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AN ENQUIRY INTO RISK ASPECTS OF BEEF FARMING IN CROP DISTRICT NUMBER THREE, MANITOBA

by

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ABSTRACT

The study investigates the impact of risk on enterprise choice and resource allocation decisions in a farm firm growth situation. Interest is focussed on comparative risk levels in beef cattle and the five major cash-cropping enterprises in southern Manitoba, and the influence of these risk levels on farmers' decisions to allocate resources to beef cattle operations over time.

Portfolio selection theory is used as the foundation for a theoretical farm decision-making model. As adapted to farm decision problems, this theory shows how a farmer can select a portfolio of farm enterprises by simultaneous evaluation of the expected return and variance in return for each enterprise, and the stochastic relationship (or covariance) between enterprises. Most relevant to decision-making is the subset of alternative efficient portfolios (those lying on a Markowitz E-V frontier).

A multi-period separable programming model is used to generate points on an E-V frontier for a hypothetical farm situation in Crop District Number Three, Manitoba. By comparing the compositions of alternative efficient portfolios, four hypotheses are tested:-

(i) that historical risk levels have been greater for beef cattle than for cash-cropping enterprises;

(ii) that accounting for risk influences resource allocation and enterprise choice decisions;

(iii) that risk levels in beef cattle enterprises inhibit their expansion;

(iv) that hypothetical programmes to stabilize beef cattle returns can affect enterprise choice decisions.

The empirical results indicate greater risk levels only in cow-calf enterprises compared with cash-crops. Feeder cattle carry less risk except for wheat, and slaughter cattle enterprises have less risk than all five cash-crops.

Significant increases in diversification of enterprises are found when total risk becomes increasingly constrained, but when risk is not constrained, intensive enterprise specialization occurs, thereby indicating the importance of accounting for risk in decision models. Also, the linear programming equivalent solution (no-risk constraint) is dominated by the constrained risk solution adjacent to it on the E-V frontier.

The effect of risk on expansion of beef cattle enterprises is indicated by a level of growth five times greater for the no-risk constraint solution than for a low (0.25 billion dollars) risk constrained solution. When risk is constrained to 0.1 billion dollars, growth ceases altogether.

The results of testing two hypothetical programmes, one to stabilize beef gross returns, the second to stabilize beef net returns, showed both are effective in reallocating resources in favour of beef feeder and slaughter cattle enterprises. Neither programme is effective enough in compensating for high risk in cow-calf enterprises to draw them into any efficient portfolio. The net returns stabilization programme is found to be more effective.

It is concluded that with further research to improve enterprise risk estimates by using primary (farm) data, and by including a survey of farmers' attitudes to risk, portfolio selection theory may find useful applications in models to assist decision-makers and policy advisors.

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THE IMPORTANCE OF RISK IN AGRICULTURE

1. Delineation of the Problem

The importance of risk in agriculture stems from the farmer's lack of control over variations in weather, product prices and many other factors. This leads to lack of certainty surrounding the possible outcomes of decision alternatives, and therefore adds complexity to the decision-making process. Through the application of risk theory, it is possible to provide the farmer with information about the probability of occurrence of possible outcomes, and thereby suggest the consequences of making alternative decisions. This may assist the farmer in making more effective decisions concerning resource allocation and enterprise choice. Estimates of risk levels and their effects on farmers' decisions may also help in the formulation of more effective programmes aimed at farm price and income stabilization.

Relatively little research has been done on the usefulness and relevance of applying risk theory to agriculture. Few studies have been done on the influence of comparative enterprise risk levels on resource allocation and enterprise choice decisions. Empirical research into comparative risk levels in Canadian farm enterprises is entirely lacking. In Manitoba recent legislation is aimed at encouraging diversification into livestock (including sheep, beef, and manufacturing milk enterprises), and reducing dependence on cash-cropping. This suggests that farmers require incentives to switch from cash-cropping to livestock operations. Research into comparative risk levels in livestock and cash-cropping enterprises may help explain the need for incentives and the relatively limited presence of these livestock types in Manitoba. An empirical study designed to compare risk levels between all livestock enterprise types and cash-cropping enterprises would be a considerable undertaking. Beef cattle enterprises were selected for this study, and risk levels were compared with those for the principal Manitoba cash crops (wheat, barley, oats, rapeseed and flaxseed).

An additional problem area concerns the relatively little attention paid to developing methodological procedures for applying risk theory to agricultural problems. This process is a complex one involving several potential difficulties. Firstly, the important elements of risk must be identified and appropriate parameters selected for measuring the degree of risk. Secondly, in order to obtain an accurate measure of total risk, the stochastic relationships between paired elements of risk must be considered, since elements of risk, when combined, may vary together negatively, positively or in a neutral fashion. Thirdly, since variability in farm prices, yields, etc., occurs both over time and between locations, the data used in measuring risk ideally should come from primary (farm) sources. Secondary data, based on averaging and aggregation, is unable to portray the full extent of risk at the individual farm level. Fourthly, decision-makers' responses to risk situations are subjective, reflecting their individual utility functions with respect to possible combinations of expected values and risk levels. Therefore, the ideal way to examine the influence of risk on decision-making is to survey farmers and construct their utility functions. In short, the best procedure for obtaining estimates of risk and for gauging decision-makers' responses to risk is to conduct a survey of farmers.

In this study, primary data were unavailable or were in a form unsuitable for application. This required the use of secondary data which portrayed variability only over time. To minimize the effects of not accounting for variability between locations, data were used for a region within which soil type and climate vary little. It was also found impractical to survey farmers' attitudes to risk. Reliance was placed instead on constructing an optimization model to assess enterprise choice decisions under risk, given the assumption that decision-makers are profit maximizers. This model was also used to test the effect of hypothetical government programmes to reduce risk in beef cattle operations on choice decisions between beef and cash-cropping enterprises.

1.1 Potential for Manitoba's Beef Cattle Industry

The Manitoba farm industry has traditionally concentrated on the production and export of primary products, mainly cereal grains and oilseeds. Secondary or livestock enterprises have catered principally to domestic, provincial market requirements. In the case of beef, Manitoba traditionally has not met its domestic market requirements, although a surplus of beef calves is produced in some years, [27,70].

It has been recognized for a long time that certain benefits may result from channelling a greater proportion of Manitoba's farm resources into the production and export of livestock products. For example in 1963, the "Committee of Manitoba's Economic Future" [23], suggested the desirability of diversification and broadening the base of Manitoba agriculture. In particular, the Committee suggested expanding livestock production, including beef. The provincial government's endorsement of this diversification approach was reflected in the 1972 Manitoba Farm Diversification Programme [28]. This programme provides incentives to expand sheep, manufacturing milk and beef production.

The benefits to be gained from expanding Manitoba's beef cattle industry can be summarized as follows:-

(i) Manitoba's farm economy would benefit from retention in the province of the value added to oilseeds and feed grains. In processing these resources through beef cattle, employment opportunities for farm resources in Manitoba would be increased.

(ii) Expanded sales of beef cattle in place of primary product (feed grains and oilseeds) sales, would enhance Manitoba's agricultural export income, assuming that the terms of trade do not unduly favour primary products over the long-run.

(iii) Multiplier effects from expansion of beef cattle operations (after accounting for loss of multiplier effects from smaller cash-crop export sales) may provide a net benefit to other sectors of the Manitoba economy. Agribusiness industries such as animal feeds processors and livestock building manufacturers would gain directly through expanded business opportunities. Indirect multiplier effects would accrue to many other less related sectors of the economy, through expanded farm income, and therefore increased purchasing power, and through an expanded tax base.

(iv) The Manitoba farm economy would benefit from greater diversification at the production level, with somewhat less reliance placed on feed grains and oilseeds export income. Extensive dependence on these sources of export income have in the past led to instability in the Manitoba farm economy, due to recurring cycles of scarcity and glut in world grain and oilseed markets. While similar cycles characterize international beef cattle markets, a wider base of export income sources would tend to make Manitoba's farm economy less vulnerable to fluctuations in any one market.

The introduction of government legislation [28] suggests that some form of incentive scheme was viewed as necessary in order to achieve greater diversification in Manitoba's farm economy. This may reflect a preference by Manitoba farmers for cash-cropping enterprises over livestock enterprises.

One possible reason for such a preference is that net returns to capital and management may be higher for cash-cropping than for livestock enterprises. Other possible reasons may be hypothesized as follows:-

(i) Farmers may be conservative by nature, preferring to follow traditional and historical patterns of cash-crop production in Manitoba;

(ii) The necessary commitment to a longer and more intensive work schedule involved in livestock production may cause livestock enterprises to appear less attractive to many farmers;

(iii) Many intending livestock farmers may lack the necessary capital outlays for additional resource inputs specific to livestock, or the necessary technical and economic management expertise;

(iv) The level of risk involved in livestock enterprises may be greater than in cash-cropping.

1.2 The Need for Analyzing Risk in Beef Cattle Enterprises

Ideally for research purposes, all the postulated reasons for Manitoba farmers' preferences for cash-cropping enterprises over livestock operations should be examined. The first two reasons submitted, concerning farmers' traditional attitudes and the effects of heavier labour requirements for livestock, would be difficult to quantify in an empirical model.

The aspects of capital and management resource scarcity and comparative risk levels lend themselves fairly readily to quantitative measurement, and therefore to hypothesis testing. While some consideration is given in this study to the area of capital and management resource scarcity, efforts are primarily concentrated upon obtaining a first approximation of the effects of comparative risk levels on enterprise choice. Previous mention has been made (pp. 1-2) of the paucity of empirical applications of risk theory to agriculture, particularly in Canada, and of the lack of attention paid to developing suitable methodological procedures in this area. Apart from wishing to explore this avenue of research simply because it has been neglected, other factors may suggest possible contributions to be made by investigating comparative risk levels in Manitoba beef cattle and cash-cropping enterprises:-

(i) Risk in an important factor in the decision-making process, but has not been considered in previous empirical studies on enterprise choice decisions in Manitoba agriculture (see, for example, L.M. Johnson [71]). A study on comparative risk levels in beef cattle and alternative enterprises may help explain the limited presence of beef cattle operations.

(ii) Diversification into beef enterprises may contribute to a more stable Manitoba farm economy and a healthier balance of payments position. Encouragement of diversification is required through incentives needed to overcome impediments to entry, one of which may be that risk levels in beef are higher than in alternative enterprises.

(iii) While government programmes to encourage beef production have dealt adequately with the problem of resource scarcity, they have not dealt in any effective way with the problem of reduction of or compensation for risk.

1.3 Government Programmes Affecting Beef Production

Compensation for the effects of shortages in resources required for starting livestock enterprises has been effectively provided in the Manitoba Farm Diversification Programme. The programme makes available grants and low-interest loans to offset capital shortages. It also offers both economic and technical training to upgrade management expertise.

A Manitoba provincial government programme introduced in 1974 also offered capital assistance through 12-month, interest-free loans of 100 dollars per cow to cow-calf operators willing to put up their beef calves and breeding stock as collateral. To the extent that the loans were interest-free during an era of high interest rates, this programme offered a direct subsidy, albeit small on a per-cow basis, to the beef farmer. The loans were also beneficial in terms of easing the liquidity shortage positions of many cow-calf operators during the winter of 1974-1975.

On the other hand, government legislation to reduce or compensate for risk in beef cattle enterprises has had little or no beneficial impact. A temporary federal government programme, introduced in 1974, established a floor price of 26 cents per pound for slaughtered beef cows classified in the lower "D3" and "D4" grades. Although this programme was designed to stem declining beef prices and thereby reduce risk, it in fact had a minimally beneficial impact. Price supports applied only to lower grade cow beef, and maximum limits were imposed on the number of beef cows eligible for subsidy, (namely five cows or five percent of the breeding herd for each producer, whichever was the lesser).

The federal government's 1958 Agricultural Stabilization Act was designed to reduce risk through supporting prices of live slaughter steers, among other designated farm commodities. The objective was to stabilize prices through deficiency payments made in years when steer prices fell below 80 percent of the average prices prevailing during the previous ten years.

So far, deficiency payments have never been made on slaughter steers, because prices never fell below 80 percent of the previous ten-year average level. Not only was the support level too low to be effective, but the programme also failed to adopt a comprehensive approach to reducing beef cattle risk levels. Piece-wise attempts to reduce isolated components of risk, such as product price variability, may not necessarily succeed in reducing total (compound) risk levels. A more effective method is suggested through attempting to reduce several separate sources of risk jointly within a more comprehensive stabilization scheme. For example, product price and input cost variability could be jointly dealt with as a means of stabilizing net returns. This should prove more beneficial to beef cattle farmers than attempts to stabilize gross incomes through concentration on product prices alone.

Another method of achieving a more comprehensive approach would be to apply stabilization schemes to all segments of the beef cattle industry. The 1958 Agricultural Stabilization Act makes provision only for slaughter steer price support, while the needs for stabilization in the cow-calf and feeder cattle segments are ignored.

1.4 Deficiencies in Previous Studies on Risk

In the relatively few past studies which have made provision for risk, attention has been most heavily directed toward a static framework of analysis. This represents a serious deficiency, since decision-making in agriculture realistically operates in a multi-period environment in which production and marketing conditions, investment opportunities, and consumption preferences are constantly changing over time. Resource allocation decisions made at one point in time necessarily affect farm operations for months or even years ahead. This is due to the long-term nature of the production cycle in farming, and to the need to invest in durable assets, whose services are given off over a number of years. Optimal resource allocation must therefore be decided not in a static, but in a multi-period setting, and agricultural economic models should likewise be cast in a multi-period framework.

Until the 1950's, the emphasis in model building in agricultural production economics research had been on the application of the classical economic theory approach to static or comparative static enterprise choice and resource allocation problems. Linear programming was the most frequently used empirical tool to derive maximum profit, net revenue or similar criterion functions.

Two problems arose with such an approach. Firstly, the preference rankings of possible portfolios (of farm enterprises) were based on a single parameter only, namely the expected returns. This was rationalized by the assumption that subjective certainty could be applied to the expected returns parameter. It is more realistic to assume that farmers operate under a regime of less than perfect knowledge. Therefore more realistic models applying portfolio choice theory to agriculture would adopt a two-parameter approach, based on expected returns plus some measure of the deviation of those returns.

Secondly, the static, or comparative static aspect neglected the farm balance sheet with intertemporal net worth and debt relationships. The importance of this aspect is reflected in the long-term nature of many farm capital investments, problems of external and internal capital rationing in farming, and the necessity for long-term as well as short-term farm planning.

Recognizing these as separate problems, research efforts since the early 1950's diverged along two principal paths. The first attempted to incorporate risk elements in a static context into the farmer's decision-making matrix by appealing to such economic theories as the Markowitz portfolio selection approach [50], Hirschliefer's criteria for investment decisions, [38], and Friedman and Savage utility analysis under risk [23]. These theories have been empirically applied through the techniques of simulation (e.g. Zusman and Amiad, [69]), stochastic linear programming (e.g. Boussard and Petit, [10]), separable programming (e.g. Thomas, et al., [62]), and quadratic programming (e.g. McFarquar, [54]).

The second path stressed dynamic aspects of farm decision-making, based on economic firm growth theories such as those of Penrose, [56], and intertemporal investment theory originating with Fisher, [20], and refined by Hirschliefer, [37], among many others. Empirical applications of such theories have found their expression in multi-period linear programming models, (e.g. Loftsgard and Heady, [46]), recursive linear programming models (e.g. Heidhues, [33]), and simulation techniques (e.g. Hutton, [39]). The programming techniques can be faulted for taking little or no account of risk, while simulation techniques are not analytical and prescriptive, the latter being generally regarded as desirable features for tools to aid in decision-making.

Several attempts have been made to integrate risk measurement with intertemporal analysis in a farm firm growth context since the mid-1960's. The analytical tool most commonly employed has been a modification or refinement of a discrete stochastic linear programming model (e.g. Johnson, et al., [43], and Maruyama, [53]). Others have turned to a more simplified approach of eliminating from the set of all possible farm growth plans those which may lead to financial failure (e.g. Yaron and Horowitz, [67]). Another approach (by Hazell [31]) linearized the objective function of a quadratic programming model by employing mean absolute deviations in place of the more usual variance and covariance elements.

The inadequacies of such methods are that only partial account is taken of the entire risk environment in which the farmer operates, as for example, with the Yaron-Horowitz approaches. Hazell's method ignores completely the covariance terms, thereby omitting from the measurement of total variance the crucial stochastic relationships between farm enterprises or activities.

On the other hand, parametric solution procedures must be used to solve stochastic linear programming problems. These procedures may become relatively inefficient as the number of required solutions necessarily increases in order to approximate more closely the curvilinear "efficient plan frontier" of the conventional quadratic programming model, or its close relative, the separable programming model.

The choice between multi-period quadratic and multi-period separable programming techniques lies principally within the realms of efficiency of solution by computer and versatility of the two techniques. Quadratic programming techniques suffer the disadvantages of extremely complex and relatively inefficient algorithms for solution by computer, and consequent high costs in terms of both time and money expended. Only one attempt (by Batterham [5]) has so far been made to build and test a multi-period quadratic programming model. Computer core storage and solution time required were both found to be exorbitant.

On the other hand, relatively efficient algorithms, such as I.B.M.'s "Mathematical Programming System" (M.P.S.), [40], have been developed with the capability of solving both multi-period and separable programming problems. In addition, the M.P.S. package confers greater flexibility through its post-optimal "range" and "parameterization" procedures, features generally lacking in quadratic programming algorithms. Thus, multi-period separable programming techniques appear to offer the best potential for analysis of risk in a farm firm growth context.

A third major deficiency in past farm planning models concerns the inadequate attention paid to the interdependent nature of relationships between choices amongst production, marketing, investment and consumption alternatives. Previous investment choices directly affect the farm firm's current production and consumption potential, while current production and marketing decisions affect the cash flow stream from which further investment and consumption are derived.

Also there exists a trade-off between the choice for current consumption and the choice for further investment which will generate increasing consumption potential at some future time. Choices between different investment alternatives and intertemporal choices in consumption levels pertain to Fisher's investment opportunity and impatience principles respectively, [20 , Chaps. 4 and 7]. Thus prescriptive models as aids to decision-making should logically include both production and investment alternatives within the same analytical framework.

Some limitations imposed by divorcing production from investment decisions have been specified by Boehlje and White, [7], as a set of conditions which are implied as a result of segregating these decisions. Treating production decisions as given in a firm growth model seems reasonable only under the following conditions:-

(i) Linear relationships throughout, so that constant returns to scale prevail;

(ii) A long-run context of analysis, so that all inputs are variable;

(iii) Unlimited availability of all factors, including credit, or if limited, all factors must be available in constant proportions;

(iv) Constant technology and relative prices over time.

Such conditions are excessively restrictive, especially in situations where decisions are being made in a long-run context under conditions of risk and uncertainty, as they typically are in farming.

This indicates a need to incorporate a comprehensive set of investment, production, marketing and consumption alternatives in any model dealing with a farmer's decision-making matrix. Thus, whole farm analysis models are deemed desirable, as opposed to partial analysis models concerned with only one sector, such as production alternatives.

In summary, the nature of the farmer's decision-making environment suggests that realistic analytical models should account for:-

(i) the conditions of imperfect knowledge surrounding the outcomes of decision alternatives;

(ii) the intertemporal relationships involved in and long-run consequences of decision alternatives;

(iii) the interdependent relationships that exist between production, marketing, investment and consumption alternatives.

It is suggested that previous decision-making models have concentrated on only some of these conditions and therefore possess inherent limitations in terms of providing the farmer or the policy advisor with only part of the required or desired information.

2. Purpose and Objectives of the Study

The purpose of the study is to develop an appropriate methodological framework for combining risk theory with multi-period analysis, and the application of this approach to an empirical situation in Manitoba agriculture. The empirical application is intended firstly to compare the risk levels in beef cattle operations with those in cash-cropping enterprises. Secondly, it is intended to ascertain the extent to which risk affects resource allocation, enterprise choice and the magnitude and direction of farm firm growth. Thirdly, it is intended to examine the effects of hypothetical government stabilization programmes on risk levels and on enterprise choice and firm growth.

The specific objectives comprise the following:-

(i) The development of a suitable methodology for assessing in a firm growth context the applicability of risk theory and the importance of accounting for risk in problems related to Manitoba's beef cattle industry;

(ii) The delineation of the principal sources of risk associated with Manitoba beef cattle and cash-cropping enterprises;

(iii) The derivation of an appropriate quantitative measure of risk levels both within and between these types of enterprise.

An empirical model is developed for testing the following hypotheses:-

(i) Risk has an important influence on resource allocation and enterprise selection decisions in Manitoba agriculture;

(ii) The historical incidence of risk has been greater for beef cattle than for cash-cropping enterprises;

(iii) Expansion of beef cattle operations is inhibited by risk more than expansion of cash-cropping enterprises;

(iv) Proposed beef stabilization programmes, based on modifications to the federal government's 1958 Agricultural Stabilization Act, would lead to a reallocation of farm resources towards beef cattle from cash-cropping, and would be capable of inducing expansion of beef cattle operations.

2.1 Hypothetical Programmes to Reduce Risk in Beef Cattle

To try to overcome the shortcomings in the 1958 Agricultural Stabilization Act, as outlined in Section 1.3 above, two alternative programmes are proposed and tested in this study. Both are extended to cover beef calves and feeder cattle as well as slaughter cattle. The first programme provides for a higher level of support by subsidizing product prices if these fall below 100 percent of the previous five-year average price.

The second programme attempts a more comprehensive approach to reducing risk through stabilizing net returns. This is achieved by indexing beef product prices to operating expenses. Included in these operating expenses are costs of feeds, livestock purchases, marketing costs and other direct livestock costs (bedding material, drugs, etc.).

Although the second proposed programme accounts for both product price and input cost variability, many other sources of risk in beef cattle (discussed in Chapter II, Section 3) are not accounted for. This is because neither primary nor secondary sources of data could be found for such sources of risk as variability in mortality rates or liveweight gain rates. Further details of these programmes are given in Chapter V.

3. Outline of the Thesis

In Chapter II the foundations of the theoretical model are laid through a discussion of economic theories relating to decision-making behaviour under imperfect knowledge and firm growth, with applications of these theories to the Manitoba farm firm.

Chapter III outlines the more important methodological aspects of developing an appropriate empirical framework for measuring risk and for assessing the effects of risk on decisions pertaining to resource allocation and enterprise choice at the farm firm level. The actual empirical techniques used are also described and justified.

The discussion of Chapter IV focusses on the structure and components of the empirical model, and in Chapter V, the empirical model results are summarized.

The final chapter sets out the conclusions drawn from the analysis, describes the scope of application of the model and limitations of the study, and offers suggestions for improvements and areas for further research.

CHAPTER II

THE THEORETICAL FRAMEWORK

1. Imperfect Knowledge in Economic Theory

The importance of imperfect knowledge in the realm of decision-making has long been recognized in economics by many prominent classical theorists. Bernouilli [6] developed a theory of risk measurement to explain the St. Petersburg Paradox (that a decision-maker may not be prepared to pay the monetary expectation of a "fair" gamble in order to participate in it). More recently, economists such as Knight [45], Keynes [44, Chap.13 and 15] and Hicks [35, Parts III and IV] have referred to imperfect knowledge as it impinges on decision-making in capital and monetary theory contexts. Hicks [34] succinctly advocated that economic choices of uncertain alternative outcomes be based on expectations of the outcomes.

Not all decision problems pertaining to uncertain outcomes can be solved by evaluating the expectations of the outcomes, however, simply because the expectations may be unknown or immeasurable. For this reason it is desirable to distinguish between different types of decision problems. Luce and Raiffa [47, p.13] have classified decision problems into three situations, which, in order of increasing complexity, are:-

(i) Certainty, where each decision leads to a specific invariant outcome;

(ii) Risk, where each decision leads to a set of possible, specific outcomes, whose probabilities of occurrence are known;

(iii) Uncertainty, where each decision leads to a set of specific outcomes, whose probabilities of occurrence are entirely **unknown**.

Such a classification implies that, for empirical studies of decision problems under imperfect knowledge, such as this one, exclusion of non-measurable situations of pure uncertainty is mandatory. Only situations dealing with certainty or risk can be considered relevant, since quantitative measurements of the decision outcomes are obtainable. It is intended in this study to demonstrate that a wide range of decision problems related to resource allocation and enteprise choice in agriculture can be appropriately classified under risk or certainty situations, as opposed to situations of uncertainty. Furthermore, it is intended to show that many decision problems in agriculture are more properly classified as risk than as pure certainty situations, an aspect frequently neglected in previous studies.

The development of economic theories of choice under conditions of risk was initiated in 1943 by von Neumann and Morgenstern [66]. They outlined a theory of maximization of expected utility conforming to a set of axioms of decision-making behaviour based on a cardinal preference ranking of risky alternatives. This expected utility maximization theory was refined in 1948 by Friedman and Savage [23] who demonstrated rational decision-making behaviour when choice is made between alternatives bearing different degrees of risk. They further showed how individuals can be both gamblers (risk preferers) and insurance takers (risk averters) simultaneously.

Markowitz [48], in 1952, applied the criterion of maximization of expected utility to a theory of portfolio selection, thereby integrating the theory of rational decision-making behaviour under risk with the theory of investment. The fundamental concept of portfolio selection is that the investor is able to cardinally rank his preferences of possible security portfolios according to his subjectively defined probability distributions of returns from alternative portfolios. The probability distributions of returns are derived from a two-parameter measure, using the expected return on each security, and the variance of returns, plus the covariance, or stochastic relationship, between securities. Given his ranked preferences, the investor can select the particular portfolio that maximizes the expected utility or value of returns.

A variant of the portfolio selection approach was devised by Tobin [64] in 1958, to demonstrate rational choices for varying degrees of liquidity preference. In this case, the portfolio consisted of cash balances bearing no risk but also earning no interest, and interest-bearing securities subject to fluctuating interest rates and to capital gains and losses. Tobin showed how different attitudes toward risk, represented by an individual's set of utility indifference curves (trading off expected returns against variance in returns), affect the choice decision for varying degrees of liquidity preference amongst risk diversifiers, risk plungers and risk choosers.

Portfolio selection theory represents a major step forward in the development of theories to explain rational decision-making behaviour under imperfect knowledge for several reasons.

Firstly, the theory permits of a logically consistent method for empirically testing various applications of decision-making behaviour under risk, using some two-parameter measure of the probability distributions of expected outcomes. Secondly, the theory is based on the expected utility maximization criterion, which is consistent with von Neumann and Morgenstern's [66] set of axioms for decision-making behaviour under risk. This criterion and its underlying set of axioms have been found to be consistent with observed behaviour of decision-makers in real world situations, through empirical tests conducted by Farrar [19]. Thirdly, portfolio selection theory appears to lend itself to a wide range of theoretical and empirical applications, including many in agriculture.

The range of applicability of portfolio selection theory is not, however, to be considered completely unlimited. As Borch [8] has pointed out, cardinal ranking of alternative portfolios with respect to the two-parameter measure of expected returns and variance in returns holds only under certain limited circumstances. These are that one or both of the following assumptions must hold:-

(i) The investor's utility function must be quadratic, or at least a polynomial function of degree no greater than two;

(ii) The investor must regard the random variables for expected returns on securities as being normally distributed.

As is indicated in Chapter IV, Section 4.2, the random variables for expected returns on the farm enterprises analyzed in this study are normally distributed. Thus the second of Borch's criteria is met, and therefore the use of portfolio selection theory can be considered appropriate.

Several empirical studies based on portfolio selection theory have already been applied to specific aspects of the farm firm. For example, Driver [17] evaluated decision-making behaviour under different regimes of land tenure with associated different levels of risk. Batterham [5] applied portfolio selection to investment and financial decision-making behaviour in a farm firm growth context. In this study, portfolio selection theory is used to compare risk levels between farm enterprises and to evaluate the effects of risk on decisions related to resource allocation and enterprise choice in a multi-period framework.

2. Portfolio Selection Theory

The possibilities for applying portfolio selection theory to decisions related to resource allocation and enterprise choice under risk for the farm firm can be illustrated by a brief examination of the method of analysis involved. Portfolio selection theory enables a decision-maker to select a portfolio of securities (or other assets or income-producing activities) representing maximum expected utility.

In the context of the present study, the theory is applied to the problem of cardinally ranking the preferences for alternative portfolios of possible farm enterprises, given knowledge of the decision-maker's utility function. As mentioned previously, this can be achieved by simultaneous evaluation of the expected return and variance of returns from each farm enterprise, and the stochastic relationship (or covariance) between the enterprises.

Sharpe [59, pp. 31-32] has suggested the process of portfolio selection can be conveniently subdivided into three major components. These are referred to as security analysis, portfolio analysis and portfolio selection.

2.1 Security Analysis

This involves the computation of the set of expected returns and variance of returns for the alternative portfolios of possible securities, and the covariance between securities. The analogous situation for resource allocation and enterprise choice problems is to calculate the expected annual net returns and variance of returns for each farm activity, and the covariance of returns between activities. The relevant set of farm activities may include resource acquisition, production, marketing, investment and consumption alternatives.

Several methods can be employed for predicting expected returns whose outcomes are not known with certainty. Heady [32, pp. 475-496] has suggested that expectation models on which to base decisions in agriculture range from the "naive" to the "sophisticated".

In the naive category, Heady includes the following methods:-

(i) Random drawings of historic outcomes,

(ii) Historic mean or model average values,

(iii) Extensions of linear trend,

(iv) Projection of current yields and prices into succeeding years,

(v) Prices and yields drawn from historic periods which appear to farmers to be "normal" or similar to the current period,

(vi) Future prices based on commodities futures markets' quotations.

Heady included in his sophisticated category such techniques as regression analysis and econometric models. While these techniques are still not readily available to individual farm entrepreneurs, they are commonly used by governments, universities and agribusiness firms in predicting expected yields, prices, costs and returns in agriculture.

2.2 Portfolio Analysis

The portfolio analysis phase involves the compilation of a set of expected net returns, along with their associated risk levels, that can be obtained from varying the proportions of the different securities in the portfolio. In the context of this study, it is the varying proportions of the farm production and marketing activities comprising the enterprise portfolio which are relevant.

Obviously, the portfolio analysis stage can become extremely cumbersome and expensive to undertake as the number of activities in the portfolio increases. This is due to the fact that the number of possible combinations of varying proportions of activities increases exponentially with the number of activities included in the model. To circumvent this problem, Markowitz [49] proposed that instead of generating the entire universe of feasible solutions from all possible portfolios, a special subset of so-called efficient portfolios be selected for systematic evaluation. This subset of efficient portfolios must be comprised of portfolios which conform to the following criteria, according to Markowitz [51, p.6]:-

(i) Each must lie on the boundary of the feasible region of the solution space, whose axes measure expected net returns and variance of returns;

(ii) For alternative feasible solutions (for example point B in Figure 1) for which greater expected net returns occur for an efficient portfolio (point A), there must also occur greater variance;

(iii) For alternative feasible solutions (for example point C in Figure 1) having smaller variances than the efficient portfolio (point A), there must also be smaller expected net returns.

Thus, the E-V frontier represents the locus of points of portfolios that are both feasible and efficient, and therefore Pareto optimal. That is, the frontier plots all possible combinations of expected returns and variance in returns from alternative possible portfolios to give an equal level of efficiency. This frontier, Markowitz [49] suggested, can be generated using quadratic programming procedures. Other solution techniques have also been proposed, including separable linear programming, dynamic programming and simulation models.

2.3 Portfolio Selection

The third stage of solving portfolio selection problems refers to the decision-maker's selection from among the set of efficient portfolios on the E-V frontier, of the one portfolio that will maximize his expected

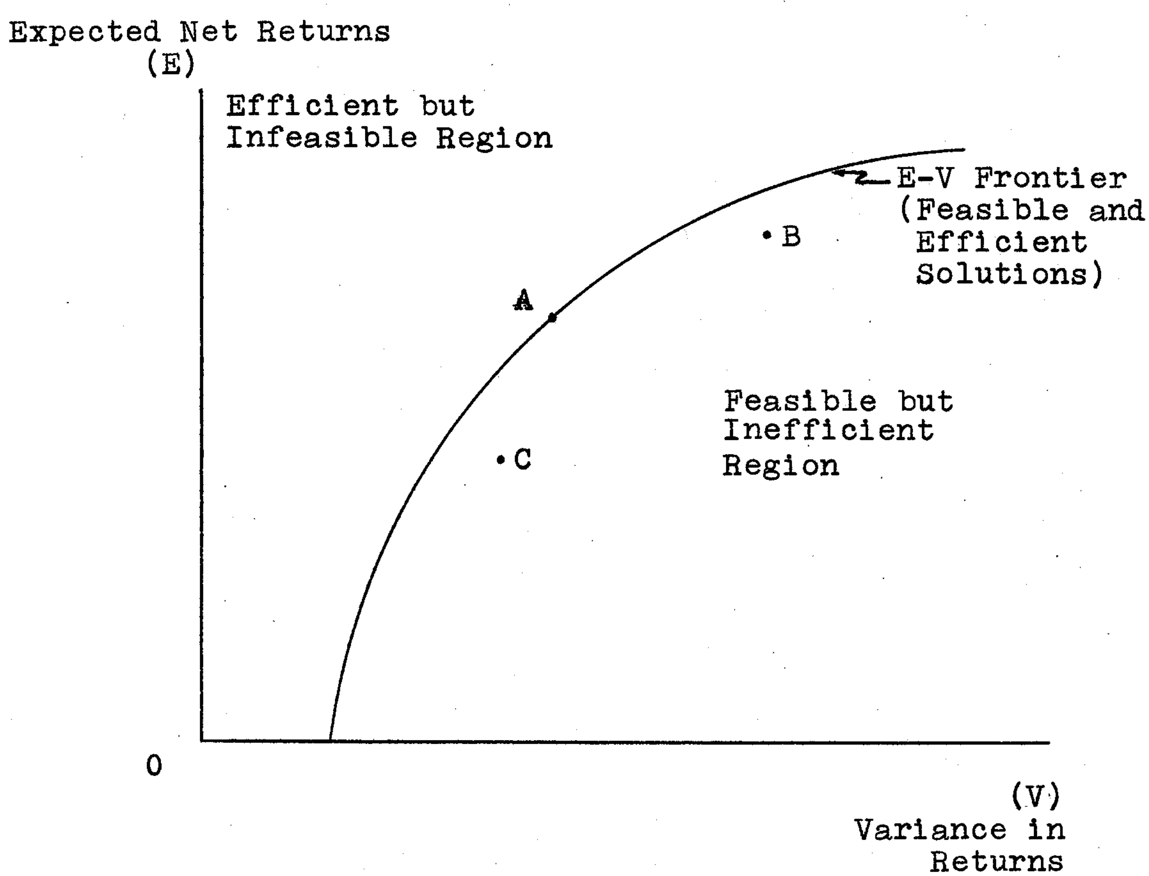

Figure 1

E-V Frontier for a Portfolio Analysis Problem utility. Although not employed in the present study, an explanation of this third stage is included in the analysis for illustrative and theoretical purposes.

The portfolio selection phase involves compiling information about the decision-maker's attitude toward risk. The conventional method used is to map out the decision-maker's expected utility function, a linear combination of the expected returns and variances of returns for the set of efficient portfolios. From the expected utility function, the decision-maker's set of indifference curves can be derived. Each curve denotes the locus of points of indifference between expected returns and variance of returns for a specified expected utility level.

By superimposing the set of indifference curves on the E-V frontier generated in the portfolio analysis stage, an optimal solution to the portfolio selection problem can be obtained at the point of tangency between an indifference curve and the E-V frontier. This optimal condition is illustrated in Figure 2 (at point A, with expected returns level at E^* and variance of returns at V^*), for the case of a decision-maker who is risk averse, denoted by a set of indifference curves with a positive slope and convex from below.

The principal reasons for excluding the portfolio selection phase from this study were:-

(i) Anticipated difficulties involved in attempting to compile expected utility functions at the individual

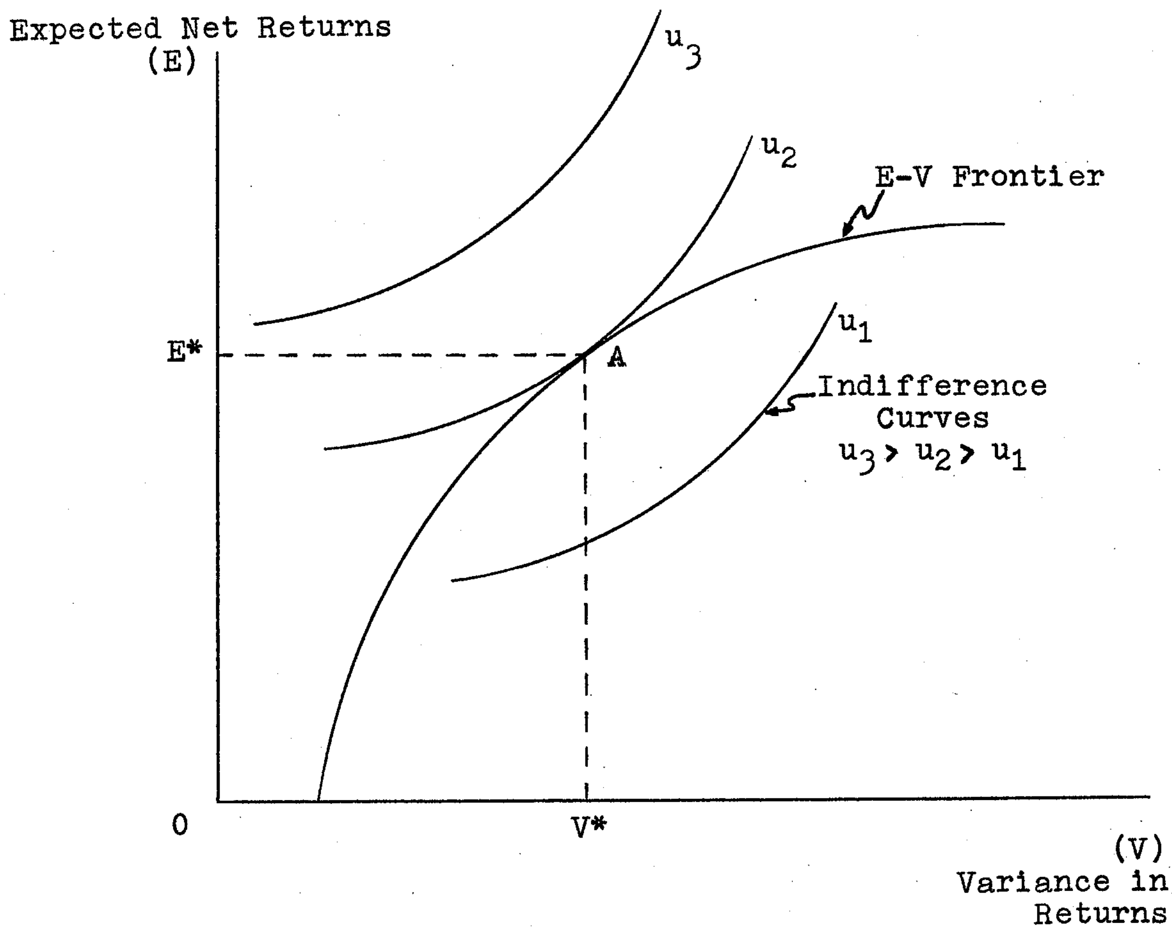

Figure 2

Optimal Solution to a Portfolio Selection Problem

for the case of a Risk Averter

farm decision-maker's level;

(ii) Knowledge of these utility functions was not essential for testing the hypotheses of the model.

Thus, the portfolio selection phase was deemed to be beyond the scope of this study.

3. <u>Sources of Risk in Agriculture</u>

One of the most crucial problems faced by the farmer is that most decisions on resource allocation, enterprise choice and investment alternatives must be made under a regime of pervasively imperfect knowledge, and therefore one of risk. Heady [32, pp. 453-455] has classified risk in agriculture into four principal component types. These are:-

(i) Market risk, or instability of product prices and resource input costs;

(ii) Technical risk, or variability of crop yields and other production coefficients for a given technique;

(iii) Technological risk, or the risk of foregone increases in efficiency by the adoption and retention of present technologies that may be rendered obsolete through further technological progress in the near future;

(iv) Institutional risk, due to changes in the sociological and legal environment in which farmers operate.

3.1 Market Risk (or Price Risk)

An important cause of price risk in agriculture is the atomistic and highly competitive nature of the industry. This results in the farmer being a price-taker with respect to both resources inputs and product outputs. Secondly, the relatively low price and income elasticities of demand for most farm products imply that for small changes in supply, product prices will change considerably. Thirdly, farm supplies of products are subject to fluctuations due to weather variations, leading directly to product price variations. Other causes of price variation include national business cycles and recurring commodity cycles for farm products. On the resource input side, since the incomes and therefore demand for inputs tend to fluctuate together for most farms, (at least in a region featuring specialized production), the prices of inputs tend to fluctuate commensurately.

The extent of market risk in Manitoba cash crops is indicated in Table 1. The data refer to average annual prices received by Manitoba farmers for the period 1954 to 1973, the last full year for which prices were available. The prices represent a weighted average of the different prices paid for the various grades in each crop, weighted by the volumes of each grade sold. The coefficient of variation, through "suggesting the percentage deviation from the mean" (Heady [32, p. 456]), gives an indication of the degree of risk involved. Using this statistic, flaxseed

Table 1

Variations in Prices of Manitoba Cash-Crops, 1954-1973[1]

Year	Wheat	Barley	Flaxseed	Rapeseed
	(dollars per bushel)			
1954	1.31	0.92	2.65	2.13
5	1.42	0.94	2.80	2.24
6	1.28	0.82	2.59	2.02
7	1.33	0.79	2.48	1.68
8	1.36	0.81	2.63	1.62
9	1.37	0.78	3.04	2.24
60	1.61	0.84	2.75	2.24
1	1.78	1.05	3.30	2.02
2	1.70	1.00	3.00	1.96
3	1.71	0.92	2.85	2.80
4	1.63	1.02	2.95	2.70
65	1.65	1.05	2.69	2.45
6	1.78	1.10	2.70	2.45
7	1.64	0.89	3.08	1.90
8	1.36	0.79	2.84	1.88
9	1.45	0.69	2.57	2.40
70	1.45	0.80	2.15	2.35
1	1.37	0.78	2.00	2.15
2	1.75	1.10	3.85	3.00
3	4.50	2.35	9.75	5.65
20-year mean	1.67	0.97	3.13	2.39
Standard Deviation	0.68	0.35	1.61	0.84
Coefficient of Variation (percent)	40.7	36.1	51.4	35.1

1. Source: Manitoba Department of Agriculture, Economics Branch, Yearbook of Manitoba Agriculture, 1964-1973.

is shown to carry the highest level of price risk. At the same time, flaxseed has the highest mean price per bushel. The lowest price risk level is indicated for rapeseed.

By themselves, mean prices and price risk levels provide only partial information on enterprise risk levels facing the decision-maker. Variability in production costs and yields must also be known. The data in Table 1, and in the three following tables, serve to illustrate the degree of risk involved in Manitoba farm enterprises by component types.

The incidence of market risk in Manitoba beef cattle operations is illustrated in Tables 2 and 3. Table 2 shows average annual prices received by Manitoba farmers for live cattle during the period 1954 to 1973. Prices are specified for the three principal liveweight categories for marketing beef cattle, namely calves, feeder cattle and slaughter cattle. Calf prices are given as a weighted average across all grades, while feeder and slaughter cattle prices refer to "good" grades. It is reasonably apparent from the coefficient of variation figures that the degree of price variability declines with increasing maturity of beef cattle. That is, calf price risk has been greater than slaughter cattle price risk during the time period considered. Price risk, however, is only one component of the total risk involved in beef cattle enterprises.

Table 3 shows average annual retail prices paid by prairie livestock farmers for a selection of feeds during

Table 2

Variations in Prices of Manitoba Beef Cattle, 1954-1973[1]

Year	Calves (400 lbs. and over)	Feeders (650 lbs. and over)	Slaughter Cattle (1,000 lbs. and over)
	(dollars per 100 pounds liveweight)		
1954	14.22	15.15	17.40
5	15.42	16.05	18.45
6	14.86	16.20	17.91
7	16.98	17.00	17.78
8	25.64	21.60	21.95
9	24.39	22.90	23.85
60	21.74	21.00	21.70
1	23.78	21.45	21.40
2	27.42	24.40	24.85
3	25.85	23.20	23.00
4	22.21	20.85	21.85
65	23.20	22.05	23.25
6	28.60	25.55	25.20
7	30.40	26.55	26.88
8	30.85	26.85	26.85
9	38.50	32.00	29.10
70	39.42	33.00	30.20
1	36.85	33.75	32.40
2	45.31	38.80	35.07
3	54.85	46.76	45.14
20-year mean	28.02	25.25	25.26
Standard Deviation	10.59	8.12	6.79
Coefficient of Variation (percent)	37.8	32.2	26.9

1. Source: Manitoba Department of Agriculture, Economics Branch, Yearbook of Manitoba Agriculture, 1964-1973.

Table 3

Variations in Retail Prices of Selected Cattle Feeds in the Prairie Region, 1954-1973[1]

Year	Oats	Corn	Linseed Meal	32 percent Supplement
		(dollars per 100 pounds)		
1954	1.46	1.59	5.01	4.96
5	1.47	1.62	5.05	5.00
6	1.51	1.75	5.19	5.14
7	1.53	2.10	5.19	5.06
8	1.56	3.22	4.84	4.85
9	1.60	2.95	4.62	4.91
60	1.64	2.90	4.77	4.84
1	1.67	2.91	4.78	4.81
2	1.89	2.83	4.85	4.97
3	1.75	2.98	5.09	5.15
4	1.74	2.58	5.53	5.06
65	1.71	2.59	5.53	5.11
6	1.71	2.67	5.45	5.03
7	1.83	2.69	5.67	5.29
8	1.83	2.67	5.90	5.34
9	1.66	2.88	6.01	5.39
70	1.50	2.78	6.01	5.34
1	1.56	2.83	6.00	5.41
2	1.56	1.86	5.97	5.25
3	2.94	3.90	6.63	6.13
20-year mean	1.74	2.48	5.41	5.15
Standard Deviation	0.32	0.59	0.55	0.30
Coefficient of Variation (percent)	18.4	23.8	10.2	5.8

1. Source: Statistics Canada, <u>Prices and Price Indexes</u>, Cat. No. 62-002, Ottawa, Table 15, "Dealer's Average Selling Prices for Feed", Prairies, 1964-1973.

the period 1954 to 1973. This selection of feeds is intended to be representative of the diverse sources and degree of processing of feeds used by Manitoba beef operators.

Oats, typical of prairie-grown cereal grain feeds, displays relatively small price variability in relation to corn, for Manitoba mainly an imported source of cattle feed. Considerably lower price variability is shown for linseed meal, while 32 percent supplement, a highly processed feed, is characterized by relative price stability. Knowledge of these differences in feed price variability could be important for those beef cattle farmers intent upon reducing input cost variability. The variability levels may be particularly influential on total cost variability in beef. When grain and oilseed prices are low relative to other costs,(for example from 1970 to 1972), feed costs typically comprise 20 percent of beef cattle operating expenses [26, pp. 101:4 ff and pp. 102:7 ff]. On the other hand, feed costs may comprise up to 50 percent of total operating expenses during periods of high grain and oilseed prices (for example 1973). This is indicative of the need not only for technical competence but also for economic and financial management expertise in the feeding of beef cattle.

3.2 Technical Risk

Several factors encompass technical risk, the most important of which is climatic instability. Its effects are felt most severely on yields of primary farm products, or crops. In Manitoba, variations in length of frost-free growing season, amount and temporal distribution of heat units, volume and seasonal distribution of rainfall, random occurrences of summer hail-storms, and depth of winter snow cover (affecting soil moisture conditions the following spring) all have significant effects on crop yield variability.

The occurrence of technical risk in Manitoba cash crops for the period 1954 to 1973 is shown in Table 4. Average annual yields are depicted for the two major cereal grains, wheat and barley, and the two principal oilseeds, flaxseed and rapeseed, for Crop District Number 3 of Manitoba. The coefficients of variation suggest that barley carries the highest level of yield risk, and rapeseed the lowest.

Of greater importance to farm decision-makers than technical risk alone is the compounded risk comprised of market, technical and other sources of risk. The compounding of different sources of risk may or may not be additive, depending on the signs of the covariances between risk types. For example if crop yields and prices vary together negatively, then compounded risk may be less than the individual sources of risk. This more crucial measure of total risk is discussed in Chapter II, Section 4.

For intensive livestock enterprises, such as poultry, hogs and dairy cattle, the production environment can be

Table 4

Variations in Yields of Manitoba Cash-Crops, Crop District Number 3, 1954-1973[1]

Year	Wheat	Barley	Flaxseed	Rapeseed
		(bushels per acre)		
1954	13.5	20.0	9.0	14.3
5	20.2	19.1	8.3	11.6
6	25.6	26.9	10.1	14.8
7	22.3	19.4	4.0	11.2
8	24.6	27.8	8.5	10.7
9	23.1	25.9	8.0	13.4
60	23.6	26.1	9.1	12.9
1	11.7	13.7	5.7	11.0
2	26.3	33.4	11.7	16.1
3	14.1	19.9	11.5	15.1
4	23.4	30.6	9.6	17.5
65	25.5	38.2	11.8	16.6
6	20.3	26.8	8.0	12.4
7	27.2	36.8	9.5	15.9
8	27.3	35.7	13.5	20.9
9	18.5	25.9	7.0	14.8
70	19.4	29.8	8.3	16.3
1	29.2	46.7	8.0	20.7
2	25.9	38.6	10.3	19.2
3	25.6	42.6	12.2	19.6
20-year mean	21.00	27.36	9.21	15.25
Standard Deviation	5.09	8.8	2.28	3.18
Coefficient of Variation (percent)	24.2	32.5	24.8	20.9

1. Source: Manitoba Department of Agriculture, Economics Branch, Yearbook of Manitoba Agriculture, 1964-1973.

substantially controlled, thereby eliminating to a large extent the effects of variable climate. To a lesser extent this is also true of extensive livestock operations such as sheep and beef cattle, for which housing facilities are generally less enclosed and controlled. Liveweight gains and feed conversion ratios would therefore tend to be adversely affected by extreme weather conditions and climatic variations. In addition to climate, such factors as random incidence of diseases and biological pests of many types affect the growth rate and health of livestock.

Unfortunately, no data were found to reveal the extent of technical risk in beef cattle in Manitoba. However, it is reasonable to assume that this source of risk is important both as an independent source, and as a part of compounded risk in beef cattle. It is technical risk that most closely reflects the level of technical competence of the beef cattle farmer, a crucial input to the profitability of beef. It should be noted that the exclusion of technical risk causes the compound risk estimates for beef cattle used in this study to be partial estimates only.

3.3 Technological Risk

This type of risk concerns decisions to invest in production techniques or durable resources when it is not known with certainty whether technological progress will render these obsolete shortly after the investment has been made. This may result in more efficient techniques resources and hence lower costs of production foregone.

This type of risk becomes intensified when viewed in the context of Johnson's "asset fixity theory" [42]. Production techniques or durable resources may become locked into the farm's asset portfolio, since the disposal criterion (that marginal value product of the services given off by the asset is less than the salvage value of the asset) may not be met for several more time periods. Meanwhile the farmer must continue production at a lower level of efficiency and at higher cost than would have been the case for techniques or resources incorporating all the latest technology. Under the heading of technological risk also is included the possibility of development of new product uses, or new competing products, or of changes in consumer tastes. All these affect demand for farm products, and therefore variability of farm receipts.

3.4 Institutional Risk

This form of risk primarily results from unpredictabilities surrounding changes in government policies. These may affect farm resource acquisition, capital rationing, production and marketing of farm products, and pricing of farm products.

There are many other potential sources of institutional risk. Abrupt and unforseen changes in agricultural marketing board regulations, contractual agreements, and tenure forms all affect the production or marketing of farm products. The occurrence of labour strikes in the non-agricultural sector may cause substantial disruption to both availability of farm resource inputs and to the marketing of farm products. Changes in consumer preferences for individual farm products also affect the variability of farmers' net returns.

4. Compound Risk Levels in Beef Cattle and Cash-Crop Enterprises

Mention has already been made (Chapter II, Section 3,) of the nature and importance of compounded risk levels in individual farm enterprise. For farm decision-makers interested in comparative levels of risk in various farm enterprises, the independent components of risk are less important than some comprehensive measure of the combined sources of risk. There are two reasons for this.

Firstly, decision-makers do not normally face the separate components of risk such as yield or price variability in isolation. Their concern lies with variability of enterprise net returns caused by the combination of all risk sources.

Secondly, compounded risk for a farm enterprise is not necessarily equal to the sum of the individual risk components. Simple additivity of the different risk sources may fail to take account of critical negative covariances between the risk sources.

For these reasons, annual net returns variability is used in this study as the criterion for assessing and comparing risk levels between beef cattle and cash-cropping enterprises in Manitoba. Ideally, the net returns variability data should reflect all four individual components of risk. At the same time, the inherent complexities and data availability problems in such an ideal approach were considered too extensive for a single study. Therefore emphasis is placed on market and technical risk sources.

To the extent that net returns variability may reflect changes in government policies affecting beef and cash crop enterprises, some aspects of institutional risk are implicitly included. Other institutional risk aspects such as land lease arrangements are explicitly held constant. No attempt is made to incorporate technological sources of risk in this study.

The historic incidence of compounded risk in Manitoba beef cattle enterprises is shown in Table 5 for the period 1954-1973. It will be recalled from Section 3.2 of this chapter that no data were available for estimating technical risk in beef cattle. Therefore, compound risk for beef cattle includes variations only in product prices and input costs. Detailed calculations of the annual net returns data are presented in Appendix B. At this juncture, it may be noted that net returns are comprised of gross income less direct operating expenses, including labour and return on operating capital.

Cow-calf enterprises represent the first chronological segment of the beef production cycle, producing beef calves usually in the spring for sale the following autumn as weaned calves of about 400 pounds liveweight. The next segment is the feeder cattle enterprise, in which calves are grown to feeder weight of 600 pounds or more. Finally,

Table 5

Variations in Annual Net Returns for

Manitoba Beef Cattle Enterprises, 1954-1973[1]

Year	Cow-Calf	Feeder Cattle	Slaughter Cattle
	(dollars per head)		
1954	-31.71	7.79	19.41
5	-26.69	8.09	23.14
6	-33.80	15.38	23.28
7	-26.55	12.57	16.60
8	8.33	6.71	23.10
9	- 2.64	14.24	31.54
60	-13.18	13.49	21.48
1	-15.44	4.62	7.61
2	- 2.64	14.03	24.96
3	- 7.31	12.80	17.46
4	-26.84	7.07	17.95
65	-26.26	6.44	15.62
6	- 3.18	7.32	6.24
7	- 1.50	7.78	25.69
8	- 5.87	14.54	31.31
9	30.63	18.66	16.45
70	33.28	24.00	28.98
1	22.06	40.11	35.85
2	40.05	25.54	5.31
3	39.68	4.83	- 8.76
20-year mean	- 2.48	13.25	19.66
Standard Deviation	24.24	8.69	10.73
Coefficient of Variation (percent)	977.4	65.5	54.6

1. Source: see Appendix B

the slaughter cattle enterprise transforms feeder cattle to slaughter status. Slaughter weight may range anywhere between 900 and 1,200 pounds liveweight, depending on such economic factors as current or expected changes in feed costs and product prices.

Although all three beef enterprises appear to have high net returns variability, the cow-calf enterprise is by far the most risky, according to the coefficient of variation statistics. Also worthy of mention is the extent to which cow-calf operations have been fairly consistent monetary losers up until 1969.

The combination of low expected returns and high variance in returns may indicate that cow-calf enterprises would be excluded from all possible efficient portfolios lying on an E-V frontier. This must be reconciled with the existence of a surplus production and net export position for beef calves in Manitoba. An explanation can be found in that most of Manitoba's cow-calf enterprises are located on the west shore of Lake Manitoba (Crop District Number 14) and in the interlake region (Crop District Number 12). In these two regions, opportunities for alternative enterprises are limited, and therefore established farmers may be expected to continue beef calf production in spite of low returns and high risk. On the other hand, in Crop District Number 3 where many alternative enterprises are possible, cow-calf operations would be expected to be excluded from efficient portfolios. This situation will be empirically examined in Chapter V.

Also of interest in discussing Table 5 is the indication that an 8 to 10-year net returns cycle applies to beef cattle enterprises in Manitoba. This becomes more apparent by referring to the troughs, or years of lowest net returns in the table. For cow-calf operations troughs appear in 1956, and again in 1964-1965. Analogous observations can be drawn for feeder cattle and slaughter cattle enterprises. The existence of net returns cycles for beef cattle has been confirmed in previous studies by Marshall [52] and has been alluded to by Tryfos [65].

One further observation should be made concerning the beef cattle net returns cycles. The peaks and troughs for each of the three beef cattle enterprises do not appear to occur in the same years. The slaughter cattle cycle apparently lags behind the feeder cattle cycle which in turn lags behind the cow-calf cycle. For example the 1964 trough in cow-calf returns is followed a year later by a trough in feeder cattle returns in 1965, and the latter is followed a further year later by a low point in slaughter cattle returns. Such a lagged sequence of troughs in part explains the reason for the existence of a beef cattle returns cycle. This is based on the physiological time lapse required for a surplus (or deficit) of beef calves to become transformed into a surplus (deficit) of slaughter cattle.

The lack of simultaneity in the occurrence of individual beef enterprises returns cycles also has significance for the portfolio theory approach to analyzing farm enterprise risk. The covariances between beef cattle enterprises may be expected to be at least small, and possibly even negative. This implies an opportunity for risk averse decision-makers to diversify among beef enterprises in order to reduce net returns variability. The actual covariance magnitudes and signs for beef enterprises are calculated in Chapter IV, Section 4.2.

In Table 6, average annual net returns data are presented for the major cereal crops (oats, wheat and barley) and the principal oilseeds (flaxseed and rapeseed) grown in Manitoba. The data refer as closely as possible to average physical and economic conditions prevailing in Crop District Number 3 for each year in the period 1954 to 1973.

Wheat appears to be the crop with least net returns variability judged by the coefficients of variation. Rapeseed seems to offer the next best opportunity for keeping net returns variability to a low level, followed by oats, barley and finally flaxseed. Apart from carrying the highest level of risk, flaxseed posed an additional problem during this twenty-year period. Negative net returns were evident in four years, plus the lowest mean average expected net return. This implies that flaxseed may be dominated by alternative cash crops in any efficient portfolio on the E-V frontier.

The data in Table 6 also reveal a tendency for cash-

Table 6

Variations in Annual Net Returns for Cash-Crops in Crop District Number 3, Manitoba, 1954-1973[1]

Year	Oats	Wheat	Barley	Flaxseed	Rapeseed
			(dollars per acre)		
1954	4.61	6.32	4.61	9.73	16.46
5	11.21	17.29	4.07	8.90	11.79
6	11.35	20.65	7.33	11.26	15.10
7	3.42	17.01	-0.02	-5.34	3.52
8	7.65	21.50	7.78	6.57	1.65
9	10.71	19.44	5.12	8.03	13.72
60	9.13	24.85	5.84	8.41	12.26
1	0.04	8.74	-0.68	1.89	5.26
2	15.73	30.86	16.69	17.22	14.20
3	6.06	12.35	1.55	14.84	24.75
4	15.22	24.20	14.19	9.95	29.18
65	22.30	27.95	22.90	13.15	22.33
6	14.91	21.62	11.92	2.47	11.84
7	18.93	29.50	14.52	9.38	10.82
8	11.67	21.56	9.51	17.86	19.28
9	4.69	10.70	-1.70	-2.73	15.32
70	3.42	12.08	4.82	-2.24	18.54
1	13.79	23.37	16.75	-4.56	24.15
2	21.70	30.12	24.10	18.28	36.08
3	56.85	98.36	79.78	94.40	86.55
20-year mean	13.17	23.92	12.45	11.87	19.64
Standard Deviation	11.99	18.93	17.49	20.74	17.82
Coefficient of Variation (percent)	91.0	79.1	140.5	174.7	90.7

1. Source: see Appendix B

crop net returns to cycle. For wheat, a six-year cycle is tentatively suggested with net returns peaking in 1962, 1967 and 1973. A less well defined cycle of eight or nine years appears for barley, while flaxseed and rapeseed display more prominently a six-year and a ten-year cycle, respectively. These net returns cycles, together with those indicated for beef cattle, have important implications for (a) the desirability for casting farm enterprise risk analysis in a multi-period framework, and (b) the choice of time horizon in the empirical model. This is elaborated upon in Chapter II, Section 6.

Based on the coefficient of variation terms in Tables 5 and 6, it appears that cow-calf enterprises carry more risk than any of the cash-crops, but all cash-crop enterprises have higher risk than either feeder or slaughter cattle operations. Not too much reliance should, however, be placed on using the statistics in Tables 5 and 6.

The coefficient of variation terms provide only a tentative index of risk by suggesting the percentage deviation from the mean average. They do not suggest the probability of obtaining negative or highly positive net returns in any one year, nor do they indicate the sequence of low or high net returns over time. According to Heady [32, p. 457], the incidence of successive years of low net returns or of high net returns influences decisions more than the comparative risk levels. This suggests the need to study the effects of risk on decision-making within a multi-period framework of analysis.

Of greater importance is the fact that the covariance terms are not accounted for in the tables. Therefore only decision-makers interested in completely specializing in a single enterprise, either a crop or a beef cattle enterprise, would obtain relevant information from comparing expected returns and variance in returns in isolation. More typical of Manitoba agriculture is some degree of diversification.

It is in this respect that portfolio analysis can provide useful guidance in comparative farm enterprise risk problems. The covariances between farm enterprises form an integral part of the security analysis stage of portfolio theory. Therefore a complete measure of total variance for combinations of enterprises is ascertained. In the portfolio analysis stage, the sets of efficient possible portfolios along the E-V frontier provide information on the combinations of farm enterprises within each portfolio. Comparison of different efficient portfolios, representing alternative combinations of expected returns and variance in returns, can then be used to assess the comparative risk levels between enterprises. This basis for assessment of enterprise risk levels is discussed in more detail in Chapter V.

5. Measures to Reduce Risk in Agriculture

Various precautions to counter risk and uncertainty can be adopted by the farmer, as outlined by Heady in 1952, [32, pp. 505-534]. These are classified broadly as:

(i) Measures to reduce variability of farm income;

(ii) Measures to prevent farm income from falling below a minimum acceptable level;

(iii) Measures to increase the farm's ability to withstand unfavourable economic outcomes.

Measures to prevent farm income from falling below some minimum level include formal insurance, forward contracts on resource input costs or product prices, and hedging operations on commodity futures markets. Forward contracts are designed to reduce risk of low incomes, but at the same time eliminate any chance of higher incomes. Insurance and hedging are designed to compensate for low incomes only. As such, they are of only passing interest to the present study, which seeks to investigate the variability of farm incomes in both upward and downward directions.

Measures to increase the farm's ability to withstand unfavourable economic outcomes include maintaining sufficient levels of liquid assets within the farm's asset portfolio, and the holding of monetary reserves. Such measures represent precautions taken by the farmer to forestall the effects of market and technical risk. They can be viewed as being relevant to studies evaluating the effects of risk on enterprise choice. They are particularly relevant to multi-period risk studies because of the impact of risk on direction and extent of farm growth.

Measures to reduce variability of farm incomes are of greater importance here. Such measures include selection of farm enterprises or products having a low subjective variability, diversification within possible portfolios of enterprises, and flexibility in the selection and use of resource inputs and product outputs. Of these, the first two measures have particular relevance to comparing risk levels on resource allocation and enterprise choice over time.

5.1 Selection of Enterprises with Low Variability

In terms of portfolio theory, farmers who are risk averse would tend to select those portfolios containing farm enterprises with, in their view, low income variability. The assessment of income variability can be based on historical analyses of income fluctuations for various enterprises. These analyses may then serve as a subjective basis for future expectations.

Portfolio analysis, or the generation of a set of efficient portfolios, can be employed as an empirical tool to assist decision-makers in this enterprise selection context. Enterprises characterized by high expected returns coupled with high variance of returns would be excluded from all efficient portfolios except those at or approaching the peak of the E-V frontier.

On the other hand, enterprises carrying low returns with low variance can be expected to appear (if at all) in portfolios toward the lower region of the E-V frontier.

That is, if extreme aversion to risk is indicated, then portfolios composed of low returns-low risk enterprises would be preferred. Obviously, such enterprise types would be completely dominated by the high expected return-low variance in returns type of enterprise. Of passing interest in the analysis of efficient portfolios are those enterprises with low expected returns and high variance of returns. These enterprises would presumably be dominated by any of the three foregoing enterprise types. This would effectively exclude them from efficient portfolios on any section of the E-V frontier.

5.2 Diversification

Diversification in factor-factor, factor-product and product-product relationships as a method of reducing risk finds direct application in portfolio theory. Portfolio analysis can be employed to indicate the extent of diversification within efficient portfolios of farm enterprises and resource uses for various combinations of expected returns and variance in returns. This in turn shows the importance that can be attached to diversification by the decision-maker.

The principal factors in determining whether diversification contributes to a lessening of income variability are the magnitudes of the variance terms, and the sign and size of the covariance terms (or the way in which expected returns from two enterprises vary together). Diversification can also occur in two different ways, as described by Heady [32, pp. 510-518].

One method of diversification involves increasing farm resource inputs in order to incorporate an additional enterprise. For the simplest case of doubling resource inputs in order to combine a second enterprise, B, with an original enterprise, A, total variance can be written as [32, pp. 512-513]:-

$$\sigma_T^2 = \sigma_A^2 + \sigma_B^2 + 2\rho\sigma_A\sigma_B \qquad (2.1)$$

where: σ_T^2 is total variance in returns from the two enterprises,

σ_A^2, σ_B^2 are the variance of returns from enterprise A, and B, respectively,

ρ is the correlation coefficient for returns from A and B,

$2\rho\sigma_A\sigma_B$ is the covariance of returns between A and B,

σ_A, σ_B are the standard deviations of returns from enterprise A and B, respectively.

Three possible cases can result, depending on the sign and size of ρ and the size of the variance and the covariance terms. These are as follows:-

(i) If $\rho = 0$, the covariance term becomes zero, and total variance must increase due to combining both enterprises;

(ii) If $\rho > 0$, the covariance term must be positive, thereby increasing total variance even moreso through diversification;

(iii) If $\rho < 0$, the resulting negative covariance term may be sufficient to reduce total variance, increase it, or leave it unchanged. This depends on whether the ratio of the covariance term to the variance of the additional enterprise is greater than one, less than one or equal to one, respectively.

The second method of diversification involves no further resource expansion, but the allocation of a portion of existing resources to an additional enterprise. This method of diversification can produce quite different results from the first. For the case where a portion $(1 - q)$ of existing resources are shifted into the production of a new enterprise, B, total variance can be expressed as:-

$$\sigma_T^2 = q^2\sigma_A^2 + (1 - q)^2 \sigma_B^2 + 2\rho q(1 - q)\sigma_A\sigma_B \qquad (2.2)$$

where: q is the proportion of resources allocated to enterprise A,

$(1 - q)$ is the proportion of resources allocated to enterprise B,

and the remaining symbols are the same as for equation (2.1).

Following Heady [32, pp. 515-516], the criterion for reduction of total variance (for the case of equal variances for enterprises A and B) is given in terms of ρ, the correlation coefficient:-

$$\rho < \frac{2q(1 - q)}{2q(1 - q)}, \text{ (or } \rho < 1.0) \qquad (2.3)$$

Thus, for the "reallocation of resources" type of diversification, where equal enterprise variances exist, total variance will be reduced for any correlation coefficient less than 1.0.

In this study, both types of diversification are applicable. Diversification through additional resources is relevant to a case where a beef cattle enterprise is added as a new enterprise to a Manitoba cash-crop farm, or where an existing beef enterprise is expanded. The additional resources comprise the specialized buildings, feeding and handling equipment, and forage crop machinery necessary for beef cattle, but non-substitutable for production of cash crops.

On the other hand, diversification through reallocation of existing resources applies to the case where an extra cash-crop enterprise is added to those already being produced, or where an additional beef cattle enterprise is appended to an already existing beef operation. An example of the latter is the addition of a new feeder cattle enterprise to an already existing cow-calf enterprise. In such cases, it may not be necessary to purchase additional resources. Diversion of previously-owned resources will suffice.

5.3 Flexibility

As a method of reducing income variability, flexibility is designed to place the farm firm in a position to take maximum advantage of actual or expected changes in product prices, market opportunities, or production conditions. In this study, flexibility is exemplified by beef cattle barns and equipment that have multiple-choice uses. These resources can be used equally well for a cow-calf operation or a feed-lot enterprise. Analogously for cash-crops, the same seeding and harvesting machinery can be used with only relatively minor adjustments for a variety of cereal grains and oilseeds production. This type of flexibility is termed factor flexibility by Heady [32, pp. 525-526].

Other types of flexibility cited by Heady, also relevant to this study, include time flexibility and product flexibility. An example of the former is given by the production of beef calves that can be sold as weaned calves, stockers, feeder cattle, or slaughter cattle, depending on actual or foreseen changes in product prices. Product flexibility can be illustrated by the production of cereal grain or oilseed crops for sale off the farm or for feeding to beef cattle. Another example is given by the production of both feeder cattle and slaughter cattle simultaneously.

The extent of flexibility as a means of reducing income variability can be assessed to some degree in portfolio analysis through the resource use patterns and product mixes associated with each efficient portfolio. In addition, portfolio analysis in a multi-period framework represents an appropriate method of measuring time flexibility.

6. Risk Analysis in a Multi-Period Setting

There are several reasons for the appropriateness of analyzing risk in agriculture within a multi-period framework. One of the reasons mentioned previously is the importance of sequences of either low or high net returns years to the decision-making process. The net returns cycles for both beef cattle and cash crops may have important implications for efficient diversification of farm enterprises over time.

A second reason concerns risk and its implications for farm firm growth. Thirdly, investment in long-term assets are an integral part of farm resource allocation and enterprise choice problems. Investment alternatives of this type are closely associated with both imperfect knowledge of the outcomes and with flows of services over several years.

6.1 Risk Analysis and Lapsed Time in Farm Production

An example of extensive lapse of time between the commitment of farm resource and receipt of proceeds from the sale of farm products is given by the vertically integrated beef cattle operation. In such an enterprise the beef calves would be produced, grown, and fattened to slaughter weight on the same farm. The time lapse may range anywhere between 24 and 36 months, depending, amongst other factors, upon feeding management, rate of liveweight gain and slaughter weight. This implies that a farmer considering the commitment of calf-production resources must have subjective expectations about slaughter cattle prices several years in the future.

Portfolio analysis can indicate the viability of such an enterprise, on the basis of expected returns and variance in returns, only if a multi-year model is used. If the basic time period in farming is accepted as one year, because of taxation, accounting procedures and crop production seasons, then a multi-year model implies the need for a multi-period model.

6.2 Risk Analysis and Farm Firm Growth

The theory of firm growth has been defined by Penrose [56, p. 31] as "...essentially an examination of the changing productive opportunity of firms." Constraints on the extent of or rate of firm growth depend on constraints on productive opportunities in any one time period. Limitations on short-run production opportunities curtail the potential for generating investment funds through savings or borrowing.

The impacts of risk on the process of firm growth are several. Firstly, the changing productive opportunities referred to by Penrose must be examined in the context of imperfect knowledge for the farm firm. Even if resource costs and product prices were assured through, for example contractual agreements, the farm decision-maker must still contend with technical sources of risk. Thus one of the constraints on productive opportunities may be the high variability of returns to that opportunity. If the best opportunities for farm firm growth are also the ones with most risk, then growth rate may be reduced or direction of growth may be influenced. Portfolio analysis in a multi-period framework would appear to offer a means of assessing the influence of risk on rate and direction of farm firm growth.

Secondly, the degree of compounded risk in farm enterprises both affects the possible rate of savings directly, and affects the farmer's borrowing capacity indirectly. Fluctuating product prices in farming are correlated with variations in resource costs, and therefore values of assets owned by the farmer. The farmer may be presented with opportunities for expansion, but may not be in a position to take full advantage because of variability in his borrowing capacity. Thus risk-related changes in farm liquidity status and net worth position over time can influence farm firm growth.

6.3 Risk Analysis and Long-term Investments

The close association between firm growth theory and investment theory, alluded to above, requires the incorporation in the proposed model of some measure of investment alternatives in a time dimension. This can be achieved through applying the classical investment theories of Fisher [20, Chap. 4] and Hirschliefer [37]. According to Fisher's fundamental principles concerning investment opportunities and impatience (for current consumption or utility rather than in the future), classical investment theory proceeds through two essential phases. In a two time-period dimension, the theory under assumed certainty can be evaluated graphically in simplified form, (Figure 3).

For the first (investment choice) phase, the decision-maker's investment opportunity curve can be represented by curve PQRS, where the amount OP is equivalent to total income available in the present time period. All of OP can be consumed in the current period, or any portion can be invested to provide for consumption in the future. The concavity of the investment opportunity curve denotes diminishing marginal productivity of investment as investment is increased. If the line MN represents the market rate of interest for both borrowing and lending, under the assumption of perfect capital markets, then the maximum rational investment in the current time period will be OP_2, since at point R, the market rate of interest equals the rate of return on investment.

Fisher's impatience principle, concerning the second (consumption choice) phase, is included for theoretical presentation only, since utility choices are not a part of this study. Consumption choices are represented by the indifference curves, IC_1 and IC_2, denoting the locus of points of equal utility from all combinations of consumption between present and future time periods. In the absence of borrowing or lending, the optimal consumption pattern occurs at point Q, where the marginal utility (from consumption over time) equals the marginal rate of return on

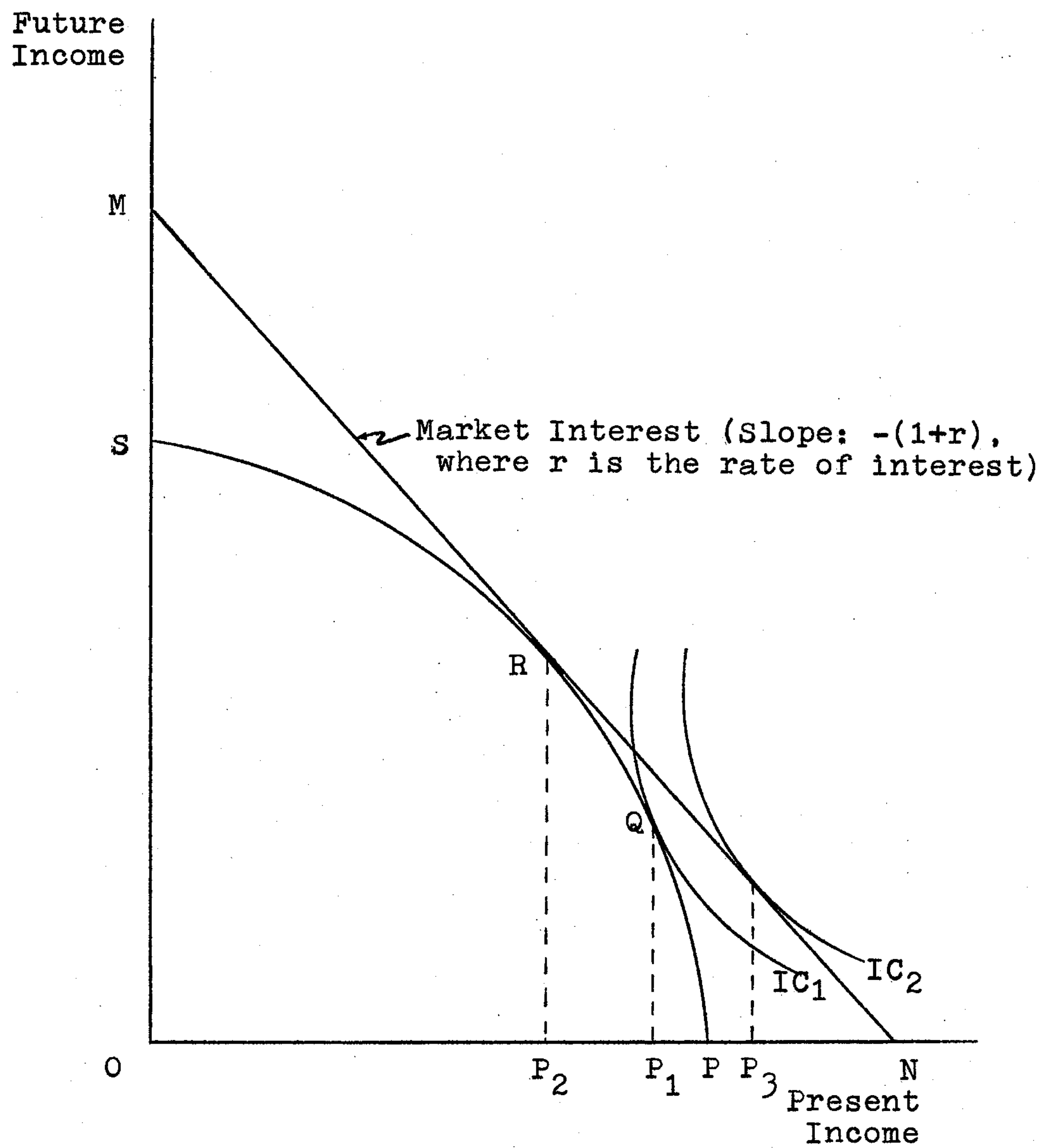

Figure 3

Classical Theory of Investment

investment. However, a higher, preferred indifference curve, IC_2, can be reached by borrowing (in this case) amount P_2P_3 for investing in the current period and subsequent repayment in the future time period.

When risk is introduced into this investment theory, the future outcomes must be evaluated in terms of expected returns and variance of returns. This is relevant for investments in agriculture, where the outcomes from investment alternatives are not generally known with certainty. In particular, investments in durable or long-term assets, such as land and buildings, should be evaluated in terms of the expected returns and variance of returns from each investment, and for each year of the life of the asset. This once again suggests the applicability of combined multi-period and risk analysis, and the use of portfolio analysis for assessing investment alternatives based on efficient portfolios of enterprises using these investments.

6.4 Risk Analysis and Net Returns Cycles

The existence of net returns cycles for the various beef cattle and cash crop enterprises in Manitoba, with a different expected net return each year, suggests that efficient enterprise portfolios may change over time. Much more information can be potentially obtained from analyzing efficient portfolios in each year of a net returns cycle than from evaluating the efficient portfolio for any one year within the cycle. For example, multi-period portfolio analysis offers opportunities for assessing whether high risk enterprises are included in the portfolio only in some years, depending on the stage in the net returns cycle.

In addition to the above set of reasons for using analysis of risk at the farm level in a multi-period framework, there are reasons peculiar to agriculture itself. For example, land rental agreements are often made for more than one year. Good land management requires a crop rotation policy. Aspects such as these may affect the efficient diversification of farm enterprises over time. This can be revealed by portfolio analysis only if a multi-period framework is employed. Having submitted the need for a multi-period setting in which portfolio analysis can be used to evaluate risk in agriculture, the choice of appropriate empirical and analytical technique is discussed next.

CHAPTER III

THE ANALYTICAL MODEL

1. Choice of Analytical and Empirical Techniques

Reference was made in the literature review (Chapter I, Section 1.4) to several available analytical and empirical techniques that have been employed for measuring and accounting for the effects of risk on agriculture. It will be recalled that these techniques included stochastic linear programming, quadratic programming, separable programming and simulation.

Each of these techniques is capable of analyzing risk situations, and each can be fairly readily adapted to account for multiple period problems. However the techniques differ in terms of their strengths and limitations in general. In particular the techniques differ with respect to their ability to meet the specific objectives and test the hypotheses outlined in Chapter I, Section 2.

Stochastic linear programming was discounted as an appropriate technique because of its relatively inefficient procedure for solving risk problems. Every alteration made to the coefficients of stochastic variables requires the generation of a new, separate solution. For relatively simple risk problems having one or at most a few stochastic variables, this solution technique may be appropriate. For a more complex type of risk problem, where many stochastic variables are considered simultaneously, the possible combinations of alternative values for the stochastic variables implies the need for an exorbitant number of solutions. In addition, difficulties are encountered in stochastic linear programming for representing the crucial covariance terms. Thus only partial account may be taken of total variance (or total risk) associated with any particular portfolio of farm enterprises.

Simulation on the other hand offers much greater scope, principally because of its flexibility and range of application. As Irwin [41] has indicated, simulation offers a particularly versatile method of solving problems characterized by multiple goals along with indivisibilities, scale effects, non-linear relations, decisions of a sequential nature, and decisions related to subjective variables and personal preferences. Of these, the characteristics of non-linear relations and decisions of a sequential nature are particularly relevant in this study. So too is the multiple goal characteristic, since the present concern is not only with the effects of risk on resource allocation, and enterprise choice, but also with the effects of risk on rate and direction of farm firm growth.

Thompson [63] has pointed out that programming models (of the types mentioned above) can be applied to problems with multiple conflicting goals only with great difficulty and at high cost. On the other hand simulation techniques offer, as one of their most commendable features, the ability to manipulate multiple-goal problems with dexterity and relatively low cost, even where such goals may be essentially incompatible. Programming models can, however, be employed to good effect on multiple-goals problems in cases where these goals are basically homogeneous, or at least non-conflicting, as has been shown by, for example, Candler and Boehlje [12].

In Chapter II, Section 2, it was argued that portfolio selection theory can be usefully applied to the resource allocation and enterprise choice problems in farming, where imperfect knowledge prevails and a measurement of risk is necessary. Interest then centres on the set of feasible and efficient solutions representing the series of possible efficient portfolios of farm enterprises along an E-V frontier. By investigating only efficient portfolios, as opposed to feasible but inefficient portfolios (see Figure 1, p.30), the effects of risk on resource allocation and enterprise choice can be assessed in a Pareto optimal context.

While it is possible to obtain efficient solutions along the E-V frontier with simulation techniques, it is by no means assured, since simulation does not contain an objective function to be optimized. That is, simulation may generate many solutions that are feasible and inefficient, but unless Pareto optimal solutions are obtained, the effects of risk on efficient diversification of resources and enterprises cannot be measured.

Candler, et al. [13] have underscored the significant differences that can occur between the "best"solution to a multiple-goal problem obtained from simulation techniques, and the optimum solution to the same problem using a programming model. The best solution value for the Monte Carlo simulation technique used by Thompson [63] was 13.26 percent lower than the objective function value derived from the programming model used by Candler, et al.

In the absence of a systemmatic search routine, simulation techniques cannot identify the Pareto optimal solutions considered vital in the empirical testing phase of this study. On the other hand, it will be recalled (from Chapter II, Section 2.2) that Markowitz suggested the set of efficient and feasible solutions along the E-V frontier can be generated using a quadratic programming model. An alternative solution technique is separable linear programming, which can be used to approximate (as closely as desired) the solution to a quadratic programming model.

The choice between quadratic and separable programming techniques in this study was based on the following criteria:-

(i) Availability of solution algorithms adaptable for use on the University of Manitoba's computer;

(ii) Suitability of the computer algorithms for solving combined multi-period and risk problems;

(iii) Flexibility of use of the techniques with respect to such aspects as availability of post-optimal procedures, and ease of making modifications to the model structure and coefficients.

Separable programming techniques were found to be preferable on the basis of all three criteria. Several solution algorithms for quadratic programming were acquired and considered, but all were found to require modifications before use on the University of Manitoba's computer would have been possible. On the other hand, a solution algorithm for separable programming was already available and ready for use, namely International Business Machine's Mathematical Programming System (M.P.S.) [40].

While no particular problems were encountered in incorporating a multi-period framework in a separable programming model, several difficulties prevented the adaptation of the quadratic programming algorithms to multi-periodicity. Problems of loading the input data were especially severe, reflecting both the complexity of quadratic programming algorithms, and the fact that they had been created with single-period models in mind.

Finally the M.P.S. separable solution technique lends itself readily to modifications to the model's structure or coefficients through the "revise" option [40, p. 30]. This easily manipulated revision feature was found to be invaluable in this study, where many model modifications were necessary for testing the various hypotheses. The quadratic programming algorithms in contrast require the entire reloading of the model for all but minor modifications such as changes to a single coefficient. A further advantage of the M.P.S. separable algorithm over quadratic programming algorithms lies in the post-optimal procedures such as "parameterization" and "range analysis" [40, pp. 87-112]. Parameterization involves inserting a sequence of values for a particular coefficient or set of coefficients in the model, with consequent changes in the solution being recorded within a single computer run. This feature was found to be particularly useful in this study, where a series of efficient portfolio solutions is required to plot the E-V frontier of the portfolio analysis phase.

Thus a multi-period separable linear programming (M.P.S.L.P.) model is deemed to be the most appropriate method for solving the multi-period portfolio analysis problems in this study. This tool is not without some disadvantages, however. These include:-

(i) The need for a large number of "special variable" activities to represent a linear segmented approximation of quadratic functions, if the quadratic programming solution is to be approximated at all closely. For large risk problems, this can result in extremely large separable programming models;

(ii) The need for extensive calculations in the creation of the coefficients for the special variables. Alternatively a coefficient generator programme can be built for computerization of this step. The latter was the approach used in this study;

(iii) Although separable programming circumvents the linearity (of functions) and the certainty (of expectations) limitations of linear programming, the restrictive assumptions of additivity (between activities) and divisibility (of activity levels) remain applicable. Divisibility was a particularly troublesome problem in this study, especially in the areas of purchasing indivisible units of resources like farm machinery, and production of indivisible units of outputs such as cattle.

2. Formulation of a Multi-Period Separable Linear Programming Model

Since separable programming represents an approximation technique for quadratic programming, it is appropriate to show how the former can be developed in a mathematically logical way from the latter. A quadratic programming problem is defined by Hadley [30, p. 212] as follows:

find a non-negative vector,

$$x \geq 0, \tag{3.1}$$

satisfying a set of m simultaneous linear equations,

$$Ax = b \tag{3.2}$$

which maximizes the objective function,

$$z = c'x + x'Vx, \tag{3.3}$$

where: x is an $n \times 1$ vector of economic or other activities,

A is an m x n matrix of input-output coefficients,
b is an m x 1 vector of resource or other constraints,
c is an n x 1 vector of prices, costs, net returns, or other financial coefficients,
V is an n x n matrix of variance-covariance elements associated with the expected net returns or prices of the x decision variables.

The above definition by Hadley represents a general definition of the problem, and as such may be used to generate a solution that is feasible but not necessarily efficient. That is, any particular solution generated may lie on or below the E-V efficient frontier portrayed in Figure 1, p. 30. In the context of applying portfolio selection theory in this study, interest is confined to the subset of solutions that are feasible and also efficient. That is, it is desired to generate only alternative efficient solutions that lie along the feasible solution boundary (or the E-V efficient frontier).

Markowitz [49] has demonstrated that any desired subset of alternative feasible and efficient solutions can be obtained through quadratic programming methods by modifying the above general definition of the quadratic programming problem. The modification requires that one of the two components, (expected net returns, $c'x$, or variance of net returns, $x'Vx$), in the objective function (equation(3.3)), be transformed into a constraint equation. This procedure gives rise to two alternative but mathematically equivalent methods of obtaining a set of solution points on the efficient E-V frontier.

The first method is to specify the objective function as the quadratic component, (consisting of the total variance of net returns, $x'Vx$),while the expected net returns component ($c'x$) is set up as a constraint equation. The objective is then to minimize total variance, or risk, subject to some minimum acceptable constraint level on expected net returns. In mathematical terms, this problem can be formulated as follows:-

find a non-negative vector,

$$x \geq 0, \tag{3.4}$$

which satisfies a set of m simultaneous linear equations,

$$Ax = b, \tag{3.5}$$

and which satisfies a minimum net returns constraint,

$$c'x \geq \lambda, \tag{3.6}$$

and which minimizes the total variance of net returns,

$$\theta = x'Vx, \tag{3.7}$$

where: λ and θ are parameters representing net returns and total variance, respectively.

The second approach involves maximizing net returns as a linear objective function, subject to some maximum acceptable level of variance in net returns inserted as a quadratic constraint row. More formally, this can be stipulated as:-

find a non-negative vector,

$$x \geq 0, \tag{3.8}$$

which satisfies a set of m simultaneous linear equations,

$$Ax = b, \tag{3.9}$$

and which satisfies a maximum variance constraint,

$$x'Vx \leq \theta, \tag{3.10}$$

and which maximizes net returns,

$$\lambda = c'x, \tag{3.11}$$

where: θ and λ are two parameters, representing total variance and net returns, respectively.

That these two approaches are mathematically equivalent can be shown to be true through invoking the Kuhn-Tucker theory as applied to duality in quadratic programming problems [30, pp. 238-240]. Either approach can be equally well employed in the portfolio analysis phase of portfolio selection theory. The first approach treats the total expected net returns as a constraint in the problem while seeking to minimize the total variance of net returns. In terms of Figure 1, p. 30, this can be interpreted as selecting a particular point on the vertical axis (expected returns) for which the minimum variance level on the horizontal axis can be found. The solution represents one possible efficient portfolio, and therefore one point on the E-V frontier. By generating a series of solutions for each of a set of different constraint levels on expected returns, the entire E-V frontier can be mapped out.

Analogously, the second approach treats total variance as the constraint in the model, while a maximum value for expected returns is sought. For different constraint levels on total variance, a series of alternative efficient portfolio solutions can be generated. Thus the two methods of obtaining solution points on the same E-V frontier are equivalent.

The separable linear programming problem, as an approximate technique for solving quadratic programming problems, can be defined as follows [30, p. 105]:-

find a non-negative vector,

$$x_j \geq 0, \; j = 1, \ldots, n, \qquad (3.12)$$

which satisfies a set of m simultaneous linear equations,

$$\sum_{j=1}^{n} a_{ij}x_j = b_i, \; i = 1, \ldots, m, \qquad (3.13)$$

and which satisfies a minimum net returns constraint,

$$\sum_{j=1}^{n} c_j x_j \geq \lambda \;, \qquad (3.14)$$

and which minimizes the total variance,

$$\theta = \sum_{j=1}^{n} f'_j(x_j), \tag{3.15}$$

where: each function, $f'_j(x_j)$, is a polygonal approximation of the true quadratic function, $f_j(x_j)$.

The second formulation, equivalent to the first through invocation of the Kuhn-Tucker conditions, is specified as follows:-

find a non-negative vector,

$$x_j \geq 0, \; j = 1, \ldots, n, \tag{3.16}$$

which satisfies a set of m simultaneous linear equations,

$$\sum_{j=1}^{n} a_{ij} x_j = b_i, \; i = 1, \ldots, m, \tag{3.17}$$

and which satisfies a maximum total variance constraint,

$$\sum_{j=1}^{n} f'_j(x_j) \leq \theta \;, \tag{3.18}$$

and which maximizes net returns,

$$\lambda = \sum_{j=1}^{n} c_j x_j. \tag{3.19}$$

The second formulation was chosen for this study, and the approximating problem was solved using the δ-form of solution derived from Hadley [30, pp. 116-119]. An explanation can be found in Appendix A for the construction of the separable functions and for the method of calculating the coefficients in these functions. Examples are also given for the separable coefficients used in the empirical model.

3. Choice of Criterion Function and Time Horizon

It will be recalled from the analysis in Chapter II, Sections 4 and 6, that attention is centred not only on the effects of risk on enterprise choice, but also on the influence of risk on the rate and direction of farm firm growth. This can best be evaluated in a model which has a composite criterion function, that is, a criterion function with multiple goals. An appropriate set of goals for this study would be the maximization of present value of expected net returns in each year over the time horizon, plus maximization of present value of net worth at the end of the horizon.

The use of a composite criterion function in multiperiod analyses has been advocated by Cocks and Carter [14], and by Boehlje and White [7]. The advisability of using such a composite function can be exemplified by some of Boehlje and White's findings. Using "maximization of present value of disposable income" and "maximization of terminal net worth" as alternative criterion functions for a farm growth model, significant differences were found in growth and investment patterns, amount of borrowing, enterprise choices and terminal size of farm. Inclusion of terminal net worth in the criterion function further ensures the existence of a planning horizon, as noted by

Boussard [9].

It should be noted that the choice of maximization of present value of net returns and net worth as a criterion function contains potential problems of double counting. This could occur because net returns generated throughout the time horizon can be dispersed as either consumption or savings. Since the savings may then be used as investments in the farm firm, thereby contributing to any increase in net worth, a simple addition of net returns and terminal net worth would include the value of savings twice. To overcome this problem, terminal net worth in this study is compiled net of the cash down-payment, or savings, portion of new capital investments over the time horizon.

A reasonable length of time horizon over which to extend this criterion function is taken to be nine years. This choice is based upon the tentatively suggested duration of the annual net returns cycle associated with beef cattle, (see Chapter II, Section 4). It may then be possible to assess the extent to which fluctuations in beef cattle annual net returns have an influence on the inclusion of beef cattle operations in efficient farm enterprise portfolios. On the other hand, since no attempt was made to use formal statistical methods to isolate the period of annual net returns cycles, the nine-year cycle may not be taken as statistically significant. Therefore the choice of a nine-year time horizon should be considered somewhat arbitrary.

The beginning and ending years for this nine-period time horizon were chosen as 1964 and 1972, respectively. The year 1973 was not included in the horizon because it was considered to be somewhat of an anomaly. Reference to Table 5, p. 46 and Table 6, p. 50, indicates that net returns for beef feeder and slaughter cattle enterprises fell to very low levels, while those for both cereal grain and oilseed cash crops rose to an unprecedented high level. This was due on the one hand to abnormally heavy international demand, with attendant high prices, for Manitoba's cash crops, and on the other hand to a world-wide glut of beef with consequent depressed beef prices. The concurrence of both these events within the one year led to an extremely wide divergence between beef cattle and cash crop net returns in 1973. Indeed the divergence in that year became so wide that cash crops completely dominated beef cattle enterprises. The consequent exclusion of beef cattle enterprises from all possible efficient portfolios for 1973 implies that little further information could be gained about the effects of risk on enterprise choice by including 1973 data in the model.

To summarize the discussion in this chapter, a multi-period separable programming model is considered to be the most appropriate analytical technique with which to apply portfolio selection theory to problems relating risk to enterprise choice in agriculture. A composite criterion function, combining maximization of the present value of expected annual net returns and the present value of terminal net worth, was selected. This permits evaluation of the effects of risk not only on enterprise choice over time, but also on rate and direction of farm firm growth. Finally, a nine-year time horizon was chosen somewhat arbitrarily on the basis on a tentatively suggested nine-year annual net returns cycle for beef cattle enterprises. Specifically the nine-year horizon from 1964 to 1972 inclusive was chosen as the time dimension for the empirical model. Detailed specification of the empirical model and data sources used follow in the next chapter.

CHAPTER IV

SPECIFICATION OF THE EMPIRICAL MODEL

1. Use of a Representative Farm Approach

The various sources of risk applicable to farm firms, (as delineated in Chapter II, Sections 3 and 4), occur both as intertemporal phenomena and as interregional and interfarm phenomena. It therefore follows that any empirical study of the effects of risk on resource allocation and enterprise choice should ideally be conducted within a combined intertemporal and interlocational framework. This implies the need to use combined cross-section and time series data derived from primary sources.

Rather than adhering to this ideal, interlocational aspects of risk were excluded from this study through the use of a "typical" or "representative" farm approach. Such an approach was adopted for the following reasons:-

(i) Suitable financial and technical data at the primary farm level were found to be generally lacking. Where appropriate cross-section data were available, it was found that each farm's records extended over an insufficient number of time periods to be of use in compiling the required set of combined cross-section and time series data.

Secondary data sources, on the other hand, are not only readily accessible, but also exist for an adequate number of time periods, deemed for purposes of this study to be not less than the length of the beef cattle net returns cycle, or nine years.

(ii) To some extent, the adverse effects of excluding interlocational sources of risk can be mitigated by judicious selection of secondary data drawn from a geographical region characterized by homogeneity of soil type and climate. This considerably reduces variations in the biological growth environment as an interlocational source of risk. In this study, data pertaining to Crop District Number Three of southern Manitoba were used as far as possible. Although differences in biological growth environment occur within this district, these differences are sufficiently small to admit of basic homogeneity.

(iii) Management variations between farms are a second principal contributing factor to interlocational sources of risk. In this model, explicit assumptions are made about managerial competence levels (see Chapter IV, Section 8), so that management variations may be considered beyond the scope of the present study.

(iv) The use of a representative farm approach permits greater concentration to be made on the intertemporal aspects of risk. This may be considered to be more germane to a study seeking to analyze the effects of risk on farm enterprise choice within a multi-period setting.

2. <u>Assumptions Made in Building the Model</u>

The more important assumptions made concerning the structure of the empirical model and the calculation of the coefficients are as follows:-

(i) The size of farm initially under consideration is 640 acres. This size of farm is typical of the larger farms having relatively younger, more progressive owners with good management capabilities. It is this type of farm operator who is typically associated with farm firm growth, and therefore of most relevance to analyses of the effects of risk on farm growth.

(ii) Other durable assets assumed in place at the beginning of the time horizon include the necessary buildings and equipment to house and feed 225 beef calves to feeder weight (750 pounds). Alternatively, the same facilities can be used to carry 360 feeders, in two consecutive lots, to slaughter weight (1,100 pounds), or to sustain a 100 beef cow herd producing calves to 400 pounds liveweight. Also included are the requisite power units and machinery for planting and harvesting 620 acres of cash crops, (the remaining 20 acres are allotted to buildings, cattle feed-lot, roads, etc.). The principal items of machinery include the following:-

(a) two 95-horse-power tractors;

(b) one 18-foot self-propelled combine with

48-inch cylinder width;

(c) one 18-foot discer with fertilizer attachment;

(d) one 18-foot self-propelled swather;

(e) one power-take-off driven baler with heavy-duty five-foot pulley.

(iii) For calculation of initial (1964) owner's net worth and debt load, the following assumptions were made:-

(a) the 640 acres of land were valued at 1964 market prices;

(b) livestock buildings had three-quarters of their useful life (of 20 years) left, and were valued at 75 percent of their 1964 market price;

(c) machinery had one half of its useful life (of 10 years) left, and was valued at 50 percent of its 1964 market price;

(d) initial debt outstanding was taken (for simplicity of calculation) as 50 percent of the 1964 values for buildings and machinery, plus 25 percent of 1964 land values.

Detailed calculations of initial net worth and debt load are given in Appendix C.

(iv) For calculation of depreciation schedules for both taxation and accounting purposes, a straight line rate of 10 percent is taken on machinery, and a straight line rate of five percent is charged to buildings.

(v) Provision for firm expansion is made through land, buildings and machinery purchase activities, with alternative methods of financing these investments included to permit greater financial and credit flexibility. In addition facilities are provided for hiring physical labour services, and for renting in land, with an assumed three-year lease prevailing for all land rental contracts.

(vi) The rate and extent of farm firm growth is assumed to be constrained by the financial status of the firm in terms of both liquid funds available and borrowing capacity generated. Borrowing capacity is in turn assumed to be closely associated with changes in farm net worth.

(vii) Net worth at the end of 1972 is calculated using 1972 market values of land, including buildings, plus depreciated values of machinery and equipment, less all outstanding debts, and less the cash downpayment (or savings) portion of all assets newly acquired since 1964. All outstanding debts in 1972 are assumed repayable without penalty in the form of additional interest charges.

This enumeration of the more significant assumptions in the model is not designed to be exhaustive. Other assumptions of lesser importance are specified and elaborated upon in the ensuing sections of this chapter.

3. Selection of the Separable Variables

The set of separable variables, those for which a measure of risk is designated, were selected so as to portray the comparative degree of compounded risk involved in the production of beef cattle and cash crops. For beef cattle, three separable activities were included. These are cow-calf, beef feeder, and beef slaughter cattle activities. The cow-calf activity produces, over a period of 420 days (including gestation), calves at 400 pounds liveweight. The beef feeder activity produces over a period of 175 days a 750-pound feeder from a 400-pound calf. The slaughter cattle activity produces over a period of 140 days an 1,100-pound slaughter animal from a 750-pound feeder. These activities were selected as a representative cross-section of marketable livestock periods typically found in Manitoba's beef cattle industry.

For cash-cropping enterprises, the three principal cereal grains, wheat, barley and oats, and the two principal oilseeds, flaxseed and rapeseed, were selected as being a representative sample for the area of southern Manitoba under consideration. Over the time horizon 1964 to 1972, these five crops have consistently comprised over 75 percent of all cash crops grown in Crop District Number Three [27].

4. Derivation of the Variance-Covariance Elements

Associated with each separable variable is an expected annual net returns figure plus a variance of net returns element, and with each pair of separable activities, a covariance element. The calculation of these elements comprises the security analysis phase of portfolio selection theory.

4.1 Calculation of Expected Annual Net Returns

Expected annual net returns for each enterprise were derived from product prices, yields, and costs of production and marketing provided by the Manitoba Department of Agriculture (M.D.A.) [26 and 27]. Detailed calculations of net returns are specified in Appendix B.

The net returns figures for the years 1965 to 1972 were discounted to present value (basis 1964) to reflect the assumption that resource allocation and enterprise choice decisions in a multi-period framework are made at the beginning of the time horizon. Normally, a farmer would have to base such decisions on subjectively defined expected returns and variance in returns from alternative possible portfolios of farm enterprises. In this study, objectively defined probability distributions of returns (based on historical data) are used to determine Pareto optimal efficient portfolios. Using the procedure of discounting to present value in 1964, the effects of risk on enterprise choice can then be assessed in a Pareto optimal context from the hypothetical point of view gained from the beginning of the time horizon. For simplicity, a discount rate equal to the average of short-term lending rates for the period 1964 to 1972 (eight percent) was used in the model. Discounting to 1964 is selected over compounding to 1972 since it is assumed the hypothetical decision-maker in the model is viewing decisions to be made on growth and enterprise choice from the beginning of the time horizon.

The discounting formula used is given by:-

$$P.V.\ (R_{ij}) = \frac{R_{ij}}{(1 + r)^{i}} \tag{4.1}$$

where: $P.V.\ (R_{ij})$ is present value of expected annual net returns in the i^{th} year for the j^{th} farm enterprise,

R_{ij} represents undiscounted expected annual net returns in the i^{th} year for the j^{th} enterprise,

r represents the discount rate.

The discounted net returns values are then entered as the elements in the objective function in the M.P.S.L.P. model.

4.2 Calculation of the Variance-Covariance Elements

To obtain estimates of the variance and covariance elements associated with the expected annual net returns, the conventional formulae were used as follows (see Snedecor and Cochran, [60, pp. 37 and 186]):-

$$\text{Variance}_j = \sigma_j^2 = \frac{\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2}{n - 1}, \tag{4.2}$$

$$\text{Covariance}_{jk} = \sigma_j \sigma_k = \frac{\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{n - 1}, \tag{4.3}$$

where: x_i represent expected annual net returns,

$\bar{x}$ represents the mean average,

n is the number of observations,

j and k subscripts denote individual cattle or crop enterprises.

No attempt was made to assign any system of distributed weights to the individual annual deviations, since this is a strictly subjective phenomenon and likely to vary between decision-makers. Instead, the simplifying assumption was made that the decision-maker is influenced equally by the deviations for each year.

The set of variance and covariance elements, expressed in dollar terms for the year 1964 is depicted in Table 7. For the years 1965 to 1972, these elements were discounted to present value (basis 1964) in order to conform to the discounted values of the expected net returns in the objective function. The variance-covariance elements thus formed were entered in the total variance constraint row of the M.P.S.L.P. model.

To ensure that the probability distributions of expected net returns met the condition of normality necessary for use in portfolio selection theory (see for example Markowitz [51 , p. 17] or Borch [8]), small sample tests of symmetry were performed. These tests concern the use of the third moment about the mean, or the average of the third powers of the deviations from the mean, based on the formulae [60, p. 200]:-

Table 7

Variance and Covariance Elements for

Expected Annual Net Returns for 1964[1]

Enterprise Type	Cow-Calf	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Flaxseed	Rapeseed
	(dollars per head)			(dollars per acre)				
Cow-Calf	645.12							
Beef Feeders	251.33	134.16						
Slaughter Cattle	47.87	66.20	122.93					
Wheat	-80.72	-19.29	-37.37	48.71				
Barley	-66.51	-1.49	-49.44	59.63	66.92			
Oats	-83.60	-25.38	-47.34	51.70	58.05	44.86		
Flaxseed	-98.91	-49.51	-47.30	47.96	47.93	44.95	78.92	
Rapeseed	32.53	39.31	-28.31	24.24	43.74	22.99	34.62	66.82

1 Based on Observations of Expected Annual Net Returns for the years 1964 to 1972 inclusive.

$$\text{Measure of skewness} = \beta = \frac{\dfrac{n\left[\sum_{i=1}^{n}(x_i - \bar{x})^3\right]}{(n-1)(n-2)}}{\sqrt{\left[\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}\right]^3}} \tag{4.4}$$

$$\text{Standard error of } \beta = s_\beta = \frac{6n(n-1)}{(n-2)(n+1)(n+3)} \tag{4.5}$$

where: if $\beta = 0$, the distribution is symmetric,

if $\beta > 0$, the distribution if right-skewed,

if $\beta < 0$, the distribution is left-skewed.

The test of significance at the one percent level, for n - 1 degrees of freedom, is given by :-

$$-3.355 < t = \frac{\beta}{s_\beta} < 3.355 \tag{4.6}$$

The results of this test confirmed that all eight values were not significantly different from zero, or that the probability distributions of expected net returns for each of the eight farm enterprises were approximately symmetrical.

5. <u>Beef Cattle and Cash-Crop Activities</u>

The three cattle and five cash-crop enterprises specified in Chapter IV, Section 3, collectively comprise the set of farm production and principal revenue-earning activities in the model. Expected net returns from each enterprise contribute to the value of the objective function, generate liquid funds with which to pay farm overheads, debts, taxes and consumer expenditures, and contribute to the taxable income base.

In addition to the (assumed non-constrained) resource inputs already included in the calculation of annual net returns, such as feeds and physical labour, production of beef cattle requires inputs in terms of management labour and livestock housing and feeding facilities. Both of these resources are entered as constraints in the model. The resource requirements per head of beef cattle produced, together with the resource constraint levels imposed at the beginning of the time horizon are shown in Table 8.

Production of cash crops requires inputs of management labour, land and machinery, in addition to the (assumed non-constrained) variable inputs already included in annual net returns calculations. These resource requirements and the associated resource constraint levels are illustrated in Table 9. It should be noted that only a selection of the more important items of machinery are inserted as constraints in the model, and that the constraint levels, particularly those segregated by season for the tractor, are intended to reflect the estimated available annual average hours (limited by weather and soil conditions) deemed suitable for conducting field operations.

6. <u>Activities Providing for Farm Firm Growth</u>

Provision is made in the model for expansion of the farm firm over the growth horizon through the inclusion of land purchase or rental alternatives, livestock housing purchase and farm machinery purchase activities. In this way, it is possible to represent the farmer's choice

Table 8

Resource Requirements per Animal

for Beef Cattle Enterprises

	Feeder Cattle (400-750 lbs.)	Slaughter Cattle (750-1100 lbs.)	Resource Constraint Level
Management Labour (hours)	3.18[1]	2.10[2]	4,000[4]
Livestock Housing (square feet)	20.0 [3]	25.0 [3]	4,500[5]

1. Based on estimate of 0.015 hours per day for 212-day production period.
2. Based on estimate of 0.015 hours per day for 140-day production period.
3. Source: Manitoba Department of Agriculture, Economics Branch, Beef Manual, pp. 102:47-48.
4. This represents a maximum figure, and not necessariy the amount of time actually worked. The figure also includes time devoted to such non-supervisory management activities as bookkeeping, farm planning, buying and selling.
5. See Chapter IV, Section 2, Assumption (ii).

Table 9

Resource Requirements per Acre

for Cash-Cropping Enterprises

Resource	Wheat	Barley	Oats	Flax-seed	Rape-seed	Resource Constraint Level
Land (acres)	1	1	1	1	1	620
Management Labour (hours)[1]	2	2	2	2	2	4,000
Tractor (Apr-Jun) (hours)[2]	0.25	0.25	0.25	0.4	0.4	300
Tractor (Jul-Sep) (hours)[2]	0.2	0.2	0.4			300
Tractor (Oct-Nov) (hours)[2]	0.123	0.123	0.123	0.123	0.123	200
Combine (hours)[2]	0.238	0.238	0.238	0.238	0.238	200
Discer (hours)[2]	0.123	0.123	0.123	0.246	0.246	200
Swather (hours)[2]	0.138	0.138	0.138	0.138	0.138	200
Baler (hours)[2]	0.2	0.2	0.4			300

1. Estimated Requirements per acre for planning, administration and marketing operations.
2. Source: Manitoba Department of Agriculture, Economics Branch, Farm Data Handbook, pp. II:27A-35

decisions on comparative enterprise growth rates and associated resource acquisition and allocation in accordance with relative net returns and risk levels between beef cattle and cash-cropping enterprises.

6.1 Land Acquisition Activities

Two land purchase alternatives are entered in each time period in the model, the first requiring 25 percent cash downpayment for use when ample liquid funds are available, but credit is restricted, the second based on the more liberal requirement of 10 percent cash downpayment for use during times of liquidity shortage but more abundant credit. For both alternatives, credit must be drawn from long-term capital sources, such as the Federal Government's Farm Credit Corporation (F.C.C.) so that the term of the loan is commensurate with the length of life of the asset. The debt is assumed for simplicity and easier structuring of the model to be repaid on the basis of equal principal instalments over the 25-year life of the loan. Interest charges, levied at variable annual rates (see Appendix B) on the debt outstanding at the beginning of each time period, must be paid fully each year. The land values (including crop storage buildings) used in the model are shown in Table 10, together with property tax rates used.

To provide further flexibility in financial planning and management, land can be rented on a cash basis under terms of a three-year lease. Rental charges per acre vary

Table 10

Land Values, Property Taxes

and Cash Rental Rates

for Crop District Number Three,

Manitoba

Year	Land Values[1]	Property Taxes[2]	Cash Rental Rates[3]
		(dollars per acre)	
1964	91.83	1.54	8.89
5	95.25	1.62	9.24
6	101.54	1.70	9.82
7	119.79	1.81	11.39
8	126.80	1.96	12.10
9	113.04	2.04	11.08
70	112.74	2.04	11.06
1	117.99	2.04	11.47
2	122.57	2.08	11.89

1. Source: (1964 to 1969 values) Roehle, R., "An Econometric Analysis of Farm Land Values in Western Manitoba", unpublished M.Sc. thesis, University of Manitoba, 1971, p.103; subsequent values derived from Statistics Canada, Farm Input Price Indexes, p. 22, Land Index.
2. Source: (for 1971) Framingham, C.F., W.J. Craddock and L.B.B.Baker, "Alternative Futures for Manitoba Agriculture: The Application of a Model for the Analysis of Agricultural Income, Employment, Price, Production and Farm Size Policy Alternatives", Research Bulletin, University of Manitoba, Dept. of Agric. Econ., to be published in 1975, Property Taxes Section, Table 2; values for remaining years indexed from Statistics Canada, Farm Input Price Indexes, p. 23, Property Tax Index.
3. Source: Derived by adding annual property tax rate to eight percent of annual land value.

in each year to reflect average cash rental rates prevailing in Crop District Number Three, Manitoba (shown in Table 10). Such rental arrangements were designed to represent in the model the increased security obtained from a reasonable length of lease. It was assumed that any land rented was cultivable land having the same soil productivity rating as the land originally owned by the decision-maker, and that no restrictions were placed on the use of the land, other than crop rotational constraints (to be discussed in Chapter IV, Section 8).

6.2 Farm Buildings Acquisition Activities

The term farm buildings in this study refers to the necessary housing, feeding and handling facilities for beef cattle enterprises. It was assumed that such buildings would not be included with the acquisition of additional land, under either land purchase or land rental arrangements. Therefore any expansion of the beef cattle enterprises beyond the level constrained by the initial set of livestock buildings would necessitate separate investment in additional buildings.

Analogous to the case of land purchase alternatives, two buildings purchase alternatives are included in the model. The first requires 25 percent down-payment, while the second requires ten percent. In each case, the loan portion is financed from long-term capital sources (assumed to be the F.C.C.), and the debt is repayable in 25 equal principal instalments.

Buildings purchase costs used in the model are shown in Table 11. These are expressed in terms of costs per animal for provision of open shed housing, concrete feeding aprons, feed storage and processing facilities, feeding and watering equipment, and cattle handling facilities.

Depreciation charges on both originally owned and newly acquired livestock buildings are rated in the model at a fixed annual five percent of the acquisition price, using the straight line method of depreciation. A depreciation rate higher than the rate of debt write-off or the life of the asset is used so that income tax payments would be reduced in the earlier years, thereby enhancing the prospects for farm firm growth.

6.3 Farm Machinery Acquisition Activities

Provision was made in the model to purchase additional farm machinery to match the expansion of cultivable land. Two financing alternatives are included for each of five different machines. Only five of the more essential machines were selected from the entire set commonly employed on Manitoba farms in order to prevent undesirable expansion of the model size. The machines selected for inclusion appear in Table 12, along with their acquisition prices. Machine size or capacity specifications are given in Chapter IV, Section 2, Assumption (ii).

The two methods of financing machinery purchases are designed to improve flexibility in financial planning

Table 11

Purchase Costs of Livestock Housing[1]

Year	Cost per Beef Feeder (400-750 lbs.)	Cost per Slaughter Animal[2] (750-1100 lbs.)
	(dollars per animal)	
1964	73.80	46.13
5	76.20	47.63
6	80.40	50.25
7	83.60	52.25
8	88.00	55.00
9	97.20	60.75
70	96.80	60.50
1	102.00	63.75
2	113.40	70.88

1. 1970 figures derived from Manitoba Department of Agriculture, Economics Branch, Beef Manual, p. 102:57; for the remaining years, 1970 figures were modified using Statistics Canada, Farm Input Price Indexes, p. 22.
2. Cost per slaughter animal is halved, since two sets of slaughter cattle can be processed through the same building in a single year.

Table 12

Purchase Costs of Principal Farm Machines[1]

Year	Tractor	Combine	Discer	Swather	Baler
		(dollars per machine)			
1964	9,172	14,107	2,690	3,289	2,652
5	9,472	14,676	2,772	3,407	2,674
6	9,891	15,548	2,892	3,574	2,704
7	10,342	15,814	2,988	3,695	2,754
8	10,643	16,282	3,074	3,804	2,865
9	10,991	16,838	3,165	3,931	2,930
70	11,291	17,204	3,249	4,031	3,010
1	11,576	17,419	3,314	4,115	3,070
2	11,900	17,925	3,405	4,230	3,165

1. Source: (for 1972) Alberta Department of Agriculture, Production Economics Branch, Farm Machinery Costs, Edmonton, Alberta, 1972, Table 4. Purchase costs for the remaining years were indexed from 1972 figures using Statistics Canada, Farm Input Price Indexes, p. 24

and credit use. One method is based on a 50 percent cash down-payment, the other on a 30 percent cash down-payment. The debt portion is obtained from medium-term loan sources, assumed in this study to be the chartered banks. Such debts are subsequently retired over a five-year period on an equal principal basis, with interest charges levied on outstanding balances at the beginning of each year falling due at the end of that year. Interest rates are based on variable prime bank lending rates (see Appendix B) plus two percent, in accordance with medium-term lending rate policies of the Royal Bank of Canada [57].

Depreciation rates for both the initial set and newly acquired farm machinery were assumed for simplicity to be based on the straight line method, using ten percent of the acquisition price in each of the ensuing ten years. It is acknowledged that different depreciation rates for different machinery types are permitted for tax write-off purposes, but the straight line, 10 percent method was chosen for greater ease of structuring the empirical model. The inclusion of a depreciation schedule for machinery permits account to be taken in the model of machinery replacement.

7. <u>Taxation, Consumption and Farm Overhead Activities</u>

Apart from the annual payment of property taxes already discussed (in Chapter IV, Section 6.1), the farm entrepreneur is liable for income taxes levied on the taxable income from farm operations in each year.

Taxable income in each year is calculated internally in the model on the basis of gross farm income, less all operating expenses and allowable fixed overhead costs. Gross farm income includes interest earned from short-term investments[1], and income from renting out land. Deductions of fixed costs allowed for tax purposes include interest charged on debts, depreciation on buildings and machinery, repairs and maintenance costs on fences and buildings, plus costs of public utilities such as water. The same depreciation figure is used for both accounting and taxation purposes, in order to simplify the structure of the model. It should be noted, however, that since 1971, Canadian farmers have been permitted to use only the reducing balance method of depreciation for tax purposes, except for farm machinery to which the straight line method had been applied prior to 1971. In this latter case, it is permitted to continue using the straight line method until the machine is fully depreciated.

Income tax payable activities were entered non-linearly into the model according to schedules prescribed by the federal government's Department of National Revenue, Taxation [24], as shown in Table 13. The assumption was made for taxation purposes, that the hypothetical farmer

1. Provision was made for investment of surplus liquid funds at the end of each year into a short-term securities activity, earning interest commensurate with prevailing prime bank lending rates less 1 percent, with both sale proceeds and interest earnings available in the year succeeding investment.

Table 13

Federal and Provincial Income Tax Rates Applicable to Manitoba Farmers, 1964-1972[1]

Year	Taxable Income Bracket			
	$2,600	$2,601-4,600	$4,601-10,600	$10,601
	(all values in percentage terms)			
1964	0	0.125	0.23	0.35
5	0	0.125	0.23	0.35
6	0	0.127	0.232	0.36
7	0	0.13	0.233	0.37
8	0	0.135	0.234	0.38
9	0	0.135	0.234	0.38
70	0	0.138	0.24	0.41
1	0	0.165	0.31	0.42
2	0	0.17	0.328	0.43

1. Source: Canada Department of National Revenue, Taxation, Individual Income Tax Return Forms, Taxation Data Centre, Ottawa, Ontario, 1964-1972.
2. Basic exemption bracket for a married man with two children increased in 1972 to $3,450, with commensurate increases in the remaining taxable income brackets.

in the model is a married man with two children, resulting in the basic tax exemption bracket shown in column 1.

7.1 Consumer Expenditures

The annual allowance for the farm family's consumer expenditures was based on the assumption that a reasonable standard of living was sought initially, rising at a rate proportional to changes in average Canadian spending habits over the time horizon. Therefore expenditures for the initial year, 1964, were based on the national average expenditures on goods and services per family. Subsequent years were revised according to the annual percentage change in personal expenditures on goods and services for Canadians as a whole. These data are illustrated in Table 14.

7.2 Farm Overheads

Farm overhead costs other than taxation were subdivided in the model into three components. These are (a) depreciation, (b) interest charges and principal repayments on outstanding debt, and (c) all other overheads.

Two activities were created for depreciation accounting. The first contained the pre-calculated payments made annually on plant and equipment already in place at the beginning of the time horizon. The second calculated internally in the model the requisite annual depreciation charges associated with newly acquired livestock housing and farm machinery.

Calculation of interest payable on capital debts

Table 14

Farm Family Consumer Expenditures, 1964-1972

Year	Annual Expenditures	Percentage Increase in Expenditures[2]
	(dollars)	
1964	4896.00[1]	
		7.4
5	5258.30	
		8.1
6	5684.22	
		8.7
7	6178.73	
		8.4
8	6697.74	
		9.3
9	7320.63	
		8.7
70	7957.52	
		5.5
1	8395.18	
		7.8
2	9050.00	

1. Dominion Bureau of Statistics, Urban Family Expenditure, 1964, Cat. no. 62-527 (Occasional), Ottawa, Ontario, Table 20, p. 91.
2. Bank of Canada, Bank of Canada Review, Series 21, Table 1, Feb. 1974.

was achieved through lodging interest costs per dollar of debt outstanding at the beginning of each time period, and per dollar of new capital borrowing in each time period, in the objective function. Three activities were created for each time period to cater for initial outstanding debt, one each for short-term, medium-term and long-term debts. These activities were replicated for new capital borrowing, again for short-, medium-, and long-term loans in each time period. Repayment of principal at the end of each time period was achieved by transferring 33 percent of all short-term debt and loans, 10 percent of total medium-term debt and loans, and four percent of combined long-term debt and loans into short-term, medium-term and long-term capital repayment activities, respectively.

All remaining farm overheads were assumed (in the initial time period) to amount to $4,000 (see Table 15), and this figure was increased for subsequent time periods using a farm inputs price index. Residual overheads were entered into a separate activity in the model.

8. Constraints on Resources

Constraints are imposed on cultivable land acreage, livestock housing space and farm machinery hours specific to the beginning of the time horizon, and repeated for each succeeding time period. These do not serve to restrict growth of the farm firm since provision is made for acquiring additional durable assets of these types. It is assumed that there is no difficulty in hiring additional (non-management)

Table 15

Residual Farm Overhead Costs[1]

Year	Annual Overhead Costs
	(dollars)
1964	4,000
1965	4,138
1966	4,374
1967	4,410
1968	4,555
1969	4,722
1970	4,762
1971	4,928
1972	5,135

1. The figure of $4,000 was derived for 1964 from assessing an overhead cost of $4.00 per cultivable acre, (or $4 x 620 acres = $2480), plus an overhead cost of $33.33 per 100 square feet of livestock building space (or $33.33 x 4500 square feet = $1500). Subsequent years indexed using Statistics Canada, <u>Farm Input Price Indexes</u>, p. 22.

farm labour, so that no constraint is imposed on this resource.[1] Simple crop rotation constraints are included in each time period of the model. These serve to proscribe monocultural oilseed cropping practices by forcing each eacre of land used for oilseeds to be followed by a cereal grain crop.

Credit availability, for each of short-, medium-, and long-term capital borrowing sources, is tied to the farm's net worth position. Initial net worth is assessed at $50,000, based on 1964 values of the undepreciated portions of durable assets, and initial debts outstanding of $1,000 short-term, $20,000 medium-term, and $23,000 long-term debt.[2]

For purposes of establishing credit limits, the constraints are initially based on arbitrarily allocating initial net worth in the proportions of $4,000 to short-term, $19,000 to medium-term, and $27,000 to long-term borrowing. Provision is made for increasing the credit limits throughout the time horizon. This is accomplished by entering the cash down-payment portions of durable asset purchases plus repayments of principal made in each time period into the requisite capital borrowing constraint rows. This permits changes in farm net worth to be reflected in corresponding changes in borrowing capacity, the procedure generally adhered to in the real world. Farm firm growth is in this way directly related to the ability of the firm to generate new capital borrowing capacity.

Management capacity is also assumed to limit firm

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1. The cost of non-management labour is included in net returns calculations for each farm enterprise (see Appendix B, Table B-2 to B-9).
2. See Appendix C for calculations.

growth. Table 16 indicates the maximum possible enterprise size if the farm operator devoted all his available time (assumed to be a maximum of 4,000 hours per annum) to management of a single enterprise. Maximum limits to growth would occur at the levels of 1,185 beef feeders, 1,777 slaughter cattle, 3,555 acres of cereal grains or 2,370 acres of oilseeds. The limits on the beef cattle enterprises, moreover, are based on the assumption that all feed requirements are purchased, thereby ignoring the management inputs associated with growing some portion of feeds on the farm.

Under a more realistic assumption that eighty percent of beef cattle feed requirements are grown on the farm, the maximum limits to growth would then be reached at 605 head for feeders and 742 head for slaughter cattle. To provide 80 percent of the feeds required for this number of feeders or slaughter cattle, cultivable land acreage would have to be increased to 817 acres and 1,040 acres, respectively.

9. Intertemporal Transfer Equations

The individual time periods in the model are interconnected through transfer equations which carry forward all stocks of durable assets, debts and liquid funds outstanding at the end of one period to the succeeding period. Stocks of durable assets comprise land, livestock buildings and machinery, including all assets purchased since the beginning of the time horizon. Land rented in

Table 16

Enterprise Size Limits due to Management Capacity

	Beef Feeders	Slaughter Cattle	Cereal Grains	Oilseeds
	(hours per head)		(hours per acre)	
Supervision of livestock or crop	3.0	2.0	1.0	1.5
Supervision of Hired Physical Labour[1]	0.375	0.25	0.125	0.188
Total Management Requirements	3.375	2.25	1.125	1.688
Maximum Enterprise Size	1185 head (605 head)[2]	1777 head (742 head)[2]	3555 acres	2370 acres

1. Based on physical labour requirements per head or per acre x 0.125 hours of management supervision per hour of labour hired.
2. Maximum size of cattle enterprise where 80 percent of feed requirements are grown on the farm.

any one period is also transferred to the two subsequent periods, in order to comply with the assumed three-year rental contract in the model.

Outstanding debts in the short-, medium-, and long-term categories are calculated at the end of each period by summing the principal outstanding at the beginning of the period and all new capital loans, then deducting repayments of principal. As already discussed in Chapter IV, Section 6, repayment schedules are formulated on an equal principal payment basis, namely four percent per annum over 25 years for long-term debt, 10 percent over 10 years for medium-term, and 33.3 percent per annum over three years for short-term debt. The amount outstanding net of repayments is then transferred into the subsequent time period for purposes of calculating the interest payments due.

Liquid funds outstanding at the end of each period can be transferred either as surplus cash earning no interest or as short-term interest-bearing investments. Surplus cash is transferred directly into liquid funds for the subsequent period. Short-term investments are channelled through a sell investments activity before being entered together with interest earnings into the liquid funds row for the succeeding time period.

9.1 Terminal Net Worth Transfer Equations

In the final time period of the model, a set of transfer equations is used to accumulate the land, livestock buildings and machinery originally-owned and purchased across the time horizon for transfer into a set of sell activities. For buildings and machinery, it is assumed that only the undepreciated portion of the acquisition price, less the cash down-payment (or savings) portion of the assets is available for sale. For land, it is assumed that sale proceeds amount to the going market price for 1973, or $122.57 per acre (see Table 10), less the cash down-payment portion. As already explained, (p.79), the cash down-payments on durable asset purchases are already included implicitly in terminal net worth. To prevent double-counting therefore, these savings portions must be deducted from the gross sales proceeds. Finally, all proceeds from sale of durable assets in the final period are discounted to present value.

It is assumed that all principal in the three categories of debt is repaid in full at the end of the final time period, and that no penalty is incurred in doing so. Principal repayments are also discounted to present value, so that terminal net worth is expressed in discounted dollar terms.

This completes the explanation of the essential structural components and coefficients in the empirical M.P.S.L.P. model. In the following chapter, the empirical procedure for testing the model and the hypotheses formulated in Chapter I, Section 2, is outlined, together with the empirical results. The empirical analysis constitues the second, or portfolio analysis phase of portfolio selection theory.

CHAPTER V

EMPIRICAL ANALYSIS AND RESULTS

1. Empirical Model Building and Testing Strategy

Since the multi-period separable linear programming (M.P.S.L.P.) model is both large and complicated, a strategy of building and testing the model in increasingly complex stages was followed. The stages involved were:-

(i) The building and solving of a static (one-period) linear programming (L.P.) model for the initial year in the time horizon. This model contained all the activities and constraints relevant to a single year, as detailed in Chapter IV, except for the special activities and equations associated with the separable variables. That is, no provision was made for analyzing risk, implying that all coefficients in the L.P. model were assumed known with certainty.

(ii) The conversion of the static L.P. model to a single-period separable linear programming model. This was accomplished by the inclusion of the grid and functional equations, the total variance constraint row, and the "special variable" activities. The latter are obtained from partitioning each of the variance and covariance functions related to the separable livestock and crop activities into 11 linear segments, so specified in Appendix A. This model was tested for accuracy by solving it as an equivalent L.P. problem (through setting the total variance row as a non-constraining type), and comparing the results against those for the L.P. model in stage (i). The results were identical to the third decimal place, indicating the equivalence of the two models when risk level is non-constraining, and therefore indicating the accuracy of the separable programming technique.

(iii) The extension of the L.P. model to a multi-period linear programming (M.P.L.P.) model. This required the addition of the A matrices for the subsequent eight time periods in the horizon, and the insertion of the intertemporal transfer equations connecting individual time periods.

(iv) The conversion of the M.P.L.P. model into the M.P.S.L.P. model by appending the sets of grid and functional equations, the total variance constraint row, and the special variable activities (associated with the eight separable variables) for each of the nine time periods. The dimensions of the M.P.S.L.P. model were 4,943 columns, including slack variables, and 771 rows. The model contained 12,547 elements, resulting in a density of 0.32. The M.P.S.L.P. model was tested for accuracy by comparing the results derived from freeing completely the constraint on total variance against the results from the

M.P.L.P. model in stage (iii). Again the two sets of results were found to be identical to three decimal places, thereby providing a measure of confidence in using the M.P.S.L.P. model for testing the hypotheses formulated in Chapter I, Section 2.

1.1 Role of Computer Facilities

Extensive use was made of a series of computer programmes in the empirical model building and testing stages. Some of these programmes were readily-available software packages, such as the M.P.S./360 package used to solve linear and separable programming problems [40]. Other programmes were specific to the study, and therefore had to be written especially for it. These programmes served the purpose of expediting the calculation of the model coefficients and the construction of the various empirical models outlines in the previous section.

The flow-chart in Figure 4 depicts the sequence of computer programmes employed in the empirical analysis. The specially-written programmes include those for calculating the variance-covariance elements and the discounted coefficient values. Both of these programmes were written in Fortran IVG. The programmes for generating the M.P.L.P. and M.P.S.L.P. models were also specifically written for this study, using Assembler and Fortran IVG computer languages.

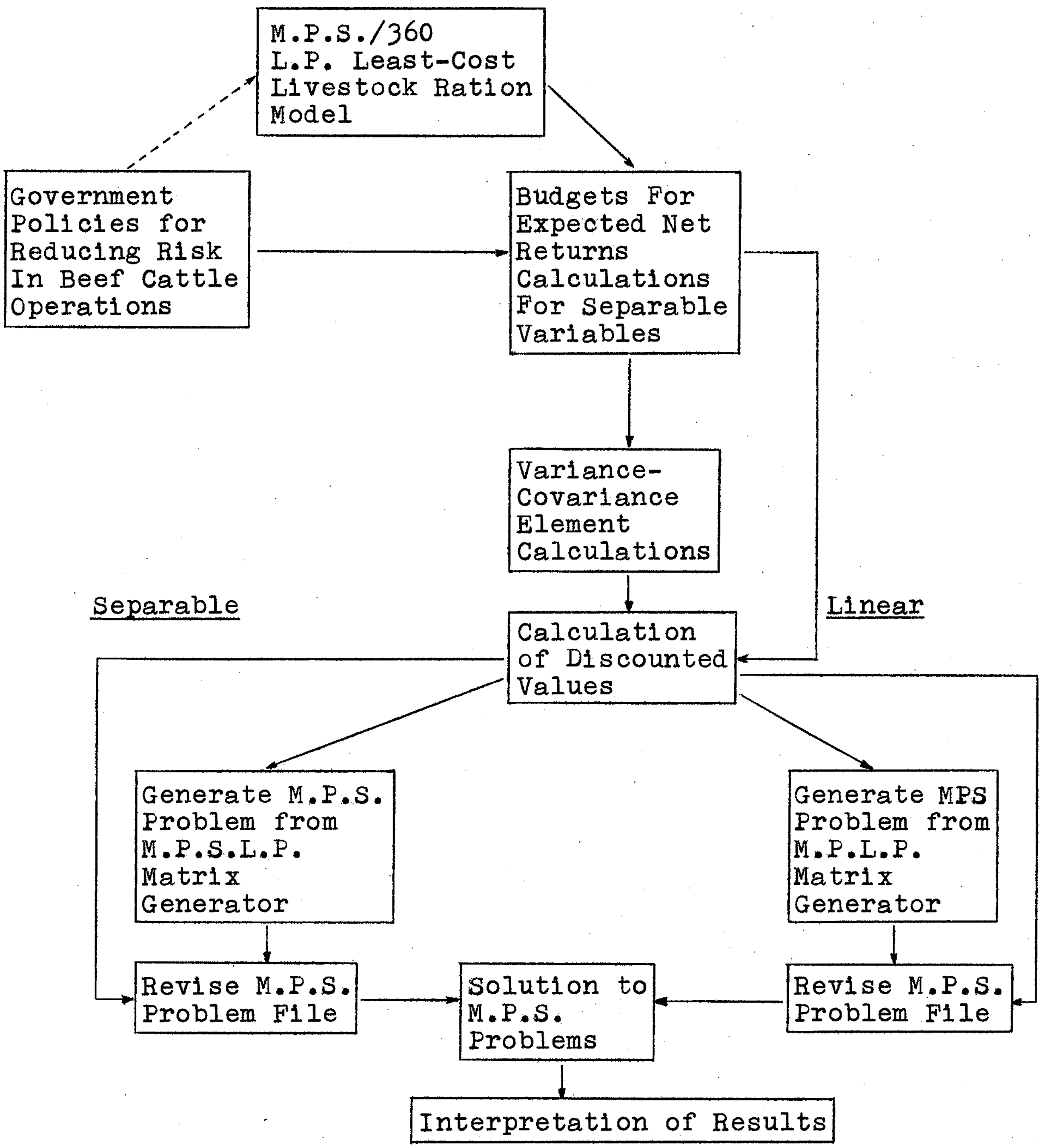

Figure 4

Computer Programmes Used in the Empirical Analysis

2. Method of Testing the Hypotheses and Presentation of Results

The empirical results obtained from the M.P.S.L.P. model comprise the portfolio analysis stage of portfolio selection theory by generating a set of alternative efficient farm enterprise portfolios along the E-V frontier shown in Figure 1, p. 30. Each point on the frontier represents one possible Pareto optimal portfolio for some given combination of the sum of expected annual net returns across the time horizon and the variance of those returns. Evaluation of the differences in constituents for alternative efficient portfolios along the same E-V frontier provides a method of testing the first three hypotheses outlined in Chapter I, p. 18. Testing the fourth hypothesis requires the generation of additional E-V frontiers, and the comparison of equivalent points between different frontiers. These methods of testing the hypotheses are explained in greater detail, and the empirical results presented in subsequent sections of this chapter.

2.1 The Importance of Accounting for Risk in Agriculture

The first hypothesis suggested that inclusion of risk measurements in analytical models has an important influence on enterprise choice and resource allocation. This hypothesis can be tested by comparing the L.P. equivalent solution with alternative efficient portfolios representing combinations of increasingly lower expected returns and variance in returns. The efficient portfolio associated with the equivalent L.P. solution is located at the apex of the E-V frontier, as shown in Figure 5. This particular solution represents the portfolio of farm enterprises which produces the maximum possible expected net returns for any level of variance in returns. That is, no constraint is placed on the total level of variance permissable, so that the objective function is maximized on the basis of complete disregard for risk level. This is equivalent to assuming all coefficients in the model are known with certainty, and hence is equivalent to the L.P. model solution.

Alternative efficient portfolios are obtained on the same E-V frontier by successively reducing the level of total variance permissable. The solutions obtained for various combinations of expected returns and variance in returns are shown in Figure 5. Note that the solution associated with a total variance constraint level of $750 million (point B in Figure 5) dominates the L.P. equivalent solution (point A), since the latter increases expected returns by only $2,234 (or a 1.4 percent increase), while variance is increased by $186,504 (or a 24.9 percent increase). Thus, for even a small degree of risk aversion for farmers in Manitoba Crop District Number Three, the preference for the risk model solution may be significantly great. The difference between solutions at points A and B in itself reveals the desirability of

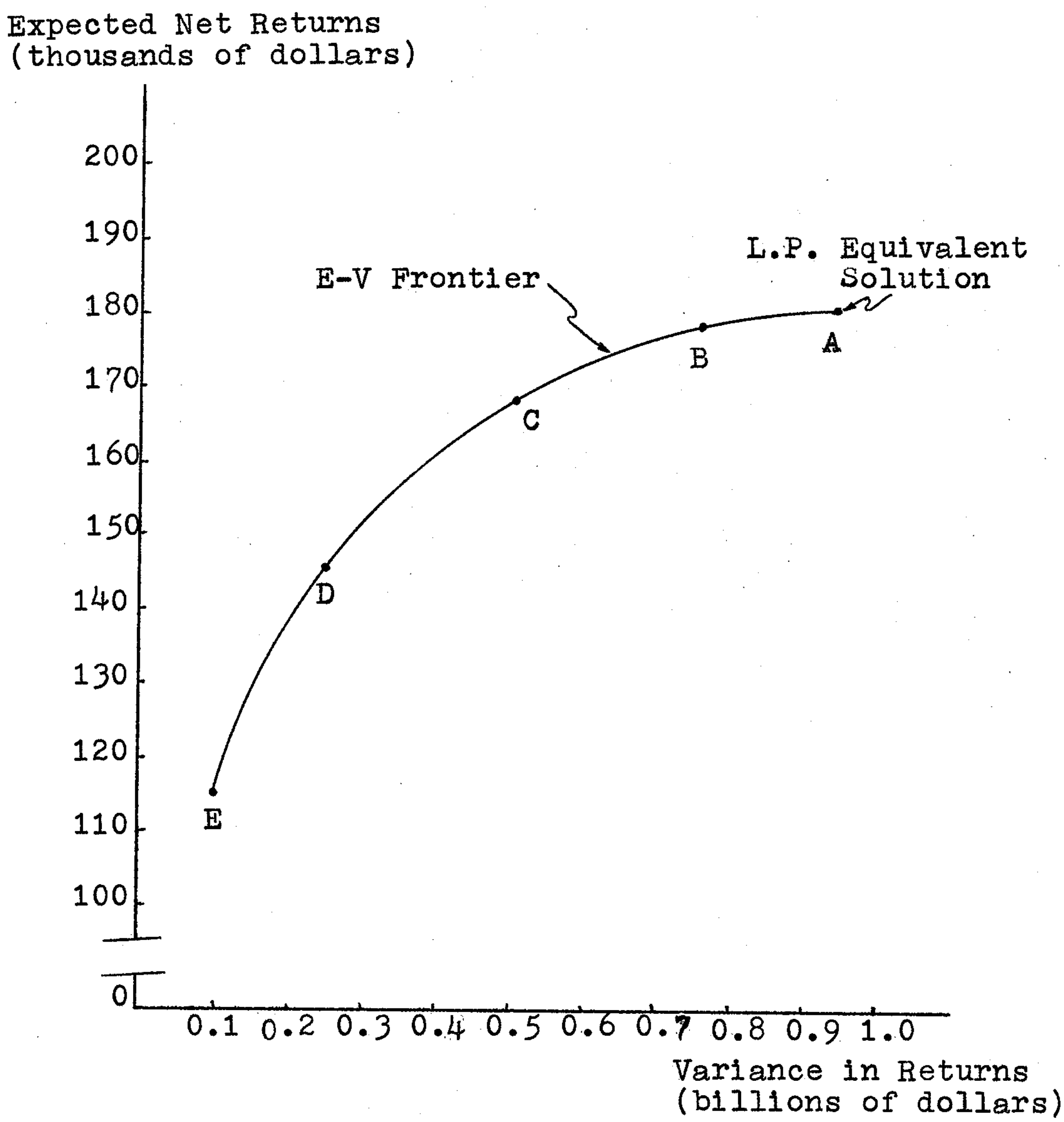

Figure 5

Efficient E-V Frontier Associated With Historical Levels of Beef Cattle and Cash-Crop Enterprise Risk in Manitoba Agriculture accounting for risk in analyses relating to agricultural problems.

The influence of risk on farm enterprise choice and resource allocation can be gleaned from Tables 17 to 21. These tables contain the efficient portfolios of farm enterprises associated with each E-V frontier point shown in Figure 5. The more significant points in these tables of results include the following:-

(i) The complete absence of both cow-calf and flaxseed enterprises from all efficient portfolios in all years indicates their dominance by the remaining six cattle and crop enterprises. This dominance phenomenon was alluded to in Chapter II, Section 4, where it was noted that both cow-calf and flaxseed enterprises were characterized by a low expected return and high variance combination during the period 1954 to 1973.

Had either of these two enterprises been forced (by minimum activity constraint levels) into the farm portfolios, the resulting solution for each risk constraint level would have been inferior (lying on a new, lower efficient frontier) to its corresponding point shown in Figure 5. That is, for the same constrained level of risk, a smaller amount of expected net returns would have been obtained. The opportunity cost of including either cow-calf or flaxseed activities in the solutions could therefore

Table 17

Efficient Annual Farm Enterprise Portfolios for the L.P. Equivalent (No Risk Constraint) Solution

Year	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
	(head)	(head)	(acres)	(acres)	(acres)	(acres)
1964	3	423	1240	-	-	648
5	3	466	1240	648	-	-
6	92	323	1240	-	434	215
7	5	609	1240	430	219	-
8	3	720	1240	-	-	648
9	3	720	944	-	-	945
70	3	720	944	-	-	945
1	58	720	1222	8	8	650
2	506	4	648	-	-	1240

Objective Function Value $182,145.68

Total Variance $936,504,927.10

Table 18

Efficient Annual Farm Enterprise Portfolios for a Constrained Variance Level of $0.75 billion

Year	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
	(head)	(head)	(acres)	(acres)	(acres)	(acres)
1964	1	647	1125	-	-	453
5	12	630	1239	6	164	169
6	181	360	1239	28	236	75
7	12	720	1239	167	167	6
8	12	720	1166	1	1	409
9	12	720	788	2	2	786
70	12	720	788	2	2	786
1	26	720	952	3	3	619
2	475	2	731	1	1	845
Objective Function Value			$179,912.18			

Table 19

Efficient Annual Farm Enterprise Portfolios for a Constrained Variance Level of $0.5 billion

Year	Beef Feeder	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
	(head)	(head)	(acres)	(acres)	(acres)	(acres)
1964	2	399	732	-	-	447
5	27	360	1052	54	20	54
6	125	203	1180	-	-	-
7	9	706	995	-	182	-
8	1	720	1175	1	1	2
9	2	717	662	-	-	517
70	2	717	662	-	-	517
1	86	583	845	7	7	320
2	449	3	661	-	-	517
Objective Function Value			$167,668.68			

Table 20

Efficient Annual Farm Enterprise Portfolios for a Constrained Variance Level of $0.25 billion

Year	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
	(head)	(head)	(acres)	(acres)	(acres)	(acres)
1964	11	341	394	5	169	394
5	11	341	783	5	5	169
6	75	163	630	8	165	158
7	9	430	777	1	182	1
8	13	425	515	8	157	281
9	11	427	394	5	169	394
70	9	431	558	4	4	394
1	25	406	558	4	4	394
2	275	5	558	4	4	394

Objective Function Value $145,551.36

Table 21

Efficient Annual Farm Enterprise Portfolios for a Constrained Variance Level of $0.10 billion

Year	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
	(head)	(head)	(acres)	(acres)	(acres)	(acres)
1964	4	206	322	7	7	281
5	12	165	281	6	165	165
6	25	134	382	18	169	49
7	11	293	439	5	169	5
8	25	294	394	15	40	169
9	25	229	328	9	25	256
70	75	229	323	8	8	280
1	109	184	394	15	40	169
2	125	32	321	7	9	281

Objective Function Value $115,200.98

be measured as the amount of expected net returns foregone for accepting some particular maximum constrained level of risk.

(ii) Much greater diversification of farm enterprise portfolios is evident for those solutions with lower maximum allowable variance (points D and E in Figure 5). Tables 20 and 21 show that for the two most constrained levels of variance, every enterprise was included in the efficient portfolio for each year in the time horizon. This may be indicative of the importance of the covariance terms (Table 7, p. 91) as a means of reducing total variance.

At the other extreme, the greatest amount of enterprise specialization is to be found in the annual portfolios for the L.P. equivalent solution. As Table 17 indicates, when risk is a non-constraining factor, specialization in wheat, beef feeders and slaughter cattle is extensive. On the other hand, rapeseed is excluded from the portfolio in two out of nine years, while barley and oats are each excluded from the portfolio in six out of nine years.

The spasmodic appearance of barley and oats over the time horizon, and the erratic behaviour of the beef feeder activity levels in different years can be explained only partly by the comparative risk aspects. Other possible explanatory factors include the comparative expected net returns levels, the type of relationship between enterprises (whether competitive, or complementary), and resource input scarcities. Reference to comparative expected net returns (see Table 5, p. 46 and Table 6, p. 50) suggests a close association between years of high net returns for beef feeders, barley and oats relative to the other enterprises, and years of high activity levels in the efficient portfolio solutions.

Secondly, a competitive relationship between beef feeders and slaughter cattle is suggested by the apparent inverse relationship of activity levels for these two enterprises over the time horizon (see Tables 17 to 21). A similar competitive relationship between the four cash-crops is apparent, and particularly between barley and oats on the one hand and rapeseed on the other. A complementary relationship between the two groups of beef cattle and cash-cropping enterprises is suggested by the relatively stable aggregate numbers of beef cattle and cash-cropping acres over the time horizon.

Thirdly, constraints on resource inputs may account in part for the distribution over time of activity levels within efficient alternative portfolios. In particular, management labour and medium- and long-term capital were found to have positive shadow prices (see Appendix E, Tables E-1 to E-5) on a consistent basis in the solutions. Availability of additional amounts of these scarce resources could alter the composition of the efficient portfolios without any adjustments being made to the comparative enterprise risk levels. Thus, the degree of diversification or specialization with efficient enterprise portfolios is dependent on (a) whether risk is accounted for, (b) the level of risk selected, and (c) other economic factors such as comparative expected net returns and types of relationship between enterprises.

(iii) Comparison of the L.P. equivalent solution (Table 17) with the dominant solution where total variance is constrained to a level of $0.75 billion (Table 18) reveals the most significant differences between disregarding and accounting for risk. Not only is the extent of enterprise diversification much higher in the dominant solution, but also there is evidence of a change in direction of specialization. The L.P. equivalent solution indicates greater emphasis on wheat and rapeseed, with wheat acreages surpassing those in the dominant solution in eight out of nine years, and rapeseed in seven out of nine years. Conversely, beef cattle tend to be more emphasized in the dominant solution. Slaughter cattle numbers are greater than those in the L.P. equivalent solution in four of the years, with no difference in another four years. Beef feeders have greater numbers in the dominant solution in six out of nine years.

From these observations, it can be said that significant differences appear in farm enterprise choice and resource allocation solutions, depending on whether risk is included, and on what level of risk is chosen. The actual degree of importance to be placed on including risk in models dealing with agricultural problems can only be assessed on the basis of individual farmers' attitudes towards risk. Since a survey of risk attitudes is beyond the scope of this study, the only claim made here is that one-parameter(or L.P.) models are appropriate only where attitudes to risk are known to be neutral. Where any degree of risk aversion is present, then a two-parameter (or risk) model would provide more informative solutions.

2.2 Comparison of Risk Levels in Beef Cattle and Cash-Crop Enterprises

Testing the hypothesis that the historical incidence of risk has been greater in beef cattle than in cash-cropping enterprises requires more than a simple comparison of enterprise variance or coefficient of variation statistics. These statistics, as mentioned in Chapter II, Section 4, do not by themselves provide a perfect index of risk, nor are the stochastic relationships between expected outcomes considered. In choosing enterprise types to make up efficient annual portfolios over some planning horizon, the decision-maker is concerned with total risk, including the contributions made by the covariance terms. Enterprise choice decisions are also influenced by the incidence of successive years of low expected returns or high expected returns [32, p. 457]. The empirical model, embodying both portfolio analysis and a multi-period framework to reflect enterprise net returns cycles, offers an appropriate technique for testing the hypothesis. This technique is particularly well suited for application to Crop District Number Three in Manitoba where both beef cattle and cash-cropping enterprises may occur on the same farm.

The method of testing is to compare alternative efficient portfolios (representing different points on the same E-V frontier) on the basis of their enterprise compositions. As the total variance level becomes increasingly constrained in the model, rational decision-making would suggest the selection of enterprises having not only lower variance levels, but also negative covariance terms. Thus, the relative contribution of each enterprise to total farm risk can be gauged from its changing proportions in alternative efficient portfolios along the E-V frontier. For the L.P. equivalent solution, where risk is non-constrained, enterprise choice decisions would be based on comparative expected returns alone, and extreme specialization may be expected. As total variance becomes more constrained, increasing diversification toward enterprises making the least contribution to total farm risk can be expected.

For greater ease and efficiency with which to apply this test criterion, a numeraire system is used to develop enterprise ratios within each efficient portfolio. To ensure that the enterprise ratios are measured in commensurable units, the ratios shown in Tables 22 to 26 are expressed in terms of expected net returns contributed by each enterprise in a given year. In choosing the numeraire, not all enterprises were found to be satisfactory. Barley, oats and rapeseed were not considered because of zero observations in some portfolios, thereby giving rise to meaningless ratios of infinity. The slaughter cattle enterprise was selected as numeraire, although beef feeders or wheat would have been equally suitable. It follows that slaughter cattle net returns are given a proportion of one in each efficient annual portfolio, and the proportions of all other enterprises are based on the ratios of their net returns to slaughter cattle net returns. This provides a system of weighting each enterprise within a portfolio, and therefore a method of judging its relative importance.

The ratios in Tables 22 to 26 indicate the following:-

(i) Cow-calf and flaxseed enterprises can be placed in the high risk category because of their complete absence from any efficient portfolio in any year. This result is obtained in spite of the negative (and in some cases considerable size of) covariance terms between each of cow-calf and flaxseed enterprises and several alternative enterprises (see Table 7, p. 91).

Table 22

Efficient Annual Portfolios of Farm Enterprise Ratios

for the L.P. Equivalent Solution

Year	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
(dollars of expected net returns)						
1964	*	1	3.95	*	*	2.49
5	*	1	4.76	2.04	*	*
6	.34	1	15.79	0.0	3.81	1.51
7	*	1	2.33	.39	.26	0.0
8	.02	1	11.85	*	*	5.54
9	*	1	.51	0.0	0.0	.73
70	*	1	.55	0.0	0.0	.84
1	.09	1	1.11	.01	.01	.61
2	581.5	1	878.58	1.08	1.0	2014.0

* Denotes a ratio of less than 0.01.

Table 23

Efficient Annual Portfolio of Farm Enterprise Ratios

for a Constrained Variance Level of $0.75 billion

Year	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
	(dollars of expected net returns)					
1964	*	1	2.35	*	*	1.14
5	*	1	3.51	.02	.37	.38
6	.6	1	14.18	.18	1.86	.47
7	*	1	2.01	.13	.17	*
8	.01	1	1.12	*	*	.35
9	.01	1	.43	*	*	.61
70	.01	1	.46	*	*	.69
1	.04	1	.86	*	*	.58
2	920.0	1	1702.0	2.86	4.29	2354.0

* Denotes a Ratio of less than 0.01

Table 24

Efficient Annual Portfolios of Farm Enterprise Ratios

for a Constrained Variance Level of $0.5 billion

Year	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
	(dollars of expected net returns)					
1964	*	1	1.64	*	*	1.82
5	.03	1	5.42	.23	.25	.22
6	.74	1	23.87	*	*	*
7	.08	1	29.86	.02	3.52	.02
8	*	1	1.12	*	*	*
9	*	1	.36	*	*	.40
70	*	1	.38	*	*	.46
1	.17	1	.94	.01	*	.37
2	668.0	1	1196.78	1.0	1.0	1121.78

*Denotes a ratio of less than 0.01

Table 25

Efficient Annual Portfolios of Farm Enterprise Ratios for a Constrained Variance Level of $0.25 billion

Year	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
	(dollars of expected net returns)					
1964	.01	1	1.56	.01	.42	1.88
5	.01	1	4.09	.02	.02	.71
6	.55	1	15.86	.12	2.88	2.18
7	.01	1	2.07	*	.31	*
8	.01	1	.83	.01	.14	.41
9	.02	1	.36	*	.07	.51
70	.02	1	.54	*	*	.59
1	.07	1	.89	*	*	.66
2	223.88	1	535.0	3.71	3.53	45.18

* Denotes a ratio of less than 0.01

Table 26

Efficient Annual Portfolios of Farm Enterprise Ratios for a Constrained Variance Level of $0.10 billion

Year	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
	(dollars of expected net returns)					
1964	*	1	2.1	.03	.03	2.21
5	.03	1	3.04	.06	1.43	1.43
6	.22	1	11.69	.31	3.56	.83
7	.09	1	13.21	.08	3.25	.06
8	.04	1	.92	.02	.05	.35
9	.07	1	.56	*	.02	.62
70	.03	1	.59	.01	*	.78
1	.66	1	1.39	.04	.09	.62
2	18.34	1	55.46	1.06	1.17	5.85

* Denotes a ratio of less than 0.01

It may be concluded that the combined contribution to total farm risk of variance and covariance terms for cow-calf and flaxseed enterprises is too great to warrant their inclusion in efficient portfolios. The continuous presence in practice of these enterprises in Crop District Number Three of Manitoba may be attributed to lack of decision-makers' knowledge of the degree of risk involved. An alternative explanation is that farmers are aware of comparative enterprise risk levels, but at the same time enough farmers are risk-takers. Since farmers' knowledge of and attitudes toward risk lie beyond the scope of the study, only hypothetical explanations are offered.

(ii) It is not possible to judge which of the cow-calf or flaxseed enterprises carries the higher risk on the basis of Markowitz efficient frontier analysis. This is because these enterprises would be included only in feasible but inefficient portfolios (below the frontier in Figure 5), and these are excluded by the Markowitz approach.

(iii) Barley and oats may be placed in the second highest category of risk. Their sporadic appearances in only some years of the L.P. equivalent solution (Table 22) suggests selection on the basis of occasional high expected net returns relative to other cash-crops competing for similar resources. Their negligible presence (proportions of less than 0.01) in many years of the risk-constrained solutions suggests their relative contribution to expected net returns is low, while their relative contribution to total variance is high. As risk becomes increasingly constrained, the higher portfolio proportions of barley and oats suggests that their negative covariance terms become increasingly more important than their high variances relative to competing enterprises. Scarcity of capital and management resources may also partly explain the intermittant appearance of barley and oats in solutions where risk is less constrained.

(iv) In the third highest category of risk may be placed beef feeders and rapeseed. Both these enterprises make fairly consistent appearances in the efficient annual portfolios. However some years of complete absence (zero proportion) or negligible presence (proportion of less than 0.01) serve to place them at an intermediate risk level between barley and oats on the one hand, and wheat and slaughter cattle on the other.

(v) Comparing feeder cattle and rapeseed, it appears tentatively that rapeseed carries thegreater risk loading. An indication of this is given by the tendency for the rapeseed proportions in most years to trend upwards as the total variance constraint is relaxed, while the feeder cattle proportions show either no trend or a slightly declining trend for increasingly higher levels of total variance. This shows that risk levels, and particularly covariance terms, are relatively more important when considering rapeseed for possible portfolio inclusion than it is for feeder cattle. This is confirmed by the positive covariance terms between rapeseed and all alternative enterprises except slaughter cattle, as shown in Table 7, p. 91.

(vi) The consistent inclusion of both slaughter cattle and wheat in all efficient annual portfolios suggests that these enterprises can be placed in the lowest risk category. This is further borne out be the relatively high proportions of both enterprises in the portfolios for all constraint levels on total variance, and for all years except slaughter cattle in 1972 (see particularly Tables 22, 23 and 24). The relatively small rising trend in proportions as total variance is permitted to increase suggests that both slaughter cattle and wheat enjoy both high expected annual net returns and relatively low contributions to total farm risk. The low contributions to risk can be gleaned from the relatively small wheat and slaughter cattle variances and the large proportion of negative or small positive covariances between these two and alternative enterprises (see Table 7, p. 91).

(vii) There appears a tendency for proportions of wheat to rise slightly faster than those for slaughter cattle as total variance constraint levels are relaxed. This tendency becomes most noticeable in comparing the change in proportions between Tables 22 and 23. The proportion of wheat increases significantly in relation to slaughter cattle when the total variance is changed from a constraint level of $0.75 billion (Table 23) to a non-constrained status (Table 22). This would suggest that wheat has a slightly higher risk loading than slaughter cattle.

From the above indications, it is possible to rank, at least on a tentative or first approximation basis, beef cattle and cash crop enterprises in order of risk loading. In descending order, starting with enterprises with heaviest risk loading, this tentative ranking is:- cow-calf and flaxseed (jointly), barley and oats (jointly) rapeseed, feeder cattle, wheat, and slaughter cattle. This ranking is claimed to be only a first approximation for two reasons. Firstly, the results were insufficiently definitive in many cases to be able to rank the enterprises categorically. For example, the differences in portfolio proportions between wheat and slaughter cattle, or between rapeseed and feeder cattle were too small in which to place any high level of confidence.

Secondly, the proportional distributions of enterprise within each efficient portfolio are a function not only of expected annual net returns and variance in returns, but also of the covariance terms. Inability to isolate the effects of sign and size of covariance terms can be interpreted as a shortcoming of Markowitz portfolio analysis as a tool for ranking farm enterprises on the basis of comparative risk loading.

Even without these difficulties in interpreting the results, the test on the hypothesis that there has historically been more risk in beef cattle than in cash-crop enterprises does not lend itself to the drawing on any definitive conclusions. This is because of the apparent discrepancies in risk loading between different segments of the beef cattle production chain. For the farmer operating the cow-calf segment alone, the results suggest there is more risk in beef cattle than in cash-crops. On the other hand, for the slaughter cattle operator, the opposite is indicated, while for feeder cattle, only wheat appears to be less risky. Thus, much depends on the beef cattle segment to which attention is being directed.

2.3 The Influence of Risk on Expansion of Beef Cattle Enterprises

To test the hypothesis that expansion of beef cattle enterprises is inhibited by risk more than expansion of cash-cropping enterprises, multi-period portfolio analysis can be employed to show efficient resource allocation over time. In a two-parameter model accounting for risk, additional beef cattle or cash-cropping production resources are acquired not only on the basis of the expected returns to those resources but also on the basis of the variance in expected returns and the covariances between each pair of returns. Thus the comparative rate of expansion over time of beef cattle and cash-cropping enterprises would be a reflection of the combined contribution made to the objective function value (increase in expected net returns) and to the total risk associated with any alternative efficient portfolio (due to the variance and covariance terms). If beef cattle enterprises can be shown to expand at a lesser rate than cash-cropping enterprises as risk becomes increasingly constrained, this would indicate that risk has more of an inhibitory effect on beef cattle than on cash-cropping enterprise expansion rates.

In Tables 27 to 30, the degree of expansion in durable farm assets, plus the extent of capital borrowing over the time horizon are portrayed for the four different levels of risk associated with the points A, B, C and D, respectively, on the E-V frontier in Figure 5. Note that the results in Tables 27 to 30 also correspond with and are part of the efficient portfolio solutions presented in Tables 17 to 20 respectively. For example, the efficient portfolio of farm enterprises shown in Table 17, and the pattern of durable asset acquisition and capital borrowing shown in Table 27, both refer to the L.P. equivalent solution (point A in Figure 5). No table of results is presented for resource acquisition and capital borrowing for the $0.10 billion total variance constraint level (point E in Figure 5), since no additional resources were purchased in this solution.

Included in Tables 27 to 30 are purchases of land and farm machinery to serve as a basis for comparing cash-crop enterprise growth. Also, land and machinery purchases

Table 27

Durable Asset Purchases and Capital Loans

for the L.P. Equivalent Solution

Year	Land	Livestock Housing	Capital Loans for Land	Capital Loans for Livestock Housing
	(acres)	(sq.ft)	(dollars)	(dollars)
1964	1270	855	104,965	2,855
5		536		1,840
6				
7		1831		6,887
8		1338		4,418
9				
70				
1		1115		4,259
2				
Total	1270	5675	104,965	20,259

Year	Tractor	Combine	Discer	Swather	Baler	Capital Loans for Machinery
		(number of machines)				(dollars)
1964	0.89	1.25	0.56	0.30		19,162
5					0.26	348
6					0.14	197
7						
8						
9	0.14		0.18			1,105
70						
1						
2	0.15		0.18			1,186
Total	1.18	1.25	0.92	0.30	0.40	21,998

Table 28

Durable Assets Purchases and Capital Loans

for a Constrained Variance Level of $0.75 billion

Year	Land	Livestock Housing	Capital Loans for Land	Capital Loans for Livestock Housing
	(acres)	(sq.ft)	(dollars)	(dollars)
1964	960	3628	79,344	12,059
5				
6				
7		1112		4,182
8				
9				
70				
1		290		1,109
2				
Total	960	5030	79,344	17,350

Year	Tractor	Combine	Discer	Swather	Baler	Capital Loans for Machinery
	(number of machines)					(dollars)
1964	0.54	0.88	0.25	0.09		9,187
5					0.05	76
6					0.11	149
7						
8						
9	0.16		0.20			1,238
70						
1						
2	0.02		0.03			238
Total	0.72	0.88	0.48	0.09	0.16	10,879

Table 29

Durable Asset Purchases and Capital Loans

for a Constrained Variance Level of $0.5 billion

Year	Land	Livestock Housing	Capital Loans for Land	Capital Loans for Livestock Housing
	(acres)	(sq.ft)	(dollars)	(dollars)
1964	560	546	46,284	1,894
5				
6				
7		3982		14,973
8				
9				
70				
1				
2				
Total	560	3982	46,284	16,867

Year	Tractor	Combine	Discer	Swather	Baler	Capital Loans for Machinery
		(number of machines)				(dollars)
1964	0.2	0.4				3,814
5						
6						
7						
8						
9	0.04		0.04			262
70						
1						
2						
Total	0.24	0.4	0.04			4,076

Table 30

Durable Asset Purchases and Capital Loans

for a Constrained Variance Level of $0.25 billion

Year	Land	Livestock Housing	Capital Loans for Land	Capital Loans for Livestock Housing
	(acres)	(sq.ft)	(dollars)	(dollars)
1964	343		28,371	
5				
6				
7		1085		4,081
8				
9				
70				
1				
2				
Total	343	1085	28,371	4,081

Year	Tractor	Combine	Discer	Swather	Baler	Capital Loans for Machinery
	(number of machines)					(dollars)
1964		0.14				1,031
5						
6						
7						
8						
9						
70						
1						
2						
Total		0.14				1,031

are relevant to beef cattle enterprises where at least some portion of cattle feeds are grown on the same farm. However, it should be noted that land and machinery purchases are directly influenced by crop enterprise risk levels, whereas the same purchases are affected only indirectly by beef cattle risk levels. This is because crop products can be sold off the farm for cash, or alternatively processed through livestock such as beef cattle.

On the other hand, acquisitions of livestock buildings are specific to beef cattle in this study. Buildings purchases therefore provide the principal indicator of the effects of beef cattle risk on enterprise expansion. By taking the ratio of capital investments in land and livestock housing to represent the ratio of expansion in cash-cropping and beef cattle enterprises, the effects of increasingly constraining risk on comparative enterprise growth rates can be measured.

From Tables 27 to 30, it should be noted that both durable asset acquisitions and capital borrowings (of all types) decline as total risk becomes increasingly constrained, and cease altogether when risk is constrained to the 0.1 billion dollar level. In particular, the ratio of cash-cropping to beef cattle enterprise expansion is 5.6 to 1 for the L.P. equivalent (no risk constraint) solution (Table 26), rising to 7.5 to 1 when risk is constrained to the 0.25 billion dollar level (Table 30).

These observations confirm the expected result that risk has a significant effect upon farm firm growth in general. In particular, the greater degree of cash-cropping enterprise expansion relative to beef cattle as risk is increasingly constrained suggests that risk has more of an inhibitory effect on the growth of beef cattle enterprises.

This conclusion is supported by an examination of the shadow prices (marginal value products) for those resources responsible for limiting expansion of the farm firm and individual enterprises.[1] For the L.P. equivalent solution (Table E-1), management labour is shown to be a limiting resource on an intermittent basis across the time horizon, while medium- and long-term capital borrowing are indicated as limiting resources consistently in all years. In most years, meduim-term capital carries a larger shadow price than long-term capital, indicating that the capacity to expand farm machinery (using medium-term capital) is more constraining than capacity to expand the land and livestock buildings base (using long-term capital loans). Comparing the shadow prices on land and livestock buildings would suggest greater potential for expanding returns to management and non-operating capital from growth in beef cattle than cash-cropping enterprises when risk is not a factor. This is indicated from the consistent appearance in all years of non-zero shadow prices for livestock housing, in constrast to the sporadic shadow prices for land.

1. See Appendix E, Tables E-1 to E-5.

When risk becomes increasingly constrained (Tables E-2 to E-5), the shadow prices indicate the following:-

(i) capital loans constrain farm growth more than management labour;

(ii) long-term capital borrowing becomes a more limiting constraint on growth than medium-term capital borrowing, suggesting that land and livestock housing purchases restrict farm growth more than farm machinery purchases;

(iii) there is an increasingly sparse appearance of non-zero shadow prices on livestock housing compared with land purchases as risk becomes more constraining. This implies that for the same additional dollar of variance in net returns accepted by the decision-maker, there is greater potential for procuring additional net returns to management and non-operating capital from investing in the principal cash-cropping resource input (land) than from investing in the principal beef cattle resource input (livestock housing). In turn, this implies that the risk attached to beef cattle enterprises has a greater inhibitory effect on growth of beef cattle enterprises than the risk in cash-cropping has on growth of cash-cropping enterprises.

Before leaving this discussion of the effects of risk on comparative enterprise growth rates, attention should be drawn to problems associated with purchases of indivisible resource inputs. Tables 27 to 30 indicate that purchases of fractional units of farm machinery resources are a consistent problem in all the alternative efficient portfolio solutions. This represents an inherent limitation of the empirical model, in that certain qualifications should be placed on the interpretation of results. Failure to impose integer value constraints on activity levels for machinery purchases possibly masked significant constraints on the potential growth rates of cash-cropping enterprises. This would have been reflected in higher shadow prices on medium-term capital borrowings used to finance machinery purchases, and most likely in lower growth rates of cash-cropping enterprises relative to beef cattle enterprises.

Integer and separable linear programming techniques were considered as alternative means of surmounting this indivisibilities problem, but were not adopted due to the degree of complexity already built into the empirical model. At the same time, the comparative risk level aspect would not have been affected, since variability in machinery operating costs were computed on a per-acre basis and would be the same for a fraction of a machine and for a whole machine. It is therefore maintained that comparative enterprise risk levels have a greater effect on expansion of beef cattle enterprises.

3. Government Programmes to Reduce Risk in Beef Cattle

It will be recalled from Chapter I that the Manitoba government is actively encouraging diversification into beef cattle operations, while at the same time, government programmes to reduce or compensate for risk in beef cattle have been ineffective. The 1958 Agricultural Stabilization Act was found to provide too low a level of support, did not cover all segments of the beef cattle industry, and failed to adopt a comprehensive approach to the beef risk problem (see Chapter I, Section 2.1). Alternative government programmes were therefore proposed, with the purpose of rectifying the shortcomings of the 1958 Stabilization Act. The objective is to find out what effect these proposed programmes have on the inclusion of beef cattle enterprises in efficient portfolios, compared with the results already obtained in Chapter V, Section 2. These alternative programmes are analyzed by means of the M.P.S.L.P. model for the period 1964 to 1972.

3.1 Programme to Stabilize Beef Cattle Gross Incomes

The first programme suggested in Chapter I, Section 2.1 called for the support of beef cattle prices at 100 percent of the previous five-year average. Support measures are applied to beef calves and feeder cattle, as well as slaughter cattle. The expected net returns under this programme are shown in Table 31 for cow-calf, feeder cattle and slaughter cattle enterprises for each year of the 1964

Table 31

Annual Net Returns for Manitoba Beef Cattle Enterprises Under the Proposed Gross Income Stabilization Programme 1964 to 1972

Year	Cow-Calf	Feeder Cattle	Slaughter Cattle
	(dollars per head)		
1964	-18.62	18.99	29.11
5	-23.46	6.41	14.62
6	- 4.18	6.32	5.24
7	- 2.50	6.78	24.69
8	- 6.87	13.54	30.31
9	29.63	17.66	15.45
70	32.28	23.00	27.98
1	21.06	39.11	34.85
2	39.05	24.54	4.31

to 1972 time horizon. These net returns may be compared with those based on historical data in Table 5, p. 46. The principal effect of the government programme is to raise net returns in 1964.

The modified variance and covariance elements due to this proposed programme are portrayed in Table 32. Comparing these elements with those in Table 7, p. 91, it should be noted that while variance levels for cow-calf and feeder cattle enterprises are reduced, the proposed programmed paradoxically increases variance for slaughter cattle. This anomaly can be explained largely by the significantly higher (supported) price for slaughter cattle in 1964, resulting in a new net return figure well above the nine-year average. Thus, even an Agricultural Stabilization Act designed to be more effective may transpire to be more of a price support scheme than a price stabilization measure. It should further be noted that while covariances between individual beef cattle enterprises are reduced, the proposed programme causes increases in covariances between beef cattle and cash crop enterprises (taking into account the signs on the covariance terms).

The effects of these changes (to the expected returns and variances and covariance coefficients) on the efficient portfolios of farm enterprises are shown in Tables 35 to 39 (pp.159-163,). These tables also illustrate the efficient portfolios of farm enterprises under the second proposed government programme to enable a direct comparison

Table 32

Variance and Covariance Elements for Expected Annual Net Returns Under the Proposed Gross Income Stabilization Programme for 1964

Enterprise Type	Cow-Calf	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Flaxseed	Rape seed
	(dollars per head)			(dollars per acre)				
Cow-Calf	545.83							
Beef Feeders	179.14	115.94						
Slaughter Cattle	-41.41	57.37	125.55					
Wheat	-75.24	-15.19	-16.11	48.71				
Barley	-59.57	1.92	-24.60	59.63	66.92			
Oats	-77.62	-21.99	-30.62	51.70	58.05	44.86		
Flaxseed	-91.39	-43.52	-27.61	47.96	47.93	44.95	78.92	
Rapeseed	44.33	53.63	- 6.50	24.24	43.74	22.99	34.62	66.82

to be made. Before evaluating these tables of results, however, the second proposed government programme is described.

3.2 Programme to Stabilize Beef Cattle Net Returns

The second programme proposed in Chapter I, Section 2.1 called for a system of indexing beef cattle product prices directly to operating expenses. This programme is designed to establish a basis for stabilizing annual net returns, and thus lay the foundation for a more comprehensive approach to reducing beef cattle risk levels. The programme indexes product prices for weaned calves, feeder cattle and slaughter cattle to the respective sets of operating expenses. Included in operating expenses are the costs of feeds, livestock inputs and other variable costs such as bedding materials and marketing expenses. Efforts to account for technical sources of risk in this programme were confounded by lack of data on technical variations in beef enterprises (see Chapter II, Section 3.2).

Having compiled a set of indexes for the three beef cattle enterprises in each of the years 1964 to 1972, the limits on a permissible range of index values were arbitrarily selected. For those years in which index values fell above (or below) the permissible range, the product prices were lowered (or raised) in order to bring the index values down to (or up to) the specified range limits.

For example, for the cow-calf enterprise, the index of historical beef calf prices to operating expenses for the period 1964 to 1972 was found to range between a low of 0.184 (in 1964) and a high of 0.296 (in 1970), with a mean average index of 0.247. The new index was arbitrarily selected to lie between the limits 0.225 and 0.275, so that the mid-point of the new index approximately equalled the mean average of the historical index. Invoking the new index limits required the upward adjustment of beef calf prices in two of the nine years, while downward adjustment was necessary in three of the years.

Similarly, the index of historical beef feeder prices to operating expenses for the period 1964 to 1972 ranged between a low of 0.147 and a high of 0.169, with a mean average index of 0.154. The new index was arbitrarily chosen to have limits of 0.150 and 0.160, so that again the mid-point of the new range approximated the mean average of the historical index. The historical index for slaughter cattle ranged between a low of 0.096 and a high of 0.106, with a mean average value of 0.102. The new index was arbitrarily allocated a lower limit of 0.100 and an upper limit of 0.105. Adopting these new indexes required feeder cattle prices to be increased in four years and reduced in one, while slaughter cattle prices were increased in three years and reduced in one.

It is assumed that increases in product prices would be achieved through the use of government deficiency payments. For years in which product prices decline, the difference between market price and indexed price would be paid by beef producers into a government administered fund. Each beef producer participating in the programme would be required to register with the government, and would be levied an arbitrary fee of one dollar per animal marketed, the proceeds to be used to help defray administrative expenses of the programme.

The merits of this comprehensive programme are three-fold. Firstly, both product price variability and operating cost variability are accounted for and related to each other within the one scheme, thereby opening possibilities for stabilizing net returns to beef cattle. Secondly, by including provision for product price declines in years of high market prices relative to operating costs, the peaks as well as the troughs in annual net returns tend to be moderated. This enhances the possibilities for reducing variance in annual net returns by a wider margin than in the first government programme proposed. Thirdly, by receiving funds in some years in addition to contributing funds in others, this government programme possesses the potential for being at least partially self-supporting financially. This should make it more publicly appealing than the first government programme proposed, which can be viewed as a direct government support scheme.

The set of annual expected net returns modified by the second programme are specified in Table 33. Comparison with the historical annual net returns data in Table 5, p. 46, shows that (apart from minor changes due to the one dollar per head levy) cow-calf returns were reduced under this programme in three years and raised in two. Beef feeder returns were raised in four years and reduced in one, while slaughter cattle returns were increased in three years and reduced in one. Note that no attempt was made to use this programme for complete elimination of negative net returns for cow-calf operations. The objective is to reduce variance in net returns rather than to supply direct financial support at a level sufficient to maintain positive net returns in every year.

The changes caused by the programme to the variance and covariance elements are recorded in Table 34. Comparison with the elements based on actual historical analysis in Table 7, p. 91, shows a significant decline in all beef cattle enterprise variances and in the covariance values between beef cattle enterprises, except for covariance between cow-calf and slaughter cattle. As expected, the covariances for most of the beef cattle and cash-crop enterprise combinations reveal increases, since beef product prices are related to feed input costs (i.e. cash-crop product prices) under this programme.

3.3 Efficient Portfolio Analysis Under the Proposed Government Programmes

To record the effects of the two proposed government

Table 33

Annual Net Returns for Manitoba Beef Cattle Enterprises

Under the Proposed Net Returns Stabilization Programme

1964 to 1972

Year	Cow-Calf	Feeder Cattle	Slaughter Cattle
		(dollars per head)	
1964	- 8.96	7.56	16.95
5	-10.96	8.41	16.04
6	- 4.18	9.88	18.16
7	- 2.50	10.35	24.69
8	- 6.87	13.54	27.25
9	23.63	17.66	15.45
70	21.60	23.00	27.98
1	21.06	25.38	34.85
2	27.65	24.54	25.98

Table 34

Variance and Covariance Elements for Expected Annual Net Returns Under the Proposed Net Returns Stabilization Programme for 1964

Enterprise Type	Cow-Calf	Beef Feeders	Slaughter Cattle	Wheat	Barley	Oats	Flaxseed	Rapeseed
	(dollars per head)			(dollars per acre)				
Cow-Calf	262.25							
Beef Feeders	123.46	51.93						
Slaughter Cattle	53.75	38.95	45.08					
Wheat	-49.69	-15.35	4.88	48.71				
Barley	-33.24	- 4.32	9.23	59.63	66.92			
Oats	-49.47	-18.07	-4.73	51.70	58.05	44.86		
Flaxseed	-72.41	-25.36	-9.21	47.96	47.93	44.95	78.92	
Rapeseed	41.92	24.25	10.56	24.24	43.74	22.99	34.62	66.82

programmes on efficient farm enterprise portfolios necessitates the generation of two new E-V frontiers using the modified expected annual net returns data (Tables 31 and 33) and the modified variance-covariance elements (tables 32 and 34). The empirical results from the alternative efficient portfolios on these new frontiers provide the basis for testing the fourth hypothesis. This hypothesis suggested that modifications to the provisions for beef cattle in the 1958 Agricultural Stabilization Act would make a significant difference to allocation of resources between beef cattle and cash-cropping enterprises in Manitoba agriculture.

To provide an accurate basis for testing this hypothesis, it is necessary for efficient portfolio solutions to be generated for identical total variance constraint levels to those used in the historical analysis (Chapter V, Section 2). The E-V frontiers associated with each of the two proposed government programmes are illustrated in Figure 6. The frontier derived from the historical data on expected net returns, and variance-covariance elements (see Figure 5, p. 120) is also included for purposes of comparison. It is important to note that any differences between the original E-V frontier and the two new ones may be attributed to changes both in expected returns and in variance and covariance terms.

The alternative efficient portfolio of farm enterprises produced under each of the proposed government

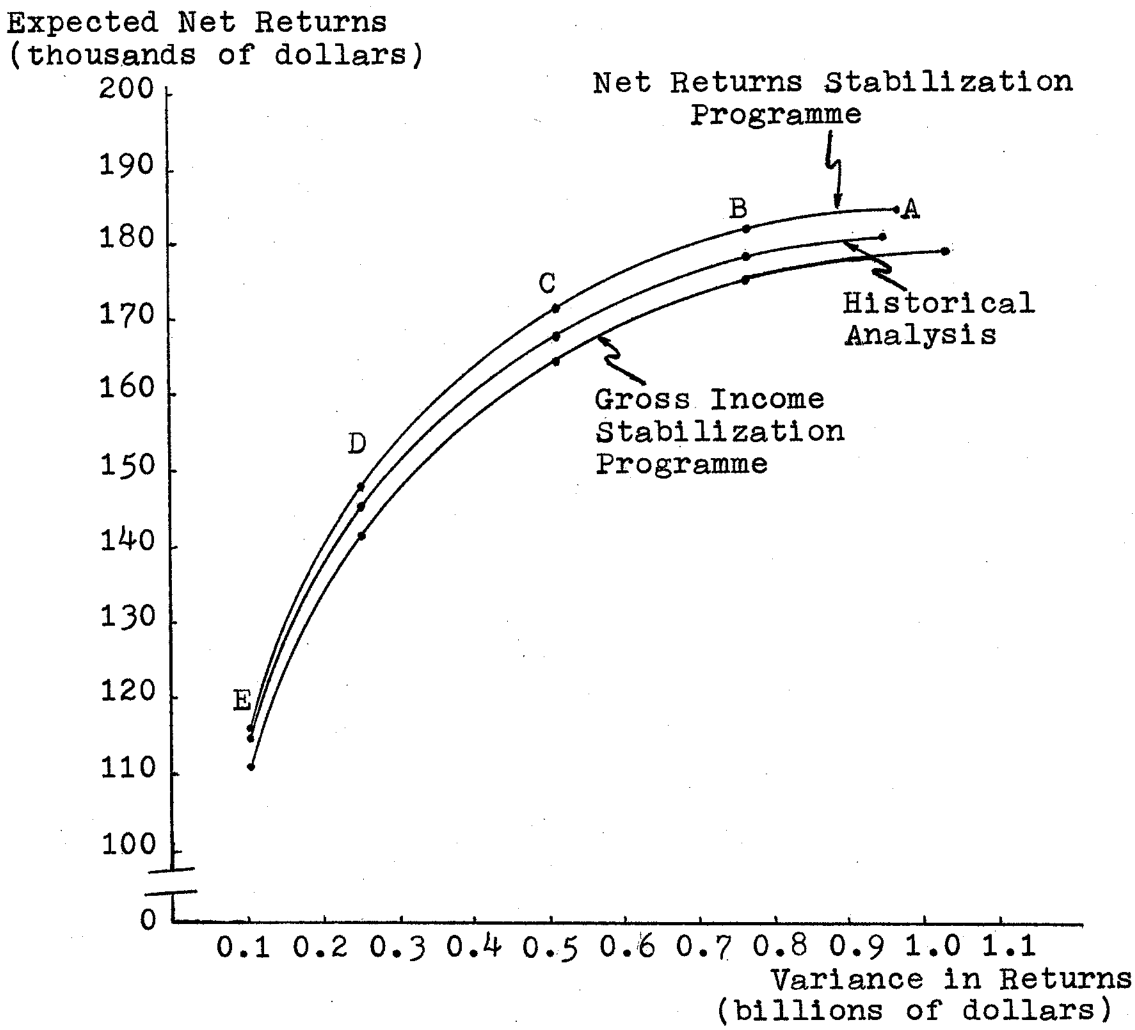

Figure 6

Efficient E-V Frontiers Associated With Historical Levels of Risk and With Proposed Government Gross Income and Net Returns Stabilization Programmes

programmes are depicted in Tables 35 to 39. Also included for ease of comparison are the efficient portfolio solutions obtained from using the historical data on expected returns and variance-covariance elements. Each table shows the three solutions for a different total variance constraint level, starting with the L.P. equivalent (no risk constraint) solution in Table 35. The tables specify only those enterprises for which positive values were indicated in any of the alternative efficient portfolios.

From these tables of empirical results, the following observations can be made:-

(i) Neither of the proposed government programmes would result in bringing cow-calf enterprises into any of the alternative efficient portfolios. This may be the result of expected returns that are still too low, or risk levels that are still too high, or a combination of these two. In any case, the exclusion of cow-calf operations is interpreted as continued dominance by other farm enterprises in spite of the government programmes. This conclusion, it should be noted, is made only for Crop District Number Three in Manitoba. The presence of cow-calf enterprises in other crop districts in Manitoba (already discussed in Chapter II, Section 4), may reflect entirely different comparative enterprise risk levels, or different production opportunities. On the other hand, the existence of cow-calf operations in Crop District

Table 35

Efficient Annual Farm Enterprise Portfolios for the L.P. Equivalent Solutions Under Alternative Government Programmes

Year	Government Programme[1]	Feeder Cattle	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
		(head)	(head)	(acres)	(acres)	(acres)	(acres)
1964	a	3	423	1240	-	-	648
	b	-	549	1239	-	-	476
	c	-	540	1239	-	-	487
1965	a	3	466	1240	648	-	-
	b	2	601	1239	-	237	237
	c	1	594	1239	-	243	243
1966	a	92	323	1240	-	434	215
	b	20	572	1239	23	345	107
	c	1	655	1239	243	243	-
1967	a	5	609	1240	430	219	-
	b	2	720	1239	237	237	-
	c	1	720	1239	243	243	-
1968	a	3	720	1240	-	-	648
	b	2	720	1239	-	-	475
	c	1	720	1239	-	-	487
1969	a	3	720	944	-	-	945
	b	382	111	1186	-	-	529
	c	381	112	1239	-	-	487
1970	a	3	720	944	-	-	945
	b	2	720	528	-	-	1186
	c	1	720	2	484	-	1239
1971	a	58	720	1222	8	8	650
	b	99	720	1239	-	-	475
	c	1	720	1239	-	-	487
1972	a	506	4	648	-	-	1240
	b	550	-	476	-	-	1239
	c	1	720	487	-	-	1239

Objective Function Values	a	$182,145	Total Variance Level	a $936,504,927
	b	$180,358		b $1,002,270,050
	c	$183,646		c $944,619,301

1. a denotes Historical Analysis
   b denotes Gross Income Stabilization Programme
   c denotes Net Returns Stabilization Programme

Table 36

Efficient Annual Farm Enterprise Portfolios for a Total Variance Constraint Level of $0.75 billion Under Alternative Government Programmes

Year	Government Programme[1]	Feeder Cattle	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
		(head)	(head)	(acres)	(acres)	(acres)	(acres)
	a	1	647	1125	-	-	453
1964	b	1	720	937	-	-	456
	c	-	719	1020	1	1	463
	a	12	630	1239	6	164	169
1965	b	56	632	1239	26	56	71
	c	19	720	1239	22	111	111
	a	181	360	1239	28	236	75
1966	b	225	163	1234	18	123	18
	c	19	720	1239	22	111	111
	a	12	720	1239	167	167	6
1967	b	17	693	1234	18	123	18
	c	19	720	1239	111	111	22
	a	12	720	1166	1	1	409
1968	b	1	720	937	-	-	456
	c	19	720	1082	2	2	397
	a	12	720	788	2	2	786
1969	b	383	109	856	1	1	535
	c	401	108	750	1	1	732
	a	12	720	788	2	2	786
1970	b	3	716	770	2	2	619
	c	19	720	747	2	2	732
	a	26	720	952	3	3	619
1971	b	89	579	856	1	1	535
	c	19	720	869	2	2	610
	a	475	2	731	1	1	845
1972	b	449	3	732	-	-	660
	c	19	720	604	2	2	875

Objective function Values	a	$179,912
	b	$176,165
	c	$183,230

1. a denotes Historical Analysis
   b denotes Gross Income Stabilization Programme
   c denotes Net Returns Stabilization Programme

Table 37

Efficient Annual Farm Enterprise Portfolios for a Total Variance Constraint Level of $0.5 billion Under Alternative Government Programmes

Year	Government Programme[1]	Feeder Cattle	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
		(head)	(head)	(acres)	(acres)	(acres)	(acres)
1964	a	2	399	732	-	-	447
	b	1	559	710	-	-	455
	c	2	719	668	-	-	448
1965	a	27	360	1052	54	20	54
	b	44	490	975	14	38	137
	c	25	682	958	25	67	67
1966	a	125	203	1180	-	-	-
	b	175	144	1165	-	-	-
	c	2	720	1115	-	-	-
1967	a	9	706	995	1	182	1
	b	9	546	980	1	182	1
	c	9	707	931	1	182	1
1968	a	1	720	1175	1	1	2
	b	2	556	870	1	1	293
	c	3	720	831	1	1	281
1969	a	2	717	662	-	-	517
	b	281	112	671	-	-	494
	c	386	105	604	2	2	507
1970	a	2	717	662	-	-	517
	b	2	557	658	-	-	507
	c	4	717	604	2	2	507
1971	a	86	583	845	7	7	320
	b	92	425	732	-	-	433
	c	2	720	662	-	-	454
1972	a	449	3	661	-	-	517
	b	355	3	657	-	-	507
	c	4	720	507	2	2	604

Objective Function Values
a $167,669
b $162,594
c $172,389

1. a denotes Historical Analysis
   b denotes Gross Income Stabilization Programme
   c denotes Net Returns Stabilization Programme

Table 38

Efficient Annual Farm Enterprise Portfolios for a Total Variance Constraint Level of $0.25 billion Under Alternative Government Programmes

Year	Government Programme[1]	Feeder Cattle	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
		(head)	(head)	(acres)	(acres)	(acres)	(acres)
1964	a	11	341	392	5	169	394
	b	3	353	439	-	-	439
	c	6	349	419	-	-	419
1965	a	11	341	783	5	5	169
	b	27	294	687	21	56	115
	c	12	360	507	7	162	162
1966	a	75	163	630	8	165	158
	b	75	98	746	21	56	56
	c	5	460	619	2	212	6
1967	a	9	430	777	1	182	1
	b	9	344	964	1	182	1
	c	9	453	654	1	182	1
1968	a	13	425	515	8	157	281
	b	4	352	561	4	4	309
	c	3	463	551	3	3	281
1969	a	11	427	394	5	169	394
	b	163	98	554	5	5	313
	c	233	95	507	7	7	318
1970	a	9	431	558	4	4	394
	b	6	349	349	7	7	515
	c	9	454	394	3	3	439
1971	a	25	406	558	4	4	394
	b	40	294	553	6	6	314
	c	2	621	542	2	2	293
1972	a	275	5	558	4	4	394
	b	221	6	413	1	1	463
	c	9	720	394	3	8	433

Objective Function Values
a $145,551
b $142,114
c $148,816

1 a denotes Historical Analysis
b denotes Gross Income Stabilization Programme
c denotes Net Returns Stabilization Programme

Table 39

Efficient Annual Farm Enterprise Portfolios for a Total Variance Constraint Level of $0.10 billion Under Alternative Government Programmes

Year	Government Programme[1]	Feeder Cattle	Slaughter Cattle	Wheat	Barley	Oats	Rapeseed
		(head)	(head)	(acres)	(acres)	(acres)	(acres)
1964	a	4	206	322	7	7	281
	b	41	292	315	6	16	281
	c	4	168	322	7	7	281
1965	a	12	165	281	6	165	165
	b	12	163	281	6	165	165
	c	75	229	283	8	169	158
1966	a	25	134	382	18	169	49
	b	27	98	361	21	169	56
	c	27	316	327	21	169	56
1967	a	11	293	439	5	169	5
	b	16	176	394	12	169	32
	c	11	341	394	5	169	5
1968	a	25	294	394	15	40	169
	b	25	229	394	5	14	169
	c	11	341	328	9	9	169
1969	a	25	229	328	9	25	256
	b	125	48	326	11	11	178
	c	219	9	312	5	15	169
1970	a	75	229	323	8	8	280
	b	75	163	324	8	13	181
	c	77	235	319	7	7	169
1971	a	109	184	394	15	40	169
	b	122	163	337	8	12	169
	c	11	341	315	6	11	169
1972	a	125	32	321	7	9	281
	b	80	7	328	9	9	180
	c	6	397	293	1	2	205

Objective Function Values		
	a	$115,201
	b	$110,524
	c	$115,913

1. a denotes Historical Analysis
   b denotes Gross Income Stabilization Programme
   c denotes Net Returns Stabilization Programme

Number Three, in the face of the empirical model results, may be accounted for by decision-makers' lack of knowledge of comparative enterprise risk levels.

(ii) Both programmes would give rise to increases in the portfolios of feeder or slaughter cattle enterprises, relative to cash-crop enterprises. This would apply to the portfolios for most risk constraint levels and for most years in the time horizon. This result may be attributed to a combination of increased returns for beef cattle, reductions in variances for beef (except for slaughter cattle variance under the gross income stabilization programme), reduction in the covariance term between feeder and slaughter cattle, and increases in the covariance terms between beef and cash-cropping enterprises. It would not be possible to isolate the individual contributions made by each of these factors, nor are estimates of these individual effects essential to the testing of the hypothesis. The net effect of both programmes would be to induce reallocation of resources from cash-cropping to beef cattle enterprises when applied to farms in Manitoba Crop District Number Three.

(iii) An evaluation of the shadow prices for those resources most limiting farm growth (see Appendix E, Tables E-1 to E-5) suggests confirmation of the observations made in (ii) above. The two government programmes would affect the shadow prices for land and livestock housing to the greatest extent, and the shadow price for medium-term capital borrowing to a somewhat lesser degree. As risk becomes increasingly constrained, a trend is tentatively suggested towards fewer and smaller non-zero shadow prices on land purchases, smaller shadow prices on medium-term capital borrowings, and larger and more frequent non-zero shadow prices on livestock housing purchases. These trends appear more consistent in the case of the net returns stabilization programme.

The implications of these shadow price changes are that for an increase of a dollar of net returns variance accepted by the decision-maker, the returns to management and non-operating capital from investing in beef cattle resources would become larger relative to returns from investing in cash-cropping resources. That is, the imposition of the government programmes would suggest a reduction in the risk associated with beef cattle relative to cash-cropping enterprises. This is more clearly indicated for the case of the net returns rather than the gross income stabilization programme.

(iv) The comprehensive programme aimed at stabilizing expected annual net returns is judged to be more effective in increasing beef enterprise proportions than the gross income stabilization programme. This may be due to the relative differences made by the two programmes to any or all of the expected returns to beef cattle, the variance levels in beef cattle, and the covariance terms involving beef cattle. Again the isolated effects are considered subordinate to the overall net effect. The implications for policy are that a net returns stabilization programme would be more effective in encouraging more beef production when applied to farms in Manitoba Crop District Number Three.

The general conclusion drawn from the above observations is that the proposed government programmes are effective in increasing the proportions of slaughter and feeder cattle enterprises in alternative efficient portfolios, but have no effect on expanding cow-calf operations. The hypothesis that the proposed government programmes significantly affect the allocation of resources between beef cattle and cash crop enterprises can therefore be accepted only with qualifications.

4. <u>Treatment of Fixed Costs in the Empirical Results</u>

The annual costs in the empirical model that are assumed fixed, no matter what constraint level of risk is imposed, include consumer expenditures, repayment of debts outstanding at the beginning of the time horizon, depreciation on the set of assets owned at the beginning of the time horizon, and overhead expenses. Since these fixed costs affect the objective function values, but do not affect the variance of net returns values, the fixed costs are not deducted from the objective function values shown in the tables of empirical results (in the preceding two sections). In this way each point on the three E-V frontiers generated is correctly evaluated in terms of a particular combination of expected net returns and the variance in net returns.

Put another way, had the total of annual fixed costs been deducted, the objective function values would then have been measured in units of expected net profits (net returns minus fixed costs). The latter would not have been directly comparable with variance in net returns.

This treatment of fixed costs is not meant to imply that these costs can be ignored. In fact, they should be included in any multi-period analysis, as advocated by Baker [2], and they form an integral part of the empirical model in this study.

The interpretation of the empirical results should then be modified to take account of the fixed costs, once the evaluation of the Markowitz efficient portfolio analysis has been completed. The total annual fixed costs, discounted to present value, amount to $113,106.40 (as shown in Appendix D). Deduction of this amount from each efficient portfolio solution would have lowered the three E-V frontiers (as measured by the vertical axis in Figure 6, p. 157), by an equal amount at every point. This is because the total fixed costs apply to every solution in an identical way, regardless of risk constraint level or whether a government programme was involved.

To illustrate the complete interpretation of the empirical results, the L.P. equivalent solution under the proposed government net returns stabilization programme recorded an objective function value of \$183,646.08 (Table 35, p.159). The standard deviation of this value, calculated from the square root of the sum of variances (or \$944,619,301) is \$30,734.65. The objective function value should then read as follows:-

$$\$183,646.08 \pm \$30,734.65 - \$113,106.40,$$

thus providing a net profit confidence interval of from \$39,805.03 to \$102,274.33 for one standard deviation. That is, for a farm decision-maker who has a risk neutral attitude, the probability is 0.6827 that net profit will fall within the interval \$39.805.03 to \$102,274.33 if the efficient annual portfolios of farm enterprises (shown in Table 35, rows c) are adopted as the nine-year farm plan.

This completes the explanation on the building and testing of the empirical model, the use of the model for testing the hypotheses, and the tabulation and interpretation of the empirical results. The final chapter reviews the study in terms of its achievement of the stated objectives, its contributions and its limitations. Some suggestions are also given for areas of further research.

CHAPTER VI

SUMMARY AND CONCLUSIONS

1. Achievement of the Stated Objectives

The first objective specified in this study was to develop a suitable methodological framework for combining multi-period analysis with the economic theory of risk in order to assess the influence of risk on Manitoba's beef cattle industry. In Chapter II, Section 2, economic theories pertaining to decision-making under imperfect knowledge were outlined, and it was indicated that possibilities exist for applying Markowitz portfolio selection theory to enterprise choice and resource allocation problems in agriculture. In particular, it was suggested that useful information can be obtained from the security analysis and portfolio analysis phases of portfolio selection theory in terms of generating Markowitz E-V frontiers of alternative efficient (Pareto optimal) portfolios of farm enterprises.

In Chapter II, Section 6, a theory of risk in a multi-period setting was developed on the basis of Hirschliefer's theory of investment choices over time, under a regime of imperfect knowledge of investment outcomes. Several reasons were stipulated for the desirability of analyzing risk problems in agriculture within a multi-period framework. This approach was found to be particularly desirable for analyzing risk problems related to beef cattle, for which a fairly definite cycle of expected net returns was established in Chapter II, Section 4.

From these economic theory foundations, several alternative empirical techniques were considered in Chapter III, on the basis of suitability for applying multi-period portfolio analysis to risk problems in agriculture. These techniques included multi-period quadratic and multi-period separable programming techniques and simulation. Of these, multi-period separable programming techniques were considered most appropriate for this study. Simulation techniques, while flexible and efficient, do not necessarily generate the required Pareto optimal efficient portfolios on the E-V frontier. Quadratic programming techniques were discounted because of their poor adaptability to a multi-period framework of analysis, and because of the relatively inefficient and costly computer solution algorithms currently available compared with those for separable programming techniques.

Problems of lack of primary (farm level) data were discussed in Chapter I, Section 1 and Chapter IV, Section 1. This lack of data prevented the empirical application of the model to analysis of individual farm risk problems as originally intended. The empirical model was then adapted to use a "representative" or "typical" farm approach based on secondary sources of data. This approach was found to be amenable to analysis of risk problems over time, as shown in the empirical results in Chapter V, but failed to account for differences in risk levels between farms. Therefore the methodological approach used in this study claims to offer only a first approximation or partial analysis of the effects of risk on enterprise choice and resource allocation on farms in Manitoba Crop District Number Three.

The second objective was to delineate the principal sources of risk associated with beef cattle and cash-cropping enterprises in Manitoba Crop District Number Three. These sources of risk were developed in Chapter II, Section 3, under the separate component headings of market, technical, technological and institutional risk. Each of these sources of risk was discussed in terms of their general applicability to agriculture, with examples provided in each case. For purposes of this study, emphasis was placed on delineating market and technical sources of risk as these affect Manitoba beef cattle and cash-cropping enterprises. Both technological and institutional sources of risk were not considered in order to simplify somewhat a problem already complex enough with two sources of risk included.

The third objective concerned the derivation of appropriate quantitative measures of risk levels within individual farm enterprises, and a measure of the contribution to total risk from combining any pair of enterprises together. Individual enterprise risk measures were developed in Chapter II on the basis of the two parameters expected annual net returns and the variance in annual net returns. The former parameter provides a measure of central tendency, a net returns figure which the decision-maker might reasonably expect to obtain, while the variance parameter provides a measure of dispersion around the expected returns figure, or a measure of risk.

Quantitative measures of some important component sources of risk were provided in Chapter II, Section 3, for individual beef cattle and cash-crop enterprises. These included measures of technical and market sources of risk in cash-crop enterprises, and of market sources of risk in beef enterprises. (No data could be found for estimating technical sources of risk in beef cattle).

In Chapter II, Section 4, quantitative measures of compound risk were specified by individual farm enterprises on the basis of expected annual net returns and variance in returns. The compound measure of risk was designed to combine individual sources of risk within a single comprehensive measure, thereby accounting for any additivity between risk components.

Contributions to risk are also made by combining pairs of enterprises. Covariance terms provide a measure of the stochastic relationship between any pair of farm enterprises, or the way in which expected annual net returns from two enterprises vary together. The complete set of variance-covariance elements for three beef cattle and five cash-crop enterprises were estimated in Chapter IV, Section 4.

The calculation of these elements represents the security analysis phase of portfolio selection theory.

The fourth objective concerned the testing of four hypotheses dealing with various aspects of the importance and influence of risk in Manitoba agriculture. The method of testing the hypotheses was to use the multi-period separable programming model (described in Chapter IV) to generate alternative efficient farm enterprise portfolio solutions for different constraint levels on total risk, and to compare these solutions for difference in portfolio content.

The first hypothesis, that enterprise choice and resource allocation decisions in Manitoba agriculture are influenced by risk, was tested by comparing the linear programming(L.P.) equivalent solution with alternative risk-constraining solutions on the same E-V frontier. The empirical results in Chapter V, Section 2.1, indicated that significant differences appeared in choice of enterprises, allocation of resources, and objective function values obtained depending on whether risk was accounted for, and on the level of risk constraint imposed. Furthermore, the solution associated with the highest risk constraint level ($0.75 billion) was found to dominate the L.P. equivalent (no risk constraint) solution adjacent to it on the E-V efficient frontier, since the L.P. solution resulted in a smaller increase in objective function value relative to increase in risk.

The second hypothesis suggested that the historical incidence of risk in Manitoba Crop District Number Three beef cattle enterprises has been greater than in cash-crop enterprises. This was tested in Chapter V, Section 2.2. The results proved to be somewhat inconclusive. Cow-calf enterprises were shown to have a higher risk level than any crop enterprise with the possible exception of flaxseed. Conversely beef feeder enterprises were shown to have a higher risk loading only than wheat, while slaughter cattle were tentatively placed in the lowest risk category. Thus the hypothesis can be accepted if cow-calf enterprises are considered, but rejected if feeder or slaughter cattle are the basis for comparison.

The third hypothesis, that beef cattle enterprise expansion has been inhibited by risk more than cash-cropping enterprises, could not be rejected on the basis of the empirical results in Chapter V, Section 2.3. These results showed the ratio of cash-cropping to beef cattle enterprise expansion rose from 5.6 to 1 for the L.P. equivalent solution to 7.5 to 1 when risk was constrained to the 0.25 billion dollar level. This implied a greater potential for increasing net returns by expanding cropping rather than beef enterprises for an additional dollar of risk, so that risk inhibited beef enterprise growth more than cash-cropping.

The fourth hypothesis concerned the testing of two amended versions of the 1958 Agricultural Stabilization Act to see if these affected efficient enterprise choice and resource allocation in Manitoba Crop District Number

Three over the 1964 to 1972 time horizon. The first proposed government programme emphasized gross income stabilization in beef cattle enterprises, through supporting product prices with deficiency payments. This programme resulted in higher expected net returns, reduced variance in returns to cow-calf and feeder cattle enterprises, increased variance in returns to slaughter cattle, smaller covariances between beef enterprises, and larger covariances between beef cattle and cash-cropping enterprises. The net effects of all these factors due to the programme were an increase in feeder cattle, a smaller increase in slaughter cattle and some reduction in cash-crops. There was no effect recorded on cow-calf enterprises, which were not selected in any of the efficient annual portfolios at any level of risk constraint.

The second proposed programme concentrated on stabilization of net returns in beef cattle enterprises, by relating product price variability to resource cost variability. This programme raised net returns in years of low returns historically, and conversely lowered net returns in years of high returns historically. Variance levels were reduced for all beef enterprises, covariances between beef enterprises declined, and covariances between beef and cash-crop enterprises increased. The net effects of these factors were an increase in slaughter cattle enterprises, and smaller increases in feeder cattle enterprises, both of these at the expense of cash-crop enterprises. The second programme also had no effect on cow-calf enterprises.

Based on these conclusions, the fourth hypothesis can be accepted as a true statement under certain conditions. The proposed programmes were effective in reallocating resources away from cash-cropping and into beef cattle, but only into the feeder and slaughter cattle segments of the beef production industry.

On the other hand, it was noted in Chapter V, Section 3.3, that cow-calf enterprises continued to operate in Crop District Number Three in spite of higher risk levels found in this study. One explanation suggested was that decision-makers may not be aware of comparative risk levels between cow-calf and alternative enterprises. This implies a less pressing need for government stabilization programmes for cow-calf enterprises in Manitoba Crop District Number Three.

Given these circumstances, the conditional acceptance of the hypothesis related to the proposed government stabilization programmes takes on added significance. The government objective of expanding Manitoba's beef cattle industry would have been assisted in Crop District Number Three through implementation of beef cattle stabilization policies aimed at feeder and slaughter cattle enterprises. This of course is true only if the following assumptions hold:-

(i) Manitoba Crop District Number Three decision-makers are profit maximizers, in the sense that only efficient portfolios of farm enterprises are rationally selected;

(ii) These farmers have a risk-averse attitude, so that by stabilizing returns to beef enterprises, more farmers are induced to enter the industry;

(iii) The remaining three factors, (of farmer conservatism, higher labour requirements for livestock, and lack of capital and management expertise) hypothesized to be impediments to entry into the beef cattle industry (see p.6) are less important to farmers than risk.

2. Scope for Application of the Model and Methodological Approach

The principal beneficiaries of the empirical model and results are intended to be government policy-makers seeking to reallocate resources from cash-cropping to livestock enterprises, and in particular to beef cattle enterprises. Although offering limited benefits due to the use of a representative farm approach in only one crop district of Manitoba, the model could be modified to permit an analysis involving several crop districts and primary farm data as availability increases. The model could then be applied to testing the effects on enterprise choice and resource allocation decisions of government programmes designed to reduce either individual components of risk or compound enterprise risk.

With appropriate modifications to the choice of structural variables in the model, plus the incorporation of necessary new data, the model could be fairly readily adapted to the testing of government programmes directed toward stabilizing returns in, and increasing efficient portfolio proportions of alternative livestock enterprises. In particular, sheep and manufacturing milk production enterprises may be considered appropriate, in the context of the Manitoba government's stated objectives in the Farm Diversification Programme [28]. In addition, possibilities exist for application of the model to policy-orientated problems concerning risk in other provinces or regions of Canada.

Should primary (farm level) data on annual production costs, yields, product prices, etc., become more readily available in Manitoba in future years, the possibilities exist for widening the scope of the model to include efficient portfolio analysis at the individual farm level. With the additional knowledge of farm decision-makers' utility functions involving risk, the model could be extended to include the portfolio selection phase of portfolio selection theory (Chapter II, Section 2.3). The model could then be employed as a farm planning and risk analysis tool, permitting farmers to select the optimum optimorum efficient portfolio of farm enterprises according to their particular attitudes toward risk.

The primary beneficiaries of the methodological approach used in the study are intended to be researchers interested in applying the economic theory of risk to practical agricultural problems related to enterprise choice and resource allocation under a regime of imperfect knowledge. The method of combining portfolio analysis and multi-period analysis may prove particularly useful in problems concerning uncertain outcomes of enterprise choice and resource allocation decisions over time. In the context of the present study, the multi-period separable programming technique would appear to offer an appropriate methodological framework for solving problems that include both time and risk elements. The method seems to be particularly will suited to solving decision problems in agriculture in which both the timeliness of decisions and the riskiness of decision outcomes are important factors.

Some justification may be claimed for using this technique as a means of evaluating the effects of comparative enterprise risk levels on decision-making in agriculture. Not only is the risk factor realistically assessed through inclusion of the covariance terms, but also the multi-period framework permits evaluation of long-term investment and financial decisions and of changing production and marketing conditions over time. In addition, the technique confines the solution space to alternative enterprise portfolios that are feasible and efficient, that is, those solutions most relevant to the decision-maker who has definite goals (for example, profit maximization) in mind.

On the other hand, the scope for using this methodological approach for evaluating the effects of government risk-reducing policies should be viewed as somewhat limited. As noted in Chapter V, considerable difficulties are involved in attempts to isolate the individual effects of changes in the expected returns, variance in returns and covariance elements on the portfolio solutions.

Separable programming methods may also provide a useful and feasible alternative to multi-period quadratic programming methods. The little information that is available on empirical applications of the latter technique suggests that some severe limitations are involved. Batterham [5] indicated (a) computer core storage problems, and (b) the need for more advanced and efficient computer solution algorithms when using these techniques.

3. Limitations of the Model and Data

Many of the limitations of the model and data used in the study have already been outlined. The purpose of this section is to summarize these limitations, include any that were previously omitted, and discuss briefly their importance.

3.1 Limitations of the Model

(i) The omission of the portfolio selection phase in the present study presented some difficulties in interpreting the empirical results. The results in ChapterV (Sections 2.2 and 3.3) showed that considerable differences were evident between alternative efficient annual portfolios for different risk constraint levels. Knowledge of risk attitudes would have made possible the selection of a particular efficient portfolio for each farmer, and would then have permitted more definitive testing of the hypothesis from a sample survey of farmers' portfolio choices.

(ii) The omission of farm enterprise options representing various combinations of the three principal beef cattle production segments resulted in only a partial assessment being made of risk in beef cattle. Since it is fairly common practice for the cow-calf, beef feeder and slaughter cattle segments to be carried out on different farms in Manitoba, these three enterprises were the only activities included in the model for representing beef cattle. Vertically-integrated beef operations do however exist. For testing the hypothesis on comparative enterprise risk levels, additional information may have been obtained from including activities representing combinations of beef cattle enterprises. These combinations, particularly the one representing a fully vertically-integrated enterprise (all three segments combined), may have revealed a degree of additivity of risk in beef enterprises different from that indicated by the covariance terms in this study.

(iii) The theoretical analysis in Chapter II focussed attention on the importance of compound measures of risk, whereby all the individual risk components, plus the additivity, effects between components, are accounted for. In the empirical analysis, only the price risk component for beef and crop enterprises and the technical risk component for crops were considered, and these in an incomplete way. For example, delivery quotas on Canadian Wheat Board drops were completely ignored. Omission of technical risk sources in beef and of technological and institutional risk components implies underestimation of the variance and covariance elements used in the model. The empirical results may therefore be viewed as providing only a first approximation of the effects of comparative risk levels on enterprise choice decisions.

(iv) The use of an annual time period in the empirical model posed several problems. The use of variations in annual, as opposed to monthly, prices, masks important seasonal variations in beef and crop prices, and also ignores the variability in quantities sold at different prices throughout the year. An annual model fails to account adequately for cash flow within a year, and the effects this can have on enterprise choice and resource allocation decisions. The farmer's liquidity position at the critical time short-term inputs are required to be purchased, plus the elapsed time before sale proceeds become available, can and frequently do affect decisions to engage in particular farm enterprises. Provision for this aspect may have considerably influenced the empirical results and their interpretation with respect to the contribution made by risk to enterprise choice decisions.

(v) Other factors which were relatively or absolutely neglected in the study may have had an important bearing on the empirical results and their interpretation. Management, a critical variable, was given only cursory consideration in the model as a constrained resource and in terms of quantitative requirements for each enterprise. In reality, different types as well as amounts of management inputs are required for different enterprises, and this may have significant effects on enterprise choice decisions. Qualitative differences in management were not considered in this study, either between farms or between enterprises on the same farm.

Competitive or complementary relationships between enterprises also have an important influence on enterprise choice and resource allocation decisions overtime. For example, the degree to which feed-crop production and beef cattle operations are complementary in terms of resource use may have as much effect on enterprise choice decisions as comparative risk levels. Such enterprise relationships were not well defined in the model.

Neglect of the indivisibilities problem associated with purchases of inputs such as machinery items also has a bearing on interpreting the empirical results. The imposition of integer constraints on these resource acquisition activities would doubtless have resulted in different compositions of the efficient portfolios.

(vi) Another limitation concerned with interpreting the empirical results is the use of a representative farm approach and the confinement of analysis to only one crop district in Manitoba. Greater credence could have been placed in the results had the analysis been conducted on the basis of actual farm situations covering several Manitoba crop districts where mixed beef and cash-crop farms occur.

(vii) Finally, the applicability and realism of the model are affected by the many simplifying assumptions that were made. An example of what may appear to be an unrealistic assumption is that expected returns and risk levels in each year are viewed by the decision-maker as independent of those in other years. In fact, decision-makers are probably influenced to at least some degree and in different ways by the events of other years. However, the magnitude and direction of these influences are subjective factors and probably differ between decision-makers. In the absence of a survey of farmers to determine how they are influenced, the simplifying assumption of equal influence by risk levels in all years was used.

3.2 Limitations of the Data

Many of the limitations of the data used can be closely associated with specific aspects of the model limitations. Most data defects have been alluded to already in Section 3.1, but are more specifically outlined here.

(i) The principal limitation is viewed as the lack of adequate data with which to estimate the expected net returns coefficients and the variance-covariance elements. Reference was made in Chapter IV, Section 1 to the need for combining cross-section and time series data in order to obtain accurate measurements of enterprise risk levels. In this way, both interlocational and intertemporal aspects of risk would be accounted for. The omission of the interlocational risk aspect in the model, due to lack of cross-section data, most likely resulted in underestimation of the variance-covariance elements.

(ii) Further underestimation of the variance-covariance elements may have been caused by using average annual net returns data, as opposed to average monthly data. Had monthly data been used, it is possible that variance estimates, particularly for beef cattle enterprises, would have been considerably higher.

(iii) Improved estimates of expected net returns and variance in returns for beef cattle enterprises could have been obtained if data had been available for technical sources of risk. These should include variations in calving rates, mortality rates within each beef enterprise type, rates of liveweight gain, and culling rates for cows.

(iv) Availability of primary farm data would have obviated the need to employ the representative farm approach and to build in the set of assumptions listed in Chapter IV, Section 2. These assumptions referred to initial farm size and other durable assets owned at the beginning of the time horizon, initial net worth and debts outstanding, depreciation schedules, availability of and financing alternatives for additional resources, and constraints on firm expansion. Since all these tend to very considerably between farmers, the assumptions may be considered restrictive.

(v) The use of primary farm data would have permitted the critical differences in management capabilities between farmers to be accounted for in the model. Secondary data, based on averaging across farms, effectively obscures the management variability factor.

From the above synthesis of the limitations of the model and data, some general observations may be made. Many of the model limitations concern omissions or inadequate treatment of factors other than risk that influence resource allocation and enterprise choice decisions in agriculture. A more thorough treatment of these factors may have suggested a less important role for risk in decision-making than has been claimed in the study, and without doubt would have conferred more realism on the application of the methodology. At the same time, providing adequately for these factors would have increased the size and solution costs of an already large and complex model. Furthermore, the stated objectives of the study were to appraise the effects of risk on decision-making, so that adequate treatment of the risk aspects in the model was permitted to take precedence over other factors, even though the importance of the other factors is acknowledged.

4. Suggestions for Further Research

Three possible avenues are suggested for conducting further research work. The first, in the context of limitations of the data used in the empirical model, is that efforts may be usefully directed toward obtaining improved estimates of variances and covariances in net returns for various farm enterprises. The use of monthly time series data and primary level cross-section data as these become available would provide better estimates of price and technical sources of risk. Inclusion of technological and institutional sources of risk would furthermore provide a more complete and accurate estimate of compound enterprise risk levels. Applying these improved risk coefficients to the empirical model would result in a more realistic assessment of the effects of comparative enterprise risk levels on enterprise choice decisions.

The second avenue suggested is to modify and improve the empirical model structure by attending to those features omitted or inadequately covered in this study. Particular features such as the use of a monthly instead of an annual time period, and the inclusion of vertically-integrated beef enterprise activities would be feasible only if a larger, more costly model could be contemplated.

The third suggested avenue concerns extending the scope of the model to include the portfolio selection phase of portfolio theory. This may prove to be the most fruitful direction for further research since it could lead to possibilities of applying the methodology to solving practical farm decision problems in which both risk and the time factor are important. It may also prove a difficult direction to take since a survey of farmers' attitudes to risk and estimates of their income/risk utility functions would be a necessary prerequisite to adopting the portfolio selection phase.

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APPENDIX A

DERIVATION OF SEPARABLE FUNCTIONS AND COEFFICIENTS

From the specification of the quadratic programming problem (pp.73-75), the total variance term, x'Vx, can be expressed in terms of its variance and covariance components as follows:-

$$x'Vx = \sum_{j=1}^{n} \sigma_j^2 x_j^2 + 2\sum_{j=1}^{n} \sum_{v=j+1}^{n} \sigma_j \sigma_v x_j x_v , \qquad (A.1)$$

where: V represents the matrix of variance terms, σ_j^2, and covariance terms, $\sigma_j \sigma_v$.

The quadratic and product terms representing the variance and covariance elements, respectively, require some arithmetical manipulation before they can be entered in the multi-period separable linear programming (M.P.S.L.P.) model. This is because linear programming models can deal only in terms of linear functional forms, whereas the variance and covariance terms represent non-linear functional forms. Any non-linear function can, however, be approximated arbitrarily closely by the technique of subdividing the function into a sufficient number of linear segments. These linear segments can then be entered in a linear programming model as a sequence of activities (columns) called "special variables".

This approximation technique is illustrated in Figure A-1 for the case of the variance terms. The smooth curve labelled $f_j(x_j)$ represents the quadratic function for each variance term,

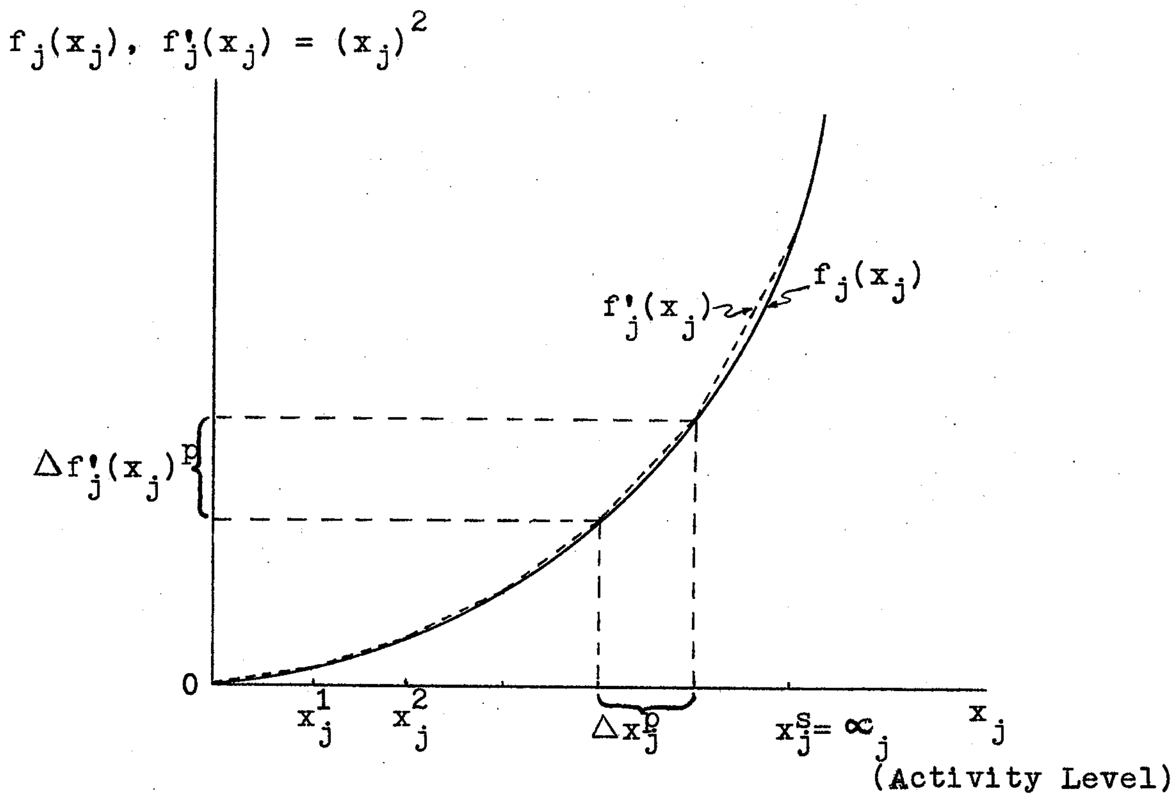

Figure A-1

Polygonal Approximation of a Quadratic Function

for Variance Terms

$$f_j(x_j) = (x_j)^2 \ , \tag{A.2}$$

while the dashed-line function labelled $f'_j(x_j)$ represents the set of linear segments approximating the quadratic function. That is,

$$f'_j(x_j) \doteqdot (x_j)^2 \ , \tag{A.3}$$

where: the function, $f'_j(x_j)$, is called the "separable function".

The separable function is formulated firstly by arbitrarily deciding the interval over which the separable variable, x_j, may take on values. In Figure A-1, the interval is shown as:-

$$0 \leq x_j \leq \alpha_j \ , \tag{A.4}$$

on the horizontal axis. This interval is then divided into s + 1 arbitrarily selected points, x_j^p,

where: $x_j^0 = 0, \ x_j^1 < x_j^2 < \ldots < x_j^s = \alpha_j$. (A.5)

Corresponding points on the function, $f_j(x_j)$, are found by drawing perpendiculars from the horizontal axis. By joining each pair of adjacent points on the function, $f_j(x_j)$, with straight lines, the desired linear segments in the function, $f'_j(x_j)$, are obtained.

Each linear segment denotes on the vertical axis the amount by which $(x_j)^2$ changes for a given change in separable variable level, x_j, as measured along the horizontal axis. For the p^{th} segment shown in Figure A-1, for a given change in the separable variable level,

$$\Delta x_j^p = x_j^p - x_j^{p-1} \ , \tag{A.6}$$

the term $f'_j(x_j) = (x_j)^2$ changes by the amount:-

$$\triangle f'_j(x^p_j) = (x^p_j)^2 - (x^{p-1}_j)^2. \qquad (A.7)$$

Then to calculate the change in the level of variance associated with a given change in the separable variable level, the term $f'_j(x_j)$ needs to be multiplied by the appropriate variance coefficient, σ^2_j. Suppose, for example, the variance coefficient for some separable variable is 100, and the separable variable level increases by 10 from 20 to 30 units, so that:-

$$\triangle x^p_j = 30 - 20 = 10 \qquad (A.8)$$

The change in the term $f'_j(x^p_j)$ is given by :-

$$\triangle f'_j(x_j) = 900 - 400 = 500 \ , \qquad (A.9)$$

and the change in the level of variance is given by:-

$$\triangle f'_j(x^p_j).(\sigma^2_j) = 500.(100) = 50{,}000 \ . \qquad (A.10)$$

In general, to obtain the total amount of variance associated with any given level of a separable variable (denoted by $\bar{x}_j$), the sum of the $\triangle x^p$ terms is first set equal to the given activity level, so that:-

$$\bar{x}_j = \sum_{p=1}^{z} \triangle x^p_j \ , \qquad (A.11)$$

where: z, the number of linear segments, may be less than or equal to s, the total number of linear segments.

The sum of the $\triangle f'_j(x^p_j)$ terms for the identical (z) number of linear segments is then calculated. This sum is multiplied by the variance coefficient to give a polygonal approximation of the variance term, $\sigma^2_j \bar{x}^2_j$. That is,

$$\sum_{p=1}^{z} \Delta f'_j(x_j^p).(\sigma_j^2) \doteq \sigma_j^2 \bar{x}_j^2 . \qquad (A.12)$$

As an example, let the basis level for a separable variable, $\bar{x}_j$, be 35, let the variance coefficient be 100, and let there be five linear segments of equal length 10, so that the interval over which x_j can take on values in the basis is given by:-

$$0 \leq x_j \leq 50 . \qquad (A.13)$$

Note that in this case, the number of segments, z, required for $\bar{x}_j = 35$ is four, (so that z is less than s, the total number of segments). Note also that a portion only of the fourth segment is required. The sum of the Δx_j^p terms is given by:-

$$\sum_{p=1}^{4} \Delta x_j^p = (35 - 30) + (30 - 20) + (20 - 10) + (10 - 0),$$
$$= 35 . \qquad (A.14)$$

The sum of the $\Delta f'_j(x_j^p)$ terms is given by:-

$$\sum_{p=1}^{4} \Delta f'_j(x_j^p) = (35^2 - 30^2) + (30^2 - 20^2) + (20^2 - 10^2) + (10^2) ,$$
$$= 1,225 , \qquad (A.15)$$

so that the total variance when $\bar{x}_j = 35$ is:-

$$\sum_{p=1}^{4} \Delta f'_j(x_j^p).(\sigma_j^2) = 1,225.(100) = 122,500 . \qquad (A.16)$$

It is the terms Δx_j and $\Delta f'_j(x_j)$ that are entered as coefficients down the special variable columns in the M.P.S.L.P. model. Examples of special variable coefficients calculated for two risk activities (beef feeders and wheat) in this study are shown in Table A.1. The interval over which beef feeders can take on values is arbitrarily taken as $0 \leq x_{BF} \leq 550$ head, and for wheat, $0 \leq x_W \leq 1240$ acres. Both intervals are divided into eleven unequal segments, as follows:-

$$\Delta x_j^p = \frac{\alpha_j}{11}, \text{ if } p = 2, 3, \ldots, 10, \tag{A.17}$$

$$\Delta x_j^p = 0.5\left(\frac{\alpha_j}{11}\right), \text{ if } p = 1 \tag{A.18}$$

$$\Delta x_j^p = 1.5\left(\frac{\alpha_j}{11}\right), \text{ if } p = 11 \tag{A.19}$$

The segment lengths were made unequal in order to improve the efficiency of the procedure for searching for a solution.

The segment values, Δx_j, and their associated functional values, $\Delta f_j'(x_j)$, are entered into the M.P.S.L.P. matrix as "grid" and "functional" [40, Chap.5] linear constraint equations, respectively. The format for entering these equations into the model is shown in Table A-2, using the coefficient values for the beef feeder cattle activity as an example. The level of the beef feeder activity in any particular basis of the M.P.S.L.P. model is determined by the value of the column entitled x_{BF}. The unitary coefficient in the x_{BF} column transfers the activity level in the basis to the special variable coefficients (see equation (A.6)), along the grid equation. Each special variable represents one of the linear segments approximating

Table A-1

Examples of Separable Variable Coefficients

for Variance Functional Relationships

Segment Number	Beef Feeders			Wheat		
p	x_{BF}	Δx_{BF}	$\Delta f'_{BF}(x_{BF})$	x_W	Δx_W	$\Delta f'_W(x_W)$
	(number of cattle)			(acres)		
1	0			0		
		25	625		56.36	3,177
2	25			56.36		
		50	5,000		112.73	25,415
3	75			169.09		
		50	10,000		112.73	50,830
4	125			281.82		
		50	15,000		112.73	76,244
5	175			394.55		
		50	20,000		112.73	101,659
6	225			507.27		
		50	25,000		112,73	127,074
7	275			619.99		
		50	30,000		112.73	152,489
8	325			732.73		
		50	35,000		112.73	177,940
9	375			845.45		
		50	40,000		112,73	203,318
10	425			958.18		
		50	45,000		112.73	228,732
11	475			1070.91		
		75	76,875		169.09	390,753
12	550			1240.00		

Table A-2

Format for Entering the Variance Separable Coefficients into the Model

Row Name	Beef Cattle Activity	Beef Cattle Special Variables				Variance Transfer Activity	
	x_{BF}	x_{BF}^{1}	x_{BF}^{2}	. . . . .	x_{BF}^{11}	x_{BF}^{T}	
Grid Equation	1	-25	-50	-.	-75		= 0
Functional Equation		625	+ 5000	+. . .	+76875	-1	= 0
Variance Constraint						134.16	$\leq \theta$[1]

1 θ is a variable representing the total variance constraint level for any particular solution.

the quadratic function given by equation (A.2). The functional equation elements (see equation (A.7)) are then summed across according to the number of special variables brought into the basis. The accumulated functional equation value is then transferred to the corresponding variance value in the variance constraint row through the -1.0 element in the variance transfer activity.

For illustrative purposes, suppose the level of the beef feeder activity, x_{BF}, in the basis is 75. The unitary coefficient in the grid equation then becomes 75, and to complete the equation, the first two special variables, x^1_{BF} and x^2_{BF}, must be brought into the basis at their upper limit levels. Summing across the functional equation row would then give an accumulated value of 5,625 (625 + 5,000). To complete this equation, the -1.0 coefficient in the transfer activity, x^T_{BF}, must become -5,625. This amount is then multiplied by the beef cattle variance coefficient of 134.16 in the variance constraint row. Thus if 75 beef feeder cattle were in the basis, they would contribute \$134.16 x 5,625, or \$745,650.00 of variance to the total portfolio variance.

The approximation by linear segmentation of the covariance functional relationships, which are given from equation (A.1) by:-

$$f_{jv}(x_j x_v) = 2x_j x_v \ , \tag{A.20}$$

necessitates the creation of two new variables. This is because product forms of variables must be transformed into separable components before they can be entered into a separable programming problem (see Hadley, [30, pp. 119-123]). By forming two new variables, defined as:-

$$g_{jv} = 0.5(x_j + x_v) \,, \qquad (A.21)$$

$$\text{and } q_{jv} = 0.5(x_j - x_v) \,, \qquad (A.22)$$

the product, $x_j x_v$, in equation (A.20) can be dealt with in a separable format, since:-

$$x_j x_v = (g_{jv})^2 - (q_{jv})^2 \qquad (A.23)$$

The interval over which $x_j x_v$ can assume values in equation (A.21) is given by:-

$$0 \leq x_j x_v \leq (0.5\alpha_j + 0.5\alpha_v) \,, \qquad (A.24)$$

while the interval for which $x_j x_v$ can take on values in equation (A.22) is given by:-

$$-0.5\alpha_v \leq x_j x_v^* \leq 0.5\alpha_j \,, \qquad (A.25)$$

where α_j and α_v represent the upper limits of the ranges of variation for x_j and x_v, respectively, (see Hadley, [30, p.119]).

It should be noted that solution procedures for separable programming problems are relatively inefficient when a set of separable variables takes on negative values. This is the case for the $x_j x_v$ term over the negative portion of the interval defined in equation (A.25). The concept is portrayed in Figure A-2. The efficiency of the solution procedure can, however, be improved by redefining the interval in equation (A.25) in such a way that $x_j x_v$ takes on strictly positive values. The interval is redefined here as:-

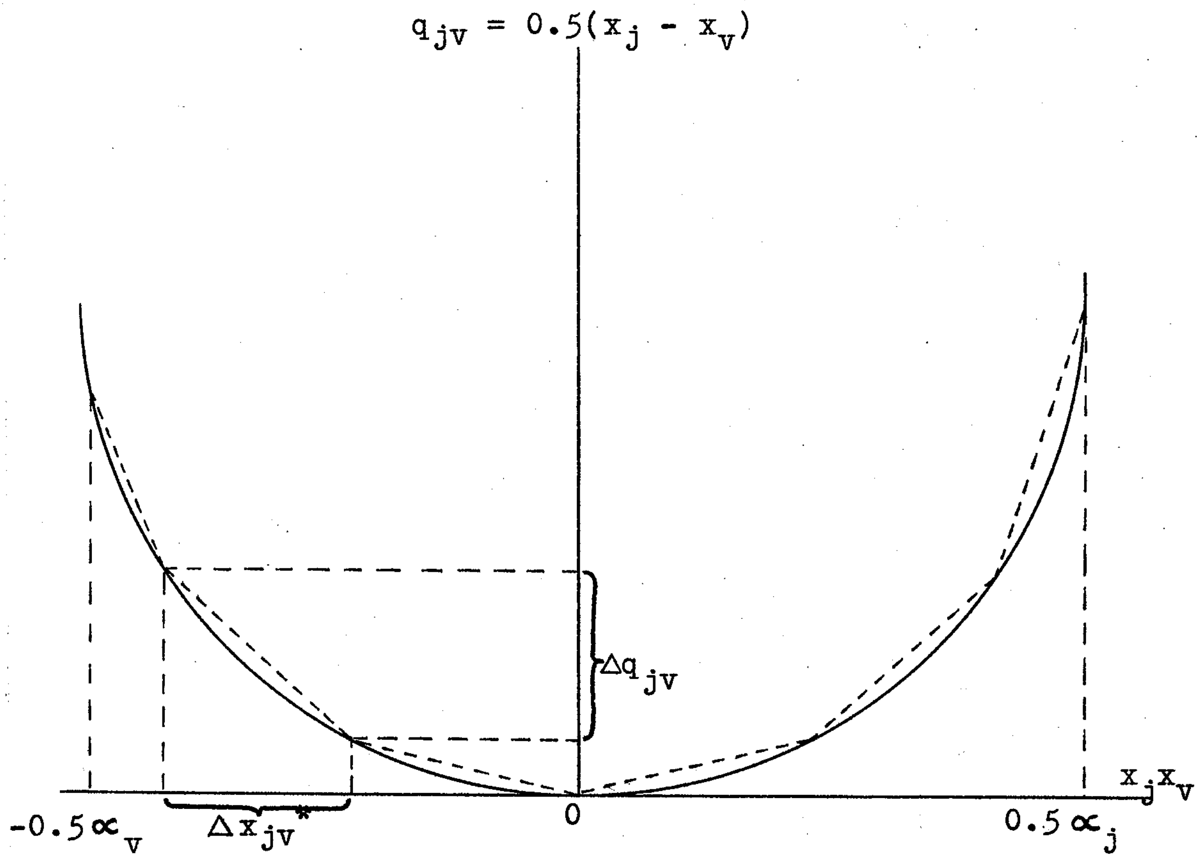

Figure A-2

Linear Approximations of the Covariance Terms, q_{jv}

$$0.5\alpha_v \leq x_j x_v^* \leq \alpha_j + \frac{\alpha_j}{5}, \qquad (A.26)$$

and is illustrated in Figure A-3.

Having defined the two intervals over which the separable components of the $x_j x_v$ terms may take on values, the procedure for specifying the separable functions and special variable coefficients is similar to that outlined previously for the variance terms, $(x_j)^2$.

Both intervals for the covariance separable functions are arbitrarily divided into eleven unequal segments in this study, as follows:-

$$\Delta x_j x_v^p = \frac{(0.5\alpha_j + 0.5\alpha_v)}{11}, \text{ if } p = 2, 10, \qquad (A.27)$$

$$\Delta x_j x_v^p = \frac{0.5(0.5\alpha_j + 0.5\alpha_v)}{11}, \text{ if } p = 1, \qquad (A.28)$$

$$\Delta x_j x_v^p = \frac{1.5(0.5\alpha_j + 0.5\alpha_v)}{11}, \text{ if } p = 11, \qquad (A.29)$$

$$\Delta x_j x_v^{*p} = 0.6\alpha_j, \text{ if } p = 1, \qquad (A.30)$$

$$\Delta x_j x_v^{*p} = 0.1\alpha_j, \text{ if } p = 2,6, \qquad (A.31)$$

$$\Delta x_j x_v^{*p} = 0.1\alpha_v, \text{ if } p = 7, 11. \qquad (A.32)$$

It should be noted that in the case of the covariance terms, it is the joint changes for every possible pair, $x_j x_v$, of separable variable levels that must be considered. Also, given two new variables, g_{jv} and q_{jv}, from which have been derived two separate separable functions, there are two sets of special variable coefficients which must

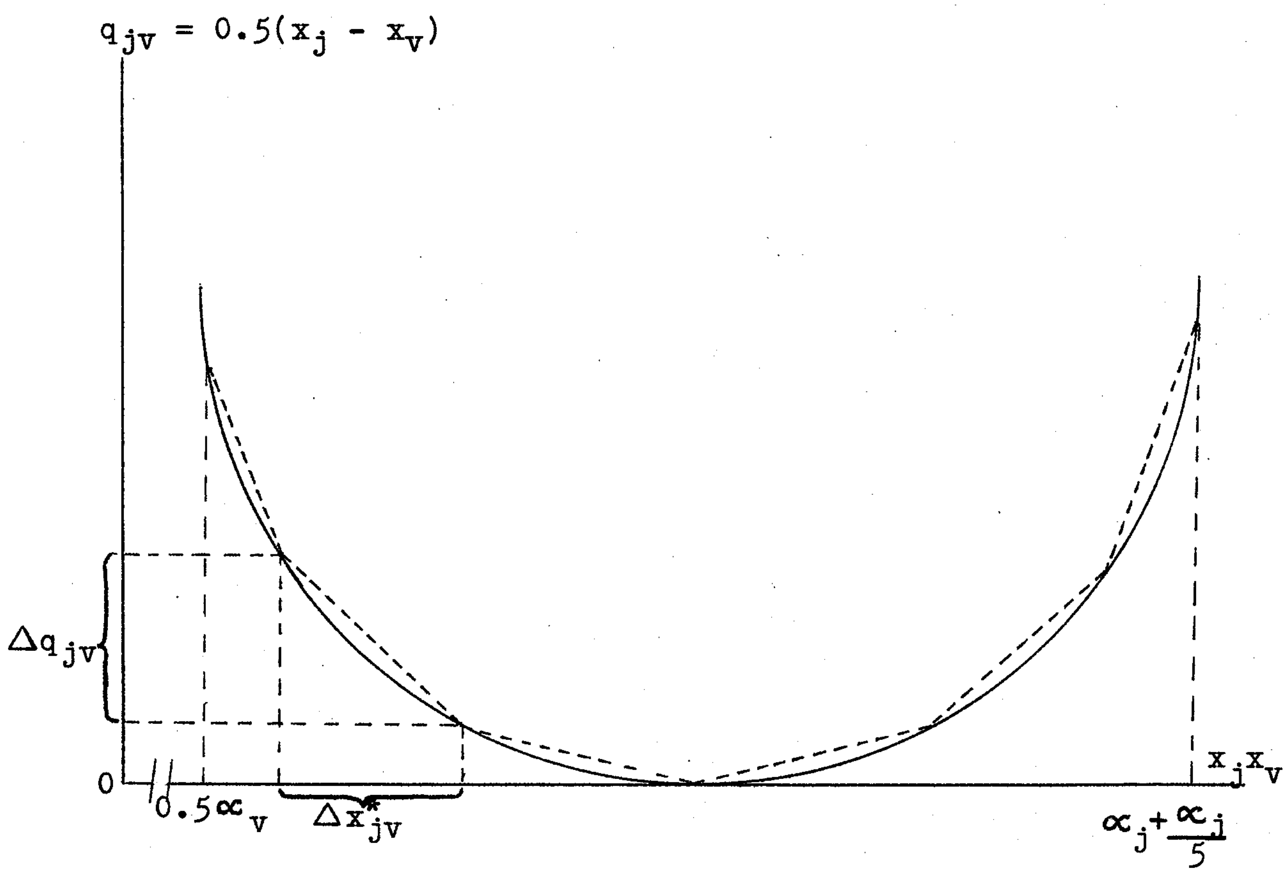

Figure A-3

Linear Approximation of the Covariance Terms, q_{jv},

Using a Different Interval.

be jointly considered for any change in either x_j, or x_v, or both. For any change in the joint activity level, $x_j x_v$, there are therefore two components, namely $\Delta x_j x_v{}^p$ coefficients associated with the new variable g_{jv}, and $\Delta x_j x_v^{*p}$ coefficients associated with the new variable, q_{jv}, where:

$$\Delta x_j x_v^p = x_j x_v^p - x_j x_v^{p-1} \tag{A.33}$$

$$\text{and } \Delta x_j x_v^{*p} = x_j x_v^{*p} - x_j x_v^{*p-1} \tag{A.34}$$

For the two coefficients defined in equations (A.33) and (A.34), there are corresponding changes to the terms g_{jv} and q_{jv}. These special variable coefficients are defined by:-

$$\Delta g_{jv}^p = (x_j x_v^p)^2 - (x_j x_v^{p-1})^2 \ , \tag{A.35}$$

$$\text{and } \Delta q_{jv}^p = (x_j x_v^{*p})^2 - (x_j x_v^{*p-1})^2 \ . \tag{A.36}$$

To obtain the contribution made to total variance by the covariance associated with some given levels of the separable activities, x_j and x_v, the appropriate number of linear segments must be summed for both the terms Δg_{jv} and Δq_{jv}. The joint sum is then multiplied by the covariance coefficient, $\sigma_j \sigma_v$, and the resulting term represents a polygonal approximation of the covariance term, $2\sigma_j \sigma_v x_j x_v$. That is:-

$$2\sum_{p=1}^{s} \Delta g_{jv}^p \, \Delta q_{jv}^p . \, \sigma_j \, \sigma_v \doteqdot 2\,\sigma_j \,\sigma_v x_j x_v \ . \tag{A.37}$$

An example of a set of special variable coefficients, $\Delta x_{BF} x_W$ and $\Delta x_{BF} x_W{}^*$, and their corresponding functional values, $\triangle g_{BF,W}$ and $\triangle q_{BF,W}$, for the covariance pair beef feeders and wheat is shown in Table A-3.

These coefficients for each covariance pair are entered into the M.P.S.L.P. matrix as two sets of "grid" equations (one for each of the terms $\triangle x_{BF}x_W$ and $\triangle x_{BF}x^*_W$) and one "functional" equation (for the combined terms $\triangle g_{BF,W}$ and $\triangle q_{BF,W}^*$) as shown in Table A-4. Again, beef feeder cattle and wheat are used as an example.

The elements +0.5 and -0.5 in Table A-4 serve to transfer the levels of the activities x_{BF} and x_W in the basis to the two grid equations. The -1 coefficient in the column $x_{BF}x^T_W$ transfers the accumulated sum of functional equation elements to the covariance element $\sigma_{BF}\sigma_W$ in the total variance constraint row.

The separable variable segments enumerated above are permitted to enter the solution basis only in a restricted segmented way (see Hadley, [30, p.117]). In general, each segment $\triangle x^p_j$ may take on values between 0 and 1, but if the p^{th} segment is to enter the basis, so that,

$$\triangle x^p_j > 0, \tag{A.38}$$

then all previous segments,

$$\triangle x^0_j,\ \triangle x^1_j,\ .\ .\ .\ \triangle x^{p-1}_j, \tag{A.39}$$

must already be in the basis at their upper bound values

Table A-3

Example of Separable Variable Coefficients

for Covariance Functional Relationships

(Beef Feeders and Wheat)

Segment Number	$0.5(x_{BF} + x_W)$			$0.5(x_{BF} - x_W)$		
p	$x_{BF}x_W$	$\Delta x_{BF}x_W$	$\Delta g_{BF,W}$	$x_{BF}x_W^*$	$\Delta x_{BF}x_W^*$	$\Delta q_{BF,W}$
1	0			605		
		40.68	1,655		330	-75,625
2	40.68			275		
		81.36	13,240		55	27,225
3	122.04			220		
		81.36	26,480		55	21,175
4	203.40			165		
		81.36	39,720		55	15,125
5	284.76			110		
		81.36	52,960		55	9,075
6	366.12			55		
		81.36	66,200		55	3,025
7	447.48			0		
		81.36	79,400		124	-15,376
8	528.84			124		
		81.36	92,680		124	-46,128
9	610.20			248		
		81.36	105,920		124	-76,880
10	691.56			372		
		81.36	119,160		124	-107,632
11	772.92			496		
		122.08	203,560		124	-138,384
12	895.00			620		

Table A-4

Format for Entering the Covariance Separable Coefficients into the Model

Row Name	Beef Feeder Activity	Wheat Activity	Special Variables for g_{jv} terms	Special Variables for q_{jv} terms	Covariance Transfer Activity	
	x_{BF}	x_W	$x_{BF}x_W^1$ . . $x_{BF}x_W^{11}$	$x_{BF}x_W^{*1}$ . . $x_{BF}x_W^{*11}$	$x_{BF}x_W^T$	
g_{jv} Grid Equation	0.5	+ 0.5	40 +. . . + 122			= 0
q_{jv} Grid Equation	0.5	- 0.5		330+. . + 124		= 605
Functional Equation			1655 +. . +203560	- 75625+. . -138384	- 1	= 0
Variance Constraint						$-19.29 \leq \theta$[1]

1. θ is the variable constraint level on total variance in the model.

of 1. Furthermore, for the p^{th} segment in the basis,

$$0 < \Delta x_j^p < 1 \ , \qquad (A.40)$$

all remaining segments,

$$\Delta x_j^{p+1}, \Delta x_j^{p+2}, \ . \ . \ . \ , \Delta x_j^s \qquad (A.41)$$

must assume their lower bound values of 0. This restricted entry rule applies to every covariance segment, $\Delta x_j x_v^p$ and $\Delta x_j x_v^{*p}$. The rule ensures that due account is taken in the solution basis of the entire section of the linear segmented variance (or covariance) function up to the point where (some portion of) the p^{th} segment is brought into the basis, and to ensure that the remaining section beyond that point is not included in that particular basis.

APPENDIX B

CALCULATION OF EXPECTED ANNUAL NET RETURNS FOR BEEF CATTLE AND CASH-CROPPING ENTERPRISES

Expected annual net returns for each enterprise are based as far as possible on data provided by the Manitoba Department of Agriculture (M.D.A.). For the beef cattle enterprises, shown in Tables B-2 to B-4, costs of production and marketing were taken from M.D.A. beef cattle budgets based on 1971 data [26, pp. 102:10-11], except for the feed costs and return on capital items. Since M.D.A. budget costs were supplied only for the year 1970, operating costs (other than feed costs and return on capital) for the remaining years were calculated using farm input price indexes for western Canada [60, p.32]. That is, 1970 operating costs were discounted by the livestock expenses index for years prior to 1970, and compounded by this index for years after 1970.

To calculate feed costs, a least-cost ration linear programming model was formulated according to Hadley [29, p.76] as follows:-

find a non-negative vector

$$x \geq 0, \qquad \text{(B.1)}$$

which satisfies the set of m linear simultaneous equations,

$$Ax = b, \qquad \text{(B.2)}$$

and which minimizes the objective function

$$z = c'x \qquad \text{(B.3)}$$

where: x is an n x 1 vector of feed purchasing-nutrient supplying activities,

A is an m x n matrix of nutrients supplied per unit weight of feed purchased,

b is an m x 1 vector of nutrients required to produce a given liveweight gain for beef cattle over a given time period,

and c is an n x 1 vector of costs per unit weight of feeds purchased.

This model was used to find the minimum cost ration for each of the years 1964 to 1972, where the vector of purchased feed costs, c, was based on average annual retail feed prices paid by Manitoba livestock producers [61, Table 15], updated for each year over the period. The coefficients in matrix A for nutrients supplied and values in vector b for nutrient requirements were unchanged over the time horizon. That is, technological progress was assumed constant, and inter-year variations, due to, for example, climatic differences, were ignored. For cow-calf enterprises, it was assumed that pasture provided the feed requirements for the five summer months (mid-May to Mid-October). Pasture was costed on the basis of two acres per cow at ten dollars per acre [26, p. 101:b]. All pasture was assumed rented.

The cost item, "return on capital" refers to the opportunity cost on the capital used to cover operating expenses. This cost was calculated by multiplying a variable interest rate by total operating costs (excluding labour) for the number of days in the production period. Labour costs were not included because return to operator's labour is already provided for in the model through consumer expenditures constraints. However, the remaining operating costs can correctly be treated as operating capital, which can be invested in interest-earning alternatives, and therefore should bear a return when invested in a farm enterprise.

The rate of return on operating capital is taken as the variable average annual prime bank lending rate (see Table B-1) less one half of one percent. This rate reflects the potential earnings to be made from investing in alternatives such as term deposits. Finally, the amount of time for which the operating capital is invested in a farm enterprise is invariably different from one year. Therefore the rate of return expressed in per annum terms must be adjusted by multiplying by the number of days taken to produce and market a particular farm product, and dividing by 365.

The net returns per acre figures for cash crops are specified in Tables B-5 to B-9, using M.D.A. data. The direct or operating costs for cash crops are available from M.D.A. sources [25 and 27] for the year 1971 only. Therefore operating costs for the remaining years from 1964 to 1972 were estimated using the annual farm input price indexes for western Canada [60, pp. 22-33]. That is, 1971 operating cost figures were discounted for years prior to 1971, and were compounded for the year 1972. Each operating expense item (shown in Tables B-5 to B-9) was discounted (or

Table B-1

Interest Rate Schedules Used in the Model

Year	Prime Bank Lending Rate[1]	Long-term (Farm Credit Corporation) Lending Rate[2]
	(average annual rates in percentages)	
1964	6.0	5.69
5	5.76	5.69
6	6.0	5.75
7	5.92	5.75
8	7.0	5.76
9	7.96	8.13
70	8.12	8.69
1	6.5	7.88
2	6.0	7.13

1. Source (for 1964 to 1971): Bank of Canada, Bank of Canada Statistical Summary, Ottawa, 1964-1971; Source (for 1972): The Bank of Canada Review, Ottawa, 1971-1972, Table 18, p.49.
2. Source: Farm Credit Corporation, Portage Avenue, Winnipeg, Manitoba, Personal Interview, October, 1974.

compounded) by the price index most closely corresponding to it in the farm input price indexes tables. For example, "machinery repairs and fuels" item was indexed by the "Machinery Operating Costs" index; the same index was used for the item "trucking grain", since a separate index for this item is not given.

Return on capital was based on the same method as that used for beef cattle operations, namely the product of operating costs (excluding labour charges), a variable interest rate, and the number of days in the production period. The variable interest rate used was the prime bank lending rate (Table B-1), less one-half of one percent.

Indexing procedures were not required for cash-crop enterprise income items, since M.D.A. sources provide crop prices and yield data on an annual basis [27].

The expected annual net returns figures for each of the three beef cattle and five cash-cropping activities over the horizon 1964 to 1972 are contained in Tables B-10 and B-11, respectively. These are shown on a present value (1964) basis.

Table B-2

Calculation of Cow-Calf Enterprise Net Returns for 1970

Income or Expense Item	Income[1]	Expenses[2]
	(dollars)	(dollars)
400 lbs. weaned calf @ $39.42 per cwt.	157.68	
1,000 lbs. culled cow @ $23.45 per cwt. x 0.20 replacement rate	46.84	
Feed and feed preparation		46.73
Bedding Straw (1200 lbs. @ $10 per ton)		6.00
Veterinary and medicines		4.60
Purchase bull replacement ($600 x 0.25 replacement rate x 0.04 share per cow)		6.00
Purchase cow replacement ($325 x 0.20 replacement rate)		65.00
Trucking, commission and yardage		4.78
Sub-total of Expenses		133.11
Return on operating capital ($133.11 x 0.0762)		10.15
Labour cost (10 hours @ $2.00 per hour)		20.00
Death loss (0.05 x $157.68 per calf)		7.88
Totals	204.52	171.14
Net return per cow (for land, management and overhead capital)	33.38	

1. Source: Manitoba Dept. of Agric., Economics Branch, Yearbook of Manitoba Agriculture, 1972, p.21.
2. Source (except Feed Cost and Return on Capital): Manitoba Dept. of Agric., Economics Branch, Beef Manual, 1972, p.101:6.

Table B-3

Calculation of Feeder Cattle Net Returns for 1970

Income or Expense Item	Income[1]	Expenses[2]
	(dollars)	(dollars)
750 lbs. feeder steer @ $33.00 per cwt.	247.50	
Purchase 400 lbs. calf @ $39.42 per cwt.		157.68
Buying commission (400 lbs. @ $0.25 per 100 lbs.)		1.00
Trucking in (400 lbs. @ $0.30 per 100 lbs.)		1.20
Veterinary and medicines		2.50
Bedding straw (212 days x 3.3 lbs. @ $10 per ton)		3.50
Feed Costs (212 days)		32.61
Feed preparation		4.00
Selling commission		3.00
Trucking out (750 lbs. @ $0.30 per 100 lbs.)		2.25
Sub-total of Expenses		207.74
Return on operating capital ($207.74 x 0.0762 x 212 days ÷ 365 days)		9.19
Labour cost (2 hours @ $2.00 per hour)		4.00
Death loss (0.01 x $247.50 per feeder steer)		2.48
Totals	247.50	223.41
Net return per feeder steer (for land, management and overhead capital)	24.09	

1. Source: Manitoba Dept. of Agric., Economics Branch, Yearbook of Manitoba Agriculture, 1972, p.21.
2. Source (except for Feed Cost and Return on Capital): Manitoba Dept. of Agric., Economics Branch, Beef Manual, 1972, p. 102:8.

Table B-4

Calculation of Slaughter Cattle Net Returns for 1970

Income or Expense Item	Income[1]	Expenses[2]
	(dollars)	(dollars)
1100 lbs. slaughter steer @ $30.20 per cwt.	332.20	
750 lbs. feeder steer @ $29.67 per cwt.		222.50
Feed costs (140 days)		46.90
Buying commission		1.87
Trucking in (750 lbs. @ $0.30 per 100 lbs.)		2.25
Veterinary and medicines		2.00
Bedding straw (560 lbs. @ $10 per ton)		2.80
Trucking out (1,100 lbs. @ $0.30 per 100 lbs.)		3.30
Selling commission		3.00
Feed milling and mixing		5.00
Sub-total of Expenses		289.62
Return on operating capital($289.62 x 0.0762 x 140 days ÷ 365 days		8.44
Labour cost (1.75 hours @ $2.00 per hour)		3.50
Death loss (.005 x $332.20 per slaughter steer)		1.66
Totals	332.20	303.22
Net return per slaughter steer (for land, management and overhead capital)	28.98	

1. Source: Manitoba Dept. of Agric., Economics Branch, Yearbook of Manitoba Agriculture, 1973, p.67.
2. Source (except Feed Cost and Return on Capital): Manitoba Dept of Agric. Economics Branch, Beef Manual, 1972, pp. 102:10-11.

Table B-5

Calculation of Net Returns per Acre for Wheat, 1971

Income or Expense Item	Income	Expenses[1]
	(dollars)	(dollars)
29.2bu.[2] wheat @ $1.37 per bu.[3]	40.00	
1 ton straw @ $10.00 per ton[4]	10.00	
Seed and seed treatment		3.51
Fertilizer		3.06
Chemical sprays		3.52
Crop insurance		1.22
Machinery repairs and fuels		6.96
Trucking		1.46
Sub-Total of Expenses		19.73
Return on operating capital ($19.73 x 0.06 x 100 days[5] ÷ 365 days)		.32
Labour (3.15 hours @ $2.09 per hour)		6.58
	50.00	26.63
Net return per acre (for land, management and overhead capital)	27.37	

1. Source (except Return on capital): Manitoba Dept. of Agric., Economics Branch, Farm Data Handbook, 1972, pp. II - 27A, 27B.
2. Source: Manitoba Dept. of Agric., Economics Branch, Yearbook of Manitoba Agriculture, 1972, p.7.
3. Source: Ibid., p. 42.
4. Estimated yield and price.
5. Return on capital calculated for production period only, i.e. from seeding to harvesting.

Table B-6

Calculation of Net Returns per Acre for Barley, 1971

Income or Expense Item	Income	Expenses[1]
	(dollars)	(dollars)
46.7 bu.[2] @ $0.78 per bu.[3]	36.43	
1 ton straw @ $10.00 per ton[4]	10.00	
Seed and seed treatment		3.32
Fertilizer		5.36
Chemical sprays		3.37
Crop insurance		1.36
Machinery repairs and fuels		7.01
Trucking grain		2.34
Sub-total of Expenses		22.76
Return on operating capital ($22.76 x 0.06 x 90 days[5] ÷ 365 days)		0.34
Labour cost (3.15 hours @ $2.09 per hour)		
Totals	46.43	29.68
Net Returns per acre (for land, management and overhead capital)	16.75	

1. Source: (except Return on capital): Manitoba Dept. of Agric., Economics Branch, Farm Data Handbook, 1972, pp. II-30, 31.
2. Source: Manitoba Dept. of Agric., Economics Branch, Yearbook of Manitoba Agriculture, 1972, p.10.
3. Source: Ibid., p.44.
4. Estimated yield and price.
5. Return on capital calculated for production period only.

Table B-7

Calculation of Net Returns per acre for Oats, 1971

Income or Expense Item	Income	Expenses[1]
	(dollars)	(dollars)
57.6 bu.[2] @ $0.53 per bu.[3]	30.53	
1.3 tons straw @ $10.00 per ton[4]	13.00	
Seed and seed treatment		4.04
Fertilizer		5.36
Chemical sprays		0.25
Crop insurance		1.36
Machinery repairs and fuels		8.51
Trucking grain		2.88
Sub-total of Expenses		22.40
Return on operating capital ($22.40 x 0.06 x 90 days[5] ÷ 365 days)		0.34
Labour cost (3.35 hours @ $2.09 per hour)		7.00
Totals	43.53	29.74
Net Return per acre (for land, management and overhead capital)	13.79	

1. Source (except Return on Capital): Manitoba Dept. of Agric., Economics Branch, Farm Data Handbook, 1972, pp. II-28,29.
2. Source: Manitoba Dept. of Agric., Economics Branch, Yearbook of Manitoba Agriculture, 1972, p.9.
3. Source: Ibid., p. 43.
4. Estimated yield and price.
5. Return on capital calculated for production period only.

Table B-8

Calculation of Net Returns per acre for Flaxseed, 1971

Income or Expense Item	Income	Expenses[1]
	(dollars)	(dollars)
8.00 bu[2] @ $2.00 per bu.[3]	16.00	
Seed and seed treatment		4.83
Fertilizer		3.60
Chemical sprays		3.18
Crop insurance		1.16
Machinery repairs and fuels		1.97
Trucking		0.40
Sub-total of Expenses		15.14
Return on operating capital ($15.14 x 0.06 x 120 days[4] ÷ 365 days)		0.30
Labour cost (2.45 hours @ $2.09 per hour)		
Totals	16.00	20.56
Net return per acre (for land, management and overhead capital)	-4.56	

1. Source (except Return on capital): Manitoba Dept. of Agric., Economics Branch, Farm Data Handbook, 1972, p. II-32, 33.
2. Source: Manitoba Dept. of Agric., Economics Branch, Yearbook of Manitoba Agriculture, 1972, p.11.
3. Source: Ibid., p.45.
4. Return on capital calculated for production period only.

Table B-9

Calculation of Net Returns per Acre for Rapeseed, 1971

Income or Expense Item	Income	Expenses[1]
	(dollars)	(dollars)
20.7 bu.[2] @ $2.15 per bu.[3]	44.51	
Seed and seed treatment		1.43
Fertilizer		4.93
Chemical sprays		3.14
Crop insurance		1.71
Machinery repairs and fuels		2.55
Trucking		1.18
Sub-total of Expenses		14.94
Return on operating capital ($14.94 x 0.06 x 120 days[4] ÷ 365 days)		0.30
Labour cost (2.45 hours @ $2.09 per hour)		5.12
Totals	44.51	20.36
Net return per acre (for land, management and overhead capital)	24.15	

1. Source: (except Return on capital): Manitoba Dept. of Agric., Economics Branch, Farm Data Handbook, 1972, pp. II-34,35.
2. Source: Manitoba Dept. of Agric., Economics Branch, Yearbook of Manitoba Agriculture, 1972, p.13.
3. Source: Ibid., p.51.
4. Return on capital calculated for production period only.

Table B-10

Present (1964) Values of Annual Net Returns for Manitoba Beef Cattle Enterprises, 1964-1972[1]

Year	Cow-Calf	Feeder Cattle	Slaughter Cattle
		(Dollars per animal)	
1964	-26.84	7.07	17.95
5	-24.32	5.96	14.46
6	- 2.73	5.41	4.49
7	- 1.19	6.17	20.39
8	- 4.31	10.68	23.01
9	20.86	12.70	18.64
70	20.97	15.12	18.23
1	12.86	23.40	20.91
2	21.63	13.79	2.86

1. Discount rate assumed equal to eight percent.

Table B-11

Present (1964) Values of Annual Net Returns for Manitoba Cash-Crop Enterprises, 1964-1972[1]

Year	Wheat	Barley	Oats	Flaxseed	Rapeseed
		(dollars per acre)			
1964	24.20	14.19	15.22	9.95	29.18
5	25.88	21.20	20.64	12.18	20.67
6	18.53	10.21	12.78	2.12	10.15
7	23.41	11.52	15.02	7.45	8.58
8	15.84	6.99	8.57	13.13	14.17
9	7.28	-1.15	3.19	-1.86	10.42
70	7.61	3.03	2.15	-1.41	11.68
1	13.64	9.77	8.04	-2.66	14.09
2	16.27	13.02	11.72	9.87	19.49

1. Discount rate assumed equal to eight percent.

APPENDIX C

CALCULATION OF INITIAL (1964) FARM NET WORTH

Table C-1

Calculation of Initial (1964) Farm Gross Worth

		(dollars)
(i) Land (640 acres @ $91.83[1])		58,771.20
(ii) Livestock Buildings[2]:-		
Beef Cattle Housing (4500 sq.ft. @ $3.35 per sq.ft.	15,075.00	
Livestock Forage Feed Storage (150 tons @ $9.27 per ton)	1,390.50	
Livestock Grain Feed Storage (10,000 bu. @ $0.17 per bu.)	1,700.00	
Total, Livestock Buildings	18,165.50	
less 25 percent depreciation	4,541.75	
1964 Value of Buildings	13,623.75	13,623.75
(iii) Farm Machinery[3]:-		
95 h.p. Tractor (2 @ $9.172)	18,344.00	
Combine (1 @ $14,107)	14,107.00	
Discer (1 @ $2,690)	2,690.00	
Swather (1 @ $3,289)	3,289.00	
Baler (1 @ $2,652)	2,652.00	
Total, Farm Machinery	41,082.00	
less 50 percent depreciation	20,541.00	
1964 Value of Machinery	20,541.00	20,541.00
(iv) Cash		2,000.00
Total 1964 Farm Gross Worth		94,935.95

1. Source: See Table 10, p.97; land value includes value of cash crop storage buildings.
2. Source: see Table 11, p. 100; average age of five years assumed, or 75 percent of useful life (of 20 years) remaining.
3. Source: see Table 12, p.101; average age of five years assumed, or 50 percent of useful life (of 10 years) remaining.

Table C-2

Initial (1964) Debts Assumed Outstanding

	(dollars)
(i) Long-term Debt:-	
25 percent of 1964 land value ($58,771.20 x 0.25)	14,692.80
50 percent of undepreciated 1964 Buildings value ($18,165.50 x 0.5)	9,082.75
Total, Long-term Debt	23,775.55
(ii) Medium-term Debt:-	
50 percent of undepreciated 1964 Farm machinery values ($41,082.00 x 0.5)	20,541.00
(iii) Short-term Debt	1,000.00
Total Debts Outstanding	45,316.55

Table C-3

Initial (1964) Farm Net Worth

	(dollars)
Total Farm Gross Worth (Table C-1)	94,935.95
less Total Debts Outstanding (Table C-2)	45,316.55
Farm Net Worth	49,619.40

APPENDIX D

CALCULATION OF TOTAL ANNUAL FIXED COSTS

Table D-1

Present (1964) Values of Annual Fixed Costs, 1964-1972

Year	Consumer Expen-ditures	Depreciation[1]	Overhead Expenses	Repayment of Out-standing Debts[2]	Total Annual Fixed Costs
		(dollars)			
1964	4896.00	2681.00	4000.00	3256.25	14833.25
5	4837.64	2482.61	3831.79	3015.28	14167.32
6	4831.59	2297.62	3748.52	2790.61	13668.34
7	4881.20	2128.71	3501.54	2320.80	12832.25
8	4889.35	1970.54	3347.93	2148.35	12356.17
9	4978.03	1825.76	3215.68	1990.51	12009.98
70	5013.24	1689.03	3000.06	1841.44	11543.77
1	4869.20	1563.02	2873.02	1704.06	11009.30
2	4887.00	1447.74	2772.90	1578.38	10686.02
Total Discounted Fixed Costs (1964-1972)					113106.40

1. Includes 10 percent depreciation of Farm Machinery and fice percent depreciation on Livestock Buildings.
2. Includes repayment of short-term, medium term and long-term debts outstanding.

APPENDIX E

SHADOW PRICES FOR SELECTED RESOURCES

Table E-1

Shadow Prices for Selected Resources for the L.P. Equivalent (No Risk Constraint) Solutions

Year	Government Programme[1]	Management Labour	Medium-term Capital Loans	Long-term Capital Loans	Land	Livestock Housing
		($/hour)	($/dollar)	($/dollar)	($/acre)	($/sq.ft)
1964	a	2.49	1.00	.86	3.77	.37
	b	8.14	1.00	.86	3.16	.37
	c	2.43	1.00	.86	3.16	.37
1965	a	1.02	.93	.81	0.00	.46
	b	0.59	.93	.81	0.58	.46
	c	2.40	.93	.81	3.33	.26
1966	a	0.00	.87	.77	0.00	.10
	b	0.00	.87	.77	0.00	.10
	c	4.22	.87	.77	10.37	.31
1967	a	4.01	.48	.73	15.62	.26
	b	0.00	.59	.73	13.86	.64
	c	0.00	.81	.73	0.00	.66
1968	a	2.31	.63	.71	0.00	.23
	b	0.00	.54	.61	0.00	.26
	c	0.00	.56	.61	0.00	.26
1969	a	0.00	.71	.61	0.00	.23
	b	0.00	.71	.61	0.00	.26
	c	0.00	.71	.61	0.00	.26
1970	a	0.00	.60	.62	6.50	.30
	b	0.00	.65	.64	2.43	.26
	c	0.00	.65	.64	2.43	.56
1971	a	1.18	.42	.60	0.00	.39
	b	0.00	.42	.60	2.14	.58
	c	0.00	.27	.60	2.14	.13
1972	a	0.00	.57	.56	0.00	.30
	b	0.00	.57	.57	3.69	.12
	c	0.00	.42	.57	3.69	.24

1. a denotes Historical Analysis
b denotes Gross Income Stabilization Programme
c denotes Net Returns Stabilization Programme

Table E-2

Shadow Prices for Selected Resources for Solutions

With a Total Variance Constraint Level of $0.75 billion

Year	Government Programme[1]	Management Labour	Medium-term Capital Loans	Long-term Capital Loans	Land	Livestock Housing
		($/hour)	($/dollar)	($/dollar)	($/acre)	($/sq.ft)
1964	a	0.06	1.00	.86	3.16	.53
	b	0.00	1.00	.86	9.04	.97
	c	1.28	1.00	.86	3.74	.37
1965	a	0.00	.93	.81	4.05	.38
	b	0.00	.93	.74	0.00	.04
	c	0.00	.47	.81	0.00	.42
1966	a	0.00	.87	.73	3.85	.02
	b	0.00	.87	.71	0.67	0.00
	c	0.00	.59	.77	6.35	.55
1967	a	0.00	.59	.73	6.08	.62
	b	0.00	.54	.71	16.14	.41
	c	0.00	.81	.63	10.19	.63
1968	a	0.00	.74	.61	0.00	.19
	b	0.00	.65	.65	0.00	.54
	c	0.00	.76	.51	0.00	.17
1969	a	0.00	.71	.61	2.90	.22
	b	0.00	.71	.55	0.00	.07
	c	0.00	.71	.58	1.90	.20
1970	a	0.00	.60	.61	0.00	.28
	b	0.00	.65	.59	0.00	.30
	c	0.00	.63	.59	0.00	.24
1971	a	0.00	.42	.60	2.14	.51
	b	0.00	.42	.60	0.00	.49
	c	0.00	.42	.60	0.00	.12
1972	a	0.00	.57	.57	3.69	.18
	b	0.00	.57	.53	0.00	.11
	c	0.00	.57	.57	3.69	.23

1. a denotes Historical Analysis
   b denotes Gross Income Stabilization Programme
   c denotes Net Returns Stabilization Programme

Table E-3

Shadow Prices for Selected Resources for Solutions with a Total Variance Constraint Level of $0.5 billion

Year	Government Programme[1]	Management Labour	Medium-term Capital Loans	Long-term Capital Loans	Land	Livestock Housing
		($/hour)	($/dollar)	($/dollar)	($/acre)	($/sq.ft)
1964	a	0.51	1.00	.86	3.74	.51
	b	0.00	1.00	.84	3.28	1.02
	c	0.00	1.00	.84	4.02	.37
1965	a	0.00	.10	.80	0.00	.40
	b	0.00	.76	.79	10.83	.02
	c	0.00	.93	.79	0.00	.36
1966	a	0.00	.26	.77	0.00	.02
	b	0.00	.87	.64	0.00	0.00
	c	0.00	.26	.75	2.08	.22
1967	a	0.00	.41	.73	16.14	.42
	b	0.00	.41	.69	0.00	.40
	c	0.00	.41	.66	1.53	.59
1968	a	0.00	.56	.67	0.00	.43
	b	0.00	.56	.63	3.82	.53
	c	0.00	.56	.60	5.94	.20
1969	a	0.00	.71	.59	0.00	.32
	b	0.00	.71	.59	0.00	.06
	c	0.00	.71	.57	8.67	.15
1970	a	0.00	.65	.61	0.15	.28
	b	0.00	.65	.61	2.37	.23
	c	0.00	.63	.59	0.00	.44
1971	a	0.00	.40	.60	2.14	.49
	b	0.00	.42	.58	4.47	.52
	c	0.00	.42	.58	0.17	.27
1972	a	0.00	.54	.57	3.69	.06
	b	0.00	.57	.53	0.00	.11
	c	0.00	.57	.55	2.38	.29

1. a denotes Historical Analysis
   b denotes Gross Income Stabilization Programme
   c denotes Net Returns Stabilization Programme

Table E-4

Shadow Prices for Selected Resources for Solutions with a Total Variance Constraint Level of $0.25 billion

Year	Government Programme[1]	Management Labour ($/hour)	Medium-term Capital Loans ($/dollar)	Long-term Capital Loans ($/dollar)	Land ($/acre)	Livestock Housing ($/sq.ft)
1964	a	0.00	1.00	.84	4.02	.40
	b	0.00	1.00	.84	7.89	1.02
	c	0.00	.81	.84	16.86	.26
1965	a	0.00	.12	.78	0.00	.23
	b	0.00	.93	.74	0.00	0.00
	c	0.00	.69	.79	0.00	.26
1966	a	0.00	.05	.73	5.99	0.00
	b	0.00	.56	.75	0.00	0.00
	c	0.00	.50	.75	0.00	.51
1967	a	0.00	.21	.72	0.00	.35
	b	0.00	.38	.63	0.62	.36
	c	0.00	.32	.66	1.99	.40
1968	a	0.00	.36	.67	7.91	.56
	b	0.00	.20	.59	4.01	.47
	c	0.00	.20	.60	1.84	.35
1969	a	0.00	.51	.59	0.00	.28
	b	0.00	.04	.65	9.77	.06
	c	0.00	.04	.57	1.70	.09
1970	a	0.00	.63	.61	4.46	.24
	b	0.00	.12	.57	0.00	.24
	c	0.00	.12	.59	0.00	.27
1971	a	0.00	.39	.56	0.00	.47
	b	0.00	.27	.56	0.00	.43
	c	0.00	.27	.58	0.00	.34
1972	a	0.00	.53	.55	2.38	.05
	b	0.00	.42	.55	2.38	.08
	c	0.00	.42	.55	2.38	.29

1. a denotes Historical Analysis
   b denotes Gross Income Stabilization Programme
   c denotes Net Returns Stabilization Programme

Table E-5

Shadow Prices for Selected Resources for Solutions with a Total Variance Constraint Level of $.10 billion

Year	Government Programme[1]	Management Labour	Medium-term Capital Loans	Long-term Capital Loans	Land	Livestock Housing
		($/hour)	($/dollar)	($/dollar)	($/acre)	($/sq.ft)
1964	a	0.00	.95	.78	5.08	0.00
	b	0.00	.96	.84	5.32	0.16
	c	0.00	.60	.66	5.35	0.00
1965	a	0.00	.75	.71	6.96	0.00
	b	0.00	.93	.77	5.78	0.00
	c	0.00	.40	.59	4.13	0.00
1966	a	0.00	.56	.61	3.65	0.00
	b	0.00	.88	.74	0.00	0.00
	c	0.00	.21	.64	0.00	0.26
1967	a	0.00	.38	.49	0.00	0.00
	b	0.00	.34	.53	0.00	0.00
	c	0.00	.02	.63	0.00	0.19
1968	a	0.00	.20	.48	1.40	0.00
	b	0.00	.51	.59	0.00	0.00
	c	0.00	.15	.63	0.00	0.14
1969	a	0.00	.04	.55	0.00	0.00
	b	0.00	.68	.65	0.00	0.00
	c	0.00	.32	.64	0.00	0.02
1970	a	0.00	.12	.57	0.00	0.00
	b	0.00	.25	.57	0.00	0.00
	c	0.00	.15	.57	0.00	0.16
1971	a	0.00	.27	.56	0.00	0.20
	b	0.00	.40	.56	0.00	0.20
	c	0.00	.30	.58	0.00	0.29
1972	a	0.00	.42	.56	0.00	0.00
	b	0.00	.54	.55	0.00	0.00
	c	0.00	.42	.55	0.00	0.29

1. a denotes Historical Analysis
   b denotes Gross Income Stabilization Programme
   c denotes Net Returns Stabilization Programme