AN ECONOMETRIC MODEL OF MANITOBA CROP SUPPLY RESPONSE UNDER RISK AVERSION AND PRICE UNCERTAINTY

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BY

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ABSTRACT

Restricted and unrestricted distributed lag models are used to investigate and establish probable lag structures in specific crop supply response to market and Canadian wheat board prices. The identified lag structures are in some respects consistent with other studies showing different lag structures for crop yield and acreage response and also give an indication of model sensitivity to numeraire choice in normalized quadratic functional forms.

Lag structures form the basis of expectation and variance proxies employed in risk neutral, linear mean-variance and non-linear mean-variance econometric model specification and estimation using duality. Linear SUR was used to estimate duality models in the cases of risk neutrality and linear mean-variance. Both a linear reduced form and a non-linear duality model were estimated in the case of non-linear mean-variance.

Price variances and expected prices are more significant in the risk aversion models which imply gains in model specification. Symmetry and CRTS are rejected in all models. Homogeneity conditions corresponding to risk neutrality is rejected in all models.

Constant absolute risk aversion and Constant relative risk aversion are also rejected.

DEDICATION

To Jenniffre, Victoria and Allan.

The debt I owe you can never be fully repaid.

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All errors of omission or commission remain my sole responsibility.

CHAPTER I

INTRODUCTION

Background

Agricultural products are generally characterized by relatively long gestation periods and perishability. These attributes together with the competitive nature of the agricultural industry implies that agricultural producers have to contend with risk in their resource allocation decisions. The effects of risk and uncertainty on efficient resource use and the decision-making process by agricultural producers therefore cannot be minimized.

Consequently an illuminating resource allocation analysis should identify and include sources of risk and uncertainty in it's formulation. The major sources of risk and uncertainty in agricultural production can be categorized into production and market uncertainty respectively. Production uncertainty involves all those risks associated with variations in yield such as weather, pests and diseases. Market uncertainty may arise out of risks mainly associated with the price variability of outputs and inputs.

Manitoba agricultural producers face both production and market uncertainty and it is critical that their resource allocation behaviour be analyzed in the context of the risks they encounter. This study attempts to provide an indication of Manitoba grain producers resource allocation behaviour in the presence of market uncertainty.

Problem Statement

Previous studies on the response efficiency of Manitoba crop output supply have often assumed that risk is unimportant or that producers are risk neutral. These analyses have proceeded to elicit resource allocation behaviour under conditions of certainty contrary to the true circumstances of Manitoba producers which is inherently uncertain. In cases where risk has been incorporated, it has either been done in an ad hoc manner that is not amenable to testing on the basis of economic theory or it has employed highly aggregated measures that obscure the effects of risk on individual crop enterprises. The formulation of analytical methods that incorporate risk and uncertainty and application to Manitoba grain producers is expected to provide an improved indication of individual crop output supply response.

There are other serious problems with earlier studies of supply response in Manitoba crops that are perhaps even more basic than risk and uncertainty. Other studies have generally analyzed wheat supply response as independent of prices of other crops, and possible lags in the production decision making process have not been considered systematically. Any reasonable model of crop supply response must consider these issues in addition to and perhaps prior to, risk and uncertainty.

Objectives of the Study

The general objective of the thesis is to develop an econometric model of crop supply response in Manitoba that attempts to address the above problems in a tractable manner. This will be the first econometric study to attempt to measure the impacts of risk aversion and price uncertainty on supply of specific crops within the framework of duality theory. Given the exploratory nature of this research, it will be necessary to pay relatively little attention to dynamics, specifics of government support programs, and yield uncertainty.

A system of crop output supply equations will be estimated as conditional on (expected) prices and input prices. The model will then be extended to incorporate risk aversion and price uncertainty. It is difficult to incorporate yield uncertainty into the analysis due to the absence of farm level data on yields. Variation in yields at the provincial level greatly underestimates variation in yields at the farm level, since weather conditions are not perfectly correlated across farms.

CHAPTER II

LITERATURE ON RISK IN PRODUCTION

Mathematical Programming and Risk

Initial studies of risk in agricultural production utilized mathematical programming based methodologies. The problem was perceived from a portfolio selection perspective in a quadratic programming framework using the mean-variance (EV) approach which minimizes portfolio variance for alternative levels of expected returns. This methodology was first applied by Freund (1956) who found that the introduction of risk into the programming model reduced both the level and standard deviation of net revenue.The Freund study inspired other investigators to use the quadratic programming methodology in risk analysis.The studies that followed included Markowitz (1959), Scott and Baker(1972), Lin et al(1974),Barry and Willman(1976), Wiens(1976), Johnson(1979), Adams et al(1980) and Musser and Stamoulis(1981). The major drawback of the quadratic programming procedures was the arbitrary way in which risk aversion was specified.

Following the quadratic programming methodology was the MOTAD approach which was basically a modified linear programming model specified as a minimization of total absolute deviations , from which the acronym MOTAD is derived. The MOTAD approach has been used in such studies as Hazell(1971), Brink and McCarl(1978), Gebremeskel and Shumway(1979), Mapp et al(1979) and Persaud and Mapp(1980). Mapp and Helmers(1984) find the MOTAD approach to be inadequate, even in comparison to the quadratic programming methodology, in view of the sensitivity of it's solutions to changes in risk measures, constraints and technical coefficients.

The general criticism of mathematical programming based methodologies has been it's inability to provide for the drawing of statistical inferences which has become central to the current study of agricultural economics.The mathematical programming methodologies developed to incorporate risk in agricultural production analysis also fails on statistical inference.

Econometric Estimation and Risk

The inability to draw statistical inferences from mathematical programming methodologies , including those designed to incorporate risk, resulted in efforts to develop econometric based estimation methods. Just(1974) conducted the pioneering study that attempted to incorporate risk in estimating an econometric acreage response model. To gauge the effects of risk on California field-crop supply response, Just estimates single equation generalized adaptive expectation models by defining quadratic lag terms to capture risk effects.

Risk Attitudes and Risk Preference Structure

The next phase of risk analysis in agricultural production consisted mainly of studies on risk attitudes and preference structures of producers, both of which form the foundations of hypothesis testing in this study. For some time, there appeared to be a distinction emerging between risk attitude and risk reference structure . The former emphasized the ordering of producers by elicited attitudes toward risk which was determined through the estimation of the coefficient of absolute risk aversion. Producers are then classified risk averse, risk neutral or risk preferring depending on the estimated coefficient of absolute risk aversion. Work on risk attitudes was initiated by Dillon and Scandizzo(1978) and then followed by Binswanger(1980, 1982), Hazzel(1982), Szpiro(1982), Robison et al(1984) and Antle(1987) . The foregoing studies employed experimental methods (games), actual economic decisions or econometric estimation.

Risk preference structure studies have concentrated on identifying the specific type or nature of existing risk aversion. Risk preference structure studies are based on the acceptance of risk aversion among agricultural producers as a stylized fact.These studies have attempted to test whether producers exhibit various properties of risk aversion for instance constant, decreasing, or increasing absolute or relative risk aversion.Risk preference structure models have been formulated and estimated by Cohn et al(1975), Landskroner (1977), Lins et al(1981), Siegel and Hoban(1982), Morin and Suarez(1983), Bellante and Saba(1986), Chavas and Holt(1990) and Pope and Just(1991). To jointly determine and measure both risk attitude and preference structure Saha et al(1994) formulate and estimate an expo-power (EP) utility function using farm level wheat production data from Kansas. The superiority and flexibility of the EP utility function cannot be contested but it's application using duality approaches is yet to be accomplished. The studies on the determination and measurement of risk attitudes and preference structures do not give a reliable indication of multicrop resource allocation behaviour under risk.

Inputs and Risk

Just and Pope(1979) while analyzing effects of inputs on output production and apply a production function specifically designed to provide for the separation of the marginal contribution of inputs to the mean and variance of output. This production function has now become the basis of most risk related studies in agricultural production involving stochastic yield. Other notable studies on inputs and risk were conducted by Farnsworth and Moffit(1981), Feder(1979), Horowitz and Lichtenberg (1993, 1994).

Relevant Empirical Research

Although numerous studies have attempted to incorporate the effects of uncertainty into econometric models of production, relatively few studies have investigated the structure of risk preferences aside from testing for risk neutrality. Pope and Just (1991) estimated reduced form models of acreage demand for potatoes in Idaho using measures of the mean and variance of prices of potatoes and sugar beets and initial wealth (value of land and buildings minus associated debts). Constant absolute risk aversion (CARA)(linear mean-variance utility) was rejected, but constant relative risk aversion(CRRA) was not rejected. Saha, Shumway and Talpaz (1994) estimated jointly a Just-Pope stochastic production function and a utility for wheat in Kansas. Both CARA and CRRA were rejected (for decreasing absolute risk aversion and increasing relative risk aversion).

Chavas and Holt (1990) estimated acreage demands for corn and soybeans in the U.S. using measures of mean prices, variances and covariances of prices and yields for outputs truncated for effects of government price supports and a proxy for initial wealth (farm equity). Input prices and random variable output prices were normalized by a consumer price index. This normalization was justified by assuming expected utility maximization of household consumption together with a budget constraint equating consumption expenditure to exogenous income plus profits from corn and soybean production. However this budget constraint was seriously mis-specified (e.g. by ignoring capital investment and returns from other enterprises, and assuming an aggregate price index for household consumption). Symmetry and negativity restrictions related to compensated demands was specified by a generalized Slutsky equation. Symmetry was not rejected, whereas CARA and CRRA were rejected. Von Massow and Weeresink (1993) estimated a similar model for various crops in Ontario. Most variances were insignificant and CARA was rejected.

Coyle (1992) estimated a duality model for Manitoba agriculture assuming a linear mean-variance utility function and price uncertainty. However the model was highly aggregated to two outputs (crops and livestock), so that the model is seriously mis-specified in this respect and can have few implications for policy. In the case of a normalized quadratic functional form, inclusion of price variances in the model increased the significance of expected prices. A homogeneity condition was imposed, and symmetry and curvature properties implied by the linear mean-variance model were rejected.

Duality Models and Risk

The importance of duality theory in modelling agricultural production has been well recognized (see Pope 1982; Lopez 1982; Chambers 1981 and for a more recent review, Shumway 1994). Perhaps the most serious remaining criticism of applied duality models has been the absence of a tractable methodology for incorporating risk aversion and uncertainty (e.g. Shumway). Proceeding within the framework of expected utility maximization, Pope has argued that duality theory works poorly in modelling risk because the objective function is non-linear in parameters when either prices or yields are uncertain. Others (Epstein 1977, 1978) have reached similar negative conclusions.

Nevertheless several tractable dual models with risk and uncertainty have been developed in recent years. Coyle (1992) incorporated risk aversion and price uncertainty into a duality model of a linear mean-variance utility function. Properties of the corresponding dual indirect utility function were characterized, and application of the envelop theorem leads to output supply and input demand equations that can be estimated by linear methods. This model has subsequently been generalized to a non-linear mean-variance utility function (Coyle 1994a; Saha 1994). The one serious complication is that corresponding output supply and factor demands cannot be estimated by linear methods. A further generalization of the model permits a non-linear mean-variance utility function and yield uncertainty as well as price uncertainty (Coyle 1994b). This further complicates estimation of the (expected) output supply equations although the model remains tractable for empirical research.

Researchers have also begun to generalize the standard theory of cost minimization to stochastic outputs (Pope 1980; Pope and Just 1991b; Pope and Chavas 1994). In contrast to the above studies, these cost minimization models attempt to explain input decisions conditional on moments of output supply decisions. Thus these models are not directly applicable to empirical studies of supply response, although such dual cost functions together with a risk preference utility function can jointly specify a model of choice regarding the probability distribution of outputs.

Risk aversion and price uncertainty have also been incorporated recently into dynamic duality models. The importance of uncertainty in modelling dynamic decisions is well known, and several studies have incorporated price uncertainty into dynamic duality models of the risk neutral firm (Stefanou 1987; Chavas 1994). More recently, dynamic duality models have been specified under risk aversion and price uncertainty (Coyle 1994b; Arnade and Coyle 1995).

Theoretical Framework of the Study

The simplest assumption regarding risk preferences is a linear relation between expected profits ($E\pi$) and profit variance ($V\pi$) i.e.

(1) $$U = E\pi - \alpha/2 \, (V\pi)$$

where α is the coefficient of absolute risk aversion. A general non-linear mean-variance utility function can be formulated by allowing the coefficient of risk aversion to vary with the mean and variance of the random variable wealth, where wealth (w) is equal to initial wealth (w_0) plus profits (π). Then the firm's utility function can be expressed as:

$$(2) \qquad U = w_0 + E\pi - \alpha (w_0 + E\pi, V\pi)/2 \; V\pi$$

where the mean and variance of wealth are $Ew = w_0 + E\pi$, $Vw = V\pi$. Given Y and X as vectors of output and input levels and P and W as vectors of output and input prices respectively, profits are defined as follows:

$$(3) \qquad \pi = PY - WX$$

With the assumption that only price risk exists Y, X and W are considered non-stochastic and the expected profits and variance of profits conditional on the output and input levels becomes :

$$(4) \qquad E\pi(Y,X) = P^eY - WX$$

$$(5) \qquad V\pi(Y,X) = Y^T V_p Y$$

where P^e and V_p represent the vector of expected output prices and covariance matrix of prices respectively.

The indirect utility function corresponding to maximization of linear mean-variance utility function (1) is

$$(6) \qquad U^* (P^e,W,V_p) = \text{Max} \;\; P^eY - WX -\alpha/2 \; (Y^T V_p Y)$$

However this expression is rather restrictive in it's empirical application due to the assumption of constant absolute risk aversion (linear mean-variance risk preference). Nevertheless the linear mean-variance model continues to be used in various applied studies involving risk and uncertainty.

A more general model can be formulated in terms of a non-linear mean-variance utility function as follows

(7) $U^* (P^e,W,V_p,w_0) = \text{Max } w_0 + P^eY - WX -$

$$\alpha(w_0 + P^e Y - WX , Y^T V_p Y)/2 \; Y^T V_p Y$$

If only one price is uncertain, then this non-linear mean-variance model is equivalent to expected utility maximization (Sinn, Meyer).

The most important properties of the dual indirect function $U^* (P^e,W,V_p)$ for the linear mean-variance model (6) can be summarized as follows (Coyle 1992): the dual is linear homogenous and convex in (P^e,W,V_p), and

(8)

$$\frac{\partial U^*(.)}{\partial P_j^e} = Y_j \qquad j=1,,M$$

$$\frac{\partial U^*(.)}{\partial W_i} = -X_i \qquad i=1,,N$$

These are analogous to properties of the dual profit function for a risk neutral competitive firm:

π (P,W)= Max PY - WX implies π (P,W) is linear homogenous

and convex in (P,W), and (Hotelling's lemma)

(9)

$$\frac{\partial U^*(.)}{\partial P_j} = Y_j \qquad j=1,,M$$

$$\frac{\partial U^*(.)}{\partial W_i} = -X_i \qquad i=1,,N$$

In addition (6) implies non-linear relations between derivatives of the dual with respect to price variances and expected prices. The coefficient of risk aversion ($\acute{\alpha}$) can be recovered from the dual, and there is a duality between the dual and technology similar to risk neutral case.

The properties of the dual indirect utility function U^* (P^e, W, V_p, w_0) for the non-linear mean-variance model (7) are more complex (Coyle 1994a). The common assumption of constant relative risk aversion (CRRA) implies

(10)

$$U^*(\lambda P^e, \lambda W, \lambda^2 Vp, \lambda w_o) = \lambda U^*(P^e, W, Vp, w_o)$$

(11)

$$Y(\lambda P^e, \lambda W, \lambda^2 Vp, \lambda w_o) = \lambda Y(P^e, W, Vp, w_o)$$

$$X(\lambda P^e, \lambda W, \lambda^2 Vp, \lambda w_o) = \lambda X(P^e, W, Vp, w_o)$$

under CRRA. Note the difference from the homogeneity conditions for the linear mean-variance model: here Vp is multiplied by the λ^2 rather than by λ. The homogeneity result (11) was derived by Pope (1988). CRRA also implies the following restriction on the risk aversion function $\acute{\alpha}$(Ew, Vw):

(12)

$$\alpha(\lambda Ew, \lambda^2 Vw) = \lambda^{-1} \alpha(Ew, Vw)$$

under CRRA. The generalized convexity relation for the dual can be stated as :

(13)

$$U^{*}_{vv}(v)+[\frac{\alpha(Ew^{*},Vw^{*})}{2}]_{vv} \quad \textit{symmetric} \quad \textit{positive} \quad \textit{definite}$$

Where V = (P^e,W,Vp,w_o).

The generalized envelope relations include

(14)

$$\frac{\frac{\partial U^{*}(.)}{\partial P^{e}_{j}}}{\frac{\partial U^{*}(.)}{\partial w_{o}}}=Y_{j} \qquad j=1,,M$$

$$\frac{\frac{\partial U^{*}(.)}{\partial W_{i}}}{\frac{\partial U^{*}(.)}{\partial w_{o}}}=Y_{i} \qquad i=1,,M$$

(15)

$$\frac{\partial U^{*}(.)}{\partial w_{o}}=1-\frac{\partial \alpha(.)}{\partial Ew}\frac{V\pi}{2} \qquad i=1,,M$$

The coefficient of risk aversion function $\acute{\alpha}(.)$ can be recovered locally from the dual given CRRA, and there is a duality between the dual and technology given the assumption of decreasing absolute risk aversion (a stylized fact in the theory of risk behaviour).

Allowing for linear mean-variance risk preferences as in (6) does not complicate or compromise the dual approach to production in any essential manner. In contrast, application of duality theory for the non-linear mean-variance model (7) is more complex than the risk neutral case in the following respects: a homogeneity property is not necessarily implied by the theory (risk preferences may not satisfy CRRA); and the estimating equations for output supply and factor demand (14) generally are non-linear in coefficients. Problems in non-linear estimation may be simplified to some extent by substituting (15) into (14) or by estimating (15) and (14) in an iterative manner.

CHAPTER III

ECONOMETRIC MODELS AND RESULTS

Initial Selection of Variables

The initial selection of variables for this research was based largely on results of earlier studies of Manitoba agriculture. Coyle (1993a) specified a risk-neutral model of crop acreage demands for Western Canada as follows:

(1)

$$Z_t^i = a_i + \sum_{J=1}^{4} a_{ij}\left(\frac{P^j}{W^1}\right)_t + a_{i5}\left(\frac{W^2}{W^1}\right)_t + a_{i6}K_t + a_{i7}A_t + a_{i8}A_{t-1} + e_t^i \qquad i=1,..,4$$

Here there are four crops(wheat, barley,rapeseed and other), Z^i is acres in crop i, A is total crop acreage, P^j is expected price of crop j, W^1 is price of variable inputs for crops, W^2 is wage rate for hired labour, K is the stock of machinery and equipment, and t is a time trend. Inclusion of current total crop acreage, A_t implies that livestock prices can be omitted from (1) to the extent that crops and livestock are weakly separable in production. Lagged total crop acreage A_{t-1} is also included in the model assuming that individual crop acreage demands depend on lags in adjustment of the overall crop rotation.

Expected prices for crops covered by the Canadian Wheat Board (including Wheat, barley and until recently oats) were defined as the sum of the most recently observed components of Canadian Wheat Board payments at planting time : current initial payments , plus adjustment and interim payments for crops marketed in the previous year, plus final payment for crop marketed two years previously. Expected prices for crops not covered by the Board were defined as market prices plus Government payments in the previous year. This specification of expected prices led to significant results for model (1) in contrast to specifications based on lagged market prices for Board crops or on rational expectations using forecasts from ARIMA models. This is somewhat consistent with the results of Sulewski, Spriggs and Schoney (1994), who concluded that simple models of expectation formation appeared to explain the stated expectations of Saskatchewan farmers for wheat and rapeseed prices better than do rational expectations models or futures prices.

The current study specifies output supply equations for wheat,barley,rapeseed and oats and a factor demand equation for variable crop inputs in terms of the above explanatory variables. The variance for farmers subjective probability distribution of prices is calculated as in Chavas and Holt and in Coyle (1992):

(2)

$$VAR(P_t^i)=0.5(P_{t-1}^i - E_{t-2}(P_{t-1}^i))^2 + 0.33(P_{t-2}^i - E_{t-3}(P_{t-2}^i))^2 + 0.17(P_{t-3}^i - E_{t-4}(P_{t-3}^i))^2$$

Here current variance equals the sum of squares of prediction errors of the previous three years, with declining weights 0.50, 0.33 and 0.17. For simplicity price covariances are assumed to be zero, as in most econometric studies of risk in production.

Annual data was collected for Manitoba agriculture for 1961 - 1990. Output price data was obtained from the Canadian Wheat Board annual reports and the Canada Grain

Trade Statistics. Input price indexes were obtained for hired labour, variable inputs for crops (e.g. fertilizer) and machinery and equipment from the Farm Input Price Indexes publication of Statistics Canada. An index of the quantity of variable inputs for crops was calculated as the current value of variable expenses for crops deflated by it's price index and an index of the stock of physical capital was in the crop sector was calculated as the current value of machinery and equipment deflated by it's price index. These current values were obtained from the Farm Net Income publication of Statistics Canada. Crop acreages sown annually for harvest were obtained from the Handbook of Field Crop Statistics published by Agriculture Canada.

It is important to note the following deficiencies in selection of variables for this study: a) the influence of crop yield uncertainty on production is ignored; b) crop output supply response is not decomposed into crop acreage and crop yield responses; c) the impact of Government programs on production is not modelled. Yield uncertainty is not considered here because it cannot be measured by province level data: to the extent that weather varies by region, the contribution of yields to revenue variation or uncertainty at the farm level will be underestimated by data aggregated over producers.

There appear to be substantial advantages to decomposing econometric models of crop output supply response into crop acreage and yield response models, although this has not been done in any published studies. There may be considerably longer lags in crop yield response than in crop acreage response to prices. For example, a preliminary study of yield response to price for wheat and barley in Manitoba suggested a 3 to 4 year lag in response Coyle (1993b). This difference in lag structures implies gains in understanding and efficiency by estimating a system of crop acreage demand and yield equations. Moreover further gains in efficiency can be obtained by the adding-up restrictions in acreage demands (Coyle 1993a) and by imposing weak separability between crops in production or yield response equations.

Unfortunately this study does not decompose crop supply response into acreage demand and yield responses. The reason for this is that duality models with risk aversion have not yet been generalized to incorporate this decomposition. Indeed duality models of production under risk neutrality have only recently endogenized crop acreage demands (Chambers and Just 1989), and even these models have not yet incorporated different lags in yield and acreage response.

Initial Regressions

Distributed lag models were estimated for individual crop outputs.The general form of the equations follows:

(3)

$$Y_t^i = a_i + \sum_{S=0}^{L} b_{ij} \left(\frac{P^i}{P^{Ni}}\right)_{t-s} + e_t^i \qquad i=1,..,4$$

Where there is a distributed lag in own price deflated by a numeraire. Both unrestricted and polynomial distributed lag (PDL) models were estimated by OLS, and both market prices and CWB prices (defined as above) were considered. Representative regression results are presented in Tables A11 to A17 of Appendix I.

It is interesting to note that the distributed lags often appear in a sense to be double peaked. For example in Table A14 (A15) for an unrestricted distributed lag for wheat output, lag lengths of 1 - 2 (2) and 4 - 5 (5 - 6) but never 3 are significant. Since coefficients do not decrease uniformly or exponentially as lag length increases, these results are not consistent with geometric lag models such as the Nerlovian adaptive expectations model or Partial adjustment in supply or indeed with most models where lags are explained in terms of expectations. Instead these results can be interpreted as being broadly consistent with the crop acreage demand and yield studies referenced in the previous section: these results suggest approximately a 1 to 2 year lag in wheat acreage response and a 4 to 6 year lag in wheat yield response. Coefficient restrictions in PDL models are inappropriate for representing such discrete or double peaked lag patterns. Similar double peaked patterns are obtained for barley but a single-peaked lag pattern is obtained for canola and oats.

The smaller (one period) lags apparently can be explained to some extent by an essentially static model (eg. Coyle 1993a), but the longer (four to six period) lags presumably should be explained by an explicitly dynamic model. Given the difficulties in modelling dynamic behaviour, it was decided that the first study of Manitoba crop output response under risk should emphasize one period lags in response. Consequently this thesis will ignore the effects of longer lags in prices on crop outputs. Of course, since prices two or three years apart are highly correlated, this mis-specification of output supply models inevitably biases the interpretation of results as short-run impacts. Unfortunately this problem is common to most models of supply response. Perhaps the most appropriate method for modelling crop supply response is to decompose it into acreage and yield response models, but this is beyond the scope of this initial study.

Another matter that was considered in initial investigations was the choice of numeraire price. The standard theory of the competitive profit maximizing (and hence risk neutral) firm assumes that output supplies and factor demands are homogeneous of degree zero in output and input prices; so the choice of numeraire price is arbitrary. However, for econometric purposes the choice of numeraire price for a normalized quadratic dual profit function implies very different specifications for output supply/factor demand equations , since the equation for the numeraire commodity is quadratic in prices whereas all other equations are linear in prices. Similarly different choices of numeraire imply different restrictions on technology. In addition this risk neutral standard homogeneity condition is likely to be an error in model specification: different homogeneity conditions apply under constant absolute risk aversion (CARA) and under constant relative risk aversion (CRRA), and there may be no simple homogeneity condition under general risk preferences. A homogeneity-like restriction involving compensated and uncompensated responses has been defined for a general duality model by Chavas and Pope (1985) and Chavas and Holt (1990).

Consequently various normalizations were considered initially and throughout this research. In principle it is possible to construct tests for discriminating between these normalizations (eg. Shumway and Gottret 1991). However, given the large number of possible specifications and the exploratory nature of this research, such testing was not attempted here.

Assuming a one period lag in supply response, output supply equations were estimated using all alternative choices of numeraire price. These equations were generally specified as:

(4)

$$Y_t^i = a_i + \sum_{J=1}^{5} a_{ij} P_{t-j}^{*j} + a_{i6} K_t + a_{i7} A_t + a_{i8} A_{t-1} + a_{i9} + e_t^i \qquad i=1,..,4$$

Here P^* denotes normalized output and input prices. The most significant own price coefficient for the wheat output supply equation was obtained using barley price as the numeraire, but this choice of numeraire led to less significant results for the oats and barley equations. The most significant own price effects for the oats and barley equations were obtained using canola and oats as numeraire respectively.

Risk Neutral Systems of Equations

A system of output supply and factor demand equations was estimated with the price of barley as the most common numeraire. Imposing the homogeneity conditions for risk neutrality, the system was generally specified as :

(5)

$$Y_t^i = a_i + \sum_{J=1}^{3} a_{ij} \frac{P^j}{P^o} + a_{i4} \frac{W^1}{P^o} + a_{i5} \frac{W^2}{P^o} + a_{i6} K_t + a_{i7} A_t + a_{i8} A_{t-1} + a_{i9} T + e_t^i \qquad i=1,2,3$$

(6)

$$X_t^1 = a_4 + \sum_{J=1}^{3} a_{4j} \frac{P^j}{P^o} + a_{44} \frac{W^1}{P^o} + a_{45} \frac{W^2}{P^o} + a_{46} K_t + a_{47} A_t + a_{48} A_{t-1} + a_{49} T + e_t^4$$

where (Y^1, Y^2, Y^3) denote the output of wheat, rapeseed and oats, respectively,(P^1, P^2, P^3) denote corresponding expected prices and P^o is the price of barley.

X^1 is a quantity index for variable crop inputs, W^1 is the corresponding price, and W^2 is a wage rate for hired labour. K is the stock of machinery and equipment, and A is total crop acreage.

Hotelling's lemma for a normalized quadratic profit function with price of barley as numeraire implies equations such as (5) and (6) but the corresponding output supply equation for barley is quadratic in parameters. Consequently the supply equation for barley is omitted from in the above model. A demand equation for labour is omitted as well because data on the amount of labour specific to crop agriculture is unavailable.

The primary method for estimation was Zellner's seemingly unrelated regressions technique (SUR). Since all equations in (5) and (6) have identical explanatory variables, in the absence of symmetry or other cross-equation restrictions this is equivalent to OLS. The model was also estimated by three stage least squares (3SLS), where current total crop acreage (A) was specified as endogenous and a livestock price index (normalized by the price of barley) was used as an additional instrument. However, with the exception of the wheat output supply equation, results were considerably less significant than in the case of SUR. In contrast, other studies of production using similar time series data sets have typically reported only minor differences between SUR and 3SLS. This present result presumably reflects at least in part, the apparent difficulty in constructing an appropriate model for total crop acreage. The total amount of land in crops presumably shows substantial lags in response with respect to many variables. Thus, since it does appear somewhat reasonable to assume that total crop acreage is predetermined, the above model was estimated primarily by SUR rather than 3SLS.

The above model was estimated without differencing or detrending the data . It has been argued that in the case of random walk data, the Durbin-Watson (d) statistic approaches zero as the sample size increases without limit, and in turn a low Durbin-Watson statistic is to be expected in models with data generated by random walks (Phillips 1986; Durlauf and Phillips 1988). Moreover the most common tests of unit roots apparently are not very powerful against relevant alternatives, i.e. these tests may often accept the null hypothesis of a unit root even though it is false (Kwiatkowski, Phillips, Schmidt and Shin, 1992). In this study the calculated Durbin-Watson statistics are usually 1.5 or higher. This suggests that data does not follow a random walk, and for the above reason the hypothesis of unit roots is not tested formally. Since the data is obviously non-stationary (by casual inspection) this conclusion would suggest that the data is trend-stationary. Rather than detrending the data, a time trend is included in regression models. Although strongly trending data violates the assumption for conventional asymptotic theory, the finite sample properties of least squares regression hold.

Several variations on the above model (5) - (6) were estimated: (a) price expectations P were based on a one year lag in market prices or the sum of the most recently observed components of CWB payments; (b) equations were specified in terms of either current price expectations P_t and current output prices W_t or in terms of a one year lag in expected prices P_{t-1} and input prices W_{t-1}; (c) the numeraire price was specified either as the barley expected price or the hired labour wage rate; (d) the standard risk neutral homogeneity condition was not always imposed and
(e) constant returns to scale (CRTS) was not always imposed. Regarding (a) - (c), the most significant coefficient estimates corresponding to priors were obtained when
(a) price expectations were defined as the sum of most recently observed components of CWB payments and a one year lag in market prices for crops not covered by the CWB,
(b) current expectations are lagged one year (so Y_t is specified as a function of P_{t-1}, W_{t-1}), and (c) the numeraire price is defined as the expected price of barley.

Table 1 in the text illustrates SUR results obtained for a representative model consistent with these specifications (a) - (c). The equations are specified as:

(7)

$$\frac{Y_t^i}{A_t}=a_i+\sum_{J=1}^{3} a_{ij}\left(\frac{P^j}{P^o}\right)_{t-1}+a_{i4}\left(\frac{W^1}{P^o}\right)_{t-1}+a_{i5}\left(\frac{W^2}{P^o}\right)_{t-1}+a_{i6}\frac{K_t}{A_t}+a_{i7}A_t+a_{i8}A_{t-1}+a_{i9}T+a_{i10}P^o_{t-1}+e_t^i$$

$$i=1,2,3$$

(8)

$$\frac{X_t^1}{A_t}=a_4+\sum_{J=1}^{3} a_{4j}\left(\frac{P^j}{P^o}\right)_{t-1}+a_{44}\left(\frac{W^1}{P^o}\right)_{t-1}+a_{45}\left(\frac{W^2}{P^o}\right)_{t-1}+a_{46}\frac{K_t}{A_t}+a_{47}A_t+a_{48}A_{t-1}+a_{49}T+a_{410}P^o_{t-1}+e_t^4$$

In contrast to model (5) - (6), the dependent variables are defined as crop outputs normalized by total crop acreage, and the numeraire price of barley (P^o_{t-1}) is added as a separate variable to the equations rather than simply as a numeraire. Here constant returns to scale in production (CRTS) implies that all coefficients of A_t are equal to zero ($a_{17} = a_{27} = a_{37} = a_{47} = 0$). The coefficient of A_t in the wheat, oats, and variable input equation are significant individually and the Wald chi-square statistic for the corresponding joint hypothesis is 44.7 (4 degrees of freedom (df)). Thus CRTS is rejected.

The standard risk neutral homogeneity condition implies that all coefficients of the numeraire price barley (P^o_{t-1}) are equal to zero ($a_{110} = a_{210} = a_{310} = a_{410} = 0$). The individual coefficients of P^o_{t-1} are not significant and the Wald chi-square statistic for the corresponding joint hypothesis is 1.59 with 4 degrees of freedom (df). Thus the standard risk neutral homogeneity property is not rejected.

In contrast the standard symmetry conditions ($a_{12} = a_{21}$, $a_{13} = a_{31}$, $a_{14} = -a_{41}$, $a_{23} = a_{32}$, $a_{24} = -a_{42}$, $a_{34} = -a_{43}$) are rejected. The Wald chi-square statistic for this joint hypothesis is 33.74 (6 df). Thus the set of differentiated equations corresponding to (7) - (8) does not integrate up to a parent profit function.

The only significant variables in the wheat output supply equation are the expected price of wheat and total cropland, but all output prices are significant and with expected signs in the rapeseed equation. Output prices are either insignificant or the sign is inconsistent with priors in the oats and variable crop input equations. Input prices are either insignificant or do not have anticipated signs. Since the major crops have similar input requirements and the one period lag effects of prices presumably reflect primarily substitution of land between different enterprises, it was anticipated that input prices would not have any significant effect on crop outputs or inputs in this model conditional on total cropland. Durbin-Watson statistics for these equations varied from 1.57 to 2.11, which does not suggest random walks.

Table 1. Risk Neutral SUR Estimation Results

INDEPENDENT VARIABLES	DEPENDENT VARIABLES			
	Wheat output	Canola output	Oats output	Input index
Intercept	3.5103	-1.5662	9.5996*	-477.4*
	(0.2839)	(-0.8402)	(3.150)	(-1.957)
	[11.18]	[-48.5986]	[108.5383]	[-33.3226]
Expected Price of Wheat	0.1178*	-0.0219*	0.028*	-3.8332*
	(2.660)	(-3.278)	(2.564)	(-4.391)
	[0.563]	[-1.0202]	[0.4753]	[-0.4022]
Expected Price of Canola	-0.0245	0.0156*	-0.0061	-0.1103
	(-1.217)	(5.130)	(-1.243)	(-0.2775)
	[-0.1951]	[1.2082]	[-0.1745]	[-0.0192]
Expected Price of Oats	-0.0346	-0.0477*	0.0051	-1.5107
	(-0.6148)	(-5.620)	(0.3656)	(-1.361)
	[-0.1017]	[-1.3657]	[0.0529]	[-0.0973]
Crop Input Price Index	-10.804	1.8292	-11.86*	637.18*
	(-0.9351)	(1.050)	(-4.165)	(2.796)
	[-0.4166]	[0.6872]	[-1.6234]	[0.5384]
Hourly Labour Wage	0.0363	0.0204	(0.1023)*	0.2528
	(0.2666)	(0.9941)	(3.045)	(0.0941)
	[0.0977]	[0.5347]	[0.9758]	[0.0149]
Total Crop Acreage	0.0002*	0.000003	0.00003*	0.0027*
	(4.371)	(0.4852)	(2.912)	(3.449)
	[4.7946]	[0.7816]	[2.7946]	[1.6354]
Lagged Total Crop Acreage	-0.000009	0.0000003	0.00003*	0.00194*
	(-0.2711)	(0.0517)	(-3.364)	(3.027)
	[-0.24]	[0.0672]	[-2.6058]	[1.1582]
Time Trend (Technology)	-0.0023	0.0008	-0.0048*	0.223*
	(-0.3598)	(0.8189)	(-3.035)	(1.761)
	[-14.5406]	[48.5969]	[-107.2922]	[30.756]

Continuation of Table 1.

INDEPENDENT VARIABLES	DEPENDENT VARIABLES Wheat Output	Rapeseed Output	Oats Output	Input Index
Capital Stock	-33.64	-4.103	-4.9829	7042*
	(-0.3955)	(-0.3200)	(-0.2377)	(4.198)
	[-0.1695]	[-0.2014]	[-0.0891]	[0.7777]
Expected barley Price	0.0001	0.00011	-0.00005	-0.0067
	(0.1)	(0.7872)	(-0.2209)	(-0.3395)
	[0.0268]	[0.3101]	[-0.0519]	[-0.0394]
R-Square	0.8431	0.9214	0.9466	0.9891
Durbin-Watson	1.5761	1.9232	1.5671	2.1103
Test for Homogeneity	WCS=1.59	DF=4	PV=0.81	
Test for Symmetry	WCS=33.74	DF=6	PV=0	
Test for CRTS	WCS=44.72	DF=4	PV=0	

* Indicates Significance at 5%

Figures in ( ) are T-Ratios

Figures in [ ] are Elasticities at means

WCS stands for Wald chi square statistic

PV Represents probability value

DF Represents degrees of freedom

Risk Averse Models: Linear Mean-Variance Preferences

The most basic properties of the dual indirect utility function corresponding to maximization of a linear mean-variance risk preference relation under price uncertainty were summarized in the theoretical framework section of the previous chapter. Of most importance for this study, output supplies and factor demands are homogenous of degree zero in mean and variance of prices (P^e,W,Vp) rather than in mean of prices (P^e,W), standard reciprocity (symmetry) relations hold, and the generalization of Hotelling's lemma is straightforward. In addition output supplies are increasing in own expected prices and factor demands are decreasing in own prices. Finally output supplies are decreasing in own price variance in the case of risk aversion ($\acute{\alpha} > 0$) (Coyle 1992).

Assuming a linear mean-variance risk preference function, the models of the previous section can be modified by including normalized variances as explanatory variables. Variances but not covariances between prices are included in the models. Omission of price covariances simplifies model specification and has some empirical support (e.g. Pope and Just 1991; Coyle 1992). Thus the general models of the previous sections were modified as follows:

$$(9) \quad Y_t^i = a_i + \sum_{J=1}^{3} a_{ij} \left(\frac{P^j}{P^o}\right)_{t-1} + a_{i4}\left(\frac{W^1}{P^o}\right)_{t-1} + a_{i5}\left(\frac{W^2}{P^o}\right)_{t-1} + a_{i6}K_t + a_{i7}A_t + a_{i8}A_{t-1} + a_{i9}T + \sum_{j=1}^{4} b_{ij}\frac{Vp_t^j}{P_{t-1}^o} + e_t^i \qquad i=1,2,3$$

$$(10)\quad X_t^1 = a_4 + \sum_{J=1}^{3} a_{4j}\left(\frac{P^j}{P^o}\right)_{t-1} + a_{44}\left(\frac{W^1}{P^o}\right)_{t-1} + a_{45}\left(\frac{W^2}{P^o}\right)_{t-1} + a_{46}K_t + a_{47}A_t + a_{48}A_{t-1} + a_{49}T + \sum_{j=1}^{4} b_{4j}\frac{Vp_t^J}{P_{t-1}^o} + e_t^4$$

and alternatively

(11)

$$\frac{Y_t^i}{A_t} = a_i + \sum_{J=1}^{3} a_{ij}\left(\frac{P^j}{P^o}\right)_{t-1} + a_{i4}\left(\frac{W^1}{P^o}\right)_{t-1} + a_{i5}\left(\frac{W^2}{P^o}\right)_{t-1} + a_{i6}\frac{K_t}{A_t} + a_{i7}A_t + a_{i8}A_{t-1} + a_{i9}T + a_{i10}P_{t-1}^o + \sum_{j=1}^{4} b_{ij}\frac{Vp_t^j}{P_{t-1}^o} + e_t^i \qquad i=1,2,3$$

(12)

$$\frac{X_t^1}{A_t} = a_4 + \sum_{J=1}^{3} a_{4j}\left(\frac{P^j}{P^o}\right)_{t-1} + a_{44}\left(\frac{W^1}{P^o}\right)_{t-1} + a_{45}\left(\frac{W^2}{P^o}\right)_{t-1} + a_{46}\frac{K_t}{A_t} + a_{47}A_t + a_{48}A_{t-1} + a_{49}T + a_{410}P_{t-1}^o + \sum_{j=1}^{4} b_{4j}\frac{Vp_t^J}{P_{t-1}^o} + e_t^4$$

As in the previous section, CWB payments were used as expected prices for CWB crops, current expectations were lagged an additional one year, and barley price was the most common numeraire.

Table 2 in the text provides SUR results for model (11) - (12) consistent with the above specifications. As in the risk neutral model, CRTS and symmetry are rejected. In contrast to the risk neutral model, the homogeneity condition ($a_{110} = a_{210} = a_{310} = a_{410} = 0$) is also rejected. Thus CARA (linear mean-variance preferences) is rejected.

These test results for homogeneity should not necessarily be interpreted as favouring risk neutrality over linear mean-variance preferences, since test results under risk neutrality may largely reflect the relative imprecision of coefficient estimates which can make it relatively difficult to reject null hypotheses.

The results in Table 2 indicate that several coefficients of price variances in output supply equations are significant, and these significant coefficients have the anticipated signs. Wheat output is decreasing in the variance of wheat price and increasing in the variance of oat price, and oats output is increasing in the variance of rapeseed price. The Wald chi-square statistic for the hypothesis of risk neutrality (zero coefficients for all price variances) is 155.59 (16 df), so the hypothesis of risk neutrality is strongly rejected.

Table 2. Linear Mean-Variance SUR Estimation Results

INDEPENDENT VARIABLES	DEPENDENT VARIABLES			
	Wheat output	Canola output	Oats output	Input index
Intercept	8.4168	-2.2611	7.1228*	-148.22
	(0.8872)	(-1.115)	(2.299)	(-0.6247)
	[26.8084]	[-70.162]	[80.5342]	[-10.3456]
Expected Price of Wheat	0.1329*	-0.0304*	0.0309*	-4.8767*
	(3.905)	(-4.182)	(2.784)	(-5.728)
	[0.6364]	[-1.4192]	[0.5261]	[-0.5116]
Expected Price of Canola	0.0202	0.0103*	-0.0111*	0.1288
	(1.129)	(2.675)	(-1.905)	(0.2869)
	[0.1613]	[0.7956]	[-0.3155]	[0.0225]
Expected Price of Oats	-0.1454*	-0.048*	-0.018	-1.4059
	(-3.159)	(-4.876)	(-1.197)	(-1221)
	[-0.4276]	[-1.3739]	[-0.879]	[-0.0906]
Crop Input Price Index	-46.676*	4.5877*	-11.118*	377.05
	(-4.092)	(1.882)	(-2.984)	(1.322)
	[-1.7998]	[1.7234]	[-1.5218]	[0.3186]
Hourly Labour Wage	0.2862*	0.004	0.0919*	3.373
	(2.606)	(0.1715)	(2.562)	(1.228)
	[0.7694]	[0.1054]	[0.8771]	[0.1987]
Total Crop Acreage	0.0002*	0.000002	0.00003*	0.0037*
	(6.025)	(0.3414)	(3.001)	(4.452)
	[5.5015]	[0.6491]	[3.1775]	[2.2281]
Lagged Total Crop Acreage	0.000002	0.0000004	0.00004*	0.00245*
	(0.1035)	(0.081)	(-4.86)	(4.143)
	[0.0675]	[0.11]	[-3.6764]	[1.4814]
Time Trend (Technology)	-0.00497	0.0012	-0.0035*	0.0487
	(-1.003)	(1.094)	(-2.164)	(0.3931)
	[-31.2587]	[70.9446]	[-78.1791]	[6.7117]

Continuation of Table 2.

INDEPENDENT VARIABLES	DEPENDENT VARIABLES Wheat Output	Rapeseed Output	Oats Output	Input Index
Capital Stock	183.86*	-16.67	-2.014	9416.8*
	(2.287)	(-0.9701)	(-0.0767)	(4.683)
	[0.9264]	[-0.8183]	[-0.036]	[1.0399]
Wheat Price variance	-0.2981*	0.00038*	-0.0014	0.0758
	(-3.089)	(1.837)	(0.432)	(0.3141)
	[-0.0945]	[0.1171]	[-0.0153]	[0.0053]
Barley Price Variance	-0.0048*	-0.0002	-0.00006	0.0762
	(-1.764)	(-0.3282)	(0.072)	(1.11)
	[-0.064]	[-0.0248]	[-0.003]	[0.0221]
Canola Price Variance	-0.00017	0.00005	0.0003*	-0.0235*
	(0.4289)	(0.5543)	(2.488)	(-2.435)
	[0.0129]	[0.0346]	[0.0864]	[-0.04]
Oats Price Variance	0.0084*	0.00013	0.00016	0.10243*
	(5.565)	(0.4109)	(0.3161)	(2.726)
	[0.1249]	[0.0192]	[0.0082]	[0.0335]
Expected barley Price	-0.1362*	0.00011	-0.0003	-0.0126
	(-1.862)	(0.7353)	(-1.21)	(-0.6899)
	[-0.3642]	[0.2994]	[-0.2745]	[-0.0739]
R-Square	0.935	0.9346	0.9612	0.9927
Durbin-Watson	2.4882	2.0596	2.0557	2.9527
Test for Symmetry	WCS=57.11	DF=6	PV=0	
Test for Homogeneity	WCS=15	DF=4	PV=0.00047	
Test for CRTS	WCS=135.1	DF=4	PV=0	

* Indicates Significance at 5%

Figures in ( ) are T-Ratios

Figures in [ ] are Elasticities at means

WCS stands for Wald chi square statistic

It is interesting to note that adding price variance terms to the risk neutral model increases the significance of expected prices to some extent. Comparing Tables 1 and 2, there is a substantial increase in the t-ratios for the coefficients of expected wheat and oats prices in the wheat output equation, and there is a moderate increase in the significance of expected prices in the oats equation. This further suggests that inclusion of price variances has improved the model specification. A similar result was noted for a more highly aggregated normalized quadratic model of Manitoba agriculture (Coyle 1992).

The impacts of price variances on supply are substantially smaller than the impacts of expected prices, as was anticipated. In the wheat output equation, the elasticities of supply response with respect to expected prices of wheat and oats were +0.63 and -0.42, respectively (at means of data). In contrast the elasticities of wheat supply response with respect to price variance of wheat and oats were -0.09 and +0.12 respectively.

In contrast to the above model where price variances were measured as a weighted sum of squared errors in expectations (2), price variances were also measured using an autoregressive conditional heteroscedasticity (ARCH) model. A regression equation was formulated whereby current output price depends on a three period distributed lag in price and the variance of the disturbance follows an ARCH (1) process:

$$\sigma_t^2=\delta_o+\delta_1 e_{t-1}^2$$

Then estimates of (δ_o, δ_1) were used to construct a corresponding measure of price variance (a GARCH, process was not considered because the measure of variance would be more complex to construct using Shazam software). However econometric results for (11) - (12) were poor using these measures of price variances, i.e. all variances were insignificant in the model. These variance terms were included in the model along with expected prices based on both CWB payments and market price forecasts from the ARCH (1) model. These poor results for price variances constructed from the ARCH model are not surprising, given the poor performance of rational expectations prices from ARIMA models that has already been noted.

Output supply and factor demand equations were also estimated using modified Generalized Leontief functional forms. Measures of price variances, constructed as in (2), were again significant.

However it is important to reiterate that the measures of expected prices and especially price variances used here are at best crude approximations. For example, note that the model with results in Table 2 assumes that current period output (Y_t) depends on lagged expectations (P_{t-1}) and current period variances (Vp_t). Using either current period expected prices on lagged variances generally led to insignificant results. The arbitrary nature of this final specification illustrates the need for a more appropriate theory of (at the least) how firms evaluate uncertainty.

Risk Averse Models: Non-Linear Mean-Variance Preferences

As noted in the theoretical section of the previous chapter, properties of the dual indirect utility function are considerably more complex when risk preferences are non-linear mean-variance rather than linear mean-mean variance. Production decisions depend on initial wealth (w_o) as well as the mean and variance of prices. In general there is no simple homogeneity condition that applies to (uncompensated) output supplies and factor demands. However in the case of constant relative risk aversion (CRRA), multiplication of initial wealth and expected prices (w_o, P^e,W) by a scalar λ and multiplication of price variances and covariances by λ^2 does not change output supplies and factor demands. Reciprocity conditions are more complex than in risk neutral or linear mean-variance models. Standard reciprocity conditions hold only if the dual U^* (w_o, P^e,W,Vp) is weakly separable as follows: $U^* = U'(\phi(P^e,W,Vp),w_o,Vp)$.

The generalized reciprocity and convexity conditions can be summarized as follows:

$$U^*_{VV}(V)+[\alpha\frac{(w_o+P^eY^*-WX^*,Y^{*T}VpY^*)}{2}Y^{*T}VpY^*]_{VV} \qquad \textit{Symetric positive semidefinite}$$

where V = (w_o, P^e,W,Vp) (Coyle 1994a). Thus comparative static properties of the model are more complex than in the standard risk neutral or linear mean-variance models.

Perhaps the major complication in empirical work due to non-linearity of the mean-variance relation concerns the generalized envelope theorem. Non-linear mean-variance implies, instead of Hotelling's lemma,

(14)

$$Y_j = \frac{\frac{\partial U^*(\,.\,)}{\partial P_j^e}}{\frac{\partial U^*(\,.\,)}{\partial w_o}} \qquad j = 1,..,M$$

$$X_i = \frac{\frac{\partial U^*(\,.\,)}{\partial W_i}}{\frac{\partial U^*(\,.\,)}{\partial w_o}} \qquad i = 1,..,N$$

and also

(15)

$$\frac{\partial U^*(\,.\,)}{\partial w_o} = 1 - \frac{\partial \alpha(Ew, Vw)}{\partial Ew} \, \frac{V\pi}{2}$$

As a result of (14), the standard approach to modelling output supplies and factor demands based on the specification of a functional form for the dual generally leads to estimating equations that are non-linear in coefficients.

Nevertheless we can define the following reduced form output supply and factor demand equations for non-linear mean-variance preferences:

$$(16)\quad \frac{Y_t^i}{A_t}=a_i+\sum_{J=1}^{3} a_{ij}(\frac{P^j}{P^o})_{t-1}+a_{i4}(\frac{W^1}{P^o})_{t-1}+a_{i5}(\frac{W^2}{P^o})_{t-1}+a_{i6}\frac{K_t}{A_t}+a_{i7}A_t+a_{i8}A_{t-1}$$
$$+a_{i9}T+a_{i10}P^o_{t-1}+\sum_{j=1}^{4} b_{ij}\frac{Vp_t^j}{(P^o_{t-1})^2}+b_{i5}\frac{w_{ot}}{P^o_{t-1}}+e_t^i \qquad i=1,2,3$$

(17)

$$\frac{X_t^1}{A_t}=a_4+\sum_{J=1}^{3} a_{4j}(\frac{P^j}{P^o})_{t-1}+a_{44}(\frac{W^1}{P^o})_{t-1}+a_{45}(\frac{W^2}{P^o})_{t-1}+a_{46}\frac{K_t}{A_t}+a_{47}A_t+a_{48}A_{t-1}$$
$$+a_{49}T+a_{410}P^o_{t-1}+\sum_{j=1}^{4} b_{4j}\frac{Vp_t^J}{(P^o_{t-1})^2}+b_{45}\frac{w_{ot}}{P^o_{t-1}}+e_t^4 \qquad i=1,2,3$$

Equations (16) - (17) are a linear approximation to the reduced forms :

$$g^j\ (\ .\ )\ =\frac{\dfrac{\partial U^*(\ .\ )}{\partial P_j^e}}{\dfrac{\partial U^*(\ .\ )}{\partial w_o}}$$

$$g^i\ (\ .\ )\ =\frac{\dfrac{\partial U^*(\ .\ )}{\partial W_i}}{\dfrac{\partial U^*(\ .\ )}{\partial w_o}}$$

SUR results for (16) - (17) are reported in Table 3. Here, the initial stock of wealth is proxied as the sum of the values of Land and buildings, machinery and equipment and a proportion of the house (data is obtained from Statistics Canada Cansim database).

Table 3.Reduced form non-Linear Mean-Variance SUR Estimation Results

INDEPENDENT VARIABLES	DEPENDENT VARIABLES			
	Wheat output	Canola output	Oats output	Input index
Intercept	-4.698	-1.5213	14.768*	-565.1*
	(-0.4553)	(-0.7786)	(4.2591)	(-1.9927)
Expected Price of Wheat	0.1332*	-0.0294*	0.0161	-3.5889*
	(3.431)	(-4.2114)	(1.3504)	(-3.8266)
Expected Price of Canola	0.0217	0.0109*	-0.0032	-0.1115
	(1.047)	(2.8891)	(-0.4804)	(-0.2149)
Expected Price of Oats	-0.1107*	-0.0492*	0.0089	-3.2106*
	(-1.8477)	(-4.7821)	(0.479)	(-2.4025)
Crop Input Price Index	-34.903*	3.0719	-2.538	-477.24
	(-2.586)	(1.1862)	(-0.5616)	(-1.2771)
Hourly Labour Wage	0.1747	0.0107	0.1621*	-0.5263
	(1.4976)	(0.48211)	(4.1017)	(-0.172)
Total Crop Acreage	0.0002*	0.00001	0.00004*	0.0034*
	(4.8165)	(0.8357)	(3.3973)	(3.4868)
Initial Wealth	-0.0106	0.0001	-0.0026*	0.1969*
	(-0.4872)	(0.2909)	(-3.5746)	(3.4277)
Time Trend (Technology)	0.0018	0.0008	0.0077*	0.2779*
	(0.3382)	(0.7523)	(-4.2474)	(1.8815)
Capital Stock	146.75	-13.368	95.599*	3178.3
	(1.3919)	(-0.639)	(2.5777)	(1.0864)
Wheat Price variance	-0.2362*	0.0363*	-0.03	1.6069
	(-2.3037)	(1.8833)	(-0.8877)	(0.6247)
Barley Price Variance	-0.1021	-0.0255	0.2552*	-10.1
	(-0.365)	(-0.4729)	(2.6897)	(-1.3644)
Canola Price Variance	-0.0065	0.0034	-0.0357*	1.9491*
	(-0.1481)	(0.4180)	(-2.426)	(1.7243)

Continuation of Table 3.

INDEPENDENT VARIABLES	DEPENDENT VARIABLES Wheat Output	Rapeseed Output	Oats Output	Input Index
Oats Price Variance	0.3224*	0.0184	0.0158	6.0952*
	(4.0468)	(1.256)	(0.6137)	(3.1565)
Expected Price of Barley	-0.0016*	0.0001	-0.0003	-0.0089
	(-2.0147)	(0.8475)	(-0.996)	(-0.4172)
R-Square	0.9227	0.9394	0.9512	0.991
Durbin-Watson	2.5739	2.1262	2.4184	2.5178
Test for CRTS	WCS=74.52	DF=4	PV=0	
Test for CRRA	WCS=17.56	DF=4	PV=0.0015	
Risk Neutrality Test	WCS=71.42	DF=16	PV=0	

* Indicates Significance at 5%

Figures in ( ) are T-Ratios

Figures in [ ] are Elasticities at means

WCS stands for Wald chi square statistic

Results are mixed for the proxy for initial wealth w_o : coefficients are significant in the wheat and rapeseed equations but are insignificant in the oats and crop input equations. Linear mean-variance utility (CARA) implies that w_o does not influence production decisions. Thus it is not entirely clear from these results whether CARA should be accepted.

The numeraire price of barley (P^o) is significant in the wheat equation and has the anticipated sign (negative). On the other hand, the numeraire is insignificant in other equations. These results are similar to the linear mean-variance model. The joint hypothesis of zero coefficients for the numeraire is rejected. Here the hypothesis of zero coefficients for the numeraire corresponds to the homogeneity condition under CRRA.

In sum, casual inspection of results for the linear mean-variance model (11) - (12) and the reduced form non-linear mean-variance model (16) - (17) does not lead to clear preference for one model over another. The homogeneity and symmetry conditions implied by linear and non-linear mean-variance preferences are rejected. In the non-linear mean-variance model, the proxy for initial wealth is significant in some equations (oats, variable crop inputs) but not in others (wheat, rapeseed). Non-zero coefficients for initial wealth suggest rejection of a linear mean-variance model. Rejection of the homogeneity hypothesis for (16) - (17) suggests that the non-linear mean-variance model does not satisfy CRRA. Price variances and expected prices are more significant in the two risk aversion models than in the risk neutral model, but results are somewhat less significant for the non-linear mean-variance model than for the linear mean-variance model.

The above models (11) - (12) and (16) - (17) are not nested, due to different normalizations of price variances.

A structural model based on a modified normalized quadratic indirect utility function and the generalized envelope relations (14) was also considered. The output supply and factor demand equations were specified as:

(18)

$$\frac{Y_t^i}{A_t} = \frac{a_i + \sum_{j=1}^{3} a_{ij}\left(\frac{P^j}{P^o}\right)_{t-1} + a_{i4}\left(\frac{W^1}{P^o}\right)_{t-1} + a_{i5}\left(\frac{W^2}{P^o}\right)_{t-1} + a_{i6}\frac{K_t}{A_t} + a_{i7}A_t + a_{i8}A_{t-1} + a_{i9}T + a_{i10}P^o_{t-1} + \sum_{j=1}^{4} b_{ij}\frac{Vp_t^j}{(P^o_{t-1})^2} + b_{i5}\frac{w_{ot}}{P^o_{t-1}}}{a_5 + \sum_{j=1}^{3} a_{5j}\left(\frac{P^j}{P^o}\right)_{t-1} + a_{54}\left(\frac{W^1}{P^o}\right)_{t-1} + a_{55}\left(\frac{W^2}{P^o}\right) + a_{56}\frac{K_t}{A_t} + a_{57}A_t + a_{58}A_{t-1} + a_{59}T + a_{510}P^o_{t-1} + \sum_{j=1}^{4} b_{5j}\frac{Vp_t^j}{(P^o_{t-1})^2} + b_{55}\frac{w_{ot}}{P^o_{t-1}}} + e_t^i \qquad i = 1,2,3$$

(19)

$$\frac{X_t^1}{A_t} = \frac{a_4 + \sum_{j=1}^{3} a_{4j}\left(\frac{P^j}{P^o}\right)_{t-1} + a_{44}\left(\frac{W^1}{P^o}\right)_{t-1} + a_{45}\left(\frac{W^2}{P^o}\right)_{t-1} + a_{46}\frac{K_t}{A_t} + a_{47}A_t + a_{48}A_{t-1} + a_{49}T + a_{410}P^o_{t-1} + \sum_{j=1}^{4} b_{4j}\frac{Vp_t^j}{(P^o_{t-1})^2} + b_{45}\frac{w_{ot}}{P^o_{t-1}}}{a_5 + \sum_{j=1}^{3} a_{5j}\frac{(P^j}{P^o)}_{t-1} + a_{54}\left(\frac{W^1}{P^o}\right)_{t-1} + a_{55}\left(\frac{W^2}{P^o}\right)_{t-1} + a_{56}\frac{K_t}{A_t} + a_{57}A_t + a_{58}A_{t-1} + a_{59}T + a_{510}P^o_{t-1} + \sum_{j=1}^{4} b_{5j}\frac{Vp_t^j}{(P^o_{t-1})^2} + b_{55}\frac{w_{ot}}{P^o_{t-1}}} + e_t^4 \qquad i = 1,2,3$$

These equations are highly non-linear in coefficients. An arbitrary normalization of coefficients is required for identification (here $a_5 = 1.0$).

In order to conduct more formal tests regarding risk preferences, within a nested framework, equations (16) - (17) for the non-linear mean-variance model were modified as follows :

all terms $b_{ij}\ Vp^j_t\ /(P^o_{t-1})^2$ in all equations were replaced by $b_{ij}\ Vp^j_t\ /(P^o_{t-1})^\delta$ where δ is a coefficient to be estimated. Replacing $(P^o_{t-1})^2$ by $(P^o_{t-1})^\delta$ as normalizations for price variances specifies a general mean-variance model that contains both CARA and CRRA as special cases. A linear mean-variance model (CARA) implies the following restrictions: $H_o\ :\ \delta = 1 \qquad b_{i5} = 0 \qquad a_{i10} = 0 \qquad i=1,2,3,4.$

Similarly, a non-linear mean-variance model with CRRA implies:

$$H_o:\quad \delta = 2 \qquad a_{i10} = 0 \qquad i=1,2,3,4.$$

This modified version of (16) - (17) was estimated as a non-linear SUR system by maximum likelihood methods using the Davidson-Fletcher-Powell non-linear algorithm as encoded in Shazam (version 7.0). Starting values for δ were 1.0 and 2.0, other starting values were obtained from Table 3. Table 4 presents results using a starting value of $\delta = 1.0$ (the value of the likelihood function was high in this case). The estimated value of δ is 0.62, and all hypotheses $\delta = 0,1,2$ are rejected. Initial wealth (w_o) is significant in the oats and variable crop input equations but not in the wheat and rapeseed equations, and the corresponding joint null hypothesis ($b_{15} = b_{25} = b_{35} = b_{45} = 0$) is rejected. The numeraire price of barley is significant in the wheat, rapeseed, and variable crop input equation, and the corresponding joint null hypothesis ($a_{110} = a_{210} = a_{310} = a_{410} = 0$) is rejected. Thus without conducting joint tests listed under H_o CARA and H_o CRRA, the results in Table 4 suggest that both CARA and CRRA should be rejected.

The price variance of wheat and oats are significant in the wheat equation and the price variance of wheat is significant in the rapeseed equation, as in Table 2 - 3. In contrast to other Tables, price variances are insignificant in the oats equation. Nevertheless the corresponding null joint hypothesis (coefficients of all price variances equal zero) is not rejected based on the Wald chi square statistic. This result is surprising since many price variance terms are significant here and the null hypothesis was rejected in Tables 2 and 3.

The above results must be interpreted with caution due to the non-linear nature of the model when δ is to be estimated and due to the limited choice of starting values for coefficients. For example a starting value of 1.0 for δ led to a an estimate for δ of 0.62 (7.83 t-ratio) as in Table 4, whereas a starting value of 2.0 for δ led to an estimate for δ of 1.08 (12.14 t-ratio). In the latter case, the coefficient of initial wealth was significant in all equations except for wheat, and the corresponding joint null hypothesis ($b_{15} = b_{25} = b_{35} = b_{45} =0$) was rejected. Thus CARA presumably is rejected in the latter case even though the hypothesis $\delta = 1$ is not rejected. When the separate price numeraire term (P^o was deleted from the model (i.e. $a_{110} = a_{210} = a_{310} = a_{410} = 0$ were imposed), then starting values for δ of 1.0 and 2.0 led to estimates of 0.23 (3.07 t-ratio) and 0.4 (4.33 t-ratio) respectively. A more complete grid search in terms of starting values is required in order to approximate the global solution to the maximum likelihood problem.

Table 4. Non-linear estimation of reduced form model nesting CARA and CRRA

INDEPENDENT VARIABLES	DEPENDENT VARIABLES			
	Wheat output	Canola output	Oats output	Input index
Intercept	7.655	-2.1882	8.2326*	-169.08*
	(0.9477)	(-1.3653)	(2.912)	(-3.1583)
Expected Price of Wheat	0.1146*	-0.0308*	0.0249*	-4.6299*
	(3.5648)	(-4.7058)	(2.1198)	(-5.2998)
Expected Price of Canola	-0.1363	0.0078*	-0.0056	0.8178*
	(0.9781)	(2.5923)	(-1.1383)	(-2.004)
Expected Price of Oats	-0.1402*	-0.0486*	0.0036	-3.0214*
	(-3.2025)	(-5.5809)	(0.2457)	(-3.0233)
Crop Input Price Index	-30.624*	4.9926*	-1.9935	-101.47*
	(-2.5454)	(2.0186)	(-0.4645)	(-3.0106)
Hourly Labour Wage	0.2993*	-0.0136	0.1032*	-0.6524
	(3.4579)	(-0.7518)	(3.248)	(-0.4043)
Total Crop Acreage	0.2142*	-0.000004	0.00001*	0.0034*
	(8.9906)	(-0.8461)	(1.9644)	(6.7293)
Initial Wealth	-0.263	0.0005	-0.0019*	0.2299*
	(-1.1334)	(1.4429)	(-2.8254)	(10.053)
Time Trend (Technology)	-0.4674	0.0012	-0.0042*	0.0766*
	(-1.1304)	(1.4107)	(-2.8973)	(2.7387)
Capital Stock	229.32*	50.589*	39.534*	9.4094*
	(3.2615)	(-3.7734)	(1.7887)	(3.3439)
Wheat Price variance	-0.0005*	0.0001*	-0.00003	0.0094*
	(-2.5951)	(2.0775)	(-0.4911)	(1.735)
Barley Price Variance	-0.0006	-0.0001	0.0001	-0.0178
	(-0.8533)	(-0.9619)	(0.4163)	(-1.0405)
Canola Price Variance	-0.00004	0.00003	0.00002	0.0058*
	(-0.3772)	(1.4963)	(-0.4749)	(1.9144)

Continuation of Table 4

INDEPENDENT VARIABLES	DEPENDENT VARIABLES			
	Wheat Output	Rapeseed Output	Oats Output	Input Index
Oats Price Variance	0.0017*	-0.00001	-0.00003	0.0191*
	(2.4459)	(-0.1508)	(-0.1887)	(1.7123)
Expected Price of Barley	-0.0014*	0.0003*	-0.0003	0.0248*
	(-2.261)	(2.2369)	(-1.1298)	(2.2976)
Delta	0.6240*			
	(7.8396)			
R-Square	0.9332	0.9304	0.9382	0.989
Durbin-Watson	2.3423	1.956	2.3594	2.3474
Test for CRTS	WCS=148.7	DF=4	PV=0	
Test for CARA	WCS=286.8	DF=5	PV=0	
Risk Neutrality Test	WCS=12.98	DF=16	PV=0.67	
Initial wealth	WCS=102.7	DF=4	PV=0	
Symmetry Test	WCS=40	DF=6	PV=0	
CRRA Test	WCS=-1.76	DF=1	PV=0	

* Indicates Significance at 5%

Figures in ( ) are T-Ratios

Figures in [ ] are Elasticities at means

WCS stands for Wald chi square statistic

The generalized envelope theorem result (15) can be used along with (14) in order to simplify non-linear estimation of the above model (18) - (19). Equation (15) can be rewritten as

(20)

$$\frac{V\pi}{2}=\frac{1-\dfrac{\partial U^{*}(\,.\,)}{\partial w_o}}{\dfrac{\partial\alpha(\,.\,)}{\partial Ew}}$$

$$\textit{where} \qquad V\pi=\sum_{j=1}^{4} Vp^{j}(Y^{j})^{2}$$

In order to simplify non-linear estimation, it is necessary to assume a functional form for $\partial\acute{\alpha}(Ew, Vw)/\partial Ew$ as well as for $U^{*}(.)$. The generalized convexity result (13) implies that economic theory places restrictions on second derivatives of $U^{*}(.)$. Consequently we should choose a functional form that provides a first order approximation to $\partial\acute{\alpha}(.)/\partial Ew$. Furthermore CRRA implies $\acute{\alpha}(\lambda Ew, \lambda^{2} Vw) = \lambda^{-1} \acute{\alpha}(Ew,Vw)$ (see (12) of chapter 2), and in turn (differentiating by Ew and noting $\partial(\lambda Ew)/\partial Ew = \lambda$)
$\lambda\partial\acute{\alpha}(\lambda Ew, \lambda^{2} Vw)/\partial(\lambda Ew) = \lambda^{-1}\partial\acute{\alpha}(Ew,Vw)/\partial Ew$ which implies
$\partial\acute{\alpha}(\lambda Ew, \lambda^{2} Vw)/\partial(\lambda Ew) = \lambda^{-2}\partial\acute{\alpha}(Ew,Vw)/\partial Ew$. Let $\lambda = 1/Vw$, so CRRA implies

(21) $$\frac{\partial\alpha(Ew,Vw)}{\partial Ew}=\frac{Vw^{-2}\ \partial\alpha(\frac{Ew}{Vw},\frac{1}{Vw})}{\partial(\frac{Ew}{Vw})}$$

Thus the following linear (in coefficients) approximation to $\partial\acute{\alpha}$(Ew, Vw)/∂Ew is consistent with CRRA ($\acute{\alpha}$ = 0 under CRRA)

(22)

$$\frac{\partial\alpha(Ew,Vw)}{\partial Ew}=\frac{(\alpha_o+\alpha_1\frac{Ew}{Vw}+\frac{\alpha_2}{Vw}+\alpha_3 Ew)}{Vw^2}$$

where Vw = Vπ. Combining functional forms in (18) - (19) and (22) the generalized envelope relation (20) implies

(23)

$$\frac{V\pi}{2}=\frac{[1-a_5-\sum_{J=1}^{3}a_{5j}(\frac{P^j}{P^o})_{t-1}-a_{54}(\frac{W^1}{P^o})_{t-1}-a_{55}(\frac{W^2}{P^o})_{t-1}-a_{56}\frac{K_t}{A_t}-a_{57}A_t-a_{58}A_{t-1}-a_{59}T-a_{510}P^o_{t-1}-\sum_{j=1}^{4}b_{5j}\frac{Vp^j_t}{(P^o_{t-1})^2}-b_{55}\frac{w_{ot}}{P^o_{t-1}}}{[(\alpha_o+\alpha_1\frac{Ew}{V\pi}+\frac{\alpha_2}{V\pi}+\alpha_3 Ew)V\pi^2]}$$

where Ew = w_o + Eπ.

One alternative to estimating (18) - (19) directly is as follows:

(a) estimate (23) to obtain predicted values MUw_{ot} of $\partial U^*(.)/\partial w_o$, (b) substitute these predictions for $\partial U^*(.)/\partial w_o$ in (18) - (19) to obtain

(24)

$$\frac{Y_t^i}{A_t}=\frac{a_i+\sum_{J=1}^{3}a_{ij}(\frac{P^j}{P^o})_{t-1}+a_{i4}(\frac{W^1}{P^o})_{t-1}+a_{i5}(\frac{W^2}{P^o})_{t-1}+a_{i6}\frac{K_t}{A_t}+a_{i7}A_t+a_{i8}A_{t-1}+a_{i9}T+a_{i10}P^o_{t-1}+\sum_{j=1}^{4}b_{ij}\frac{Vp_t^j}{(P^o_{t-1})^2}+b_{i5}\frac{w_{ot}}{P^o_{t-1}}}{MUw_{ot}}$$

$$+\ e_t^i \qquad i\ =\ 1,2,3$$

(25)

$$\frac{X_t^1}{A_t}=\frac{a_4+\sum_{J=1}^{3}a_{4j}(\frac{P^j}{P^o})_{t-1}+a_{44}(\frac{W^1}{P^o})_{t-1}+a_{45}(\frac{W^2}{P^o})_{t-1}+a_{46}\frac{K_t}{A_t}+a_{47}A_t+a_{48}A_{t-1}+a_{49}T+a_{410}P^o_{t-1}+\sum_{j=1}^{4}b_{4j}\frac{Vp_t^j}{(P^o_{t-1})^2}+b_{45}\frac{w_{ot}}{P^o_{t-1}}}{MUw_{ot}}$$

$$+e_t^4 \qquad i\ =\ 1,2,3$$

and then (c) estimate (24) - (25) . The advantage of this approach is that only one equation is (23) is non-linear in coefficients, and there are fewer coefficients in (23) than in equation (18) -(19). Equations (23) are non-linear in endogenous variables ($E\pi$,$V\pi$) as well as in coefficients.

A second alternative to estimating (18) - (19) directly is as follows: substitute (22) for $\partial U^*(.)/\partial w_o$ in (18) - (19) using (15). The resulting system of equations is

(26)

$$\frac{Y_t^i}{A_t}=\frac{a_i+\sum_{J=1}^{3}a_{ij}(\frac{P^j}{P^o})_{t-1}+a_{i4}(\frac{W^1}{P^o})_{t-1}+a_{i5}(\frac{W^2}{P^o})_{t-1}+a_{i6}\frac{K_t}{A_t}+a_{i7}A_t+a_{i8}A_{t-1}+a_{i9}T+a_{i10}P^o_{t-1}+\sum_{j=1}^{4}b_{ij}\frac{Vp_t^j}{(P^o_{t-1})^2}+b_{i5}\frac{w_{ot}}{P^o_{t-1}}}{1-\frac{(\alpha_o+\alpha_1\frac{Ew}{V\pi}+\frac{\alpha_2}{V\pi}+\alpha_3 Ew)}{(2V\pi)}}$$

$$+\ e_t^i \qquad i\ =\ 1,2,3$$

(27)

$$\frac{X_t^1}{A_t}=\frac{a_4+\sum_{J=1}^{3}a_{4j}(\frac{P^j}{P^o})_{t-1}+a_{44}(\frac{W^1}{P^o})_{t-1}+a_{45}(\frac{W^2}{P^o})_{t-1}+a_{46}\frac{K_t}{A_t}+a_{47}A_t+a_{48}A_{t-1}+a_{49}T+a_{410}P^o_{t-1}+\sum_{j=1}^{4}b_{4j}\frac{Vp_t^j}{(P^o_{t-1})^2}+b_{45}\frac{w_{ot}}{P^o_{t-1}}}{1-\frac{(\alpha_o+\alpha_1\frac{Ew}{V\pi}+\frac{\alpha_2}{V\pi}+\alpha_3 Ew)}{2V\pi}}$$

$$+\ e_t^4 \qquad i\ =\ 1,2,3$$

This approach reduces the number of coefficients in the denominator of the estimating equation relative to (18) - (19), although the equations are non-linear in endogenous variables. Equation (23) may also be estimated in order to obtain coefficient estimates $\partial U^*(.)/\partial w_o$.

Although the second approach involves a somewhat more complex non-linear estimation problem than in the first case (coefficients for $\partial U^*(.)/\partial w_o$ appear in the denominator of more than one equation), it has the advantage of directly estimating all the coefficients $\partial U^*(.)/\partial P^e$,$\partial U^*(.)/\partial W$,$\partial \acute{\alpha}(.)/\partial Ew$ in the equations of primary intent (output supply and factor demands).

Whereas the first approach led to meaningless results, the second approach appeared to offer some promise in improving over estimates from direct estimation of (18) - (19). Non-linear SUR estimates of (26) - (27) are presented in Table 5. The difficulty in estimation of (26) - (27) is indicated by the fact that even after 273 iterations to "convergence", the estimates of several coefficients of $\partial\acute{\alpha}(.)/\partial Ew$ are still at the starting values of 1.0. Nevertheless estimates of other coefficients are not obviously meaningless, in contrast to direct estimates of (18) -(19). This may suggest that, given a detailed grid search of starting values for coefficients of $\partial\acute{\alpha}(.)/\partial Ew$, this second approach can provide a feasible method for estimating a non-linear mean-variance structural model (14) - (15).

Non-linear 3SLS of (26) - (27) and non-linear 2SLS of (23) (using squares of exogenous variables for the risk neutral model as additional instruments) were unsuccessful.

Table 5. Non-linear estimation of modified structural model

INDEPENDENT VARIABLES	DEPENDENT VARIABLES				
	Wheat output	Canola output	Oats output	Input index	DEN.
Intercept	2.149	-3.3608	17.067*	-812.48*	
	(0.1624)	(-1.6295)	(4.9556)	(-3.1443)	
Expected Price of Wheat	0.1516*	-0.0299*	0.0231*	-3.3927*	
	(3.5816)	(-4.1004)	(1.8491)	(-4.0068)	
Expected Price of Canola	0.0178	0.0112*	-0.0033	-0.3301	
	(0.7911)	(2.7777)	(-0.4873)	(-0.6986)	
Expected Price of Oats	-0.0891	-0.0506*	0.0182	-1.7455	
	(-1.3987)	(-4.6086)	(0.9062)	(-1.2246)	
Crop Input Price Index	-38.401*	3.8421	-4.0612	-374.85	
	(-2.261)	(1.3204)	(-0.8273)	(-1.1158)	
Hourly Labour Wage	0.1172	-0.0045	0.14788	-6.0916*	
	(0.7658)	(-0.1852)	(3.587)	(-1.7942)	
Total Crop Acreage	0.0001*	0.000003	0.00004*	0.0018*	
	(2.7626)	(0.4017)	(2.8145)	(1.7463)	
Initial Wealth	0.0022	0.00005	-.0018*	0.2668*	
	(0.914)	(0.136)	(-2.6295)	(5.1846)	
Time Trend (Technology)	-0.0014	0.0017	-0.0088*	0.4122*	
	(-0.2034)	(1.5845)	(-4.8528)	(3.0247)	
Capital Stock	-61.001	-9.6633	47.921	-9.3443	
	(-0.525)	(-0.5468)	(1.5489)	(-0.0042)	
Wheat Price variance	-0.2043*	0.0266	-0.0268	0.2958	
	(-1.7295)	(1.361)	(-0.8256)	(0.1275)	

Continuation of Table 5:

INDEPENDENT VARIABLES	DEPENDENT VARIABLES				
	Wheat Output	Rapesseed Output	Oats Output	Input Index	DEN.
Barley Price Variance	-0.4621	-0.0321	0.1373	-18.059*	
	(-1.2633)	(-0.5785)	(1.3245)	(-2.5193)	
Canola Price Variance	0.0256	0.0051	-0.0261*	2.5894*	
	(0.4663)	(0.5814)	(-1.7208)	(2.4158)	
Oats Price Variance	0.2158*	0.0155	-0.0258	2.7139	
	(2.4309)	(1.0233)	(-0.8859)	(1.2769)	
G10					0.9296
					(.5252)
G11					1
					1
G12					1
					1
G13					-875*
					(-2.73)
R-Square	0.8905	0.9385	0.9576	0.9935	
Durbin-Watson	2.5658	2.1033	2.4627	2.4004	
Test for CRTS	WCS=33.6	DF=4	PV=0		
Test for Numeraire	WCS=18.3	DF=4	PV=0		
Risk Neutrality Test	WCS=46.9	DF=16	PV=0		

* Indicates Significance at 5%

Figures in () are T-Ratios

Figures in [] are Elasticities at means

WCS stands for Wald chi square statistic

DEN. Refers to the denominator in the model

CHAPTER IV

CONCLUSION

Summary of Results

In recognising that Manitoba grain producers operate in a risky and uncertain environment, this study attempts to develop a relevant econometric model based on duality theory. The developed models are estimated to provide measures of the impacts of risk aversion and price uncertainty on the supply of specific crops.

Estimates of restricted and unrestricted distributed lag models were used to investigate both market and CWB prices. Results supported previous studies which suggested that the two components of crop supply response (crop acreage and yield response) involve different time lags.

In comparing risk neutral models to linear mean-variance models using a simple static structure, it is observed that the inclusion of risk proxies generally improves model specification as indicated by the marked increases in the significant and appropriately signed coefficient estimates. While CRTS and symmetry conditions are rejected in both models, homogeneity is not rejected in the risk neutral case although this may be due to imprecise coefficient estimates. The rejection of homogeneity in the linear mean-variance model implies rejection of CARA. Risk neutrality (insignificance of price variances) is strongly rejected. In view of the crude measures employed in approximating expectations and consequently variances, the ARCH model was tried as an alternative but yielded comparatively poorer estimates.

A reduced form non-linear mean-variance model was also specified and estimated through the inclusion of an initial wealth proxy yielding mixed results. The reduced form non-linear mean-variance model performed better than the risk neutral model but not necessarily better than the linear mean-variance model. The proxy for initial wealth was significant in two of four equations, but price variances and expected prices were somewhat less significant in the non-linear mean-variance model than in the linear mean-variance model. CRRA was rejected for the non-linear mean-variance model.

Efforts to estimate a structural model of non-linear mean-variance directly were discouraging given the unreasonable results obtained despite convergence. However one of two modified versions of the structural model suggests reasonable estimates for the structural model are attainable.

Further Research

There are many obvious serious weaknesses of this study that need to be addressed in future research. Methods of modelling expected prices and price uncertainty are ad hoc; yield uncertainty is not considered; relevant specific aspects of government support programs are ignored; and the dynamics of crop agriculture is not addressed. Unfortunately these problems are not unique to this study.

One serious weakness of this research that can perhaps be addressed with some success in the near future concerns the difference in lag structure between crop acreage and yield responses. Preliminary estimates of distributed lag models for crop outputs were interpreted here as consistent with such a distinction, but this distinction was not incorporated into the systems models of crop output supply.

However it now appears possible to decompose crop output supply response into acreage and yield responses in a tractable manner as follows. Assuming that crop yields are predetermined, an essentially static crop acreage allocation model can be formulated using an elementary modification of static duality models of production under risk aversion and price uncertainty. The adding up properties of acreage demands conditional on total crop acreage facilitate estimation of these joint acreage allocations.

Assuming disjoint technologies, disjoint investment decisions related to crop yields can be specified using dynamic duality models of production under risk aversion and price uncertainty. These dynamic duality models for crop yields can be formulated in terms of optimal control (if current acreage for a particular crop provides an adequate index of the intertemporal plan) or in terms of discrete time calculus of variations. An obvious weakness of standard dynamic models is reliance on ad hoc cost of adjustment mechanisms, but to at least some extent this can be mitigated by incorporating non-static expectations and irreversibility.

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APPENDIX A

Initial Unrestricted and restricted Distributed Lag Models

Table A11. Distributed lag estimation using labour wage as numeraire

	DEPENDENT	VARIABLES		
INDEPENDENT VARIABLES	Wheat	Barley	Canola	Oats
Own Price Lags normalized by:	Output	Output	Output	Output
Labour Wage				
0	-935.49	-253.37	-91.3*	-4.9*
	(317.9)	(268.5)	(25.3)	(1.2)
1	-414.29	-20.813	11.9	6.1
	(405.8)	(363.4)	(36.8)	(7.9)
2	-94.451	22.744	-36.1	2.6*
	(416.1)	(362.8)	(39.6)	(0.8)
3	-221.58	-246.52	-33.2	2.2*
	(414.2)	(359.7)	(40.7)	(0.5)
4	-551.67	161.62	-9.6	7.5
	(421.3)	(363.4)	(39.8)	(6.10
5	42.091	30.481	84.6*	-6.2
	(406.6)	(348.8)	(36.2)	(5.9)
6	-490.5	-80.164	-50.1*	-7
	(347.7)	(272.6)	(26.2)	(8)

* Indicates significance at 5%
Figures in parentheses are standard errors

Table A12. Distributed lag estimation using input price index as numeraire

		DEPENDENT	VARIABLES		
INDEPENDENT VARIABLES		Wheat	Barley	Canola	Oats
own price lags normalized by:		Output	Output	Output	Output
Input Price Index					
	0	-12.9	-3	-1	-5.4*
		(4)	(3.2)	(0.4)	(1.4)
	1	-7.4	-1.7	-0.2	1.5*
		(4.7)	(3.9)	(0.6)	(0.9)
	2	-3	-1	-0.4	3.3*
		(4.8)	(3.9)	(0.7)	(0.8)
	3	-4	-2.8	-0.6	2.3*
		(4.8)	(3.9)	(0.7)	(0.5)
	4	-7.4	1.7	-0.5	1.4
		(4.9)	(4)	(0.7)	(7.5)
	5	-1.2	0.7	1.1	-1.3*
		(4.8)	(3.8)	(0.6)	(0.7)
	6	-7.3	-2.2	-0.5	-0.3
		(4.2)	(3.1)	(0.4)	(1.0)

* Indicates significance at 5%
Figures in parentheses are standard errors

Table A13. Distributed lag estimation using Barley market price as numeraire

		DEPENDENT	VARIABLES	
INDEPENDENT VARIABLES		Wheat	Canola	Oats
own price lags normalized by:		Output	Output	Output
Barley Price				
	0	1574.1	80.6	-4.3
		(1156)	(76.2)	(0.40
	1	-1187.4	228.9*	-3.3*
		(1159)	(70.2)	(1.9)
	2	1069.2	43.4	1.1*
		(1137)	(90.3)	(0.2)
	3	3407.9*	77.1	8.5*
		(1087)	(83.3)	(0.9)
	4	1009.2	16.8	-0.3
		(1175)	(99.5)	(11)
	5	2082.2*	5.2	-5.3*
		(1169)	(78.1)	(1.0)
	6	330.49	-29.1	2.7
		(1141)	(83)	(1.8)

* Indicates significance at 5%
Figures in parentheses are standard errors

Table A14. Distributed lag estimation using Rapeseed market price as numeraire

INDEPENDENT VARIABLES own price lags normalized by:		DEPENDENT VARIABLES Wheat Output	Barley Output	Oats Output
Canola Price				
	0	219	-1277.5	-11.8*
		(1969)	(1628)	(1.4)
	1	4872*	2478.1*	-1.3
		(1991)	(1487)	(0.9)
	2	3349*	1866.4	3.2*
		(1997)	(1939)	(0.7)
	3	1733	292	3.5*
		(2109)	(1760)	(0.5)
	4	4883*	193.1	1.5*
		(2131)	(2005)	(0.6)
	5	6415*	4146.8*	-7.6
		(2203)	(1527)	(8.60
	6	709	149	-1.6*
		(2005)	(1670)	(0.90

* Indicates significance at 5%
Figures in parentheses are standard errors

Table A15. Distributed lag estimation using Oats market price as numeraire

	DEPENDENT	VARIABLES	
INDEPENDENT VARIABLES	Wheat	Barley	Canola
own price lags normalized by:	Output	Output	Output
Oats Price			
0	-6	-253.4	-91.3*
	(2.4)	(268.5)	(25.3)
1	3.1	-20.8	11.9
	(2.4)	(363.4)	(36.8)
2	7.8*	22.7	-36.1
	(2.5)	(362.8)	(39.6)
3	-0.4	-246.5	-33.2
	(3)	(359.7)	(40.7)
4	1.5	162.6	-9.6
	(3.2)	(363.4)	(39.8)
5	5.2*	30.5	84.6*
	(3.1)	(348.8)	(36.2)
6	5.3*	-80.2	-50.1
	(2.6)	(272.6)	(26.2)

* Indicates significance at 5%
Figures in parentheses are standard errors

Table A16. Restricted distributed lag model using CWB Barley price as numeraire

INDEPENDENT VARIABLE own price lags normalized by:		DEPENDENT VARIABLE Wheat Output
Barley		
	0	442.9
		(619.3)
	1	1908*
		(365.9)
	2	2624.4*
		(440.1)
	3	2592*
		(448.4)
	4	1810.9*
		(423.5)
	5	281
		(733)

* Indicates significance at 5%
Figures in parentheses are standard errors

Table A17. Unrestricted distributed lag model using CWB barley price as numeraire

INDEPENDENT VARIABLES own price lags normalized by:		Wheat Output
Barley		
	0	346
		(611)
	1	1521.4*
		(624.4)
	2	2212.5*
		(739.7)
	3	3445*
		(742.7)
	4	1661.6*
		(796.3)
	5	569.8
		(796.3)

* Indicates significance at 5%
Figures in parentheses are standard errors

APPENDIX B

Linear Mean-Variance Models

Table B11. Linear mean-variance model with current CWB prices

INDEPENDENT VARIABLES	DEPENDENT VARIABLES			
	Wheat output	Canola output	Oats output	Input index
Intercept	20.03	-2.051	-1.5756	-237.64
	(25.17)	(3.156)	(5.58)	(609.6)
Expected Price of Wheat	-0.0917	-0.0261*	0.0298	-1.2807
	(0.09049)	(0.0113)	(0.02)	(2.191)
Expected Price of Canola	0.00345	0.0187*	0.0183*	0.9936
	(0.03358)	(0.0042)	(0.0074)	(0.8131)
Expected Price of Oats	0.07212	-0.0394*	0.0543*	-4.9676
	(0.1391)	(0.0174)	(0.0308)	(3.367)
Crop Input Price Index	2.0223	-0.0049	5.2137	-903.76
	(23.53)	(2.951)	(5.216)	(569.9)
Hourly Labour Wage	-0.03832	0.0317	-0.1291	13.274
	(0.3591)	(0.045)	(0.0796)	(8.695)
Total Crop Acreage	0.0002*	0.000006	0.00002*	0.00365*
	(0.00006)	(0.000007)	(0.00001)	(0.0014)
Lagged Total Crop Acreage	0.00004	-0.000009*	-0.00003*	0.0013
	(0.00004)	(0.000005)	(0.000009)	(0.0009)
Time Trend (Technology)	-0.0111	0.0011*	0.00094	0.00991
	(0.01305)	(0.0016)	(0.00289)	(0.3161)
Capital Stock	186.5	10.214	8.5522	12087*
	(142.4)	(17.85)	(31.55)	(3448)
Wheat Price variance	-0.0004	-0.00027	0.0005*	-0.0988*
	(0.0014)	(0.00017)	(0.0003)	(0.0329)
Barley Price Variance	-0.00069	0.00005	0.0005	0.0463

	(0.00512)	(0.00064)	(0.0011)	(0.124)
Canola Price Variance	-0.0005	-0.00002	0.0003*	-0.0205
	(0.0008)	(0.0001)	(0.00017)	(0.0194)
Oats Price Variance	0.7906*	0.00009	0.00047	0.1323*
	(0.00285)	(0.00036)	(0.00063)	(0.06924)
Expected barley Price	-0.00107	0.00009	-0.001*	-0.015
	(0.0017)	(0.0002)	(0.00037)	(0.0405)
R-Square	0.809	0.9338	0.9474	0.98
Durbin-Watson	1.6676	1.7992	2.057	1.6328

* Indicates significance at 5%
Figures in parentheses are standard errors

Table B12. Linear mean-variance model with current ARCH prices and ARCH variances

INDEPENDENT VARIABLES	DEPENDENT VARIABLES			
	Wheat output	Canola output	Oats output	Input index
Intercept	13.956	-3.6797	2.6422	-309.44
	(14.44)	(3.11)	(4.532)	(486.6)
Expected Price of Wheat	-0.2714*	0.0032	-0.6162	7.3313*
	(0.12)	(0.0025)	(0.0377)	(4.044)
Expected Price of Canola	0.0078	0.0176*	0.0012	0.3518
	(0.0181)	(0.0039)	(0.0057)	(0.6108)
Expected Price of Oats	-0.6042*	-0.0431	-0.0189	2.809
	(0.1319)	(0.0284)	(0.0414)	(4.444)
Crop Input Price Index	-1.6024	-3.6474	-2.3767	-650.89
	(12.73)	(2.74)	(3.994)	(428.8)
Hourly Labour Wage	0.2674	0.0109	-0.0652	8.1097
	(0.1907)	(0.0411)	(0.0598)	(6.424)
Total Crop Acreage	0.002*	0.00002*	0.00005*	0.003*
	(0.00004)	(0.000009)	(0.00001)	(0.0014)
Lagged Total Crop Acreage	0.00002	-0.1405	-0.00001	0.0013
	(0.00003)	(0.000006)	(0.00001)	(0.0009)
Time Trend (Technology)	-0.0073	0.0184	-0.0013	0.1297
	(0.0074)	(0.0031)	(0.0023)	(0.2495)
Capital Stock	0.1977	47.465*	53.739*	8620.8*
	(92.43)	(19.90)	(29)	(3114)
Wheat Price variance	-0.0089	0.0018	0.0059	0.1945
	(0.0145)	(0.0031)	(0.0046)	(0.4896)
Barley Price Variance	-0.0533*	-0.0105*	-0.0163*	0.0524

	(0.0296)	(0.0064)	(0.0093)	(0.9970)
Canola Price Variance	0.0209	0.0046	0.0088*	0.1734
	(0.0139)	(0.0030)	(0.0044)	(0.4697)
Oats Price Variance	0.0427*	0.0023	-0.0026	-0.0362
	(0.0152)	(0.0032)	(0.0048)	(0.5104)
Expected Price of Barley	-0.0034	-0.0013*	-0.00248*	0.0283
	(0.0026)	(0.0006)	(0.0008)	(0.0887)
R-Square	0.8932	0.891	0.9411	0.9784
Durbin-Watson	1.386	1.3914	1.2617	1.3091

Test for Symmetry	WCS=67.301	DF=6	PV=0
Test for CRTS	WCS=172.43	DF=4	PV=0

APPENDIX C

UNITS OF MEASUREMENT

Acreage	Thousands of acres
Output	Thousands of metric tonnes
Prices/Value	Canadian dollars per metric tonne

YEAR	WHEAT ACREAGE	WHEAT OUTPUT	WHEAT MARKET PRICE	OATS ACREAGE
v1	v2	v3	v4	v5
1961	2914	926	65	1300
1962	3042	2177	62	1794
1963	3153	1660	63	1620
1964	3385	2313	60	1635
1965	3240	2150	61	1525
1966	3255	2150	65	1530
1967	3520	2449	60	1600
1968	3400	2477	48	1580
1969	2500	1742	46	1530
1970	1400	830	52	1260
1971	2519	2014	50	1395
1972	2600	1878	68	1140
1973	3000	2095	158	1300
1974	2800	1605	147	1200
1975	3100	2123	130	1100
1976	3800	2803	103	1250
1977	3200	2749	98	1050
1978	3400	2831	133	750
1979	3000	2041	170	450
1980	3300	1905	203	450
1981	3900	3326	174	600
1982	4000	3701	165	550
1983	4600	3410	174	550
1984	4450	3742	172	550

1985	4850	5226	160	480
1986	4950	4478	130	450
1987	4850	3946	134	450
1988	4820	2401	197	400
1989	5170	4063	172	500
1990	5430	5865	135	430

OATS OUTPUT	OATS MARKET PRICE	RYE ACREAGE	RYE OUTPUT	RYE MARKET PRICE
v6	v7	v8	v9	v10
370	41	79	22	43
1373	38	119	76	41
956	36	95	53	49
1126	42	133	71	41
1141	46	133	76	41
987	49	100	61	42
1018	45	141	67	43
1249	32	120	63	39
1064	35	163	75	34
817	37	130	72	34
1172	34	128	83	29
848	58	81	46	51
972	106	82	54	105
663	99	102	63	89
771	93	117	76	99
941	73	92	68	85
894	64	110	84	86
632	64	125	99	102
308	89	125	79	146
278	119	150	75	171
463	105	190	175	140
524	81	230	213	87
401	110	210	163	107
432	109	220	195	93
494	96	193	167	79
463	90	77	61	62
416	112	64	46	85
224	145	121	58	113
339	108	230	198	111
409	80	220	193	85

FLAX ACREAGE	FLAX OUTPUT	FLAX MARKET PRICE	RAPESEED ACREAGE	RAPESEED OUTPUT
v11	v12	v13	v14	v15
748	109	130	29	8
667	198	118	32	13
820	236	112	45	17
1025	269	116	84	33
1350	411	106	145	54
1107	266	106	170	47
660	144	121	145	52
820	264	112	91	43
1100	259	101	196	79
1100	292	87	400	163
566	149	86	581	272
500	149	161	470	192
600	193	374	400	174
700	167	376	500	192
750	213	258	750	283
525	160	267	250	102
750	330	207	500	290
750	317	268	1050	578
1250	444	280	1350	567
800	210	320	800	294
700	261	322	600	306
900	436	244	850	399
750	297	323	950	397
1050	439	317	1200	544
1050	559	257	1000	635
1030	572	176	1000	578
800	406	212	1000	585
700	198	351	1550	612
700	221	374	1150	399

800	422	231	950	499
RAPESEED MARKET PRICE	BARLEY ACREAGE	BARLEY OUTPUT	BARLEY MARKET PRICE	CWB EXPECTED WHEAT PRICE
v16	v17	v18	v19	v20
79	655	196	48	58.42
77	629	457	44	69.63
110	584	348	45	70.19
119	497	348	48	68.86
108	601	479	48	72.54
108	875	610	50	69.34
85	970	718	41	80.72
83	1170	936	36	80.35
106	1200	914	32	59.31
102	1500	1110	34	57.8
95	2052	2047	32	57.58
137	2100	1851	58	59.93
258	2100	1807	115	98.68
312	1800	1154	102	152.26
225	1500	1110	105	168.21
257	1600	1459	92	191.95
278	1900	2047	76	118.72
280	1750	1851	78	117.15
267	1450	1263	103	157.04
280	2000	1568	137	215.65
275	2350	2330	119	255.11
259	2000	2373	91	200.12
383	1750	1589	120	195.12
351	1800	1938	121	187.84
268	1850	2526	110	183.98
201	1550	1851	80	146.37
273	1700	1938	74	110
307	1400	1089	124	160

304	1600	1546	124	204.02
288	1550	2014	90	157.14

CWB EXPECTED BARLEY PRICE	CWB EXPECTED OATS PRICE	EXPECTED PRICE OF RAPESEED	EXPECTED PRICE OF RYE
v21	v22	v23	v24
41.02	77.1	88	34
44.04	74.2	79	43
55.33	77.2	77	41
48.22	71.8	110	49
50.43	69.2	119	41
54.43	56.1	108	41
59.66	66.65	108	42
60.07	57.25	85	43
38.95	50.2	83	39
37.2	38.9	106	34
42.8	45.3	102	34
39.5	44.1	95	29
68.43	82.73	137	51
129.27	97.33	258	105
121.36	124.73	312	89
102.46	120.91	225	99
97.17	107.9	257	85
87.12	87	278	86
92.06	68	280	102
144.61	79.1	267	146
148.89	130	280	171
125.55	127.3	275	140
102.07	75	259	87
125	134.5	383	107
153.02	106.6	351	93
86.3	96.63	268	79

60	85.9	201	62
125	112	273	85
94.08	145	307	113
109.23	108	304	111

TOTAL GRAIN ACREAGE	CROP INPUT PRICE INDEX	CROP INPUT QUANTITY INDEX	COST OF FERTILIZER	COST OF PESTICIDE
v26	v28	v29	v30	v31
5725	0.43397	24098.58	3494	1981
6283	0.44894	23259.49	3237	2184
6317	0.45731	28105.7	4558	2276
6759	0.46827	32966.31	5925	2406
6994	0.46925	36283.63	7564	2666
7037	0.47349	55015.24	15302	3516
7036	0.49059	59739.77	18521	3995
7181	0.50221	73545.09	24772	4704
6689	0.48622	52441.12	13050	4150
5790	0.46308	49143.18	10906	3716
7241	0.47376	61092.01	15780	4001
6891	0.4982	69769.29	18903	5253
7482	0.55286	85589.87	26708	8399
7102	0.77987	84820.68	36732	13209
7317	1	97624	59390	19988
7517	1.09343	99890.72	65274	24376
7510	1.06835	117058.52	73516	30264
7825	1.11279	153853.65	101402	42928
7625	1.24923	183428.07	133540	58263
7500	1.46991	164436.73	145361	57093
8340	1.74204	164860.81	162959	82298
8530	1.67882	180056.69	167540	89400
8810	1.62485	207086.25	188069	97786
9270	1.68445	224403.84	211073	110034

9423	1.71805	225766.41	218748	115301
9057	1.54769	231819.7	199373	106100
8864	1.45692	238019.9	188943	107310
8991	1.43846	258051	197141	119542
9350	1.53692	229894.2	183307	115739
9380	1.53076	234173.2	186594	115908

COST OF SEED	COST OF TWINE	TOTAL VARIABLE COST (CROPS)	VALUE OF LAND	VALUE OF MACHINERY
v32	v33	v34	v35	v36
2911	2072	10458	601	272
3044	1977	10442	615	279
3352	2667	12853	670	297
3990	3116	15437	766	322
4521	2275	17026	889	350
4736	2495	26049	984	380
4340	2452	29308	1132	405
5274	2185	36935	1247	424
6208	2090	25498	1197	426
5872	2263	22757	1184	420
6996	2166	28943	1175	411
8345	2258	34759	1253	424
9740	2472	47319	1502	455
11591	4617	66149	1962	584
13793	4453	97624	2320	787
16349	3225	109224	2782	959
18074	3206	125060	3302	1067
23675	3202	171207	4104	1203
33352	3988	229143	4858	1403
34968	4286	241708	5917	1618
36984	4953	287194	6883	1823
39542	5800	302282	6206	2015
44985	5644	336484	6038	2132

51483	5407	377997	5686	2177
48267	5562	387878	5401	2101
47449	5893	358785	5596	2231
44428	6095	346776	5204	2197
48387	6126	371196	4788	2243
47983	6300	353329	5077	2274
49359	6602	358463	5471	2290

LIVESTOCK PRICE INDEX	LABOUR WAGE	CAPITAL STOCK	SUMMERFALLOW ACREAGE	CROP ACREAGE WITH SUMM.FALLOW
v42	v43	V44	V50	
	22.4	8.8	1307	7032
	22.9	8.6	1281	7564
	23.3	8.9	1319	7636
	23.8	9.3	1165	7924
	25	9.8	1133	8127
	27	10.2	1080	8117
	29	10.5	1072	8108
	30	10.7	1097	8278
	31.4	10.4	1295	7984
	32.5	10.0	1619	7409
31.9	33.9	9.6	1074	8315
38.7	37	9.7	1174	8065
53.6	42	10.0	1012	8494
54.6	50.7	11.3	1093	8195
59.7	61.8	13.2	1012	8329
59.1	70.5	15.6	931	8448
59	78.2	16.1	931	8441
74.4	82.5	16.1	809	8634
84.3	87.2	16.8	850	8475
84	93.9	17.7	890	8390
91.8	100	18.2	607	8947

97.1	106.6	18.2	587	9117
90.3	110.9	17.9	546	9356
95.8	114.4	17.7	405	9675
91.5	117.6	17.8	405	9828
100	120.1	17.7	506	9563
103	123	17.6	526	9390
96.4	128.5	17.0	465	9456
96.9	135	16.6	384	9734
101.7	136.9	16.2	384	9764