THE UNIVERSITY OF MANITOBA

EFFECT OF SHEAR STRENGTH PARAMETERS ON SLOPE STABILITY ANALYSES IN THE WINNIPEG AREA

by

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SUMMARY

Several previous investigators including SUTHERLAND (1966), MISHTAK (1964) and BARACOS (1961) have shown that slope stability analyses in the Winnipeg area, using laboratory test results for the Winnipeg clays, will substantially overestimate the factor of safety. The majority of the slope failures occur in the stratum of highly plastic lacustrine clay of glacial Lake Agassiz which varies in thickness from 35.0 to 55.0 feet and overlies glacial till or limestone bedrock. The clay is noticeably varved.

Consideration is given to various methods of both total and effective stress analysis and different sets of shear strength parameters as determined by triaxial and direct shear tests. All analyses are applied to the best documented case history which is the P.F.R.A. test pit constructed for the Greater Winnipeg Red River Floodway.

Using conventional shear strength parameters as determined by conventional triaxial tests, all methods of total and effective stress analysis substantially overestimate the factor of safety. Bishop's method of analysis used with various sets of peak shear strength parameters determined by triaxial tests and direct shear tests with the failure plane perpendicular to the varves also substantially overestimates the factor of safety. Residual shear strength parameters substantially underestimate the factor of safety. Peak shear strength parameters determined by direct shear tests with the failure plane parallel to the varves used in conjunction with Bishop's Method of Analysis produced safety factors of 1.09 and 1.20.
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SYMBOLS AND ABBREVIATIONS

\( \tau'_s \) - effective shear strength at failure
\( \tau' \) - effective shear stress
\( \nu'_f \) - effective normal stress at failure
\( \nu' \) - effective normal stress
\( \tau'_r \) - effective residual shear strength
\( \tau'_f \) - effective friction component of shear strength
\( c_p \) - ultimate cohesion component of shear strength
\( c_r \) - rheological component of shear strength
\( \tau_o \) - surface energy component of shear strength
\( d \tau \) - time interval
\( d y \) - change in thickness during time interval, \( d \tau \), at failure
\( d x \) - lateral displacement during time interval, \( d \tau \)
\( c' \) - effective cohesion
\( \varphi' \) - effective angle of internal friction
\( \varphi'_r \) - residual angle of internal friction
\( v'_p \) - preconsolidation pressure
\( v'_1 \) - maximum principal effective stress
\( v'_3 \) - minimum principal effective stress
\( R \) - residual factor
\( \beta \) - parameter
\( c_s \) - parameter
\( p.s.f. \) - pounds per square foot
PSI - pounds per square inch
IN. - inch
FT. - foot
% - percent
° - degrees
in./min. - inches per minute
sq. ft./ton - square feet per ton
cm./sec. - centimeters per second
CHAPTER I

INTRODUCTION

The slope stability problem as it exists in Winnipeg has several facets that have not been explained by previous investigators. SUTHERLAND (1961)\(^1\), MISHTAK (1964), and BARACOS (1961) have all investigated the problem to some extent and have noted the inadequacies of design procedures based on laboratory test results. The problem of slope stability is of major proportions in Winnipeg as the city is situated at the confluence of the Red and Assiniboine Rivers. Approximately fifty miles of river banks varying in height up to fifty feet, exist in the Metropolitan Winnipeg area alone. Therefore a realistic and reliable method for investigating the stability of these slopes, as well as excavations that occur in the area, would be very useful to the local soils engineers.

The soil profile in the Winnipeg area and the Red River Valley in general is fairly consistent. The area is overlain by six to twelve inches of topsoil and up to ten feet of clayey silts and silty clays. Under these relatively recent deposits are the highly plastic, lacustrine clays of glacial

\(^1\) Items indicated thus, SUTHERLAND (1966), refer to the corresponding entries arranged alphabetically, in the Appendix A - References.
Lake Agassiz. These clays extend down to a depth of between thirty-five and fifty-five feet depending on the depth to limestone bedrock in the area. Between the bedrock and the lacustrine clays is a stratum of very wet, ungraded, gravel or clay till which may or may not be cemented. This layer of till varies in thickness from zero to sixteen feet.

Of prime interest are the glacio-lacustrine clays as this is the stratum in which almost all slope failures occur. The clay deposit itself can be broken down into a brown upper layer, an intermediary layer of brown and blue-grey mixed clays and finally a layer of blue-grey clay. The intermediate mixed layer is often not discernible and in most cases can be neglected. As would be expected with glacio-lacustrine deposits, the clays are noticeably varved. The varves in the brown clay layer are very noticeable while those in the blue-grey are not as discernible but nevertheless exist. In the section dealing with laboratory investigations, a major point will be to discover if the peak shear strength on a failure plane parallel to the varves is the same as on a failure plane perpendicular to the varves.

**PREVIOUS INVESTIGATIONS**

The most common method of analysis that is currently being used is a total stress analysis. It entails substantial modification of laboratory test values in order for the analysis to be compatible with observed failures. BARACOS (1961) and SUTHERLAND (1966) both recommended that an undrained shear
strength of approximately five hundred p.s.f.² be used in conjunction with a total stress analysis to give reliable safety factors. The actual undrained shear strength has been measured by several investigators and found to lie between one thousand p.s.f. and one thousand two hundred p.s.f. Obviously, if the laboratory test results were used, safety factors would be grossly overestimated. Both SUTHERLAND (1966) and BARACOS (1961) have made attempts to explain this anomaly, but all proposals have remained strictly postulatory.

BJERRUM and KJAERNSLI (1956) published evidence that very unreliable safety factors are obtained when it is attempted to apply total stress analysis to long term failures in cuts and natural slopes. For this reason alone it is imperative that effective stress analysis be applied to the problem. A well documented case history of a failed slope analyzed on the basis of effective stress was that performed by the Water Control and Conservation Branch of the Province of Manitoba in conjunction with the Prairie Farm Rehabilitation Administration. The location was a specially designed and constructed test pit at the site of the Greater Winnipeg Red River Floodway. After failure, both total and effective stress analyses produced safety factors on the unsafe side. Although the safety factor of the test slope was obviously one, effective stress analysis produced safety

² All abbreviations are listed in the List of Symbols and Abreviations at the beginning of the report.
factors ranging from 1.28 to 2.08. The variation in results was caused by variations in the assumptions for shear strength parameters and pore pressure measurements. Working backwards, i.e., assuming a safety factor of one, John Mishtak of W.C.C.B. was unable to come up with a combination of measured shear strengths and pore pressures that would give the desired result. The total stress analysis gave safety factors of approximately two.

In short, the extent of previous investigations can be summed up by Sutherland's three conclusions in his report to the River and Streams Control Authority of Winnipeg. (SUTHERLAND 1965)

1. The $\phi = 0$ method of analysis substantially overestimates the stability of slopes when applied to both the "end of construction" condition and the long term stability condition.

2. A check on the effective stress method had only been possible in one case, viz. the end of construction condition in the Floodway test. This method also overestimated the factor of safety. There has been no check on the effective stress method in long term stability problems.

3. Both methods of analysis imply that the shear strength of the clay at failure is substantially less than the shear strengths obtained by laboratory tests on undisturbed samples. This loss of strength applies to the short term (end of construction) and to the long term stability cases.

SCOPE OF THE THESIS

If the geometric cross section and the safety factor of a
slope are known, the remaining three variables in a slope
stability analysis are:

1. method of analysis
2. shear strength parameters
3. groundwater conditions

Groundwater conditions have a very definite effect on safety
factors calculated with an effective stress analysis. Generally,
pore pressures are very difficult to predict, but instrumentation
has now progressed to the point where they can be measured with
reasonable accuracy. For the purpose of this thesis, pore
pressure conditions are fixed since they have been measured for
the slopes that will be analyzed.

The solution to the local problem then must lie in the
method of analysis and the choice of shear strength parameters.
Enough previous work has been done to indicate the advantages of
an effective stress analysis employing a reduced shear strength
value. The objectives of this thesis, then, are the following:

1. To evaluate the shear strength parameters of the Winnipeg
   clays employing various laboratory techniques and failure
   criteria.

2. To analyze the existing case history of a failed slope using
   conventional shear strength parameters and different methods
   of analysis. The only well documented local case history
   available is the previously mentioned Floodway Test Pit and will
   therefore be used for all analyses. This is an "end of
construction" case so that both total and effective stress analyses will apply.

3. To analyze the Floodway Test Pit failure using different sets of shear strength parameters determined by the laboratory investigation.
CHAPTER II
REVIEW OF SHEAR STRENGTH CHARACTERISTICS
OF OVERCONSOLIDATED CLAYS

COMPONENTS OF SHEAR STRENGTH

HVORSLEV (1960) proposed that the shear strength of a saturated clay could be represented by the following expression:

\[ \tau'_c = \tau'_\phi + c_p + c_R + J_d, \]

where, \( \tau'_c \) = shear strength at failure,
\( \tau'_\phi \) = effective friction component,
\( c_p \) = ultimate cohesion component,
\( c_R \) = rheological component,
\( J_d \) = surface energy component.

This relationship may be represented on a constant void ratio diagram as shown in Figure 1.

![Figure 1. Components of shear strength at constant void ratio](image_url)
The effective friction component, \( \tau' \), is a function of the effective stress \( \sigma' \). If the cohesion and rheological components are constant and the surface energy correction is applied, a straight shear strength line at an inclination, \( \phi' \), is obtained. This is the effective angle of internal friction. The angle, \( \phi' \), depends on composition and orientation of clay minerals, is independent of void ratio, and is assumed to be independent of time or rate of deformation.

The surface energy component is the result of addition or expenditure of energy due to a volume change. In a direct shear test this component may be calculated from the expression:

\[
\tau_d = \tau'_t \cdot \frac{dy}{dx}
\]

where,  
\( dy \) = change in thickness during time interval, \( dt \), at failure,
\( dx \) = lateral displacement during the same time interval,
\( \tau'_t \) = effective normal stress at failure,
\( \tau_d \) = surface energy component of shear strength

The correction factor, \( \tau_d \), is positive for highly overconsolidated clays where swelling takes place. Effect of the correction is to slightly increase \( \phi' \).

The ultimate cohesion and rheological components are constant when: (1) the void ratio or water content of saturated clays is constant, (2) the rate of deformation or test duration is constant, (3) there is no significant difference in the geometric structure of the clays. The two components combine to form the effective cohesion, \( c' \). The ultimate cohesion, \( c_p \), is usually a constant but
the rheological component, $c_R$, decreases with time. The ultimate cohesion component, $c_p$, is however, affected by the constituents and structure of the clay.

Later work by BOROWICKA (1963) attempts to show that the effective cohesion caused by overconsolidation should be considered as a variable in the shear strength equation. This is consistent with HVORSLEV (1960) as one of his conditions for a constant cohesion value is a unique clay structure. Borowicka's variable cohesion value is based on a rearrangement of the clay domains which is a change in structure.

FAILURE MECHANISM OF AN OVERCONSOLIDATED, SATURATED CLAY

A typical stress-strain curve for a Winnipeg clay is as shown in Figure 2. This type of curve which includes strength at large strains can best be determined from a direct shear test or some type of ring shear apparatus.

![Diagram of stress-strain curve]

Figure 2. Typical stress-strain curve for a Winnipeg clay
All discussion will be based on the assumption of a constant rate of strain. In a drained test, the effective shear strength is measured directly while in an undrained test the pore pressure must be measured and subtracted from the total stress. Consider a drained direct shear test.

Peak strength is developed quite quickly. The effective cohesion value comes into effect almost immediately. The effective friction component is also developed quite rapidly needing only a small displacement to interlock the grains and provide full friction. The friction component can only be changed by a change in effective normal stress. In a direct shear test, the normal load is constant but effective normal pressure is slightly increased due to a decrease in area.

For a lightly overconsolidated clay, the surface energy component is usually small but increases with the length of duration of the test. Its relative magnitude depends on the ratio of vertical consolidation to horizontal displacement and magnitude of \( \gamma' \). This increase is usually quite small.

BOROWICKA (1963) and SKEMPTON (1964) put forth the idea that for a large number of clays, domains form in which the flakey clay particles are oriented in the direction of shear. GOLDSTEIN (1961) presented evidence that these domains begin to form at relatively small strains. This would have the result of gradually decreasing the effective cohesion until the structure was completely destroyed and the cohesion component was zero. This phenomenon along with an increase in water content along the plane of failure,
results in a decrease of shear strength from the peak to the residual value.

If the normal load is held constant a small change in normal stress will occur due to a reduction in area. This increase in normal stress will cause an increase in effective friction, $\tau_\varepsilon$, and a corresponding increase in surface energy. Therefore, the residual strength will remain approximately constant since the effective cohesion is now zero and the negative surface energy component offsets the positive change in effective friction.

**INTERPRETATION AND ERRORS IN LABORATORY RESULTS**

SEED and MITCHELL (1963) summarized the problems and difficulties in correlating laboratory strength results with actual field conditions. The conditions under which the testing for this thesis was done are outlined in Chapter III. The principal problems involved can be summarized as follows:

1. **Condition of Specimens.** This takes into account strength variations due to different methods of sampling.
2. **Testing Procedure.** The problem is one of simulating laboratory tests to field conditions.
3. **Method of Stress Application.** This includes rate of loading.
4. **Measurement of Porewater Pressures.**
5. **Strength Criteria.**

Effects of the first four problems are very difficult to assess and have been studied by a large number of investigators. A good summary of their effects is put forth by SEED and MITCHELL.
(1963), COATES and McROSTIE (1963) and CHAN and RIVARD (1963). Since this discussion is beyond the scope of this thesis it will be dropped at this point. The problem of choosing a strength criteria will be discussed in the following paragraphs as all strength results are particular to the strength criteria used.

The oldest and most widely used expression for shear strength is still the Coulomb failure criterion:

$$
\tau'_f = c' + \nu'_f \tan \phi',
$$

where,

- $\tau'_f$ = effective shear stress on plane of failure,
- $\nu'_f$ = effective normal stress on plane of failure,
- $c'$ = effective cohesion,
- $\phi'$ = effective angle of internal friction.

HVORSLEV (1960) proposed a modification to take into account the preconsolidation load. He refers to it as the Krey-Tiedeman failure criterion.

![Coulomb shear strength diagram](image)

Figure 3. Coulomb shear strength diagram
The shear strength equation then may be written as:

\[ \tau' = \nu' \tan \phi' + \nu \tan \phi' , \]

where,

- \( \nu' \) = preconsolidation pressure,
- \( \phi' \) = effective residual angle of internal friction,
- \( \tau' \) = effective peak shear strength at failure,
- \( \nu' \) = effective normal stress,
- \( \phi' \) = angle for cohesion.

For different preconsolidation pressures, lines parallel to BA would be obtained. Laboratory testing, however, has shown that the failure envelope below the preconsolidation pressure is not straight but curved as shown in Figure 3. Shear strength lines with double curvature are obtained. It is still common practice to express shear strength as a function of \( \nu' \), \( c' \), and \( \phi' \), thus neglecting the preconsolidation pressure. The main problem is to define, \( c' \), and \( \phi' \), in the range of stresses that will be encountered in the field.

Other failure criteria have been proposed and experimental work has been done on them. The main ones are the octahedral shear stress theory and the maximum shear stress theory. These theories have not achieved any significant use for practical problems mainly because until recently there has been insufficient test data to justify their use for a variety of conditions.

For the direct shear tests, the conditions at failure are easily defined. For peak strength they are simply maximum shear stress (on the failure plane), and the corresponding normal stress. Residual shear strength is the final constant shear stress value
obtained. The residual normal stress is chosen as the normal stress corresponding to the point where the stress-strain curve first becomes horizontal.

Failure conditions in a triaxial test may be defined in two ways:

1. Minimum principal effective stress ratio - $\frac{\sigma_1'}{\sigma_3'}$ where,
   $\sigma_1'$ = maximum principal effective stress
   $\sigma_3'$ = minimum principal effective stress.


SIMONS (1963) gave evidence that if the maximum principal effective stress ratio is used as the failure criteria, the same results are obtained for both drained and undrained triaxial tests. For the maximum principal stress difference criteria, undrained tests give results eight percent lower than drained tests. For sensitive clays this difference could be larger. Simons proposes that the maximum principal effective stress ratio be used for both types of triaxial tests. In accordance with this and general practise, the principal effective stress ratio will be used to interpret laboratory results. CRAWFOR D (1960) offers criticism of Simons paper but his criticisms apply mainly to very sensitive clays.

**Residual Shear Strength**

SKEMPTON (1964) proposed that the shearing resistance of an overconsolidated, fissured, clay slope would attain a reduced value over a long time interval, equivalent to the ultimate or residual shear strength. As has been shown earlier, the residual
strength is the constant shear strength that most clays exhibit after failure and under large strains.

One of the main reasons for the reduction in shear strength from the peak to residual value is the alignment of the clay particles in the direction of movement. It follows, that if rounded sand or silt particles are present, this reduction will not be as great. This relationship was illustrated by Skempton in the following Figure based on a number of different clays.

![Figure 4](image)

**Figure 4.** Decrease in $\phi'$ with increasing clay fraction

This point is of particular significance in the case of Winnipeg clays as they are all of a varved nature containing minute silt layers.

In all of the case histories given by Skempton, the peak strength obtained from the direct shear tests corresponded quite well with the strengths obtained from triaxial tests. For a varved clay the peak strength measured on a failure plane parallel to the
varves could be quite different from the strength measured on a failure plane perpendicular to the varves. The residual angle of internal friction on a failure plane along one of the minute silt layers would be very little reduced from the peak strength due to the high percentage of silt particles. It is anticipated that there also could be a difference in residual strengths along a failure plane through a silt layer and one that is basically through the clay.

Assuming the existence of a residual strength, there still must be some mechanism that will cause the peak shear strength of the clay to be exceeded in an embankment or cut section. SKEMPTON (1964) proposed a number of possibilities for producing the reduced strength:

1. Stress concentration along fissures and cracks which cause local overstress and a progressive decrease in average strength.
2. Seasonal fluctuations in moisture content and temperature.
3. Shear creep which reduces the shearing resistance over the long term.
4. Tectonic movements.

Tectonic movements associated with construction or earthquake vibrations seldom apply to slope stability problems in the Winnipeg area. Seasonal changes affect soil strength parameters to a relatively shallow depth of approximately ten feet as revealed by tests performed at test plots at the University of Manitoba. Skempton attributes the strength reduction in London clays to the presence of fissures which act as stress concentrators. Very little
is known about the effects of creep on long term strength. It is known that for some clays, creep strength is significantly less than peak strength. As an example, CASAGRANDE and WILSON (1951) reported that the creep strength of Bear Paw Shale was 80 percent of the peak strength.

Skempton's mathematical treatment of residual strength as applied to slope stability is as follows:

![Diagram](image)

**Figure 5. Definition of residual factor**

Peak strength is represented by the expression $\tau_f = c' + \sigma' \tan \phi'$. Residual strength is represented by the expression $\tau_R = c'_R + \sigma' \tan \phi'_R$ which can be reduced to $\tau'_R = \sigma' \tan \phi'_R$ since cohesion is almost totally destroyed with the mobilization of residual strength. For the purposes of analysis, Skempton introduced a residual factor "$R$" defined as:

$$R = \frac{\tau_f - \tau}{\tau_f - \tau_R}$$
where, \( \tau' \) = peak shear strength,
\( \bar{\tau} \) = average shear strength,
\( \tau'_R \) = residual shear strength.

For a failed slope where the safety factor is one, \( R \) will equal zero if no reduction in strength has occurred and all the clay along the slip surface is at peak strength. If the average strength has reached the residual value, \( R \) will equal one. Skempton claims that in physical terms, \( R \) is the proportion of the total slip surface in the clay along which its strength has fallen to the residual value. There is also the possibility that since \( R \) is based on average shear strength along the slip surface, that all the clay along the slip surface is at some reduced value between the peak and residual strengths. This would be the case if reduced strength was due to creep rather than overstress due to cracks and fissures.
CHAPTER III
LABORATORY INVESTIGATIONS OF SHEAR STRENGTH

As mentioned in Chapter I, the soil stratum of major interest in dealing with slope stability problems in the Winnipeg area is the highly plastic lacustrine clay. The laboratory testing described in the following sections was conducted entirely on samples taken from this lacustrine clay stratum. It was not possible to obtain samples directly from the site of the Floodway Test Pit, with the result that representative samples from other sites in the Greater Winnipeg area had to be used. This will obviously introduce some error into the analysis of the Test Pit slopes, but the uniformity of the clays in the area is such that the error should not be large.

The bulk of the testing was performed on samples obtained from various depths at a site near the University of Manitoba which is approximately ten miles from the test slope. One set of direct shear tests was performed on chunk samples obtained from a depth of thirty-two feet at a site in Transcona approximately four miles from the test slope. Included in the summary of triaxial tests are results obtained by a fellow graduate student Prasop Krasaesindhu. These samples were also obtained from locations near the university.

A small number of classification tests were performed on the samples from the University location. The average results of these tests are listed in Table 1.
TABLE 1
RESULTS OF LABORATORY CLASSIFICATION TESTS

<table>
<thead>
<tr>
<th></th>
<th>Brown Clay</th>
<th>Grey Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Limit</td>
<td>106.5</td>
<td>94.6</td>
</tr>
<tr>
<td>Plastic Limit</td>
<td>30.7</td>
<td>29.5</td>
</tr>
<tr>
<td>Plastic Index</td>
<td>75.8</td>
<td>75.1</td>
</tr>
<tr>
<td>Clay Content (%)</td>
<td>83</td>
<td>76</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>2.68</td>
<td>2.70</td>
</tr>
</tbody>
</table>

A summary of the classification properties of the three clay layers was prepared by Baracos in 1961 and was later verified by the numerous tests conducted on the Red River Floodway. Baracos' results are tabulated in Table 2. A more exhaustive description of soil properties and profile is contained in papers by BARACOS (1961), RIDDELL (1950), and MacDONALD (1937).
TABLE 2

PROPERTIES OF GREATER WINNIPEG GLACIAL LAKE CLAYS

AFTER BARACOS - 1969

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max.</td>
<td>Av.</td>
<td>Min.</td>
<td>Max.</td>
</tr>
<tr>
<td>Depth to top of stratum - ft.</td>
<td>16</td>
<td>11</td>
<td>2</td>
</tr>
<tr>
<td>Depth to bottom of stratum - ft.</td>
<td>40</td>
<td>25</td>
<td>11</td>
</tr>
<tr>
<td>Moisture Content - %</td>
<td>57</td>
<td>48</td>
<td>27</td>
</tr>
<tr>
<td>Dry Density - p.s.f.</td>
<td>99</td>
<td>77</td>
<td>64</td>
</tr>
<tr>
<td>Moist Density - lb/cu.ft.</td>
<td>125</td>
<td>109</td>
<td>95</td>
</tr>
<tr>
<td>Saturation - %</td>
<td>100</td>
<td>97</td>
<td>86</td>
</tr>
<tr>
<td>Unconfined Compression Strength - p.s.f.</td>
<td>4570</td>
<td>2054</td>
<td>865</td>
</tr>
<tr>
<td>Plastic Limit</td>
<td>40</td>
<td>30</td>
<td>14</td>
</tr>
<tr>
<td>Liquid Limit</td>
<td>117</td>
<td>89</td>
<td>37</td>
</tr>
<tr>
<td>Plasticity Index</td>
<td>88</td>
<td>59</td>
<td>23</td>
</tr>
</tbody>
</table>

Note: Numbers rounded to nearest integer.
CONSOLIDATION TESTS

Two consolidation tests were performed on the University samples using a 2.5 inch diameter fixed ring consolidometer. Tests were performed using the no swell procedure. Permeabilities were calculated for the 2500 p.s.f. load increment at fifty percent consolidation. The following results were obtained.

TABLE 3

RESULTS OF CONSOLIDATION TESTS

<table>
<thead>
<tr>
<th>Depth (Ft.)</th>
<th>Compressive Index (Sq. Ft/Ton)</th>
<th>Swelling Pressure (P.S.F.)</th>
<th>Preconsolidation Pressure (P.S.F.)</th>
<th>Permeability (Cm./Sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>0.38</td>
<td>800</td>
<td>5600</td>
<td>$1.3 \times 10^{-8}$</td>
</tr>
<tr>
<td>15</td>
<td>0.80</td>
<td>700</td>
<td>4200</td>
<td>$0.8 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

TRIAXIAL TESTS

Six consolidation drained triaxial tests were performed with a loading frame as shown in Figure 6. The samples were brought to failure by increasing the axial load while maintaining a constant cell pressure. Additional load increments were not applied until there was no further volume change under the previous load increment. Filter strips were placed around the samples to facilitate drainage. The failure conditions were determined by the minimum effective principal stress ratio ($V'/V'_s$). No membrane or piston friction corrections were applied. However, a trial calculation using a piston friction correction was made and it was found that the effect on the final result was very small.
Figure 6. Triaxial Test Apparatus

The results of the tests are plotted on Figure 7 in the form of \((\sigma'_1 + \sigma'_2)\) versus \((\sigma'_1 - \sigma'_3)\). Included are the results of similar consolidated drained triaxial tests obtained by KRASAESINDHU (1965). His samples were tested under identical conditions with the exception of some samples where he varied both principal stresses while keeping the octahedral normal stress
FIGURE 7: SUMMARY OF TRIAXIAL TEST RESULTS
constant. His remaining samples were brought to failure in the conventional manner of increasing the major principal stress only.

A best fit line (upper line on Figure 7) was drawn through the points and the angle of inclination, \( B \), and the intercept, \( C_S \), were converted to the parameters, \( \phi' \), and \( c' \), using the following conversions:

\[
\sin \phi' = \tan B, \quad c' = \frac{C_S}{2 \cos \phi'},
\]

where,

\( \phi' \) - effective angle of internal friction,

\( c' \) - effective cohesion.

This resulted in an effective angle of internal friction of thirteen degrees and an effective cohesion value of 5.0 p.s.i.

SKEMPTON (1964) proposed a method for determining residual strength from a consolidated drained triaxial test. He proposed that if the failure plane was precut with a fine wire saw, then all cohesion along the plane would be destroyed and the strength measured would be the residual shear strength. The failure plane is precut at an angle of \( (45^\circ + \phi'/\sqrt{2}) \) degrees to the major principal axis with, \( \phi' \), being the residual angle of internal friction. Two sets of consolidated drained triaxial tests were conducted using the same procedures as previously described with failure planes cut at fifty-two and fifty-four degrees with the minor principal axis. The results of these tests are also plotted on Figure 7 (lower line) and reveal a markedly reduced cohesion value with an effective angle of internal friction of thirteen
degrees. Varying the angle of the precut failure plane from fifty-two to fifty-four degrees had very little effect on results.

**DIRECT SHEAR TESTS**

Direct shear tests were performed using a constant strain machine developed by Dr. Bishop of Imperial College in England. The machine is illustrated in the accompanying photographs, Figures 8 and 9. The dimensions of the specimens were 6.0 cm. by 6.0 cm. by 2.0 cm. thick. Special four inch diameter Shelby spoon samples were recovered in order to provide a large enough sample from which the specimens could be cut.

A constant rate of strain of 0.000096 in. per min. was chosen for all tests. The value was arrived at after computing the time to failure for a consolidated drained test as shown by GIBSON and KENKEL (1954). The actual rate of strain of 0.000096 in. per min. is in fact somewhat slower than the calculated rate. This was done because of an absence of any previous experimental evidence to corroborate the computed rate and the fact that a completely drained result was desired. Consolidation readings were taken after the application of the vertical load. A time versus consolidation curve was then plotted to determine when one hundred percent consolidation had been obtained. No horizontal loads were applied until one hundred percent consolidation had been obtained.
Figure 3. Direct Shear Apparatus

Figure 9. Direct Shear Apparatus
No shear reversal technique was possible with this machine. There was, however, provision made to insert a spacer block in the shear box at the mid-point of the test. The sample had to be unloaded in order to insert the spacer block. The samples were then reloaded and carried to a total strain of up to twenty-five percent. This unloading and reloading did not seem to seriously affect results. All samples were carried to residual strength if possible.

One set of tests was performed with the failure planes parallel to the stratification of the soil and one set was performed with the failure planes perpendicular to the varves. This enabled the determination of four sets of shear strength parameters as both peak and residual strengths were defined for each case. The stress-strain curves for both cases are plotted on Figures 10 and 11.

It is difficult to evaluate the corrections to be applied to the area of shear. The walls of the shear box are approximately 0.5 in. thick. As the test progressed, the soil along the completely sheared off portions of the failure plane passed over the thick metal edge of the shear box. It was difficult to tell if the soil and the metal were in contact and if so to estimate what percentage of shear strength was being developed between the two surfaces. To further investigate the effects of applying an area correction, the results of the tests conducted parallel to the natural soil stratification were computed using both a full area correction and no area correction. The remaining
FIGURE 10: STRESS-STRAIN CURVES FROM DIRECT SHEAR TESTS WITH THE FAILURE PLANE I TO THE VARVES AND FULL AREA CORRECTION
FIGURE II: STRESS-STRAIN CURVES FROM DIRECT SHEAR TESTS WITH THE FAILURE PLANE // VARVES AND FULL AREA CORRECTION.
results were computed using full area correction only.

The results of each set of tests are plotted on Figures 12, 13, and 14. They are further summarized in Table 4.

<table>
<thead>
<tr>
<th>Description of Test</th>
<th>Area Correction</th>
<th>$\phi'$</th>
<th>$c'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak strength - // varves</td>
<td>Full</td>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>Peak strength - // varves</td>
<td>None</td>
<td>15.5</td>
<td>2</td>
</tr>
<tr>
<td>Residual strength - // varves</td>
<td>Full</td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td>Residual strength - // varves</td>
<td>None</td>
<td>12.7</td>
<td>0</td>
</tr>
<tr>
<td>Peak strength - ⊥ varves</td>
<td>Full</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>Residual strength - ⊥ varves</td>
<td>Full</td>
<td>13</td>
<td>0</td>
</tr>
</tbody>
</table>

DISCUSSION OF RESULTS

As can be seen from Table 4 the effect of applying full area correction or no area correction to direct shear test results has a minimal effect on the shear strength parameters. Both shear stress and normal stress are inversely proportional to the area. Thus any change in area will affect both stresses proportionately. A comparison of Figures 13 and 14 reveals that the results of individual tests plot in different locations, but the failure envelopes are approximately the same.

Peak shear strength parameters obtained from direct shear tests with the failure plane parallel to the varves were $\phi' = 15^\circ$
FIGURE 13: DIRECT SHEAR TEST RESULTS WITH THE FAILURE PLANE // VARVES AND FULL AREA CORRECTION
\( \phi' = 15.5^\circ \quad C' = 0 \text{ P.S.I.} \)

\( \phi'' = 12.7^\circ \quad C'' = 0 \text{ P.S.I.} \)

**Figure 14: Direct Shear Test Results with Failure Plane // Varves and No Area Correction**
and $c' = 2.0$ p.s.i., while corresponding values for the failure plane perpendicular to the varves are $\phi' = 13.0$ degrees and $c' = 4.0$ p.s.i. The difference in cohesion values can be explained by the fact that in the first instance the samples failed along one of the minute silt layers where the increased silt content lowered the cohesion from 4.0 p.s.i. to 2.0 p.s.i. Also, the added silt content along the failure plane might possibly tend to increase the $\phi'$ value from 13.0 to 15.0 degrees. The residual angle of internal friction, however, was measured as 13.0 degrees. In order to assess the effect that this increase would have on stability analysis an assumed set of parameters ($\phi' = 13.0$ degrees and $c' = 2.0$ p.s.i.) were also used in the stability analysis in Chapter V.

Theoretically, the peak shear strength along a failure plane at some angle to the natural stratification of the soil (or basically through the clay portion of the stratum) should be the same regardless of the method of the test. Direct shear tests produced parameters of $\phi' = 13.0$ degrees and $c' = 4.0$ p.s.i. while triaxial tests produced values of $\phi' = 13.0$ degrees, and $c' = 5.0$ p.s.i. These results compare favourably with triaxial results reported by MATYAS (1967) ($\phi' = 16.7^\circ$ and $c' = 4.3$ p.s.i.), but are in sharp disagreement with results published by CRAWFORD (1964) ($\phi' = 9.0^\circ$ and $c' = 8.3$ p.s.i.). The slower rate of loading used by Crawford for his triaxial tests is believed to be the reason for this discrepancy. The values published by Crawford will also be used in the stability analyses in Chapter V.
The residual shear strength parameters as shown on Figures 12, 13 and 14 were $\phi' = 13.0^\circ$ and $c' = 0$ p.s.i.
CHAPTER IV
ANALYSIS OF THE FLOOD WAY TEST PIT
USING DIFFERENT METHODS OF ANALYSIS

In order to assess the various methods of stability analysis available at the present time, the Red River Floodway test slope will be analyzed using the various methods of analysis and keeping all other variables constant. Since numerous papers have been published describing and evaluating all methods in use at the present time, no rigorous development will be presented. Each method will be analyzed as follows:

1. Discussion and evaluation of theoretical basis.
2. Computation of safety factors for the test slope.
3. Discussion and evaluation of results.

RED RIVER FLOODWAY TEST PIT

The Red River Floodway Test Pit was constructed in the fall of 1961 to provide information for the design of side slopes for the Red River Floodway. The Floodway was designed to carry the flood waters of the Red River around the City of Winnipeg and prevent the serious spring flooding which had always plagued the city. A detailed description of the test pit, field installations, laboratory investigations and slope stability analyses performed at that time are contained in an unpublished report prepared by the Canada Department of Agriculture (P.F.R.A. branch) in 1962. Figure 15 contains a plan view and cross-section of the entire excavation.
FIGURE 15: PLAN AND CROSS-SECTION OF THE TEST PIT
The original failure occurred on the North slope on October 8, 1962. This slope had been excavated at an angle of approximately 45 degrees and was 34 feet high prior to failure. Figure 16 shows cross sections and plan views of the slope both before and after failure. Slide plane detectors in the area indicated that the failure plane was at a depth of 48 feet and definitely not through the glacial till. Other evidence contained in the report proved that the possibility of a failure arc through the glacial till could be neglected. Average depth to bedrock in the area was 58 feet.

A line of Casagrande piezometers in the area ran directly through the slide area. The water levels from these piezometers were used to predict pore pressures at the time of failure. The P.F.R.A. analyzed two different cross-sections, one on centerline and one 75 feet West of centerline. Since the line 75 feet West of centerline runs through the actual slide area, it will be used for all analyses in this paper.

There was some difficulty in choosing piezometer readings to represent pore pressures at the time of failure. This was due to the fact that there were significant changes in water levels right up to the time of failure. Originally water levels were 6.0 to 7.0 feet below ground surface. For the first 20 feet of excavation the water levels dropped an amount equal to the depth of soil excavated. On October 2, 1962 when the depth of excavation was slightly less than that on October 8, all piezometers showed a rise in water level.
FIGURE 16: CROSS-SECTIONS BEFORE AND AFTER SLIDE

PLAN OF NORTH SLOPE BEFORE SLIDE

SECTION A-A BEFORE SLIDE OCT. 5, 1961

PLAN OF NORTH SLOPE AFTER SLIDE

SECTION A-A AFTER SLIDE OCT. 8, 1961

SCALE 1" = 20'
Water levels then proceeded to drop once more until failure occurred. The authors of the P.F.R.A. report believe that failure actually occurred October 2, but that slight movements caused the water pressure to be relieved and complete failure did not occur till later. Initial signs of movement were first noted at this earlier date. For this reason the water levels of October 2, along with the depth of excavation at that time were used for all analyses.

The geometrics of the slope that will be used for all stability analysis is illustrated on Figure 17.

SAFETY FACTOR

Factor of safety as it applies to slope stability can be defined in the following manner: That factor of safety by which the strength parameters may be reduced to bring the soil mass into a state of limiting equilibrium along a given slip surface. The main limitation of this definition is that it does not account for the stress path taken to failure i.e. Safety factor for total stresses does not equal the safety factor for effective stresses.

If the shear strength of a cohesive soil is represented by $\tau' = c' + \gamma' \tan \phi'$, then the portion of shear strength mobilized along any slip surface will be:

$$\tau' = \frac{c'}{F} + \frac{\gamma' \tan \phi'}{F}$$

where,
- $\tau'$ = shear strength mobilized,
- $F$ = factor of safety
- $c'$ = effective cohesion
FIGURE 17: TEST SLOPE
\[ \phi' = \text{effective angle of internal friction} \]
\[ \gamma' = \text{effective normal stress} \]

**PORE PRESSURES**

Bishop and Bjerrum (1960) state that all problems involving pore pressure prediction can be divided into two classes:

(a) Problems where pore pressure is an independent variable and is controlled by either ground water level or by flow pattern of impounded or underground water.

(b) Problems in which the magnitude of the pore pressure depends on the magnitude of the stresses tending to lead to instability, as in rapid construction or excavation in soils of low permeability.

Due to instrumentation at the test site, the actual pore pressures at or very near the instant of failure were recorded. Therefore the piezometric surface is defined eliminating the necessity of predicting pore pressures. As has been mentioned previously, two different sets of piezometer readings were investigated by the P.F.R.A. For this analysis, the readings of October 2 were used throughout.

**SHEAR STRENGTH PARAMETERS**

The shear strength parameters used are average, \( \phi' \), and, \( c' \), values obtained from consolidated undrained triaxial tests performed by the P.F.R.A. and other investigators. At the time
these analysis were performed the laboratory testing program had not been completed. The parameters used were \( c' = 5.0 \) p.s.i. and \( \phi' = 16 \) degrees.

No tests were performed to determine the unconfined compressive strength. In the course of their laboratory investigations the P.F.R.A. tested a large number of samples from the test site itself and investigated results from a number of other projects. All strengths compiled are in close agreement and are accepted as being valid. The actual values will be listed with their corresponding safety factors in the next section on total stress analysis.

**TOTAL STRESS METHOD OF ANALYSIS**

The total stress \( (\phi = 0) \) method applies to homogeneous, isotropic, saturated clays. If saturated clays are tested under conditions of no water content change they will exhibit a \( \phi = 0 \) characteristic at failure. The method as applied to slope stability has been used since the early 1900's and is well documented by Skempton (1948). The expression for the factor of safety using the slip circle analysis is:

\[
F = \frac{c \tan \phi}{\gamma W \sin \alpha}
\]

Note: All terms are defined in Figure 18.

Tables for the solution of this simple relationship have been published by a number of different investigators including Terzaghi, Taylor and Janbu. May published a
\( E_n, E_{n+1} \) - resultants of the total horizontal forces on the sections \( n \) and \( n + 1 \) respectively

\( X_n, X_{n+1} \) - the vertical shear forces

\( W \) - total weight of the slice of soil

\( P \) - total normal force acting on its base

\( h \) - height of element

\( S \) - shear force acting on its base

\( b \) - width of element

\( l \) - length of arc BC

\( l_a \) - length of arc AD

\( \alpha \) - angle between BC and horizontal

\( \gamma \) - horizontal distance from slice to centre of rotation

\( C_v \) - cohesion value determined from unconfined compressive strength test

\( U \) - component of pore pressure normal to failure surface

**Figure 18:** Symbols Used In Slope Stability Analyses
graphical solution. For their analysis, the P.F.R.A. used Janbu's tables and May's graphical method of solution. Two different unconfined compressive strengths were used:

(i) strength measured from a test hole in the actual slide area.
(ii) an average strength from samples throughout the general area. The results were:

<table>
<thead>
<tr>
<th></th>
<th>S.F. (Measured c)</th>
<th>S.F. (Average c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>May's Method</td>
<td>1.79</td>
<td>1.39</td>
</tr>
<tr>
<td>Janbu's Method</td>
<td>1.84</td>
<td>1.44</td>
</tr>
</tbody>
</table>

Each method of analysis gives essentially the same result. The P.F.R.A. also investigated the effect of assuming a ten foot deep tension crack. Safety factors were only slightly lower for this more severe condition. The unconfined compressive strength measured from samples obtained in the actual slide area was much higher than the average for the surrounding district. In any case, the total stress analysis substantially overestimated the factor of safety.

Two months elapsed from the beginning of construction until failure occurred. The coefficient of permeability calculated from the consolidation tests was in the order of 10^-8 cm/sec or approximately 1/8 inch per month. Because of this relatively impervious structure it is unlikely that any significant moisture content
changes would have occurred due to the stress change prior to failure.

**FRICION CIRCLE METHOD**

The friction circle method is a convenient approach for both graphical and mathematical solutions. The graphical solution is presented in detail by TAYLOR (1948). The analysis performed was actually the modified friction circle approach. The radius of the friction circle was taken as KR sin \( \theta \). The \( K \) values were obtained from Figure 16.16 in TAYLOR (1948). There is a possible source of error in this calculation as Figure 16.16 had to be extrapolated to meet the requirements of this problem. The factor of safety defined was the factor of safety with regard to strength.

The factor of safety obtained by this analysis was 1.95 as shown on Figure 19. BISHOP and BJERRUM (1960) give evidence that the results obtained by this method should be in close agreement with results from Bishop's Method of Slices.

**SWEDISH METHOD OF SLICES**

This method proposed by Fellenius around the turn of the century is based on the following assumptions:

1. Safety factor is constant along the failure surface
2. The shear strength mobilized is defined as:
   \[
   \tau = \frac{c'}{E} + \frac{\sigma}{E} \tan \phi'
   \]
3. The resultant of the forces on the sides of the slices has no component in the direction normal to the failure arc.
   The standard solution is expressed as:
FIGURE 19: STABILITY ANALYSIS BY FRICITION CIRCLE METHOD ($\theta = 16^\circ$, $C' = 5.0 \text{ O.S.I.}$)
\[ F = \frac{R \cos \phi + R_U (P-U) \tan \phi}{R \sin \phi} \]

Note: Symbols defined on Figure 18.

This solution satisfies two conditions of equilibrium namely: summation of moments about the centre and summation of forces normal to the failure arc. The result obtained from a partial graphical and mathematical solution was a safety factor of 1.75 as shown on Figure 20.

BISHOP'S METHOD OF SLICES

This method introduced by BISHOP (1954) is similar to the Swedish Method of Slices but takes full account of both horizontal and vertical forces between slices. The factor of safety "F" is given by:

\[ F = \frac{1}{W \sin \alpha} \left\{ \sum \left[ \frac{c b + \tan \phi [W (1-r_U) + (X_n - X_n*)]}{1 + \tan \phi - \tan \alpha} \right] \right. \]

Note: All symbols defined on Figure 18.

Mathematical and graphical solutions are extremely laborious but the solution can be programmed for the computer. For the case in question, a computerized solution yielded a safety factor of 1.94 as shown on Figure 21.

DISCUSSION OF RESULTS

Table 6 summarizes the results of the various methods of analyses. In order to compare the locations of the centres of the failure circles a set of co-ordinates has been set up with the origin at the crest of the slope with x values increasing positively to the left and y values increasing positively upwards.
FIGURE 20: STABILITY ANALYSIS BY SWEDISH METHOD OF SLICES ($\phi' = 16^\circ, C' = 5.0$ P.S.I.)
Figure 21: Stability analysis by Bishop's method of slices ($\phi' = 16^\circ$, $c' = 5.0$ psi.)

S.E. = 1.94
The following conclusions can be drawn from Table 6.

1. All methods of analysis substantially overestimate the factor of safety.

2. As was first proven by BISHOP and BJERRUM (1960), the factor of safety determined by Bishop's Method of Slices agrees within 1.0 percent with the value obtained by the friction circle method of analysis.

3. The Swedish Method of Slices produces a lower factor of safety than either of the other effective stress analyses, implying that it is a more conservative method of design than the other two.

Conclusions Number 2 and 3, were to be expected from the
work of other investigators, BISHOP and BJERRUM (1960),
BISHOP (1954). Of major importance, however, is the fact that
using typical shear strength parameters obtained from standard
laboratory triaxial tests, all methods of analysis substantially
overestimated the factor of safety. The investigation also
revealed that, as would be expected, the fastest and most
accurate method of performing an effective stress analysis is to
use computer programming. If a computer solution is to be
used it can be applied to a more rigorous and correct solution
(Bishop's Method of Slices) with very little additional time
and cost.
CHAPTER V

ANALYSIS OF FLOODWAY TEST PIT USING DIFFERENT
SETS OF SHEAR STRENGTH PARAMETERS

The test slope was analyzed using six different sets of shear strength parameters which are listed as follows:

1. Residual shear strength parameters as determined by direct shear tests - $\phi' = 13.0^\circ$, $c' = 0$ p.s.i.

2. Peak shear strength parameters as determined by direct shear tests with the failure plane parallel to the varves - $\phi' = 15.0^\circ$, $c' = 2.0$ p.s.i.

3. Peak shear strength parameters obtained by combining results of various direct shear tests both perpendicular and parallel to the varves - $\phi' = 13.0^\circ$, $c' = 2.0$ p.s.i.

4. Peak shear strength parameters as determined by direct shear tests with the failure plane perpendicular to the varves - $\phi' = 13.0^\circ$, $c' = 4.0$ p.s.i.

5. Peak shear strength as determined by consolidated drained triaxial tests - $\phi' = 13.0^\circ$, $c' = 5.0$ p.s.i.

6. Peak shear strength parameters as determined by triaxial tests performed by Crawford of the National Research Council $\phi' = 9^\circ$, $c' = 8.3$ p.s.i.

The method of analysis used was the computerized solution of Bishop's Method of Slices. For cases Number 2 and 3 a large number of circles were analyzed which provided enough information to completely plot a number of safety factor contours. In the other four cases, only enough circles were analyzed to locate
the worst failure arc. This search procedure was simplified by the fact that the position of the centre of the worst failure circle was only slightly affected by a change in shear strength parameters. The safety factors obtained are listed in the following table:

**TABLE 7**

SAFETY FACTORS FOR VARIOUS SHEAR STRENGTH PARAMETERS

<table>
<thead>
<tr>
<th>Shear Strength Parameters (Degrees)</th>
<th>(p.s.i.)</th>
<th>Type of Test</th>
<th>Definition of Shear Strength</th>
<th>Safety Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>0</td>
<td>Direct Shear</td>
<td>Residual</td>
<td>0.64</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>Direct Shear</td>
<td>Peak (// to Varve)</td>
<td>1.20</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>Empirical*</td>
<td></td>
<td>1.09</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>Direct Shear</td>
<td>Peak (⊥ to Varve)</td>
<td>1.60</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
<td>Triaxial</td>
<td>Peak</td>
<td>1.91</td>
</tr>
<tr>
<td>9</td>
<td>8.3</td>
<td>Triaxial</td>
<td>Peak</td>
<td>2.36</td>
</tr>
</tbody>
</table>

*Formulated by combining $\theta$' value obtained from shear tests perpendicular to the varves and $c'$ from shear tests parallel to the varves.

**CASE 1**

The factor of safety obtained using residual shear strength parameters was 0.64, thus substantially under-estimating the factor of safety. The worst failure circle is shown on Figure: 22. Depending on the definition of peak shear strength the following values were obtained for Skempton's residual factor "R"
S.F. = 0.64

FIGURE 22: STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES (\(\phi' = 13^\circ\), \(c' = 0\) P.S.I.)
TABLE 8
RESIDUAL FACTORS

<table>
<thead>
<tr>
<th>Definition of Peak Shear Strength</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Shear // Varves</td>
<td>0.36</td>
</tr>
<tr>
<td>Theoretical Direct Shear // Varves</td>
<td>0.23</td>
</tr>
<tr>
<td>Direct Shear ⊥ Varves</td>
<td>0.62</td>
</tr>
<tr>
<td>Triaxial Test (after MUIR)</td>
<td>0.71</td>
</tr>
</tbody>
</table>

In Skempton's terminology, these factors represent the portion of the failure arc along which the clay has been reduced to its residual strength. The residual factor for this test slope would be expected to be very low. The slope was a cut section which failed during excavation and approximately two months after excavation began. Chapter III lists four mechanisms that tend to reduce peak strength to residual strength in a clay slope. Assuming any or all of these mechanisms were present, the time interval of approximately two months appears too short to have permitted any substantial reduction in shear strength. Skempton reports that for cut sections in weathered London clay a period of 20 to 30 years is required to develop a residual factor of 0.5.

CASES 2 and 3

Using the laboratory defined values of peak strength parallel to the varves, a factor of safety of 1.20 was obtained. As mentioned in Chapter III, there is some question as to whether the $\phi'$ value of 15 degrees actually should have been 13 degrees
as was obtained from all other strength determinations. Using 
\[ \phi' = 13 \text{ degrees} \] and \[ c' = 2 \text{ p.s.i.} \] a reduced factor of safety 
of 1.09 was obtained. The "safety factor contours" for these two 
cases are plotted on Figures: 23 and 24.

Taking into account all assumptions there were made in the 
analysis, both methods closely approximate the desired factor 
of safety of one. Some verification of this result is given in 
Fundamentals of Soil Mechanics by TAYLOR (1948) as follows:
"an approximate method that may be used in stratified soils 
consists, first, of laboratory tests to obtain the shearing 
strength on planes parallel to the strata; then the stratification 
is ignored, the shearing strengths that have been obtained are 
assumed to be valid for all surfaces through the embankment, the 
assumption of circular failure arcs is used, and the analysis 
is carried through according to the usual procedures". Taylor 
contends that the shearing strength assumed is the correct one 
for over 50 percent of the failure arc and that the portions of 
the failure arc that do not follow the stratification are the 
shallower parts where pressures and shear strengths are relatively 
low.

Based on both the theoretical and statistical evidence 
it appears that these shear strength parameters can be used to 
predict safety factors for these conditions. As explained by 
Taylor, however, the method is at best an approximate one and 
cannot be relied upon for extremely accurate predictions of 
safety factors.
FIGURE 23: STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES ($
\phi' = 15^\circ$, $C' = 2.0$ P.S.I.)
FIGURE 24: STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES (\(\phi' = 13^\circ\), \(c' = 2.0\) P.S.I.)
CASES 4, 5, and 6

The three sets of shear strength parameters listed in the bottom of Table 7 all are defined as peak shear strength along a failure plane at some angle to the natural stratification of the soil. The discrepancy between the three sets of parameters has been discussed in Chapter III. In any event all three substantially over-estimate the factor of safety as values range from 1.60 to 2.36. The worst failure circle for the three cases are plotted on Figures: 25, 26 and 27.

In light of the preceding discussion of cases 2 and 3, and previous investigations in the Winnipeg area, these results were to be expected. The results conclusively prove that an analysis using peak shear strengths determined on a failure plane at some angle to the natural stratification of the soil will substantially overestimate the factor of safety.
FIGURE 25: STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES ($\phi^\prime = 13^\circ$, $c^\prime = 4.0$ P.S.I.)
FIGURE 26: STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES (\(\phi' = 13^\circ, C' = 5.0 \text{ P.S.I.}\))
FIGURE 27: STABILITY ANALYSIS BY BISHOP'S METHOD OF SLICES (\( \phi' = 9^\circ \), \( C' = 8.3 \text{ P.S.I.} \))
CONCLUSIONS

1. Using shear parameters determined by standard triaxial testing procedures, all total and effective methods of stability analysis substantially overestimated the factor of safety.

2. The residual angle of internal friction for the clays tested was basically the same as the angle of internal friction for peak strength. As expected the effective cohesion was zero for residual strength.

3. Peak shear strength along a failure plane parallel to the natural soil stratification was significantly less than peak shear strength along a failure plane at some angle to the varves.

4. Stability analyses of new cut sections using residual shear strength parameters were found to substantially underestimate the factor of safety. A period of many years would be required for the shear strength of the clay in a cut section to be substantially reduced and approach residual strength. Residual shear strength parameters are likely in affect along the failure arc of any previously failed slope. This is likely the case for the majority of the river banks in the Winnipeg area as most of these banks have undergone failures at some time in their past history.
5. For the cut section analyzed, an approximately correct safety factor was obtained by using Bishop's effective stress method of analysis and shear strength parameters determined by a direct shear test with the failure plane parallel to the soil stratification. The method of analysis assumed a circular failure arc tangent to the underlying glacial clay till.

RECOMMENDATIONS FOR FURTHER INVESTIGATION

1. Further constant rate of strain direct shear tests of the Winnipeg clays are necessary. The tests are rather time consuming requiring approximately five days for each of the 25 tests performed for this thesis. Tests should be performed with the failure plane both perpendicular and parallel to the varves.

2. Studies should be made of existing case histories and any future case histories to determine the exact shape and location of the failure arc. It would be especially valuable to know if the failure arc is tangent to the underlying glacial clay till.
APPENDIX A - BIBLIOGRAPHY


MATYAS (1967) Shear Strength Characteristics of Two Clays From Western Canada.


