

NONLINEAR STRESS-DEFORMATION ANALYSES OF LAKE AGASSIZ CLAYS
USING FINITE ELEMENT METHOD

by

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LIST OF SYMBOLS

Symbol	Meaning
a,b	Experimental constants
a,b	Soil constants determined by iteration
A,B	Semilogarithmic regression coefficients
$[A^{-1}]$	Displacement transformation matrix
b	Reciprocal of the asymptotic value of resultant deviatoric stress
b	Diameter of anchor shaft
B	Diameter of anchor base
$[B_{\alpha}]$	Strain displacement matrix
c	Cohesion (Mohr Coulomb strength parameter)
c'	Effective cohesion
$[C]$	Stress-strain matrix
C,D	Constants relating regression coefficient A and plasticity index
d	Slope of the transformed hyperbolic stress-strain relationship
D	Depth of anchor base
e	Element
e_i	Initial void ratio
e_{ic}	Void ratio after isotropic consolidation and expansion
E	Young's modulus
E	Number of elements in the global assemblage
E_i	Initial tangent modulus of elasticity
E_o	Initial tangent modulus of elasticity of soil when stress variant is zero

Symbol	Meaning
m	Modulus number
M	Tangent modulus
M	Number of increments
n	Experimentally determined nonlinear parameter
n	Shape parameter
n	Sample size
NC	Normally consolidated clays
NCQ	Normally consolidated quick clays
OC	Overconsolidated clays
OCQ	Overconsolidated quick clays
p	Atmospheric pressure
PI	Plasticity index
PL	Plastic type of soils
q	Uniform applied (base) pressure
{q}	Vector of nodal displacements
Q_u	Critical uplift load
{ Q_α }	Element load vector
r	Radial coordinate
r	correlation coefficient
{R}	Load vector for the whole continuum
{ R_e }	Expanded load vector for an element
R_f	Failure ratio
S	Shape factor
SS	Sandy soils
S_d	Resultant deviatoric stress

Symbol	Meaning
S_{df}	Resultant deviatoric stress at failure
S_{dult}	Asyptotic value of resultant deviatoric stress
S_i	Sum of squares between observed data and predicted values in an iteration
S_{ij}	Deviatoric stress component
T	Thickness of anchor base
$\{\bar{T}\}$	Vector known surface traction intensities for an element
u	Displacement in r-direction
UCK	Unconsolidated undrained triaxial test
v	Displacement in z-direction
V_c	Volume of sample after isotropic consolidation and expansion
ΔV	Change in volume of sample
$\{w\}$	Vector of displacement functions
W_a	Self weight of an anchor
W_E	External work
W_I	Internal work
$\{\bar{X}\}$	Vector of known body force intensities
$\Delta x^2, \Delta x^3$	Error terms
X, Q	Variables in three parameter relationship
X_y, Q_y	Characteristic values in three parameter realationship
y	Observed value
y'	Value predicted by regression
\bar{y}	Mean of observed value

Symbol	Meaning
z	Depth below surface
$z,$	Vertical coordinate
α	Positive constant indicating degree of deviation from linearity
α	Nonlinear coefficient of soil
α	Failure criterion parameter
$\{\alpha\}$	Vector of generalized displacements
β	Rate of increase of Young's modulus with depth
β	Nonlinear coefficient of soil
β	Failure criterion parameter
γ	Shear strain
γ	Unit weight of soil
γ_h	Hyperbolic strain
γ_{oct}	Octahedral shear strain
γ_r	Reference strain
δ	Uniform displacement
δ_{ij}	Kronecker delta
ϵ	Axial strain
ϵ_d	Resultant deviatoric strain
ϵ_{ij}	Strain components
ϵ_{kk}	Cubical dilation
ϵ_m	Isotropic strain
e_m	Mean-normal strain
ϵ_{oct}	Octahedral normal strain
$\left. \begin{matrix} \epsilon_r, \epsilon_z \\ \epsilon_\theta, \epsilon_{rz} \end{matrix} \right\}$	Strains in cylindrical coordinates
	Strains in cylindrical coordinates

Symbol	Meaning
ϵ_v	Volumetric strain
ϵ_v	Isotropic strain
ϵ_{vc}	Characteristic isotropic strain
ϵ_{vd}	Normalized volumetric strain
ϵ_1	Axial strain
$\epsilon_1, \epsilon_2, \epsilon_3$	Principal strains
θ	Angle
ν	Poisson's ratio
σ	Vertical stress
$\sigma_1, \sigma_2, \sigma_3$	Principal stresses
$\sigma_1 - \sigma_3$	Principal stress difference
σ_c	Preconsolidation pressure
σ_{ij}	Stress component
σ_m	Isotropic stress (effective)
σ_{mc}	Mean normal stress Effective-mean-normal stress
σ_{mc}	Characteristic isotropic stress
$\frac{\sigma_m}{\sigma_c}$	Reduced consolidation ratio
$\left(\frac{\sigma_m}{\sigma_c}\right)_{cr}$	Critical reduced consolidation ratio
$\left\{ \frac{\sigma_m}{\sigma_c \cdot e_{ic}} \right\}$	Combined variable
σ_{oct}	Octahedral normal stress

Symbol	Meaning
$\bar{\sigma}_{\text{oct}}$	Effective octahedral normal stress
σ_{ult}	A measure of shear strength
τ_{oct}	Octahedral shear stress
ϕ	Angle of internal friction (Mohr Coulomb strength parameter)
ϕ'	Effective angle of internal friction
$[\phi]$	Interpolation functions
Π	Potential energy of the global assemblage

ABSTRACT

For nonlinear stress-deformation analyses of soils, it is necessary to define the stress-strain behaviour of the soil in quantitative terms, and to develop techniques for incorporating this behaviour in the analyses. The stress-strain behaviour of Lake Agassiz clays was separated into mean normal and deviatoric components and the stresses and strains were interrelated through bulk and shear moduli.

Drained isotropic compression tests were used for the evaluation of bulk modulus. A three-parameter stress-strain relationship was used to represent the isotropic stress-strain data. Solutions were obtained for bulk modulus as a function of soil type, void ratio, and imposed stress system.

The shear modulus was investigated by drained constant-mean-normal-stress triaxial compression tests. The basic two parameter hyperbolic relationship in a modified form was used to represent the resultant deviatoric stress-strain data and an analytical expression was determined for the shear modulus. The solution for shear modulus was obtained as a function of soil type, previous stress history, failure criterion, void ratio, and imposed stress system.