ECONOMIC EVALUATION OF ON-FARM IRRIGATION IN THE MORDEN-WINKLER AREA OF SOUTHERN MANITOBA

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by
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ABSTRACT

Scope and Method of Study: The feasibility of building a dam on the Pembina River (running across northern North Dakota and southern Manitoba) is being considered by the International (U.S.A.-Canada) Joint Commission. The general objective of this study is to investigate the economic feasibility of on-farm use of irrigation water which would become available for the Morden-Winkler Area of southern Manitoba with the completion of the dam.

The methodology involves the use of mathematical programming models to evaluate various economic factors which may influence the feasibility of irrigation in this area. The study deals primarily with representative individual farm units. The regional dimensions such as the total demand for water are aggregated from the micro-unit analysis.

The first phase of this study (Model I) involves an assessment of irrigation feasibility on the basis of long run average conditions and prices. In this aspect, "economic feasibility" of irrigation implies the possibility of increasing farm incomes, but does not explicitly take into account the income-stabilizing effect of irrigation. Mixed integer programming and conventional linear programming are employed to determine the profit maximizing use of water and other resources and corresponding farm organization.

The second phase of this study (Model II) involves the recognition of different risk preferences on the part of potential irrigators and an assessment of the impact on the overall feasibility of irrigation. Risk programming techniques are incorpolated into the economic models developed for the first phase to accommodate the refinement. Five levels of risk aversion are defined in terms of probabilities attached to various sizes of guaranteed incomes.

Findings and Conclusions from the First Phase of Study: Mixed integer programming methods are more advantageous than the conventional linear programming technique in this study because the economic feasibility conditions of irrigation and the projected demand for water are significantly affected through consideration of purchasing specialized machines at integral units.

Following conclusions are drawn up from the analysis of optimal solutions obtained under various economic conditions. (1) Specialty crops (sugar beets and potatoes) can be irrigated profitably under wide ranges of water prices, total holdings

and capital loan limits, whereas small grains can not be irrigated profitably except at very low prices of water. (2) Feed grains are not irrigated at any price of water, although fodder corn is irrigated at water prices lower than $2.14 per acre-inch. (3) Barley, oats, sunflowers and field peas can not enter the optimal solution at any combination of water price, total holdings and operating capital loans. (4) Unless an operating capital loan is available, no physically irrigable land can be profitably developed for irrigation even though a development capital loan is available.

A distinct shift of the aggregate demand curve derived from the non-integer linear programming solutions for small (60 acre), medium (250 acre) and large (500 acre) farms occurs at a water price of $3.36 per acre-inch. The price elasticity of demand for water is less than one for the water prices lower than $3.36 and is larger than one in the higher price range. Demand for water at prices higher than $3.36 is very small. With the prices of water from $.74 to $2.62, 55.7 to 75.8 percent of total irrigable land can be irrigated profitably.

If the purchasing of specialized machines is considered at integral units, no crop can be irrigated profitably at any price of water on small (60 acre) farms. On medium and large farms, irrigation water can be used profitably over the price range of zero to $2.62 per acre-inch. The aggregate demand curve estimated by mixed integer programming lies below that obtained by non-integer linear programming. Approximately 18,000 to 187,700 acre-inches could be used to irrigate about 35 percent of the total irrigable land over the price range extending from $1.17 to $2.62 per acre-inch. The price elasticity of demand for water is smaller than one over the entire price range.

Findings and Conclusions from the Second Phase of Study: When there is the possibility of irrigation, the optimal solutions are not sensitive to the changes of a risk aversion coefficient within the medium to high levels. However, the optimal solutions pertaining to the low to low-medium levels of risk aversion differ significantly from those mentioned above. Under dryland conditions, optimal solutions show more sensitive response to change in the level of risk aversion.

Expected and guaranteed incomes obtainable under irrigation conditions are respectively higher than those under dryland for all levels of risk aversion. Irrigation would increase the utilities of the two extreme types of risk averters (ie., the low and high risk averters) more than those of medium risk averters. The importance of flax and sow-hog operations increases significantly with irrigation for all levels of risk aversion, except the low and the low-medium. A higher level of expected utility is attainable under irrigation conditions than under dryland. This is indicated by the fact that the utility possibility curve for irrigation conditions lies above that for dryland conditions.

Under both dryland and irrigation conditions, all risk averters can reach higher level utility indifference curves by choosing optimal plans developed by stochastic programming, than by choosing those given by linear programming. Under these two conditions, the optimal stochastic programming plans differ significantly from the optimal linear programming plans, especially when these plans are developed for the medium to high levels of risk aversion. Without consideration of risk, field peas, sunflowers and sow-hog operations do not enter any optimal plan, and flax enters only at very low price of water. Taking risk into consideration, however, these activities in many cases are found in optimal plans, under dryland and irrigation conditions, especially for medium to high levels of risk aversion.

The economic feasibility of irrigation and demand for water would be increased by taking into account the income stabilization effect of irrigation. When the price of water is lower than $3.25 per acre-inch, all demand curves derived from the stochastic programming solutions lie above the one from the linear programming solution.

The alternative optimal cropping systems obtained for various levels of risk aversion parameters under dryland conditions are compared with the actual one. From this comparison, it is inferred that the farmers in the project area have, on the average, a high level of risk aversion.

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CHAPTER I

INTRODUCTION

The feasibility of building a dam on the Pembina River is being considered by the International (America-Canada) Joint Commission. In 1962 a benefit-cost study of the proposed joint project was made by the International Pembina River Engineering Board.[1] If a dam is constructed on the Pembina River according to the proposed plans, then an average annual flow of about 112,000 acre-feet can be supplied to a storage reservoir located near Walhalla, North Dakota. From this amount must be subtracted dead storage, reservoir losses, 12 per cent of the flow for upstream withdrawals and an annual volume of 10,000 acre-feet for municipal and industrial uses. The remaining water is sufficient to irrigate about 36,000 acres and, by agreement, Canada is entitled to half of this amount. Therefore, approximately 18,000 acres of farm land will become physically irrigable in the Morden-Winkler irrigation project area with completion of the dam.

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[1] International Pembina River Engineering Board, Joint Investigation for Development of the Water Resources of the Pembina River Basin, Vol. I-III, The International Joint Commission, December 1964.

The technical feasibility of irrigation, however, is not necessarily compatible with the economic feasibility of irrigation. Therefore, the economic feasibility of on-farm use of irrigation water supplied from the storage reservoir must be studied. This study entails the investigation of the economic conditions under which the supplied irrigation water can be utilized profitably at the farm level. If irrigation water can be utilized profitably at the farm level, then a substantial change of cropping system will occur in the project area when the irrigation water becomes available.

I. PROBLEM AND NATURE OF THE STUDY

Scope and nature. Because the dam, once it is constructed, will be able to serve multiple purposes such as irrigation, municipal and industrial uses, recreation, flood control, wildlife protection and others, the benefit of the dam should be calculated by taking these aspects into consideration. The purpose of this study, however, is not to estimate the total benefits and costs of the proposed dam-construction project; it is to investigate the economic feasibility of on-farm use of irrigation water which will become available with completion of the proposed project.

This study assumes that irrigation water can be purchased at the gates of the farm supply laterals which are connected with the main supply canal. The water would then be taken into the farms by building farm lateral distribution systems (permanent head ditches and supply laterals), annual cross ditches and farm drainage systems between the main canal and drain. Therefore, the price of water which is considered in this study is the one available at the gates of the laterals. Accordingly, the on-farm irrigation costs include the annual costs of farm laterals and farm drains, but not those of the main canal and drain. The costs of the latter items are included in the price of water.

Limitations of existing studies. There have been several studies of irrigation feasibility using budgeting and linear programming techniques.[1] These studies have indicated that linear programming methods are, in general, more useful for this type of problem than budgeting. Some reasons for this finding are:

1. Linear programming is capable of handling a larger number of production alternatives than budgeting.

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[1] These studies include; W.I.R.Johnson, A Micro-economic Analysis of Irrigation in the Morden-Winkler Area of Manitoba, Unpublished M. Sc. Thesis, Univ. of Manitoba, Aug. 1963; B.H. Sonntag, Supplemental Irrigation in the Proposed South Saskatchewan River Irrigation Project, Economic Branch, C.D.A., 1965 and others (see the bibliography).

2. In a complex problem where there are many restrictions and production alternatives, it is unlikely that all of them would be fully considered by budgeting.

3. The effect of varying the levels of certain variables can be examined systematically by the methods of parametric resource and price programmings, whereas budgeting does not lend itself to these types of adjustments.

4. The marginal value productivity of limiting resources can be estimated in terms of shadow prices in the linear programming method.

In spite of the many advantages of linear programming compared to budgeting, it has some significant limitations. The types of problems encountered using linear programming in irrigation feasibility analysis are outlined below.

1. Development of irrigation often entails a large investment in specialized machines and other equipment. In practice, purchase of these machines are feasible only at integral units. Consideration of the purchasing units of them at integral units in the linear programming model will result in a substantial difference in the optimal plans developed for irrigation conditions compared to the non integer solutions. Conventional linear programming techniques can not directly deal with integer variables. Accordingly, past studies of irrigation have assumed that

the purchasing units of these machines are infinitesimally divisible. Such an assumption would reduce, to a considerable extent, the applicability of the results of such studies for actual farm planning. It could be hypothesized that higher levels of irrigated crops might enter the optimal plans developed by the conventional linear programming method than would have entered if purchasing units of these machines had been considered at integral units. The projected demand for irrigation water might therefore also be over-estimated.

2. Many farmers take risk into account in their decision-making. Naturally, they will take into account the fact that irrigation not only increases the average yields of crops, but also gives an income-stabilization effect. To date, little recognition has been given to the attitudes of farmers toward risks in their decision-making. The effect of irrigation upon risk reduction has either been ignored or taken into consideration only implicitly.

Due to these limitations, optimal irrigation plans developed by conventional linear programming methods are less useful than those which would be obtained with the consideration of both integral purchasing units of machines and risk.

New approaches. This study attempts to overcome the forestated limitations of linear programming. A mixed-integer programming model and a stochastic programming model are applied in determining the economic feasibility of irrigation, estimation of demand for irrigation water and the development of optimal plans for farms under irrigation conditions. Unfortunately, however, the above problems can not be solved simultaneously by these new models. Only one problem can be solved by each and not both at the same time. The first difficulty can be overcome by using a mixed-integer programming method and the second by stochastic programming techniques.

This study will demonstrate how the optimal plans obtained by the conventional linear programming method differ from those obtained by mixed-integer programming as well as by stochastic programming, and how the latter plans are more applicable. In this study, an emphasis is put on methodological aspects of economic feasibility of irrigation and the validity of the alternative programming models is tested by using the data obtained for the project area. Nevertheless, the results of this study should also provide some useful information for farm planning under irrigation conditions as well as the estimation of demand for irrigation water in the project area.

II. THE OBJECTIVES OF THE STUDY

The general objective of this study is to investigate the economic feasibility of irrigation and to project the potential demand for irrigation water in the Morden-Winkler Irrigation Project area in southern Manitoba. Two models having different sets of assumptions are adopted for the investigation and projection. These models are referred to as Model I and Model II.[1]

The specific objectives of the study associated with Model I are:

1. to find the economic conditions under which irrigation water can be used by the farmers in the project area,

2. to estimate the amount of farm land which would be developed for irrigation under these various conditions,

3. to derive the static normative demand function for irrigation water,

4. to estimate the economic value of irrigation water,

5. to compare the present farm organization under dryland conditions with the optimum farm organizations under irrigation.

6. to find the minimum increases in yields from the dryland levels required for major crops to be economically irrigable,

7. to analyze the optimum investment in the specialized

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[1]These models are defined and discussed in detail in chapters, II and III.

machines which would be required when irrigation is introduced,

8. to estimate the demand for irrigation development capital and operating capital,

9. to contrast the mixed-integer programming solutions with those of non-integer programming, and

10. to propose the use of mixed-integer programming with a non-integer parametric-cost programming of irrigation water.

The specific objectives of the study on the basis of Model II are:

1. to find the economic conditions under which the farmers in the area can utilize irrigation water so as to stabilize their farm incomes as well as to increase them to some extent,

2. to compare and contrast the economic feasibility conditions of irrigation of Model I and Model II,

3. to develop the optimum production plans under dry-land and irrigation conditions for farmers having different levels of risk aversion,

4. to investigate the sensitivity of optimum plans to the variation of a risk aversion index,

5. to analyze the complementary relationships among alternative enterprises for the reduction of risk,

6. to estimate and compare the potential demand for irrigation water and the acreage of land developed for

irrigation under various levels of risk aversion,

7. to evaluate crop insurance as a method of income stabilization in comparison with irrigation,

8. to estimate the level of risk aversion revealed by 250-acre farms in the project area, and

9. to contrast an ordinary linear programming solution with that of stochastic programming, compare Van Moeseke's model with Freund's and Heady-Candler's models and finally to examine the theoretical relationships among these three models.

III. DESCRIPTION OF STUDY AREA[1]

The study area (ie., the Morden-Winkler Irrigation Project Area) is located in the southern end of Manitoba adjacent to the Pembina River. The main supply canal will be located on the western boundary of the project area,

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[1]Most of the information was obtained from International Pembina River Engineering Board, Joint Investigation For Development of the Water Resources of the Pembina River Basin, Vol. III, International Joint Commission, December 1964, pp. 313-367.

running nearly eight miles north from the U.S.A. boundary. It empties into the Hespeler Floodway just south of Winkler. Farm distribution laterals are taken off at appropriate locations of the main canal. Several types of structures will be required on the main canal to control the flow of irrigation water. The farm laterals carry the irrigation water from the main canal to the farms. The small farm drains take the waste water from the farms to the project drains. The project area is shown on FIGURE I with the main canal, the farm lateral distribution system and the drainage system.

Climate and soil types. The average temperature in the summer months is above 50 degrees. The average length of the frost free period is about 120 days. Annual precipitation at Morden during the 43-year period has varied from 10.85 inches to 27.12 inches with an average of 19.57 inches. Moisture deficiencies occur every crop year.

The soils in the project area are dominantly Black types and are under good drainage conditions. Six soils, Rheinland, Neuenberg, Aschfeld, Guadenthal sand substrate phase, Deadhorse sand substrate phase and Alluvium, occupy more than 70 percent of the total area. About 29 percent of the soils are coarse to moderately coarse textured, about 55 percent are medium to moderately fine textured and about 16

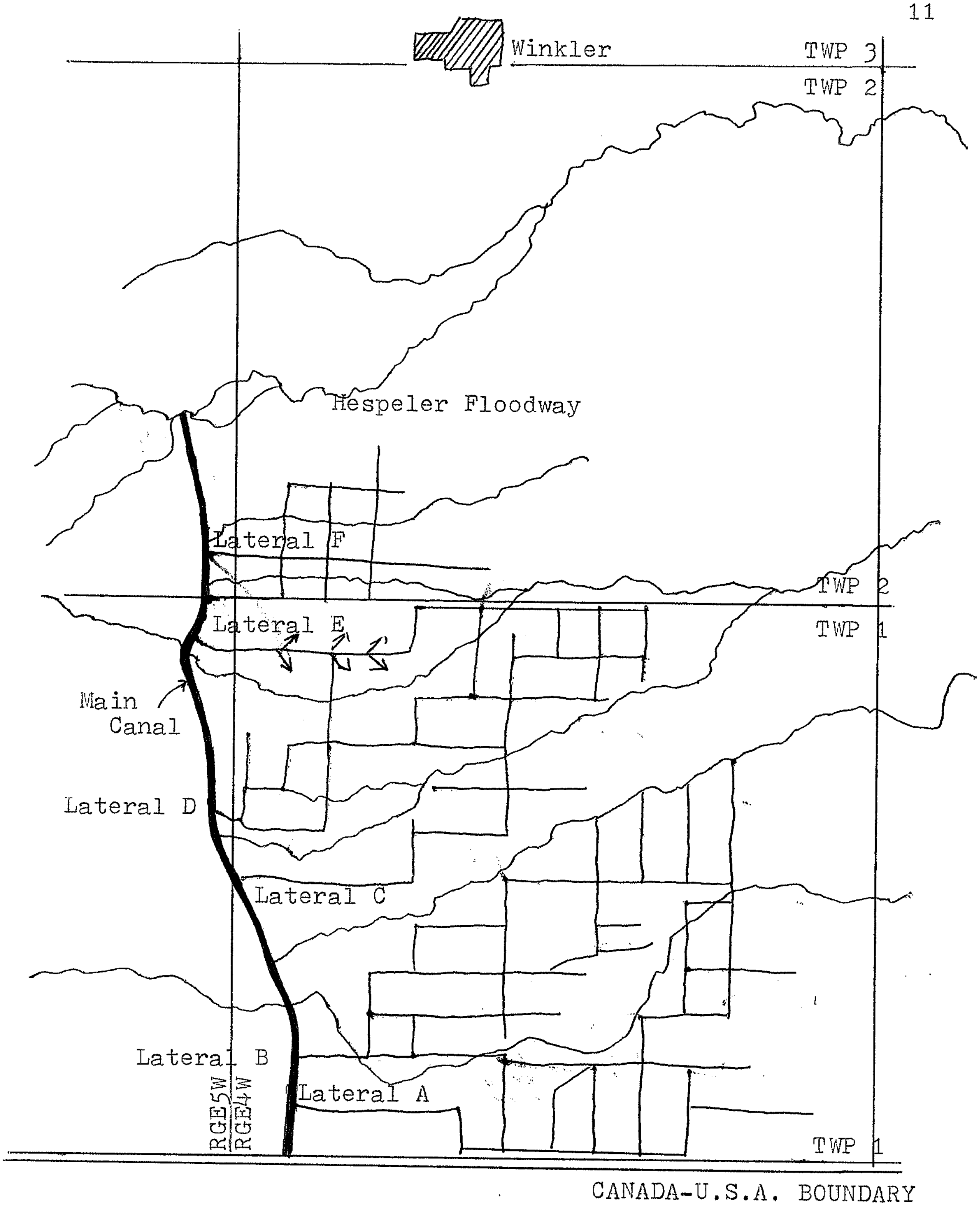

FIGURE 1

MAP OF THE PROPOSED MORDEN-WINKLER IRRIGATION PROJECT AREA percent are fine textured.

There are 368 farms in the project area and they occupy about 71,040 acres of total farm land. The farm business study conducted by the Prairie Farm Rehabilitation Administration excluded 32 farms smaller than 10 acres and 7 farms larger than 759 acres from the study. Therefore, 329 farms are used as the total sample in calculating the percentages of small, medium and large farms.

Approximately 70 percent of total farm land in the project area is physically irrigable. About 80 percent of the physically irrigable land is classified as T_1 land which requires from 0 to 200 cubic yards per acre of soil to be moved at the initial levelling in irrigation development. The remaining 20 percent is classified as T_2 land in which 200 to 350 cubic yards of soil per acre should be moved at the initial levelling of land.

<u>The existing farm organization</u>. The number of farms and their average sizes according to size of farm are as shown in Table I. Percentages of different categories of land according to size of farm are presented in Table II.

In the project area, crop production dominates the farm enterprises with cereal crops and flax occupying most of the acreage. Specialty crops such as sugar beets, sunflowers and potatoes have been introduced to the area in the

TABLE I

NUMBER OF FARMS AND THEIR AVERAGE SIZES ACCORDING TO SIZE OF FARM[1], 1962

	Average Size	No. of Farms
Small farm (10 - 149 acres)	59.8 acres	131
Medium farm (150 - 399 acres)	242.0 acres	158
Large farm (400 - 759 acres)	498.0 acres	40
		329

[1]The source of data is the International Pembina River Engineering Board, Ibid., Vol. III, p. 337.

TABLE II

PERCENTAGES OF DIFFERENT CATEGORIES OF LAND ACCORDING TO SIZE OF FARM[2], 1962

	Av.size	Improved land	Physically irrigable land	T_1 land	T_2 land	Unimproved land
	ACRES					
Small	60	51(85%)	42(70%)	34(57%)	8(13%)	9(15%)
Medium	250	225(90%)	170(70%)	136(56%)	34(14%)	25(10%)
Large	500	453(91%)	350(70%)	280(56%)	70(14%)	45(9%)

[2]Source: Ibid., pp. 343-4.

past 15-20 years. More recently processing vegetables such as peas, beans and corn have been grown. Only a few farmers grow fresh vegetables and on a limited acreage. Forage crops are also produced on a minor acreage. The growing season is a little too short for grain corn. Summerfallow acreage is almost negligible.

Nearly all farmers have small herds of cattle. Cattle operations have mainly provided milk products and beef for home use and cream and live animals for sale. Cattle finishing is still a minor enterprise, but there is some evidence of an increase. Small hog and poultry enterprises can be found on nearly half of the farms in the area.

CHAPTER II

FARM DECISION MODELS UTILIZED IN STUDY

This chapter deals with the theoretical models which are utilized in this study. Two different types of models, Model I and Model II, are employed for the investigation of on-farm irrigation feasibility. Model I is based on the methods of a mixed-integer and a conventional linear programming, while Model II is a stochastic programming application.

Model I assumes that:

1. The prices of farm products and input factors as well as the yields of crops are known perfectly.
2. Technical coefficients are given.
3. The resources available for an individual farm are limited and the limited amounts are known, respectively.

This model is characterized by the perfect knowledge of all related variables as well as its static nature. Hence, all variables appearing in the model are of deterministic nature and are not "dated". "Economic feasibility" of irrigation implies the possibility of increases in farm incomes under irrigation conditions, but the income-stabilizing effect of irrigation is not explicitly taken into account. A mixed-integer programming technique is adopted for this model with the consideration of purchasing indivisible units of specialized machines.

In Model II, some of the assumptions of Model I are relaxed. The assumptions relaxed are those of perfect knowledge with respect to yields and prices of crops. Instead, the risks attributable to the selection of irrigated and dryland crops having different patterns of year-to-year variation in yields and prices are taken into consideration. In other words, Model II takes into account the fact that the economic feasibility of irrigation is concerned not only with increasing farm incomes, but also with stabilizing them. In this model, therefore, two variables, yields and prices of farm products, are treated as stochastic, while other variables such as technical coefficients and resource limitations are defined as deterministic.

A detailed explanation of the applied models based on these theoretical models are presented in the next chapter.

I. MIXED-INTEGER PROGRAMMING METHOD

One of the disadvantages of a conventional linear programming model is the assumption of continuity of all variables both in the objective function and constraint inequalities. Sometimes this assumption cannot be satisfied in actual farm planning and a normative analysis because some variables can take only discrete values. For instance, the purchase of capital assets such as land, machinery and livestock is feasible only for discrete units, mostly integer. Sometimes a capital loan is also available only in lump sums. Services of these capital assets are divisible and continuous, but they can be purchased only for integer units. If the level of an activity entering the optimal solution is large enough in comparison with the unit of the activity, then we may treat the variable (activity) as a continuous one. This results because the "rounding off" of the decimal part in the optimal solution will not substantially affect the optimal solution. Land, livestock and capital loans can satisfy this condition in many cases, but investments in machinery and buildings will not satisfy it.

One possible solution of this problem is to relax the continuity condition on the variables. A mixed or all discrete variable programming technique can be used for such a case.[1]

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[1] Y. Kubo, classified the problematic situations into three categories for which discrete variable programming can be effectively adopted.

1. Some activities have a minimum, indivisible physical unit mainly because of technical or physical reasons, and each unit has constant technical coefficients and a given net price.
2. A production process cannot satisfy the linearity assumptions of a production function of which the return to scale is variable at discrete levels of that activity.
3. An activity is inevitably accompanied by an item of cost which takes a constant value regardless of the level of activity.

Y. Kubo, "A Study of Application of Discrete Variable Programming", Nokei Ronso, Vol. 17, Hokkaido University, Japan.

Several different models of discrete variable programming have been presented by R.D. Gomory, E.M.L. Beale, A.H. Land, A.G. Doig, H.M. Markowitz, G.B. Dantzig, and others.[1] A combination of an integer programming model with a dynamic feature (multi-period) was attempted by D. Coyler[2], using a simple problem as an example.

Land-Doig model. In this study, the Land-Doig model is adopted. This model is a modification of the conventional linear programming model and the solution can be obtained by using a conventional simplex method and widely available computer programs for linear programming. The variables can be either all discrete or mixed discrete with continuous

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[1]R.E. Gomory, and W.J. Baumol. "Integer Programming and Pricing", Econometrica, Vol. 28, July 1960, pp. 521-550.

A.H. Land, and A.G. Doig. "An Automatic Method of Solving Discrete Programming Problem", Econometrica, July 1960, pp. 497-520.

E.M.L. Beale. "A Method of Solving Linear Programming Problem When Some But Not All of the Variables Must Take Integral Values", Statistical Technique Research Group Technical Report, No. 19, July 1958.

H.M. Markowitz, and A.S. Manne. "On the Solution of Discrete Programming Problem", Econometrica, Vol. 25, No. 1, 1957, pp. 84-110.

G.B. Dantzig. "On the Significance of Solving Linear Programming Problem with Some Integer Variables", Econometrica, No. 1, 1960, pp. 30-44.

[2]D. Colyer. "A Capital Budgeting Mixed Integer, Temporal Programming Model", Canadian Journal of Agricultural Economics, Vol. 16, No. 1, 1968, pp. 1-7.

variables and the unit of a discrete variable can be an integer or any other discrete unit. In practice, an integer unit is most commonly used. In this chapter, therefore, only an integer programming is presented.

Generally, a mixed integer variable programming problem can be formulated as follows.

$$(\text{II} - 1)\quad \max \dashrightarrow Z = \sum_{j=1}^{n} C_j X_j + \sum_{j=n+1}^{h} C'_j X'_j$$

subject to

$$(\text{II} - 2)\quad \sum_{j=1}^{n} A_{ij} X_j + \sum_{j=n+1}^{h} A'_{ij} X'_j \leq b_i \qquad (i = 1,2,3.......m)$$

$$(\text{II} - 3)\quad X_j \geq 0 \text{ and is integral} \qquad (j = 1,2.........n)$$

$$(\text{II} - 4)\quad X'_j \geq 0 \qquad (j = n+1,n+2,n+3..h)$$

where:

C_j is the net price of the jth and integer activity,

C'_j is the net price of the jth and non-integer activity,

X_j is the level of the jth and integer activity,

X'_j is the level of the jth and non-integer activity,

A_{ij} is the technical coefficient of the jth and integer activity for the ith constraint inequality,

A'_{ij} is the technical coefficient of the jth and non-integer activity for the ith constraint inequality,

b_i is the right-hand-side value of the ith constraint inequality, and

n number of variables X_j out of h number of variables ($n \leq h$) are assumed to be the variables which should take only integral values, and the other variables, X_j' are non-integer, i.e., continuous variables. The underlying assumptions are the same as for a conventional linear programming except for the relaxation of continuity and divisibility conditions.

An optimal solution of a simple two variable linear programming problem without discrete variables can be illustrated diagrammatically (see Figure 2). Variables, X_1 and X_2, measures the levels of the two activities. Line A B C D indicates a production possibility frontier in the linear programming sense when X_1 and X_2 are both continuous variables. Line $Z_0 - Z_0'$ shows the functional line or iso-income line in the linear programming sense at an initial level. The optimal solution for this particular problem as indicated in the diagram is found at the corner point B. At B, the levels of X_1 and X_2 are $X_2 = 2.85$ and $X_1 = 5.75$.

In the case of an integer programming problem, however, point B is not feasible. The feasible region is confined to a set of lattice points of which both co-ordinates are integers rather than the entire region enclosed by line O A B C D. In this example, the optimal solution is found at one of these lattice points which reaches the maximum

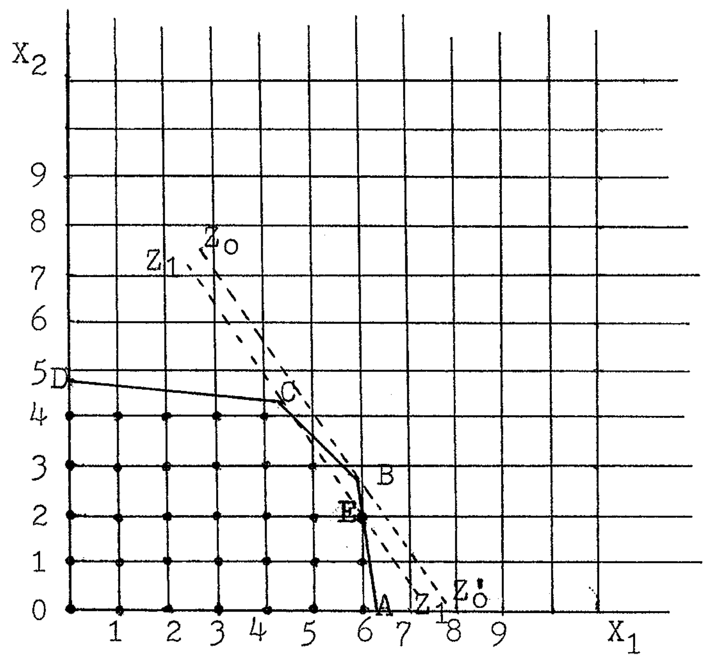

FIGURE 2

ILLUSTRATION OF SOLVING A LINEAR AND AN INTEGER PROGRAMMING PROBLEM WITH TWO VARIABLES functional level $Z_1 - Z_1'$. The point is denoted by E. The procedure for arriving at this point could be described as "pushing down the functional line until it touches the first integral point".[1] At the optimal solution, X_1 and X_2, have integer values, respectively, of $X_1 = 5$ and $X_2 = 3$.

The general method for solving an integer programming problem may be described as:

> ..systematic parallel shifts in the functional in the direction of reducing the maximand of the functional until a point within the ordinary linear programming convex set is found which has integral co-ordinates in the n dimension.[2]

The two dimensional case was illustrated in Figure 2. The three dimensional case is illustrated in FIGURE 3. Consider the convex set of feasible solutions of a conventional linear programming problem. The related variables, X_1, X_2 and X_3 can take any non-negative value respectively in the first step. The triangle plane ABC indicates the functional plane. The polygonal cube, a b c d e shows the convex set of feasible solution for the non-integral linear programming problem. Now, suppose that as the functional plane shifts parallel to itself toward the origin, the corner point of the convex set d touches the functional plane first. This point d indicates the optimal solution for the ordinary non-integer

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[1] A.H. Land and A.G. Doig, "An Automatic Method of Solving Discrete Programming Problem", Econometrica, July 1960, P. 499.

[2] Ibid., P. 500.

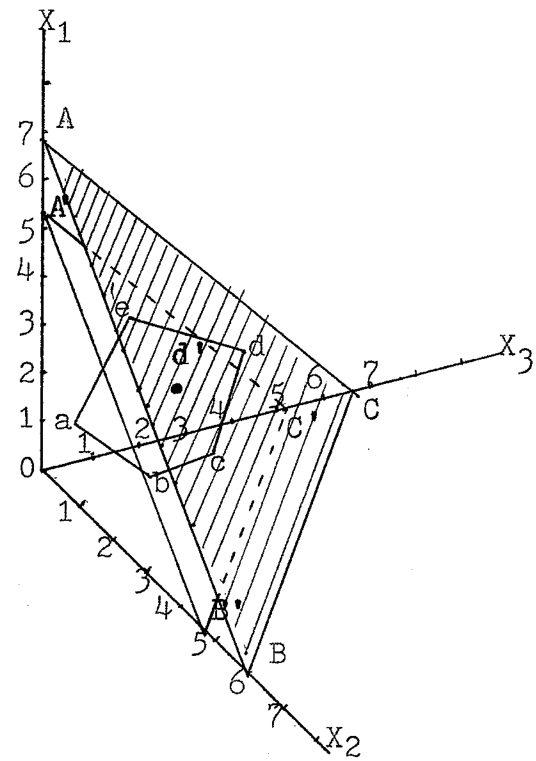

FIGURE 3

ILLUSTRATION OF SOLVING AN INTEGER PROGRAMMING PROBLEM WITH THREE VARIABLES linear programming problem with three variables, X_1, X_2 and X_3. If we keep moving the functional plane parallel to itself, the point d' will be the first lattice point which is touched by the functional plane and has integral co-ordinates. This point d' indicates the optimal integral solution if the functional plane did not pass any other integer point within the convex set of feasible non-integer solutions before the point d' is reached. The point d corresponds to the maximum feasible functional for the ordinary linear programming problem, while the point d' corresponds to the maximum feasible functional for the integer programming problem. The point d can also be obtained by solving the ordinary, non-integer linear programming problem by a simplex method.

If more than three variables are involved, the optimal integral solution can not be shown by the diagrammatic method. In this case, the simplex method provides us with a practical procedure for solving a mixed-integer programming problem. The details of this procedure appear in Appendix I.

II. STOCHASTIC PROGRAMMING MODELS

Basic Concept

A risk (or stochastic) programming model may be defined, in a very broad sense, as a mathematical programming model in which at least one of the variables in the objective functional, constraint inequalities, or the right-hand-side elements[1] of constraint inequalities is a stochastic variable.

Different types of stochastic programming models have been developed by Charnes, Cooper, Madansky, Evers, Dantzig, Tinter, Heady, Candler, Freund, Van Moeseke and others[2] which handle a part of the conventional L.P. model as a stochastic variable while the other variables are treated as determinestic variables. In this chapter, only one type of risk programming model which is characterized by the probabilistic treatment of net-price coefficients in the objective function is discussed.

In agricultural production, crop yields and product prices are subject to significant year-to-year fluctuations, whereas other factors such as technical coefficients, resource restrictions and input costs have increased over time with little variation from the trend.[3] This statement is supported

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[1]The right-hand-side elements comprise of resource restrictions, zeros, an expected income parameterized, etc..

[2]See the bibliography in the appendix.

[3]See Heady-Candler Linear Programming Method (Iowa State Univ. Press), 1958, p. 556-7 and Van Moeseke "Stochastic Linear Programming", Yale Economic Essay, Vol. 5, No. 1. 1965 p. 211; D.O. Anderson, The Value of Irrigation Water In the Washita River Basin of Roger Mills County, Oklahoma, (Unpublished Ph.D. Thesis), 1965, pp. 1-2. (the Oklahoma State Univ.)

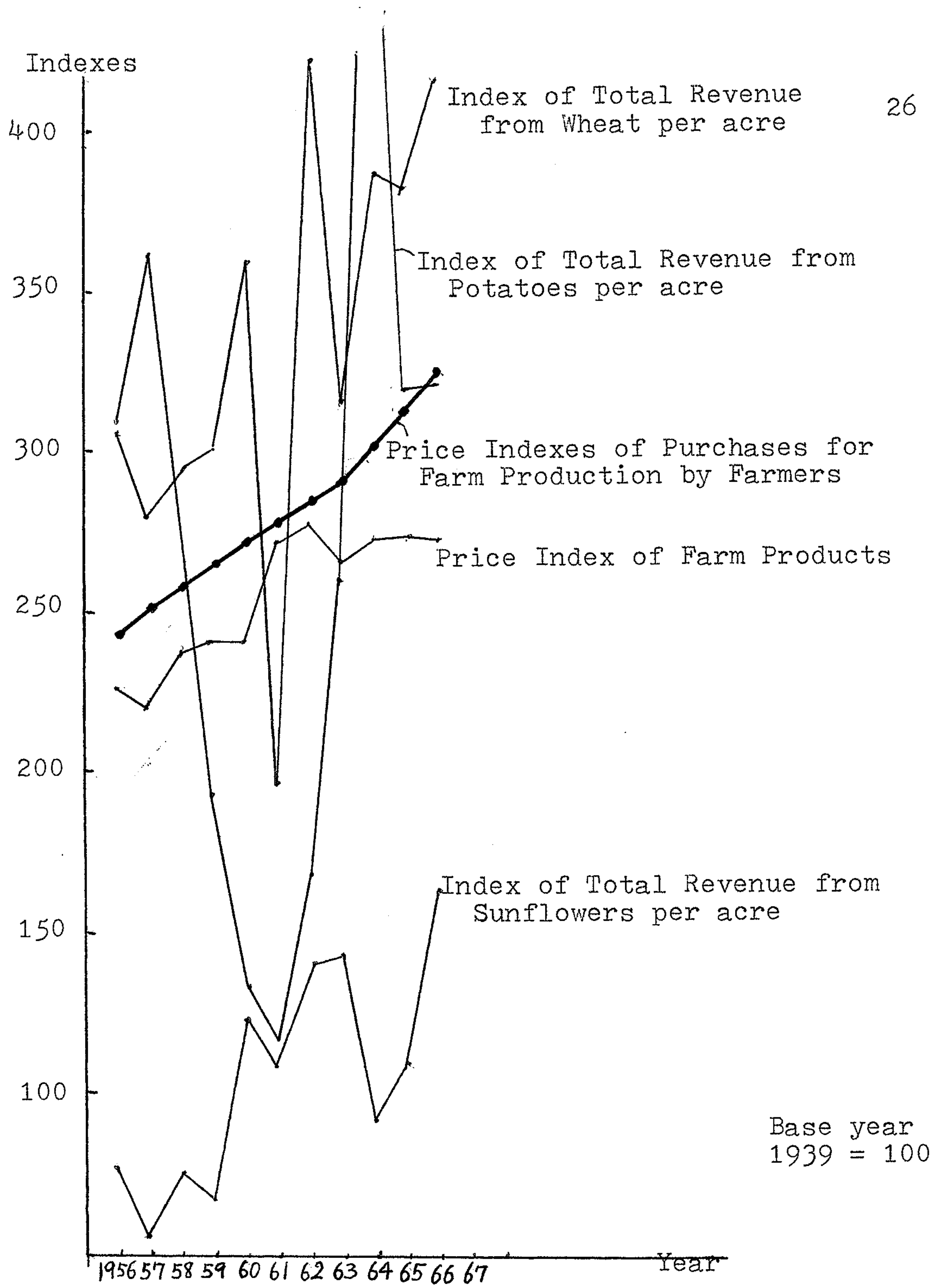

FIGURE 4

PRICE INDEX OF PURCHASES FOR FARM PRODUCTION AND THE INDECES OF TOTAL REVENUES OF CROPS PER ACRE, 1956-66, MANITOBA (Source: "Yearbook of Manitoba Agriculture", 1966.)

by empirical studies made by O.E. Heady, W.G. Aanderud and others.[1] In Manitoba, there has been a steady increase in the price index of purchases for farm production between 1956 and 1966 whereas, the price index of farm products and the indices of total revenues of crops per acre show greater fluctuations (FIGURE 4). The significant year-to-year variation of product prices is caused mainly by the interaction of weather and inelastic demands for food. Consideration of the stochastic nature of product prices is necessary to provide a farm decision-maker with a more realistic farm plan, if stability as well as profitability of the farm are simultaneously required.

In this case, a decision maker will try to maximize the net utility obtained by subtracting the dis-utility due to the variation of income from the positive utility derived from the expected value of farm income. Under such a circumstance, a programming model should also be formulated in a stochastic form.

Most of the risk programming models with a stochastic objective function assume that all of the stochastic net-price

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[1] E.O. Heady "Diversification in Resource Allocation and Minimization of Income Variability", Journal of Farm Economics, Vol. 34, 1952; W.G. Aanderud "Income Variability of Alternative Plans, Selected Farm and Ranch Situation", Unpublished Ph.D. Thesis, Oklahoma State University, 1964; E.O. Heady and others, Economic Instability and Choices Involving Income and Risk in Primary or Crop Production, Agricultural Experiment Station, Iowa State College, 1954.

coefficients and the weighted sum of these coefficients are normally distributed.[1] The Heady-Candler risk programming model does not necessarily require the assumption of normality. The assumption of normal distribution of net prices requires that no weather or price cycles occur and that there be no trends in prices or technology.[2]

This type of stochastic programming model may be classified into three categories.

1. Freund type

The utility function is built into the programming model itself and the expected utility which depends upon the expected income and income variance is maximized subject to a set of constraints. The risk programming model is complete by itself. An unique optimal solution associated with a given risk aversion coefficient is obtainable directly from the programming solution.

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[1] Suppose that C_j $(j=1,2,3,\ldots.n)$ is independently and normally distributed respectively with mean, $\bar{C}_j$ and variance S_j^2. Then, the weighted sum, $\sum_{j=1}^{n} C_j \cdot X_j$ will also be normally distributed with the mean equal to $\sum_{j=1}^{n} \bar{C}_j \cdot X_j$ and the variance equal to $X' \cdot V \cdot X$ where V is the variance-covariance matrix of $C_j (j=1,2,\ldots n)$ and X is a column vector of activity levels.

[2] R.B. How. Use of Quadratic Programming in Farm Planning Under Uncertainty, 1968 p. 16; G. Tintner, Econometrics, (Wiley) 1952, p. 186; Y. Maruyama. "The Production Planning under Instability", Journal of Rural Economics, Vol. 38, No. 1, p. 1; M. Yeh "Premium Ratemaking in All Risk Crop Insurance Program", J.F.E., Vol. 48, No. 5, 1966, p. 1582.

2. Heady-Candler type

In this model, only the locus of the income-variance possibility curve is obtained. A utility function (ie. an income-risk indifference contour map) should be added to find the optimal solution.

3. Van Moeseke type

The lower-bound income guaranteed at a prescribed level of probability is maximized subject to a set of constraints. The decision-maker's satisfaction (utility) depends upon the lower-bound income and the prescribed level of confidence attached to it.

Heady-Candler Model[1]

Under the assumption that a decision maker will avert risk while trying to maximize expected income, a set of solution vectors corresponding to a locus of expected incomes associated with the minimum income variances will be found with a given set of constraints. By using the results from the computation of a risk programming problem, we can draw an income-variance possibility curve as in FIGURE 5. All possible combinations of income and income variance obtainable from the feasible solutions of the problem will

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[1] E.O. Heady and W. Candler. Linear Programming Methods (Iowa State Univ. Press), 1958, Chapt. 17.

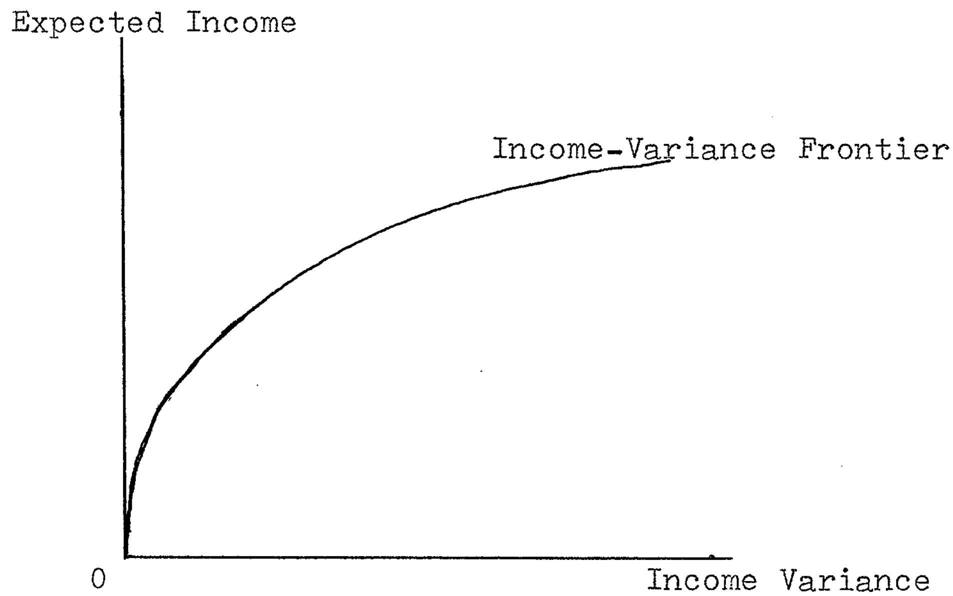

FIGURE 5

INCOME-VARIANCE POSSIBILITY CURVE

fall in the area under the income-variance frontier curve.

It should be noted that while the Heady-Candler risk programming model can provide an income-variance possibility curve, there is no unique optimal solution for the risk programming problem. We need the assistance of an income-risk utility function to reach an optimal equilibrium. In this sense, the Heady-Candler risk programming model in itself is incomplete. On the other hand, no specification of the probability distribution is required to derive an income-variance possibility curve.

The Heady-Candler risk programming model can be formulated in a matrix form as:

(II - 5) min. $\rightarrow$ $F(X) = X' \cdot V \cdot X$

subject to

(II - 6) $\bar{C}' \cdot X + Yo = \bar{Z} \leftarrow \bar{Z}$ is parameterized.

(II - 7) $P \cdot X + Y = B$

$X \geq 0$, $Y \geq 0$ and $Yo = 0$

where:

X is a column vector of activity levels,

X' is the transposed vector of X,

V is the variance-covariance matrix of net prices,

$\bar{Z}$ is the expected income,

$\bar{C}$ is the column vector of expected net prices,

P is the matrix of technical coefficient,

Y is the column vector of slack variables,

B is the column vector of right-hand-side elements of constraint inequalities.

The expected income, $\bar{Z}$, is varied successively from zero to the maximum value attainable with the constraint, $P \cdot X \leq B$.

For computational convenience, we may change the sign of $X' \cdot V \cdot X$ and multiply it by $\frac{1}{2}$. Then, the model becomes:

(II - 8) $\text{Max} \dashrightarrow F(X) = -\frac{1}{2} X' \cdot V \cdot X$

subject to

(II - 9) $\bar{C}' \cdot X + Yo = \bar{Z} \dashleftarrow \bar{Z}$ is parameterized.

(II - 10) $P \cdot X + Y = B$

$X, Y, \geq 0$ and $Yo = 0$

<u>Freund Model</u>[1]

Freund defines a deterministic utility function

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[1] R.J. Freund, "The Introduction of Risk into a Programming Model", <u>Econometrica</u>, Vol. 24, No. 3, 1956, pp. 253-263.

depending upon the size of income, Z as follows:

$$(II - 11) \qquad U = 1 - e^{-a \cdot z}$$

where:

U is the total utility,

Z is the level of income,

e is a constant (the base of natural logarithm), and

a is a constant and indicates a subjective risk aversion index.

This utility function indicates a relative level of utility because U varies from zero to one as income, Z changes. This utility function is characterized by a diminishing marginal utility of income because the second derivative of U is negative. The constant, a, indicates a subjective risk aversion coefficient which shows the subjective preference to an income-risk situation as expressed by a combination of income and the income variance attached to it. As the value of risk aversion coefficient, a, becomes greater, the utility curve shifts toward the left.

Now, we assume that the total income Z is a stochastic variable which is normally distributed. Then, U also becomes a stochastic variable which has a normal distribution. Therefore, the expected value of U, (E(U)) can be obtained by

$$(II - 12) \quad E(U) = \int_{-\infty}^{\infty} (1-e^{-az}) \cdot \frac{1}{\sqrt{2\pi} \cdot \sigma_z} \cdot e^{-\frac{(Z - \bar{Z})^2}{2\sigma_z^2}} \cdot dZ$$

$$= 1 - e^{-a(\bar{z} - \frac{a}{2} \cdot \sigma_z^2)}$$

where:

σ_z^2 is the variance of total income, and

$\bar{Z}$ is the expected value of Z.

Our objective is to maximize E(U). It is maximized when $\bar{Z} - \frac{a}{2}\sigma_z^2$ is maximized. Let this be equal to F(Z):

$$F(Z) = \bar{Z} - \frac{a}{2}\sigma_z^2 \tag{II - 13}$$

where:

F(Z) can be conceived as an utility function of income, Z, taking account of risk. The function, F(Z), becomes large as $\bar{Z}$ becomes large and becomes smaller as σ_z^2 and the risk aversion coefficient, a, become greater. In other words, the more conservative with respect to risk a decision maker is, the greater is the discounting of income, $\bar{Z}$, due to variance, σ_z^2. The decision maker is assumed to maximize an utility function F(Z) which depends upon the mean income and income variance. Thus, our problem is:

$$\max \rightarrow F(Z) = \bar{Z} - \frac{a}{2}\sigma_z^2 \tag{II - 14}$$

subject to

$$P \cdot X \leq B \tag{II - 15}$$

$$X \geq 0$$

where:

$\bar{Z} = \bar{C}' \cdot X$, and

$\sigma_z^2 = X' \cdot V \cdot X$

Thus, a stochastic programming problem is converted to a deterministic quadratic programming problem.

The relationship between Freund's risk programming model and a conventional linear programming (hereinafter abbreviated as L.P.) model is explained below. Freund's utility function is a function of expected income and income variance, and may be written as $E(U) = F(\bar{Z}, \sigma_z^2)$. On the other hand, in a conventional L.P. model, total utility depends solely upon income, and the income is deterministic (ie., variance is zero), or variation of income is ignored in a decision maker's utility function (a = zero). Therefore, we can relate Freund's objective function to a conventional L.P. objective function either by letting the variance, σ_z^2, in Freund's objective function equal zero or the subjective risk aversion coefficient, a, equal zero. In the latter case, there may be income-variation, but a decision maker is not concerned with it. The two objective functions are:

(II - 16) Freund's objective function -- $E(U)=1-e^{a(\frac{a}{2}\overset{2}{Z}-\bar{Z})}$

(II - 17) L.P. objective function in terms of utility ---$F(Z)= 1-e^{-a\bar{Z}}$

The objective function $U(\bar{Z})$ can be obtained from (II - 16) by letting $\sigma_Z{}^2 = 0$. Maximization of (II - 17) is equivalent to maximization of $\bar{Z}$ because $U(\bar{Z})$ depends solely upon $\bar{Z}$. The linear programming objective function, $\bar{Z}$, can be obtained from $F(Z) = \bar{Z} - \frac{a}{2}\sigma_Z^2$ by letting a equal zero.

Van Moeseke Model[1]

Van Moeseke also assumes that all the stochastic net-price coefficients, Cj, are independently and normally distributed. Consequently, the weighted sum of these stochastic and deterministic net prices is also normally distributed with the mean equal to $\sum_j \bar{C}_j \cdot x_j$. and the variance equal to $X' \cdot V \cdot X$.

The lowest-bound of stochastic farm income guaranteed at a prescribed level of confidence, γ, is denoted as Z_*. The guaranteed income, Z_*, can be expressed in terms of the mean and standard deviation of income by:

$$\text{(II - 18)} \quad Z_* = \bar{Z} - q \cdot \sigma_Z.$$

where:

q is a constant and indicates a risk preference coefficient. The probability that the farm income falls below Z_* will be given by $1 - \gamma$. The meaning of the lowest-bound income, Z_*,

[1] P. Van Moeseke. "Stochastic Linear Programming", Yale Economic Essay, Vol. 5, No. 1, Spring 1965, pp. 197-225.

is that incomes lower than Z_* may possibly occur every n years as calculated by $1 - \gamma = \frac{1}{n}$.

This model is formulated in matrix form as:

$$\text{(II - 19)} \quad \max. \leftrightarrow Z = C' \cdot X$$

subject to

$$\text{(II - 20)} \qquad P \cdot X \leq B$$

$$\text{(II - 21)} \qquad \Pr.\{ Z \geq Z_* \} = \gamma$$

$$X \geq 0, \; Z_* = \bar{\bar{Z}} - q \cdot \sigma_Z$$

where:

Z is the total income,

C is the column vector of net prices,

X is the column vector of activity levels,

P is the matrix of technical coefficient,

B is the column vector of right-hand-side elements of constraint inequalities,

σ_Z is the standard deviation of Z, and C and Z are stochastics variables which are normally distributed. Equation (II - 19) can be converted into a deterministic form by using "certainty equivalent" of Z at a prescribed level of probability γ. This can be written as:

$$\text{(II - 22)} \quad \text{Max.} \rightarrow Z_* = \bar{C}' \cdot X - q \cdot (X' \cdot V \cdot X \cdot)^{\frac{1}{2}}$$

subject to

(II - 23) $$P \cdot X \leq B$$

$$X \geq o$$

where:

q is the confidence coefficient with a given level of probability, and

V is the variance-covariance matrix of net prices.

The constant, q is named the "risk preference (aversion) coefficient" by Van Moeseke. The probability $Pr.\{Z \geq Z_*(= \bar{Z} - q \cdot \sigma_Z)\} = \gamma$ is given by the oblique-lined area under the normal distribution curve (FIGURE 6) or:

(II - 24) $$\int_{Z_*}^{\infty} \frac{1}{\sqrt{2\pi} \cdot \sigma_Z} \cdot e^{-\frac{(Z - \bar{Z})^2}{2\sigma^2_Z}} \cdot dZ$$

The objective function is called "the truncated minimax criterion" by Van Moeseke.[1] This is "a risk preference functional in terms of E(Z) and σ_Z that possesses a confidence limit interpretation".

Van Moeseke's objective function has the following characteristics. For a low level of confidence (i.e., small γ), linear programming gives us the same result as stochastic

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[1]P. Van Moeseke "Stochastic L.P.", <u>Yale Economic Essay</u>, Vol. 5, No. 1, 1965, pp. 207-210.

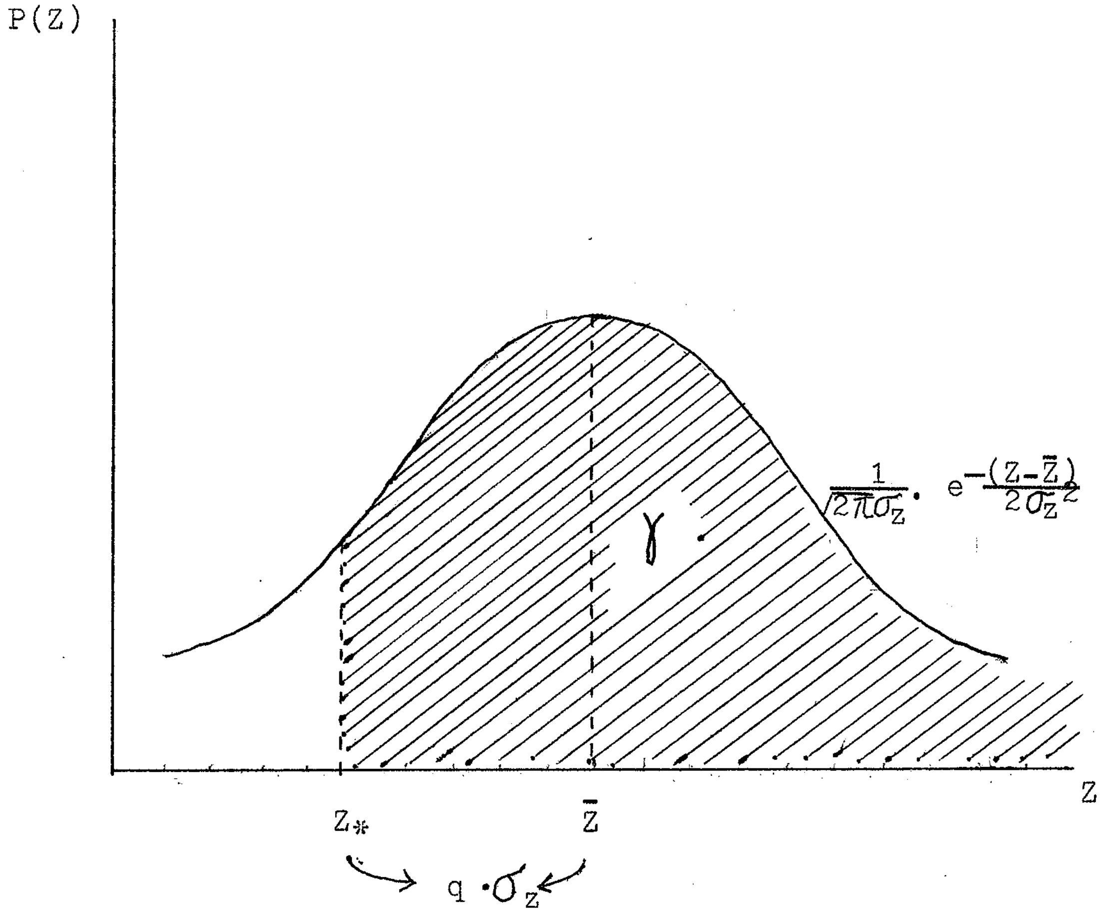

FIGURE 6

PROBABILITY DISTRIBUTION OF INCOME, Z

programming, but as the level grows, there appears to be a difference between them. At too high a level of confidence, (i.e. too small $1 - \gamma$ and too large q), a trivial solution $X^* = 0$ would be obtained. This is reasonable because too severe a restriction would make any activity impossible.

It should be noted that Van Moeseke's model is not a quadratic programming form because a root sign of $X' \cdot V \cdot X$ is involved. Therefore, his model can not be directly solved by a quadratic programming method. Some modifications are required to solve this model as a quadratic programming problem.

One computational procedure for these types of problems is presented by Kataoka and is described below.[1]

A stochastic programming problem (Problem 1) stated as:

$$\text{(II - 25)} \quad \max. \rightarrow Z*_1 = \bar{C} \cdot X - q \cdot (X' \cdot V \cdot X)^{\frac{1}{2}}$$

subject to

$$\text{(II - 26)} \quad P \cdot X \leq B$$

$$X \geq 0$$

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[1] S. Kataoka. "Stochastic Programming Model", Econometrica Vol. 31, Jan. - April 1963, pp. 181-196.

has only one maximum (or degenerate maximum) because the objective function is concave and its constraints are linear. Then, an optimal solution vector X* can be obtained by solving a subsidiary quadratic programming problem (Problem II) formulated as:

$$\text{(II - 27) Max.} \dashrightarrow Z_{*2} = \bar{C}' \cdot X - \frac{q}{R} X' \cdot V \cdot X$$

subject to

$$\text{(II - 28)} \qquad P \cdot X \leq B$$

$$X \geq 0$$

where:

R is a positive constant and should satisfy the following equation when an optimal solution vector X* (R) under a given value of R is given.

$$\text{(II - 29)} \qquad R = \sqrt{X^*(R)' \cdot V \cdot X^*(R)}$$

Theorem: Suppose that an optimal solution vector X* (R) of Problem II satisfies the criterion.

$$R = \sqrt{X^*(R)' \cdot V \cdot X^*(R)}$$

Then, X*(R) is also an optimal solution for Problem I and vice versa.[1]

For computational convenience, Problem II can be slightly changed, by multiplying $\frac{q}{R} \cdot (X' \cdot V \cdot X)$ by $\frac{1}{2}$. Then, the subsidiary quadratic programming problem becomes

$$\text{(II - 30)} \quad \max. \rightarrow Z_{*2} = \bar{C}' \cdot X - \frac{q}{2R} X' \cdot V \cdot X$$

subject to

$$\text{(II - 31)} \qquad P \cdot X \leq B$$

$$X \geq 0$$

The theorem stated above can also apply to Problem II. The theorem can be proved by Kuhn-Tucker conditions applicable to both problems.[2]

A stochastic programming problem formulated like Problem I can be solved indirectly by using the Heady-Candler model. Firstly, obtain the income-variance possibility curve by solving the Heady-Candler risk programming problem for different levels of parameterized expected income $\bar{Z}$. Then, feed back pairs of income and standard deviation of income

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[1]The implications of the theorem and the detailed procedure of solving a stochastic programming problem by this method appear in Appendix II.

[2]Ibid., P. 187.

into the Van Moeseke model objective function with a given value of q and find successively Z_{*1} values. If we select the maximum value of calculated Z_{*1} values, the corresponding solution in the Heady-Candler type of risk programming problem is the optimal solution. This method is based on the mathematical relation between the Van Moeseke and Heady-Candler models.

A stochastic programming problem of the Van Moeseke type can also be solved through the solution of risk programming problems of Freund's type. The indirect solution of Van Moeseke's type of problem is based on the mathematical relationship as discussed in the following section. A practical application of this method appears in Chapter III. (see pages 94-7).

Descriptive Comparison of Three Stochastic Programming Models

1. Freund's model has a built-in utility function so that the expected utility may be maximized at the same time as the objective functional of his risk programming problem is maximized.

With a given value of risk preference coefficient, q, an unique optimal solution is obtainable for a Moeseke formulation of the risk programming problem.

In contrast to these models, the Heady-Candler risk programming model provides no unique solution, but traces out a locus of expected incomes with the minimum income variances subject to a given set of constraints. A decision maker must choose one point on the income-variance frontier according to his income-risk preference contours.

2. In the case of Freund and Heady-Candler models, the utility of stochastic income is conceived in terms of expected income and income variance.

Van Moeseke's decision (choice) criterion in a risk situation is based on a lowest-bound income and a level of probability attached to it.

3. As recognized by Freund himself, the determination of risk aversion coefficient, a, is a purely subjective task.

In the case of Van Moeseke's model, the choice of a prescribed level of confidence, q, is purely a subjective matter and there is no objective basis for the determination of that level.

4. In Freund's and Van Moeseke's models, each stochastic net price is assumed to have an independent and normal distribution.

Normality is not required, however, for Heady-Candler's model unless we want to know the lowest-bound income at a prescribed level of probability corresponding to each point on the income-variance possibility curve.

If we consider an expost-choice of "risk aversion coefficient" or "risk preference parameter" in Freund's or Van Moeseke's model, neither of these two models is self-complete. Subjective choice of a risk preference parameter, q, for instance, may depend both on the size of a lowest-bound income and the level of probability attached to it. Therefore, the value of q which must be assigned by a particular decision maker can not be determined prior to solving a risk programming problem. We may, however, determine the value of q arbitrarily and vary it to find a locus of lowest-bound incomes maximized and prescribed levels of probability. By letting a decision maker choose, according to his own income-risk preference, one maximized lowest-bound income and the prescribed level of probability attached to it, we can detect the value of q which corresponds to that pair. Thus, we need the assistance of the decision-maker's income-risk preference function in terms of lowest-bound income and prescribed level of probability attached to it. It is recognized that there is no essential difference between these models even though two of them declare a self-complete model while the other does not.

Mathematical Relationships Between Three Stochastic Programming Models

Heady-Candler and Van Moeseke models. In order to introduce a probabilistic statement into a risk programming model, specification of the probability distribution of net prices is necessary. If we assume a normal distribution of net prices, then the mathematical relationship between these two models is that specified below.

Suppose $\Pr. (Z \geq \bar{Z} - q \cdot \sigma_Z) = \gamma$ is given. Then, the value of q is determined:

Heady-Candler model;

$$\text{(II - 32)} \quad \min. \rightarrow X' \cdot V \cdot X$$

subject to

$$\text{(II - 33)} \qquad \bar{C}' \cdot X = \bar{Z} \leftarrow \bar{Z} \text{ is parameterized.}$$

$$\text{(II - 34)} \qquad P \cdot X \leq B$$

$$X \geq 0$$

Van Moeseke model I;

$$\text{(II - 32)'} \quad \text{Max.} \rightarrow \bar{C}' \cdot X - q(X' \cdot V \cdot X)^{\frac{1}{2}}$$

subject to

$$\text{(II - 33)'} \qquad P \cdot X \leq B$$

$$X \geq 0$$

With a given q and a given value of $\bar{C}' \cdot X$ in the Van Moeseke model, $X' \cdot V \cdot X$ must be minimized if $\bar{C}' \cdot X - q(X' \cdot V \cdot X)^{\frac{1}{2}}$ is maximized. Now, add another constraint equation, $\bar{C}' \cdot X = \bar{Z}$ ($\leftarrow$--$\bar{Z}$ is parameterized), to the Van Moeseke model I. This gives the Van Moeseke model II. Maximizing (II - 32) subject to the new constraints is equivalent to minimizing $X' \cdot V \cdot X$ subject to the same constraints. In other words, it is the same as solving a Heady-Candler formulation of the risk programming problem. It is obvious, therefore, that, if we vary $\bar{Z}$ and solve the Van Moeseke model II successively, then we can find an optimal solution vector X^* and $\bar{C}' \cdot X^*$ which is identical to those in the optimal solution of the Van Moeseke model I. This means that the optimal solution of the Van Moeseke model I with a given q, is on the locus of optimal solutions (accordingly on the income-variance possibility curve) obtained by the Heady-Candler model. This relationship between the two models enables us to solve a Van Moeseke formulation of the risk programming problem by substituting Van Moeseke's objective function with the solution vectors obtained from the Heady-Candler model.

Heady-Candler model and Freund model. Normal distributions are also assumed in the Heady-Candler model.

Heady-Candler model:

(II - 35) $\min. \rightarrow X' \cdot V \cdot X$

subject to

(II - 36) $\bar{C}' \cdot X = \bar{Z} \leftarrow$ $\bar{Z}$ is parameterized.

(II - 37) $P \cdot X \leq B \qquad X \geq 0$

Freund model I

(II - 38) $\max. \rightarrow \bar{C}' \cdot X - \frac{a}{2} X' \cdot V \cdot X$

subject to

(II - 39) $P \cdot X \leq B \qquad X \geq 0$

Freund model II:

(II - 40) $\max. \rightarrow \bar{C} \cdot X - \frac{a}{2} X' \cdot V \cdot X$

subject to

(II - 41) $\bar{C}' \cdot X = \bar{Z} \leftarrow$ $\bar{Z}$ is parameterized.

$P \cdot X \leq B \qquad X \geq 0$

If the risk aversion coefficient, a, is a given positive value, then solving the Heady-Candler model is equivalent to solving Freund model II. Furthermore, if we parameterize $\bar{Z}$ and solve Freund model II successively, one of these optimal solutions should be identical with the optimal solution of Freund model I. Thus, the optimal solution of Freund model I is also on the locus of optimal solutions obtained by the Heady-Candler model.

Van Moeseke and Freund models. The objective function of Freund's risk programming model can be rewritten as

$$\text{(II - 41)} \quad \max. \rightarrow \bar{Z} - \frac{a}{2} \sigma_z^2$$

$$\text{(II - 42)} \quad = \bar{Z} - \frac{a\sigma_z}{2} \cdot \sigma_z$$

This is equivalent to the objective function of Van Moeseke model with $q = \frac{a\sigma_z}{2}$. In the Freund model, the confidence coefficient (ie., the risk preference parameter in Van Moeseke's sense) is equal to $\frac{a\sigma_z}{2}$. Now, suppose that a risk programming problem of the Freund formulation with a given value of its risk aversion coefficient, a, is solved and the expected income and income variance in the optimal solution are obtained. They are denoted respectively by $\bar{Z}_*$ and σ_{z*}^2. In this case, the decision maker has indirectly chosen the confidence coefficient q (ie., the risk preference parameter in Van Moeseke's sense) at the level equal to $\frac{a\sigma_{z*}}{2}$. Conversely, we can calculate indirectly the value of a (ie., the risk aversion coefficient in Freund's sense) for a Van Moeseke's type risk programming problem when an optimal solution of the problem is obtained. From $q = \frac{a \cdot \sigma_{z*}}{2}$, we can derive the relationship:

$$(II - 43) \qquad a = 2 \cdot q / \sigma_{z*}$$

where:

$\sigma_{z*} = \sigma_z^*$ and σ_z^* is the standard deviation of income in the optimal solution of the Van Moeseke type of problem with a given value of q.

The same relationship can also be derived from the subsidiary quadratic programming problem of Van Moeseke's stochastic programming problem.

By examining the mathematical relationships between these models, we can see that there is no essential difference between them. If we apply one of these models to a set of alternative activities and constraints and solve the problem, then we can estimate the risk parameters of other models which will provide the same optimal solution. For instance, if we solve a risk programming problem of Freund's formulation with a given value of its risk aversion coefficient, a, then we can find the value of q in Van Moeseke's model so as to provide the same optimal solution. The reverse case is also possible.

We can also evaluate the risk preference parameter in the Van Moeseke sense in terms of the subjective risk aversion coefficient in the Freund sense, and vice versa.

If we have obtained an income-variance possibility curve by the Heady-Candler model, then we can determine a particular optimum point on the curve by using Freund's objective function with given value of a, or Van Moeseke's objective function with a given q. These two optimal points may differ from each other because the risk aversion coefficients, a and q are determined independently, but we can interpret one optimal solution in terms of the other criterion.

Limitations of Risk Programming Models

Introduction of risk consideration into a programming analysis no doubt provides a more realistic planning and analytical model both for the decision maker and the economist. There are, however, some limitations in these risk programming models which pertain mainly to the assumptions about models and the estimation of parameters. The most significant limitations are related to (1) the determination of risk aversion (or preference) parameters (q and a) and, (2) the underlying assumptions of probability distribution and estimation of variance-covariance matrices by time series data.

Concerning the determination of risk aversion parameters, there is no objective method for determing a value of the risk aversion parameter, especially before solving a risk programming problem. The difficulty in determining a value of the risk aversion parameter is due to the subjective nature of decision-making criteria in a risk situation. Concerning this point, Freund himself stated that "a (ie., subjective risk aversion coefficient) is a purely subjective task and any chosen value is exceedingly difficult to defend".[1]

Variation of net prices is considered as a probability distribution in these stochastic programming models. In this case, net prices are assumed to be random variables which have no trends and no cyclical movements. For the estimation of variances and covariances of these net prices, time series data are utilized. Time series data can be used for such estimation if actual annual net prices have no trends and cyclical movements. Actually, however, time series data of net prices used to estimate the variance-covariance matrix involve trends and cycles as well as random variation. Therefore, these variables are not completely random. Thus, we can not directly use time series data to estimate a variance-covariance matrix.

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[1]R. J. Freund. "The Introduction of Risk into a Programming Model", Econometrica, Vol. 24, No. 3, 1956, p. 263.

Attempts to Overcome Limitations of Stochastic Models

In spite of the forestated limitations, there are several approaches which may lead to improved stochastic models. These modifications are outlined in the following sections.

Discounting rate of expected income for the associated income variance. The nature of Freund's risk aversion coefficient, a, can be examined in more detail and interpreted more concretely in terms of a subjective discounting rate of expected income for the income variance attached to it. Suppose, that a decision-maker's risk aversion coefficient, a, is known. Then, we can derive his utility indifference curves of expected income and the associated standard deviation of income from the expected utility function:

$$\text{(II - 44)} \quad E(U)^* = 1 - e^{a(\frac{a}{2}\sigma_z^2 - \bar{z})}$$

where:

E(U)* and a are given. His utility indifference curves may be drawn on a graph which has expected income on the vertical axis and the associated standard deviation on the horizontal axis. The utility indifference function is given by: (Figure 7)

$$\text{(II - 45)} \qquad \bar{z} = \frac{a}{2}\sigma_z^2 - \frac{\ln K}{a}$$

where:

$K = 1 - E(U)^*$.[1]

The intersection of the E(U)* curve with the vertical axis is given by:

$$(II - 46) \qquad \bar{Z}_{(o)} = -\frac{\ln K}{a}$$

Since $E(U) < 1$, $\ln K$ is negative with range $0 < E(U) < 1$. Then, $\bar{Z}_{(o)}$ is positive. If $E(U) < 0$, then $\bar{Z}_{(o)}$ is also negative. As E(U)* increases, the utility indifference curves shift toward the upper-left corner of the graph. Since the slope of an indifference curve is obtained by:

$$(II - 47) \qquad \frac{d\bar{Z}}{d\sigma_z} = a \cdot \sigma_z$$

various levels of indifference curves have a constant slope with a given value of σ_z. If the risk aversion coefficient, a, increases with a given value of σ_z, then the slope of the

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[1] Equation (II - 45) is derived from equation (II -44), as follows.

$$e^{a(\frac{a}{2}\sigma_z^2 - \bar{Z})} = 1 - E(U)^*$$

$$\therefore \quad a(\frac{a}{2}\sigma_z^2 - \bar{Z}) = \ln(1 - E(U)^*)$$

$$\therefore \quad a \cdot \bar{Z} = \frac{a^2}{2}\sigma_z^2 - \ln(1 - E(U)^*)$$

Let $K = 1 - E(U)^*$, then

$$\bar{Z} = \frac{a}{2}\sigma_z^2 - \frac{\ln K}{a}$$

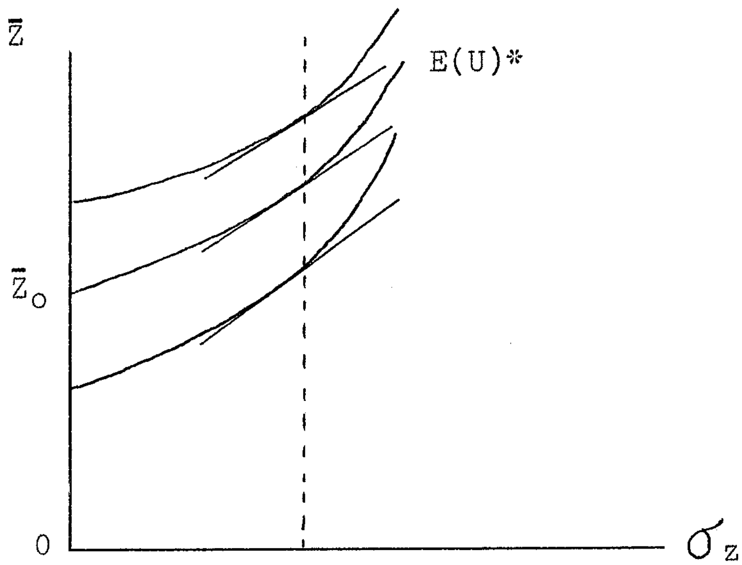

FIGURE 7

UTILITY INDIFFERENCE CURVES

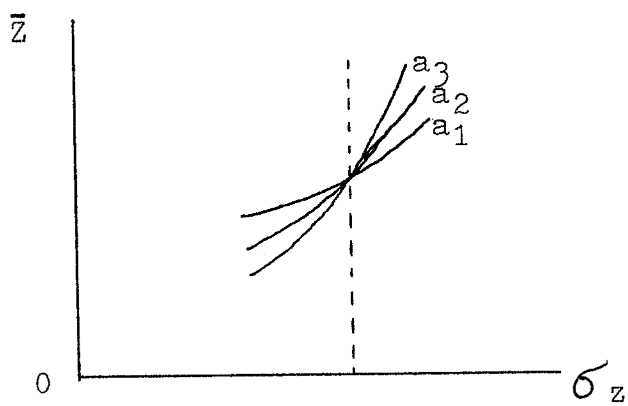

FIGURE 8

CHANGE IN THE SLOPE OF A UTILITY INDIFFERENCE CURVE WITH VARIED RISK AVERSION COEFFICIENT indifference curve also increases, (Figure 8). This indicates that the marginal rate of substitution of $\bar{Z}$ for σ_z increases over the whole range of an indifference curve. In other words, the marginal rate of discounting Z for σ_z rises with increasing values of a.

For a given value of a, the average rate of discounting Z for σ_z is obtained by

$$\text{(II - 48)} \quad R_{(a)} = \frac{\bar{Z}(a) - \bar{Z}(o)}{\sigma_z(a)} = \frac{\bar{Z}(a) + \frac{\ln K}{a}}{\sigma_z(a)}$$

where:

$\bar{Z}_{(a)}$ and $\sigma_z(a)$ indicate respectively the expected income and the corresponding standard deviation associated with the maximized E(U). As the value of a increases, R(a) also increases. Thus, a given value of the risk aversion coefficient, a, can be interpreted in terms of the average rate of discounting an expected income for a dollar of increased standard deviation of income.

Debt/equity ratio. An approach to the determination of the risk preference parameter, q, was made by using the debt/equity ratio for q on an individual farm. The use of debt/equity ratio for q, however, has no empirical grounds unless a high correlation between the debt/equity ratio and q values actually chosen by the decision-makers having these ratios, is verified. The risk preference parameter as defined by Van Moeseke indicates that a decision maker will choose a prescribed level of probability such that Pr. $\{Z \geq \bar{Z} - q \cdot \sigma_Z\} = \gamma$ where Z is normally distributed. If q is 2, for example, then the prescribed level of probability chosen by the decision maker is approximately 0.975. If q is one, the probability is 0.84. Now, suppose that a debt/equity ratio equal to 1.2 is used for q. This means that the decision maker with q equal to 1.2 has chosen the prescribed level of probability equal to 0.885 because Pr. $\{Z \geq \bar{Z} - 1.2\sigma_Z\} = 0.885$. There is, however, no theoretical ground for the statement that the decision maker who has a debt/equity ratio equal to 1.2 will or must choose the prescribed level of probability equal to 0.885. Why should a decision maker with a debt/equity ratio equal to 1.2 choose a particular prescribed level of probability equal to 0.885?

Expost-determination of the risk aversion parameter. One method of determination of the risk aversion coefficient as suggested by Maruyama and Freund is an "expost determination" method.[1] First, a series of risk programming problems

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[1] Y. Maruyama, R.J. Freund. "Prediction, Instability and Production Planning", Journal of Rural Economics, Vol. 39, 1967.

having various values of risk aversion coefficient, a, is solved. With respect to each optimal solution, we can find an expected income and income variance pair. Varying the value of a, we can draw an income-variance possibility locus on an income variance diagram (FIGURE 9). Finally, we ask a particular decision maker to choose a $\bar{Z}$ and σ_Z^2 pair on the locus according to his own income-risk preference. His choice of a $\bar{Z}$ and σ_Z^2 pair determines indirectly the value of a as well as the corresponding optimal solution vector. Difficulties would likely occur when a decision maker attempted to choose an income and income variance pair because the concept of income variance or standard deviation of income would not likely be grasped clearly by him.

Van Moeseke's truncated minimax criterion can provide a more concrete concept of risk for a decision maker who is asked to choose an income-risk combination. An income-risk combination expressed by $\Pr.\{Z \geq Z_*\} = \gamma$ can be stated more concretely as; "such and such amount of income (Z_*) is guaranteed with a probability or reliability of such and such percent (γ)" or "the chance of income falling below Z_* is 1/n; ($n = \frac{1}{1-\gamma}$)" or "income will fall below Z_* once in n years". On the basis of these income-risk statements, we can make an "income-risk preference table" which will be easily understood by a farmer. An example of such a Table

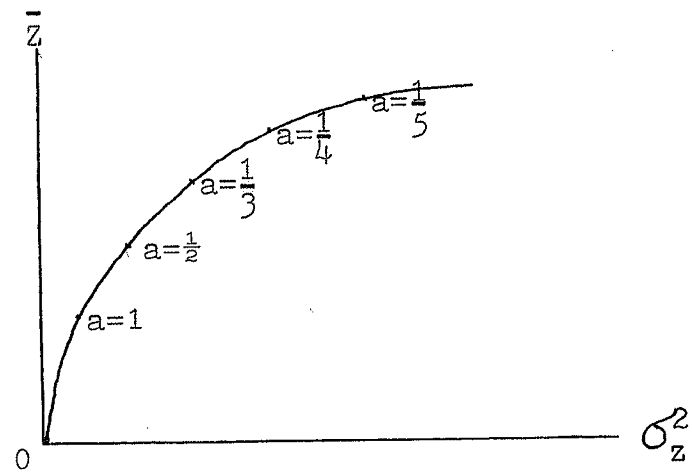

FIGURE 9

INCOME-VARIANCE POSSIBILITY LOCUS OBTAINED BY VARYING THE VALUE OF "A"

TABLE III

INCOME-RISK PREFERENCE TABLE

Income-risk Combinations →	1	2	3	4	----→
Guaranteed income $	Z_{1*}	Z_{2*}	Z_{3*}	Z_{4*}	----→
Degree of sureness given to a guaranteed income %	γ_1	γ_2	γ_3	γ_4	----→
Chance of failure (income would fall once in n years below the guaranteed levels)	$1/n_1$	$1/n_2$	$1/n_3$	$1/n_4$	----→

is given below (see Table III). A farmer would be requested to choose three income-risk combinations with priority according to his own income-risk preference.

As mentioned earlier, there is no essential difference between these models from a theoretical point of view. A given value of a can be interpreted in terms of q and vice versa. However, from a practical point of view, Van Moeseke's model is more relevant to determination of a risk parameter. By the "expost determination" method, we can determine a risk preference parameter, q, for Moeseke's risk programming problem. The procedure is to vary the value of q as well as the prescribed level of probability γ over a range, and solve the risk programming problems of Van Moeseke's formulation. Then find the lowest-bound income $Z*$'s corresponding to various levels of γ and/or q. From these data, we can construct an "income-risk preference table". Then, a decision-maker is required to select one of these alternative income-risk situations. A chosen income-risk combination determines indirectly the value of q and the corresponding optimal solution.

"Residuals" method of the variance-covarance matrix estimation. As to the second point of limitations, randomness of net prices can be achieved if trends and cycles are eliminated from time series data. The residual variation may be random and have a normal distribution. We can use these "residual" data for the estimation of a variance-covariance matrix.

CHAPTER III

ANALYTICAL MODELS FOR THE INVESTIGATION OF ON-FARM IRRIGATION FEASIBILITY

The analytical framework is constructed in relation to the objectives of the study as described in Chapter I. It is divided in two parts. The first part is devoted to a deterministic analysis of irrigation development under assumptions of perfect knowledge. The other part is a probabilistic analysis taking into account the stochastic nature of yields and prices of farm products. In part one, an ordinary linear programming model and a mixed-integer programming model are utilized. Stochastic programming models of Van Moeseke's and Freund's types are adopted for the second part. Both analyses are static in that the process of adjustment over time is not considered. Although machinery has a useful lifespan and land developed for irrigation can be used for many years, the utilization and costs of these factors are not treated dynamically in this study, but rather in a stationary manner (ie., in terms of depreciation or amortization).

The major components of linear and non-linear programming models are (1) activities, (2) constraint inequalities or equations and (3) an objective function to be optimized.

The alternative activities under consideration are determined by the objectives of the study. They also depend on the actual production alternatives available for producers under irrigation and existing dryland conditions. A group of constraints is determined by the actual restrictions of production and management such as limited levels of resources available for a producer and technical and managerial restrictions. Another group of constraints may include those of a purely logical and institutional nature such as; (1) a constraint inequality which regulates the relation between production and selling activities, (2) a regulation of specified acreage quota and (3) a constraint equation stating that the sum of net prices in the optimal solution should equal an amount of expected income determined exogenously.

The type of objective function to be optimized should be determined by the behavior of the relevant producing unit, the objectives of the study, the underlying assumptions, etc. If an objective of study is to develop production plans for a selected farm firm, then the behavior of the farm manager toward risk should be taken into account in determining the type of objective function. For example, if he is interested in an optimum production plan based on the long-run-average conditions of prices, yields and technologies, one may simply maximize the expected value of total income. If he is a risk averter to some extent, then the objective function should be formulated in such a manner that the kind of utility depending both on the size of income and the risk attached to it is maximized. If a relevant producing unit is a region, a quadratic type of objective function considering a declining demand curve faced by regional producers would be more realistic than a linear type.

I. ANALYTICAL FRAMEWORK UNDER PERFECT KNOWLEDGE

The major objective of the first part of the analysis is to investigate the economic feasibility of irrigation under long-run-average conditions for prices, crop yields and technologies as observed in past years. More specific objectives of the study are described in Chapter I. These objectives will be embodied in the deterministic framework of the analysis based on linear programming and mixed-integer programming models.

Economic analysis of irrigation is confined to a 250 acre representative farm.[1] Projection of aggregate demand for irrigation water, however, is based on the weighted sum of water utilized on small, medium and large representative

[1]The characteristics and resource endowment of the 250 acre representative farm are described on page 88 and in Appendix Table 14. The representative farm means the farm which has the average size of medium farms in the area (pp.12-3).

farms. For this purpose only, non-integer and mixed-integer programming problems associated with the parameterized price of water are solved with respect to 60-acres-average of small farms and 500-acres-average of large farms.

Two types of cropping alternatives, "dryland" and "irrigated", are considered. Two fertilizing alternatives, (ie., "fertilized" and "not fertilized") are also considered for each dryland small grain crop. All irrigated crops are fertilized. A few livestock alternatives are also taken into consideration together with alternative feed-supply activities.

The irrigated and dryland crops and forages are placed on a competitive basis in one linear programming matrix, so that the optimal mixture of irrigated and dryland crops and forages can be selected so as to maximize the objective functional.

The irrigated crops are accompanied by purchasing activities of irrigation machines and some special machines to be used exclusively for specialty crops. This model assumes that these machines can be purchased only at integral units. Therefore, the model includes seven integer variables (ie., X_{129} to X_{135}) together with non-integer variables.

Setting Up the Mixed-Integer Programming Framework

The mixed-integer programming model is constructed as follows:

1. Firstly, the production activities having the possibility of using the specialized machines purchased in integral units should be set up in the programme. If other production methods are available, (ie., other machinery or hand labor), then the production activities employing these alternatives should be included. In this model, irrigated-crop activities require at least one full set of irrigation machines, (ie., X_{135}). These activities range from X_{11} to X_{74}. Sugar beet activities, both on dryland and irrigated land, have two alternatives; hiring hand labor or buying a thinner for thinning operations. There are also two alternatives in the harvesting operation; using a sugar beet harvester or custom harvested. Potato activities require at least one seed cutter and there are two alternatives in the harvesting operation; using a potato digger or a custom digger. Fodder corn can be harvested either by a forage harvester or by custom harvester. Three alternatives are considered for harvesting hay. They are using a "hay-baler", "custom-baler", or "hay-loader".

2. With the assumption that the use of services of these machines (purchased in integral units) is divisible and continuous, the requirement coefficients of the production activities for these machines can be calculated. These

coefficients are denoted by $\frac{1}{K_h}$ (h = 129 to 135), where K_h is the maximum operating capacity of the h^{th} machine to be purchased. The method of calculating K_h for each machine is presented later.

3. Purchasing activities of the specialized machines are set up. These activities are treated as integer variables.

4. The constraint inequalities should be set up to specify the relationship between the services of machines purchased and the services used by production activities. The constraint inequalities are formulated as follows:

$$(III - 1) \sum_j \frac{1}{K_h} \cdot X_{hj} \leq X_h \qquad h = 129 \text{ to } 135$$

or

$$(III - 2) \sum_j \frac{1}{K_h} \cdot X_{hj} - X_h \leq 0 \qquad h = 129 \text{ to } 135$$

where:

X_h is the level of the h th machine purchased.

X_{hj} indicates the level of the j th production activity which uses the services of the h th machine. It should be noted that X_h is treated as a non-integer variable here. In this model, the coefficients in these inequalities are calculated by multiplying both sides of the inequalities by K_h such as:

$$(III - 3) \sum_j X_{hj} - K_h \cdot X_h \leq 0$$

5. When the net prices of production activities are

calculated, the operating costs of special machines should be included in the variable cost items.

6. The purchasing activities of the special machines should include the annual depreciation, repair and maintenance costs and interest in their net prices. The operating costs of the special machines, however, are not included in these net prices. The annual depreciations, repair-and-maintenance costs and interest on these machines appear in Appendix Table 12.

7. Seven integer constraint equations are added to the system of constraint inequalities and equations so as to enter the integer variables into the final basis at integral values.

The selling activities and the rows of yields of major crops, (ie., wheat, barley, oats, sugar beets and potatoes) are set up such that the minimum increase in yields due to irrigation required for major irrigated crops to be selected for entry in the optimal basis can be investigated.

The cost of irrigation water is separated from the variable costs of irrigated crops through a "buying irrigation water" activity. This enables a parametric variation of the cost of irrigation water.

The land distribution activity is added so as to enable one to vary parametrically the land consisting of a proportion of specific categories of land.

In this model, rotational activities are not used. Instead the upper limits and rotational relationships among certain crops are specified by using special constraint inequalities. Otherwise any crop can be grown either repeatedly or after any other crop.

Maximum Operating Capacities of Specialized Machines

Hay Baler with a full-time crew. Under the most favorable conditions, it is possible to bale 5 to 7 tons of hay per hour with automatic field balers; but the average daily capacity is usually about 4 tons per hour with a seasonal capacity of 2.5 to 3.25 tons per hour. The maximum operational capacity is calculated as follows;

4 tons/hour = 0.25 hrs./ton or 0.5 hrs./acre (dry matter)

3 tons/hour = 0.33 hrs./ton or 0.66 hrs./acre

It is assumed that about ten days in July are available for harvesting hay in favorable conditions. If a baler with a full time crew operates ten hours a day, approximately one hundred work hours are feasible for a crew with a machine. Therefore,

$$\frac{100}{50} = 200 \text{ acres}$$

or

$$\frac{100}{0.66} = 150 \text{ acres}$$

is harvestable. Thus, the maximum operating capacity K of a hay baler with a full-time crew range from 150 to 200 acres.

The maximum operation capacities of a corn forage harvester, sugar beet thinner, sugar beet harvester and potato digger are calculated in the same manner as for a hay baler. These capacities are shown in Appendix Table 6.

Assumptions For This Model

1. Farm firms aim to maximize farm incomes.

2. One of the conventional assumptions of linear programming, that is, the continuity and divisibility condition of purchasing some special machines, is relaxed.

3. The use of services of special machines is divisible and continuous.

4. The prices of products and factors, the technical coefficients and the amounts of available resources on the farms are perfectly known. All variables related to the prices of products and factors, yields, technical coefficients and resource availabilities are treated as deterministic variables rather than stochastic variables.

5. The durable lifetimes of specialized machines are known perfectly and the efficiencies of these machines do not change in their lifetimes.

6. A perfectly competitive market is faced by all farmers

in the project area.

7. Total land consists of a fixed proportion of specific categories of land, (ie., physically irrigable T_1 and T_2 land, crop land and unimproved land).

8. Irrigation water is readily available at the farm area level by constructing the farm lateral distribution system fed by the Pembina River.

9. The rainfall condition under consideration is a twenty year average for the project area.

10. Mature irrigation conditions are reached.

11. Irrigation development capital loans are available to a maximum amount of $20,000 for an individual farm.

12. Annual operating capital loans are also available to an upper limit of $10,000 per farm.

13. Short-term credit or loans are available for buying feeders for cattle operations.

14. Relatively cheap labor (ie., for thinning and potato-harvesting by hand) can be hired only for these specific purposes but can not be used for operating machines. Operating labor can be hired at a higher wage rate.

15. Custom work is also available for harvesting special crops such as sugar beets, potatoes and forage crops. Harvesting of small grains and sunflowers, however, is done only by the farmer's own combine.

16. Sugar beets and potatoes can be grown repeatedly on the same land with an interval of at least four years. Repeated use of the same land should be avoided in the case of field peas unless they are planted only at intervals of at least six years.

17. The natural environment in the Lower Yellow Stone Irrigation Project Area in North Dakota resembles that in the Morden-Winkler Project Area; so the ratios of irrigated crop yields over dryland crop yields calculated in the Lower Yellow Stone Project Area may be applied to the projection of irrigated-crop yields in the Morden-Winkler project area.

18. The same type and efficiency of machinery is owned and purchased by all farms of medium size. Techniques and the productivity of soil are homogeneous on all farms in this class.

Activities

The alternative crop and livestock activities considered in this model are based on the existing crops and livestock in the project area and in the neighboring areas. Some of them currently occupy very small percentages of the acreage in the project area; however, if irrigation water were to become available, they have the possibility of expansion. Vegetables are also promising crops when irrigation water becomes available, but they are not considered here for two reasons; (1) data for labor requirements and yields of vegetables, especially on irrigated land, are not available in the project area or any area under similar natural conditions, and (2) vegetable operations require highly skillful handling, special techniques and special capital equipments which can not be easily obtained by the farmers in the area.

A description of all activities under consideration is given in Appendix Table 15. Only a few noteworthy activities are explained here:

Tame hay and tame pasture. Two years stand is assumed here. Both hay and pasture are seeded every three years with a companion crop of wheat. The companion crop is harvested in the first year, but hay and pasture are not available for livestock in the seeded year. They become available after the second summer. Therefore, one acre of hay or pasture activity provides for two-thirds of an acre of hay or pasture and produces one third of an acre of wheat.

Irrigation. The irrigation method under consideration is one which is usually called "flood method" or "irrigation by gravity" as opposed to "sprinkler irrigation". Levelling of land is necessary for this type of irrigation. Usually, heavy movement of sub-soil (ie., heavy levelling) is done by a bulldozer in the beginning of irrigation development and is followed by light levelling by a leveller every spring. The total costs of light levelling can be included in the annual costs of both irrigated crop activities and "purchasing irrigation machines" activity in the forms of the leveller operating cost and depreciation and repair cost, respectively. In a static or stationary model, the cost of heavy levelling must be amortized. The initial cost of land development for irrigation is amortized on the basis of compound interest for a thirty year repayment period. The amortization formula in equation (III - 4) is used.[1] The capital for land development

$$\text{(III - 4)} \qquad R = \frac{A \cdot i}{1 - 1/(1+i)^n}$$

and investments in irrigation and special machines in the beginning of irrigation development is assumed to be supplied by an irrigation development capital loan at a 5.5 percent interest rate. The amortized cost of land development is included in the variable cost of each irrigated crop activity.

Purchase of specialized machines. Purchase of these machines requires a considerable amount of capital in the beginning of irrigation development. This type of capital is assumed to be supplied by a special loan (ie., an irrigation

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[1]The principle of amortizing cost is presented in Appendix III.

development loan up to a maximum of $20,000). Purchase of these machines also requires large amounts of annual operating capital because they will depreciate every year.

Constraint Inequalities and Equations

The first inequality states that the total land used can not exceed the given amount of total land in the right hand side. The given land can be used only as one of the four distinct categories of land (unirrigable crop land, irrigable T_1 crop land, irrigable T_2 crop land and unimproved land). Hence, the given total of land must be distributed to each category according to the presumed fixed ratio. For this purpose the land distribution activity is used. This constraint condition may be rewritten in a manner such that the total land distributed to the four categories can not exceed the given amount. If all of the land is not productively used, the remaining amount is allowed to be in "disposal". Therefore, the constraint is given by the inequality,

(III - 5) $X_{82} \leq b_1$ (Inequality no. 1)

The second inequality is concerned with the restrictive condition that crop land is assumed to occupy ninety percent of the total land. This category of land can be used for crops and forage crops either under irrigation or dryland condition. The constraint inequality specifies that the sum of land used for dryland or irrigated crops can not exceed the amount which is distributed to this category by the land distribution activity and must occupy ninety percent of the total land. This constraint can be used in the case of either variable or fixed total land. The constraint inequality is given as;

$$\text{(III - 6)} \qquad \sum_{j=1}^{61} X_j \leq 0.9\ X_{82}$$

or

$$\text{(III - 7)} \qquad \sum_{j=1}^{61} X_j - 0.9\ X_{82} \leq 0 \qquad \text{(inequality no. 2)}$$

The third inequality specifies that the amount of land used for irrigated crops on T_1 land can not be larger than the amount supplied to this category by the land distribution activity. T_1 land is assumed to be fifty-six percent of the total land. Therefore, the inequality is given as;

$$\text{(III - 8)} \quad X_5 + X_7 + X_9 + X_{11} + X_{13} + X_{14} + X_{15} + X_{16} + X_{21} + X_{22} + X_{25} + X_{27} + X_{28} + X_{31} + X_{33} + X_{35} + X_{36} + X_{37} + X_{41} \leq 0.56\ X_{82}$$

or

$$X_5 + X_7 + X_9 + X_{11} + X_{13} + X_{14} + X_{15} + X_{16} + X_{21} +$$

$$X_{22} + X_{25} + X_{27} + X_{28} + X_{31} + X_{33} + X_{35} + X_{36} + X_{37}$$

$$+ X_{41} - 0.56\ X_{82} \leq 0 \qquad \text{(constraint no. 3)}$$

In the same manner the fourth and fifth inequalities are given as;

$$\text{(III - 9)}\quad X_6 + X_8 + X_{10} + X_{12} + X_{17} + X_{18} + X_{19} + X_{20} + X_{23}$$

$$+ X_{24} + X_{26} + X_{29} + X_{30} + X_{32} + X_{34} + X_{38} + X_{39} +$$

$$X_{40} + X_{42} - 0.14\ X_{82} \leq 0 \qquad \text{(constraint no. 4)}$$

$$\text{(III - 10)}\quad X_{62} - 0.10\ X_{82} \leq 0 \qquad \text{(constraint no. 5)}$$

Inequality 6 specifies that the total acreage devoted to sugar beet production both on irrigated-land and dryland cannot be greater than one fourth of the total crop land. This means that the same land cannot be used successively for sugar beet production in four years because of disease problems. This inequality is given as;

$$\text{(III - 11)}\quad \sum_{j=13}^{20} X_j + \sum_{j=47}^{50} X_j \leq \frac{0.9}{4}\ X_{82}$$

or

$$\sum_{j=13}^{20} X_j + \sum_{j=47}^{50} X_j - \frac{0.9}{4}\ X_{82} \leq 0 \qquad \text{(Inequality no. 6)}$$

For the same reason, total acreage devoted to potato production is also limited to one fourth of the total crop land. The inequality is stated as:

$$\text{(III - 12)} \quad \sum_{j=21}^{24} X_j + \sum_{j=51}^{52} X_j - \frac{0.9}{4} X_{82} \leq 0 \quad \text{(inequality no. 7)}$$

Inequalities eight through twelve are concerned with relationships between demand for, and supply of, labor on a seasonal basis. The constraints are specified so that the total demand for labor within each season can not be greater than the total of residential and seasonally hired labor. These constraints are expressed as:

$$\text{(III - 13)} \quad \sum_{j=1}^{68} L_{8j} X_j - X_{76} \leq 861 \quad \text{(inequality no. 8)}$$

$$\text{(III - 14)} \quad \sum_{j=1}^{68} L_{9j} X_j - X_{77} - X_{80} \leq 1218 \quad \text{(inequality no. 9)}$$

$$\text{(III - 15)} \quad \sum_{j=1}^{68} L_{10j} X_j - X_{78} - X_{81} \leq 634 \quad \text{(inequality no. 10)}$$

$$\text{(III - 16)} \quad \sum_{j=1}^{68} L_{11j} X_j - X_{79} \leq 289 \quad \text{(inequality no. 11)}$$

$$\text{(III - 17)} \quad \sum_{j=1}^{68} L_{12j} X_j \leq 929 \quad \text{(inequality no. 12)}$$

Coefficients L_j's indicate the labor requirement coefficients.

Inequality thirteen specifies that the hand-thinning-weeding labor for sugar beets, hired at a comparatively low rate of wage, cannot be used for other purposes, especially for operational work. Since one acre of sugar-beet-with-hand-thinning activity requires 16.66 hours of hand-thinning summer labor, the constraint inequality which specifies this relationship is given as;

(III - 18) $X_{80} \leq 16.66\ X_{15} + 16.66\ X_{16} + 16.66\ X_{19} + 16.66\ X_{20} + 16.66\ X_{49} + 16.66\ X_{50}$

or

$$X_{80} - \sum_{\substack{j=15 \\ j \neq 17, 18}}^{20} 16.66\ X_j - 16.66\ X_{49} - 16.66\ X_{50} \leq 0$$

(inequality no. 13)

Inequality fourteen states a similar constraint with respect to potato-harvesting manual labor:

(III - 19) $X_{81} - 13.0\ X_{21} - 13.0\ X_{22} - 13.0\ X_{23} - 13.0\ X_{24} - 13.0\ X_{51} - 13.0\ X_{52} \leq 0$ (inequality no. 14)

The fifteenth inequality indicates the relationship between the amount of wheat supplied from wheat production activities and the amount of wheat sold. This relation is expressed by an inequality:

(III - 20) $X_{69} - 28.0\ X_1 - \sum_{j=5}^{6} 45\ X_j - 25\ X_{43} \leq 0$

(inequality no. 15)

In the same manner, four inequalities, sixteen through nineteen, are specified respectively for barley, oats, sugar beets and potatoes:

(III - 21) $X_{70} - 45X_2 - 55X_7 - 55X_8 - 29X_{44} \leq 0$

(inequality no. 16)

(III - 22) $X_{71} - 60X_3 - 80X_9 - 80X_{10} - 45X_{45} \leq 0$

(inequality no. 17)

(III - 23) $X_{72} - \sum_{j=13}^{20} 15X_j - \sum_{j=47}^{50} 10X_j \leq 0$ (inequality no. 18)

(III - 24) $X_{73} - \sum_{j=21}^{24} 180X_j - \sum_{j=51}^{52} 100X_j \leq 0$

(inequality no. 19)

The twentieth inequality provides for concentrated feed in terms of T.D.N. (total digestible nutrients) produced on the farm or purchased, to be allocated among the various livestock feeding activities. Feed grain is produced on the farm to feed livestock, but not for sale. The constraint is expressed as:

(III - 24) $310X_{63} + 750X_{64} + 2594X_{65} + 3377X_{66} + 4009X_{67} + 8900X_{68} -$

$1450X_4 - 1924X_{41} - 1924X_{42} - 74X_{74} \leq 0$

(inequality no. 20)

The twenty-first inequality provides for both tame pasture and native pasture on the farm to be allocated among the various feed consuming activities. Community or rented pasture is not considered here. The inequality is given as:

(III - 25) $2602X_{63} + 1865X_{64} - 1560X_{33} - 1560X_{34} - 878X_{87} -$

$1000X_{62} \leq 0$ (inequality no. 21)

The twenty-second inequality specifies the relationship between consumption and supply of T.D.N. derived from roughage. The constraint inequality is:

$$\text{(III - 26)}\quad 2404X_{63} + 1304X_{64} + 1297X_{65} + 1523X_{66} + 2005X_{67}$$

$$-\sum_{j=27}^{30} 4000X_j - \sum_{j=54}^{55} 1260X_j - \sum_{j=35}^{40} 2180X_j - \sum_{j=58}^{60} 1128X_j$$

$$- 940X_{75} \leq 0 \qquad \text{(inequality no. 22)}$$

Inequalities 23 through 29 state respectively that total seasonal use of services of each special machine which is required by each special crop and forage crop and is measured in terms of non-integral values, can not exceed the maximum operating capacity of the machine purchased. These constraints are expressed as:

$$\text{(III - 27)}\quad X_{27} + X_{29} + X_{54} - 40X_{83} \leq 0 \qquad \text{(inequality no. 23)}$$

$$\text{(III - 28)}\quad X_{35} + X_{38} + X_{58} - 150X_{84} \leq 0 \qquad \text{(inequality no. 24)}$$

$$\text{(III - 29)}\quad X_{13} + X_{14} + X_{17} + X_{18} + X_{47} + X_{48} - 130X85 \leq 0 \qquad \text{(inequality no. 25)}$$

$$\text{(III - 30)}\quad X_{21} + X_{23} + X_{51} - 50X_{86} \leq 0 \qquad \text{(inequality no. 26)}$$

$$\text{(III - 31)}\quad X_{13} + X_{15} + X_{17} + X_{19} + X_{47} + X_{49} - 50X_{87} \leq 0 \qquad \text{(inequality no. 27)}$$

$$\text{(III - 32)}\quad \sum_{j=21}^{24} X_j + \sum_{j=51}^{52} X_j - 300X_{88} \leq 0 \qquad \text{(inequality no. 28)}$$

$$\text{(III - 33)} \sum_{j=5}^{42} X_j - 250X_{89} \leq 0 \qquad \text{(inequality no. 29)}$$

Inequality thirty states that the total consumption of irrigation water by the irrigated crop activities in the optimal basis is supllied by the purchase of irrigation water:

$$\text{(III - 34)} \sum_{j=5}^{10} 10.3X_j + \sum_{j=11}^{12} 13.2X_j + \sum_{j=13}^{20} 10.7X_j + \sum_{j=21}^{24} 9X_j + \sum_{j=25}^{26} 10.6X_j + \sum_{j=27}^{30} 12.1X_j + \sum_{j=31}^{32} 5.2X_j + \sum_{j=33}^{40} 16.4X_j - X_{90} \leq 0 \qquad \text{(inequality no. 30)}$$

Three kinds of capital loans are considered in this model. Development of irrigation requires a large amount of capital. A fairly long-term capital loan is necessary to meet the demand for this kind of capital. Irrigation also requires a larger amount of annual operating capital because irrigated crops need more machinery operations, more fertilizer, specialized machines and the use of irrigation water. A third category of capital loan is considered for livestock activities in which the purchase of feeders requires a large amount of short-term capital. In this model, the upper limit of capital loans is considered in the case of the irrigation development capital loan and the annual operating capital loan, but not for the purchase of feeder. Inequality 31 specifies the upper limit of the irrigation development capital loan and inequality thirty-two restricts the upper limit of the operating capital loan. These inequalities are:

(III - 35) $\sum_{j} 60 \cdot X_j + \sum_{j} 90X_j + 4000X_{83} + 2400X_{84} + 250X_{85} +$

j=5,7,9,11, 13-16,21, 22,25,27, 28,31,33, 35-7,41   j=6,8,10,12, 17-20,23,24, 26,29,30,32, 34,38-40,42

$1600X_{86} + 7000X_{87} + 2000X_{88} + 2500X_{89} \leq 20{,}000$

(inequality no. 31)

(III - 36) $\sum_{j=1}^{90} dj \cdot X_j -$ [illegible] $X_{91} \leq 5{,}000$ (inequality no. 32)

$j \neq 62, 69\text{--}73, 82$

(d_j is the annual operating capital requirement of the j th activity.)

The thirty-third inequality is concerned with the upper limit of sunflower acreage. Half of the total crop land is available for sunflower planting each year because sunflowers can be grown every other year on the same land. This inequality is:

(III - 37) $\sum_{j} X_j - 0.45\ X_{82} \leq 0$ (inequality no. 33)

$j = 25, 26, 53$

Inequalities 34 and 35 provide for special constraints on field peas. Inequality 34 restricts the acreage available for field peas in each year to an upper limit of one sixth of the total crop land. This restriction is due to potential disease problems. Inequality 35 states that field peas can be grown only after row crops other than field peas, sod-breaking and summerfallow. These constraints are expressed as:

$$\text{(III - 38)} \sum_{j} X_j - 0.15\, X_{82} \leq 0 \qquad \text{(inequality no. 34)}$$

$$j = 31, 32, 56$$

$$\text{(III - 39)} \sum_{j=31}^{32} X_j + X_{56} - \sum_{j=13}^{26} X_j - \sum_{j=33}^{40} X_j - \sum_{j=47}^{53} X_j - \sum_{j=57}^{60} X_j \leq 0$$

(inequality no. 35)

The upper limit of selling small grains through the Wheat Board on the basis of specified acreage quota is restricted by the 36 th inequality. Nine bushels per specified acre is used as a basis for calculating the maximum quantity of small grains to be sold through the Wheat Board. The specified crops include small grains, (except flax) forage crops, pasture and fallow land. The constraint is:

$$\text{(III - 40)} \sum_{j=114}^{116} X_j \leq \sum_{j} 9 \cdot X_j \qquad (j=1,..,10,27,...,30,33,...,45, 54,55,57,..,62.)$$

or

$$\sum_{j=114}^{116} X_j - \sum_{j} 9X_j \leq 0 \qquad \text{(inequality no. 36)}$$

Inequality thirty-seven restricts the annual operating capital loan to an upper limit:

(III - 41) $X_{91} \leq$ upper limit of operating capital loan. (inequality no. 37)

Equations 38 through 44 are used to enter the integer variables into the final basis at integral values:

(III - 42)	X_{83} =	an integral value or zero	(equation no. 38)
(III - 43)	X_{84} =	" "	(" 39)
(III - 44)	X_{85} =	" "	(" 40)
(III - 45)	X_{86} =	" "	(" 41)
(III - 46)	X_{87} =	" "	(" 42)
(III - 47)	X_{88} =	" "	(" 43)
(III - 48)	X_{89} =	" "	(" 44)

When the integer variables are treated as non-integer variables, the problem may be solved without integer constraint equations 38 through 44.

II. ANALYTICAL FRAMEWORK UNDER IMPERFECT KNOWLEDGE

The major objectives of the stochastic analysis is to examine how the consideration of risk attributable to the fluctuation of crop yields and product prices would affect the economic evaluation of irrigation feasibility. In addition, the optimum farm organizations associated with various levels of risk aversion will be compared with those of existing farms and with the optimum solutions as derived by linear programming problems.

In order to render the optimum solutions of stochastic programming problems comparable with those of ordinary linear programming, the same activities and constraints should be considered in both programming problems. Due to lack of data available for computing variances and covariances, however, some of the activities used in the previous analytical framework are not available for the stochastic programming model. Therefore, another linear programming problem excluding these activities was solved for comparison purposes. The activities excluded from these two programming frameworks are "irrigated sunflower" and "irrigated field peas".

Activities

The activities considered in the analytical frameworks comprise (1) those for which the net prices are of deterministic nature and (2) the others having stochastic net prices. The former includes purchase of specialized machines, hiring of seasonal labour and capital loans. All of the production activities for crops and livestock are classified in the latter category. The input-output coefficients and net prices of these activities are adjusted to one-acre basis. Selling activities of wheat, sugar beets and potatoes are eliminated because the revenues of all crop production activities are calculated on a net-of-variables-costs basis. Year-to-year variation in the yields of intermediate products should be counted in the stochastic variation of net price of an activity which utilizes those intermediate products. In this analysis, the stochastic variations of feed grain and forage crop are included in the net prices of livestock activities. Description of the activities considered for the stochastic programming model and the linear programming model used for comparison appears in Appendix Table 18.

Constraints

Constraints are essentially the same as those in Section I, except for a few points of modification. In the present frameworks, constraints of selling wheat, sugar beets and potatoes are eliminated. Integer constraints are not considered because infinitesimal purchase units of specialized machinery are assumed. The constraints regulating the specified acreage quota for small cash grains are modified as below. A "specified acreage constraint inequality" and "available specified acreage" activity are added. The "specified acreage constraint inequality" is defined as:

$$\text{(III - 49)} \quad \sum_{j} X_j \geq X_{73}$$

where:

X_j indicates the acreage of the j th specified crop, and

X_{73} denotes the "available specified acreage" activity.

Inequality (49) can be converted to:

$$(\text{III} - 50) \; -\sum_j X_j + X_{73} \leq 0$$

The specified acreage quota constraint based on nine bushels per acre is set up as:

$$(\text{III} - 51) \sum_j Y_j \cdot X_j \leq 9X_{73} \quad \text{or} \quad \sum_j Y_j \cdot X_j - 9X_{73} \leq 0$$

where X_j is the level of j th small cash grain activity and Y_j indicates the associate yield per acre.

Resource Endowment of the Representative 250-acre Farm

The present analytical frameworks are confined to the 250 acre representative farm. As mentioned in Chapter I, all farms in the project area are classified into three sizes small, medium and large farms. The small farm is defined by a farm size of 10 to 149 acres; similarly, medium from 150 to 399 acres; and the large farm from 400 to 759 acres. The medium sized farms represent 48.0 percent of the total farms. The average size of medium farms is 242.0 acres. The "250-acre representative farm" represents this class of farms. Its resource endowment is based on the average quantities of resources owned by these farms. The 250-acre representative farm is typified by:

1. 250 acres of total farm land comprising 56% of irrigable T_1 land, 14% of irrigable T_2 land and 10% of unimproved land,

2. $5,000 of the farmer's own operating capital,

3. 3931.06 hours of annual family labour supply and

4. 2 tractors, and one combine, swather and truck.

Assumptions For This Model

The specific assumptions of the stochastic analysis are:

1. The net prices of crop production and livestock activities are stochastic variables having normal distributions. Accordingly, the total income associated with these activities is also distributed normally. If net prices have trends over time, then the residuals from the trend lines have normal distributions.

2. Total variable cost of each activity is either constant over the planning period or it increases on a trend line with very little variation from the trend. The random variation from the trend line is assumed negligible. Hence, variable cost may be treated as a deterministic variable.

3. Resource requirement coefficients of each activity

and the quantities of resources available for individual farms are given (known).

4. The assumptions described in Section I of the present chapter are valid except for (1), (2) and (4).

5. Farm firms aim to optimize their farm incomes taking account of both expected incomes and the associate risks.

Estimation of Variances and Covariances of Net Prices

Variances and covariances of net prices are essential elements in a stochastic programming framework in addition to technical coefficients, net prices and right-hand-sides. The variances and covariances of net prices are estimated by using the time series data of net prices for the period of 1954 to 1965. A trend line was fitted to each of 53 time series of net prices by least squares regression and the differences (residuals) between the time series observations and the predicted values of regressand variable are calculated. These residuals are employed to estimate the variances and covariances of 53 sets of net prices. The regression coefficients of the trend lines, results of Student's t tests and R^2 values appear in Appendix Table 21 . The t-tests indicate that most of the net prices have significant trends (one exception is sugar beets). It is therefore desirable to use the residuals of the time series observations from the trend lines for the estimation of variances and covariances rather than to use the differences from their means.

Actual Procedure of Solving the Van Moeseke Type of Stochastic Programming Problems

A stochastic programming problem of the Van Moeseke type can not be solved immediately by a quadratic programming procedure. Three alternative methods of solving this problem by a quadratic programming method are discussed in Chapter III.

The first approach, is to use the subsidiary quadratic programming problem of an original stochastic programming problem is used. In the second approach, a Heady-Candler type of model is applied to obtain the locus of income-variance minimized subject to a parameterized level of expected income and a set of linear constraints. The combinations of expected income and standard deviation of income on the locus are substituted successively into an objective function of the Van Moeseke type in order to find the maximum value of the functional. Thirdly, a Freund type risk programming model can be used to solve indirectly a stochastic programming problem of the Van Moeseke type. The second and third methods provide an approximation of the optimum solution for the problem. With the practical application of a stochastic programming model for the development of optimal plans for a decision-maker, it is not always necessary to assign an exact value of the risk aversion parameter. Essentially, the same optimal solution may be obtained for a range of risk aversion parameters. Therefore, we may assign a range of risk aversion parameter (q or a). Furthermore, a decision-maker may have so inexact a parameter that it may be defined by only a particular combination of guaranteed income and associated probability. A certain range of combinations would be indifferent for his choice.

Definition of risk aversion levels. In the present study, five different levels of risk aversion are considered. They are defined in Table IV.

The level of high risk aversion is defined by the probabilities (attached to a guaranteed income) in the range 0.95 and over. The level of high-medium risk aversion is represented by probabilities of 0.85 to 0.95. The other levels are defined in the same way. The high risk aversion coefficients of q may vary in the range 1.645 and over. This means that income in the optimum plan is guaranteed at a probability greater than 0.95. In other words, there would be a chance for him to have a lower income than that guaranteed, once in 22-and-over years. Hence, the farmer's income in this optimal plan is guaranteed with certainty for most of his farming career. The medium risk aversion coefficient may vary from 0.675 to 1.037 of q, that is probabilities 0.75 to 0.85 attached

TABLE IV

FIVE LEVELS OF RISK AVERSION

Level of risk aversion	Probabilities attached to guaranteed Income	Value of q	Value of "a" used for calculation	Chance of failure (income falls below the guaranteed level once in ......)
High risk aversion	0.95-1.00	1.645 and over	0.0039	22 and over years
High-medium risk aversion	0.85-0.95	1.037-1.645	0.0018	8-14 years
Medium risk aversion	0.75-0.85	0.675-1.037	0.0011	5-6 years
Low-medium risk aversion	0.65-0.75	0.385-0.675	0.00066	3-4 years
Low risk aversion	0.55-0.65	0.126-0.385	0.0002	2-3 years

to a guaranteed income. An income lower than the guaranteed level in the optimal plan may occur once in 5 to 6 years. The low risk aversion coefficients range from 0.126 to 0.385 and are associated with a range of probability, 0.55 to 0.65. In this case, a decision-maker should be prepared to accept an income lower than the guaranteed level, once in 2 to 3 years.

For practical purposes, it should be satisfactory if an optimal solution of a stochastic programming problem is obtained with respect to a value of q in each range as described above.

Practical procedure of solving the problem. In this study, the second approach was attempted first to solve the stochastic programming problems. Solutions were obtained with the computer programme developed by D.J. Soults and J.J. Zurbec.[1] Difficulties were encountered in the process of practical computation by this approach because:

1. Many "infeasible" solutions were obtained for various levels of expected incomes.

2. It was difficult to judge whether or not the derived maximum point was the global maximum. This would present serious problems if the linear constraints were complex and linear constraint equations were involved.

3. Negative levels of activities often appeared in an optimal solution which ignored the non-negativity condition of X vectors. Negative values also appeared in the Lagrange multipliers in the optimal solution and the non-negativity condition was violated.

The third approach was, in practice, a more efficient method of solving the same stochastic programming problem by

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[1] D.J. Soults & J.J. Zurbeck. Zorilla - An Algorithm for the Optimization of a Quadratic form Subject to Linear Restraints, Iowa State Univ., May, 1966.

the same computer programme. The principle of this approach is summarized below.

An objective function of a stochastic programming problem of the Van Moeseke type is:

(III - 52) $$f(X) = \bar{C}' \cdot X - q(X' \cdot V \cdot X)^{\frac{1}{2}}$$

or

$$Z_* = \bar{Z} - q \cdot \sigma_Z$$

When it is maximized, the function is denoted by

(III - 53) $$Z_*^* = \bar{Z}^* - q \cdot \sigma_Z^*$$

Equation (III - 52) can be rewritten as

(III - 54) $$Z_* = \bar{Z} - (a/2 \cdot \sigma_Z) \cdot \sigma_Z$$

where a is a constant satisfying $a/2 \cdot \sigma_Z = q$.

Furthermore, equation (III - 54) can be written as

(III - 55) $$Z_* = \bar{Z} - a/2 \cdot (\sigma_Z)^2$$

where a should be equal to $2q/\sigma_Z$.

Obviously, equations (III - 54) and (III - 52) are maximized if equation (III - 55) is maximized. In other words, if a Freund type objective function associated with a given value of its risk aversion coefficient, a, is maximized, then the objective function of the Van Moeseke type associated with q and equal to $a/2\sigma_z$ is maximized simultaneously.

Using this relation, a Van Moeseke stochastic programming problem can be solved through the Freund type of stochastic programming procedure.

The following steps are taken in the present study:

1. An arbitrary value of a is chosen for each level of risk aversion as defined in this section.

2. The estimated variance-covariance matrix, V, is multiplied by the risk aversion coefficient, a, in order to convert the Freund objective function:

$$\bar{C}'\cdot X - (a/2)(X'\cdot V\cdot X) \qquad \text{(III - 56)}$$

to the form specified in "Zorilla"!

$$\bar{C}'\cdot X - \tfrac{1}{2}X'\cdot V^{*}\cdot X \qquad \text{(III - 57)}$$

where:

$V^{*} = a\cdot V$.

3. The stochastic programming associated with a given a is then solved by using the computer programme in "Zorilla".

4. The corresponding values of q and a are calculated by the maximized functional:

$$U^{*} = \bar{C}'^{*}\cdot X^{*} - (a/2)\cdot(X'^{*}\cdot V\cdot X^{*}) \qquad \text{(III - 58)}$$

where:

X^{*} is the optimal solution vector, and

U^{*} denotes the maximized functional value of (III-56).

Since:

$$\text{(III - 59)} \qquad U^* = \bar{C}'^* \cdot X^* - (a/2) \cdot \sigma_z^{2^*},$$

$$\text{(III - 60)} \qquad \sigma_z^{2^*} = 2 \cdot (\bar{C}'^* \cdot X^* - U^*) / a$$

$$\text{(III - 61)} \qquad \sigma_z^{\ *} = (2(\bar{C}'^* \cdot X^* - U^*) / a)^{\frac{1}{2}} \quad ,$$

the corresponding value of q is obtained by

$$\text{(III - 62)} \qquad q^* = (a \cdot \sigma_z^*) / 2 \quad .$$

5. The value of a is adjusted until the calculated value of q* falls in the specified range of risk aversion.

<u>Livestock</u> <u>Production</u> <u>Activities</u>

The input-output coefficients of livestock activities are computed on a one-acre basis. In order that this may be done, the livestock-maintaining capacity of one acre of land must be estimated.

The maintaining capacity varies, depending on year-to-year variations in the yields of feed crops. On the other hand, livestock operations have the advantage that a fairly constant size herd can be maintained per acre regardless of yield fluctuation because purchased feed is available. Of course, the prices of purchased feed are high in a bad year for feed crops. However, even in an average crop year, purchased feed is more expensive than home-grown feed.

Therefore, stochastic variation of net revenue per unit livestock activity depends upon variation in the yields of feed crops as well as on variation of livestock prices.

Livestock-maintaining capacity on one-acre basis. The size of herd which can be maintained constantly per acre of land is estimated by the procedure in the following paragraphs.

A beef cattle operation may require three kinds of feed, feed grain, forage and pasturage. Feed-lot enterprises require only feed grain and forage. Sow-hog activities need feed grain and supplementary feed. Long run average yields of feed crops are used to determine the maintaining capacity. In this case, however, the capacity can not be determined simply by dividing the amount of T.D.N. produced per acre by the T.D.N. requirement per unit of livestock activity. This results because three kinds of feed elements are required to provide the T.D.N. A unit of land should be allocated for the most efficient production of these three kinds of feed crops.

An acre of land would be used in the most efficient manner if the ratios of T.D.N.'s from these three kinds of feeds equaled that of T.D.N. requirements. The feed requirements ratio is assumed to be adjustable to some extent to the annual yield situation of feed crops. In a bumper year of forage crops, for instance, livestock producers will substitute forage for feed grain to a permissible extent. Keeping this sort of adjustability in mind, the long run average yields of feed crops are used to determine the ratio at which the land for feed crops is allocated among these feed crops. Let W_1, W_2 and W_3 denote the fraction of each of these crops in one acre of land. It is obvious that $W_1 + W_2 + W_3 = 1$. The long run average yields under both irrigation and dryland conditions are shown in Table V.

TABLE V

AVERAGE YIELDS OF FEED CROPS IN TERMS OF T.D.N.

	feed grain	hay[1]	pasture[1]
	----------------pounds----------------		
Under dryland condition	1046 per acre	1128 per 2/3 acre	878 per 2/3 acre
Under irrigation	1924 per acre	2180 per 2/3 acre	1560 per 2/3 acre

[1]Recall that one third of forage crop land is seeded every year and is not available for harvesting.

The dryland case for a cow-calf operation is illustrated first. The average yields under dryland conditions should be converted to a ratio having the yield of feed grain as a basis (equal to one). The ratio is:

(III - 63) grain: hay: pasture = 1: 1.078 : 0.839

On the other hand, the ratio of T.D.N. requirements is:

(III - 64) grain: hay: pasture = 1: 7.754 : 8.393

An acre of land devoted to a cow-calf operation would be allocated most efficiently to these three crops if the following relationship holds:

(III - 65) $$1W_1 : 1.078W_2 : 0.839W_3 = 1 : 7.754 : 8.393$$

From (III - 65), we may derive the following:

(III - 66) $$W_2 = \frac{7.754}{1.078} \cdot W_1 \text{ and } W_3 = \frac{8.393}{0.839} \cdot W_1$$

On the other hand, the equation $W_1 + W_2 + W_3 = 1$ should be satisfied. Therefore:

(III - 67) $$W_1 + \frac{7.754}{1.078} W_1 + \frac{8.393}{0.839} W_1 = 1$$

and,

(III - 68) $$W_1 = 0.054,\ W_2 = 0.388 \text{ and } W_3 = 0.540$$

The same procedure is applicable for all other livestock activities under both dryland and irrigation conditions. The most efficient allocation of an acre of land among these crops as determined by the above procedure, appears in Table VI.

TABLE VI

ALLOCATION RATIOS OF FEED CROP LAND TO THREE CATEGORIES OF FEEDS

Livestock operations[1]	Under dryland conditions			Under irrigation condition		
	feed grain	hay[2]	pasture	feed grain	hay[2]	pasture
	acres					
Cow-calf (1:7.75:8.39)	0.05	0.39	0.56	0.075	0.415	0.510
Feeder-calf (1:1.74:2.49	0.176	0.283	0.541	0.235	0.292	0.473
Feed-lot 400 (1:0.5:0.00)	0.684	0.316	0.00	0.736	0.264	0.00
Feed-lot 600 (1:0.456:0.0)	0.703	0.297	0.00	0.754	0.246	0.00
Feed-lot 800 (1:0.50:0.00)	0.684	0.316	0.00	0.736	0.264	0.00
Sow-hog (1:0.0:0.0)	1.00	0.00	0.00	1.00	0.00	0.00

[1]The ratios in brackets indicate the T.D.N. requirements ratios for alternative livestock operations.

[2]Two third of the land used for hay productions or pasture is available for harvesting as hay, whereas, the remaining one third produces a companion crop.

Once the allocation ratios are determined, the number of livestock maintainable per acre of land can be estimated in terms of the sum of T.D.N.'s produced per acre. The levels of livestock activities maintained on one acre of land are presented in Table VII.

Since the sum of T.D.N.'s produced per acre fluctuates from year to year, the sum of T.D.N.'s produced will exceed the sum of T.D.N.'s required by a given number of livestock in one year and will be short of that in another year. In a year of shortage, the shortage should be supplemented by purchased feed. In a bumper year, the surplus can be sold. Thus, a constant number of livestock per acre can be maintained. The estimated revenue from selling the surplus feed in a bumper year is included in the net price of a livestock activity. The sum of T.D.N.'s required by each livestock activity on a one-acre basis appears in Table VII. The T.D.N.'s produced per acre as well as the surplus or shortage of feed in terms of T.D.N. in the years of 1954 to 1965, are presented in Appendix Table 37.

Linear Programming Framework for Crop Insurance Alternatives

One of the specific objectives relating to stochastic analysis is to evaluate some alternative plans of crop insurance in the light of stochastic programming framework. Various criteria and approaches may be available for the

TABLE VII

LEVELS OF ALTERNATIVE LIVESTOCK ACTIVITIES PER ACRE OF LAND

	Level of activity maintainable on an acre		T.D.N. requirements		
		Total	Feed grain	Hay	Pasture
		------------pounds------------			
1. Under dryland conditions;					
Cow-calf	0.185 cow	983.90	57.36	444.72	481.13
Feeder-calf	0.250 head	987.32	186.86	325.78	465.68
Feed-lot 400	0.344 "	1071.91	714.96	356.95	0.00
Feed-lot 600	0.357 "	1070.36	735.34	335.02	0.00
Feed-lot 800	0.458 "	1071.91	714.96	356.95	0.00
Sow-hog	0.118 sow	1046.00	0.00	0.00	0.00
2. Under irrigation conditions;					
Cow-calf	0.347 cow	1844.60	107.54	833.76	902.01
Feeder-calf	0.466 head	1826.58	348.88	608.25	869.45
Feed-lot 400	0.638 "	1991.58	1328.38	663.20	0.00
Feed-lot 600	0.662 "	1986.98	1365.06	621.92	0.00
Feed-lot 800	0.851 "	1991.58	1328.38	663.20	0.00
Sow-hog	0.216 sow	1924.00	0.00	0.00	0.00

economic evaluation of crop insurance. In the present study, crop insurance is compared with irrigation and diversification of farm operation as a means of reducing risk. In this case, the same criterion of utility measurement should be adopted for the comparison of utilities obtainable from these methods in a state of risk.

Manitoba crop insurance programme. Buying crop insurance means that the insured is willing to pay a prescribed amount of premium for a minimum return to the insured crop guaranteed at the probability of unity. Under the Manitoba Crop Insurance Programme (1967), eight crops can be insured. They include wheat, oats, barley, flax, sugar beets, fall rye, etc. In the study area, wheat, oats, barley, flax and sugar beets are considered for crop protection. With the all-risk crop protection plan offered by Manitoba Crop Insurance Corporation, farmers in Manitoba have three choices of bushel coverages and two options of crop prices for an insurable crop. The three bushel coverages are determined, respectively, at 60, 70, and 80 percent of the 25 year average yield of each insurable crop within each risk area. Two options of crop prices are available only for wheat, oats, and barley. In the same risk area, levels of bushel coverage and premiums differ slightly among various soil zones. In this study, however, the average figures of Stanley municipality are used to calculate bushel coverages, guaranteed revenues and premiums per acre.

Analytical framework. Two alternative approaches may be considered for the evaluation of crop insurance plans in a risk programming framework. Firstly, "insured dryland crop" activities are included in the programming framework. The expected net price of an insured crop activity is calculated in the same way as the non-insured crops. However, the estimation of variance and covariances with respect to the insured crop differs from that of a non-insured crop. For instance, variance and covariances of an insured crop having 60 percent of bushel coverage are estimated as; (1) all yields less than 60 percent of the 25 year average yield appearing in the time series data of 1954 to 1965 are converted to the level of bushel coverage, and (2) the time series observations of net price calculated on the basis of bushel coverage are used for the estimation of variance and covariances of the insured crop. This approach, however, can not fully satisfy the normality assumption because the distribution of yield of an insured crop lacks the part corresponding to the range of yield less than 60 percent of the 25 average yield.

The second approach takes account of two cases. From a practical point of view, crop insurance may be more useful for a decision-maker who devotes a high proportion of crop land to a specialized crop yielding a high expected net revenue associated with a high risk. At the same time, a decision-maker who buys crop insurance may have a fairly strong risk aversion because he wants to insure a certain amount of income at 100 percent of reliability.

Two types of decision-makers who are interested in crop insurance are presumed here. One has a very conservative attitude towards risk. It is assumed that he insures all crops he grows, and in addition he chooses a combination of crops such that the total "insured" revenue may be maximized. The other type of decision-maker would maximize, subject to a set of constraints, the "expected" total income accruing from a few specialized enterprises of high expected net prices, while insuring all of the insurable crops entering the optimal plan. The former is concerned only with the insured revenue regardless of level of expected revenue; whereas, the latter is concerned with both expected revenue and the insured revenue.

A decision-maker of the first type will try to maximize the guaranteed income net of variable costs and premium of crop insurance subject to a set of constraints. Therefore, his objective function to be maximized is the summation of insured net revenues of individual crops for which alternative crop insurance plans are available. On the other hand, the production of insured crops are subject to a set of technical and operating constraints. Thus, his optimization problem can be formulated as a standard linear programming problem where the objective function is the summation of "insurable" net revenues rather than expected net revenues. Activities considered in this linear programming problem should include all alternative protection schemes of insurable crops. Since five insurable crops having, respectively, three bushel coverage alternatives associated with two price options are considered in this study, (no price option available for flax and sugar beets), twenty-three crop protection activities should be taken into account. Moreover, four different methods of harvesting sugar beets, the hiring of seasonal labour, acquiring operating capital loan, and purchasing specialized machinery are also considered. Therefore, a total of 43 activities are considered in this linear programming problem. The constraint inequalities are essentially the same as those in the previous section except for excluding those of irrigation and those regulating the uninsured crops. The description of these activities and constraints appears in Appendix Table 20.

Regarding the optimum solution, the maximum functional provides a combination of minimum guaranteed income and the associated level of probability (Z_* and γ). Obviously γ equals unity. The combination of guaranteed income and probability obtained by this linear programming solution is comparable with any combination of those obtained by the solution of stochastic programming problems of the Van Moeseke type.

CHAPTER IV

ECONOMIC EVALUATION OF IRRIGATION UNDER PERFECT KNOWLEDGE ASSUMPTION

This chapter presents the results based on the first section of the analytical framework. The actual procedure of solving the mixed-integer programming problem associated with the parameterized price of irrigation water is presented in Appendices IV and V. The analysis in this chapter is based on the optimal solutions of the mixed-integer programming problem unless otherwise specified. The economic analysis of irrigation except the projection of aggregate demand for water is confined to the representative 250-acre farm.

The first section of this chapter analyses the economic conditions under which irrigation water can be used optimally. In the second section, the optimal farm organizations are compared with the existing ones. The third section is devoted to an analysis of the static-normative demand function for irrigation water. Marginal values of irrigation water on 250-acre farms are also investigated. The last section deals with optimal investment in the specialized machines.

I. ECONOMIC FEASIBILITY OF IRRIGATION UNDER VARIOUS CONDITIONS

Change in conditions such as prices of products and factors, technical coefficients and resource constraints, would affect the optimal farm organization. Some optimal combinations of activities, however, are fairly stable for changes in these conditions. In other words, the optimality of some farm organizations is valid for fairly wide ranges of varied conditions. For instance, the optimal farm organization on 250-acre farms with the price of water fixed at $2.00 per acre-inch is optimal in the range of total land from 206 acres to 353 acres. This range covers approximately 60 percent of the total range in acres of middle size farms (150-400 acres). The optimal solution does not change significantly for changes in the price range of water between $o.888 to $2.139 per acre inch. The optimal solution, however, is sensitive to a change in the upper limit of operating capital loan. A change in the specified acreage quota of small grains from 6 to 11 bushels per acre does not significantly affect the optimal solutions which are obtained under the assumption of the price of water equal to $2.00 per acre inch.

The optimal solution obtained with the price of hogs fixed at $27.00 per 100 lbs. of carcass weight is not severely affected by changes in the price of hogs until the hog price exceeds $27.70.

The optimal solutions which are obtained by varying one of these conditions while holding the others constant are given in Appendix Tables 22 to 25.

Specialty crops, sugar beets and potatoes are irrigated economically under wide ranges of change in the price of water, total holdings and upper limit of operating capital loan; while small cash grains are not irrigated profitably under almost any of these conditions. Flax is irrigated optimally on T_1 land only when the price of water is lower than \$0.69. This result is associated with the maximum amount of operating capital loan of \$10,000. Fodder corn is also optimally irrigated only on T_1 land in the low-to-middle price range of water, (ie., from zero to \$2.139). Hay is not irrigated under any range of price of water, operating capital loan and total holdings. Land of T_2 quality can be irrigated optimally if the price of water is lower than \$0.69. At this price of water, all of the physically irrigable land can be optimally irrigated. While the annual operating capital loan is utilized up to the upper limit for all prices of water, the irrigation development loan is not fully used even when the price of water is zero. If the annual operating capital loan is not available at all, none of the physically irrigable land is irrigated optimally when the price of water is fixed at \$2.00 and the operating capital at \$5,000.

Optimal Plans With Varied Prices of Water

Some major changes occurring in the optimal solutions are investigated for the 250 acre farms when the price of water is varied. When the price of water is lower than \$0.69 per acre-inch, all of the physically irrigable land (both T_1 and T_2) are devoted to the irrigated specialty crops and forage crop; that is, sugar beets, potatoes, flax and fodder corn. Dryland wheat fertilized and cattle accompanied by feed grain and fodder corn are also in the optimal solution, but they occupy a relatively small percentage of total land. This optimal solution remains the same in the price range from zero to \$0.69 per acre inch. The optimal functional value, however, changes from \$15,771 to \$14,490 in inverse proportion to the increasing price of water. This optimal solution includes the most diversified combination of irrigated crops found among all optimal solutions. One unit each of irrigation machines, sugar beet thinner, potato digger and potato seed cutter are purchased. When the price of water is greater than \$0.69 and less than \$0.887, T_2 land is not irrigated in the optimal solutions. The acreage of irrigated flax falls from 66.2 acres to 27.3 acres while the acreage of irrigated sugar beets increases up to the maximum limit. Dryland wheat fertilized and dryland feed grain with fertilizer also increases. All of T_2 land is taken over by the increase in these dryland activities. When the price of water is less than \$0.887, summer labour, fall labour and potato-harvesting manual labor are hired.

As the price of water rises to the lower limit of the price range, \$0.887 to \$1.710, the proportion of dryland activities increases significantly. A part of the sugar beets begins to be produced on dryland and the acreage of irrigated sugar beet is cut by nearly a half. Both dryland wheat with fertilizer and dryland feed grains increase.

As the price of water rises from \$0.888 to \$2.139, almost the same optimal solution continues while the functional value falls inversely and proportionate with the climbing price of water. With this wide price range of water, nearly equal proportions of crop land are devoted to dryland wheat, dryland feed grain for livestocks, irrigated potatoes and sugar beets under both irrigated and dryland conditions.

In the final price range from \$2.14 to \$2.62 per acre inch, the dryland activities such as wheat and feed grain occupy a higher proportion of crop land. Emphasis is put on feed grain for feeding cattle. Sugar beets are produced only on the irrigated land and the dryland devoted to sugar beets in the lower price range of water is shifted to dryland feed grains. Potatoes on irrigated land are slightly decreased.

When the price of water exceeds \$2.62, no irrigated crop enter the optimal solution of the mixed-integer programming problem in which purchase of irrigation and other specialized machines is considered at integral units. In this price range, only one optimal solution exists and it is not affected by the change in the price of water. Total crop land is devoted to only two dryland crops, ie., sugar beets and feed grains. This indicates that the optimal farm organization without irrigation but with sufficient operating capital would be specialized in a specialty crop and cattle operations. Under irrigation, the optimal farm organization includes more diversified enterprises.

Limiting Resources and Their Shadow Prices

Through the entire range of water prices, fall labor and harvesting labor limit the optimal solutions with approximately 100-150 hours of fall labor and 180-380 hours of potato-harvesting manual labor hired. Annual operating capital is also a limiting resource and used up to the upper limit of operating capital loan. Land is another limiting resource. On the other hand, family labor in spring and winter and irrigation development loan are not totally used.

The shadow prices of these limiting resources under variable prices of water appear in Table VIII. The amounts of unused resources are presented in Table IX.

Effect of Change in Hog Price Upon Optimal Plans

Changes in hog prices affect the optimal solution and optimal level of irrigation development when the hog price exceeds $27.70 per 100 pounds of carcass weight. When the price of hog is less than $27.70, no hog activities come into the optimal solution. When the price of hogs is higher than $27.70 but less than $28.80, hog activities become optimal while the optimal level of the activity "feed lot 400"[1] is reduced to approximately one third of the level (28 lots) attainable at the hog price $27.70. About half of the land devoted to fodder corn is switched to the productions of feed grain for hogs when the price of hog lies in the range from $27.70 to $28.80. Dryland sugar beets activity is switched completely to irrigated sugar beets. Potatoes decrease slightly. When the price of hogs exceeds $28.8, wheat production is stopped and total crop land is allocated to 13.9 acres of feed grain, 62.5 acres of irrigated sugar beets and 20.2 acres of irrigated potatoes. The optimal level of hog production is increased from 8.8 sows to 13.4 sows. The activity "feed lot 400" also increase to 21.6 lots with the increased feed grain production. If the price of hog becomes higher than $29.40, hogs begin to compete again with "feed lot 400" which is decreased to 13.4 lots. Demand for irrigation water, however, does not change substantially when the price of hog varies from $27.00 to $29.40. If the price of hog becomes higher than that, the demand for irrigation water decreases by approximately 200 acre-inches because irrigated

[1]This beef cattle operation is such that fall-purchased 400 lbs. calves are finished in feed-lots with feed grains in the following September.

TABLE VIII

SHADOW PRICES OF LIMITING RESOURCES UNDER VARIOUS PRICES OF IRRIGATION WATER AS OBTAINED FROM THE MIXED-INTEGER PROGRAMMING SOLUTIONS (250 ACRE FARM)

	Price of Water ($ Per Acre-inch)						
	0 ~ 0.688	0.689~ 0.887	0.888~ 1.710	1.711~ 1.983	1.984~ 2.134	2.140~ 2.620	2.621[1] and over
	------dollars------						
Fall labor (per hour)	2.3	2.31	2.24	2.25	2.25	2.25	1.94
Summer labor (per hour)	2.3	2.31	2.24	1.87	1.83	1.63	0
Harvest labor (per hour)	1.9	1.70	1.64	1.65	1.65	1.66	0
Operating capital (per a dollar)	0.36	0.36	0.32	0.32	0.32	0.33	0.14
Total land (per acre)	32.6	31.9	30.6	30.6	30.7	30.9	41.7

[1]When the price of water is higher than$2.62, no irrigated crop activities are included in the optimal solution.

TABLE IX

AMOUNTS OF UNUSED RESOURCES UNDER VARIOUS PRICES OF IRRIGATION WATER (OBTAINED FROM THE MIXED-INTEGER PROGRAMMING SOLUTIONS, 250 ACRE FARM)

	Price of Water ($ Per Acre-inch)						
	0 ~ 0.688	0.689 ~ 0.887	0.888 ~ 1.710	1.711 ~ 1.983	1.984 ~ 2.139	2.140 ~ 2.620	2.621 and over
Irrigation develop. capital loan ($)	2100	5250	7944	7944	7944	8192	19750
Winter labor (hr)	770.3	682.4	638.9	6.389	6.389	542.7	0
Spring labor (hr)	118.7	143.1	207.7	207.7	207.7	186.6	185.6
Irrigable T_1 land (per acre)	0	0	44.9	44.9	44.9	49.1	140.0
Irrigable T_2 land (per acre)	0	35.0	35.0	35.0	35.0	35.0	35.0

potatoes and irrigated fodder corn are replaced largely by feed grains for hogs.

Effect of Varied Quota on Wheat Selling

As the specified acreage quota is increased from six to eleven bushels, wheat production becomes more competitive with feed grains for crop land. When the quota is open, 111.2 acres of wheat are cultivated as contrasted with only 6.9 acres of feed grain. In the open quota situation, no "feed lot 400" is in the optimal solution. Only a small number of cattle are fed on the native pasture supplemented by additional grains. Different quota levels do not, however, have a significant impact on the acreage of irrigated crops and the quantity of water utilized in the optimal plans.

Effect of Change in the Available Operating Capital Loan

The optimal solutions are sensitive to changes in the amount of operating capital loan available. Particularly affected are the optimal levels of irrigated crops and the demand for irrigation water. In this sub-section, rather than being forced into the optimal basis at one unit, all integer variables were treated as non-integer variables rendering optimal levels of irrigated crops more adjustable to the change. The optimal solutions having various upper limits of operating capital loan are shown in Appendix Table 23. When a loan is not available or at less than $1,000 per farm, no irrigation is undertaken. When $1,500 to $2,500 of loan are available, very little irrigated crops come into the optimal solution. Even when $3,000 to $4,500 of loan are available, no crop can be optimally irrigated if the purchase of the specialized machines is considered in integral units. At least $5,000 of operating capital loan are necessary to make irrigation optimal: in other words optimal irrigation requires in addition to the irrigation development capital, at least twice as much annual total operating capital as the amount actually owned by an average middle sized farm.

It can also be observed from Appendix Table 23 , that when the upper limit of operating capital loan is less than $6,500, dryland wheat is not fertilized, but is fertilized once the maximum availability of loan exceeds $6,500.

Minimum Increases in Crop Yields Required

Minimal increases in yields of major crops(as induced by irrigation)required to render irrigation profitable are studied in the following part of this section. This entails an examination of the minimal increases in yield coefficients of major irrigated crop activities required for such activities to be selected for entry into the optimal basis. Minimal required increases in yield-coefficients are calculated in two ways. Firstly, all yield coefficients of irrigated crop activities are set at dryland levels and the minimal increase in the yield of one specific crop required for that crop to be included in the optimal solution is calculated. Secondly, with all yield coefficients of irrigated crop activities evaluated at irrigation levels, the minimal increases in the yield of a specified non-basic irrigated crop activity required for that activity to be selected for entry into the optimal basis is investigated. In both cases, the availability of annual operating capital loan is the maximum amount of $10,000. The theoretical framework for this type of investigation is presented in Appendix VI.

These minimal required increases in yields of major crops are estimated by using the shadow prices in the final stages of simplex tableaus computed for the non-integer linear programming problem as well as the mixed-integer programming problem. The results of these estimations appear in Tables X, XI and XII.

In this analysis, the price of irrigation water is fixed at $2.00 per acre inch unless otherwise specified.

With other crop yields held at dryland levels. Wheat can be grown profitably on irrigated T_1 land in competition with other dryland crop activities if yields are at least 21.4 percent greater than dryland yields. The purchase of specialized machines is considered in terms of infinitesimal units.

TABLE X

MINIMUM INCREASES IN CROP YIELDS REQUIRED TO BE ECONOMICALLY IRRIGABLE (PROJECTED BY SHADOW PRICES OF THE NON-INTEGER PROGRAMMING SOLUTIONS WITH OTHER CROP YIELDS HELD AT THE DRYLAND LEVELS)

0 ↓	0 th Activity	(1) Z_{hr}	(2) α_0	(3) Min. Req'd In-creases	(4) Dry-land Yield	(5) (3)/(4) x100	(6) Minimal Yields Required (3)+(4)
				......bushels		per-cent	bushels
5.	Wheat T_1	1.81	10.84	6.00	28.0	21.4	34.0
6.	Wheat T_2	1.81	13.06	7.22	28.0	25.8	35.22
7.	Barley T_1	1.11	11.40	10.27	45.0	22.8	55.27
8.	Barley T_2	1.11	13.62	12.27	45.0	27.3	57.27
9.	Oats T_1	0.70	20.34	29.06	60.0	48.4	89.06
10.	Oats T_2	0.70	22.63	32.33	60.0	53.9	92.33
21.	Potatoes T_1	1.45	44.68	30.81	111.6	30.8	142.41
23.	Potatoes T_2	1.45	46.90	32.34	111.6	32.3	143.94
				....tons.....			tons
14.	Sugar beets T_1	14.96	33.85	2.26	10.0	22.6	12.26
18.	Sugar beets T_2	14.96	36.07	2.41	10.0	24.1	12.41

NOTE: The price of water is fixed at \$2.00 per acre-inch. Symbols, Z_{hr} and α_o denote the shadow price of the r th disposal activity (r = 15,---,19) and that of the o th real activity (0 = 5,---10, 14, 18, 21, 23), respectively, in the final stage of simplex tableaus. The minimum required change in the r th technical coefficient of the o th activity (a non-basic activity) is given by α_o / Z_{hr}. The r th coefficients represent the yield coefficients of irrigated crops adjusted to the dryland levels.

TABLE XI

MINIMUM INCREASES IN CROP YIELDS REQUIRED TO BE ECONOMICALLY IRRIGABLE (PROJECTED BY SHADOW PRICES OF THE MIXED INTEGER PROGRAMMING SOLUTIONS WITH OTHER CROP YIELDS HELD AT THE DRYLAND LEVELS)

0 ↓	0 th Activity	(1) Z_{hr}	(2) α_0	(3) Min. Req'd In-creases	(4) Dry-land Yield	(5) (3)/(4) x100	(6) Minimal Yields Req'd
				....bushels..		per-cent	bushels
5.	Wheat T_1	1.74	16.44	9.45	28.0	33.8	37.45
6.	Wheat T_2	1.74	18.79	10.80	28.0	38.6	38.80
7.	Barley T_1	1.23	9.55	7.76	45.0	17.2	52.76
8.	Barley T_2	1.23	11.90	9.67	45.0	21.5	54.67
9.	Oats T_1	0.70	24.03	34.33	60.0	57.2	94.33
10.	Oats T_2	0.70	26.46	37.80	60.0	63.0	97.80
21.	Potatoes T_1	1.45	50.43	34.78	100.0	34.8	134.78
23.	Potatoes T_2	1.45	52.79	36.41	100.0	36.4	136.41
				....tons.....			tons
14.	Sugar beets T_1	14.96	35.71	2.39	10.0	23.9	12.39
18.	Sugar beets T_2	14.96	38.06	2.54	10.0	25.9	12.54

NOTE: The price of water is fixed at $2.00 per acre-inch. Symbols, Z_{hr} and α_o denote the shadow price of the r th disposal activity (r = 15,--19) and that of the 0 th real activity (0=5,--10, 14, 18, 21, 23), respectively, in the final stage of simplex tableaus. The minimum required change in the r th technical coefficient of the 0 th activity is calculated by α_0/Z_{hr}. The r th coefficients represent the yield coefficients of irrigated crops adjusted to the dryland levels.

Also on T_1 land, yields of barley, oats, sugar beets and potatoes should increase by more than 22.8, 48.4, 22.6 and 30.8 percent, respectively, in order to be irrigated profitably. If specialized machines are considered in integral purchasing units, the yield of wheat on irrigated T_1 land must exceed the dryland yield by at least 33.8 percent.

In general, the minimum required increases in yields obtained from the non-integer linear programming problem are lower than those from the mixed-integer programming problem. This means that, should specialized machine services be shared among a few farms, then irrigated crop activities would enter the optimal plans with lesser increases in yields.

Among the crops considered, barley on T_1 land requires the lowest "minimum percent increase in yield" when integer constraints are utilized, and, wheat on T_1 land, when they are not. Sugar beets have the second lowest required increases both in the non-integer linear programming and the mixed-integer programming analyses. Oats has the highest required increases. These are 48.4 and 57 percent on T_1 land for the non-integer and mixed integer models, respectively. This means that the yield of oats on irrigated T_1 land must increase by nearly 50 percent or more over the dryland yield level in order to be selected for entry into the optimal solution while yields of other crops are held at dryland levels. The yield coefficient of irrigated barley initially assumed is 55 bushels per acre.

This yield is slightly higher than that minimally required. However, the relative competitive position of barley is lowered when all crops' yield coefficients are considered at irrigation levels: hence, the irrigated barley activity is excluded from the optimal solution. Given its projected yield increase, barley would have been included in the optimal solution, if it were the only irrigated crop.

With other crop yields given at irrigated levels. The minimal required increases in yields are projected on the basis of the mixed-integer programming solutions. These results are presented in Table XII.

For wheat to be irrigated profitably on T_1 land in competition with other irrigated and dryland crops, the irrigated yield of wheat should be more than 15 percent higher than the yield-coefficient used in this study, and more than 85 percent greater than the dryland yield. Similarly, irrigated yields of barley and oats on T_1 land and sugar beets and potatoes on T_2 land should be higher than their assumed yield coefficients by more than 52, 78, 1 and 1 percent, respectively. Sugar beets and potatoes can be irrigated on T_2 land if their yields under irrigation conditions are only slightly higher than their assumed yield coefficients.

A comparison of the results in Table XII with those in Table XIII, indicates that the minimal increases in yields

TABLE XII

MINIMUM INCREASES IN CROP YIELDS REQUIRED TO BE ECONOMICALLY IRRIGABLE (PROJECTED BY SHADOW PRICES OF THE MIXED INTEGER PROGRAMMING SOLUTIONS WITH OTHER CROP YIELDS HELD AT THE IRRIGATED LEVELS)

0 ↓	0 th Activity	(1) Z_{hr}	(2) α_0	(3) Min. Req'd Increases	(4) (3)/(5) x100	(5) Irrigated Yield (projected)	(6) (3)+(5)	(7) Minimal Inc. Req'd above Dryland Level	(8) $\frac{(7)\times 100}{\text{Dryland Yield}}$
				bushels	percent	-----bushels	-----------	-----------	percent
5.	Wheat T_1	1.646	11.136	6.77	15	45	51.77	23.77	85
6.	Wheat T_2	1.646	13.715	8.33	19	45	53.33	25.33	90
7.	Barley T_1	1.016	19.125	28.67	52	55	83.67	38.67	86
8.	Barley T_2	1.016	31.704	31.20	55	55	86.20	41.20	92
9.	Oat T_1	0.606	37.670	62.16	78	80	142.16	82.16	137
10.	Oat T_2	0.606	40.328	66.55	83	80	146.55	86.55	144
23.	Potatoes T_2	1.450	2.579	1.78	1	180	181.78	81.78	82
				tons		------tons	-----------	-----------	
18.	Sugar beets T_2	14.960	2.579	0.17	1	15	15.17	5.17	52

NOTE: The price of water is fixed at $2.00 per acre-inch. Symbols, Z_{hr} and α_0 denote the shadow price of the r th disposal activity (r=15,--19) and that of the 0 th real activity (0=5, ---10, 18, 23), respectively, in the final stage of simplex tableaus. The minimum required change in the r th technical coefficient of the 0 th activity is calculated by α_0 / Z_{hr}. The r th coefficients represent the yield coefficients of irrigated crops.

required for small grains to be irrigated profitably in competition with other irrigated crops are much larger than those required for them in competition with other dryland crops only. The relative competitive position of small grains vis-a-vis specialty crops is lowered when the possibilities of irrigating all alternative crops are considered simultaneously.

The yield coefficients of major irrigated crops used in this study are presented in Table XIII. These are compared with dryland yields under fertilization.

TABLE XIII

COMPARISON OF MAJOR IRRIGATED CROP YIELDS WITH DRYLAND YIELDS

	(1) Dryland Yield (fertilized)	(2) Assumed Irrigated Yield	(3) Differences	(4) (3)/(1)x100
	------------bushels------------			percent
Wheat	28	45	+17	60.7
Barley	45	55	+10	22.2
Oats	60	80	+20	33.3
Potatoes	100	180	+80	80.0
	------------tons------------			
Sugar beets	10	15	+ 5	50.0

Source: Appendix Tables 2 and 3.

Comparison of Table XIII with Table XII shows that the yield coefficients of irrigated small grains are much lower than the minimal yields required for these crops to be included in the optimal solution, competing with other irrigated and dryland crops. Small grains, therefore, have very little chance of being irrigated profitably with the given prices, technical coefficients and resource availabilities.

Effect of varied water prices on minimal-required increases with other crop yields given at irrigated levels. To this point, the price of water has been held constant at $2.00 per acre inch. If the price of water varies, then so do the minimal required yields. When the price of water rises, required yields increase also; required yields diminish with lowered water prices. While variation in other factors, such as product prices and resource restrictions, will also affect the sizes of yields required, it is assumed here that product prices are fixed at the 10 year average and that all resource restrictions remain unchanged.

Table XIV indicates the minimum required yields of irrigated crops associated with varied prices of water, the feasibility of irrigation being considered simultaneously for all crops. Table XV shows the minimum required increases above dryland yield levels. Both are calculated from the parametric-cost programming of water associated with mixed-

TABLE XIV

THE MINIMUM REQUIRED YIELDS OF IRRIGATED CROPS ASSOCIATED WITH VARIED WATER PRICES (SIMULTANEOUS CONSIDERATION OF POSSIBLE IRRIGATION FOR ALL CROPS)

	Activity	Price of Water (Dollar Per Acre-inch) 0.0 ~ 0.69			0.77 ~ 1.71			2.13 ~ 3.19			3.19 ~ 3.27		
		Min. Req'd Inc.	Per-cent	Min. Yield Req'd	Min. Req'd Inc.	Per-cent	Min. Yield Req'd	Min. Req'd Inc.	Per-cent	Min. Yield Req'd	Min. Req'd Inc.	Per-cent	Min. Yield Req'd
5.	Wheat T_1	1.06	2.4	46.06	5.88	13.1	50.88	7.18	16.0	52.18	9.95	22.1	54.95
6.	Wheat T_2	1.06	2.4	46.06	7.46	16.6	52.46	8.74	19.4	53.74	11.46	25.5	56.46
7.	Barley T_1	19.09	34.7	74.09	27.83	50.6	82.83	29.03	52.8	84.03	29.84	54.3	84.84
8.	Barley T_2	19.09	34.7	74.09	30.41	55.3	85.41	31.55	57.4	86.55	32.17	58.5	87.17
9.	Oats T_1	44.98	56.2	124.98	62.64	78.3	142.41	61.85	77.3	141.85	57.96	72.5	137.96
23.	Potatoes T_2	0	-	-	1.77	1.0	181.77	1.78	1.0	181.78	1.81	1.0	181.81
					tons		---tons-----			----tons-----			tons
18.	Sugar Beets T_2	-	-	-	0.17	1.1	15.17	0.17	1.1	15.17	0.18	1.1	15.18

NOTE: The minimum required yields are calculated by the shadow prices of the mixed integer programming solutions.

TABLE XV

THE MINIMUM REQUIRED INCREASES IN YIELDS OF IRRIGATED CROPS ABOVE THE DRYLAND LEVELS (PROJECTED BY THE SHADOW PRICES OF THE MIXED INTEGER PROGRAMMING SOLUTIONS WITH OTHER CROP YIELDS HELD AT IRRIGATED LEVELS)

	Price of Water (Dollar Per Acre-inch)								
		0 ~ 0.69		0.77 ~ 1.71		2.13 ~ 3.19		3.19 ~ 3.27	
	Activity	Min. Req'd Increase Above Dryland Level		Min. Req'd Increase Above Dryland Level		Min. Req'd Increase Above Dryland Level		Min. Req'd Increase Above Dryland level	
		bushels	percent	bushels	percent	bushels	percent	bushels	percent
5.	Wheat T_1	18.06	64.5	22.88	81.7	24.18	86.4	26.95	96.3
6.	Wheat T_2	18.06	64.5	24.56	87.7	25.74	91.9	28.46	101.6
7.	Barley T_1	29.09	64.6	37.83	84.1	39.03	86.7	39.84	88.5
8.	Barley T_2	29.09	64.6	40.41	89.8	41.55	92.3	42.17	93.7
9.	Oats T_1	64.98	108.3	82.41	137.4	81.85	136.4	77.96	129.9
10.	Oats T_2	65.27	108.8	87.14	145.2	86.17	143.6	81.82	136.4
23.	Potatoes T_2	---	---	81.77	81.77	81.77	81.8	81.81	81.8
				tons		tons		tons	
18.	Sugar beets T_2	---	---	5.17	51.7	5.17	51.7	5.18	51.7

integer variables.

When the price of water is less than \$0.69, wheat on T_1 and T_2 land can be irrigated without great losses in the optimal functional values; the minimal yield of wheat required in this price range approximates the yield of irrigated wheat utilized in this study. Even in this low price range, however, other small grains can not be profitably irrigated; the gaps seperating minimal required yields from assumed yields are simply too large. When the price of water exceeds \$0.69 per acre inch, no small grains can be irrigated profitably. The yield coefficients of irrigated wheat and barley must be nearly doubled in order for them to enter the optimal basis in the price range above \$3.19. The yield of irrigated oats must increase by more than 100 percent over the dryland yield level throughout the enter price range of water.

II. COMPARISON OF THE OPTIMAL WITH ACTUAL FARM ORGANIZATIONS

The present organization of middle-sized farms in the project area is characterized by:

1. Firstly, crop land is allotted to cash small grains to the limit of the specified acreage quota or even further. The remaining land, which is a fairly large acreage, is used to a significant extent in the production of flax. Small acreages of specialty crops such as sunflower, buckwheat, sugar beets, peas, and potatoes are grown. A part of the

remaining crop land is also used for the production of feed grain and forage crops.

The percentage use of total crop land is given in Table XVI.

TABLE XVI

PERCENTAGE DISTRIBUTION OF TOTAL CROP LAND AMONG ALTERNATIVE CROPS, 1962

	Acreage	Percentage
Cash small grains (wheat, oats, barley)	87.0	38.8
Flax	71.0	31.6
Feed grain	3.0	1.3
Forage crops	17.5	7.8
Specialty crops (sugar beets, potatoes, peas, beans, etc.)	26.0	11.6
Others	20.0	8.9
Total crop land	224.5	100.0

Source: International Pembina River Engineering Board, Joint Investigation for Development of the Water Resources of the Pembina River Basin, Vol. III, Dec. 1964, p. 345.

2. The present livestock organization is typified by small herds of cattle that provide beef and dairy products for home use. Only small hog herds and flocks of poultry are present in the area. Table XVII shows the average sizes of

herds and flocks according to farm size.

TABLE XVII

AVERAGE SIZE OF LIVESTOCK ENTERPRISE ACCORDING TO SIZE OF FARM, 1962

	Small Farm (70 to 149 ac.)	Middle Farm (150 to 399 ac.)	Large Farm (400 to 760 ac.)
		Number	
Cows	5.4	7.45	8.7
All Cattle	10.4	12.45	16.7
Sows	2.3	2.2	3.5
All Hogs	10.0	10.05	23.8
Sheep	-	-	-
Hens & Chickens	59.0	125.05	166.0

Source: Ibid., p. 346.

Comparison of the Actual With Optimal Farm Organizations Under Dryland Conditions

Two optimal farm organizations for the 250 acre farm obtained by linear programming are compared with present farm organizations. The first represents the optimal solution of the linear programming problem without irrigation and operating capital loan. This solution should be closer to the present farm organization than that with irrigation. The optimal solution appears in Appendix Table 23. A large part of crop land is devoted to the production of wheat (61 acres), feed grain (69 acres) and hay (55 acres) and the remaining land is used for the production of sugar beets (39 acres). Firstly, wheat is grown up to the upper limit of the specified acreage quota. However, this is not the maximum acreage of wheat attainable by allocating total crop land only to wheat and specified crops or summerfallow. (If the total crop land were allocated only to wheat and specified crops or summerfallow so as to maximize the acreage of wheat, then the maximum acreage of wheat would have been about 80 acres.) This maximum is not reached because wheat competes with specialty crop and livestock for crop land. The largest proportion of crop land is used for the production of feed grain and hay to feed cattle, whereas, a fairly small portion is devoted to specialty crops. The percentage distribution of total crop land among these crops and forages on dryland appears in Table XVIII.

Both in the actual and optimal farm organizations under dryland conditions, cash grains hold the first priority in the use of crop land, but the manner of utilizing the remaining crop land differs in the two organizations. In the actual farm organization, flax has the second priority, whereas, in the optimal farm organization, no flax is produced. In the latter, the position of flax is replaced by the livestock enterprise utilizing the remaining crop land through their need for the production of feed grains and hay. In the optimal farm organization, a specialty crop, sugar beets, occupy a fairly important place in the use of crop land. In the actual situation, however, both livestock and specialty crops are merely minor enterprises. The optimal plan suggests that it would be profitable to increase livestock operations which utilize the winter family labor more efficiently, and to increase the proportion of sugar beets which would make fuller use of available labor.

What must be examined, however, is whether or not there is a large difference in income between the farm organization having flax as the second largest crop and the other farm organization in which livestock is the second most important enterprise. If seventy acres of flax are forced into the final basis so as to make the solution more like the present farm organization, then the functional value will decrease by $255. At the same time 20.1 units of "feed lot 400", 34.8 acres of feed grain, 22.4 acres of hay production and 23.0 acres of wheat would be taken away from the final basis. On the other hand large amounts of winter labor (ie., 136.2 hours) would be released. The percentage distribution of total crop land among the various crops would be as shown in Table XIX. The activity, "feed lot 400", is reduced from 36.43 to 14.54 lots. While flax is increased from zero to 31 percent, wheat is decreased from 27.2 to 17.0 percent. This

TABLE XVIII

DISTRIBUTION OF TOTAL CROP LAND USE UNDER DRYLAND CONDITIONS (OPTIMAL FARM ORGANIZATION ON 250 ACRE FARM)

	Acreage	Percentage
Cash small grain	61.1	27.2
Feed grain	68.9	30.6
Hay	55.3	24.6
Specialty crops	39.7	17.6
Total Crop Land	225.0	100.0

TABLE XIX

DISTRIBUTION OF TOTAL CROP LAND USE UNDER DRYLAND CONDITIONS, 70 ACRES OF FLAX FORCED INTO FINAL BASIS (OPTIMAL FARM ORGANIZATION ON 250 ACRE FARM)

	Acreage	Percentage
Cash grain	38.15	17.0
Feed grain	34.10	15.2
Hay	32.92	14.6
Specialty crops	49.83	22.1
Flax	70.00	31.1
Total crop land	225.00	100.00

occurs because the specified acreage quota is also reduced as flax acreage increases. In any case, flax can be increased by seventy acres without a significant reduction in income.

Comparison of the Actual With Optimal Farm Organizations Under Irrigation Conditions

The introduction of irrigation generates a different picture than that present earlier. In this case, the availability of an operating capital loan to a maximum limit of $10,000 is also taken into account. Without an operating capital loan, no irrigation is developed despite the availability of development capital loan. If irrigation becomes available, the competitive position of specialty crops in terms of profitability can be improved.[1] Naturally this causes increase both in the acreages of specialty crops and the number of the specialty crops included in the optimal plans. With irrigation, hay is replaced by fodder corn in the production of forages. The number of beef cattle decreases slightly in comparison with that under dryland conditions, while the proportion of specialty crops increases. Wheat also decreases. The utilization of total crop land is given in Table XX. In

[1] See the minimum percentage increases in yields over dryland levels required for major irrigated crops to be selected for entry into the optimal basis. These percentages are small for specialty crops.

this case, the price of water is in the range of $0.89 to $2.41 per acre-inch.

In comparison with the actual farm organization, the percentage of specialty crops is much higher in the irrigated optimal farm organization. The percentage of wheat, however, is relatively low and flax is reduced to a zero level in the optimal organization. Livestock operation associated with feed grains and forage crop production are also important under irrigation conditions. Specialty crops and forage crops are irrigated, but wheat and feed grain are not.

TABLE XX

DISTRIBUTION OF TOTAL CROP LAND USE UNDER IRRIGATION CONDITION WITH WATER PRICES RANGED FROM $.89 TO $2.41 PER ACRE-INCH (OPTIMUM FARM ORGANIZATION ON 250 ACRE FARM)

	Acreage	Percentage
Cash grain	45.3	20.1
Feed grain	57.1	25.4
Forages	13.5	6.0
Specialty crops	109.1	48.5
Total crop land	225.0	100.0

If seventy acres of flax irrigated on T_1 land are forced into the final basis, the functional value will be decreased by $910. This reduction in income corresponds to the elimination of 14 acres of irrigated sugar beets, 24 lots of cattle and 24.5 acres of wheat. If the same acres of dry-land flax are forced into the final basis, the functional value will decrease by $434. This results in 24 lots of cattle and 24 acres of wheat being removed from the final basis. Thus, the relative profitability of flax is lower with the advent of irrigation; it can not be produced without incurring a fairly large amount of estimated loss in possible income.

III. ANALYSIS OF DEMAND FOR IRRIGATION WATER AND ITS VALUE

An economic appraisal of irrigation water and projection of the quantity needed in the project area entails the estimation of the demand curve for irrigation water. The quantity of irrigation water to be used on farms in the project area varies according to its price. Price-quantity relationships, therefore, should be derived for the entire range of possible prices of water.

A Derived Demand Function for an Input Factor

Two kinds of approaches are available for the derivation of a demand function for an input factor of production. The first approach is an econometric approach where macro-time-series data or macro-cross-sectional data are utilized to determine the parameters of a demand function. This approach is based on the assumption that future patterns and practices of demand for an input factor are determined by their past trends. This implies that the factors which determine the patterns and the practices of demand will continue in the future. This assumption would be satisfied if there were no significant change in the underlying conditions such as technology, institutional factors, the prices of substitute goods, population, etc. However, if a significant change in technology, for example, is expected in the near future, this approach is not relevant to estimate the demand for the input in the future. Furthermore, this approach is not capable of estimating the demand function for a recently prevailing input factor. The second approach is the "derived demand" method in which a demand function for an input factor is derived, on the assumption of profit-maximization, from a production function or the optimal solutions of linear parametric price programming analysis. Because the first approach does not require the assumption of profit maximization while the second does, the first is sometimes called "positive", whereas, the second is referred to as "normative".

Input factors are purchased for the sake of the contribution they make to production. Accordingly, the demand for an input factor is determined in the process of production adjustment where the particular input is used profitably with varied prices. Such a production adjustment would be affected by several factors including the prices of products and inputs, the amounts of fixed resources, technology, etc. If the input factor is utilized in agricultural production, its contribution to production would also be affected by weather and other natural conditions. The demand for the input, therefore, will be a function of these factors. If given technology and amounts of fixed resources are assumed, then the demand for an input would depend upon its price, the prices of other inputs and the price of the output.

The general type of demand function of the i th firm for a particular input can be derived from a production function on the assumption of profit maximization.[1] The derived demand function is given as:

$$\text{(IV - 1)} \qquad Di = Di(r_1, r_2, \ldots\ldots r_n, P)$$

where:

r_1 is the price of the particular input,

$r_2, \ldots\ldots, r_n$ are the prices of other variable inputs,

P is the price of the output, and

Di is the i th firm's quantity demand for the particular input.

---

[1] J.M. Henderson and R.E. Quandt, Microeconomic Theory - A Mathematical Approach, (McGraw-Hill Book Company), 1958, pp. 107-8.

Furthermore, assuming that the prices of output and all other inputs are constant, the i th firm's demand function for a particular input factor is:

$$(IV - 2) \qquad Di = Di(r_1)$$

where r_1 is the price of the input factor. Then, the aggregate demand function is obtained by

$$(IV - 3) \qquad D = \sum_{i=1}^{m} Di(r_1) = D(r_1)$$

where there are m firms in the industry.

Given the prices of alternative outputs, the amounts of fixed resources, technical coefficients and the prices of inputs except the input whose price is varied, the linear parametric price programming method can be used for the derivation of a normative demand function for an input. Under these assumptions the quantity of the input used by the i th firm to maximize its profit depends solely upon the price of the input. The linear programming method will be superior to the production function method if the firm operates multi-enterprises with a complex constraint system of resource use.

In this study, the linear programming method is adopted for the estimation of the demand function for irrigation water in the project area. The linear programming method is used because (1) irrigation water is a new input factor available for agricultural production in the project area; and (2) many alternative activities of production should be considered for the possible use of irrigation water. Furthermore, the farm adjustment to the varied water prices is made subject to a complex constraint system of resource use. This method is based on the assumption that, when irrigation water becomes available, all farms will in the long run adjust their operations to the new conditions so that their farm incomes may be maximized. The estimation of the normative demand curve for irrigation water is based upon this type of normative farm adjustment.

The Linear Programming Approach to the Estimation of Demand for Water

The general procedure of estimating a normative demand curve is first to separate the cost of irrigation water from other variable cost items for each irrigated crop activity. An activity for buying irrigation water is added to the matrix. By parametrically varying the price of irrigation water, the quantities of it used in the optimal solutions are traced through the entire range of possible prices. The price-quantity relationships obtained from the series of optimal solutions and weighted by the number of farms in each class are plotted on a diagram with the price of water on the vertical axis and the quantity of water on the horizontal axis. The normative demand curve is drawn as a stepped curve sloping-downwards. Then, the mid-points of the steps are located and plotted on the diagram with a few lines drawn through the plots by free hand. In many cases, the entire curve can be divided into a few parts having different slopes. A regression equation could be fitted to each portion.

In this study, two different demand curves for irrigation water are estimated for each of (1) the entire project area and (2) a representative 250 acre farm. The first set of demand curves is obtained from a non-integer parametric-cost programming solution in which the variables for the purchase of specialized machines are treated as continuous. The other is derived from a parametric-cost programming solution with mixed-integer variables. The demand function of the project area for irrigation water is estimated by aggregation of individual demand schedules projected for the representative small, medium and large farms.

The first part of this section deals with the estimation of demand for irrigation water by a representative 250 acre farm and the second part, with the aggregate demand for irrigation water.

Analysis of Demand for Water by a Representative 250 acre Farm

Non-integer linear programming approach. The results of the analysis in the first part of this sub-section is based on the optimal solutions of non-integer parametric-cost programming. The price of irrigation water is varied parametrically

TABLE XXI

QUANTITIES OF WATER USED AND THE ACREAGES OF LAND IRRIGATED UNDER VARIOUS PRICES OF WATER (OPTIMAL NON-INTEGER PROGRAMMING SOLUTION FOR 250 ACRE FARM)

Price of Water Dollar Per Acre-inch	Quantity of Water used (acre-inches)	Irrigated Acreage				Unused Irrigable Land		
		Total		T_1	T_2	T_1	T_2	Total
		Acres	Percent	acres				
0 ~ 0.478	1966.9	175	100	140	35	0	0	0
0.479 ~ 0.591	1499.1	140	80	128.6	11.4	0	35	35
0.683 ~ 1.177	1221.7	119.2	68	105.3	13.9	20.8	35	55.8
1.177 ~ 1.843	986.5	97.8	56	97.8	0	42.2	35	77.2
1.888 ~ 2.053	907.2	90.9	52	90.9	0	49.1	35	84.1
2.146 ~ 3.339	790.7	78.6	45	78.6	0	61.4	35	96.4
3.339 ~ 3.863	382.5	42.5	25	42.5	0	97.5	35	132.5
3.863 ~ 5.903	85.2	9.5	5.4	9.5	0	130.5	35	165.5
5.903 ~ 7.204	50.7	5.6	3.2	5.6	0	134.4	35	169.4
7.204 ~	0	0	0	0	0	140.0	35	175.0

NOTE: The total physically irrigable land is 175 acres. This is made up of 140 acres of T_1 and 35 acres of T_2 land.

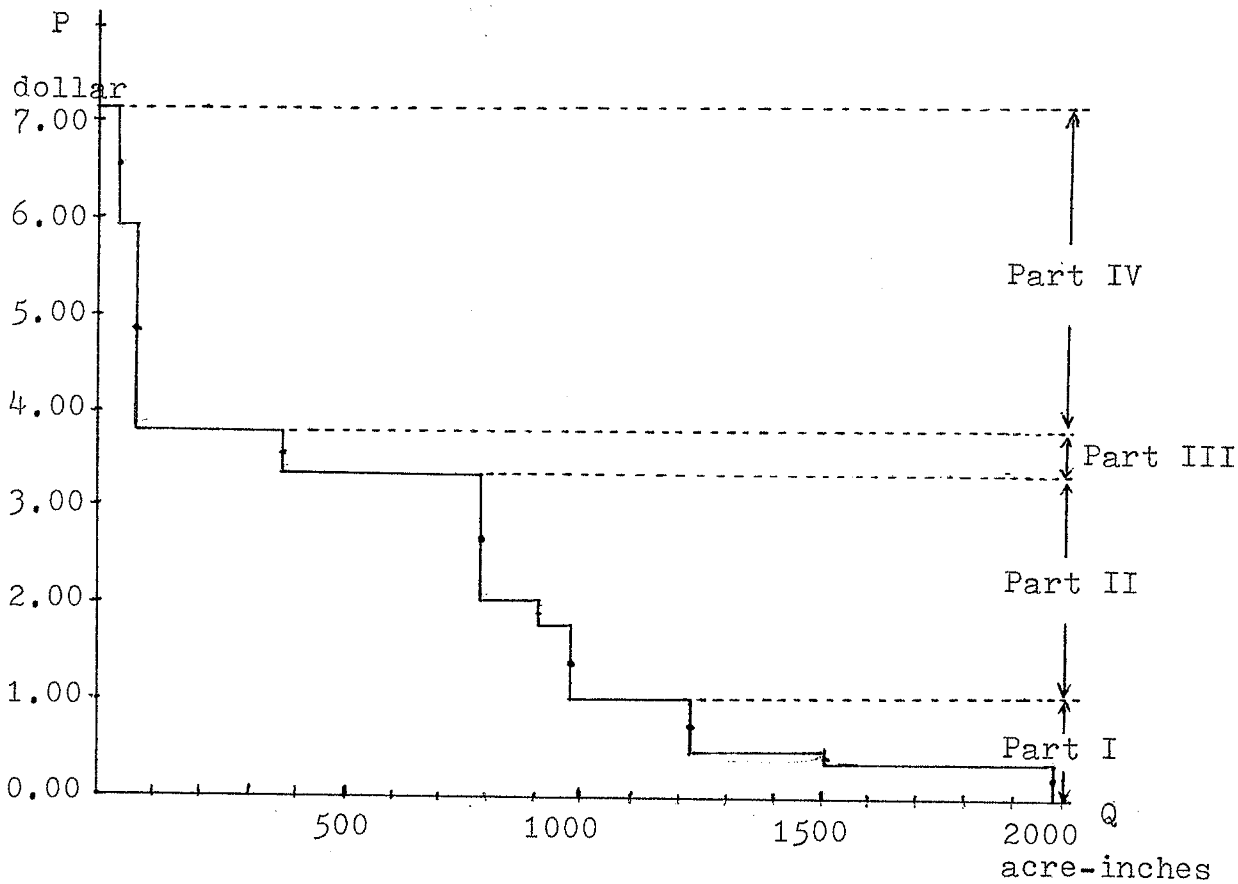

FIGURE 10

DEMAND CURVE FOR IRRIGATION WATER DERIVED FROM THE NON-INTEGER PROGRAMMING SOLUTIONS (250 ACRE FARM) from zero to $7.204 per acre-inch to derive the price-quantity schedule. This price-quantity schedule appears in Table XXI. Figure 10 shows the stepped demand curve for the 250 acre farm.

When the price of water increases from zero to $0.478, the quantity demanded remains the same, that is, at about 2,000 acre-inches. Similarly, when the price of water declines to zero from $0.478, quantity demanded does not increase.

As the price of water rises from $.478 to $.591, the optimal quantity of water used decreases by 500 acre-inches. As the price of water rises from $0.591 to $0.683, the projected demand for water decreases by approximately 310 acre-inches. The same quantity of water (ie., 1222 acre-inches) is used over the entire price range from $0.683 to $1.177 per acre-inch. The same quantity of water, 987 acre-inches is used in the price range from $1.177 to $1.843. As price rises to the lower limit of the price range, $1.888 to $2.053, the demand for water decreases slightly (by 80 acre-inches). About 790 acre-inches of water are used consistently in the wide price range extending from $2.146 to $3.339. As the price of water exceeds $3.339, the demand for water falls rapidly from 790 down to 380 acre-inches. The same quantity of water is demanded in the range of prices from $3.339 to $3.863, but a further rise in the price causes demand to decrease rapidly to the very small quantity of 85 acre-inches. As the price exceeds $7.204, the demand for water becomes zero. The quantity of water demanded varies from zero to 2,000 acre-inches while the price of water varies from $7.00 to zero. Near the middle point of the quantity (ie., 1,000 acre-inches), the price of water ranges from $1.177 to $1.843. At $2.00, about 910 acre-inches of water are demanded.

The total physically-irrigable land per farm is 175 acres. When the price of water ranges between zero and $0.478, all of the physically-irrigable land is economically irrigable. When the price of water rises to the price range, $0.478 to $0.591, 80 percent of the total irrigable land can be optimally irrigated. In the price range between $0.683 and $1.177 per acre-inch, 68 percent of total irrigable land is irrigated. In the price range between $1.177 and $1.843, approximately half of the total irrigable land is irrigated optimally. In the wide price range extending from $2.146 to $3.339, 45 percent of total irrigable land can be optimally irrigated. As the price of water rises to the range $3.339 to $3.863, the economically irrigable acreage is reduced to 25 percent of total physically-irrigable land. If the price exceeds $3.863, only five percent of total irrigable land is irrigated. As long as the prices of water remains lower than one dollar per acre-inch, both T_1 and T_2 land can be profitably irrigated.

The marginal change in demand for irrigation water is great both in the range of low prices, zero to $1.77, and in the range of high prices, $3.339 to $3.863; it is small in the range of medium prices, \$1.177 to \$3.339. The marginal change is also small in the higher price range extending from \$3.863 to \$7.204. The nearly constant demand for water in the price range from \$1.177 to \$3.339 corresponds to the stability of the optimal solutions for the price change of water in this price range. If the irrigation water can be supplied in this price range and specialized machines can be considered in infinitesimal purchasing units, then the supply of water will be met by a fairly stable normative demand.

The price elasticity of demand ($\mathcal{E}$) at a point on a demand curve is generally defined as the absolute value obtained by the rate of percentage change in quantity divided by the rate of percentage change in price:

$$\mathcal{E} = - \frac{dQ}{dP} \cdot \frac{P}{Q}$$

where:

Q is the quantity of demand, and

P is the price.

With a given demand curve, the total revenue as calculated by P x Q is maximized where the price elasticity of demand ($\mathcal{E}$) is unitary.

In the low price range, that is Part I in Figure 10, the point price elasticity of demand for water at the mean is 0.284. The elasticity at the mean in the middle price range, Part II, is 0.290. In the high price range from \$3.339 to

$3.863, Part III, the price elasticity of demand for water at the mean is 4.95. The elasticity in Part IV is 1.19.

The slopes of four different portions of the demand curve are estimated separately by fitting a linear equation to each of the parts. Because the data used here do not satisfy the usual assumptions of regression analysis such as normality, statistical testing and setting confidence intervals can not be employed. Regression coefficients only will be determined. Pertinent results of the regression analysis are summarized below.

1. Part I (Price of water ranges from zero to $1.177 per acre-inch)

   (IV - 4) $\quad Q = 2022.4 - 894\ P$

   Price elasticity of demand for water at the mean = 0.284.

2. Part II. (Price of water ranges from $1.177 to $3.339)

   (IV - 5) $\quad Q = 1151.59 - 120.09P$

   Price elasticity of demand at the mean = 0.290.

3. Part III. (Price of water ranges from $3.339 to $3.863)

   (IV - 6) $\quad Q = 2442.59 - 561.35P$

   Price elasticity of demand at the mean = 4.95

4. Part IV. (Price of water ranges from $.863 to $7.204)

(IV - 7) $Q = 155.13 - 15.0P$

Price elasticity of demand at the mean = 1.19

where:

Q denotes the quantity of demand, and

P is the price of water.

Mixed-integer programming approach. The second portion of this section is devoted to the analysis of optimal demand for irrigation water based upon the mixed-integer programming solutions in which the price of water is varied parametrically. In this case, a combination of irrigation and special machines is forced into the optimal bases at one unit each. The combination of machines includes a set of irrigation machines, a potato digger, a potato seed cutter and a sugar beet thinner. With this fixed number of machines, the price of irrigation water is varied parametrically from zero to $2.62 per acre-inch at which the use of irrigation water and its associated investment in these machines at integral units becomes non-optimal.[1] The parametric-cost programming of water with a set of special machines forced into the final bases can still proceed by varying the price of water beyond $2.62. It is useless, however, to do so, because investment in the irrigation machines, if purchase is considered at integral units, becomes zero at

[1]See Appendix V. Quantities of water used at various prices are presented in Table XXII. The demand curve appears in Figure 11.

TABLE XXII

QUANTITIES OF WATER USED AND THE ACREAGES OF IRRIGATED LAND UNDER VARIOUS PRICES OF WATER (OPTIMAL MIXED INTEGER PROGRAMMING SOLUTION FOR 250 ACRE FARM)

Price of Water Dollar Per Acre-inch	Quantity of Water Used (acre-inch)	Total Irrigated Acreage		Irrigated Acreage		Unused Irrigable Land		
		acres	percent	T_1	T_2	T_1	T_2	Total
				acres				
0 — 0.688	1982	175	100	140	35	0	0	0
0.689 — 0.887	1516	140	80	140	0	35	0	35
0.888 — 1.710	957	95	54	95	0	80	45	35
1.711 — 1.983	957	95	54	95	0	80	45	35
1.984 — 2.139	957	95	54	95	0	80	45	35
2.140 — 2.62	921	91	52	91	0	84	49	35

NOTE: The total physically irrigable land is 175 acres. This is made up of 140 acres of T_1 and 35 acres of T_2 land.

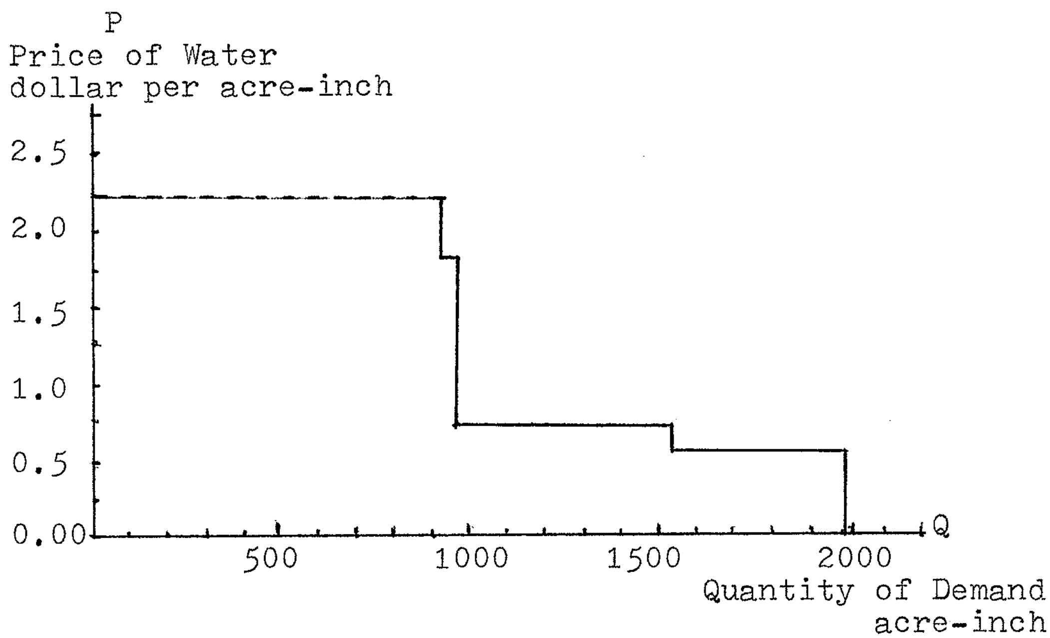

FIGURE 11

STATIC NORMATIVE DEMAND CURVE FOR IRRIGATION WATER DERIVED FROM THE MIXED INTEGER PROGRAMMING SOLUTIONS (250 ACRE FARM) this price of water. So the parametric-cost programming terminates at \$2.62. The normative demand curve for irrigation water is discontinuous at the point, P = 2.62 and Q = 921.

The demand curve derived from the optimal solutions developed by the mixed-integer programming demonstrates a much simpler response to changes in the water price than that obtained by the non-integer linear programming. In the price range from zero to \$0.69, the quantity of water used is 1982 acre-inches per farm and the quantity does not change over that price range. Under these conditions, both T_1 land and T_2 land are irrigated and no irrigable land is in "disposal". As the price goes up to the range, \$0.69 to \$0.89, 1,516 acre-inches of water are demanded and all T_1 land is irrigated. In the wide price range extending from \$0.89 to \$2.14, a consistent quantity of water can irrigate 95 acres of T_1 land. A slightly reduced quantity of water (921 acre-inches) is used in the higher price range, \$2.14 to \$2.62, with 91 acres of T_1 land being irrigated. As explained above, when the price exceeds \$2.62, the demand for water becomes zero.

If the price of water rises from zero to 0.89 dollars, the demand for irrigation water decreases rapidly, but remains nearly constant over a wide range of prices, \$0.89 and \$2.62 per acre-inch. The stable-demand range on this demand curve is much wider than that on the demand curve derived from the non-integer linear programming solutions. Marginal change in demand for water in response to change in the price of water is large in the price range, zero to $1.00, but becomes very small (nearly zero) in the price range, $1.00 to $2.62.

Usually, the value of a scarce resource is measured in terms of its marginal or average value productivity. The marginal net revenue product of a scarce resource in the linear programming sense can be measured by the shadow price of its disposal activity. If the relevant resource can be obtained only through purchase in the factor market, then its shadow price becomes exactly equal to its unit price. This results because the resource is purchased up to the point where the shadow price of this resource equals its unit cost. The marginal value products of irrigation water at various levels of use can be derived from the linear programming solutions where the price of irrigation water is varied. The marginal value product curve can be traced out by finding the prices of water at various levels of use in those solutions. It is conceptually identical with a farm's demand curve for irrigation water. A minimum size of irrigation development is required for a farm to make economic use of irrigation machines purchased at integral units. The minimum size as such determines the minimum quantity of irrigation water which is used profitably on a farm with a set of irrigation machines purchased at integral units. In the case of the 250 acre farm, the minimum quantity is 921 acre-inches. The marginal value of irrigation water is \$2.62 when 921 acre-inches are used. The marginal value of water falls rapidly when the quantity of water used on a 250 acre farm exceeds 921 acre-inches. The marginal value of water is constant (\$0.887) for the quantities of water used within the range, 957 to 1,516 acre-inches; it is \$0.69 over the range of 1,516 to 1,982 acre-inches.

Analysis of Aggregate Demand for Water

Non-integer linear programming approach. In the first part of this sub-section, the aggregate demand function derived from non-integer linear programming solutions is analyized. The numbers of farms in the small, medium and large classes are used as weights to derive the aggregate demand. There are 131, 158 and 40 farms in the small, medium and large categories, respectively. Individual demand schedules for water for each size class appear in Appendix Tables 26-B & C and text Table XXI. The aggregate demand schedule is presented in Appendix Table 26. Figure 12 indicates the "stepped" aggregate demand curve of the entire irrigation project area.

A non-linear equation was fitted to the aggregate demand schedules by least squares. The midpoints of the price ranges are assumed to be the most stable for price change. These midpoints, therefore, were used as observations to which the regression equations were fitted. The type of demand equations is specified below.

$$\text{(IV - 8)} \qquad \log Q = \alpha - \beta \cdot \log P$$

where:

Q is the aggregate quantity of demand for water, and

P is the price of water.

α and β are constant.

The equation (IV - 8) can be rewritten as

$$\text{(IV - 9)} \qquad Q = \alpha^{*} \cdot P^{-\beta} = \frac{\alpha^{*}}{P^{\beta}}$$

By differentiating the equation (IV - 9), we have

$$\text{(IV - 10)} \quad \frac{dQ}{dP} = -\beta \cdot \alpha^{*} \cdot P^{-\beta-1} = -\beta \cdot \alpha^{*} \cdot \frac{1}{P^{\beta+1}}$$

If $\beta > 0$, then $\frac{dQ}{dP} < 0$ and the graphs of (IV - 8) and (IV -9) are downwards sloping.

By differentiating (IV - 10), we obtain

$$\text{(IV - 11)} \quad \frac{d^2Q}{dP^2} = -\beta \cdot -(\beta + 1) \cdot \alpha^{*} \cdot P^{-\beta-2} = \beta \cdot (\beta+1) \cdot \alpha^{*} \cdot \frac{1}{P^{\beta+2}}$$

The right hand side of equation (IV - 11) is positive if β is positive. Then, the graphs of (IV - 8) and (IV - 9) are concave upwards. The aggregate demand curve drawn by free hand through the midpoints of steps indicates that the aggregate curve is sloping downwards and is concave upwards. Equation (IV - 8), therefore, would be expected to give a better fit to the data than a linear function.

Figure 12 shows that a distinct shift occurs when the price of water exceeds $3.35 per acre-inch. In other words, the aggregate demand curve is discontinuous at a price of about $3.35. To take the shift of the demand curve into account, the entire demand schedule is divided into two parts and a separate regression equation is fitted to each part. Using the predicted values of quantity, the estimated aggregate demand curve is drawn in Figure 13.

The highest price of water at which some quantity of irrigation water could be used in the project area is $7.39 per acre-inch. At this price, a very small amount of water, 9,589 acre-inches would be used in the entire area. The quantity of water demanded in the area will not increase substantially unless the price of water declines to $3.86 per acre-inch. When the range of price varies downwards from $7.39 to $3.36, an increase in irrigation should occur such that the quantity of water used increases from about 10,000 to about 70,000 acre-inches. When the price of water declines to the range, $3.26 to $3.36, the demand for water increases substantially from 70,000 to 164,000 acre-inches. In the wide range of price between $3.36 to $0.74, there is no particular point at which the quantity of demand rises distinctly in response to a change in price. As the price of water varies

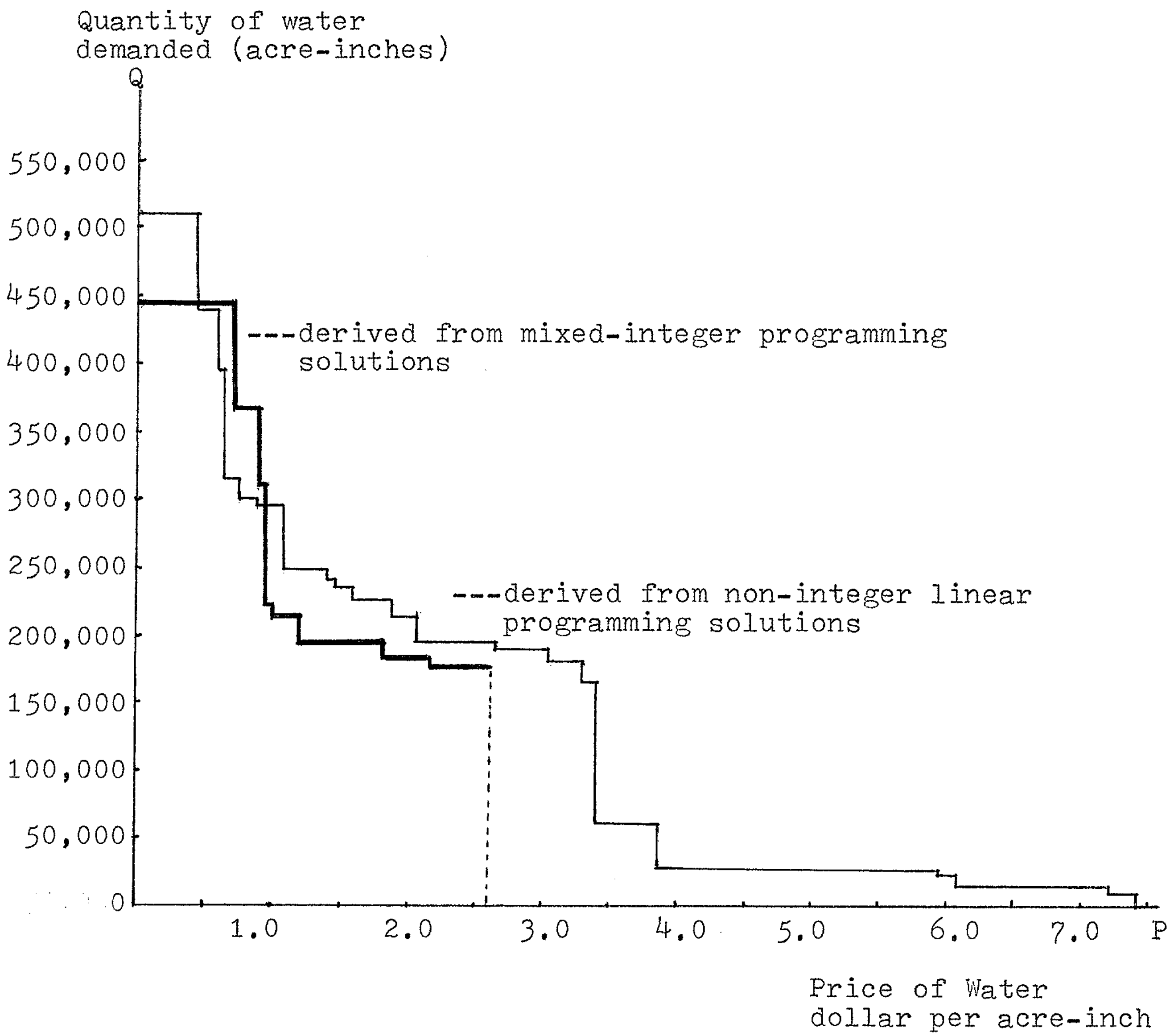

FIGURE 12

"STEPPED" AGGREGATE DEMAND CURVE FOR IRRIGATION WATER

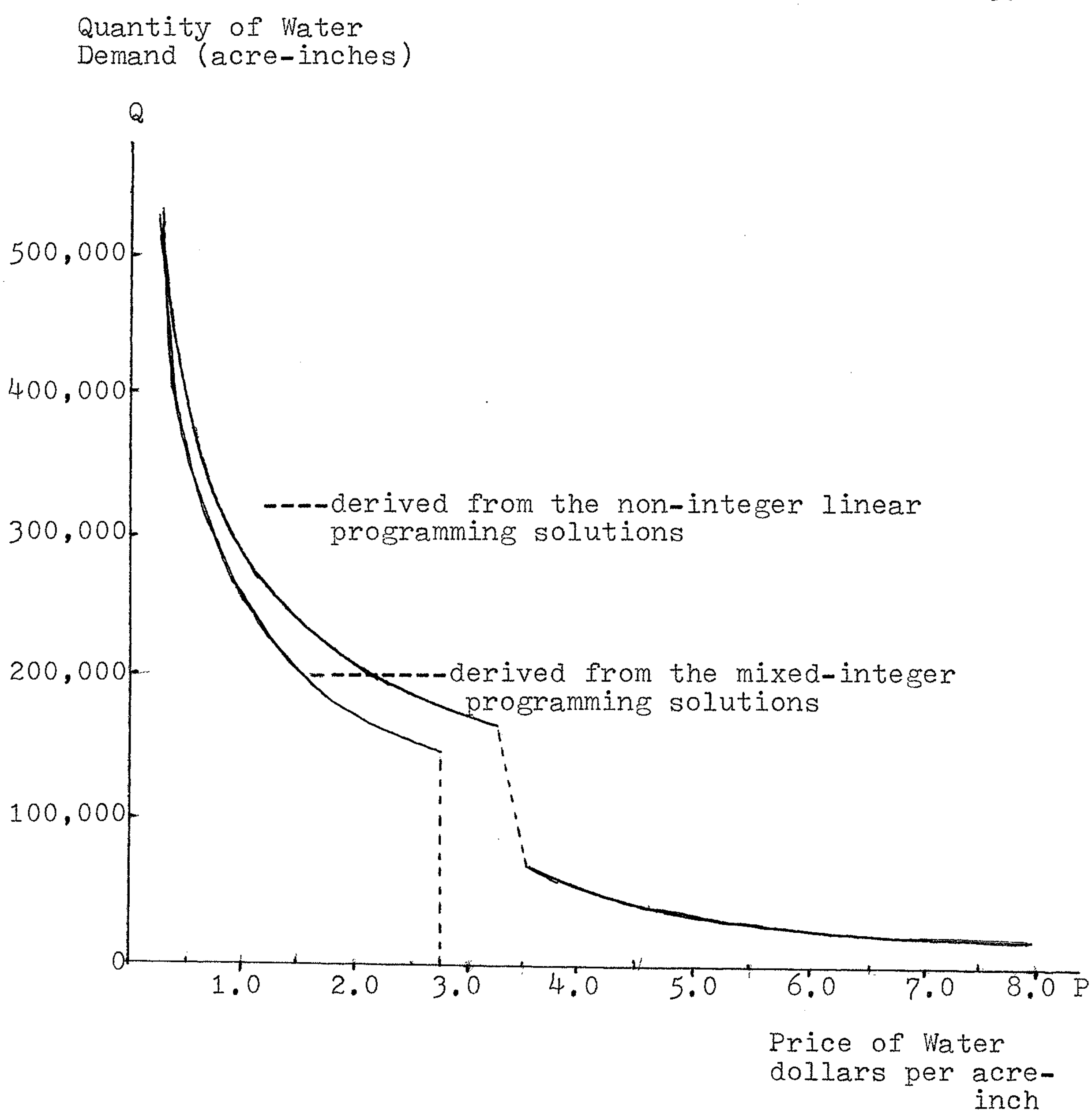

FIGURE 13

AGGREGATE DEMAND CURVE FOR IRRIGATION WATER
ESTIMATED BY LEAST SQUARES

downwards from \$3.36 to \$2.05, the quantity demanded increases gradually from 163,441 to 183,176 acre-inches. Approximately 195,000 acre-inches are used consistently in the wide range of price from \$2.62 to \$2.06. Within the range of \$2.06 to \$0.74, changes in the price of water at small intervals are met by successive small changes in the quantity demanded. When the price falls to the range, \$0.74 to \$0.63, the quantity demanded again increases substantially. About 124,000 acre-inches of an increase (from 314,000 to 438,000 acre-inches) in demand would be induced by lowering the price to the range of \$0.63 to \$0.59. Further reduction of price to the range of \$0.48 to \$0.00 would result in about 75,000 acre-inches of increase (438,000 to 513,000 acre-inches) in water use. Any reduction of price beyond \$0.48 would not affect the quantity of demand. A small reduction of price within the range of \$0.74 to \$0.48 would effect a large increase in demand. Because a substantial increase in demand occurs at the prices, \$3.36 and \$0.74, any water supply agency which might be associated with this project area would be recommended to set the price of water slightly lower than \$3.36 to \$0.74. This would be expected to stimulate a large demand for water in the project area.

At the midpoint (\$3.69 per acre-inch) of the entire price range, approximately 70,000 acre-inches are used. At the midpoint (\$1.68) of the lower price range, \$0.00 to \$3.36, about 226,000 acre-inches are demanded while at the midpoint ($5.38) of the higher price range, $3.36 to $7.39, about 3,100 acre-inches are used.

The aggregate demand functions were fitted separately to the low and high price ranges, in logarithmic form. The logarithmic and exponential results are estimated as follows:

1. In the low price range ($0.00 to $3.36),

(IV - 12) $\log Q = 5.4579 - 0.454 \log P \qquad R^2=0.97$

or

(IV - 13) $Q = \dfrac{287,000.00}{P^{0.454}}$

2. In the high price range ($3.36 to $7.39),

(IV - 14) $\log Q = 5.8581 - 1.8884 \log P \quad R^2=0.94$

or

(IV - 15) $Q = \dfrac{721,333.33}{P^{1.8884}}$

In both cases, a good fit is indicated by high R^2 values. Since this type of function has a constant elasticity over its entire range, we need not calculate point elasticities. The constant elasticity is indicated by the exponent of P. Demand equation (IV - 13) has a constant price elasticity of demand of 0.454, while equation (IV - 15) has the value, 1,884. Hence, equation (IV - 15) is more price elastic than equation (IV - 13). Because the price elasticity of demand is larger than unity in equation (IV - 15), and smaller in equation (IV - 13), total revenue of the water supply agency could be increased either by reducing or raising the price of water to $3.36. Total revenues at various prices were calculated by, TR = P X Q, where P is varied from $0.00 to $7.40 at five cent intervals and Q is obtained by demand equations (IV - 13) and (IV - 15). The results are presented in Appendix Table 26-D. The water supply agency would receive the maximum total revenue, $555,336.50, per annum by selling 165,772.13 acre-inches of water at $3.36 per acre-inch.

Mixed-integer programming approach. The second portion of this section is devoted to the analysis of the aggregate demand function derived from the optimal solutions of three mixed-integer programming problems associated with varied prices of water. In the case of small farms, no irrigation enters the optimal solution at any price level of water when purchase of specialized machineries is considered at integral units. Therefore, the demand for water on small farms is zero. On large farms, irrigation water is used optimally in the price range extending from $0.00 to $2.62 per acre-inch. The individual demand schedule derived from the optimal solutions of mixed-integer programming problems for the medium and large farms appears in text Table XXII and Appendix Table 26-C. The aggregate demand schedule is presented in Appendix Table 27.

Graphic presentation of the aggregate demand derived by mixed-integer programming produces a "stepped" demand curve as indicated in Figure 12.

The aggregate demand curve estimated by mixed-integer programming lies below that obtained by non-integer linear programming. This results because irrigation is not profitable for small farms with purchase of specialized machinery considered at integral units.

Approximately 180,000 acre-inches of water is used consistently over the range of $1.83 to $2.62 per acre-inch. As the price of water declines to the range, $1.18 to $1.83, the quantity of water demanded rises to about 187,700 acre-inches. This quantity is also stable for the range of price from $1.18 to $1.83. The demand indicates highly sensitive response to small changes in the price of water throughout the entire range of $0.69 to $1.18. An additional approximately 150,000 acre-inches would be used if the price was lowered by $0.33 from $1.18 to $0.85, and, about 70,000 acre-inches for a decrement of $0.17 from $0.85 to $0.69. Therefore, in this range, reduction in the price of water is effective in enlarging the aggregate demand. In the price range $0.00 to $0.69, a constant quantity of water (441,644 acre-inches) is used.

With the price of water lower than \$0.69, all physically irrigable land (37,650 acres), including both T_1 and T_2 land, is optimally irrigated. When the price of water rises to the range, \$0.69 to \$0.85, T_2 irrigable land becomes idle for irrigation, but all T_1 land is optimally irrigated. At the midpoint (\$1.31) of the entire price range, approximately 188,000 acre-inches would be utilized, irrigating 36.5 percent of total irrigable land. In the highest price range, \$1.17 to \$2.62 per acre-inch, 33 to 36 percent of total irrigable land would be optimally irrigated. These acreages and the percentages of land developed for irrigation at various levels of water price appear in Appendix Table 27.

The demand function fitted by regression analysis to the entire range of price quantity schedule is estimated as:

$$\text{(IV - 16)} \qquad \log Q = 5.4080 - 0.5461 \log P \qquad R^2=0.86$$

in the logarithmic form, and

$$\text{(IV - 17)} \qquad Q = \frac{287,000.00}{P^{0.5461}}$$

in the exponential form.

The price elasticity of demand of this equation is 0.5461 which is larger than that of aggregate demand equation (IV -13). Since the price elasticity of demand is less than one, total revenue of the water supply agency would be increased by raising the price of water to \$2.62 per acre-inch. Total revenues under various prices of water are calculated at 5 cent intervals and presented in Appendix Table 26-D. The annual maximum total revenue, $397,083.19 is reached at a price of $2.62.

IV. ANALYSIS OF OPTIMAL INVESTMENT IN SPECIAL MACHINES

Irrigation is accompanied by investment in specialized machines. The specialized machines under consideration include the new machines required for the irrigation operation, and for specialty crops such as sugar beets and potatoes whose relative profitability increases through irrigation. These purchases are considered at integral units. The possibility exists for acquiring seven such specialized machines: a set of irrigation machines, potato seed cutter, potato digger, sugar beet thinner, sugar beet harvester, hay baler and corn forage harvester.

Optimal investment in these machines may be affected by changes in such conditions as prices of products and factors, technical coefficients, and resource restrictions, etc. It is assumed here, however, that prices of products, technical coefficients, prices of input factors other than water, and resource restrictions except operating capital loan remain as given. The price of water and the upper limit of operating capital loan are varied separately to determine the effect of their change upon optimal investment in the machines.

With the Water Price Fixed at $2.00 Per Acre-inch

The first part of this section presents the optimal investment in machines with the price of water and the upper limit of operating capital loan fixed at $2.00 per acre-inch and $10,000, respectively (see Table XXIII).

While the purchases of these machines are not divisible, the use of their services is divisible and continuous. It was found that even though a set of irrigation machines is purchased at one unit, only a part of its possible services is used in an optimal plan. Only 12 percent of the annual available potato seed cutter service is used. The potato digger is used to nearly full capacity operation. Therefore, if seasonal use of these machines do not severely compete among farmers who use them, then it is preferable that these specialized machines with the exception of the potato digger be owned co-operatively by two or more farmers. Because the operating capacity of each machine is calculated in terms of acreage which can be handled with a full time crew in a favorable time of each operation, there should not be severe competition between farms in the use of these machines if they were owned by several farmers.

The purchase of sugar beet harvesters enters the optimal solution neither in integral nor non-integer units despite the inclusion of sugar beet activities in the optimal basis. Sugar beets are custom harvested. This may result from the relatively

TABLE XXIII

THE OPTIMAL LEVELS OF SPECIALIZED MACHINERY

	Optimal Non-Integral Level	Optimal Integral Level of Investment
	No.	No.
A set of irrigation machines	0.364	1.0
Potato seed cutter	0.128	1.0
Potato digger	0.770	1.0
Sugar beet thinner	0.481	1.0
Sugar beet harvester	0.0	0
Corn forage harvester	0.0	0
Hay baler	0.0	0

small size of the sugar beet operation coupled with the high cost of the machine. How large a loss in income would ensue if one unit of sugar beet harvester were forced into the final basis on farms of this size? When one unit of sugar beet harvester was forced into the final basis along with four other specialized machines each at one unit, the optimal functional value became $13,686.71. This is $525.5 less than the optimal functional value without purchasing a sugar beet harvester. Therefore, if a custom harvester is available at a favourable time, harvesting sugar beet by custom work becomes more profitable than buying one unit of harvester for sugar beet operations of this size (sixty acres).

Effect of Varied Water Prices

The second part of this section presents optimal investment in special machines under different prevailing water prices. As discussed in detail in Appendix V, one combination of specialized machines including one set of irrigation machines, seed cutter, potato digger and sugar beet thinner is optimal in the price range of water extending from zero to \$2.26. When the price of water exceeds \$2.26 per acre-inch, only one unit of sugar beet thinner to be used for dryland sugar beet operations is purchased. No investment either in irrigation machines or in irrigation development is optimal with this price range. Thus, while changes in the price of water affects the optimal investment pattern for specialized machines, one optimal pattern remains stable throughout a wide range of water prices. This is especially true when investment in these machines is considered at integral units. Optimal levels of investment under various prices of water appear in Table XXIV. In the optimal plans, the hay baler, sugar beet harvester and forage corn harvester are not purchased at any price of irrigation water.

Effect of Varied Operating Capital Loan

Changes in the upper limit of operating capital loan also affect the optimal investment levels for the specialized machines. The non-integral and integral levels of optimal investment under varying limits of operating capital loan appear in Table XXV. It is assumed that the irrigation development capital loan associated with an upper limit of $20,000 at 5.5 percent interest can be used to buy the specialized machines.

If operating capital loan is not available at all and the purchase of specialized machines is considered at integral units, then none of these machines is purchased in the optimal plans. It is interesting to note that the purchase of specialized machines as well as irrigation development are infeasible without the availability of annual operating capital loan even though loans for the purchase of these machines and land development for irrigation are feasible.

With the consideration of integral purchasing units for the machines, at least $5,500 of operating capital loan is required to render the annual operation of irrigation machines feasible. This loan is required in addition to the irrigation development capital loan. This means that about twice as much annual operating capital as the amount actually owned will be needed for farm operation under irrigation conditions. When more than $9,000 of operating capital loan is available, one unit each of irrigation machines, sugar beet thinner, seed-

TABLE XXIV

OPTIMAL LEVELS OF THE SPECIALIZED MACHINES PURCHASED UNDER VARIOUS PRICES OF WATER (250 ACRE FARM)

Price of Water Dollar Per Acre Inch	Non-Integral Levels: Irrig. Mchne	Seed Cutter	Potato Digger	S.B. Thinner	Hay Baler	S.B. Harvester	Corn Harvester	Integral Levels: Irrig. Mchne	Seed Cutter	Potato Digger	S.B. Thinner	Hay Baler	S.B. Harv.	Corn Harv.
0 ~ 0.478	0.70	0.14	0.85	0.48	0	0	0	1	1	1	1	0	0	0
0.478~ 0.591	0.56	0.14	0.86	0.48	0	0	0	1	1	1	1	0	0	0
0.683~ 1.177	0.48	0.14	0.86	0.48	0	0	0	1	1	1	1	0	0	0
1.177~ 1.843	0.42	0.12	0.71	0.48	0	0	0	1	1	1	1	0	0	0
1.888~ 2.146	0.42	0.12	0.71	0.48	0	0	0	1	1	1	1	0	0	0
2.146~ 2.62	0.36	0.12	0.70	0.48	0	0	0	1	1	1	1	0	0	0
2.62~ 3.339	0.36	0.12	0.70	0.48	0	0	0	0	0	0	1	0	0	0
3.339~ 3.863	0.19	0.16	0.96	0.35	0	0	0	0	0	0	1	0	0	0
3.863~ 5.903	0.07	0.16	0.32	0.35	0	0	0	0	0	0	1	0	0	0
5.903~ 7.204	0.06	0.05	0.22	0.29	0	0	0	0	0	0	1	0	0	0
7.204 over	0	0	0	0.28	0	0	0	0	0	0	1	0	0	0

TABLE XXV

OPTIMAL LEVELS OF THE SPECIALIZED MACHINES PURCHASED UNDER VARIOUS OPERATING CAPITAL LOAN (250 ACRE FARM)

Upper Limit of Loan	Non-Integral Levels							Integral Levels						
	Irrig, Mchne	Seed Cutter	Potato Digger	S.B. Thinner	Hay Baler	S.B. Harv	Corn Harv	Irrig. Mchne	Seed Cut-ter	Potato Digger	S.B. Thinner	Hay Baler	S.B. Harv	Corn Harv
dollars														
0	0	0	0	0.17	0	0	0	0	0	0	0	0	0	0
500	0	0	0	0.24	0	0	0	0	0	0	0	0	0	0
1,000	0	0	0	0.32	0	0	0	0	0	0	0	0	0	0
1,500	0.035	0	0	0.33	0	0	0	0	0	0	0	0	0	0
2,000	0.092	0	0	0.34	0	0	0	0	0	0	0	0	0	0
2,500	0.155	0	0	0.39	0	0	0	0	0	0	0	0	0	0
3,000	0.218	0	0	0.45	0	0	0	0	0	0	1	0	0	0
3,500	0.280	0	0	0.51	0	0	0	0	0	0	1	0	0	0
4,000	0.33	0	0	0.54	0	0	0	0	0	0	1	0	0	0
4,500	0.35	0	0	0.55	0	0	0	0	0	0	1	0	0	0
5,000	0.36	0.033	0.2	0.57	0	0	0	0	0	0	1	0	0	0
5,500	0.37	0.046	0.278	0.58	0	0	0	1	0	0	1	0	0	0
6,000	0.39	0.060	0.360	0.59	0	0	0	1	0	0	1	0	0	0
6,500	0.38	0.065	0.391	0.48	0	0	0	1	0	0	1	0	0	0
7,000	0.38	0.068	0.363	0.48	0	0	0	1	0	0	1	0	0	0
7,500	0.37	0.067	0.404	0.48	0	0	0	1	0	0	1	0	0	0
8,000											1	0	0	0
8,500	0.35	0.073	0.437	0.48	0	0	0	1	0	0	1	0	0	0
9,000	0.36	0.095	0.552	0.48	0	0	0	1	1	1	1	0	0	0
9,500	0.36	0.111	0.667	0.48	0	0	0	1	1	1	1	0	0	0
10,000	0.36	0.116	0.697	0.48	0	0	0	1	1	1	1	0	0	0

NOTE: The price of water is fixed at 2.00 dollars per acre-inch.

potato cutter and potato digger are purchased in the optimal plan. Purchase of a sugar beet thinner will be profitable if $3,000 of operating capital loan is available. Purchases of hay baler, sugar beet harvester and forage corn harvester are not profitable on this size of farm (250 acres) even though both an annual operating capital loan and loans for the purchase of machines are available.

CHAPTER V

ECONOMIC EVALUATION OF IRRIGATION UNDER ASSUMPTION OF IMPERFECT KNOWLEDGE

This chapter deals with the results based on the analytical framework as presented in the second section of Chapter III.[1] The stochastic programming problem, as described in that section, is solved with respect to various levels of risk aversion. Under each level of risk aversion, the problem is solved for the prices of water varied from zero to the border price, at which demand for irrigation water becomes zero. These optimum solutions are shown in Appendix Tables 30 to 34. The other series of optimal solutions under various levels of risk aversion are obtained by solving the above-stated stochastic programming problem excluding all irrigated activities. These optimum solutions are presented in Appendix Table 28. The latter solutions are utilized to estimate the level of risk aversions revealed in the actual enterprise combination of middle-sized farms in the project area. Also, an ordinary linear programming problem comprising the same set of activities, net prices, constraints and right-hand-side elements of constraint inequalities as those in the above stochastic

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[1]All analyses in this chapter are confined to the 250 acre representative farm.

programming problem is solved for comparison purposes. The optimum solutions obtained by varying the price of water appear in Appendix Table 29.

I. ANALYSIS OF OPTIMAL PLANS DEVELOPED UNDER VARIOUS LEVELS OF RISK AVERSION

Sensitivity of Optimal Solutions to Varied Risk Aversion

Under irrigation conditions. Optimum solutions are not sensitive to the change of risk aversion coefficient within the medium to high levels of risk aversion. Under the medium through high levels of risk aversion, no essential difference is found between the optimum solutions. These optimum solutions are similar to each other with respect to (1) activity combinations, (2) major and minor enterprises, (3) acreage allocation of land to these activities, (4) irrigation water utilization, (5) development of land for irrigation, and (6) the way in which the optimum solutions respond to change in the price of water. Comparisons of these optimum solutions are made in Table XXVI.

The price ranges of water are again defined as; (1) high, which includes prices higher than about \$2.50 per acre-inch at which demand for irrigation water indicates a distinct decline; (2) medium, prices between approximately \$1.00 and \$2.50; (3) low, prices lower than \$1.00.

When the level of risk aversion is in the range of

TABLE XXVI

UTILIZATION OF CROP LAND ON 250-ACRE FARM WITH THE MIDDLE RANGE OF WATER PRICES AND THE MEDIUM TO HIGH LEVELS OF RISK AVERSION

	Price of Water (Dollar Per Acre-inch)									
	1.00			1.50		2.00			2.50	
	Level of Risk Aversion									
Activity	Med.	Hi-Med.	High	Hi-Med.	High	Med.	Hi-Med.	High	Hi-Med.	High
	acres									
Wheat (D)	28.52	24.48	25.36	25.99	24.40	25.84	25.11	26.52	22.68	27.60
Sugar beets (D)	31.39	39.22	35.29	42.06	35.57	32.38	44.12	35.82	44.66	30.84
Potatoes (D)	-	4.71	8.65	3.94	7.85	-	-	6.28	-	-
Sunflower (D)	2.77	-	9.77	11.32	19.64	26.26	23.36	33.31	33.66	46.23
Sow-hog (D)	-	-	-	-	-	16.64	-	-	41.86	29.89
Flax (I)	58.07	61.89	60.33	53.92	55.79	39.16	46.83	48.51	37.28	36.52
Sugar beets (I)	31.24	23.25	16.03	20.45	14.42	30.12	18.38	11.94	17.84	11.06
Potatoes (I)	-	19.74	16.03	19.64	15.82	23.24	19.64	15.49	21.00	16.64
Sow-hog (I)	50.69	51.68	53.53	47.69	51.50	31.36	44.65	47.14	6.02	19.17
Land developed	140.0	156.56	145.62	141.70	137.53	123.88	129.50	123.08	82.14	83.39
	acre-inches									
Water used acre inch	1823	1773	1661	1596	1561	1370	1443	1391	934	947

Source: Appendix Tables 32 to 34.

NOTE: A "D" in brackets indicates a dryland activity, while an "I" refers to an irrigated activity.

medium to high and the price of water is in the low to middle range, the major enterprises in terms of acreage and contribution to total net revenue are "sow-hog with irrigated feed crops" and irrigated flax. In these ranges of price and risk aversion, minor enterprises are sugar beet and potatoes under both irrigated and dryland conditions, dryland wheat, and dryland sunflowers. In the high price range associated with the stated levels of risk aversion, the sow-hog or "feed-lot 400" activity associated with dryland feed crop and dryland sugar beets and sunflower, instead of "sow-hog with irrigated feed crops" and irrigated flax, occupy positions as major enterprises.

For the low to low-medium levels of risk aversion, optimum solutions differ to a considerable extent from those obtained under the medium to high levels of risk aversion. When the level of risk aversion is in the low to low-medium ranges, the optimum solutions include larger acreage of irrigated sugar beets in the low to middle price ranges, and higher proportion of "feed-lot 400 with dryland feed crops" in the high price range, than in the medium to high levels of risk aversion. For low risk aversion, "feed-lot 400 with dryland feed crops"[1] is the major enterprise throughout the entire

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[1]The activities "feed-lot 400 with dryland feed crops", "sow-hog with irrigated feed crops" and "sow-hog with dryland feed crops" are hereinafter abbreviated as "feed-lot 400 (D)", "sow-hog(I)" and "sow-hog(D)", respectively.

range of water price.

Under dryland conditions. Optimum solutions show a more sensitive response to change in the level of risk aversion. These optimum solutions differ from each other particularly in their activity levels. The optimum solutions for different levels of risk aversion are compared in Table XXVII.

TABLE XXVII

UTILIZATION OF CROP LAND ON 250-ACRE FARM UNDER DRYLAND CONDITIONS AND VARIOUS LEVELS OF RISK AVERSION

	Levels of Risk Aversion			
	Low	Medium	High-Medium	High
	acres			
Wheat (D)	35.46	34.66	33.27	32.82
Sugar beets (D)	62.50	62.50	58.79	22.65
Potatoes (D)	21.78	-	-	-
Flax (D)	-	-	-	-
Sunflower (D)	-	34.17	55.29	70.77
Feed-lot 400 (D)	105.27	63.93	46.02	38.07
Sow-hog (D)	-	29.74	31.63	33.31
Field peas (D)	-	-	-	27.38

Source: Appendix Table 28.

NOTE: A "D" in the bracket indicates a dryland activity.

The acreage of small cash grain is nearly constant under all levels of risk aversion. The acreage of specialty crops (sugar beets, potatoes and sunflower), however, vary sensitively with various levels of risk aversion. The level of the activity "feed-lot 400 (D)" also shows considerable variation for different levels of risk aversion.

Diversification and Risk Aversion Levels

Under both dryland and irrigation conditions, the number of activities entering the optimum solutions increases as the level of risk aversion rises. This indicates that diversification is effective to reduce risk with a proper combination of enterprises. Under irrigation conditions (price of water at $2.00), two major and two minor enterprises are combined in the optimum solution associated with low risk aversion. As the level of risk aversion rises to the medium and high levels, the number of enterprises increases to two major and four minor enterprises.

Major and Minor Activities Appearing in Various Optimal Plans

The irrigated flax and "sow-hog (I)" activities maintain their dominant position for many levels of risk aversion and water prices except for that of low risk aversion and high water prices. Thirty to fourty acres of dryland wheat are included in the optimum solutions obtained under all levels of risk aversion and all prices of water considered. With low to medium water prices and a medium level of risk aversion, about thirty acres of dryland sugar beets enter the optimum solution. A slightly lower acreage of irrigated and dryland sugar beets is included in the optimum solutions associated with the entire range of water prices and the high-medium to high levels of risk aversion. Relatively small acreages (15 to 30 acres) of potatoes, both under irrigated and dryland conditions, appear in almost all optimum solutions related with various levels of risk aversion and water price. For low level risk aversion, sunflowers on dryland do not enter the optimum solution at all, but do become one of the minor crops in the middle range of water price under the low-medium level of risk aversion. It becomes a major enterprise in the high price range under medium to high levels of risk aversion.

Risk-Reducing Combinations of Activities

The following several pages are devoted to an analysis of complementary relationships for risk reduction between activities, and to a comparison of single enterprise with diversified operations.

In traditional production economics,[1] diversification

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[1] E.O. Heady: Economics of Agricultural Production and Resource Use, (Englewood Cliffs: Prentice-Hall Inc.), 1952 pp. 439-464.

of farm operation has been considered as one of the means of reduction of risks encountered by farmers. Advantages of diversification are: (1) possible utilization of complementary relationships between crops, (2) reduction of risk (reduction of income variance), (3) efficient utilization of intermediate products, (4) even allocation of labour throughout a year, and (5) more intensive use of resources.

When n crops are combined, the variance of total income (S_z^2) is given by the summation of variances of individual crops' incomes per acre (s_j^2, j=1,2--n) weighted by their squared values of cropped acreage (X_j^2) and income covariances (s_j^2k, $j \neq k$; j and k =1,2--n) weighted by their cross-products of acreage ($Xj \cdot Xk$, $j \neq k$):

$$(V - 1) \qquad S_z^2 = \sum_{j=1}^{n} \sum_{k=1}^{n} s_j^2k \cdot Xj \cdot Xk$$

The total income variance, S_z^2, is small if (1) s_j^2's combined are small and (2) s_j^2k's (covariances) for combined crops are negative.

The effectiveness of diversification is demonstrated by the increased number of activities included in the optimum solutions of stochastic programming problem associated with higher levels of risk aversion coefficients. Optimum solutions obtained by stochastic programming contains more activities than linear programming solutions. The relationships between variances and covariances of major activities and those of minor activities entering the optimum solutions are examined in the following section. The variances and covariances of alternative activities are presented in Appendix Table 35.

Grouping of activities. When an activity is selected for entry into an optimum solution of a risk programming problem, at least one of the following conditions must be satisfied: (1) high expected net revenue, (2) relatively small variance of net revenue, or (3) negative covariances between this and other significant activities entering the optimum solution. On the basis of these criteria, the activities under consideration may be classified in the following way. The first group would include the sugar beet and potato activities, both under irrigated and dryland conditions, which have high expected net prices associated with large variances. The second group is characterized by above-average expected net prices and relatively small variances. It would include livestock activities ("sow-hog (I)" and "feed-lot 400 (D)"), both dryland and irrigated wheat, and dryland oats and barley. The third group comprises such activities as have average or below-average expected net prices associated with small variances. Cow-calf, sunflower, flax and field peas are included in this group.

Risk-reducing combinations with medium to high risk aversions. As mentioned earlier, in the optimum solutions of risk programming problems having medium-to-high levels of risk aversion and low-to-middle range of water prices, "sow-hog (I)" and irrigated flax are major enterprises. To be selected as a major enterprise, an enterprise will be required to have a fairly high net price associated with a relatively small variance. Also, its covariances with other major enterprises should be large and negative. Minor enterprises should have large negative covariances with respect to major enterprises. The irrigated sugar beet activity has a high expected net price, but its variance is also very high (ie., \$1,774.20 per acre). Consequently, when the level of risk aversion is in the medium to high range, irrigated sugar beets can not be a major enterprise. For the same reason, potatoes can not be a major enterprise under any level of risk aversion. The "sow-hog (I)" activity (No. 47) has the third highest expected net price (\$42.20). Also, its variance is very low, (\$198.81 per acre). Thus, with the medium to high levels of risk aversion and the low to middle range of water prices, the "sow-hog (I)" activity satisfies the condition of a primary, major enterprise. Once the "sow-hog (I)" activity is selected as a major enterprise, the activities to be selected as the second major or important minor enterprises should have large negative covariances with the net price of the "sow-hog (I)" activity. Irrigated potatoes and flax satisfy this condition. The covariances of irrigated potatoes and flax for the "sow-hog (I)" activity are \$-236.4 and \$-115.91. The variance of the irrigated potato activity, however, is the highest of all activities (\$7916.14 per acre), while that of irrigated flax is considerably lower than irrigated potatoes (\$464.31 per acre). Hence, the irrigated flax activity (No. 10) is selected as the second major enterprise despite its medium expected net price (\$30.27).

With the low to middle range of water prices and the medium to high levels of risk aversion, dryland wheat and sunflowers, and sugar beets and potatoes both on irrigated land and dryland, are included in the associated minor enterprises. Dryland wheat has the fourth highest expected net price (\$37.52) coupled with a small variance and a small positive covariance with the "sow-hog (I)" activity as well as for irrigated flax. The expected net price of dryland sunflower is about average, being associated with a small variance, a negative covariance with "sow-hog (I)", and a small positive covariance with irrigated flax. Sugar beets and potatoes both under dryland and irrigation conditions enter the minor group because they have distinctly high expected net prices; but they have large variances. Potatoes have large negative covariances with these two major enterprises, while the sugar beet activity has large positive covariances with them.

<u>Risk-reducing combinations with low and low-medium risk</u>

aversion. When the level of risk aversion ranges from "low" to "low-medium", activity combinations differ substantially from those under the medium to high levels of risk aversion. In this range of risk aversion, variances and covariances do not play a significant role, but expected net prices do. Firstly, the importance of irrigated sugar beets increases remarkably. Under these levels of risk aversion and the low to middle range of water prices, irrigated and dryland sugar beets are selected as major enterprises, and 30 to 50 acres each of dryland wheat, "feed-lot 400 (D)", and potatoes, flax and "sow-hog (I)", all under irrigation conditions, are selected as minor enterprises. The irrigated potato activity has large negative covariances ($-2307.5 and $-1096.36) with respect to these major enterprises, while irrigated flax has a fairly large negative covariance ($-108.5), only with dryland sugar beets. "Sow-hog (I)" as well as dryland wheat and "feed-lot 400 (D)" show small positive covariances with irrigated and dryland sugar beet activities. When the price of water is in the high range and the level of risk aversion in the low to low-medium ranges, "feed-lot 400 (D)" is a major enterprise and is coupled with a second major enterprise, dryland sugar beets. Minor enterprises are dryland wheat and irrigated potatoes. "Feed-lot 400 (D)" has small positive covariances with dryland sugar beets as well as with dryland wheat and a negative covariance with irrigated potatoes.

Comparison of Single Enterprise With Diversified Operations

In the Morden-Winkler Irrigation Project Area, the typical 250 acre farm is a mixed-enterprise operation. Nevertheless, a farm specializing in a single enterprise has some advantages as described below. In the following, therefore, the guaranteed incomes (Z_*) and expected incomes ($\bar{Z}$), of a farm specializing in a single crop, are compared with those of a mixed-enterprise operation. The advantages of specialization are: (1) proficiency in a single enterprise operation, (2) advantages of selling farm products and purchasing productive factors in quantity, and (3) efficient use of specialized machinery.

In this analysis, it is assumed that a 250 acre farm specializes in a single enterprise, devoting all crop land (225 acres) to its production; twenty five acres of unimproved pasture is used for a feeder-calf activity. An open quota is assumed for wheat production, and successive re-use of crop land for sugar beets, potatoes or sunflower is permitted. Expected incomes as well as those guaranteed at prescribed levels of probabilities under various risk aversion coefficients are calculated for different kinds of specialized operations. The results of these calculation are presented in Tables XXVIII and XXIX. They are also compared with those of a mixed-enterprise operation. Here, the mixed-enterprise

TABLE XXVIII

GUARANTEED AND EXPECTED INCOMES WITH VARIOUS RISK AVERSION COEFFICIENTS UNDER DRYLAND CONDITIONS

	Level of Risk Aversion					
	Low q=0.24		Medium q=0.88		High q=1.92	
	Z_*	$\bar{Z}$	Z_*	$\bar{Z}$	Z_*	$\bar{Z}$
	dollars					
Opt. mixed-enterprises	10,373.05	10,881.40	8,957.46	10,647.89	5983.16	7,785.80
σ_z →		2,254.66		1,753.14		961.47
Wheat only (open quota)	8,372.93	8,735.25	7,406.73	8,735.25	5,836.66	8,735.25
σ_z →		1,509.68		1,509.68		1,509.68
Flax only	5,672.72	6,069.17	4,615.52	6,069.17	2,897.58	6,069.17
σ_z →		1,651.87		1,651.87		1,651.87
Sunflower only	5,849.44	6,264.75	4,751.95	6,264.75	2,942.29	6,264.75
σ_z →		1,730.45		1,730.45		1,730.45
Sugar beets only	17,490.09	18,833.54	13,907.56	18,833.54	8,085.94	18,833.54
σ_z →		5,597.71		5,597.71		5,597.71
Potatoes only	2,916.94	6,109.52	0.0	6,109.52	0.0	6,109.52
σ_z →		12,155.92		12,155.92		12,155.92
Feed-lot 400 only	7,095.55	7,548.15	5,888.61	7,548.15	3,927.33	7,548.15
σ_z →		1,885.84		1,885.84		1,885.84
Sow-hog only	6,157.96	6,792.78	4,465.12	6,792.78	1,714.25	6,792.78
σ_z →		2,645.07		2,645.07		2,645.07

NOTE: These values are obtained from the stochastic programming solutions under dryland conditions. The guaranteed incomes, Z_*'s, are calculated by Moeseke's objective function: $Z_* = \bar{Z} - q \cdot \sigma_z$. Symbols, σ_z, denote standard deviations of income. A symbol, $\bar{Z}$, represents an expected income. The incomes appearing in the columns of Z_* are guaranteed at the prescribed levels of probability as described on page 93. Overhead costs (about $4,000) should be subtracted from the both types of incomes to derive net incomes available for household consumption.

TABLE XXIX

GUARANTEED AND EXPECTED INCOMES WITH VARIOUS RISK AVERSION COEFFICIENTS UNDER IRRIGATION CONDITIONS

	Levels of Risk Aversion					
	Low q=0.24		Medium q=0.88		High q=1.92	
	Z_*	$\hat{Z}$	Z_*	$\hat{Z}$	Z_*	$\hat{Z}$
	dollars					
Wheat (I) T_1	7,039.54	8,203.12	6,744.09	8,203.12	5,019.78	8,203.12
σ_z -->		1,657.99		1,657.99		1,657.99
Flax (I) T_1	5399.29	6,562.87	2296.39	6,562.87	0.0	4,848.27
σ_z -->		4,848.27		4,848.27		
Sugar beet (I) T_1	18,328.89	20,603.39	12,263.57	20,603.39	2,407.42	20,603.39
σ_z -->		9,477.07		9,477.07		9,477.07
Potatoes (I) T_1	8,695.38	13,499.90	0.0	13,499.90	0.0	13,499.90
σ_z -->		20,018.85		20,018.85		20,018.85
Feed-lot 400 (I) T_1	5,229.45	5,894.44	3,456.14	5,894.44	664.52	5,894.44
σ_z -->		2,700.79		2,700.79		2,700.79
Sow-hog (I) T_1	6,240.42	7,002.01	4,209.51	7,002.01	909.27	7,002.01
σ_z -->		3,173.30		3,173.30		3,173.30
Opt. mixed enterprises	12,340.63	13,114.73	10,549.70	11,691.28	8,563.42	10,315.04
σ_z -->		2,782.36		1,440.69		947.77

NOTE: The price of water is fixed at \$2.00 per acre-inch. These values are obtained from the stochastic programming solutions under irrigation conditions. The guaranteed incomes, Z_*'s, are calculated by: $Z_* = \bar{Z} - q \cdot \sigma_z$. Symbols, $\bar{Z}$ and σ_z, denote expected incomes and standard deviations of income, respectively. The incomes indicated by Z_* are guaranteed at the prescribed levels of probability as described on page 93. Overhead costs should be subtracted from these incomes.

operation is assumed to follow the optimum plans obtained by the stochastic programming analysis.

Under dryland conditions. The sugar beet enterprise shows the highest guaranteed and expected incomes with respect to the entire range of risk aversion coefficients. Dryland wheat ranks second, both with guaranteed and expected incomes, under all levels of risk aversion coefficients. Both guaranteed and expected incomes of sugar beets as well as of wheat are substantially higher than other enterprises. In the case of specialization under dryland conditions, therefore, sugar beets and wheat would be most advantageous. Under the low to medium levels of risk aversion, a mixed-enterprise operation would produce higher guaranteed and expected incomes than any single enterprise except sugar beets. With its high risk aversion coefficients, the mixed-enterprise operation produces a slightly higher guaranteed income, but a lower expected income, than that of a specialized wheat operation.

Under irrigation conditions. The advantage of a mixed-enterprise operation increases. (In this case, the price of water is fixed at $2.00 per acre-inch.) When the level of risk aversion is in the high range, a mixed-enterprise operation produces the highest guaranteed income. For low to medium risk averters, specialized sugar beet operations would be preferable to mixed-enterprise operations. All risk averters should prefer mixed-enterprise operations to single-enterprise operations except when the single enterprise is sugar beets.

With the consideration of acreage constraints. To this point, open quotas on wheat sales and the possibility of single-crop operations on stubble have been assumed. If the specified acreage quota were taken into account, specialized wheat operations under dryland conditions would becomes less profitable than dryland mixed-enterprise operations because large acreages of summerfallow land would be required to provide the specified acreage, reducing both guaranteed and expected incomes. If sugar beets can be grown on the same crop land only once every four years, specialized sugar beet operations (three-quarters of total crop land should be devoted to summerfallow) would be less profitable in terms of guaranteed and expected incomes, than, the mixed-enterprise operation.

Irrigation vs. Dryland Conditions

In this sub-section, it is examined whether or not irrigation could increase the utility of farmers in the project area. The optimal plans developed for irrigation conditions under various levels of risk aversion are compared with those for dryland conditions in Table XXXI. The optimal plans associated with the price of water fixed at $2.00 per acre-inch are used for the comparison.

<u>Expected and guaranteed incomes under irrigation and dryland</u>. Table XXX shows the differences of expected and guaranteed incomes between dryland and irrigation conditions.

TABLE XXX

DIFFERENCES OF EXPECTED AND GUARANTEED INCOMES BETWEEN IRRIGATION AND DRYLAND CONDITIONS (INCOME UNDER IRRIGATION MINUS INCOME UNDER DRYLAND).

	Linear Programming	Level of Risk Aversion				
		Low	Low-Med.	Medium	High-Med.	High
		dollars				
Difference of Expected Income	+2,233	+2,233	+1,426	+1,124	+1,500	+2,559
Difference of Guaranteed Income	+2,233	+1,968	+1,622	+1,742	+2,401	+2,580

Irrigation increases both expected and guaranteed incomes by considerable amounts for all levels of risk aversion. Increases in the guaranteed incomes are larger for all levels (except the lowest) of risk aversion than those in the expected incomes. The differences of expected and guaranteed incomes are large with respect to the low, high-medium and high levels of risk aversion and relatively small for the low-medium and medium levels. This means that irrigation would increase the utilities of two extreme types of risk averters more than those of medium risk averters. Increases in utility due to irrigation will be especially large for the high level risk averters; this follows from the fact that they have the largest values of increases in the expected and guaranteed incomes.

Activity combinations under irrigation and dryland conditions. The optimal enterprise combinations under irrigation conditions differ from those under dryland conditions except for the one which is coupled with the low level of risk aversion. This results because the expected net prices, as well as the variances and covariances of alternative activities estimated for dryland conditions, are changed by irrigation at different rates. With the low level of risk aversion, the optimal cropping systems, under irrigation are nearly identical with those under dryland conditions. They comprise approximately 62 and 100 acres each of sugar beets and "feed-lot 400", 30 to 35 acres of wheat and 22 to 30 acres of potatoes. As the level of risk aversion increases to low-medium, the optimal cropping systems under these two conditions become distinctively different. These two cropping systems differ substantially except that the same acreage of sugar beets is included under both systems. Under irrigation conditions, 29 and 34 acres each of flax and sunflower, respectively, are included, while none should be produced under dryland conditions. Livestock operation under irrigation conditions also differs from that

TABLE XXXI

COMPARISON OF OPTIMAL PLANS UNDER DRYLAND WITH IRRIGATION CONDITIONS FOR VARIOUS LEVELS OF RISK AVERSION

	Level of Risk Aversion									
	Low		Low-med.		Medium		Hi-med.		High	
	Dry	Irrig.	Dry	Irrig.	Dry	Irrig.	Dry	Irrig.	Dry	Irrig
	acres									
Flax (I)	-	-	-	28.72	-	39.16	-	46.83	-	48.51
Sugar beets(I)	-	62.5	-	49.90	-	30.12	-	18.38	-	11.94
Potatoes (I)	-	32.73	-	27.36	-	23.24	-	19.64	-	15.49
Sow-hog (I)	-	-	-	5.59	-	31.36	-	44.65	-	47.14
Sugar beets(D)	62.5	-	62.5	12.59	62.5	32.38	58.79	44.12	22.65	35.82
Potatoes (D)	21.78	-	-	-	-	-	-	2.91	-	6.28
Wheat (D)	35.46	32.70	38.81	23.18	34.66	25.84	33.27	25.11	32.82	26.52
Sunflower (D)	-	-	-	34.31	34.17	26.26	55.29	23.36	70.77	33.31
Field peas (D)	-	-	-	-	-	-	-	-	27.38	-
Sow-hog (D)	-	-	20.69	43.34	29.74	16.64	31.63	-	33.31	-
Feed-lot 400	-	-	-	-	-	-	-	-	-	-
(D) acre	105.27	97.07	103.00	-	63.93	-	46.02	-	38.07	-
	dollars									
Expected income	10,881	13,115	10,717	12,143	10,567	11,691	9,853	11,353	7,786	10,315
Guaranteed income	10,373	12,341	9,475	11,097	8,807	10,550	7,519	9,920	5,983	8,563
Standard deviat.	2,255	2,782	1,915	1,780	1,742	1,441	1,610	1262	961	948
Q value	0.225	0.278	0.625	0.587	0.956	0.787	1.45	1.14	1.88	1.85
probability	0.59	0.61	0.74	0.722	0.83	0.794	0.93	0.87	0.97	0.97

Source: Appendix Tables 28 and 30 to 34.

NOTE: The price of irrigation water is fixed at $2.00 per acre-inch. "I"'s in the brackets indicate "irrigated" activities while "D"'s in the brackets, "dryland" activities.

under dryland conditions (ie., about 100 acres of "feed-lot 400" enters the optimal cropping system under dryland conditions whereas about 50 acres of "sow-hog" activity, under irrigation conditions). As mentioned earlier, under irrigation, there is not a substantial difference between the alternative optimal cropping systems associated with the medium to high levels of risk aversion, but there is, under dryland conditions. With the medium level of risk aversion, 62.5, 30 to 35 and 26 to 34 acres each of sugar beets, wheat and sunflower, respectively, are included under both conditions, but the acreages of other activities under these conditions is different. Approximately 40 acres of flax is included in the optimal cropping system under irrigation, but none under dryland conditions. About 64 acres of "feed-lot 400" and 30 acres of "sow-hog" enter the optimal plan under dryland conditions, whereas, 48 acres of the "sow-hog" activity only, under irrigation. When the level of risk aversion is high-medium, 58 to 62, 25 to 33 and 32 to 45 acres each of sugar beets, wheat and the "sow-hog" activity, respectively, are included in the optimal plans for irrigated and dryland conditions. However, the acreages of "feed-lot 400", potatoes and sunflower under irrigation are different from those under dryland conditions. While 46 acres of "feed-lot 400" and 55 acres of sunflower are included in the optimal plan under dryland conditions, none of "feed-lot 400" and only 23 acres of sunflower appear under irrigation conditions. The optimal plan under irrigation includes 23 and 47 acres each of potatoes and flax, respectively, but under dryland conditions does not include any of them. The optimal plan developed for irrigation conditions under consideration of high risk aversion, contrasts with that for dryland conditions as described below. The acreage of sugar beets is very small (22.65 acres) under dryland conditions, but it is fairly large under irrigation (47.7 acres). On the other hand, however, larger acreages of crop land are devoted to sunflowers (71 acres) and field peas (27.4 acres) under dryland conditions than under irrigation. Under dryland conditions, 38 and 33 acres each of "feed-lot 400" and "sow-hog", respectively, are included in the optimal plan, while the "sow-hog" activity only, under irrigation conditions.

With the medium to high levels of risk aversion, the profitability of flax and the "sow-hog" activity is increased remarkably by irrigation, and that of the sugar beet activity maintained even with the high level of risk aversion. Throughout all levels of risk aversion, the profitability of potato activity is increased. The "feed-lot 400" activity appears promising under dryland conditions for all risk averters, but not, under irrigation conditions.

<u>Risks and attainable utilities under irrigation and dryland conditions</u>. As discussed in Chapter II, the various

TABLE XXXII

UTILITY INDIFFERENCE CONTOURS AND AVERAGE DISCOUNTING RATES OF $\bar{Z}$ FOR ONE DOLLAR OF SD

	Level of Risk Aversion				
	Low	Low-Med	Medium	High-Med	High
	(1)	(2)	(3)	(4)	(5)
Under Dryland Conditions					
Value of a	0.0002	0.00066	0.0011	0.0018	0.0039
Value of q	0.225	0.635	0.964	1.449	1.875
Prob. Attached	0.5910	0.7389	0.8315	0.9265	0.9700
Expected Income $\bar{Z}$	\$ 10,881.39	10,916.52	10,647.89	9,852.73	7,785.80
SD of Income, σ_Z	\$ 2,254.66	1,924.61	1,753.14	1,610.28	961.47
Expected Utility E(U)	0.87439	0.99834	0.99995	0.99999	0.99999
Income at $\sigma_Z=0$, $\bar{Z}_o$	\$ 10,373.03	9,694.16	8,957.58	7,524.71	5,983.36
$\bar{Z} - \bar{Z}_0$	\$ 508.36	1,222.36	1,690.31	2,328.01	1,802.44
$\bar{Z} - \bar{Z}_0$ /SD[1]	\$ 0.2255	0.63512	0.96416	1.44572	1.8746
Under Irrigation Conditions[2]					
	(6)	(7)	(8)	(9)	(10)
Value of a	0.0002	0.00066	0.0011	0.0018	0.0039
Value of q	0.278	0.587	0.787	1.136	1.848
Prob. Attached	0.6103	0.7224	0.7938	0.8729	0.9678
Expected Income $\bar{Z}$	\$13,114.73	12,142.90	11,691.27	11,352.70	10,315.04
SD of Income σ_Z	\$ 2,782.26	1,780.11	1,440.69	1,261.72	947.77
Expected Utility E(U)	0.91526	0.99934	0.99999	0.99999	0.99999
Income at $\sigma_Z=0$ $\bar{Z}_o$	\$12,340.63	11,097.23	10,550.09	9,920.28	8,558.50
$\bar{Z} - \bar{Z}_o$	\$ 774.10	1,045.67	1,141.19	1,432.42	1,756.54
$\bar{Z} - \bar{Z}_o$/SD	\$ 0.27823	0.58742	0.792.11	1.1352	1.8533

[1] This is interpreted as "average discounting rate of $\bar{Z}$ for one dollar of SD".

[2] Price of water is 2.00 dollars per acre-inch.

NOTE: Expected utilities are calculated by $E(U) = 1-e^{a(\frac{a}{2}\sigma_Z^2-\bar{Z})}$. SD is the standard deviations of income.

risk aversion coefficients used in the Freund objective function can be interpreted in terms of average discounting rates of expected incomes for the associated standard deviation of income. These discounting rates are calculated for the optimum solutions of risk programming problems obtained under five different risk aversion coefficients. Under irrigation conditions, the price of water is fixed at $2.00 per acre-inch. The results of this analysis are shown in Table XXXII.

Indifference curves of expected utility, E(U), derived under the five levels of risk aversion are drawn in Figure 14. The vertical axis measures an expected income, and the horizontal axis the associated standard deviation of income. Each indifference curve is tangent to the same utility possibility curve derived from the optimum solutions of Heady-Candler type of stochastic programming problems.

The indifference curves, $E(U)_1$ to $E(U)_5$, indicate five levels of expected utilities derived under dryland conditions, and $E(U)_5$ to $E(U)_{10}$ those derived under irrigation conditions. The line F_1, shows the utility possibility (frontier) curves of the Heady-Candler type obtained under dryland conditions and, F_2, that obtained under irrigation conditions. Tangency points, $\bar{\bar{Z}}_1$ to $\bar{\bar{Z}}_{10}$, coordinate the expected income and associated standard deviation of income pairs found in the optimum solutions of risk programming problems associated with the low to high levels of risk aversion.

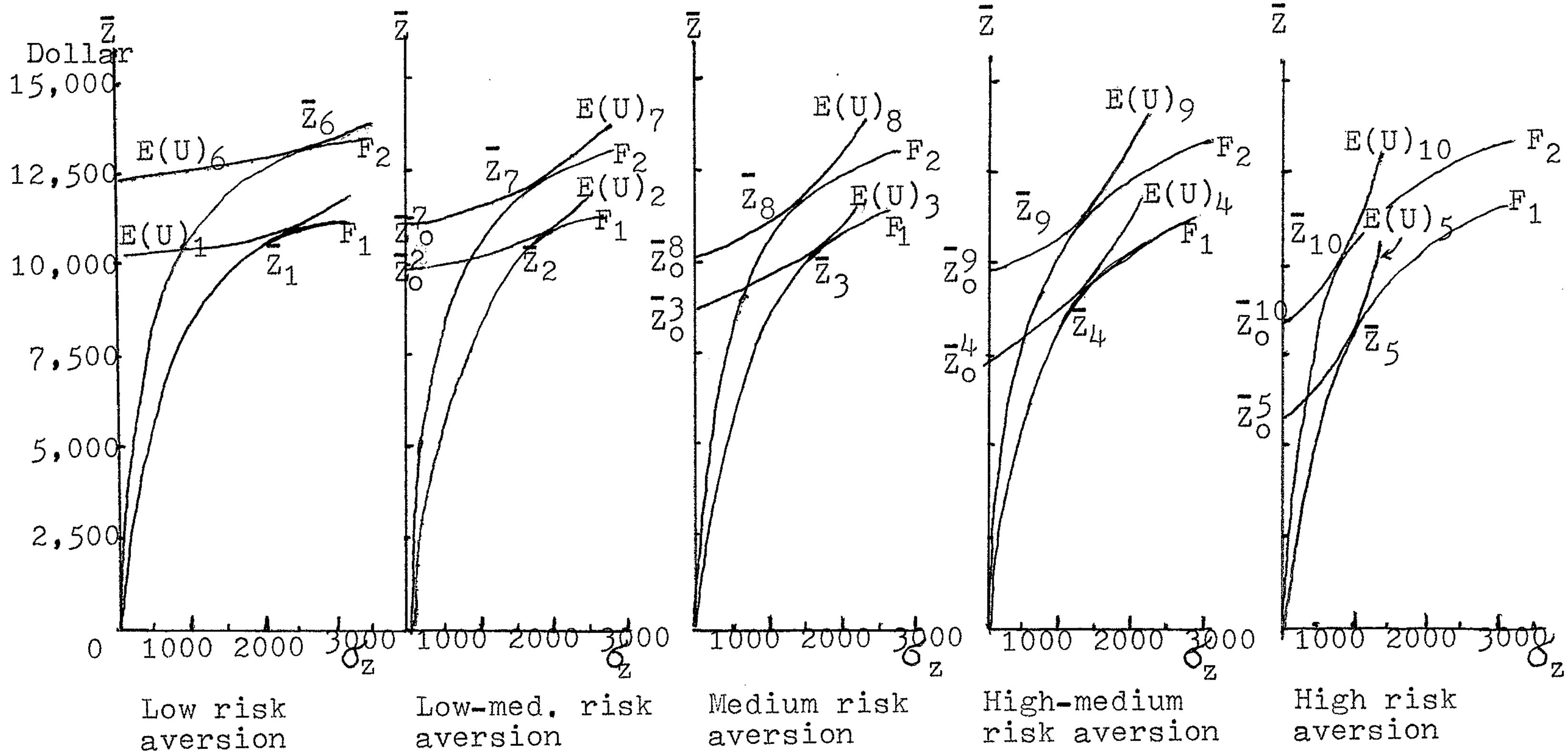

FIGURE 14

UTILITY INDIFFERENCE CURVES AND UTILITY POSSIBILITY CURVES UNDER VARIOUS LEVELS OF RISK AVERSION

A higher level of expected utility is attainable under irrigation conditions than under dryland conditions. This is indicated by the utility possibility curve for irrigation conditions, F_2, which lies above that for dryland conditions, F_1.

It is observed from the graphs that the slopes of utility indifference curves at the same levels of income standard deviation, σ_z, increase as the level of risk aversion increases. This indicates that the marginal substitution rate of expected income for standard deviation of income, rises with the level of risk aversion. In other words, the marginal discounting rate of expected income for standard deviation increases in proportion to the level of risk aversion.

Expected utility, E(U), is calculated by:

$$(V - 2) \qquad E(U) = 1 - e^{a(\frac{a}{2}\sigma_z^2 - \bar{Z})}$$

where:

a, σ_z and $\bar{Z}$ are indicated in Table XXXII. The expected utility, E(U), can take on any fractional value between one and minus one including zero. If $\sigma_z{}^2$ is very large relative to $\bar{Z}$, then E(U) may be a negative value.

Under irrigation conditions, expected utilities derived using medium to high levels of risk aversion indicate high and nearly identical values. Also, expected utilities under all levels of risk aversion vary less than those under dryland conditions. These results coincide with those from the comparison of optimum combinations of activities derived under various levels of risk aversion. (See pages 174-5.)

The term, Z_0, (ie., an income at σz = zero) can be interpreted as an income having the associated standard deviation of income equal to zero and which provides a decision-maker with the same level of utility as that accruing from the combination of expected income and standard deviation in an optimum solution. In other words, Z_0, is determined at the intersection point of the vertical axis and an utility indifference curve indicating the same utility as that provided by the combination of expected income and standard deviation in a optimum solution of the risk programming problem.

Under dryland conditions, a low level risk averter is indifferent in choosing between the expected income $10881.39, associated with the standard deviation of $2254.66 and the expected income $10373.00, having zero standard deviation. For him, a $2254.66 reduction in the standard deviation of income is equivalent to an increase in his expected income of only $508.36. This means that he does not pay much attention to a reduction in the standard deviation of income. In the other extreme, a high risk averter evaluates an increased standard deviation of income of $969.47 as equivalent to a decrease in his expected income of $1802.44. The average discounting rate of expected income for a one-dollar-increase in the standard deviation of income is $0.2255 for a low risk averter; it is $1.8746 for a high risk averter.

All levels of risk averter, except the lowest, should have less fear of risk under irrigation conditions, than under dryland conditions. This follows from the average discounting rates for one dollar of increased standard deviation of income. Only the low risk averters do not care for irrigation as a method of reducing risks.

II. COMPARISON OF STOCHASTIC PROGRAMMING SOLUTIONS WITH THOSE OF LINEAR PROGRAMMING

In this section, a comparison is made between the optimal solutions of the linear programming and of the stochastic programming problems at various levels of risk aversion for both dryland and irrigation conditions. The results are presented in Appendix Tables 29 and 30 to 34.

Comparison of Optimal Solutions Under Dryland Conditions

For the low level of risk aversion, the optimal solution obtained by the stochastic programming is identical with that from the linear programming. These two optimal solutions include about 62 and 105 acres of sugar beets and the "feed-lot 400" activity, respectively. These major enterprises are accompanied by 35 acres of wheat and 22 acres of potatoes.

When the level of risk aversion is low-medium, the major enterprises included in the optimal solutions of stochastic programming are identical with those of linear programming, but the minor enterprises are not. The potato activity (about 22 acres) enters the optimal solution of linear programming, but not that of stochastic programming. Instead, the "sow-hog" activity is included.

The optimal solution of stochastic programming for a medium level of risk aversion is substantially different from that of linear programming. Firstly, the level of the "feedlot 400" activity entering the former is much lower than that in the latter. Instead, 34 acres of sunflowers are included in the former solution. Furthermore, about 30 acres of the "sow-hog" activity appears in this optimal solution in contrast with 22 acres of potatoes in the latter one.

The comparison of the optimal linear programming solution with that of stochastic programming associated with the high-medium and medium level of risk aversion, shows a similar contrast. This occurs because there are insignificant differences between the optimal solutions of the stochastic programming analyses for the medium and high-medium levels of risk aversion.

With a high level of risk aversion, the difference between the two optimal solutions is more significant. The major enterprises in the optimal linear programming solution are sugar beets and the "feed-lot 400" activity, whereas, sunflowers are the only major enterprise in the stochastic programming analysis.

The sugar beet activity is one of the most promising activities for the low to high-medium levels of risk averters as well as for the expected-income maximizers. It is, however, less important for the high level risk averters. In contrast, sunflowers, which are not included in the optimal linear programming solution at all, is the most important activity for the high risk averters. About 33 to 38 acres each of wheat, "feed-lot 400" and "sow-hog" activities, as well as 27 acres of field peas and 23 acres of sugar beets are included as the minor activities in the optimal stochastic programming solution. None of the "sow-hog" activity and field peas is in the linear programming solution. However, the latter solution includes a small acreage of potatoes which is not included in the former.

The optimal plan developed by linear programming tends to concentrate more on a few activities at a higher rate than does the stochastic programming plan. In the case of linear programming, approximately 25 and 42 percent of the total land are occupied by, respectively, the sugar beet and the "feed-lot 400" activities (about 67 percent of the total land is devoted solely to their production). In the optimal stochastic programming analysis for the high level risk averters, about 28 percent of the total land is used for sunflower, but the remainder is distributed among four activities in nearly equal proportions.

From these comparisons, we may draw the conclusion that, if the farmers in the project area are in the medium to high levels of risk aversion range, then an optimal solution obtained by the linear programming method would not provide adequate information for their farm planning. Usually, an optimal functional value of linear programming tends to be unrealistically high in comparison with the actual farm incomes. This results primarily because an optimal plan obtained by the linear programming method has a tendency to concentrate on a few activities having high expected net prices, neglecting the stability of obtainable incomes. The actual farm incomes, however, result from a farm operation where stability of income is also taken into consideration. The expected and guaranteed incomes projected by the stochastic programming method are more like the actual incomes. In this study, the maximum functional value of the linear programming solution is $10,881.40, whereas, the expected and guaranteed incomes obtained by the stochastic programming for the high risk averters are $7,785.80 and $5,983.16. (About $4,000 of overhead costs should be subtracted from these amounts to obtain the net incomes available for household consumption.) The actual farm income in the project area comparable with the derived incomes is $7,458.

Comparison of Optimal Solutions Under Irrigation Conditions

Under irrigation conditions, there is not a significant difference between the optimal plans derived from the stochastic programming solutions under medium or high levels of risk aversion. Therefore, the comparison of optimal solutions between the linear programming and the stochastic programming is made with respect to the low and the high levels of risk aversion.

Low level of risk aversion. At a water price of $2.00 per acre-inch, the optimal stochastic programming solution is identical with that of linear programming. However, a change in the price of water produces different patterns of responses in these two optimal solutions. In the price range of water lower than $0.67, the optimal linear programming solution includes a large acreage of irrigated flax (81 to 97 acres), while the stochastic one has only a moderate acreage (48 to 59 acres). In the same price range, the latter solution contains 26 to 30 acres of the irrigated "sow-hog" activity, but the former solution excludes this activity. The concentration ratio of the optimal linear programming solution is higher than that of the stochastic one. Approximately 74 percent of the total crop land is used for the three major activities in the former solution, while about 60 percent is required in the latter solution. With the prices of water from $0.67 to $3.00, the optimal solutions are nearly identical with each other except for a low level of irrigated "sow-hog" activity which is included in the optimal stochastic programming solution associated with water prices of $0.67 to $1.50. In the price range higher than $3.00, the optimal linear programming solution includes a larger acreage of irrigated potatoes than that of stochastic programming. The optimal stochastic programming solution is more sensitive to the change in the price of water in its high range than is that of linear programming.

With a high level of risk aversion. Throughout the entire range of water prices, the optimal solutions of stochastic programming, being more diversified, differ substantially from those of linear programming (Appendix Tables 29 and 34). When the price of water is $0.50 per acre-inch, irrigated flax, irrigated sugar beets and the "feed-lot 400 (D)" activity are important in the optimal linear programming solution. However, "feed-lot 400 (D)" activity is not included at all in the other program. Instead, the "sow-hog (I)" activity enters the optimal solution of the stochastic program as one of the important activities. Sugar beets are irrigated in the optimal linear programming solution, but not in the other. Both optimal solutions include irrigated flax as one of the important activities in this price range.

Given the water price at $1.00 or $2.00, the difference between the two optimal solutions is more significant than with a price of $0.50. This results because the optimal linear programming solutions at these two prices are concentrated more than at $0.50, while those of stochastic programming are more diversified. At $2.00, about 71 percent of the crop land is devoted to only two activities (dryland sugar beets and the "feed-lot 400 (D)" activity) in the optimal solution of the linear programming, while about 42 percent (the irrigated flax and the "sow-hog (I)" activity) is used in the stochastic analysis. The expected income, $13114.74, of the linear programming optimal solution is about $2,000 higher than that of stochastic programming. This high expected income is associated with a high standard deviation of income, $2,782.26; the standard deviation pertaining to the stochastic programming is lower, $974.77. Under dryland conditions, the linear programming expected income of $10881.40, is higher by about $3,100 than the value for the stochastic programming analysis.

<u>Some conclusive remarks</u>. These comparisons reveal that, under irrigation conditions, the optimal solutions of linear programming coupled with the varied prices of water are significantly different from those of stochastic programming. This is especially true for the medium to high levels of risk aversion parameters. Even with the low level of risk aversion, these two kinds of optimal solutions show different patterns

of responses to varying prices of water. Therefore, if the farmers in the project area are to some extent risk averters, the stochastic programming method will provide them with more useful information on the optimal adjustment of their farm organization to irrigation conditions. Projection of aggregate demand for irrigation water requires taking the farmers' attitude toward risk into consideration. The projection of the aggregate demand function for water should be based upon the "weighting" by the number of farms having each level of risk aversion as well as the number in each farm size class. A shift of the demand curve will occur at a lower price of water and be larger for the stochastic programming estimation than for the linear programming (see pp.218-20).

<u>Comparison of the utilities obtainable by the two optimal plans</u>. The optimal plans developed by the linear programming method should be evaluated by various levels of risk averters, as inferior to those developed by the stochastic programming method. This would be demonstrated by the level of expected utility derived from an optimal linear programming solution for various risk averters, as calculated by the Freund type of expected utility function. This level is lower than that from the optimal stochastic programming solutions. In the following discussion, the expected utilities from these two types of programming are compared with respect to various

levels of risk aversion parameters (Table XXXIII).

Under irrigation conditions, the expected utility of the optimal linear programming solution calculated for high level risk averters is negative. The optimal plan obtained by the linear programming method involves too high a risk for high level averters. Therefore, the optimal plan gives them "disutility". Such an optimal plan coupled with a high risk could be accepted by these farmers only with the assistance of all-risk crop insurance.

Under both dryland and irrigation conditions, the expected utilities of the optimal stochastic programming solutions are higher, at all levels of risk aversion parameters, than those of the linear programming optimal solutions. In other words, every risk averter can reach a higher level of utility indifference curve on the utility possibility curve by choosing an optimal plan developed by the stochastic programming, than he can by choosing a linear programming solution. This can be illustrated on a graph as in Figure 15. Let us consider the case of a high risk averter under dryland conditions. The utility indifference curve, obtained from the expected income and the associated standard deviation of income obtained through the stochastic programming, lies at a higher utility level (E(U)=0.99999) than that obtained from linear programming (E(U)=0.97712). Points, E_1 and E_2, are determined by the expected income and its associated standard

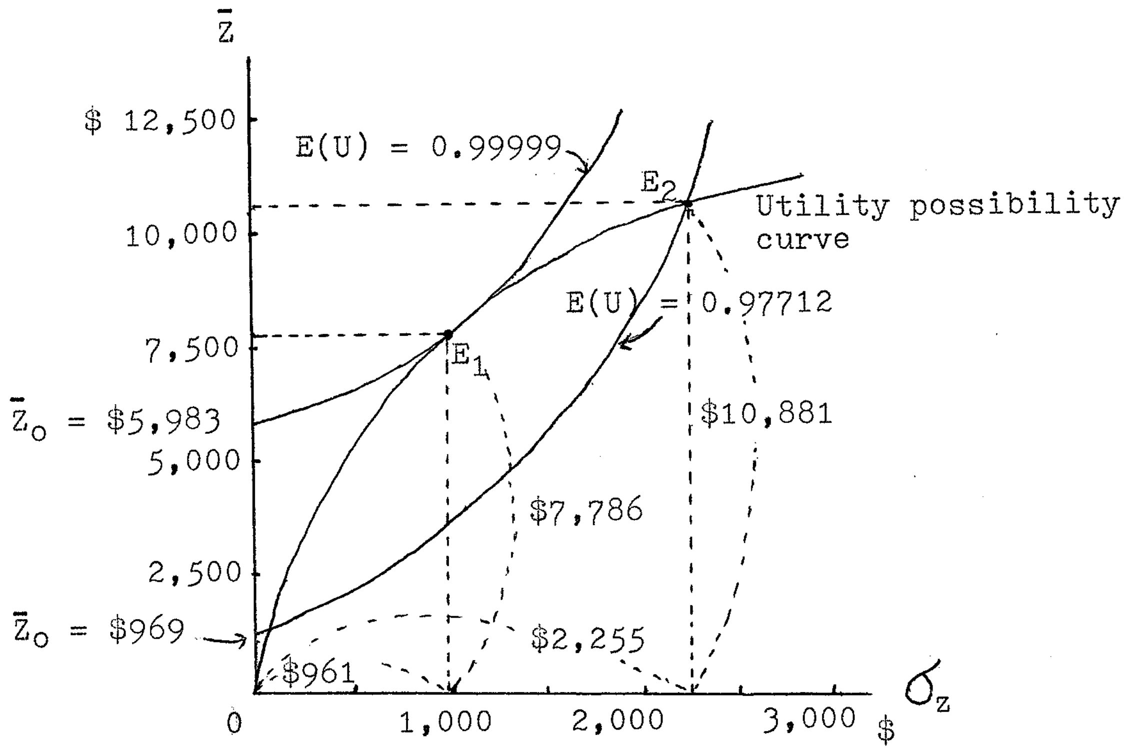

FIGURE 15

COMPARISON OF UTILITY LEVELS OBTAINABLE BY LINEAR AND STOCHASTIC PROGRAMMING

TABLE XXXIII

COMPARISON OF EXPECTED UTILITIES DERIVED FROM THE LINEAR PROGRAMMING SOLUTION FOR VARIOUS LEVELS OF RISK AVERSION WITH THOSE FROM THE STOCHASTIC PROGRAMMING SOLUTIONS

	Level of Risk Aversion				
	Low	Low-Med	Medium	Hi-Med	High
Under Dryland Conditions:					
Value of a	0.0002	0.00066	0.0011	0.0018	0.0039
$\bar{Z}$ of LP. Solution	10,881.39	10,881.39	10,881.39	10,881.39	10,881.39
σ_z of LP. Sol. $	2,254.66	2,254.66	2,254.66	2,254.66	2,254.66
E(U) from LP. Solution[1]	0.87439	0.99770	0.99986	0.99999	0.97712
E(U) from stochastic prog. sol.	0.87439	0.99834	0.99995	0.99999	0.99999
$\bar{Z}o$ with LP. sol $	10,373.03	9,203.86	8,085.85	6,306.85	968.59
$\bar{Z}o$ with SP. sol $	10,373.03	9,694.16	8,975.58	7,524.71	5,983.36
Av. discount rates with LP. $	0.23	0.74	1.24	2.03	4.40
Av. discount rates[2] with SP. $	0.23	0.64	0.96	1.45	1.87
Under Irrigation Conditions:[3]					
Value of a	0.0002	0.00066	0.0011	0.0018	0.0039
$\bar{Z}$ of LP Sol. $	13,114.73	13,114.73	13,114.73	13,114.73	13,114.73
σ_z of LP Sol. $	2,782.26	2,782.26	2,782.26	2,782.26	2,782.26
E(U) from LP. Sol.	0.91526	0.99906	0.99994	0.99998	-2,257.88
E(U) from SP. Sol.	0.91526	0.99934	0.99999	0.99999	0.99999
$\bar{Z}o$ with LP. Sol $	12,340.63	10,560.29	8,858.09	6,148.44	-1,980.16
$\bar{Z}o$ with SP. Sol $	12,340.63	11,097.23	1,0550.09	9,920.28	8,558.50
Av. discount rates with LP.	0.28	0.92	1.53	2.50	5.43
Av. discount rates with SP.	0.28	0.59	0.79	1.14	1.85

[1]The values of "E(U) from L.P. solution" is calculated by:

$$E(U)=1-e^{a(\frac{a}{2}\sigma_z^2-\bar{Z})}$$

where the expected income and the associated standard deviation of income obtained by the linear programming solution are substituted into Z and σz, respectively. This value shows the expected utility of a risk averter obtainable from the optimal L.P. solution.

[2]Average discounting rates of $\bar{Z}$ for a one-dollar-increase in σz are calculated by $\frac{\bar{Z} - \bar{Z}o}{\sigma z}$.

[3]The price of water is fixed at $2.00 per acre-inch.

deviation of income as derived from the optimal solutions of stochastic and linear programming. The expected utility of the optimal stochastic programming solution is 0.99999, while that of the optimal linear programming solution is 0.97712. Also, the $\bar{Z}_0$ value of the former is higher (\$5983) than that of the latter (\$969). The expected income of the same high risk averter is discounted at a higher average rate for a one-dollar-increase in the standard deviation of the income with the optimal plan developed by the linear programming, than it is by stochastic programming. With the former plan, his average discounting rate is \$4.40, whereas, it is \$1.86 in the latter plan. This means that his subjective aversion for an income variation is greater in the case of a linear programming than it would be for stochastic programming. In other words, using a linear programmed plan makes him more cautious than does using a stochastic plan.

III. IRRIGATION VERSUS ALL RISK CROP INSURANCE

<u>In Case of the Conservative Decision-maker</u>

The purchase of crop insurance by two types of decision-makers is taken into consideration in the analysis of this section (see Section 2, Chapter III). In the first part of this section, the results of the linear programming solution obtained for a decision-maker who maximizes the insured net revenue are presented. The maximum insured income is compared with the maximum incomes, guaranteed at the prescribed levels of probabilities, which are obtainable by the stochastic programming plans.

Table XXXIV shows the optimal solution of a linear programming problem in which the activities comprise the insurable crop alternatives and the insured total income is to be maximized subject to a set of constraints. One optimal solution is obtained with the assumption of an open quota, and the other with the consideration of specified acreage quota of nine bushels.

In both cases, the insured net incomes are calculated by subtracting variable costs and crop insurance premiums from the total revenue. The overhead cost (about $4,000) should be subtracted from these incomes to obtain the net incomes available for household consumption.

With open quota assumption on wheat selling. Firstly, sugar beets are grown to the upper limit (62.5 acres) of the constraint regulating the repeated use of crop land for sugar beets (see constraint No. 15 in Appendix Table 20). The remaining crop land is used exclusively for wheat production. Both crops are insured to the maximum possible amount of revenue protection, 80 percent of the long-run-average yields associated with the high price options. The total insured revenue

TABLE XXXIV

OPTIMAL SOLUTIONS OF LINEAR PROGRAMMING PROBLEM COMPRISING ALTERNATIVE INSURABLE CROPS

Activity[1] No.	Activity[1] Description	Units	Quota: Open	Quota: 9 bus. specified acreage
3	Wheat with crop insurance scheme 3	acre	162.50	-------
21	Flax with crop insurance scheme 3	acre	------	162.50
31	Sugar beets, scheme 3, thinner + custom	acre	8.87	8.87
33	Sugar beets, scheme 3, hand-thinner + custom	acre	53.63	53.63
37	Hire fall labor	hours	53.88	55.50
39	Total land	acre	280.00	280.00
40	Operating capital loan	$	1117.06	854.95
41	Purchase of thinner	Unit	0.068	0.068
43	Specified acreage	acre	-----	0.00
44	Summerfallow activity	acre	-----	0.00
	Expected income	$	11055.26	9825.21
	Guaranteed income (insured)	$	5403.88	4572.16
	Probability		1.00	1.00

[1]A more complete description of these activities appears in Appendix Table 20 .

of wheat is $4,004.00 and that of sugar beets is $7,579.69; the maximum insurable revenue totals $11,583.69. On the other hand, the total variable costs including insurance premiums are $6,179.29. Thus, the maximum insurable income net of variable costs and the premium is $5,403.88. The expected income from the combination of these activities is $11,055.26. The maximum insurable income is enough to cover the total farm expenses including variable and overhead costs, producing about $1,400 net income. However, this amount is not enough to guarantee the income required to meet the minimum household consumption.[1] If the overhead cost is not paid, then $5.403.88 is available for household consumption. Therefore, the maximum insurable total revenue is enough to cover the total variable cost plus either the overhead cost or the farm household expenditures. In the optimal plan, $1,117.06 of operating capital is borrowed and 53.63 hrs. of fall labor is hired. The insured net prices of alternative crop activities appear in Appendix Table 20-B.

[1] According to the study of farm household expenditures based on 226 Manitoba Farm Account records by J.A. MacMillan and R.M.A. Loyns, the total consumption expenditure is estimated as $4485.92. The minimum household consumption expenditure is calculated by including the items: food, clothing, health, personal, education, furniture, car and home-grown food value. Investment and house depreciation are excluded. The minimum household consumption expenditure is $3,555.82; "A Cross-sectional Analysis of Farm Household Expenditures", Canadian Journal of Agricultural Economics, Vol. 17, No. 2, pp. 99-104.

The insured income, $5,403.88, is obtainable by maximizing the objective functional value of insured net prices, subject to the same set of constraints as applied to the stochastic programming problem, and it is guaranteed with a probability of one. This insured income is comparable with the guaranteed incomes obtained, under irrigation conditions, by the solution of the stochastic programming problem having the high risk aversion parameter and water prices lower than $2.25 per acre-inch. This is because the prescribed levels of probabilities given to these guaranteed incomes are also close to one. When the price of water is lower than $2.25, the probabilities attached to the guaranteed incomes fall in the range 0.965 to 0.988. Given these prescribed levels of probabilities, an income lower than the guaranteed incomes may occur only once in 30 to 80 years. In other words, the incomes obtainable by the optimal stochastic programming plans are guaranteed for the entire managerial span of a farmer, except for one year.

With the price range of water extending from zero to $1.50 per acre-inch, the guaranteed incomes lie between $9,344.52 and $12,186.09, and the expected incomes are between $11,306.94 and $14,576.74. Thus, with the price of water lower than $1.50, higher levels of expected and guaranteed incomes are obtainable by employing the optimal plans developed for high risk averters under irrigation conditions than are obtainable by the optimal plan associated with crop insurance under dryland conditions.

With the consideration of nine bushel specified acreage quota. In this case too, crop land is used for the sugar beet activity up to the upper limit of the constraint (62.5 acres), and the remaining 162.50 acres is allocated to flax. Sugar beets and flax entering the optimal plan are insured to 80 percent of the long-run-average yields. A higher level of insured income can be obtained by utilizing 162.50 acres of crop land for flax, than is possible by producing wheat on 66.3 acres coupled with 96.2 acres by summer-fallow activity. With this optimal plan, $9,825.21 of expected income and $4,572.16 of insured income would be obtained. On the other hand, the expected incomes and guaranteed incomes obtainable by the optimal plan developed by the stochastic programming for high risk aversion and water prices lower than $2.25 fall in the ranges $9,891.29 to $14,576.74 and $8,205.90 to $12,816.09, respectively. With the price of water in this range, both the expected and guaranteed incomes obtainable under irrigation, are higher than those obtainable by the optimal plan coupled with crop insurance. Hence, irrigation associated with water prices lower than $2.25 per acre-inch is preferable to crop insurance as a method of farm income protection.

The second type of decision-maker under consideration is the expected income maximizer who insures all insurable crops produced to the maximum feasible amount of revenue protection. The optimal linear programming solution obtained under dryland conditions includes 35.5 acres of wheat, 62.5 acres of sugar beets, 21.78 acres of potatoes and 105.3 acres of the "feed-lot 400" activity. The maximum expected income is $10,881.40. The insurable crops included in this optimal solution are wheat, sugar beets and a feed grain (oats). The acreage of feed grain is 71.6 acres. The total insurable income due to these crops, given maximum bushel coverages and high price options, is $4,125.73. When the water price is lower than $1.50 per acre-inch, the employment of the optimal plans developed for high risk aversion and irrigated conditions would produce $11,306.94 to $14,596.74 expected income, and $9,344.52 to $12,186.09 guaranteed income. Therefore, so long as the price of water is lower than $1.50, the expected and guaranteed incomes obtainable by the optimal stochastic programming plans developed for irrigation conditions would be higher than those obtainable by linear programming with crop insurance for dryland conditions. Thus, both types of decision-makers under consideration would prefer irrigation, coupled with water prices lower than $1.50 per acre-inch, to crop insurance under dryland conditions.

IV. ANALYSIS OF DEMAND FOR IRRIGATION WATER

In the beginning of this study, it was hypothesized that the economic feasibility of irrigation and the demand for irrigation water would be increased when the income stabilizing effect of irrigation was taken into account. If this hypothesis is true, then a demand curve derived from the stochastic programming solutions would lie above that derived from the linear programming solutions. In this section, the demand curves derived from the stochastic programming problems for various levels of risk aversion are compared with the ones obtained by the linear programming method. The demand schedules derived from the stochastic and linear programming solutions are presented in Appendix Table 36. The quantities of water demanded as derived by the stochastic programming technique, are given at 50 cent intervals. In order to detect, as precisely as possible, the shifting points of the demand curves, the price of water is varied at 25 cent intervals in some parts of the demand schedules. In the case of the linear programming results, the price of water is varied parametrically and a "stepped" demand curve is derived. Because the parametric price programming technique can not be applied to a stochastic programming problem, the demand curves obtained by this programming method are not "stepped". It should be noted that the demand curves themselves obtained by the stochastic programming method are not of a stochastic nature, but of a deterministic nature. This is because none of the variables in the demand functions, such as the price of water and the quantity of demand for water, is treated as a stochastic variable. However, the stochastic nature of yields and prices of farm products is taken into consideration indirectly, in the determination of demand for irrigation water. The effect of irrigation upon the reduction of risk, being different for various levels of risk aversion, reflects upon the estimation of these demand curves. The linear programming method applied to the estimation of the demand functions for irrigation water assumed that the adjustment of production to the varied prices of water on a farm can be made under perfect knowledge of all related variables including product prices and crop yields. The stochastic programming method, however, assumes that some of these variables (ie., the product prices and crop yields) are stochastic; and that the adjustment of production to the varied water prices is made so as to maximize the producer's utility depending upon both the size of income and the certainty attached to it. All demand curves are estimated only for the 250 acre representative farm with no aggregation. The same type of demand function as applied to the static-normative demand curve estimation is fitted to each set of price-quantity data derived by the stochastic and the linear programming techniques.

Diagrammatic Analysis

Figure 16 shows the demand curves obtained for various levels of risk aversion. The predicted values are used to plot the points on the graph. Demand curve IV was derived from the linear programming solutions. The three other demand curves relate to the stochastic programming analysis: demand curve I is associated with low risk aversion; demand curve II with the high-medium risk aversion; and demand curve III, with the high risk aversion. Distinct shifts of all demand curves occur when the price of water is within the range \$2.00 to \$3.25 per acre-inch. For prices of water higher than this range, the quantities of demand for irrigation water are very small.

When the price of water is lower than \$3.25, all demand curves derived from the stochastic programming solutions lie above demand curve IV. With the price of water higher than \$3.25, however, demand curve IV lies above all the others. This indicates that consideration of the income stabilizing effect of irrigation would result in an upward shift of demand curves in the low to middle price range, and a downward shift in the high price range. In other words, sensitivity of demand to price change is increased by taking the income stability effect of irrigation water into account. Marginal response of demand to the change in the water price is noticeably large

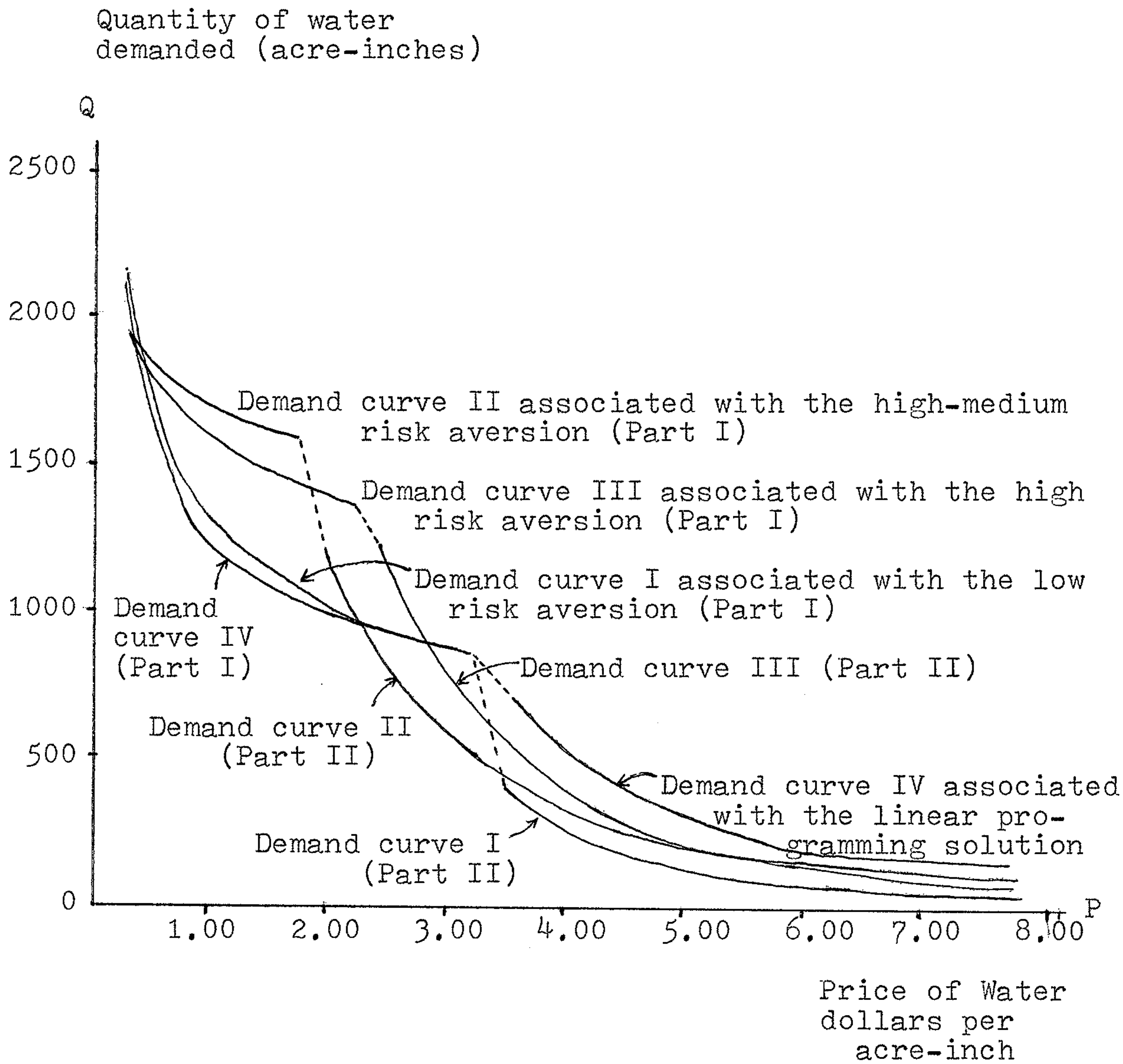

FIGURE 16

DEMAND CURVES OBTAINED FOR VARIOUS LEVELS OF RISK AVERSION
(250 ACRE REPRESENTATIVE FARM)

at high-medium risk aversion coupled with water prices in the range of \$1.75 to \$3.25, and at high aversion coupled with prices of \$2.25 to \$3.25. When the level of risk aversion is low, the marginal response in these price ranges is small. A distinct shift of demand curve I occurs at a water price of about \$3.25. If the farmers in the project area are high risk averters, then a small decrease in the water price within the range \$2.25 to \$3.25 will be responded to by a large increase in the quantity of demand for water. If they are low risk averters or expected-income maximizers, then the reduction in water price, within the range \$1.00 to \$3.25, will not induce a larger demand for water.

The differences between the quantities of demand for water projected by linear programming and those projected by stochastic programming are especially large with respect to water prices in the range \$0.75 to \$3.25. The gaps between demand curves, I, II, and III are also large in this price range. Therefore, when the price of water lies between \$0.75 and \$3.25, a change in the level of risk aversion will have a substantial effect upon the quantity of demand for water. With water prices higher than \$3.25, however, the different attitudes of decision-makers toward risk will not significantly affect this quantity. The importance of irrigation will decline for all decision-makers when the price is higher than \$3.25 per acre-inch. When the water price is about \$1.50 to \$1.75 per acre-inch, the quantity of water demanded by the high-medium to high risk averters is larger by approximately 50 percent than that demanded by the expected income maximizers. At the same price of water, even the low risk averter will demand about 10 percent more water than the expected-income maximizer. When the price of water is lower than $0.50 per acre-inch, the quantities of water demanded by various levels of risk averters and the expected-income maximizer will differ very little from each other, because, with these low prices, almost all physically irrigable land is developed for irrigation regardless of the level of risk aversion.

When the water price is lower than $0.50, approximately 1,950 to 1,980 acre-inches can be used optimally on the 250 acre farm, regardless of the level of risk aversion. In this case, almost all irrigable land (about 175 acres) is developed for irrigation. At a price of $0.75, about 1,750 acre-inches are used for irrigation at the high-medium level of risk aversion; similarly 1,670 acre-inches, are used at the high level of risk aversion; 1,450 acre-inches, at the low level of risk aversion; and 1,300 acre-inches by the expected-income maximizer. At $1.00, the gaps between the quantities of water demanded by various levels of risk averters are widened. At this price, about 1,700, 1,600, 1,320 and 1,200 acre-inches are demanded by the high-medium, high and low risk averters and the expected-income maximizer, respectively. If irrigation water is available at $1.50 per acre-inch, approximately 1,600, 1,480, 1,120 and 1,050 acre-inches will be demanded by these four classes of producers, respectively. The differences between these quantities become largest at the water price of $1.75 per acre-inch. Here, the difference between the quantities of water demanded by the high-medium, and the high risk averters is about 150 acre-inches; similarly between the high and the low risk averters it is 380 acre-inches; between the low risk averter and the expected-income maximizer, 60 acre-inches; and between the high risk averter and the expected-income maximizer, 440 acre-inches. The expected-income maximizer utilizes about 1,000 acre-inches at this water price. If we can assume that all farmers in the project area are high risk averters, then the demand for water, supplied at the prices of $0.75 to $2.50 per acre-inch, will be larger by 25 to 50 percent than those projected by the linear programming method.

Irrigated land as a proportion of the total physically irrigable land is calculated for various combinations of water prices and levels of risk aversion (Appendix Table 30-4). When the price of water is lower than $0.50, almost all irrigable land (ie., 175 acres) is developed for irrigation on the representative 250 acre farm, regardless of the level of risk aversion.

For high levels of risk aversion, 89 percent of the total irrigable land is developed for irrigation at a water price of \$0.75 per acre-inch; similarly 83 percent at \$1.00; 79 percent at \$1.50; 70 percent at \$2.00; 65 percent at \$2.50; and 48 percent at \$2.50. When the water price exceeds \$3.50, the percentages are remarkably lower (less than 25 percent). In contrast to this, the percentages of irrigated land projected by the linear programming method with the low to medium range of water prices are as follows: 80 percent at prices of \$0.67 to \$0.84; 55 percent at \$0.84 to \$1.35; and 54 percent at \$1.35 to \$3.33. At prices of water higher than \$3.50, no more than 25 percent of the total irrigable land can be optimally irrigated.

From these results we can see that for the price range of water extending from \$0.84 to \$2.25 per acre-inch, the irrigated land as projected by the stochastic programming method for the high risk averter is 20 to 44 percent more than that projected by the linear programming method.

Estimations of Demand Functions

Because of distinct shifts of all demand curves, two separate demand functions are fitted to each of demand curves I, II, III, and IV. The part of each demand curve corresponding to water prices lower than the price at the shifting point is denoted as Part I and the other, as Part II. The demand functions of various levels of risk averters are estimated as follows:

1. Demand curve I, Part I (price range, \$0.50 to \$3.25)

   $\log Q = 3.11806 - 0.36478 \log P \qquad R^2 = 0.86$

   or

   $Q = 1313.3 / P^{0.36478}$

   Price elasticity of demand = 0.36478

   Demand curve I, Part II (price range, \$3.25 to \$7.00)

   $\log Q = 4.28526 - 3.08165 \log P \qquad R^2 = 0.85$

   or

   $Q = 19287.0 / P^{3.08165}$

   Price elasticity of demand = 3.08165

2. Demand curve II, Part I (price range, \$0.50 to \$1.75)

   $\log Q = 3.2289 - 0.1255 \log P \qquad R^2 = 0.85$

   or

   $Q = 1694.0 / P^{0.1255}$

   Price elasticity of demand = 0.1255

Demand curve II, Part II (price range, \$1.75 to \$8.00)

$\log Q = 3.60851 - 1.78225 \log P \qquad R^2 = 0.92$

or

$Q = 4060.01 / P^{1.78225}$

Price elasticity of demand = 1.788225

3. Demand curve III, Part I (price range, \$0.50 to \$2.25)

$\log Q = 3.200197 - 0.176636 \log P \qquad R^2 = 0.88$

or

$Q = 1585.56 / P^{0.176636}$

Price elasticity of demand = 0.176636

Demand curve III, Part II (price range, \$2.25 to \$8.00)

$\log Q = 4.0022 - 2.3313 \log P \qquad R^2 = 0.94$

or

$Q = 10051.2 / P^{2.3313}$

Price elasticity of demand = 2.3313

4. Demand curve IV, Part I (price range, \$0.50 to \$3.25 per acre-inch)

$\log Q = 3.1035 - 0.34069 \log P \qquad R^2 = 0.75$

or

$$Q = 1269.12 / P^{0.34069}$$

Price elasticity of demand = 0.34069

<u>Demand curve IV</u>, Part II (price range, \$3.25 to \$6.10)

$$\log Q = 4.404 - 2.771 \log P \qquad R^2 = 0.73$$

or

$$Q = 25352.40 / P^{2.7706}$$

Price elasticity of demand = 2.7706.

where:

Q is the quantity of water demanded, and

P is the price of water.

As mentioned earlier, each of the estimated demand functions has a constant elasticity of Q with respect to P, (ie., a constant price elasticity of demand). Because the price elasticity of the demand curve I in Part II is larger than one, the total revenue of the water supply agency accruing from the expected-income maximizer, will be increased by reducing the price of water to a level as low as possible within the price range of Part II; ie., to \$3.25 per acre-inch. By the same token, the total revenue from the risk averters will be increased by lowering the water price to $3.25 for the low risk averter, to $2.25 for the high risk averter and to $1.75 for the high-medium risk averter. The maximum total revenues are; $2776.7 for the expected-income maximizer, $2776.66 for the low risk averter, $2763.44 for the high-medium risk averter, and $3091.41 for the high risk averter. Note that these are the values projected for the 250 acre representative farm, but not aggregated values for the entire project area.

A Criterion For the Determination of Water Price

Suppose that the aggregate demand curve for water in the project area is similar to that drawn as line DD' in Figure 17. The consumers' surplus can be increased by lowering the price of water to the level Pe which simultaneously increases the total revenue of the water supply agency. Therefore, the reduction of water price to the level Pe will benefit both parties, ie., the farmers in the project area and the water supply agency. Further reduction in the price of water will increase the consumers' surplus, but decrease the total revenue of the water supply agency. Points such as E may be considered as a kind of "Pareto optima"[1] because the welfare of the farmers

[1] "Pareto optima(lity)" or the "Pareto criterion" is defined by Pareto as; "Any change which harms no one and which makes some people better off must be considered to be an improvement"; W.J. Baumel "Economic Theory and Operation Analysis", (Prentice-Hall), 1965, p. 376.

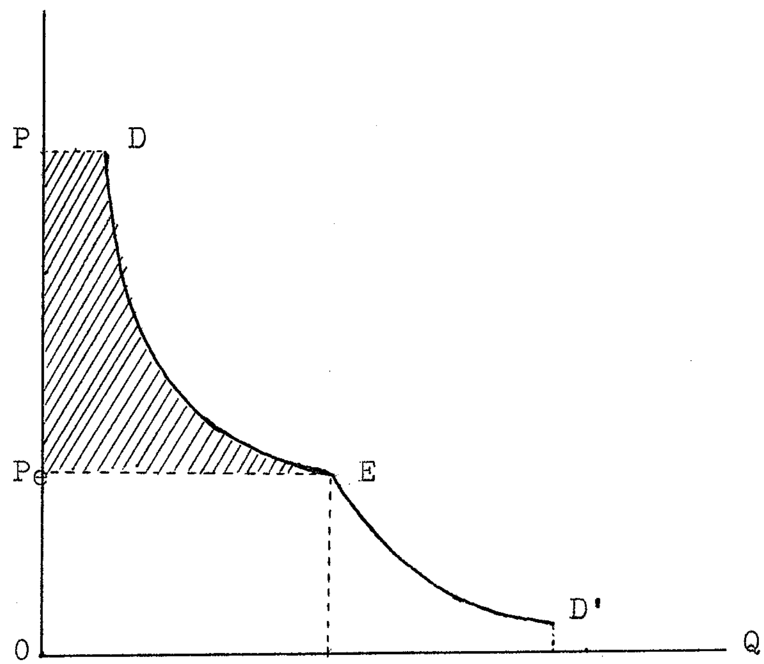

FIGURE 17

A HYPOTHETICAL AGGREGATE DEMAND CURVE DERIVED FROM THE STOCHASTIC PROGRAMMING SOLUTIONS in the area can be increased by reaching the point E, while the welfare of the water supply agency is not decreased. The point E is variable, depending upon the level of risk aversion revealed by the farmers in the area; it tends to shift downwards with respect to the water price, being associated with an increase in the quantity of demand, as the level of risk aversion rises.

If we can obtain the number of farms in the project area classified according to their level of risk aversion, then an aggregate demand function weighted by the number of farms in each size of farm having each level of risk aversion can be estimated. With the estimated aggregate demand curve, we can find the shifting point, E, at which the total aggregate revenue of the water supply agency is maximized. This type of information is especially important if the water supply agency is a non-profit-making public organization, because the estimated maximum total revenue would determine the maximum scale of the water distribution project. The maximum total cost of the project which can be repaid by the users of water should be equal to the maximum total revenue unless a part of the project cost is subsidized by the government. The maximum total cost which the agency can spend, on the basis of the estimated maximum total revenue from the users of water, will determine the scale of the project. It is recommended that, for the benefit of both parties, the price of water be determined at a level not higher than the price at the shifting point, E. Of course, the point E may not be the profit-maximizing point for the water supply agency. At least, however, the estimated maximum total revenue would provide information which will be useful for determining the scale of the water distribution project.

The study in this section indicates that an individual's demand curve for irrigation water changes significantly as the level of risk aversion varies. Therefore, a more accurate aggregate demand curve for irrigation water than the one obtained in Chapter IV can be obtained by considering, for the determination of "weights", the number of farms classified according to the level of risk aversion as well as to the size of farm.

V. AN APPROACH TO THE DETERMINATION OF "RISK AVERSION PARAMETER, q"

The studies in the preceding sections of this chapter indicate that both the optimal plans and the projected demand for irrigation water would be affected significantly by varying the levels of the risk aversion parameters along with other parameters included in the programming framework. Therefore, we must know the levels of risk aversion of the farmers in the project area, so that the applicability of these optimal plans, and the projection of demand for water, may be assessed.

A useful approach to the determination of risk aversion parameters is discussed in Chapter II (see pages57-60). This method is a "survey" method, based on the "expost determination" principle of risk aversion parameters, in which farmers are required to select a combination of guaranteed income and its associated level of probability according to their income-risk preference criteria. Unfortunately, however, no time was available to conduct this kind of survey in the project area. Instead, another approach was attempted to estimate an average level of risk aversion of the farmers on the medium farms in the area. In this approach, it is assumed that a farmer's attitude toward risk in his decision-making is revealed in the enterprise combination actually chosen by him. The cropping system adopted for his actual farm operation resulted from decision-making (associated with the implicit or explicit consideration of risk) based on his knowledge as well as past experiences concerning the income variations of various crops. This method of estimating a level of risk aversion is applicable for a particular decision-maker as well as for a group of decision-makers.

The actual cropping system in Table XXXV shows the average acreages of crop land distributed to alternative dry-land crops as observed on 158 farms of medium size (averaged to 250 acres). Accordingly the actual cropping system is derived from the aggregation of individual decisions of farmers in the project area. Therefore the level of risk aversion revealed in such a cropping system will indicate an average level for the medium sized farms in the area. The actual cropping system is compared with the optimal solutions obtained through the stochastic programming method at various levels of risk aversion (Table XXXV). Such comparisons will enable us to find an optimal solution which provides the cropping system most similar to the actual one. In order to render the optimal cropping systems comparable with the actual system, a stochastic programming problem set up for dryland conditions is solved at varied levels of risk aversion.

The actual cropping system is characterized by large acreages of small grains and flax, associated with small acreages of specialty crops (sugar beets, potatoes and others), feed grains, and forage crops. In contrast to this, the optimal cropping systems developed for the low to high medium levels of risk aversion comprise large acreages of specialty crops and feed grains, moderate acreages of wheat, and small acreages of forage crops. The optimal cropping system pertaining to a high level of risk aversion includes large acreages of sunflowers and feed grains coupled with moderate acreages of wheat, and small acreages of specialty crops (sugar beets and potatoes). All optimal cropping systems appear to be different from the actual one, but the one obtained at high levels of the risk aversion parameter is very similar. In this case,

TABLE XXXV

COMPARISON OF OPTIMAL CROPPING SYSTEMS UNDER VARIOUS LEVELS OF RISK AVERSION WITH AN ACTUAL SYSTEM

	Actual Cropping System	L.P. Solution	Level of Risk Aversion			
			Low	Medium	Hi-Medium	High
	---acres---					
Cash small grain	87.00	46.44	46.44	41.33	38.07	36.46
Feed grain for livestock	3.00	72.00	72.00	73.47	63.11	60.35
Forage crop	17.50	22.11	22.11	13.53	9.74	7.39
Flax	71.00	-----	-----	-----	-----	-----
Sunflowers	-----	-----	-------	34.17	55.29	70.77
Specialty crops and others[1]	46.00	84.28	84.28	62.50	58.79	50.03
Total	224.00	225.00	225.00	225.00	225.00	225.00
Expected income $		10,878.24	10,878.24	10,567.45	9,852.73	7,785.80
Guaranteed income $		10,878.24	10,373.05	8,807.28	7,519.04	5,983.16
Value of q		0.00	0.225	0.956	1.449	1.875
Probability		0.5	0.591	0.83	0.927	0.97
Chance of failure		1/2	1/2.4	1/6	1/13	1/33

[1]This item includes sugar beets, potatoes, buckwheat, peas, beans and summerfallow for the actual cropping system and sugar beets, potatoes and peas for the optimal cropping systems.

71 acres of flax is replaced by sunflowers and 51 acres of cash small grains by feed grains. Therefore, if:

1. the characteristics of sunflowers pertaining to risk resemble those of flax, and

2. about 50 acres out of 87 acres of cash small grains included in the actual cropping system are cash feed grain,

then we may consider this optimal cropping system as being similar to the actual one.

The variances of the net prices of sunflower and flax are small, having values similar to each other (ie., \$59.15 and \$53.90, respectively). Both are very stable crops. On the other hand, the expected net price of sunflowers (\$32.77) is higher than that of flax (\$27.91). As regards their covariances with other dryland crops, sunflowers have negative covariances with wheat, barley, oats, potatoes and the "sow-hog" activity, whereas, flax has positive covariances with all dryland activities except potatoes. Therefore, under dryland condition, sunflowers are superior to flax from the point of view of profitability and reduction of risk. Hence, with a sufficient knowledge of the characteristics of these two crops, farmers in the project area having high levels of risk aversion would prefer sunflowers to flax. It seems, however, that they have actually chosen flax rather than sunflowers. This could be because farmers pay attention only to the stability of each crop and overlook the interaction of crops for the stability of income. They might also be impressed with the high stability of flax relative to that of sunflowers. This difference can be seen from a comparison of variances between flax and sunflowers. (Note that flax has a slightly smaller variance than sunflowers).[1]

The actual cropping system included 87 acres of cash small grains, whereas, the optimal one has 36.46 acres. It must be recognized, however, that 36.46 acres of small grain in the optimal plan is all wheat, handled by the Wheat Board, while 87 acres of cash small grains includes feed grains. The question then arises, "How many acres out of 87 can be used for wheat production, assuming a specified acreage quota restriction?" Let us assume that 9 bushels of wheat per acre of specified crops can be sold to the Wheat Board. The actual cropping system includes 17.5 acres of forage crops, 3.0 acres of feed grain for livestock operations and 87 acres of wheat and other feed grains. The summerfallow activity is negligible. The average yield of wheat per acre in the project area is 25 bushels under dryland conditions. On the basis of these specified acreages how many acres of wheat can be grown? The

[1]Another important reason for the fact that farmers in this area have chosen flax rather than sunflowers is immaturity of markets for sunflowers.

total specified acreage is about 108 acres. Let X be the acreage of wheat:

Then,

$$X = \frac{108 \times 9}{25} = 38.88 \text{ acres}$$

Hence, wheat can be grown on 39 out of 87 acres. The remaining 48 acres must be used for cash feed grain production. The acreage of small grain included in the actual cropping system comparable with that in the optimal one is about 39 acres. The actual and the optimal cropping systems on a comparable basis are shown in Table XXXVI.

From the point of view of risk, these two cropping systems may be considered to be similar. At least, we may conclude that the optimal cropping system obtained by the stochastic programming method, at a high level of risk aversion parameter, is more realistic (ie., more similar to the actual one than that obtained by the linear programming method, in which sugar beets and potatoes, having high expected net prices coupled with high risks, occupy a large proportion of the total acreage). The optimal plan associated with the medium level of risk aversion combines the mixed characteristics of the linear programming, and the actual cropping systems. The farmers in the project area can be seen, through their actual cropping systems, to have a high average level of risk aversion.

TABLE XXXVI

ACTUAL AND OPTIMAL CROPPING SYSTEMS ON COMPARATIVE BASIS

	Actual Cropping System	Optimal Cropping System
	----------acres----------	
Wheat handled by the Wheat Board	39.00	36.46
Feed grains for livestock operation or sale	51.00	60.53
Flax or sunflowers	71.00	70.77
Specialty crops (other than flax and sunflower) and others	46.00	50.03
Forage crops	17.50	7.39

CHAPTER VI

SUMMARY AND CONCLUSIONS

I. SUMMARY AND SOME IMPLICATIONS OF FINDINGS

Nature and Objectives of the Study

The feasibility of building a dam on the Pembina River is being considered by the International (U.S.A.-Canada) Joint Commission. The dam site is near Walhalla, North Dakota several miles south of the Canada-U.S.A. boundary. When a dam is constructed on the river, irrigation water will be available to farms in the Morden-Winkler Irrigation Project Area. The irrigation project area lies between Winkler, and the Canada-U.S.A. boundary and includes about 71,040 acres of farm land cultivated by about 368 farmers.

The general objective of this study was to investigate the economic feasibility of on-farm use of irrigation water which will become available in the project area with completion of the dam. In this study, it is assumed that irrigation water can be purchased at the gates of the farm supply laterals connected from the main canal. More specific objectives are: (1) to provide a useful information for the irrigation development plan in the project area; and (2) to test the methodological superiority of mixed-integer programming and stochastic programming to conventional linear programming when applied to such a study.

To provide such information for the irrigation development plan, the following was studied: (1) the economic conditions under which irrigation water can be utilized so as to enhance the economic utility of farmers in the project area; (2) the optimal farm plans developed under these various conditions; and (3) the potential demand for irrigation water and the estimated amount of farm land which would be developed for irrigation.

Theoretical and Analytical Models

Two models (Model I and Model II) having different sets of assumptions are adopted for the investigation of irrigation feasibility. Model I assumed the perfect knowledge of all related variables, whereas, in Model II, risk is taken into account. In Model I, "economic feasibility" of irrigation implies the possibility of increasing farm incomes, but does not explicitly take into account the income-stabilizing effect of irrigation. Model II, however, recognizes that "economic feasibility" of irrigation is concerned not only with increasing farm incomes but also with stabilizing them. In Model II, therefore, two variables, yields and prices of farm products are assumed to be stochastic. Model I is based on methods of a mixed-integer programming and linear programming, while Model II is based on stochastic programming.

Production alternatives included in the programming frameworks for Models I and II are small grain crops, specialty crops, forage crops and livestock operation. These crops appear in the existing dryland cropping system in various proportions. The possibility of irrigation is considered for all of these crops. Some of the specialty crops and livestock operation are relatively unimportant in the existing farm organization, but the possibility of expansion under irrigation conditions is considered. Fresh vegetables are not taken into consideration because of data deficiencies.

Five levels of risk aversion are defined for the study of irrigation development in Model II. They are the high, high-medium, medium, low-medium and low levels of risk aversion. These levels of risk aversion are defined in terms of probabilities attached to various sizes of guaranteed incomes. Optimal plans are developed for these levels of risk aversion by using stochastic programming methods. A decision-maker is assumed to have his own income-risk preference criterion to which one of the alternative optimal plans corresponding to various income-risk pairs can be chosen.

High risk averters are defined as decision-makers who try to maximize incomes guaranteed at prescribed levels of probability larger than .95. High-medium, medium, low-medium and low risk averters are defined as decision-makers who try to maximize their incomes guaranteed at the prescribed levels of probability of .85 to .95, .75 to .85, .65 to .75 and .55 to .65, respectively. A decision-maker, who tries to maximize his expected income, is referred to as an expected-income maximizer and can be considered as trying to maximize his guaranteed incomes at a prescribed level of probability of .50. An income insured under the Manitoba Crop Insurance Programme is assumed to be guaranteed at a prescribed level of probability of 1.00. Given a prescribed level of probability chosen by a decision-maker, the larger the guaranteed income, the larger the utility associated with it.

Economic Evaluation of Irrigation Under Perfect Knowledge

Advantages of mixed-integer programming method. As mentioned in Chapter III, a mixed-integer programming model with the assistance of a parametric-cost programming method is applied as the main analytical tool for the study of economic feasibility of irrigation under the perfect knowledge assumption. The results are compared with those obtained from a non-integer linear programming method. Through this comparison, it is recognized that the economic feasibility conditions of irrigation are significantly affected through consideration of the possibility of purchasing specialized machines at integral units. This same consideration also significantly affects the nature of the normative demand curve for irrigation water and the maximum water price payable by farmers in the project area. Therefore, if continuity and divisibility can not be assumed for the purchasing activities of specialized machines, then a mixed integer programming model is more appropriate than the ordinary linear programming model. One of the shortcomings attributable to the mixed-integer programming model is that a parametric price or cost programming method can not be applied directly. This shortcoming, however, can be overcome to some extent by the use of a mixed-integer programming method combined with the non-integer parametric price or cost programming method.

Analysis of optimal plans under various conditions. Although changes in such conditions as prices, technical coefficients and resource constraints will affect an optimal solution, an optimal solution obtained with the assumptions of average prices of products and factors, given technical coefficients and given resource constraints is fairly stable for any variation of these conditions within certain ranges. Specialty crops (ie., sugar beets and potatoes) can be irrigated under a wide range of water prices, total holdings and capital loan limits, whereas, small grains are not irrigated profitably under some combinations.

With the price of water lower than \$0.69 and the availability of operating capital loans at the upper limit of \$10,000, flax on T_1 land can be irrigated optimally. Under irrigation conditions, the irrigated fodder corn activity, rather than the hay activity, is included in the optimal farm organization as a forage crop for beef cattle operations. Feed grains are not irrigated at any price of water, although fodder corn is irrigated at water prices lower than $2.14 per acre-inch. Barley, oats, sunflowers, and field peas can not enter the optimal solution at any combination of water prices, total holdings and operating capital loans.

When the price of hogs exceeds $27.70 per 100 pounds of carcass weight, hog enterprises begin to compete with beef cattle operations. The change in hog price does not significantly affect the demand for irrigation water, especially when the purchase of the specialized machines is considered at discrete units.

Increases in the specified acreage quota of small grains generate competition between dryland wheat and dryland feed grains, but do not significantly affect the acreage of irrigated crops and the demand for irrigation water.

Unless an operating capital loan is available, no physically-irrigable land can be profitably developed for irrigation. At least $10,000 of operating capital is necessary to make irrigation feasible. With irrigation, at least twice as much annual operating capital will be required as actually available on the 250 acre farms.

<u>Existing versus irrigation conditions</u>. Without irrigation and operating capital loans, dryland wheat holds the first priority in the use of crop land both in the present and the optimal farm organizations, but the manner of using the remaining crop land in the optimal farm organization is different from that in the present farm organization. In the present farm organization, flax occupies the place of second importance in the use of crop land, but, in the optimal case, the position of flax is taken over by two livestock activities which utilize about 70 acres of crop land (31 percent of the total crop land) for home-grown feed grains.

When irrigation is coupled with the feasibility of an annual operating capital loan, the competitive position of specialty crops (sugar beets and potatoes) in terms of profitability increases substantially, and these crops occupy a large percentage, (about 50 percent of the total crop land in the optimal farm organization. The relative competitive position of dryland wheat is lowered to some extent by irrigation. Neither dryland nor irrigated flax enter the optimal farm organization when water prices are higher than $0.89 per acre-inch. Under both dryland and irrigation conditions, beef cattle operations are major enterprises in the optimal farm organization, although they are of minor importance on present farms in the study area. Under dryland conditions, the importance of beef cattle operations increases significantly with the availability of operating capital loans.

Analysis of demand for irrigation water. Consideration of purchasing integral units of specialized machines generates a demand curve for irrigation water which is of a different nature from that obtained under the assumption of continuity and divisibility of these units. The results obtained by analysis of the static-normative demand curve for the 250 acre representative farm are summarized below. (1) The maximum price of water payable by the farmers in the project area is $2.62 per acre-inch if the purchase of the specialized machines is considered at integral units; it is $7.20 without this consideration. (2) The "stepped" demand curve for water, derived from the non-integer linear programming solutions, has a distinct shifting point at a water price of $3.34 per acre-inch. The quantity of demand for water declines substantially when the price of water exceeds this level. Between the prices $3.34 - $7.20 per acre-inch, little water is demanded, in fact a negligible amount. The maximum price of water payable by the farmers is then $3.34 per acre-inch. The sensitivity of demand to change in the price of water is higher if continuity and divisibility of purchasing units of the specialized machines are assumed. The stable price range for water, is much wider with the consideration of purchasing integral units of specialized machines than without such a consideration. The range corresponds to prices from \$0.88 to \$2.62 per acre-inch for integral units, whereas, it is from \$2.14 to \$3.34 per acre-inch when non integral units can be purchased. Within these ranges, the demand for water will not change when the price of water changes. In the former range, about 950 acre-inches are consistently used, irrigating 54 percent of the total irrigable land. A smaller quantity of water, about 800 acre-inches is used within the latter range. (45 percent of the total irrigable land can be economically irrigated.) Both demand curves have large marginal changes in the quantity of water demand in the low price ranges of \$0.69 to \$0.89, and \$0.48 to \$1.18. For the demand curve derived from the non-integer programming solutions, the marginal changes of demand are also large in the high price range of \$3.34 to \$3.86 per acre-inch.

An aggregation of individual demand schedules was obtained for the representative small, medium and large sized farms using the number of farms in each size class as weights. Two aggregate demand curves are derived, one from the non-integer linear programming solutions, and the other from the mixed-integer programming solutions.

The results of analysis based on the aggregate demand curve derived from the non-integer programming solutions are summarized below. A distinct shift of the demand curve occurs at a water price of \$3.36 per acre-inch. When the price of water is higher than this, the quantity demanded is very small (less than 70,000 acre-inches). When the price declines to the range \$3.26 to \$3.36, about 164,000 acre-inches could be profitably used. As the price of water varies downwards from \$3.36 to \$2.05, the demand increases gradually from 163,441 to 183,176 acre-inches. Approximately 195,000 acre-inches will be used consistently over the price range extending from \$2.06 to \$2.62. Marginal changes of demand are large in the low price range, \$0.48 to \$0.74. An additional 216,000 acre-inches (from 297,000 to 513,000 acre-inches) of water use could be induced by lowering the price of water from \$0.74 to \$0.47. With the price of water lower than \$0.48 per acre-inch, 97.4 percent of the total irrigable land in the project area could be profitably developed for irrigation. When the price is higher than \$3.36, only 9.10 to 26 percent of the total irrigable land can be economically developed for irrigation. With the price of water \$2.06 to \$2.62, about 52 percent of the total irrigable land will be irrigated profitably; similarly in the range \$0.74 to \$2.06, 55.7 to 75.8 percent can be irrigated; and at \$3.36 per acre-inch the corresponding figure is 40.3 percent. The price elasticity of demand for water is less than one for prices lower than \$3.36, and is larger than one in higher price ranges. Therefore, the total revenue of the water supply agency is a maximum at a water price of \$3.36 per acre-inch. A maximum annual total revenue of $555,336.50 could be received from users in the project area by selling about 166,000 acre-inches. About 40 percent of the total irrigable land could be optimally developed for irrigation at this price.

If the purchasing of specialized machines is considered at integral units, no crop can be irrigated profitably at any water price on small farms. On medium and large farms, irrigation water is used profitably over the price range extending from zero to $2.62 per acre-inch. In this case, the maximum price of water payable in the project area is $2.62 per acre-inch. The aggregate demand curve estimated by mixed-integer programming lies below that obtained by non-integer linear programming. This happens because irrigation is not profitable for small farms if they must purchase specialized machines at integral units. Approximately 180,000 to 187,700 acre-inches could be used to irrigate about 35 percent of the total irrigable land over the price range extending from $1.17 to $2.62 per acre-inch. With consideration of purchasing integral units of specialized machines, the stable range of demand for water is much wider than that obtainable without such consideration. Marginal changes of demand are large in the low price range from $0.69 to $1.17. At $0.48 reduction in the price of water is met by an increase of about 254,000 acre-inches in the quantity of demand. The demand will be more than doubled by reducing the price from $1.17 to $0.69. At prices of water lower than \$0.69, about 441,000 acre-inches will be demanded, irrigating profitably 75.7 percent of the total irrigable land in the project area. The price elasticity of demand for water is smaller than one over the entire price range. Therefore, the total revenue of the water supply agency will be maximized at a price of \$2.62 per acre-inch. The maximum total revenue is \$397,083 per annum.

Minimal increases in crop yields required for profitable irrigation. The minimal increase in the yields of major crops from dryland levels which is required to render irrigation feasible economically is significantly affected by consideration of integral purchasing units for specialized machines. The minimal required increases are larger in this case. (see Tables 10 and 11.) The minimal required increase estimated for a specified crop with the yields of the other crops held at the dryland levels is different, in every case, from when their yields were held at irrigation levels. With the consideration of simultaneous opportunities for irrigation of small grains and specialty crops, the minimal required increases of small grains become much larger than those estimated with the yields of specialty crops held at the dryland levels. This means that the competitive position of small grains relative to the specialty crops is lowered under irrigation conditions. When the specialized machines are purchased at integral units and the price of water is fixed at $2.00 per acre-inch, the yields of wheat, barley, oats, sugar beets, and potatoes must be increased by at least 33.8, 17.2, 57.2, 23.9 and 34.8 percent, respectively, from the dryland levels in order to be profitably irrigated, assuming the other crops are grown under dryland conditions. If the possibility of irrigation is considered simultaneously for all crops (at initial assumed yields), then the yields of wheat, barley and oats must be increased by at least, 85, 86 and 137 percent, respectively, from the dryland levels so as to be irrigated profitably. The initial yields of wheat, barley and oats assumed for irrigation conditions, respectively, are higher than the dryland levels by 60.7, 22.2 and 33.3 percent. Hence, small grains have little chance of being irrigated profitably in competition with the irrigated specialty crops and fodder corn used by cattle. These minimal required yields vary as the price of water changes. If the price of water is lower than $0.69, wheat can be irrigated without much decrease in the optimal functional value, but other small grains and flax can not.

Optimal investment in specialized machines. When considering the purchase of integral units of the specialized machines, only two combinations of the specialized machines appear in the optimal solutions with varied prices of water.

When the price of water is lower than $2.62, one unit each of irrigation machines, sugar beet thinner, potato digger and potato seed cutter is purchased. When the price of water is higher than $2.62, the optimal investment in the specialized machines includes only one unit of sugar beet thinner. A sugar beet harvester may be purchased without a very large amount of loss in the functional value.

Marginal values of irrigation water. The value of irrigation water per acre-inch near the middle point (957 acre-inches) of the range of water demand, varies from $0.90 to $2.50. Given these prices of water, about 54 percent of the total irrigable land can be economically irrigated.

Economic Evaluations of Irrigation Under Imperfect Knowledge

Optimal plans under various risk aversion levels and water prices. In the first section of Chapter IV, the impact of varied levels of risk aversion and price of water upon the optimal solutions is investigated. The findings in that section are summarized below.

When there is the possibility of irrigation, the optimal solutions are not sensitive to change from the medium to high levels of risk aversion. The optimal solutions obtained are essentially the same for these levels of risk aversion. However, the optimal solutions pertaining to the low to low-medium levels of risk aversion differ significantly from those mentioned above.

Under the assumption of medium to high levels of risk aversion and the low to medium range of water prices, flax and feed grains used for the "sow-hog" activity are irrigated, occupying a large proportion of the total crop land (32 to 50 percent). Medium acreages of sugar beets and potatoes are also irrigated under these risk and water price assumptions. Small to medium acreages of sunflowers, sugar beets and wheat, all under dryland conditions, are also included in the optimal solutions. When the price of water is in the high range, irrigated activities no longer occupy a major acreage; sugar beets, the "feed-lot 400 (D)" or "sow-hog(D)" activity and sunflowers, all under dryland conditions, are more significant.

Given the low to low-medium levels of risk aversion and the low to medium range of water prices, the optimal solutions include a large acreages of irrigated sugar beets and the "feed-lot 400 (D)" activity or the "sow-hog (I)" activity. Medium acreages of flax, potatoes and the part of the feed grain reserved for the "sow-hog" activity, all irrigated, are also included in these optimal solutions. If the water price is high, then the optimal solution is highly concentrated on the "feed-lot 400 (D)" activity and dryland sugar beets.

Wheat is not irrigated under any combination of risk aversion and water price, but enters all optimal solutions at a moderate acreage.

Under dryland conditions, optimal solutions show more sensitive responses to change in the level of risk aversion.

If risk is taken into account in the development of optimal farm plans, not only expected net prices but also variances and covariances of alternative activities will have an impact upon the optimal solutions. When the level of risk aversion is low, variances and covariances do not play important roles for the determination of optimal activity combinations, but they are important at high levels of risk aversion. The importance of these elements is demonstrated by relatively small variances and large negative covariances of the important activities appearing in optimal solutions obtained under high levels of risk aversion.

Under both dryland and irrigation conditions, the number of activities entering optimal solutions increases as the level of risk aversion rises. This indicates that diversification is effective in reducing risk when the enterprises are combined properly. The expected and guaranteed incomes obtainable by a single enterprise operation are compared with those from a mixed-enterprise operation. If there are a specified acreage quota of nine bushels per acre on wheat sales and restrictions on the repeated use of crop land for sugar beets, under both irrigation and dryland conditions, a mixed-enterprise operation produces larger expected and guaranteed incomes than any single enterprise operation. When irrigation water is available, the advantage of a mixed-enterprise operation increases.

Comparison of expected and guaranteed incomes between dryland and irrigation conditions shows that for all levels of risk aversion and for the maximizer of expected income, these incomes obtainable under irrigation conditions are higher than those under dryland. Differences in these incomes between dryland and irrigation conditions are particularly large for the low and the high risk averter. This means that irrigation would increase the utilities of the two extreme types of risk averters more than those of medium risk averters. The increase in utility due to irrigation will be largest for the highest level risk averter. Furthermore, the comparison of optimal plans between the two conditions indicates that the importance of flax and the "sow-hog" activity increases significantly with irrigation for all levels of risk aversion, except the low and the low-medium.

Various levels of risk aversion can be interpreted in terms of average discounting rates of expected incomes for the associated standard deviations of income. The discounting rates calculated for various levels of risk aversion are high in proportion with the level of risk aversion, and they are lower under irrigation conditions than under dryland conditions.

This follows from an analysis of the average discounting rates for one dollar of increased standard deviation of income. The utility possibility curves are also derived for dryland and irrigation conditions. A higher level of expected utility is attainable under irrigation conditions than under dryland conditions. This is indicated by the fact that the utility possibility curve for irrigation conditions lies above that for dryland conditions.

Comparison of stochastic with linear programming solutions. A comparison of the optimal stochastic programming solutions with the linear programming ones yields the conclusions described as follows.

Under dryland conditions:

1. For a low level of risk aversion, the optimal solutions for the stochastic and linear programming analyses are identical with each other, both of them including sugar beets and beef cattle operations (the "feed-lot 400" activity) as major enterprises.
2. For the low-medium level of risk aversion, major activities are the same in both optimal solutions, but minor activities are different.
3. When the level of risk aversion is medium, these two solutions are remarkably different from each other. The level of the "feed-lot 400" activity included in the stochastic

programming solution is significantly lower than that in the linear programming solution, while sunflowers and the "sow-hog" activity are included in the stochastic solution, but not in the linear programming.

4. Under a high level of risk aversion, sunflowers are the only major activity in the stochastic programming solution and the importance of sugar beets is much less than in the linear programming solution. The optimal linear programming solution has fewer activities than does the stochastic solution. In the former 67 percent of the total crop land is used exclusively for the two major activities, while, in the latter, 28 percent is used for the major activity, the remaining crop land being distributed to four crops in equal proportions.

Under irrigation conditions:

1. When the level of risk aversion is low and the price of water is about $2.00 per acre-inch, the optimal solutions for the stochastic and linear programming analyses are identical with each other. However, these two optimal solutions show different patterns of response to change in the price of water.

2. Given a high level of risk aversion, the optimal stochastic programming solutions are substantially different from those of the linear programming analysis. The former is more diversified than the latter. At a water price of $2.00, about 71 percent of the total crop land is devoted solely to two activities in the linear programming analysis (dryland sugar

beets and the "feed-lot 400 (D)" activity), while 42 percent is used for the irrigated flax and the "sow-hog (I)" activities in the stochastic analysis. The expected income obtained by the linear programming solution is higher than that from the stochastic programming solution, but the standard deviation of income is also high in the former.

Under both dryland and irrigation conditions, the expected utilities derived from the optimal linear programming solutions for various levels of risk aversion are lower than those derived from the stochastic programming solutions. In other words, all risk averters can reach higher level utility indifference curves by choosing optimal plans developed by stochastic programming, than by choosing those given by the linear programming method. Every level of risk aversion shows a larger average discounting rate of expected income for a one dollar increase in the standard deviation of income for the linear programming optimal plan than for the stochastic programming plan. This means that a producer would be more cautious with employment of the former plan than with the latter. Under irrigation, the expected utility derived from the optimal linear programming solution for the high level risk aversion is negative.

Under both dryland and irrigation conditions, these two optimal plans differ significantly from one another, especially when they are developed for the medium to high levels of risk aversion. If the farmers in the project area were to have these levels of risk aversion, then the linear programming method would not provide adequate information for them on their farm planning. Without consideration of risk (ie., in the linear programming approach), field peas, sunflowers, and the "sow-hog" activity do not enter any optimal plan, and flax enter only at very low prices of water. Taking risk into consideration, however (ie., in the stochastic programming approach), these activities in many cases are found in optimal plans, both under dryland and irrigation conditions, especially for medium to high levels of risk aversion. In the linear programming approach, the competitive position of sugar beets and potatoes is considerably enhanced by irrigation, whereas, in the stochastic programming approach, that of flax and the "sow-hog" activity is increased substantially by irrigation.

Irrigation versus all-risk crop insurance. Section 3 of Chapter V deals with the comparison of all-risk crop insurance to irrigation as a method of farm income protection. Two types of decision-makers are assumed to exist; one who utilizes crop insurance in the most conservative manner and the other, in the most aggressive manner. Then, the expected and insured incomes attainable by the linear programming optimal plans coupled with crop insurance under dryland conditions, are compared with the expected and guaranteed incomes obtainable by the stochastic programming analysis under irrigation and the assumption of high risk aversion.

With the price of water lower than $1.50 per acre-inch, the linear programming paired incomes are lower for both types of decision makers than are the stochastic paired incomes. Therefore, the two types of decision-makers will prefer irrigation, associated with these water prices as a method of farm income protection, over crop insurance designed for dryland conditions.

Analysis of demand for irrigation water. The results of the analysis of demand for irrigation water, projected under various levels of risk aversion are discussed in Section 4 of Chapter V. In the beginning of this study, it was hypothesized that the economic feasibility of irrigation and the demand for water would be increased by taking into account the income stabilization effect of irrigation. If so, a demand curve derived from the stochastic programming solutions should lie above those for linear programming. In order to test this hypothesis, the demand curve derived from the linear programming solutions is compared with those derived from the stochastic programming solutions under various levels of risk aversion.

Distinct shifts of all demand curves occur at prices from $2.00 to $3.25 per acre-inch. If the price is higher than this range, the demand for water becomes very small. When the price of water is lower than $3.25, all demand curves derived from the stochastic programming solutions lie above the one from the linear programming solution. When the price is higher than $3.25, the positions of these demand curves are reversed. In the low to medium price range, every risk averter utilizes much larger quantities of water than does the expected-income maximizer, while in the high price range, the latter uses larger quantities than the former. From this consideration, it is also recognized that the risk averters are more sensitive to change in the price of water than is the expected-income maximizer. When the price is lower than $0.75, the same quantities of water are used by all risk averters and by the expected-income maximizer. This occurs because, at such low prices, all irrigable land can be economically developed for irrigation, regardless of the level of risk aversion. The gaps between the demand curves are especially large in the price range, $0.75 to $3.25. Therefore, when the price of water lies in this range, a change in the level of risk aversion will affect significantly the demand for water. Given prices of $1.00, $1.50 and $2.00 per acre-inch, the high risk averter utilizes, respectively, about 1,590, 1,480 and 1,400 acre-inches, while the expected-income maximizer uses about 1,150, 1,050 and 950 acre-inches. With these prices, the high risk averter will develop 80, 79 and 70 percent of the total irrigable land for irrigation, whereas the expected-income maximizer use 55, 54 or 54 percent. At water prices from \$0.84 to \$2.25, the former will irrigate acreages of land from 20 to 40 percent larger than the latter.

The estimated demand functions indicate that the upper sections of all demand curves have constant price elasticities of demand smaller than one while those of the lower sections are larger than one. Therefore, the total revenue of the water supply agency accruing from the various levels of risk aversion of producers, can be increased by reducing the price of water to the lowest levels in the upper section of the curves (ie., \$3.25 per acre-inch for the low risk averter as well as for the expected-income maximizer, \$1.75 for the high-medium risk averter and \$2.25 for the high risk averter). If we can obtain the numbers of farms classified according to level of risk aversion as well as by size of farm, then an aggregate demand function weighted by these numbers can be estimated. With the estimated aggregate demand curve, we can find a shifting point, E, at which the total revenue of the water supply agency is maximized. By reducing the water price to the level, Pe, at the shifting point, both the farmers in the project area and the water supply agency will be benefitted simultaneously. If the water supply agency is a non-profit-making public organization, this sort of information will be useful, because a projected maximum total revenue will provide information for the determination of the optimal scale of the water supply project. The demand for water is affected significantly by varied levels of risk aversion. Therefore, if we use the numbers of farms classified by the level of risk aversion as well as by the size of farm, as weights in aggregation, we will be able to obtain a more accurate aggregate demand curve than is obtainable by the linear programming method.

Determination of risk aversion parameter, q. The last section of Chapter V is devoted to an approach to estimate the risk aversion parameter, q. One useful method is the application of the "expost determination" principle of the parameter as discussed in Chapter II. Because of time limitation, however, no interviews could be conducted with the farmers in the project area for this purpose. Instead, another approach is attempted here. Under the assumption that the risk aversions of farmers are revealed in their actual choices of enterprise combinations, the alternative optimal cropping systems obtained for various levels of risk aversion parameters are compared with the actual ones. It is found that the optimal cropping system pertaining to a high level of risk aversion is essentially similar to the actual one. From this, it is inferred that the farmers in the project area have on the average a high level of risk aversion.

II. SOME RECOMMENDATIONS FOR POLICY

On the basis of this study, the following may be recommended to the policy makers who are concerned with the irrigation project.

1. The farmers in the project area may be recommended to place highest priority on irrigation of specialty crops, including sugar beets, potatoes and flax, if irrigation becomes available at low to medium water prices. If livestock operations are developed under irrigation conditions, then fodder corn for the feed-lot operation and feed grain for the "sow-hog" activity may also be irrigated. However, none of the cash small grains can be recommended for profitable irrigation except at very low water prices.

2. Development of livestock operation should be promoted along with the development of irrigation.

3. In order to render irrigation feasible, not only irrigation development capital loans, but also opportunities for getting operating capital loans should be given to the farmers in the project area.

4. In order that the farms of small size can utilize irrigation profitably, it is recommended that the water supply agency provides the services of the specialized machines at low rates, not higher than the costs of depreciation, maintenance

and operation of the machines. This follows from the fact that irrigation would become economically feasible even on small farms if infinitesimal divisibility of purchasing units can be assumed for the purchases of specialized machines. In other words, if the services of these machines which are required in the optimal solutions for small farms are available at the costs of depreciation, maintenance and operations of the machines, then irrigation would become economically feasible for these farms.

5. The price of water should be set at a level not higher than the price at the shifting point of the demand curve where the total revenue of the water supply agency becomes a maximum.

According to the aggregate demand function derived from the mixed-integer linear programming solutions, the maximum price of water payable by the farmers in the project area is $2.62 per acre-inch. This price coincides with the price at which the projected total revenue of the water supply agency becomes a maximum. If an aggregate demand function were derived from the stochastic programming solutions, the price at the shifting point of the demand curve would be lower than this, say, $2.00 to $2.25 per acre-inch. A feasible scale of the water supply project may be determined on the basis of the annual maximum total revenues from the users, as projected by these methods.

6. In order to develop economically all irrigable land for irrigation, the price of water should be lower than $0.69 per acre-inch.

III. SUGGESTIONS FOR FURTHER STUDIES

1. Because of data deficiency, irrigated peas, beans and buckwheat as well as fresh vegetables are excluded from this study. A further study including these crops, especially

fresh vegetable crops, should be undertaken because fresh vegetables have been relatively profitable in other irrigation areas.

2. As mentioned in earlier chapters, the value of the risk aversion parameter plays a significant role in the development of optimal plans under risk consideration. One of the useful methods for determination of these values is the "expost-determination" method which is discussed in Chapter II. Due to time limitation, however, no interview could be conducted with farmers in the project area to investigate what income-risk combinations they choose. An income-risk preference table or income-risk preference statements as suggested in Chapter II may be utilized to find the values of the risk aversion parameter revealed by farmers in the project area. These values may vary depending upon the size of farm, age, education, type of farm, etc. It might also be revealing to examine a correlation between the values of the risk aversion parameter actually chosen by farmers and the corresponding debt/equity ratios.

3. An aggregate demand function for irrigation water, weighted by numbers of farms classified according to levels of risk aversion as well as to farm sizes, should be derived from stochastic programming solutions. For this purpose, all farms in the project area must be classified by risk aversion levels.

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APPENDIX

APPENDIX I

SOLUTION OF A MIXED-INTEGER PROGRAMMING PROBLEM

Suppose that our objective is to solve a mixed-integer programming problem including n integer variables and h-n non-integer variables. The theoretical procedure of solving a mixed-integer programming problem is as follows:

Step 1.

Firstly, an optimal solution must be obtained treating all integer variables as non-integer. The optimal functional value is denoted by Z^o. A decision is made to constrain to an integral value one of the integer variables, say, X_1 which is in the optimal basis at a non-integral level, X_1^o. With X_1 forced into the optimal basis at levels of the maximum integral value smaller than X_1^o and the minimum integral value larger than X_1^o, respectively, optimal solutions of the problem are obtained. The maximum integral value of X_1 is denoted by $[X_1^o]^*$ and the minimum integral value, by $[X_1^o]^* + 1$. The functional values of these two optimal solutions are denoted by $Z_1([X_1^o]^*)$ and $Z_1([X_1^o]^* + 1)$. At this stage, it is convenient for the next step to obtain two other optimal solutions containing X_1 at $[X_1^o]^* - 1$ and $[X_1^o]^* + 2$, respectively. Their optimal functionals are denoted by $Z_1([X_1^o]^* - 1)$ and $Z_1([X_1^o]^* + 2)$. If $[X_1^o]^*$ is zero, then we do not need calculate of $Z_1([X_1^o]^*-1)$.

The two optimal functionals, $Z_1([X_1^o]^*)$ and $Z_1([X_1^o]^* + 1)$ are compared with each other and the integral level of X_1 producing a larger functional value is selected. If $Z_1([X_1^o]^*) > Z_1([X_1^o]^*+1)$, then X_1 is forced into the optimal basis at the integral level, $[X_1^o]^*$, in the next step. The variable X_1 can be forced into the basis by using an integer constraint equation, $X_1 = [X_1^o]^*$.

In each step, the results of the calculation may be diagrammatized in a treee graph whose branches indicate comparative levels of functional values corresponding to selected integral values. In Step 1 the tree graph would be constructed as in Figure 18.

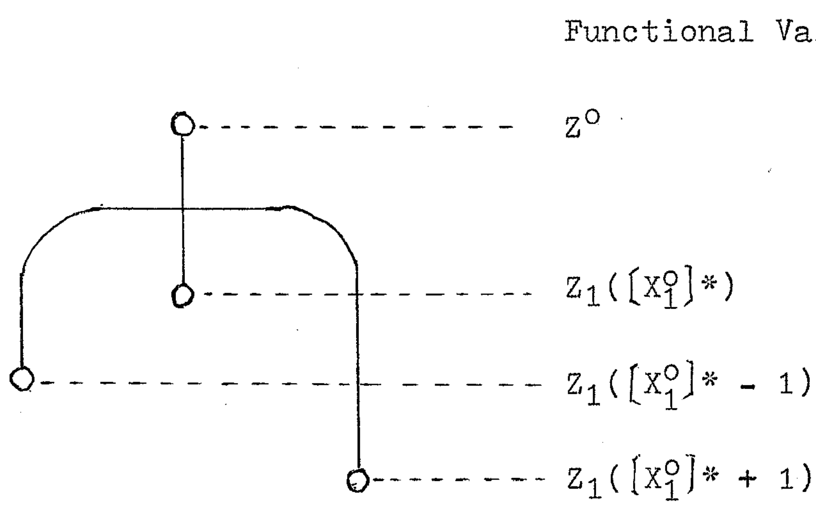


FIGURE 18

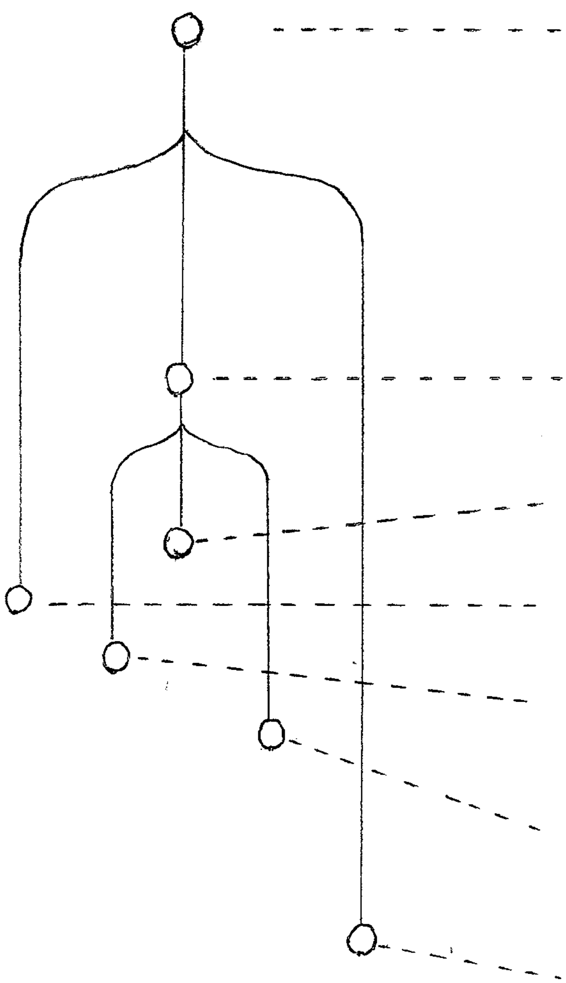
TREE GRAPH FOR STEP 1

The positions of small circles in the tree graph indicate the comparative levels of functionals derived from optimal solutions including the specified integer variables at various integral values.

Step 2.

The second integral variable, say, X_2 is selected and forced to take alternative integral values. The non-integral level of X_2 entering the optimal solution associated with the integer variable, X_1, fixed at $([X_1^o]^*)$ is denoted by X_2^1. The maximum integer smaller than X_2^1 and the minimum integer larger than X_2^1 are denoted by $[X_2^1]^*$ and $[X_2^1]^* + 1$. Given X_2 at $[X_2^1]^*$ and $[X_2^1]^* + 1$, respectively, optimal solutions of the mixed-integer programming problem are derived and their functionals are denoted by $Z_{1,2}([X_2^1]^*)$ and $Z_{1,2}([X_2^1]^* + 1)$. If $Z_{1,2}([X_2^1]^* + 1) < Z_{1,2}([X_2^1]^*)$, then X_2 is forced into the optimal basis at the integral level, $[X_2^1]^*$, in the 3rd step, holding X_1 at $[X_1^o]^*$. In this step, $Z_{1,2}([X_2^1]^*)$ must be compared with $Z_1([X_1^o]^* + 1)$ and $Z_1([X_1^o]^* - 1)$. If $Z_{1,2}([X_2^1]^*)$ is larger than these two functionals, then we may proceed to Step 3.

Integer variables, X_1 and X_2 are forced into the basis at $[X_1^o]^*$ and $[X_2^1]^*$ in the next step. At this stage, the tree graph is constructed as follows:



Functionals	Levels of Integer Variables
Z^o	All variables are non-integers.
$Z_1([X_1^o]^*)$	$X_1 = [X_1^o]^*$ Other variables are non-integer.
$Z_{12}([X_2^1]^*)$	$X_1 = [X_1^o]^*;\ X_2 = [X_2^1]^*$ Other variables are non-integers.
$Z_1([X_1^o]^*-1)$	$X_1 = [X_1^o]^* - 1$ Other variables are non-integers.
$Z_{12}([X_2^1]^*-1)$	$X_1 = [X_1^o]^*;\ X_2 = [X_2^1]^*-1$ Other variables are non-integer.
$Z_{12}([X_2^1]^*-1)$	$X_1 = [X_1^o]^*;\ X_2 = [X_2^1]^*-1$ Other variables are non-integer.
$Z_1([X_1^o]^*-1)$	$X_1 = [X_1^o]^* -1$ Other variables are non-integer.

FIGURE 19

TREE GRAPH IN STEP 2

NOTE: The levels of small circles on the tree indicate the comparative levels of the functionals for those optimal solutions.

The same procedure as step 1 and 2 is repeated until no integer variable having a fractional value is found in an optimal basis. At this point, the optimal solution obtained in the last step provides us with the optimal solution of the mixed-integer programming problem. The integer levels at which the selected integer variables are forced into the optimal basis in the last step are the optimal integral levels of these variables. All the other integer variables have zero levels.

If in the k th step a selected functional, $Z_{1,2---k}$ $(\{X_k^{1,2,--k-1}\}^*)$, is smaller than one of those calculated in the preceding step, then the procedure must be repeated, starting from the optimal solution having that larger functional.

APPENDIX II

SOLUTION OF A STOCHASTIC PROGRAMMING PROBLEM

A stochastic problem, which will be referred to as Problem I, can be formulated in a general, matrix form as;

$$\text{(II - 1)} \quad \text{Max.} \dashrightarrow Z_{*1} = \bar{C}'X - q(X' \cdot V \cdot X)^{\frac{1}{2}}$$

Subject to:

$$P.X \leq B$$
$$X \geq 0$$

where:

Z_{*1} is an income guaranteed at a prescribed level of probability,

$\bar{C}$ is a column vector of expected net prices,

X is a column vector of activity levels,

V is a matrix of variances and covariances of net prices,

P is a matrix of technical coefficients,

B is a column vector of right-hand-side elements of constraint inequalities, and

q is a constant which indicates a risk preference coefficient.

A subsidiary quadratic programming problem of Problem I is formulated as;

(II - 2) Max. $\dashrightarrow$ $Z_{*2} = \bar{C}' \cdot X - \frac{q}{R} X' \cdot V \cdot X$

Subject to

$$P \cdot X \leq B$$
$$X \geq 0$$

where R is a positive constant and equal to $\sqrt{X^*(R)' \cdot V \cdot X^*(R)}$ with a given solution vector X*. This problem is referred as Problem II.

Suppose that an optimal solution vector, X*, is given for Problem II and the condition:

(II - 3) $$R = \sqrt{X^*(R) \cdot V \cdot X^*(R)}$$

is satisfied for the given R. Then, we can substitute R by $\sqrt{X^*(R)' \cdot V \cdot X^*(R)}$ in the maximized objective function of Problem II. For computational convenience, $\frac{q}{R} X' \cdot V \cdot X$ is multiplied by $\frac{1}{2}$. Then, we have the maximized objective function as;

(II - 4) $$Z^*_{*2} = \bar{C}' \cdot X^* - \frac{q}{2\sqrt{X^*(R)' \cdot V \cdot X^*(R)}} \cdot X^*(R) \cdot V \cdot X^*(R)$$

Multiplying $$\frac{q}{2\sqrt{X^*(R)' \cdot V \cdot X^*(R)}} \cdot X^*(R) \cdot V \cdot X^*(R)$$

by $$\frac{\sqrt{X^*(R)' \cdot V \cdot X^*(R)}}{\sqrt{X^*(R)' \cdot V \cdot X^*(R)}}$$

we can obtain

(II - 5) $$Z^*_{*2} = \bar{C}' \cdot X^* - \frac{q}{2} \sqrt{X^*(R)' \cdot V \cdot X^*(R)}$$

It is obvious that if equation (II - 5) is maximized, then

$$\bar{C}'\cdot X^* - q\sqrt{X^*(R)'\cdot V\cdot X^*(R)}$$

is also maximum.

Thus, if we can find an optimal solution for Problem II, and $R = \sqrt{X^*(R)'\cdot V\cdot X^*(R)}$ is satisfied with respect to the given R, then the optimal solution of Problem II is also the optimal solution of Problem I.

Now, consider

$$\text{(II - 6)} \qquad r = \sqrt{X^*(R)'\cdot V\cdot X^*(R)}$$

If R changes, then r also changes because the solution vector X* varies as R changes. Therefore, we may consider that r is a function of R. We can write

$$\text{(II - 7)} \qquad r = r(R) = \sqrt{X^*(R)'\cdot V\cdot X^*(R)}$$

where R is variable.

We can diagrammatize this functional relationship as in Figure 20.

If q changes, the r(R) curve will shift. The vertical axis measures r(R) values and the horizontal axis, R values. The 45 degree line passing through the origin shows all possible points where r = R is satisfied. A hypothetical r(R) curve is drawn by line, OB. Point, E, indicates the equilibrium point where r(R) equals R on a given r(R) curve. At point, E,

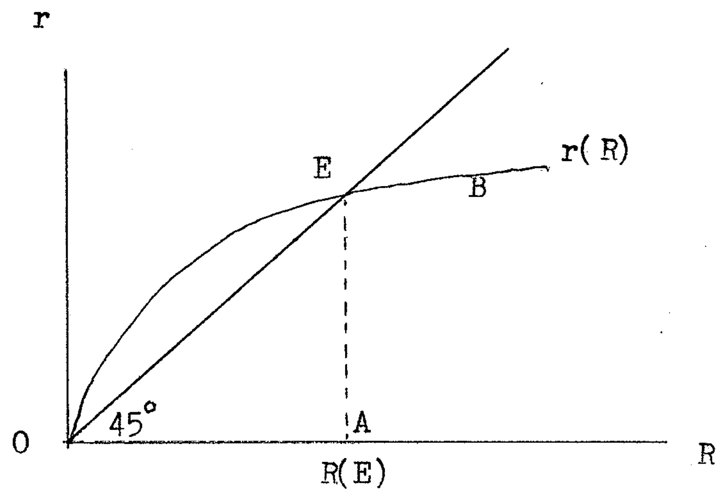


FIGURE 20

HYPOTHETICAL r(R) CURVE

OA = AE. This value of R is denoted by R(E). We can not know exactly what the r(R) curve looks like. We can know, however, some characteristics of the r(R) function,

1.

(II - 8) $$\lim_{R \to \infty} r(R) = \sqrt{X_0^{*\prime} \cdot V \cdot X_0^*}$$

where X_0^* is the optimal solution vector for a conventional linear programming problem having the same sets of activities, expected net prices, constraint inequalities and resource limitations as Problem I. If R approaches infinity in the objective function of Problem II, then $\frac{q}{2R}$ approaches zero and Z_{*2}^* approaches $\bar{C}' \cdot X$. Therefore, an optimal solution for

Problem II obtained as R approaches infinity becomes identical with the optimal solution of the linear programming problem. By substituting X_o^* into the r(R) function, we can calculate:

$$r_o = \sqrt{X_o^{*\prime} \cdot V \cdot X_o^*} \qquad \text{(II - 9)}$$

2. The function, r(R), is monotone and non-decreasing in R and has a finite limit, r_o.

3. If the slope of the r(R) curve is greater than one for a sufficiently small R, then Problem I has a non-zero solution. This condition is satisfied if the r(R) curve lies above the 45 degree line for a range of sufficiently small R's.

4. The value of R which is equal to r_o is denoted by R_o. On the diagram R_o can be found at the intersection of the r_o line and the 45 degree line (see Figure 21).

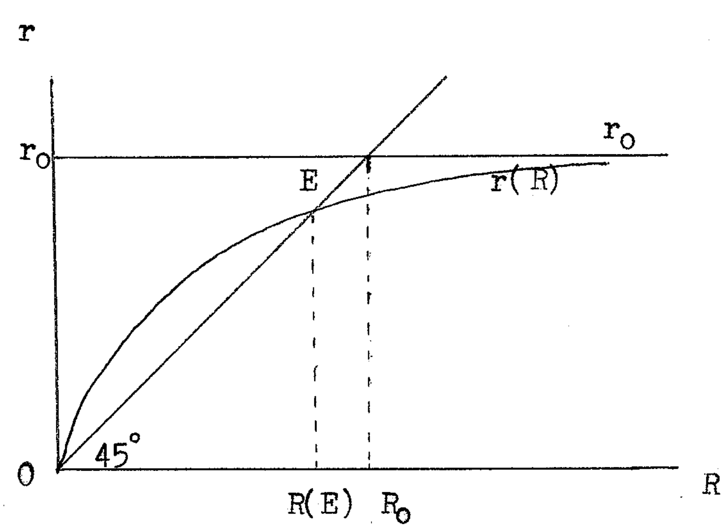


FIGURE 21

AN ILLUSTRATION OF $R(E) < R_o$

Because the $r(R)$ curve approaches the r_o line when R approaches infinity, and the $r(R)$ curve is a monotonically non-decreasing, the $r(R)$ curve should lie below the r_o line when R equals R_o. The equilibrium point E and $R(E)$ must exist somewhere to the left of R_o. In other words, $R(E)$ should be smaller than $R_o(=r_o)$.

On the basis of these characteristics of the $r(R)$ function, a stochastic programming problem formulated as Problem I can be solved through solution of its subsidiary quadratic programming problem. The procedure is as follows;

Step 1.

Start by solving its linear programming problem

$$\text{Max.} \dashrightarrow \bar{C}' \cdot X$$

Subject to

$$P \cdot X \leq B$$
$$X \geq 0$$

and derive an initial value of R, ie., R_o, from $\sqrt{X_o^{*\prime} \cdot V \cdot X_o^*}$ and store it in R.

Step 2.

Using R_o obtained in Step 1, solve Problem II. If $\sqrt{X^*(R_o)' \cdot V \cdot X^*(R_o)}$ is equal to R_o, then the solution vector $X^*(R_o)$ is the optimal solution for Problem I. If $r(R_o) < R_o$, then store the computed $r(R_o)$ in R and iterate step 2. Repeat the procedure until $r(R) = R$ is found.

APPENDIX III

AMORTIZATION OF AN INITIAL DEVELOPMENT COST

Suppose that a large amount of investment in capital equipment or the development of a fixed resource such as land is required in the beginning of a planning period. How can we deal with a cost like this in a static or stationary model? The initial cost of investment should be converted to a constant annual cost in some way. Annual costs of machinery and buildings having limited life spans can be calculated by depreciation. Land developed for irrigation, however, has a nearly permanent life-span, and does not need to be depreciated. In such a case, amortization is a useful method by which an initial development cost and its compound interest can be converted to a constant annual cost.

Now, assume that a total development cost, A, is repaid annually by a constant amount, R, in n years. If R is not repaid in the t th year, then the sum of principal and compound interest of that amount, R, in the n th year will be:

$$\text{(III - 1)} \qquad R(1 + i)^{n-t}$$

where i is the interest rate. For example, if t is the n-1 th year, then the sum of principal and compound interest of R in the n th year is given by:

$$\text{(III - 2)} \qquad R(1 + i)^{n-(n-1)} = R(1 + i)$$

The sum of principal and compound interest can be calculated for all R's of n years. The grand total of principal and compound interest is given by

$$(\mathrm{III} - 3) \quad R(1+i)^{n-1}+R(1+i)^{n-2}+ \cdots\cdots\cdots\cdots R(1+i)+R=R\cdot\frac{(1+i)^n-1}{i}$$

This total should equal the sum of principal and compound interest of the initial development cost, A, which is repaid after n years. Thus, we have:

$$(\mathrm{III} - 4) \quad A(1+i)^n = R\cdot \frac{(1+i)^n-1}{i}$$

Therefore,

$$(\mathrm{III} - 5) \quad R = A(1+i)^n \cdot \frac{i}{(1+i)^n-1} = \frac{A(1+i)^n\cdot i}{(1+i)^n-1} = \frac{A\cdot i}{1-\frac{1}{(1+i)^n}}$$

where R is the annual amortized value of the initial development cost, A, based on n years of repayment period.

If n approaches infinity, then

$$R \dashrightarrow A\cdot i$$

as $\quad n \dashrightarrow \infty$

APPENDIX IV

SOLUTION OF AN ACTUAL MIXED-INTEGER PROGRAMMING PROBLEM

The theoretical procedure of solving an integer programming problem was presented in Chapter II and Appendix I. The same method is applied for the study of actual problem in the irrigation project area. In this example, the mixed-integer programming problem includes seven integer variables, X_{129} to X_{135}.

In the first step, an optimal non-integer solution is obtained for the problem without setting integer constraints on these variables. All levels of integer variables are given at fractional values or zeros in the optimal basis. In this example, they are;

		Levels
one set of irrigation machines	, X_{135}	0.364
sugar beet thinner	, X_{131}	0.480
potato digger	, X_{132}	0.770
seed potato cutter	, X_{134}	0.130
sugar beet harvester	, X_{133}	0.000
hay baler	, X_{130}	0.000
forage harvester	, X_{129}	0.000

The result reveals that purchases of a sugar beet harvester, a hay baler and a forage harvester are not profitable on this size of farm even though their purchases are considered at fractional units. Other machines are purchased at levels less than one.

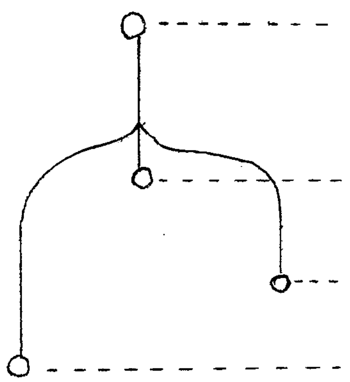
In the second step, one of the machines entering the optimal basis at non-zero levels is selected and constrained to integral levels. Two integral levels of the machine are specified at the maximum integral value smaller than its non-integral level and the minimum integral value larger than that, respectively. In this case, X_{135} having its fractional level, 0.364, in the optimal basis is selected firstly. The maximum integer less than 0.364 is zero and the minimum integer greater than that is one. Therefore, two optimal solutions are obtained for the mixed-integer programming problems including X_{135} at zero and one, respectively. The two functionals are compared to each other and the larger one is selected. An optimal solution is also derived from the problem containing X_{135} at two units. The functionals of these three alternative solutions are as follows;

	Functionals
1. $X_{135} = 0$	$13,670.08
All the other variables are non-integers.	
2. $X_{135} = 1$	$14,798.05*
All the other variables are non-integers.	

	Functionals
3. $X_{135} = 2$	\$14,125.42

All the other variables are non-integers.

The optimal solution corresponding to the combination of integer variables (2.) has the highest functional value. The functional value is marked by *. At this stage, the tree graph is constructed as below:



	Functionals \$		The levels of integer variables
	$Z' = 15,175.44$		
	$Z_1^2 = 14,789.05$*	---	$X_{135} = 1$
	$Z_2^2 = 14,125.42$	---	$X_{135} = 2$
	$Z_0^2 = 13,670.08$	---	$X_{135} = 0$

In the third step, X_{135} is forced into the final basis at one unit. One of the other integer variables, ie., X_{134} is selected and forced into the optimal bases at three integral levels, respectively. Three alternative solutions having X_{134} and X_{135} at three pairs of integral levels (X_{135} = 1 and X_{134} = 0; X_{135} = 1 and X_{134} = 1; X_{135} = 1 and X_{134} = 2) are obtained and their functionals are compared. The alternative functionals are as follows:

	Functionals $
4. $X_{135} = 1$	
$X_{134} = 0$	14,181.12
All the other variables are non-integers.	
5. $X_{135} = 1$	
$X_{134} = 1$	14,435.35*
All the other variables are non-integers.	
6. $X_{135} = 1$	
$X_{134} = 2$	14,019.11
All the other variables are non-integers.	

The optimal solution obtained for a combination of integral levels, (5.), has the highest functional value. Therefore, the functional is marked by * and compared with the unmarked functionals in the previous step. If the marked functional is larger than both of the unmarked functionals obtained in the preceding step, then X_{135} and X_{134} are forced into the final basis at these integral levels in the next step.

In the second step, only one variable, X_{135}, is forced into the optimal basis at three integral levels, 0, 1 and 2, respectively. The corresponding functionals are denoted by $Z(X_{135}=0)$, $Z(X_{135}=1)$ and $Z(X_{135}=2)$. If another variable, ie.,

X_{134} is forced into the optimal basis at 0, 1 and 2, respectively, holding X_{135} at zero level, all functionals obtainable with these integral levels of X_{134} should be smaller than $Z(X_{135}=0)$. A similar conclusion can be drawn up with respect to each integral level of X_{135}.

Now, in the third step, $Z(X_{135}=1, X_{134}=1)$ has the largest functional value. If this functional is larger than $Z(X_{135}=0)$, then it should also be larger than $Z(X_{135}=0, X_{134}=0)$, $Z(X_{135}=0, X_{134}=1)$ and $Z(X_{135}=0, X_{134}=2)$. Similarly, $Z(X_{135}=1, X_{134}=1)$ should be larger than $Z(X_{135}=2, X_{134}=0)$, $Z(X_{135}=2, X_{134}=1)$ and $Z(X_{135}=2, X_{134}=2)$, if it is larger than $Z(X_{135}=2)$.

Generally, if, in the i th step, an optimal solution including a combination of integral levels has the largest functional and that functional value is larger than all unmarked functionals appearing in the i-1 th step, then the functional should be largest among functionals obtainable for all possible combinations of integral levels in the i th step.

In the third step, all possible combinations are: $X_{135}=0$ and $X_{134}=0$; $X_{134}=0$ and $X_{134}=1$; $X_{135}=0$ and $X_{134}=2$; $X_{135}=1$ and $X_{134}=0$; $X_{135}=1$ and $X_{134}=1$; $X_{135}=1$ and $X_{134}=2$; $X_{135}=2$ and $X_{134}=0$; $X_{135}=2$ and $X_{134}=1$; $X_{135}=2$ and $X_{134}=2$. The functional obtained for the combination, $X_{135}=1$ and $X_{134}=1$, is largest among functionals obtainable for all of these combinations because $Z(X_{135}=1, X_{134}=1)$ is the largest functional in the third step and is larger than all unmarked functionals in the second step.

In the fourth step, X_{135} and X_{134} are forced into the optimal basis at one unit each. The third integer variable X_{131} is selected and constrained to 0, 1 and 2, respectively. The functionals obtained for the alternative combinations of these integer levels are as follows:

		Functionals $
7. $\left\{\begin{array}{l} X_{135}=1 \\ X_{134}=1 \\ X_{131}=0 \end{array}\right.$	All other variables are non-integers.	12,573.492
8. $\left\{\begin{array}{l} X_{135}=1 \\ X_{134}=1 \\ X_{131}=1 \end{array}\right.$	All other variables are non-integers.	14,408.424*
9. $\left\{\begin{array}{l} X_{135}=1 \\ X_{134}=1 \\ X_{131}=2 \end{array}\right.$	All other variables are non-integers.	14,356.573

The functional obtained for (8) has the maximum value so it is marked by *. The functional value must be compared with all unmarked functionals derived in the third step. This functional is greater than all unmarked functionals in the preceeding step. Therefore, the combination (8) is selected.

Similarly, the combination of integer variables, $X_{135}=1$,

$X_{134}=1$, $X_{131}=1$ and $X_{132}=1$, is selected in the fifth step. This combination is numbered by (10). The functional value with this combination of integer constraints, however, is slightly lower than one of the unmarked functionals, ie., the one derived for (9) in the fourth step. In such a case, we must repeat the similar procedure, starting from the combination, $X_{135}=1$, $X_{134}=1$, and $X_{131}=2$. However, the functional value coupled with the combination (10) should be larger than that of the optimal solution including two units of X_{131}. This occurs because even one unit of X_{131} is not used to its full capacity in the optimal solution obtained in the fifth step. The functional value coupled with the combination (9) is slightly larger than that associated with the combination (10), simply because X_{132} is treated as a non-integer variable in the fourth step. Once X_{132} is also forced to an integer value, the functional obtained for the combination including two units of X_{131} would be lower than that derived for the combination (10). The optimal basis including these four integer variables at one unit each contains no other integer variable at a fractional level. All other integer variables are at zero levels; so the calculation terminates here.

Thus, the optimal mixed integer programming solution is the one including these four integer variables at one unit each.

APPENDIX V

SOLVING A MIXED-INTEGER PROGRAMMING PROBLEM WITH THE AID OF NON-INTEGER PARAMETRIC PRICE PROGRAMMING

One of the shortcomings of the integer programming method is that direct application of a parametric price or cost programming for an integer programming problem is infeasible. An integer programming solution should be obtained separately for every possible price of an activity whose price is varied. This is time-consuming because all possible prices of this activity must be considered at small intervals. The purpose of this appendix is to demonstrate that a non-integer parametric price or cost programming technique can be utilized to cover up such a shortcoming of the integer programming method. In this example, the price of water is varied.

Firstly, an integer programming problem must be solved with a low fixed price of water. This problem is referred as the first integer programming problem.

Secondly, all integer variables included in the relevant programming problem are treated as non-integer variables; optimal non-integral solutions are obtained for various prices of water by the parametric price programming technique. The non-integral (fractional) levels of integer variables appearing in the optimal solutions obtained with varied water prices are traced out until a water price at which one of these non-integer levels indicates a notable change is found. With this price of water, another integer programming problem is solved. Then, the optimal integral levels of these variables are compared with those appearing in the first optimal solution of the first integer programming problem. If identical integral levels are found in both solutions, then the fractional levels of these integer variables must be traced out until another water price where a notable change in one of these levels occurs is reached. Another integer programming problem is solved with the price of water fixed at this level. The optimal integral solution is compared with that obtained for the first integer programming problem. If these two optimal solutions differ from each other, then the third one is valid for a range of water prices including this particular price. At this stage, another integer programming solution should be obtained for a water price slightly lower than this particular price. If the optimal solution is identical with that of the first integer programming problem, then the first optimal solution is valid for the entire range of water prices lower than this particular one.

The same procedure is repeated for the entire range of water prices for which non-integer parametric price programming solutions are obtained. In this study, the price of water is varied from zero to $7.20 at which the demand for irrigation water becomes zero. In this case, all integer variables are treated as non-integer variables. With the price of water fixed at $2.00 per acre-inch, a mixed-integer programming problem is solved and the optimal integral levels of these variables are noted. At this price of water, the combination of one set irrigation machines, one unit each of sugar beet thinner, potato digger and seed potato cutter enter the optimal solution. All the other integer variables take zero values. Non-integer levels of the machines purchased for irrigated crops decline as the price of water rises. When the price of water becomes high enough, they approach zeros with decreasing levels of irrigated crops. If these machines are forced into an optimal solution at one unit each, optimal solutions coupled with high prices of water may differ from the true integral optimal solutions. In this example, the irrigated sugar beet activity is switched to the dryland sugar beet operation as the price of water rises. Therefore, even though one unit of sugar beet thinner is forced into the final bases throughout the entire range of varied water prices, the optimal solutions will not be distorted by such an enforcement. However, if one unit of irrigation machinery and a potato digger are forced into the final bases for the entire range of water prices, the optimal solutions obtained in the high price range will be somehow distorted.

Fractional levels of the irrigation machinery fall distinctly when the price of water exceeds $3.34 per acre-inch. Therefore, another integer programming problem is solved with the price of water fixed at $3.34. In the optimal solution of this mixed-integer programming problem, only one unit of sugar beet thinner is included, but all the other integer variables are at zero levels. Thus, the optimal combination of machines differs from that appearing in the optimal solution of the first mixed-integer programming problem (see Table XXXVII). However, $3.34 of water price is not the exact border price at which the optimal combination of integer variables changes distinctly. Another mixed-integer programming problem associated with the price of water fixed at $2.15 is solved. At this price, the optimal combination of integer variables is identical with that of the first mixed-integer programming solution. In order to detect the exact border price, therefore, several mixed-integer programming problems must be solved with the price of water varied from $2.15 to $3.34 at small intervals. The exact border price exists somewhere between these two prices of water.

A short-cut method can be used here instead of solving a number of mixed-integer programming problems at various prices of water within this range. We know that, with the prices of water higher than $3.34, irrigated crops, irrigation machines and irrigation water no longer enter the optimal solutions of mixed-integer programming problems. In this

TABLE XXXVII

NON-INTEGER LEVELS OF SPECIALIZED MACHINES UNDER VARIOUS RANGES OF WATER PRICES (250 ACRE FARM)

Prices of Water (Dollar Per Acre-inch)	Non-integral Level of Irrigation Machines	Non-integral Level of Seed Cutter	Non-integral Level of Potato Digger	Non-integral Level of Sugar Beet Thinner
0 ~ 0.478	0.700	0.153	0.851	0.481
0.478 ~ 0.591	0.560	0.143	0.858	0.483
0.683 ~ 1.177	0.477	0.143	0.856	0.481
1.177 ~ 1.843	0.416	0.118	0.706	0.481
1.888 ~ 2.053	0.364	0.128	0.770	0.481
2.053 ~ 2.146	0.346	0.116	0.697	0.481
2.146 ~ 3.339	0.346	0.116	0.697	0.481
3.339 ~ 3.863	0.192	0.160	0.958	0.349
3.863 ~ 5.903	0.065	0.160	0.322	0.349
5.903 ~ 7.204	0.062	0.052	0.311	0.290
7.204	0	0	0	0.281

price range, therefore, an optimal solution and its optimal functional value will not be affected by a change in the price of water. Only one optimal mixed-integer solution exists in this range of water prices. It must be noted, however, that $3.34 is not the exact border price at which this optimal solution becomes valid. We do not know exactly at what price of water this optimal solution becomes valid. We must detect it. Whatever the border price may be, the optimal solution will not be affected by a change in the water price when it exceeds this exact border price. To find the exact border price of water, a parametric price programming problem must be solved for the water prices, \$2.14 to \$3.34, with the integer variables fixed at the integral levels as appearing in the optimal solution of the first mixed-integer programming problem. Several optimal solutions associated with ranged water prices will be derived within this price range. As the price of water rises, optimal functional values will decrease. Even within a range of water prices in which the same optimal solution is valid, the optimal functional value decreases as the price of water increases. In this example, four optimal solutions associated with ranged water prices are obtained in the price range, \$2.14 to \$3.34. These functional values are indicated in Table XXXVIII.

TABLE XXXVIII

OPTIMAL FUNCTIONALS OBTAINED FOR THE WATER PRICES, \$2.14 TO \$3.34 PER ACRE-INCH

Price Ranges Dollar Per Acre-inch	2.14 ~ 3.19	3.19 ~ 3.27	3.27 ~ 3.27	3.27 ~ (5.22)
Functional Values	\$14,079.00	13,111.00	13,047.00	13,044.00

On the other hand, the optimal functional value of the mixed-integer programming solution obtained for the price of water fixed at \$3.34 per acre-inch is \$13,635.00. The comparison of this functional value with those appearing in Table XXXVIII reveals that the exact border price lies somewhere between \$2.14 and \$3.19. Where the price of water is higher than \$3.19, all functionals shown in the table are smaller than \$13,635.00. Thus, we must detect the exact border price existing between \$2.14 and \$3.19. The optimal functional value of the mixed-integer programming solution obtained for the first problem must equal that of the mixed-integer programming solution obtained for the second one at the exact border price of water. As mentioned earlier, the optimal functional value of the mixed-integer programming solution obtained with the water price, \$3.34, is valid for all water prices higher than the exact border price. On the other hand, the optimal functional value of the mixed-integer programming solution derived for the first problem varies in inverse proportion to the increasing prices of water. Only the price of water increases from \$2.14 to \$3.19, holding the activity combination the same. As the water price increases by \$1.05, approximately 925 acre-inches of water are used consistently. This means that, as the water price rises from \$2.14 to \$3.19, the optimal functional value declines by about \$967.00 (from \$14,079 to \$13,111). Therefore, the optimal functional values falling within this price range can be calculated as;

$$(V - 1) \quad Z = \$14{,}079 - \frac{967}{1.05} X$$

where X indicates the price increase of water on $2.14.

At the exact border price of water, the following relation must be satisfied;

$$(V - 2) \quad \$14{,}079 - \frac{967}{1.05} X = \$13{,}635$$

Therefore,

$$(V - 3) \quad X = (14{,}079 - 13{,}635) \cdot \frac{967}{1.05} = \$0.48$$

Thus, the exact border price is

$$(V - 4) \quad \$2.14 + \$0.48 = \$2.62$$

As the price of water exceeds $2.62 per acre-inch, the optimal functional of the mixed-integer programming solution obtained for the first problem becomes lower than that obtained for the second one. The mixed-integer programming solution obtained for the latter includes one unit of sugar beet thinner only and no irrigation machine is purchased. Therefore, if the purchasing units of specialized machines are considered at integral units, then irrigation would be economically infeasible with the prices of water higher than $2.62 per acre-inch.

APPENDIX VI

MINIMAL REQUIRED CHANGES IN TECHNICAL COEFFICIENTS

The shadow prices appearing in the final stage of simplex tableaus can be utilized to find how much, at least, a technical coefficient of a non-basic activity must be changed in order that the activity is selected for entry into the optimal basis. In the following, the formula which can be used for calculation of minimal required changes in a technical coefficient is derived from the final stage of simplex tableaus.

Now consider a standard linear maximization problem comprising m number of real activities and n number of constraints. It is assumed that the o th activity, Po, is not included in the optimal solution. Our attempt is to render Po enter the final basis by changing a technical coefficient a_{ro} (the r th coefficient of the o th activity) in a minimum amount. The notations used in this derivation are defined as below;

a_{io} is the i-th technical coefficient of the o th activity $(i = 1,2,3,\cdots n)$,

A_o is the column vector comprising of a_{io} $(i = 1,2,\cdots n)$,

H is the matrix containing all column vectors of disposal activities appearing in the final stage of simplex tableaus,

h_{ii} denotes the elements of H $(i = 1,2,3,\cdots n)$, Ko is the column vector of the o th real activity, Po, in the final stage of simplex tableaus,

k_{io} denotes the elements of the K_o vector (i = 1,2,3,···n), and

π_i is the net price of the i th activity entering the final basis.

The Ko vector and the H matrix can be presented as below;

TABLE XXXIX

THE FINAL STAGE OF SIMPLEX TABLEAUS

B	P_o	P_{m+1}	P_{m+2}	P_{m+i}	P_{m+r}	P_{m+n}
π_1	k_{10}	h_{11}	h_{12}	h_{1i}	h_{1r}	h_{1n}
π_2	k_{2o}	h_{21}	h_{22}	h_{2i}	h_{2r}	h_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
π_i	k_{io}	h_{i1}	h_{i2}	h_{ii}	h_{ir}	h_{in}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
π_r	k_{ro}	h_{r1}	h_{r2}	h_{ri}	h_{rr}	h_{rn}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
π_n	k_{no}	h_{n1}	h_{n2}	h_{ni}	h_{nr}	h_{nn}
Z	Z_o	Z_{h1}	Z_{h2}	Z_{hi}	Z_{hr}	Z_{hn}
Z-C	α_o	Z_{h1}	Z_{h2}	Z_{hi}	Z_{hr}	Z_{hn}

The shadow price of the P_o activity (α_o) can be calculated by:

(VI - 1) $\alpha_o = (k_{1o} \cdot \pi_1 + k_{2o} \cdot \pi_2 + \ldots + k_{io} \cdot \pi_i + \ldots + k_{ri} \cdot \pi_r + \ldots + k_{no} \cdot \pi_n)$

$- C_o$

where:

C_o is the net price of P_o activity.

Since

(VI - 2) $$K_o = H \cdot A_o,$$

we can obtain the following results;

(VI - 3) $k_{ro} = h_{r1} \cdot a_{1o} + h_{i2} \cdot a_{2o} + \ldots + h_{ri} \cdot a_{io} + \ldots + h_{rr} \cdot a_{ro} + \ldots + h_{rn} \cdot a_{no}$

Therefore, α_o can be calculated by

$$\alpha_o = \left\{ \sum_{\substack{i=1 \\ i \neq r}}^{n} k_{io} \cdot \pi_i + k_{ro} \cdot \pi_r \right\} - C_o$$

(VI - 4) $$= \left\{ \sum_{\substack{i=1 \\ i \neq r}}^{n} k_{io} \cdot \pi_i + h_{ri} \cdot a_{1o} \cdot \pi_r + h_{r2} \cdot a_{2o} \cdot \pi_r + \ldots + h_{ri} \cdot a_{io} \cdot \pi_r + \ldots + h_{rr} \cdot a_{ro} \cdot \pi_r + \ldots + h_{rn} \cdot a_{no} \cdot \pi_r \right\} - C_o$$

Now, suppose that the shadow price of P_o (denoted by α_o) in the last stage of simplex tableaus becomes just equal to zero when the r th technical coefficient of P_o (a_{ro}) changes to a'_{ro}.

Then, the new shadow price of P_o is given by:

(VI - 5) $$0 = \left\{ \sum_{\substack{i=1 \\ i \neq r}}^{n} k_{io} \cdot \pi_i + h_{ri} \cdot a_{1o} \cdot \pi_r + h_{r2} \cdot a_{2o} \cdot \pi_r + \ldots + h_{ri} \cdot a_{io} \cdot \pi_r + \ldots + h_{ri} \cdot a'_{ro} \cdot \pi_r + \ldots + h_{rn} \cdot a_{no} \cdot \pi_r \right\} - C_o$$

By (VI - 4) minus (VI - 5),

$$\text{(VI - 6)}\quad \alpha_o = h_{rr}\cdot a_{ro}\cdot \pi_r - h_{rr}\cdot a'_{ro}\cdot \pi_r = (a_{ro}-a'_{ro})h_{rr}\cdot \pi_r$$

Therefore:

$$\text{(VI- 7)}\quad a_{ro} - a'_{ro} = \frac{\alpha_o}{h_{rr}\cdot \pi_r}$$

If a_{ro} is a negative technical coefficient, then the equation (VI - 7) becomes

$$\text{(VI - 8)}\quad - a_{ro} - (-a'_{ro}) = a'_{ro} - a_{ro} = \frac{\alpha_o}{h_{rr}\cdot \pi_r}$$

Furthermore:

$$\text{(VI - 9)}\quad Z_o = a_{1o}\cdot Z_{h1}+a_{2o}\cdot Z_{h2}+\ldots+a_{io}\cdot Z_{hi}+\ldots+a_{ro}\cdot Z_{hr}+\ldots+a_{no}\cdot Z_{hn}$$

where:

Zo is the o th element of the Z row in Table XXXIX and $Z_{hi}(i=1,2,\cdots n)$ is the m+i th element of the Z-C row.

Therefore:

$$\text{(VI - 10)}\quad \alpha_o = Z_o-C_o = \left\{\sum_{\substack{i=1\\ i\neq r}}^{n} a_{io}\cdot Z_{hi}+a_{ro}\cdot Z_{hr}\right\} - C_o < 0$$

Now, suppose that the shadow price α_o becomes just equal to zero when a_{ro} changes to a'_{ro} holding other input-ouput coefficients the same.

Then we have

$$\text{(VI - 11)}\quad 0 = \left\{\sum_{\substack{i=1\\ i\neq r}}^{n} a_{io}\cdot Z_{hi} + a'_{ro}\cdot Z_{hr}\right\} - C_o$$

By (VI - 10) minus (VI - 11),

$$\alpha_o = a_{ro} \cdot Z_{hr} - a'_{ro} \cdot Z_{hr} = (a_{ro} - a'_{ro}) \cdot Z_{hr}$$

Therefore:

$$\text{(VI - 12)} \quad a_{ro} - a'_{ro} = \frac{\alpha_o}{Z_{hr}}$$

If a_{ro} is a negative technical coefficient, then

$$\text{(VI - 13)} \quad a'_{ro} - a_{ro} = \frac{\alpha_o}{Z_{hr}}$$

Since α_o and Z_{hr} are positive, a_{ro} must be greater than a'_{ro} when a_{ro} is positive and a'_{ro} should be greater than a_{ro} when a_{ro} is a negative coefficient. In other words, if a_{ro} is a positive technical coefficient, then a_{ro} should be reduced, at least, by $\frac{\alpha_o}{Z_{hr}}$ in order to activate a non-basic activity, P_o. If a_{ro} is a negative coefficient, then the absolute value of a_{ro} should be increased, at least, by $\frac{\alpha_o}{Z_{hr}}$ in order to render P_o enter the optimal solution.

APPENDIX VII

BASIC DATA USED IN THE STUDY

Linear programming usually requires three kinds of data which are used respectively for calculating technical coefficients, net prices and restrictions on resource use. In this study, three major groups of activities are considered. They are irrigated crop activities, dryland crop activities and livestock activities. The information used for calculation of labour requirements and machine times for irrigated crop operations is obtained from a survey report[1] published by the Agricultural Experiment Station, South Dakota State College. Reference is also made to a study[2] of the Lower Yellow Stone Project Area conducted by the North Dakota State College. The information on labor requirements and machine time is based on the cultivation and management practices actually observed on the irrigated farms in those project areas. The cultivation practices will be more or less the same in the Morden-Winkler

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[1]J. Ulvilden, Farm Labour, Power and Machinery Performance for Selected Operations Under Dryland and Irrigated Conditions in Central South Dakota, Agricultural Experiment Station, South Dakota State College, 1953.

[2]Rex Helfinistine, Management Practices and Yields on the Lower Yellow Stone Project, 1949, North Dakota Agricultural Experiment Station, Agricultural Economics Department.

Project area, too, because natural conditions of farming, size of farm, type of enterprise and major crops are approximately the same in these areas. Information used for calculating the labor requirements and machine time for dryland crop activities were obtained mainly from "How Labor is Used on Red River Valley Farms".[1] Reference is also extended to "Economic Aspects of Farm Machinery Use in Crop Productions"[2] and a few other publications.

To calculate net prices, we need information about the prices of factors and products and the amounts of productive agents used per unit of activities. In this case, only variable factors should be taken into consideration. The prices of products used for the calculation of expected net prices assume the 10 year averages observed in 1956-66 unless specified in Appendix Tables. The prices of factors are the current prices paid by farmers in Winnipeg area. The prices of products and factors used in this study appear respectively in Appendix Tables 4, 5, 9 and 10. Quantities of variable factors needed on a unit of activity basis are shown in Appendix Table 17.

Information on custom rates and wage rates of hire labour was

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[1]MacKenzie, J.G. and J.C. Brown, How Labor is Used on Red River Valley Farms, 1954, Economic Division, Canada Dept. of Agriculture.

[2]Dubois, M.J., Economic Aspects of Farm Machinery Use in Crop Production, 1965, Economics Branch, C.D.A.

obtained directly from the farmers and also from "Custom Charges for Farm Machinery", 1967, Manitoba Dept. of Agriculture. These rates used in this study appear in Appendix Tables 10 and 11. The costs of irrigation development are taken from Part 8 of the "Pembina Report".[1] The costs of irrigation development include the items; initial levelling of land, farm lateral distribution systems, farm drains, etc. The initial costs of irrigation development on T_1 and T_2 land are amortized respectively and included in the variable costs of irrigated crop activities per acre. The method of amortization is presented on page 74 and also in Appendix Table 11 and Appendix III.

In linear programming studies, the cost of each activity is based on the variable factors of which the levels of inputs increase in proportion with the level of activity. Naturally, over-head costs are not subtracted from the total revenue usually denoted by functionals Z. In the present study, depreciation, repairs and maintenance of all machines (except the special machines as specified on page 165), depreciation of buildings, taxes, insurance, farm share of hydro and telephone costs, etc. are treated as over-head costs. The over-head costs amount to approximately $4,000. This amount was calculated by an increase of 30 percent of the amount

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[1]International Pembina River Engineering Board, Joint Investigation For Development of the Water Resources of the Pembina River Basin, Manitoba and North Dakota, Vol. III, Part 8, PP.202-208, International Joint Commission, Dec. 1964.

obtained from the Tables IV-11 and IV-13 in the "Pembina Report".[1] This amount should be subtracted from the functionals to obtain the net return to operator and family labor, own capital and own land.

Information of resources available on the farms in the project area was obtained from the 329 farm records collected by the Prairie Rehabilitation Administration's Economic Division in 1962.

Reliable information on crop yields is most difficult to obtain, especially when the information is required directly for a particular area. No information on crop yields on irrigated land is immediately available in the project area. The only possibility is to use the crop yields projected by some methods. In this study, yields of dryland crops were obtained from a report on a survey conducted by the Manitoba Crop Insurance Corporation, Crop Canner and Winkler and Delmonte Canners at Morden in 1964-1966. Three year averages are used. The yields of irrigated crops were projected as follows. Ten year averages of crop yields on irrigated land as observed in the Lower Yellow Stone Irrigation Project Area in 1956-1965 were compared with ten year averages of dryland crop yields observed in the adjacent area. The ratios of crop yields, (ie., irrigated crop yields over dryland crop yields)

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[1]Pembina Report, Vol. IV, P. IV-28. (a preliminary draft of Ibid.)

were calculated from these data. The ratios appear in Appendix Table 1. The three year average yields on dryland in the Morden-Winkler project area were multiplied by these ratios to estimate the projected crop yields on irrigated land in the same area. The projected yields may be justified by comparisons of the natural conditions of farming such as precipitation, temperature, daytime hours and soils, in the two project areas. These conditions are very similar for the two project areas, except for the soil types.

Total water requirements and irrigation water requirements of major crops and forage crops are calculated by the method appearing in "Determining Consumptive Use and Irrigation Water Requirements".[1] These figures are also compared with the figures recommended for irrigating crops in Alberta.

Fairly old data were used for the estimation of labour requirements and machine times for irrigation conditions as well as resource restrictions in this study. Unfortunately, no other data were available for the calculation of these input-output coefficients at the time of the study. The cultivation practices under irrigation conditions using gravity methods, however, have not changed considerably during the past ten or fifteen years. As to the resource data, the

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[1]H. F. Blaney, Determining Consumptive Use and Irrigation Water Requirements, Tech. Bulletin No. 1275, 1962, pp. 16-19, Agricultural Research Service, U.S.D.A.

following points were taken into consideration. (1) The amount of annual operating capital owned by farmers was adjusted to the current level by increasing 30 percent of the amount owned at the time of the survey (1962). (2) It is assumed that the supply of family labour on an average farm in the area has not changed significantly in the past ten years.

APPENDIX TABLES

APPENDIX TABLE 1

CROP YIELDS[1] UNDER DRYLAND AND IRRIGATION CONDITIONS AND THEIR IRRIGATED/DRYLAND RATIOS IN THE LOWER YELLOW STONE IRRIGATION PROJECT AREA, NORTH DAKOTA

	Irrigated land (1)	Dryland[2] (2)	Ratio(1)/(2)
	---bushels per acre---------		
Wheat	36.0	17.5	2.06
Barley	43.7	24.1	1.81
Oats	57.8	32.0	1.81
Flax	22.4	6.4	3.50
Potatoes	221.2	107.4	2.06
Sunflower[3]	721.2 lbs.	518.0 lbs.	1.39
	------tons per acre---------		
Corn Silage	10.2	3.64	2.80
Alfalfa-Broome	2.56	1.28	2.00
Sugar Beets	14.15	-	-

[1]Yields are the ten year average of 1956-65. These data were obtained from the Dept. of Agricultural Economics, North Dakota State University.

[2]Dryland yields data are taken from Dunn County in which the Project Area is located.

[3]Sunflower yields are for Southern Alberta.

APPENDIX TABLE 2

PROJECTED YIELDS[1] OF CROPS PER ACRE ON STUBBLE WITH AND WITHOUT FERTILIZER FOR THE MORDEN-WINKLER IRRIGATION PROJECT AREA, MANITOBA (DRYLAND)

	Fertilized			Not Fertilized
	Kind of Fertilizer	Amount Applied	Estimated Yield	
		pounds	bushels	bushels
Wheat	11-48-0	60	28.0	25.0
Barley	23-23-0	60	45.0	29.0
Oats	23-23-0	80	65.0	45.0
Flax	-	-	-	11.0
Potatoes[2]	16-20-0	400	148.83	-
Sunflowers[3]	11-48-0	30	720.0 lbs.	720.0 lbs.
Field peas	-	-	-	18.0 lbs.
Sugar Beets	11-48-0	60	10.0 ton	-
Corn Silage	-	-	-	3.5 tons
Tame Hay (Dry matter)	-	-	-	1.8 tons
Tame Pasture[4]	-	-	-	Equivalent to 1.4 or 1.5 ton drymatter.

[1]Information is obtained from Manitoba Crop Insurance Corporation and Year Book of Manitoba Agriculture (Crop District No. 3), 1966. Crop yields are the ten year average of 1956-65 except for wheat, barley and oats. Four year average yields (1963-66) are used for these three crops.

[2]Three quarters of the total yield (111.62 bushels) are sellable.

[3]A small amount of fertilizer (30 lbs.) is applied to shorten the growing period.

[4]This yield is equivalent to 1400-1500 pounds of T.D.N. or 3.5-3.6, AUM.

APPENDIX TABLE 3

PROJECTED YIELDS[1] OF CROPS AND FORAGES PER ACRE ON STUBBLE WITH FERTILIZER IN THE MORDEN-WINKLER PROJECT AREA (IRRIGATED LAND)

	Kind of Fertilizer	Amount Applied	Estimated Yield
		pounds	bushels
Wheat	11-48-0	100	45
Barley	23-23-0	100	55
Oats	11-48-0	100	80
Flax	16-20-0	50	23
Field Peas	11-48-0	100	33
Sunflower	11-48-0	80	17 (1008 lbs.)
Potatoes[2]	16-20-0	400	240 (7.2 tons)
			tons
Sugar Beets	11-48-0	100	15
Corn Silage (finished as silage)	16-20-0	100	12.0
Tame Hay (Dry matter)	11-48-0	100	3.5
Tame Pasture[3]	11-48-0	100	Equivalent to 2.5 ton dry matter

[1]These yields are calculated by dryland yields not fertilized in the Morden-Winkler area multiplied by ratios of irrigated yields over dryland yields obtained for the Lower Yellow Stone Project Area.

[2]Three quarters of this yield are sellable.

[3]This yield is equivalent to 2350 pounds of T.D.N. or approximately to 6.0 AUM.

APPENDIX TABLE 4

THE PRICES OF BEEF CATTLE AND HOGS (AVERAGE OF 1962-6)

	dollars
all slaughter steer	23.00 per 100 lbs.(live weight)
all calves	22.50 per 100 lbs.(live weight)
all market hog	27.00 per 100 lbs.(carcass)

APPENDIX TABLE 5

PRICES OF FARM PRODUCTS AT THE MARKET OR FACTORY LEVEL

	10 year average (1955-66)
	dollars
Wheat (No. 2, No. 3)	1.74 per bushel
Barley (No. 3 CW 6 row)	1.11 per bushel
Oats (No. 3 CW)	0.70 per bushel
Flax	3.06 per bushel
Potatoes	1.45 per bushel
Field peas	2.06 per bushel
Sunflowers	4.89 per 100 lbs.
Sugar Beets	14.96 per ton

NOTE: Shipping (freight, trucking, container, etc.) and other marketing charges should be subtracted to obtain the prices of farm product at the farm level. The price of wheat is calculated by: final payment + intermediate payment + initial payment - P.F.A.A. levy. The price of barley is calculated by: initial payment + final payment - P.F.A.A. levy. The price of sugar beets is the factory level in Winnipeg. The potato price is the three year average price of the Red Dry No. 2 sold by the Manitoba Vegetable Marketing Commission at the Winnipeg market.

APPENDIX TABLE 6

THE MAXIMUM OPERATING CAPACITIES OF SPECIALIZED MACHINES WITH FULL-TIME CREWS FOR OPERATION

-----one unit-------	acres	------one unit-------	acres
Hay Baler	150~200	Sugar Beet Harvester	50
Forage Corn Harvester	40	Seed Potato Cutter	300
Sugar Beet Thinner	130	Irrigation Machineries (one set)	250
Potato Digger	50		

APPENDIX TABLE 7

FEED REQUIREMENTS FOR BEEF CATTLE AND SOW-HOG OPERATIONS (T.D.N.)

	Unit	Hay or Silage	Pasture	Grain	Total T.D.N.	Feed Supplements
		lb.	lb.	lb.	lb.	dollar
Cow-calf	cow	2404	2602 (6.2 AUM)	310	5316	-
Cow-calf-feeder	cow	3245	3488 (8.3 AUM)	640	7373	-
Feeder calf	head	1304	1865 (4.4 AUM)	750	3919	-
Feed-lot[1] (400-1000 lb)	head	1040	-	2080	3120	3.0
Feed-lot (650-1100 lb)	head	940	-	2060	3000	2.8
Feed-lot (800-1200 lb)	head	780	-	1560	2340	2.5
Sow-hog (2 litters)[2]	sow	-	-	8900	8900	58.0

NOTE: In case of cow-calf and cow-calf-feeder operations, the feed requirements for cow and the heifer for replacement are also included in these figures.

[1] In feed-lot operations, calves, yearlings and steers are purchased and finished in feed-lots in 330, 300 and 150 days, respectively.

[2] Sow-hog operation assumes 2 litters in a year and produces 14 hogs per sow. Feed requirements include those of a sow and 2 litters of hogs fattened up to 240 lbs.

APPENDIX TABLE 8

THE WATER REQUIREMENTS AND THE ESTIMATED AMOUNT OF IRRIGATION WATER FOR VARIOUS CROPS AND FORAGES UNDER THE AVERAGE RAINFALL CONDITIONS IN THE MORDEN-WINKLER AREA

	(1) All Possible Daytime Hours in a Year	(2) Growing-Seasonal Possible Daytime (hrs.)	(3) Average Temperature of Growing Season	(4) $\frac{(2)\text{x}100}{(1)}$	(5) $\frac{(4)\text{x}(3)}{100}$ = f	(6) (5)xk's =(k.f)	(7) The amount to be Irrigated
	hours					---inches---	---------
Small Grains	2172	902 hrs.(May-Aug.15)	62.1	41.5	25.77	19.33	10.25
Flax	2172	1609 hrs.(April-Oct.)	55.0	74.1	40.76	28.53	13.23
Corn	2172	1072 hrs.(May-Aug.)	62.1	49.4	30.68	23.01	12.06
Potatoes	2172	1072 hrs.(May-Aug.)	62.1	49.4	30.68	19.94	8.99
Sugar Beets	2172	1459 hrs.(April-Sept.)	56.9	67.2	38.24	24.86	10.69
Field Peas	2172	802 hrs.(May-July)	60.8	36.9	22.43	13.46	5.18
Alfalfa	2172	1459 hrs.(April-Sept.)	56.9	67.2	38.24	30.59	16.42
Pasture	2172	1459 hrs.(April-Sept.)	56.9	67.2	38.24	30.59	16.42
Sunflowers	2172	1459 hrs.(April-Sept.)	56.9	67.2	38.24	24.77	10.60

NOTE: In this study, 29 year average rainfalls are assumed for various growing seasons. The average rainfalls are; 9.08 inches for May-Aug.15, 15.30 inches for April-Oct., 10.95 inches for May-Aug. 31, 14.17 inches for April-Sept. and 8.28 inches for May-July, respectively. Water requirements for various crops during their growing seasons were calculated by using the formula developed by: Blaney H.F., Determining Consumptive Use and Irrigation Water Requirements, Technical Bulletin No. 1275, U.S.D.A., 1964. K's are constants calculated by the same author.

APPENDIX TABLE 9

THE PRICES OF FEED, FUEL, GREASE AND FERTILIZERS

		Unit Prices
Feed		----dollars------
Wheat		1.55 per bushel (0.026 per lb.)
Barley		0.95 per bushel (0.020 per lb.)
Oats		0.65 per bushel (0.018 per lb.)
Pig Starter 16%		0.041 per lb.
Sow Supplement 35%		0.054 per lb.
Hog Grower, Finishing Supplement 35%		0.050 per lb.
Alfalfa Meal		0.044 per lb.
Creep Ration 16%		0.046 per lb.
Wild Hay		9.00 per ton
Tame Hay (mix)		15.00 per ton
Alfalfa Hay		16.00 per ton
Fuel and Grease		
Tractor Fuel	No. 2 Gas	0.22 per gallon
	No. 1 Diesel	0.19 per gallon
Truck Fuel	No. 2 Gas	0.22 per gallon
Grease		9.85 per 35 lb.
Fertilizer		
11-48-0		0.054 per lb.
16-20-0		0.04 per lb.
23-23-0		0.05 per lb.
27-14-0		0.042 per lb.

APPENDIX TABLE 10

WAGE RATES OF HIRED LABOR, CUSTOM RATES, FREIGHT RATES AND STORAGE COST

1.	Sugar Beet Operation	dollars
	Hand Thinning and Weeding	25. or 30. per acre
	Custom Harvesting (Only Rotobeating and Lifting)	1.5 per ton or 20 per acre.
	Trucking Rate (Winkler Area)	1.5 per ton
	Trucking Rate (to the Station)	2.5 per ton per 50 miles
	Train (Winkler to the Factory in Winnipeg)	1.85 per ton
2.	Potato Operation	
	Custom Potato Digger (Digger and Operator)	22 per acre
3.	Small Grains	
	Custom Combine	4.36 per acre
	Custom Swather	1.00 per acre
4.	Custom Hay Baler	8.00 per acre
5.	Custom Forage Corn Harvesting	4.50 per acre
6.	Operating Labor	1.70 per hour
7.	Hand-labor for Harvesting Potatoes	1.25 per hour
8.	Freight (Container) Rate of Potatoes from Winkler to Winnipeg	0.30 per bushel
9.	Freight Rates of Small Grains (Morden to Lakehead)	
	Wheat, Oats, Barley	0.15 per 100 lbs.
	Flax	0.165 per 100 lbs.
	Note: 18,000 lb. is the minimum amount per car via Canadian Pacific Railway.	
10.	Storage Cost of Potatoes on a Farm (Winkler)	0.05 per 100 lbs. for 3 months

NOTE: Information was obtained from; (1)potato grower, (2)sugar beet growers, (3)Mr. Stone, (4)Custom Charges for Farm Machinery, 1967, Manitoba Department of Agriculture and (5)Canadian Pacific Railway Company.

APPENDIX TABLE 11

AMORTIZED COST OF LAND-DEVELOPMENT BASED ON 30 YEAR REPAYMENT PERIOD

1. T_1 Land

The total initial cost of development is \$60 per acre. The amortized annual cost with the interest rate of 5 percent is:

$$\frac{A \cdot i}{1-\left(\frac{1}{1+i}\right)^t} = \frac{60x0.05}{1-\left(\frac{1}{1+0.05}\right)30} = \$3.9 \text{ per acre}$$

where:

The notation, i, indicates an interest rate,

A denotes the initial cost of land development, and

t indicates the period of repayment.

2. T_2 Land

The initial cost of development is \$90 per acre.

The amortized cost is \$5.85 per acre.

NOTE: T_1 land requires zero to 200 cubic yard per acre of soil to be removed for land-levelling. T_2 land requires to remove 200 to 350 cubic yard per acre of soil for the initial levelling to make flood irrigation possible.

APPENDIX TABLE 12

PRICES AND ANNUAL COSTS OF THE SPECIALIZED MACHINES

	Price Per Unit	Life	Depreciation	Maintenance and Repair		Interest Paid (5.5%)	Annual Total Costs
	dollars	percent	dollars	percent	-----dollars	-----	-----
Sugar Beet Thinner (one row)	200-250	8	20.	3	7.5	14.	42.
Sugar Beet Harvester (3 row)	6800-7800	12	640.	4	280.	385.	1315.
Corn Harvester	3500-4000	8	320	4	160.	220.	700.
Potato Digger	1,600	12	196	4	64.	88.	348.
Combine 12' (AM Auxuary)	5,700	8	456.	4	228.	314.	998.
Hay Baler (AM)	2,400	8	192	4	96.	152.	420.
Potato Harvester	10,000-12,000	8					
Seed Potato Cutter	1500-4000	8	160.	3	60.	110.	330.
One set of irrigation machines	2,500	12	300.	4	100.	138.	538.

NOTE: Information was obtained from John Deere Limited and the Alberta Department of Agriculture. The annual repair cost is calculated by using the percentages of inventory. One set of irrigation machines includes the following items:

two-way plow	$650.	levellers	$850.	syphons	$400.
ditcher	370.	ditch-filler	210.		

APPENDIX TABLE 13

COSTS OF BEEF CATTLE AND SOW-HOG OPERATIONS (1966)

	Feed-lot Operations (Housing) lbs. 400-1,000	lbs. 650-1,100	lbs. 800-1,200	Cow-Calf Operation (No Housing)	Feeder-Calf Operation (No Housing	Sow-Hog Operation (2 litters)
	---dollars per head---			dollars per cow	dollars per head	dollars per sow
Feeder	108.00	145.00	194.00	-	108.00	-
Freight and Marketing	7.00	7.00	8.00	2.50	7.00	21.00
Feed Supplements	-	-	-	-	-	62.00
Salts and Minerals,etc	0.50	0.50	0.50	2.00	0.50	-
Veterinary and Drugs	3.30	3.00	3.00	1.50	3.30	14.00
Breeding	-	-	-	5.00	-	-
Beddings	2.00	2.00	2.00	2.50	2.00	14.00
Machinery Use	-	-	-	-	-	14.00
Depreciation and Repair (Housing and Facilities) etc)	6.20	6.20	6.20	0.50	0.50	33.60
Depreciation of Sow and Baar	-	-	-	-	-	16.80
Death Loss and Breeding Failure	3.24	2.90	3.88	1.60	3.24	14.00
Others	-	-	-	-	-	20.00
Total	130.24	167.10	217.58	15.60	124.54	209.40

NOTE: The costs of home-grown feed and unpaid family labour are not included in these cost items. Information was obtained from: (1) Farm Records taken by the Engineering Dept., University of Manitoba and (2) Outlook, Saskatchewan Dept. of Agriculture, 1967.

APPENDIX TABLE 14

FAMILY LABOR AVAILABLE FOR VARIOUS SEASONS, 1962, (HOURS)

Seasons		Farm Small	Farm Medium	Farm Large
		----------hours---------		
Spring (April to May)	Operator	565.23	681.00	681.00
	Full-time Family	53.48	110.78	244.48
	Other Family	47.08	69.55	78.11
	Total	667.79	861.33	1003.59
Summer (June to Aug. 15)	Operator	738.80	923.50	923.50
	Full-time Family	79.87	147.18	365.12
	Other Family	92.66	146.90	164.98
	Total	911.33	1217.58	1453.60
Harvesting (Aug.16 to Sept.20)	Operator	383.68	467.90	467.90
	Full-time Family	41.31	85.58	188.86
	Other Family	50.68	80.34	90.23
	Total	475.67	633.82	755.99
Fall (Sept.21 to Oct.)	Operator	182.25	225.00	225.00
	Full-time Family	17.64	36.54	80.64
	Other Family	18.38	27.99	31.21
	Total	218.27	289.33	336.85
Winter (Nov. to March)	Operator	748.00	787.0	787.0
	Full-time Family	47.5	98.3	217.0
	Other Family	29.6	43.7	49.1
	Total	825.1	929.0	1053.1
TOTAL		3096.16	3931.06	4603.13

NOTE: "Full-time family" includes sons and brothers of the operator who usually contributes a full day's work. Other family is converted to the M.E.(Man Equivalent).

SOURCES: 1. How Labor is Used on Red River Valley Farms 1954, PP. 13-14 Economics Division, Canada Department of Agriculture.

2. The survey data of Morden-Winkler Project Area conducted by P.F.R.A.

APPENDIX TABLE 15

DESCRIPTION OF ACTIVITIES USED FOR THE NON-INTEGER AND MIXED-INTEGER PROGRAMMING PROBLEMS -SMALL(60 ACRE), MEDIUM(250 ACRE) AND LARGE(500 ACRE) FARMS-

Act. No.	Description of Activity	Unit
1	Dryland wheat fertilized	acre
2	Dryland barley fertilized	acre
3	Dryland oats fertilized	acre
4	Dryland feed grains fertilized	acre
5	Irrigated wheat on T_1 land	acre
6	Irrigated wheat on T_2 land	acre
7	Irrigated barley on T_1 land	acre
8	Irrigated barley on T_2 land	acre
9	Irrigated oats on T_1 land	acre
10	Irrigated oats on T_2 land	acre
11	Irrigated flax on T_1 land	acre
12	Irrigated flax on T_2 land	acre
13	Irrigated sugar beets on T_1 land; thinned and harvested by the farmer's own machines.	acre
14	Irrigated sugar beets on T_1 land; thinned by the farmer's own thinner and custom-harvested.	acre
15	Irrigated sugar beets on T_1 land; hand-thinned and harvested by the farmer's own harvester	acre
16	Irrigated sugar beets on T_1 land; hand-thinned and custom-harvested.	acre
17	Irrigated sugar beets on T_2 land; thinned and harvested by the farmer's own machines.	acre
18	Irrigated sugar beets on T_2 land; thinned by the farmer's own thinner and custom-harvested.	acre
19	Irrigated sugar beets on T_2 land; hand-thinned and harvested by the farmer's own harvester.	acre
20	Irrigated sugar beets on T_2 land; hand-thinned and custom-harvested.	acre
21	Irrigated potatoes on T_1 land; harvested by the farmer's digger.	acre
22	Irrigated potatoes on T_1 land; custom-harvested.	acre
23	Irrigated potatoes on T_2 land; harvested by the farmer's digger.	acre
24	Irrigated potatoes on T_2 land; custom-harvested.	acre
25	Irrigated sunflowers on T_1 land.	acre
26	Irrigated sunflowers on T_2 land.	acre
27	Irrigated fodder corn on T_1 land; harvested by the farmer's own corn harvester.	acre
28	Irrigated fodder corn on T_1 land; custom-harvested.	acre

(continued)

APPENDIX TABLE 15 (CONTINUED)

Act. No.	Description of Activity	Unit
29	Irrigated fodder corn on T_2 land; harvested by the farmer's own corn harvester.	acre
30	Irrigated fodder corn on T_2 land; custom-harvested.	acre
31	Irrigated field peas on T_1 land.	acre
32	Irrigated field peas on T_2 land.	acre
33	Irrigated tame pasture on T_1 land.	acre
34	Irrigated tame pasture on T_2 land.	acre
35	Irrigated tame hay on T_1 land; harvested by the farmer's own hay baler.	acre
36	Irrigated tame hay on T_1 land; hay loader.	acre
37	Irrigated tame hay on T_1 land; custom baler.	acre
38	Irrigated tame hay on T_2 land; harvested by the farmer's own hay baler.	acre
39	Irrigated tame hay on T_2 land; hay loader.	acre
40	Irrigated tame hay on T_2 land; custom baler.	acre
41	Irrigated feed grains on T_1 land.	acre
42	Irrigated feed grains on T_2 land.	acre
43	Dryland wheat not fertilized.	acre
44	Dryland barley not fertilized.	acre
45	Dryland oats not fertilized.	acre
46	Dryland flax not fertilized.	acre
47	Dryland sugar beets not fertilized; thinned and harvested by the farmer's own machines.	acre
48	Dryland sugar beets not fertilized; thinned by the farmer's own thinner and custom-harvested.	acre
49	Dryland sugar beets not fertilized; hand-thinned and harvested by the farmer's own harvester.	acre
50	Dryland sugar beets; hand-thinned and custom-harvested.	acre
51	Dryland potatoes fertilized (400 lbs. per acre); harvested by the farmer's own digger.	acre
52	Dryland potatoes fertilized (400 lbs. per acre); custom-digger.	acre
53	Dryland sunflowers not fertilized.	acre
54	Dryland fodder corn not fertilized; harvested by the farmer's own corn harvester.	acre
55	Dryland fodder corn not fertilized; custom-harvested.	acre
56	Dryland field peas not fertilized.	acre
57	Dryland tame pasture not fertilized.	acre
58	Dryland tame hay not fertilized; harvested by the farmer's own hay baler.	acre
59	Dryland tame hay not fertilized; hay loader.	acre

(continued)

APPENDIX TABLE 15 (continued)

Act. No.	Description of Activity	Unit
60	Dryland tame hay not fertilized; custom baler.	acre
61	Dryland feed grains not fertilized.	acre
62	Natural Pasture	
63	Cow-calf; spring-born calves are sold as stockers (450 lb.) in the fall; cow and calves are grazed.	cow
64	Feeder-calf; fall purchased 400 pound calves are finished on pasture in the next fall (850 lbs.).	head
65	Feed-lot 400; fall-purchased 400 pound calves are finished in feed-lots within 330 days (1000 lbs.).	lot
66	Feed-lot 650; fall-purchased yearlings (650 lbs) are finished in feed-lots within 300 days.(1,100 lbs.)	lot
67	Feed-lot 800; steers purchased at 800 lbs. in Nov. and May are finished at 1100 lbs. in feed-lots in March and Sept., respectively.	lot
68	Sow-hogs; a year-round hog operation is allowed through this activity; Sows are bred to farrow twice a year; hogs are sold at 400 lbs. (150 lbs. carcass weight).	sow
69	Selling wheat to the Wheat Board.	bushel
70	Selling barley to the Wheat Board.	bushel
71	Selling oats to the Wheat Board	bushel
72	Selling sugar beets to the Winnipeg factory.	ton
73	Selling potatoes through the Winnipeg market.	bushel
74	Purchase of feed grains.	100 lbs.
75	Purchase of hay.	ton
76	Hiring seasonal labour, spring.	hour
77	Hiring seasonal labour, summer.	hour
78	Hiring seasonal labour, harvesting season.	hour
79	Hiring seasonal labour, fall.	hour
80	Hiring special manual labour for sugar beets thinning.	hour
81	Hiring special manual labour for harvesting potatoes.	hour
82	Land-distribution activity.	acre
83	Purchase of forage corn harvester;	an integer unit
84	Purchase of hay baler.	an integer unit
85	Purchase of sugar beet thinner.	an integer unit
86	Purchase of potato digger.	an integer unit
87	Purchase of sugar beet harvester;	an integer unit
88	Purchase of seed-potato cutter;	an integer unit
89	Purchase of irrigation machines;	an integer unit
90	Purchase of irrigation water;	acre-inch
91	Operating capital loan	dollar

APPENDIX TABLE 16

DESCRIPTION OF CONSTRAINTS USED FOR THE NON-INTEGER AND MIXED-INTEGER PROGRAMMING PROBLEMS (SMALL, MEDIUM AND LARGE FARMS)

Constraint Inequality No.	Description of Constraints	Unit
1	Total land.	acre
2	Total crop land; 90% of the total land.	acre
3	Irrigable land, T_1; 56% of the total land.	acre
4	Irrigable land, T_2; 14% of the total land.	acre
5	Unimproved land; 10% of the total land.	acre
6	Restriction of repeated use of crop land for sugar beets.	acre
7	Restriction of repeated use of crop land for potatoes.	acre
8	Spring labour, Aprul to May.	hour
9	Summer labour, June to Aug. 15.	hour
10	Harvesting season labour, Aug.16 to Sept.20.	hour
11	Fall labour, Sept. 21 to October.	hour
12	Winter labour, November to March.	hour
13	Hand-thinning labour for sugar beets.	hour
14	Hand labour for potato harvest.	hour
15	Wheat	bushel
16	Barley	bushel
17	Oats	bushel
18	Sugar beets	ton
19	Potatoes	bushel

continued ....

APPENDIX TABLE 16 (continued)

Constraint Inequality No.	Description of Constraints	Unit
20	Feed grains (T.D.N.)	pound
21	Pasture (T.D.N.)	pound
22	Hay and corn silage (T.D.N.)	pound
23	Use of fodder corn harvester	acre
24	Use of hay baler	acre
25	Use of sugar beet thinner	acre
26	Use of potato digger	acre
27	Use of sugar beet harvester	acre
28	Use of seed potato cutter	acre
29	Use of irrigation machines	acre
30	Irrigation water	acre-inch
31	Loan available for irrigation-land-development and purchase of specialized machines	dollar
32	Annual operating capital	dollar
33	Upper limit of sunflower, $\frac{1}{2}$ of the total crop land	acre
34	Upper limit of field peas, 1/6 of the total crop land	acre
35	Special constraint of field peas; can be grown only after row-crops	acre
36	Constraint of quota on wheat sale; $\sum X_j - \sum 9X_s \leq 0$	bushel
37	The upper limit of operating capital loan	dollar
38	Integer constraint (Corn harvester)	an integer unit
39	Integer constraint (hay baler)	an integer unit

continued......

APPENDIX TABLE 16 (continued)

Constraint Inequality No.	Description of Constraints	Unit
40	Integer constraint (sugar beet thinner)	an integer unit
41	Integer constraint (potato digger)	an integer unit
42	Integer constraint (seed-potato cutter)	an integer unit
43	Integer constraint (sugar beet harvester)	an integer unit
44	Integer constraint (a set of irrigation machines)	an integer unit

APPENDIX TABLE 17

INPUT-OUTPUT COEFFICIENTS USED FOR THE NON-INTEGER AND MIXED-INTEGER PROGRAMMING PROBLEMS(250 ACRE FARM)

Constraint Number	Unit	Right Hand Sides	Activity Number 1	2	3	4	5	6
1	acre	250.						
2	acre	0.	1.	1.	1.	1.	1.	1.
3	acre	0.					1.	
4	acre	0.						1.
5	acre	0.						
6	acre	0.		1.				
7	acre	0.						
8	hour	861.	1.16	1.16	1.24	1.18	2.91	2.91
9	hour	1218.	.14	.14	.19	.14	2.97	2.97
10	hour	634.	1.46	1.46	1.53	1.47	2.07	2.07
11	hour	289.	1.46	.13	1.43	1.56	1.61	1.61
12	hour	929.						
13	hour	0.						
14	hour	0.						
15	bushel	0.	-28.				-45.	-45.
16	bushel	0.		-30.				
17	bushel	0.			-60.			
18	ton	0.						
19	bushel	0.						
20	pound	0.				-1450.		
21	pound	0.						
22.	pound	0.						
23	acre	0.						
24	acre	0.						
25	acre	0.						
26	acre	0.						
27	acre	0.						
28	acre	0.						
29	acre	0.					1.	1.
30	acre-inch	0.					10.3	10.3
31	dollar	0.					60.	90.
32	dollar	20,000.	11.20	10.89	13.07	7.55	20.14	22.09
33	acre	5,000.						
34	acre	0.						
35	acre	0.						
36	bushel	0.						
37	dollar	10,000.						
38	integer=							
39	integer=							
40	integer=							
41	integer=							
42	integer=							
43	integer=							
44	integer=							
Net Price	dollar		-11.20	-10.89	-13.07	-7.55	-20.14	-22.09

continued......

Constraint Number	Activity Number 7	8	9	10	11	12	13	14	15
1									
2	1.	1.	1.	1.	1.	1.	1.	1.	1.
3	1.		1.		1.		1.	1.	1.
4		1.		1.		1.			
5									
6							1.	1.	1.
7									
8	2.91	2.91	2.91	2.91	2.91	2.91	4.17	4.17	4.17
9	2.97	2.97	2.97	2.97	2.97	2.97	12.21	12.21	29.52
10	2.07	2.07	2.07	2.07	2.07	2.07	5.48	1.00	5.48
11	1.61	1.61	1.61	1.61	1.61	1.61	1.62	1.62	1.62
12									
13									-16.66
14									
15									
16	-55.	-55.							
17			-80.	-80.					
18							-15.	-15.	-15.
19									
20									
21									
22									
23									
24									
25							1.	1.	
26									
27							1.		1.
28									
29	1.	1.	1.	1.	1.	1.	1.	1.	1.
30	10.3	10.3	10.3	10.3	13.2	13.2	10.7	10.7	10.7
31	60.	90.	60.	90.	60.	90.	60.	60.	60.
32	19.99	21.94	20.86	22.87	13.71	15.66	71.66	93.55	75.51
33									
34									
35							-1.	-1.	-1.
36									
37									
38									
39									
40									
41									
42									
43									
44									
	-19.99	-21.94	-20.86	-22.87	56.67	54.72	-71.66	-93.55	-71.51

continued.......

Constraint No.	Activity Number							
	16	17	18	19	20	21	22	23
1								
2	1.	1.	1.	1.	1.	1.	1.	1.
3	1.					1.	1.	
4		1.	1.	1.	1.			1.
5								
6	1.	1.	1.	1.	1.			
7						1.	1.	1.
8	4.17	4.17	4.17	4.17	4.17	4.02	4.02	4.02
9	29.52	12.21	12.21	29.52	29.52	9.55	9.55	9.55
10	1.	5.48	1.	5.48	1.	16.14	13.00	16.14
11	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
12								
13	-16.66			-16.66	-16.66			
14						-13.	-13.	-13.
15								
16								
17								
18	-15.	-15.	-15.	-15.	-15.			
19						-18.	-18.	-18.
20								
21								
22								
23								
24								
25		1.	1.					
26						1.		1.
27		1.		1.				
28						1.	1.	1.
29	1.	1.	1.	1.	1.	1.	1.	1.
30	10.7	10.7	10.7	10.7	10.7	9.	9.	9.
31	60.	90.	90.	90.	90.	60.	60.	90.
32	93.4	68.61	95.5	73.46	95.35	127.7	148.61	129.65
33								
34								
35	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.
36								
37								
38								
39								
40								
41								
42								
43								
44								
	-93.40	-68.61	-95.50	-73.46	-95.35	-127.7	-148.61	-129.65

continued.....

Constraint No.	Activity Number								
	24	25	26	27	28	29	30	31	32
1									
2	1.	1.	1.	1.	1.	1.	1.	1.	1.
3		1.		1.	1.			1.	
4	1.		1.			1.	1.		1.
5									
6									
7	1.								
8	4.02	2.91	2.91	3.26	3.26	3.26	3.26	2.91	2.91
9	9.55	3.65	3.65	7.61	7.61	7.61	7.61	3.10	3.10
10	13.			4.29		4.29		3.52	3.52
11	1.62	5.42	5.42	1.61	1.61	1.61	1.61	1.38	1.38
12									
13									
14	-13.								
15									
16									
17									
18									
19	-18.								
20									
21									
22				-4000.	-4000.	-4000.	-4000.		
23				1.		1.			
24									
25									
26									
27									
28	1.								
29	1.	1.	1.	1.	1.	1.	1.	1.	1.
30	9.	10.6	10.6	12.1	12.1	12.1	12.1	5.2	5.2
31	90.	60.	90.	60.	60.	90.	90.	60.	90.
32	150.56	13.14	15.09	13.23	17.03	15.18	18.98	23.31	25.26
33		1.	1.						
34								1.	1.
35	-1.	-1.	-1.	-1.	-1.	-1.	-1.	1.	1.
36									
37									
38									
39									
40									
41									
42									
43									
44									
	-150.56	36.25	34.30	-13.23	-17.03	-15.18	-18.98	44.67	42.72

continued......

Constraint	Activity Number							
No.	33	34	35	36	37	38	39	40
1								
2	1.	1.	1.	1.	1.	1.	1.	1.
3	1.		1.	1.	1.			
4		1.				1.	1.	1.
5								
6								
7								
8	1.17	1.17	1.17	1.17	1.17	1.17	1.17	1.17
9	5.35	5.35	11.24	12.73	3.73	11.24	12.73	3.73
10	.50	.50	.50	.50	.50	.50	.50	.50
11	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
12								
13								
14								
15	-12.4	-12.4	-12.4	-12.4	-12.4	-12.4	-12.4	-12.4
16								
17								
18								
19								
20								
21	-1560.	-1560.						
22			-2180.	-2180.	-2180.	-2180.	-2180.	-2180.
23								
24			.66			.66		
25								
26								
27								
28								
29	.33	.33	.33	.33	.33	.33	.33	.33
30	16.4	16.4	16.4	16.4	16.4	16.4	16.4	16.4
31	60	90.	60.	60.	60.	90.	90.	90.
32	16.36	18.31	18.07	17.89	24.06	20.02	19.84	26.01
33								
34								
35	-.33	-.33	-.33	-.33	-.33	-.33	-.33	-.33
36								
37								
38								
39								
40								
41								
42								
43								
44								
	-16.36	-18.31	-18.07	-17.89	-24.06	-20.02	-19.84	-26.01

continued......

APPENDIX TABLE 17 (continued)

Constraint No.	Activity Number 41	42	43	44	45	46	47	48	49
1									
2	1.	1.	1.	1.	1.	1.	1.	1.	1.
3	1.								
4		1.							
5									
6							1.	1.	1.
7									
8	2.91	2.91	1.16	1.18	1.24	1.46	1.94	1.94	1.94
9	2.97	2.97	.14	.14	.19	.14	4.16	4.16	21.6
10	2.07	2.07	1.46	1.47	1.53	1.44	5.30	1.	5.3
11	1.61	1.61	1.46	1.56	1.43	1.47	1.69	1.69	1.69
12									
13									-16.66
14									
15			-25.						
16				-29.					
17					-45.				
18							-10.	-10.	-10.
19									
20	-1924.	-1924.							
21									
22									
23									
24									
25							1.	1.	
26									
27							1.		1.
28									
29	1.	1.							
30	10.3	10.3							
31	60.	90.							
32	14.41	16.36	7.58	7.65	7.68	5.75	43.66	64.18	44.04
33									
34									
35							-1.	-1.	-1.
36									
37									
38									
39									
40									
41									
42									
43									
44									
	-14.41	-16.36	-7.58	-7.65	-7.68	27.91	-43.66	-64.18	-44.04

Continued.......

Constraint No.	Activity Number								
	50	51	52	53	54	55	56	57	58
1									
2	1.	1.	1.	1.	1.	1.	1.	1.	1.
3									
4									
5									
6	1.								
7		1.	1.						
8	1.94	5.17	5.17	1.33	1.61	1.61	1.43	.45	.45
9	21.6	2.98	2.98	.60	1.65	1.65	.44	.80	4.06
10	1.	15.30	13.		2.86		3.39	.46	.40
11	1.69	1.37	1.37	4.76	.96	.96	1.38	.56	.56
12									
13	-16.66								
14		-13.	-13.						
15								-6.6	-6.6
16									
17									
18	-10.								
19		-100.	-100.						
20									
21								-878.	
22					-1260.	-1260.			-1128.
23					1.				
24									.66
25									
26		1.							
27									
28		1.	1.						
29									
30									
31									
32	63.52	89.29	111.29	3.06	3.95	7.87	13.05	5.93	6.08
33				1.					
34							1.		
35	-1.	-1.	-1.	-1.	-1.	-1.	1.	-.33	-.33
36									
37									
38									
39									
40									
41									
42									
43									
44									
	-63.52	-89.29	-111.29	32.15	-3.95	-7.87	24.03	-5.93	-6.08

continued......

Constraint No.	Activity Number								
	59	60	61	62	63	64	65	66	67
1									
2	1.	1.	1.						
3									
4									
5				1.					
6									
7									
8	.45	.45	1.18		3.70	1.80	2.80	2.80	2.80
9	4.94	.80	.14		3.80	1.18	3.50	3.50	3.50
10	.40	.40	1.47		2.10	.60	1.63	1.63	1.63
11	.56	.56	1.56		2.50	.73	1.87	1.87	1.87
12					11.70	7.00	7.00	7.00	7.00
13									
14									
15	-6.6	-6.6							
16									
17									
18									
19									
20			-1046.		310.	750.	2594.	3337.	4009.
21				-1000.	2602.	1865.			
22	-1128.	-1128.			2402.	1304.	1297.	1523.	2005.
23									
24									
25									
26									
27									
28									
29									
30									
31									
32	5.99	9.83	4.29		16.	17.	27.73	34.99	60.6
33									
34									
35	-.33	-.33							
36									
37									
38									
39									
40									
41									
42									
43									
44									
	-5.99	-9.83	-4.29	0.0	80.00	70.96	124.4	139.16	150.14

continued.......

Constraint No.	Activity Number										
	68	69	70	71	72	73	74	75	76	77	78
1											
2											
3											
4											
5											
6											
7											
8	7.50								-1.		
9	4.70									-1.	
10	10.15										-1.
11	10.90										
12	16.75										
13											
14											
15		1.									
16			1.								
17				1.							
18					1.						
19						1.					
20	8900.						-74.				
21											
22								-940.			
23											
24											
25											
26											
27											
28											
29											
30											
31											
32	200.						2.1	16.	1.7	1.7	1.7
33											
34											
35											
36		1.	1.	1.							
37											
38											
39											
40											
41											
42											
43											
44											
	357.	1.74	1.11	.70	14.96	1.45	-2.1	-16.	-1.7	-1.7	-1.7

continued........

Constraint No.	Activity Number 79	80	81	82	83	84	85	86	87	88
1				1.						
2				-.9						
3				-.56						
4				-.14						
5				-.10						
6				-.25						
7				-.25						
8										
9		-1.								
10			-1.							
11	-1.									
12										
13		1.								
14			1.							
15										
16										
17										
18										
19										
20										
21										
22										
23					-40.					
24						-150.				
25							-130.			
26								-50.	-50.	
27									-50.	
28										-300.
29										
30										
31					4000.	2400.	250.	1600.	7000.	2000.
32	1.7	1.4	1.25		700.	420.	42.	348.	1315.	330.
33	-.45									
34	-.15									
35										
36	-9.									
37										
38										
39										
40										
41										
42										
43										
44										
	-1.7	-1.4	-1.25	0.0	-700.	-420.	-42.	-348.	-1315.	-330.

continued.......

APPENDIX TABLE 17 (continued)

Constraint Number	Activity Number 89	90	91
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29	-250.		
30		-1.	
31	2500.		
32	538.	2.	-1.
33			
34			
35			
36			
37			1.
38			
39			
40			
41			
42			
43			
44			
	-538	-2.	-.055

APPENDIX TABLE 18

DESCRIPTION OF ACTIVITIES[1] USED FOR THE STOCHASTIC PROGRAMMING PROBLEM (250 ACRE FARM UNDER IRRIGATION CONDITIONS)

Activity No.	Description of Activities
1	Dryland wheat fertilized.
2	Dryland barley fertilized.
3	Dryland oats fertilized.
4	Irrigated wheat on T_1 land.
5	Irrigated wheat on T_2 land.
6	Irrigated barley on T_1 land.
7	Irrigated barley on T_2 land.
8	Irrigated oats on T_1 land.
9	Irrigated oats on T_2 land.
10	Irrigated flax on T_1 land.
11	Irrigated flax on T_2 land.
12	Irrigated sugar beets grown on T_1 land; the farmer's own thinner is used for thinning operation and his own harvester, for harvesting operation.
13	Irrigated sugar beets on T_1 land; thinned by the farmer's own thinner and custom-harvested.
14	Irrigated sugar beets grown on T_1 land; hand-thinned and harvested by the farmer's own harvester.
15	Irrigated sugar beets grown on T_1 land; hand-thinned and custom-harvested.
16	Irrigated sugar beets grown on T_2 land; thinned by the farmer's own thinner and harvested by his harvester.
17	Irrigated sugar beets grown on T_2 land; thinned by the farmer's own thinner and custom-harvested.
18	Irrigated sugar beets grown on T_2 land; hand-thinned and harvested by the farmer's own harvester.
19	Irrigated sugar beets grown on T_2 land; hand-thinned and custom-harvested.
20	Irrigated potatoes grown on T_1 land; harvested by the farmer's own digger.
21	Irrigated potatoes grown on T_1 land; custom-harvested.
22	Irrigated potatoes grown on T_2 land; harvested by the farmer's own digger.
23	Irrigated potatoes grown on T_2 land; custom-harvested.

[1]Units of all activities are "one area" unless otherwise specified.

continued.....

APPENDIX TABLE 18 (Continued)

Activity No.	Description of Activities
24	Dryland wheat not fertilized.
25	Dryland barley not fertilized.
26	Dryland oats not fertilized.
27	Dryland flax not fertilized.
28	Dryland sugar beets; thinned by the farmer's own thinner and harvested by his own harvester.
29	Dryland sugar beets; thinned by the farmer's own thinner and custom-harvested.
30	Dryland sugar beets; hand-thinned and harvested by the farmer's own harvester.
31	Dryland sugar beets; hand-thinned and custom-harvested.
32	Dryland potatoes with fertilizer; harvested by the farmer's own digger.
33	Dryland potatoes with fertilizer; custom-harvested.
34	Dryland sunflowers not fertilized.
35	Dryland field peas not fertilized.
36	Dryland cow-calf; beef cows are grazed and spring-born calves are sold as stockers in the fall; feed grains and forages are grown on dryland.
37	Dryland feeder calf; fall-purchased calves (400 lbs.) are finished on pasture with feed grains and sold at about 850 lbs. next fall; no housing; feed grains and forages are grown under dryland conditions.
38	Dryland feed-lot 400; calves purchased at 400 lbs. in the fall are fattened in feed-lots and sold at about 1000 lbs. after about 330 days; feed grains and forages are grown under dryland conditions.
39	Dryland feed-lot 650; yearlings purchased at 650 lbs. in the fall are fattened in feed-lots and sold at about 1100 lbs. after about 300 days; feed grains and forages are grown under dryland conditions.
40	Dryland feed-lot 800; steers purchased at 800 lbs. in the fall and spring, are fattened in feed-lots and sold at about 1200 lbs. after about 150 days; feed grains and forages are grown under dryland conditions.
41	Dryland sow-hogs; a year-round hog operation is allowed through this activity; sows are bred to farrow in October and April; market hogs are sold at 240 lbs. (150 lbs. carcus weight); feed grains are grown under dryland conditions.
42	Irrigated cow-calf, T_1 land; Same as the activity, no. 36 except that feed grains and forages are grown under irrigation conditions.

continued.....

APPENDIX TABLE 18 (Continued)

Activity No.	Description of Activities	
43	Irrigated feeder-calf, T_1 land; same as activity 37 except that feed grains and forages are grown under irrigation conditions.	
44	Irrigated feed-lot 400, T_1 land; same as activity 38 except that feed grains and forages are grown under irrigation conditions.	
45	Irrigated feed-lot 650, T_1 land; same as activity 39 except that feed grains and forages are grown under irrigation conditions.	
46	Irrigated feed-lot 800, T_1 land; same as activity 40 except that feed grains and forages are grown under irrigation conditions.	
47	Irrigated sow-hogs, T_1 land; same as activity no. 41 except that feed grains are grown under irrigation.	
48	Irrigated cow-calf, T_2 land; same as activity no. 42.	
49	Irrigated feeder-calf, T_2 land; same as activity no. 43.	
50	Irrigated feed-lot 400, T_2 land; same as activity no. 44.	
51	Irrigated feed-lot 650, T_2 land; same as activity no. 45.	
52	Irrigated feed-lot 800, T_2 land; same as activity no. 46.	
53	Irrigated sow-hogs, T_2 land; same as 47.	
54	Hire of seasonal labour, spring (April to May)	one hour unit
55	Hire of seasonal labour, summer (June to Aug. 15)	one hour unit
56	Hire of seasonal labour, harvesting (Aug. to Sept. 20)	one hour unit
57	Hire of seasonal labour, fall (Sept. 21 to October)	one hour unit
58	Hire of special manual labour, hand-thinning	one hour unit
59	Hire of special manual labour, potato harvesting	one hour unit
60	Land distribution activity.	
61	Purchase of sugar beet thinner,	at a non-integral purchasing unit
62	Purchase of potato digger,	at a non-integral purchasing unit
63	Purchase of sugar beet harvester,	at a non-integral purchasing unit
64	Purchase of seed-potato cutter,	at a non-integral purchasing unit
65	Purchase of irrigation machines,	at a non-integral purchasing unit
66	Buying irrigation water,	acre-inch unit
67	Annual operating capital loan	dollar unit
68	Feeder-calf on unimproved land; feed grain and hay are purchased.	
69	Specified acreage supply activity.	

APPENDIX TABLE 19

CONSTRAINT INEQUALITIES OF THE STOCHASTIC PROGRAMMING PROBLEM (250 ACRE FARM)

Constraint Inequality No.	Description of Constraint Inequality	Unit
1	Constraint of the total farm land	acre
2	Constraint of the total crop land (90% of the total farm land)	acre
3	Constraint of irrigable T_1 land (56% of the total farm land)	acre
4	Constraint of irrigable T_2 land (14% of the total farm land)	acre
5	Constraint of unimproved land (10% of the total farm land)	acre
6	Restriction of repeated use of crop land for sugar beet production (The total acreage of sugar beets can not exceed $\frac{1}{4}$ of the total crop land.)	acre
7	Restriction of repeated use of crop land for potato production (Its total acreage can not exceed $\frac{1}{4}$ of the total crop land.)	acre
8	Constraint of spring labour, April to May	hour
9	Constraint of summer labour, June to Aug. 15	hour
10	Constraint of harvesting labour, Aug. 16 to Sept. 20	hour
11	Constraint of fall labour, Sept. 21 to Oct.	hour
12	Constraint of winter labour, Nov. to March	hour
13	Constraint of hand-thinning labour used for sugar beet operation	hour
14	Constraint of hand labour used for potato harvesting	hour
15	Constraint of sugar beet thinner utilization	acre
16	Constraint of potato digger utilization	acre
17	Constraint of sugar beet harvester utilization	acre
18	Constraint of seed-potato cutter utilization	acre
19	Constraint of use of irrigation machines	acre
20	Constraint of use of irrigation water	acre-inch
21	The upper limit of loan available for irrigation land-development and the purchase of specialized machines.	dollar
22	The use of operating capital	dollar
23	The upper limit of sunflower production ($\frac{1}{2}$ of the total crop land)	acre
24	The upper limit of field peas production (1/6 of the total crop land)	acre
25	Special constraint of field peas, (Field peas can be grown only after the row crops.)	acre
26	The upper limit of operating capital loan.	dollar
27	The specified acreage constraint inequality	acre
28	The constraint of quota on wheat sale	bushel

APPENDIX TABLE 20

DESCRIPTION OF ACTIVITIES AND CONSTRAINTS USED FOR LINEAR PROGRAMMING ANALYSIS OF CROP INSURANCE ALTERNATIVES (250 ACRE FARM UNDER DRYLAND CONDITIONS)

Activity Number	Description of Activity Name of Activity	Bushel Guaranteed Per Acre	Insured Price	Premium Per Acre
		bushels	dollars per bu.	dollars
1.	Wheat with Insurance Scheme 1	12.59	1.47	1.14
2.	Wheat with Insurance Scheme 2	14.73	1.47	1.84
3.	Wheat with Insurance Scheme 3	16.76	1.47	2.70
4.	Wheat with Insurance Scheme 4	12.59	1.28	1.00
5.	Wheat with Insurance Scheme 5	14.73	1.28	1.60
6.	Wheat with Insurance Scheme 6	16.76	1.28	2.35
7.	Oats with Insurance Scheme 1	22.50	0.51	0.85
8.	Oats with Insurance Scheme 2	26.25	0.51	1.35
9.	Oats with Insurance Scheme 3	30.00	0.51	2.03
10.	Oats with Insurance Scheme 4	22.50	0.46	0.77
11.	Oats with Insurance Scheme 5	26.25	0.46	1.22
12.	Oats with Insurance Scheme 6	30.00	0.46	1.83
13.	Barley with Insurance Scheme 1	14.97	0.84	1.39
14.	Barley with Insurance Scheme 2	17.52	0.84	2.01
15.	Barley with Insurance Scheme 3	19.90	0.84	2.71
16.	Barley with Insurance Scheme 4	14.97	0.75	1.24
17.	Barley with Insurance Scheme 5	17.52	0.75	1.79
18.	Barley with Insurance Scheme 6	19.90	0.75	2.42

continued.....

APPENDIX TABLE 20 (Continued)

Activity Number	Name of Activity	Bushel Guaranteed Per Acre	Insured Price	Premium Per Acre
		bushels	dollars per bu.	dollars
19.	Flax with Insurance Scheme 1	5.10	2.60	1.25
20.	Flax with Insurance Scheme 2	5.90	2.60	1.94
21.	Flax with Insurance Scheme 3	6.85	2.60	2.90
		tons	dollar per ton	
22.	Sugar Beets with Insurance Scheme 1; thinned by the farmer's own thinner and harvested by his own harvester.	7.4	12.25	1.65
23.	Sugar Beets with Insurance Scheme 1; thinned by the farmer's own thinner and custom-harvested.	7.4	12.25	1.65
24.	Sugar Beets with Insurance Scheme 1; thinned by manual labour and harvested by the farmer's own harvester.	7.4	12.25	1.65
25.	Sugar Beets with Insurance Scheme 1; thinned by manual labour and custom-harvested.	7.4	12.25	1.65
26.	Sugar Beets with Insurance Scheme 2; thinned by the farmer's own thinner and harvested by his harvester.	8.7	12.25	3.17
27.	Sugar Beets with Insurance Scheme 2; thinned by the farmer's thinner and custom-harvested.	8.7	12.25	3.17
28.	Sugar Beets with Insurance Scheme 2; hand-thinned and harvested by the farmer's own harvester.	8.7	12.25	3.17
29.	Sugar Beets with Insurance Scheme 2; hand-thinned and custom-harvested.	8.7	12.25	3.17
30.	Sugar Beets with Insurance Scheme 3; thinned by the farmer's own thinner and harvested by his harvester.	9.9	12.25	6.02

continued.....

APPENDIX TABLE 20 (Continued)

Activity Number	Name of Activity	Bushel Guaranteed Per Acre	Insured Price	Premium Per Acre
		tons	dollar per ton	dollars
31.	Sugar Beets with Insurance Scheme 3; thinned by the farmer's own thinner and custom-harvested.	9.9	12.25	6.02
32.	Sugar Beets with Insurance Scheme 3; hand-thinned and harvested by the farmer's own harvester.	9.9	12.25	6.02
33.	Sugar Beets with Insurance Scheme 3; hand-thinned and custom-harvested.	9.9	12.25	6.02
34.	Hiring Spring Labor			
35.	Hiring Summer Labor			
36.	Hiring Harvesting Labor			
37.	Hiring Fall Labor			
38.	Hiring Hand-thinning Labor			
39.	Land-distribution Activity			
40.	Annual Op. Capital Loan			
41.	Purchasing Sugar Beet Thinner			
42.	Purchasing Sugar Beet Harvester			
43.	Specified Acreage Supply Activity			

NOTE: Crop insurance scheme 1 to 6, are defined as follows:

Insurance Scheme No.	Bushel Coverage	Price Option
	percent	
1	60	high
2	70	high
3	80	high
4	60	low
5	70	low
6	80	low

With the insurance scheme 1, for example, 60 percent of the long run average yield of a crop coupled with the high price option is guaranteed. Sugar beets and flax have no price options. Net price coefficients of insured crop activities are calculated by:

$$\begin{pmatrix}\text{Insured}\\\text{Price}\end{pmatrix} \times \begin{pmatrix}\text{Long Run}\\\text{Average Yield}\\\text{Per Acre}\end{pmatrix} \times \begin{pmatrix}\text{Percent}\\\text{Coverage}\end{pmatrix} - \begin{pmatrix}\text{Variable Costs and}\\\text{Insurance Premium}\\\text{Per Acre}\end{pmatrix}$$

continued.........

APPENDIX TABLE 20 (Continued)

Constraint Number	Description of Constraint
1	Total land
2	Crop land
3	Unimproved land
4	Spring labour
5	Summer labour
6	Harvesting labour
7	Fall labour
8	Sugar beet hand-thinning labour
9	Sugar beet thinner
10	Sugar beet harvester
11	Ann. operating capital
12	Upper limit of annual operating capital loan
13	Specified acreage constraint equation
14	Quota
15	Upper limit of sugar beet acreage
16	Integer constraint
17	integer constraint

APPENDIX TABLE 20-B

NET PRICE COEFFICIENTS OF INSURABLE CROPS USED FOR LINEAR PROGRAMMING ANALYSIS (DOLLAR)

Activity Number	Net Price	Activity Number	Net Price	Activity Number	Net Price	Activity Number	Net Price
1	9.79	12	4.11	23	24.83	34	- 1.7
2	12.23	13	3.53	24	44.96	35	- 1.7
3	14.35	14	5.06	25	25.48	36	-1.7
4	7.45	15	6.36	26	59.75	37	- 1.7
5	9.67	16	2.34	27	39.23	38	- 1.4
6	11.52	17	3.70	28	59.37	39	0
7	2.76	18	4.86	29	39.89	40	- .055
8	4.18	19	6.26	30	71.60	41	-42.00
9	5.41	20	7.66	31	51.08	42	- 1315.00
10	1.72	21	9.16	32	71.22	43	0
11	2.99	22	45.34	33	51.74	44	0

APPENDIX TABLE 21

THE RESULTS OF REGRESSION FUNCTIONS FITTED TO THE TIME SERIES OF NET PRICES (53 ACTIVITIES) 1954-1965

No of Act	Regress Coefficient,b	Constant terms	Student's t	R^2	No of Act	Regress Coefficient,b	Constant terms	Student's t	R^2
1	1.744	24.523	2.963	0.47	28	3.857	79.61	1.768	0.24
2	2.047	23.732	4.369	0.66	29	3.857	59.09	1.768	0.24
3	1.567	21.740	3.018	0.48	30	3.857	79.23	1.768	0.24
4	3.337	34.290	5.164	0.73	31	3.867	59.75	1.768	0.24
5	3.337	32.340	5.164	0.73	32	1.040	-25.96	2.194	0.32
6	2.634	23.986	5.286	0.74	33	1.072	-49.23	2.174	0.32
7	2.634	22.038	5.285	0.74	34	9.951	23.46	1.475	0.18
8	1.505	24.696	3.326	0.53	35	1.573	11.92	2.723	0.43
9	1.505	22.686	3.326	0.53	36	7.669	9.76	2.254	0.34
10	1.164	51.436	6.159	0.04	37	1.657	0.40	3.509	0.55
11	1.164	49.486	6.159	0.04	38	2.909	11.08	3.957	0.61
12	4.156	123.64	1.125	0.11	39	3.182	4.16	4.108	0.63
13	4.153	101.77	1.124	0.11	40	4.057	-10.16	4.114	0.63
14	4.153	123.81	1.124	0.11	41	2.104	30.57	2.041	0.29
15	4.153	101.92	1.124	0.11	42	1.994	12.61	4.192	0.64
16	4.153	126.71	1.124	0.11	43	3.524	2.97	5.641	0.76
17	4.153	99.82	1.124	0.11	44	4.459	13.30	4.129	0.63
18	4.153	121.86	1.124	0.11	45	4.907	1.02	4.134	0.63
19	3.839	102.84	1.035	0.10	46	6.166	-20.88	3.789	0.59
20	1.693	8.82	2.162	0.32	47	2.126	41.50	1.719	0.23
21	1.693	-12.09	2.162	0.32	48	1.994	10.66	4.192	0.64
22	1.693	6.84	2.163	0.32	49	3.419	2.54	4.995	0.71
23	1.725	-16.92	2.205	0.33	50	4.460	11.35	4.127	0.63
24	1.169	23.89	2.709	0.42	51	4.907	-0.93	4.134	0.63
25	2.089	11.20	3.768	0.59	52	6.166	-22.83	3.789	0.59
26	1.431	14.00	3.023	0.48	53	2.222	39.25	1.801	0.25
27	6.062	25.60	9.412	0.08					

NOTE: degree of freedom = 10

Level of significant	0.10	0.05	0.025
Student's t value	1.372	1.812	2.228

APPENDIX TABLE 22

OPTIMAL SOLUTIONS OF THE MIXED-INTEGER PROGRAMMING PROBLEM WITH VARIOUS PRICES OF WATER (250 ACRE FARM)

Activity No.	Activity	Unit	Price of Water (dollar per acre-inch) 0 ~ 0.688	0.689~ 0.887	0.888~ 1.710	1.711~ 1.983	1.984~ 2.139	2.140~ 2.620	2.621~
1	Wheat (D F)	acre	26.5	39.0	45.3	45.3	45.3	50.5	0
4	Feed grain (D F)	acre	23.5	46.0	57.1	57.1	57.1	81.7	162.5
11	Flax T_1 (I F)	acre	66.2	27.3	0	0	0	0	0
14	Sugar beet T_1 (I F)	acre	27.5	62.5	35.0	35.0	35.0	60.6	0
48	Sugar beet (D F)	acre	0	0	27.5	27.5	27.5	1.9	62.5
21	Potato T_1 (I F)	acre	38.7	38.7	46.7	46.7	46.7	30.3	0
28	Fodder corn T_1 (I F)	acre	7.4	11.4	13.5	13.5	13.5	0	0
18	Sugar beet T_2 (I F)	acre	35.0	0	0	0	0	0	0
62	Natural pasture	acre	25.0	25.0	25.0	25.0	25.0	25.0	25.0
64	Feeder calf	No.	13.4	13.4	13.4	13.4	13.4	13.4	13.4
65	Feed lot 400	lot	9.3	21.8	28.0	28.0	28.0	41.8	106.8
69	Sell wheat	bu.	741.4	1,093.0	1,267.2	1,267.2	1,267.2	1,414.9	0
72	Sell sugar beets	ton	937.5	937.5	800.0	800.0	800.0	928.2	625.0
73	Sell potatoes	bu.	7,005.3	6,972.8	8,405.2	8,405.2	8,405.2	5,452.4	0
89	Purchase irrigation machines	No.	1	1	1	1	1	1	0
85	Purchase sugar beet thinner	No.	1	1	1	1	1	1	1
86	Purchase potato digger	No.	1	1	1	1	1	1	0
88	Purchase potato seed cutter	No.	1	1	1	1	1	1	0
90	Irrigation water	acre-inch	1982.2	1516.5	957.1	957.1	957.1	921.5	0
77	Hire summer labor	hr.	224.8	187.4	0	0	0	0	0
78	Hire harvest labor	hr.	0	0	0	0	0	0	0
79	Hire fall labor	hr.	96.2	116.7	128.8	128.8	128.8	150.6	336.7
81	Hire potato-harvest hand labor	hr.	290.1	278.4	385.9	385.9	385.9	187.4	0
91	Loan ann. operating capital	$	10,000	10,000	10,000	10,000	10,000	10,000	10,000
	Loan Irrigation Develp. capital	$	17,900	14,750	12,056	12,056	12,056	11,806	250
75	Buy hay	ton	0	0	0	0	0	76.2	165.9
74	Buy feed grain	bu.	0	0	0	0	0	0	1324.5
	Functional value	$	15,771	15,647	14,490	14,241	14,212	14,079	13,635

NOTE: Functionals are calculated for the lowest price of water in each price range. "DF" and "IF" in brackets indicate "Fertilized on dryland" and "Fertilized under irrigation", respectively.

APPENDIX TABLE 23

OPTIMAL SOLUTIONS OF THE NON-INTEGER LINEAR PROGRAMMING PROBLEM WITH VARIOUS UPPER LIMITS OF OPERATING CAPITAL LOAN (250 ACRE FARM)

Act. No.	Activity	Unit	Operating Capital Loan (dollars) 0	500	1000	1500	2000	2500	3000
1	Wheat (D F)	acre	0	0	0	0	0	0	0
43	Wheat (D)	acre	61.1	58.4	55.6	61.1	67.5	67.5	67.5
4	Feed grain (D F)	acre	68.9	64.7	60.5	68.4	77.8	77.8	77.8
48	Sugar beets (D)	acre	39.7	49.3	58.9	62.5	56.7	41.0	25.3
14	Sugar beets (I F)	acre	0	0	0	0	5.8	21.5	37.2
28	Fodder corn (I F)	acre	0	0	0	8.6	17.2	17.2	17.2
21	Potatoes (I F)	acre	0	0	0	0	0	0	0
62	Natural Pasture	acre	25.0	25.0	25.0	25.0	25.0	25.0	25.0
60	Hay (D)	acre	55.3	52.6	49.9	24.4	0	0	0
75	Buying Hay	ton	0	0	0	0	0	0	0
65	Feed lot 400	lot	34.6	32.3	30.0	34.4	39.6	39.6	39.6
64	Feeder calf	head	13.4	13.4	13.4	13.4	13.4	13.4	13.4
69	Sell wheat	bu.	1892.9	1806.1	1719.4	1687.5	1687.5	1687.5	1687.5
72	Sell sugar beets	ton	396.8	493.2	589.6	625.0	654.1	732.6	811.1
73	Sell potatoes	bu.	0	0	0	0	0	0	0
79	Hire fall labor	hour	80.3	80.1	80.0	114.1	147.7	146.6	145.5
81	Hire potato hand labor	hour	0	0	0	0	0	0	0
85	Pur. S.B.thinner	No.	0.169	0.244	0.320	0.332	0.338	0.393	0.449
89	Pur. irrigation machine	No.	0	0	0	0.035	0.092	0.155	0.218
86	Pur. potato digger	No.	0	0	0	0	0	0	0
88	Pur. seed cutter	No.	0	0	0	0	0	0	0
91	Loan ann. oper. capital	$	0	500	1000	1500	2000	2500	3000
	Loan development capital	$	7	28	48	656	1667	2779	3891
90	Irrigation water	acre-inch	0	0	0	104.5	270.6	438.6	606.5
	Functional	$	10,514	10,932	11,351	11,739	12,088	12,399	12,709

NOTE: "DF", "D" and "IF" in brackets indicate"fertilized on dryland", "on dryland with no fertilizer" and "fertilized under irrigation", respectively. The price of water is fixed at $2.00 per acre-inch.

continued......

APPENDIX TABLE 23 (Continued)

Operating Capital Loan (dollars)											
3500	4000	4500	5000	5500	6000	6500	7000	7500	8500	9000	9500
0	0	0	0	0	0	54.0	43.5	27.1	0	48.0	49.5
67.5	54.3	19.6	26.4	40.5	55.0	0	0	0	0	0	0
77.8	89.0	118.4	108.2	91.0	73.7	73.5	89.7	111.2	147.2	86.9	79.6
9.6	0	0	0	0	0	1.6	0	0	0	0	4.9
49.5	62.5	62.5	62.5	62.5	62.5	60.9	62.5	62.5	62.5	62.5	57.6
17.2	19.2	24.6	22.7	19.6	16.5	15.4	11.2	7.7	0	0	0
0	0	0	5.1	11.2	17.3	19.6	18.1	16.4	14.3	27.6	33.4
25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	4.4	34.7	66.2	122.9	80.3	74.6
39.6	45.9	12.3	56.6	47.0	37.4	37.2	46.2	58.3	78.4	44.7	40.6
13.4	13.4	13.4	13.4	13.4	13.4	13.4	13.4	13.4	13.4	13.4	13.4
1687.5	1357.2	489.3	660.2	1017.5	1374.7	1511.4	1217.4	759.6	0	1342.8	1387.3
889.3	937.5	937.5	937.5	937.5	937.5	937.5	937.5	937.5	937.5	937.5	912.8
0	0	0	925.0	2015.9	3106.8	3522.3	3265.1	2959.0	2580.2	4969.5	6004.9
144.4	156.8	191.3	180.2	161.0	141.8	141.8	159.3	183.2	223.1	156.0	148.9
0	0	0	0	0	0	0	0	0	12.2	152.8	230.5
0.505	0.543	0.554	0.565	0.576	0.588	0.481	0.481	0.481	0.481	0.481	0.481
0.280	0.327	0.348	0.362	0.373	0.385	0.383	0.381	0.365	0.384	0.364	0.364
0	0	0	0.200	0.278	0.360	0.391	0.363	0.404	0.437	0.552	0.667
0	0	0	0.033	0.046	0.060	0.065	0.068	0.067	0.073	0.095	0.111
3500	4000	4500	5000	5500	6000	6500	7000	7500	8500	9000	9500
5000	5829	6213	6645	7085	7525	7585	7251	6821	6118	7495	7775
773.9	901.5	966.0	990.0	1006.8	1023.5	1013.7	967.7	910.0	809.0	917.2	916.1
13,017	13,302	13,557	13,785	14,008	14,231	14,432	14,561	14,682	14,920	15,017	15,096

APPENDIX TABLE 24

OPTIMAL SOLUTIONS OF THE MIXED-INTEGER PROGRAMMING PROBLEM WITH VARIED PRICE OF HOG (250 ACRE FARM)

Act. No.	Activity	Unit	Price of Hog (dollar per 100 carcass weight) (< 27.7) 27.50	(27.7~28.8) 27.75	(28.8~29.4) 29.0	(29.4~30.9) 30.0
1	Wheat (D F)	acre	45.3	50.3	0	0
4	Feed grain (D F)	acre	57.1	73.9	130.9	148.3
14	Sugar beet (I F)	acre	35.0	62.5	62.5	62.5
48	Sugar beets (D)	acre	27.5	0	0	0
21	Potatoes (I F)	acre	46.7	31.1	20.2	9.8
28	Fodder corn (I F)	acre	13.5	7.2	11.4	4.4
62	Natural pasture	acre	25.0	25.0	25.0	25.0
64	Feeder calf	No.	13.4	13.4	13.4	13.4
65	Feed lot 400	lot	28.0	8.8	21.6	13.4
68	Sow-hog	sow	0	8.4	13.9	23.0
75	Buy hay	ton	0	0	0	0
74	Buy feed grain	bus.	0	0	0	0
69	Sell wheat	bus.	1,267.2	1,407.8	0	0
72	Sell sugar beets	ton	799.8	937.5	937.5	937.5
73	Sell potatoes	bus.	8,405.2	5,594.3	3,644.2	1,765.6
89	Pur. irrigation machine	No.	1	1	1	1
85	Pur. S.B. thinner	No.	1	1	1	1
86	Pur. potato digger	No.	1	1	1	1
88	Pur. seed cutter	No.	1	1	1	1
90	Irrigation water	acre-inch	957.1	1035.7	988.5	809.9
79	Hire fall labor	hour	128.8	180.3	269.2	327.4
81	Hire potato-har.hand labor	hour	385.9	219.4	132.0	46.7
91	Operating capital loan	$	10,000	10,000	10,000	10,000
	Irrig. development loan	$	12,056	12,398	11,997	10,951
	Functional values	$	14,212	14,239	14,480	14,890

NOTE: "DF", "IF" and "D" in brackets indicate "fertilized on dryland", "fertilized under irrigation" and "on dryland with no fertilizer", respectively. The price of water is fixed at $2.00 per acre-inch.

APPENDIX TABLE 25

OPTIMAL SOLUTIONS OF THE MIXED-INTEGER PROGRAMMING PROBLEM OBTAINED UNDER VARIOUS AMOUNTS OF QUOTA ON WHEAT SALE (250 ACRE FARM)

Act. No.	Activity	Unit	Specified Acreage Quota (bushels per acre)					
			6	7	8	10	11	open
1	Wheat (D F)	acre	30.2	35.2	40.2	50.3	55.3	111.2
4	Feed grain (D F)	acre	69.9	65.7	61.4	52.8	48.5	6.9
14	Sugar beet (I F)	acre	29.8	31.5	33.2	36.7	38.4	62.5
21	Potatoes (I F)	acre	46.6	46.6	46.7	46.7	46.8	43.8
28	Fodder corn (I F)	acre	15.8	15.0	14.2	12.7	11.9	0.6
48	Sugar beets (D)	acre	32.7	31.0	29.3	25.8	24.1	0
62	Natural pasture	acre	25.0	25.0	25.0	25.0	25.0	25.0
64	Feeder calf	head	13.4	13.4	13.4	13.4	13.4	13.4
65	Feed lot 400	lot	35.2	32.8	30.4	25.6	23.2	0
69	Sell wheat	bushel	845.6	986.3	1,126.8	1,407.6	1,547.9	3,113.8
72	Sell sugar beets	ton	774.1	782.7	791.2	808.3	816.8	937.5
73	Sell potatoes	bushel	8,381.2	8,389.2	8,397.2	8,413.2	8,421.2	7,882.2
79	Hire fall labor	hour	144.3	139.1	134.0	123.7	118.6	67.1
81	Hire hand pot.harv.labor	hour	392.4	390.2	388.1	383.7	381.6	315.9
89	Pur. irrigation machine 1	set	1	1	1	1	1	1
85	Pur. S.B. thinner 1	unit	1	1	1	1	1	1
86	Pur. potato digger 1	unit	1	1	1	1	1	1
88	Pur. seed cutter 1	unit	1	1	1	1	1	1
91	Operating capital loan	$	10,000	10,000	10,000	10,000	10,000	10,000
	Develop. capital loan	$	11,880	11,940	11,998	12,196	12,174	12,761
90	Irrigation water	acre-inch	929.2	938.5	947.8	966.4	975.7	1069.7
75	Buy hay	ton	0	0	0	0	0	16.2
	Functional	$	14,173	14,186	14,199	14,225	14,239	14,382

NOTE: "DF", "IF" and "D" in brackets indicate "fertilized on dryland", "fertilized under irrigation" and "on dryland with no fertilizer", respectively. The price of water is fixed at $2.00 per acre-inch.

APPENDIX TABLE 26

AGGREGATE DEMAND FOR IRRIGATION WATER PROJECTED BY NON-INTEGER LINEAR PROGRAMMING

Price Ranges (dollars per acre-inch)	Quantity of Water Used (acre-inch)	Price Ranges (dollars per acre-inch)	Quantity of Water Used (acre-inch)
0.0 ~ 0.48 (0.24)	512,889.9	2.05~2.62 (2.34)	194,965.1
0.48~0.59 (0.54)	438,977.5	2.62~3.01 (2.82)	183,175.9
0.59~0.63 (0.62)	395,148.3	3.01~3.25 (3.14)	174,045.1
0.63~0.74 (0.69)	313,576.3	3.25~3.36 (3.31)	163,641.1
0.74~0.89 (0.82)	296,656.3	3.36~3.86 (3.62)	69,653.5
0.89~1.18 (1.04)	284,680.3	3.86~5.90 (4.89)	31,212.1
1.18~1.41 (1.30)	247,518.7	5.90~6.03 (5.97)	25,761.1
1.41~1.47 (1.45)	235,154.9	6.03~7.20 (6.62)	17,757.8
1.47~1.57 (1.53)	232,558.9	7.20~7.39 (7.30)	9,589.2
1.57~1.84 (1.71)	225,901.5	7.39 over	0.0
1.84~2.05 (1.95)	213,372.1		

NOTE: The figures in brackets indicate the mid-points of "steps" of the aggregate demand curve. Total farm land in the project area is 71,040 acres. Total irrigable land is 49,728 acres. Irrigable T_1 land is 39,782. Irrigable T_2 land is 9.946 acres.

APPENDIX TABLE 26-B

DEMAND SCHEDULE FOR IRRIGATION WATER ESTIMATED BY NON-INTEGER LINEAR PROGRAMMING FOR SMALL FARMS (60 ACRE)

Price of Water	Quantity Used	Irrigated Land T_1	Irrigated Land T_2	Irrigated Land Total	Per-cent	Unused Irrigable Land T_1	Unused Irrigable Land T_2	Unused Irrigable Land Total
$/acre-inch	acer-inch	--- acres	---------			------	acres	-------
0 ~ .78	440.7	34.2	7.8	42.0	100	0	0	0
.78 ~1.41	440.7	34.2	7.8	42.0	100	0	0	0
1.41~1.57	346.32	34.2	0	34.2	81	0	7.8	7.8
1.57~3.36	295.5	30.0	0	30.0	71	4.2	7.8	12.0
3.36~3.94	135.0	15.0	0	15.0	36	19.2	7.8	27.0
3.94~6.03	135.0	15.0	0	15.0	36	19.2	7.8	27.0
6.03~7.39	73.16	8.13	0	8.13	19	26.07	7.8	33.87

APPENDIX TABLE 26-C

DEMAND SCHEDULE FOR IRRIGATION WATER ESTIMATED BY NON-INTEGER AND MIXED INTEGER LINEAR PROGRAMMING FOR LARGE FARMS (500 ACRE)

Price of Water, Dollar Per Acre-inch	Quantity Used Acre-inch	Irrigated Land T_1	Irrigated Land T_2	Irrigated Land Total	Per-cent	Unused Irrigable Land T_1	Unused Irrigable Land T_2	Unused Irrigable Land Total
		----- acres	--------			------	acres	-------
Estimated by non-integer programming:								
0~0.63	3609.7	277.26	0	277.26	79	2.74	70.0	72.74
0.63~0.74	1570.4	127.80	0	127.80	37	152.20	70.0	222.20
0.74~0.89	1147.4	96.17	0	96.17	27	183.83	70.0	253.83
0.89~1.47	848.0	74.67	0	74.67	21	205.33	70.0	275.33
1.47~2.62	783.1	68.65	0	68.65	20	211.35	70.0	281.35
2.62~3.01	488.37	42.62	0	42.62	12	237.38	70.0	307.38
3.01~3.25	260.1	21.50	0	21.50	6	258.50	70.0	328.50
3.25 over	0	0	0	0	0	280.00	70.0	350.00
Estimated by mixed integer programming:								
0~0.85	3212.2	250.00	0	250.0	71	30.00	70.00	100.00
0.85~0.97	1760.3	140.95	0	140.95	40	139.05	70.0	209.05
0.97~1.17	1678.0	134.73	0	134.73	38	145.26	70.0	215.26
1.17~1.83	886.74	78.17	0	78.17	22	201.83	70.0	271.83
1.83~2.62	640.93	55.36	0	55.36	16	224.64	70.0	294.64
2.62 over	0	0	0	0	0	280.00	70.0	350.00

APPENDIX TABLE 26-D

TOTAL REVENUES OF THE WATER SUPPLY AGENCY PROJECTED BY NON-INTEGER AND MIXED INTEGER LINEAR PROGRAMMING(CALCULATED BY AGGREGATE DEMAND EQUATIONS FOR WATER)

Price of Water Dollar per Acre-inch	Projected by Non-integr Programming		Projected by Mixed Integer Programming	
	Quantity Used	Total Revenue	Quantity Used	Total Revenue
	acre-inches	dollars	acre-inches	dollars
0.30	495757.0	148727.0	495624.6	148687.3
0.40	435057.5	174022.9	423203.3	169281.3
0.50	393142.0	196571.0	374399.9	187199.9
0.60	361910.5	217146.3	338732.8	203239.6
0.70	337448.4	236213.8	311240.9	217868.6
0.80	317598.9	254079.1	289236.7	231389.3
0.90	301061.9	270955.6	271112.5	244010.2
1.00	287000.0	287000.0	255882.3	255882.3
1.10	274846.3	302330.7	242835.1	267118.4
1.20	264200.5	317040.5	231505.8	277806.8
1.30	254772.0	331203.4	221551.1	288016.3
1.40	246342.8	344879.8	212716.6	297803.1
1.50	238746.2	358119.3	204808.8	307213.1
1.60	231852.3	370963.6	197677.9	316284.4
1.70	225558.0	383448.5	191205.6	325049.5
1.80	219780.0	395603.8	185297.7	333535.7
1.90	214450.9	407546.6	179877.4	341766.9
2.00	209514.6	419029.1	174881.8	349763.5
2.10	204924.6	430341.6	170258.8	357543.3
2.20	200642.1	441412.4	165964.8	365122.4
2.30	196633.5	452256.9	161962.8	372514.3
2.40	192870.5	462889.1	158221.8	379732.1
2.50	189329.0	473322.5	154714.6	386786.6
2.60	185987.6	483567.5	151418.3	393687.3
2.65	184386.2	488623.3	149842.8	397083.2
2.70	182828.0	493635.6		
2.80	179834.1	503535.4		
2.90	176991.8	513276.2		
3.00	174288.6	522865.7		
3.10	171713.2	532310.8		
3.20	169255.8	541618.6		
3.30	166907.8	550795.6		
3.35	165772.1	555336.5		
3.40	71530.6	243204.1		
3.50	67720.3	237020.9		

APPENDIX TABLE 27

AGGREGATE DEMAND FOR IRRIGATION WATER PROJECTED BY MIXED-INTEGER PROGRAMMING

Price Ranges (dollar per acre-inch)		Quantity of Water used	Total Land irrigated	% of Total	Irrigated T_1 land	Irrigated T_2 land	Unused Irrigated Land
		acre-inch	acre	%	acre	acre	acre
0.0 ~ 0.69	(0.35)	441,644.0	37,650.0	75.7	32,120.0	5,530.0	12,078.0
0.69 ~ 0.85	(0.78)	368,016.0	32,120.0	64.6	32,120.0	0.0	17,608.0
0.85 ~ 0.89	(0.88)	309,940.0	27,758.0	55.8	27,758.0	0.0	21,970.0
0.89 ~ 0.97	(0.94)	221,618.0	20,648.0	41.5	20,648.0	0.0	29,080.0
0.97 ~ 1.17	(1.08)	218,326.0	20,399.0	41.0	20,399.0	0.0	29,329.0
1.17 ~ 1.83	(1.51)	187,674.0	18,136.0	36.5	18,136.8	0.0	31,591.0
1.83 ~ 2.14	(1.99)	176,842.0	17,224.4	34.6	17,224.4	0.0	32,504.0
2.14 ~ 2.62	(2.39)	171,154.0	16,592.4	33.4	16,592.4	0.0	33,136.0
2.62 over		0.0	0.0	0.0	0.0	0.0	49,728.0

NOTE: The figures in the brackets indicate the mid-points of "steps" (price ranges) of the aggregate demand curve.

APPENDIX TABLE 28

OPTIMAL SOLUTIONS OF THE STOCHASTIC PROGRAMMING PROBLEM WITH VARIOUS LEVELS OF RISK AVERSION (FOR 250 ACRE FARM UNDER DRYLAND CONDITIONS)

Act. No.	Activity	Unit	Linear Programming	Levels of Risk Aversion				
				Low	Low-med	Medium	Hi-med	High
1	Wheat (DF)	acre	35.46	35.46	38.81	34.66	33.27	32.82
24	Wheat Compan. (D)	acre	11.46	11.46	10.46	6.71	5.01	4.14
29	Sugar beets (D)	acre	21.57	21.57	26.38	24.92	18.44	–
31	Sugar beets (D)	acre	40.93	40.93	36.12	37.58	40.35	22.65
32	Potatoes (D)	acre	21.78	21.78	–	–	–	–
34	Sunflowers (D)	acre	–	–	–	34.17	55.29	70.77
35	Field peas (D)	acre	–	–	–	–	–	27.38
38	Feed-lot 400 (D)	acre	105.27	105.27	103.00	63.93	46.02	38.07
41	Sow-hog (D)	acre	–	–	20.69	29.74	31.63	33.31
37	Feeder calf (D)	acre	25.00	25.00	25.00	25.00	25.00	25.00
57	Hire fall labor	hour	105.4	105.4	129.1	242.6	309.6	345.0
61	Pur. S.B. thinner		0.166	0.166	0.203	0.192	0.142	–
62	Pur. potato digger		0.436	0.436	–	–	–	–
64	Pur. potato seed cutter		0.073	0.073	–	–	–	–
69	Specified acreage		140.7	140.7	145.6	111.9	97.2	96.2
67	Oper. capital loans	$	4,095	4,095	5,309	5,173	4,650	1,983
	Expected income	$	10,881	10,881	10,717	10,567	9,853	7,786
	Standard deviation	$	2,255	2,255	1,915	1,742	1,610	961
	Guaranteed income	$	10,878	10,373	9,475	8,807	7,519	5,983
	"q" value		0.00	0.225	0.625	0.956	1.45	1.88
	Probability		0.50	0.59	0.74	0.83	0.93	0.97

NOTE: "DF" and "D" in brackets indicate "fertilized on dryland" and "on dryland with no fertilizer", respectively.

APPENDIX TABLE 29

THE OPTIMAL SOLUTION OF LINEAR PROGRAMMING COMPARABLE WITH THOSE OF STOCHASTIC PROGRAMMING (250 ACRE FARM UNDER IRRIGATION CONDITIONS)

Act. No.	Activity	Unit	Price of Water (dollars per acre-inch) 0.0-0.40 (0.0)	0.40-0.67 (.50)	0.67-0.84 (.80)	0.84-1.35 (1.0)
10	Flax T_1 (I)	acre	96.6	80.8	54.0	—
13	Sugar beets (I) T_1	acre	43.8	59.2	62.5	62.0
15	Sugar beets (I) T_1	acre	—	—	—	0.5
16	Sugar beets (I) T_2	acre	8.5	—	—	—
17	Sugar beets (I) T_2	acre	10.7	3.3	—	—
20	Potato (I) T_1	acre	—	—	23.5	33.6
22	Potato (I) T_2	acre	15.9	19.1	—	—
21	Wheat (DF)	acre	—	—	21.4	—
24	Wheat (D)	acre	14.3	17.9	—	36.8
29	Sugar beets (D)	acre	—	—	—	—
31	Sugar beets (D)	acre	—	—	—	—
32	Potatoes (D)	acre	—	—	—	—
38	Feed-lot 400 (D)	acre	35.7	44.8	63.6	92.1
68	Feeder calf (D)	acre	—	—	—	—
55	Hire summer labour	hour	32.6	28.1	16.3	—
57	Hire fall labour	hour	76.5	77.2	79.2	80.7
59	Potato harv. labour	hour	—	—	51.4	169.5
61	Pur. Sugar beets thinner		.481	.481	.481	.477
62	Purchase potato digger		.318	.381	.470	.671
63	Pur. Sugar beets harvester		.169	—	—	—
64	Pur. seed-potato cutter		.053	.064	.117	.134
65	Pur. irrigated machines		.700	.649	.560	.384
66	Irrig. water used	acre-inch	2087	1907	1593	971
67	Oper. capital loan	$	10,000	10,000	10,000	10,000
69	Specified acreage	acre	50.00	62.67	85.00	128.94
	Expected income	$	15,873	14,847	14,315	14,081
	Standard deviation	$	—	—	—	—
	Total land irrigated	acre	175.00	162.34	140.00	96.06
	T_1 land irrigated	acre	140.00	140.00	140.00	96.06
	T_2 land irrigated	acre	35.00	22.34	—	—

NOTE: The functional values are calculated for the water prices in the brackets. "I" and "D" in brackets indicate "irrigated" and "dryland", respectively.

APPENDIX TABLE 29 (continued)

Price of Water (dollar per acre-inch)					
1.35-3.33 (1.5)	3.33-4.77 (3.5)	4.77-4.96 (4.8)	4.96 - 6.09 (5.0)	6.09-over	2.00
—	—	—	—	—	—
61.9	—	—	—	—	61.9
0.6	—	—	—	—	0.6
—	—	—	—	—	—
—	—	—	—	—	—
32.7	51.5	47.7	20.5	—	32.7
—	—	—	—	—	—
32.7	28.0	28.9	35.8	35.5	32.7
—	—	—	—	—	—
—	42.2	42.5	29.2	21.6	—
—	20.3	20.0	33.3	40.9	—
—	—	—	—	21.8	—
97.1	83.1	85.9	106.2	105.3	97.1
—	—	25.0	25.0	25.0	—
—	—	—	—	—	—
81.9	85.4	109.6	110.9	105.4	81.9
157.9	431.4	396.7	—	—	157.9
.477	.326	.327	.224	.166	.477
.655	1.03	.955	.411	.436	.655
—	—	—	—	—	—
.134	.185	.169	.073	.073	.134
.381	.206	.191	.082	—	.381
963	463	429	185	—	963
10,000	10,000	10,000	5171	4094	10,000
129.77	111.02	114.79	141.98	140.72	129.77
13,596	11,756	11,155	11,079	10,881	13,115
2782	—	—	—	2255	2782
95.23	51.48	47.71	20.52	—	95.23
95.23	51.48	47.71	20.52	—	95.23

APPENDIX TABLE 30

OPTIMAL SOLUTIONS OF THE STOCHASTIC PROGRAMMING PROBLEM WITH THE LOW LEVEL OF RISK AVERSION FOR 250 ACRE FARMS UNDER IRRIGATION (a=0.0002)

Act. No.	Activity	Unit	Price of Water (dollar per acre-inch) 0.0	0.20	0.50	1.00	1.50	2.00
10	Flax T_1 (I)	acre	47.59	49.76	58.97	41.85	4.80	-
11	Flax T_2 (I)	acre	3.06	4.75	-	-	-	-
13	Sugar beets T_1(I)	acre	62.50	62.50	55.48	62.50	62.50	61.89
15	Sugar beets T_1(I)	acre	-	-	-	-	-	0.61
17	Sugar beets T_2(I)	acre	-	-	7.02	-	-	-
20	Potatoes T_1(I)	acre	-	-	-	15.97	29.07	32.73
22	Potatoes T_2(I)	acre	31.94	30.25	27.98	10.99	-	-
1	Wheat (DF)	acre	23.19	22.43	-	-	33.93	32.70
24	Wheat (D)	acre	-	-	24.55	29.05	-	-
29	Sugar beets (D)	acre	-	-	-	-	-	-
31	Sugar beets (D)	acre	-	-	-	-	-	-
32	Potatoes (D)	acre	-	-	-	-	-	-
38	Feed-lot 400 (D)	acre	26.80	27.57	25.45	44.95	79.89	97.07
47	Sow-hog T_1 (I)	acre	29.91	27.74	25.56	19.68	14.81	-
68	Feeder calf unimp. land	acre	-	-	-	-	-	-
57	Hire fall labor	hour	144.5	139.6	133.8	121.9	114.3	81.9
55	Hire summer labor	hour	158.3	145.9	126.2	68.4	5.5	-
59	Hire Potato harv. labor	hour	525.5	224.1	187.0	147.6	146.5	157.9
61	Pur. S.B.thinner		0.481	0.481	0.481	0.481	0.481	0.476
62	Pur. potato digger		0.639	0.605	0.560	0.539	0.581	0.655
64	Pur.seed potato cutter		0.106	0.101	0.093	0.090	0.097	0.109
65	Pur.irrig.machines		0.580	0.589	0.598	0.525	0.386	0.381
67	Oper.capital loan	$	10,000	10,000	10,000	10,000	10,000	10,000
69	Specified acreage	acre	79.91	77.74	75.56	93.69	128.62	129.77
	Water used	acre-inch	1931	1945	1961	1666	1146	963
	Expected income	$	16,375	15,941	15,282	14,168	13,577	13115
	Standard deviation	$	2,191	2,164	2,158	2,144	2,591	2,782
	Guaranteed income	$	15,895	15,473	14,816	13,708	12,906	12,341
	Probability		0.587	0.587	0.587	0.583	0.603	0.610
	Value of "q"		0.291	0.216	0.216	0.214	0.259	0.278
	Chance of failure (once in n years)		1/2.4	1/2.4	1/2.4	1/2.4	1/2.5	1/2.6
	Total irrigated land	acre	175	175	175	151	111	95
	T_1 irrigated land	acre	140	140	140	140	111	95
	T_2 irrigated land	acre	35	35	35	11	-	-

continued......

APPENDIX TABLE 30 (continued)

Price of Water (dollar per acre-inch)								
2.50	3.00	3.25	3.50	4.00	5.00	6.00	7.00	8.00
-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-
60.59	56.17	45.27	20.60	-	-	-	-	-
1.91	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-
30.09	29.74	28.90	23.90	20.52	20.52	12.44	3.65	-
-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-
-	1.34	8.86	19.69	29.14	29.14	26.15	22.90	21.55
-	4.99	8.37	22.20	33.36	33.36	36.35	39.60	40.95
-	-	-	-	-	-	8.57	17.90	21.78
99.05	99.31	99.94	103.7	106.2	106.2	105.8	105.4	105.3
-	-	-	-	-	-	-	-	-
-	-	-	25.00	25.00	25.00	25.00	25.00	25.00
82.1	82.5	107.3	109.3	110.9	110.9	108.7	106.4	105.4
-	-	-	-	-	-	-	-	-
119.3	114.3	122.2	49.2	-	-	-	-	-
0.466	0.442	0.416	0.310	0.224	0.224	0.201	0.176	0.166
0.602	0.595	0.578	0.478	0.410	0.410	0.420	0.431	0.436
0.100	0.099	0.096	0.080	0.068	0.068	0.070	0.072	0.073
0.370	0.344	0.297	0.178	0.082	0.082	0.050	0.015	-
10,000	10,000	10,000	7521	5541	5725	5195	4406	4094
132.41	132.76	133.60	138.60	141.98	141.98	141.48	140.95	140.72
940	869	744	436	185	185	112	33	-
12,548	12,001	11,717	11,417	11,243	11,048	10,863	10,839	10,881
2,714	2,631	2,499	2,197	2,079	2,079	2,016	2,142	2,257
11,811	11,309	11,093	10,934	10,811	10,616	10,457	10,380	10,373
0.606	0.603	0.599	0.587	0.583	0.583	0.579	0.583	0.591
0.271	0.263	0.250	0.220	0.208	0.208	0.202	0.214	0.225
1/2.50	1/2.5	1/2.5	1/2.4	1/2.4	1/2.4	1/2.4	1/2.4	1/2.4
93	86	74	45	21	21	12	4	-
93	86	74	45	21	21	12	4	-

APPENDIX TABLE 31

OPTIMAL SOLUTIONS OF THE STOCHASTIC PROGRAMMING PROBLEM WITH THE LOW-MEDIUM LEVEL OF RISK AVERSION FOR 250 ACRE FARMS UNDER IRRIGATION (a=0.00066)

Act. No.	Activity	Unit	Price of Water (dollar per acre-inch) 0.00	0.50	1.00	2.00	3.00	4.00	5.00
1	Wheat (DF)	acre	25.84	-	-	23.18	29.18	38.54	38.81
10	Flax T_1 (I)	acre	55.52	56.40	55.79	28.72	-	-	-
13	Sugar beets T_1 (I)	acre	31.73	32.64	44.39	49.90	27.15	-	-
17	Sugar beets T_2 (I)	acre	8.99	9.69	-	-	-	-	-
20	Potatoes T_1 (I)	acre	-	-	-	27.36	22.55	18.65	16.89
22	Potatoes T_2 (I)	acre	26.01	25.31	23.62	-	-	-	-
24	Wheat (D)	acre	-	28.67	25.19	-	-	-	-
29	Sugar beets (D)	acre	21.78	20.17	16.32	4.73	13.17	27.17	26.38
31	Sugar beets (D)	acre	-	-	1.79	7.86	22.18	35.33	36.12
32	Potatoes (D)	acre	.58	-	-	-	-	-	-
34	Sunflowers (D)	acre	-	1.16	11.12	34.31	36.83	-	-
38	Feed-lot 400 (D)	acre	-	-	6.97	-	42.70	82.93	86.11
41	Sow-hog (D)	acre	1.81	-	-	43.34	31.24	22.38	20.69
47	Sow-hog T_1 (I)	acre	52.75	50.97	39.83	5.59	-	-	-
68	Feeder-calf, unimproved land	acre	-	-	-	-	25.00	25.00	25.00
57	Hire fall labor	hour	197.47	194.67	201.12	237.42	249.95	130.49	129.11
59	Hire potato harv. labor	hour	238.90	212.65	142.57	94.26	-	-	-
61	Pur. Sugar beet thinner		.481	.481	.467	.420	.310	.209	.203
62	Pur. potato digger		.532	.506	.472	.547	.451	.373	.338
64	Pur. seed-potato cutter		.089	.084	.079	.091	.075	.062	.056
65	Pur. irrig. machines		.489	.496	.495	.424	.199	.075	.068
66	Irrig. water used	acre-inch	1943	1948	1832	1217	494	168	152
67	Op. capital loan	$	9,391	10,000	10,000	10,000	7,653	5,491	5,309
69	Specified acreage	acre	80.4	79.6	72	72.1	103.1	143.9	145.6
	Expected income	$	16,031	15,006	13,848	12,143	11,359	11,118	10,917
	z (S.D.)	$	1,632	1,647	1,682	1,780	1,882	1,950	1,943
	Income guaranteed	$	15,152	14,111	12,914	11,097	10,190	9,863	9,694

APPENDIX TABLE 52

OPTIMAL SOLUTIONS OF THE STOCHASTIC PROGRAMMING PROBLEM WITH THE MEDIUM LEVEL OF RISK AVERSION FOR 250 ACRE FARMS UNDER IRRIGATION (a=0.0011)

Act. No.	Activity	Unit	Price of Water (dollar per acre-inch) 0.00	0.50	1.00	2.00	3.00	4.00	5.00
1	Wheat (DF)	acre	23.53	24.33	-	6.20	24.42	30.53	31.24
10	Flax T_1 (I)	acre	60.52	61.35	58.07	39.16	16.28	-	-
13	Sugar beets, T_1 (I)	acre	29.81	27.29	31.24	30.12	21.62	1.22	-
17	Sugar beets, T_2 (I)	acre	11.34	11.26	-	-	-	-	-
20	Potatoes T_1 (I)	acre	-	-	-	23.24	22.02	17.72	16.42
22	Potatoes T_2 (I)	acre	23.86	23.74	22.45	-	-	-	-
24	Wheat (D)	acre	-	-	28.52	19.64	-	-	-
29	Sugar beets (D)	acre	21.55	22.50	24.82	19.62	16.69	24.76	24.92
31	Sugar beets (D)	acre	-	1.45	6.37	12.76	24.19	36.52	37.58
32	Potatoes (D)	acre	4.92	1.73	-	-	-	-	-
34	Sunflowers (D)	acre	-	-	2.77	26.26	46.77	36.43	34.17
38	Feed-lot 400 (D)	acre	-	-	-	-	5.07	46.28	50.93
41	Sow-hog (D)	acre	-	-	-	16.64	47.94	31.54	29.74
47	Sow-hog T_1 (I)	acre	49.66	51.36	50.69	31.36	-	-	-
68	Feeder calf, unimp. land	acre	3.19	-	-	-	25.00	25.00	25.00
57	Hire fall labor	hour	190.68	192.41	199.87	246.04	294.65	251.01	242.63
59	Hire potato harv. labor	hour	262.75	219.33	168.68	97.14	-	-	-
61	Pur. Sugar beet thinner		0.481	0.470	0.432	0.383	0.295	0.200	0.192
62	Pur. potato digger		0.576	0.509	0.449	0.465	0.440	0.354	0.328
64	Pur. seed-potato cutter		0.096	0.085	0.075	0.077	0.073	0.059	0.055
65	Pur. Irrig. machines		0.501	0.495	0.447	0.370	0.240	0.076	0.066
66	Water used (I)	acre-inch	1,961	1,962	1,823	1,370	644	173	148
67	Op. capital loan	$	9,487	10,000	10,000	10,000	8,064	5,360	5,173
69	Specified acreage	acre	73.19	75.68	79.21	73.83	77.43	108.35	111.91
	Expected income	$	15,945	14,887	13,571	11,691	10,829	10,813	10,648
	z (S.D.)	$	1,565	1,563	1,464	1,441	1,532	1,755	1,753
	Income guaranteed	$	14,598	13,543	12,392	10,550	9,540	9,119	8,957
	Prob. attached to		0.805	0.805	0.791	0.794	0.780	0.834	0.832
	Value of "q"		0.861	0.860	0.805	0.787	0.842	0.965	0.964
	Chance of failure		1/5.1	1/5.1	1/5.0	1/5.0	1/5.0	1/6.0	1/5.9
	Total land irrigated	acre	175	175	162	124	60	19	16
	T_1 irrigated	acre	140	140	140	124	60	19	16
	T_2 irrigated	acre	35	35	22.45	-	-	-	-

NOTE: "I" and "D" in the brackets indicate "irrigated" activity and "dryland" activity, respectively.

APPENDIX TABLE 33

OPTIMAL SOLUTIONS OF THE STOCHASTIC PROGRAMMING PROBLEM WITH THE HIGH-MEDIUM LEVEL OF RISK AVERSION FOR 250 ACRE FARMS UNDER IRRIGATION (a=0.0018)

Act. No.	Activity	Unit	Price of Water (dollar per acre-inch) 0.0	0.20	0.50	1.00	1.50
4	Wheat T_1 (I)	acre	7.15	7.20	7.47	-	-
10	Flax T_1 (I)	acre	65.37	65.04	65.24	61.89	53.92
13	Sugar beets T_1 (I)	acre	16.93	16.76	15.52	23.25	20.45
17	Sugar beets T_2 (I)	acre	14.37	14.34	14.44	-	-
20	Potatoes T_1 (I)	acre	-	-	-	3.18	17.95
22	Potatoes T_2 (I)	acre	20.64	20.66	20.56	16.56	1.69
1	Wheat (DF)	acre	10.39	10.52	10.38	24.48	4.44
24	Wheat (D)	acre	-	-	-	-	21.55
25	Barley (D)	acre	-	-	-	-	-
29	Sugar beets (D)	acre	31.19	31.40	28.22	29.22	28.44
31	Sugar beets (D)	acre	-	-	4.32	10.03	13.38
32	Potatoes (D)	acre	8.41	8.08	7.08	4.71	3.94
34	Sunflowers (D)	acre	-	-	-	-	11.32
38	Feed-lot 400 (D)	acre	-	-	-	-	-
41	Sow-hog (D)	acre	-	-	-	-	-
47	Sow-hog T_1 (I)	acre	50.55	51.00	51.77	51.68	47.69
57	Hire fall labor	hour	215.5	209.4	194.8	193.5	219.9
59	Hire potato harv.labor	hour	291.0	281.9	255.0	203.1	158.5
61	Pur. S.B. thinner		0.468	0.463	0.448	0.404	0.378
62	Pur. potato digger		0.581	0.575	0.553	0.489	0.472
64	Pur. seed-potato cutter		0.097	0.096	0.092	0.081	0.079
65	Pur. irrig. machines		0.498	0.496	0.493	0.419	0.376
	Oper. capital loan	$	9,825	10,000	10,000	10,000	10,000
	Water used	acre-inch	1,975	1,974	1,974	1,773	1,596
	Expected income	$	15,579	15,160	14,513	13,265	12,234
	Standard deviation	$	1,400	1,400	1,395	1,307	1,277
	Guaranteed income	$	13,814	13,395	12,762	11,727	10,766
	Probability		0.90	0.90	0.90	0.88	0.87
	Value of "q"		1.260	1.260	1.255	1.176	1.149
	Chance of failure, once in n years		1/9.6	1/9.6	1/9.6	1/8.4	1/8.0
	Specified acreage	acre	68.09	68.72	69.62	76.16	73.68
	Feeder calf unimp.land	acre	-	-	-	-	-
	Total irrigated land	acre	175.00	175.00	175.00	156.56	141.70
	T_1 irrigated land	acre	140.00	140.00	140.00	140.00	140.00
	T_2 irrigated land	acre	35.00	35.00	35.00	16.50	1.70

continued.......

APPENDIX TABLE 33 (continued)

Price of Water (dollar per acre-inch)								
1.75	2.00	2.50	3.00	4.00	5.00	6.00	7.00	8.00
-	-	-	-	-	-	-	-	-
50.70	46.83	37.28	30.66	15.07	-	-	-	-
19.31	18.38	17.84	14.54	5.07	-	-	-	-
-	-	-	-	-	-	-	-	-
19.62	19.64	21.00	20.61	18.55	16.18	15.42	14.58	13.69
-	-	-	-	-	-	-	-	-
-	-	22.68	23.24	-	-	-	-	-
25.94	-	-	-	27.89	29.23	29.83	28.51	23.63
-	-	-	-	-	-	-	1.77	6.64
28.44	28.00	22.06	21.56	23.66	23.82	22.34	20.37	18.44
14.75	16.12	22.60	26.41	33.77	38.68	39.19	39.79	40.35
3.21	2.91	-	-	-	-	-	-	-
16.90	23.36	33.66	38.93	49.23	57.71	57.11	56.41	55.29
-	-	-	--	7.56	25.69	27.94	31.04	35.33
-	-	41.86	49.06	44.20	33.69	33.17	32.53	31.63
46.12	44.65	6.02	-	-	-	-	-	-
234.1	250.4	261.1	271.1	299.5	317.9	315.5	313.1	309.6
133.6	112.6	269.8	-	-	-	-	-	-
0.367	0.357	0.307	0.278	0.221	0.183	0.172	0.157	0.142
0.457	0.451	0.420	0.412	0.371	0.324	0.308	0.292	0.274
0.076	0.075	0.070	0.069	0.062	0.054	0.051	0.049	0.046
0.359	0.339	0.304	0.263	0.155	0.065	0.062	0.058	0.055
10,000	10,000	8,569	8,034	6,619	4,950	4,890	4,782	4,650
1,525	1,443	934	746	420	146	139	131	123
11,768	11,353	11,230	10,468	10,141	-	10,249	10,042	9,853
1,264	1,262	1,478	1,345	1,471	1,668	1,651	1,629	1,610
10,329	9,920	9,265	8,840	8,194	7,946	7,796	7,653	7,519
0.87	0.87	0.91	0.89	0.91	-	0.93	0.93	0.93
1.138	1.136	1.330	1.211	1.323	-	1.486	1.466	1.449
1/7.9	1/7.9	1/10.9	1/8.8	1/10.7	-	1/14.0	1/14.0	1/13.6
72.07	69.76	70.56	72.30	79.65	88.61	90.55	93.85	97.23
-	-	25.00	25.00	25.00	25.00	25.00	25.00	25.00
135.75	129.50	82.14	65.81	38.69	16.18	15.42	14.58	13.69
135.75	129.50	82.14	65.81	38.69	16.18	15.42	14.58	13.69

APPENDIX TABLE 34

OPTIMAL SOLUTIONS OF THE STOCHASTIC PROGRAMMING PROBLEM WITH THE HIGH LEVEL OF RISK AVERSION FOR 250 ACRE FARMS UNDER IRRIGATION (a=0.0039)

Act. No.	Activity	Unit	Price of Water (dollar per acre-inch) 0.0	0.20	0.50	0.75	1.00	1.50
4	Wheat T_1 (I)	acre	11.23	11.26	12.33	3.39	-	-
5	Wheat T_2 (I)	acre	2.97	2.91	0.24	-	-	-
10	Flax T_1 (I)	acre	69.46	69.20	68.35	64.54	60.33	55.79
13	Sugar beets T_1(I)	acre	-	-	-	15.93	16.03	14.42
15	Sugar beets T_1(I)	acre	-	-	-	-	-	-
17	Sugar beets T_2(I)	acre	16.62	16.65	16.67	0.75	-	-
20	Potatoes T_1 (I)	acre	-	-	-	-	10.10	15.82
22	Potatoes T_2(I)	acre	15.41	15.44	15.55	16.00	5.93	-
24	Wheat (D)	acre	-	-	-	-	-	-
1	Wheat (DF)	acre	1.19	1.32	4.30	20.17	25.36	24.40
29	Sugar beets (D)	acre	27.18	27.15	26.73	23.70	20.48	18.54
31	Sugar beets (D)	acre	9.08	9.06	9.51	12.39	14.81	17.03
32	Potatoes (D)	acre	12.55	12.47	12.01	9.90	8.65	7.85
34	Sunflowers (D)	acre	-	-	-	2.10	9.77	19.64
35	Field peas (D)	acre	-	-	-	-	-	-
38	Feed-lot 400 (D)	acre	-	-	-	-	-	-
41	Sow-hog (D)	acre	-	-	-	-	-	-
47	Sow-hog T_1 (I)	acre	59.31	59.52	59.32	56.13	53.53	51.50
68	Feeder calf unimp. land	acre	25.00	25.00	25.00	25.00	25.00	25.00
57	Hire fall labor	hour	236.7	237.2	236.4	233.6	251.1	277.8
59	Hire potato harv. labor	hour	307.3	307.1	300.4	257.9	218.6	182.2
61	Pur. S.B.thinner		0.337	0.337	0.334	0.311	0.281	0.254
62	Pur. potato digger		0.559	0.558	0.551	0.518	0.494	0.473
64	Pur.seed potato cutter		0.093	0.093	0.092	0.086	0.082	0.079
65	Pur. irrig.machines		0.463	0.462	0.453	0.402	0.370	0.344
	Water used	acre-inch	1,988	1,987	1,958	1,785	1,661	1,561
	Op. capital loan	$	9,090	9,493	10,000	9,821	9,636	9,871
	Specified acreage	acre	74.70	75.03	76.18	79.69	78.89	75.90
	Expected income	$	14,577	14,159	13,521	12,941	12,323	11,307
	Standard deviation	$	1,107	1,108	1,105	1,086	1,045	1,003
	Guaranteed income	$	12,186	11,767	11,139	10,642	10,192	9,345
	Probability		0.99	0.99	0.99	0.98	0.98	0.98
	Value of "q"		2.159	2.160	2.155	2.117	2.039	1.956
	Chance of failure once in n years		1/84	1/84	1/84	1/58.8	1/48.5	1/40.0
	Total irrigated land	acre	175	175	172	156	146	138
	T_1 land irrigated	acre	140	140	140	140	140	138
	T_2 land irrigated	acre	35	35	32	17	6	-

continued......

APPENDIX TABLE 34 (continued)

Price of Water (dollar per acre-inch)								
2.00	2.25	2.50	3.50	4.00	5.00	6.00	7.00	8.00
-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-
48.51	44.87	36.52	27.78	23.15	11.12	-	-	-
11.94	10.88	11.06	2.39	-	-	-	-	-
-	-	-	8.52	8.68	2.45	-	-	-
-	-	-	-	-	-	-	-	-
15.49	15.46	16.64	16.29	15.60	13.12	11.04	10.61	10.17
-	-	-	-	-	-	-	-	-
26.52	26.06	27.60	26.39	26.81	29.30	31.13	31.24	31.34
-	-	-	-	-	-	-	-	-
14.90	12.53	1.01	-	-	-	-	-	-
20.92	23.04	29.82	27.47	27.94	27.74	23.90	23.27	22.65
6.28	5.31	0.67	-	-	-	-	-	-
33.31	40.53	46.23	43.85	53.24	67.19	74.52	72.64	70.77
-	-	6.37	25.38	21.92	17.99	20.87	23.05	25.24
-	-	-	-	-	13.90	28.36	29.94	31.51
-	2.98	29.89	46.92	47.67	42.21	35.18	34.25	33.31
47.14	43.35	19.17	-	-	-	-	-	-
25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00	25.00
310.6	327.8	315.8	276.4	307.5	348.0	364.5	357.2	350.0
120.0	86.3	-	-	-	-	-	-	-
0.206	0.180	0.093	0.018	-	-	-	-	-
0.435	0.415	0.346	0.326	0.312	0.262	0.221	0.212	0.203
0.073	0.069	0.058	0.053	0.052	0.044	0.037	0.035	0.034
0.304	0.285	0.257	0.220	0.190	0.107	0.044	0.042	0.041
1,391	1,292	947	630	539	291	99	95	92
9,430	9,122	7,051	5,857	5,500	3,752	2,043	2,017	1,983
73.66	72.38	76.67	73.31	74.48	85.40	94.67	95.42	96.17
10,315	9,891	9,458	8,565	8,266	7,980	8,058	7,918	7,786
948	930	889	860	863	909	980	970	962
8,563	8,206	7,916	7,123	6,815	6,367	6,185	6,082	5,983
0.97	0.96	0.96	0.96	0.96	0.96	0.97	0.97	0.97
1.848	1.813	1.734	1.677	1.682	1.773	1.911	1.892	1.875
1/31.1	1/28.4	1/24.0	1/22.0	1/22.0	1/26.0	1/35.6	1/34.0	1/33.2
123	115	83	55	47	27	11	11	10
123	115	83	55	47	27	11	11	10
-	-	-	-	-	-	-	-	-

APPENDIX TABLE 35

TABLE OF VARIANCE-COVARIANCES OF NET PRICES FOR SELECTED ACTIVITIES

Activity No.	1	3	2	27	29
Act. No. Activity	Wheat (DF)	Oats (DF)	Barley (DF)	Flax (D)	Sugar beets(D)
1 Wheat (DF)	40.02	32.10	21.85	16.46	38.21
3 Oats (DF)	32.10	35.04	22.49	14.79	13.03
2 Barley (DF)	21.85	22.49	28.54	12.84	-2.68
27 Flax (D)	16.46	14.79	12.84	53.90	118.24
29 Sugar beets (D)	38.21	13.03	-2.68	118.24	618.95
32 Potatoes (D)	31.91	112.67	104.64	-121.80	-253.06
34 Sunflowers (D)	-5.28	-0.79	-6.10	20.87	79.02
35 Field peas (D)	-0.96	4.67	14.79	13.55	36.16
36 Cow-calf (D)	19.18	12.03	4.28	14.95	48.51
37 Feeder calf (D)	20.89	15.39	2.96	20.21	57.09
38 Feed-lot 400 (D)	28.92	22.73	4.92	31.29	87.93
39 Feed-lot 650 (D)	25.15	20.25	1.75	30.45	80.61
40 Feed-lot 800 (D)	18.36	16.08	-6.01	31.75	73.16
41 Sow-hogs (D)	45.93	36.69	34.88	8.08	97.38
4 Wheat (I)	2.69	-8.80	-10.51	2.62	56.72
8 Oats (I)	2.60	13.57	3.16	-0.22	-37.96
6 Barley (I)	0.67	8.09	8.33	-1.55	9.42
10 Flax (I)	-2.59	0.96	5.88	66.77	-108.47
13 Sugar beets (I)	64.62	-10.24	4.93	154.54	793.89
20 Potatoes (I)	-265.80	-98.81	-147.31	-328.39	-1096.34
42 Cow-calf (I)	- 5.62	- 9.02	- 12.02	6.03	18.26
43 Feeder-calf (I)	- 2.23	- 1.94	- 15.90	18.98	27.91
44 Feed-lot 400 (I)	1.19	1.15	-16.68	27.89	55.29
45 Feed-lot 650 (I)	0.36	1.14	-19.08	31.51	63.66
46 Feed-lot 800 (I)	1.36	1.89	-26.84	48.95	99.00
47 Sow-hogs (I)	8.62	6.35	14.70	-20.94	52.30
31 Sugar beets (D)	38.21	13.03	118.24	118.24	618.95
15 Sugar beets (I)	64.42	-10.24	154.54	154.54	793.89
24 Wheat (D)	45.69	32.54	16.25	16.25	37.95
$\bar{C}_j$(Expected Net Prices)	37.52	32.43	37.67	27.91	85.10[1]

[1]Depreciation costs of sugar beet thinner and potato digger per acre are subtracted, respectively. Cost of hired labour for potato harvesting is also subtracted. Depreciations of sugar beet thinner and potato digger per acre as $0.32, and $6.69, respectively. Manual labour for potato harvesting is $16.25 per acre.

NOTE: Figures of Cj in brackets are net of variable costs and cost of water for irrigation.

continued.....

APPENDIX TABLE 35 (continued)

Act. No.	32 Potatoes (D)	34 Sunflower (D)	35 Field peas (D)	36 Cow-Calf (D)	37 Feeder-calf (D)	38 Feed-lot 400 (D)
1	31.91	-5.28	-0.96	19.18	20.90	28.92
3	112.67	-0.79	4.67	12.03	15.39	22.73
2	104.64	-6.10	14.79	4.28	2.98	4.92
27	121.84	20.88	13.55	14.95	20.21	31.29
29	253.06	79.02	36.16	48.51	57.09	87.93
32	2918.84	-138.48	83.51	-35.02	-66.63	-106.34
34	-138.48	59.15	18.22	4.82	7.55	10.92
35	83.51	18.22	40.94	-2.78	-6.30	-10.54
36	-35.02	4.82	-2.78	15.04	19.88	29.58
37	-66-63	7.55	-6.30	19.88	29.03	44.67
38	-106.34	10.92	-10.54	29.58	44.67	70.25
39	-142.95	12.78	-12.98	29.43	46.22	73.41
40	-242.78	19.06	-19.75	32.12	54.58	88.22
41	118.16	-18.73	11.94	14.48	12.38	17.72
4	250.97	29.86	-16.20	8.11	11.44	19.19
8	-41.52	0.26	-7.46	0.87	7.49	14.47
6	-17.77	7.54	-1.81	-6.47	-8.20	-9.94
10	-725.95	39.58	-20.25	-6.89	-1.45	-0.43
13	-984.41	115.22	-29.88	81.73	102.92	173.85
20	2727.23	-369.00	-159.96	-121.53	-106.51	-145.87
42	-219.33	16.32	-15.10	6.07	13.08	22.93
43	-249.79	20.19	-18.15	13.51	26.12	41.98
44	-359.24	15.87	-27.50	23.22	46.90	80.00
45	-399.34	16.86	-32.35	26.66	52.66	89.31
46	-584.02	29.93	-39.67	37.60	73.16	122.78
47	-41.22	-24.33	2.85	-22.23	-3.80	-2.18
31	-253.06	79.02	36.16	48.51	57.09	87.93
15	-984.41	115.22	-29.88	81.73	102.92	173.85
$\bar{2}4$	36.42	-6.75	-1.83	19.55	21.43	29.79
$\bar{C}_j$	32.77	32.15	24.03	15.45	15.98	32.72

continued......

APPENDIX TABLE 35 (continued)

	39	40	41	4	8	6
Act. No.	Feed-lot 650 (D)	Feed-lot 800 (D)	Sow-hogs (D)	Wheat (I)	Oats (I)	Barley (I)
1	25.15	18.36	45.93	2.69	2.60	0.67
3	20.25	16.08	36.69	-8.80	13.57	8.09
2	1.75	-6.01	34.88	-10.51	3.16	8.33
27	30.45	31.75	8.08	2.62	-0.22	-1.55
29	80.61	73.16	97.38	56.72	-37.96	9.42
32	-142.95	-242.78	118.16	-250.97	-41.52	-17.77
34	12.78	19.04	-18.73	29.86	0.26	7.53
35	-12.98	-19.75	11.94	-16.20	-7.46	-1.81
36	29.43	32.12	14.48	8.11	0.87	-6.48
37	46.22	54.58	12.38	11.44	7.49	-8.20
38	73.41	88.22	17.72	19.19	14.46	-9.94
39	78.04	96.73	10.80	22.43	18.07	-10.29
40	96.73	126.43	-4.73	32.12	27.81	-12.64
41	10.80	-4.73	138.20	-3.11	3.51	29.41
4	22.43	32.12	-3.11	54.30	1.44	11.20
8	18.08	27.81	3.51	1.44	26.64	12.66
6	-10.29	-12.64	29.41	13.20	12.66	32.28
10	8.54	29.16	-104.40	33.70	37.02	20.37
13	175.44	196.12	92.63	227.24	-41.29	46.55
20	-130.35	-103.93	-350.96	-388.87	37.61	-174.16
42	28.14	43.05	-23.49	31.87	9.13	1.24
43	48.66	69.49	-28.93	24.34	16.36	-7.34
44	92.04	129.23	-28.32	38.01	35.56	-7.18
45	102.36	142.93	-33.48	42.92	37.53	-9.71
46	140.56	196.12	-45.77	58.43	50.51	-14.64
47	-3.22	-6.47	134.74	10.72	14.02	43.95
31	80.61	73.16	97.38	56.72	-37.96	9.42
15	175.44	196.12	92.63	227.24	-41.29	46.55
24	25.99	19.20	46.36	2.30	2.74	0.26
$\bar{C}j$	28.94	25.21	31.05	58.16 (37.56)	35.14 (14.54)	41.06 (20.46)

continued......

APPENDIX TABLE 35 (continued)

Act. No.	10 Flax (I)	13 Sugar beets(I)	20 Potatoes (I)	42 Cow-calf (I)	43 Feeder calf (I)	44 Feed-lot 400 (I)
1	-2.59	64.42	-265.90	-5.62	-2.23	-1.17
3	0.96	-10.24	-98.81	-9.02	-1.94	1.15
2	5.88	4.93	-147.31	-12.02	-15.90	-16.68
27	66.77	154.54	-328.39	6.03	18.98	27.89
29	-108.47	793.89	-1096.34	18.24	27.91	55.29
32	-725.95	-984.41	2727.23	-219.33	-249.79	-359.24
34	39.58	115.22	-369.00	16.32	20.19	15.87
35	-20.25	-29.88	-159.96	-15.10	-18.15	-27.50
36	-6.89	81.73	-121.53	6.07	13.51	23.22
37	-1.45	102.92	-106.51	13.03	26.12	46.90
38	-0.43	173.85	-145.87	22.93	41.98	80.00
39	8.54	175.44	-130.35	28.14	48.66	92.04
40	29.16	196.12	-103.93	43.05	69.49	129.23
41	-104.40	92.63	-350.96	-23.49	-28.93	-28.32
4	33.70	227.24	-388.87	31.87	24.34	38.01
8	37.02	-41.29	37.61	9.13	16.36	35.56
6	20.37	46.55	-174.16	1.24	-7.34	-7.19
10	464.31	41.88	-710.94	48.31	64.23	68.16
13	41.88	1774.12	-2307.49	129.21	93.22	209.27
20	-710.94	-2307.49	7916.14	-139.38	-61.55	-52.07
42	48.31	129.21	-139.38	29.41	31.75	55.12
43	64.23	93.32	-61.55	31.75	50.71	78.47
44	68.16	209.27	-52.07	55.12	78.47	151.65
45	76.93	239.31	-61.07	61.23	87.08	166.92
46	117.06	322.64	-151.81	83.40	121.96	227.42
47	-115.91	72.13	-236.40	-8.49	-21.99	-4.69
31	-108.47	793.89	-1096.34	18.26	27.91	55.29
15	41.88	1774.12	-2307.49	129.21	93.32	209.27
24	-4.35	65.22	-255.84	-5.68	-2.22	-0.38
$\bar{C}j$	56.67	130.53[1]	94.24[1]	25.45	34.68	53.55
	(30.27)	(109.13)	(76.24)	(-6.45)	(4.80)	(29.81)

[1]Depreciation costs of sugar beet thinner and potato digger per acre subtracted, respectively. Hired manual labour for potato harvesting is also subtracted.

continued.......

APPENDIX TABLE 35 (continued)

Act. No.	45 Feed-lot 650 (I)	46 Feed-lot 800 (I)	47 Sow-hogs (I)	31 Sugar beets (D)	15 Sugar beets (I)	24 Wheat (D)
1	0.36	1.36	8.63	38.21	64.22	45.69
3	1.14	1.89	6.35	13.03	-10.24	32.54
2	-19.08	-26.84	14.70	-2.68	4.93	21.98
27	31.51	48.95	-20.94	118.24	154.54	16.25
29	63.66	99.00	52.30	618.95	793.89	37.95
32	-399.34	-584.02	-41.22	-253.06	-984.41	36.42
34	16.86	29.93	-24.33	79.02	115.22	-6.75
35	-32.35	-39.67	2.85	36.16	-29.88	-1.83
36	26.66	37.60	-2.23	48.51	81.73	19.55
37	52.66	73.16	-3.80	57.09	102.92	21.43
38	89.31	122.78	-2.18	87.93	173.85	29.79
39	102.36	140.56	-3.22	80.61	175.44	25.99
40	142.93	196.12	-6.46	73.16	196.12	19.20
41	-33.48	-45.77	134.74	97.38	92.63	46.36
4	42.92	58.43	10.72	56.72	227.24	2.30
8	37.53	50.51	14.03	-37.96	-41.29	2.74
6	-9.71	-14.64	43.95	9.42	46.55	0.26
10	76.93	117.06	-115.91	-108.47	41.88	-4.35
13	239.31	322.64	72.13	793.89	1774.12	65.22
20	-61.07	-151.81	-236.40	-1096.34	-2307.49	-255.88
42	61.23	83.40	-8.49	18.26	129.21	-5.68
43	87.08	121.96	-21.99	27.91	93.32	-2.22
44	166.92	227.42	-4.69	55.29	209.27	-0.38
45	184.19	250.85	-10.69	63.66	239.31	1.30
46	250.85	344.23	-16.85	99.00	322.64	2.37
47	-10.69	-16.85	198.91	52.30	72.13	8.56
31	63.66	99.00	52.30	618.95	793.89	37.95
15	239.31	322.64	72.13	793.89	1774.12	65.22
24	1.30	2.37	8.56	37.95	65.22	46.44
$\bar{C}_j$	46.46 (22.92)	39.56 (15.82)	62.70 (42.20)	62.76[1]	107.68[1] (86.28)	35.92

[1]Labour cost of hand-thinning is subtracted. Labour cost of hand-thinning is $23.32 per acre.

APPENDIX TABLE 36

QUANTITIES OF DEMAND FOR IRRIGATION WATER AT VARIOUS PRICES, ON THE 250 ACRE FARM, PROJECTED BY THE STOCHASTIC AND THE LINEAR PROGRAMMING METHOD (ACRE-INCH)

Price of Water	Levels of Risk Aversion: Low	High-Med	High	From the Linear Programming Solutions: Price range	Midpoint	Quantity
dollar per acre-inch	-------acre-inches-----			dollar per acre-inch	dollar per acre-inch	acre-inches
0.00	1931.36	1975.23	1987.59	0.00~0.40	0.20	2087.04
0.20	1944.84	1974.35	1986.80	0.40~0.67	0.535	1906.55
0.50	1960.85	1974.35	1958.03	0.67~0.84	0.755	1593.03
0.75	-	-	1784.78	0.84~1.35	1.095	970.82
1.00	1665.56	1773.03	1561.04	1.35~3.33	2.34	963.36
1.50	1145.64	1596.01	1561.04	-	-	-
1.65	-	1559.96	-	3.33~4.77	4.05	463.29
1.75	-	1525.31	-	4.77~4.96	4.865	429.37
2.00	963.36	933.69	1390.68	4.96~6.09	5.525	184.69
2.25	-	-	1292.16	6.09~over	-	0.00
2.50	939.52	933.69	946.75			
3.00	868.68	745.72	752.49			
3.25	744.40	-	-			
3.50	435.53	-	630.03			
4.00	184.69	420.08	538.77			
5.00	184.69	145.65	291.09			
6.00	111.98	138.76	99.39			
7.00	32.89	131.23	95.46			
8.00	0.00	123.24	91.53			

APPENDIX TABLE 37

T.D.N.'s PRODUCED PER ACRE FOR ALTERNATIVE LIVESTOCK ACTIVITIES, SURPLUS OR SHORTAGE OF HOME-GROWN FEED PER ACRE (1954 to 1956)

Year	Alternative Livestock Activities With Home-grown Feed											
	Under Dryland						Under irrigation					
	Feeder calf		Feed-lot 400		Sow-hog		Feeder calf		Feed-lot 400		Soe-hog	
	Production	Surplus or Shortage	Prod.	Surp. or Shortage	Prod.	Surp. or Shortage	Prod.	Surp. or Shortage	Prod.	Surp. or Shortage	Prod.	Surp. or Shortage
	pound (T.D.N.)											
1954	1060	176	1069	- 3	1033	- 13	1742	- 85	1760	-232	1650	-274
1955	1013	26	1176	104	1245	201	1905	81	1936	- 56	1822	-102
1956	1244	257	1493	421	1595	549	1735	- 82	1850	-142	1794	-130
1957	1003	16	1165	93	1237	191	1729	- 98	1943	- 49	1943	19
1958	858	-129	1198	126	1403	357	1963	136	2229	237	2242	318
1959	1026	39	1213	141	1299	253	2010	183	2152	160	2092	168
1960	977	- 10	1227	154	1358	312	1887	60	1991	- 1	1918	- 6
1961	620	-367	886	-186	1068	22	2028	200	2096	104	1993	69
1962	1234	247	1512	440	1634	588	1934	107	2137	145	2117	193
1963	999	12	1223	151	1336	290	2016	188	2170	178	2117	193
1964	998	11	1410	338	1647	601	1987	160	2079	87	1993	69
1965	1028	41	1467	395	1719	673	2110	283	2240	248	2167	243

NOTE: "Surplus" or "shortage" is calculated by T.D.N.'s produced per acre minus feed requirements for alternative livestock activities based on one acre of home-grown feed. Feed requirements for alternative livestock activities per acre are presented in Table VII, P. 103.