THE UNIVERSITY OF MANITOBA

THE EFFECT OF COMPUTER-ASSISTED INSTRUCTION ON HIGH SCHOOL MATHEMATICS ACHIEVEMENT

by

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J.R.B.

ABSTRACT

The purpose of this study was to determine whether students who use a computer to study mathematics attain a higher level of achievement than students who do not use a computer. The study was conducted during April and May of 1972 at Fort Richmond Collegiate, a senior high school in Winnipeg, Manitoba.

Students in two grade 10 university entrance algebra classes participated in the study: 21 boys and five girls in the experimental group which used a computer as a teaching and learning tool; 16 boys and 11 girls in the control group which did not use a computer. Both groups were taught by the same teacher for a period of four weeks, and except for the use of a computer in the experimental class, the course objectives, methods, techniques, and instructional materials were the same for both groups.

A computer terminal linked to a large computer at the University of Manitoba was installed in the school for use with the experimental group. For teacher demonstration purposes, computer output at this terminal was displayed via closed circuit television.

The computer was used in three ways: (1) by the instructor as a teaching aid, (2) for problem solving by students, and (3) for student experimentation. The programing language used was FORTRAN IV, and it was introduced to students only as required.

To determine the level of mathematical and general mental ability of both groups, two pretests were administered. Analysis of variance performed on the results of these tests confirmed that significant pre-treatment differences existed between the experimental

and the control group, and scores from the pretests were subsequently used as covariates in the analysis of a post-treatment achievement test. This posttest was designed to measure only the achievement of mathematical objectives and was in no way computer oriented.

As a result of this study and the concomitant statistical analysis, the following conclusions were derived:

1. After treatment in a unit of grade 10 mathematics, there was no significant difference in mean achievement between students who used a computer and students who did not, when post-treatment results were analyzed using (a) mathematical ability scores as a covariate, and (b) both mathematical and mental ability scores as covariates.

2. After unit treatment, there was a significant difference in mean achievement between students who used a computer and students who did not, when results were analyzed using mental ability scores alone as a covariate. The difference was barely significant at the five percent level of confidence and was in favor of the group which did not use a computer.

3. There was no significant treatment-sex interaction in any of the analyses, and thus, males and females did not react differently to the experimental treatment.

4. There is a definite need for further revision and development of instructional methods and materials relevant to this type of computer-assisted learning in mathematics.

5. Further studies of this type are required to identify those areas of the high school mathematics curriculum in which this kind of computer-assisted learning can offer a significant contribution.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter I

INTRODUCTION

Importance of Computers in Society

The appearance in 1951 of the world's first commercial digital computer, UNIVAC I, marked the beginning of the modern computer age - an age which historians may one day call the second Industrial Revolution. Since that time, the number of computers at work throughout the world has increased exponentially to an estimated 100,000 in 1971, and this number is expected to double by 1975.[1] Technical improvements have proceeded in a similar manner. Today's computers are literally 1000 times larger (in terms of memory capacity), 10,000 times faster, and yet, relatively cheaper than UNIVAC I.[2] In fact, computers have progressed from infancy through three generations[3] in less than two decades.

The number of applications has kept pace with these developments, so that today the computer is involved in virtually every field of human endeavor. Not only is there no doubt that "we are,"

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[1] Nigel Hawkes, The Computer Revolution (London: Thames and Hudson Ltd., 1971), pp. 35-37.

[2] Ned Chapin, Computers: A Systems Approach (New York: Van Nostrant Reinhold Co., 1971), pp. 616-17.

[3] Computers are termed as being of the first, second, or third generation according to whether certain components are vacuum tubes, transistors, or integrated circuits, respectively. Third generation computers were first marketed in 1965.

as Alvin Toffler so succinctly states, "in the midst of the super-industrial revolution,"[4] but also that the tempo of this revolution is ever increasing. We are indeed "hurtling into the computer age at a pace which makes the Industrial Revolution look like a funeral procession."[5]

This rapid development of computers in recent years is regarded by many as the embodiment of an acceleration of change which has become so ubiquitous in present day society. It is precisely this kind of change that is responsible for the modern, very real phenomenon of "future shock." Alvin Toffler coined this term in 1965 "to describe," as he said, "the shattering stress and disorientation that we induce in individuals by subjecting them to too much change in too short a time."[6]

The fact that society is indeed suffering from this "disease of change" is evidenced by the almost irrational resistance to change displayed by many individuals and groups. Nowhere is this perhaps more evident than with regard to the computer. People, including many educators, are bewildered by, fearful of, and more often than not, misinformed about these "gentle machines." They fear that computers will create a race of mindless, standardized, and conforming creatures. In fact, nothing could be further from the truth. On the contrary, automation "frees the

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[4]Alvin Toffler, Future Shock (Toronto: Bantam Books of Canada Ltd., 1971), p. 186.

[5]James Martin and Adrian Norman, The Computerized Society (Englewood Cliffs, N. J.: Prentice-Hall Inc., 1970), p. 16.

[6]Toffler, op. cit., p. 2.

path to endless, blinding, mind-numbing diversity."[7]

The computerized assembly systems of today, for example, are beginning to offer such a wide range of consumer goods as to make choice, rather than the lack of it, a problem. Nor is this move away from standardization confined to material things. In education, computers:

> . . . make it easier for a large school to schedule more flexibly. They make it easier for the school to cope with independent study, with a wider range of course offerings and more varied extra-curricular activities. More important, computer-assisted education, programmed instruction and other such techniques, despite popular misconceptions, radically enhance the possibility of diversity in the classroom. They permit each student to advance at his own purely personal pace. They permit him to follow a custom-cut path toward knowledge, rather than a rigid syllabus as in the traditional industrial era classroom.[8]

One of the fears often expressed by anti-computer protagonists is that computers will eventually usurp human decision making and virtually enslave mankind. Once again, this fear is based on misconception and even ignorance. Study after study has clearly demonstrated that even junior high school students, when given the opportunity, readily learn to program computers, that is to command computers to do <u>their</u> bidding. It is true, perhaps, that those who do remain ignorant of computers and the processes involved may well become enslaved, and pass from bewilderment to frustration - to future shock. They might indeed become the "proles" of George Orwell's 1984.

If computers and their involvement in human affairs continue to develop as they are at present, then it is clear that, as

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[7]Toffler, op cit., p. 266. [8]Ibid., p. 275.

Martin and Norman have stated:

> Children starting school today are going to . . . spend almost their entire working lives in a world markedly different from today, in which it will be at least as important to understand and communicate with computers as driving a car is today.[9]

The basic question to be posed now is - are children being prepared for life in such a world, or are present day educational leaders like the old generals who persisted in preparing their troops for the last war? If society is to avoid the disruptions of future shock, it is fundamental that an education system be developed that is geared to the super-industrial revolution. It must be a future-facing system in which students not only learn about computers per se, but also use them in pursuit of other knowledge. Although a great deal has already been accomplished in this direction, much work remains to be done, not only in the research area but also in incorporating existing knowledge into functioning classroom curricula.

Modes of Computer Use in Education

More than a decade ago a few concerned educators foresaw the potential of the computer as a powerful teaching and learning aid, particularly in mathematics. In 1961 The National Council of Teachers of Mathematics initiated a project known as the Computer Oriented Mathematics Project (COMP). Some other groups were already doing work which emphasized how computers operate and how to prepare computer programs for solving simple problems. Project COMP, on the

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[9] Martin and Norman, op. cit., p. 452.

other hand, attempted to capitalize on the immense appeal of computers to motivate students in the study of mathematics. The use of computers was seen as a means rather than an end in itself. An early result of the project was a book - Computer Oriented Mathematics[10] - which was intended primarily as a background for junior and senior high school teachers. The material could be used either as enrichment for regular mathematics courses or as background for new, computer oriented courses. At that time there was no clear conception of how computers could be used to the best advantage, either in mathematics or in education as a whole.

Since then, however, numerous applications have been identified. One recent report for example, listed ten areas of computer use in the process of education. These included, in part:

(1) Computer-assisted instruction, drill and practice, tutorial and dialogue modes using programmed instructional techniques.

(2) Computer used as a computational aid to problem solving in classes and laboratories for science, mathematics, accounting, economics, etc.

(3) Gaming and simulation of real life situations.

(4) Computer-mediated instruction involving TV, films, etc.[11]

All of these as well as other uses are commonly lumped together under the often confusing title of "Computer Assisted Instruction," or CAI. Some leaders in the field have endeavored

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[10]Harley E. Tillitt et al., Computer Oriented Mathematics (Washington, D. C.: National Council of Teachers of Mathematics, Inc., 1963)

[11]Lee Lewellen (ed.), Computers in the Classroom (San Carlos, Calif.: Technica Education Corp., 1971), p. 3.

to change this acronym to CAL (Computer Assisted Learning),[12] while still others have grouped the uses into subsets. K. L. Zinn groups seven or eight modes of use which he has identified into three types: author control, simulation and gaming, and scholarly aids for learning and problem solving.[13]

The most publicized applications, and the ones normally associated with the term CAI, are the drill and practice and tutorial modes of programmed learning. Zinn includes these in his author control group, since students using them have no control over the material or the pace and order in which it is presented. An extensive and on-going development of this type of computer-based instruction began at Stanford University in early 1963. Much of the work there has been focussed on arithmetic, reading, and spelling at the elementary school level, although some work has been done with junior high mathematics.

The drill and practice system is one in which the instruction is supplementary to the regular curriculum taught by the classroom teacher, and as the name suggests, provides practice in basic skills. The tutorial system on the other hand, is intended to provide as much of the actual instruction as possible. Both systems employ one computer terminal[14] for each student undergoing

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[12] R. W. Gerard, "Computers In Mathematics And Other Education," <u>Computer-Assisted Instruction And The Teaching Of Mathematics</u>, ed. R. T. Heimer ([n.p.] : National Council of Teachers of Mathematics, Inc., 1969), p. 1.

[13] Karl L. Zinn, "Implications of Programming Languages For Mathematics Instruction Using Computers," Ibid., p. 82.

[14] Terminals are remote devices controlled by a computer

instruction. The terminals are all connected to a central computer whose speed is such that it can handle 100 or more students simultaneously. Lessons or drills are presented at the optimum rate, sequence, and level of difficulty, which the computer calculates for each student from past and present performance. Immediate attention is given to every response of each student as well as immediate correction if it is wrong. Apart from recording a continuous evaluation of each student, the computer also provides data which is useful in analyzing student learning difficulties.

In these two modes the computer appears to offer a long sought method of individual learning and true continuous progress. Despite these advantages, however, the tremendous costs involved seem likely to limit the wide use of these two modes, at least for the present.

Another less costly, but no less effective use of computers began to emerge in the middle 1960's. It deals with junior and senior high school mathematics and is the area of CAI of concern to this study. Karl Zinn calls it "problem solving with computation and display tools,"[15] and includes it in his aids for learning and problem solving category. The terminology is misleading here, since in this mode the computer is used not only to

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and connected to it, usually by an ordinary telephone line. There are two basic types of terminal - cathode ray tube (CRT) and teletype. They provide a two-way communication link between user and computer.

[15]Karl L. Zinn, "Computer Technology For Teaching And Research On Instruction," Review of Educational Research, XXXVII, No. 5 (1967), p. 622.

solve problems, but also to aid in the teaching and learning of mathematical concepts and in the broadening of students' understanding of basic principles.

Early attempts to use the computer in high school mathematics tended to regard it as an end in itself and generally treated it in isolation from existing courses. A fundamental feature of the present approach, however, is that the computer is used to strengthen existing courses without disrupting or changing their objectives or contents. It is a learning aid.

No one who has ever taught a course of any description will deny the truth of the old saying, "the best way to learn something is to teach it." Therein lies the real usefulness of computers in problem solving. When students use a computer to solve a particular problem they must literally "teach" the computer how to do it. The following quotation describes the process:

> In solving any kind of problem, a computer is helpless until it has been given a detailed set of instructions. The computer can't figure out how to solve the problem. All it can do is slavishly follow directions step by step until the job is completed. This is true for any problem we try to solve with a computer, whether it's simply adding a column of numbers or solving the complicated equations necessary to send a missile to the moon. These detailed instructions are called the computer program.[16]

In preparing computer programs, students need not be concerned with the inner workings of the machine or how their instructions are executed electronically. On the other hand, they are not allowed to do a bad job of teaching. The machine

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[16] General Electric Company, You And The Computer (Schenectady, N. Y.: Education Publications, 1965), p. 9.

cannot "understand" illogical instructions, nor can it carry out illegal operations such as division by zero. If such things are attempted, students receive a printed error message, from which they can diagnose the trouble and repair the program for another try. As Richardson concluded in his report on Project H-212:

> Programming work facilitates the acquisition of rigorous thinking and expression. Children impose the need for precision on themselves through attempting to make the computer understand and perform their algorithms.[17]

A further advantage arising from the use of computers in problem solving is the ease with which many problems can be generalized. In dealing with systems of numeration for example, students might first be required to write a program that converts numerals from base 2 to base 10. Succeeding assignments might then consist of changing the program so that it converts any given base to base 10, and finally, any given base to any target base. Such a general radix-conversion program would undoubtedly give students a better grasp of this topic than they could have obtained without the use of a computer.[18] Such generalizing techniques need not be part of the curriculum, but can be assigned to bright students as enrichment, while the teacher assists slower members of a class with the basic program.

Experience has shown that creativity and individuality are

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[17] Jesse O. Richardson, Teaching Mathematics Through The Use Of A Time-Shared Computer, Report prepared under a grant from the U. S. Department of Health, Education and Welfare, Washington, D.C.: Office of Education, Bureau of Research, 1968. p. 1.

[18] Walter Hoffman, et al., "Computers For School Mathematics," The Mathematics Teacher, LVIII (May, 1965), p. 396.

two other qualities which the use of a computer helps to develop in students. Most problems can be programed for the computer in a myriad of ways, and assignments are often completed with as many different methods as there are students in a class. Some are shorter, faster, or more elegant than others, but all are equally valid. Many students who would not otherwise be interested are also led to experiment with problems, and often discover techniques which, although not always original, are none the less thrilling for them. In fact, many students become more adept at programing than their teacher, particularly in senior grades.

The second, but no less important, aspect of this mode of use is the computer's ability to aid in imparting concepts. In the course of developing a new topic a teacher can, for example, call upon previously prepared programs which have been stored in the computer system. The output from such programs appears on a terminal installed in the classroom and is displayed to an entire class via closed circuit TV. The example outlined below is based on one reported by Berry, Falkoff, and Iverson, and serves to illustrate this technique.[19]

After introducing linear expressions in two variables, a teacher might cause the computer to evaluate an expression as each of two unknowns varies systematically. The behavior of an expression such as, x + 2y - 12, would be displayed as shown in table 1,

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[19] P. C. Berry, A. D. Falkoff, and K. E. Iverson, Using The Computer To Compute, Philadelphia Scientific Center Technical Report No. 320-2988 (Philadelphia: Philadelphia Scientific Center IBM Corporation, 1970), p. 14.

when supplied with the values:

$$x = 0,1,2,3,4,5,6,7,8$$
$$y = 8,7,6,5,4,3,2,1,0$$

Table 1

Evaluations of the Expression x + 2y - 12

	8	4	5	6	7	8	9	10	11	12
	7	2	3	4	5	6	7	8	9	10
	6	0	1	2	3	4	5	6	7	8
	5	-2	-1	0	1	2	3	4	5	6
Y Values	4	-4	-3	-2	-1	0	1	2	3	4
	3	-6	-5	-4	-3	-2	-1	0	1	2
	2	-8	-7	-6	-5	-4	-3	-2	-1	0
	1	-10	-9	-8	-7	-6	-5	-4	-3	-2
	0	-12	-11	-10	-9	-8	-7	-6	-5	-4
		0	1	2	3	4	5	6	7	8

X Values

The meaning of the values in this table would be discussed with the class, and attention focussed on the zeros. That is, what pairs of x,y values make the expression equal to zero? Are there other pairs besides those shown? How many? Other tables could quickly and effortlessly be produced with x,y values supplied by the class. The concept of linear equations in two unknowns and their graphical representation would follow naturally from such tables. The graph of an equation would result from the second display of a table in which all entries were blanked out and the zeros replaced by dots. In order to complete an assignment students could then use this stored program or write one of their own.

Using such an approach, Berry, Falkoff, and Iverson claim, ". . . the machine is not an individual's tutor, but the arbiter of proposals made by a class; mathematics becomes a laboratory science, open to experiment, conjecture, and discovery."[20] Although such claims for the foregoing mode of CAI are extremely promising, they are as yet based largely on conjecture and feasibility studies. Before 1968 there was no reported research evidence on the effectiveness of this type of CAI in mathematics education.[21] Since then a few experiments have shown that it is an effective teaching and learning aid in some areas of mathematics, and it is the purpose of this study to add to this evidence in one small part of the curriculum.

The Problem

The main purpose of this study centered on the question of whether the use of a computer would have any significant effect on student achievement in a unit of grade 10 Mathematics. A sub-purpose related to this was to determine if there was any significant difference in achievement between a computer and non-computer group for male and female students.

In relation to these purposes the following null hypotheses were tested:

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[20] Berry, Falkoff, and Iverson, op. cit., p. iii.

[21] Thomas E. Kieran, "The Computer as a Teaching Aid For Eleventh Grade Mathematics: A Comparison Study" (unpublished Ph.D. dissertation, University of Minnesota, 1968), p. 10.

1. There is no significant difference in mean achievement between students in a unit of grade 10 mathematics who use a computer and those who do not.
2. There is no significant difference in mean achievement between male students in computer and non-computer groups.
3. There is no significant difference in mean achievement between female students in computer and non-computer groups.

Significance of the Problem

The computer, ". . . for the first time gives the possibility of bringing education out of the completely artistic stage into a science built upon an art - which is what most of our sciences have been."[22] If this possibility is attainable, then educators are clearly remiss if they do not strive to implement it. On the other hand, a science cannot endure on untested hypotheses alone. Thousands of schools in the United States alone are already using computers in the instructional process with very little solid research evidence to justify the additional expense. In this age of accountability, where most people still think of the computer in terms of cartoons, and jokes about million dollar pension cheques, educators cannot afford to jeopardize CAI with premature and unfruitful implementations.

A few experiments in recent years have already shown that

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[22] R. W. Gerard, "Computers In Mathematics And Other Education," Computer-Assisted Instruction And The Teaching Of Mathematics, ed. R. T. Heimer ([n.p.] : National Council of Teachers of Mathematics, Inc., 1969), p. 1.

CAI produces significant gains in student learning in certain areas of mathematics. It is important that studies such as this present one continue to investigate further areas of the curriculum, and in particular, to develop new materials and techniques.

Description of the Study

In order to test the hypotheses of the study, two comparable grade 10 algebra classes were selected and randomly assigned to experimental and control groups containing 26 and 27 students respectively. Both groups received instruction in the same unit of mathematics from the same teacher for a period of four weeks. Except for instruction and assignments in the experimental group pertaining to computer programing and problem solving, as well as the use of a computer terminal, the unit objectives, content, and instructional materials were the same for both groups. Thus, the experimental group used a computer as an instructional aid to learn algebra, while the control group did not.

Two pretests designed to measure mathematical and general mental ability were administered to both groups prior to commencing the study, and on completion of the unit of instruction both groups were given the same achievement test. The post-treatment achievement test was designed to test only unit objectives and made no reference to computer methods. Finally, to determine the effect of the computer treatment on student achievement, results of these tests were subjected to analyses of variance and covariance.

Limitations of the Study

Like most research, the study described in this report has some limitations, the most serious of which appears in the assignment of participating students. In the school where the study was conducted, existing, intact classes could not be broken up in the middle of the school year, and it was not possible to randomly assign students to experimental and control groups. Instead, the two participating classes were assigned to computer and non-computer treatments by flipping a coin. The method used by the school to form classes was such that those used in the study did not contain comparable students. Consequently, even though statistical methods may allow for pre-treatment differences, the generalization of the findings of this study is severely limited.

The short duration of the study is also considered to be quite a serious limitation. The four week period may have been too short, not only to overcome any novelty effect of the computer, but also for students to become proficient enough in computer programing to allow the problem solving mode to be fully exploited.

Still another factor which may have influenced results, is the fact that the school in which the study was conducted has been subjected to a number of educational and other studies over the past few years. There were undoubtedly some students in the study who resented the administration of further standardized tests and who did not perform honestly.

Finally, the problem solving and demonstration materials used were adapted and devised by the experimenter and had not been previously tested. Once again, the need for such materials that have proven to be both valid and reliable, cannot be overemphasized.

Chapter II

REVIEW OF THE RELATED LITERATURE

After more than a decade of debate there is still some disagreement among educators as to the best mode of computer use in mathematics instruction, and very little research evidence relating to the effectiveness of the various methods. This chapter contains a review of the literature pertaining to the nature and extent of computer use in education, and describes what research has been conducted to date in the mode of computer-assisted problem solving that is of concern to this study. The chapter has been organized into the following sections:

- Nature and extent of computer use in education
- Research in computer-assisted problem solving
- Summary

Nature And Extent of Computer Use in Education

Although schools using computers are still in the minority, the instructional use of computers is growing rapidly, particularly in the United States. In 1970, a nationwide survey conducted in U. S. public secondary schools reported that of the schools responding, 3,776 (30.5%) were using computers for administrative purposes, while 1,559 (12.9%) reported some instructional use of computers, with 75% of these applications in mathematics courses.[1] The following is a summary of the findings resulting from this survey:

(1) The instructional applications listed in order of frequency of mention were: Electronic Data Processing (EDP) skills training, problem solving, guidance/counseling, gaming/simulation, computer assisted instruction, management of instruction, other classroom instructional applications.

(2) A variety of patterns of combinations of applications emerged, clustered around guidance-administrative and problem solving-EDP skills.

(3) The most frequently mentioned student activity was writing and running programs with teachers assisting.

(4) Although the overall purposes of the applications varied widely, there was a general emphasis on using the computer as a tool to accomplish subject matter goals rather than on learning about the computer as an end in itself.

(5) Applications of computers to mathematics instruction dominated. Almost three quarters of all computer applications were involved with mathematics instruction.

(6) There was very little formal integration of computers into the curriculum except in mathematics where there was a little more integration.

(7) The model grade in which instructional computer applications were introduced was grade 10 with some variation across applications within grades 9-11.

(8) Overall the most frequently mentioned programing language used for instructional applications was FORTRAN; second was BASIC.[2]

It is significant to note that even though the professed

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[1] Lee Lewellen (ed.), Computers in the Classroom (San Carlos, Calif.: Technica Education Corp., 1971), pp. 3-4.

[2] Ibid., pp. 4-5.

aim of most schools is to use the computer as an aid in attaining subject matter goals, there is still very little integration into existing curricula. What is even more significant, is that after more than a decade of increasing computer use of one form or another in mathematics education, there still exists uncertainty as to which role is the most useful. In 1969, seven levels at which a computer might be used in conjunction with secondary school mathematics programs were listed by Charles Zoet in terms of the following objectives:

1. To let students <u>know about computers</u> - their impact on modern society and on mathematics.

2. To enable a student to <u>understand the concepts involved in the design of a computer</u>. This also consists of knowing about computers - but from a design point of view.

3. To develop a student's <u>ability to use a particular computer in a manner than emphasizes its basic concepts and its potential for use</u>.

4. <u>To develop a student's ability to use a computer to solve some problems in mathematics</u> in order that he can become aware of its potential and the strategies enlisted in using it to solve mathematical problems.

5. To develop the student's ability to use the computer <u>to provide additional insight into some of the problems studied in classes such as advanced algebra</u>.

6. To use the computer as an integral part of the teaching of advanced algebra (or other classes) - with all students required to use it in certain phases of the class.

7. To teach mathematics by programing the computer to interact with individual students in a fashion designed to achieve some particular behavioral objective.[3]

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[3]Charles J. Zoet, "Computers In Mathematics Education," ed. Walter Koetke, <u>The Mathematics Teacher</u>, LXII (Nov., 1969), pp. 565-566.

The many hundreds of schools now using the computer in mathematics instruction undoubtedly range along a continuum from those that use the machine only superficially, to those that use it as the basis for entire courses. In view of the previously mentioned survey however, it is apparent that most are using it in pursuit of objective four above: that is, for problem solving in conjunction with, but not an integral part of, existing curricula. While many claims have been made for the effectiveness of this type of problem solving, very little has been done to substantiate them. On the other hand, most of the research studies conducted to date have concentrated on a combination of objectives four, five and six above, and have shown this to be a most effective mode - a mode in which the computer is used as a learning <ins>and teaching</ins> aid and is completely integrated into the curriculum. This mode of problem solving is the one of concern to this study and the relatively few experiments already conducted in this area will be discussed below.

Research In Computer-Assisted Problem Solving

The first controlled investigation into the effectiveness of computers on mathematics achievement began in 1965 at the University of Minnesota High School. This two-year study - called the Computer-Assisted Mathematics Project (CAMP) - encompassed grades 7 to 12 and was designed to:

1. Identify appropriate sections of existing curricula that students might learn more effectively by writing computer programs,

2. Develop materials and methods for using the computer

as part of the regular mathematics curriculum, and

3. Evaluate the effectiveness of the devised materials and methods on student achievement.

One of the most useful results arising from this study is a series of six books entitled, Computer Assisted Mathematics Program, which contain topics that are usually found in secondary school mathematics programs. As the authors state:

> In short, the books in the CAMP series use the computer as a problem-solving tool. Nearly all the lessons focus on teaching the students to design and test algorithms (programs) for the kinds of problems they are likely to encounter in their regular textbooks.[4]

It is important to note that even in this early study, experimenters realized that the computer is not an effective aid in every part of a given mathematics curriculum, as the following quotation emphasizes:

> However, not all topics from secondary-school mathematics are equally appropriate for computer-assisted learning. The authors have attempted to identify important topics that will be better understood and appreciated by students if they have the opportunity to examine the ideas by writing computer programs and studying the output.[5]

In fact one of the major contributions of this and other such studies is the identification of those areas of school mathematics in which the computer offers significant advantages over traditional methods.

Two important studies within the CAMP project have been

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[4]Pamela W. Katzman, Geometry, Vol. IV of Computer Assisted Mathematics Program (Glenview, Ill.: Scott, Foresman and Co., 1970), p. vi.

[5]Ibid., p. v.

reported separately by Kieren[6] and Hatfield.[7] The study conducted by Kieren took place during the 1965-66 and 1966-67 school years at the University of Minnesota High School, and involved 97 grade 11 students in the course called Intermediate Mathematics. Both an experimental (computer) and a control group were taught the same material by one teacher, using identical course objectives and philosophies, but different methods. The computer class had access to a remote computer through a teletype terminal located in the school. This group used computer materials which contained mathematical expositions, sample computer programs, and many exercises that directed students to devise and run their own programs. In some topics, programs were developed by the teacher and students together, either individually or as a group. In any case, the output from computer programs was used to help develop mathematical concepts and understandings.[8] In contrast, the control group did not have access to a computer and was taught the same subject matter by traditional methods.

To determine the effect of using a computer in this way, all participating students were pretested with a standard achieve-

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[6]Thomas E. Kieren, "The Computer as a Teaching Aid For Eleventh Grade Mathematics: A Comparison Study," (unpublished Ph.D. dissertation, University of Minnesota, 1968).

[7]Larry L. Hatfield, "Computer-Assisted Mathematics: An Investigation of the Effectiveness of the Computer Used as a Tool to Learn Mathematics," (unpublished Ph.D. dissertation, University of Minnesota, 1969).

[8]A complete description of the materials and mode of computer use for the unit on quadratic equations was given by Kieren in The Mathematics Teacher, Apr., 1969.

ment test in mathematics at the start of each year. Unit achievement tests were also given at intervals throughout the year and a final post-treatment test administered in May of 1966 and 67. Treating each year as a separate experiment, analysis of variance and covariance methods revealed that during the second year, the mean achievement of students in the computer group was significantly better in the final, post-treatment achievement test. Also in 1966-67, it was found that the null hypothesis of no treatment differences had to be rejected in favor of the control class in two unit tests on trigonometry. This last detrimental result dramatically points out the need for experimentation to determine the areas of the mathematics curriculum for which computer methods are effective.

> In addition, the proportions of correct responses were examined for 348 test items used during the second year.
> ....
> Kieren's study revealed that the null hypothesis of no difference in the proportion of students correctly responding to a test item was rejected for 43 of the 348 items included. A thorough study of the items indicated that the computer had little positive effect on simple skills such as computation with complex numbers and geometrical treatments of trigonometry.[9]

In contrast, "from the evidence of this study," Kieren states, "it seems to make its strongest contributions in the areas of complex skills, organization of data and drawing conclusions therefrom, and the study of infinite processes."[10]

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[9] Franklin D. Ronan, "A Study Of The Effectiveness Of A Computer When Used As A Teaching And Learning Tool In High School Mathematics," (unpublished Ph.D. dissertation, University of Michigan, 1971).

[10] Kieren, op. cit., pp. 127-28.

Another two-year study, similar to Kieren's both in design and procedures, was conducted by Hatfield using seventh grade students. The second year in particular showed some significant gains in mean achievement for the computer class:

> During Year 2, the means analysis of treatment effect revealed significance on one (Elementary Number Theory) of the six unit tests and two (contemporary Mathematics Test and Thought Problems) of the six post-treatment tests. These significant differences all favored the computer treatment. . . . The number theory unit was recognized as a particularly relevant setting for the use of the computer. The emphasis of this unit was on exploration and inquiry with problems involving many laborious calculations. The orientation was to use the computer as a laboratory tool to explore a number of interesting number theory settings.[11]

One of the earliest large scale research studies in the field of computer-assisted problem solving was entitled, "Teaching Mathematics Through the Use Of A Time-Shared Computer," (Project H-212) and was conducted by the Massachusetts State Department of Education from 1965 to 1968. It might best be described by quoting directly from the final project report:

> The design of Project H-212 envisaged making use of the computer as the basis for a laboratory approach to the presentation of mathematics. Classroom instruction was to be augmented by student experiments in devising and testing mathematical algorithms on the computer.
> ....
> The students participating in Project H-212 were taught to program a computer--or, more precisely, to write programs in a particular programing language. The primary goal was not to impart facility in programing for its own sake, but rather to exploit it for the presentation of mathematical ideas through classroom instruction and individual student laboratory work.
> The work performed in Project H-212 sought to show that the teaching of the set of concepts related to computing, programing, and information processing could

[11] Hatfield, op. cit., p. 19.

be used to facilitate and enhance the presentation of standard school mathematical curricular material, including arithmetic, algebra, and elementary calculus. . . . From the work carried out by the project at grades 6 through 12, the following conclusions are drawn:

(1) It is possible to construct programing languages of great expressive power yet so simple to learn that they can be effectively taught to elementary school children.

(2) Children are easily motivated to write programs at computer consoles. This kind of mathematical activity is immensely enjoyable to children generally, including those not in the top levels of mathematical ability.

(3) Programing work facilitates the acquisition of rigorous thinking and expression. Children impose the need for precision on themselves through attempting to make the computer understand and perform their algorithms.

(4) A series of key mathematical concepts such as variable, equation, function, and algorithm, can be presented with exceptional clarity in the context of programing.

(5) The use of a programing language effectively provides a working vocabulary, an experimental approach, and a set of experiences for discussing mathematics. Mathematical discussion among high school students, relatively rare in the conventional classroom, was commonplace in this laboratory setting.

(6) Computers and programing languages can be readily used in either of two ways in the mathematics classroom,
   (a) By individual students for independent study on extracurricular problems or special projects.
   (b) As a laboratory facility to supplement regular classroom lecture and discussion work. In this mode students are given assigned problems to work out at the computer. . . .[12]

A third and equally important use of the computer in the mathematics classroom - teaching through demonstration programs - was also pioneered in Project H-212. The numerous teacher demonstration programs used in the study were pre-written and stored

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[12]Jesse O. Richardson, Teaching Mathematics Through The Use Of A Time-Shared Computer, Report prepared under a grant from the U.S.Department of Health, Education and Welfare, Washington, D.C.: Office of Education, Bureau of Research, 1968. pp. 1-2.

in the computer system according to the following criteria:

1. Error-free calculations with a minimum of computation time in class.

2. Increased plausibility of abstract mathematical concepts through inductive support.

3. Difficult programs with output that allows students to concentrate on mathematical concepts while avoiding non-relevant or cumbersome student programing.

4. Motivation and challenge to a broader study of mathematics.[13] Some examples from the many such programs used, were those designed to show: (1) the density of rational numbers, (2) the effect of a, b, and c on the graph of $y = ax^2 + bx + c$, and (3) the convergence of a hyperbola to its asymptote. Whenever required by the teacher, the output of such programs was instantly available on the computer terminal, which in turn could be displayed to the class via closed circuit television. Bolt Beranek and Newman, the firm which supplied technical assistance to the project, has subsequently developed an inexpensive projection system for displaying terminal outputs.

Although Project H-212 was primarily a methodological rather than an empirical study, it brought to light many areas for further research, and has been, perhaps more than any other single study, responsible for the growing interest in this mode of computer use. Interest in this project was so widespread that during the 1965-66 school year more than 100 individuals from 32

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[13] Ibid., pp. 16-19.

states and 10 foreign countries observed some project activities. In addition to these visitations, project newsletters were also widely distributed.[14] As a direct result of this interest at least six major projects similar in nature to H-212 were initiated by 1967.[15] One such project, initiated by Dartmouth College in New Hampshire, resulted in the installation of computer terminals in 18 secondary schools in five New England States by 1968.

Another study stemming from H-212 was Project LOCAL (Laboratory For Computer Assisted Learning), which involved grade 10, 11, and 12 students from five Massachusetts school systems, and provided intensive in-service courses for teachers. This project also investigated the use of teletype terminals in the "off-line" mode, in which students prepare their programs on punched paper tape with no connection to the computer. In order to run a program, the contents of students' tapes can be transmitted to the computer in seconds instead of the long periods required when using the terminal keyboard. This results in a great reduction in the time a terminal is actually connected to the computer and achieves a significant saving of money. Project LOCAL also provided some empirical results. In one participating school for example, three groups of students taking second year algebra were taught the same material by the same teacher using three different methods.

> . . . Students in two groups received instruction in flow-charting and used this technique of designing and representing problem solutions graphically in doing their homework. Students in one of these two groups also learned computer programing and did homework problems on the

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[14]Ibid., p. A-4-2. [15]Ibid., pp. A-4-4 - A-4-7.

computer. The third group, which served as a control group, was taught in the traditional fashion, using lecture, classroom discussion, and ordinary pencil and paper homework assignments.

Over the school year, the group which worked with the computer improved more than either of the other groups in general scholastic and reasoning abilities, as measured by standardized tests. As can be seen in the following figure, the computer group improved more than twice as much as the control group on a test of general scholastic ability and almost four times as much on a reasoning test.[16]

Percent Change in Group Mean
From Pre-Test to Post-Test

	Abstract Reasoning Test	Scholastic Aptitude Test
Control Group Scores	4.6	2.9
Flowchart Group Scores	9.7	5.1
Computer Plus Flowchart Group Scores	17.2	7.5

These results are particularly significant if it is considered that abstract reasoning is not confined to mathematics alone, but will carry over into other disciplines, and indeed, to many everyday problems and situations.

More recently, a few researchers such as Berry, Falkoff, and Iverson, have been "troubled," as they say, "by the marked tendency for the use of a computer to divert attention from the topics at hand to other quite different ones. . . ."[17] One of the main causes of this diversion is that:

[16] Computers In The Classroom, op. cit., p. 2.

[17] P. C. Berry, A. D. Falkoff, and K. E. Iverson, Using The Computer To Compute, Philadelphia Scientific Center Technical Report No. 320-2988 (Philadelphia Scientific Center IBM Corp., 1970), p. 1

> . . . Much of the work in educational computing has been done by people who narrowly construe "computer aided instruction" as an extension of programed instruction, and the computer as a successor to the Skinnerian teaching machine. This approach has discouraged the development of direct contributions of computing to the elucidation of mathematical topics.[18]

Consequently, Berry, Falkoff, and Iverson have concentrated on developing computer techniques to elucidate mathematical topics within the framework of existing courses. They have made extensive use of the programing language called APL (A Programing Language), mainly because of the following features:

1. The ease with which expressions are generalized to arrays, which makes it much easier to appreciate patterns and structure in results--a fundamental objective in teaching.

2. The treatment of functions, which facilitates examination of the properties of algebraic expressions.

3. APL's rich set of primitive operations, which provides a full range of functions for mathematical and logical operations and for the manipulation of arrays.[19]

In their report, Using The Computer To Compute, Berry, Falkoff, and Iverson describe most lucidly the use of this language in the teaching of several algebraic topics, including functions and equations. A series of such experimental teaching units was developed during the summer of 1969 for students in the first year of high school algebra. The report concluded that:

1. Effective use of the computer can be based on the introduction of only three simple ideas. Further notation need be introduced only as called for by the topic of study.

2. APL permits the treatment of mathematical topics in a notation that is close to that already in use

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[18]Ibid.

[19]Ibid., p. 2.

in mathematics, and differs from it only in the direction of increased consistency, generality, or power, and a minimum of concession to the inner working of the machine. Notation used in the text or classroom is directly executable by the machine.

3. It is possible to extend use of the computer to all areas of the mathematics curriculum, and not just to topics involving extensive computation.

4. Use of the computer for this purpose does not require the teaching of "programing" in the sense it is usually understood. Much of the instruction involves the behavior of functions which can be defined in a single statement, involving neither sequencing nor branching.

5. Effective use of the computer can be made on a collective basis, at a cost far below that of systems requiring individual interaction with each student.[20]

While this work has undoubtedly been invaluable in the production of new computer materials and techniques, conclusions such as (3) above are misleading. Although it is true that computer materials can be devised for all areas of the mathematics curriculum, every empirical study conducted to date has shown that, in their present form at least, these methods are not as effective as traditional ones in some topics. The instructional use of computers is so new, however, and the techniques so untried, that topics which have not proved adaptable in one instance should not be discarded categorically from further studies, particularly if results were inconclusive. In an additional study very similar to those conducted by Kieren and Hatfield,[21] Franklin Ronan stated in his findings that, "there is widespread need for revision and further development of instructional materials relevant to computer-

[20] Ibid., p. 19.

[21] Supra, p. 22.

assisted problem-solving in mathematics."[22] Statements to this effect have also been contained in every study of this type reviewed by this author. This fact, plus the knowledge that the computer methods already described do assist students to achieve significantly better in certain areas of mathematics, makes further experimentation in this mode of computer use not only relevant, but also imperative.

Summary

It is apparent that even though there is a wide range of computer use in present day education, many schools are using it as a problem solving aid to achieve subject matter goals, but with little integration into existing curricula. In addition, virtually no empirical evidence exists to support the effectiveness of this mode of use. On the other hand, some experiments have been conducted with the computer completely integrated into existing courses; not only as a problem solving tool, but also to provide students with additional insight into some areas of study, and to assist in the teaching process itself. These studies have shown that this mode of computer use does significantly increase achievement in certain topics of high school mathematics, but not in others. All stress the importance of identifying those areas where the computer is effective, and voice the need for developing new computer materials and techniques.

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[22] Ronan, op. cit., p. 3.

Chapter III

SETTING, PROCEDURES, AND DESIGN OF THE STUDY

This study was designed to answer the question: Do students who use a computer to study a unit of grade 10 mathematics achieve significantly better than students who do not use a computer? This chapter contains a description of the design and procedures used in the investigation to answer this question. It is divided under the following headings:

- Description of community and school
- Selection of participants
- Subject-matter
- Design of the study
- Instructional materials, methods, and techniques
- Evaluative instruments

Description of Community and School

The community of Fort Garry - a suburb of Winnipeg, Manitoba - was selected as the locale for the study, because of the pioneering and long standing interest of the Fort Garry School Division in computer science; and also because the University of Manitoba, whose computer was used, is located there.

Fort Garry is mainly a residential area although it does contain the university and some diversified industry in an expanding industrial park. Fort Richmond, the actual section of Fort Garry where the study was conducted, is very new and completely residential. It is located between the university and the perimeter highway on the extreme southern edge of Winnipeg. The area is growing rapidly, with the construction of new homes in constant evidence. The population contains a large number of families with young children and includes many university staff members.

The Fort Garry School Division contains 13 schools which include seven elementary, four junior high, and two senior high schools. In 1972-73 there were 6,439 students enrolled in kindergarten through grade 12, and 331 teachers. The courses taught are largely academic, although some business education and industrial arts courses are offered.

Fort Richmond Collegiate, the school in which the study was conducted, was completed in 1967 and at that time included students from kindergarten to grade 12. By 1970, with the opening of new junior high and elementary schools in the district, Fort Richmond became a senior high school exclusively, with students enrolled in grades 10, 11, and 12. Since then the student population has increased by about 50 to 75 students annually, and at the time of this study in 1972, the enrollment at Fort Richmond Collegiate was 375.

Two-thirds of the courses offered at the school are of the university entrance type, which are required by the universities in Manitoba for entrance into various faculties. The remainder are general, business education and industrial arts courses. A large percentage of students graduating from Fort Richmond Collegiate go on to post secondary education, either at university or community college.

Selection of Participants

Since every grade 11 and 12 mathematics class at Fort Richmond contained some students who were also taking a computer science course, the experimenter decided to use grade 10 students for the study rather than attempting to allow for this additional variable. Because the study took place in the middle of the spring term, students could not be randomly selected for participation, and existing, intact, grade 10 mathematics classes were used. The use of university entrance classes was predetermined, as two were required for the study and only one of the six grade 10 classes in the school was taking the general course.

Due to time-table arrangements of the school, one participating group had to be selected from three existing classes and the other from two. Since comparable groups were desired, selection was based on class means in mathematics from the previous term. This criterion for selection, however, did not guarantee completely comparable groups, because the two classes had been taught by different teachers and evaluated separately. On the other hand, any pretreatment differences arising from this factor were allowed for in the design of the study and analysis of data.

Finally, assignment of the participating classes to experimental and control groups was done by flipping a coin. The experimental and control groups thus chosen contained 26 students (21 boys and 5 girls) and 27 students (16 boys and 11 girls) respectively.

Subject-Matter

The subject-matter selected for the study was the unit of the grade 10 university entrance algebra course concerning systems of linear equations as outlined in the Manitoba high school program of studies,[1] and contained in chapter 10 of the authorized text.[2] The following outline identifies the relevant content of this chapter in detail:

1. Equations in two variables
2. Independent, inconsistent, and dependent systems
3. Solving systems of equations by graphing
4. Equivalent systems of equations
5. Solving systems of equations by addition, comparison, and substitution

In addition to this material, a little time was spent in reviewing coordinate systems on a plane.

In this study the computer was used with the experimental group to help attain the current behavioral objectives of the unit, and evaluation of the computer's effectiveness was based upon student achievement in relation to those objectives. No new material was added, and nothing was deleted from the content of the course with either the experimental or control group.

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[1] Manitoba High School Program of Studies: Grades 9-12, 1971-72, Department of Education, p. 31.

[2] E. D. Nichols, R. T. Heimer, and E. H. Garland, Modern Intermediate Algebra (Toronto: Holt, Rinehart and Winston of Canada, Ltd., 1966), pp. 289-302.

Design of the Study

The design of this study corresponds with the one that Campbell and Stanley identify in their taxonomy of experimental designs, as the Pretest-Posttest Control Group Design.[3] It has the form:

$$\begin{array}{cccc} R & O_1 & X & O_2 \\ R & O_3 & & O_4 \end{array}$$

The top and bottom rows represent experimental and control groups respectively while the two R's symbolize the random assignment of classes to these groups. The X represents the exposure of an experimental group to a treatment, the effects of which are to be measured. The purpose of the O's is to signify a process of measurement. The O's vertical to one another, such as O_1O_3 and O_2O_4, indicate simultaneous measurement, whereas the left to right dimension represents temporal order. In other words, O_1O_3 are the pretesting, and O_2O_4 the posttesting of two groups, separated in time by the exposure of the experimental group to some treatment. It should be noted that the comparison of X in the experimental group with no X in the control group is an over-simplification. "The comparison," Campbell and Stanley state, "is actually with the specific activities of the control group which have filled the time period corresponding to that in which the experimental

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[3]D. T. Campbell, J. C. Stanley, "Experimental and Quasi-Experimental Designs for Research on Teaching," Handbook of Research on Teaching, ed. N. L. Gage (Chicago: Rand McNally & Co., 1965), pp. 171-246.

group receives the X."[4] In this study, the computer treatment corresponds to the X, while the conventional method of instruction represents the no X.

Two pretests, consisting of an IQ test and a standardized mathematics achievement test, were administered simultaneously to both groups, one week prior to commencement of the study. The purpose of these tests was to determine to what extent the two groups were comparable. If significant pretreatment differences existed, then the scores from these tests would be used as co-variates in the analysis of post-treatment data. In this way, any significant differences in post-treatment scores could be attributed to the treatment.

In order to eliminate the effect of different teachers, the experimenter taught both groups. Following 15 fifty minute periods of instruction over a period of four weeks, a teacher constructed achievement test was administered to both classes. It tested only the mathematical objectives of the unit and was in no way computer oriented.

Instructional Materials, Methods, and Techniques

As in many present day mathematics classrooms, the experimenter not only made use of the blackboard, but also relied heavily on overhead projector transparencies in presenting new material - both in the control and experimental groups. Also, in addition to the authorized text, examples, demonstrations and illustrations

[4] Ibid., p. 183.

from several supplementary sources were used. Great care was exercised in ensuring that identical content was presented to both groups.

To minimize curiosity and antagonism, or other forms of contamination between classes, both groups were told that they were part of an experiment to help improve the mathematics curriculum for future students. Towards the end of the experiment, the control group was also given a demonstration of the computer terminal. This consisted of interactive games played with the computer and did not include any mathematical content.

Throughout the experiment both groups met at the regularly scheduled times, with the control group in their usual classroom and the experimental group in the computer science room. For the study, this room was equiped with a telephone, an IBM 2741 typewriter terminal, a closed circuit television camera and viewing set, plus a special overhead projector, designed for use with the terminal. Since teacher demonstration programs formed an integral part of the computer method, it was extremely important that all students receive an immediate and clear view of the output of such programs from their seats. Figure 1 illustrates the arrangement of equipment to accomplish this. When demonstrating a program, the experimenter was seated at the terminal in front of the room and facing the class, with the camera focussed on the output over his shoulder. Program outputs were then displayed on the TV set to the right of the terminal. This set was mounted on a wheeled stand and could be easily positioned for optimum viewing. The overhead projector was mounted on the terminal and when in use

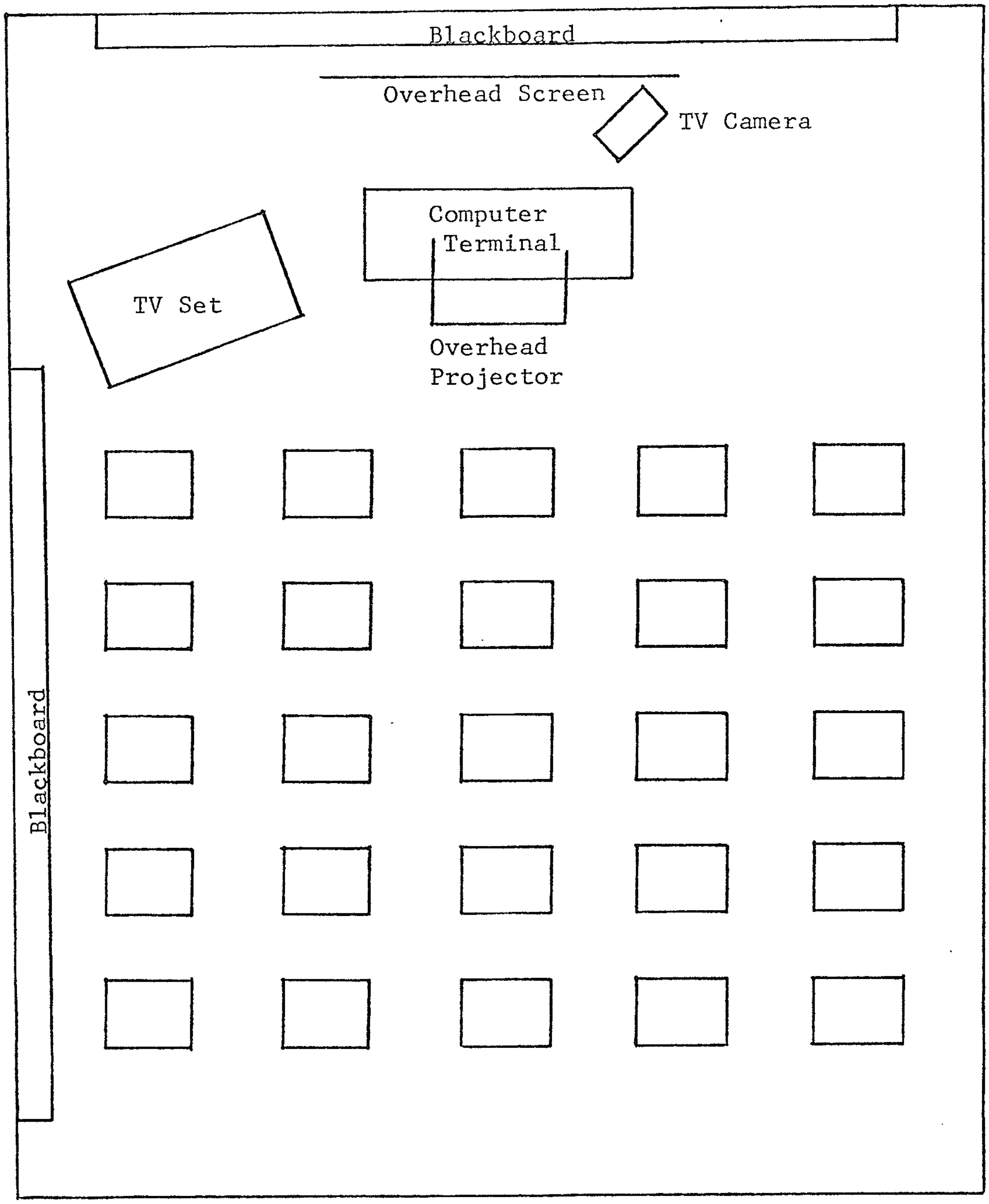

Figure 1

Arrangement of Viewing Equipment in Experimental Group Classroom displayed outputs on a large portable screen directly behind the instructor. The projector was also used for ordinary transparencies. For blackboard work, the screen and camera were moved aside or the board along one side of the room was used.

Use of the computer with the experimental group was really threefold. It was used for: (1) demonstration programs, (2) student assignments, and (3) student experimentation. The demonstration programs were teaching aids developed by the experimenter and stored in the computer system, where they were continuously available for display as required. The actual computer programs used in the study, together with descriptions of their use and sample outputs, are contained in appendix A. In many cases, only the output was of importance to the students, and they never saw the actual programs. At other times, simple problem solving programs were developed in class by teacher and students together. In this way, mathematical concepts did not take second place to computer programing per se, the language and techniques of which were introduced only when required.

FORTRAN IV was the actual programing language used, with an IBM 360 series computer and WATFIV compiler.[5] Two other languages, BASIC and APL, were also available, but for various reasons were not considered feasible for this study.

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[5]Unlike human languages such as English, programing languages are extremely formal and unambiguous, and are thus interpretable by machine. A compiler, itself a computer program, translates languages such as FORTRAN into a form that is executable by computer. The WATFIV compiler was designed especially for students and provides excellent error diagnostics for the programer.

In using the computer for assignments, students sometimes used the demonstration programs to solve given problems, while at other times they were required to develop and run their own programs. Some conventional, non-computer assignments were also given. The computer room was available to the students both before and during the regular school day, as well as in the evenings. Some class time was also reserved for students to use the terminal.

One IBM Selectric typewriter terminal was available for the study, and along with the computer time used, was donated without charge by the University of Manitoba. Although in almost constant use for four weeks, this terminal gave excellent service and was lost to the study for only one day. Use of the terminal was governed by the Manitoba University Monitor (MUM) system, which is a program developed at the University of Manitoba.[6] Under this system, programs can be created, edited and run, either immediately, or stored for later use. Response is virtually instantaneous and each programer has the distinct impression that he is the sole user of the computer.

The first period of instruction with the experimental group was spent in demonstrating various procedures for using the terminal, and a board containing these was placed on the terminal table for student reference. This board is shown in appendix B. Similar boards with a description of the purpose and method of use for each demonstration program were also provided.

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[6]R. J. Collens (ed.), The University of Manitoba MUM User's Guide (Winnipeg: [nn], 1971)

An example of a demonstration program, teacher/student developed program, and student assignment, as was used in teaching the solution of systems of equations by comparison, is outlined below.

A system of equations of the form: $\begin{matrix} Ax + By + C = 0 \\ Dx + Ey + F = 0 \end{matrix}$, where $A = D$, or $B = E$, lends itself to solution by the comparison method. This method was introduced with a parallel development of the theory and a concrete example such as the following.

Given a system where the coefficients of x are the same in both equations - we have:

1) $Ax + By + C = 0$	1) $10x + 10y + 3 = 0$
2) $Ax + Ey + F = 0$	2) $10x - 10y - 5 = 0$
3) $Ax = -By - C$	3) $10x = -10y - 3$
4) $Ax = -Ey - F$	4) $10x = 10y + 5$
5) $-By - C = -Ey - f$	5) $-10y - 3 = 10y + 5$

What is required now is to find the value of y which will make the left and right hand sides of equations (5) equal. This value can be found using a trial and error method which evaluates both sides of the equations separately (but uses the same y value in each side) for a wide range of different y values. Eventually, if enough different y values are tried, the correct one will be found. The program titled Trial and Error Comparison Program 1, contained in appendix A, was designed to illustrate this procedure. The above example can be evaluated for values of y starting at any given initial value and increasing in steps of one, as illustrated in the computer output of table 2. If none of the values tried makes the two sides equal, then either y is not in this range, or it is a fraction lying between two values. If, as in this case, it is a fraction, then the difference between the left and right hand sides reaches a minimum and then begins to increase. The required y value must lie between the two values that make the difference closest to zero, and can be found by repeating the evaluation process, starting with the smallest of these values and increasing in steps of 0.1. This process is repeated until the value of y is found exactly, or at worst, correct to four decimal places.

Now that the value of y has been found to be -0.4, this value is substituted in the original equations (1) and (2), and the value of x found by trial and error in the same manner. The Trial and Error Program 2 contained in appendix A accomplishes this. The correct value of x is that which makes both original equations equal to zero, as shown by the computer output of table 3.

Table 2

Computer Output of Trial and Error Comparison Program 1 for the System

$$10x + 10y + 3 = 0$$
$$10x - 10y - 5 = 0$$

Y	-By-C	-Ey-F	DIFFERENCE
-5.0000	47.0000	-45.0000	92.0000
-4.0000	37.0000	-35.0000	72.0000
-3.0000	27.0000	-25.0000	52.0000
-2.0000	17.0000	-15.0000	32.0000
-1.0000	7.0000	-5.0000	12.0000
0.0000	-3.0000	5.0000	8.0000
1.0000	-13.0000	15.0000	28.0000
2.0000	-23.0000	25.0000	48.0000
3.0000	-33.0000	35.0000	68.0000
4.0000	-43.0000	45.0000	88.0000
-1.0000	7.0000	-5.0000	12.0000
-0.9000	6.0000	-4.0000	10.0000
-0.8000	5.0000	-3.0000	8.0000
-0.7000	4.0000	-2.0000	6.0000
-0.6000	3.0000	-1.0000	4.0000
-0.5000	2.0000	0.0000	2.0000
-0.4000	1.0000	1.0000	0.0000

Thus, the solution set of the system is $\{(0.1, -0.4)\}$. There are always some students who attempt to solve equations and systems of equations by trial and error, but unlike a computer program, they often do so in a random manner and fail to find the solution. The power of such a demonstration program is that calculations that would not ordinarily be attempted can be done without error, in a few minutes. The method is illustrated here to emphasize its inefficiency, particularly if it is done by hand and the solution is a fractional one.

Table 3

Computer Output of Trial and Error Comparison Program 2 for the System
$10x + 10y + 3 = 0$
$10x - 10y - 5 = 0$

X	Y	Ax+By+C	Ax+Ey+F
-5.0000	-0.4000	-51.0000	-51.0000
-4.0000	-0.4000	-41.0000	-41.0000
-3.0000	-0.4000	-31.0000	-31.0000
-2.0000	-0.4000	-21.0000	-21.0000
-1.0000	-0.4000	-11.0000	-11.0000
0.0000	-0.4000	-1.0000	-1.0000
1.0000	-0.4000	9.0000	9.0000
2.0000	-0.4000	19.0000	19.0000
3.0000	-0.4000	29.0000	29.0000
4.0000	-0.4000	39.0000	39.0000
0.0000	-0.4000	-1.0000	-1.0000
0.1000	-0.4000	0.0000	0.0000

The comparison method normally used was now developed. As before:

1) $Ax + By + C = 0$ $\qquad 10x + 10y + 3 = 0$
2) $Ax + Ey + F = 0$ $\qquad 10x - 10y - 5 = 0$

3) $Ax = -By - C$ $\qquad 10x = -10y - 3$
4) $Ax = -Ey - F$ $\qquad 10x = 10y + 5$

5) $-By - C = -Ey - F \qquad -10y - 3 = 10y + 5$

6) $Ey - By - C = -F \qquad -10y - 10y - 3 = 5$

7) $Ey - By = C - F \qquad -10y - 10y = 3 + 5$

8) $y(E - B) = C - F \qquad y(-10 - 10) = 3 + 5$

9) $y = \frac{C - F}{E - B} \qquad y = \frac{3 + 5}{-10 - 10} = \frac{8}{-20} = -0.4$

From equation (3) or (4):

10) $x = \frac{-By - C}{A}$, or $\qquad x = \frac{-10y - 3}{10}$

11) $x = \frac{-Ey - F}{A}$

The value of x is found by substituting the value of y from (9) into equation (10) or (11). $\qquad x = \frac{-10(-0.4) - 3}{10} = 0.1$

The computer program called Comparison Method Program 3, shown in appendix A, was developed at this point with the students. It is actually a series of four small programs - the first three of which were done in class and the fourth given as an assignment. The first one simply uses equations (9) and (11) above. When supplied with the coefficients A,B,C,E,F of a system, the output is: X = ..., Y = ... The second program merely adds a verification feature, while the third allows for inconsistent systems which have no solution.

Systems in which the coefficients of y are the same in both equations can also be solved by the comparison method. The assignment given the students consisted of writing a program with the same features as the third one mentioned above, to solve a number of given systems. In order to do this, the students had to develop formulae similar to equations (9) and (10) above, and solve for X first. This development is very similar to that already done in class and provided excellent practice in formalizing the comparison method. The fourth program listed under Comparison Method Program 3 in appendix A is an example of this type.

Unknown to the students, the last system given in the assignment was a dependent one, with infinite solutions. Unless they foresaw this and allowed for it, their program printed out incorrectly that the system had no solutions. This was pointed out when the assignments were reviewed and served to bring out the relationship between the coefficients in a dependent system. A method of allowing for this type of system in a program was discussed in the next lesson.

Other demonstration programs and assignments were used in a similar manner with the remaining topics of the study.

Evaluative Instruments

The two pretests administered to the control and experimental groups for the purpose of determining pretreatment differences were: (1) the Otis-Lennon Mental Ability Test, Advanced Level, Form J, and (2) the Canadian New Achievement Test in Mathematics, Form A (CNATM). The Otis-Lennon test is designed to measure broad reasoning abilities, with the advanced level recommended for use with typical pupils in grades 10 through 12. Table 4 contains details of the reliability of this test by grade and age.[7] The mean age of all students in the study was 15. The

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[7] A. S. Otis and R. T. Lennon, Otis-Lennon Mental Ability Test: Manual for Administration (New York: Harcourt, Brace & World, Inc., 1967), pp. 20-22.

CNATM test was developed by the Ontario Mathematics Commission to provide a criterion by which to judge student achievement in mathematics at the end of grade nine. It has a reliability of .70 calculated using the Kuder-Richardson Formula 20.[8]

Table 4

Reliability of the Otis-Lennon Mental Ability Test by Grade and Age

	Reliability			
	Split-Half[1]	K-R #20	Alternate-Forms[2]	Standard Error of Measurement[3]
Grade 10	.95	.94	.94	4.0
Age 15	.94	.94	.92	4.5

[1]Corrected by Spearman-Brown Prophesy Formula

[2]Computed by using weighted Fisher's Z transformation

[3]Computed using alternate forms reliability

On completion of instruction, a post-treatment achievement test, constructed by the experimenter and contained in appendix C, was administered to both groups. It tested only mathematical objectives with no reference to computers or computer methods, and was approved by three mathematics teachers for content vali-

[8]V. R. D'Oyley, S. M. Avital, and H. R. Russell, Canadian New Achievement Test in Mathematics Technical Manual (Toronto: The Ontario Institute for Studies in Education, 1965), p. 4.

dity and thoroughness. Prior to the study the test was piloted by two grade eleven classes and an item analysis performed on the results. From this analysis, forty items were selected, some of which were relatively easy and some relatively difficult, but with the majority having a difficulty index of between .5 and .6. Following administration of the test to participants of the study, a further item analysis was performed and a reliability coefficient of .81 calculated using the Kuder-Richardson Formula 20. The standard error of measurement calculated using this coefficient was found to be 2.58.

Chapter IV

ANALYSIS OF DATA

It is the purpose of this chapter to present detailed results of the investigation described in the preceding chapter. More specifically, the hypotheses set out in chapter I and restated below will be tested and either accepted or rejected.

1. There is no significant difference in mean achievement between students in a unit of grade 10 mathematics who use a computer and those who do not.
2. There is no significant difference in mean achievement between male students in computer and non-computer groups.
3. There is no significant difference in mean achievement between female students in computer and non-computer groups.

To facilitate the reporting of results, this chapter is divided into the following two sections:

- Pre-treatment testing
- Post-treatment testing.

Pre-Treatment Testing

In order to avoid mistakes in recording, scoring, and calculating, all test scores in this study were checked several times and a computer used for all calculations.[1] Because of the relatively short time period involved, students who missed four or more of the 15 classes were dropped from the study, leaving a total of 42 participants - 21 in the experimental group (16 male and 5 female), and 21 in the control group (12 male and 9 female).

Although participating classes were selected for the study on the basis of comparable means in mathematics from the previous school term, it was suspected that the two classes were not truly "alike." Consequently, the two pre-treatment tests were administered to ascertain the general and mathematical ability levels of both groups. Results from these tests were submitted to analysis of variance and subsequently used to analyze post-treatment test scores.

The first test administered was the Canadian New Achievement Test in Mathematics (CNATM), which is designed to measure mathematical ability. A factorial analysis of variance computer program was used on the scores, which were arranged by sex in the experimental and control groups.[2] Table 5 shows the means resulting from the analysis. The mean of the control group was 15.48, which is considerably higher than 12.05 for the experimental group, while those of the total male and female participants were very similar at 13.64 and 14.00 respectively.

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[1] "Statistical Package: Part B of the Programer's Guide" (Winnipeg: University of Manitoba Computer Centre, 1972). (Computer print out.)

[2] Ibid., pp.37-40.

Table 5

Means of the Pre-Treatment CNATM Test

Group	N	Mean
Control	21	15.48
Experimental	21	12.05
Male (both groups)	28	13.64
Female (both groups)	14	14.00
Male (control)	12	15.25
Female (control)	9	15.78
Male (experimental)	16	12.44
Female (experimental)	5	10.80

Analysis of variance results of the computer program included calculated F variance ratios for the main effects and their interaction, based on the within-cells mean square as the error term. The program also adjusted the sum of squares for unequal subclass numbers. Table 6 summarizes the results for the CNATM data and indicates significant differences between calculated and expected F values. The calculated value of F for the main effect of treatment was 16.61 compared to the expected value of 7.31 at the one percent level of confidence. Thus, a significant pre-treatment difference in mathematics ability existed between the two groups, with the control group performing significantly higher than the experimental group. Calculated F values of 0.30 and 1.07 for the effects of sex and treatment/sex interaction respectively, were not significant.

Table 6

Results of Two-Way Analysis of Variance for the Pre-Treatment CNATM Test

Source of Variation	df	Sum of Squares	Mean Square	F
Treatment	1	159.3069	159.3069	16.61*
Sex	1	2.8735	2.8735	0.30
Treatment Sex	1	10.2603	10.2603	1.07
Within Cells	38	364.5427	9.5932	
Total	41	499.6184		

*Significant at .01 level

A second pretest, the Otis-Lennon Mental Ability Test, was administered to students to determine whether the two groups differed in general mental ability, and the computer program previously described was used to perform a two-way analysis of variance on the results. Table 7 shows that the mean of the experimental group was 56.48 compared to 65.57 for the control group. As with the CNATM test, means for males and females were virtually the same at 61.00 and 61.07 respectively. Table 8 contains details of the calculated F ratios for the main effects of treatment and sex, as well as treatment/sex interaction. Since the calculated F for treatment was 10.54 and the expected value at the .01 level was 7.31, a significant pre-treatment difference in general mental ability

Table 7

Means of the Pre-Treatment Otis-Lennon Test

Group	N	Mean
Control	21	65.57
Experimental	21	56.48
Male (both groups)	28	61.00
Female (both groups)	14	61.07
Male (control)	12	67.00
Female (control)	9	63.67
Male (experimental)	16	56.50
Female (experimental)	5	56.40

Table 8

Results of the Two-Way Analysis of Variance
for the Pre-Treatment Otis-Lennon Test

Source of Variation	df	Sum of Squares	Mean Square	F
Treatment	1	828.5925	828.5925	10.54*
Sex	1	27.5048	27.5048	0.35
Treatment sex	1	22.8791	22.8791	0.29
Within Cells	38	2987.2000	78.6105	
Total	41	3912.9761		

*Significant at .01 level

also existed between the two groups; again in favor of the control group. Calculated values of 0.35 for the effect of sex and 0.29 for treatment/sex interaction were not significant.

Thus, it was determined that the two classes participating in the study were not comparable in general mental or mathematical ability, and the results of the two pretests would have to be used as covariates in the analysis of post-treatment data.

Post-Treatment Testing

On completion of the unit of instruction on systems of equations, a teacher constructed post-treatment achievement test was administered to the control and experimental groups. Since analyses of pre-treatment tests had indicated significant differences between the groups in mathematical as well as general mental ability, post-treatment scores were subjected to analyses of covariance. In this way, pre-treatment differences were allowed for, and post-treatment differences in achievement could be attributed to the experimental computer treatment.

Once again, a computer program was used for the analyses which provided F ratios, as well as means and adjusted means for each level (control/experimental and male/female) in the main effects of treatment and sex, plus interactions.[3] Six separate analyses were made; three without, and three with a sex factor. In each of these two groups of analyses, one was done using CNATM test results as a covariate, one with Otis-Lennon results, and

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[3]Statistical Package, pp. 50-52.

one with both. Results of the three analyses without a sex factor are summarized in tables 9 and 10. As seen in table 9, the

Table 9

Results of Analyses of Covariance With No Sex Factor for Post-Treatment Achievement Test

Covariate	Source of Variation	df	Sum of Squares	Mean Square	F
	Treatment	1	84.862	84.862	3.87
CNATM	Error	39	855.900	21.946	
	Total	40	1133.614		
	Treatment	1	103.197	103.197	4.70*
Otis-Lennon	Error	39	855.901	21.946	
	Total	40	1133.615		
	Treatment	1	44.969	44.969	2.00
CNATM and Otis-Lennon	Error	38	855.900	22.524	
	Total	39	1133.614		

*Significant at .05 level

calculated F value using the Otis-Lennon Mental Ability pre-test as a covariate was 4.70, compared to an expected value of 4.08 at the five percent level of confidence. The adjusted means of the post-treatment achievement test for this covariate were 28.478 and 25.713 for the control and experimental groups respectively. Thus, a significant difference in post-treatment mathematics achievement existed, in favor of the control group, and the null hypothesis of no significant difference was rejected. Calculated F values for the other two analyses were not significant and the null hypothesis was accepted in these cases.

Table 10

Means and Adjusted Means of Post-Treatment Achievement Test With CNATM, Otis-Lennon, CNATM and Otis-Lennon Used as Covariates

Covariate	Group	Mean	Adjusted Mean
CNATM	Control	29.667	28.329
	Experimental	24.524	25.862
Otis-Lennon	Control	29.667	28.478
	Experimental	24.524	25.713
CNATM and Otis-Lennon	Control	29.667	27.949
	Experimental	24.524	26.241

Results of the three analyses of covariance having sex as a factor are given in tables 11 to 16. Table 11 contains the results of the analysis using CNATM pretest scores as a covariate, and shows the calculated F values for the effects of treatment, sex, and treatment/sex interaction to be 3.69, 0.41, and 1.46 respectively. Since the expected F ratio is 4.08 at the .05 level, none of the calculated values were significant. This caused the null hypotheses of no significant difference in achievement between groups and

Table 11

Results of Analysis of Covariance With a Sex Factor and Covariate CNATM for Post-Treatment Achievement Test

Source of Variation	df	Sum of Squares	Mean Square	F
Treatment	1	81.494	81.494	3.69*
Sex	1	9.038	9.038	0.41*
Treatment sex	1	32.241	32.241	1.46*
Error	37	817.439	22.093	
Total	40	1133.615		

*Not significant

Table 12

Means and Adjusted Means of Post-Treatment Achievement Test With a Sex Factor and Covariate CNATM

Group	Mean	Adjusted Mean
Control	29.667	28.299
Experimental	24.524	25.891
Male (both groups)	27.357	27.423
Female (both groups)	26.571	26.440
Male (control)	30.750	29.927
Female (control	28.222	27.107
Male (experimental)	24.813	25.545
Female (experimental)	23.600	25.238

between groups for male and female students to be accepted.

Results of the analysis with Otis-Lennon pretest scores used as a covariate are contained in tables 13 and 14. The calculated F ratio for the effect of treatment was 4.92 compared to the expected value of 4.08 at the five percent level of confidence. Table 14 shows that the adjusted means of the post-treatment achievement test for the control and experimental groups were 28.512 and 25.679 respectively. Thus, as in the case with no sex factor, the control group achieved significantly higher than the experimental group in the posttest and the null hypothesis was rejected. F values of 0.27 and 0.88 were calculated for the effect

Table 13

Results of Analysis of Covariance With a Sex Factor and Otis-Lennon Covariate for Post-Treatment Achievement Test

Source of Variation	df	Sum of Squares	Mean Square	F
Treatment	1	108.798	108.798	4.92*
Sex	1	5.934	5.934	0.27
Treatment sex	1	19.343	19.343	0.88
Error	37	817.439	22.093	
Total	40	1133.615		

*Significant at .05 level

of sex and treatment sex interaction respectively, as seen in table 13. These values were not significant and the null hypotheses of no significant differences between groups for male or female students were accepted.

Table 14

Means and Adjusted Means of Post-Treatment Achievement Test With a Sex Factor and Otis-Lennon Covariate

Group	Mean	Adjusted Mean
Control	29.667	28.512
Experimental	24.524	25.679
Male (both groups)	27.357	27.361
Female (both groups)	26.571	26.564
Male (control)	30.750	29.705
Female (control)	28.222	27.760
Male (experimental)	24.813	25.603
Female (experimental)	23.600	24.408

The final analysis, with CNATM and Otis-Lennon pretests used as covariates, did not show any significant differences in achievement; although the adjusted mean of the control group, as shown by table 16, was once again higher than the experimental group. None of the calculated F ratios contained in table 15 were significant when compared to the expected value of 4.08 at the five percent level. Consequently, the null hypotheses of no

Table 15

Results of Analysis of Covariance With a Sex Factor And Covariates CNATM and Otis-Lennon for Post-Treatment Achievement Test

Source of Variation	df	Sum of Squares	Mean Square	F
Treatment	1	47.108	47.108	2.07*
Sex	1	8.505	8.505	0.37*
Treatment sex	1	21.176	21.176	0.93*
Error	36	817.439	22.707	
Total	39	1133.614		

*Not significant

Table 16

Means and Adjusted Means of Post-Treatment Achievement Test With a Sex Factor and Covariates CNATM and Otis-Lennon

Group	Mean	Adjusted Mean
Control	29.667	27.967
Experimental	24.524	26.223
Male (both groups)	27.357	27.413
Female (both groups)	26.571	26.460
Male (control)	30.750	29.339
Female (control)	28.222	27.000
Male (experimental)	24.813	25.968
Female (experimental)	23.600	25.489

significant differences in post-treatment achievement between groups for male or female students, were accepted.

Summary

In summary, six separate analyses using pretest scores as covariates were performed on post-treatment achievement scores; three with sex as a factor and three without. Of these analyses, only the two with the Otis-Lennon Mental Ability Test scores as a covariate showed any significant differences in achievement, and these differences were in favor of the control group.

Chapter V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Purpose of the Study

The main purpose of this study was to determine whether grade 10 students who use a computer to study mathematics attain a higher level of achievement than other comparable grade 10 students who do not use a computer. Since it is possible that boys and girls might react differently to computer hardware, a sub-purpose of the study was to determine if male students in a computer group achieve higher than male students in a non-computer group, and similarly, whether female students who use a computer reach a higher achievement level than females who do not.

Interest in pursuing the experiment was aroused by the feeling of many educators that computers can play an important part in the learning of mathematics, and the lack of empirical research to support this feeling; as well as by the need to introduce young people to the machine that is playing an increasingly important role in human affairs.

Research Procedures

Two grade 10 university entrance classes were selected for the study on the basis of previously demonstrated mathematics ability, and randomly assigned to an experimental and a control group. In order to confirm the general mental ability and mathematics achievement levels of these groups, two pretests were administered prior to commencing the experiment. A unit on systems of linear equations was then taught to both groups by the experimenter, in 15 fifty-minute classes over a period of four weeks. The same mathematical content was presented to both groups, but with the experimental group using a computer as an instructional tool, while the control group did not. Following the unit of instruction, an achievement test that contained no reference to computers was administered to all participants.

Throughout the study students in the experimental group had direct access to a computer through the teletype terminal located in their classroom, and they were encouraged to use it as often as possible, both to complete assignments and to experiment. During class time the terminal was used mainly by the teacher, not only to display previously prepared demonstration programs, but also to run programs developed jointly by teacher and students. Students were assigned problems to solve with the computer, using their own programs as well as the demonstration ones.

Although two other programing languages were available with the University of Manitoba terminal system, FORTRAN IV was the one chosen for the study. It is a powerful language, well suited to mathematical problem solving.

As the study progressed it soon became apparent that the IBM 2741 typewriter terminal was not completely satisfactory, especially for demonstration purposes where the typing of some outputs took several minutes. While this is not a long time in real terms, to a class of expectant students it was at times like a teacher with a speech impediment. Although cathode ray tube (CRT) terminals have the disadvantage of not providing hard copies, their instantaneous output make them preferable for demonstration programs. As it was also found that one terminal is not sufficient to give students enough "hands on" time, three terminals, including one CRT, could have been used.

It also became apparent that the four week duration of the study was too short for students to become proficient enough in computer programing to allow the problem solving mode to be fully exploited. This factor, as well as the fact that only one terminal was available, may also have hampered experimentation by students.

Design of the Study

The Pretest-Posttest Control Group Design used in this study affords good control over the internal validity. The factors of internal validity are those that directly affect test scores and could themselves cause changes which might be confused with the effects of the experimental treatment. Campbell and Stanley list the following eight factors relevant to internal validity:

1. History
2. Maturation
3. Testing
4. Instrumentation
5. Statistical Regression
6. Selection of Respondents
7. Experimental Mortality
8. Selection-Maturation Interaction[1]

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[1] D. T. Campbell and J. C. Stanley, "Experimental and Quasi-Experimental Designs for Research on Teaching," Handbook of Research on Teaching, ed. N. L. Gage (Chicago: Rand McNally & Co., 1965), pp. 175-76.

Only four of these factors are relevant to this study: that is, History, Maturation, Selection of Respondents, and Selection-Maturation Interaction.

The history factor refers to specific events that occur during a study in addition to the experimental treatment. These events, such as the change of season or approaching examinations, for example, might produce a pretest-posttest change confusable with the effect of the experimental variable. Maturation processes, on the other hand, are ones within the participants operating as a function of the passage of time per se (not specific to the particular events), including growing older, growing hungrier, growing more tired, and the like. The effects of both these factors are controlled in this study since, as Campbell and Stanley state, ". . . they would be manifested equally in experimental and control groups."[2]

A third factor among the threats to internal validity is the effect of biases resulting in the differential selection of respondents for the comparison groups. This means simply that any bias on the part of the researcher might cause him to place some subjects into the experimental rather than the control group, or vice versa; making the comparison groups unequal in some respect before treatment. Although the design normally controls this effect by the random assignment of subjects to experimental and control groups, it could not be done in the case of this study since existing classes had to be used. The original placing of

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[2]Ibid., p. 184.

students into these classes by the school was not random, and in fact, was based on student course selection. Thus, the selection of respondents for this study was indeed biased, and as both pretests showed, the comparison groups were not similar either in general mental ability or in mathematical achievement. To the extent that it is possible with statistical methods, these pretreatment differences were identified and then controlled using analysis of covariance techniques.

The final effect listed by Campbell and Stanley as being a factor of internal validity, and relevant to this study, is that of selection-maturation interaction. This effect arises when one of the comparison groups has a naturally higher rate of maturation than the other, which could result in differential pretest-posttest gains regardless of any experimental treatment. The design normally controls this effect once again by ensuring the equality of comparison groups through random assignment. Although not controlled in this study, the short duration of the experiment very likely reduced any such effect to a minimum.

The remaining factors which can affect an experimental design are those pertaining to external validity, which is concerned with generalizability. That is, to what population and with what degree of confidence can the effect of a specific experiment be generalized? This is a difficult question that can seldom be satisfactorily answered, and is not resolved in the pretest-posttest control group design. The threats to external validity are referred to by Campbell and Stanley as interactive effects, involving the experimental treatment and some other variable.

The first of these is the interactive effect of testing, in which a pretest might increase or decrease a respondent's sensitivity or responsiveness to the experimental variable. This could make the results obtained for a pretested group unrepresentative of the effects of the experimental variable for the population from which the respondents were selected. This might well have been a factor in this study, as several students expressed open hostility to the standardized pretests.

Another threat to external validity is the interaction effect of selection biases and the experimental treatment. Because of certain selection biases, the sample used in an experiment might possess some unique characteristics causing it to be unrepresentative of the parent population. These characteristics might then cause the experimental variable to be more or less effective than it would be in the target population. In this study there was no guarantee that the experimental group used was representative of all the grade 10 university entrance classes in the school.

The final factor affecting external validity is the reactive effects of experimental arrangements. The artificiality and novelty of some experimental settings, as well as the awareness of participation, cause students to react differently to treatment. These reactions are not representative of normal classroom situations and seriously hamper generalization. The short duration of this study may have allowed the novelty effect of the computer terminal and other equipment to be a factor. Although the presence of a strange teacher in many experiments also contributes to this factor, it did not in this study, as the experimenter taught both groups.

In summary, the experimental design of this study gave good control over internal validity, thus allowing post-treatment changes to be attributed to the effects of treatment. External validity, on the other hand, was not well controlled, and results of the study cannot be generalized, at least not beyond the participating school.

Statistical Procedures

In order to determine whether pre-treatment differences existed between the comparison groups, results of both pretests were subjected to a two-way analysis of variance. Calculations of the F ratios for the main effects of group and sex, plus their interaction, were based on the within cells mean square as the error term. Adjustments to the sums of squares for unequal sub-class numbers were also made. Since these analyses both showed significant differences, analyses of covariance were performed on posttest scores using pretest scores as covariates. Six separate analyses were made; three without, and three with a sex factor as shown in table 17.

A reliability coefficient for the teacher constructed posttest was calculated using the Kuder-Richardson Formula 20, but as the test appeared to be slightly too long for the allotted time, this estimate is probably somewhat inflated.

Table 17

Posttest Analyses of Covariance Configuration
With and Without a Sex Factor, and CNATM,
Otis-Lennon Pretests as Covariates

	Covariate		
With a Sex Factor	CNATM	Otis-Lennon	CNATM and Otis-Lennon
No Sex Factor	CNATM	Otis-Lennon	CNATM and Otis-Lennon

Results of Testing Hypotheses

Analyses of results of the post-treatment achievement test on systems of linear equations showed no significant differences between the two groups at the five percent level of confidence, either with CNATM scores as a covariate, or with CNATM and Otis-Lennon together. This was the case both with sex as a factor and without. Accordingly, the null hypothesis concerning the similarity of the experimental and the control group was accepted:

> There is no significant difference in mean achievement between students in a unit of grade 10 mathematics who use a computer and those who do not.

As no significant treatment-sex interactions were found, the two null hypotheses depicting the similarity of the groups for both sexes were also accepted:

There is no significant difference in mean achievement between male students in computer and non-computer groups.

There is no significant difference in mean achievement between female students in computer and non-computer groups.

When the Otis-Lennon pretest was used alone as a covariate, however, a significant difference in achievement did exist at the five percent level, in favor of the control group. Again this was true with and without sex as a factor. In this case the null hypothesis was rejected. No significant treatment-sex interactions were discovered here, and once again the null hypotheses concerning sex were accepted.

Conclusions

As a result of this investigation and the concomitant statistical analysis, several conclusions were derived:

1. Prior to treatment there were significant differences between the experimental and the control group in general mental ability and in mathematics achievement.

2. After treatment in a unit involving systems of linear equations, there was no significant difference in mean achievement between students who used a computer during treatment and students who did not, in the cases where the covariates used were: (a) mathematics achievement scores, and (b) both mathematics achievement and mental ability scores.

3. After unit treatment there was a significant difference between the mean achievement of students who did not use a computer during treatment and the mean achievement of students who

did use a computer, when results were analyzed using mental ability scores alone as a covariate. The difference was in favor of the group which did not use a computer, and it was only slightly significant at the five percent level of confidence.

4. There was no significant treatment-sex interaction in any of the analyses, and thus, males and females did not react differently to the experimental treatment.

5. The above findings apply only to one unit of the grade 10 university entrance mathematics curriculum, for two classes in a particular high school.

6. The findings are not conclusive and the slight adverse affect of the experimental treatment was due, in part, to several limitations of the study.

7. There is a definite need for revision and further development of instructional materials and techniques relevant to this type of computer-assisted learning in mathematics.

Implications and Recommendations for Further Study

The statistical evidence obtained from this study did not provide evidence to support the use of computer-assisted learning in high school mathematics. On the other hand, it did indicate that the unit on systems of linear equations is perhaps not one of the areas of mathematics in which the use of a computer as an instructional tool can favorably influence student learning behavior. This was an implied purpose of the study, and as such, provides useful information for further projects of this type.

It is evident that a great deal of additional research remains to be done, not only to further examine the hypotheses of this study, but also to answer such questions as the following:

1. Is there a relationship between the programing language used and achievement?

2. Would a short unit on computer programing per se prior to a study affect achievement?

3. Does the use of a computer benefit low or average achievers differently than high achievers?

4. What effect does the amount of time that a student uses a computer have on his achievement in a course?

5. Does the use of a computer as an instructional tool change the attitude of students towards mathematics?

6. Do students who use a computer receive beneficial learning experiences that are not identified or measured by traditional achievement tests?

7. Is there a transfer effect of computer methods to other courses?

8. Is there an optimum number and combination of types of computer terminals?

Quite apart from the above questions, the following procedural recommendations are also pertinent to future studies of this type:

1. In order to preclude any effects of testing on treatment, pretests should be administered at some time prior to commencement of the study.

2. All students in the school who are enrolled in the course under investigation should be randomly assigned to classes at the

beginning of the school year. Entire comparable classes could then be randomly selected for participation, allowing conclusions of the study to be safely generalized.

3. Duration of the study should be at least eight weeks, and preferably one term or the entire school year.

4. A CRT type computer terminal should be used for teacher demonstrations.

In conclusion, the study reported here gathered some empirical evidence concerning the educational value of computer-assisted learning in high school mathematics. Every similar study conducted to date has noted not only the deplorable lack of such evidence but also the need for new computer materials and techniques. This study attempted to fulfill this need in one section of the curriculum. Many additional questions concerning the use of computer-assisted instruction remain to be answered, however, and the need for further studies of this type is apparent.

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APPENDIX A

Descriptions and Procedures for use of Programs,
With Program Listings and Sample Outputs

PROGRAM NAME - COORD

This program is designed to plot up to 361 points in a coordinate system when given the X and Y coordinates.

Input Data

First line - A single integer representing the number of points to be plotted.

Second line - Up to 24 X coordinates.
- A blank must be left between coordinates.
- If more than 24 points are to be plotted, use as many lines as necessary, but only 24 coordinates per line.

Next line - Up to 24 Y coordinates.
- There must be the same number of Y as X coordinates.

Example - To plot the points (9,4), (9,5), (-9,6), the data would be:

```
3
9 9 -9
4 5  6
```

Procedure for Use

- In the example below, user's typing is underlined.
- Assume that you have signed on.
- Hit the RETURN key after typing each line.

```
o coord
00610  $$1 580,100
00580  3
00590  3 4 5
00600  2 5 7
00610  $$r 580,600 ... This number (600) will be the
00580  5               same as that of the last line
00590  1 3 5 7 9       of data typed by the computer.
00600  7 8 9 0 -2
00610  $$s
FILE SUBMITTED
ENTER ACCOUNT #, PGM NAME
218.xx print i ben
899 AWAITING PRINT
f 899 prtl 1,100 ..... Results will now be printed out.
         .
         .
         .
e  .................. After all typing stops.
```

Listing of Program COORD

```
      INTEGER*2 XCOORD(361),YCOORD(361)
      CHARACTER CHAR(19,19)/361*' '/
      CHARACTER*2 NUM(19)/'-9','-8','-7','-6','-5','-4','-3','-2','-1',
     *' ',' 1',' 2',' 3',' 4',' 5',' 6',' 7',' 8',' 9'/
      READ, N
C
C  N=O RESULTS IN OUTPUT OF AXES ONLY.
C  N=-1 RESULTS IN AXES AND A PLOT OF ALL POSSIBLE POINTS.
C  N=1 TO 361 INDICATES THE NUMBER OF POINTS TO BE PLOTTED.
C
      PRINT 500, N
      PRINT 1000
      IF(N.GT.361) GO TO 80
      IF(N.GE.O) GO TO 4
      DO 3 I=1, 19
      DO 2 J=1, 19
    2 CHAR(I,J)='*'
    3 CONTINUE
      GO TO 50
    4 DO 5 I=1, 19
      CHAR(I,10)='+'
    5 CHAR(10,I)='+'
      IF(N.EQ.O) GO TO 50
      READ, (XCOORD(I),I=1,N)
      READ, (YCOORD(I),I=1,N)
      PRINT 600, (XCOORD(I),I=1,N)
      PRINT 900
      PRINT 700, (YCOORD(I),I=1,N)
      DO 20 I=1,N
      I1=XCOORD(I)
      I2=YCOORD(I)
      IF(IABS(I1).GT.9.OR.IABS(I2).GT.9) GO TO 90
   20 CHAR(10-YCOORD(I), 10+XCOORD(I))='*'
   50 PRINT 100
      DO 60 I=1,10
   60 PRINT 200, (CHAR(I,J),J=1,19), NUM(20-I)
      NUM(10)=' 0'
      PRINT 300, (NUM(I),I=1,19)
      DO 70 I=I1,19
   70 PRINT 400, (CHAR(I,J),J=1,19), NUM(20-I)
      STOP
   80 PRINT, 'FATAL ERROR - THIS PROGRAM WILL NOT PLOT MORE THAN 361 PO
     *INTS. ADJUST YOUR DATA AND RESUBMIT THE PROGRAM.'
      PRINT 800
      STOP
   90 PRINT, 'THAT''S A NO NO - THIS PROGRAM WILL NOT PLOT POINTS WHOSE
     *X OR Y COORDINATE IS GREATER THAN +9 OR LESS THAN -9.'
      PRINT, 'CHECK YOUR DATA AND RESUBMIT THE PROGRAM.'
      PRINT 800
```

```
      STOP
 100  FORMAT(//45X,'Y AXIS')
 200  FORMAT (1X//19A5,T48,A2)
 300  FORMAT(19A5,4X,'X AXIS'/)
 400  FORMAT(19A5,T48,A2//)
 500  FORMAT(//1X,'DATA'/1X,'N=',I3)
 600  FORMAT(1X,'X COORDINATES ARE ',24I3)
 700  FORMAT(1X,'Y COORDINATES ARE ',24I3)
 800  FORMAT(////)
 900  FORMAT(/)
1000  FORMAT('REMINDER - IF YOU PUT MORE THAN 24 COORDINATES ON A LIN
     *E AN ERROR WILL RESULT. PUT THE REST ON NEXT LINE.')
      END
```

```
                                             Y AXIS
  *    *    *    *    *    *    *    *    *   9 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *   8 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *   7 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *   6 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *   5 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *   4 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *   3 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *   2 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *   1 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *     *    *    *    *    *    *    *    *    *    *
 -9   -8   -7   -6   -5   -4   -3-  -2   -1     0    1    2    3    4    5    6    7    8    9 X AXIS
  *    *    *    *    *    *    *    *    *  -1 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *  -2 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *  -3 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *  -4 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *  -5 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *  -6 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *  -7 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *  -8 *    *    *    *    *    *    *    *    *    *
  *    *    *    *    *    *    *    *    *  -9 *    *    *    *    *    *    *    *    *    *
```

Figure 2

Output of Program COORD - Plot of All Possible Points

```
DATA
N=17
REMINDER - IF YOU PUT MORE THAN 24 COORDINATES ON A LINE AN ERROR WILL
RESULT. PUT THE REST ON NEXT LINE.
X COORDINATES ARE 2 2 2 2 2 3 4 5 5 5 5 5 8 8 8 8 8

Y COORDINATES ARE -3 -4 -5 -6 -7 -5 -5 -3 -4 -5 -6 -7 -3 -4 -5 -6 -7

                                 Y AXIS

                                    9+

                                    8+

                                    7+

                                    6+

                                    5+

                                    4+

                                    3+

                                    2+

                                    1+

 +   +   +   +   +   +   +   +   +   +   +   +   +   +   +   +   +   +   +
-9  -8  -7  -6  -5  -4  -3  -2  -1   0   1   2   3   4   5   6   7   8   9 X AXIS
                                   -1+

                                   -2+

                                   -3+       *           *           *

                                   -4+       *           *           *

                                   -5+       *   *   *   *           *

                                   -6+       *           *           *

                                   -7+       *           *           *

                                   -8+

                                   -9+
```

Figure 3

Output of Program COORD - Plot of the Letters HI

PROGRAM NAME - GRAPH

This program is designed to evaluate one or more linear expressions in two variables for a given number of values of the variables. The output can be any of the following; depending on the value of a control value supplied.

	Control value	Output
a)	1	A table of evaluations for each expression.
b)	2	A graph of each expression.
c)	3	A table and a graph of each expression.
d)	4	A table for each expression and a combined graph of all expressions.
e)	5	A combined graph of all expressions.

Input Data

First line - Two natural numbers, separated by a blank, representing the number of expressions and control value respectively.

Second line - Five integers, separated by blanks. The first three are the coefficients of an expression. The last two represent the number of X and Y values and the smallest X or Y value respectively.
NOTE: THE MAXIMUM NO. OF X AND Y VALUES ALLOWED IS 20.

Third line - Same as second line but for a second expression.

Fourth line - Same as second line but for a third expression.

.
.
.

Example - To obtain a combined graph of the expressions:

$$-2x + 2y - 2$$
$$x + y - 7$$

evaluated from X and Y = -3 to +8, the data would be:

```
2 5
-2 2 -2 12 -3
1 1 -7 12 -3
```

Procedure for Use

- In the example below, user's typing is underlined.
- Assume that you have signed on.
- Hit the RETURN key after typing each line.

```
0 graph

01010  $$1 940,100

00940  2 5

00950  -2 2 -2 12 -3

00960  1 1 -7 12 -3

01010  $$r 940,960 ...... This number (960) should be the
                          same as the last line of data
00940  3 5                typed by the computer

00950  2 -4 12 19 -9

00960  1 -2 2 19 -9

01010  1 -2 -4 19 -9

01020  $$s

FILE SUBMITTED

ENTER ACCOUNT #, PGM NAME

218.xx print i ben

915 AWAITING PRINT

f 915 prtl 1,100 ........ Output will now be printed.
          .
          .
          .
e ....................... After all typing stops.
```

Listing of Program - GRAPH

```
      INTEGER*2 X(20),Y(20),Z(20,20),A,B,C
      CHARACTER CHAR(20,20)/400*' '/
      READ, NUM,M
      PRINT 800, NUM,M
      IF(M.GT.5.OR.M.LT.1) GO TO 40
C
C  NUM=NUMBER OF EXPRESSIONS.
C  M IS AN INTEGER WITH VALUES 1,2,3,4 OR 5.
C  M=1 RESULTS IN NUM TABLES ONLY.
C  M=2 RESULTS IN NUM GRAPHS ONLY
C  M=3 RESULTS IN NUM TABLES AND NUM GRAPHS.
C  M=4 RESULTS IN NUM TABLES AND A MERGED GRAPH OF ALL EXPRESSIONS.
C  M=5 RESULTS IN A MERGED GRAPH ONLY, OF ALL EXPRESSIONS.
C  IF A MERGED GRAPH IS REQUESTED THE VALUES OF X AND Y MUST BE THE
C  SAME FOR ALL EXPRESSIONS.
C
      DO 1 L=1,NUM
      READ, A,B,C,N,LL
      PRINT 900, L,A,B,C,N
      PRINT 1000, LL
      IF(N.GT.20) GO TO 50
C
C  N IS THE NUMBER OF X AND Y VALUES.
C  LL IS THE SMALLEST VALUE OF X AND Y.
C  NOTE - ONLY INTEGERS MAY BE USED FOR X AND Y VALUES AND THUS FOR LL.
C
      K=(N+1)/2
      L1=LL
      DO 4 I=1,N
      X(I)=LL
      Y(I)=L1+(N-I)
    4 LL=LL+1
      DO 10 L=1,N
      DO 5 J=1,N
    5 Z(I,J)=A*X(J)+B*Y(I)+C
   10 CONTINUE
      IF(M.EQ.1) GO TO 11
      DO 13 I=1,N
      DO 12 J=1,N
      IF(L.GT.1.AND.M.GE.4) GO TO 12
      IF(X(1).EQ.0) CHAR(J,I)='+'
      IF(Y(I).EQ.0) CHAR(I,J)='+'
   12 IF(Z(I,J).EQ.0) CHAR(I,J)='*'
   13 CONTINUE
      IF(M.EQ.5.AND.L.EQ.NUM.OR.M.EQ.2) GO TO 21
      IF(M.EQ.5) GO TO 1
```

```
 11 PRINT 700
    DO 20 I=1,N
    IF(I.EQ.K) GO TO 15
    PRINT 100, Y(I), (Z(I,J),J=1,N)
    GO TO 20
 15 PRINT 200, Y(I), (Z(I,J),J=1,N)
 20 CONTINUE
    PRINT 300, (X(I),I=1,N)
    PRINT 400
    IF(M.EQ.4.AND.L.LT.NUM.OR.M.EQ.1) GO TO 1
 21 PRINT 700
    DO 30 I=1,N
    IF(I.EQ.K) GO TO 25
    PRINT 500, Y(I), (CHAR(I,J),J=1,N)
    GO TO 30
 25 PRINT 600, Y(I), (CHAR(I,J),J=1,N)
 30 CONTINUE
    PRINT 300, (X(I),I=1,N)
    PRINT 400
  1 CONTINUE
    STOP
 40 PRINT,'CONTROL VALUE MUST BE BETWEEN 1 AND 5. ENTER A CORRECT VAL
   *UE AND RESUBMIT THE PROGRAM.'
    PRINT 1100
    STOP
 50 PRINT,'NAUGHTY NAUGHTY - ONLY 20 X AND Y VALUES ALLOWED. ENTER A
   *CORRECT NUMBER AND RESUBMIT THE PROGRAM.'
    PRINT 1100
    STOP
100 FORMAT(//12X,13,6X,20I5)
200 FORMAT(//1X,'Y VALUES    ',I3,6X,20I5)
300 FORMAT(////21X,20I5)
400 FORMAT(/41X,'X VALUES'///)
500 FORMAT(//12X,I3,6X,20A5)
600 FORMAT(//1X,'Y VALUES    ',I3,6X,20A5)
700 FORMAT('1')
800 FORMAT(//1X,'DATA'/1X,'NUMBER OF EXPRESSIONS IS ',I3/1X,'CONTROL
   *VALUE IS ',I3)
900 FORMAT(1X,'COEFFICIENTS OF EXPRESSION 'I2,' ARE ',3I4/1X,'NUMBER
   *OF X AND Y VALUES IS ',I3)
1000 FORMAT(1X,'SMALLEST X AND Y VALUES ARE ',I3//)
1100 FORMAT(////)
    END
```

```
DATA
NUMBER OF EXPRESSIONS IS  2
CONTROL VALUE IS 5
COEFFICIENTS OF EXPRESSION 1 ARE  1  -1  1
NUMBER OF X AND Y VALUES IS  17
SMALLEST X AND Y VALUES ARE  -8

COEFFICIENTS OF EXPRESSION 2 ARE  1   1  -7
NUMBER OF X AND Y VALUES IS  17
SMALLEST X AND Y VALUES ARE  -8
```

```
          8                             *   +                           *

          7                                 *                       *

          6                                 +   *               *

          5                                 +       *       *

          4                                 +           *

          3                                 +       *       *

          2                                 +   *               *

          1                                 *                       *

Y VALUES  0 +   +   +   +   +   +   +   *   +   +   +   +   +   +   +   *   +

         -1                         *       +                               *

         -2                     *           +

         -3                 *               +

         -4             *                   +

         -5         *                       +

         -6     *                           +

         -7 *                               +

         -8                                 +

            -8  -7  -6  -5  -4  -3  -2  -1  0   1   2   3   4   5   6   7   8

                                        X VALUES
```

Figure 4

Output of Program GRAPH - Merged Graph of Expressions X + Y - 7 and X - Y + 1

PROGRAM NAME - TRIAL AND ERROR COMP1

This program is designed to determine the value of Y in a system of linear equations $Ax + By + C = 0$, $Ax + Ey + F = 0$ by trial and error.

Input Data

One line - 8 integers with a blank between each representing the coefficients B,C,E,F, the first trial value of Y, the amount by which Y is to be incremented, the number of Y values to be tried, and a control value, respectively. Students are to use a control value of 0.

Example - For the system $10x + 10y + 3 = 0$, $10x - 10y - 5 = 0$ the data would be:

10 3 -10 -5 -2 1 5 0

Output - The output for the above data would be:

Y	-BY-C	-EY-F	DIFFERENCE
-2.0000	-17.0000	-15.0000	32.0000
-1.0000	7.0000	-5.0000	12.0000
0.0000	-3.0000	5.0000	8.0000
1.0000	-13.0000	15.0000	28.0000
2.0000	-23.0000	25.0000	48.0000

If the difference between -BY-C and -EY-F is never equal to zero, then the user must find the two Y values in the output for which the difference is closest to zero. The smallest of these values then becomes the new initial Y value and the program is resubmitted, using 11 increments of 0.1.

New data - For the above example, the new data would be:

10 3 -10 -5 -1 0.1 11 0

New output -

Y	-BY-C	-EY-F	DIFFERENCE
-1.0000	7.0000	-5.0000	12.0000
-0.9000	6.0000	-4.0000	10.0000
-0.8000	5.0000	-3.0000	8.0000
-0.7000	4.0000	-2.0000	6.0000
-0.6000	3.0000	-1.0000	4.0000
-0.5000	2.0000	0.0000	2.0000
-0.4000	1.0000	1.0000	0.0000

THE VALUE OF Y IS -0.4000

NOTE: -0.4000 is the value of Y that is now used to find X using program COMP2.

If the value of Y had still not been found, the procedure would be repeated with values of Y increasing in steps of 0.01, again 11 times. In all, this procedure can be repeated up to four times, with values of Y increasing in steps of 0.1, 0.01, 0.001, and 0.0001.

Procedure for Use

- In the example below, user's typing is underlined.
- Assume that you have signed on.
- Hit the RETURN key after typing each line.

```
o compl

00540  $$r 530

00530  10 3 -10 -5 -1 0.1 11 0

00540  $$s

FILE SUBMITTED

ENTER ACCOUNT #, PGM NAME

218.xx print i ben

471 AWAITING PRINT

f 471 prtl 1,100  ..... Output will now be
                        printed
       .
       .
       .
 e  .................... After all typing stops.
```

Listing of Program - TRIAL AND ERROR COMP1

```
      DOUBLE PRECISION YINC,Y,Y2
      CHARACTER*120 MSG/'Y NOT IN THIS RANGE. EXAMINE OUTPUT (DIFFEREN
     *CE) AND SELECT A NEW INITIAL Y VALUE.'/
      READ, B,C,E,F,Y,YINC,N,M
      TRUEY=(C-F)/(E-B)
      PRINT 500, B,C,E,F,Y,YINC,N,M
      IF(N.LT.1.OR.N.GT.20) GO TO 50
C
C  B,C,E AND F ARE COEFFICIENTS OF THE SYSTEM.
C  Y IS THE INITIAL VALUE OF Y.
C  YINC IS THE INCREMENT FOR Y.
C  N IS THE NUMBER OF VALUES OF Y.
C  M IS A CONTROL VALUE WHICH IS EITHER 0 OR 1.
C     M=0 RESULTS IN Y INCREASING FROM THE GIVEN INITIAL VALUE IN INCRE
C     MENTS OF YINC FOR A GIVEN NUMBER OF TIMES (N).
C
C     M=1 RESULTS IN THE AUTOMATIC SELECTION OF Y VALUES BETWEEN WHICH
C     DIFFERENCES ARE SMALLEST.  THE PROGRAM THEN RUNS AGAIN STARTING
C     WITH SMALLEST OF THESE VALUES AND YINC=YINC/10. THIS PROCESS REP
C     EATS UNTIL Y IS FOUND, OR AT MOST 4 TIMES.
C
      DO 20 J=1,5
      PRINT 100
      Y2=10.D9
      DO 10 I=1,N
      Z1=-B*Y-C
      Z2=-E*Y-F
      DIFF=ABS(Z1-Z2)
      PRINT 200, Y,Z1,Z2,DIFF
      IF(DIFF.LT.0.00001) GO TO 30
      IF(TRUEY.LT.Y.AND.Y2.EQ.10.D9) Y2=Y-YINC
   10 Y=Y+YINC
      IF(Y2.EQ.10.D9) GO TO 40
      IF(M.EQ.0) STOP
      Y=Y2
      YINC=1.D9/(10.D0**J)
   20 N=11
      PRINT 400, Y2+0.00005
      STOP
   30 PRINT 300, Y
      STOP
   40 PRINT 600, MSG
      STOP
   50 PRINT 700
      STOP
  100 FORMAT(//10X,'Y',7X,'-BY-C',5X,'-EY-F',3X,'DIFFERENCE'/)
  200 FORMAT(f14.4,2F10.4,F11.4)
```

```
300 FORMAT(/10X,'THE VALUE OF Y IS',F8.4///)
400 FORMAT(/10X,'THE VALUE OF Y CORRECT TO 4 DEC PLACES IS',F9.5///)
500 FORMAT('1'/1X,'DATA ( ',4F8.1,2F10.5,2I3,' )'//)
600 FORMAT(/1X,A120//)
700 FORMAT(///1X,'NO WAY - THE NUMBER OF Y VALUES TRIED MUST BE BETWEEN
   *1 AND 20. INSERT CORRECT DATA AND RESUBMIT THE PROGRAM.'///)
    END
```

PROGRAM NAME-TRIAL AND ERROR COMP2

This program is designed to determine the value of X by trial and error in a system $\begin{matrix} AX + BY + C = 0 \\ AX + EY + F = 0 \end{matrix}$, when the value of Y is known.

Input Data

One line - 11 integers with a blank between each, representing the coefficients A,B,C,D,E and F, the initial value of X, the amount by which X is to be incremented, the value of Y (obtained from program COMP1), the number of X values to be tried, and the control value respectively. Students are to use a control value of 0.
NOTE: The number of X values tried cannot exceed 20.

Example - For the system $\begin{matrix} 10X + 10Y + 3 = 0 \\ 10X - 10Y - 5 = 0 \end{matrix}$, with Y=-0.4, data would be:

10 10 3 10 -10 -5 -2 1 -0.4 5 0

Output - The output for the above data would be:

X	Y	AX + BY + C	DX + EY + F
-2.0000	-0.4000	-21.0000	-21.0000
-1.0000	-0.4000	-11.0000	-11.0000
0.0000	-0.4000	-1.0000	-1.0000
1.0000	-0.4000	9.0000	9.0000
2.0000	-0.4000	19.0000	19.0000

The procedure is now the same as in program COMP1, except that we look for values of X between which AX+BY+C and DX+EY+F are closest to zero, and use the smallest of these as the new initial X value.

New data - For the above example, the new data would be:

10 10 3 10 -10 -5 0 0.1 -0.4 11 0

New output

X	Y	AX + BY + C	DX + EY + F
0.0000	-0.4000	-1.0000	-1.0000
0.1000	-0.4000	0.0000	0.0000

THE SOLUTION SET OF THE SYSTEM IS (0.1000, -0.4000)

Procedure for Use

- In the example below, user's typing is underlined.
- Assume that you have signed on.
- Hit the RETURN key after typing each line.

```
0 comp2

00340 $$r 330

00330 10 10 3 10 -10 -5 -2 1 -0.4 5 0

00340 $$s

FILE SUBMITTED

ENTER ACCOUNT #, PGM NAME

218.xx print i ben

172 AWAITING PRINT

f 172 prtl 1,100 ....... Output will now be printed
        .
        .
        .
e   .................... After all typing stops
```

Listing of Program - TRIAL AND ERROR COMP2

```
      DOUBLE PRECISION X,XINC,X2
      CHARACTER*120 MSG/'X NOT IN THIS RANGE. EXAMINE OUTPUT (DX+EY+F)
     *AND SELECT A NEW INITIAL X VALUE.'/
      READ, A,B,C,D,E,F,X,XINC,Y,N,M
      TRUEX=(-B*Y-C)/A
      PRINT 400, A,B,C,D,E,F,X,XINC,Y,N,M
      IF(N.LT.1.OR.N.GT.20) GO TO 50
      DO 30 J=1,5
      PRINT 100
      X2=10.D9
      DO 10 I=1,N
      Z=D*X+E*Y+F
      PRINT 200, X,Y,Z,Z
      IF(ABS(Z).LT.0.00001) GO TO 20
      IF(TRUEX.LT.X.AND.X2.EQ.10D9) X2=X-XINC
   10 X=X+XINC
      IF(X2.EQ.10.D9) GO TO 40
      IF(M.EQ.0) STOP
      X=X2
      XINC=1.DO/(10.DO**J)
   30 N=11
   20 PRINT 300, X,Y
      STOP
   40 PRINT 500, MSG
      STOP
   50 PRINT 600
      STOP
  100 FORMAT(//10X,'X',9X,'Y',6X,'AX+BY+C',5X,'DX+EY+F'/)
  200 FORMAT(F13.4,F10.5,2F12.5)
  300 FORMAT(//10x,'THE SOLUTION SET OF THE SYSTEM IS (',F11.5,',',F11
     *.5,')'//)
  400 FORMAT('1'/1X,'DATA (',6F7.1,3F10.5,2I3,' )'//)
  500 FORMAT(/1X,A120//)
  600 FORMAT(///1X,'NO WAY - THE NO. OF X VALUES TRIED MUST BE BETWEEN
     *1 AND 20. INSERT CORRECT DATA AND RESUBMIT THE PROGRAM.'///)
      END
```

PROGRAM NAME - COMP

This series of four programs is designed to solve systems of the form $\begin{matrix} AX + BY + C = 0 \\ DX + EY + F = 0 \end{matrix}$, by the comparison method. The first three are of increasing sophistication for demonstration purposes only, and not for student use. The fourth is a student written program given as an assignment.

Input data - For all four programs: one line containing six real numbers, representing the coefficients A,B,C,D,E, and F.

Example 1 - For the system $\begin{matrix} 8X + 2Y - 1 = 0 \\ 8X + 3Y - 2 = 0 \end{matrix}$, the data would be:

8 2 -1 8 3 -2

Output 1 - Output from the second program would be:

```
X=  -0.12500
Y=   1.00000

AX+BY+C= 0.0
DX+EY+F= 0.0
```

Example 2 - For the system $\begin{matrix} 3X - 2Y + 17 = 0 \\ 3X - 2Y - 4 = 0 \end{matrix}$, the data would be:

3 -2 17 3 -2 -4

Output 2 (a) Output from the second program would be:

```
***ERROR***  LIMIT EXCEEDED FOR FLOATING POINT DIVISION
             BY ZERO
PROGRAM WAS EXECUTING LINE 2 IN ROUTINE M/PROG WHEN
TERMINATION OCCURED
```

(b) Output from the third program would be:

SYSTEM HAS NO SOLUTION

Example 3 - For the system $\begin{matrix} 2X + 3Y + 4 = 0 \\ 2X + 3Y + 4 = 0 \end{matrix}$, the data would be:

2 3 4 2 3 4

Output 3 - The output of program three or four would be:

SYSTEM HAS NO SOLUTION

This is an incorrect answer because no provision was made in the programs for dependent systems.

Listing of Program - COMP

1. Basic Program:

```
   READ, A,B,C,D,E,F
   Y=(C-F)/(E-B)
   X=(-E*Y-F)/A
   PRINT 1, X,Y
 1 FORMAT(/1X,'X=',F10.5/1X,'Y=',F10.5//)
   STOP;END
```

2. Verification feature added. Lines 4,5,8 and 9 are additions to the basic program.

```
   READ, A,B,C,D,E,F
   Y=(C-F)/(E-B)
   X=(-E*Y-F)/A
   Z1=A*X+B*Y+C
   Z2=D*X+E*Y+F
   PRINT 1, X,Y
 1 FORMAT(/1X,'X=',F10.5/1X,'Y=',F10.5//)
   PRINT 2, Z1,Z2
 2 FORMAT(1X,'AX+BY+C=',F4.1/1X,'DX+EY+F=',F4.1//)
   STOP;END
```

3. Allowance for inconsistent systems added. Lines 2,12 and 13 are additions to program 2.

```
   READ, A,B,C,D,E,F
   IF(B.EQ.E) GO TO 3
   Y=(C-F)/(E-B)
   X=(-E*Y-F)/A
   Z1=A*X+B*Y+C
   Z2=D*X+E*Y+F
   PRINT 1, X,Y
 1 FORMAT(/1X,'X=',F10.5/1X,'Y-',F10.5//)
   PRINT 2, Z1,Z2
 2 FORMAT(1X,'AX+BY+C=',F4.1/1X,'DX+EY+F=',F4.1//)
   STOP
 3 PRINT 4
 4 FORMAT(//1X,'SYSTEM HAS NO SOLUTION'//)
   STOP;END
```

4. Same as program 3 but with X calculated before Y. Coefficient B=E.

```
      READ, A,B,C,D,E,F
      IF(A.EQ.D) GO TO 3
      X=(C-F)/(D-A)
      Y=(-A*X-C)/B
      Z1=A*X+B*Y+C
      Z2=D*X+E*Y+F
      PRINT 1, X,Y
    1 FORMAT(/1X,'X=',F10.5/1X,'Y=',F10.5//)
      PRINT 2, Z1,Z2
    2 FORMAT(1X,'AX+BY+C=',F4.1/1X,'DX+EY+F=',F4.1//)
      STOP
    3 PRINT 4
    4 FORMAT(//1X,'SYSTEM HAS NO SOLUTION'//)
      STOP;END
```

PROGRAM NAME - SUB

These two programs are designed to solve systems of equations by the substitution method. The first is a demonstration program and not intended for student use. It calculates X first and then substitutes this value in the Y-form of one of the original equations to find Y. There is no allowance made for dependent systems.

The second is a student written program for a given assignment and does provide for dependent systems. It calculates Y first and then substitutes to find X. It should be noted that the formulas used here for calculating X and Y can also be developed using the addition method.

Input Data

The data for both programs consists of six real numbers with a blank between each, representing the coefficients A,B,C,D,E, and F of the system.

Listing of Program - SUB

1. Basic Program:

```
    READ, A,B,C,D,E,F
    IF(B*D.EQ.E*A) GO TO 3
    X=(E*C-B*F)/(B*D-E*A)
    Y=(-A*X-C)/B
    Z1=A*X+B*Y+C
    Z2=D*X+E*Y+F
    PRINT 1, X,Y
  1 FORMAT(1X,'X=',F10.5/1X,'Y=',F10.5//)
    PRINT 2, Z1,Z2
  2 FORMAT(1X,'AX+BY+C=',F4.1/1X,'DX+EY+F=',F4.1//)
    STOP
  3 PRINT 4
  4 FORMAT(//1X,'SYSTEM HAS NO SOLUTION'//)
    STOP;END
```

2. Basic program with Y calculated before X and allowance made for dependent systems. Lines 2, 16 and 17 are additions to the basic program.

```
    READ, A,B,C,D,E,F
    IF(B*D.EQ.E*A.AND.C/F.EQ.A/D) GO TO 5
    IF(B*D.EQ.E*A) GO TO 3
    Y=(D*C-A*F)/(A*E-D*B)
    X=(-B*Y-C)/A
    Z1=A*X+B*Y+C
    Z2=D*X+E*Y+F
    PRINT 1, X,Y
  1 FORMAT(/1X,'X=',F10.5/1X,'Y=',F10.5//)
    PRINT 2, Z1,Z2
  2 FORMAT(1X,'AX+BY+C=',F4.1/1X,'DX+EY+F=',F4.1//)
    STOP
  3 PRINT 4
  4 FORMAT(//1X,'SYSTEM HAS NO SOLUTION'//)
    STOP
  5 PRINT 6
  6 FORMAT(//1X,'SYSTEM HAS INFINITE SOLUTIONS'//)
    STOP;END
```

APPENDIX B

Procedures for Switching on the Terminal and Signing on/off

PROCEDURES

I) SWITCHING ON

a) Switch phone coupler on.

b) Switch terminal on.

c) Dial 284-7352

d) Wait for the high pitched tone and then insert the phone into the coupler.

e) Press the RETURN key.

f) If the keyboard does not unlock you may have to power-off and power-on.

II) SIGNING ON

- User's typing is underlined.
- Hit the RETURN key after typing each command.

a) Type: <u>on 218.xx edit</u>

b) Type: <u>??time</u>

The computer will respond with the message:

TIME IS 14.06.11

c) Enter your name and the time given, in the log book.

d) Type: <u>o name</u> (the name that is typed here is that of the program you want to use)

e) Enter data and run the program. (See procedure for use under specific program names)

III) SIGNING OFF

a) Type: <u>?? time</u>

b) Enter the time given by the computer in the log book.

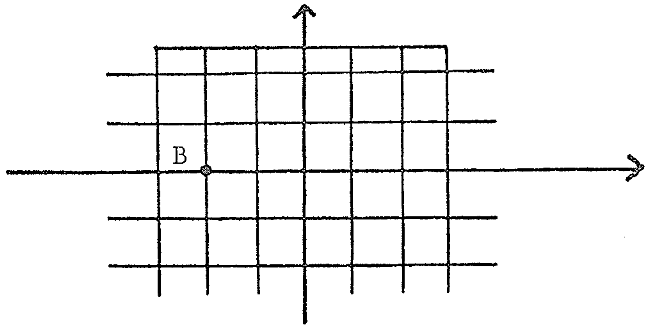
c) Type: <u>off 218.xx</u>

APPENDIX C

Post-Treatment Achievement Test

1) The ordered pair of real numbers corresponding to point B in the figure below is:

A. (0,2)
B. (2,0)
C. (0,-2)
D. (-2,0)
E. (-2,2)



2) In the equation, $2x - 7y + 1 = 0$, if x is o then y = ?

A. -1/7
B. 1/7
C. 1/2
D. -1/2
E. 7/2

3) Which of the following equations has a graph that is the same straight line as the graph of $3x + y - 3 = 0$?

A. $6x + 2y - 3 = 0$
B. $6x + 2y - 6 = 0$
C. $6x + 2y - 9 = 0$
D. $4x + 2y - 6 = 0$
E. $4x + 2y - 9 = 0$

4) Graphing is not always an accurate method of solving systems of equations because:

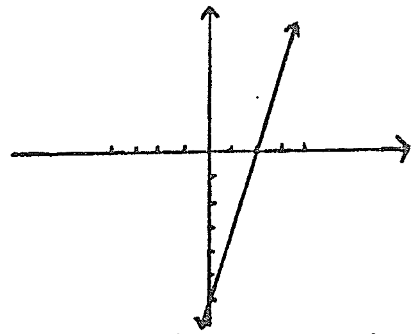
A. The graphs might be parallel.
B. The graphs might be the same line.
C. Every system does not have a solution.
D. The graphs might never intersect.
E. It is difficult to read the exact value of coordinates from a graph if the values are fractions.

5) In the equation, $3x + 2y - 5 = 0$, the real numbers 3, 2, -5 are called:

A. coordinates.
B. Coefficients.
C. Exponents.
D. Verifications.
E. Variables.

6) The figure below shows the graph of a linear equation in two variables. On this graph, where y is 0, x is ?

A. 3
B. -6
C. 6
D. 0
E. 2



7) The solution set of a system of two linear equations is:

A. The solution set of either equation.
B. The solution set of both equations.
C. The union of the solution sets of both equations.
D. The intersection of the solution sets of both equations.
E. C or D.

8) The solution set of the system $\begin{array}{l} 4x + y = 0 \\ 6x - y = 5 \end{array}$ contains:

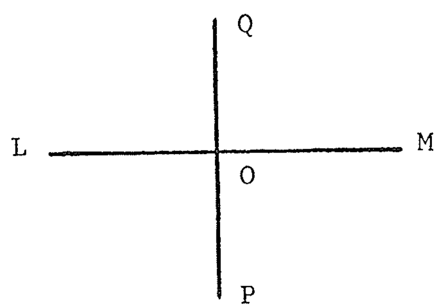
A. (1/2,2)
B. (1/2,-2)
C. (1,-4)
D. (3/2,-6)
E. (-1,4)

9) How many ordered pairs from the solution set of a linear equation are sufficient to graph the equation?

A. An infinite number.
B. At least three.
C. Four.
D. One.
E. Two.

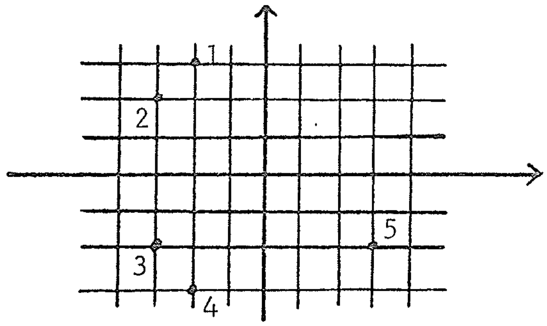
10) On the coordinate system shown below, the Y axis is the line segment:

A. OM
B. PQ
C. LO
D. OP
E. LM



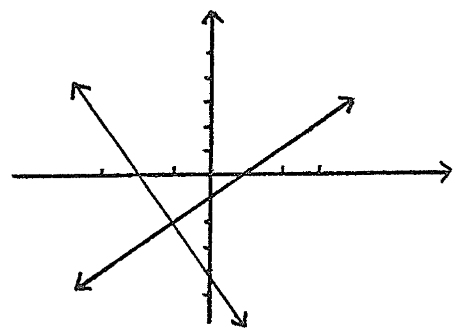
11) Which of the points in the coordinate system shown below has coordinates (-2,3) ?

A. 5
B. 4
C. 3
D. 2
E. 1



12) The solution set of the system in the figure below contains:

A. (-2,-1)
B. (-1,-2), (0,-1), (1,0)
C. (-1,-2), (0,-4), (-2,0)
D. (2,1), (0,-1)
E. (-1,-2)



13) Which method of solution would be the quickest for solving the system

$$x + 4y = 0$$
$$-x - 3y = 1$$

A. Substitution.
B. graphing.
C. Comparison.
D. Addition.
E. A or C

14) Which of the following is the solution set of the system

$$2x + y = 4$$
$$x + 2y = 5$$

A. The empty set.
B. (3,-2)
C. (1,2)
D. (2,0), (1,2)
E. (2,0), (3,-2)

15) Solve the following system for Y.

$$\begin{aligned} 2x + y &= 4 \\ 2x - y &= 2 \end{aligned}$$

A. 0
B. 3/2
C. 4
D. 3
E. 1

16) In a system of two linear equations in x and y, we have found that x = -1. The value of y could be found by:

A. Plotting x.
B. Replacing x by -1 in the first equation.
C. Replacing x by -1 in the second equation.
D. Replacing x by -1 in either equation.
E. The value of y cannot be found.

17) How many ordered pairs are in the solution set of a system if the graphs of the equations are parallel straight lines?

A. One
B. None
C. Two
D. An infinite number.
E. The number cannot be determined from the information given.

18) Which of the following systems has an infinite number of solutions?

A. $\begin{aligned} x + y - 3 &= 0 \\ 2x - 2y + 6 &= 0 \end{aligned}$

B. $\begin{aligned} x + y - 3 &= 0 \\ 2x + 2y - 3 &= 0 \end{aligned}$

C. $\begin{aligned} x + y - 3 &= 0 \\ 2x + 2y - 6 &= 0 \end{aligned}$

D. $\begin{aligned} x + y - 3 &= 0 \\ x + y - 6 &= 0 \end{aligned}$

E. $\begin{aligned} x + y - 3 &= 0 \\ 2x + 2y + 6 &= 0 \end{aligned}$

19) Which of the following ordered pairs are (is) in the solution set of the system

$$\begin{aligned} x + y &= 3 \\ 3x + 2y &= 6 \end{aligned}$$

A. (2,1) and (-1,4)
B. (5,-2)
C. (3,0)
D. (0,3)
E. (3,0) and (0,3)

20) In the system $2x + y = 12$, $3x - 2y = 11$ $\quad y = ?$

A. -14
B. 2
C. -2
D. 5
E. 58/7

21) To verify that (1/2,-1) is in the solution set of the system $4x + y = 1$, $2x - 3y = 4$ you would:

A. Replace y by -1 in the first equation and by 1/2 in the second.
B. Replace x by 1/2 and y by -1 in both equations.
C. Replace x by 1/2 in the first equation and by -1 in the second.
D. Replace x by 1/2 and y by -1 in the first equation.
E. Replace x by -1 and y by 1/2 in both equations.

22) If y is replaced by $-4x + 1$ in the equation $2x - 3y = 4$ then $x = ?$

A. 4/7
B. -2
C. 1/2
D. -1/2
E. 2

23) In the system $2x + 3y = -14$, $-x + y = 3$ the coefficients of x can be made additive inverses of each other by:

A. Multiplying the second equation by -2.
B. Multiplying the second equation by 2.
C. Adding the two equations.
D. Subtracting 1 from the first equation.
E. Adding 2 to the second equation.

24) A coordinate system on a plane is a system that:

A. Consists of two number lines on a plane.
B. Establishes a one-to-one correspondence between all the points on a line and all real numbers.
C. Establishes a one-to-one correspondence between all the points on a plane and all ordered pairs of real numbers.
D. Consists of two number lines that intersect at the origin.
E. Consists of two number lines on a plane that intersect at right angles.

25) The solution set of one linear equation in two variables contains how many ordered pairs of real numbers?

A. An infinite number if the equation is inconsistent.
B. An infinite number.
C. One.
D. Two.
E. At least three.

26) In the system $\begin{matrix} 2x + y = 1 \\ -3x + y = 2 \end{matrix}$ the coefficients of x can be made additive inverses of each other by:

A. Multiplying the first equation by 3 and the 2nd by -2.
B. Multiplying the first equation by -3 and the 2nd by 2.
C. Multiplying the first equation by 3 and the 2nd by 2.
D. Adding the two equations.
E. Adding 1 to the first equation.

27) Which of the following is an equation whose graph is parallel to the graph of $2x + 3y = 7$?

A. $4x + 6y = 14$
B. $4x + 6y = 7$
C. $3x + 2y = 14$
D. $2x - 3y = 7$
E. $12x + 18y = 42$

28) If $\begin{matrix} 2x = y + 1 \\ 2x = 2y - 1 \end{matrix}$ then:

A. $y + 1 = 2y - 1$
B. $y + 1 \neq 2y - 1$
C. The two equations are equivalent.
D. The two equations are inconsistent.
E. Nothing can be said about the equivalence of the two equations.

29) In the system $\begin{matrix} Ax + By + C = 0 \\ Dx + Ey + F = 0 \end{matrix}$ B and E can be made additive inverses of each other by:

A. Multiplying the first equation by E and the 2nd by B.
B. Multiplying the first equation by B and the 2nd by E.
C. Multiplying the first equation by -B and the 2nd by -E.
D. Multiplying the first equation by -E and the 2nd by B.
E. It cannot be done unless the values of B and E are known.

30) If $8x + 2y = 1$ then:
$8x + 3y = 2$

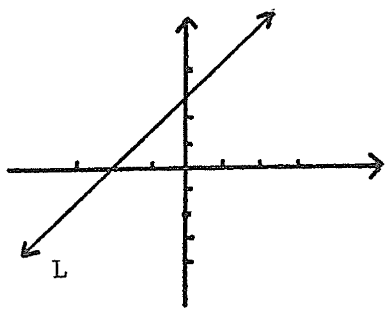
A. $2y + 1 = 3y + 2$
B. $-1 - 2y = -2 - 3y$
C. $2y + 1 = 3y - 2$
D. $1 - 2y = 2 - 3y$
E. $2y - 1 = 3y + 2$

31) A linear equation in two variables has the form:

A. $Ax + By + C = 0$
B. $Ax + By = -C$
C. $y = \frac{-Ax - C}{B}$
D. $x = \frac{-By - C}{A}$
E. Any of the above.

32) Line L in the figure below is the graph of:

A. $3x - 2y = 0$
B. $3x - 2y = -6$
C. $3x + 2y = 6$
D. $3x - 2y = 6$
E. $2x + 3y = 6$



33) If the system $\begin{matrix} Ax + By + C = 0 \\ Dx + Ey + F = 0 \end{matrix}$ has no solution, then:

A. A=D and B=E and $C \neq F$
B. A=D and B=E and C=F
C. C=F
D. A=D and C=F
E. B=E and C=F

34) The system $\frac{x - y}{2} = 1$ has how many solutions?

$3x = 6 + 3y$

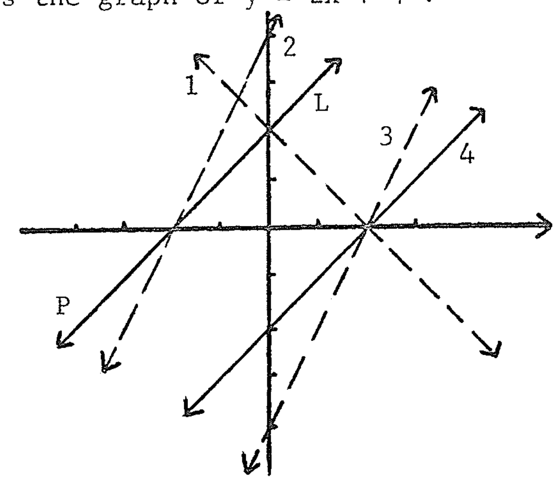
A. The number cannot be determined because the system is not linear.
B. The number cannot be determined because the system does not contain equivalent equations.
C. None.
D. An infinite number.
E. One.

35) Which of the equations below is equivalent to $x - y = 1$?

A. $\frac{x - 1}{y} = 0$
B. $-y = 1/x$
C. $x - 1 = y + 2$
D. $x + 1 = y + 2$
E. None of the above.

36) In the figure below, PL is the graph of $y = x + 2$.
Which of the dotted lines is the graph of $y = 2x + 4$?

A. 4
B. 3
C. 2
D. 1
E. None of these.



37) In solving systems of equations by comparison, the first step is to:

A. Compare the graphs of the two equations.
B. Compare the constant terms of the two equations.
C. Compare additive inverses.
D. Find an equivalent system of equations.
E. Find the additive inverses of the coefficients of one variable.

38) In solving the system $\begin{matrix} Ax + By + C = 0 \\ Dx + Ey + F = 0 \end{matrix}$ by substitution, the first step is to obtain:

A. $x = \frac{-By - C}{A}$
B. $x = \frac{-Ey - F}{D}$
C. $y = \frac{-Ax - C}{B}$
D. A or C
E. A, B or C

39) In developing a formula for solving systems of the form $\begin{array}{l} Ax + By + C = 0 \\ Ax + Ey + F = 0 \end{array}$ by comparison, we would write:

A. $-By - C = -Ey - F$
B. $By + C = Ey + F$
C. $-Ax - C = -Ax - F$
D. A or B
E. A or C

40) Which of the following is the formula for finding y in any system of the form $\begin{array}{l} Ax + By + C = 0 \\ Ax + Ey + F = 0 \end{array}$ by comparison?

A. $\frac{C + F}{D - A}$

B. $\frac{C - F}{D - A}$

C. $\frac{C - F}{E - B}$

D. $\frac{E - B}{C - F}$

E. $\frac{F - C}{E - B}$