The Manifold and Intention Curriculum Model:

A way to create and evaluate curriculum

by

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Abstract

Using a version of Elliot Eisner's connoisseurship, this paper examines the current Manitoba Framework document for high school mathematics, finds it wanting, and proposes a new way of considering curriculum, using high school mathematics as an example space. Drawing inspiration from the writing of John Dewey, the model makes use of complexity theory as described by Brent Davis and Elaine Simmt (2003) to contribute to curriculum theorizing.

The Manifold & Intention Model uses major themes, termed Manifolds, as organizing devices for creating and using curricula. The underlying social, mathematical and educational assumptions surrounding the curriculum are opened to scrutiny in the Intention. The content that appears in a curriculum must meet the Content Evaluation Criteria and in the Common Content and Local Content, specific learning outcomes are eschewed, replaced by exemplars. The Manifold & Intention Model is a social and generative way to create curriculum.

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Finally, I wish to acknowledge all my students over the years for helping me learn how to teach, and who are helping me still.

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1. Concerning Curriculum

The goal of this thesis is to offer a new way in which curriculum may be considered by teachers and by all interested in creating curriculum. Such a choice may be useful to vary the experiences of curriculum for pre-service teachers, and to provide teachers working with students another means to conceptualize and enact their pedagogy.

I will engage in educational criticism and theorizing in two principal ways. I shall create some criteria for evaluating curricula, apply those criteria to existing curriculum documents, and I shall outline a new approach to considering curriculum, exemplified by creating some exemplars for high school mathematics.

Madeline Grumet (2009) cites Franklin and Johnson's desire that the dissatisfaction with No Child Left Behind in the United States will "offer an opening for reasserting the important and contentious issues of how the curriculum should be organized and what schools should teach" (p. 233). This thesis is intended to contribute in some way to fulfilling such a hope and I demonstrate connoisseurship by the following actions:

1. I present some of the research that serves as an anchor for my theorizing. I discuss my methodology of educational critique based on Elliot Eisner's connoisseurship, supplemented by questions suggested by Ted Aoki (1986/2005) to engage in curriculum theorizing through the development of two critical criteria or lenses. Other influences I cite, particularly that of John Dewey (1902/1990,1938/1997, 1916/2005) and Brent Davis and Elaine Simmt (2003) will also play a role in the development of these criteria.

2. I exemplify critique by developing criteria for investigating curriculum documents, and then considering portions of the Manitoba Math Framework document with the criteria.

3. I explain my Manifold & Intention Model for curriculum and generate sample components of each element of that model. Through this I invite others to see how the Model may connect with the world of the classroom, and how it may function as a generative structure in a broader sense. The discussion of the Model will show how it matches more closely to my criteria than the current Manitoba curriculum documents do.

4. I will summarize some elements of my journey as an act of *currere*.

I begin with "What *is* curriculum?" and "What is curriculum theorizing?"

The term 'curriculum' can carry a variety of connotations. For some, perhaps, it refers to the document still shrink-wrapped on the side counter, or on an unexamined website. For others, such as William Pinar (1975), the notion of curriculum has been seen as a hearkening back to its Latin root '*currere*', a reflective examination of the course of an individual's life. Several images of curriculum include: a list of content, an intended program of planned activities, a mode of cultural reproduction and/or hegemony of the dominant culture, a way of reconstructing society, social efficiency, and a host of others.

Not only is the term 'curriculum' multi-faceted and open, so is the project of formulating theories of curriculum. In 1971, James B. MacDonald begins an article: "Curriculum theory and theorizing may be characterized as being in a rather formative condition, as there are no generally accepted and clear-cut criteria to distinguish curriculum theory and theorizing from other forms of writing in education" (p. 196). Eleven years later, Herbert Kliebard opens *his* article with "One of the surest ways to kill conversation on the subject of curriculum theory is to ask someone to name one" (1982, p. 11). At the time, there was no dearth of writing on curriculum, and no shortages of new ideas; Pinar launched the notion of *currere* in the seventies, and with him in the lead a group of thinkers termed "reconceptualists" emerged. Still, the notion of what counted as a theory of curriculum was hazy.

Grumet (2009) examined the state of curriculum theory in a review essay, and she had no clear definition either. In 40 years, curriculum theorists have not been able to make a decision about precisely what curriculum theorizing is, particularly on how practical their theorizing should be. "Ah, the practical, the defensive shibboleth of curriculum studies" (p. 222). One thing that Grumet does provide is a structure that seems to encompass many of the concepts used to investigate curriculum, and that can be useful for examining curriculum:

> One, *the study of the curriculum phenomena as a cultural object*.
>
> Two, *the study of the curriculum object as an event*.
>
> Three, *the study of curriculum in the perspective of the researcher*.
>
> It seems clear to me that both Pinar's and my own schema for curriculum theory must involve this address to curriculum as an event that happens in time and place and politics, as well as a rhetoric that will carry this work back to a public forum if curriculum theory is to contribute to the project of social reconstruction (p. 232).

The nature of curriculum theory is still very much open-ended and undecided, as Kliebard (1982) and Grumet (2009) have shown. This could leave the novice curriculum theorist at sea. Possibly curriculum is such a complex and interactive space that single focused intellectual notions cannot perceive enough of it to be useful. Maybe curriculum writ large resists abstraction in this way. If theory transcends the particular, a theory of curriculum would leave any particular curriculum and its social situation behind. Perhaps we cannot transcend the particular because curriculum only has affiliated, human and interconnected meaning. The particular *is* the integral whole, and to leave the people behind leaves the curriculum behind.

So, we have no clear, agreed upon understanding of curriculum. But if we have not yet some Grand Unified Theory, that should not prevent us from seeking to understand curriculum. Hence, we need to consider and introduce new models for curriculum theory. The quality of the lives of children and, ultimately, the quality of our society depends upon our attempts to understand and improve education.

At this stage in my development, I see curriculum theorizing as requiring action. Curriculum theorizing must create, discuss, challenge, re-view or explicate ideas or patterns of thought that may be used when considering the making or evaluating of curriculum. Curriculum should go some way to help change schooling into education.

Given the possibilities in curriculum and curriculum theorizing, there is room for a new model for mathematics curriculum in Manitoba. 'Curriculum' carries some of the meanings already mentioned. However, to ground our discussion, in this thesis I suggest that two major connotations may be useful when I use the term 'curriculum'. At its most fundamental, I see curriculum as a term for the formal expression of a desire to guide the development and noticing of members of a society. (I use 'noticing' to mean the objects, ideas and values to which we want people to attend. But when one brings some things forward, others recede and become less visible.) Such a vision permits many ways of interpreting the nature and legitimacy of that societal desire as in Grumet's first strand. This use of the term typically will take center stage when I wish to point at the intentions embedded in a practice. In another connotation, I see 'curriculum' as a term that can mean the objects/ structures and belief systems provided to teachers for use in guiding their planning. This fits with Grumet's idea of curriculum as event. In this case, the primary use of 'curriculum' is used for displays intended to direct teachers. In this vein, unless otherwise stated, the phrase 'the Manitoba curriculum' will refer to documents released in print or in electronic form.

The Manifold and Intention Model I propose for curriculum, (here, both as a noticing structure and as an manifestation of what may be offered to teachers and students), is sufficiently open to deal with issues and decisions made at the fundamental level of curriculum, while having the potential to encourage and support teachers in changing their beliefs and practice at the second level, given their experiences and reflective understanding, Grumet's third strand.

2. Academic Influences and Methodology

Grumet is critical of inquiry as compared to theorizing, concerned that reframing curriculum theory as practical inquiry sidesteps some intellectual rigour. "Inquiries, I have found, are weak tea compared to theories" (p. 221).

Instead of supporting her stated position, Grumet (2009) fails to cite examples to justify her taste of theory over inquiry, nor does she get to a 'theory' either when she gives Pinar's laundry list of areas of legitimate curriculum research.

> What follows are sketches of curriculum theory discourses, indicating their salient themes and citing many of their contributors. …Pinar addresses curriculum history and curriculum theory discourses related to politics, multiculturalism, gender, phenomenology, postmodernism and post structuralism, autobiography, aesthetics, theology, institutionalism, and finally, internationalization (p. 231).

Perhaps she senses that Pinar is closest to what she wants when she later says he "identifies three terms – 'academic knowledge, 'subjective meaning', and 'social reconstruction' – necessary to the dynamism of curriculum theory" (2009, p. 231). But this is not a theory in itself. Despite her apparent hunger for theory, it isn't clear that she finds it from the passages she selects. Further, no one in her citations defines curriculum theory, and neither does she. We might struggle with someone who professes to value theory and who does not find or provide one. However, Grumet (2009) *does* provide a useful structure to those who consider curriculum in all its senses. I have mentioned the three strands already. Here, she provides more detail:

> One, *the study of the curriculum phenomena as a cultural object*. This means that the topic whether it is whole language literacy, arts integration, or hands-on science, is recognized as cultural object with a social history, anchored in ideology and nested in layers of meaning that call for clarification and interpretation arguing that not only do they have

> to be present but that each strand's claims need to be considered and challenged from the perspective of the other two.
>
> Two, *the study of the curriculum object as an event*. This means that curriculum happens, in schools, every day. It is a transaction that takes place among teachers and students, administrators and school boards, legislators and federal and state agencies. This is a strand of ethnographic research that strives to grasp the lived experience and meaning of curriculum to these actors.
>
> Three, *the study of curriculum in the perspective of the researcher*. This means that the consciousness of any scholar who has been schooled is itself saturated and shaped by curriculum. Curriculum inquiry requires a recapitulation of the researchers own history of experience and associations with the object to be studied.
>
> It seems clear to me that both Pinar's and my own schema for curriculum theory must involve this address to curriculum as an event that happens in time and place and politics, as well as a rhetoric that will carry this work back to a public forum if curriculum theory is to contribute to the project of social reconstruction (p. 232).

Grumet (2009) further claims that curriculum theory should have some public dimension. My own research will meet the expressed view of inviting curriculum to be more public, as Grumet requires. Further, it offers some method to connect to the world, a position suggested by Schwab in 1969, and explicit in Grumet's second strand. The Manifold & Intention Model, detailed in Chapter Four, is intended for teachers and it can be analyzed and considered using current tools for curriculum theorizing.

I outline several of the key notions and authors that have influenced my view of curriculum, and led to my Manifold & Intention Model.

2.1 Ralph Tyler – An Influential Model for Constructing Curriculum

Ralph W. Tyler wrote *Basic Principles of Curriculum and Instruction* (1949). Tyler raises four questions that he claims must be answered in developing curriculum:

1. What educational purposes should the school seek to attain?

2. What educational experiences can be provided that are likely to attain these purposes?

3. How can these educational experiences be effectively organized?

4. How can we determine whether these purposes are being attained? (p. 1)

Tyler's four questions about curriculum are stated directly in the introduction to his book, and Tyler even goes on to say that no attempt is made at a direct answer, but that "instead of answering the questions, an explanation is given of procedures by which these questions can be answered. This constitutes a rationale by which to examine problems of curriculum and instruction" (Tyler, p. 2). Thus, Tyler himself suggests that this may not be the only way to draft curriculum. However, in the work that follows his introduction, Tyler suggests a model that is generally instrumental and reductionist in its methodology, and seems to conflate education with schooling. This behaviourist approach has been taken to heart so broadly and deeply by those who followed, that it is often referred to as the dominant or traditional approach to creating curriculum. This is the part of his work that generates the disparaging epithet 'Tylerian' in some curricular circles. Consider Ted Aoki's (1986/2005) critical analysis of Tyler's work, described as an end-means orientation: "But what does this orientation imply in terms of cognitive interests and assumptions held tacitly? I suggest that underneath the avowed interest in efficiency, effectiveness, predictability and certainty…is a more deeply rooted interest – that of *control*" (p.141).

However, we can adapt Tyler's questions, to make them frame a curricular structure that is ecologically and socially just and meaningful. At the heart of Tyler's questions are the same questions that seem to drive all human activity:

"What do I want"?

"What might I do to get what I want?"

"Of these choices, what is a good way to get what I want?"

and, after taking action,

"Did I get what I wanted?"

Notice that in the case of almost every human behavior, if the answer to the final question is "No" then the cycle of questions is often repeated and, assuming the desire remains, further elaboration and change takes place in the answers to the second and third questions. Because of my experience in the classroom, and what I have learned in my reading, I want there to be a different possible response to these two central questions as regards curriculum than what we have.

2.2 Systems and Complexity Theory

We are, in part, products of our culture, for our culture guides our perceptions (Davis, Sumara, & Luce-Kapler, 2000). One powerful element that is created by culture – and that is co-created with it – is the structure of the organizations and systems within which we work and live.

The organization of a curriculum – its structure – affects teaching. The structure of any organization is not, as may be commonly supposed, simply an inert frame. The complex relationship between teachers and curriculum is a structure where changes in one leads to changes in the other, and vice versa, sometimes rapidly, sometimes with long delays. The structure of any organization where there is the potential for one element to affect the behaviour of some other element is the type of organization that may be considered using systems theory.

Peter Senge (1990) in *The Fifth Discipline* discusses many aspects of systems theory and its application to natural phenomena, business and even to the nuclear arms race during the Cold War. The holistic approach is important for healing wounded views of school mathematics.

> Systems thinking is a discipline for seeing wholes. It is a framework for seeing interrelationships rather than things, for seeing patterns of change rather than static "snapshots.' It is a set of general principles – distilled over the course of the twentieth century…During the last thirty years, these tools have been applied to understand a wide range of corporate, urban, regional, economic, political, ecological, and even physiological systems. And systems thinking is a sensibility – for the subtle interconnectedness that gives living things their unique character (p. 68).

Evidence for the power of structure in education comes from Cooney (1994). Cooney has described how a critical part of how teachers teach has to do with whether they see authority as being primarily internal or external. An increase in a sense of internal authority increases the likelihood of a teacher acting in a more flexible and responsive way. Such a teacher is less likely to be dogmatic, more likely to be open to the connections and humanity of mathematics.

> The dogmatic teacher teaches mathematics from a cut-and-dried perspective, thus indoctrinating students with certain beliefs about mathematics. This may account for the fact that many students in the United States think of mathematics as a subject best learned through memorization (p. 628).

Cooney (1994) claims that even though schools are authoritarian structures, an individual teacher's locus of authority is not necessarily fixed. If the locus of authority changes for a teacher, Cooney describes the result:

> [There may be] a certain point at which a person sees authority as an internal agent rather than as an external agent. At this point, truth is seen as eventuating from a personal perspective. It is here that one begins crossing the bridge from a submissive orientation to a position in which one's voice is a significant determiner of what one believes. Once a person or teacher begins to accept herself or himself as a legitimate authority, she or he begins to lay the groundwork for the acceptance of contextuality and relativism that fosters a sense of self and control – expunging the notion that authority (a professor, a textbook, guidelines from the school district, etc.) is omnipotent (p. 628).

But until this change comes, if it ever does, the teacher is likely to remain dogmatic, his or her view of mathematics remains potentially sterile, and the importance of sources of authority, such as curricula, remains unquestioned. Thus, the structure of a curriculum can play a significant role in how students and teachers behave. Changing the structure strongly influences the behaviours.

Brent Davis and Elaine Simmt (2003) helped me to crystallize some of the underlying features that may be desirable in a system. Over time humankind has shown remarkable adaptation to climate, circumstance and ways of thinking. Given the certain and possibly rapid change that our descendents will face, it is the duty of a wise education to help people to be able to recognize current and potential change, understand the implications of that change and seek measured, nuanced and subtle approaches to interceding in the physical and social world. A curriculum that understands balance and change is more likely to help students be ecologically and socially aware.

Balance and change are subtle and complex things, and if curriculum development is to have a chance at producing this power, the conditions for fostering their emergence and growth should be examined. I view attention to complexity as a central requirement for curricula. By viewing education from the perspective of complexity theory, Davis and Simmt (2003) describe individuals, social situations and cultures as being complex systems. We desire our complex educational systems to allow individuals and the society they create to be able to persist, adapt and organize itself. We want a rich and satisfying life for those who live now, and those who will live. A curriculum is a method people use to increase the probability that society will endure, will create deep and meaningful connections for its citizens, and will permit that society the greatest chance to make necessary changes. Such a system is complex, and requires adaptability and interaction for it to arise and flourish. Davis and Simmt (2003) describe five necessary principles for a complex system to exist:

(a) Internal diversity
(b) Redundancy
(c) Decentralized control
(d) Organized randomness
(e) Neighbour interactions (p. 147)

Since these principles are required for a complex system to display emergent, self-organizing behaviour of the sort that will dignify the enterprise of mathematics education, these principles should be present in the structure of any curriculum.

Davis and Simmt (2003) connect the adaptability, the *intelligence*, of a system to the range of options it has, which is a direct function of the internal diversity of its parts, and they describe intelligence "as the capacity of a system to respond not just appropriately, but innovatively to novel circumstances.… a system's intelligence is linked to its range of possible innovations, which in turn is rooted in the diversity represented among its agents" (p. 148). Each person in a classroom has something to offer, something unique in his or her insights. The curriculum model I propose has great capacity for individual insight to add to the collective through the vehicle of the class-created content.

Davis & Simmt (2003) state that "a system's capacity to maintain coherence is tied to the redundancy expressed among its agents…. redundancy among agents is an important quality for coping with stress, sudden injury or other impairments" (p. 150). Resilience or redundancy means a way of describing commonalities among agents. The shared parts are what permits cooperation and interaction, and the assumption of the duties of one agent by another if the first agent should fail. This is a complement to diversity. "Whereas internal diversity is more outward-oriented in that it enables novel actions and possibilities in response to contextual dynamics, internal redundancy [in a system] is more inward-oriented, enabling the moment-to-moment interactivity of the agents" (p. 150).

A concern about using the term 'redundancy' is that there is a connotation of uselessness and waste associated with it, and Davis and Simmt recognize the connotations of the word. I prefer David Orr's term, 'resilient'. In *Ecological Literacy,* David Orr (1992) describes the ecological meaning of redundant or resilient systems:

> Resilient systems absorb shock more gracefully and forgive human error, malfeasance, or acts of god. Resilience does not imply a static condition, but rather flexibility that permits a system to survive unexpected stress; not that it achieve the greatest possible efficiency all the time, but that it achieve the deeper efficiency of avoiding failures so catastrophic that afterwards there is no function left to be efficient (Orr, p. 34).

Accordingly, when I cite the five principles necessary for complexity, I have chosen to replace ‘redundancy’ with ‘resiliency’, while still crediting Davis and Simmt for the concepts and most of the wording.

In the case of curriculum resilience, I suggest that common terms, definitions and notations play a role in permitting the agents to interact. It is a way for an individual’s own conceptions to move from the utterly individual and idiosyncratic toward a more public space, where these ideas can be communicated and used to influence and be influenced by others. Without this interactivity, a complex structure like a mathematical community in a classroom cannot develop. Hence, curriculum critique and design must create a link between the individual and the community so that participation and interaction may take place as students move from one community – the classroom – to other communities and types of communities in the larger world.

Decentralized control is required to permit a system to display behaviours that no single individual may display. As an example, the Internet was conceived of as a way to avoid complete shutdown of military computers if any one part was destroyed. It started for one reason; now consider how the world of communication has changed since its advent. Davis and Simmt (2003) show that, besides safety, this emergent organization may occur and that it operates on a deep and ecologic level.

> The whole does behave as a unit and as if there were a coordinating agent present at its center… [A coherent global pattern] emerges from the activity of simple local components, which seems to be centrally located, but is nowhere to be found, and yet is essential as a level of interaction for the behavior of the whole (p. 152).

Davis and Simmt (2003) describe organized randomness as how a system may be proscribed from certain behaviours, but permitted wide latitude of action inside the given limits, this structure serving as a mediator between unresponsive rigid order and utter chaos. “[Organized randomness] is a structural condition that helps to determine the balance between redundancy and diversity among agents” (p.154). By this measure, a curriculum must permit innovation and interaction while supplying a context that provides support for emerging order and restricts aimlessness.

Finally, for a system to operate in a complex and emergent fashion, parts of the system must have an influence on other parts of the system, and the authors make the point that we must not take for granted the exact nature of the word ‘neighbour’. “… [W]e realized that the ‘neighbour’s in mathematical communities are not physical bodies or social groupings… Rather, in mathematics, these neighbours that must ‘bump’ against one another are ideas, hunches, queries, and other manners of representation” (Davis & Simmt, 2003, p. 156).

Some may insist that a complex system can be developed or instilled in any classroom with *any* curriculum structure if the teacher plans carefully and is both artful and strategic about trying to make these conditions the backbone of their class climate. I have my doubts. Every classroom, school, or district is a system – elements influence each other – but for *emergent* systems or *complex* systems to arise require the five necessary conditions, and the structure of the surroundings affect the degree to which the five conditions can be encouraged. For instance, consider a field of monoculture barley that has been sowed with *Round-Up* resistant seed and chemically fertilized. Both biotic and abiotic factors interact here - we have a system. But given the reduction in individual diversity and the lack of resiliency in this system, and given the lack of a strong selection mechanism and the harvest for food instead of seed, both neighbour interactions and organized randomness are low. Hence, the system has little *intelligence*; its capacity to evolve or adapt has been drastically reduced. Thus, we do not expect complex or adaptive behaviour over human time scales.

In the curricular realm, we may have similar circumstances. Given the strength of influence of systemic structure on human behaviour, I think two key points are important to consider. First, are the five conditions likely to emerge or be persistent themselves in the curriculum structure we have currently? Second, does the curriculum document encourage this type of collaboration and innovation among teachers? In my experience, the answer for both points for our senior years mathematics curriculum document is no. The model we have certainly does not obviate the possibility for a teacher to introduce the five conditions, but there is no positive support for it either. In the model I propose, we will see that decentralized control is increased over the existing Manitoba curriculum model, there are more possibilities for agents to interact with each other, the capacity for organized randomness is higher and respect for internal diversity is much higher. Eisner (1998) says the current system does not provide the opportunity for the careful introspection needed.

> The result of a packed schedule and the paucity of time and space for reflection leaves teachers in a position of trying to find out, on their own, how they are doing….Americans have designed schools that from a structural perspective make it difficult for teachers to improve at what they do, except by dint of their own reflectivity. Schools make little place for reflectivity (pp. 114-115).

In the busy world of a teacher, the probability of a teacher happening upon the readings that might lead to the ideas surrounding complexity theory seems remote, and for one who may happen to deal with skeptical colleagues, the lack of support in the documents will not assist their cause.

The case for teachers to have similar opportunities for themselves and their peers is perhaps a little better. Some school divisions encourage or permit teachers from differing schools to meet or cooperate. In my experience, while very welcome, these occasions tend to be too infrequent to create the levels of trust and productiveness that are required. But suppose the curricular structure created strong support for teachers to share activities. Suppose a province-wide electronic meeting-hall / conference were on-going, whose function was to provide support for ideas and interactions. Such electronic clearinghouses of ideas are possible now, indeed, they exist. But the widespread use of such instruments is not likely, because of the insidious notion that the provincial curricular document already has all that one needs to teach. Imagine the greatly increased potential for ideas to bump against each other in a curriculum structure that insists that this happens, rather than the current structure whose outcomes are so specific, and so isolated from each other that the notion of teacher decision-making at the curricular level seems unimaginable. The model I propose contains the space for exchange and debate, a place where stale drill, pet ideas and offbeat activities can be taken up and metamorphosed by the emergent community of teachers into deep and powerful exemplars of the underlying components of mathematics, or any other discipline.

In light of the history of curriculum, my personal story of having been presented with one model of curriculum is now understandable, but not acceptable. I knew I wanted change - and more than just two fives for a ten - but I needed a more thorough grounding in what education was, and what it was supposed to do. In this, the writings of John Dewey provided me with a way to think about curricula and to help me effectively question what is behind the curtain of the current curriculum.

2.3 John Dewey - Inspiration

The central figure for developing my philosophical views of education is John Dewey. To me, Dewey developed, as nobody else has done, a coherent and useful notion of the purpose of education, along with what did not constitute education. His critique showed existing educational models were not intellectually sound and his resolution of some of the tensions between arguments for social efficiency versus personal development had a profound effect on me. One of his seminal works is *Democracy & Education* (1916/2005) and herein Dewey develops his definition of education:

> …the ideal of growth results in the conception that education is a constant reorganizing or reconstructing of experience…the direct transformation of the quality of experience….We thus reach a technical definition of education: It is that reconstruction or reorganization of experience which adds to the meaning of experience, and which increases ability to direct the course of subsequent experience. (1) The increment of meaning corresponds to the increased perception of the connections and continuities of the activities in which we are engaged…. (2) The other side of an educative experience is an added power of subsequent direction or control (1916/2005, p. 47).

Education is the reconstruction or reorganization of experience that adds to meaning and increases the ability to direct later activity. This definition reads as powerfully now as in the modern times of which Dewey was a part. The notion of experience is central to Dewey's work, and from this idea springs the notion of the progressive school movement and such a definition is consonant with constructivism, a widely held idea today. A measure of the utility of the definition is its applicability to other behaviours such as habits:

> ...[Education]…means prolongation of plasticity, or power of acquiring variable and novel modes of control. Hence it provides a further push to social progress. ... Habits reduce themselves to routine ways of acting, or degenerate in ways of action to which we are enslaved just in the degree in which intelligence is disconnected from them. Routine habits are unthinking habits: "bad" habits are habits so severed from reason that they are opposed to the conclusions of conscious deliberation and decision…..Routine habits, and habits which possess us instead of our possessing them, are habits which put an end to plasticity. They mark the close of power to vary (1916/2005, pp. 29-31).

In our current educational structures, how often do our habits of focusing on having students doing some particular *thing*, rather than on particular ways of thinking, mark a push toward the close of the power to vary?

Dewey seemed to have a sense of systems theory as well. He remarks on the systems that are in place for education in his time. He discusses how thoughtless and rigid control of behaviours may in fact undo the very thing that the control was designed to curb.

> The control afforded by the customs and regulations of others may be short-sighted. It may accomplish its immediate effect, but at the expense of throwing the subsequent action of the person out of balance. A threat may, for example, prevent a person form doing something to which he is naturally inclined by arousing fear of disagreeable consequences if he persists. But he may be left in the position which exposes him later on to influences which will lead him to do even worse things. His instincts of cunning and slyness may be aroused, so that things henceforth appeal to him on the side of evasion and trickery more than would otherwise have been the case. Those engaged in directing the actions of others are always in danger of overlooking the importance of the sequential development of those they direct (1916/2005, p. 18).

His criticisms of the traditional models of instruction prevalent at the time contributed much to his definition of education and to the central role of the experience of the student in creating and recreating his or her own thoughts. He shows an acute awareness of the illogical circularity of the arguments used to support a fixed canon of material. The idea that a set canon of knowledge sharpens the mind has been sacrosanct for a long time.

Dewey points out the problem with such a claim.

> The root of the error long prevalent in the conception of training of mind consists in leaving out of account movements of things…in which an individual shares….[The error] consists in regarding the mind as complete in itself….In historic practice, the error … has screened and protected traditional studies and methods of teaching from intelligent criticism and needed revisions. To say that they are 'disciplinary' has safeguarded them from all inquiry….that they were 'disciplinary' stifled every question, subdued every doubt, and removed the subject from the realm of rational discussion. By its nature, the allegation could not be checked up. Even when discipline did not accrue as matter of fact, when the pupil even grew in laxity of application and lost power of intelligent self-direction, the fault lay with him, not with the study or the methods of teaching. His failure was but proof that he needed more discipline, and thus afforded a reason for retaining the old methods. The responsibility was transferred from the educator to the pupil because the material did not have to meet specific tests; it did not have to be shown that it fulfilled any particular need or served any specific end (1916/2005, p. 80).

Dewey in effect answers Tyler's questions 33 years before they were asked. As to the aims and purposes of education, he provides the following as the criteria of good aims:

> (1) The aim set up must be an outgrowth of existing conditions. It must be based upon a consideration of what is already going on; upon the resources and difficulties of the situation. Theories about the proper end of our activities-educational and moral theories-often violate this principle. They assume ends lying *outside* our activities; ends foreign to the concrete makeup of the situation; ends which issue from some outside source…. They are something for which we *ought* to act. In any case such 'aims' limit intelligence; they are not the expression of mind in foresight, observation, and choice of the better among alternative possibilities. They limit intelligence because, given ready-made, they must be imposed by some authority external to intelligence, leaving the latter nothing but a mechanical choice of means (1916/2005, p.63).
>
> The aim must always represent a freeing of activities…The only way in which we can define an activity is by putting before ourselves the objects in which it terminates - as one's aim in shooting is the target. But we must remember that the *object* is only a mark or sign by which the mind specifies the *activity* one desires to carry out. Strictly speaking, not the target but *hitting* the target is the end-in-view; one *takes* aim by means of the target…. The object is but a phase of the active end, - continuing the

> activity successful. This is what is meant by the phrase, used above, 'freeing activity' (1916/2005, p. 64).

Dewey indicates not only what aims should be, but also how they should evolve. Aims for education are envisioned as feedback systems; they are to be organically adaptive structures:

> We have spoken as if aims could be completely formed prior to the attempt to realize them... The aim as it first emerges is a mere tentative sketch. The act of striving to realize it tests its worth….[T]he aim has to be added to or subtracted from. An aim then must be *flexible*; it must be capable of alteration to meet circumstances. An end established externally to the process of action is always rigid. Being inserted or imposed from without, it is not supposed to have a working relationship to the concrete conditions of the situation. What happens in the course of action neither confirms, refutes, nor alters it. Such an end can only be insisted upon…The aim, in short, is experimental, and hence constantly growing as it is tested in action (1916/2005, p. 63).

One can ask whether the aims of the current Manitoba mathematics curriculum could meaningfully meet the standards that follow:

> An educational aim must be founded upon the intrinsic activities and needs (including original instincts and acquired habits) of the given individual to be educated…An aim must be capable of translation into a method of cooperating with the activities of those undergoing instruction. It must suggest the kind of environment needed to liberate and to organize *their* capacities. Unless it lends itself to the construction of specific procedures, and unless these procedures test, correct, and amplify the aim, the latter is worthless….The vice of externally imposed ends has deep roots. Teachers receive them from superior authorities; these authorities accept them from what is current in the community. The teachers impose them upon children. As a first consequence, the intelligence of the teacher is not free; it is confined to receiving the aims laid down from above. Too rarely is the individual teacher so free from the dictation of authoritative supervisor, textbook on methods, prescribed course of study, etc., that he can let his mind come to close quarters with the pupil's mind and the subject matter. Educators have to be on their guard against ends that are alleged to be general and ultimate. Every activity, however specific, is, of course, general in its ramified connections, for it leads out indefinitely into other things. So far as a general idea makes us more alive to these connections, it cannot be too general. But 'general' also means 'abstract', or detached from all specific context. And such abstractedness means remoteness…. A truly general aim broadens the outlook; it stimulates one

> to take more consequences (connections) into account. This means a wider and more flexible observation of means (1916/2005, p. 65-66).

To me, it is clear that a provincially mandated curriculum with no apparent mechanism for taking students' intrinsic needs and activities into account cannot meet the criterion Dewey sets forth. The existing curriculum does better with translating the general aim into specifics through specific objectives. And certainly, the teacher is not free from the dictation of authorities.

Dewey uses several terms, including 'environment', for the objective or external conditions in which a student finds herself. Dewey sees two components – continuity and interaction – as being not separate but inter-related axes of two dimensions of experience that "intercept and unite" (1938/1997, p.44) and that his view of experience explains why traditional education has fared badly.

> The trouble with traditional education was not that the educators took upon themselves the responsibility for providing an environment. The trouble was that they did not consider the other factor in creating an experience; namely, the powers and purposes of those taught (1938/1997, p. 45).

Dewey describes true education as growth, and the purpose of growth is the capacity for future growth. It all sounds quite circular, and Dewey might be criticized for statements like this "…the educational process has no end beyond itself; it is its own end…" (1916/2005, p. 50). These critics complained that simply valuing growth is insufficient; they claim the direction of the growth must be specified. In *Experience & Education* (1938), Dewey responds to these people saying that when experience leads to personal growth, with an increase in the power of an individual to contribute meaningfully and generally to his or her own life and to society, then this growth is educative. He does not deny that other experiences may be less useful:

> The belief that all genuine education comes about through experience does not mean that all experiences are genuinely or equally educative. Experience and education cannot be directly equated to each other. For some experiences are miseducative. Any experience is miseducative that has the effect of arresting or distorting the growth of further experience (1938/1997, p. 25).

Dewey further describes experiences that may be miseducative. Experiences that increase callousness may reduce a person's capacity to be sensitive to others, or experiences that increase specialization may land a person in a rut. Or

> [the experience may] be immediately enjoyable and yet promote the formation of a slack and careless attitude; this attitude then operates to modify the quality of subsequent experiences so as to prevent a person from getting out of them what they have to give" (1938/1997, p. 26).

He also cautions against seeing education solely as preparation. For he claims, if the role of schooling is simply to prepare for the future, then

> motive power is not utilized. Children proverbially live in the present; that is not only a fact not to be evaded, but it is an excellence. The future just as future lacks urgency and body. To get ready for something, one knows not what nor why, is to throw away the leverage that exists, and to seek for motive power in a vague chance. ... [Thus, preparation] fails most just where it thinks it is succeeding-in getting a preparation for the future.
>
> The mistake is not in attaching importance to preparation for the future need, but in making it the mainspring of present effort. Because the need of preparation for a continually developing life is great, it is imperative that every energy should be bent to making the present experience as rich and significant as possible. Then as the present merges insensibly into the future, the future is taken care of (1916/2005, pp.34-35).

Therefore, for Dewey, it is the *quality* and richness of experience that is the central and animating idea. He regards the value of education as being an increase in capacity, and more growth just leads to greater capacity to adapt and foresee. His cycle can be a virtuous and democratic one that extends to all stages of life.

> Since life means growth, a living creature lives as truly and positively at one stage as at another, with the same intrinsic fullness and the same absolute claims. Hence education means the enterprise of supplying

> conditions which insure growth, or adequacy of life, irrespective of age (1916/2005, p. 51).

It is hard to overstate Dewey's influence on the experiential model of curriculum. In Dewey's time (as well as our own), many people felt that this essence of growth has been lost; that school has become a (usually) well-meaning instrument of dogmatic training. His descriptions of traditional modes of education are telling, and struck a strong chord with many people in the early 1900's. Dewey was a beacon to many and a huge influence on the progressive school movement that flourished then, as well as on the later experiential model of curriculum. Some who believed the instincts of the child are 'natural' and therefore not to be interfered with wanted to claim kinship with Dewey. They would have had to ignore statements like this from *The Child and the Curriculum*:"Any power, whether of child or adult, is indulged when it is taken on its given and present level in consciousness. Its genuine meaning is in the propulsion it affords toward a higher level" (1902/1990, p.193).

2.3 Methodology - Elliot Eisner, Connoisseurship, Creation

As well as theorizing *about* curriculum models, I will theorize *with* my curriculum model. Such theorizing is important if curricula are to adapt, for theorizing opens habitat, a niche for ideas, movements, and new considerations of curricula to flourish. J. J. Schwab in 1969 famously claimed that the curriculum field was moribund and I mean to contribute in a little way to the voices that claim that curriculum should not be solely theoretical, unanchored to the lives of students. Curriculum must be constructed that matters to teachers and to students.

As described previously, part of my methodology will be the creation of two criteria or requirements for curriculum that are informed by the influences I've cited. These requirements will be used to analyze both the current Manitoba curriculum and to serve as touchstones in my Manifold & Intention Model. The influence of Dewey and Davis and Simmt I have discussed in detail. It is time to bring forth more of the ideas surrounding Elliot Eisner's connoisseurship.

2.3.1 Eisner's notion of connoisseurship

Eisner has written about *connoisseurship* in curriculum, in popular parlance a term more often connoted with art or gastronomy than with education. The essential skills of being able to notice and appreciate are vital, if one is to appraise and recommend. Eisner distinguishes connoisseurship from criticism in *The Enlightened Eye: Qualitative Inquiry and the Enhancement of Educational Practice* (1998):

> Connoisseurship, unlike criticism is a private act. Its aim is to appreciate the qualities that constitute some object, situation, or event. To be a connoisseur in some domain means to notice or experience the significant and often subtle qualities that constitute an act, work, or object and. typically, to be able to relate these to the contextual and antecedent conditions. But connoisseurship imposes no obligation upon the connoisseur to articulate or justify, to explain or persuade (p. 85).

He uses wine-tasting to elucidate how a connoisseur functions. Eisner shows how the role of understanding the processes and factors of wine-making as well as sensual perception are important, and he describes the importance (and limitations!) of prior knowledge in being a connoisseur. His fundamental argument is that connoisseurs are those with a large and growing schema for many aspects of their particular field, and he notes particularly the importance of subtle and nuanced perception. Eisner suggests that becoming a connoisseur of Olympic diving – surely a daunting prospect – pales beside that of becoming an educational connoisseur, in complexity of action and actors, and interplay of factors. Despite the distinction he makes between connoisseurship and critique, his use of the term ‘connoisseurship’ often suggests or implies critique, perhaps wanting to separate his notions from the very broad field of writing that might be called critique. So, though this thesis is an example of educational criticism, I will follow his lead in using the term ‘connoisseurship’.

Eisner posits that one way to consider educational connoisseurship is through five dimensions of the ‘ecology of schooling’: (1) the intentional, (2) the structural, (3) the curricular, (4) the pedagogical, and (5) the evaluative (p. 73). Of these, the intentional and the curricular are of current interest to me. One suggestion he makes for a possible structure for engaging in connoisseurship is to examine these dimensions through Description, Interpretation, Evaluation and Thematics. He is by no means adamant that this is how connoisseurship must take place, but it does provide a measure of organization for critique.

For him, description is a way of permitting meaningful access to the tone of the school for the audience. Far from a didactic and bloodless account (what he calls ‘emotionally eviscerated’), Eisner wants enough detail and sense of the qualities of the dimension apparent that the reader can place herself there and begin to experience the circumstances, at least vicariously.

> The “trick” in writing...is to create…the sense of discovery and excitement that pervades a classroom…. [T]he function is not mere embellishment or ornamentation…it is epistemic. Its aim is to help the reader know. One source of knowing is visualization. Another is emotion. How a situation feels is not less important than how it looks. The descriptive dimension of educational criticism makes both possible (1998, p.89).

For Eisner, if description makes images, and gives an account *of*, then interpretation involves accounting *for*.

> Educational critics are interested not only in making vivid what they have experienced, but in explaining its meaning; this goal frequently requires putting what has been described in a context in which its antecedent factors can be identified. It also means illuminating the potential consequences of practices observed… (1998, p.90).

Eisner is attuned to the complexity of the educational process saying that "there are so many contingencies and interactive relationships among variables in classrooms that is more reasonable to regard theories as guides to perception than as devices that lead to the tight control or precise prediction of events" (p. 95). The role of a theory for interpreting the events of a classroom he says is to "satisfy rationality, to deepen the conversation, to raise fresh questions" (p. 95). For him, interpretation situates the dimension being considered, putting it in a larger context.

Evaluation attends to setting a value on the phenomena under discussion. It is the critic's role to appraise, and as such, to make comparisons with values. He quotes Sir Herbert Read who says that the greatest good in education is that it assists people in becoming what they have the potential to be. If this is the value we hold for schools, Eisner claims that comparing children, each of whom have their own distinct talents, is not a useful exercise.

> Since these potentialities are uniquely configured, at its best the educational experience the school engenders will lead to increased individuation. Within increased individuation, the appropriateness of comparing students becomes increasingly problematic…. We are, and we must be, guided by an image of virtue for this child, and as I have already indicated, that image is always as contestable as it is fallible (1998, p.101).

But the complexity and inherent ambiguity of the task of evaluating does not relieve the connoisseur of its burden. Eisner does recognize the utility of an end-means criterion referenced system, where the number of factors and their interactions is small or can be well controlled. But the number of factors, and the complexities of those factors, in the lives of students and teachers are not small, and can only be ‘well-controlled’ with methods that tend to be extremely restrictive, punitive or simplistic. For connoisseurs of curriculum who recognize the snares and potentials of a particular value system such as the one given by Read, we accept the job of evaluating with caution.

By Thematics, Eisner means that specific instances of classes, students, teachers or curricula can have general themes or aspects that they often share, and that these themes may help to guide our noticing. Indeed, Eisner contends that such a search for theme is an example of *naturalistic generalization* (1998, p. 103), contrasted with *formal generalization* of the sort sought by statisticians. To me, this description reintroduces John Mason’s (1985) *generalizing*, that powerful mode of thinking that helps people to seek and to find patterns to guide them. An educational connoisseur must consider the themes.

> The formulation of themes *within* an educational criticism means identifying the recurring messages that pervade the situation about which the critic writes. Themes are the dominant features of the situation or person, those qualities of place, person, or object that define or describe identity…. They also provide clues or cues to the perceptions of other situations like the situation from which the themes were extracted (Eisner, 1998, p. 104).

If curriculum connoisseurship is to have an impact on the field of curriculum theorizing, it must carry weight among those who attend to it. Eisner contends that there are no unassailable understandings, since all that we perceive and understand is mediated by our own idiosyncrasies and experiences. Hence, even quantitative methods applied to the social sciences cannot generate some absolute Truth. Therefore, in qualitative research, we cannot expect to, and have no need to, require that our methods find certainty. Rather, credibility is enhanced by attention to the evidence that forms the basis for our interpretations and the overall ‘rightness’ of those interpretations. This rightness is not some frivolous maundering, not some sly, mendacious shell-game of argument, but the kind of rightness that inspires, that leads to new thoughts and richer perspectives.

Eisner argues that three types of evidence can be used to secure credibility for educational criticism. He terms them *structural corroboration, consensual validation,* and *referential adequacy* (1998, p.110). Structural corroboration is defined as a collecting of data from a variety of sources, including, perhaps, interviews, textbooks used, types of assignments and quantitative results: “a mustering of evidence” (p.111). The process is also known as triangulation. Eisner describes qualities of such evidence should have to be persuasive “…matters of weight and coherence appeal to aesthetic criteria….The *tight* argument, the *coherent* case, the *strength* of evidence are terms that suggest rightness of fit” (p.111). He does not apologize for the potentially diverse points of view or interpretations that come out of such data; rather, he considers such open diversity a virtue. Insisting on a uniform method generates a single point of view, and may lead to important omissions. Even as he makes this point, Eisner cautions against intentional bias.

> To point this out is not to sanction or justify the intentional neglect of evidence contrary to one’s vested interests or educational values. On the contrary, because qualitative methods are vulnerable to such effects, *it is especially important not only to use multiple types of data, but also to consider disconfirming evidence and contradictory interpretations or appraisals* when one presents one’s own conclusions (p. 111).

This is not a call to include every possible permutation of ideas and theories; it is a requirement to provide other credible possibilities to broaden the conversation, not to abandon one's position. "Like criticism in the arts and humanities, the manner in which criticism is written should bear the signature of the writer" (p. 111).

The second way of demonstrating credibility is consensual validation. At heart, this is simply that there is some measure of agreement between competent others that there is a rightness about the work in question. Eisner is careful not to look for unanimity or even some kind of 'average' result in other's work to justify having credibility. The differences in interpretations come from different contexts and different worldviews; with vastly different orientations, the criticism offered by others may differ markedly from our own. "Consensual validation in criticism is typically won from the reader who are persuaded by what the critic has to say, rather than consensus among several critics" (1998, p.113). Still, there is the framing of possibilities that comes from considering the work of others, and the credibility of the work in question can be in part located and valued through these various frames.

Referential adequacy is the third of Eisner's three modes of securing credibility. He claims that this is the most empirical of the three modes, because one can observe the effect on people.

> If criticism does not bring about more complex and sensitive human perception and understanding, it fails in its primary aim. It is this aim that underlies *referential adequacy*. Criticism is referentially adequate to the extent to which a reader is able to locate in its subject matter the qualities the critic addresses and the meanings he or she ascribes to them….An educational critic's work is referentially adequate when readers are able to see what they would have missed without the critic's observations (1998. p. 114).

If a work can be generative and rich, this very fecundity can illuminate its referential adequacy. The final assessment must come from the reader, but it is likely that a critical work that can generate ideas and meanings and new thoughts will resonate with its audience.

2.4 My Lenses for Curriculum

As a means of considering Interpretation, Evaluation and Thematics, I will use two lenses. These lenses are influenced by the work of Dewey, complexity theory and some questions raised by Ted Aoki. In examining the Manitoba curriculum documents, I will use these lenses as a way to provide critique and to display connoisseurship.

2.4.1 The Lens of Complexity

Central to the reimagining of environmental, social, economic and political realities a society must be capable of emergent or intelligent behaviour Such behaviour permits wise adaptation, which is at the centre of an enduring and conscious society. Curriculum has always had a role in influencing the thought patterns of the young; it is time for a model that encourages an understanding of complexity. Therefore, the structure, organization and content of the curriculum creation process, and its published form, should be sensitive to the five principles necessary for emergent behaviour, (a) Internal diversity, (b) Redundancy, (c) Decentralized control, (d) Organized randomness and (e) Neighbour interactions (Davis & Simmt, p. 147).

To move towards this goal of considering complexity, one cultural and structural factor to consider is the capacity for mutability in the curriculum. In other words, is it presented as adaptable or absolute? Is the curriculum a general guide, which is to be interpreted by a professional in the context of the classroom? Or is the curriculum a mandated set of standards, an unquestionable (and therefore an unquestioned) authority? When a curriculum is absolute, the potential for discussion about what could be or should be taught is sharply limited, or non-existent. For what is the point of even considering changes to what is known to be unchangeable? To ask teachers to change or adopt "best practice" or implement a curriculum without allowing them deep autonomy in the decision-making of their classroom is unrealistic, and disrespectful of the profession. In Pinar's interpretation of Aoki (2005), 'implementation' is the term "that did the dirty work of erasing academic, which is to say, intellectual freedom, in the name, presumably, of institutional efficiency." (p.3)

In considering the Criterion of Complexity, the curriculum as expressed should describe how it is open to change and reinterpretation. That reinterpretation should show elements of the five necessary principles for complexity and further, the structure and content of the curriculum should show evidence for the five principles as well on a fairly explicit level.

2.4.2 The Lens of Perspective

The field of hermeneutics suggests that we are products of our culture, and a facet of hermeneutics is concerned with the social and political conditions that make a culture possible. We may consider the hermeneutics of curriculum to be an examination of the contextual background and implications of the interactions of people with curriculum. Thus, we may inquire into how the content and culture of a curriculum, and its institutional surroundings influences the beliefs, expectations and actions of teachers.

The culture that a teacher functions within is composed of a huge variety of interacting components. These may include, among many others, society's general view on the role of children in the world, a teacher's personal confidence in the material he or she teaches, the nature of the beliefs and ideals of the teacher's colleagues, and even how that teacher views the nature of his or her subject. An important subset of this entire culture, one part of the context of teaching, is the culture of a curriculum. A curriculum's culture is the set of interactions, traditions and worldviews held by the people associated with that curriculum and transmitted by all parts of that curriculum. Of course, this culture is far from monolithic: nothing like a sharply delineated single thing. As in any culture, there are individuals with in it that may have different – even radically different – points of view. Still, there are important shared metaphors and expectations that arise from, and that are expressed by a curriculum. What are the themes that run through the Manitoba documents?

In what structure are the teachers in Manitoba immersed? The roots of many modern curricula were set down in the early part of the last century and a pervasive mode of thought then was that human beings could - and should - make 'progress' through scientific and practical inquiry. In describing this early period, Eisner (1969/ 2005) quotes Thorndike, who aptly expresses modernist thought:

> What Thorndike sought was a precise, exact, objective science of human behaviour, one without spiritual or metaphysical bogey-men.
>
> > In the third stage, behaviour will be defined in terms of events in the world which any impartial observer can identify and, with the proper facilities, verify…. Science of this sort, by giving perfect identifiability and fuller knowledge, leads to completer and finer prophecy and control of human nature (Thorndike, 1921).
>
> The significance of these views about the nature of science of psychology and education cannot in my opinion be overemphasized (2005, p. 27).

Dewey's more holistic view of the purpose and means of education stand in sharp relief to Thorndike's, and thus I am most taken with the following quote: "One cannot understand the history of education in the United States during the twentieth century unless one realizes that Edward L. Thorndike won and John Dewey lost" (Lagemann, in Gibboney, 2006, p. 171).

Through the last century there has been, and continues to be, a search for a magic bullet, a final solution, to the issue of how to teach. In the three decades preceding Thorndike's comments, there were many developments in science, including the discovery of the electron, X-rays, discovery of radioactivity, the photo-electric effect, special relativity, general relativity and the mathematics that predicted the existence of black holes. In several parts of the world, universal suffrage was beginning. In this atmosphere of critical breakthroughs and progress in science and in some social matters, it is easy to imagine the enthusiasm for the application of some of the principles of science to education; an analysis of detailed particulars, sewn into a synthesis of how schools and curriculum should function. The management of teaching and even the research about teaching in the past century were concerned with reductionism, and little has changed today:

> American models of supervision have been rooted in the industrial world. The cult of efficiency, as Callahan (1962) reminds us, was America's early effort to reduce teaching to a practical routine. The task of the supervisor, like that of a boss on an assembly line, was to see to it that the job was done right, and this meant according to specifications…. [The American] research tradition is more in keeping with Taylorism, the scientific management of human behaviour, than with interpretive or qualitative orientations to the improvement of teaching….The American educational research tradition emanates from the work of Edward L. Thorndike. It is specific, focused on individuals, and concerned with learning and the shaping of behaviour. It is oriented more toward control and less toward interpretation (Eisner, 1998, pp. 12-13).

Given the concerns of the structural raised here we need to examine the perspectives and viewpoints displayed by curriculum documents. Ted Aoki provides six questions that may help us gain perspectives:

> 1. What are the perspectives underlying curriculum X? (What are underlying root interests, root assumptions, and root approaches?)
> 2. What is the implied view of the student or the teacher held by the curriculum planner?
> 3. At the root level, whose interests does Curriculum X serve?
> 4. What are the root metaphors that guide the curriculum developer, the curriculum implementer, or the curriculum evaluator?
> 5. What is the basic bias of the publisher/author/developer of prescribed or recommended resource materials?
> 6. What is the curriculum's supporting worldview? (1986/ 2005, p.145)

In my lens of Perspective, the notion of the supporting worldview is important. This world view is represented through the types of topics selected by the curriculum planners, the imagery, metaphors or symbols used, the view of the environment and the role of education as a means of social efficiency, developmentalism, humanism or social meliorism. Another component of Perspective is the variety and sophistication of the perspectives that are offered or supported. Paolo Freire insists on "Education as the practice of freedom – as opposed to education as the practice of domination" (2006/1970, p.81). This sense of education as liberation rather than domestication is central to the Lens of Perspective.

Also in this lens will be some of the ideas from Dewey: Is the component examined regarded as 'disciplinary' or proof against examination? Is there evidence of dogmatic thinking? Are false dichotomies introduced or perpetuated? Is there a focus on positive growth and freeing of the mind of the pupil so that she becomes more powerful and more adaptable?

I will use the lens of Perspective and the lens of Complexity as tools for discussing thematics and for interpreting and evaluating curriculum documents.

3. The Manitoba Curriculum Documents

In 2012, Manitoba is in the process of implementing new senior high Math curricula. The 1999 curricula are being replaced, with the Grade 9 course being mandatory as of September 2009, the two Grade 10 courses as of September 2010, the three Grade 11 Courses implemented beginning September 2011 and the three Grade 12 courses to be in place for September 2012. Through Eisner's connoisseurship, and considering the Lenses I have generated for curricula, I will examine a few components of the current Manitoba high school curriculum documents. The analysis is not intended to be extremely detailed; as far as description goes, the documents themselves are freely available online, and in dealing with Interpretation, Thematics and Evaluation, the use of a few sections of the documents will be sufficient for me to illustrate the salient points. I will consider the introductory part of the curriculum Framework document in one section, then the section containing the general outcomes, specific outcomes and the achievement indicators.

In 1996, the western provinces established the Western Canadian Protocol to develop common frameworks to guide the authoring of each province's curriculum documents Originally the Western Canadian Protocol, the WNCP is a creation of the Western Provinces of Canada (and later the northern territories) as described in the Background of the Manitoba framework document.

The entire high school Framework document, with the background, introduction, specific outcomes and achievement indicators is available online from the Manitoba Education and Youth website: http://www.edu.gov.mb.ca/k12/cur/math/framework_9-12/index.html and part of the index page is shown below.

Layout of the Index for the Manitoba Education, Citizenship and Youth

Document Sections:

- Cover (24 KB)
- Title Page & ISBN (34 KB)
- Contents (72 KB)
- Acknowledgements (98 KB)
- Background (95 KB)
- Introduction (105 KB)
- Conceptual Framework for Grades 9 to 12 Mathematics (175 KB)
- Instructional Focus (94 KB)
- General and Specific Learning Outcomes by Strand (94 KB)
    - Applied Mathematics (106 KB)
    - Essential Mathematics (109 KB)
    - Pre-Calculus Mathematics (114 KB)
- General and Specific Learning Outcomes with Achievement Indicators by Course
    - Grade 9 (10F) (169 KB)
    - Grade 10 Essential Mathematics (20S) (156 KB)
    - Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)
    - Grade 11 Applied Mathematics (30S) (143 KB)
    - Grade 11 Essential Mathematics (30S) (156 KB)
    - Grade 11 Pre-Calculus Mathematics (30S) (234 KB)
    - Grade 12 Applied Mathematics (40S) (150 KB)
    - Grade 12 Essential Mathematics (40S) (138 KB)
    - Grade 12 Pre-Calculus Mathematics (40S) (316 KB)
- Bibliography (123 KB)

(March 2012, http://www.edu.gov.mb.ca/k12/cur/math/framework_9-12/index.html)

The Framework document is the document that most teachers consider when they want to know 'what to teach' and thus it is the document I will reference. It is what most teachers would understand as 'the curriculum'. I shall define the Introductory section of the Manitoba High School Framework document as including the following sections:

- Cover (24 KB)
- Title Page & ISBN (34 KB)
- Contents (72 KB)
- Acknowledgements (98 KB)
- Background (95 KB)
- Introduction (105 KB)
- Conceptual Framework for Grades 9 to 12 Mathematics (175 KB)
- Instructional Focus (94 KB)

I shall define the Learning Outcomes section as the section containing these elements:

- General and Specific Learning Outcomes with Achievement Indicators by Course
    - Grade 9 (10F) (169 KB)
    - Grade 10 Essential Mathematics (20S) (156 KB)
    - Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)
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- Bibliography (123 KB)

(March 2012, http://www.edu.gov.mb.ca/k12/cur/math/framework_9-12/index.html)

The entire introductory section of the document is in a two-column text format with some use of bullet points. Most of the subsections within each named section are about a page or less in length, with exceptions that will be noted. There are two diagrams used in the introductory section, with several pull-quotes. I will describe the structure to some degree, making liberal use of quotes when these are helpful for the analysis of thematics, interpretations and evaluation that follows the description each subsection.

3.1 Describing the Background

As text, the Background is in the common style of the rest of the Introductory section as just described. A portion of the Background is given here:

> In December 1993, the Ministers of Education from Alberta, British Columbia, Manitoba, Northwest Territories, Saskatchewan, and Yukon Territory signed the Western Canadian Protocol (WCP) for collaboration in Basic education (Kindergarten to Grade 12). In February 2000, following the addition of Nunavut, the protocol was renamed the Western and Northern Canadian Protocol (WNCP) for collaboration in education (Kindergarten to Grade 12). In 2005, the Ministers of education from all the WNCP jurisdictions unanimously concurred with the rationale of the original partnership because of the importance placed on:
>
> - common educational goals
> - the ability to collaborate to achieve common goals
> - high standards in education
> - planning an array of educational opportunities
> - removing obstacles to accessibility for individual learners
> - the optimum use of limited educational resources
>
> (p. 1, 2009, http://www.edu.gov.mb.ca/k12/cur/math/framework_9-12/background.pdf).

In Canada, we are used to provinces guarding their independence jealously. What pervading worldview helps them to agree on education? We apply the Lenses.

3.2 The Lenses on the Background

The Lens of Perspective

The central theme in the Background is of efficiency. Now, no one is in favor of inefficiency. But the efficiency suggested by the Background – and we shall see it again later – is a social efficiency designed to make people serve the existing state of affairs. The themes are of consistency, and of compensating for scarcity, shown by the points stressing commonality. There is just the single perspective of how to get on in the current world. As for imagery or metaphor, one of the points mentions increasing standards (a word that suggests uniformity as well as high achievement). The real question is whether the type of curriculum given has a reasonable chance to produce these standards. And while one of the points is about removing obstacles (by having the students fitting the system rather than the system fitting the student) by the Background, this is a curriculum of domestication.

Lens of Complexity

Under this Lens, the five principles of complexity should be evident.

(a) Internal diversity - Consider the phrase "the importance placed on common educational goals" (p.1). The reason for the importance is not stated, but the desire to create people with very similar skills suggests that the variation between people in what they learn should be reduced or eliminated. This makes people more homogeneous, more predictable and therefore a more uniform 'input' for the next round of processing, whatever that may be. The tone of this document suggests that diversity is something to be reduced or overcome. On this principle the Background doesn't support complexity very well.

b) Resiliency - One particularly strong area demonstrated by the current Manitoba Framework document is the principle of resiliency. The focus on commonalities and similarity and the potential for elements of the system (students) to be afforded the opportunity to move elsewhere in the systems demonstrates a concern with being sure that the system is not deeply dependent on a few, and that the roles played by some can be taken by others.

c) Decentralized control – There is little evidence of decentralized control. To the contrary, control has been centralized more than ever in the past fifteen or twenty years. While there are a number of parties signatory to the WNCP, the document that emerges serves as the template for all the departments of education to use. There is a little control at the provincial or territorial level, but none at all at the level of individual school boards or schools or classrooms. The control is very centralized. Consider the case of textbooks. In the seemingly worthy desire for efficiency – the optimum use of educational resources – the control of the textbook production has been left to three companies, and currently each company has a monopoly on the approved textbook for a given pathway. Thus at the level of student/teacher resources, the control has been centralized and diversity reduced.

(d) Organized randomness – the notion of freedom within bounds is not addressed in any meaningful way.

(e) Neighbour interactions – Having high resiliency should help increase the likelihood of neighbour interactions to some degree, as a common language makes exchange of ideas easier. But a key feature of neighbour interaction in complexity theory is that neighbours should have influence on each other. In the focus on efficiency shown in the background, there is little room for influence on the content of the curriculum or on the process of curriculum making, so there is little point in teachers or divisions collaborating to gain new capabilities; only to further regiment and constrain.

The Background to the Framework Document does not suggest that the principle of complexity was central to the construction of the document.

3.3 Describing the Introduction

The Introduction describes the Purpose of the document (about ¼ of one page), Beliefs about Students and Learning (about ¾ of a page, including a short paragraph on assessment), First Nations, Métis, and Inuit Perspectives (about ½ page), Affective Domain (about ½ page), and the Goals for Students (about ¾ page). There is a two column approach used, and there is some highlighting of sections of the text by use of pull-quotes, and in the Goals for Students section, extensive use of bullet points. No illustrations or diagrams appear in this section. All quotes from the introduction come from http://www.edu.gov.mb.ca/k12/cur/math/framework_9-12/introduction.pdf (2009) and the page number in that section will be referenced.

3.3.1 Purpose

The Purpose of the Framework document is given as follows:

> This document provides sets of outcomes to be used as a common base for defining mathematics curriculum expectations that will be mandated in Grades 9, 10, 11, and 12. This common base should result in consistent student outcomes in mathematics across the WNCP jurisdictions and enable easier transfer for students moving from one jurisdiction to another. This document is intended to clearly communicate high expectations for students' mathematical learnings in Grades 9, 10, 11, 12 to all education partners across the jurisdictions, and to facilitate the development of common learning resources. (2009, p.3)

The Beliefs about Students and Mathematics Learning section first describes students as "…curious, active learners with individual interests, abilities, needs, and career goals" (p.3). There is discussion of the differences that students bring to their school life and an assertion that mathematical meaning, "is best developed when learners encounter experiences that proceed for simple to complex and from the concrete to the abstract" (p.3).

As well, there is a paragraph describing the use of diagnostic, formative and summative assessment. "Assessment *for* learning, assessment *as* learning and assessment *of* learning are all critical to helping students learn mathematics. A variety of evidence and a variety of assessment approaches should be used in the mathematics classroom" (p.3).

3.3.2 First Nations, Métis, and Inuit Perspectives

In the section First Nations, Métis, and Inuit Perspectives, the framework document describes the variety of settings in which these students go to school. There is a general statement made about learning for such students.

> First Nations, Métis, and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning and learn best when it is contextualized and not taught as discrete content. (2009, p.4)

There is reference to the traditional emphasis placed on oral rather than written communication.

3.3.3 Affective Domain

Affective Domain is the next subsection of the Introductory section. In this short section, two pull-quotes are used to emphasize the ideas that students "must be taught to set achievable goals and assess themselves as they work towards these goals (p. 4), and that "curiosity about mathematics is fostered when students are actively engaged in their environment" (p.4).

The final subsection in the Introduction to the 2009 Manitoba Framework document is Goals for Students:

> The main goals of mathematics education are to prepare students to:
> - solve problems
> - communicate and reason mathematically
> - become mathematically literate
> - appreciate and value mathematics
> - make informed decisions as contributors to society
>
> Students who have met these goals
> - gain an understanding and appreciation of the role of mathematics in society
> - exhibit a positive attitude toward mathematics
> - engage and persevere in mathematical problem-solving
> - contribute to mathematical discussions
> - take risks in performing mathematical tasks
> - exhibit curiosity about mathematics and situations involving mathematics
>
> In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through
> - taking risks
> - thinking and reflecting independently
> - sharing and communicating mathematical understanding
> - solving problems in individual and group projects
> - pursuing greater understanding of mathematics
> - appreciating the value of mathematics throughout history (p.5).

3.4 The Lenses on the Introduction

The Lens of Perspective

In the Introduction, there is a general open and positive regard for mathematics, tempered by a still instrumentalist viewpoint and a difficult to interpret section on Aboriginal students. The Purpose states that students are active and curious learners, and this suggests an appreciation for diversity and possibility, and the sense that people may be motivated intrinsically. This is undermined slightly by including career goals in the same sentence. In particular, in the Goals for students the goals are given ‘to prepare’ students and we have seen Dewey’s cautions about preparation as a aim of education.

In the section on affective domain, most of the statements are positive or affirming. The statement that students should make informed choices as "contributors to society" (2009, p.5) may bear some scrutiny. Does the students’ role as contributors imply that they have no role in creating and shaping society? If so, the perspective of pure social efficiency rises again. Or perhaps the term 'contribution' suggests that society is a kind of charity case? Instead of ‘contributor’, I suggest ‘participant in society’ or ‘shaper of society’, with the latter being the more consciously evocative of neighbour interactions.

A further recognition of the diversity inherent in the people who study mathematics comes from a section on First Peoples. Other learners do not have a distinct section in the Introduction to the Framework document, so there must be a reason to declare differences. The perspectives taken here are difficult to tease out. The statement that First Peoples often have a holistic view of the world has the tang of paternalism, is not supported in the document by research, and also suggests that other students don’t learn this way. Is this mere stereotyping, or an effective attempt to assist students? The claim that learning must be contextualized is not very helpful; the residential school system was started by people operating within the context of their times, and that system has proven to be very damaging for some. And for a mathematician, solving a particular problem in a single context is seen as less powerful than generalizing to a larger number of situations. I claim that one of the reason mathematics has been chosen by people to apply to very many circumstances is because of the very portability its decontextualization provides.

Part of the difficulty of writing such a section is that there are political landmines. It may be hard to point out that First Peoples' children are at educational risk, without some blaming the victim. And if the document criticizes the system, it undermines itself, because the system that leads to those risks is promulgated by the very document that makes the critique. Given the challenges of this topic in particular there is an attempt to display several perspectives of different people, and the themes within are largely focused on valuing and appreciating mathematics.

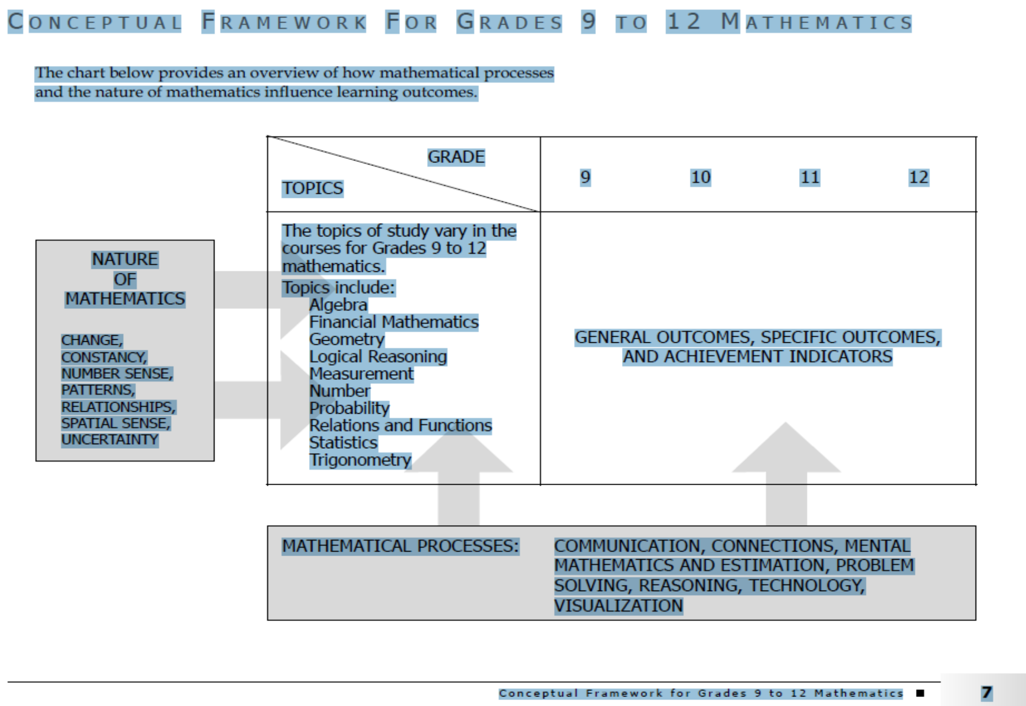
The document in the section on ‘Affective Domain’ has seen fit to discuss the attitudes, demeanor and motivations of students, very important in the oft-maligned area of teaching math. But the discussion ends here. Later in the curriculum, these challenging, complicated and hard to measure ideas have been cleansed from the Specific Outcomes. Some may argue that in the Analysis of Games and Numbers there has been an official display of considering the affective realm. (Of course, it’s only the Essential Math and part of the Applied Math programs that have this. The students in Pre-Calculus, who might be imagined to be the most interested in this sort of work don’t have this in their curriculum document). But some teachers leave these areas out. Perhaps it’s because when time is precious, the least pragmatic things have to go. Or perhaps deep down, there is an unconscious agreement with many of the students that we should not use math class to take the fun out of games that aren't much fun to begin with.

The Lens of Complexity

The document recognizes the diversity of learners, both as a whole and as members of First People's communities, but this is strongly trumped by the statement in the Purpose that "this document provides sets of outcomes to be used as a common base for defining mathematics curriculum expectations that will be mandated in Grades 9, 10, 11, and 12. This common base should result in consistent student outcomes" (p.3). As before discussed, such commonality has a chance to increase resiliency and perhaps neighbour interaction, but strongly mitigates against decentralized control, organized randomness and puts the lie to the words about diversity that are in the Introduction. Overall, this is not a curriculum that suggests complexity.

3.5 Describing the Conceptual Framework for Grades 9 to 12 Mathematics

The conceptual Framework section consists of 12 pages. The first page of the section is a chart describing "how mathematical processes and the nature of mathematics influence learning outcomes" (2009, p.7).



Source: Manitoba Education, Citizenship and Youth. *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes.* Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2009. Available online at <www.edu.gov.mb.ca/k12/cur/math/framework_9-12/index.html>. Reprinted with permission. All rights reserved. August 16, 2012.

The other subsections of the Conceptual Framework consists of Mathematical Process (4 ½ pages), Nature of Mathematics(about 3 pages), Pathways and Topics (about 2 pages) and Outcomes and Achievement Indicators (about ¾ page) followed by a one paragraph Summary. The Pathways and Topics section contains a diagram of which courses lead to others.

3.5.1 Mathematical Processes

The Mathematical Processes are described as Communication [C], Connections [CN], Mental Mathematics and Estimation [ME], Reasoning [R], Technology [T] and Visualization [V] and this section first introduces us to the processes (shown here) then describes each in more detail.

> The seven mathematical processes are critical aspects of learning, doing and understanding mathematics. Students must encounter these processes regularly in a mathematics program in order to achieve the goals of mathematics education.
>
> The common curriculum framework incorporates the following interrelated mathematical processes. It is intended that they permeate the teaching and learning of mathematics.
>
> Students are expected to:
>
> - use *communication* in order to learn and express their understanding
> - make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences and other disciplines
> - demonstrate fluency with *mental mathematics and estimation*
> - develop and apply new mathematical knowledge through *problem-solving*
> - develop and use *technology* as a tool for learning and solving problems
> - develop *visualization* skills to assist in processing information, making connections, and solving problems
>
> All seven processes should be used in the teaching and learning of mathematics. Each specific outcome includes a list of relevant mathematical processes. All seven processes should be incorporated in to learning experiences but the identified processes are to be used as a primary focus of instruction and assessment (2009, p. 8).

Under Communication, communication of mathematical ideas is discussed both in terms that represent both the intrapersonal and the interpersonal so that "these opportunities allow students to create links among their own language and ideas, the language and ideas of others, and the formal language and symbols of mathematics" (2009, p. 8).

It will be useful to have the Connections section in full:

> Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Through connections, students begin to view mathematics as useful and relevant.

> Learning mathematics within contexts and making connections relevant to learners can validate past experience and increase student willingness to participate and be actively engaged.

> The brain is constantly looking for and making connections. “Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding … Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine & Caine, 1991, p. 5) (2009, p.9).

Mental Mathematics and Estimation is the third of the seven processes. The general discussion is on the improvement of computational fluency, and how frequent use of mental math and estimation are helpful in decision making: "Even more important than performing computational procedures or using calculators is the greater facility that students need – more than ever before – with estimation and mental math" (National Council of Teachers of Mathematics, 2005). Here also is the language of freedom and individuality. “Students proficient with mental mathematics ‘become liberated from calculator dependence, build confidence in doing mathematics…’” (Rubenstein, 2001, p. 442) and in the next paragraph “Mental mathematics…’ provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers’ (Hope, 1988, p. v)"(2009, p.9).

Problem-solving rates an entire page in the Framework document, and there is the strong statement that “Learning through problem-solving should be the focus of mathematics at all grade levels. …Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics” (p. 10).

The framework document describes the two types of problem solving as solving problems that are inherently mathematical in nature and solving problems situated in a context outside of mathematics: “Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem" (p. 10).

"Mathematical reasoning helps students think logically and make sense of mathematics" (p.11). This is the lead sentence and is the text of the pull-quote as well. Reasoning is thus described as a process that *aids* thinking rather than *being* thinking. A significant portion of the text describes the importance of deductive and inductive reasoning in making and justifying conjectures and finding patterns. The value of such reasoning is described: "The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines" (p. 11).

Technology rates about a half page, with extensive use of bullets. The assertion is made that technology can help students learn many of the specific outcomes. Further, "Technology enables students to explore and create patterns, examine relationships, test conjectures, and solve problems" (p.12). Listed among the potential uses of calculators and computers are:

- explore and demonstrate mathematical relationships and patterns
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of basic facts
- model situations
- develop number and spatial sense.

The pull-quote in the Technology section reads, "The use of technology should not replace mathematical understanding" (p. 11).

The last of the seven processes is Visualization. The description quotes Armstrong to describe "thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (p. 12).

Present in this section are several descriptions of the role in visualization in supporting and connection other processes and ideas.

> Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representation of numbers…
>
> Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw & Cliatt, p.1989, p. 150).
>
> Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations. It is through visualization that abstract concepts can be understood by the student. Visualization is a foundation to the development of abstract understanding, confidence, and fluency (p.12).

3.5 The Lenses on the Mathematical Processes

The Lens of Perspective

This part of the Framework document has several places where some powerful ideas are expressed. However, we are warned immediately against becoming too hopeful. The first sentence of the Conceptual Frame subsection exposes how instrumental the overall Framework document is when it states that “The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes” (p.7). The weakness of the word ‘influence’ is damning. In a mathematics curriculum document where you are discussing what mathematics is, and what it should be, how is it that the processes and the nature of math do not direct or create outcomes, but merely influence them? The question arises: If the Nature and Processes of Math are not generating or directing the selection of outcomes, then what is? Not the future of the children, or even of society, but a straight and rigid recreation of what has already passed us by. The structure of the rest of the document is consistent with the dominance of an outcome-based linear and non-organic structure. In the chart given, the topics (a relic from the structure of previous documents) are boxed off from the Nature of Mathematics and the mathematical Processes (though faint arrows penetrate the box) and the outcomes are in another section entirely, kept well away from such messy things as what mathematics itself may be. We will see examples of powerful and useful statements in this section of the Framework document, but underlying it all is the theme of instrumentalism and social efficiency.

The seven (not more, nor less!) mathematical processes given are generally unexceptional. An interesting statement creeps in when the document mentions the processes should be ‘incorporated in learning’ and used as a ‘focus’ for instruction. Here is a reference to mathematics made flesh and the use of the word ‘focus’ also references the organic sense rather than an industrial image. Regrettably, this embodied sense of mathematics does not remain consistent.

The Connections process is vital; our brain thinks through connecting. Here is described the connection of mathematical ideas to ‘real-world phenomena’ thus revealing again that it is the utility of mathematics in our current social circumstances that is most valuable. This is at odds with Dewey’s view of preparation, indeed educational in general. Further, Dewey’s notion of growth must be considered when the document says that student ‘begin’ to view mathematics as utilitarian. Is the curriculum designed to help the students grow from this beginning? So while the Connections process is vital, the language belies it.

While I don't consider Technology a 'process' of mathematics, this section talks about creating patterns and modelling, two powerful things people do with mathematics. The claim that technology can reinforce the learning of basic facts could do with some support, especially in light of the Mental Math section that talked about calculator dependence. Still, here and in Visualization, the seventh Process, there is language use that suggests students will grow in power and capability, the essence of education.

The world view of the curriculum shows up strongly in the Problem Solving process, when the example given of a contextual problem is to find the maximum profit given manufacturing constraints. Of course mathematics can apply to this part of our world, and there is much right in using math to reduce waste or inefficiency in how we live. But where is another example that shows respect for the interrelatedness of our biosphere? Or the effects on societies of offshore manufacturing? Such examples could be included as well to give a broader set of perspectives on how we do live, and how we might live. Again, there is an implicit message that we should reproduce the existing ways of thinking, viewing and acting. A theme that recurs in the Framework document is that Mathematics is about doing things the way they already being done. Even if a person holds the view that education should be heavily skewed to practical issues in the here and now, this document does little in terms of encouraging innovation. This curriculum is about unconsciously reproducing what we are already doing, regardless of the implications of that action.

The Lens of Complexity

The five principles of complexity fare better in this section than in the previous. Internal diversity and organized randomness are supported to some degree by the statements in

Mental Mathematics where students are to be liberated from reliance on calculators and that they should be able to used 'nonstandard' techniques. It also makes a good show of representing decentralized control when it states "educators need to *orchestrate the experiences*" (p.9). Compare this to the deeply centralizing language of the Purpose earlier in the Framework document! As well, the Communication Process usefully references both interpersonal and intrapersonal communications which are important for fully utilizing neighbour interactions.

3.6 Describing the Nature of Math, Pathways, Outcomes and Instructional Focus

For the next four sections, several of which are brief, I shall consolidate the description, and then examine them through the Lenses of Perspectives and Complexity.

3.6.1 The Nature of Mathematics

The next major subsection in the Introductory Section is the Nature of Mathematics. This subsection, like the one before it, is divided into several parts. We might consider the nature of something to be a discussion of its inherent qualities and this section begins,

> Mathematics is one way of understanding, interpreting and describing our world. There are a number of characteristics that define the nature of mathematics, including changes, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.
>
> Change
>
> It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.
>
> Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions,

> students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change….
>
> Students best experience change to their understanding of mathematical concepts as a result of mathematical play (p.12-13).

There is a quote from Steen included and the pull-quote states,” Change is an integral part of mathematics and the learning of mathematics” (p.12).

Constancy is the next component of the Nature of Mathematics section. It is short and I reproduce it here.

> Many important properties in mathematics do not change when conditions change. Examples of constancy include:

- the conservation of equality in solving equations
- the sum of the interior angles of any triangle
- theoretical probability of an event

> Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables student to solve problems such as those involving constant rates of change, lines with constant slope, or direct variation situations (p. 13).

The pull-quote is the first sentence following the bullet points.

Number sense receives several paragraphs of attention including a quote from the British Columbia Ministry of Education about the primacy of number sense in numeracy. The second paragraph begins “A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms” (p. 13) and in the third paragraph,

> “Number sense develops when students connect numbers to real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about number. Evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing mathematically rich tasks that allow students to make connections” (p.13 - 14).

The nature of mathematics – what it is – is most directly stated in the first line of the mathematic process, Patterns. “Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics…” (p.14). The order of the descriptors in this sentence about patterns is interesting, about which more later: “Students need to learn to recognize, extend, create, and apply mathematical patterns (p.14). ‘Relationships’ is another component of the Nature of Mathematics, and like Patterns before, its first sentence says something of what mathematics may be, “Mathematics is used to describe and explain relationships. Within the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables, and concepts” (p. 14). It is mentioned that technology will be used to help find relationships.

Spatial sense is given as “the representation and manipulation of 3-D objects and 2-D shapes” (p.14). This section describes how spatial sense is increased through experiences with concrete and physical models. An interesting choice made this section is that the pull-quote is not selected from the text but states, “Spatial sense offers a way to interpret and reflect on the physical environment” (p. 14), a summation of the main text.

Uncertainty is the final component given of the Nature of Mathematics. “In mathematics, interpretations of data and the predictions made from data inherently lack certainty” (p.15). There is elaboration on how data generated by experiments is uncertain, and on the importance for students to make decisions about the reliability of interpretations. There is the assertion that, “the language of mathematics becomes more specific and describes the degree of uncertainty more accurately” (p.15). Presumably, ‘accurately’ means that probabilities for more complex situations are accessible.

3.6.2 Pathways and Topics

This section is meant to describe the possible choices for mathematics program for students. Very roughly, we can describe this as grade 9, the Essentials Math stream, the Applied Math stream and the Pre-calculus math stream. It will be useful to have a considerable amount of this text available for later discussion; accordingly, I will quote a significant portion of it.

> The *Grades 9 to 12 Mathematics Manitoba Curriculum Framework of Outcomes* includes topics rather than strands as in the *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes*. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.
>
> There is one course available for students in Grade 9. In Grade 10, students may chose between two courses or may choose to take both courses. In Grade 11 and 12, students have four choices for courses and may take one or multiple courses.
>
> Note: Accounting 30S and Accounting 40S are not referenced in this framework but can be used as a mathematics credit for graduation in Manitoba (p. 15).

There follows a diagram of the expected course progression as one selects the Essential, Applied or Pre-Calculus stream, and the goals and design of the Pathways

> Goals of Pathways
>
> The goals of all pathways are to provide prerequisite attitude, knowledge, skills, and understandings for specific post secondary programs or direct entry into the workforce. All three pathways provide students with mathematical understanding and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. …Students, parents, and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.
>
> Design of Pathways

> Each pathway is designed to provide students with the mathematical understandings, rigour, and critical-thinking skills that have been identified for specific post-secondary of study and for direct entry into the workforce.
>
> The content of each pathway has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their for High School Mathematics: Final Report on Findings* and on consultation with mathematics teachers (pp. 15,16).

Following this are three paragraphs describing the basics of the Applied, Essential and Pre-calculus Mathematics pathways.

3.6.3 Outcomes and Achievement Indicators

This section contains paragraphs describing the nature of the General Outcomes, the Specific Outcomes and the Achievement Indicators that are given in the Learning Outcomes section of the Framework document. Following the paragraph about Achievement indicators are bullet points intending to clarify the meaning of words. Again, a substantial portion of the text will be useful.

> The common curriculum framework is stated in terms of general outcomes, specific outcomes and achievement indicators.
>
> **General outcomes** are overarching statements about what students are expected to be able to learn in each course. They remain consistent through several years of schooling.
>
> **Specific outcomes** are statements that identify the specific knowledge, skills, and understandings that students are required to attain by the end of a given course. Some outcomes will be revisited several times during a course to allow for connections to be made to other outcomes in the course.
>
> **Achievement indicators** describe the depth and scope of each specific learning outcome. They are not presented in any particular order and need not be specifically addressed in the classroom. However, students need to understand the outcomes at least to the depth indicated by the indicators. Therefore, the achievement indicators are sufficient as a basis for

> instructional design and assessment, and will form the basis for provincial assessment as appropriate.
>
> In the specific outcomes and achievement indicators, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome.
>
> In the specific outcomes and achievement indicators, the phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome. (pp.17, 18).

3.6.4 Summary

The final component of the Conceptual Framework is the Summary. It is brief and quoted in full:

> The Conceptual Framework for Grades 9 to 12 Mathematics describes the nature of mathematics, the mathematical processes, the pathways and topics, and the role of outcomes and achievement indicators in Grades 9 to 12 mathematics. Activities that take place in the mathematics classroom should be based on a problem-solving approach that incorporates the mathematical processes and leads students to an understanding of the nature of mathematics (p. 18).

3.6.5 Instructional Focus

This section is one page in length and uses bullet quotes extensively, several of which require scrutiny.

Learning in Mathematics is richer and more engaging when instruction and assessment develop a direct relationship between conceptual and procedural understanding. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful.

- Teachers should consider the following points when planning for instruction and assessment.
- Outcomes need to be organized into units of study. Each course suggests at least one possible order but teachers need to decide which order works best in their unique context.
- The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches, and should support the intent of the outcomes.

- Whenever possible, meaningful contexts should be used in examples, problems, and projects. …

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related (p. 19).

Having described these sections of the Framework document, I turns to the interpretation and evaluation of the these.

3.7 The Lenses on the Nature of Math, Pathways, Outcomes and Instructional Focus

The Lens of Perspective

The Nature of Mathematics is a core piece of a mathematics curriculum. By taking some care to decide what is being studied, we have a better chance to focus on helping students to recognize appreciate and work with it themselves. The lead paragraph in the Manitoba Framework document is not very useful for determining what mathematics is. But there are several powerful ideas that can be used by a teacher of mathematics, and some of these I use in my curriculum model later.

There is one remark to be made regarding the components of the Nature of mathematics. In Patterns, application is listed after extending and creating. This doesn't fit Bloom's taxonomy and if inadvertent, it may represent the bias of the entire document to an instrumental perspective.

In Pathways and Topics, the term 'pathway' suggests a linear perspective. That we are bound in time is unassailable, and any set of actions over time may plot a path. But the metaphor suggests that there is only one way to accomplish the learning of mathematics in schools. Here also is the statement that "topics rather than strands" (p. 15) are used in high school. While the metaphor of curriculum as strands carries various associations, (a weaving together, a flow in a direction, and an entanglement of ideas) it is still fundamentally a linear construct. In one respect, this change moves us away from the linear image of strands. But the replacement is not something of a richer dimension but topics, a series of discrete stopping points, broken apart relics from previous curricula.

Also in Pathways and Topics is the information that the Accounting courses are not referenced in this document, but are accepted as math credits. What are the implications of this? One perspective could be that such a circumstance means that the curriculum document does not value accounting because it is not part of mathematics, and doesn't rate description here. Another view is that, (since credit is given despite accounting not being mathematics), a view of business or practicality trumps all else. That the latter view is meant is supported by the statement in the Goals of Pathways that asserts that the goals of *all* pathways is to provide for *specific* post secondary programs or direct entry to the working world. No subtle shadings of meaning here.

Under the heading of Outcomes and Achievement Indicators the Framework document begins to show its teeth. As one moves from general goals to Specific outcomes to achievement indicators the description becomes fuller and more detailed. So as the ideas become less wide ranging and important, the support and information about them increases. The specific outcomes want "specific knowledge, skills and understandings that students are required to attain." (p.17) and the Achievement Indicators warn that, "the achievement indicators are sufficient as a basis for instructional design and assessment, and will form the basis for provincial assessment as appropriate (p.17). This is a powerful statement, given that none of the General or Specific outcomes earn this status, and not even the processes or the Nature of Mathematics itself is referred to as *sufficient for instructional design* and assessment.

Instructional Focus has two items of interest. First is the statement that outcomes need to be organized into units of study. That organization is required is a commonplace, more important is that the only way to do this is through ‘units’. The term is sufficiently vague as to be defendable perhaps, but given that teachers have been referring to discrete masses of content as unit for ages, this is scarcely a way to support teachers in changing their external locus of control. Secondly, this section shows the dominance of the specific outcome mindset saying that the seven processes must be integrated and “*should support the intent of the outcomes*” (p.19) [Italics mine]. To me, this is the exact reverse of how a curriculum should be conceptualized!

There is much that seeks to engage and support students and teachers at the centre of the Conceptual Framework, but the very first sentence and the warnings near the end show the middle to be so much filler.

The Lens of Complexity

The Nature of Mathematics provides the potential for teachers and students to view their mathematics from a variety of perspectives, thus increasing the organized randomness of the system. That is, within the boundaries of what we call mathematics, there are many ways to see and create, and the capacity for individuals to contribute to the conversations based upon their own diverse strengths is notable. Here at least, we see a role for internal diversity and for organized randomness. Along with Communication in the Processes of math, we have strong support for resiliency and some support for neighbour interactions (at the levels of teachers and students at least). The hope for decentralized control is dashed completely with the statements about provincial assessment and the fact that the outcomes are merely influenced by what mathematics is. Having said that a few of the necessary conditions for complexity thinking may exist, the ideas themselves are not featured at all, and one must be familiar with the notions and be willing to take the time to seek out these vague potentialities. So far, despite areas of promise, the Introductory section of the Framework document cannot be considered conducive to seeing the world or mathematics from the perspective of complexity.

3.8 Describing the Outcomes section

These portions of the Framework document provides sets of outcomes to be used as a common base for defining mathematics curriculum expectations that will be mandated in Grades 9, 10, 11, and 12. For teachers wanting to know 'what to teach' , and the expected depth of topics as represented by the achievement indicators, one can go to the link for any given course, thereby bypassing the background, introduction and the conceptual framework.

I will reproduce a few pages from three different courses to serve as samples. The first example is from the Grade 10 Introduction to Applied and Pre-Calculus mathematics, and the second regarding quadratics from the Grade 11 Pre-Calculus.

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 10 – Introduction to Applied and Pre-Calculus Mathematics

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Measurement	**General Outcome:** Develop spatial sense and proportional reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators* ***may*** *be used to determine whether students have met the corresponding specific outcome.*
10I.M.1. Solve problems that involve linear measurement, using ▪ SI and imperial units of measure ▪ estimation strategies ▪ measurement strategies [ME, PS, V]	▪ Provide referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard, and mile, and explain the choices. ▪ Compare SI and imperial units, using referents. ▪ Estimate a linear measure, using a referent, and explain the process used. ▪ Justify the choice of units used for determining a measurement in a problem-solving context. ▪ Solve a contextual problem that involves linear measure, using instruments such as rulers, tape measures, trundel wheels, micrometers, or calipers. ▪ Explain a personal strategy used to determine a linear measurement such as the circumference of a bottle, the length of a curve, or the perimeter of the base of an irregular 3-D object, and explain why it works.
10I.M.2. Apply proportional reasoning to problems that involve conversions within and between SI and imperial units of measure. [C, ME, PS, T]	▪ Explain how proportional reasoning can be used to convert a measurement within or between SI and imperial systems. ▪ Solve a contextual problem that involves the conversion of units within or between SI and imperial systems. ▪ Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.

General and Specific Outcomes ■ **101**

Source: Manitoba Education, Citizenship and Youth. *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes.* Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2009. Available online at <www.edu.gov.mb.ca/k12/cur/math/framework_9-12/index.html>. Reprinted with permission. August 16, 2012.

Grade 11 Pre-Calculus Mathematics

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Relations and Functions *(continued)*	**General Outcome:** Develop algebraic and graphical reasoning through the study of relations.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators* ***may*** *be used to determine whether students have met the corresponding specific outcome.*
11P.R.4. Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including ▪ vertex ▪ domain and range ▪ direction of opening ▪ axis of symmetry ▪ *x*- and *y*-intercepts [C, CN, PS, R, T, V]	▪ Explain the reasoning for the process of completing the square as shown in an example. ▪ Write a quadratic function given in the form $y = ax^2 + bx + c$ as a quadratic function in the form $y - a(x - p)^2 - q$ by completing the square. ▪ Identify, explain and correct errors in an example of completing the square. ▪ Determine the characteristics of a quadratic function given in the form $y = ax^2 + bx + c$, and explain the strategy used. ▪ Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$. ▪ Verify, with or without technology, that a quadratic function in the form $y = ax^2 - bx + c$ represents the same function as a quadratic function in the form $y = a(x - p)^2 - q$. ▪ Write a quadratic function that models a situation, and explain any assumptions made. ▪ Solve a problem, with or without technology, by analyzing a quadratic function.

General and Specific Outcomes ■ **149**

Source: Manitoba Education, Citizenship and Youth. *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes.* Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2009. Available online at <www.edu.gov.mb.ca/k12/cur/math/framework_9-12/index.html>. Reprinted with permission. All rights reserved. August 16, 2012.

And the third example is drawn from Grade 12 Applied Mathematics.

Grade 12 Applied Mathematics

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

Logical Reasoning *(continued)*	**General Outcome:** Develop logical reasoning.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators* ***may*** *be used to determine whether students have met the corresponding specific outcome.*
12A.L.2. Solve problems that involve the application of set theory. [CN, PS, R, T, V]	■ Explain how set theory is used in applications such as Internet searches, database queries, data analysis, games, and puzzles. ■ Provide examples of the empty set, disjoint sets, subsets and universal sets in context, and explain the reasoning. ■ Organize information such as collected data and number properties, using graphic organizers, and explain the reasoning. ■ Explain what a specified region in a Venn diagram represents. ■ Determine the elements in the complement, the intersection or the union of two sets. ■ Identify and correct errors in a solution to a problem that involves sets. ■ Solve a contextual problem that involves sets, and record the solution
12A.L.3. Solve problems that involve conditional statements. [C, CN, PS, R, T]	■ Analyze an "if-then" statement, make a conclusion, and explain the reasoning. ■ Make and justify a decision, using "what if?" questions, in contexts such as probability, finance, sports, games, or puzzles, with or without technology. ■ Determine the converse, inverse and contrapositive of an "if-then" statement; determine its truth; and, if it is false, provide a counter-example. ■ Demonstrate, using examples, that the truth of any statement does not imply the truth of its converse or inverse. ■ Demonstrate, using examples, that the truth of any statement does imply the truth of its contrapositive. ■ Identify and describe contexts in which a biconditional statement can be justified. ■ Analyze and summarize, using a graphic organizer such as a truth table or Venn diagram, the possible results of given logical arguments that involve biconditional, converse, inverse, or contrapositive statements.

160 ■ Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes

Source: Manitoba Education, Citizenship and Youth. *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes.* Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2009. Available online at <www.edu.gov.mb.ca/k12/cur/math/framework_9-12/index.html>. Reprinted with permission. All rights reserved. August 16, 2012.

In each case, there is one or more Specific Outcomes listed on a page and Achievement Indicators are given on the right beside the outcomes. Underneath each Specific outcome in square brackets are the mathematical processes that teachers are to employ.

3.9 Lenses on the Outcomes Section

The Lens of Perspective

The Manitoba high school curricula are composed mostly of specific outcomes. We have seen the Introductory section has 19 pages for describing the nature of mathematics and the processes that lead to greater understanding. By contrast this is followed by approximately 170 pages giving 'Prescribed Learning Outcomes' with associated

Achievement Indicators. The introduction is meant to describe what mathematics is in its nature and processes, and to change instruction by indicating areas of increased and decreased emphasis. However, teachers understand the tacit message sent by the disproportion of content to philosophy, and the separation of the content from the philosophy. Then there is the far less tacit message sent by exams that focus on the specific outcomes.

Because of the importance of the curriculum as a source of ideas, guidance, cultural norms and authority, examining curriculum is a meaningful part of improving the experience of learners of mathematics. At the heart of my own dissatisfaction with the current high school curricula, is that I often feel there is insufficient time to develop ideas more fully for students, or for students to develop their own particular lines of interest. The reasons for this feeling of constraint come from the narrow objectives, and cultural and pragmatic beliefs that I should be 'covering' the curriculum.

Let us concern ourselves with specific details. Consider the sample from Grade 10 Introduction to Mathematics and Pre-Calculus course. The intention of the curriculum is revealed by a lack of vision. Here, the purely economic focus is shown by the inclusion of the Imperial system, which is not the official measurement system of our country, in a curriculum ostensibly aimed at those who will go on in science, where world-wide the Imperial system is eclipsed by the International System of Units (SI). Clearly, the principles of measurement can be extended beyond current economic utility to many applications, including social equity and the environment, but by excluding such mentions from the part of the curriculum that most teachers read as 'the content', we miss this opportunity to educate our students about ways of thinking that are not purely based on production and consumption. This focus on current economy so pervades all the documents that even in the Grade 12 Applied mathematics section on Set Theory, finance is suggested as a legitimate context, but environment and social justice issues are not. (And here I pass over the other topics included in various courses such as Personal Finance, Home Finance, Vehicle Finance and Business Finance).

In the second sample from Grade 11 Pre-Calculus, I have selected a topic that does not directly mention matters economic or financial. Yet instrumental thinking still holds sway. The Achievement indicators are in the main directed to performing computations and operations. There is one that asks for reasoning about a computation, one that suggests finding errors, where presumably reasoning and analysis would be involved and the last one, as a kind of throwaway, says that we should solve problems. The emphasis on particulars leaves behind the more general and ostensibly the longer lasting and most portable notions.

Consider as well the negligible support a teacher receives for integrating the processes of math, just a list of letters given under the specific outcomes. This is the section that teachers use for planning lessons, and the biggest and most important ideas are relegated to hieroglyphics, while the minutia of particular skills is enlarged upon. If the Processes of mathematics and the Nature of Mathematics are truly worthy of consideration, very much more is expected.

An even more general concern I have is the focus on Specific Outcomes as the only possible apparent means of communicating to teachers what students can learn. Carol Wien and Curt Dudley-Marling (1998) have shared their criticisms of the effect of systems and curricula on teaching and learning as they examine a move in Ontario to outcomes-based learning.

> Outcomes-based learning, as described in recent policy documents in Ontario, offers a narrow, controlling vision of teachers and learners, and of diversity and ecology. Although the documents profess an integrative vision, they provide lists of "expectations" that students are required to reach. … [We] criticize these documents for their failure to acknowledge the experiences of either learners or teachers, for superficial treatment of diversity and ecology, and for an internal contradiction that imposes a rigid, lock-step system upon an integrative vision of teaching and learning. Their attempt to describe education using the metaphor of lists, as in lists of parts to be assembled in a machine, is outdated (p.406).

To them, the preamble of the Ontario curriculum has some sense of a holistic discussion of learning, but this sense is undermined by the structure of what follows. "[T]he channelling of an integrative philosophy through the device of lists of outcomes or expectations sets up a rigid, static system" (p. 405). This vision they deplore is essentially a corporate model:

> If knowledge and skills should be common to all students, how does a society decide which (or whose) knowledge and skills will be privileged? Cultural values are embedded in ways of doing things, and an instrumental focus privileges corporate values. Why should education be restricted to the values of the corporate elite? The recent Ontario documents convey no doubt about whose knowledge is important, nor do they acknowledge that this could be a site of debate (p.411).

In this Framework document, to some degree in the introductory section, then wholly at the 'business end' the principal thrust is for mathematics to be a useful way for graduates to participate in the economy, and whether that economy is based on heavy industry or on nano-technology is perfectly immaterial. It is the narrowness of the vision, a vision of uncomplicated social efficiency, of math external to the student, which makes this a document of the past, not the future. It is a way of thinking that has produced slavery, CFCs, gridlock and global warming. Now, it would be ludicrous to suggest that we don't want students to be able to provide for themselves in the here and now, but there is more in mathematics than economic utility. Where is the understanding of social justice? Where is the conception of the beauty, and history of mathematics – notions that have spurred on the development of mathematics, people and societies for millennia? And where too is the understanding of the interconnected systems that make up the ecosphere? The fact that a curriculum meant to deal with the future only really reflects the past shows that the current curriculum is at cross purposes with itself. The inconsistency in the introductory section demonstrates that the current Manitoba mathematics curriculum has not really been examined as to its true role in what it brings to students and teachers. It has what I shall later define as a default intention.

The Lens of Complexity

As I do, Wien and Dudley-Marling (1998) reject Outcomes-based for curricula as being linear and fragmented. The curriculum does not show an understanding of the relationships of the subject matter and the humans who have come together to consider it.

> In other words, documents like The Ontario Curriculum, Grades 1-8 describe but a tiny part of the whole and do not sufficiently recognize schools as ecologically interrelated to their surroundings or as feedback systems (p.412).

From isolating the course content into topics, to making it linear with outcomes and by not responding to feedback, this curriculum document cannot be considered as one that will encourage spontaneous adaptable and emergent growth.

In the matter of internal diversity, components of the system recognize that diversity exists, and then proceeds to insist that every student be responsible for every outcome to the same degree. This is not about high standards; this is about producing human silage to be fed to the existing economic feedlot. This system does not use the diversity in the system to increase its range of possible responses.

The continued effective working of a system depends on resiliency, the capacity for some elements to take over the functions of others in the case of disaster. The insistence on rigid conformity ensures a strong type of resilience. This conformity sees students as stem cells with capacity to fill in wherever required by the existing social body. The system shows very high resiliency.

Decentralized control is not exhibited in any great degree. There are places where teachers are called upon to exercise their professional understanding, but it is clear from the specific outcomes, the highly specified Achievement Indicators and the direct warnings about provincial assessments that this is not to go further than the classroom. There is no intention to have the students and teachers deeply and meaningfully help to create and recreate what mathematics can be.

Organized randomness requires that there be a great deal of latitude to achieve some end inside of boundaries. The boundaries are more to describes what shouldn't be done as opposed to what should, thereby permitting the other principles to recombine in novel ways. This is not prevented by the current curriculum system – teachers may meet with each other for example – but there is scarcely the support or encouragement for it either. This system does not display organized randomness to any significant degree.

Neighbour interactions are necessary to permit different components of the system, whether ideas or people of organisms to interact and influence each other's behaviour. The focus on commonalities means that if elements should meet there is a good chance they will be able to communicate. There is little opportunity for the system as a whole to be changed by these interactions. There is a moderate degree of neighbour interaction in this system.

Taken together, the Manitoba High school Mathematics curriculum, as represented by the Framework document, fails to show multiple or progressive perspectives and lacks most of the requirements for becoming an intelligent system.

3.10 Conclusion

Carol Rodgers (2002) describes a "triangle of factors (i.e., teacher/ teaching, learner/learning, and content - what Hawkins (1974) called the "I-Thou-It)" (p. 858). One word in Hawkins' description suddenly crystallized the sense I want in a curriculum. For me, the word *Thou* has sacred connotations, an elevating, serious and respectful tone. *Thou* has been lost from everyday speech and, to me, *Thou* retains an exquisitely tender formality, an awakened sense of grandeur with an intense personal intimacy. When teachers and students commune in mathematics, it is this sense of tenderness and personal strength that I want the teachers and students in our province to feel. When I speak of a dignified or abundant curriculum, it is a curriculum that provides the choices and structures and that recognizes the individuality of each person. A dignified curriculum respects the depth of connections we can see in mathematics, it promotes creative thought and encourages a sense of joy and of personal efficacy. I want a curriculum that carries the dignity of *Thou*, the immediacy of *I*, and the beauty and life of all three. I want a curriculum that contributes to a vital experience for all who dwell in it, so while my focus is on the *It* as it affects the *I*, both are in the service of *Thou*. As we provide and help teachers understand a curriculum of exuberance, their own initiative and sense of personal efficacy will change. With an expectation to develop their own ideas and those of students, the locus of authority clearly shifts toward students and teachers, and this shift may promote the type of flexibility of thought, curiosity and eagerness that will dignify mathematics and those who seek to understand it.

The integrity of this system is paramount. Just because we can learn valuable things from examining individual parts of a system doesn't mean that each component is independent. The *I–Thou–It* are pedagogical quarks. They exist as separate entities for describing the educational system, but individually they are not the vital electron. The electricity of learning only happens when teachers, students and significant mathematics cohabit the same space. Indeed, the absence of one denies the existence of the others. Is a teacher required? Yes. Consider the girl who learns something on her own. She is her own teacher; she employs strategies, knowledge and principles to effect change in herself as learner. Could she not do so, she could not learn.

Not everyone feels a change in curriculum is required to restore dignity and exuberance to the *I*, *Thou* and *It*. Thomas Falkenberg, (personal communication, November 10, 2007), has suggested that there is flexibility in existing documents that make it entirely possible for teachers to create liberating, socially just and mathematically effective experiences for their students. Jardine, Friesen and Clifford (2006) argue for abundance in a similar way. They cite examples from an existing curriculum in Alberta, in an existing school, to show that freedom and abundance is more a function of the capacity of teachers to let go of a mindset of scarcity than actual paucity in the material in the curriculum. They first describes their apprehension of a 'culture of scarcity'. They suggests the current situation in many classrooms is far from one of plenty. For many teachers there is a feeling that there is never enough time; there is a hunger for something *more*. "Once we concede, willingly or otherwise, to education understood as a regime of scarcity, *the desire for more must be maintained if the ravenous sway of scarcity is to be maintained*….Imagined from within the bounds of scarcity, abundance becomes near-monstrous" (p.5). They claim that this regime of scarcity is promoted by the current culture of education, (and of the ubiquitous marketing we all face), but that there is another way of having a rich experience for students, independent of a curriculum, that depends more on how we think, rather than what is thought about. That is to say, the way to his desired abundance is one of *being* more than one of *knowing*:

> We are suggesting … that the topics entrusted to schools *are* abundant, and, therefore, suggestions of multiplicity and diversity are not opulent educational *options* regarding how we might come to know topics that are in reality simple and manageable. Rather, multiplicity, diversity and abundance define the way in which things *are,* and therefore, the great array of the ways of traversing a place that students bring to the classroom *is precisely what living things require if they are to be "adequately" understood in their abundance.* In short, abundance is an ontological issue, not an epistemological one (Jardine et al., p. 88).

In this light, *any* curriculum may be dignified and abundant, and just to the extent that it is interpreted as such by the teacher and students who use it. Fundamentally, it is the teacher and students of a class that co-create the dignity and joy, the *being* of a curriculum, with the teacher having the lead role. The question becomes, how is a particular curriculum *likely* to be interpreted? Is it *likely* to bring to its users a sense of possibility, interconnection and efficacy? How does the structure of a curriculum and the environment it creates around itself further create its own interpretation?

Through my own experience and the experience and anecdotes of colleagues over the years, I suggest that many teachers see the curriculum in terms of a near absolute authority, immutable, and the type of abundance in the existing curricula as suggested by Jardine and by Falkenberg seems to be unavailable to many teachers in the current circumstances. The perspectives and beliefs of teachers are important. Created by culture in which a teacher works, his or her beliefs, both conscious and unconscious, play a role in the actions a teacher takes. Indeed, Davis, Sumara and Luce-Kapler (2000) discuss the role of the unconscious on all human perception and learning and argue that teacher beliefs, their ‘embodied sensibilities’ are important:

> For the most part, teaching actions are not consciously considered and deliberately selected. There are simply so many demands on the teacher’s limited attentions that most of what they do has to be “automatic”. This is not to say that teaching is thoughtless. Rather, it is an assertion that teaching is an ongoing enactment of embodied sensibilities, as opposed to a sequence of conscious decisions (p. 41).

It is for the redirection and nurturing of these ‘embodied sensibilities’ that a different curriculum structure is most needed. By focusing with specificity on content and algorithms, the Manitoba curricula carry the sensibilities of an instrumental understanding far more powerfully than that of relational understanding. Richard Skemp (1976) describes understanding as being relational (involving primarily conceptual understanding) or instrumental (involving primarily sets of procedures). If a curriculum focuses on one at the expense of another, it seems reasonable that teachers will view mathematics, or at least the pedagogy of mathematics, as being mostly relational or instrumental as the case may be. Particularly if a curriculum is focused on the acquisition of algorithms or specific pieces of content, the understanding may be limited:

> Instrumental understanding I would until recently not have regarded as understanding at all. It is what I have in the past described as ‘rules without reasons’, without realizing that for many pupils *and their teachers* the possession of such a rule, and ability to use it was what they meant by ‘understanding” (p.9).

Pinar (2005) tells how Ted Aoki condemns this piecewise, mechanical, fundamentally economic way of seeing any subject, or of implementing any curriculum:

> Aoki is clear that traditionally the dominant way of understanding curriculum implementation was instrumentally. In 1974 he suggested: “A basic problem in implementation of programs may be found in the producer-consumer paradigm underlying the view of implementation.” He understood that instrumentalism was “a business metaphor,” and expression of our cultural “intoxication” with the technical power of science and technology. So understood, curriculum implement [sic] amounted to a subset of business management techniques, “one in which curriculum producers offer something to curriculum consumer”. (p.2)

The loosely connected, highly specific outcomes in the high school mathematics curricula reveal their instrumental culture. This focus on specifics is reinforced by the presence of high stakes exams in Grade 12 that focus on content.

Others are concerned about a broad 'coverage' of topics without much depth. In the U.S., the National Mathematics Advisory Panel released *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*:

> There seem to be two major differences between the curricula in top-performing countries and those in the U.S.—in the number of mathematical concepts or topics presented at each grade level and in the expectations for learning. U.S. curricula typically include many topics at each grade level, with each receiving relatively limited development, while top-performing countries present fewer topics at each grade level but in greater depth. In addition, U.S. curricula generally review and extend at successive grade levels many (if not most) topics already presented at earlier grade levels, while the top-performing countries are more likely to expect closure after exposure, development, and refinement of a particular topic. These critical differences distinguish a spiral curriculum (common in many subjects in U.S. curricula) from one built on developing proficiency—a curriculum that expects proficiency in the

> topics that are presented before more complex or difficult topics are introduced.
>
> The Singapore standards (Singapore Ministry of Education, 2006) provide an established example of curriculum standards designed to develop proficiency in a relatively small number of important mathematics topics, as validated by a recent analysis (Ginsburg et al., 2005). The desirability of emphasizing fewer important mathematics topics in greater depth has also been recognized by some U.S. educators (2008, p. 20).

It is not just those who seek higher international scores that lead this charge for depth in curriculum. Gardner (2000) talks about how the structure of existing cultures of curriculum and instruction are lacking.

> There is that old devil "coverage". So long as one is determined to get through the book no matter what, it is virtually guaranteed that most students will not advance toward genuine understanding of the subject at hand.
>
> This state of affairs constitutes the strongest set of arguments in favor of a curriculum that examines a limited number of topics in depth. For only rich, probing, and multifaceted investigation of significant topics is likely to male clear the inadequacy of early misconceptions, and only further exploration of those topics, …makes it reasonably certain that more sophisticated understandings will emerge (p. 122-123).

By focusing with great specificity on content and algorithms, along with the practice of naming the units after the content material within, the Manitoba curricula carry the culture of an instrumental understanding far more powerfully than that of relational understanding.

The structure of the Manitoba mathematics curricula – completely mandated, content focused, bound by behaviourist outcomes thinking – creates what I perceive as a propensity to creating passive teachers and students, who do not see fully the nature of the discipline they teach or take. The current Manitoba curricula certainly mention, even encourage, problem-solving and some processes of mathematics in the introduction, but in the fundamental construction of these curricula, one sees that the documents are based on a narrow Tylerian model, with a bias to content acquisition. The promise of the rationale and introduction are belied by the very large list of specific content objectives. In this model, the content is king, and people – indeed mathematical thinking itself – must serve the content.

We have seen Dewey's rejection of an unexamined canon. Gardner shows the necessity for a curriculum deep in meaning, if more limited in scope. There must be time to engage more fully with some significant ideas. The curriculum is a central structure in current educational systems in Canada, and systems theory shows that the structure of a system is a major determinant of how well the system performs. Using Eisner's connoisseurship as exemplified by the Lenses of Complexity and Perspective I have demonstrated a system that is, within that framework, inherently flawed. The Manifold & Intention model I envision attempts to bypass this passive, unthinking approach to curriculum by having the teachers and students be, in part, a creator of curriculum. A curriculum with a clear and living philosophy and an opportunity for choice can influence both teacher and student views of authority, and how they view mathematics. Further, this type of curriculum can change the experience of the students who may be the mathematics teachers of the future.

4. The Manifold & Intention Model

Eisner (1998) seems to suggest the idea that those engaged in connoisseurship be capable of nuanced noticing. "Educational connoisseurship gives access to the complex and subtle aspects of educational phenomena, and it is through such access that educational critics secure the content they need to function as critics" (p. 86). For me, one powerful way to demonstrate the capacity for noticing is to be creative. I think it is important to share the burden of contributing to the area in which one comments. This is not normally required of critics or connoisseurs. Not every oenophile needs to pick grapes in an award winning winery; nor is every literary critic also an author. Still, if one can come to grips with the difficulties and challenges of the underlying art, and yet be able to stand adjacent to the work in order to comment upon that art, one must develop these nuanced noticings. To meet my own requirement, I have devised the Manifold & Intention model of curriculum. I introduce and elaborate on the model with examples of the five components.

Eisner's call for validity is satisfied by this elaboration of my Model along with its exemplars. For a conceptual document like this, references in the literature present the principal component of structural corroboration. But by demonstrating some of the possibilities of the Model, I present a different form of data, further solidifying this claim to validity. I also establish referential validity, as I use the model to bring an enlarged understanding and the possibility for enhanced conversations to the discussion of curriculum for mathematics teachers. Consensual validity is ultimately left to the reader, but the instantiation of portions of the model is an existence proof that something different can be done.

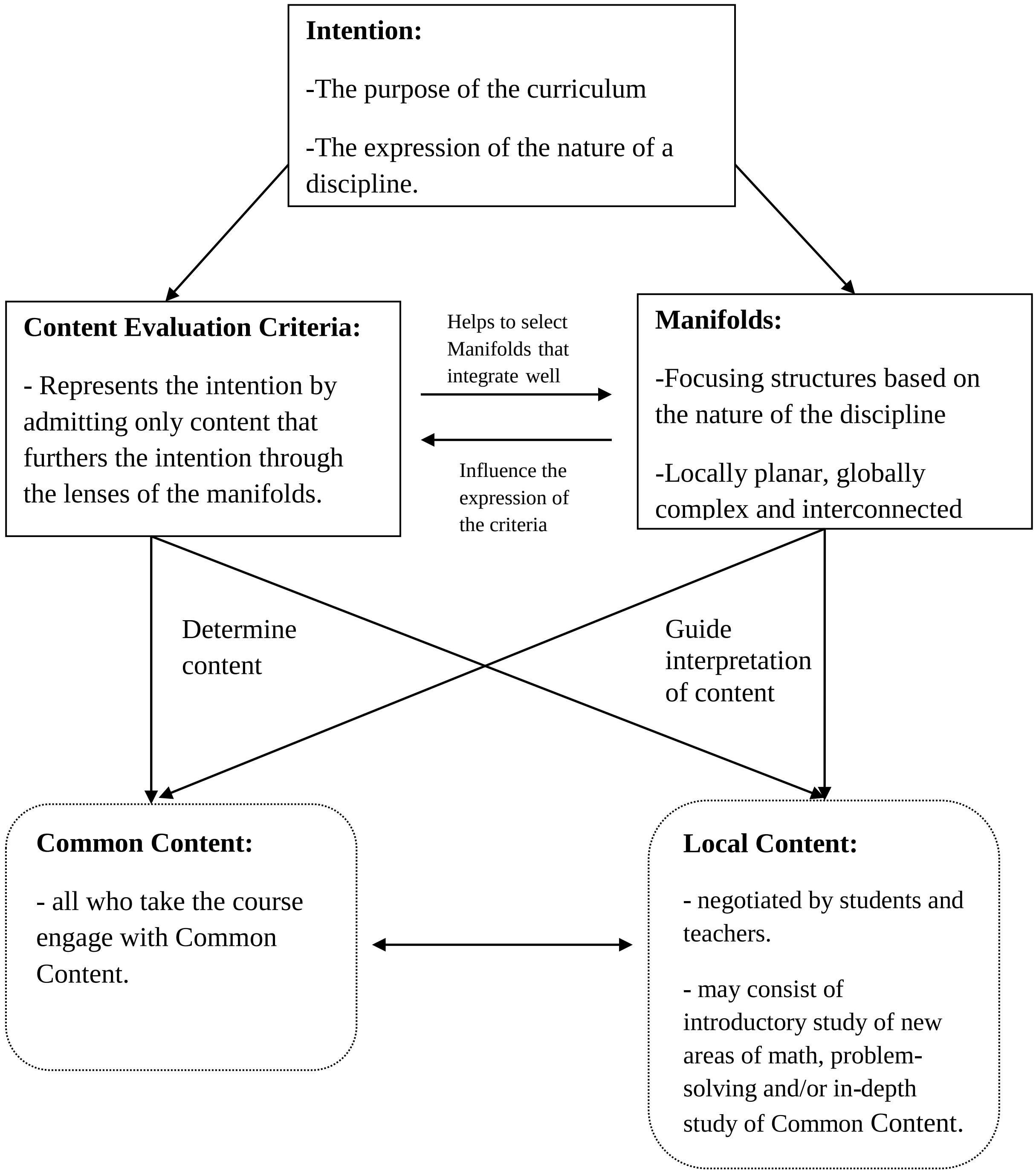
For me, a model is a way of seeing that has intended practical applications. Models may be as diverse as metaphors or sets of equations or computer programs. I want my considerations of curriculum to help teachers, students and authors of curriculum see ways of doing, and so I have been turning in my mind the broad tenets of a model for curriculum. Teachers' actions are influenced by the culture of the curriculum, and the existing Manitoba curriculum seems to me to be narrow, with an air of immutability. I propose this new curricular structure to overcome the concerns of too strong a focus on matters algorithmic and to give teachers the right and responsibility to decide some aspects of curriculum, thereby addressing issues of locus of authority, and of professional growth. Its genesis was a Eureka moment in a course on historical and contemporary curricula, but over time, I have viewed and reviewed the notions that surround it and its implications for my own curriculum theorizing, and potentially, that of others. To me, curriculum theorizing is important, certainly as an academic discipline and it fills a necessary role in contextualizing the present curriculum. But there is more. A valuable curriculum theory must influence for the better the lives of students. It must have some form of expression that can affect teachers' actions. It may be inspirational, struggled with, liberating, or provocative, but it must be able to touch people. It must be, in Grumet's terms, an event.

My model uses important ideas - here termed Manifolds - as a way to notice the world, with content selected to illuminate these ideas. Exemplars rather than outcomes provide scaffolding for teachers. The model can adapt itself well to a variety of roles: pondering the essential nature of a discipline; creating new curriculum documents; evaluating existing curricula; helping people see how the discipline is meaningful; and creating lessons. Manifolds are intellectual focusing structures to guide perception in some area. While this thesis focuses on mathematics, there is great scope for the model to be used in any discipline.

i) The underlying purpose – the Intention – of any curriculum should be made plain. The purpose of education (in my case, mathematics education) and the nature of the subject (mathematics, here) are expressed in the Intention. The Intention will be used as a touchstone in the development of the Content Evaluation Criteria and the Manifolds of the discipline.

ii) The Manifolds are perspectives for areas of content, processes, attitudes and ways of seeing the world in terms of some major component of the discipline. They are a way of perceiving experience. As themes, the Manifolds help to organize without fragmenting. Manifolds are mental structures with which to sense, and with which to make sense. Manifolds can be generated and used for any discipline: henceforth my focus shall be on the Manifolds for high school mathematics. In the highest realization of the Model, teachers have a direct say in the discussion about what the Manifolds will be. Figure 4.1 gives a schematic diagram of the model:

Figure 4.1- Schematic of the Manifold & Intention Model



iii) The Content Evaluation Criteria are a set of questions or requirements designed to have the specifics of the curriculum document – the 'content' – reflect the Intention and thus contribute to an integrated whole. The aim is to have an explicit structure that constantly directs attention to what mathematics is and our intended use of that mathematics when choosing content.

iv) The Manifolds and Content Evaluation Criteria together are used to develop the specifics of the Common Content. It is anticipated that all students taking the course will engage with the Common Content. The Common Content is presented as a set of linking diagrams/ exemplars/ activities, rather than as specific outcomes. It is anticipated that teachers will share the lessons and ideas they create via the Internet, adapting and innovating as time goes on.

v) A key feature of the Manifold & Intention Model is the presence of classroom-created content. This Local Content is decided upon by the students and teacher in collaboration. Here, teachers and students together will select/develop individual and small group plans for broader and/or deeper study. It is intended that teachers will share their locally developed content via the Internet, and that a presentation of exemplars and diagrams will be used there, along with the author's rationale for how the content fits the Manifolds.

The dignity of this structure for students and teachers comes from the purposes of the intention and from the freedom entrusted to teachers and students through the mechanism of the model. First, they have the opportunity and the responsibility to enact the Intention throughout the course. An insistence on returning to the nature of the subject and the purpose of teaching mathematics provides a legitimate opportunity for synthesis and evaluation in the thought process of teachers. This could give a sense of coherence and connection to their understanding and lessons. Second, when teachers consider all the possibilities for Manifolds, the interconnections and the question of what mathematics is, and how that may be best expressed, is open for discussion. Analytical thought, personal development, possibility and autonomy are supported. To choose which notions of mathematics are best illustrated by which Manifolds is a challenging and engaging task. In the third place, when teachers and students are integral in developing the details of the Local Content, they may be better able to experience both the content and the nature of the Manifolds in a holistic and creative fashion. People interacting this way, as agents of their own learning and development can occasion a sense of dignity. The dignity for mathematics comes from the power of people seeing the discipline not as a collection of minutiae, external to themselves, but as a way of interpreting our world as focused by the Manifolds. The joy and exuberance of this curriculum may be occasioned by people interacting in a participatory, active and potentially emergent fashion, due to a structure more organic and complex than the existing curriculum structure or curriculum documents. The mutability of the Manifold & Intention curriculum Model allows this complexity - in particular the shared responsibility of the jurisdiction, the students and the teachers in creating mathematics. Having the time and permission to examine mathematics in a conceptually powerful way permits a community of ideas to grow in ways that are hard to imagine in the current circumstances.

4.1 Intention in the Manifolds & Intention model

> What we should aim at producing is men [sic] who posses both culture and expert knowledge in some special direction. Their expert knowledge will give them the ground to start from, and their culture will lead them as deep as philosophy and as high as art.
>
> Alfred North Whitehead, 1967.

The conception of education as a social process and function has no definite meaning until we define the kind of society we have in mind.

John Dewey, 1916

The scholar who has most influenced my thinking in respect the notion of the Intention of a curriculum is John Dewey. From him I see a curriculum that seeks social ameliorization, which is not in opposition to social efficiency. He is the one that points out so clearly the necessity of increasing capacity as a fundamental aim of education. A democratic curriculum, both the guiding principles and the method of its creation, is one that invites dialogue and cooperation.

The first part of the Manifold & Intention curriculum model, the Intention, comes from the perspective of wanting to engage those involved with curriculum, particularly teachers, in a growth mindset. One way I seek to achieve this is to be more democratic in terms of providing openness and transparency - so much as a person can - in writing the introduction for a curriculum. To me, the writers of any curricular preamble or introduction must do more than explain how to use the rest of the document; they must give some account of the nature of the subject as seen by the writers, and the purpose of the curriculum. By expressing the nature of the subject and the curricular purposes as Lenses or options, teachers will be led to wonder what other options are available. Once people begin to inquire, the type of intellectual activity needed for creativity and adaptation is begun. The principal roles of a curriculum's preamble or introduction are to display modes of perceiving, and to awaken readers to the fact that other modes of perceiving exist. Central to my ideal for a curriculum is that all who use it will gain a greater capacity to adapt and foresee. In this way, there is some decentralization of control. No longer is it *the* curriculum document, but *a* curriculum document.

The Intention is used to help us determine the Content Evaluation Criteria and the Manifolds. Consider the aim(s) of schooling as expressed in the worldview of the Intention. If the aim is uncomplicated social efficiency then the Intention will state that, and the type of criteria we select may require the content to be focused on the utility of math in current economic contexts, such as buying or leasing fossil fuel powered vehicles. This is quite different than if we consider self-actualization as the primary aim of school. Here, the type of criteria we select may require the content to focus on the most beautiful mathematics we know, perhaps more geometry or number theory. The choice of Manifolds too is influenced by the Intention. If we think about the nature of math, we gain insight into what some Manifolds might be, say, Proof. If we think about the life of our students in math class, that gives us other insights into what some Manifolds may be, say, Patterns or Modelling.

Over time, the Intention will adapt and change. As a social construct, this is expected. One key feature of the Intention & Manifold Model, is that there is a place and an expectation that teachers and members of faculties of Education will contribute to the discussion. Currently in Manitoba, teams of teachers are invited to discuss details of the content, but the overall intent and structure of the curriculum is fixed beforehand.

4.1.1 Educational Purpose

Intention is a response made plain to Tyler's first question; what educational purposes should the school seek to attain? To encourage an examination of the role a curriculum may play in a teacher's perspective, a curriculum must declare its intention – its view of the purpose of education, of the nature of the region of study under consideration, and how it supports the development of that signature human strength: adaptability. The intention is a discussion of the assumptions and beliefs the initiators of the curriculum hold and wish to espouse. While no intention can be perfectly complete, making the intention clear asks all who use the curriculum to examine its assumptions and beliefs – and their own! With their underlying beliefs, - what Kessels and Korthagen (2001) call the 'gestalt' - now exposed, teachers can become more conscious of how and why they teach as they do. What prevents there being both an overt and covert intention? It is not only the integrity of those drafting the curriculum. The declared purpose of an intention is to attract attention to its assumptions and axioms, making it that much more difficult for the covert or unspoken to remain in the shadows.

Whether or not a curriculum has a formally expressed intention, a curriculum is not an apolitical, neutral construction. A curriculum is an expression of the members of a society to guide the noticing of its (usually young) members. Who has the privilege of determining what society deems worthy, and how that expression is enacted carries with it the potential for many unintended interpretations. These unintentionalities may be so inimical to the purposes sought that they slow down, undermine or even destroy what was originally desired. Our desire for a deep understanding of mathematics can be derailed by the structures comprising and surrounding the curriculum, including instrumental understanding and narrow outcomes. When the intention is undisclosed to the users of a curriculum, it may be unnoticed but it is still present, and still affects thought and action, but not in the conscious way that John Dewey would call meaningful. "When things have meaning for us, we *mean* (intend, propose) what we do: when they do not, we act blindly, unconsciously, unintelligently" (1916/2005, p. 20).

I define any curriculum that substantially ignores the depth of Tyler's first question to be a curriculum that has a *default intention*. A default intention is a position with a philosophy of education, of mathematics and of learning left unstated and unclear to the receivers (and possibly - in the haste of producing an inoffensive document - even to the framers!) of the curriculum, or a position that is contradictory, internally inconsistent or unexamined. The intention then is likely to be unwittingly and feebly established in the enactment of the courses - by personalities who either happen to be in the classroom or on some committee, by the current economic trends or by the vagaries of the existing political situation - without the guidance of coherent, examined public statement. For much of the last century, and even before, one main default intention has been what is called the Dominant or Traditional scheme (Dewey, 1938/ 1997, p.18): a curriculum of clearly stated objectives, selected with great input from subject area specialists that train people to perpetuate the existing social, economic and political conditions.

> The traditional school could get along without any consistently developed philosophy of education. About all it required in that line was a set of abstract words like culture, discipline, our great cultural heritage, etc., actual guidance being derived not from them but from custom and established routines (p.28).

In general, then, an effective intention will describe the purpose of education. It will state as clearly as possible the nature of the discipline. It will portray how the curriculum intends to lead teachers to help students to see patterns and relationships in an adaptive, balanced and complex way. Such a perspective is required for society to be able to deal with our foreseeable concerns: environmental degradation and change, models of greed or scarcity and cultural and religious intolerance. When the intention is expressed, and in the context of the rest of the model, the debate is opened and can lead to questioning of custom and established routines. A move away from a static (stagnant?) curriculum structure in the minds of teachers is more likely than the current situation to help teachers move to complex and nuanced decisions. This thinking, embodied in the person, the art and the lessons of the teacher may have to power to help students reduce dogmatic thinking. Adaptability is also likely to provide the best hope for those issues we cannot yet imagine.

4.1.2 The nature of mathematics

Math has been represented to me by some students, parents and even other teachers in terms that suggest it is boring, lifeless and remote; that in the essence of mathematics itself there exists only meagreness and perhaps a vaguely anti-social obsession. Others have noticed and commented in a similar vein. Jardine, Friesen and Clifford (2006), in the aptly titled book *Curriculum in Abundance*, envision an approach to mathematics that is rich, connected and abundant, and is urgent about the need for a living mathematics.

> [F]or me it is quite literally a matter of life and death, of liveliness and deadliness, not only for myself but for the teachers and students I often witness laboring under the terrible burden of the belief in a world that doesn't fit together and that therefore must be doled out in well-monitored, well-managed, well-controlled packages, one lifeless fragment, one lifeless worksheet, one lifeless objective at a time (p. 100).

The description given by Jardine, Friesen and Clifford is both so different from the mathematics I know and care for, and so similar to what I see, that an alternate structure for mathematics curricula, one that dignifies by liberating is required. The study of mathematics will be freed from a strictly content-based focus, and teachers and students will be liberated to follow mathematics as it might be.

Is there value in considering mathematics as a distinct discipline? The argument Davis, Samara and Luce-Kapler (2000) put forth in describes how seeing a part is a helpful adjunct to, and not a replacement for, the viewing and integrating of all ways of knowing:

> There is value in examining the part. In the example of the fern leaf, one could learn a great deal about the fern plant by looking only at the structure within [part of the leaf]. It is also important to note that such partial viewings could never be adequate to understanding the nature of the larger form. Rather, the point being made is that such partial studies are no simpler and no less informative than studies of the whole (p. 72).

In knowing your perfect squares for example, a small detail assumes its importance when used as a way to perceive patterns in other places such as recognizing differences of squares or the nature of even powers.

Gardner in *The Disciplined Mind* (2000) reaffirms the real contribution that a part may make to a whole, if one is careful of not ignoring the whole. In speaking of a passage in Mozart's *The Marriage of Figaro,* Gardner describes how moving 'back and forth between part and whole' – really a changing of scale (forgive the pun) – aids in the appreciation of both.

> The trio is in no way a substitute for the work; and yet, if the trio is apprehended as part of the entire work, one can move back and forth between part and whole…As one oscillates between the trio and the rest of the work, one can appreciate numerous resonances of character, gesture, melodic motifs, and orchestration (p. 174).

By interrogating ourselves about the nature of mathematics and its contributions to human knowing, then, we do not necessarily atomize mathematics. A rich and joyful curriculum will have a purposeful changing of perspectives. We move in to see its subtle features, and move outward to see its utility and role in the functioning of the natural world and of the moral and aesthetic realms of humanity. There is expected to be a changing of scale as we deliberately shift the balance between delightful detail, and grand vista. So long as this is clearly understood, then studying mathematics will not mean its fragmentation, but a way to gain a deeper understanding as well as a broader one. This notion of changing scale is important for math teachers to recognize, from a mathematical and pedagogical viewpoint, so I include this quotation again in my sample Intention yet to come.

Considering the nature of mathematics means considering the What, Why and How of mathematics. The examination of the 'What' of math (the content as represented by the exemplars) as well as the 'Why' (the intention) and the 'How' (the curriculum structure) will be used to create a broad and powerful view of the learner's entire world, and not just specifics of content in one subject area.

What follows is an example of a statement of Intention for a high school math curriculum. (This exemplar is not intended to comprise the entire explanation of how to use the Manifold & Intention Model, nor how it develops and is renewed. All this and other information and direction would be provided for practising teachers.) There is repetition in here; some of the core understandings and passages I have used other places in this work belong in the kind of Intention I expect. To highlight exemplars in this chapter, I have used italics.

4.1.3 An Exemplar of a Statement of Intention for High school mathematics

This statement of Intention is to make explicit the underlying views about students, learning and mathematics, and to express the worldview that is promoted by this curriculum model. To emphasize the notion that the assumptions and ideas of this curriculum document are not law; many positions will be framed in the first person. As a teacher, you are expected to examine and critique the stated and implicit perspectives of this Intention and their own perspectives as well. Without this examination of perspectives, you may have a blindness to some personally held positions that affect your role as a teacher, or remain unaware of the inevitable tacit positions of this document.

The purpose of a curriculum is to provide an organizing and focusing structure to permit those termed 'teachers' to devise meaningful, deep and rich experiences for those termed 'students', who do not currently have the attitudes, information, maturity or background to reliably organize these experiences for themselves. These experiences are intended to "prompt...learners to notice certain aspects of their worlds and to interpret those elements in particular ways" (Davis, Sumara and Luce-Kapler, 2000, p.2).

Worldview

Humans are part of the planet's environment and our future success as individuals and as a species are reliant on becoming more adaptable. At least some of this adaptability may be expressed via changes in behaviour occasioned by conscious thought. Education through schooling may play a large role in increasing the scope and complexity of people's thought.

Of the many ways one can view the purposes of schooling, one way is to consider the dimensions of social efficiency, humanism, developmentalism and social meliorism (Kliebard, 2004).

Social Efficiency – the general aim is to help students to find a place in the existing world. At its best, in my view, a social efficiency model helps students to find meaningful role in society, and permits society to benefit from their contributions. At the other end of the spectrum, one could see social efficiency models as an institutionalized method of preserving the status quo and having little regard for individual gifts or the aspirations of the under classes.

Humanism – the general aim is to improve the thinking capacity of students by exposure to fine works; creations that are the acme of human thought. In this capacity, a student might be excited, emboldened and ennobled by her thoughts and reactions to such experiences. Through this betterment of self, a better society must follow. But whose thoughts are at the apex? The pursuit of humanism, taken to extremes, may lead to the material, not the student, becoming the reason for schooling, and the selection of classics reinforces the cultural hegemony of the colonial powers.

Developmentalism – the general aim is to lead to the self-actualization of the individual; to focus on what the student can be, given his aims and powers. One can imagine a system focused on each child, encouraging that child to stretch and grow as an individual. One could also imagine a system designed to determine the talents of students by a variety of tests, and to assign them to a variety of schools depending on their scores, places where what already is precludes what might be. Alternatively, one *could imagine such a perfect focus on the individual that the nature of humans as communities is fragmented or lost.*

Social Meliorism - the aim is to educate students with a view to change the existing social order. In a world of poverty, inequity, racism, hunger and environmental degradation, who can honestly argue against the need for citizens to be able to make this world a better place? Although, who is to decide what 'better' means? Might this lead to a cultural colonialism as pernicious as any other in the past?

In this curriculum, I take the viewpoint that each of these purposes of curriculum has some merit and some troublesome points, and that the perfect exclusion or heavy overweighting of any is not desirable. However, I have a bias toward social meliorism. All things economic are a proper subset of things social, and all things social are a proper subset of all things human. Humans require a place and sustenance, so all things human are proper subsets of the ecology of the world. Consequently, without caring for the ecology of the planet, no other consideration can be cared for over the medium and long term.

I claim that people in a democratic, open, and just society must concern themselves with the purpose of education. In this document, I take the position that the aim of education is to help people in society become less dogmatic, more respectful of others, themselves and the environment, and more creative, fulfilled and adaptable. This may be done by building capacity in students' senses of personal efficacy and deep appreciation. When a student learns to grow in such a way that she can direct wisely her future growth, education may be said to be effective.

Please note your responses to this worldview:

[More space will be provided in a published document].

Another perspective central to effective and responsive education is the notion of complexity. A curriculum is a method we use to increase the probability that society will endure, will create deep and meaningful connections for its citizens, and will permit members of that society the greatest chance to make necessary changes. Such a system is complex, and requires adaptability and interaction for it to arise and flourish. People in systems that have the five principle of complexity, and particularly those who understand those principles are more likely to be able to be capable of evolving in ways that suit the circumstances, and are more likely to be able to adapt short-term behaviours towards a longer term view. These principles are neighbour interactions, decentralized control, internal diversity, organized randomness and resilience.

Davis and Simmt (2003) connect the adaptability of a system to the range of options it has, which is a direct function of the internal diversity of its parts, and they describe intelligence "as the capacity of a system to respond not just appropriately, but innovatively to novel circumstances…. a system's intelligence is linked to its range of possible innovations, which in turn is rooted in the diversity represented among its agents" (p. 148). Each person in a classroom has something to offer, something unique in his or her insights.

Resilience or redundancy means a way of describing commonalities among agents. The shared parts are what permits cooperation and interaction, and the assumption of the duties of one agent by another if the first agent should fail. This is a complement to diversity. “Whereas internal diversity is more outward-oriented in that it enables novel actions and possibilities in response to contextual dynamics, internal redundancy [in a system] is more inward-oriented, enabling the moment-to-moment interactivity of the agents” (p. 150).

In his 1992 work, Ecological Literacy, David Orr describes the ecological meaning of resilient systems:

> Resilient systems absorb shock more gracefully and forgive human error, malfeasance, or acts of god. Resilience does not imply a static condition, but rather flexibility that permits a system to survive unexpected stress; not that it achieve the greatest possible efficiency all the time, but that it achieve the deeper efficiency of avoiding failures so catastrophic that afterwards there is no function left to be efficient (p. 34).

In the case of curriculum resilience, I suggest that common terms, definitions and notations play a role in permitting the agents to interact. It is a way for an individual’s own conceptions to move from the utterly individual and idiosyncratic toward a more public space, where these ideas can be communicated and used to influence and be influenced by others. Without this interactivity, a complex structure like a mathematical community in a classroom cannot develop.

Decentralized control is required to permit a system to display behaviours that no single individual may display. As an example, the Internet was conceived of as a way to avoid complete shutdown of military computers if any one part was destroyed. It started for one reason; now consider how the world of communication has changed since its advent. Davis and Simmt (2003) show that this emergent organization may occur and that it operates on a deep and ecologic level.

> *The whole does behave as a unit and as if there were a coordinating agent present at its center… [A coherent global pattern] emerges from the activity of simple local components, which seems to be centrally located, but is nowhere to be found, and yet is essential as a level of interaction for the behavior of the whole (p. 152).*

They describe organized randomness as how a system may be restricted from certain behaviours, but permitted wide latitude of action inside the given limits, this structure serving as a mediator between unresponsive rigid order and utter chaos. "[Organized randomness] is a structural condition that helps to determine the balance between redundancy and diversity among agents" (p.154).

Finally, for a system to operate in a complex and emergent fashion, parts of the system must have an influence on other parts of the system, and the authors make the point that we must not take for granted the exact nature of the word 'neighbour'. "… [W]e realized that the 'neighbours' in mathematical communities are not physical bodies or social groupings… Rather, in mathematics, these neighbours that must 'bump' against one another are ideas, hunches, queries, and other manners of representation" (Davis & Simmt, 2003, p. 156).

This curriculum model relies on neighbour interactions. Each element of this curriculum is expected to be challenged and rethought, with the voices of many interacting to create the great ideas or the great determination that is needed to change the quality of people's lives. Feedback systems of dialogue and innovation will continue to shape the curriculum. Some elements of the material taught are to be decided by the teacher and students together, thus explicitly and greatly increasing the amount of decentralized control. The Manifolds, described later, help with creating organized randomness; they are the bounding structures that help to reduce wild or inappropriate responses in the *system, while still encouraging great latitude. The expectation for teachers and students to develop a huge variety of activities and approaches frees the vital internal diversity that produces the range of responses that permits the system to adapt. Finally, there is enough coherence in the Manifolds and the Common Content to develop resiliency. People will be able to share through commonalities and will be able to take the part of others so our society can continue to change.*

Mathematics can play an important role in help students see environmental, social and cultural circumstances. Mathematics not only can be the result of a complex approach, but it can be a tool in examining complexity itself. Math can assist students with fresh and vital perspectives, leading to truly educative growth.

How does the culture of your classroom display the elements of complexity?

[More space will be provided in a published document].

Learning

I posit that greatest barriers to student learning are a lack of engagement and a lack of rich rehearsal. Engagement is the result of sufficient motivation and sufficient opportunity. Sufficiency varies from child to child, and from one occasion to another. Lessons, activities, tasks or surroundings that have little novelty or room for creativity coupled with ideas that are insufficiently arresting, noble or exciting reduce motivation. Reduced motivation reduces the chance of new thoughts and also reduces the likelihood of rehearsal. When students have a feeling of legitimate input and control over choices that affect them, and when there is an authentic audience for their activities there is an increase in motivation. Tasks or lessons that showcase individual strengths are more *motivating than those focused on repairing deficits, although the latter is important for ultimate growth.*

Learning consists of persistent thought connections that are different in nature or degree from what was connected before. New thought connections are formed by a combination of expanded experience (novelty, by a sudden feeling of insight, or by trauma) and by rehearsal. Novelty creates attention where there was automatic (dogmatic) functioning, and attention is required to initiate and sustain thought. Novelty without rehearsal, insight or without trauma creates transient, not permanent patterns, and thus doesn't produce learning. As trauma is always unethical to use, we have insight, that feeling of suddenly increased personal efficacy, and rehearsal.

"Rehearsal" is often associated with preparation for some performance, and perhaps the tests and exams so frequently used in the school system are auditions of a sort. (More probably they are used as audits for sorting students). In many cases, homework or practice is intended to ingrain patterns of behaviour, to develop competence in the post novelty of a lesson or activity. But the word 'rehearse' has an interesting past. The Etymology Dictionary [Online] says it comes from the Old French from "to rake over," from re- *"again +* hercier *"to rake, harrow". In a growth-minded view of education, it is perhaps fitting that rehearsal prepares the ground for learning. So let the rehearsal truly prepare the ground by being rich and productive.*

We may associate rehearsal with pure repetition, the parroting of a set of phrases or actions. But learning requires more than this. The rehearsal must keep a person's attention on particular concept, or set of concepts, branching out, and gradually *illustrating the power and potential of the concept, until the person can recognize, apply, and become fluent with the concept or skill. The more or less permanent change to behavior and capacity perhaps can only be claimed when the person can be creative; they can synthesize and build and adjudicate the application of the concept within the context of other notions and skills. This process is meaning-making. Learning is constructed by integrating meaningful experiences.*

Rehearsal can make new thought patterns stable and persistent so that attention can be used on new patterns and situations to search for still further patterns and generalities. Comprehensive rich rehearsal will guide students in where the idea applies and where it doesn't. It will foreground the skills and concepts that need to be available to permit the subtle trying and varying that the mind needs to generalize particulars. The aim of the generalization is not just to generalize. Mathematics should be portable. That is, students must feel and understand that when they generalize, they have the capacity to use and create with the concept in any other circumstances they may encounter. Therefore, rich rehearsal asks students to replicate, to some degree, but also to respond and create. As familiarity with a concept grows, and consonant with students developing capacities, the creative component should increase.

Motivation is literally the motive force that provides the psychic energy to engage in and maintain the discipline to persist in discovering or rehearsing. We infer a person's motivation by their engagement; the things they do. Motivation is affected by internal and external factors. External factors include marks, money, praise, competition, fear or shame, and clearly, some external motivators are unethical to apply to people. Internal factors such as guilt, pleasure, sense of deeper personal capacity, duty, loss, curiosity *and altruism are also powerful. Motivation is enhanced when we cultivate individual strength and identities.*

Humans are susceptible to immediate feedback, both positive and negative. To increase motivation for the important delight of getting better, activities designed for repairing deficits should show immediate change to the student.

Given this belief system, this curriculum document will focus on asking teachers to develop experiences for students that build and extend towards further capacity. Experiences of beauty, surprise, and mystery create novelty and invite people to explore. Increased capacity helps them maintain the determination to persevere. A sense of expanded experience, of increased capacity is intrinsically motivating, and usually leads to adaptive growth.

What counts as learning for you? How long must something persist before you consider it 'learned'? Does the homework you assign consciously build up your students' adaptability?

[More space will be provided in a published document].

Students

Students come from a variety of backgrounds with regard to prior experience and family circumstances and they differ in their predilections and natural and trained talents but they do share certain commonalities:

- Students have a multiplicity of needs. For example, almost all students wish to find a place in the world and are social creatures. It is harder (but not impossible!) to

learn when you are hungry and some otherwise baffling behaviours may make sense if the underlying needs are considered.

-Most students can learn most things. Differences in ability, while real, are less important than deep engagement, meaningful conceptual understanding and rich rehearsal, and the effort expended by the student, teachers and parents. These areas, rather than ability differences, are where focus should be applied.

As teachers are key components in the interplay between parts of the system, we must be alert to examining our practice to see that the school is helping, rather than harming. The most important areas to examine for dogma and subtle long-term damage are those areas that appear to require no examination at all.

All the components and subtleties of teaching and curriculum come down to whether each student is growing in efficacy, capacity to direct further growth, creativity, and adaptability. All practices, institutional structures, programs of study and curricular perspectives are to be redesigned, restudied, and re-imagined toward this purpose.

What do you expect from your students? In what areas of teaching are you dogmatic?

[More space will be provided in a published document].

Mathematics

Another component of this Intention is a description of the nature of the field of study. This demonstrates what society, through the curriculum developer, sees as the discipline's fundamental contribution to the human world. Again, the purpose here is so that anyone may question and ruminate on whether the description of the subject *accords with their own beliefs and how each of these may be enlarged, and whether that description is likely to bring that understanding of the subject to life in the stated fashion. Additionally, teachers can use the view of the subject to guide, support or question their own understanding as they prepare lessons. We must ask, what is the nature of mathematics, as mediated by the curriculum?*

What is Mathematics?

One attraction for some people is the idea that mathematics is somehow certain and absolute. Its truths are fixed and its method irresistible. The notion of the ideal nature of mathematical objects has been around in the western world since the time of Pythagoras and Plato. Perhaps the attraction of Platonist absolutism for those people is a sense of connection with eternity. If mathematics is eternal and infinite, maybe some people's participation in it is moved by a sacred sense. Perhaps in times of unpredictable events on both personal and social scales, mathematics gave people a way to manage uncertainty.

Paul Ernest (1998) and Reuben Hersh (1997) argue that mathematics is not an absolute, separate from society. Ernest critiques the absolutism, in the forms of logicism, formalism and constructivism that has been a central fixture of mathematical philosophy for much of the previous century, dispensing first with logicism, "the axioms of mathematics are not eliminable in favor of those of logic. Mathematics is a science with a definite content, and mathematical theorems depend on an irreducible set of mathematical assumptions. Thus the second claim of logicism is refuted" (1998, p.16). then with formalism, citing Gödel's two incompleteness theorems, and he also dismantles *constructivism (as a foundation for mathematical certainty, not as a pedagogical theory) and Platonism.*

If mathematics isn't absolute, if what is taken for mathematics has changed (chaos theory and abstract algebra were not 'mathematics' to Pythagoras), if the notion of proof has changed (we now have more rigorous proofs of theorems that had already been 'proven') and if even the axioms that are accepted have changed (the parallel postulate is not a necessary axiom for geometry), then what remains? What is mathematics?

Drawing on Hersh (1997) and Ernest (1998), synthesized with Davis and Simmt (2003), I see mathematics is an emergent, social phenomenon. Having a society does not guarantee the emergence of mathematics (the Pirahã people) but it is necessary for it to develop. What is studied by mathematicians, what passes for proof and what are the axioms - those unavoidable articles of faith - are decided and understood by convention, that is, by social agreement. However, these conventions are not haphazard nor random. They arose, and still arise, from complex interplay in the social-cultural-technical-historical realm. This realm provides organized randomness (great freedom within bounds of what is desired), internal diversity (millions of minds have contemplated at least some part of mathematics), resilience (the commonality in the conventions and in the program of schooling), decentralized control (different cultures, problems from different spheres of influence, each individual mind) and neighbour interactions (schools and universities, schools of thought, journals, books and letters). One of the most famous quotes in mathematics, attributed to Newton, is a testament to the social nature of even private inspiration. "If I have seen farther than others, it is because I have stood on the shoulders of giants".

What is its method?

Math is different than other social constructions such as religion or hockey. In the case of mathematics, the conventions place an extremely high value on internal consistency and on the notion of proof. The pursuit of mathematics according to Hersh (1997) is one of conjecture and proof. No other field of human endeavour follows the notion of rigorous proof nor abstraction so closely.

What are its objects of study?

"Mathematics is the art and *science of abstraction; it is the systematic study of quantity, structure, space, and change" (2012,* http://academics.adelphi.edu/artsci/math/*). At a more specific level, we can talk of symmetries or number or groups. My best current explanation is that number, geometric forms or functions and others attain object-status by:*

1. In simple cases, appearing to accord with the five senses and the sense of time: This sensing is largely in the individual (private) mind, but is socially (publically) encouraged.

2. Being useful for reducing uncertainty in predicting or organizing: Uncertainty is an issue for both individuals and society, and so it is one motivator for sharing ideas and granting object-status. In these contexts of sharing, codifying and creating, the conditions of neighbour interaction, resilience, organized randomness and decentralization are created.

3. Being beautiful and interesting: What constitutes beauty and interest is a dance between individual epiphanies and esthetics (internal diversity) and fads or movements *(resilience, organized randomness, neighbour interaction). Therefore this component is, like the second point, an interplay of the public and private mind.*

Object-status grants these socially constructed ideas a sort of permanence, and a place apparently outside the private mind, but to which, through socialization processes, the private mind has access.

If mathematics has a large socio-cultural component, how can it be so similar across cultures?

It isn't, especially when the cultures are in relative isolation. The Babylonians base sixty differs from the base twenty of the Mayans and from our bases ten and two, and they're all different from the wretched Roman numerals. Social negotiation, probably through trade, as happened when Fibonacci brought Hindu-Arabic numerals to Western Europe, might account for the relative homogeneity we see now.

In the way we reason, and in what is (often) considered, mathematics is different from any other discipline. But while viewing the world mathematically can be distinct from how we perceive the world musically, politically or historically, at the same time mathematics informs and is informed by these and other views. Accordingly, we must examine both that which is customarily termed 'Mathematics' and its interrelationships with other ways of human knowing.

Should we then refuse to separate one part of perceiving from all the others? Is there value in setting math up as a separate subject in school? Howard Gardner in The Disciplined Mind (2000) reaffirms the real contribution that a part (in our case mathematics) may make to a whole, if one is careful of not ignoring the whole. In speaking of a passage in Mozart's The Marriage of Figaro, Gardner describes how *moving 'back and forth between part and whole' – really a changing of scale (forgive the pun) – aids in the appreciation of both.*

> *The trio is in no way a substitute for the work; and yet, if the trio is apprehended as part of the entire work, one can move back and forth between part and whole…As one oscillates between the trio and the rest of the work, one can appreciate numerous resonances of character, gesture, melodic motifs, and orchestration (p. 174).*

By inviting the examination of the nature of mathematics and its contributions to human knowing, then, we do not necessarily atomize mathematics. A rich and joyful curriculum will have a purposeful changing of perspectives. We move in to see subtle features of mathematics, and move outward to see its utility and role in the functioning of the natural world and of the moral and aesthetic realms of humanity. There is expected to be a changing of scale as we deliberately shift the balance between delightful detail, and grand vista. So long as this is clearly understood, then studying mathematics will not mean its fragmentation, but a way to gain a deeper understanding as well as a broader one. So, it is appropriate to consider mathematics as one mode of human perception. If we want to see, we must spend time looking.

Do you define mathematics as I have done? Do you change your scale when considering mathematics?

[More space will be provided in a published document].

End of Statement of Intention Exemplar

This exemplar is meant to demonstrate that we can have a statement of Intention that draws attention to itself as it points to some key areas of teaching. What is pointed at is not new; there are similar elements in the Introduction of the Manitoba Framework document. What is different is the invitation to view the biases and perspectives inherent in any Intention. This is done explicitly in the introduction, and the stating of the Intention in the first person is radical enough to be mentioned. I am not so naive as to think that every teacher will agonize over his or her answers to the questions above, but even the presence of the questions says something about the invitation central to the Manifold & Intention Model.

4.2 Manifolds in the Manifolds & Intention Model

Once the authors of the curriculum have been explicit about its intention, they can move to organization. The organization of curriculum should facilitate the purposes proposed and guide the actions of people involved in the endeavor. It should be sensitive to the Lenses of Perspective and Complexity. Since Tyler's *Basic Principles of Curriculum and Instruction* attempted to suggest an organizing structure, the use of objectives as a way of focusing attention on specific behaviours has been extremely common. There are advantages to the use of specific learning outcomes. Because they are narrow and reductionist, they help to focus and isolate specific behaviours. If the stance held is that humans are fundamentally stimulus-response machines, the desire to focus on improving the machine by improving its component parts makes sense. I do not hold this stance. Because I believe people construct and integrate their thoughts and seek meaningful agency, a mechanistic approach is inimical to my curriculum. Yet neither is it appropriate, effective or fair to provide no guidance whatsoever to teachers or students. Such a *laissez-faire* approach of letting students purely follow their own interests is irresponsible to the degree that the worthwhile purposes are left to chance, and opportunities for growth may be missed. Dewey (1902/1990) said as much when he warned, "Any power, whether of child or adult, is indulged when it is taken on its given and present level in consciousness. Its genuine meaning is in the propulsion it affords toward a higher level" (p. 145).

The challenge in the organization of a topic is that to organize we must sequence, and classifications are required to create a meaningful sequence. Classify without care, and these classifications may be limited or disconnected. I have selected the term 'Manifold' to represent one of the main organizing structures for curriculum.

4.2.1 Manifold as a Mathematical Entity

Consider first the mathematical definition of a manifold:

> A ... manifold can be described as a topological space that on a small enough scale resembles the Euclidean space of a specific dimension, called the dimension of the manifold. Thus, a line and a circle are one-dimensional manifolds, a plane and sphere (the surface of a ball) are two-dimensional manifolds, and so on into high-dimensional space. More formally, every point of an *n*-dimensional manifold has a neighborhood homeomorphic to an open subset of the *n*-dimensional space $\mathbf{R}^n$.
>
> Although manifolds resemble Euclidean spaces near each point ("locally"), the global structure of a manifold may be more complicated. ...The concept of manifolds is central to many parts of geometry and modern mathematical physics because it allows more complicated structures to be expressed and understood in terms of the relatively well-understood properties of simpler spaces (Manifold, 2012).

Imagine a crumpled ball of paper. Certainly, from our view, a complicated structure! Folds, hollows, and a convex surface. But at any point on the paper, even on folds, we can imagine, at least on a microscopic level, a little flat disk of paper. Since every point has this property, the ball of paper is a two manifold. [A real sheet of paper has a thickness, and thus is not truly a two manifold.] Another example is that of a 'flat' farmer's field on the oblate spheroid two-manifold that is our globe. With manifolds, we have an underlying structure that can be characterized by certain characteristics demonstrated at a local level. By associating power and importance to all points in the space, we have a connection to complexity theory's decentralized control. How may we take advantage of this metaphor in the curricular realm?

4.2.2 Manifold as an Image of Curriculum

We have a 'world' of mathematics too, with complexities and concavities, with lumps and bumps and bevel-edged pieces. This concept of mathematics as terrain or topography is familiar, and one that is nicely subsumed in the topological concept of Manifolds, and the metaphor can lead us to an environmental and global perspective. The attraction of the metaphor of Manifold in curriculum is that it respects the complex discipline that is mathematics, imagined perhaps as a rugged mountain or free-running brook, and yet simplifies and clarifies the surroundings - makes the rough places plain (plane?), a clear invitation to the novice mathematicians that attend our schools. At the same time, the idea of rigor is important in mathematics and the metaphor of Manifold – this disc or platform that defines the Manifold – is stable enough to support meaningful mathematics done in a meaningful way. Therefore, a two-Manifold carries the connotations of the support under each region of study that can be accessible to teachers and students and with each disc being part of a larger, entire, organic and more complex structure. Using Manifolds as our metaphor, we have a more integrative and ecologic model than that of strands, for this world is not a linear place, and the notion of Manifolds may conjure the image of views of mathematics from different conceptual territories of experience, each of these views being crisscrossed and interrelated with the mathematics students learn. Such a metaphor shows the interconnectedness of the world, and the paradoxical strength and fragility by which it is held together. (I leave it to others to develop three or four Manifolds as curriculum structures, with perhaps pedagogical knowledge – the mathematics-of-teaching perhaps, or curriculum critique as other dimensions. The potential generativity of the notion of Manifold in this context is immense.)

For the purposes of my curriculum model, a Manifold is a focus on experience. Manifolds are ways of noticing the world from a mathematical perspective (or from some other perspective in some other discipline.) A Manifold is a view from a particular angle or a touch of a particular texture that has shown meaning to others in the past, and that can serve as a focusing/centering metaphor for a huge range of mathematical ideas, and not just a collection of content of a certain type. Manifolds are themes, those ubiquitous abstract notions that help us to make sense of the world. Thus, Manifolds serve an integrative purpose. A Manifold is an organic focusing structure that provides coherence to the discipline of mathematics while permitting access. A Manifold is a metaphor that simultaneously directs attention to certain particulars, and is re-created by those particulars in the minds of the children. A Manifold is a perspective from which to view the landscape of mathematics. It is a lens to examine experience: a well-handled familiar tool, a textured and open organizer.

4.2.3 Manifolds and Complexity

Manifolds serve as lenses, and also potentially as boundaries. The idea of organized randomness permits wide variation within some limits. The limits are necessary to prevent the long term spread of pathological variations which will inevitably occur over time in an loose, unbounded system. The Manifolds can help to steer teachers toward productive areas of mathematics and give gentle guidance for teachers and students in planning Local Content. Quite often in Nature, beautiful solutions occur when a system is placed under some constraints.

At the same time, the Manifolds themselves are open to question, modification and change. In my model, it is one of the roles of teachers, professors and members of Departments of Education to discuss, argue and decide upon such a selection. Such interactivity seeks views from a variety of sources and strongly contributes to decentralized control and neighbour interaction. In this case, the debate over what should become Manifolds for the next round of curriculum refreshment will require a forum and create neighbours where none before existed. I anticipate the Internet will be used to dissolve the spatial distance of this neighborhood, and this permits people from remote locations in Manitoba (and interested people throughout the world) to contribute ideas, research and questions.

4.2.4 Manifolds for High School Mathematics

The number of choices for Manifold in the broad world of mathematics is large. These choices should help us see mathematics as a way of considering the physical world we inhabit, and the intellectual worlds that inhabit us. The first step in deciding the Manifolds for a curriculum is to collect some ideas of how the discipline may be seen. From the Nature of Mathematics in the Manitoba Framework document, we have Constancy, Change, Number Sense, Pattern, Relationships, Spatial Sense and Uncertainty. In *Fundamental Constructs in Mathematics Education* (2004), John Mason contributes as well. "The study of Mathematics may be helped by noticing the underlying themes which weave across topics. Recurring themes include Doing and Undoing, Invariance amidst Change, Freedom and Constraint, Ordering and Classifying, Stressing and Ignoring and Constructing Meaning" (p.196). Finally, I quote the extensive list from *On the Shoulders of Giants: New approaches to Numeracy* (1990), edited by Lynn Arthur Steen:

> School tradition has it that arithmetic, measurement, algebra, and a smattering of geometry represent the fundamentals of mathematics. But there is much more to the root system of Mathematics – deep ideas that nourish the growing branches of mathematics. One can think of specific mathematical structures:
>
> Numbers Shapes
> Algorithms Functions
> Ratios Data
>
> or attributes:
>
> Linear Random
> Periodic Maximum
> Symmetric Approximate
> Continuous Smooth
>
> or actions:
>
> Represent Model
> Control Experiment
> Prove Classify
> Discover Visualize
> Apply Compute
>
> or abstractions:
>
> Symbols Equivalence
> Infinity Change
> Optimization Similarity
> Logic Recursion
>
> or attitudes:
>
> Wonder Beauty
> Meaning Reality
>
> or behaviours:
>
> Motion Stability
> Chaos Convergence
> Resonance Bifurcation
> Iteration Oscillation
>
> or dichotomies:
>
> Discrete vs. continuous
> Finite vs. infinite
> Algorithmic vs. existential
> Stochastic vs. Deterministic
> Exact vs. approximate
>
> (pp.3-4).

A lengthy list. Such a list is a powerful reminder that there are many perspectives to consider. In the book, Steen (1990) has selected six deep ideas to be worthy of further notice, and he and the other authors consider Pattern, Dimension, Quantity, Uncertainty, Shape, and Change.

For a single person or entity to dispense Manifolds as finished products without input from many sources is antithetical to the tenets of the Manifold & Intention Model, but I provide a starting point for such a discussion.

I suggest

i) Change & Invariance

ii) Patterns

iii) Modelling

iv) Doing & Undoing

v) Certainty & Uncertainty

vi) Equivalence.

There is no special significance to selecting six (although it *is* a perfect number…), but we shall try to keep the number reasonably small, for the sake of unity and remembering. Twenty-eight Manifolds might do a wonderful job of representing Mathematics, but they would be too unwieldy to recall and interrelate effectively. In the following exemplar I give some brief reasoning why these may be used as a place to begin.

4.2.5 An Exemplar of a set of Manifolds

i) ***Change & Constancy:*** *We are surrounded by change, from our own growth to the cycles of nature. Mathematics has become a powerful way to characterize change, and interestingly enough, the absence of change. Many interesting properties such as the sum of the interior angles of a triangle have interest precisely because they do not vary when other properties of the triangle do. I have claimed that adaptability is a signature human characteristic. To adapt implies the capacity to change. At the most fundamental levels, humans consider change.*

ii) ***Pattern:*** *Whenever we talk of generalizations or structures, we are talking about Pattern. What's more, Pattern certainly extends beyond the realm of mathematics. Themes are considered in literature and film, history considers historical antecedents and science is based utterly on repeatable observations. That link to the broader human experience, coupled with the powerfully effective ways mathematics has of imagining and describing patterns make it an excellent candidate for inclusion.*

iii) ***Modelling:*** *Inside us all is the desire to know the future, or at least something outside our current experience. When we are running Monte Carlo simulations for ecosystems, fitting curves to data or 'translating' language to mathematics, modelling is used to connect the outer physical world to the inner intellectual world. In this inner world, experiments may be done with no harm and little cost, and the understanding that is gained can then be turned into wiser action.*

iv) ***Doing & Undoing:*** *All the notions of functions and inverse function are subsumed in this notion. Order of operations and school algebra are now linked together, instead of being separate studies, and the familiar connection of factoring and multiplication can* *now be named and compared to other instances of this Manifold. Recursion, experiments in chaos theory or infinite sequences are captured in the ideas of repeated doing without undoing. Also, this Manifold asks us to be flexible in our thinking. If such and such is done, what process(es) can be used to undo this effect? The notion of consequences is mathematized to some degree.*

v) ***Certainty & Uncertainty:*** *These two describe a yin/yang paradox of mathematics. Steen selects Uncertainty as one of the deep ideas and Uncertainty is evident in the Nature of Math. Uncertainty is handled with more surety and elegance in mathematics that in any other discipline. The entire fields of probability and statistics are built upon working with uncertainty, and what can be expected. In this, we have a nice connection to Modelling. The paradox is that only in mathematics does one really have the notion of proof. It's true that proof depends on the audience, and that we start from postulates and axioms as articles of faith, but no other discipline can even come close to the concept of having proven something; of being certain. Given the role that proof plays in all of mathematics, and given the unique contribution mathematics makes to dealing with uncertainty, this is a powerful candidate for a Manifold.*

vi) ***Equivalence:*** *This idea encapsulates all the notions of the equal sign, and in school mathematics, that says a great deal. But it goes further than that. We can include congruence in geometry, and proportionality and its many connections to geometry, algebra and rational numbers. Modular arithmetic can be used to display different types of equivalence. Further, the idea of equivalence extends easily to transforming between different representations of numbers or equations or modes of expression. A table of* *values, an equation and a graph may be seen as equally valid ways to represent the same thing, each with their own advantages for particular situations.*

End of Manifolds Exemplar

The fundamental aim of the Manifolds is for students (and teachers) to recognize underlying patterns in mathematics and that these patterns may be to help them organize, connect and direct their thought. Imagine integrating these with other disciplines. What of Change in history or in the way the history is perceived? Manifolds are useful to draw attention to the metacognition essential to lasting learning.

There are many possibilities for Manifolds for high school mathematics, and there is intended in my model a robust discussion on what the Manifolds should be. Steen selects Dimension as a deep idea in mathematics, and it has much to recommend it as a Manifold, including links in geometry, in Chaos theory and potentially in examining systems. Currently, I don’t consider Quantity and Shape to be good candidates for Manifolds. Even though mathematics might be about the study of quantity and shape, using these as Manifolds points too directly at the objects of math, and not at how math is done. If we use Shape or Quantity, I feel the focus will be on particulars, as it is now, whereas we want to encourage people thinking about how the particulars interact and interconnect. Of course, we will discuss numbers, shapes, or functions in a math class, but the objects of math should serve to illuminate the Manifolds, even as the Manifolds reflect the light of scrutiny back on to them. The choices for Manifolds need to be more general than objects.

4.3 The Content Evaluation Criteria (CEC)

> It is the business of the school environment to eliminate, so far as possible, the unworthy features of the existing environment from influence upon mental habitudes. It establishes a *purified medium of action*. Selection aims not only at simplifying but at weeding out what is undesirable. Every society gets encumbered with what is trivial, with dead wood from the past, and with what is positively perverse. The school has the duty of omitting such things from the environment which it supplies, and thereby doing what it can to counteract their influence in the ordinary social environment. (Dewey, 1916/2005, p. 15) [Emphasis mine]

> Knowledge is humanistic in quality not because it is *about* human products in the past, but because of what it *does* in liberating human intelligence and human sympathy. Any subject matter which accomplishes this result is humane, and any subject matter which does not accomplish it is not even educational. (Dewey, 1916/2005, p. 135)

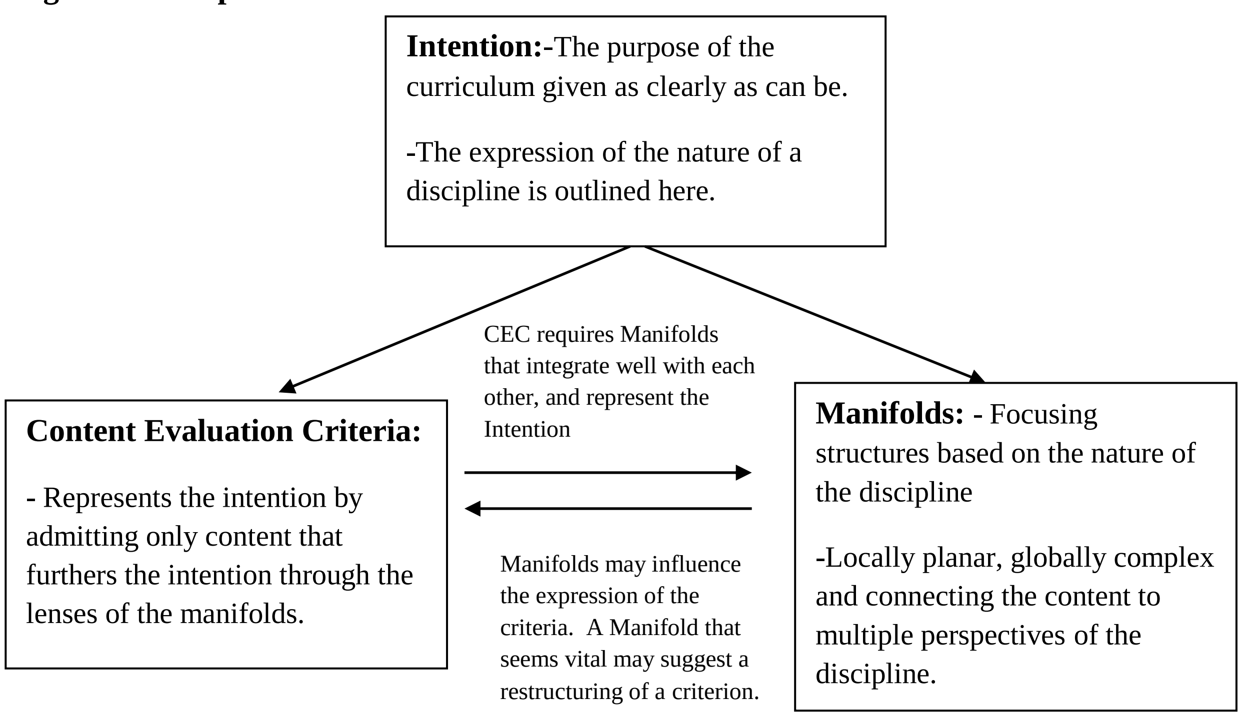
The Content Evaluation Criteria are the vehicle through which the Intention is implemented. These criteria ensure that all the specifics written into the curriculum are there, not simply by tradition, but by dint of their contribution to the Intention. These specifics will include what is traditionally known as content – specific understandings and processes, along with explicit links to other content, attitudes, historic and social concerns, the environment and other Manifolds. Collectively, I shall refer to all these as materials as 'content'.

Again, John Dewey is the person to whom I trace the development of this element of the Model. His caveats against an established canon and the need for a ‘purified medium of action’ led me to devise a system where content is not in a curriculum because it is desired by a small branch of the academy, nor by what jobs are currently open, nor by simply having been in the previous curriculum documents by historical accident. Some express mechanism is required to help balance these forces with what will help students understand the nature and possibilities of mathematics.

The Content Evaluation Criteria are a set of questions or requirements designed to have the specifics of the curriculum reflect the Intention and thus contribute to an integrated whole. So, choosing whether there is to be discrete mathematics or algebra or calculus in a curriculum document will depend on the degree to which these areas or objects of study reflect the Intention, as indicated by the Criteria.

The Intention, with its focus on the nature of mathematics, helps to determine what the candidates for the selected Manifolds may be. The ideas of Dimension, Pattern and Change might be considered candidates for Manifolds, for example. But the Content Evaluation Criteria is another structure that influences the choice of Manifolds. If the material of the proposed curriculum cannot satisfy the Content Evaluation Criteria, then the choice of Manifolds must be rethought. Note in this expanded detail from the previous diagram how all three of the components interconnect.

Figure 4.2 - Expanded Detail of the Manifold & Intention Model



The Intention, Manifolds and the Content Evaluation Criteria provide a coherent and responsive basis for selecting the content that is to be taught. Using the Manifold & Intention Model to create a new curriculum for students should be an interactive, recursive and dynamic process that invites discussion, permits new ideas to come forward, and adapts and evolves. The Content Evaluation Criteria play a role in organized randomness, and in resilience.

4.3.1 An Exemplar of Content Evaluation Criteria

The Content Evaluation Criteria is designed to draw attention to all components of the content.

i) Are the concepts and material developmentally appropriate? *We know that mathematics is richly connected, but for some material, those connections may be best displayed by concepts currently beyond the grasp of students. Whatever is included in the Common Content must be developmentally appropriate. The Local Content is chosen with the student to be appropriate.*

ii) Are there significant and explicit connections to the Manifolds? *All material selected should be excellent exemplars of the Manifolds. That is to say, the Common Content and the Local Content should give a very clear view of the perspective of the Manifolds. Only material that is clearly and explicitly linked to more than one Manifold is permitted.*

iii) Does the material have a rich history, and/or does it contribute to an understanding of the lives of people in different cultures, ages and genders? *While never discounting the significant accomplishments of the ancient Greeks, care must be taken to include information about many cultures through history, including obstacles that people have faced and still face today. The material should permit a way to view* *historical, current and potential environmental, economic, technological and social problems.*

iv) Does the material have associated experiences that are within the reach of most teachers, and is there a mechanism to increase neighbour interactions and internal diversity? *Based on a constructivist perspective, learning experiences are to be constructed through guided inquiry, discussion, problem-solving and teacher input. It is understood that developing as a teacher is a lifelong task. For inexperienced teachers, I propose a web-based set of optional activities that they may use while they are learning their craft, particularly for Local Content. As teachers become more knowledgeable, they will contribute to the refinement and innovation online. This is an example of decentralized control and it shows respect for the interests, knowledge and growth of teachers.*

v) Does the material lend itself to modelling or understanding the natural world, its changes and challenges? *Without a planet to live upon, the future of mathematics and all other human activities is jeopardized. The material selected will adapt to modelling of the natural world, with deep regard for systems theory, complexity theory and chaos theory.*

vi) Do the Manifolds and the Content that passes the first criteria lead to a deeper understanding of the human activity that is mathematics? Is the Intention of the curriculum manifest in its details? *Does the richness and integrity of mathematics become more apparent to students and teachers? Are the ideals of the Intention present and enacted?*

End of Content Evaluation Criteria Exemplar

There are other concerns and criteria that can – and should – be discussed. As with every other component of the Manifold & Intention Model, there is the expectation that the criteria be examined and subject to evolution. Together, these criteria will help to focus students, pre-service teachers, teachers and academic researchers on the intentions and perspectives of mathematics, while encouraging an active and curious mindset, and concern for the vital social and environmental factors in our life.

4.4 Common Content

> In order to have a large number of values in common, all the members of the group must have an equable opportunity to receive and to take from others. There must be a large variety of shared undertakings and experiences. Otherwise, the influences which educate some into masters, educate others into slaves... A separation into a privileged and a subject-class prevents social endosmosis. The evils thereby affecting the superior class are less material and less perceptible, but equally real. (Dewey, 1916/2005, p. 51)

While Dewey has just given us ample reason to consider content common to all, the work of Davis and Simmt (2003) provide yet more. For a system to be dynamic and intelligent, it is necessary that components of a system be able to affect each other. In human societies, that interaction is handled largely through communication. Without some commonalities of, say, language or mathematical conventions, people may be unable to influence each other in the manner required for a complex system to occur. Thus, it is necessary for some parts of the content to be shared.

The Manifolds and Content Evaluation Criteria together are used to develop the material of the Common Content. It is anticipated that all students taking the course will engage with the Common Content.

In terms of complexity, Common Content expresses the ideas of resilience and contribution to neighbour interactions. The primary role of the Common Content is to provide the common language and common experiences so that students may have the capacity to share ideas among themselves and with the larger community. The Common Content is also important for building the class climate, the sense of discovery, togetherness and mutual support that is in itself a model for human organization. The selection of Common Content will prove a very great trial, as there is so much of interest, meaning and tradition amongst educators. However, we shall hew to the advice of Alfred North Whitehead (1967), noted philosopher and mathematician in *The Aims of Education*: "We enunciate two educational commandments. 'Do not teach too many subjects,' and again, 'What you teach, teach thoroughly' (p.1)" and "The result of teaching small parts of a large number of subjects is the passive reception of disconnected ideas, not illumined with any spark of vitality" (p.1).

So while the selection will be difficult, we shall include the smallest amount of material we can in the Common Content section consonant with making it possible for people both inside the classroom and out to communicate their ideas.

There is another aspect of the discipline of mathematics that clearly belongs in Common Content, and that is how we pursue mathematics. The Common Content is the place for establishing the notions and practice of solving problems, and of analyzing and developing heuristics for dealing with problems. It is to be clearly understood with any mathematical content being learned that the *way* of pursuing the understanding of mathematics is of greater importance than any particular piece of content, any particular algorithm. This is so for the most basic reason. As students develop their own methods or algorithms, they will be doing their mathematics instead of aping the math of someone else. The power and delight of making meaning in mathematics leads to greater flexibility, adaptability and perseverance. It is just these qualities that lead to a richer life. With understanding and a way of thinking about problems and mathematics, there is the hope that any content or algorithm may be invented if needed. It is not nearly so likely that having access to a rote set of steps will allow one to conceive a way to tackle other problems.

But life is complex, and simple prescriptions can lead to unwonted excess in some direction. Intending that our students create, invent and adapt ideas and algorithms does not mean that there is no need for students to become technically adept at skills such as factoring, or graphing. The fundamental approach taken in teaching the Common Content is based on the creation of experience, and so the paradigm for this curriculum is neither a mechanistic "This is the standard and only way." nor is it "Let the children do whatever they want." The key is an approach that maximises students' capacity to continue. It means that teachers must consider carefully if and where their students should seek for automaticity. The question to be answered is: "*Will this approach increase this child's capacity to organize his or her own growth now and later, or will it hinder it*"? If not knowing one's multiplication tables means that patterns and larger ideas are not accessible, then one must learn them. If teaching fraction operations by rote means that they have to be retaught every year, and that students aren't creative or fluent with the ideas of rational numbers - and how many high school math teachers have come across *that*? - then emphasis must be placed on conceptual understanding through experience. Seeking research at the university level and from the student, the teacher must try to re-answer this question with every topic and with every child.

The Common Content is necessary not because it has more importance than other content but because it contributes to neighbour interactions and resiliency. The selection process will be difficult, but the choosing of Common Content will be guided by the content Evaluation Criteria, in turn reflecting the Manifolds and the Intention.

4.5 Local Content

> The vice of externally imposed ends has deep roots. Teachers receive them from superior authorities; these authorities accept them from what is current in the community. The teachers impose them upon children. As a first consequence, the intelligence of the teacher is not free; it is confined to receiving the aims laid down from above. Too rarely is the individual teacher so free from the dictation of authoritative supervisor, textbook on methods, prescribed course of study, etc., that he can let his mind come to close quarters with the pupil's mind and the subject matter (Dewey, 1916/2005, p.65).

Once again, Dewey has inspired me to examine the nature of what is included in a curriculum document for teachers and students. And once again, the precepts of complexity theory from Davis and Simmt (2003) give yet more credence to the notion as seen below. One central component of an intelligent system is that it is adaptive. For systems to adapt there must be internal diversity; the source of natural variation that permit adaptation in the first place. Consequently, including a component that respects and fosters internal diversity is very important.

Hence, a key feature of the Manifold & Intention Model is the presence of classroom-created content. This Local Content is to be decided upon by the students and teacher in collaboration. Here, teachers and students together will select/develop individual and small group plans for broader and/or deeper study. The solving of problems and inquiry-based methods are the vital core of instruction, and much of the specific substance of the content areas will be achieved through discussion and the attempted resolution of problems. In addition, links to other content and via other Manifolds will be suggested, creating a more holistic and integrative construct than purely outcome-based learning. The capacity for individual relevance of the Manifold & Intention Model is a great strength. By limiting the amount of prescribed content, and guiding students to learn what is termed ‘Local Content’, we can increase the relevance and engagement of students. Eisner (2002) calls for just such an approach:

> The kind of schools we need …would embrace the idea that good schools increase the variance in student performance and at the same time escalate the mean….What one would have at the end of the school year is wide difference in student’s performance. At the same time, since each program is ideally suited to each youngster, the mean for all students in all of the areas would be higher that it would be in a more typical program of instruction. Such a conception of the aims of education would actually be instrumental to the creation of a rich culture… (p. 580).

A second argument in favor of Local Content comes from complexity science discussed earlier. Davis and Simmt (2003) describe the potential importance of involving students and teachers in the community of discourse of those involved with mathematics:

> Complexity science might be interpreted to offer critique of an effort to understand mathematics teaching that is not attentive to both the particular contextual conditions and the broader cultural circumstances that give shape to the project of school mathematics. Further, complexity science renders problematic those discourses that focus on peripheries, fringes, border spaces, novice and other notions that suggest that complex forms might have clear centers, boundaries, and origins. In fact, considerable research has be undertaken to better understand the vital roles played by those agents that are commonly imagined to exist on the edges of complex unities – including, significantly for educators, persons who are popularly classed as novices and initiates (p.143).

Consider the potential richness of the dialogue in the discussion and creation of mathematics just in deciding the Local Content. Examining the options through the lens of each Manifold and justifying its inclusion through the Content Evaluation Criteria would be a wonderful opportunity to understand mathematics. Perhaps something akin to Freire's 'thematic fan' (1970/2006) would unfold before us, a rich panoply of options that teacher and student would offer each other.

Such discussions could also occur amongst educators. Deep professional development could come about through a website, an 'e-coffeehouse', a wiki, where ideas about which Manifolds could be selected, which how different types of content demonstrated the roles of the Manifolds and specific activities/ exemplars for teachers to mull over, to experiment with and to present. How exciting might it be if, at least at the high-school level, interested students could join in the discussions? How much authentic mathematics might be done that way? Such a presentation encourages teachers to examine all the parts of the exemplar, and its non-linear structure invites one to explore, adding new links, considering alternatives and entering discussion. By sharing their ideas via the Internet, such an interactive, and open but bounded structure will address several of the requirements that Davis and Simmt (2003) have described as necessary for an organic entity to emerge: (a) Internal diversity, (c) Decentralized control, (d) Organized randomness and (e) Neighbour interactions.

4.6 Expressing Content - Exemplars

How shall we direct our attention? In the limitless universe of mathematics, how may we give guidance and support to teachers about their teaching? In my view, the pattern in

Manitoba has been to over-specify; the students must know this specific thing, and we will mandate textbooks to frame the extent of that thing, thus removing the responsibility of teachers to think carefully about the subject. Wong (2006) describes what is occurring in China, something that has long plagued our province. “The responsibility of curriculum design is now moving out of the hands of teachers to control by a few outside experts” (p. 19). When teachers have their complex tasks separated and simplified the result has been described as ‘deskilling’. The lists of specific outcomes we have seen play a role in reducing the decision-making and capacities of teachers.

In my Manifold & Intention Model, the Common Content and the Local Content will be expressed differently than in the current system. Objectives are not required to illustrate what is powerful or useful mathematics, and they are not required for teachers to be able to teach. They are reductionist relics from behaviourism. Behavioural outcomes represent centralized control, a linear and suspicious view of what people are able to achieve together. By contrast, in the Manifold & Intention Model, the specifics traditionally known as content - knowledge, understandings, techniques and processes - shall be presented in a variety of ways as concept maps/ activities/ exemplars/ Linking diagrams or text, along with explicit links to other content, attitudes, historic and social concerns, the environment and the other Manifolds. A Linking diagram is a set of branching possibilities that connect in some way to the featured concept. Linking diagrams can be carried on nearly indefinitely. An example of a part of a Linking diagram is shown in a few of the exemplars that follow. These diagrams are implemented best on the Internet, as that medium is well suited to permit students and teachers the capacity to explore and add different branches to the concept or topic in the fashion of a wiki.

I present a few exemplars to demonstrate the potential generativity of this method. The exemplars here have been selected without regard to a traditional grade level or course, and without necessarily declaring that such-and-so must be in the Common Content, so the exemplars presented here may be well suited to either Common or Local Content. The style and presentation of each of the exemplars is deliberately varied. Sometimes there are notes to the student or teacher, and sometimes not. The scope for elaboration and refocus is immense; there is no claim that the exemplars are complete; in fact, quite the contrary, and this is one of the strengths of the Model.

Content Exemplar #1: Introduction to Cosine Law

For the Teacher

The Cosine Law Activity is meant to activate the schema that students have for creating models, and to show that this process can be problematic. It is meant to help students wonder what technique will work to predict the side length, when a linear model for side length ultimately fails to make adequate predictions. Some prerequisite skills and processes are useful or required for this activity:

Measurement: *Students should be able to measure to the nearest millimeter with a ruler. Encourage estimating before measuring. (Estimating helps reduce errors in measuring. It helps get a sense of the magnitude of numbers, thus helping with other reasoning and estimating.)*

Technology*: Students should be able to make a scatter plot and do a linear regression with a calculator or other device. Students should be able to use the linear model to interpolate and extrapolate. Students should understand that regression is one way to model behaviour, in this case, change of a side length.*

Concepts: A right angle is not necessary to solve a triangle.

- *Formulas can be proven.*
- *Models are useful, but have their limits (Appropriate domain)*
- *Not all situations are linear.*

Affective: Not everything that looks promising always works out. That's OK.

- *Proofs give us something that measurements never can. Notice the connection to the Certainty Manifold.*

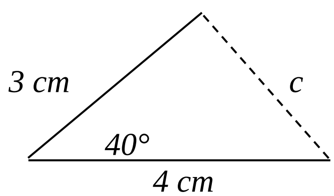
Content Exemplar #1: Cosine Law Student Activity

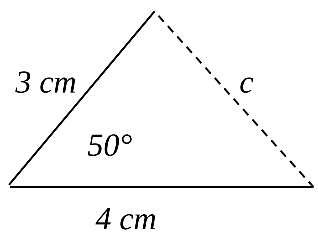
This activity is intended to provide experience for your understanding of how the effect of a change in an angle manifests in change of the length in the side opposite by creating and testing a model.

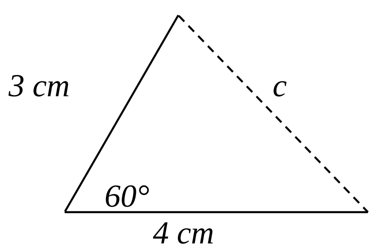
Part 1: Measuring to collect data: *You are given several triangles below. Each one has a leg of 3 cm, and a base 4 cm long. The angle in between these two legs will be called theta (θ). Measure the length of the dashed side opposite theta and record the angle and the length in cm.*

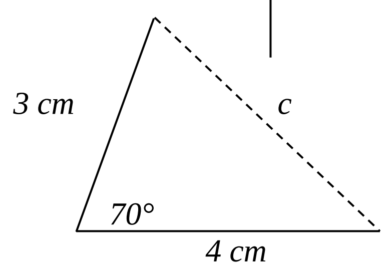
Measure as carefully as possible and record the length of the unknown (dashed) side, with the angle as the domain and the length as the range.

Angle (θ)	*Length (cm)*









Part 2: Change and Representing Data

Use your TI to make a scatter plot of the data. Does the length of the side change linearly, or nearly so, as a function of θ? ________________

Sketch a copy of your scatter plot: *(Use $X_{min} = 35$, $X_{max} = 75$, $Y_{min} = 2$, $Y_{max} = 4.5$)*

Part 3: Modelling:

"As the angle gets larger, the length of the dashed line gets ______________________."

If the data appears to be linear, let's model the change using a linear regression. (Use c for the length of the dashed side and θ for the independent variable.)

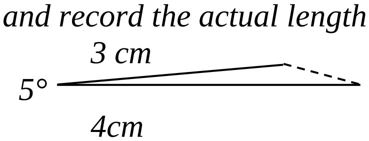
c = ________________________

Now reset your window so we can make some predictions: (Use X_{min} = 0, X_{max} = 150, Ymin = 0, Ymax = 8)

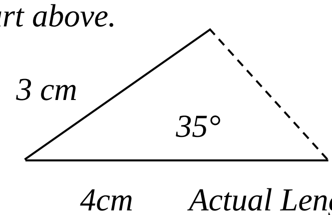
*Use your regression to **predict** the length of side c when there is an angle of 5°, 35° and 140°. Calculate the lengths predicted by your model and record in this chart under predicted length:*

Angle (θ)	*Predicted length (cm)*	*Actual length (cm)*
5°		
35°		
140°		

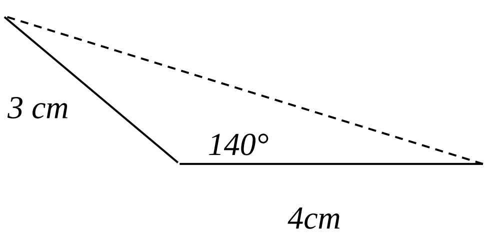
Part 4: Testing the Model: *The actual triangles are below. Measure each one carefully and record the actual length in the chart above.*



Actual Length: ___________



Actual Length:__________



Actual Length: __________

b) What should be the distance between the ends if you close the angle between the 3 cm and 4 cm line down to 0°? __________. What does the model predict? ________________

c) What should be the distance between the ends if we open the angle to180°? _______.

What does the model predict? ________________

Part 5: Concluding and Reflecting

a) Given the measurements above and your calculations here, do you believe mathematicians accept the linear model as a 'good enough' fit? _______________________

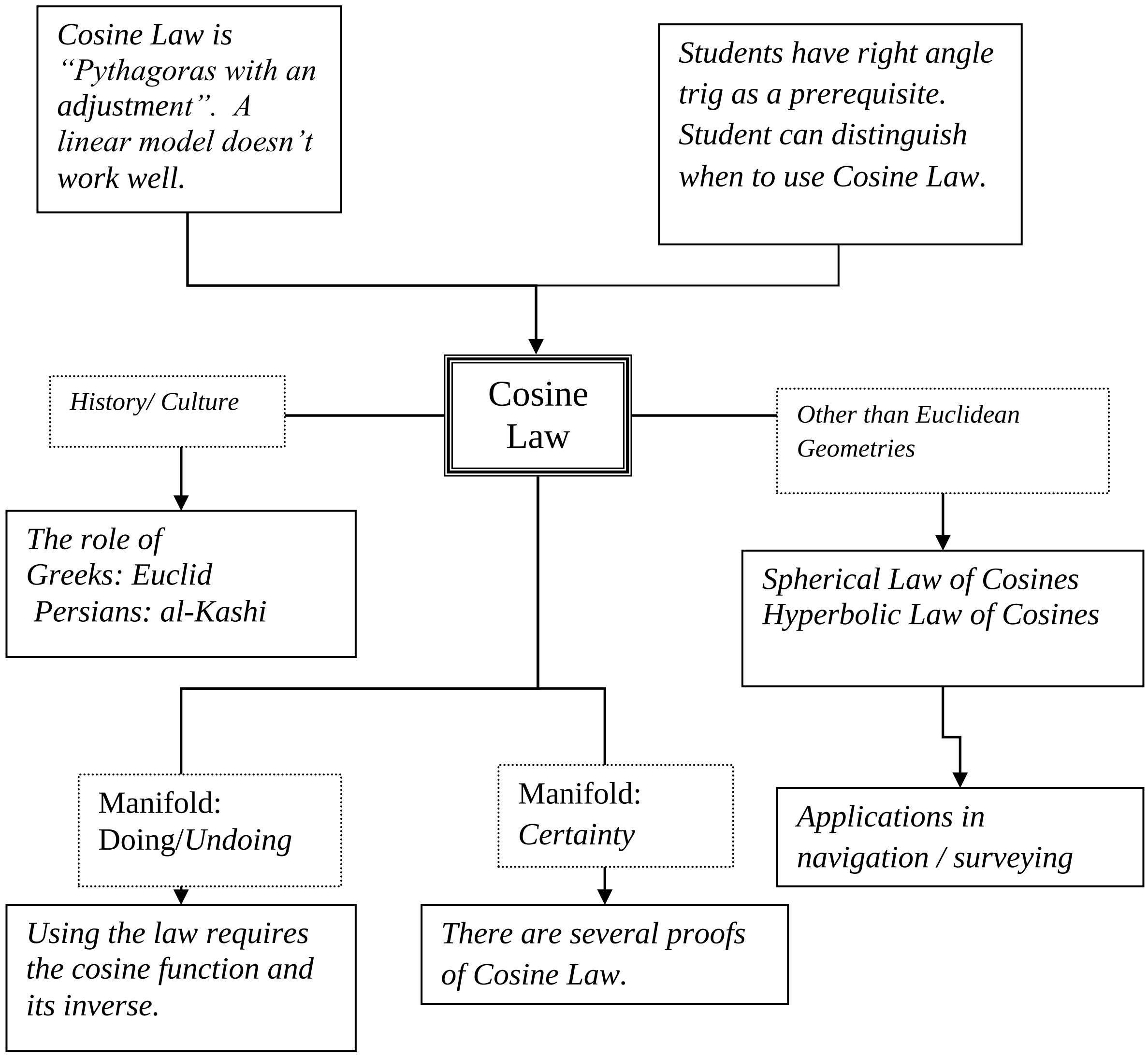
b) Two Manifolds mentioned in this activity are Change and Modelling. How are these apparent in the activity? Do we use other Manifolds that aren't mentioned? Record your group's ideas here:

c) To find these side lengths, we need another model that isn't linear. You will learn the Cosine Law.

End of Student Activity

Exemplar continued...

A Linking Diagram for Cosine Law



Some potential sources: http://en.wikipedia.org/wiki/Law_of_cosines
http://en.wikipedia.org/wiki/Al-Kashi
http://www.cut-the-knot.org/pythagoras/cosine.shtml

End of Cosine Law Exemplar

In the exemplar there are direct references to two of the Manifolds, and further, students are asked to examine the activity for other Manifolds. They might mention uncertainty in measurement or patterns in looking at any linear data they may find. The connections to the major organizing structures of the Manifold & Intention Model are directly stated.

Content Exemplar #2: Soda Pop Activity Exemplar

Student Activity

When fresh cola is poured into a clean glass, it often foams up. It would be interesting to investigate whether we can view this situation mathematically. You will probably want movies/still pictures and a stop watch to investigate this. (Use your phone). We can examine this phenomenon from a couple of perspectives. You will receive some cola and a glass to perform the activity.

Change

a) How fast does the level of the foam decrease from its maximum?

b) How does the average bubble size change?

c) How does the number of total bubbles in the foam change?

d) ____

Modelling

a) Each person in your table group selects one of these situations of change. Join another group that is modelling the same thing as you. Collect data by pouring some cola into a glass and measuring the results. Together, create an equation that is a mathematical model of your situation. Sketch a graph of the data you have collected, and *draw on it the function(s) you used to model the situation. You should include the original data and your model graphed on the same coordinate plane.*

Uncertainty

a) What are the lower and upper limits for your measurements?

b) How could the uncertainty in the measurements affect your model? Refine your model, attempting to take into account the uncertainty in your measurements.

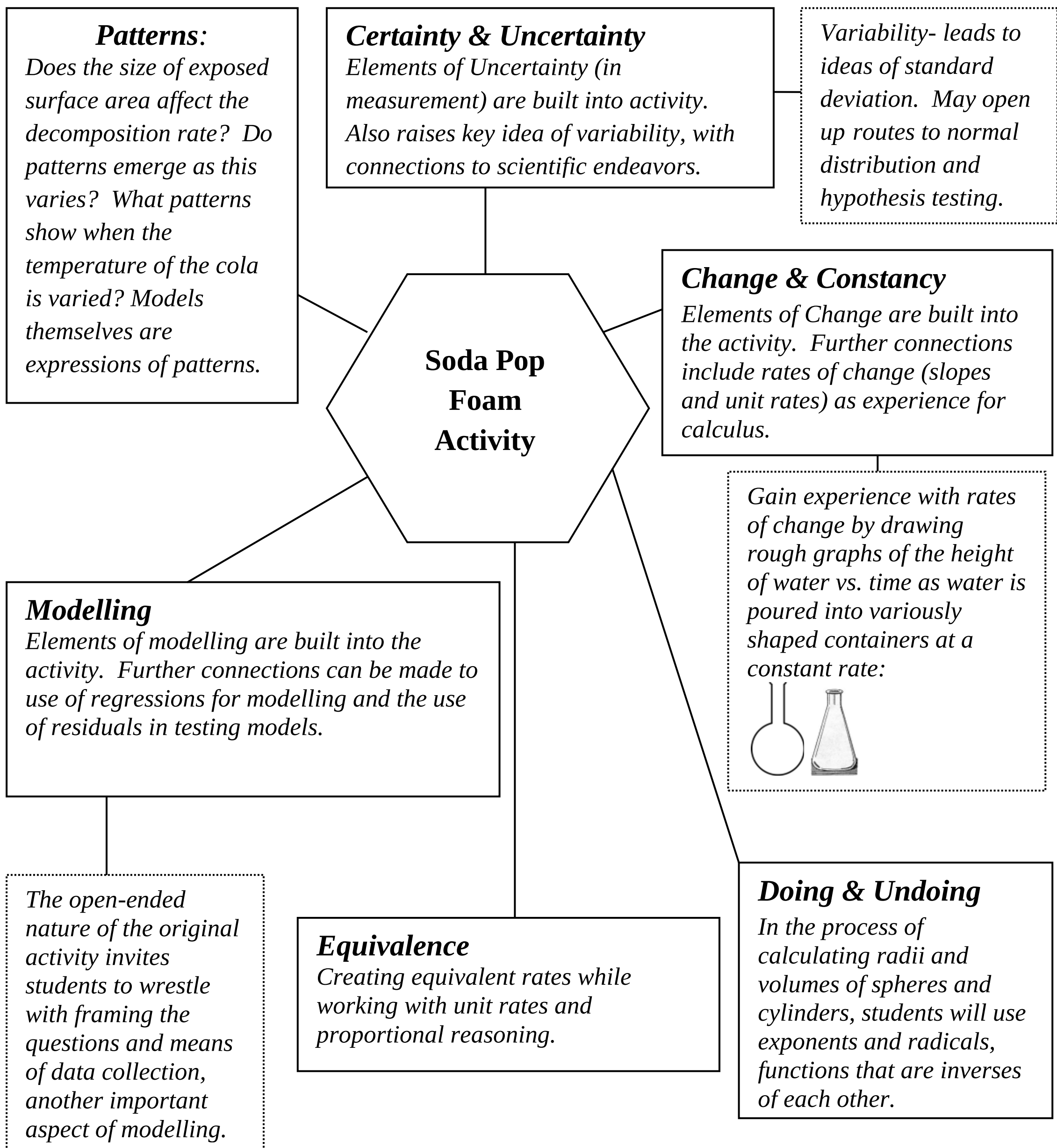
c) In your new model(s), how long will it take the foam to disappear? (A range of times is expected).

b) Pour two new part glasses of cola, to see if your model predicts the disappearance time well. That is, did the foam disappearance time fall inside the range of times you created in part c)?

End of student activity

Exemplar continued...

Content Exemplar #2: A Linking Diagram for Soda Pop Foam Activity



End of Exemplar #2

In this exemplar, and in the linking diagram, the Manifolds are clearly displayed. Further the notion of decentralized control is apparent when the open-ended suggestions are made in the linking diagrams. The invitation to organized randomness and internal diversity are built into the expression of the curriculum model.

Content Exemplar #3: Powers and Exponents Exemplar

For the Teacher

The notion of powers and exponents is very important and serve very well to illustrate and exemplify the Manifold of Change & Constancy, Equivalence, Patterns, Doing & Undoing, Modelling and Certainty & Uncertainty. As such, powers and exponents are very important in a math program. Here are some connections to the Manifolds with ideas for activities or connections to activate or consolidate meaning.

Change & Constancy: *Graph $y = x$, $y = x^2$, $y = x^3$ and $y = x^4$, first on [-4, 4], then zoom in on [-1.2, 1.2], experimenting with the range to obtain clear results. What stays constant in the behaviour of the graphs? How do the graphs differ?*
Now graph $y = 1^x$, $y = 2^x$, $y = 3^x$ and $y = 4^x$, first on [-2, 2], then on [1, 4], experimenting with the range to obtain clear results. What stays constant in the behaviour of the graphs? How do the graphs differ? Explain this behaviour.
Make and test conjectures about the graphs of $y = x^5$, $y = x^6$ and of $y = 5^x$ and $y = 6^x$.
Equivalence: *The Exponent Laws are description of how certain expressions are equivalent to each other. Focus on the utility of different representations for imagining the world or seeing patterns. For example, -8 is more convenient than $\frac{-1}{2^{-3}}$ in most situations while 2^{6000} is typically more convenient than its standard form representation.*

Patterns *: One model for considering exponentiation is that of repeated multiplication. This model works well for natural number exponents. (What's the last digit of 2^{6000}?) The patterns obtained from action can also be examined. For example, obtain a length of* *toilet paper with, say, 64 squares. Repeatedly fold in half, comparing the total number of folds, the length of the remaining paper and the number of layers formed. What changes? What stays constant?*

Consider the problem of defining exponentiation as repeated multiplication for rational exponents. We can remove this problem if we adopt a Pattern model meaning for exponents and use this reasoning for the meaning of zero and negative exponents.

Another activity could be to model successive powers of ten with Base 10 blocks: 1 is a cube, 10 is a rod, 10^2 *is a flat,* 10^3 *is a cube, and* 10^4 *will be a rod... From this we can examine higher powers and see how* 10^6 *is a square and a cube.*

Also consider the old favourite of painting the outer surface of a cube made of n^3 *smaller cubes, then disassembling the large cube. How many of the smaller cube will have paint on all faces? On just five faces? On four faces? On three faces and so on? Finding patterns in the numbers that result ties in extremely nicely to exponents and to surface area and volume.*

Certainty & Uncertainty*: Arguments with natural number exponents, while not rigorous, may give students an appreciation for the idea of proof or justification in math. Asking 'What is the probability of rolling 4 consecutive sixes on a single die?' shows the potential connection in probability.*

continued...

Content Exemplar #3: Powers and Exponents - continued

Modelling*: Modelling may be seen as attempting to fulfill one of human's oldest desires: to predict the future. Exponential functions, radicals and the logarithmic functions can be used to model many situations. From the toilet paper example above, students could write models for the change in the number of layers and for the length of the strip, in squares. We could also take the temperature of hot tap water over the course of five or ten minutes in an uninsulated can every 30 seconds. Can we model the temperature vs. time with a linear model? What type of model works better? Predict the shape of the graph if the can was insulated. What effect would that have on the base of the power that models the temperature?*

Exponents can also be connected to the notion of dimension, one of Steen's (1990) choices for a deep idea in math. Take an air pump and several balloons that inflate to a near spherical shape. Put a number of pumps of air in the balloon, and then measure the diameter. Release the balloon to fly about, timing the length of the flight. Repeat several more times, using more pumps of air each time. Compare the how the diameter and the length of flight changes with respect to the number of pumps. Are these well modelled with a linear function? How does the dimension of a sphere related to the linear dimension of the number of pumps? Perform calculations with the volume formula to suggest reasons for the observations.

Doing/ Undoing*: The repeated multiplication model for exponentiation is an example of repeated doing. The notion of geometric progression is such an example. Consider the role that repeated application of a given percent increase generates. Exponential functions can be used to create models for examining half-lives of radioactive elements, exponential growth of species unchecked by limiting factors, or examples of inflation or compound interest. Exponents and radicals demonstrate one notion of Doing & Undoing, while exponents and logarithms display another. (Exactly how many digits are in* 2^{6000}*?)*

End of Content Exemplar #3

In this exemplar, there is a discussion of all the Manifolds, with some descriptions of individual problems or activities for students. The fact that there are so many connections with the Manifolds and so many activities and questions would strongly recommend powers and exponents to be part of the Common Content.

Content Exemplar #4: Wolves and Deer Exemplar

For the teacher

In this exemplar, a spread sheet is used to model a simple idealized ecosystem interaction between predator and prey. A few rules and a random component to represent weather changes and disease simulate an ecosystem. Students run the simulations several times to notice any similarities or difference between the runs. Then they can alter various parameters to examine the effect of their changes.

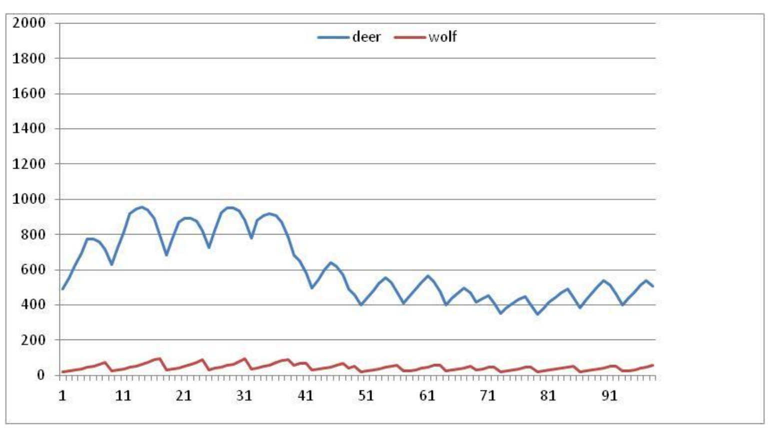
***Change & Constancy**: What variation and what trends appear in the first twenty or thirty simulations?*

***Doing & Undoing**: Notice that the effect of doing and redoing the same simple rules may give rise to interesting outputs.*

***Modelling**: What effect does changing numbers such as the increasing the starting number of deer by 20% have on the 'ecosystem'? Do the final totals also go up by 20%? How sensitive is the system to initial conditions?*

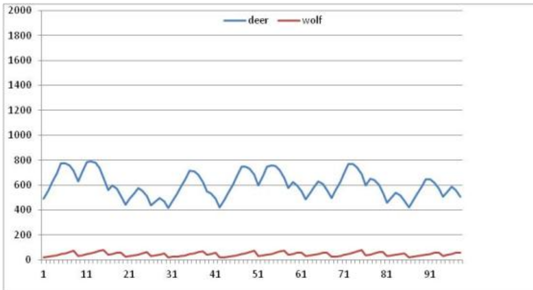
Below are seven outputs demonstrating some of the variation generated by the model. The top line refers to the population of deer and the lower line the population of the wolves over time in years: a) Deer/ Wolf simulation #1

Population

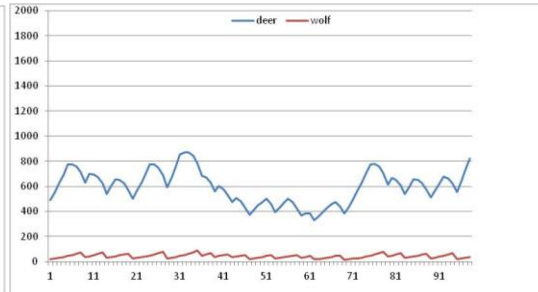


# of years elapsed

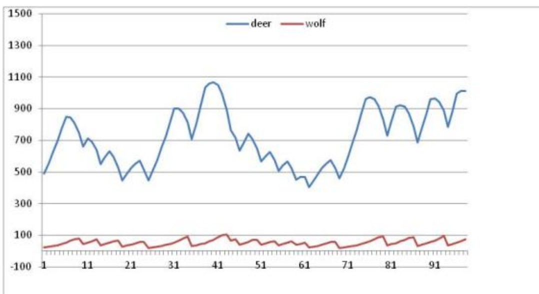
b) Deer/ Wolf simulation #2



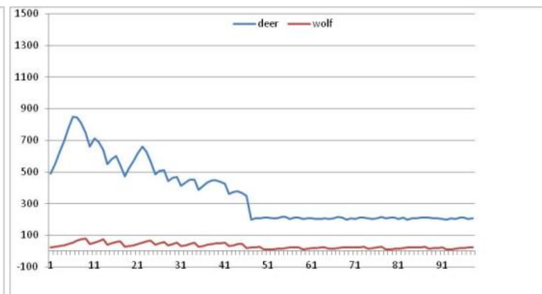
c) Deer/ Wolf simulation #3



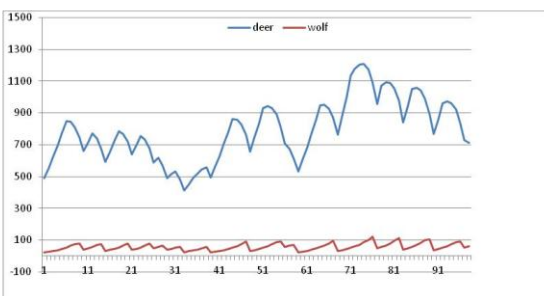
d) Deer/ Wolf simulation #4



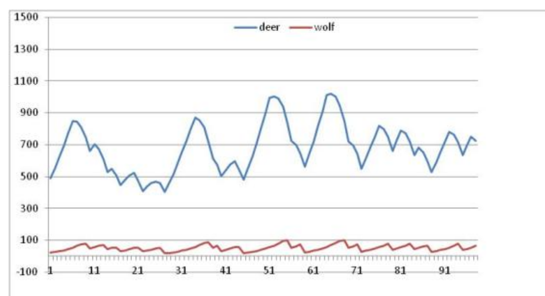
e) Deer/ Wolf simulation #5



f) Deer/ Wolf simulation #6



g) Deer/ Wolf simulation #7



The number of deer always starts at 500, and wolves at 20.

End of Content Exemplar #4

A major theme of this activity is to introduce chaos theory, showing that sensitive feedback systems may be complex and difficult to predict, and that therefore caution should be observed when making decisions about complex phenomena, as the results may vary significantly from what a simple model might suggest. If humans are to have any subtlety or humility in making alterations to the natural world (or any other complex) system we must be made aware of potential ramifications. Such discussions of complex feedback systems are, in my view, dangerously and negligently omitted from the existing Manitoba Curriculum Framework document.

The final exemplar I offer here is a version of an activity I have used with very good success over the past 15 or 20 years. In some respects, it is a forerunner of the notion of Local Content. It arose from my desire to have students have some influence over the mathematics they worked with. We can scarcely imagine the level of contempt we would feel for a language arts program that didn't provide an opportunity for students to write about their world and tell their own stories, from the earliest stages right through high school. Now, consider the circumstances that many students find themselves in mathematics class. Where is the same expectation that students will be creative in the mathematical realm? Why is the math that is taught always someone else's math? To be fair, the early years curriculum in Manitoba does support the notion of invented algorithms and individual understandings. However, I suspect that in many cases, mathematics is not done *by* students, but *to* students. We need activities that demonstrate to students and teachers alike that mathematics is a beautiful, connected and human activity.

Content Exemplar #5 The Exploration Inquiry Project Student Exemplar

The Exploration is a long-term major project that invites you to investigate and explore areas of mathematics in which you have an interest. ***The principal aim is for you to use your creativity and imagination, to be in the position of creating mathematics.*** *The final project, due ____________, must be of display quality. This is a very challenging and open-ended assignment designed to give you the maximum opportunity to use your talents.* ***It will require much time and preparation to produce a project of high quality. Do not procrastinate!***

You are to submit a project in which you explore or invent mathematics that is new to you. You must ***invent and solve a mathematical problem****. Make up a problem that you currently do not know how to do. This problem may arise from any area of mathematics, or any other subject. Consider geometry, algebra, number theory, statistics, probability, chaos theory, computer science and many more areas as a source of problems. The internet should only be used as a source of data, not for problems or solutions. Be sure to reference all information taken from any source, where you use it in the project, and again in a reference list.* *Any research done requires a reference list at the end the project, and a citation in your text at the place you use the data, diagram, quote etc.*

i) Make a ***Problem Statement*** *that clearly states the problem you are trying to solve. If ideas are gained from an outside source, the Problem Statement must declare this and you must provide the source. Include your reasoning and the RUBRIC writing when writing up your solution(s).*

ii) In the ***Solution section****, write up as many solutions to your problem as you can, explaining your thinking and your major steps. Use the Stuck /Aha/ Check/ Extend RUBRIC.* *Keep all your rough work**; it must be available when I ask to see it. At the end of this section, include a thoughtful analysis of how your problem/solution uses, or relates to, or exemplifies the six aspects of math: Change/Constancy, Patterns, Doing/Undoing, Modelling, Equivalence, and Certainty/Uncertainty.*

iii) At the end of your solution, please include ***a process paper*** *that indicates the decisions you made regarding the content of your piece (how you thought) the organization of your solution (how you chose to plan the writing of your solution) and decisions you made regarding Style and Mechanics. Also, include the effect of the process on you as a mathematician and you as a student. This* ***reflective writing*** *should be of such depth that you profit from the exercise.*

The criteria for the evaluation of your problem-solving skills in this section will come from the challenge of the problem, the evidence for persistence and innovation, the correctness of the solutions, the variety of solution methods and the level of extension or generalization of the problem. Consider that a worthy question that is partially answered may be worth as much or more as a lesser question answered completely. In order to display very good performance, strive to generalize your results (create formulas or rules) and connect your ideas to other areas of study.

Here are some comments to help you in your writing:

Communication: *The quality of your communication will be assessed on how well your document shows purpose, style, unity and mechanics. Your audience is at least one* *mathematics teacher, and your peers. Your aim is to have your Exploration of near publication quality, so it may be used as an exemplar for other students. I may show your work to other teachers and students.*

Style: Writing will be clear and concise. New or invented terms will be defined and explained, and there will be no heavy repetition. Charts/ graphs/ tables will be labeled effectively and included at the place in the text where they are discussed. Projects are to be written in the first person, as it is your *Exploration.*

Displaying Challenge and Innovation: *Good projects describe how you have developed new, interesting ways to view or solve your problem, and are more than rehearsal of a previously learned idea. Some elements of this will come from the degree of generalization that you make, the depth of new material you must learn, and new ways to apply or extend your idea(s).*

Solutions: *Good projects will explain precisely what the problem is, and will use diagrams, graphs, charts etc. if appropriate. It may be useful to specialize: to work out specific examples of your problem, or simpler/partial versions of your main problem. High quality solutions may have more than one way to tackle a problem.*

Generalizations/ Extensions: *Manifolds: Describe how your thinking shows one or more of the Manifolds: Change & Constancy, Equivalence, Modelling, Certainty & Uncertainty, Doing & Undoing or Patterns. The connections to the Manifolds are not just simple mentions; you must show how the idea, concept, or application you are using is a good example of one or more of the Manifolds.*

Generalizations are rules, patterns, and formulas etc. that show how ideas can be used in circumstances other than the few that may be attempted in your specializing.

Extensions are circumstance where you attempt to use what you've learned, discovered, or conjectured to tackle other problems or circumstances that were not part of your original work.

End of Content Exemplar #5

In such an open-ended project, there is great scope for individual variation, and when students discuss their work in class, the discussions are frequently of a high level: thoughtful, curious and respectful.

4.7 Inspecting The Manifold & Intention Model

The Lens of Complexity

The entire Model is built upon the notions of complexity, particularly the five necessary conditions given by Davis and Simmt (2003). Every element is interdependent, and open to adaptation and refinement, and solicits input from a broad number of people to create an adaptive system. The system is generated with the principles that are necessary for emergent behaviour. But there's more. The principles of complexity theory are explicitly referenced regularly in the elements, thus drawing attention to the centrality of this important component. There is support for neighbour interaction (all five elements), resiliency (Common Content, to some degree the Manifolds), organized randomness (Manifolds, Content Evaluation Criteria, Common Content and to some degree the Intention), internal diversity (Local Content and the processes that affect any instantiation of the Model), and finally decentralized control (all the elements).

The Lens of Perspective

Far from having a default intention, this model expressly attempts to draw attention to its underlying assumptions and perspectives, and thus make those positions problematic. In addition, the expected role of hundreds or even thousands of people in the evolution of the model mean that a broad spectrum of environmental, political, educational and social perspectives are not just possible, but are expected and expressed. Environmental and social views are foremost in the model, and while this is the stated bias there is room to discuss expectations of our current societal structure including issues of economics and short-term social efficiency. The Model requires perspectives to be solicited, expressed openly, debated and examined.

Thus, we have an overview of the Manifold & Intention Model for curriculum. Each of the five major elements has been described and some exemplars given for each. As with any complex adaptive system, the essence of this Model is that its elements, though the agency of teachers, experts and academics, interrelate and are intended to adapt and evolve. Such an intelligent system is the type most likely to be emergent: to come up with ways of acting that are not simple additions of the actions of the parts. It is vital to have an educational system that is able to generate possibilities that are not imagined, and to have the fecundity and versatility to deal with the expected, such as climate change, the expected unexpected, such as catastrophic economic restructuring and deep social change and - the most challenging of all - the unexpected unexpected.

There are disadvantages to any system. At the beginning, the number and scope of the exemplars may be small, the Manifolds and Content Evaluation Criteria may be centrally controlled, and some teachers may not wish to engage with, or have the skills for, the project of building and evolving with the elements of the Model. But this Model, unlike the existing Manitoba situation, invites, encourages, and expects evolution. It is a mark of its organic nature that it is perfectly capable of growing lushly where conditions are right and of adapting to the environment while it adapts the environment to suit it in places less well suited at the beginning. Even more importantly, the model is capable of handling the subsequent growth that results. Thus, it perseveres, leads and follows.

Ultimately, the Manifold & Intention Model uses the products and strength of human reasoning and creativity in a structure with a complex, open and evolutionary nature. It nurtures human capacity in an organically potent and respectful form.

5. Some thoughts on my path through curriculum – an instance of *currere*

The final component of this act of scholarship will be to reflect upon my growth as an act of *currere*. From my perspective at the end of this process, I look at what I have tried to achieve through the construction and examination of a model for curriculum and how this has influenced me as a person and as a teacher.

1n 1975, William Pinar presented a paper entitled *The Method of Currere*. In it he asks people to reconceptualize their consideration of their lived and academic life, to understand more of the general by 'experimenting' on oneself. By means of regression (a recounting of the past), progression (a free association of ideas and selves of the future), analysis (a snapshot of the present) and synthesis (a way of integrating and merging all three to make the present less overhung by the images of past and future), Pinar suggests we can reduce the distance between our temporal academic and personal selves and thus potentially reduce the distance between ourselves and others in academic research. For me to document this process completely would be a major work in itself, and it would be rife with the most tedious banalities.

In his paper, however, Pinar (1975) suggests that the methods are adaptable and so I will provide a very condensed version of me as a teacher and student of curriculum. In the spirit of the free association Pinar recommends, I shall then include some musings about challenges I faced or questions that are problematic for me still, a sort of bricolage or pastiche of questions, reflections and ideas.

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Regression

I am a high school math teacher in a large urban high school in Manitoba. My experiences have shaped how I view curriculum, and have lead me to inquire about finding a new way of interpreting existing curriculum and of developing new curricula in mathematics.

I began my career believing that curriculum was supreme. As a young teacher, I thought that the given curriculum wert and art, and evermore should be. The supremacy of The Curriculum was driven home to me not only during my time in faculty, but also as a student teacher. In fact, one of my cooperating teachers told me directly that any teacher who did not cover the curriculum would put his job in jeopardy.

My own awakening started, as things may do, with a question from a student. This question – actually, the same question twice repeated – truly began my understanding of the need to question curriculum. This awakening continued with my experience as a member of the 5-8 Provincial Negotiating team for the Western Canada Protocol – now the Western and Northern Protocol - and my experience in my Masters of Education program. Each of these has acted as a frame for re-viewing the world of what students are supposed to learn.

For a number of years I taught both Grade 8 and Grade 9 science (as well as mathematics). In Grade 8 we studied astronomy, an area of interest for me, and one many students seemed to enjoy as well. In Grade 9, one of my students who had enjoyed astronomy in Grade 8 asked if we could take it again this year. I said, politely, that it wasn't in the curriculum. A few weeks later, she asked again. "Can we learn Astronomy this year?" and I replied "No."

Right then, I knew something was wrong. I called myself a teacher, but when a student asked if they could learn something, I said "No". To address this injustice, I created a system where students were able to work more independently on their curriculum 'responsibility' and therefore permitted class time for students to work on topics of personal interest and to perform experiments they thought up. It was important to me that students had an opportunity to use school time to learn science that was meaningful to them.

At about the same time, I was a member of the 5-8 Provincial Mathematics Negotiating team for the Western Canada Protocol. Manitoba had recently finished a major curriculum overhaul and part of our instructions as a team was 'not to let Alberta get everything.' I also recall that an activity that I used successfully with students was criticized openly by the Assessment leader during the meeting as being too difficult to write specific outcomes for or to mark in a Standards exam. The members of the 5-8 team worked well and diligently, but the politics behind what I had thought previously as a purely pedagogical exercise disturbed me. Perhaps it would be well to add curriculum to the list of the law and sausages as the kinds of things that are better enjoyed when one has not seen them made.

Finally, my experience in my Master of Education program has about understanding and appreciating that curriculum may be viewed from a variety of perspectives, and that these perspectives are influenced by personalities, philosophies and politics. The reading and discussions have influenced me deeply, and have helped me create a context for understanding and completely reimagining what curriculum is and can be.

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Organized Randomness - The Role of Creativity

At one point in my coursework, a professor required our final piece to be anything but a formal paper. I admitted to some confusion at the openness; what would be expected? In a conversation I was reassured that open-ended meant open-ended. This freeing of my creativity was a watershed for me. I enjoyed the process immensely, which was for me deeply synthetic, and that artifact was a precursor to this work.

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Dewey

It is hard for me to express precisely the effect that Dewey's work has had on me. My extensive use of quotations signals both a desire to let the reader see the level of the impact he has had on me as well to combat the possibility that small snippets might be taken out of context. One of his major contributions to my conceptualization of my curriculum model is the idea and nature of growth. I want a curriculum model to be one that respects the dignity of each person, their 'intrinsic fullness' and at the same time understands that individuals make - and are made in - a society. To me, a curriculum model requires at least some way for teachers and students to influence its content, and understand, if not agree with, its direction or meaning. This is Dewey's rejection of the potential dichotomy between of personal development and social efficiency in the aims of school. My hope for my work in teaching is sustained by Dewey's view of humanity, and the balanced and reasoned notions he puts forth in bringing life to that view.

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Eureka - The origin of Manifolds

An image that came to my mind during a 15 minute break in one of my courses was that of a platform. I was analyzing the nature of possible organizing structures for curriculum and I was considering the linear imagery evoked by the term ‘strand’. Previous versions of the mathematics curricula and documents put forth by NCTM referred to ‘Strands’, such as Number, Shape and Space and so forth. While the metaphor of curriculum as strands carries various associations, (a weaving together, a flow in a direction, and an entanglement of ideas) it is still fundamentally a linear construct. Additionally, the Strands were identified with specific areas of content, rather than as mathematical processes. I didn’t (and don't) believe that the somewhat disjoint content and the Strand metaphor behind existing curricula is rich enough to direct teachers away from a focus on procedural or instrumental understanding. So, ultimately, I rejected the term 'Strand' for its linearity and associations with prior states. Therefore, I was left with the concern of providing firm support for the mathematics we wish to do, without being so narrow that integration and exploration cannot practically occur. I wanted something more multidimensional. The word ‘Platform’ sprang to mind. A platform connotes a stable, open area, capable of bearing the metaphorical weight of a region of study, and giving solid footing to the student, while being open enough to move around. But here too, the nature of the metaphors offered must be examined. Platforms may be seen as flat, and devoid of feature. If richness and texture are being sought for our curricula (and they are!), the flat nature of a platform may be questioned. The term I invented to represent the dimensionality of the underlying mathematics and the simplification of a small region of math that is so helpful for a novice's introduction was 'Manifold'.

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Does the model have perseverance with me?

Sometimes, in retrospect, ideas we have had seem to be simplistic or to lack the strength to carry new ideas. It has been close to six years that I have worked with ideas of some form for this Model for curriculum, and I am happy to say that I continue to be comfortable with the Model and its potential. I have been using a version of the Manifolds in my classes for the past few years, attempting to create an integrated view for students. I have a poster on the wall of my room displaying the six Manifolds I have suggested in the exemplars in Chapter Four, although for the students I refer to them as Aspects of Math rather than Manifolds. While I refer to it often in my explanations and sometimes in my written notes, it is only occasionally that students spontaneously see the Manifolds in the work we do together. Still, the references to the Manifolds brings up the chance to discuss how mathematics is a way of becoming personally powerful, and now I am seldom asked "When will we use this?" which happened somewhat more frequently before this.

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Honorable Intentions?

One of the struggles I had was in generating my sample Intention in section 4.1.3. This section has few references. It was my goal in this section to bring forward my own embodied sensibilities. It was here that I had to confront myself.

It's easy enough to lay out the broad strokes; say what you believe in, say what learning is and what students are and then describe the field of study. I was prepared to be challenged - the most fundamental things are often difficult to pin down. I was not prepared to find that the most difficult part was not expressing a worldview, nor in describing mathematics, but in discussing learning and students. I have taught over three thousand students, but describing my understanding of these vital facets was very hard to do.

For example, how would I deal with beliefs about ability differences in students? To me, they seem to exist. But what are the implications of such a belief? Are children fundamentally prisoners of their genetic heritage and early experience? If we cannot really help students change beyond what would in living in the world, then schooling itself is an empty enterprise. Expensive in time, money and human spirit, it should be abolished. But, if *all* students *can* learn just about anything, then how can I reconcile my lack of success in helping them all?

After consideration, I think the position I took in the exemplar was realistic and positive. Students vary, but they are not fated by their variance. Indeed, one of the powerful concomitants of the Manifold and Intention Model is that its use of complexity theory means that the variety in students is an excellence, not a fault. For instance, the conversation and questions that enliven and enrich a classroom can only arise because people differ in what they see and how they see. A broader set of perspectives can be offered, the life of internal diversity.

Another area of concern for me is in the section on learning, and how my considerations there affect my own practice. In reminding myself of the importance of rich rehearsal, I have to make this become more true for the students I teach. I need a more integrated and more deliberate rehearsal plan for my students. Thus, the writing of a sample Intention leads to the consideration of myself in the role of teacher-as-student, a useful position, paradoxically humbling and elevating. Curriculum as *currere*, indeed.

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Thinking with Manifolds

Manifolds are mental structures with which to think. [Not only specific to mathematics!]

-Deciding on Manifolds may help us to ponder the sacred.

- What is the essence of the discipline?

We may critique and evaluate existing curricula with a specific set of Manifolds and an Intention.

- We may examine the choice of material, what's in and what's out.

-We may examine the cultural situation of the curriculum. (What is its expressed/implied intention? Does the material selected support that belief system?)

Manifolds can help to create good lessons.

- It pushes one to search for meaningful connections and therefore more strategic discovering. Lessons may be more coherent, more beautiful. In searching there is more chance for cross-curricular work

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Metaphoric Modalities

In writing this thesis I have noticed my reliance on visual metaphors. I think I have used them because of their cultural weight; they are powerful and varied and familiar. A counterpoint to my visual modalities is expressed in the title of Ted Aoki's (2005) *Curriculum in a New Key* where the auditory is featured. In this modality, we have all the power of music and tone as imagery and a notion of remembrance and the past as we 're-call' things. In *The method of Currere*, Pinar (1975) seeks to replace the notion of the mind as all with the notion that mind is part of the body, thus refuting Descartes. I'm still teaching the intellect. This discussion of the metaphoric modalities in my thesis makes me consider the senses of lived experience, the embodiment of us as learners, and in this, I'm still not very far along in my teaching *through* the senses, *with* the body. I know about constructivism, and yet I often make the assumptions that everyone can abstract from their prior experience, or create analogies. But being able to do this is the mark of one who is practised and proficient, not that of the novice who is in my class. So, perhaps the dominance of just one modality points out another important journey I have still to make as a teacher.

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Elements of Religiosity

In several places I have used language that suggests or arises from religious contexts. And yet I strongly support the separation of church and state. What leads me to select words that I might consider loaded when used by another? Perhaps it's because I suspect dogma in another, whereas I believe I'm choosing words that have – or at least are intended to have – an uplifting, positive appeal, a sacred rather than religious sense, if such can exist. Even as I consider this, I am led to ideas surrounding a lack of trust and a culture of suspicion or scarcity as being operative in me. Yet typically, I'm very trusting, particularly of my students. I have no suitable explanation for any of this.

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From Truth to Efficiency, Radford (2004):

> I myself fear that if we do not find good reasons quickly, the place of 'pure' mathematics will become smaller and smaller in the school curricula. For example, deductive geometry has practically vanished from the Ontario mathematics school program. Perhaps we will have to learn to react to our contemporary societies, where questions about *truth* have been traded in for questions about *efficiency*. *Important* questions, indeed, seem to be no longer about whether something is true but if it works (Lyotard, 1979).
>
> In talking about truth, I am not referring to Plato's aristocratic Athenian and reactionary ontology of unchanging forms. I am talking about truth in small capitals, as it is formed by individuals in their reflection on the world—not exactly a relativist version of truth, though … We might need to find ways of making mathematics a relevant part of our cultural critical reflections of the world, where mathematics can go hand in hand with other cultural manifestations, such as art and literature. Perhaps, we need to understand ourselves and, in turn, help our students to understand that mathematics is a narrative genre too and that behind a geometric proof may be an aesthetic experience as rich and enjoyable as the experience behind a poem or a painting (p. 555).

I agree with Radford (2004) in the concern regarding truth and efficiency. The beauty of pure mathematics does not appear to be a concern anymore. (A little set theory is in Grade 12 *Applied* mathematics and absent from the Pre-Calculus course. Hmmm...). I have shown in Chapter Three that the fundamental ethos behind the Manitoba document is of efficiency, and the notion of truth is simply absent. The discussion about making mathematics a cultural manifestation is interesting but problematic. First, mathematics already carries enormous cultural weight as something that is powerful, worthy of derision, inaccessible, awe-inspiring and socially damaging, all at the same time.

Second, if math could be rehabilitated into cultural status with poetry and painting, we may still have a concern. How many students don't value paintings and claim to dislike poetry? Given this, am I fooling myself when I seek for a way to have others enjoy mathematics as much as I have? Though I want this, from experience I know this will be difficult to achieve. At the back of my mind, how do I really view those who haven't yet 'seen the light'? (Elitism and snobbery. Such wonderful attributes in a teacher…). Worse, am I seeking to recreate myself through others? Am I seeking to remake them in my own image? (And the religious imagery issue recurs.)

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T'was brillig… Lewis Carroll

Through the Common and Local Content, we can meet all five necessary criteria for an emergent, participatory and fundamentally effective set of experiences to be possible. The Intention sets out the world to be examined, and the reasons for so doing. The Content Evaluation Questions serve to winnow the huge range of choices to those that are rich and nourishing. The Manifolds provide the lenses, the different vantage points that display the essences of mathematics: interplay, connection, power, beauty, abundance and utility. The curriculum development structure I propose is adaptive, open and balanced, able to meet the stringent requirements Dewey would have it meet:

> There is incumbent upon the educator the duty of instituting a much more intelligent, and consequently more difficult, kind of planning. He must survey the capacities and needs of the particular set of individuals with whom he is dealing and must at the same time arrange the conditions which provide the subject-matter or content for experiences that satisfy these needs and develop these capacities. The planning must be flexible enough to permit free play for individuality of experience and yet firm enough to give direction towards continuous development of power (1997/1938, p. 58).

It is possible that, for some people, almost any curriculum will be abundant in the way I seek. However, I believe that the structures and culture of the existing curriculum makes abundance unlikely for most teachers. And when a teacher does not have a spirit of abundance, it is hard to imagine the majority of his or her students seeing mathematics that way. The Manifold & Intention Model can be a route to that enlightened end. This new structure is a way to promote dignity for teachers and students along the road to such levels of understanding, to such imaginings as may redeem mathematics in the eyes of everyone in our society. It is not the only way, but it is *a* way: a way station, a wayside chapel, a way beyond what we have now.

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