{1-n}$]. The development of analytical solutions in the form of equations (1.1) and (1.2) required the calculation of K via equation (4.16).

$$K = K' Q^{n} \qquad \text{.....(4.16)}$$

where Q = the reference discharge for the data, taken from Table 5-2, Run No. 1, 9, and 10.

Analytical solutions are provided for either end of the observed range of bed sediment concentration for the data base (i.e., 670 ppm and 2800 ppm; see Tables 5-2 and 5-4), as well as for an intermediate concentration (i.e., 1440 ppm). Figures 5.7, 5.9 and 5.11 reveal that the range in the observed data for any particular width or unit discharge, is explained by Brownlie's analytical solutions using the calculated range of sediment concentrations, X_{bm} (i.e., 670 ppm to 2800 ppm). This statement is supported by the plotting of analytical

solutions for X_{bm} = 10 ppm [47], in Figures 5.7, 5.9 and 5.11. A bed sediment concentration, X_{bm} = 10 ppm is well outside the range of observed sediment concentrations for this data base, so that one would not expect any observed data to plot close to it. Clearly, this is the case in Figures 5.7, 5.9 and 5.11, supporting the conclusion that the data conform to the analytical solutions.

Comparison of Figures 5.7, 5.9 and 5.11 to the results of the empirical regression analyses presented in Figures 5.8, 5.10, and 5.12, further confirms the similarity in the observed data and the analytical solutions of Brownlie (1981b). Figure 5.8 is a plot of the empirical regression results from Table 5-4 (Run No. 1, 8, 9, 10) in the form of equation (3.28) [$d = K' q^{n}$]. Direct comparison of Figures 5.8 and 5.7 show that the empirical regression results (Figure 5.8) generally concur with the analytical solutions of Brownlie for the sediment concentrations modelled. This concurrance is exemplified by the similar shape of the curves, the relative position of the curves with respect to the flow condition, X_{bm}, and the direction of the curves. Figures 5.10 and 5.12 plot the empirical regression results from Table 5-3 (Run No. 1, 4, 5, 6, 7) in the form of equations (1.1) [$d = K W^{-n}$] and (1.2) [$A = K W^{1-n}$]. The sediment concentrations corresponding to the discharge conditions shown can be read from Table 5-3 (e.g., Run No. 1, $Q = 6.7\ m^3/s$ corresponds to X_{bm} = 670 ppm). Direct comparison of Figures 5.10 and 5.12 to Figures 5.9 and 5.11, respectively, again show that the empirical regression results generally concur with the analytical solutions using Brownlie's equations.

The width:area relationship helps explain how sediment equilibrium is maintained in a reach with varying top widths. The width:area relationships developed using discharge, Q, as the reference flow condition, show an increase of width with area (i.e., Figure 5.12). This confirms the analytically predicted trend for equilibrium sediment concentration conditions plotted in Figure 5.11. Moreover, the exponent of the width versus area curve [i.e. (1-n) in equation (1.2)] is generally greater than the 95% confidence limit on the same value (see

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47 Since X_{bm} = 10 ppm is a hypothetical case for this data base (corresponding approximately to static equilibrium), it was necessary to assume a discharge; $Q_{dom} = 9.9\ m^3/s$ is assumed.

Table 5-3, part 5-3-2). As indicated in Section 4.5.6, this demonstrates that area is not constant along a reach, but increases with width (i.e., (1-n) - 95% limit > 0). The increase of area with width corresponds to a decrease in velocity, and as will be shown in Chapter 6 a decrease in unit sediment load. Equilibrium sediment transport is maintained by the fact that the total transport is balanced by the increasing width and the decreasing unit sediment transport (i.e., $Q_{bm} = q_{bm} W$).

RIO GRANDE RIVER

A similar analysis to that for the Niobrara River is presented for the Rio Grande River. The conformance of the observed Rio Grande River data to the analytical solutions using Brownlie's equations is best illustrated by considering Figures 5.13 to 5.24. Figures 5.13, 5.14, 5.19 and 5.20 employ the format of equation (3.28) [$d = K' q^{n}$], to compare the entire observed data base to the selected analytical solutions calculated using Brownlie's (1981b) equations (i.e., Figures 5.13, 5.19). They also allow for comparison to empirically derived regression equations (i.e., Figures 5.14 , 5.20). The equations plotted in these Figures can be found in Tables 5-11 and 5-12, for the Bernallilo and Casa Colorada reaches, respectively. Analytical solutions are provided for either end of the observed range of bed sediment concentration for the data base (see Tables 5-6 and 5-8 or 5-11 and 5-12), as well as for selected intermediate concentrations. For the Bernallilo reach the range of bed sediment concentration is 370 ppm to 4360 ppm, while for the Casa Colorada reach the range is 280 ppm to 5715 ppm.

Figures 5.15, 5.16, 5.21, and 5.22 employ the format of equation (1.1) [$d = K W^{-n}$], and Figures 5.17, 5.18, 5.23 and 5.24 the format of equation (1.2) [$A = K W^{1-n}$], to compare the observed data base to the selected analytical solutions (i.e., Figures 5.15, 5.17, 5.21, 5.23) and allow for comparison to the empirical regression equations (i.e., Figures 5.16, 5.18, 5.22, and 5.24). The empirical regression equations plotted in Figures 5.16, 5.18, 5.22 and 5.24 can be found in Tables 5-9.A and 5-10.A, for the Bernallilo and Casa Colorada reaches, respectively. The development of analytical solutions in the form of

equations (1.1) and (1.2) required the calculation of K via equation (4.16).

$$K = K' \, Q^n \qquad \text{.................................(4.16)}$$

where Q = the reference discharge for the data, taken from Tables 5-5 to 5-8.

Analytical solutions are provided for either end of the observed range of the reference discharge, Q, for the data base (see Tables 5-9.A and 5-10.A), as well as for selected intermediate discharges. For the Bernallilo reach the range of discharge modelled was 39 m^3/s to 232 m^3/s while for the Casa Colorada reach the range was 30 m^3/s to 224 m^3/s.

Figures 5.13 to 5.24 show that the empirical regressions and Brownlie's analytical results do not always agree, in absolute terms. However, the analytical results generally explain the observed range of the data, and the relative shape, position with respect to the flow condition, and the direction of the curves is consistent between the analytical and empirical solutions. The overlap of the regression results for the Casa Colorada reach (i.e., Figure 5.20) and to a lesser extent the Bernallilo reach (i.e., Figure 5.14) are attributed to the meagre number of data available at certain reference flow conditions.

Again the width:area relationships (Figures 5.18 and 5.24) help explain how sediment equilibrium is maintained in a reach with varying top widths. These width:area relationships, developed using discharge, Q, as the reference flow condition, show an increase of width with area. This confirms the analytically predicted trend for equilibrium sediment concentration conditions plotted in Figures 5.17 and 5.24. Moreover, the exponent of the width versus area curve [i.e. (1-n) in equation (1.2)] is generally greater than the 95% confidence limit on the same value (see Table 5-9.A and 5-10.A), confirming the increase of area with width. This increase of area corresponds to a decrease in velocity, and a decrease in unit sediment load. The total sediment transport is balanced by the increasing width and the decreasing unit sediment transport (i.e., $Q_{bm} = q_{bm} W$).

In conclusion, although absolute agreement has not been attained the *general* concurrance of the analytical and empirical solutions for the Niobrara River and Rio Grande River data, verifies the ability of equation (3.28) to represent equilibrium conditions, even through the transition across flow regimes. The specific requirements of equilibrium as proposed in Section 5.2 are considered next.

5.3.7 Conditions Required for a Continous Equation (3.28)

In Section 5.2 it was concluded that in order for equation (3.28) [$d = K' q^{n}$] to fit the data without obvious discontinuities, then the flow regime must progress from lower to transition to upper regime as q increases. That it to say that, for any given concentration, X_{bm}, each q should have only one feasible flow regime (and depth) associated with it, in order that multiple solutions, and therefore multiple equations, do not exist. In addition it was speculated that for any given reference flow condition, one should rarely observe all three flow regimes, and more typically only two (i.e., lower and transition or transition and upper) or one regime should be observed. These analytically derived conclusions are tested herein.

Consider first the latter point; with regards to the number of flow regimes observed for a given reference flow condition. For the Niobrara and Rio Grande Rivers this observation holds relatively true. Tables 5-2 and 5-5 to 5-8 reveal that out of a total of 51 flow conditions, only two exhibit all three regimes. All other flow conditions include data which follow one of the combinations: lower only or; lower and transition or; transition and upper or; upper only flow regimes. On the basis of this observation, potential outliers in data sets might be identified on the basis of flow regime[49].

Next consider the requirement for progression from lower to transition to upper regime as q increases. Figures 5.25 is a plot of observed unit discharge versus depth data versus the empirical regression equation of the form of equation (3.28) [$d = K' q^{n}$], for a flow condition, X_{bm} = 1440 ppm. On Figure 5.25 the regime is denoted by F-flat (i.e., upper) and T-Transition. This Figure illustrates that for

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49 For example if one had 49 lower regime and one upper regime points in a 50 point data set, at a given reference flow condition, the upper regime point might be suspect.

the Niobrara River, the data do not conform to the requirement of one regime per unit discharge. However, the flow regime attribute is suspect since the gauging station bed condition has been assumed to apply to all of the cross sections (i.e., C-1 to C-10, see Section 5.3.1) regardless of the width variation. Based on the analytical solutions this is likely incorrect. On the other hand the bed form observations do allow for general trends in the bed forms to be discerned ensuring that the data base does in fact cover all three flow regimes, and that the conclusions drawn in Section 5.3.6 are valid.

The Rio Grande River data does include flow regime (i.e., bed form) observations at each cross-section. Figure 5.26 (Casa Colorada reach) presents a flow condition which has a forward progression from transition to upper flow regime as q increases. However, this is the exception rather than the rule. Figures 5.27 to 5.35 are plots of observed unit discharge versus depth data versus the empirical regression equation of the form of equation (3.28) [$d = K' q^n$], for a range of flow conditions, X_{bm}, on the Rio Grande River. In Figures 5.27 to 5.35 the regime is denoted by L-Lower, T-Transition and U-Upper. These Figures show that the observed flow regimes often overlap at any given unit discharge, for the modelled reference flow conditions. The reason for this overlap is unknown. However, since there does not appear to be a discontinuity in the data as a result of the overlap, it is expected that the range in the reference flow conditions assumed in the data selection process, the subjective nature of the observations, and other local variations in temperature and flow could lead to these deviations from the ideal analytical case. There is in fact a general progression from lower to upper regime as concentration, X_{bm}, increases and the unit discharge concurrently increases. This last point is illustrated by considering Figures 5.27 to 5.30 for the Casa Colorada and 5.31 to 5.36 for the Bernallilo reaches. Figure 5.36 shows that for the Bernallilo reach the analytical and empirical regression results agree reasonably well for the upper flow regime. It is recommended that a controlled laboratory experiment is the best means of confirming or denying the ability of rivers to meet this requirement. Observations in a laboratory are more objective due to the environment and

accessability, and the controlling variables (e.g., discharge) can adjusted and monitored closely.

For the Niobrara River, Figures 5.25 and 5.3 to 5.6 do not have any obvious discontinuities that could be attributed to the jump between flow regimes. The correlation coefficients reported in the Table 5-3 indicate that the data are modelled reasonably well by equation (3.28). The analysis of the Rio Grande River data generally agree with this position, as illustrated by Figures 5.26 to 5.36. Two deviations from this conclusion present themselves. In Figure 5.27 (Casa Colorada), the single lower regime data point is outside the 95% confidence interval for the regression, indicating a discontinuity. It is impossible to draw any conclusions from one data point, but it is expected that this point is an outlier to the regression. In Figure 5.31 (Bernallilo) the two transition regime points plot outside the 95% confidence interval. Again the data are inconclusive as to the probable cause.

5.3.8 Sensitivity to Reference Flow Condition

The empirically derived regression results reported in Tables 5-3, 5-9 and 5-10 and partially summarized in Tables 5-4, 5-11 and 5-12 confirm that the exponent, n and coefficient, K' of equation (3.28) [$d = K' q^n$] vary with the reference flow condition.

For the Niobrara River the empirically derived exponent, n varies from 0.70 to 0.50, generally decreasing with increasing discharge or concentration (see Table 5-4). The lower value of n= 0.5 exceeds the lower limit for n given by any of the analytical methods (see Table 5-4)[50]. Furthermore, the 95% confidence limits on Run No. 1 (i.e., Table 5-3, part 5-3-3; n = 0.71 +- .18) encompases the n suggested by Tamburi for static equilibrium (i.e., n = 0.86). Thus it is impossible to assess whether this reach is in dynamic equilibrium, from an empirically derived value of n alone. The empirically derived coefficient K', on the other hand varies from 1.07 to 0.65 in relatively close agreement with the range of K' developed via the analytical solutions for Brownlie's equations (see Table 5-4). Moreover, these empirical values of K' are significantly different from the analytically

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50 n=0.5 is also lower than Tamburi's suggested n value for dynamic equilibrium (i.e., 0.64)

derived value of K' at static equilibrium. Using Brownlie's equations, the value of K' for static equilibrium (i.e., approximated by X_{bm} = 1 ppm) is 2.6, which is much larger than the empirically derived range of 1.07 to 0.65. Therefore, based on K', one can conclude that the reach is in dynamic equilibrium. This conclusion is verified by Figure 5.7[51].

For the Rio Grande River both reaches have a wide range of empirically determined n values (see Tables 5-11 and 5-12) encompasing and occasionally exceeding the range proposed by Tamburi from static to dynamic equilibium (i.e., 0.86 to 0.64). Part of the wide range in n is no doubt attributable to the small number of data available for some flow conditions (e.g., Table 5-10). The empirically derived coefficient K' on the other hand, varies from 1.16 to 0.67 (see Table 5-11) for the Bernallilo reach and 1.34 to 0.63 (see Table 5-12) for the Casa Colorada reach. These empirically derived values of K' are in relatively close agreement with analytically derived values of K' developed using Brownlie's equations (see Table 5-11 and 5-12). As with the Niobrara River, the range in the empirically derived K' is substantially different from the analytical value of K' at static equilibrium (i.e., approximately 2.6). Therefore, based on K', one can conclude that these reaches are in dynamic equilibrium.

Both rivers examined support the conclusion from Chapter 4, that K' is a better indicator of the equilibrium state than n. Furthermore, this conclusion is extended to include all flow regimes.

5.4 Summary

In this Chapter, analytical solutions to the rational width:depth equation (4.8a) [$d = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v)$], which are valid for all flow regimes, have been presented. The solution follows the method of Sections 4.4.1 and 4.4.2, and employs the sediment transport and flow resistance equations of Brownlie (1981b). The solution data are used to show that the width:depth equation (3.28) [$d = K' q^{n}$] adequately models equilibrium channel geometry (i.e., constant X_{bm}), in a continuous fashion over the lower, transition and upper flow regimes. A general requirement for maintaining the continuity of the relationship is a forward progression from lower to transition to upper flow regime with

51 See Section 5.3.6.

increasing unit discharge, for a given reference flow condition. As such only one flow regime (and depth, d) is feasible at any given unit discharge and flow condition.

The analytical solutions developed using Brownlie's equations are also employed to quantify the relationship given by equation (4.11a), for all flow regimes.

$$n, K, K', = f(X_{bm};D_g,\sigma_g,g,p,p_s,k_v) \quad \ldots\ldots\ldots\ldots\ldots (4.11a)$$

The results are presented graphically. The relationship $K' = f(X_{bm}, D_{50})$ does not show any signs of discontinuity associated with transition between flow regimes. The relationship $n = f(X_{bm}, D_{50})$ is somewhat convoluted but never the less continuous. The range of K' with X_{bm} makes it a better indicator of channel equilibrium state than n.

Empirical investigations on the Niobrara and Rio Grande Rivers were used to verify the applicability of the analytical results, to present discharge as an alternative to concentration for the purpose of defining the reference flow condition, and to evaluate the sensitivity of width:depth analysis to the reference flow condition. While absolute concurrance between the data and the analytical solutions is not obtained for all cases, the trends in the relations are similar. Furthermore, the overall variation in the data bases is generally explained by the analytical solutions. Lastly, the empirical regression results for K' follow the analytical solutions relatively closely. The confirmation of increasing area with top width also provides a means of maintaining sediment equilibrium.

The specific requirement of a forward progression from lower to transition to upper regime, was not observed in the data bases considered. For the Rio Grande River there is an overall progression from lower to upper regime with increasing concentration and unit discharge, but at any given reference flow condition the flow regimes overlap. Further study is required to identify the reason for this observation.

The empirical studies support the conclusion that K' is a better indicator of equilibrium than n.

Chapter 6

Sediment Transport

6.1 Introduction

Public Works (1978) expressed the philosophy behind the relationship between equations (1.1) [$d=K\ W^{-n}$] and (1.3) [$q_b=C_b\ p\ V^m$]:

> "The intimate nature of the equilibrium between channel form and sediment transport also makes it possible to determine sediment transport conditions by analyzing sounding data"

Equation (1.4) [$m = 1/(1-n)$], which partially quantifies this relationship, is a byproduct of Tamburi's analytical basis for width:depth analysis (i.e., Appendix A). To date, no comparable equation for relating the coefficient C_b in equation (1.3) to equation (1.1) has been determined, although methodologies have been proposed (Tamburi et al., 1983).

A premise to the relationship of equations (1.1) and (1.3) was expressed by Public Works (1978):

> "... bedload has the primary impact on channel bottom geometry...(and) ..... Velocity is the most significant variable influencing bedload transport"

This rationale explains the form of equation (1.3) and the fact that only bedload is considered. Suspended load is said to be important if delta growth, dredging, major scour or aggradation are being considered (Public Works, 1978).

In this Chapter the relationship between sediment transport and width:depth analysis is briefly reexamined, in light of the preceeding Chapters. It is proposed that total bed material load, rather than bedload, should be used in equation (1.3), leading to equation (6.1).

$$q_{bm} = C_{bm}\ p\ V^m \quad \text{.....(6.1)}$$

q_{bm} = unit bed material load (e.g., kg/m/s)
C_{bm} = coefficient for bed material load
m = bed material load exponent (superceeds equation 1.3)

Equation (6.1) is confirmed as a valid and practical sediment transport formula. The relationship between the sediment transport equation and the rational forms of the width:depth equation is evaluated, and an equation is proposed for the calculation of C_{bm}. Lastly, a proposal for determining C_{bm} from width:depth data is presented.

6.2 Bed Material Load

The total sediment load of a stream can be divided into: *bedload*, which refers to that load moving on or near the bed by rolling, sliding or saltating, and *suspended load*, which refers to the remainder. Alternately, the total sediment transport load can also be subdivided into: *bed material load* and *wash load*, referring respectively to the sediment discharge of those materials found in appreciable quantities and very small quantities in the bed material.

Tamburi's notion that bedload is of primary importance in determining hydraulic geometry can be traced to WCHL (1974a,b; 1978) studies on borrow pit migration. The relative importance of suspended load in infilling borrow holes on the Fraser River was dependent on the length of the borrow hole and the amount of turbulence, whereas bedload reacted almost immediately to changes in channel shape.

It is suggested here that width:depth relations are in dynamic equilibrium with bed material load, not just bedload. Simons and Lagasse (1976) and Maddock and Leopold (1953) observed that as width/depth ratio decreases, corresponding to a decrease in width, then bedload decreases and suspended bed material load increases. The distinction between suspended bed material and bedload is more a computational or data collection convenience than a real-life barrier. Bedload and suspended bed material load are intimately linked at this artificial barrier. Mathematical derivations of bed material transport, require the bedload concentration as a boundary condition for determination of the suspended bed material load concentration (Van Rijn, 1982). Width:depth relations are therefore dependent on bed material load, not just bedload. In support of this statement, dynamic equilibrium for the Rio Grande River near Bernallilo reach involves total bed material load (Nordin ,1977; Figure 3.29).

As a result of the above discussion, an alternative to equation (1.3) is proposed here as equation (6.1).

$$q_{bm} = C_{bm}\, p\, V^{m} \qquad \text{.....(6.1)}$$

Also, equilibrium conditions in a reach apply to total bed material load, which is by definition, given by equation (6.2).

$$Q_{bm} = q_{bm}\, W \qquad \text{.....(6.2)}$$

Q_{bm} = total bed material load

Equations (6.1) and (6.2) imply that equilibrium can only be maintained if velocity decreases with increasing width. This supports the conclusion that area increases with width (see Section 4.5.4, Figure 4.8; and Figure 3.11).

6.3 Validity of Sediment Transport Equation (6.1)

WCHL (1976, 1977) presented a rationale for acceptance of equation (1.3) as a valid bedload equation (see Appendix B). ASCE (1975) suggested that a continuous, single-valued relation also exists between bed material load and velocity, on the basis of Figures 6.1 and 3.38. Figure 3.38 especially lends itself to the form of equation (6.1), while Figure 6.1 cannot be modelled exactly by equation (6.1), unless a piece-wise fit is used. The acceptance of equation (6.1) as a valid sediment transport formula for use in deriving hydraulic geometry relations is furthered by its use in several recent mathematical models for alluvial rivers (Lopez-Garcia, 1977; Simons et al., 1980; Ribbernink et al., 1985).

Consider briefly the analytical alternatives to equation (6.1), in order to better understand its value in engineering applications. A large number of sediment transport formulae have been developed over the years (ASCE, 1975) ranging from the theoretical derivations of Einstein to the empirical methods such as Colby, and equations (6.1) and (1.3). Figure 2.10 illustrates the fact that many of these methods do not correlate well with field observations. Nakato et al. (1979) considered the formulae of Einstein-Brown, Inglis-Lacey, Engelund-Hansen, Colby and Toffalletti for application to the Mississippi River and found none to be satisfactory. Power relations relating bedload, Q_b and suspended load, Q_S, to velocity, V, and discharge, Q were found to be the best predicting tools. WCHL (1978) also found statistical methods, including equations similar to equation (6.1), to be superior to theoretical and empirical sediment transport formulas in predicting observed sediment rating curves on the Fraser River. Thompson (1985) found that the Meyer-Peter-Mueller, Brown and Ackers-White formulae only predicted well if observed velocity, V, was input rather then slope, S, and depth, d.

Furthermore, he noted a strong relationship of the form in equation (6.3).

$$q_{bm} = n_s V^{4.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.3)$$

Single-variate equations such as equation (6.1) are not without their detractors. Yang (1972) studied equations of the form $q_{bm} = f(Q)$, $f(\tau)$, $f(S)$, $f(V)$; concluding that the first three were multivalued and therfore inappropriate, and the power exponent associated with the velocity relation was so high as to make it insensitive to deviations from the curve. Yang (1972) proposed that instead of simple velocity, V, bed sediment load should be related to unit stream power VS. Yang's equations have met with mixed reviews (Vanoni, 1977; Nordin, 1977; Hubbell, 1977; Jordan, 1977) and have not garnered any wide recognition (Vanoni, 1984). Nordin (1977) concluded that since slope is virtually constant for natural streams that increases in VS are basically increases in V. Figure 6.2 (Yang, 1972) supports Nordin's contentions.

Despite lack of acceptance for Yang's (1972) bivariate equation for sediment transport, there have been strong arguments in support of abandoning a single-variate approach. Parker (1977), in a generalized treatment of alluvial river flow, proposed equation (6.4) for bed material load.

$$q_* = f\ (x,y\ ;\ R_p, R_b, p_i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.4)$$

where $q_* = q_{bm}/(R_b\ g\ D_s)^{0.5}\ D_s$

x,y = substituted by two independent variables from the particular solution (e.g., V*,S; or q~,R; etc.)

Parker (1977) criticized relations such as the Meyer-Peter bedload equation for reducing sediment transport to a single variate relation, when intrinsically a multivariate relation is indicated. The same criticism might be levelled against equation (6.1).

Colby's (1964) investigations revealed that depth influenced the relation of velocity and sediment discharge, confirming the multivariate approach. The general form of his graphical solutions (i.e., Figure 6.3; Colby, 1964) is given by equation (6.5)

$$q_{bm} = f(V,d,D_g) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.5)$$

Moreover, Colby's (1964) approach included corrections for the effects of temperature and wash load concentration (i.e., viscosity). However, Colby (1961) concluded that for typical velocities found in natural

streams (e.g., 3-4 fps), depth has little or no effect on the relationship of bed material load and velocity.

In the final analysis one must conclude that observation is the strongest indicator of the validity of equations of the form of equation (6.1). The reliability of these equations is exemplified in their application to the solution of real-world problems (e.g., Simons et al., 1980). However, when applying or developing such an equation the temperature, concentration of wash load, and depth variations for which it is applicable should be documented. Furthermore, equation (6.1) is only valid for velocities greater than the critical velocity for initiation of motion and does not allow for the case of armouring (Lopez-Garcia, 1977). Caution should be exercised not to extrapolate equation (6.1) beyond the conditions it is developed for.

6.4 Relationship to Width:depth Analysis

Equation (4.8c) is the rational form of the sediment transport equation (6.1), given that equation (4.8a) holds true.

$$d = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v) \quad \text{.....(4.8a)}$$

$$V = f(q, X_{bm}; D_g, \sigma_g, g, \rho, \rho_s, k_v) \quad \text{.....(4.8c)}$$

This is demonstrated as follows:

Step 1

- consider the equations for continuity of water (Parker, 1977) and conservation of mass (Nordin, 1989) given by equations (4.9) and (4.3) respectively:

$$q = V\,d \quad \text{.....(4.9)}$$

$$X_{bm} = 10^6 (R_b+1)\, q_{bm} / q \quad \text{.....(4.3)}$$

- combine to give:

$$V = 10^6 (R_b+1)\, q_{bm} / X_{bm}\, d$$

- represent this equation as

$$V = f(q_{bm}, d, X_{bm};\ \text{sediment and fluid properties})$$

Step 2

- but $d = f\ (q, X_{bm};$ sediment and fluid properties) ..(4.8a)

- substitution for d, and noting $q_{bm} = f(q, X_{bm})$ gives:

$$V = f\ (q, X_{bm};\ \text{sediment and fluid properties}) \qquad ..(4.8c)$$

- therefore once $d = f(W, X_{bm};$ fluid and sediment properties) is known then one can solve for a sediment transport equation

Appendix F demonstrates this logic using the specific width:depth equation (3.28) and sediment transport equation (6.1). Equation (1.4) is confirmed and an equation (F.4) relating C_{bm} and K is developed.

$$m = 1 / (1-n) \qquad .....(1.4)$$

$$C_{bm} = X_{bm}/10^6 * K'^m \qquad .....(F.4)$$

where X_{bm} is the sediment concentration corresponding to K' Rigourous verification of equations (1.4) and (F.4) with observations from natural streams has not been undertaken to date (Tamburi, 1976, 1979, 1983). Verification of equations (1.4) and (F.4) will serve as secondary proof to the validity of equation (3.28). For example, Lopez-Garcia (1977) indicates that the range of m in equation (6.1), is $3 < m < 7$. Substituting these values into equation (1.4) produces corresponding n values of $0.666 < n < 0.857$. These values correspond to the range for dynamic and static equilibrium as proposed for long constrictions.

Time limitations prevented further analysis of these relations. It is recommended that the Middle Loup River, Niobrara River and the Rio Grande River be investigated in this regard. These rivers have well defined total bed material load relations to velocity, as indicated in Figure 6.4 (ASCE, 1975).

6.5 Alternate Proposal for Determining C_{bm}

Tamburi, Shewfelt and Pomalaza (1983) proposed a method to determine C_{bm} in equation (6.1). With reference to Figure 6.5, the logic is as follows:

1. consider a discharge defined reach of river, assuming channel shape to be equilibrium with the dominant discharge.

2. the reach is described by a series of cross-sections, characterized by average depth and top width at dominant discharge.

3. width:depth analysis is used to derive equation (1.1) for the reach.

4. the exponent m is calculated from equation (1.4)

5. the program initializes C_{bm} (i.e., K in Figure 6.5) thereby producing a bed material load (bedload in Figure 6.5) function.

6. a bed transient model (e.g., Lopez-Garcia, 1977) is used to calculate bed and water level pertubations along the reach over discrete time intervals. The starting condition is defined by the actual field cross-sections.

7. the model reiterates the calculation of bed and water levels at dominant discharge, until a dynamic equilibrium is approached, and the bed stabilizes, giving a set of calculated cross-sections.

8. statistical comparisons are made between actual (i.e., field data) and the calculated stable bed elevations; and C_{bm} (i.e., K in Figure 6.5) is incremented.

9. the model stops at some point where incrementing C_{bm} (i.e., K) leads to a decrease in statistical fit to the actual data. This is considered the best fit C_{bm} value.

This methodology has not been tested to date, and is considered second priority to the testing of equations (1.4) and (F.4).

6.6 Summary

This Chapter has concluded that a relationship exists between width:depth equations (e.g., equation (3.28)) and sediment transport equations of the form of equation (6.1). The relationship is formalized by equations (1.4) and (F.4), and has a basis in the rational regime relations developed in Chapter 4. Equation (6.1) is an accepted sediment transport formula, which must not be employed outside the conditions for which it is developed.

Equations (1.4) and (F.4) require further verification with field data. The Middle Loup, Niobrara and Rio Grande Rivers are recommended for this purpose. An alternate method has been proposed for the derivation of C_{bm}, but has not been tested.

CHAPTER 7
CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The major conclusions of this thesis are presented in this Chapter. The conclusions demonstrate that simple non-linear width:depth equations [e.g., equation (1.1); $d = K W^{-n}$] can be used to represent equilibrium channel geometries for wide, straight alluvial rivers (i.e., equilibrium sediment transport). Therefore, these equations represent *regime* conditions for a discharge defined reach of river, whose top width varies due to geologic constraints. Accordingly, the engineering applications claimed for width:depth equations appear to be valid, subject to the sensitivities and limitations described herein.

The conclusions are subdivided into the problem areas defined in Chapter 1. A separate section is provided for conclusions derived from the literature review. The conclusions have been numbered consecutively and the reference section/chapter is provided in *italics* for each conclusion, or set of conclusions.

7.2 Literature Review

A literature review was undertaken to establish how width:depth analysis relates to other methods for studying hydraulic geometry, and to qualitatively assess the problems outlined for this thesis. The conclusions are subdivided by topic below.

7.2.1 Regime and Analytical Methods *(Section 3.3 and 3.4)*

Conclusions with respect to other regime and analytical methods for hydraulic geometry are as follows:

1. The traditional regime methods (i.e., $d,W,S,V = f(Q)$) appear to be at odds with width:depth analysis until one examines the influence of bank stability. Modified regime equations that account for bank stability (Blench, 1969; Thorne and Hey, 1986) imply that significant width variations (e.g., up to 200 %) are possible within a discharge defined reach. Solution to Blench's (1969) regime equations provides data which

conform to equation (1.1) [$d = K\ W^{-n}$], implying that equation (1.1) can be employed to define regime conditions. *(Section 3.3)*

2. The *rational regime* methods of Parker and Anderson (1977) and Peterson (1975) relate dimensionless unit discharge, q~ to dimensionless depth, R, which for a discharge defined reach are dimensionless equivalents to width and depth. An alternate form of the width:depth equation (1.1) was drawn from their results and formalized by equation (3.21). *(Section 3.3)*

$$R = K\sim\ q\sim^{\ n} \quad \text{.....................................(3.21)}$$

3. Paris's (1984) analytical solutions for hydraulic geometry indicate an alternate form of the width:depth equation given by equation (3.28) (for lower flow regime). Again, the variable q (=Q/W) is analogous to width in a discharge defined reach. *(Section 3.4)*

$$d = K'\ q^{\ n} \quad \text{.....................................(3.28)}$$

4. The results associated with Conclusions 3 and 4 suggest that $n, K', K\sim = f(X_{bm}, D_g)$, and imply equilibrium sediment transport, by definition. The implication is that equations (3.21) and (3.28) [and therefore equation (1.1)] can be employed to represent equilibrium conditions. Equations (3.21) and (3.28) represent two-dimensional models of an alluvial river. *(Section 3.4)*

5. The comparison of width:depth analysis to other regime and analytical methods for hydraulic geometry has been formalized. Ten existing methods for analyzing hydraulic geometry are classified into two groups, namely *(Section 4.2)*;

a. *Optimal width* methods (White, 1980; Chang, 1979, 1980b, 1985c; Parker 1979) predict optimal hydraulic parameters (i.e., S, d, W). A conditional formula (e.g., maximizing sediment transport) is required in order to define the optimal state. No allowance is made for bank hetrogeniety.

b. *Equilibrium* methods (Parker, 1977; Paris, 1984; Peterson, 1975; Staub, 1935; Komura, 1963; Width:Depth Analysis) provide results for a range of hydraulic parameters (i.e., S, d, q) which can

maintain equilibrium sediment transport. Width is employed as an independent variable, and reflects bank competency.

Optimal width methods and traditional regime methods assume that regime slope has been everywhere attained, and that bed and bank materials are similar. Width:depth analysis, on the other hand, recognizes that the optimal width may not be attainable due to constraints dealing with bank competency and does not require strict adjustment to regime slope.

7.2.2 Observed River Behavior *(Section 3.5)*

Several rivers are presented in order to illustrate the potential for width:depth equations to describe observed conditions. Conclusions were as follows:

6. Observations on several rivers confirm that width:depth equations have potential to be a useful engineering tool in describing field conditions. Examples are provided for the Middle Loup, Missouri, North Canadian, South Saskatchewan, Hundred Foot, Rio Grande, and Niobrara Rivers.

7. In general, width adjustments are relatively slow (i.e., 1% per year), indicating ample time for complementory depth adjustment. Width is therefore also a suitable independent variable for regime analysis, which rely on instantaneous measurements to study channel equilibrium.

7.2.3 Sensitivity to Channel Pattern *(Section 3.6)*

The literature review included a qualitative assessment of the sensitivity of width:depth analysis to changes in the channel pattern state. Conclusions in this regard included:

8. Channel pattern should be identified at the outset of any width:depth investigation, and reaches subdivided into similar channel types. Width:depth analysis should not be undertaken in ignorance of other controlling factors, such as thresholds between channel pattern states. This thesis does not consider the case of steep braided channels.

7.3 Theoretical Underpinnings

The theoretical underpinnings of width:depth analysis are evaluated in this thesis, using analytical methods. Conclusions in this regard are as follows:

9. Tamburi's (WCHL, 1974; Tamburi, 1979) proposed analytical bases for width:depth analysis are not generalized or thorough enough to explain the underlying assumptions and limitations and are therefore inadequate for their stated purpose. *(Section 3.2)*

10. A statement of case is a part and parcel to the development of hydraulic geometry relationships. For this thesis the following statement of case applies:

- straight, wide, alluvial river, with steady quasi-uniform flow
- **variables**: $W,d,f,V,S,Q_{bm},Q,X_{bm},D_g,o_g,g,p,p_s,k_v$
- **process** equations:

  1. continuity of water [equation (3.1)]
  2. conservation of sediment mass [equation (4.3)]
  3. momentum; [equation (4.2)]
  4. sediment transport
  5. flow resistance
  6. bank competency equation

- **resultant** equation :

  d,f,V,S,Q,Q_{bm} = function $(W,X_{bm};D_g,o_g,g,p,p_s,k_v)$

The variable q is optional, contingent on the inclusion of the equation defining $q = Q/W$. *(Section 4.2)*

11. *Rational*[9] forms of the width:depth equation, derived from the statement of case, are given by equations (4.6a), (4.8a), (4.9a).

$$d = f(W,X_{bm};D_g,o_g,g,p,p_s,k_v) \quad \text{.....(4.6a)}$$

$$d = f(q,X_{bm};D_g,o_g,g,p,p_s,k_v) \quad \text{.....(4.8a)}$$

$$R = f(q\sim,X_{bm};R_p,R_b,o_g) \quad \text{.....(4.9a)}$$

These equations detail the underlying variables in the specific width:depth equations (1.1), (3.21) and (3.28), respectively.

$$d = K\ W^{-n} \quad \text{.....(1.1)}$$

$$d = K'\ q^{n} \quad \text{.....(3.28)}$$

$$R = K\sim\ q\sim^{n} \quad \text{.....(3.21)}$$

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9 See footnote 9 page 6 for explanation of *rational*

Therefore, equation (4.11) indicates the variables which control the values of n, K, K', and K~.

$$n, K, K' = f\ (X_{bm}; D_g, o_g, g, p, p_s, k_v) \quad \text{.....(4.11a)}$$

$$n, K\sim = f\ (X_{bm}; R_p, R_b, o_g) \quad \text{.....(4.11b)}$$

(Section 4.3)

12. Using a two-dimensional (i.e., vertical/downstream) approach, the variable list can be reduced to; $q, d, V, f, S, X_{bm}, D_s, p_s, p, k_v, g, o_g$, and the *rational* equations (4.8a) and (4.9a) can be quantified through the simultaneous solution of the following equations:

1. continuity 2. momentum 3. sediment transport and 4. flow resistance

This is a simplification of a very complex three-dimensional flow phenomena. *(Section 4.4.1)*

13. Solution of the equations listed in Conclusion 12 are provided herein, employing the sediment transport and flow resistance equations of: a. Engelund (1967); b. Ackers-White (1973) and White (et.al., 1980); and c. Brownlie (1981a) (see Appendix C,D,E, respectively). Equations (3.21) and (3.28) adequately represent the analytically derived solutions to the rational regime equations (4.8a) and (4.9a), for the lower flow regime (i.e., a. Engelund; b. Ackers-White), and for constant sediment and fluid properties and bed sediment concentration (i.e., Figures C-1, C-4, D-1 to D-16).

$$R = K\sim\ q\sim^{n} \quad \text{.....(3.21)}$$

$$d = K'\ q^{n} \quad \text{.....(3.28)}$$

Since the analytical solutions are given for a constant bed material concentration, X_{bm}, which in turn defines equilibrium, then equations (3.21) and (3.28) represent equilibrium channel geometries (i.e., regime conditions). *(Section 4.4.2, 5.2)*

14. Contrasting width:depth analysis to the *optimal width* methods (minimum stream power - Chang, 1979; 1980a,b; 1985b,c; maximum sediment transport - White et al., 1982; Griffiths, 1939) indicates that the conditions for maintaining equilibrium sediment transport in a reach

with varying top widths include local slope adjustments (see Conclusion 5). Adjustments of local slope required to maintain sediment equilibrium have been quantified in graphical format using Brownlie's equations (see Figures E-13,15,17,19,21,23). *(Section 3.4, 5.2)*

7.4 Sensitivity of Width:Depth Analysis

The sensitivity of width:depth analysis to flow condition and flow regime was quantified using analytical methods. The conclusions have been subdivided to flow condition and flow regime.

7.4.1. Sensitivity to Flow Condition

The sensitivity of width:depth analysis to reference flow condition (X_{bm}) has been quantified using the analytical methods described under Conclusions 12 and 13. Sediment concentration, X_{bm}, is employed to define the reference flow condition.

15. The exponent, n, and coefficients ,K' and K~, in equations (3.21) and (3.28) are functions of D_g , X_{bm} (see Conclusion 11), thereby impacting the use of these equations to define the state of equilibrium (i.e., static or dynamic).

$$n, K, K' = f\ (X_{bm};D_g,o_g,g,p,p_s,k_v) \quad \text{.....(4.11a)}$$

$$n, K\sim = f\ (X_{bm};R_p,R_b,o_g) \quad \text{.....(4.11b)}$$

The variation of n, K' and K~ with sediment gradation, sediment and fluid density, and fluid viscosity has not been accounted for (i.e., constants assumed). Results are provided herein in graphical and tabular format (see Figures C-2, D-11 to D-16, and E-24 to E-27; Tables C-1 to C-3, D-1 to D-2, and E-9 to E-10). *(Sections 4.4.2 , 5.2, Appendices C, D, E)*

16. The computed values of n for all concentrations and sediment sizes fall within the range of $0.9 < n < 0.6$, with the lower limit reached asymtotically as X_{bm} increases, in agreement with Tamburi (1979). The exponent, n and the coefficients, K' and K~ both decrease with increasing X_{bm}. *(Sections 4.4.2 , 5.2, Appendices C, D, E)*

17. The range in the computed values of n and K' indicate that K' is a better indicator of channel equilibrium state than n. *Static*

equilibrium is indicated by the value of K' as X_{bm} tends to 0 PPM. *(Section 4.4.2, 5.2, Appendices C, D, E)*

7.4.2 <u>Sensitivity to Flow Regime</u> *(Section 5.2)*

The sensitivity of width:depth equations to the transition between flow regimes was explored using analytical methods. Theorectical conditions exist which permit the chosen simple non-linear equation (i.e., equation (3.21)) to be continuous over the transition regime.

18. Analytical methods employing Brownlie's (1981a) sediment transport and resistance equations (see Conclusion 13) were used to define the impact of transition between flow regimes on width:depth equations. The solution data conform to the *rational* equation (4.8a), and are adequately described by equation (3.28), for any equi-concentration line and constant sediment and fluid properties.

$$d = K' \, q^{n} \qquad \text{(3.28)}$$

The results indicate that equation (3.28) is continuous over all three regimes ($S<0.006$), and therefore adequately describe the solution data. It is recommended that width:depth analysis not be used for slope > 0.006 (i.e., steep braided streams) at this time.

19. The analytical results require that a forward progression exist from lower to transition to upper regime as q increases at any given reference flow condition. This requirement ensures that only one flow regime exists at any given q and X_{bm} combination.

20. The relationship of n, K' and K~ with X_{bm} is continuous, albeit somewhat convoluted in the case of n, despite the transition between flow regimes (Figures E-24 and E-26). The variation of n and K' with X_{bm} and D_g are remarkably similar to those determined for the lower flow regime by way of Paris's equations.

7.5 Empirical Investigations

Empirical investigations were undertaken in order to provide verification of the analytically derived relations and conclusions. Field data from the Middle Loup, Niobrara and Rio Grande Rivers were considered. The conclusions have been subdivided by topic.

7.5.1 Related to Equilibrium Channel Geometry *(Sections 4.5.4, 5.3.6)*

21. The analytical solutions to the rational width:depth equations (see Conclusions 12 and 13) are verified by the general trends in the Niobrara and Rio Grande River data and at the dominant flow conditions for the Middle Loup data. Thereby, the hypothesis that width:depth equations (3.21) and (3.28) represent equilibrium channel geometries is also verified.

22. Regression on the data using equation (1.2) confirms that area increases with width (i.e., (1-n)-95% confidence limit > 0).

$$A = K\,W^{(1-n)} \quad \cdots\cdots\cdots\cdots\cdots (1.2)$$

This increase in area allows the channel to maintain equilibrium in total sediment transport, through a reduction in velocity and therefore in unit sediment discharge (see Conclusion 29).

7.5.2 Related to Flow Condition *(Sections 4.5, 5.3)*

23. The sensitivity of the width:depth equations to the choice of reference flow condition is illustrated by the field data. The range in the observed data for any particular width or unit discharge is explained by the analytical solutions using the observed range of sediment concentrations. Moreover, the observed data are significantly different from the case of static equilibrium, as approximated by Paris's theory at X_{bm} = 10 ppm, and Engelund's theory at X_{bm} = 0 ppm, and Brownlie's theory at X_{bm} = 1 PPM. Therefore the rivers studied are in dynamic equilibrium with their sediment load.

24. The analysis confirms that K' is a better indicator of channel equilibrium state (i.e., static or dynamic) than n (see Conclusion 17). The range of n values from regression results includes the analytically

predicted n for the case of static equilibrium. On the other hand, the value of K' from the regressions generally agree with the analytical solutions, and the observed range in K' is substantially different from that for static equilibrium (see conclusion 23).

25. Discharge, Q, is a practical alternative to concentration, X_{bm}, for purposes of defining the reference flow condition. The relationships between X_{bm} and Q developed for the Middle Loup, Niobrara and Rio Grande Rivers permit discharge to be substituted for sediment concentration. An alternative form of the width:depth equation (4.6.a), given by equation (4.10) is possible if a relationship exists between Q and X_{bm}.

$$d = f(W,Q;D_g,\sigma_g,g,p,p_s,k_v) \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.10)$$

7.5.3 Related to Transition Between Flow Regimes *(Section 5.3.5)*

The flow regimes on the Niobrara and Rio Grande Rivers have been observed to range from lower to transitional to upper. Empirical analysis of this data, lead to the following conclusions:

26. The analytical solutions represented by equation (3.28) (see Conclusion 18) are verified by the general trends in the Niobrara and Rio Grande data . Therefore, the data confirm Conclusion 18.

$$d = K' q^{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.28)$$

The values of K' determined by regression on the data generally follow those predicted analytically and there are no obvious discontinuities in the relations resulting from transition between flow regimes.

27. The requirement of the forward progression from lower to transition to upper regime as unit discharge increases (see Conclusion 19) are not evident in the data of the Niobrara or Rio Grande Rivers. For the Niobrara River the observed bed forms were taken only at one section, and are therefore not suitable for this purpose. For the Rio Grande River the flow regimes overlap at several concentration and unit discharge combinations. There is however a general trend to progress from lower to upper flow regime as unit discharge and concentration increase. Further study is required to identify the potential reasons for these observations (see section 7.7).

7.6 Relationship of Width:Depth Analysis and Sediment Transport *(Chapter 6)*

The relationship between width:depth equations and sediment transport was examined in a qualitative manner. The following conclusions are drawn:

28. Width:depth relations are in dynamic equilibrium with bed material load, Q_{BM} ($=q_{BM}$ W), not just bedload, Q_B ($=q_B$ W).

29. In accordance with conclusion 28, equation (1.3) is replaced by equation (6.1).

$$q_b = C_b \; p \; V^{m} \quad \text{.....................................(1.3)}$$

$$q_{bm} = C_{bm} \; p \; V^{m} \quad \text{.....................................(6.1)}$$

Based on a review of sediment transport literature, equation (6.1) is considered to be a valid sediment transport equation. Given that equation (6.1) is true, then equilibrium sediment transport can only be maintained if velocity decreases with increasing width. This supports the conclusion that area increases with increasing width (see Conclusion 22).

30. A relationship exists between the width:depth equation (3.28) and the sediment transport equation (6.1), as formalized by equations (1.4) and (F.4) and supported by the the form of the rational regime relations developed in Chapter 4.

$$m = 1 / (1-n) \quad \text{.....................................(1.4)}$$

$$C_{bm} = X_{bm} / 10^{6} * K'^{m} \quad \text{........................(F.4)}$$

31. An alternative means of determining C_{bm}, through mathematical modelling is proposed.

7.7 Recommendations for Further Study

Several topics are recommended for further study as a direct result of this thesis.

a. Exploration of the impact of σ_g on the variation of n and K' with the reference flow condition. Brownlie's (1981a) equations could be used.

b. Further verification of the impact of the transition between flow regimes on the applicability of equation (3.28), including:

   b1. using the analytical methods of Van Rijn (1984) and White et al. (1987).

   b2. using controlled flume studies, aimed at testing the existance of multiple bed forms at a given q and X_{bm}.

c. Extended testing of the analytical solutions with field data.

d. Testing of equation (6.1), (1.4) and (F.4) with field data.

e. Development and testing of the program proposed in Chapter 6.4.

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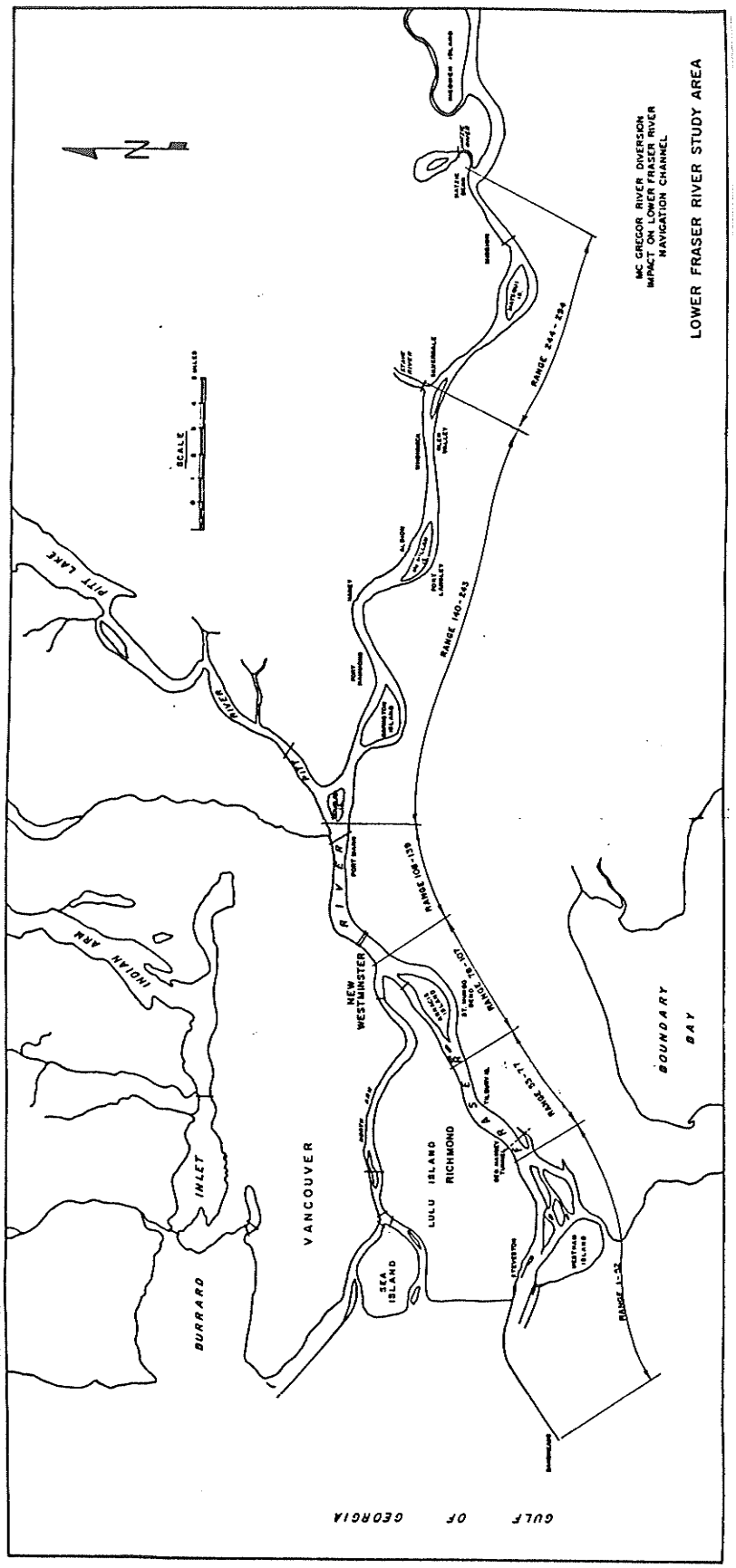
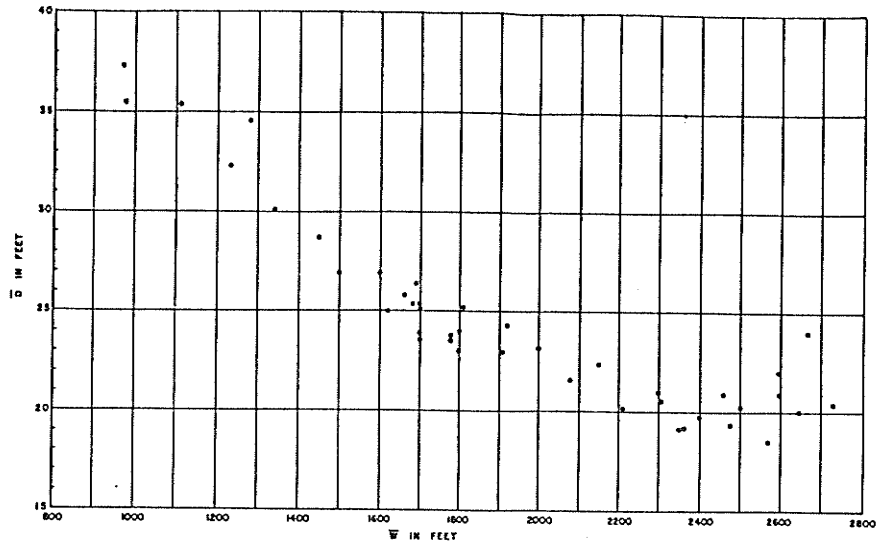


Figure 2.1 Lower Fraser River; Width:Depth Sub-Reaches (WCHL, 1978)



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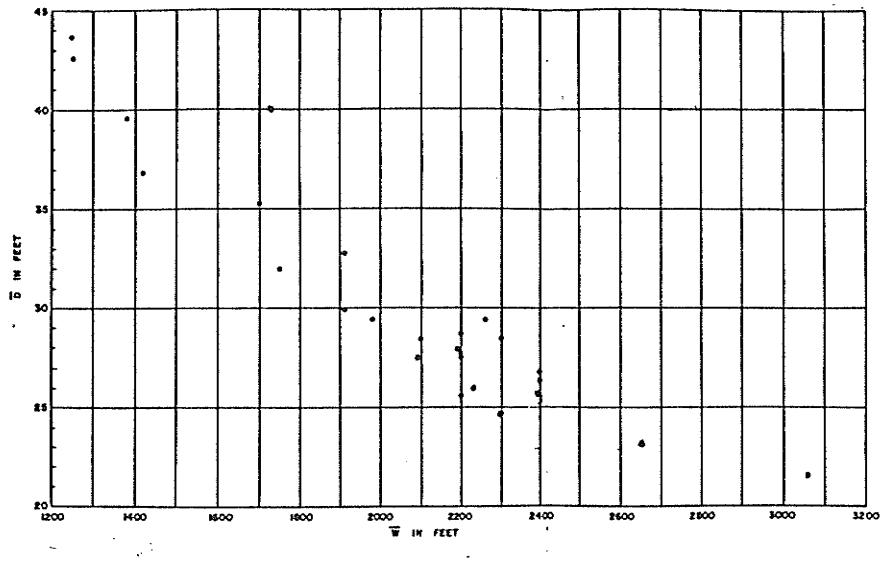

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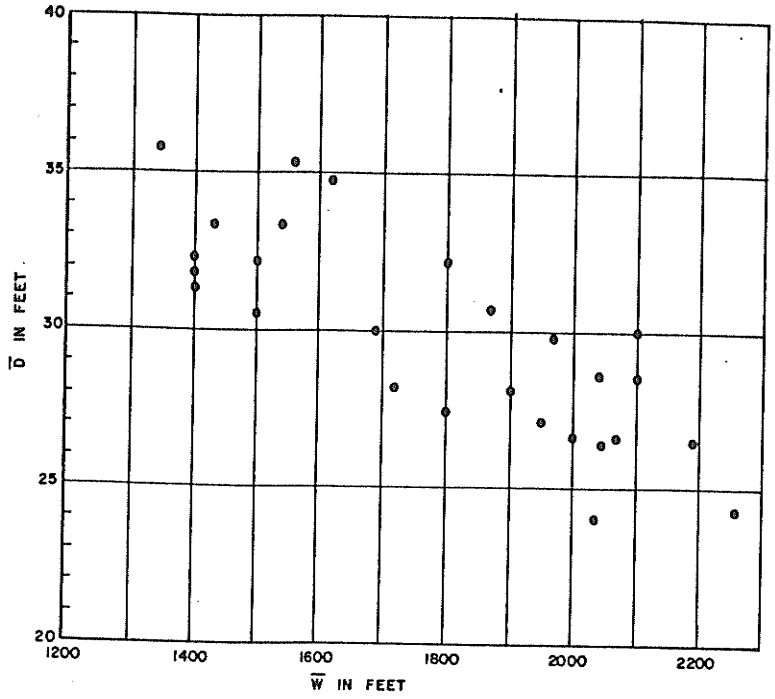

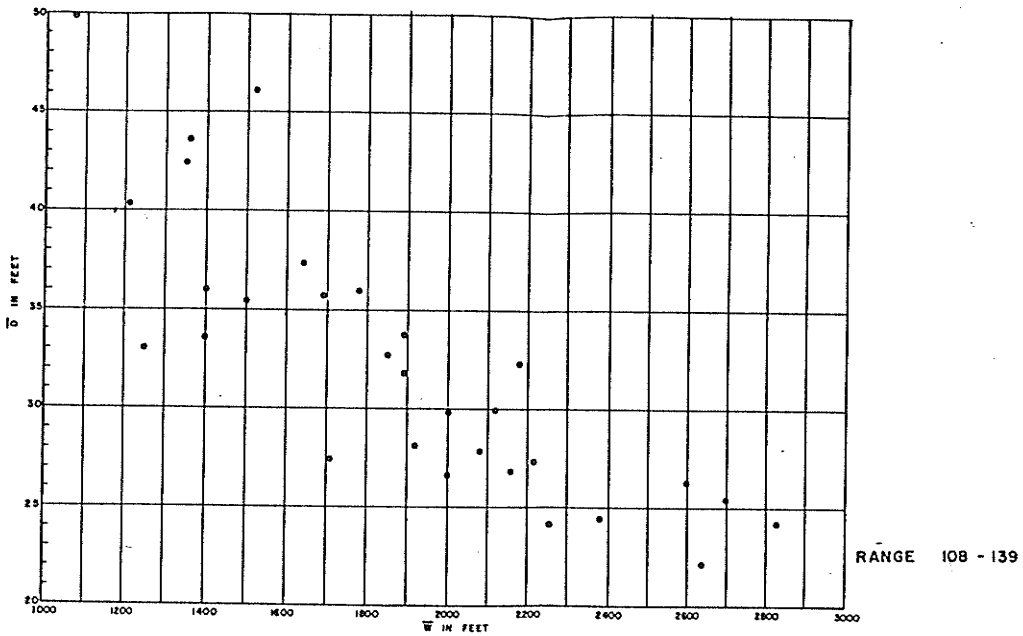

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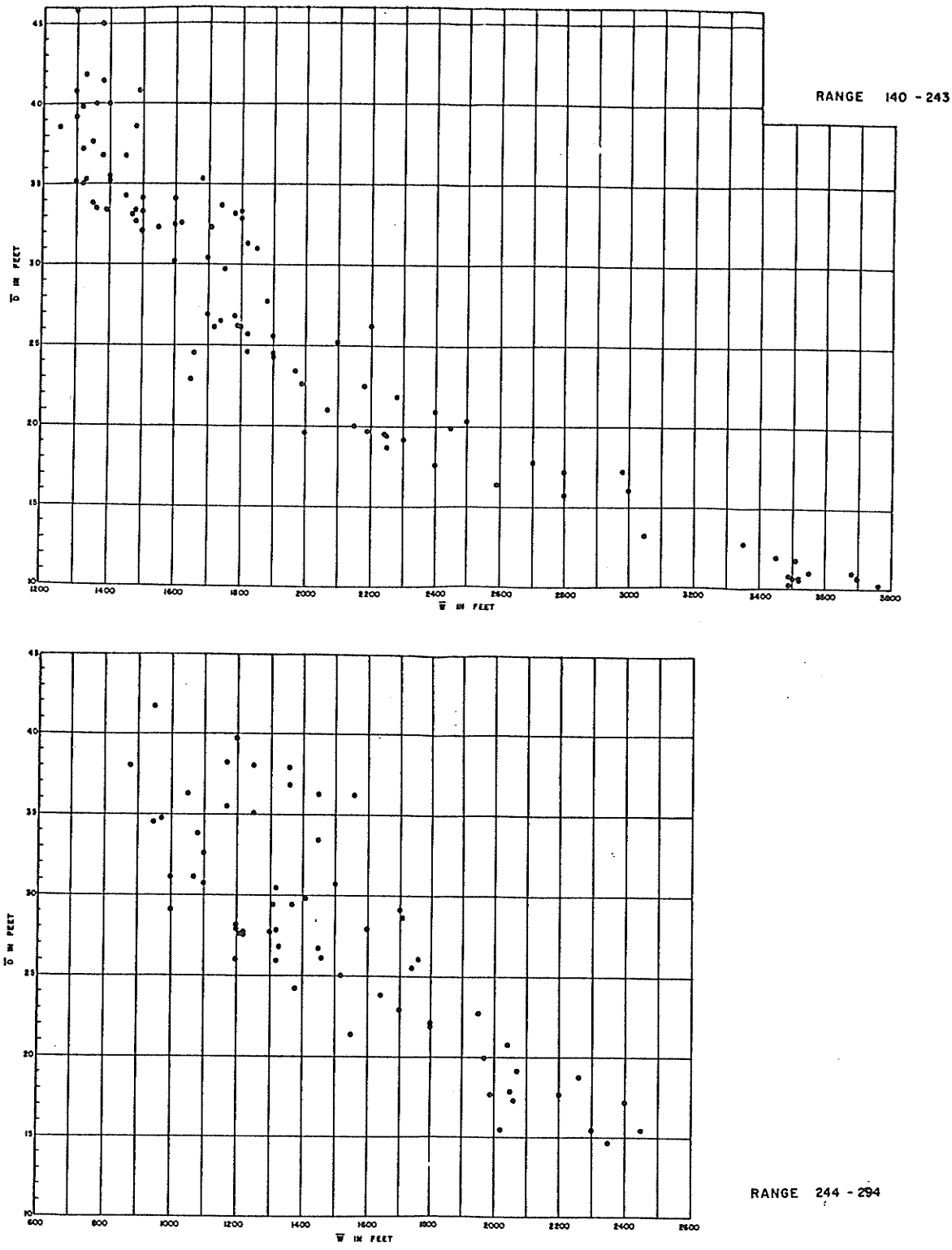

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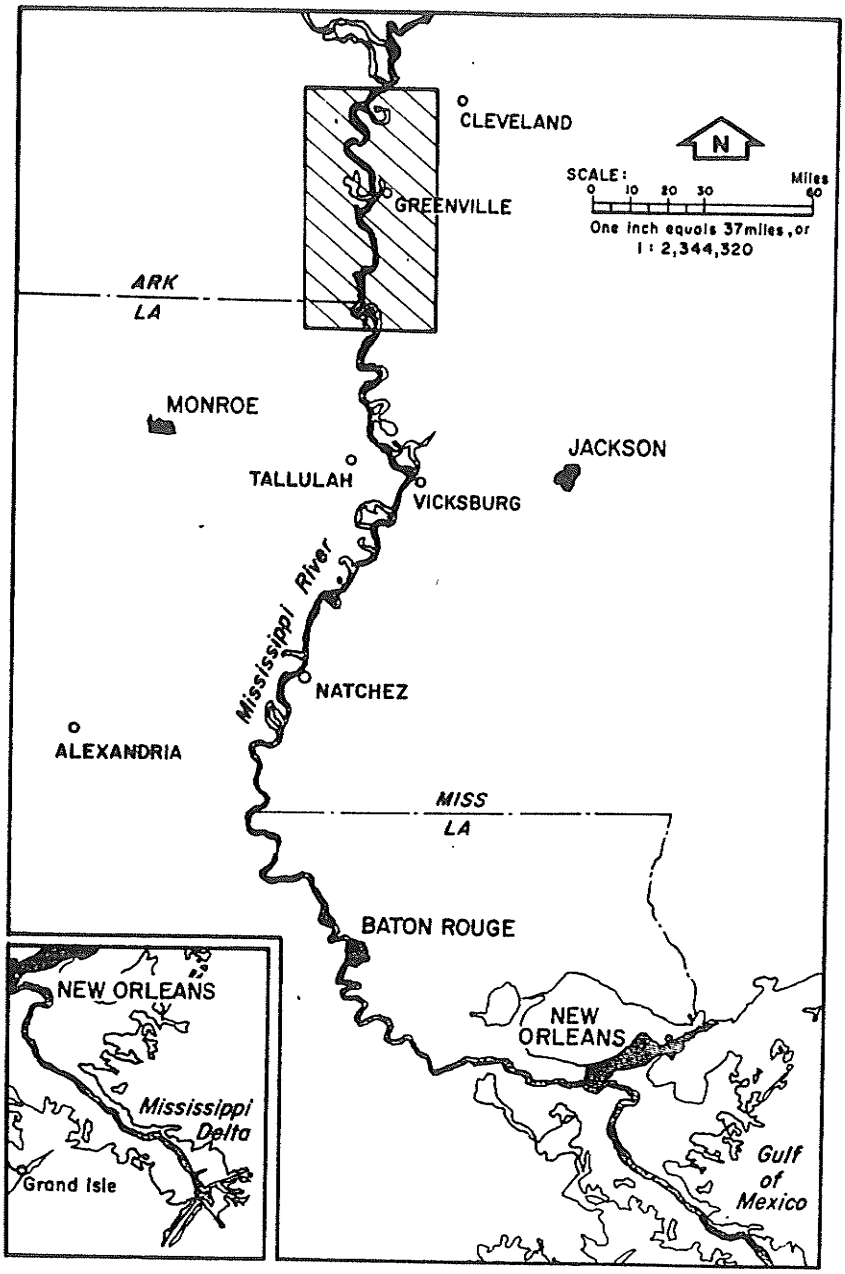


Figure 2.5 Mississippi River; Location of Study Reach (Tamburi et al., 1983)

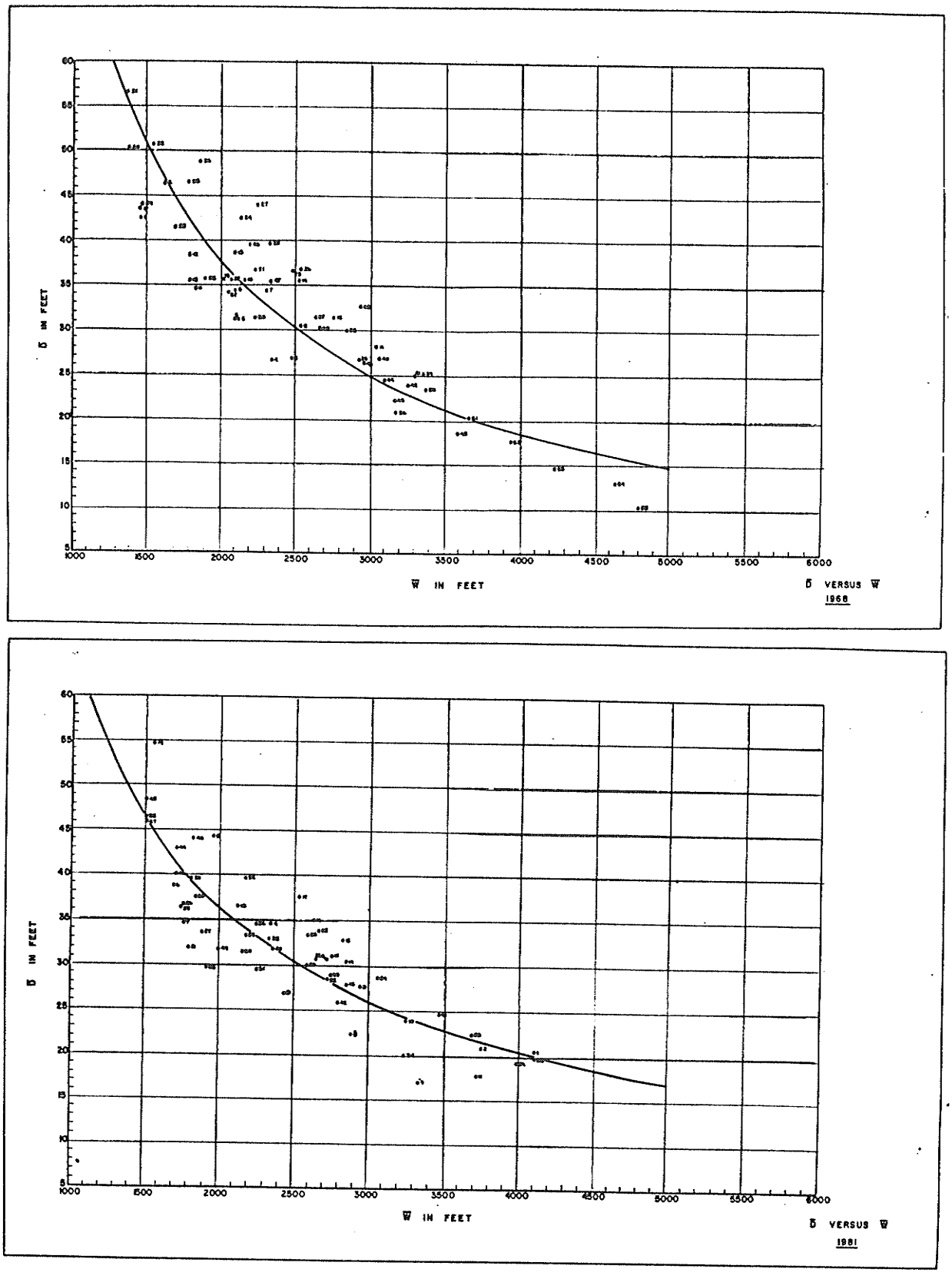


Figure 2.6 Mississippi River; Width:Depth Relationships (Shewfelt et al., 1981)

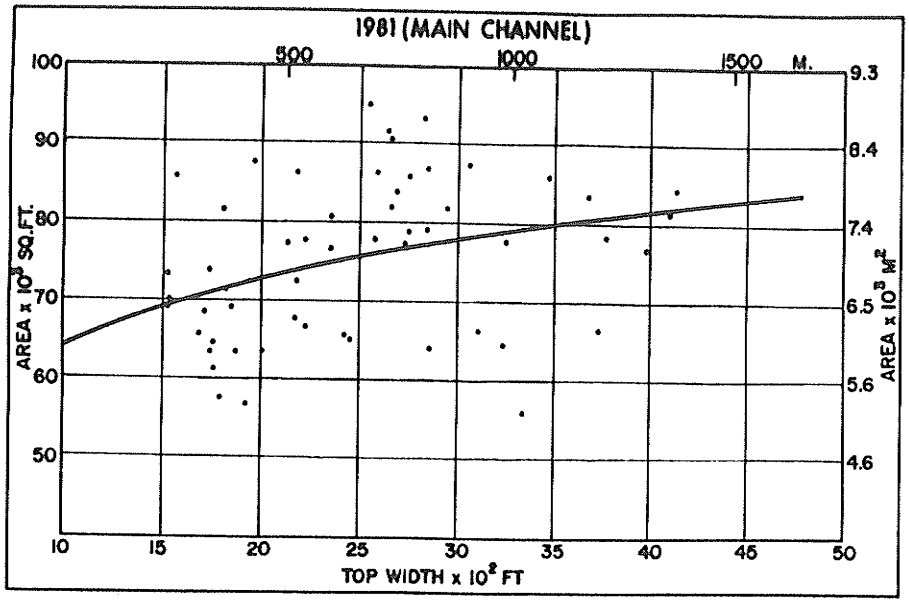


Figure 2.7 Mississippi River; 1981 Width:Area Relationship (Tamburi et al., 1983).

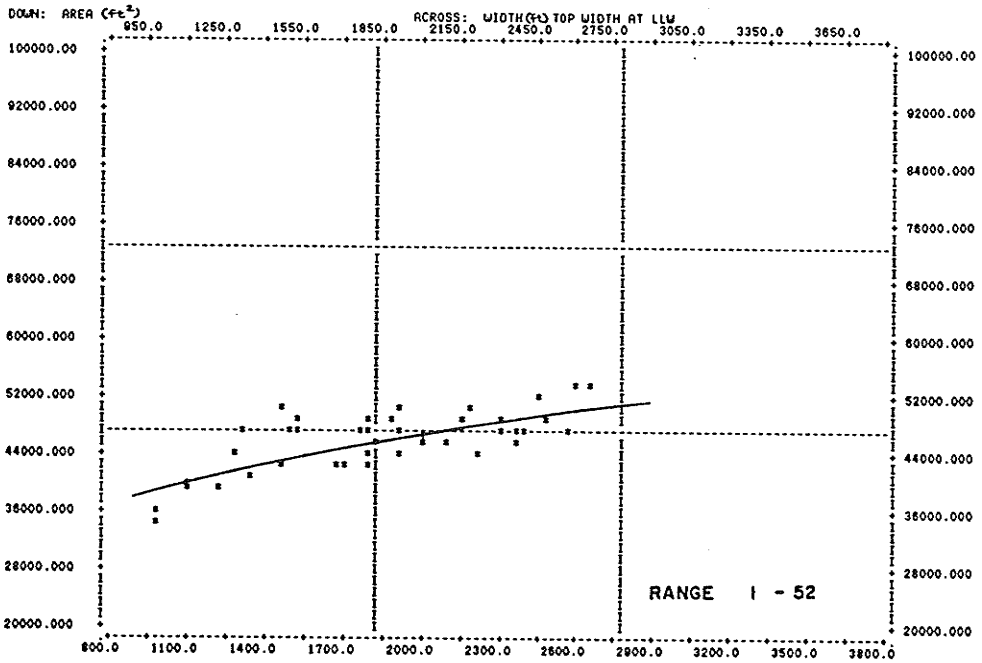
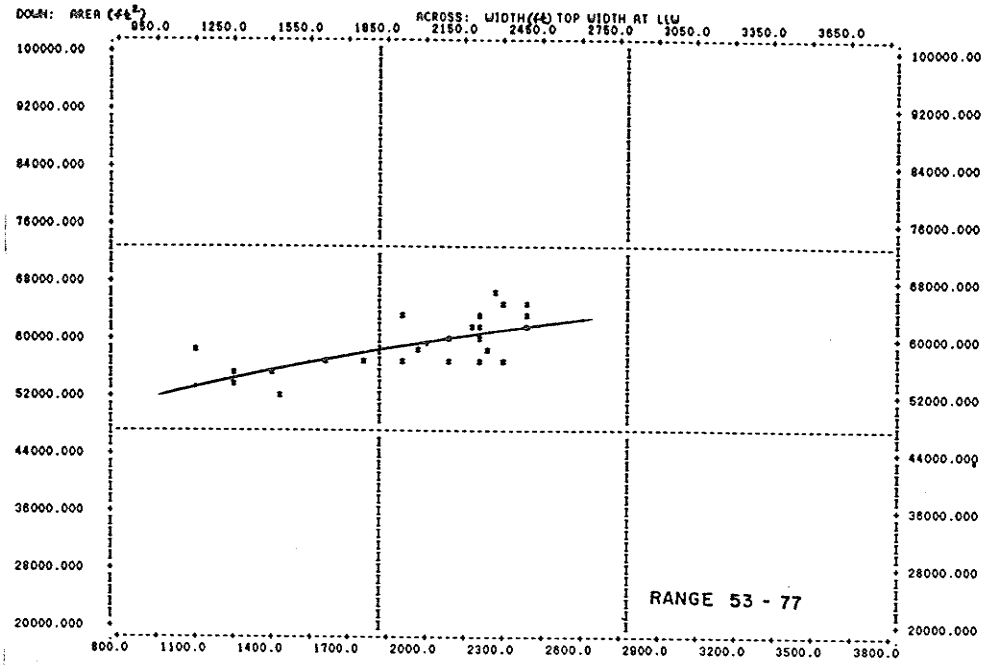


Figure 2.8 Lower Fraser River; Width:Area Relationships



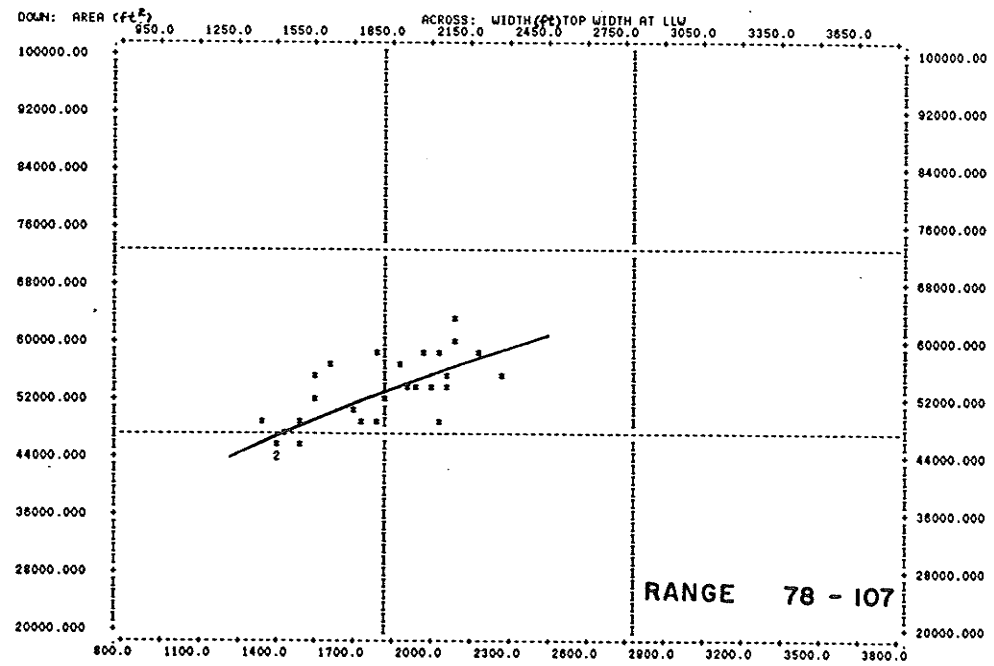


Figure 2.9 Lower Fraser River; Width:Depth Relationships (WCHL, 1978)

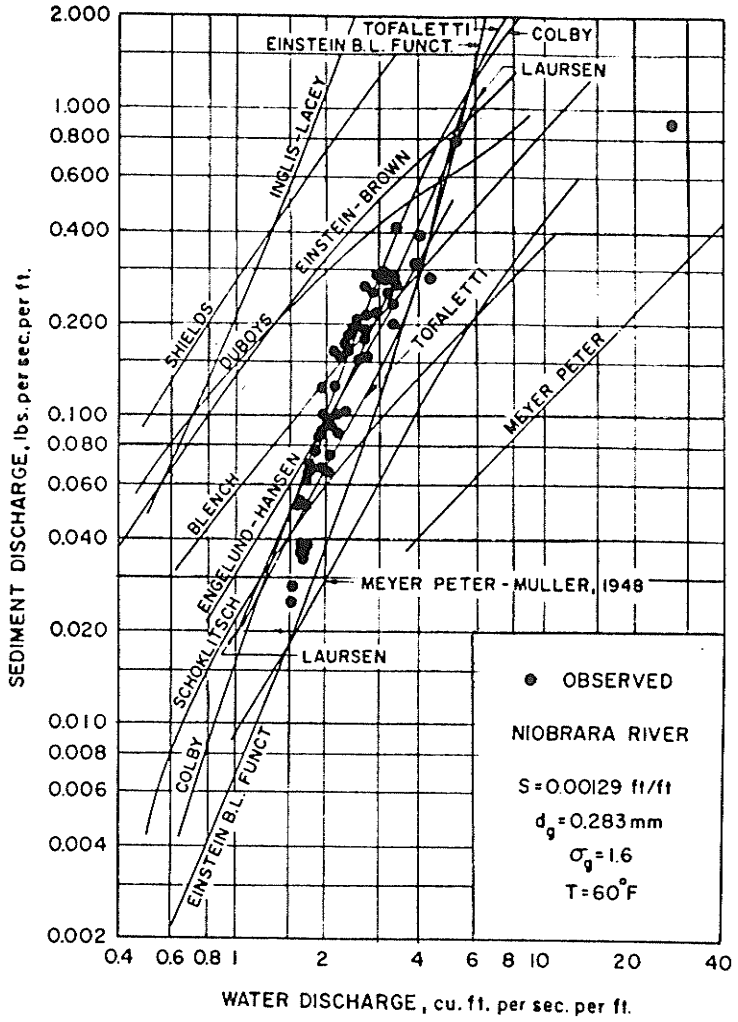


Figure 2.10 Observed versus Computed Sediment Discharge, Niobrara River near Cody, Nebraska (ASCE, 1975)

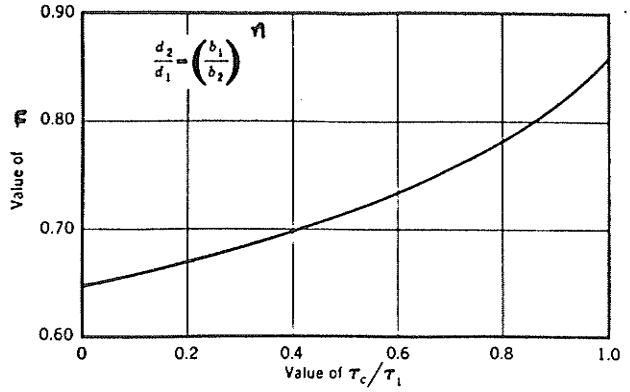


Figure 3.1 Variation of Exponent n with Relative Transport Rate (ASCE, 1975)

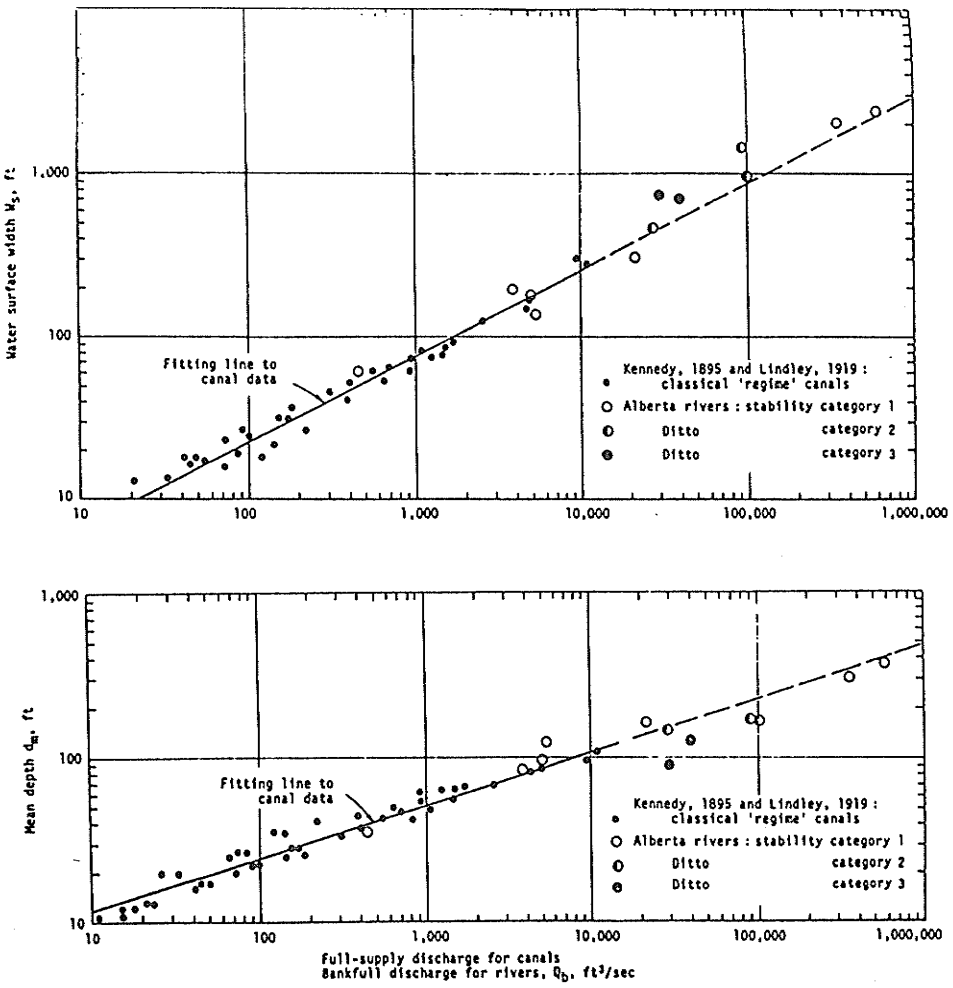


Figure 3.2 Hydraulic Geometry versus Channel Forming Discharge for Sand-Bed Canals and Rivers (Blench, 1986; Neill, 1973)

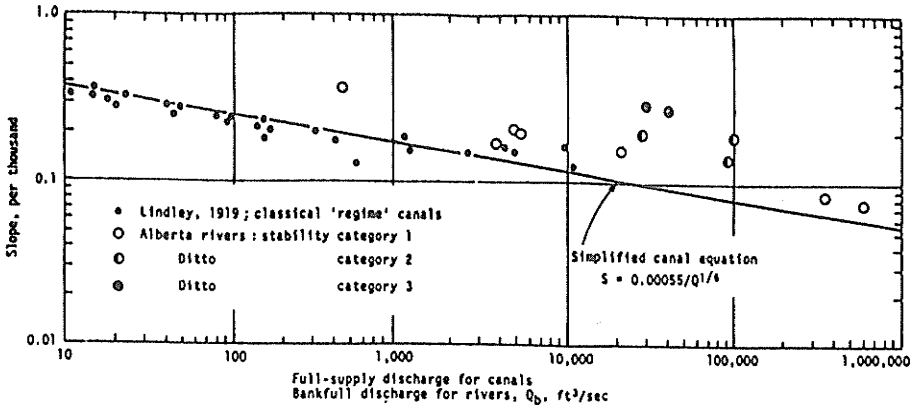


Figure 3.2 Hydraulic Geometry versus Channel Forming Discharge for Sand-Bed Canals and Rivers (Blench, 1986; Neill, 1973)

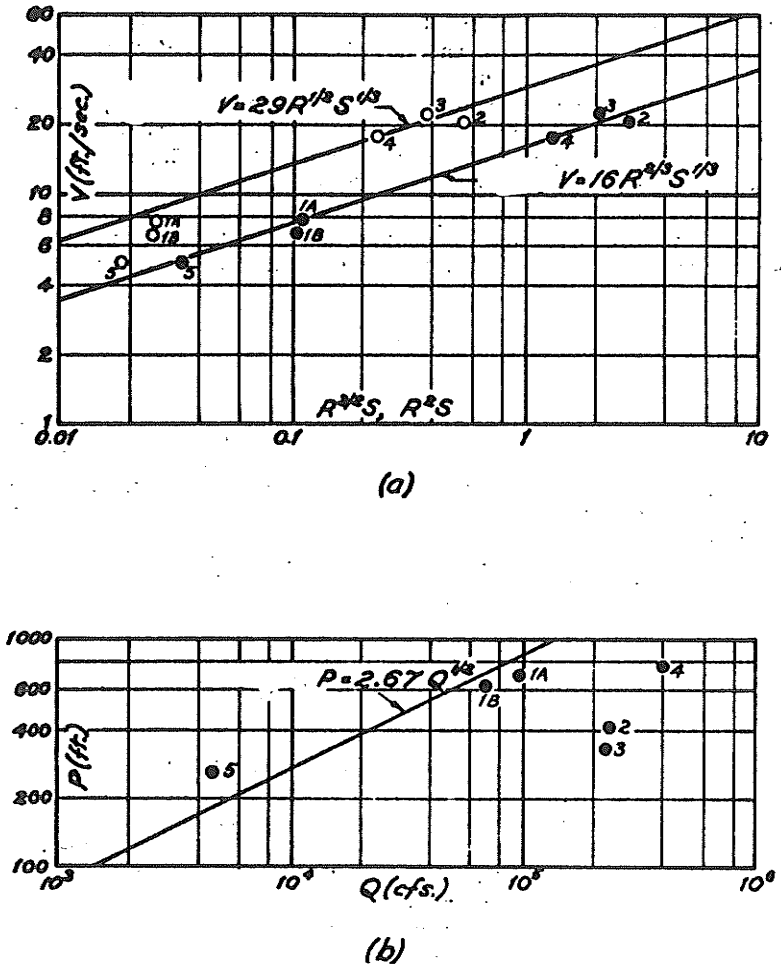


Figure 3.3 Henderson's (1963) New Zealand River Data

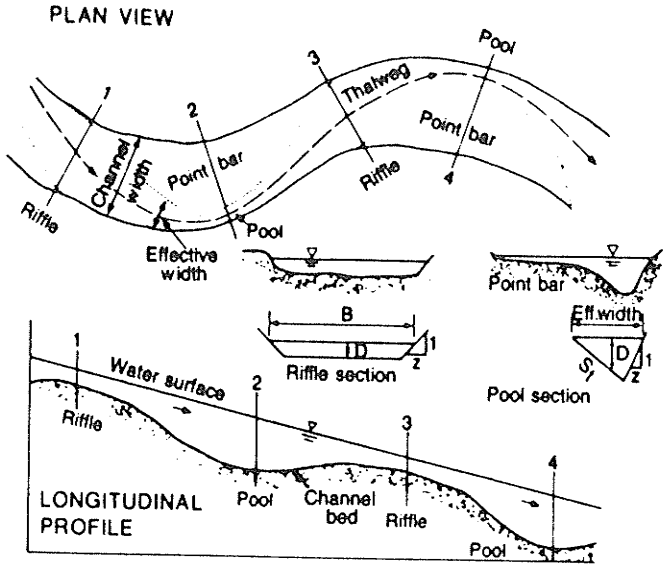


Figure 3.4 Riffle-Pool Sequence (Chang, 1985c)

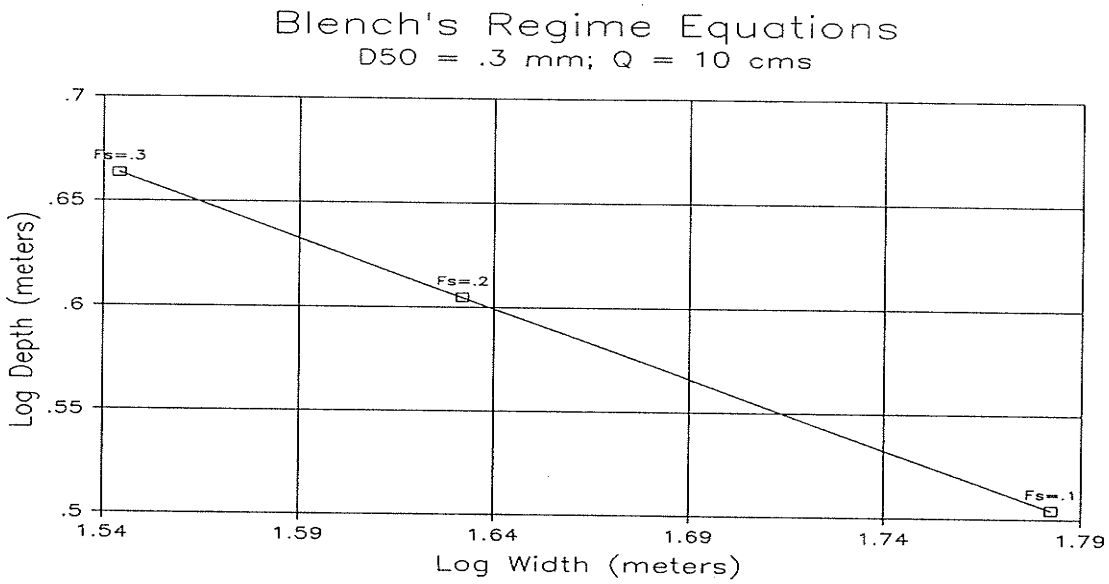


Figure 3.5 Width:Depth Plot using Blench's Equations (3.13) to (3.17)

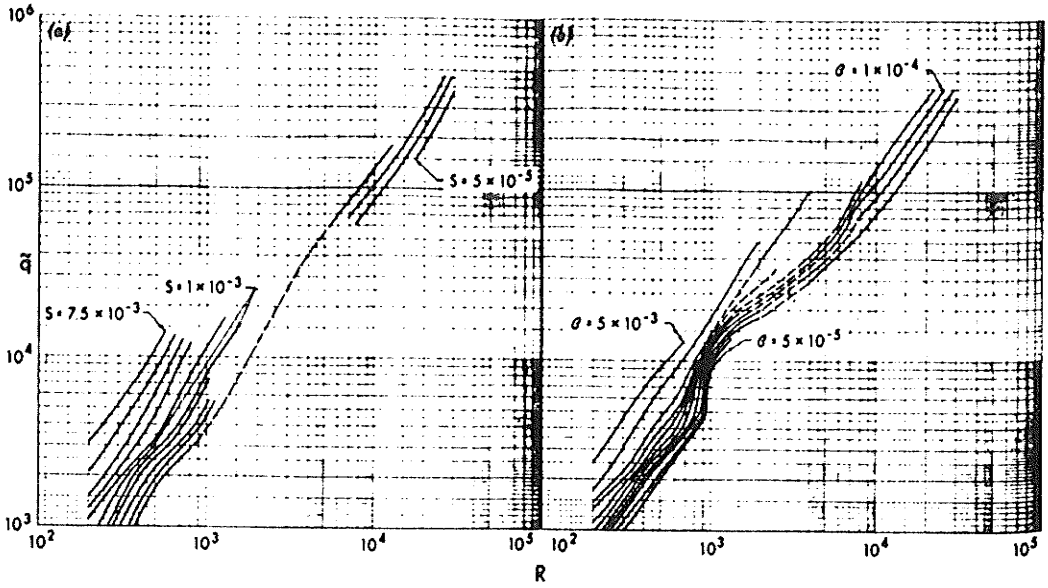


Figure 3.6 Parker's (1977) Empirical q~ versus R Relations for: (a) Constant Slope; (b) Constant Concentration [$20 < R_p < 40$]

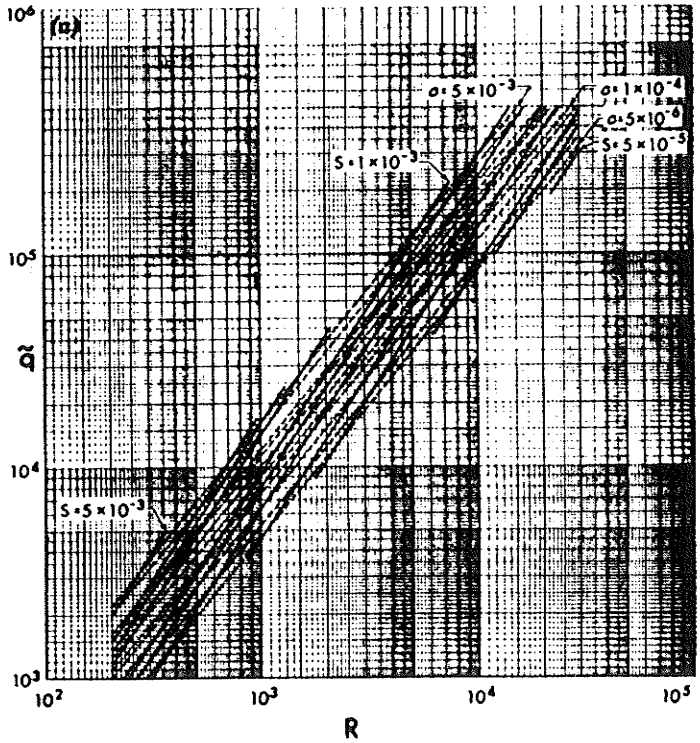


Figure 3.7 Peterson's (1975) Empirical q~ versus R Relations [0.3 mm $< D_s <$ 0.4 mm] (Parker and Anderson, 1977)

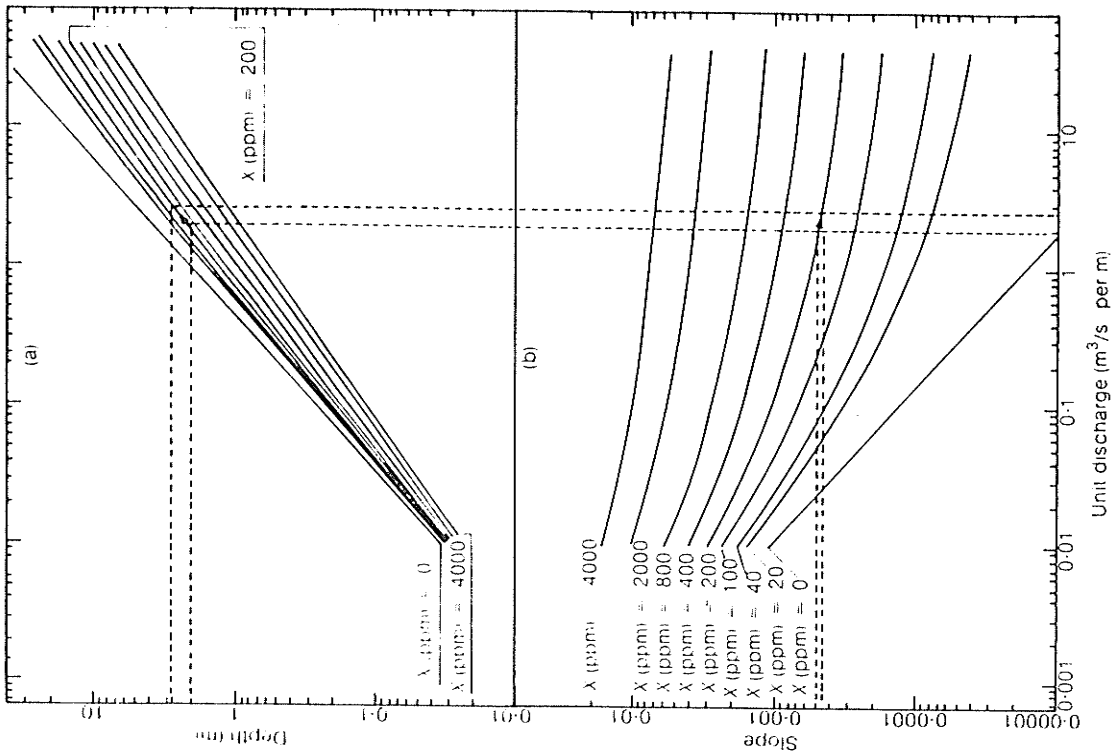


Figure 3.8 Paris's (1984) Analytical q versus d and S Relations; D_{35} = 0.5 mm

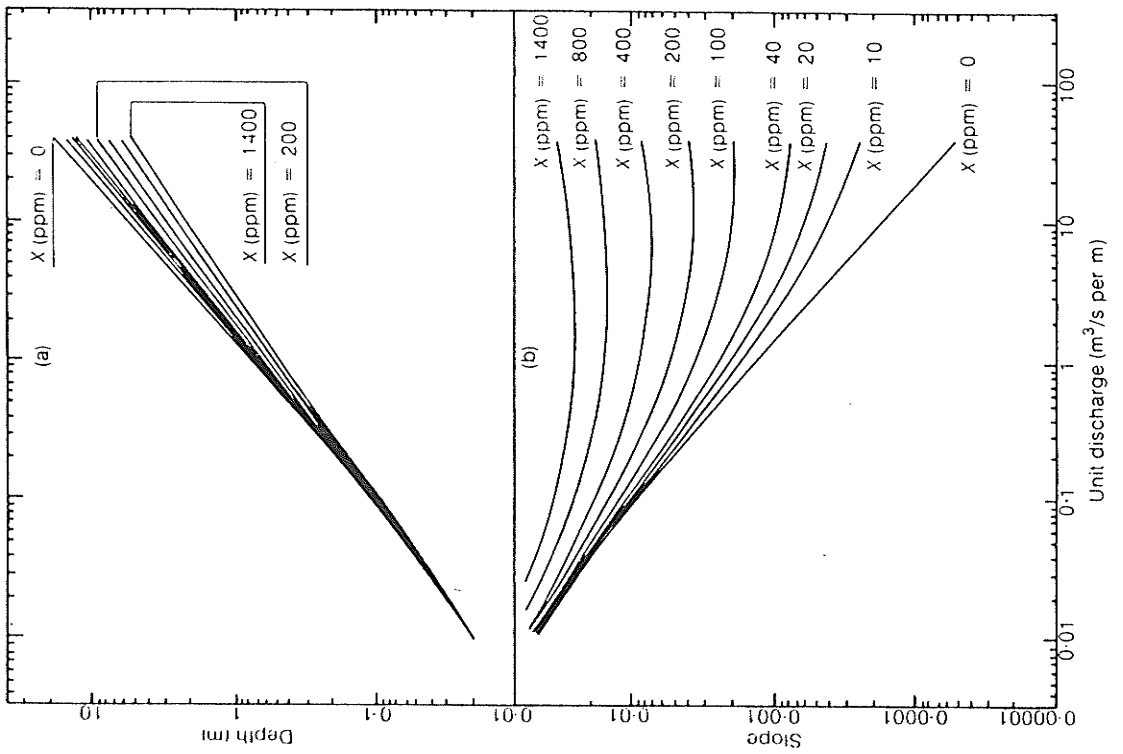


Figure 3.9 Paris's (1984) Analytical q versus d and S Relations; D_{35} = 20 mm

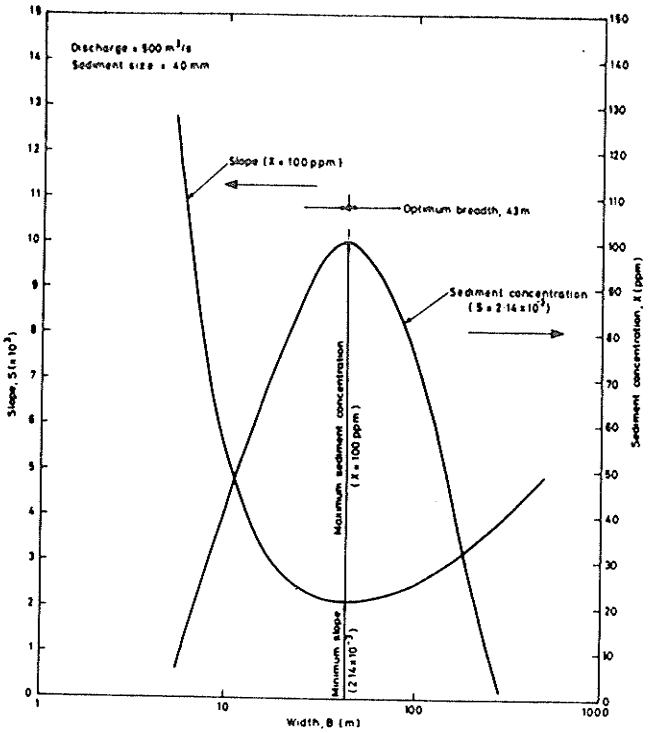


Figure 3.10 White et al. (1984) Analytical Solution for Optimal Width; D_s = 40 mm and Q = 500 m^3/s

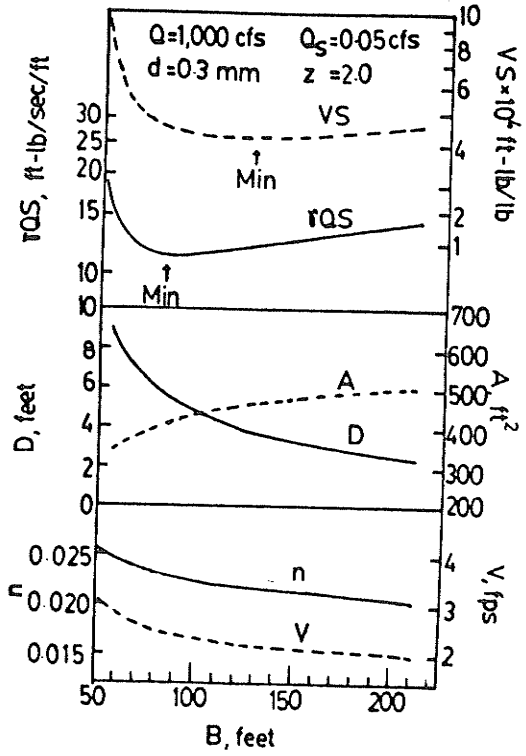


Figure 3.11 Chang's (1978) Relations between Hydraulic Parameters and Channel Width for Specified Water and Sediment Discharge

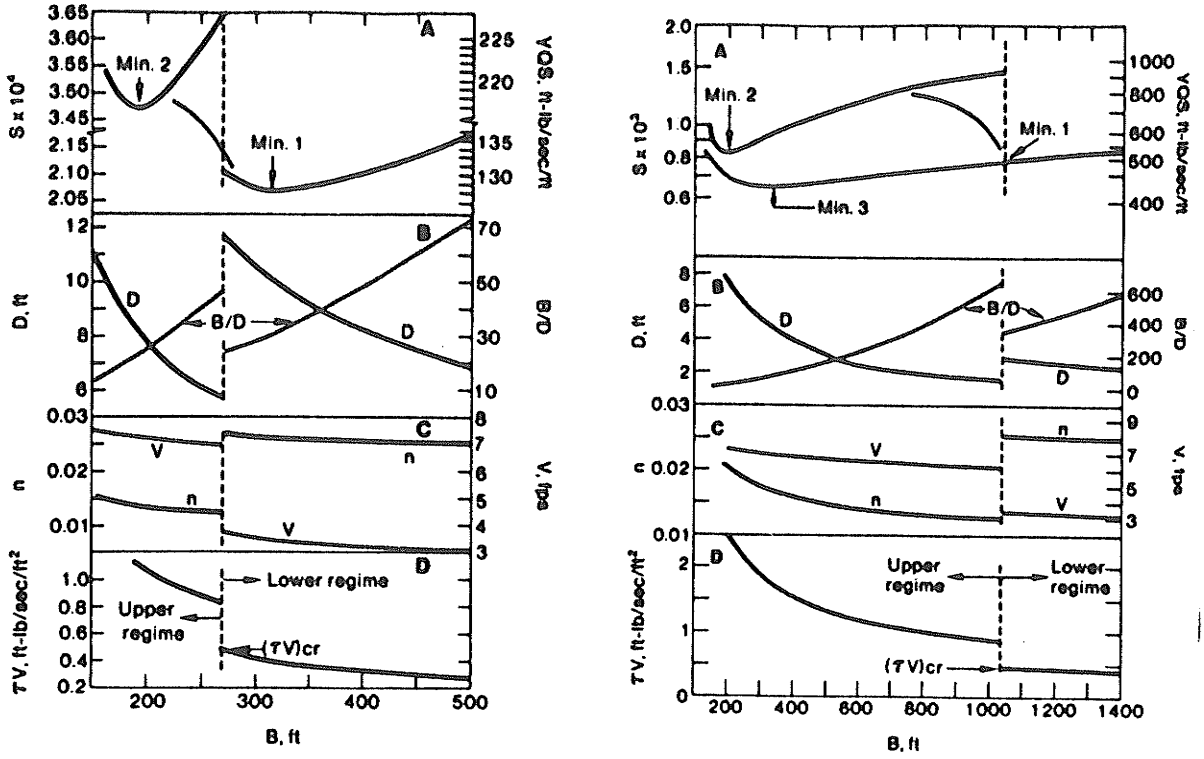


Figure 3.12 Chang's (1979) Relations between Hydraulic Parameters and Channel Width

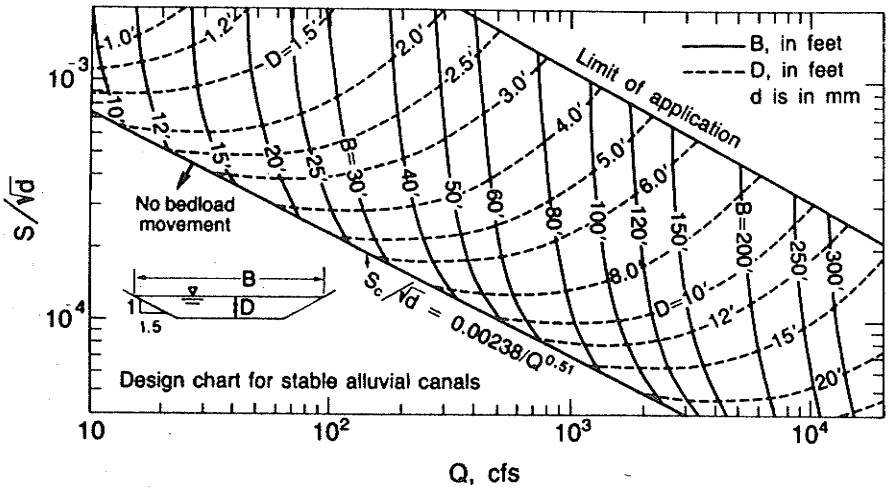


Figure 3.13 Chang's (1985b) Regime Channel Geometry for Stable Alluvial Canals with Sand Beds

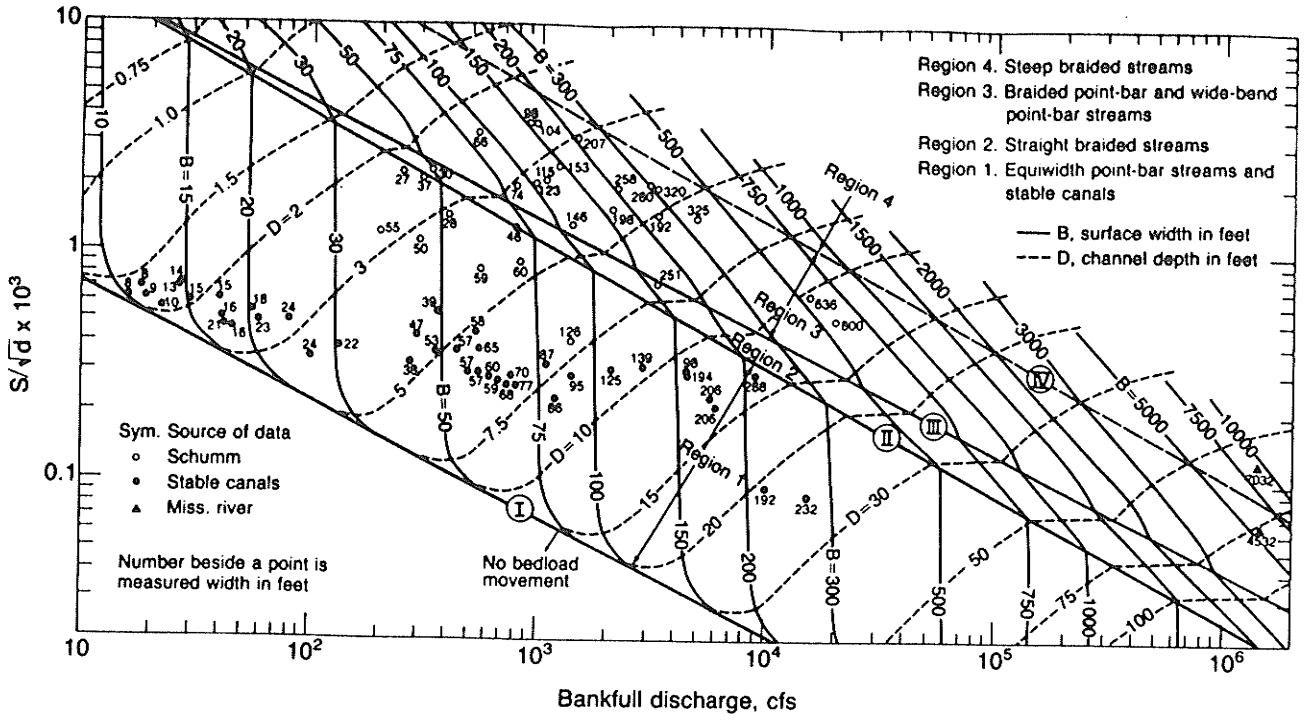


Figure 3.14 Chang's (1985c) Regime Channel Geometry for Sand-Bed Rivers

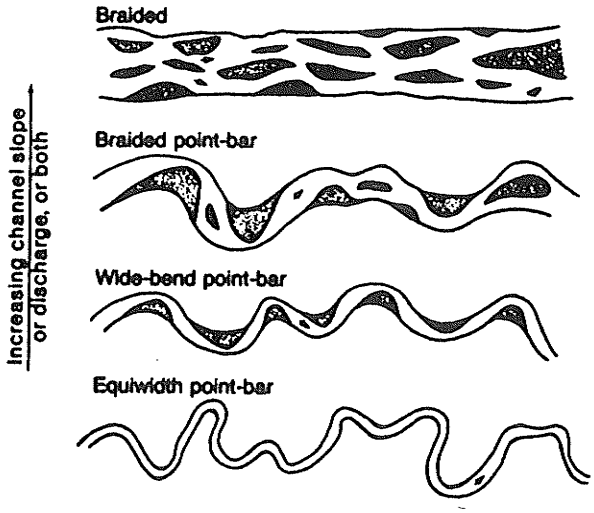
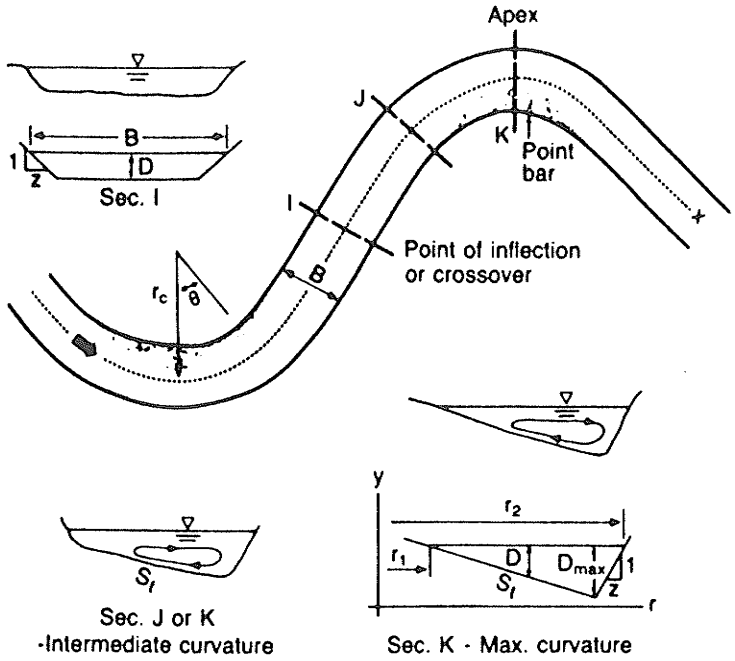


Figure 3.15 Chang's (1985c) Classification of Channel Types



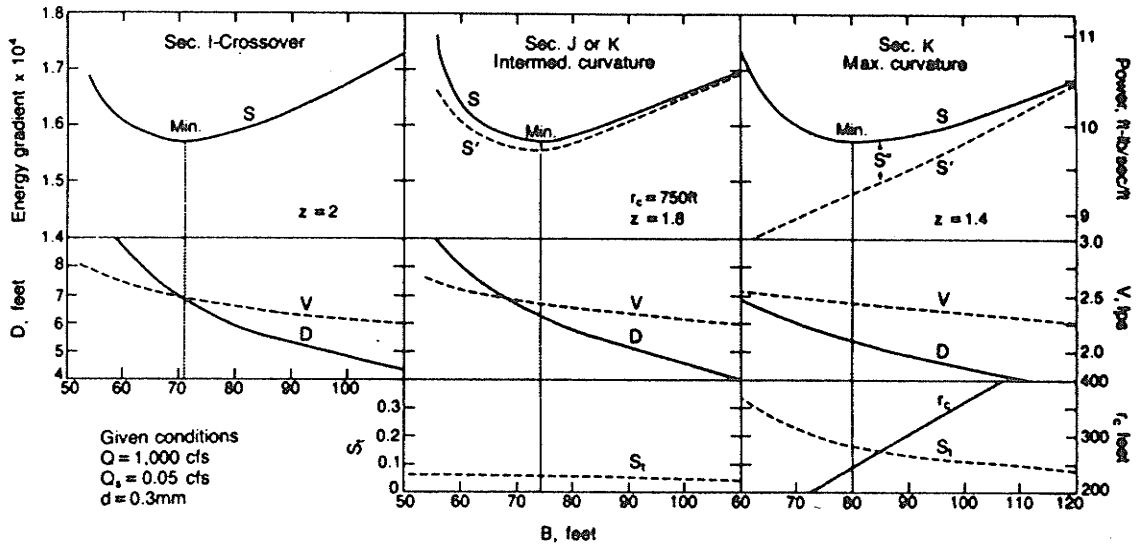
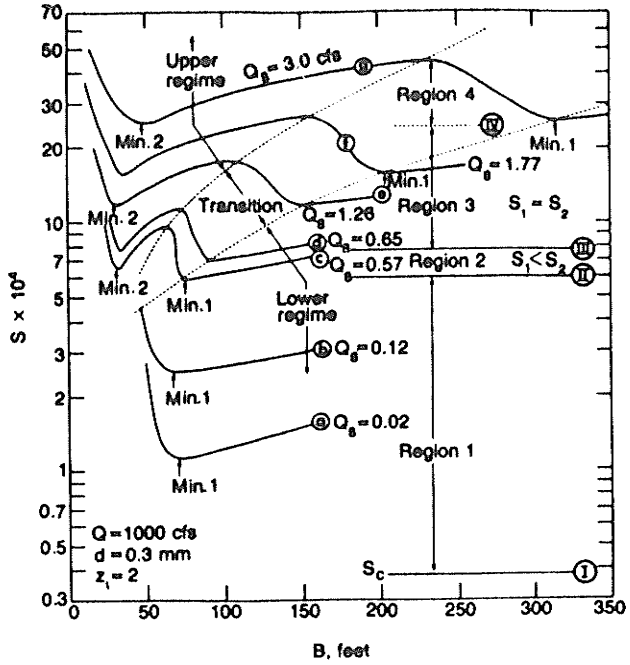


Figure 3.16 Chang's (1984a) Relations between Hydraulic Parameters and Channel Width and Channel Curvature



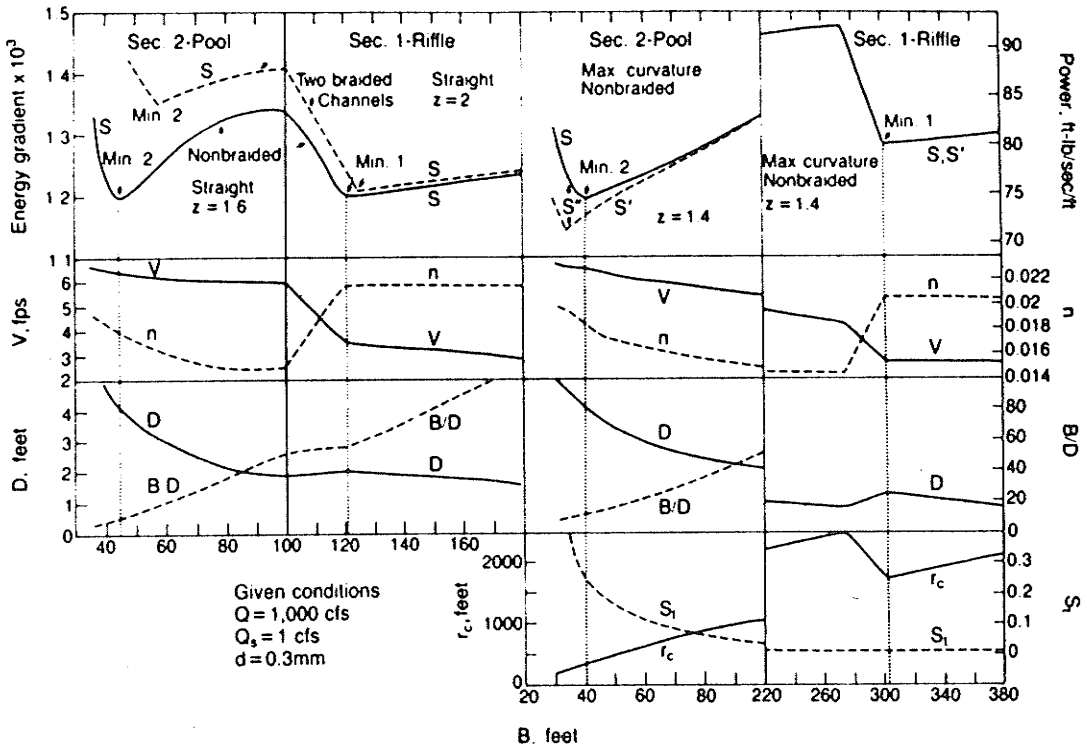


Figure 3.17 Chang's (1985c) Relations between Hydraulic Parameters and Channel Width, Curvature and Bank Slope

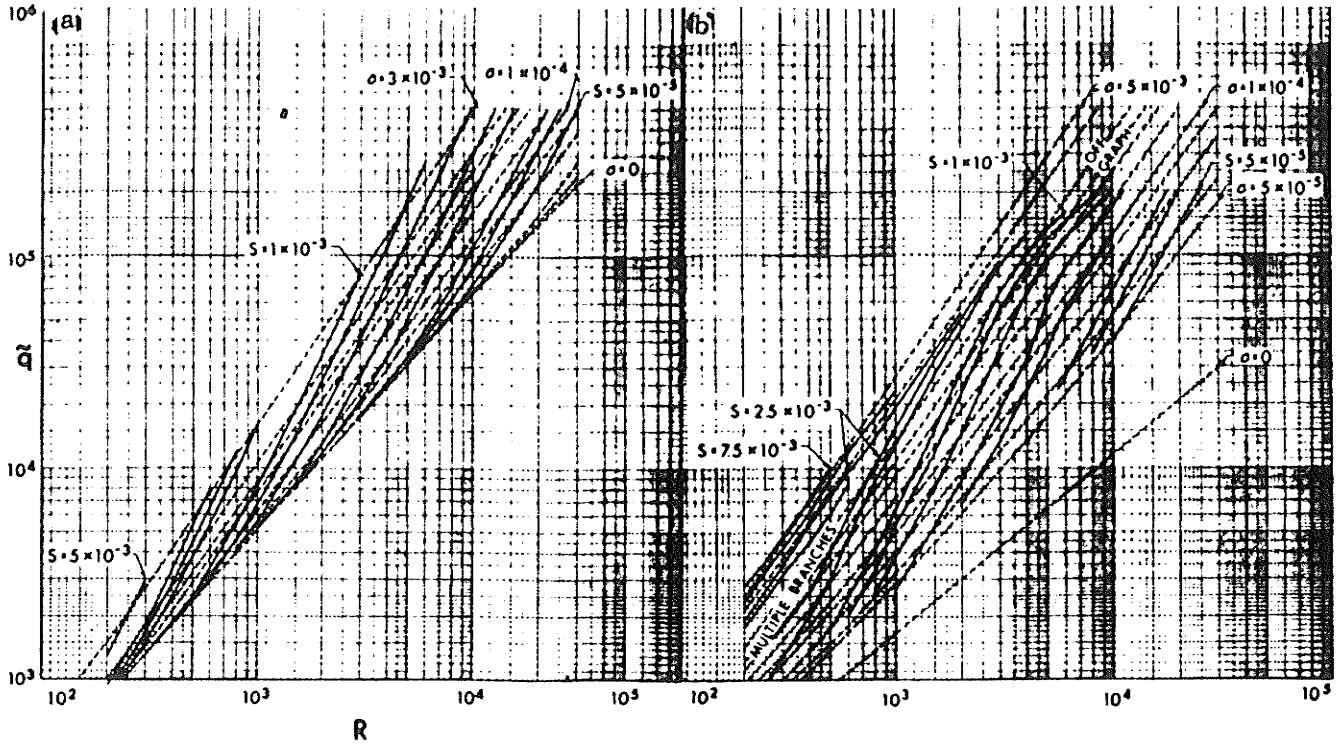


Figure 3.18 Analytical Hydraulic Geometry Relations; (a) Engelund's (1967) Resistance and Sediment Transport (b) Alam et al. (1969) Resistance and Rottner's (1959) Sediment Transport [Parker and Anderson, 1977]

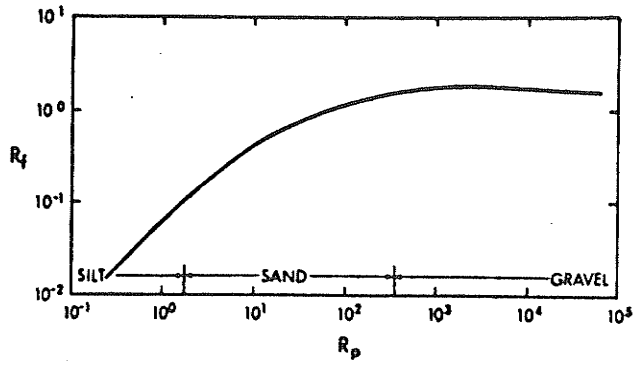


Figure 3.19 Dimensionless Fall Velocity R_f as a Function of Particle Reynolds Number R_p (Parker, 1977)

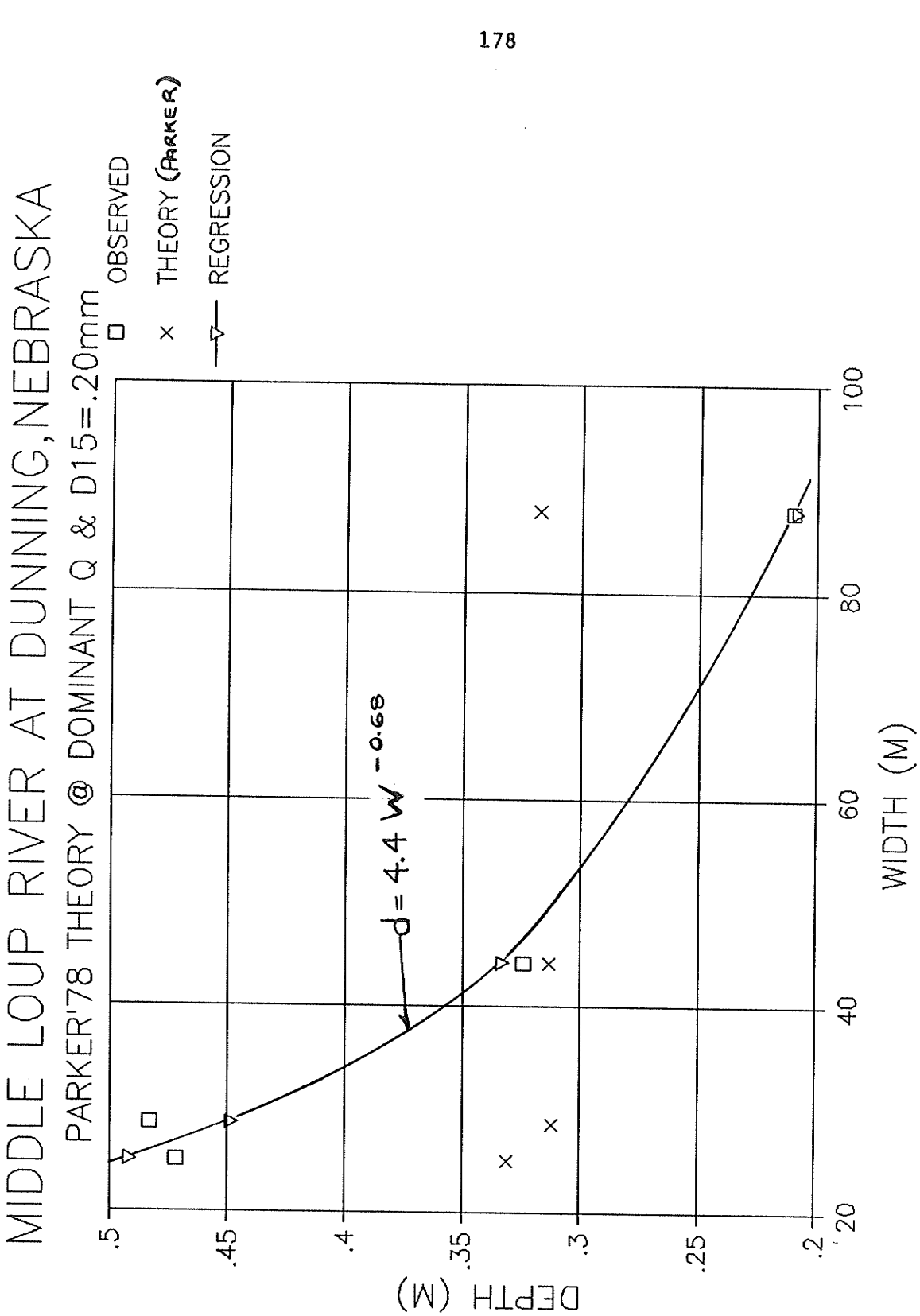


Figure 3.20 Middle Loup River at Dunning, Nebr.; Observed Width:Depth Relations versus Parker's (1978a) Theory.

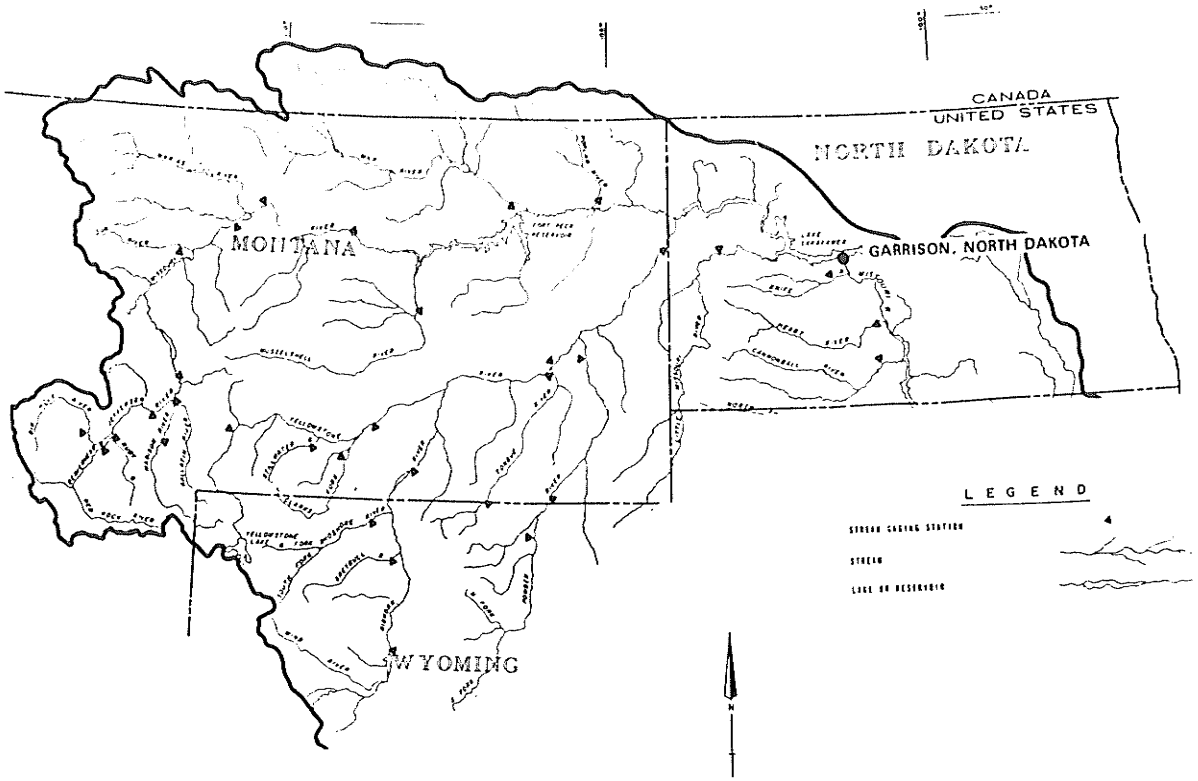


Figure 3.21 Missouri River below Garrison Dam; Study Reach.

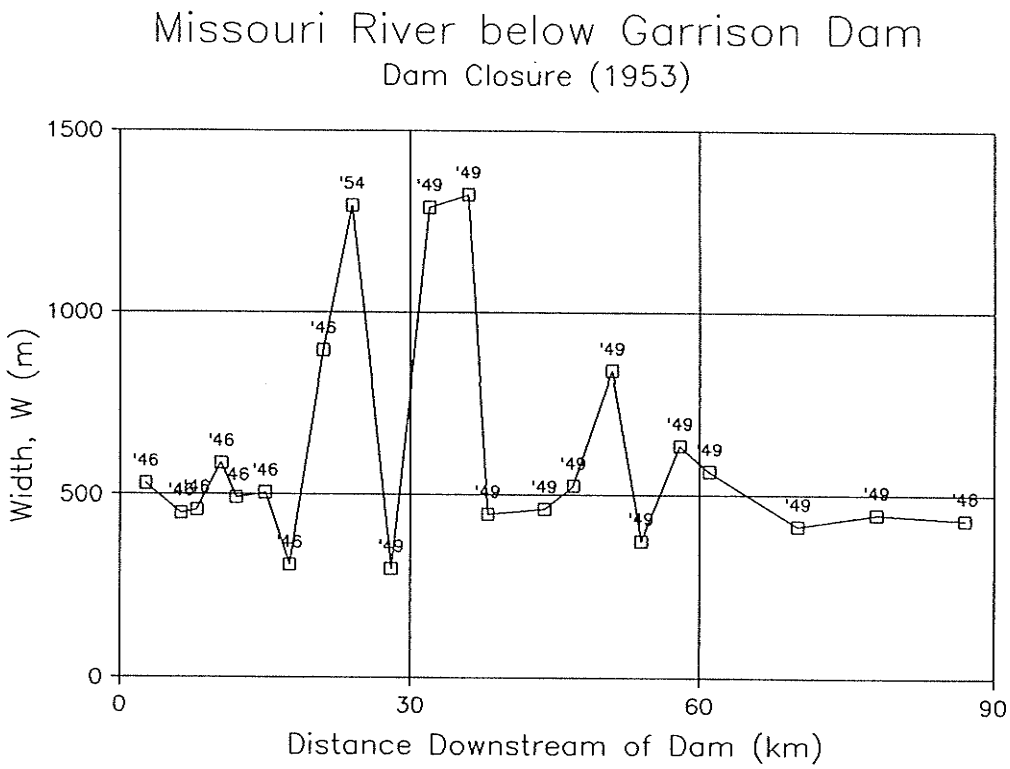


Figure 3.22 Missouri River below Garrison Dam; Variation in Channel Width

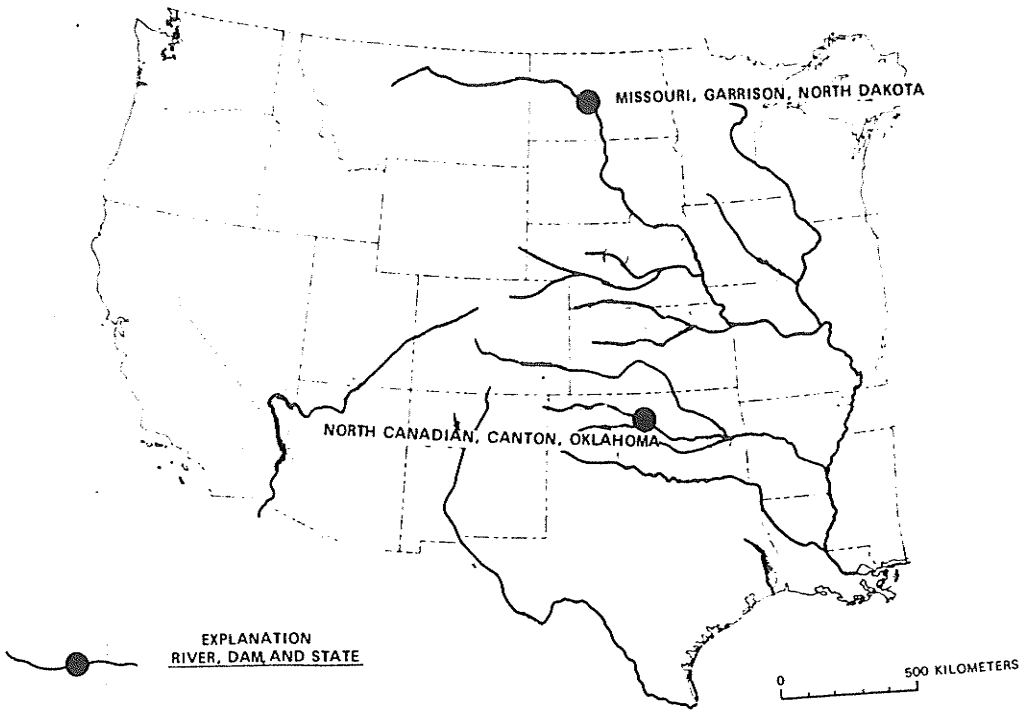


Figure 3.23 North Canadian River below Canton Dam; Study Reach

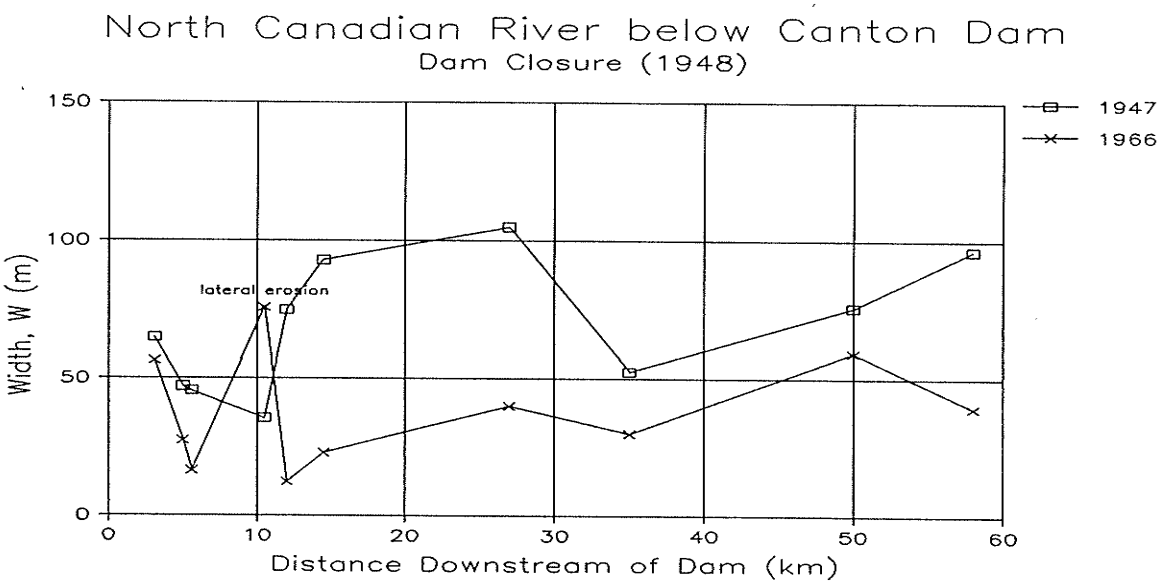


Figure 3.24 North Canadian River below Canton Dam; Variation in Channel Width

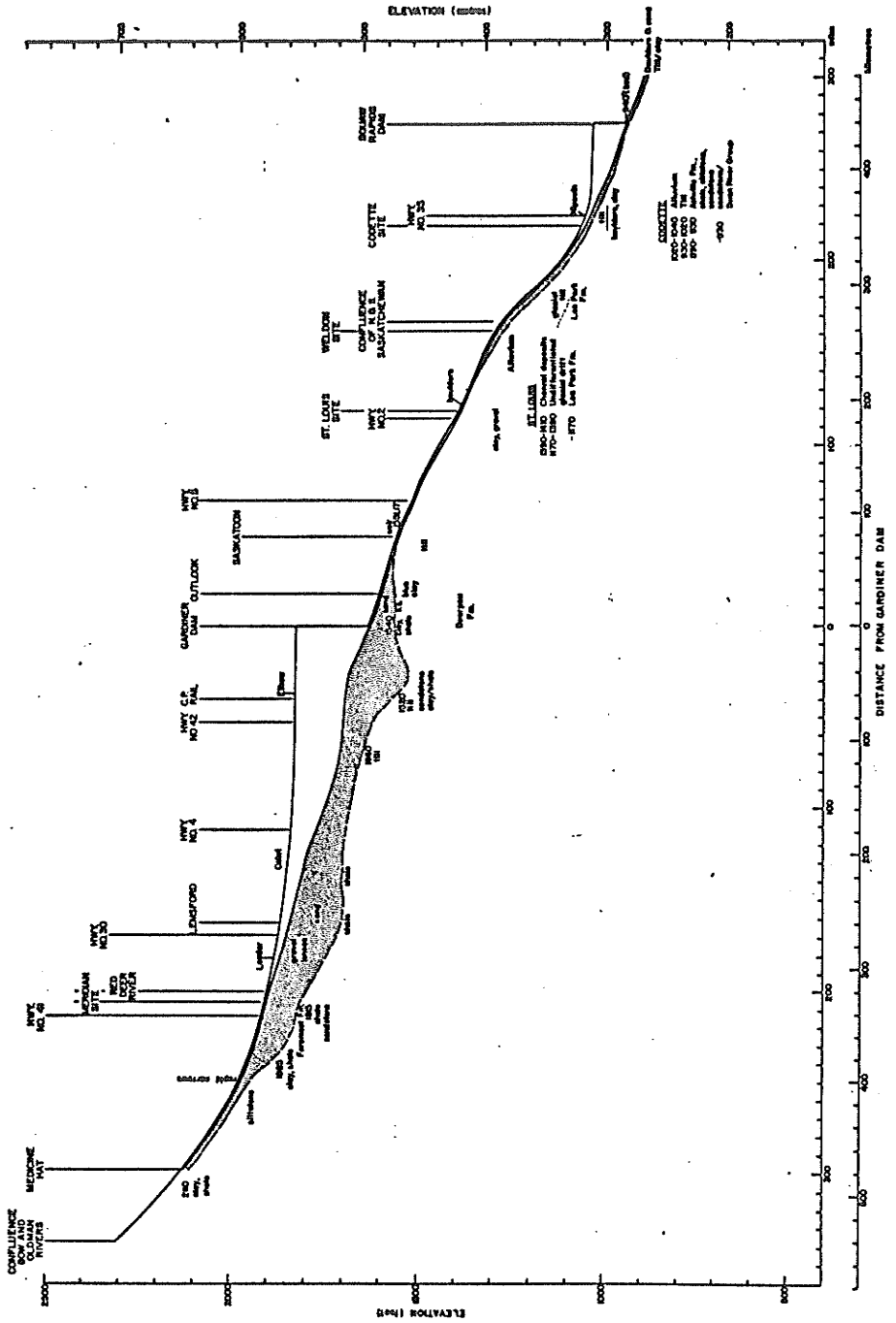


Figure 3.25 South Saskatchewan River; Longitudinal Profile of Sand-Bed Reach (NWHC,1980)

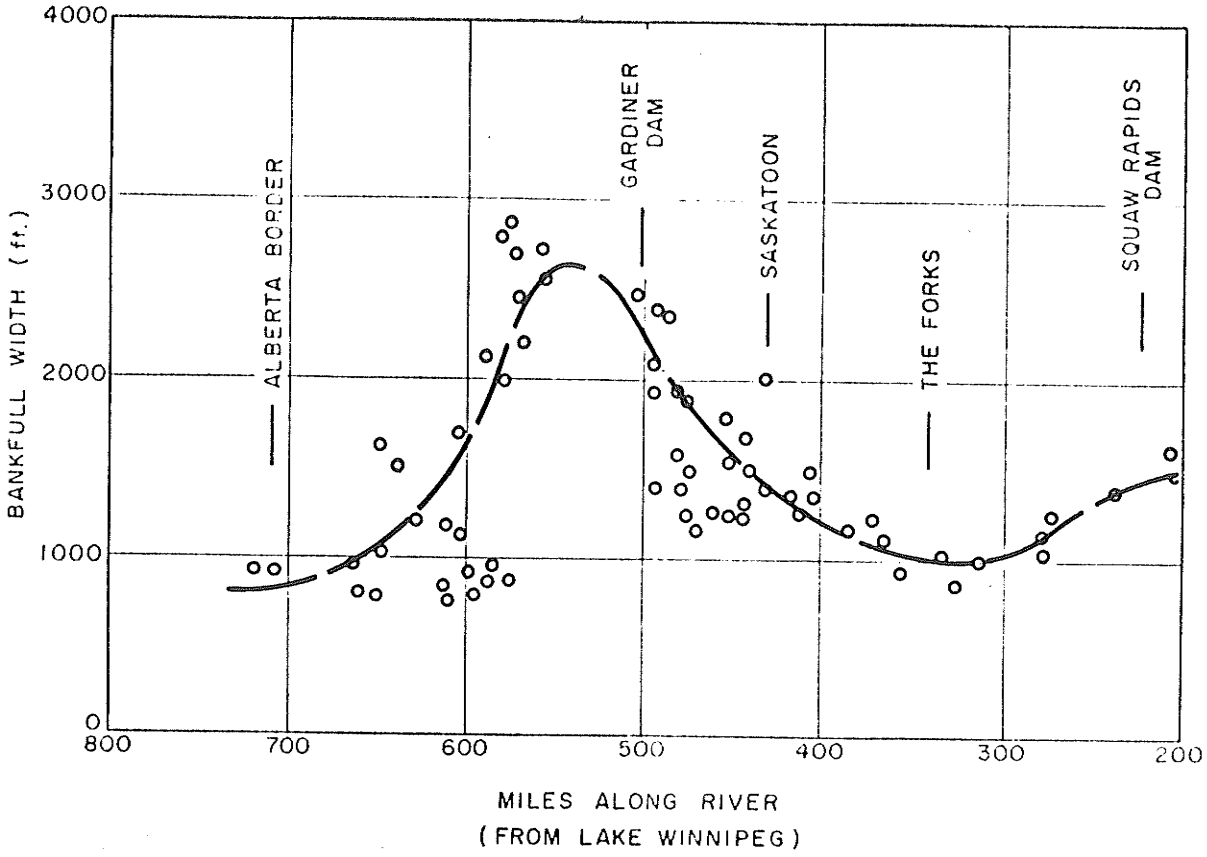


Figure 3.26 South Saskatchewan River; Variation in Bankfull Channel Width (Kuiper and Galay, 1972)

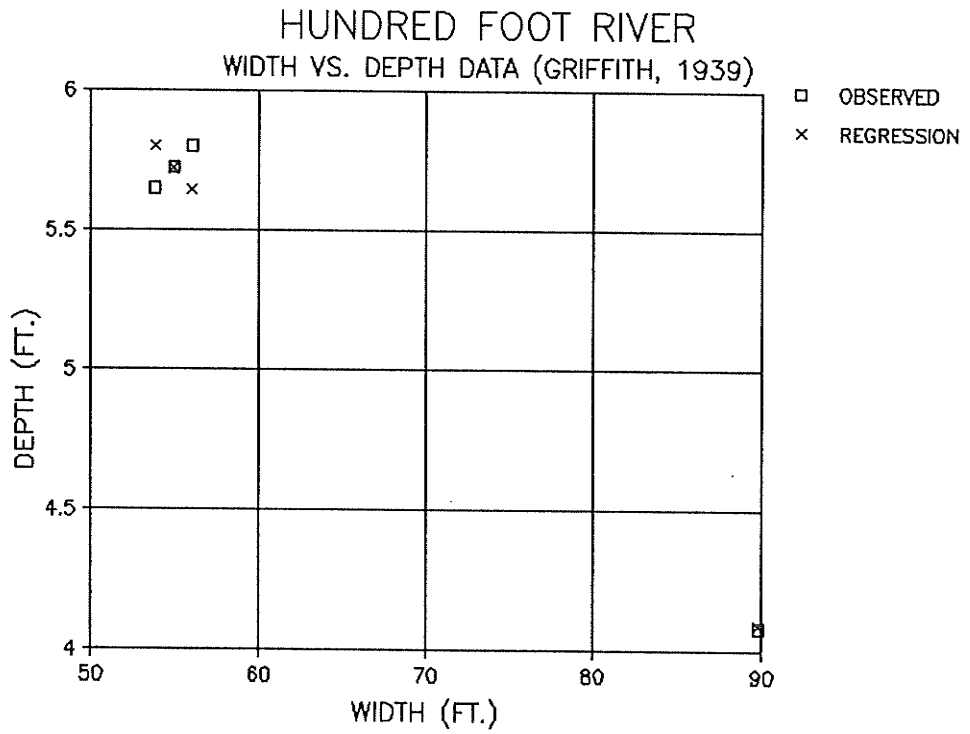


Figure 3.27 Hundred Foot River; Width:Depth Plot of Griffith's (1939) Data

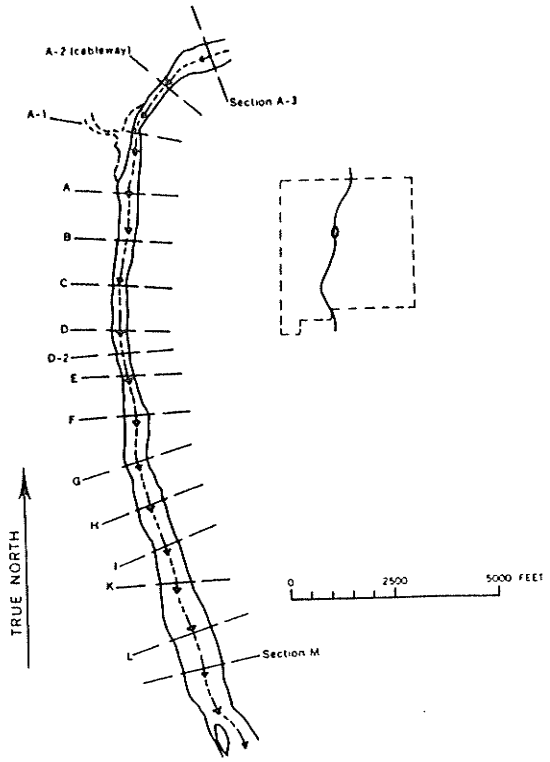


Figure 3.28 Rio Grande River near Bernalillo, New Mexico; Plan View of Study Reach (Nordin, 1964)

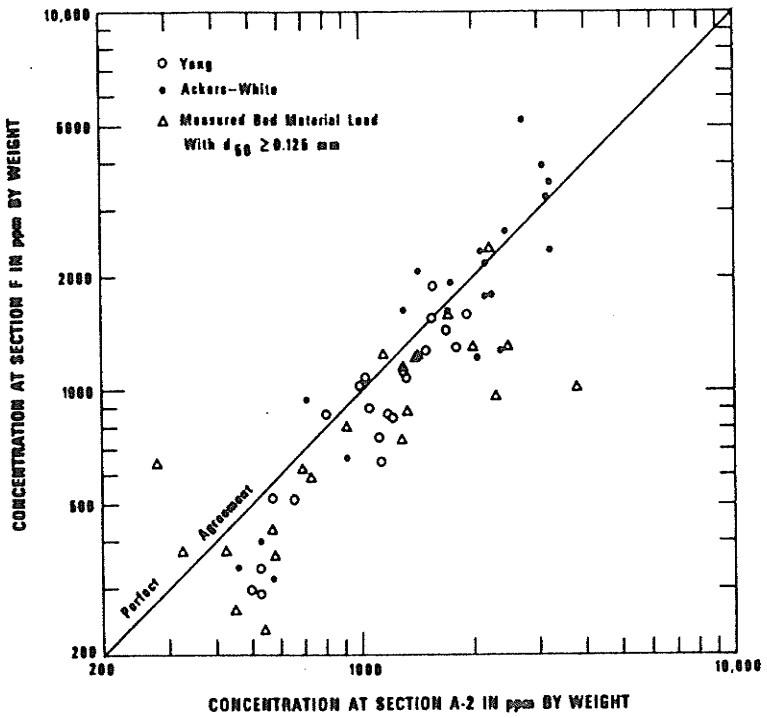


Figure 3.29 Rio Grande River near Bernalillo, N.M.; Comparisons of Measured Bed Material Concentrations and Computed Results by Yang's and Ackers-White Equations at Sections A-2 and F (Yang, 1978b)

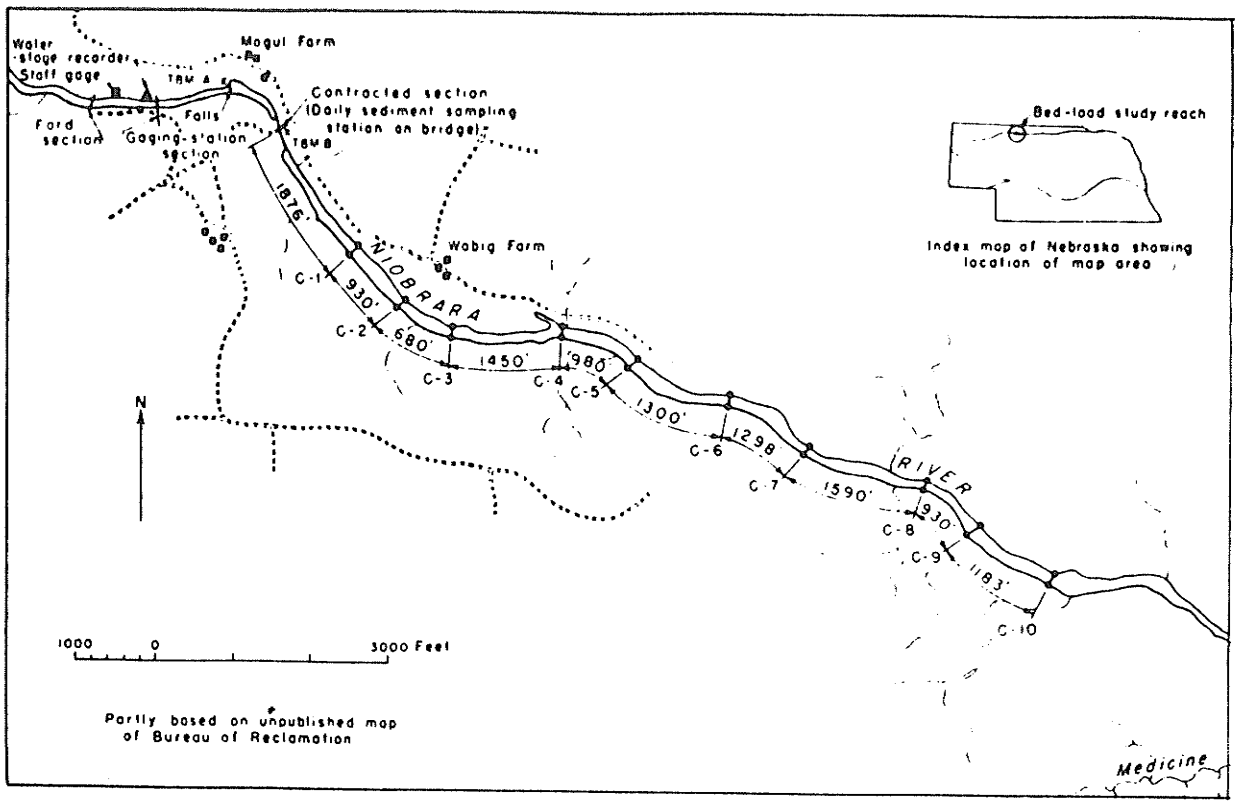


Figure 3.30 Niobrara River near Cody, Nebraska; Study Reach (Colby and Hembree, 1955)

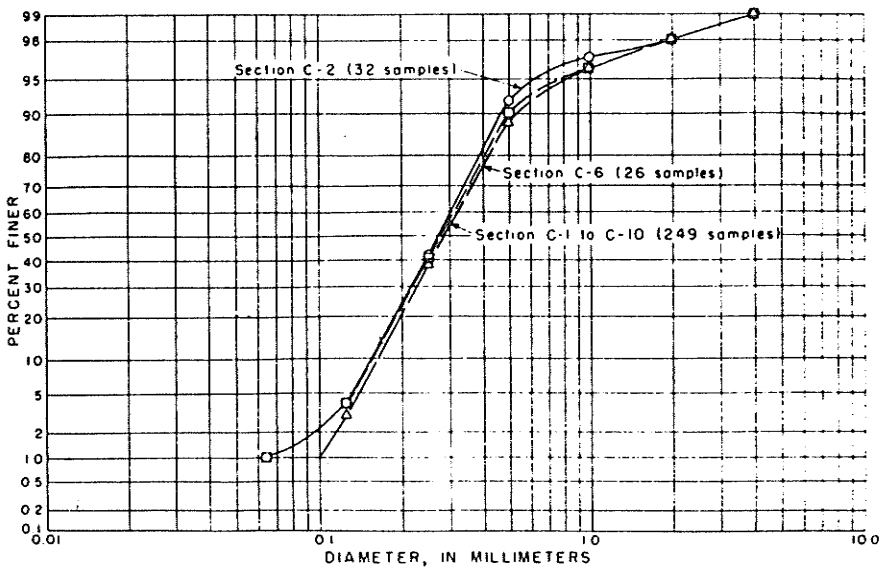


Figure 3.31 Niobrara River near Cody, Nebr.; Computed Average Analyses of Bed Material Samples at Sections C2, C6 and C1-C10 (Colby and Hembree, 1955)

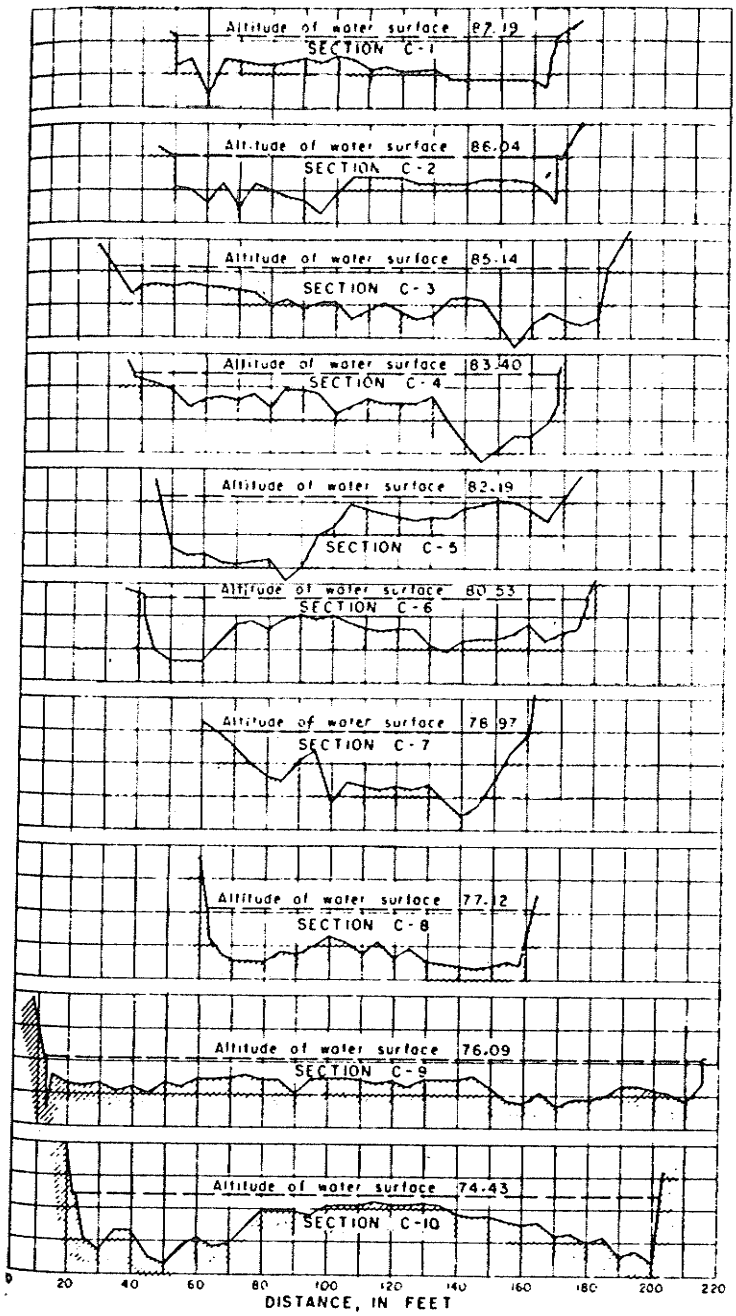


Figure 3.32 Niobrara River near Cody, Nebr.; Variation of Channel Geometry, Sections C1 to C10. [Colby and Hembree, 1955]

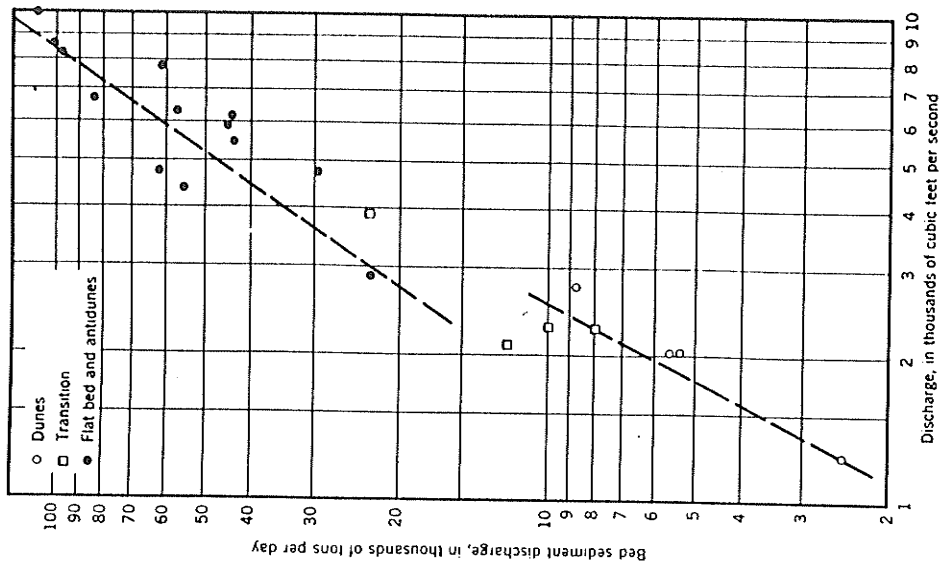


Figure 3.33 Rio Grande River near Bernalillo, N.M.; Sediment Discharge versus Water Discharge at Section F (Nordin, 1964)

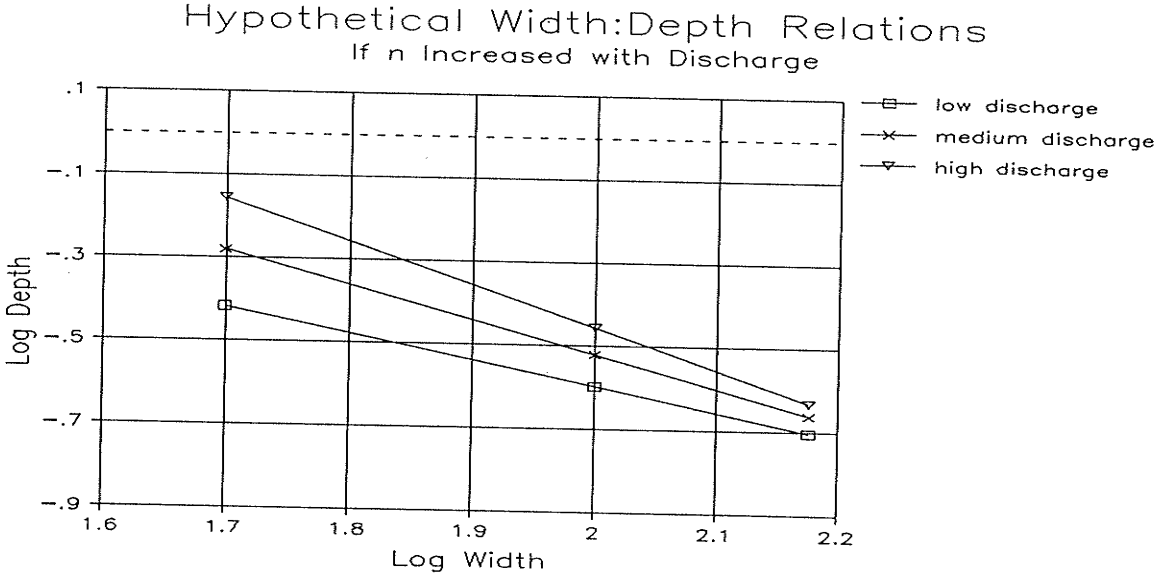


Figure 3.34 Hypothetical Width:Depth Relations if Exponent n Increased with Discharge

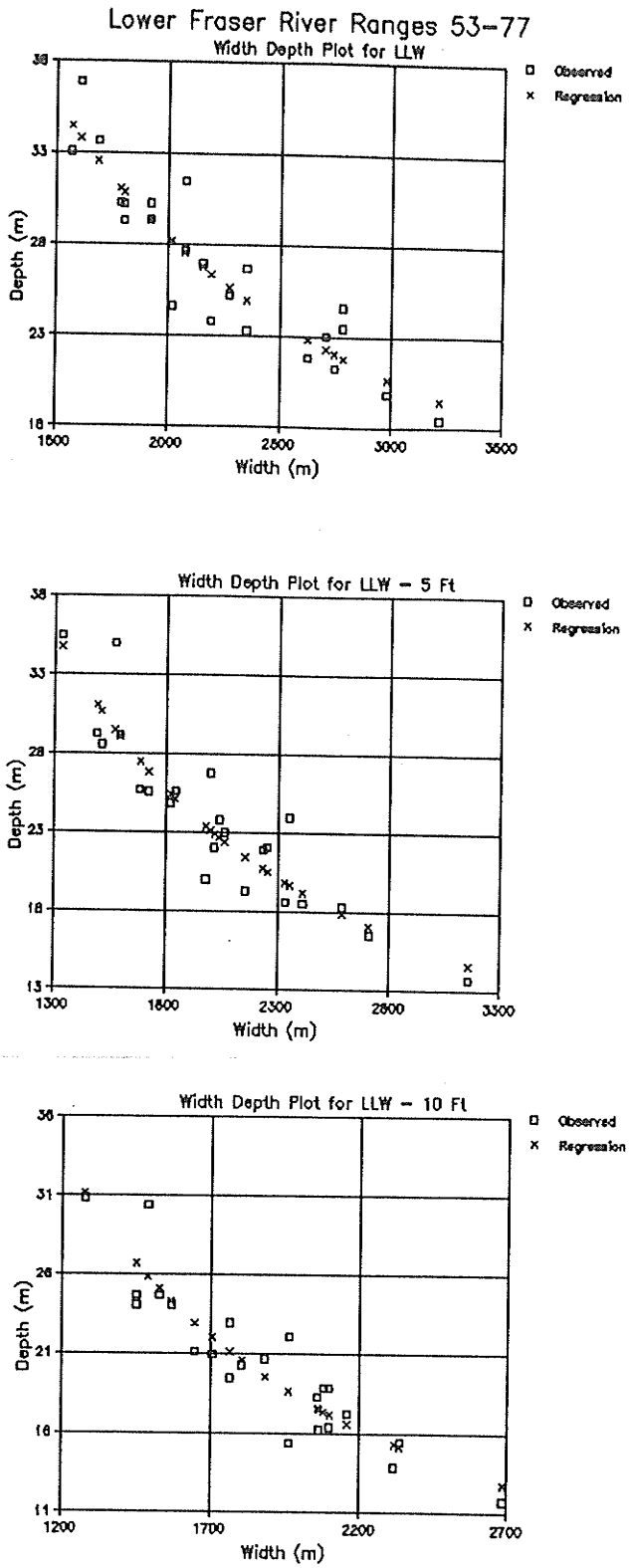


Figure 3.35 Lower Fraser River; Width:Depth Plots Ranges 53-77, for LLW, LLW - 5 feet, LLW - 10 feet

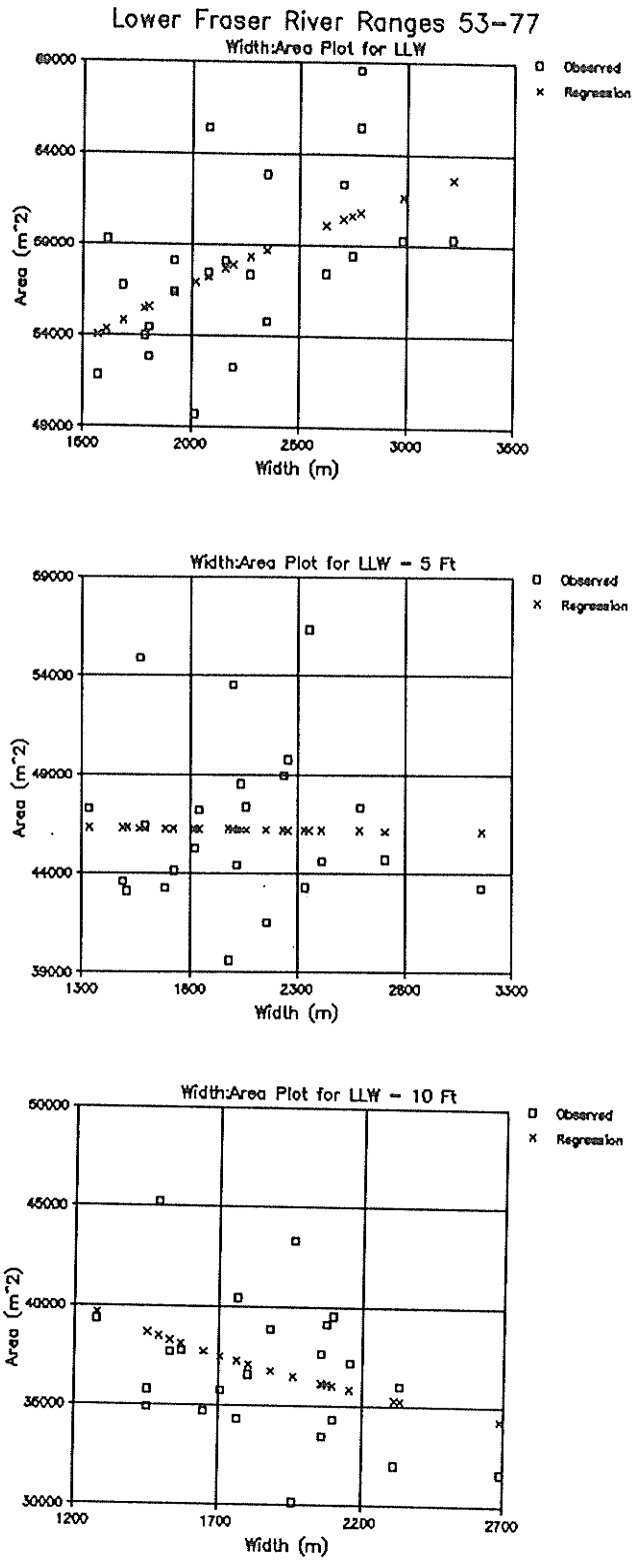


Figure 3.36 Lower Fraser River; Width:Area Plots, Ranges 53-77, for LLW, LLW - 5 feet, LLW - 10 feet

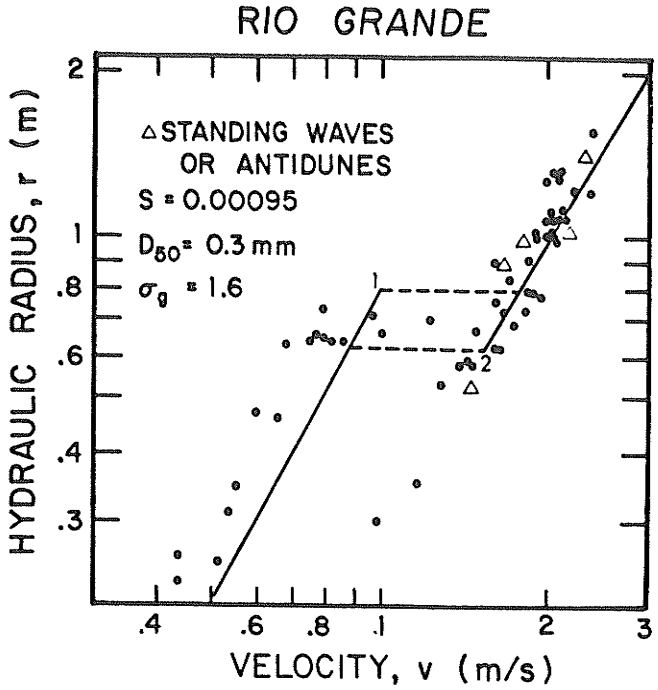


Figure 3.37 Rio Grande River near Bernalillo, N.M.; Discontinuous Rating Curve (Brownlie, 1981a)

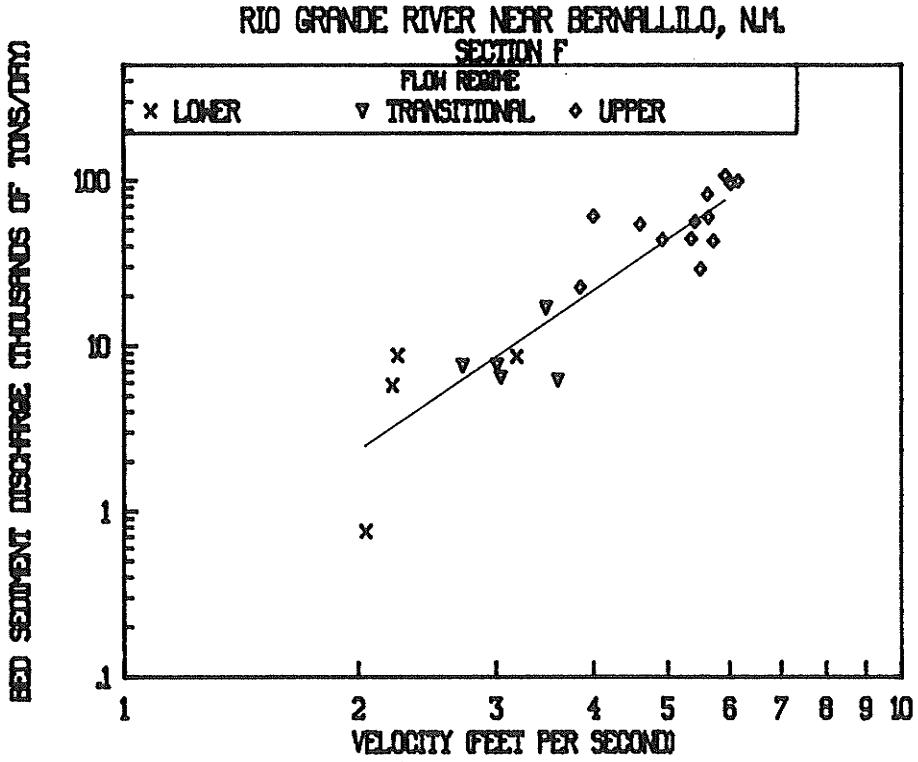
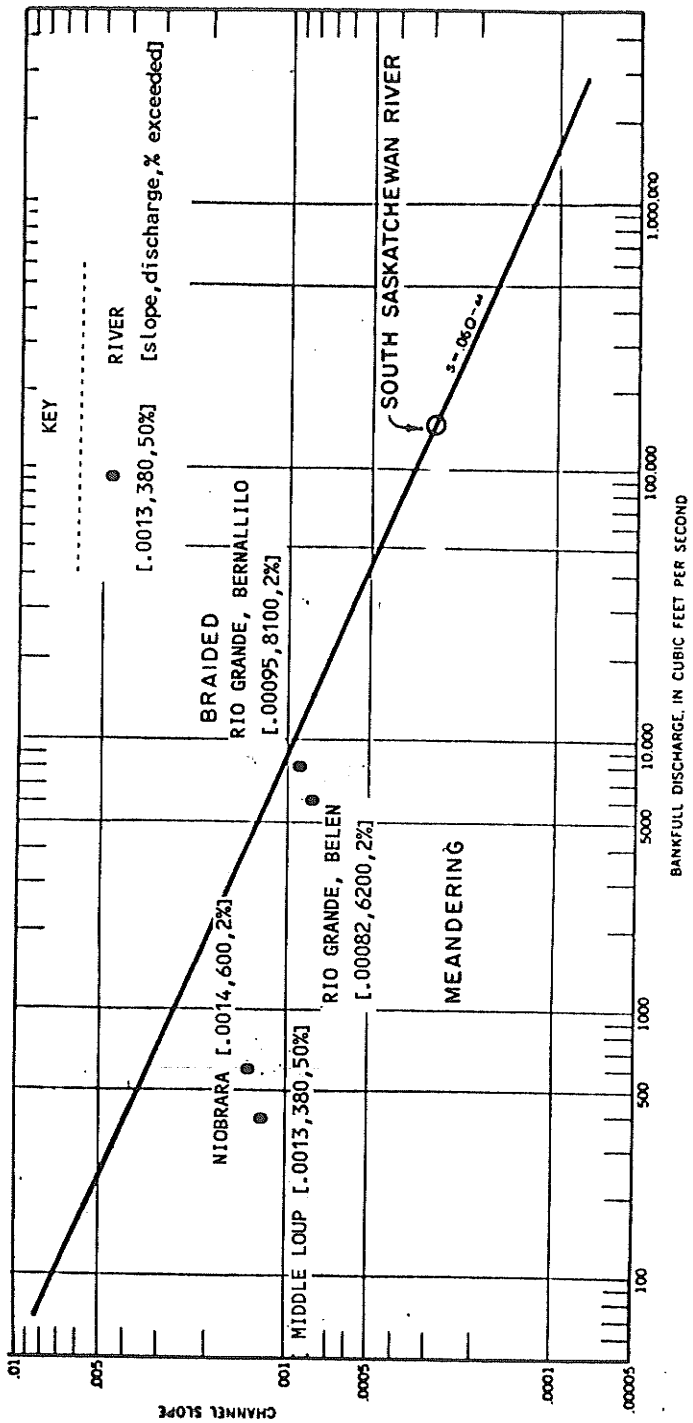


Figure 3.38 Rio Grande River near Bernalillo, N.M.; Sediment Discharge per Unit Width versus Mean Velocity at Section F



NOTE:
CHART DEVELOPED BY LEOPOLD AND WOLMAN (1957) TO DISTINGUISH BRAIDED FROM MEANDERING CHANNELS.

Figure 3.39 Leopold and Wolman's (1957) Chart for Channel Pattern

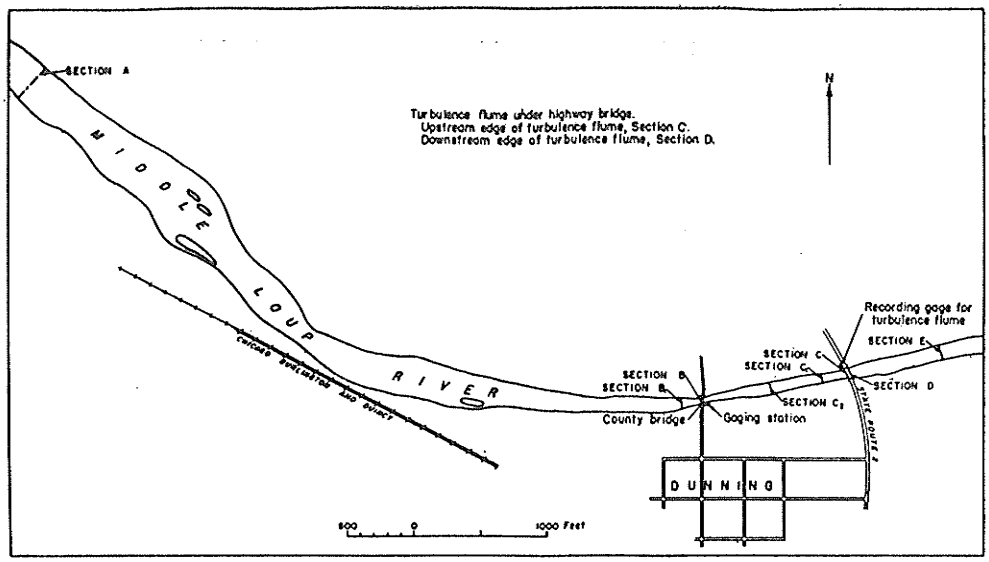


Figure 4.1 Study Reach of the Middle Loup River at Dunning, Nebraska (Hubbell and Majteka, 1959)

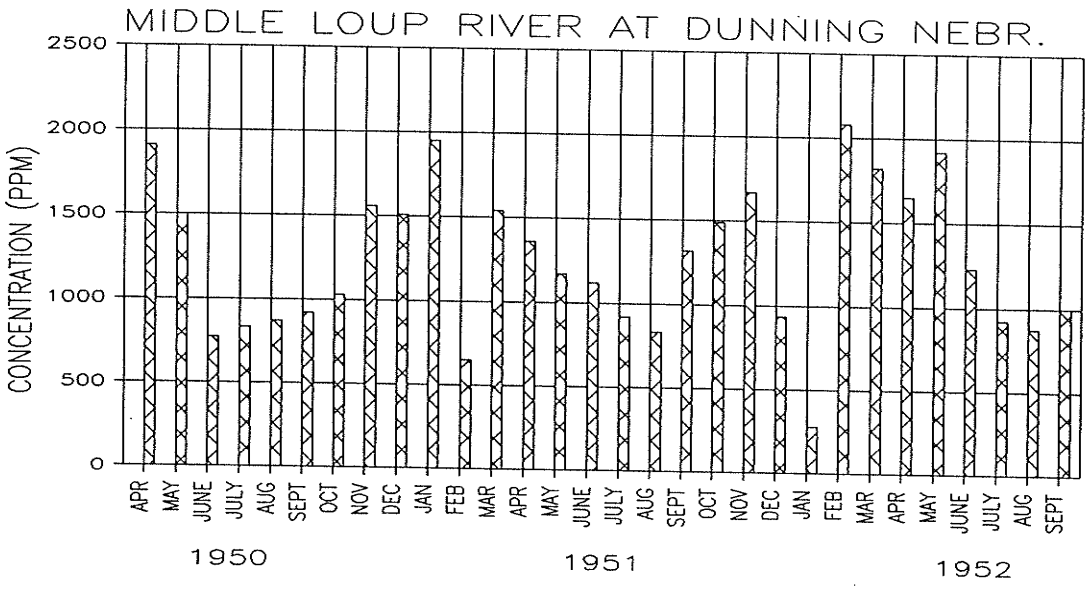


Figure 4.2 Middle Loup River at Dunning, Nebr.; Variation of Weighted Mean Sediment Concentration (PPM), April, 1950 to September, 1952

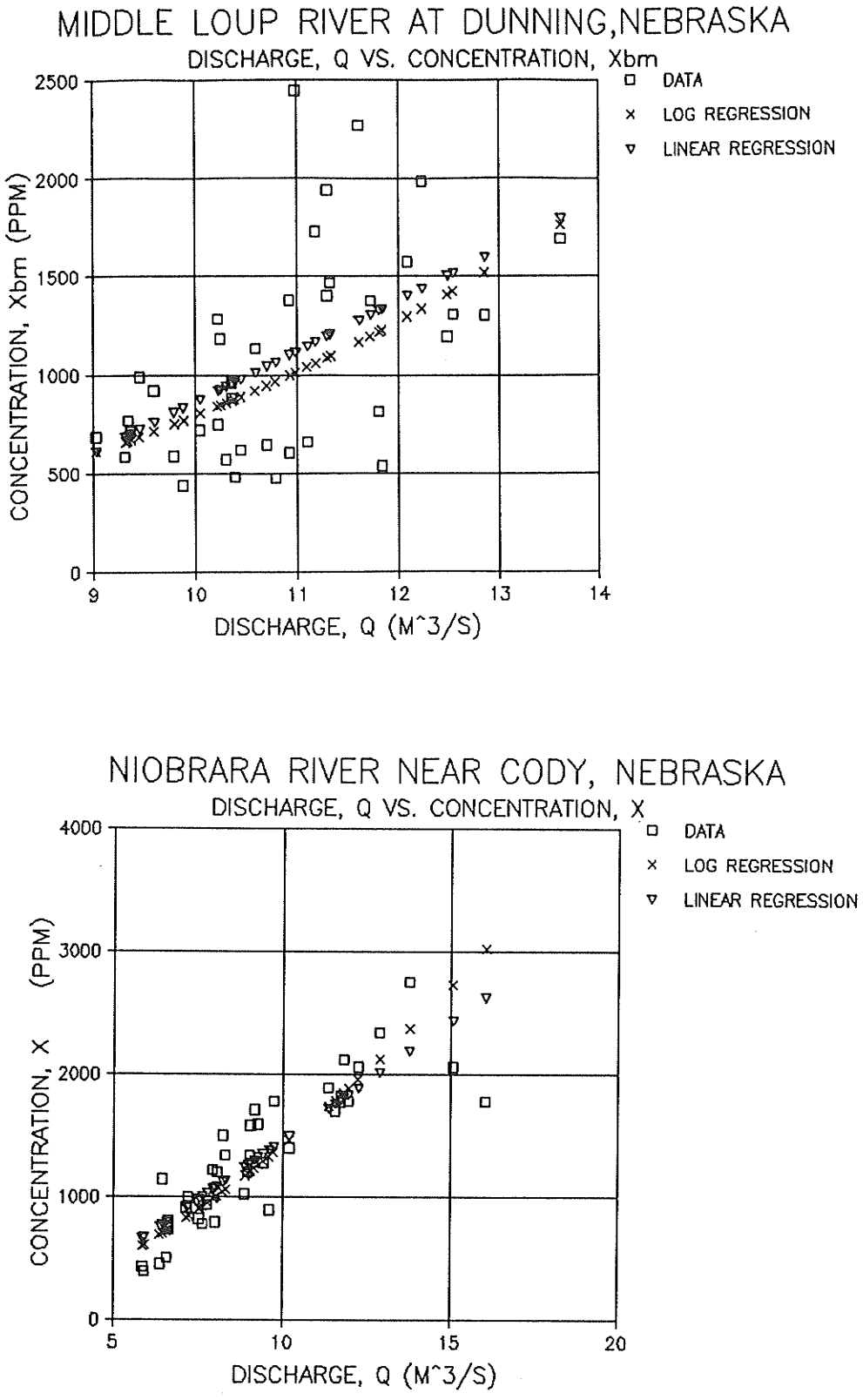


Figure 4.3 Discharge versus Bed Sediment Concentration: a) Middle Loup River at Dunning, Nebr.; b) Niobrara River near Cody, Nebr.

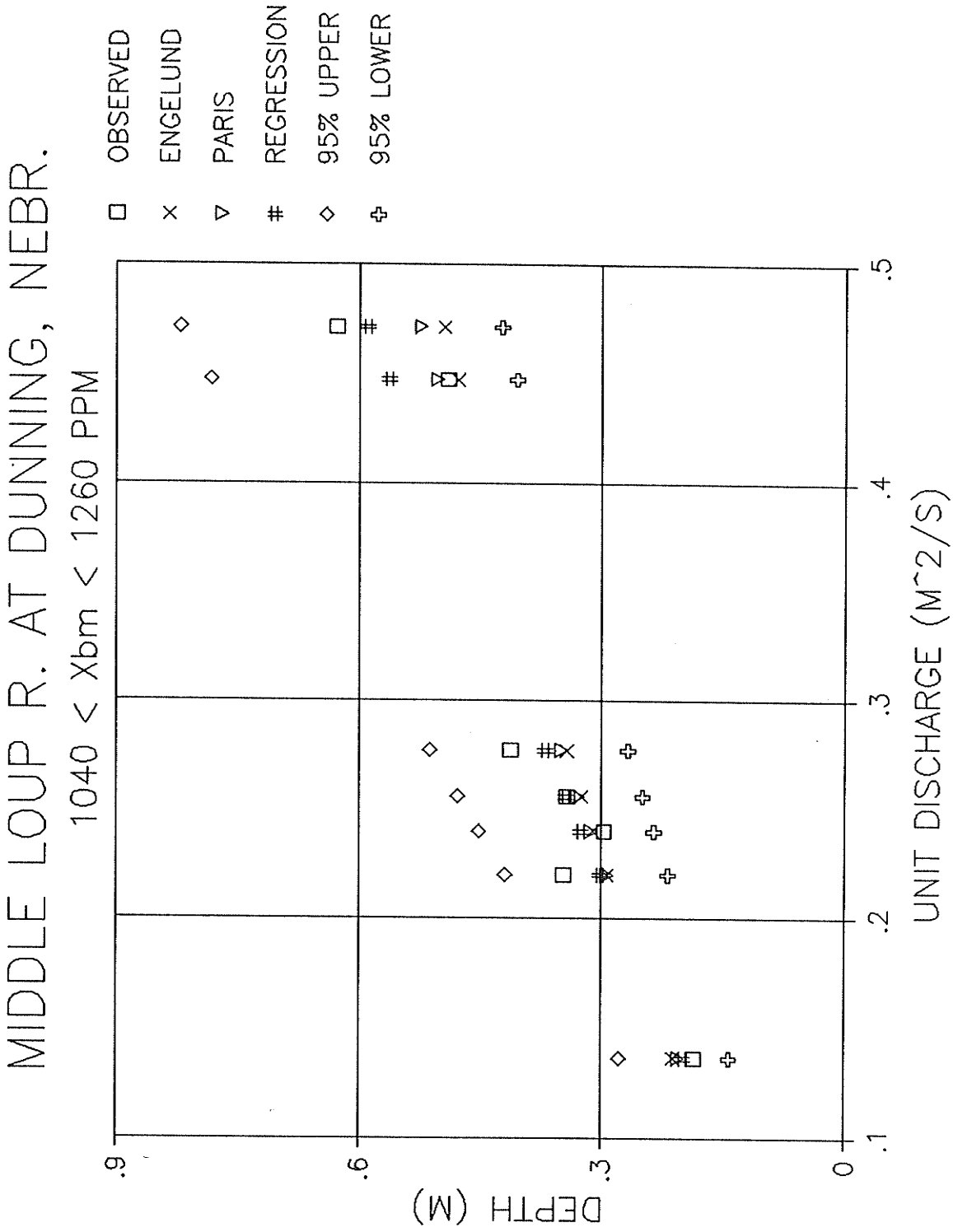


Figure 4.4 Middle Loup River at Dunning, Nebr.; Unit Discharge vs. Depth at Dominant Concentration, X_{bm} = 1150 PPM, Observed vs. Regression and Analytical Solutions

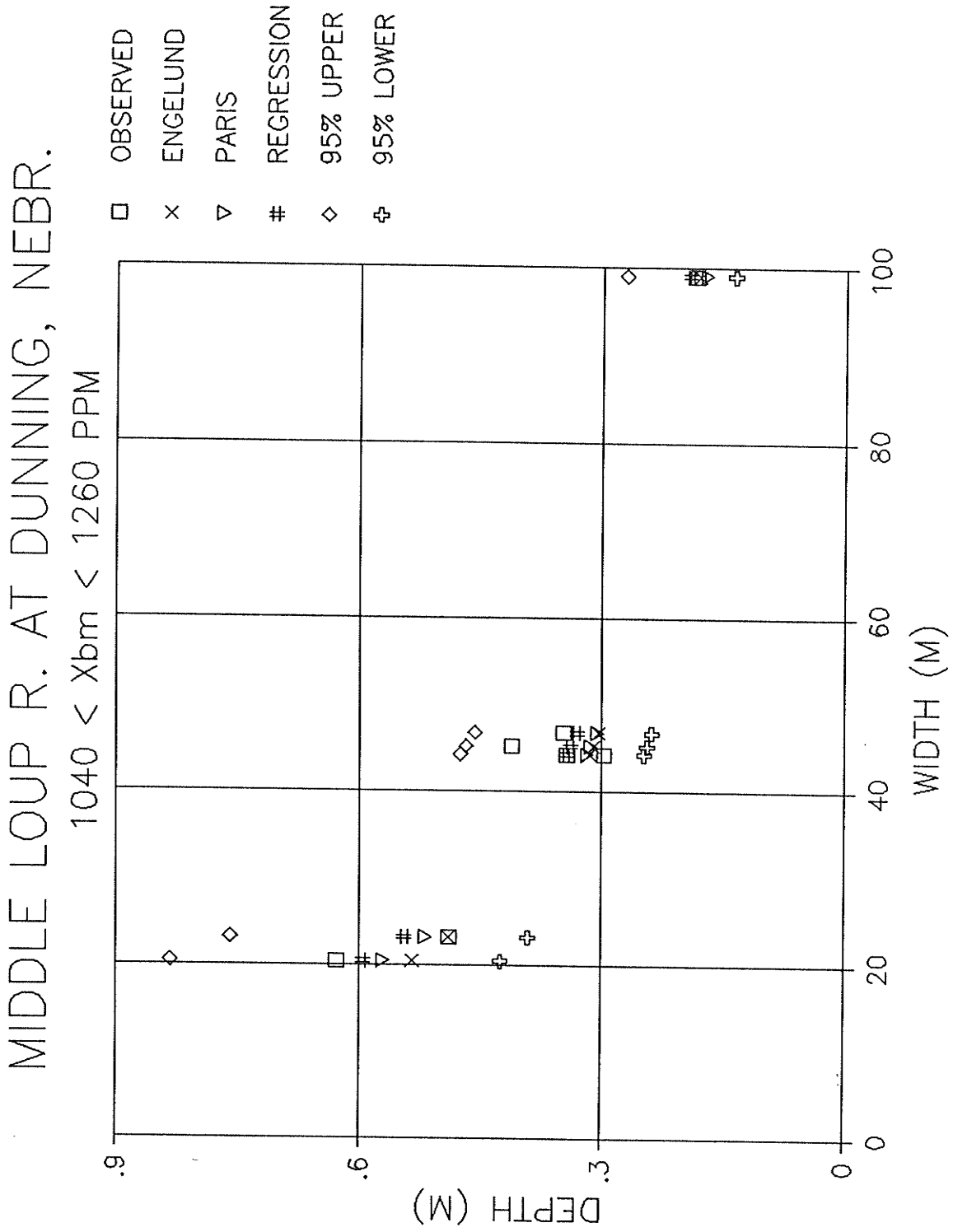


Figure 4.5 Middle Loup River at Dunning, Nebr.; Width versus Depth at Dominant Concentration, X_{bm} = 1150 PPM, Observed vs. Regression and Analytical Solutions

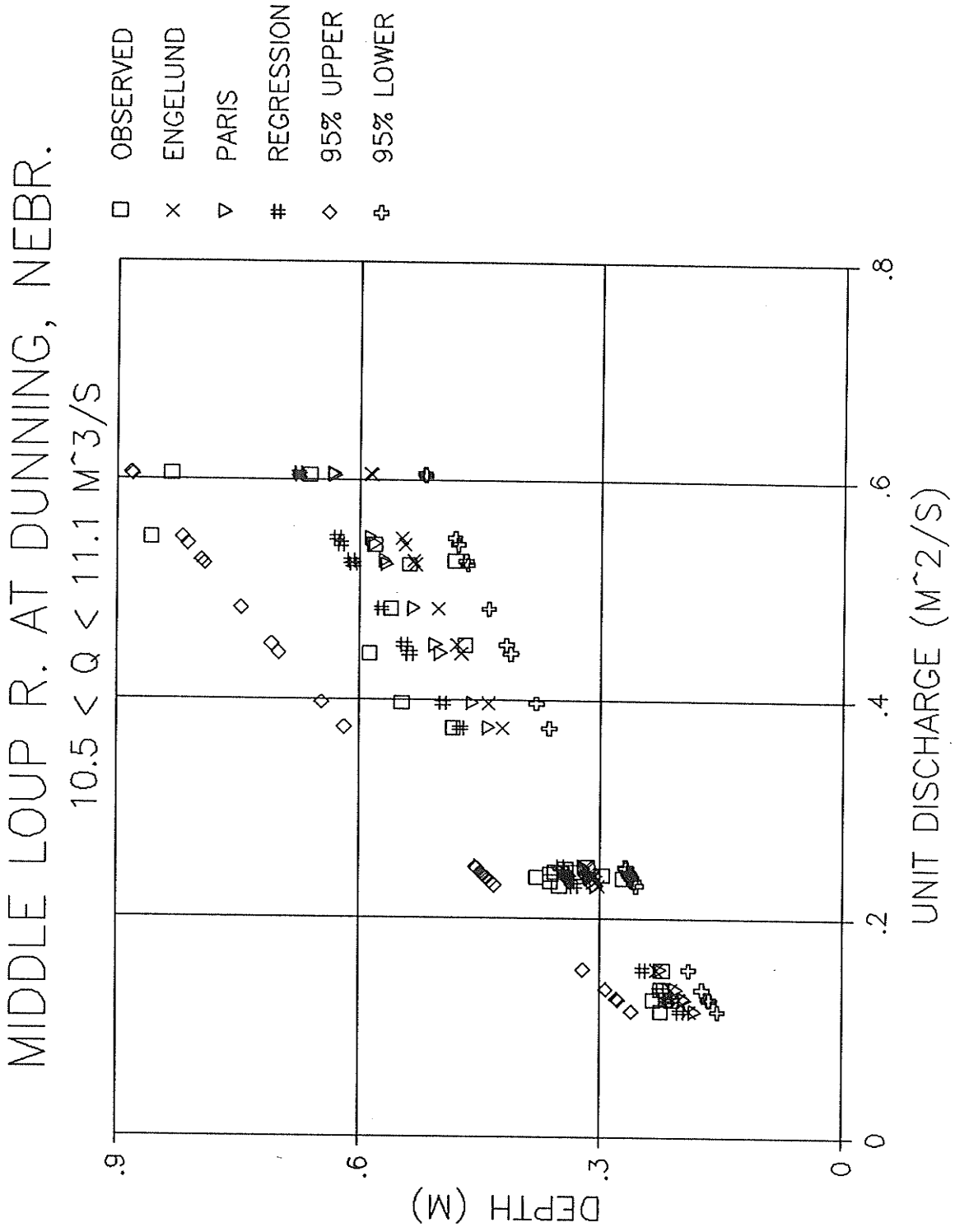


Figure 4.6 Middle Loup River at Dunning, Nebr.; Unit Discharge vs. Depth at Dominant Discharge, Q= 10.8 m^3/s, Observed versus Regression and Analytical Solutions

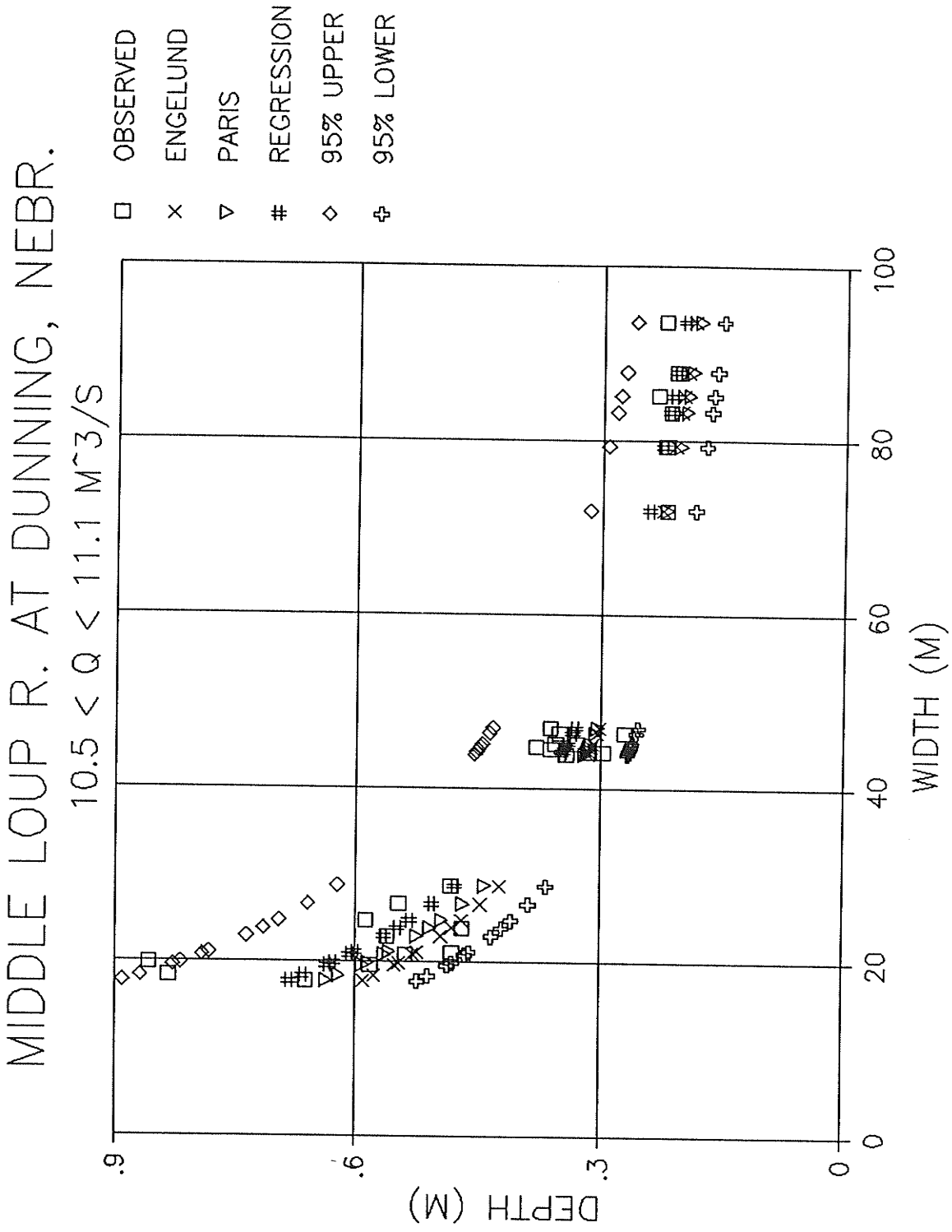


Figure 4.7 Middle Loup River at Dunning, Nebr.; Width versus Depth at Dominant Discharge, Q= 10.8 m^3/s, Observed versus Regression and Analytical Solutions

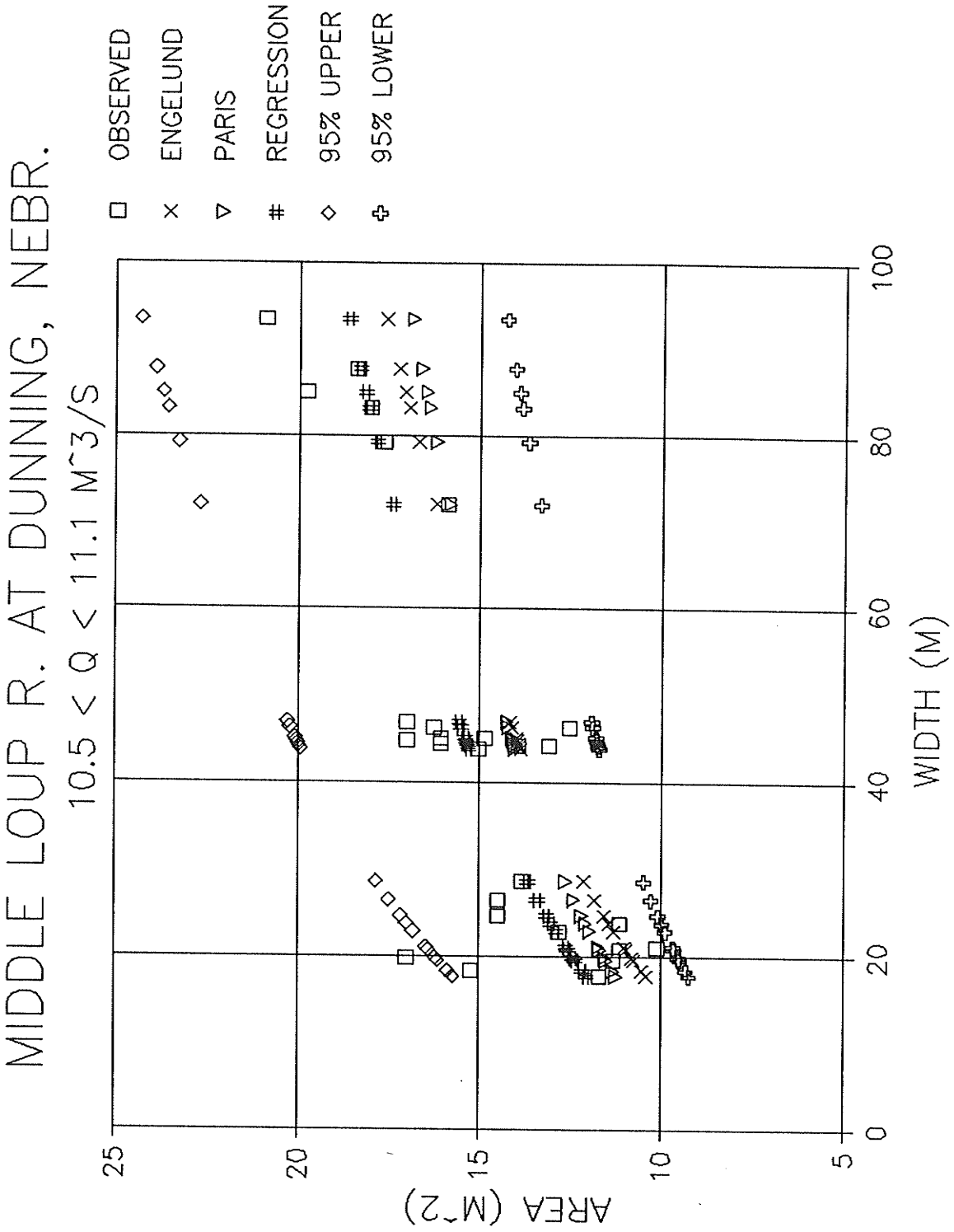


Figure 4.8 Middle Loup River at Dunning, Nebr.; Width versus Area at Dominant Discharge, Q= 10.8 m^3/s, Observed versus Regression and Analytical Solutions

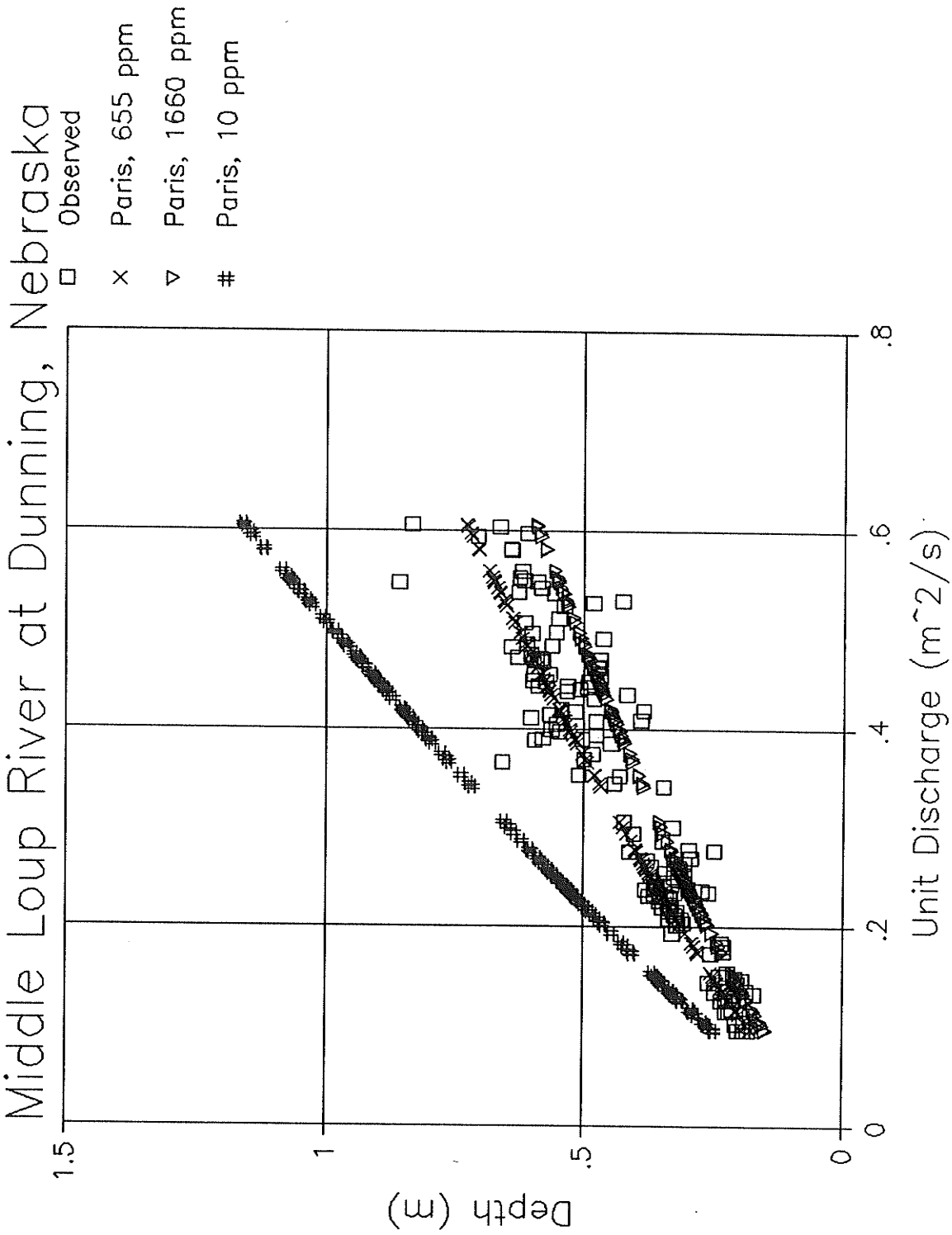


Figure 4.9 Middle Loup River at Dunning, Nebr.; Unit Discharge versus Depth, Observed Data Base versus Paris's Analytical Solutions

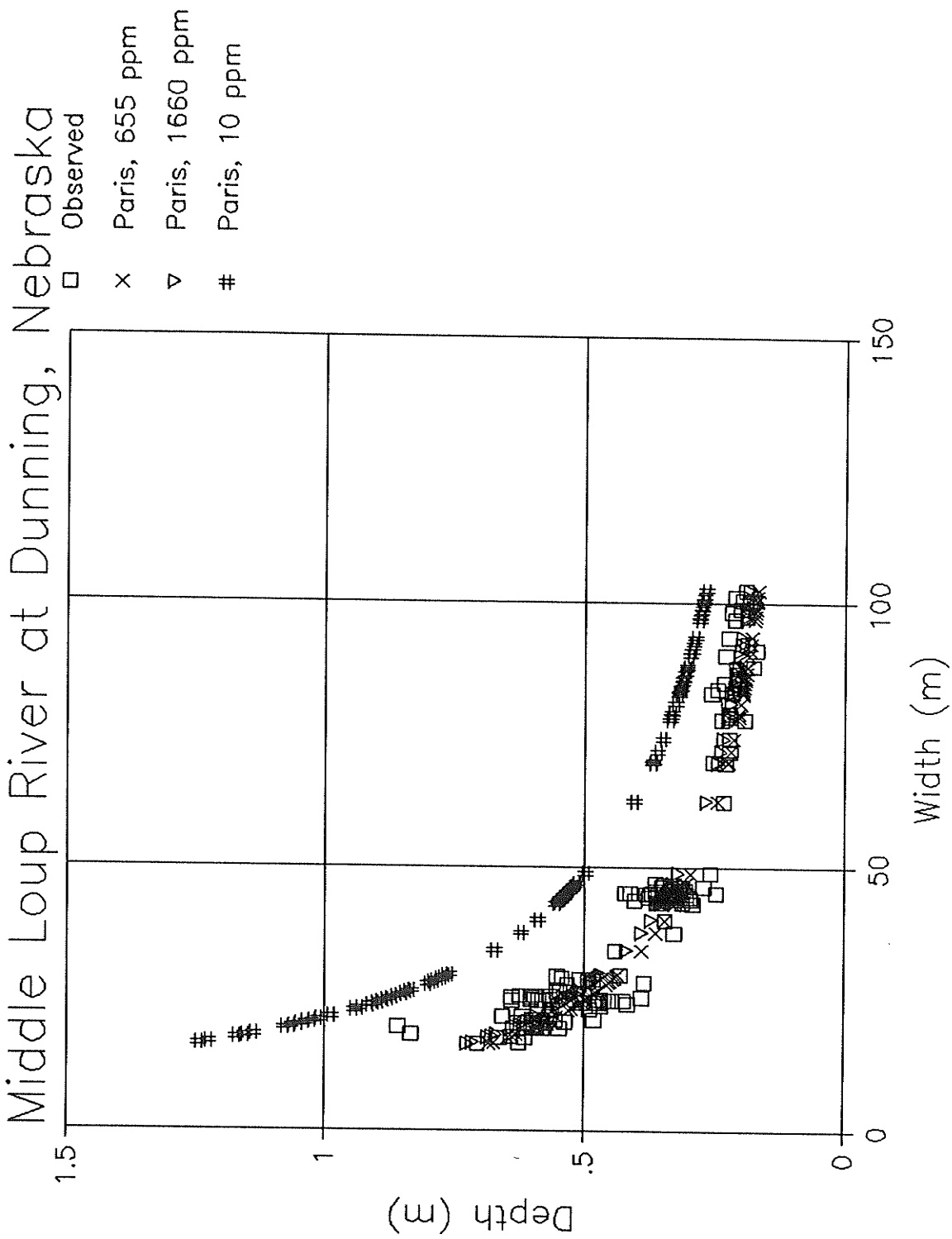


Figure 4.10 Middle Loup River at Dunning, Nebr.; Width versus Depth, Observed Data Base versus Paris's Analytical Solutions

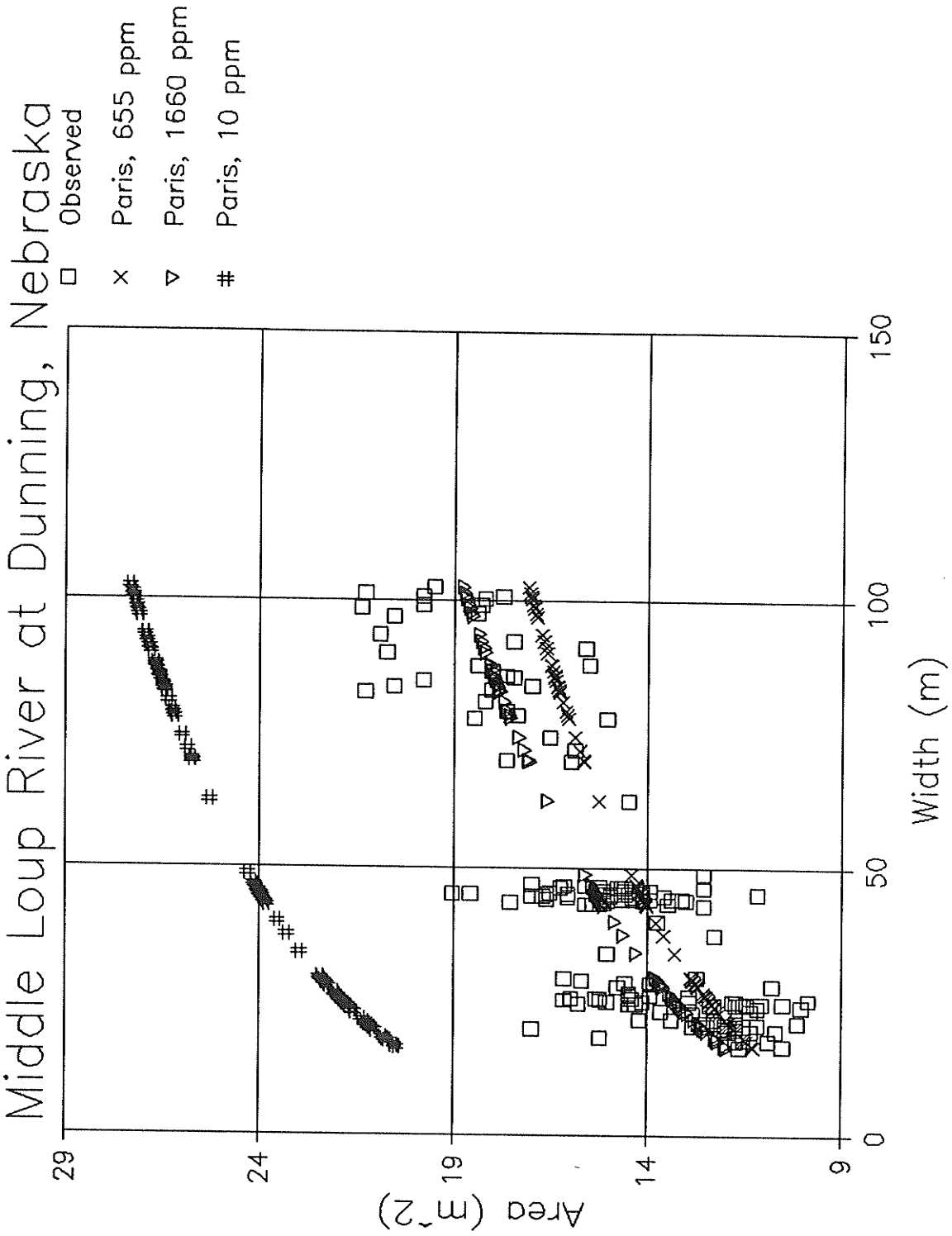


Figure 4.11 Middle Loup River at Dunning, Nebr.; Width versus Area, Observed Data Base versus Paris's Analytical Solutions

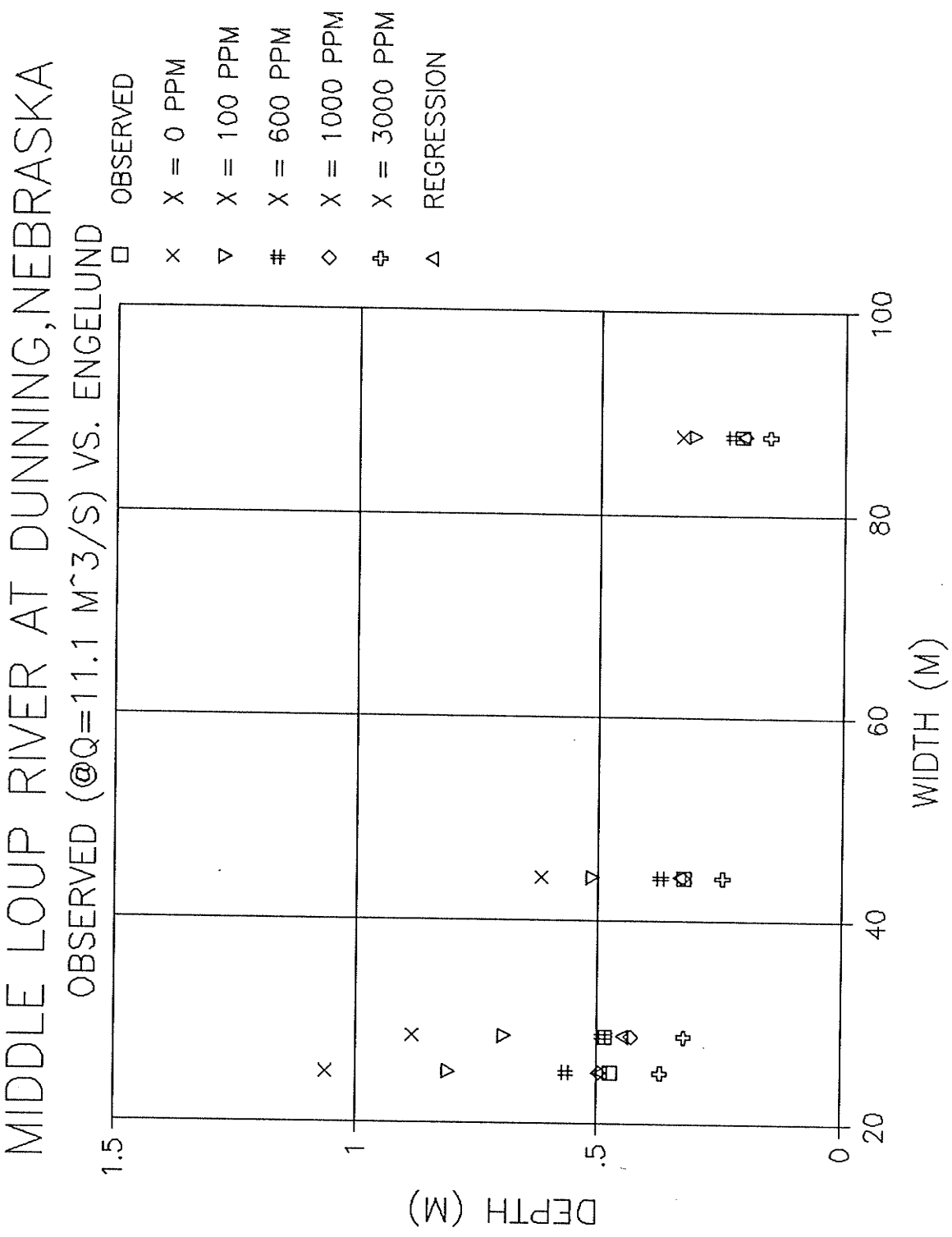


Figure 4.12 Middle Loup R. at Dunning, Nebr.; Observed Data (Parker, 1978a) vs. Engelund (1967), X_{bm}=0 to 3000 PPM, Q= 11.0 m^3/s

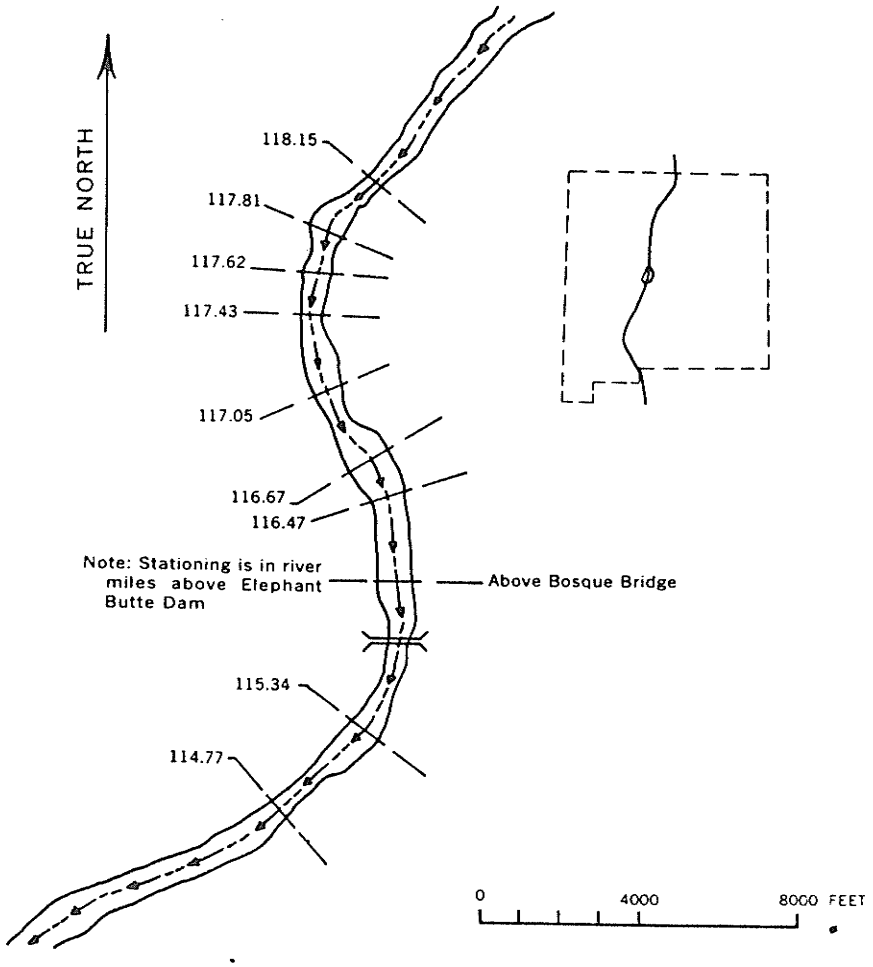
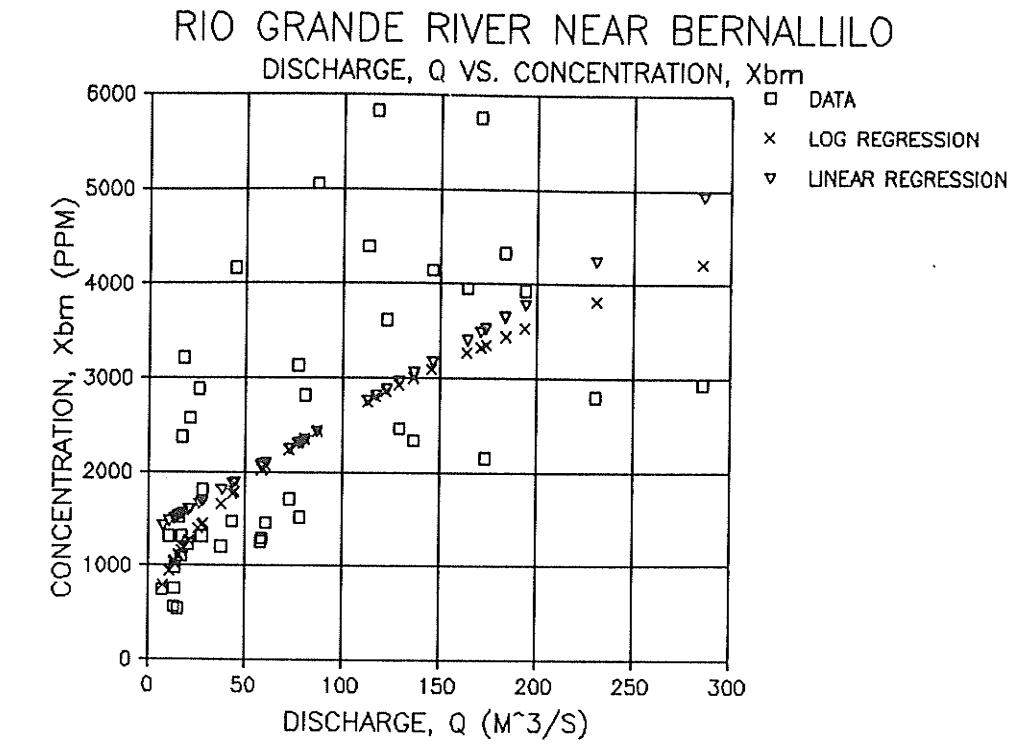


Figure 5.1 Study Reach of the Rio Grande River, Casa Colorada, New Mexico (Culbertson and Dawdy, 1964)



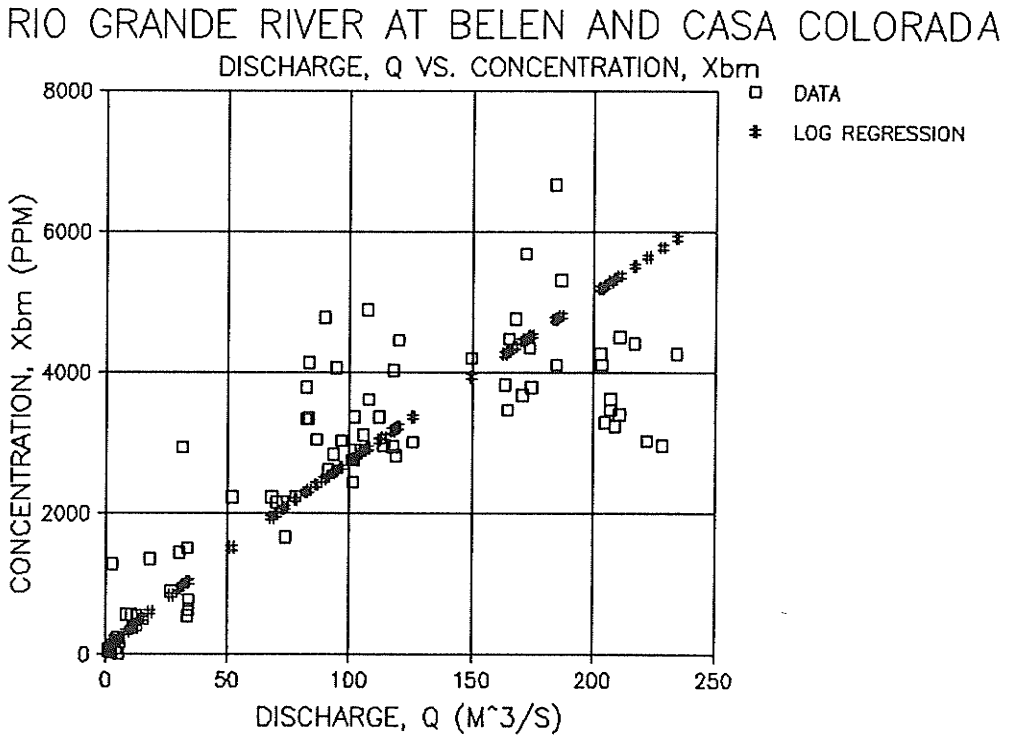


Figure 5.2 Discharge versus Bed Sediment Concentration for the Rio Grande R. at: a) Bernallilo, N.M. ; b) Casa Colorada, N.M.

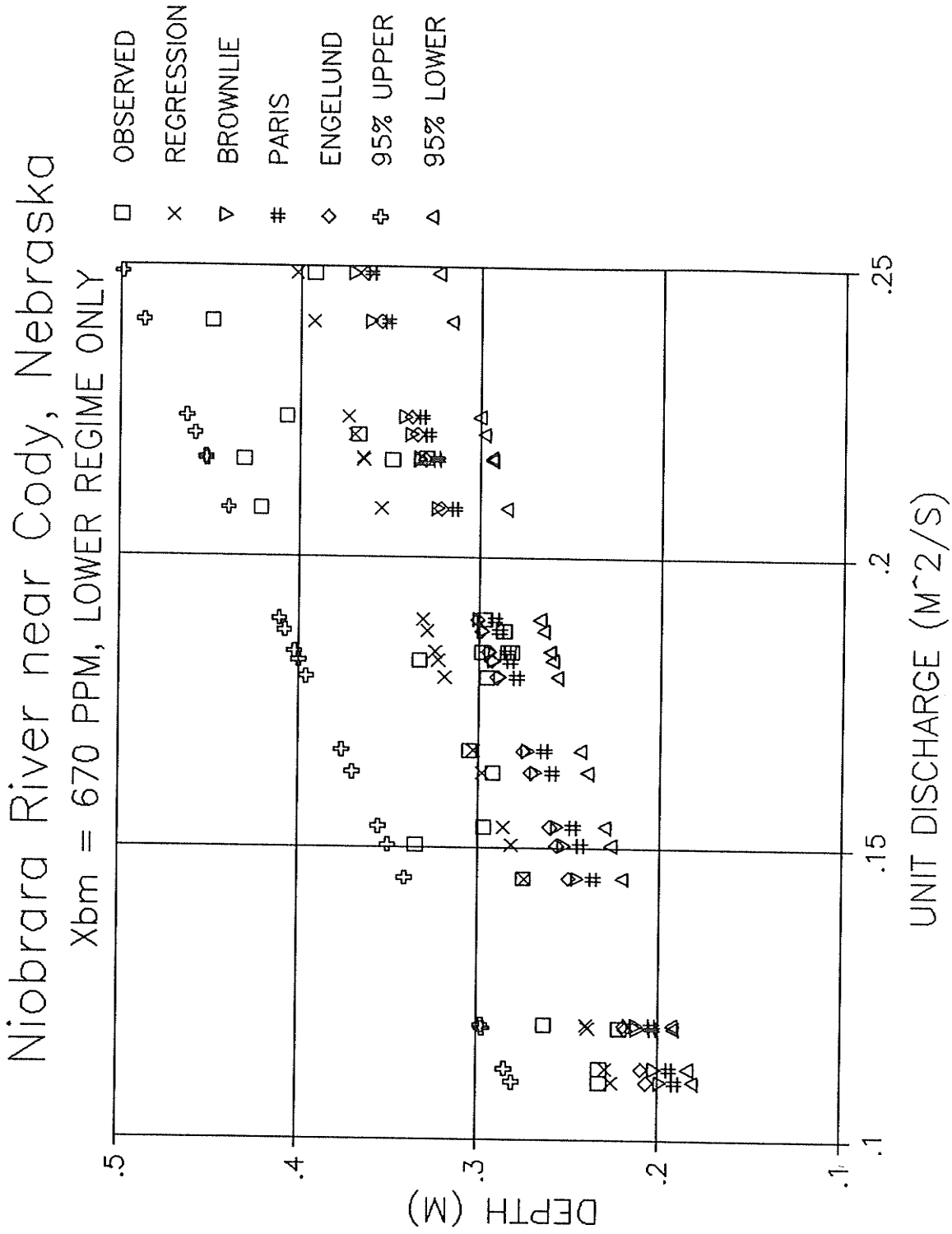


Figure 5.3 Niobrara River near Cody, Nebr.: Unit Discharge versus Depth, Observed versus Analytical: X_{bm} = 670 ppm; Lower Flow Regime

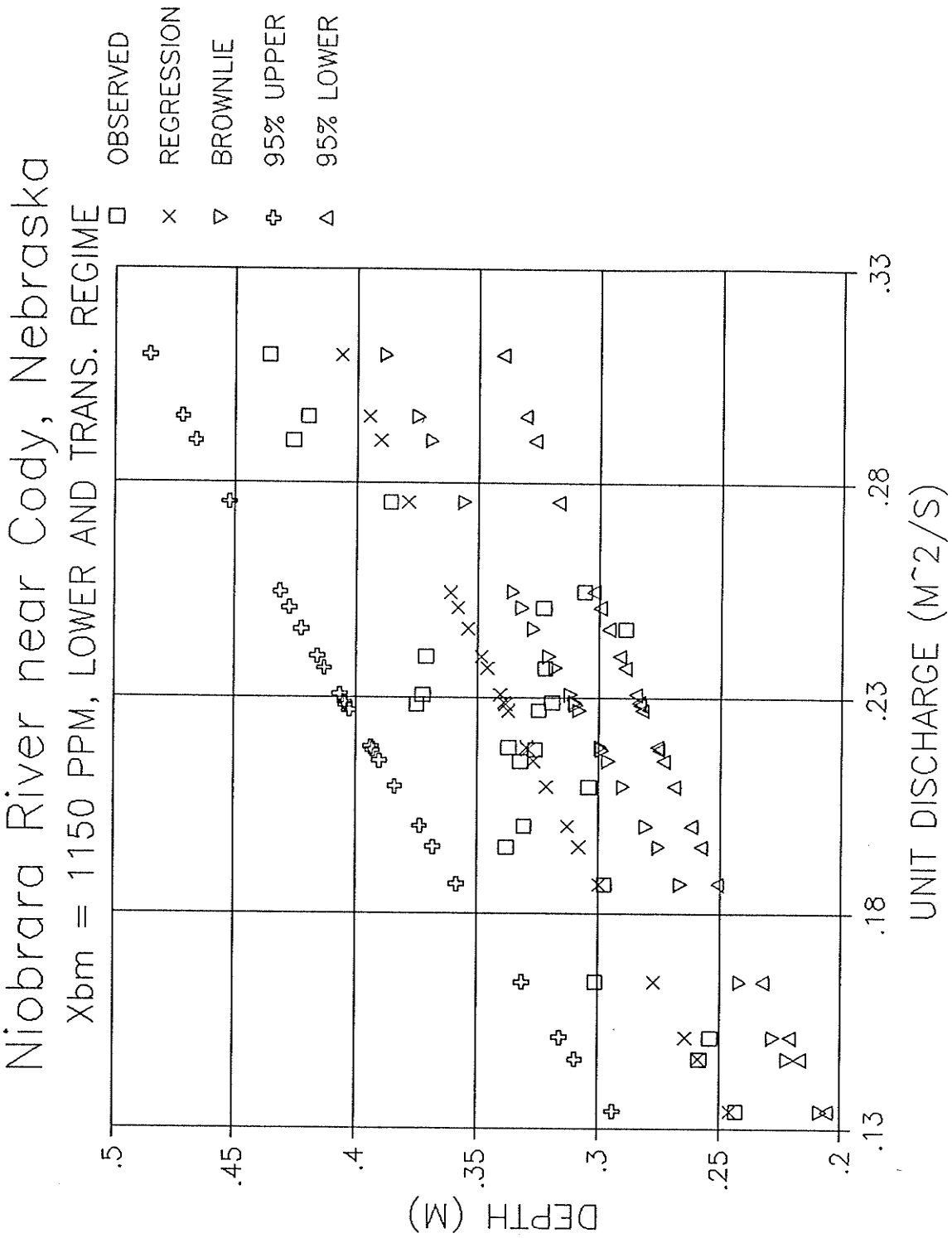


Figure 5.4 Niobrara River near Cody, Nebr.: Unit Discharge versus Depth, Observed versus Analytical: X_{bm} = 1150 ppm; Lower and Transition Flow Regimes

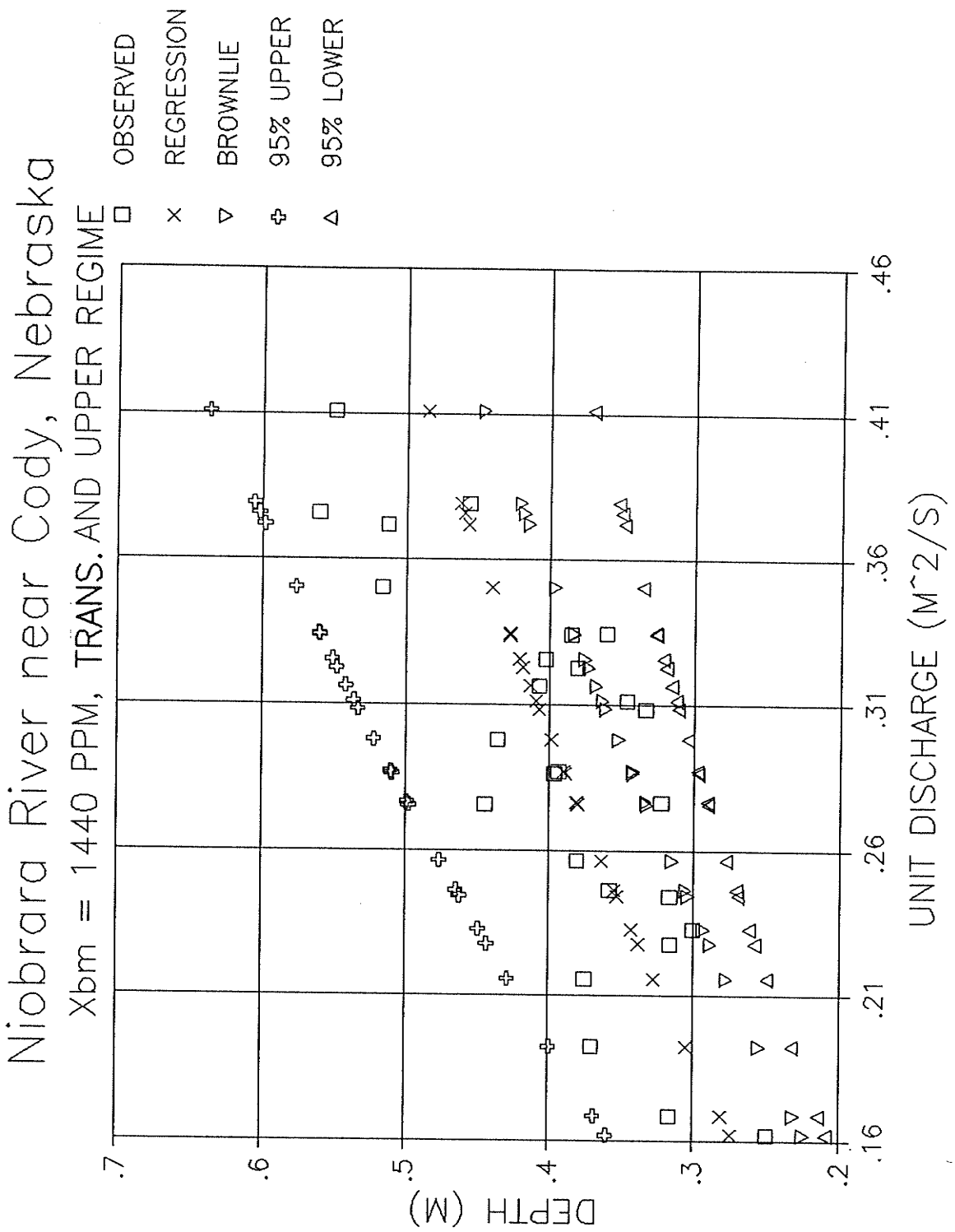


Figure 5.5 Niobrara River near Cody, Nebr.: Unit Discharge versus Depth, Observed versus Analytical: X_{bm} = 1440 ppm; Transition and Upper Flow Regimes

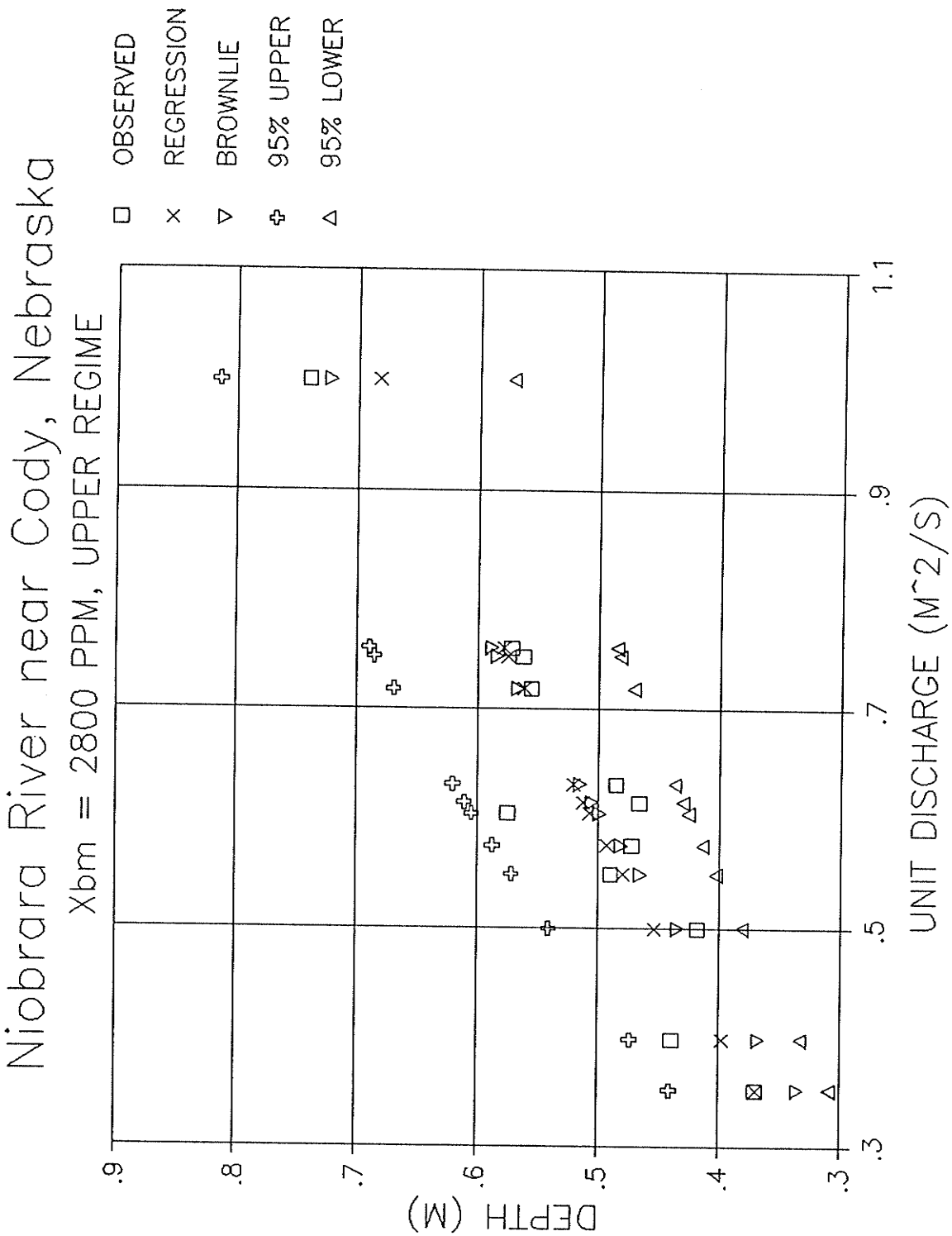


Figure 5.6 Niobrara River near Cody, Nebr.: Unit Discharge versus Depth, Observed versus Analytical: X_{bm} = 2800 ppm; Upper Flow Regime

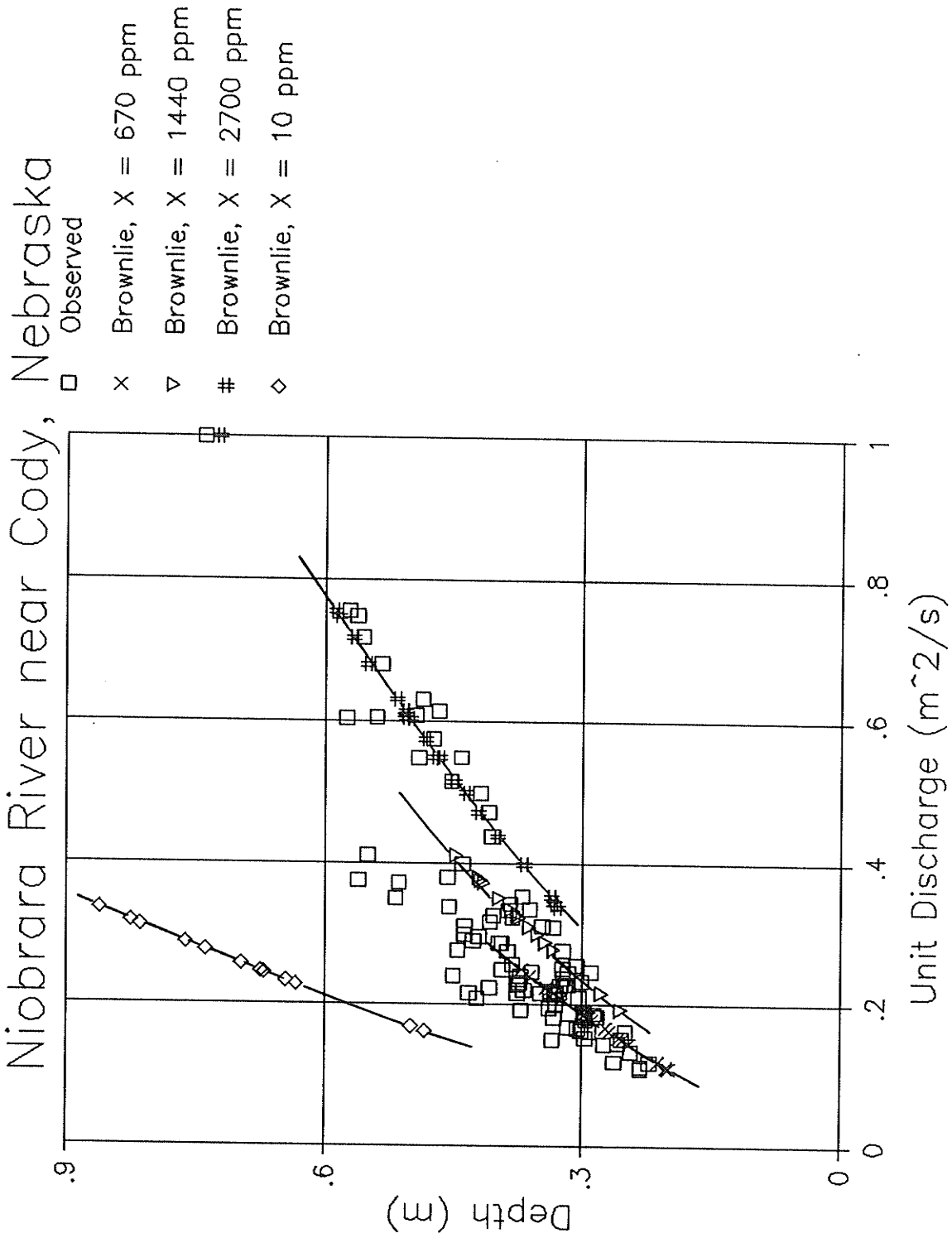


Figure 5.7 Niobrara River near Cody, Nebr.: Unit Discharge versus Depth, Observed Data Base versus Brownlie's Equations

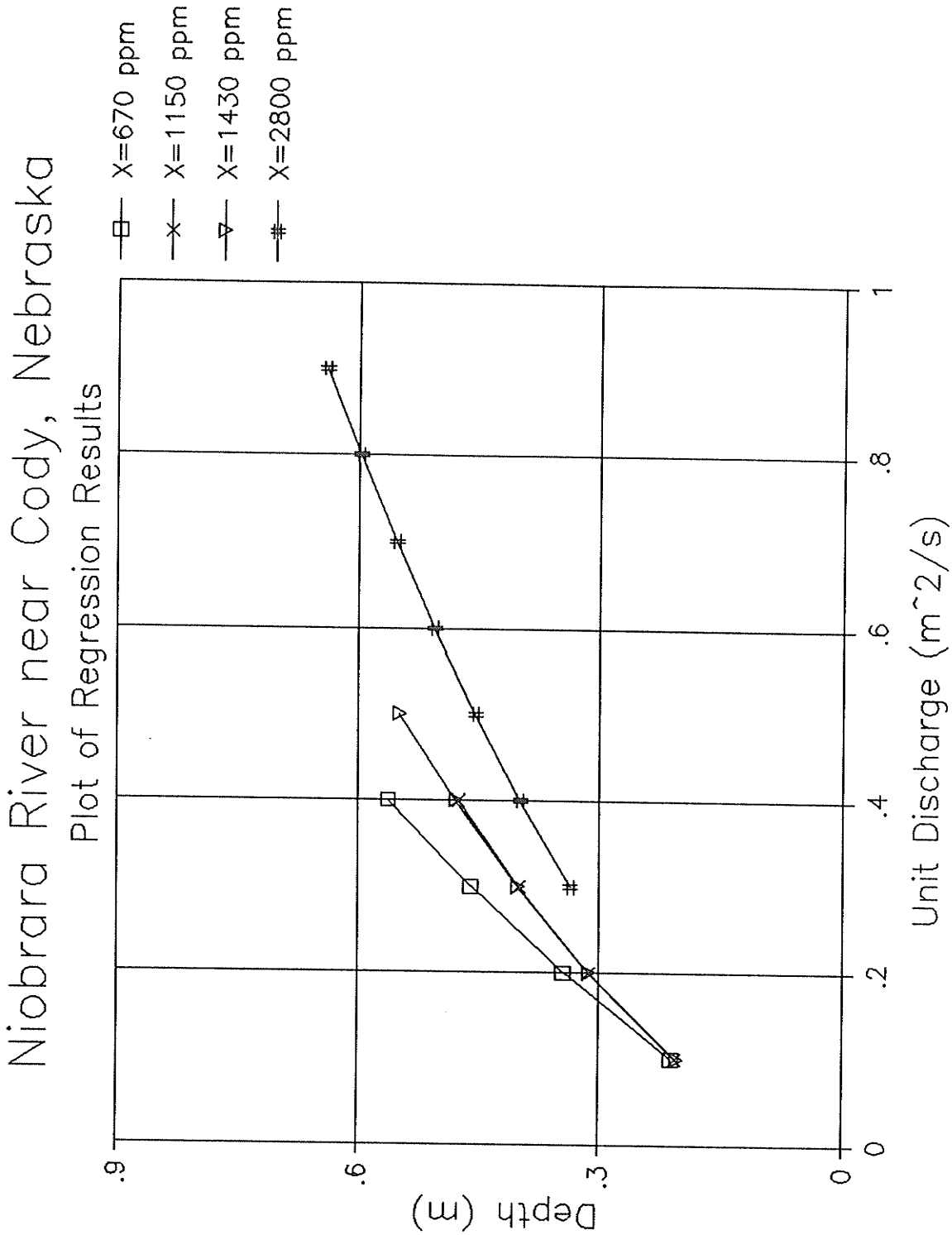


Figure 5.8 Niobrara River near Cody, Nebr.: Unit Discharge versus Depth, Plot of Regression Results for Observed Data Base

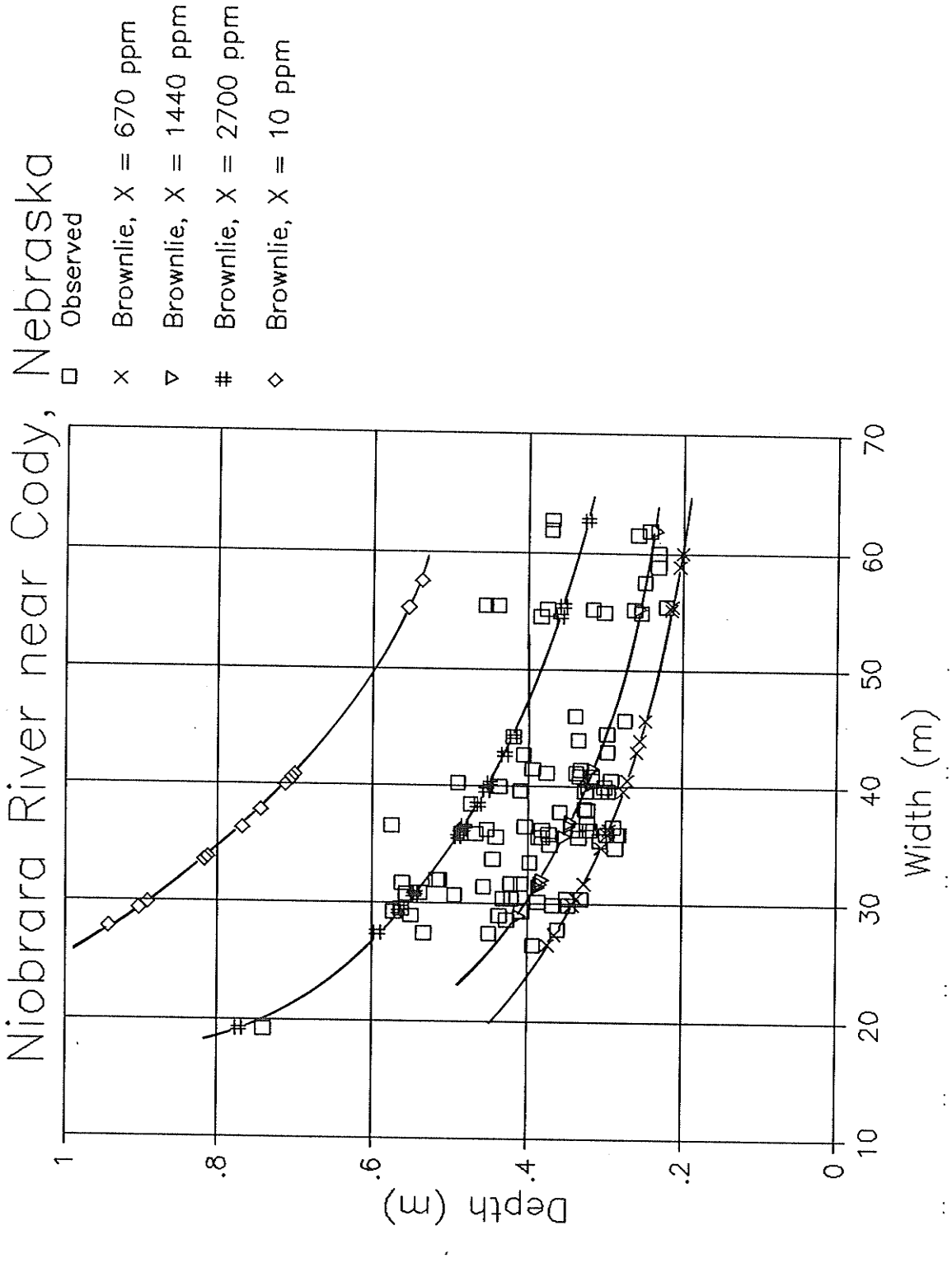


Figure 5.9 Niobrara River near Cody, Nebr.: Width versus Depth, Observed Data Base versus Brownlie's Equations

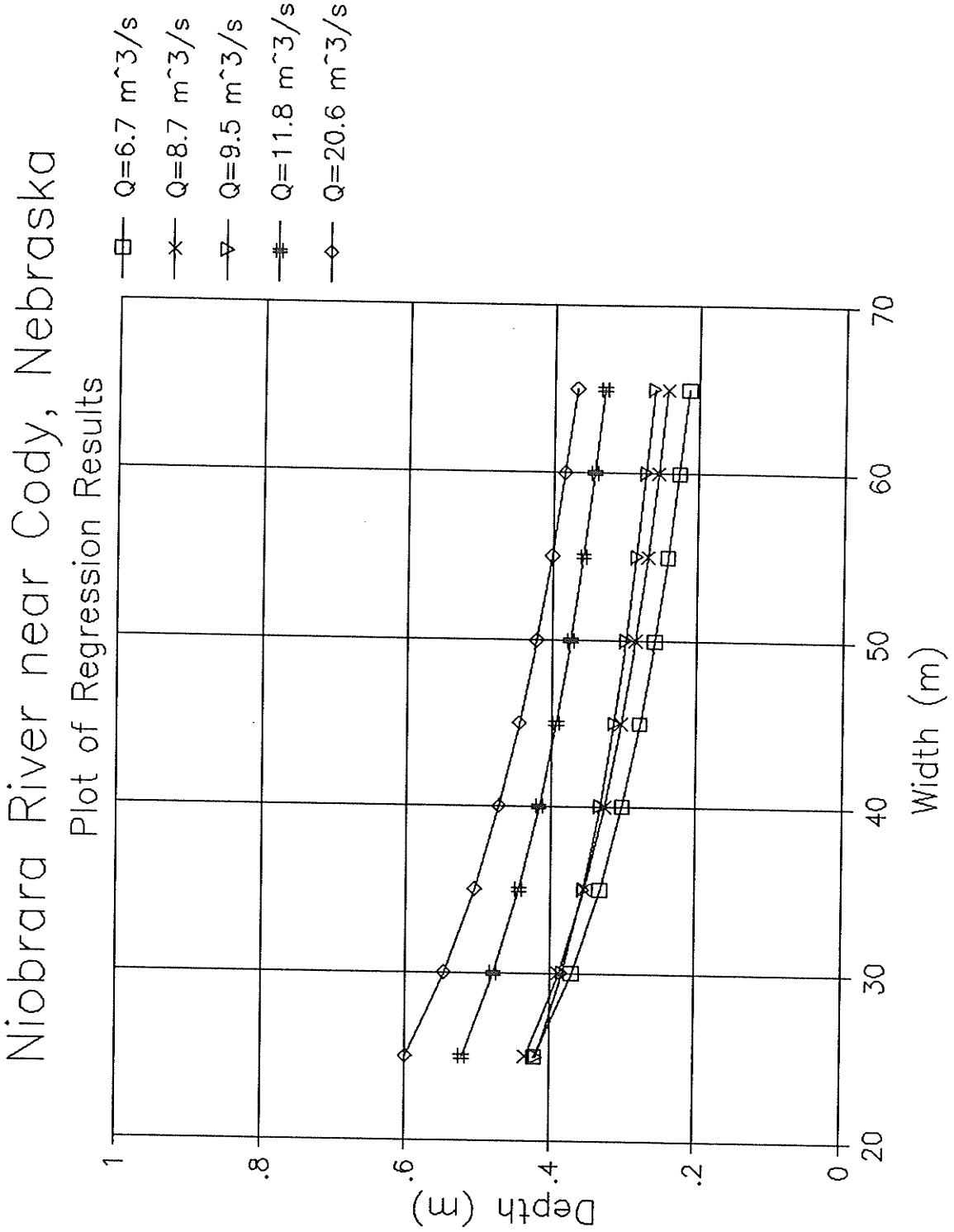


Figure 5.10 Niobrara River near Cody, Nebr.: Width versus Depth, Plot of Regression Results for Observed Data Base

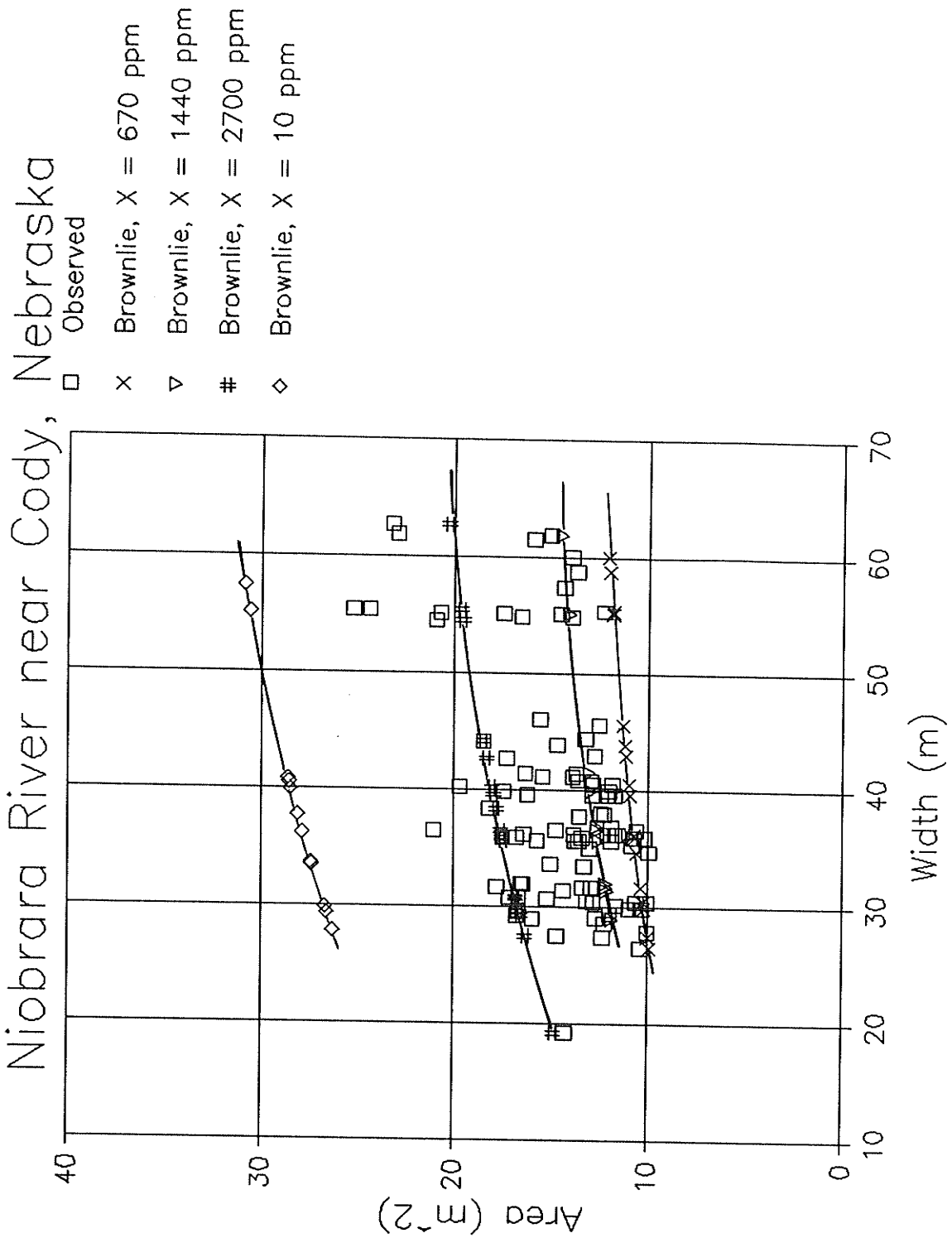


Figure 5.11 Niobrara River near Cody, Nebr.: Width versus Area, Observed Data Base versus Brownlie's Equations

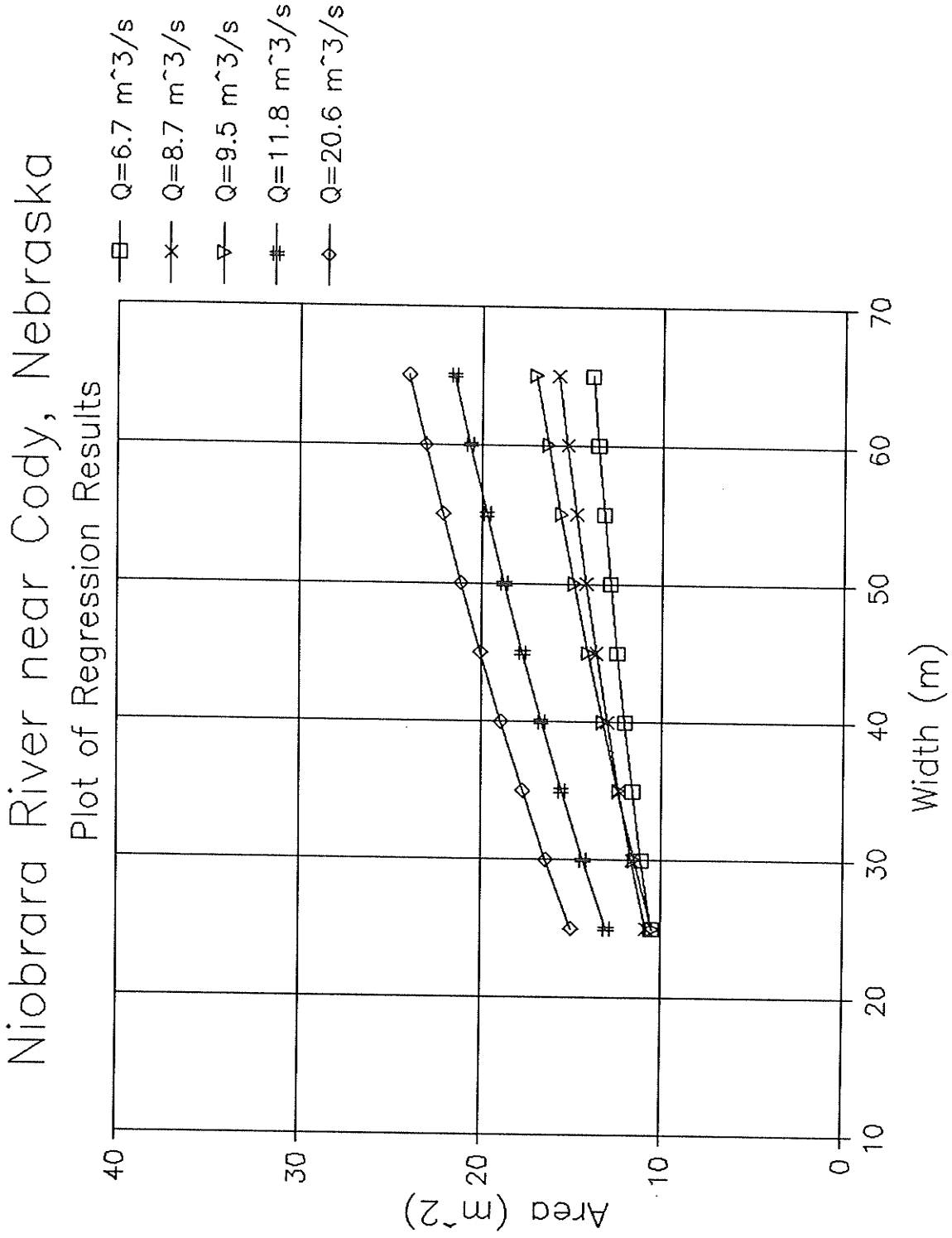


Figure 5.12 Niobrara River near Cody, Nebr.: Width versus Area, Plot of Regression Results for Observed Data Base

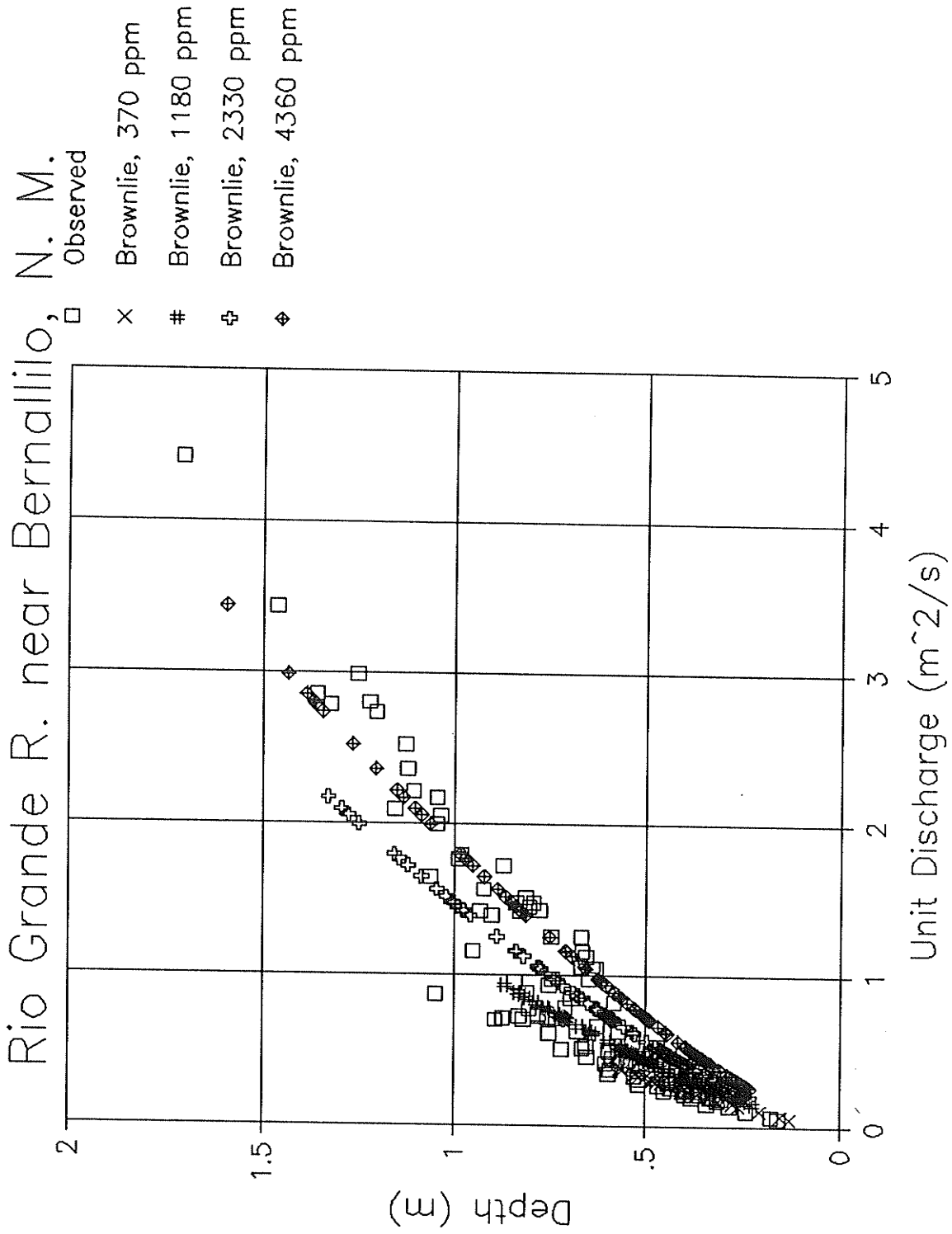


Figure 5.13 Rio Grande R. near Bernallilo, N.M.: Unit Discharge versus Depth, Observed Data Base versus Brownlie's Equations

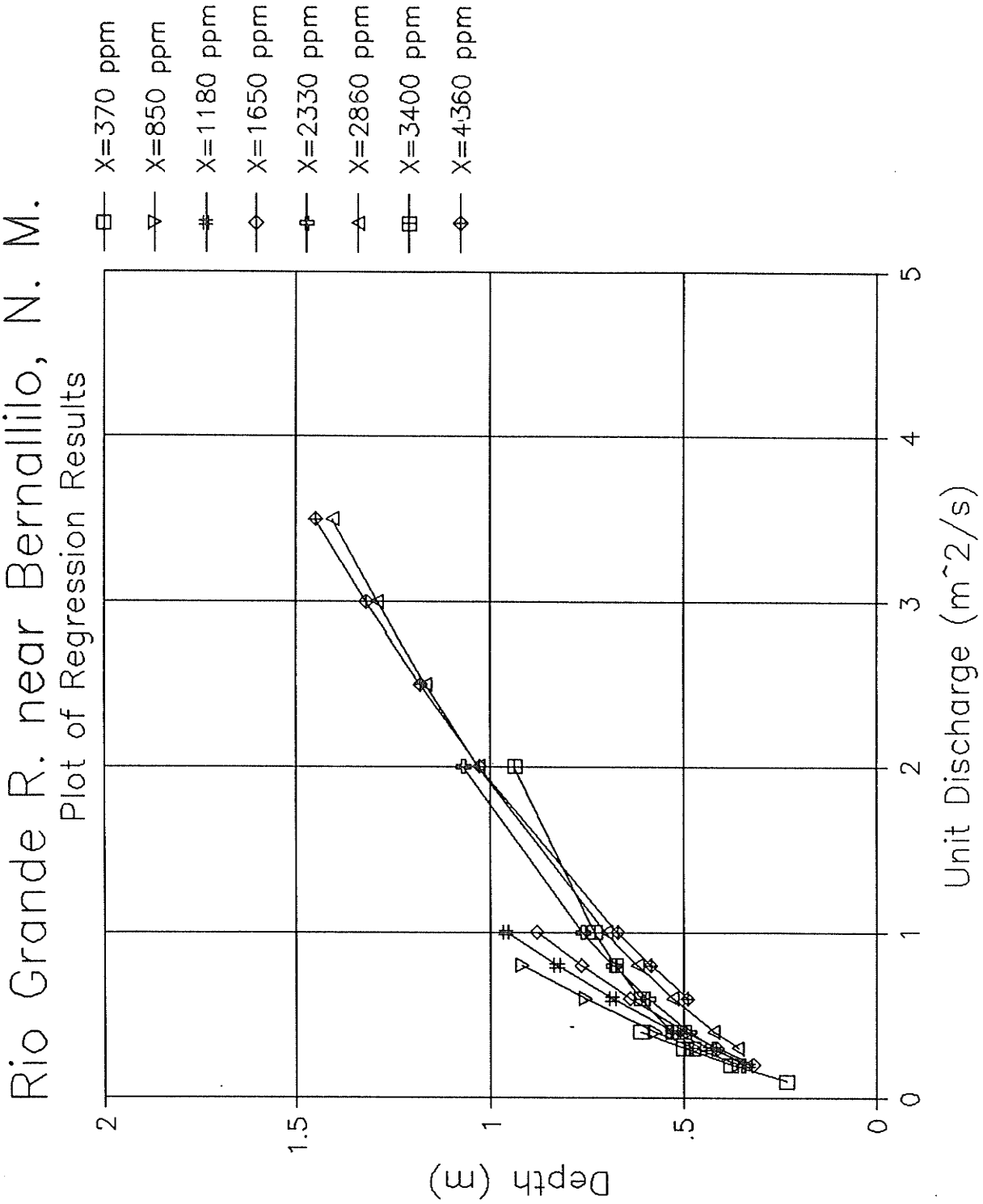


Figure 5.14 Rio Grande R. near Bernallilo, N.M.: Unit Discharge versus Depth, Plot of Regression Results for Observed Data Base

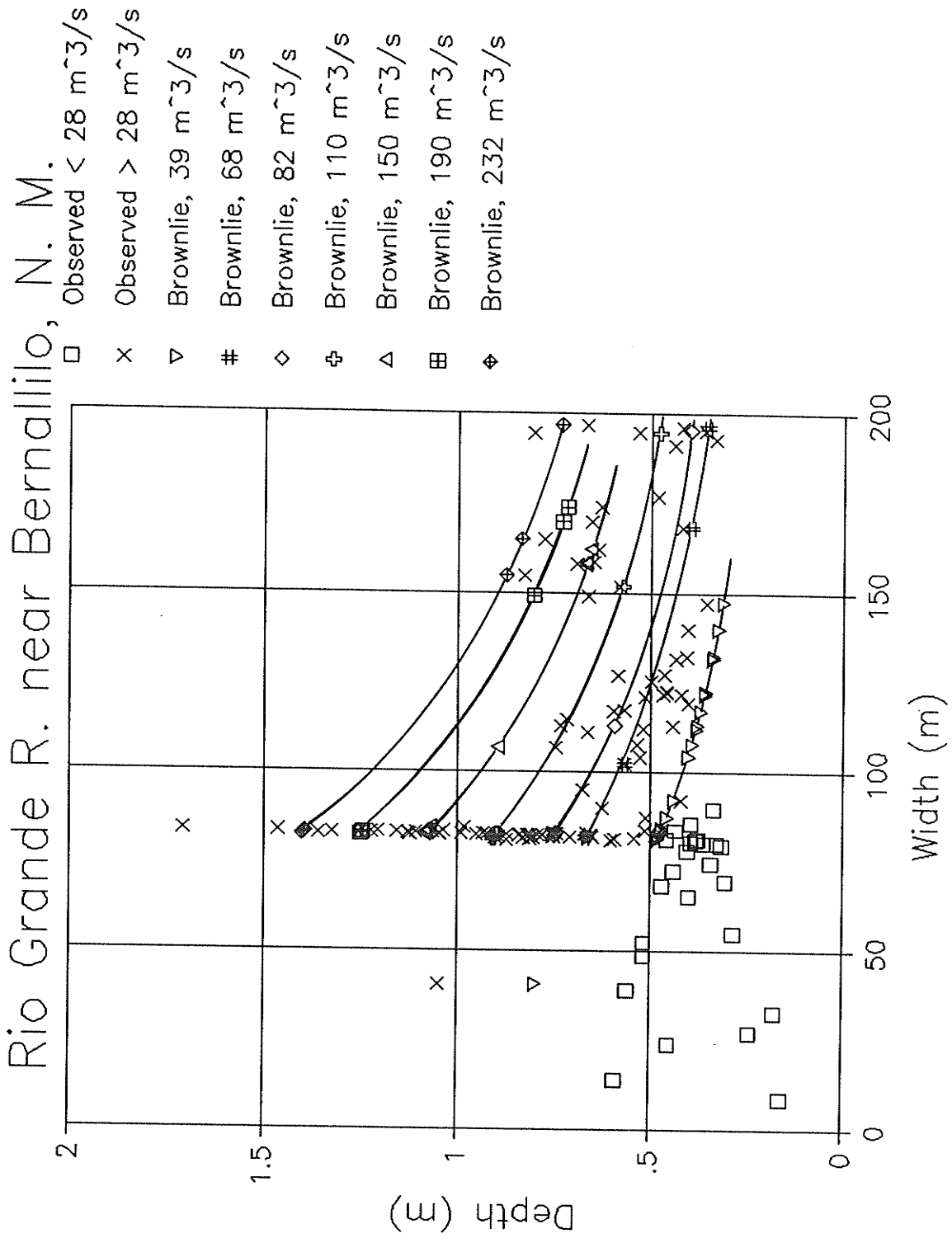


Figure 5.15 Rio Grande R. near Bernallilo, N.M.: Width versus Depth, Observed Data Base versus Brownlie's Equations

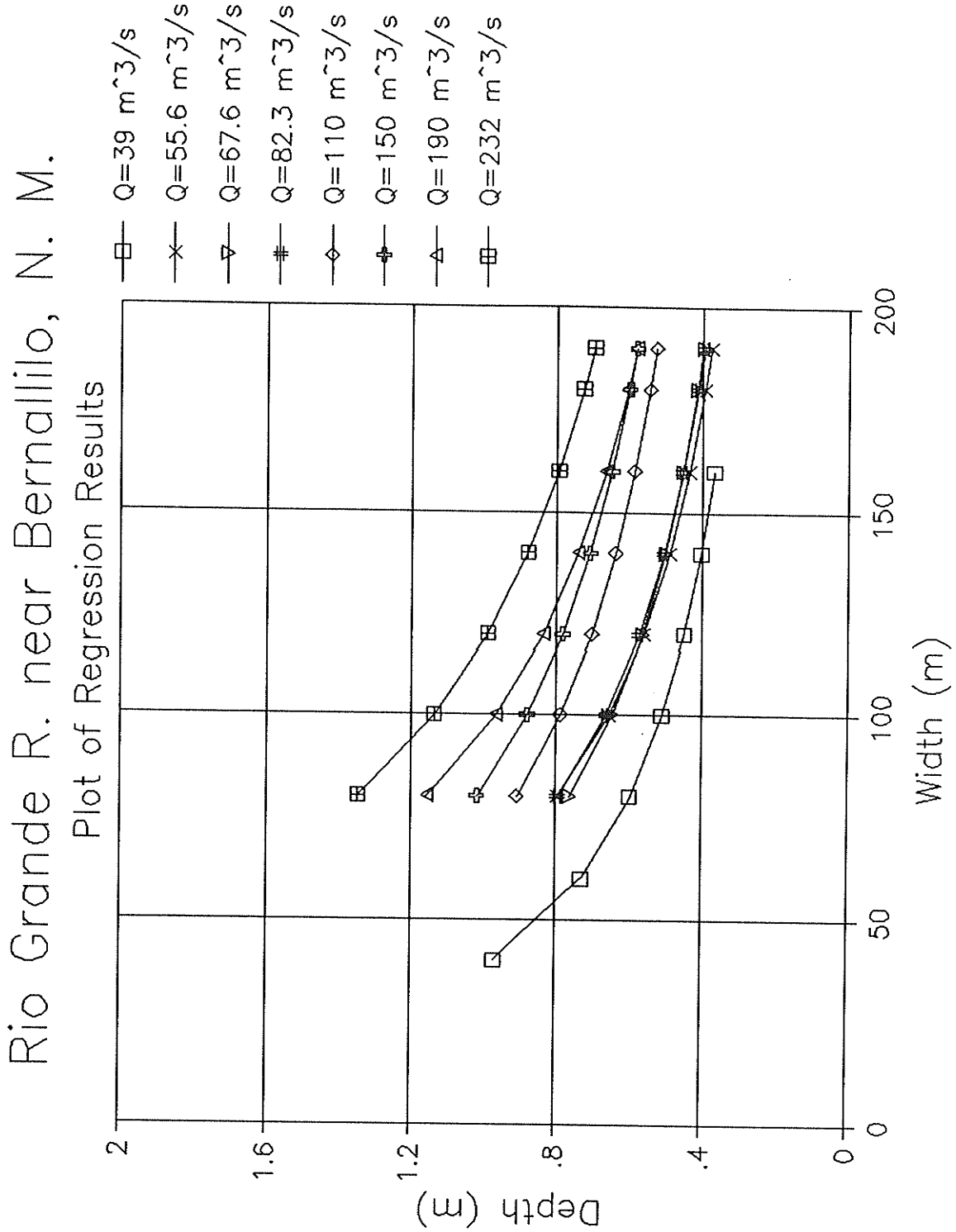


Figure 5.16 Rio Grande R. near Bernallilo, N.M.: Width versus Depth, Plot of Regression Results for Observed Data Base

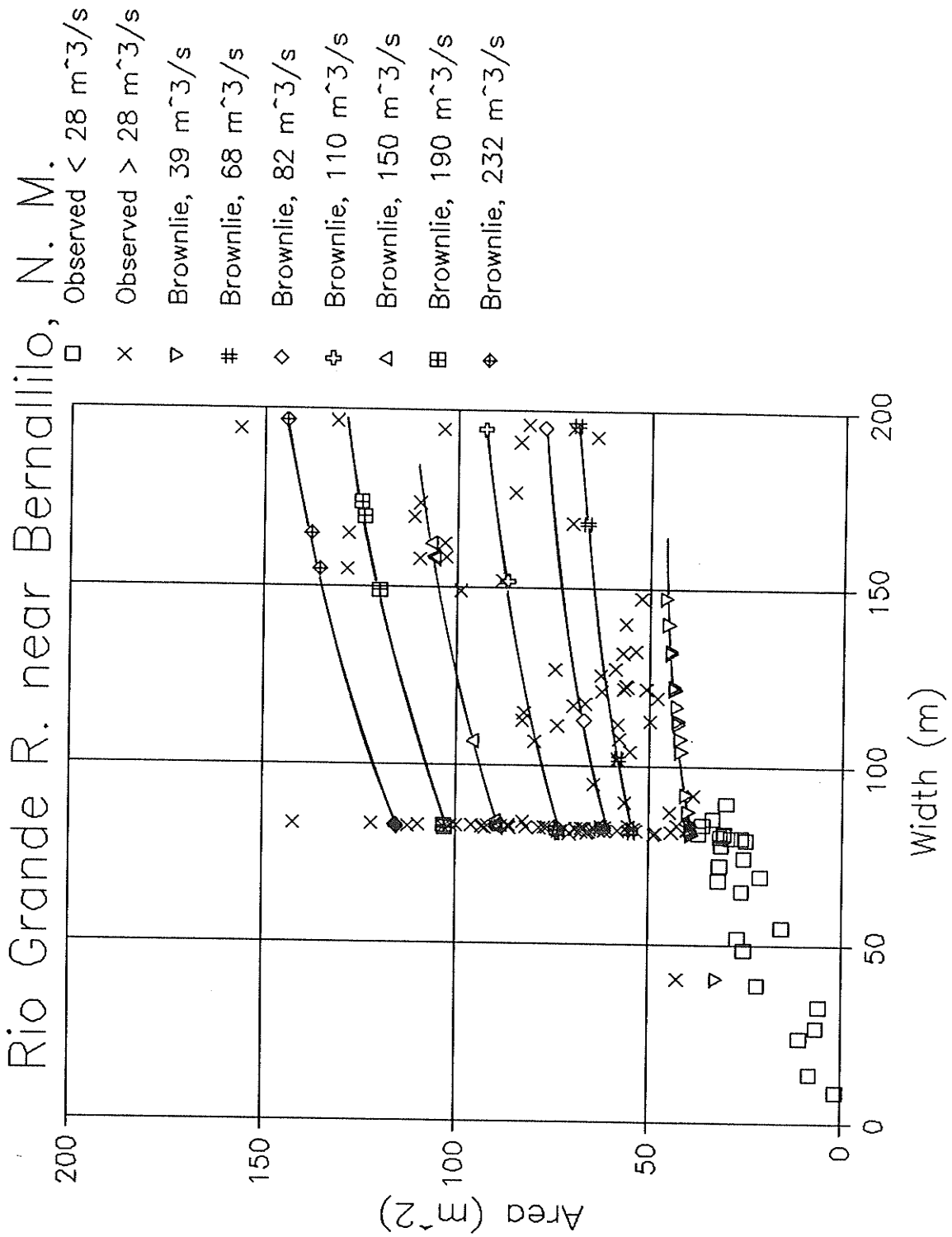


Figure 5.17 Rio Grande R. near Bernallilo, N.M.: Width versus Area, Observed Data Base versus Brownlie's Equations

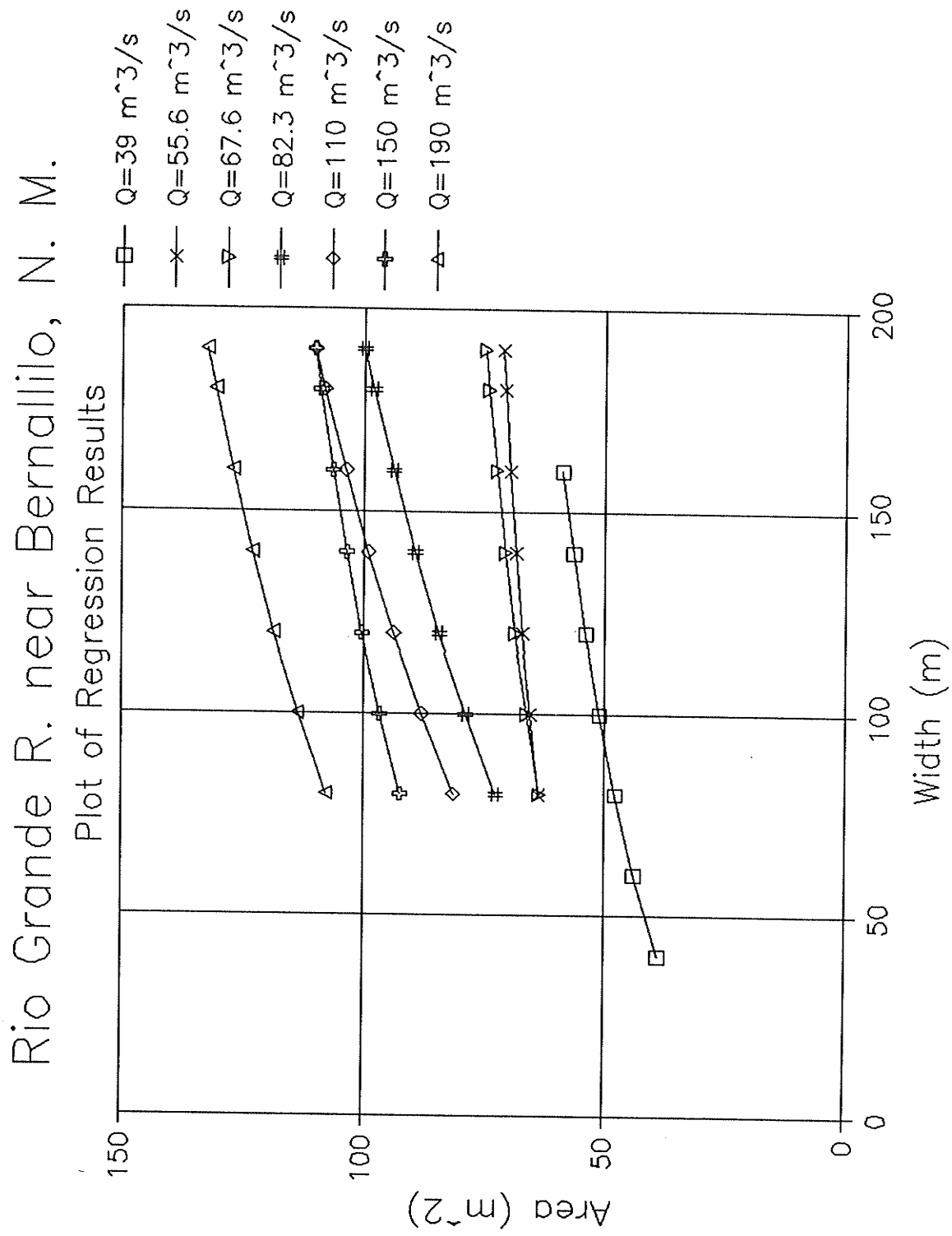


Figure 5.18 Rio Grande R. near Bernallilo, N.M.: Width versus Area, Plot of Regression Results for Observed Data Base

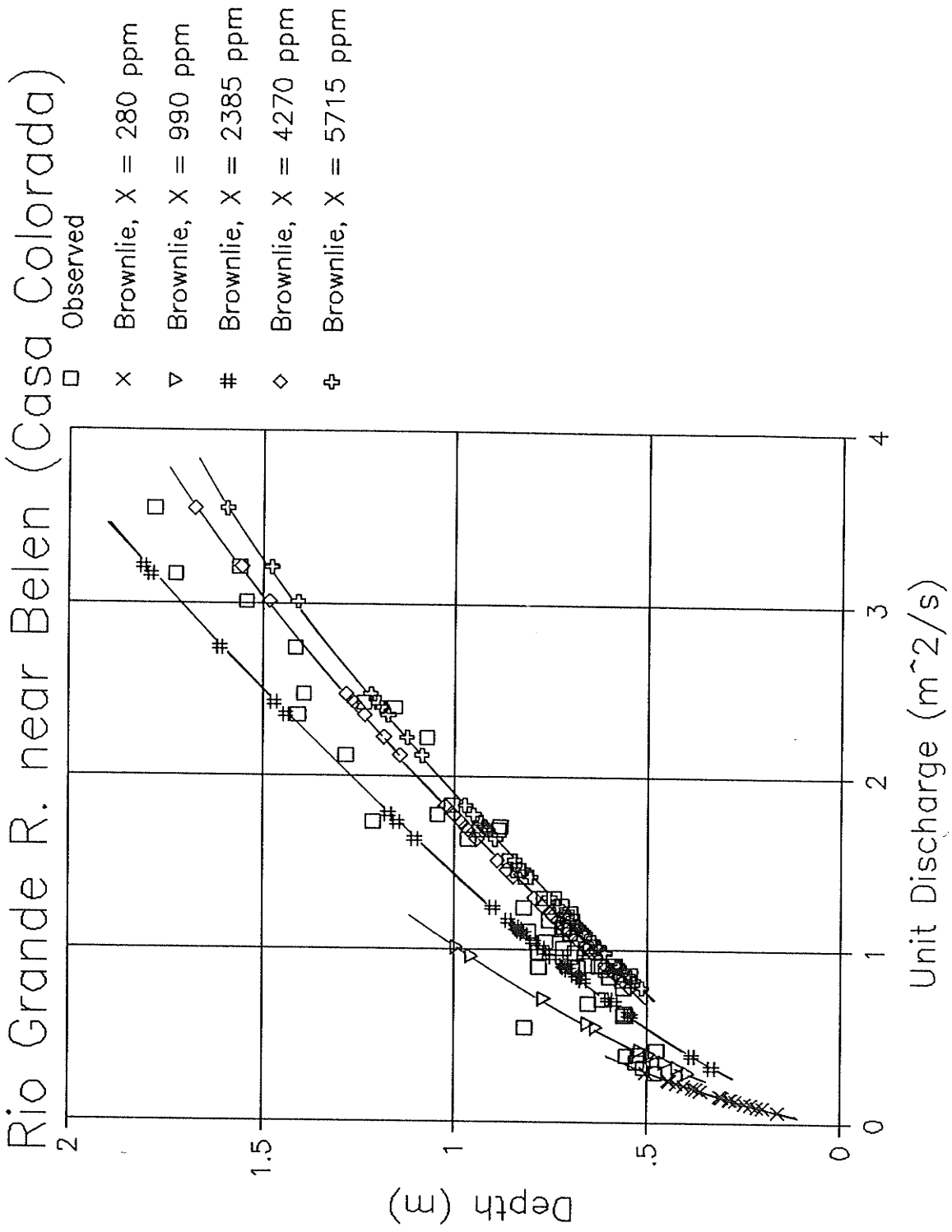


Figure 5.19 Rio Grande R., Casa Colorada, N.M.: Unit Discharge versus Depth, Observed Data Base versus Brownlie's Equations

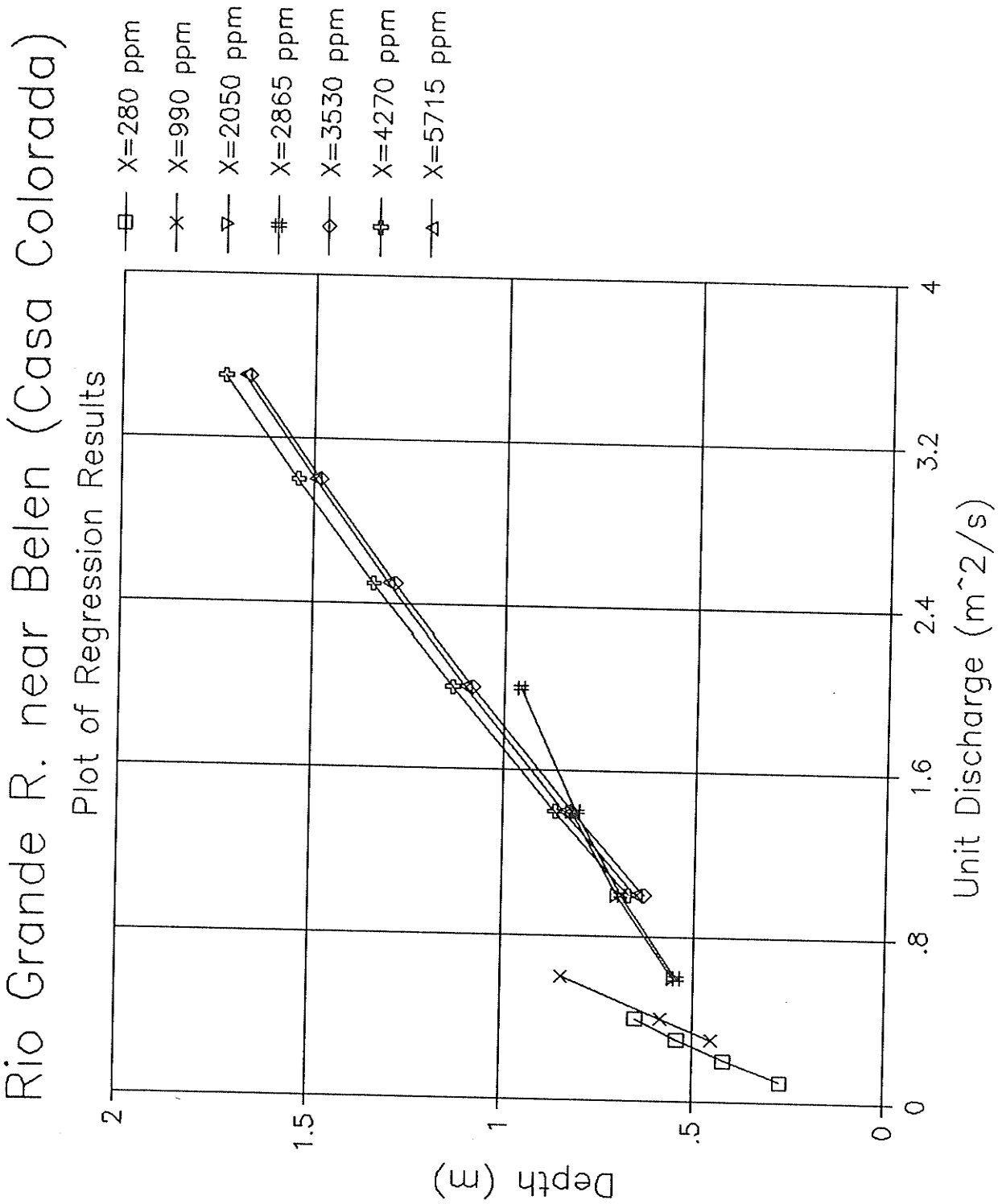


Figure 5.20 Rio Grande R., Casa Colorada, N.M.: Unit Discharge versus Depth, Plot of Regression Results for Observed Data Base

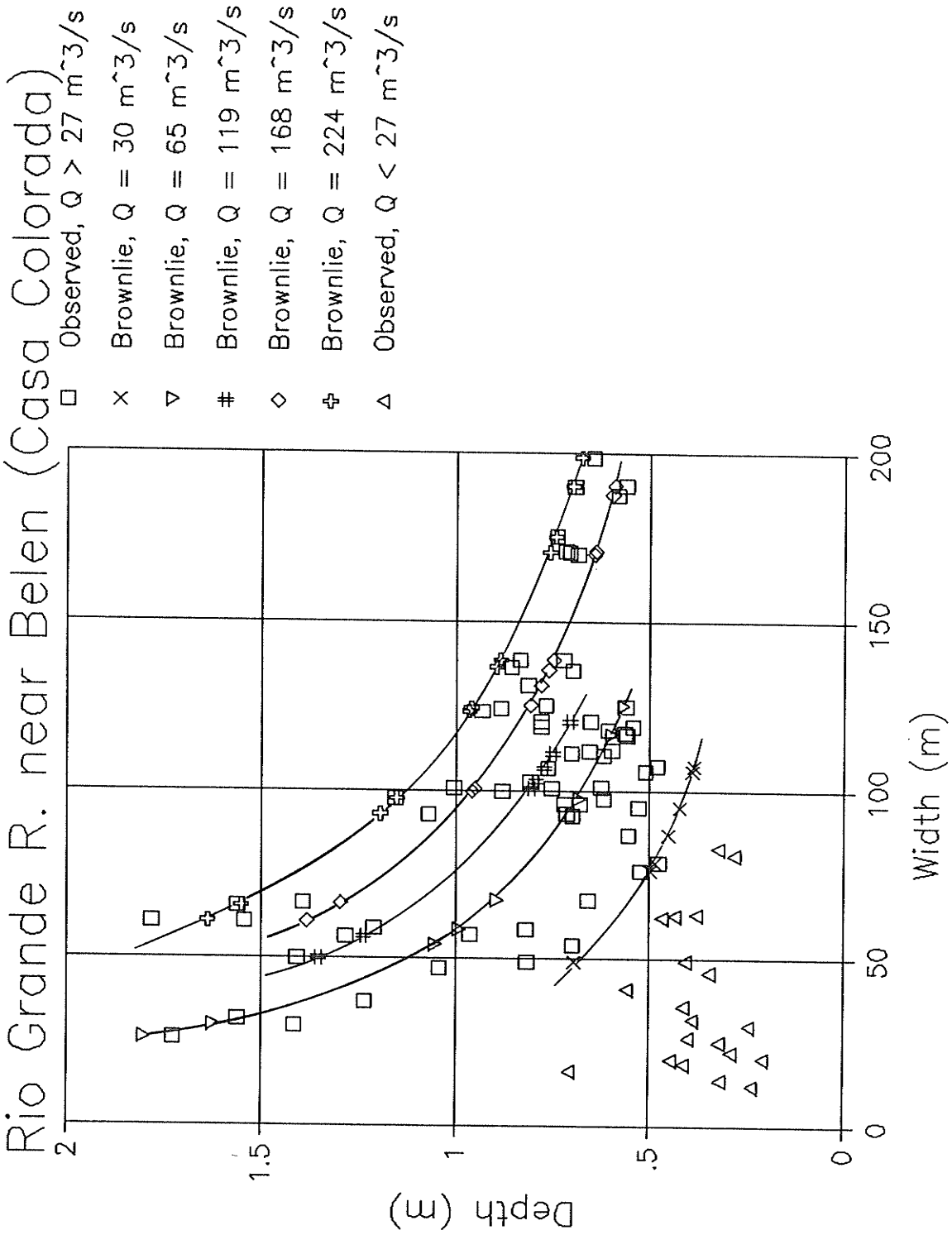


Figure 5.21 Rio Grande R., Casa Colorada, N.M.: Width versus Depth, Observed Data Base versus Brownlie's Equations

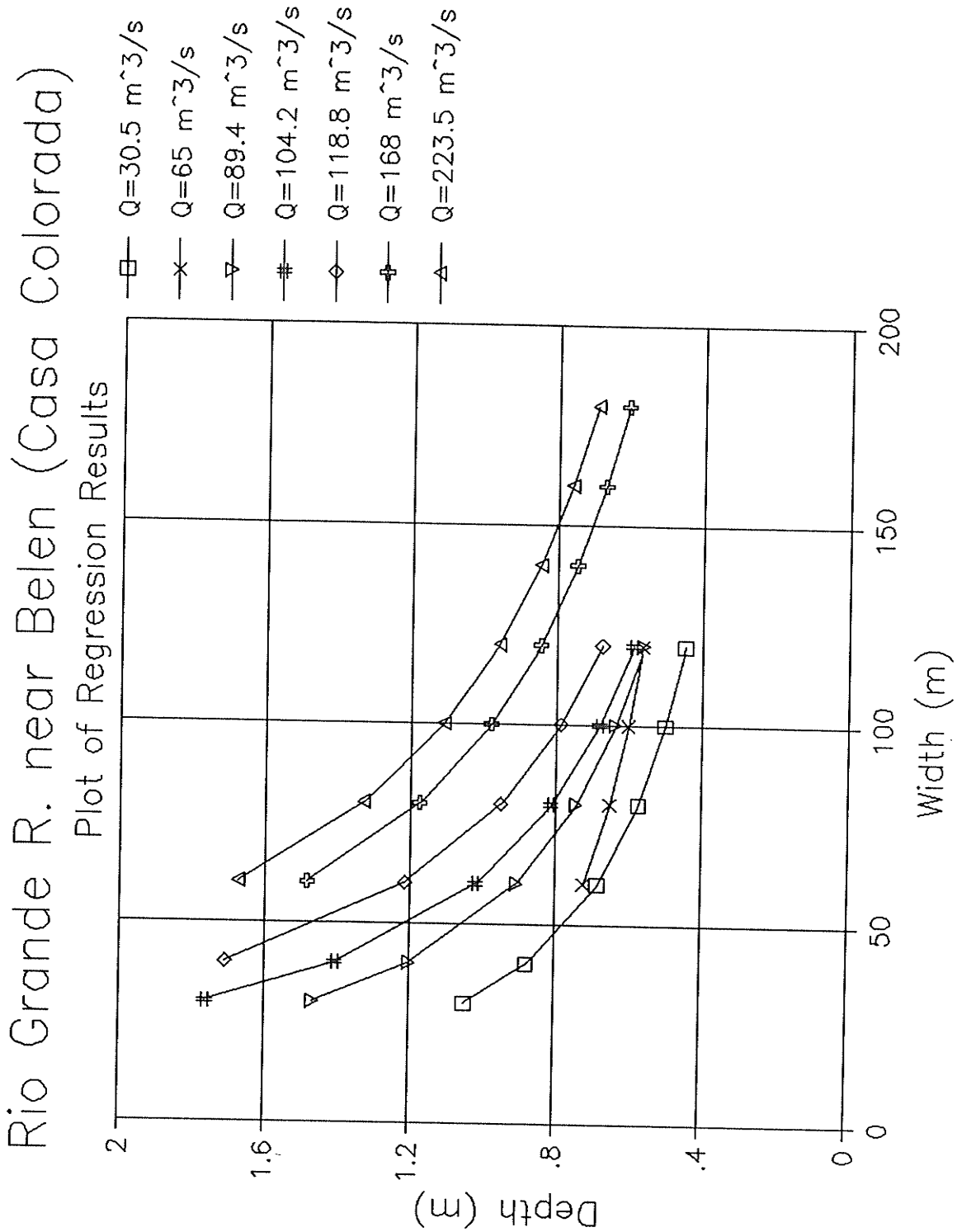


Figure 5.22 Rio Grande R., Casa Colorada, N.M.: Width versus Depth, Plot of Regression Results for Observed Data Base

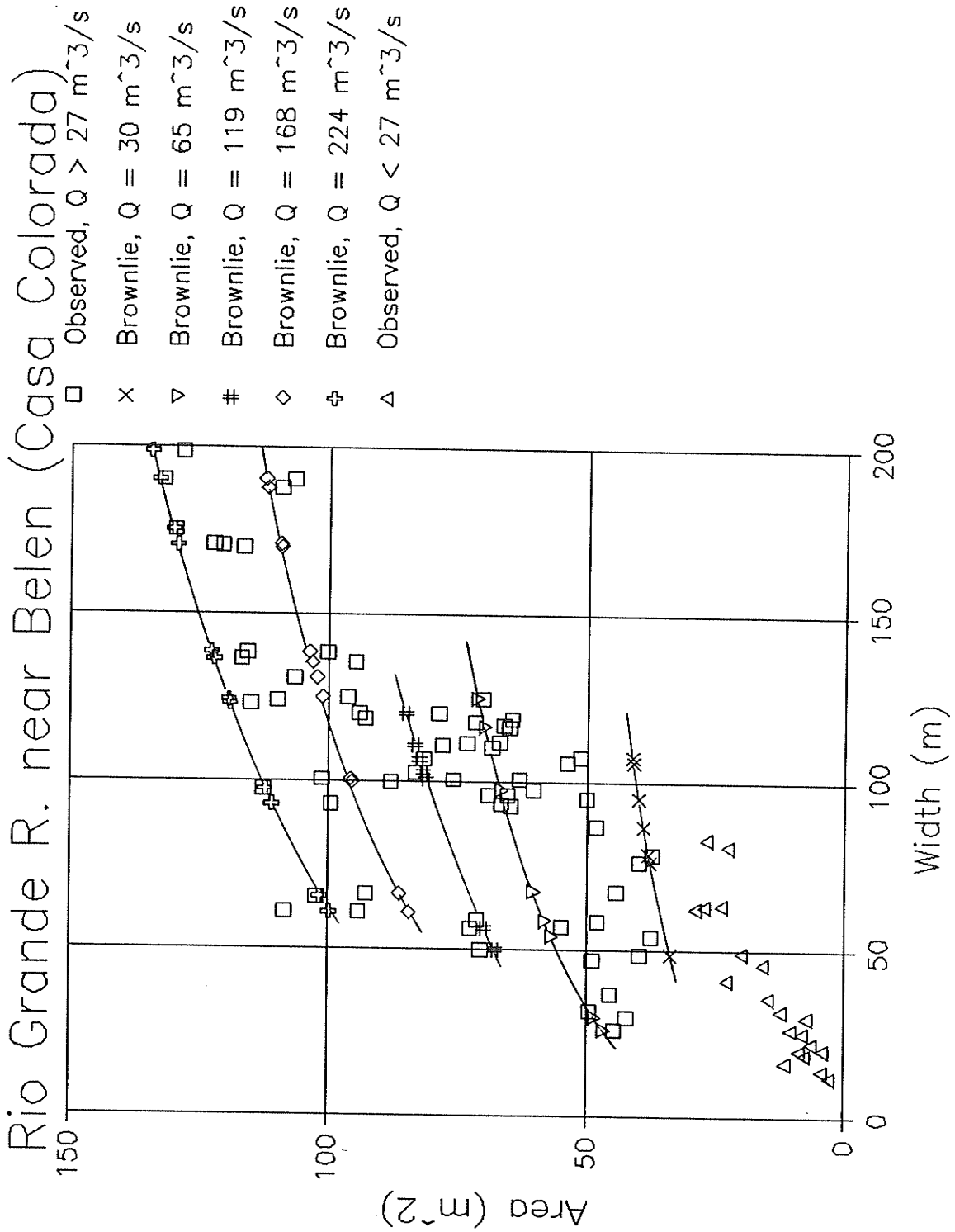


Figure 5.23 Rio Grande R., Casa Colorada, N.M.: Width versus Area, Observed Data Base versus Brownlie's Equations

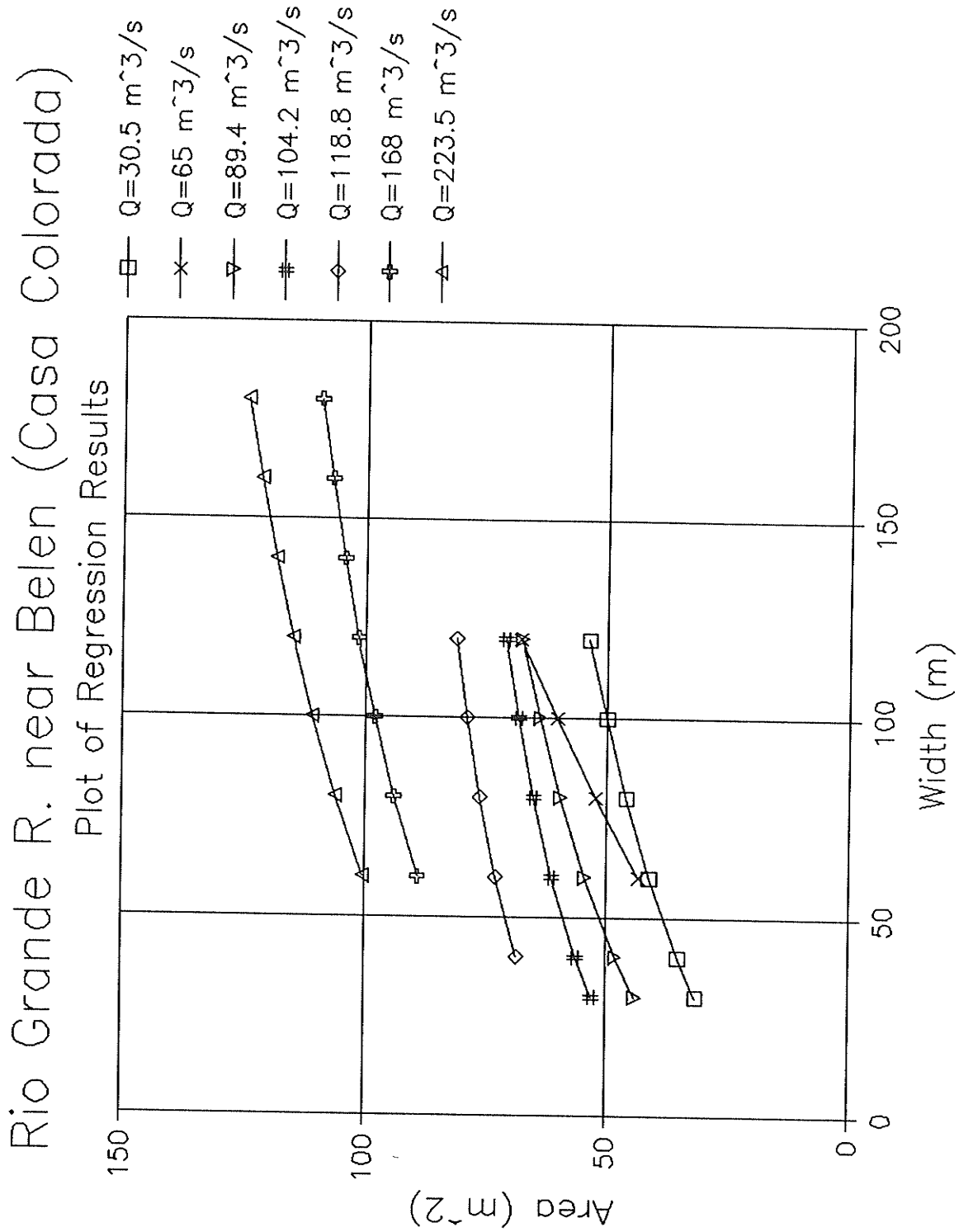


Figure 5.24 Rio Grande R., Casa Colorada, N.M.: Width versus Area, Plot of Regression Results for Observed Data Base

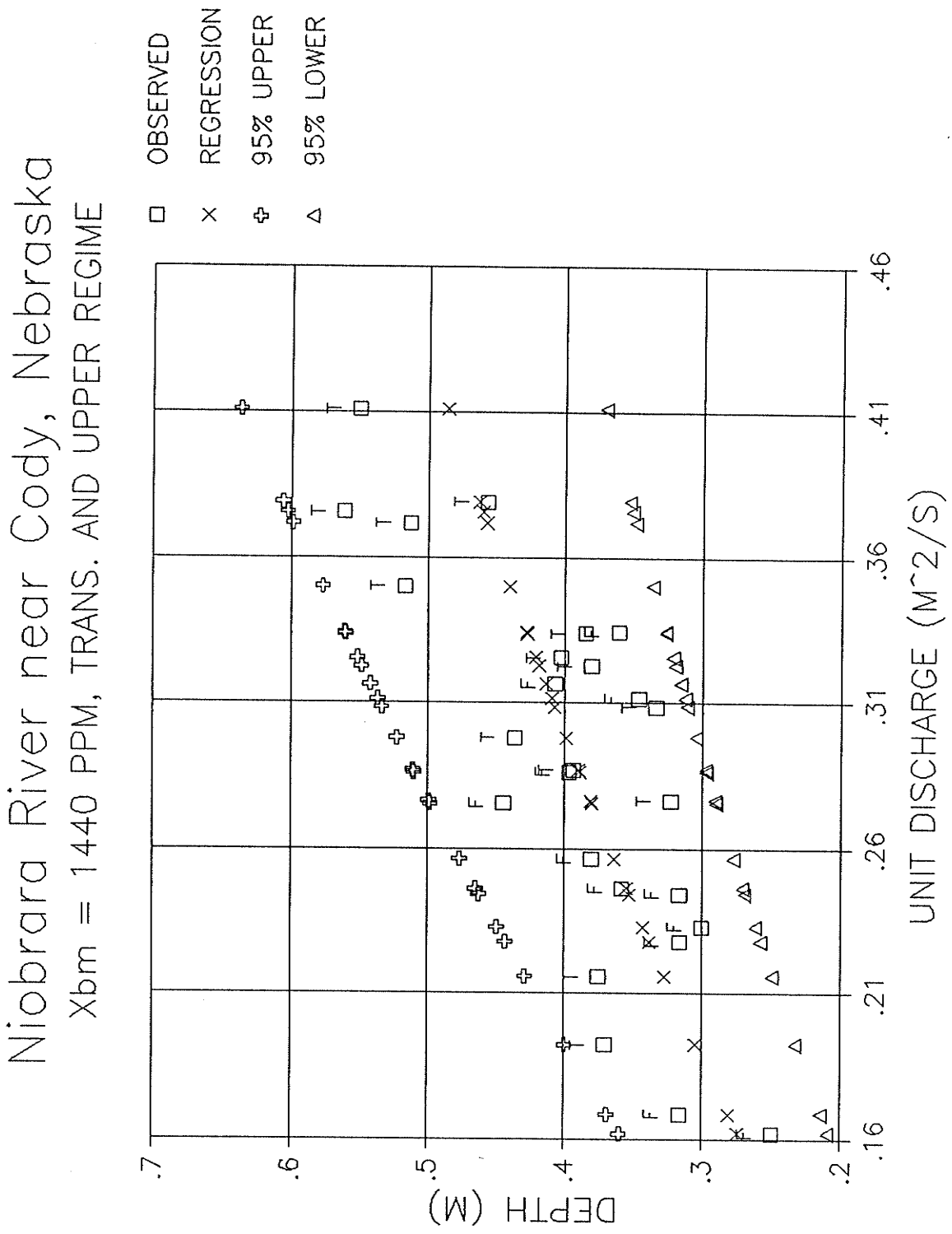


Figure 5.25 Niobrara River near Cody, Nebr.: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 1440 ppm;

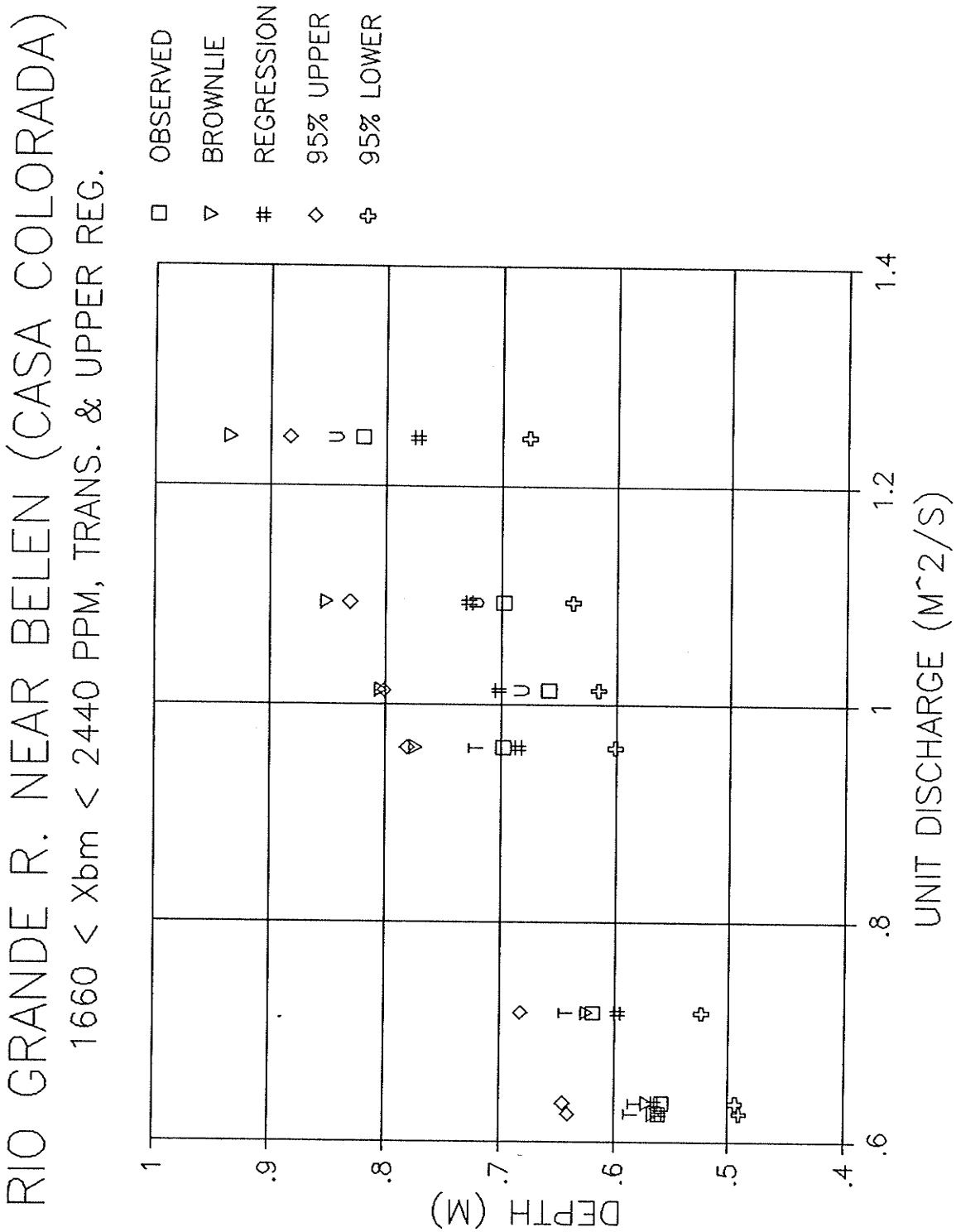


Figure 5.26 Rio Grande River, Casa Colorada: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 2050 ppm;

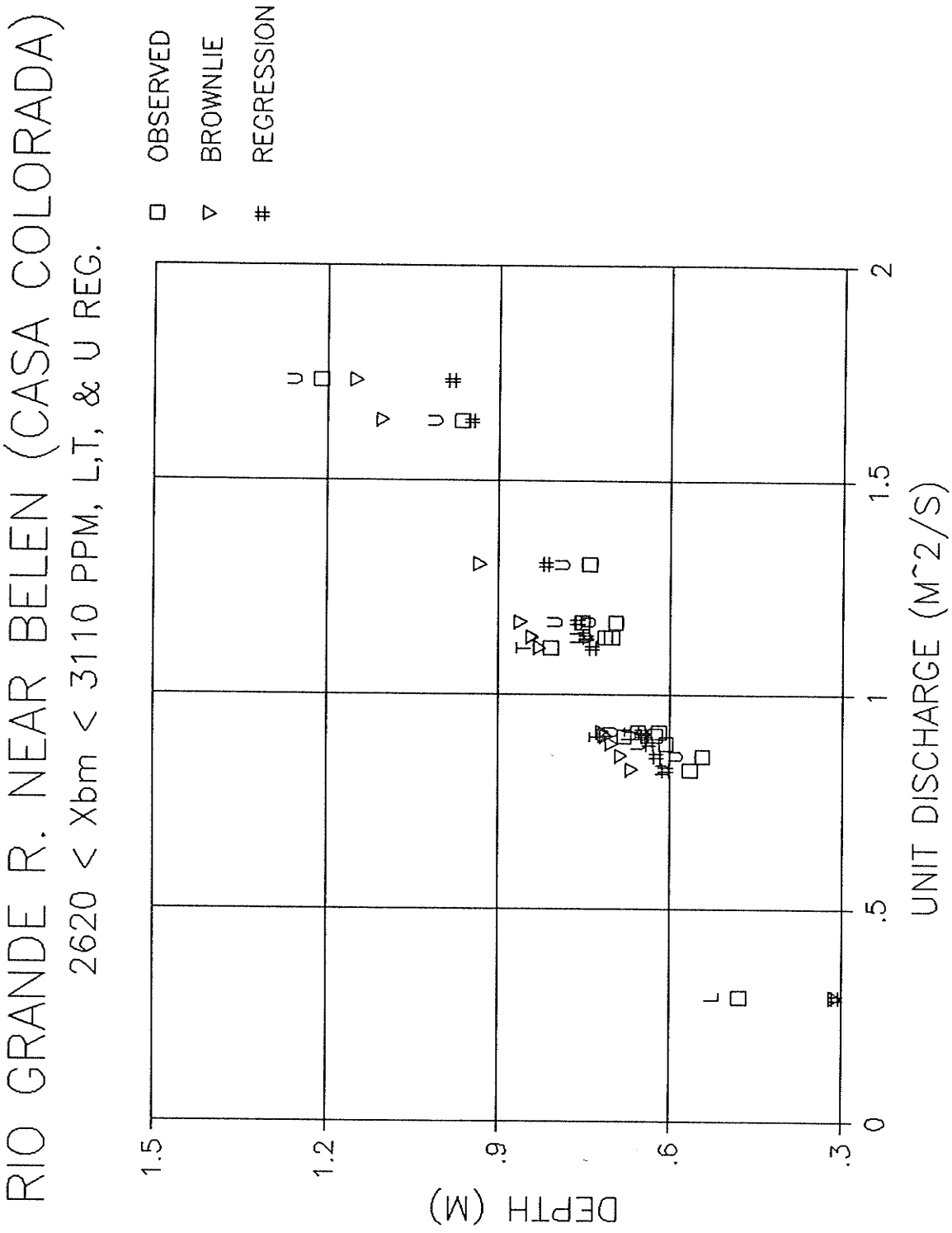


Figure 5.27 Rio Grande River, Casa Colorada: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 2865 ppm;

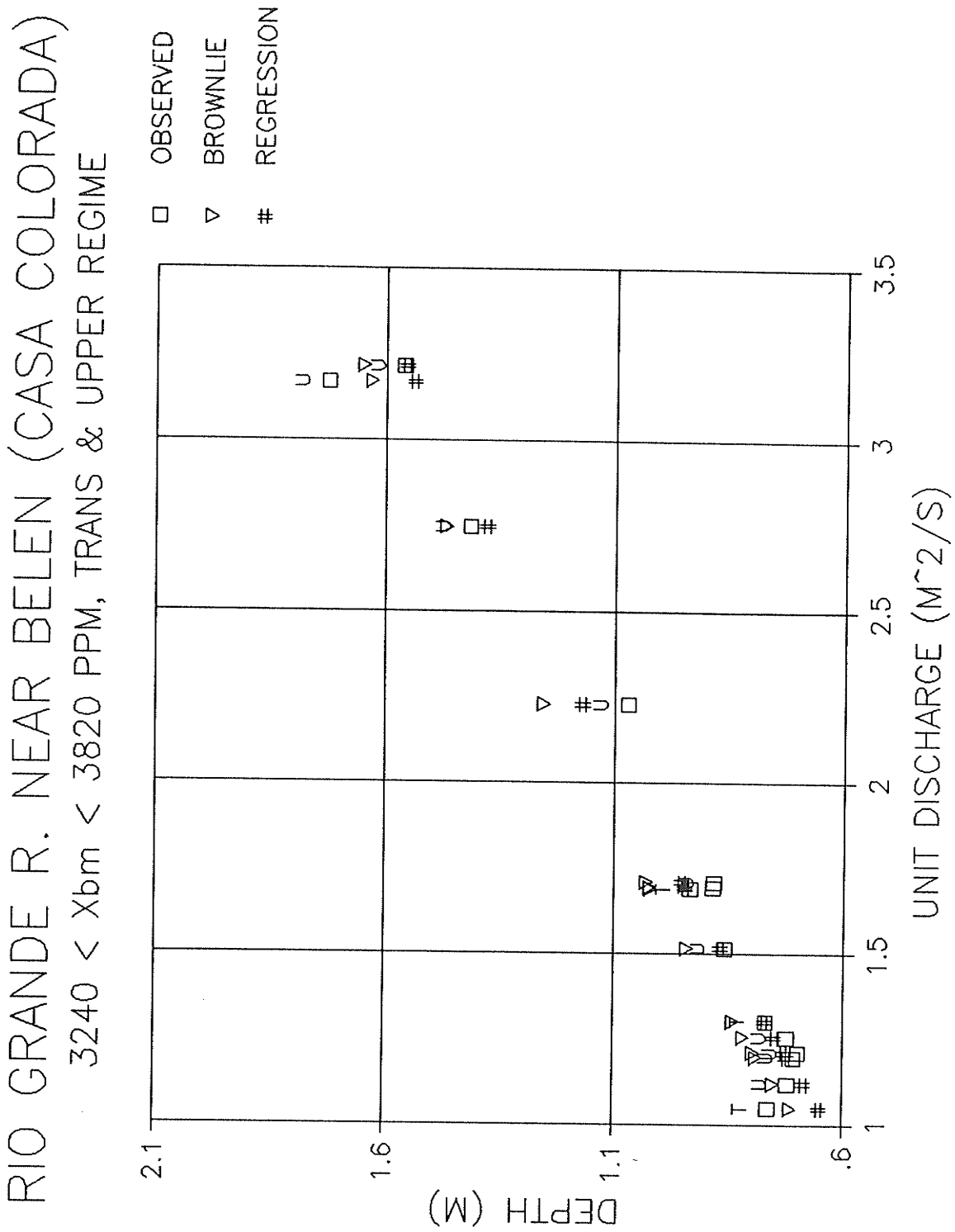


Figure 5.28 Rio Grande River, Casa Colorada: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 3530 ppm;

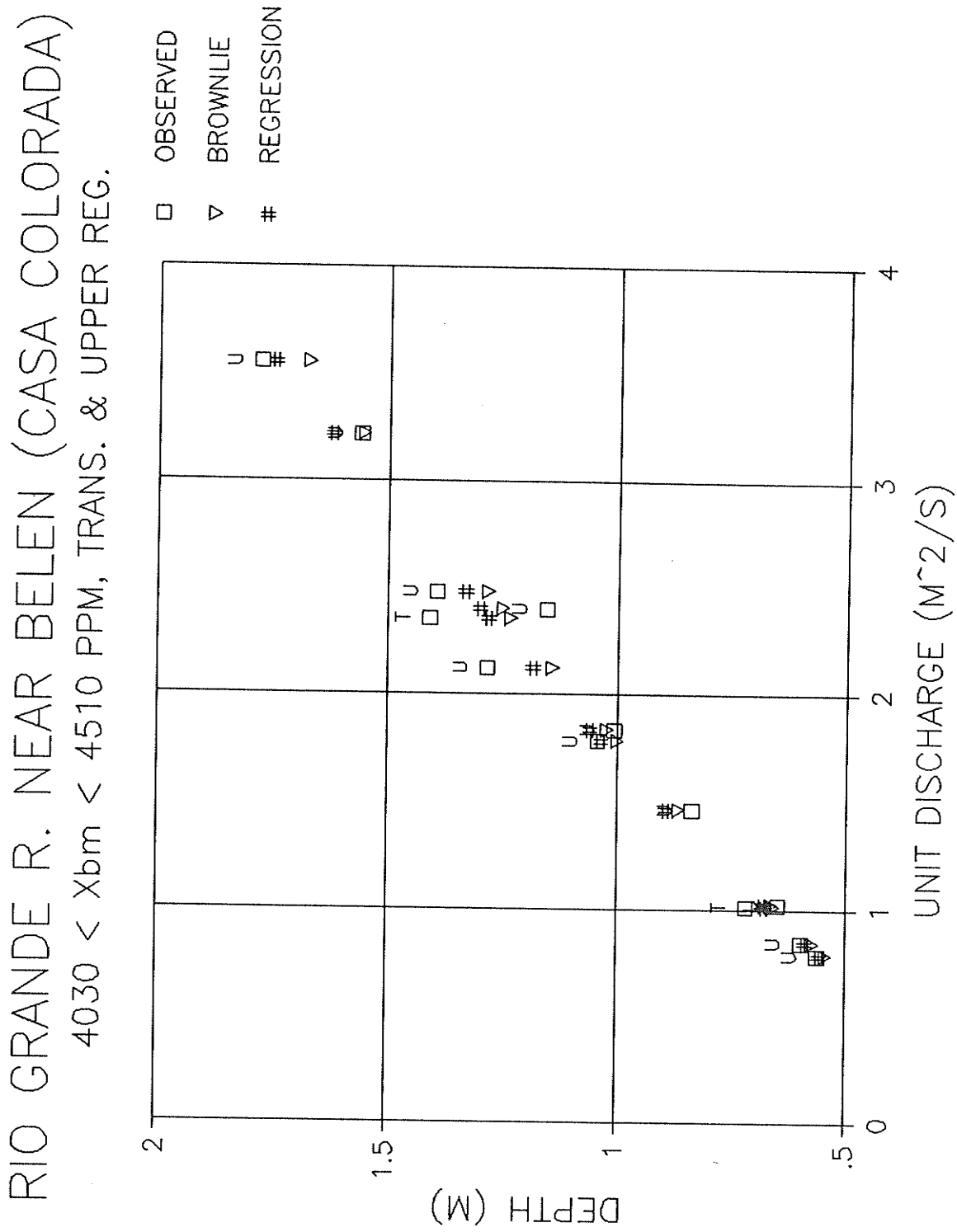


Figure 5.29 Rio Grande River, Casa Colorada: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 4270 ppm;

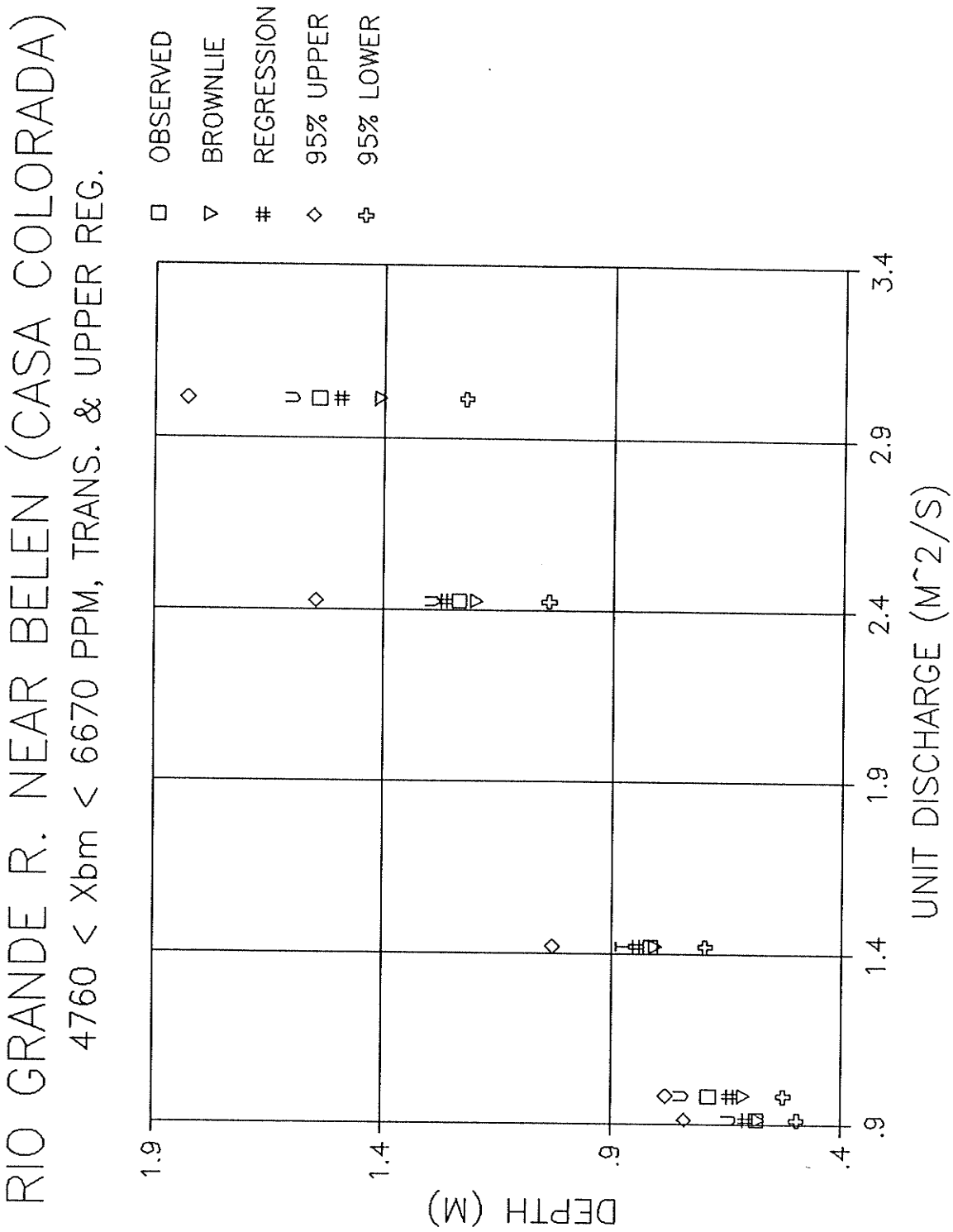


Figure 5.30 Rio Grande River, Casa Colorada: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 5715 ppm;

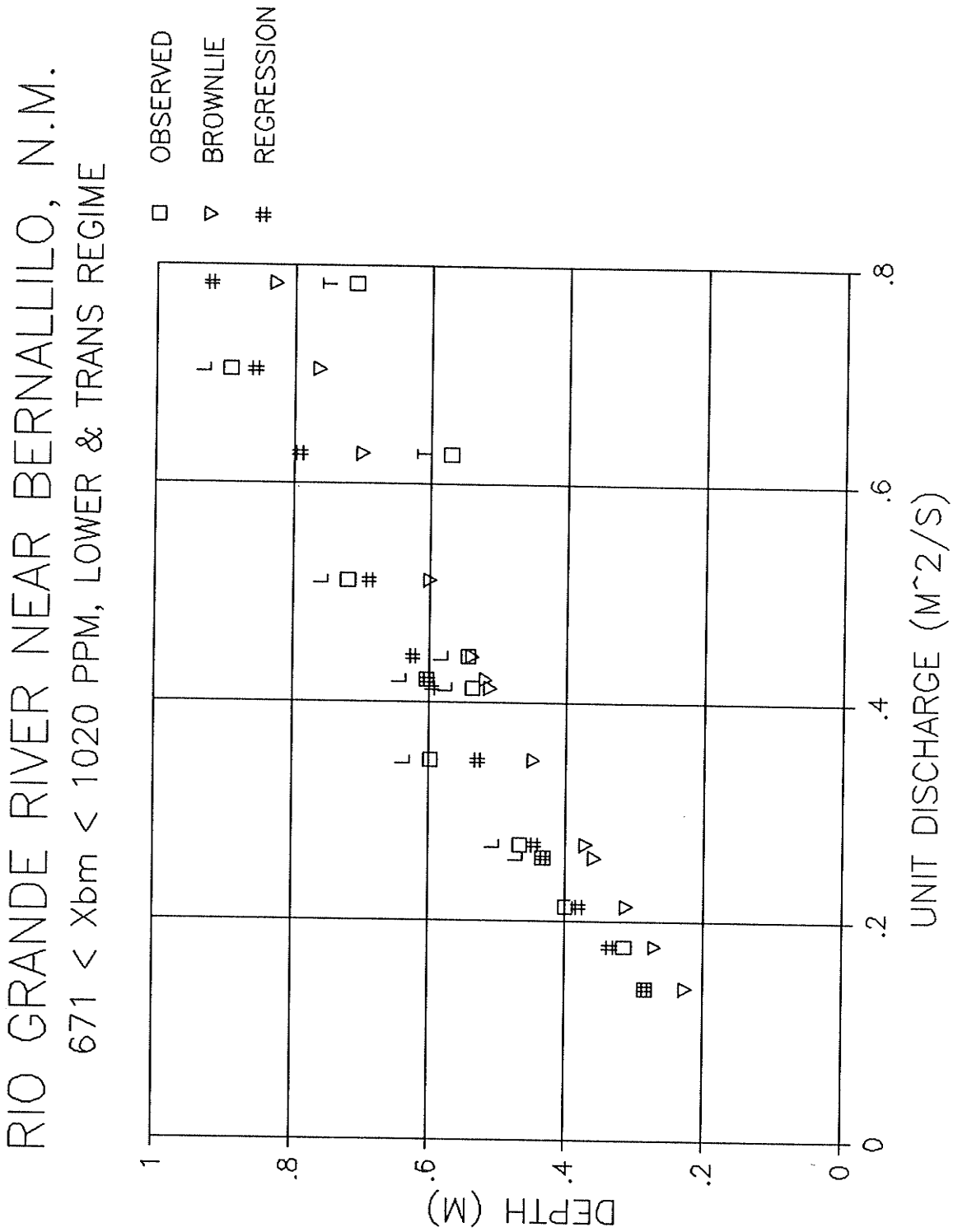


Figure 5.31 Rio Grande River near Bernallilo: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 850 ppm;

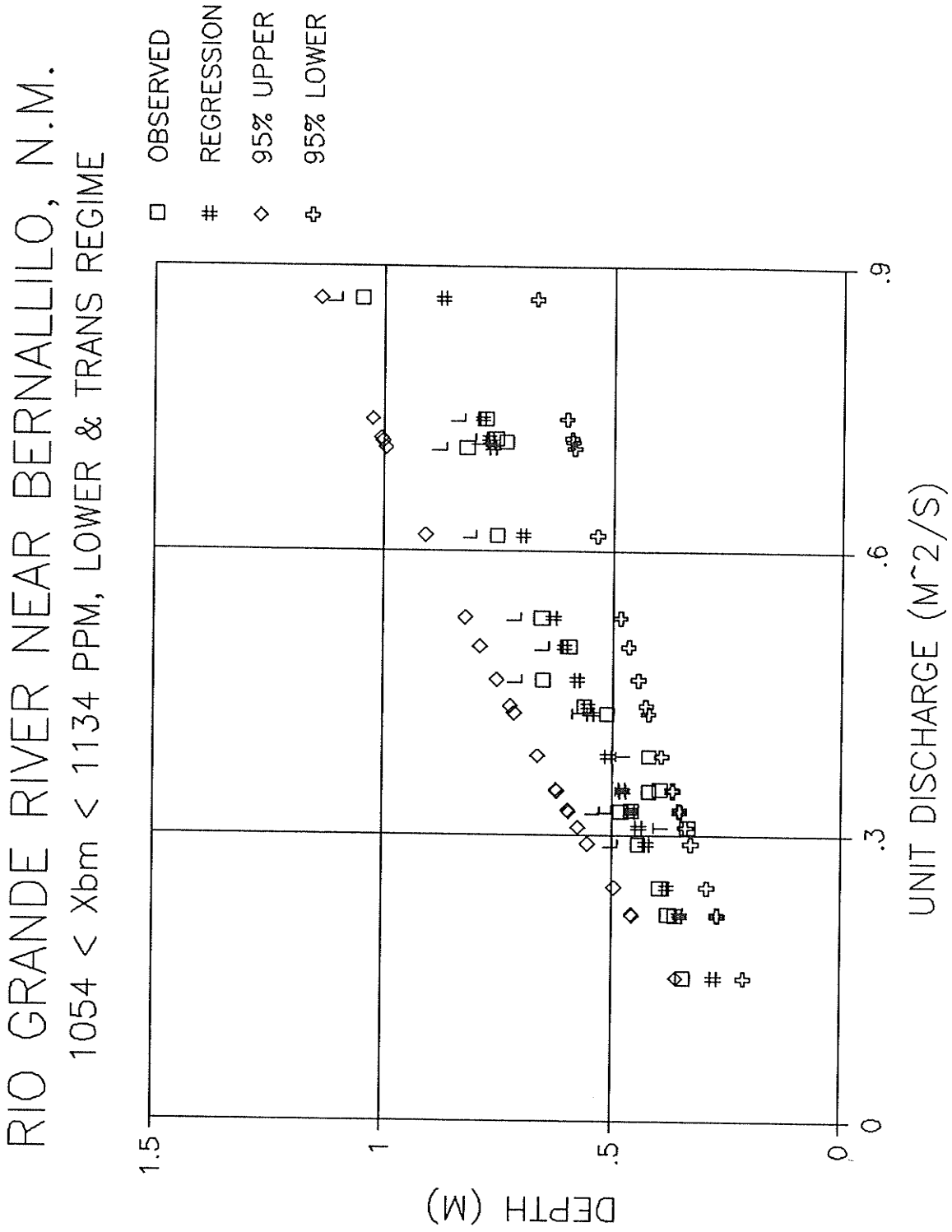


Figure 5.32 Rio Grande River near Bernallilo: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 1180 ppm;

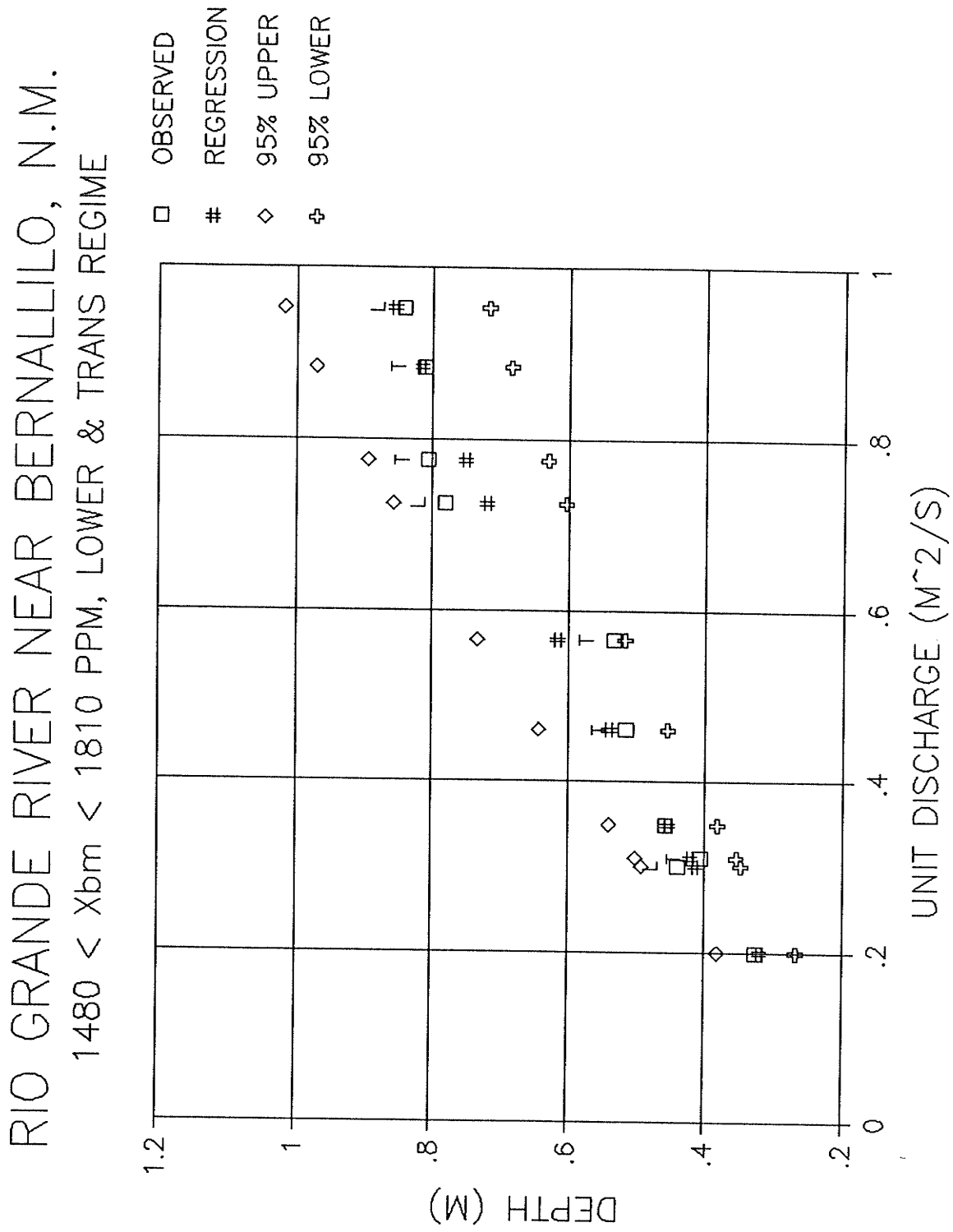


Figure 5.33 Rio Grande River near Bernallilo: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 1650 ppm;

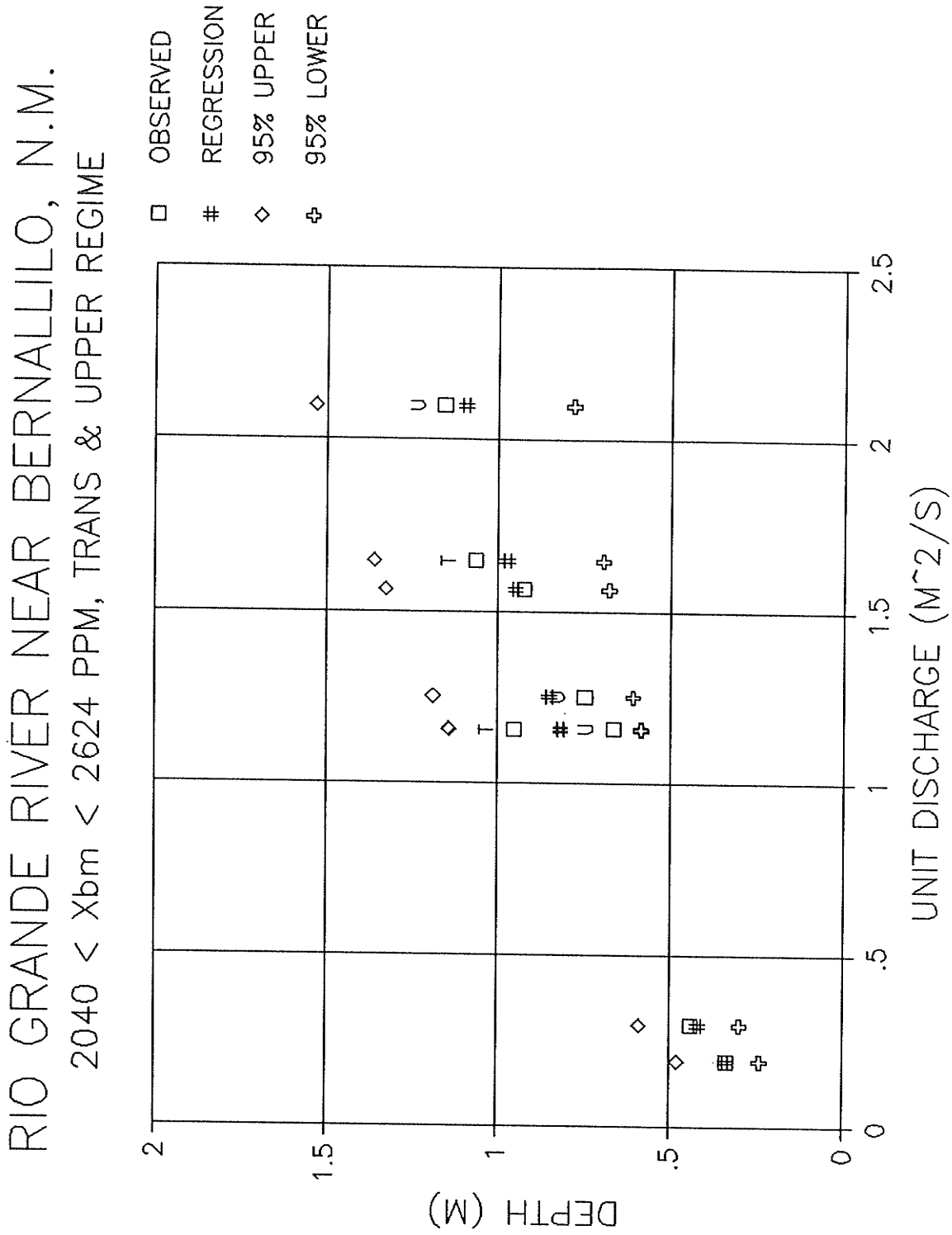


Figure 5.34 Rio Grande River near Bernallilo: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 2330 ppm;

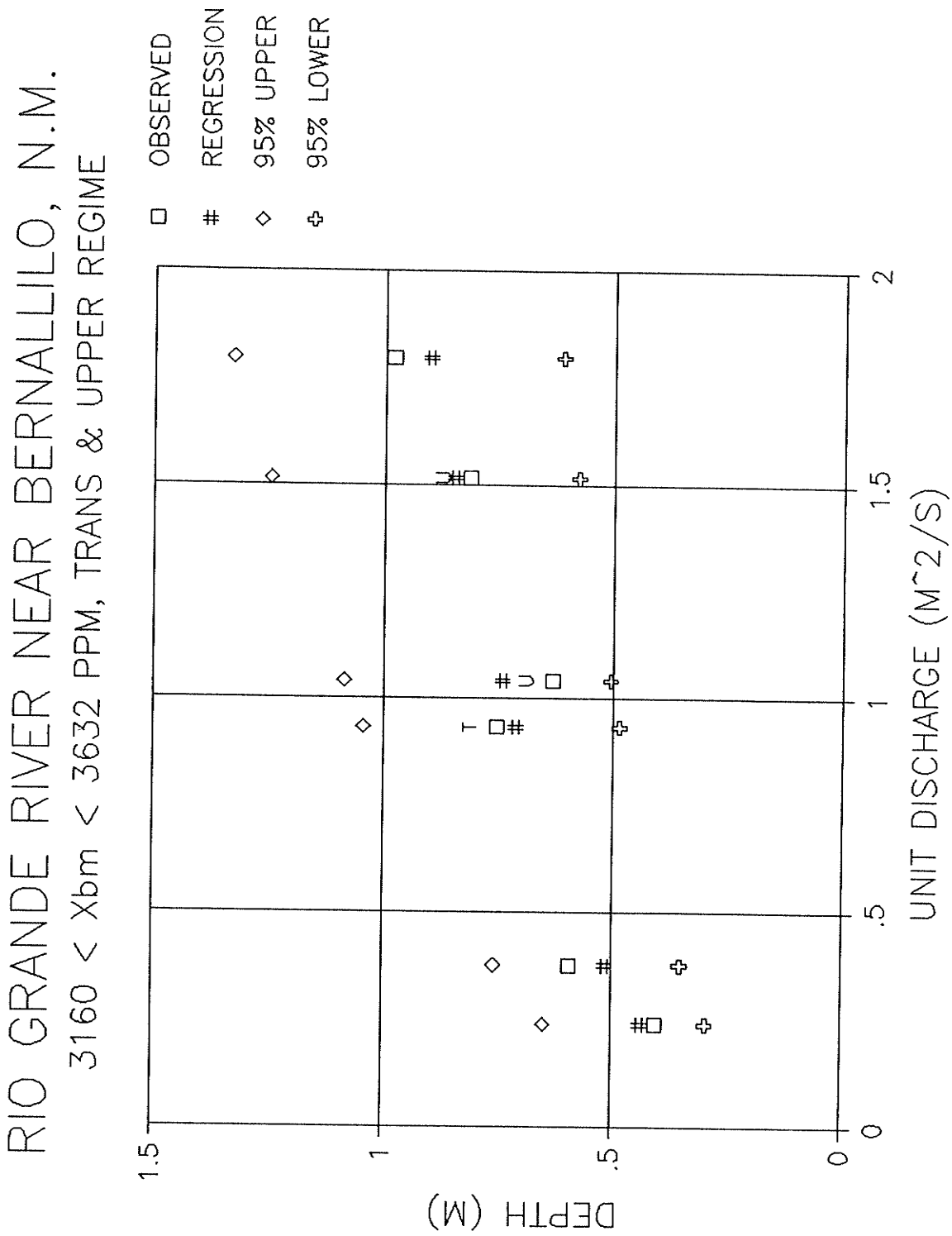


Figure 5.35 Rio Grande River near Bernallilo: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 3530 ppm;

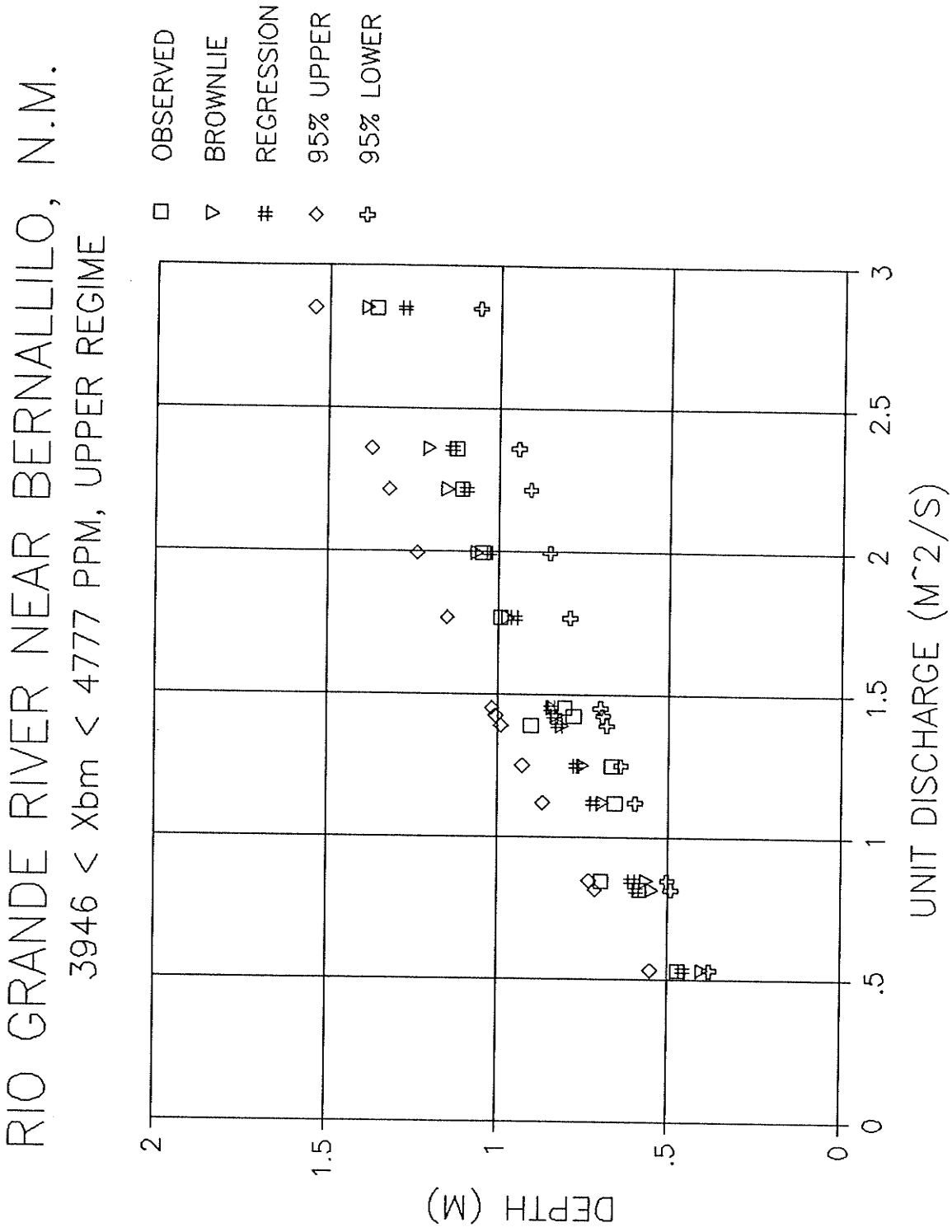


Figure 5.36 Rio Grande River near Bernallilo: Unit Discharge versus Depth, Observed Flow Regimes at X_{bm} = 4360 ppm;

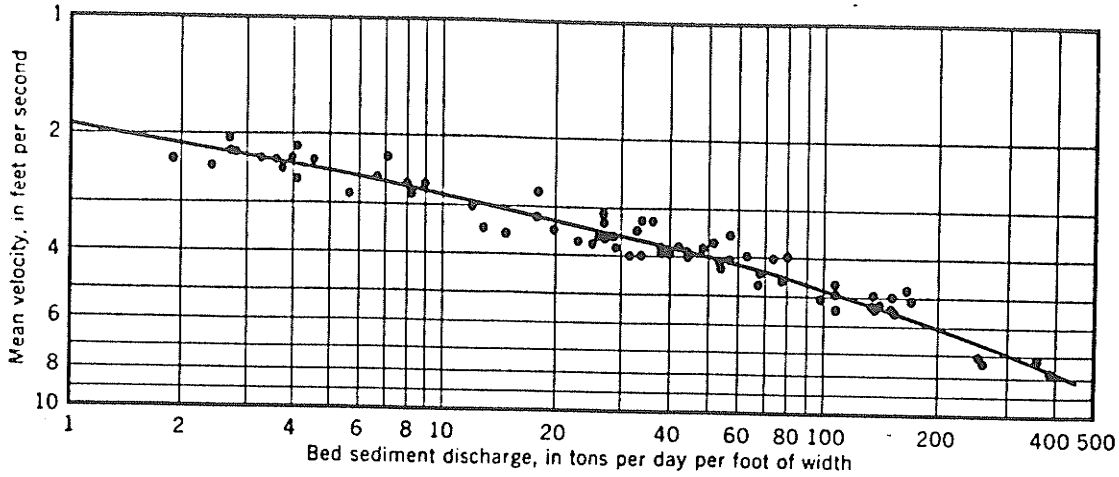


Figure 6.1 Mississippi River at St. Louis, Mo.; Sediment Discharge per Unit Width versus Mean Velocity (Colby, 1964)

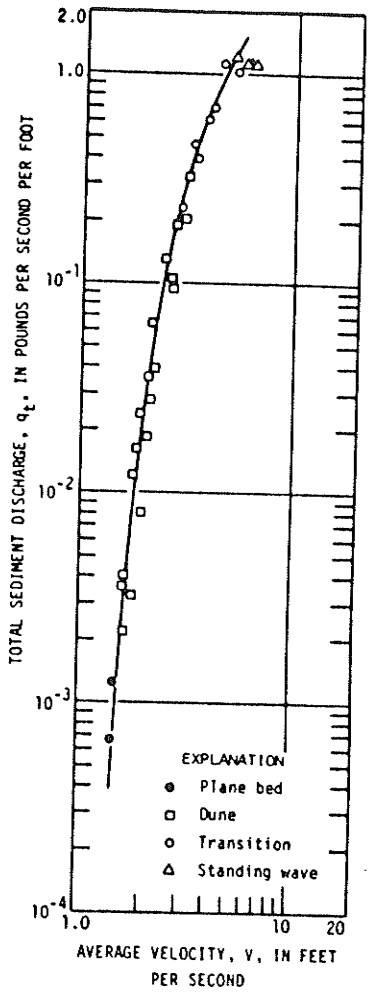


Figure 6.2 Relationship between Total Sediment Discharge and Mean Velocity; for 0.93 mm Sand in an 8-foot wide flume (Yang, 1972)

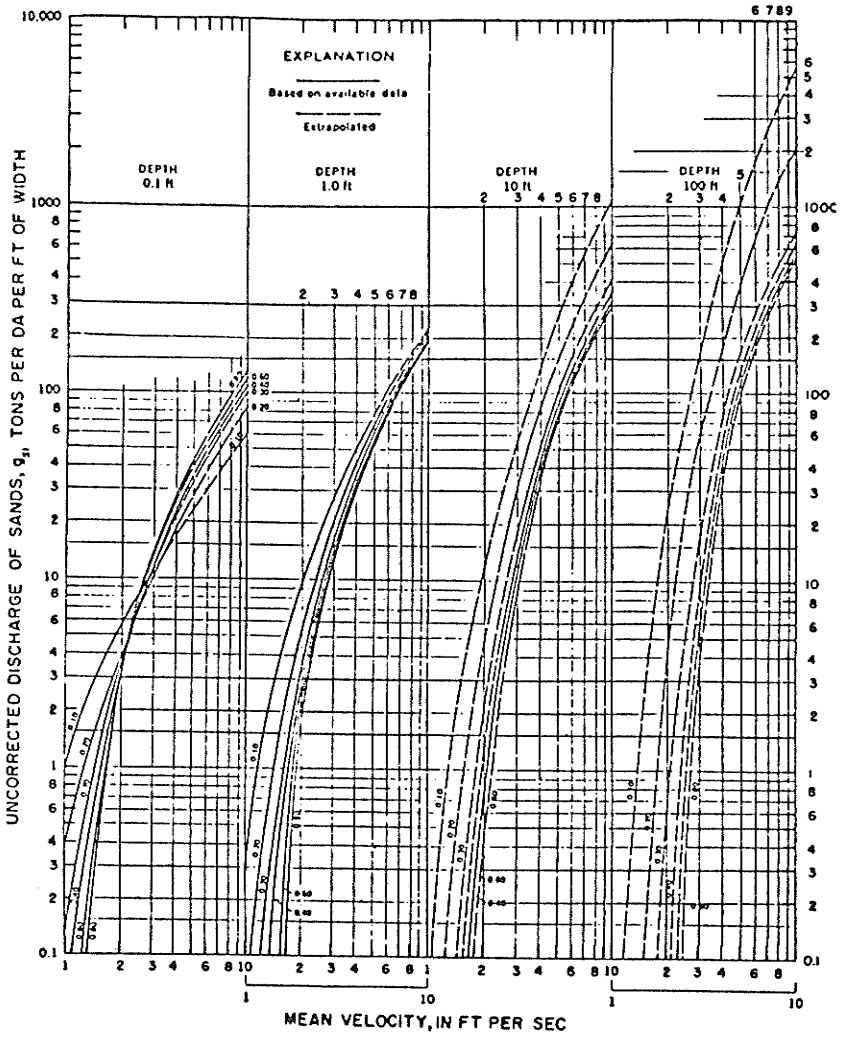


Figure 6.3 Colby's (1964) Relationship for Sediment Discharge of Sands versus Mean Velocity (@ 60 F)

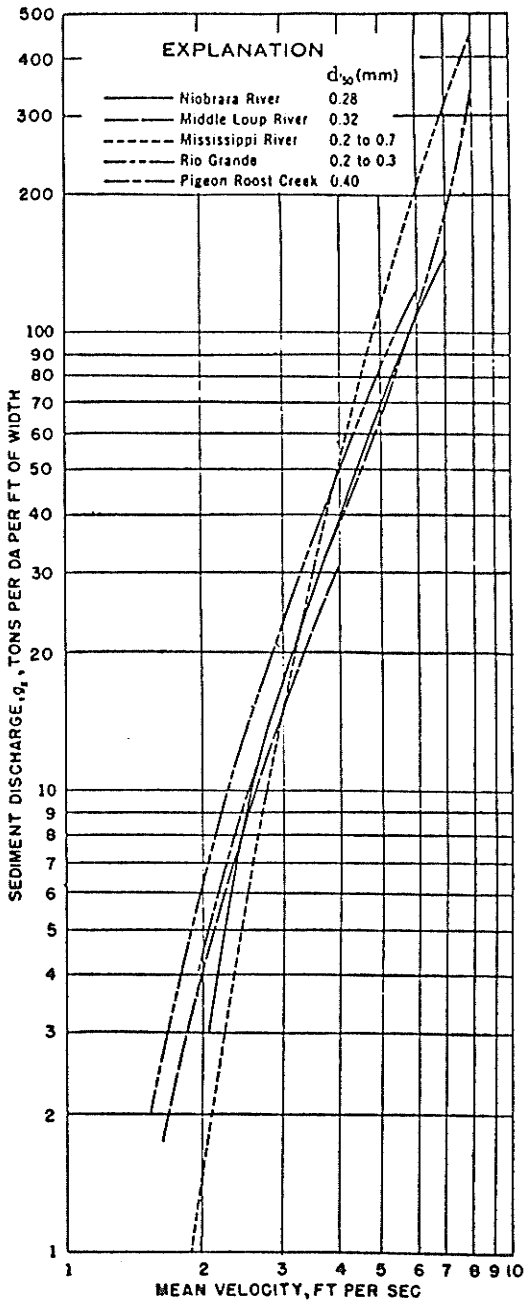


Figure 6.4 Relationship between Observed Sediment Discharge and Mean Velocity for Five Sand-Bed Streams (@ 60°F) (Colby, 1964)

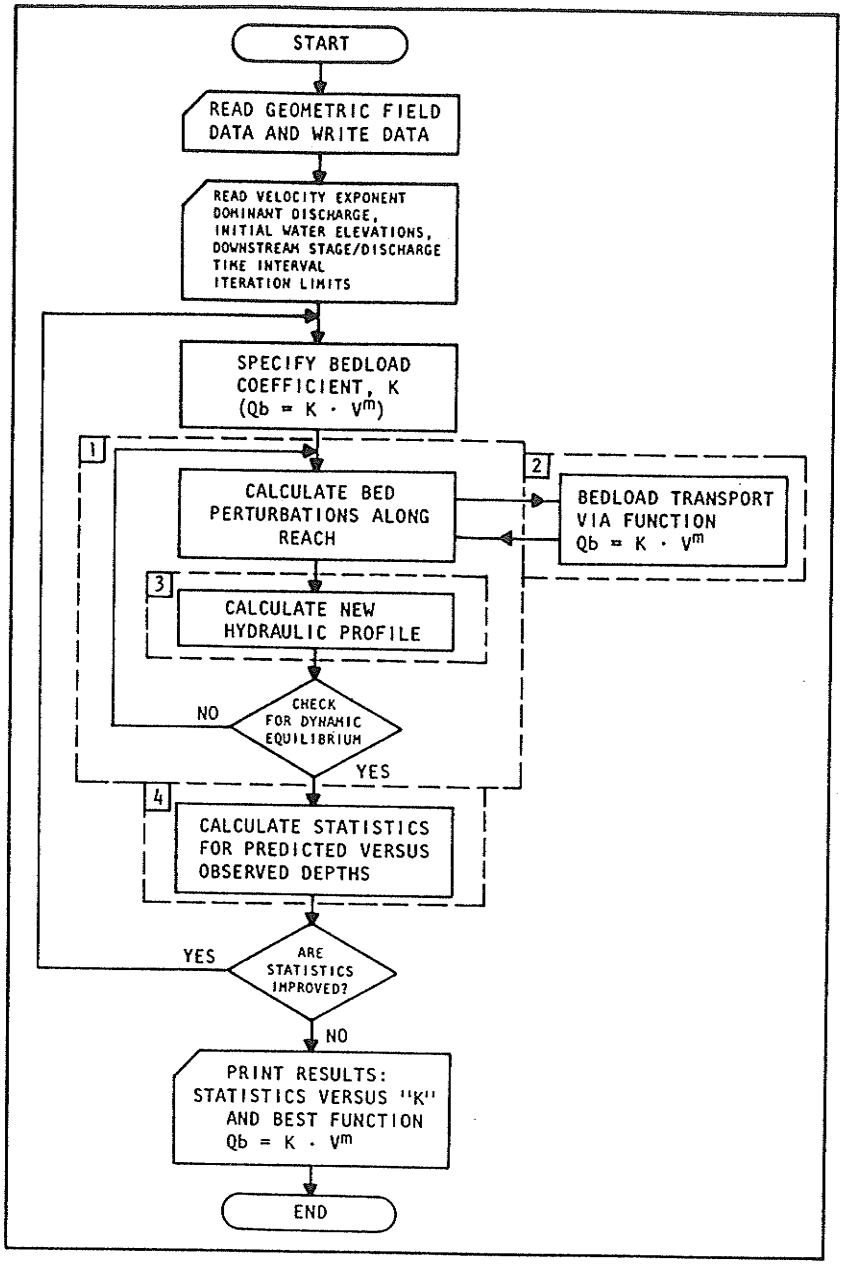


Figure 6.5 Flowchart for Bed Function Program (Tamburi et al., 1983)

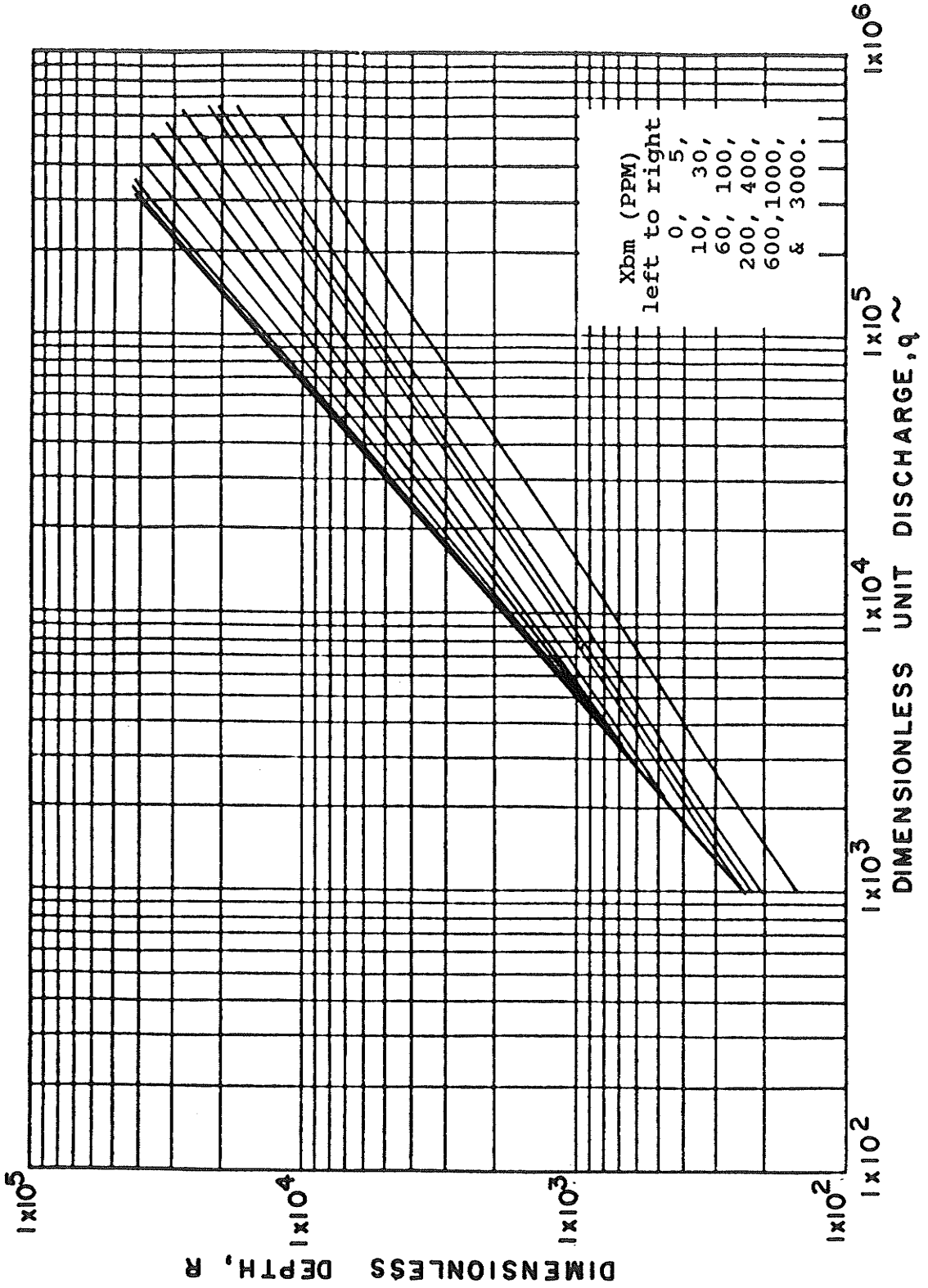
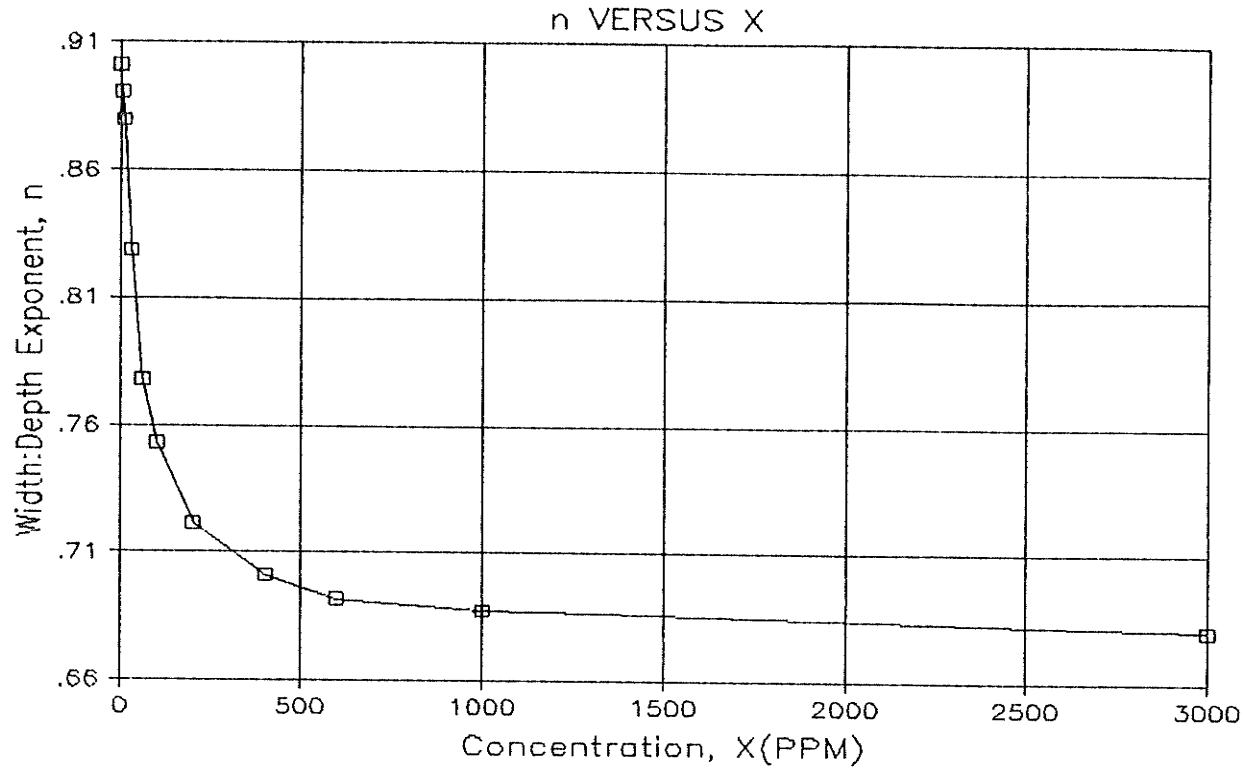


Figure C-1 Engelund's (1967) Sediment Transport and Resistance Equations; Least-Square Relations, Log_{10} (q~) versus Log_{10} (R)



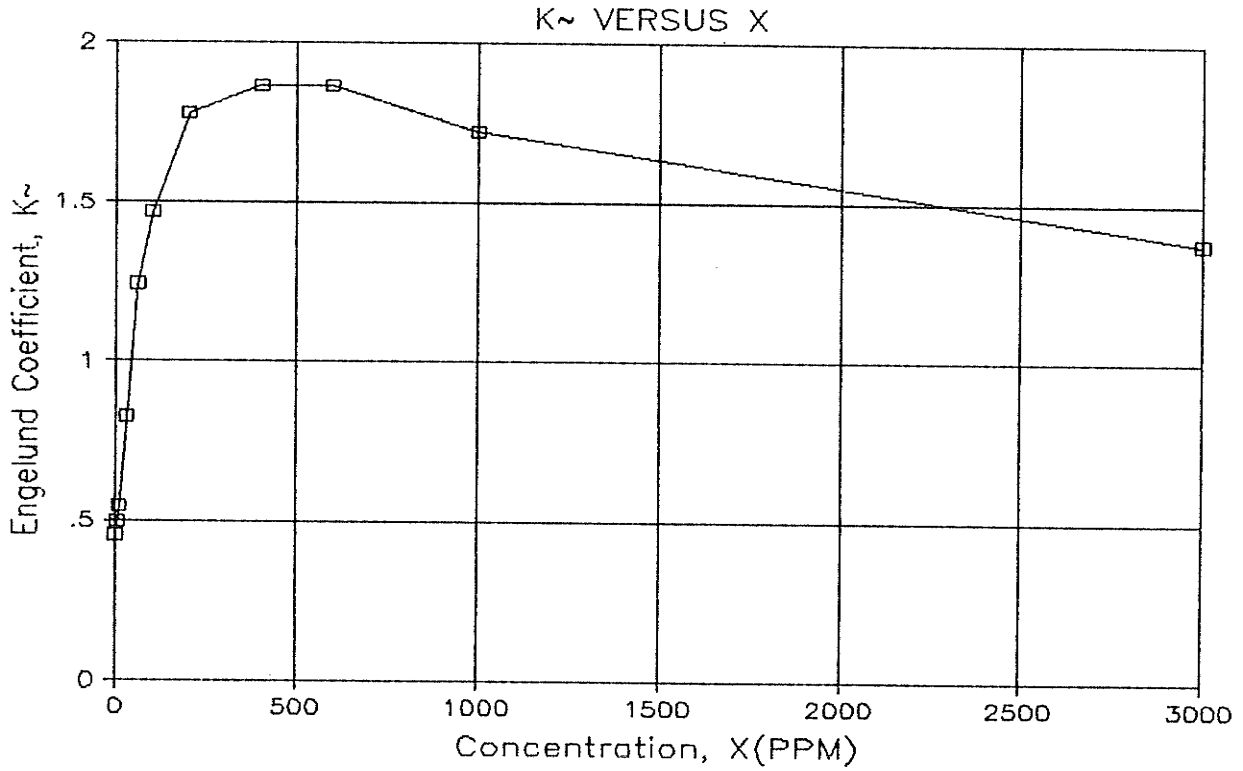


Figure C-2 Engelund's (1967) Sediment Transport and Resistance Equations; Variation of Exponent, n, and Coefficient, K~, of Equation (3.28) with Concentration, X_{bm}

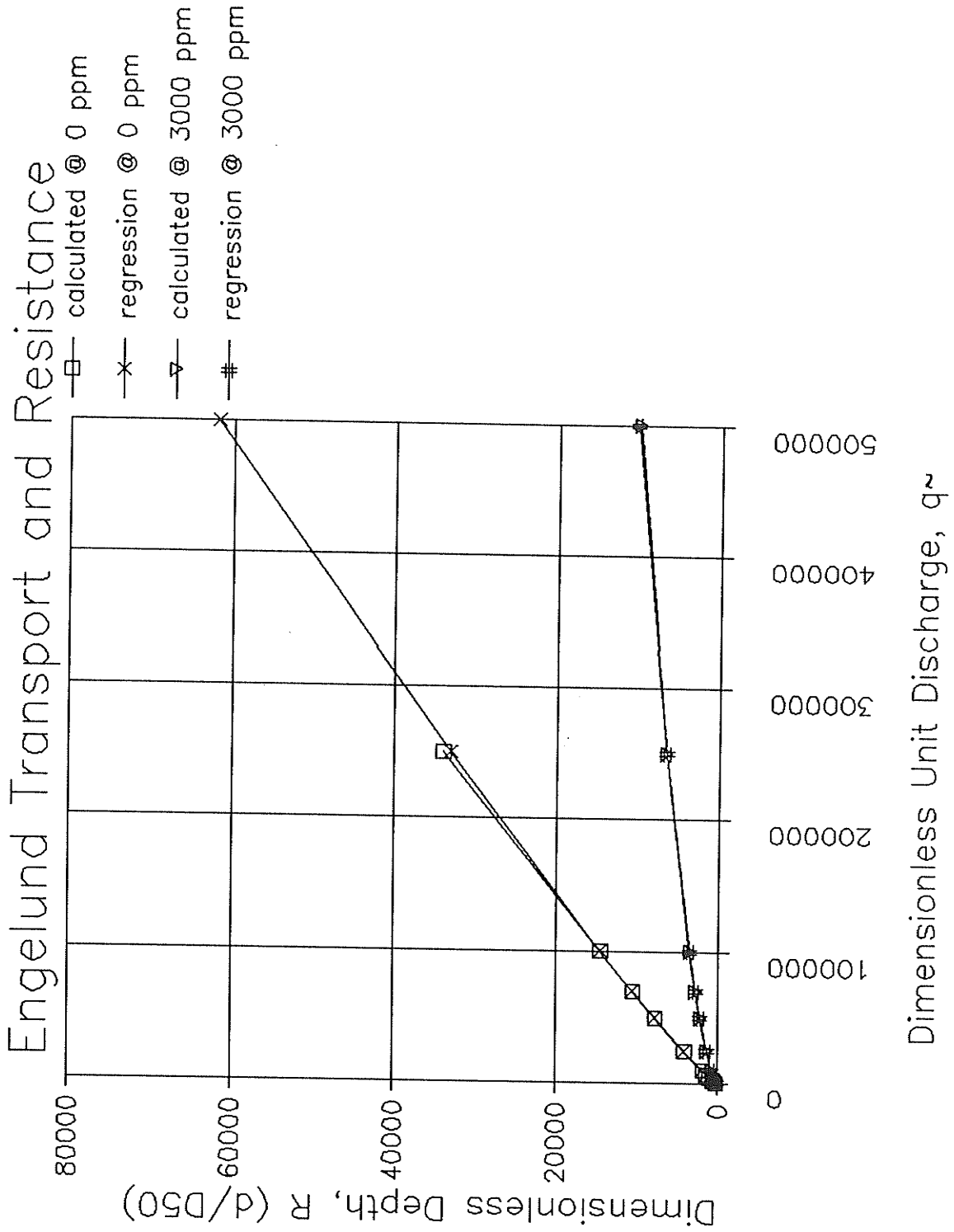


Figure C-3 Engelund's (1967) Sediment Transport and Resistance Equations; Least-Square Linear Relationship versus Analytical Solutions; Log_{10} (q~) versus Log_{10} (R);

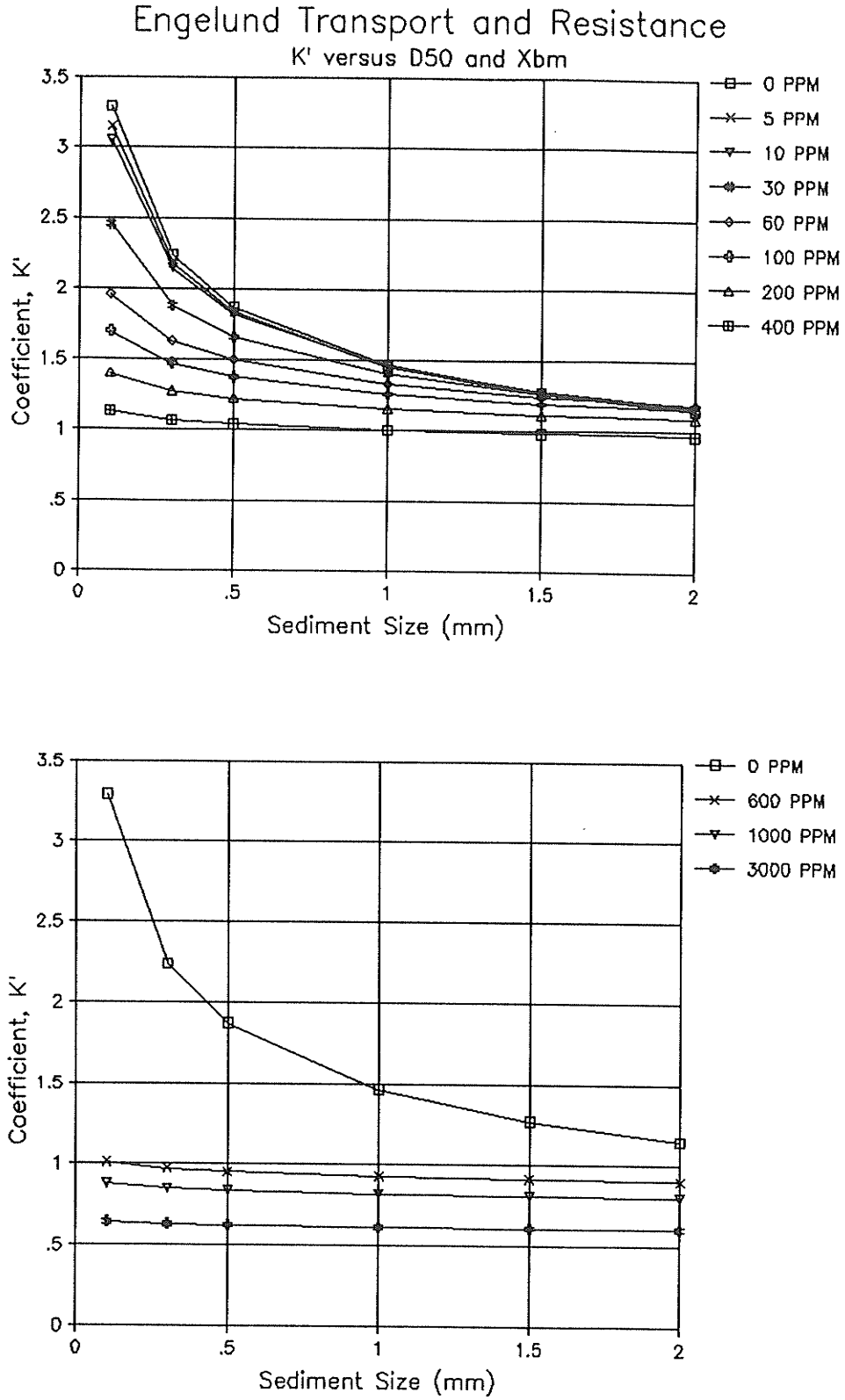


Figure C-4 Engelund's (1967) Sediment Transport and Resistance Equations; Variation of Coefficient, K', with Concentration, X_{bm}

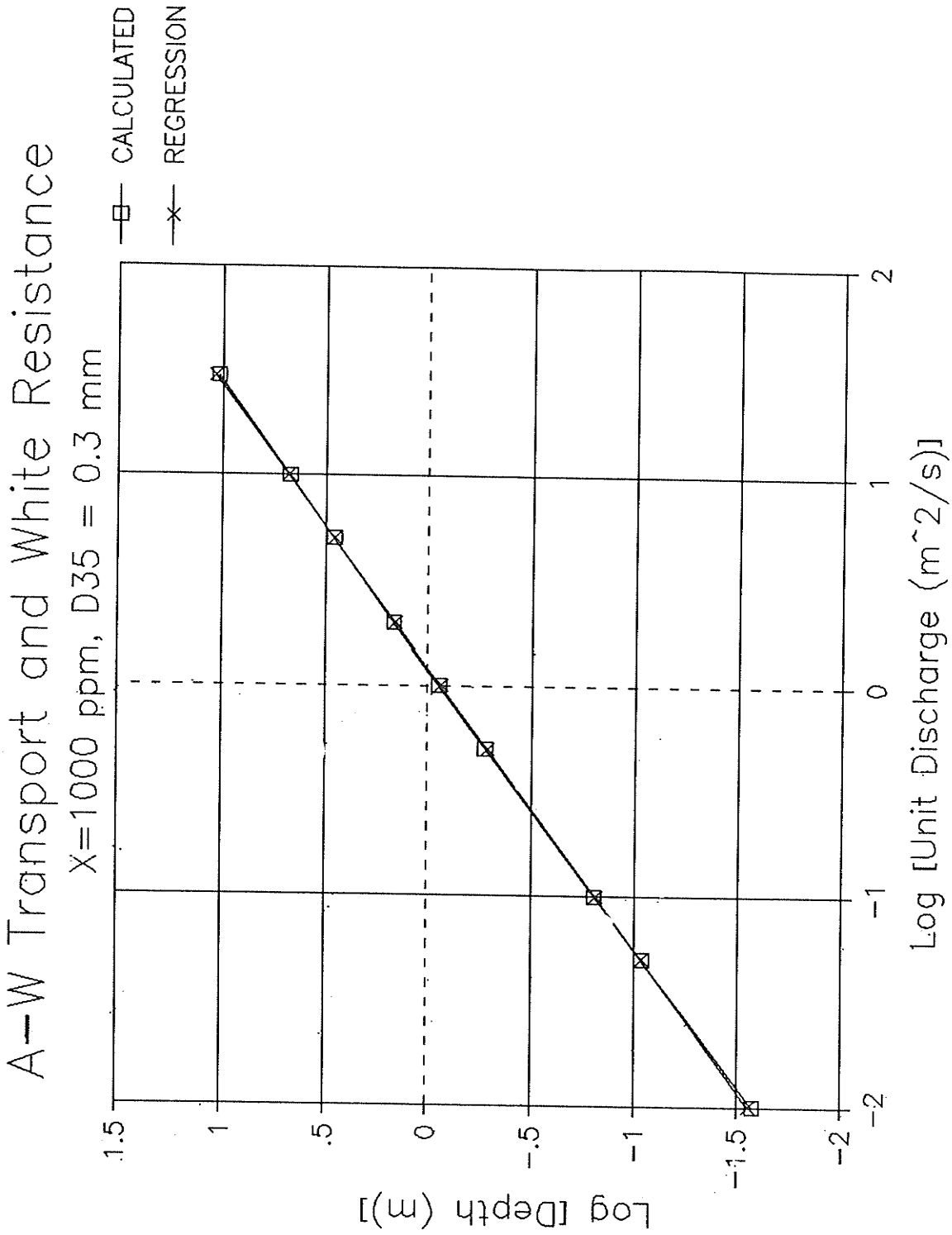


Figure D-1 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Linear Relationship versus Analytical Solutions; Log_{10} (q) versus Log_{10} (d); X_{bm} = 1000 PPM, D_{35} = 0.30 mm

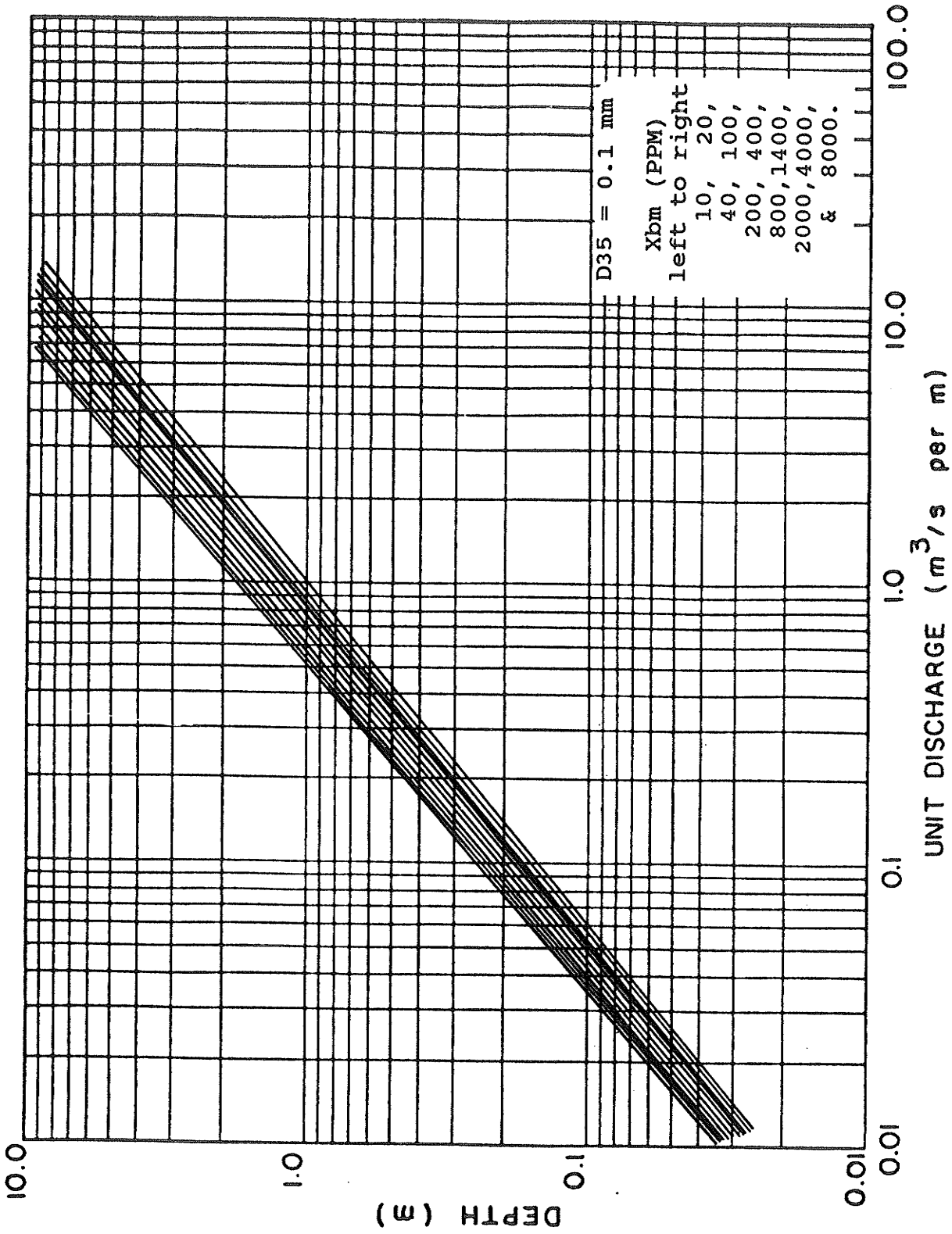


Figure D-2 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 0.1 mm

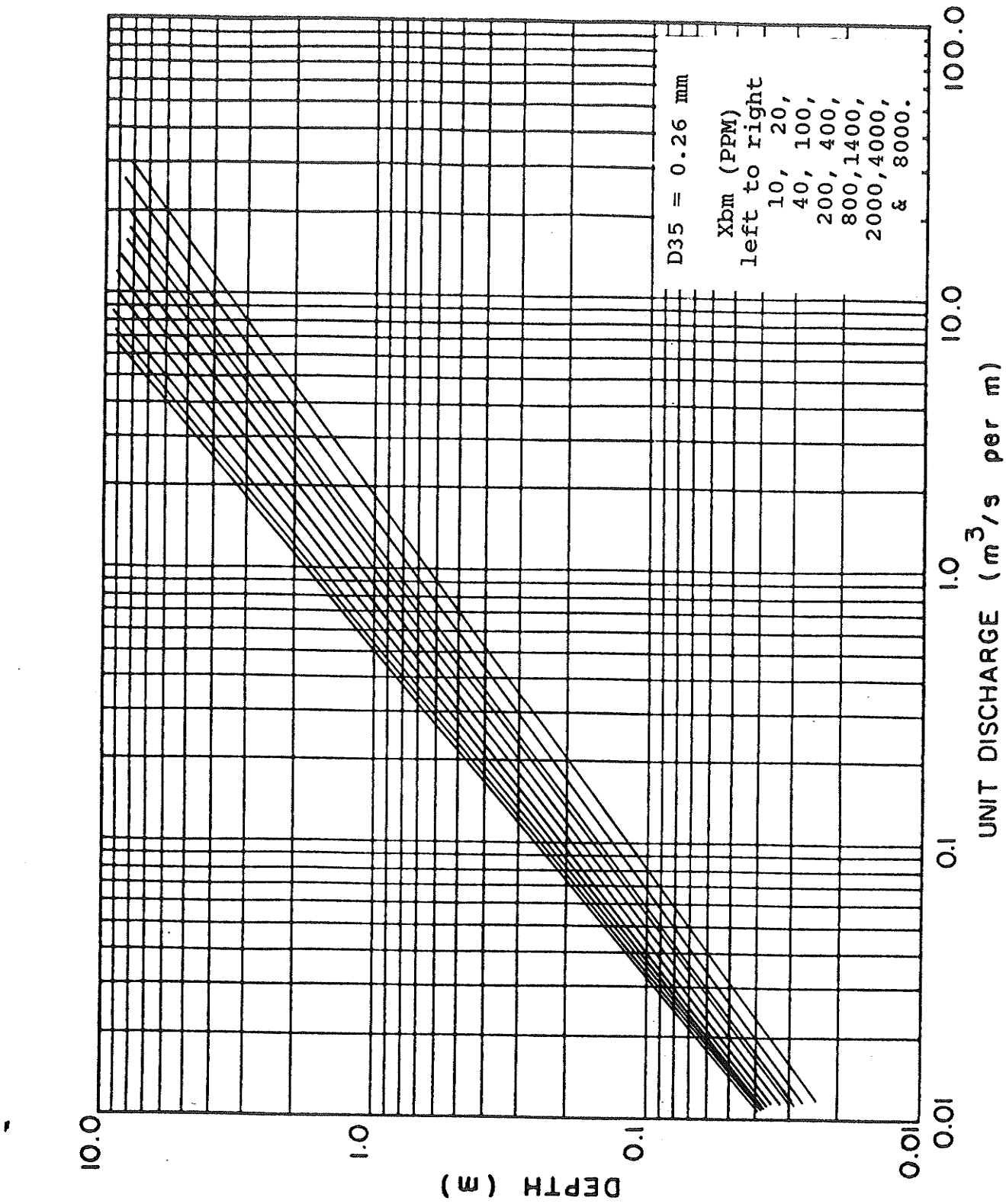


Figure D-3 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 0.26 mm

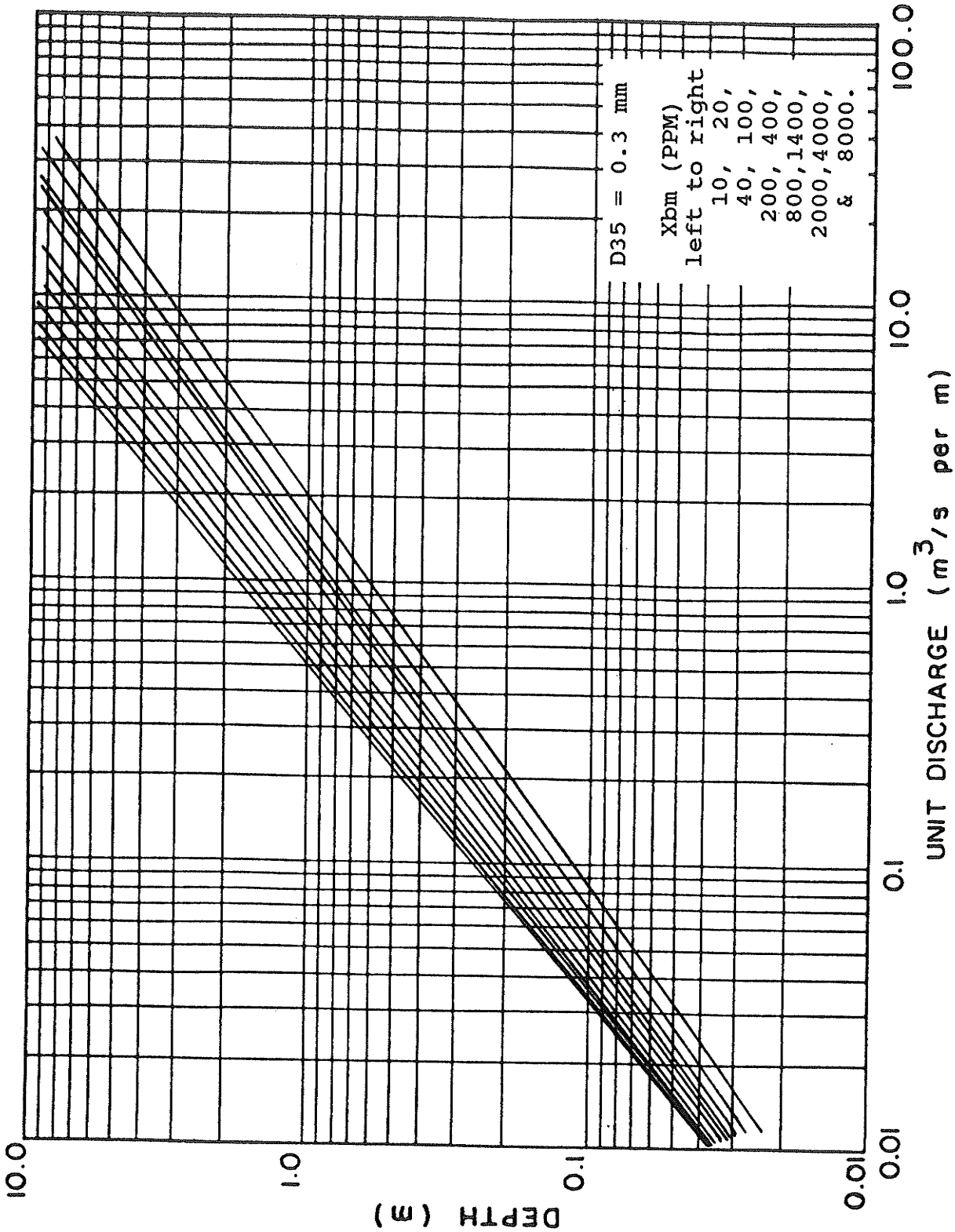


Figure D-4 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 0.3 mm

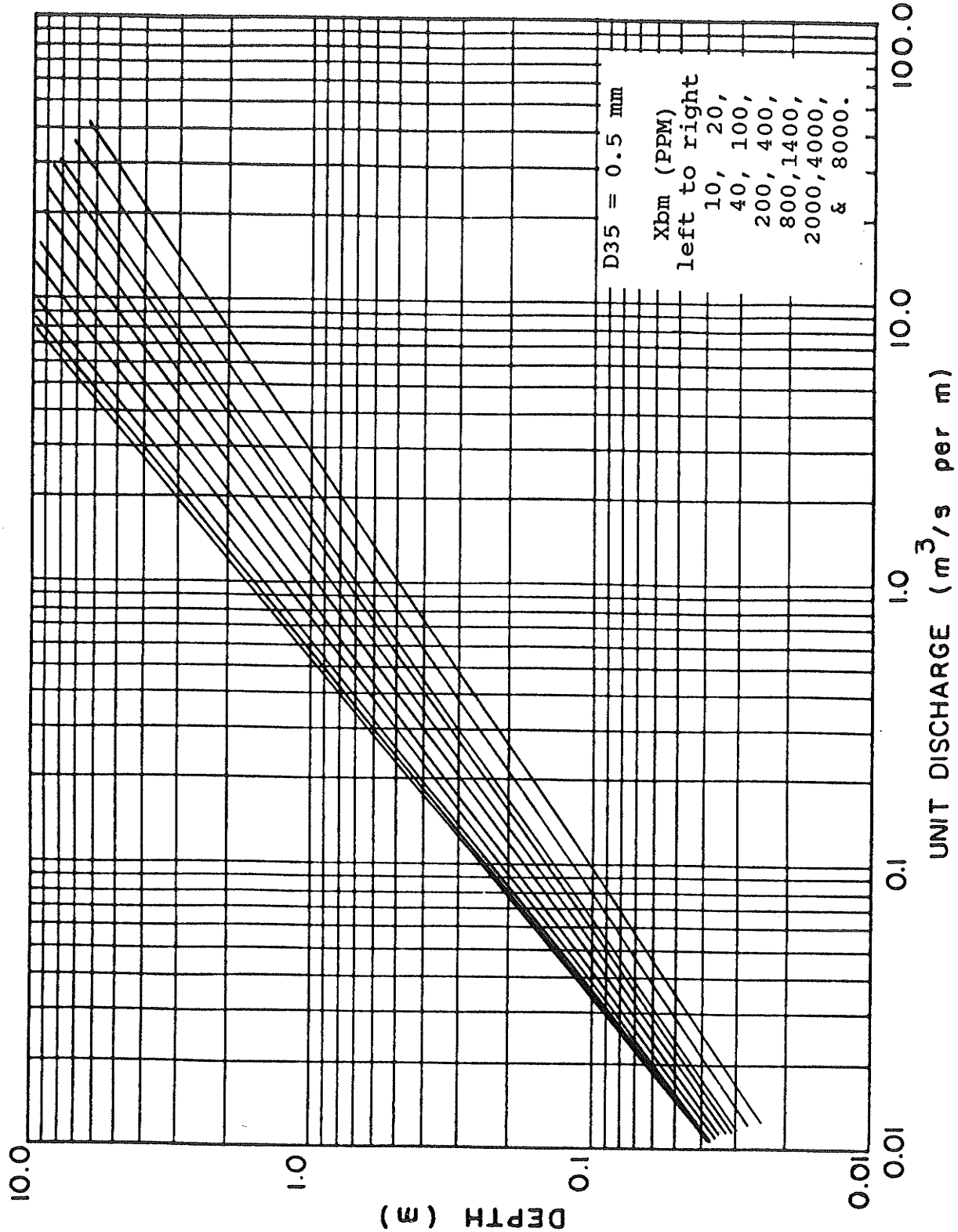


Figure D-5 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 0.5 mm

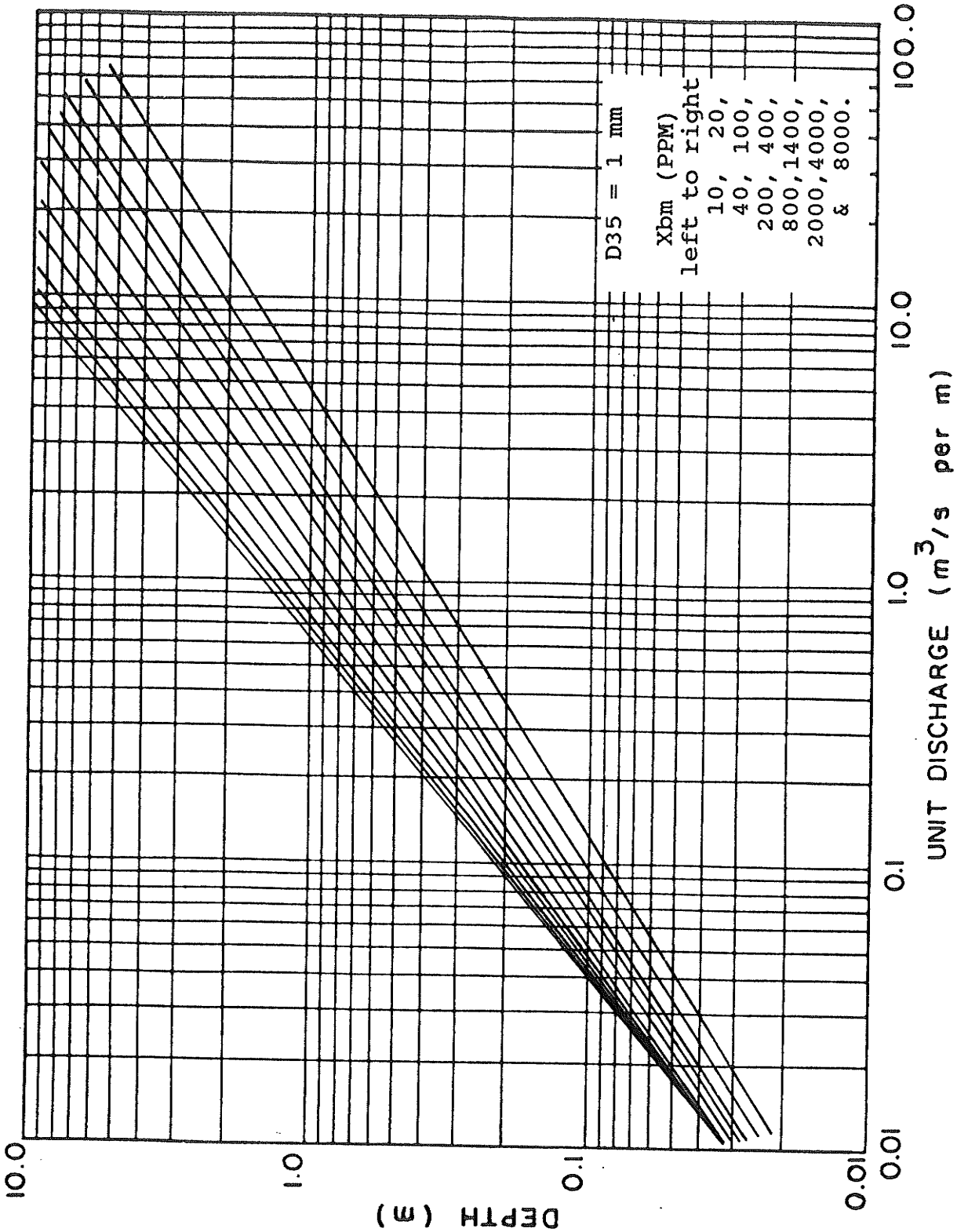


Figure D-6 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 1.0 mm

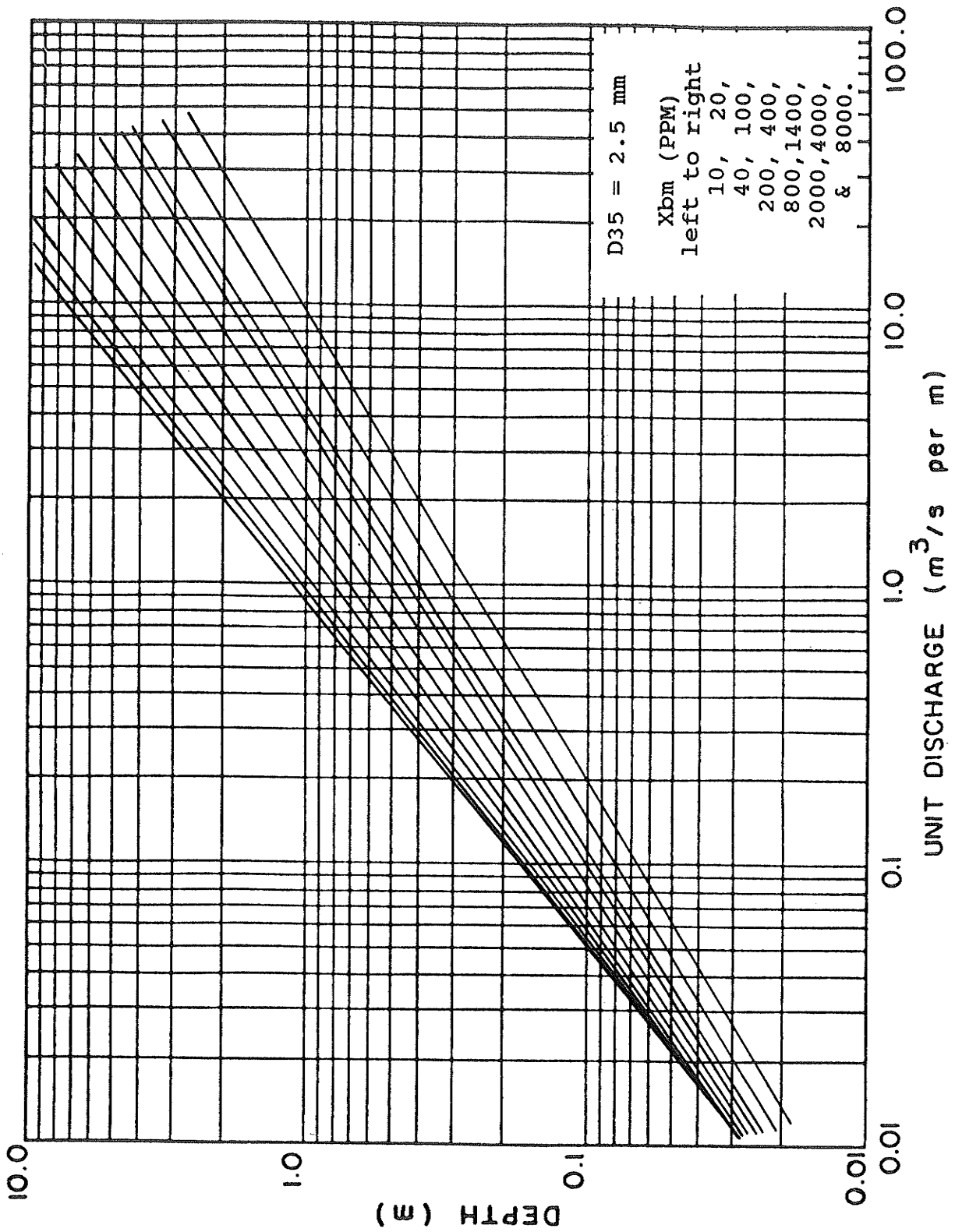


Figure D-7 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 2.5 mm

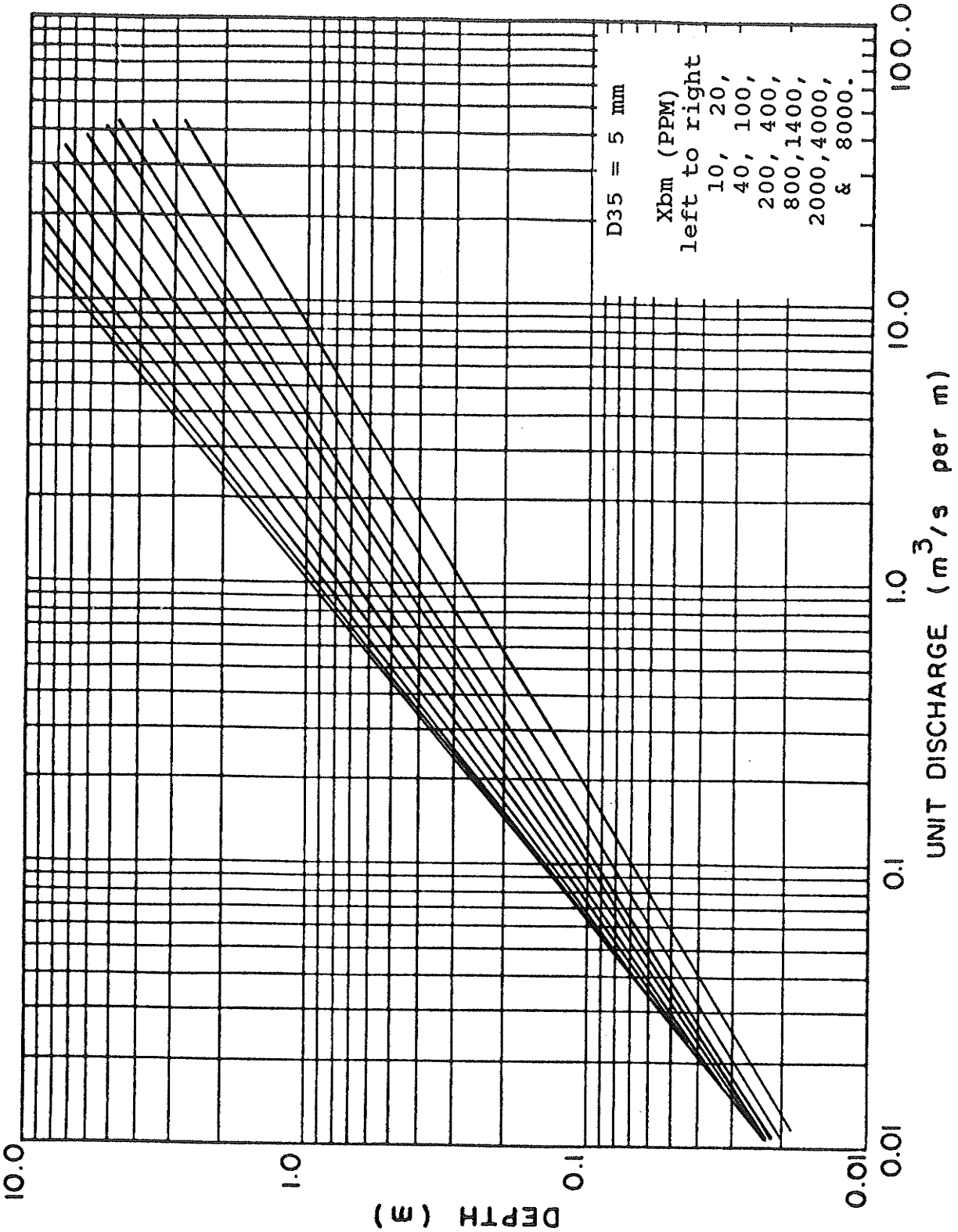


Figure D-8 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 5.0 mm

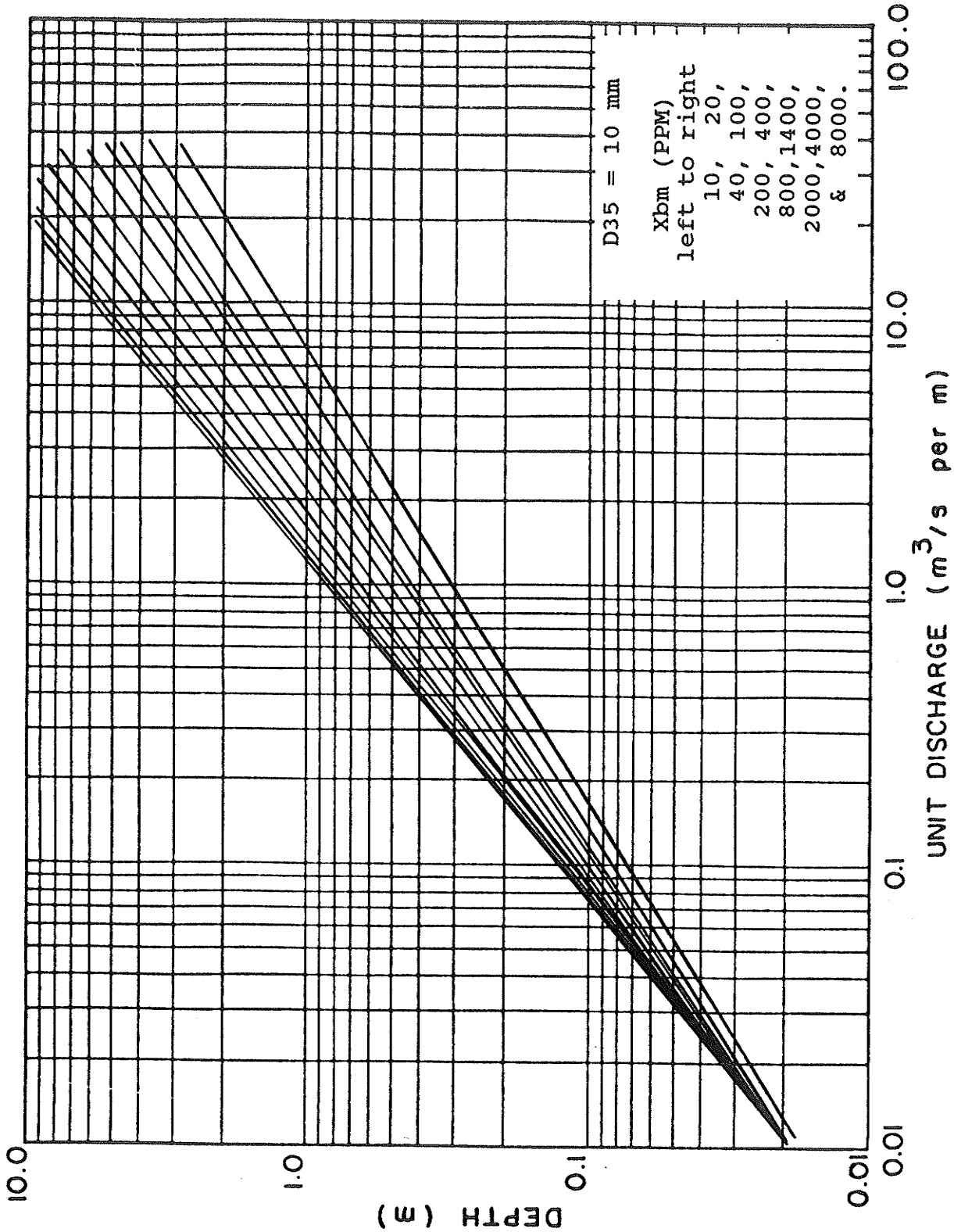


Figure D-9 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 10.0 mm

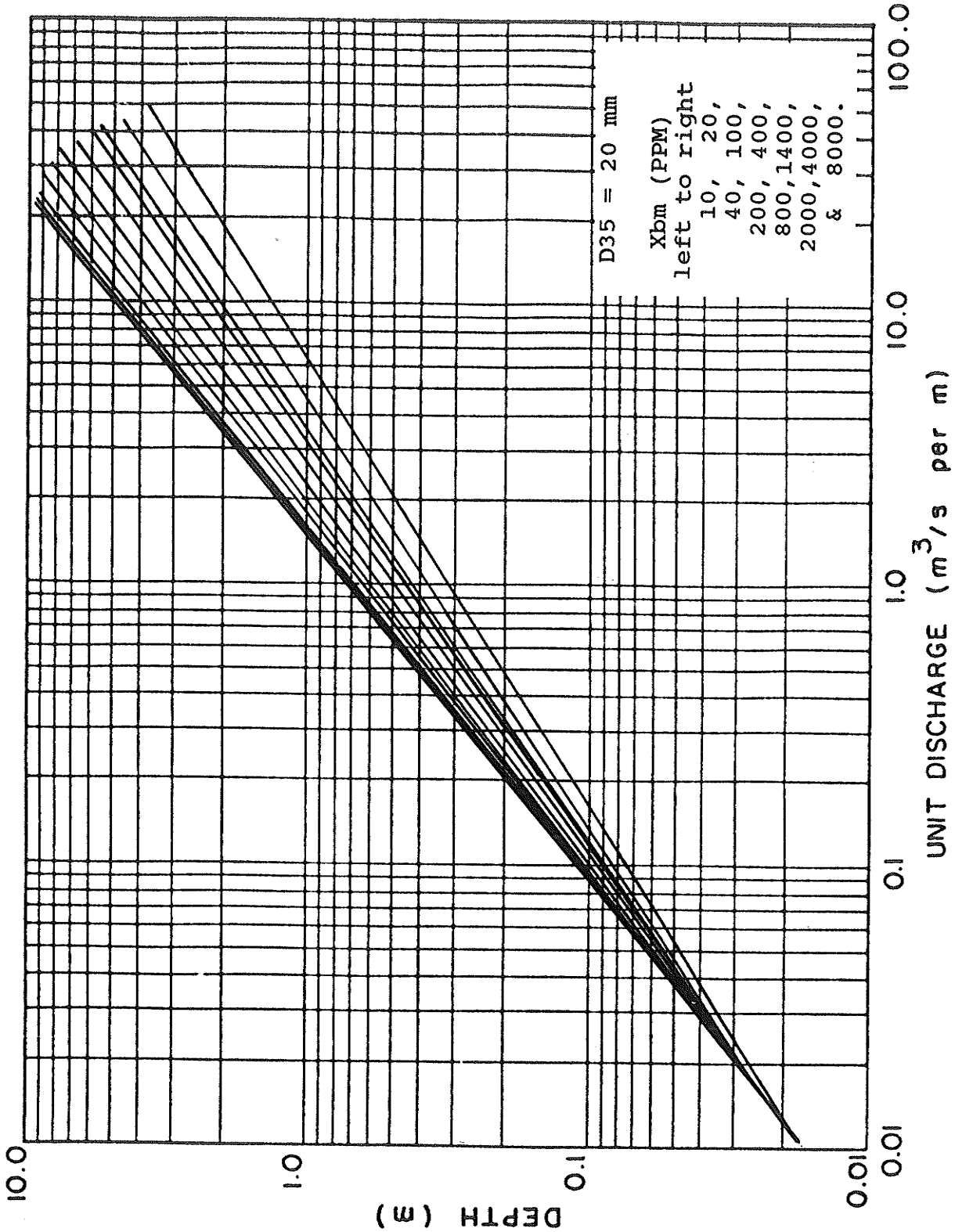
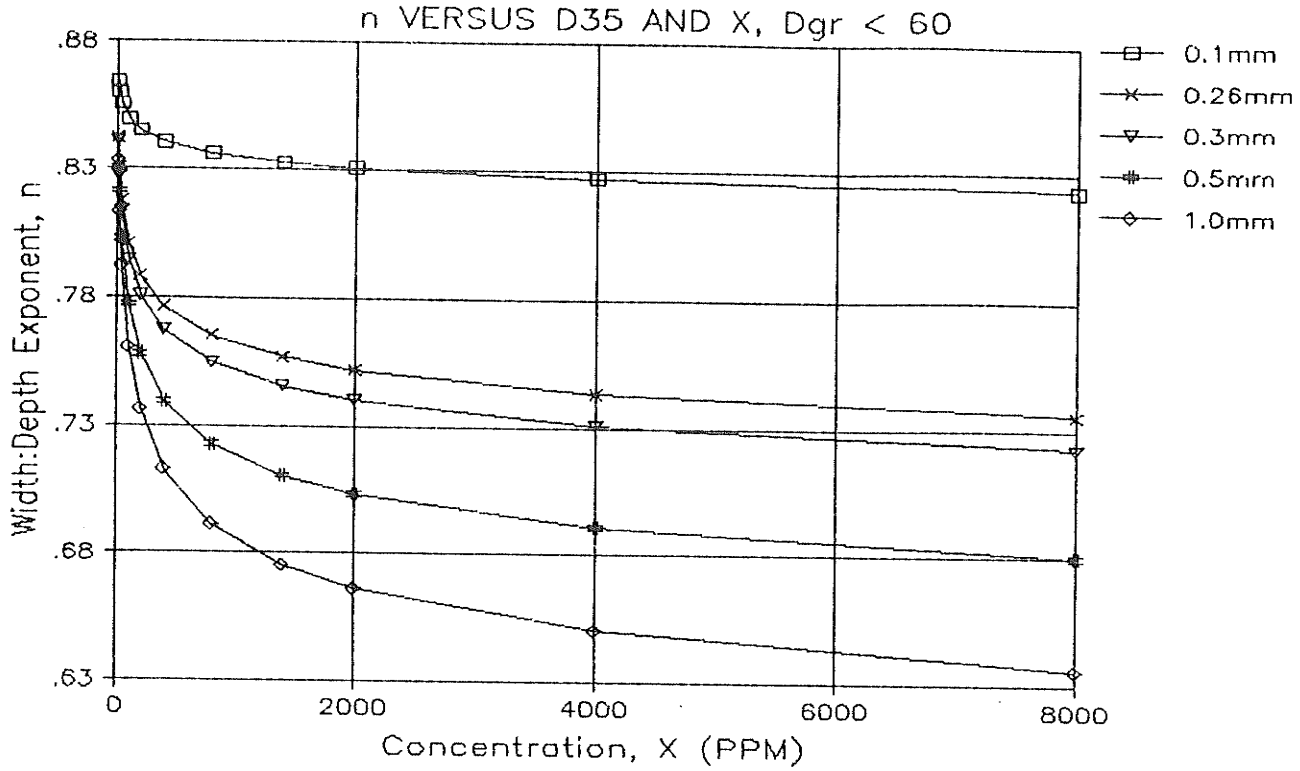


Figure D-10 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 20.0 mm



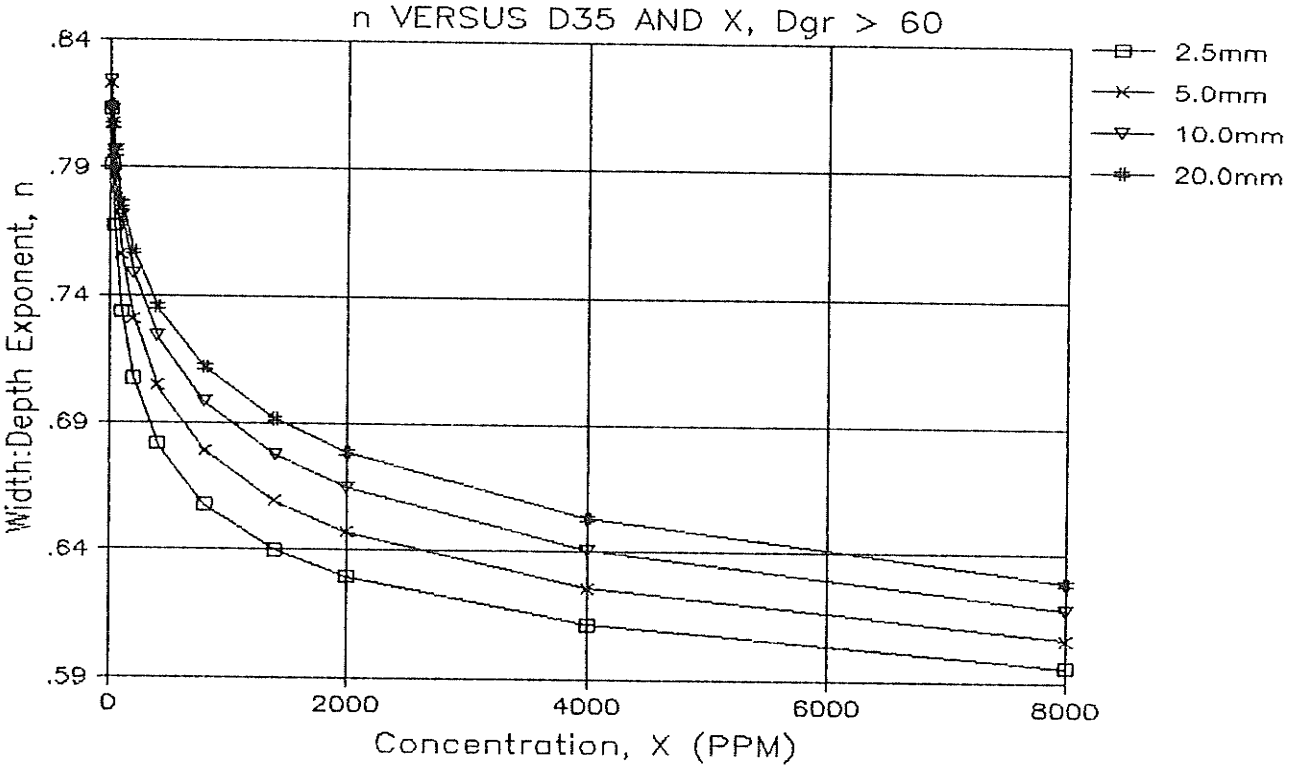
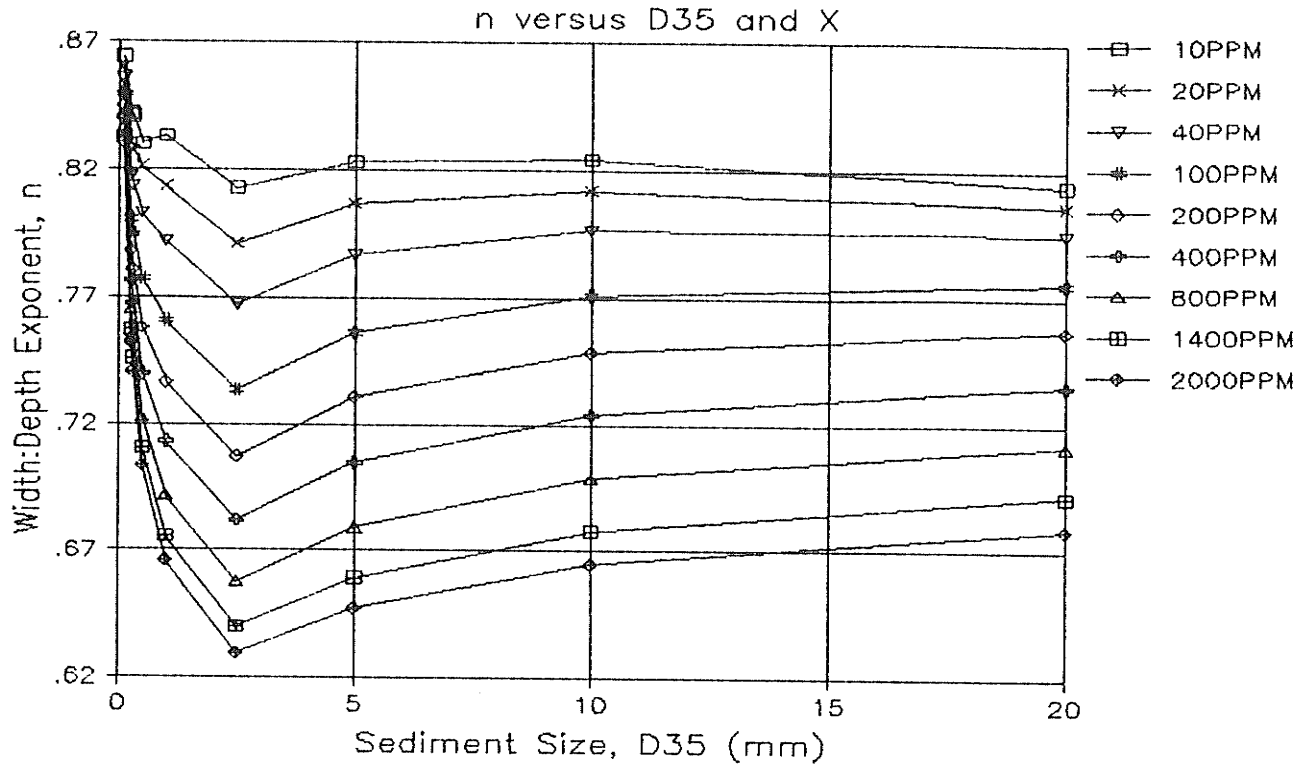


Figure D-11 Ackers-White (1973) Transport and White et al. (1980) Resistance Equation; Variation of Exponent, n, with Concentration, X_{bm}, Sediment Size, D_{35}



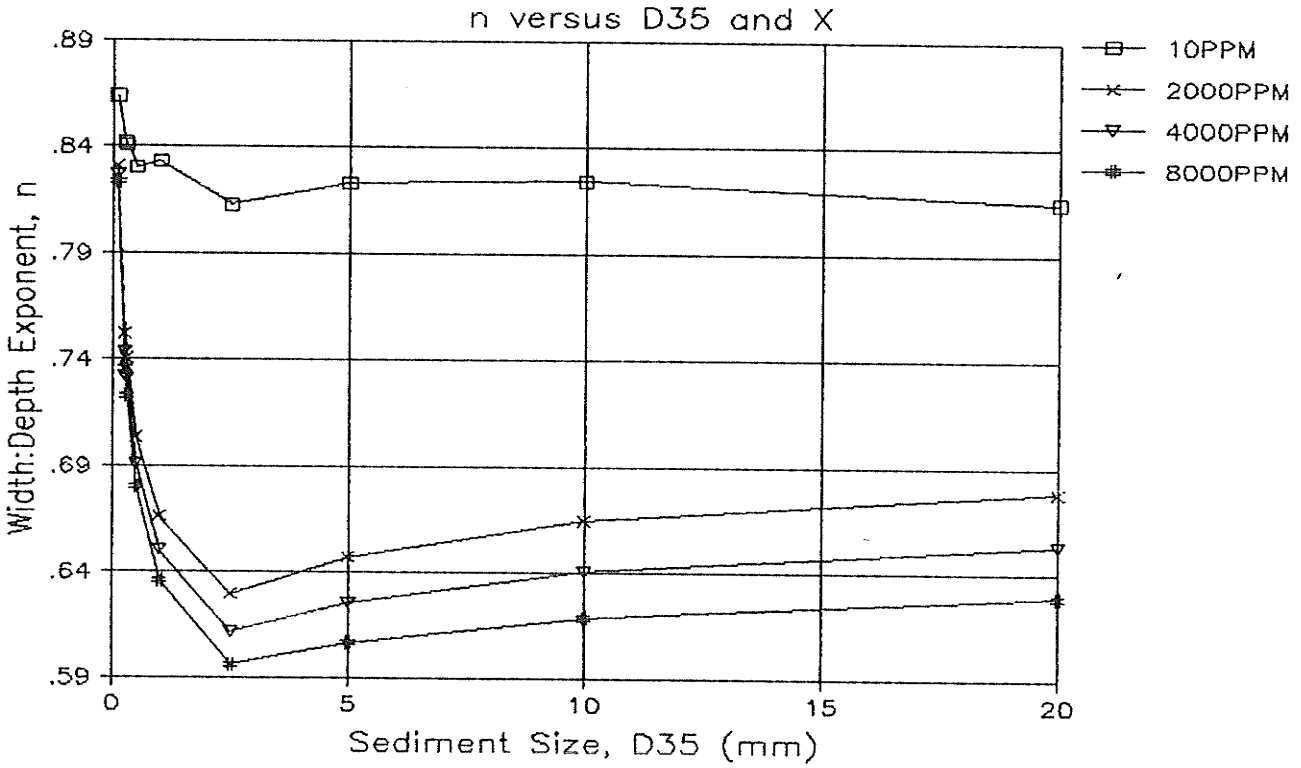
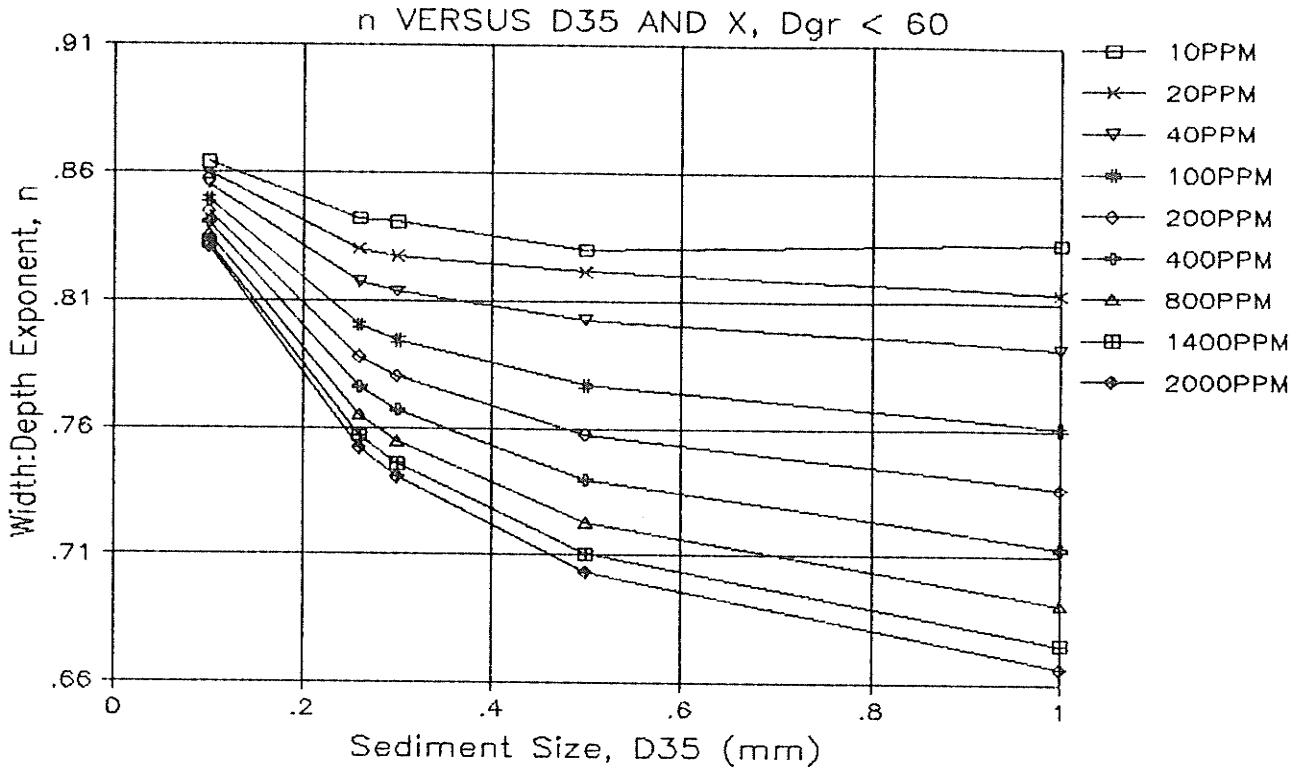


Figure D-12 Ackers-White (1973) Transport and White et al. (1980) Resistance Equation; Variation of Exponent, n, with Sediment Size, D_{35}, and Concentration, X_{bm}



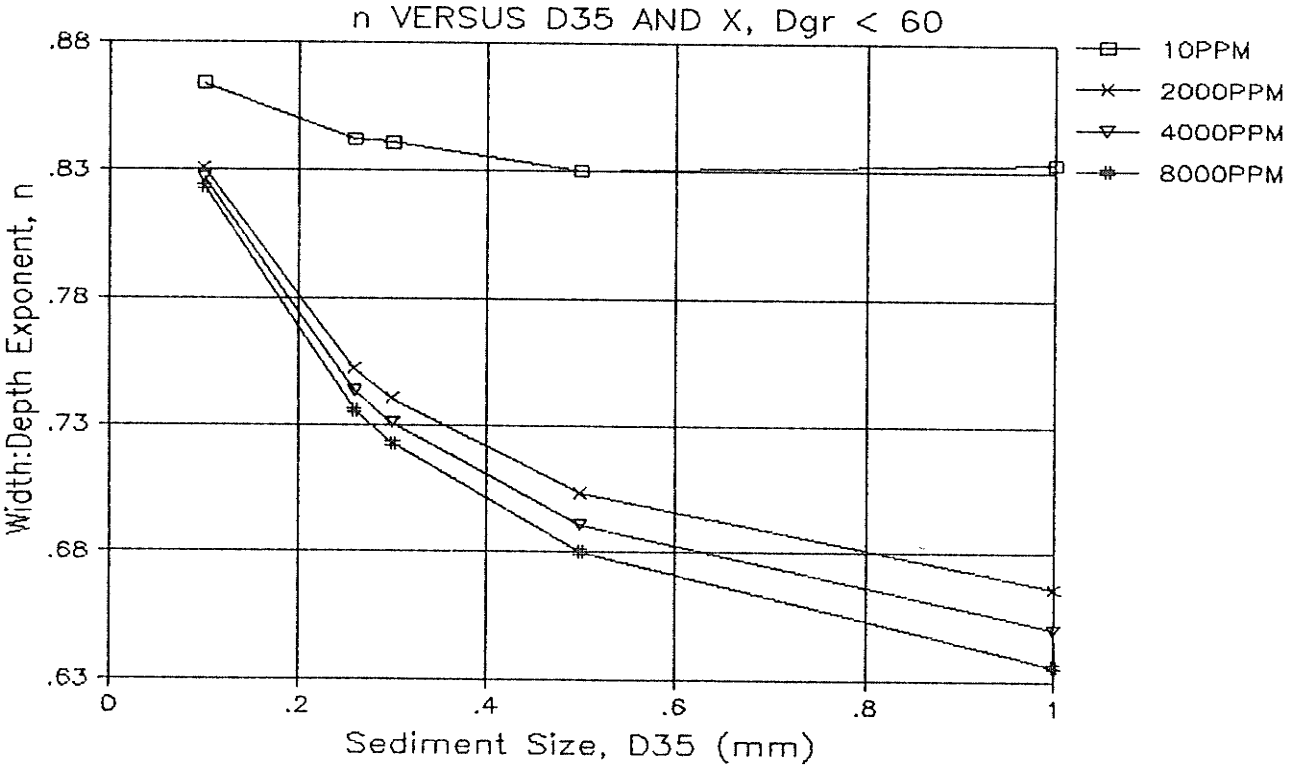
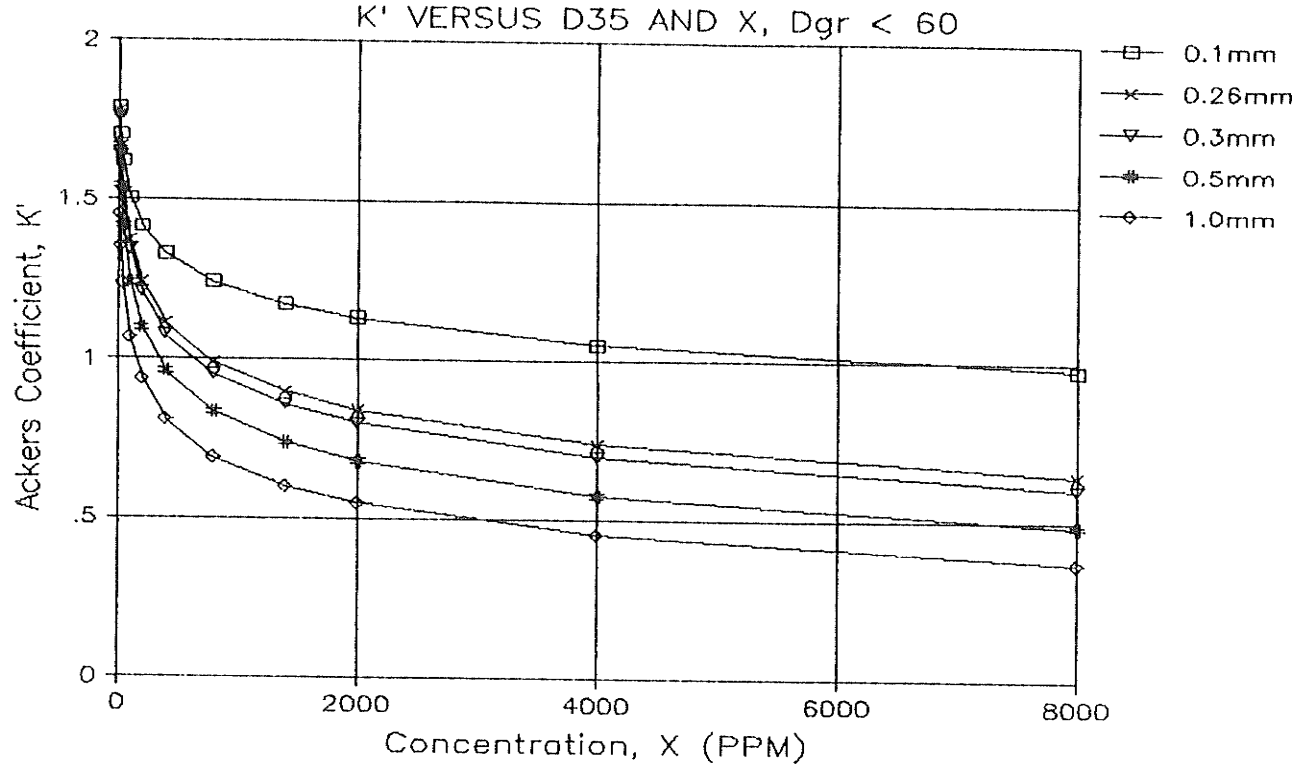


Figure D-13 Ackers-White (1973) Transport and White et al. (1980) Resistance Equation; Variation of Exponent, n, with Sediment Size, D_{35}, and Concentration, X_{bm}, for $D_{gr} < 60$



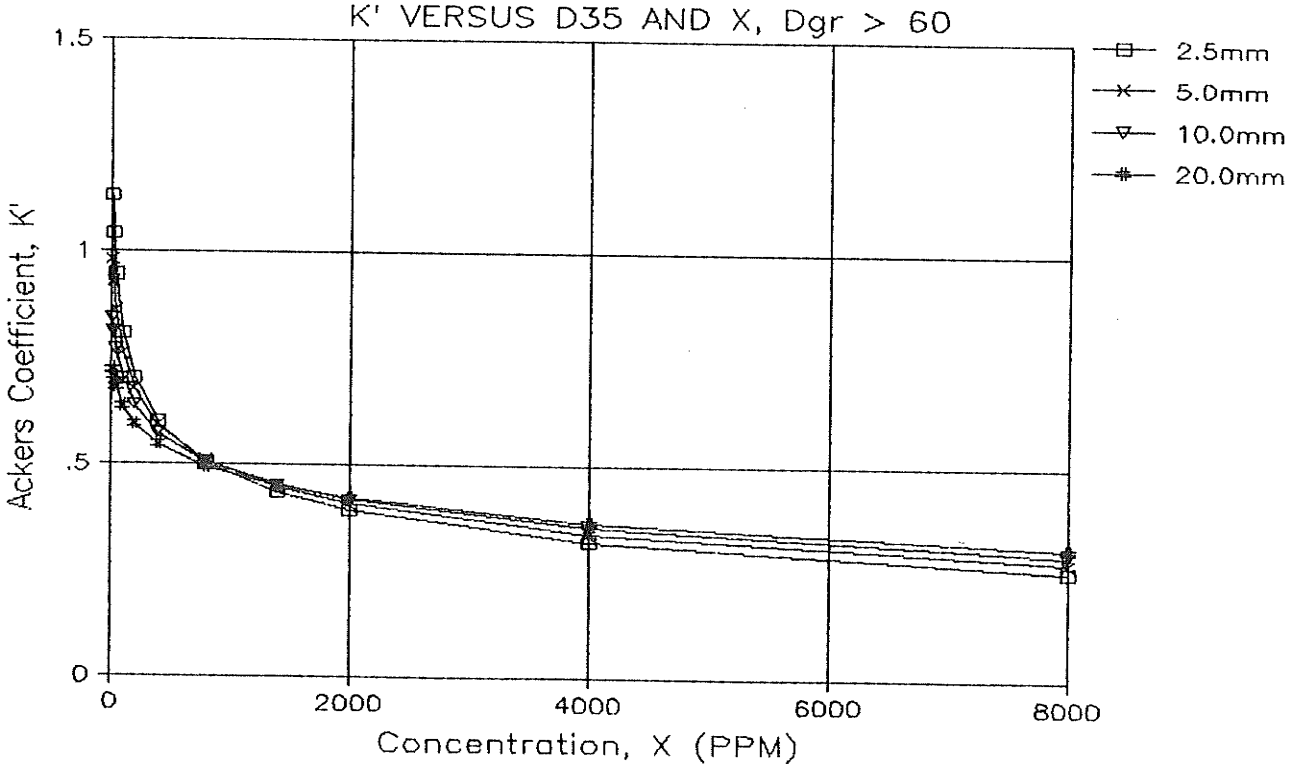


Figure D-14 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Variation of Coefficient, K', with Concentration, X_{bm}, and Sediment Size, D_{35}

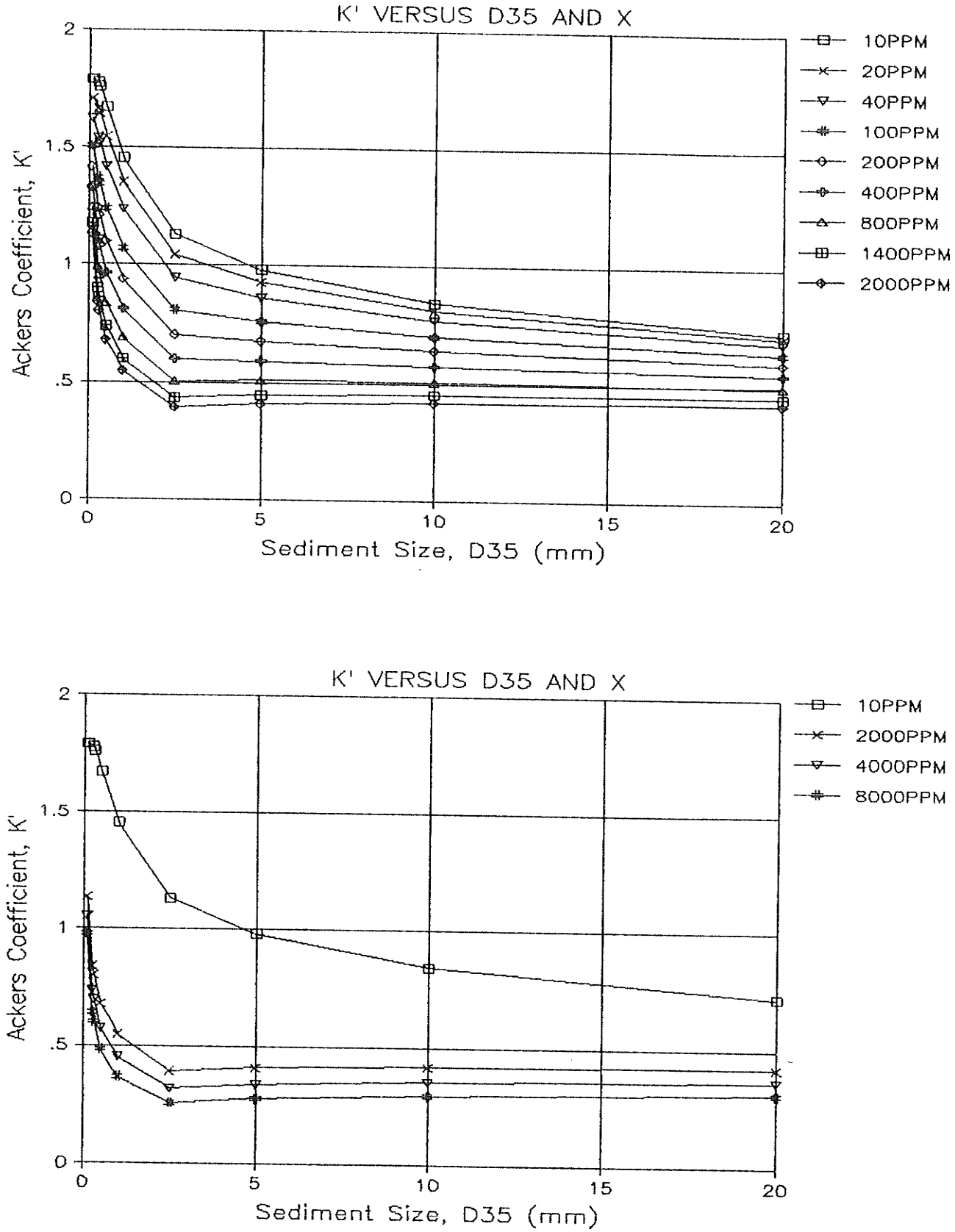
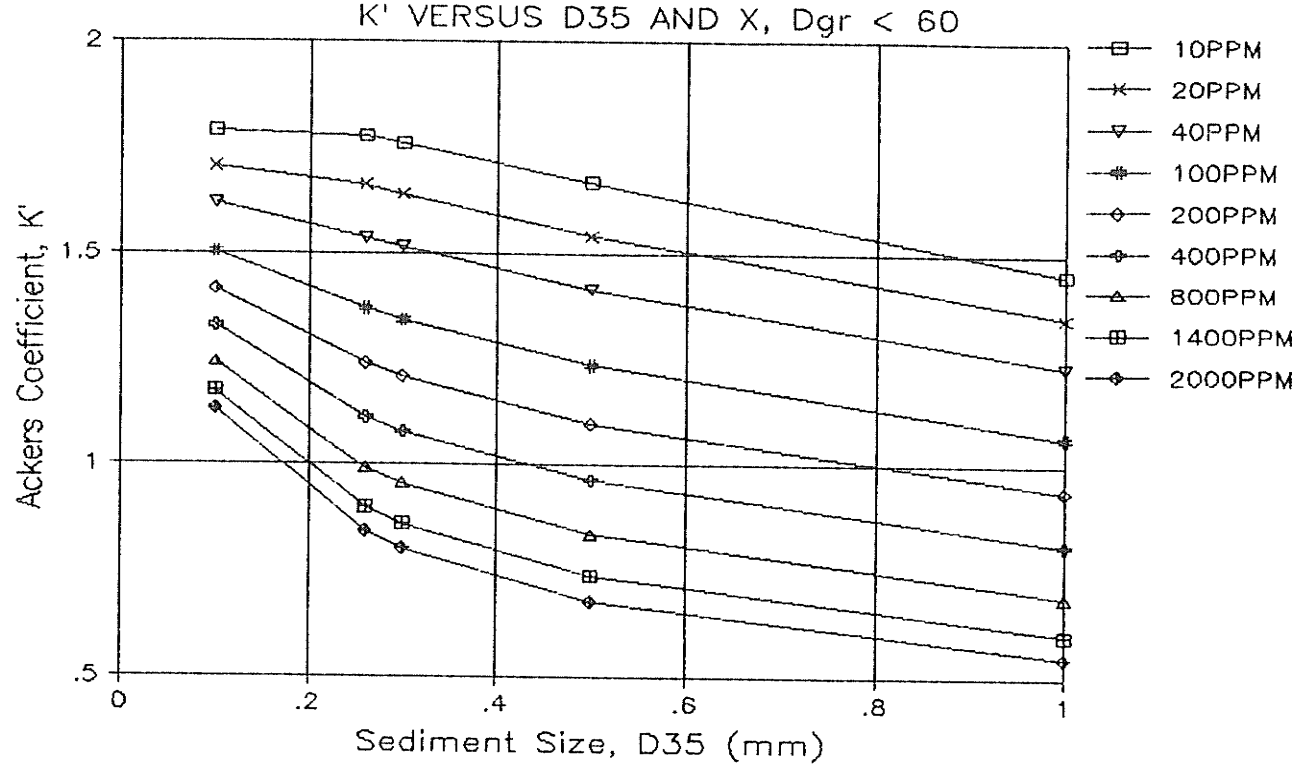


Figure D-15 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Variation of Coefficient, K', with Sediment Size, D_{35}, and Concentration, X_{bm}



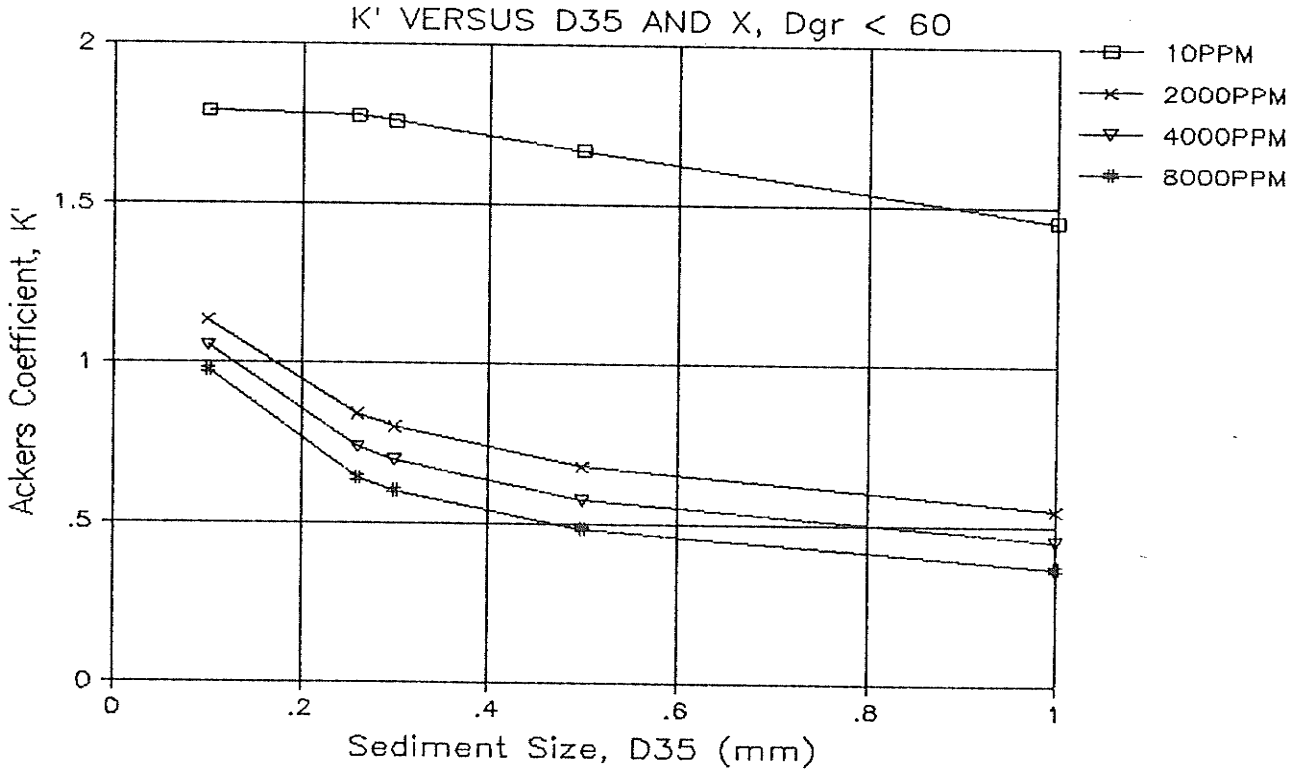


Figure D-16 Ackers-White (1973) Transport and White et al. (1980) Resistance Equation; Variation of Coefficient, K', with Sediment Size, D_{35}, and Concentration, X_{bm}, for $D_{gr} < 60$

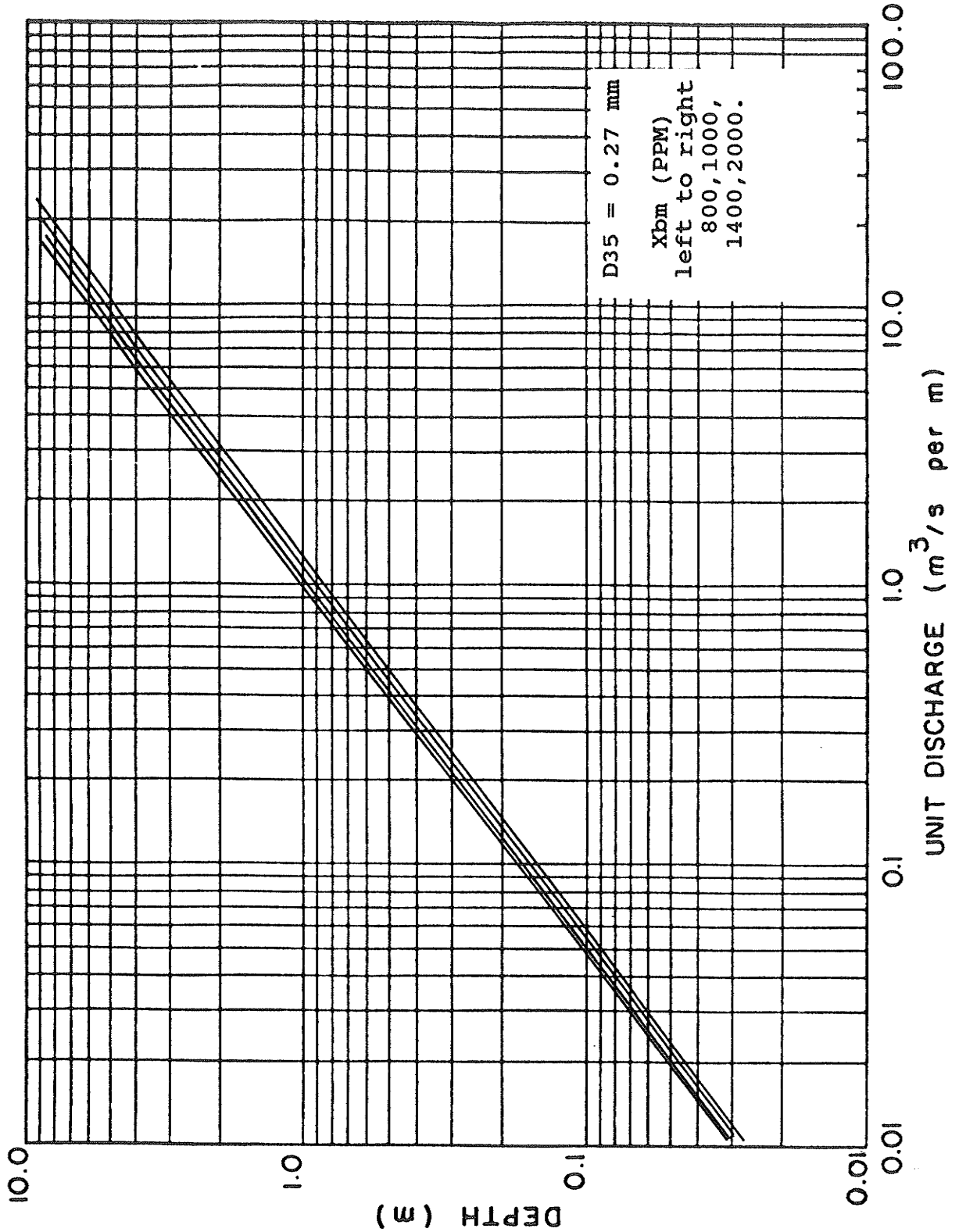


Figure D-17 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 0.27 mm

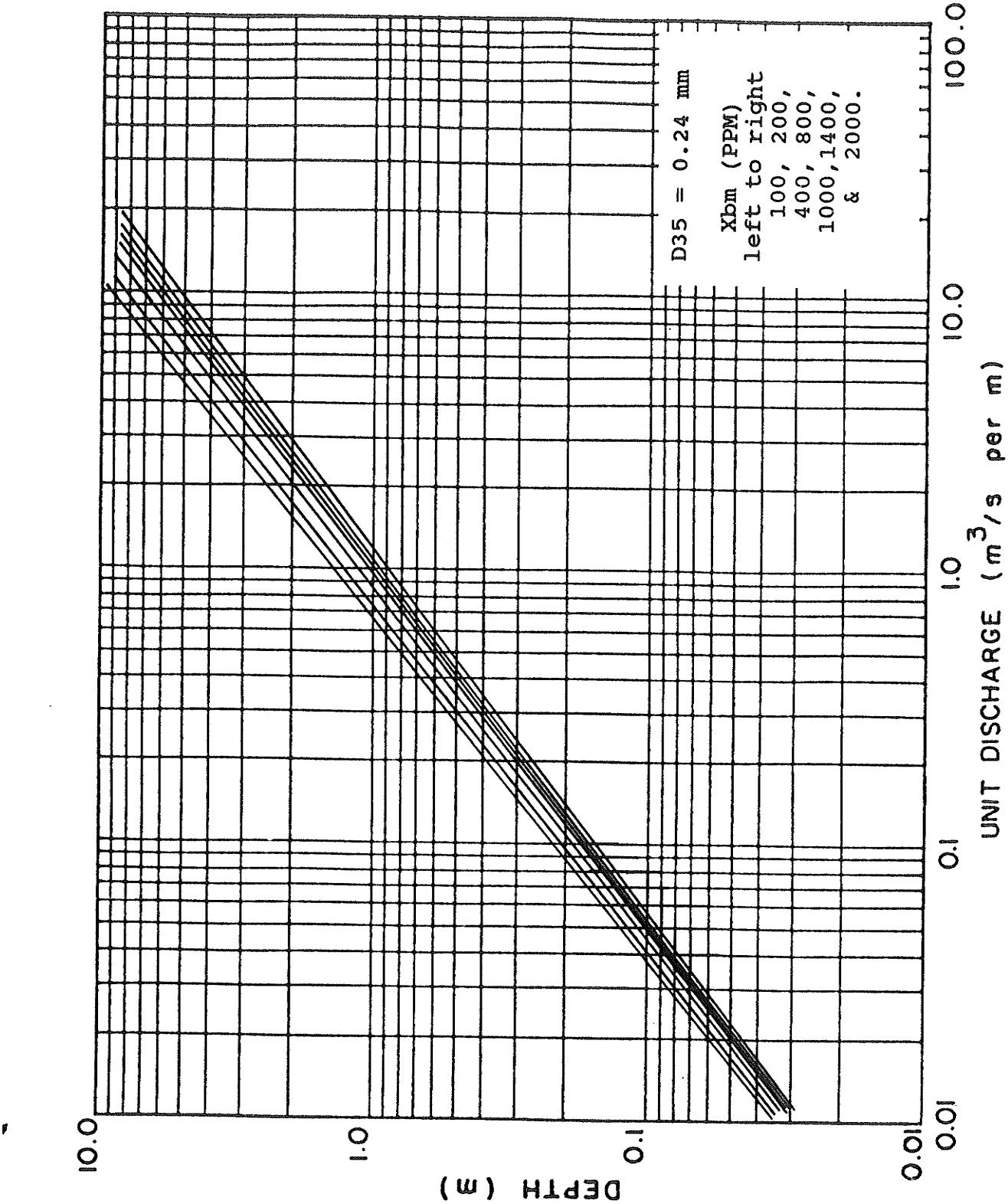


Figure D-18 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 0.24 mm

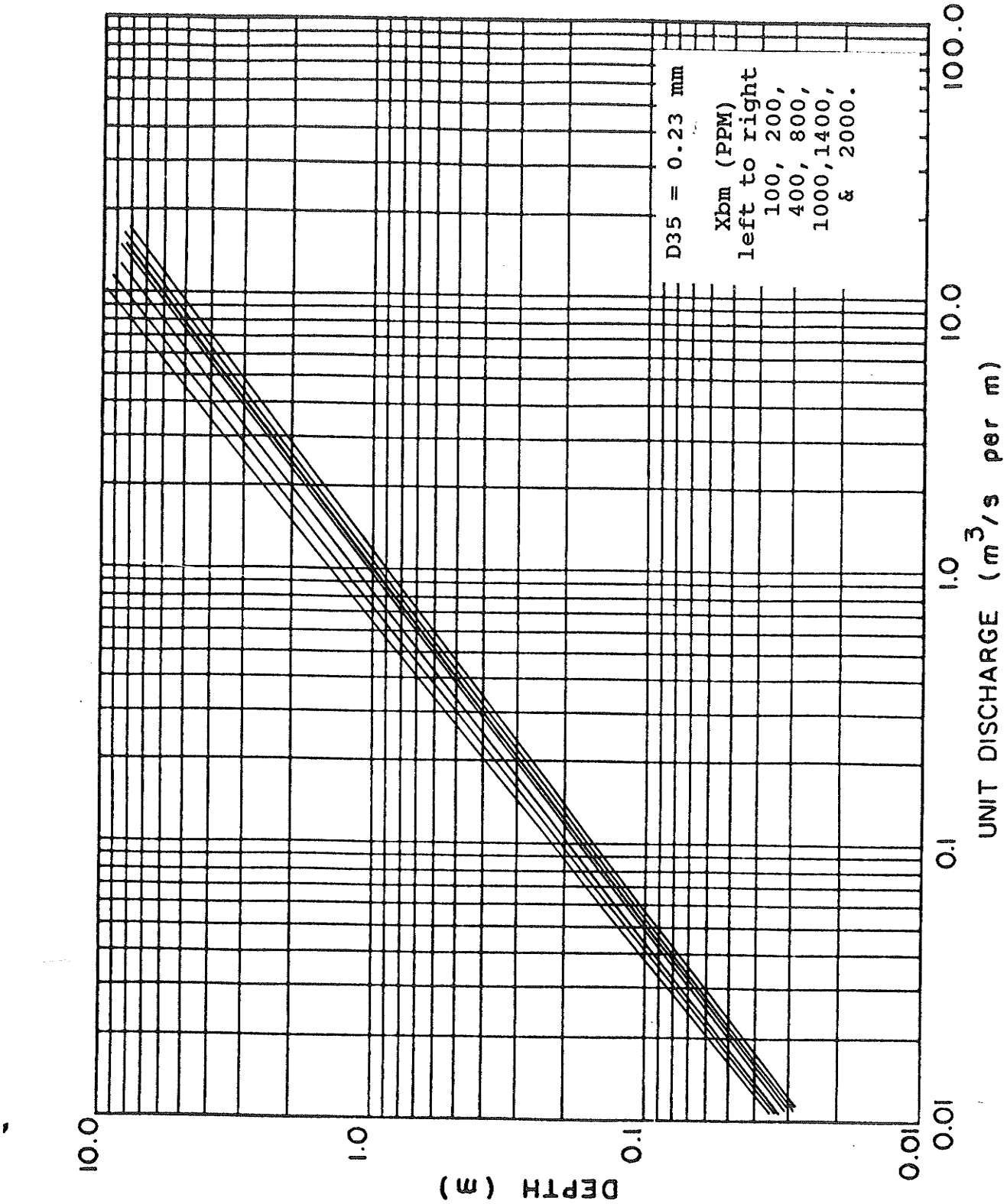


Figure D-19 Ackers-White (1973) Transport and White et al. (1980) Resistance Equations; Least-Square Relations, Log_{10} (q) versus Log_{10} (d); D_{35} = 0.23 mm

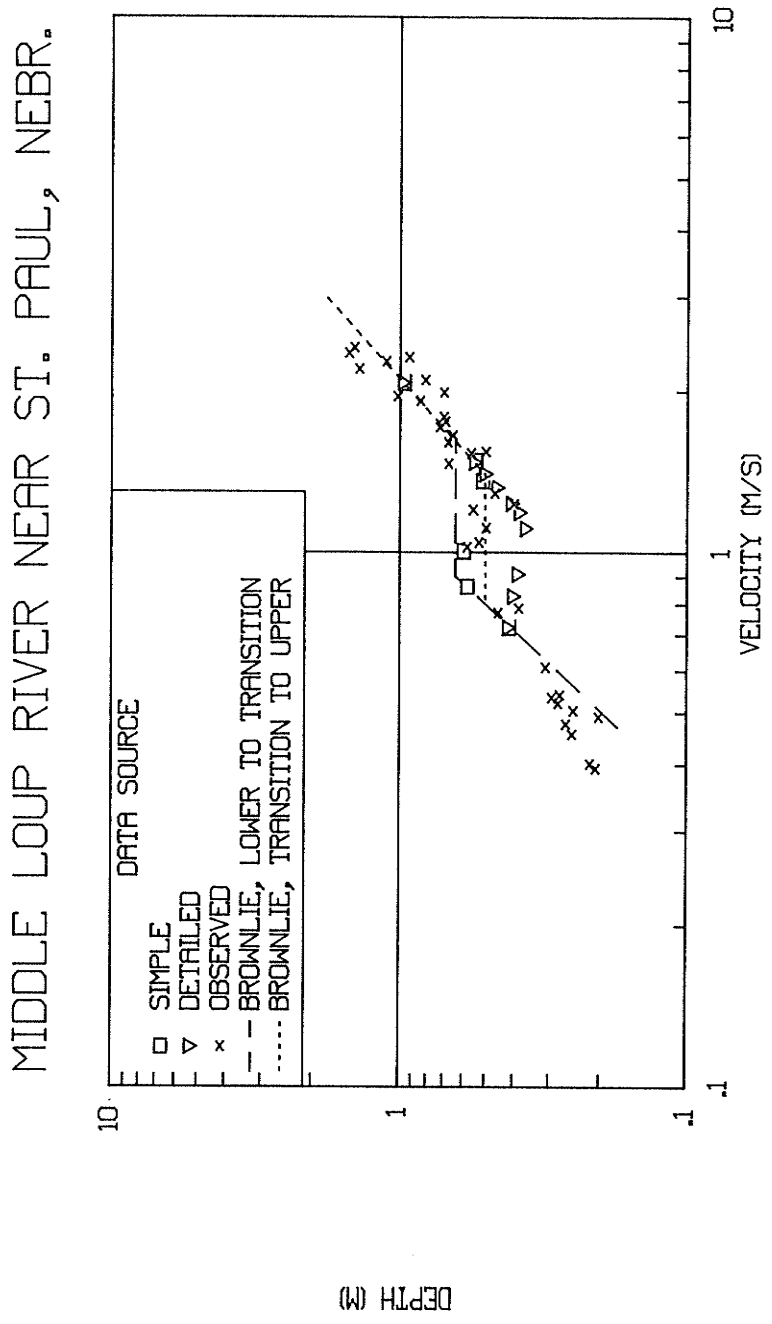


Figure E-1 Brownlie (1981a) and Brown3 Detailed and Simplified Flow Depth Computations For Observed Rating Curves Middle Loup R. at St. Paul, Nebr.

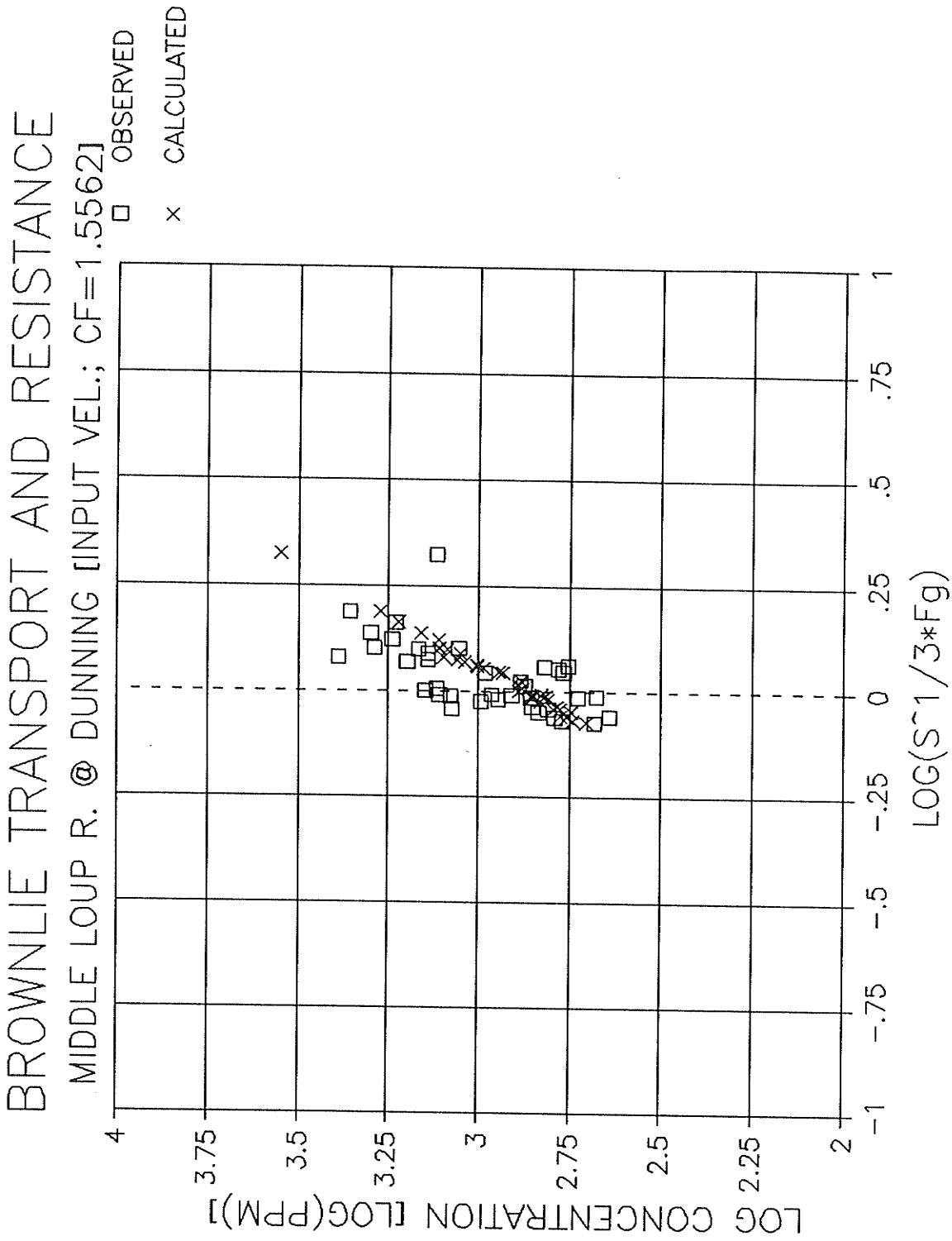


Figure E-2 Brown3 Sediment Transport Results For Observed Sediment Concentration Curves, Middle Loup R. near Dunning, Nebr., C_F = 1.5662; Observed Velocity and Depth.

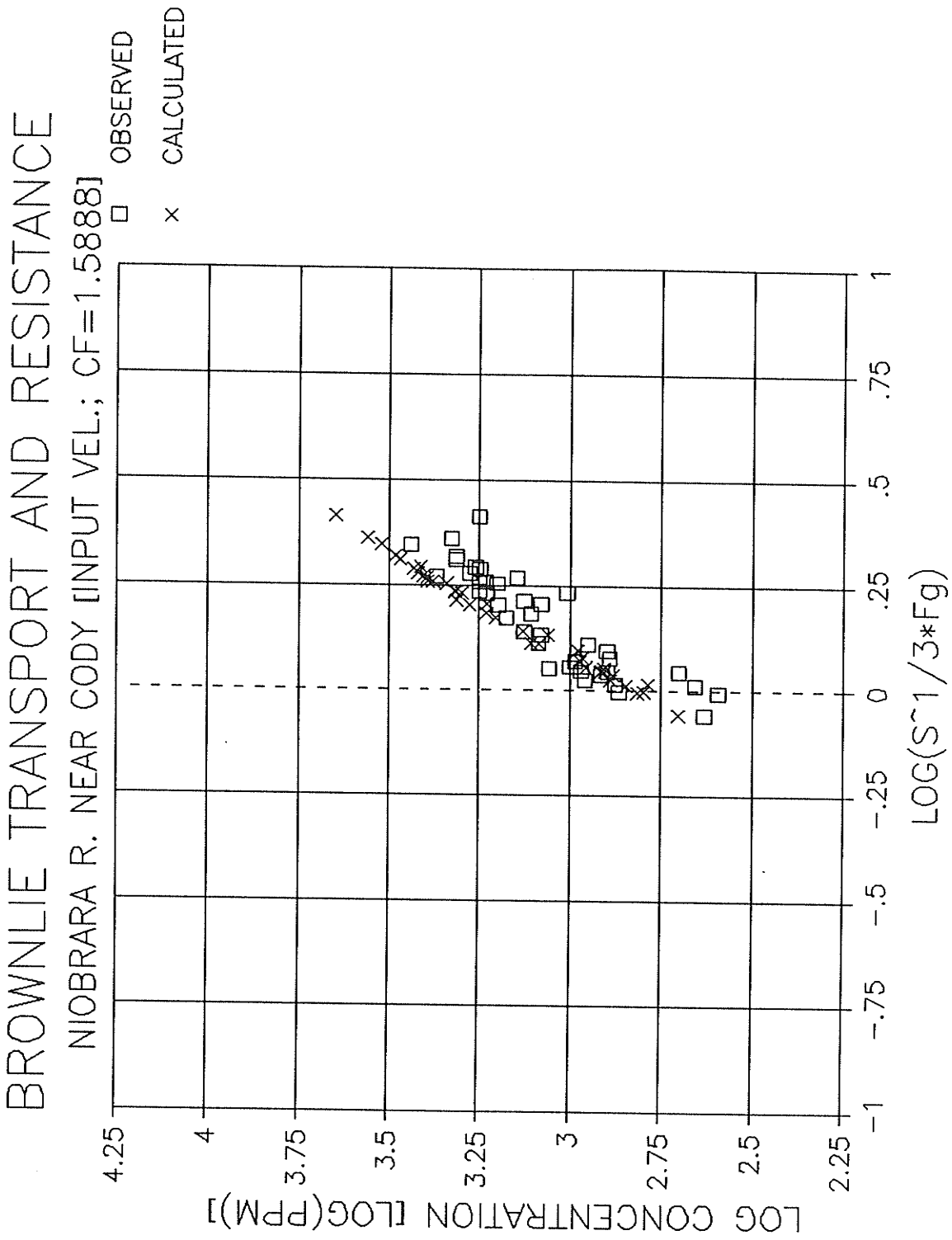


Figure E-3 Brown3 Sediment Transport Results For Observed Sediment Concentration Curves, Niobrara R. at Cody, Nebr., C_F = 1.5888; Observed Velocity and Depth.

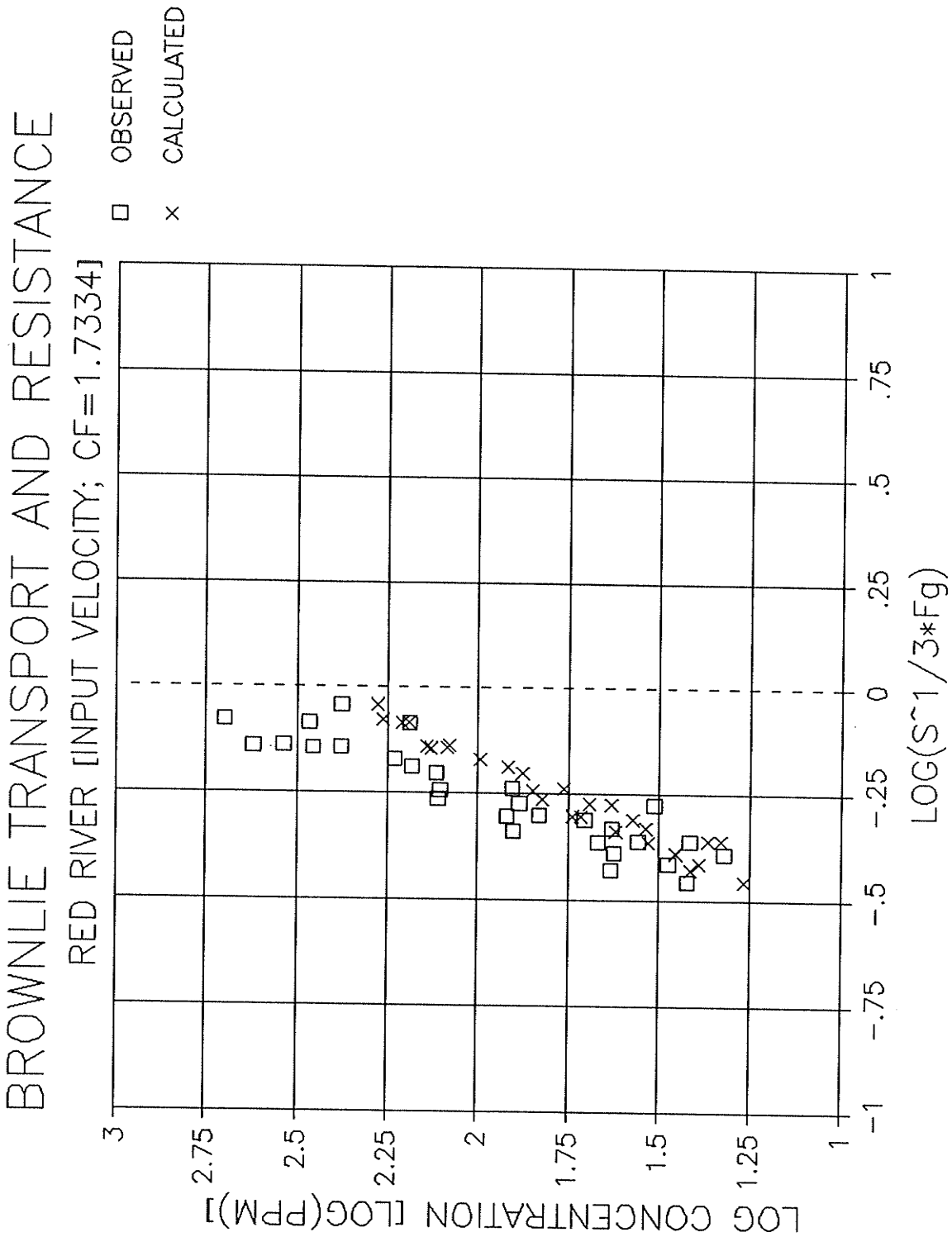


Figure E-4 Brown3 Sediment Transport Results For Observed Sediment Concentration Curves, Red River at Alexandria, La., C_F = 1.7334; Observed Velocity and Depth.

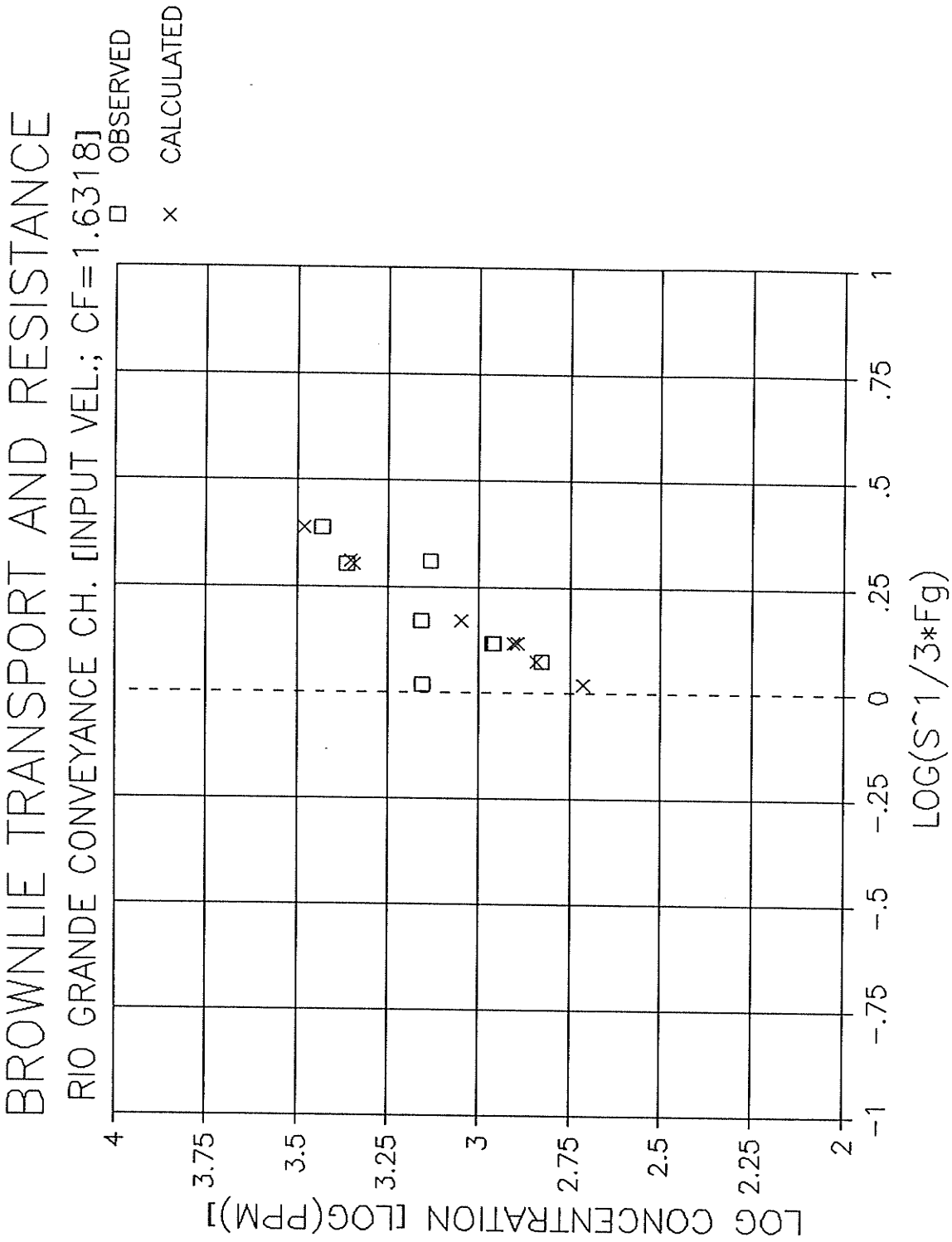


Figure E-5 Brown3 Sediment Transport Results For Observed Sediment Concentration Curves, Rio Grande Conveyance Channel, C_F = 1.6318; Observed Velocity and Depth.

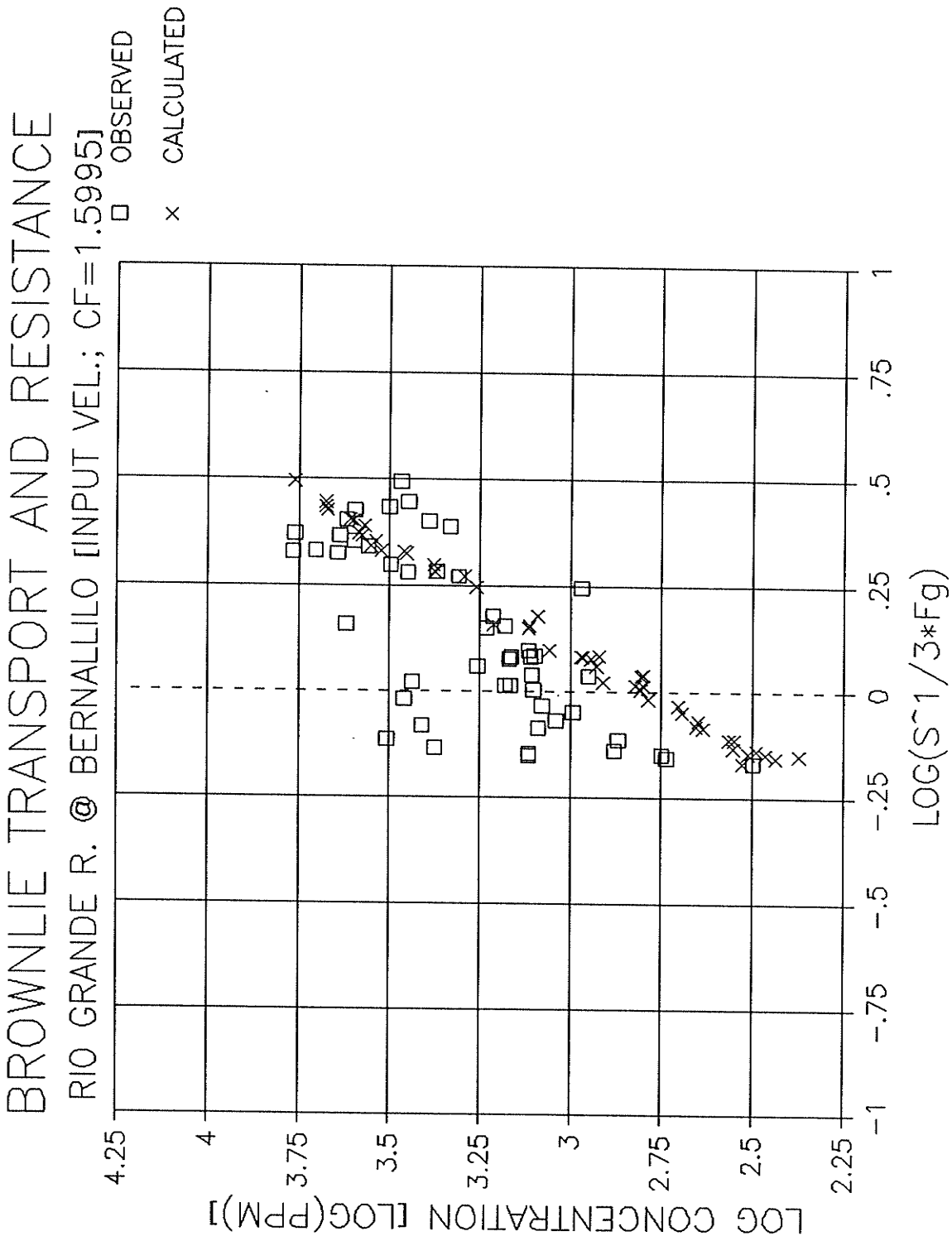
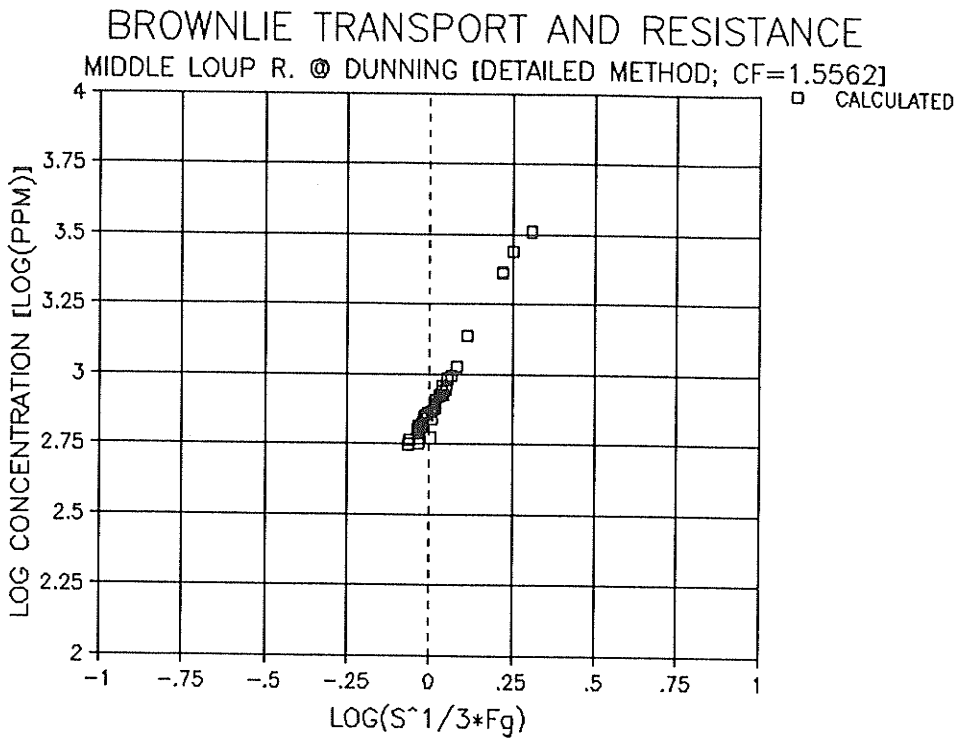


Figure E-6 Brown3 Sediment Transport Results For Observed Sediment Concentration Curves, Rio Grande R. near Bernallilo, N.M. C_F = 1.5995; Observed Velocity and Depth.



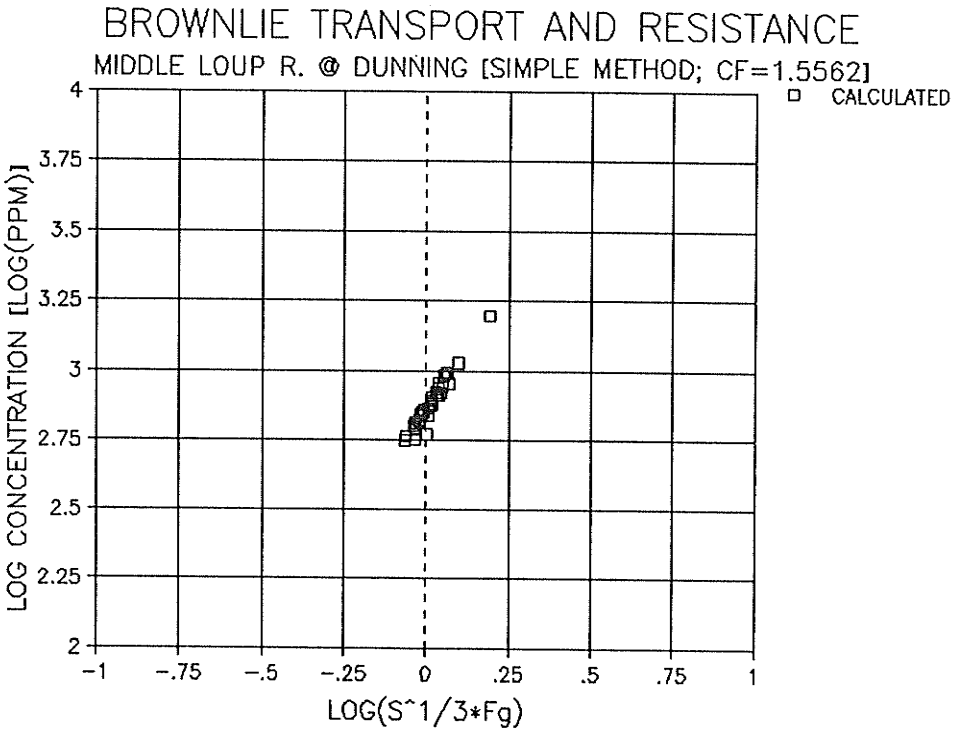
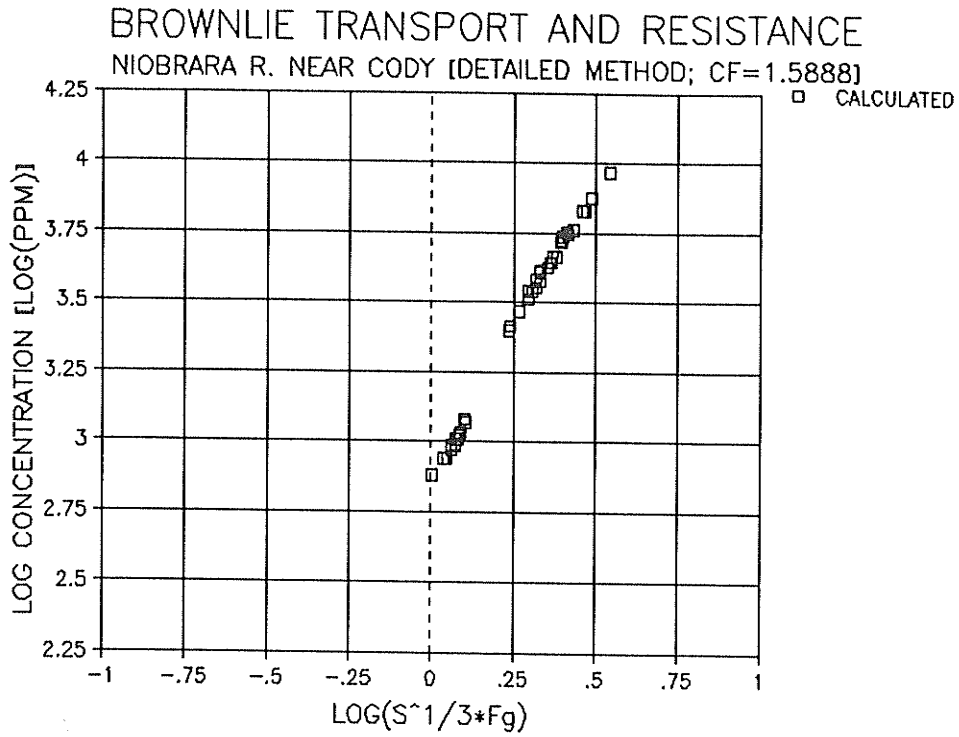


Figure E-7 Brown3 Sediment Transport Results For Observed Sediment Concentration Curves, Middle Loup R. near Dunning, Nebr., C_F = 1.5662; a) Detailed and b) Simplified Methods.



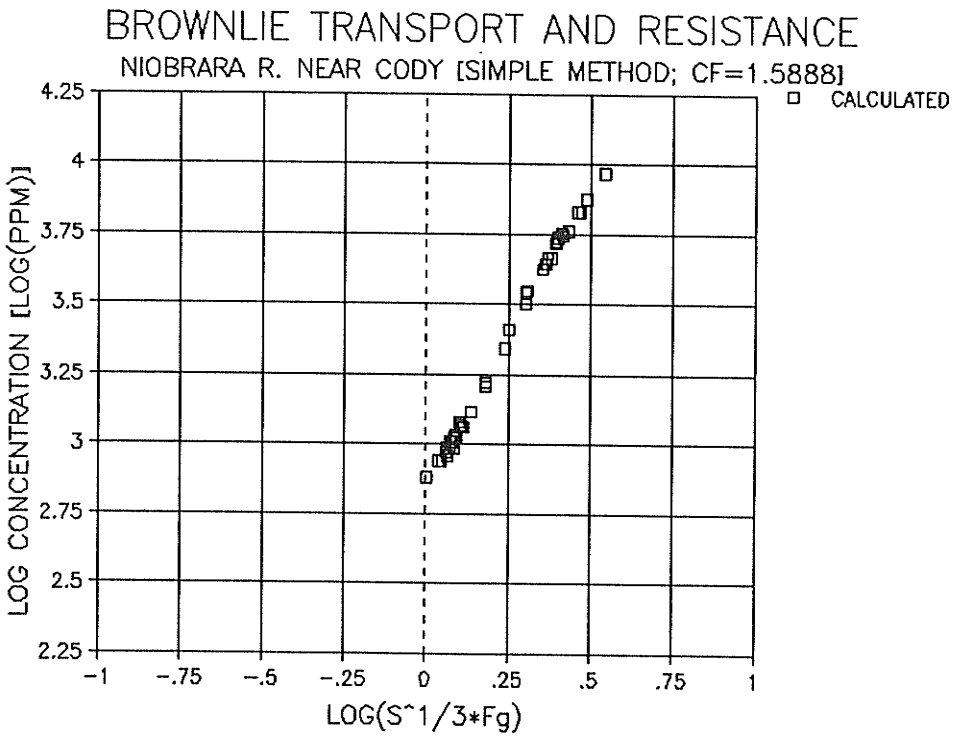
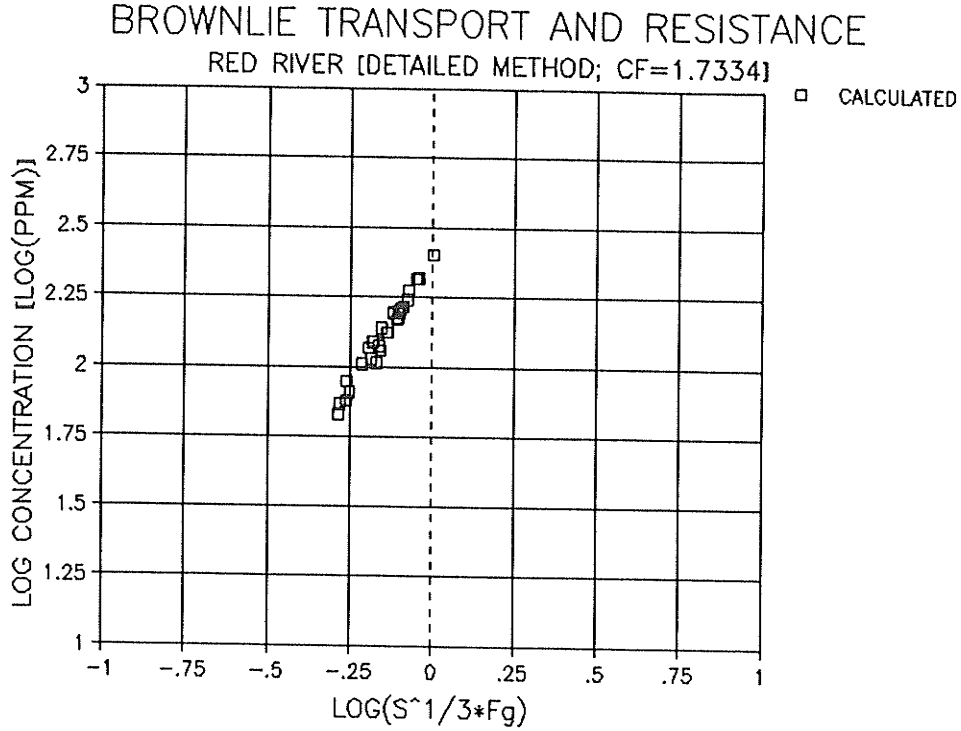


Figure E-8 Brown3 Sediment Transport Results For Observed Sediment Concentration Curves, Niobrara R. at Cody, Nebr., C_F = 1.5888; a) Detailed and b) Simplified Methods.



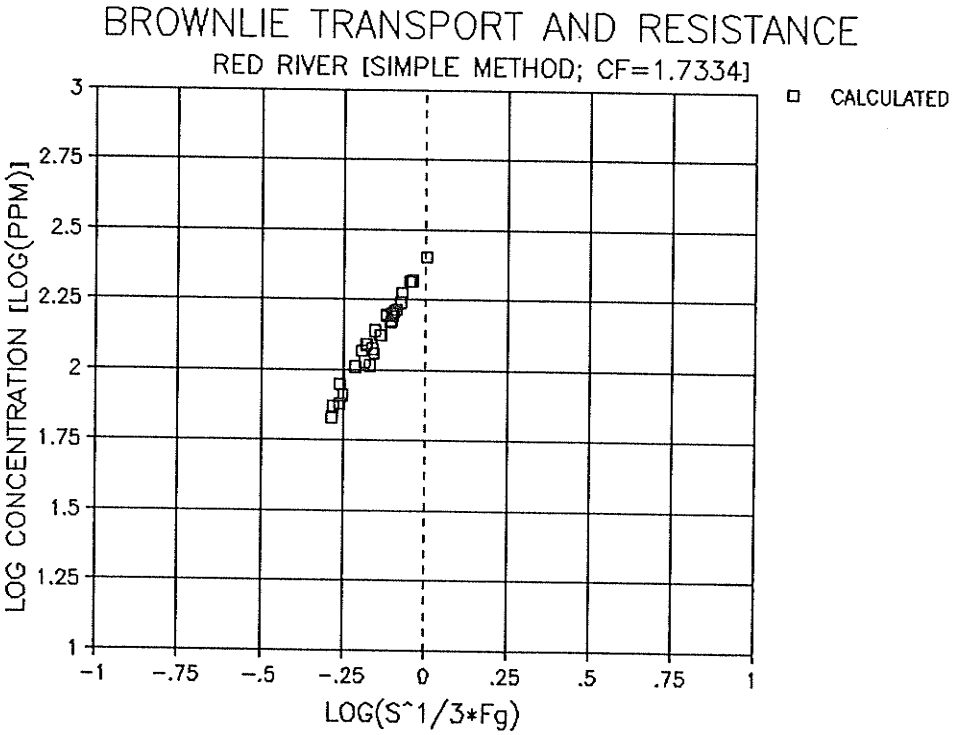
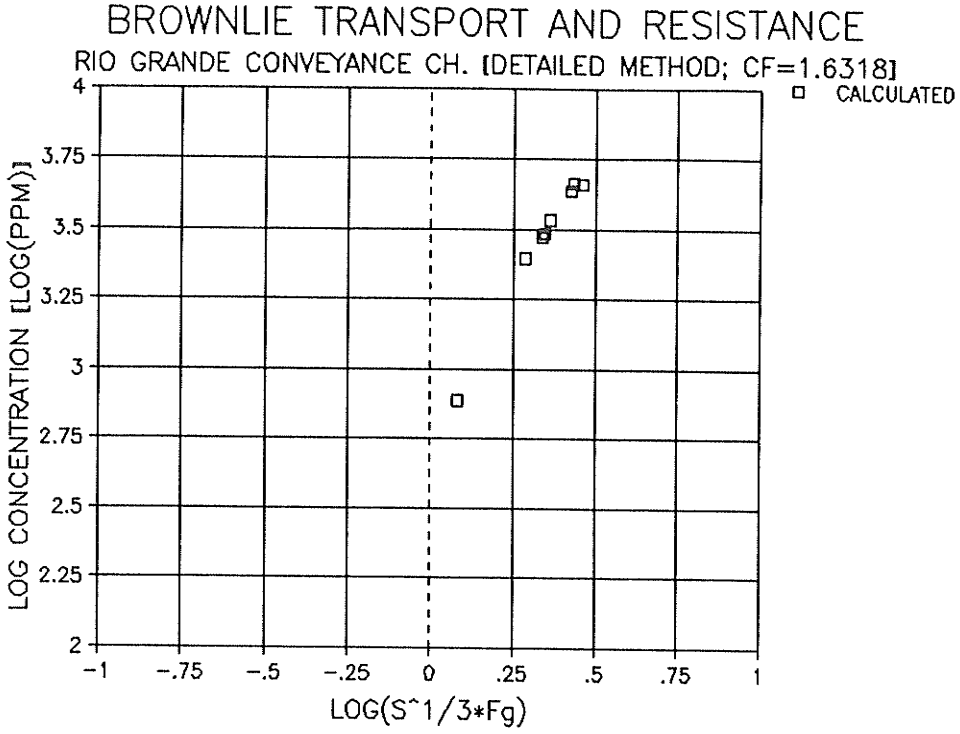


Figure E-9 Brown3 Sediment Transport Results For Observed Sediment Concentration Curves, Red River at Alexandria, La., C_F = 1.7334; a) Detailed and b) Simplified Methods.



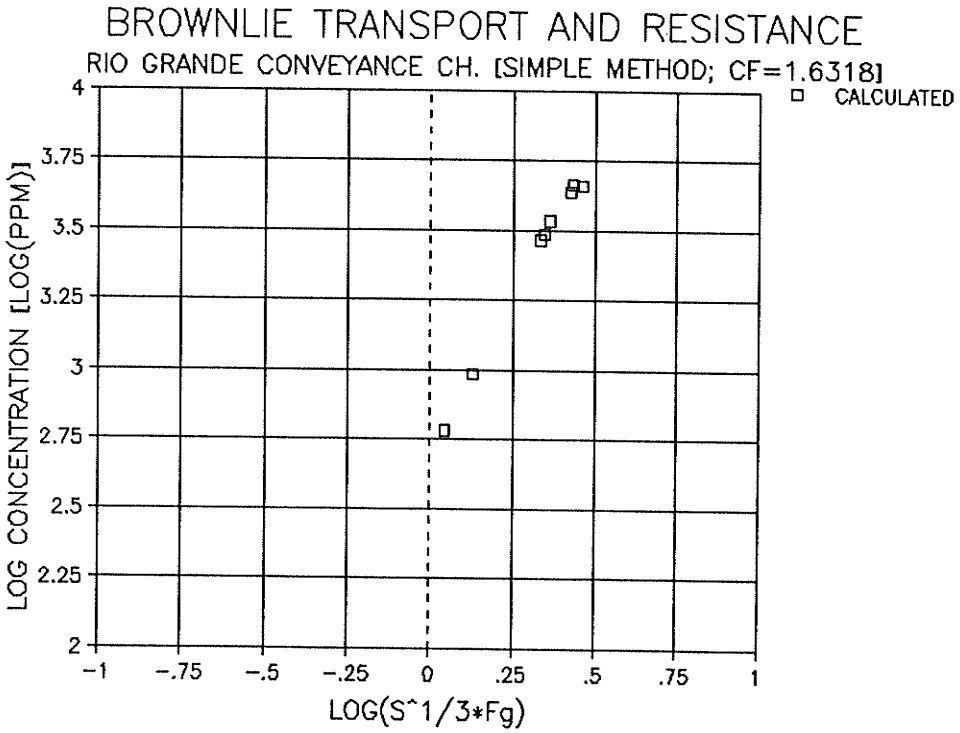
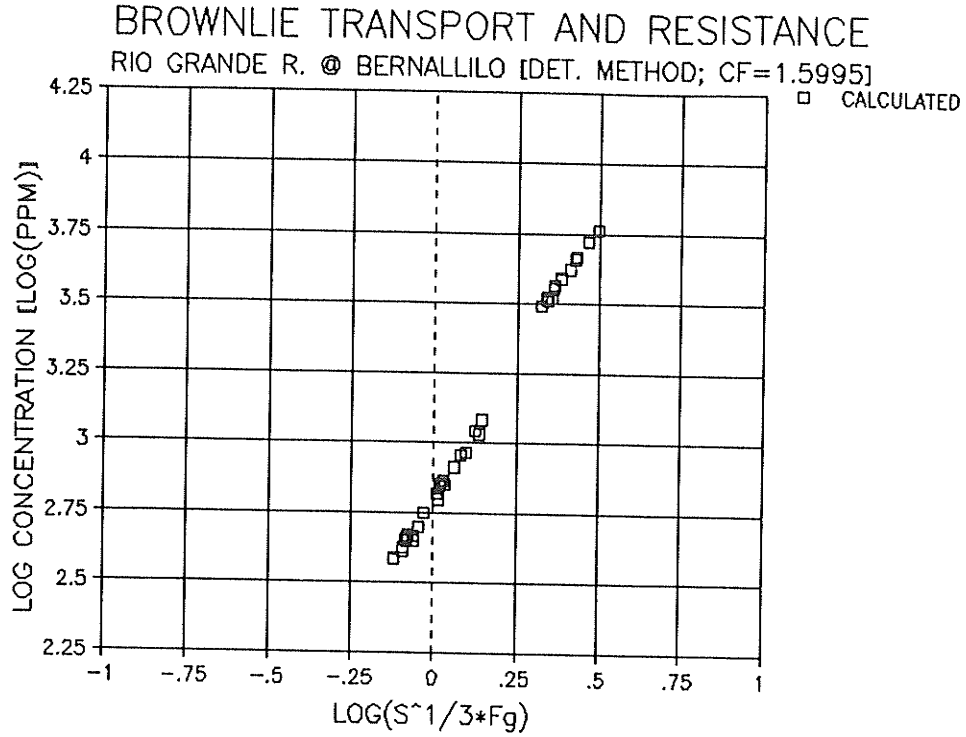


Figure E-10 Brown3 Sediment Transport Results For Observed Sediment Concentration Curves, Rio Grande Conveyance Channel, C_F = 1.6318; a) Detailed and b) Simplified Methods.



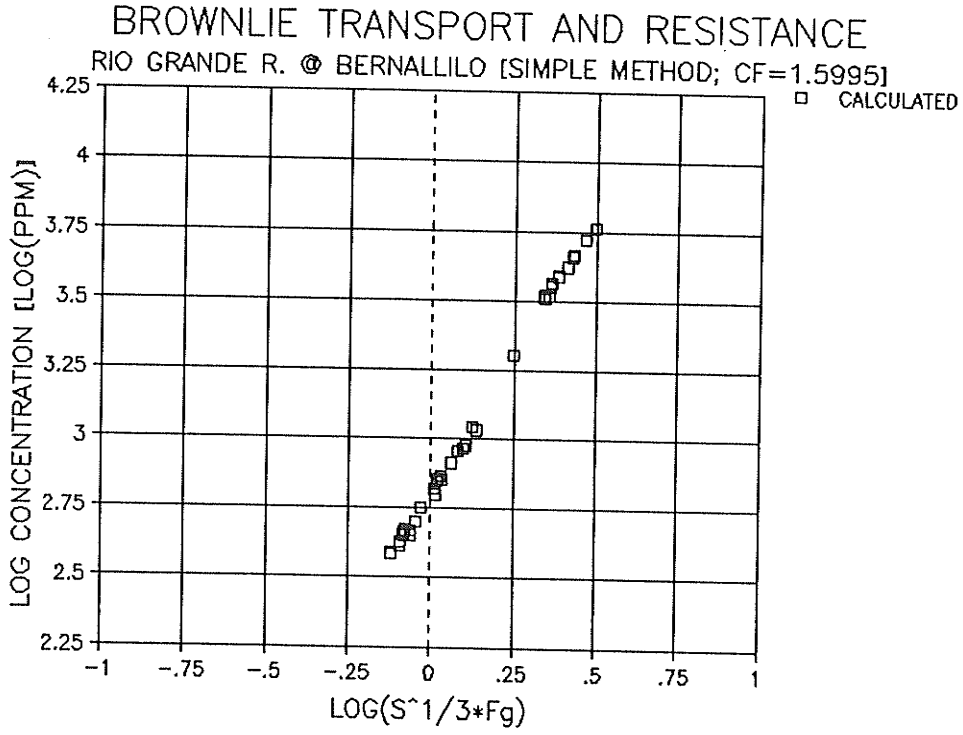


Figure E-11 Brown3 Sediment Transport Results For Observed Sediment Concentration Curves, Rio Grande R. near Bernallilo, N.M. C_F = 1.5995; a) Detailed and b) Simplified Methods.

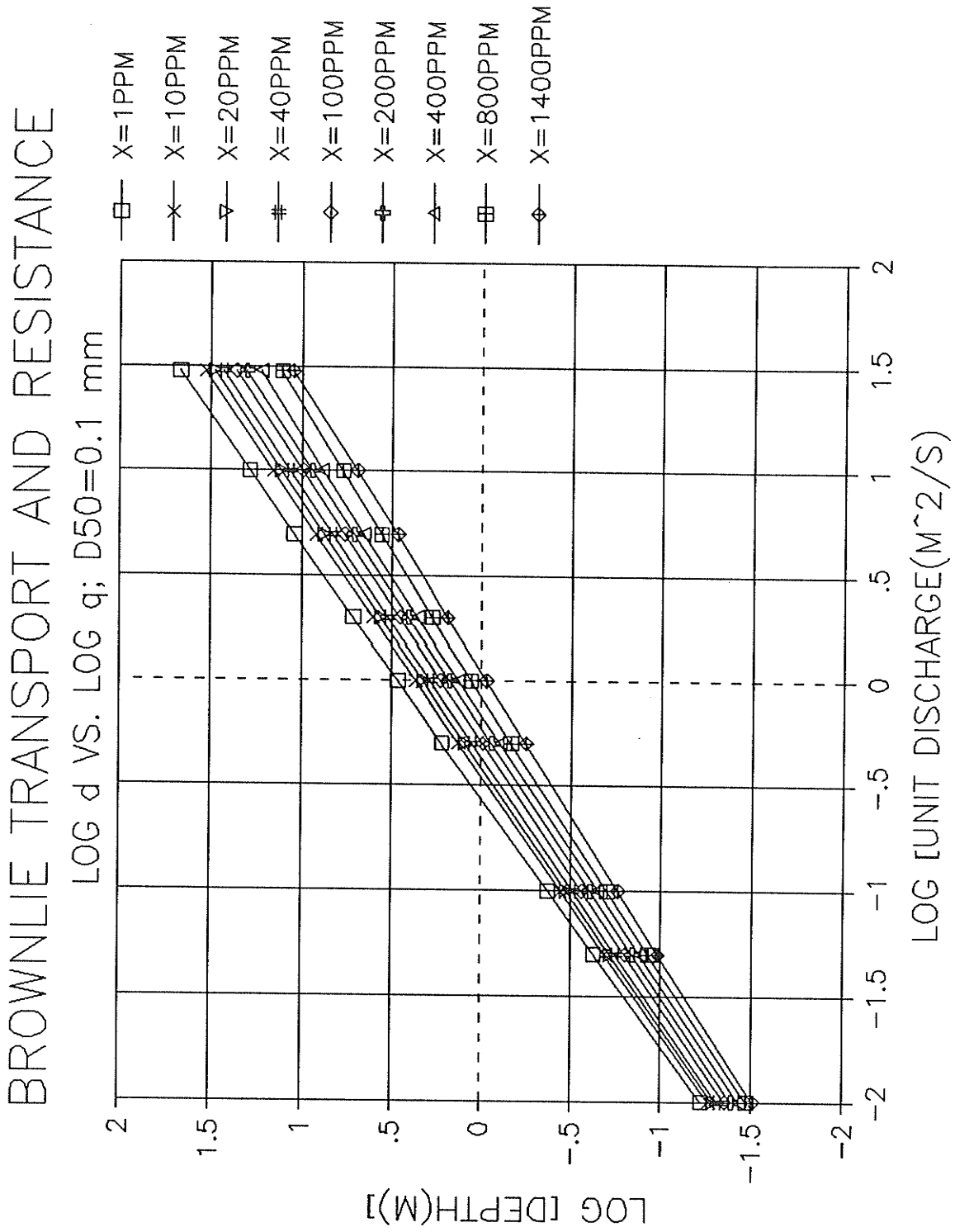


Figure E-12 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 0.1 mm; Lower, Transitional and Upper Flow Regimes

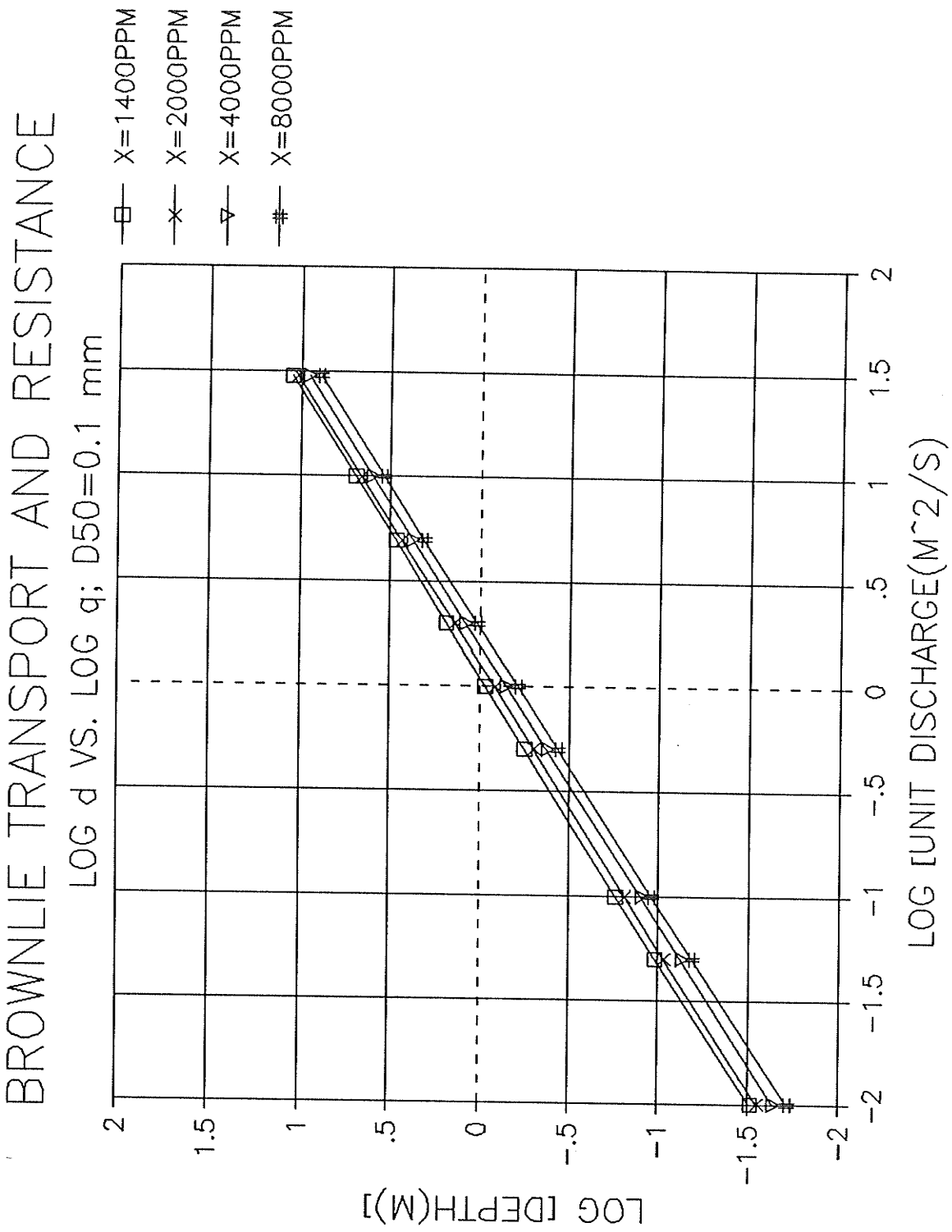


Figure E-12 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 0.1 mm; Lower, Transitional and Upper Flow Regimes

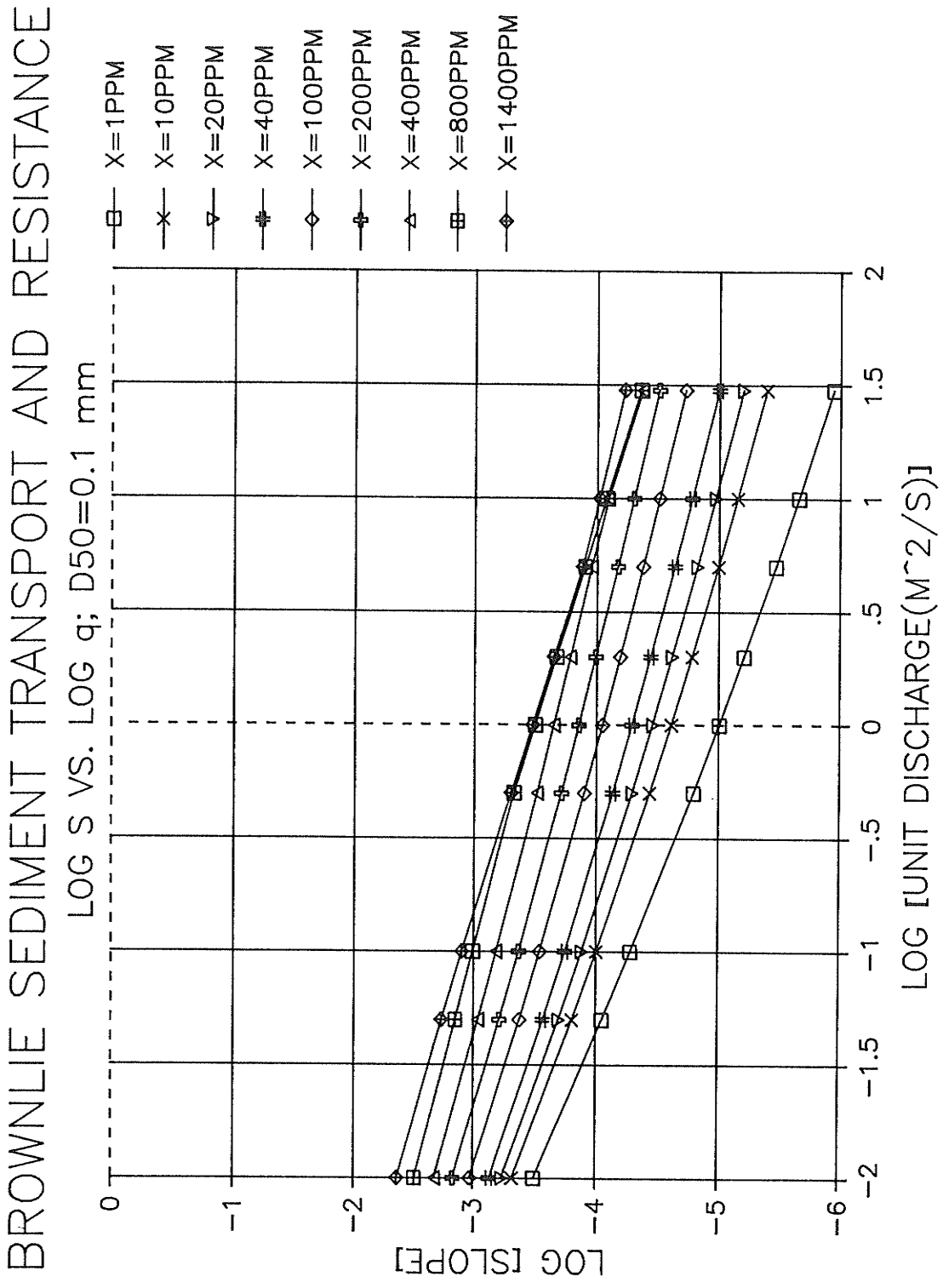


Figure E-13 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 0.1 mm; Lower, Transitional and Upper Flow Regimes

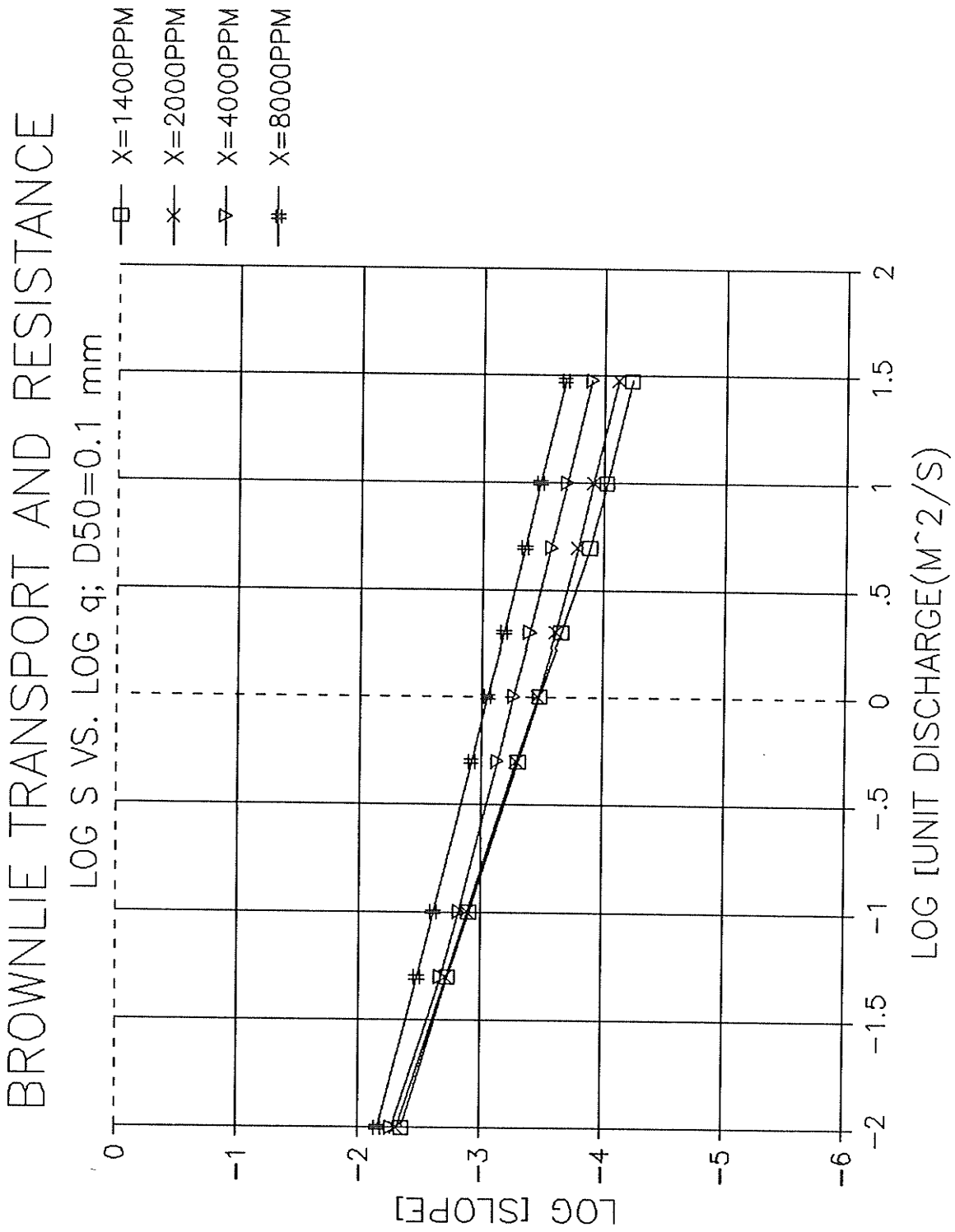


Figure E-13 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 0.1 mm; Lower, Transitional and Upper Flow Regimes

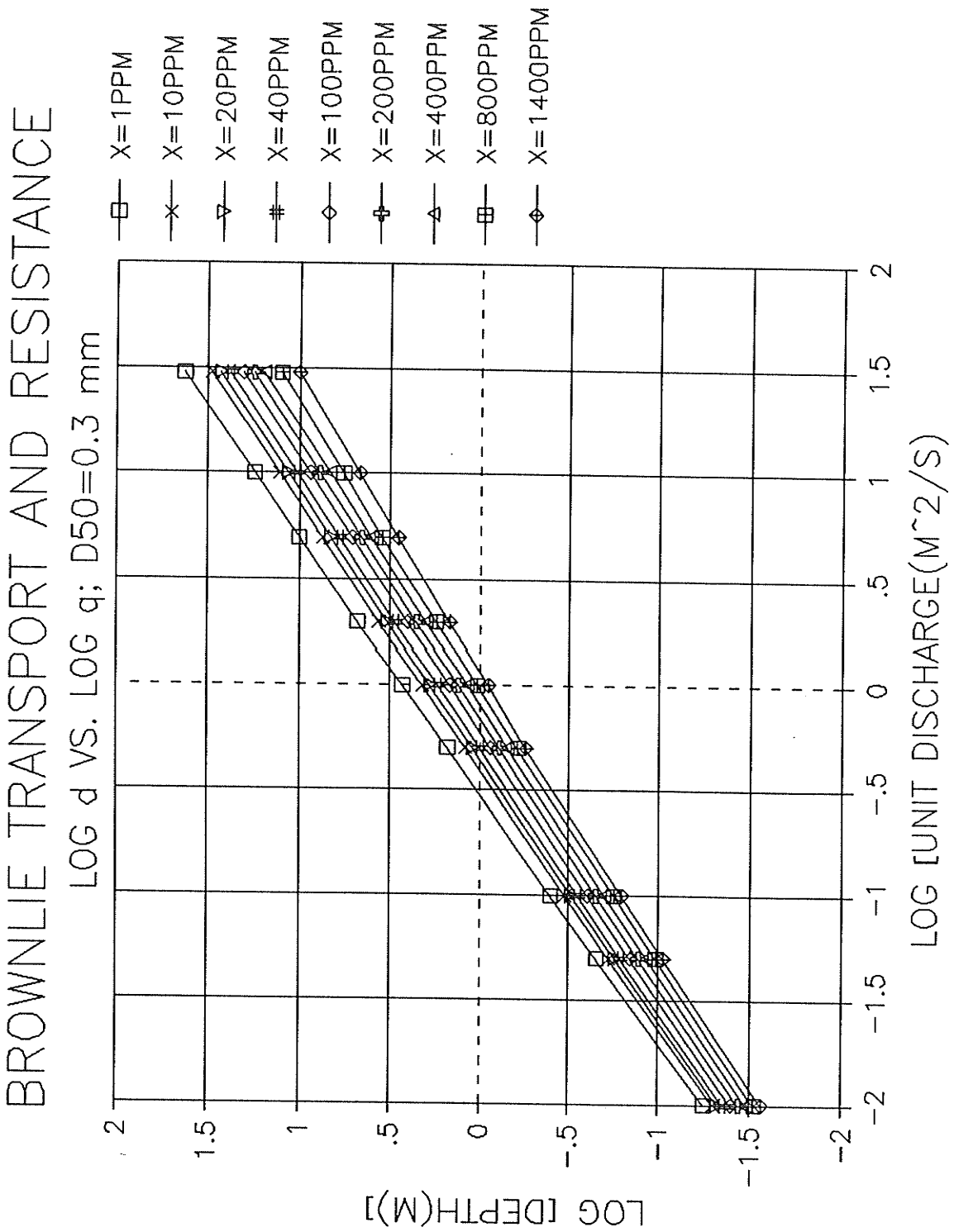


Figure E-14 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 0.3 mm; Lower, Transitional and Upper Flow Regimes

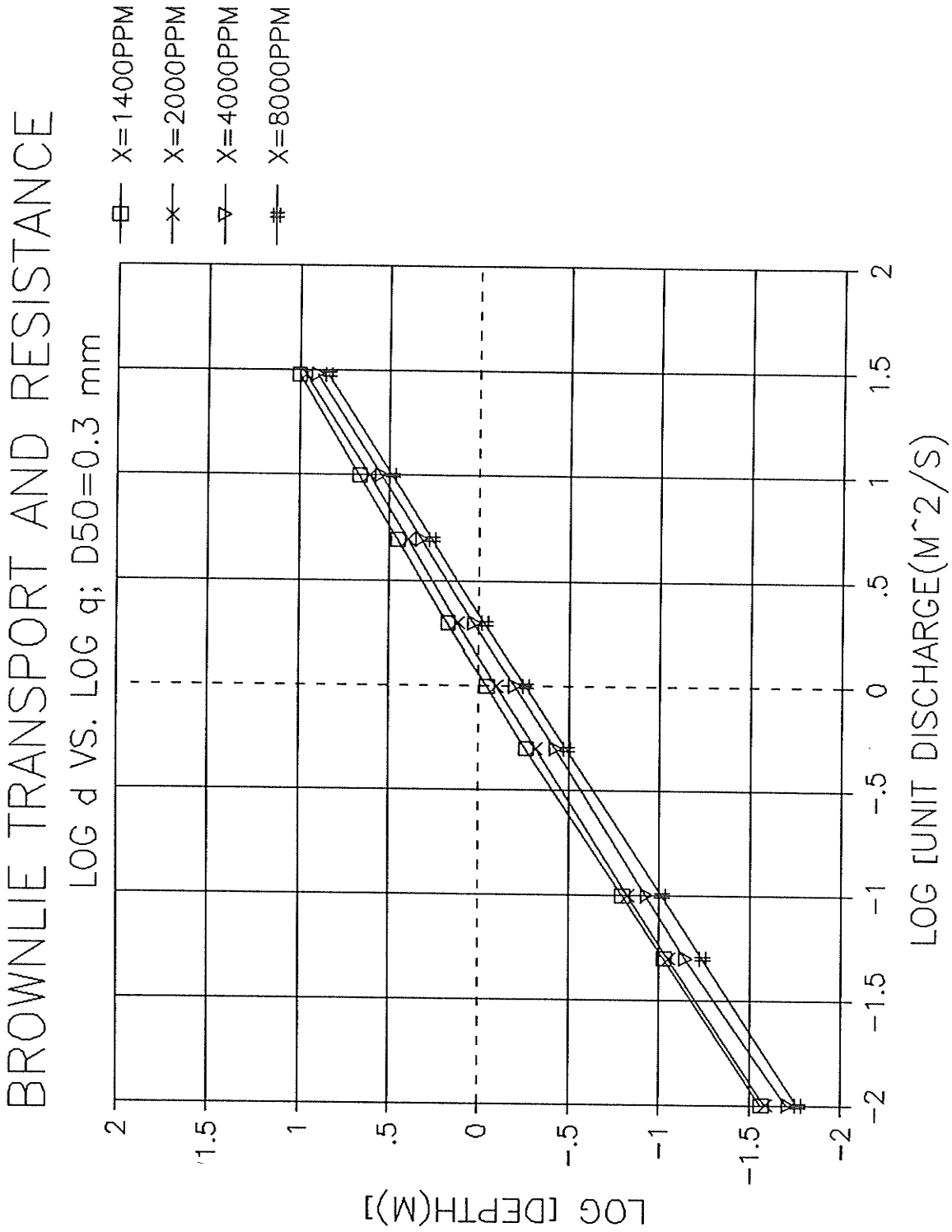


Figure E-14 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 0.3 mm; Lower, Transitional and Upper Flow Regimes

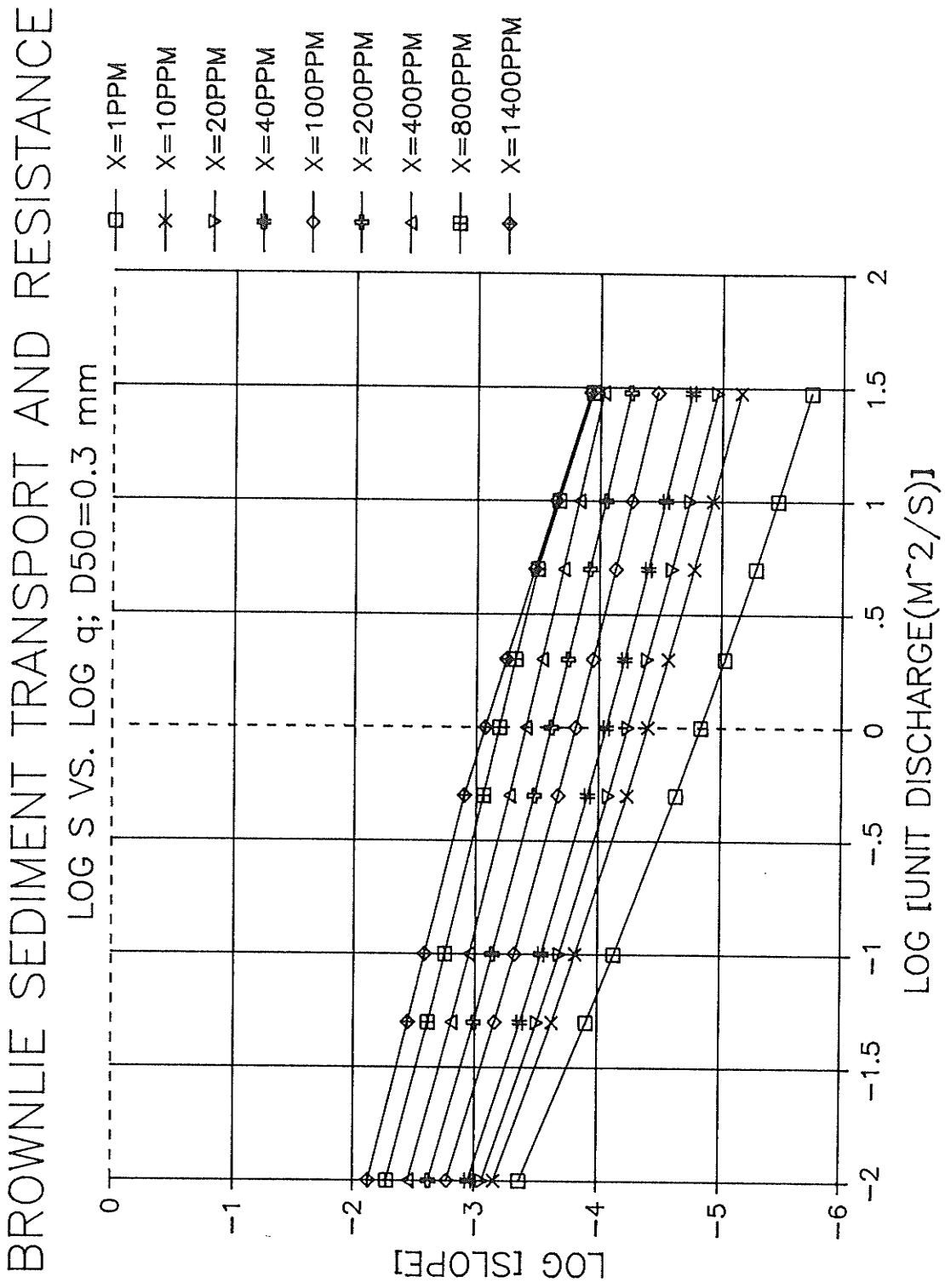


Figure E-15 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 0.3 mm; Lower, Transitional and Upper Flow Regimes

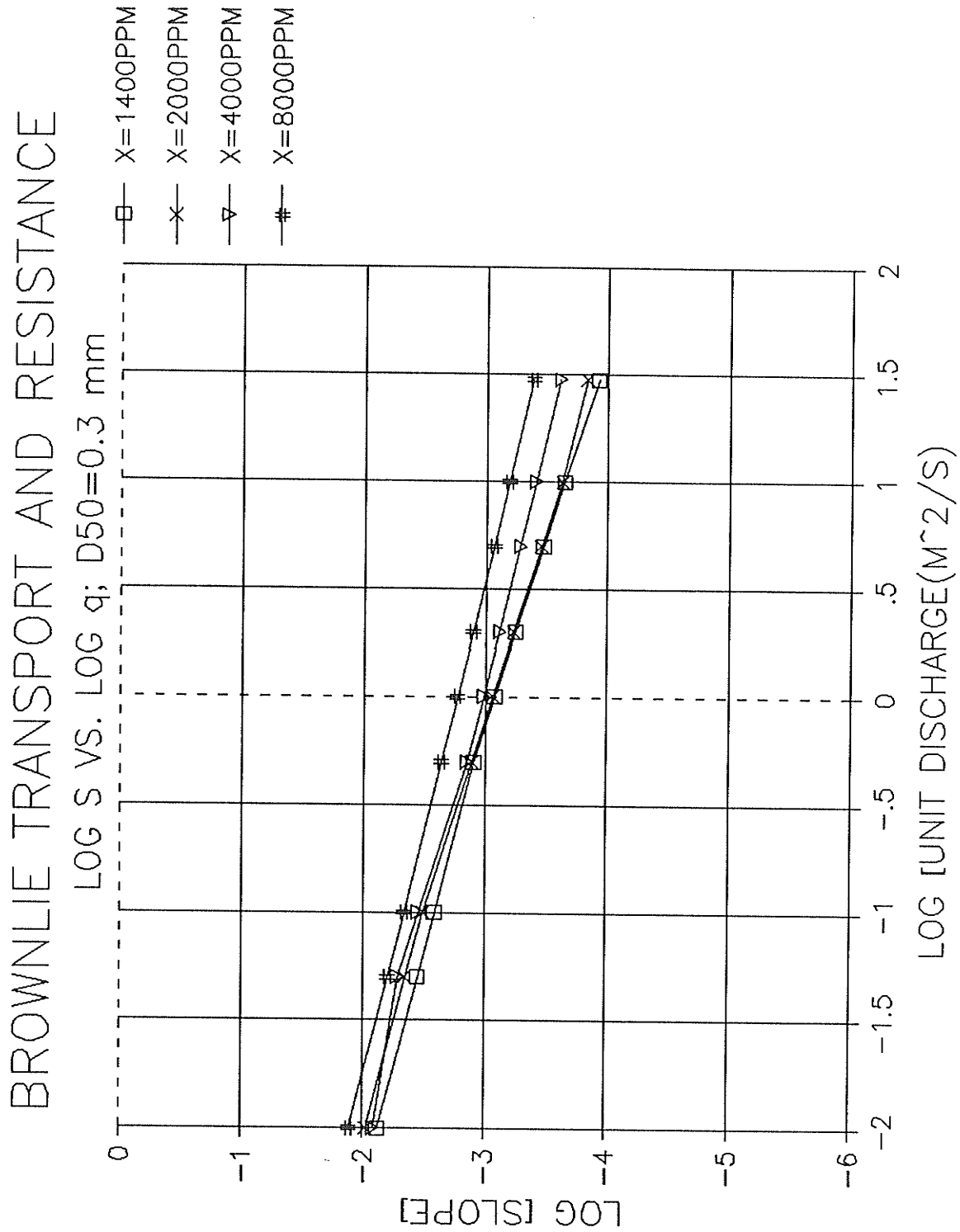


Figure E-15 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 0.3 mm; Lower, Transitional and Upper Flow Regimes

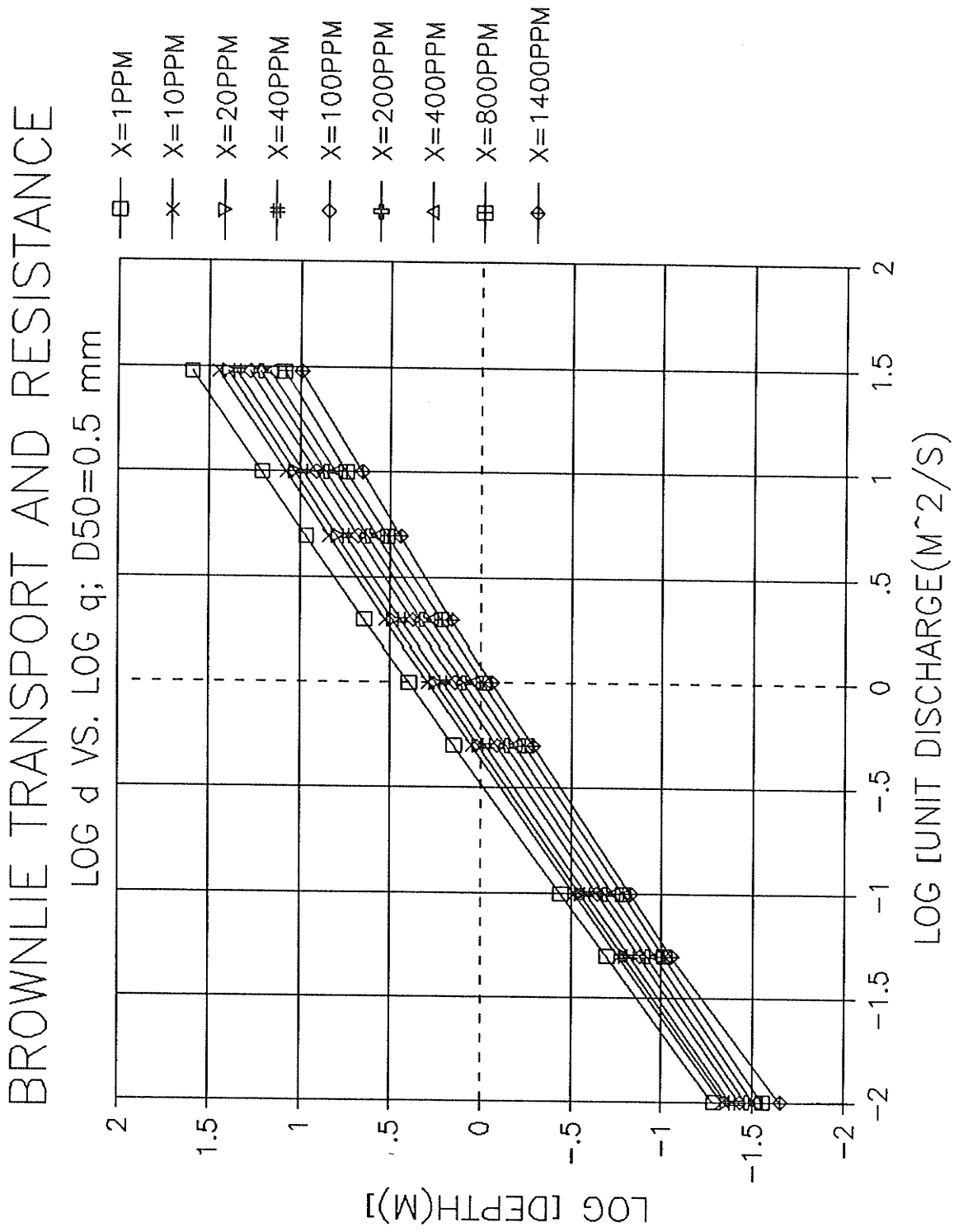


Figure E-16 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 0.5 mm; Lower, Transitional and Upper Flow Regimes

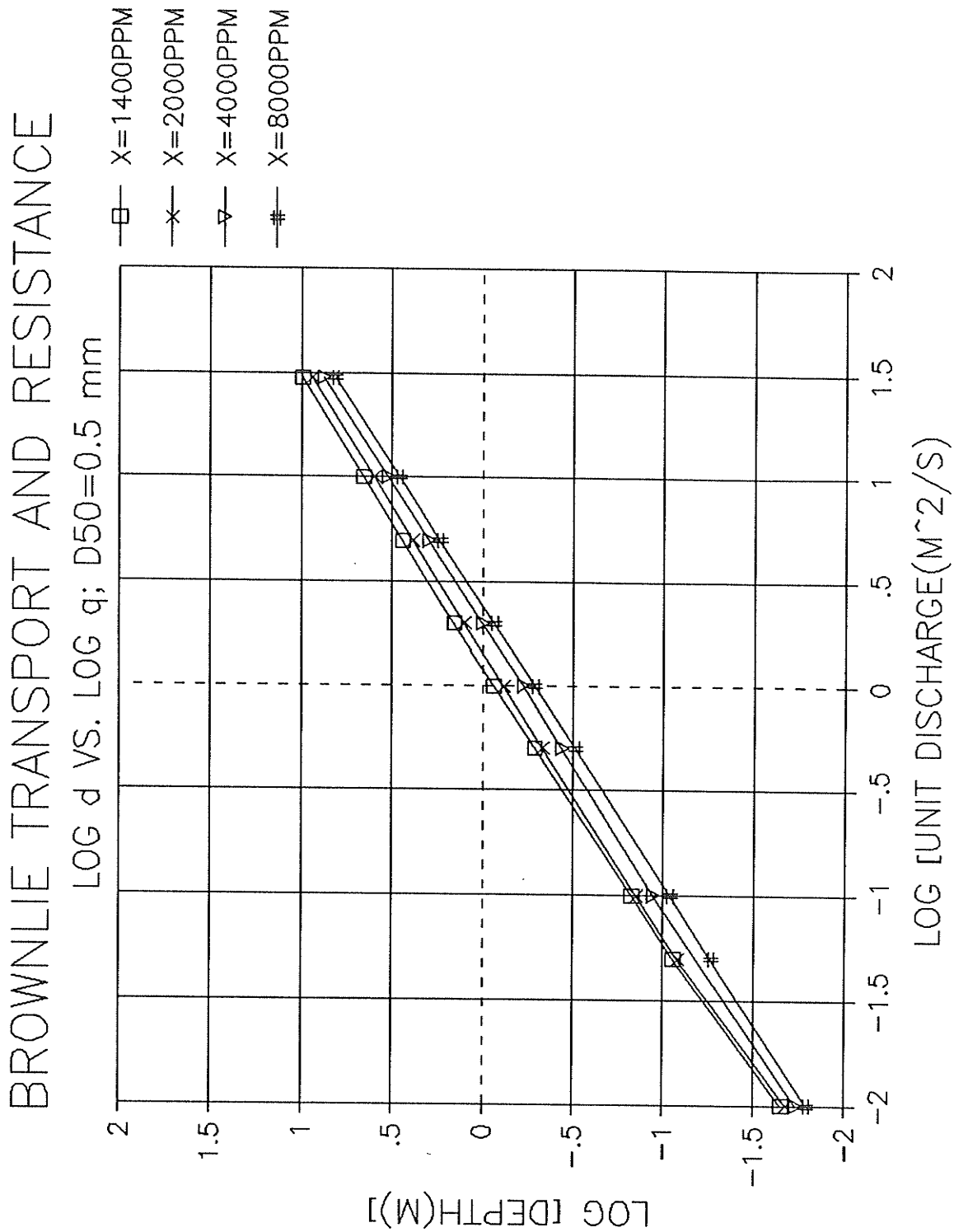


Figure E-16 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 0.5 mm; Lower, Transitional and Upper Flow Regimes

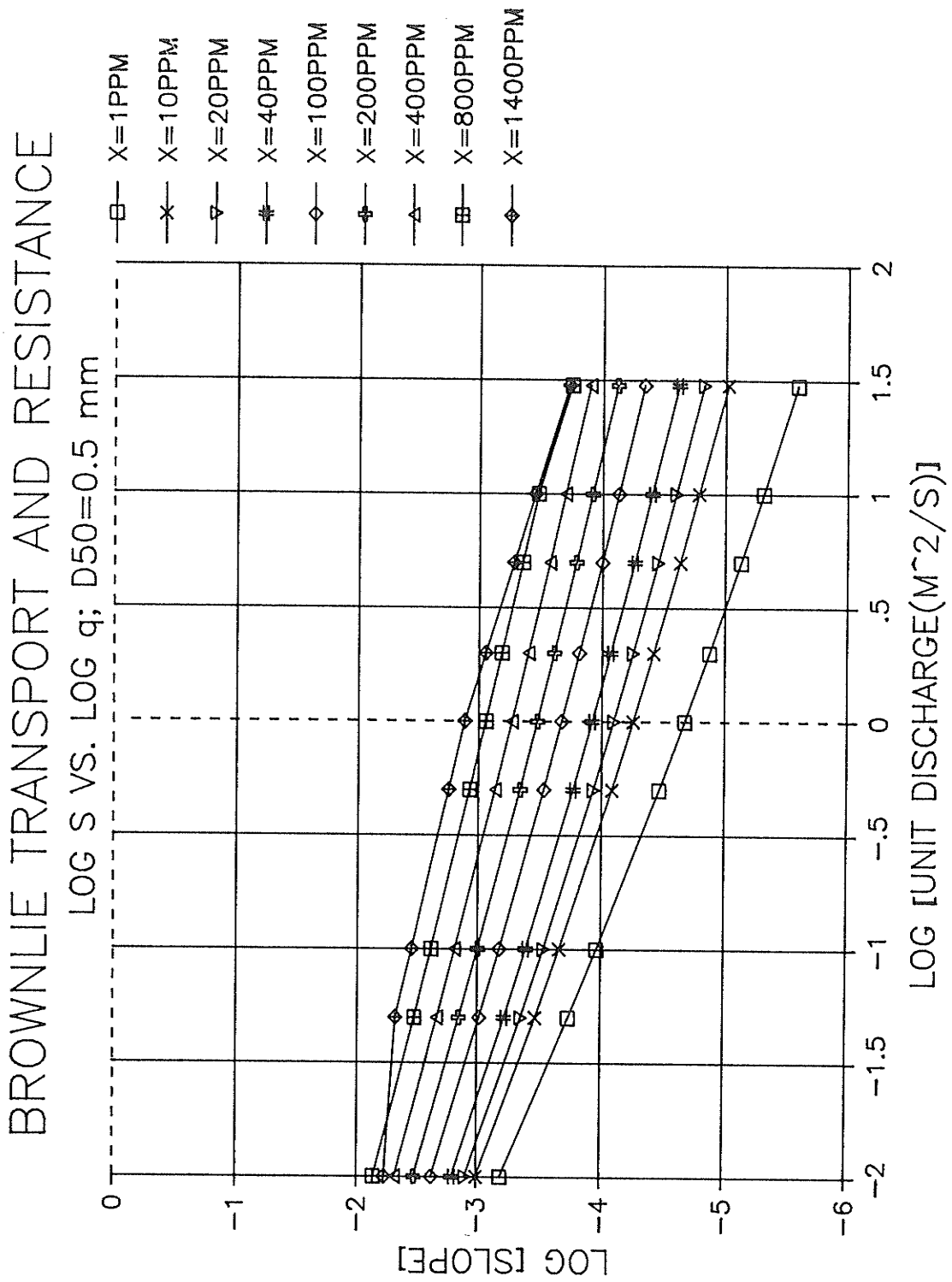


Figure E-17 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 0.5 mm; Lower, Transitional and Upper Flow Regimes

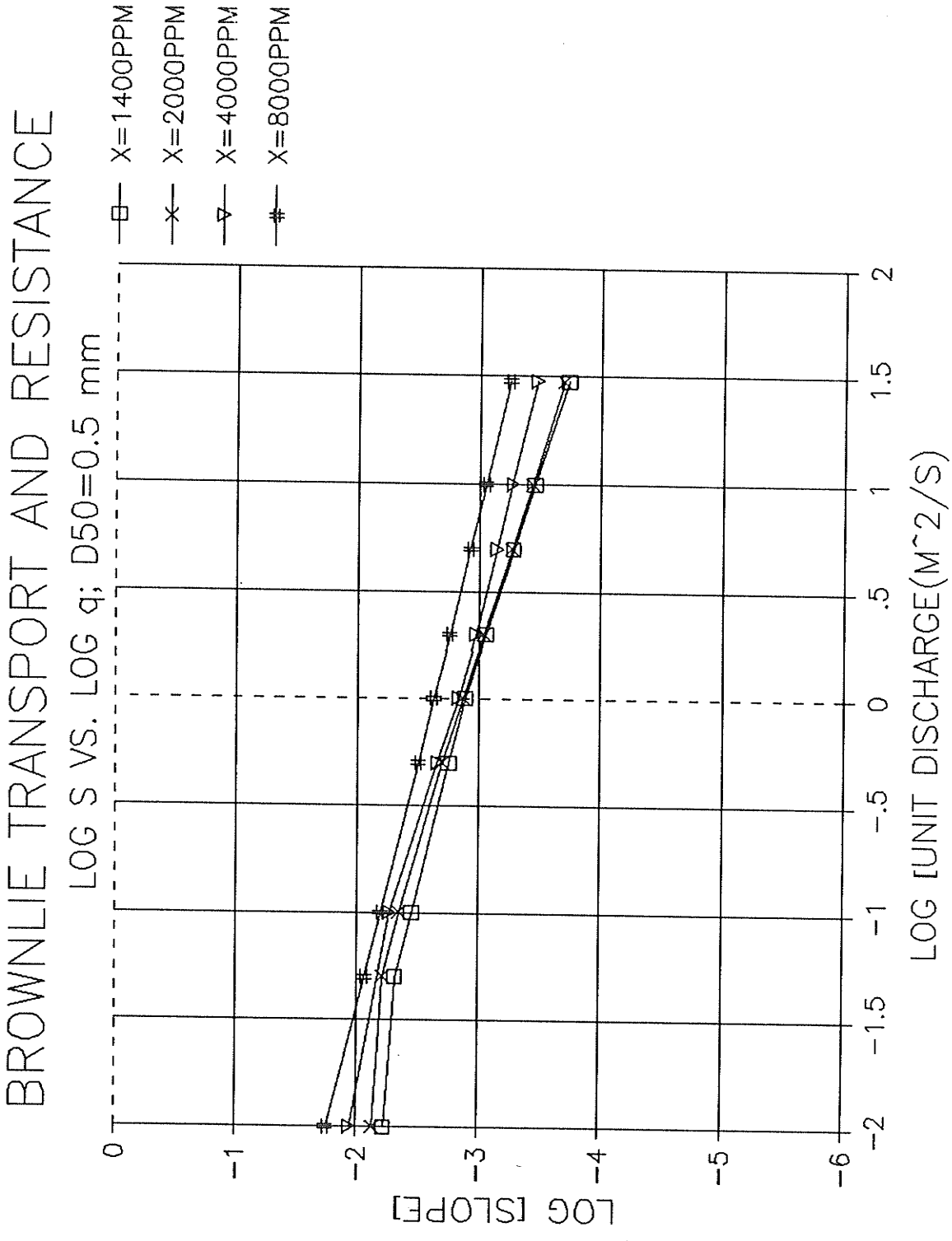


Figure E-17 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 0.5 mm; Lower, Transitional and Upper Flow Regimes

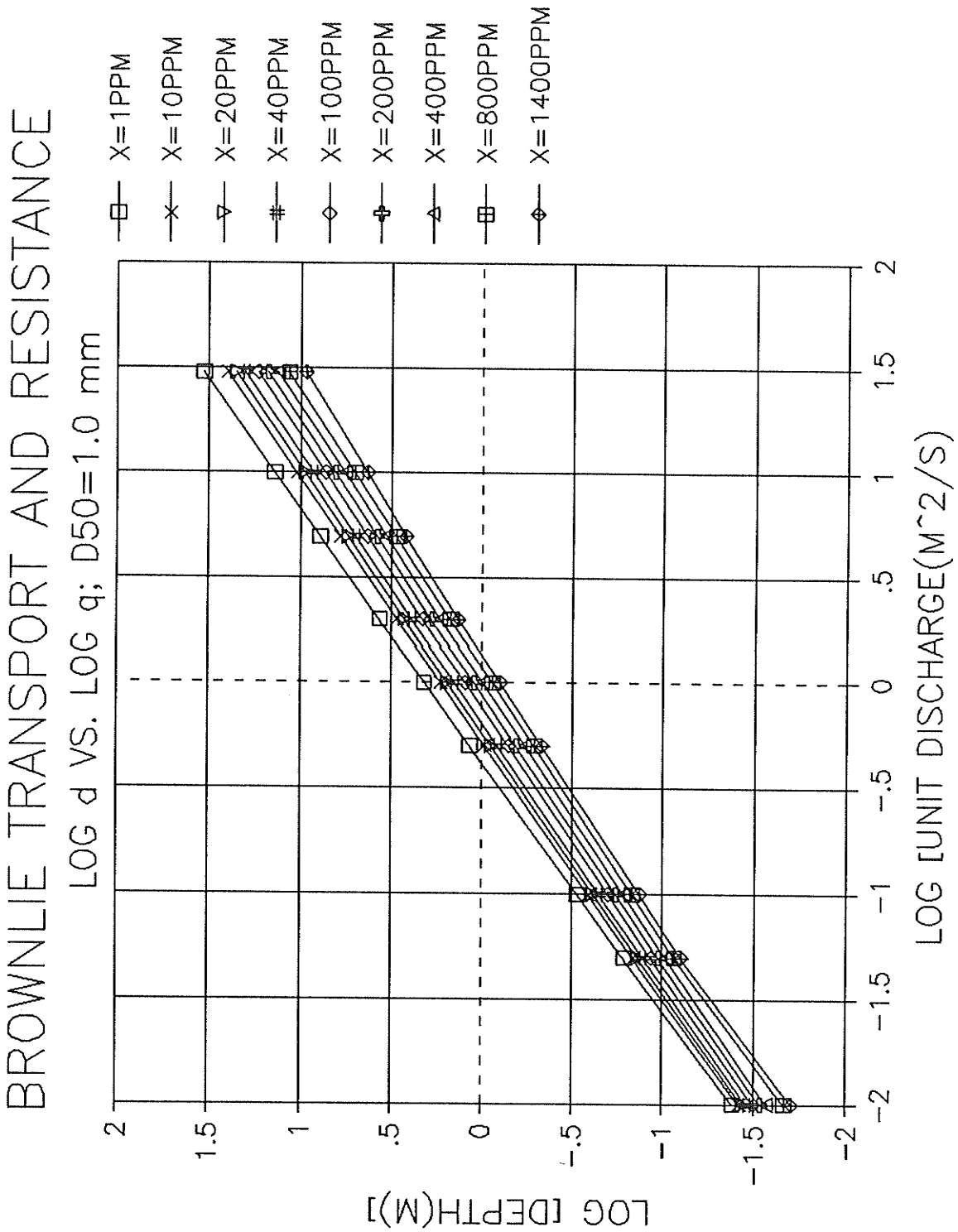


Figure E-18 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 1.0 mm; Lower, Transitional and Upper Flow Regimes

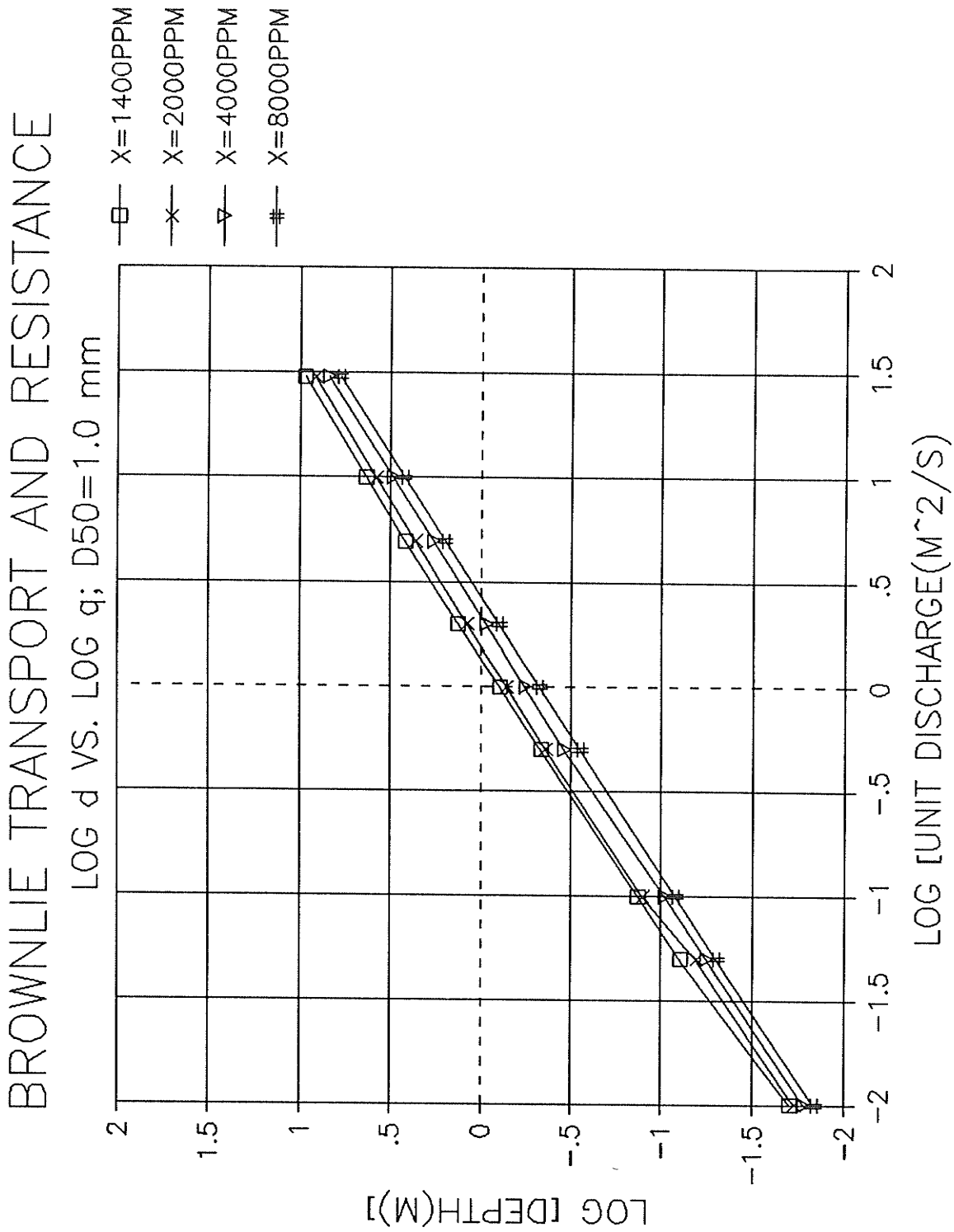


Figure E-18 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 1.0 mm; Lower, Transitional and Upper Flow Regimes

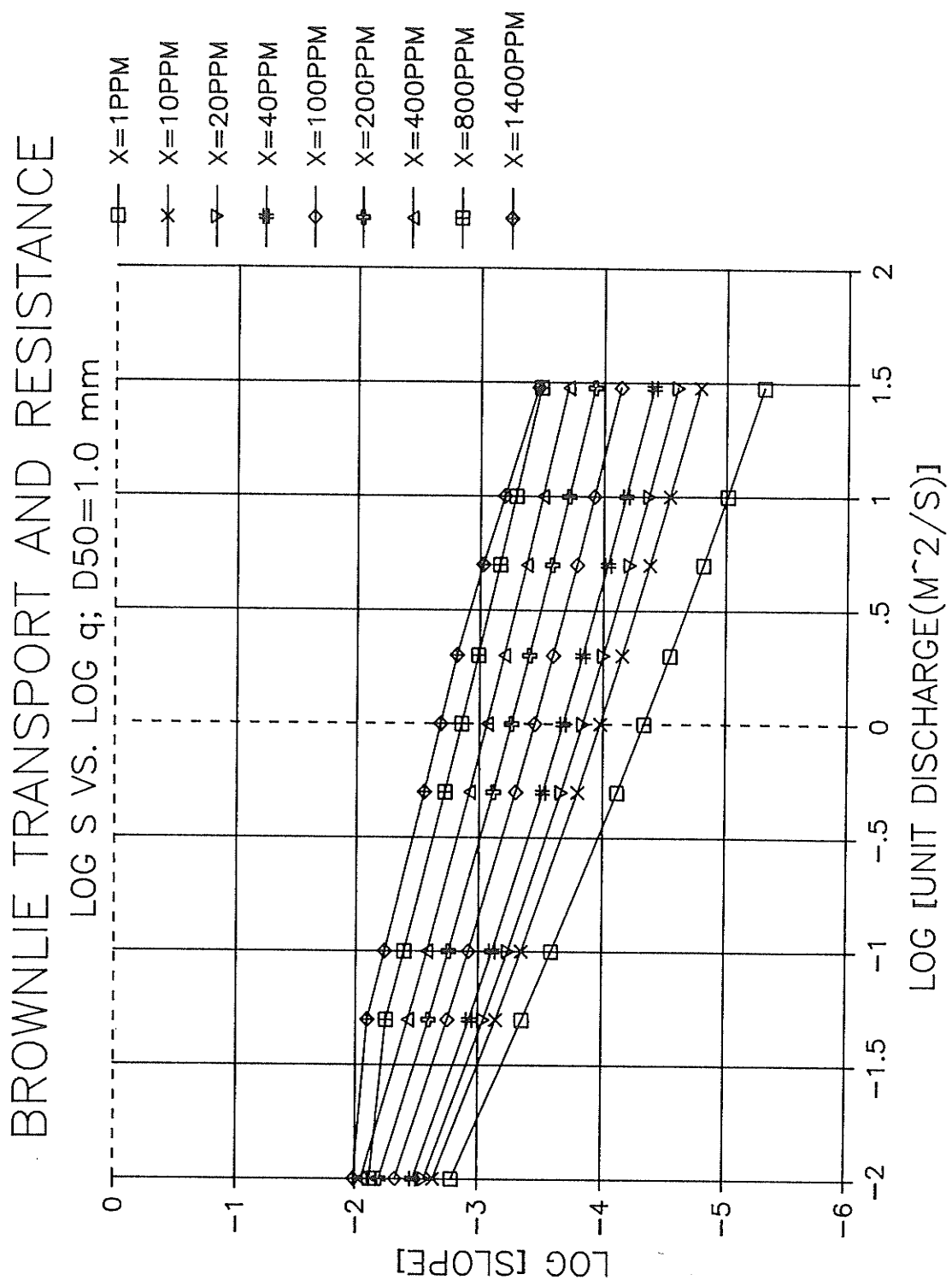


Figure E-19 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 1.0 mm; Lower, Transitional and Upper Flow Regimes

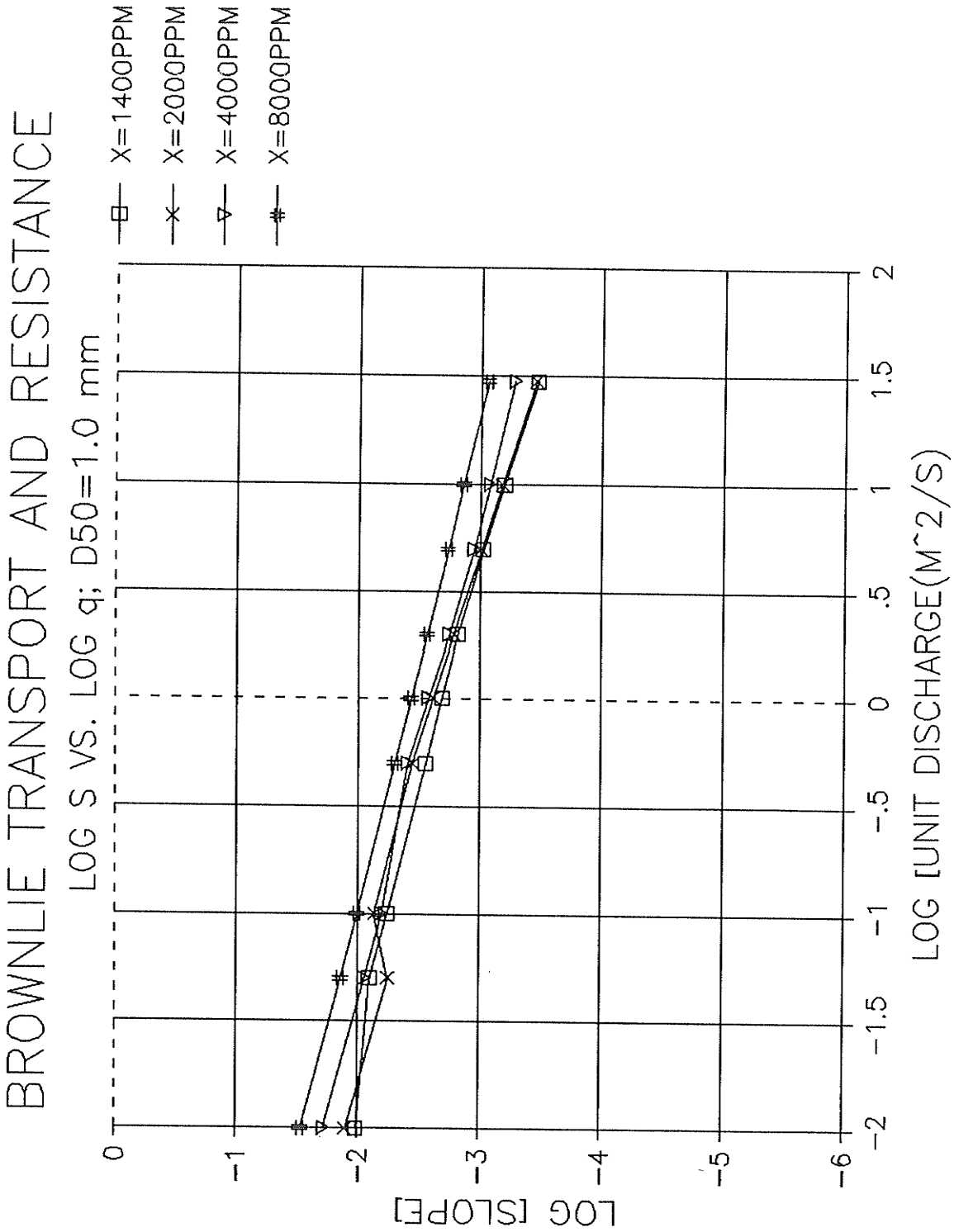


Figure E-19 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 1.0 mm; Lower, Transitional and Upper Flow Regimes

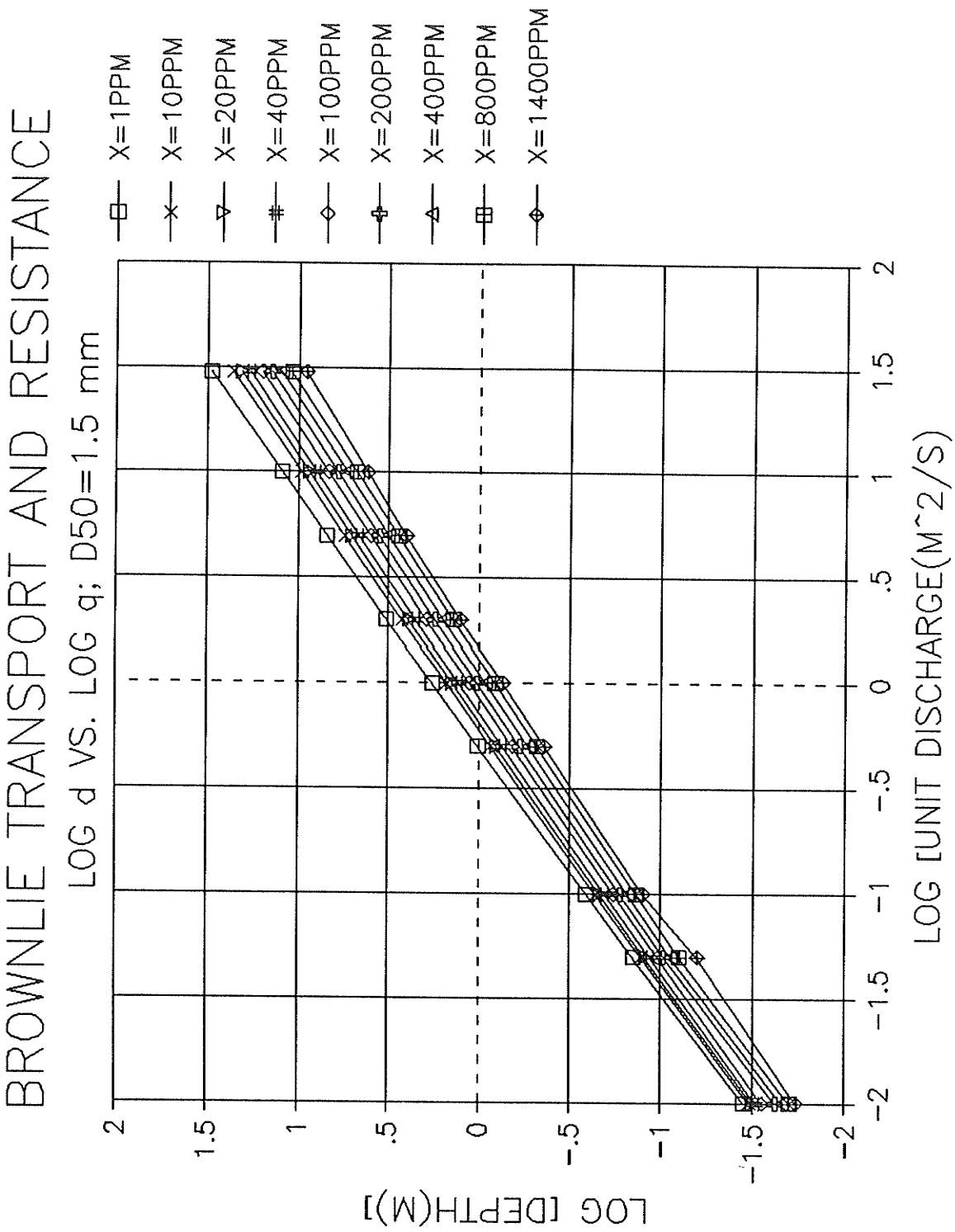


Figure E-20 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 1.5 mm; Lower, Transitional and Upper Flow Regimes

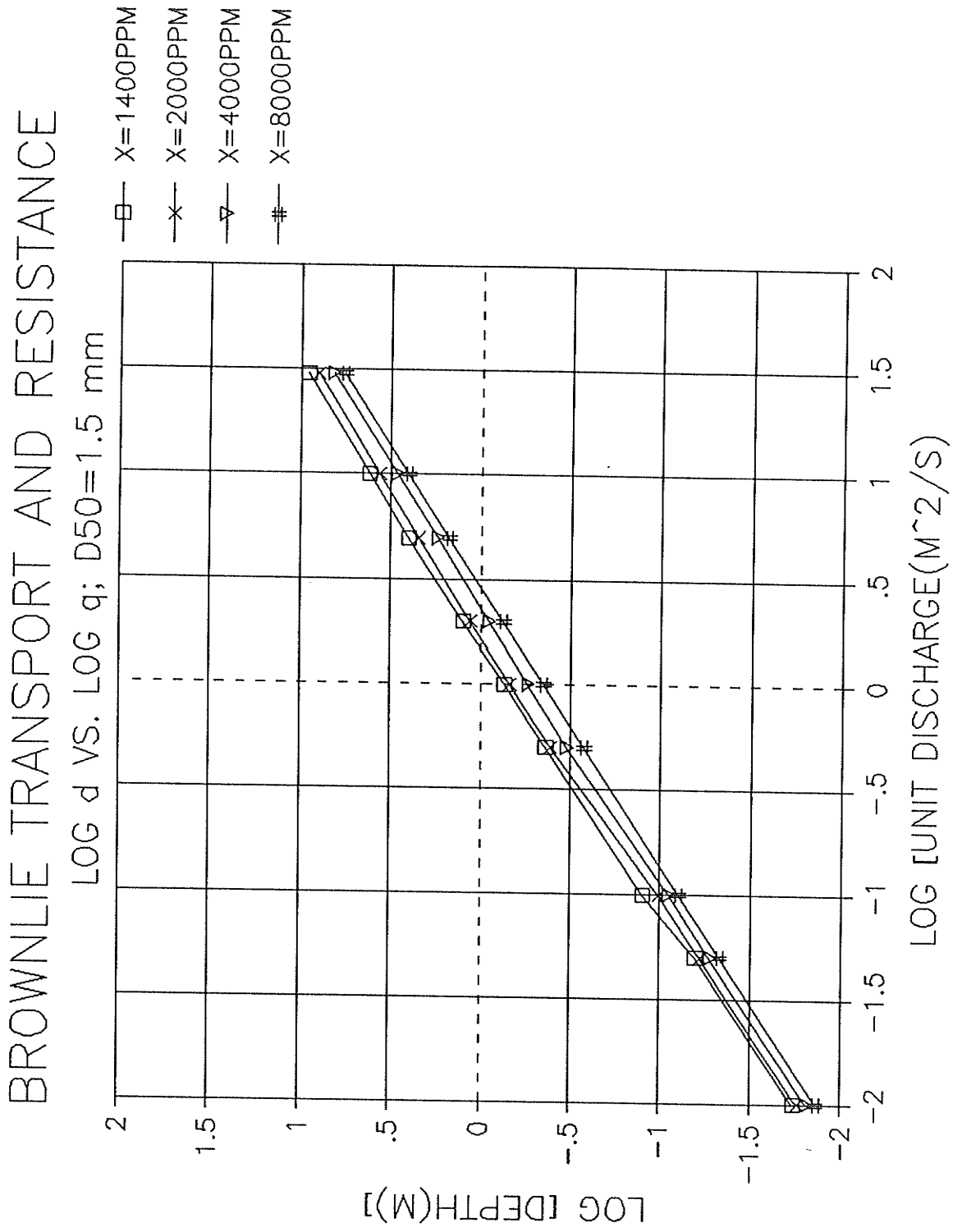


Figure E-20 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 1.5 mm; Lower, Transitional and Upper Flow Regimes

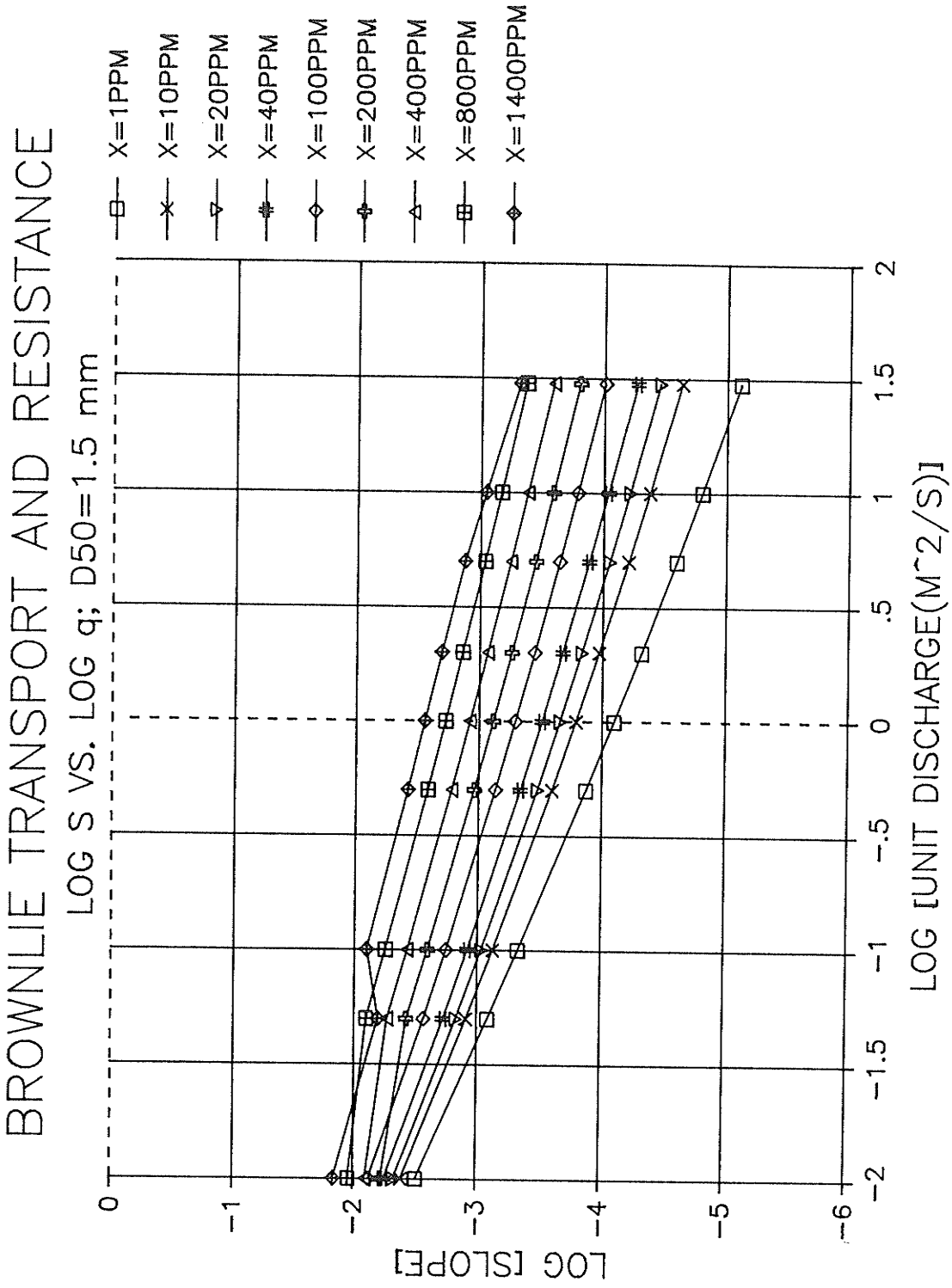


Figure E-21 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 1.5 mm; Lower, Transitional and Upper Flow Regimes

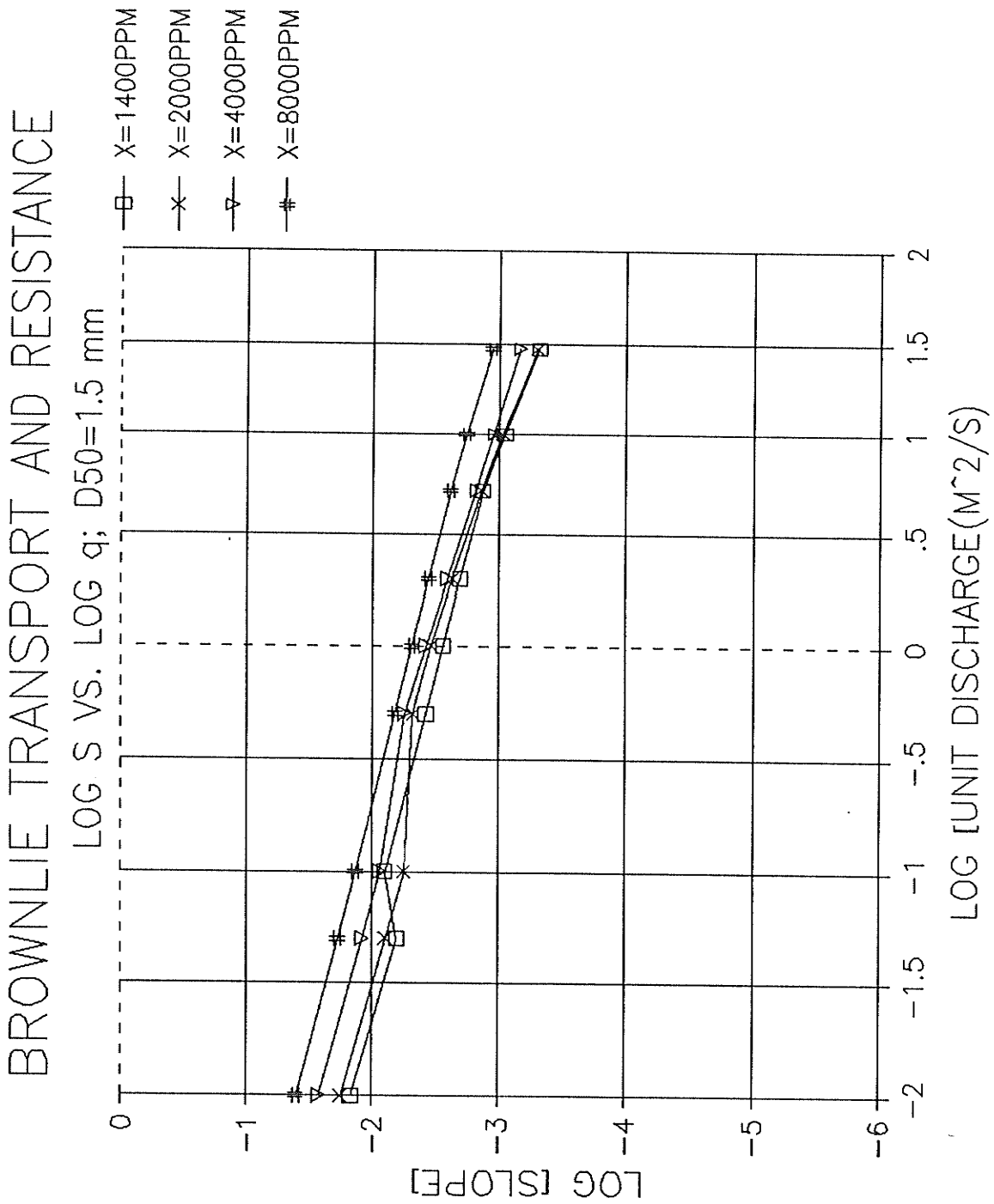


Figure E-21 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 1.5 mm; Lower, Transitional and Upper Flow Regimes

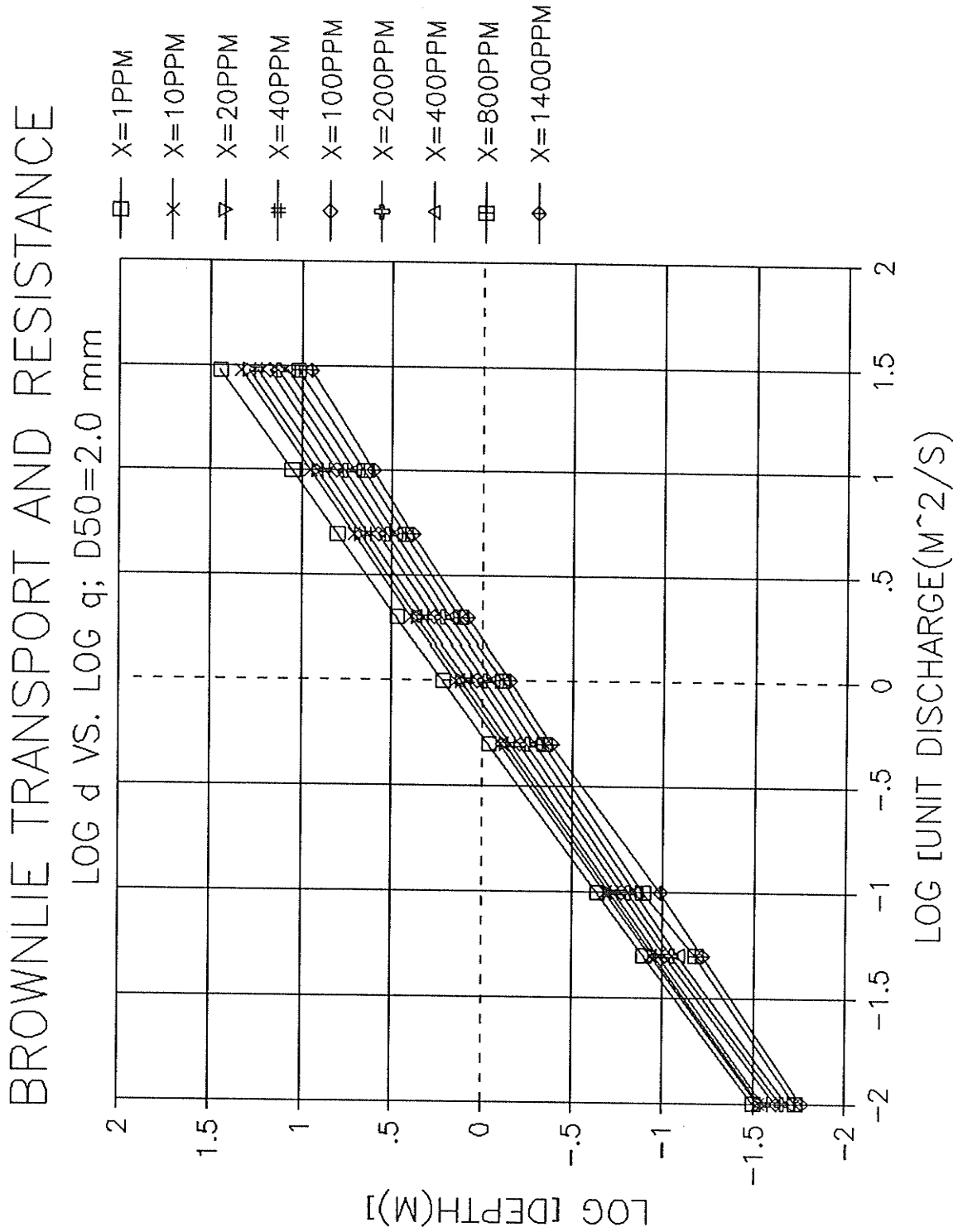


Figure E-22 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 2.0 mm; Lower, Transitional and Upper Flow Regimes

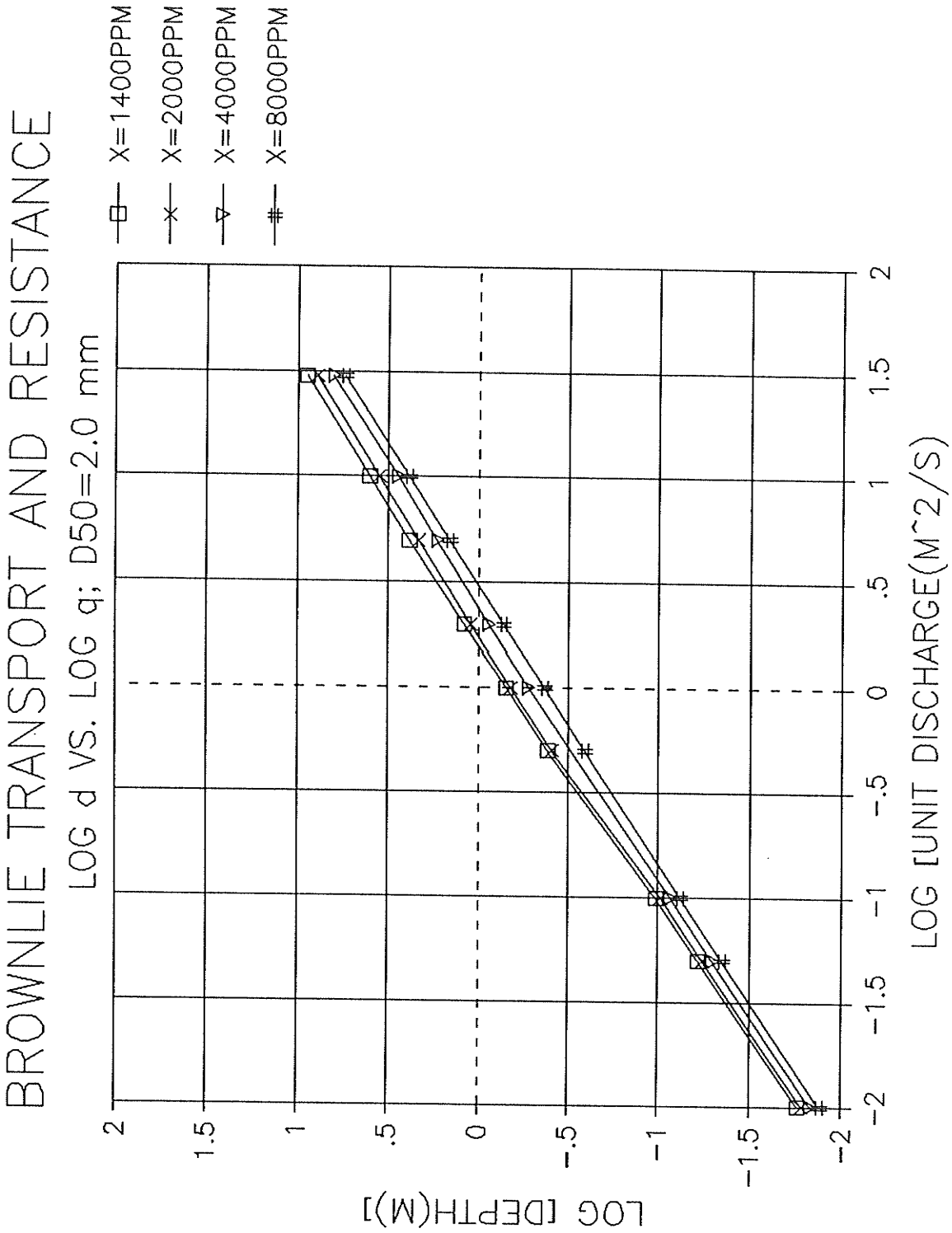


Figure E-22 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (d); D_{50} = 2.0 mm; Lower, Transitional and Upper Flow Regimes

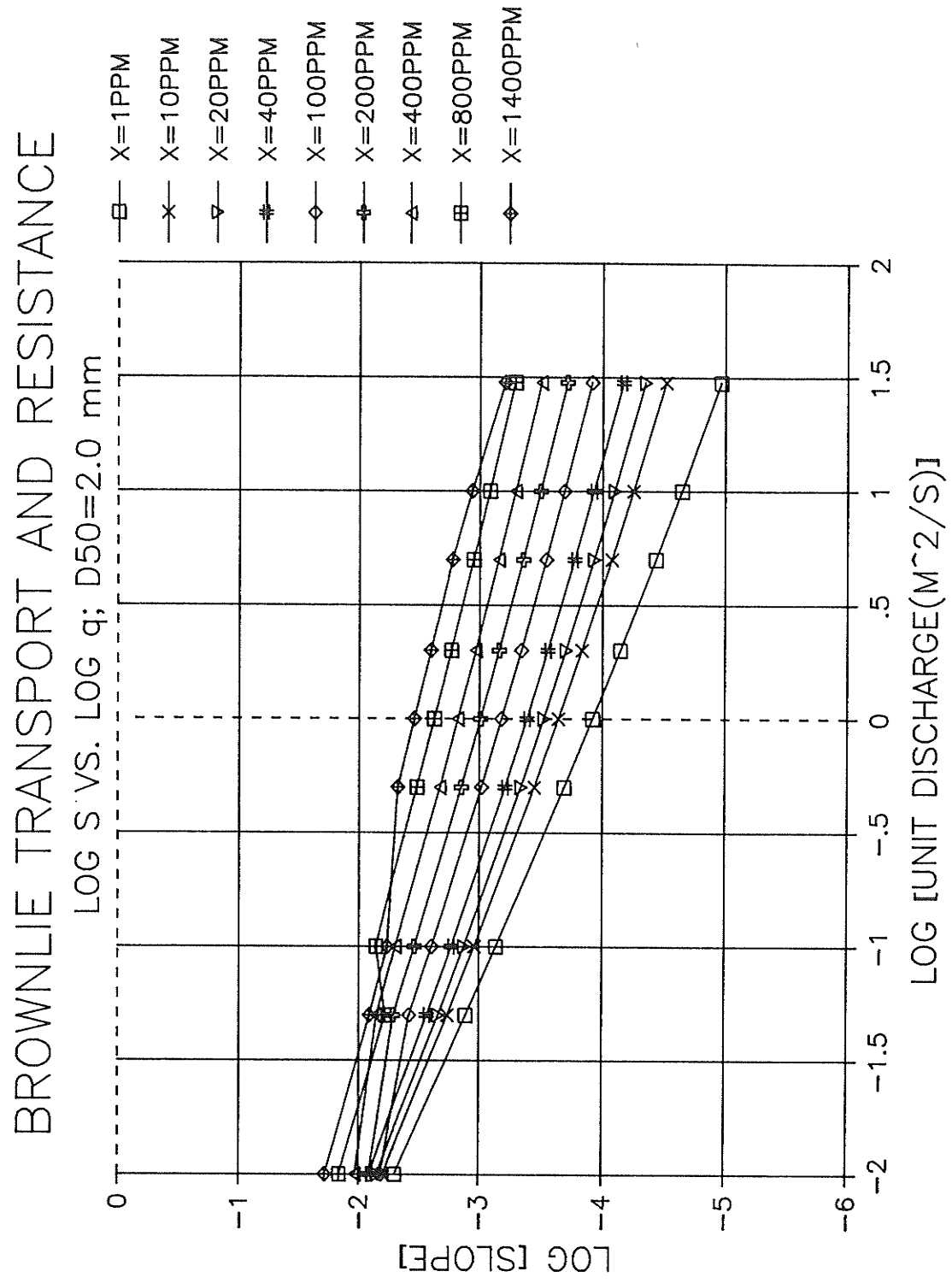


Figure E-23 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 2.0 mm; Lower, Transitional and Upper Flow Regimes

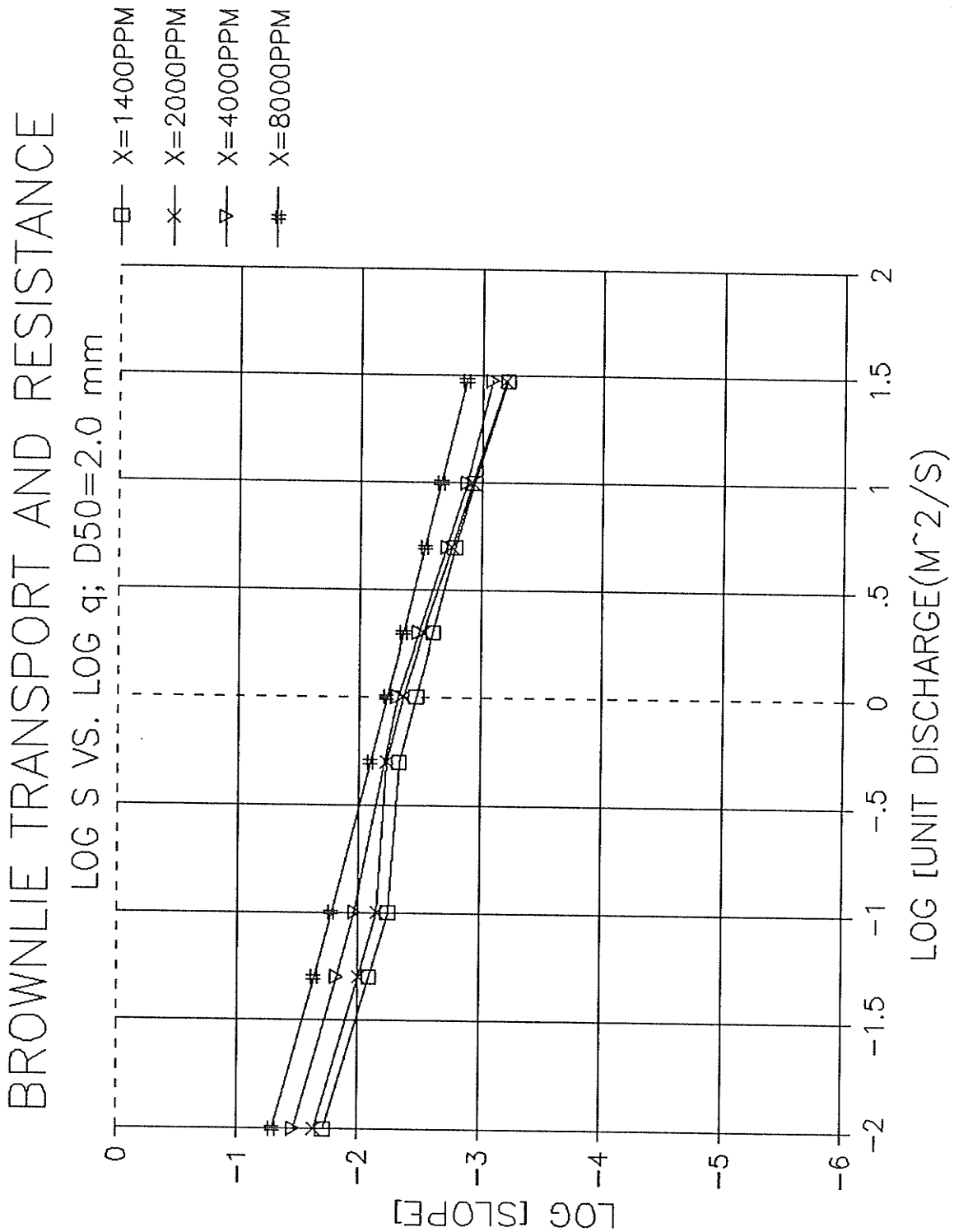


Figure E-23 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Solution Data For Simplified Method; Log_{10} (q) versus Log_{10} (S); D_{50} = 2.0 mm; Lower, Transitional and Upper Flow Regimes

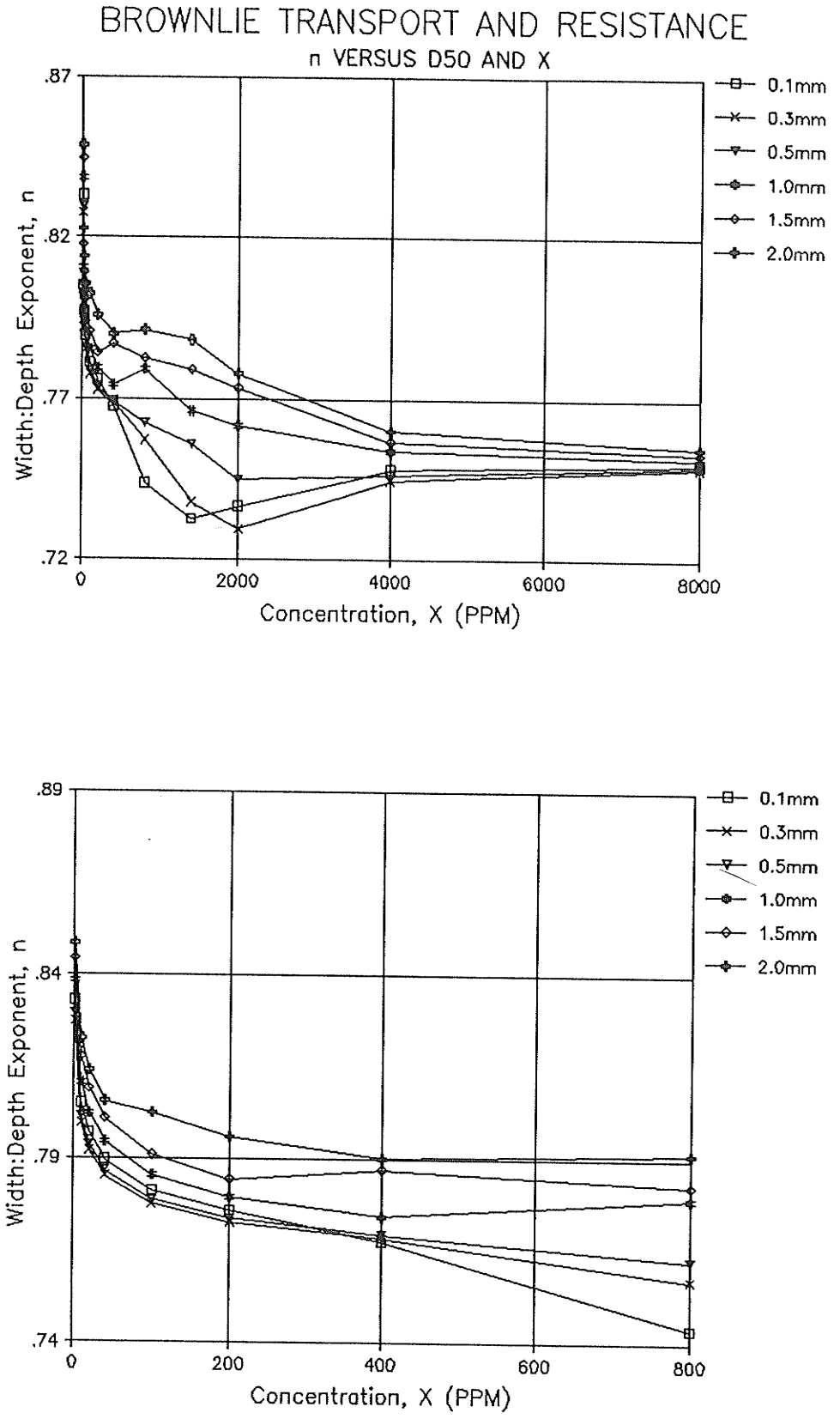


Figure E-24 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Variation of Exponent, n, with Concentration, X_{bm}, Sediment Size, D_{50}

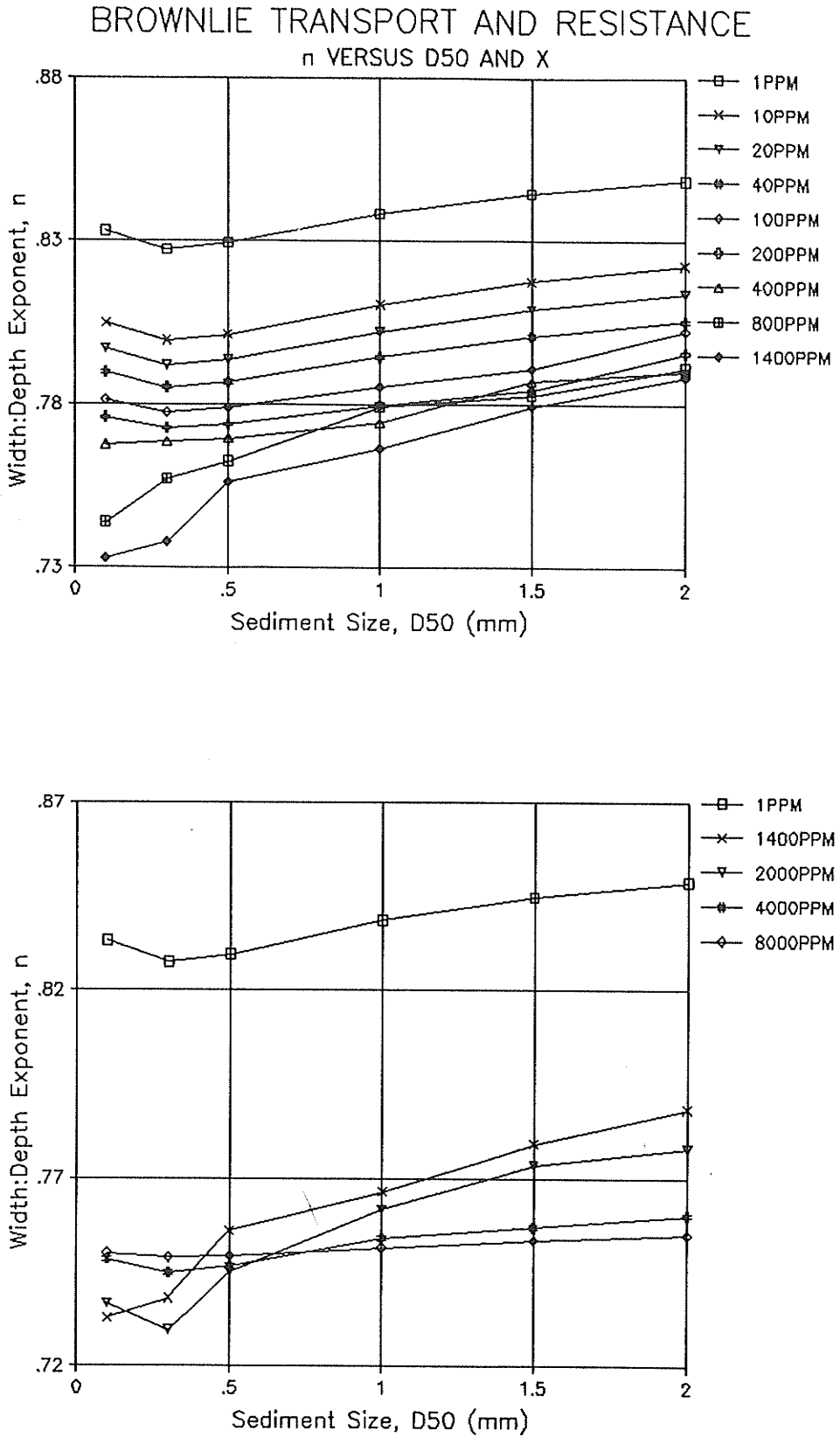


Figure E-25 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Variation of Exponent, n, with Sediment Size, D_{50} and Concentration, X_{bm}

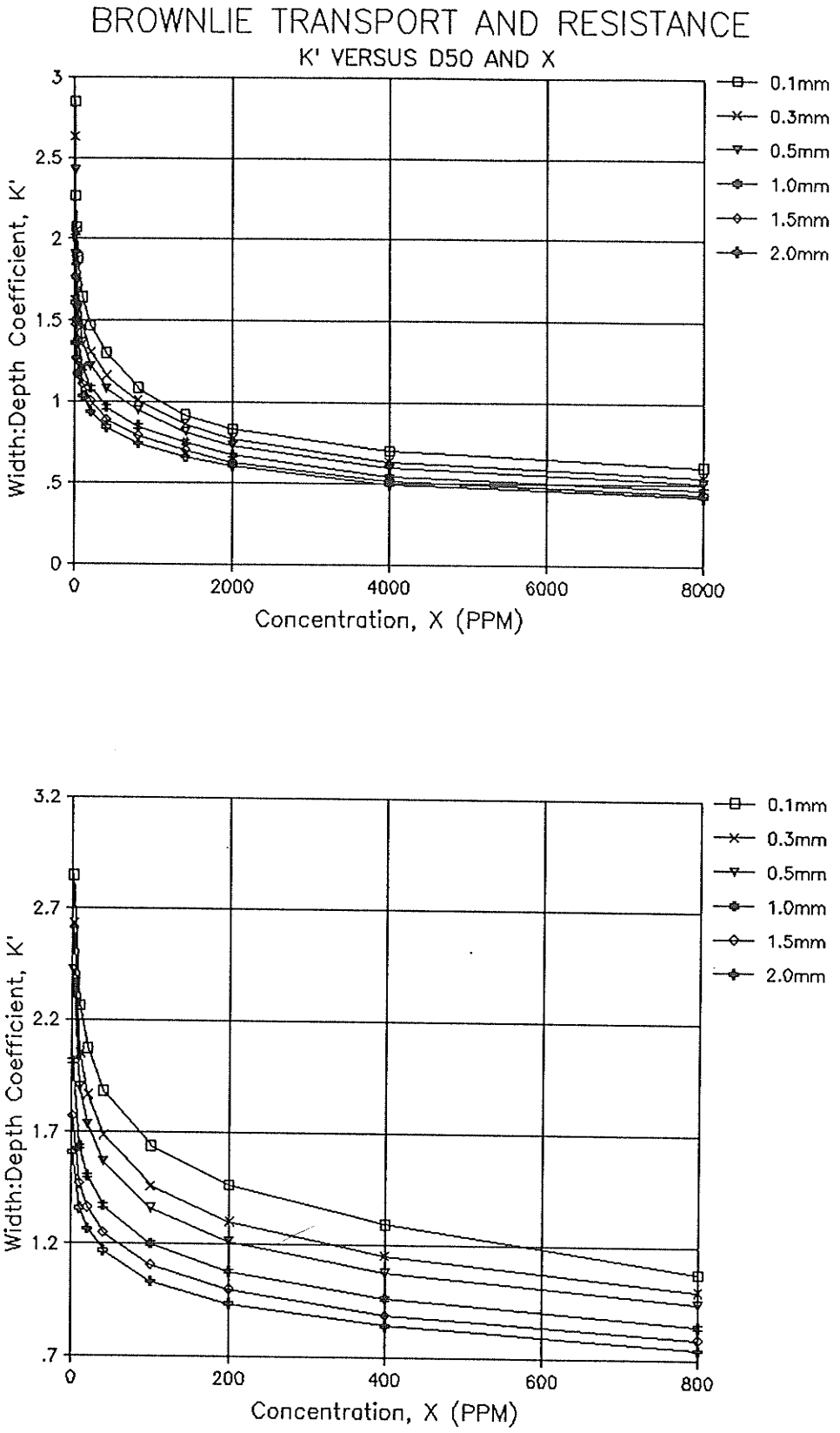


Figure E-26 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Variation of Coefficient, K', with Concentration, X_{bm}, Sediment Size, D_{50}

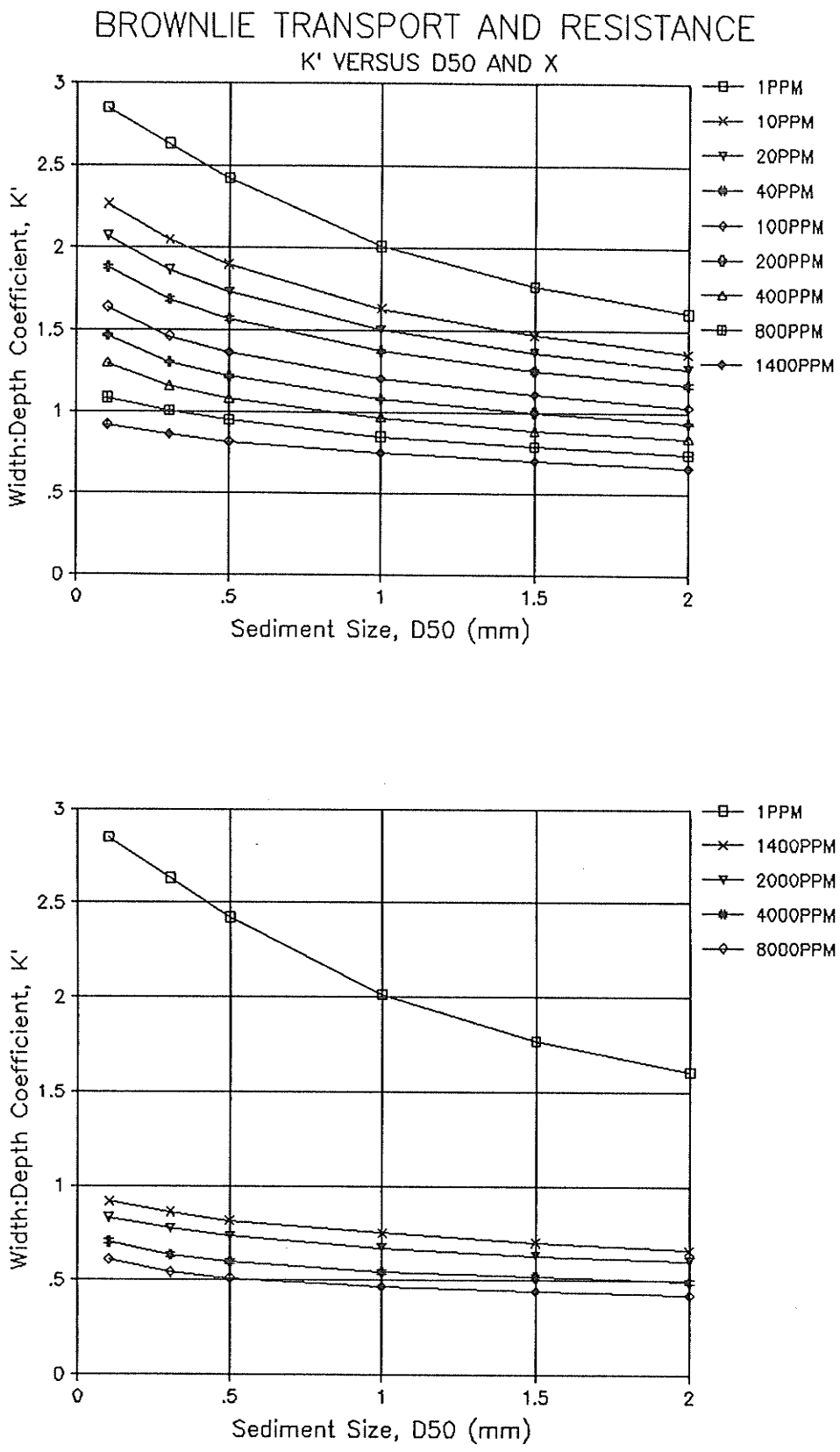


Figure E-27 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Variation of Coefficient, K', with Sediment Size, D_{50} and Concentration, X_{bm}

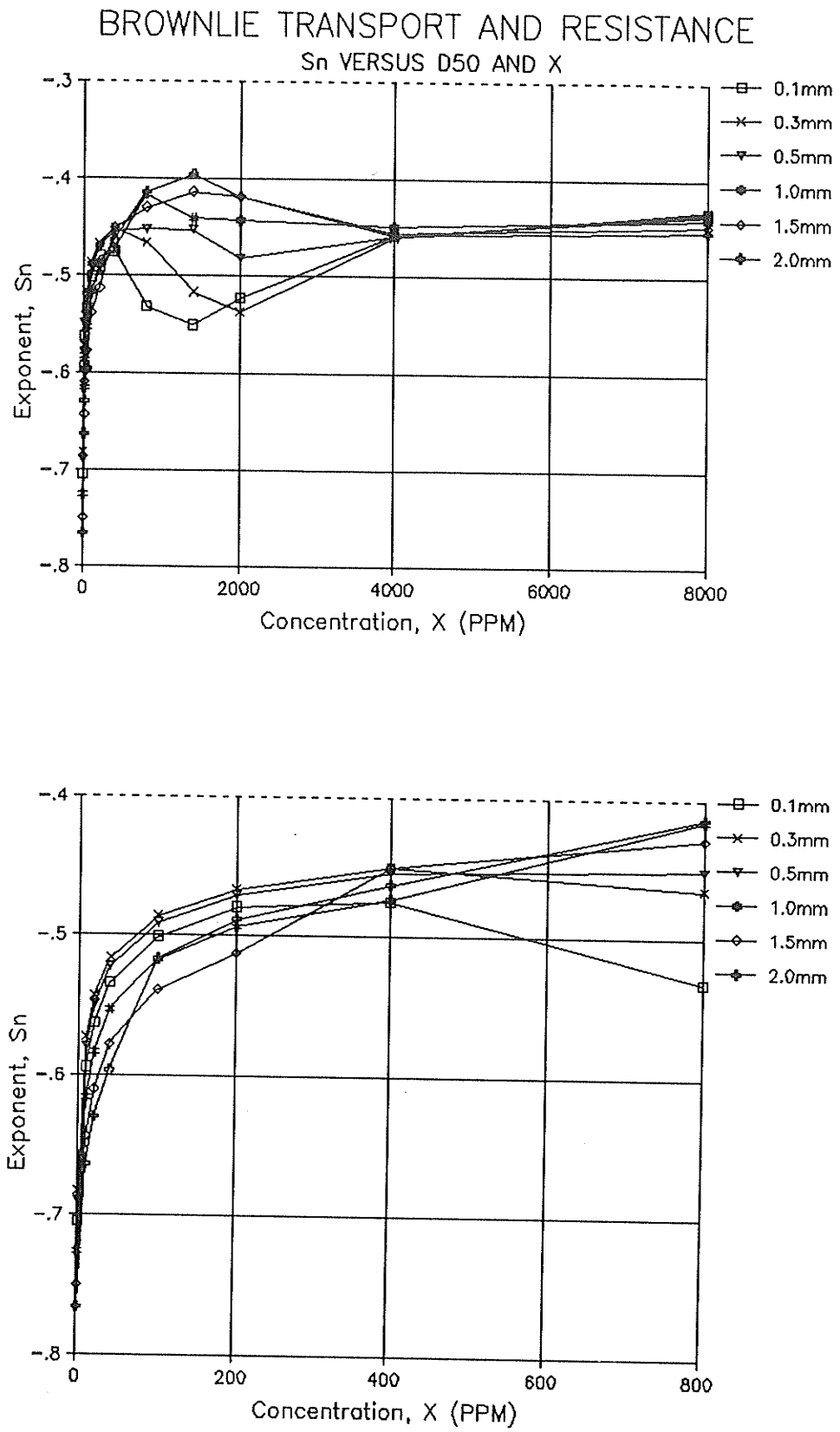


Figure E-28 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Variation of Exponent, Sn, with Concentration, X_{bm}, Sediment Size, D_{50}

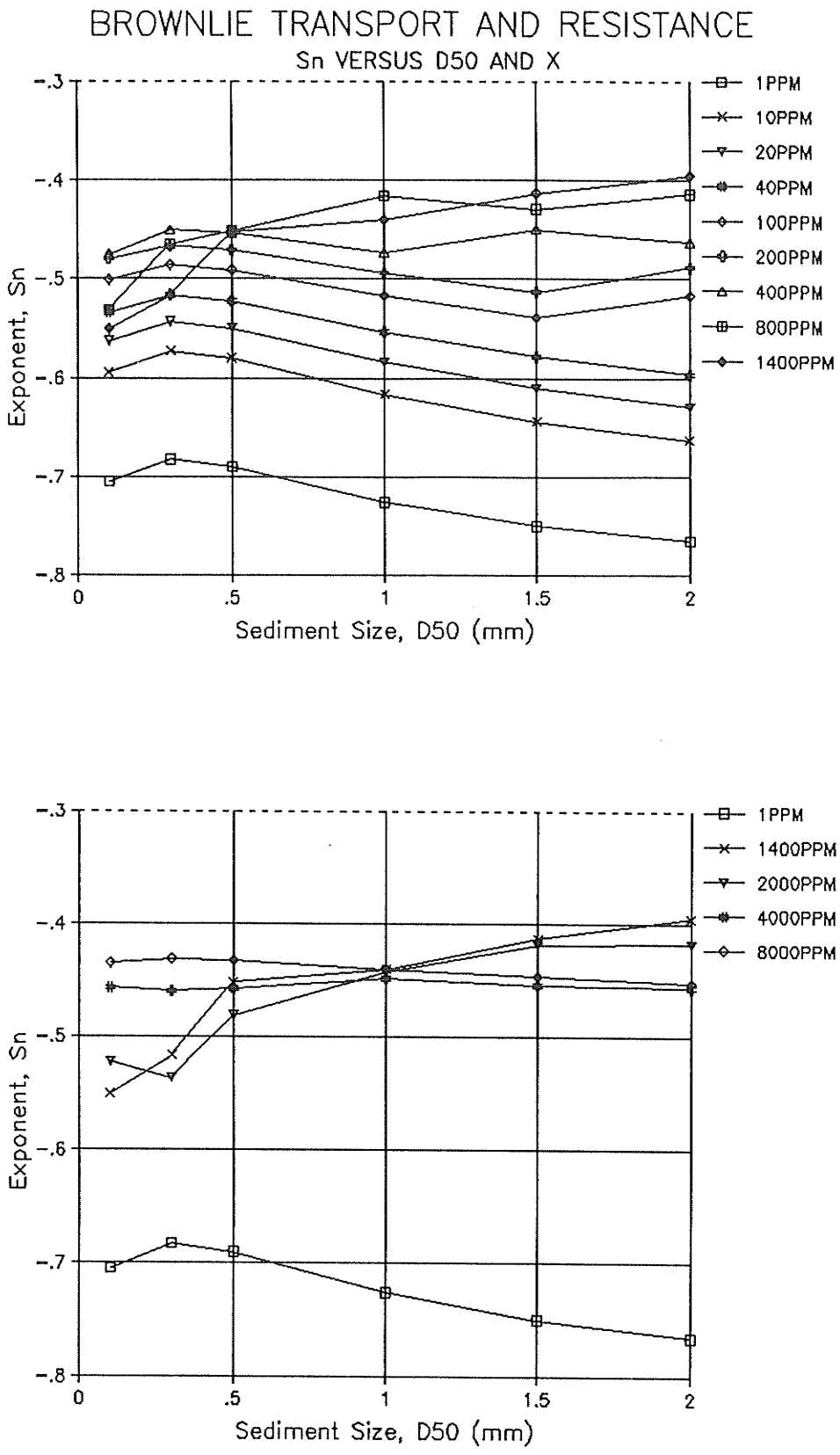


Figure E-29 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Variation of Exponent, Sn, with Sediment Size, D_{50} and Concentration, X_{bm}

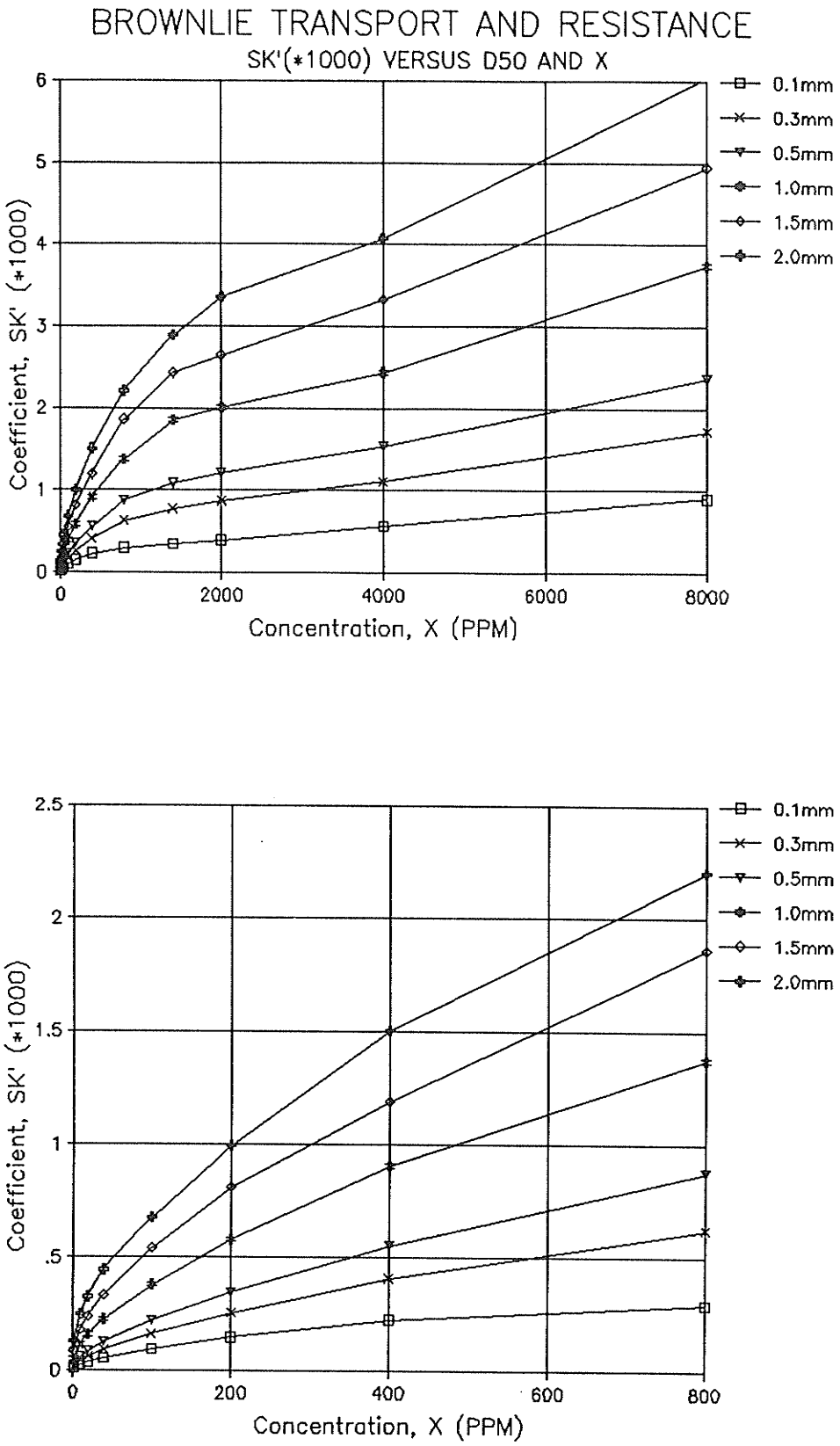


Figure E-30 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Variation of Coefficient, SK', with Concentration, X_{bm}, Sediment Size, D_{50}

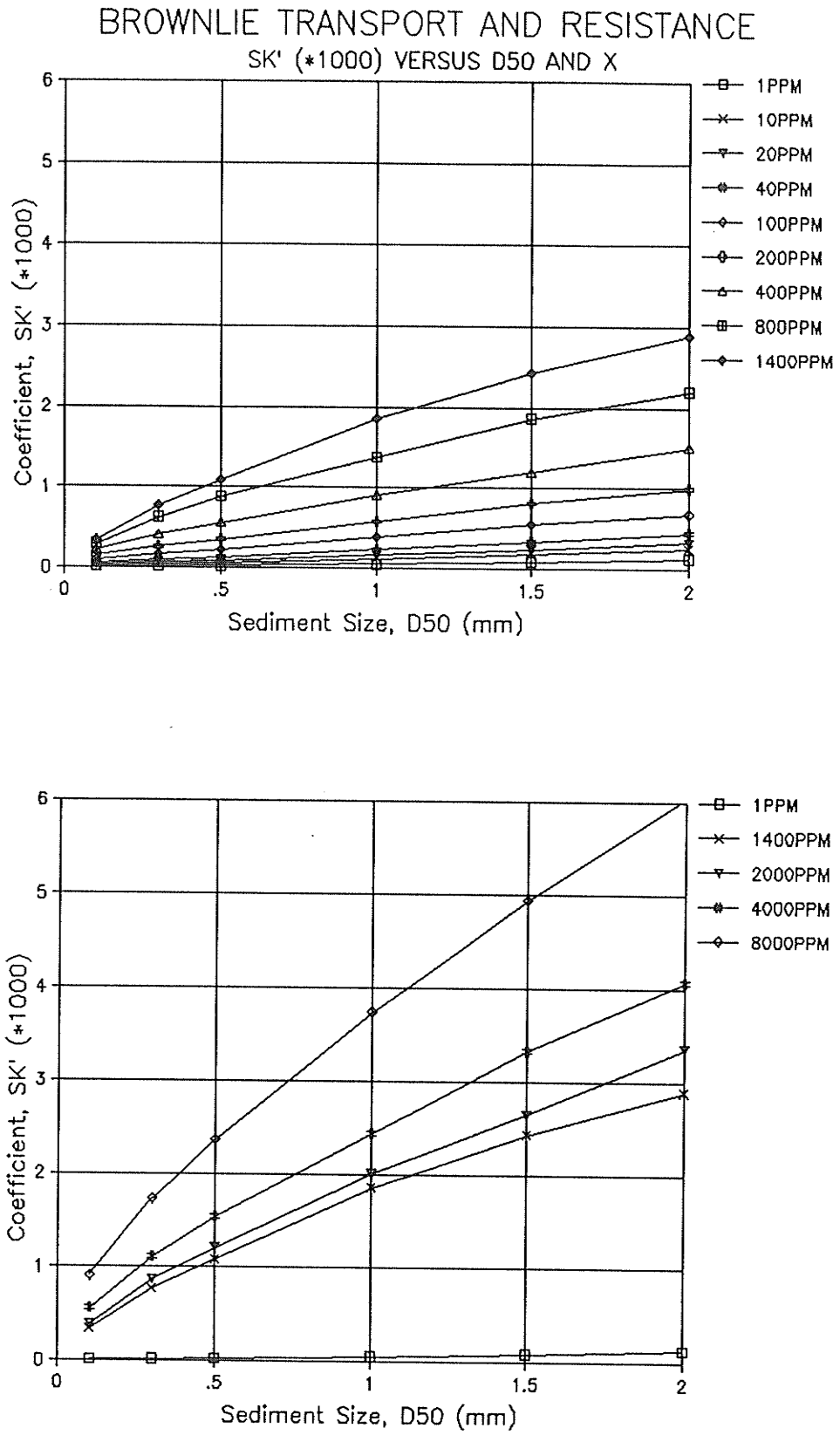


Figure E-31 Brownlie (1981a) Sediment Transport and Flow Depth Equations; Variation of Coefficient, SK', with Sediment Size, D_{50} and Concentration, X_{bm}

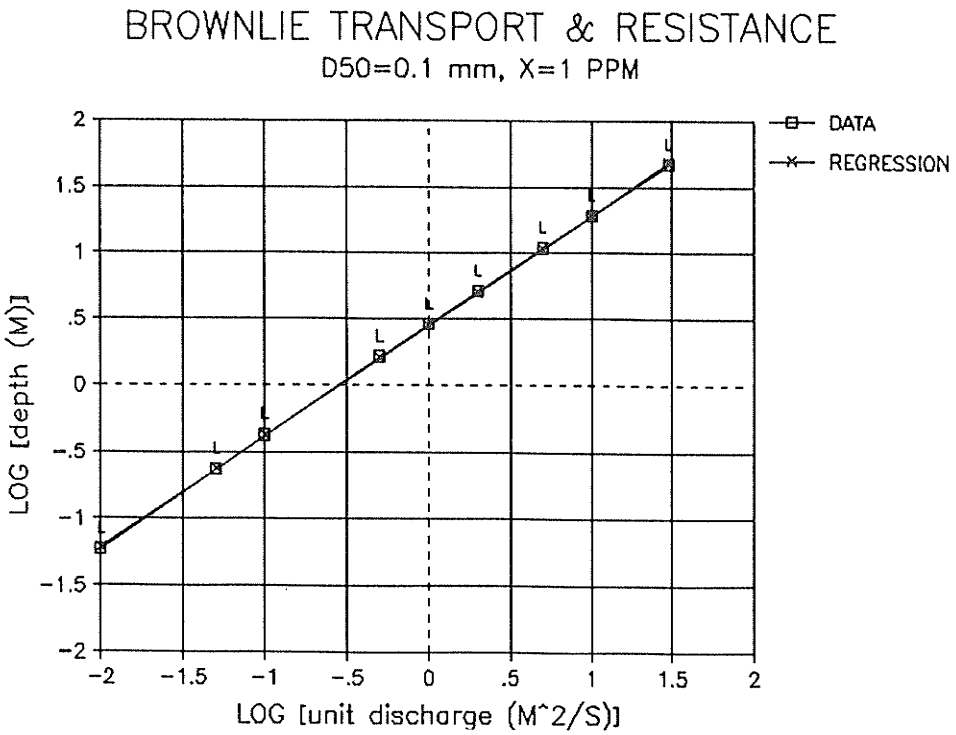


Figure E-32 Brownlie (1981a) Sediment Transport and Flow Depth Equations Least-Square Relationship versus Solution Data; $Log_{10}(q)$ vs. $Log_{10}(d)$; D_{50} = 0.1mm, X_{bm} = 1 PPM

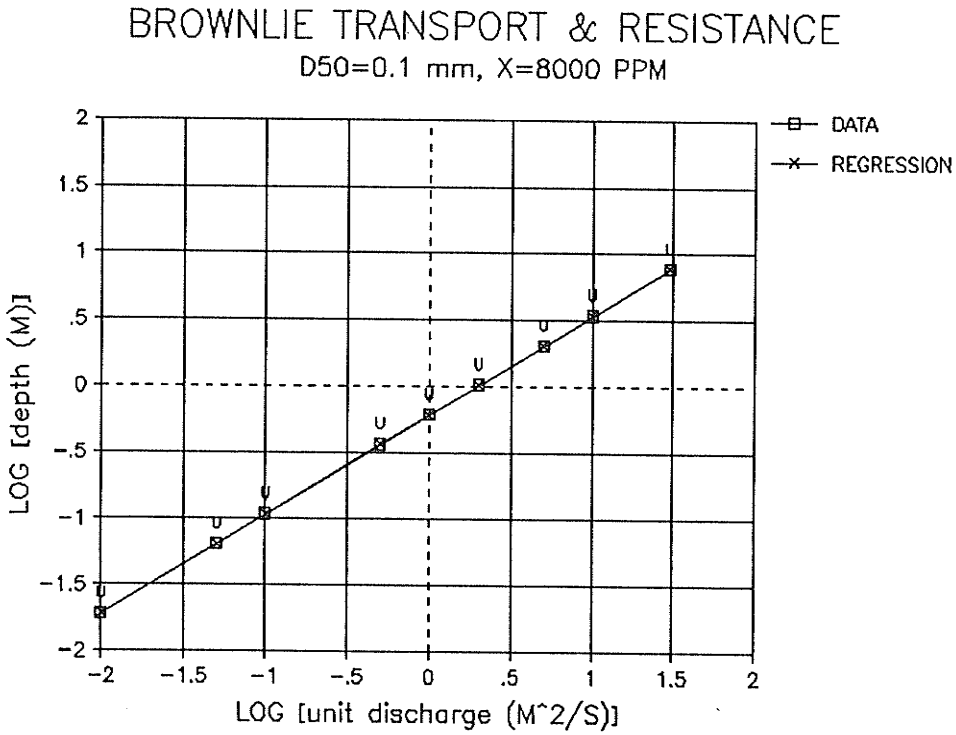


Figure E-33 Brownlie (1981a) Sediment Transport and Flow Depth Equations Least-Square Relationship versus Solution Data; $Log_{10}(q)$ vs. $Log_{10}(d)$; D_{50} = 0.1mm, X_{bm} = 8000 PPM

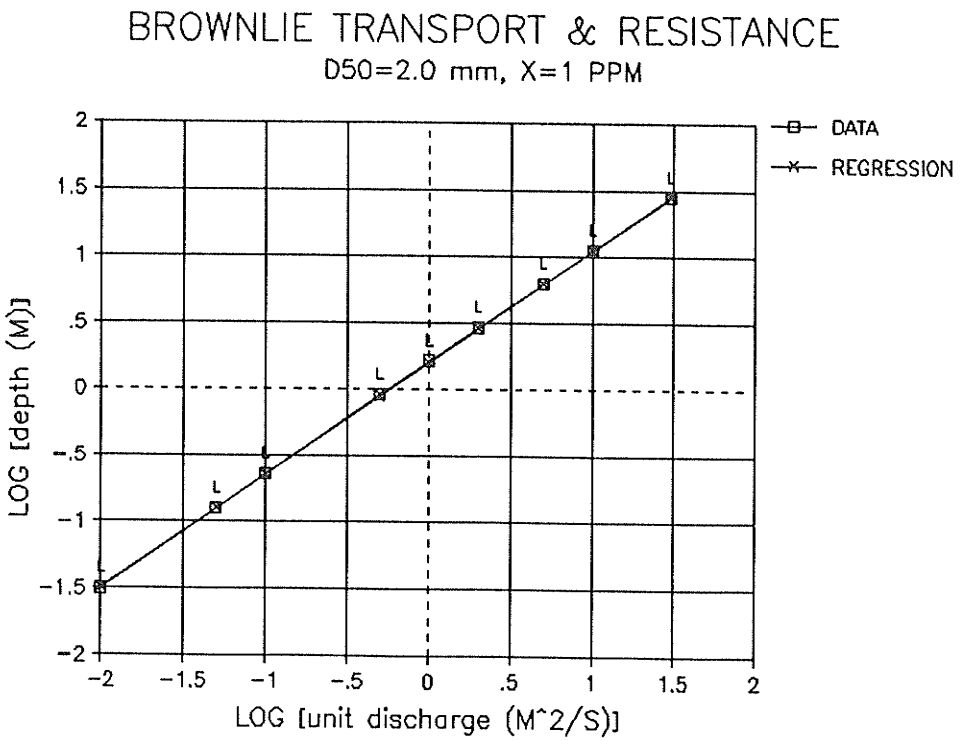


Figure E-34 Brownlie (1981a) Sediment Transport and Flow Depth Equations Least-Square Relationship versus Solution Data; $Log_{10}(q)$ vs. $Log_{10}(d)$; D_{50} = 2.0mm, X_{bm} = 1 PPM

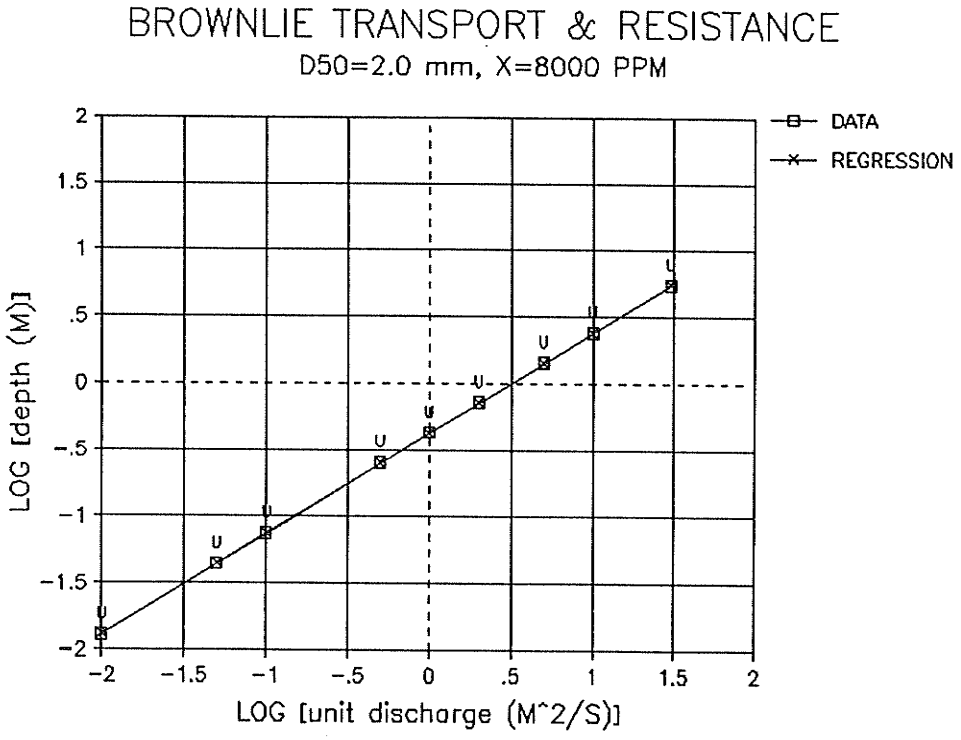


Figure E-35 Brownlie (1981a) Sediment Transport and Flow Depth Equations Least-Square Relationship versus Solution Data; $Log_{10}(q)$ vs. $Log_{10}(d)$; D_{50} = 2.0mm, X_{bm} = 8000 PPM

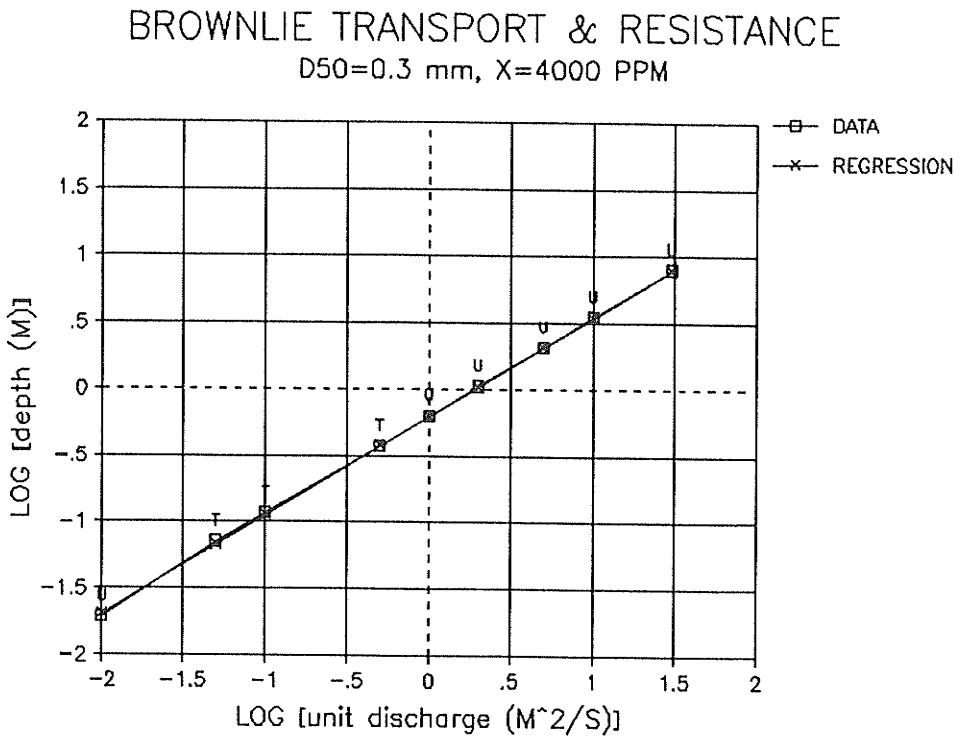


Figure E-36 Brownlie (1981a) Sediment Transport and Flow Depth Equations Least-Square Relationship versus Solution Data; $Log_{10}(q)$ vs. $Log_{10}(d)$; D_{50} = 0.3mm, X_{bm} = 4000 PPM

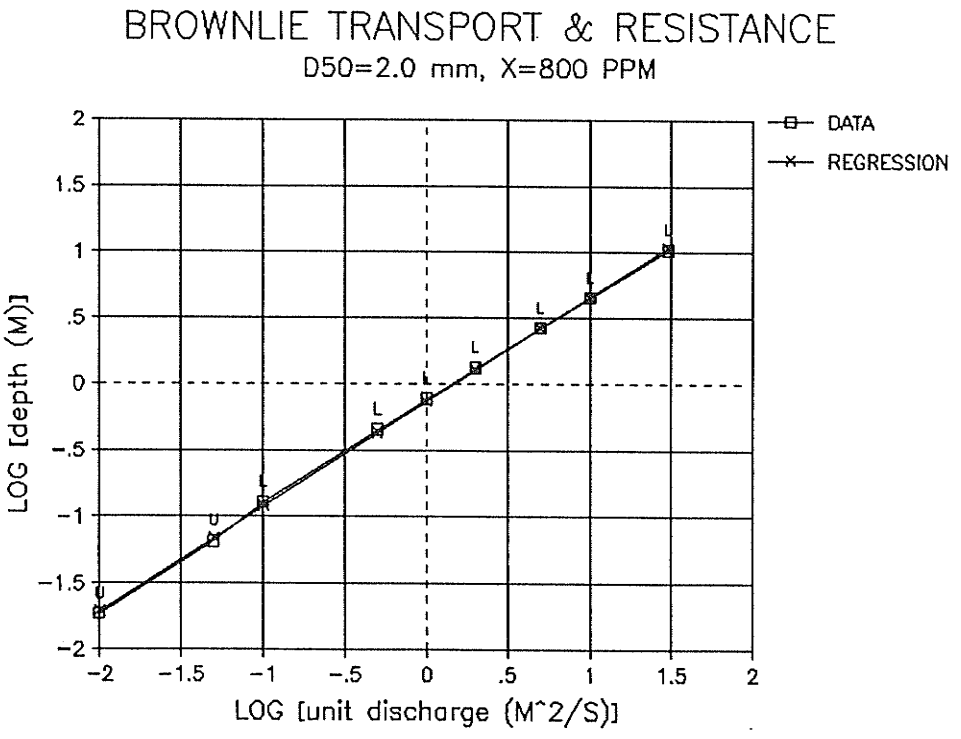


Figure E-37 Brownlie (1981a) Sediment Transport and Flow Depth Equations Least-Square Relationship versus Solution Data; $Log_{10}(q)$ vs. $Log_{10}(d)$; D_{50} = 2.0mm, X_{bm} = 800 PPM

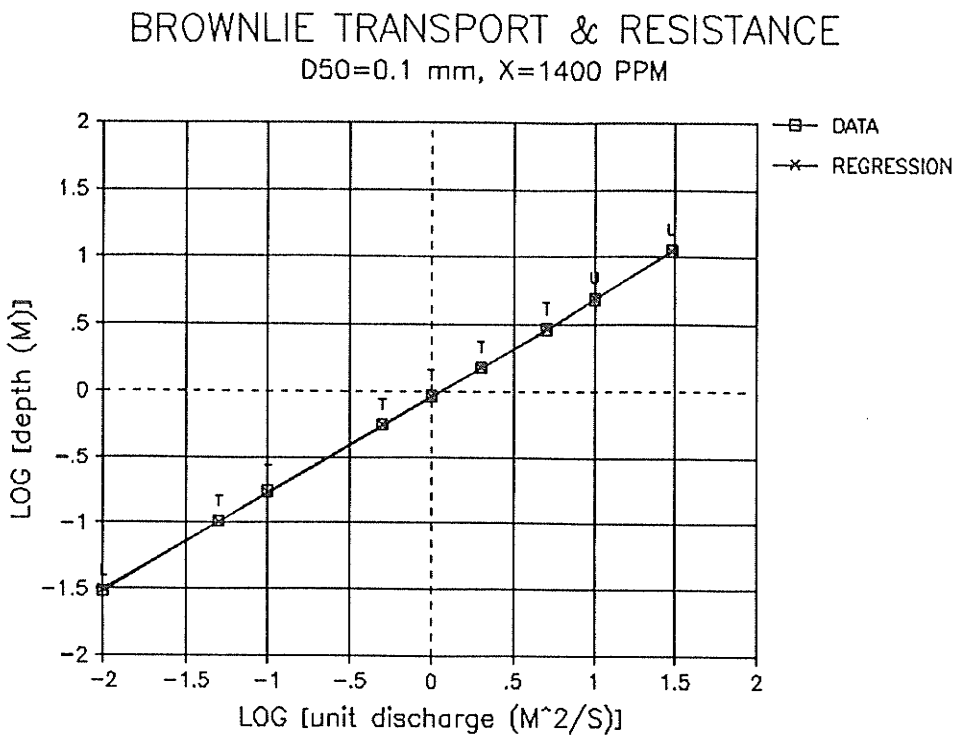


Figure E-38 Brownlie (1981a) Sediment Transport and Flow Depth Equations Least-Square Relationship versus Solution Data; $Log_{10}(q)$ vs. $Log_{10}(d)$; D_{50} = 0.1mm, X_{bm} = 1400 PPM

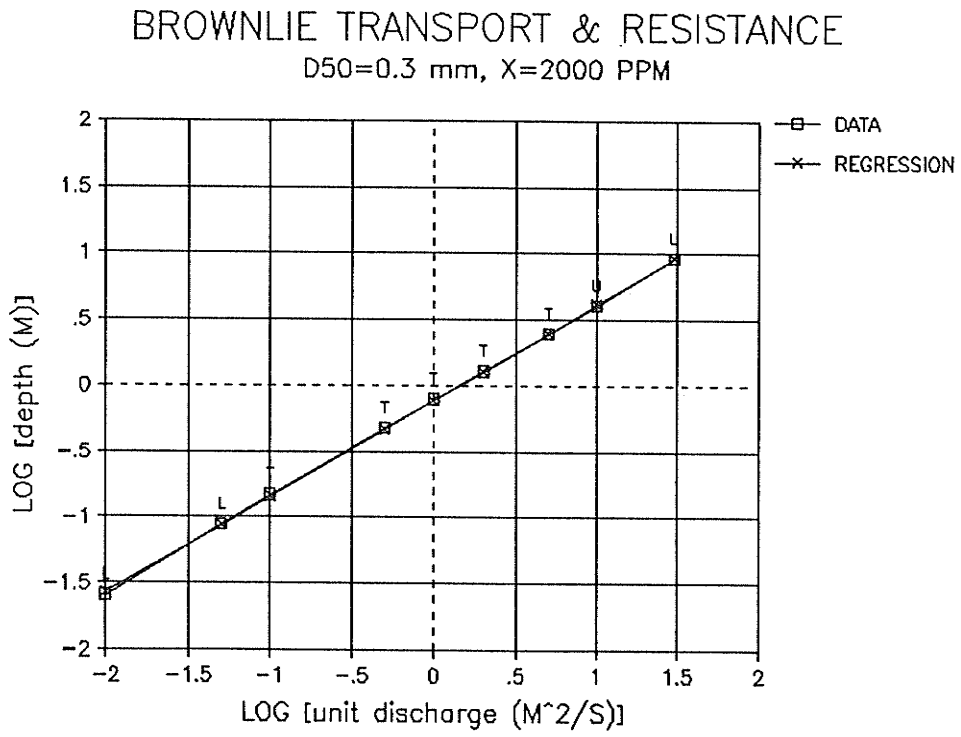


Figure E-39 Brownlie (1981a) Sediment Transport and Flow Depth Equations Least-Square Relationship versus Solution Data; $Log_{10}(q)$ vs. $Log_{10}(d)$; D_{50} = 0.3mm, X_{bm} = 2000 PPM

APPENDIX A

DERIVATION OF A WIDTH DEPTH RELATION FOR THE FRASER (WCHL,1976)

CONTINUITY WATER $V_1 d_1 W_1 = V_2 d_2 W_2$ (1)

CONTINUITY SEDIMENT $q_{b1} W_1 = q_{b2} W_2$ (2)

SEDIMENT TRANSPORT[1] $q_b = \text{const } V^4$ (3)

Solving equations 1-3 as follows:

a. substitute 3 into 2

$$\text{const } V_1^4 W_1 = \text{const } V_2^4 W_2 \quad (4)$$

or

$$\frac{V_1^4}{V_2^4} = \frac{\text{const } W_2}{\text{const } W_1}$$

b. rewrite 1 as

$$\frac{V1}{V2} = \frac{W2\ d2}{W1\ d1} \quad (1)$$

c. substituting 1 into 4

$$\frac{(W_2)^4}{(W_1)} \frac{(d_2)^4}{(d_1)} = \frac{\text{const } W_2}{\text{const } W_1} \quad (5)$$

or

$$\frac{d_2}{d_1} = \text{const } \frac{(W_1)}{(W_2)}^{3/4} \quad (5)$$

which is the dimensionless form of the width depth equation

d. Tamburi claimed without formal proof, that the dimensional form was therefore:

$$d = K\ W^{-.75} \quad (6)$$

---

1 TAMBURI ASSUMED THAT CHANNEL SHAPE WAS A FUNCTION OF BED LOAD ONLY

APPENDIX B

DERIVATION OF A REGIME BEDLOAD FORMULA (WCHL, 1976)

Tamburi (citing Garde and Albertson, 1961) claimed that the bedload equations developed by Meyer-Peter, Kalinske, and Eistein can all be expressed in the general form:

$$q_B^* = f(T^*)$$

where; $$T^* = \frac{\tau_o}{(\check{S}_s - \check{S}_f)\, D_{50}}$$

and where τ_o = shear stress
$\check{S}_s$ = unit weight of sediment
$\check{S}_f$ = unit weight of fluid

$$q_B^* = \frac{qB}{v^* \check{S}_s\, D_{50}}$$

and where v^* = shear velocity
q_B = unit bedload

so that ;

$$\frac{\tau_o}{(\check{S}_s - \check{S}_f)\, D_{50}} \alpha \frac{qB}{v^* \check{S}_s\, D_{50}}$$

solving for q_B;

$$q_B \alpha \frac{\tau_o\, v^*\, \check{S}s\, D50}{(\check{S}_s - \check{S}_f)\, D_{50}}$$

substituting for T_o and v^*;

$$q_B \alpha \frac{\check{S}f\ r\ S\ (g\ r\ S)^{.5}\ \check{S}s\ D50}{(\check{S}_s - \check{S}_f)\, D_{50}}$$

simplifying;

$$q_B \alpha \frac{\check{S}f\ r^{.66}\ S^{.66}\ g^{.5}\ \check{S}s}{(\check{S}_s - \check{S}_f)}$$

$$q_B \alpha K\, r^{.66}\, S^{.66}$$

and generalizing;

$$q_B \alpha K\, r^N\, S^M$$

Manning's formula relates the rhs of this equation to velocity

APPENDIX C

SOLUTION

TO

ENGELUND (1967)

SEDIMENT TRANSPORT

AND

FLOW RESISTANCE

EQUATIONS

C.1 INTRODUCTION

This appendix documents the soution of the Engelund's (1967) sediment transport and flow resistance equations. The relationship of the rational width:depth equation (4.9a) and the specific width:depth equation (3.21) is quantified using the solution data.

C.2 EQUATIONS

Engelund's (1967) equations are given by C.1 to C.5 (Brownlie, 1981b).

Resistance equations:

$$v/u_*' = 5.76 \log (5.51\ r' / D_{65}) \quad \text{.....(C.1)}$$

Lower Regime

$$\tau_* = 1.581\ (\tau_*' - 0.06)^{0.5} \quad \text{.....(C.2)}$$

Upper Regime

$$\tau_* = \tau_*' \qquad (\tau_*' < 1) \quad \text{.....(C.3)}$$

$$\tau_* = (1.425\ \tau_*'^{-1.8} - 0.425)^{-1/1.8} \qquad (\tau_*' > 1) \quad \text{.....(C.4)}$$

Sediment transport equation:

$$X = 0.05\ (\rho_s/\ \rho_s-\rho)\ (vS/\ (\rho_s-\rho/\ \rho)\ g\ D_{50})\ \tau^{*0.5} \quad \text{.....(C.5)}$$

C.3 METHODOLOGY

Simultaneous solution of equations C.1 to C.5 along with the dimensionless forms of the continuity, and the momentum and mass equations (i.e., C.6, C.7) led Parker (1977) to non-dimensional plots of q~ versus R and X_{bm}, as shown in Figure 3.8a.

Continuity : $$q\sim = \tau_*^{0.5}\ R / C \quad \text{.....(C.6)}$$

Momentum : $$\tau_* = RS / R_b \quad \text{.....(C.7)}$$

In comparison to equation (4.9a), sediment gradation, σ_g, is not accounted for in the above equations. Furthermore, it is obvious that k_v has been assumed to be constant, even though the value of this variable is not given.

Fitting equations of the form in equation (3.21) to the graph presented in Figure 3.8a, allows the value of n and K~, as predicted by Engelund's (1967) sediment transport and resistance equations, to be determined.

$$R = K\sim\ q\sim^{n} \quad \text{.....(3.21)}$$

Since Engelund's (1967) equations reduce to a singular non-dimensional graph, there will be no variation of n with D_{50}.

Rather than solving numerically the above equations, the available graphs as presented in Parker and Anderson (1977) were used to obtain data points for use in the regression. The inaccuracies in reading the graph did not significantly affect the regression results. A series of data points were obtained relating R to q~ and concentration, X_{bm}. Equations of the form in equation (C.8) were then fitted to the data by means of the Supercalc4 spreadsheet LEASTSQ, from which the values of n and K~ and relevent statistics were determined.

$$LOG_{10}(R) = n * LOG_{10}(q\sim) + LOG_{10}(K\sim) \qquad (C.8)$$

C.4 RESULTS

The data digitized from Figure 3.8a (Parker, 1977) are presented in Table C-1.

TABLE C-1 : ENGELUND (1967) SEDIMENT TRANSPORT AND RESISTANCE RELATIVE ROUGHNESS R VERSUS UNIT DISCHARGE q~ AND CONCENTRATION X_{bm}

	R					
q~	5e5	2.5e5	1e5	7e4	5e4	2.5e4
X(PPM)						
0		3.4e4	1.5e4	1.1e4	7.7e3	4.1e3
5		3.1e4	1.5e4	1.1e4	7.7e3	4.1e3
10		2.8e4	1.4e4	1.1e4	7.7e3	4.1e3
30		2.2e4	1.1e4	8.6e3	6.9e3	4e3
60	3e4	1.9e4	9.9e3	7.5e3	6e3	3.5e3
100	2.7e4	1.7e4	8.6e3	6.6e3	5.2e3	3.2e3
200	2.2e4	1.4e4	7.1e3	5.6e3	4.5e3	2.7e3
400	1.8e4	1.1e4	6e3	4.8e3	3.7e3	2.3e3
600	1.6e4	1e4	5.4e3	4.3e3	3.4e3	2.1e3
1000	1.4e4	8.9e3	4.6e3	3.8e3	3.0e3	1.9e3
3000	1e4	6.5e3	3.5e3	2.8e3	2.2e3	1.4e3

TABLE C-1 (CONT'D)

	R				
q~	1e4	7e3	5e3	2.5e3	1e3
X(PPM)					
0	1.8e3	1.3e3	9.6e2	5.3e2	2.4e2
5	1.8e3	1.3e3	9.6e2	5.3e2	2.4e2
10	1.8e3	1.3e3	9.6e2	5.3e2	2.4e2
30	1.8e3	1.3e3	9.6e2	5.3e2	2.4e2
60	1.7e3	1.3e3	9.6e2	5.3e2	2.4e2
100	1.6e3	1.2e3	9.3e2	5.3e2	2.4e2
200	1.5e3	1.1e3	8.4e2	5.1e2	2.4e2
400	1.2e3	9.5e2	7.5e2	4.5e2	2.3e2
600	1.1e3	8.6e2	7.0e2	4.2e2	2.2e2
1000	9.7e2	7.5e2	6.1e2	3.5e2	2.1e2
3000	7.2e2	5.7e2	4.5e2	2.8e2	1.5e2

Table C-2 details the results of the regressions on the data using equation (C.8). The strong correlations suggest that the method of obtaining the data did not lead to large random errors and is therefore adequate.

TABLE C-2 : ENGELUND (1967) SEDIMENT TRANSPORT AND RESISTANCE REGRESSION RESULTS : EQUATION (C.8)

X(PPM)	n	$Log_{10}(K\sim)$	K~	R^2
0	.901	-.343	.453	1.00
5	.890	-.304	.497	1.00
10	.880	-.264	.545	1.00
30	.829	-.083	.826	1.00
60	.778	.094	1.243	1.00
100	.753	.167	1.469	1.00
200	.722	.250	1.778	1.00
400	.701	.271	1.867	1.00
600	.692	.271	1.867	1.00
1000	.687	.236	1.723	1.00
3000	.680	.139	1.378	1.00

Figures C-1 and C-2 graphically display the least square solutions and the variation of n and K~ with concentration X_{bm}. Figure C-3

illustrates the results of a typical regression analysis on the data, in this case for X_{bm} = 600 ppm.

The coefficient K~ of equation (3.21) can be converted to an equivalent coefficient K' for equation (3.28) using the equation (C.9). Table C-3 summarizes the results of this conversion for various sediment sizes and Figure C-4 illustrates the same.

$$K' = D_{50}\, K\sim \,/\, ((R_b\, g\, D_{50})^{0.5} D_{50})^n \quad \ldots\ldots\ldots\ldots\ldots\ldots (C.9)$$

TABLE C-3: CONVERSION OF K~ TO K' FOR ENGELUND (1967)

			Width:depth Coefficient, K'					
D50 (MM)			.1	.3	.5	1	1.5	2
X(PPM)	n	K~						
0	.901	.453	3.29	2.24	1.87	1.46	1.27	1.15
5	.890	.497	3.16	2.19	1.84	1.46	1.27	1.16
10	.880	.545	3.03	2.14	1.81	1.45	1.28	1.17
30	.829	.826	2.44	1.87	1.65	1.40	1.26	1.18
60	.778	1.243	1.96	1.63	1.50	1.33	1.25	1.19
100	.753	1.469	1.70	1.47	1.38	1.26	1.20	1.15
200	.722	1.778	1.39	1.27	1.22	1.15	1.11	1.09
400	.701	1.867	1.13	1.07	1.04	1.01	.98	.97
600	.692	1.867	1.01	.97	.95	.93	.91	.90
1000	.687	1.723	.88	.85	.84	.82	.81	.80
3000	.680	1.378	.65	.63	.62	.62	.61	.61

APPENDIX D

PARIS'S (1984) NUMERICAL SOLUTION

TO

ACKERS AND WHITE (1973) SEDIMENT TRANSPORT EQUATIONS

AND

WHITE, BETTESS, AND PARIS (1980) FLOW RESISTANCE EQUATIONS

D.1 OBJECTIVE

This appendix documents the numerical simultaneous solution to the Ackers-White (1973) sediment transport equations and the White, Paris, and Bettess (1980) flow resistance equations. The results of the numerical solution are employed to quantify the relationship of the rational width:depth equation (4.8a) and the specific width:depth equation (3.28).

D.2 EQUATIONS

The Ackers-White (1973) sediment transport and White et al. (1980) flow resistance equations can be reduced to a set of three equations in three unknowns as follows (personal correspondance with E.Paris):

$$F_{gr} - (V_{shear}/CON_1 - a) * CON_3 - a = 0 \quad \text{.....(D.1)}$$

$$F_{gr} - (V_{shear}^{N}/CON_1) * (q/(d*5.65*Log_{10}(10d/D)))^{(1-N)} = 0 \quad \text{.....(D.2)}$$

$$X_{bm} - C_a*CON_4^{M} * S_g * D/d * (q/(D*V_{shear}))^{N} = 0 \quad \text{.....(D.3)}$$

where:

$$CON_1 = (g*D*(S_g-1))^{0.5} \quad \text{.....(D.4)}$$

$$CON_3 = 1 - 0.76\ (1 - 1/(EXP(Log_{10}(D_{gr}))^{1.7}) \quad \text{.....(D.5)}$$

$$CON_4 = F_{gr}/a-1 \quad \text{.....(D.6)}$$

$$D_{gr} = D(g(S_g-1)/K_v^2)^{0.333} \quad \text{.....(D.7)}$$

and for $1 <= D_{gr} <= 60$

$$N = 1.0 - 0.56\ Log_{10}(D_{gr}) \quad \text{.....(D.8)}$$

$$M = 9.66/Dgr + 1.34 \quad \text{.....(D.9)}$$

$$a = 0.23/(Dgr)^{0.5} + 0.14 \quad \text{.....(D.10)}$$

$$C_a = 10^{(2.86Log_{10}(D_{gr})-(Log_{10}(D_{gr})\hat{}2-3.53))} \quad \text{.....(D.11)}$$

and conversely for $D_{gr} > 60$

$$N = 0.0\ ,\ M = 1.50\ ,\ C_a = 0.025\ ,\ a = 0.17$$

For $D_{gr} > 60$ an exact solution exists, as documented in Paris (1983). For $1 < D_{gr} < 60$ (where $D_{gr} = D(g(S_g-1)/K_v^2)^{0.333}$) an exact solution does not exist (personnal correspondance Paris), and a numerical solution is required.

A discrepency exists between White et al.(1982) and Ackers et al.(1973) with regards to the variable N. White et al.(1982) specify N equal to 0.1 for Dgr >60 , whereas it is said to be 0.0 in Ackers et al.(1973). Further review of the equations revealed that an exact solution is possible for $D_{gr} > 60$ only if N = 0.

The equations of momentum and continuity, given by equations (3.27) and (D.12), are necessary only after the fact in order to investigate the variation of V and C.

Momentum $C = V / u_*$ .....(3.27)

Continuity $q = V\ d$ .....(D.12)

D.3 METHODOLOGY

The following observations are made from review of equations (D.1) to (D.11):

- the independent variables are :

  D , representitive sediment size
  K_v , kinematic viscosity
  S_g , specific gravity of the sediment
  q , unit discharge
  X_{bm} , sediment concentration

- specifying D, K_v, and S_g allows D_{gr} to be calculated and therefore also N, M, a, C_a, CON_1, and CON_3

- once the constants relating to sediment size and fluid are established, specifying q and X_{bm} leaves d, S, and F_{gr} as unknowns. Also, $CON_4 = f(F_{gr})$ and $V_{shear} = f(d,S)$.

Proceed to simplify the equations by reducing equations (D.1) and (D.2) to one equation by subtraction:

$$(V_{shear}/CON_1 - a)*CON_3 + a = V_{shear}^{N}/CON_1*(q/(5.65d*Log_{10}(10d/D)))^{(1-N)} \quad .....(D.13)$$

Paris's (1983) direct solution for Dgr > 60 stems from the fact that N=0 for Dgr > 60, which allows V_{shear} to drop out of the right hand side of equation (D.13). Substituting $(gdS)^{0.5}$ for V_{shear} allows equation (D.13) to be reduced to $S = f(q,d)$. For $1 <= D_{gr} <= 60$ do not substitute $(gdS)^{0.5}$ for V_{shear}, since the equation can be solved as $V_{shear} = f(q,d)$, as per (D.14):

$$V_{shear}^{(1-N)}*CON_3 - CON_1*a*V_{shear}^{(-N)} = (q/(5.65d*Log_{10}(10d/D)))^{(1-N)} \quad .....(D.14)$$

Equation (D.14) can be solved by numerical methods. In particular, if q, D, a, N, CON_1, and CON_3 are assumed to be known we have an equation of the form $V_{shear} = f(d)$. Assuming a value for d sets the right-hand side of equation (D.14), and the left-hand side of the equation is a polynomial in V_{shear}. A Newton iterative technique is used to converge on the value of V_{shear} corresponding to the chosen depth ,d. Back substitution into the

other equations allows solution of the other variables. To illustrate, consider equation (D.3) in which the unknowns are X, d, q, Vshear, and CON_4= f(Fgr). Assuming d and q one can calculate V_{shear} from equation (D.14). Then from equation (D.1) or (D.2) F_{gr} can be calculated and therefore so can CON_4. Finally X can be calculated from equation (D.3).

If one is interested in calculating d for a particular q and X, an extra iteration must be used to converge on the required value of X.

D.4 PROGRAM

The problem was programmed on an Zenith PC using the Supercalc4 spreadsheat package , a product of SORCIM/IUS Corp.

The solution of the equations was programmed into a single column and then duplicated so that several values of q could be investigated simultaneously. A sample column of the Supercalc program listing is given in D.4.1. Sample results are included in sections D.4.2, D.4.3 and D.4.4.

The iteration on equation (D.14) was accomplished within one spreadsheet operation. The iteration on d to obtain X required reloading the results of rows 34 and 35 (new increment and new d) back to rows 18 and 19 (inc and guess d). Execution files (i.e., macros) were set up to calculate several distinct values of X for any given D_{35}. In the resulting program D_{35} is specified the program calculates values of depth ,d , for set values of concentration, X, and unit discharge, q.

The execution file also loaded the results into a leastsquare regression template, which in turn calculated the best fit straight line of the form (D.15).

$$Log_{10}\ (d) = n\ \ Log_{10}\ (q) + log_{10}(K') \qquad \ldots(D.15)$$

The program listing, and sample results follow immediately. Furthermore, a listing of the required computer program files, execution files, data files, and output files is documented in section D.4.5.

D.4.1 PROGRAM LISTING

The following is a listing of the spreadsheet program, comments have been inserted where required to explain the program.

Row	Column A	Column B
1	Ackers White Transport & White, Paris, Bettess Resistance Equations	
	**** initialize the variables ****	
	**** choose K_v, and D_{35}; S_{min} is adjusted to get an initial solution, and provide an initial guess for V_{shear} ****	
	**** g, and S_g chosen to be standard values ****	
	**** q, and X were standardized for this investigation ****	
2	g=	9.81
3	S_g=	2.65
4	K_v=	.000001068
5	q=	.01
6	X=	.004
7	S_{min}=	.0002
8	D_{35}=	.00024
	**** calculate the following constants for a specific temperature and sediment size (Q can be input if so desired) ****	
9	D_{gr}=	B8*(B2*(B3-1)/B4^2)^(1/3)
10	N=	IF(B9>60,0.1,1-.56*LOG(B9))
11	M=	IF(B9>60,1.50,(9.66/B9)+1.34)
12	a=	IF(B9>60,0.17,.23/((B9)^.5)+.14)
13	C_a=	IF(B9>60,.025,10^(2.86*LOG(B9)-((LOG(B9))^2)-3.53))
14	Q =	
15	CON_1=	(9.81*B8*(B3-1))^.5
16	CON_3=	1-.76*(1-(1/EXP((LOG(B9))^1.7)))
	**** initialize the difference (old dif) for iteration of d on X ****	
	**** guess the values of the initial increment (inc) and depth (d). Some trial and error is ussually required to obtain an initial feasible solution. Once a solution is found for each column (i.e., value of q) then the entire set of equations for a particular sediment size can ussually be calculated. Further trial and error may be required for those concentrations which show an infeasible solution (i.e.,ERROR). ****	
17	old dif=	0
18	inc=	B5/5
19	guess d=	B5*1.5
	**** compute the RHS of equation (D.14) in text above ****	
20	RHS=	(B5/(5.65*B19*LOG(10*B19/B8)))^(1-B10)

```
**** guess U* and increment if ITER > 1 ****

21  guess U*=  IF(ITER=1,((B7*B2*B19)^.5),B21+B28)

**** compute difference between RHS and LHS of equation (D.14) ****

22  RHS-LHS=  B20-(B21^(1-B10)*B16-B15*B12/(B21^B10)*(B16-1))

****  Newton iteration scheme with the goal being 0 (i.e.,RHS=LHS) ****

23  Goal seek 0
24            B23-B22
25            IF(MOD(ITER,2)=1,B24,B25)
26            IF(B27=B25,B28*.1,B24*B28/(B27-B25))
27            IF(MOD(ITER,2)=0,B24,B27)
28            IF(ITER=1,.02,IF(ISERROR(B26),B28*.5,IF(MOD(ITER,2)=1,B26,
              -B26))
29            ADD CELL ABOVE ME TO X

**** calculate Fgr from equation (D.1), CON4 from equation (D.6), and  X
from equation (D.3) ****

30  Fgr=      ((B21/B15)-B12)*B16+B12
31  CON4=     B30/B12-1
32  X calc    B13*B31^B11*B3*B8/B19*(B5/(B19*B21))^B10

**** compare the Xcalc to the target X as pre-chosen above ****

33  RHS-X=    B32-B6

**** reset the increment and the new depth based on row 33 ****

34  new inc   IF(ABS(B33)<.000001,0,B18*B33/(B17-B33))
35  new d=    B19-B34

**** at this point an external execution file reinserts rows 34 and 35 into
rows 18 and 19 . The execution file contains a set number of iterations (16)
to allow convergence towards the required solution X.  If convergence does
not occur retry with a more appropriate depth, d. The execution file is
called the SEDI?  series were ? is replaced by a number. ****
***** calculate the pertinent output for use in the regressions ****

36  SLOPE=    B21^2/(B2*B19)
37  LOGd      LOG(B19)
38  LOGq      LOG(B5)
39  XPPM      B32*1000000
40  D35MM     B8*1000
41  q         B5
42  VELOCITY  B5/B19
43  CHEZY C   B42/B21
44  WIDTH     B14/B5
```

D.4.2 SAMPLE COLUMN OUTPUT FOR Dgr < 60

NOTES:
- D35 = .24mm, q = .01m^3/s/m and a goal of X=800 ppm assumed
- results are printed after 16 iterations to within 1 ppm.
- results are d = .02797m, S = .00468, V = .35m/s, C = 9.7

row	column A	column B
1	Ackers White Transport & White, Paris, and Bettess Resistance	
2	g=	9.81
3	Sg=	2.65
4	Kv=	.0000011
5	q=	.01
6	X=	.0008
7	Smin=	.0002
8	D35=	.00024
9	Dgr=	5.810516
10	N=	.5720398
11	M=	3.002503
12	A=	.2354158
13	Ca=	.0117932
14	Q=	
15	CON1=	.0623278
16	CON3=	.6435205
17	old dif=	.0000003
18	inc=	0
19	guess d=	.0279747
20	RHS=	.1899713
21	guess v=	.0358497
22	RHS-LHS=	-1.9e-15
23	Goal seek	0
24		1.94e-15
25		1.94e-15
26		-1.5e-15
27		5.23e-10
28		-1.5e-15
29		ADD CELL
30	Fgr=	.4540611
31	CON4=	.9287619
32	X calc	.0008003
33	RHS-X=	.0000003
34	new inc	0
35	new d=	.0279747
36	SLOPE=	.0046831
37	LOGd	-1.55323
38	LOGq	-2
39	XPPM	800.3454
40	D35MM	.24
41	q	.01
42	VELOCITY	.3574654
43	CHEZY C	9.971215
44	WIDTH	0

D.4.3 SAMPLE COLUMN OUTPUT FOR Dgr > 60

NOTES:
- D35=20mm, q=.01m^3/s/m and a goal of X=1400 ppm assumed
- results are printed after 16 iterations to within 1 ppm.
- results are d = .01824m, S = .08, V = .55m/s, C = 4.4

row	column A	column B
1	Ackers White	Transport & White, Paris, and Bettess Resistance
2	g=	9.81
3	Sg=	2.65
4	Kv=	.0000011
5	q=	.01
6	X=	.0014
7	Smin=	.01
8	D35=	.02
9	Dgr=	484.2097
10	N=	.1
11	M=	1.5
12	A=	.17
13	Ca=	.025
14	Q=	
15	CON1=	.5689728
16	CON3=	.2435705
17	old dif=	.0000003
18	inc=	0
19	guess d=	.0182436
20	RHS=	.1270812
21	guess v=	.1225653
22	RHS-LHS=	2.78e-17
23	Goal seek	0
24		-2.8e-17
25		-1.8e-11
26		-1.4e-16
27		-2.8e-17
28		1.41e-16
29		ADD CELL
30	Fgr=	.1810618
31	CON4=	.0650692
32	X calc	.0014003
33	RHS-X=	.0000003
34	new inc	0
35	new d=	.0182436
36	SLOPE=	.0839376
37	LOGd	-1.73889
38	LOGq	-2
39	XPPM	1400.302
40	D35MM	20
41	q	.01
42	VELOCITY	.5481388
43	CHEZY C	4.472219
44	WIDTH	0

D.4.4 SAMPLE CONCENTRATION OUTPUT

NOTES:
- D_{35}=.24mm assumed,and therefore D_{gr} < 60
- given 0.01 <= q <= 5 m^3/s/m and a goal of X=2000 ppm
- results are printed after 16 iterations to within 1 ppm.

row	column A	B	C	D	E	F	G	H
1	Ackers White transport & White, Paris, and Bettess Resistance							
2	g=	9.81	9.81	9.81	9.81	9.81	9.81	9.81
3	Sg=	2.65	2.65	2.65	2.65	2.65	2.65	2.65
4	Kv=	.0000011	.0000011	.0000011	.0000011	.0000011	.0000011	.0000011
5	q=	.01	.05	.1	.5	1	2	5
6	X=	.002	.002	.002	.002	.002	.002	.002
7	Smin=	.0002	.0002	.0002	.0002	.0002	.0002	.0002
8	D35=	.00024	.00024	.00024	.00024	.00024	.00024	.00024
9	Dgr=	5.810516	5.810516	5.810516	5.810516	5.810516	5.810516	5.810516
10	N=	.5720398	.5720398	.5720398	.5720398	.5720398	.5720398	.5720398
11	M=	3.002503	3.002503	3.002503	3.002503	3.002503	3.002503	3.002503
12	A=	.2354158	.2354158	.2354158	.2354158	.2354158	.2354158	.2354158
13	Ca=	.0117932	.0117932	.0117932	.0117932	.0117932	.0117932	.0117932
14	Q=							
15	CON1=	.0623278	.0623278	.0623278	.0623278	.0623278	.0623278	.0623278
16	CON3=	.6435205	.6435205	.6435205	.6435205	.6435205	.6435205	.6435205
17	old dif=	.0000003	.0000007	.0000002	1.601e-8	.0000009	.0000002	.0000005
18	inc=	0	0	0	0	0	0	0
19	guess d=	.0254405	.0891505	.1522133	.5203731	.8788974	1.480503	2.938201
20	RHS=	.1990002	.2158509	.2248209	.2500788	.2630164	.2772614	.2982599
21	guess v=	.0429709	.0570580	.0650611	.0897039	.1036031	.1199527	.1461140
22	RHS-LHS=	0	2.78e-17	2.78e-17	-5.6e-17	0	0	0
23	Goal seek	0	0	0	0	0	0	0
24		0	-2.8e-17	-2.8e-17	5.55e-17	0	0	0
25		7.28e-12	5.55e-17	-3.1e-16	-1.7e-13	-2.6e-13	-2.2e-13	-8.5e-14
26		0	-1.6e-17	-2.8e-17	5.77e-17	0	0	0
27		0	-2.8e-17	-2.8e-17	5.55e-17	0	0	0
28		0	1.61e-17	2.79e-17	-5.8e-17	0	0	0
29		ADD CELL	ADD CELL	ADD CELL	ADD CELL	ADD CELL	ADD CELL	ADD CELL
30	Fgr=	.5275855	.6730313	.7556614	1.010093	1.153598	1.322405	1.592515
31	CON4=	1.241079	1.858904	2.209900	3.290675	3.900259	4.617314	5.764687
32	X calc	.0020003	.0020007	.0020002	.0020000	.0020009	.0020002	.0020005
33	RHS-X=	.0000003	.0000007	.0000002	1.601e-8	.0000009	.0000002	.0000005
34	new inc	0	0	0	0	0	0	0
35	new d=	.0254405	.0891505	.1522133	.5203731	.8788974	1.480503	2.938201
36	SLOPE=	.0073987	.0037225	.0028348	.0015763	.0012449	.0009907	.0007407
37	LOGd	-1.59447	-1.04988	-.817547	-.283685	-.056062	.1704094	.4680815
38	LOGq	-2	-1.30103	-1	-.301030	0	.3010300	.6989700
39	XPPM	2000.303	2000.667	2000.192	2000.016	2000.895	2000.193	2000.524
40	D35MM	.24	.24	.24	.24	.24	.24	.24
41	q	.01	.05	.1	.5	1	2	5
42	VELOCITY	.3930738	.5608493	.6569728	.9608491	1.137789	1.350892	1.701721
43	CHEZY C	9.147441	9.829463	10.09779	10.71134	10.[illegible]20	11.26187	11.64653
44	WIDTH	0	0	0	0	0	0	0

D.4.5 DISKETTE FILES

The following is a listing of the required computer files.

1. PROGRAM FILES (DISK B)

-PARIS4 : PROGRAM FILE FOR SIMULTANEOUS SOLUTION OF THE ACKERS WHITE TRANSPORT AND WHITE PARIS AND BETTESS RESISTANCE EQUATIONS (SEE PROGRAM LISTING ABOVE FOR DESCRIPTION)

-LEASTSQ: PROGRAM FILE FOR LEASTSQUARE REGRESSION ON LOG(d) VS. LOG(q); COMPUTES SLOPE AND INTERCEPT OF STRAIGHT LINE AS WELL AS STATISTICS

2. EXECUTION FILES (DISK B)

For each of the following files a .CAL and a .XQT file are required

-MULT1(I) : EXECUTES LOADING OF PARIS4 FOR EACH SEDIMENT CONCENTRATION, X, AND INSERTS THE NEXT SEDIMENT CONCENTRATION FROM XFILE INTO PARIS4. CONTROL PASSES TO SEDI1(I). SEE LISTING BELOW.

```
****************** LISTING **********************
        /LB:PARIS4,A
        /BB6:J6
        /LB:XFILE,PB1:J1,B6,V
        !
        /XB:SEDI11
**************  END LISTING ********************
```

-SEDI1(I) : EXECUTES 16 ITERATIONS ON d BY RE-INSERTING THE RESULTS FOR THE NEW INCREMENT AND DEPTH, INTO THE ROWS FOR THE OLD INCREMENT AND GUESS DEPTH. CONTROL PASSED TO MULT2(I). SEE LISTING BELOW.

```
 ***************  LISTING  **********************
        /SC:ACKERS,OV
        /LC:ACKERS,PB33:K35,B17
        !
        /SC:ACKERS,OV
        /LC:ACKERS,PB33:K35,B17
        !
   ***                                        ***
        REPEAT ABOVE 3 ROWS 14 MORE TIMES
   ***                                        ***
        ....
        /XB:MULT21
 ***************  END LISTING  *****************
```

-MULT2(I) : EXECUTES PRINTING OF RESULTS, LOADING OF RESULTS INTO LEASTSQ, CALCULATING THE REGRESSION RESULTS AND PRINTING, AND SAVING ALL RESULTS TO DISK B. CONTROL PASSES TO MULT1(I+1), EXCEPT FOR CASE OF LAST SEDIMENT CONCENTRATION. SEE LISTING BELOW.

********** LISTING **********

```
/SA:X10,OV
/ODA1:H44,P
/ODH1:J44,P
/ZY
/LB:LEASTSQ,A
/LA:X10,PK47:L55,A6,V
!
/LB:TITLE,PA1,A15
/ODA1:B19,P
/ODA44:E85,P
/SA:LS10,OA
/ZY
/XB:MULT12
```

********* END LISTING **********

3. DATA FILES (DISK B)

-TITLE : DATA REQUIRED BY LEASTSQ FILE SO THAT GRAPHS HAVE PROPER D35 AND X VALUES.

-XFILE : DATA REQUIRED BY MULT1(I) EXECUTEABLE. (I) MUST EQUAL THE NUMBER OF ROWS IN THIS FILE. EACH ROW MUST HAVE ONE AND ONLY ONE CONCENTRATION REPEATED IN EACH COLUMN AS REQUIRED BY PARIS4. EACH SUCCESSIVE MULT1(I) FILE READS ONE ROW STARTING FROM THE TOP.

4. OUTPUT FILES (DISK A)

-X???? : OUTPUT FILE CONTAINING THE SUPERCALC3 RESULTS OF PROGRAM PARIS4 FOR ONE PARTICULAR D35 AND X. THE ???? IS THE SEDIMENT CONCENTRATION IN PPM.

-LS???? : OUTPUT FILE CONTAINING THE SUPERCALC3 RESULTS OF PROGRAM LEASTSQ FOR ONE PARTICULAR D35 AND X. THE ???? IS THE SEDIMENT CONCENTRATION IN PPM. THIS FILE ALSO CONTAINS A GRAPH OF LOG(q) VS. LOG(d).

D.5 RESULTS

Solution to the equations are provided for all combinations of the following independent variables:

X_{bm}= 10, 20, 40, 100, 200, 400, 800, 1400, 2000, 4000, and 8000 ppm
q=.01, .05, .1, .5, 1.0, 2.0, 5.0, 10.0, 30.0 m^2/s
D_{35} = .1, .26, .30, .50, 1.0, 2.5, 5.0, 10.0, and 20.0 mm

Regressions of the data were undertaken by the program LEASTSQ to arrive at equations of the form (3.28).

$$d = K' * q^{n} \qquad \text{.....(3.28)}$$

Figure D-1 illustrates typical results from the regression analysis of $\log_{10}(d)$ versus $\log_{10}(q)$, for a concentration X_{bm} = 1000 ppm, and D_{35} = 0.30 mm. Figures D-2 to D-10 present the least square linear relations for the solution data as calculated using equation (D.15), for the various sediment sizes and concentrations stated above.

The numerical results of the regression analyses are given in Tables D-1 and D-2. In general the regression correlation coefficients were extremely high, with r^2 in the order of .99, as demonstrated by Figure D-1.

TABLE D-1: VARIATION OF n WITH SEDIMENT SIZE, D_{35} AND CONCENTRATION, X

Width:depth exponent, n

D_{35}(mm)	.1	.26	.3	.5	1	2.5	5	10	20
X(PPM)									
10	.864	.842	.841	.830	.833	.813	.823	.824	.814
20	.860	.830	.827	.821	.813	.791	.807	.812	.806
40	.855	.817	.813	.803	.792	.767	.787	.797	.795
100	.849	.800	.794	.777	.761	.734	.756	.771	.776
200	.845	.788	.780	.758	.736	.707	.731	.749	.757
400	.840	.776	.767	.740	.713	.682	.705	.724	.736
800	.836	.765	.755	.723	.691	.658	.679	.698	.712
1400	.833	.757	.746	.710	.675	.640	.659	.678	.692
2000	.831	.752	.740	.703	.666	.630	.647	.665	.679
4000	.827	.744	.731	.691	.650	.611	.626	.641	.653
8000	.823	.736	.723	.680	.636	.596	.607	.618	.629

TABLE D-2: VARIATION OF K' WITH SEDIMENT SIZE, D_{35}, AND CONCENTRATION, X

Width:depth Coefficient, K'

D_{35}(mm)	.1	.26	.3	.5	1	2.5	5	10	20
X(PPM)									
10	1.790	1.776	1.760	1.670	1.454	1.130	.9810	.8429	.7231
20	1.706	1.663	1.641	1.544	1.351	1.043	.9278	.8110	.7053
40	1.621	1.539	1.518	1.417	1.235	.9456	.8613	.7695	.6811
100	1.504	1.369	1.343	1.237	1.068	.8083	.7606	.7011	.6375
200	1.416	1.240	1.210	1.098	.9380	.7025	.6776	.6405	.5961
400	1.329	1.113	1.079	.9629	.8109	.6001	.5930	.5747	.5480
800	1.243	.9911	.9540	.8340	.6904	.5044	.5100	.5063	.4950
1400	1.176	.8976	.8587	.7369	.6003	.4338	.4461	.4514	.4500
2000	1.134	.8407	.8010	.6787	.5468	.3923	.4075	.4170	.4209
4000	1.054	.7369	.6961	.5746	.4520	.3197	.3382	.3534	.3649
8000	.9777	.6424	.6014	.4827	.3683	.2580	.2772	.2951	.3113

Figure D-11 shows the trends of Table D-1 graphically. Figures D-12 and D-13 present the same data from slightly different perspectives. The results of Table D-2 are displayed graphically in Figures D-14 to D-16. In addition to the results of the regressions, three additional graphs are included as Figures D-17, D-18, and D-19, which show the relationship of d to q and X for three different sediment sizes.

For X_{bm} = 10 ppm the exponent in n does not strictly decrease for Dgr<60 and increase for Dgr>60, with sediment size. This can be explained by the accuracy limits placed on the solutions. The program performed iterations on X_{bm} until it was within 1 ppm. Whereas this is suitable for X = 8000 ppm, it appears that a finer convergence is required for X = 10 ppm. The difference in the results did not warrent refinement.

APPENDIX E
NUMERICAL SOLUTION
TO
BROWNLIE (1981A)
SEDIMENT TRANSPORT
AND
FLOW RESISTANCE
EQUATIONS

E.1 INTRODUCTION

This appendix documents the simultaneous solution of Brownlie's (1981a) equations for sediment transport and flow depth, for the purpose of investigating the relationship of the rational width:depth equation (4.8a) and specific width:depth equation (3.28). Attention is given to the behavior of the relations during transition between flow regimes.

E.2 EQUATIONS

Brownlie (1981a) derived empirical relations for flow depth and sediment concentration as functions of several other hydraulic variables. Some 22 seperate data sources were employed to develop his flow depth equations and 31 different sources were used in developing the sediment transport equations (Brownlie, 1981b). Over 900 data points were used in developing the equations.

Brownlie's (1981a) equations for flow depth, in lower and upper regimes, and sediment concentration are given by equations (E.1 a,b) and (E.2) respectively.

Lower regime depth

$$r = d_l = 0.3724\ D_{50}/S\ (q^*\ S)^{0.6539}\ S^{0.09188}\ \sigma_g^{0.1050} \qquad \ldots(E.1a)$$

Upper regime depth

$$r = d_u = 0.2836\ D_{50}/S\ (q^*\ S)^{0.6248}\ S^{0.08750}\ \sigma_g^{0.08013} \qquad \ldots(E.1b)$$

Sediment concentration

$$X_{bm}(ppm) = 7115\ C_F\ (F_g - F_{go})^{1.978}\ S^{0.6601}\ (r/D_{50})^{-0.3301} \qquad \cdots(E.2)$$

where C_F = 1.0 for lab data, and 1.268 for field data

$$F_g = v/\ (g\ D_{50}(SD-1))^{0.5} \qquad \ldots\ldots\ldots(E.3)$$

$$F_{go} = 4.596\ t_{*o}^{0.5293}\ S^{-0.1405}\ \sigma_g^{-0.1606} \qquad \ldots\ldots(E.4)$$

$$t_{*o} = f(g\ D_{50}/k_v) \qquad \ldots\ldots(E.5)$$

d_l = lower regime depth (m)

d_u = upper regime depth (m)

$q^* = q/(g\ D_{50}^3)^{0.5}$

r = hydraulic radius = mean depth for wide rivers

The restrictions placed on the data set used for developing equation (E.2) are given in Table E-1.

TABLE E-1: RESTRICTIONS ON DATA USED TO DEVELOP EQUATION (E.2)

DATA	RESTRICTION	REASON
median grain size	$.062mm<D_{50}<2.0mm$	sand size only
geometric stan dev.	$o_g<5$	avoid bimodal
width/depth	$W/d > 4$ (lab)	side-wall effects
relative roughness	$r/D_{50} > 100$	low flow depth
concentration	$X_{bm} > 10$	low concentration

The range of the data used in development of both the sediment transport and flow depth equations is presented in Table E-2.

TABLE E-2: RANGE OF DATA USED IN DEVELOPING EQUATIONS (E.1) AND (E.2)

VARIABLE		MINIMUM	MAXIMUM
q (m^2/s)		0.012	40
S		3.0×10^{-6}	3.7×10^{-2}
r(m)		0.025	17.282
T(°c)		0	63
X_{bm}(ppm)	lab:	10.95	39623
	field:	11.70	5830
D_{50}(mm)		0.088	2.8

Equations describing the lower limit of the upper regime and the upper limit of the lower regime are given and are reproduced here as (E.6 a,b) and (E.7 a,b).

Lower limit of upper regime

- if S>.006 then upper regime is predicted, otherwise:

$$\log F_g/F_g' = -0.02469 + 0.1517 \log (D_{50}/\check{S}) + 0.8381(\log (D_{50}/\check{S}))^2 \quad [\text{for } D_{50}/\check{S} < 2] \qquad \ldots(E.6a)$$

$$= \log 1.25 \quad [\text{for } D_{50}/\check{S} \geq 2] \qquad \ldots(E.6b)$$

Upper limit of lower regime

$$\log F_g/F_g' = -0.2026 + 0.07026 \log (D_{50}/\check{S}) + 0.9330 (\log (D_{50}/\check{S}))^2 \quad [\text{for } D_{50}/\check{S} < 2] \qquad \ldots(E.7a)$$

$$= \log 0.80 \quad [\text{for } D_{50}/\check{S} \geq 2] \qquad \ldots(E.7b)$$

where $\check{S} = 11.6\ k_v/u_*'$ ...(E.8)
[$u_*' = (g\ d\ S)^{0.5}$; calculated using the upper regime depth]

$F_g' = 1.74\ S^{-1/3}$ ...(E.9)
[for $S<.006$]

If temperature effects are not important then equations (E.6b) and (E.7b) can be used for all $D_{50}/\check{S}$. Ignoring temperature effects will be referred to herein as the simplified or simple method. Inclusion of the temperature effects requires consideration of equations (E.6a) and (E.7a), and will be referred to herein as the detailed method.

E.3 METHODOLOGY

Brownlie's equations (E.1a,b) and (E.2), can be generalized as per equations (E.10 a,b) and (E.11).

$$d_l = f_1\ (S,\ q;\ D_{50},\ \sigma_g,\ p,\ p_s,\ k_v,\ g) \quad ....(E.10a)$$
$$d_u = f_2\ (S,\ q;\ D_{50},\ \sigma_g,\ p,\ p_s,\ k_v,\ g) \quad ....(E.10b)$$
$$X_{bm} = f_1\ (C_F,\ q,\ d;\ D_{50},\ \sigma_g,\ p,\ p_s,\ k_v,\ g) \quad ....(E.11)$$

The method for determination of the flow regime and depth is as follows:

1. set q, S, D_{50}, σ_g, p, p_s, k_v, and g
2. calculate d_l and d_u (equations E.1 a,b)
3. using continuity, calculate upper regime velocity, U_u and lower regime velocity, U_l
4. calculate the upper limit of F_g for the lower regime from equation (E.7), and the upper limiting velocity for the lower regime, U_{lcr}.
5. calculate the lower limit of F_g for the upper regime from equation (E.6), and the lower limiting velocity for the upper regime, U_{ucr}.
6. determine the flow regime as follows:
   - *if $s>.006$ the upper regime, and $d=d_u$*
   - *if $U_u>U_{ucr}$, and $U_l>U_{lcr}$ then upper regime, and $d=d_u$*
   - *if $U_u<U_{ucr}$, and $U_l<U_{lcr}$ then lower regime, and $d=d_l$*
   - *if $U_u<U_{ucr}$, and $U_l>U_{lcr}$ then transition regime and $d=d_t$*
   - *if none of the above conditions prevail, then multiple solutions exist*
7. determine the velocity, U, by dividing q by the chosen depth, d.

Since Brownlie (1981a) did not present a method of determining the flow depth in the transition region, a method is proposed here. The method takes advantage of the observation that $U_{lcr} < U_{ucr}$ and $q_l = U_{lcr}\, d_l < q_u = U_{ucr}\, d_u$. The proposed equation for the transitional velocity is given by (E.12).

$$U_t = (q - q_l) / (q_u - q_l)\ (U_{ucr} - U_{lcr}) + U_{lcr} \qquad \ldots(E.12)$$

Continuity is then used to derive the transitional flow depth d_t.

Once the appropriate regime, flow depth and velocity is determined then equation (E.2) can be used to calculate the sediment concentration X_{bm} directly. For equations of the form (4.8a) and (3.28) it was necessary to specify q and X rather then q and S. A Newton iterative scheme was used, employing S an the iterating variable, to converge on the solution for a given q and X.

E.4 PROGRAM DETAILS

The problem was programmed on a Zenith Pc making use of the Supercalc4 software package. The simultaneous solution of the equations was first programmed into a single column and then duplicated in adjacent columns to allow simultaneously consideration of several independent variables. Macros were employed to print and graph the results, and fit equations of the form (3.28) to the data, using a least square regression routine.

A sample column listing is provided in section E.4.1. Input to the program includes T, g, D_{50}, σ_g. p, p_s, q, S(initial), C_F, simple or detailed method, and X (optional). The specification of X =-1 allows direct calculation of d and X given q and S. Specifying X > 0 requires the program to iterate on S (i.e., S_{cur}), to find the value of S and d that produce the desired X. A sample column of the program output is given under section E.4.2.

E.4.1 Program Listing

The following is a listing of program *Brown3*, which simultaneously solves Brownlie's (1981a) sediment transport and flow depth equations.

```
      column
row   |  A ||       B       ||     C     ||   D   ||   E   ||   F   ||
1     BROWNLIE SEDIMENT TRANSPORT AND RESISTANCE  (FILE : BROWN3T)
2..4
5   INPUT (I=INPUT, I(O)=OPTIONAL INPUT, C=CALCULATED, INT=INSTRUCTIONS)
6   ------
7           CONSTANTS
8           ---------
9     I     TEMPERATURE          TE (C) = 20
10    C     K.VISCOSITY       kv (m2/s) = (1.14-.031*(D9-15)+.00068*(D9-
                                         15)^2)*10^-6
11    I     GRAV. CONST.            g = 9.81
12
13          SEDIMENT/H20
14          ------------
15    I     SED. SIZE 50%      D50 (um) = 2000
16    C                        D50  (m) = D15*10^-6
17    I     GEOM.STAN.DEV.          og = 1.6
18    I     DENSITY SED.     ps (kg/m3) = 2650
19    I     DENSITY H2O      p  (kg/m3) = 1000
20    C     SPECIFIC DENS.          SD = D18/D19
21    C     RELATIVE DENS.       (SD-1) = D20-1
22
23          INDEPENDENT VARIABLES
24          ---------------------
25    I     UNIT DISCHARGE    q (m^2/s) = 2
26    I     SLOPE (INITIAL)         s = .00297
27    C     CURRENT SLOPE         Scur = IF(D31=-1,D26,IF(ITER=1,D26,
                                         D27+D113))
28
29          OPTIONS
30          -------
31    I   **TARGET CONC. ,     Cbm (PPM) = 2000
32    INT     IF Cbm = AS CALCULATED FOR INITIAL SLOPE, CHOOSE = -1
33    INT     IF Cbm= SPECIFIC VALUE (EG. 200 PPM), CHOOSE=VALUE (EG. 200)
34    NOTE    IF CHOOSE > 0  THEN PROGRAM WILL ITERATE ON SLOPE TO GET Cbm
35    C     TARGET CONC. ,    Cbm (vol.) = D31/D18/1000
36
37    I   **PPM FACTOR ,              Cf = 1.6
38    INT     CHOOSE CF VALUE (EQUATION 6.8 REPORT KH-R-43A)
39
40    I   **LIMITING VELOCITIES, METHOD = 0
41    INT   IF TEMPERATURE EFFECTS IMPORTANT THEN CHOOSE = 1 (RIGOUROUS
            METHOD)
42    INT   IF TEMPERATURE NOT IMPORTANT THEN CHOOSE = 0 (SIMPLIFIED
            METHOD)
43
```

```
44   CALCULATIONS
45   ------------
46         UPPER AND LOWER REGIME DEPTHS
47         -----------------------------
48         DIMENSIONLESS q            q* = D25/(D11*D16^3)^.5
49         DEPTH(LOWER R.)       dL (m) = .3724*D16/D27*(D48*D27)^.6539
                                          *D27^.09188*D17^.105
50         DEPTH(UPPER R.)       dU (m) = .2836*D16/D27*(D48*D27)^.6248
                                          *D27^.0875*D17^.08013
51
52         TRANSITION ZONE HYDRAULICS
53         --------------------------
54         VELOCITY(LOWER)     uL (m/s) = D25/D49
55         VELOCITY(UPPER)     uU (m/s) = D25/D50
56         SHEAR VELOCITY  U*'=U* (m/s)= (D11*D50*D27)^.5
57         LAMINAR SUBLAYER    DELTA(m) = 11.6*D10/D56
58         REL.ROUGH          D50/DELTA = D16/D57
59         GRAIN FROUDE ',          Fg' = 1.74*D27^(-1/3)
60         Fg/Fg' LOWER LIMIT UPPER REG. IF(D40=0,1.25,IF(D58>=2,1.25,
                                          10^(-.02469+.1517*LOG(D58)+
                                          .8381*(LOG(D58))^2)))
61         Fg/Fg' UPPER LIMIT LOWER REG. IF(D40=0,.8,IF(D58>=2,.8,10^(-
                                          .2026 +.07026*LOG(D58)+.933*
                                          (LOG(D58))^2)))
62         uUcr.  LOWER LIMIT UPPER REG. (D11*D21*D16)^.5*D59*D60
63         uLcr.  UPPER LIMIT LOWER REG. (D11*D21*D16)^.5*D59*D61
64   ------------------------------------------------------------------
65..68
69   CALCULATIONS (CONTINUED)
70   ------------------------
71         CALCULATE FLOW REGIME (***LOWER,TRANSITION,UPPER***)
72         --------------------
73   NOTE  FOR CHOOSING u,  0 = NOT CHOSEN
74   NOTE  uT = (q-qL/qL-qU)(uUcr-uLcr) + uLcr
75   NOTE  where: qL=dL*uLcr  & qU=dU*uUcr
76                                    qU = D50*D62
77                                    qL = D49*D63
78         IF S>.006                u=uU  = IF(D27>.006,D55,0)
79         IF uL<uLcr & uU<uUcr, u=uL  = IF(D54<=D63,IF(D55<D62,
                                          D54,0),0)
80         IF uL>uLcr & uU>uUcr, u=uU  = IF(D54>D63,IF(D55>=D62,
                                          D55,0),0)
81         IF uL>uLcr & uU<uUcr, u=uT  = IF(D54>D63,IF(D55<D62,IF(D55>=
                                          D54,(D25-D77)/(D76-D77)*(D62-
                                          D63)+D63,("WARNING1")),0),0)
82         IF uL<uLcr & uU>uUcr, u=?   = IF(D54<=D63,IF(D55>=D62,
                                          ("WARNING2"),0),0)
83         CHOSEN VELOCITY     u (m/s) = MAX(D78,D79,D80,D81,D82)
84         REGIME                       IF(D78>0,("UPPER"),IF(D79>0,
                                          ("LOWER") ,IF(D80>0,("UPPER"),
                                          IF(D81>0, ("TRANS"),IF(D82>0,
                                          ("MULT"),0)))))
85         EXCEEDS BOUNDARIES(NO)       ("NO")
86
```

```
87          SEDIMENT CONCENTRATION
88          ----------------------
89          CHOSEN DEPTH          dCH (m) = D25/D83
90          GRAIN REYNOLDS            Rg  = (D11*D16^3)^.5/D10
91          GRAIN FROUDE               Fg = D83/(D21*D11*D16)^.5
92          Y FACTOR                    Y = (D21^.5*D90)^(-.6)
93          CR. SHEAR STRESS           ~*0 = .22*D92+.06*10^(-7.7*D92)
94          CR. GRAIN FROUDE           Fgo = 4.596*D93^.5293*D27^(-.1405)
                                           *D17^(-.1606)
95          CONCENTRATION      Cbm (PPM) = IF(D94-D91>0,0,7115*D37*(D91
                                           -D94)^1.978*D27^.6601 *
                                           (D89 /D16) ^(-.3301))
96
97
98          MISCELLANEOUS OUTPUT
99          --------------------
100         Rg*og^.1067/ S^.1608 =         D90*D17^.1067/D27^.1608
101         S^1/3 * Fg =                   D27^(1/3)*D91
102         CONCENTRATION      Cbm (PPM) = D95
103
104         ITERATION ROUTINE
105         -----------------
106         INITIAL INCREMEN     INCinit = D26/100
107         Xcalc - Xtarg =                IF(D31=-1,0,D95-D31)
108         Goal seek                      0
109                                        D108-D107
110                                        IF(MOD(ITER,2)=1,D109,D110)
111                                        IF(D112=D110,D113*.1,D109
                                           *D113/(D112-D110))
112                                        IF(MOD(ITER,2)=0,D109,D112)
113                                        IF(ITER=1,D106,IF(ISERROR(D111)
                                           ,D113*.5,IF(MOD(ITER,2)=1,
                                           D111,-D111)))
114                                        ADD CELL ABOVE ME TO X
115
116         OUTPUT FOR REGRESSION
117         ---------------------
118         TEMPERATURE             TE (C) = D9
119         SED. SIZE 50%         D50 (um) = D15
120         GEOM.STAN.DEV.              og = D17
121         UNIT DISCHARGE      q (m^2/s) = D25
122         LOG UNIT Q            LOG (q) = LOG(D121)
123
124         CONCENTRATION      Cbm (PPM) = D95
125         DEPTH                  d (m) = IF(D85=("YES"),("N/A"),D89)
126         VELOCITY             V (m/s) = IF(D85=("YES"),("N/A"),D83)
127         SLOPE                      S = D27
128         LOG DEPTH            LOG (d) = IF(D125=("N/A"),("N/A"),
                                           LOG(D125))
129         LOG SLOPE             LOG(S) = LOG(D127)
130         EXCEEDS BOUNDARIES(NO)         D85
131         REG.(L=LOWER,U=UPPER,T=TRANS) IF(D78>0,("U"),IF(D79>0,("L"),
                                           IF(D80>0,("U"),IF(D81>0,("T"),
                                           IF(D82>0,("M"),0)))))
```

The program was found to be very sensitive to the choice of the initial slope. Therefore the following adjustments to the program code were incorporated to enable calculation of lower and upper regime solutions seperately. The solutions for the three flow regimes were merged after calculation by the appropriate program (i.e., Brown3t, Brown3l, or Brown3u).

```
     column
row  |  A ||       B       ||      C      ||   D   ||   E   ||   F   ||   G
1      BROWNLIE SEDIMENT TRANSPORT AND RESISTANCE  (FILE : BROWN3L)
2
3      *****ITERATING PROGRAM FOR LOWER REGIME*******
.
.
.
69   CALCULATIONS (CONTINUED)
70   -----------------------
71         CALCULATE FLOW REGIME (***LOWER REGIME ASSUMED***)
72         --------------------
73   NOTE  FOR CHOOSING u,  0 = NOT CHOSEN
74   NOTE  uT = (q-qL/qL-qU)(uUcr-uLcr) + uLcr
75   NOTE  where: qL=dL*uLcr  & qU=dU*uUcr
76                                  qU = D50*D62
77                                  qL = D49*D63
78         IF S>.0006             u=uU  =N/A
79         IF uL<uLcr & uU<uUcr, u=uL  =N/A
80         IF uL>uLcr & uU>uUcr, u=uU  =N/A
81         IF uL>uLcr & uU<uUcr, u=uT  =N/A
82         IF uL<uLcr & uU>uUcr, u=?   =N/A
83         CHOSEN VELOCITY     u (m/s) = D54
84         REGIME (ASSUMED)              ("LOWER")
85         EXCEEDS UPPER BOUNDARY        IF(D83>D63,("YES"),("NO"))
.
.
.
116        REGRESSION OUTPUT (***LOWER REGIME ASSUMED***)
117        ----------------
.
.
.
130        EXCEEDS UPPER BOUNDARY        D85
131        REGIME (L=LOWER,U=UPPER)      ("L")
```

```
    column
row |  A ||       B       ||      C      ||   D   ||   E   ||   F   ||   G
1       BROWNLIE SEDIMENT TRANSPORT AND RESISTANCE  (FILE : BROWN3U)
2            (LATEST UPDATE:       )
3     *****ITERATING PROGRAM FOR UPPER REGIME*******
.
.
.
69    CALCULATIONS (CONTINUED)
70    -----------------------
71          CALCULATE FLOW REGIME (***UPPER REGIME ASSUMED***)
72          --------------------
73    NOTE  FOR CHOOSING u,  0 = NOT CHOSEN
74    NOTE  uT = (q-qL/qL-qU)(uUcr-uLcr) + uLcr
75    NOTE  where: qL=dL*uLcr  & qU=dU*uUcr
76                                    qU = D50*D62
77                                    qL = D49*D63
78          IF S>.0006              u=uU  = IF(D27>.006,D55,0)
79          IF uL<uLcr & uU<uUcr, u=uL  =N/A
80          IF uL>uLcr & uU>uUcr, u=uU  =N/A
81          IF uL>uLcr & uU<uUcr, u=uT  =N/A
82          IF uL<uLcr & uU>uUcr, u=?   =N/A
83          CHOSEN VELOCITY     u (m/s) = D55
84          REGIME (ASSUMED)              ("UPPER")
85          BELOW LOWER BOUNDARY          IF(D27>.006,("NO"),IF(D83<D63,
                                          ("YES"),("NO")))
.
.
.
116         REGRESSION OUTPUT (***UPPER REGIME ASSUMED***)
117         ----------------
.
.
.
130         BELOW LOWER BOUNDARY          D85
131         REGIME (L=LOWER,U=UPPER)      ("U")
```

E.4.2 **SAMPLE OUTPUT**

```
    BROWNLIE SEDIMENT TRANSPORT AND RESISTANCE  (FILE : BROWN3T)
    INPUT (I=INPUT, I(O)=OPTIONAL INPUT, C=CALCULATED, INT=INSTRUCTIONS)
    ------
          CONSTANTS
          ---------
    I     TEMPERATURE          TE (C) =       20
    C     K.VISCOSITY       kv (m2/s) = .0000010
    I     GRAV. CONST.             g =     9.81

          SEDIMENT/H20
          ------------
    I     SED. SIZE 50%      D50 (um) =     2000
    C                        D50  (m) =     .002
    I     GEOM.STAN.DEV.           og =      1.6
```

```
I     DENSITY SED.      ps (kg/m3) =     2650
I     DENSITY H2O       p  (kg/m3) =     1000
C     SPECIFIC DENS.            SD =     2.65
C     RELATIVE DENS.         (SD-1) =     1.65

      INDEPENDENT VARIABLES
      ---------------------
I     UNIT DISCHARGE    q (m^2/s) =        2
I     SLOPE (INITIAL)           s =   .00297
C     CURRENT SLOPE          Scur = .0029631

      OPTIONS
      -------
I   **TARGET CONC. ,     Cbm (PPM) =     2000
INT    IF Cbm = AS CALCULATED FOR INITIAL SLOPE, CHOOSE = -1
INT    IF Cbm = SPECIFIC VALUE (EG. 200 PPM), CHOOSE=VALUE (EG. 200)
NOTE   IF CHOOSE > 0  THEN PROGRAM WILL ITERATE ON SLOPE TO GET Cbm
C     TARGET CONC. ,    Cbm (vol.) = .0007547
I   **PPM FACTOR ,              Cf =      1.6
INT    CHOOSE CF VALUE (EQUATION 6.8 REPORT KH-R-43A)
I   **LIMITING VELOCITIES, METHOD =        0
INT   IF TEMPERATURE EFFECTS IMPORTANT THEN CHOOSE = 1 (RIGOUROUS
      METHOD)
INT   IF TEMPERATURE NOT IMPORTANT THEN CHOOSE = 0 (SIMPLIFIED
      METHOD)

CALCULATIONS
------------
      UPPER AND LOWER REGIME DEPTHS
      -----------------------------
      DIMENSIONLESS q          q* = 7139.216
      DEPTH(LOWER R.)      dL (m) = 1.137906
      DEPTH(UPPER R.)      dU (m) = .8039539

      TRANSITION ZONE HYDRAULICS
      --------------------------
      VELOCITY(LOWER)    uL (m/s) = 1.757614
      VELOCITY(UPPER)    uU (m/s) = 2.487705
      SHEAR VELOCITY  U*'=U* (m/s)= .1528707
      LAMINAR SUBLAYER   DELTA(m) = .0000760
      REL.ROUGH         D50/DELTA = 26.30441
      GRAIN FROUDE ',         Fg' = 12.11434
      Fg/Fg' LOWER LIMIT UPPER REG.     1.25
      Fg/Fg' UPPER LIMIT LOWER REG.       .8
      uUcr.  LOWER LIMIT UPPER REG. 2.724590
      uLcr.  UPPER LIMIT LOWER REG. 1.743738
-----------------------------------------------------
CALCULATIONS (CONTINUED)
-----------------------
      CALCULATE FLOW REGIME (***LOWER,TRANSITION,UPPER***)
      ---------------------
NOTE  FOR CHOOSING u,  0 = NOT CHOSEN
NOTE  uT = (q-qL/qL-qU)(uUcr-uLcr) + uLcr
NOTE  where: qL=dL*uLcr  & qU=dU*uUcr
```

```
                                qU = 2.190445
                                qL = 1.984210
IF S>.006                    u=uU  =         0
IF uL<uLcr & uU<uUcr,        u=uL  =         0
IF uL>uLcr & uU>uUcr,        u=uU  =         0
IF uL>uLcr & uU<uUcr,        u=uT  = 1.818835
IF uL<uLcr & uU>uUcr,        u=?   =         0
CHOSEN VELOCITY          u (m/s) = 1.818835
REGIME                                 TRANS
EXCEEDS BOUNDARIES(NO)                    NO

SEDIMENT CONCENTRATION
----------------------
CHOSEN DEPTH            dCH (m) = 1.099605
GRAIN REYNOLDS              Rg  = 279.5837
GRAIN FROUDE                Fg  = 10.10885
Y FACTOR                    Y   = .0292987
CR. SHEAR STRESS           ~*0  = .0421361
CR. GRAIN FROUDE           Fgo  = 1.806539
CONCENTRATION         Cbm (PPM) =      2000

MISCELLANEOUS OUTPUT
--------------------
Rg*og^.1067/ S^.1608 =                749.6080
S^1/3 * Fg =                          1.451949
CONCENTRATION         Cbm (PPM) =      2000

ITERATION ROUTINE
-----------------
INITIAL INCREMEN        INCinit = .0000297
Xcalc - Xtarg =                       -9.8e-11
Goal seek                                    0
                                      9.78e-11
                                      .0014742
                                      -1.3e-17
                                      9.78e-11
                                      1.31e-17
                                   ADD CELL ABOVE ME TO X

OUTPUT FOR REGRESSION
---------------------
TEMPERATURE              TE (C) =          20
SED. SIZE 50%          D50 (um) =        2000
GEOM.STAN.DEV.               og =         1.6
UNIT DISCHARGE       q (m^2/s) =           2
LOG UNIT Q             LOG (q) = .3010300
CONCENTRATION         Cbm (PPM) =        2000
DEPTH                     d (m) = 1.099605
VELOCITY                V (m/s) = 1.818835
SLOPE                         S = .0029631
LOG DEPTH               LOG (d) = .0412366
LOG SLOPE                LOG(S) = -2.52825
EXCEEDS BOUNDARIES(NO)                    NO
REG.(L=LOWER,U=UPPER,T=TRANS)              T
```

E.5 VERICATION

E.5.1 Depth

Brownlie's (1981a) Table 4.3 provided discrete results for given input variables. Table E-3 compares the results computed via the program listed in E.4.1 to those of Brownlie's Table 4.3. Actual observed data are provided for reference purposes.

TABLE E-3: VERIFICATION OF PROGRAM *BROWN3* [BROWNLIE (1981A) TABLE 4.3]

River	$q(m^2/s)$	S*1000	D_{50}(mm)	σ_g	T(c)	Depth d(m) Table 4.3	*Brown3*	actual
Rio	.225	.50	.25	1.48	20	.41	.408	.403
Grande	.693	.55	.19	1.40	11	.82	.823	.769
Conveyance	2.01	.60	.23	1.36	13	1.10	1.100	1.19
Mississippi	4.74	.0183	.31	1.66	21	7.1	7.06	7.59
	10.4	.0266	.25	1.81	24	11	10.79	10.7
	26.0	.0382	.30	1.63	18	18	17.78	16.7

notes:
1. all data are in lower flow regime except q=2.01 Rio Grande Conveyance Channel, which is upper regime
2. all flow regimes were modelled correctly by *Brown3*.

A second verification is provided by Figure E-1. Calculated values are plotted on these figures for both the simplified and detailed methods. Figure E-1 illustrates that the program *Brown3* computes values of d_l and d_u which match those provided by Brownlie (1981a). Brownlie apparently plotted only the simplified method, since the data suggest that the simple method follows the observed data better. Figures E-1 show that the proposed transitional equation (E.12) provides a smooth transition between the lower and upper regimes, falling in between the horizontal lines representing the upper limit of the lower regime and the lower limit of the upper regime.

The trend of the transitional regime flow depth to decrease with increasing velocity (Figure E-1), might be explained by the following hypothesis:

> "Consider the upper limit of the lower flow regime which is typified by a duned bed and therefore a high resistance to the flow. At some small increment in q the dune bed starts to wash out, being replaced by flat bed with lower total resistance. It seems possible that the increased depth attributable to the small increment of q is not as large as the decrease in depth caused by dropping resistance, whereby the total or net effect is a slight decrease in depth. This effect will proceed up until the point of the bed being totally washed out, perhaps coinciding with the lower limit of the upper regime, after which depth will again increase."

As pointed out by Brownlie (1981a), further investigation is required to establish the behavior of rivers in the transition region. It is not the intent of this thesis to tackle this rather large problem, but rather to illustrate the possible impact of transition from lower to upper regime on the flow depth. The hypothesis presented above is considered reasonable for this purpose.

E.5.2 Sediment Concentration

Verification of the concentration equation (E.2) was accomplished indirectly using Figure 6.5 and Table 6.3 in Brownlie (1981a). Review of Brownlie (1981a) revealed that observed velocities were employed in testing equation (E.2), as opposed to calculated velocities. Five sets of field data were chosen for verification purposes, as follows:

1. Middle Loup River at Dunning, Nebraska (Hubbel and Matejka, 1959)
2. Niobrara River near Cody, Nebraska (Colby and Hembree, 1955)
3. Rio Grande River near Bernallilo, N. M. (Nordin and Beverage, 1965)
4. Rio Grande Conveyance Channel (Culbertson, Scott and Bennett, 1976)
5. Red River (Toffaleti, 1968)

In applying equation (E.2), the suggested value of C_F=1.268 did not reproduce either the graphical or statistical results presented in Brownlie (1981a). Figures E-2, E-3, E-4, E-5 and E-6 illustrate that a range of C_F values are required to reproduce Brownlie's results. Tables E-4, E-5, E-6, E-7 and E-8 reveal that the chosen C_F values reproduce the statistical results from Brownlie (1981a).

TABLE E-4: RATIO OF PREDICTED TO OBSERVED BED MATERIAL CONCENTRATION MIDDLE LOUP RIVER; COMPARISON OF STATISTICAL RESULTS USING BROWN3 TO REPORT KH-R-43A (BROWNLIE, 1981a)

DATA SOURCE	NO. OF DATA	GEOM. MEAN	GEOM. STAND. DEV.	MINIMUM	16 %ILE	MEDIAN	84 %ILE	MAXIMUM	AVE. VALUE
A. KH-R-43A	38	.908	1.453	.504	.625	.854	1.319	2.721	
B. ACTUAL VEL. [CF=1.268]	38	.740	1.459	.411	.507	.697	1.080	2.208	
C. RATIO A./B.		1.226	.996	1.225	1.232	1.225	1.221	1.232	1.227
D. IMPLIED CF [C.*1.268]		1.555	N/A	1.554	1.562	1.554	1.548	1.562	1.556
E. ACTUAL VEL. [CF=1.556]	38	.909	1.459	.505	.623	.855	1.326	2.710	
F. RATIO A./E.		.999	.996	.998	1.003	.998	.995	1.004	1.000

TABLE E-5: RATIO OF PREDICTED TO OBSERVED BED MATERIAL CONCENTRATION NIOBRARA RIVER; COMPARISON OF STATISTICAL RESULTS USING BROWN3 TO REPORT KH-R-43A (BRONWLIE, 1981a)

DATA SOURCE	NO. OF DATA	GEOM. MEAN	GEOM. STAND. DEV.	MINIMUM	16 %ILE	MEDIAN	84 %ILE	MAXIMUM	AVE. VALUE
A. KH-R-43A	40	1.230	1.295	.706	.950	1.210	1.592	2.518	
B. ACTUAL VEL. [CF=1.268]	40	.989	1.305	.567	.758	.962	1.290	1.992	
C. RATIO A./B.		1.244	.992	1.246	1.254	1.258	1.234	1.264	1.253
D. IMPLIED CF [C.*1.268]		1.577	N/A	1.580	1.590	1.595	1.564	1.603	1.589
E. ACTUAL VEL. [CF=1.5888]	40	1.239	1.305	.710	.949	1.205	1.617	2.496	
F. RATIO A./E.		.993	.992	.994	1.001	1.004	.985	1.009	1.000

TABLE E-6: RATIO OF PREDICTED TO OBSERVED BED MATERIAL CONCENTRATION RIO GRANDE CONVEYANCE CHANNEL; COMPARISON OF STATISTICAL RESULTS TO REPORT KH-R-43A (BROWNLIE, 1981a)

DATA SOURCE	NO. OF DATA	GEOM. MEAN	GEOM. STAND. DEV.	MINIMUM	16 %ILE	MEDIAN	84 %ILE	MAXIMUM	AVE. VALUE
A. KH-R-43A	8	.890	1.501	.358	.593	.886	1.336	1.661	
B. ACTUAL VEL. [CF=1.268]	8	.690	1.539	.281	.448	.686	1.063	1.288	
C. RATIO A./B.		1.289	.975	1.276	1.322	1.292	1.257	1.290	1.287
D. IMPLIED CF [C.*1.268]		1.635	N/A	1.618	1.677	1.639	1.594	1.635	1.632
E. ACTUAL VEL. [CF=1.5562]	8	.888	1.539	.361	.577	.882	1.368	1.657	
F. RATIO A./E.		1.002	.975	.992	1.028	1.004	.977	1.002	1.000

TABLE E-7: RATIO OF PREDICTED TO OBSERVED BED MATERIAL CONCENTRATION RIO GRANDE RIVER; COMPARISON OF STATISTICAL RESULTS USING BROWN3 TO REPORT KH-R-43A (BROWNLIE, 1981A)

DATA SOURCE	NO. OF DATA	GEOM. MEAN	GEOM. STAND. DEV.	MINIMUM	16 %ILE	MEDIAN	84 %ILE	MAXIMUM	AVE. VALUE
A. KH-R-43A	50	.608	1.917	.112	.317	.657	1.166	2.005	
B. ACTUAL VEL. [CF=1.268]	50	.480	1.922	.091	.250	.522	.923	1.558	
C. RATIO A./B.		1.266	.998	1.235	1.268	1.258	1.263	1.287	1.261
D. IMPLIED CF		1.605	N/A	1.566	1.608	1.595	1.602	1.632	1.599
E. ACTUAL VEL. [CF=1.5995]	50	.606	1.922	.114	.315	.659	1.164	1.965	
F. RATIO A./E.		1.003	.998	.979	1.005	.997	1.001	1.020	1.000

TABLE E-8: RATIO OF PREDICTED TO OBSERVED BED MATERIAL CONCENTRATION RED RIVER; COMPARISON OF STATISTICAL RESULTS USING BROWN3 TO REPORT KH-R-43A (BROWNLIE, 1981a)

DATA SOURCE	NO. OF DATA	GEOM. MEAN	GEOM. STAND. DEV.	MINIMUM	16 %ILE	MEDIAN	84 %ILE	MAXIMUM	AVE. VALUE
A. KH-R-43A	29	.642	1.394	.332	.461	.622	.895	1.370	
B. ACTUAL VEL. [CF=1.268]	29	.470	1.396	.244	.337	.457	.656	.995	
C. RATIO A./B.		1.366	.999	1.363	1.369	1.362	1.364	1.377	1.367
D. IMPLIED CF		1.732	N/A	1.729	1.735	1.727	1.730	1.747	1.733
E. ACTUAL VEL. [CF=1.7334]	29	.643	1.396	.333	.460	.624	.897	1.360	
F. RATIO A./E.		.999	.999	.997	1.001	.996	.998	1.008	1.000

Table E-9 summarizes the variance in the implied C_F values that were required to reproduce the statistics given in Brownlie (1981a).

TABLE E-9: SUMMARY OF IMPLIED C_F VALUES FOR EQUATION E.4

River	Implied C_F
Middle Loup R.	1.5662
Niobrara R.	1.5888
Rio Grande R.	1.5995
Rio Grande Conv.	1.6318
Red R.	1.7334

Sediment concentration could be calculated by using the equations for flow depth prediction rather than the observed velocity as reported in Brownlie (1981a). The statistics for the detailed and simplified methods have also been computed using the implied C_F values but are not reported here. Figures E-7, E-8, E-9, E-10, and E-11 show the output from these calculations allowing direct comparison to Figures E-2 to E-6. The general slope and position of the observed sediment concentration data are relatively consistent with the slope and position of the data from the detailed and simple methods. However the statistics revealed that the method is not particular accurate in predicting actual singular values. Accordingly where actual velocity measurements exist, they are preferred input for equation (E.2).

E.6 RESULTS

For the purposes of this thesis it was decided to employ a value of $C_F = 1.6$ for use in equation (E.2). This value closely follows four of the five sets of field data presented. Since it is the intent here to investigate the transitional phenomena the choice of C_F is not critical, although the absolute values of the width:depth coefficients were impacted by the choice of C_F.

Solutions to the Brownlie (1981a) sediment transport and flow depth equations were calculated, using the simplified method, for the following unit discharges, q and bed material concentrations, X_{bm} and sediment sizes, D_{50}:

q = .01, .05, .1, .5, 1, 2, 5, 10, 30 m^3/s, and;

X_{bm} = 1, 10 , 20, 40, 100, 200, 400, 800, 1400, 2000, 4000, and 8000 ppm, and;

D_{50} = .1, .3, .5, 1, 1.5, and 2 mm

Furthermore, the following assumptions were made regarding the sediment and fluid properties:

T = 20°C; $o_g = 1.6$; specific density = 2.65; g = 9.81 m^2/s

Figures E-12 to E-23 present the results of the analysis. The data include a full range of lower, transitional and upper regime flow, with some equi-concentration lines including data from all three regimes. The deviations of the results from straight lines are

generally due to Brownlie's overiding criteria for upper regime, given by S > .006, which creates a slight discontinuity in the relations.

The results from *Brown3* were fit to regression equations of the form (E.15) and (E.16).

$$\log_{10}(d) = n * \log_{10}(q) + \log_{10}(K') \quad .....(E.15)$$

$$\log_{10}(S) = Sn * \log_{10}(q) + \log_{10}(SK') \quad .....(E.16)$$

Figures E-32 to E-39 are typical results of the regression analysis of $\log_{10}(q)$ versus $\log_{10}(d)$ for several sediment size and concentration combinations. The correlation coefficients (r^2) for equation (E.15) were above 0.98. Figures E-36 to E-39 illustrate the impact of the transition between flow regimes. Tables E-9 to E-12 present the pertinent results of the regression analysis. The tables have been labelled to illustrate the relevent flow regime conditions pertaining to any given combination of D_{50} and X_{bm}. Figures E-24 to E-31 illustrate the relationship of the regression analysis exponents and coefficients to the sediment concentration and size.

TABLE E-9: n VERSUS D_{50} AND X

		Width:Depth Exponent, n					
	D_{50}(mm)	.1	.3	.5	1	1.5	2
	X(PPM)						
	1	.833	.827	.829	.838	.845	.849
LOWER	10	.805	.799	.801	.811	.818	.823
REGIME	20	.797	.792	.794	.802	.809	.814
ONLY	40	.790	.785	.787	.795	.801	.806
	100	.781	.777	.779	.785	.791	.795
	200	.776	.773	.774	.779	.784	.796
LOWER	400	.767	.768	.769	.774	.787	.790
TRANS	800	.744	.757	.762	.779	.783	.791
& UPPER	1400	.733	.738	.756	.766	.779	.788
REGIMES	2000	.737	.729	.745	.762	.773	.778
	4000	.748	.745	.746	.754	.757	.760
UPPER	8000	.750	.749	.749	.751	.753	.755

TABLE E-10: K' VERSUS D_{50} AND X

	D_{50}(mm) / X(PPM)	Width:Depth Coefficient, K' .1	.3	.5	1	1.5	2
	1	2.848	2.631	2.422	2.013	1.768	1.608
LOWER	10	2.263	2.049	1.899	1.633	1.470	1.359
REGIME	20	2.072	1.865	1.732	1.503	1.363	1.267
ONLY	40	1.882	1.686	1.568	1.372	1.253	1.171
	100	1.639	1.460	1.361	1.203	1.107	1.041
	200	1.466	1.302	1.215	1.080	1.000	.935
LOWER	400	1.295	1.155	1.079	.965	.887	.841
TRANS	800	1.083	1.003	.948	.847	.792	.743
& UPPER	1400	.917	.860	.815	.750	.698	.661
REGIMES	2000	.831	.773	.731	.671	.630	.605
	4000	.703	.632	.597	.545	.517	.494
UPPER	8000	.608	.541	.510	.467	.442	.425

TABLE E-11: Sn VERSUS D_{50} AND X

	D_{50}(MM) / X(PPM)	Slope Exponent, Sn .1	.3	.5	1	1.5	2
	1	-.705	-.682	-.690	-.726	-.750	-.766
LOWER	10	-.594	-.573	-.580	-.616	-.644	-.663
REGIME	20	-.563	-.543	-.550	-.584	-.610	-.630
ONLY	40	-.534	-.516	-.522	-.553	-.578	-.596
	100	-.501	-.486	-.491	-.517	-.539	-.555
	200	-.480	-.467	-.471	-.494	-.513	-.528
LOWER	400	-.475	-.451	-.454	-.473	-.490	-.503
TRANS	800	-.531	-.465	-.452	-.457	-.470	-.481
& UPPER	1400	-.551	-.517	-.495	-.482	-.482	-.484
REGIMES	2000	-.523	-.537	-.526	-.514	-.510	-.509
	4000	-.457	-.460	-.458	-.493	-.498	-.501
UPPER	8000	-.435	-.431	-.433	-.440	-.447	-.452

TABLE E-12: SK' (*1000) VERSUS D_{50} AND X

	D_{50}(MM) / X(PPM)	Coefficient, SK' .1	.3	.5	1	1.5	2
	1	.011	.016	.023	.050	.086	.128
LOWER	10	.027	.043	.060	.114	.178	.248
REGIME	20	.038	.062	.086	.158	.240	.326
ONLY	40	.055	.092	.127	.226	.334	.445
	100	.095	.162	.222	.380	.543	.678
	200	.147	.255	.347	.580	.812	.995
LOWER	400	.226	.408	.554	.906	1.189	1.503
TRANS	800	.294	.624	.878	1.374	1.862	2.203
& UPPER	1400	.342	.771	1.084	1.858	2.427	2.891
REGIMES	2000	.390	.866	1.273	2.005	2.642	3.357
	4000	.563	1.110	1.542	2.564	3.498	4.319
UPPER	8000	.903	1.726	2.367	3.743	4.946	6.049

APPENDIX F

RELATIONSHIP OF WIDTH:DEPTH EQUATION TO SEDIMENT TRANSPORT

The purpose of this appendix is to demonstrate the relationship between the sediment transport and the width:depth relations.

First, derive the units of the variable C_{bm} in equation (6.1):

$$q_{bm} = C_{bm}\, p\, V^m \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.1)$$

units of q_{bm} = mass(W)/time(t)/length(l)
units of $p = W / l^3$
units of $u = l / t$

Assuming that the units of m are dimensionless, leads to the following:

$$q_{bm} = W/t/l = C_{bm} * p * V^m = \text{units of } C_{bm} * W/l^3 * l^m/t^m$$

therefore the units of $C_{bm} = \underline{t^{(m-1)} / l^{(m-2)}}$

Next, develop a sediment transport equation from the width:depth equation as follows.

Step 1 First consider the continuity equations:

$$q = V\, d \quad \ldots(F.1) \qquad q_{bm} = X_{bm}/10^6\, q\, p \quad \ldots(F.2)$$

where units of X_{bm} = ppm
q = m^2/s
q_{bm} = $kg/s/m$
p = kg/m^3

Step 2 Introduce the width:depth equation representing regime conditions at the reference flow, X_{bm},

$$d = K'\, q^{\,n} \qquad \ldots\ldots\ldots (3.28)$$

$$d = K'\, (V\, d)^n$$

solving for d get:

$$d = K'^{\,(1/1-n)}\, (V)^{(n/1-n)}$$

and substituting into (F.2) get:

$$q_{bm} = X_{bm}/10^6\, p\, K'^m\, V^m \qquad \ldots\ldots\ldots (F.3)$$

where $m = 1/(1-n)$

equation (F.3) can be arranged to fit equation (4.8c)

Equation (F.3) and (6.1) are equivalent, and comparison results in:

$$C_{bm} = X_{bm}/10^6\, K'^m \qquad \ldots\ldots\ldots\ldots\ldots (F.4)$$

Check the units of C_{bm} as follows:

$$K'^m = t^{n\,m} / l^{\,(2n-1)m} = \underline{t^{(m-1)} / l^{(m-2)}} = \text{UNITS } C_{bm} \text{ AS ABOVE}$$