

THE UNIVERSITY OF MANITOBA
EFFECT OF SHEAR STRENGTH PARAMETERS ON
SLOPE STABILITY ANALYSES IN THE WINNIPEG AREA

by

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A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE DEGREE
OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

WINNIPEG, MANITOBA

May, 1971



ACKNOWLEDGEMENTS

The author wishes to express his gratitude to Professor A. Baracos, Department of Civil Engineering, University of Manitoba, for his guidance and direction during the preparation of this thesis. Gratitude is also due Professor M. Mindess of the University of Manitoba whose inspiration led to the undertaking of the study, and Dr. L. Domaschuk, University of Manitoba, for his guidance during preparation of the thesis.

Gratitude is also expressed to the University of Manitoba and Division of Building Research of the National Research Council for the financial assistance they provided to complete computer programming for the final phase of the work.

SUMMARY

Several previous investigators including SUTHERLAND (1966), MISHTAK (1964) and BARACOS (1961) have shown that slope stability analyses in the Winnipeg area, using laboratory test results for the Winnipeg clays, will substantially overestimate the factor of safety. The majority of the slope failures occur in the stratum of highly plastic lacustrine clay of glacial Lake Agassiz which varies in thickness from 35.0 to 55.0 feet and overlies glacial till or limestone bedrock. The clay is noticeably varved.

Consideration is given to various methods of both total and effective stress analysis and different sets of shear strength parameters as determined by triaxial and direct shear tests. All analyses are applied to the best documented case history which is the P.F.R.A. test pit constructed for the Greater Winnipeg Red River Floodway.

Using conventional shear strength parameters as determined by conventional triaxial tests, all methods of total and effective stress analysis substantially overestimate the factor of safety. Bishop's method of analysis used with various sets of peak shear strength parameters determined by triaxial tests and direct shear tests with the failure plane perpendicular to the varves also substantially overestimates the factor of safety. Residual shear strength parameters substantially underestimate the factor of safety. Peak shear strength parameters determined by direct shear tests with the failure plane parallel to the varves used in conjunction with Bishop's Method of Analysis produced safety factors of 1.09 and 1.20.

TABLE OF CONTENTS

	<u>PAGE</u>
FRONTISPIECE	i
ACKNOWLEDGEMENTS	ii
SUMMARY.	iii
TABLE OF CONTENTS	iv
LIST OF TABLES	vi
LIST OF FIGURES.	vii
LIST OF SYMBOLS AND ABBREVIATIONS.	ix
CHAPTER:	
I INTRODUCTION	1
Previous Investigations.	2
Scope of the Thesis.	4
II REVIEW OF SHEAR STRENGTH CHARACTERISTICS OF OVERCONSOLIDATED CLAYS.	7
Components of Shear Strength	7
Failure Mechanism of an Overconsolidated Clay	9
Interpretation and Errors in Laboratory Results	11
Residual Shear Strength.	14
III LABORATORY INVESTIGATION FOR SHEAR STRENGTH. .	19
Consolidation Tests.	22
Triaxial Tests.	22
Direct Shear Tests.	26
Discussion of Results.	31

IV	ANALYSIS OF FLOODWAY TEST PIT USING DIFFERENT METHODS OF ANALYSIS	37
	Red River Floodway Test Pit.	37
	Safety Factor.	41
	Pore Pressures	43
	Shear Strength Parameters.	43
	Total Stress Method of Analysis.	44
	Friction Circle Method of Analysis	47
	Swedish Method of Slices	47
	Bishop's Method of Slices.	50
	Discussion of Results.	50
V	ANALYSIS OF FLOODWAY TEST PIT USING DIFFERENT SETS OF SHEAR STRENGTH PARAMETERS	54
	Case 1 - Residual Strength	55
	Case 2 & 3 - Peak Strength Parallel to Varves.	57
	Case 4, 5 & 6 - Peak Strength Perpendicular to Varves.	61
VI	CONCLUSIONS AND RECOMMENDATIONS	65
	Conclusions.	65
	Recommendations for Further Investigation.	66
	APPENDIX A - BIBLIOGRAPHY.	67

LIST OF TABLES

<u>NUMBER</u>	<u>TITLE</u>	<u>PAGE</u>
1	RESULTS OF LABORATORY CLASSIFICATION TESTS	20
2	PROPERTIES OF GREATER WINNIPEG GLACIAL LAKE CLAYS	21
3	RESULTS OF CONSOLIDATION TESTS	22
4	SUMMARY OF DIRECT SHEAR TEST RESULTS	31
5	SAFETY FACTORS BY TOTAL STRESS ANALYSIS	46
6	SAFETY FACTORS USING TYPICAL TRIAXIAL TEST RESULTS	52
7	SAFETY FACTORS FOR VARIOUS SHEAR STRENGTH PARAMETERS	55
8	RESIDUAL FACTORS	57

LIST OF FIGURES

<u>NUMBER</u>	<u>TITLE</u>	<u>PAGE</u>
1	COMPONENTS OF SHEAR STRENGTH AT CONSTANT VOID RATIO	7
2	TYPICAL STRESS - STRAIN CURVES FOR A WINNIPEG CLAY	9
3	COULOMB SHEAR STRESS DIAGRAM	12
4	DECREASE IN ϕ' WITH INCREASING CLAY FRACTION	15
5	DEFINITION OF RESIDUAL FACTOR	17
6	TRIAXIAL TEST APPARATUS	23
7	SUMMARY OF TRIAXIAL RESULTS	24
8	DIRECT SHEAR APPARATUS	27
9	DIRECT SHEAR APPARATUS	27
10	STRESS - STRAIN CURVES	29
11	STRESS - STRAIN CURVES	30
12	DIRECT SHEAR TEST RESULTS WITH FAILURE PLANE ⊥ TO THE VARVES AND FULL AREA CORRECTION	32
13	DIRECT SHEAR TESTS RESULTS WITH FAILURE PLANE // TO THE VARVES AND FULL AREA CORRECTION	33
14	DIRECT SHEAR TEST RESULTS WITH FAILURE PLANE // TO THE VARVES AND NO AREA CORRECTION	34
15	RED RIVER FLOODWAY TEST PIT	38
16	CROSS - SECTIONS BEFORE AND AFTER SLIDE	40
17	TEST SLOPE	42
18	SYMBOLS USED IN SLOPE STABILITY ANALYSIS	45

<u>NUMBER</u>	<u>TITLE</u>	<u>PAGE</u>
19	STABILITY ANALYSIS BY FRICTION CIRCLE METHOD ($\phi' = 16^\circ$, $c' = 5.0$ p.s.i.)	48
20	STABILITY ANALYSIS BY SWEDISH METHOD OF SLICES ($\phi' = 16.0^\circ$, $c' = 5.0$ p.s.i.)	50
21	STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES ($\phi' = 16.0^\circ$, $c' = 5.0$ p.s.i.)	51
22	STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES ($\phi' = 13.0^\circ$, $c' = 0$ p.s.i.)	56
23	STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES ($\phi' = 15.0^\circ$, $c' = 2.0$ p.s.i.)	59
24	STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES ($\phi' = 13^\circ$, $c' = 2.0$ p.s.i.)	60
25	STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES ($\phi' = 13^\circ$, $c' = 4.0$ p.s.i.)	62
26	STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES ($\phi' = 13^\circ$, $c' = 5.0$ p.s.i.)	63
27	STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES ($\phi' = 9^\circ$, $c' = 8.3$ p.s.i.)	64

SYMBOLS AND ABBREVIATIONS

- τ'_f - effective shear strength at failure
- τ' - effective shear stress
- σ'_f - effective normal stress at failure
- σ' - effective normal stress
- τ'_R - effective residual shear strength
- τ'_ϕ - effective friction component of shear strength
- c_p - ultimate cohesion component of shear strength
- c_R - rheological component of shear strength
- τ_o - surface energy component of shear strength
- dt - time interval
- dy - change in thickness during time interval, dt ,
at failure
- dx - lateral displacement during time interval, dt
- c' - effective cohesion
- ϕ' - effective angle of internal friction
- ϕ'_R - residual angle of internal friction
- σ'_p - preconsolidation pressure
- σ'_1 - maximum principal effective stress
- σ'_3 - minimum principal effective stress
- R - residual factor
- β - parameter
- c_s - parameter
- P.S.F. - pounds per square foot

(x)

P.S.I. - pounds per square inch

IN. - inch

FT. - foot

% - percent

° - degrees

IN./MIN. - inches per minute

SQ. FT./TON - square feet per ton

CM./SEC. - centimeters per second

CHAPTER I
INTRODUCTION

The slope stability problem as it exists in Winnipeg has several facets that have not been explained by previous investigators. SUTHERLAND (1961)¹, MISHTAK (1964), and BARACOS (1961) have all investigated the problem to some extent and have noted the inadequacies of design procedures based on laboratory test results. The problem of slope stability is of major proportions in Winnipeg as the city is situated at the confluence of the Red and Assiniboine Rivers. Approximately fifty miles of river banks varying in height up to fifty feet, exist in the Metropolitan Winnipeg area alone. Therefore a realistic and reliable method for investigating the stability of these slopes, as well as excavations that occur in the area, would be very useful to the local soils engineers.

The soil profile in the Winnipeg area and the Red River Valley in general is fairly consistent. The area is overlain by six to twelve inches of topsoil and up to ten feet of clayey silts and silty clays. Under these relatively recent deposits are the highly plastic, lacustrine clays of glacial

¹ Items indicated thus, SUTHERLAND (1966), refer to the corresponding entries arranged alphabetically, in the Appendix A - References.

Lake Agassiz. These clays extend down to a depth of between thirty-five and fifty-five feet depending on the depth to limestone bedrock in the area. Between the bedrock and the lacustrine clays is a stratum of very wet, ungraded, gravel or clay till which may or may not be cemented. This layer of till varies in thickness from zero to sixteen feet.

Of prime interest are the glacio-lacustrine clays as this is the stratum in which almost all slope failures occur. The clay deposit itself can be broken down into a brown upper layer, an intermediary layer of brown and blue-grey mixed clays and finally a layer of blue-grey clay. The intermediate mixed layer is often not discernible and in most cases can be neglected. As would be expected with glacio-lacustrine deposits, the clays are noticeably varved. The varves in the brown clay layer are very noticeable while those in the blue-grey are not as discernible but nevertheless exist. In the section dealing with laboratory investigations, a major point will be to discover if the peak shear strength on a failure plane parallel to the varves is the same as on a failure plane perpendicular to the varves.

PREVIOUS INVESTIGATIONS

The most common method of analysis that is currently being used is a total stress analysis. It entails substantial modification of laboratory test values in order for the analysis to be compatible with observed failures. BARACOS (1961) and SUTHERLAND (1966) both recommended that an undrained shear

strength of approximately five hundred p.s.f.² be used in conjunction with a total stress analysis to give reliable safety factors. The actual undrained shear strength has been measured by several investigators and found to lie between one thousand p.s.f. and one thousand two hundred p.s.f. Obviously, if the laboratory test results were used, safety factors would be grossly overestimated. Both SUTHERLAND (1966) and BARACOS (1961) have made attempts to explain this anomaly, but all proposals have remained strictly postulatory.

BJERRUM and KJAERNSLI (1956) published evidence that very unreliable safety factors are obtained when it is attempted to apply total stress analysis to long term failures in cuts and natural slopes. For this reason alone it is imperative that effective stress analysis be applied to the problem. A well documented case history of a failed slope analyzed on the basis of effective stress was that performed by the Water Control and Conservation Branch of the Province of Manitoba in conjunction with the Prairie Farm Rehabilitation Administration. The location was a specially designed and constructed test pit at the site of the Greater Winnipeg Red River Floodway. After failure, both total and effective stress analyses produced safety factors on the unsafe side. Although the safety factor of the test slope was obviously one, effective stress analysis produced safety

² All abbreviations are listed in the List of Symbols and Abbreviations at the beginning of the report.

factors ranging from 1.28 to 2.08. The variation in results was caused by variations in the assumptions for shear strength parameters and pore pressure measurements. Working backwards, i.e., assuming a safety factor of one, John Mishtak of W.C.C.B. was unable to come up with a combination of measured shear strengths and pore pressures that would give the desired result. The total stress analysis gave safety factors of approximately two.

In short, the extent of previous investigations can be summed up by Sutherland's three conclusions in his report to the River and Streams Control Authority of Winnipeg. (SUTHERLAND 1965)

1. The $\phi = 0$ method of analysis substantially overestimates the stability of slopes when applied to both the "end of construction" condition and the long term stability condition.
2. A check on the effective stress method had only been possible in one case, viz. the end of construction condition in the Floodway test. This method also overestimated the factor of safety. There has been no check on the effective stress method in long term stability problems.
3. Both methods of analysis imply that the shear strength of the clay at failure is substantially less than the shear strengths obtained by laboratory tests on undisturbed samples. This loss of strength applies to the short term (end of construction) and to the long term stability cases.

SCOPE OF THE THESIS

If the geometric cross section and the safety factor of a

slope are known, the remaining three variables in a slope stability analysis are:

1. method of analysis
2. shear strength parameters
3. groundwater conditions

Groundwater conditions have a very definite effect on safety factors calculated with an effective stress analysis. Generally, pore pressures are very difficult to predict, but instrumentation has now progressed to the point where they can be measured with reasonable accuracy. For the purpose of this thesis, pore pressure conditions are fixed since they have been measured for the slopes that will be analyzed.

The solution to the local problem then must lie in the method of analysis and the choice of shear strength parameters. Enough previous work has been done to indicate the advantages of an effective stress analysis employing a reduced shear strength value. The objectives of this thesis, then, are the following:

1. To evaluate the shear strength parameters of the Winnipeg clays employing various laboratory techniques and failure criteria.
2. To analyze the existing case history of a failed slope using conventional shear strength parameters and different methods of analysis. The only well documented local case history available is the previously mentioned Floodway Test Pit and will therefore be used for all analyses. This is an "end of

construction" case so that both total and effective stress analyses will apply.

3. To analyze the Floodway Test Pit failure using different sets of shear strength parameters determined by the laboratory investigation.

CHAPTER II
REVIEW OF SHEAR STRENGTH CHARACTERISTICS
OF OVERCONSOLIDATED CLAYS

COMPONENTS OF SHEAR STRENGTH

HVORSLEV (1960) proposed that the shear strength of a saturated clay could be represented by the following expression:

$$\tau'_f = \tau_\phi + c_p + c_R + \tau_d,$$

where,

τ'_f = shear strength at failure,

τ_ϕ = effective friction component,

c_p = ultimate cohesion component,

c_R = rheological component,

τ_d = surface energy component.

This relationship may be represented on a constant void ratio diagram as shown in Figure 1.

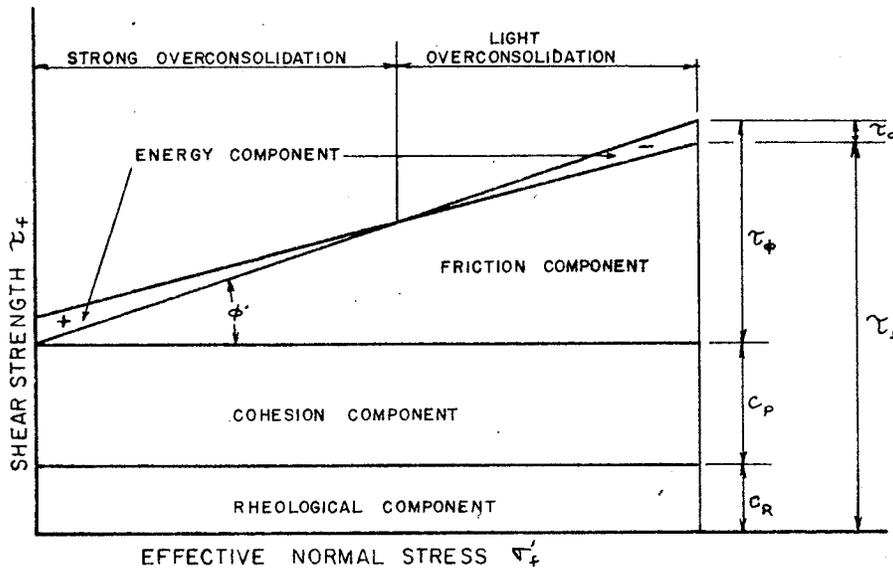


Figure 1. Components of shear strength at constant void ratio

The effective friction component, τ_f , is a function of the effective stress σ'_f . If the cohesion and rheological components are constant and the surface energy correction is applied, a straight shear strength line at an inclination, ϕ' , is obtained. This is the effective angle of internal friction. The angle, ϕ' , depends on composition and orientation of clay minerals, is independent of void ratio, and is assumed to be independent of time or rate of deformation.

The surface energy component is the result of addition or expenditure of energy due to a volume change. In a direct shear test this component may be calculated from the expression:

$$\tau_d = \sigma'_f \cdot dy/dx$$

where, dy = change in thickness during time interval, dt ,
at failure,
 dx = lateral displacement during the same time interval,
 σ'_f = effective normal stress at failure,
 τ_d = surface energy component of shear strength

The correction factor, τ_d , is positive for highly overconsolidated clays where swelling takes place. Effect of the correction is to slightly increase ϕ' .

The ultimate cohesion and rheological components are constant when: (1) the void ratio or water content of saturated clays is constant, (2) the rate of deformation or test duration is constant, (3) there is no significant difference in the geometric structure of the clays. The two components combine to form the effective cohesion, c' . The ultimate cohesion, c_p , is usually a constant but

the rheological component, c_R , decreases with time. The ultimate cohesion component, c_p , is however, affected by the constituents and structure of the clay.

Later work by BOROWICKA (1963) attempts to show that the effective cohesion caused by overconsolidation should be considered as a variable in the shear strength equation. This is consistent with HVORSLEV (1960) as one of his conditions for a constant cohesion value is a unique clay structure. Borowicka's variable cohesion value is based on a rearrangement of the clay domains which is a change in structure.

FAILURE MECHANISM OF AN OVERCONSOLIDATED, SATURATED CLAY

A typical stress-strain curve for a Winnipeg clay is as shown in Figure 2. This type of curve which includes strength at large strains can best be determined from a direct shear test or some type of ring shear apparatus.

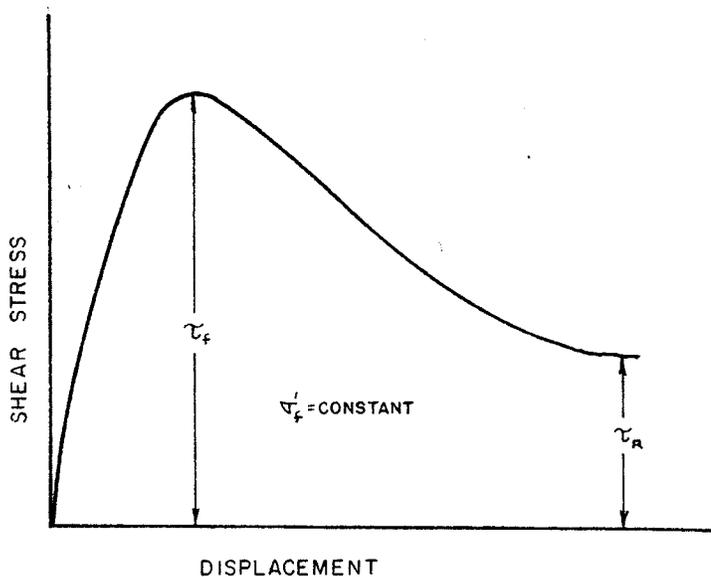


Figure 2. Typical stress-strain curve for a Winnipeg clay

All discussion will be based on the assumption of a constant rate of strain. In a drained test, the effective shear strength is measured directly while in an undrained test the pore pressure must be measured and subtracted from the total stress. Consider a drained direct shear test.

Peak strength is developed quite quickly. The effective cohesion value comes into effect almost immediately. The effective friction component is also developed quite rapidly needing only a small displacement to interlock the grains and provide full friction. The friction component can only be changed by a change in effective normal stress. In a direct shear test, the normal load is constant but effective normal pressure is slightly increased due to a decrease in area.

For a lightly overconsolidated clay, the surface energy component is usually small but increases with the length of duration of the test. Its relative magnitude depends on the ratio of vertical consolidation to horizontal displacement and magnitude of σ'_v . This increase is usually quite small.

BOROWICKA (1963) and SKEMPTON (1964) put forth the idea that for a large number of clays, domains form in which the flakey clay particles are oriented in the direction of shear. GOLDSTEIN (1961) presented evidence that these domains begin to form at relatively small strains. This would have the result of gradually decreasing the effective cohesion until the structure was completely destroyed and the cohesion component was zero. This phenomenon along with an increase in water content along the plane of failure,

results in a decrease of shear strength from the peak to the residual value.

If the normal load is held constant a small change in normal stress will occur due to a reduction in area. This increase in normal stress will cause an increase in effective friction, τ'_f , and a corresponding increase in surface energy. Therefore, the residual strength will remain approximately constant since the effective cohesion is now zero and the negative surface energy component offsets the positive change in effective friction.

INTERPRETATION AND ERRORS IN LABORATORY RESULTS

SEED and MITCHELL (1963) summarized the problems and difficulties in correlating laboratory strength results with actual field conditions. The conditions under which the testing for this thesis was done are outlined in Chapter III. The principal problems involved can be summarized as follows:

1. Condition of Specimens. This takes into account strength variations due to different methods of sampling.
2. Testing Procedure. The problem is one of simulating laboratory tests to field conditions.
3. Method of Stress Application. This includes rate of loading.
4. Measurement of Porewater Pressures.
5. Strength Criteria.

Effects of the first four problems are very difficult to assess and have been studied by a large number of investigators. A good summary of their effects is put forth by SEED and MITCHELL

(1963), COATES and McROSTIE (1963) and CHAN and RIVARD (1963). Since this discussion is beyond the scope of this thesis it will be dropped at this point. The problem of choosing a strength criteria will be discussed in the following paragraphs as all strength results are particular to the strength criteria used.

The oldest and most widely used expression for shear strength is still the Coulomb failure criterion:

$$\tau'_f = c' + \sigma'_f \tan \phi',$$

where, τ'_f = effective shear stress on plane of failure,
 σ'_f = effective normal stress on plane of failure,
 c' = effective cohesion,
 ϕ' = effective angle of internal friction.

HVORSLEV (1960) proposed a modification to take into account the preconsolidation load. He refers to it as the Krey-Tiedeman failure criterion.

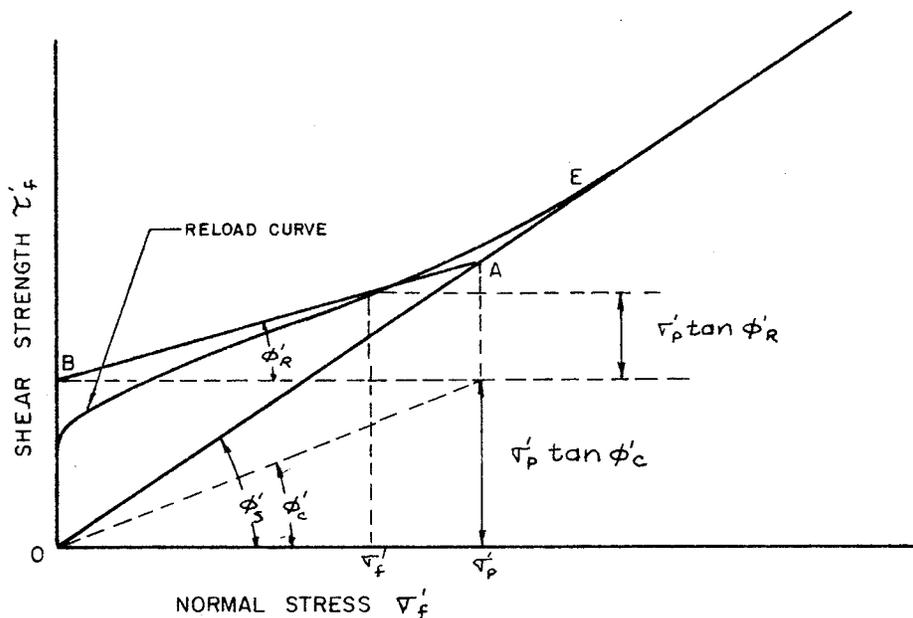


Figure 3. Coulomb shear strength diagram

The shear strength equation then may be written as:

$$\tau'_f = v'_p \tan \phi'_c + v'_f \tan \phi'_R,$$

where, v'_p = preconsolidation pressure,
 ϕ'_R = effective residual angle of internal friction,
 τ'_f = effective peak shear strength at failure,
 v'_f = effective normal stress,
 ϕ'_c = angle for cohesion.

For different preconsolidation pressures, lines parallel to BA would be obtained. Laboratory testing, however, has shown that the failure envelope below the preconsolidation pressure is not straight but curved as shown in Figure 3. Shear strength lines with double curvature are obtained. It is still common practise to express shear strength as a function of v'_f , c' , and ϕ' , thus neglecting the preconsolidation pressure. The main problem is to define, c' , and ϕ' , in the range of stresses that will be encountered in the field.

Other failure criteria have been proposed and experimental work has been done on them. The main ones are the octahedral shear stress theory and the maximum shear stress theory. These theories have not achieved any significant use for practical problems mainly because until recently there has been insufficient test data to justify their use for a variety of conditions.

For the direct shear tests, the conditions at failure are easily defined. For peak strength they are simply maximum shear stress (on the failure plane), and the corresponding normal stress. Residual shear strength is the final constant shear stress value

obtained. The residual normal stress is chosen as the normal stress corresponding to the point where the stress-strain curve first becomes horizontal.

Failure conditions in a triaxial test may be defined in two ways:

1. Minimum principal effective stress ratio - σ_1'/σ_3' where,
 σ_1' = maximum principal effective stress
 σ_3' = minimum principal effective stress.
2. Maximum principal stress difference.

SIMONS (1963) gave evidence that if the maximum principal effective stress ratio is used as the failure criteria, the same results are obtained for both drained and undrained triaxial tests. For the maximum principal stress difference criteria, undrained tests give results eight percent lower than drained tests. For sensitive clays this difference could be larger. Simons proposes that the maximum principal effective stress ratio be used for both types of triaxial tests. In accordance with this and general practise, the principal effective stress ratio will be used to interpret laboratory results. CRAWFORD (1960) offers criticism of Simons paper but his criticisms apply mainly to very sensitive clays.

RESIDUAL SHEAR STRENGTH

SKEMPTON (1964) proposed that the shearing resistance of an overconsolidated, fissured, clay slope would attain a reduced value over a long time interval, equivalent to the ultimate or residual shear strength. As has been shown earlier, the residual

strength is the constant shear strength that most clays exhibit after failure and under large strains.

One of the main reasons for the reduction in shear strength from the peak to residual value is the alignment of the clay particles in the direction of movement. It follows, that if rounded sand or silt particles are present, this reduction will not be as great. This relationship was illustrated by Skempton in the following Figure based on a number of different clays.

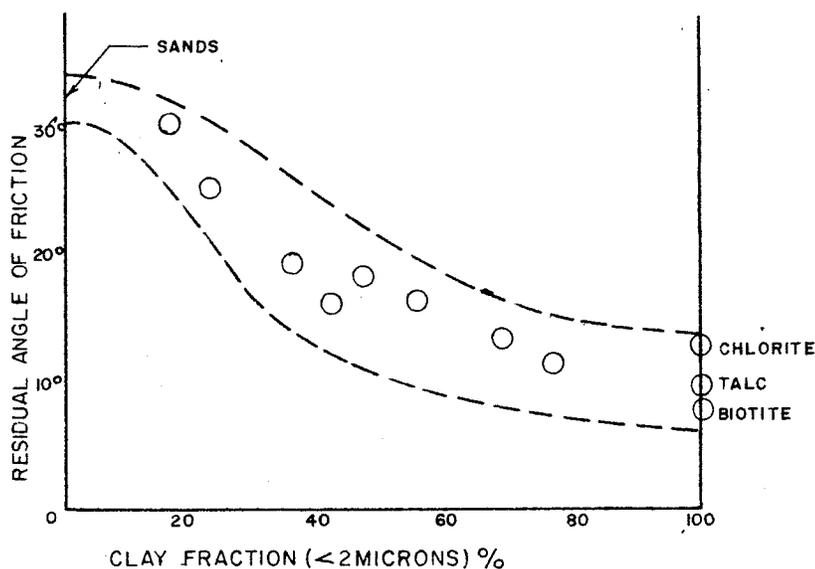


Figure 4. Decrease in ϕ' with increasing clay fraction

This point is of particular significance in the case of Winnipeg clays as they are all of a varved nature containing minute silt layers.

In all of the case histories given by Skempton, the peak strength obtained from the direct shear tests corresponded quite well with the strengths obtained from triaxial tests. For a varved clay the peak strength measured on a failure plane parallel to the

varves could be quite different from the strength measured on a failure plane perpendicular to the varves. The residual angle of internal friction on a failure plane along one of the minute silt layers would be very little reduced from the peak strength due to the high percentage of silt particles. It is anticipated that there also could be a difference in residual strengths along a failure plane through a silt layer and one that is basically through the clay.

Assuming the existence of a residual strength, there still must be some mechanism that will cause the peak shear strength of the clay to be exceeded in an embankment or cut section.

SKEMPTON (1964) proposed a number of possibilities for producing the reduced strength:

1. Stress concentration along fissures and cracks which cause local overstress and a progressive decrease in average strength.
2. Seasonal fluctuations in moisture content and temperature.
3. Shear creep which reduces the shearing resistance over the long term.
4. Tectonic movements.

Tectonic movements associated with construction or earthquake vibrations seldom apply to slope stability problems in the Winnipeg area. Seasonal changes affect soil strength parameters to a relatively shallow depth of approximately ten feet as revealed by tests performed at test plots at the University of Manitoba. Skempton attributes the strength reduction in London clays to the presence of fissures which act as stress concentrators. Very little

is known about the effects of creep on long term strength. It is known that for some clays, creep strength is significantly less than peak strength. As an example, CASAGRANDE and WILSON (1951) reported that the creep strength of Bear Paw Shale was 80 percent of the peak strength.

Skempton's mathematical treatment of residual strength as applied to slope stability is as follows:

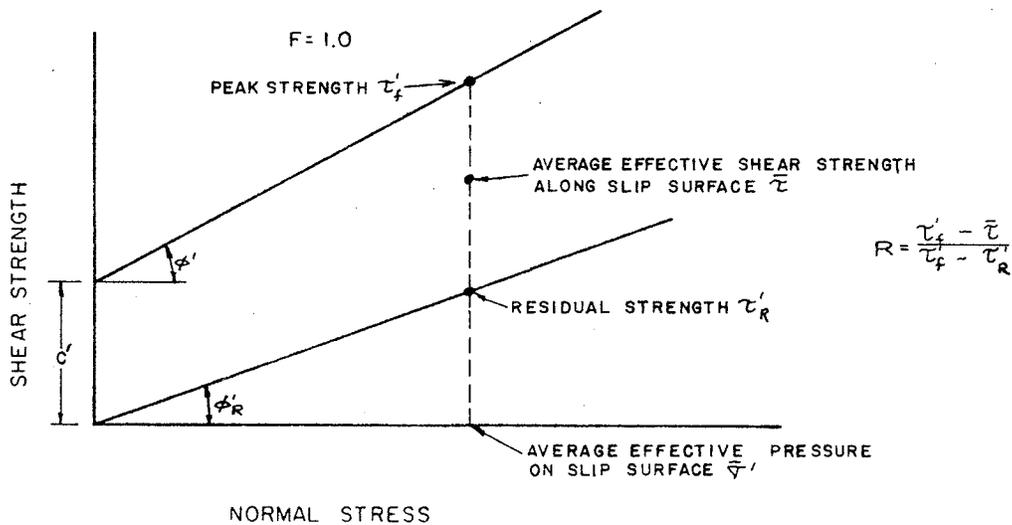


Figure 5. Definition of residual factor

Peak strength is represented by the expression $\tau'_f = c' + \sigma'_f \tan \phi'$. Residual strength is represented by the expression $\tau'_R = c'_R + \sigma'_f \tan \phi'_R$ which can be reduced to $\tau'_R = \sigma'_f \tan \phi'_R$ since cohesion is almost totally destroyed with the mobilization of residual strength. For the purposes of analysis, Skempton introduced a residual factor "R" defined as:

$$R = \frac{\tau'_f - \bar{\tau}}{\tau'_f - \tau'_R}$$