OPTIMAL TWK-NOP DETERMINATION IN JOB SHOP SCHEDULING

by

Margaret Ley Shen Sim B.Sc(Hons)

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Department of Actuarial and Management Sciences
University of Manitoba
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ABSTRACT

This dissertation is concerned with the study of the operating characteristics of the combined TWK-NOP due date assignment method in a dynamic job shop. The due date for each job is established by adding multipliers of the job's total processing time and number of operations to its arrival time at the shop. Penalty costs will be incurred if the shop quotes excessively long due dates compared with those of its competitors. The objective is to minimize the expected aggregate cost per job subject to restrictive assumptions on the priority discipline and the penalty functions. This aggregate cost includes two kinds of opportunity costs for each job; (i) the cost of quoting long due dates, and (ii) the cost of missed due dates. Both of these costs are opportunity costs because they represent possible gain of profit should an alternative policy be adopted. Combined together, they constitute a total cost function, the expected value of which is to be minimized by optimal choice of the multipliers representing the processing time α^* and number of operations β^*. We first used an analytical approach to explore two cost models, a general and a special one, for examining TWK-NOP and found that the linear cost model under TWK-NOP due dates is computationally intractable, so that any attempt to tackle it would fail analytically. Hence, simulation experiments were carried out to find the optimal multipliers, α^* and β^*.

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Chapter I
INTRODUCTION

Scheduling is one of the most challenging problems of production or operations management. It concerns activities that are involved in an operating system. Typically, a schedule shows a set of tasks to be carried out within a specified set of criteria. These criteria may involve job due dates, routings with standardized processing times, flexibility of due dates and others. For instance, at a shop floor level, the scheduling problem is concerned with organizing the flow and sequencing of raw materials for processing through the various assembly work centres. Ideally, what is required is an algorithm that would, within all the myriad of constraints on the amount and availability of human labor and monetary resources, establish an optimal overall schedule for the specified planning horizon.

The work "optimal" here, strictly speaking, would mean that the algorithm will generate either the maximum profit or incur the minimum cost. To attain this ideal is a formidably complex global problem to solve which, current research suggests, may forever be beyond our capability. It is the recognition of this complexity that has forced researchers in this field to pursue a piecemeal approach in which the constituent parts of the problem were considered separately under the generic titles of aggregate scheduling and machine scheduling.

1.1 Machine Scheduling

Machine scheduling involves procedures for setting processing priorities which specify the manufacturing timetable. In general, a shop is a place where jobs have to be performed by several types of work have to be routed through a number of machines. The term job may be applied to describe a single item or a batch of items that require processing on the machines. Many of the jobs will be waiting at work stations to be worked on while work is progressing on other jobs. The shop configuration depicted in Figure 1 indicates an example of the flow pattern of jobs. A job shop can be visualized as a set of work stations with waiting lines, or queues, in front of them and a set of random flow paths connecting the work stations. The two conventional approaches to the shop scheduling problem are (i) flow shop scheduling and (ii) permutation scheduling. In the former incoming jobs may require any machine in the shop and may visit the same machine more than once, as illustrated in Figure 2; in the latter the same job sequence may be in force on every machine.

A rudimentary assumption of machine scheduling is that no item can be processed by two machines at the same time. In addition, the routings of items through the machines are predetermined by the operations that are required to be performed. One of the special cases of the scheduling problem is one where a single machine is used. The fundamental features of a single machine are that it can demonstrate a variety of scheduling topics in a adaptable model and make provision for the examination of alternative performance measures and solution techniques. To help us to comprehend the behaviour of a multifarious system, considering single machine problems first enables us to capture the operations of its components as rudimentary constituents in a larger scheduling problem. Often, the embedded single machine problem is solved independently and then fitted into the larger problem. For instance, a bottleneck stage in a multiple-operation process can be considered as a single machine which, in turn determines the characteristics of the schedule as a whole. Essentially, this simplified approach can be extended to multiple-resource systems as one aggregated facility.

Particularly important results arising from the study of the single machine scheduling problem are the shortest processing time (SPT) rule and the earliest due date (EDD) rule. The SPT requires the jobs to be scheduled in order of increasing processing time, beginning with the job with the shortest time. Operating this rule will minimize average completion times, which minimize average waiting time and flowtime. The EDD rule, in which jobs are scheduled in order of successive due dates beginning with the job with the earliest due date, minimizes maximum tardiness. Tardiness is defined as positive lateness, that is, completion time minus due date. Evidently, the groundwork to be done concerns single machine problems.

Nonetheless, the study of single machine problems has its limitations. The model tends to concentrate on the macro-performance of the system. It overlooks the micro-performance of the individual resources. Hence, it is virtually impossible to distinguish individual resource performance. In addition, despite its simplicity, the scheduling of a single aggregated facility is by no means an easy problem. In contrast to single resource scheduling theory, multiple-resource scheduling is the study of constructing schedules of sequencing the resources for a set of assignments in order to ensure the termination of all assignments within specified time. The degree of complexity is magnified because of the allocation of resources. This can be demonstrated as follows. For instance, scheduling ten tasks on one resource can be done in 10!. However, with one additional resource of the same type, the number of possible permutations becomes $(10!)^2$, a net increase of 3,628,800. By comparison, we see that the overall complexity of the multiple resource scheduling problem is magnified exponentially as the number of resources, or tasks, or both simultaneously, increases. One of the advantages of having more than one machine of each type accessible that we may avoid being held up owing to failure of a machine. Other advantages include flexible rates of production and routing, the unlikeliness of bottleneck situations and others. The major issue of multiple-resource scheduling is the determination of the processing sequence of assignments to each resource in order to minimize the total cost of processing.

1.2 Aggregate Production Scheduling

The aggregate production scheduling problem can be defined as establishing a minimum total cost production schedule that will meet the forecast demand within the planning horizon. The solution lies in striking a balance between the many conflicting requirements.

For instance, the current demand should not exceed the sum of current production and inventory, if any, carried over from an earlier period. The former entails setup costs whereas the latter entails inventory holding costs. Given that there are available alternatives to meet demand each month in the planning horizon and the accompany costs are linear, the aggregate scheduling problem can be formulated on as linear program. Furthermore, the optimal solution attained is within the available capacity and resource constraints. In fact, besides the linear programming, linear decision rule and computer search are the most frequently used tools for aggregate planning.

Linear programming, because of its efficiency of computer codes and ability to handle large scale problems, is used extensively for generating optimum aggregate production schedules. To illustrate, Hanssmann and Hess (1960) apply linear programming to determine the optimum aggregate production and work-force decisions for minimizing total costs within a specified planning horizon. Their research is based on the well-known essential study of Holt et al. (1960), in which they have given a method of simultaneously smoothing aggregate production and work-force requirements over any specific future planning horizon. Although they assumed that the various cost components were in quadratic, in the process of using derivatives to determine the minimum costs, the assumption of quadratics reduced the decision rules to linear form.

Attempts have been made to further develop the method introduced by Holt et al. For modelling the simple decision rules, one has to know (i) the size of the work force in the foregoing period, (ii) the aggregate inventory remaining at the end of that period, and (iii) the aggregate demand forecasted for each period in the planning horizon. Therefore, as a rule, the decision rules can be employed on a continuous basis and modifications can be made when changes in costs arise which in turn require reevaluation of the coefficients.

1.3 Classification of Due Date Assignment Methods

Panwalkar and et al. (1982) have approached the job scheduling and sequencing problem from two directions, the shop that will process the job and the customer placing the order for the job. The objective of the former is the minimization of one or more cost factors; that of the latter is related to the due date. In a job shop production system, the due date for delivery, must be determined for each job, prior to processing. For the last thirty years, research on scheduling concerning due dates has been growing in popularity among researchers. Much of the early research focuses on due dates that are either generated arbitrarily or based on job characteristics. Conway and et al. (1965) were probably the first to study due date assignment in a systematic manner. They have compared the effectiveness of some common due date assignment methods by means of a number of performance measures. In the early years, due date scheduling research to assess the relative effectiveness of various due date assignment rules was undertaken primarily using computer simulation. A comprehensive review on scheduling research involving due date determination decisions is provided by Cheng and Gupta (1989). Other pertinent surveys are presented by Ragatz and Mabert (1984) and Sen and Gupta (1984). There are many decision functions involved in due date assignment, in this paper, we shall classify the scheduling in according to both dynamic and static scheduling problems, with determination of due date decision variables as our primary interest.

In a dynamic production setting, jobs continuously arrive for processing. As for static production settings, the jobs available for processing are coming in at a fixed rate. For ease of understanding, a brief classification of the due date assignment procedures are discussed under the following categories. Tables 1, 2 and 3 provide the notational form of the assignment methods.

1. Exogenous Assignment Procedures

Externally-imposed due dates are set by some independent external agency and are announced upon arrival of the job.

There are two types of due date decisions that could be applied to the assignment methods.

(i) Constant allowance due date (CON)

The exact same flow allowance is allotted by all jobs.

(ii) Random allowance due date (RAN)

Each job is assigned a flow allowance at random.

These methods disregard all information about arrival of jobs, status of jobs, or the framework of the shop. The CON due dates are representative of common practice, in which a standard lead time is quoted, whereas RAN is intended to represent arbitrary deadlines assigned by an external agency.

2. Endogenous Assignment Procedures

Internally-set due dates are established by the scheduler as each job arrives (Ragatz and Mabert, 1985). The scheduler determines the due date for each job on the basis of various factors, in particular the job characteristics and the status of other jobs in the facility. As each job arrives, the scheduler estimates the job flowtime for each job and sets the due date according to the job characteristics and shop status information.

Some methods of due date assignment, grouped according to job characteristics, are as follows:

(i) Total-work-content due date (TWK)

Due dates are based on total work content.

(ii) Slack due date (SLK)

Due dates are based on the time remaining before the job due date less all remaining processing time for the job.

(iii) Number-of-operations due date (NOP)

Due dates are based on the number of operations to be performed on the job.

TWK method is commonly used for jobs needing more processing time, whereas NOP provides a greater allowance for jobs needing a large number of operations. These two methods provide a reasonable assignment and attainable due dates.

Another set of due date assignment methods under the endogenous category relates to shop status. Many researchers have indicated that due date assignment based on shop status information provides more attainable due dates than methods based solely on job characteristics.

(iv) Job in queue (JIQ)

Due dates are based on information of the queue lengths in the system.

(v) Job in system (JIS)

Due dates are based on information on the number of jobs in the system.

(vi) Processing-time-plus-wait (PPW)

Due dates are based on information on the waiting times in the system.

For each of the above categories, scheduling takes place by assigning a priority to each job. The relevant priority rules are the following:

(i) First come, first served (FCFS)

Job priorities are assigned in the order in which they arrive.

(ii) Due date (DDATE)

Jobs are scheduled in order of their due dates, that is, a job with earlier due date is given priority over a job with a later due date.

(iii) Operation due date (OPNDD)

At the time the job arrives at the shop, the allowance is divided equally among the operations and an internal due date is assigned to each operation. The job with the earlier operation due date is given priority.

(iv) Slack time (SLACK)

The job with the lesser slack time is given priority.

(v) Slack per operation (S/OPN)

The slack time of each job is divided by the number of operations remaining and the job with the smallest ratio has priority.

(vi) Shortest processing time (SPT)

The job with the shortest processing time for the imminent operation is given priority.

1.4 Dimensions of Scheduling Problems

The essential elements in identifying a specific scheduling problem are production, objectives, constraints, and decision variables. Considering a scheduling problem which consists of a set of jobs and machines, denoted respectively as $J = \{J_i | i = 1, 2, ..., n\}$ and $M = \{M_j | j = 1, 2, ..., m\}$. In addition, schedules are generally evaluated by various performance measures that require information about job parameters such as cost functions, the number of machines, set-up times of machines, due dates of jobs, release (ready) times, processing times and the number of early or late jobs.

Each job J_i is characterized by the processing time, p_i, the release time, r_i, and due date, d_i. The release time r_i is the point in time at which job $i \in J$ is available for processing. In static production settings, it is assumed that release times are known and fixed. As a result of scheduling decisions, job i will be assigned a completion time. We define C_i the time at which the processing of the last operation is finished and F_i, the flowtime of job as the amount of time job i spends in the system from order to completion. These variables, describing the solution to a scheduling problem, are functions of $W_{i,j}$, defined as the waiting time preceding the jth operation of job i. The total waiting time for all operations on job i is

$$W_i = \sum_{j=1}^{n} W_{i,j}. \tag{1.1}$$

Therefore, the completion time and flowtime of job i can be expressed as

$$C_i = r_i + p_i + W_i \tag{1.2}$$

and

$$F_i = p_i + W_i = C_i - r_i, \tag{1.3}$$

respectively. In final analysis, every comparison of schedules is based on a comparison of different sets of Wi, j, the quality of a schedule is completely determined by the values of the Wi, j.

To compare, for a given scheduling problem, the actual completion time with the desired completion time, we introduce a new set of variables, each describing one aspect of the solution in a different way. The following variables that describe the solution to a scheduling problem are different ways of comparing the actual completion time with the desired completion time.

In this category, each job has been assigned a due date, d_i, by some external agency. The objective of sequencing is to determine for each job its completion time, C_i. We define the lateness of job i, L_i as the difference between its completion time and due date: $L_i = C_i - d_i$; note that this may be positive, negative or zero. If a job is completed after its due date, that is if $L_i > 0$ and $C_i > d_i$, the job is said to be tardy. T_i, tardiness of job i is the lateness of job i if it fails to meet its due date, and zero otherwise: $T_i = \max\{L_i, 0\}$, $i \in J$. On the other hand, if the completion of the job occurs before its due date; $d_i > C_i$, the job is said to be early and the earliness of the job, E_i, is defined as $E_i = \{0, -L_i\}$.

In general, schedules are evaluated by aggregate quantities that involve information about all jobs. According to Baker (1974), this results in one-dimensional performance measures. Given that n jobs are to be scheduled, the aggregate performance measures are defined as follows:

Mean flowtime: $$\bar{F} = \frac{1}{n}\sum_{j=1}^{n} F_j, \tag{1.4}$$

Mean tardiness: $$\bar{T} = \frac{1}{n}\sum_{j=1}^{n} T_j, \tag{1.5}$$

Maximum flowtime: $$F_{max} = \max_{1 \le j \le n} \{F_j\}, \tag{1.6}$$

Maximum tardiness: $$T_{max} = \max_{1 \le j \le n} \{T_j\}. \tag{1.7}$$

1.5 Cost Based Criteria Used in Job Shop Scheduling

Interest in costs or penalties incurred by early completion of jobs as well as late has accelerated in recent years. The main objective in many industrial systems is to minimize total costs and this objective is considered mostly in recent studies, for example, Aggarwal and McCarl (1974), Berry (1972), Jones (1973), Shue and Smith (1978) and Ulgen (1979). A recent and complete review of this literature is provided by Baker and Scudder (1990). Literature reviews on this topic include costs such as

(i) Cost of idle machines.

These are out-of-pocket costs of the workers standing by and opportunity costs of business foregone.

(ii) Costs of carrying work-in-process inventory.

These costs are invariant with respect to due date coefficients and dispatching rules.

(iii) Costs of long promise.

This cost represents the potential loss of sales associated with quoting long delivery dates.

(iv) Costs of missed due dates.

These are associated with jobs that cannot be finished on their assigned due date. Costs of tardiness and earliness are included in the more recent discussions.

(a) Tardiness cost.

This cost is experienced each time an order is delivered after its due date.

(b) Earliness cost.

This cost is associated with finished goods inventory carrying costs when jobs cannot be shipped out at time of completion.

There are other costs based on criteria such as finished goods investments, storage costs of finished goods, average $ days of queue, and setup costs. The most frequently used criteria in simulation and analytical studies of stochastic-dynamic shops are based on job completion times, in-process inventory, jobs-utilization due dates and costs.

1.6 Queueing Theory

A methodology commonly utilized to investigate scheduling problems is queueing theory. Conway et al. (1967) have proposed valuable queueing models that provide a more parsimonious approach to the general job shop scheduling problems. One of the main decision factors involved in the dynamic job shop scheduling is the dispatching rule. Scheduling includes selection of a dispatching rule, in conjunction with a due date decision to ensure maximum performance, given the shop load condition. The first come first served (FCFS) rule is often a convenient and natural selection discipline of queueing theory. In addition, its simplicity contributes to a norm for the comparison of other disciplines such as SPT, MWKR, LWKR, LPT, LCFS and RANDOM. Conway et al. (1967) use the Laplace transform of the distribution of flowtime under FCFS. Based on this Laplace transform, Cheng (1985c) further broadened the study of FCFS an provided an ingenious technique for determining the precise values for mean and standard deviation of job flowtime. These results are often embedded as a secondary development for other research. Various methods have in this respect been unsuccessfully tried, for instance, an algebraic manipulation by Miyazaki (1981) only provided a closer estimation of standard deviation of job flowtime.

Queueing theory is intended to provide the expected queueing schemes for the system and to determine the expected time of an average job spent in the system. This objective differs prominently from the traditional goal of scheduling theory which is to determine the most suitable processing sequence for the set of assignments. A parallel can be found in several studies concerning queueing systems with relaxed preemption assumptions. Preemption is said to occur when processing is inserted into the sequence before its turn, so as to allow higher priority jobs to preempt lower priority jobs. However, this restricts the liberty of the scheduler to choose the dispatching discipline. Therefore, it poses an obstacle to study scheduling problems as queueing problems.

1.7 Organization of the Thesis

The balance of this dissertation is organized as follows. The next chapter reviews previous research addressing due date assignment method, test performance of various dispatching rules, possible methods of solution, sequencing with earliness and tardiness penalties with topics classified according to different performance measures and finally, investigation of cost-based rules for job shop scheduling. We have introduced some classical methodologies to solve for optimal due dates. In addition, an introduction of simulation is given. Computer simulation has been used intensively for research regarding due date setting. The compatibility of simulation to incorporate realistic job shop operating characteristics are profound.

The analytical approach in modelling a general cost model for examining the operating characteristics, its structure and features of TWK-NOP due date assignment method are explored in Chapter 3. The simplicity of procedure for assigning due date demonstrates the customary practice of designating due dates in industries where manufacturers are accountable for inveterating the due date. If shop quotes excessively long due dates measured up to its competitors and cannot complete the jobs exactly on their assigned due date, penalty costs will be incurred. We extend the results of Cheng (1985b), provide a detailed mathematical cost-based analysis which is subsequently validated by comparison to a set of simulation experiments.

The result of an extensive computational comparison is presented in Chapter 4. We generalized the method to simple networks of multiple workcentres, and test it on a hypothetical production network. In the last chapter, concluding remarks and some future directions will be highlighted.

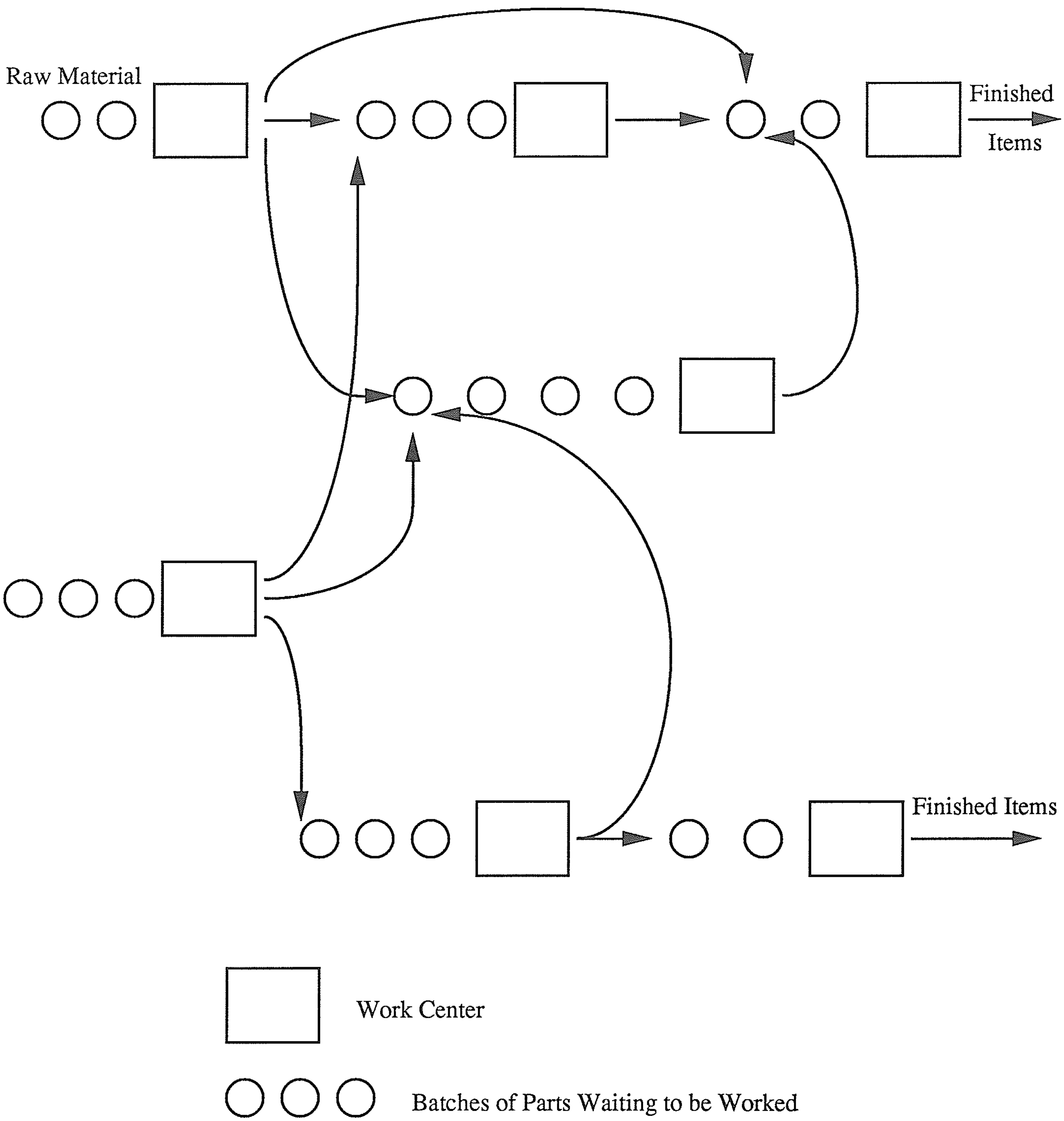

Figure 1 The Shop Configuration (Dilworth, 1989)

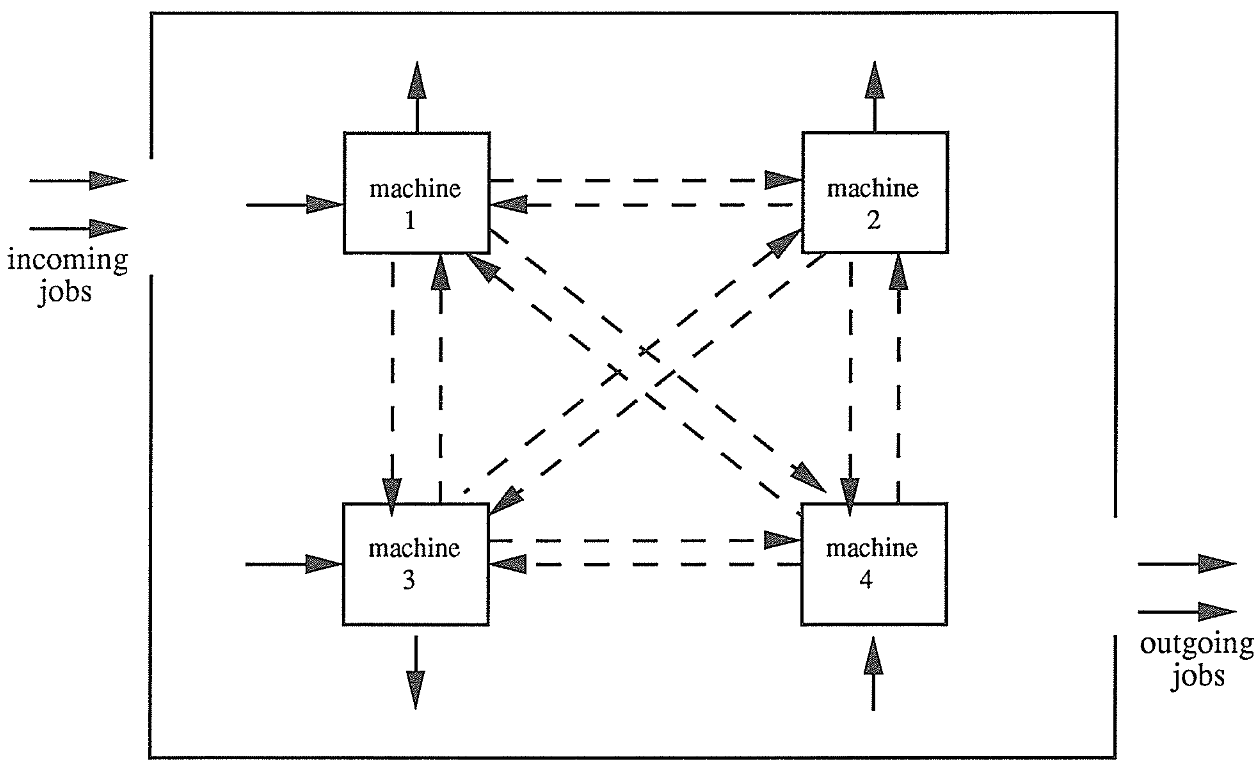

Figure 2 Diagram of the Flow Pattern of Jobs (Dar-El and Wysk, 1982)

Table 1 Notational Form Under Exogenous Category

Due Date Assignment Method	Notation
Constant Allowance Due Date (CON)	$d_i = r_i + k$
Random Allowance Due Date (RAN)	$d_i = r_i + e_i$

where k is a constant and e_i is a random number.

Table 2 Notational Form Under Endogenous Category

Due Date Assignment Method	Notation
Total-Work-Content Due Date (TWK)	$d_i = r_i + kp_i$
Slack Due Date (SLK)	$d_i = r_i + p_i + k$
Number-of-Operations Due Date (NOP)	$d_i = r_i + kn_i$

where n_i is number of operations of job i.

Table 3 Notational Form Under Endogenous Category: Shop Status Information

Let k_1 and k_2 be constants; Q_i is number of jobs in queue at machines job i will visit; D is the mean waiting time in the system; σ_D is the standard deviation of waiting time in the system; J_i is number of jobs in the system when job i arrives, and $a(J_i)$is defined as

$$a(J_i) = \begin{cases} -1 & \text{if } J_i < J - \sigma_J \\ 0 & \text{if } J - \sigma_J < J_i < J + \sigma_J \\ 1 & \text{if } J_i \geq J + \sigma_J. \end{cases}$$

where J and σ_J are the mean and standard deviation of number of jobs in the system respectively and m_j is number of operations of job i.

Due Date Assignment Method	Notation
Job in Queue (JIQ)	$d_i = r_i + k_1p_i + k_2Q_i$
Job in System (JIS)	$d_i = r_i + p_i + D + a(J_i)\sigma_D$
Processing-Time-Plus-Wait (PPW)	$d_i = r_i + p_i + k_1m_j$

Chapter II

REVIEW OF LITERATURE

An intermittent production system or job shop is designed to provide more versatility. In this type of production system the production equipment or work stations are grouped and organized according to the function or the process they execute. The complexity of job shop scheduling, as compared to other production scheduling, has prompted many researchers to study both analytical and heuristic approaches. Analytical approach attempts to quest optimal solutions to job shop scheduling problem. Inexpediently, such approaches are disadvantageous on a more realistic scenario, although more valid theoretically. The scheduling of a versatile, dynamic environments is a member of a class of problems whose optimal solutions are too complex to be tractable (Newman, 1988). Nonetheless, it provides inceptive approximate results which lead to optimal solutions, particularly real size problems. Contrarily, heuristic approach relinquishes optimality but computational wieldable. This approach has been widely employed in a dynamic job shop environment.

2.1 The Dynamic Job Shop Situation

A generic dynamic job shop situation a number of jobs, varying over time is available for processing. There is a perpetual stream of random arrivals and departures of jobs at the shop. For each job, a job file is created which contains information such as job arrival time, routing, due date and processing time of each operation. This information is determined on the arrival of the jobs at which time the due date is either externally or internally, set. The following parameters present a framework for assessing job shop scheduling research.

1. The arrival time distribution.
2. The work flow characteristics.
3. The processing time distribution.
4. The due date distribution.
5. The shop configuration.
6. The shop "rules" (overtime, processing by several machines, etc.).
7. The performance measure(s).
8. How the performance measures are determined.
9 The priority rule selected.
10. The shop load level.

(Dar-El and Wysk, 1982)

The composition of dynamic and stochastic behaviour of jobs arrival in the theoretical model delivers the solutions, hence; acquired more applicability in practical situation. If the dynamic model is complex, computer simulation techniques may become a suitable procedure in that they provide a heuristic optimum among alternative sequencing strategies for the dynamic job shop model. This sequencing decision is termed as dispatching or priority rule. The objectives of scheduling are often multi-dimensional, and aforementioned, there are numerous possible measures of scheduling performance. Generally, dispatching rules, due date assignments, and shop load level are three essential decision variables in the investigation of the job shop scheduling. A basic measure of a shop's performance at turning around orders is the mean job flowtime. This is used as an endeixis of success in acknowledging immediately to customers. It is also a measure of the average work-in-process level is expected to effect a reduction of mean flowtime.

Incorporating SPT as the priority rule (Conway, 1965). Achieving due dates tends to be a more significant factor than minimal shop time. Unfortunately, the study of due date performance is more problematic since the existence of a single, universally-accepted measure of effectiveness on this dimension is unavailable. The literature on scheduling theory and priority dispatching rules recommends several suggestions, such as the proportion of late jobs, the mean tardiness for all jobs and the conditional mean tardiness, to quantify the level of performance. Another concern to be addressed is the non-existence of a single priority rule that governs performance comparisons, as opposed to the case with respect to mean flowtime.

2.2 Review of Due Date Assignment Methods and Test Performance of Various Dispatching Rules

There is no dearth of published literature dealing with due date determination which is the primary factor in testing job shop implementation as well as the performance of various dispatching rules. Extensive surveys of the effect of due date assignment methods on the performance of various dispatching rules are provided by Conway (1965). In addition, he affirms that NOP is the most effective procedure of asssigning due dates at high levels of shop utilization. The success of this process is due to the large proportion of job flowtime spent in waiting for service, and waiting time is proportional to the number of operations of a job. Baker (1984) also attests that NOP yields due dates that are efficient in averting tardiness entirely. The interaction between dispatching rules and methodology of assigning due dates has been a subject of extensive study. A comprehensive treatment of the study can be found in the work of Baker (1984). A set of simulation experiments demonstrates how average flow allowances and due date assignment methods interact with the dispatching rule. The experimental results suggest which combinations are most effective in a scheduling system. Among other results, Baker (1984) proves that TWK is the best due date rule. This implies that due dates should reflect work content.

Eilon and Chowdhury (1976) compare of the two traditional methods of assigning due dates; assigning due date as a function of job characteristics and assigning due date as a function of job characteristics and current shop status. They claim that the second method performs exceedingly better than the first only when used in concurrence with due-date-oriented dispatching rules. Ragatz and Mabert (1985) further corroborate these results through simulation analysis of due date assignment rules. Their study has led to the following conclusions. Dispatching rules used to sequence jobs at work centres influence shop performance. Information regarding work centre congestion along a job's routing is more superior to information concerning general shop conditions. The use of more specific information in forecasting flowtime only contributes to an infinitesimal improvement in performance over other rules that employ more aggregate inputs.

Relatively few studies have incorporated discretionary due dates (see, for example, Eilon and Chowdhury (1976), Elvers (1973) and Weeks (1979)) and rules intended for due date selection have infrequently been developed from normative, analytic results. Baker and Bertrand (1981a) contrast three basic rules, CON, SLK and TWK, and demonstrate that a rule for determining the flow allowance of a job should be based on the job's length. They introduce two workload scenarios and make the observation that the parallelism between the TWK rule and SLK rule is more atypical under random workloads than under controlled workloads. Taking this into account, they conclude that in complex production control systems it might be desirable to develop a strategy for due date selection that depends on the strategy for order releasing, since the latter will affect workload behaviour.

Subsequently, Baker and Bertrand (1981b) propose an improved due date selection rule which operates adequately in association with internally-set deadlines and can easier be adapted to both tight and loose conditions. They have designed a two-level model of such a system. At the level of due date assignment, the average tightness of due dates imposes on the operations constraint. On the other hand, at the level of job scheduling, a priority dispatching rule determines a processing sequence, of which the performance is assessed by average tardiness. There is a conflict between the objectives targeted at the two levels. As for the former, tight due dates are preferred to loose due dates. The reason is that, if tight due dates are achieved, this tends to reinforce more feedback from customer services and to reduce in-process inventory levels. Nonetheless, in the latter case, tight due dates are harder to attain compared to loose due dates. Tight due dates tend to result in more tardiness in job completion and to inhibit scheduling flexibility. The main inference from their study is that, when due dates are extremely tight, few jobs can be completed on time. This scenario depicts the reduction of mean tardiness as being proportional to mean flowtime. It is apparent that this property allows using a flowtime-oriented priority rule, such as SPT. Some attributes of their approach merit being noted. The authors promote the use of analytically based due date assignment rules. The tightness parameters and the configuration of workload-dependent forms are being identified as analytic rather than empirical. The workload-dependent forms of due date assignment rules are prevalent.

In all such studies (for example, Conway et al. (1967), Eilon and Chowdhury (1976), Adam et al. (1978) and Weeks (1979)), aggregate, or non-time-phased, workload information is used to strike a balance between the predicted flowtime of new jobs and the actual workload in the shop. The new approach introduced by Bertrand (1983a) reaffirms the established way, he advocates the use of time-phased workload information for determining job schedules and setting attainable job due dates. The relevance of time-phased workload information was recognized at a very early stage in industrial practice. The contemporary work of Adam and Surkis (1977), Adam et al. (1978), Heard (1976) and Weeks (1979) have substantiated the significance of the workload in the shop for predicting job flowtimes. Mean and variance of lateness relative to the internal job due dates are employed as a measure of due date performance. The due date assignment rule should generate a constant mean lateness, independent of the level of the shop load and a small variance of lateness. One of the managerial implications of the research is that the reliability delivery to customers can be controlled by setting the external due date equal to the internal due date plus the sum of mean lateness and a safety time related to the variance of lateness. Consequently, a small variance of lateness reduces the quoted external job lead times. This implies that the reliability of delivery is increased. Bertrand (1983b) performs another empirical study exploring the due date performance of job shop control systems, which based job due dates on a time-phased representation of the workload and the machine capacity in the shop. The performance is measured by the mean and the standard deviation of the lateness. Two parameters, (i) a minimum allowance for waiting and (ii) a maximum fraction of the available capacity allowed for loading, are used to vary the functioning of the due date assignment system. Simulation experiments are used to evaluate the performance of the assignment system.

2.3 Review of the Possible Methods of Solution

Occasionally, the scheduler will encounter a situation where he is unable to find the best schedule for the problem within a prescribed time. Instead of abandoning all analysis and selecting a schedule at random, he should use his knowledge and experience to find a schedule which, if not ideal, may at least be expected to perform better than average (French, 1982). This type of approach, known as heuristic approach although optimality is sacrificed, is still feasible since it is found empirically to perform fairly well. Among the techniques for heuristic approach are simulation, statistic, linear programming, stochastic processes, network analysis, decision theory, queueing theory and inventory control (Schriber, 1990).

As forementioned, simulation has been widely employed in dynamic scheduling. This is because simulation techniques have a comprehensive objective of using the scientific method to help solve complex decision problems in organizational settings. Generally, simulation is a process in which experiments are conducted on a computer model of a system with the intention to determine, in its composition or surroundings, how the system would react to different circumstances.

Simulation is not a panacea. Although it is very general and ranks very well among other operation research techniques in terms of importance, it has its shortcomings. Among them are failure to produce exact results, difficulty to generalize empirical results and failure to optimize. It is not exceptional for simulation experiments to run millions of simulated jobs through the model. In such cases, simulation can be prohibitively time consuming is evidence that simulation, and similar approaches relying on trial and error often lack in efficiency. The fact that reasonably accurate estimates of the parameters may only be achieved when millions of simulation runs are carried out, at the cost of hours and hours of CPU time. On this account, it is expedient to consider some analytical approaches.

2.3.1 Analytical Approach

The analytical approach as a way of providing optimal solutions to job shop scheduling problems is a powerful tool. Often, it is used for initial approximations and for providing optimal solutions for the simpler cases of real size problems. In scheduling, different analytical approaches have been proposed. The most commonly used are the following:

- (i) Network Analysis
- (ii) Branch and Bound Method
- (iii) Hierarchical Approach
- (iv) Dynamic Programming Approach
- (v) Integer Programming Approach
- (vi) Alternate Routing Combination Approach

A considerable amount of research on the static single machine problem has been solved analytically with multifarious degree of success. Accounts of some of this research are provided in Smith (1956), Jackson (1955), Conway et al. (1967), Moore (1968), Sturm (1970), Maxwell (1970), Baker (1974) and Hodgson (1977).

Dynamic programming is one of the prime techniques considered in scheduling modelling. In a dynamic job shop environment, a scheduler may be faced with problems in which it may be possible to decompose decisions into smaller components and then recombine the previous decisions in some form to obtain a desired solution. This approach is called multistage problem solving, and dynamic programming is a systematic technique for reaching an answer in problems of this nature (Ravindran et al., 1987). A general dynamic programming scheme has wide applications (see, for example, Rothkopf (1966), Lawler and Moore (1969)). Reinitz (1963) and Heard (1970) perceive shop operations as a Markov process and employ dynamic programming to designate optimal due dates in machine-constrained shops. Dynamic programming has always been limited in its feasibility by the computational complexities of the problems. Hence, the need for other more efficient analytical approaches is imperative. The study by Seidmann and Smith (1981) determines the optimal lead time based on analytical formulation of a dynamic single-machine scheduling problem. Subsequent work on this topic is presented in Cheng (1985). He contrasts the novelty of the aggregate cost per job to the more traditional cost function associated with a multifaceted due date assignment method. Cheng (1983) continues his investigation and considers an analytical model to ascertain the optimal processing time and number of operations multipliers for the TWK and TWK+NOP due date assignment methods, both subject to restrictive assumptions on queue discipline and processing time distribution. He evaluates the disparity between the analytical results on the one hand and experimental results achieved from simulation of a hypothetical job shop under various conditions on the other. Should the disparity be minute, the agreement of the results discloses the cogency of the analytical model. The development of TWK+NOP method exhibits its efficacy in minimizing missed due date costs in a job shop. Miyazaki (1981) postulates an alternative approach by considering a total scheduling system which combines the due date assignment and job sequencing procedures to reduce job tardiness in a job shop. Cheng (1986c) develops an innovative method of assigning due dates in a single machine shop using the SPT dispatching rule. He has also proposed a heuristic approach to obtain the optimal due dates which in turn minimizes the average amount of missed due dates. This is accomplished by simulating, under different job conditions a hypothetical job shop having various processing characteristics. Despite its simplicity, the heuristic approach enables the scheduler to assign precise due dates efficiently.

2.3.2 Computer Simulation Approach

Simulation analysis is not at all specific and, hence, is ultimately capable of modelling almost ultimately any system. Since simulation permits the user to determine in advance which rules should be used in order to achieve desired changes in operating conditions (Melnyk et al. 1985), simulation is extensively used in industry to determine the effectiveness of designed scheduling strategies. Simulation research on the scheduling of job shops has a long tradition. In an effort to investigate rules that led to effective performance, much of the empirical study focus on priority dispatching. Notwithstanding this, the ensuing research has been aimed at reviewing some of those results from a broader perspective and perceiving new avenues of inquiry.

Interest in a composition of the combinatorial nature of the problem has burgeoned in recent years. It poses a theoretical challenge and, furthermore, the results can be widely used in a multitude of applications. Since the complexity of the scheduling problem is generally astronomical, the analytical approach is inappropriate; on the other hand, with the improvement in technology, computer simulation is one of the most viable approaches accessible to researchers. On the scheduling problem, much research has been published in the last decade including simulation models. The earliest of these papers is Jackson's (1957). His achievements have influenced most of the research in the dispatching aspect of the scheduling problem. Furthermore, he has made the comparison of various dispatching rules in terms of some prescribed performance measures has become the crux of the research. The standard academic vehicle for examining the job shop problem has been a computer simulation of job shop. The characteristics of the shop have been well defined in Gere (1966).

Eilon and Hodgson (1967) consider a simulation model of a machine-constrained shop to determine a multiple of the estimated job processing time is to be used in assigning due dates. This in turn minimizes several lateness penalty functions for a variety of shop loads and dispatching rules. Jones (1973) attempts to provide an economic framework to assess heuristic dispatching rules, the amount of work-in-process inventory and due date lengths in the classic job shop scenario. Past research, for example Eilon and Hodgson (1967), Jones (1973) and Nelson (1967 and 1970), has been predominantly concerned with the dispatching aspect of single-constrained job shops. Deviating from the conventional approach, Weeks and Fryer (1976) present a method which considers the importance of labour assignment decision rules, dispatching rules in dual constrained job shops and the significance of due date assignment rules. The parameters used to measure the performance of these decision rules are mean flowtime, variance of flowtime, mean lateness, variance of lateness, proportion of jobs late, and total labour transfers. Simulation is used to generate data to test the hypothesis that decision rules for dispatching, labour and due date assignments correlate to shop performance. Weeks and Fryer (1976) propose multiple regression analysis techniques to review the simulation results. This tool provides both linear and nonlinear multiple regression coefficients and analysis of variance statistics that facilitate the estimation of response measures of shop performance and multiplier of total processing time. Cheng (1988a) uses a similar technique to estimate the functional relationship between the performance measure and the shop decision variables: (i) the job dispatching rule, (ii) the due date assignment method and (iii) the shop load ratio. The regression model formulated by Cheng (1988a) addresses the following issues:

1. Given a shop load ratio, a specific combination of dispatching rules and the due date assignment method, what should the percentage of late jobs be?
2. Given an objective to yield the least estimated percentage of late jobs at a given shop load ratio, what should the most ideal combination of dispatching rule and due date

assignment method be?

As well, Cheng (1988a) further expands his approach to reviewing the simulation results by taking the regression equation as the objective function of a mixed-integer programming problem to explore the optimal combination of dispatching rule and due date assignment method. Weeks (1979) further enriches the literature on this problem by examining simulation studies of assigning predictable due dates in hypothetical labour and machine constrained job settings of varying size and structure. The predictable due date assignment rules are established through conditional estimates of individual flowtime derived from simulation runs and shop congestion information. Another simulation study conducted by Cheng (1988b) is to investigate the effects on missed due dates and job flowtime. He constructs a simple "semi-local" dispatching rule that is capable of monitoring the progress of jobs in the shop. A "semi-local" dispatching rule is one that takes into consideration information about the workload of the immediate successor and/or predecessor on the machine concerned in addition to the job characteristics, and then calculates priority values for the jobs (Cheng, 1988b). The integration of priority dispatching with due date assignment brings about an advancement in performance of both the dispatching and due date assignment rules.

2.4 A Review of Sequencing with Earliness and Tardiness Penalties

In most industrial scheduling problems, costs arising from both earliness and tardiness (E/T) of the individual jobs being scheduled must be accounted for. It is evident that the failure of completing a job on its promised delivery date gives rise to various penalty costs. Recently, much attention has been given to scheduling problems in which costs or penalties are incurred by early as well as late completion of job. For instance, for jobs that finish ahead of schedule, there will be inventory carrying costs associated with finished goods or from product deterioration. For jobs that finish behind schedule, there are tardiness costs caused by possible losses of goodwill and of future sales; furthermore, contractual penalties may be incurred. Consideration of these types of costs in scheduling is in accordance with the fundamental premises of JIT inventory control systems which accentuates curtailing the in-process inventories and related costs. Most literature on E/T problems deal with static scheduling which is the set of available jobs to be scheduled in advance. The objective is to minimize the total E/T penalty.

There have been many studies carried out (Sidney 1977, Lakshiminarayan et al. 1978, Kanet 1981a and others) to investigate the models and solution techniques for E/T problem. The author knows of no other works published on the general Early/Tardy problem until the last decade. An attempt is made by Sidney (1977) to present, for the first time, the problem of minimizing the maximum job penalty (early or tardy), where all jobs have the same early and tardy cost functions, and idle-time inserted was polynomially bounded (Ow and Morton, 1989). He suggests that one may represent the cost of completing a project early by including an earliness cost in the objective function, for example in PERT-CPM analyses. Table 4 presents a background for prior research on the general early/tardy problem. In most of the literature, scheduling research focuses on measures such as mean flowtime, mean lateness, percentage of job tardy, and mean tardiness. These are the standard ways of measuring compliance with due date, in particular mean tardiness. With this standardization, the consequences of jobs completed early have been overlooked. Nevertheless, this has changed with the recent attention on Just-In-Time (JIT) production. The compatibility of the E/T objective with the JIT production principle is evident, in that it minimizes the deviation of job completion times around the due dates.

In a JIT scheduling environment, jobs that are finished early must be retained in finished goods inventory up to the time of their due date, while jobs that finish after their due dates may cause customers to hold up their operations. In an ideal schedule one would expect all jobs to be completed precisely on their assigned due dates. This has led to introduction of new method when non-regular performance measures were to be considered. Hence, the concept of penalizing both earliness and tardiness has spawned a new and rapidly developing line of research in the scheduling field. In the next section, we shall review the literature and bring together the main results on scheduling models with earliness and tardiness (E/T) penalties.

2.4.1 The Earliness and Tardiness Model

Consider a generic E/T model with n jobs to be scheduled. Job j is characterized by its processing time, p_j, and a due date, d_j. With the assumption of a static scheduling, job j will be allotted a completion time, C_j. Let the, E_j and T_j, denote the earliness and tardiness of job j:

$$E_j = \max\left\{0,\ d_j - C_j\right\} = \left(d_j - C_j\right)^+, \tag{2.1}$$

$$T_j = \max\left\{0,\ C_j - d_j\right\} = \left(C_j - d_j\right)^+. \tag{2.2}$$

Accompanying each job is a unit earliness penalty $\alpha_j > 0$ and a unit tardiness penalty $\beta_j > 0$. This also determines the weight given to these quantities.

In a comprehensive survey on sequencing with E/T penalties by Baker and Scudder (1989), the objective function, f(s), for a schedule s is written as

$$f(s) = \sum_{j=1}^{n}\left[\alpha_j\left(d_j - C_j\right)^{+} + \beta_j\left(C_j - d_j\right)^{+}\right], \tag{2.3}$$

$$= \sum_{j=1}^{n}\left(\alpha_j E_j + \beta_j T_j\right). \tag{2.4}$$

Thus, they assume linear penalty functions. A generalization of the above objective function could introduce completion time penalty $\theta_j C_j$ and a due date penalty $\gamma_j d_j$ as well, it can be expressed as follows,

$$f(s) = \sum_{j=1}^{n}\left(\alpha_j E_j + \beta_j T_j + \theta_j C_j + \gamma_j d_j\right). \tag{2.5}$$

In some formulations of the E/T problem, the due date is given, while in others the problem is to optimize the due date and the job sequence simultaneously. Some of the simplest results for E/T problems have been obtained for models in which all jobs have a common due date. A more general model allows distinct due dates. Likewise, some models prescribe common penalties, while others allow differences among jobs or differences between the earliness and tardiness penalties. Based on Baker and Scudder's (1989) survey, they have distinguished two classes of models. One class requires a common due date for all jobs (see, for example, Bagchi et al. (1986), Szwarc (1989)) whereas the other class allows different due dates (see, for example, Kanet (1981a), Cheng (1991)). Problems in the first class tend to be NP-hard whereas many problems in the second class are solvable in polynomial time. Each class evidently allows both exogenous and endogenous due date determination. If the due date is determined by some external agency, it may have been fixed at such a very large value that, often we categorize it in the second class since the value of such due dates does not alter the scheduling of the jobs. Aforementioned, the foremost of penalty functions is to guide solutions toward the target of meeting all due dates exactly (Baker and Scudder, 1989).

More research is required in addressing on other problems confronting practising managers. We categorize job shop research by four performance criteria:

(i) mean absolute lateness (MAL)

(ii) squared lateness

(iii) sum total of earliness and tardiness

(iv) total aggregate costs.

Topics of job shop research, classified into different categories are displayed in Table 5. Cheng and Gupta (1989) surveyed the two-dimensional analysis by performance measure and due date assignment, and showed that most research efforts have been concentrated on CON, which they attributed to the simplicity of the problems addressed.

The most commonly used performance criteria are MAL and total aggregate costs. Almost all research efforts with the exception of Sundararaghavan and Ahmed (1984), have been geared towards the single-machine problem. The most realistic case of job shops with different weights ascribed to the jobs, has been less thoroughly explored. Multiple-machine problems should be studied and previous results suggest that the combination of TWK and NOP methods performs well.

2.4.2 Mean Absolute Lateness

Customarily, research directed at job scheduling was confined to problems involving penalty functions increasing with job completion times. Such functions are regarded as regular performance measures by Conway and et al. (1965) and Baker (1974).

Notwithstanding this, there are numerous cases for which non-regular criteria are pertinent. Irrespective of the importance of such non-regular measures, relatively few authors have addressed particular problems which arise in this area. The problem of minimizing completion time variation has been studied extensively by Merton and Muller (1972), Schrage (1975) and Eilon and Chowdhury (1977). The scope of their research is based on the relation between the flowtime and waiting time variables. They fail to show techniques to minimize non-regular performance measures but establish that if some schedule S minimize a set of measures, the corresponding antithetical schedule S′ minimize the rest. The significance is that the minimum value of both measures is equivalent. They also emphasized that a schedule with minimum completion time variance is V-shaped. This characterization of the schedule is unique. Their work emphasizes the significance of non-regular measures for specific scheduling environments.

Kanet (1981a) reviews several applications and research efforts aimed at non-regular measures which is an average absolute deviation of completion time from a common due date so that penalties occur at the same rate whether the jobs are completed early or late. His objective is to minimize the total penalty subject to restrictive assumptions. He formulated his problem for a single machine with n jobs instantly available for processing. Each job i has its own required processing time, p_i. All jobs have a due date, exceeding the makespan of the job set. Hence,

$$MS = \sum_{i=1}^{n} p_i . \tag{2.6}$$

The objective function is

$$\text{MAD} = f(s) = \frac{1}{n}\sum_{j=1}^{n}\left[\left(d - C_j\right)^+ + \left(C_j - d\right)^+\right] = \frac{1}{n}\sum_{j}\left|C_j - d\right|. \tag{2.7}$$

Here MAD is the mean absolute lateness (MAL). Kanet (1981a) presented a constructive algorithm for finding the optimal schedule.

A similar problem is studied by Sundararaghavan and Ahmed (1984) in which a modification of the algorithm is proposed. They have presented an algorithm for determining multiple optimal schedules. Aside from being optimal, it may satisfy other additional requirements. They have also proposed an implicit enumeration procedure in case the restrictive due date assumption is relaxed. Heuristics and exact algorithms for this problem lay a foundation focusing non-regular measures of performance in additional research on job scheduling.

2.4.3 Squared Lateness

Whenever a job cannot be completed punctually on its due date, irrespective of whether it is early or tardy, costs will inevitably be incurred. Thus, it seems justifiable to minimize the total missed due dates as our main objective. As a case in point, Cheng (1986a and 1984) has adopted the expected value of the total job lateness square, $E\left[{L_T}^2\right]$ and total squared of lateness, L^2 as objective functions to be minimized for various scenarios. His paper considers the problem of optimal due date determination and scheduling of n independent jobs on a single machine and assigned due date based on total work content.

In Cheng's 1984 paper, he defines σ as any arbitrary sequence of X. He lets d_i denote the assigned due date of job i so that $d_i = r_i + kt_i$ for TWK due dates. His goal was to find the optimal value k^* of the processing-time multiple, the optimal sequence σ^* and to minimize a lateness cost function. He considered both deterministic processing times and random processing times. With former he assumes that all processing times t_i are known before processing commences, whereas with the latter this assumption has been relaxed. He has proven that the optimal sequence is in SPT and the optimal processing-time multiple for the deterministic case is

$$k^* = \frac{\sum_{i=1}^{n} t_{[i]} \sum_{j=i}^{i} t_{[j]}}{\sum_{i=1}^{n} t_{[i]}^2}. \tag{2.8}$$

and for the random processing times, they are assumed to have means μ_i and common coefficient of variation c. This random processing multiple is

$$k^* = \frac{\sum_{i=1}^{n} \left(c^2 \mu_{[i]}^2 + \mu_{[i]} \sum_{j=1}^{i} \mu_{[j]} \right)}{\left(1 + c^2\right) \sum_{i=1}^{n} \mu_{[i]}^2}. \tag{2.9}$$

These two results are obtained when total squared value of lateness, L^2 is used. Cheng (1984) has proposed this as the performance measure to be used.

Denoting the lateness, completion time, and assigned due date of the job in position [i] respectively, by $L_{[i]}$, $C_{[i]}$, and $d_{[i]}$, he formulated objective function as

$$L^2 = \sum_{i=1}^{n} L_{[i]}^2, \tag{2.10}$$

$$= \sum_{i=1}^{n} \left(C_{[i]} - d_{[i]} \right)^2, \tag{2.11}$$

$$= \sum_{i=1}^{n} \left(\sum_{j=1}^{i} t_{[j]} - k t_{[i]} \right)^2. \tag{2.12}$$

We obtain k^* by differentiating (2.12) with respect to k and equating the result to zero. He has also proven that k^* is constant and independent of the way in which the jobs are sequenced for processing. Moreover,

$$L^2 = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} t_{[j]} \right)^2 + \left(k \sum_{i=1}^{n} t_{[i]} \right)^2 - 2k \sum_{i=1}^{n} t_{[j]} \sum_{j=1}^{i} t_{[j]}, \tag{2.13}$$

is minimized by arranging the job in the SPT order $t_{[1]} \leq t_{[2]} \leq \ldots \leq t_{[n]}$.

In the case of random processing times, $E(\cdot)$ is the expected value operator. The objective function to be minimized is

$$E\left[L^2\right] = E\left(\sum_{i=1}^{n} L_{[i]}^2 \right), \tag{2.14}$$

$$= E\left[\sum_{i=1}^{n} \left(\sum_{j=1}^{i} t_{[j]} - k t_{[j]} \right)^2 \right], \tag{2.15}$$

$$= \sum_{i=1}^{n}\left[E\left(\sum_{j=1}^{i} t_{[j]}\right)^2 + k^2E\left(t_{[i]}^2\right) - 2kE\left(t_{[j]}\sum_{j=1}^{i} t_{[j]}\right)\right], \tag{2.16}$$

$$= \sum_{i=1}^{n}\left[\left(\sum_{j=1}^{i}\mu_{[j]}\right)^2 + c^2\sum_{j=1}^{i}\mu_{[j]}^2 + \left(1+c^2\right)k^2\mu_{[i]}^2 - 2k\left(c^2\mu_{[i]}^2 + \mu_{[i]}\sum_{j=1}^{i}\mu_{[j]}\right)\right]. \tag{2.17}$$

The optimal processing-time multiple k^* found by differentiating (2.17) with respect to k and equating the result to zero. In Cheng's 1988 paper, he used a similar technique is; however, a more generalized expression resulted. Given that $c^2\mu_{[i]}^2$ is equivalent to $\sigma_{[i]}^2$, the expected value of the total job lateness squared is

$$E\left[L_T^2\right] = \sum_{i=1}^{n}\left[\left(\sum_{j=1}^{i}\mu_{[j]}\right)^2 + \sum_{j=1}^{i}\sigma_{[j]}^2 + k^2\left(\mu_{[i]}^2 + \sigma_{[i]}^2\right) - 2k\left(\sigma_{[i]}^2 + \mu_{[i]}\sum_{j=1}^{i}\mu_{[j]}\right)\right]. \tag{2.18}$$

The above is a brief recapitulation of the models developed by Cheng, which provide a major contribution. In certain production environments in which completion times of jobs can be anticipated, they can be used to obtain optimal due dates and optimal sequences.

2.4.4 Sum Total of Earliness and Tardiness

To determine the optimal due date multiple factor so as to minimize a cost function based on due date multiple and job earliness and tardiness value, linear programming (LP) is proposed by Cheng (1985a) and Quaddus (1987a). In Cheng's problem formulation, a weighting factor W_i ($0 < W_i < 1$) and the CON due date assignment method ($d_i = a_i + k$) are considered. The objective function, in terms of the constant follow allowance k, is

$$f(k) = \sum_{i=1}^{n} W_{[i]} \; |L_{[i]}|, \tag{2.19}$$

$$= \sum_{i=1}^{n} W_{[i]} \; |C_{[i]} - d_{[i]}|, \tag{2.20}$$

$$= \sum_{i=1}^{n} W_{[i]} \; |C_{[i]} - k|. \tag{2.21}$$

A special case of Cheng's formulation, $W_{[i]} = 1$, is presented by Quaddus (1987a). He proposed total value of lateness as the measure of performance . The objective is to find an optimal CON due date, d^*, as well as the optimal job sequence to minimize the total value of lateness. [i] denotes the job occupying the ith position in any specified sequence σ. His LP model is

$$\min f(d, \sigma) = \sum_{i=1}^{n} \left(E_{[i]} + T_{[i]}\right) \tag{2.22}$$

subject to

$$d + T_{[i]} - E_{[i]} = C_{[i]}, \tag{2.23}$$

$$d, T_{[i]}, E_{[i]} \geq 0.$$

Quaddus and Cheng proposed duality approach to solve for d^*. They introduced dual variables $y = (y_1, y_2, ..., y_n)$, and transformed the primal to

$$\max g(y,\sigma) = \sum_{i=1}^{n} C_{[i]} y_{[i]} \tag{2.24}$$

subject to

$$\sum_{i=1}^{n} y_i \le 0, \tag{2.25}$$

$$|y_i| \le 1 \quad \forall i, \tag{2.26}$$

$$y_i \text{ restricted } \forall i.$$

Duality theory states that, at optimality, $f(d^*, \sigma) = g(y^*, \sigma)$ where d^* and y^* are optimal solutions to the primal and dual problems respectively. In both Cheng's and Quaddus' paper, d^* is proven to be equal to $C_{[r]}$ (here r is the smallest integer such that $r \ge \frac{n}{2}$). Also, d^* is proven to be independent of the sequence σ. The main merit of their results is that it is also applicable to a better measure of performance; in terms of P, constant positive penalty which is the same for all jobs, this measure is $f(d, \sigma) = \sum_{i=1}^{n} P\,|L_{[i]}|$.

Interest in scheduling due dates has grown in recent years. The reason for this is that the problem is theoretically challenging,and solutions of great importance in the real world. Cheng (1986d) has given a generalization of the basic model where earliness and tardiness should be penalized at different rates. The fundamental distinction between this model and previous models is that each job is assigned a due date using the SLK due date assignment method. In terms of α, β_i, γ_i, and k which are the due date assignment cost per unit time, the earliness cost per unit time, the tardiness cost per unit time of job i and slack allowance respectively, the objective function is

$$f(k, \sigma) = \sum_{i=1}^{n} \left(\alpha k + \beta_{[i]} E_{[i]} + \gamma_{[i]} T_{[i]} \right). \qquad (2.27)$$

The LP formulation for this is

$$\min f(k, \sigma) = n\sigma k + \sum_{i=1}^{n} \beta_{[i]} E_{[i]} + \sum_{i=1}^{n} \gamma_{[i]} T_{[i]} \qquad (2.28)$$

subject to

$$k + T_{[i]} - E_{[i]} = C_{[i-1]} \qquad \forall\, i \in N, \qquad (2.29)$$

$$k,\ E_{[i]},\ T_{[i]} \geq 0 \qquad \forall\, i \in N.$$

In like manner, to solve for the optimal value of the slack allowance k^* and the optimal job sequence σ^*, minimizing the cost function based on the slack allowance and the job earliness and tardiness values, the dual problem is considered. The primal constitutes equality constraints, and hence, its dual must be asymmetric. In Cheng's dual problem, the vector of dual variables is denoted by $W = (W_1, W_2, ..., W_n)$. Hence, the problem can be expressed as

$$\max g(w, \sigma) = \sum_{i=1}^{n} C_{[i-1]} W_{[i]} \qquad (2.30)$$

subject to

$$\sum_{i=1}^{n} W_{[i]} \leq n\alpha, \qquad (2.31)$$

$$-\beta_{[i]} \le W_{[i]} \le \gamma_{[i]} \qquad \forall\, i \in N, \tag{2.32}$$

and $W_{[i]}$, $\forall\, i \in N$ is unrestricted in sign.

An analogous problem, but with a different due date assignment was also studied. In Cheng's (1987a) paper, based on TWK-P, he introduced a due date multiple factor k, an exponent m of the processing time, a_i is the available time of job i and assigned due date, $d_i = a_i + kt_i^m$. He noted two cost components that have predominated in the cost function. The first component is the cost of assigning due dates and the second the cost of missing due date. Then LP problem becomes

$$\min f(k, \sigma) = n\sigma k + \sum_{[i]\in N}\left[E_{[i]} + T_{[i]}\right] \tag{2.33}$$

subject to

$$k + T_{[i]} - E_{[i]} = C_{[i]} \qquad \forall\, i \in N, \tag{2.34}$$

$$k, E_{[i]}, T_{[i]} \ge 0 \qquad \forall\, i \in N.$$

where k is the due date multiple factor, α is the cost per unit value of k and $d_{[i]} = kt_{[i]}{}^{*}$. Here it is assumed that all jobs are available for processing at the same time. Thus, without loss of generality, a_i is set to equal to 0. With the application of LP duality theory, the optimal solution k^* is found to be $\dfrac{C_{[r]}}{t_{[r]}{}^{m}}$. Furthermore, the optimal solution for the dual problem is

$$x_{[r]}^{*} = \frac{\left\{ n\alpha - \sum_{\substack{[i]\in N \\ i\neq r}} t_{[i]}^{m} x_{[i]}^{*} \right\}}{t_{[r]}^{m}}. \tag{2.35}$$

These noteworthy attempts by Cheng confirm the value of analytical determination of optimal due date multiples, given the job sequence.

2.4.5 Total Aggregate Cost

As an alternative to the fundamental E/T measure, other performance measures can be considered. Panwalkar, Smith and Seidmann (1982) propose a two-dimensional criterion to evaluate due date and flowtime penalties. The formulation of their model undertake the common due date d as a decision variable. This is logical; for instance, in the case of a firm offering a due date to a customer during sales negotiations, but having to offer a price reduction if the due date is set too late (Baker and Scudder, 1990).

The most elementary form of setting due dates is a constant lead time, that is the lead time that a customer might expect between time of placing the order and time of delivery, so that the due date is the time of order plus a constant lead time. Hence, if the shop status is relatively stable, all jobs may be given due dates based upon the constant lead time regardless of job content. The per unit costs functions of due date, earliness and tardiness are assumed linear. These cost function with linearity property proffer an argument that is increasingly conformable compared to nonlinear costs. The forementioned costs can be observed as opportunity costs.

Seidmann and Smith (1981) investigates a commonly used industrial policy for assigning minimum cost due dates in a dynamic job production system. The model considered in the study assumes that the distribution of the total time in the shop is common to all jobs. The objective is to minimize the expected aggregate cost per job subject to restrictive assumptions on the priority discipline and the penalty functions. This aggregate cost includes (i) a cost that increases with increasing lead times (ii) a cost for jobs that are delivered after the due dates (the cost is proportional to tardiness) and (iii) a cost proportional to earliness for jobs that are completed prior to the due dates. They have presented an algorithm that employs analytical procedures for solving this problem. The optimal solution obtained is found to be independent of specific probability distribution function of both the interarrival job times and the total shop time. It is shown that the optimal lead time is a unique minimum point of strictly convex functions. This simple structure of the optimal solution has two significant implications. The unimodal function simplifies a numerical search to obtain the minimum cost lead time and having a single solution makes it easier to implement it in the industrial environment. The result is that the approach is flexible, since no specific distributions need to be assumed and the procedure is computationally tractable. The analysis of the basic scheduling model has provided new insights that may foster improved guidelines for the design of production control systems for more complex scenarios. For management, perhaps the key contribution of the results is that they demonstrate how, in certain production environments, an optimal due date assignment rule can easily be derived.

In a similar problem, studied by Cheng (1985b), he has proposed a general cost model for analyzing the operating characteristics of the TWK due date assignment method in a job shop environment. The cost model is general, since no specific distributions have been imposed on the underlying random processes involved. The total cost function is composed of two opportunity cost components: (i) the cost of quoting long due dates, and (ii) the cost of missed due dates. The objective is to find the optimal processing time multiple, k_p^*, that will minimize the expected total cost per job. He presented an analytical procedure for deriving the optimal solution and to show that k_p^* is a unique absolute minimum point of the strictly convex cost functions included in the cost model. It is also shown that determination of the optimal processing time multiple requires only information readily accessible in the shop. Under certain circumstances, k_p^* can even be exclusively expressed in terms of the shop parameters, such as the number of machines in the shop, mean job arrival rate and processing time. The main value of Cheng's analysis is that determination of k_p^* becomes a simple process requiring only a modest modest amount information and can easily be implemented in actual practice.

Panwalkar, Smith and Seidmann (1982) were of the opinion that, due date and tardiness are associated with customer objectives, whereas earliness cost can be viewed as an essential element that pertains the production shop. Let N denote the set of n jobs and t_i the processing time of job i, in the order $t_1 \leq t_2 \leq \ldots \leq t_n$, that is in the SPT sequence. Consider the total penalty function $f(d, \sigma)$ related to a given parameter d and specified sequence, σ, it follows that

$$f(d, \sigma) = \sum_{i=1}^{n}\left(P_1 d + P_2 E_{[i]} + P_3 T_{[i]}\right). \qquad (2.36)$$

P_1 represents the due date assignment cost per unit of time, P_2 and P_3 are the earliness and tardiness costs per unit of time, respectively. The objective is to ascertain the optimal value of this due date and an optimal sequence to minimize a total penalty function. They have presented with scheduling algorithm with a polynomial bound scheduling of the order

0(nlog(n)) for the solution of this problem along with proof of optimality. Their results show that the optimum due date can be selected if one can anticipate completion times of different jobs. A similar conclusion is drawn by Weeks (1979) in his multi-machine job shop simulation study. The close agreement of the results confirms the validity of the model. In addition, the determination of the number of non-tardy jobs does not require the values of the processing times of the n jobs.

Weighted objective functions are difficult to implement in practice for at least two reasons. First, it is difficult to estimate proper weights. Second, the penalties implied by these weights is notational and may not easily be used in the calculation of savings resulting from good scheduling.

2.5 Review of Cost-Based Rules for Job Shop Scheduling

A frequent problem in evaluating priority rules has been the need to balance a myriad of performance measures. In dynamic-stochastic scheduling, priority rules are used for selecting the next job from a queue to be processed on a particular machine. Priority rules may be classified according to their time dependency, what type of data they use, or both (Moore and Wilson, 1967), whether they are static or dynamic (Jackson, 1957), and local or global (Conway, 1967). A static rule determines only one priority value for each operation of a job during its existence in the shop. It resolves the conflict whenever two or more jobs are waiting for the services of a single machine. A dynamic priority value, on the other hand, changes over time. It facilitates recognizing, among those waiting for service at a particular work centre, the job with maximum priority in relation to a given performance measure. Priority values of jobs must be updated before each decision is made. Therefore, more calculation is involved. Nonetheless, in most managerial problems, cost is a more global and homogeneous criterion than time-based criteria such as flowtime and lateness. Therefore, it seems intuitively reasonable that a priority rule based on a composite of the most relevant costs, such as inventory, setup, processing and lateness, should perform as well as, if not better than, time-based rules.

Aggarwal and McCarl (1974) develop a scheduling rule that will optimize the overall operating costs of processing jobs through a shop without adversely affecting the other measures of performance. They have stressed four major performance criteria; in-process inventory, use of facilities, lateness and mean setup time. Each criterion represents a different component of operational costs and each may be translated into a cost index. The four cost indices are combined into a single composite expression for the priority rule. With the following notations

i = Index for the job i.

j = Index for operation j of job i.

n = Index for machine group n.

C_1 = Daily inventory cost per dollar of inventory.

V_i = Estimated mean dollar value of the job.

D_{ijn} = Due date of operation j of job i in machine group n.

T = The current day on which priority ratings Z_{ijn} are being calculated in machine group n.

C_{2n} = Cost per hour of processing in machine group n.

K_{1n} = Mean processing time through the nth machine group.

P_{ijn} = Processing time of the job i, operation j, on machine group n.

C_3 = Daily cost of lateness per dollar value of the job

$$d = \begin{cases} 0 & \text{if job is not late} \\ 1 & \text{if job is late.} \end{cases}$$

C_{4n} = Mean setup cost per hour for machine group n.

K_{2n} = Mean setup time per operation for machine group n.

S_{ijn} = Required setup time by operation j of job i in machine group n.

Z_{ijn} = Priority value for the job i, operation j, machine group n.

the priority rule becomes,

$$Z_{ijn} = C_1 V_i (D_{ijn} - T) + C_{2n} K_{1n} P_{ijn}{}^{-1} + dC_3 V_i (T - D_{ijn})^2 + C_{4n} K_{2n} S_{ijn}{}^{-1}. \qquad (2.37)$$

The Z_{ijn} represents the priority number of a job waiting for a machine group on a particular day. Priority is given to the jobs with the highest value of Z_{ijn} first. The salient properties of this rule may be summarized as follows:

Inventory cost. As time draws near to the due date for completion of an operation, the inventory cost of the job decreases. Therefore, less priority is given to the job since it will soon be out of the shop.

Setup, processing costs. Jobs that require shorter setup and processing time get higher priorities.

Lateness cost. The penalty for late completion of individual jobs represents opportunity losses due to locked-up capital. Late deliveries cause a loss of future orders. As the average amount of lateness increases, the probability of obtaining repeat orders decreases rapidly. It is assumed that the lateness cost index of an operation increases in direct proportion to the square of the number of days a job is late (Aggarwal and McCarl, 1974).

The cost-based composite scheduling rule is evaluated in comparison with three other well-researched scheduling rules. They are SPT, S/OPN and SST. Cost rule (2.37) allows the optimization of more than one performance measure at a time. A factorial experimental design involving three factor levels of loads, three factor levels of cost and three factor levels of mean time is conducted. Analysis of variance is performed on each of the five output measures to study the effects of each of the three factors on each individual rule. The results affirm that most of the criteria are satisfied by SPT and these cost rules.

Jones (1973) provides an economic framework for evaluating the cost structure and dispatching rules of various job shops. It exemplifies the relative advantage of the SPT rule in gaining increased utilization of the shop facilities and the relative advantage of minimum slack rules in meeting promised commitments. He graphs each of four kinds of costs, (i) costs of long promises, (ii) costs of missed promises, (iii) costs of idle resources and (iv) costs of carrying inventory, against two independent variables, (i) the amount of work-in-process inventory and (ii) the tightness of the promises. The graphs show two curvilinear relationships, one of which depicts missed promise costs against promise length and the other idle machine costs against work-in-process inventory. A model including both kinds of costs would present a surface of costs graphed against two independent variables. The examination of the shape of such a cost surface is presented. This analysis demonstrates the kind of cost structure which causes a MS rule to be superior to a SPT rule.

Ulgen (1979) considers the effectiveness of simple priority rules in relation to setup cost, in-process inventory cost, storage cost, penalty cost and various cost functions comprised of two or more of these four costs. His study further validate the results of other researchers in that (i) SPT and JV reduce in-process inventory costs, (ii) S/OPN gives best results for earliness and tardiness costs and (iii) SST is best for setup cost among simple priority rules.

Shue and Smith (1978) have taken another perspective in investigating the costs structures. They used a sequential application of simple priority rules. The sequential approach examines all waiting jobs; the first simple rule allows a subset of jobs to be considered by the second rule; the third rule selects one job for the machine (Kiran and Smith, 1984). Their studies have shown that each sequential rule performs better than its constituents. S/OPN-JV-SST appear to be superior for setup, in-process inventory and late penalty costs. In a broadly parallel approach, Hollier (1968) and Berry (1972) consider cost functions and priority rules in batch type manufacturing shops. They conclude that SPT is not successful in batch type manufacturing if it includes in-process inventory cost. This is due to the high flowtime variance which causes high in-process inventory costs.

The subsequent literature in this area has concentrated on the theoretical aspects of the methods. The consequences changes in customer requirements for due dates are studied by Eilon and Chowdhury (1976), Elvers (1974), Hottenstein (1970) and Ulgen (1979). In general, customer-requested due date changes adversely affect due date performance of all priority rules. However, the rules which use due date information are more responsive to due date changes.

Table 4 Objective Functions From Prior Research

Alternative Criteria	References
$f(s)=\sum_j (d-C_j)^+ + \sum_j (C_j-d)^+$	Kanet (1981a) Sundararaghavan and Ahmed (1984) Bagchi, Sullivan and Chang (1986) Hall (1986) Emmons (1987) Szwarc (1989) Baker and Chadowitz (1989) Hall, Kubiak and Sethi (1989)
$f(s)=\sum_i \sum_j \lvert C_j-C_i \rvert$	Kanet (1981b)
$f(s)=\alpha\sum_j (d-C_j)^+ + \beta\sum_j (C_j-d)^+$	Panwalkar, Smith and Seidmann (1982) Emmons (1987) Bagchi, Chang and Sullivan (1987) Baker and Chadowitz (1989)
$f(s)=\sum_j (C_j-d)^2$	Bagchi, Sullivan and Chang (1987) De, Ghosh and Wells (1989a,b)
$f(s)=\sum_j (C_j-C)^2$	Eilon and Chowdhury (1977) Kanet (1981b) Vani and Raghavachari (1987)
$f(s)=\sum_i \sum_j (C_j-C_i)^2$	Kanet (1981b)

Alternative Criteria	References
$f(s) = \alpha \sum_j \left[\left(d - C_j\right)^+ \right]^2 + \beta \sum_j \left[\left(C_j - d\right)^+ \right]^2$	Bagchi, Chang and Sullivan (1987)
$f(s) = \sum_j \alpha_j \left(d - C_j\right)^+ + \sum_j \beta_j \left(C_j - d\right)^+$	Bagchi (1985) Cheng (1987c) Emmons (1987) Quaddus (1987b) Bector, Gupta and Gupta (1988) Baker and Scudder (1989) Hall and Posner (1989)
$f(s) = \sum_j \alpha_j \left(d_j - C_j\right)^+ + \sum_j \beta_j \left(C_j - d_j\right)^+$	Ahmadi and Bagchi (1986a, b) Davis and Kanet (1988) Fry, Darby-Dowman and Armstrong (1988) Fry, Armstrong and Blackstone (1987) Fry and Leong (1987) Fry, Leong and Rakes (1987) Yano and Kim (1986) Chand and Schneeberger (1988) Abdul-Razaq and Potts (1988) Szwarc (1989) Ow and Morton (1988, 1989)
$f(s) = \sum_j \alpha_j \left[\left(d_j - C_j\right)^+ \right]^2 + \sum_j \beta_j \left[\left(C_j - d_j\right)^+ \right]^2$	Gupta and Sen (1983) Cheng (1984)

Table 5 Classification of Performance Measures Versus Due Date Assignment Methods

Performance Measures	Due Date Assignment Methods			
	CON	TWK	SLK	Others
Mean Absolute Lateness	Ashour & Vaswani (1972) Kanet (1981a) Karla & Bagga (1983) Sundararaghavan & Ahmed (1984) Ahmed & Sundararaghavan (1984) Bagchi et al. (1986) Cheng (1987d)			
Squared Lateness		Cheng (1984, 1986a)		
Sum Total of Earliness and Tardiness	Cheng (1985a, 1986d,1988c) Quaddus (1987a, 1987b) Bector et al. (1988)		Cheng(1986b)	Cheng(1988f)
Total Aggregate Costs	Seidmann et al. (1981) Panwalker et al. (1982) Ragatz & Mabert (1984) Cheng (1986e)			

Chapter III

ANALYTICAL DETERMINATION OF OPTIMAL TWK-NOP DUE DATE WITH PERFORMANCE MEASURES INCLUDING COST FACTORS

Considerable research has been conducted in the area of industrial scheduling characterized by random assignments of due dates to jobs, or by some subjective preference of a due date assignment policy and its parameter. Of obvious practical importance, an optimal assignment of due dates has become the subject of growing interest over the last years. The problems where due dates are not a priori specified and have to be assigned in decision making are considered in the papers of Cheng (1984, 1987b, 1989c, and 1991), Cheng and Li (1989), Gordon (1991), Panwalkar et al. (1982), Seidmann et al. (1981), Seidmann and Smith (1981) and Van de Velde (1990). The increasing availability of large digital computers has allowed researchers to enquire into such problems by way of computer simulation. Citations of the review have been given in Chapter 2. Even though this standard approach is widely taken to specify due date, research aimed towards striving optimality of some policies by means of analytical formulation are scarce. Up-to-date analytical formulations of this research can be found in Seidmann and Smith (1981) and Cheng (1985b). This chapter purports to exploit a detailed mathematical model similar to the state-of-the-art achieved in the mentioned area.

3.1 Cost Model

In this chapter, it is proposed that for each job there are two kinds of opportunity costs which represent possible increase of profit should an optional policy be taken up; (i) the cost of quoting long due dates, and (ii) the cost of missed due dates. The former represents the potential loss of sales. Customers will divert their orders to competitors who can furnish them with a more appealing delivery promise. Hence, demand will decrease. To recapture business by providing supplementary selling effort, it is inevitable for the shop to convert the volume loss to a cost penalty. The due date allowance cost is assumed to be independent of the actual job completion time, and its value is determined by the specific due date allocated to each individual order when it arrives at the shop (Seidmann and Smith, 1981). As noted by Jones (1973), due date allowance cost is applicable only if the promise is approaching to that of the competitor's. The introduction of cost of quoting long due dates by Jones (1973) has led Weeks and Fryer (1976) to incorporate this cost in their hypothetical dual constrained job shop scenario. The cost of missed due dates represents the costs associated with jobs that are unable to be finished in their allotted time. This is an agglomeration of tardiness and earliness costs. The two possible cases depict jobs that finish ahead of schedule or behind schedule. Inventory carrying costs have to be considered when jobs cannot be shipped out at their juncture of completion. Additionally, this includes interest and stockholding costs. On the contrary, should the order be delivered unpunctually, that is, after its due date, costs representing potential loss of future sales, customer good will and contractual penalties will be incurred.

3.2 Problem Formulation

3.2.1 Due Date Assignment Model

This section considers the problem of scheduling a set of n independent, nonpreemptive jobs on a set of multiple machines. A feasible schedule is one in which all job due dates are met. Let N be a set of n independent jobs to be processed on a multiple machine. Each job requires P_i processing time, which is deterministic, on the machine that cannot simultaneously process more than one job. The underlying procedure of each due date assignment rule comprises an aggregate of estimating the flowtime of job i and r_i, the time epoch at which the job arrives to the shop. The estimated flowtime of job i is a function of the total processing time, P_i, with parameter α.

With total work and number of operations (TWK-NOP) due date procedure, each arriving job i, $\forall\ i \in N$, will be assigned due date, d_i, which is denoted by

$$d_i = r_i + \alpha\, P_i + \beta\, N_i \tag{3.1}$$

where r_i, d_i, P and N are the arrival time, assigned due date, total processing time and the required number of operations of a job respectively; α and β are the respective processing time and number of operations multipliers. TWK-NOP method establishes for each job, a due date proportional to both the total processing time and the required number of operations. It is our objective to find a procedure in which determines the most appropriate values of these parameters, and essentially the optimal processing time multiplier, α, and the number of operations multiplier, β, will minimize the expected total cost per job.

The principal advantage in selecting TWK-NOP due date assignment policy is its ability to address the most general case due date assignment policy and its parameters. Processing time and number of operations are utilized simultaneously to estimate flowtime. It is clear that we can divert our focus on the operating characteristics of TWK or NOP assignment rule individually, in the generic job shop environment. Furthermore, processing-time-plus-wait (PPW) due date is a special case of TWK-NOP where α takes on the value 1. It is assumed that all jobs are available for processing at the same time; without loss of generality, we take $r_i = 0$, $\forall\ i \in N$. We further assume that both job splitting and inserted-idleness are not allowed.

3.2.2 Due Date Allowance Cost

The due date allowance cost is a spline or piecewise continuous functions. Each function depends upon the processing multiplier, α and number of operations multiplier, β. The due date allowance cost for a job with multipliers, α and β, can be modelled as

$$C_D(\alpha, \beta) = \begin{cases} 0 & \alpha_1 \leq \alpha_a \text{ and } \beta_1 \leq \beta_a \\ \phi_D(\alpha_2/\alpha_a) & \alpha_2 > \alpha_a \text{ and } \beta_2 \leq \beta_a \\ \phi_D(\beta_3/\beta_a) & \alpha_3 \leq \alpha_a \text{ and } \beta_3 > \beta_a \\ \phi_D(\alpha_4/\alpha_a) + \phi_D(\beta_4/\beta_a) & \alpha_4 > \alpha_a \text{ and } \beta_4 > \beta_a \end{cases} \tag{3.2}$$

where α_a and β_a, are the average processing-time and number of operations multipliers respectively. These multipliers are quoted in the particular industry and representing some predetermined base value. $C_D(\alpha, \beta)$ is governed by four different forms. Each form depends upon various ranges of parameter.

Scenario I: Since the flowtime of a job and the minimal flowtime allowance per operation of a job will not be less than its processing time and number of operations, there exists no penalty when the quoted multipliers are less than α_a and β_a.

Scenario II: An opportunity cost, $\phi_D(\alpha_2/\alpha_a)$ will be incurred if the quoted processing time multiplier is greater than the industry's average value. In conjunction, the quoted number of operations multiplier is less than the mean value.

Scenario III: By the same token, given the quoted number of operations multiplier is greater than average value and quoted processing time multiplier is less than the industry's average value, an opportunity cost, $\phi_D(\beta_3/\beta_a)$ is resulted.

Scenario IV: Given both the quoted multipliers are greater than the average value, opportunity cost $\phi_D(\alpha_4/\alpha_a) + \phi_D(\beta_4/\beta_a)$ is incurred.

3.2.2.1 An Illustration

The preceding scenarios have demonstrated the existence of four possible cases. Theoretically, this categorization is highly sensible. However, the successful design and implementation of the model becomes the primary objective. To validate the due date allowance cost model, we consider the following numerical examples which illustrate the derivation of the industry's average processing time and number of operations multipliers, α_a. and β_a respectively.

Given an manufacturing industry with four work centres, A, B, C, and D, the work centres process the same type of job with a set of five operations. The job consists of a single component routed randomly through the work centre to various processing facilities on which operations are performed. The processing times for each operation are, $p_1 = 1$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$ and $p_5 = 10$. The total processing time is 26 units. The preassigned due date method is TWK. The assumption and calculation of flowtimes and processing time multipliers, α_i, where i = 1, 2, 3, and 4, correspond to each work centre is presented in the following table.

Table 6 Assumption and Calculation of Flowtimes and Processing Time Multipliers

Work centre	Flowtime (Units)	α_i	α_i > or < α_a
A	145	145/26 = 5.58	$\alpha_1 < \alpha_a$
B	176	176/26 = 6.77	$\alpha_2 > \alpha_a$
C	169	169/26 = 6.50	$\alpha_3 > \alpha_a$
D	153	153/26 = 5.88	$\alpha_4 < \alpha_a$
		Total: 24.73	
		α_a = 24.73/4 = 6.18	

The average processing time multiplier quoted in the industry, α_a is 6.18.

Consider the previous example with the exception of NOP due date assignment method, we modify the problem by considering the number of operations. The assumption and calculation of flowtimes and number of operations multipliers, β_i, where i = 1, 2, 3, and 4, correspond to each work centre is presented in the following table.

Table 7 Assumption and Calculation of Flowtimes and Number of Operations Multipliers

Work centre	Flowtime (Units)	β_i	β_i > or < β_a
A	145	145/5 = 29.0	$\beta_1 < \beta_a$
B	176	176/5 = 35.2	$\beta_2 > \beta_a$
C	169	169/5 = 33.8	$\beta_3 > \beta_a$
D	153	153/5 = 30.6	$\beta_4 < \beta_a$
		Total: 128.6	
		β_a = 128.6/4 = 32.2	

The average processing time multiplier quoted in the industry, β_a is 32.2.

The average flowtime, F_{avg} for the TWK-NOP due dates is expected to be 169.75. Given that the total processing time is 26 units and 5 number of operations performed for each job, the following formula must hold.

$$F_{avg} = 26\alpha + 5\beta \tag{3.3}$$

From the above two versions of the industrial example, α_a is found to be 6.18 and β_a is equal to 32.2. However, by substituting the averages, α_a and β_a, quoted by the particular industry into equation (3.3), F_{avg} fails to remain valid. Should TWK and NOP due dates assignment methods be considered separately, F_{avg} remains valid. With the intention of having (3.3) to hold and the existence of four scenarios, mentioned previously, we deduce that different weights must be allotted to α_a and β_a depending on the given circumstances. If we were to assign the following weights to (3.3),

$$F_{avg} \cong 26 * (0.67\alpha) + 5 * (0.37\beta), \tag{3.4}$$

F_{avg} holds. In the later chapter, we apply our methodology to a more realistic production setting representing an actual industrial production network.

3.2.3 Missed Due Date Cost

The review of literature on some of the problems encountered by practising manager is presented in Chapter 2. The scheduling decision have been classified into four different performance measures. In view of previous discussion, it is possible to estimate the cost of a missed due date for each job as a function of squared lateness of the job. The justification is that lateness is a measure of the amount of deviation from the due date, both positive and negative lateness will give rise to costs. Furthermore, disregarding whether the job is early or tardy, costs will inevitably be contracted whenever a job misses its due date. As follows, the missed due date cost for a job with lateness L is expressed as

$$C_M(L) = \phi_M(L^2). \tag{3.5}$$

Lateness, L, is the difference between the completion time C and the due date D of a job. A job with lateness L is given by

$$L = C - D. \tag{3.6}$$

Completion time, C, is the sum of the ready time, r, and flowtime, F. The flowtime constitutes the total waiting time, W and the total processing time, P, so,

$$F = W + P. \tag{3.7}$$

Together with the TWK-NOP due date, given in equation (3.1), L can be explicitly written as

$$L = W - (\alpha - 1)P - \beta N. \tag{3.8}$$

By taking the square of both sides of (3.8), we obtain

$$L^2 = W^2 - 2W[(\alpha - 1)P + \beta N] + \left[(\alpha - 1)^2 P^2 + 2\beta(\alpha - 1)PN + \beta^2 N^2\right]. \tag{3.9}$$

The resulting missed due date cost for a job is

$$C_M(W, P, N, \alpha_1, \beta_2) = \phi_M\Big\{W^2 - 2W[(\alpha - 1)P + \beta N] +$$

$$\left[(\alpha-1)^2P^2+2\beta(\alpha-1)PN+\beta^2N^2\right]\Big\}, \qquad (3.10)$$

a spline function $\phi_M(\cdot)$ of multipliers α and β and the random variables W, P, and N. Although it is generally true that costs will inevitably be incurred whenever jobs cannot be completed exactly on their assigned due dates, be they early or tardy, it appears more appropriate to employ a missed due date rather than tardiness cost function as a performance measure. In addition, it is reasonable to assume a functional relationship between the missed due date cost and $\phi_M(L^2)$, which is the adopted objective function to be minimized in this study. Both of these cost functions, $C_D(\cdot)$ and $C_M(\cdot)$ quantify the costs in terms of monetary quantities per time unit.

3.3.4 General Assumptions

1. ϕ_D and ϕ_M a are monotone increasing and strictly convex functions in α and β. As well, twice continuously differentiable.
2. Due date allowance cost is independent of the dispatching rule, the total processing time, flowtime of the job and the number of operations of a job.

The validation of the monotonicity and convexity assumptions is justified by Jones (1973). He advocates that the cost of long promise is a steep slope in the neighbourhood of competitive promise lengths. The validations affirm the fact that costs of long promises fit simple differentiable functions. Furthermore, he observes that in production operations, both tardiness and inventory carrying costs display exponential growth and always contribute steep slopes.

The following additional assumptions are made.

3. There are m single non-identical machines in the shop to perform the various types of operations required by the jobs.

4. Jobs arrive at the shop randomly and follow a Poisson process with mean arrival rate λ.

5. The processing times p_i, $i = 1, 2, \ldots$, for each machine are common random variables and equal to p, which is negative-exponentially distributed with mean μ_p.

6. The routing of the jobs through the machines are determined by a fixed probability transition matrix which asserts that each machine is equally likely to be chosen to process the next operation of a job, except that two consecutive operations on the same machine are not permitted.

7. The queue disciplines at each machine are first-come-first-served (FCFS).

8. The machine utilization for each individual machine is equal to a common value $\rho = \lambda\mu_p$ and is less than unity.

9. The shop load ratio ρ_s, which is defined as the ratio of the total work load to the total production capacity available in the shop, remains unchanged throughout.

10. The machine set-up times are included in the processing times and sequence independent and transportation times between machines are ignored.

11. Job preemption is not allowed.

Assumptions (4) through (7) are essentially the sufficient conditions of Jackson's decomposition principle (Jackson, 1963), which states that under such conditions decomposition of a queueing system into a network of independent individual machine systems is possible. In addition, assumption (5) ensures that the job arrival rate at each individual machine is equal to the mean job arrival rate at the system λ, and assumptions (4) and (5) together assert the validity of assumption (8).

3.2.5 The Aggregate Cost

The total cost for each job as a function, $C_T(W, P, N, \alpha, \beta)$ comprised of the due date allowance cost $C_D(\alpha, \beta)$ and the missed due date cost $C_M(W, P, N, \alpha_i, \beta_i)$, i = 1, 2, 3, and 4 for each job is given by;

$$C_T(W, P, N, \alpha, \beta) = \begin{cases} C_1(\cdot) = C_M(W, P, N, \alpha_1, \beta_1) & \alpha_1 \le \alpha_a \text{ and } \beta_1 \le \beta_a \\ C_2(\cdot) = \phi_D(\alpha_2/\alpha_a) + \\ \qquad C_M(W, P, N, \alpha_2, \beta_2) & \alpha_2 > \alpha_a \text{ and } \beta_2 \le \beta_a \\ C_3(\cdot) = \phi_D(\beta_3/\beta_a) + \\ \qquad C_M(W, P, N, \alpha_3, \beta_3) & \alpha_3 \le \alpha_a \text{ and } \beta_3 > \beta_a \\ C_4(\cdot) = \phi_D(\alpha_4/\alpha_a) + \\ \qquad \phi_D(\beta_4/\beta_a) + C_M(W, P, N, \alpha_4, \beta_4) & \alpha_4 > \alpha_a \text{ and } \beta_4 > \beta_a \end{cases} \tag{3.11}$$

where $C_i(\cdot)$, i = 1, 2, 3, and 4 are the total cost per job for α_i and β_i to be less than or greater than α_a and β_a.

The optimal processing time multiplier, α_i^* and the number of operations multiplier, β_i^* should minimize the expected total cost per job. The cost per job is a function of random variables P, W, and N. Subsequently, the expected total cost per job $H(\alpha, \beta)$ is expressed as

$$E[C_T(W, P, N, \alpha, \beta)] = H(\alpha, \beta)$$

$$= \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} C_T(W, P, N, \alpha, \beta)\, f(W, P, N)\, dW\, dP\, dN \tag{3.12}$$

$$= \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} C_i(\cdot)\, f(W, P, N)\, dW\, dP\, dN \tag{3.13}$$

where i = 1, 2, 3, and 4 and together with its perspective boundaries. The optimal α_i^* and β_i^* satisfy the condition that $H(\alpha, \beta) \leq H(\alpha_a, \beta_a)$ will minimize the expected total cost per job. Inspite of that, determination of the optimal α_i^* and multipliers β_i^* for the discontinuous function $H(\alpha_i, \beta_i)$ requires investigation of the cost function $H(\alpha_i, \beta_i)$ over four real intervals of α_i and β_i, namely,

$$u = \{\alpha_1, \beta_1 : (\alpha_1 \leq \alpha_a) \cap (\beta_1 \leq \beta_a)\} \tag{3.14}$$

$$v = \{\alpha_2, \beta_2 : (\alpha_2 > \alpha_a) \cap (\beta_2 \leq \beta_a)\} \tag{3.15}$$

$$y = \{\alpha_3, \beta_3 : (\alpha_3 \leq \alpha_a) \cap (\beta_3 > \beta_a)\} \tag{3.16}$$

$$z = \{\alpha_4, \beta_4 : (\alpha_4 > \alpha_a) \cap (\beta_4 > \beta_a)\}. \tag{3.17}$$

Let $H_i(\alpha_i, \beta_i)$, i = 1, 2, 3, and 4 denote the expected cost functions for the intervals u, v, y and z respectively. It follows that the expected total cost per job is

$$
H(\alpha, \beta) = \begin{cases}
H_1(\cdot) = \int_0^\infty\int_0^\infty\int_0^\infty C_1(W, P, N, \alpha_1, \beta_1)\, f(W, P, N)\, dW\, dP\, dN & \text{if } \alpha_1 \le \alpha_a \text{ and } \beta_1 \le \beta_a \\
H_2(\cdot) = \int_0^\infty\int_0^\infty\int_0^\infty C_2(W, P, N, \alpha_2, \beta_2)\, f(W, P, N)\, dW\, dP\, dN & \text{if } \alpha_2 > \alpha_a \text{ and } \beta_2 \le \beta_a \\
H_3(\cdot) = \int_0^\infty\int_0^\infty\int_0^\infty C_3(W, P, N, \alpha_3, \beta_3)\, f(W, P, N)\, dW\, dP\, dN & \text{if } \alpha_3 \le \alpha_a \text{ and } \beta_3 > \beta_a \\
H_4(\cdot) = \int_0^\infty\int_0^\infty\int_0^\infty C_4(W, P, N, \alpha_4, \beta_4)\, f(W, P, N)\, dW\, dP\, dN & \text{if } \alpha_4 > \alpha_a \text{ and } \beta_4 > \beta_a
\end{cases} \tag{3.18}
$$

3.3 Optimal Multipliers Under General Cost Model

This section considers the general aggregate cost model. Procedures are provided for calculation of the four different expected cost functions. Define f(W, P, N) as the probability density function of continuous random variables, W, P, and N where f(W, N, P) > 0, $\{W, P, N\} \in S$ and $S = \{W, P, N : 0 < W, P, N, < \infty\}$. The construction of f(W, P, N) can be based upon some theoretical considerations of the underlying processes, or it can be estimated statistically from historical operational data. Assuming that the shop is using a priority discipline such as FCFS, where the distribution of time-in-shop is common to all jobs.

3.3.1 Minimization of Expected Cost Function Under General Cost Model

For α_1 and β_1 in interval u, the expected total cost per job shown in (3.18) is

$$H_1(\alpha_1, \beta_1) = \int_0^\infty\int_0^\infty\int_0^\infty C_1(W, P, N, \alpha_1, \beta_1)\, f(W, P, N)\, dW\, dP\, dN$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty \phi_M\Big\{W^2 - 2W[(\alpha_1 - 1)P + \beta_1 N] + \Big[(\alpha_1 - 1)^2 P^2 + 2\beta_1(\alpha_1 - 1)PN + \beta_1{}^2 N^2\Big]\Big\}\, f(W, P, N)\, dW\, dP\, dN \tag{3.19}$$

The first derivative of $H_1(\alpha_1, \beta_1)$ with respect to α_1 is

$$\frac{dH_1(\alpha_1, \beta_1)}{d\alpha_1} = 2\int_0^\infty\int_0^\infty\int_0^\infty\Big[(\alpha_1 - 1)P^2 + \beta_1 PN - WP\Big]\, \phi'_M\, f(W, P, N)\, dW\, dP\, dN \tag{3.20}$$

and first derivative of $H_1(\alpha_1, \beta_1)$ with respect to β_1 is

$$\frac{dH_1(\alpha_1, \beta_1)}{d\beta_1} = 2\int_0^\infty\int_0^\infty\int_0^\infty\Big[(\alpha_1 - 1)PN + \beta_1 N^2 - WN\Big]\, \phi'_M\, f(W, P, N)\, dW\, dP\, dN \tag{3.21}$$

The second derivative of $H_1(\alpha_1, \beta_1)$ with respect to α_1 is

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\alpha_1{}^2} = 4\int_0^\infty\int_0^\infty\int_0^\infty\Big[(\alpha_1 - 1)P^2 + \beta_1 PN - WP\Big]^2\, \phi''_M\, f(W, P, N)\, dW\, dP\, dN$$

$$+ 2\int_0^\infty\int_0^\infty\int_0^\infty P^2 \phi'_M \, f(W, P, N) \, dW \, dP \, dN \tag{3.22}$$

The second derivative of $H_1(\alpha_1, \beta_1)$ with respect to β_1 is

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1{}^2} = 4\int_0^\infty\int_0^\infty\int_0^\infty \left[(\alpha_1 - 1)PN + \beta_1 N^2 - WN\right]^2 \phi''_M \, f(W, P, N) \, dW \, dP \, dN$$
$$+ 2\int_0^\infty\int_0^\infty\int_0^\infty N^2 \phi'_M \, f(W, P, N) \, dW \, dP \, dN \tag{3.23}$$

By differentiating (3.19) twice with respect to α_1 and β_1, we obtain

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1 d\alpha_1} = 4\int_0^\infty\int_0^\infty\int_0^\infty \left\{\left[(\alpha_1 - 1)P^2 + \beta_1 PN - WP\right]\left[(\alpha_1 - 1)PN + \beta_1 N^2 - WN\right]\right\}$$
$$\phi''_M \, f(W, P, N) \, dW \, dP \, dN + 2\int_0^\infty\int_0^\infty\int_0^\infty PN \, \phi'_M \, f(W, P, N) \, dW \, dP \, dN$$
$$(3.24)$$

and

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\alpha_1 d\beta_1} = 4\int_0^\infty\int_0^\infty\int_0^\infty \left\{\left[(\alpha_1 - 1)P^2 + \beta_1 PN - WP\right]\left[(\alpha_1 - 1)PN + \beta_1 N^2 - WN\right]\right\}$$
$$\phi''_M \, f(W, P, N) \, dW \, dP \, dN + 2\int_0^\infty\int_0^\infty\int_0^\infty PN \, \phi'_M \, f(W, P, N) \, dW \, dP \, dN.$$
$$(3.25)$$

Given the function $H_1(\alpha_1, \beta_1)$ differentiable partially in a neighborhood of $(\alpha_1^{\nabla}, \beta_1^{\nabla})$, a relative minimum for the function occurs at a point $(\alpha_1^{\nabla}, \beta_1^{\nabla})$ if the following conditions are met. The conditions (3.26) must hold simultaneously

$$\frac{dH_1(\alpha_1, \beta_1)}{d\alpha_1} = 0 \qquad \text{and} \qquad \frac{dH_1(\alpha_1, \beta_1)}{d\beta_1} = 0. \tag{3.26}$$

Since (3.22) and (3.23) are equal, we can calculate the Hessian Determinant,

$$\left[\frac{d^2H_1(\alpha_1, \beta_1)}{d\alpha_1{}^2}\right]\left[\frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1{}^2}\right] - \left[\frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1 d\alpha_1}\right]^2. \tag{3.27}$$

Then if

$$\left[\frac{d^2H_1(\alpha_1, \beta_1)}{d\alpha_1{}^2}\right]\left[\frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1{}^2}\right] - \left[\frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1 d\alpha_1}\right]^2 > 0 \tag{3.28}$$

at $(\alpha_1^{\nabla}, \beta_1^{\nabla})$, the function, $H_1(\alpha_1, \beta_1)$, is either a maximum or a minimum at the point. If

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\alpha_1{}^2} > 0 \qquad \text{and} \qquad \frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1{}^2} > 0 \tag{3.29}$$

at $(\alpha_1^{\nabla}, \beta_1^{\nabla})$, the value of the function at the point is a minimum and vice versa. The same procedure is applied to the rest of the expected cost function in determining $(\alpha_2^{\nabla}, \beta_2^{\nabla})$, $(\alpha_3^{\nabla}, \beta_3^{\nabla})$, and $(\alpha_4^{\nabla}, \beta_4^{\nabla})$. Table 8 provides a summary of partial derivatives used in determining the local minimum points.

3.4 Optimal Multipliers Under Linear Cost Model

The special case of our general cost model is linear having four component cost functions in linear forms. The most discernible situation to which linearity is applicable is the case where a product has to be delivered at a certain time and at a specified price and where a fixed amount is deducted from this price for each unit of time that delivery is beyond the promised time. Among researcher, Jones (1973), Eilon and Chowdhury (1976), Cheng (1985b) and Seidmann et al. (1981), to mention a few, have discussed the applicability of linear cost functions in the context of job shop scheduling. As well, it actual practice, it is not uncommon to obtain reasonably accurate linear approximations of the cost components constituting the total cost function to be minimized. For the linear cost model, the due date allowance cost is

$$C_D(\alpha, \beta) = \begin{cases} 0 & \alpha_1 \le \alpha_a \text{ and } \beta_1 \le \beta_a \\ K_1(\alpha_2/\alpha_a) & \alpha_2 > \alpha_a \text{ and } \beta_2 \le \beta_a \\ K_2(\beta_3/\beta_a) & \alpha_3 \le \alpha_a \text{ and } \beta_3 > \beta_a \\ K_3(\alpha_4/\alpha_a) + K_3(\beta_4/\beta_a) & \alpha_4 > \alpha_a \text{ and } \beta_4 > \beta_a \end{cases} \tag{3.30}$$

and it follows that the missed due date cost is given by

$$C_M(W, P, N, \alpha_1, \beta_2) = K_M\Big\{W^2 - 2W[(\alpha-1)P + \beta N] + \Big[(\alpha-1)^2P^2 + 2\beta(\alpha-1)PN + \beta^2N^2\Big]\Big\}, \tag{3.31}$$

where K_M and K_i, i = 1, 2, and 3 are known non-negative constants.

3.4.1 Minimization of Expected Linear Cost Function $H_1(\alpha_1, \beta_1)$

Based on previous assumptions, the expected cost for $H_1(\alpha_1, \beta_1)$ over the interval u for the linear model is

$$= \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} K_M\Big\{W^2 - 2W[(\alpha_1 - 1)P + \beta_1 N] + \Big[(\alpha_1 - 1)^2 P^2 + 2\beta_1(\alpha_1 - 1)PN + \beta_1{}^2 N^2\Big]\Big\} f(W, P, N)\, dW\, dP\, dN. \tag{3.32}$$

To determine the minimum point $(\alpha_1^{\nabla}, \beta_1^{\nabla})$ of the unconstrained cost function $H_1(\alpha_1, \beta_1)$, the first derivatives of (3.33) with respect to α_1 and β_1, are set to zero. Thus,

$$2K_M \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \Big[(\alpha_1 - 1)P^2 + \beta_1 PN - WP\Big]\, f(W, P, N)\, dW\, dP\, dN = 0 \tag{3.33}$$

$$2K_M\Big\{(\alpha_1 - 1)\, E(P^2) + \beta E(PN) - E(WP)\Big\} = 0 \tag{3.34}$$

and

$$2K_M \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \Big[(\alpha_1 - 1)PN + \beta_1 N^2 - WN\Big] f(W, P, N)\, dW\, dP\, dN = 0. \tag{3.35}$$

$$2K_M\Big\{(\alpha_1 - 1)\, E(PN) + \beta E(N^2) - E(WN)\Big\} = 0. \tag{3.36}$$

Given two equations, (3.34) and (3.36), and two unknowns, $(\alpha_1^{\nabla}, \beta_1^{\nabla})$ is determined. It follows that

$$\alpha_1^{\nabla} = 1 + \frac{E(WP)\,E(N^2) \; - \; E(WN)\,E(PN)}{E(P^2)\,E(N^2) \; - [E(PN)]^2} \tag{3.37}$$

$$\beta_1^{\nabla} = \frac{E(WP)\,E(PN) \; - \; E(WN)\,E(P^2)}{[E(PN)]^2 \; - \; E(N^2)\,E(P^2)}. \tag{3.38}$$

To ensure that $(\alpha_1^{\nabla}, \beta_1^{\nabla})$ is global minimum over the interval u, we shall compute the partial derivatives of $H_1(\alpha_1, \beta_1)$.

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\alpha_1{}^2} = E(P^2) > 0, \qquad \frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1{}^2} = E(N^2) > 0, \tag{3.39}$$

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1 d\alpha_1} = \frac{d^2H_1(\alpha_1, \beta_1)}{d\alpha_1 d\beta_1} = E(PN). \tag{3.40}$$

The Hessian Determinant is $E(P^2)\,E(N^2) \; - \; [E(PN)]^2 > 0$, and (3.40) holds, $(\alpha_1^{\nabla}, \beta_1^{\nabla})$ is a global minimum point over the interval u.

3.4.2 Minimization of Expected Linear Cost Function $H_2(\alpha_2, \beta_2)$

For interval v, the expected cost function is given by

$$H_2(\alpha_2, \beta_2) = K_1(\alpha_2 / \alpha_a) + \int_0^\infty\!\int_0^\infty\!\int_0^\infty K_M\Big\{W^2 - 2W[(\alpha_2 - 1)P + \beta_2 N] + \Big[(\alpha_2 - 1)^2 P^2 + 2\beta_2(\alpha_2 - 1)PN + \beta_2{}^2 N^2\Big]\Big\} f(W, P, N)\, dW\, dP\, dN. \tag{3.41}$$

Similarly, the minimum point $(\alpha_2^{\nabla}, \beta_2^{\nabla})$ of $H_2(\alpha_2, \beta_2)$ over the interval $u \cup v$ has to satisfy the following condition,

$$\frac{K_1}{\alpha_a} + 2K_M \int_0^\infty\int_0^\infty\int_0^\infty \Big[(\alpha_2 - 1)P^2 + \beta_2 PN - WP\Big] f(W, P, N)\, dW\, dP\, dN = 0 \tag{3.42}$$

and

$$2K_M \int_0^\infty\int_0^\infty\int_0^\infty \Big[(\alpha_2 - 1)PN + \beta_2 N^2 - WN\Big] f(W, P, N)\, dW\, dP\, dN = 0. \tag{3.43}$$

The equations which determine the optimal processing time and number of operations multipliers are

$$\frac{K_1}{\alpha_a} + 2K_M\Big\{(\alpha_2 - 1)\, E(P^2) + \beta E(PN) - E(WP)\Big\} = 0 \tag{3.44}$$

and

$$2K_M\Big\{(\alpha_2 - 1)\, E(PN) + \beta E(N^2) - E(WN)\Big\} = 0. \tag{3.45}$$

By solving for $(\alpha_2^\nabla, \beta_2^\nabla)$ from (3.44) and (3.45), we obtain the following results,

$$\alpha_2^\nabla = 1 + \frac{\alpha_a\Big\{E(WP)\, E(N^2) \ - \ E(WN)\, E(PN)\Big\} - K_1 E(N^2)}{\alpha_a\Big\{E(N^2)\, E(P^2) - [E(PN)]^2\Big\}} \tag{3.46}$$

and

$$\beta_2^\nabla = \frac{\alpha_a\Big\{E(WP)\, E(PN) \ - \ E(WN)\, E(P^2)\Big\} - K_1 E(PN)}{\alpha_a\Big\{[E(PN)]^2 \ - \ E(N^2)\, E(P^2)\Big\}}. \tag{3.47}$$

3.4.3 Minimization of Expected Linear Cost Function $H_3(\alpha_3, \beta_3)$

The expected cost function, $H_3(\alpha_3, \beta_3)$ over the interval y is given by

$$H_3(\alpha_3, \beta_3) = K_2(\beta_3 / \beta_a) + \int_0^\infty\int_0^\infty\int_0^\infty K_M\Big\{W^2 - 2W[(\alpha_3 - 1)P + \beta_3 N] + \Big[(\alpha_3 - 1)^2 P^2 + 2\beta_3(\alpha_3 - 1)PN + \beta_3{}^2 N^2\Big]\Big\} f(W, P, N)\, dW\, dP\, dN. \quad (3.48)$$

The minimum point of $(\alpha_3^\nabla, \beta_3^\nabla)$ over the interval $u \cup v \cup y$ of the unconstrained function $H_3(\alpha_3, \beta_3)$ can be determined by differentiating (3.48) with respect to α_3 and β_3 and setting the result equal to zero. Hence,

$$2K_M \int_0^\infty\int_0^\infty\int_0^\infty \Big[(\alpha_3 - 1)P^2 + \beta_3 PN - WP\Big] f(W, P, N)\, dW\, dP\, dN = 0 \quad (3.49)$$

$$2K_M\Big\{(\alpha_3 - 1)\, E(P^2) + \beta E(PN) - E(WP)\Big\} = 0 \quad (3.50)$$

and

$$\frac{K_2}{\beta_a} + 2K_M \int_0^\infty\int_0^\infty\int_0^\infty \Big[(\alpha_3 - 1)PN + \beta_3 N^2 - WN\Big] f(W, P, N)\, dW\, dP\, dN = 0 \quad (3.51)$$

$$\frac{K_2}{\beta_a} + 2K_M\Big\{(\alpha_2 - 1)\, E(PN) + \beta E(N^2) - E(WN)\Big\}. \quad (3.52)$$

Subsequently, $(\alpha_3^\nabla, \beta_3^\nabla)$ is ascertained by solving (3.50) and (3.52) and so,

$$\alpha_3^{\nabla} = 1 + \frac{\beta_a\left\{E(WP)\,E(N^2) \;-\; E(WN)\,E(PN)\right\} - K_2 E(PN)}{\beta_a\left\{E(N^2)\,E(P^2) - [E(PN)]^2\right\}} \qquad (3.53)$$

$$\beta_3^{\nabla} = \frac{\beta_a\left\{E(WP)\,E(PN) \;-\; E(WN)\,E(P^2)\right\} - K_2 E(P^2)}{\beta_a\left\{[E(PN)]^2 \;-\; E(N^2)\,E(P^2)\right\}}. \qquad (3.54)$$

3.4.4 Minimization of Expected Linear Cost Function $H_4(\alpha_4, \beta_4)$

Finally, the expected total cost per job for α_4 and β_4 in interval z is

$$H_4(\alpha_4, \beta_4) = K_3(\alpha_4 / \alpha_a) + K_3(\beta_4 / \beta_{a4}) + \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} K_M\left\{W^2 - 2W[(\alpha_4 - 1)P + \beta_4 N] + \left[(\alpha_3 - 1)^2 P^2 + 2\beta_3(\alpha_4 - 1)PN + \beta_4{}^2 N^2\right]\right\}. \qquad (3.55)$$

The first derivative of $H_4(\alpha_4, \beta_4)$ can be obtained by differentiating (3.55) once with respect to α_4 and β_4. This gives

$$\frac{K_3}{\alpha_a} + 2K_M \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\left[(\alpha_4 - 1)P^2 + \beta_4 PN - WP\right] f(W, P, N)\; dW\, dP\, dN \qquad (3.56)$$

and

$$\frac{K_3}{\beta_a} + 2K_M \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\left[(\alpha_4 - 1)PN + \beta_4 N^2 - WN\right] f(W, P, N)\; dW\, dP\, dN. \qquad (3.57)$$

Setting the results equal to zero, the minimum point $(\alpha_4^{\nabla}, \beta_4^{\nabla})$ of $H_4(\alpha_4, \beta_4)$ over the interval $u \cup v \cup y \cup z$ is found.

Consequently,

$$\alpha_4^{\nabla} = 1 + \frac{\alpha_a\,\beta_a\left\{E(WP)\,E(N^2)\ -\ E(WN)\,E(PN)\right\} + \alpha_a K_3 E(PN)\ -\ \beta_a K_3 E(N^2)}{\alpha_a\,\beta_a\left\{E(N^2)\,E(P^2) - [E(PN)]^2\right\}}$$

(3.58)

$$\beta_4^{\nabla} = \frac{\alpha_a\,\beta_a\left\{E(WP)\,E(PN)\ -\ E(WN)\,E(P^2)\right\} + \alpha_a K_3 E(P^2) - \beta_a K_3 E(PN)}{\alpha_a\,\beta_a\left\{[E(PN)]^2\ -\ E(N^2)\,E(P^2)\right\}}.$$

(3.59)

Table 9 provides a summary of the minimum points under various scenarios.

$E(P^2)$ is the sum of the variance and the squared mean total processing time of the jobs. For any job shop, it is relatively easy to estimate the value of $E(P^2)$. However, the information on E(WP) is not readily available. Therefore, to obtain a reasonable estimate of E(WP), we record the values of the product of the waiting time and processing time of jobs over a period of time. Moreover, if the processing time independent dispatching rule is utilized, for example, FCFS, LCFS, SPT, in the shop, estimation of E(WP) will become much easier. This is due to the independence property where W and P are independent and it follows that E(WP) can be expressed as a product of E(W) * E(P). E(W) and E(P) can either be theoretically determined by considering the underlying random processes or empirically constructed from historical shop data.

Table 8 The Results of Minimization of Expected Cost Function, $H_i(\alpha_i, \beta_i)$

$H_1(\alpha_1, \beta_1)$

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\alpha_1^2} = 4\int_0^\infty\int_0^\infty\int_0^\infty\left[(\alpha_1-1)P^2+\beta_1 PN-WP\right]^2 \phi''_M\, f(W, P, N)\, dW\, dP\, dN + 2\int_0^\infty\int_0^\infty\int_0^\infty P^2\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1^2} = 4\int_0^\infty\int_0^\infty\int_0^\infty\left[(\alpha_1-1)PN+\beta_1 N^2-WN\right]^2 \phi''_M\, f(W, P, N)\, dW\, dP\, dN + 2\int_0^\infty\int_0^\infty\int_0^\infty N^2\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1 d\alpha_1} = 4\int_0^\infty\int_0^\infty\int_0^\infty\left\{\left[(\alpha_1-1)P^2+\beta_1 PN-WP\right]\left[(\alpha_1-1)PN+\beta_1 N^2-WN\right]\right\} \phi''_M\, f(W, P, N)\, dW\, dP\, dN + 2\int_0^\infty\int_0^\infty\int_0^\infty PN\,\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$

$H_2(\alpha_2, \beta_2)$

$$\frac{d^2H_2(\alpha_2, \beta_2)}{d\alpha_2^2} = \frac{1}{\alpha_{a2}^2}\phi''_D(\alpha_2/\alpha_{a2}) + 2\int_0^\infty\int_0^\infty\int_0^\infty P^2\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$
$$4\int_0^\infty\int_0^\infty\int_0^\infty\left[(\alpha_2-1)P^2+\beta_2 PN-WP\right]^2 \phi''_M\, f(W, P, N)\, dW\, dP\, dN$$

$$\frac{d^2H_2(\alpha_2, \beta_2)}{d\beta_2{}^2} = 4\int_0^\infty\int_0^\infty\int_0^\infty\left[(\alpha_2 - 1)PN + \beta_2N^2 - WN\right]^2 \phi''_M\, f(W, P, N)\, dW\, dP\, dN$$
$$+ 2\int_0^\infty\int_0^\infty\int_0^\infty N^2\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$

$$\frac{d^2H_1(\alpha_1, \beta_1)}{d\beta_1 d\alpha_1} = 4\int_0^\infty\int_0^\infty\int_0^\infty\left\{\left[(\alpha_2 - 1)P^2 + \beta_2PN - WP\right]\left[(\alpha_2 - 1)PN + \beta_2N^2 - WN\right]\right\}$$
$$\phi''_M\, f(W, P, N)\, dW\, dP\, dN + 2\int_0^\infty\int_0^\infty\int_0^\infty PN\,\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$

$H_3(\alpha_3, \beta_3)$

$$\frac{d^2H_3(\alpha_3, \beta_3)}{d\alpha_3{}^2} = 4\int_0^\infty\int_0^\infty\int_0^\infty\left[(\alpha_3 - 1)P^2 + \beta_3PN - WP\right]^2 \phi''_M\, f(W, P, N)\, dW\, dP\, dN$$
$$+ 2\int_0^\infty\int_0^\infty\int_0^\infty P^2\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$

$$\frac{d^2H_3(\alpha_3, \beta_3)}{d\beta_3{}^2} = \frac{1}{\beta_{a3}^2}\phi''_D(\beta_3 / \beta_{a3}) + 2\int_0^\infty\int_0^\infty\int_0^\infty N^2\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$
$$+ 4\int_0^\infty\int_0^\infty\int_0^\infty\left[(\alpha_3 - 1)PN + \beta_3N^2 - WN\right]^2 \phi''_M\, f(W, P, N)\, dW\, dP\, dN$$

$$\frac{d^2H_3(\alpha_3, \beta_3)}{d\beta_3 d\alpha_3} = 4\int_0^\infty\int_0^\infty\int_0^\infty\left\{\left[(\alpha_3 - 1)P^2 + \beta_3PN - WP\right]\left[(\alpha_3 - 1)PN + \beta_3N^2 - WN\right]\right\}$$
$$\phi''_M\, f(W, P, N)\, dW\, dP\, dN + 2\int_0^\infty\int_0^\infty\int_0^\infty PN\,\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$

$H_4(\alpha_4, \beta_4)$

$$\frac{d^2H_4(\alpha_4, \beta_4)}{d\alpha_4{}^2} = \frac{1}{\alpha_{a4}^2}\phi''_D(\alpha_4 / \alpha_{a4}) + 2\int_0^\infty\int_0^\infty\int_0^\infty P^2\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$

$$4\int_0^\infty\int_0^\infty\int_0^\infty\left[(\alpha_4 - 1)P^2 + \beta_4 PN - WP\right]^2 \phi''_M\, f(W, P, N)\, dW\, dP\, dN$$

$$\frac{d^2H_4(\alpha_4, \beta_4)}{d\beta_4{}^2} = \frac{1}{\beta_{a4}^2}\phi''_D(\beta_4 / \beta_{a4}) + 2\int_0^\infty\int_0^\infty\int_0^\infty N^2\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$

$$4\int_0^\infty\int_0^\infty\int_0^\infty\left[(\alpha_4 - 1)PN + \beta_4 N^2 - WN\right]^2 \phi''_M\, f(W, P, N)\, dW\, dP\, dN$$

$$\frac{d^2H_4(\alpha_4, \beta_4)}{d\beta_4 d\alpha_4} = 4\int_0^\infty\int_0^\infty\int_0^\infty\left\{\left[(\alpha_4 - 1)P^2 + \beta_4 PN - WP\right]\left[(\alpha_4 - 1)PN + \beta_4 N^2 - WN\right]\right\}$$

$$\phi''_M\, f(W, P, N)\, dW\, dP\, dN + 2\int_0^\infty\int_0^\infty\int_0^\infty PN\,\phi'_M\, f(W, P, N)\, dW\, dP\, dN$$

Table 9 A Summary of the Minimum Points for the Four Scenarios Under Linear Cost Model

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Scenario #1

$$\alpha_1^{\nabla} = 1 + \frac{E(WP)\,E(N^2) - E(WN)\,E(PN)}{E(P^2)\,E(N^2) - [E(PN)]^2}$$

$$\beta_1^{\nabla} = \frac{E(WP)\,E(PN) - E(WN)\,E(P^2)}{[E(PN)]^2 - E(N^2)\,E(P^2)}$$

Scenario #2

$$\alpha_2^{\nabla} = 1 + \frac{\alpha_a\left\{E(WP)\,E(N^2) - E(WN)\,E(PN)\right\} - K_1 E(N^2)}{\alpha_a\left\{E(N^2)\,E(P^2) - [E(PN)]^2\right\}}$$

$$\beta_2^{\nabla} = \frac{\alpha_a\left\{E(WP)\,E(PN) - E(WN)\,E(P^2)\right\} - K_1 E(PN)}{\alpha_a\left\{[E(PN)]^2 - E(N^2)\,E(P^2)\right\}}$$

Scenario #3

$$\alpha_3^{\nabla} = 1 + \frac{\beta_a\left\{E(WP)\,E(N^2) - E(WN)\,E(PN)\right\} - K_2 E(PN)}{\beta_a\left\{E(N^2)\,E(P^2) - [E(PN)]^2\right\}}$$

$$\beta_3^{\nabla} = \frac{\beta_a\left\{E(WP)\,E(PN) - E(WN)\,E(P^2)\right\} - K_2 E(P^2)}{\beta_a\left\{[E(PN)]^2 - E(N^2)\,E(P^2)\right\}}$$

Scenario #4

$$\alpha_4^{\nabla} = 1 + \frac{\alpha_a \beta_a \left\{E(WP)\,E(N^2) - E(WN)\,E(PN)\right\} + \alpha_a K_3 E(PN) - \beta_a K_3 E(N^2)}{\alpha_a \beta_a \left\{E(N^2)\,E(P^2) - [E(PN)]^2\right\}}$$

$$\beta_4^{\nabla} = \frac{\alpha_a \beta_a \left\{E(WP)\,E(PN) - E(WN)\,E(P^2)\right\} + \alpha_a K_3 E(P^2) - \beta_a K_3 E(PN)}{\alpha_a \beta_a \left\{[E(PN)]^2 - E(N^2)\,E(P^2)\right\}}$$

Chapter IV

SIMULATION STUDY OF JOB SHOP SCHEDULING WITH TWK-NOP DUE DATES

This chapter presents a study of a hypothetical job shop by computer simulation. The purpose is to investigate the optimal multipliers, α^* and β^* for the TWK-NOP due date. Since the linear cost model under TWK-NOP due date are computationally intractable, any attempt to tackle it fails analytically. The only feasible method to deal with this job shop scheduling problem is a computer simulation. The simulation model is written in discrete simulation language SIMCRIPT II.5 (Russell, 1983).

4.1 The Hypothetical Job Shop Model

The hypothetical job shop in this study consists of m = 5 single non-identical machines. The characteristics of each job are generated on its arrival at the shop. Arrival of jobs at the shop follows a Poisson process with a mean arrival rate λ determined by:

$$\lambda = \frac{m\rho_s}{\mu_p \mu_N} \tag{4.1}$$

where λ is the mean job arrival rate; m is the number of machines in the shop; μ_p is the mean processing time on a machine; μ_N is the mean number of operations per job. The interarrival rate is determined by each shop utilization setting. In the present case, the number of operations per job is m. Therefore, (4.1) becomes

$$\lambda = \frac{\rho_s}{\mu_p} \tag{4.2}$$

which means that the shop load ratio ρ_s is equal to the individual machine utilization ρ. The required number of operations for each job is uniformly distributed ranging from 1 to 2m - 1. This makes E(N) = m and $E(N^2) = m(4m-1)/3$. The routing of each job is determined by a fixed-probability transition matrix which has all equal entries except for zeros on the principal diagonal. The distribution of the processing times is the same for each machine is negatively exponentially distributed with mean 1.0. Each machine is equally likely to perform a job's next operation. A machine could perform more than one operation for one job, but no consecutive operations are allowed to perform on the same machine.

By adjusting λ appropriately, we are able to obtain the desired shop load ratio ρ_s ($\rho_s = 0.9$) to be used in our experiment. This implies that the degree of congestion in the shop can be controlled by varying the mean job arrival rate. For the experiment, the shop is allowed a run-in period to reach steady state which is the time from the start of the simulation run to the completion of the 1,000th job. Then, the statistics for the next 4,000 jobs are collected for analysis. The simulation run stops at the completion of the last job in the 4,000 job set. A total of 100 experiments are performed, representing all possible combinations of the two multipliers involved with four replications in each combination.

4.2 Model Assumptions

In addition to the general assumptions listed in Chapter 3, other explicit assumptions about the model are as follows.

1. The machine setup times are included in the processing times and are sequence independent.
2. Transportation times between machines are neglected.
3. Job pre-emption is not permitted.

4. Each machine is continuously available for production.

5. Machines will never break down.

6. Only one job may be processed on one machine at a given time.

7. Only one machine type is required for any given operation.

8. Alternate routing is not considered.

9. There are no labour constraints.

10. The dispatching rule is First-Come-First-Served.

With the above characteristics, Cheng (1983) derived the following theoretical results

$$E(W) = \frac{m\lambda\mu_p^2}{(1-\lambda\mu_p)} \tag{4.3}$$

$$E(P) = m\mu_p \tag{4.4}$$

$$E(P^2) = \frac{2}{3}m(2m+1)\mu_p^2. \tag{4.5}$$

Therefore, α_i^∇ and β_i^∇, i = 1, 2, 3, and 4 from Table 9 for a linear cost model, can be expressed in terms of the known shop variables. Accordingly, the optimal processing time multiple, α^* and optimal number of operations, β^* can be determined.

4.3 Experimental Results

Simulation runs are performed to search for the optimal multipliers, α^* and β^*. The results of the simulation for the optimal processing time and number of operations multipliers with respect to mean squared lateness is displayed in Tables 10 - 13. Assuming the averages of processing time multiplier and number of operations are 4 and 5 units respectively, and the non-negative constants of K_i, i = 1, 2, and 3, are 1500, 700 and 1100 units respectively, we can calculate the expected aggregate cost function.

Figure 3 shows of symmetry of mean absolute lateness for missed due date cost for a job. Figures 4 - 7 depicts the expected total cost per job versus α_i and β_i, where i = 0, 1, 2, ..., 9, for the TWK-NOP due date. The difference in the graphs show the significance of due date allowance cost affecting the total cost. The global minimum point occurs at α^* = 3 and β^* = 5. Therefore, given the average processing time and number of operations multipliers quoted in the particular industry are known, the optimal processing time and number of operations should be set at 3 and 5 units respectively, to ensure the minimization of expected total cost per job.

Table 10 Simulation Runs to Search for the Optimal Processing Time and Number of Operations Multiples, α^* and β^* for TWK-NOP due dates.

First Replication

α	β	$E(L^2)$	Expected Total Cost
0	0	2822.77	2823
	1	2335.60	2336
	2	1912.21	1912
	3	1552.60	1553
	4	1256.77	1257
	5	1024.72	1025
	6	856.44	1696
	7	751.95	1732
	8	711.23	1831
	9	734.29	1994
1	0	2336.62	2337
	1	1912.25	1912
	2	1551.66	1552
	3	1254.85	1255
	4	1021.82	1022
	5	852.57	853
	6	747.09	1587
	7	705.40	1685
	8	727.48	1847
	9	813.34	2073
2	0	1921.87	1922
	1	1560.31	1560
	2	1262.52	1263
	3	1028.51	1029
	4	858.28	858
	5	751.82	752
	6	709.15	1549
	7	730.25	1710
	8	815.13	1935
	9	963.79	2224
3	0	1578.53	1579
	1	1279.76	1280
	2	1044.78	1045
	3	873.57	874
	4	766.13	766
	5	**722.48**	**722**
	6	742.61	1583
	7	826.51	1807
	8	974.19	2094
	9	1185.65	2866

α	β	$E(L^2)$	Expected Total Cost
4	0	1306.59	1307
	1	1070.62	1071
	2	898.44	898
	3	790.03	790
	4	745.39	745
	5	764.54	765
	6	847.46	1688
	7	994.17	1974
	8	1204.65	2325
	9	1478.91	2739
5	0	1106.05	1106
	1	932.89	1308
	2	823.50	1574
	3	777.89	1903
	4	796.06	2296
	5	878.00	2753
	6	1023.73	3738
	7	1233.23	4088
	8	1506.51	4501
	9	1843.57	4978
6	0	976.92	977
	1	866.55	1242
	2	819.96	1567
	3	837.15	1962
	4	918.12	2418
	5	1062.87	2938
	6	1271.39	4361
	7	1543.70	4773
	8	1879.78	5250
	9	2279.64	5789
7	0	919.19	919
	1	871.62	1247
	2	887.83	1638
	3	967.82	2093
	4	111.59	2612
	5	1319.14	3194
	6	1590.46	5055
	7	1925.56	5530
	8	2324.45	6069
	9	2787.11	6672

α	β	$E(L^2)$	Expected Total Cost
8	0	932.86	933
	1	948.09	1023
	2	1027.10	1777
	3	1169.89	2295
	4	1376.46	2876
	5	1646.81	3522
	6	1980.93	5820
	7	2378.83	6358
	8	2840.52	6960
	9	3365.98	7626
9	0	1017.93	1018
	1	1095.96	1471
	2	1237.78	1988
	3	1443.36	2568
	4	1712.73	3213
	5	2045.88	3921
	6	2442.80	6658
	7	2903.51	7258
	8	3427.99	7923
	9	4016.25	8651

Table 11 Simulation Runs to Search for the Optimal Processing Time and Number of Operations Multiples, α^* and β^* for TWK-NOP due dates.

Second Replication

α	β	$E(L^2)$	Expected Total Cost
0	0	2951.49	2952
	1	2425.96	2426
	2	1963.50	1964
	3	1564.11	1564
	4	1227.79	1228
	5	954.53	955
	6	744.35	1584
	7	597.23	1577
	8	513.18	1633
	9	492.19	1752
1	0	2421.14	2421
	1	1958.85	1959
	2	1559.63	1560
	3	1223.48	1223
	4	950.40	950
	5	740.38	740
	6	593.43	1433
	7	509.55	1490
	8	488.74	1608
	9	531.00	1791
	0	1963.85	1964
	1	1564.80	1565
	2	1228.82	1229
	3	955.91	956
	4	746.07	746
	5	599.29	599
	6	515.58	1356
	7	494.94	1475
	8	537.37	1657
	9	642.86	1903
3	0	1579.62	1579
	1	1243.81	1244
	2	971.07	971
	3	761.40	761
	4	614.79	615
	5	**531.25**	**531**
	6	510.78	1350
	7	553.38	1533
	8	659.05	1779
	9	827.78	2088

α	β	$E(L^2)$	Expected Total Cost
4	0	1268.45	1269
	1	995.88	996
	2	786.38	786
	3	639.94	640
	4	556.58	557
	5	536.28	536
	6	579.05	1419
	7	684.88	1664
	8	853.79	1974
	9	1085.76	2346
5	0	1030.34	1030
	1	821.01	1196
	2	674.75	1424
	3	591.55	1716
	4	571.42	2867
	5	614.36	2489
	6	720.37	3435
	7	889.44	3744
	8	1121.59	3747
	9	1416.80	4591
6	0	865.29	865
	1	719.20	1094
	2	636.17	1386
	3	616.21	1741
	4	659.32	2955
	5	765.50	2640
	6	934.75	4024
	7	1167.06	4397
	8	1462.45	4830
	9	1820.90	5330
7	0	773.29	773
	1	690.44	1065
	2	670.65	1420
	3	713.94	1838
	4	820.29	2376
	5	989.70	2864
	6	1222.19	4687
	7	1517.74	5122
	8	1876.36	5621
	9	2298.05	6183

α	β	$E(L^2)$	Expected Total Cost
8	0	754.36	754
	1	734.74	1109
	2	778.20	1528
	3	884.72	2009
	4	1054.31	3350
	5	1286.96	3161
	6	1582.69	5423
	7	1941.48	5922
	8	2363.34	6484
	9	2848.27	7109
9	0	808.48	808
	1	852.11	1227
	2	958.80	1708
	3	1128.56	2253
	4	1361.38	3657
	5	1657.28	3532
	6	2016.24	6231
	7	2438.28	6793
	8	2923.38	7418
	9	3471.54	8106

Table 12 Simulation Runs to Search for the Optimal Processing Time and Number of Operations Multiples, α^* and β^* for TWK-NOP due dates.

Third Replication

α	β	$E(L^2)$	Expected Total Cost
0	0	2992.92	2992
	1	2453.16	2453
	2	1976.56	1976
	3	1563.10	1563
	4	1212.78	1213
	5	925.61	926
	6	701.59	1581
	7	540.71	1521
	8	442.98	1563
	9	408.39	1668
1	0	2435.98	2436
	1	1961.33	1961
	2	1549.83	1550
	3	1201.48	1201
	4	916.27	916
	5	694.21	694
	6	535.29	1375
	7	439.52	1419
	8	406.89	2015
	9	437.41	1697
2	0	1956.43	1956
	1	1546.89	1547
	2	1200.50	1201
	3	917.25	917
	4	697.15	697
	5	540.19	540
	6	446.38	1286
	7	415.72	1369
	8	448.20	1568
	9	543.83	1804
3	0	1554.27	1554
	1	1209.84	1210
	2	928.56	929
	3	710.42	710
	4	555.42	555
	5	**463.57**	**462**
	6	434.87	1275
	7	469.31	1449
	8	566.90	1687
	9	727.63	1988

α	β	$E(L^2)$	Expected Total Cost
4	0	1229.51	1230
	1	950.19	950
	2	734.01	734
	3	580.98	581
	4	491.09	491
	5	464.34	464
	6	500.75	1341
	7	600.30	1580
	8	726.99	1882
	9	988.83	2249
5	0	982.14	982
	1	767.93	1142
	2	616.85	1366
	3	528.93	1653
	4	504.15	2800
	5	542.51	2417
	6	644.02	3359
	7	808.67	3663
	8	1036.48	4031
	9	1327.42	4462
6	0	812.17	812
	1	663.06	1038
	2	577.09	1327
	3	554.27	1679
	4	594.60	2890
	5	698.07	2573
	6	864.68	3954
	7	1094.45	4324
	8	1387.35	4757
	9	1743.41	5253
7	0	719.58	719
	1	635.58	1010
	2	614.72	1364
	3	657.01	1782
	4	762.44	3050
	5	931.02	2806
	6	1162.74	4627
	7	1457.61	5062
	8	1815.62	5560
	9	2236.78	6121

α	β	$E(L^2)$	Expected Total Cost
8	0	704.39	704
	1	685.50	1060
	2	729.74	1479
	3	837.14	1962
	4	1007.68	3303
	5	1241.36	3116
	6	1538.19	5379
	7	1898.17	5879
	8	2321.29	6442
	9	2807.55	7068
9	0	766.60	766
	1	812.81	1187
	2	922.16	1672
	3	1094.66	2219
	4	1330.31	3626
	5	1629.10	3504
	6	1991.03	6206
	7	2416.12	6771
	8	2904.34	7399
	9	3455.72	8090

Table 13 Simulation Runs to Search for the Optimal Processing Time and Number of Operations Multiples, α^* and β^* for TWK-NOP due dates.

Fourth Replication

α	β	$E(L^2)$	Expected Total Cost
0	0	4015.03	4015
	1	3372.50	3373
	2	2795.25	2795
	3	2283.28	2283
	4	1836.57	1837
	5	1455.14	1455
	6	1138.99	1979
	7	888.11	1868
	8	702.51	1822
	9	582.17	1842
1	0	3374.22	3374
	1	2796.48	2796
	2	2284.00	2282
	3	1836.80	1837
	4	1454.88	1455
	5	1138.23	1138
	6	886.85	1726
	7	700.75	1681
	8	579.92	1700
	9	524.37	1784
2	0	2807.25	2807
	1	2294.28	2294
	2	1846.58	1847
	3	1464.16	1464
	4	1147.01	1147
	5	895.14	895
	6	708.54	1550
	7	587.22	1567
	8	531.17	1651
	9	540.39	1800
3	0	2314.11	2314
	1	1865.92	1866
	2	1483.00	1483
	3	1165.35	1165
	4	912.99	912
	5	**752.79**	**630**
	6	604.07	1444
	7	547.52	1528
	8	556.25	1676
	9	630.25	1890

α	β	$E(L^2)$	Expected Total Cost
4	0	1894.80	1895
	1	1511.39	1511
	2	1193.25	1193
	3	940.38	940
	4	725.89	753
	5	630.47	725
	6	573.43	1413
	7	581.66	1562
	8	655.17	1776
	9	793.95	2054
5	0	1549.33	1549
	1	1230.69	1605
	2	977.33	1727
	3	789.24	1914
	4	666.43	2932
	5	608.89	2483
	6	616.62	3331
	7	689.63	3544
	8	827.91	3822
	9	1031.47	4166
6	0	1277.69	1277
	1	1023.83	1398
	2	835.24	1585
	3	711.93	1836
	4	653.90	2949
	5	661.13	2536
	6	733.65	3823
	7	871.43	4101
	8	1074.50	4444
	9	1342.83	4852
7	0	1079.88	1079
	1	890.80	1265
	2	766.99	1516
	3	708.46	1833
	4	715.20	3011
	5	787.22	2662
	6	924.51	4389
	7	1127.07	4732
	8	1394.91	5139
	9	1728.02	5613

α	β	$E(L^2)$	Expected Total Cost
8	0	955.90	955
	1	831.60	1206
	2	772.57	1522
	3	778.82	1903
	4	850.34	3146
	5	987.13	2862
	6	1189.20	5030
	7	1456.54	5437
	8	1789.16	5910
	9	2187.05	6448
9	0	905.76	905
	1	846.24	1221
	2	851.99	1601
	3	923.01	2048
	4	1059.31	3355
	5	1260.88	3135
	6	1527.72	5742
	7	1859.84	6214
	8	2257.24	6752
	9	2719.91	7354

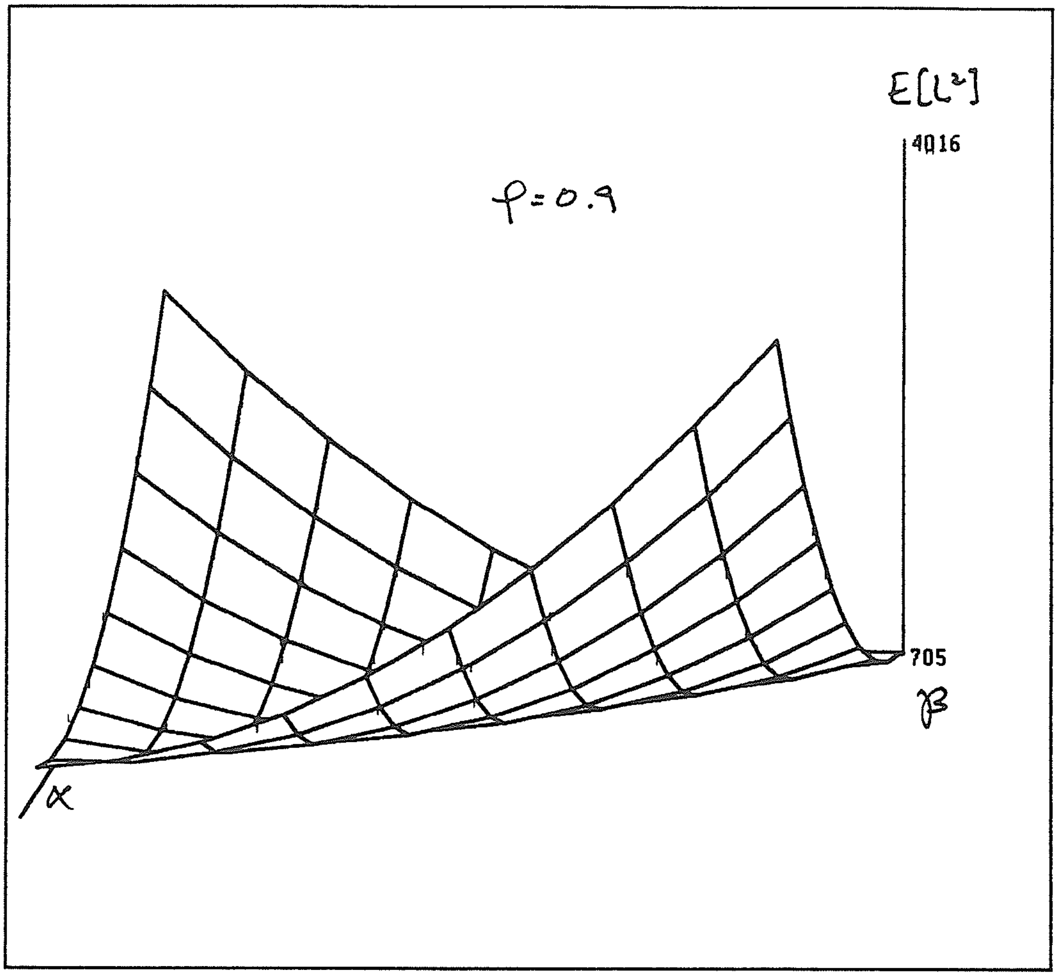

Figure 3 A Graph of $E(L^2)$ Versus α and β at $\rho = 0.9$ for the TWK-NOP Due Dates with Replication No. 1

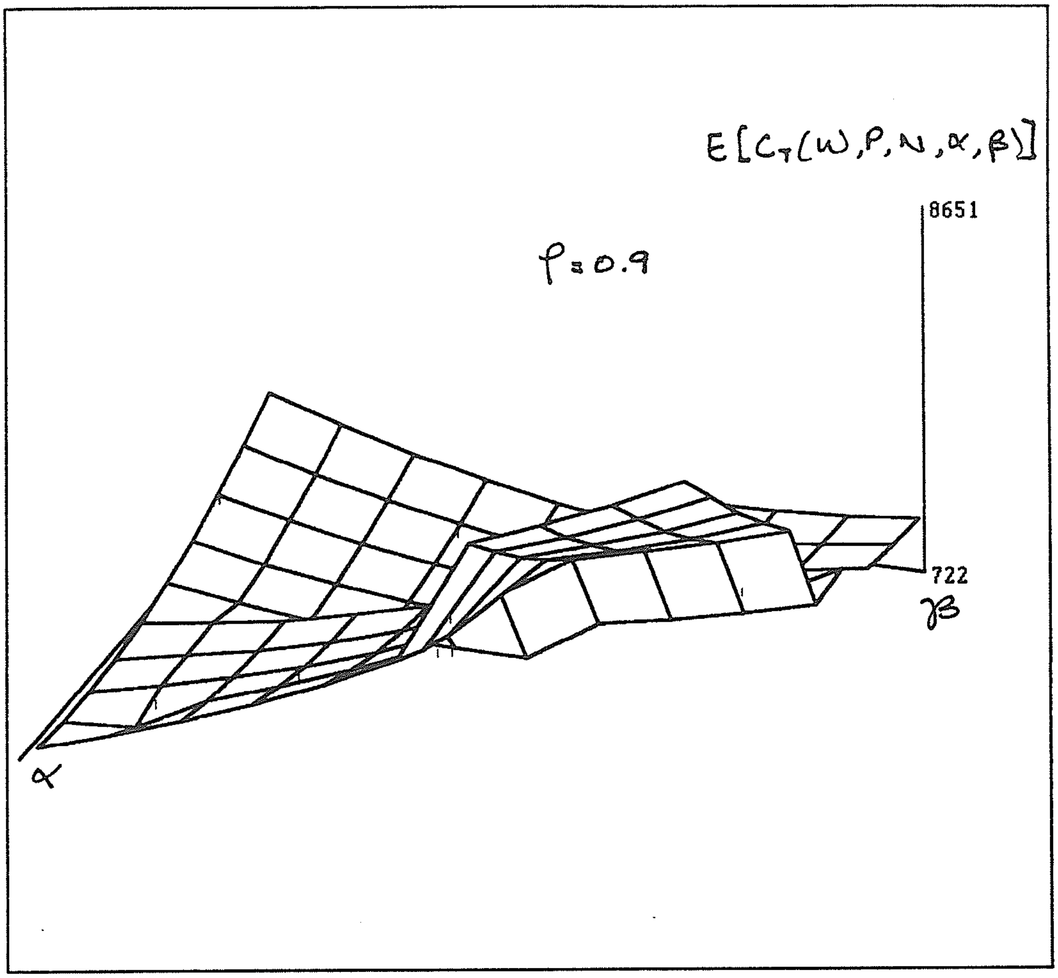

Figure 4 A Graph of C_T(W, P, N, α, β) Versus α and β at $\rho = 0.9$ for the TWK-NOP Due Dates with Replication No. 1

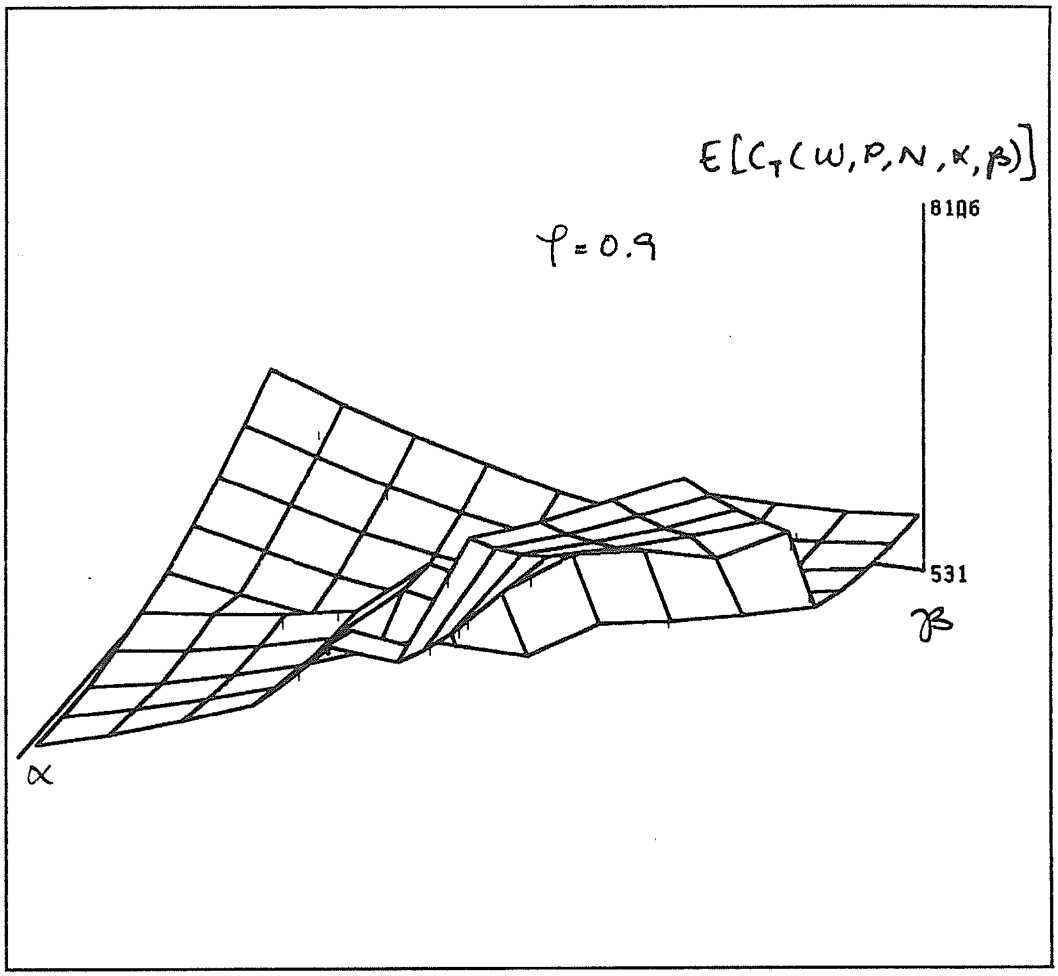

Figure 5 A Graph of $C_T(W, P, N, \alpha, \beta)$ Versus α and β at $\rho = 0.9$ for the TWK-NOP Due Dates with Replication No. 2

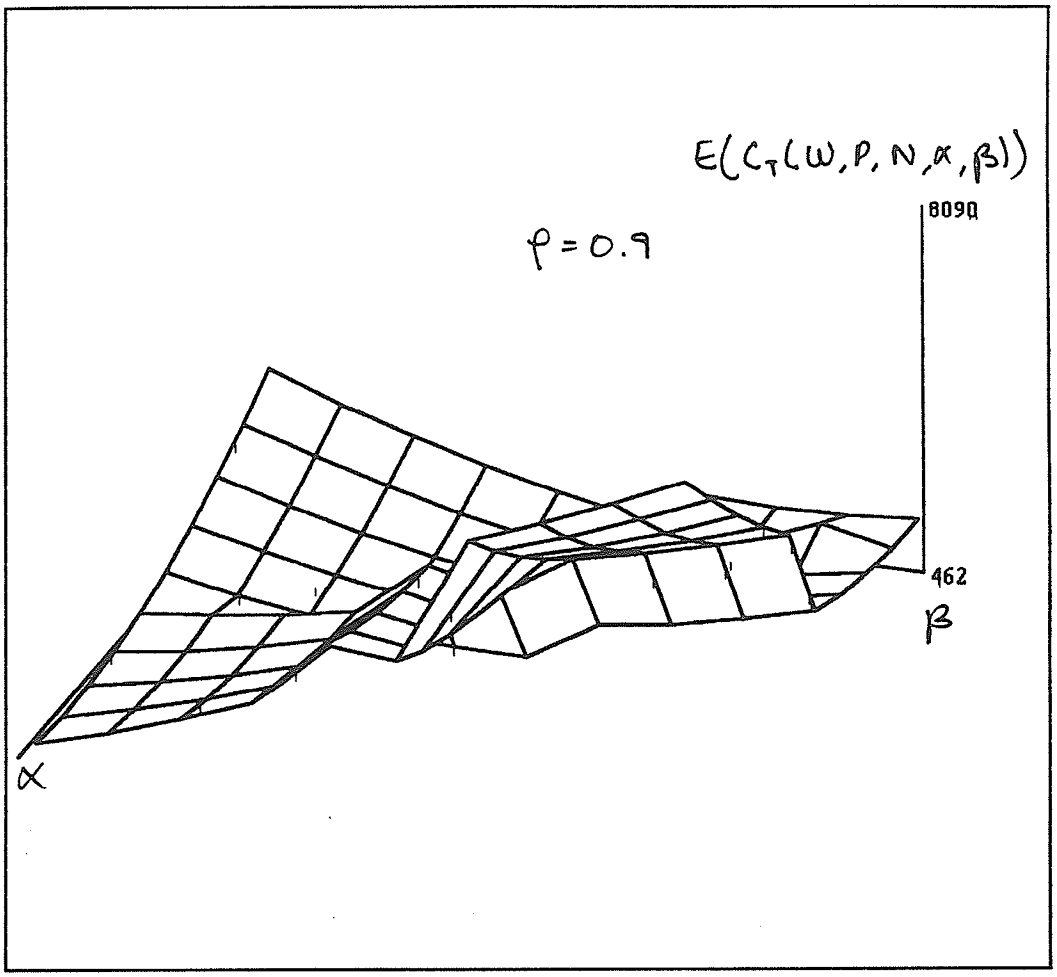

Figure 6 A Graph of $C_T(W, P, N, \alpha, \beta)$ Versus α and β at $\rho = 0.9$ for the TWK-NOP Due Dates with Replication No. 3

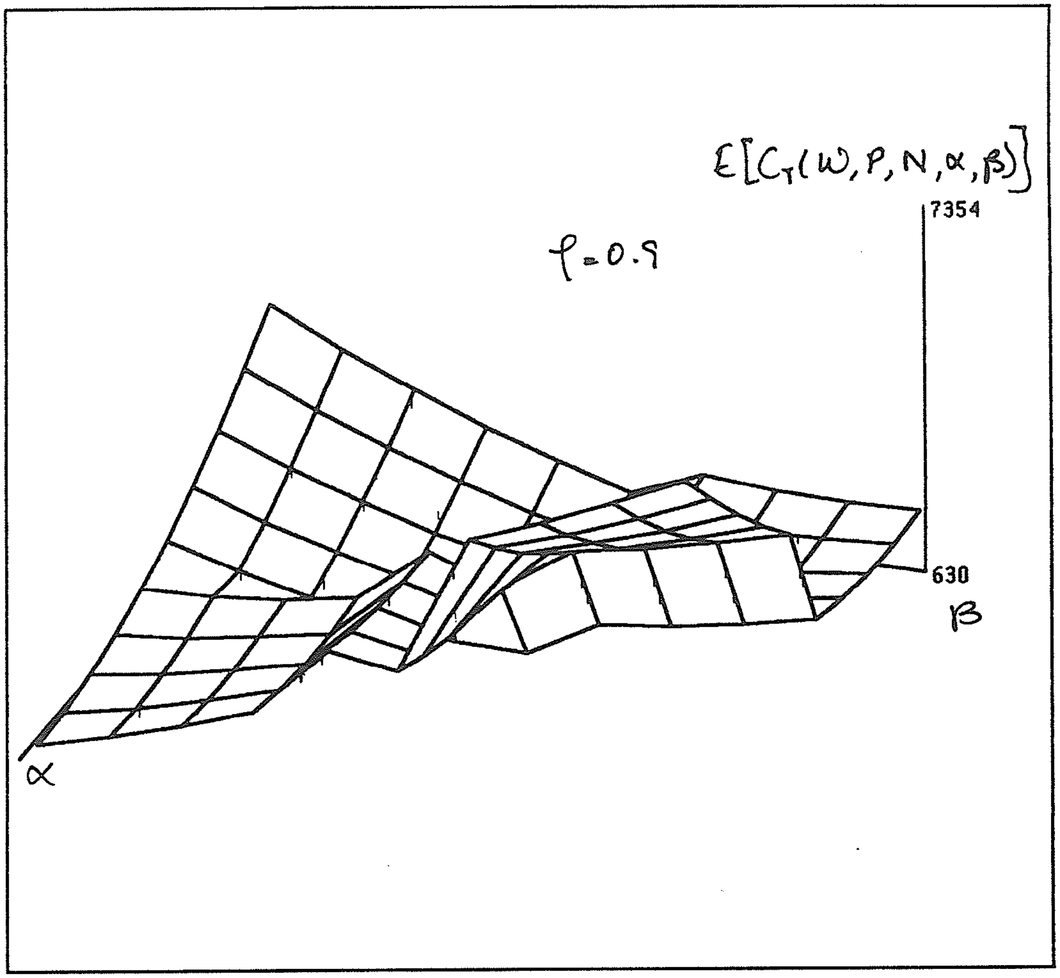

Figure 7 A Graph of $C_T(W, P, N, \alpha, \beta)$ Versus α and β at $\rho = 0.9$ for the TWK-NOP Due Dates with Replication No. 4

Chapter V

CONCLUDING REMARKS AND FUTURE RESEARCH

5.1 Concluding Remarks

This dissertation has reported an extensive review of analytical and simulation approaches in determining the optimal processing time and number of operations multipliers for the NOP and TWK-NOP methods of assigning due dates subject to restrictive assumptions of FCFS queue discipline. We have proposed a general cost model for analyzing the operating characteristics of the TWK-NOP due date assignment method in a job shop environment. The cost model is general since no specific distributions have been imposed on the underlying random processes involved. We then consider a linear cost model and able to show the local minimum points for each of the four scenarios. However, due to the complexity of the cost function, we employ simulation approach to determine the overall minimum point which will minimize the expected total cost per job.

5.2 Research Potential and Future Directions

It is believed that a number of extensions are possible to the dynamic job shop scheduling problems. The general cost model presented in Chapter 3 can be modified to accommodate different cost based criteria such as finished goods investments, storage costs of finished goods, percent of deviation in penalty cost associated with optimal schedule, setup costs and others.

It is expected that our analytical and simulation models under general cost function should open avenues for its successfully addressing more sophisticated due date assignment procedures such as TWK-NOP. It is believed that the simulation model given in Chapter 4 that enables us to determine the optimal multipliers will have many potential applications. On the contrary, given a specific joint probability function of the continuous random variables W, P and N, we are able to graph the surface on a domain which is an ordered pair (α, β, and expected aggregate cost per job) (3-tuple). Assuming the number of operations and the number of machines are the same, the possible distributions of processing time and the number of operations for the hypothetical job shop in Chapter 4 are $f(P) = 5\mu_p e^{-\mu_p P}$ and $f(N) = \frac{1}{N}$. However, the distribution of waiting time is not as straightforward and requires statistical estimation from historical operational data. It also depends on the processing time and arrival distributions. With the aid of advance graphical softwares, the interaction of the surfaces can be studied. Perhaps, an analytical determination of the optimal multipliers is possible.

We can further extend our analysis by studying the interaction of the due date assignment methods with various dispatching rules under various shop conditions. Past research has shown that reasonable TWK and NOP due dates in conjunction with the SPT dispatching rule achieved the best overall result with respect to a number of performance measures. As of the present, the only known waiting time under SPT discipline is derived by Conway et al. (1967) through busy-period concept application for a single machine shop. Since no analytical model has been developed for multi-machine, one should investigate the theoretical results of mean flowtime for a job shop. With this, simulation results can be used to validate the accuracy of prediction based on the analytical model.

Simplistic as it may seem, the model in this study represent an initial research effort to assess the optimal multipliers of TWK-NOP due date in job shop. Future empirical research is needed to test the applicability of our model under various conditions.

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APPENDIX

Simulation Program for the Hypothetical Job Shop

```
PREAMBLE ''
  LAST COLUMN IS 72  ''
  NORMALLY, MODE IS INTEGER
  PERMANENT ENTITIES
    EVERY MACHINE HAS A STATUS AND OWNS A QUEUE
    DEFINE STATUS AS AN INTEGER VARIABLE
  TEMPORARY ENTITIES
    EVERY JOB HAS AN ARRIVAL.TIME, A DUE.DATE, A PRIORITY,
      A TOT.PROC.TIME,MAY BELONG TO A QUEUE, OWNS A ROUTING
      DEFINE DUE.DATE AND ARRIVAL.TIME AS REAL VARIABLES
      DEFINE TOT.PROC.TIME AS A REAL VARIABLE
      DEFINE PRIORITY AS A REAL VARIABLE
      DEFINE ROUTING AS A FIFO SET
      DEFINE QUEUE AS A SET
  ''
    EVERY OPERATION HAS A MACHINE.DESTINED AND A PROCESS.TIME
      AND BELONGS TO A ROUTING
      DEFINE PROCESS.TIME AS A REAL VARIABLE
      DEFINE MACHINE.DESTINED AS AN INTEGER  VARIABLE
  ''
  EVENT NOTICES INCLUDE JOB.ARRIVAL AND END.OF.SIMULATION
    EVERY END.OF.PROCESS HAS AN ITEM AND A PRODUCER
  PRIORITY ORDER IS END.OF.PROCESS, JOB.ARRIVAL
    AND END.OF.SIMULATION
  ''
  BEFORE DESTROYING JOB CALL STAY.TIME
    DEFINE STAY,LATENESS,ABS.LATENESS AS REAL VARIABLES
''  DEFINE TARDINESS AS REAL VARIABLE
  TALLY AVG.STAY AS THE MEAN OF STAY
  TALLY AVG.LATENESS AS THE MEAN OF LATENESS
  TALLY AVG.ABS.LATENESS AS THE MEAN OF ABS.LATENESS
  TALLY AVG.SQR.LATENESS AS THE MEAN.SQUARE OF LATENESS
''TALLY AVG.TARDINESS AS THE MEAN OF TARDINESS
''TALLY STD.DEV.STAY AS THE STD.DEV OF STAY
''
  DEFINE SHOP.LOAD,MEAN.PROC.TIME,INTER.ARRIVAL AS REAL VARIABLES
''
  DEFINE TIGHT.FACTOR AS AN INTEGER VARIABLE
  DEFINE IDLE TO MEAN 0
  DEFINE BUSY TO MEAN 1
  DEFINE HOURS TO MEAN UNITS
  DEFINE NUM.OPN AS AN INTEGER VARIABLE
  DEFINE NUM.JOB,DISPATCH.RULE AS INTEGER VARIABLES
```

```
  DEFINE TOT.MACHINE,MAX.OPN AS INTEGER VARIABLES
  DEFINE FINISHED.JOB AS AN INTEGER VARIABLE
  DEFINE OPN.TIME,TOT.FLOWTIME AS REAL VARIABLES
  DEFINE TREATMENT,DUEDATE.RULE AS INTEGER VARIABLES
  DEFINE TIGHTNESS AS REAL VARIABLES
  TALLY AVG.TIGHTNESS AS THE MEAN OF TIGHTNESS
  DEFINE TWK.MULT,NOP.MULT AS  INTEGER VARIABLES
  DEFINE THIS.JOB AS INTEGER VARIABLES
  DEFINE TOT.IN.SHOP,TOT.IN.QUEUE AS INTEGER VARIABLES
END
  ''
  ''
MAIN
  PERFORM INITIALIZATION
  PRINT 2 LINE WITH SHOP.LOAD THUS
    JOB  SHOP  SIMULATION  WITH  SHOP  LOAD  =  .**     TWK+NOP

=======================================
===========
  FOR TWK.MULT=0 TO 9 BY 1,DO
   SKIP 1 OUTPUT LINE
    PRINT 1 DOUBLE LINE THUS
DISP. TREAT.  M.FLOWTIME  M.LATE   M.ABS.LATE   M.SQR.LATE
''M.TARDINESS    STD.DEV.FLOWTIME M.TIGHTNESS
    FOR NOP.MULT=0 TO 9 BY 1,DO
    LET SEED.V(1)=3413904823 LET SEED.V(2)=98328751
    LET SEED.V(3)=02844503  LET SEED.V(4)=617452421
     FOR TREATMENT=1 TO 1, DO
       FOR EACH MACHINE,  DO
         LET STATUS=IDLE
         FOR EACH JOB IN QUEUE, DO
           FOR EACH OPERATION IN ROUTING, DO
              REMOVE THE FIRST OPERATION FROM ROUTING
              DESTROY THE OPERATION
           LOOP
           REMOVE THE JOB FROM QUEUE
           DESTROY THE JOB
         LOOP
  ''   RESET THE TOTALS OF STATUS
       LOOP
       RESET TOTALS OF STAY,LATENESS,ABS.LATENESS
       LET TIME.V=0
       LET NUM.JOB=0
       LET FINISHED.JOB=0
       LET TOT.FLOWTIME=0
       SCHEDULE A JOB.ARRIVAL NOW
```

```
    START SIMULATION
   LOOP
  LOOP
 LOOP
 STOP
 END
 ''
 ROUTINE FOR INITIALIZATION
  READ N.MACHINE,SHOP.LOAD,MEAN.PROC.TIME
  LET TOT.MACHINE=N.MACHINE
  LET MAX.OPN=TOT.MACHINE*2-1
  LET INTER.ARRIVAL=(1+MAX.OPN)/(2*TOT.MACHINE*SHOP.LOAD)
                 *MEAN.PROC.TIME
  LET TOT.FLOWTIME=0
  LET FINISHED.JOB=0
  CREATE EVERY MACHINE
  FOR EACH MACHINE, DO
   LET STATUS=IDLE
  LOOP
  LET NUM.JOB=0
  RETURN
END
''
EVENT JOB.ARRIVAL
 DEFINE PRE.MCH,OPN.MCH AS INTEGER VARIABLES
 CREATE A JOB
 LET NUM.JOB=NUM.JOB+1
 LET NUM.OPN=RANDI.F(1,MAX.OPN,2)
 LET PRE.MCH=0
 LET ARRIVAL.TIME(JOB)=TIME.V
 LET TOT.PROC.TIME(JOB)=0
 FOR I=1 TO NUM.OPN, DO
'OPN'  LET OPN.MCH=RANDI.F(1,TOT.MACHINE,3)
     IF OPN.MCH=PRE.MCH,
        GO TO OPN
     ALWAYS
     LET PRE.MCH=OPN.MCH
   CREATE AN OPERATION
   LET MACHINE.DESTINED(OPERATION)=OPN.MCH
   LET PROCESS.TIME(OPERATION)=EXPONENTIAL.F(MEAN.PROC.TIME,4)
   LET OPN.TIME=PROCESS.TIME(OPERATION)
   ADD OPN.TIME TO TOT.PROC.TIME(JOB)
   FILE THIS OPERATION IN ROUTING
 LOOP
 ''
 LET THIS.JOB=JOB
```

```
  PERFORM ASSIGN.DUE.DATE
  LET JOB=THIS.JOB
  PERFORM ATTEND.TO.JOB
  SCHEDULE A JOB.ARRIVAL IN EXPONENTIAL.F(INTER.ARRIVAL,1) HOURS
  ''
  RETURN
END
''
ROUTINE TO ASSIGN.DUE.DATE
   LET DUE.DATE(JOB)=TIME.V+TWK.MULT*TOT.PROC.TIME(JOB)+NOP.MULT
                  *NUM.OPN
   RETURN
END
ROUTINE TO ATTEND.TO.JOB
  LET MACHINE=MACHINE.DESTINED(F.ROUTING(JOB))
  IF STATUS(MACHINE)=IDLE,
     LET STATUS(MACHINE)=BUSY
     PERFORM ALLOCATION
     RETURN
  ELSE
     FILE THIS JOB IN QUEUE
     RETURN
  ''
END
''
ROUTINE ALLOCATION
  REMOVE THE FIRST OPERATION FROM THIS ROUTING
  SCHEDULE AN END.OF.PROCESS GIVEN JOB AND MACHINE
    IN PROCESS.TIME HOURS
  DESTROY THE OPERATION
  RETURN
END
''
EVENT END.OF.PROCESS GIVEN TASK AND MCH
  LET JOB=TASK
  IF ROUTING IS EMPTY,
     DESTROY THIS JOB
  ELSE
     CALL ATTEND.TO.JOB
  ALWAYS
  LET MACHINE=MCH
  IF QUEUE IS EMPTY,
     LET STATUS(MACHINE)=IDLE
     RETURN
  ELSE
   ''CALL SELECT.JOB
```

```
    REMOVE THE FIRST JOB FROM QUEUE
    PERFORM ALLOCATION
    RETURN
  '' ALWAYS
END
''
ROUTINE SELECT.JOB
   DEFINE MIN.PROC.TIME AS A REAL VARIABLE
   DEFINE IMMINENT.PROC,REST.TIME,OPN.DUE.DATE,
     TOP.PRIORITY,SLACK.TIME AS REAL VARIABLES
   DEFINE TOP.JOB AS AN INTEGER VARIABLE
  GO TO L(DISPATCH.RULE)
'L(1)'
     ''  FCFS RULE
   REMOVE THE FIRST JOB  FROM QUEUE
   RETURN
'L(2)'
     ''    SPT rule
   FOR EVERY JOB IN QUEUE,
     COMPUTE TOP.JOB AS THE MINIMUM (JOB) OF
       PROCESS.TIME(F.ROUTING(JOB))
     LET JOB=TOP.JOB
     REMOVE THIS JOB FROM QUEUE
   RETURN
END
''
ROUTINE FOR STAY.TIME GIVEN JOB
   ADD 1 TO FINISHED.JOB
 IF FINISHED.JOB>500
   LET STAY=TIME.V-ARRIVAL.TIME(JOB)
   LET LATENESS=TIME.V-DUE.DATE(JOB)
   LET ABS.LATENESS=ABS.F(TIME.V-DUE.DATE(JOB))
  ALWAYS
  IF FINISHED.JOB=3000,
     SCHEDULE AN END.OF.SIMULATION NOW
  ALWAYS
   RETURN
END
''
EVENT FOR END.OF.SIMULATION
  FOR EACH JOB.ARRIVAL IN EV.S(I.JOB.ARRIVAL),
    DO
     CANCEL THE JOB.ARRIVAL
     DESTROY THE JOB.ARRIVAL
  LOOP
  FOR EACH END.OF.PROCESS IN EV.S(I.END.OF.PROCESS),
```

```
    DO
     CANCEL THE END.OF.PROCESS
     FILE ITEM IN QUEUE(PRODUCER)
     DESTROY THE END.OF.PROCESS
  LOOP
 SKIP 1 OUTPUT LINE
  PRINT 1 DOUBLE LINE WITH DISPATCH.RULE,TREATMENT,AVG.STAY,
     AVG.LATENESS,AVG.ABS.LATENESS,AVG.SQR.LATENESS
      THUS
*     *     ****.***     ****.***   ****.***    *******.**

   RETURN
END
```