ANALYSIS OF PORE WATER STRESS IN LAKE AGASSIZ CLAY THROUGH A SEPARATION OF THE VOLUMETRIC AND DEVIATORIC COMPONENTS OF STRESS

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LIST OF SYMBOLS

Symbol	Definition
σ	Normal Stress
τ	Shear Stress
ℓ, m, n	Direction cosines
J_1, J_2, J_3	First, Second and Third Stress Invariants
$\sigma_x, \sigma_y, \sigma_z$	Normal Total Stresses on Faces of Elemental Unit Volume; Normal Stresses at a Point in Coordinate Directions x, y, z
$\sigma_1, \sigma_2, \sigma_3$	Principal Stresses
σ_o	Octahedral Normal Stress ($\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$)
τ_o	Octahedral Shear Stress ($\frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}$)
$\epsilon_1, \epsilon_2, \epsilon_3$	Principal Strains
ϵ_o	Octahedral Normal Strain ($\frac{1}{3}(\epsilon_1 + \epsilon_2 + \epsilon_3)$)
γ_o	Octahedral Shearing Strain ($\frac{2}{3}\sqrt{(\epsilon_1-\epsilon_2)^2+(\epsilon_2-\epsilon_3)^2+(\epsilon_1-\epsilon_3)^2}$)
A_c	Area of Sample after Isotropic Compression
A	Area of Sample During Shearing Test
P	Axial Load Increment
d	Initial Diameter of Sample
d_c	Diameter of Sample after Isotropic Compression
L_o	Initial Height of Sample

Symbol	Definition
L_c	Height of Sample after Isotropic Compression
V_o, V	Initial Volume of Sample
V_c	Volume of Sample after Isotropic Compression
U	Pore Pressure
B, A	Skempton's Pore Pressure Parameters
β, α	Pore Pressure Parameters

SUMMARY

The maximum resistance to shear on any plane in the soil is a function, not of the total normal stress acting on the plane, but of the difference between the total normal stress and the pore pressure so that the study of the relationship between stress and pore pressure is very important. Skempton established the concept of pore pressure parameters to relate changes in pore pressure with changes in stress. He related the change in pore pressure to the minor principal stress and to the maximum principal stress difference. This approach does not take into account the intermediate principal stress nor does it unequivocally separate effects of normal and shearing stresses.

This paper describes a method for determining the pore pressure at any state of stress. This is done by separating the mechanics of the soil behavior into a volumetric and a deviatoric component. These components refer to the stress-strain relationship involving changes in the mean principal stress and changes in the deviatoric components of stress. The mean principal stress is equal to the octahedral normal stress and the resultant deviator stress is equal to the octahedral shear stress. The change in pore pressure is thus directly related to the changes in the octahedral normal and octahedral shear stresses.

The relationship between the octahedral normal stress and the pore pressure was determined by means of isotropic compression tests. The coefficient of proportionately β was experimentally determined to be 0.86 for stresses in excess of those imposed by the weight of the existing overburden.

The relationship between the induced pore pressure and the octahedral shear stress, or the pore pressure parameter α, was investigated by means of constant octahedral normal triaxial compression tests. The values of parameter α were found to vary from about 1.0 to values in the negative range depending on the magnitude of the over-consolidation ratio of the soil as well as the magnitude of shear stress.

To assess the validity of this method, a comparison was made between measured pore pressure in the conventional triaxial compression tests and pore pressure computed on the bases of changes in volumetric and deviatoric stresses. Very good agreement was found between the predicted and the observed pore pressures. These results indicate that predicting pore pressure through a separation of the volumetric and deviatoric components appears to be valid.

CHAPTER I

INTRODUCTION

The strength and deformation characteristics of soil are understood by visualizing it as a compressible skeleton of solid particles enclosing continuous void space. Terzaghi (1) was the first one to express the normal stress on any plane of soil as the sum of two components. The first component is the stress carried by the solid particles which is called the effective stress and the second component is the pressure in the fluid in the void space which is called the pore pressure. Therefore, the higher the pore pressure, the less will be the effective stress for a given total normal stress. The shear stress of course can only be carried by the skeleton of solid particles. Generally a change either in the normal stress or in shear stress carried by the solid skeleton of the soil results in a tendency for a volume change within the soil mass. An increase in normal stresses will induce a volume reduction. Unless, the fluid in the pore space can be freely expelled, an excess pore pressure will temporarily result from the stress change. The rate at which this excess pore pressure will dissipate depends on the permeability of the soil. If for example, building foundations are placed on a clay which possesses a very low permeability, the excess pore pressure will dissipate very slowly. Consequently there exists the danger of a shear failure occuring in the clay due to the increase in shear stresses without a corresponding increase in shearing resistance. By knowing the relation of change in pore pressure to load, an engineer can stop construction in order not to increase the stresses in the clay until the critical pore pressure has been dissipated to a safe value. The construction can then proceed.

The above description has shown that the relationship between stress and pore pressure is very important for all soil mechanics works. It will be valuable to all soil problems whenever engineers can predict and control the change in pore pressure. Skempton (2) tried to solve this problem by using the concept of pore pressure parameters to relate changes in pore pressure with changes in stress. He showed that the change in pore pressure depended on the change in the all-round stress and some measure of the shearing stress. Details of his solution are given in chapter II. There would be some error in applying the results of laboratory tests to estimate the pore pressure in the field because in many practical problems such as slope stability of embankments and foundation design, the principal directions of the components of the stress change do not correspond to those of the stresses under which the specimen was consolidated in the triaxial cell. The rotation of the planes of principal stresses is very hard to reproduce in the triaxial apparatus so that this error has been usually ignored.

A suggested method of obtaining an accurate pore pressure prediction at any state of stress is to separate the mechanics of the soil behavior into a volumetric and a deviatoric component. The volumetric component refers to the stress-strain relationship involving changes in the mean normal stress only. The deviatoric component refers to the stress-strain relationship involving deviatoric components of stress deviates from the mean. The mean normal stress is identically equal to the octahedral normal stress and the resultant deviator stress, is identically equal to the octahedral shear stress. For this reason the state of stress at a point will be defined by the octahedral normal stress and the octahedral shear stress components. The relationship between the octahedral components of stress and the principal stress components is given in Chapter II.

CHAPTER II

THEORETICAL CONSIDERATIONS

Stress Theory

Stress at a Point in Two Dimensional Problems

Consider an elemental volume of material of triangular shape and a unit thickness perpendicular to the paper as O A B in Figure 1.

Assume a normal stress, σ and a shear stress τ to act on a plane AB which makes an angle α with the Z-axis. Let the cosines of angles between the normal to the plane AB and the X- and Z-axes be ℓ, and n, respectively. The relationship between normal and shear stresses and stresses in the X- and Z-directions are found by means of equilibrium condition as follow:

Resolve forces in the direction normal to the plane AB (or X'-direction).

$$\sigma_{AB} = \sigma_x \ell^2 \cdot AB + \sigma_z \cdot n^2 \cdot AB + \tau_{xz} \cdot \ell \cdot n \, AB + \tau_{zx} \, \ell \cdot n \cdot AB \quad \ldots (1)$$

By substituting $\tau_{xz} = \tau_{zx}$ and dividing throughout by AB, the above equation becomes

$$\sigma = \sigma_x \ell^2 + \sigma_z n^2 + 2 \tau_{xz} \cdot \ell n \quad \ldots \ldots (2\text{-}a)$$

Resolve forces in the direction parallel to the plane AB (or Z'-direction).

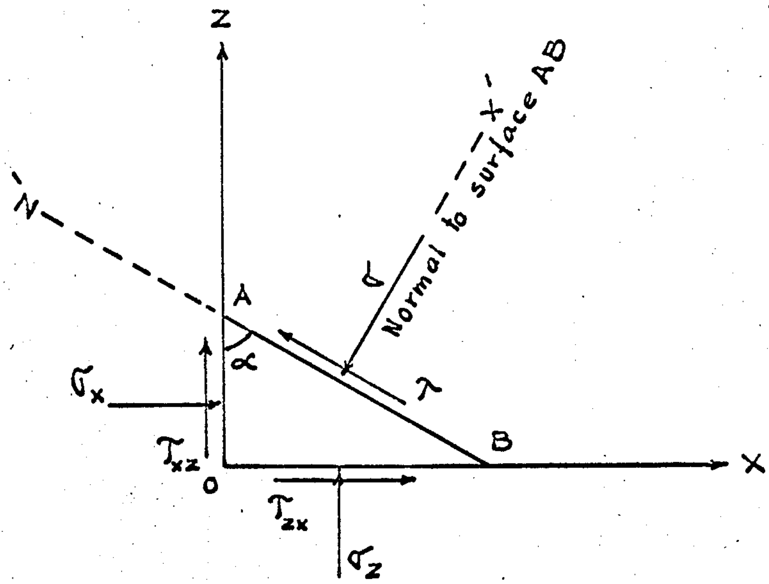

Figure 1. Stresses on a Soil Element in Two Dimensions

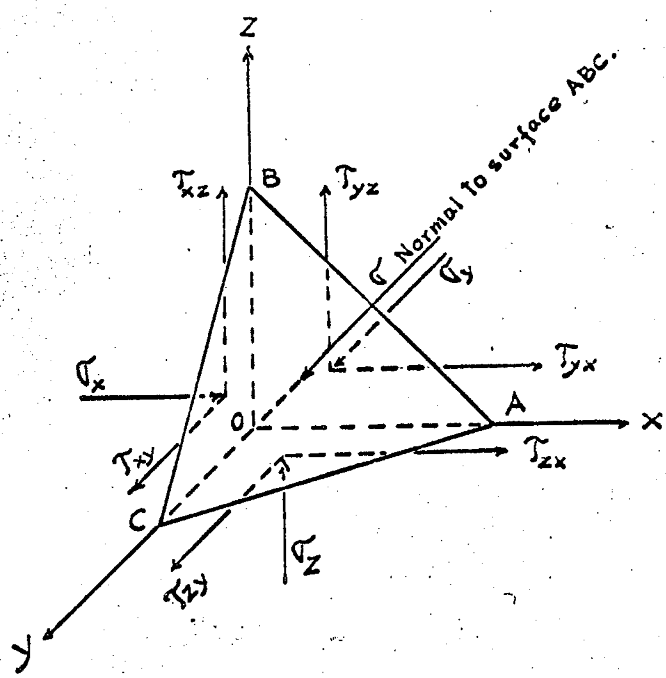

Figure 2. Stresses on a Soil Element in Three Dimensions

$$\tau \cdot AB = \sigma_z \cdot ln \cdot AB - \sigma_x \cdot ln \cdot AB - \tau_{xz} \cdot l^2 \cdot AB + \tau_{zx} \cdot n^2 \cdot AB .$$

By substituting $\tau_{zx} = \tau_{xz}$, and dividing throughout by AB, the equation becomes

$$\tau = (\sigma_z - \sigma_x)\, ln + \tau_{xz}(n^2 - l^2) \quad \text{.......} \quad (2\text{-b})$$

The X'- and Z'-axes as shown in Figure 1 can be taken as a new set of orthogonal axes in which the cosine of the angle between the X'- and X- axes is l and the cosine of the angle between X'- and Z axes is n. In this case, equations (2) become transformation equations for the streeses σ'_x and $\tau'_{x'z'}$ in the new framwork. In equation (2-b), τ can be made zero, if

$$\tan 2\alpha = \frac{2\tau_{xz}}{(\sigma_x - \sigma_z)} \quad \text{------} \quad (3)$$

where $\cos \alpha = n$. as shown in Figure 1. The angle between the X- and Z- axes and two planes at right angles to each other on which no shear stresses act is therefore defined by equation (3). These two planes are called principal planes and the two axes are also principal axes. The definition of the principal stress is therefore a normal stress acting along the principal axis and the plane perpendicular to the principal axis is a principal plane.

Stress at a Point in Three Dimensional Problems

The transformation equations in the three dimensional situation can be derived as follow (3).

ABC in Figure 2 is assumed to be a plane having direction cosines equal to ℓ, m and n with the X-, Y- and Z- axes respectively. If the element is in equilibrium by only a normal stress σ acting on this plane, from above definition, ABC is a principal plane. Three equations are obtained by balancing forces in all directions

$$(\sigma_x - \sigma)\,\ell + \tau_{yx}\cdot m + \tau_{zx}\cdot n = 0 \quad \ldots \quad (4\text{-}a)$$

$$\tau_{xy}\cdot \ell + (\sigma_y - \sigma)\cdot m + \tau_{zy}\cdot n = 0 \quad \ldots \quad (4\text{-}b)$$

$$\tau_{xz}\cdot \ell + \tau_{yz}\cdot m + (\sigma_z - \sigma)\cdot n = 0 \quad \ldots \quad (4\text{-}c)$$

The values of ℓ, m and n are solved from these three equations and the location of a principal plane is determined. These equations can be solved by determinants, to give for example, in the case of ℓ :

$$\ell = \frac{\begin{vmatrix} 0 & \tau_{yx} & \tau_{zx} \\ 0 & (\sigma_y - \sigma) & \tau_{zy} \\ 0 & \tau_{yz} & (\sigma_z - \sigma) \end{vmatrix}}{\begin{vmatrix} (\sigma_x - \sigma) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - \sigma) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - \sigma) \end{vmatrix}}$$

and similar expressions for m and n, all possessing the same denominator. However, the numerator in each case is zero , and since $\ell^2 + m^2 + n^2 = 1$. so that ℓ , m and n cannot be zero simultaneously , the common denominator must also be zero.

Expanding the denominator and equating it to zero, the following equation is obtained.

$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + (\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2)\sigma$$
$$-(\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2) = 0 \quad \text{- - - (5)}$$

The coefficients of σ are called the first, second and third stress invariants and are given the symbols J_1, J_2 and J_3.

$$J_1 = \sigma_x + \sigma_y + \sigma_z \quad \text{- - - (6-a)}$$
$$J_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 \quad \text{- - - (6-b)}$$
$$J_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2 \quad \text{(6-c)}$$

If principal stresses are considered no shearing stresses act and the invariants become

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad \text{- - - (7-a)}$$
$$J_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \quad \text{- - - (7-b)}$$
$$J_3 = \sigma_1\sigma_2\sigma_3 \quad \text{- - - (7-c)}$$

Ofcourse , any functions of the invariants are also invariant.

The stresses in a plane are determined by the orientation of the X-, Y, and Z- axes (Figure 2) to the directions of the principal stresses, σ_1, σ_2, and σ_3 since the stresses are orthogonal.

The direction cosines of the new axis X' in Figure 3., which is assumed to be the direction normal to the plane ABC and to be a straight line through the origin are l, m, and n with respect to the X -, Y - and Z - axes (or σ_1, σ_2, and σ_3 directions). Two other axes, Y' and Z' are also orthogonal to the X' - axis and oriented. as shown. For convenience, the Y' - axis would lie in the plane of X- and Y- axes and is normal to the Z - axis.

Since the plane ABC is normal to the X' axis, there is one normal stress σ_x', and two shearing stresses acting on this surface. These sbresses can be computed in terms of direction cosines and principal stresses and also expressed by resolving the forces on the element OABC of Figure 4. in the X' - ,Y' - and Z' - directions.

$$\sigma_{x'} = \sigma_1 l^2 + \sigma_2 m^2 + \sigma_3 n^2 \qquad \text{------ (8-a)}$$

$$\tau_{x'y'} = \frac{l m}{\sqrt{1-n^2}} (\sigma_2 - \sigma_1) \qquad \text{----- (8-b)}$$

$$\tau_{x'z'} = -\sigma_1 \frac{l^2 n}{\sqrt{1-n^2}} - \sigma_2 \frac{m^2 n}{\sqrt{1-n^2}} + \sigma_3 n \sqrt{1-n^2} \qquad \text{--- (8-c)}$$

By this orientation of the coordinate system $X'Y'Z'$, the principal stress σ_3 does not contribute to the stress in the Y' - direction.

The sum of the squares of $\tau_{x'y'}$ and $\tau_{x'z'}$ from equations (8) will be the resultant tangential stress τ' on the plane ABC as follow :

$$\tau'^2 = (\sigma_1 - \sigma_2)^2 l^2 m^2 + (\sigma_2 - \sigma_3)^2 m^2 n^2 + (\sigma_3 - \sigma_1)^2 n^2 l^2 \quad \text{.... (9)}$$

Figure 4 , which represents a view along the X' - axis normal to the plane ABC, shows the stresses $\tau_{x'z'}$,

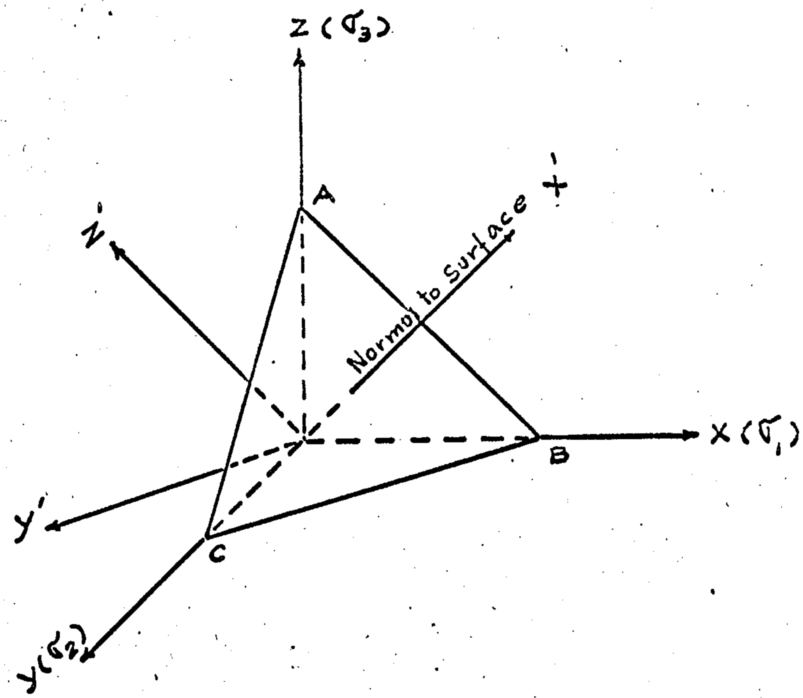

Figure 3. Transformation of Coordinates

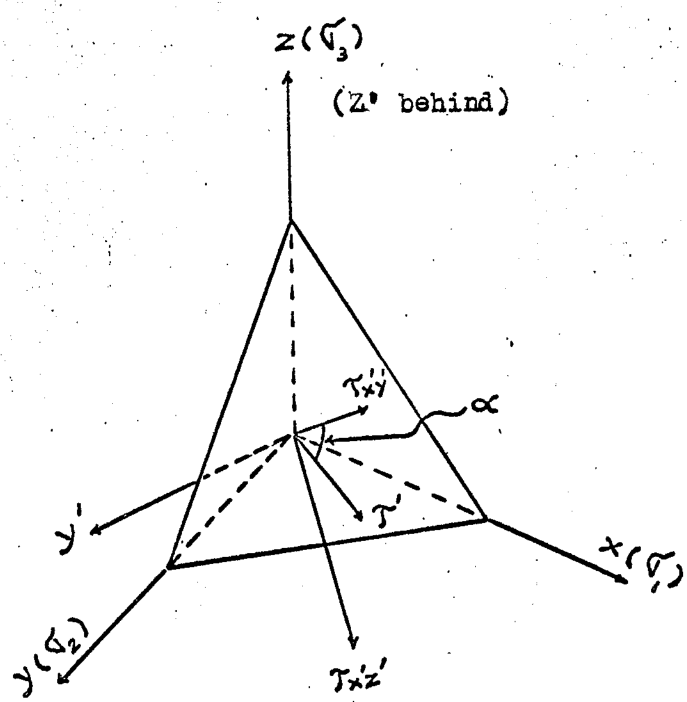

Figure 4. View Along X'-axis Perpendicular to the Plane

$\tau_{x'y'}$, and τ'. The direction of orientation of the resultant is given by the angle α whose tangent is the ratio $\tau_{x'z'}/\tau_{x'y'}$.

$$\tan \alpha = \frac{\tau_{x'z'}}{\tau_{x'y'}} = \frac{n}{lm} \left[\frac{(\sigma_1 - \sigma_3) l^2 + (\sigma_2 - \sigma_3) m^2}{(\sigma_1 - \sigma_2)} \right] \quad \cdots (10)$$

From the above equations, it is seen that stresses at any point not only depend on the amount of principal stresses but also on the values of the direction cosines.

In any particular plane, the direction cosines should be specific values. If they are substituted in the above equations, the new equations for a particular plane will be obtained.

Octahedral Stresses

The planes whose normal direction makes equal angles with the directions of the three principal axes are called octahedral planes and the stresses acting on these planes are octahedral stresses. The letters l, m and n are assumed to be the values of the normal directions cosines. For this particular plane, these values are equal to $1/\sqrt{3}$.

In order to find the relationships between the octahedral stresses and principal stresses, these values should be substituted in the equations (8).

The octahedral normal stress is obtained from equation (8-a).

$$\sigma_0 = 1/3(\sigma_1 + \sigma_2 + \sigma_3) \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The octahedral shear stress is obtained from