AN ANALYSIS OF PROGRAMMED INSTRUCTION AND STUDENT ATTITUDE TOWARD IT AT THE SECONDARY SCHOOL LEVEL

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ABSTRACT

The basic purpose of this study was to examine the effectiveness of programmed instruction as an alternative teaching method and compare its results with those achieved using traditional teaching methods. The questions raised in this regard were:

i) Is there a significant difference in the relative achievement of students exposed to programmed instruction as compared to those taught by conventional classroom procedures?

ii) Is there a significant difference in attitude toward mathematics of students exposed to programmed instruction as compared to those taught by conventional classroom method?

iii) Is there a significant difference in the attitude toward mathematics, the teacher, and programmed instruction by secondary school students as a result of programmed instruction?

A sample of eighty-two Seven Oaks School Division #10 grade XII students, taking either mathematics 300 or 301, was involved in the study. Four intact classroom groups, two taking mathematics 300 and the other two taking mathematics 301, were used. One of the groups from each level, randomly chosen as an experimental group, was taught

logarithms by the use of programmed materials. The remaining group from each level was designated as the control group and received instruction on the same topic using the conventional classroom methods.

The following independent variables were obtained--I.Q. scores, chronological ages in months, sex, grade IX mathematics achievement scores, grade XII 1971 first term mathematics scores, socio-economic index scores, and experimental pre-test scores of achievement (LAT) and attitude (MAS). The experimental groups were also given semantic differential attitude scales to measure their attitude toward mathematics (AMT), attitude toward teacher (TAT) and attitude toward programmed instruction (PIAT). These five tests were administered again at the end of the experimental period to determine possible differences in achievement and attitude.

The analysis of covariance statistical design was employed to test for significance between mean differences of achievement (LAT) and attitude scores (MAS) on initial and final administrations of the tests. The paired t-test was used to test for significant differences in the AMT, TAT and PIAT attitude mean scores of the experimental groups on pre- and post-administrations.

Mean achievement scores did not differ significantly between the treatment groups of the two different courses. Method of instruction was found to have no significant effect on the mean scores of attitude of students toward mathematics. The mean scores of attitude of the experimental groups toward mathematics and the teacher did not differ significantly. There was a decrease in the mean scores of attitude toward programmed instruction expressed by the experimental groups with the students taking mathematics 300 showing a significant difference.

Several conclusions appear warranted:

1. Sufficient evidence was given that learning did take place regardless of method of instruction.

2. No one method was superior to the other in terms of better student achievement or fostering better attitudes toward mathematics.

3. Student attitude toward mathematics appeared to be in the positive direction.

4. During the time lapse of approximately two and one half weeks, student attitude toward programmed instruction declined.

5. It appeared that the programmed unit on logarithms was time saving as far as coverage of the topic was concerned.

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THE PROBLEM

Statement of the Problem

The purpose of the study was to examine the effectiveness of programmed instruction as a teaching method as it relates to student achievement in secondary mathematics. The study was also designed to measure student attitude toward mathematics and programmed instruction.

More specifically, the problems of major interest were:

i) Is there a significant difference in the relative achievement of students exposed to programmed instruction as compared to those taught by the conventional classroom methods?

ii) Is there a significant difference in the attitude toward mathematics of students exposed to programmed instruction as compared to those taught by the conventional classroom method?

iii) Is there a significant difference in the attitude toward mathematics, the teacher and programmed instruction by secondary school mathematics students as a result of programmed instruction?

Significance of the Study

Each student is a unique individual. Many differences which exist from one student to the next are significant to the teaching and learning of mathematics.

Students vary in their mental ability, their ability to reason logically, their ability to solve problems, their ability to use mathematical symbols, and their ability to compute. They vary in their knowledge of mathematical concepts, structures and processes which are related to their previous educational experiences and largely determine the readiness of the learner for further mathematics courses. Some students are more highly motivated than others. They differ in their interests. Their attitude toward mathematics varies. Some possess special creative talents. They have varying levels of self-discipline. They display differences in their attention and retention spans.

Each student develops these characteristics at his own rate. It is a continuous process which proceeds at varying physical, mental, and social rates.

The Seven Oaks School Division #10, in which the study was done, has adopted this philosophy of continuous growth and progress to provide for these varying rates. Its philosophy is stated in the Progress Report and is summarized by G. H. Nicholls, Superintendent of Seven Oaks Schools (1968).

The Core Committee on the Reorganization of the Secondary Schools in Manitoba (1970) also makes mention of the need for programs to be designed for students to develop their interests and abilities at their own pace.

School Divisions in Manitoba are beginning to divide the school year into trimesters or semesters. This involves different distributions of time during the school day. In order to meet the demands of such a system and at the same time allow for individual development, the mathematics teacher will be required to have a great variety of teaching methods, aids, and materials.

One such method which requires further experimentation in Manitoba is programmed instruction. An experiment in grade IX programmed algebra was conducted in the Winnipeg School Division #1 (1964) and in the Portage la Prairie School Division #24 (1962-63). Kristal (1966) conducted a short term experiment in grade XI geometry at St. Paul's High School. To the writer's knowledge, these are the only three experiments on which reports have been written.

These three experiments concentrated mainly on differences in achievement between traditional approaches to the teaching of mathematics and programmed mathematics materials. This writer investigated this aspect as well as attitude change toward programmed instruction.

If one subscribes to the responsibility of the school to generate attitudes that will stimulate interest and thus continued learning beyond the confines of the school structure, one must recognize the need for some information regarding the degree to which the classroom organization under programmed instruction can contribute to the formation of such attitudes.

Students are beginning to challenge traditional classroom procedures. They are demanding that teaching methods and the learning environment be shaped to meet their goals and requirements. "Teacher centred programs must give way to learner centred schools." (Bushnell, 1969, p. 96).

It was the intention of the writer to attempt to see if programmed instruction was a suitable variation to traditional teaching methods as far as differences in achievement were concerned and to see if it affects the attitude of the student significantly.

DESIGN OF THE STUDY

The subjects for the experiment were chosen from grade XII mathematics students at West Kildonan Collegiate. They were enrolled in either mathematics 300 or mathematics 301. The writer was assigned two mathematics 300 classes and two mathematics 301 classes. From each classification, one class was randomly chosen as the control group and the other was designated as the experimental group.

The control group was taught by the traditional teacher-lecture, question and answer periods, teacher-class discussions, and correction of homework assignments. The experimental group was instructed by the sole use of programmed materials.

The mathematical topic that was used in the experiment was an introduction to logarithms, a common topic to both curricula.

Experimental Design

In carrying out the investigation, the following data were obtained: IQ scores, chronological age in months, sex, grade IX mathematics achievement score, grade XII 1971 first term mathematics achievement score, and an indication of socio-economic status.

Pre- and post-tests of achievement on the unit of logarithms were given as well as pre- and post-attitude tests. In addition, the subjects of the experimental groups were given a semantic differential scale designed to measure the subject's attitude toward mathematics, teacher, and programmed instruction.

Statistically, the analysis used to test significant differences in mean achievement and mean attitude was the analysis of covariance design. The paired t-test was used to compare differences in attitude toward mathematics, teacher, and programmed instruction of the experimental group.

HYPOTHESES

The hypotheses tested were the null hypotheses:

Hypothesis I

That there is no significant difference in mean scores of achievement of students exposed to programmed instruction as compared to those taught by the conventional classroom methods.

Hypothesis II

That there is no significant difference in mean scores of attitude toward mathematics of students exposed to programmed instruction as compared to those taught by the conventional classroom methods.

Hypothesis III

That there is no significant difference in mean scores of pre- and post-attitude toward mathematics on the part of students using programmed instruction.

Hypothesis IV

That there is no significant difference in mean scores of pre- and post-attitude toward teacher on the part of students using programmed instruction.

Hypothesis V

That there is no significant difference in mean scores of pre- and post-attitude toward programmed instruction on the part of students using programmed instruction.

ASSUMPTIONS AND LIMITATIONS OF THE STUDY

Assumptions

It was assumed that all students in the study had little experience, if any, in handling programmed materials. It was further assumed that, since the writer taught both the experimental and the control groups, results were not seriously affected by the teacher variable.

It was further assumed that the IQ scores and past mathematics achievement test scores were valid.

A further assumption that was made was that students answered the attitude tests items according to how they felt, not according to how they should have felt.

Limitations

The study was limited by the way in which the sample was chosen. The students were timetabled according to the options that they wished. It was impossible to have statistical random sampling.

A further limitation was that other independent variables were not used. It may have been that other achievement scores such as English achievement scores or reading test scores would contribute to successful achievement.

It may have been that the topic of logarithms could have been a bias which would affect the results.

DEFINITION OF TERMS

AMT is the abbreviation for the attitude toward mathematics scale.

Attitude is a composite of the intellectual appreciation of the subject and emotional reactions to it.

Constructed or linear response is a technique of programming whereby the student must proceed from one frame to the next. This was the type of programming used in the study.

Feedback informs the student about the correctness of his answer and occurs immediately after he has responded.

Frame is a single step in the program. It presents a small amount of information to which the learner must respond in programmed instruction.

General mathematics is the course set by the Department of Education which gives the student a high school credit in mathematics. It is a course requirement for some fields of instruction at the technical colleges and one of the alternatives toward admission to certain faculties at the universities. It is designated mathematics 301.

Intrinsic or branched programming is a technique of programming whereby the student answers a frame and the answer chosen tells him which sequence to follow. Not everyone follows the same sequence.

LAT is the abbreviation used for the logarithm achievement test.

MAS is the abbreviation of Mathematics Attitude Scale.

Mathematics 300 is the university entrance mathematics course.

Mathematics 301 is the general mathematics course.

PIAT is the abbreviation of the attitude toward programmed instruction scale.

Programmed instruction is a teaching method which uses systematically arranged materials. It presents small bits of information to which the learner responds. He gets immediate feedback about the correctness of his answer.

Programmed material is subject matter arranged into a sequence of steps.

Response is the student's answer to the question posed in the frame.

Stimulus is the technical name given to the information presented in the frame.

TAT is the abbreviation for the attitude toward teacher scale.

Teaching machine is a mechanical device used to run a program.

TEMAC materials are programmed materials published by Encyclopedia Britannica.

Traditional instruction or conventional instruction includes the teacher-lecture, student-teacher class discussions, and correction of assignments to check whether the students had learned the specific concept.

University entrance mathematics is the course set out by the Department of Education which gives a student credit for mathematics in order that he may take mathematics at university or gain admission to other courses requiring this subject. It is designated mathematics 300.

ORGANIZATION OF THE THESIS

The remainder of the thesis will follow the format given below. Chapter II will present a review of the literature related to the topic. Chapter III will contain information about the sample, evaluative instruments and experimental procedures. Chapter IV will contain the presentation of data and the statistical treatment of the data. Chapter V will present the findings and conclusions.

CHAPTER II

THE REVIEW OF THE LITERATURE

The purpose of this chapter is to summarize the immense amount of literature that has been written on programmed instruction. The review will include the historical aspect, the nature of programmed instruction, research in the field of programmed instruction as it applies mainly to the field of mathematics and research on student attitudes toward the learning of mathematics.

HISTORICAL SETTING

Programmed instruction has been regarded as a recent development because its practical application has only become apparent in the past decade. However, an examination of the history of education has revealed that many of the early educators were on the threshold of programmed instruction.

Socrates (Lysaught, 1963, p. 3), Quintilian (Lawson, 1969, p. 94), Plutarch (Ulich, 1963, p. 96), Comenius (Ulich, 1963, p. 345), Pestalozzi (Monroe, 1905, p. 611) and Montessori (Saettler, 1967, p. 537) have stressed certain principles involved in programmed instruction. The man who did pioneer work in the field of developing a mechanical means to control the stimulus-response situation was Pressey. (Saettler, 1967, p. 537). In 1926, he unveiled his model of a teaching machine. However, programmed instruction received its greatest impetus from the research carried on by B. F. Skinner at Harvard University. The publishing of his paper "The Science of Learning and the Art of Teaching" provided the basis for a revolution in the area of instructional methods. (Skinner, 1954, pp. 86-97). Skinner felt that the development of a teaching machine embodied the principle of reinforcement in terms of rewarding the student. It permitted him to learn the validity of his answer as soon as he had given it.

His basic approach to programming was based on the idea of operant behaviour. The learner was presented with small units of information called frames which were to be read by the learner. The content was shaped into a question. A frame was presented to the learner as a stimulus. The learner was required to make a response by answering the statement. Through a feedback system he was informed as to whether his answer was correct. Each time a new frame was presented, the stimulus-response cycle was repeated.

During the decade following, many different programs were produced following Skinner's pattern. He has been credited with reviving the concept of programmed instruction and setting up a closer relationship between the behavioural sciences and instructional technology.

Although most programmers agreed on the basic principles of program construction, different techniques were developed to apply these principles. Skinner developed the constructed response linear program whereby students wrote in the correct response and it was immediately reinforced. Each student followed the same sequence of steps from beginning to end.

A second technique was developed by Norman Crowder who used intrinsic or branched programming. (Crowder, 1959, pp. 109-116). In each step the student was given a small paragraph of information to read. He was then required to answer a multiple-choice question. The answer to this question would determine the sequence of material to be seen next. The identifying feature was that each student determined the pattern that he would follow by his response to a multiple-choice question. Not all students would follow the same programmed steps.

Although the earlier types of programs were written for machine use, the format was adapted to include presentation through textbook, pamphlets, folders, film or television. There was a choice between multiple-choice response or constructed response programs.

One of the most important results of the growth of programmed instruction has been its implementation in the field of computer-assisted instruction. In the United States, it has grown in less than ten years to the point where during the 1967-68 school term, several thousand students ranging from elementary school to university received a significant portion of their instruction in at least one subject area under computer control. (Atkinson and Wilson, 1969, p. 9).

The future development of programmed instruction will be closely connected with the development and application of computer-based learning systems which will maintain control over individualized learning situations and optimize their progress in accord with a model of the instructional process. Computer systems will be expected to analyze constructed responses and requests from students.

Considering that the movement of programmed instruction started after 1954, the amount of research conducted has been voluminous. One of the best indications has been the large number of books and articles that have been written on the subject of teaching machines and programmed instruction. The Education Index first listed these two headings in its 1959-61 volume. Since that time there have been 1025 entries on programmed teaching and 347 entries on teaching machines.

Nature of Programmed Instruction

In most of the literature reviewed, there was general agreement on the basic psychological principles underlying programmed instruction. These basic principles came from the realm of experimental psychology and were based on the following:

1. The learner is active.

2. The learner gets frequent and immediate feedback on his performance.

3. Learning proceeds gradually from the less complex toward the more complex in an orderly sequence.

4. The learner is allowed to develop his own best pace of learning.

5. The teacher's strategies are constantly reappraised on the basis of any objective analysis of the learner's activity. (Komoski, 1963, p. 292).

The literature also pointed out that there were certain characteristics common to programmed instruction.

The behavioural objectives of any programmed learning situation must be defined. It is important to state what terminal behaviour is expected to be achieved. This is important, since only with these clearly stated can the content, materials, and methods to obtain these objectives be selected. A list of behavioural objectives which could be used as a guideline has been prepared by Johnson. (Johnson, 1971, pp. 109-115).

A second characteristic is the orderly arrangement of subject material. The material is broken down into small units or steps and then arranged in a sequence that gradually increases in complexity. Careful sequencing embodies the gradual leading of a student toward the desired behavioural objectives. With each small unit of information, a question is posed or a statement has to be completed. This is known as the stimulus.

This arrangement requires the student to complete the statement or answer the question. An interaction has to take place between the student and the program. He must make a response before he can proceed through the sequence.

The student then receives reinforcement by immediate knowledge of his results. By this kind of feedback, his response is reinforced. If he has answered incorrectly, he has to find out why or be given information about how to proceed. If he is correct, the stimulus-response reinforcement idea is repeated with the second frame of information.

These characteristics are similar in nature to the teaching principles that have been known in conventional instruction. However with large classes, the textbook and lecture method provides for little student response. The student may remain passive during the lecture period and there is no guarantee that he is grasping the concepts or paying attention. In programmed instruction, he has to become active and supply responses before he can proceed.

In the traditional method, a more extensive presentation of information is given before the student has a chance to respond or question. It is also possible that the material has not been arranged sequentially. The student has to do his own filling in and it is possible that the student could miss a concept early in the presentation. If this happens, he is proceeding without really understanding what happened earlier in the lesson. With programmed instruction, a response is usually required after each step.

With conventional instruction, there is little provision made for the correction of errors at the most crucial time of the learning situation. These errors may not be detected by the teacher until a test has been written and the deficiency is then revealed. With programmed materials, appropriate measures are taken to reinforce corrective responses automatically. They allow the student to work at his own rate. They allow more latitude for individual differences than group instruction does. They enable the teacher to evaluate the student's progress by locating his position in the program. They also enable the instructor to pinpoint areas of difficulty more readily.

Programmed instruction has another advantage in that it is possible to develop a well-constructed program through a series of try-outs on students. The program is repeatedly refined with effective frames being retained and faulty ones being discarded. It is possible to modify the traditional method somewhat but not with the same precision.

Those who are less enthusiastic about the method of programmed instruction find that student-teacher rapport which can play such a significant role in the learning process is eliminated. Both oral and written responses are almost eliminated. A high degree of motivation is necessary to keep the poor student working. Some find it very easy to pursue a false economy of time simply by looking ahead at the answers. Good students find the small steps lack challenge and miss the teacher-student discussions. Such students usually become bored with the method rather quickly. Sometimes the cost of programmed materials prohibits their use in the classroom.

Use of Programmed Instruction in the Mathematics Classroom

The use of programmed materials has been hindered by several factors. Producers of programmed materials have been issuing claims that their courses could be completed in half the teaching time and that students scored in the upper ten percent on national achievement tests. Such claims have misled and misinformed administrators and teachers.

The reaction of some individuals to programmed instruction has been stirred by the fear that eventually programmed instruction would replace the teacher in the classroom. For this reason, some teachers avoided using such materials. Some have used programs when they were not fully prepared to do so and this ineffective utilization has led to undesirable results.

Others feared that the innovation would steal some of the vitality of their classes and have a sterilizing effect on both innovation and instruction. Perhaps some of the people who could have provided leadership in this area took the attitude that if they ignored programs, the idea would soon disappear.

Another apparent setback that programming received was caused by the commercial publishers. Low investment, fast production and low quality were three terms which sometimes characterized such efforts. Many of the programs produced were nothing more than poorly organized books.

However, there is no single recipe for the proper use of programmed materials any more than there is for proper teaching procedure in mathematics education. What works for one teacher in one classroom may not work for the same teacher with a different group of students or for another teacher.

The role of the classroom teacher becomes increasingly important as he must determine the objectives that the program is to fulfill in his teaching environment. The teacher may find programmed materials useful for the home-bound student, the transient, or some subset of a conventionally taught class. The materials may also be used for enrichment purposes or for remedial work on a specific topic. Some teachers have used programs with groups of students to teach individual topics and skills. Many have used programs for research for the specific reason that programmed instruction does provide a way of controlling the stimuli and noting the effect of variation in the teaching method.

L. W. Smith (1965, p. 708) reminds the reader that it is the teacher's responsibility to become aware of available materials, acquire the ability to evaluate the content of programs, read and discuss the uses practised by other teachers, and remain open-minded in the search for new teaching strategies. He suggests that a primary goal should be the provision of successful learning experiences in mathematics at all levels, using--not abusing--the best materials of instruction. May (1965, p. 4) has done one of the more significant pieces of research to support the fact that programmed learning is not intended to replace the classroom teacher but rather to supplement, complement and augment the teacher's efforts.

RESEARCH IN PROGRAMMED INSTRUCTION IN MATHEMATICS

The teaching of mathematics represents the largest of all subject matter areas in the research of the use of self-instructional programs. Although most of the programs are based on topics from the secondary school level, the actual range is from elementary school through college. Topics covered include arithmetic, algebra, geometry, sets and number theory, trigonometry, calculus and vectors as well as areas of applied mathematics such as statistics. They range in length from a few pages to several hundred pages, from single units to full length courses and from twenty minutes to two hundred hours. (Smith, 1965, p. 706).

Coverage of the experimental literature on programmed instruction using mathematics is divided into two major sections:

(a) presentation of the results of what may be termed methods studies in which the practical effectiveness of programmed instruction is compared with that of conventional methods of teaching;

(b) the description of findings in experimental studies in which some of the issues in the theory and technique of effective programming have been explored.

The first group of studies is generally concerned with the practical value of programmed instruction as a classroom procedure and with the role of the teacher in such procedures. The second group pertains to experimental attempts to identify factors that make programmed instruction effective and workable. This review will concentrate mainly on the former group--the more pertinent to this study.

These studies as a group are not characterized by elaborate experimental design or control of extraneous variables. However, the majority of the investigators did use some form of statistical test to assess significant differences on criterion test scores. Findings of no significant differences were reported more often than were significant differences.

Thus of the 190 or so research reports examined in his survey of programmed instruction, Schramm (1964, pp. 17-107) made no citation of studies attempting to identify the conditions under which programs may be most effectively used. At least 80 percent of the research of the previous three years had been concerned with presentation and response mode variables. The remaining studies had been on a variety of issues, particularly comparison between programmed and conventional instruction and the effectiveness of programmed instruction among various learner groups.

The following review will pertain to research on programmed instruction in mathematics in the United States, Canada, and Manitoba.

Review of the Research on Programmed Instruction in Mathematics in the United States

Slaichert and Stephens (1964, pp. 542-544) compared achievements of groups using programmed materials and groups using similar materials in a conventional classroom situation with two above average and three average classes of plane geometry. Both teacher-made and standardized results showed no indication of one method being superior to another.

In another plane geometry study, Jordy (1964, pp. 472-477) used high ability eleventh grade students and compared their achievements with two groups of eleventh and twelfth grade students instructed by the conventional lecture method. He recommended that programmed materials be used with lower-ability students who showed correlations between ability and reading levels. He also found that boredom could be reduced by using units of programmed materials in conjunction with a textbook and classroom discussions.

Beane (1962, pp. 310-326) compared linear and branching programs for a unit on parallel and perpendicular lines in plane geometry. Two experimental groups and one control group comprised of sixty-five students at the high school level were used in the study. The control group was given conventional classroom teaching. The experimental group was divided into the following sections according to the use of the programs--one used linear programs entirely; one used branching programs entirely; the remaining two switched from one format to the other midway through the experiment. All five groups indicated that a significant amount of learning had taken place. The branching program was more efficient timewise than the linear program, but students had a more favourable attitude toward the latter.

McGarvey (1962, pp. 576-579) and Henderson (1963, pp. 248-251) used programmed materials for the purpose of providing remedial work in algebra. Both investigators noted that programmed materials were beneficial as an aid for teaching slow learners, not as a replacement for normal teaching.

Glaser, Reynolds, and Fullick (1963, pp. 1-49) compared various ways of combining programmed and teacher instruction--teacher instruction followed by programmed materials, programmed instruction followed by teacher explanations, and programmed instruction alone. No significant differences were found with respect to any of the above combinations.

Goldbeck *et al*. (1962, p. 70) found that a combination of teacher and programmed materials was more effective than when each was used individually. Although Sneider (1968, pp. 62-64) demonstrated that programmed instruction was not a better method of teaching algebra, she suggested that different methods of partial programmed instruction be interwoven with conventional materials to achieve the maximum potential of programmed instruction.

Brown (1964, pp. 3-35) designed an experiment where the "pure" group received only programmed instruction materials, the "anticipating" group received programmed materials preceded by teacher development of the lesson, and the "control" group received no programmed materials. The results showed that the control group and the anticipating group achieved significantly better than the pure group. There were no significant differences reported between the anticipating group and the control group. He suggested that topic units may be more useful than a year's worth of work.

Kellems (1965, pp. 434-436) conducted a three semester experiment in college algebra. During the first semester, the experimental group received instruction by a programmed text to be used in class time only. In the second semester, they were given copies of the programmed text to be used at their discretion. During the third semester, the experimental group was exposed to a combined lecture-discussion method of instruction as well as the programmed text. In each semester, the control group was taught by the traditional lecture-discussion method. Significant results for the experimental group of the third semester were obtained when considered with respect to each of the other groups in the study. Student boredom appeared in the group using only programmed texts.

Carpenter and Greenhill (1963, pp. 11-14) experimented for a three-year period using an experimental group taught mathematical concepts by teaching machines, programmed texts, and filmstrips and a control group taught similar material by a lecture-discussion method. No significant difference measured by criterion test was noted on unit and course tests for the two groups, but on unit tests the programmed group's combined score was significantly higher than that of the control group.

Furno et al. (1970, pp. 1-56) compared the effects of programmed instruction and convention instruction in the teaching of senior high mathematics over a period of three years in Baltimore schools. During the first and third years, significant differences in favour of the control group were noted on criterion tests. During the second year, there was no significant difference. It should be noted that during the second year significant differences in favour of the programmed instruction groups were found on the Progressions and Logarithm subtests.

Meadowcraft (1965, pp. 422-425) considered programmed texts as an aid to improving mathematical achievement and providing favourable attitudes toward mathematics. Two teaching methods were used. The first used a programmed text seventy percent of the time and teacher instruction the remainder of the time. A second method utilized teacher instruction with a programmed text being used for homework. Five groups were divided into high and low achievers. The results of the study indicated that the first method group showed greater mean scores in arithmetic achievement than did those in the second method group.

Feldhusen (1962, pp. 8-10), N. H. Smith (1962, pp. 417-420) and Biddle (1966, p. 3356-A) did similar studies comparing the effects of programmed instruction to conventional instruction methods. No significant results were noted.

Review of the Research on Programmed Instruction in Mathematics in Canada

The history of programmed instruction in Canada ran parallel to the history of its development in the United States. From 1954-1960, educators did little more than read literature on the topic. From 1960-1962, enthusiasm in many circles was aroused and experimentation began. The time period from 1962 to the present has become a period of quiet consolidation in which people evaluated with care the programs that they used and did some research on the results.

The Canadian Council for Research in Education (1965) sent out a survey to certain organizations which revealed the following:

(1) considerable and varied activity was being conducted in Canada using programmed instruction;

(2) several activities were suggested to be undertaken at the national level:

a) setting up an information centre in Canada on all matters pertaining to programmed instruction;

b) promoting basic research into programmed instruction;

c) developing high quality programs suitable for Canadian needs;

d) holding seminars for those working in programmed instruction;

e) continuing the promotion of programmed instruction;

f) providing liaison among industry, government and formal education in the field of programmed instruction.

Little of the above has been implemented.

In a report of the Research and Information Division of the Canadian Education Association (1965) as a result of a survey done with Departments of Education and school systems in Canada, agreement was reached that programmed instruction has a contribution to make to education. Results showed that programmed instruction was suitable with small groups of students for remedial work, enrichment purposes, correspondence courses, advanced individualized study, and make-up work. It was not intended to replace the teacher in the classroom but it was viewed as being an aid or supplement which would improve the teacher's effectiveness in meeting the needs of the individual students.

In order to ascertain the effectiveness of programmed instruction as compared to teacher instruction and to evaluate the quality of classroom research done by teachers, the Canadian Teachers' Federation offered two instruction programs in mathematics and chemistry to teachers throughout Canada and asked them to evaluate the effectiveness of the programs in 1964. A total of two hundred teachers ordered programs and eleven of them wrote research reports. The results of the ones pertaining to programmed instruction using mathematics are summarized below.

Naka used programmed instruction to teach the topics of signed numbers and set theory to grade X mathematics students. He obtained 14 matched pairs of students. The experimental group used programmed instruction and the control group was taught by the regular classroom teacher. Comparisons in achievement favoured the control group.

Francis conducted an investigation into the use of programmed mathematics materials with lower ability grade X students. When these results were compared with their counterparts who had been teacher instructed, their achievement was much lower.

Richards in an experiment involving 106 students equally divided into experimental and control groups concluded that short units of programmed instruction were as effective as teacher instruction when the means of the achievement test in grade IX mathematics were compared.

It was suggested that the programmed instruction provided for individual differences in pupils better than teacher instruction did.

Price and De Paoli formed groups of paired students and used as aids to instruction in mathematics either programmed materials or the authorized textbook. Both groups received supplementary teacher-made materials. After thirteen hours of instruction, it was concluded that both methods of instruction were equally effective when no statistical difference between the two groups in net gain from pre-test to post-test situations were considered. It was implied that programmed materials were best suited to the role of teaching aids rather than independent teaching devices.

Feir concluded from an investigation involving grade IX students that the programmed instruction group completed their work more quickly than the teacher instructed group but their overall achievement results were lower.

Clark conducted an experiment using programmed instruction at the grade IX level for several months of the year. The study revealed many problems that were associated with programmed instruction when used over a longer period of time.

In an investigation conducted by Robinson, an experimental group used a programmed text in algebra from October to May while a control group received regular classroom instruction. A teacher-made test of achievement in algebra showed no significant difference between the groups after instruction by the two methods. However, comparison of gains by the groups on a standardized test of algebra achievement revealed a significantly greater gain in achievement for the experimental group.

Review of the Research on Programmed Instruction in Mathematics in Manitoba

There have been only three experimental projects involving programmed instruction reported.

During the 1962-1963 school year, the Portage-Oakville experiment was conducted using TEMAC algebra materials at the grade IX level. At the Portage and Oakville Collegiates, control groups using conventional teaching methods and experimental groups using programmed materials were established. A progress report issued at the end of November stated that the students in the experimental group were learning the content at least as well as those in the control group. Teacher help was provided to the experimental group. It was impossible to compare the results of the experimental groups with those of the control groups because a different examination had been written. However, the teacher evaluation stated that the students in the experimental group learned the material at least as well, if not better than the control group. The progress report released in May stated that learning was taking place with those using programmed materials but the more teacher assistance that was given, the better were the test marks.

A five-month experiment in the Winnipeg School Division was conducted in 1963-1964 by Duncan and Sigurdson to evaluate programmed materials at the grade IX level. Two experimental and two control groups numbering ninety students were chosen with the experimental groups using programmed materials and the control group using the traditional textbook. At the end of the experimental period, the pupils who were taught grade IX algebra by the traditional teaching methods scored significantly higher on achievement tests than did those who used programmed instruction. The final results of the Manitoba Department of Education Grade IX Mathematics Examination showed that there was no significant difference in achievement between the experimental and control groups. The results of the experiment showed that where programmed learning materials were used, there may have been real differences in the rate of achievement while the material was being used. However after a complete and thorough review, the mathematical competence of the pupils seemed to be developed to a desired level.

Kristal (1966, pp. 95-99) conducted an experiment on a programmed unit on area of polygons in grade XI geometry over a three week period. It involved fifty students who were paired and distributed randomly into two classrooms. There was no significant difference in mean achievement between the experimental group taught by programmed instruction and the control group taught by the investigator. The amount of time saved by the programmed instruction group was statistically significant.

The review of the literature on programmed instruction indicated that very few experiments had been conducted at the grade XII level. Many of the studies had been associated with grades IX and X students or with college students. Many of the studies were conducted over time periods of one or more years. The writer felt that there was a need to investigate further the use of programmed materials for certain mathematical topics extending over a shorter period of time at the secondary level while at the same time controlling specified variables.

ATTITUDES TOWARD MATHEMATICS

The attitudes of students toward mathematics play a very important role in their learning. Corcoran and Gibb (1961, p. 106) stated that a student's attitude toward mathematics is a composite of intellectual appreciation of the subject and emotional reactions to it.

Most studies of student attitude toward mathematics have been concerned with the direction (Does a student generally like or dislike the subject?) and intensity (How strongly does the student feel about this attitude?) of attitude toward mathematics in general. Few attitude studies have been directed toward specific areas such as special mathematics courses, specific aspects of mathematics such as problem solving, the mathematics teacher, mathematics methods or student reaction to areas of difficulty.

Methods of Attitude Appraisal

Corcoran and Gibb (1961, pp. 106-118) outlined different methods that could be used in appraising attitudes. Briefly, they include:

1. Self-report Methods:

a) Questionaires - Opinion questionaires, rating forms and check lists of various types have been devised to provide standard situations in which attitudes may be studied.

b) Attitude Scales - Each item on the scale consists of a statement to which the student is asked to indicate varying degrees of agreement or disagreement.

c) Incomplete Sentences - These are open-ended statements designed to stimulate responses in specific areas of attitude while allowing the individual considerable freedom in his choice of response. The reply may be partly controlled by the content and grammatical structure of the stem of the sentence.

d) Essays - Topics may be chosen to elicit reactions to specific aspects of mathematics learning or they may be quite general so that a student may choose a topic which concerns him most.

2. Observational Methods

Classroom teachers can observe the student as he proceeds with the learning situation. Sometimes checklists are devised to note particular activities. The intensity of observed student attitudes may be rated if the instrument provides an opportunity for indicating the degree of feeling expressed.

3. Interviews

Valuable information may be obtained from skillfully handled interviews designed to provide the student with an opportunity for free expression of his feelings about the class, the subject, and other related topics.

Review of the Research on Student Attitudes Toward Mathematics

It is generally recognized that attitudes toward mathematics in adults can be traced to their childhood. (Morrisett and Vinsonhaler, 1965, p. 21). There was evidence that very definite attitudes toward arithmetic may be formed as early as the third grade, but these attitudes tend to be more positive than negative in the elementary school. (Stright, 1960, pp. 280-286: Dutton, 1960, pp. 418-424; Smith, 1964, pp. 474-477). Dutton (1968, pp. 259-264) suggested that the junior high school grades seem to be a critical area for the determination of attitude toward mathematics. Anttonen (1969, pp. 467-471) examined the relationships between mathematics attitude and mathematics achievement over a six-year period from late elementary to the late secondary school level. He found the correlation between attitude toward mathematics in the elementary and secondary school was $r = .30$ for the group. Roberts (1970, pp. 785-793) suggested that attitudes toward mathematics, once adopted, may be relatively stable over the years.

The assessment of attitudes toward mathematics would be of less concern if attitudes were not thought to affect performance in some way. Neale (1969, pp. 631-640) and Husen (1967, pp. 46-48) have reported that correlations between attitude and achievement are consistent between .20 and .40. Brown and Abel (1965, pp. 547-549) found that the correlation between pupil attitude toward a subject and achievement in that subject was higher for arithmetic than for spelling, reading or language. Anttonen (1969, p. 469) stated that achievement was greater for students whose attitudes had remained favourable or had become favourable since elementary school. Roberts (1970, pp. 785-793) revealed that students and teachers showed a tendency to take the middle-of-the-road position with respect to attitudes about the difficulties of learning mathematics and the place of mathematics in society.

Cross (1968, p. 141) measured the student's attitude toward geometry and the effect of student attitude on achievement in geometry. At no place in the data was there an indication of a strong positive acceptance of geometry specifically or of mathematics in general.

ATTITUDES TOWARD PROGRAMMED INSTRUCTION

Questionaires, attitude scales, teacher observation and interviews were some of the methods employed to determine student attitudes toward programmed instruction.

Review of the Research on Student Attitudes Toward Programmed Instruction in Mathematics in the United States

In a study designed specifically to measure student attitudes toward programmed instruction, Eigen (1963, pp. 282-285) used a linear programmed text on sets, relations and functions with thirty-three high school students and a teaching machine with a group of thirty-nine students. Students scoring high on the achievement test given at the end of the unit were significantly more likely than low scoring students to say that programmed instruction was the best method of learning. The student's overall attitude toward automated teaching bore no relation to the amount learned by the respective methods. About all that can be concluded from the study was that it was difficult to measure student reaction to controversial statements about programmed instruction after only their first exposure to such a procedure.

Meadowcraft (1965, pp. 422-425) reported more favourable attitudes using a programmed text seventy percent of the time and teacher instruction the remainder of the time as opposed to teacher instruction with a programmed text used for homework.

Noble and Gray (1968, pp. 271-282) reported that if programmed instruction was used solely as the means of instruction for mathematics during three periods a week, the children became very bored. Favourable initial attitudes declined steadily and significantly. The reaction of individual children to programmed instruction was significantly different. More favourable attitudes were displayed by mild, adventurous, and uncontrolled children who did not score highly on the post-test. Girls displayed more favourable attitudes to programmed instruction and their attitudes did not decline as rapidly as the boys.

Noble (1966, p. 8) experimented with teacher-programmed machine integration as one method and solely programmed instruction as a second method. He found that the attitudes of the integrated group were significantly more favourable than attitudes of the non-integrated group.

Devine (1968, pp. 296-301) found that student attitudes toward mathematics and programmed instruction are not affected by the approach when students are under the direction of experienced teachers. However their attitudes are significantly affected in a negative direction by inexperienced teachers. Attitudes toward programmed materials are not significantly decreased by using those materials for the duration of a year if the teacher involved is an average or above average teacher.

Frey, Shimabukuro, and Woodruff (1965, pp. 297-301) did a study on attitude change in programmed instruction as related to achievement and performance. They conducted their experiment over three semesters. There was a significant decline in learner attitude toward programmed instruction during the second semester as a result of prolonged use. There was also a corresponding drop in achievement. It appeared however that when learners were taken as a group, there was a positive relationship between attitude toward programmed instruction and subject matter achievement.

Sarason (1958, p. 339-344) cited that the attitude and other personal characteristics that one brings into a particular situation are important to the success or failure of the desired goal. In programmed learning, the desired goal is mastery of the material presented in the program. If negative attitudes or expectations are held toward programmed learning as a teaching method, then one would expect that these factors would influence the effectiveness of the program.

Davies and Banning (1960, p. 13) found that students of lower academic ability displayed better attitudes toward mathematics, teacher, and programmed instruction after using programmed materials. It was observed that their attitude toward school and education shifted negatively during this time period.

Biddle (1966, p. 3356-A) designed a questionnaire to obtain information from the programmed classes concerning attitude toward the material, the course, their work habits in and out of class, and their opinion of their performance in geometry relative to their performance in the subject in the conventional situation. It was concluded that programmed instruction did not affect the student's desire to continue in mathematics.

Little (1964, p. 5154) studied the attitude of college algebra students over an eleven-week period. He found that their attitudes toward programmed instruction, ease of learning, and extra study time gained from programmed instruction were significantly more positive than those of the traditional control group in the third week. However by the eleventh week, the experimental group's attitude had shifted significantly in the negative direction. Alton (1964, p. 4488) found similar results.

Robson (1965, p. 85-A) used an attitude scale to determine changes in the attitudes of students toward mathematics when they were subjected to programmed instruction or the conventional method. There was no significant difference in attitude when the different teaching methods were used. Rafig (1964, p. 4581) reached similar conclusions.

Feldhusen et al. (1962, pp. 8-10) found that there was a greater concentration of responses favouring programmed texts than favouring the conventional method.

Review of the Research on Student Attitudes Toward Programmed Instruction in Mathematics in Canada

Feir reported that pupil reaction to programmed instruction varied. Some were bored, some liked working alone on it and others thought that a combination of programmed instruction and teacher instruction would be excellent.

Clark administered attitude tests at various intervals and the final test indicated that 37 out of 133 students were deeply dissatisfied with programmed instruction. A group of adults who used the same program had a highly favourable attitude toward the program.

Robinson administered an attitude survey in January and again in June. It showed an overall change in favour of traditional instruction. Ten of twenty-four students agreed that they would like to learn other subjects by programmed instruction as well. Fourteen students felt that programmed instruction was boring. In many cases it was the better student who rejected the use of programmed materials.

Review of the Research on Student Attitude Toward Programmed Instruction in Mathematics in Manitoba

In the first progress report issued with the Portage-Oakville experiment (1962-1963), an opinion survey revealed that most of the pupils liked programmed instruction. Many had found mathematics more difficult using conventional teaching methods. Most thought that the teacher had more time for individual attention. Only a few pupils felt that they were getting bored. They liked the idea of having no homework. No other information was given regarding student attitude at the end of the experiment.

In the Winnipeg experiment (1963-1964) no attitude survey was taken. It was indicated however, that the students worked well and only a small minority complained of boredom.

Kristal (1966, p. 88) administered an attitudinal questionnaire to his experimental group which indicated the majority of students favoured programmed instruction as compared to teacher instruction. However, only 60% favoured programmed instruction for further mathematics courses; 37% did not want to work with it again and 3% were undecided.

The review of the literature on attitudes toward mathematics and programmed instruction indicated that little research had been done in this area. A need for further research into the general area of attitude toward mathematics and more specifically attitude toward programmed instruction prompted the writer to include this aspect in the study.

SUMMARY

When the writer began to examine the literature on programmed instruction, it soon became apparent that all aspects of programmed instruction neither could be reviewed nor were pertinent as background for this study. Most of the literature could be divided into two parts:

(a) review of the techniques of programming and program construction;

(b) comparison of the effectiveness of programmed instruction to traditional teaching methods.

The writer reviewed the historical growth of programmed instruction from the time of Socrates to B. F. Skinner's historical document. The nature of programmed instruction was also reviewed to give some idea of the differences and similarities between conventional instruction and programmed instruction.

In the review of the literature on the research on programmed instruction in the United States, programmed materials were used for different purposes. Some considered it as a means of providing supplementary material for above average and below average students, as a method of providing remedial work, as material to augment, supplement and complement existing materials, as an aid to improving mathematical attitude and providing favourable attitudes toward mathematics and as a means for research design. In most of the studies cited, a comparison between instruction using programmed materials and the conventional classroom methods was utilized. Despite the differences in methods of experimental design, the majority of the reports indicated no significant differences in achievement. Some reports indicated conventional methods were superior to programmed instruction and a small number reported favourable significant differences toward programmed instruction.

In Canada, research indicated that a majority of the studies favoured conventional instruction as compared to programmed instruction. The minority of the reports showed no differences in mean achievement or favourable differences toward programmed instruction.

In Manitoba, two of the three studies reviewed indicated that there was no difference in achievement between the classes taught by the use of programmed instruction and those taught by the conventional methods. The third study was not statistically designed but the teacher evaluation stated that the students using programmed materials learned as well as those that were conventionally taught.

In the review of the literature on attitudes toward mathematics, it was indicated that attitudes toward mathematics may be formed as early as the third grade. The measurement of attitudes in the junior high grades was very difficult to assess. It was felt that attitudes toward the subject area do affect the students' performance and achievement.

In Manitoba, the only attitude testing done in mathematics was done in the area of geometry. At no place in the data was there an indication of a strong positive acceptance of geometry or of mathematics generally.

The majority of the research studies on student attitude toward programmed instruction indicated favourable attitudes when teacher instruction and the use of programmed materials were used over a period of three weeks. If used longer than three weeks, student attitude toward them declined significantly.

The writer felt the need to investigate further the use of programmed instruction at the senior high school level over a relatively short period of time. Because of contrasting results between the American and Canadian research on the effectiveness of programmed instruction, it seemed that another Canadian study might contribute to resolving the contrast. Since the area of attitudes is a continuing concern to mathematics educators, it seemed appropriate to include this aspect in the study.

CHAPTER III

EXPERIMENTAL DESIGN

The purpose of this chapter is to outline and describe the study in detail. It will include a description of the school, population and sample, course description and treatment, the hypotheses restated, data collection procedures and method of analysis.

EXPERIMENTAL SETTING

The School

The subjects for the experiment were grade XII students at West Kildonan Collegiate, one of the two senior high schools in the Seven Oaks School Division #10. The school timetable operates on the basis of a six day cycle with eight periods of forty minutes a day. Each class has seven periods of mathematics per cycle at the grade XII level.

Population and Sample

The total enrolment for West Kildonan Collegiate was six hundred and thirty-eight. Two hundred and ten were enrolled in grade XII, of whom one hundred and forty enrolled in a grade XII mathematics course. Sixty-three were taking mathematics 300 which is the university orientated mathematics course, and seventy-seven were taking mathematics 301 which is the general mathematics course. This is summarized in Table I.

TABLE I

SUMMARY OF ENROLMENT IN GRADE XII

Classification	Number
Students enrolled in mathematics 300	63
Students enrolled in mathematics 301	77
Grade XII students not taking mathematics	70
Total	210

The grade XII students, apart from grouping dictated by course selection, were randomly assigned to sections. There were three sections of mathematics 300 and four sections of mathematics 301. Two teachers were assigned to teach the mathematics 300 sections and two were assigned to teach the mathematics 301 sections. Two of the three mathematics 300 sections were assigned to the writer. Two of the four mathematics 301 sections were assigned to the writer.

One of the writer's mathematics 300 classes was chosen as the control group which was taught by the traditional teaching methods, and the other class which was called the experimental group was subjected to programmed instruction. The same arrangement was used with the two mathematics 301 classes. The enrolment of each of these classes is summarized in Table II.

TABLE II

SUMMARY OF ENROLMENT OF CLASSES IN THE SAMPLE

Class	Control	Experimental	Total
Mathematics 300	26	20	46
Mathematics 301	16	27	43
Totals	42	47	89

Course Description and Treatment

The topic that was used in the experiment was an introduction to logarithms. The unit on logarithms was chosen for the experiment as it is a common topic to the mathematics 300 and mathematics 301 curricula.

For the members of the control group in mathematics 300, the text used was Mathematics 12 by E. P. Vance (1968). For those in the control group taking mathematics 301, the text used was Foundations of Mathematics by J. E. Dean and G. E. Moore (1964).

The writer investigated the feasibility of obtaining a programmed unit on logarithms. Any that were available commercially had been written for use with four place logarithm tables. The programmed unit that the writer used was an adaptation of two available programmed texts--Logarithms by George Sackheim (1964) and Calculation and Slide Rule by Thomas J. McHale and Paul T. Witzke (1971).

The first program did not allow for any review of decimals; exponents and the nature of squares, square roots, cubes, and cube roots. It consisted of 158 frames.

The second program was more detailed. The chapter on logarithms consisted of 271 frames. It provided for a review of exponents and powers of ten form.

There was no mention of program preparation accompanying the program by Sackheim. The second book mentioned was the product of a five year project whose goal was the development of mathematical skills needed in basic science and technology for a wide range of student ability. It is a highly organized and highly assessed system which was designed to be used by the regular classroom teacher without any special training in the use of programmed materials. The following results from its five-year period of use include:

(a) drop out rate in the course was reduced by fifty percent;

(b) average scores on examinations increased from fifty-five percent to eighty-five percent;

(c) rate of class absenteeism decreased;

(d) student motivation and attitudes were favourable.

The course has had wide use. Field tests with over four thousand students have provided many constructive comments by teachers and students, plus a wealth of test data which has been item analyzed and error analyzed. This has led to program revision.

The course series was written for a two-semester technical mathematics course but it is so flexible that it can be used at the college level or secondary school level for teaching purposes.

In the writer's adaptation which appears in Appendix E, questions were used from the conventional texts in an attempt to make sure that the subject matter for the two different methods was comparable. The writer also adapted the program for five place tables which is the requirement of the Manitoba Department of Education.

The program was worked through by a second year mathematics major at the University of Manitoba and it was compared to the course syllabus by the Mathematics Department at West Kildonan Collegiate.

The conventional or traditional method consisted of the teacher-lecture and question and answer periods, teacher-class discussion, worksheet periods and homework correction periods.

Those in the group using programmed materials were given an overview of what was expected. They were instructed to proceed at their own pace. There was no teacher assistance given to the mathematics 300 group and very minimal assistance was given to the mathematics 301 group.

Hypotheses

The hypotheses tested were the null hypotheses:

Hypothesis I

That there is no significant difference in mean scores of achievement of students exposed to programmed instruction as compared to those taught by the conventional classroom methods.

Hypothesis II

That there is no significant difference in mean scores of attitude toward mathematics of students exposed to programmed instruction as compared to those taught by the conventional classroom methods.

Hypothesis III

That there is no significant difference in mean scores of pre- and post-attitude toward mathematics on the part of students using programmed instruction.

Hypothesis IV

That there is no significant difference in mean scores of pre- and post-attitude toward teacher on the part of students using programmed instruction.

Hypothesis V

That there is no significant difference in mean scores of pre- and post-attitude toward programmed instruction on the part of students using programmed instruction.

Data Sources and Collection

The following were used as independent variables in the study:

1. IQ scores

Each student who participated in the experiment had been given the Otis Quick Scoring Mental Ability Test in grade X. Those scores were obtained from the students' cumulative folders.

2. Chronological age in months

The subjects' birthdates were obtained from their registration forms and the writer converted their ages into months as of January 31, 1972.

3. Sex

This was determined by the writer's own judgment. No biological tests were administered.

4. Socio-economic status

In early February, each subject was given the Home Index Scale which was obtained from the Planning and Research Division of the Department of Education. The scale was administered directly to the student by the writer. A self-report was made by answering the twenty-one questions of the yes-no type. A re-test reliability of $r = .989$ is reported and a Kuder-Richardson reliability of $r = .74$ is given for high school students. The test relies heavily on the acquisition of material goods as an indicator of status. A copy of the scale is found in Appendix A. The scale was scored by the writer with the "yes" answer being given a weight of one and the "no" answer a zero weight.

5. Pre-test of attitude toward mathematics

The Mathematics Attitude Scale (MAS) which was developed by Aiken and Dreger (1961) was administered to each class of subjects. It was given as a pre-test to determine the student's attitude toward mathematics before experimentation. The MAS scale is composed of twenty items in all--ten items denoting positive attitudes and ten denoting negative attitudes. Investigations by Aiken and Dreger (1961) found its reliability to be $r = .94$ for test-retest. The MAS was scored using a Likert method of summated ratings. The student reacted to each statement on a five point scale indicating that he strongly agreed, agreed, neither agreed nor disagreed, disagreed, or strongly disagreed. The score was computed by weighting the responses from one to five for a positive statement beginning with strong disagreement. Values were assigned in reverse order for unfavourable statements. The scores from each of the items were added to obtain a score representative of their attitude toward mathematics. Consequently on the test, the maximum favourable score was 100, a maximum unfavourable score was 20, and a neutral score was 60. The scoring and computing were done by the writer. The MAS scale appears in Appendix B.

6. Grade IX mathematics achievement test scores

This mark represented the student's achievement in grade IX mathematics as measured by the Department of Education Grade IX Mathematics Examination. These were available from the students' cumulative folders.

7. First term grade XII mathematics achievement test scores

The writer was responsible for setting, administering and correcting the tests which had been written during the first term. These were chapter tests which were written at the completion of each chapter. They were then averaged to obtain a first term mathematics achievement score.

8. Logarithm achievement pre-test score

Prior to the beginning of the experiment, each class was administered the logarithm achievement test (LAT) to determine the subject's prior knowledge of logarithms.

Since there was no standardized achievement test available from the testing agencies, the writer developed his own logarithm achievement test. It was composed of test items which had appeared on old copies of the Department of Education mathematics examinations. The test was validated by the Mathematics Department at West Kildonan Collegiate.

To check its reliability, it was administered to a sample of the population not involved in the experiment. This sample was a section of mathematics 300 students who did the section on logarithms using programmed instruction. By using the split half method of calculating reliability, its $r = .75$.

The writer marked and recorded the results of the achievement pre-test scores. The same test was used as an achievement post-test.

The following were considered to be the dependent variables:

(a) logarithm achievement post-test (post-LAT)

(b) mathematics attitude post-scale score (post-MAS)

For the experimental groups, the following variables were considered as variables for a pre-test post-test situation:

(a) attitude toward mathematics test (AMT)

(b) attitude toward teacher (TAT)

(c) attitude toward programmed instruction (PIAT)

The attitude scale used was a semantic differential scale. Using Osgood's Semantic Differential Scale (1957) as a model, fifteen bi-polar items were written for each of the three areas under consideration. A seven point Guttman-type scale for each item enabled the subject to indicate the degree of feeling toward each item. Positive and negative choice positions were varied to avoid a subject checking all items of either a positive or negative nature in order to express general approval or disapproval. An example of the scale used is contained in Appendix D. Scoring of the attitude scale was done as follows: scores ranged from one to seven--completely negative responses were assigned a score of one, completely positive a score of seven. A mark halfway between the two extremes was given a score of four. Cassel (1970) on scales similar to those used by the writer reported reliabilities of $r = .42$ to $.61$ for part scores and $r = .93$ to $.96$ for the total score. McCallon and Brown (1971) developed a similar semantic differential scale which had a correlation with the MAS of $r = .90$.

The classroom learning situation began with the control group in each classification receiving the conventional instruction composed of lecture, teacher-class discussion, questions and answer periods, and the doing and correcting of assignments. This group kept a record of the amount of time that they spent doing their assignments.

The experimental groups were introduced to the programmed instruction booklet. The writer outlined the objectives of the method. The writer read the instructions at the beginning of the program. The first four frames were read through aloud by the writer to make certain that there were no misunderstandings or difficulties being encountered. The writer then told the subjects that no assistance from the teacher could be sought.

Each subject of the experimental group was given a sheet to keep a record of the number of frames completed each period and the time spent. Table III shows a copy of the outline of the sheet.

TABLE III

RECORD SHEET

Date	Frame Started	Time	Frame Ended	Time

The experimental group could not take the programmed material from the classroom. If the student was absent from a class and wanted to make up the time, he was allowed to do so in the writer's office.

When the program was completed, the Semantic Differential Scales on attitude toward mathematics, teacher and programmed instruction was administered to the experimental groups for the second time.

When both methods of instruction had been completed, the Logarithm Achievement Test was administered for the second time. The following day, the Mathematics Attitude Scale was administered for the second time. This was done before the students were given any indication of their success on the Logarithm Achievement Test.

Method of Analysis

Statistically, the analysis used to test for significant differences in mean achievement and mean attitude was the analysis of covariance design. It provided a method of statistical control over the variables that were considered relevant to the study. It is a technique which adjusts for initial differences in the relevant variables. This removes the effect of such independent variables as I.Q., chronological age in months, sex, socio-economic status, grade IX mathematics achievement scores and the logarithm and attitude pre-test scores. This made it possible to analyze the logarithm achievement post-test and mathematics attitude post-scale for significant differences while simultaneously adjusting these scores of initial covariate differences (I.Q., age, sex, etc.).

The analysis of covariance technique allowed the researcher to equate statistically the independent variable groups with respect to one or more variables which are relevant to the dependent variable. In other words, analysis of covariance allowed the researcher to study the performance of several groups which were unequal with regard to an independent variable as though they were equal in this respect. The research design can be described as in Table IV.

TABLE IV

RESEARCH DESIGN FOR ACHIEVEMENT AND ATTITUDE

Group	Pre-test	Treatment	Post-test
Experimental	Yes	Prog. Inst. & MAS	Yes
Control	Yes	Trad. Inst. & MAS	Yes

The paired t-test for correlated measures was the statistical model used with the semantic differential attitude scales administered to the experimental groups. It was used to determine whether there were any significant differences in the means of the three scales between the pre-test and post-test administrations.

SUMMARY

A sample of eighty-nine students was drawn from West Kildonan Collegiate grade XII population. These subjects were divided into four sections on the basis of their choice of mathematics course. Two of the sections took mathematics 300 and the other two took mathematics 301. In each classification, an experimental and control group were designated.

The experimental groups learned logarithms solely by programmed materials and the control groups were conventionally instructed by the writer on the same topic. The length of time was approximately three weeks.

The dependent and independent variables were described. Achievement pre- and post-tests on logarithms and attitude pre- and post-tests were written by all groups. The experimental groups had further attitude scales administered to them in order to measure subjects' attitude change toward: (a) mathematics; (b) teacher; and (c) programmed instruction. The various measurement instruments were described.

The statistical treatments in the study were outlined and hypotheses to be tested were listed.

CHAPTER IV

ANALYSIS OF THE DATA

Before presenting the analysis of the data, a review of the experimental procedures is outlined below. Specific procedures were:

1. Collection of IQ, age, sex, grade IX mathematics achievement scores and first term grade XII mathematics achievement scores from the students' cumulative folders;
2. Administration of Home Index Scale;
3. First administration of the Mathematics Attitude Scale (MAS);
4. First administration of the Logarithm Achievement Test (LAT);
5. Commencement of the experimental instruction period on the topic "Introduction to Logarithms";
6. First administration of the semantic differential scales on attitude toward mathematics (AMT), attitude toward teacher (TAT), and attitude toward programmed instruction (PIAT);
7. End of experimental instruction period;
8. Second administration of the semantic differential scales (AMT, TAT and PIAT);
9. Second administration of LAT;
10. Second administration of MAS.

The purpose of the present chapter is to present the analysis of the data.

Descriptive Data Concerning the Sample

Certain members of the original sample were excluded from the analysis. They were either school drop-outs during the period of the experiment or pupils who had failed to get credit for their mathematics course the previous year and were repeating instruction in the course material.

Table V gives the enrolment of students included in the data analysis.

TABLE V

SUMMARY OF ENROLMENT OF STUDENTS FOR DATA ANALYSIS

Group	Control	Experimental	Total
Math. 300	26	19	45
Math. 301	14	23	37
Totals	40	42	82

Table V reveals that the experimental 300 group was reduced by one subject and the 301 experimental group was reduced by four subjects when it is compared to the enrolment figures given in Table II.

The means of each independent variable of each group are summarized in Table VI.

TABLE VI

SUMMARY OF MEANS OF INDEPENDENT VARIABLES FOR EXPERIMENTAL AND CONTROL GROUPS

Group	I.Q.	Age in mos.	Socio-Econ Status	Pre MAS	Grade IX	Grade XII	Pre LAT
300C *	118	211	14	73	79	66	4
300E **	117	213	14	82	81	70	5
301C	104	213	12	71	63	55	3
301E	110	214	13	65	68	68	1

* C-Control

** E-Experimental

The mean age of each group was the average of the students' chronological ages converted into months. A range of scores from 12-14 on the socio-economic index scale would indicate middle-class status. The Mathematics Attitude Scale (MAS) mean score was an average of the student scores out of a maximum of 100. The grade IX mathematics mean was the average of the students' percentage scores on the departmental examination. The grade XII mathematics score was obtained by averaging the chapter tests given during the first term. The resulting scores were averaged. The Logarithm Achievement Test (LAT) was scored by the writer with the scores being converted to percentage scores and the mean was then calculated. The distribution of girls to boys in the mathematics 300 groups existed in the ratio of seven to twelve. In the mathematics 301 groups, the boys and girls were evenly distributed.

The data in Table VI illustrates that there are differences in some of the independent variables in the various groups. For instance, the mean scores of the grade IX mathematics scores range from a low of 63 for mathematics 301C to a high of 81 for the mathematics 300E. To compensate for such differences as well as to accommodate intact classroom groups, the analysis of covariance design was used. The raw data as collected was transferred to computer cards and the analyses were conducted using the University of Alberta Ancova 10 program.

Differences between the means of the control and experimental groups were considered significant if they fell into that range of differences which by chance could occur less than five times out of one hundred. The .05 level was also considered significant for the differences in means of initial and final attitude tests.

ANALYSIS OF COVARIANCE FOR ACHIEVEMENT

Tables VII and VIII present the analysis of covariance significance tests for the mathematics 300 and mathematics 301 covariates.

TABLE VII

SUMMARY OF ANALYSIS OF COVARIANCE SIGNIFICANCE TESTS FOR MATHEMATICS 300 COVARIATES

Source of Variation	F-Value	Degrees of Freedom		Probability
		Between	Within	
I.Q.	0.06	1	43	0.81
Age	1.08	1	43	0.30
Sex	0.13	1	43	0.72
Socio-Econ. Status	0.01	1	43	0.93
Pre-MAS	4.04	1	43	0.05 *
Grade IX Math.	0.88	1	43	0.36
Grade XII Math.	1.15	1	43	0.29
Pre-LAT	0.11	1	43	0.75

* Significant at 0.05 level

TABLE VIII

SUMMARY OF ANALYSIS OF COVARIANCE SIGNIFICANCE TESTS FOR MATHEMATICS 301 COVARIATES

Source of Variation	F-Value	Degrees of Freedom		Probability
		Between	Within	
I.Q.	5.34	1	35	0.03 *
Age	0.04	1	35	0.84
Sex	0.40	1	35	0.53
Socio-Econ. Status	1.33	1	35	0.26
Pre-MAS	1.06	1	35	0.31
Grade IX Math.	2.27	1	35	0.14
Grade XII Math.	6.94	1	35	0.01 **
Pre-LAT	0.43	1	35	0.52

* Significant at .05 level

** Significant at .01 level

The data listed in Table VII revealed that the MAS pre-test score for the mathematics 300 group was significant at the .05 level.

The independent variables of I.Q. and the first term mathematics achievement score for the mathematics 301 group were significantly different at the .05 and .01 levels respectively as outlined in Table VIII.

Tables IX and X contain the analysis of covariance significance tests for the post-LAT scores with the experimental and control sections of mathematics 300 and mathematics 301.

TABLE IX

ANALYSIS OF COVARIANCE FOR POST-LAT SCORES USING EXPERIMENTAL 300 AND CONTROL 300 GROUPS

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F Value	P
Between	1	3.25	3.25	0.03	0.86
Within	43	4732.75	110.06		
Totals	44	4736.00			

TABLE X

ANALYSIS OF COVARIANCE FOR POST-LAT SCORES USING EXPERIMENTAL 301 AND CONTROL 301 GROUPS

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F Value	P
Between	1	24.25	24.25	0.12	0.74
Within	35	7291.88	208.34		
Totals	36	7316.13			

The analyses as outlined in Tables IX and X indicate that the differences were not significant at the .05 level.

Summaries of the adjusted analyses of variance for the post-LAT scores of the mathematics 300 and mathematics 301 groups are found in Tables XI and XII. This tests for mean differences by identifying the amount of variation resulting from differences between the groups.

TABLE XI

ADJUSTED ANALYSIS OF VARIANCE FOR POST-LAT SCORES OF EXPERIMENTAL 300 AND CONTROL 300 GROUPS

Source of Variation	Degrees of Freedom	Mean Square	Adjusted F	P
Between	1	56.35	1.10	0.30
Within	35	51.14		

TABLE XII

ADJUSTED ANALYSIS OF VARIANCE FOR POST-LAT SCORES OF EXPERIMENTAL 301 AND CONTROL 301 GROUPS

Source of Variation	Degrees of Freedom	Mean Square	Adjusted F	P
Between	1	0.80	0.01	0.94
Within	27	126.58		

No significant differences exist at the .05 level between the control groups and the experimental groups of either mathematics 300 or mathematics 301 as shown in Tables XI and XII.

Table XIII gives a summary of the unadjusted and adjusted means of the post-LAT scores for the sample.

TABLE XIII

SUMMARY OF UNADJUSTED AND ADJUSTED MEANS OF POST-LAT SCORES FOR THE SAMPLE

Group	Unadjusted Mean	Adjusted Mean
Math. 300C *	88	89
Math. 300E **	88	87
Math. 301C	71	73
Math. 301E	73	72

* C-Control

** E-Experimental

Very little change in the means can be noted. These negligible differences were reinforced by the fact that the analysis of covariance revealed significant differences at the .05 level for only one independent variable for the 300 groups and only two independent variables for the 301 groups.

Hypothesis Relating to Achievement

There is no significant difference in mean scores of achievement of students exposed to programmed instruction as compared to those taught by conventional classroom methods.

1. Mathematics 300 groups

Pre-test and post-test data were treated statistically by the analysis of covariance design. From Tables XI and XIII, no statistically significant difference attributable to the experimental treatment was found at the .05 level. Therefore, the null hypothesis is accepted.

2. Mathematics 301 groups

Using the same statistical design as above and the evidence from Tables XII and XIII, no significant difference is indicated at the .05 level. The null hypothesis is accepted.

It can be concluded from this that learning did take place regardless of the method of instruction used. The students who used programmed materials were not handicapped in their acquistion of knowledge of logarithms. They seemed to acquire a sense of accomplishment working with the program for a limited period of time.

ANALYSIS OF COVARIANCE FOR ATTITUDE TOWARD MATHEMATICS

Tables XIV and XV contain the analysis of covariance significance tests for the post-MAS scores using the experimental and control sections of the sample.

TABLE XIV

ANALYSIS OF COVARIANCE FOR POST-MAS SCORES USING EXPERIMENTAL 300 AND CONTROL 300 GROUPS

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F Value	P
Between	1	684.56	684.56	2.97	.09
Within	43	9911.13	230.49		
Totals	44	10595.69			

TABLE XV

ANALYSIS OF COVARIANCE FOR POST-MAS SCORES USING EXPERIMENTAL 301 AND CONTROL 301 GROUPS

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F Value	P
Between	1	87.38	87.38	0.38	0.54
Within	35	8083.88	230.97		
Totals	36	8171.26			

No significant difference at the .05 level is indicated for the data found in Tables XIV and XV. Notice should be made that the analysis of covariance for post-MAS scores using experimental 300 and control 300 showed significant differences at the .10 level.

Summaries of the adjusted analysis of variance for the post-MAS scores of the mathematics 300 and mathematics 301 groups are found in Tables XVI and XVII.

TABLE XVI

ADJUSTED ANALYSIS OF VARIANCE FOR POST-MAS SCORES OF EXPERIMENTAL 300 AND CONTROL 300 GROUPS

Source of Variation	Degrees of Freedom	Mean Square	Adjusted F	P
Between	1	4.15	0.08	0.78
Within	35	50.54		

TABLE XVII

ADJUSTED ANALYSIS OF VARIANCE FOR POST-MAS SCORES OF EXPERIMENTAL 301 AND CONTROL 301 GROUPS

Source of Variation	Degrees of Freedom	Mean Square	Adjusted F	P
Between	1	18.27	0.33	0.57
Within	27	54.69		

No significant difference exists at the .05 level between the control groups and the experimental groups of either mathematics 300 or mathematics 301 as shown in Tables XVI and XVII.

A summary of the unadjusted and adjusted means of the post-MAS scores of the experimental and control groups is contained in Table XVIII.

TABLE XVIII

SUMMARY OF UNADJUSTED AND ADJUSTED MEANS OF POST-MAS SCORES FOR THE SAMPLE

Group	Unadjusted Mean	Adjusted Mean
Math. 300C *	74	77
Math. 300E **	81	77
Math. 301C	70	67
Math. 301E	67	69

* C-Control

** E-Experimental

Table XVIII indicates that a difference was evident in the unadjusted means of each group. The unadjusted means of the mathematics 300 groups ranged from 74 to 81 whereas the unadjusted of the mathematics 301 groups ranged from 67 to

70. However, when the means were adjusted statistically, no significant differences at the .05 level were revealed.

Hypothesis Relating to Attitude Toward Mathematics

There is no significant difference in mean scores of attitude toward mathematics of students exposed to programmed instruction as compared to those taught by conventional classroom procedures.

1. Mathematics 300 groups

Pre-test and post-test data obtained from the two administrations of the MAS scale were treated statistically by the analysis of covariance design. Data reported in Tables XVI and XVIII indicated that no statistical difference at the .05 level was found. Therefore, the null hypothesis is accepted.

2. Mathematics 301 groups

Using the same statistical design as above and the data from Tables XVII and XVIII, no significant difference is indicated at the .05 level. The null hypothesis is accepted.

ANALYSIS OF THE SEMANTIC DIFFERENTIAL SCALE ON ATTITUDE TOWARD MATHEMATICS USING THE PAIRED T-TEST

The paired t-test for correlated measures was the statistical model used with the semantic differential attitude scales administered to the experimental 300 and 301 groups. For purposes of this study, a difference was declared significant at the five percent level or less.

Tables XIX and XX contain a summary of the attitude toward mathematics scores (AMT) for the experimental 300 and 301 respectively.

TABLE XIX

SUMMARY OF AMT SCORES FOR THE EXPERIMENTAL 300 GROUP

No.	Pre-test Mean	Post-test Mean	S.D. of Diff.	Cal. t	Degrees of Freedom
19	84	83	6.56	0.52	18

At .05 level, theoretical t = 2.10.

TABLE XX

SUMMARY OF AMT SCORES FOR THE EXPERIMENTAL 301 GROUP

No.	Pre-test Mean	Post-test Mean	S.D. of Diff.	Cal. t	Degrees of Freedom
23	71	70	9.09	0.41	22

At .05 level, theoretical t = 2.07.

The initial and final means of each group differed by one unit during the time lapse between the two administrations.

Hypothesis Regarding Attitude Toward Mathematics for the Experimental Groups

There is no significant difference in mean scores of pre- and post-attitude toward mathematics on the part of students using programmed instruction.

1. Experimental 300 group

This hypothesis was tested using the paired t-test for significant difference between the means of the pre-test and post-test AMT scores. The results from Table XX indicated that there was no significant difference in the mean scores at the .05 level from the initial administration to the final administration. Therefore, the null hypothesis is accepted.

2. Experimental 301 group

The data in Table XX derived by using the same statistical design showed no significant difference in the mean scores at the .05 level of AMT pre-tests and post-tests. The null hypothesis is accepted.

It might be noted that similar results for both experimental and control groups were obtained when the pre- and post-MAS scores were analyzed using the paired t-test.

ANALYSIS OF THE SEMANTIC DIFFERENTIAL SCALE ON ATTITUDE TOWARD TEACHER USING THE PAIRED T-TEST

Summaries of the attitude toward teacher scores (TAT) for the experimental groups are found in Tables XXI and XXII.

TABLE XXI

SUMMARY OF TAT SCORES FOR THE EXPERIMENTAL 300 GROUP

No.	Pre-test Mean	Post-test Mean	S.D. of Diff.	Cal. t	Degrees of Freedom
19	82	84	3.72	-0.65	18

At the .05 level, theoretical t = 2.10.

TABLE XXII

SUMMARY OF TAT SCORES FOR THE EXPERIMENTAL 301 GROUP

No.	Pre-test Mean	Post-test Mean	S.D. of Diff.	Cal. t	Degrees of Freedom
23	80	80	7.78	0.11	22

At the .05 level, theoretical t = 2.07.

The data from Table XXI indicates a slight increase in the attitude scores between the initial and final administrations for the experimental 300 group. Table XXII revealed no change in the attitude of the experimental 301 group.

<u>Hypothesis Regarding Attitude Toward Teacher for the Experimental Groups</u>

There is no significant difference in mean scores of pre- and post-attitude toward teacher on the part of students using programmed instruction.

1. Experimental 300 group

This hypothesis was tested using the paired t-test, and, as indicated in Table XXI, there was no significant difference at the .05 level in the pre- and post-TAT mean scores of the experimental 300 group. The null hypothesis is accepted.

2. Experimental 301 group

The data was exposed to the same statistical design as in (1.), and as indicated in Table XXII, no statistical significant difference existed at the .05 level between the two administrations of the TAT scale. Therefore, the null hypothesis is accepted.

ANALYSIS OF THE SEMANTIC DIFFERENTIAL SCALE ON ATTITUDE TOWARD PROGRAMMED INSTRUCTION USING THE PAIRED T-TEST

Tables XXIII and XXIV contain summaries of the attitude of the experimental groups toward programmed instruction.

TABLE XXIII

SUMMARY OF PIAT SCORES FOR THE EXPERIMENTAL 300 GROUP

No.	Pre-test Mean	Post-test Mean	S.D. of Diff.	Cal. t	Degrees of Freedom
19	82	76	11.23	2.13 *	18

* Significant at .05 level since theoretical t = 2.10.

TABLE XXIV

SUMMARY OF PIAT SCORES FOR THE EXPERIMENTAL 301 GROUP

No.	Pre-test Mean	Post-test Mean	S.D. of Diff.	Cal. t	Degrees of Freedom
23	74	71	18.17	0.67	22

At the .05 level, theoretical t = 2.07.

The data revealed in Tables XXIII and XXIV showed that the means of both groups decreased between the initial and final administrations of the PIAT scales.

Hypothesis Regarding Attitude Toward Programmed Instruction for the Experimental Groups

There is no significant difference in mean scores of pre- and post-attitude toward programmed instruction on the part of students using programmed instruction.

1. Experimental 300 group

As indicated in Table XXIII, the decrease in attitude toward programmed instruction from the pre-test to the post-test situation for the experimental 300 group was significant at the .05 level. The null hypothesis is rejected.

2. Experimental 301 group

Although there was a slight decline in the means of the pre-test to post-test scores on the PIAT scores for the experimental 301 group, the paired t-test revealed that the difference was not significant at the .05 level. The null hypothesis is accepted.

TIME FOR COMPLETION OF PROGRAM

Although there were no hypotheses formulated regarding the time spent becoming proficient with logarithms under the two different methods, Table XXV shows a record of the average time spent on the topic.

TABLE XV

TIME RECORD

Group	Time to Complete
Math. 300C *	348 minutes
Math. 300E **	287 minutes
Math. 301C	455 minutes
Math. 301E	358 minutes

* C-Control

** E-Experimental

The time of the 300C and 301C groups included only the time spent doing assignments. It did not include the teacher presentation or discussion of questions. Absenteeism was evident among the control groups and this was why class time spent on teacher presentation could not be included. There were some subjects in the control groups who lost their original time records so the average stated is not as accurate as in the case of the experimental groups. The time variation for the experimental 300 group was from 195 minutes to 403 minutes and in the other experimental group, the time ranged from 206 minutes to 521 minutes.

There does, however, seem to be considerable time saved by those who used programmed materials.

SUMMARY OF FINDINGS OF THE STUDY

This study was designed to investigate the use of programmed instruction as an additional aid and specifically as an alternative to the traditional teaching methods on logarithms. Tentative answers are presented here.

Findings on Achievement Measures (LAT)

No significant differences were found for the mathematics 300 and mathematics 301 groups when the achievement score means were adjusted using the analysis of covariance design.

Findings on Attitude Measures for Mathematics (MAS)

No significant differences were found for the mathematics 300 and mathematics 301 groups when the attitude score means were adjusted using the analysis of covariance design.

Findings on Attitude Measures for Mathematics of Experimental Groups (AMT)

No significant t values were found in the statistical analysis.

Findings on Attitude Measures Toward Teacher for Experimental Groups (TAT)

The paired t-test revealed no significant difference in attitude mean scores for attitude toward teacher scales.

Findings on Attitude Measures Toward Programmed Instruction for Experimental Groups (PIAT)

A significant t value was found for the experimental mathematics 300 group. It indicated a decline in attitude toward programmed instruction on the part of the students. No significant t value was obtained for the experimental mathematics 301 group.

CHAPTER V

SUMMARY AND CONCLUSION

SUMMARY OF DESIGN AND PROCEDURE

The basic purpose of this study was to examine the effectiveness of programmed instruction as an alternative teaching method and compare its results with the results achieved using traditional teaching methods. The questions raised in this regard were:

i) Is there a significant difference in the relative achievement of students exposed to programmed instruction as compared to those taught by the conventional classroom methods?

ii) Is there a significant difference in the attitude toward mathematics of students exposed to programmed instruction as compared to those taught by the conventional classroom method?

iii) Is there a significant difference in the attitude toward mathematics, the teacher and programmed instruction by secondary school mathematics students as a result of programmed instruction?

A review of literature indicated that experimentation had been done in this field. However many of the studies involved varying amounts of teacher help with the programmed materials. The literature concerning attitude toward mathematics indicated that much more research should be carried out in this area. Many of the studies concerning attitude and programmed instruction were not experimentally evaluated.

A sample of eighty-two Seven Oaks grade XII students, taking either mathematics 300 or 301, were involved in the study. Four intact classroom groups, two taking mathematics 300 and the other taking mathematics 301, were used. An experimental group which was randomly chosen out of each level was taught logarithms by the use of programmed materials. The remaining group from each level was designated as the control group and received instruction on the same topic using the conventional classroom methods.

The following independent variables were obtained--I.Q. scores, chronological age in months, sex, grade IX mathematics achievement scores, grade XII 1971 first term mathematics scores, socio-economic index score, and pre-experimental data on achievement and attitude. The two pre-experimental tests administered were the Logarithm Achievement Test (LAT) and the Mathematics Attitude Scale (MAS). The experimental groups were also given semantic differential attitude scales to measure their attitude toward mathematics (AMT), attitude toward teacher (TAT) and attitude toward programmed instruction (PIAT). These five tests were re-administered at the conclusion of the experimental period to determine possible differences in achievement and attitude.

To allow for the variation that existed among the independent variables and the intact classroom groups, the analysis of covariance statistical design was employed to test for significance between mean differences of achievement scores and attitude scores on initial and final administrations of the tests. The paired t-test was used to test for significant differences in the AMT, TAT, and PIAT attitude mean scores of the experimental groups on pre- and post-administrations.

SUMMARY OF FINDINGS

1. Achievement scores did not differ significantly between the treatment groups of each of the two different courses.

2. Method of instruction was found to have no significant effect on the attitude of students toward mathematics.

3. The attitude of the experimental groups toward mathematics and teacher did not differ significantly during the experimental period.

4. There was a decline in attitude toward programmed instruction expressed by the experimental groups with students taking university entrance mathematics showing a significant difference.

LIMITATIONS

As mentioned in Chapter I, generalizations from the findings of this study are limited by the non-random sampling and intact classroom groups, the population from which the sample was chosen, the independent variables that were used, and the mathematical topic chosen. The validity and reliability of the measuring instruments used must also be considered as a limitation. These limitations must be considered with respect to any conclusions based on the findings.

CONCLUSIONS

There was sufficient evidence to show that programmed instruction can be used as a suitable variation to teaching a specific unit in mathematics. Both experimental groups worked through the programmed unit efficiently. There was little time wasted in commencing their work each day. The student seemed to like the immediate feedback of answers provided. The method seemed to provide the student with a feeling of satisfaction or accomplishment.

The results of this study show no level of superiority for either the experimental program instructed groups or the traditionally taught groups in terms of student achievement and mathematical attitude. In both comparison groups, equivalent results were obtained by both groups for achievement and attitude. There was sufficient evidence to show that learning had taken place regardless of which method was used.

A comparison of the mean scores of the MAS revealed that the university entrance students exhibited a positive attitude toward mathematics whereas the general mathematics students exhibited a slightly less positive attitude toward the subject. This could probably be attributed to the fact that the people taking the general mathematics course were not taking it because they liked the subject but rather they were taking it for a credit only. The university entrance students were for the main part probably going to take university courses in mathematics whereas those taking the general mathematics course would likely pursue an arts orientated course.

Data from the initial and final administrations of the semantic differential scales revealed that the experimental groups' attitudes toward mathematics and teacher were positive and did not change during the experimental period. Their attitude did not change as a result of a change in the method of instruction.

Data obtained on the attitude scale for programmed instruction revealed a decrease in mean scores for both experimental groups. Typical high enthusiasm toward programmed instruction at the start of the experimental period may have resulted in high unrealistic initial test scores.

A significant decrease in attitude by the university entrance group's attitude could be attributed to the fact that after a three-week period they preferred to have teacher-class discussion to eliminate the boredom of programmed instruction. Discussion that took place with the students after the experiment ended indicated that they would prefer some integration of teacher-class discussion with the programmed instruction.

It could also be concluded that there is a time saving factor when programmed materials are used as a unit topic. Since each student worked at his own pace, he developed a sense of achievement more rapidly than under conventional teaching methods.

It could also be concluded that the teacher variable did not affect the experimental results. Each control group was instructed by the writer and each experimental group was supervised by the writer. No significant difference was noted in the achievement results, and attitude change toward mathematics did not occur. The attitude toward the teacher of the experimental groups did not change significantly over the time span of the experiment.

RECOMMENDATIONS FOR FURTHER RESEARCH

Replication of the present study extending the number of units taught using programmed materials should be of value. Perhaps some of the trends suggested would be substantiated or clearly negated by such a study. This may tend to limit bias toward certain topics taught by programmed materials.

The study could be repeated using a larger sample drawn from a larger population. It could possibly be used as a study involving enrichment materials for lower grade level, mathematically superior students or for remedial work with students who have been conventionally taught.

Research work could be done involving students completing matriculation requirements by the use of correspondence courses. Programmed units may be helpful for the more mature student.

Studies may also be set up to investigate the use of programmed materials integrated with teacher help. No studies have been done to reveal what ratio of programmed material and teacher help approaches the ideal learning situation.

The study could be replicated by allowing the students to work at the program in pairs or in some other size of group.

Another study could be set up using other variables such as reading levels and English achievement scores as independent variables.

The study indicated the need for research into the formation of mathematical attitudes and their effect upon achievement. The mathematics 300 groups tended to have more positive attitudes toward mathematics than did the mathematics 301 groups. It may be speculated that the mathematics 301 group would likely have achieved a higher achievement score if they had exhibited a more positive attitude toward mathematics. Different methods and techniques will have to be found to increase the student's acceptance of mathematics. Further research should also be undertaken to discover relationships between attitude toward mathematics and such factors as teacher personality, student personality, and the student's past experience with mathematics. This would probably indicate how student achievement in and attitude toward mathematics could be improved.

Studies may also be set up to investigate any change in attitude toward programmed instruction when varying amounts of teacher help is available.

Experimental work may also be done with intrinsic programming to see if any differences in results occur with a different technique of programming.

Further studies may be done to provide experimental evidence of the effectiveness of programmed instruction at certain grade levels for certain topics. This evidence may be used by the curriculum designer along with the teacher's subjective evaluation in the formation of new course supplementary materials.

Studies may be done to provide evidence as to whether programmed instruction seems better suited for male or female.

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APPENDIXES

APPENDIX A

Home Index Scale

HOME INDEX SCALE

Circle either YES or NO for your answer.

1. Does your family own a car? YES NO
2. Does your family have a garage or carport? YES NO
3. Did your father go to high school? YES NO
4. Did your mother go to high school? YES NO
5. Did your father go to university? YES NO
6. Did your mother go to university? YES NO
7. Is there a writing desk in your home? YES NO
8. Does your family have a hi-fi record player or stereo? YES NO
9. Does your family own a piano? YES NO
10. Does your family get a daily newspaper? YES NO
11. Do you have your own room at home? YES NO
12. Does your family own its own home? YES NO
13. Is there an encyclopedia in your home? YES NO
14. Does your family have more than 100 hard covered books? (e.g. 4 shelves 3 feet long) YES NO
15. Did your parents borrow any books from the library last year? YES NO
16. Does your family leave town each year for a holiday? YES NO
17. Do you belong to any club where you have to pay fees? YES NO
18. Does your mother belong to any clubs or organizations such as study, church, art or social clubs? YES NO
19. Does your family own a color TV set? YES NO
20. Have you ever had lessons in music, dancing, art, swimming, etc., outside of school? YES NO
21. Does your family have a tape recorder? YES NO

APPENDIX B

Mathematics Attitude Scale

MATHEMATICS ATTITUDE SCALE

GENERAL DIRECTIONS:

Please fill in the personal information on the answer sheet before reading the directions.

This questionnaire contains a set of statements about the mathematics you have been taking this year. You are to answer them according to the directions given. This questionnaire asks about your own attitudes and judgments. It is not a test, so it is very important that you answer the questions according to your own feelings and judgments. After you read each statement carefully, it is best to answer by giving your first impression or reaction and then go on to the next item. Remember this questionnaire is concerned with your attitudes, and it is important that you answer according to your own feelings. Feel free to answer honestly and frankly, as your answers will be kept confidential and will not be used by anyone in your school.

Below are a number of statements pupils have made about mathematics. Indicate how much you agree or disagree with each of these statements. Blacken the space on the answer sheet which corresponds with the letter which represents one of the following expressions:

SD - Strongly Disagree
D - Disagree
N - Neither Agree nor Disagree
A - Agree
SA - Strongly Agree

1. I am always under a terrific strain in a mathematics class.

2. I do not like mathematics, and it scares me to have to take it.

3. Mathematics is very interesting to me, and I enjoy mathematics courses.

4. Mathematics is fascinating and fun.

5. Mathematics makes me feel secure, and at the same time it is stimulating.

6. My mind goes blank, and I am unable to think clearly when working at mathematics.

7. I feel a sense of insecurity when attempting mathematics.

8. Mathematics makes me feel uncomfortable, restless, irritable and impatient.

9. The feeling that I have toward mathematics is a good feeling.

10. Mathematics makes me feel as though I'm lost in a jungle of letters and diagrams and can't find my way.

11. Mathematics is something which I enjoy a great deal.

12. When I hear the word, "mathematics", I have a feeling of dislike.

13. I approach mathematics with a feeling of hesitation, resulting from a fear of not being able to do it.

14. I really like mathematics.

15. Mathematics is a course in school which I have enjoyed studying this year.

16. It makes me nervous to even think about having to do a mathematics question.

17. I have not liked mathematics this year, and it is my most dreaded subject.

18. I am happier in a mathematics class than in any other class.

19. I feel at ease in mathematics, and I like it very much.

20. I feel a definite positive reaction to mathematics; it is enjoyable.

ANSWER SHEET

	Strongly Disagree	Disagree	Neither Agree or Disagree	Agree	Strongly Agree
1.	/ /	/ /	/ /	/ /	/ /
2.	/ /	/ /	/ /	/ /	/ /
3.	/ /	/ /	/ /	/ /	/ /
4.	/ /	/ /	/ /	/ /	/ /
5.	/ /	/ /	/ /	/ /	/ /
6.	/ /	/ /	/ /	/ /	/ /
7.	/ /	/ /	/ /	/ /	/ /
8.	/ /	/ /	/ /	/ /	/ /
9.	/ /	/ /	/ /	/ /	/ /
10.	/ /	/ /	/ /	/ /	/ /
11.	/ /	/ /	/ /	/ /	/ /
12.	/ /	/ /	/ /	/ /	/ /
13.	/ /	/ /	/ /	/ /	/ /
14.	/ /	/ /	/ /	/ /	/ /
15.	/ /	/ /	/ /	/ /	/ /
16.	/ /	/ /	/ /	/ /	/ /
17.	/ /	/ /	/ /	/ /	/ /
18.	/ /	/ /	/ /	/ /	/ /
19.	/ /	/ /	/ /	/ /	/ /
20.	/ /	/ /	/ /	/ /	/ /

APPENDIX C

Logarithm Achievement Test

LOGARITHM ACHIEVEMENT TEST

VALUES

1 1. Express in logarithmic notation: $243 = 3^5$

1 2. Express in exponential form: $\log_3 81 = 4$

1 3. Given log 34.7 = 1.54033. What is the log .00347?

1 4. Which part of the logarithm is always positive?

1 5. Determine the characteristic of the common log of the number 0.02345.

2 6. With the aid of logarithmic tables, express the following numbers as powers of ten:

(a) 0.08124 (b) 3474.7

2 7. With the aid of mathematical tables, express the number corresponding to the following logarithms:

(a) 1.78721 (b) $\bar{3}.76711$

6 8. Without the use of tables, find the value of the unknown in each of the following:

(a) $x = \log_5 125$ (b) $\log_8 x = 7/3$

(c) $\log_x 8 = -3$

9. Solve using logarithms:

4 (a)

$$\frac{10}{\sqrt[3]{.024796}}$$

6 (b)

$$\frac{(3.214)^4}{\sqrt{0.374} \times 45.23}$$

25

APPENDIX D

Semantic Differential Scales

Name ______________________ Room _____ Date ___________________

DIRECTIONS:

These sheets are given to find out how certain words make you feel. When you fill out the sheet, decide how the word at the top of the page makes you feel, and then mark the scales below the word. If the word at either end of the scale very strongly describes your feeling about the word at the top of the page, place your check-mark as shown below:

GOOD _X_/___/___ <___> ___/___/___ BAD
OR
GOOD ___/___/___ <___> ___/___/_X_ BAD

If the word at either end of the scale gives a fairly good description of the way you feel about the word at the top of the page (but you don't feel quite as strongly about the word) mark the scale as follows:

EASY ___/_X_/___ <___> ___/___/___ DIFFICULT
OR
EASY ___/___/___ <___> ___/_X_/___ DIFFICULT

If the word at either end of the scale only slightly describes your feeling about the word, mark the scale as follows:

TERRIFIC ___/___/_X_ <___> ___/___/___ TERRIBLE

OR

TERRIFIC ___/___/___ <___> _X_/___/___ TERRIBLE

If neither word seems to describe your feelings about the word at the top of the page, you should mark the scale as shown below:

SILLY ___/___/___ <_X_> ___/___/___ WISE

Try this sample:

FISHING

GOOD ___/___/___ <___> ___/___/___ BAD

INTERESTING ___/___/___ <___> ___/___/___ BORING

STUPID ___/___/___ <___> ___/___/___ SMART

DISLIKE ___/___/___ <___> ___/___/___ LIKE

IMPORTANT:

1. Be sure to check every scale.

2. Don't take too much time for any one item. We are interested in how you feel when you first look at the words.

3. Wait for further instructions.

Note to teacher - Answer any questions. Then start the test by saying, "Ready - Begin".

MATHEMATICS

GOOD	___/___/___ <___> ___/___/___	BAD
INTERESTING	___/___/___ <___> ___/___/___	BORING
EASY	___/___/___ <___> ___/___/___	DIFFICULT
MEANINGLESS	___/___/___ <___> ___/___/___	MEANINGFUL
TERRIFIC	___/___/___ <___> ___/___/___	TERRIBLE
SUCCESSFUL	___/___/___ <___> ___/___/___	UNSUCCESSFUL
ACTIVE	___/___/___ <___> ___/___/___	PASSIVE
DISLIKE	___/___/___ <___> ___/___/___	LIKE
FAST	___/___/___ <___> ___/___/___	SLOW
NEGATIVE	___/___/___ <___> ___/___/___	POSITIVE
SATISFACTORY	___/___/___ <___> ___/___/___	UNSATISFACTORY
STRONG	___/___/___ <___> ___/___/___	WEAK
USELESS	___/___/___ <___> ___/___/___	USEFUL
CLEAR	___/___/___ <___> ___/___/___	VAGUE
CONSERVATIVE	___/___/___ <___> ___/___/___	PROGRESSIVE

TEACHER

GOOD	___/___/___<___>___/___/___	BAD
INTERESTING	___/___/___<___>___/___/___	BORING
EASY	___/___/___<___>___/___/___	DIFFICULT
MEANINGLESS	___/___/___<___>___/___/___	MEANINGFUL
TERRIFIC	___/___/___<___>___/___/___	TERRIBLE
SUCCESSFUL	___/___/___<___>___/___/___	UNSUCCESSFUL
ACTIVE	___/___/___<___>___/___/___	PASSIVE
DISLIKE	___/___/___<___>___/___/___	LIKE
FAST	___/___/___<___>___/___/___	SLOW
NEGATIVE	___/___/___<___>___/___/___	POSITIVE
SATISFACTORY	___/___/___<___>___/___/___	UNSATISFACTORY
STRONG	___/___/___<___>___/___/___	WEAK
USELESS	___/___/___<___>___/___/___	USEFUL
CLEAR	___/___/___<___>___/___/___	VAGUE
CONSERVATIVE	___/___/___<___>___/___/___	PROGRESSIVE

PROGRAMMED INSTRUCTION

GOOD	___/___/___ <___> ___/___/___	BAD
INTERESTING	___/___/___ <___> ___/___/___	BORING
EASY	___/___/___ <___> ___/___/___	DIFFICULT
MEANINGLESS	___/___/___ <___> ___/___/___	MEANINGFUL
TERRIFIC	___/___/___ <___> ___/___/___	TERRIBLE
SUCCESSFUL	___/___/___ <___> ___/___/___	UNSUCCESSFUL
ACTIVE	___/___/___ <___> ___/___/___	PASSIVE
DISLIKE	___/___/___ <___> ___/___/___	LIKE
FAST	___/___/___ <___> ___/___/___	SLOW
NEGATIVE	___/___/___ <___> ___/___/___	POSITIVE
SATISFACTORY	___/___/___ <___> ___/___/___	UNSATISFACTORY
STRONG	___/___/___ <___> ___/___/___	WEAK
USELESS	___/___/___ <___> ___/___/___	USEFUL
CLEAR	___/___/___ <___> ___/___/___	VAGUE
CONSERVATIVE	___/___/___ <___> ___/___/___	PROGRESSIVE

APPENDIX E

Introduction to Logarithms

INTRODUCTION TO LOGARITHMS

One major objective of this section is to show that any number can be written in power-of-ten form and that calculations can be performed by converting numbers to power-of-ten form. Since the exponents of the power-of-ten form of most numbers is a decimal, we will begin by showing that decimal exponents make sense. And since calculations by the power-of-ten method are based on the laws of exponents, we will show that the laws of exponents can be used with decimal exponents.

A second major objective of this chapter is to discuss an alternate notation to power-of-ten notation. This alternate notation is called logarithmic notation. We will show how logarithms can be used to perform multiplications and divisions, and to find powers and roots. Though logarithms are used less frequently for calculations than they formerly were before the increasing use of the slide rule and desk calculators, logarithms are still the only way to calculate many powers and roots. These calculations are based on the laws of logarithms which parallel the laws of exponents used with powers-of-ten.

A thorough understanding of the meaning of logarithms and their use in calculations is necessary for basic science and technology.

notebook paper will do. When you answer a frame, run the mask down the page until you come to the horizontal line drawn across the page. With the mask in this position you can not see the printed response. Read the whole frame and do all the thinking and acting that the frame wants you to do. Write down your answer on a separate sheet of paper. Then move the mask down to uncover the printed responses to compare with your own response.

You may take it for granted that the printed response is accurate. You may make occasional errors. Your errors may run as high as one error for each ten frames. But if you check back and find out where the error is, and why you made it you will learn just as much as if you had made the right response in the first place.

Errors are made usually because learners try to go too fast, or read carelessly. Take your time and remember that there are no unimportant frames. You must understand the content of each frame before you can go ahead with the next one. You can not skip anything in the program.

Again, do not try to hurry. Go at a pace comfortable for you.

HOW TO LEARN WITH THIS PROGRAM

A program consists of a series of numbered learning situations called "frames". Each frame consists of a bit of information, or a problem or a challenge to you, or an incomplete statement, or whatever will help you to learn what you are supposed to learn at that point.

Somewhere in each frame there is a mark like this: __________. That shows the place at which you are supposed to do something. Maybe the __________ will ask you to solve a problem or part of a problem or to supply a correct word or phrase or to report on the kind of thinking that you are doing at that moment.

The __________ in every case calls for some sort of response from you. Whatever the program is about you learn by making responses not by reading the frames.

You are always ready to make the right response. You will be led to do so by what the program has taught you earlier or by certain hints and cues that are part of the wording of the frame. Whenever you see this mark __________ you already know all you need to know in order to make the right response. The correct responses are printed toward the right of the page in a separate column and under the bottom line of the frame. The best way to learn is to have a mask ready--just a half sheet of

TABLE OF CONTENTS

1-1 A REVIEW OF THE LAWS OF EXPONENTS

In this section, we will review the meaning of exponents and the laws of exponents and we will generalize their use to other bases besides "10".

1. Expressions like 10^2, 10^3, 10^{-1} and 10^{-4} are called base-exponent expressions. For example: In 10^2: "10" is called the base; "2" is called the exponent. Other numbers besides "10" can be the base in base-exponent expressions. For example: 2^3, 5^4, 3^{-1}, 6^{-5} are also called base-exponent expressions. In 5^4: (a) The base is ____. (b) The exponent is ____.	
2. Any base-exponent in which the exponent is a positive whole number stands for a multiplication in which the factors are identical. The base is the number used as each factor.	(a) 5 (b) 4

(Continued on following page.)

The exponent tells you how many times the base is used as a factor. Just as 10^3 means (10) (10) (10) so 6^4 means ________	
3. Write each multiplication below in base-exponent form: (a) (7) (7) (7) (7) ________ (b) (9) (9) (9) ________	(6)(6)(6)(6)
4. Any base-exponent expression with a positive whole number exponent can be converted to a regular number by performing the multiplication. For example: $2^3 = (2)(2)(2) = 8$ Convert each of the following to a regular number: (a) 2^4 = ________ (b) 3^3 = ________ (c) 2^5 = ________ (d) 4^3 = ________	(a) 7^4 (b) 9^3
5. Any base-exponent expression with a negative whole number exponent stands for a fraction in which	(a) 16 (b) 27 (c) 32 (d) 64

(Continued on following page.)

The numerator is "1"

The denominator is a base-exponent expression with a positive whole-number exponent.

That is $10^{-2} = \frac{1}{10^2}$ $\quad 4^{-3} = \frac{1}{4^3}$

Write each of the following as a fraction whose denominator is a base-exponent expression:

(a) 2^{-1} = ______ (b) 3^{-2} = ______

(c) 8^{-4} = ______

---

6. If a base-exponent has a negative whole number, it can be converted to a regular fraction in the following way:

$$2^{-3} = \frac{1}{2^3} = \frac{1}{(2)(2)(2)} = \frac{1}{8}$$

Convert each of the following to a regular fraction:

(a) 5^{-2} = ________

(b) 3^{-3} = ________

(a) $\frac{1}{2}$
(b) $\frac{1}{3^2}$ or $\frac{1}{9}$
(c) $\frac{1}{8^4}$ or $\frac{1}{4096}$

7. The law of exponents for multiplying powers of ten is: $$(10^a)\ (10^b) = 10^{a+b}$$ That is, to multiply two powers of ten, we simply add their exponents. Using this law complete each of the following: (a) $(10^5)\ (10^2) = 10$ ___ (b) $(10^{-3})\ (10^2) = 10$___ (c) $(10^{-4})\ (10^{-1}) = 10$ ___	(a) $\frac{1}{25}$ (b) $\frac{1}{27}$
8. The law of exponents for multiplication can be generalized to base-exponent expressions <u>with any base</u>. The general law is: $$\boxed{(b^x)\ (b)^y = b^{x+y}}$$ That is, to multiply two base-exponent expressions with the same base, we simply add their exponents. For example: $$(2^3)\ (2^2) = 2^5 \quad (3^{-1})\ (3^{-2}) = 3^{-3}$$ To show that this law makes sense, we have converted each multiplication	(a) 10^7 (b) 10^{-1} (c) 10^{-5}

(Continued on the following page.)

above to regular number form below.

$(2^3)\ (2^2) = 2^5$ $\qquad$ $(3^{-1})\ (3^{-2}) = 3^{-3}$

$(8)\ (4) = 32$ $\qquad$ $\frac{1}{3}\ \frac{1}{9} = \frac{1}{27}$

(a) Does $2^5 = 32$? ______

(b) Does $3^{-3} = \frac{1}{27}$? ______

---

9. The general law of exponents for multiplication applies <u>only if the bases are identical</u>.

For example:

$(2^3)\ (5^2)$ doesn't equal 2^5 or 5^5 or 10^5

Since $(2^3)\ (5^2) = (8)\ (25) = 200$

and $2^5 = 32$

$5^5 = 3125$

$10^5 = 100{,}000$

In these (1) Write the product in base-exponent form if the law applies,

(2) Write "doesn't apply" if the law does not apply

(a) $(8^3)\ (8^4) =$ ______

(b) $(5^{-4})\ (5^2) =$ ______

(c) $(7^3)\ (3^7) =$ ______

(d) $(4^{-3})\ (3^{-3}) =$ ______

(a) yes

(b) yes

10. The law of exponents for multiplication can be extended to multiplications with more than two factors. As usual, we simply add the exponents. Complete the following:

(a) $(10^3)\ (10^2)\ (10^1) =$ __________

(b) $(2^4)\ (2^{-5})\ (2^{-2}) =$ __________

(c) $(5^3)\ (3^2)\ (7^4) =$ __________

(a) 8^7

(b) 5^{-2}

(c) doesn't apply

(d) doesn't apply

---

11. The law of exponents for dividing powers of ten is

$$\frac{10^a}{10^b} = 10^{a-b}$$

That is to divide two powers of ten, we simply subtract the exponent of the denominator from the exponent of the numerator.

Using this law, complete each of the following:

(a) $\frac{10^5}{10^2} =$ __________

(b) $\frac{10^{-3}}{10^4} =$ __________

(c) $\frac{10^{-5}}{10^{-2}} =$ __________

(a) 10^6

(b) 2^{-3}

(c) doesn't apply as bases are different

12. The law of exponents for division can be generalized to base-exponent expressions with any base. The general law is:

$$\boxed{\frac{b^x}{b^y} = b^{x-y}}$$

That is, to divide two base-exponent expressions with the same base, we subtract the exponent of the denominator from the exponent of the numerator.

For example:

$$\frac{3^4}{3^2} = 3^{4-2} = 3^2$$

$$\frac{2^{-1}}{2^{-3}} = 2^{-1-(-3)} = 2^2$$

The general law of exponents for division also applies only if the bases are identical. Use the general law to complete each of these, if it apply.

(a) $\frac{3^8}{3^3} =$ ____________

(b) $\frac{13^1}{12^2} =$ ____________

(c) $\frac{6^4}{4^3} =$ ____________

(d) $\frac{7^4}{7^{-3}} =$ ____________

(a) 10^3

(b) 10^{-7}

(c) 10^{-3}

13. In 10^2, the exponent "2" tells you to "take the quantity 10 as a factor two times". That is

$$10^2 = (10)\ (10) = 100$$

Similarly $(10^3)^2 = (10^3)\ (10^3) = 10^6$

This operation is called "raising a quantity to the second power".

Do these:

(a) $(10^5)^2 = (\quad)\ (\quad) =$ ________

(b) $(10^{-2})^2 = (\quad)\ (\quad) =$ ________

(a) 3^5

(b) doesn't apply

(c) doesn't apply

(d) 7^7

---

14. In 10^3, the exponent "3" tells you to "take the quantity 10 as a factor three times".

That is $10^3 = (10)(10)(10) = 1000$

This operation is called "raising a quantity to the <u>third</u> <u>power</u>.

Do these:

(a) $(10^4)^3 = (\quad)(\quad)(\quad) =$ ________

(b) $(10^{-1})^3 = (\quad)(\quad)(\quad) =$ ________

(a) $(10^5)(10^5) = 10^{10}$

(b) $(10^{-2})(10^{-2}) = 10^{-4}$

15. You have learned these two laws of exponents for raising powers-of-ten to the second or third power.

$$(10^a)^2 = 10^{2a} \qquad (10^a)^3 = 10^{3a}$$

That is, to raise a power of ten to the <u>second</u> or <u>third</u> power, we simply multiply its exponent by "2" or "3".

Using these laws, complete these:

(a) $(10^{-4})^2 =$ ________

(b) $(10^5)^3 =$ ________

(a) $10^4 \cdot 10^4 \cdot 10^4 = 10^{12}$

(b) $10^{-1} \cdot 10^{-1} \cdot 10^{-1} = 10^{-3}$

---

16. The general law for raising powers-of-ten to any power is:

$$(10^a)^b = 10^{ab}$$

That is, to raise a power-of-ten to a power, we simply multiply its exponent by that power.

For example:

$$(10^2)^4 = 10^8 \qquad (10^{-1})^5 = 10^{-5}$$

Using the law above, complete:

(a) $(10^{-2})^4 =$ ________

(b) $(10^2)^3 =$ ________

(a) 10^{-8}

(b) 10^{15}

17. Base-exponent expressions with bases other than 10 can also be raised to powers.

For example:

$(3^4)^2$ means "raise 3^4" to the second power

(a) $(2^5)^3$ means "raise 2^5" to the ________ power

(b) $(5^{-1})^7$ means "raise 5^{-1}" to the ________ power

(a) 10^{-8}

(b) 10^6

---

18. The law of exponents for raising a power-of-ten to any power can be generalized to base-exponent expressions with any base. The general law is:

$$\boxed{(b^x)^y = b^{xy}}$$

That is to raise any base-exponent expression to a power, we simply multiply its exponent by that power.

For example:

$(3^4)^2 = 3^8 \qquad (2^5)^3 = 2^{15} \qquad (5^{-1})^7 = 5^{-7}$

Using these, complete the following:

(a) $(5^3)^6 =$ ______ (b) $(2^{-4})^3 =$ ______

(a) third

(b) seventh

19. Square roots and cube roots must also be mentioned. There is some terminology about roots which you must know. In $\sqrt[3]{27}$: (1) The small "3" is called the <u>index</u>. (2) The symbol $\sqrt[3]{}$ is called the <u>radical sign</u>. (3) $\sqrt[3]{27}$ is called the <u>radical</u>. In $\sqrt[3]{81}$ (a) "3" is called the ______ (b) $\sqrt[3]{}$ is called the ______ (c) $\sqrt[3]{81}$ is the ______	(a) 5^{18} (b) 2^{-12}
20. In $\sqrt{64}$, there is no written index. When a radical is without an index, the operation called for is <u>square root</u>. Therefore, $\sqrt{64}$ means "the square root of 64." $\sqrt{64}$ means "find one of two equal factors whose product is 64." Since (8) (8) = 64, 8 is the number which when taken as a factor two times, equals the product 64. Therefore $\sqrt{64}$ = 8. Since (12) (12) = 144, $\sqrt{144}$ = ______	(a) index (b) radical sign (c) radical

21. $\sqrt[3]{27}$ means "the cube root of 27". 12

$\sqrt[3]{27}$ means "find one of <u>three</u> equal factors whose product is 27".

Since (3) (3) (3) = 27 $\sqrt[3]{27}$ = _____

---

22. There are other terms for "square" root and "cube" root. <u>Square</u> root is sometimes called <u>second</u> root. <u>Cube</u> root is sometimes called <u>third</u> root. We can also find roots higher than the <u>second</u> root or <u>third</u> root. 3

Here are some examples:

$\sqrt[4]{81}$ asks you to find the fourth root. Notice the index of the fourth root is <u>4</u>. $\sqrt[4]{81}$ means "find one of 4 equal factors whose product is 81".

(a) Since (3)(3)(3)(3) = 81

$\sqrt[4]{81}$ = __________

(b) Since (2)(2)(2)(2)(2) = 32

$\sqrt[5]{32}$ = __________

23. Two laws of exponents for finding <u>square</u> roots and <u>cube</u> roots of powers-of-ten:

$$\sqrt{10^a} = 10^{a/2} \qquad \sqrt[3]{10^a} = 10^{a/3}$$

Using these laws, complete:

(a) $\sqrt{10^6}$ = _____ (b) $\sqrt{10^{-8}}$ = _____

(c) $\sqrt[3]{10^9}$ = _____

(a) 3

(b) 2

---

24. We can find roots of powers of ten which are higher than <u>second</u> or <u>third</u> roots.

$\sqrt[4]{10^8}$ means "find one of <u>four</u> equal factors whose product is 10^8".

Since $(10^2)(10^2)(10^2)(10^2) = 10^8$

$$\sqrt[4]{10^8} = 10^2$$

$\sqrt[5]{10^{-15}}$ means "find one of <u>five</u> equal factors whose product is 10^{-15}".

Since $(10^{-3})(10^{-3})(10^{-3})(10^{-3})(10^{-3})$

$$= 10^{-15}$$

$$\sqrt[5]{10^{-15}} = \_\_\_\_\_\_$$

(a) 10^3

(b) 10^{-4}

(c) 10^3

25. The general law for finding roots of powers-of-ten is $$\sqrt[b]{10^a} = 10^{a/b}$$ That is, to find a root of a power-of-ten, we simply divide its exponent by that root. For example: $\sqrt[4]{10^8} = 10^{8/4} = 10^2$ $\sqrt[5]{10^{-15}} = 10^{-15/5} = 10^{-3}$ Complete: (a) $\sqrt[3]{10^{-12}} =$ ______ (b) $\sqrt[9]{10^{27}} =$ ______	10^{-3}
26. We can also find roots of base-exponent expressions with other bases besides ten. For example: (a) $\sqrt{2^6}$ means "find one of <u>two</u> equal factors whose product is 2^6". (b) $\sqrt[4]{5^{-8}}$ means "find one of <u>four</u> equal factors whose product is 5^{-8}". Since $(5^{-2})(5^{-2})(5^{-2})(5^{-2}) = 5^{-8}$, $\sqrt[4]{5^{-8}} =$ ______	(a) 10^{-4} (b) 10^3

27. The general law for finding roots of base-exponent expressions is:

$$\boxed{\sqrt[y]{b^x} = b^{x/y}}$$

That is, to find the root of a base-exponent expression, we simply divide its exponent by the root.

eg. $\sqrt{2^6} = 2^{6/2} = 2^3$

$$\sqrt[4]{5^{-8}} = 5^{-8/4} = 5^{-2}$$

Find:

(a) $\sqrt[3]{5^{15}} =$ _____ (b) $\sqrt[6]{9^{-18}} =$ _____

(a) 2^3

(b) 5^{-2}

---

28. Expressions like 9^4, 7^8, and 5^{-4} are in base-exponent form. Expressions in base-exponent form are generally referred to as being in <u>exponential</u> form.

Convert the following radical forms to exponential form:

(a) $\sqrt{6^{14}} =$ _____ (c) $\sqrt[3]{5^{12}} =$ _____

(b) $\sqrt[4]{10^{-8}} =$ _____ (d) $\sqrt[7]{9^{-14}} =$ _____

(a) 5^5

(b) 9^{-3}

29. We have seen the following four general laws of exponents.

I. $b^x \cdot b^y = b^{x+y}$

II. $\frac{b^x}{b^y} = b^{x-y}$

III. $(b^x)^y = b^{xy}$

IV. $\sqrt[y]{b^x} = b^{x/y}$

These four laws can also be used with base exponent expressions <u>in which the base is a letter</u>.

(a) $(x^5)(x^{-3}) =$ ______ (b) $\frac{y^{10}}{y^4} =$ ______

(c) $(m^4)^3 =$ ______

(d) $\sqrt[4]{d^{-8}} =$ ______

(a) 6^7
(b) 10^{-2}
(c) 5^4
(d) 9^{-2}

(a) x^2
(b) y^6
(c) m^{12}
(d) d^{-2}

SELF TEST I

Frames 1 - 29

Work these problems using the laws of exponents.

Write each answer in exponential form:

1. $(13)^4(13)^2 =$ ______ 2. $\sqrt[3]{4^9} =$ ______

3. $(9^6)^7 =$ ______ 4. $\frac{4^9}{3^4} =$ ______

5. $\sqrt{14^{10}} =$ ______ 6. $\frac{8^5}{8^4} =$ ______

7. $\sqrt[5]{7^{30}} =$ ______ 8. $(5^3)\;5 =$ ______

9 $(r^{-2})^3 =$ ______

Answers

1. 13^6 2. 4^3 3. 9^{42}

4. no law applies because bases are different

5. 14^5 6. 8^1 7. 7^6

8. 5^{15} 9. r^{-6}

1-2 THE MEANING OF FRACTIONAL AND DECIMAL EXPONENTS

This section deals with exponents which are fractional or in decimal form.

---

30. The general law for converting from a radical to exponential form is

$$\sqrt[y]{b^x} = b^{x/y}$$

In $\sqrt{8^6}$, $= 8^{6/2} = 8^3$

In $\sqrt{9} = 9^{1/2}$

Though $9^{1/2}$ has a fractional exponent, it does stand for a regular number.

That is:

Since $9^{1/2} = \sqrt{9} = 3$

$8^{1/3} = \sqrt[3]{8} =$ ________

---

31. When converting from radical form to exponential form, in many cases, the exponent of the exponential form is a fraction. Complete each of the following conversions to exponential form: 2

(a) $\sqrt{25}$ (b) $\sqrt[3]{27}$

32. Convert to radical form:

(a) $23^{3/4}$ = ______ (b) $5^{11/6}$ = ______

(a) $25^{1/2}$
(b) $27^{1/3}$

---

33. Just as any radical form can be converted to exponential form, any exponential form with a fractional exponent can be converted to an exponential form with a "decimal" exponent.

For example: $\sqrt{4} = 4^{1/2} = 4^{0.5}$

(a) Write $\sqrt[4]{16}$ in exponential form with a fractional exponent = ______

(b) Write $\sqrt[4]{16}$ in exponential form with a decimal exponent = ______

(a) $\sqrt[4]{23^3}$
(b) $\sqrt[6]{25^{11}}$

---

34. (a) Write $\sqrt[4]{12^5}$ in exponential form with a fractional exponent = ______

(b) Write $\sqrt[4]{12^5}$ in exponential form with a decimal exponent = ______

(a) $16^{1/4}$
(b) $16^{0.25}$

---

35. $\sqrt[3]{9} = 9^{1/3} = 9^{0.33333}$

$\sqrt[3]{9^2} = 9^{2/3} = 9^{0.66667}$

(a) $12^{5/4}$
(b) $12^{1.25}$

(Continued on the following page.)

In the above conversions from fractional to decimal exponents, the decimal exponent was carried to five places beyond the decimal point. In this course we expect you to write all decimal exponents with five digits after the decimal point. They will then be in the required form for the tables you will soon be using.

Should you write $4^{1/2}$ as $4^{0.5}$ or as $4^{0.50000}$?

$4^{0.50000}$

36. Convert the following to exponential form with decimal exponents:

(a) $4^{3/4} =$ _____ (b) $10^{5/3} =$ _____

(c) $\sqrt[5]{11^4} = 11^{4/5} =$ _____

(d) $\sqrt[5]{8^7} =$ _____

37. Convert the following exponential forms with decimal exponents to radical form.

(a) $6^{0.40000} = 6^{2/5} =$ _____

(b) $9^{0.75000} = 9^{3/4} =$ _____

(c) $10^{1.25000} =$ _____

(a) $4^{0.75000}$
(b) $10^{1.66667}$
(c) $11^{0.80000}$
(d) $8^{1.40000}$

---

38. The same procedure can be used to convert a radical to an exponential and vice versa when letters are involved. For example:

$\sqrt{x} = x^{1/2}$ $\quad$ $p^{3/4} = \sqrt[4]{p^3}$

Convert each radical to an exponential and each exponential to a radical:

(a) $y^{1/2} =$ _____ (b) $\sqrt[5]{x^4} =$ _____

(c) $\sqrt[3]{d} =$ _____ (d) $m^{7/4} =$ _____

(a) $\sqrt[5]{6^2}$
(b) $\sqrt[4]{9^3}$
(c) $\sqrt[4]{10^5}$

(a) $\sqrt{y}$
(b) $x^{4/5}$
(c) $d^{1/3}$
(d) $\sqrt[4]{m^7}$

1-3 LAWS OF EXPONENTS WITH FRACTIONAL AND DECIMAL EXPONENTS

In this section, we will briefly show that the laws of exponents hold for both fractional and decimal exponents. To do so, we will give only one numerical example of each type. Some practise frames in using the laws with fractional and decimal exponents are included.

---

39. The law of exponents for multiplication applies to both fractional and decimal exponents. Here is an example:

Using the following facts:

$8^{0.33333} = 8^{1/3} = \sqrt[3]{8} = 2$

$8^{0.66667} = 8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

We will perform the multiplication $2 \times 4 = 8$ with decimal and fractional exponents:

$$2 \quad \times \quad 4 \quad = \quad 8$$

1. $8^{0.33333} \cdot 8^{0.66667} = 8^{1.00000}$

   or 8^1 or 8

2. $8^{1/3} \cdot 8^{2/3} = 8^{1/3 + 2/3}$

   or 8^1 or 8

Notice that the addition of the decimal and fractional exponents gave the correct product.

40. Using the law of exponents for multiplication, write the following product in exponential form: (a) $10^{2/5} \times 10^{1/2} =$ ______ (b) $7^{0.33333} \cdot 7^{2} =$ ______ (c) $5^{0.40000} \cdot 5^{1.12500} =$ ______	
41. The <u>law</u> of <u>exponents for division</u> also holds for both fractional and decimal exponents. Here is an example using these facts:	(a) $10^{9/10}$ (b) $7^{2.33333}$ (c) $5^{1.52500}$

$$81^{0.25000} = 81^{1/4} = \sqrt[4]{81} = 3$$

$$81^{0.50000} = 81^{1/2} = \sqrt{81} = 9$$

We will perform the division $\frac{9}{3} = 3$ with decimal and fractional exponents:

$$\frac{9}{3} = 3$$

1. $\dfrac{81^{0.50000}}{81^{0.25000}} = 81^{0.50000 - 0.25000} = 3$

2. $\dfrac{81^{1/2}}{81^{1/4}} = 81^{1/2 - 1/4} = 81^{1/4} = 3$

Notice that the subtraction of the decimal and fractional exponents gave use to the correct quotient.

42. Using the law of exponents for division, write each quotient in exponential form.

(a) $\frac{4^{1/3}}{4^{1/6}}$ = ________

(b) $\frac{15^{0.66667}}{15^{0.50000}}$ = ________

(a) $4^{1/6}$

(b) $15^{0.16667}$

43. The law of exponents for raising to a power also holds for fractional or decimal exponents. Here is an example using these facts:

$9^{0.50000} = 9^{1/2} = \sqrt{9} = 3$

$9^{1.50000} = 9^{3/2} = \sqrt{9^3} = \sqrt{729} = 27$

We will perform the problem $3^3 = 27$ with decimal and fractional exponents.

$$3^3 = 27$$

1. $(9.^{0.50000})^3 = 9^{(3)(0.50000)} = 9^{1.50000} = 27$

2. $(9^{1/2})^3 = 9^{3/2} = 27$

Notice that we got the same answer by raising the exponentials with decimal and fractional exponents to the third power.

44. Raise each of these to the indicated power, and write the answer in exponential form:

(a) $(3^{3/4})^5$ = ______

(b) $(9^{0.80000})^2$ = ______

(a) $3^{15/4}$

(b) $9^{1.60000}$

45. The law of exponents for finding roots also holds for fractional and decimal exponents. Here is an example using these facts:

$$64^{0.16667} = 64^{1/6} = \sqrt[6]{64} = 2$$

$$64^{0.50000} = 64^{1/2} = \sqrt{64} = 8$$

We will perform the problem $\sqrt[3]{8} = 2$ with decimal and fractional exponents:

$$\sqrt[3]{8} = 2$$

1. $\sqrt[3]{64^{0.50000}} = 64^{\frac{0.50000}{3}}$
$= 64^{0.16667} = 2$

2. $\sqrt[3]{64^{1/2}} = 64^{\frac{1/2}{3}} = 64^{(1/2)(1/3)}$
$= 64^{1/6} = 2$

46. Using the law of exponents for roots, write each root in exponential form:

(a) $\sqrt[4]{3^{0.75000}} =$ ______

(b) $\sqrt[4]{5^{1/4}} =$ ______

---

47. Using the appropriate laws of exponents, write each answer in exponential form:

(a) $5^{0.33333} \times 5^{1.66667} =$ ______

(b) $\sqrt[4]{10^{3.75000}} =$ ______

(a) $3^{0.18750}$

(b) $5^{1/16}$

---

48. The laws of exponents also hold for fractional and decimal exponents whose base is a letter. For example:

$(x^{1/3})(x^{1/3}) = x^{2/3}$

$(t^{1/2})^5 = t^{5/2}$

$\dfrac{y^{1.50000}}{y^{0.75000}} = y^{0.75000}$

$\sqrt[4]{m^{2.50000}} = m^{0.62500}$

Using the appropriate laws of exponents, write each answer in exponential form:

(a) $x^{0.50000} \cdot x^{0.33333} =$ ______

(b) $\dfrac{d^{4/5}}{d^{1/5}} =$ ______ (c) $(t^{0.75000})^5 =$ ______

(a) 5^2

(b) $10^{0.93750}$

(a) $x^{0.83333}$

(b) $d^{3/5}$

(c) $t^{3.75000}$

SELF TEST III

Frames 39 - 48

Using the laws of exponents, simplify each of the following:

1. $\dfrac{10^{0.66667}}{10^{0.42500}} = 10^{?}$

2. $(7^{0.60000})^3 = 7^{?}$

3. $\sqrt{12^{3/8}} = 12^{?}$

4. $2^{4.33333} \times 2^{0.500000} = 2^{?}$

5. $\sqrt[7]{6^{0.88550}} = 6^{?}$

6. $\sqrt{10^{0.80000}} = 10^{?}$

Answers

1. $10^{0.24167}$
2. $7^{1.80000}$
3. $12^{3/16}$
4. $2^{4.83333}$
5. $6^{0.12650}$
6. $10^{0.40000}$

1-4 THE MEANING OF THE TERM LOGARITHM

In the preceding section we were dealing with the laws of exponents. We were dealing with exponential expressions and the laws of exponents. In this section we shall show the relationship between exponential form and logarithmic form.

49. In the general expression $b^x = N$, b represents the base and x represents the exponent. $b^x = N$ may also be read "The logarithm of N to the base b is x." In other words, a logarithm is simply another name for an exponent. Its abbreviation is "log". Thus since $5^2 = 25$, the logarithm of 25 to the base ________ is ________. Similarly since $3^4 = 81$, the logarithm of 81 to the ________ 3 is ________.	
50. Convert to logarithmic form. (a) $2^6 = 64$ (b) $3^5 = 243$ (c) $10^3 = 1000$ (d) $10^{-3} = .001$	5, 2 base, 4

51. If $\log_2 32 = 5$, by using the definition of logarithm, it is possible to put it into exponential form. Since $\log_2 32 = 5$ is read the log of 32 to the base 2 is 5. In exponential form it means $2^5 = 32$

Convert the following to equivalent exponential form.

(a) $\log_5 125 = 3$ (b) $\log_{10} 1000 = 3$

(c) $\log_3 81 = 4$ (d) $\log_{10} .001 = -3$

(a) $\log_2 64=6$

(b) $\log_3 243=5$

(c) $\log_{10} 1001=3$

(d) $\log_{10} .001=-3$

---

52. To generalize

$b^x=N$	$\log_b N=x$
Exponential Form	Logarithmic Form

In this program, the base will be 10, unless otherwise stated.

When $10^3 = 1000$, it is written as $\log_{10} 1000 = 3$. It will be commonly written as log 1000 = 3 without writing 10 in as the base.

(a) $5^3=125$

(b) $10^3=1000$

(c) $3^4=81$

(d) $10^{-3}=.001$

SELF TEST IV

Frames 49 - 52

1. Convert to logarithmic form.

   1. $10^2 = 100$ 2. $2^4 = 16$

2. Convert to exponential form.

   3. $\log_9 81 = 2$ 4. $\log 0.1 = -1$

Answers

1. $\log_{10} 100 = 2$ 2. $\log_2 16 = 4$
3. $9^2 = 81$ 4. $10^{-1} = 0.1$

1-5 LOGARITHMIC FORM OF NUMBERS GREATER THAN 1

One of the useful facts of mathematics is: Any positive number can be written in power-of-ten form. For example:

$5.83 = 10^{0.76567}$ $\qquad$ $37.6 = 10^{1.57519}$

$812 = 10^{2.90956}$ $\qquad$ $6380 = 10^{3.80482}$

Notice these two features of the exponential forms above:

(1) "10" is the base in all of them

(2) The exponents are decimals.

The purpose of this section is to teach you to write numbers greater than one in logarithmic form.

---

53. The numerical values of some powers-of-ten which have decimal exponents can be computed. In the brief table at the right, the power-of-ten form of some numbers between 1 and 10 is shown. Referring to the table, it is apparent that since the number

$10.0 = 10^{1.00000}$
$4.64 = 10^{0.66652}$
$3.16 = 10^{0.49969}$
$2.15 = 10^{0.33244}$
$1.00 = 10^{0.00000}$

(Continued on the following page.)

"4" lies between 3.16 and 4.64, its exponent in power-of-ten form must lie between 0.49969 and 0.66652.

Based on this table, in power-of-ten form:

(a) "3" would have an exponent which lies between 0.33244 and ________

(b) "7" would have an exponent which lies between ________ and 1.00000.

(c) "1.75" would have an exponent which lies between ________ and ________

(d) "4.5" would have an exponent which lies between ________ and ________

(e) Any number between 1 and 10 would have an exponent between ________ and ________.

---

54. In frame 49, we used an incomplete power-of-ten table. In the Mathematical Tables that you have been given, you will find a table

(a) 0.49969

(b) 0.66652

(Continued on the following page.)

entitled "Logarithms". Look at the table. Notice these points:

1. The table covers both pages.
2. There is an "N" in the upper left hand corner. "N" stands for number.
   (a) On the left side, there is an "N" column.
   (b) Across the top, there is an "N" row.
3. All the five-digit numbers in the body of the table are exponents. Their base is 10.

Using the table, the procedure for writing 1.32 in power-of-ten form is:

(1) Since the first two digits of 1.32 are"13" encircle the "13" in the N-column and draw a horizontal arrow to the right.

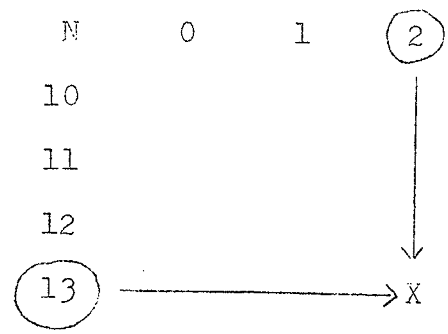
(2) Since the last digit is "2", encircle the "2" in the N-row and draw a vertical arrow downwards.

(c)
0.00000 &
0.33244

(d)
0.49969 &
0.66652

(e)
0.00000 &
1.00000

(Continued on the following page.)



(3) Now look at the actual table. Record the number "x" which appears where the two arrows intersect. It is ___________.

---

55. The "12057" recorded in the table is really the decimal exponent 0.12057. For convenience, the decimal point is not shown in the table. Therefore, in power-of-ten: $1.32 = 10^{0.12057}$. Using the same procedure, write each of these in logarithmic form:

(a) 1.31 = __________

(b) 1.30 = __________

12057

56. From the table, verify that $2.47 = 10^{0.39270}$. Use the table to write each of these in power-of-ten form. (a) $4.13 =$ ______ (d) $7.02 =$ ______ (b) $9.81 =$ ______ (e) $5.19 =$ ______ (c) $1.78 =$ ______ (f) $8.38 =$ ______	(a) $10^{0.11727}$ (b) $10^{0.11394}$
57. To find the exponent for a two digit number such as 2.5, simply add a "0" in the hundredths place and look up 2.50. Therefore $2.5 =$ ________	(a) $10^{0.61595}$ (b) $10^{0.99167}$ (c) $10^{0.25042}$ (d) $10^{0.84634}$ (e) $10^{0.71517}$ (f) $10^{0.92324}$
58. To find the exponent for a one-digit number such as 7, add "0's" in the tenths and hundredths places and look up 7.00 Therefore $7 =$ ________	$10^{0.39794}$

(Continued on the following page.)

Convert to power-of-ten form:

(a) 6.9 = ________

(b) 4.2 = ________

(c) 8 = ________

---

59. We can use the table to find the exponent for a four digit number. At the right hand side of the log table is a section labelled Mean Differences with columns headed 1 through 9. We will use the Mean Differences table to find the value for the fourth digit in our real number.

For example: convert 4.698 to power-of-ten form.

Go down the column to 46 and across to the column under 9. We get 67117. Now keep going across into the Mean Differences table to the column under 8 because 8 is our fourth digit. The number in the row

$10^{0.84510}$

(a) $10^{0.83885}$

(b) $10^{0.62325}$

(c) $10^{0.90309}$

(Continued on the following page)

we are concerned with in the 8 column is 74. We then add this to the reading for the first three digits. Thus 67117 + 74 = ________ Therefore 4.698 = _________	
60. Convert the following to power-of-ten form: 1. 6.197 4. 1.155 2. 7.023 5. 6.007 3. 1.145	67191 $10^{0.67191}$
61. Since our tables are five place tables, then it is possible to calculate the exponent for 5 digits. For example: 4.6984 To do this conversion you follow the steps in frame 59. Look up 4 6 9 = 67117 Then 8 in the mean differences 74 Then you record the reading for 4 in the mean differences column and take 1/10 of it.	1. $10^{0.79219}$ 2. $10^{0.84653}$ 3. $10^{0.05883}$ 4. $10^{0.06255}$ 5. $10^{0.77865}$

(Continued on the following page.)

Therefore $\frac{1}{10} \times 37$ is 3.7. The answer is rounded off to the nearest whole number and added on to the other numbers.

Therefore 67117 + 74 + 4 = 67195

Therefore 4.6984 = ________

Similarly, to convert 6.8179

681	=	83315
mean difference for 7	=	44
1/10 of the mean diff. for 9	=	6
		.83365

Therefore 6.8179 = ________

	$10^{0.67195}$
	$10^{0.83365}$

62. Convert to power-of-ten:

1. 4.3829
2. 6.5747
3. 8.6426
4. 2.2727
5. 5.6293

63. In the preceding frames, you have converted numbers to power-of-ten. In other words, you have been finding the logarithms of numbers. If for instance $6.8179 = 10^{0.83365}$ is in exponential form, in log form it is written $\log_{10}6.8179 = 0.83365$ with the base 10 to be understood. When you use the logarithm tables you are changing the given number into logarithmic form.

To express 7.1882 in logarithmic form look up: 718 = 85612

the 8 under the mean diff.= 49

Then record the reading for 2 in the mean differences, take 1/10 of it, and round off to the nearest whole number. Therefore

$$\begin{array}{r} 85612 \\ 49 \\ 1 \\ \hline 85662 \end{array}$$

Therefore log 7.1882 = 0.85662

Express the following numbers in

1. $10^{0.64176}$

2. $10^{0.81788}$

3. $10^{0.93664}$

4. $10^{0.35656}$

5. $10^{0.75045}$

(Continued on the following page.)

logarithmic form using the tables:

1. 3.40
2. 6.192
3. 7.8746

---

64. Consider the following table.

Number	Power	Log
1,000,000	$10^{6.00000}$	6.00000
100,000	$10^{5.00000}$	5.00000
10,000	$10^{4.00000}$	4.00000
1,000	$10^{3.00000}$	3.00000
100	$10^{2.00000}$	2.00000
10	$10^{1.00000}$	1.00000
1	$10^{0.00000}$	0.00000

You can see that we have been examining numbers that are between 1 and 10. Now from the tables find log 4.5 = ______

1. 0.53148
2. 0.79183
3. 0.89622

---

65. Because logarithms are exponents, we can write this logarithm as an exponent of the power of 10

0.65321

(Continued on the following page.)

Example: $10^{0.65321}$

$10^{0.65321}$ expresses the number 4.5 as a power of 10 just as 10^2 expresses the number 100 as a power of 10.

No.	Power	Log
100	$10^{2.00000}$	2.00000
4.5	$10^{0.65321}$	0.65321

From this we can work out the log of 45. We know that 45 is ten times bigger than 4.5.

Therefore $45 = 10 \times 4.5$

$$= 10^{1.00000} \times 10^{0.65321}$$

$$= 10^{1.65321}$$

Therefore log 45 = 1.65321

Notice $100 = 10^{2.00000}$

$45 = 10^{1.65321}$

$10 = 10^{1.00000}$

Therefore any number between 10 and 100 will have a logarithm between ______ and ______.

66.	So any number that lies between 100 and 1000 (eg. 450) lies between 10^2 and 10^3 and so must have a log between ________ and ________.	1.00000 and 2.00000
67.	Since $10^{2.65321}$ lies between 10^2 and 10^3, $10^{2.65321}$ must stand for a number between ________ and ________.	2.00000 and 3.00000
68.	Since 450 lies between 10^2 and 10^3 it must have a log between ________ and ________. Since 450 is 100 times bigger than 4.5, $450 = 100 \times 4.5$ $= 10^2 \times 4.5$ $= 10^2 \times 10^{0.65321}$ $= 10^{2.65321}$	100 and 1000

69. So we have

Number	Power	Log
4.5	$10^{0.65321}$	0.65321
45.0	$10^{1.65321}$	1.65321
450.0	$10^{2.65321}$	2.65321
4500.0	$10^{3.65321}$	3.65321

Notice that the decimal part of the exponent is always the same but the whole number part varies. These two parts of a logarithm have special names:

The whole number part is called the characteristic.

The decimal part is called the mantissa.

Therefore the characteristic of any number between 1 and 10 is "0".

eg. $1.73 \doteq 10^{0.23805}$

Its mantissa is .23805.

What is characteristic of:

(a) $10^{4.47991}$ ______

(b) $\log N = 1.76554$ (c) $10^{0.53681}$

2.00000 and 3.00000

70. The following rule should be learned to find the characteristics of logarithms. <u>If the number is greater than one, its characteristic will be one less than the number of digits on the left hand side of the decimal.</u> Give the characteristics of the following: eg. 21.76 = 1 14,470.7 = 4 (a) 276 = ______ (b) 21.9 = ______ (c) 2004.5 = ______ (d) 326.7 = ______ (e) 12.245 = ______ (f) 7.88 = ______	(a) 4 (b) 1 (c) 0
71. <u>To find the logarithm of a number, we must first write down the characteristic.</u> Characteristics are never found in the tables. They must be written down first. We then find the mantissa in the tables.	(a) 2 (b) 1 (c) 3 (d) 2 (e) 1 (f) 0

(Continued on the following page.)

eg. to find the log of 276, since there are 3 digits on the left hand side of the decimal so the characteristic is one less than the number of digits.

Therefore characteristic is 2.

The mantissa is found from the log table. Go down the "N" column to 27 and then across to column headed 6.

The mantissa is .44091

Therefore log 276 = 2.44091

The log of 27.6 would have the same mantissa but its characteristic would be one less. Therefore the

log 27.6 = 1.44091

log 2.76 = 0.44091

Review frames 56 and 58 if necessary.

Find the logs of the following:

(a) log 35 (b) log 676

(c) log 1.467 (d) log 54796

(a) 1.54407
(b) 2.82995
(c) 0.16641
(d) 4.73876

SELF TEST V

Frames 53 - 71

Find the logs of:

1. 1.81
2. 42.7
3. 867.4
4. 8.674
5. 901.45
6. 1.0067

Answers:

1. 0.25768
2. 1.63043
3. 2.93822
4. 0.93822
5. 2.95493
6. 0.00284

1-6 LOGARITHMIC FORM OF NUMBERS BETWEEN 0 and 1

Up to this point we have avoided numbers between 0 and 1 becuase they have negative exponents. In this section we shall deal with these numbers.

72. We have seen the common logarithms are numbers rewritten to the base 10.

$100 = 10^2$ and log of $100 = 2.00000$

$10 = 10^1$ and log of $10 = 1.00000$

$1 = 10^0$ and log of $1 = 0.00000$

We have also seen that logarithms have two parts: a characteristic (whole number part) and a mantissa (decimal part). To find the characteristic of a number greater than one, count the number of digits on the left hand side of the decimal and subtract one.

Find the characteristic of

(a) 276.2 (b) 1.463

73. Notice that the characteristic of the examples were zero or greater. To show that powers-of-ten with negative exponents make sense, we have listed some familiar powers-of-ten in the table below: | 2, 0

$$100 = 10^{2.00000}$$
$$10 = 10^{1.00000}$$
$$1 = 10^{0.00000}$$
$$1/10 = 0.1 = 10^{-1.00000}$$
$$1/100 = 0.01 = 10^{-2.00000}$$
$$1/1000 = .001 = 10^{-3.00000}$$

Convert from exponential form to logarithmic form:

log 100 = 2.00000

(a) log 10 = ________

(b) log 1 = ________

(c) log 0.1 = ________

(d) log 0.01 = ________

(d) log 0.001 = ________

74. So the log of any number between 0.1 and 1.0 eg. (.1995) will have a characteristic of -1. Any number between .01 and .1 eg. (.0246) will have a characteristic of -2 and so on. From this, it is possible to formulate a rule for writing the characteristics of the log of a number less than 1. <u>The characteristic of the logarithm of a number less than 1 is negative and one more than the number of zeros between the decimal and the first significant digit.</u> Thus if the number is 0.46 then, its characteristic will be -1 since it is always one more than the number of zeros between the decimal and the first significant digit.

0.46 has a characteristic of -1
0.046 has a characteristic of -2
0.0046 has a characteristic of -3
0.00046 has a characteristic of _____

(a) 1.00000

(b) 0.00000

(c) -1.00000

(d) -2.00000

(e) -3.00000

75. To escape the difficulty of having to add negative characteristics to positive mantissas, we will use a special way of writing the characteristics of logarithms of decimals. We will write the minus sign over the top of the characteristic and leave the mantissa as it is. -4

Thus we will write the characteristic of 0.46 as $\bar{1}$

0.046 as $\bar{2}$ and so on.

The minus or bar sign as we now call it, tells us that it is only the characteristic that is negative; the mantissa is still positive.

Thus to find the log of 0.46 we would first determine the characteristic = $\bar{1}$. Look up the mantissa in the log table in the usual manner by tracing down to 46 and across to column zero.

log 0.46 = $\bar{1}.66276$

(Continued on the following page.)

Find the logs of the following numbers:

(a) .0078 = _____ (b) 0.0195 = _____

(c) .00052 = _____ (d) .01436 = _____

---

76. <u>We now have two rules for finding the characteristics of numbers:</u>

1. The characteristic of the logarithm greater than one is positive and one less than the number of digits to the left of the decimal.
2. The characteristic of the logarithm of a number less than one is negative and one more than the number of zeros between the decimal and the first significant digit.

Find the logs of the following numbers:

(a) 64 = _____ (b) .64 = _____

(a) 3.89209

(b) 2.29003

(c) 4.71600

(d) 2.15717

(Continued on the following page.)

(c) .0064 = ________

(d) 6.4 = ________

(e) 640 = ________

(f) 64000 = ________

Remember you must calculate the characteristic first before you use the table. It is only the mantissa portion of the log that is found in the table.

(a) 1.80618

(b) $\bar{1}.80618$

(c) $\bar{3}.80618$

(d) 0.80618

(e) 2.80618

(f) 4.80618

SELF TEST VI

Frames 72 - 76

State the characteristic of each of the following:

1. log 0.938 = __________
2. log 28,300 = __________
3. log 1.29 = __________
4. log 0.00517 = __________

Write the log of the following numbers:

5. log 0.00427 = __________
6. log 0.427 = __________
7. log 4270 = __________

Answers:

1. $\bar{1}$ 2. 4 3. 0 4. $\bar{3}$

5. $\bar{3}.63043$ 6. $\bar{1}.63043$ 7. 3.63043

1-7 ANTILOGARITHMS - CONVERTING LOGS BACK TO REAL NUMBERS

The purpose of this section is to learn how to convert a number expressed in logarithmic force back into a real number.

77. In the last section we learned how to express numbers as powers-of-ten or in logarithmic form. In this section we will learn how to find a real number if we know the logarithm of a number. To do this conversion other tables have been written in which you look up the mantissa part of the log in the same way that you look up ordinary numbers in the log tables. These tables are called Antilogarithms. You will find them on the next page to your logarithm tables. Turn to them now.

78. Antilogs like logs deal solely with the mantissa part of the logarithm.

(Continued on the following page.)

Remember characteristics are never found in the tables. We use the characteristics of a logarithm to tell us where to place the decimal point in the ordinary number that we obtain.

---

79. Find the logarithm of 56.976.

Step 1 Find the characteristic ______

Step 2 In the log tables find the row with 569

In the same row go to mean difference column for 7

Go to the mean difference column for 6 in the same row and take 1/10 of the number there. Round off to the nearest whole number ______

Add them up to obtain a mantissa of

____

Then the log 56.976 = ______________

	1

---

80. If we were given the number $10^{1.75600}$, notice that the number lies between

75511
54
5
75570
1.75570

(Continued on the following page.)

10^1 and 10^2. This means that the log of the same number is 1.75600. If we were asked to find what real number this corresponded to, we would then use the tables called

______________.

81. To use them we deal with the decimal part or the mantissa first. Trace down the left hand column until you come to the row containing .75. Go across the row to column 6. The reading is ________. Then we use the characteristic to tell us where to place the decimal. Since the characteristic is 1, and the number lies between ______ and ______. Therefore the number is __________.	antilogs
82. Note that there are no decimals in front of the numbers in the antilog	57016 10^1 & 10^2 57.016

(Continued on the following page.)

tables. These numbers are the ordinary numbers that make the log. We use the characteristic of the log later to see where to place the decimal. However, the first two figures that we look up in the left hand column usually have a decimal point in front of them. This reminds us that we are looking for the ________ part of the log.	
83. So finding a log is like going on a journey and finding the antilog is like coming back. Going we find first the characteristic and then the mantissa. Coming back we look up the mantissa; then the characteristic is used last to tell us where to place the decimal point intthe figures obtained. <u>Caution: Make sure you are using the correct tables</u>.	mantissa or decimal

84. Find the number whose log is 2.54438.

Look up the mantissa in the antilog tables. In the horizontal row thro' 54 and the vertical column headed 4 we find ________

Add the mean difference for 3 ________

Add 1/10 the mean difference for 8 rounded off ________

Total ________

Since the characteristic is 2 we know the number lies between ______ and ______.

The number then must have ___ digits to the left of the decimal point.

Therefore the number is ________.

---

85. Find the number whose log is $\bar{2}.54628$

Looking up 546 in antilog table we get ________

Add mean difference of 2 ________

Add 1/10 mean difference of 8 rounded off ________

Total ________

34995
24
7
35026

10^2 & 10^3

3

350.26

(Continued on the following page.)

Since the characteristic is negative, the required number is less than one. Since the characteristic is $\bar{2}$, there must be ______ zero between the decimal and the first significant number. Therefore the required number is __________.	
	35156 16 7 35179 1 0.035179

SELF TEST VII

Frames 77 - 85

1. Given: $\log 5280 = 3.72263$
   (a) The logarithm is ________.
   (b) The mantissa is ________.
   (c) The characteristic is ________.

2. Given: $\log 0.0873 = \bar{2}.94101$
   (a) The log is ________.
   (b) The mantissa is ________.
   (c) The characteristic is ________.

3. Find the antilogs of these logs:
   (a) 0.87724 (b) $\bar{1}.09912$
   (c) 2.39138 (d) $\bar{3}.21184$
   (e) 1.84159

Answers:

1. (a) 3.72263
   (b) .72263
   (c) 3

2. (a) $\bar{2}.94101$
   (b) .94101
   (c) $\bar{2}$

3. (a) 7.5378 (b) 0.12564
   (c) 246.26 (d) 0.0016287
   (e) 69.439

1-8 LAWS OF LOGARITHMS

Logarithms are exponents. Therefore for every law of exponents, there is a corresponding law of logarithms.

1-9 LAW OF LOGARITHMS FOR MULTIPLICATION

In this section we will learn how to multiply using logarithms. We will see that there is a correspondence between the law of exponents for multiplication and the law of logarithms for multiplication.

---

86. When you multiplied $10^2 \times 10^3$, we added the exponents of the base and wrote 10------. We stated the law of exponents for multiplication as $b^x \times b^y =$ ________ THE EXPONENT OF THE PRODUCT EQUALS THE SUM OF THE EXPONENTS OF THE FACTORS.	
87. Since the exponents are logarithms, we can write a corresponding law of	10^5 b^{x+y}

(Continued on the following page.)

logarithms for multiplication.

THE LOGARITHM OF THE PRODUCT EQUALS THE SUM OF THE LOGARITHMS OF THE FACTORS

Let $M = b^x$ then in log form:

$\log_b M =$ ________

Let $N = b^y$ then in log form:

$\log_b N =$ ________

$MN = b^x \times b^y$

$= $ ________

Change $MN = b^{x+y}$ into log form:

$\log_b MN =$ ________

but $x = \log_b M$

$y = \log_b N$

Therefore $\boxed{\log_b MN = \log_b M + \log_b N}$

Similarly $\log_b MNP = \log_b M + \log_b N + \log_b P$

	x
	y
	b^{x+y}
	$x+y$

---

88. The procedure for multiplying by using logs is:

1. change the numbers into logs
2. add the logs
3. change the answer back into a real number by finding the antilog

(Continued on the following page.)

Since our study of logarithms uses 10 as the base our law becomes

$\log_{10} MN = \log_{10} M + \log_{10} N$

or simply

$\underline{\log MN = \log M + \log N}$

Let us apply this method to solve this problem.

79.6 x 6.87

Let x = 79.6 x 6.87

log x = log 79.6 + log 6.87

Step 1= ________ + ________

Step 2= ______________

Step 3 x = antilog 2.73787

= ________________

This gives an answer correct to five figures.

---

89. It is important to remember that the mantissa of any logarithm is positive. The characteristic of the log can be positive, negative, or zero depending on the position	1.90091 0.83696 2.73787 546.85

(Continued on the following page.)

of the . It is also important to follow the above form. Multiply using logs. $27.29 \times .004421$ Let $x = 27.29 \times .004421$ $\log x = \log 27.29 + \log .004421$ Step 1 $= 1.43599 + \bar{3}.64552$ If we add 1.43599 $\bar{3}.64552$ Step 2 $\bar{1}.08151$ Notice that you add the characteristics as positive and negative numbers. $\log x = \bar{1}.08151$ $x = \text{antilog } \bar{1}.08151$ Step 3 $=$ ______	
90. Multiply $= 0.00415 \times 0.8856 \times 186.3$ Let $x = 0.00415 \times 0.8856 \times 186.3$ Step 1 $\log x = \log 0.00415$ $+ \log 0.8856$ $+ \log 186.3$	0.12064

(Continued on the following page.)

$= \_\_\_\_ + \_\_\_\_ + \_\_\_\_$ Step 2 $= \_\_\_\_\_\_\_\_$ $x = \text{antilog } \bar{1}.83548$ Step 3 $x = \_\_\_\_\_\_\_\_$	
91. Multiply using logs. 1. 0.768 x 865.4 2. 5638 x 0.00653	$\bar{3}.61805$ $\bar{1}.94723$ 2.27020
N. B. Caution: make sure you are using the correct tables.	$\bar{1}.83548$ 0.68467
	664.63 36.816

SELF TEST VIII

Frames 86 - 91

Multiply using logs.

1. 9.456 x 8.216

2. 0.007628 x 0.4792

Answers:

1. 77.692

2. 0.0036554

1-10 LAW OF LOGARITHMS FOR DIVISION

In this section we will learn how to divide using logarithms. We will see that there is a correspondence between the law of exponents for multiplication and the law of logarithms for multiplication.

---

92. When you divided 10^4 by 10^2 you subtracted the exponents of the base and wrote 10——.

We stated the law of exponents for division as

$$\frac{b^x}{b^y} = \_\_\_\_\_\_\_$$

TO OBTAIN THE EXPONENT OF THE QUOTIENT WE SUBTRACT THE EXPONENT OF THE DENOMINATOR FROM THE EXPONENT OF THE NUMERATOR.

---

93. Since the exponents are logarithms, we can write a corresponding law of logarithms for division.

TO OBTAIN THE LOGARITHM OF THE QUOTIENT, WE SUBTRACT THE LOGARITHM

Answers: 10^2 ; b^{x-y}

(Continued on the following page.)

OF THE DENOMINATOR FROM THE LOGARITHM OF THE NUMERATOR.

Let $M = b^x$ then in log form: $\log_b M =$ ____

$N = b^y$ then in log form: $\log_b N =$ ____

$$\frac{M}{N} = \frac{b^x}{b^y}$$

$=$ ________

Change $\frac{M}{N} = b^{x-y}$ into log form

$\log \frac{M}{N} =$ __________

but $x = \log_b M$

$y = \log_b N$

$\boxed{\text{Therefore } \log_b \frac{M}{N} = \log_b M - \log_b N}$

	x
	y
	b^{x-y}
	$x-y$

94. The procedure for dividing by using logs is:

1. change the numbers into logs
2. subtract the log of the denominator from that of the numerator
3. find the antilog of the difference

Since our study of logarithms uses 10 as the base our law becomes

(Continued on the following page.)

$$\log_{10}\frac{M}{N} = \log_{10}M - \log_{10}N$$

or simply

$\boxed{\log M - \log N}$

Let us apply this method to solve the following division. Remember our form will be the same as for multiplication except we will subtract the logs.

$$\frac{36.28}{5.146}$$

Let $x = \frac{36.28}{5.146}$

Step 1 $\log x =$ _____ - _____

Step 2 $=$ ________ - ________

$=$ ________

$x =$ antilog 0.84820

Step 3 $=$ ________

	log36.28 - log 5.146
	1.55966 - 0.71146
	0.84820
	.70501

95. You must write a characteristic as a part of every logarithm. This is especially important when the characteristic is zero. In the

(Continued on the following page.)

log 0.34782 the zero at the left of the decimal does not mean "nothing". It is the characteristic of the log and it means as much as if it were a 4 or a 7 or a 9.

Notice in the first example the subtracting of the logarithms was done just as any ordinary subtraction.

Suppose you had the following logs to subtract:

$$\begin{array}{l} 1.49273 \\ \underline{\bar{3}.62419} \\ 3.86854 \end{array}$$

You subtract the mantissas.

Notice that when you borrowed from 1, that left 0 and then the question reads $0 - \bar{3} = 0 + 3 = 3$

Complete the following:

$$\begin{array}{l} \bar{1}.49221 \\ \bar{3}.62400 \end{array}$$

Note: 1 borrowed from $\bar{1} = \bar{2}$

Then you proceed as in subtraction with positive and negative numbers.

96. Divide: $\frac{73.57}{0.005837}$

Let $x = \frac{73.57}{0.005837}$

log x = __________ - __________

= __________ - __________

= ________________________

x = antilog 4.10051

= ______________

*Remember when either or both of the characteristics are negative, the standard rules of algebra regarding the subtraction of negative numbers are applicable.

---

log 73.57 - log 0.0058[illegible]

1.86670 - $\bar{3}$.76619

= 4.10051

= 12604

---

97. Divide 0.7428 by 0.008372

Let $x = \frac{0.7428}{0.008372}$

Then log x = log 0.7428 - log 0.008372

= __________ - __________

= ____________________

x = antilog 1.94803

= ____________________

98. Divide: $\frac{1}{73.852}$

Remember log 1 - 0.00000 since $10^0 = 1$

$x = \frac{1}{73.852}$

log x = ________ - ________

= ________ - ________

= ____________________

x = antilog $\bar{2}.13163$

= ____________________

Notice:
$$\begin{array}{r} 0.00000 \\ \underline{1.86837} \\ \bar{2}.13163 \end{array}$$

$\bar{1}.87086$ - $\bar{3}.92283$

= 1.94803

= 88.722

---

99. Do the following divisions by logarithms:

1. 2.2703 ÷ 0.003862
2. 100 ÷ 0.07238
3. 36.813 ÷ 656.27

log 0.00000- log 73.852

$$\begin{array}{r} 0.00000 \\ \underline{1.86837} \\ \bar{2}.13163 \end{array}$$

.013541

---

1. 587.85
2. 1381.7
3. 0.056095

SELF TEST IX

Frames 92 - 99

Divide using logarithms:

1. $\frac{876.39}{73.487}$

2. $\frac{0.1}{513.27}$

3. $\frac{0.04382}{0.6563}$

Answers to self test.

1. 11.926

2. .00019482

3. .066769

1-11 SOLVING PROBLEMS IN MULTIPLICATION AND DIVISION BY USING LOGARITHMS

In this section, we will find that a combination question involving multiplication and division can be solved using the laws of logarithms for multiplication and division.

---

100. There are problems that call for both multiplying and dividing. This is an example:

$$\frac{16.21 \times 82.61}{37.04}$$

To solve such problems:

a. find the logs of all the numbers

b. Do the multiplication by _____ the logs

c. Do the division by _____ the logs

d. Find the _____

---

101. To solve the problem in frame 100

Let $x = \dfrac{16.21 \times 82.61}{37.04}$

$\log x = \log 16.21 + \log 82.61 - \log 37.04$

$= \_\_\_\_\_ + \_\_\_\_\_ - \_\_\_\_\_$

$= \_\_\_\_\_$

Answers to frame 100: adding; subtracting; antilog

(Continued on the following page.)

x = antilog 1.55815 = ________ Notice you found the antilog only <u>once</u>.	
102. Evaluate: $\frac{54.6 \times 83.75}{6.752 \times 184.6}$ Let $x = \frac{54.6 \times 83.75}{6.752 \times 184.6}$ log x = log 54.6 + log 83.75 - (log 6.752 + log 184.6) = ____ + ____ - (____ + ____) = ______ - ______ = ________ x = antilog 0.56450 = ________	1.20978 + +1.91703 - -1.56866 1.55815 36.153
103. Find the value of: 1. $\frac{4.385 \times 243.91}{65.837}$ 2. $\frac{(62.85)(176.42)}{(9327.6)(0.08356)}$	1.73719 +1.92299 - (0.82943) + 2.26625 3.66018 - 3.09568 0.56450 3.6686
	16.244 14.226

SELF TEST X

Frames 100 - 103

Find the value of: using logarithms

1. $\dfrac{0.0853 \times 0.06721}{0.004163}$

Answer

1. 1.3771

1-12 THE LAW OF LOGARITHMS FOR POWERS

In this section we will learn how to raise a number to a specified power using logarithms. We will see that there is a correspondence between the law of exponents for raising numbers to certain powers and the law of logarithms for raising numbers to powers.

---

104. In the section on exponents we showed that

$10^2 = 10 \times 10$

$10^3 = 10 \times 10 \times 10$

When you square a number, you raise it to the second power. When you cube a number, you raise it to the third power and so on.

When we had $(10^2)^4$ we multiplied the 2×4 and wrote it as 10----

We stated the law of exponents for powers as

$(b^x)^y =$ __________

TO OBTAIN THE EXPONENT OF "A QUANTITY RAISED TO A POWER", WE MULTIPLY THE EXPONENT OF THE QUANTITY BY THE POWER.

105. Since exponents are logarithms, we can write a corresponding law of logarithms for "raising to a power."	10^8
	b^{xy}

TO OBTAIN THE LOGARITHM OF "A QUANTITY RAISED TO A POWER", WE MULTIPLY THE LOGARITHM OF THE QUANTITY BY THE POWER.

Let $M = b^x$ then $\log_b M =$ ________

$(M)^n = (b^x)^n =$ ________

Written in log form:

$\text{Log}_b M^n = nx$

$= n \log_b M$

$\boxed{\text{Log}_b M^n = n \log_b M}$

eg. $(4.2)^3$

$\log (4.2)^3 = 3 \log 4.2$

---

106. To raise a number to a power with logarithms:

1. find the log of the number
2. multiply the log by the power to which the number is to be raised
3. find the antilog

Solve $(3.2728)^4$

Answers: x; b^{xn}

(Continued on the following page.)

Let x = ______

log x = ______ (log 3.2728)

= ______ (______)

= ______

x = antilog 2.05972

= ______

107.	If we took $(0.9)^2$ and solved it by logarithms, we would find that it has a ______ characteristic but its mantissa is always ______.	$(3.2728)^4$ 4 4(0.51493) 2.05972 114.74
108.	To multiply a logarithm that has a negative characteristic, multiply the mantissa first remembering that it is positive and add what you have "to carry" which is also positive to your answer to the multiplication of the negative characteristic. eg. $\begin{array}{r} \bar{1}.46143 \\ 3 \\ \hline \bar{2}.38429 \end{array}$	negative positive

(Continued on the following page.)

Notice that $3 \times \bar{1} = \bar{3}$ but you carry a positive 1 over from the multiplication of the mantissa and it changes to $\bar{2}$.

Complete the following:

a. $\bar{2}.60213$
$\underline{\qquad 2}$
______

b. $\bar{3}.84213$
$\underline{\qquad 3}$
______

---

109. Evaluate $(0.0735)^5$ using logs

Let $x = (0.0735)^5$

then $\log x =$ _____(log 0.0735)

$=$ _____(__ ______)

$=$ ____________

$x =$ antilog $\bar{6}.33145$

$x =$ ____________

a. $\bar{3}.20426$

b. $\bar{7}.52639$

---

110. Evaluate using logs

1. $(16.72)^3$
2. $(0.0637)^3$
3. $(0.39237)^4$

5

$5(\bar{2}.86629)$

$\bar{6}.33145$

0.00000 21452

---

a. 4674.3

b. 0.0000 25848

c. 0.023703

SELF TEST XI

Frames 104 - 110

Evaluate using logs

1. $(0.16273)^5$
2. $(.0385)^4$

Answers

1. 0.00011410
2. .0000021971

1-13 LAW OF LOGARITHMS FOR ROOTS

In this section we will learn how to raise a number to a specified power using logarithms.

111.	We know that $3 \times 3 = 9$ or 3^2. If in turn we ask, what number multiplied by itself or what number squared would make 9, the answer is 3 because $3 \times 3 = 3^2$ or 9. This number 3 is called the <u>square root</u> of 9. Thus the square root of 49 is ________. The square root of 81 is ________.	
112.	This mark is called a radical $\sqrt{}$. It stands for the root of a number. When the radical alone encloses a number, it stands for the square root of that number. Therefore: $\sqrt{100} = 10$ Thus $\sqrt{9} =$ ________ $\sqrt{36} =$ ________	7 9

113. But when roots are something other than than square roots, there is a small number written with the radical. | 3 | 6

eg. $\sqrt[3]{8}$ means the cube root of 8

$\sqrt[5]{32}$ means the fifth root of 32

$\sqrt[4]{81}$ = ______

$\sqrt[3]{27}$ = ______

---

114. THE LOGARITHM OF A ROOT OF A NUMBER IS EQUAL TO THE LOGARITHM OF THE NUMBER DIVIDED BY THE INDEX OF THE ROOT. | 3 | 3

Let $M = b^x$ then $\log_b M = x$

$\sqrt[n]{M} = M^{\frac{1}{n}} = (b^x)^{\frac{1}{n}} = b^{\frac{x}{n}}$

by def^n $\log_b \sqrt[n]{M} = \frac{x}{n}$

but since $x = \log_b M$

then $\boxed{\log_b \sqrt[n]{M} = \frac{1}{n} \log_b M}$

eg. $\log \sqrt{64} = \frac{1}{2} \log 64$

$\log \sqrt[3]{27} = \frac{1}{3} \log 27$

115. To find the root of a number 1. divide the log of that number by the given root 2. look up the antilog of the quotient Solve $\sqrt{98.38}$ Let $x = \sqrt{98.38}$ $\log x = \frac{1}{2} \log 98.38$ $= \frac{1}{2}(1.99290)$ $=$ ________ $x =$ antilog 0.99645 $=$ ________	
116. Find $\sqrt[3]{47.62}$ $\log x =$ ____ $\log 47.62$ $=$ ____(______) $=$ ________ $x =$ antilog ________ $=$ ________	1.99290 0.99645 9.9185
117. Now suppose we want to divide into negative characteristics, that is, suppose we want to find the roots of decimals.	1/3 1/3(1.67779 0.55926 0.55926 3.6245

(Continued on the following page.)

This is quite easy provided that again we remember that the characteristic of the log of a decimal is __________. Whereas the mantissa is always ________.

Here is the log of a decimal

$\bar{2}.40682$

The characteristic is negative and the mantissa is positive.

$\bar{2}.40682$ divided by 2 = $\bar{1}.20341$

The division is straightforward when the characteristic is exactly divisible by the number we wish to divide with.

$\bar{3}.63423 \div 3$

$\bar{2}.45762 \div 2$

$\bar{4}.88486 \div 2$

negative

positive

---

118. But suppose we have a log like $\bar{3}.20412$ and we want to divide it by 2. The $\bar{3}$ is not exactly divisible by 2. When this is so we meet the problem that one part of $\bar{3}.20412$ is negative and the other part positive. The characteristic

$\bar{1}.21141$

$\bar{1}.22881$

$\bar{2}.44243$

(Continued on the following page.)

is negative and the mantissa is always positive. We cannot divide a negative characteristic and leave any remainder to be carried over to the positive mantissa. So we make the characteristic exactly divisible by the number we are dividing with. What is the smallest negative number we can add to $\bar{3}$ to make it exactly divisible by 2? ________

---

119. It is $\bar{1}$ because $\bar{3} + \bar{1} = \bar{4}$ which is evenly divisible by 2. But if we add a minus quantity to the negative part of the log ($\bar{3} + \bar{1} = \bar{4}$), to cancel out the effect this has on the size of the logarithm, we must add the same quantity in plus form to the positive side.
Therefore: $\bar{3}.20412 = \bar{4} + 1.20412$
Now we can quite easily divide $\bar{4} + 1.20412$ by 2. It equals $\bar{2} + 0.60216$ or $\bar{2}.60205$.
Another example.

$\bar{1}$

(Continued on the following page.)

To divide $\bar{3}.69022$ by 2, the first thing to do is to make the characteristic exactly divisible by 2. To do this we add $\bar{1}$ to $\bar{3}$ to make it $\bar{4}$. To balance this we add 1 to the mantissa

Therefore $\bar{3} + \bar{1} + 1.69022$

$\bar{4} + 1.69022$

Divide by 2 $= \bar{2} + .84511$

$= \bar{2}.84511$

Complete the following. Remember make the characteristic exactly divisible by the number you are dividing with and adjust the mantissa accordingly.

a. $\bar{1}.40453 \div 3$

b. $\bar{1}.40462 \div 2$

c. $\bar{2}.40462 \div 2$

d. $\bar{2}.40436 \div 3$

e. $\bar{2}.40448 \div$ [illegible]

---

120. Find $\sqrt{.000049}$

Let $x = \sqrt{.000049}$

$\log x = \frac{1}{2} \log .000049$

$= \frac{1}{2}\ \bar{5}.69020$

a. $\bar{1}.80151$

b. $\bar{1}.70231$

c. $\bar{1}.20231$

d. $\bar{1}.46812$

e. $\bar{1}.60112$

(Continued on the following page.)

$= \frac{1}{2}\ \bar{6} + 1.69020$

$= \bar{3} + 0.84510$

$= \bar{3}.84510$

$x = \text{antilog } \bar{3}.84510$

$= 0.007$

Find $\sqrt[3]{.276}$

log x = ____ log .276

= ____ ________

= $\frac{1}{3}$ (_____ + _________)

x = antilog ___________

= _________________

$\frac{1}{3}$ log .276

$= \frac{1}{3}(\bar{1}.44091)$

$= \frac{1}{3}(\bar{3} + 2.44091)$

$= \bar{1}.81364$

$= .65109$

---

121. Find

a. $\sqrt[3]{0.07236}$

b. $\sqrt{0.002983}$

c. $\sqrt[4]{0.00275}$

a. .41670

b. .054618

c. .22900

SELF TEST XII

Frames 111 - 121

Evaluate using logs.

a. $\sqrt[3]{3.6145}$

b. $\sqrt{.006107}$

Answers

a. 1.5347

b. .078150

1-14 PROBLEMS WITH MULTIPLYING, DIVIDING, POWERS AND ROOTS

122	In this program we have covered 4 types of problems. 1. to multiply numbers we change them into logs and ________ the logs 2. to divide ordinary numbers we change them to logs and __________ the logs 3. to raise a number to a particular power we can ________ its log by the power required 4. to find a particular root of a number we can _______ by the root required	
123.	Some problems combine two or more of these processes. For example: $$\frac{42.61 \times 0.923}{7.8}$$ It consists of 2 processes: (1) add the logs of the top numbers together	add subtract multiply divide

(Continued on the following page.)

and (2) subtract the log of the bottom. Then use the antilog tables to convert it to a number.

Let $x = \frac{42.61 \times 0.923}{7.8}$

Then $\log x = \log 42.61 + \log 0.923 - \log 7.8$

$= \_\_\_\_\_\_ + \_\_\_\_\_\_ - \_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_\_\_$

$x = \text{antilog } 0.70262$

$= \_\_\_\_\_\_\_\_\_\_\_\_$

1.62951
+
$\bar{1}.96520$
-
0.89209
0.70262
5.0422

---

124. $\sqrt[3]{\frac{227.32}{416.15}}$

Let $x = \sqrt[3]{\frac{227.32}{416.15}}$

$\log x = \frac{1}{3}\left[\log 227.32 - \log 416.15\right]$

$= \frac{1}{3}\left[\_\_\_\_\_\_ - \_\_\_\_\_\_\right]$

$= \frac{1}{3}\left[\_\_\_\_\_\_\right]$

$= \frac{1}{3}\left[\_\_\_\_ + \_\_\_\_\_\_\right]$

$= \_\_\_\_\_\_\_\_\_\_$

$x = \text{antilog } \bar{1}.91247$

$= \_\_\_\_\_\_\_\_\_\_$

125. Solve $\dfrac{1}{\sqrt{0.82637}}$

$$x = \frac{1}{\sqrt{0.82637}}$$

$\log x = \log 1 - \frac{1}{2} \log 0.82637$

$= \_\_\_\_\_\_ - \frac{1}{2}\_\_\_\_\_\_$

$= \_\_\_\_\_\_ - \_\_\_\_\_\_$

$= \_\_\_\_\_\_$

$x = \text{antilog } 0.04141$

$= \_\_\_\_\_\_$

2.35665

$2.6\bar{1}924$

$\frac{1}{3}(\bar{1}.7374)$

$= \frac{1}{3}(\bar{3} + 2.7374$

$= \bar{1}.91247$

$= 0.81747$

0.00000
$-\frac{1}{2}(\bar{1}.91718)$

0.00000
$-\bar{1}.95859$

0.04141

1.10000

SELF TEST XIII

Frames 122 - 125

Solve:

a. $(8.637)^2 \times \sqrt[3]{189.6}$

b. $$\frac{\sqrt{.1627}}{3.154\ (.982)^2}$$

Answers

a. 428.55

b. 0.13261

1-15 CONCLUSION

By working through the previous frames you can see that logarithms are an aid to calculation.

126. The four theorems that were used are:

$$\log_b MN = \log_b M + \log_b N$$

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b M^n = n \log_b M$$

$$\log^n M = \frac{1}{n} \log_b M$$

Special logs:

$$\log_b 1 = 0$$

$$\log_b b = 1_0$$

If you need additional exercises before the final evaluation, see the instructor.