THE UNIVERSITY OF MANITOBA

A COMPARISON OF METHODS OF TEACHING PROBLEM SOLVING IN MATHEMATICS AT THE HIGH SCHOOL LEVEL

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BY

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THESIS ABSTRACT

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The Problem

The purpose of the study was to compare two methods of teacher presentation of problems in high school mathematics. The differences in effectiveness of the methods were measured by means of pupil achievement during the year of teaching and by subsequent pupil performance in the next grade.

The methods compared were a traditional teacher demonstration method used with the control group of students and an analytic-discovery method used with the experimental group. Because of certain evidence noted during the "pilot" study, the writer also sought to determine if either method

proved beneficial for use with students of a particular ability level.

The Experimental Population

The experimental group consisted of a class of 35 Grade X students at St. John's High School. The writer taught this group mathematics during the time of the experiment. The control group was made up of two classes: a class of 35 students at Sisler High School, and a class of 40 students at Churchill High School.

The majority of the students of both the experimental and control groups had come through the Major Work Programme in elementary and junior high school in the Winnipeg School System. Equivalence between the groups was established on the basis of chronological age, intelligence and mathematical achievement in Grade IX. All the students remained in the respective high schools until completion of Grade XI.

The Experiment

The experimental teaching period extended over the school year 1961-62. The control group was taught problem solving by a traditional method in which the teacher demonstrated a particular type of problem and the students were given experience and practice, using problems from the textbook. The experimental group was introduced to problem solving by means of an analytic-discovery technique, in which the students analyzed many problems using a problem analysis form. Complete algebraic solutions were not required once a suitable algebraic equation had been established.

Experimental equivalence between the groups was retained by controlling teaching time, length of assignments and amount of out-of-class help.

In order to carry on a statistical comparison of the results of the teaching methods, a "pretest-posttest" design was employed. A criterion test in problem solving was administered prior to the teaching period and again upon its completion. Further comparisons were carried on using the results of the mathematics examinations given during the school year.

To determine if a significant difference existed between the achievement of the groups, the mean scores were compared at various stages in the investigation. The null hypothesis of the equality of the means was tested. F-tests were used to show homogeneity of the variances, and these were followed by t-tests in order to determine the significance of the differences in the mean scores.

Specific Findings

(1) There is a significant difference between the achievement in problem solving of students of high academic ability taught by the analytic-discovery method and that of students of similar ability taught by the traditional method as indicated by scores on a problem solving criterion test.

(2) There is no significant difference between the achievement in problem solving of students of average academic ability taught by the analytic-discovery method and that of students of similar ability taught by the traditional method as indicated by scores on a problem solving criterion test.

(3) There is a significant difference between students of high academic ability taught by the analytic-discovery method and similar students taught by the traditional method as indicated by examination marks in Grade X science and mathematics.

(4) Differences in mathematical achievement of the students of the experimental group and that of the students of the control group increased steadily in favour of the experimental group during the time of the final investigation.

(5) There is no significant difference between students of high academic ability taught in the tenth grade by the

analytic-discovery method and students of similar ability taught by the traditional method as indicated by the examination marks in Grade XI mathematics.

Conclusions

The results of the investigation indicate the following conclusions:

(1) In teaching problem solving to high school students of superior academic ability, the emphasis on the fundamental concepts required by the analytic-discovery method yielded better results than lack of such emphasis.

(2) The analytic-discovery method proved to be superior for students of high academic ability. For students of average ability the traditional method produced equivalent or superior results.

(3) Conducting of educational research, even on a limited scale, produced beneficial results in the classes involved in the experiment. Interest in mathematics was sparked among the class members in the experimental group and reports from the control group instructors indicated that a similar lift was noted.

(4) Improvement in performance produced by use of special teaching methods in one area may have beneficial results in a related area. Although no suggestion is made

in this report to indicate that the superior results in science were a direct result of the improved achievement in mathematics it is possible that such was the case.

(5) Because of the limited size of the samples used in the study, generalizations are hazardous. The difference in performance between the control and experimental group could be accounted for by chance factors, but the data indicate that considerable confidence may be placed in the assertion that real differences exist.

ACKNOWLEDGEMENTS

The experimental teaching and statistical comparisons reported in this thesis were carried on under the supervision of Dr. W.H. Lucow, Associate Professor at the University of Manitoba. Dr. Lucow assisted in developing the design of the experiment and in giving instruction in the operation of the Bendix G-15 computer.

It would have been impossible to arrange the experimental classes without the kind co-operation of Mr. J.E. Ridd and Mr. A.J. Ryckman of St. John's High School.

Mr. C. Martin of Churchill High School and Mr. C. Unruh of Sisler High School contributed greatly by providing the control classes. Their interest and co-operation added much to the investigation.

The report of the experiment was developed with the help of Dr. E. Boyce and Miss L.D. Baker. Without their invaluable guidance, encouragement, and constructive criticism, the compilation of this report would not have been possible.

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THE NATURE AND THE BACKGROUND OF THE INVESTIGATION

The Purpose of the Investigation

This study is concerned with the relative effectiveness of two methods of teacher presentation of problems in high school mathematics, as measured by pupil achievement during the year of teaching and by subsequent pupil performance in the next grade. These methods are, briefly, a traditional problem demonstration approach on one hand, and an analytic-discovery technique on the other. The achievement of the groups taught by each method was measured in Grade X by a criterion test, by an external examination in mathematics and by an external examination in a related field, in this instance, science. In Grade XI the achievement measures were limited to the external examination in mathematics.

Because of the variation in results between a "pilot" study which was carried on with students of average or low academic ability and the final study which involved students of superior ability, a secondary purpose developed within the time of the experiment. This purpose was to determine which method proved better for superior students and which for slow average students. Although no attempt was made to show whether the differences appearing were significant, it seemed important from the pedagogical standpoint to consider this secondary purpose.

The Setting of the Study

The experimental population.-- The one hundred and ten students selected for study were enrolled in Grade X classes at three Winnipeg high schools in 1961-62. Although no special name was used to describe these classes, they were regarded at each school as the academically superior Grade X group. The majority of the students had come through the Major Work programmes in elementary and junior high school and all continued in the same school until the completion of Grade XI.

The control group.-- For purposes of comparison, a control group consisting of seventy-five students in two classes was set up. This group consisted of the classes from Churchill High School and from Sisler High School.

Classes for the control group were selected on the basis of teachers who indicated a willingness to cooperate in the experiment. Subsequent statistical comparisons showed that the control group and the experimental group were very close in academic ability.

The experimental group.-- The group of students chosen for experimental treatment was a Grade X class at St. John's High School. The selection of students for this class was carried on by the high school Principal with assistance from the two senior guidance Counsellors. As with the classes which constituted the control group, no special selection was made other than high general ability throughout the junior high grades and at least eighty percent achievement in the Grade IX year end tests.

The matriculation course.-- The Matriculation or General Course[1] is designed for students desiring a programme which will lead to entrance to the Manitoba Teachers College or to admission to the University of Manitoba. Details for the year in question (1961-62) may be found in the Programme of Studies.[2] Mathematics is one of the core or compulsory subjects outlined in this programme. Thus all students enrolled in the General Course are required to take the same mathematics course.

The time allotted for the study of mathematics is twelve percent of the total school day or approximately three hours per week. This amount of time does not permit one

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[1] Programme of Studies for the Schools of Manitoba Senior High School 1961-62 (Department of Education, The Queen's Printer for Manitoba 1961), p. 7.

[2] Ibid.

period of mathematics per day as was the custom in many former programmes. In the schools involved in this study, mathematics classes were scheduled on each of five days of a six day cycle.[3] As the mathematics course at the tenth grade included algebra and geometry, the time of instruction was further divided into two relatively equal portions. In the classes involved in this study, algebra and geometry were taught in alternate class periods.

The students enrolled in this course were permitted to follow a similar programme through grades eleven and twelve. Departmental standing could be secured by means of writing examinations given by the Manitoba High School Examination Board, at the end of Grade XI (Junior Matriculation) or upon the completion of Grade XII (Senior Matriculation).

The schools.-- The schools selected for this study were all operated by the Winnipeg School Division #1. All three were of approximately the same size serving areas of the city having similar general populations. Each school was a combined junior-senior high administrative unit, but in each case some students in the Grade X classes came from smaller junior high schools.

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[3] Winnipeg School Division #1, Superintendent's Bulletin (September 1961).

The Major-Work programme in Winnipeg[4].-- During the past fifteen years increasing emphasis on the need to recognize the gifted members of society has resulted in the establishment of special classes to provide for the education and training of gifted children. In Winnipeg, an enrichment programme was started in 1954 consisting of three classes at the grade four level. At the present time these special classes, referred to as Major Work classes, number approximately sixty and include Grades IV - IX.

Although in the past some provision was made for children of superior ability to be gathered together, mainly through grouping by ability and choice of electives, a more carefully planned programme was required. The Major Work programme is designed to create an atmosphere in which creativity, curiosity, initiative and imagination may flourish.

Enrichment is the heart of the programme. Provision is made for the able child to probe more deeply , range more widely, and accomplish more than the average child in intellectual, social and cultural experiences. Activities are extended in art, language and literature, oral reporting, reviewing books, studying a foreign language and speaking before the class.

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[4]Naomi Louise Hersom, A Follow-up Study of the High School Performance of Students who were Members of the Inaugural Major Work Classes in Winnipeg, Unpublished Master's Thesis, University of Manitoba, 1962.

Children are considered for placement in these classes on the basis of preliminary screening by elementary school principals, results of Primary Mental Abilities test, and consultations with teachers. Final selection rests with the Director of Special Education. Parents' consent is required for special class placement. These gifted children represent five percent of the school population.

The teachers of Major Work classes are selected from staff members with qualifications such as: academic training, outstanding teaching ability, and personal qualities of flexibility and ability to deal with individual differences.

The augmented programme for Grades X and XI.-- The students involved in this experiment were all enrolled in classes taking a special course known as the Augmented Programme. The essential difference from the regular course was that the students in the Augmented Programme took an extra option in Grades X and XI.

Several years ago, a number of Winnipeg high schools began experimenting with methods of enriching the programmes for better students.[5] The plan to offer an extra option has proved popular with many students and staff members. When the first of the Major Work classes moved into high school, the need for enrichment became apparent and the number of schools offering the Augmented Programme increased.

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[5]John Douglas MacFarlane, A Follow-up Study to Determine the Effect of Enrichment Programs in a High School Upon Achievement at University, Unpublished Master's Thesis, University of Manitoba, 1961.

In order to accommodate the extra subject in the students' timetable, one less class period is scheduled in each of the major subjects in the six day cycle. This leaves sufficient time for the extra option to be included.

The Limitations of the Study

While the writer is aware that in conducting a scientific experiment, all pre-experimental factors are kept constant, a discrete variable is introduced and the reactions recorded; in the experiment reported here certain factors limited the nature of the findings. These were:-

1. The experimental class was given instruction in the solving of problems by an analytical method which was novel. It is known that new or novel approaches do produce enthusiasm which could produce biased results. The "Hawthorne" or "halo" effect would be noticeable during the first year of use, but whether the enthusiasm generated by the experimental method of instruction can be maintained cannot be determined. It should be noted, however, that although the experimental class showed enthusiasm because of the novelty of the approach, the control classes also became aware of their part in the experiment and probably developed competitive enthusiasm during the term of the experimental work.

2. Teacher competence and ability enter into the success of any teaching procedure and these cannot be measured accurately. In this experiment all teachers had approximately

the same number of years of teaching experience, and in all cases had been selected to teach the classes of highest academic ability in their school. These men had all taught the augmented programme in previous years. It should be noted, moreover, that the teachers of the control classes were using instructional techniques which are traditional and with which they were familiar, while the instructional techniques used in the experimental situation were new. This inequality would tend to modify the results.

3. The length of instructional time is a major factor in securing valid evidence in an experiment of this nature. Two aspects of time should be noted in this study. Although the actual periods of instruction were equated as closely as possible, no effort was made to record or regulate the time of outside-of-class help which some students would receive from their teacher. Since the classes were of similar academic backgrounds, it was thought that the need and requests for such assistance would be approximately equal. If certain students of either group were to receive considerable additional help, then the results would be affected.

The experiment recorded lasted one year. If the time of instruction had been extended to take in the total high school careers of these students, then perhaps the effectiveness of the methods could be evaluated more accurately.

4. The treatment of the data in this experiment was limited to an application of the t-test.[6]

[6]Henry E. Garrett, Statistics in Education and Psychology (New York: Longmans, Green and Co. 1953) p. 233.

CHAPTER II

A REVIEW OF THE LITERATURE

This chapter contains a review of some of the literature on modern methods of teaching students to solve mathematical problems. Evidence, in the form of results of previous studies and opinions of experts, will be presented in the following areas:

1. the importance of the study,
2. the place of problems in the history of mathematics,
3. methods of teaching problem solving.

The Importance of the Study

The ability to solve problems has long been recognized as a primary aim of teaching mathematics. A search of the literature on the teaching of mathematics reveals, however, a limited number of tested suggestions for the improvement of this vital function. In this study, two methods of instruction are used and the writer seeks to compare the relative merits of these approaches to the problem.

In the list of objectives of mathematics instruction as given in the Programme of Studies, the following objectives are stated:

1. to develop competence in problem solving,
2. to develop ability to apply mathematical information, concepts, principles and skills in various aspects of social life,
3. to help students develop a method of logical thinking.[1]

These three objectives are closely related but, separately or collectively, they point to the importance of teaching students sound methods of solving problems.

Thirty-five years ago, Morton stated:

> It is the chief purpose of arithmetic instruction to teach pupils to solve problems. The operations (addition, subtraction, multiplication and division) are taught in order that the pupils may be able to solve problems which they encounter in their school days and in their after-school experiences. Skill in the fundamental operations are not ends in themselves;- the end is the ability to solve problems.[2]

Since that time (1927) the theories on the teaching of mathematics have changed. The popularity of the Social Utility Theory[3] with its emphasis on problem situations, has waned and other theories have gained prominence; however, more recent writers have indicated a shifting emphasis on the part to be played by problems in mathematics instruction.

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[1] Programme of Studies for the Schools of Manitoba, Senior High Schools 1961-62, (Department of Education, The Queen's Printer for Manitoba, 1961), p. 46.

[2] Robert Lee Morton, Teaching Arithmetic in the Intermediate Grades, (New York: Silver Burdett Company, 1927), p. 27.

[3] Wilbur H. Dutton, Evaluating Pupils' Understanding of Arithmetic (Englewood Cliffs, N.Y: Prentice-Hall Inc., 1964), p. 6.

Dr. Evenson of the University of Alberta describes this change of emphasis in this way:

> Before a problem can be solved by mathematical methods it must be translated into mathematical symbols. Therefore, students must have a clear and complete understanding of all of the mathematical symbols they need to use. Finally they must be skilled in computational processes through understanding of mathematical techniques and adequate related practice.
>
> In this age of rapid change, careful attention must be given to sound problem solving techniques. We cannot hope to teach children how to solve all the problems they may encounter, so we must be sure to teach them the basic principles of mathematics and problem solving in order that students may apply them correctly in varying situations.[4]

While exhaustive bibliographies may be compiled for research which has been carried on in regard to algebra, the studies dealing specifically with the solution of problems are few in number.[5] Most studies are of the prognostic type dealing with general achievement (including computation) in algebra, and are not applicable to the more restricted study of methods of teaching problem solving.

McLeod and McIntyre[6] note that verbal problems are

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[4]A.B. Evenson, Modern Mathematics (Toronto: W.J.Gage & Co., 1963), p. 7.

[5]National Council of Teachers of Mathematics, "The Learning of Mathematics", Twenty-first Yearbook, (N.C.T.M. Washington, D.C., 1953), pp. 268-270.

[6]C. McLeod and C.I. McIntyre, "Problem Solving in Algebra", Mathematics Teacher XXX (1937), pp. 371-373.

the chief stumbling block in algebra. Although these authors present no experimental evidence, they suggest sources of difficulty. These difficulties point up the necessity for responsibility and ingenuity on the part of the student rather than any stereotyped method of approach by the teacher. The pupil must select, translate, relate-all in one question. Rote memory or the teaching of one general rule will not suffice. The formation of the essential equation will require instead, clear and perhaps original thinking. Again, the language of algebra may not be adequately understood. Thus the facts of the problem may not be properly translated in the establishing of the equation itself in correct algebraic form and content.

Hawkins[7] likewise stresses the need for precise reading, the ability to express in algebraic symbols the words and phrases of the problem, the ability to recognize the relationship involved, to form the necessary equations and to solve them. Hawkins demonstrates that while practice in translating English expressions into algebraic symbols may not raise test scores, yet training in problem analysis has a favourable influence on achievement in the solution of problems.

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[7] G.E. Hawkins, "Teaching Verbal Problems in Algebra", School Science and Mathematics, XXXII (1932), pp. 655-660.

Stright[8] found that training in the reading of algebra problems led to a significant increase in the correct solution of verbal problems.

Buckingham[9] noted similar relationships between reading ability and problem solving competence, but concluded that so many other factors were involved in problem solving that reading ability alone would not guarantee ability to solve problems.

The teachers of elementary grades in the schools of Manitoba are given some guidance in the methods of teaching students to solve problems. In the secondary grades, however, no guidance is given. The Programme of Studies[10] does not indicate any technique by which problems may be presented to students, and the texts[11] in use at present simply illustrate an example or two followed by exercises. This lack of guidance is most striking at the

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[8] I.L. Stright, "The Relation of Reading Comprehension and Efficient Methods of Study to Skill in Solving Algebraic Problems", Mathematics Teacher, XXXI (1938), pp. 368-372.

[9] Guy E. Buckingham, "Relationship Between Vocabulary and First Year Algebra", Mathematics Teacher, XXX (1937),

[10] Programme of Studies for the Schools of Manitoba, Senior High Schools 1961-62 (Department of Education, The Queen's Printer for Manitoba, 1961).

[11] Ibid, p. 47.

junior high level where most concepts are introduced through the medium of problems.

The lack of definite methods of teaching problems as evidenced in the outlines for teaching is common in many areas as well as in Manitoba. A review of the doctoral dissertations [12] of recent years shows relatively few papers dealing with the comparative efficiency of different procedures in teaching problems. In the decade 1950-1960, a list of sixty doctoral theses reveals only four or five topics bearing on this important part of mathematics education.

Many of the authors of method texts in the teaching of mathematics do stress the importance of good problem solving techniques. Teachers are given elaborate lists of factors which are causes of difficulty in problem solving and various methods of teaching are outlined. Dr. Hartung, writing in a mathematical yearbook, describes the importance of a good problem solving approach in another way:

> Genuine interest in mathematics probably depends basically upon the problem solving aspect of the subject. Problems once recognized or sensed leave an individual in a state of perplexity, uneasiness, or tension until they are solved. When a solution has been found, tension-reduction and satisfaction results. If mathematics is properly taught, it presents the student with an abundance of problems and it also provides him with certain general modes of thought and a supply of techniques which

[12] Edward G. Summers and James E. Stochl, "A Bibliography of Doctoral Dissertations Completed in Elementary and Secondary Mathematics 1950-60", School Science and Mathematics, LXI (June, 1961), pp. 431-439.

enable him to attack these problems successfully. With each successful solution he receives a divident of satisfaction-he feels good when he gets the answer. As a result he seeks more experiences of the same kind, and displays other desirable types of behaviour which are important in defining interest.

As the student grows in mathematical maturity, he obtains satisfaction also from contemplation of the power of his methods and the sharpness and beauty of his tools. The term "appreciation" is often used in this connection. The behaviour is relevant to interest, however, because it leads the student to seek more experience with mathematics, to discuss it favorably with other people, and to value it for what it does to his personal life.

A good many of the devices recommended for arousing interest seem to be based on the assumption that mathematics is uninteresting and hence learning must be encouraged by extraneous methods such as the mystery of a puzzle or the competition of a game.[13]

Butler and Wren[14] discuss the causes of difficulty in problem solving and then continue with a description of specific methods of teaching. These methods will be discussed in detail later in this chapter. As to the relative merits of the various approaches, their opinions are stated as follows:

> The truth is that the solution of verbal problems requires intellectual activity of a higher order and more complex nature than that

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[13]Maurice L. Hartung, "Motivation for Education in Mathematics", Twenty-first Yearbook (N.C.T.M. Washington, D.C., 1953), pp. 51-52.

[14]C.H. Butler and F.L. Wren, The Teaching of Secondary Mathematics (3rd. ed; New York: McGraw-Hill, 1960), pp. 302-305.

involved in sheer computation. It requires conceptual understanding, insight, originality, independence of thought and self-reliance. Some students are much more richly endowed with these characteristics than others are. The characteristics are not specific but complexes, and they can usually be developed only slowly, with painstaking, patient, and unremitting effort.

Efforts have been made and experiments carried out to try to determine the best method for teaching verbal problems. Each of the methods mentioned has given fair results. Teachers ought to become familiar with all the suggested methods and make use of those which seem to be best suited to the requirements of each immediate situation, adapting them as needed and perhaps devising other ways of helping students learn to clarify the elements and relations involved in verbal problems.[15]

Previous studies point in a general way to the difficulties faced by classroom teachers in developing in their students abilities to solve problems. Various methods are described and varied appraisals of the relative strengths of these methods are given. Further investigation in the field should prove valuable, particularly if such investigation should give evidence as to the suitability of methods for particular levels.

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[15] Ibid, pp. 303-305.

The Place of Problems in the History of Mathematics

The history of mathematics can be described as a study of a long series of mathematical problems and the solutions which man has devised for these problems. The majority of the problems with which the earlier mathematicians worked were practical in nature, while many of the later efforts involved theoretical problems. It is noted, however, that at all stages in history many mathematicians devised solutions to problems which did not have any utilitarian value at the time. At least one modern author (Eves) has attempted to teach the history of mathematics by means of a survey of the great problems with which mathematicians have wrestled.

> And rather than just tell a student that the ancient Greeks solved quadratic equations geometrically, let him solve some by the Greek method; in so doing he will not only thoroughly understand the Greek method, but he will achieve a deeper understanding of the Greek mathematical accomplishment.[16]

Archeologists working in the Near East have unearthed, since the middle of the nineteenth century, thousands of tablets bearing inscriptions of historical value. Even the

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[16]Howard Eves, An Introduction to the History of Mathematics (New York: Holt, Rinehart and Winston, 1961), p. 1.

oldest of the mathematical tablets show a high level of computational ability and make clear that certain number systems were common to large areas of the known world of these early times. The computation which appears most frequently in these records of stone is directed to solutions of problems involving agriculture and trade.

Tablets of later eras indicate that geometric problems became more common as man became interested in surveying land for permanent habitation and in construction of dwellings. It is interesting to note that many of the solutions offered for very simple surveying problems were outlined in geometric terms but were essentially algebra problems. Eves describes the problem solving of the Babylonians in this way:

> In summary, we conclude that the ancient Babylonians were indefatigable table makers, computers of high skill, and definitely stronger in algebra than in geometry. One is certainly struck by the depth and the diversity of the problems which they considered.[17]

Other evidence of the importance of problem solving in the development of mathematics is found in written records such as the early papyri. Two such documents, the Moscow and Rhind papyri, are simple lists of problems which proved challenging to the best minds of the time. The Moscow papyrus contains one hundred and ten problems of

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[17] Ibid, p. 33.

which twenty-six are geometric in nature.

The Rhind papyrus proved very difficult to decipher[18], but is known because of some of the unique types of problems it contains. One such problem contains a curious set of data involving the belongings of a wealthy man:

Houses	7
Cats	49
Mice	343
Heads of wheat	2401
Hekat measures	16807
	19607

Various authors throughout the ages have given explanations for this problem. One such explanation was included in an old English poem:

As I was going to St. Ives
I met a man with seven wives;
Every wife had seven sacks;
Every sack had seven cats;
Every cat had seven kits;
Kits, cats, sacks and wives,
How many were going to St. Ives?[19]

The Greek mathematicians contributed not only more problems and solutions but also the first attempts at demonstrating the solutions to other people. Thales, Pythagoras and Euclid represent the three major periods of Greek mathematical thought. Although the problems which were developed in these periods are too numerous to list here,

[18] Several more recent papyri such as the Dead Sea Scrolls proved impossible to decipher until modern scholars received assistance from computers. Time, (April 2, 1965).

[19] D.E. Smith, "On the Origin of Certain Typical Problems", American Mathematics Monthly, XXIV (February, 1917), pp. 64-71.

a few well-known examples will be given to illustrate the nature of the contributions of these scholars. Thales worked on the problem of determining the height of a pyramid using shadows and similar triangles. The Pythagoreans dealt with many practical and theoretical problems; the best known of them being the solution of the right angled triangle which led to the work on the length of the hypotenuse and thus to irrational numbers. Euclid's contribution is usually associated with the demonstrative proof in geometry problems; however, the scholars of his Alexandrian school did study many practical problems of the time.

During the Dark Ages, the only real evidence of mathematical thought seems to centre around several problems of a practical nature. Alcuin of York (ca. 775) is credited with compiling a collection of mathematical puzzles called "Problems for the Quickening of the Mind".[20] Samples from this collection will challenge most mathematical students even today. Fibonacci or Leonardo de Pisa, writing in the thirteenth century, used rules and number patterns which are unfamiliar to us. As puzzles and recreational exercises, the works of Fibonacci prove fascinating.[21]

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[20] Eves, op. cit., p. 227.

[21] "Leonardo de Pisa", Encyclopaedia Britannica, 18th ed., Vol. XIII, p. 939.

With the dawn of modern mathematics in the seventeenth century, the importance of great practical problems waned as mathematics became more theoretical in nature. Certain mathematicians did, however, develop and isolate certain problems, and write dissertations on these. Pascal is an example of such a scholar. His concern with taxes and interest rates led to suggestions for computation in these areas. Newton used age old problems and the historical solutions for these as a background for several of his studies. The construction necessary for the duplication of a cube is an example of Newton's interest in such a problem. It should also be noted that the mechanics problems of mathematical descriptions of rate of change, led Newton to begin his work in calculus. The feelings of his contemporaries with respect to this work in calculus were expressed by Pope in the lines:

> Nature and Nature's laws lay hid in night;
> God said, 'Let Newton be,' and all was light.[22]

During the latter part of the nineteenth century the interest of mathematicians in problems and problem solving divided into two separate streams. The traditional approach to great problems continued and modern mathematicians did a useful job in developing new theoretical and practical problems which will continue to

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[22]Eves, op. cit., p. 337.

be a challenge in the years to come. The methods used to solve these problems have, in the main, been individual techniques which best serve the scholar dealing with them. In contrast to this situation, the advent of large schools with great numbers of students being introduced to mathematics has forced the development of a teaching approach to problem solving. The differences in the background and mathematical knowledge of the teachers, along with the wide range of student ability in school classes, lead to difficulties in developing teaching methods which are universally acceptable. As the writer is concerned primarily with the teaching of problem solving, the remainder of this section will deal with this aspect.

The stress and importance of problem solving have varied greatly in the school programmes over the last half century. As indicated by Morton[23] in the nineteen twenties, problem solving constituted the major objective of high school mathematics programmes. The school curricula have changed since that time but still the ability to solve problems remains one of the chief outcomes of school programmes. With the advent of the computer, such ability on the part of the average graduate of high schools might appear unnecessary, but modern literature does not bear this out. A

[23] Morton, op. cit., p. 454.

recent article in Time makes the following point on problem solving:

> When someone wishes to solve a problem, he defines the problem in computer language-a combination of letters, numbers, punctuation marks and mathematical symbols. This is part of a computer science called programming, which is a way of telling machines what to do with information in order to achieve the desired result.[24]

But what kind of mathematics is necessary for a computer-centered society? Beberman states his view in this way:

> While our high school graduates are not expected to be ready for computer programming, the applications of mathematics through these machines are going to effect the lives of these students in numerous ways. Today's world demands that in addition to teaching the basic skills, we must teach our children to think and reason mathematically and to develop the ability to apply known concepts to new situations.[25]

This ability is the essence of problem solving.

Thus, not only has problem solving been the challenge which has produced mathematics, but it is also the skill which makes mathematics one of the major disciplines in an education system.

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[24]"Technology", Time (April 2, 1965), p. 69.

[25]Max Beberman, Time (January 16, 1965), p. 75.

The Teaching of Problem Solving

> A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.[26]

This statement made by Polya in his text on problem solving, focuses on the essential element necessary for success in this important part of mathematics, namely:- "if you can solve it yourself". A teacher cannot solve a problem for a student; the best a teacher can do is to present the methods by which the student can develop a solution.

The ability of a student to solve mathematical problems depends upon the depth of his understanding of mathematics. It also depends upon his interests, understanding and skills concerning problem solving methods. The teacher of mathematics must understand mathematics as well as the psychological processes of problem solving in order to be an effective teacher.

Before examining methods by which students may be taught to solve problems, one must have a clear

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[26]Geo. Polya, How to Solve It (New York: Doubleday Anchor Books, 1947), Preface p. v.

understanding of what constitutes a problem. The definition of a problem is itself a problem of the first order.

Georges states:

> An example denotes a situation wherein the mathematical operations for its solution are indicated. A problem, on the other hand, denotes a mathematical situation wherein the necessary operation(s) for its solution must be determined.[27]

Georges gives his working definition of a mathematical problem as:

> A stated mathematical relationship between two or more quantities in which the value of one or more than one quantity is to be determined in terms of the values of the other quantity.[28]

A further element now arises in the definition; a question only becomes a problem to an individual when he accepts the challenge to solve it. What is a problem to one is not necessarily a problem to all. A realization of this point is most important in teaching problem solving.

Thorndike[29] in his Psychology of Algebra states:

> A problem is a task in which the individual has to select his tools and processes. A problem necessarily involves novel elements, or a novel situation to which familiar elements must be applied in a novel way.

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[27] J.S. Georges, "Learning to Solve Problems Intelligently", School Science and Mathematics (December, 1956), pp. 701-707.

[28] Ibid.

[29] E.L. Thorndike, Psychology of Algebra, (New York: McMillan and Company, 1923), p. 190.

Later he analyzes the process of problem solving and identifies three necessary conditions which must exist in an individual in order to complete a solution. These conditions are:

1. The individual has a clearly defined goal of which he is consciously aware and whose attainment he desires.
2. Blocking of the path toward the goal occurs, and the individual's fixed patterns of behaviour or habitual responses are not sufficient for removing the block.
3. Deliberation takes place. The individual becomes aware of the problem, defines it more or less clearly, identifies various possible hypotheses (solutions), and tests these for feasibility.[30]

Dewey[31] describes a problem as "a forked road" situation in which the individual is forced into the elaboration of suitable hypothesis and testing these hypotheses.

Unfortunately, clear understanding of the meaning and nature of problems does not guarantee that the students will be able to solve problems. Every question that is proposed for solution is not a problem. As Cronbach[32] points out, "...it is not the posing of the question that makes the problem, but a person's accepting it as something he must try to solve." It really makes no

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[30] Ibid.

[31] John Dewey, How to Think (Boston: D.C. Heath and Co., 1933), p. 107.

[32] Lee J. Cronbach, The Meaning of Problems, Arithmetic, 1948, Supplementary Educational Monographs, Number 66. (Chicago: University of Chicago, 1948), p. 42.

difference whether the student poses the problem for himself or whether the teacher poses it for him. The crucial factor is the extent to which the student becomes involved with the problem. The teacher's task becomes one of setting the stage in such a manner that the students of the class will become involved. Henderson and Pingry describe the teacher's part in this operation as follows:

> It remains to be proven that a mathematics teacher who is highly enthusiastic, has an exciting personality, and is a student of psychology, cannot involve students in more problems than can a teacher who waits for the spirit to move the students.[33]

The wide range of mathematical problems and the even wider range of life's problems, make it impossible to follow any set plan for each and every one. The problems are individual and the approach to them must necessarily be different. Our aim must, therefore, be limited to a broad method of solution adaptable to meet the various problems encountered but at the same time having flexibility.

Four methods that appear in the literature on problem solving are:- the formal analysis method, the method of analogies, the individual method and the graphic method.

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[33]K.B. Henderson and R.E. Pingry, "Problem Solving in Mathematics", N.C.T.M. 21st Year Book (Washington: National Council of Teachers of Mathematics, 1953), p. 232.

Formal analyses method.-- Where there has been any conscious attempt to develop a method in teaching problem solving, the method of formal analyses or conventional method has been the most frequently used. In its simplest form, this method consists of teaching the pupil to ask and answer three questions if he is not sure that he knows how to solve the problem: (1) What am I to do? (2) What facts (numbers) are given? (3) What shall I do with the numbers? Sometimes these questions are elaborated and others are added. For example, Durell describes a method which includes six steps, as follows:

1. State what is given.
2. State what is to be found.
3. Make a list of the operations to be performed.
4. Estimate the answer.
5. Make the computations.
6. Check the answer.[34]

The method of formal analyses seems to have an advantage in that it requires the pupil to read the problem critically and to think specifically of what is required and of what facts he is given. The difficulty with the method lies in the fact that there is a tremendous gap between seeing what is given and what is to be found, and deciding what steps should be taken to solve problems.

Interpretation of the problem by this method depends upon the students' understanding of the words and symbols

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[34]Fletcher Durell, Mathematical Adventures, (Boston: Bruce Humphries, Inc., 1938), pp. 47-59.

used in the statement of the problem. In addition he must be able to identify what is given and what is required. Duncker[35] points out the relationship of the given and the solution: "a solution always arises out of the demands made by what is required and what is given." If students are made aware of this notion early in their experiences with the formal analyses method, the gap between the given and required in the solution of a problem may be bridged satisfactorily.

Analogies.-- Another method which has been widely used by teachers is the method of analogies. It consists of giving the pupil an easy problem which is similar to the difficult problem. It is presumed that once the pupil has solved the easy problem he will see the relationship of the difficult problem to the easy one. An analogous situation often helps to clarify the situation at hand since it implies a very similar idea. It is generally considered that this method does not stimulate thinking, nor does it provide guidance in overcoming difficulties in logical thinking.

Because of the extensive use of the method of analogies in everyday life, modern psychologists have become interested in more extensive classroom use of this method. The particular concept which has been found useful in considering the thought processes of everyday life is that

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[35]Karl Duncker, On Problem Solving, Psychological Monograph Number 58, 1945, pp. 1-111.

of "set". A person is said to have a "set" toward a problem when, because of past experience, he is predisposed to a certain hypothesis or plan of action. If a student develops a "set" not in agreement with the given material, in attempting to use the method of analogies, a solution might be almost impossible. Luchins[36] used the name "Einstellung" for a "set" which "immediately predisposes an organism to one type of conscious act". Under the impact of an Einstellung, a person does not look at a problem on its own merits, but tries mechanically to employ a previously learned method. It is the belief of the writer that many mathematics teachers feel that multi "Einstellung" frequent their classrooms.

Graphic method.-- The use of diagrams, charts, and other visual aids in the teaching of problems is referred to as the graphic method. The diagram may be a sketch of a situation, a complete working model, or it may be a symbolic representation. This method is especially helpful because it exhibits the problem visually and then makes relationships easier to detect and formulate. Caution must be taken in repeated use of this method lest the students lean too heavily upon the diagram and neglect more abstract forms of reasoning.

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[36]A.S. Luchins, Mechanization in Problem Solving, the Effect of Einstellung, Psychological Monographs Number 54, 1942, pp. 1-95.

<u>Individual method</u>.-- Pupils left to their own devices develop systems of solving problems which are usually classified as individual methods. Butler and Wren[37] suggest that the good teacher should be familiar with many methods and should make use of those which seem best suited to the requirements of the immediate situation. They also indicate that the teacher who assists the student to develop sound individual methods of attacking many problems will be equipping them to meet the problems of after-school life.

<u>Aids to effective problem solving</u>.-- Literature on problem solving describes many helpful techniques and recommendations to supplement methods employed in teaching.
Techniques suggested most frequently include the following:

(1) "The mathematics teacher is a reading teacher".[38] To solve problems consistently the student must be retaught how to read. Ordinary reading matter is diffuse, with often only one idea in a whole paragraph. Mathematical reading, however, is condensed. Therefore, the good student who has learned to skim his geography text for ideas must be slowed down and encouraged to read mathematical problems several times to glean all the information that they contain.

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[37] Butler and Wren, <u>op. cit</u>., p. 305

[38] Henderson and Pingry, <u>op. cit</u>., p. 250.

(2) "Students should be advised to abandon temporarily the attempt to solve a problem on which they have worked unsuccessfully for a long time, and return to it later."[39] A pupil may be tired and consequently a rest and a shift of attention may be all that is needed to restore his perspective.

(3) "The technique of testing hypotheses by prediction and verification should be explained to students."[40] Students should be taught both to formulate an estimate of the answer before solving a problem and then to compare it with the answer obtained; they should also be taught to check their answer by substitution in the equation used or in the original formulation of the problem.

(4) "To help students improve in problem-solving ability, the teacher should try to create a climate in the class friendly to questions."[41] Not only should the atmosphere be such that the student feels free to ask questions and thus verbalize the problem and clarify the search model, but also the teacher should "ask many questions that require thinking and then give the students an opportunity to think".[42] Often teachers seem to be embarrassed if a period

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[39] _Ibid._, p. 258.

[40] _Ibid._, p. 267.

[41] _Ibid._, p. 254.

[42] _Ibid._, p. 255.

of silence occurs subsequent to their asking of a question. Obviously, if the question posed is of such a nature as to require thought, then ample time must be allowed for that thought to take place.

(5) "Recognition should be given to the student who solves a problem in more than one way, and to the student who is able to find a particularly neat solution."[43]

(6) "Teachers can help students improve in problem-solving abilities by not requiring step-by-step procedures to be followed."[44] Teaching students to attack all problems by filling in a box or table, or by any method which can only be used for a particular problem may help them to obtain answers for the particular problem but will not help them in their efforts to solve other problems.

(7) "One very useful technique that the teacher can use and encourage the student to use is the heuristic method."[45] By careful questioning the teacher can introduce some new factor, or approach the subject from another angle and hence change the student's field of thought on the problem and enable the student to arrive at a solution.

De Vault, writing on the dangers of mechanization in problem solving, warns mathematics teachers to beware

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[43] Ibid., p. 263.

[44] Ibid., p. 265.

[45] Ibid., p. 256.

of developing "habituated behaviour" in answering problems as all too often the habit masters the individual instead of the individual mastering the habit.

On teaching methods of solving problems, De Vault[46] says:

> Methods are needed which will teach the child to stand on his own feet, to face the world freely and act through intelligent thinking rather than by force of blind habit.

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[46] M. Vere De Vault, Improving Mathematics Programmes (Columbus, Ohio: Charles E. Merrill Books, Inc., 1961), p. 205.

CHAPTER III

THE EXPERIMENT

The Design of the Experiment

In order to carry on a statistical comparison of the results of two methods of teaching, the writer selected the "Pretest-posttest control group design".[1] This design calls for the establishment of two comparable groups, one to be known as the control group, the other as the experimental group. In the experiment herein described, the groups were treated in similar fashion with respect to the time of teaching and the content to be taught; only the methods of presentation differed. Thus the difference in teaching method was the only variable the effect of which upon pupil achievement was measured.

A criterion test was administered prior to the teaching period, i.e., before the introduction of the variable, and again after the teaching period. Comparison, by means of a t-test on the posttest data, formed the basis of evaluation of the teaching methods.

Good states that the "Pretest-posttest control group

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[1]Carter V. Good, Introduction to Educational Research (New York: Appleton-Century-Crofts Inc., 1959), p. 366.

design" seeks to eliminate the main effects of history, maturation, testing instrument decay, regression and selection.[2] Any differences observed for the experimental group which are due to the above-mentioned extraneous variables would also be evident in the control group.

There was, however, no way to make certain that all these extraneous variables were eliminated entirely. In the matching of classes any of these variables might effect either group abnormally.

The Teaching Methods

As the study was concerned with comparisons of teaching methods, it was essential that all other factors be controlled as closely as possible. In establishing the two groups, a simple randomized design,[3] in which all students under each treatment are handled in an intact group, was used. Because of the nature of this design replication errors (or Errors of Type R)[4] are likely unless suitable uniformity is employed in teaching and testing.

Before the actual teaching began, conferences were

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[2] Ibid., p. 367.

[3] J. Francis Rummel, An Introduction to Research Procedures in Education (New York: Harper and Brothers, 1958), p. 211.

[4] Ibid., p. 211.

held with teachers involved in the experiment. These meetings ensured common understanding of the experiment and control of factors such as time and assignments. To add to the uniformity, the number of class periods devoted to each section of the course and to testing were also arranged.

The traditional or control group method. - The mathematics text used in Grades X and XI introduces problem solving by means of several sample problems followed by one or more solutions developed in detail.[5] These problems are closely related to the computation in algebra which precedes the problem section. With this approach to problem solving described in the text, it has become common for teachers to use this technique in teaching problems. For purposes of this study, the writer called this method of teaching the traditional approach.

Thus for the control group, the method involved giving a class a problem from the text or on the blackboard, followed by detailed discussion or illustrations of the problem and then a suitable solution. The pupils were then given similar or analogous problems to solve independently.

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[5] Daniel W. Snader, Algebra - Meaning and Mastery (Toronto: John C. Winston Co., 1954), pp. 142-146.

The assumption was, that the students would see the similarities between the problems and then be able to transfer reasoning and method to the new situation.

Since this method involved working on specific types of problems in any one class period, it necessitated sets of work problems in which the individual example matched the general type. Following the class work the students continued to work on these "type" problems, working out the necessary solutions in their notebooks. Each day a specific number of problems was assigned and those not completed were taken for homework.

In the following class period the solutions for difficult problems were placed on the blackboard, corrected and explained. After necessary remedial work, the teacher then continued with the next type of problem.

<u>The analytical-discovery technique</u>.-- The teaching method used with the experimental class was based on the premise that the ability to solve problems depends upon several component abilities each of which must be used in logical order.

These component abilities are:-

(1) reading for understanding,

(2) picking out essential details and unknowns,

(3) using graphs and charts to analyze situations,

(4) writing algebraic equations from descriptive relationships,

(5) using ratios as a means of estimating and solving,

(6) estimating answers,

(7) efficiency in arithmetic and algebraic computation.

Problem solving was introduced to the experimental class by means of simple verbal problems which are analyzed carefully. The stress was not on solution of the whole, but upon recognition of the essentials which would lead to solution. In the succeeding class period, each student was given a set of ten problems and a problem analysis sheet.[6] This sheet required that the students fill in the essential components of the problem in the properly designated column.

In order to complete this task, each student had to read and understand the nature and purpose of the problem. He, however, was not required to take time to go through the necessary calculations to secure a final answer - the estimate of the answer completed the operation. The

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[6]Problem analysis sheet - see Appendix B.

algebraic computation necessary had been completed in previous exercises and did not add appreciably to the understanding of problem solving.

The pupils were then allowed to take the problem analysis sheets home with them and were assigned a limited number of the problems to work completely. This homework assignment proved easier because of the previous experience with the problems and of the challenge to see how closely the estimate matched the true solution.

The Achievement Criteria

As stated in the design of the experiment the teaching methods were compared by student achievement as measured by tests administered at intervals after the teaching was completed. In employing this design, it was necessary to be certain that the control and experimental groups were of equivalent status before teaching began.

The pre-experimental population.-- The class involved in the experiment had completed the Grade IX course in mathematics in June, 1961. This course is based on the text "Intermediate Mathematics, Book III"[7] and includes

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[7] P.A. Petrie, et al. Intermediate Mathematics, Book III (Toronto: Copp Clark Publishing Co., 1956).

social arithmetic, introductory algebra and experimental geometry. This is a comprehensive course, the aim of which is to provide a firm background for high school mathematics courses.

Three sets of data were available for comparing the abilities of the two groups upon the completion of the Grade IX course. These were provided from the following sources.

(1) During the final year in junior high school all students in Manitoba schools are given an intelligence or mental ability test. This test (Dominion) is usually administered at the time of the Grade IX departmental examinations, thus the results are available as a guide to principals in their assignment of students to high school classes. The I.Q. rating as secured from this test was the first and prime basis for establishing equivalence of groups.

(2) The second pre-experiment comparison used was based on the marks in mathematics as recorded from the Grade IX departmental examination of June, 1961.

(3) The final comparison involved the scores from the School and College Aptitude Test. All Winnipeg schools administered this test to students in Grade IX during the month of February, 1961.

From these data the equivalence of the control group and experimental group was established before the experiment began.

<u>The criterion test</u>.-- The most important single item in an investigation such as this, is the measuring instrument used to evaluate the achievement of the two groups of students. The criterion test or measuring device used is an achievement test developed by the writer over a two year period preceding the time of the final study.

The test, "An Achievement Test in Problem Solving in High School Mathematics" was designed to measure ability to solve problems of the types taught in junior and senior high school. In order to make this test suitable for classroom use, it was designed for a working time of forty-five minutes.

During the school year 1959-60, the items were written and administered to Grade X and Grade XI students. Following the procedures outlined by Thorndike and Hagen,[8] an item analysis was made; this analysis necessitated rewriting and rearranging items for the final form of the test. This final draught of the criterion test was used in the pilot study which was conducted in 1960-61.

The mathematical concepts and materials examined in the test are: (a) the meaning and language of algebra,

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[8] R.L. Thorndike and E. Hagen, <u>Measurement and Evaluation in Psychology and Education</u> (New York: John Wiley and Sons Inc., 1958), pp. 74-78.

(b) fundamental operations of algebra, (c) solving equations, (d) deriving equations from problems, (e) functional relationships, (f) graphical relationships and (g) solving problems. The test is divided into three sections, each section representing one of the three objectives upon which measurements of achievement were made. These sections are:

(1) Recall of basic formulae and definitions,

(2) Application of formulae and basic concepts,

(3) Interpretation and comprehension.

The instructions to the student indicate that the test is a timed test and that it is designed to measure some of the abilities, skills and understandings necessary to solve mathematical problems.

The external tests.-- Prior to the time of the investigation, all students had written a set of external examinations, these being the Grade IX Inspectors' Tests of June, 1961. As stated earlier in this chapter, the results of these examinations were used in the original placement of students in the classes employed in the study.

The regular examinations in mathematics were given for Grade X in June, 1962 and for Grade XI in June, 1963. Each student in the classes in this investigation received marks based on the Departmental standards for these grades. These marks were recorded in the office of the Registrar for the Department of Education, and were used in this study for comparisons between the control and experimental groups.

In order to measure any improvement in problem solving ability in a related discipline, namely science, the Departmental marks in this subject were recorded also. These marks were based on Departmental tests and had been determined in a manner similar to that used for mathematics.

<u>The status of the classes after the experiment</u>.--Upon completion of the experimental teaching period the criterion test and the Grade X Departmental tests were administered. The composite Departmental mark for each student was used for promotion to, and for placement in, Grade XI. The results of the criterion test were used for this study only.

All of the students involved in the experiment enrolled in Grade XI in the following school year. Due to the varied selection of options, it was not possible to have the classes remain intact for Grade XI; however, the fact that they did enroll for Grade XI in the same high school as for Grade X made possible a follow-up of achievement.

Tables 15 16 and 17 indicate the appropriate data and statistics for all of the students in their Grade XI year.

In the school year following Grade XI the students of the classes involved in the study, with a few exceptions, continued into Grade XII or into First Year University classes. Because of the diversity of options and of interests at this level, no attempt was made to continue the study beyond the eleventh grade.

The Experiment

The pilot study.-- Upon reviewing reports on studies of the teaching of problem solving, the writer noted that certain methods were recommended for slow students while others were considered satisfactory for all. As no research findings were available giving specific details of the experimental teaching technique, it was decided that a pilot study should precede the final experiment.

The purpose of the pilot study was two-fold:- to provide an opportunity for the writer to perfect the experimental teaching plans, and to determine the general ability level for which this technique would best be suited.

In the fall of 1960, the writer was assigned sixty students in St. John's High School for experimental work in Grade XI mathematics. These students were of approximately similar ability (average) and had indicated a desire to proceed into a programme which included French and Biology options. Using a table of random numbers,[9] the writer divided the students into two groups of thirty. The experimental class was determined by the toss of a coin.

During the 1960-61 school year the writer taught these two classes, employing the traditional technique with the control group and the analytical-discovery technique in the experimental classroom. The results of the pilot study which are to be found in Tables 4 and 5, were not as anticipated. A brief survey of the results will reveal that the experimental class did not achieve as well as the control class. The differences in achievement were not significant; nevertheless, the experimental class was not in a favourable position. Analysis of the results did show, however, that the better students showed superior growth under the analytical-discovery approach.

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[9]W.J. Dixon and F.J. Massey, Introduction to Statistical Analysis, (New York: McGraw Hill Book Co., 1957), Table of Random Numbers - Table IA Appendix pp. 366-370.

For this reason it was decided that the final study should be conducted using students of above-average ability.

The final study.-- During the summer of 1961, the pattern of the investigation was finalized and the study was carried on during the 1961-62 school term. As described earlier the experimental group at St. John's High School and the control group at Sisler and Churchill High Schools, were established from the Major Work classes of the previous year.

Before beginning the experiment the teachers of the classes met and the design of the experiment was formulated and explained. The teaching schedules were compared and efforts were made to ensure equal teacher time with the pupils of the experimental and control groups. At this time, teaching plans were checked for similarity and dates for testing established as closely as possible.

Soon after the commencement of classes in September, all students in the investigation were informed of the experiment and of their part in it. In this way it was hoped that the students of the control group would feel a part of the experiment and that this would counterbalance the novelty or "halo" effect expected in the experimental class. Following this introductory period, the pretest (Criterion test) was administered to both groups.

During the period of time when the experiment was being carried on, the teachers made no special effort to have problem solving become the course of study for either group. Each instructor proceeded to teach problem solving whenever problems appeared in the yearly plan for teaching Grade X mathematics. In so far as individual school departmental differences would allow, the participating classes were given identical time on solving problems.

The control group classes were taught to solve problems by means of the traditional techniques which have been described. These classes were aware of their part in the investigation, but beyond this, no attempt was made to develop a competitive type of motivation. The experimental class was taught by the analytic-discovery technique. In this class extreme care was taken to guard against the development of attitudes detrimental to the experimental results.

The school examinations in December and March were designed to test the same content in all schools and were administered at the same time. These examinations were marked by the instructor in each school after consultation among the teachers involved in the investigation and with the departmental head in each school. The results of these examinations are recorded in Appendix B.

The formal teaching for the year was completed in all classes in early May. The posttest (Criterion test) was given immediately following, that is, at the end of the second week in May. The remainder of the class time was spent in review of the year's work and in discussion of enrichment topics. Although this review period is of major importance in preparing the classes for their final examinations, it was not considered essential that the Criterion test be delayed until the completion of the review, since the Criterion test was designed to compare teaching methods not review techniques.

The June final marks in all subjects were determined using the Departmental standards.[10] These are composite marks calculated by averaging the June mark with the school mark for the year. This means that the final mark is a measure of achievement of the year's work. The final results in mathematics and science were recorded for this study.

During the following school year 1962-63, the students involved in the investigation were enrolled in Grade XI classes at their respective schools. In order to determine if any difference in achievement at the Grade X level would carry over into succeeding years, the achievement in mathematics was measured and compared at the end of Grade XI. These comparisons completed this investigation.

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[10] Departmental Test Score Sheets Grade X, (Manitoba Department of Education, June 1962).

The Statistical Techniques

The test scores.-- At each stage in the investigation the students of the experimental and control groups were given similar tests. The scores were determined using the same standards and recorded in the same manner in all schools. For this reason the raw scores from each test were used in the statistical treatment.

The use of Z scores rather than marks would tend to nullify any bias that might have been introduced had all students not written identical examinations. In view of the uniformity of testing at all stages, this sophistication in scores was not deemed necessary.

The sampling.-- The groups under study in the investigation were not so chosen as to be considered random samples. The possibility existed that they did, however, belong to the same original population, since all the students were selected from Major Work classes in Grade IX. This possibility was tested by means of the ratio of the variances which will be discussed under the F test.

Probability.-- The differences in mean scores were examined with a view to determining which were sufficiently large to be considered real, rather than merely the result of fluctuations in sampling. Since any difference, no matter how large, could conceivably be attributed to chance, a method of expressing the probability of such a difference being due to chance has been used.[11]

If a difference is so large that it would be obtained by chance only once in one hundred similar samplings, then the probability of the difference being due to chance is 0.01. Stated in another way, it may be said that, at the 1 per cent level of significance, the difference is real.

The 5 per cent level, used frequently, expressing a slightly lower degree of significance, may also be accepted. It is important to note that a statement of a difference being significant in these terms indicates only that the obtained difference is not due entirely to chance fluctuations in sampling; it does not indicate what does account for the difference.

The null hypothesis.-- The most convenient way of testing the significance of the difference between two means is to state that the means are equal, and then to accept or reject this hypothesis of equality or of no

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[11] E.F. Lindquist, A First Course in Statistics (New York: Houghton Mifflin Co., 1942), p. 136.

significant difference is known as the null hypothesis.

<u>The F-test</u>.-- The test of the significance of the difference of means of two samples is a t-test. Before the appropriate t-test can be applied, it is necessary to know whether or not the variances of the groups are sufficiently homogeneous. The F-test is used to compare two variances.

The variance is a statistic used to express the variability of a sample. It is the square of the standard deviation, and is found by means of the formula:[12]

$$s^2 = \frac{\Sigma(X - M)^2}{N - 1}$$

where s^2 is the variance, X is a raw score, M is the mean of the sample, $\Sigma(X - M)^2$ is the sum of the squared deviations of raw scores from the mean, N is the number of cases, and N - 1 is the number of degrees of freedom.

While $\Sigma(X - M)^2$ may be calculated directly, its value is found more easily by use of the formula:[13]

$$\Sigma(X - M)^2 = x^2 - \frac{(\Sigma X)^2}{N}$$

where N is the number of cases, ΣX^2 is the sum of the squared scores in the group, $(\Sigma X)^2$ is the square of the sum of the scores.

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[12] James E. Wert, Charles O. Neidt, and J. Stanley Ahmann, <u>Statistical Methods in Educational and Psychological Research</u>, (New York: Appleton-Century-Crofts Inc., 1954), p. 59.

[13] <u>Ibid.</u>, p. 57.

The comparison of variances is made by means of the variance ratio, also known as the F-ratio:[14]

$$F = \frac{s_1^2}{s_2^2}$$

where s_1^2 and s_2^2 are the variances of the samples, s_1^2 always being the larger.

If the F-ratio for a pair of samples exceeds a critical value, then it can be stated with a high degree of confidence, that the two samples do not possess sufficiently homogeneous variance to warrant use of the t-test employed in this thesis. Critical values for the F-ratio at the 1 and 5 per cent significance levels may be found by referring to a table such as that in Wert, Neidt, and Ahmann's standard text.[15]

The t-test.-- The statistic, t, is designed to test the significance of the difference in means of a pair of samples, and may be found by means of the formula:[16]

$$t = \frac{M_1 - M_2}{\sqrt{\left(\frac{\Sigma_1 x_1^2 + \Sigma_1 x_2^2}{N_1 + N_2 - 2}\right)\left(\frac{N_1 + N_2}{N_1 N_2}\right)}}$$

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[14]*Ibid.*, p. 134.

[15]*Ibid.*, p. 419.

[16]P.O. Johnson, *Statistical Methods in Research* (New York: Prentice-Hall, 1949), p. 74.

where the number of degrees of freedom is given by: $N_1 + N_2 - 2$. The Wert, Neidt and Ahmann text contains a table of critical values of t.[17] If an obtained value of t exceeds the value shown in the table at the desired level of significance for the appropriate number of degrees of freedom, then the null hypothesis of equal means may be rejected at that level of significance.

Tests of Significance

In order to determine if a significant difference existed between the achievement of the groups, the mean scores were compared at various stages in the investigation. As a result of recording the various test and examination scores, and averaging these scores for each group, seven mean scores were obtained.

The following hypotheses were tested:

1. The mean chronological age in months of the control and experimental groups were not significantly different.

2. The mean scores of the control and experimental groups in Grade IX mathematics were not significantly different.

3. The mean scores of the control and experimental groups in general intelligence were not significantly different.

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[17] Wert et al., <u>op. cit.</u>, p. 418.

4. The mean scores of the control and experimental groups on the pretest were not significantly different.

5. The mean scores of the control and experimental groups in Grade X mathematics in October, were not significantly different.

6. The mean scores of the control and experimental groups in Grade X mathematics in December were not significantly different.

7. The mean scores of the control and experimental groups in Grade X mathematics in March were not significantly different.

8. The mean scores of the control and experimental groups on the posttest were not significantly different.

9. The mean scores of the control and experimental groups on the June final mathematics test were not significantly different.

10. The mean scores of the control and experimental groups on the June Grade X Science test were not significantly different.

11. The mean scores of the control and experimental groups on the June Grade XI test the following year were not significantly different.

CHAPTER IV

STATISTICAL ANALYSIS OF THE DATA

In this chapter the data and analyses are presented for: (1) equivalence of the experimental and control groups, (2) mathematical achievement by both groups on the Grade X school examinations (December and March), (3) mathematical achievement by both groups as measured at the completion of the teaching period (Criterion test scores and June final marks), (4) achievement of both groups in a related subject area (Science) on the Grade X June examination, (5) mathematical achievement of both groups as indicated by Grade XI final mathematics marks the following year.

The Equivalence of the Experimental and Control Groups

The status of the experimental and control groups before the experimental teaching began is illustrated in Table 6 , 7 , and 8 , in Appendix C.

In order to establish the equivalence of the two groups, null hypotheses were stated for each set of scores. These hypotheses were tested using the t-test for significant differences between the means of the group scores. Before using the t-test, all variances were compared by means of the F test. The obtained values of F did not exceed the tabled values,[1] that is, were not found significant at the one per cent level. (See Appendix C, Table 18) The null hypothesis states that the difference between the means of the groups is zero. For purposes of this study a difference was declared significant at the five per cent level or less.

The formula for the t-test was used and in each case M_1 is the mean for the experimental group, and M_2 is the mean for the control group. All calculations were made using a desk calculator and checked using a programme for the Bendix computor.[2]

<u>Chronological age in months</u>.-- The ages of the students as recorded in Table 6 , were substituted in the formula for t and the result was 0.7027 with 108 degrees of freedom.

$$t = \frac{181.429 - 180.547}{\sqrt{\left(\frac{912.571 + 3146.587}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$$t = 0.7027$$

$$d.f. = 108$$

$$p > .40$$

The null hypothesis that there is no significant difference between the groups in chronological age in months was accepted.

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[1]Allen L. Edwards, <u>Statistical Analysis</u> (New York: Rinehart and Company, 1959), <u>Table V Appendix</u> pp. 221-224.

[2]Wm.H. Lucow, <u>Intercom Programme 1000D U.M. 21</u>, t-test. November 26, 1961.

<u>The Dominion Intelligence test</u>.-- For the scores on the intelligence test the value of t was 1.0445 with 108 degrees of freedom.

$$t = \frac{124.743 \quad - \quad 126.440}{\sqrt{\left(\frac{4366 \ - \ 9483}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$$t = 1.0445$$

$$d.f. = 108$$

$$p > .30$$

The null hypothesis that there is no significant difference between groups as measured by the Dominion Intelligence test is accepted.

<u>Co-operative School and College Ability test</u>.-- Using the Quantitative section scores of the intelligence test, the value for t was - .7763 with 108 degrees of freedom.

$$t = \frac{313.2571 \quad - \quad 314.7600}{\sqrt{\left(\frac{3734.6857 \ - \ 5923.6800}{35 \ + \ 75 \ - \ 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$$t = -.7763$$

$$d.f. = 108$$

$$p > .40$$

The null hypothesis that there is no significant difference between groups in intelligence as measured by the Quantitative section of the School and College Aptitude test is accepted.

<u>The Grade IX achievement in mathematics</u>.-- Using the marks obtained on the Grade IX Inspectors' test given in June 1961, the value for t was 2.592 with 108 degrees of freedom.

$$t = \frac{77.486 - 83.080}{\sqrt{\left(\frac{4500.743 + 7507.520}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$$t = -2.592$$

$$\text{d.f.} = 108$$

$$p > .01$$

The null hypothesis that there is no significant difference between the groups in mean achievement on Grade IX Inspectors' test in mathematics is rejected.

This rejection of the hypothesis indicates that a difference between the groups in mean achievement in Grade IX mathematics exists. This difference is, however, in favour of the control group.

<u>The pretest (Criterion test)</u>.-- Using the means of the scores on the Criterion test administered prior to the teaching period, the value for t was 1.7942 with 108 degrees of freedom.

$$t = \frac{28.9714 - 26.8133}{\sqrt{\left(\frac{1136.9714 + 2591.3866}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$$t = 1.7942$$

$$\text{d.f.} = 108$$

$$p > .05$$

The null hypothesis that there is no significant difference between groups in mean achievement on the pretest is accepted.

Summary.-- Table 1 on the following page gives a summary of the statistics related to the equivalence of the groups.

Achievement in Mathematics as Measured
by School Term Examinations.

The effects of the teaching methods on class achievement in mathematics were first examined by comparisons of scores obtained on school examinations. These term examinations were given in October, December and March.

The October tests were classroom tests while the December and March sets were scheduled examinations. These examinations were designed in the schools and marked by the individual instructors; only the content examined was uniform over the experimental and control groups. Tables 9 , 10 , and 11 show the results of these examinations.

Grade X October tests.-- Using the scores obtained on the October tests in mathematics, the value of t was 1.9668 with 108 degrees of freedom.

TABLE 1

SIGNIFICANCE OF DIFFERENCES IN MEANS
OF EXPERIMENTAL AND CONTROL GROUP STUDENTS
FOR AGE, GENERAL INTELLIGENCE, MATHEMATICAL
ABILITY AND PROBLEM SOLVING ABILITY

Test	Experimental		Control		t	p	Hyp.
	N	M	N	M			
Chron. Age in Months	35	181.4	75	180.5	.7027	> .40	Accept
Intelligence Dominion test	35	124.7	75	126.4	1.0445	> .30	Accept
Mathematics Ability SCAT Quantitative	35	313.3	75	314.8	-.7763	> .40	Accept
Achievement Gr.IX Maths.	35	77.5	75	83.1	2.592	< .01	Reject*
Problem Solving Pretest	35	28.9	75	26.8	1.7942	> .05	Accept

*Significant differences in means in favour of the control group.

$$t = \frac{77.771 - 81.000}{\sqrt{\left(\frac{5098.171 + 5714.000}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$t = 1.9668$*

$d.f. = 108$

$p > .05$

The null hypothesis that there is no significant difference in mean achievement on Grade X October tests is accepted.

*Although this t value is of insufficient size to indicate a significant difference between means, it is approaching the 5 per cent level but in favour of the control group.

Grade X school examinations (December).-- Using marks obtained on December school examinations, the value of t was 1.2405 with 108 degrees of freedom.

$$t = \frac{86.229 - 83.427}{\sqrt{\left(\frac{3540.172 + 9606.347}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$t = 1.2405$

$d.f. = 108$

$p > .20$

The null hypothesis that there is no significant difference between groups in mean achievement on Grade X December examinations is accepted.

<u>Grade X School examinations (March)</u>.-- Using marks obtained on the March school examinations, the value of t was 1.7657 with 108 degrees of freedom.

$$t = \frac{78.857 - 73.840}{\sqrt{\left(\frac{7170.286 + 13636.080}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$$t = 1.7657^{*}$$

$$d.f. = 108$$

$$p > .05$$

The null hypothesis that there is no significant difference between groups in mean achievement on the Grade X March examinations is accepted.

*This value of t is below the value necessary to indicate a significant difference in mean achievement, however, it is approaching the 5 per cent level in favour of the experimental group.

<u>Summary</u>.-- Table 2 gives a summary of the statistics related to development as indicated by school examinations. Table 2 is presented on the following page.

Mathematical Achievement Upon Completion of the Teaching Period

The immediate effects of the teaching methods were examined by comparisons of scores of year end examinations. Two tests were given in mathematics; the first being the posttest immediately after the teaching period and the second being the June final examination. Tables 12,13 , and 14 in the Appendix give the scores for these examinations.

TABLE 2

SIGNIFICANCE OF DIFFERENCES IN MEAN SCORES OF EXPERIMENTAL AND CONTROL GROUPS FOR SCHOOL EXAMINATIONS

Test	Experimental		Control		t	p	Hyp.
	N	M	N	M			
October test	35	77.7	75	81.0	1.9668	$> .05$	Accept
December examination	35	86.2	75	83.4	1.2405	$> .20$	Accept
March examination	35	78.8	75	73.8	1.7657	$> .05$	Accept

<u>The posttest (Criterion test)</u>.-- Using the means of the scores on the Criterion test administered following the teaching period, the value for t was 2.3825 with 108 degrees of freedom.

$$t = \frac{36.429 - 33.453}{\sqrt{\left(\frac{1466.571 + 2552.587}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$$t = 2.3825$$

$$d.f. = 108$$

$$p < .02$$

With 108 degrees of freedom, the t-value is significant at the 2 per cent level of significance. The null hypothesis that there is no significant difference between groups on the posttest is rejected.

<u>Grade X June achievement test</u>.-- Using the scores in mathematics calculated by the standards of the Department of Education for June 1962, the value of t was 2.1663 for 108 degrees of freedom.

$$t = \frac{82.571 - 76.827}{\sqrt{\left(\frac{5002.571 + 13120.747}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$$t = 2.1663$$

$$d.f. = 108$$

$$p > .02$$

The null hypothesis that there is no significant difference between groups in mean achievement in Grade X mathematics

(June) is rejected (accepted at 2 per cent level p .02).

Summary.-- The achievement of the experimental and control groups upon completion of the teaching period is summarized in Table 3.

TABLE 3

SIGNIFICANCE OF DIFFERENCES IN MEAN SCORES OF EXPERIMENTAL AND CONTROL GROUPS FOR GRADE X MATHEMATICS (JUNE)

Test	Experimental		Control				Hyp.
	N	M	N	M	t	p	
Criterion test	35	36.4	75	33.5	2.3825	$<.02$	Reject
June Final Departmental Marks	35	82.6	75	76.8	2.1663	$<.05$	Reject

Achievement in a Related Subject - Science

Tables 15, 16, and 17 in Appendix C show the scores obtained by the students of the experimental and control groups in science. These scores were recorded at the Department of Education in June, 1962. A t-value of 2.3900 for 108 degrees of freedom resulted when these scores were used.

$$t = \frac{81.114 - 76.373}{\sqrt{\left(\frac{2763.543 + 7377.547}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$$t = 2.3900$$

$$d.f. = 108$$

$$p < .02$$

The null hypothesis that there is no significant difference between groups in mean achievement in Grade X science is rejected.

Grade XI Mathematics Achievement

Having compared the effects of the teaching methods at the tenth grade level, further measures of the effects were made in the following school year. Tables 15, 16, and 17 in Appendix C give these marks. Using the final marks in Grade XI mathematics, the value of t was 1.5643 for 108 degrees of freedom.

$$t = \frac{78.971 - 76.547}{\sqrt{\left(\frac{4480.971 + 15258.587}{35 + 75 - 2}\right)\left(\frac{1}{35} + \frac{1}{75}\right)}}$$

$$t = 1.5645$$

$$d.f. = 108$$

$$p > .10$$

The null hypothesis that there is no significant difference between groups in mean achievement in Grade XI mathematics is accepted.

CHAPTER V

SUMMARY AND IMPLICATIONS OF THE STUDY

General Summary

The problem.-- The mathematics programme in the matriculation course for Manitoba high schools outlines many computational procedures which students must master and apply to the solution of problems. Many students, although able to perform these computations, are unable to make the necessary application of this learning, that is, to solve problems.

The present study was concerned with the relative effectiveness of two distinct me hods of teaching students to solve problems and with the ability level at which any such effectiveness was most apparent. In order to carry on the investigation by statistical means, the problem was stated as follows:- to compare the mathematical achievement of equivalent groups after these groups had been given instruction in problem solving by two distinct methods.

The two methods of teaching are known as the traditional or control group method and the analytic-discovery method. In order to perfect the experimental teaching technique, to carry on suitable analyses of the criterion test to be used, and to investigate the ability level best served by each method of teaching, a pilot study was designed and carried on for one year prior to the final investigation.

The pilot study showed no significant advantage in the use of the analytic-discovery method of teaching students of average ability. An analysis of the individual scores indicated that students of higher academic ability in the experimental group had shown greater achievement in problem solving than students of equivalent ability in the control group. For this reason, a final study involving students of high academic ability was planned for the following year.

The experimental population.-- The control and experimental groups involved in the pilot study were composed of sixty Grade XI students of average academic ability. These students were selected from a total Grade XI population of approximately four hundred students at St. John's High School. The division of the original sixty students into experimental and control groups was done by random selection. Thus any differences in ability between the two classes could be prescribed to pure chance.

These students would, in general, not be referred to as "college capable" but some of this group did later gain university entrance and continued their academic training. As the purpose of the pilot study had been fulfilled by the end of the first year, no follow-up study was attempted with these students.

For the final study, students of high academic ability were selected. All had come through the Major Work programme of the Winnipeg Schools or had demonstrated a high level of performance in the junior high grades.

The members of the experimental group were enrolled as a class at St. John's High School. The control group consisted of two classes; one at Sisler High School and the other at Churchill High School. The selection of the classes for the investigation was done on the basis of teacher willingness to co-operate in such a study. The selection of students for the classes in each school was performed by the principal and his counselling staff, with special assistance from the Director of Special Education.

The groups did not differ significantly with respect to chronological age, intelligence or ability to solve algebraic problems. On the basis of the Grade IX marks in mathematics there was a significant difference in favour of the control group. Thus, although the potential ability of the experimental and control groups was equivalent, the achievement of the control group prior to the experiment was slightly higher than that of the experimental group.

<u>The achievement measures</u>.-- The equivalence of general ability between the groups was established using marks obtained on the Grade IX mathematics examination (Inspectors' tests), scores of the Dominion Intelligence test and the scores on the Quantitative section of the School and College Ability test (SCAT). The raw scores for the Inspectors' test were used since all students in Grade IX wrote the same examination which was marked by a central committee.

The Criterion test which was used as a pretest and posttest in problem solving was designed by the writer over a period of two years prior to the time of the final study. The details of the development of this test have been given earlier. In the final study it was administered by the three instructors prior to the beginning of the teaching period (October) and immediately after the teaching was completed (May). The scores from the Criterion test form one of the bases of evaluating the teaching methods.

Other scores which were used at the conclusion of the experiment included the Grade X departmental scores in mathematics and science and the Grade XI departmental scores in mathematics.

The data: collection and recording.-- In the pilot study the writer had access to all records of test scores of mathematical achievement in the school files for the experimental and control groups. In the final study, however, this was not the case, since the control classes were located in other high schools. The instructors of the control classes supplied the data from these schools and all Departmental results were secured from Department files.

Analysis of data.-- The significance of the differences between the mean scores for the control and experimental group students was computed by the t-test. The homogeneity of the variances was checked in each case bymeans of the F test. The level of significance for each of these tests was determined by use of tables for t and F. Only those t values at the five per cent level of significance or less were accepted as statistically significant.

Results.== Significant differences between the achievement of the experimental group students and that of the control group students, ascertained by the t-test, appeared in Grade X mathematics. These differences in favour of the analytic-discovery method were, however, evident only when the experimental populations consisted of students of high academic ability. For experimental populations made up of students of average ability no significant differences in mathematical achievement were observed.

Use of the analytic-discovery method of teaching problem solving produced a greater improvement in achievement than did the traditional method during the year of the investigation. Achievement in science was also significantly better for the experimental group at the end of the teaching period.

The t-test showed no significant differences between the achievement in mathematics of the students of the experimental class and that of the control group students, at the end of the school grade following the year of the experiment.

Specific Findings

(1) There is a significant difference between the achievement in problem solving of students of high academic ability taught by the analytic-discovery method and that of students of similar ability taught by the traditional method as indicated by scores on a problem solving criterion test.

(2) There is no significant difference between the achievement in problem solving of students of average academic ability taught by the analytic-discovery method and that of students of similar ability taught by the traditional method as indicated by scores on a problem solving criterion test.

(3) There is a significant difference between students of high academic ability taught by the analytic-discovery method and similar students taught by the traditional method as indicated by examination marks in Grade X science and mathematics.

(4) Differences in mathematical achievement of the students of the experimental group and that of the students of the control group increased steadily in favour of the experimental group during the time of the final investigation. These differences were noted by comparisons of the t-values and of probabilities obtained from term examinations.

(5) There is no significant difference between students of high academic ability taught in the tenth grade by the analytic-discovery method and students of similar ability taught by the traditional method as indicated by the examination marks in Grade XI mathematics.

Implications of the Study

The findings of the present study suggest a need for teachers to concern themselves with the importance of methods of teaching problem solving. The analytic-discovery method proved advantageous for students of superior academic ability; other methods may prove more suitable for classes of other ability levels. Hence the need for teachers to give careful consideration to the manner in which problems are taught.

If teachers should be more deeply concerned about methods of presenting problem solving to students of varied abilities, perhaps text-book writers should be concerned about this matter also. It was noted early in the study that the traditional method was the method of the text being used. Since the findings suggest a need for more than one method, perhaps text books would be of greater use if several methods were presented. An alternative to this suggestion would be provision of a variety of texts for the teacher's use.

The fact that there was no significant difference between the achievement of the experimental and control group in Grade XI mathematics indicates that in order for the experimental group to retain an advantageous position with respect to achievement in mathematics, the teacher must continue in Grade XI with the method employed in Grade X.

Conclusions

The results of the investigation indicate the following conclusions:

(1) In teaching problem solving to high school students of superior academic ability, the emphasis on the fundamental concepts required by the analytic-discovery method yielded better results than lack of such emphasis.

(2) The analytic-discovery method proved to be superior for students of high academic ability. For students of average ability the traditional method produced equivalent or superior results.

(3) Conducting of educational research, even on a limited scale, produced beneficial results in the classes involved in the experiment. Interest in mathematics was sparked among the class members in the experimental group and reports from the control group instructors indicated that a similar lift was noted.

(4) Improvement in performance produced by use of special teaching methods in one area may have beneficial results in a related area. Although no suggestion is made in this report to indicate that the superior results in science were a direct result of the improved achievement

in mathematics, it is possible that such was the case.

(5) Because of the limited size of the samples used in the study, generalizations are hazardous. The difference in performance between the control and experimental group could be accounted for by chance factors, but the data indicate that considerable confidence may be placed in the assertion that real differences exist.

BIBLIOGRAPHY

BIBLIOGRAPHY

Beberman, Max. Quoted in "Education", Time, Canadian Edition, (January 16, 1965), p. 75.

Buckingham, Guy E. "Relationship Between Vocabulary and First Year Algebra," Mathematics Teacher, XXX (1937), pp. 76-79.

Butler, C.H., and F.L. Wren. The Teaching of Secondary Mathematics (3rd. ed.), New York: McGraw-Hill, 1960.

Cronbach, Lee J. The Meaning of Problems. Arithmetic, 1948, Supplementary Educational Monographs, Number 66. Chicago: University of Chicago, 1948, p.42.

De Vault, M. Vere. Improving Mathematics Programmes. Columbus, Ohio: Charles E. Merrill Books, Inc., 1961.

Dewey, John. How to Think. Boston: D.C. Heath and Co., 1933.

Dixon, W.J., and F.J. Massey. Introduction to Statistical Analysis. New York: McGraw Hill Book Co., 1957.

Duncker, Karl. On Problem Solving, Psychological Monograph Number 58, pp. 1-111.

Durell, Fletcher. Mathematical Adventures. Boston: Bruce Humphries, Inc., 1938.

Dutton, Wilbur H. Evaluating Pupils' Understanding of Arithmetic. Englewood Cliffs, New York: Prentice-Hall Inc., 1964.

Edwards, Allen L. Statistical Analysis. New York: Rinehart and Company, 1959.

Evenson, A.B. Modern Mathematics. Toronto: W.J. Gage and Co., 1963.

Eves, Howard. An Introduction to the History of Mathematics. New York: Holt, Rinehart and Winston, 1961.

Garrett, Henry E. Statistics in Education and Psychology. New York: Longmans, Green and Co., 1953.

Georges, J.S. "Learning to Solve Problems Intelligently," School Science and Mathematics, (December, 1956), pp. 701-707.

Good, Carter V. Introduction to Educational Research. New York: Appleton-Century-Crofts Inc., 1959.

Hartung, Maurice L. "Motivation for Education in Mathematics" Twenty-first Yearbook, Washington D.C. National Council of Teachers of Mathematics, 1953.

Hawkins, G.E. "Teaching Verbal Problems in Algebra," School Science and Mathematics, XXXII (1932), pp. 655-660.

Henderson K.B., and R.E. Pingry. "Problem Solving in Mathematics," N.C.T.M. 21st Year Book. Washington D.C. National Council of Teachers of Mathematics, 1953, p. 232.

Hersom, Naomi Louise. "A Follow-up Study of the High School Performance of Students who were Members of the Inaugural Major Work Classes in Winnipeg," Unpublished Master's Thesis, University of Manitoba, 1962.

Johnson, P.O. Statistical Methods in Research. New York: Prentice-Hall, 1949.

"Leonardo de Pisa", Encyclopaedia Britannica, 18th ed., Vol. XIII, p. 939.

Lindquist, E.F. A First Course in Statistics. New York: Houghton Mifflin Co., 1942.

Luchins, A.S. Mechanization in Problem Solving, the Effect of Einstellung, Psychological Monographs Number 54, 1942, pp. 1-95.

Lucow, William H. Intercom Programme 1000D U.M. 21, t-test. November 26, 1961.

MacFarlane, John Douglas. "A Follow-up Study to Determine the Effect of Enrichment Programs in a High School upon Achievement at University," Unpublished Master's Thesis, University of Manitoba, 1961.

McLeod, C., and C.I. McIntyre. "Problem Solving in Algebra," Mathematics Teacher, XXX (1937), pp. 371-373.

Morton, Robert Lee. Teaching Arithmetic in the Intermediate Grades. New York: Silver Burdett Company, 1927. p. 27.

National Council of Teachers of Mathematics. "The Learning of Mathematics," Twenty-first Yearbook, Washington D.C. 1953. pp. 263-270.

Petrie, P.A. Intermediate Mathematics Book III. Toronto: Copp Clark Co., 1956.

Polya, George. How to Solve It. New York: Doubleday Anchor Books, 1947.

Programme of Studies for the Schools of Manitoba. Senior High School 1961-62. Department of Education. The Queen's Printer for Manitoba, 1961.

Rummel, J. Francis. An Introduction to Research Procecures in Education. New York: Harper and Brothers, 1958.

Smith, D.E. "On the Origin of Certain Typical Problems," American Mathematics Monthly, XXIV (February, 1917), pp. 64-71.

Snader, Daniel W. Algebra - Meaning and Mastery. Toronto: John C. Winston Co., 1954.

Stright, I.L. "The Relation of Reading Comprehension and Efficient Methods of Study to Skill in Solving Algebraic Problems," Mathematics Teacher, XXXI (1938), pp. 368-372.

Summers, Edward G., and James E. Stochl. "A Bibliography of Doctoral Dissertations Completed in Elementary and Secondary Mathematics 1950-60," School Science and Mathematics, LXI (June, 1961), pp. 431-439.

Thorndike R.L., and E. Hagen. Measurement and Evaluation in Psychology and Education. New York: John Wiley and Sons Inc., 1958.

Thorndike, E.L. Psychology of Algebra. New York: McMillan and Co., p. 190.

"Technology", Time, Canadian Edition, (April 2, 1965), p. 69.

Wert, James E., Charles O. Neidt, and J. Stanley Ahmann. Statistical Methods in Educational and Psychological Research. New York: Appleton-Century-Crofts Inc., 1954.

Winnipeg School Division #1, Superintendent's Bulletin, (September, 1961).

APPENDIX A

SAMPLE SETS OF EXAMINATIONS

THE DOMINION TESTS

GROUP TEST OF LEARNING CAPACITY

INTERMEDIATE—GRADES 7, 8, 9

CAT. NO. 139 **(1950 OMNIBUS EDITION)** FORM **A**

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

Fill in the blanks below, giving your name, age, etc., and when you have done so, read the rest of this cover page. Only a short time will be given for this so you will need to work rapidly.

Name .... Boy or Girl....................
(IN CAPITALS) LAST FIRST

Age .... Birthdate .... Grade ....
MONTH DATE YEAR

School .... Teacher .... Today's Date ....

City, Town, or Municipality .... Province ....

Five sample questions are given below to show you what this test is like. In questions such as 1, 2, and 3, you must in each case select the best answer from the five choices presented, and write the number of your choice in the brackets following the question. In questions in which no choices are given, such as 4 and 5 below, it will be quite clear what you are expected to do. The sample questions have all been answered for you. The questions in the test must be answered in the same manner.

In doing this test you must work as rapidly as possible, since you are not likely to do all the questions in the 30 minutes allowed for it. Each question is worth one point. Skip any questions which appear to be too difficult, or which take up too much of your time, and return to them later if you have any time left. Spend your time now studying the samples below. **Do not open the booklet until you are told to do so.**

1. Which word does not belong in this list?
   (1) green (2) purple (3) red (4) sweet (5) yellow ....(4)
2. **Fish** is to **Swim** as **Bird** is to
   (1) feathers (2) fly (3) nest (4) chirp (5) egg ....(2)
3. Which word means the opposite of **Come?**
   (1) late (2) home (3) run (4) ride (5) go ....(5)
4. What number comes next in this list?
   12, 11, 10, 9, 8, ...(7)
5. Jim spent half of his money and has 15 cents left. How much did he have at first? ....(30)

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SCORE......... **M.A.**.......... **C.A.**.......... **I.Q.**...............

A

PAGE 1

1. Which word means the opposite of **Create?**
   (1) acquire (2) disband (3) destroy (4) resume (5) finish................( )

2. **Teacher** is to **Pupil** as **Doctor** is to
   (1) patient (2) medicine (3) nurse (4) sick (5) hospital....................( )

3. What number comes next in this list?
   1, 8, 2, 7, 3, 6, 4..( )

4. If Sam had 5 cents more he would have twice as much money as Bill. Bill has 30 cents. How many cents has Sam?..( )

5. Which word does not belong in this list?
   (1) stoop (2) bow (3) jump (4) bend (5) curtsy................................( )

6. **Sheep** is to **Flock** as **Bee** is to
   (1) sting (2) flowers (3) shepherd (4) honey (5) swarm....................( )

7. I had 9 apples and John had 10 apples. I gave him 7 of mine. How many more has he than I now?..( )

8. What number comes next in this list?
   2, 3, 3, 3, 4, 3, 5, 3,...( )

9. Which word means the opposite of **Uncertain?**
   (1) possible (2) doubtful (3) careful (4) positive (5) hopeful..............( )

10. It is 76 yards around a square lawn. How many yards is it along each side?............( )

11. **Fish** is to **Water** as **Bird** is to
    (1) nest (2) egg (3) air (4) feather (5) fly..( )

12. Which word does not belong in this list?
    (1) valley (2) hill (3) gully (4) ravine (5) gorge...............................( )

13. What fraction comes next in this list?
    $\frac{11}{5}$, $\frac{10}{7}$, $\frac{9}{9}$, $\frac{8}{11}$,...( )

14. What is the smallest number that may be subtracted from 77 to make the remainder exactly divisible by 9?...( )

15. Which word means the opposite of **Hasten?**
    (1) tarry (2) quiet (3) return (4) hurry (5) late ....( )

16. Which word does not belong in this list?
    (1) girl (2) maid (3) damsel (4) lass (5) child ....( )

17. **Spade** is to **Earth** as **Spoon** is to
    (1) fork (2) soup (3) table (4) silver (5) bread ....( )

18. What number added to 6 gives a number 2 more than half of 16?..........................( )

19. What fraction comes next in this list?
    $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{9}$, ...( )

GO ON TO PAGE 2

20. Which word means the opposite of **Generous?**
(1) wicked (2) miserly (3) rich (4) careless (5) poor............................( )

21. Which word does not belong in this list?
(1) measure (2) gauge (3) disagree (4) reckon (5) estimate................( )

22. What number comes next in this list?
29, 30, 28, 29, 27, 28,..( )

23. **Torrid** means the same as
(1) ugly (2) hostile (3) gloomy (4) rainy (5) hot ....( )

24. What number added to 7 gives a number 2 less than one-third of 36?........................( )

25. To **Predict** is to
(1) recall (2) describe (3) remind (4) foretell (5) prevent.................( )

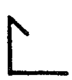
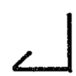
26. What number comes next in this list?
1, 2, 4, 5, 7, 8,..( )

27. **Wheat** is to **Granary** as **Books** are to
(1) pages (2) print (3) read (4) paper (5) library...............................( )

28. Bill is taller than Joe and Joe is shorter than Harvey. Therefore of the three boys
(1) it is certain that Bill is the tallest
(2) it is certain that Joe is the tallest
(3) it is certain that Harvey is the tallest
(4) it is impossible to tell just who is the tallest ....( )

29. Which word does not belong in this list?
(1) shrink (2) contract (3) enlarge (4) reduce (5) diminish.................( )

30.  is to  as  is to

(1)  (2)  (3)  (4)  (5)  ....( )

31. What number comes next in this list?
4, 2, 5, $2\frac{1}{2}$, 6, 3, 7, $3\frac{1}{2}$, 8, ...( )

32. Which word does not belong in this list?
(1) tremble (2) taunt (3) mock (4) jeer (5) jibe..................................( )

33. **Mouse** is to **Cat** as **Fly** is to
(1) moth (2) kitten (3) insect (4) spider (5) cheese...........................( )

34. What number comes next in this list?
25, 20, 16, 13, 11,..( )

GO ON TO PAGE 3

35. Which word means the opposite of **Depart?**
(1) meet (2) walk (3) embark (4) journey (5) return........................(

36. A horse walks 4 miles per hour and trots 12 miles per hour. How many hours will it take to go 24 miles if it trots half the distance? .... (

37. What number comes next in this list?
6, 21, 8, 19, 10, 17,......... .... (

38. A prize is to be given to the most proficient pupil in the class. Mary is more proficient than Alice; Alice is in advance of the rest of the class. Therefore
(1) Alice will get the prize
(2) Mary will get the prize
(3) One of the other girls will get the prize
(4) Mary will not get the prize
(5) We do not know who should get the prize ....(

39. Which word means the opposite of **Probable?**
(1) unlikely (2) possible (3) certain (4) never (5) always.....................(

40. What must I divide 32 by in order to get twice 4? ....(

41. What fraction comes next in this list?
$\frac{15}{3}$, $\frac{13}{6}$, $\frac{11}{9}$, $\frac{9}{12}$, ... (

42. Which word means the opposite of **Double?**
(1) enlarge (2) halve (3) decrease (4) couple (5) treble......................(

43. Which word does not belong in this list?
(1) swamp (2) slough (3) river (4) bog (5) marsh...............................(

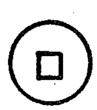
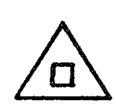
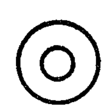
44. What number, if halved, gives us one-third of 24?...(

45. What number comes next in this list?
3, 9, 27, 81,.. (

46. Which word means the opposite of **Answer?**
(1) inquire (2) dictate (3) explain (4) retort (5) reply........................(

47. Which word does not belong in this list?
(1) bark (2) yelp (3) growl (4) bay (5) purr.......................................(

48.  is to  as  is to

(1)  (2)  (3)  (4)  (5)  ....(

9. What number comes next in this list?
3, 5, 13, 15, 23, 25, ... ( )

0. Which word means the opposite of **Vengeance?**
(1) disgust (2) gratitude (3) justice (4) forgiveness (5) jealousy .... ( )

1. It rained yesterday. Tomorrow is Thursday. Therefore
(1) it will rain tomorrow (2) Tuesday was wet
(3) it rained on Wednesday (4) it is raining today
(5) yesterday was Wednesday .... ( )

2. What number is 2 more than the number which 3 is one-half of? .... ( )

3. What number comes next in this list?
2, 3, 5, 8, 12, ... ( )

4. Which word does not belong in this list?
(1) seven (2) nine (3) three (4) four (5) five .... ( )

5. **Room** is to **Door** as **Field** is to
(1) gate (2) farm (3) wheat (4) fence (5) plough .... ( )

6. What number comes next in this list?
92, 97, 72, 77, 52, 57, ... ( )

7. What is the number one-third of which is 9? .... ( )

8. I have three packets of mixed seed—L, M, N. All the varieties of seeds in packet M are also in packet L, but L has varieties that M does not contain. Packet N has seeds that are in neither L nor M. If I wish to grow as many varieties of seeds as possible I can give away
(1) L (2) M (3) N (4) none .... ( )

9. Which word means the opposite of **Acquire?**
(1) lose (2) borrow (3) accept (4) receive (5) detain .... ( )

0. **Was** is to **Now** as **Yesterday** is to
(1) tomorrow (2) hour (3) after (4) today (5) soon .... ( )

1. It is 16 feet around the edge of my table. If it is 3 feet wide, how many feet long is it? .... ( )

2. **Gaudy** means the same as
(1) worthless (2) expensive (3) showy (4) noisy (5) clumsy .... ( )

3. How many sheets of tin 3 inches by 5 inches can be cut from a sheet 15 inches by 12 inches? .... ( )

4. **Stand** is to **Sit** as **Sit** is to
(1) fly (2) walk (3) rest (4) run (5) lie .... ( )

5. What number comes next in this list?
1, 4, 9, 16, 25 .... ( )

GO ON TO PAGE 5

A **PAGE 5**

66. Multiply each of the numbers, 9, 8, by a number 7 less than itself. What is the sum of the two products?.. (

67. **Mouse** is to **Mice** as **He** is to
(1) they (2) we (3) us (4) him (5) she..(

68. The faces of a cube are numbered, 1, 2, 3, 4, etc. What is the sum of all the numbers on the faces?.. (

69. An **Adversary** is
(1) a misfortune (2) an opponent (3) a gossip (4) a counsellor
(5) a superior .... (

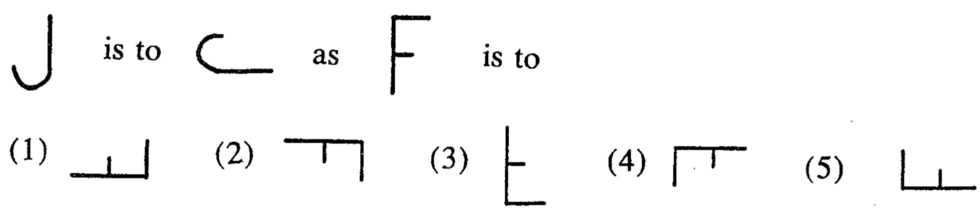
70. Jack is one year older now than Jim was 2 years ago. Jack is 7. How old is Jim?....(

71. Which word means the opposite of **Cautious?**
(1) rash (2) confident (3) severe (4) quick (5) angry ....(

72. What number comes next in this list?
5, 6, 8, 12, 20.. (

73. I spent half of my money and one-third of the remainder. How many cents have I left if I had 84 cents at first?.. (

74.



...................(

75. If the day before yesterday was the day after Tuesday, the day after tomorrow will be
(1) Thursday (2) Friday (3) Saturday (4) Sunday (5) Monday...........( )

END OF TEST

RADE X　　　　　　　　MATHEMATICS　　　　　　　　December, 1961.

alues: - $2\frac{1}{2}$ hours

Name ______________________ Rm.____

Subj. Teacher ____________________

Answer Question 1 in the spaces provided on this paper. Questions 2 - 7 will be answered on Foolscap. Show all work necessary to get each answer in Questions 2 - 7.

1. (a) Add: $x^2 + xy + 2y^2$ and $2x^2 - 3xy - 2y^2$ ____________.

   (b) Subtract: $a^2 - 2a + 1$ from $4a - 1$ ____________.

   (c) Multiply: $+3x^2 + 6x - 5$ by $-2x^2$ ____________.

   (d) Divide: $a^4b - 3a^3b^2$ by $-a^2b$ ____________.

   (e) Collect like terms: $6x - 2y - 7x + 12y - 2x$ ____________.

   (f) Expand: $(2a - 5)^2$ ____________.

   (g) Remove parenthesis and combine like terms:

   $$4x + [3x - (y + 6)] - [x - (3y - 2)]$$ ____________.

   (h) Solve for t: $A = p + prt$ ____________.

   (i) Multiply: $3y(-2x^3y^2)^2(-xy)^3$ ____________.

   (j) If $x = y - \frac{1}{y^2}$, and y is a positive number, state whether x increases or decreases as y increases. ____________.

2. Write algebraic equations for the following. Do <u>not</u> solve.

2　(a) The product of a certain number and 7 is equal to 5 more than twice the number.

   (b) The square of the sum of "p" and "q" is 10 less than "r".

   (c) If the square root of "m" is subtracted from "a", the result is the same as the sum of "a" and "3b".

3. (a) Two variables are related as shown in the following table of values. Find the equation which relates these.

3

x	0	2	4	6	8
y	2	5	8	11	14

5　(b) On graph paper, draw a graph for $y = 2x + 1$. Use only positive values of x.

4\. (a) If $x = -2$, $y = 3$ and $z = -1$, find the value of

x2 $$3x - 2xy - xyz$$

(b) If $u = 10$, $a = -32$ and $t = 3$, find d in

$$d = ut - \frac{1}{2}at^2$$

5\. Solve the following equations: <u>Solve for "x"</u>

3 (a) $2(x - 3) - 4(x - 2) - 6(x - 1) = 0$

3 (b) $S = \frac{ab - x}{a - 1}$

3 (c) $\frac{3x}{4} - 4 = \frac{2x}{3} + 6$

3 (d) $2x - 2m = -5(m - x)$

6\. A man invested \$5000, part at 8% and part at 5%. The interest on the 8% investment exceeded the interest on the 5% investment by \$205.00. How much did he invest at each rate?

7\. If a telegram of 13 words costs 73¢ while a telegram of 20 words between the same cities costs \$1.15 what is the rate for the first ten words and the rate for each additional word?

8\. If the first angle of a triangle is three times the second while the third is 5 degrees more than the first what is the number of degrees in each angle?

GEOMETRY

Answer questions 1 - 6 on this paper in the spaces provided. Question 7 will be answered on foolscap.

1\. (a) Lines perpendicular to the same line are __________ to each other.

(b) The acute angles of a right triangle are ____________________

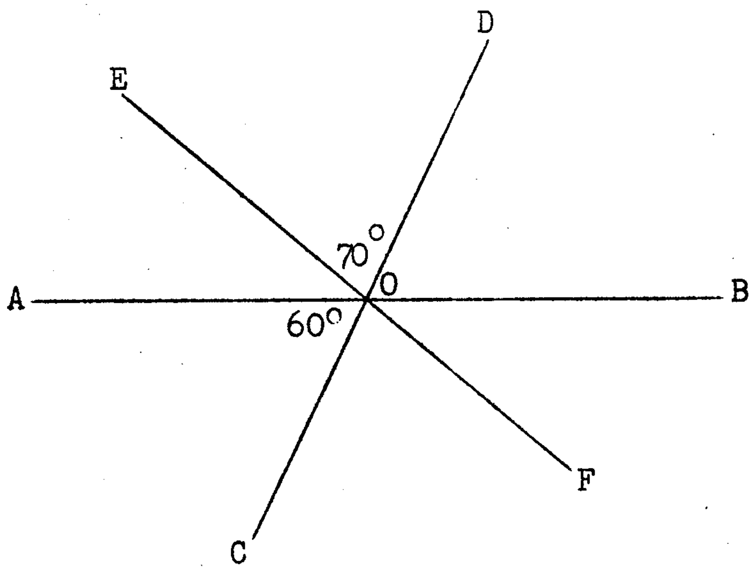
x1 (c) The angles opposite the equal sides of an isosceles triangle are _________________________.

(d) Any four sided figure is called a _________________.

(e) Each angle of an equilateral triangle is equal to _____ degrees.

alues:

2.(c)

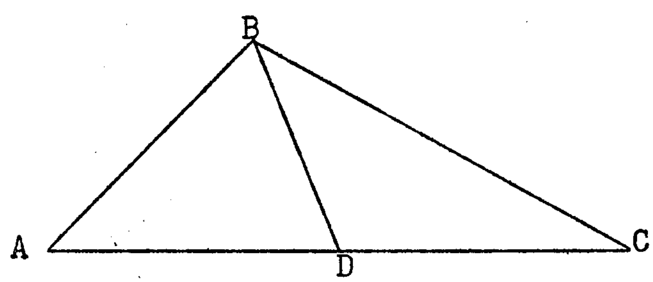


$\angle AOC = 60^{\circ}$

$\angle EOD = 70^{\circ}$

$\angle BOF =$ __________

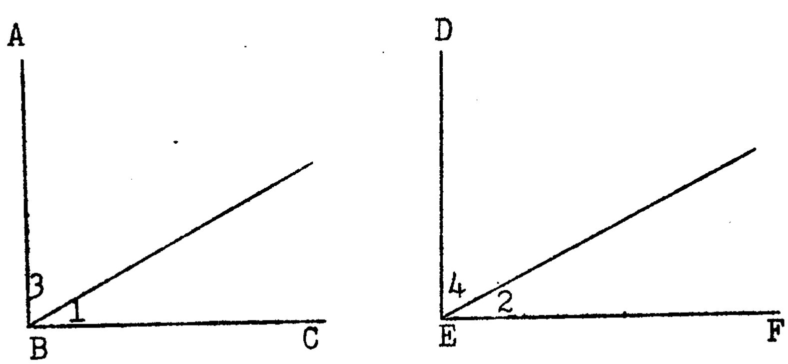
(d) What will be true in the diagram, if BD is a median?



Answer: ________________________

3.

4



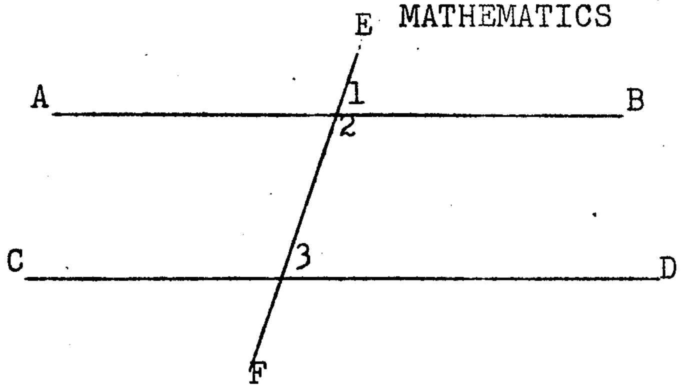
Given: $AB \perp BC$, $DE \perp EF$

$\angle 3 = \angle 4$

Required: to prove $\angle 1 = \angle 2$

alues:

4.



Given: AB, CD and EF straight lines in adjoining figure

$\angle 1 = \angle 3$

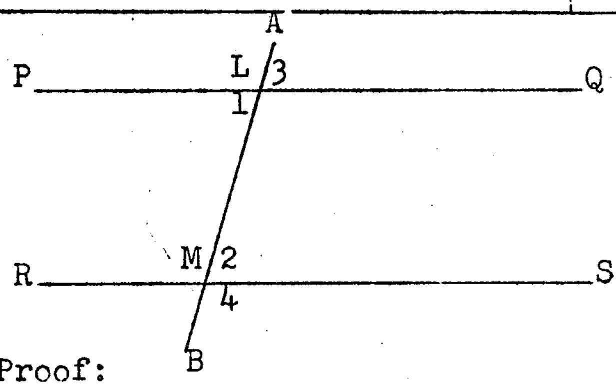
Required to prove

$\angle 2 + \angle 3 = 180^\circ$

Proof:

Statements	Reasons

5.



Given: $\angle 1 = \angle 2$

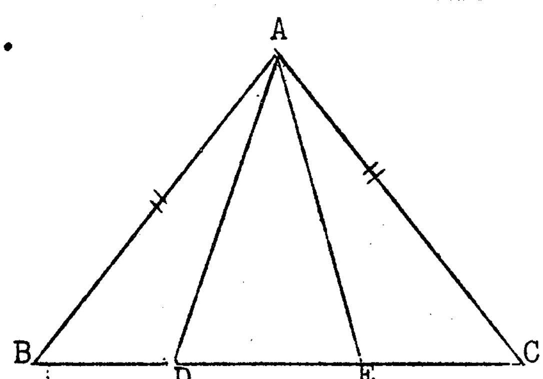
Required: to prove

$\angle 3 + \angle 4 = 180^\circ$

Proof:

Statements	Reasons

6.



Given: $\triangle$ ABC having

AB=AC, BE=DC

Required: to prove

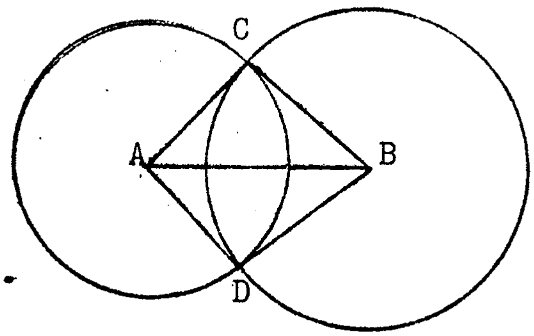
$\angle ADC = \angle AEB$

Proof:

RADE X | MATHEMATICS | December, 196 .

alues:

7.

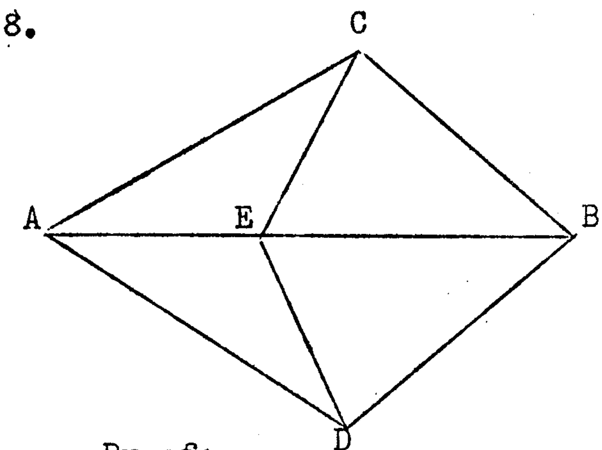


5

Given: Two circles with centres A and B intersect each other at C and D.

Required: to prove △ ABC equals △ ABD in area.

8.



Given: The adjoining figure with AEB a straight line.

AC=AD, BC=BD

Required: to prove EC=ED

Proof:

Statements	Reasons

7

7 9. If two triangles have two sides and the included angle of one respectively equal to two sides and the included angle of the other, then the two triangles are congruent. (DO THIS QUESTION ON FOOLSCAP).

END OF QUESTION PAPER

Grade X Mathematics (Matric) **SISLER HIGH SCHOOL** December 1961

Time 2 hours
Pages 4

Name____________________ Rm. _____

Date____________________

VALUES ALGEBRA

Work all questions neatly on foolscap.

I. Remove brackets and combine terms where possible.

3 1) $2a + [5b - (c - 3a)] - [a - (2b - 3c)]$

2 2) $2g + 3(g - 2h) - 2(2g - h) - 3h$

II Perform the indicated operation in each of the following:

1 1) $-3cd\ (-2c^2d^3)(4c^3d^2)$

2 2) $2a(-3b^2c^3)^2\ (-2a^2b)^3$

2 3) $-2/3b\ (12b^3 - 6b^2 + 9)$

2 4) $(a - b + c)^2$

2 5) $(9m^3 - 19m - 10) \div (3m^2 - 2m - 5)$

3 6) $(5ab^2 + 3a^3 - 3b^3 - 5a^2b) \div (2a - 3b)$

III Find the numerical value of each of the following expressions when

$a = 3$ and $b = 5$

3 1) $\dfrac{6a^2 - 2b}{(b - a)^2}$

2 2) $\sqrt{b^3 - 2a^3 + 2b}$

ade X Mathematics (Matric) -2- December 1961
gebra cont.

Name ___________________________ Rm. _______

ALUES IV FACTOR COMPLETELY

1) $4p^2 + 13p - 12$

14 2) $45a^2 - 20a^2d^2$

3) $r^2 - 2/3r + 1/9$

4) $2a^2 - 7ab + 6b^2$

5) $b^3 + 27$

6) $6t^2 + 11t + 4$

7) $3m^4 - 6m^2 - 45$

4 V The edge of a cube is $(a - b)$ inches long.
Find its total area and its volume.

VI If a train travels $(9t^2 - 3t)$ miles in $3t$ hours, what is its
2 average speed in miles per hour?

VII The area of a rectangle is $15c^2d^3$. The length of the rectangle is $5c^2d$.
4 Find the width and the perimeter of the rectangle.

VIII
The total area of a circular cylinder is represented by
4
$2\pi r^2 - 2\pi rh$. Factor this expression and use the factored form to find the total area of a cylinder whose radius is 8 inches and height 17 inches.
Use $\pi = 3.14$

Grade X Mathematics (Matric) December 1961

Name________________________________ Rm ____

- 3 -

GEOMETRY

VALUES

DO ALL WORK ON FOOLSCAP

1. The path of a point which moves according to a certain restriction or condition is--------------------------.

2. A line drawn through the mid point of another line, at an angle of 90° is called--------------------------.

3. A statement of an important fact which requires proof by a chain of sound reasoning is ----------------------.

10 x 1 4. If a straight line meets another straight line, the adjacent angles are-----------------------.

5. In congruent triangles corresponding angles are opposite-----------.

6. When two straight lines intersect, the two pairs of angles which are not adjacent angles are-----------------------.

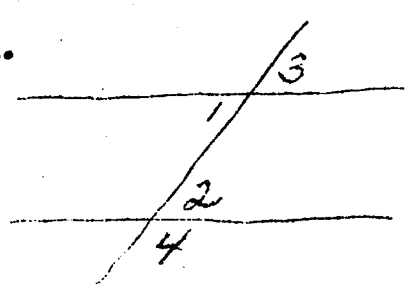
7. The supplement of 2/3 of a right angle is-----------------.

8. If a = 3 and y = 3, why is "a" equal to "y"?---------------.

9. What is equal when $\angle 1 + \angle 4 = 90^\circ$ and $\angle 1 + \angle 5 = 90^\circ$?

10. Two complementary angles differ by 20°. How many degrees are there in each angle?----------

3 11.

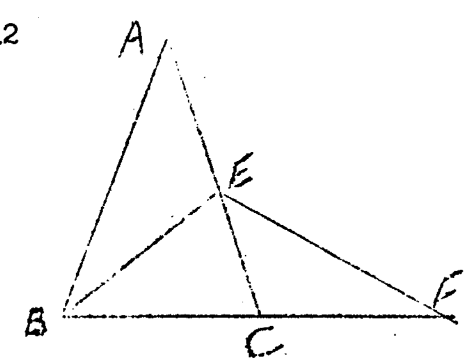


Given $\angle 1 = \angle 2$

Required to prove:

$\angle 3 + \angle 4 = 180^\circ$

5 12

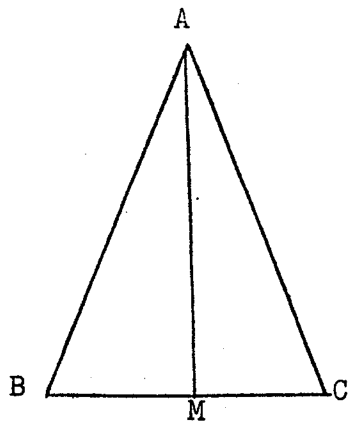


Given: $\triangle$ EBC with
BC = BE, BC = CF
and CE = AE
Required to prove: AB = FE

Grade X Mathematics (Matric) - 4 - December, 1961.

Geometry cont. Name ______________________ Rm. ________

Values 13



Given: $\triangle$ ABC with
AB = AC and
median AM
Required to prove
6 1) $\angle$ BAM = $\angle$ CAM
2) AM $\perp$ BC

14. Prove the following theorems:

6 a) If two triangles have two sides and the included angle of one respectively equal to two sides and the included angle of the other, then the two triangles are congruent.

5 b) The angles at the base of an isosceles triangle are equal.

1 & 2 15. State the two corollaries of the theorem in question 14b.

6 16. PN = PR in $\triangle$ PNR and PW bisects $\angle$ P meeting NR at W. If S is any point on PW, prove $\triangle$ SNR is isosceles.

6 17. AB is a diameter of a circle whose centre is O, and C is any point on the circumference. Join CA, CB, CO and prove $\angle$ ACB = $\angle$ A + $\angle$ B.

CHURCHILL HIGH SCHOOL

NAME: ______________________ RM: ________

No. of Question Papers - 3

TEACHER OF SUBJECT: ______________

Grade X - $2\frac{1}{2}$ hours

DECEMBER, 1961.

VALUES

MATHEMATICS

Part A - Geometry

1. Complete in spaces provided

a) The supplement of 67^{o} is ______________

b) If two a ngles are adjacent, equal and supplementary, each is ______________

c) The equal angles always formed by two intersecting straight lines are called ______________

d) If two triangles are equal in all respects they are ______________

e) A conclusion following readily from a theorem is called a(n) ______________

10x1 f) If AC = AB and AB = 4, then AC = 4. The axiom used here is the axiom of ______________

k

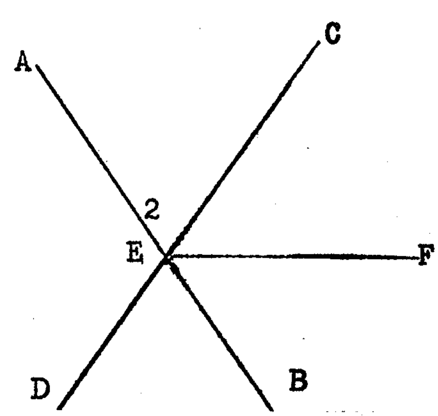
g) A triangle with two equal sides is ______________

h) Angles of 28^{0} and 62^{o} are said to be ______________

i) The path made by a point moving according to certain conditions is called a(n) ______________

j) A self-evident truth is called a(n) ______________

2.



Given: Two intersecting straight lines AB, CD. At E the point of intersection, EF is drawn bisecting $\angle$BEC.

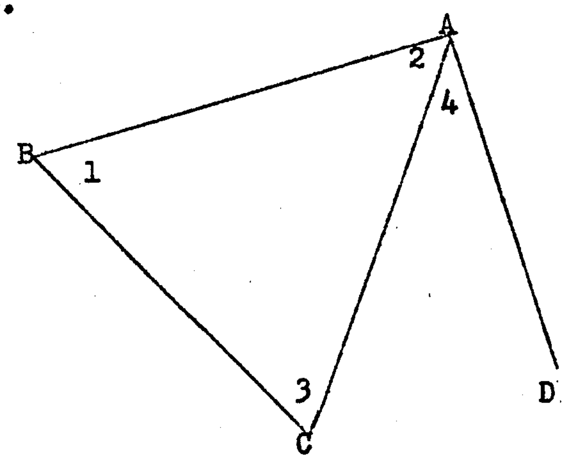
$\angle BEF = 55^{o}$

2x2

$\therefore \angle 2 =$ ______________

$\therefore \angle AED =$ ______________

3.



3x1

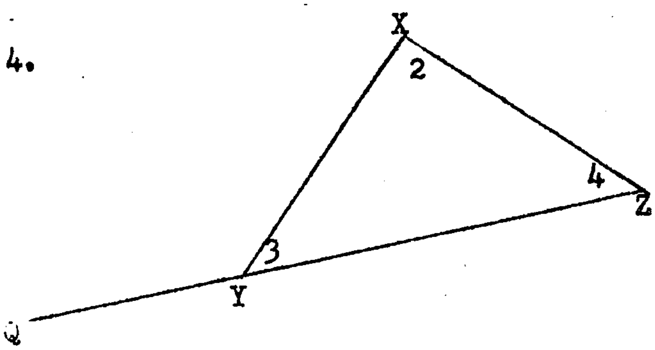
Given: $\triangle ABC$ with $AB = AC$
$\angle 1 + \angle 2 + \angle 3 = 180^\circ$
$\angle 1 = 62^\circ$
$DA \perp AB$

$\therefore \angle 3 =$ ________

$\angle 2 =$ ________

$\angle 4 =$ ________

4.



2+1

Given: $\triangle XYZ$ with ZY produced to Q. $XY = XZ$.
ZX is perpendicular to XY
$\angle 2 + \angle 3 + \angle 4 = 180^\circ$

$\angle 3 =$ ________

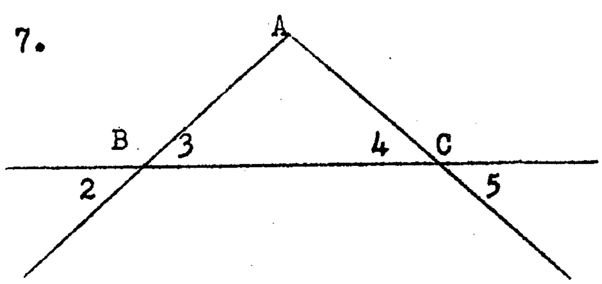
$\angle XYZ =$ ________

5. <u>Remainder of Questions on Foolscap.</u>
<u>Theorem</u>

6 Prove that the base angles of an isosceles triangle are equal.

6. <u>Problem</u> <u>Constuction</u>
Given the acute angle ABC and any straight line with a point E in it.
Construct at E the angle DEF equal to angle ABC.
8 Describe your <u>construction</u> and give a <u>formal</u> <u>proof</u>.

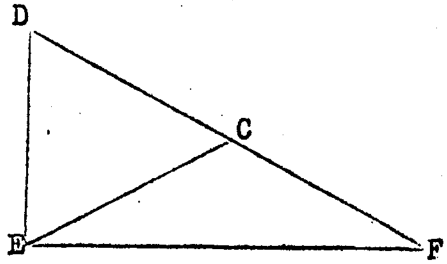
7.



4

Given: $\triangle ABC$ with sides produced as shown.
$\angle 3 = \angle 5$

Required to Prove
$\angle 2 = \angle 4$

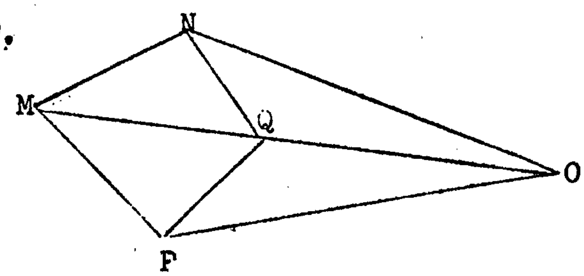
8. 

5

Given: $\triangle$ DEF with C the midpoint of DF
EC = DC

Required to Prove:

$\angle DEF = \angle D + \angle F$

9. 

7

Given: All lines straight
NO = PO
MN = MP

Required to Prove:

1) $\angle NOM = \angle POM$
2) $\angle NQO = \angle PQO$

Part B. Algebra

1. Write the following in algebraic symbols:

a) Subtract three times the square of x from y.

b) The perimeter of a square whose side is b inches.

c) The present age of a boy who x years ago was five years old.

5x1 d) If x-1 is an odd number write the next higher consecutive odd number.

e) The average speed of a plane which flies the three stages of a flight, x, y and z miles long respectively, in h hours.

2.

a) Add $-3x$, $7x$, $-4x$, x.

b) From $-5ab$ take $-6ab$

c) Find the quotient of $16x^2y$ and $-8xy$

d) Multiply $-3mn^2$ by $-7m$

e) Simplify $-4x^2 + 3xy - 2x^2 + y^2 - 2y^2$

f) Simplify $\dfrac{12a^2 b - 8\,ab}{-4a}$

10x1

g) Remove brackets and combine: $-10 - 2(6x \div 15)$

h) Solve: $3 = -y + 2$

i) Solve: $\dfrac{x}{4} = 1$

j) Solve for P if $K = PM - 2$

3. (a) Subtract: $6a^2 - 5ab + 2b^2$ from $7\,ab - 3b^2$

4x2 (b) Add: $2x + 3xy - 4y$, $-7xy + 2y$, $-4x + xy$ and $-x + 3xy$.

(c) Remove brackets and combine

$$4x - \left[3 + 2x - (6x + 7)\right] - (5 - x)$$

(d) When $x = 1$, $y = 2$, $Z = -3$ and $C = 0$

evaluate $\dfrac{\sqrt{xy^2\,z^2}}{xZ - cy}$ (Root sign is for numerator only)

4. Perform the operations as indicated.

a) $(3x^2y)(2x)(-4xy^2z)$

3x2 b) $-5a(-3a^2b + 4b^2)$

2x3 c) $(4x^4y^3 - 2x^3y^4) \div (x^2y^2)$

d) $(3ab^2 - a^2b)^2$

e) $(12x^3 - 14x^2y + 8xy^2 - 2y^3) \div (4x - 2y)$

1x2 5. Solve the following equations

a) $7x - 3 = 8x + 7 - 4x$

3x3 b) $2(a+4) - 6(a-4) = 0$

c) $\dfrac{3x}{4} - 5 = \dfrac{x}{3}$

d) $S = \dfrac{n(a+R)}{2}$ Solve for a

6. Find three consecutive numbers such that the sum of the second and the third is eight less than three times the first. (4)

St. John's High School.
GRADE X
Values:

MATHEMATICS

Easter, 1962.

Name ______ ____________________ Rm.____

Subj.Teacher ____________________

NOTE: DO ALL WORK ON FOOLSCAP

1 I.(a) Simplify: $3a - (5a) + (-a) - (1-a)$

1 (b) Subtract: $a - 25$ from $2a - 1 - a^2$

1 (c) Reduce to lowest terms: $\dfrac{-2x^3y^5}{30x^7y^2}$

1 (d) Simplify: $\left(\frac{1}{3}x^3y^2\right)^3$

1 (e) Expand: $(3p - 7g)^2$

1 (f) Simplify: $(2a^2)^3 + \sqrt{9a^{12}} + (2a^4)(4a^2)$

1 (g) State whether the following set of equations are simultaneous, inconsistent, or dependent:

$$\begin{aligned} 2x + y &= 3 \\ 6x + 3y &= 7 \end{aligned}$$

3 II.(a) Solve for "F" : $C = \frac{5}{9}(F - 32)$

3 (b) Solve for "y" : $\frac{y}{5} - \frac{y}{3} = y - 17$

3 (c) Solve for "x" : $\frac{1}{3}(x + 5) - 4 = \frac{1}{4}(x - 10)$

III.(Factor completely:

3 (a) $16x^3 - 2$

2 (b) $x^2 - 9x - 52$

2 (c) $3x^3y + 9x^2y - 3xy$

2 (d) $9a^4 - 16$

4 IV. Solve graphically: $2y - x = 0$
$3y = x - 1$

Do all the work pertaining to this question on graph paper.

4 V. Solve algebraically: $\frac{1}{2}x + \frac{1}{4}y = 8$
$x - \frac{1}{3}y = 6$

3 VI.(a) Reduce to lowest terms: $\dfrac{a^2 + 2a - 24}{2a^2 - 72}$

4 (b) Simplify: $\dfrac{x^2 + 9x + 18}{6x + 12} \cdot \dfrac{x^2 - 4}{x^2 + 4x - 12} \div \dfrac{x + 3}{36x}$

VII. Solve the following problems algebraically showing all necessary statements:

(a) In 10 years a man will be twice as old as his son, but 8 years
5 ago the man was 8 times as old as his son. Find their present ages.

(b) Mr. Brown wishes to invest part of $10,000 at 6% and the re-
5 mainder at 3% in order that his yearly income will be equal to $4\frac{1}{2}$% of his entire investment. How much should he invest at each rate?

Part 'B' - Geometry

NOTE: Do questions 3 and 8 on foolscap. All other questions are to be done on the test papers in the spaces provided.

1. Fill in the blanks:

(a) Two angles which have the same vertex and are on opposite sides of a common arm are ______________ angles.

(b) An __________ angle of a triangle is equal to the sum of the two
5 interior opposite angles.

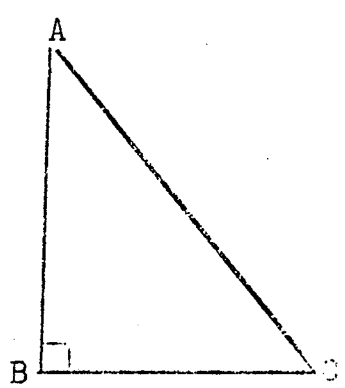
(c) The definition of a parallelogram is: ______________________________

____

(d) The straight line drawn from a vertex of a triangle perpendicular to the opposite side is called ____________________.

(e) If an angle equals its supplement then the angle equals ______ degrees.

2.(a)



2 Given: $\triangle$ ABC having $\angle B = 90^\circ$

$\angle A = (4x - 20)^\circ$

$\angle C = (x + 10)^\circ$

$\therefore \angle A =$ ______$^\circ$

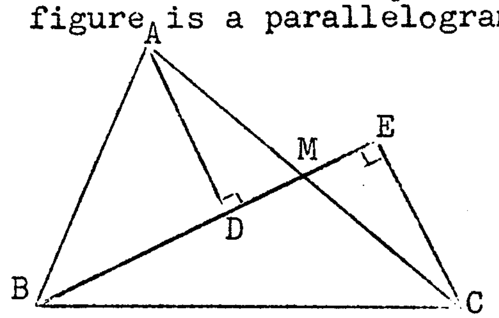
Part 'B' - Geometry cont'd.

3. Prove the following theorems:

(a) If two triangles have two angles and any side of one respectively equal to two angles and the corresponding side of the other, they are congruent. (a.a.s.)

(b) If two sides of a quadrilateral are equal and parallel, the figure is a parallelogram.

4.



Given: $\triangle$ ABC with BM a median produced to E.

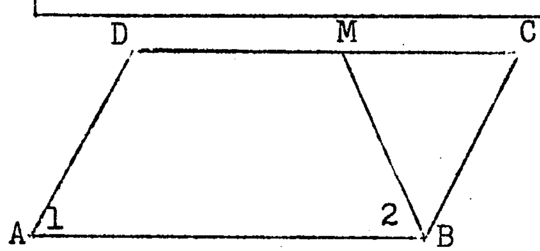
CE $\perp$ BE, AD $\perp$ BE

Prove: AD = CE

Proof:

Statements	Reasons

5.



Given: ABCD a //gm.

$\angle 1 = \angle 2$

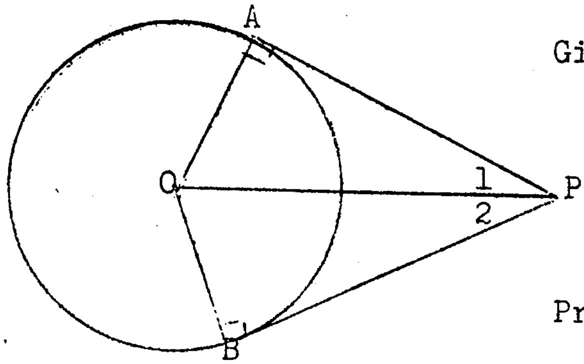
Prove: AD = MB

Proof:

Statements	Reasons

Part 'B' - Geometry cont'd.

6.



Given: PA and PB are straight lines touching the circle, centre O, at A and B.

OA, OB, and OP are joined.

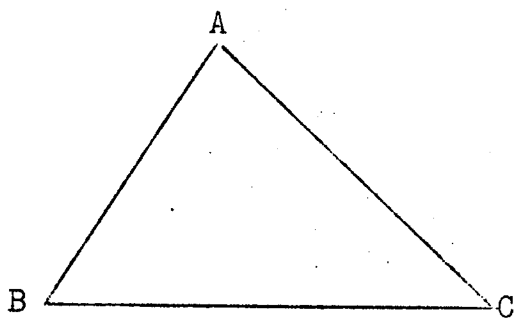
OA $\perp$ AP, OB $\perp$ BP

Prove: $\angle 1 = \angle 2$

Proof:

Statements	Reasons

7.



Given: $\triangle$ ABC

Req'd: to construct a perpendicular from A to BC
Use compasses and straight edge only. Leave all construction lines and arcs. Write out construction but do not prove.

Construction:

____

____

____

____

8. ABCD is a parallelogram. DX bisects angle D meeting AB at X
4 and CB produced at E. Prove AB = BE.

SISLER HIGH SCHOOL

Grade X — MATHEMATICS — Easter, 1962

Time: 2 hours

Values: Name________________________ Rm.___

ALGEBRA

Note: All work in Algebra must be done on foolscap.

10 1. a) What is the difference between $8y$ and $3a$?

b) If $a = 3$ and $b = 4$, find the value of $\frac{3b^2 - 6a^3}{a}$

c) Add $6y - 2z$ and $-5y + 3z$

d) Simplify: $(\frac{1}{3} a^2)^3$

e) How many inches are there in m ft. and 3n inches?

f) Divide: $-15a^2b^5$ by $3ab^3$

g) Multiply: $(3a-b)(2a+b)$

h) Subtract: $2a-5b$ from $7a-3b$

i) The point at which the axes of a graph meet is known as the ___ ___________.

j) Solve for a: $-3 = \frac{20}{3a}$

3 2. a) Divide $12a^3 - a^2 + 18a - 9$ by $4a-3$

3 b) Multiply: $(3p + 4q - 7)(2p + 3q)$

3. Factor fully:

3 a) $9b^2 - 36$

2 b) $8c^3 + 27d^3$

2 c) $x^2 - xy - 20y^2$

2 d) $2x^2 - 5xy - 3y^2$

2 e) $a^6 - \frac{1}{8} b^3$

2 4. Find the equation showing the relation between r and s in the following tables:

r	1	2	3	4	5
s	4	7	10	13	16

3 5. Solve for m: $\frac{2(m+1)}{3} = 5 - \frac{3m}{2}$

4 6. Solve graphically: $2x - y = 7$
$3x + y = 3$

4 7. Solve, but not graphically, for x and y: $3x + 4y = -1$
$2x - y = -8$

2 8. Find the value of s in the formula $g = \frac{2s}{t^2}$, when $g = 32$ and $t = 7$.

4 9. Mr. Jones invested $8000, some at 6% and the remainder at 4%. If the total income from the investments was $370, how much money did he invest at each rate of interest?

4 10. A merchant mixed some candy priced at 90 cents a pound with candy priced at 40 cents a pound. He sold 50 pounds of this mixture for $35.00. How many pounds of each kind of candy did he use?

GEOMETRY:

I. All work and answers in Geometry must be shown on foolscap.

5x1 I. a) In a right angled triangle, the side opposite the right angle is the ____________________.

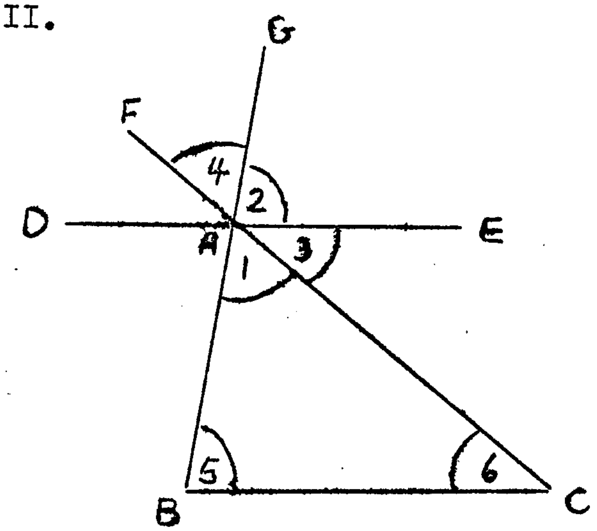
b) A four sided figure with only two sides parallel is a(n) ____________________.

c) A straight line drawn from a vertex of a triangle perpendicular to the opposite side is a(n) ____________________.

d) A triangle with unequal sides is a(n) ________________ triangle.

e) Lines perpendicular to the same straight line are ________________ to each other.

3x½ II.



Given: $\triangle ABC$
DAE $\parallel$ BC
$\angle 5 = 80°$
$\angle 4 = 60°$

Required: Find the size of the following angles and give reasons for your answers.

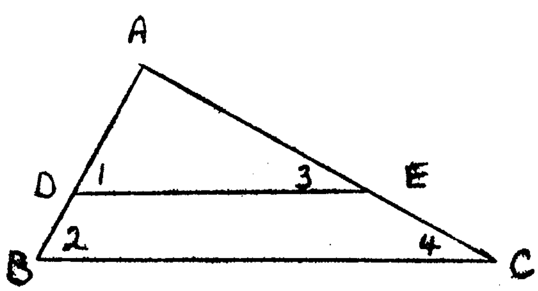
Name of Angle	Size	Reason
$\angle 1$	______°	________________________
$\angle 6$	______°	________________________
$\angle 3$	______°	________________________
$\angle 2$	______°	________________________

III. "If two triangles have two angles and any side of one respectively equal to two angles and the corresponding side of the other, they are congruent. (a.a.s.)"

Write out fully as a theorem.

6 IV. "To draw a perpendicular to a given straight line from a given point on the straight line." Write up this Problem under the four headings: Given, Required, Construction, and Proof.

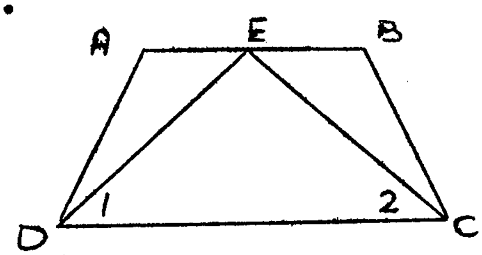
3 V.



Given: $\triangle$ ABC Points D and E are joined.
$\angle 1 = \angle 2$

Required: To prove $\angle 3 = \angle 4$

5 VI.



Given: Quadrilateral ABCD with $\angle A = \angle B$, $\angle AEC = \angle DEB$
E is a midpoint of AB

Prove: $\angle 1 = \angle 2$

6 VII. Two medians of a given triangle are equal and intersect so as to form an isosceles triangle with the base. Prove that the given triangle is isosceles.

7 VIII. <u>NOTE</u>: Mr.   classes will answer VIII (b) only.
All other classes answer VIII (a) only.

(a) Prove that the bisectors of the opposite angles of a parallelogram are equal.

ISLER HIGH SCHOOL, Easter 1962 (Page 4)

(b) PQ = PR in $\triangle$PQR. If the bisectors of $\angle$'s Q and R meet at W, prove that $\triangle$WQR is isosceles.

7 IX. Prove that any point equidistant from the arms of an angle, must lie on the bisector of the angle.

CHURCHILL HIGH SCHOOL
No. ~~of~~ Question Papers 3

NAME ______________________ RM ____
SUBJECT TEACHER ______________________

EASTER 1962

GRADE X
Time: 2 hours
Values:

MATHEMATICS
GEOMETRY

1. Each indicated blank requires a word or number to complete the statement.

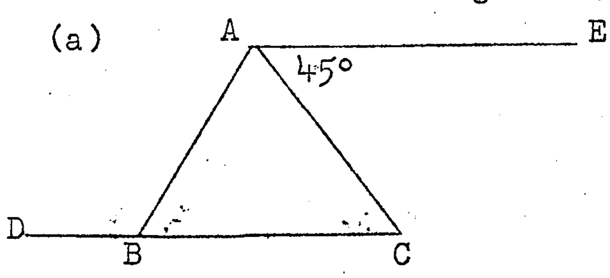
(a) A(n) __________ is a straight line joining the opposite vertices of a polygon.
(b) A parallelogram with one right angle is a __________
(c) Two triangles are congruent if three __________ of one equal three __________ of the other.
(d) A(n) __________ is a straight line from the vertex of a triangle perpendicular to the opposite side.

10 x 1

(e) The exterior angle of a triangle is __________ __________ either of the two __________ __________ angles.
(f) The supplement of a 79° angle is __________ °
(g) Lines which are the same distance apart throughout their length ~~line~~ are __________.
(h) The median drawn from the right angle of a right triangle to the hypotenuse __________ __________ the hypotenuse.
(i) The acute angles of a right triangle are __________.
(j) All points on the bisector of an angle are __________ from the arms of the angle.

2. Calculate the angle sizes as required. No reasons required.

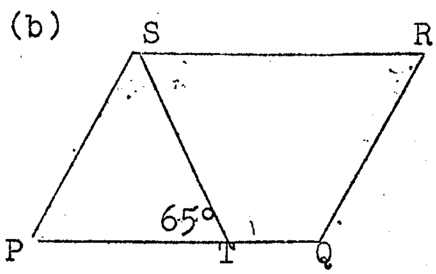
(a)



Given: △ABC
AB = AC
AE // DC
∠EAC = 45°
∴ ∠ABD = °

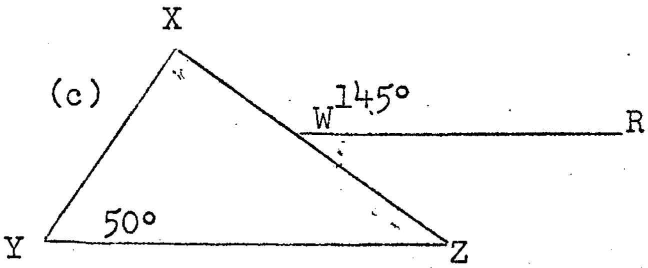
3 x 2

(b)



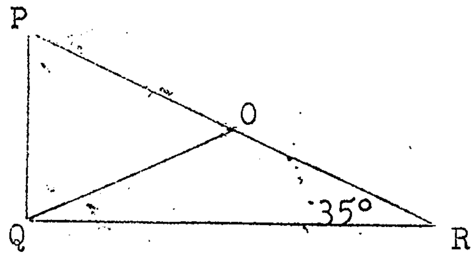
Given: PQRS a parallelogram
SP = ST
∠STP = 65°
∴ ∠RQT = °

(c)



Given: $\triangle XYZ$
$WR \parallel YZ$
$\angle XYZ = 50°$
$\angle XWR = 145°$
$\therefore \angle YXZ = \quad °$

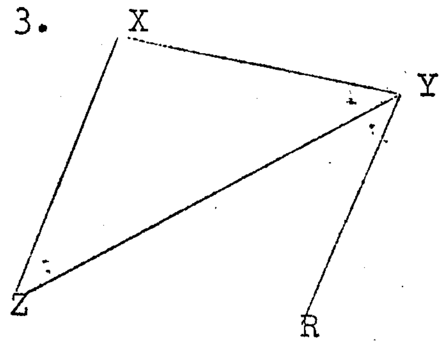
(d)



Given: PQR a right triangle
QO is a median
$\angle ORQ = 35°$
$\therefore \angle POQ = \quad °$

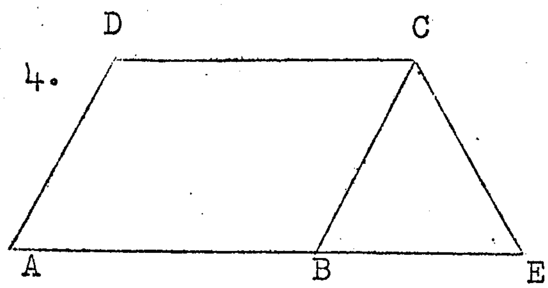
The remaining problems require statements and reasons.
To be done on foolscap.

4 3.



Given: $\triangle XYZ$
$XZ = XY$
$XZ \parallel YR$
Required to prove ZY bisects $\angle XYR$

4 4.



Given: Quadrilateral ABCD
with $AB = DC$
and $AB \parallel DC$
$\angle CBE = \angle CEB$
Required to prove: $AD = EC$

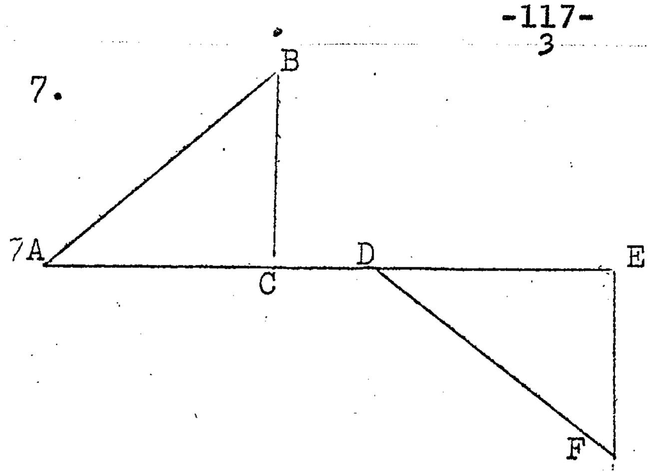
5. Given any straight line AB and P a point outside AB. With ruler and compasses only construct a straight line through P parallel to AB.

4 All necessary construction marks are to be shown clearly. You are not required to describe the construction, but prove that the line you have made is parallel to AB.

8 6. If two right triangles have the hypotenuse and a side of one respectively equal to the hypotenuse and a side of the other, the triangles are congruent (H.S.) Prove.

7.

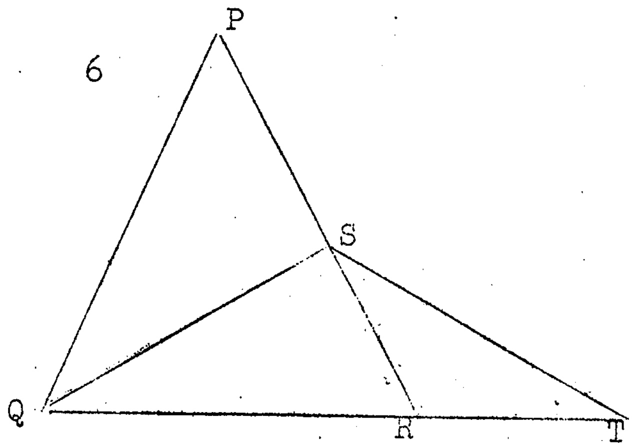
5



Given: ACDE a straight line
FE $\perp$ AE
FE // CB
AB = DF
AD = CE
Prove $\triangle$ ABC $\cong \triangle$ DEF

8.

6



Given: $\triangle$ PQR in which
PQ = PR
$\triangle$ RST in which
RS = RT
QS bisects $\angle$ PQR

Required to Prove:
$\triangle$ QST is isosceles

Algebra

Values: - 3 -

All questions to be done on foolscap.

1. (a) From $2a^2 - 5b + 6$ take $-a^2 + 6$ ✓

(b) $A = \frac{1}{2}bh$. Find value of b when A = 15, h = 5

(c) Remove brackets and combine $-4x - [2 - (7x + 3)]$

(d) If $x - 1$ is an even integer write the next higher even integer.

(e) Simplify $\dfrac{8x^2y - 4xy}{-4xy}$

10 1 (f) Multiply: $-3m^2n(5m - 2n^2 + 1)$

(g) A man's present age is x years. One year from now he will be six times older than his son. Write an algebraic expression for his son's present age.

(h) Expand $(\frac{1}{3}x^2y^3)^2$

(i) Express in feet the sum of 2x yards, 3x feet and 12 inches.

(j) Solve for x if $\dfrac{6}{x} = 12$

4

2. Solve the following:

2 (a) $\frac{4x}{3} + 4 = 3(x-2)$

3 (b) $\frac{m-5}{4} - \frac{4m}{3} = 2$

2 3. (a) When $x = -2$, $y = 1$, $z = o$ Evaluate: $\frac{3x^2y - 4zxy}{xy^2}$

2 (b) Multiply $(3y-x)(y + 7x)$

3 (c) Divide $(2a^3 - 11a^2 + 16a - 6)$ by $(2a - 3)$

5 4. Solve graphically

$$3x - y = -5$$
$$x + y = 1$$

4 5. Solve the simultaneous equations

$$2x - 3y = 9$$
$$5x - y = 16$$

6. Factor the following

2 x 2 (a) $32m^2 - 18$

(b) $y^2 - 6xy + 9x^2$

2 x 3 (c) $3p^2 - 10\ pq - 8q^2$

(d) $x^3 - 8$

4 7. \$4500 was invested, part at 5% and the remainder at 4%. If the annual income from both investments is \$205, how much was invested at each rate?

8. 3 bushels of wheat and 5 bushels of barley cost \$5.30. At the same price 2 bushels of wheat and 3 bushels of barley cost \$3.30. Find the cost per bushel of each grain.

GRADE X — MATHEMATICS — JUNE, 1962.

ALGEBRA AND GEOMETRY

VALUES

15 1. One mark each.

a. Combine similar terms:
$4a + 2b - c + 2a + 5c - 3b + 2c$.

b. Subtract $12x^2$ from $-3x^2$.

c. Multiply $7\,ab$ by $-3a^2$.

d. State the number of yards in x yards and y feet.

e. Evaluate $P^3 + 8$, when $P=4$.

f. Divide $\frac{3}{y}$ by $\frac{6}{y}$.

g. Reduce to lowest terms $\frac{M^3}{M^3 + M^2}$.

h. Combine $\frac{1}{2z} - \frac{1}{z^2}$.

i. Simplify $\frac{-24a^2b}{4a}$.

j. Simplify $\left(\frac{3a^2}{2b^3}\right)^3$

k. Solve for X: $\frac{M}{Y} = \frac{M}{Y}$.

l. If $a = -2$, what is the value of $2a^3 + 16a^0$.

m. If $n - 2$, is an odd integer what is the next high integer?

n. Expand $(3x - 2y)^2$.

o. Simplify $3m - [-(2m - n) + (m + 2n)] - m$.

2. Factor completely:

2 a. $16 - 6y - y^2$.

2 b. $4x^2 - 9y^2$.

3 c. $40x^3 - 5$.

2 d. $2a^3y - 4a^2xy + 2axy$.

3. Simplify:

3 a. $\frac{m^2 + 2mn + n^2}{m - n} \cdot \frac{m^2 - 2mn + n^2}{m + n} \div \frac{m^2 - n^2}{mn}$.

4 b. $\frac{3}{x^2 - 4x + 4} - \frac{2}{x^2 - 5x + 6} - \frac{1}{x^2 - 4}$.

VALUES

4. Solve:

2 a. $\dfrac{R - 12}{10} + \dfrac{3R - 6}{2} = \dfrac{3(2R + 11)}{5}$.

2 b. $2(2 - b)^2 = b(4 - 2) + 2b^2$.

4 5. Solve graphically:

$b = 2a + 3$

$b + a = 6$

3 6. Divide $24x^3 + 28 - x - 47x^2$ by $4 - 3x$.

4 7. A box contains 30 coins which have a total value of $4.50. If the coins are all quarters and dimes how many are there of each kind?

4 8. Divide $300 among A, B, and C so that B receives $20 more than A and C receives two-thirds as much as A and B together.

G E O M E T R Y

5 1. One mark each. Fill in the blanks.

a. The ________________________ of a pair of vertically opposite angles form a straight angle.

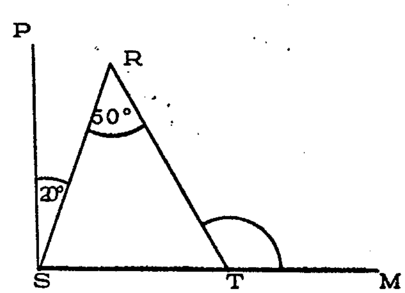
b. A ________________________ is a statement of important fact which requires proof by a chain of sound reasoning.

c. The diagonals of a rhombus ________________________ each other.

d. A line perpendicular to one of two parallel lines is ________________________ to the other.

e. A pair of complementary angles are equal. They are each equal to ________________________ degrees.

3 2. In the diagram $PS \perp STM$



$\angle R = 50°$
$\angle PSR = 20°$

Find the size of $\angle RTM$ ________________________°
(Show all work.)

VALUES

3 3. In $\triangle ABC$, $\angle BAC=90°$

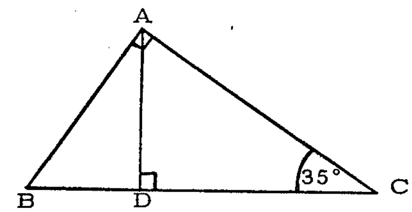
$AD \perp BC$, $\angle C=35°$

Find: a. $\angle B=$ ________°

b. $\angle BAD=$ ________°

c. $\angle CAD=$ ________°

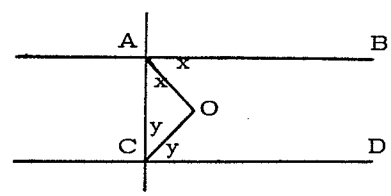
(Show all work.)



3 4. Given AB//CD

$\angle AOC=$ ____________°

(Show all work.)



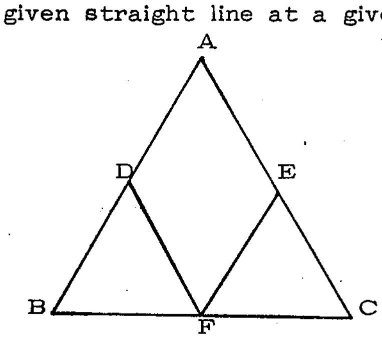
7 5. If two triangles have two angles and any side of one respectively equal to two angles and the corresponding side of the other, they are congruent. (a.a.s.)

5 6. To draw a perpendicular to a given straight line at a given point in it.

5 7. Given $\triangle ABC$
AB=AC

D, E, and F are the midpoints of AB, AC, and BC respectively.

Prove DF=EF

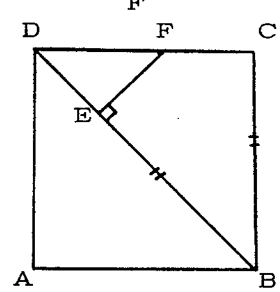


4 8. Given ABCD is a square

BE=BC

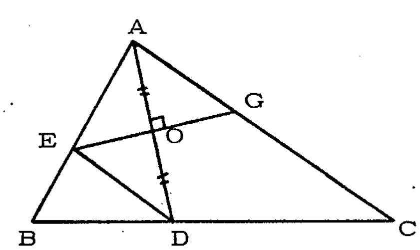
$EF \perp DB$

Prove FC=EF.



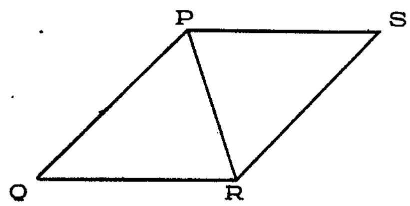
6 9. Given $\triangle ABC$, with AD the bisector of $\angle BAC$ and EOG the right bisector of AD.

Prove ED//AC.



VALUES

6 10. Given // grm PQRS with diagonal PR bisecting ∠ QPS and ∠ QRS.



Prove PQRS is a rhombus.

3 11. Given a diagonal 2" in length, draw a square. Use ruler and compass. (Do not use a protractor).

SCIENCE

GRADE X JUNE, 1962

VALUES

PART A

20 x 1 Complete the following by writing the proper word or words in the blank or blanks provided:

1. All matter is composed of minute particles called ____________.

2. A stretched rubber band is an example of ____________________ energy.

3. A half horsepower motor will do ____________ ft. lbs. of work per minute.

4. The door knob is an application of the ____________________.

5. To melt 10 grams of ice will require ____________ calories.

6. When 10 grams of steam at 100 degrees Centigrade condense to boiling water ____________ calories of heat are liberated.

7. Purifying a liquid by boiling and condensing its vapour is called ______________.

8. When oxygen unites with substances it produces compounds called ____________.

9. The head of a match is made up of a compound called ____________.

10. The gas Argon is used in incandescent bulbs to prevent the filament from _____.

11. A fuse wire should have a ________ resistance and a ________ melting point.

12. The device that makes it possible for a 6-volt battery to produce 15,000 volts across the spark plug gaps in an automobile is called the __________.

13. Alcohol is used in preference to mercury in some thermometers because it has a (n) ______________.

14. When a liquid evaporates, its temperature ________.

15. A substance found to contain only one kind of material is called __________.

16. In an open dish a liquid cannot be heated above its __________.

17. A cubic foot of water at 22 degrees Centigrade weighs _____ than a cubic foot of water at 17 degrees Centigrade.

18. The loss of top soil caused by wind and other weather agents is called _______.

19. The type of vibration produced in one body by another body with the same frequency of vibration is called ______________.

20. The apparatus connected with the toilet that controls the water supply is called the ____________.

2 21. a. Two advantages of a hot water heating system are : ____.

2 b. Two disadvantages of the hot water heating system are: ____.

2 22. What are the two major reasons why minors should not drink alcoholic beverages?

9 23. Match the following word or words in column B with the most appropiate word or words in column A by placing the designating letters of column B in the Parentheses of column A.

VALUES

23.

	Column A			Column B
1.	balloon	( )	a.	cardinal points
2.	log	( )	b.	sextant
3.	latitude	( )	c.	Johnson bar
4.	hypoid gears	( )	d.	nautical mile
5.	locomotive drive wheels	( )	e.	transmission
6.	compass card	( )	f.	differential
7.	knot	( )	g.	cam
8.	mechanical brakes	( )	h.	fire tubes
9.	locomotive boiler	( )	i.	slide valve
			j.	air brakes
			k.	trail rope
			l.	rotator

PART B

10 24. Answer any <u>five</u> of the following:

1. The following procedures were carried out in the laboratory. State what was observed in each case and also give a brief explanation for your observation.

   a. To a test tube half filled with alcohol iodine crystals were added.
   b. Two powders, iron and sulphur, are mixed and highly heated.
   c. A bright tin cup is half filled with ice and then taken into a warm room.
   d. A cold beaker is held, mouth downward, over the flame of a candle for a short time.
   e. A thermometer is placed in a large beaker of ice and water. Heat is applied to the beaker and the temperature is recorded several times until the ice has melted.
   f. A few drops of water are placed in the centre of a large flat cork. A watch glass containing ether is placed on the water. The ether is fanned rapidly.

25. Assume that you have been given the following objective to an experiment:
OBJECTIVE: To discover what happens to the air in which iron rusts.
2 a. State the observations you have made in the experiment.
2 b. State the conclusions to the experiment.

PART C

4 26. Explain with the aid of a labelled diagram how the simple wet cell operates on closed circuit. Indicate with an arrow the direction of the current in the circuit.

3 27. Draw a labelled diagram of the telephone transmitter. Explain briefly how it operates.

3 28. Draw a labelled diagram to illustrate the power stroke of an internal combustion gasoline engine. Explain what happens on this stroke.

1 29. a. Draw a labelled diagram to indicate how a force pump would operate on a downward stroke of the piston.
1 b. What two factors must be considered if the pump is going to pump water?
1 c. What is the essential difference between the structure of a force pump and that of a lift pump?

PART D

In this part ALL CALCULATIONS must be shown.

2 30. A Fahrenheit thermometer reads -4° F. What will be the equivalent reading on a Centigrade thermometer?

VALUES

1 31. Ruth weighs 108 lbs. while Ann weighs 50 Kilograms. Which girl weighs more? How much more? (in lbs.)

1 32. a. Upon what two things does the loudness of a sound depend?
2 b. The sound from a steamship horn was heard 12 seconds after the condensed steam cloud was seen rising above the ship. How far away was the ship?

4 33. William pulls a loaded cart weighing 240 lbs. up an inclined plane 18 feet long. The higher end of the plank is 3 feet above the ground.
a. What force or effort does he exert? (Neglect friction.)
b. If it actually requires a force or pull of 50 lbs. what is the efficiency of the inclined plane?
c. What is the actual mechanical advantage of the machine?
e. What is the theoretical mechanical advantage of the machine?

4 34. John weighs 165 lbs. He can run up a flight of stairs in 4 seconds. The vertical height of the stairs is 16 feet.
a. What power does he develop in Watts?
b. What Horse Power does he develop?

3 35. Fred ran the new motor on his saw 20 hours last month. The motor uses 3 amperes of current on a 110 volt circuit. At a price of 5¢ per kilowatt-hour what did it cost to operate the motor?

1 36. A rowboat weighs 250 libs. What volume of water does it displace whan afloat?

2 37. An anchor from a steamship on the Great Lakes has a volume of 2 cubic feet, and weighs 1000 lbs. What is its weight when submerged?

PART E

1 38. Why are the discoveries made known by careful scientists likely to be true?

3 39. a. What are the three essential ideas regarding the structure of matter according to the "Molecular Theory"?
2 b. How does the following facts support this theory?
2 1. Water evaporates from an open container.
2. Ice melts when heated.

6 40. Explain what is being done to conserve our forests through the following practices:
a. Fire prevention.
b. Disease control.
c. Selective cutting and reforestation.

4 41. Young people consume alcoholic beverages for the following reasons:
a. to be like others.
b. It is considered to be a symbol of maturity.
c. Fear of disapproveal by their friends.
Discuss TWO of these reasons to show that they are not valid for consuming alcoholic beverages.

MATHEMATICS

GRADE XI — (ALGEBRA & GEOMETRY) — JUNE, 1963.

VALUES

PART A — ALGEBRA

1 1. a. Simplify: $(-3x^2y^3)^3$.

1 b. Evaluate: $a^3 - 2a^2 + 2a$ when $a = -2$.

1 c. If $x^2 - 3x + k$ is a perfect square trinomial what is the value of k?

2. Factor completely:

2 a. $(x + 3)(x - 2) + 3x + 9$.

3 b. $a^2 - 9b^2 + 30bc - 25c^2$.

3 c. $x^3 + 6x^2 - x - 30$.

5 3. Simplify: $\dfrac{5x + 1}{10x + 5} - \dfrac{x}{3 - x} - \dfrac{3x^2 - 2x}{2x^2 - 5x - 3}$

3 4. Find the square root of: $34a^2 - 20a + 9a^4 - 12a^3 + 25$.

5. Simplify:

2 a. $3^{-2} - (5a^0)^0 + 2(\frac{1}{2})^2$.

3 b. $\left(\dfrac{49a^{-6}b^8}{81y^4}\right)^{-\frac{1}{2}}$ (Express the answer with positive indices.)

3 c. $3\sqrt{32} + 5\sqrt{98} - \frac{2}{5}\sqrt{50}$.

3 6. Express with a rational denominator and simplify: $\dfrac{\sqrt{3} + \sqrt{2}}{5 + 2\sqrt{6}}$.

7. Solve the following equations:

3 a. Solve for x: $\frac{2}{3}(x - 2) - \frac{3}{5}(2x - 3) = 1 - \frac{1}{2}(x - 1)$.

4 b. Solve for x:

$$\frac{3}{2x} + \frac{2}{3y} = \frac{1}{12}$$

$$\frac{2}{x} - \frac{3}{2y} = \frac{5}{2}.$$

4 c. Solve for x: (Leave your answer in simplest surd form.)

$$\frac{2}{x - 4} - \frac{1}{x - 2} = 2.$$

4 8. Solve for x graphically: $x^2 - 2x - 3 = 0$.

5 9. Solve the following problem algebraically:
Jack bought a number of copies of a particular book and spent \$30 for them. Henry bough a certain number of copies of another book and spent \$30 also. On comparing notes they found that Henry paid \$1 more per book than Jack and that Jack got one book more than Henry. How much did each man pay per book?

PART B — GEOMETRY

1. Complete the following statements:

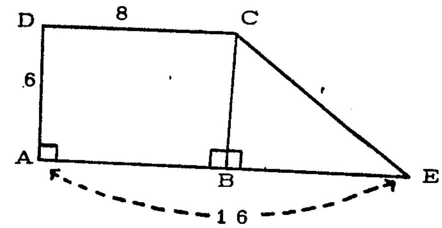
1 a. A median of any triangle divides it into two triangles which are __________

1 b. An exterior angle of a cyclic quadrilateral is equal to the __________

1 c. If a line divides two sides of a triangle proportionally it is __________

VALUES

2. ABCD is a rectangle with AB produced to E. C is joined to E.

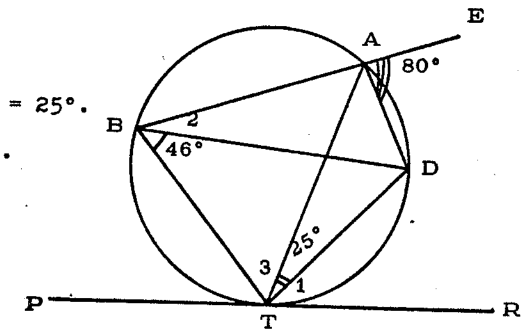


AD = 6 units
DC = 8 units
AE = 16 units

3 x 1 Then:

a. BE = __________ units.
b. Area of $\triangle$BCE = ____ sq. units.
c. CE = __________ units.

3. PTR is a tangent at T. TD, TB, DA, AT, BD and BA are chords as shown, BA is produced to E. $\angle$DAE = 80°, $\angle$DBT = 46°, $\angle$ATD = 25°.



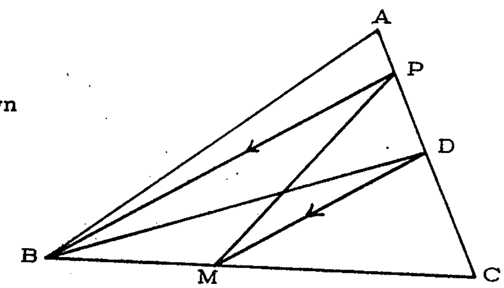
Find the sizes of angles 1, 2, and 3. Give a reason for each answer.

3 x 1 Then:

a. $\angle$1 = __________°
b. $\angle$2 = __________°
c. $\angle$3 = __________°

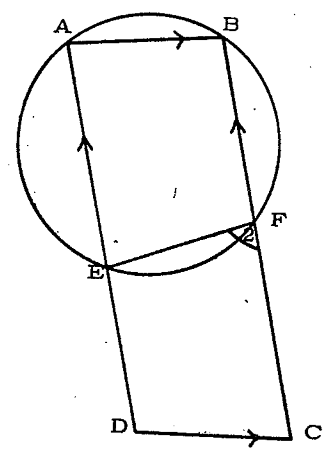
5 4. Given: Triangle ABC with BD a median from B meeting AC in D. P is any point in AD. DM is drawn parallel to PB and meets BC in M. P is joined to M.

Prove: That $\triangle$PMC is equal in area to one-half of $\triangle$ABC.



4 5. Given: ABCD is a parallelogram. A circle is drawn passing through A and B and cutting AD and BC at E and F respectively. E is joined to F.

Prove: That EFCD is a cyclic quadrilateral.



PART C — GEOMETRY

6. Prove the following theorems:

6 a. Equal chords of a circle are equidistant from the centre.

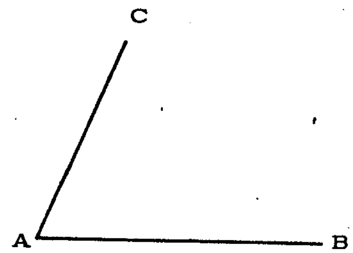
7 b. A straight line parallel to one side of a triangle cuts the other sides proportionally.

GRADE XI MATHEMATICS Continued - 3 - JUNE, 1963.

VALUES

5 7. BAC is an acute angle with arm AB 3 units in length. Locate a point P which will be equidistant from the arms AB and AC of the angle BAC and which, when joined to A and to B will make angle APB a right angle.

(Use compasses and straight edge only. Show all construction lines. Describe the construction but omit any proof.)



7 8. Given: AB is a chord of a circle and C is the midpoint of the major arc AB. P is any point on AB. Join CP and produce it to meet the circle at M.

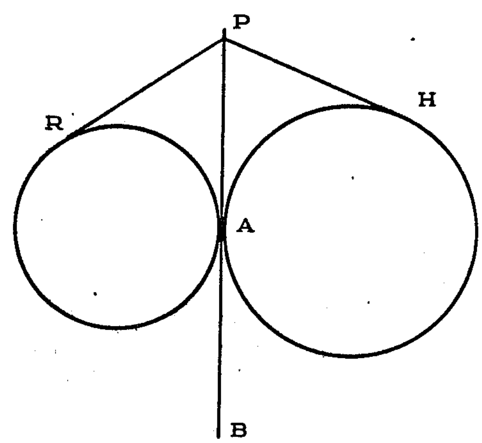
Join AC, AM and BC.

Prove: $\frac{CM}{AC} = \frac{AC}{CP}$.

3 9. Given: Two circles which touch each other externally at A. P is any point on the common tangent at A .

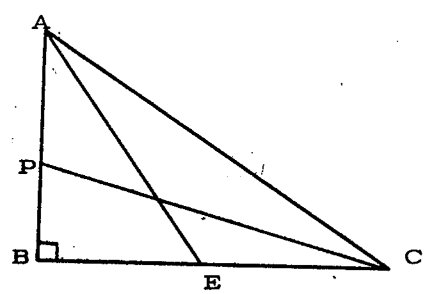
PR is a tangent drawn from P to one circle and PH is a tangent to the other circle.

Prove: That PR = PH.



4 10. Given: $\triangle ABC$ with $\angle B = 90°$. P is any point in AB and E is any point in BC. CP is joined and AE is joined.

Prove: That $\overline{AE}^2 + \overline{CP}^2 = \overline{AC}^2 + \overline{BE}^2 + \overline{BP}^2$.



MATHEMATICS EXAMINATION

An Achievement Test in Problem Solving in High School Mathematics [1]

TEST I

The material examined in Test I deals with: (a) the meaning and language of algebra; (b) fundamental operations of algebra; (c) solving equations; (d) deriving equations from problems; (e) functional relationships; (f) graphical relationships; and (g) solving problems.

The test is divided into three parts, each part represents one of the three objectives upon which measurement of achievement will be made.

Part I: Recall of basic formulae and definitions
Part II: Application of formulae and basic concepts
Part III: Interpretation and comprehension.

DIRECTIONS

This is a test of some of the abilities, skills and understandings necessary to solve mathematical problems. The following instructions will help you secure your best score.

1. Each of the test questions is followed by five suggested answers. Read each question carefully and answer by blackening the space on the answer sheet which corresponds with the answer you selected.

2. Your score will be the number of correct answers you mark. Do NOT spend too much time on any question which seems difficult, make the best selection you can and proceed to the next question.

3. If you finish before time is called go back and reread any questions which seemed difficult.

4. If you change your mind about any answer place an "X" through your original mark and blacken in your new space.

5. Fill in your name, room, school, etc. on the answer sheet, but DO NOT BEGIN THE TEST UNTIL THE EXAMINER TELLS YOU TO DO SO.

Please DO NOT write or put any marks in this booklet.

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1 Based on the following publications:
Daniel W. Snader. Algebra - Meaning and Mastery. John C. Winston and Company, Toronto. 1957. 456 pp.

P. A. Petrie, V. E. Baker, W. Darbyshire, J. R. Levit, W. B. MacLean. Intermediate Mathematics. Copp-Clark Publishing Company, Toronto, 1956. 364 pp.

1. The result obtained in a multiplication question is the:

   A. quotient

   B. remainder

   C. sum

   D. product

   E. addend

2. If the sum of two angles is 90^{o}, the angles are:

   F. adjacent

   G. complementary

   H. equal

   I. supplementary

   J. vertically opposite

3. If the length of a rectangle is represented by "a" and the width by "b", then the area is:

   A. $2a + 2b$

   B. ab

   C. $\frac{1}{2}ab$

   D. $\frac{a}{b}$

   E. $a + b$

4. If each side of a square is "s" units, then the perimeter is:

   F. $4s$

   G. $\frac{s}{4}$

   H. s^2

   I. $\frac{s}{2}$

   J. $2s$

5. A rectangle measures 10 feet by 7 feet. Its perimeter is:

   A. 17 feet

   B. 27 feet

   C. 24 feet

   D. 37 feet

   E. 34 feet

6. A statement whose truth is accepted without proof is:

   F. an axiom

   G. an assumption

   H. a theorem

   I. a conclusion

   J. a corollary

7. A man invests \$100. at 6% per annum for a period of 6 months. The interest on his investment is:

   A. \$ 6.00

   B. \$106.00

   C. \$ 3.03

   D. \$ 3.00

   E. \$ 9.00

8. If two equations in two unknowns have one and only one solution, they are said to be:

   F. dependent

   G. common

   H. parallel

   I. quadratic

   J. simultaneous

9. The formula which gives the rate of a moving object in terms of its time and distance is:

   A. $r = dt$

   B. $r = \frac{t}{d}$

   C. $r = \frac{2d}{t}$

   D. $r = \frac{d}{t}$

   E. $r = dt^2$

10. In the algebraic expression $5x^2$ the 5 is known as:

   F. an exponent

   G. a term

   H. an index number

   I. a power

   J. coefficient

Form A: PART I Page 2

11. A circle has a radius of "r". Its circumference will be:

A. πr
B. πr^2
C. $2\pi r$
D. $2r$
E. $(\pi r)^2$

12. The area of a triangle is "A" square units and the base is "b" units. The altitude will be:

F. Ab
G. $\frac{1}{2}Ab$
H. 2Ab
I. $\frac{2b}{A}$
J. $\frac{2A}{b}$

13. A cube measures "e" units on each side. The area of the faces will be:

A $6e^2$
B. $4e^2$
C. e^3
D. $8e^3$
E. 4e

14. The point where the X axis and the Y axis of a graph meet is called the :

F. the abscissa
G. the origin
H. the coordinate
I. solution
J. vertex

15. If eggs cost "c" cents per dozen, the cost of "x" eggs will be:

A. xc
B. $\frac{x}{c}$
C. $\frac{12x}{c}$
D. $\frac{xc}{12}$
E. 12xc

16. The formula for the volume of a cone is:

F. $V = B.h$
G. $V = r.h$
H. $V = \frac{1}{3}B.h$
I. $V = l.w.h$
J. $V = \frac{1}{4}r.h$

17. The area of a trapezium or trapezoid is expressed by:

A. $\frac{1}{2}\ b.h$
B. $2\ b.h$
C. $\frac{b + b_1}{2}$
D. $\frac{1}{2}\ b.a$
E. $\frac{1}{2}\ h(b + b_1)$

18. When two variables are so related that a change in one produces a change in the other they are called:

F. reciprocals of each other
G. equals
H. functions of each other
I. constants
J. identities

Proceed to PART II

The vertical angle of an isosceles triangle is 70°. Each base angle will be?

A. 65° D. 75°

B. 110° E. 56°

C. 55°

The product of a certain number and 5 is equal to the difference between 7 and the number. The equation which represents this statement is:

F. $x + 5 = x - 7$

G. $5x = x - 7$

H. $x - 5 = 7x$

I. $5x = 7 - x$

J. $5x = 7(x - 7)$

The complement of a certain angle is 36°. The supplement of this angle is:

A. 54° D. 144°

B. 164° E. 216°

C. 126°

If $a\,b\,c = 0$, then:

F. $a = b = c = 0$

G. $a\,b = 0$

H. a or b or c $= 0$

I. $a = 0$

J. none of the letters $= 0$

A classroom of n pupils donated $(x + 10)$ dollars to the Community Chest. The average donation per pupil will be:

A. $\dfrac{x + 10}{2}$ B. $\dfrac{n}{x + 10}$

D. $\dfrac{n}{2}$ E. $\dfrac{x + 10}{n}$

C. $n(x + 10)$

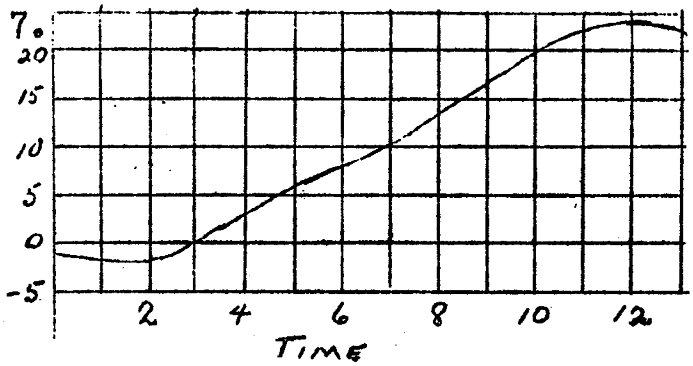
6. For what two positive values of x is the expression $x^2 - 5x + 4$ equal to zero?

F. 5, 2 I. 2, 2

G. 2, 4 J. 4, 1

H. 3, 6

7.



In the temperature graph above how many degrees did the temperature rise between 3 and 10 o'clock?

A. 10° D. 30°

B. -5° E. 23°

C. 20°

8. The width of a rectangle is W and the perimeter is 24. The algebraic expression for the length of this rectangle is:

F. $W - 12$ I. $2(W - 12)$

G. $\dfrac{24 - W}{2}$ J. $24\,W$

H. $12 - W$

9. In $\triangle ABC$, the angle at A is 34°, and angle B is 65°. How large is angle C ?

A. 81° D. 99°

B. 9° E. 261°

C. 91°

10. The product of a^x and a^y is:

F. a^{xy}
I. a^{x+y}
G. $a^x + a^y$
J. a^{2xy}
H. $(x+y)^a$

11. The formula for the distance S travelled by a freely falling object in time t is

$$S = 16t^2$$

How many seconds will an object take to fall 144 feet?

A. 9
D. 48
B. 3
E. 3/4
C. 12

12. The circumference of a circle is 88 inches. Its radius is:

F. 44
I. 277
G. 1078
J. 14
H. 34

13. The quotient of $6a^5b^2$ divided by $2a^3b^2$ is:

A. $3a^2b$
D. $3a^2$
B. $3ab$
E. $3a^3$
C. $4a^3$

14. There are m pupils in a class, of these n are boys. The % of the class who are girls would be represented by:

F. $\frac{m-n}{m} \times 100$

G. $\frac{m}{n} \times 100$

H. $\frac{n-m}{m} \times 100$

I. 100 mn

J. $\frac{n}{m} \times 100$

15. If t is the ten's digit and u is the unit's digit of a certain two digit number, then the value of this number is:

A. tu
D. $\frac{t}{10} + u$
B. $10t + u$
E. cannot be expressed
C. $10(t + u)$

16. The temperature on the Centigrade scale reads 70° C. The reading on the Fahrenheit scale will be:

F. 36°
I. 273°
G. 100°
J. 158°
H. 126°

17. If a chord is drawn in a circle so that its length is equal to the radius, then the triangle formed by this chord and the two radii will be:

A. isosceles

B. equilateral

C. right angled

D. scalene

E. inscribed

18. If two scouts can paddle a canoe up a river a distance of $(d-4)$ miles in one hour, how many hours will it take them to paddle 28 miles?

F. 7 hours

G. $\frac{d-4}{28}$

H. $\frac{28}{d-4}$

I. $28\ (d-4)$

J. $d - 7$

PART III

Page 5

1. If air is a mixture of 4 parts nitrogen and 1 part oxygen, how many cubic feet of nitrogen are there in a room 30 feet by 20 feet and 10 feet high?

   A. 1200
   B. 1500
   C. 6000
   D. 4800
   E. 120

2. The coordinates of the point at which the graph of $2x + y = 11$ meets the graph of $x - y = -2$ are:

   F. 2, -4
   G. 3, 5
   H. 4, 3
   I. 7, -3
   J. 5, -7

3. A man invests $200. and at the end of one year receives $9.00 interest. If the ratio between the amount invested and the interest remains the same how much interest would he receive if he invested $360?

   A. $12.60
   B. $126.00
   C. $16.20
   D. $72.00
   E. $160.00

4. A storekeeper offers an overcoat at a price of $36.00 which is 20% less than the regular price. What is the regular price?

   F. $43.20
   G. $42.00
   H. $50.40
   I. $45.00
   J. $28.80

5. The sum of a number and the number squared is equal to four times the number increased by 4. The equation for this statement is:

   A. $x + x^2 = 4x + 4$
   B. $x + x^2 = 4(x+4)$
   C. $(x + x)^2 = 4x + 4$
   D. $(2x)^2 = 4(x + 4)$
   E. $x + x^2 = 4x - 4$

6. The sum of two numbers is 32. The ratio of these numbers is 3 : 5 . The smaller number is:

   F. 13
   G. 15
   H. 20
   I. 16
   J. 12

7. On a five hour trip, Mr. Smith travels for two hours at a certain rate of speed and for the last three hours at a rate which is 5 miles per hour faster. The total distance hr travels is 150 miles. To find his original speed which of the following equations would you use?

A. $2x = 3x + 5$

B. $2x + 3x + 5 = 150$

C. $2x + 3(x + 5) = 150$

D. $x + (x + 5) = 150$

E. $\frac{150}{x} + \frac{150}{x+5} = 1$

8. The difference between one third and one fifth of a certain number is 11 less than one half of the number. What is the number?

F. 12
G. 16
H. 24
I. 30
J. 48

9. A boat can travel 6 miles per hour in still water. If the rate of the current in a river is "x" miles per hour, the speed that the boat travels upstream will be:

A. $6 - x$ m.p.h.
B. $x - 6$ m.p.h.
C. $6x$ m.p.h.
D. $60(x - 6)$ m.p.h.
E. $x + 6$ m.p.h.

10. If x and y are related as in the following table of values

x	1	2	3	4	5
y	5	7	9	11	13

the equation for their relationship is:

F. $y = 3x + 2$
G. $y = 2x + 3$
H. $y = 6x - 1$
I. $y = 2x - 3$
J. $y = x + 4$

11. A bank contains nickels, dimes and quarters. There are twice as many dimes as nickels but only one half as many quarters as nickels. The total value of the coins is $3.00. How many nickels are there?

A. 6
B. 16
C. 10
D. 5
E. 8

12. One parallel side of a trapezium is 4 feet greater than the altitude while the other parallel side is 2 feet greater than the altitude. Find the altitude of a trapezium whose area is 54 square feet.

F. 8 ft.
G. 9 ft.
H. 12 ft.
I. 6 ft.
J. 10 ft.

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13. The cost of operating a light bulb was found to be directly proportional to the length of time it was in use and to the rate at which it uses electricity. Here is a chart for 3 bulbs for one week:

25 watt bulb for 96 hours
40 watt bulb for 60 hours
100 watt bulb for 24 hours

Which bulb would cost the most to operate for 1 week?

A. 25 watt bulb
B. 40 watt bulb
C. 100 watt bulb
D. cannot tell
E. all cost the same

14. Jack's grandmother is now three times as old as Jack. Ten years ago she was four times as old as Jack. Their present ages are:

F. 10 and 30 years
G. 30 and 90 years
H. 20 and 60 years
I. 15 and 45 years
J. 19 and 57 years

15. An acid bottle in the laboratory contains 4 ounces of 15% acid solution. How much pure acid must be added in order to change the solution to 32% strength?

A. 6 ounces
B. 10 ounces
C. 1 ounce
D. 2 ounces
E. 8 ounces

If you are finished before TIME is called, go back and reread any question that you have found difficult.

THE END

APPENDIX B

PROBLEM ANALYSIS FORMS

PROBLEM ANALYSIS SHEET

No.	The Unknown	Related Unknown	Geometric Diagram	Formula (if any)	Algebraic Equation	Estimate of Answer

APPENDIX C

TABLES OF TEST SCORES

AND

STATISTICAL SUMMARIES

TABLE 4

ACHIEVEMENT OF EXPERIMENTAL GROUP
ON MATHEMATICS TESTS AND PROBLEM SOLVING
(PILOT STUDY)

Student	I.Q. Dom.Test	Gr.IX Maths.	S.T.E.P. Conv.Score	Criterion Posttest
1.	90	67	278	18
2.	93	55	275	19
3.	103	55	260	16
4.	108	69	283	28
5.	119	64	284	25
6.	99	55	278	19
7.	109	50	284	23
8.	106	67	278	27
9.	140	60	286	28
10.	96	60	268	11
11.	111	59	274	26
12.	111	64	284	21
13.	82	69	282	23
14.	100	55	289	22
15.	112	54	265	18
16.	111	93	283	28
17.	114	59	282	19
18.	92	50	274	25
19.	102	79	284	23
20.	105	50	282	20
21.	106	50	278	23
22.	138	72	278	20
23.	96	68	282	25
24.	107	65	290	18
25.	115	64	275	28
Mean Score:	106.60	62.12	279.04	22.12

TABLE 5

ACHIEVEMENT OF CONTROL GROUP
ON MATHEMATICS TESTS AND PROBLEM SOLVING
(PILOT STUDY)

Student	I.Q. Dom.Test	Gr.IX Maths.	S.T.E.P. Conv.Score	Criterion Posttest
1.	120	51	282	19
2.	122	55	290	20
3.	122	64	283	29
4.	85	59	280	22
5.	118	72	283	23
6.	90	47	275	19
7.	110	72	285	24
8.	118	54	280	20
9.	113	63	283	25
10.	111	79	285	24
11.	86	70	274	15
12.	125	50	291	21
13.	112	62	297	34
14.	100	57	280	23
15.	99	51	260	19
16.	105	50	287	29
17.	92	66	284	23
18.	99	78	284	23
19.	140	97	296	37
20.	108	76	282	28
21.	110	58	296	30
22.	111	75	283	24
23.	98	55	263	15
24.	108	46	292	19
25.	113	72	284	22
Mean Score	108.60	63.16	283.16	23.48

TABLE 6

THE STATUS OF THE EXPERIMENTAL GROUP
BEFORE THE TEACHING PERIOD

Student	Age in Months	I.Q. Dom.Test	S.C.A.T. Quan.Score	Gr.IX Maths.	Criterion Pretest
1.	181	115	322	76	19
2.	179	117	312	82	30
3.	179	123	320	82	39
4.	186	116	315	92	35
5.	185	117	306	69	26
6.	186	122	308	56	24
7.	182	142	320	96	40
8.	175	150	339	96	37
9.	175	117	302	94	25
10.	183	120	298	62	28
11.	189	103	300	68	21
12.	183	134	330	87	34
13.	186	103	296	79	23
14.	188	105	324	78	27
15.	186	128	330	96	34
16.	182	125	320	93	33
17.	186	118	308	77	26
18.	183	119	298	65	29
19.	177	124	304	58	23
20.	187	110	330	76	33
21.	185	125	318	70	30
22.	178	131	318	72	26
23.	178	129	316	77	33
24.	167	154	308	92	35
25.	178	132	312	68	29
26.	177	134	320	95	39
27.	181	137	316	64	33
28.	178	127	302	70	22
29.	186	112	300	74	19
30.	183	135	298	68	29
31.	187	126	318	70	31
32.	188	115	310	57	24
33.	179	137	310	78	26
34.	177	142	318	80	31
35.	170	122	308	84	21

TABLE 7

THE STATUS OF THE CONTROL GROUP A (SISLER)
BEFORE THE TEACHING PROCESS

Student	Age in Months	I.Q. Dom.Test	S.C.A.T. Quan.Score	Gr.IX Maths.	Criterion Pretest
1.	181	116	320	78	22
2.	188	116	310	89	35
3.	180	131	304	62	31
4.	181	125	322	89	20
5.	187	120	313	81	28
6.	175	122	309	80	23
7.	181	115	318	83	23
8.	181	118	316	78	25
9.	196	94	302	83	23
10.	186	117	308	80	23
11.	182	145	326	86	44
12.	174	129	306	50	22
13.	187	115	314	64	26
14.	193	109	322	85	29
15.	170	133	314	77	25
16.	184	131	322	88	26
17.	180	112	308	81	22
18.	184	137	320	78	25
19.	184	109	316	89	21
20.	183	116	314	67	28
21.	185	114	320	79	26
22.	160	124	298	59	25
23.	187	113	310	85	31
24.	184	137	312	71	27
25.	179	101	302	58	13
26.	183	126	320	78	29
27.	184	120	330	68	13
28.	178	110	312	78	27
29.	182	129	316	78	21
30.	187	137	326	91	33
31.	199	135	312	88	33
32.	177	122	316	88	28
33.	187	110	312	69	24
34.	182	130	324	85	29
35.	182	140	308	61	29

TABLE 8

THE STATUS OF THE CONTROL GROUP B (CHURCHILL)
BEFORE THE TEACHING PERIOD

Student	Age in Months	I.Q. Dom.Test	S.C.A.T. Quan.Score	Gr.IX Maths.	Criterion Pretest
1.	180	117	314	81	26
2.	181	133	314	90	39
3.	176	128	298	84	24
4.	189	124	318	91	27
5.	182	120	306	82	27
6.	177	135	308	93	29
7.	177	118	330	90	29
8.	182	116	308	76	23
9.	187	134	302	80	28
10.	180	116	318	84	31
11.	170	130	318	95	32
12.	182	114	314	83	24
13.	170	144	324	95	27
14.	171	136	330	96	35
15.	180	138	318	90	27
16.	185	116	302	80	12
17.	170	142	314	81	26
18.	172	144	318	98	22
19.	182	137	318	97	22
20.	181	112	302	81	22
21.	186	121	304	90	24
22.	185	115	316	83	29
23.	184	152	324	97	37
24.	183	137	320	82	30
25.	181	149	322	88	37
26.	184	139	339	95	37
27.	184	127	296	76	14
28.	179	140	330	97	34
29.	170	125	308	87	25
30.	184	117	316	85	18
31.	171	156	326	96	36
32.	175	137	308	86	26
33.	180	123	320	88	20
34.	176	138	308	81	29
35.	170	131	318	94	26
36.	173	135	314	93	35
37.	174	147	324	88	30
38.	170	144	332	92	30
39.	180	120	298	90	26
40.	185	118	308	92	27

TABLE 9

ACHIEVEMENT OF EXPERIMENTAL GROUP
AS MEASURED BY SCHOOL TERM EXAMINATIONS

Student	October Exam	December Exam	March Exam
1.	63	74	61
2.	91	97	82
3.	73	92	95
4.	69	88	94
5.	76	81	80
6.	71	91	87
7.	94	100	96
8.	98	99	97
9.	88	99	88
10.	68	77	53
11.	76	70	80
12.	89	98	95
13.	56	58	55
14.	74	88	87
15.	95	98	98
16.	98	97	91
17.	65	76	54
18.	59	92	81
19.	56	77	50
20.	75	76	81
21.	64	85	76
22.	84	88	93
23.	76	86	65
24.	86	89	90
25.	82	86	64
26.	94	96	90
27.	83	90	84
28.	79	93	84
29.	69	68	55
30.	88	86	83
31.	97	87	74
32.	70	70	58
33.	67	90	82
34.	76	91	84
35.	73	85	73

TABLE 10

ACHIEVEMENT OF CONTROL GROUP A (SISLER)
AS MEASURED BY SCHOOL TERM EXAMINATIONS

Student	October Exam	December Exam	March Exam
1.	83	81	72
2.	84	80	70
3.	85	89	90
4.	89	75	69
5.	88	86	83
6.	75	75	77
7.	93	78	78
8.	74	85	69
9.	72	89	59
10.	82	88	86
11.	95	88	84
12.	65	50	50
13.	80	66	64
14.	76	95	84
15.	84	83	71
16.	86	93	79
17.	76	85	72
18.	94	94	89
19.	91	90	82
20.	80	80	90
21.	74	92	90
22.	78	60	61
23.	86		62
24.	63	63	59
25.	55	75	31
26.	85	83	67
27.	87	70	73
28.	72	55	54
29.	81	94	76
30.	97	95	91
31.	86	95	92
32.	83	77	88
33.	75	66	73
34.	80	56	56
35.	66	50	42

TABLE 11

ACHIEVEMENT OF CONTROL GROUP B (CHURCHILL)
AS MEASURED BY SCHOOL TERM EXAMINATIONS

Student	October Exam	December Exam	March Exam
1.	68	79	60
2.	92	93	85
3.	86	83	74
4.	88	96	87
5.	74	76	66
6.	88	89	66
7.	82	87	81
8.	78	77	60
9.	74	87	67
10.	96	95	83
11.	88	79	77
12.	66	85	54
13.	84	80	83
14.	76	82	67
15.	80	89	67
16.	80	83	60
17.	78	89	74
18.	90	93	79
19.	90	97	96
20.	78	71	60
21.	70	79	55
22.	70	70	64
23.	92	97	88
24.	92	84	65
25.	88	95	93
26.	75	94	89
27.	66	79	70
28.	92	91	94
29.	80	87	70
30.	86	87	78
31.	92	96	92
32.	92	87	71
33.	80	93	84
34.	80	78	57
35.	90	93	94
36.	92	97	94
37.	86	92	89
38.	82	96	76
39.	84	89	73
40.	90	87	63

TABLE 12

MATHEMATICAL ACHIEVEMENT OF EXPERIMENTAL GROUP
UPON COMPLETION OF TEACHING PERIOD

Student	Criterion Test Scores	June Final Mathematics
1.	22	62
2.	40	90
3.	42	94
4.	41	91
5.	29	81
6.	34	89
7.	45	98
8.	47	98
9.	38	94
10.	35	65
11.	26	75
12.	43	97
13.	23	56
14.	31	88
15.	46	98
16.	43	94
17.	28	64
18.	39	87
19.	37	64
20.	37	79
21.	46	81
22.	34	91
23.	38	76
24.	39	90
25.	36	75
26.	41	93
27.	41	87
28.	36	89
29.	26	61
30.	37	85
31.	34	81
32.	28	64
33.	36	86
34.	42	88
35.	35	79

TABLE 13

MATHEMATICAL ACHIEVEMENT OF CONTROL GROUP A (SISLER) UPON COMPLETION OF TEACHING PERIOD

Student	Criterion Test Scores	June Final Mathematics
1.	34	77
2.	38	68
3.	36	89
4.	37	66
5.	39	84
6.	29	72
7.	37	81
8.	31	75
9.	24	63
10.	29	87
11.	47	86
12.	28	47
13.	27	65
14.	31	88
15.	27	77
16.	36	85
17.	30	76
18.	36	91
19.	32	85
20.	27	82
21.	33	91
22.	33	64
23.	39	95
24.	31	51
25.	22	51
26.	35	69
27.	25	76
28.	37	51
29.	27	86
30.	43	93
31.	38	93
32.	33	82
33.	27	78
34.	37	59
35.	23	43

TABLE 14

MATHEMATICAL ACHIEVEMENT OF CONTROL GROUP B (CHURCHILL) UPON COMPLETION OF TEACHING PERIOD

Student	Criterion Test Scores	June Final Mathematics
1.	26	68
2.	42	88
3.	31	78
4.	35	90
5.	27	70
6.	35	75
7.	30	69
8.	28	55
9.	34	62
10.	39	88
11.	38	80
12.	30	61
13.	39	82
14.	37	73
15.	32	75
16.	22	57
17.	39	80
18.	35	85
19.	37	96
20.	24	55
21.	36	65
22.	25	66
23.	46	92
24.	36	65
25.	40	94
26.	40	91
27.	25	74
28.	45	93
29.	32	77
30.	30	82
31.	40	94
32.	35	77
33.	29	88
34.	34	65
35.	33	94
36.	41	95
37.	40	90
38.	37	84
39.	27	79
40.	32	73

TABLE 15

ACHIEVEMENT OF EXPERIMENTAL GROUP
IN GRADE X SCIENCE AND GRADE XI MATHEMATICS

Student	Grade X Science	Grade XI Mathematics
1.	57	71
2.	93	97
3.	90	78
4.	91	89
5.	68	69
6.	83	67
7.	96	100
8.	96	100
9.	84	78
10.	80	94
11.	80	77
12.	88	76
13.	64	70
14.	78	80
15.	81	87
16.	82	83
17.	69	82
18.	83	65
19.	84	89
20.	78	69
21.	77	64
22.	76	87
23.	76	67
24.	91	86
25.	78	88
26.	86	90
27.	90	78
28.	81	82
29.	67	62
30.	90	60
31.	82	75
32.	76	68
33.	74	66
34.	90	98
35.	80	72

TABLE 16

ACHIEVEMENT OF CONTROL GROUP A (SISLER)
IN GRADE X SCIENCE AND GRADE XI MATHEMATICS

Student	Grade X Science	Grade XI Mathematics
1.	81	84
2.	76	88
3.	86	85
4.	77	64
5.	90	78
6.	70	66
7.	56	93
8.	76	87
9.	80	20
10.	81	91
11.	89	83
12.	53	67
13.	69	59
14.	80	76
15.	84	72
16.	78	
17.	76	73
18.	77	92
19.	82	82
20.	86	92
21.	81	82
22.	73	62
23.	92	82
24.	68	61
25.	60	67
26.	73	90
27.	75	71
28.	69	51
29.	63	67
30.	94	100
31.	93	84
32.	77	88
33.	70	67
34.	65	47
35.	63	

TABLE 17

ACHIEVEMENT OF CONTROL GROUP B (CHURCHILL)
IN GRADE X SCIENCE AND GRADE XI MATHEMATICS

Student	Grade X Science	Grade XI Mathematics
1.	66	77
2.	78	85
3.	81	73
4.	83	98
5.	79	74
6.	80	89
7.	79	81
8.	69	79
9.	72	68
10.	86	84
11.	52	94
12.	76	72
13.	86	86
14.	68	84
15.	64	88
16.	58	73
17.	69	71
18.	72	75
19.	87	81
20.	61	72
21.	64	70
22.	80	77
23.	92	95
24.	74	63
25.	83	83
26.	84	72
27.	66	41
28.	92	97
29.	66	97
30.	81	82
31.	91	83
32.	86	58
33.	78	69
34.	88	78
35.	81	84
36.	87	81
37.	81	80
38.	80	91
39.	80	62
40.	67	75