STUDENT ACHIEVEMENT AND ATTITUDE

IN A MODERN AND A TRADITIONAL

GRADE TEN GEOMETRY PROGRAM

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STUDENT ACHIEVEMENT AND ATTITUDE IN A MODERN AND A TRADITIONAL GRADE TEN GEOMETRY PROGRAM

Statement of the Problem

Instruction in secondary school geometry has been undergoing significant changes in approach and content for the last decade. Many new programs have been developed for the purpose of providing a curriculum which was more appropriate than traditional programs for the needs of secondary students. Because of the lack of appropriate experimental evidence about the effect of such programs on students, the purpose of this study was to compare the effect of a modern geometry program with the effect of a traditional geometry program on student achievement and attitude at the grade ten level.

Selection of Samples

Pilot class samples were selected on a geographic basis from schools which were authorized to use the modern text, *Geometry*, by Moise and Downs. The respondents were compared on a class to class basis with traditional classes using *A First Course in Plane Geometry* by Oliver *et al*. selected, with one exception, from the same schools. The eight classes in this study provided three experimental comparison groups which included: (1) five class to class comparison samples; (2) a matched sample; and (3) a total pilot-traditional sample.

Pre-Experimental Data

Pilot and traditional groups were matched on five variables obtained from the administration of two pre-tests, *Mathematics Attitude Scale* and *Sequential Tests of Educational Progress - Form 2A*, and the collection of information about the sex, age, mental ability and grade nine final mathematics mark of each student. These measurements were analyzed by using a stepwise discriminant function analysis program.

Treatment of Experimental Data

Four post tests, *Cooperative Geometry* (*COOP*), *Geometrical Achievement Measure Experiment* (*GAME*), *Mathematics Attitude Scale* (*MAS*), and *Geometrical Attitude Scale* (*GAS*) were administered to all participants. The results from these tests provided information about nine experimental variables which were analyzed by using a correlation matrix program and a stepwise discriminant function program to test two of the three experimental hypotheses. In addition, an item analysis with appropriate tests of significance was carried out on the *Cooperative Geometry* (*COOP*) and the *Geometrical Achievement Measure Experiment* (*GAME*) tests.

Summary of Findings

1. There is no significant difference, with one exception, between the achievement of the pilot and traditional groups on the total *GAME* test. The exception revealed that one pilot class had outperformed the traditional class in terms of achievement. An item

analysis of the GAME test also disclosed that all pilot groups responded better than their traditional counterparts to questions relating to the structure of a miniature geometry and items concerning congruence situations.

2. There is no significant difference, with one exception, between the achievement of the pilot and traditional groups on the total COOP test. A discriminant analysis of the results on this test revealed that the exception occurred in a comparison group where the pilot class achieved superior results to the traditional class. In addition, an item analysis of the COOP test revealed several significant divergent responses between the pilot and traditional groups. This examination revealed that the pilot groups performed better than the traditional on items referring to congruence situations, while the traditional groups produced results superior to the pilot groups on items relating to angle measures and the properties of isosceles triangles.

3. There are significant correlations among the total sample population, the total pilot sample, and the total traditional sample on the following eleven variables: (1) MAS (2) GAME (3) COOP (4) GAS-Text (5) GAS-Course (6) GAS-Needs (7) GAS-Interest (8) GAS-Involvement (9) GAS-Total (10) SCAT (11) 9MATH. An examination of three correlation matrices revealed that all the experimental tests were measuring consistently. In addition, almost all the intercorrelations were significant. There was a highly significant correlation between grade nine mathematics final mark

and the other ten variables with an advantage in favour of the total pilot group. One interpretation may be that the increase in significant correlations for the pilot group may indicate that previous success in grade nine mathematics helped a student accept the pilot geometry course better than it helped a student accept the traditional geometry program.

4. There was no significant difference, with one exception, between the pilot groups and the traditional groups on the nine experimental variables. The exception revealed that the pilot class had achieved better results on Cooperative Geometry and the Geometrical Achievement Measure Experiment tests than the traditional class. However, the traditional class in this group accepted the text and course content more favourably and were more interested and involved in their geometry program than the pilot class.

Conclusions

The results of this study show no consistent pattern of superiority for either the pilot or the traditional geometry programs in terms of student achievement and attitude. In all comparison groups, except one, equivalent results were obtained by both treatments. This supported and verified the findings of similar studies which have concluded that students taking a modern geometry course do as well as students taking a traditional geometry course. However, nowhere in the data of this study was there any indication of a strongly positive acceptance of geometry by either the pilot or the traditional groups. Consequently, further research is recommended to study methods which will increase student enjoyment and acceptance of geometry.

In the judgement of Manitoba mathematics teachers and University professors the modern geometry program contained in Geometry by Moise and Downs was more mathematically sound and more compatible with the present and future needs of university-oriented students. The fact that the modern course students learned significant geometrical concepts not usually treated in the traditional course, but which should be elements of a modern course, was an added advantage for them. Consequently, the modern geometry program contained in Geometry should be considered as an acceptable alternative to the traditional secondary geometry program contained in A First Course In Plane Geometry.

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CHAPTER I

THE NATURE OF THE INQUIRY

The traditional first course in geometry has been Euclidean-based for several thousand years. Since the turn of this century the pedagogical and logical defects of Euclid's approach have been recognized and criticized. As a result, instruction in geometry has been undergoing significant changes to the present day. There is no longer any need to rely completely upon Euclid's theorems and methods because several acceptable alternative programs have been written and are being used in schools across the continent. However, there is a need to evaluate such programs on a local experimental basis to help determine whether their stated objectives satisfy the needs of our secondary students in Manitoba. Part of the intent of this study was to supply appropriate experimental evidence which would assist provincial curriculum planners in making decisions about a new geometry program.

Effective instruction in geometry is required by our young people, and appropriate geometry programs must be authorized by the Department of Education to help provide that instruction. Nevertheless, it was not the intent of this study to provide material upon which a course outline could be based.

I. THE PROBLEM

Statement of the Problem

The basic purpose of this study was to evaluate how well grade ten students were achieving in a modern and in a traditional geometry program, and to what extent they were accepting each. This study did not propose to decide whether grade ten students should or should not study geometry of either a traditional or a modern kind. The answer to this question can not be provided by the usual methods of empirical educational research. The answer must be given by the collective decision of curriculum planners and professional educators and based on educationally sound evidence.

A Need for Investigation

The need for such an investigation was evident from a study of reports in current publications. Johnson notes that these are critical times in mathematics education. This is so because much of the criticism being made of new mathematics programs is extremely difficult to answer. Johnson maintains that we do not now have enough information about the effectiveness of new mathematics programs. It is both necessary and possible, according to Johnson, to design studies that will give valid information to help compare the mathematics programs.[1] He describes the present situation in the following way:

> Although several important studies have been made, we have not established criteria, we have not devised measuring instruments; and we have not completed research that would give us valid information about the effectiveness of a mathematics program.[2]

These comments may also be applied to the current state of affairs for new geometry programs. There is an urgent need for the appraisal of geometry programs in terms of student achievement and attitude.

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[1] Donovan A. Johnson, "A Pattern for Research in the Mathematics Classroom," The Mathematics Teacher, 59:418-425, May, 1966.

[2] Ibid., p. 148.

The Report of the Secondary School Curriculum Committee emphasizes the present concern as follows:

> Increasing emphasis is being given to the significance of mathematics as a substantial part of our culture and to the rapidity with which it is changing and is producing change. As a consequence, both lay opinion and professional opinion have become quite concerned over the curriculum content and the instructional practices which characterize the current mathematics program in the schools of our nation. The significance of this concern and the serious nature of its implications are underscored by the questions which teachers and administrators are asking.[3]

They are also concerned about the changing emphasis on geometry in the secondary school curriculum. Presently, this emphasis is a source of confusion to many educators. The issues which are central to determining an effective and appropriate secondary geometry program must be clarified to alleviate that confusion. Geometry is a significant subject in the secondary mathematics curriculum because it presents an excellent opportunity for a student to learn about the nature of mathematics and how it is created. Thus, provincial curriculum planners must arrive at some valid answers to the question, "What shall we teach in high school geometry?"[4]

The Report of the Secondary School Curriculum Committee, describing the importance of geometry in the secondary school mathematics curriculum, noted:

> Those who teach geometry in our schools should be assured that they are making a vital contribution to the education of their pupils. Synthetic plane geometry taught from a suitable text by

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[3]Secondary School Curriculum Committee: N.C.T.M., The Secondary Mathematics Curriculum (Washington: National Council of Teachers of Mathematics, 1959), p. 389.

[4]Irving Adler, "What Shall We Teach in High School Geometry," The Mathematics Teacher, 61:226-238, March, 1968.

> a competent teacher provides magnificent opportunities for bringing to life in the minds of the pupils latent powers of analysis, insight, and reasoning. No other mathematics course contains such a wealth of nontrivial original exercises of all degrees of difficulty. Pupils have the experience first of proving fairly simple (almost immediate) consequences of various theorems, and later of solving more sophisticated originals based on a more miscellaneous set of theorems. The solving of such originals will involve considerable thought, trial-and-error experience, and a gradual development of an awareness of essential needs. When a pupil has, after considerable effort, solved any original of the more challenging type, he has gained a great victory. The experience can make a significant contribution toward the increase of his power over any situation, mathematical or otherwise, requiring sustained and creative thinking.[5]

As a result, in addition to investigating the basic purpose, this study was considered to be significant for several practical reasons. First, the importance of geometry in the secondary mathematics curriculum and its changing emphasis required that appropriate value judgements on new geometry programs be made by provincial curriculum planners. Secondly, there was a definite lack of experimental evidence about the effectiveness of a proposed pilot geometry program. Consequently, it became a subsidiary purpose of this study to supply some evidence to assist the planners.

II. DESIGN OF THE STUDY

This study was concerned with two different groups of students: one group taking a modern geometry course and denoted as the pilot group; the other taking the conventional geometry course and denoted as the traditional group. The pilot group followed the geometry program contained in the text, Geometry, by Moise and Downs. The

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[5] Secondary School Curriculum Committee: N.C.T.M., The Secondary Mathematics Curriculum (Washington: National Council of Teachers of Mathematics, 1959), p. 404.

traditional group used the program contained in _A First Course in Plane Geometry_, by Oliver, Winters and Hodgkinson. Both the pilot and traditional students were enrolled in the University Entrance Course which included a second language and limited electives.

Selection of Samples

Pilot samples were selected from schools where pilot classes were actually being conducted. Traditional classes were selected from the same schools with one exception. This exception was a traditional class which was selected from a matching geographic and socio-economic school area as the pilot class area.

The total experimental or pilot group of 98 grade ten students was obtained from three secondary schools selected on a geographic basis. This group was composed of four classes of students selected from the total pilot population of grade ten students using _Geometry_ by Moise and Downs. A control sample of 114 grade ten students was selected from comparable classes and schools. It contained four classes of students using the traditional text, _A First Course in Plane Geometry_. In the total pilot sample, 37 students were enrolled in a Winnipeg school, 25 in a suburban school, and 36 in a rural school. The total traditional sample consisted of 67 students from a Winnipeg school, 26 from a suburban school and 21 from a rural school.

Experimental Comparison Groups

Three kinds of experimental comparison groups were provided by the eight classes in this study. These included: (1) five class-to-class comparison samples, (2) a matched sample, and (3) a total pilot-traditional sample. The five class-to-class comparison groups were structured as follows: Groups I and II were composed of three classes, one pilot and two traditional from a large Winnipeg high school, arranged so that the pilot class was compared with each of the traditional classes; Group III consisted of two classes, one pilot and one traditional, taught by the same teacher and selected from the same large suburban high school; Groups IV and V were composed of three classes, two pilot and one traditional, selected from two different composite rural high schools and arranged so that each pilot class was compared with the traditional class.

In addition, a matched sample was selected in the following way. A table of random numbers was used to select thirty students from the total pilot group. They were then matched in terms of the variables, sex, age, mathematical ability and achievement, with thirty students selected from the total traditional class. Finally, the total pilot group was compared with the total traditional group.

The basic experimental structure used in this investigation was a post test only design with matched control or traditional group samples based on pre-experimental characteristics. As a result, the pilot and traditional groups were matched on five variables that are correlated with the experimental variables of achievement and attitude. The variables selected were student age, mental ability, mathematics achievement, mathematical ability and attitude towards mathematics.

Administration of Pre-Tests

Two pre-tests were administered by the participating teachers to all students during the second week of September, 1966. One of these was a standardized mathematics achievement test entitled Sequential Tests of Educational Progress, Mathematics - Form 2A, developed by the Educational Testing Service, Cooperative Test Division, Berkely, California. The other was a mathematics attitude test entitled Math Attitude Scale (MAS). It was developed by Aikens of the University of North Carolina to measure a positive or negative attitude towards mathematics.

Experimental Procedure

The experimental period continued from the second week of September, 1966 until the second week of May, 1967. During this time both the pilot and traditional classes followed the programs presented in their texts. Four post tests were administered to each student to provide answers to the following questions: (1) Is there a difference in the relative achievement of students taking a conventional geometry program as compared to those taking a pilot geometry program? and (2) Is there a difference in the degree of acceptance of grade ten geometry by both groups of students? Student attitudes toward the two geometry programs were measured by two instruments, one developed under the title Mathematics Attitude Scale (MAS), and the other developed under the title Geometrical Attitude Scale (GAS). To evaluate student achievement under both programs the following two tests were used: (1) Cooperative Geometry: Form A -- a standardized test designed to measure the student's comprehension of the basic concepts, techniques and unifying principles in geometry, and (2) Geometrical Achievement Measure Experiment (GAME) -- a criterion achievement test developed by the writer.

All post tests are included in the Appendix and described in detail in Chapter IV. Results from these tests were scored by the writer and then recorded on punch cards for processing by a computer.

III. ASSUMPTIONS AND LIMITATIONS OF THE STUDY

Assumptions

It was assumed that the participating teachers followed the test directions for administration and maintained effective testing conditions. An accepted condition of this study was also that the tests were kept secure and that the students had not practiced similar items. It was further assumed that all participating students had the necessary skill to deal adequately with multiple choice test items, and that all teachers had the necessary training to administer such tests effectively.

Limitations

This study may be limited by the ability and competence of the teachers to instruct the pilot and traditional courses, although several replications were drawn from different school areas to minimize this effect. For the same reason an attempt was made to include in the study teachers of similar academic and professional qualifications.

The inexperience of pilot course teachers with the new geometry program may have limited the results of this study. Traditional course teachers had the advantage of prior instructional practice with their course.

The length of the experimental period may have been a third limitation. It is quite possible that long term results of either the pilot or the traditional approach may have occurred if the experiment had continued further. However, the measurement of either the retention of, or increase in, achievement standards and positive attitudes was not an objective of the study.

Finally, the procedures used to select the pilot and traditional groups may have influenced the results. As a result, any inferences made about the samples must be done with caution. In effect, the comparison samples which were selected were those that were available in terms of the scarcity of pilot classes.

IV. HYPOTHESES TESTED

The following null hypotheses were tested in this investigation:

Hypothesis I

No significant differences exist between the percentage correct responses of students of the pilot groups and students of the traditional groups on each of the items of the two tests *Geometrical Achievement Measure Experiment* and *Cooperative Geometry*.

Hypothesis II

No significant correlations exist among the total sample population, the total pilot groups and the total traditional group on the following eleven variables: (a) student attitude towards mathematics; (b) student achievement on a criterion test; (c) student achievement on a standardized test; (d) student attitude towards the text; (e) student attitude towards course content; (f) student needs; (g) student interest in geometry; (h) student involvement in geometry; (i) total score of five GAS sub-tests; (j) mental ability test; and (k) prior mathematical achievement.

Hypothesis III

No significant mean differences exist between the students of the pilot group and the students of the traditional group on the following nine experimental variables: (a) student attitude towards geometry; (b) student achievement on a criterion test; (c) student achievement on a standardized test; (d) student attitude towards text; (e) student attitude towards course content; (f) student needs; (g) student interest in geometry; (h) student involvement in geometry; and (i) total score of five GAS sub-tests.

CHAPTER II

A REVIEW OF THE LITERATURE

The literature on evaluation of secondary mathematics programs was examined for pertinent studies in geometry. This review revealed that very little published research on the evaluation of modern geometry courses was on the record. Not only was there a scarcity of such research, but also many researchers reported conflicting results in the investigations that were completed and published. Some studies concluded that modern geometry programs were more effective than traditional programs, while other studies concluded the opposite.

For the past decade some form of modern geometry program has been tried in different American schools. However, in most places the changes have occurred so recently that research projects have not been carried out. Consequently, published research on comparative geometry studies is quite limited.

I. EXPERIMENTAL EVALUATION OF GEOMETRY PROGRAMS

The greatest volume of research data comparing modern and traditional mathematics programs generally and geometry programs specifically has been published and sponsored by the School Mathematics Study Groups.[1] They have undertaken a variety of studies. Two which are pertinent for this investigation were carried out for the SMSG by

[1] School Mathematics Study Group, "SMSG Publications," SMSG Newsletter, 26:5-13, April, 1967.

the Educational Testing Service[2] and the Minnesota National Laboratory.[3] They were evaluations of student achievement in the SMSG courses and not evaluations of the courses themselves. The studies evaluated student achievement in a course in terms of the analysis of student scores on specified standardized tests at the end of the course.

Educational Testing Service

The first study, conducted by the Educational Testing Service, found that students in SMSG classes do about as well as students in conventional classes on standardized tests of achievement. In addition, they discovered that students in SMSG classes learned substantial amounts of mathematics not included in traditional courses.[4] Part of the research program carried on by the Minnesota National Laboratory and the Educational Testing Service included a comprehensive evaluation of the SMSG geometry test at the grade ten level. Because this text is quite similar in presentation and organization to the text, Geometry, by Moise and Downs, the results of this area of their research are particularly significant.

At the grade ten level, Payette reports, for the Educational Testing Service, that the average achievement of a traditional grade

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[2] School Mathematics Study Group, "Reports on Student Achievement in SMSG Courses," SMSG Newsletter, 10:1-26, November, 1961.

[3] Paul Rosenbloom, "Minnesota National Laboratory," SMSG Newsletter No. 10 (November, 1961), pp. 12-26.

[4] Roland F. Payette, "Educational Testing Service," SMSG Newsletter, 10:5-11, November, 1961.

ten geometry group was 6.25 raw score points higher than the modern group in plane geometry. The advantage of the traditional group in this study over the modern group was both practically and statistically significant. On the other hand, all other modern groups scored as well as the traditional groups in this study at the secondary level, with this one exception in grade ten plane geometry.[5]

In a second set of comparisons, reported in the same study, the traditional group, exposed to conventional geometry, had neither a pronounced nor a consistent advantage over the modern group with respect to the learning of traditional mathematical skills.[6] The achievement of the traditional group was compared with the achievement of the modern group who were taught by teachers who previously taught SMSG mathematics. Although the grade ten modern geometry group had higher average achievement scores on common tests of traditional mathematics than the traditional group the difference was not statistically significant. A major conclusion of the Educational Testing Service Study was that the SMSG curriculum at all grade levels does not detract from student achievement with respect to traditional mathematical skills.

To determine whether the SMSG curriculum resulted in a measurable extension of developed mathematical ability beyond that of traditional mathematics instruction was another aim of the ETS study. Consequently, the traditional group was compared with two sets of modern groups on the basis of their performance on SMSG tests. Both modern

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[5] _Ibid._, p. 7.

[6] _Ibid._, p. 8.

groups had higher achievement scores than the grade ten plane geometry traditional group. Thus, Payette concludes:

> ... students exposed to SMSG instruction acquire pronounced and consistent extensions of developed mathematics ability beyond that developed by students exposed to conventional mathematics instruction.[7]

Minnesota National Laboratory

For the Minnesota National Laboratory, Rosenbloom reported an experimental study dealing with the evaluation of student achievement in grade ten SMSG geometry. He indicated that students in the SMSG classes did at least as well as could be expected in achievement. The highest-ability students (top quartile) did slightly worse than expected in the post test, but the following fall they did slightly better than expected on retention tests. Data from the study also revealed that low ability students do slightly better in the SMSG geometry course than would be expected. Because there were no control classes available in this study, the comparisons made were on the basis of publisher's data for STEP norms on the performance of students in conventional classes.[8] In summary, Rosenbloom states:

> We did not find any ability group in the SMSG course for any grade that did much worse than would be expected according to national norms. In all cases the students did at least as well as would be expected and in some cases much better.[9]

Thus, the major conclusion of both the Educational Testing Service

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[7] Ibid., p. 10.

[8] Paul Rosenbloom, "Minnesota National Laboratory" SMSG Newsletter No. 10 (November, 1961), pp. 12-26.

[9] Ibid., p. 24.

and the Minnesota National Laboratory studies is that the modern geometry program contained in the SMSG text, Geometry, is as effective as the traditional approach in developing student achievement.

United States Office of Education

A summary analysis of research in mathematics education for 1961 and 1962 was prepared by Brown and Abell for the United States Office of Education.[10] A review of this material revealed that some researchers were interested in the evaluation of the "new" programs in mathematics at the secondary level. Their studies indicated that high school students using a modern geometry program do as well on standardized tests as students using a traditional program. Although few research studies on "new" geometry programs were reported for 1961 and 1962, Brown and Abell do mention the following two. First, Kraft reports that grade ten geometry classes using SMSG materials did as well as students from all parts of the United States had done on STEP standardized tests.[11] Secondly, Campo reveals that he compared the effectiveness of the modified geometry program proposed by the College Entrance Examination Board (CEEB) with a traditional plane geometry program.[12] The comparison was made on the basis of achievement on the ACE Cooperative Plane Geometry Test and the Otis

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[10]Kenneth E. Brown and Theodore L. Abell, Analysis of Research in the Teaching of Mathematics (Washington: Bureau of Educational Research and Development, 1965).

[11]Charles H. Kraft, "Evaluation of SMSG Grades 7-12," cited by K. E. Brown and T. L. Abell, op. cit., p. 62.

[12]David V. Campo, "An Evaluation of the Effectiveness of the Modified Geometry Program Proposed by the CEEB as compared with the Traditional Geometry Program," cited by Brown and Abell, op. cit., p. 36.

Quick Scoring Mental Ability Test. Campo notes that there was no significant difference in achievement between the modern and traditional groups.

Bell reports similar results in his study of a modern geometry course based largely upon the Birkhoff-Beatley concepts; the same approach used in Geometry by Moise and Downs.[13] He tested a modern and a traditional grade ten geometry group with the Shaycoft Plane Geometry Test. His observations were that the modern group did as well as the traditional group, even though the former had much less opportunity to practice items of the sort on the Shaycoft test.

Research Studies Between 1959 and 1965

A survey of research in secondary mathematics education prepared by Dessart provided an interesting summary of mathematical investigations between 1959 and 1963.[14] He describes the trend in secondary mathematics research as follows:

> The revolution in secondary mathematics education was reflected in the types of studies carried out during 1960-1963. Although most experimental investigations were of limited scope and duration, one began to see indications of a trend toward rather exhaustive studies of wider scope and longer duration than those of previous three-year periods. Such studies, which are likely to provide valid conclusions upon which to base curricular decisions, must become standard rather than unusual if future research in mathematics education is to make exceptional contributions to the improvement of instruction.[15]

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[13] Max S. Bell, "High School Geometry via Ruler-and-Protractor Axioms - Report on a Classroom Trial," The Mathematics Teacher, 54: 353-360, May, 1961.

[14] Donald J. Dessart, "Mathematics in The Secondary School," Review of Educational Research, 34:298-312, June, 1964.

[15] Ibid., p. 298.

However, before such studies can be carried out it will be necessary for the researchers to come to some fundamental agreement about the goals of mathematics education. Dessart suggests that additional research should be undertaken to delineate and agree on the objectives for all secondary school students.[16] In addition, the goals should be defined in terms of expected changes in behaviour patterns so that an effective evaluation can be made. This conclusion indicates a direction for future mathematics education research to take.

At present, Dessart discovered that most studies faced a basic problem caused by the teacher variable. He concluded that most comparative investigations were probably better measures of the effectiveness of teachers under particular sets of conditions than of the effectiveness of methods.[17] Another basic problem confronted by most comparative studies was the inadequacy of their measuring instruments. Dessart reported that more often than not the measuring techniques selected did not adequately measure the objectives of the course or topics.[18] Consequently, basic research on test design is required to improve this aspect of comparative curriculum investigations. In this study, the second basic problem was confronted by the writer in the design of the two instruments, _Geometrical Attitude Scale_ (_GAS_) and _Geometrical Achievement Measure Experiment_ (_GAME_). The techniques used to validate both instruments are discussed in detail in Chapter IV.

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[16] _Ibid._, p. 299.

[17] _Ibid._, p. 301.

[18] _Ibid._, pp. 302-303.

Further evidence of the increased interest in secondary mathematics programs is noted by Dessart in the organization of the National Longitudinal Study of Mathematical Abilities (NLSMA). He concludes that this study has been structured to compensate for the weaknesses, already pointed out by him, in current evaluation studies.[19] In the intent of the NLSMA study, Dessart recognizes a concern for the identification of factors having a significant long range effect on success in mathematics.[20]

Cahen also recognized this concern, and he describes the objectives of the NLSMA study as follows:

> A major purpose in undertaking this large scale, five year research program was to gather necessary achievement data for the constantly evolving processes of mathematics curriculum revision and modification.
>
> A second purpose of the study was to learn a great deal more about the nature of mathematical abilities. The study will also, hopefully, provide sound leads for tightly controlled experimentation in the future. A highly important by-product of the research endeavour will be the creation and field testing of many evaluative instruments and techniques for assessing achievement in mathematics.
>
> Lastly, the study may provide information that will contribute to a better understanding of the processes of learning in school environments with special relevance to the learning of mathematics.[21]

At the present time, the initial stages of the above study have been completed, but unfortunately no comprehensive reports will be made available on it until early 1969.

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[19] Ibid., p. 306.

[20] Ibid., p. 307.

[21] Leonard S. Cahen, "An Interim Report on the National Longitudinal Study of Mathematical Abilities," The Mathematics Teacher, 58: 522-523, October, 1965.

Burns and Dessart have emphasized the need for an effective evaluation of traditional and modern mathematics programs in their summary of mathematics studies for 1965. Their report disclosed that the past decade has witnessed the development of new curricular materials in secondary mathematics education and that it appears that the next decade will produce a large number of evaluation studies. Many of the new curricular changes may or may not have been actual improvements and without sound evaluations, propaganda rather than sound reasoning may dictate innovations in the curricula of schools.[22]

Wick identifies one of the dangers in comparative evaluation studies in the following:

> When interpreting the results of ... any ... comparison of traditional and experimental mathematics program materials, it must always be recognized that there is great variation among "traditional" school mathematics programs. Undoubtedly there are specific mathematics programs of a traditional type which are more effective, and others which are less effective, in a given school situation than any experimental program. Nevertheless, it is still important to know how a new or experimental mathematics program compares with traditional programs in general.[23]

The National Council of Teachers of Mathematics

The National Council of Teachers of Mathematics (NCTM) has also expressed some concern about the quality and credibility of current research in mathematics education. Its experience has been that many teachers are asking for evidence to support or deny the current crop

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[22]Donald J. Dessart and Paul C. Burns, "A Summary of Investigations Relating to Mathematics in Secondary Education: 1965," School Science and Mathematics, 67:135-144, February, 1967.

[23]Marshall E. Wick, "A Study of the Factors Associated With Success in First Year College Mathematics," The Mathematics Teacher, 58:648, November, 1965.

of claims demanding changes in curriculum and pedagogy for geometry and algebra programs. This organization recognizes that valid and effective research is necessary to provide the answers, but that much of it is not meeting the demands that seem to be emerging.[24] After consideration of its role in providing answers to some of the questions about mathematics research, the National Council of Teachers of Mathematics published Research in Mathematics Education for the following purposes:

> (1) to provide a rationale for both basic and applied research in mathematics education; (2) to exhibit significant research efforts; (3) to clarify the complementary nature of "information-oriented" (basic) and "product-oriented" (applied) research; (4) to demonstrate the potential impact of research and the implementation of research on the teaching of mathematics; and (5) to sample the reactions of members of the profession to a research-oriented journal in mathematics education.[25]

The eleven papers reported in this research publication indicate that many different kinds of research and development are at present underway in mathematics education. In one paper, the case is made for basic research in mathematics education with theory construction as an essential guide to data collection. In three other papers, the authors report studies designed to increase understanding about the teaching and learning of mathematics. Other studies reported are concerned with the importance of prior learning in the acquisition of mathematical knowledge, and the significance of new technologies for constructing materials and curricula. The studies reported are

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[24] Joseph M. Scandura (ed.), Research in Mathematics Education (Washington: National Council of Teachers of Mathematics, 1967).

[25] *Ibid.*, pp. iii-iv.

characterized as either "information-oriented" or "product-oriented." The first phrase is used to describe studies which seek information leading to the development of theory about mathematics learning, teaching, and/or curricula. The phrase, "product-oriented," is used to describe studies which may use the theory or technology to devise a new process or product, and then evaluate the process or product with an eye to its improvement.

In conclusion, Scandura notes the future pattern of research in mathematics education:

> ... if mathematics education is to improve fundamentally beyond its present state more will be required than simply teaching more mathematics at an earlier age. We, as mathematics educators, will have to turn our attention more and more towards the development of improved technologies for preparing materials and for instructing students. Such advanced technologies, in turn, may be expected to depend increasingly on a more complete understanding of how mathematical knowledge is organized, learned, taught, measured, and created.
>
> Information-oriented research, product-oriented research, and development are all necessary.[26]

The Present Research Situation

In this thesis the research may be described as product-oriented in the sense that it is evaluative in nature. It is similar to the evaluative work of the School Mathematics Study Group (SMSG). This group did not strive to study particular methods of teaching the subject matter that was selected. Instead, they allowed the teachers who were using their textual materials to employ any methods they wished, so that the "methods" variable was randomized. Consequently,

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[26] *Ibid.*, pp. 124-125.

they were more concerned with the relation between subject matter taught a student and his behaviour subsequent to having been taught it.

On this continent, the present state of affairs of research on geometry programs could be summarized as follows: First, some isolated research work has been carried out on evaluating geometry programs on the part of individuals engaged in doctoral programs. The results of such research has been inconclusive for the most part. Secondly, there has been a major effort by the School Mathematics Study Group to promote their program from K-12 by carrying out exhaustive evaluation studies that compare modern and traditional courses. As for geometry, the results show that the modern program does as well as the traditional program when student achievement is compared. Finally, researchers in mathematics education recognize the need for large scale comprehensive studies to be carried out to evaluate not only the new emerging geometry programs but also all mathematics programs. Some of the dangers inherent in such research have already been identified in areas where research safeguards may be employed.

II. THE EMERGING PROBLEMS

A review of the literature of evolving mathematics programs indicates that many new geometry courses have been developed and will be developed to provide for the current and future mathematical needs of today's young people. The traditional programs are being replaced by new courses, content and materials. It appears that the majority of current new programs are designed to develop the processes and concepts of mathematics rather than the rote memorization of factual information. The content of the new mathematics courses is the "hardware" of what many authors have described as the "revolution in school mathematics," and the emerging programs in geometry are one part of this revolution.

The Revolution in Mathematics

Baley Price in "Progress in Mathematics and Its Implications for the Schools" recognized the revolution as a fact and described its three basic causes as: (1) extensive research in basic mathematics; (2) automation; and (3) automatic digital computing machines.[27] He notes:

> The changes in mathematics in progress at the present time are so extensive, so far-reaching in their implications, and so profound that they can be described only as a revolution.[28]

As a result of the revolution in mathematics the total mathematics curriculum from K to 12 has been forced to change to insure that the mathematics education provided by the schools was adequate for the present and the future.

Allendoerfer in "The Second Revolution in Mathematics" has also noted the fact of the revolution in school mathematics, and he has described several of the significant features of its development. In particular, he noted that the first revolution in mathematics education began in the smaller colleges in the United States after

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[27]G. Baley Price, "Progress in Mathematics and Its Implications for the Schools," The Revolution in School Mathematics (Washington: National Council of Teachers of Mathematics, 1961), pp. 1-14.

[28]Ibid., p. 1.

World War II where new curricular college materials were being produced by mathematics professors interested in revitalizing an old, dull and sterile curriculum. This situation came to the attention of the College Entrance Examination Board which appointed a Commission on Mathematics to study how changes could be made in the school mathematics curriculum. The Commission consulted with other groups, such as that at the University of Illinois and examined mathematics programs for university-bound young people. They published a report of their recommendations which prepared the way for later events in the revolution.[29]

With the launching of Sputnik I in 1957, the revolution entered its active phase and after this real progress began to be made. This took the form of comprehensive mathematics programs organized and developed by influential groups such as the School Mathematics Study Group.

Characteristics of New Mathematics Programs

A review of the literature discloses that there are now many improved mathematics programs available at the elementary and secondary levels. The majority of those at the secondary level are geared for college-capable young people, although a few of the program developers, notably SMSG, have begun developing programs for general course students. However, very little of the curriculum construction and preparation of text material has been based upon any theory of

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[29] Carl B. Allendoerfer, "The Second Revolution in Mathematics," _The Mathematics Teacher_, 58:690-695, December, 1965.

learning or educational research. Most of the curriculum material represents what the writers thought to be best in terms of their own knowledge of mathematics and their experience in the classroom.

Nevertheless, there has been a great deal of activity in the development of new mathematics curricula in many areas, particularly the universities. Unfortunately, most of the new programs suffer from two basic deficiencies. First, most of the programs ignore the problem of what mathematics should be taught to the lower seven-eighths of the ability group, and in particular the lower third. This group includes the culturally deprived. Secondly, few of the new programs are based on the knowledge of how students learn mathematics. In spite of some attention to the above problems by the NCTM and the SMSG, much work remains to be done by program planners to cope adequately with the problems.

Brown describes some of the general characteristics of the new mathematics programs in the following:

> There are many similarities among the different programs, but there are also differences - both in development and in emphasis. One program may have involved many more persons in its development than another; one program may have been tried in many cities in the United States and another confined to a local region; one program may have material for grades 7 through 12 and another for only some of these grades; one program may emphasize the discovery approach and another the historical approach.[30]

In any case, the major programs reviewed for this study were American in origin. It is quite apparent that programs and texts from the United States have had more influence than programs and texts from

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[30] Kenneth E. Brown, "The Drive To Improve School Mathematics," _The Revolution in School Mathematics_ (Washington: National Council of Teachers of Mathematics, 1961), pp. 16-17.

other provinces and countries on the kind of secondary algebra and geometry programs developing in the schools of Manitoba. For this reason the American programs were given the emphasis in this study. However, later in this study, the characteristics of the geometry program in Ontario and two of the new geometry programs in Great Britain are noted. These have definite merit and could be considered as acceptable alternatives to the American programs.

All of the comprehensive mathematics programs discussed in the literature are aimed at the improvement of mathematics instruction. They share certain common characteristics while still retaining a variety of unique qualities. Generally, they avoid the development of new material as a string of unrelated topics. Rather, they emphasize certain unifying concepts in mathematics such as set theory, structure and logical deductions.

Developing Geometry Programs

The basic changes that have been going on in the mathematics programs from K - 12 are reflected in the specific changes that have occurred in the developing geometry programs. A number of these have been developed in the United States and Great Britain for secondary and elementary instruction. At the secondary level, the changes which are occurring appear to be freeing geometry from its previous isolated position, and encouraging the student's understanding and appreciation of the subject as a mathematical system. However, "the changing role of geometry is a source of confusion," according to

Meserve.[31]

The United States. In the United States this confusion has been caused by the following factors: (1) the present emphasis upon geometry as a postulational system independent of physical representations, which requires the inclusion of postulates of existence in our mathematical system; (2) the subtle distinction between equality and congruence; (3) the distinction between a geometrical figure and its model or representation; (4) the emphasis upon generality and interrelations among figures with the overlapping present in definitions and; (5) the differences in the usage of terms in different texts.[32] Meserve concludes that most of the new geometry programs are doing little to reduce this confusion and congestion when they should be doing much. On the other hand, he views the synthesis of algebraic concepts with geometrical concepts in the new programs as a healthy and encouraging trend.[33] The School Mathematics Study Group also supports this as a device which will help to clarify the role of geometry as new programs are being developed.

In considering the new geometry programs Meserve pleads for a flexible approach as follows:

> We need divergent views. We need to explore drastically different approaches and methods. Above all, we need to maintain a flexibility in our curriculum which allows us to modify our

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[31] Bruce E. Meserve, "Geometry in the United States," Geometry in the Secondary School (Washington: National Council of Teachers of Mathematics, 1967), pp. 1-7.

[32] Ibid., pp. 3-4.

[33] Ibid., p. 4.

courses to take advantage of changes for students who can profit from them, without placing all students in a straight-jacket of either new or old curriculum material.[34]

Ontario. A number of operational geometry programs have considered drastic revisions of secondary school geometry which have a different emphasis than most American programs. An example of one of these is contained in the Ontario K - 13 Geometry Report which places the emphasis on visual and intuitive work. Instead of the axiomatic approach, with rules and definitions, they have recommended an intuitive interest approach through problems that are significant to the student. In this way, geometry can be taught for its interesting results and as an exercise in informal reasoning.[35] In a paper on the Ontario Report, Coxeter notes its significant characteristics in:

> Instead of the axiomatic approach, with rules and definitions, we recommend the intuitive "interest" approach through problems significant to the student. Certain properties of simple figures are assumed. These lead to short chains of easy deductions. Later a more ambitious use of assumptions can be made, so that a wider range of problems is accessible, and some of the old tentative assumptions become theorems. This method minimizes the laying down of authority and the making of apparently arbitrary rules at the outset.
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> The systematic use of axioms in geometry is admissible only after the students have already had several years of experience with simple deductions.[36]

This report also recommends the early introduction of geometric

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[34] Ibid., p. 5.

[35] G. F. D. Duff (ed.), Geometry, Kindergarten to Grade Thirteen (Toronto: The Ontario Institute for Studies in Education, 1967).

[36] H. S. M. Coxeter, "The Ontario K - 13 Geometry Report," Geometry in the Secondary School (Washington: National Council of Teachers of Mathematics, 1967), p. 9.

transformations, and so follows the British and Russian proposals.

Great Britain. A second example of geometry programs that have a different emphasis than the current American programs is found in the school mathematics of Great Britain. In that country, a number of programs have been developed. The most important of these are the Nuffield Mathematics Teaching Project and the Leicestershire Experiment and they share many of the characteristics of the other programs. In both programs, the traditional Euclidean geometry course does not exist, and they place little emphasis on analytic geometry. They also specify that geometry must be a continuous program from K - 12, with the emphasis placed on both two dimensional and three dimensional space.[37]

To achieve their objectives both programs indicate that symmetry, tessellations and classification of two and three dimensional shapes must play a major role in the program. The content of the program includes the use of an experimental approach which involves the manipulation of a variety of visual aids and models. The final aim of the new British programs at the elementary level is to provide a basis for a high school geometry of vectors, translations, rotations and reflections with co-ordinate methods as one way of writing these topics. In contrast to the operational approaches in the United States, the British programs have made no attempt in grades one to ten to introduce any axiomatic deductive

[37] Andrew Elliot, "Geometry in Great Britain," Geometry in the Secondary School (Washington: National Council of Teachers of Mathematics, 1967), pp. 13-17.

systems. Recently, the British programs have had some influence on the developing programs and ideas in the United States secondary geometry program.

However, the major emphasis in almost all American geometry programs is still to give a mathematically sound treatment of the Euclidean geometry that is now taught in the tenth grade. An examination of the development of certain American secondary geometry programs, and their influence on the geometry program in Manitoba schools will be the subject of Chapter III.

III. SUMMARY

A review of the literature indicated that there has been very little published comparative research on traditional and modern geometry programs. Most of the reported investigations were concerned with the various programs of the SMSG, and they reported that secondary students using a modern program do as well as students using a traditional program. The need for effective and valid curriculum research based on sound statistical models was recognized as a pressing one.

New secondary geometry programs developed in the United States were designed to produce a mathematically sound treatment of Euclidean geometry for the tenth grade. They evolved to meet the needs of students in a changing world of mathematics. Foreign geometry programs de-emphasized the axiomatic structure of plane geometry and placed more stress on interesting geometrical constructions and models and transformations. Some research attempts have been made to evaluate the American geometry programs but little has been done in this area in Great Britain or in Canada.

CHAPTER III

THE REVOLUTION IN GEOMETRY AND THE SITUATION IN MANITOBA

I. INTRODUCTION

Since 1958 many groups like the National Council of Teachers of Mathematics, the School Mathematics Study Group and the Commission on Mathematics of the C.E.E.B. have been vitally interested in improving geometry instruction in the secondary schools. They have been encouraging curriculum planners, textbook writers and teachers to abandon the traditional Euclidean organization of geometry in favour of a new structure. In a review of the School Mathematics Study Group position, Moise reveals that such a change is necessary. He notes:

> When the SMSG geometry group met at Yale two years ago, we began by considering the Commission's recommendations. You will recall that the first and primary objections that the Commission made to synthetic geometry was that the logic of the treatment was not up to modern standards. In the last century the level of rigor in modern mathematics rose past that of Euclid and so modern mathematicians did the foundations of geometry all over again. In Hilbert's book, you will find a "very" rigorous treatment. The Commission thought - and we in SMSG agree - that Hilbert is not the answer for the tenth grade. In response to these troubles, the Commission proposed, first to reduce the number of synthetic theorems and second, to avoid the problems in foundation by the use of analytic methods.[1]

In the following, four general topics related to the revolution in geometry in the secondary school will be discussed. These topics include: the reasons for the revolution, the objectives and procedures

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[1]Edwin E. Moise, "The SMSG Geometry Program," The Mathematics Teacher, 53:438, October, 1960.

of the major influence groups, the traditional approach to instruction in geometry and the modern approach.

II. EUCLID'S PURPOSE

When Euclid wrote his series of geometry books entitled Elements, he was writing for scholars and not school students. His purpose was to write an introduction to the study of geometry in a philosophical manner. As a result, he emphasized deduction and logic, not geometrical thinking, so that his subject matter was limited by his philosophical orientation. He was more interested in the collection and systematization of results, than in their discovery, and his treatment gave no clue to the underlying geometrical thought patterns involved. However, this does not in any way detract from the significance of his achievements. These are noted by Daus in the following:

> Euclid unified the work of many scholars and systematized the known mathematics of the day. His outstanding contribution lies in his extended application of the mathematical method - the hypothetico-deductive method of modern mathematics. He set himself the task of finding an adequate and universally acceptable set of postulates for geometry and at the same time avoiding a proliferation of assumptions, many of which would be repetitions in the sense of non-independence. He entertained the ideal of placing mathematics on an unimpeachably logical basis. He demonstrated how much knowledge can be derived by reasoning alone, and it was through his Elements that later civilizations learned the power of reason.[2]

Euclid achieved his purpose with a great measure of success. Thus, two thousand years later, groups like the SMSG, who have developed comprehensive new geometry texts containing an approach different

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[2]Paul H. Daus, "Why and How We Should Correct the Mistakes of Euclid," The Mathematics Teacher, 53:576, November, 1960.

from the traditional Euclidean, find it necessary to expend a great deal of money and effort to gain acceptance. Because of certain shortcomings in the traditional treatment of plane geometry in the secondary school several curriculum groups have proposed suitable alternative approaches. In addition, like the SMSG, they have written adequate and effective textbook series which show how these approaches may be applied. It is essential that such groups gain widespread acceptance of their programs in order to improve the instruction in geometry in our schools.

III. WEAKNESSES IN EUCLID'S APPROACH

There are certain weaknesses in Euclid's approach to geometry that make it inadequate for present or future students. Felix Klein pointed out the three principal weaknesses in 1908. These are summarized by Adler in:

1. Euclid lacked Archimedes' sense for numerical calculations, his interest in applications, and his heuristic approach.
2. The inadequate arithmetic and algebra of the Greeks compelled Euclid to use cumbersome geometric substitutes as, for example, in the theory of ratio and proportion.
3. There are defects in the logical structure of Euclid's Elements. These include: (a) his failure to realize the need for undefined terms, (b) the presence of gaps in the postulate system (the complete absence of axioms of order, for example) and (c) the use of circular reasoning in proofs by super position.[3]

In addition to the above Adler supplies a fourth weakness to indicate why Euclid is not adequate for teaching geometry to present day students:

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[3]Irving Adler, "What Shall We Teach in High School Geometry?" The Mathematics Teacher, 61:226, March, 1968.

4. Geometry has developed considerably since Euclid's time. There are new approaches to the study of Euclidean geometry as for example, in the coordinate geometry of Descartes. There are also new branches of geometry such as projective geometry, hyperbolic geometry, and others.[4]

These are the major weaknesses in Euclid's treatment that have spurred on curriculum makers and planners to introduce changes into the geometry program for secondary students. Groups such as the Commission on Mathematics, the Illinois Study Group and the SMSG, and individuals such as Birkhoff and Hilbert were aware of the inadequacies of geometry programs patterned on abridged editions of Euclid's Elements. They realized that a new approach was required to make Euclidean plane geometry teachable in a mathematically appropriate way. In this regard, Daus notes:

> For the past sixty years mathematicians have been trying to rewrite Euclid's Elements in a form satisfactory for use in the schools, and today success seems near but not certain. This writing is done in the spirit of Euclid - that is, it involves logical deduction from a relatively small number of explicitly stated assumptions - but an effort is made not to repeat the mistakes.[5]

IV. A SYNTHETIC TREATMENT

Hilbert introduced the needed assumptions to correct the logical weaknesses in Euclid's approach to plane geometry. His main intent was to fill in the gaps left by Euclid, and to stay as close to the Euclidean structure as possible. As a result, Hilbert supplied precise postulates of incidence, of order of points, of congruent

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[4] Ibid., p. 226.

[5] Paul H. Daus, "Why and How We Should Correct the Mistakes of Euclid," The Mathematics Teacher, 53:580, November, 1960.

segments and of congruent angles. In this way, he contributed the required ideas to correct the defects in Euclid and provide a basis of correct proofs for all the propositions. The system was completed with a parallel postulate and a continuity postulate.[6]

However, most geometry curriculum groups recognize that Hilbert's synthetic treatment is far too sophisticated and rigorous for a beginning course in formal plane geometry. In addition, the logical gaps left by any teachable presentation of Hilbert's approach at the secondary level would only be filled by a thorough course in the foundations of geometry. Such a course could only come several years later, if it came at all. Thus, much of the immediate relevancy of any secondary geometry course based on Hilbert's axioms would be ineffective.

The School Mathematics Study Group has reported that any grade ten course based on Hilbert's postulates would be almost unteachable because of the level of rigor required.[7] Experimental evidence from the Illinois Study Group also supported this view.[8] At first, the Illinois program did not capitalize on the student's own experience. Later modifications of the program recognized this, and also provided the students with more initial algebraic experience. Reports from pilot teachers in Manitoba who used the text, Geometry, by Brumfiel,

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[6]Nathan A. Court, Mathematics in Fun and in Earnest (Toronto: The New American Library, 1964), pp. 56-58.

[7]Edwin E. Moise, "The SMSG Geometry Program," The Mathematics Teacher, 53:437-442, October, 1960.

[8]Daus, op. cit., p. 580.

Eicholz and Shanks,[9] revealed results similar to that of the Illinois Study Group. The text used by the pilot classes presented a purely synthetic approach to the study of plane geometry based on a modified version of Hilbert's postulates. It was concluded from this experience that any available plane geometry programs using a synthetic approach were too rigorous and sophisticated for grade ten students.

V. A METRIC APPROACH

The School Mathematics Study Group was among the first curriculum planners to recognize that the problem of approach in plane geometry could be solved by the use of the Birkhoff postulates. According to Moise, they make geometry teachable to grade ten students by arithmetizing it. In addition, they bring out the basic unity in geometry where it really exists and use it to improve understanding.[10] Use of a modified version of the Birkhoff metric postulates (which involve the idea of measurement by real numbers of distances, angles, and areas) do permit a teachable presentation at an appropriate level of logical rigor. However, the basic reason for using them appears to be to allow the student to use the ideas from the postulates in the solution of problems. Thus, the ideas used in both the postulates and the theorems can reinforce and be reinforced by the ideas presented in the extended examples. In this way, Moise has argued that the

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[9] Charles F. Brumfiel, Robert E. Eicholz and Merrill E. Shanks, Geometry (Palo Alto: Addison-Wesley, 1960).

[10] Moise, op. cit., pp. 439-440.

theory in a plane geometry course can serve a useful purpose.[11] He summarizes the advantages of using an abridged edition of the Birkhoff postulates in the following:

> ... in Birkhoff's treatment, the connection between lines and real numbers is made immediately. This means that geometry and algebra can immediately begin making their natural contributions to each other in the mind of the student and it means that the gaps that we leave in the logic of the geometry are gaps of a sort that are going to be filled when the student learns more algebra.[12]

In addition, the Birkhoff postulates are adequate to prove the theorems, so that a modern treatment can be presented which is close to being logically complete and also teachable to grade ten students.

VI. SOME PROPOSALS FOR IMPROVING INSTRUCTION IN GEOMETRY

Commission on Mathematics

Definite proposals to improve geometry instruction made by certain curriculum groups have had a profound influence on the direction of that improvement. One of these groups is the Commission on Mathematics of the College Entrance Examination Board established in 1955 to consider broadly the college preparatory mathematics curriculum in the secondary schools and make recommendations looking toward its modernization, modification and improvement.[13] This group prepared an outline of a feasible geometry program with illustrative

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[11] Ibid., p. 440.

[12] Ibid., p. 439.

[13] Scott, Foresman and Company (ed.), Studies in Mathematics Education (Chicago: Scott, Foresman, 1960), p. 28.

text material in short fragments, so that an operational program was left to be written out by other interested people.

A primary objection raised by the Commission on Mathematics in the beginning was that current synthetic geometry programs did not contain an approach that was up to modern logical standards. Because programs using modified versions of Hilbert's postulates were considered to be too rigorous for grade ten students the Commission made two basic proposals. First, they recommended a reduction in the number of synthetic theorems, and second they proposed the use of analytic methods to avoid the problems in foundation.

Among the best of the Commission's proposals was to incorporate some of the essentials of solid geometry with geometry of the plane. It was felt that this could provide a genuine integration which would lend the whole subject an extra advantage. The advantage was that three dimensional problems could be used as exercise material if solid geometry was started early enough. As a result, a student is given a lot of intuitive experience with figures in space, without taking on the burden of a full deductive treatment.

In describing the general approach of the Commission on Mathematics to instruction in geometry Moise notes:

> In all of its main features, the Commission's program is surprisingly conventional. It is true that they undertake to integrate synthetic and analytic geometry; they propose to treat solid geometry intuitively; and they propose various innovations and improvements in matters of detail. Nevertheless, we can summarize the situation fairly well by saying that the Commission's tenth grade program simply fits together truncated versions of existing conventional courses. In particular, the basic approach to plane synthetic geometry is the old approach, retaining whatever plaws the old approach may have. Only the duration of these flaws is reduced when the course is shortened. And the analytic

> geometry comes much too late to be of any help in the problem of the foundations ...[14]

Consequently, the program proposed by the Commission on Mathematics is certainly feasible in Moise's view. Furthermore, even though he has been quite critical of the program proposed by the Commission, he and Downs have subscribed to the recommendations in their text, Geometry.

In the Preface to this text they acknowledge their debt to two groups by stating:

> An examination of the Table of Contents for this book will indicate that we have closely followed the recommendations of the Commission on Mathematics ... and have been strongly influenced by the text entitled Geometry, written by the School Mathematics Study Group ...[15]

Most of the new geometry texts published for secondary students do owe an immense debt to both of these groups. This becomes obvious when the content and approach of the new programs are examined. They reveal many similar patterns in the exercise material, the arrangement of topics and the development of concepts.

School Mathematics Study Group

The School Mathematics Study Group is the second organization that has had a deep influence on the improvement of geometry instruction from K - 12. It was formed in 1958 to improve the teaching of mathematics in the schools at all levels. Specifically, the sample

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[14] Edwin E. Moise, "The SMSG Geometry Program," The Mathematics Teacher, 53:442, October, 1960.

[15] Edwin E. Moise and Floyd L. Dawns, Geometry, (Palo Alto: Addison-Wesley Publishing Co. Ltd., 1965), p. v.

textbook writers in the geometry section of the SMSG were responsible for the design of a program which would be integrated, rigorous, and modern. It had to be organized so that it could be operational by 1960.[16]

Initially, the writers of the program began by considering the recommendations of the Commission on Mathematics. However, they were not convinced that the logical flaws in Euclid were serious for grade ten geometry students. Although they wished to increase the level of rigor in the program, they did not consider that this was a central problem. Instead they were more concerned about different flaws in the traditional treatment. They believed that geometry was artificially isolated from the rest of mathematics, and that the language of geometry was used in ways strange to the context of modern mathematics. Eventually, the SMSG concluded that the ancient material of Euclid could be developed in a significant modern sense.

As a result, a writing team of mathematicians and mathematics teachers sponsored by the SMSG developed a prototype text for teaching geometry to tenth grade students. The revised edition of the SMSG text, Geometry, differs from conventional ones in context, postulational scheme, and manner of treatment. Because this text is similar to Geometry by Moise and Downs in format, presentation, arrangement of topics, diagrams, exercises, etc., it is pertinent to note some of its basic characteristics. First, no artificial distinction is made between plane and solid geometry, and a considerable

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[16] Moise, op. cit., p. 438.

amount of the latter is included. In addition, an introduction to analytic plane geometry is provided, and the treatment is a modern one. Second, the postulate system is a modification of Birkhoff's and is complete. Real numbers are used freely throughout the text, both in the theory and in the problems. According to the SMSG, this integrated approach brings out the basic unity in mathematics where it really exists, and uses it to improve the student's understanding. Third, the accuracy in the statement and use of postulates, definitions, and theorems is emphasized so that the treatment can be as close to being logically complete as possible. The SMSG approach is a rigorous one then.

Although the SMSG text in geometry is different from conventional ones in the above ways it is still basically a treatment of the topics of Euclidean geometry. The usual subjects are included: congruences, similarity, parallelism and perpendicularity, area, circles, and construction with straight edge and compass. There is a main sequence of proved theorems, some minor stated theorems left as exercises and a number of originals. An appeal to the student's intuition motivates the basic postulates, definitions and theorems. Almost all of the topics recommended by the Commission on Mathematics as well as many subjects that were not proposed have been included in the SMSG text. As a result, the program presented in this text is quite ambitious in comparison to the Commission's plan.

The Cambridge Conference

The third major influence on new developing geometry programs resulted from the Cambridge Conference of 1963, organized to consider the shape and content of the pre-college mathematics curriculum during the next several decades.[17] Findings of the conference were published in a report entitled Goals for School Mathematics.[18] For geometry programs, the report proposed an earlier start, in the elementary school, so that they could be extended in the future to include geometrical transformations and axiomatic geometry in the Junior High grades. In summary form, the report outlined the following areas of innovation that represent, for the most part, a departure from the traditional curriculum:

1. The objectives of mathematics instruction from K - 6 should include the development of familiarity with the main ideas of geometry.

a) Cartesian coordinates should be introduced through appropriate games.

b) Elementary children should be given experiences with the symmetrics of certain plane and solid figures.

c) Each child should be given abundant opportunity to manipulate suitable physical objects.

2. Other topics proposed for K - 6 include vectors, truth tables for the simplest connectives, and the concept of indirect proof.

3. Two outlines are proposed for grades 7 - 12.

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[17]Irving Adler, "The Cambridge Conference Report: Blueprint or Fantasy?" The Mathematics Teacher, 59:210, March, 1966.

[18]Goals for School Mathematics (Boston: Houghton-Mifflin Co., 1963), pp. 1-102.

a) Both suggest that the study of geometry should include the study of geometric transformations.

b) One outline starts the study of axiomatic geometry in grades seven and eight, the other one starts it in grade nine.

c) All the traditional geometry content is included as well, reorganized and reintegrated with the equivalent of three additional years of work.

d) Beginning with the earliest grades, there should be a parallel and integrated development of algebra and geometry. One of the basic aims of geometry at this level is to develop a growing awareness of the nature of logical reasoning. However, in the development of postulational thinking excessive delicacy and austerity should be avoided.[19]

The program proposed by the Cambridge Report does not neglect the traditional content of elementary and secondary school mathematics. Instead, all the conventional topics are recommended by the Report. In addition, the subjects are reorganized and integrated with the equivalent of three additional years of work, moved from college programs down into the elementary and secondary school. Consequently, the Report has outlined a very ambitious program that has implications for the present. Whether or not the major proposals of this document can be implemented in any foreseeable period remains an open question.

The Secondary School Curriculum Committee

The fourth major influence on developing geometry programs has

[19] *Ibid.*, pp. 33-48.

been the work of the National Council of Teachers of Mathematics. This organization of mathematics professors and teachers has been interested in the improvement of mathematics instruction in the schools and colleges of North America. Because of effective professional support and adequate financing the NCTM has been able to exert considerable influence on developing mathematics programs through its recognized journal, pamphlets, books and periodic mathematics conventions. Accordingly, in 1958, the NCTM organized the Secondary School Curriculum Committee to make a study of mathematics curriculum and instruction. The Committee prepared a report which recommended definite proposals for strengthening and improving mathematics education in the secondary schools.

As far as geometry is concerned this report contained two major proposals which depart from traditional programs.[20] First, the report has recommended that the elements of geometry be taught continuously from grades seven to twelve. However, it emphasized that one year should be devoted to the study of synthetic geometry with the stress on geometry of the plane. Coordinate geometry material in the secondary sequence was also proposed by the report, so that some practice could be given in the use of analytic methods to provide alternative proofs for certain theorems.

The second major proposal of the Committee Report was concerned with the inclusion of solid geometry in the basic plane geometry

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[20] Secondary School Curriculum Committee: NCTM., The Secondary Mathematics Curriculum (Washington: National Council of Teachers of Mathematics, 1959), pp. 404-408.

program for capable students. Consequently, the Report recommended that superior classes would find it profitable to study solid geometrical concepts in a systematic way and not just as a counterpoint for plane geometry. The amount of solid geometry to be included was left to the discretion of the teacher. To guide program planners in the selection of appropriate geometrical topics the report of the Secondary School Curriculum Committee (SSCC) has listed the elements of geometry which it considered proper at the secondary level.

Most major geometry program innovators in the United States have accepted the proposals of the SSCC to improve instruction in the subject. This acceptance has been reflected in the choice of geometrical content and presentation of material by the following three curriculum groups: the University of Illinois Committee on School Mathematics, the School Mathematics Study Group and the Ball State Group. Although these groups have agreed in principle with the objectives of the Secondary School Curriculum Committee to improve the content and organization of grade ten plane geometry, they have differed in terms of approach.

An examination of the minutes of the Manitoba Mathematics Curriculum Revisions Committee (MMCRC) revealed that the programs of two of these groups, the Ball State Group and the SMSG have exerted some influence on the thinking and recommendations of the Committee.[21] Because the work of the Revisions Committee has been central to the

[21]Manitoba Department of Education, Curriculum Branch, Minutes of Manitoba Mathematics Curriculum Revisions Committee, October 17, 1963 - January 17, 1968.

problem of improving geometry instruction in Manitoba Schools its role and procedures are described in the next section.

VII. MANITOBA MATHEMATICS CURRICULUM REVISIONS COMMITTEE

The Manitoba Mathematics Curriculum Revisions Committee was organized in 1963 to consider the mathematics curriculum in Manitoba and to make specific recommendations as to textual material and content that would strengthen, improve, modify and modernize the curriculum. It was composed of curriculum consultants, university professors and mathematics teachers. In the beginning, the Committee recognized a need for a fundamental change in the secondary schools mathematics curriculum, a curriculum which was entirely dissociated from current mathematical research. It further recognized that the changes should not be merely a reorganization of existing material but a reorientation of approach for all topics at all levels. Later, the major objective of the Committee became the responsibility for developing a continuous curriculum of mathematical experiences from K - 12. They considered that such a program should be suitable for Manitoba students and also oriented to the needs of mathematics in the second half of the twentieth century.

<u>Need for a New Geometry Program</u>

As a part of the required changes in the mathematics curriculum, the Revisions Committee recognized the definite need for a new geometry program at the secondary level. Their concern was that the traditional geometry program was not in keeping with the intent of modern programs and the future needs of Manitoba young people. Thus, the committee was motivated to carry out an extensive evaluation and improvement of the secondary geometry program. They were quite aware of the obvious defects in Euclid's treatment of plane geometry which made it incomplete and not precise enough for modern needs. The logical and pedagogical defects in the traditional grade ten and eleven geometry text also made it necessary to provide a geometry program for Manitoba students which was up to modern standards.

Activities of the Committee

To accomplish their objectives for geometry the Manitoba Mathematics Curriculum Revisions Committee engaged in a number of procedures and activities with the main emphasis on secondary geometry instruction. These activities included numerous discussion sessions which were held to determine the philosophical and mathematical objectives of proposed grade ten and eleven geometry programs, and ways to accomplish them. To assist the Committee reports were delivered by several university professors who were concerned with the approach to new geometry material. In addition, Committee members became responsible for reviewing, evaluating and reporting on suitable texts for improving secondary geometry instruction. The two criteria of suitability considered as necessary were: (1) the texts had to be mathematically sound; and (2) they had to be compatible with teacher and student needs. In this regard, at least eight geometry texts were considered and reported on by Committee members. Eventually, pilot classes in three modern geometry texts were tried at different times from 1964 - 1968 to experiment with and determine the teachability of their material.

The Experimental Texts

The three texts selected for experimentation in pilot classes were Geometry, by Brumfiel, Eicholz, and Shanks, Geometry, by Moise and Downs, and Geometry, by Clarkson *et al*. A purely synthetic treatment of traditional plane geometry was presented in the Brumfiel text by using a modified version of Hilbert's postulates. On the other hand, the two other texts presented a metric approach to the same material by use of an edited version of the Birkhoff postulates. At the present time, the Brumfiel text will be discontinued at the end of the 1968 term and the Moise text will be discontinued at the end of the 1969 term. Further experimentation with new geometry programs will be carried on in the 1968-1969 term with the introduction of two new geometry texts.

Throughout the period from 1964 - 1968 the Revisions Committee had made provisions for a number of in-service training sessions to prepare pilot teachers for instruction in the experimental geometry texts. Outlines of content material to be covered were prepared by Committee members. In addition, during each experimental period a meeting of pilot teachers was held to discuss their reports on the evaluation of the experimental geometry texts, the acceptance of the material by students and the achievement of the students.

Many other school systems in the United States and Canada have also been concerned about improving instruction in secondary geometry programs. Those who do desire a modern geometry program have usually adopted the metric approach as presented in either Geometry, by Moise and Downs or Modern Geometry, by Jurgenson, Donnelly and Dolciani. It has been recognized and stated by the School Mathematics Study Group that the use of a purely synthetic approach like that in the Brumfiel text is only possible in the hands of an expert teacher with superior classes. Most textbook writers have agreed with this view, so that the trend in the improvement of geometry programs has been away from the synthetic approach and towards a metric approach. However, the Manitoba Mathematics Curriculum Revisions Committee has been concerned that this approach is really not the answer for curriculum improvement. Their view has been that geometry should be a discipline in its own right without any assistance from metric concepts. This view and the current unavailability of a suitable text in geometry using a modern synthetic approach have helped to prevent the authorization of a new province-wide geometry program in Manitoba.

At the present time, there is only one modern geometry program, contained in Geometry, by Moise and Downs, being tried out on an experimental basis in grade ten and eleven classes in Manitoba. This program will not be extended to any more grade ten classes in the 1968-1969 term. In addition, there has been no expressed intention by the Revisions Committee to recommend this as the official program for Manitoba schools at the secondary level. Consequently, most Manitoba schools are using the traditional geometry text, A First Course in Plane Geometry, by Oliver, Winters and Hodgkinson and will probably be doing so for several years to come.

VIII. THE TRADITIONAL GEOMETRY PROGRAM

The traditional plane geometry text authorized for use in Manitoba Schools treats most of Euclid's basic axioms. However, the number of theorems has been minimized so that only the basic Euclidean propositions which are frequently used are presented in a continuous manner. In addition, the logical structure is weaker than that in Euclid. To some extent the approach used in the traditional text, unlike Euclid, depends upon a metric treatment similar to that proposed by the School Mathematics Study Group. However, little attempt was made by the authors of the traditional text, _A First Course in Plane Geometry_, to clarify their treatment in terms of a comprehensive statement of objectives and approach.

In spite of this omission by the authors of the traditional text, there are certain objectives implicit in the Preface of the text and in a section entitled "Why Study Mathematics?" These imply that any geometry course should be designed to help accomplish the following six objectives:

1. To make the synthetic plane geometry of Euclid more attractive to the average student by developing the material so that it is less confusing, requires less memorizing, and is easier to master than other geometry texts.
2. To assist the student in acquiring a clear understanding of the principles and facts of geometry.
3. To promote self-activity and help the student work alone.
4. To encourage students to appreciate the unique historical contribution of geometry as one of the outcomes of man's effort to understand and enjoy the world in which he lives.
5. To provide students with a sound geometrical foundation which will assist them in mastering college courses.
6. To provide training in logical reasoning and clear thinking so that students may acquire the ability to examine all things

critically and act intelligently as good citizens.[22]

An examination of these objectives reveals that three main areas are being considered. These include the development of knowledge, skills and habits, and appropriate attitudes in the student -- all worthy items. However, whether the traditional course or any geometry course can develop all of the six objectives above in full, remains an open question at this point.

It is unfortunate that the authors of the traditional text did not see fit to declare their basic philosophy of teaching geometry or attempt to clarify their operational approach to the subject. This lack of direction is a basic weakness of the traditional text which is not found in the other text under study, Geometry, by Moise and Downs.

The content of the traditional geometry text treats Euclidean geometry synthetically and presents the following familiar topics: angles, loci problems, logical reasoning, congruences, parallelism and perpendicularity, construction problems, area, circles and similarity. In the development of material the sequence of proved theorems follows Euclid's presentation with some minor theorems contained in the exercises and left to the students to prove. One of the strengths of this text has been the quality of the graded exercises which have assisted average students in the learning of geometry. Another strength has been that the basic theorems are usually established inductively first before a formal deductive proof is presented.

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[22] W. J. Oliver et al., A First Course in Plane Geometry (Toronto: School Aids and Text Book Publishing Co., 1954), p. 8.

However, a close examination of the content of the traditional text indicates that there are several basic defects in the program. First, the traditional program suffers from the same defects in logic contained in Euclid's Elements. No attempt was made by the authors of the traditional text to clarify the logical structure of the subject. Consequently, many of the basic assumptions of the geometrical content in the text are left unstated, and most of the definitions are inexact. Thus, the geometry approach characterized by the traditional text, A First Course in Plane Geometry, lacks rigor. A second basic defect in the traditional program is the omission of topics which have been recognized by most program planners as important for a truly modern approach. These include topics on set theory, the effective use of algebra, the integration of plane and solid geometry, and an introduction to coordinate geometry. The significance of such topics to a treatment of geometry appropriate for secondary students is recognized by Jurgenson, Donnelley and Dolciani in the following:

> Before many of us were born, some mathematicians and educators recommended that important parts of solid geometry be integrated with plane geometry. There were also recommendations that the algebra not available to Euclid be used in geometry and that coordinate geometry be given a place in the course. These recommendations have been set forth strongly during the last decade - along with the newer recommendations that sets be used as a unifying concept.[23]

The Manitoba Mathematics Curriculum Revisions Committee also recognized the need for a new and an effective approach to instruction

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[23] Ray C. Jurgensen et al., Teacher's Manual: Modern Geometry (Boston: Houghton Mifflin Co., 1965), p. 1.

in secondary geometry. They looked for a solution to the problem of acquiring a mathematically sound geometry approach which was also compatible to secondary student and teacher needs in the metric treatment contained in Geometry, by Moise and Downs.

IX. THE PILOT COURSE

In 1966 pilot classes were selected by the Manitoba Department of Education to experiment with the modern geometry course contained in Geometry by Moise and Downs. This course was developed by the authors to provide a basic modern plane geometry course at the tenth grade level for the university-oriented student. The authors explain the spirit and method of their text by acknowledging their debt to the School Mathematics Study Group and the Commission on Mathematics. They recognize this debt in the following way:

> ... Thus, in our choice of topics we have been guided by ideas which these groups and others have commonly accepted ... our view on fundamentals has not changed very much since the summers of 1958, 1959 and 1960, the philosophy of the SMSG books seems as valid now as then; and we considered that our task was primarily to improve its execution.[24]

Thus, the authors of Geometry, Moise and Downs, have subscribed to the philosophy of the SMSG described earlier in this chapter. An examination of the "Preface," "Table of Contents," and "Teacher's Manual" of Geometry revealed that the main intent of the authors has been to develop a mathematically sound treatment of elementary plane geometry which is integrated, rigorous and modern. In this regard,

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[24] Edwin E. Moise and Floyd L. Downs, Geometry (Palo Alto: Addison-Wesley Publishing Co., 1965), p. v.

they have followed the recommendations of the SMSG by integrating algebraic and geometric concepts, introducing intuitive solid geometry early, using a modified version of the Birkhoff metric postulates, and using set ideas in the theory and in the exercises. By adopting this particular approach they have tried to avoid the fundamental weaknesses of most traditional geometry texts. As a result, the content of Geometry has been arranged and structured in a particular way. The pattern and style of the text has been explained quite clearly as the authors proceed in the appropriate sections of the "Teacher's Manual."

Content of the Pilot Text

The content of Geometry is basically a treatment of Euclidean geometry, covering the usual topics: congruence, similarity, parallelism and perpendicularity, area, circles and constructions. However, other topics such as geometric inequalities, coordinate geometry, solid geometry and the language of sets have also been included so that the text is different from conventional ones in content. Significant features of the text can be described in the following way:

1. Space geometry is introduced early so that the students have much intuitive experience with it before studying it systematically.
2. Coordinate systems are introduced early as well to enable algebraic concepts to make their natural contributions to the geometry.
3. Area concepts are introduced in the middle of the course because of their usefulness in developing the rest of the theory such as the Pythagorean Theorem and the basic proportionality theorem.
4. Concepts are usually discussed intuitively before being formally defined.
5. Clear and informative figures are used to assist in the development of the proofs.

6. Many theorems are given names to make them easier to remember.
7. The language of geometry is used in a consistently clear and accurate manner so that students can be encouraged to learn to use the language in an appropriate manner.
8. Motivation and interest is provided by frequent intuitive explanations, complete expositions, liberal use of figures, avoidance of overuse of abbreviations, and short biographical sketches of eminent mathematicians.[25]

Although much of the content of Geometry is quite conventional, the spirit in which the presentation has been developed is far from being traditional. To make certain that teachers using this text will understand the approach used the authors have described very carefully six general considerations of the approach in the "Teacher's Manual." In addition, they have discussed the specific reasons for the treatment given to each topic in the appropriate sections of the manual. As a result, it is possible to understand the approach being used in the text and then to apply it in a practical way in the classroom.

The six general considerations of the approach described by Moise and Downs include: (1) the metric postulates; (2) mathematical descriptions and models; (3) clarity and accuracy of language; (4) the language of congruence; (5) problems and (6) course schedules. The first of these, the metric postulates, are the most original feature of the text. They involve the idea of measurement by real numbers of distances, angles and areas, and permit a higher level of logical rigor in the treatment. Also, they have allowed the ideas presented in the theory to be the same ones that the student

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[25] Ibid., pp. v- vi.

will use in solving the exercises. Consequently, the metric approach has strengthened the whole geometrical treatment in the text because the theory serves a real purpose.

In the discussion of mathematical descriptions and models the authors have maintained that, at the tenth grade level, geometry should be thought of as a concrete deductive system or a mathematical model of the physical world, studied by deductive methods. Thus, the general approach used is to examine a physical situation and observe that some of its essential features can be described by mathematical conditions. Another concern of the general approach used by Moise and Downs is the clarity and accuracy of the language. In their text, terms are used and introduced when and where required and in the form in which they are used later. In general, then, the usage of these terms is explicit, and the language of geometry is brought into closer agreement with the language of modern mathematics.

An examination of the exercises in the text revealed original problem sets which included a variety of different types of questions structured in terms of difficulty and enrichment. The exercises also contain simple direct applications of definitions and theorems, challenging originals, honours problems and a number of minor theorems. Finally, course schedules are suggested in the "Teacher's Manual," by the authors, to assist teachers in organizing their program and assessing the progress of the class on a topical basis. Specific comments have also been made on the appropriate progress of an average class and the emphasis to be placed on various topics. As a result, the discussion in this section of the "Teacher's Manual" and the detailed commentary in the chapter outlines have made the presentation in Geometry an effective and adequate one.

In summary, because of the emphasis in Geometry on the structure of a geometry, students can be taught a better understanding of the basic concepts and applications of the geometrical material. This text can enable all students to be participants as they are led through the intuitive processes that establish a conjecture, and then to the formal proof.

Response to the Pilot Program

Manitoba. In 1967, the Manitoba teachers who were conducting pilot classes in Geometry met to discuss their responses to a questionnaire prepared by the Manitoba Department of Education. Fourteen reports were received from the pilot teachers of seventeen classes containing 448 students. In general, the Manitoba teachers agreed that the new geometry program was a superior treatment to the traditional program. They expressed approval of the results and progress of their students as measured by unit and final tests. In addition, the majority concluded that more of their students could read this text than the traditional one, and that they seemed to enjoy it more. It was determined that the readability of the text was due to the clearly illustrated format and the thorough presentation of content material. As strong features of the text the pilot teachers listed; the logical deductive approach, the correlation of geometry and algebra, clear definitions, adequate graded problems and readable content. A sizeable majority reported that all of their students accepted the material in Geometry as well as the traditional material, whereas some of the above average students reacted more favourably than similar students had done before.

Certain weaknesses in the pilot program were noted by some of the teachers. The first concerned the inadequate time allowance for the completion of outlined topics, particularly for below average pilot classes. This was probably a result of the teacher's unfamiliarity with the modern program and an attempt to make the approach too rigorous for the below average. Consequently, it is necessary to adjust classroom instruction of the content in Geometry by teaching some material intuitively to some students, and teaching the same material rigorously to other students. A second weakness concerned the considerable amount of time that had to be spent on introductory material before reaching the traditional content of plane geometry --the congruence proofs and the originals. The third weakness recognized by all pilot teachers was the excessive number of theorems, postulates and definitions required by the modern approach. Finally, they agreed that the approach in Geometry was too rigorous and contained too much verbiage for the below average student. In terms of the Manitoba experience most pilot teachers concluded that the Moise and Downs text, Geometry, was more suitable for students of average to above average mental and mathematical ability. However, there has been no concrete experimental evidence to support this conclusion. In any case the failure rate in Manitoba in secondary plane geometry has usually been high regardless of whether the program is new or old. Both the traditional and pilot programs in secondary plane geometry ideally are suitable for the top 30 per cent of the high school population, whereas practical instruction is usually given to 60 - 70 per cent of all students enrolled at a particular grade level. Some definite and immediate reorganization of school structure is required to phase students of varying mathematical ability into different levels and courses of mathematical instruction, so that the needs of all students can be met.

British Columbia. The text, Geometry, by Moise and Downs was introduced into all British Columbia schools as the authorized text in 1964. It is worth noting the experience of British Columbia teachers with the text. This has been described by J. Clark of the British Columbia Association of Mathematics Teachers in the following way:

> I have no hesitation in saying that our good teachers find the Moise and Downs text an excellent book. The development of the subject, the problems asked, the explanations given, and so on are of much greater mathematical interest than has been the case with more traditional texts. It is probably true to say also that after the first year or two most teachers (who favour the course) find it reasonably easy to cover most of the material in the text. This does not mean that the pupils obtain a reasonable chance to see deductive aspects and are frequently given a less rigorous intuitive exposition to speed up the amount of coverage ... I have avoided the question of what happens when the teacher is not interested in revamping his approach ... Unfortunately, he can survive by simply ignoring the fine points of the Moise text and carry on his teaching in a traditional way. There is little doubt that this is happening and no one is able to do much about it.[26]

The program contained in Geometry, was developed to provide a

[26] Letter from Jim F. Clark, President, B. C. Association of Mathematics Teachers to G. Kris Breckman, Manitoba Teachers' Society, May 12, 1966. (Mimeographed).

mathematically sound treatment of plane geometry and an introduction to solid geometry at the grade ten level, and to present the treatment so that it would meet the needs of university-oriented students in the future. Classroom experience in Manitoba and British Columbia has indicated that this program can be effective in the hands of an enthusiastic teacher with average to above average students. Although the key to the success of any program, traditional or modern, is the teacher, a text book can be a facilitative device which presents that program in an influential way. This device may encourage or hinder the development of a program in geometry for either the student or the teacher. Because of the significant influence that a text has on a geometry program, it was considered to be a worthwhile object of investigation. Consequently, it became the purpose of this study to evaluate how well grade ten students were achieving in a traditional and a modern geometry program, as presented by the particular texts, and to what extent they were accepting each.

X. SUMMARY

Many different curriculum groups have recognized the need for a change in the traditional Euclidean organization of plane geometry at the secondary level. Thus, they have tried to overcome the inadequacies of geometry courses based on edited versions of Euclid's Elements by introducing a number of changes in structure and by developing certain comprehensive programs. Their programs have included, in various combinations, changes such as the use of the Birkhoff postulates, the development of plane and solid geometry together, the use of the Hilbert postulates, and the introduction of coordinate geometry.

At the present time in the United States, a metric approach to grade ten geometry courses has been adopted by the majority of school systems interested in improving geometry programs. The purely synthetic treatment has been recognized as being too rigorous and sophisticated for most grade ten students.

In Manitoba, the Manitoba Mathematics Curriculum Revisions Committee has the responsibility for developing a continuous mathematics program from K - 12. They have recognized the need for developing a geometry program for Manitoba students which will be more effective in terms of approach and content than the present traditional program. Thus, the Committee has carried on many discussion sessions, reviewed numerous geometry texts and introduced several pilot programs to experiment with modern geometry texts. This latter process of experimentation is still going on at the present time.

CHAPTER IV

THE APPRAISAL PROCESS

I. INTRODUCTION

In a report on educational research in science and mathematics Belanger stated that the principle of the multiple determination of behaviour has been accepted generally by all researchers in psychology. In addition, he reported that most researchers in education have also agreed that the research problems in their field are concerned with multidimensional phenomena. Consequently, his conclusion was that it becomes a major concern of anyone researching educational phenomena to develop a procedure which is capable of handling the multivariate nature of the problem.[1]

Multivariate Methods

In recent years, a number of appropriate multivariate models and procedures have been developed for mathematics education research problems. Cahen recognizes the use of and need for such models in the following:

> The utilization of multivariate methodologies reflects the findings of other research in mathematics education that mathematical abilities and multi-dimensional rather than unidimensional. This way of conceptualizing mathematical abilities ... is reflected in the overall design as well as in the tests developed to measure mathematical performance.[2]

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[1]Maurice Belanger, "Methodology and Educational Research in Science and Mathematics," Review of Educational Research, 34:385, June, 1964.

[2]Leonard S. Cahen, "An Interim Report on the National Longitudinal Study of Mathematical Abilities," The Mathematics Teacher, 58:524, October, 1965.

Cooley and Lohnes have also described certain multivariate statistical procedures which are capable of making an appropriate multidimensional analysis of mathematics education research problems. In particular they have mentioned multiple discriminant analysis or discriminant function analysis as a valuable procedure for multidimensional analysis. According to these authors the two major advantages of such analysis are: the dramatic reduction in predictor space dimensionality without substantial loss of information, and the satisfaction of the important assumptions of a multivariate normal distribution by the discriminant scores better than the original test scores.[3]

The Discriminant Function

Johnson has described the formulation and solution of the mathematical problem of the discriminant function quite clearly.[4] He indicates the nature of the function in:

> The essential property of this function, which is a linear function of the observations is that it will distinguish better than any other linear function between the specified groups on whom common measurements are available. The principle upon which the discriminant function rests is that the linear functions of the measurements will maximize the ratio of the difference between the specific means to the standard deviations within classes.[5]

The discriminant function makes it possible to evaluate the relative

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[3]William W. Cooley and Paul R. Lohnes, Multivariate Procedures for the Behavioral Sciences (New York: John Wiley and Sons, 1962), p. 116.

[4]Palmer O. Johnson, Statistical Methods in Research (New York: Prentice-Hall Co., 1949), pp. 344-347.

[5]Ibid., p. 344.

amount of information for differentiation between two groups provided by the multi-measurements with a minimum of overlapping. Thus, it can become a highly efficient though sophisticated statistical device to use in the analysis of data when the problem is to discriminate between different populations on the basis of several measurements.

One of the basic problems of this study was to discriminate between students enrolled in a modern geometry program and those enrolled in a traditional geometry program on the basis of available measures of attitude and achievement. An examination of the characteristics of the discriminant function and the multidimensional features of the problem influenced the selection of a particular experimental design to study the problem. This design can be described best as a post test only structure with pre-test information available on control and experimental groups.

In addition to the selection of an appropriate design to study the problem of this thesis it also was necessary to develop certain testing devices for the investigation of the characteristics of traditional and modern geometry groups. Tests for the measurement of student acceptance of their geometry course and student achievement of recognized common objectives of secondary plane geometry were not available commercially. Thus, several testing devices were developed by the writer as part of the study of the problem of this thesis. These tests, together with the only available and appropriate commercial test, are described later in this chapter.

II. TEACHER PARTICIPATION IN THE EXPERIMENT

All experimental schools were contacted by means of a form letter which outlined the purposes and procedures of the investigation. The schools contacted agreed to cooperate in the study, and the participating teachers were interviewed personally by the writer. This initial meeting provided an opportunity to discuss the procedures of the experiment. At this time it was agreed that the length of the instructional time would be equated as closely as possible. No groups of students received more than normal additional help. The instructional periods were alternated so that all students received algebra one period and geometry the next. With an authorized time allotment of 12 per cent set out in the program of studies for Manitoba schools, both pilot and traditional groups were exposed to the same amount of class time.

Several meetings were held with individual teachers during the course of the experiment. Two of these meetings were sponsored by the Department of Education and all participating teachers were present except for two who taught traditional classes. In addition to the meetings, the pilot teachers were provided with lists of postulates, definitions and theorems which were on mimeograph copies prepared by the writer. Standard testing material was also given to each pilot group. The above techniques gave a measure of uniformity to the teaching and testing in the pilot program. Individual conferences held with the traditional teachers served the same purpose to a limited extent. However, the qualifications and interest of the participating teachers ensured that they would strive to do as effective a job as possible during the course of the experiment.

III. TEST SELECTION AND DEVELOPMENT

It has already been noted elsewhere that one of the basic problems confronting most comparative curriculum studies has been the inadequacy of their measuring instruments. Accordingly, the selection and development of appropriate measuring instruments became a major concern of this study.

Cooperative Geometry (*COOP*)

During the school term, 1966-1967, student achievement in and attitude towards tenth grade plane geometry were measured under two approaches: modern and traditional. Because of the several differences between the two approaches in terms of objectives and content, it was recognized that valid and reliable measuring devices were of the utmost importance. As a result, the writer visited the Minnesota National Laboratories in St. Paul, Minnesota, to examine the tests being used there to evaluate the School Mathematics Study Group and traditional grade ten plane geometry in a five state area. The experience of the Minnesota National Laboratory revealed that the *Cooperative Mathematics Test*, *Geometry*, *Form A* (*COOP*) was an appropriate standardized instrument to measure student achievement of common objectives of traditional and modern plane geometry courses.[6]

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[6] *Cooperative Mathematics Tests*, Geometry (Berkeley: Educational Testing Service, 1964).

In general, the COOP test measured achievement in three areas: comprehension of the basic concepts, techniques and unifying principles of plane geometry, ability to apply an understanding of geometrical ideas to new situations, and the ability to reason with insight in geometry.[7] An item analysis of the content of this test revealed that it sampled shared elements of both the modern and traditional programs. The test was examined carefully for items that would significantly bias the results of the experiment in favour of either program. No item was discovered which was biased in favour of either the modern or traditional approach to an extent which would invalidate the test. A majority of the items was determined to be highly discriminating both by the writer and two experienced secondary mathematics teachers, who were asked to evaluate the items in Part I of COOP in terms of their appropriateness. The two external judges selected the test as an appropriate one for this study to measure common elements of the course content of both the modern and traditional programs. Consequently, it was selected as one of the achievement instruments.

National high school norms were available for Part I separately and Parts I and II together of Cooperative, Geometry. These were based on 129 schools with 3303 students, and they indicated a mean of 150 with a standard deviation of 10 on Part I. Content validity of the test was insured because it was constructed by many highly qualified mathematics teachers, specialists and scholars. The reliability

[7] Handbook: Cooperative Mathematics, Tests (Berkeley: Educational Testing Service, 1964), p. 7.

coefficient reported for *Cooperative, Geometry* was .80 which was computed using the Kuder-Richardson Formula 20. In addition, the correlation between this test and SCAT-quantitative was highly significant.

Geometrical Achievement Measure Experiment (*GAME*)

The *Geometrical Achievement Measure Experiment* (*GAME*), Form 2 was developed by the writer to supplement the achievement measure obtained from *Cooperative, Geometry*. It was designed to measure student achievement of geometrical objectives in content areas not examined by any other available published test. Information from the Minnesota National Laboratories revealed the development of several appropriate tests to measure student achievement in either a modern or a traditional geometry course. Four geometry achievement tests developed at the Minnesota National Laboratories and administered to a five state area were obtained. An examination of the items on these tests and a study of geometry items from the available literature led to the development of the test, *Geometrical Achievement Measure Experiment*. During this development over two hundred items were written, examined and modified to prepare the first edition of *GAME*.

In the initial planning stages of the test consideration was given to the basic need for an appropriate measuring instrument to assess student achievement that would evaluate both a modern and a traditional geometry program. Thus, the need for listing the significant objectives of grade ten plane geometry was realized. In addition, evaluation items were selected so that they would be appropriate to the domains and levels of the objectives. To provide a frame of reference for the test items of GAME the following general objectives were considered to be significant to both modern and traditional programs:

1. Knowledge and understanding of basic geometrical facts, concepts and processes and their relationships.

2. Ability to use a variety of problem-solving techniques based on the concepts of congruence, parallelism, inequality and algebra.

3. Development of an understanding of the nature of a geometry as a mathematical model.

4. Ability to read and understand geometrical content material.

5. Ability to visualize in two and three dimensional space.

The clarification of each of these five objectives was obtained by listing the secondary aims related to abilities and understandings in terms of Bloom's Taxonomy.[8] The objectives which were developed to serve as an outline for GAME were discussed with several members of the Manitoba Mathematics Curriculum Revisions Committee and individual experienced mathematics teachers. To evaluate the attainment of the objectives by both the pilot and traditional groups over two hundred test items were written and then analyzed in the following content areas:

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[8]Benjamin S. Bloom (ed.), Taxonomy of Educational Objectives, Handbook I: Cognitive Domain (New York: David McKay Co., 1964).

1. the properties of lines, angles and triangles,
2. the properties of a miniature geometry,
3. congruences in a plane and in space,
4. the nature of a proof,
5. overlapping of figures,
6. inequalities in geometrical models,
7. parallelism,
8. altitudes and hypotenuses in space,
9. the application of geometric skills to a novel situation,
10. algebraic applications to geometrical situations.

These were considered appropriate and sufficient for the development of a test supplement to Cooperative, Geometry.

Validation of the first test draft was obtained by submitting all items for examination by three experienced secondary mathematics teachers. This examination concluded in the selection and modification of twenty-four items, and these comprised the first form of the GAME test. A pre-test of this form was then carried out on a representative sample of secondary students at a large Winnipeg high school.

Two basic characteristics of each item were examined in an item analysis: the difficulty of the item for grade ten students, and the discriminating power of the item to distinguish between high scorers and low scorers. In addition, the pre-test provided information about the effectiveness of the distractors for each item and the amount of testing time required.

The results of the pre-test with GAME, Form I, led to the development of additional items and the modification of the most effective ones of the preliminary form. A revised test, GAME, Form 2, was then developed and administered in May, 1967, as one of the achievement criteria tests. Subsequently, an item analysis of percentage correct responses was performed on Form 2 to determine whether there was any significant response differences between the modern and traditional groups and the level of both groups on each item. This analysis is discussed in Chapter 5.

Geometrical Attitude Scale

Student performance and achievement in any geometry program are affected by psychological variables such as attitudes and motives. This has been supported by the research of Alpert[9] who recently directed a study of student attitude towards mathematics and its influence on achievement. A major conclusion of that investigation was that highly significant positive correlations exist between a student's performance and his mathematical attitudes.

The development of favourable attitudes towards geometry should be included as an essential objective of any secondary geometry program. For this study, that objective implied the question: "Is there any difference in the degree of acceptance of grade ten geometry between students following an experimental program and those following a traditional program?" To answer this question it was necessary to evaluate how well the two groups were accepting their geometry programs.

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[9] R. Alpert, G. Stellwagon, and D. Becker, "Psychological Factors in Mathematics Education," SMSG Newsletter No. 15 (April, 1963), pp. 17-24.

The major concern of this evaluation was the differences in student attitude rather than the factors which contributed to the development of the attitudes. Thus, an instrument entitled the Geometrical Attitude Scale was prepared by the writer to obtain a reaction from the students that would indicate what they thought about their geometry course. This device was developed from an instrument designed by Hedley and entitled Student Attitude Towards Science.[10]

Hedley has noted that the limitations of attitude tests are usually caused by individuals who may conceal their real attitudes, or by other individuals who may not know their true feelings or may not have thought about them in a particular way before, or by still other individuals who may not be able to abstract their attitudes and predict their reaction to a particular situation. Consequently, the description and measurement of attitude becomes extremely difficult. However, the measure of opinion is closely related to real attitudes. Because it is, and because it can be obtained in a valid and reliable way from a test device, it was considered appropriate to develop such a device and measure geometrical attitudes in an indirect way.[11]

The following is a description of the development of the test entitled Geometrical Attitude Scale designed to assess a student's reaction to his geometry program. First, a number of statements of

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[10] Robert T. Hedley, "Student Attitudes and Achievements in Science Courses in Manitoba Secondary Schools" (Doctoral thesis, Michigan State University, East Lansing, 1966), pp. 80-89.

[11] Ibid., pp. 80-81.

opinions about geometry were collected and devised from four basic sources: (1) Hedley's Student Attitude Towards Science test, (2) student comments about SMSG geometry programs contained in Revolution in School Mathematics,[12] (3) Aids for Evaluators of Mathematics Textbooks,[13] and (4) The Secondary Mathematics Curriculum.[14] In all, one hundred and twenty items were developed, some connoting positive attitudes and some connoting negative ones. Initial classification by the writer placed the items into five categories: textbook, course content, student needs, student interest and student involvement. The items were structured to determine student response in the five areas. To determine item validity and quality, the unclassified individual statements were given to four teachers, two elementary and two secondary, for an item analysis.

As a result of the judges' ratings the items were modified and edited so that the revised form of the attitude scale comprised sixty items, thirty connoting negative attitudes and thirty connoting positive attitudes. For each of the five areas, text, course content, needs, interest and involvement, the final edition of the Geometrical Attitude Scale contained twelve items, six connoting positive attitudes

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[12] Frank B. Allen (ed.), The Revolution in School Mathematics (Washington: National Council of Teachers of Mathematics, 1964), pp. 30-36.

[13] William Chinn et al., Aids For Evaluators of Mathematics Textbooks (Washington: National Council of Teachers of Mathematics, 1965), pp. 1-7.

[14] Secondary School Curriculum Committee: NCTM., The Secondary Mathematics Curriculum (Washington: National Council of Teachers of Mathematics, 1959), pp. 389-416.

and six connoting negative ones for a total of sixty items.

A Likert method of summated ratings was used to measure student opinion.[15] Table I illustrates the five point scale used to score the positive and negative items, and also how each student was asked to respond to each item. Included in the appendix is a form of the Geometrical Attitude Scale.

TABLE I

RESPONSE STATEMENTS AND VALUES FOR GAS

Response Statements	Scale Value	
	Positive	Negative
(a) Strongly Disagree	1	5
(b) Disagree	2	4
(c) Neither Agree or Disagree	3	3
(d) Agree	4	2
(e) Strongly Agree	5	1

An examination of the Geometrical Attitude Scale in the Appendix discloses the following characteristics. Category one of the test contained both positive and negative statements about reading level, clearness of explanation, special terms used, level of difficulty of problems, and the diagrams in the geometry text being used

[15] Hedley, *op. cit.*, pp. 86-87.

by the student.

The second category considered twelve statements made about course content. These included items about the future use of the course, its significance, amount of homework, precision required, the difficulty of the course, and the amount of material covered.

Category three of the test related to student needs. Twelve statements were made concerning the usefulness of the program, now and in the future, its satisfying of needs, its worth, and the importance of studying geometry.

The statements developed for the fourth and fifth categories, student interest and student involvement respectively, proved to be the most difficult items to edit with the minimum amount of overlapping of content. The twelve statements developed for category four were concerned with the attention paid in class, the interest in taking future geometry courses, the desire to spend more time on geometry, the willingness to read other related texts, and the wish to study geometry in greater depth. The fifth category developed for the Geometrical Attitude Scale was designed to measure the degree of student participation in the geometry course. Students were asked to respond to statements about the amount of self responsibility attained, the development of the ability to think like a mathematician, the degree of inner satisfaction achieved in presenting their own solutions, and the ability to apply geometrical ideas.

In each category, the maximum positive score was sixty, the minimum negative score was twelve and the neutral score was thirty-six. On the whole test, the maximum positive score was three hundred, the minimum negative score was sixty, and the neutral score was one hundred and eighty. An analysis of student response to their particular geometry program on the basis of the Geometrical Attitude Scale is developed in Chapter V.

Mathematics Attitude Scale (Geometry)

To obtain information about student attitude towards geometry in addition to that supplied by the Geometrical Attitude Scale, the writer was given permission to revise and edit the attitude scale developed by Aiken and Dreger entitled Math Attitude Scale.[16] Specifically, the editing involved the following changes:

1. The word "mathematics" was replaced by "geometry."

2. Statement 10 -- "Mathematics makes me feel as though I am lost in a jungle of numbers and can't find my way," was changed to "Geometry makes me feel as though I'm lost in a jungle of letters and diagrams and can't find my way."

3. In Statement 16, "math problem" was changed to "geometry question."

4. Statement 17 -- "I have never liked math, and it is my most dreaded subject," was replaced with "I have not liked geometry this year, and it is my most dreaded subject."

Both forms of the Math Attitude Scale are included in the Appendix. An examination of each revealed that the changes from one form to

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[16] Lewis R. Aiken and Ralph M. Dreger, "The Effect of Attitudes on Performance in Mathematics," Journal of Educational Research, 52: 19-24, February, 1961.

another did not invalidate the test.

The test consists of twenty statements about geometry, ten connoting positive attitudes and ten connoting negative ones. The scoring procedure for this instrument was exactly the same as that used for the _Geometrical Attitude Scale_, a Likert method of summated ratings. Use of the five point scale to score student response yielded a single score. Consequently, on this test, the maximum positive score was one hundred, the minimum negative score was twenty, and the total neutral score was sixty. Results from the test are discussed in Chapter V.

III. DATA COLLECTION AND PROCESSING

Processing of Pre-Test Data

During the second week of September, 1966, two pre-tests were administered by the participating class teachers to the pilot and traditional groups. After administration the tests were returned to the writer who scored them and recorded the results on data sheets according to groups and individual. These tests, _Math Attitude Scale_ and _Sequential Tests of Educational Progress_ - _Form 2A_, together with the information supplied by the participating teachers provided data on five pre-tests variables: age, general intelligence, mathematical ability, mathematical achievement, and mathematical attitudes for each student. These data were considered sufficient to determine whether the individual pilot and traditional groups differed significantly in terms of those characteristics which were basic to the study. Later analysis revealed that one pilot-traditional group was too divergent in terms of pre-experimental characteristics, so that it had to be dropped from the class to class comparisons. However, some individual members of this group were retained for the matched sample comparison group. To determine the pre-experimental compatibility of traditional and pilot groups the available data were analyzed in terms of five class to class comparisons, one matched sample comparison, and a total pilot-traditional comparison.

Preliminary examination of the pre-test data and consideration of the multidimensional aspects of the problem resulted in the selection of a discriminant function analysis as the statistical model for the analysis of data to be used in the comparison of pilot and traditional groups. Although the predictor variables used in this study resemble those used in regression schemes, the criterion, whether there is a significant difference between a traditional and a pilot geometry group after experimentation, is considered a dichotomous variable rather than the usual single numerical variable. Thus, the usual regression techniques were considered to be inappropriate and an adaptation of Johnson's discriminant analysis was used. His concept of discriminant analysis was developed to determine the appropriate weights for a series of variables yielding maximum separation into two groups.[17] Consequently, the pre-test data were examined by a discriminant function analysis to determine whether the modern geometry group could be differentiated from the traditional geometry

[17]Palmer O. Johnson, Statistical Methods in Research (New York: Prentice-Hall Co., 1949), pp. 343-347.

group in terms of age, general intelligence, mathematical ability, achievement and attitude.

To carry out the analysis of data all pre-test information was accumulated by the writer and then transferred to punch cards at the Computer Centre, University of Manitoba. The data were processed by computer using a stepwise discriminant analysis program.[18]

In the actual operation of the program the stepwise procedure entered all the variables at the beginning into the discriminant equation. The discriminant function analysis program selected this equation by using appropriate weights for utilizing the five variables in the prediction scheme. From the discriminant equation a multiple serial correlation is obtained. The variable which reduced the variance in the dependent variable (criterion) the least in a single step was eliminated from the comparison. This revealed the effect of the remaining variables, thus leaving a truer picture of the relationship between the independent variables and the criterion. The procedure continued step by step until all the variables were eliminated. A complete discussion of the discriminant analysis of the pre-experimental characteristics of pilot and traditional groups is included in Chapter V.

Processing of Experimental Data

After the experimental period, four post tests were administered by the participating teachers and then returned to the writer

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[18] Rex T. Hurst and Gwen Wiser, Stepwise Discriminant Function Analysis For Two Groups (Computer Program, Utah State University, (n.d.)).

for scoring. The data from these tests provided nine variables. To test Experimental Hypothesis IV the nine variables were processed by a computer using a discriminant analysis program. The results of this analysis in terms of the three comparison groups are discussed in Chapter V.

Further treatment of the experimental data involved an item analysis of the percentage correct student responses on the Geometrical Achievement Measure Experiment and Cooperative, Geometry, and the development of a correlation matrix for the total pilot and traditional groups. A selection of eleven variables was made from the pre-test and post test data so that the matrix could be computed for each of the total group of two hundred and twelve students, the total traditional group of one hundred and fourteen students and the total pilot group of ninety-eight students. The data were processed by computer using a simple correlations program. Results from the item analysis and the correlation matrices are also analyzed in Chapter V.

IV. SUMMARY

Because of the multivariate nature of the problem being investigated, it was necessary to select an appropriate statistical model for processing the experimental data. Consequently, a discriminant function analysis was chosen as an effective device for testing the experimental hypotheses. In addition, further treatment of the data involved the development of an item analysis of the achievement tests used and the use of a correlation matrix of eleven experimental variables.

The design of certain achievement and attitude tests was made necessary by the lack of available instruments. Three tests, _Geometrical Achievement Measure Experiment_, _Geometrical Attitude Scale_ and _Math Attitude Scale_ (Geometry), were developed by the writer to measure the experimental characteristics of the pilot and traditional groups. These were validated and made reliable by the use of external judges, prior testing and item analysis.

Data from the four post tests provided nine experimental variables which were processed by a computer program using a discriminant function analysis. Finally, the experimental hypotheses were tested in terms of five class to class comparisons, one matched sample comparison, and one total pilot traditional comparison.

CHAPTER V

AN ANALYSIS OF DATA

In this chapter the effects of the processing of data are studied. An investigation of five pre-experimental variables for each comparison group was completed by means of a discriminant function analysis. In addition, the results of the examination of the experimental data were reported on the basis of a correlation matrix, an item analysis of the Geometrical Achievement Measure Experiment and Cooperative Geometry, and a discriminant analysis of experimental hypothesis III.

I. PRE-EXPERIMENTAL CHARACTERISTICS OF THE SAMPLES

To determine the pre-experimental compatibility of pilot and traditional samples in terms of characteristics central to this study, pre-tests were administered. Data from these indicated that one sample pair of classes was too divergent in characteristics to be useful as a comparison group. Consequently, this pair was eliminated from the class comparisons, and the investigation was carried on in terms of five class to class comparison groups, two matched comparison groups, and a total pilot-traditional comparison group. Included in the analysis of the pre-experimental characteristics of these groups are the five variables: (1) age (X_1); (2) SCAT -- mental ability (X_2); (3) 9 MATH -- mathematical achievement (X_3); (4) MAS -- attitude towards mathematics (X_4); and (5) STEP -- mathematical ability (X_5).

Class To Class Comparison Groups

Table II shows the mean scores on five pre-experimental variables for five class to class comparisons of the pilot and traditional groups. Groups I and II represented pilot and traditional classes selected from a Winnipeg secondary school; group III was selected from a suburban school and Groups IV and V were selected from rural schools. The range in age from 182.2 months to 185.4 months indicates that the total sample was typical of students who have entered grade ten for the first time. In the total sample there were only twelve students who were required to repeat grade ten.

The SCAT scores listed in Table II refer to a measure of general intelligence. This variable showed a mean range from 291.0 to 294.6 which was higher than the mean of 282.0 reported in the publisher's norms. Therefore, this sample was composed of students who were average to above average in intelligence.

The variable, 9MATH in Table II refers to the final grade nine mathematics score obtained on the departmental examinations of June, 1966. The mean score on this variable ranged from 69.5 to 85.9 in the five class to class comparison groups. Thus, these groups were characteristic of average to above average secondary students in terms of prior mathematics achievement.

The fourth variable in Table II, MAS, is a measure of student acceptance of mathematics in terms of prior experience. The mean score range on this variable was from 60.7 to 76.3. On the basis of the twenty statements in the instrument (the neutral position of each item has a value of 3) a total value of 60 characterizes a neutral

TABLE II

PRE-EXPERIMENTAL MEANS OF FIVE VARIABLES FOR FIVE CLASS TO CLASS COMPARISON SAMPLES

VARIABLE	I		II		III		IV		V	
	PILOT N=37	TRAD. N=34	PILOT N=37	TRAD. N=33	PILOT N=25	TRAD. N=26	PILOT N=18	TRAD. N=21	PILOT N=18	TRAD. N=21
AGE in Mo. (X_1)	182.4	182.2	182.4	185.4	183.6	183.5	182.2	183.4	183.9	183.4
SCAT (X_2)	293.9	294.2	293.9	291.1	291.0	292.4	294.6	293.1	294.2	293.1
9MATH (X_3)	69.5	79.1	69.5	69.9	73.7	75.6	85.9	83.9	85.1	83.9
MAS-1 (X_4)	62.7	60.8	62.7	61.2	63.9	70.1	60.7	76.3	69.0	76.3
STEP (X_5)	281.1	280.9	281.1	278.3	280.1	283.6	286.5	286.8	288.2	286.8

attitude towards mathematics. Student response on the Math Attitude Scale indicated that group III traditional and groups IV and V traditional expressed mildly positive attitudes towards mathematics, while the rest of the groups were quite neutral in their acceptance of mathematics.

Finally, Table II shows the STEP score as the fifth pre-experimental variable. This variable, which refers to a measure of mathematical ability, ranged from a mean score of 278.3 to 288.2. The entire range was above the mean of 274.0 reported in the publisher's norms. Differences which occurred between the pilot and traditional groups' scores on STEP are discussed below. The range in the mean scores of this variable emphasized that this study was representative of secondary students who were average to above average in mathematical achievement and ability. At the same time, the groups' mean scores on the attitude instrument were typical of students who were not particularly interested in mathematics. Whether or not this attitude is representative of the attitude towards mathematics of the average student in Manitoba remains unanswered. However, the student attitude response on the Mathematics Attitude Scale does indicate the need for further research in the area of student acceptance of mathematics.

To test the hypothesis that there is no significant differences between the means of the pilot and traditional groups on the five pre-experimental variables the data were processed by means of a stepwise discriminant analysis program. The program entered all five variables at the beginning into the discriminant equation, and then eliminated the variables, one at a time, which reduced the variance the least, until all variables were eliminated. This stepwise procedure continued for all five variables for each of the five comparison groups. The results are presented in Table III. This includes the variable combinations of age (X_1), SCAT (X_2), 9MATH (X_3), MAS (X_4), and STEP (X_5) arranged in the best hierarchal rank and in terms of the significance of the contribution of each variable combination at a particular F - level.

Reference to Table III reveals that comparison groups I, II, and IV contained significant pre-experimental differences on certain variables. Groups III and V did not differ significantly on the five variables processed. In order to determine the nature of the significant differences which did occur between the pilot and traditional class groups, the results in Table III must be studied with reference to the summary of mean scores in Table II. An examination of the variable combinations in Table II for group I reveals that three combinations were significant beyond the .01 level and two combinations were significant beyond the .05 level. On the basis of the variables selected and the mean scores for group I it was concluded that the pilot group was a little older than the traditional group and had a more favourable, although neutral, attitude towards mathematics. On the other hand, the traditional group appeared to be more intelligent than the pilot group. In addition, the traditional group had considerably surpassed the pilot group in mathematical achievement, even though the pilot group had slightly better mathematical ability. Consequently, the pilot group may be underachieving in terms of their natural ability.

TABLE III

SUMMARY OF RESULTS OF ELIMINATING VARIABLES FROM STEPWISE DISCRIMINANT ANALYSIS-CLASSES

Group	Predictor Combination	F	Group	Predictor Combination	F
I	$X_3X_4X_5X_2X_1$	2.444**	IV	$X_4X_3X_5X_1X_2$	2.017
	$X_3X_4X_5X_2$	3.078**		$X_4X_3X_5X_1$	2.587
	$X_3X_4X_5$	4.138*		$X_4X_3X_5$	3.486**
	X_3X_4	5.818*		X_4X_3	5.113**
	X_3	8.429*		X_4	7.602*
II	$X_1X_2X_3X_5X_4$	.716	V	$X_4X_5X_2X_3X_1$	.788
	$X_1X_2X_3X_5$	2.171		$X_4X_5X_2X_3$	1.013
	$X_1X_2X_3$	2.648		$X_4X_5X_2$	1.312
	X_1X_2	3.654**		X_4X_5	1.886
	X_1	4.231**		X_4	2.145
III	$X_4X_5X_2X_3X_1$	.716			
	$X_4X_5X_2X_3$	.915			
	$X_4X_5X_2$	1.203			
	X_4X_5	1.704			
	X_4	2.613			

** indicates $p < .05$

* indicates $p < .01$

However, comparison group I was left in the experimental analysis to determine whether the experiment would have any effect on the pre-experimental differences of both pilot and traditional classes.

Further examination of Table III reveals that group II contains two variable combinations significant beyond the .05 level. An analysis of these results together with the mean scores in Table II showed that the traditional class in group II was slightly older than the pilot class when age was considered by itself. It was thus not unexpected to observe that the pilot group was more intelligent than the traditional group when the SCAT score was considered together with age. However, when all five variables were considered together in combination in the discriminant analysis, no significant difference between the pilot and traditional classes of group II was evident. Thus, group II was considered to be an appropriate comparison group.

An analysis of the data contained in Table II and Table III for group IV reveals that two combinations of variables were significant beyond the .05 level and one combination beyond the .01 level. The results of the analysis of data in Table III showed that when the variable attitude MAS was considered alone for group IV the traditional class had a more positive attitude towards mathematics than the pilot class. On the other hand, if the three variables, MAS, 9MATH and STEP were considered together then the pilot class achieved better than the traditional, even though the mathematical ability of the pilot group was not as good as the mathematical ability of the traditional group. In the analysis, if all five variables were taken together in combination then there was no significant difference between the pilot and traditional classes in group IV. This group was also considered to be an appropriate comparison sample. Thus, on the basis of the discriminant analysis of the pre-experimental date it was considered appropriate to include groups I, II, III, IV and V in the experiment.

The Matched Sample

Because of the differences which were revealed in the preliminary analysis it was also considered necessary to utilize a matched traditional and pilot group as part of the study design. Table IV reveals the mean scores of the five pre-experimental variables for this matched group. The pilot and traditional samples in this group were matched on the basis of sex, age, mathematical achievement and ability. An inspection of the mean scores in Table IV reveals that the two samples were similar in these respects. However, the traditional group has a more positive attitude towards mathematics than the pilot group. Further inspection of Table IV shows that the matched comparison group was above average in achievement, general intelligence and mathematical ability. When the SCAT and STEP mean scores were compared to the publisher's norms, they fell into the upper quartile. Consequently, it appeared that the matched group was representative of an above average group of secondary students.

An examination of Table V indicates that three combinations of variables were significant at the .05 level. Although the traditional

TABLE IV

PRE-EXPERIMENTAL MEANS OF FIVE VARIABLES
FOR TWO MATCHED SAMPLES

VARIABLE		PILOT N=30	TRAD. N=30
AGE in Mo.	(X_1)	182.3	182.4
SCAT	(X_2)	297.3	294.5
9MATH	(X_3)	81.5	81.6
MAS-1	(X_4)	63.9	73.9
STEP	(X_5)	286.4	286.4

TABLE V

SUMMARY OF RESULTS OF ELIMINATING VARIABLES
FROM STEPWISE DISCRIMINANT ANALYSIS

GROUP	PREDICTOR COMBINATION	F
Matched Sample	$X_4X_2X_1X_3X_5$	1.818
	$X_4X_2X_1X_3$	2.307
	$X_4X_2X_1$	3.068**
	X_4X_2	4.539**
	X_4	6.673**

** Indicates $p < .05$

sample in this group had a more positive attitude towards mathematics than the pilot sample they achieved at the same level. On the other hand, the pilot sample was a little younger and more intelligent than the traditional sample in the matched comparison group. When all five pre-experimental variables were considered together in combination there were no significant differences between the two samples in the matched group. Therefore, they were also included in the analysis of experimental data.

The Total Pilot-Traditional Sample

Table VI shows the mean scores of five pre-experimental variables for the total pilot-traditional sample. These scores indicate that the total groups were above average in mathematical ability, achievement and intelligence. In terms of age and interest, however, the total group sample was characteristic of average students. Table VII presents the various combinations of variables arranged in a hierarchal pattern. The results of the discriminant analysis disclose that there were no significant differences between the total pilot-traditional sample in terms of the pre-experimental variables. Thus, the total pilot-traditional sample was used as one of the comparison groups.

II. A SIMPLE CORRELATION MATRIX

Intercorrelations between each pair of eleven variables were computed for the total sample group, the total pilot group, and the total traditional group. This was done to determine how consistently

TABLE VI

PRE-EXPERIMENTAL MEANS OF FIVE VARIABLES FOR TOTAL PILOT AND TRADITIONAL GROUPS

VARIABLE		PILOT N=98	TRAD. N=114
AGE in Mo.	(X_1)	182.9	183.6
SCAT	(X_2	293.3	292.7
9MATH	(X_3)	76.5	76.5
MAS-1	(X_4)	63.8	65.9
STEP	(X_5)	283.1	281.9

TABLE VII

SUMMARY OF RESULTS OF ELIMINATING VARIABLES FROM STEPWISE DISCRIMINANT ANALYSIS

GROUP	PREDICTOR COMBINATION	F
Total Pilot-Traditional Groups	$X_5X_4X_1X_3X_2$	.809
	$X_5X_4X_1X_3$	1.016
	$X_5X_4X_1$	1.315
	X_5X_4	1.635
	X_5	1.130

the developed instruments were measuring and to ascertain whether any discernible differences existed between the total pilot and traditional groups and the total sample. Tables VIII, IX and X were constructed to present the correlation matrices for the above three groups.

In Table VIII the intercorrelations for the total sample are presented. An inspection of Table VIII reveals that 51 correlation coefficients were significant at the .01 level, two were significant at the .05 level, and two were not significant for the total group. The range in correlation coefficient value between all the Geometrical Attitude Scale tests and the Geometrical Achievement Measure Experiment was from .285 to .366. Although the coefficients were significant at the .01 level, they were much lower than the correlation reported between GAME and COOP of .557. Similarly, the correlation coefficients between the Geometrical Attitude Scale tests and COOP were low, although significant at the .01 level, with values ranging from .254 to .379. At the same time, the correlation coefficients between SCAT and GAS-N (needs), and GAS-In (interest) sub-tests was not significant. Very low but significant correlations were also shown between the 9MATH variable and the GAS sub-tests with a range of values from .195 to .291. The low correlations between the achievement and the attitude instruments may indicate that different characteristics are being measured by each set of instruments. However, the fact that the correlation coefficients between the attitude and achievement instruments were significant and positive suggests two conclusions. First, it may be concluded that certain positive developed

TABLE VIII

CORRELATION COEFFICIENTS FOR ELEVEN VARIABLES IN THE TOTAL SAMPLE
(N = 212)

	MAS-2	GAME	COOP	GAS-T	GAS-C	GAS-N	GAS-It	GAS-In	GAS-Tot	SCAT
GAME	.338*									
COOP	.381*	.557*								
GAS-T	.763*	.285*	.333*							
GAS-C	.814*	.338*	.341*	.839*						
GAS-N	.788*	.297*	.254*	.760*	.848*					
GAS-It	.813*	.238*	.267*	.723*	.833*	.826*				
GAS-In	.806*	.366*	.379*	.775*	.828*	.785*	.773*			
GAS-Tot	.869*	.331*	.342*	.896*	.951*	.923*	.908*	.903*		
SCAT	.246*	.364*	.420*	.182*	.201*	.105	.133	.169**	.172**	
9MATH	.346*	.403*	.390*	.201*	.237*	.195*	.212*	.291*	.246*	.343*

** Indicates $r > .135$, $p < .05$

* Indicates $r > .178$, $p < .01$

attitudes towards geometry supported a student's achievement in his particular program. Secondly, the evidence from the correlation coefficients suggests that satisfactory achievement in both geometry programs, pilot and traditional, helped to promote positive attitudes towards the subject.

Considerably higher correlation coefficients were noted in Table VIII between the two achievement instruments, GAME and COOP and also between the sub-tests of GAS and the GAS total. Between GAME and COOP the correlation was .557, while the range between the sub-tests of GAS and the GAS total was .896 to .951. The highly significant correlations for GAS indicated that the sub-tests of this instrument were measuring consistently and thus were quite reliable.

The intercorrelations for the total pilot and total traditional groups, reported in Tables IX and X respectively supported the conclusion that the tests used were measuring consistently. An examination of Table IX discloses that forty-one coefficients were significant at the .01 level, eight were significant at the .05 level, and six were not significant. There was a higher correlation, significant at the .01 level, between the Math Attitude Scale and the four sub-tests of the Geometrical Attitude Scale for the pilot than either the traditional or total groups. In addition, Table IX shows a higher significant correlation between the sub-tests of the Geometrical Attitude Scale for the pilot group than for either the total sample or the total traditional groups.

Intercorrelations for the total traditional group are reported

TABLE IX

CORRELATION COEFFICIENTS FOR ELEVEN VARIABLES IN THE TOTAL PILOT SAMPLE
(N = 98)

	MAS-2	GAME	COOP	GAS-T	GAS-C	GAS-N	GAS-It	GAS-In	GAS-Tot	SCAT
GAME	.348*									
COOP	.300*	.551*								
GAS-T	.791*	.253**	.224**							
GAS-C	.833*	.357*	.291*	.873*						
GAS-N	.798*	.278*	.114	.789*	.875*					
GAS-It	.854*	.249**	.164	.809*	.878*	.865*				
GAS-In	.802*	.398*	.307*	.807*	.860*	.827*	.792*			
GAS-Tot	.874*	.326*	.233**	.919*	.964*	.936*	.932*	.913*		
SCAT	.306*	.262**	.480*	.230**	.264*	.152	.194	.228**	.229**	
9MATH	.411*	.442*	.473*	.275*	.289*	.199	.271*	.415	.305*	.420*

** Indicates $r > .199$, $p < .05$

* Indicates $r > .259$, $p < .01$

in Table X. The data in Table X revealed fewer significant correlations between the variables than that found in Tables VIII and IX. There were only forty-three correlations which were significant, two at the .05 level and forty-one at the .01 level, and twelve correlations which were not significant. An analysis of Table X revealed that the correlations between COOP and each of the sub-tests of the _Geometrical Attitude Scale_ were higher for the traditional group than the total pilot group in each case. All these coefficients were significant at the .01 level with a range in values from .346 to .444. A possible interpretation of the higher correlations in this area may be that the traditional course content, perceived student needs, and textual material were more oriented towards the objectives being measured by the _Cooperative Geometry_ test.

Further examination of Tables VIII, IX and X discloses certain characteristics of the data relating to the grade nine mathematics score and the other variables. Table VIII reveals that there was a significant correlation between achievement in grade nine mathematics and each of the other ten variables studied. One interpretation may be that certain attitudes and skills were being developed in prior grades which were compatible with the characteristics necessary for success in a grade ten geometry course. The response of the pilot group also revealed a similar pattern in terms of 9MATH. Nine variables of the pilot group were positively correlated to prior mathematics achievement. However, in the case of the traditional group only four variables were significantly correlated to prior achievement in mathematics as measured by 9MATH. None of these four, as is shown in

TABLE X

CORRELATION COEFFICIENTS FOR ELEVEN VARIABLES IN THE TOTAL TRADITIONAL SAMPLE
(N = 114)

	MAS-2	GAME	COOP	GAS-T	GAS-C	GAS-N	GAS-It	GAS-In	GAS-Tot	SCAT
GAME	.340*									
COOP	.450*	.553*								
GAS-T	.735*	.318*	.430*							
GAS-C	.799*	.332*	.398*	.800*						
GAS-N	.780*	.325*	.386*	.726*	.813*					
GAS-It	.774*	.233**	.346*	.640*	.792*	.793*				
GAS-In	.811*	.352*	.440*	.747*	.803*	.748*	.756*			
GAS-Tot	.866*	.345*	.444*	.870*	.935*	.907*	.888*	.898*		
SCAT	.200**	.430*	.379*	.141	.147	.063	.085	.125	.125	
9MATH	.279*	.381*	.334*	.119	.176	.190	.154	.168	.179	.281*

** Indicates $r > .185$, $p < .05$

* Indicates $r > .242$, $p < .01$

Table X, referred to variables measured by the Geometrical Attitude Scale. Consequently, one may speculate that the attitudes being encouraged and developed in previous grades were more pertinent to the modern pilot geometry program than the traditional program. Also, previous success in grade nine mathematics appeared to help some students accept their pilot geometry better than it helped other students accept the traditional course geometry. This may be a result of the approach used in the pilot geometry text which borrowed from and used many of the algebraic skills developed in the new grade nine mathematics program.

Reference to Table VIII also discloses that the more intelligent a student was, the more positive was his response to the textual material, the course content and participation in either the traditional or the pilot program. In terms of the correlation between SCAT and student attitude, three correlation coefficients were significant at the .01 level and two were significant at the .05 level. The correlations were low but one may speculate that there was a tendency for intelligence factors to affect a student's acceptance of his geometry program. Table XI reveals that a similar pattern was evident in the responses of the pilot group, and in each case the correlation coefficients were higher than for the total group. On the other hand, the information in Table X for the traditional group reveals that there was only one significant positive correlation coefficient between intelligence and student acceptance of course content, and this was quite low at .200. On the basis of these results one may speculate that the more intelligent a student was then the more the modern program appeared to appeal to him.

In summary, the parts of the Geometrical Attitude Scale measure consistently as is shown by the high correlations between the sub-tests and the total test. The low correlation coefficients between the attitude and achievement instruments indicated that each set of tests was probably measuring different characteristics so that there was a minimum of overlap in test content.

III. ITEM ANALYSIS OF ACHIEVEMENT TESTS

To determine how well the cognitive objectives of both the pilot and traditional geometry courses were being achieved an item analysis of both the Geometrical Achievement Measure Experiment and Cooperative, Geometry was carried out. The significant features of student response to the items on both achievement tests were analyzed by determining the percentage correct response to each item by the pilot and traditional groups. Differences between percentage correct responses on each item for both groups were analyzed by means of a proportionality test of significance from Walker and Lev, which used a z-statistic.[1] The information obtained from this analysis is summarized in Tables XI and XII and Figures 1 and 2. The raw data which was used to construct these figures and tables has been included in the Appendix. However, in the following discussion only those items have been included which appeared to illustrate the significant

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[1]Helen M. Walker and Joseph Lev, Elementary Statistical Methods (New York: Henry Holt and Company, 1958), p. 255.

characteristics of the responses of the pilot and traditional groups. Consequently, this discussion has been limited to items which obtained the highest percentage correct responses, those which received the lowest percentage, and those on which the difference between the group responses was statistically significant.

Geometrical Achievement Measure Experiment

Figure 1 illustrates the percentage of correct responses of the total pilot group (N=98) and the total traditional group (N-114) to the twenty-five items of GAME. The broken line stands for the pilot group while the solid line represents the traditional group. A preliminary examination of Figure 1 reveals that both groups, for the most part, responded to the items of the test in a remarkably similar manner. The shape of the graph reveals a consistency of response which may be dependent upon factors other than the particular geometry course that the pilot and traditional groups have been following. However, divergent responses, which were statistically significant did appear and seemed to give evidence of a differential treatment occurring between the two groups. The divergent responses occurred on items 2, 11, 13, 20, 24 and 25. These are analyzed below.

Highest and Lowest Percentage Correct Responses

Further examination of Figure 1 discloses that on eighteen items the pilot group scored below 50 per cent correct responses, while on twenty-two items the traditional group scored below 50 per cent correct responses. Thus, the GAME test appeared to be quite

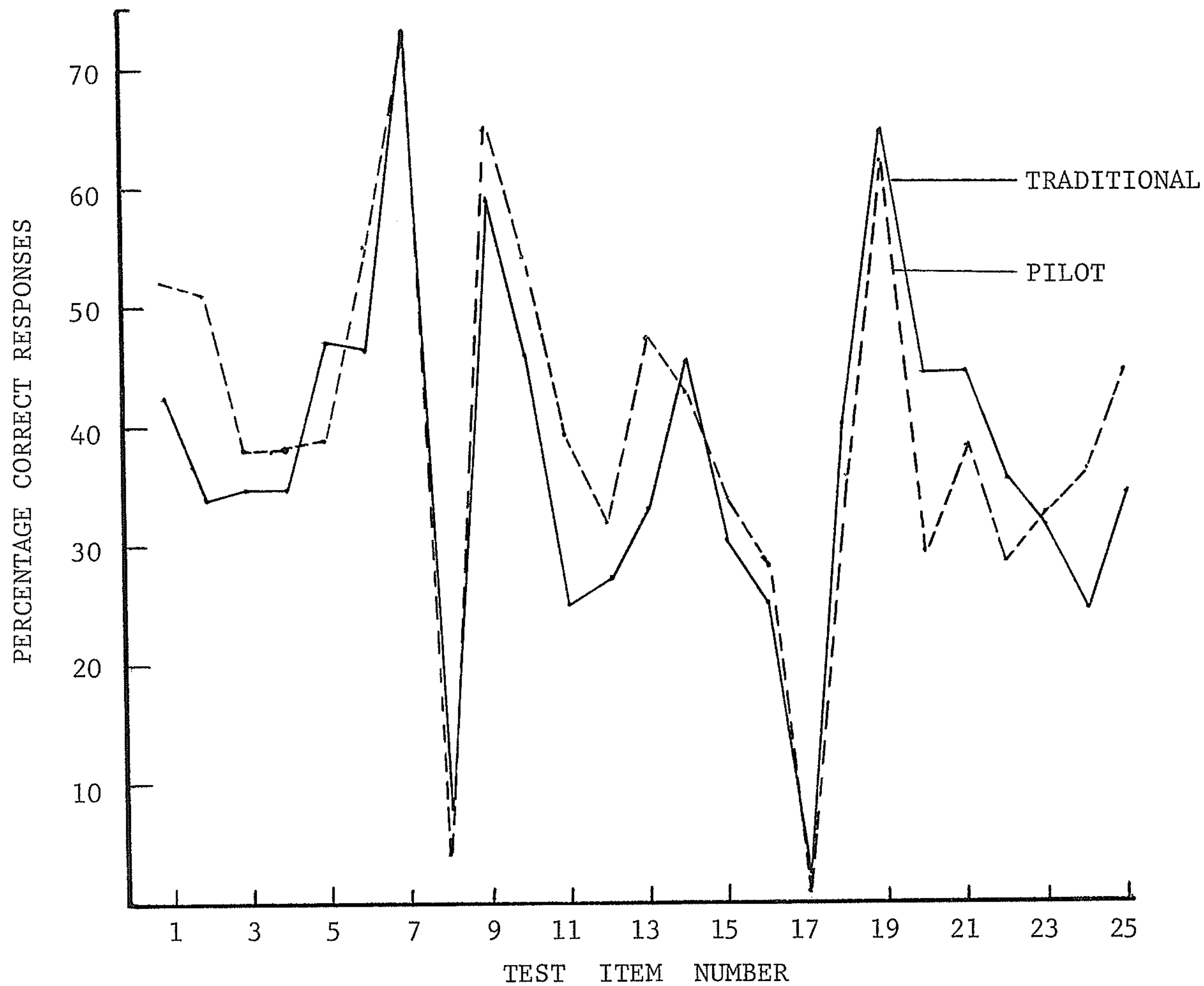


FIGURE 1

PERCENTAGE CORRECT RESPONSES ON GAME BY PILOT AND TRADITIONAL GROUPS difficult for both groups with the advantage favouring the total pilot sample. Item 17 of the test obtained the lowest percentage of correct response from both groups with 1 per cent of the pilot sample scoring it correctly and 2 per cent of the traditional group scoring it correctly. This item was:

17. If x, y, a, and b are measures of the angles indicated in the diagram, which of the following is a correct statement?

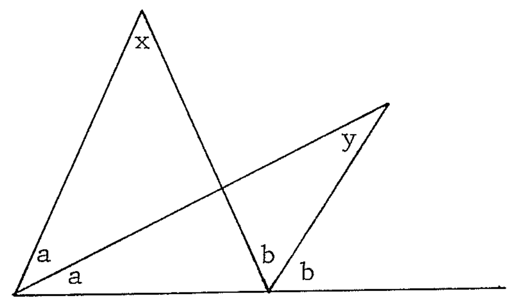
a) $x + y = 2b - a$

b) $x + 7 = a + b$

c) $x + y = 2a + b$

d) $x + y = b - a$

e) $x + y = 3b - 3a$



It required a response dependent upon knowledge of the exterior angle of a triangle theorem. However, the basic solution method demanded algebraic equation techniques which may have been too sophisticated for either group at the grade ten level.

On the other hand, both groups obtained their highest percentage of correct responses on Item 7 which was:

7. The figure below represents a three dimensional cube. What is the hypotenuse of right triangle GFC?

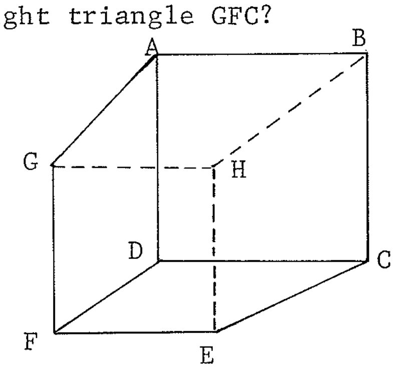
a) segment GF

b) segment GC

c) segment FC

d) segment DH

e) none of the above



Correct responses were scored by 73 per cent of each group on this item. It required a three dimensional visual response in terms of the definition of hypotenuse. On the basis of the amount of three dimensional geometry questions and applications included in the modern geometry text it was expected that the pilot group should score significantly higher than the traditional group on this item. The similar results of both groups on Item 7 may have been caused by the fact that the pilot course teachers were experimenting with the course for the first time, and thus did not utilize as effectively as possible the included textual material for space geometry, although it is difficult if not impossible to draw a truly firm conclusion on the basis of this single item.

However, the response pattern of both the pilot and traditional groups on Item 8, which also required a three dimensional visual response, was also similar. Item 8 was:

8. The figure below represents a three dimensional cube. Segment GC is an internal diagonal which is perpendicular to DX. What is the altitude of triangle GDC?

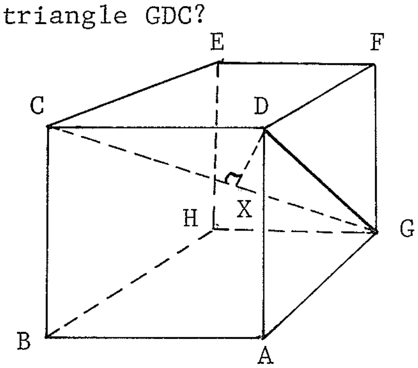
a) segment GD

b) segment DC

c) segment DX

d) segments GD, DC, and DX

e) none of the above



To answer this item successfully a student had to apply the definitions of altitude to a right triangle within a three dimensional experience. Both pilot and traditional groups scored very low on this item with the pilot group scoring 4 per cent correct responses and the traditional group scoring 6 per cent correct responses. More three dimensional situations occurred in Items 21, 22, and 23, which are congruence questions. Again, both groups responded in such a similar manner that it appeared neither the modern nor the traditional program had an advantage on items which required a three dimensional response.

Divergent Response Pattern

Table XI only summarizes differences which were statistically significant and which did occur in the pilot and traditional group responses to the items of the Geometrical Achievement Measure Experiment. Six Items (2, 11, 13, 20, 24 and 25) revealed significant difference at the .05 level, and only one of these, Item 20, was in favour of the traditional group. Three of the Items (2, 24 and 25) showed that the pilot group responded much better to questions relating to the structure of a miniature geometry than the traditional group. For example, Item 25 was:

25. Suppose that the following two assumptions describe a "new" geometry:
   I There exist exactly three distinct points.
   II For any two distinct points, there exists a unique line such that the two points are on the line.

   Which of the statements below do not "fit" the description?

   a) There exist three and only three distinct lines.

   b) Each point lies on one and only one distinct line.

   c) For each distinct line there exists a point not on the line.

   d) At least two points lie on the same distinct line.

   e) For any two distinct lines, there exists one and only one point that is on both lines.

On this item the pilot group scored 44.2 per cent correct responses, and the traditional group scored 33.6 per cent correct responses. The difference between the two was significant at the .05 level.

Item 24 was:

24. Consider making a geometry which differs from plane geometry. In this geometry, the terms <u>alpha</u>, <u>beta</u>, and <u>outersection</u> are undefined terms (as point, line and plane are undefined terms in plane geometry). The following are three possible statements of this geometry:
    I. Every beta contains at least two alphas.
    II. The outersection of two betas consists of the collection of all alphas on either of the two betas.
    III. Two betas outersect in but one alpha.

    Which one of the statements below is true?

    a) Statement II contradicts statement I.

    b) Statement III is not necessary; it can be deduced from statements I and II.

    c) Statements I and II say the same things in different ways.

    d) Statement III contradicts statements I and II.

    e) Statement III says the same thing as statement II.

On this item the pilot group scored 35.8 per cent correct responses, and the traditional group scored 23.6 per cent correct responses. The difference, in favour of the pilot group, was significant at the .05 level.

Item 2 also dealt with the characteristics of a miniature geometry. This item was:

2. What may be said about the lines of longest length of the new "geometry" of question 1? Line AB is shown.

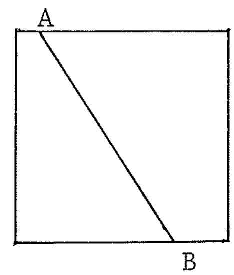
    a) A line is infinite in length

    b) There is one line longer than any others

c) There are four lines longer than any others

d) There are two lines longer than any others

e) No line or lines can be considered the longest in this system of geometry



It required a response dependent upon the application of the stated definition of a line to a model of a miniature geometry. In this geometry, it was essential for the respondent to conceive that the corner points were special elements which would maximize the length of any line. On this item the pilot group scored 50.5 per cent correct responses while the traditional group scored 33.6 per cent correct responses. The difference was significant at the .05 level.

Student response to Items 11 and 13 revealed that the pilot group responded better than the traditional group to questions related to the transfer of concepts learned in the geometrical approach the students were following to novel situations requiring an effective method of attack. Item 11 was:

11. Consider the angles 1, 2, 3, 4, 5 and 6 shown in the diagram. What is the sum of the measures of these angles?

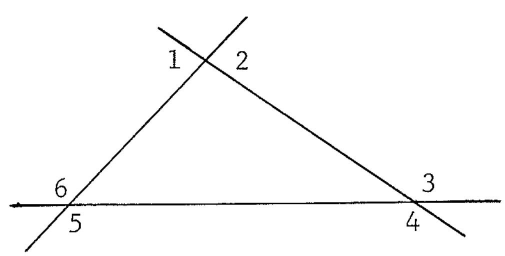
a) 720

b) 540

c) 900

d) 1080

e) 360



It required an organized algebraic method of solution using two facts: the sum of the angles of a triangle is equal to one hundred and eighty degrees, and two supplementary angles add up to one hundred and eighty degrees. Reference to Table XI indicates that the pilot group scored 38.9 per cent correct responses on Item 11, whereas the traditional group scored 25.5 per cent correct responses. The difference between the two was significant at the .05 level.

TABLE XI

SIGNIFICANCE OF DIFFERENCES BETWEEN PERCENTAGE CORRECT RESPONSES OF PILOT AND TRADITIONAL GROUPS ON GAME

Item	PILOT % Correct	TRADITIONAL % Correct	% Difference
2	50.5	33.6	16.9**
11	38.9	25.5	13.4**
13	47.4	32.7	14.7**
20	29.5	43.6	14.1**
24	35.8	23.6	12.2**
25	44.2	33.6	10.6**

** Indicates $p < .05$

Item 13 was:

13. Suppose that you were given an outline map with 6 countries, L, M, N, O, P and Q as shown in the diagram. Suppose that you were asked to paint the map with 4 different colours so that no two countries with a common

border were painted with the same colour. Which of the countries listed below could possibly have the same colour?

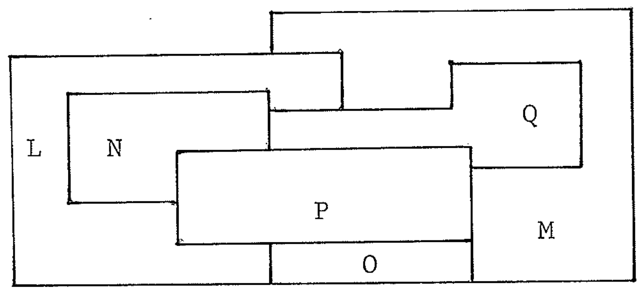
a) L and O

b) L and M

c) N and O

d) N and Q

e) O and Q



This item also required a logical method of analysis in order to select the correct response. The solution to it can be obtained by following three primary elimination steps in terms of the five responses shown. Then by applying a notation devised to distinguish the possible four colour combinations of the six countries it was possible for the respondent to eliminate all answers except "(e)", the correct solution.

An examination of Table XI reveals that the pilot group scored 47.4 per cent correct responses on Item 13, while the traditional group scored 32.7 per cent correct responses. The difference between the two was significant at the .05 level.

Finally, on Item 20 the traditional group scored significantly higher than the pilot group. This item was:

20. In triangle ABC, AB = AC, angle A = 46, and segment BD is perpendicular to AC. Find the measure of angle DBC.

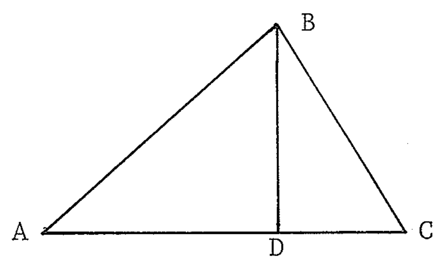
a) 67

b) 90

c) 46

d) 23

e) 44



Its solution required an application of the concepts concerning the base angles of an isosceles triangle, the sum of the angles of a triangle is equal to one hundred and eighty degrees, and the definition of a perpendicular. On this item the pilot group scored 29.5 per cent correct responses, and the traditional group scored 43.6 per cent correct responses. The difference was significant at the .05 level. It was suspected that the traditional group had more extensive experience with such questions, and this may have helped them to achieve significantly higher on this item.

Later, discriminant analysis of the test, Geometrical Achievement Measure Experiment disclosed that it was the fourth best discriminator between the two groups, pilot and traditional, in this study. In addition, a discriminant function analysis of data revealed that there was a significant difference at the .01 level in favour of one pilot class group in terms of achievement on GAME. Although no other statistically significant differences were found between any of the other groups, the analysis also disclosed a tendency for the pilot class groups to score higher on this test than the traditional class groups.

Cooperative Geometry

Figure 2 illustrates the percentage correct responses of the total pilot (N=98), the total traditional (N=114), and the total norms group to the forty items on Part I of Cooperative Geometry. Pilot group responses are represented by a broken line, traditional group responses by a solid line, and the norms group by a dotted line.

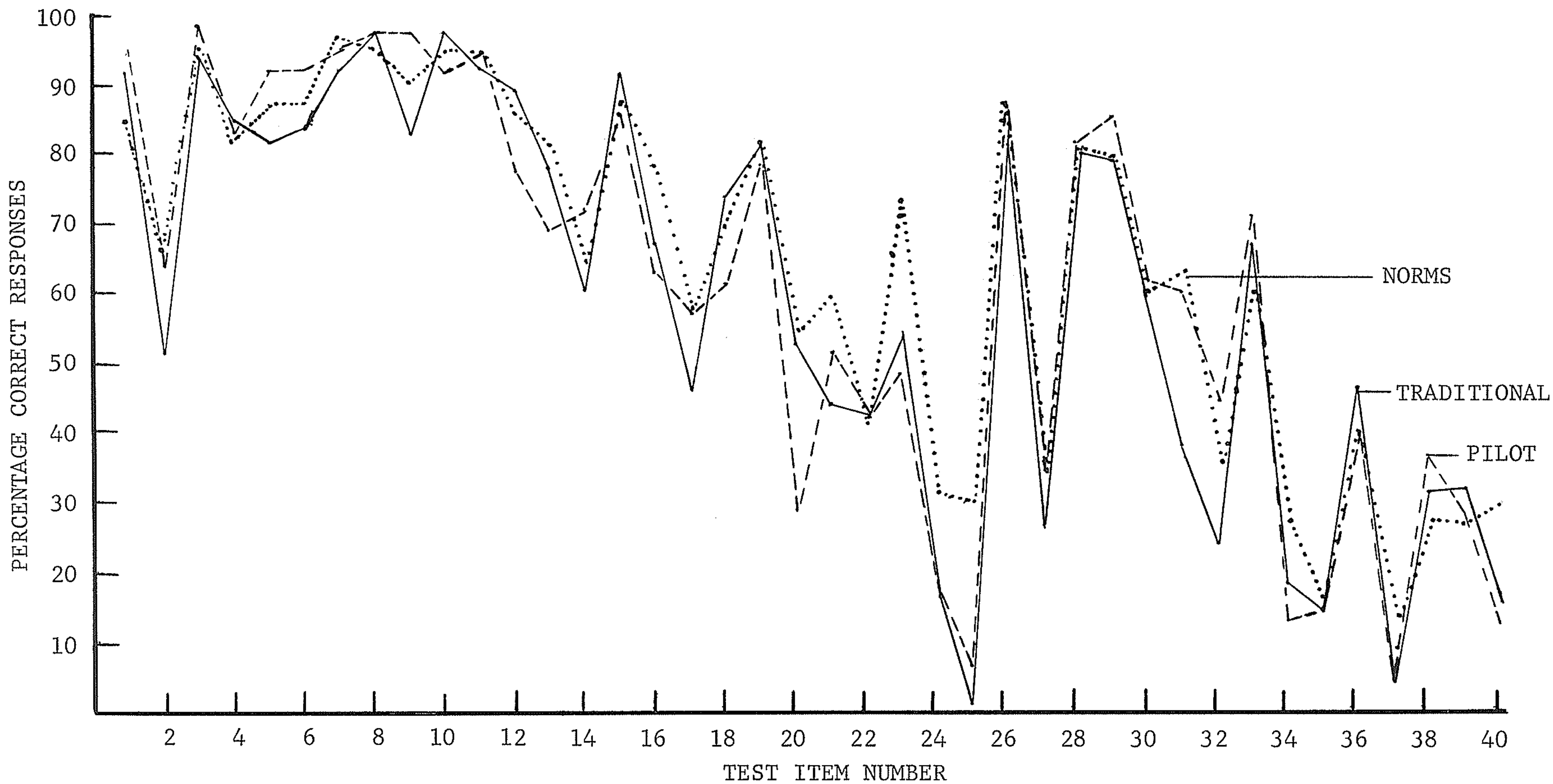


FIGURE 2

PERCENTAGE CORRECT RESPONSES ON COOP BY PILOT AND TRADITIONAL GROUPS

Although the graph showed a similarity of response by all groups for most of the items, there was a significant difference in response on Items 5, 9, 12, 14, 18, 20, 31 and 32 between the pilot and traditional groups.

Highest and Lowest Percentage Correct Responses

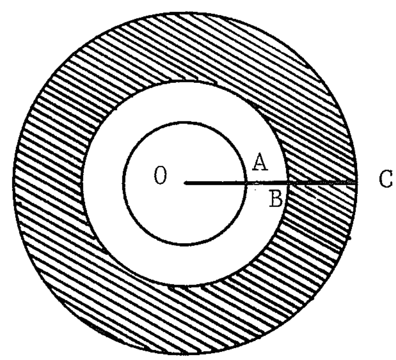
An examination of Figure 2 revealed that 80 per cent of both pilot and traditional groups scored correct responses on thirteen items. These items included from 3 to 11, 15, 26, 28, and 29. In addition, 50 per cent of both groups scored correct responses on twenty-five items. Consequently, compared to the results on the Geometrical Achievement Measure Experiment, the COOP test was easier for both groups. This was probably because of the fact that it had been structured as a standardized test for a broad population of secondary students. As a result, many of the beginning items were quite easy to give most students some encouragement to attempt the test. The content of the initial items was concerned with angle relations and measures in triangles and in parallel lines. Several items dealing with congruence properties were also included in the beginning items. Specifically the thirteen high-scoring items related to supplementary angles, base angles of an isosceles triangle, sum of the angles of a triangle, vertical angles, and the congruence properties for triangles. The success of both the pilot and traditional groups in the above concept areas indicated that these aspects of both programs were being achieved very well.

Students of both the pilot and traditional groups obtained their lowest scores on Items 25 and 37. Item 25 was:

25. A line is drawn from the origin through each of the following points. The steepest line goes through which of these points?
a) (2,7)
b) (4,7)
c) (3,3)
d) (6,2)
e) (10,1)

Item 37 was:

37.



In the figure above, AO = AB = BC = 1. What is the area of the shaded ring?
a) 9π
b) 5π
c) 4π
d) 3π
e) π

Both these items included geometrical concepts which were absent from the pilot and traditional programs in geometry. There was also little content in the items which would help the students in the solution. Because of the lack of appropriate experience with coordinate geometry and the area of circles it was not surprising that both groups did poorly on these items.

Divergent Response Pattern

Table XII summarizes the statistical significance of the difference between the percentage correct responses on eight items by the total pilot and total traditional groups on Cooperative, Geometry. Three of the items were significant at the .05 level, while five of them were significant at the .01 level. Items 5, 9, 14, 31 and 32 revealed significant differences in favour of the pilot groups, while items 12, 18 and 20 showed significant differences in favour of the traditional groups. An analysis of the content of the above eight items disclosed that concepts relating to

TABLE XII

SIGNIFICANCE OF DIFFERENCES BETWEEN PERCENTAGE CORRECT RESPONSES OF PILOT AND TRADITIONAL GROUPS ON COOP

Item	PILOT % Correct	TRADITIONAL % Correct	% Difference
5	91.6	81.9	9.7**
9	96.8	82.9	13.9**
12	77.9	88.6	10.7*
14	71.6	60.0	11.6**
18	61.1	74.3	13.2**
20	29.5	53.3	23.8**
31	60.0	37.1	22.9*
32	44.2	23.8	20.4*

* Indicates $p < .01$

** Indicates $p < .05$

properties of triangles, congruences, angle relations, parallel lines, properties of polygons, and the logic and nature of geometrical proof were involved. For example, Item 20 was:

> 20. The statement, "A figure is a triangle if and only if it is a closed broken line figure having three sides," is
>   f) a definition
>   g) a theorem
>   h) an axiom
>   j) a conclusion
>   k) a falsehood

On this item, 29.5 per cent of the pilot group scored correct responses, while 53.3 per cent of the traditional group scored correct responses. The significant differences between the responses of the two groups may have been caused by the different definitions of triangle used in the traditional text and the modern text. Response (f); the one designated as correct by the answer key; was likely selected by the majority of the traditional group because of its similarity to the non rigorous definition of the traditional text, _A First Course in Plane Geometry_. On the other hand, the rigorous definition of a triangle contained in _Geometry_ probably caused the pilot group to eliminate (f) as a correct response and select one of the remaining four responses.

Item 32 was:

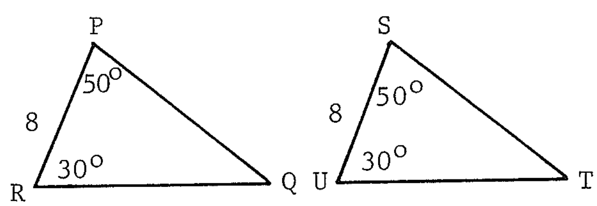
> 32. Which of the following statements most directly supports the assertion, "The hypotenuse of a right triangle is longer than either leg?"
>   f) Two distinct points determine one and only one straight line.
>   g) The distance from a point to a line is the length of the perpendicular from the point to the line.
>   h) The shortest line segment from a point to a line is the perpendicular from the point to the line.
>   j) The shortest distance between two points is a straight line.

k) There is one and only one perpendicular from a point to a line.

Correct responses were scored by 44.2 per cent of the pilot group and 23.8 per cent of the traditional group. The difference between the two percentages was significant at the .01 level. Both items were concerned with logic and the nature of proof, so that the results of the performance of the two groups were inconclusive in this concept area on the two items.

However, on two other related items of COOP, the pilot group outperformed the traditional group. These were Items 9 and 10. Item 9, which concerned the necessary and sufficient conditions for congruent triangles, was:

9.



If only the facts above are given, by what authority is △ PQR congruent to △ STU?
a) SAS
b) ASA
c) SSS
d) AAA
e) SSA

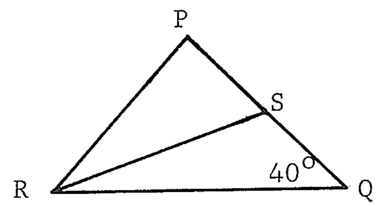
Item 31, which was also concerned with the conditions for congruence, was:

31. Which of the following should be proved equal in order to show that two parallelograms are congruent?
a) one pair of corresponding angles
b) one pair of corresponding sides
c) two pairs of adjacent sides and the included angles
d) a pair of diagonals
e) two pairs of diagonals

On the basis of student response to these two items, significant at the .01 level, it may be conjectured that the pilot group was able to respond better than the traditional group to certain items relating to congruence situations.

On the other hand, the traditional group responded much better than the pilot group to Item 12 which was:

12.



In △ PQR above, PR = PQ, angle Q = 40°, and RS bisects angle PRQ. ∠ SRQ = (?)
f) 10°
g) 20°
h) 25°
j) 40°
k) 50°

In addition, the traditional groups also outperformed the pilot group on Item 18 which was:

18. If two angles of a quadrilateral are supplementary, the other two angles are:
f) acute
g) obtuse
h) complementary
j) supplementary
k) equal and supplementary

The difference between the responses of both groups to these items as shown in Table XII, was significant at the .01 and .05 levels respectively. The solution to Item 12 required an application of information about the properties of an isosceles triangle which also included an angle bisector. The response of the traditional group on this item supported an earlier conjecture made in regard to Item 20 of the Geometrical Achievement Measure Experiment. It was speculated that the traditional group was able to handle questions dealing with angle measurement and isosceles triangles much better than the pilot group. This also appeared to be the case on Item 12, with 77 per cent of the pilot group and 88.6 per cent of the traditional group scoring correct responses.

To answer Item 18 successfully required knowledge about the sum of the angles of a quadrilateral and also about supplementary angles. On this item 74.3 per cent of the traditional group and 61.1 per cent of the pilot group scored correct responses as shown in Table XIII.

Finally, on Items 5 and 14 the pilot group outperformed the traditional group. Item 5 was concerned with the area of a rectangle, while Item 14 involved angle relations and parallel lines. Although the differences in responses to the items were significant at the .05 level, as indicated in Table XII, in favour of the pilot group, both groups achieved quite well on the two items. At the same time, there was no apparent explanation for the differences in response on the items.

In conclusion, further examination of Figure 2 revealed that the two groups, pilot and traditional, shared a similar response pattern on most items to the norms group. This similarity indicated that both groups were satisfactorily achieving those objectives which were measured by the COOP test when they were compared to the published norms. Therefore, the results of Cooperative, Geometry paralleled the results on Geometrical Achievement Measure Experiment in terms of the consistency of response for both the pilot and traditional groups. One interpretation may be that factors other than course content were operating to produce the similar results. However, on those items where the responses diverged significantly the above discussion has attempted to reveal several strengths and weaknesses of both programs.

IV. DISCRIMINANT ANALYSIS OF EXPERIMENTAL HYPOTHESIS III

To determine if significant differences existed between the achievement and attitude of the pilot and traditional groups four experimental tests were administered in May of 1967 to both groups. The results of these tests provided information about nine experimental variables. The data were processed by means of a stepwise discriminant analysis in the manner which has already been described to test the experimental hypothesis.

Specifically, the hypothesis tested was: No significant differences exist between the means of each pilot and traditional group on the following nine variables:

1. MAS-2 (X_1) - student attitude towards geometry
2. GAME (X_2) - student achievement on a criterion test
3. COOP (X_3) - student achievement on a standardized test
4. GAS-T (X_4) - student attitude towards the text
5. GAS-C (X_5) - student attitude towards course content
6. GAS-N (X_6) - student needs in geometry

7. GAS-It (X_7) - student interest in geometry
8. GAS-In (X_8) - student involvement in geometry
9. GAS-Tot (X_9) - total score of the five sub-tests

Class Comparison Groups

Table XIII summarizes the post experimental means of the nine variables for the five class to class comparison samples. Table XV and XVII were constructed to summarize the same information for the two matched samples and the total group samples, respectively. It is necessary to refer to the mean scores in Tables XIII, XV, and XVII when analyzing the results of the discriminant function analysis on the three sets of comparison groups.

Table XVI was prepared to reveal the results of the discriminant analysis for the five class to class comparison groups. In this Table, as in Tables XVI and XVIII, the data were arranged in the best hierarchal rank of discrimination. In addition, the significance of the contribution of each variable combination was shown in terms of an F value which was tested at the .01 and .05 levels.

An examination of Table XVI indicates that only one comparison class sample, group II, contained significant experimental differences. On the basis of the evidence the null hypothesis must be rejected for Group II and accepted for groups I, III, IV and V in terms of the nine variables. An inspection of the variable combinations for group II in Table XVI shows one combination group was significant beyond the .05 level, and eight variable combinations were significant beyond the .01 level. However, in terms of the five pre-experimental

TABLE XIII

EXPERIMENTAL MEANS OF NINE VARIABLES FOR FIVE CLASS TO CLASS COMPARISON SAMPLES

VARIABLE	I PILOT N=37	I TRAD. N=34	II PILOT N=37	II TRAD. N=33	III PILOT N=25	III TRAD. N=26	IV PILOT N=18	IV TRAD. N=21	V PILOT N=18	V TRAD. N=21
MAS-2 (X_1)	61.4	63.6	61.4	63.3	70.5	67.9	64.2	69.2	67.2	69.2
GAME (X_2)	8.8	9.3	8.8	6.9	10.0	10.3	11.1	10.8	11.3	10.8
COOP (X_3)	147.7	147.0	147.7	143.4	146.0	149.4	152.1	152.9	151.8	152.9
GAS-T (X_4)	41.3	40.6	41.3	42.8	47.2	45.5	42.2	45.0	43.4	45.0
GAS-C (X_5)	37.7	36.6	37.7	39.6	41.8	40.3	37.4	40.7	40.6	40.7
GAS-N (X_6)	38.5	38.0	38.5	38.4	41.3	40.7	36.5	40.1	40.3	40.1
GAS-It (X_7)	34.8	34.8	34.8	36.4	39.1	37.3	34.6	37.3	37.4	37.3
GAS-In (X_8)	36.6	37.0	36.6	37.3	40.0	40.1	36.9	40.1	40.2	40.1
GAS-Tot (X_9)	188.9	187.0	188.9	194.7	209.5	203.9	187.6	203.1	201.9	203.1

TABLE XIV

DISCRIMINANT ANALYSIS OF EXPERIMENTAL RESULTS - CLASSES

GROUP	VARIABLE COMBINATION	F	GROUP	VARIABLE COMBINATION	F	GROUP	VARIABLE COMBINATION	F
I	$X_5X_1X_2X_3X_9X_6X_4X_7X_8$	.521	II	$X_3X_9X_6X_2X_7X_1X_5X_8X_4$	2.676**	III	$X_3X_9X_8X_6X_1X_4X_2X_5X_7$	.717
	$X_5X_1X_2X_3X_9X_6X_4X_7$	.596		$X_3X_9X_6X_2X_7X_1X_5X_8$	3.060*		$X_3X_9X_8X_6X_1X_4X_2X_5$	.827
	$X_5X_1X_2X_3X_9X_6X_4$	.686		$X_3X_9X_6X_2X_7X_1X_5$	3.555*		$X_3X_9X_8X_6X_1X_4X_2$	.967
	$X_5X_1X_2X_3X_9X_6$	.782		$X_3X_9X_6X_2X_7X_1$	4.205*		$X_3X_9X_8X_6X_1X_4$	1.149
	$X_5X_1X_2X_3X_9$	.931		$X_3X_9X_6X_2X_7$	5.062*		$X_3X_9X_8X_6X_1$	1.401
	$X_5X_1X_2X_3$	1.151		$X_3X_9X_6X_2$	6.236*		$X_3X_9X_8X_6$	1.733
	$X_5X_1X_2$	1.198		$X_3X_9X_6$	7.321*		$X_3X_9X_8$	1.846
	X_5X_1	1.307		X_3X_9	7.221*		X_3X_9	2.168
	X_5	.335		X_3	8.796*		X_3	2.609
IV	$X_8X_2X_3X_1X_6X_5X_9X_7X_4$	.316	V	$X_4X_8X_3X_2X_1X_7X_5X_9X_6$	.307			
	$X_8X_2X_3X_1X_6X_5X_9X_7$	.368		$X_4X_8X_3X_2X_1X_7X_5X_9$	.357			
	$X_8X_2X_3X_1X_6X_5X_9$	.431		$X_4X_8X_3X_2X_1X_7X_5$	.420			
	$X_8X_2X_3X_1X_6X_5$	.495		$X_4X_8X_3X_2X_1X_7$	.498			
	$X_8X_2X_3X_1X_6$	.609		$X_4X_8X_3X_2X_1$	.558			
	$X_8X_2X_3X_1$	.716		$X_4X_8X_3X_2$	.680			
	$X_8X_2X_3$	.909		$X_4X_8X_3$	.711			
	X_8X_2	1.154		X_4X_8	.992			
	X_8	1.981		X_4	.769			

** Indicates $p < .05$

* Indicates $p < .01$

characteristics there were no significant differences between the pilot and traditional classes of group II when all of the variables were considered in their total interaction on each other. One interpretation is that a differential treatment has occurred between the two groups because of the particular geometry program being followed.

Reference to group II experimental mean scores in Table XIII and the discriminant analysis of data in Table XVI reveals several interesting statistically significant differences in characteristics between the pilot and traditional classes. First, the pilot group class attained highly significant better results on Cooperative, Geometry and the Geometrical Achievement Measure Experiment than the traditional class. The differences in achievement between the classes were significant at the .01 level and occurred on the two tests which appeared to be the best discriminators for the experimental data. However, the traditional class accepted the text in their course and the course content more favourably than the pilot class accepted their program. They also showed that they were more interested and involved in their geometry course than the pilot class was.

There were also certain changes which occurred in the characteristics of the classes in group I and group IV during the course of the experiment. At the beginning of the experiment, the pilot class of group I had generally better mathematical ability and a more positive attitude towards mathematics than the traditional class. However, the traditional class of group I was significantly more intelligent and had achieved better results in school mathematics than the pilot class. Also, an examination of Table III showed the grade nine mathematics mean score to be the best single discriminator between the pilot and traditional classes of group I. Consequently, the fact that there were no significant differences as far as GAME and COOP variables were concerned may be evidence that the pilot class has improved in performance over the traditional class during the experimental period. This conclusion is given added weight because of the highly significant correlation between 9MATH, GAME and COOP as noted in Table VIII, Table IX, and Table X. Therefore, although the traditional class in group I had superior prior knowledge of mathematics as measured by 9MATH, the pilot class in group I achieved just as well on GAME and COOP. Consequently, it may be inferred that the pilot geometry program did not do any harm to the students in terms of achievement, and may have been very effective for some of them.

With regard to the pilot and traditional classes of group I there was no significant difference between their acceptance of the program being followed as measured by the Math Attitude Scale and the Geometrical Attitude Scale. The pilot class did respond to mathematics more favourably than the traditional class at the beginning of the experiment, and this tendency, although not significant, was noted on the sub-tests of the Geometrical Attitude Scale at the end of the experiment. However, both classes still responded to their geometry texts and programs in a similar neutral manner. Therefore, neither program appeared to be encouraging a significantly favourable student response to geometry.

In group IV if the five pre-experimental variables are considered together, as noted in Table III, then there are no significant differences between the pilot and traditional classes. However, an examination of Table III revealed that a significant difference between the two classes did occur when the three variables of attitude, achievement and mathematical ability were considered as a combination of discriminators. In this combination it was the difference in attitude which contributed most to the variance between the two classes. If this variable is taken by itself without the effect of the others then it reveals a highly significant favourable attitude on the part of the traditional class of group IV at the beginning of the experiment.

Reference to Table XVI shows that there were no significant differences on the nine variables between the pilot and traditional classes of group IV. It was also apparent from an inspection of Table XIII that the pilot class mean score on the MAS variable has improved while the traditional class mean score has declined over the experimental period. There was a difference in favour of the traditional class on each of the sub-tests of the Geometrical Attitude Scale, but it was not significant. Consequently, both geometrical treatments have produced equivalent results for the pilot and traditional classes in group IV.

For groups III and V, Table XVI shows no significant differences in terms of attitude and achievement. Both pilot and traditional classes in each group responded to their texts and courses in the same mildly favourable way. Similarly, the mean achievement scores of the four classes on the Geometrical Achievement Measure Experiment and the Cooperative, Geometry tests were characteristic of an average reaction to the items of each. At the beginning of the experiment groups III and IV revealed no discernible differences between the five discriminator variables for each of the pilot and traditional classes. Thus, the classes of both groups were compatible at the start. In addition, the classes of group III were taught by the same highly qualified and experienced teacher. Therefore, it may be concluded that both geometry treatments produced equivalent results for the students in the group III and V comparison samples.

Matched Comparison Group

Tables XV and XVI summarize the experimental mean scores and the discriminant analysis of results, respectively, for the two matched samples selected from the total pilot and traditional groups. No significant differences in attitude or achievement were recorded between the two groups. However, if reference is made to the pre-experimental characteristics of the two, a definite trend is noticed in the data analysis of Table XV. When the five pre-experimental characteristics were considered together, there were no significant differences between the pilot and traditional matched samples. However, if the three variables, attitude toward mathematics, general intelligence and age are considered in combination, there was a significant difference at the .05 level, as noted in Table V. The pilot sample was a little younger and more intelligent than the traditional sample, while the latter had

TABLE XV

EXPERIMENTAL MEANS OF NINE VARIABLES FOR TWO MATCHED SAMPLES

VARIABLE		PILOT N=30	TRADITIONAL N=30
MAS-2	(X_1)	66.3	69.6
GAME	(X_2)	10.6	10.3
COOP	(X_3)	151.0	148.9
GAS-T	(X_4)	43.6	43.4
GAS-C	(X_5)	38.7	40.3
GAS-N	(X_6)	40.9	39.8
GAS-It	(X_7)	37.8	37.5
GAS-In	(X_8)	39.4	40.0
GAS-Tot	(X_9)	200.4	200.9

a significantly more favourable attitude towards mathematics in the beginning. At the end of the experimental period there were no significant differences between the two matched groups in terms of the nine variables. The analysis of data in Table XVI reveals equivalent results for both groups, so that it may be conjectured that the experimental geometry program has helped to improve the attitude of the matched pilot group towards geometry.

TABLE XVI

DISCRIMINANT ANALYSIS OF EXPERIMENTAL RESULTS

Group	Variable Combination	F
Matched Sample	$X_3X_5X_9X_8X_1X_7X_4X_2X_6$	1.216
	$X_3X_5X_9X_8X_1X_7X_4X_2$	1.394
	$X_3X_5X_9X_8X_1X_7X_4$	1.609
	$X_3X_5X_9X_8X_1X_7$	1.901
	$X_3X_5X_9X_8X_1$	2.155
	$X_3X_5X_9X_8$	2.419
	$X_3X_5X_9$	2.502
	X_3X_5	1.880
	X_3	1.458

Total Pilot-Traditional Comparison

Tables XVII and XVIII were constructed to show the experimental mean scores on nine variables and the discriminant analysis of results, respectively, for the total pilot and total traditional groups. No significant differences were found in terms of the analysis of data for the two groups at the beginning or the end of the experimental period. Therefore, it must be concluded that both geometrical treatments produce equally effective results when the total pilot and the

TABLE XVII

EXPERIMENTAL MEANS OF NINE VARIABLES FOR TOTAL PILOT AND TRADITIONAL GROUPS

VARIABLE		PILOT N=98	TRADITIONAL N=114
MAS-2	(X_1)	65.3	65.5
GAME	(X_2)	10.0	9.1
COOP	(X_3)	148.8	147.6
GAS-T	(X_4)	43.4	43.2
GAS-C	(X_5)	39.2	39.1
GAS-N	(X_6)	39.2	39.1
GAS-It	(X_7)	36.3	36.3
GAS-In	(X_8)	38.2	38.4
GAS-Tot	(X_9)	196.3	196.0

total traditional groups are considered together.

V. SUMMARY

In Chapter V, the data were analyzed to determine whether any significant differences in attitude and achievement existed between the pilot and traditional groups on five pre-experimental variables and nine experimental variables. Preliminary analysis of the pre-experimental variables resulted in the selection of three representative comparison groups, five class to class comparison groups, one

TABLE XVIII

DISCRIMINANT ANALYSIS OF EXPERIMENTAL RESULTS

Group	Variable Combination	F
Total Pilot and Traditional	$X_2X_9X_4X_7X_6X_5X_3X_1X_8$	.606
	$X_2X_9X_4X_7X_6X_5X_3X_1$	.686
	$X_2X_9X_4X_7X_6X_5X_3$	.747
	$X_2X_9X_4X_7X_6X_5$	.836
	$X_2X_9X_4X_7X_6$	.910
	$X_2X_9X_4X_7$	1.082
	$X_2X_9X_4$	1.350
	X_2X_9	1.968
	X_2	3.590

matched sample group, and one total pilot and total traditional comparison group.

The experimental data used to compare the groups were analyzed by means of a correlation matrix, an item analysis of the two achievement tests and a discriminant function analysis of the major experimental hypothesis. An examination of the correlation matrix revealed that the testing devices measured consistently, and that many significant correlations existed between the nine experimental variables for the total pilot-traditional sample. The item analysis of the Geometrical Achievement Measure Experiment and Cooperative, Geometry disclosed that both groups were achieving the cognitive objectives of their geometry programs very well. The results on both tests for each group were so similar that it was suspected factors other than course content were operable. However, an analysis of the percentage correct responses to several items on each achievement test revealed certain strengths and weaknesses of both the pilot and traditional groups.

In addition, the basic experimental hypothesis was analyzed by means of a stepwise discriminant function statistic to determine whether there were any significant differences in the mean scores of nine variables between the three pilot and traditional comparison groups. The results of this analysis showed that both geometry treatments produced equivalent results at the end of the experiment for all comparison groups except one. In this exception the pilot class achieved better than the traditional one, while the traditional group possessed a more favourable attitude towards geometry than the pilot. No other significant differences were discovered, although there was a general tendency for the pilot groups to improve in attitude towards geometry over the course of the experiment.

CHAPTER VI

SUMMARY AND CONCLUSIONS

I. SUMMARY

Purpose of the Study

The basic purpose of this investigation was to compare the effect of a pilot and a traditional geometry program on student achievement and attitude at the grade ten level. An examination of current publications revealed that there was a need for such investigations because there was a lack of experimental information about the effectiveness of new geometry programs. Consequently, it was a secondary purpose of this study to supply appropriate experimental evidence which would assist provincial curriculum planners in making a decision on a new geometry program.

Part of the scarcity of information about the effect of new geometry programs was caused by the unavailability of adequate testing materials. Consequently, it became a subsidiary purpose of the study to develop certain achievement and attitude instruments to test the experimental hypotheses.

Design of the Study

Pilot and traditional geometry classes were selected on a representative geographical basis from Manitoba schools who were participating in the Moise and Downs geometry program. This selection resulted in the formation of five class to class comparison groups with ninety-eight students in the total pilot group and one hundred and fourteen in the total traditional group. Available pre-experimental information on five variables, age, mental ability, mathematical achievement, mathematical attitude and mathematical ability, was used to establish the compatibility of the experimental and control groups. In addition, two other comparison groups, a matched sample, and the total pilot and total traditional groups, were used in the analysis of nine experimental variables.

Two preliminary tests, _Sequential Tests of Educational Progress-Form 2A_ and _Mathematics Attitude Scale - Form 1_ were administered to the class samples by the participating teachers. These tests provided data on pre-experimental achievement and attitude, and also familiarized the participants with the purpose and format of the experiment. During the course of the experiment from September, 1966 to May, 1967, the participating teachers attempted to provide an educational class environment which was compatible with the objectives of either the modern or the traditional geometry treatment contained in the prescribed texts. At the end of the experimental period, four tests were administered to the classes by the teachers. Two of these, _Mathematics Attitude Scale Form 2_ and _Geometrical Attitude Scale_ were designed to measure student acceptance of the geometry program in terms of the text, the course content, student interest, student involvement and student need. The other two tests, _Geometrical Achievement Measure Experiment_ and _Cooperative_, _Geometry_, were used to measure student achievement in the geometry program.

In total, nine experimental variables were obtained from the tests to compare the attitude and achievement of the three pilot and traditional comparison groups.

Hypotheses Tested

In this section, the experimental hypotheses are restated and the pertinent conclusions are summarized below each one.

Hypothesis I

No significant differences exist between the percentage of correct responses of students of the pilot group and students of the traditional group on each of the items of the two tests Geometrical Achievement Measure Experiment and Cooperative, Geometry.

Test of Hypothesis I

Hypothesis I was tested by the application of a proportionality test of significance using a z-statistic at the .01 and .05 levels of significance. This statistic was applied to each of the percentage differences between the percentage correct responses of both pilot and traditional groups to the items of the Geometrical Achievement Measure Experiment and Cooperative Geometry. On the basis of this test Hypothesis I was rejected for Items 2, 11, 13, 20, 24 and 25 of the Geometrical Achievement Measure Experiment at the .05 level and accepted for the remaining nineteen items. Application of the same test of significance to the items of Cooperative Geometry resulted in the acceptance of Hypothesis I for Items 5, 9, 14, 18 and 20 at the .05 level and Items 12, 31 and 32 at the .01 level. Hypothesis I was rejected for the remaining thirty-two items of COOP.

Discussion

Both the pilot and traditional groups experienced difficulty with the items of GAME, a device to test achievement level in terms of content which included the features of a miniature geometry, the structure of a geometry, the nature of a proof and several congruence concepts. The majority of items revealed that both groups were responding in a similar manner. Nevertheless, certain divergent responses did appear between the two, and these differences which were statistically significant, are summarized below. It was noted that the pilot group appeared to have a better understanding about items relating to the structure of a miniature geometry than the traditional group. There was also a significant difference between the pilot and traditional groups on items relating to the application of geometrical concepts to unique situations in favour of the pilot group. However, on one item which required an application of the concepts concerning the base angles of an isosceles triangle, the sum of the angles of a triangle, and the definition of perpendicular, the traditional group outperformed the pilot group. Although no other significant differences were revealed, it was noted that all pilot groups tended to score higher than the traditional groups on the items of GAME.

The pilot and traditional groups also showed a similarity of response on Cooperative, Geometry. On the total test, both groups achieved as well as each other, and much better than they had done on GAME. Compared to the norms group, the pilot and traditional groups produced equivalent results which indicated that both were satisfactorily achieving certain content goals of their geometry programs. Nevertheless, significant differences in response between the groups were noted on eight items of COOP. An examination of these revealed that the pilot group yielded significantly higher results on items referring to congruence situations. On items dealing with angle measurement and the properties of isosceles triangles the traditional groups produced significantly higher scores.

However, the responses of the pilot and traditional groups to all the items of GAME and COOP were remarkably consistent. Both responded to the items in so similar a manner that it appeared as if all students performed equally well.

Hypothesis II

No significant correlations are to be found among the total sample population, the total pilot sample, and the total traditional sample on the following eleven variables: (1) student attitude towards geometry; (2) student achievement on a criterion test; (3) student achievement on a standardized test; (4) student attitude towards the text; (5) student attitude towards course content; (6) student needs in a geometry; (7) student interest in geometry; (8) student involvement in geometry; (9) total score of five subtests; (10) mental ability; and (11) student achievement in grade nine mathematics.

Test of Hypothesis II

Hypothesis II was tested by the application of a null hypothesis about for each correlation coefficient, "r," on the eleven variables at the .01 and .05 levels. On the basis of this test Hypothesis II was rejected for the total sample population, the total pilot sample and the total traditional sample. The application of the test revealed 53 significant correlation coefficients for the total sample, 49 for the pilot group and 43 for the traditional group.

Discussion

An inspection of the three correlation matrices for the total pilot, total traditional, and total group revealed that almost all the intercorrelations were significant. The highly significant correlations among the sub-tests of the Geometrical Attitude Scale indicated that it measured consistently. The intercorrelations between the two achievement tests, COOP and GAME, and the two attitude tests, GAS and MAS-2 were significant. They were very low correlations so that different characteristics were probably being measured by each set of tests.

The correlation coefficients for the pilot and traditional groups were examined for differences. The intention was to determine whether or not an underlying trend in the developed characteristics of both groups could be discovered. There was a highly significant correlation between grade nine mathematics achievement and the other ten variables examined for each group. Consequently, it may be speculated that certain attitudes and skills were developed in the mathematics programs of prior grades which helped to lead to success in the grade ten geometry program. In addition, there appeared to be a difference between the pilot and traditional groups in terms of the total effect that prior achievement had on their attitude to geometry. It was conjectured that the increase in significant correlations for the pilot group revealed that previous success in grade nine mathematics helped a student accept the pilot geometry course better than it helped a student accept the traditional geometry program. Finally, the significant correlation coefficients between each of the ten variables and mental ability showed that the more intelligent students responded to text material and course content of both geometry programs more favourably than students of lower ability.

Hypothesis III

No significant mean differences exist between the students of the pilot groups and the students of the traditional groups on the following nine experimental variables: (1) student attitude towards geometry; (2) student achievement on a criterion test; (3) student achievement on a standardized test; (4) student attitude towards the text; (5) student attitude towards course content; (6) student needs in a geometry; (7) student interest in geometry; (8) student involvement in geometry; (9) total score of five sub-tests.

Test of Hypothesis III

Hypothesis III was tested by application of an F-test at the .01 and .05 levels of significance. The null hypothesis was accepted for class to class comparison groups I, III, IV and V, the matched sample and the total pilot-traditional sample. However, Hypothesis III was rejected for comparison group II. In this group a combination of nine variables was significant at the .05 level, and a combination of eight variables was significant at the .01 level.

Discussion

Only comparison group II in the study contained significant differences between the pilot and traditional classes. In this group, the pilot class achieved better results on COOP and GAME than the traditional class. The pilot class also perceived that their course content met their needs better than the traditional class. On the other hand, the traditional class in this group accepted the text and course content more favourably than the pilot class. They were also more interested and involved in their program than the pilot class.

Although no other comparison groups revealed any significant differences in the experimental characteristics, one group showed that certain characteristics had changed during the experiment. The pilot class in this group improved their achievement over the experimental period, and achieved as well as the traditional class on GAME and COOP. In addition, the matched sample comparison group revealed that the pilot sample had improved in attitude towards geometry over the experimental period, so that the students in the pilot sample accepted their text and course content as well as the traditional sample.

Because no other significant differences were discovered except the one already indicated, the null hypothesis must not be rejected for the other comparison groups. Consequently, it may be speculated that both geometry treatments, for the most part, have produced equally effective results.

II. CONCLUSIONS

The results of this study show no consistent pattern of superiority for either the pilot or traditional geometry program in terms of student achievement and attitude. In all comparison groups except one, equivalent results were obtained by both treatments. The exception revealed that the pilot class achieved better results than the traditional class on GAME and COOP, while the traditional group accepted their geometry program more readily. Other comparison groups showed that the pilot group's attitude scores increased in value over the experimental period, while the traditional group's attitude scores declined. This fluctuation in attitude scores was not statistically significant. In addition, the total pilot group achieved higher scores on GAME and COOP than the total traditional group. However, the differences between the two groups was not significant.

The results of this study support and verify the findings of similar studies which have concluded that students taking a modern geometry course do as well as students taking a traditional geometry program. In the judgement of Manitoba mathematics teachers and university professors the modern geometry program contained in Geometry by Moise and Downs was also more mathematically sound and more compatible with the present and future needs of university-oriented students. The fact that the modern course students learned significant geometrical concepts not usually treated in the traditional course, but which should be elements of a modern course, was an added advantage for them. Consequently, the modern geometry program contained in Geometry should be considered as an acceptable alternative to the traditional University Entrance geometry program in Manitoba.

III. SUGGESTIONS FOR FURTHER RESEARCH

This study indicated that there is need for extensive research into the formation of attitudes towards geometry and the effect of attitude on geometrical achievement. Both groups in the investigation displayed neutral to slightly positive attitudes towards geometry. Nowhere in the data was there any indication of a strong positive acceptance of geometry specifically, or mathematics in general. It may be speculated that both pilot and traditional groups would most likely have achieved a higher standard if they had a more positive attitude towards geometry. Consequently, it is suggested that to improve instruction in geometry methods must be discovered to increase student enjoyment and acceptance of it. Further investigation should be undertaken, then, to explore the relationship between attitude towards geometry and factors such as teacher personality, student personality, parental aspirations and encouragement, and student past experience with mathematics. Results from such research may indicate how student achievement in and attitude towards geometry can be improved.

In terms of achievement the pilot groups in this study achieved equal competence with the traditional groups in one year's time. However, further research will have to be undertaken in this area to determine whether both groups' competence can be retained or improved upon when the participating teachers have had more experience with the pilot geometry.

There is also a need for pilot programs to be evaluated in terms of experimental evidence so that it can be determined whether or not their stated objectives are satisfied by their programs in terms of student needs. Although it is not possible to use a single test or even a series of tests to measure the quality of a geometry program, it is very helpful to use these measures to support the subjective judgement required. Experimental evidence will help the curriculum planner in his decision-making process, but it will not make the decision for him. In the final analysis, a subjective judgement based on careful consideration of criteria which are relevant to the evaluation is necessary.

IV. RECOMMENDATIONS TO CURRICULUM PLANNERS

The need for experimental evidence on which to base that judgement is obvious in view of the rapidly changing nature of geometry programs. Other modern geometry programs must be considered and subjected to experimental scrutiny. To improve geometry instruction in our schools the experimentation with pilot programs must be carried on in a continuous manner and with it appropriate evaluation. This study has attempted to show that it is possible to design an experimental investigation which is capable of supplying some evidence to assist in such an evaluation.

In the future, a sense of perspective should be maintained when different geometry programs are being considered as suitable alternatives to current programs. There is a need for developing an appropriate and thorough evaluation program in this and other areas so that when changes are made in the program they are educationally sound. It is recommended to the Manitoba Mathematics Curriculum Revisions Committee that initially they should concentrate on at least three aspects of evaluation. First, educational goals appropriate to geometry must be identified by reference to some structure such as <u>Bloom's Taxonomy of Educational Objectives</u>.[1] Secondly,

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[1]Benjamin S. Bloom (ed.), <u>Taxonomy of Educational Objectives Handbook I: Cognitive Domain</u> (New York: David McKay, 1964).

evaluation procedures, such as attitude scales and achievement devices suited to specific outcomes must be examined for their appropriateness. Finally, ways to facilitate and sustain the movement towards improved evaluation on the part of teachers, the university and the Department of Education must be considered. Such a program requires concerted effort on the part of many educators to develop adequate evaluation procedures. This is necessary now and in the near future to provide effective guidelines for curriculum evaluation and revision so that developing geometry programs will be able to meet the needs of young people.

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APPENDIX

TABLE I

SIGNIFICANCE OF DIFFERENCES BETWEEN PERCENTAGE CORRECT RESPONSES OF PILOT AND TRADITIONAL GROUPS ON THE ITEMS OF GAME

Item	PILOT % Correct	TRADITIONAL % Correct	% Difference
1	52	44	8
2	51	34	17**
3	38	35	3
4	38	35	3
5	39	47	8
6	55	46	9
7	73	73	0
8	4	6	2
9	65	59	6
10	54	46	8
11	39	25	14**
12	32	27	5
13	47	33	14**
14	43	45	2
15	34	31	3
16	28	25	3
17	1	2	1
18	36	40	4
19	62	64	2
20	29	44	15**
21	38	44	6
22	28	35	7
23	33	32	1
24	36	24	12**
25	44	34	10**

** Indicates $p < .05$

TABLE II

SIGNIFICANCE OF DIFFERENCES BETWEEN PERCENTAGE CORRECT RESPONSES OF PILOT AND TRADITIONAL GROUPS ON THE ITEMS OF COOP

Item	PILOT % Correct	TRADITIONAL % Correct	% Difference
1	94	92	2
2	64	51	13
3	98	94	4
4	83	85	2
5	92	82	10**
6	92	84	8
7	95	92	3
8	97	97	0
9	97	83	14**
10	93	97	4
11	95	93	2
12	78	89	11*
13	69	77	8
14	72	60	12**
15	86	91	5
16	63	67	4
17	57	46	11
18	61	74	13**
19	79	81	2
20	29	53	24**
21	51	44	7
22	42	42	0
23	49	54	5
24	17	16	1
25	6	1	5
26	87	82	5
27	35	26	9
28	81	80	1
29	85	79	6
30	61	58	3
31	60	37	23*
32	44	24	20*
33	71	67	4
34	12	18	6
35	14	14	0
36	39	46	7
37	4	4	0
38	36	31	5
39	28	31	3
40	13	15	2

* Indicates $p < .01$

** Indicates $p < .05$

MATHEMATICS ATTITUDE SCALE

Name ______________________ Sex _______ Class _____________

School ____________________________________ Grade _____________

GENERAL INSTRUCTIONS

Please fill in the information above before reading the instructions.

This questionnaire contains a set of statements about the mathematics subjects you may be taking this year. You are to read carefully the directions for the following statements on page 2, and answer them according to the directions given. This questionnaire asks about your own attitudes and judgements. It is not a test, so it is very important that you answer the questions according to your own feelings and judgements. After you read each statement carefully, it is best to answer by giving your first impression or reaction and then go on to the next item. Remember, this questionnaire is concerned with your attitudes, and it is important that you answer according to your own feelings. Feel free to answer honestly and frankly, as your answers will be kept confidential and will not be used by anyone in your school.

NOW TURN THE PAGE AND READ THE INSTRUCTIONS FOR THE SET OF STATEMENTS WHICH FOLLOW.

Below are a number of statements pupils have made about mathematics. Indicate how much you agree or disagree with each of these by circling the letter which represents one of the following expressions.

Strongly Disagree (SD)
Disagree (D)
Neither Agree nor Disagree (N)
Agree (A)
Strongly Agree (SA)

1.	I am always under a terrific strain in a math class.	SD	D	N	A	SA
2.	I do not like mathematics, and it scares me to have to take it.	SD	D	N	A	SA
3.	Mathematics is very interesting to me, and I enjoy math courses.	SD	D	N	A	SA
4.	Mathematics is fascinating and fun.	SD	D	N	A	SA
5.	Mathematics makes me feel secure, and at the same time it is stimulating.	SD	D	N	A	SA
6.	My mind goes blank, and I am unable to think clearly when working math.	SD	D	N	A	SA
7..	I feel a sense of insecurity when attempting mathematics.	SD	D	N	A	SA
8.	Mathematics makes me feel uncomfortable, restless, irritable and impatient.	SD	D	N	A	SA
9.	The feeling that I have toward mathematics is a good feeling.	SD	D	N	A	SA
10.	Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way.	SD	D	N	A	SA
11.	Mathematics is something which I enjoy a great deal.	SD	D	N	A	SA
12.	When I hear the word math, I have a feeling of dislike.	SD	D	N	A	SA
13.	I approach math with a feeling of hesitation, resulting from a fear of not being able to do math.	SD	D	N	A	SA
14.	I really like mathematics.	SD	D	N	A	SA

15.	Mathematics is a course in school which I have always enjoyed studying.	SD	D	N	A	SA
16.	It makes me nervous to even think about having to do a math problem.	SD	D	N	A	SA
17.	I have never liked math, and it is my most dreaded subject.	SD	D	N	A	SA
18.	I am happier in a math class than in any other class.	SD	D	N	A	SA
19.	I feel at ease in mathematics, and I like it very much.	SD	D	N	A	SA
20.	I feel a definite positive reaction to mathematics; it is enjoyable.	SD	D	N	A	SA

MATHEMATICS ATTITUDE SCALE

(Geometry)

GENERAL DIRECTIONS:

Please fill in the personal information on the answer sheet before reading the directions.

This questionnaire contains a set of statements about the geometry you have been taking this year. You are to answer them according to the directions given. This questionnaire asks about your own attitudes and judgements. It is not a test, so it is very important that you answer the questions according to your own feelings and judgements. After you read each statement carefully, it is best to answer by giving your first impression or reaction and then go on to the next item. Remember this questionnaire is concerned with your attitudes, and it is important that you answer according to your own feelings. Feel free to answer honestly and frankly, as your answers will be kept confidential and will not be used by anyone in your school.

Below are a number of statements pupils have made about geometry. Indicate how much you agree or disagree with each of these statements. Blacken the space on the answer sheet which corresponds with the letter which represents one of the following expressions:

SD - Strongly Disagree
D - Disagree
N - Neither Agree nor Disagree
A - Agree
SA - Strongly Agree

1. I am always under a terrific strain in a geometry class.

2. I do not like geometry, and it scares me to have to take it.

3. Geometry is very interesting to me, and I enjoy geometry courses.

4. Geometry is fascinating and fun.

5. Geometry makes me feel secure, and at the same time it is stimulating.

6. My mind goes blank, and I am unable to think clearly when working at geometry.

7. I feel a sense of insecurity when attempting geometry.

8. Geometry makes me feel uncomfortable, restless, irritable and impatient.

9. The feeling that I have toward geometry is a good feeling.

10. Geometry makes me feel as though I'm lost in a jungle of letters and diagrams and can't find my way.

11. Geometry is something which I enjoy a great deal.

12. When I hear the word, "geometry," I have a feeling of dislike.

13. I approach geometry with a feeling of hesitation, resulting from a fear of not being able to do it.

14. I really like geometry.

15. Geometry is a course in school which I have enjoyed studying this year.

16. It makes me nervous to even think about having to do a geometry question.

17. I have not liked geometry this year, and it is my most dreaded subject.

18. I am happier in a geometry class than in any other class.

19. I feel at ease in geometry, and I like it very much.

20. I feel a definite positive reaction to geometry; it is enjoyable.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

GEOMETRICAL ATTITUDE SCALE

EXPERIMENTAL FORM A

GENERAL DIRECTIONS:

The following statements are related to your work in the geometry course you are taking this year. These statements are presented as opinions rather than facts. As opinions, they are neither right nor wrong. This is not a test but a device to determine how you feel about your course in geometry. In the items that follow you are asked to give your honest opinion by scoring the appropriate section. Score the item as it first impresses you.

Indicate what you believe rather than what you think you should believe. Blacken the space on the answer sheet which corresponds with the letter which represents one of the following expressions:

SD - Strongly Disagree
D - Disagree
N - Neither agree nor Disagree
A - Agree
SA - Strongly Agree

The above five expressions are levels of agreement and disagreement.

Remember, there are no right or wrong answers. The purpose of this test is to obtain your opinion. All statements refer to the geometry course you are taking this year. If you are really undecided as to your feelings on a statement, score it with an N.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

1. I can read my geometry text with no difficulty. Most of the vocabulary used is easy to understand and use.

2. The topics I have studied this year in my geometry will be of little use to me in the future.

3. I pay more attention in geometry classes than in other classes because I am interested in the topics we are studying.

4. I think that my geometry textbook helps to give adequate explanations, so I know how to solve the exercises which follow.

5. Most of the questions and problems in my geometry text are useful and beneficial. They help me understand the course.

6. I would rather have taken another geometry course this year than the course of study we had.

7. When I study a topic or section in my geometry course, I can usually see why it is important for me to study it.

8. I have done only a few of the geometry questions and problems on my own or with the help of fellow students or parents, etc. this past year. My geometry teacher has had to work out most of the solutions, before I really understood them.

9. I am not interested in taking a geometry course like this one next year, but would rather take almost any other subject.

10. I think we spent too much time in class on some topics in the course this year and rushed too quickly over other topics.

11. Too much time is devoted to the study of geometry and not enough time to the study of other subjects.

12. Because of my interest in geometry, I normally spend more time on my geometry homework than in other subjects.

13. Because of the difficulty of this geometry course, I find that I have to spend more time on geometry homework than in other subjects.

14. I wish those who develop courses and select texts would ask me what I thought I needed to learn in geometry. I think I know what I would like to study for my future work.

15. I think the course in geometry is too difficult for me.

16. In general, I think I am learning things from my geometry course that will be of value to me.

17. I think the geometrical work that I have done this year has begun to make me think as I imagine a mathematician thinks.

18. I am confused by special geometrical terms, such as hypothesis, conclusion, converse, etc., in my course.

19. I dislike doing the supplementary problems suggested in the text.

20. Most of the topics I am taking in my geometry course are those I would like to study more deeply at some future time.

21. I spend more time studying geometry than I do any other subject.

22. The geometry course covers too much material. We do not spend enough time on any one topic for me to understand it.

23. I would like to help present solutions to theorems and problems to my classmates on the topics we study in geometry.

24. I have to be forced to do my geometry homework.

25. I get little satisfaction from doing geometry.

26. My geometry text is very informative. Enough information is given on most topics, so that I can understand the main ideas.

27. The author(s) of my text has made the content interesting and easily understood.

28. The geometry course that I am taking is more difficult than the course in geometry that other students are taking.

29. I think the text is too compact and too congested, making for heavy reading.

30. I often notice in things around me applications of some of the geometrical concepts I have studied this year.

31. I think the exercises in the next serve no useful purpose and are merely busy work.

32. I frequently read other texts and reference books in order to understand the material in my geometry course.

33. I seldom know what I am supposed to do, after the assignment has been made, during work periods in a geometry class.

34. I would like to have my geometry course organized so that I could do more "discovery" work on my own.

35. I usually look forward to my geometry classes.

36. The knowledge I have gained in my geometry course gives me a real feeling of accomplishment.

37. I believe my vocabulary of geometrical terms has improved considerably this year.

38. I find the questions and problems in my geometry text too difficult.

39. There is not enough time given in class to really understand things firmly.

40. I like the way the material is presented in my geometry course so that, often, I can make guesses and then test them to see whether or not I'm right.

41. I can usually follow the steps in the example proofs in my text.

42. I think the use of pictures and diagrams in the text helped me to understand the ideas of geometry.

43. There are far too many unimportant definitions that we have to learn.

44. I dislike having to use geometrical language in such a precise and exact way.

45. I consider my geometry course dull and uninteresting.

46. I am stimulated by geometry because I can ask "off-beat" questions or disagree with accepted reasons for doing things.

47. Most of the time we are allowed to find the answers for ourselves in geometry class.

48. I really became interested in geometry this year because it introduced me to an "adventure" that was enticing and satisfying.

49. There was little chance to become bored with the routine in my class. Something different was always "popping - up".

50. This year I resisted any attempts to get my interest aroused in geometry, and refused to work hard so that I performed poorly.

51. I had particular difficulty this year with geometry, because I could not merely rely on my memory as much to get me through.

52. I see little reason for giving anyone a geometry course.

53. I still hate math. Geometry is really a bore.

54. In my previous years of math, as limited as they are, I have never had the opportunity to think out the answers for myself as I have had in geometry this year.

55. This year I felt as if I were really participating in geometrical "discoveries."

56. The geometry course aroused my interest so that I want to know more about geometry.

57. In geometry, this year, I encountered a systematic way of thinking which helped to make me aware of the power of my own mind.

58. I found little worth knowing in geometry.

59. My text requires too much detailed reasoning to prove the questions.

60. This year I learned that geometry is a living, growing product of man's mind, not a "bag of tricks" thought up centuries ago.

R.W.C.

GEOMETRICAL ACHIEVEMENT MEASURE EXPERIMENT
Form - 2

General Instructions:

This is a test of how well you understand and can apply what you have learned about mathematics. You are to read and try to answer each of these questions, even though some of the questions may be about things you have not yet studied.

Do not make any marks on the test booklet itself.

You are to indicate your answer only on the Answer Sheet.

Do your figuring only on the scratch paper you have been given.

First, print your name, room number, school and birthdate in the spaces provided on the Answer Sheet.

Now, read carefully the test procedures given below.

Test Procedures:

1. Read each item carefully and completely, think about the problem and do any figuring you want on the scratch paper.

2. Read the alternative answers carefully and select the one correct answer for each question.

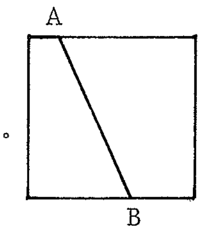
3. Indicate your answer on the Answer Sheet by blackening the space which corresponds with the answer you selected. Be sure that the row in which you mark the answer has the same number as the question. If you wish to change an answer, be sure to erase the previous answer completely.

4. Since your score depends upon the number of correct answers that you mark, you should try to answer each question. However, do not spend too much time on any one tiem. If you have difficulty determining the correct answer, make the most careful guess you can and go on to the next item. If you finish before time is called, you can go back and work on the items that were more difficult. Try to do as well as you can.

If you have any question about what you are to do, ask your teacher now.

Do not begin working until the teacher tells you to start.

1. Consider a "geometry" which is slightly different from the one you have studied. In this geometry all points in space must lie on or within a square, and not outside it. A line is <u>defined</u> as a segment which has its end points on opposite sides of a square. Line AB is shown below. What may be said about the lines of shortest length in this geometry?



   (a) All lines are infinitely long in length.

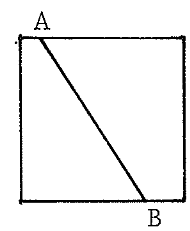
   (b) One line is shorter than any others.

   (c) There are an infinite number of shortest lines.

   (d) There are four lines shorter than any others.

   (e) All lines are of equal length in the square.

2. What may be said about the lines of longest length of the new "geometry" of question 1. Line AB is shown.



   (a) A line is infinite in length

   (b) There is one line longer than any others.

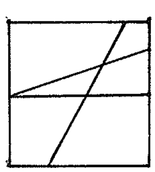
   (c) There are four lines longer than any others.

   (d) There are two lines longer than any others.

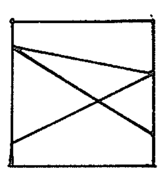
   (e) No line or lines can be considered the longest in this system of geometry.

3. Consider the "geometry" of questions 1 and 2. All points in the space must lie on or within a square, and not outside it. A line is defined as a segment which has its end points on opposite sides of a square. A triangle is defined as the union of three distinct lines, each of which intersects the other two. Which of the figures below <u>does</u> <u>not</u> represent a triangle?

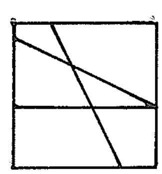
(a)



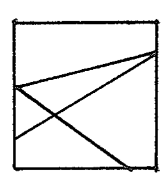
(b)



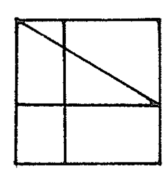
(c)



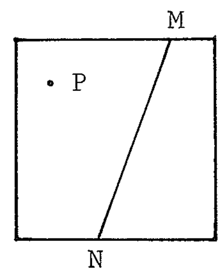
(d)



(e)



4. Consider the "geometry" of questions 1, 2 and 3. In the diagram below MN is a line as defined in this geometry. In this same geometry, two lines are defined as parallel if they do not intersect one another. How many lines can be drawn through point P which are parallel to line MN?



(a) An infinite number

(b) Only one

(c) Exactly two

(d) Exactly four

(e) None

5. Consider the following four statements:
1) If a triangle has two sides equal, then it is isosceles
2) If a triangle has two angles equal, then it is isosceles.
3) Statement 1 is a theorem if one can prove it using statement 2 as the definition of an isosceles triangle.
4) Statement 2 is a theorem if one can prove it using statement 1 as the definition of an isosceles triangle.

Which one of the assertions below is correct?

(a) Statements 1 and 2 say the same thing.

(b) Both statements 3 and 4 are true.

(c) Both statements 3 and 4 are false.

(d) Statement 3 is true, but statement 4 is false.

(e) Statement 4 is true, but statement 3 is false.

6. Triangles ABC and BCD contain a common side, segment BC. Segments AB and BC are equal sides of triangle ABC, and segments BC and BD are equal sides of triangle BCD. Which of the following statements correctly expresses the relationship between triangle ABC and triangle BCD?

(a) They are congruent by SAS.

(b) They are congruent by SSS.

(c) They are congruent by ASA.

(d) They are congruent by SSA.

(e) They are not necessarily congruent.

7. The figure below represents a three dimensional cube. What is the hypotenuse of right triangle GFC?

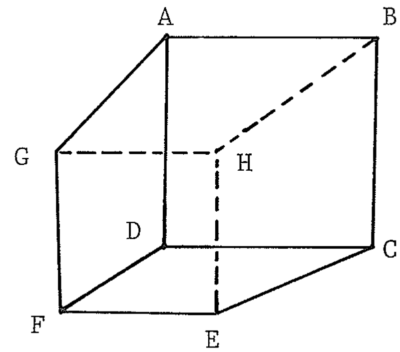
(a) segment GF

(b) segment GC

(c) segment FC

(d) segment DH

(e) None of the above



8. The figure below represents a three dimensional cube. Segment GC is an internal diagonal which is perpendicular to DX. What is the altitude of triangle GDC?

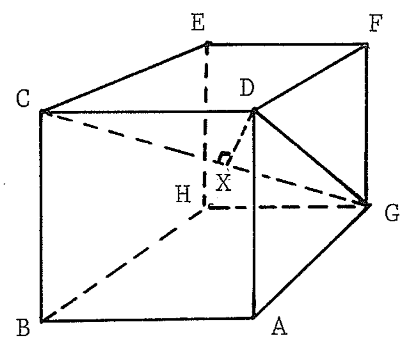
(a) segment GD

(b) segment DC

(c) segment DX

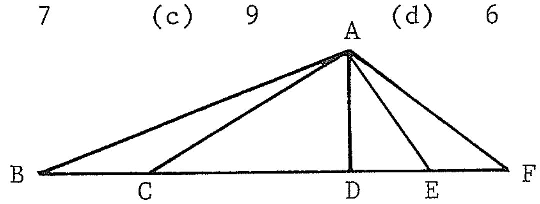
(d) segments GD, DC, and DX

(e) None of the above



9. How many triangles are there in the figure below?

(a) 8 (b) 7 (c) 9 (d) 6 (e) 10



10. The numbers below represent the measures of each of the three sides of different triangles. Which set of measures could not be used to construct a triangle?

(a) 1, 1, 1 (b) 3, 4, 5 (c) 2, 2, 1

(d) 1, 2, 1 (e) 4, 2, 3

11. Consider the angles 1, 2, 3, 4, 5 and 6 shown in the diagram. What is the sum of the measures of these angles?

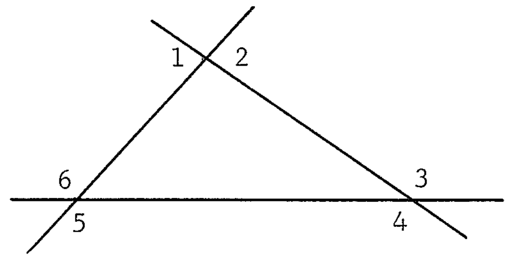
(a) 720

(b) 540

(c) 900

(d) 1080

(e) 360



12. In the figure segment AB is equal in length to segment AD. What additional information is <u>sufficient</u> to prove that angles 1 and 2 are equal in measure?

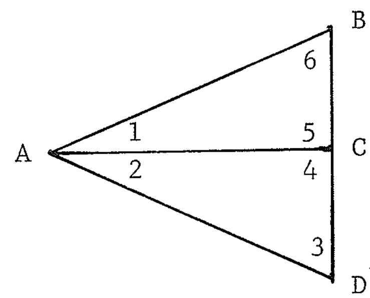
(a) Angle 5 is a right angle.

(b) Angles 1 and 6 are equal.

(c) Angles 4 and 5 are both right angles.

(d) Angles 3 and 6 are equal.

(e) No additional information is needed.



13. Suppose that you were given an outline map with 6 countries, L, M, N, O, P and Q as shown in the diagram. Suppose that you were asked to paint the map with 4 different colours so that no two countries with a common border were painted with the same colour. Which of the countries listed below could possibly have the same colour?

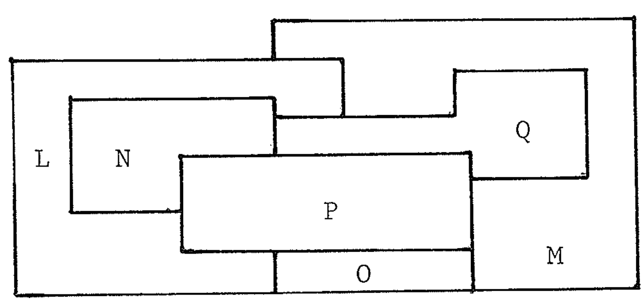
(a) L and O

(b) L and M

(c) N and O

(d) N and Q

(e) O and Q



14. Where on the earth's surface can a person go one mile North, then one mile East, then one mile South to return to his starting place? Assume that the earth is a sphere.

(a) At the equator

(b) Anywhere along the Arctic Circle

(c) At the North Pole

(d) At the South Pole

(e) Anywhere along the Tropic of Capricorn

15. If a triangle has 2 acute angles, then the third angle is not a right angle.

(a) The statement is true, but its converse is false.

(b) The statement is true, and the converse is true.

(c) The statement is false, but the converse is true.

(d) The statement is false, and the converse is false.

(e) None of the above are correct.

16. Given the following figure, which statement is correct?

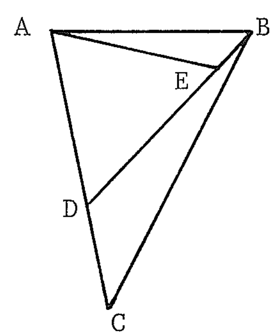
(a) Angle C $>$ Angle ADE

(b) Angle AEB $<$ Angle C

(c) Angle ADE $<$ Angle C

(d) Angle AED $>$ Angle ABD

(e) Angle ADE = Angle AED in measure



17. If x, y, a, and b are measures of the angles indicated in the diagram, which of the following is a correct statement?

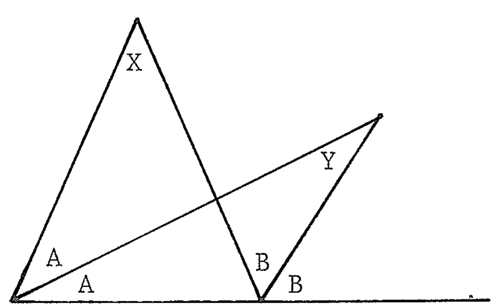
(a) $x + y = 2b - a$

(b) $x + y = a + b$

(c) $x + y = 2a + b$

(d) $x + y = b - a$

(e) $x + y = 3b - 3a$



18. In the figure if $\angle x - \angle y < \angle z$, which of the following statements is correct? (x, y, and z are measures of angles)

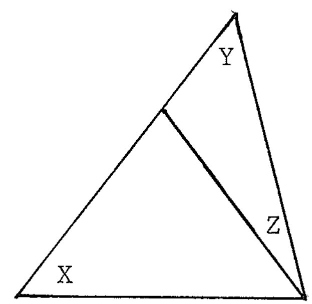
(a) $\angle y + \angle z > \angle x$

(b) $\angle y + \angle x < \angle x$

(c) $(-1)(\angle x - \angle y) < (-1)(\angle z)$

(d) $\angle y - \angle x < \angle z$

(e) $\angle x > \angle z + \angle y$



19. In the figure below, the measure of Angle B is 4 times the measure of its supplement Angle A. What is the measure of Angle B?

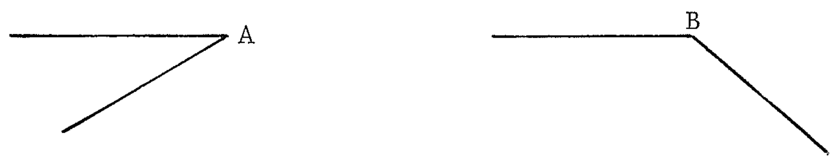
(a) 36

(b) 72

(c) 108

(d) 144

(e) 180



20. In triangle ABC, AB = AC, angle A = 46, and segment BD is perpendicular to AC. Find the measure of angle DBC.

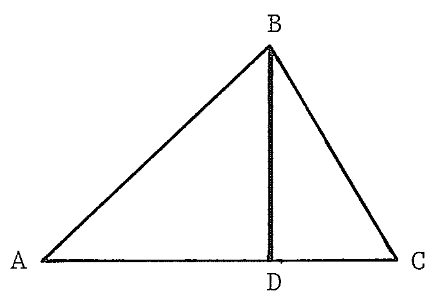
(a) 67

(b) 90

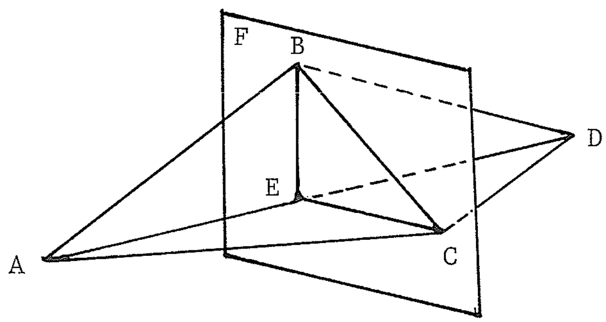
(c) 46

(d) 23

(e) 44



Questions 21, 22 and 23 refer to the three dimensional figure below. Points B, E, and C are coplanar, that is they lie in plane F. A and D lie on <u>opposite sides</u> of plane F, segment CE is perpendicular to segment AD, AE = ED and AB = BD.



21. Which of the following set of statements can be proved from the information given?

    I. Triangle EDB is congruent to Triangle EAB.

    II. Angle BEA is the same size as Angle BED.

    III. The measure of Angle BED = 90.

    (a) I (b) I and II (c) I, II and III

    (d) I and III (e) None of them

22. Which statements can be proved from the given information?

    I. The measure of Angle AEC = the measure of Angle DEC.

    II. The measure of Angle CAE = the measure of Angle DCE.

    III. Triangle AEC is congruent to Triangle DEC.

    (a) I and II (b) I and III (c) I, II and III

    (d) II and III (e) None of them

23. Which of the following set of statements can be proved from the information given?

    I. AC = DC.

    II. Triangle ABC is congruent to Triangle DBC.

    III. The measure of Angle BAC = the measure of Angle BDC.

    (a) I, II and III (b) I (c) I and II

    (d) I and III (e) None of them.

24. Consider making a geometry which differs from plane geometry. In this geometry, the terms alpha, beta, and outersection are defined terms. (as point, line and plane are undefined terms in plane geometry). The following are three possible statements of this geometry.

I. Every beta contains at least two alphas.

II. The outersection of two betas consists of the collection of all alphas on either of the two betas.

III. Two betas outersect in but one alpha.

Which one of the statements below is true?

(a) Statement II contradicts statement I.

(b) Statement III is not necessary; it can be deduced from statements I and II.

(c) Statements I and II say the same things in different ways.

(d) Statement III contradicts statements I and II.

(e) Statement III says the same thing as statement II.

25. Suppose that the following 2 assumptions describe a "new" geometry.

I. There exist exactly 3 distinct points.

II. For any 2 distinct points, there exists a unique line such that the two points are on the line.

Which of the statements below do not "fit" the description.

(a) There exist three and only three distinct lines.

(b) Each point lies on one and only one distinct line.

(c) For each distinct line there exists a point not on the line.

(d) At least two points lie on the same distinct line.

(e) For any two distinct lines, there exists one and only one point that is on both lines.

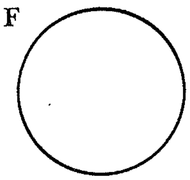
Geometry | 40 minutes

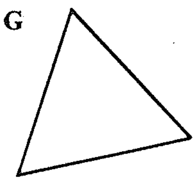
PART I

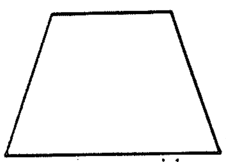
1 Which of the following are measures of a pair of supplementary angles?

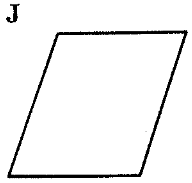
A 180°, 180°
B 40°, 50°
C 60°, 60°
D 60°, 120°
E 90°, 270°

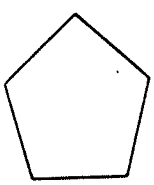
2 The area formula $A = bh$, where b is the base and h is the height, applies to which of the following figures?

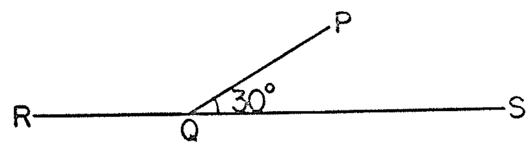
F

circle

G

triangle

H

trapezoid

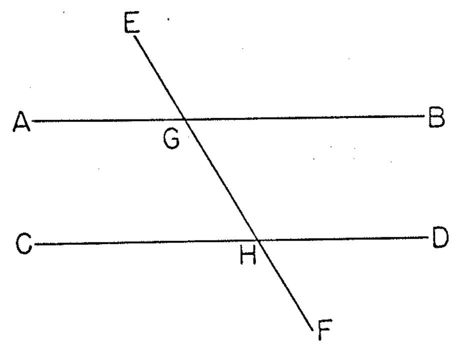
J

parallelogram

K

pentagon

3


In the figure above, if RS is a straight line and $\angle PQS = 30°$, then $\angle PQR =$ (?)

A 15°
B 30°
C 60°
D 90°
E 150°

4


In the figure above, lines AB and CD are crossed by line EF. $\angle EGB$ and $\angle EHD$ are known as

F vertical angles
G complementary angles
H alternate interior angles
J corresponding angles
K exterior angles

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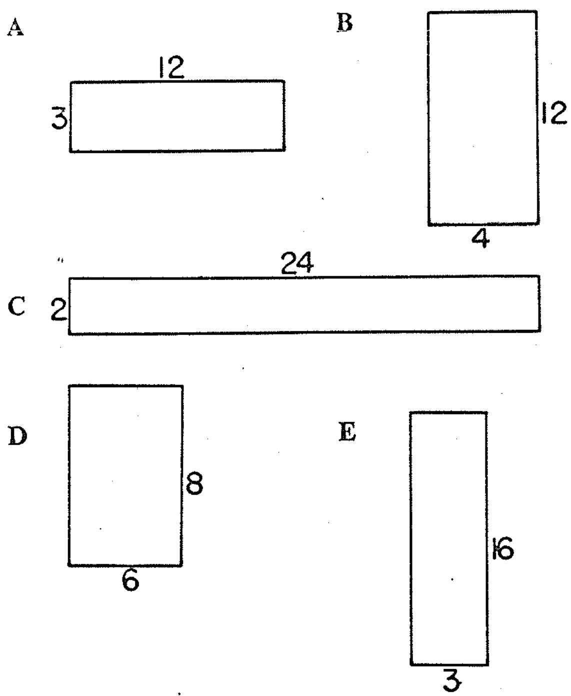
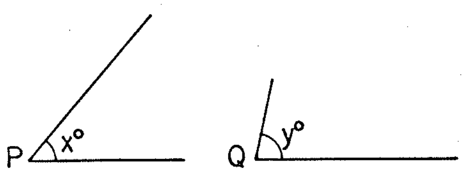
All of the following rectangles have equal areas except

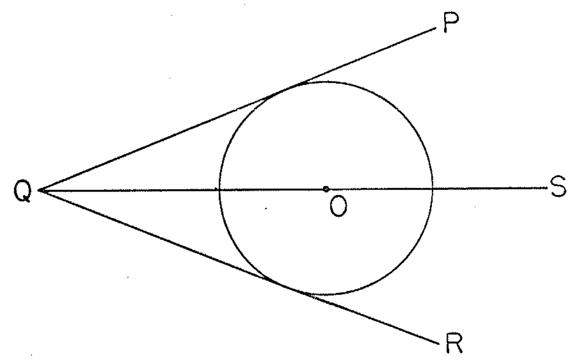




If, in the figure above, the measure of ∠Q is 3 times the measure of ∠P and if x = 24, then y = (?)

F 8
G 12
H 24
J 36
K 72

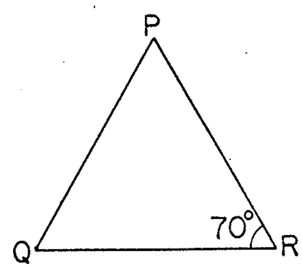
7



In the figure above, QP and QR are tangent to a circle with center at O. If ∠PQR = 70°, then ∠SQR = (?)

A 20°
B 30°
C 35°
D 45°
E 140°

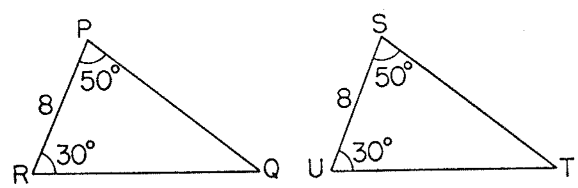
8



In △PQR above, PQ = PR, and ∠R = 70°. ∠Q = (?)

F 20°
G 35°
H 40°
J 70°
K 110°

9



If only the facts above are given, by what authority is △PQR congruent to △STU?

A SAS
B ASA
C SSS
D AAA
E SSA

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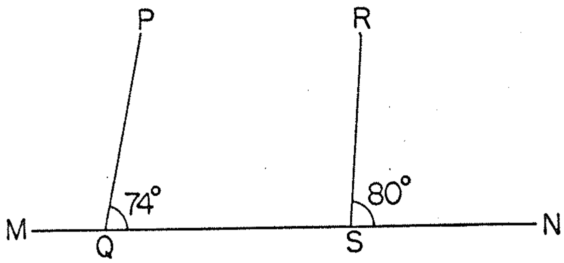
10



On straight line MN above, ∠RSN = 80° and ∠PQN = 74°. By which of the following amounts must angle PQN be increased in order that PQ will be parallel to RS?

F 6°
G 10°
H 16°
J 80°
K 106°

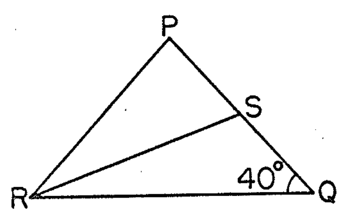
11 Following are the distances, in inches, of five points from the center of a circle:

Point A - 1.75
Point B - 2.01
Point C - 1.01
Point D - 2.00
Point E - 1.50

If the radius of the circle is 2 inches, which point lies outside the circle?

A Point A
B Point B
C Point C
D Point D
E Point E

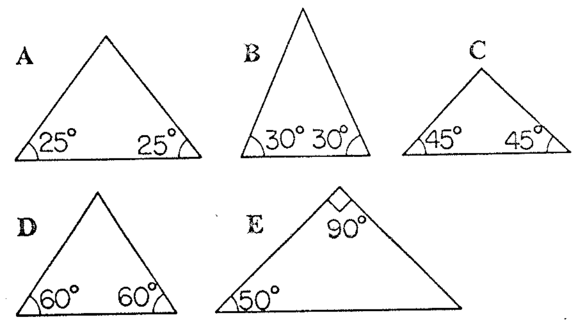
12



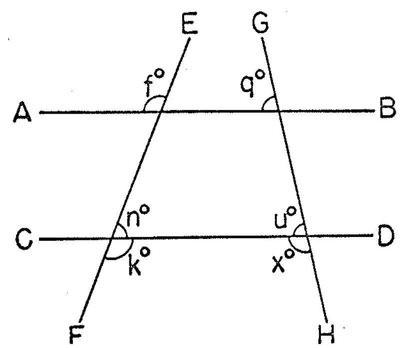
In △PQR above, PR = PQ, angle Q = 40°, and RS bisects angle PRQ. ∠SRQ = (?)

F 10°
G 20°
H 25°
J 40°
K 50°

13 Which of the following is an isosceles right triangle?



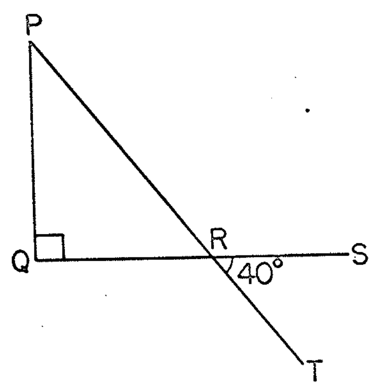
14



In the figure above, AB || CD, and EF and GH are straight lines. Which of the following is true?

F f = q
G f = u
H f = n
J f = x
K f = k

15



In the figure above, ∠Q = 90°, QS and PT are straight lines, and ∠SRT = 40°. ∠P = (?)

A 40°
B 50°
C 80°
D 90°
E 140°

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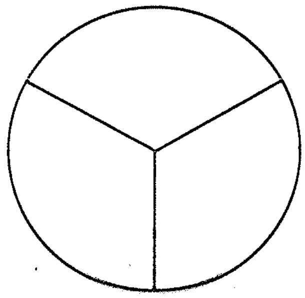
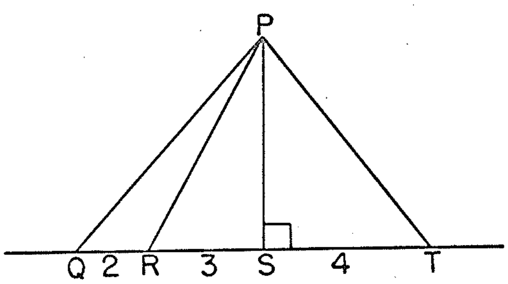
16



Shown above are three spokes from the center of a wheel. The sum of the lengths of these spokes is how many times the length of the diameter of the wheel?

F 1
G 1.5
H 2
J 2.5
K 3

17



In the figure above, PS $\perp$ QT, QR = 2, RS = 3, and ST = 4. Arrange PQ, PR, and PT in order of size, beginning with the shortest.

A PQ, PT, PR
B PT, PQ, PR
C PR, PT, PQ
D PR, PQ, PT
E PT, PR, PQ

18 If two angles of a quadrilateral are supplementary, the other two angles are

F acute
G obtuse
H complementary
J supplementary
K equal and supplementary

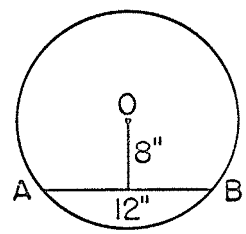
19 At 4 o'clock, the size of the angle formed by the minute hand and the hour hand of a clock is

A 30°
B 45°
C 60°
D 90°
E 120°

20 The statement, "A figure is a triangle if and only if it is a closed broken line figure having three sides," is

F a definition
G a theorem
H an axiom
J a conclusion
K a falsehood

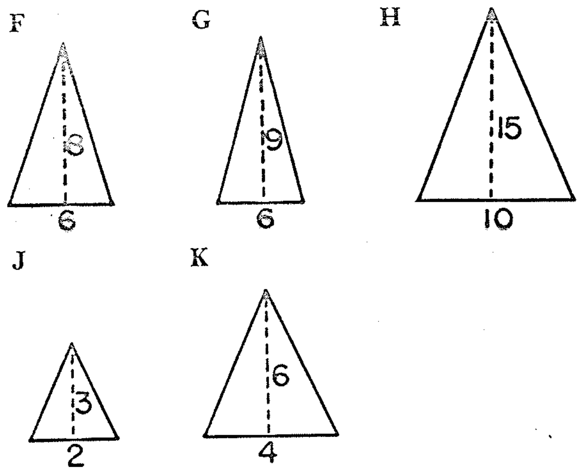
21



In the circle above, chord AB is 12 inches long and 8 inches from center O. What is the length, in inches, of the radius of the circle?

A $\sqrt{80}$
B 10
C $\sqrt{208}$
D 16
E 20

22 The length of the base and of the altitude is given in each of the following isosceles triangles. The vertex angles in all the triangles are equal **except** in

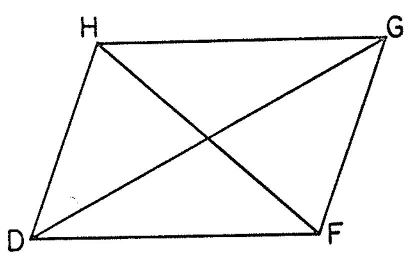


23 Which of the following statements concerning the diagonals of a square is (are) true?

I. The diagonals are equal.
II. The diagonals are perpendicular.
III. The diagonals bisect each other.

A II only
B I and II only
C I and III only
D II and III only
E I, II, and III

24



In the figure above, FGHD is a parallelogram. Which of the following statements is a condition which implies that FGHD is a rectangle?

F DF = GH
G ∠HDG = ∠DGF
H ∠HDF = ∠DHG
J ∠HDF and ∠DHG are supplementary.
K HF and DG are perpendicular bisectors of each other.

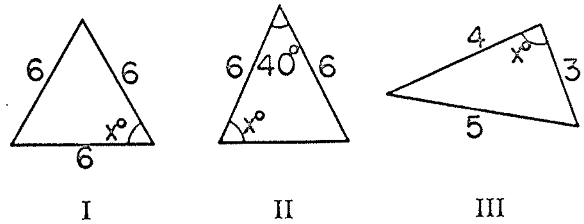
25 A line is drawn from the origin through each of the following points. The steepest line goes through which of these points?

A (2, 7)
B (4, 7)
C (3, 3)
D (6, 2)
E (10, 1)

26 What is the perimeter of a rectangle if the distance around three of its sides is 8?

F 6
G 8
H 9
J 12
K It cannot be determined from the information given.

27 For which of the following triangles can the value of x be determined?



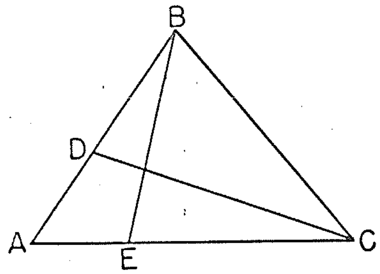
A I only
B II only
C III only
D I and II only
E I, II, and III

28 Major premise: Two lines in the same plane are parallel if and only if they have no point in common.

Minor premise: Line AB is parallel to line CD.

Conclusion: ?

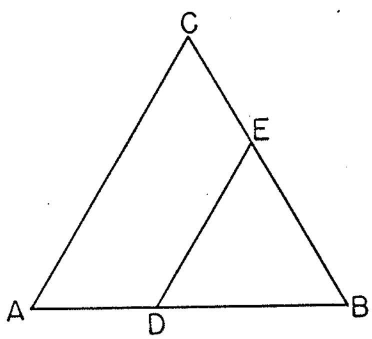
F AB and CD have no point in common.
G AB and CD have only one point in common.
H AB and CD have two points in common.
J If another line RQ crosses AB, then RQ cannot be parallel to CD.
K There are many lines in space parallel to CD.



In the figure above, ABC is a triangle and BD = CE. Triangles BCD and CBE are

A congruent by SSS
B. congruent by SAS
C congruent by ASA
D similar by SAS
E not necessarily congruent or similar

30



In the figure above, if CA = CB and ED = EB, then which of the following can be concluded?

F CA must be parallel to ED
G CA cannot be parallel to ED
H △ABC is equilateral
J △BDE is equilateral
K AD = CE

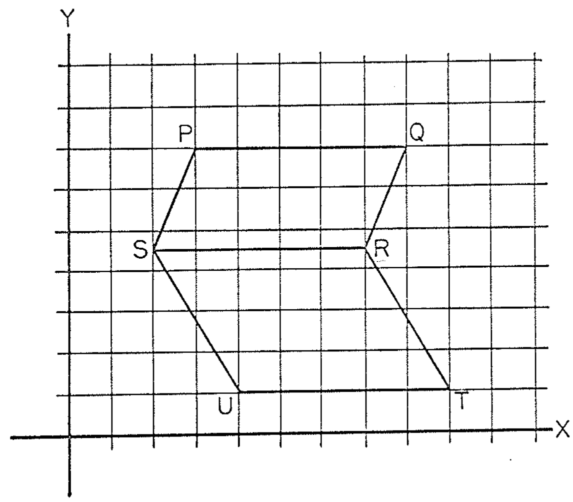
31 Which of the following should be proved equal in order to show that two parallelograms are congruent?

A One pair of corresponding angles
B One pair of corresponding sides
C Two pairs of adjacent sides and the included angles
D A pair of diagonals
E Two pairs of diagonals

32 Which of the following statements most directly supports the assertion, "The hypotenuse of a right triangle is longer than either leg"?

F Two distinct points determine one and only one straight line.
G The distance from a point to a line is the length of the perpendicular from the point to the line.
H The shortest line segment from a point to a line is the perpendicular from the point to the line.
J The shortest distance between two points is a straight line.
K There is one and only one perpendicular from a point to a line.

33



If each division of the grid in the figure above represents one foot and if SR is parallel to the X-axis, what is the area, in square feet, of PQRTUS?

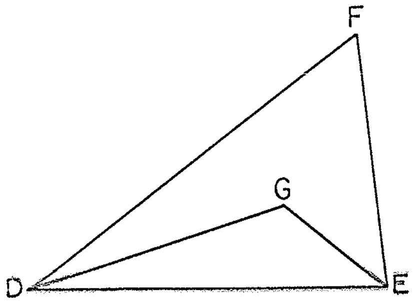
A 15
B 30
C 36
D 42
E 60

34 Two regular polygons having the same number of sides have areas whose ratio is 9 to 4. What is the ratio of their perimeters?

F 3 to 2
G 9 to 4
H 27 to 12
J 81 to 16
K It cannot be determined from the information given.

Go on to the next page.

5



In the figure above, the bisectors of angles EDF and FED intersect at G. If the number of degrees in ∠F is n, then the number of degrees in ∠G is

A $\frac{n}{2}$

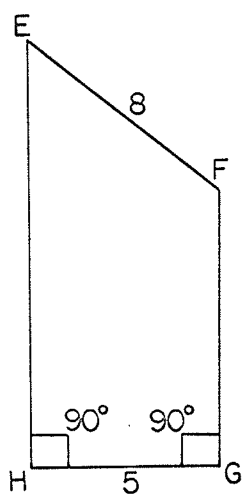
B $90 - \frac{n}{2}$

C $90 + n$

D $90 + \frac{n}{2}$

E $180 - \frac{n}{2}$

36



In the trapezoid above, the perimeter equals 37, EF = 8, and HG = 5. Find the area.

F $\frac{13}{2}$

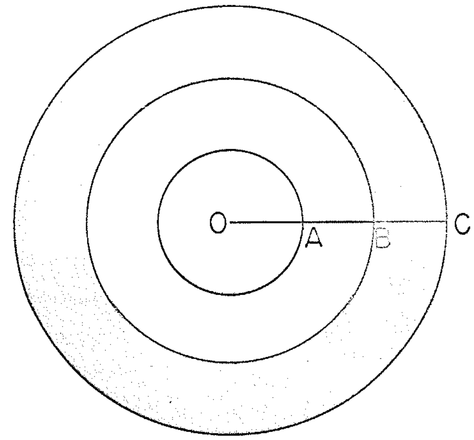
G 12

H 60

J 120

K 370

37



In the figure above, OA = AB = BC = 1. What is the area of the shaded ring?

A 9π

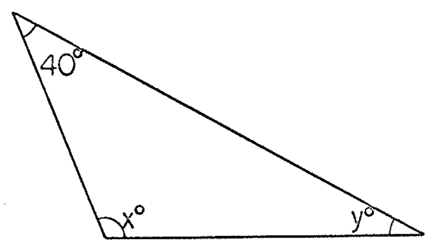
B 5π

C 4π

D 3π

E π

38



In the triangle above, if $60 \leqq y \leqq 100$, then

F $0 < x < 60$

G $40 \leqq x \leqq 80$

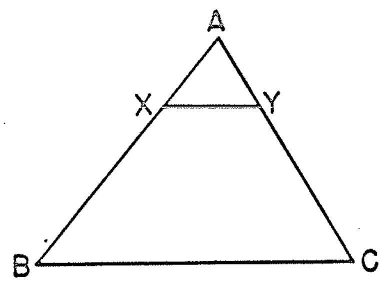
H $60 < x < 100$

J $60 \leqq x \leqq 100$

K $80 < x < 120$

Go on to the next page.

39



In the figure above, $AX = \frac{1}{3}AB$ and $AY = \frac{1}{3}AC$.

Which of the following statements are true?

I. $XY = \frac{1}{3}BC$

II. XY is parallel to BC

III. Area $\triangle AXY = \frac{1}{3}$ area $\triangle ABC$

IV. Area $\triangle AXY = \frac{1}{9}$ area $\triangle ABC$

A I and II only
B II and III only
C I and III only
D I, II, and III only
E I, II, and IV only

40 How many sides has a regular polygon if each of its interior angles has a measure of 170°?

F 10
G 34
H 36
J 144
K 170

STOP!

If you finish before time is called, look over your work on this part. Do not go on to Part II until you are told to.