

THE PROBABILITY OF DROUGHTS IN THE ASSINIBOINE
BASIN

BY

KOFI AWUMAH

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the University of Manitoba in partial fulfillment of the requirements
of the degree of

MASTER OF SCIENCE

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ABSTRACT

Accumulated Basin Storage (ABS) which is an estimate of soil moisture levels in a river basin is used to quantify agricultural droughts. The relevance of ABS to agricultural droughts is investigated by means of its correlation with crop yield within the basin. It is observed that crop yield is more strongly correlated with ABS than with other parameters such as precipitation and runoff and thus the ABS will be the better drought estimating parameter. In the method, agricultural drought is related to soil moisture deficit, when the ABS falls below a reference base level.

The ABS is a random variable and its annual sequence is a random time series with a specific correlation structure. A statistical experimental method of generating a large number of time series samples can be used to estimate probability distributions of characteristic drought variables. The drought variables considered here are the number of droughts occurring in a given time interval, the magnitude of soil moisture deficits, the rate at which soil moisture deficit progresses (or drought intensity), and the duration of drought events. Also, the largest drought deficit, intensity and longest durations are considered. The

theory of the maximum of a random number of random variables is used to interpret the experimental results. This approach is based on the assumption that drought is independent identically distributed random variable whose occurrence follows the Poisson probability law. These assumptions are checked and found to be satisfied. The comparison between the experimental and theoretical results are shown graphically and statistical tests were conducted to show their goodness of fit.

TABLE OF CONTENTS

ACKNOWLEDGEMENT	ii
ABSTRACT	iii
LIST OF TABLES	viii
LIST OF FIGURES	x

SECTION I

THE RELEVANCE OF ACCUMULATED BASIN STORAGE TO DROUGHT MEASUREMENT

	<u>Page</u>
CHAPTER 1	INTRODUCTION
1.1	Introduction 1
1.2	The Accumulated Basin Storage (ABS) . . . 3
1.3	Assiniboine River Basin Characteristics 4
CHAPTER 2	ESTIMATING THE HISTORICAL ABS
2.1	Data Requirements. 7
2.2	Precipitation And Runoff 9
2.3	Potential And Actual Evapotranspiration 16
2.4	Fitting The Historical ABS 21
2.5	Statistical Properties Of The ABS Series 27
CHAPTER 3	THE RELEVANCE OF ABS TO AGRICULTURAL DROUGHTS
3.1	What Are Droughts? 37
3.2	Types Of Droughts 38
3.3	The Relevance of ABS to Agricultural Drought 41

	<u>Page</u>
3.4 Definition Of Agricultural Drought For This Study	43
3.5 Drought Variables Relevant To Agriculture	49

SECTION II

STATISTICAL PROPERTIES OF DROUGHT VARIABLES

CHAPTER 4	ABS DATA GENERATION MODEL	
4.1	The Need For Simulation of The ABS	51
4.2	Stochastic Model Selection	53
4.3	Physical Justification For The Selected Model	58
CHAPTER 5	PROBABILITY DISTRIBUTION OF GENERATED DROUGHT VARIABLES	
5.1	The Distribution Of The Number Of Droughts	71
5.2	The Distribution Of Drought Deficits	89
5.3	Distribution Of Drought Intensity	96
5.4	The Distribution Of Drought Duration	99
CHAPTER 6	PROBABILITY DISTRIBUTION OF THE LARGEST DEFICIT, DURATION AND INTENSITY	
6.1	The Distribution Of The Largest Drought Deficit	108
6.2	The Distribution Of The Longest Drought Duration	111
6.3	The Distribution Of The Largest Drought Intensity	115

	<u>Page</u>
CHAPTER 7	CONCLUSION AND RECOMMENDATION
7.1	Conclusion 119
7.2	Recommendation 120
REFERENCES 122
APPENDIX A 124

LIST OF TABLES

<u>Table</u>		<u>Page</u>
1	Mean Daily Maximum Temperatures	10
2	Precipitation Data For Stations In The Assiniboine River Basin	11
3	Mean Monthly Flow Data For Assiniboine River At Brandon	13
4	Field Evapotranspiration Rates Of Some Major Crops In Southern Manitoba	20
5	Statistical Parameters Of ABS Series	28
6	Parameter Correlation With May ABS	36
7	Relation Between Crop Yield And Other Factors	42
8	Historical Droughts	47
9	Statistics Of The ABS Record	54
10	Correlation Matrix	63
11	Theoretical And Observed CDF Of Number Of Droughts In 30 Years	67
12	Relative Frequencies For Number Of Droughts In Period 30 Years	78
13	Theoretical And Simulated Relative Frequencies Of The Number Of Droughts In 30 Year Interval	79
14	Theoretical And Simulated Cumulative Frequencies Of The Number Of Droughts In 30 Year Interval	79
15	Theoretical And Simulated Cumulative Frequencies For The Number Of Droughts In 30 Years	80
16	Relative Frequencies For Number Of Droughts In Period 40 Years	83

<u>Table</u>	<u>Page</u>
17	Theoretical And Simulated Cumulative Frequencies And Test Of Fitness 84
18	Relative Frequencies Of The Number Of Droughts In 50 Year Interval 87
19	Cumulative Frequencies Of The Number Of Droughts In 50 Year Intervals 88
20	Theoretical And Simulated Relative Frequencies For Drought Deficits 93
21	Cumulative Frequencies Of Drought Deficit And Test Of Fitness 95
22	Relative Frequencies Of Drought Intensity 98
23	Cumulative Frequencies Of Drought Intensity 98
24	Relative Frequencies Of Drought Duration 103
25	Cumulated Frequencies Of Drought Duration 105
26	Cumulative Frequencies Of The Largest Drought Deficit 110
27	Cumulative Frequencies Of Drought Duration 114
28	Cumulative Frequencies Of Drought Intensity 117

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	Assiniboine River	5
2.1	Normal Probability Plot Of Minimum Winter Flows ABS With The 95% Confidence Limits	14
2.2	Normal Probability Plot Of Log Of Minimum Winter Flows ABS With The 95% Confidence Limits	15
2.3	Potential Evapotranspiration Of The Basin. .	19
2.4	Relationship Between ABS And Coefficient Of Evaporation	24
2.5	Plots Of Standardised ABS And Standardised Log Of Minimum Flow	26
2.6	Normal Probability Plot Of May ABS With The 95% Confidence Limits	30
2.7	Normal Probability Plot Of June ABS With The 95% Confidence Limits	31
2.8	Normal Probability Plot Of July ABS With The 95% Confidence Limits	32
2.9	Normal Probability Plot Of August ABS With The 95% Confidence Limits	33
2.10	Normal Probability Plot Of September ABS With The 95% Confidence Limits	34
2.11	Normal Probability Plot of Average Growing Season ABS With The 95% Confidence Limits	35
3.1	Standardised ABS And Crop Yield Per Hectare	44
3.2	Historical Drought Definition Using ABS . .	48
4.1	Plot Of Correlogram Of Average ABS	55
4.2	Plot Of Partial Autocorrelation Function Of Seasonal ABS	55
4.3	Plot Of Correlogram Of Residuals	59

<u>Figure</u>		<u>Page</u>
4.4	Normal Probability Plot Of Residuals With The 95% Confidence Limits	60
4.5	Plot Of Correlogram Of Summer Precipitation	65
4.6	Plot Of Correlogram Of Winter Precipitation	65
4.7	Plot Of Correlogram Of Evapotranspiration. .	66
4.8	Probability Distribution Of The Hurst Statistic For The Generated ABS Series	69
4.9	Probability Distribution Of Correlation Coefficient For The Generated ABS Series	70
5.1	Correlogram Of Drought Deficits	74
5.2	Partial Autocorrelation Function Of Drought Deficits	74
5.3	Plot Of Correlogram Of Drought Duration . .	75
5.4	Plot Of Partial Autocorrelation Function Of Drought Duration	75
5.5	Plot Of Correlogram Of Drought Intensity . .	76
5.6	Plot Of Partial Autocorrelation Function Of Drought Intensity	76
5.7	Relative Frequencies Of The Number Of Droughts In Periods of 30 Years	81
5.8	Cumulative Frequency Distribution Of The Number Of Droughts In Periods of 30 Years	82
5.9	Relative Frequencies Of The Number Of Droughts In Periods Of 40 Years	85
5.10	Cumulative Frequency Distribution Of The Number Of Droughts In Periods of 40 Years	86
5.11	Relative Frequencies Of The Number Of Droughts In Periods Of 50 Years	90
5.12	Cumulative Frequency Distribution Of The Number Of Droughts In Periods Of 50 Years	91
5.13	Theoretical Cumulative Freq. Distribution For Different Time Intervals	92

<u>Figure</u>		<u>Page</u>
5.14	A Plot Of Relative Frequency Of Drought Deficits	94
5.15	A Cumulative Frequency Distribution Of Drought Deficits In Periods Of 50 Years	97
5.16	A Plot Of Relative Frequency of Drought Intensities	100
5.17	Cumulative Frequency Distribution of Drought Intensity In Periods Of 50 Years	101
5.18	A Plot Of Relative Frequency Of Drought Duration	102
5.19	Cumulative Frequency Distribution Of Drought Durations	107
6.1	Cumulative Frequency Distribution Of Maximum Drought Deficit	112
6.2	Cumulative Frequency Distribution Of The Longest Drought Duration	116
6.3	Cumulative Frequency Distribution Of The Maximum Drought Intensity	118

S E C T I O N I

THE RELEVANCE OF ACCUMULATED BASIN STORAGE TO DROUGHT MEASUREMENT

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The objective of this study is to use a physically derived basin parameter, called Accumulated Basin Storage (abbreviated ABS), to analyse agricultural droughts in the Assiniboine River Basin.

In Assiniboine River Basin, one of the Prairie river basins, droughts as well as floods occur. Both events have serious economic consequences in the region and pose threats to the people who settle in the area. Several hydrologic studies have therefore been conducted aimed at alleviating the adverse effects of floods and droughts through physical control structures or planning and managerial measures.

The Assiniboine valley is of great economic importance to the province, because of its contribution to the economy in the agricultural sector. Agriculture is the main-stay of the economy here as oppose to the

manufacturing industries found in the east and oil in the west.

Floods and droughts however are persistent nuisances to farmers in the area. Much of the hydrologic research has been directed towards floods. All that has been done about droughts deals with streamflow droughts. This is because both floods and streamflow droughts concern streamflow magnitudes which are easily quantifiable, measurable, and records on them are available. Agricultural drought on the other hand has no universally accepted measure. It involves not only meteorological factors such as temperature, humidity, precipitation, but also soil drainage properties, evapo-transpiration rate, and other factors such as type of vegetation cover and even agricultural economics.

A single parameter, which combines at least all the hydro-meteorological factors would greatly simplify the problem of agricultural drought analysis.

The ABS, which will be defined in the next chapter, is such a parameter. The mean ABS level within the growing season from May to September was used in this study to define agricultural drought conditions in the basin. In addition, mathematical model of the mean seasonal ABS was developed and used to conduct statistical studies.

1.2 THE ACCUMULATED BASIN STORAGE (ABS)

The Accumulated Basin Storage was developed [Booy & Lye, 1986] as a wetness index in an explanation of the clustering observed in peak flows of the Red River. It is a physically based parameter that measures the average soil moisture conditions in the drainage basin.

The river basin acts as a reservoir, storing precipitation and releasing it in the form of runoff and evapotranspiration. The ABS is obtained as a time series by taking a hydrologic inventory of all inputs and outputs for this reservoir with respect to time.

Since no historical record of soil moisture exists for the basin, the ABS series must be calculated from other meteorological and hydrological time series for which records are available. This process is discussed in the next chapter.

Accumulated Basin Storage is indicative of the soil moisture levels in the upper layers of the soil where the roots of vegetation reach to extract water for transpiration. Since plants also depend on this stored moisture for their existence, very low ABS levels will signify agricultural drought conditions. Therefore ABS, which is a basin wide parameter that averages the soil moisture level over the whole basin, can be expected to be a very good parameter for quantifying agricultural drought.

1.3 ASSINIBOINE RIVER BASIN CHARACTERISTICS

The Assiniboine River is the second major river in the Province of Manitoba, takes its source in Saskatchewan and confluences with the Red River in the heart of the city of Winnipeg.

Its location is between latitudes 49.5° N and 51.5° N and longitudes 97° W and 103° W, as shown in Figure 1. The basin area under this study, upstream of the city of Brandon, covers 86,000 square kilometers. It lies at an altitude of 610 m above sea level at its source to 400 m above sea level at Brandon, and has a length of about 1250 kilometers. Thus it has a very flat gradient of between 0.09 m to 0.19 m per kilometer.

Its main tributaries are the Qu' Appelle and Minnedosa Rivers upstream of Brandon, and the Souris River downstream of Brandon. The average natural flow at Brandon is about 32.0 cumecs with a low flow of 0.15 cumecs and high flow of 435 cumecs. The natural flow has been changed by the construction of Shellmouth dam on the Assiniboine between 1968 and 1972, the Rivers Dam on the Minnedosa River in 1960 and the Qu' Appelle River diversion since 1970. The Shellmouth Reservoir is primarily for flood control but also augments low flows to 0.7 cumecs. The Rivers Dam also adds another 0.15 cumecs to the low flows.

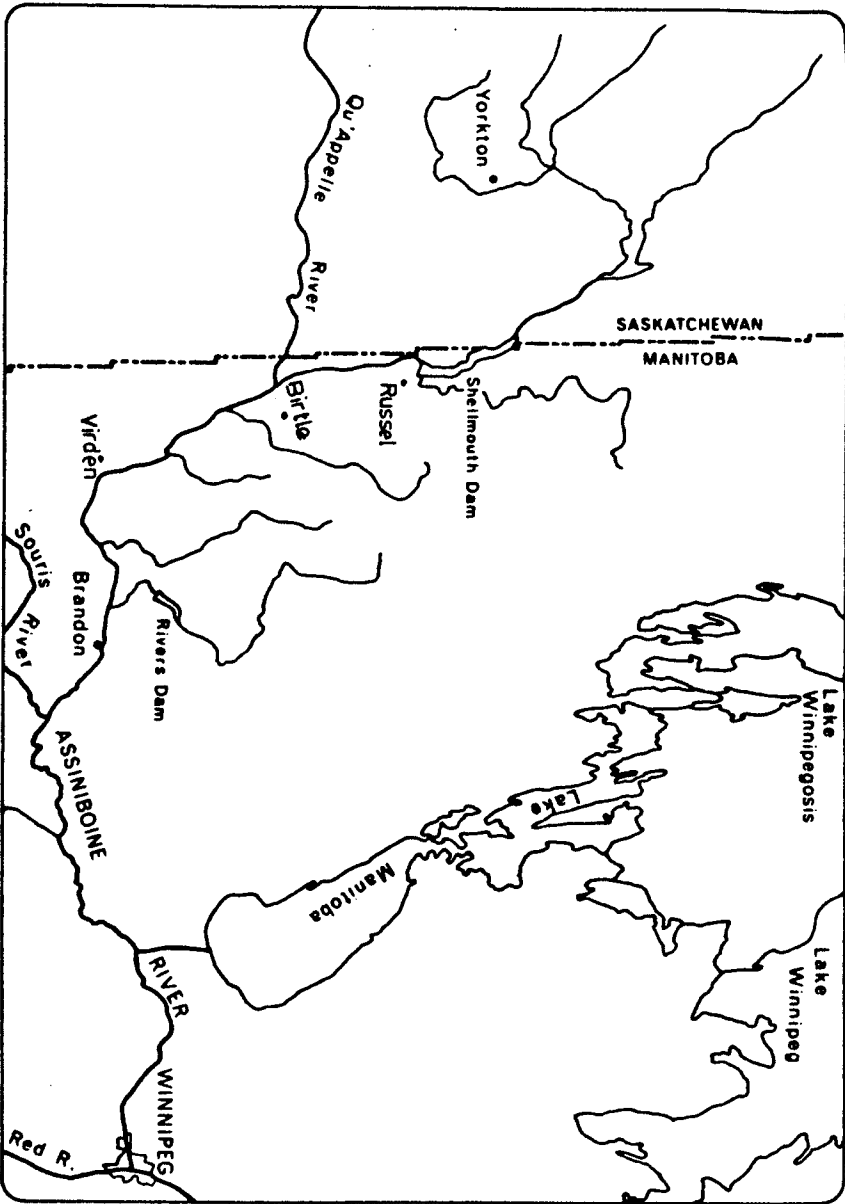


Figure 1 : Assiniboine River

The Assiniboine River valley above Brandon is wide and deep. The banks of the valley are stable and the river has lost its erosive power. The bottom of the valley consists of mainly alluvial deposits.

Studies for groundwater resources for the region show that only a small patch of blanket sand aquifer exists in the Assiniboine valley around the Shellmouth dam and also downstream of Brandon up to the confluence with the Souris River. [Ref: Saskatchewan-Nelson Basin Board, 1972].

CHAPTER 2

ESTIMATING THE HISTORICAL ABS

2.1 DATA REQUIREMENTS

ABS is the soil moisture accumulation obtained after taking a hydrological inventory of the hydrological cycle. Therefore all hydrometeorological variables should be included in the balancing if possible, and any that are not included should not have any significant effect on the magnitude of the estimated ABS.

These hydrometeorological variables include precipitation in the form of snow and rain, basin drainage as indicated by streamflow, evapotranspiration, and groundwater flow. Data will also be needed for natural minimum streamflow.

Records for all the above are available or can be constructed except groundwater flow into and out of the basin. However, in Section 1.3, it was noted that the basin has a very flat gradient. It can be assumed the variations of movement of groundwater in and out of the basin are small. A constant inflow or outflow will automatically be incorporated in the necessary coefficients of the simulation model, hence it will be ignored in the moisture balancing process.

The water year will be taken as the period between April 30th in one calendar year to March 31st the

following calendar year. This is to enable a distinction to be made between winter (when precipitation is snow and no infiltration takes place) and summer (when precipitation is rain, and infiltration takes place).

All data used in this study are obtained from four recording stations within the river basin. The data are averaged over the basin. More stations could not be used because the other stations have either very short records or a lot of missing records. Relatively little error is expected by this limitation since there is not much variation in seasonal precipitation from station to station as will be observed in subsequent tables. The length of records obtained was 63 years, covering a period from 1913 to 1975 at all four stations in the basin.

Some of the incoming precipitation in the basin is either intercepted or stored. Interception occurs when vegetation canopy or foliage traps some of the rainfall before it can infiltrate into the soil. Some of this intercepted rainfall will be evaporated directly and the rest will find its way onto the soil. Storage includes snow pack and storage of rain in depressions in the form of ponds and lakes. Evaporation from snow is insignificant and will therefore be neglected. Apart from the artificial lakes of Shellmouth and Rivers Dams, practically no natural ponds or lakes exist in the basin. All these minor evaporations will be lumped together with the basin evapotranspiration.

2.2 PRECIPITATION AND RUNOFF

Precipitation is an input into the hydrologic model and runoff or drainage is its output.

Monthly precipitation data was obtained from four recording stations and averaged over the basin using the Thiessen Polygon method. Precipitation occurs in two physical states as snow and as rain. A distinction is therefore made between winter precipitation and summer precipitation. Winter precipitation is what occurs between November 1st and March 31st and what occurs from April 1st to October 30th is summer precipitation.

From the temperature records shown in Table 1, it can be observed that winter precipitation usually melts sometime in April. Therefore it will be assumed that the winter precipitation will not contribute to the ABS during the winter months until April when it will be lumped to the April precipitation.

Table 2 shows the mean values of precipitation obtained for the stations and the basin averages.

TABLE 1

MEAN DAILY MAXIMUM TEMPERATURES

Station	Mean Daily Maximum Temperature (°C)				
	Jan	Feb	Mar	Apr	May
Brandon	-13.4	-10.1	-3.1	9.7	18.2
Birtle	-14.4	-10.5	-3.1	7.9	16.9
Russel	-15.0	-11.8	-4.3	7.7	16.6
Virden	-14.5	-10.9	-3.5	7.8	15.9

TABLE 2
 PRECIPITATION DATA FOR STATIONS IN THE
 ASSINIBOINE RIVER BASIN

Station	Mean Winter Ppt (mm)	Mean Summer Ppt (mm)	Mean Annual Ppt (mm)
Brandon	134.1	338.0	472.1
Russel	135.0	315.0	450.0
Birtle	128.5	337.0	465.9
Virden	131.6	331.0	462.6
BASIN AVG.	132.6	329.6	462.2

There are two types of streamflow used in the study. First is the mean monthly streamflow which will be used for developing the monthly ABS series. Since the problem involves taking the moisture balance, it is very important that the streamflows used are the natural flows which has not been augmented or controlled by upstream reservoirs. Data of natural streamflows are available for the period of record to be used at the location of Brandon. These natural flows were derived from historical observations which were modified to remove the effects of minor regulations that took place. The second type of streamflow needed is the minimum winter streamflows. This should also be natural, unaugmented flow. From the record of natural streamflows, the minimum winter flows were obtained as the minimum monthly flows in each year, and occurred mostly in February.

Table 3 is a summary of the streamflow record obtained for the Assiniboine River.

The distribution of the annual minimum winter flows is important in the estimation of the historical ABS. Therefore its normal probability plot was made and shown on Figure 2.1 with 95% confidence band. It can be observed that the data departs very much from normality. After taking logarithms of the data however, the normal plot in Figure 2.2 approximates to the normal distribution. Therefore the logarithm of the minimum winter flows will be used for the ABS fitting process.

TABLE 3

MEAN MONTHLY FLOW DATA FOR ASSINIBOINE
RIVER AT BRANDON

Month	Mean Flow (m ³ /s)
January	4.99
February	4.33
March	7.81
April	84.7
May	114.0
June	60.3
July	44.6
August	20.0
September	13.2
October	13.7
November	11.5
December	6.9

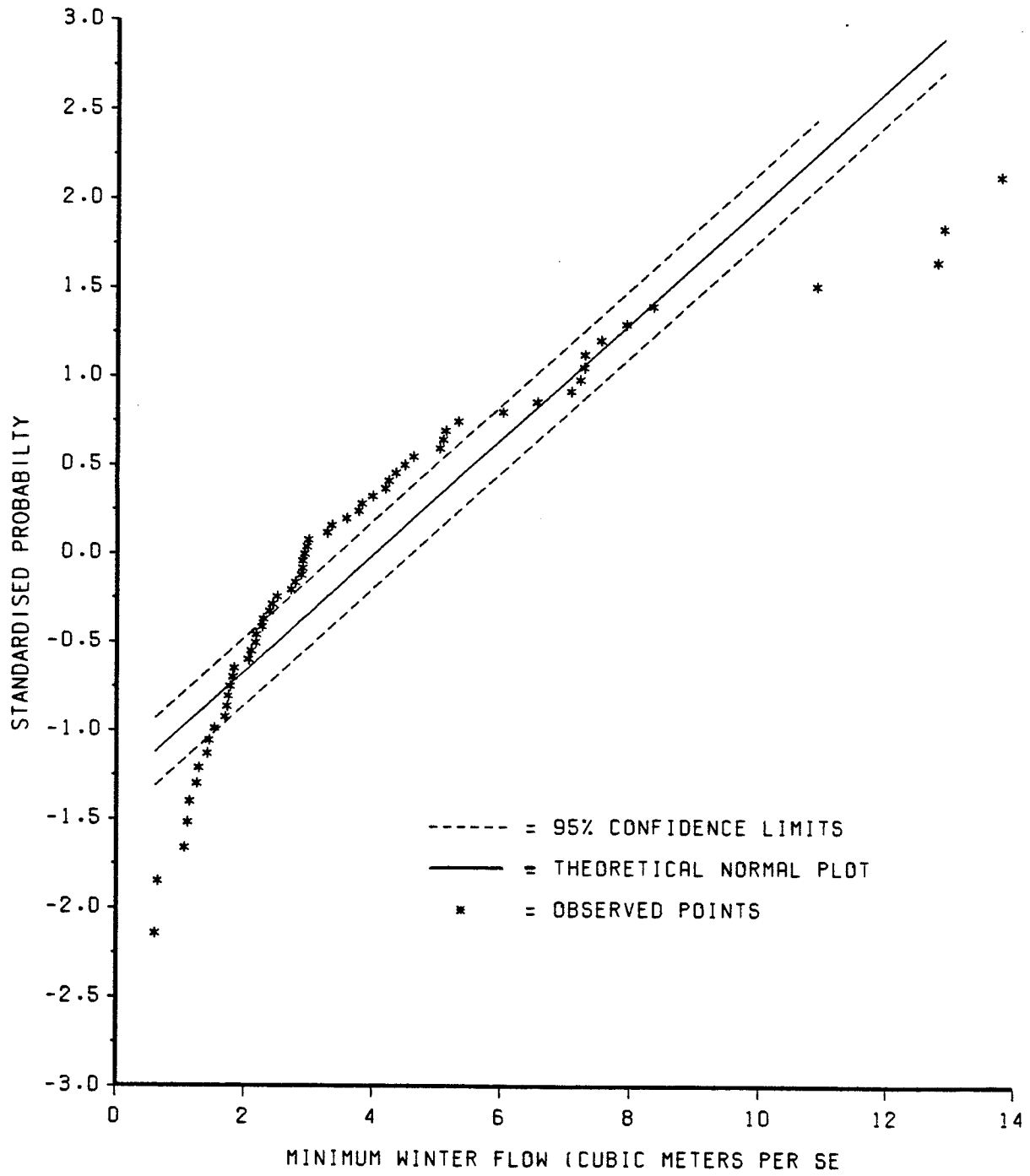


FIG. 2.1 NORMAL PROBABILITY PLOT OF MINIMUM WINTER FLOWS WITH THE 95% CONFIDENCE LIMITS

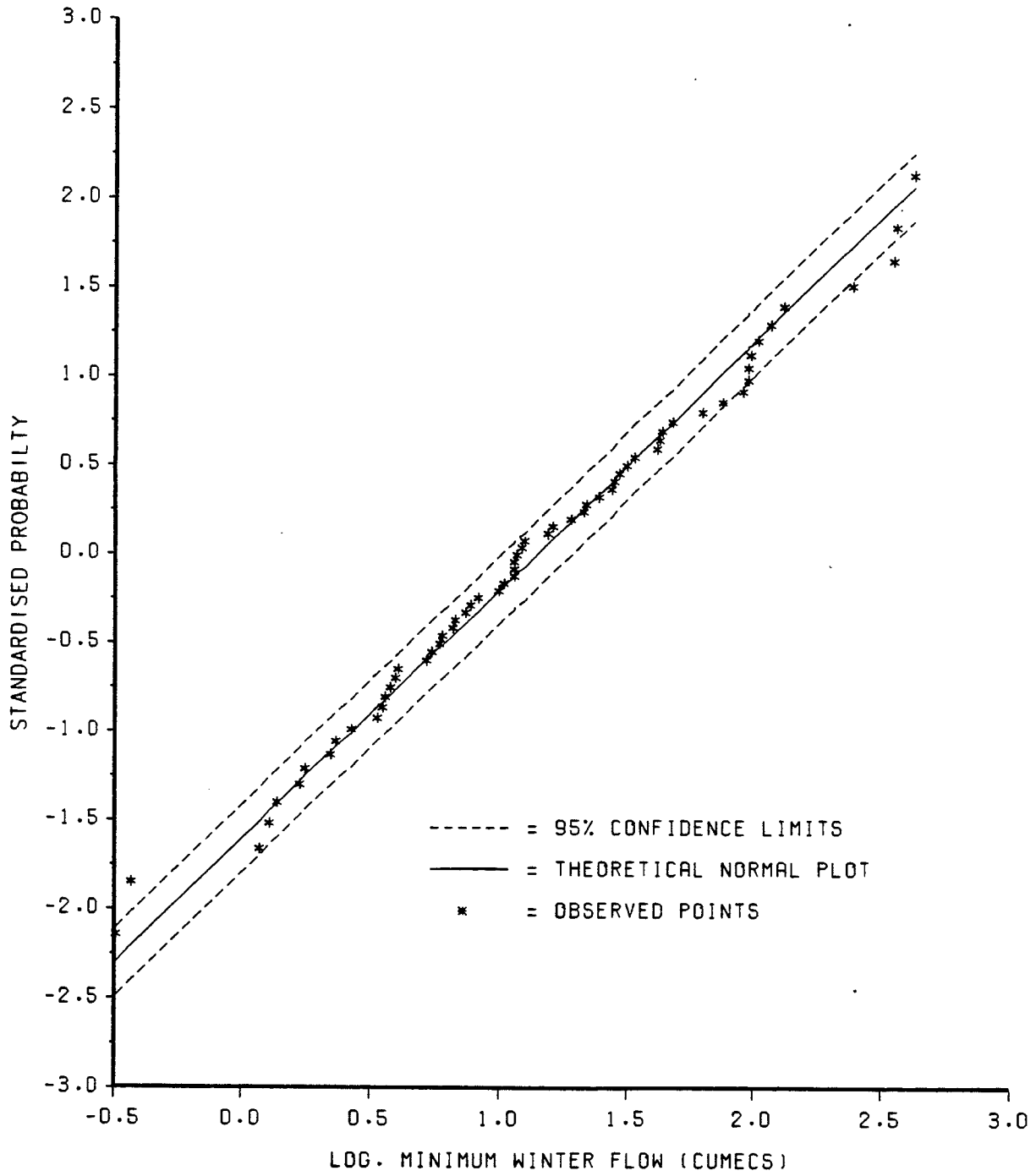


FIG. 2.2 NORMAL PROBABILITY PLOT OF LOG OF MINIMUM WINTER FLOWS WITH THE 95% CONFIDENCE LIMITS

2.3 POTENTIAL AND ACTUAL EVAPOTRANSPIRATION

Evapotranspiration is one of the required variables. All basin moisture is removed through evapotranspiration and runoff. Of the two, evapotranspiration alone removes about 90% of the total precipitation, therefore its accurate estimation is very important in the estimation of the ABS.

Evapotranspiration is the process whereby soil moisture is removed by plants and vaporized into the atmosphere through transpiration. The rate of evapotranspiration depends on the energy available for vaporisation, the amount of moisture available in the soil, and the type of vegetative cover.

For the short term evapotranspiration rate, vegetative cover type and growing stage are very important factors. However, since the vegetative type and growing patterns do not change much from year to year, if basin wide average is considered and from year to year, differences in vegetation cover can be ignored. Therefore, evapotranspiration can be determined on the basis of available energy and soil moisture alone.

The concept of potential evapotranspiration gives the evapotranspiration that is obtained if the available energy alone is considered, without the limiting factor of amount of moisture available for evapotranspiration of the type of vegetation. If this potential evapotranspiration

could be estimated, the the actual field evapotranspiration could be estimated by multiplying it by a coefficient to account for the other limiting factors.

Many complicated relationships have been proposed for deriving the potential evapotranspiration, but the one which is suitable for application to the study is that of Lowry and Johnson (1942). Their method is an empirical relationship between temperature and potential evapotranspiration developed for the Mid-Western U.S.A. Their formula is given as:

$$PE = 0.8 + 0.156F \quad \dots (2.1)$$

where the potential evapotranspiration PE is in feet of water per year over the growing season (April to October), and F is the sum of the effective heat units per month.

$$F = \sum f \quad \dots (2.2)$$

where,

$$f = (t_m - 32^\circ F) n / 1000 \quad \dots (2.3)$$

t_m = mean maximum monthly temperature in °F

n = the number of days in which the daily maximum temperature exceeds 32°F.

For any month, i, the evapotranspiration will be given as:

$$PE_i = (f_i / F) \times PE \quad \dots (2.4)$$

Temperature records within the Assiniboine River Basin show that the potential evapotranspiration for the winter months between November to March is zero because

the daily maximum temperatures are all less than 32°F.

Using the Lowry-Johnson method, the average annual potential evapotranspiration for the basin was computed and a plot of it shown on Figure 2.3. Having a mean of 572 mm and a standard deviation of only 18 mm, it can be observed that the potential evapotranspiration does not change much from year to year.

The potential evapotranspiration values obtained using the Lowry-Johnson method compares very well with the evapotranspiration rates obtained within the region for some of the major crops, as shown in Table 4 (F. Penkava, 1977). Comparing the Lowry-Johnson method mean potential evapotranspiration rate of 572 mm with actual evapotranspiration of the major crops within the basin shows the method gives a good estimate.

Having obtained the potential evapotranspiration, the actual year to year or month to month evapotranspiration can be obtained by using a coefficient that will account for the other factors.

$$E = C \times PE \quad \dots (2.5)$$

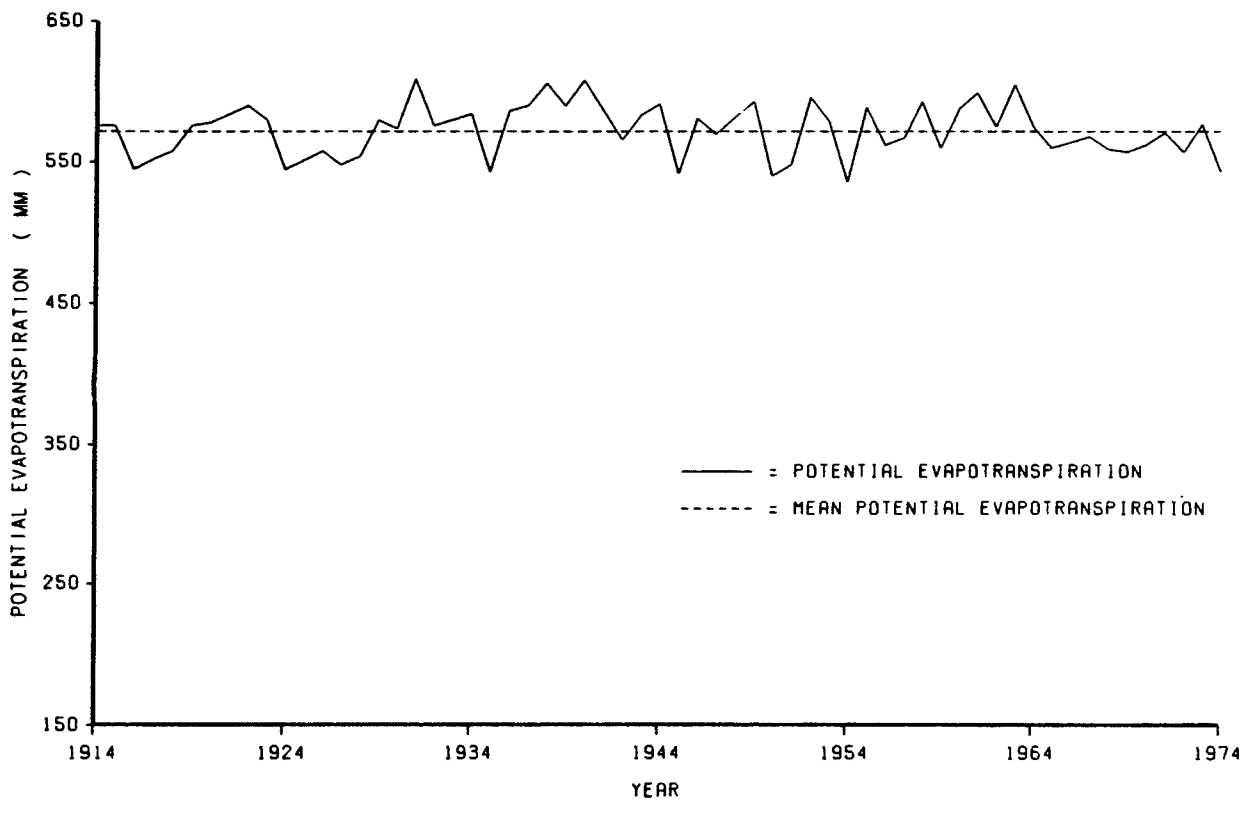


FIG. 2.3 POTENTIAL EVAPOTRANSPIRATION OF THE BASIN

TABLE 4

FIELD EVAPOTRANSPIRATION RATES OF SOME MAJOR
CROPS IN SOUTHERN MANITOBA

(Penkava, F. "Principles and Practices of Commercial Farming". Irrigation and Drainage, University of Manitoba, 1977).

Crop	Evapotranspiration (mm) Total for May to Sept
Alfalfa	525
Pastures and Meadows	540
Potatoes	455
Wheat	340 (May to August)
Cabbage and Lettuce	600
Strawberries	525

The coefficient can be considered constant from year to year, or can vary with the amount of soil moisture available.

A constant coefficient of evapotranspiration will be justified if on the average, the basin can be considered to have a vegetation cover with uniform root depth. On the other hand, a variable coefficient will be valid for a

mixture of deep and shallow root systems. In this case the coefficient will vary with the amount of soil moisture available, from a minimum value corresponding to very dry conditions, to a maximum value corresponding to very wet conditions. It is difficult to check which of these conditions will apply. Consequently the two conditions will be used to derive the ABS and the one that gives the better results will be taken. This process is explained in more detail in the next section.

2.4 FITTING THE HISTORICAL ABS

Since the study is concerned with the soil moisture storage within the growing season, the ABS will be fitted on monthly basis enabling the months of the growing season to be identified.

In general, at any time t , the basin moisture balance could be defined mathematically by:

$$ABS_t = ABS_{t-1} + PW_{t-1} + PS_t - E_t - R_t \quad \dots(2.6)$$

where,

ABS = the accumulated basin moisture storage

PW = winter precipitation

PS = summer precipitation

E_t = actual evapotranspiration

R_t = the stream runoff

$E_t = C \times PE$ where C is a coefficient and PE is the potential evapotranspiration.

All the above variables have been obtained from records except C and the initial ABS (at the beginning of the historical period).

In order to estimate the coefficient C and the initial ABS, use can be made of the fact that the base flow of a river is highly correlated to the soil moisture level in the basin. The minimum winter flow is only the base flow because in winter only stored water in the soil contributes to any flow that takes place in the river. Therefore the ABS at the end of the winter season and the minimum winter flow should be highly correlated. The technique to be used is to find, by trial and error, the values of the initial ABS and C , that will give the best fit between the minimum winter flow series and the ABS series.

On monthly basis, we have the moisture balance as:

$$ABS_{j,t} = ABS_{j-1,t} + P_{j,t} - C_j \times PE_{j,t} - R_{j,t} \dots (2.7)$$

where, j refers to the month in year t , and $P_{j,t}$ refers to the total precipitation available for infiltration in month j . P will therefore be zero for the winter months, and will be the sum of all winter precipitation plus April precipitation for the month of April.

The best way to compare the two time series, the normalised minimum winter flows (more precisely the logarithms of minimum flows) and the ABS, is by comparing their standardised values. This is done by subtracting their respective means from their values and dividing them by their standard deviations. Since the minimum winter flow occurs mostly in February, the February ABS series is fitted to the logarithm of the minimum flows. The best fit is the one for which the SUM OF THE SQUARED DEVIATIONS (SSE) between the two series is minimum.

The fitting was done for the two types of coefficients of evapotranspiration. For the uniform coefficient the coefficient was varied in increments from a small value to a maximum of 1. For each increment the corresponding SSE was calculated. The coefficient that gives the minimum value of the SSE (with a corresponding initial ABS) gives the best fit.

In the case of variable coefficient, it was assumed that a linear relationship exists between the coefficient of evapotranspiration and the ABS within an interval between some minimum and maximum values for the coefficient and the ABS (see diagram).

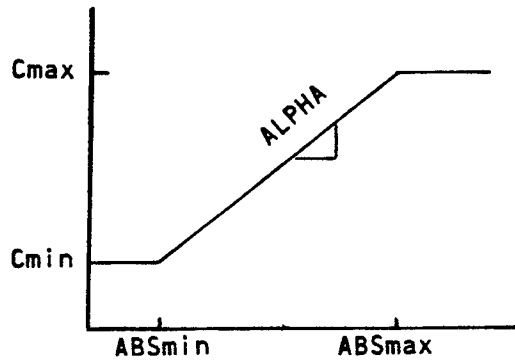


FIG. 2.4 RELATIONSHIP BETWEEN ABS AND COEFFICIENT OF EVAPOTRANSPIRATION

By varying the four key points, C_{max} , C_{min} , ABS_{max} and ABS_{min} , together with the initial ABS, the combination that gave the best fit was obtained. If C_t is the coefficient of evapotranspiration in month t , then

$$C_t = C_{min} \quad \text{for } ABS_{t-1} < ABS_{min} \quad \dots(2.8)$$

$$C_t = C_{min} + ALPHA \times ABS_{t-1} \quad \text{for } ABS_{min} < ABS_{t-1} <= ABS_{max} \quad \dots (2.9)$$

$$C_t = C_{max} \quad \text{for } ABS_{t-1} > ABS_{max} \quad \dots (2.10)$$

where ALPHA is $(C_{max} - C_{min}) / (ABS_{max} - ABS_{min})$.

From the results shown below, the varying coefficient of evapotranspiration gave the better result.

Constant Coefficient:

$$\begin{aligned}C &= 0.76 \\ \text{Initial ABS} &= 350 \text{ mm} \\ \text{SSE}_{\min} &= 51.67\end{aligned}$$

Variable Coefficient:

$$\begin{aligned}C_{\max} &= 1.0 \\ C_{\min} &= 0.6 \\ \text{ABS}_{\max} &= 500 \text{ mm} \\ \text{ABS}_{\min} &= 0.0 \text{ mm} \\ \text{Initial ABS} &= 200 \text{ mm} \\ \text{SSE}_{\min} &= 43.4\end{aligned}$$

A graphical comparison of the two time series is shown on Figure 2.5 and from it we can observe that both follow the same pattern.

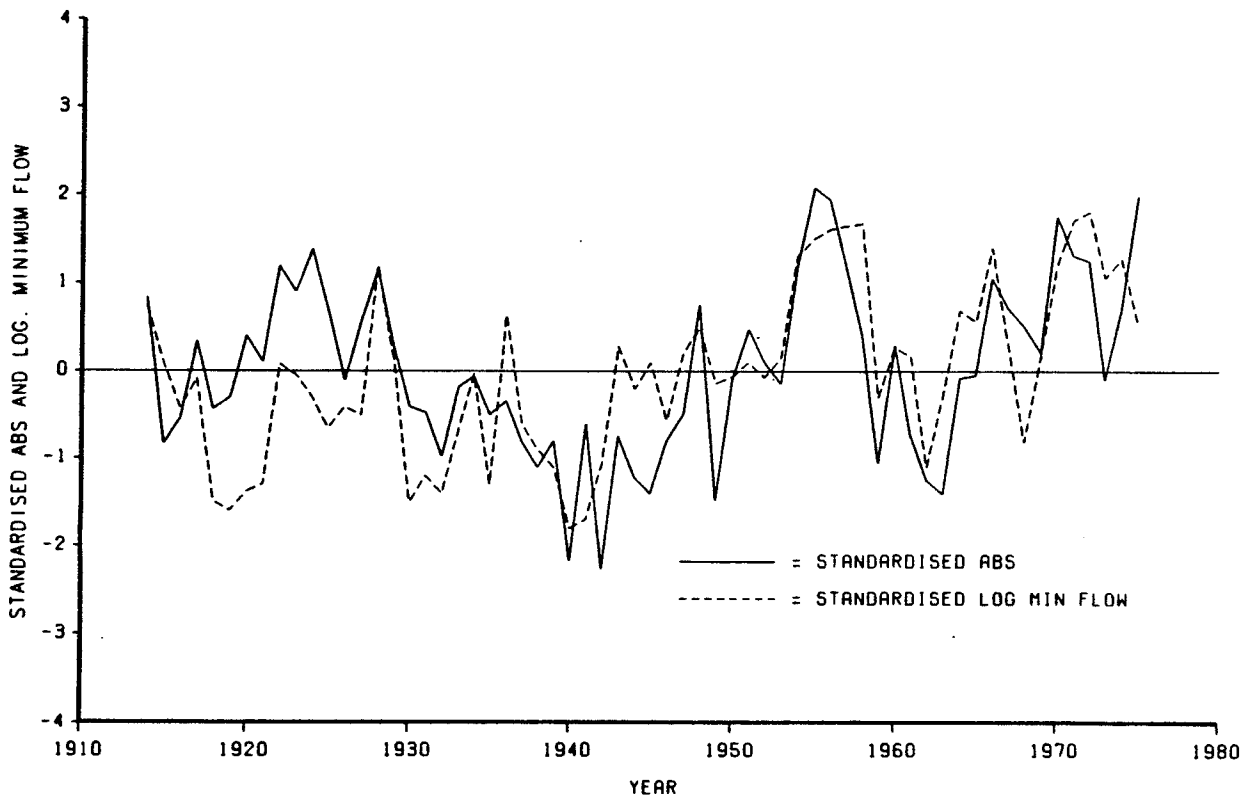


FIG. 2.5 PLOTS OF STANDARDISED ABS AND STANDARDISED LOG OF MINIMUM FLOW

2.5 STATISTICAL PROPERTIES OF THE ABS SERIES

Table 5 is a summary of some of the statistical parameters obtained for the ABS series.

From the table of the mean monthly ABS values, one can observe that on the average, within a complete water year, the largest ABS occurs in April during spring melt. In the months of summer, the high summer precipitation is removed by the high evapotranspiration rate and so the ABS falls off. In October at the beginning of winter, the ABS falls sharply from the summer values and thereafter gradually tapers to a minimum in March. Since there is practically no evapotranspiration and precipitation does not contribute to the ABS in winter, the ABS remains almost constant, with the gradual decrease being due to its depletion by the river.

It can also be observed that all the monthly ABS series exhibit very low skewness which implies that they are close to a normal distribution. They also have high first order autocorrelation coefficient showing that they are serially dependent series.

TABLE 5

STATISTICAL PARAMETERS OF ABS SERIES

Month	Mean ABS (mm)	Standard Deviation	Skewness Coeff	Lag One Ar Coeff	Hurst Stat
April	266.6	93.5	0.21	0.68	0.90
May	246.6	91.7	0.11	0.63	0.92
June	248.4	92.4	0.24	0.63	0.88
July	222.5	92.2	0.24	0.55	0.81
August	192.2	85.5	0.08	0.58	0.87
September	180.3	86.3	0.12	0.62	0.86
October	142.5	79.5	0.18	0.63	0.86
November	141.9	79.2	0.18	0.63	0.86
December	141.5	79.0	0.18	0.63	0.86
January	141.1	78.8	0.18	0.63	0.86
February	140.7	78.6	0.18	0.63	0.86
March	138.0	76.5	0.11	0.65	0.86

The ABS of the months of the farming season from May to September are of interest in this study. Their ABS series are checked for normality by plots of their normal probability shown in Figures 2.6, 2.7, 2.8, 2.9 and 2.10. The mean seasonal ABS series obtained by taking their averages is also checked for normality and shown in Figure 2.11. From the plots, in each case, the number of points outside the confidence band is less than the 5% maximum required for the 5% significance level.

Table 6 shows the correlation between the ABS at the beginning of the growing season (May) with some of the parameters of interest.

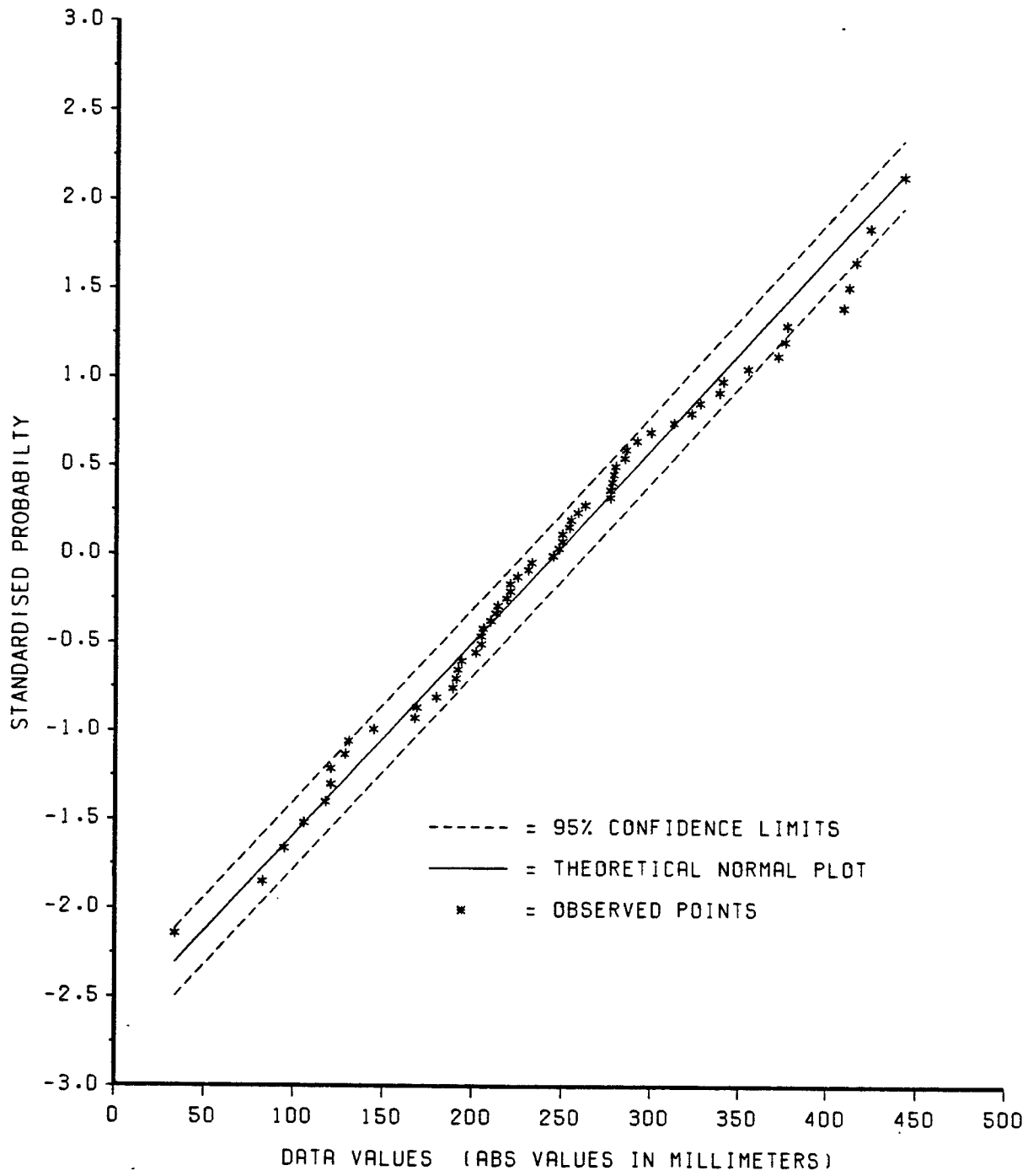


FIG. 2.6 NORMAL PROBABILITY PLOT OF MAY ABS WITH THE 95% CONFIDENCE LIMITS

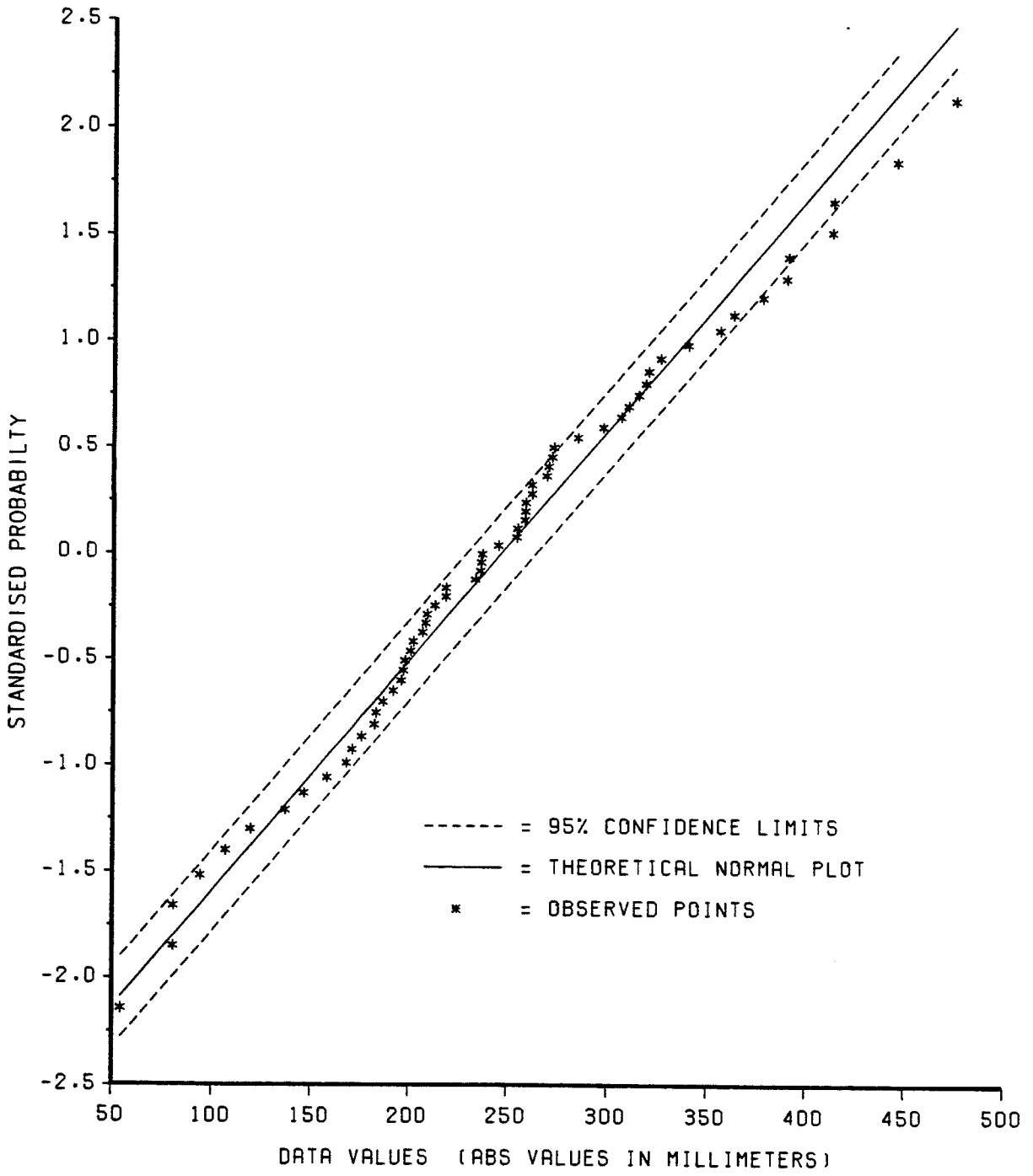


FIG. 2.7 NORMAL PROBABILITY PLOT OF JUNE ABS WITH THE 95% CONFIDENCE LIMITS