Computationally Efficient Simulation-based Sensitivity Analysis Method for Power Electronic Circuits

By

Lalin Shasipriya Kirihen Kothalawalage

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Abstract

In a proper circuit design procedure, it is important to consider the performance of a circuit when its elements are expected to vary from their nominal values due to various internal and external factors. Performing a sensitivity analysis on circuit provides deep insight to such a requirement.

The conventional sensitivity analysis methods catering power electronic circuits need lengthy and computationally demanding simulation effort when the circuit is complex and the number of circuit elements involved is large.

This thesis presents a computationally efficient sensitivity analysis method which utilizes the salient feature of network-based sensitivity analysis methods, i.e. less simulation effort. To overcome the applicability limitations of network-based methods on complex power electronic circuits, the proposed method performs sensitivity analysis on linearized average model of the circuit instead of its original circuit.

The resulting sensitivities derived from proposed method were validated against those derived from a conventional method.

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Finally, my sincere thanks go to my wife Chathuri and son Seniru for their patience, understanding and encouragements.

Lalin Kothalawala

Dedication

To my wife Chathuri and son Seniru

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α – Step size in search direction (in optimization)

β – Current gain

ω_s – Angular velocity corresponds to switching frequency

ΔY – Incremental quantity of Y

f – Function of variables

i_C – Current through capacitor

i_L – Current through inductor

v_C – Voltage across capacitor

v_L – Voltage across inductor

C – Capacitance

D – Duty cycle

I – Branch current

L – Inductance

N – Original network

N_i – Incremental network

N_p – Perturbed network

$\hat{N}$ – Adjoint network

P – Active power

R – Resistance

S_x^f – Sensitivity of f against x

T – Switching time interval

V – Branch voltage

X – Reactance

Z – Impedance

$\mathbf{A}$ – Incident matrix

$\mathbf{A}_{1,2}$ – State matrix

$\mathbf{B}$ – Fundamental loop matrix

$\mathbf{B}_{1,2}$ – Input matrix

$\mathbf{C}_{1,2}$ – Output matrix

$\mathbf{D}_{1,2}$ – Feed-through matrix

$\mathbf{I}_b$ – Vector of branch currents

$\mathbf{I}_p$ – Vector of port currents

$\mathbf{I}_E$ – Vector of currents by independent voltage sources

$\mathbf{I}_J$ – Vector of currents by independent current sources

$\mathbf{V}_b$ – Vector of branch voltages

$\mathbf{V}_p$ – Vector of port voltages

$\mathbf{V}_E$ – Vector of voltages by independent voltage sources

$\mathbf{V}_J$ – Vector of voltages by independent current sources

$\mathbf{H}$ – Hybrid parameter matrix

$\mathbf{Z}_b$ – Branch impedance matrix

$\mathbf{Z}_{oc}$ – Open-circuit impedance matrix

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Chapter 1

Introduction

Electrical circuit elements, such as resistors, inductors and capacitors, are fabricated to their nominal values within a given range of tolerances, i.e. their values can only be nominally specified and actual values may slightly, within a certain tolerance band, differ. Additionally with external factors such as temperature, aging and electromagnetic fields, those values can be further deviated over time. Therefore, analysis of the performance of a circuit with the nominal values of elements does not reflect the exact behaviour of the circuit particularly when the circuit ages or operates under varying environmental conditions. So in a proper design procedure, it is important to consider the performance of a circuit when its elements are expected to vary from their nominal values.

The change in the performance of a circuit due to the changes in circuit element values can be evaluated and the process involved in doing so is called sensitivity analysis of circuits. The IEEE Standard Dictionary of Electrical and Electronics Terms [1] defines the sensitivity analysis as, *"an analysis that determines the variation of a given function caused by the changes in one or more [of its] parameters about a selected reference value"*. The given function in the context of this work is a circuit performance index and parameters are the circuit element values. A circuit performance index is a mathematical representation of the expected performance condition of a circuit in terms of measurable quantity and as a function of its elements. Such sensitivity analysis ultimately involves in evaluation of first order partial derivatives of the performance index with respect to circuit parameters and hence the resulting sensitivities are called first order sensitivities.

The initial stage of the sensitivity analysis involves capturing the essential aspects of the performance of the circuit in the form of mathematical functions referred to performance indices. Once such functions are formed, their deviations from nominal figures are assessed when circuit element values are allowed to vary within their tolerances. Such undertakings are particularly difficult when power electronic circuits are concerned. Since many of the power electronic circuits are nonlinear complex systems; their complexity leads to it being prohibitively difficult to find an explicit form for their performance indices. In such a case, a computer simulation-based sensitivity analysis method is to be used. Conventional simulation-based analysis methods rely on numerical calculation of the derivatives of performance index and require a large number of simulations when number of parameters involved is high. This leads to lengthy and computationally demanding simulation effort. Furthermore numerical inaccuracies may tend to impact the outcomes.

Some of the existing sensitivity analysis methods for linear circuits, namely incremental network approach (INA) and adjoint network approach (ANA), have salient features in sensitivity analysis. Both of them require only two simulations for the original circuit and the derived companion circuit to define the sensitivities of a performance index against all the parameters of the circuit (i.e. elements with tolerance levels). Hence, for circuits with large number of parameters, such network-based approaches for sensitivity analysis drastically reduce the required simulation effort. A limitation in applying network based methods is that they can be used when the underlying circuits are linear. This luxurious condition is not available in the analysis of power electronic systems. Therefore to deduce the equivalent linearized average model of complex nonlinear power electronic system, available circuit averaging techniques are to be used. Sensitivity analysis can then be done on the equivalent average circuit.

1.1 Sensitivity Analysis of Power Electronic Circuits

In the path of investigating a possible efficient method to perform sensitivity analysis of power electronic circuits, existing sensitivity analysis methods are to be studied firstly and their merits and drawbacks are to be identified. The computational effort required by those methods and their applicability to a variety of circuits are considered here as key factors for the evaluation.

For a simple circuit where the performance index is available in an explicit mathematical form, the calculation of sensitivities of performance index is straight-forward by performing direct differentiation. The required condition is that the mathematical function describing the performance index is differentiable at the considered operating point. Then the resulting partial derivatives are exactly the sensitivities of performance index. As mentioned in the previous section, most of the power electronic circuits are complex and non-linear in nature. One of the reasons for their non-linearity is the presence of circuit-embedded switching elements such as thyristors and diodes. Additionally the changing of operational states of those elements adds complexity to the circuit when the number of switching elements involved is high. Hence it is difficult to form an explicit mathematical expression for the desired performance index of those circuits. In such a case, one of the simulation based sensitivity analysis methods mentioned below is to be considered:

1. Simulation-based numerical method
2. Incremental network approach
3. Adjoint network approach

The simulation-based numerical method is a conventional method in sensitivity analysis where the change in performance index is measured through circuit simulation as one of the parameters is changed by a small quantity. Then the ratio of change in performance index to change in parameter is assumed to be the sensitivity of performance index against the particular parameter. The same procedure is to be repeated for every interested parameter in the circuit and hence the number of circuit simulations required by such a method is high as the number of parameters involved is high. To overcome this inherent computational burden and numerical inaccuracies involved in the simulation-based numerical method, incremental network approach and adjoint network approach can be used as alternative methods in sensitivity analysis.

These two network-based sensitivity analysis methods first involve in deriving a companion circuit called incremental circuit or adjoint circuit (depending on the method) based on the underlying circuit. For an example, the adjoint circuit is synthesized using pre-defined adjoint branch elements corresponding to each branch element of the underlying circuit. A separate excitation is to be introduced to the adjoint circuit depending on the desired performance index of the underlying circuit. Then the sensitivity of performance index is calculated using the results of two simulations done for the underlying and adjoint circuits. For each desired performance index, the excitation of adjoint circuit is to be set separately and hence the total number of required simulations depends on the number of performance indices considered. In practise, the number of desired performance indices is much lower than the number of circuit parameters and hence the network-based methods significantly reduce the computational effort required for sensitivity analysis compared with numerical method.

In case of network-based methods, to overcome the constraint of applicability in linear circuits only, the sensitivity analysis can be performed on linearized models developed based on the piecewise-linear approximation of power electronic switching circuits [18], [19]. Then the number of linear circuits defined in-between switching instances becomes large as the number of switches in the circuit is high. This results in analyses of large number circuits and hence it becomes a burden to simulation effort again.

To utilise the salient feature of network-based methods, i.e. two circuit simulations per performance index in sensitivity analysis, this research investigates the possibility of combining the network-based methods with linearized average models of the power electronic circuits, instead of its original circuit, in sensitivity analysis. Two widely used techniques in circuit averaging, namely

1. State space circuit averaging
2. Generalized circuit averaging, are considered.

The comparative advantage of using generalized circuit averaging technique over state space circuit averaging technique is also discussed.

1.2 Thesis Organization

The content of thesis is organized in following manner. Chapter 2 provides a deep insight to the theoretical background of simulation-based sensitivity analysis methods. Moreover the relative merits and drawbacks of those methods in application to power electronic circuits are highlighted.

An in-depth analysis of the two averaging techniques mentioned in previous section is presented in Chapter 3. The important features of both techniques applicable to power electronic circuits are also discussed.

The case studies in Chapter 4 provide an investigation into the main objectives of this research. i.e. the process of combining a sensitivity analysis method as discussed with linearized average model of a power electronic circuit derived by a circuit averaging technique to obtain sensitivities. The sensitivities of a given performance index against circuit parameters resulting from such analysis are then validated against those calculated in a brute-force manner. Additionally, the practical limitations of above process are identified and the means to overcome those are also investigated.

Chapter 5 presents the conclusions and contributions of this research work. Furthermore the possible future works based on the outcomes of research are also discussed.

Chapter 2

Sensitivity Analysis Methods for Circuits

Sensitivity analysis of a circuit considered here is a process of determining first order approximations of the variation of a performance index against circuit parameter deviations around their nominal values. For a proper circuit design procedure, such information is vital in performance and reliability analyses of the circuit. Additionally the derived sensitivities can be used in optimization process of circuit parameters as will be described later in Chapter 4.

Sensitivity analysis methods are well developed in literature. In the following section, the theory behind the sensitivity analysis and simulation-based methods developed for sensitivity analysis of circuits are discussed in detail. The main features of those methods are compared in terms of the required computational effort and applicability to various circuits.

2.1 Sensitivity Analysis Theory

To quantitatively define the change in the performance of a network with respect to the changes in network parameters, a mathematical representation for the performance can be developed. Let $f(\mathbf{x})$ be such a performance index and $\mathbf{x} = (x_1, x_2, ..., x_n)$ be the vector of parameters associated with network elements. For example $f(\mathbf{x})$ can be output to input voltage or current gain of a network while $\mathbf{x}$ can be the impedances of network elements. The un-normalized and normalized sensitivities of the $f(\mathbf{x})$ with respect to a network parameter x_i are defined as follows [2]:

$$\text{Un-normalized sensitivity} = \frac{\partial f}{\partial x_i} \tag{2.1}$$

$$\text{Normalized sensitivity} = S_{x_i}^{f} = \frac{\partial f}{\partial x_i} \cdot \frac{x_i}{f} = \frac{\partial \ln(f)}{\partial \ln(x_i)} \tag{2.2}$$

Once the normalized sensitivities $S_{x_i}^{f}$ are calculated with respect to the parameters $\mathbf{x} = (x_1, x_2, ..., x_n)$, change in the performance index Δf due to changes in all circuit parameters is defined as follows (this is derived from Taylor series expansion of $f(\mathbf{x})$ when second and higher-order terms are neglected):

$$\frac{\Delta f}{f} \approx S_{x_1}^{f} \frac{\Delta x_1}{x_1} + S_{x_2}^{f} \frac{\Delta x_2}{x_2} + ... + S_{x_n}^{f} \frac{\Delta x_n}{x_n} \tag{2.3}$$

The sensitivity analysis of a circuit is based on the above formulation; sensitivity analysis methods differ based on the technique that is used to calculate the partial derivatives associated with $S_{x_i}^{f}$ in (2.3).

When the performance index $f(\mathbf{x})$ is readily available, the partial derivatives can be directly evaluated by differentiating the performance index with respect to desired parameters provided that $f(\mathbf{x})$ is differentiable at the operating point considered. A brute-force method can also be used to approximate the partial derivatives by calculating the change in network function against the change in desired parameter. Here it is assumed that the change in parameter is small enough to consider the ratio of change in network function to change in parameter as the partial derivative. For further clarification, consider the following example where the performance index is readily available for calculation of sensitivity.

Example 2.1: Sensitivity analysis by direct differentiation

Consider the circuit shown in Fig. 2-1, where the circuit parameters are $\beta = 2$, $R_S = 1\Omega$, $R_B = 2\Omega$, $R_E = 1\Omega$ and $R_L = 4\Omega$. The input voltage V_{in} is 10V. Suppose that the voltage gain (V_o/V_{in}) is the desired performance index and its sensitivities with respect to circuit parameters R_S, R_B, R_E, R_L and β are to be calculated. By solving the above circuit, the performance index f can be expressed as follows:

$$f = \frac{V_o}{V_{in}} = \frac{\beta R_L}{(R_S + R_B) + (1+\beta)R_E} \tag{2.4}$$

By differentiating f with respect to its parameters and substituting the values, explicit representation for partial derivatives can be obtained as shown below.

$$\frac{\partial f}{\partial R_S} = \frac{-\beta R_L}{\left[(R_S + R_B) + (1+\beta)R_E\right]^2} = -0.2222$$

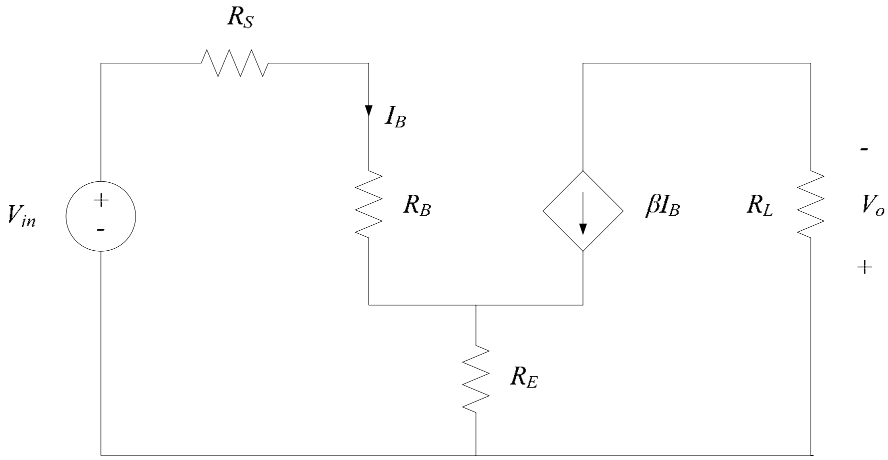


Fig. 2-1. Circuit to be analyzed for sensitivity

Similarly, $\frac{\partial f}{\partial R_B}$ = -0.2222 $\quad \frac{\partial f}{\partial R_E}$ = -0.6667 $\quad \frac{\partial f}{\partial R_L}$ = 0.3333 $\quad \frac{\partial f}{\partial \beta}$ = 0.4444

Hence the normalized sensitivity can be calculated as follows:

$$S_{R_S}^{f} = \frac{\partial \ln(f)}{\partial \ln(R_S)} = \frac{\partial f}{\partial R_S} . \frac{R_S}{f} = (-0.2222). \frac{1}{1.3333} = -0.1667$$

Similarly, other normalised sensitivities;

$$S_{R_B}^{f} = -0.3333 \qquad S_{R_E}^{f} = -0.5000 \qquad S_{R_L}^{f} = 1.0000 \qquad S_{\beta}^{f} = 0.6667$$

Then the change in performance index is derived as follows:

$$\frac{\Delta f}{f} \approx -0.1667\frac{\Delta R_S}{R_S} - 0.3333\frac{\Delta R_B}{R_B} - 0.5\frac{\Delta R_E}{R_E} + 1.0\frac{\Delta R_L}{R_L} + 0.6667\frac{\Delta \beta}{\beta}$$

For complex networks, the performance index $f(\mathbf{x})$ may not be explicitly available and hence the use of partial derivatives as sensitivities is not readily possible. In that case a simulation-based method can be used to determine the partial derivatives. The simulation results for network function against the change in desired parameter can be used in a brute-force manner to approximate the partial derivatives.

The numerical method considered below for sensitivity analysis is a simulation-based brute-force approach. The incremental network and adjoint network approaches [2] for sensitivity analysis are also discussed next and these methods are applied for sensitivity analysis of linear circuits in frequency domain. Furthermore, adjoint network approach can be extended for transient sensitivity analysis of linear circuits.

2.1.1 A Simulation-Based Numerical Method for Sensitivity Analysis

Let $f(\mathbf{x})$ be a network function where $\mathbf{x} = (x_1, x_2, ..., x_n)$ is the vector of network parameters x_i. Using the Taylor series expansion for the network function, it is seen that

$$f(x_1 + \Delta x_1, ..., x_n + \Delta x_n) = f(x_1, ..., x_n) + \frac{\partial f}{\partial x_1}\Delta x_1 + ... + \frac{\partial f}{\partial x_n}\Delta x_n + \frac{1}{2}\frac{\partial^2 f}{\partial x_1^2}\Delta x_1^2 + ... + \frac{1}{2}\frac{\partial^2 f}{\partial x_n^2}\Delta x_n^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_1 \partial x_2}\Delta x_1 \Delta x_2 + ... + \frac{1}{2}\frac{\partial^2 f}{\partial x_{n-1}\partial x_n}\Delta x_{n-1}\Delta x_n + ... \quad (2.5)$$

With the assumption that the changes in parameters (Δx_i) are sufficiently small, the second and higher-order terms can be neglected and the above expansion can be approximated as follows.

$$f(x_1 + \Delta x_1, ..., x_n + \Delta x_n) \approx f(x_1, ..., x_n) + \frac{\partial f}{\partial x_1}\Delta x_1 + ... + \frac{\partial f}{\partial x_n}\Delta x_n \quad (2.6)$$

The first-order partial derivative of the network function with respect to i-th parameter of the network can be easily approximated as shown below.

$$f(x_1, ..., x_i + \Delta x_i, ..., x_n) \approx f(x_1, ..., x_i, ..., x_n) + \frac{\partial f}{\partial x_i}\Delta x_i \quad (2.7)$$

$$\frac{\partial f}{\partial x_i} \approx \frac{f(x_1, ..., x_i + \Delta x_i, ..., x_n) - f(x_1, ..., x_i, ..., x_n)}{\Delta x_i} \tag{2.8}$$

From (2.8) the derivatives of network function with respect to each parameter can be calculated by carrying out (n+1) simulations by incrementing one parameter at a time where n is the number of network parameters involved.

Since the above method relies on a numerical approximation to estimate the derivatives, the network function need not be explicitly available. Once the derivatives are evaluated by simulations, the normalized sensitivities can be derived. The main drawback of this method is the increase in the number of simulations as the number of parameters increases. Since (2.8) has been derived through approximations, the accuracy of the sensitivity indices calculated thereby is also compromised. Even though the accuracy of estimation can be improved by performing both a positive and a negative increment for each parameter as shown in (2.9), then number of simulations involved is even higher.

$$\frac{\partial f}{\partial x_i} \approx \frac{f(x_1, ..., x_i + \Delta x_i, ..., x_n) - f(x_1, ..., x_i - \Delta x_i, ..., x_n)}{2\Delta x_i} \tag{2.9}$$

Example 2.2: Sensitivity analysis by numerical method

Consider the circuit in Example 2.1 again, and suppose that the sensitivity of its voltage gain is to be calculated against current gain β. Using a simulation method as described, the change in voltage gain is evaluated against the change in β from 2.0 to 2.1. i.e. 5% change in β from it's original value.

$$f\big|_{\beta_0=2.0} = \frac{\beta_0 R_L}{(R_S + R_B) + (1+\beta_0)R_E} = 1.3333$$

$$f\Big|_{\beta_0+\Delta\beta=2.1} = \frac{(\beta_0+\Delta\beta)R_L}{(R_S+R_B)+(1+(\beta_0+\Delta\beta))R_E} = 1.3770$$

If $\Delta\beta$ is considered to be sufficiently small,

$$\frac{\partial f}{\partial \beta} \approx \frac{f\big|_{\beta_0+\Delta\beta} - f\big|_{\beta_0}}{(\beta_0+\Delta\beta)-\beta_0} = 0.4372 \qquad \text{Hence,} \qquad S_\beta^f = \frac{\partial f}{\partial \beta}\cdot\frac{\beta}{f} = 0.6557$$

Similarly, the sensitivity of voltage gain against all the parameters can be calculated by changing one parameter at a time and then simulating the circuit. The results are as follows for the 5% change in each parameter from their respective original values.

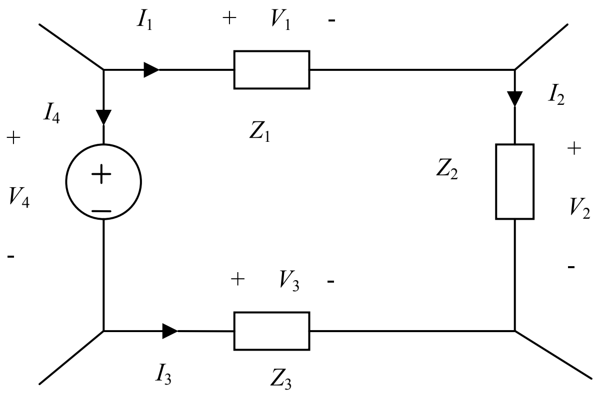
$S_{R_S}^f$ = -0.1650 $\quad$ $S_{R_B}^f$ = -0.3270 $\quad$ $S_{R_E}^f$ = -0.4875 $\quad$ $S_{R_L}^f$ = 1.0000

Note that the numerically evaluated sensitivities above are close to the analytically calculated ones in Example 2.1.

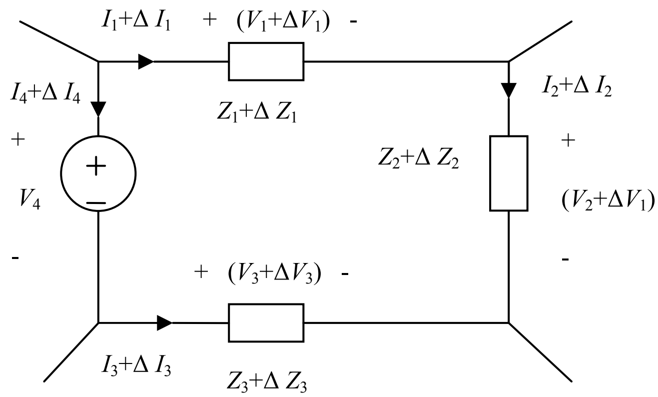
To overcome the problem of higher number of evaluations in the above method, two alternative methods with less number of evaluations, namely the incremental network approach and the adjoint network approach are available. The following descriptions of two methods are based on the presentation in [2].

2.1.2 The Incremental Network Approach

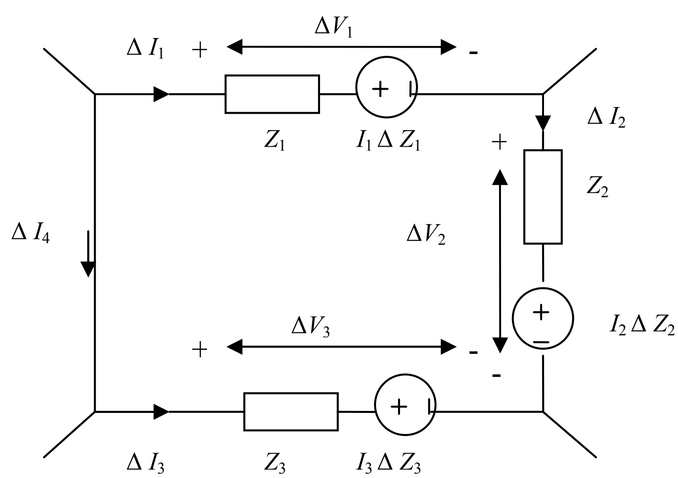
The application of incremental network approach in sensitivity analysis is discussed in [5]. To explain this network-based approach theoretically, consider a portion of a linear network *N* as shown in Fig. 2-2(a) and take **V** and **I** as the vectors of branch voltages and currents respectively.



(a) Original linear network (N)



(b) Perturbed network (N_p)



(c) Incremental network (N_i)

Fig. 2-2. Derivation of incremental network

The branch impedance matrix of the network, **Z** is incrementally changed by **ΔZ** to obtain the perturbed network N_p as shown in Fig. 2-2(b). The changes in impedances results in changes in branch voltages and currents and let those voltage and current changes to be **ΔV** and **ΔI** respectively. The aim of the incremental network approach is to determine the voltage and current changes for deriving the sensitivities.

For the original network *N,*

Kirchhoff's Current Law (KCL): $\mathbf{AI=0}$ (2.10)

Kirchhoff's Voltage Law (KVL): $\mathbf{BV=0}$ (2.11)

The (2.10) and (2.11) are the matrix representation of Kirchhoff's current and voltage laws for the topology of network where, the matrices **A** and **B** are the incident matrix and the fundamental loop matrix, respectively. Since the perturbed network N_p has the same topology as the original network *N*, KCL and KVL with the same incident and fundamental loop matrices apply to N_p as well.

KCL: $\mathbf{A\,(I+\Delta I)=0}$ (2.12)

KVL: $\mathbf{B(V+\Delta V)=0}$ (2.13)

By simplifying equations (2.10) – (2.13),

$$\mathbf{A\Delta I=0} \quad (2.14)$$

$$\mathbf{B\Delta V=0} \quad (2.15)$$

The equation (2.14) reveals that both the branch currents **I** and branch incremental currents **ΔI** have the same constraints imposed by the KCL. The branch voltages **V** and branch incremental voltages **ΔV** are also subjected to the same constraints imposed by the

KVL. It can therefore be concluded that **ΔI** and **ΔV** are branch currents and voltages of an incremental network N_i having the same topology as the original network N. By synthesising and then analysing the incremental network N_i, **ΔI** and **ΔV** can be evaluated.

Consider a branch element in the original network N with impedance Z, then

$$V=ZI \tag{2.16}$$

For the same element in perturbed network N_p,

$$V+\Delta V = (Z+\Delta Z)\,(I+\Delta I) = ZI + Z\Delta I + I\Delta Z + \Delta Z\,\Delta I \tag{2.17}$$

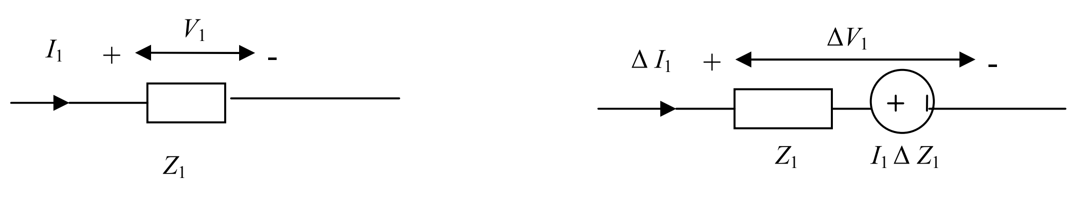
Substituting (2.16) in (2.17),

$$\Delta V= Z\Delta I + I\Delta Z + \Delta Z\,\Delta I \tag{2.18}$$

When the perturbation is infinitely small, the higher-order terms can be neglected and the above equation simplifies to,

$$\Delta V \approx Z\Delta I + I\Delta Z \tag{2.19}$$

The above equation shows that the branch in incremental network N_i has the same impedance Z as the corresponding branch in original network N in series with a voltage source $I\Delta Z$ as shown in Fig. 2-3. This results in having same admittance matrix for both the networks and hence great savings in computational effort is achieved in solving the networks in computer-based simulation program.



(a) Branch in the original network N (b) Branch in the incremental network N_i

Fig. 2-3. Deriving the incremental network N_i from the original network N

By investigating all the branch elements in the original network N, the whole incremental network N_i can be derived as shown in Fig. 2-2(c). The incremental network N_i is then analysed to determine the incremental voltages and currents and thereby the sensitivities.

In summary sensitivity analysis using the incremental network approach involves the following steps.

1) Analyzing of the original network N;

2) Deriving the incremental network N_i by observing the original network N;

3) Analysing the incremental network N_i and thereby determining the sensitivities.

Example 2.3: Sensitivity analysis by incremental network approach

Consider the same circuit in Example 2.1 for sensitivity analysis using the incremental network approach. The original circuit and its incremental circuit are as shown in Fig. 2-4.

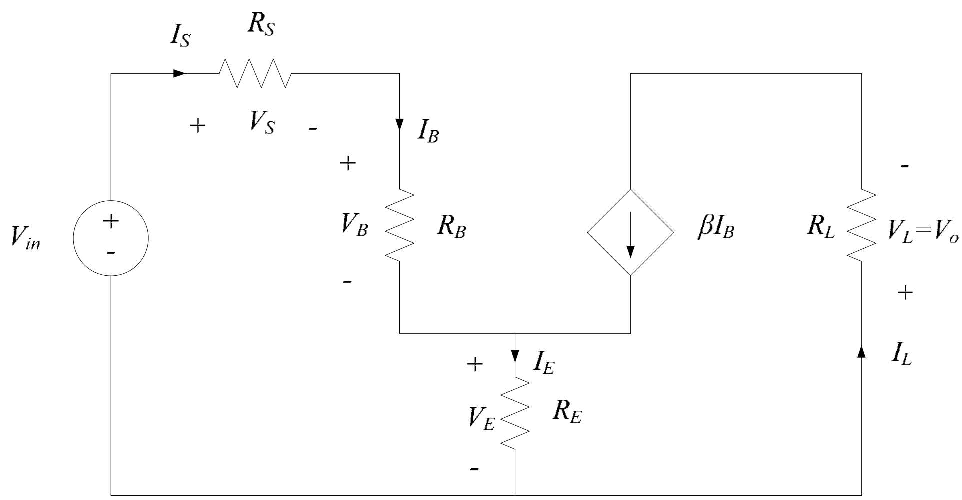
The input voltage here is assumed to be constant and the sensitivities to element values are considered. By solving the original circuit branch voltages and currents can be obtained as follows.

$$I_S = 1.6667\text{ A} \qquad I_B = 1.6667\text{ A} \qquad I_E = 5.0000\text{ A} \qquad I_L = 3.3333\text{ A}$$

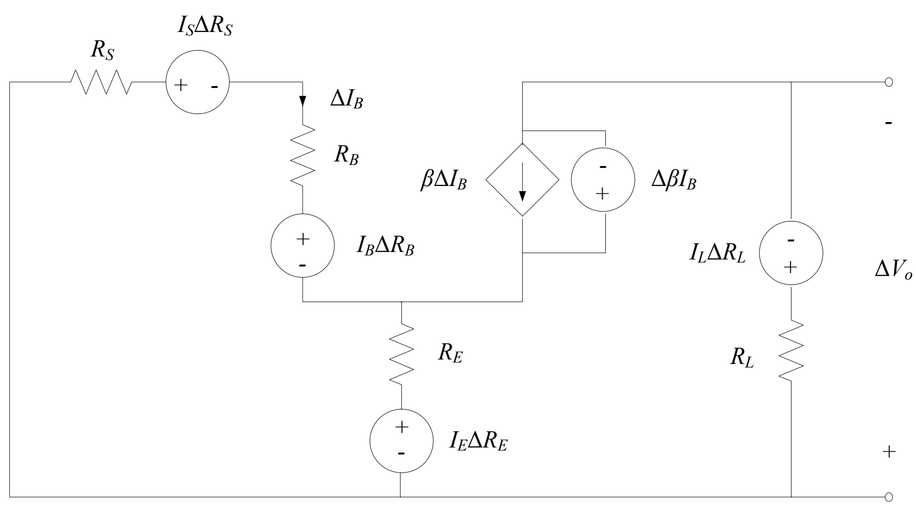
$$V_S = 1.6667\text{ V} \qquad V_B = 3.3333\text{ V} \qquad V_E = 5.0000\text{ V} \qquad V_L = 13.3333\text{ V}$$

By replacing the corresponding branch voltages and currents for the operating point in incremental circuit, the solution for the incremental circuit is obtained as follows.

$$\Delta V_o = -2.2222\Delta R_S - 2.2222\Delta R_B - 6.6667\Delta R_E + 3.3333\Delta R_L + 4.4444\Delta\beta$$



(a) Original network



(b) Incremental network

Fig. 2-4. Sensitivity analysis using incremental network approach

Assuming that the change in R_S, i.e. ΔR_S, is sufficiently small,

$$\frac{\partial V_o}{\partial R_S} \approx \frac{\Delta V_o}{\Delta R_S}$$

$$\frac{\partial f}{\partial R_S} = \frac{\partial \left(V_o / V_{in} \right)}{\partial R_S} = \frac{1}{V_{in}} \cdot \frac{\partial V_o}{\partial R_S} = \frac{1}{10} \cdot (-2.2222) = -0.2222$$

$$S_{R_S}^{f} = \frac{\partial \ln(f)}{\partial \ln(R_S)} = \frac{\partial f}{\partial R_S} \cdot \frac{R_S}{f} = (-0.2222) \cdot \frac{1}{1.3333} = -0.1667$$

Similarly the sensitivity of voltage gain against other parameters can be calculated as follows.

$$S_{R_B}^{f} = -0.3333 \qquad S_{R_E}^{f} = -0.5000 \qquad S_{R_L}^{f} = 1.0000 \qquad S_{\beta}^{f} = 0.6667$$

The above results are exactly equal to those obtained using direct differentiation method in section 2.1. Additionally, the following advantages of incremental network approach are achieved:

1) Since the admittance matrices for the original and incremental networks are the same, there is great savings in computational effort in solving the networks in computer-based simulation program;
2) Since the change in a given parameter is represented as a part of an independent source, the effect of parameter change can be visualized. For example, a parameter change ΔR is represented as a voltage source of $I \cdot \Delta R$ and can be measured in the circuit;
3) Once the incremental circuit is analyzed, all the incremental branch currents and voltages are available as additional information.

2.1.3 The Adjoint Network Approach

The adjoint network approach in sensitivity analysis is widely discussed in literature for its theoretical background [3], [4] and practical applications [7], [8]. To have a theoretical insight to this approach first consider a network N whose voltage and current vectors in time domain are denoted by $\mathbf{v}$ and $\mathbf{i}$ respectively. Equation (2.20) can be easily proven since the sum of the power to all branches in network N is equal to zero. This is the essentially a statement of the law of conservation of power.

$$\mathbf{v}^t\mathbf{i} = \mathbf{i}^t\mathbf{v} = \mathbf{0} \tag{2.20}$$

The Tellegen's theorem states that for branch voltages and currents of two networks N and $\hat{N}$ with the same network topology, the above equation is still valid as follows.

$$\mathbf{v}^t\hat{\mathbf{i}} = \mathbf{i}^t\hat{\mathbf{v}} = \hat{\mathbf{v}}^t\mathbf{i} = \hat{\mathbf{i}}^t\mathbf{v} = \mathbf{0} \tag{2.21}$$

where, $\hat{\mathbf{v}}$ and $\hat{\mathbf{i}}$ are the time-domain voltage and current vectors of network $\hat{N}$ respectively.

When the impedance matrix of network N is disturbed, it results a perturbed network N_p which has the same topology as network N. Then applying the Tellegen's theorem for N_p and $\hat{N}$, it can be shown that:

$$\hat{\mathbf{i}}^t\mathbf{v}^p = \hat{\mathbf{i}}^t(\mathbf{v} + \Delta\mathbf{v}) = \hat{\mathbf{i}}^t\mathbf{v} + \hat{\mathbf{i}}^t\Delta\mathbf{v} = \mathbf{0}$$

$$\hat{\mathbf{v}}^t\mathbf{i}^p = \hat{\mathbf{v}}^t(\mathbf{i} + \Delta\mathbf{i}) = \hat{\mathbf{v}}^t\mathbf{i} + \hat{\mathbf{v}}^t\Delta\mathbf{i} = \mathbf{0}$$

where, $\mathbf{v}^p$ and $\mathbf{i}^p$ are the voltage and current vectors of perturbed network N_p respectively in time domain.

Simplifying above equations with (2.21):

$$\hat{\mathbf{i}}^t\Delta\mathbf{v} = \mathbf{0} \quad \text{and} \quad \hat{\mathbf{v}}^t\Delta\mathbf{i} = \mathbf{0} \tag{2.22}$$

Combining the two equations in (2.22):

$$\hat{\mathbf{i}}^t\Delta\mathbf{v} - \hat{\mathbf{v}}^t\Delta\mathbf{i} = \mathbf{0} \tag{2.23}$$

By repeating the same procedure, it can be shown that the relationship (2.23) is still valid when **i** and **v** are replaced by their phasor domain transforms **I** and **V** respectively [2].

$$\hat{\mathbf{I}}^t \, \Delta\mathbf{V} - \hat{\mathbf{V}}^t \, \Delta\mathbf{I} = \mathbf{0} \tag{2.24}$$

The above expression is important in calculating sensitivities using the adjoint network approach as will be discussed later. In the adjoint network formulation, the independent sources are extracted from the network to form a multiport network as shown in Fig. 2-5 where the voltages and currents of those branches are denoted by $\mathbf{V}_p$ and $\mathbf{I}_p$ (port quantities). The voltages and currents of remaining branches which do not have independent sources themselves are denoted by $\mathbf{V}_b$ and $\mathbf{I}_b$ (branch quantities). This type of network division is required to derive a mathematical formulation for the sensitivity analysis of the network.

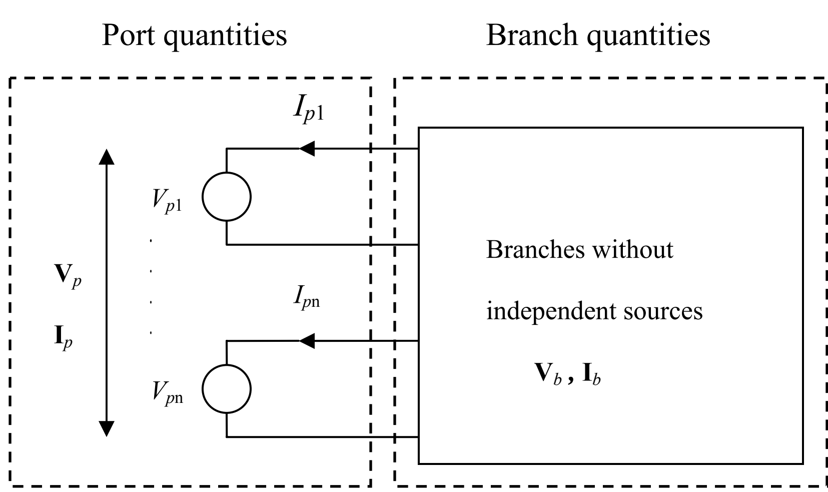


Fig. 2-5. Extraction of independent sources to a multiport

The two networks N and $\hat{N}$ are considered to be adjoint networks when the following three conditions are satisfied:

1. Both networks should have the same network topology;

2. a) If the branches of N and $\hat{N}$ which do not have independent sources hold the branch impedance matrices $\mathbf{Z}_b$ and $\hat{\mathbf{Z}}_b$ respectively, those matrices should be related as,

$$\mathbf{Z}_b^t = \hat{\mathbf{Z}}_b \tag{2.25}$$

where, $\mathbf{V}_b = \mathbf{Z}_b \mathbf{I}_b$ and $\hat{\mathbf{V}}_b = \hat{\mathbf{Z}}_b \hat{\mathbf{I}}_b$

b) If the branches of N and $\hat{N}$ which do not have independent sources are described by admittance matrices $\mathbf{Y}_b$ and $\hat{\mathbf{Y}}_b$ respectively, those matrices should be related as,

$$\mathbf{Y}_b^t = \hat{\mathbf{Y}}_b \tag{2.26}$$

Where, $\mathbf{I}_b = \mathbf{Y}_b \mathbf{V}_b$ and $\hat{\mathbf{I}}_b = \hat{\mathbf{Y}}_b \hat{\mathbf{V}}_b$

c) If the networks N and $\hat{N}$ are described by hybrid parameters, those parameters of two networks should be related as (2.29).

$$\begin{bmatrix} \mathbf{I}_{b1} \\ \mathbf{V}_{b2} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11b} & \mathbf{H}_{12b} \\ \mathbf{H}_{21b} & \mathbf{H}_{22b} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{b1} \\ \mathbf{I}_{b2} \end{bmatrix} \tag{2.27}$$

$$\begin{bmatrix} \hat{\mathbf{I}}_{b1} \\ \hat{\mathbf{V}}_{b2} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{H}}_{11b} & \hat{\mathbf{H}}_{12b} \\ \hat{\mathbf{H}}_{21b} & \hat{\mathbf{H}}_{22b} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{V}}_{b1} \\ \hat{\mathbf{I}}_{b2} \end{bmatrix} \tag{2.28}$$

$$\begin{bmatrix} \hat{\mathbf{H}}_{11b} & \hat{\mathbf{H}}_{12b} \\ \hat{\mathbf{H}}_{21b} & \hat{\mathbf{H}}_{22b} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11b}^t & -\mathbf{H}_{21b}^t \\ -\mathbf{H}_{12b}^t & \mathbf{H}_{22b}^t \end{bmatrix} \tag{2.29}$$

3. Corresponding independent sources in both networks should be the same in nature. i.e. a voltage (current) source in one network corresponds to a voltage (current) source in the other.

When the networks N and $\hat{N}$ satisfy the above conditions, the port quantities of both networks can be represented as follows. The impedance matrix $\mathbf{Z}_{oc}$ is the open circuit impedance as seen by each port. Then the port voltages and currents in both networks can be expressed as follows:

$$\mathbf{V}_p = -\mathbf{Z}_{oc}\mathbf{I}_p \tag{2.30}$$

$$\hat{\mathbf{V}}_p = -\hat{\mathbf{Z}}_{oc}\hat{\mathbf{I}}_p \tag{2.31}$$

Since $\hat{\mathbf{Z}}_b = \mathbf{Z}_b^t$ and then as derived in [2] the open circuit impedance matrices of both networks are related by,

$$\hat{\mathbf{Z}}_{oc} = \mathbf{Z}_{oc}^t \tag{2.32}$$

Equation (2.24) can then be expanded to its port quantities and branch quantities as shown below:

$$\hat{\mathbf{I}}^t \Delta\mathbf{V} - \hat{\mathbf{V}}^t \Delta\mathbf{I} = (\hat{\mathbf{I}}_p^t \Delta\mathbf{V}_p + \hat{\mathbf{I}}_b^t \Delta\mathbf{V}_b) - (\hat{\mathbf{V}}_p^t \Delta\mathbf{I}_p + \hat{\mathbf{V}}_b^t \Delta\mathbf{I}_b) = \mathbf{0}$$

$$-(\hat{\mathbf{I}}_p^t \Delta\mathbf{V}_p - \hat{\mathbf{V}}_p^t \Delta\mathbf{I}_p) = (\hat{\mathbf{I}}_b^t \Delta\mathbf{V}_b - \hat{\mathbf{V}}_b^t \Delta\mathbf{I}_b) \tag{2.33}$$

Due to the small changes in elements of $\mathbf{Z}_b$, the voltage change is approximated as,

$$\Delta\mathbf{V}_b = \Delta\mathbf{Z}_b\mathbf{I}_b + \mathbf{Z}_b\Delta\mathbf{I}_b \tag{2.34}$$

As a result, (2.30) is modified as follows:

$$\mathbf{\Delta V}_p = -\mathbf{\Delta Z}_{oc}\mathbf{I}_p - \mathbf{Z}_{oc}\mathbf{\Delta I}_p \tag{2.35}$$

By substituting (2.34) and (2.35) in (2.33), the following equation (2.36) can be derived.

Right hand side of (2.33):

$$\hat{\mathbf{I}}_b^t \mathbf{\Delta V}_b - \hat{\mathbf{V}}_b^t \mathbf{\Delta I}_b = (\hat{\mathbf{I}}_b^t \mathbf{\Delta Z}_b\mathbf{I}_b + \hat{\mathbf{I}}_b^t \mathbf{Z}_b\mathbf{\Delta I}_b) - \hat{\mathbf{I}}_b^t \mathbf{Z}_b^t \mathbf{\Delta I}_b = \hat{\mathbf{I}}_b^t \mathbf{\Delta Z}_b\mathbf{I}_b$$

Left hand side of (2.33):

$$-(\hat{\mathbf{I}}_p^t \mathbf{\Delta V}_p - \hat{\mathbf{V}}_p^t \mathbf{\Delta I}_p) = (\hat{\mathbf{I}}_p^t \mathbf{\Delta Z}_{oc}\mathbf{I}_p + \hat{\mathbf{I}}_p^t \mathbf{Z}_{oc}\mathbf{\Delta I}_p) - \hat{\mathbf{I}}_p^t \mathbf{Z}_{oc}^t\hat{\mathbf{\Delta I}}_p = \hat{\mathbf{I}}_p^t \mathbf{\Delta Z}_{oc}\mathbf{I}_p$$

Then, $$\hat{\mathbf{I}}_p^t \mathbf{\Delta Z}_{oc}\mathbf{I}_p = \hat{\mathbf{I}}_b^t \mathbf{\Delta Z}_b\mathbf{I}_b \tag{2.36}$$

In case of single port network, $\mathbf{Z}_{oc}$ becomes the input impedance $\mathbf{Z}_{in}$ of the network.

Then, $$\hat{\mathbf{I}}_p^t \mathbf{\Delta Z}_{in}\mathbf{I}_p = \hat{\mathbf{I}}_b^t \mathbf{\Delta Z}_b\mathbf{I}_b \tag{2.37}$$

Similarly, the following equations can be derived when the impedance matrices are replaced by corresponding admittance matrices. $\mathbf{Y}_{sc}$ is the short circuit admittance matrix for the multiport and $\mathbf{Y}_b$ is the admittance matrix of branches without independent sources.

$$\hat{\mathbf{V}}_p^t \mathbf{\Delta Y}_{sc}\mathbf{V}_p = \hat{\mathbf{V}}_b^t \mathbf{\Delta Y}_b\mathbf{V}_b \tag{2.38}$$

For single port network,

$$\hat{\mathbf{V}}_p^t \mathbf{\Delta Y}_{in}\mathbf{V}_p = \hat{\mathbf{V}}_b^t \mathbf{\Delta Y}_b\mathbf{V}_b \tag{2.39}$$

By selecting the suitable excitation for the networks N and $\hat{N}$, i.e. $\mathbf{I}_p$ and $\hat{\mathbf{I}}_p$ values, the left-hand side of (2.36) can produce the term to be analysed for sensitivity and the right hand side produces the sensitivities of corresponding parameters. The equations

(2.36) and (2.38) correspond to impedance matrix representation and admittance matrix representation of the network respectively. In case of hybrid parameter representation of the network, the corresponding can be derived as follows.

The port quantities of networks N and $\hat{N}$ can be extracted and written as,

$$\begin{bmatrix} \mathbf{I}_E \\ \mathbf{V}_J \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{EE} & \mathbf{H}_{EJ} \\ \mathbf{H}_{JE} & \mathbf{H}_{JJ} \end{bmatrix} \begin{bmatrix} \mathbf{V}_E \\ \mathbf{I}_J \end{bmatrix} \tag{2.40}$$

$$\begin{bmatrix} \hat{\mathbf{I}}_E \\ \hat{\mathbf{V}}_J \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{H}}_{EE} & \hat{\mathbf{H}}_{EJ} \\ \hat{\mathbf{H}}_{JE} & \hat{\mathbf{H}}_{JJ} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{V}}_E \\ \hat{\mathbf{I}}_J \end{bmatrix} \tag{2.41}$$

Where, the subscripts E and J are to represent the independent voltage sources and independent current sources respectively. By following the same procedure for deriving equation (2.36), equation (2.42) can be obtained for the case of hybrid parameter representation as shown below.

Left hand side of (2.33) is solved for hybrid parameters as follows:

$$-(\hat{\mathbf{I}}_p^t\, \Delta\mathbf{V}_p - \hat{\mathbf{V}}_p^t\, \Delta\mathbf{I}_p) = \begin{bmatrix} \hat{\mathbf{V}}_E^t & -\hat{\mathbf{I}}_J^t \end{bmatrix} \begin{bmatrix} \Delta\mathbf{H}_{EE} & \Delta\mathbf{H}_{EJ} \\ \Delta\mathbf{H}_{JE} & \Delta\mathbf{H}_{JJ} \end{bmatrix} \begin{bmatrix} \mathbf{V}_E \\ \mathbf{I}_J \end{bmatrix}$$

Right hand side of (2.33) is solved for hybrid parameters as follows:

$$(\hat{\mathbf{I}}_b^t\, \Delta\mathbf{V}_b - \hat{\mathbf{V}}_b^t\, \Delta\mathbf{I}_b) = \begin{bmatrix} -\hat{\mathbf{V}}_{b1}^t & \hat{\mathbf{I}}_{b2}^t \end{bmatrix} \begin{bmatrix} \Delta\mathbf{H}_{11b} & \Delta\mathbf{H}_{12b} \\ \Delta\mathbf{H}_{21b} & \Delta\mathbf{H}_{22b} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{b1} \\ \mathbf{I}_{b2} \end{bmatrix}$$

By combining above two equations:

$$\begin{bmatrix} \hat{\mathbf{V}}_E^t & -\hat{\mathbf{I}}_J^t \end{bmatrix} \begin{bmatrix} \Delta\mathbf{H}_{EE} & \Delta\mathbf{H}_{EJ} \\ \Delta\mathbf{H}_{JE} & \Delta\mathbf{H}_{JJ} \end{bmatrix} \begin{bmatrix} \mathbf{V}_E \\ \mathbf{I}_J \end{bmatrix} = \begin{bmatrix} -\hat{\mathbf{V}}_{b1}^t & \hat{\mathbf{I}}_{b2}^t \end{bmatrix} \begin{bmatrix} \Delta\mathbf{H}_{11b} & \Delta\mathbf{H}_{12b} \\ \Delta\mathbf{H}_{21b} & \Delta\mathbf{H}_{22b} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{b1} \\ \mathbf{I}_{b2} \end{bmatrix} \tag{2.42}$$

The sensitivity analysis using the adjoint network approach involves the following steps.

1) Analyzing the original networks N;

2) Deriving corresponding adjoint network $\hat{N}$;

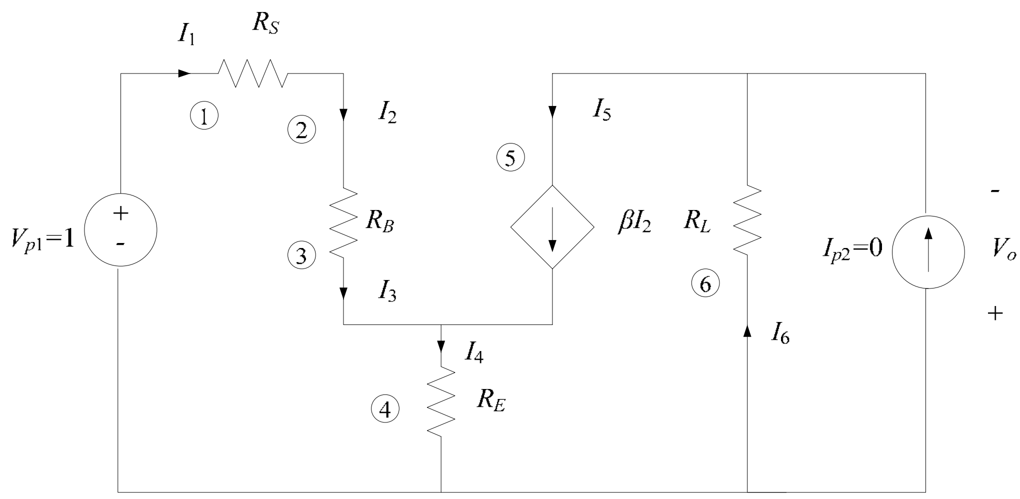
3) Selecting the suitable excitations for both networks depending on the desired performance index;

4) Solving both the networks for branch currents and voltages;

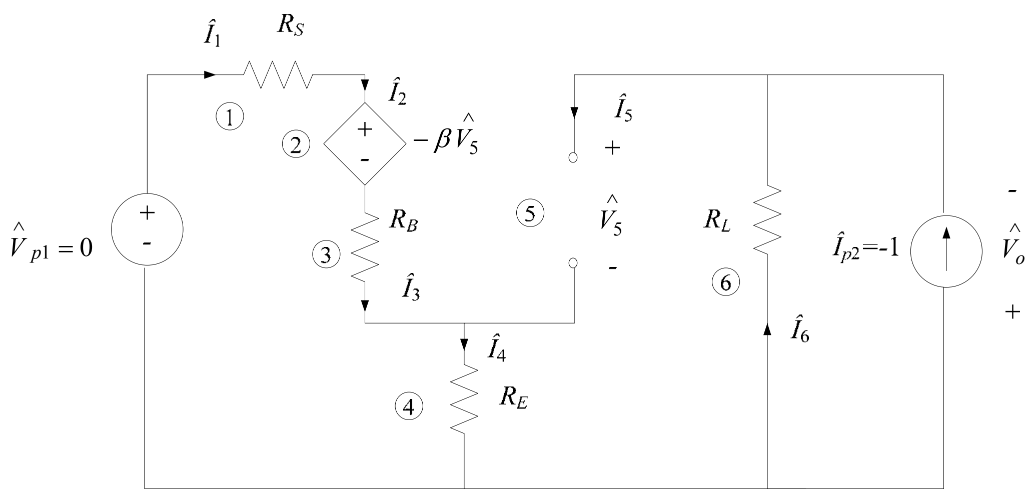
5) Using (2.36), (2.38) or (2.42) calculation of sensitivities of desired performance index.

Example 2.4: Sensitivity analysis by adjoint network approach

Consider the same circuit in Example 2.1, for sensitivity analysis using the adjoint network approach. The original circuit is reconfigured as shown in Fig. 2-6(a) to generate the corresponding adjoint network shown in Fig. 2-6 (b). The independent source branch is taken as a separate branch and all the other branches in circuit are numbered. An independent source branch is introduced to the desired performance index (here V_o). V_b and I_b represent the branch voltages and branch currents respectively where b denotes the branch number.



(a) Original network, *N*



(b) Adjoint network, $\hat{N}$

Fig. 2-6. Sensitivity analysis using adjoint network approach

The hybrid parameter representation of the circuit according to (2.27) is as follows:

$$\begin{bmatrix} I_5 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & R_S & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_B & 0 & 0 \\ 0 & 0 & 0 & 0 & R_E & 0 \\ 0 & 0 & 0 & 0 & 0 & R_L \end{bmatrix} \begin{bmatrix} V_5 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_6 \end{bmatrix}$$

Then, $\mathbf{V}_{b1} = \begin{bmatrix} V_5 \end{bmatrix}$, $\mathbf{V}_{b2} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_6 \end{bmatrix}$, $\mathbf{I}_{b1} = \begin{bmatrix} I_5 \end{bmatrix}$, $\mathbf{I}_{b2} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_6 \end{bmatrix}$

From (2.40), the interested parameter corresponding to voltage gain is $\mathbf{H}_{JE}$**.** So, the independent sources are to be selected in such a way that the left hand side of (2.42) contains only $\Delta\mathbf{H}_{JE}$ component.

The selected independent sources are as follows:

$$\hat{\mathbf{V}}_E^t = \begin{bmatrix} \hat{V}_{p1} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix},\ \hat{\mathbf{I}}_J^t = \begin{bmatrix} -\hat{I}_{p2} \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix},\ \mathbf{V}_E = \begin{bmatrix} V_{p1} \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix},\quad \mathbf{I}_J = \begin{bmatrix} -I_{p2} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

For the above selected excitations, two networks can be solved for branch quantities.

$$\mathbf{V}_{b1} = \begin{bmatrix} V_5 \end{bmatrix} = \begin{bmatrix} -1.8333 \end{bmatrix},\qquad \mathbf{I}_{b1} = \begin{bmatrix} I_5 \end{bmatrix} = \begin{bmatrix} 0.3333 \end{bmatrix}$$

$$\mathbf{I}_{b2} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_6 \end{bmatrix} = \begin{bmatrix} 0.1667 \\ 0.1667 \\ 0.1667 \\ 0.5000 \\ 0.3333 \end{bmatrix},\qquad \mathbf{V}_{b2} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0.1667 \\ 0.0000 \\ 0.3333 \\ 0.5000 \\ 1.3333 \end{bmatrix}$$

$$\hat{\mathbf{V}}_{b1} = \begin{bmatrix} \hat{V}_5 \end{bmatrix} = \begin{bmatrix} -2.6667 \end{bmatrix},\qquad \hat{\mathbf{I}}_{b1} = \begin{bmatrix} \hat{I}_5 \end{bmatrix} = \begin{bmatrix} 0.0000 \end{bmatrix}$$

$$\hat{\mathbf{I}}_{b2} = \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{I}_3 \\ \hat{I}_4 \\ \hat{I}_6 \end{bmatrix} = \begin{bmatrix} -1.3333 \\ -1.3333 \\ -1.3333 \\ -1.3333 \\ 1.0000 \end{bmatrix}, \qquad \hat{\mathbf{V}}_{b2} = \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \hat{V}_3 \\ \hat{V}_4 \\ \hat{V}_6 \end{bmatrix} = \begin{bmatrix} -1.3333 \\ 5.3333 \\ -2.6667 \\ -1.3333 \\ 4.0000 \end{bmatrix}$$

By substituting above values in (2.42):

$$\Delta\mathbf{H}_{JE} = \begin{bmatrix} 2.6667 \\ -1.3333 \\ -1.3333 \\ -1.3333 \\ -1.3333 \\ 1.0000 \end{bmatrix}^t \begin{bmatrix} 0 & 0 & \Delta\beta & 0 & 0 & 0 \\ 0 & \Delta R_S & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta R_B & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta R_E & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta R_L \end{bmatrix} \begin{bmatrix} -1.8333 \\ 0.1667 \\ 0.1667 \\ 0.1667 \\ 0.5000 \\ 0.3333 \end{bmatrix}$$

By simplifying above matrix multiplication, the change in performance index, i.e. voltage gain, is resulted as follows:

$$\Delta\mathbf{H}_{JE} = \left[\Delta f\right] = \left[-0.2222\Delta R_S - 0.2222\Delta R_B - 0.6667\Delta R_E + 0.3333\Delta R_L + 0.4444\Delta\beta\right]$$

$$\Delta f = -0.2222\Delta R_S - 0.2222\Delta R_B - 0.6667\Delta R_E + 0.3333\Delta R_L + 0.4444\Delta\beta \qquad (2.43)$$

With the assumption of $\dfrac{\partial f}{\partial R_S} \approx \dfrac{\Delta f}{\Delta R_S}$, the normalized sensitivity of the voltage gain against R_S can be calculated as follows:

$$S_{R_S}^f = \frac{\partial \ln(f)}{\partial \ln(R_S)} = \frac{\partial f}{\partial R_S} . \frac{R_S}{f} = (-0.2222) * \frac{1}{1.3333} = -0.1667$$

Similarly, the sensitivities of voltage gain against other parameters can be calculated and the results are shown below.

$$S_{R_B}^f = -0.3333 \qquad S_{R_E}^f = -0.5000 \qquad S_{R_L}^f = 1.0000 \qquad S_{\beta}^f = 0.6667$$

The above results are the same as the sensitivities derived from direct differentiation method and incremental network approach.

When (2.43) is written in symbolic form:

$$\Delta f = \hat{I}_1 I_1 \Delta R_S + \hat{I}_3 I_3 \Delta R_B + \hat{I}_4 I_4 \Delta R_E + \hat{I}_6 I_6 \Delta R_L + (-\hat{V}_5) I_5 \Delta\beta \qquad (2.44)$$

The above equation reveals that the sensitivity of performance index against a particular branch parameter is always involved in product of branch quantities (either voltage or current) of corresponding branches in original and adjoint networks. Depending on selection of the performance index, only the values of the branch quantities are varied. The advantages of the adjoint network approach over conventional simulation-based method are as follows.

1) The sensitivity analysis using this method is an entirely simulation- based approach and only two network simulations for the original and adjoint networks are required to derive all partial derivatives of the performance index. This offers great saving in simulation time.
2) Similar to the incremental network approach, this method also has the same admittance matrix for original and adjoint networks, which further reduces the computational effort significantly.

2.1.4 Time-Domain Sensitivity Analysis Using the Adjoint Network Method

Time-domain sensitivity is important when the transient behaviour of a network is to be analyzed. The sensitivities of objective function during the transient period can be determined by extending the adjoint network method described earlier for time domain sensitivity analysis of linear dynamic networks. The partial derivatives of the performance index with respect to network parameters are determined for a specified time interval.

Recalling the Tellegen's theorem for time domain quantities of branch voltages and currents of two networks N and $\hat{N}$ with same network topology, it is observed that

$$\mathbf{v}^t(t)\,\hat{\mathbf{i}}(\tau) = \mathbf{i}^t(t)\,\hat{\mathbf{v}}(\tau) = \hat{\mathbf{v}}^t(\tau)\,\mathbf{i}(t) = \hat{\mathbf{i}}^t(\tau)\,\mathbf{v}(t) = \mathbf{0} \tag{2.45}$$

where, t and τ are arbitrary time instants for networks N and $\hat{N}$, respectively.

When the perturbed network N_p is derived from the original network N, the following equation can be easily deduced from (2.45).

$$\hat{\mathbf{i}}^t(\tau)\,\Delta\mathbf{v}(t) = \mathbf{0} \qquad \hat{\mathbf{v}}^t(\tau)\,\Delta\mathbf{i}(t) = \mathbf{0}$$

$$\hat{\mathbf{i}}^t(\tau)\,\Delta\mathbf{v}(t) - \hat{\mathbf{v}}^t(\tau)\,\Delta\mathbf{i}(t) = \mathbf{0} \tag{2.46}$$

When the independent source branches are extracted from (2.46), the resultant equation with remaining branches is as follows. Subscript p denotes the independent source branches (port quantities) while b denotes remaining branches (branch quantities).

$$-\hat{\mathbf{i}}_p^t(\tau)\,\Delta\mathbf{v}_p(t) + \hat{\mathbf{v}}_p^t(\tau)\,\Delta\mathbf{i}_p(t) = \hat{\mathbf{i}}_b^t(\tau)\,\Delta\mathbf{v}_b(t) - \hat{\mathbf{v}}_b^t(\tau)\,\Delta\mathbf{i}_b(t) \tag{2.47}$$

The above equation is valid when the time instants t and τ are related by $\tau = t_f - t$, where $t_f > 0$ is the time instance at which the analysis is interested. The original circuit response is obtained for time interval $0 \leq t \leq t_f$ while adjoint network response is obtained for time interval $0 \leq \tau \leq t_f$. By integrating both sides of (2.47) from $t = 0$ to $t = t_f$, the following equation resulted.

$$\int_0^{t_f} [-\hat{\mathbf{i}}_p^t(\tau)\, \Delta\mathbf{v}_p(t) + \hat{\mathbf{v}}_p^t(\tau)\, \Delta\mathbf{i}_p(t)]_{\tau=t_f-t}\, dt = \int_0^{t_f} [\hat{\mathbf{i}}_b^t(\tau)\, \Delta\mathbf{v}_b(t) - \hat{\mathbf{v}}_b^t(\tau)\, \Delta\mathbf{i}_b(t)]_{\tau=t_f-t}\, dt \qquad (2.48)$$

It is worth noting that the response of adjoint network is analyzed in backward time in above integration resulted from the relationship, $\tau = t_f - t$. Assuming that there are n branches which do not possess independent sources, the right hand side of (2.48) can be written in scalar notation as follows.

$$\sum_{k=1}^{n} \int_0^{t_f} [\hat{i}_{bk}(\tau)\, \Delta v_{bk}(t) - \hat{v}_{bk}(\tau)\, \Delta i_{bk}(t)]_{\tau=t_f-t}\, dt \qquad (2.49)$$

The individual branches in (2.49) can be integrated separately and added up together to give the right side of (2.48).

For a resistive branch, the integral in (2.49) can be evaluated as follows.

$$\begin{aligned} &\int_0^{t_f} [\hat{i}_R(\tau)\, \Delta v_R(t) - \hat{v}_R(\tau)\, \Delta i_R(t)]_{\tau=t_f-t}\, dt \\ &= \int_0^{t_f} \{\hat{i}_R(\tau)[\, R\, \Delta i_R(t) + i_R(t)\, \Delta R] - R\, \hat{i}_R(\tau)\, \Delta i_R(t)\}_{\tau=t_f-t}\, dt \qquad (2.50) \\ &= \{\int_0^{t_f} [\hat{i}_R(\tau)\, i_R(t)\,]_{\tau=t_f-t}\, dt\}\Delta R \end{aligned}$$

For a capacitive branch, the integral in (2.49) can be evaluated to following approximation as detailed in [2].

$$\int_0^{t_f} [\hat{i}_C(\tau)\,\Delta v_C(t) - \hat{v}_C(\tau)\,\Delta i_C(t)]_{\tau=t_f-t}\,dt = \{-\int_0^{t_f} [\hat{v}_C(\tau)\,\dot{v}_C(t)\,]_{\tau=t_f-t}\;dt\}\Delta C \qquad (2.51)$$

Similarly, for an inductance branch, the integral in (2.49) can be evaluated to following approximation.

$$\int_0^{t_f} [\hat{i}_L(\tau)\,\Delta v_L(t) - \hat{v}_L(\tau)\,\Delta i_L(t)]_{\tau=t_f-t}\,dt = \{-\int_0^{t_f} [\hat{i}_L(\tau)\,\dot{i}_L(t)\,]_{\tau=t_f-t}\;dt\}\Delta L \qquad (2.52)$$

Equations (2.50), (2.51) and (2.52) consist of changes in corresponding branch elements and hence the right hand side of (2.49) can be written as an expression of those incremental values. By a proper selection of the port quantities (or excitations) for the original and adjoint networks, the term interested for sensitivity analysis can be produced at left hand side of (2.49). It is always necessary to keep the interested term as a port quantity for sensitivity analysis.

The time domain sensitivity analysis using adjoint network approach also preserves all the advantages those mentioned in frequency domain sensitivity analysis using the same approach. A detailed application of this method is presented in case studies in Chapter 4.

Although the network-based methods described in sections 2.1.2, 2.1.3 and 2.1.4 offer significant savings in the computational intensity of sensitivity analysis, they are only applicable to linear circuits. Many of the power electronic circuits are non-linear in nature and hence intermediate steps are to be taken to prepare them for sensitivity analysis using the adjoint network and incremental network approaches. The circuit averaging techniques described in Chapter 3 present such a provision in linearizing the power electronic circuits which facilitate the applicability of network-based sensitivity analysis methods on linearized average models.

Chapter 3

Circuit Averaging Methods for Power Electronic Converters

3.1 Introduction to Circuit Averaging Techniques

The inherent switching operation of power electronic converters leads to periodic changes in the circuit configuration and results in a non-linear dynamical system. For each combination of the states of switching devices, a separate set of equations is to be defined to describe the circuit. The Fig. 3-1 illustrates such a configuration change in a circuit due to switching operation. Given that each switching state typically lasts for a short while and is followed by another state, the number of sets of equations describing the dynamics of the circuit becomes large and their successive solution requires massive and time-consuming calculations. Hence the application of the sensitivity analysis methods such as adjoint network approach in these types of circuits becomes complicated and time consuming. If an averaging technique can be used to define the average behaviour of those circuits, the complication and computational effort in sensitivity analysis can be drastically reduced with little less of accuracy.

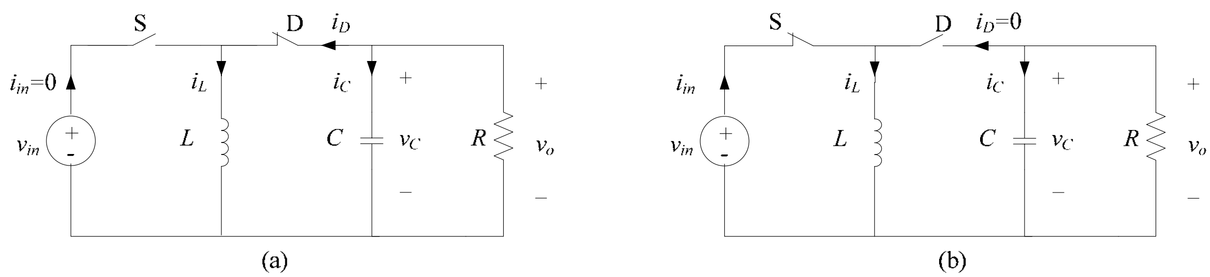


Fig. 3-1. Two typical circuit configurations due to operation of switch ‘S’ and diode ‘D’

In circuit averaging techniques, a single set of linear equations is defined by taking a linearly weighted average of the separate equations for each switching configuration of the circuit. Solution of power electronic circuit equations often reveals two components of response, namely

(a) A slowly varying component that results from the natural frequencies of the circuit and emerges due to the response of the circuit to the average impact of the switching function;

(b) A fast varying component that is due to the high-frequency switching action.

For a well-designed circuit the latter is typically much smaller in magnitude than the former. Unless the designer has specific interest in studying the small high-frequency component of the response, it is often the underlying low-frequency component that is of interest. Circuit averaging techniques provide a computationally efficient approach to obtaining the low-frequency components of the response of a switching circuit. In these techniques, a single equation is defined by taking weighted average of the separate equations for each switching configuration. It is worth noting that one of the averaging method explained in this chapter can be extended to include above mentioned high-frequency components in the average circuit model.

In the following two sections, two widely used circuit averaging techniques, namely state space averaging method and generalized averaging method, are discussed.

3.2 State Space Circuit Averaging Method

The state space averaging method is a well-developed theory and its applications in power electronic converter modeling are thoroughly discussed in [9] and [10]. In this method, a set of state equations are defined for each switching configuration of the circuit. Then the equations are combined by giving a weighing factor for each switching configuration, which represents the time based operational fraction of that configuration.

For simplicity, consider a circuit with a single controlled switch. In continuous conduction mode, the switching circuit is divided into two operational configurations depending on the state of the switching element being ON or OFF. The switching function, $q(t)$ defines the state of the switching element as:

$$q(t) = \begin{cases} 1 & \text{If switching element is ON} \\ 0 & \text{If switching element is OFF} \end{cases}$$

The average value of the switching function d over one switching cycle period T and its complement d' are defined as follows.

$$d = \overline{q(t)} = \int_{t-T}^{t} q(\tau)d\tau \qquad \text{and} \qquad d' = 1 - d$$

The averaging time interval T is assumed to be sufficiently smaller than the smallest time constant of the circuit. Even though d is constant during one cycle, it may be a time function over a large time period. A graphical view of switching function, $q(t)$ and its average value, $d(t)$ is as shown in Fig. 3-2.

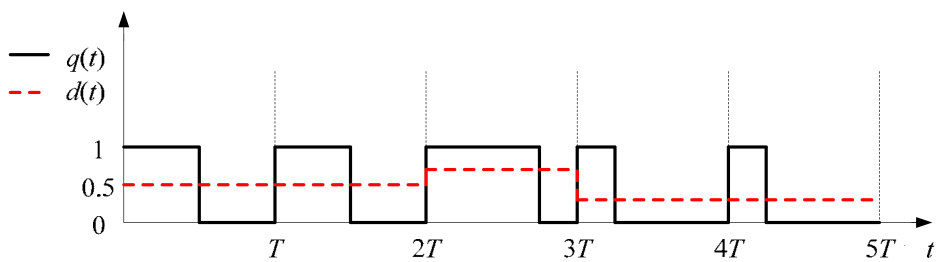


Fig. 3-2. Switching function and its average value

The two sets of state equations for ON and OFF status of the switching element are defined as follows.

$q(t) = 1$, (ON state)

$$\dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1\mathbf{u} \tag{3.1}$$

$$\mathbf{y} = \mathbf{C}_1\mathbf{x} + \mathbf{D}_1\mathbf{u} \tag{3.2}$$

$q(t) = 0$, (OFF state)

$$\dot{\mathbf{x}} = \mathbf{A}_2\mathbf{x} + \mathbf{B}_2\mathbf{u} \tag{3.3}$$

$$\mathbf{y} = \mathbf{C}_2\mathbf{x} + \mathbf{D}_2\mathbf{u} \tag{3.4}$$

The two sets of state equations are combined by means of the switching function as follows:

$$\dot{\mathbf{x}} = (\mathbf{A}_1 q(t) + \mathbf{A}_2 q'(t))\mathbf{x} + (\mathbf{B}_1 q(t) + \mathbf{B}_2 q'(t))\mathbf{u} \tag{3.5}$$

$$\mathbf{y} = (\mathbf{C}_1 q(t) + \mathbf{C}_2 q'(t))\mathbf{x} + (\mathbf{D}_1 q(t) + \mathbf{D}_2 q'(t))\mathbf{u} \tag{3.6}$$

where, $\mathbf{x}$ - state variables, $\mathbf{u}$ - input parameters, $\mathbf{y}$ - output parameters

$\mathbf{A}_1$- state matrix, $\mathbf{B}_1$- input matrix , $\mathbf{C}_1$- output matrix, $\mathbf{D}_1$- feed-through matrix

$q'(t)$ – complement of $q(t)$

By averaging the above equation over one switching cycle period T, the following set of equations is resulted.

$$\dot{\overline{\mathbf{x}}} \approx (\mathbf{A}_1 d + \mathbf{A}_2 d')\overline{\mathbf{x}} + (\mathbf{B}_1 d + \mathbf{B}_2 d')\overline{\mathbf{u}} \tag{3.7}$$

$$\overline{\mathbf{y}} \approx (\mathbf{C}_1 d + \mathbf{C}_2 d')\overline{\mathbf{x}} + (\mathbf{D}_1 d + \mathbf{D}_2 d')\overline{\mathbf{u}} \tag{3.8}$$

The above equations were derived with the following assumption. i.e. the average value of multiplication of q and x is equal to multiplication of average values of those. Since $\mathbf{x}(t)$ varies slowly relative to $q(t)$ such an assumption is valid.

$$\overline{q \cdot \mathbf{x}} = \overline{q} \cdot \overline{\mathbf{x}}$$

If the duty cycle can be kept constant, equations (3.7) and (3.8) become a set of linear equations. Consider the following example for further clarification of the method.

Example 3.1: **State space model for buck-boost converter**

Consider the buck-boost DC-DC converter shown in Fig. 3-3. The operation of the switching element, S is defined by the above mentioned switching function $q(t)$. Then the circuit equations can be developed for different switching states as follows.

$q(t)=1$, (*ON* status)

$$\frac{di_L}{dt}=\frac{1}{L}v_L=\frac{1}{L}v_{in} \tag{3.9}$$

$$\frac{dv_C}{dt}=\frac{1}{C}i_C=\frac{-1}{RC}\cdot v_C \tag{3.10}$$

$q(t)=0$, (*OFF* status)

$$\frac{di_L}{dt}=\frac{1}{L}v_L=\frac{1}{L}\cdot v_C \tag{3.11}$$

$$\frac{dv_C}{dt}=\frac{1}{C}i_C=\frac{-1}{RC}\cdot\left(Ri_L+v_C\right) \tag{3.12}$$

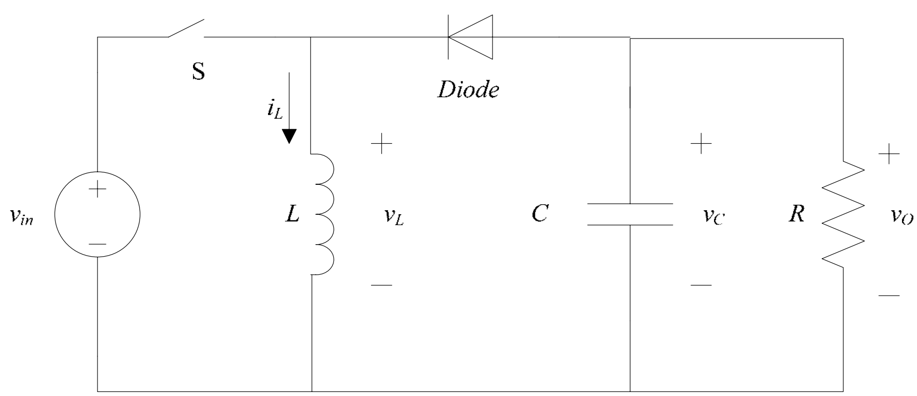


Fig. 3-3. Buck boost DC-DC converter

The equations (3.9) and (3.10) are combined with (3.11) and (3.12) respectively by means of $q(t)$ and its complement $q'(t)$ as follows.

$$\frac{di_L}{dt} = \frac{1}{L} \cdot (v_C q'(t)) + \frac{1}{L} \cdot (v_{in} q(t)) \tag{3.13}$$

$$\frac{dv_C}{dt} = \frac{-1}{RC} \cdot \left(R i_L q'(t) + v_C \right) \tag{3.14}$$

With the assumption of a sufficiently small switching period, the average model of the buck-boost converter is approximated to following equations.

$$\frac{d\overline{i_L}}{dt} \approx \frac{1}{L} \cdot (\overline{v_C} d'(t)) + \frac{1}{L} \cdot (\overline{v_{in}} d(t)) \tag{3.15}$$

$$\frac{d\overline{v_C}}{dt} \approx \frac{-1}{RC} \cdot \left(R\overline{i_L} d'(t) + \overline{v_C} \right) \tag{3.16}$$

The above set of equations, (3.15) and (3.16) can be linearized by either keeping the duty cycle a constant or using the perturbation technique as explained in [9]. The resulting linearized low frequency model from perturbation technique is not applicable to sensitivity analysis methods such as adjoint network approach. Hence for the linearization of above set of equations to apply in sensitivity analysis methods, d is to be kept constant or to be defined using a different set of equations as d changes.

The application of state space averaging is limited within the satisfaction of conditions of small ripple approximation and linear ripple approximation [12]. The small ripple approximation requires that the Fourier series expansion for a finite length segment of a circuit waveform to be dominated by its dc component. In case of linear ripple approximation, the circuit waveform is to be a linear function of time when examined over a time interval in between switching instants. In the following section, a more general averaging procedure is discussed, which is potentially applicable to a much broader class of power electronic circuits.

3.3 Generalized Circuit Averaging Method

The development of generalized average model for a periodically switched circuit is discussed with great detail in [11]. The method consists of two steps, firstly taking one-cycle averaging for each branch variable of the circuit and secondly synthesizing the 'average circuit element' corresponding to each branch variable.

The one-cycle average of branch variable, $x(t)$ is as follows:

$$\langle x \rangle (t) = \frac{1}{T} \int_{t-T}^{t} x(s) ds \tag{3.17}$$

where, T is the switching period.

The underlying circuit element and average circuit element can be graphically illustrated as shown in Fig. 3-4. Note that functions f and F correspond to underlying and average elements respectively are two different functions. The detailed derivation of such average circuit follows.

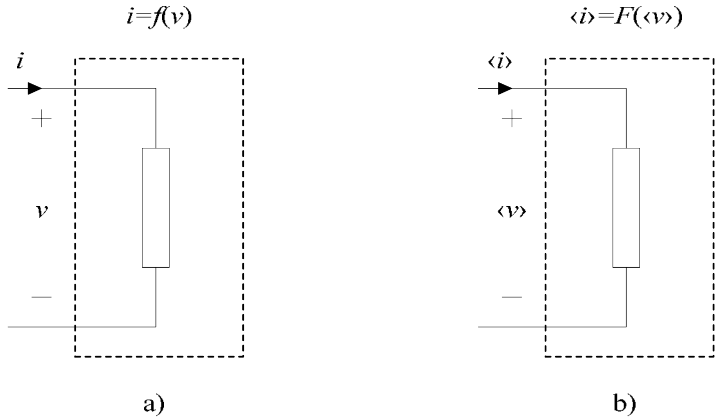


Fig. 3-4. a) Underlying circuit element and b) Averaged circuit element

The one-cycle average mentioned above is extended through Fourier series averaging method to achieve greater degree of accuracy. Any circuit waveform (of a branch variable), $x(t)$ can be approximated over time interval $(t\text{-}T,\ t]$ with arbitrary accuracy using the Fourier series representation.

$$x(t-T+s)=\sum_{k=-\infty}^{+\infty}\langle x\rangle_k(t)e^{jk\omega_S(t-T+s)} \tag{3.18}$$

where, $\langle x\rangle_k(t)$ are the complex Fourier coefficients

k – integer, $\omega_S=\dfrac{2\pi}{T}$, $s\in(0,T]$,

The Fourier coefficients are time-dependent since the waveform representation may vary depending on the selected time interval. The k^{th} Fourier coefficient or index-k average is determined by the following equation.

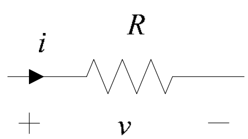
$$\langle x\rangle_k(t)=\frac{1}{T}\int_0^T x(t-T+s)e^{-jk\omega_S(t-T+s)}ds \tag{3.19}$$

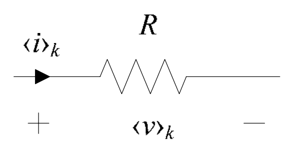
The average model of the waveform consists of several Fourier coefficients depending on dominant harmonics of the given waveform. The time derivative of the k^{th} coefficient of the Fourier series is given by the following equation [13].

$$\frac{d}{dt}\langle x\rangle_k(t) = \langle\frac{d}{dt}x\rangle_k(t) - jk\omega_S\langle x\rangle_k(t) \tag{3.20}$$

This equation derives the averaged branch variables for the basic circuit elements and consequently synthesizes the physical models as shown below.

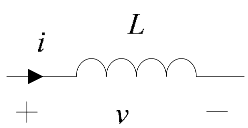
For a linear resistive element;

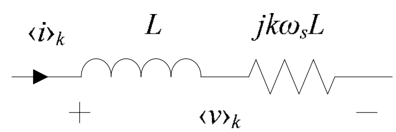




$v = Ri$ $\qquad$ $\langle v\rangle_k = R\langle i\rangle_k$

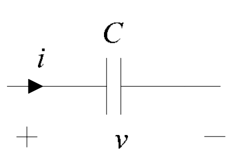
For an inductive element;

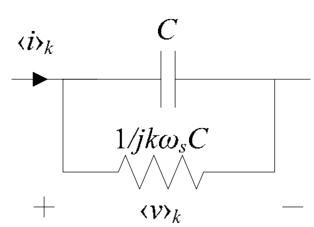




$v(t) = L\frac{d}{dt}i(t)$ $\qquad$ $L\frac{d}{dt}\langle i\rangle_k = \langle v\rangle_k - jk\omega_S L\langle i\rangle_k$

For a capacitive element;





$i(t) = C\frac{d}{dt}v(t)$ $\qquad$ $C\frac{d}{dt}\langle v\rangle_k = \langle i\rangle_k - jk\omega_S C\langle v\rangle_k$

The averaged model for switching element is also derived using the same technique as explained in [11]. Considering the underlying switching element in Fig. 3-5 (a), the following equations can be developed.

$$i_a = qi_c \qquad v_{cp} = qv_{ap} \qquad i_p = (1-q)i_c$$

where, $q = \begin{cases} 1 \\ 0 \end{cases}$ depending on the switching configuration

Fig. 3-5 (b) shows its equivalent of the switching element developed using the dependent sources. By taking the controlling port variables as v_{ap} and i_c and the non-controlling port variables as v_{ac} and i_a, the switch can be represented as a two port network. Then the switch is embedded in to the interested main circuit and the above mentioned port variables are determined in terms of main circuit parameters.

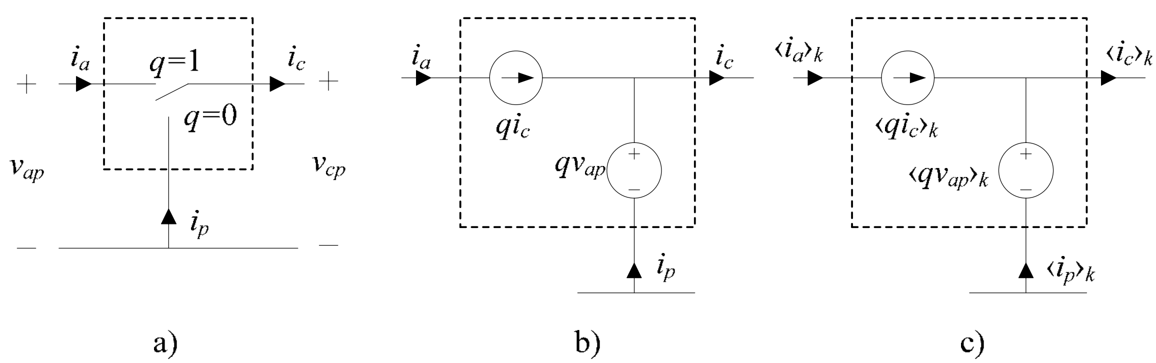


Fig. 3-5. Developing averaged model of switching element

To determine the averaged model of switching element, $\langle qv_{ap} \rangle_k$ and $\langle qi_c \rangle_k$ are to be evaluated for each index – k along with the other circuit parameters. The convolution formula (3.21) is used to determine the quantities, $\langle qv_{ap} \rangle_k$ and $\langle qi_c \rangle_k$.

The convolution formula for variables, x and y;

$$\langle xy\rangle_k = \sum_i \langle x\rangle_{k-i}\langle y\rangle_k \tag{3.21}$$

The following example provides a further clarification about the generalized averaging technique.

Example 3.2: **Generalized average model of buck-boost converter**

Consider the same buck boost converter circuit in Example 3.1. As shown in Fig. 3-6, the index – *k* averaged model or index - *k* sub-circuit of the converter can be synthesized by replacing all branch components with the corresponding averaged models defined before. The behaviour of the original circuit can be restored to an arbitrary accuracy level by taking summation of sufficient number of sub-circuits for different *k* values.

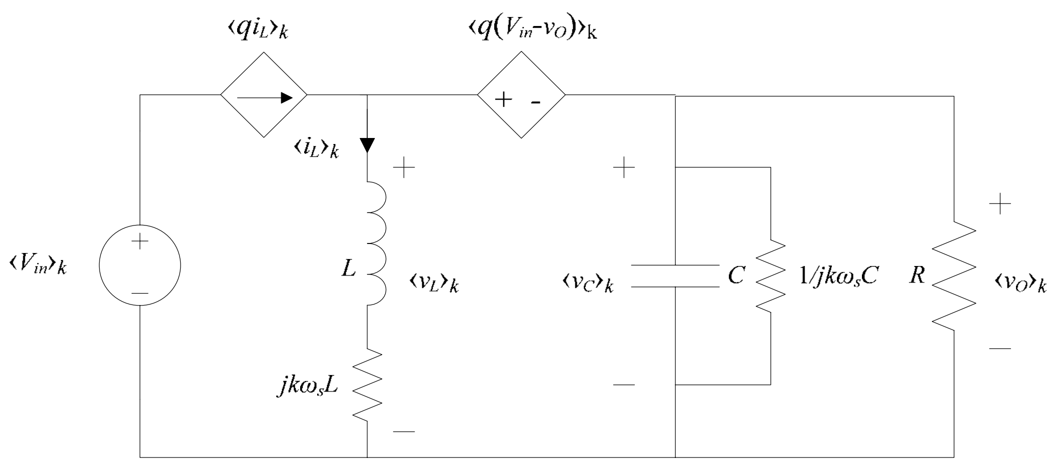


Fig. 3-6. Index-*k* averaged model of buck boost converter

The index-0 average circuit gives the simplified averaged DC model of the converter and it is same as the model resulted from state space averaging approach. Note that the dc average model is a suitable representation of the low-frequency behaviour of the circuit.

It contains the dynamics of the circuit response except for its ripple contents. Since a well-designed power electronic circuit will have small ripple it is often adequate to consider its low-frequency behaviour. Fig. 3-7 shows the index-0 model and it is worth noting that the frequency dependent component of the branch element is omitted here.

To determine the averaged quantities, $\langle qi_L \rangle_0$ and $\langle q(V_{in} - v_o) \rangle_0$ the convolution theorem is used.

From (3.21), $\langle qi_L \rangle_0 = \langle q \rangle_1 \langle i_L \rangle_{-1} + \langle q \rangle_0 \langle i_L \rangle_0 + \langle q \rangle_{-1} \langle i_L \rangle_1$

Since $\langle q \rangle_0 \langle i_L \rangle_0$ is the dominating part, by neglecting the other terms:

$$\langle qi_L \rangle_0 \approx \langle q \rangle_0 \langle i_L \rangle_0$$

Similarly, $\langle q(V_{in} - v_o) \rangle_0 \approx \langle q \rangle_0 \langle (V_{in} - v_o) \rangle_0$

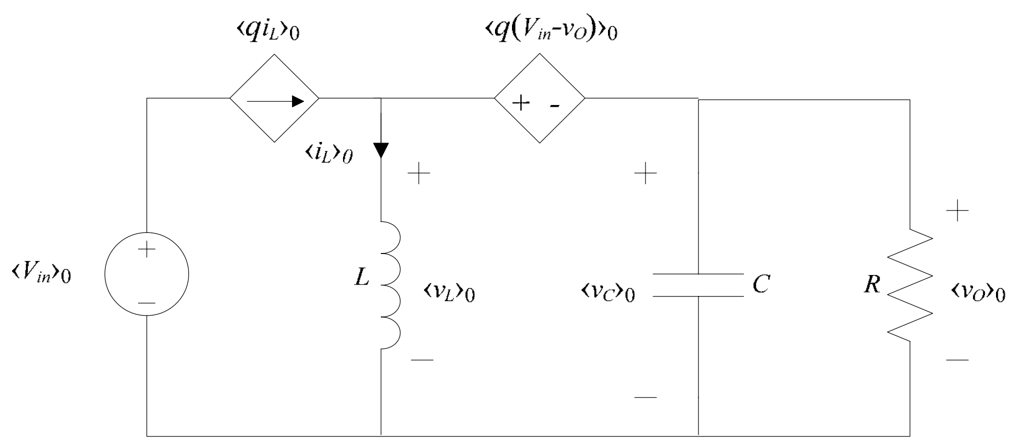


Fig. 3-7. Index-0 average model of the converter

Let $\langle q \rangle_0 = D_0$ and other average values as shown in Fig. 3-8.

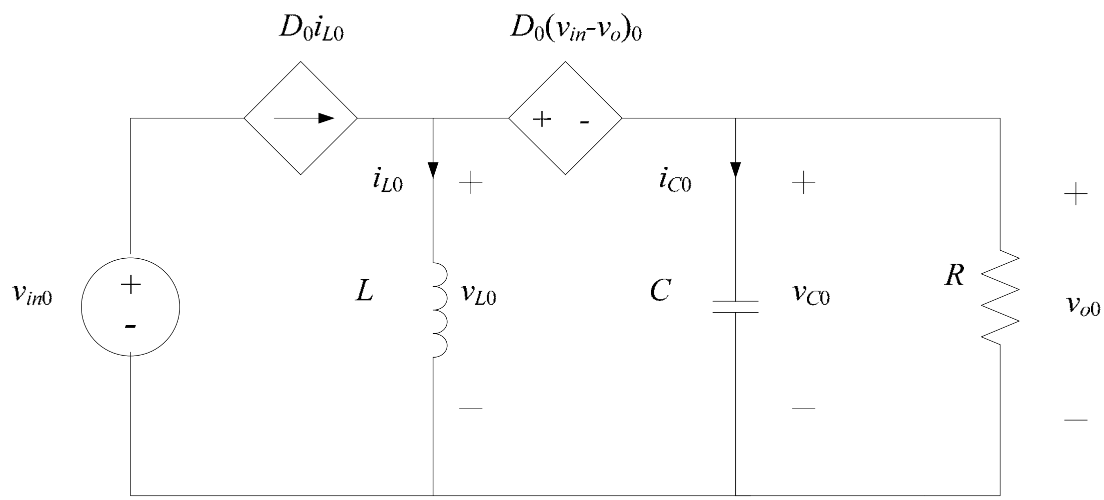


Fig. 3-8. Index-0 average model with revised notation

To validate the above model, the circuit was simulated for out-put voltage v_{o0} in the PSCAD-EMTDC circuit simulation software with its original converter circuit and the results are as shown in Fig. 3-9. The transient behaviour of the averaged circuit is almost following that of the original converter circuit.

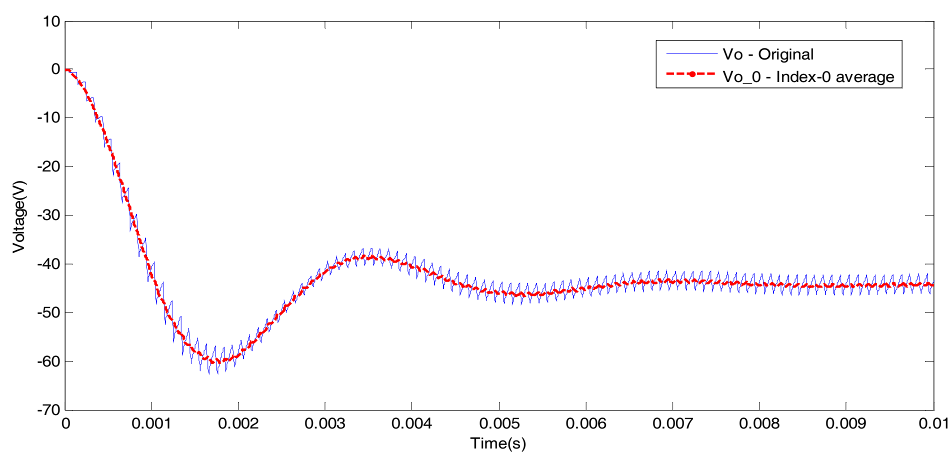
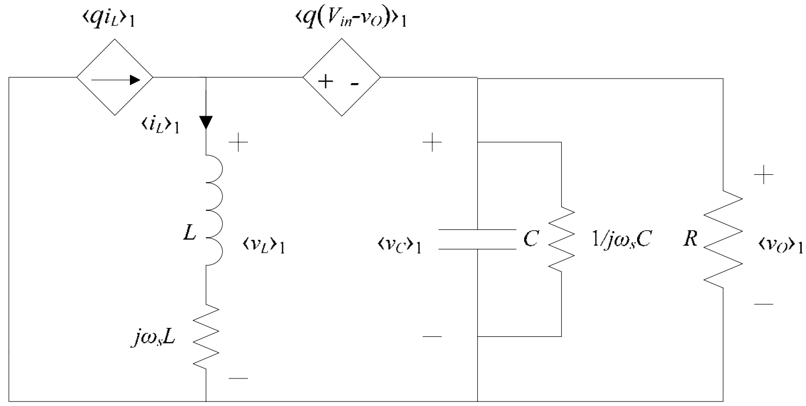


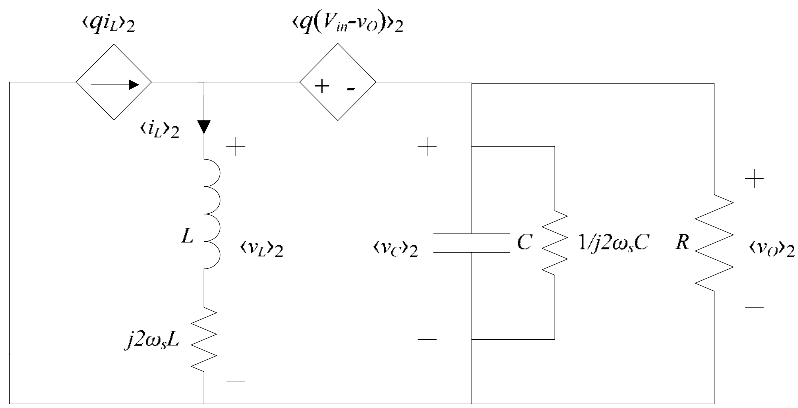
Fig. 3-9. Output voltage waveforms of original and index-0 averaged circuits

To achieve a greater degree of accuracy for the converter model, the index-0 sub-circuit can be augmented with index-1 and index-2 average models as shown in Fig. 3-10. The resulting steady state wave forms are as shown in Fig. 3-11.

It can be concluded that the combination of higher order index models with index-0 model gives acceptable approximation to the steady state voltage ripple of the original circuit. Depending on the portion of the waveform that is of interest, i.e. either the transient behaviour or the steady state ripple behaviour, selection of required sub-circuits should be done.



(a)



(b)

Fig. 3-10. Index-1, 2 sub-circuits of the converter

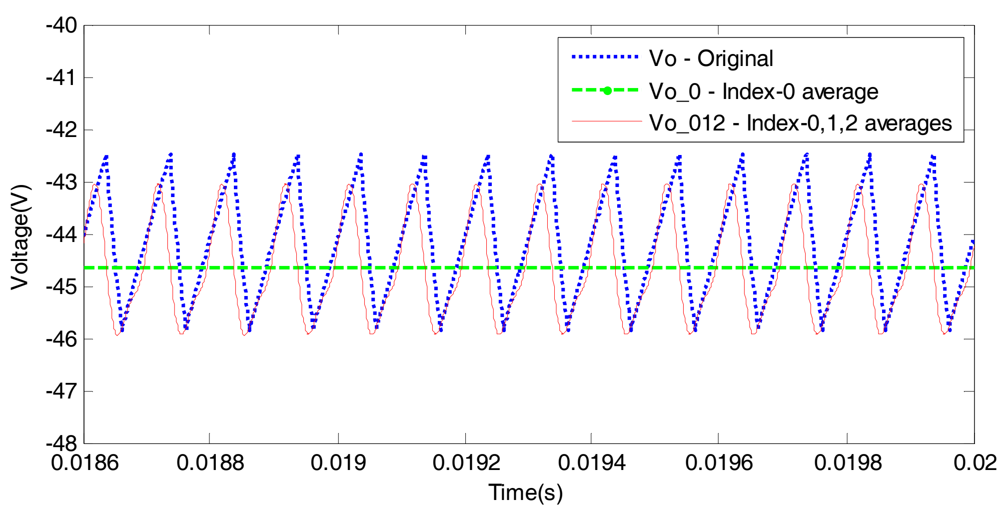


Fig. 3-11. Output voltage waveforms of original and averaged circuits

The advantages of generalized averaging method are as follows:

1) The generalized averaging method achieves a greater degree of accuracy by allowing higher order average models;
2) This method also allows direct analysis of the given circuit rather than in state space method, since all the branch variables are to be dealt with in synthesizing of the whole circuit. The consequential advantage is the possibility of using a circuit simulation software such as PSCAD-EMTDC in analyzing the circuit. Additionally, the generalized averaging method facilitates to automate the synthesis of averaged circuit model from the original circuit in simulation software. This is achievable through pre-defining of ‘averaged’ circuit elements itself in the software;
3) The less simulation time taken by averaged model when compared with that of underlying circuit is another consequential advantage of the generalized circuit averaging method [11].

Chapter 4

Case Studies

In sensitivity analysis, adjoint network approach has salient features over the other methods as explained in Chapter 2. The first case study discussed here, i.e. maximum power transfer circuit, is to demonstrate the practical usage of sensitivities derived using such a method. The second case study demonstrates the main objectives of this research. It combines the adjoint network sensitivity analysis method with circuit averaging technique to derive sensitivities of a non-linear switching circuit. Additionally, to achieve higher degree of accuracy in sensitivities, including of a high indexed averaged model analysed by incremental network sensitivity method is also discussed.

4.1 Maximum Power Transfer Circuit

Assume that the main objective of the circuit shown in Fig. 4-1 is to deliver maximum power to the load by optimizing the load side parameters. The circuit is then analyzed for sensitivities of transferred power to the load against the load side parameters and subsequently uses them for the optimization of power transfer to the load.

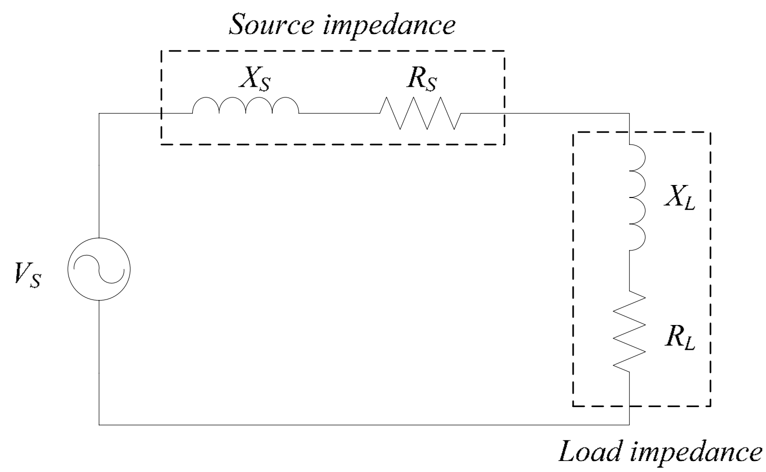


Fig. 4-1. Circuit for maximum power transfer

The maximum power transfer theorem asserts that the load impedance, Z_L should be equal to conjugate of source impedance, Z_s for delivering maximum power to the load.

where, $Z_L = R_L + jX_L$ and $Z_S = R_S + jX_S$ (4.1)

Hence the optimization of power transfer to load would result load impedance to satisfy the following conditions.

$$R_L = R_S \quad \text{and} \quad X_L = -X_S \tag{4.2}$$

The following section describes the use of adjoint network approach to derive the sensitivities of power transfer in above circuit and subsequently use them in optimization of power transfer to demonstrate above results.

4.1.1 Adjoint Network Sensitivity Analysis

The sensitivity of load power against the parameters R_L and X_L can be calculated using adjoint network approach as follows. The branches of the circuit without independent sources are numbered as shown in Fig. 4-2 and the branch voltages and branch currents are represented by V_b and I_b respectively where b denotes the branch number.

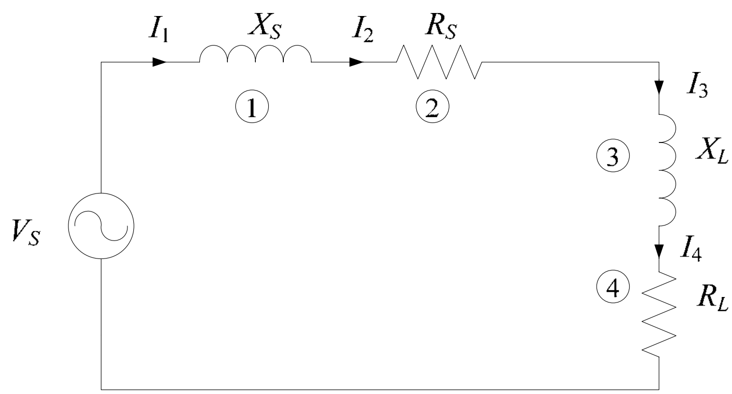


Fig. 4-2. Circuit for maximum power transfer

Recalling (2.27),

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} jX_S & 0 & 0 & 0 \\ 0 & R_S & 0 & 0 \\ 0 & 0 & jX_L & 0 \\ 0 & 0 & 0 & R_L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \tag{4.3}$$

where, $\quad \mathbf{I}_{b1} = 0 \qquad \mathbf{V}_{b2} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \qquad \mathbf{V}_{b1} = 0 \qquad \mathbf{I}_{b2} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$

$$\mathbf{H}_{11b} = 0 \qquad \mathbf{H}_{12b} = 0 \qquad \mathbf{H}_{21b} = 0 \qquad \mathbf{H}_{22b} = \begin{bmatrix} jX_S & 0 & 0 & 0 \\ 0 & R_S & 0 & 0 \\ 0 & 0 & jX_L & 0 \\ 0 & 0 & 0 & R_L \end{bmatrix}$$

The derivation of sensitivities of transferred power to load against network parameters is not straight-forward here. First it is necessary to derive the sensitivities of input admittance and then those of input current. Finally, the sensitivities of transferred power are derived. The steps involved in calculation are as follows.

Let, $I_1 = I_2 = I_3 = I_4 = I$

The power transferred to the load is given as: $P_{load} = |I|^2 R_L$ (4.4)

Then, the sensitivity of power against any network parameter x is in the following form:

$$\frac{\partial P_{load}}{\partial x} = |I|^2 \frac{\partial R_L}{\partial x} + 2|I| R_L \frac{\partial I}{\partial x} \tag{4.5}$$

The value of $\frac{\partial R_L}{\partial x}$ is either 1 or 0, while the value of $\frac{\partial I}{\partial x}$ is calculated in following manner. The circuit was modified to include input admittance, Y_{in} as shown in Fig. 4-3.

$$\frac{\partial I}{\partial x} = -\frac{\partial I_{in}}{\partial x} = \frac{\partial (V_S Y_{in})}{\partial x} = V_S \frac{\partial Y_{in}}{\partial x} \tag{4.6}$$

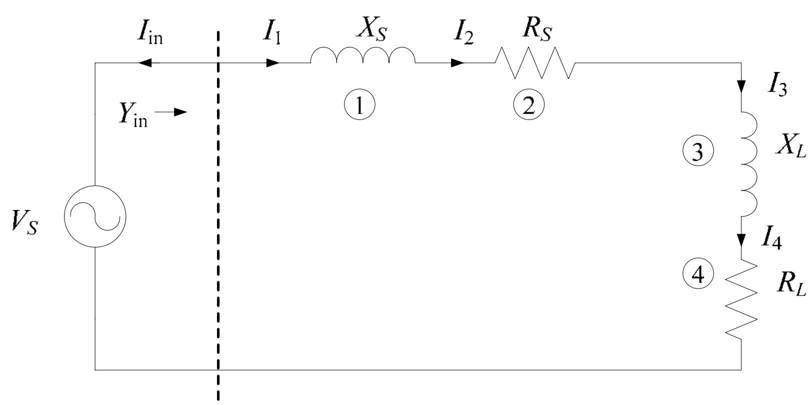


Fig. 4-3. Original network

From (2.40) the parameter interested corresponding to input admittance is $\mathbf{H}_{EE}$.

$$\frac{-\Delta \mathbf{H}_{EE}}{\Delta x} \approx \left[\frac{\partial Y_{in}}{\partial x} \right] \tag{4.7}$$

For clarity of the explanation, (2.42) is recalled as follows:

$$\begin{bmatrix} \hat{\mathbf{V}}_E^t & -\hat{\mathbf{I}}_J^t \end{bmatrix} \begin{bmatrix} \Delta\mathbf{H}_{EE} & \Delta\mathbf{H}_{EJ} \\ \Delta\mathbf{H}_{JE} & \Delta\mathbf{H}_{JJ} \end{bmatrix} \begin{bmatrix} \mathbf{V}_E \\ \mathbf{I}_J \end{bmatrix} = \begin{bmatrix} -\hat{\mathbf{V}}_{b1}^t & \hat{\mathbf{I}}_{b2}^t \end{bmatrix} \begin{bmatrix} \Delta\mathbf{H}_{11b} & \Delta\mathbf{H}_{12b} \\ \Delta\mathbf{H}_{21b} & \Delta\mathbf{H}_{22b} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{b1} \\ \mathbf{I}_{b2} \end{bmatrix}$$

So the independent sources are to be selected in such a way that the left hand side of above equation contains only $\Delta\mathbf{H}_{EE}$ component.

Therefore; $\hat{\mathbf{V}}_E^t = \begin{bmatrix} \hat{V}_S \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$ $\hat{\mathbf{I}}_J^t = \begin{bmatrix} 0 \end{bmatrix}$ $\mathbf{V}_E = \begin{bmatrix} V_S \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$ $\mathbf{I}_J = \begin{bmatrix} 0 \end{bmatrix}$

For the above selected excitations, both the original and adjoint networks have the exactly same configuration as the circuit shown in Fig. 4-3 and hence only one simulation is sufficient. This is another advantage of adjoint network approach. The circuit can be solved for branch quantities and results are as follows.

$$\hat{\mathbf{V}}_{b1}^t = \mathbf{V}_{b1} = \begin{bmatrix} 0 \end{bmatrix} \qquad \hat{\mathbf{I}}_{b2}^t = \mathbf{I}_{b2} = \begin{bmatrix} I \\ I \\ I \\ I \end{bmatrix}$$

Substituting above values;

$$\Delta\mathbf{H}_{EE} = \begin{bmatrix} 0 & I & I & I & I \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & j\Delta X_S & 0 & 0 & 0 \\ 0 & 0 & \Delta R_S & 0 & 0 \\ 0 & 0 & 0 & j\Delta X_L & 0 \\ 0 & 0 & 0 & 0 & \Delta R_L \end{bmatrix} \begin{bmatrix} 0 \\ I \\ I \\ I \\ I \end{bmatrix} \tag{4.8}$$

$$\Delta\mathbf{H}_{EE} = \begin{bmatrix} I^2 j\Delta X_S + I^2\Delta R_S + I^2 j\Delta X_L + I^2\Delta R_L \end{bmatrix} \tag{4.9}$$

From (4.6) and (4.7),

$$\frac{\partial I}{\partial X_S} = -jV_S I^2 \qquad \frac{\partial I}{\partial R_S} = -V_S I^2 \qquad \frac{\partial I}{\partial X_L} = -jV_S I^2 \qquad \frac{\partial I}{\partial R_L} = -V_S I^2 \tag{4.10}$$

The above derived sensitivities can be replaced in (4.5) to define the partial derivatives of load power.

$$\frac{\partial P_{load}}{\partial X_S} = 2|I| R_L(-jV_S I^2) \qquad \frac{\partial P_{load}}{\partial R_S} = 2|I| R_L(-V_S I^2)$$

$$\frac{\partial P_{load}}{\partial X_L} = 2|I| R_L(-jV_S I^2) \qquad \frac{\partial P_{load}}{\partial R_L} = |I|^2 + 2|I| R_L(-V_S I^2) \tag{4.11}$$

Then the load impedance parameters, R_L and X_L can be optimized to have maximum power transfer to load side. For the optimization process, let the objective function, f_0 to be (- P_{load}) which is to be minimized. Then the sensitivities of objective function against the network parameters are as follows.

$$\frac{\partial f_0}{\partial X_L} = -\frac{\partial P_{load}}{\partial X_L} \qquad \text{and} \qquad \frac{\partial f_0}{\partial R_L} = -\frac{\partial P_{load}}{\partial R_L} \tag{4.12}$$

The optimized R_L and X_L values are determined through following equations.

$$X_{L,new} = X_{L,old} - \alpha_X \left(\frac{\partial f_0}{\partial X_L} \right)_{old}$$

$$R_{L,new} = R_{L,old} - \alpha_R \left(\frac{\partial f_0}{\partial R_L} \right)_{old} \tag{4.13}$$

where, α_X and α_L – step sizes in search direction

The optimization was done with a fixed α value and a limited number of iterations for the circuit parameters of source impedance, Z_S = 0.5 +j0.04 and the source voltage V_S = 10 V. The code developed in Mathcad software for the optimization process is attached in Appendix 1. The resulting optimized values for load side parameters are close to the expected values from maximum power transfer theorem as shown below.

Table 4-1. Optimized load impedance for maximum power transfer

Circuit parameter values (Ω)			
R_S	R_L	X_S	X_L
0.5000	0.5009	0.0400	-0.0399

Once the power delivered to the load is optimized considering the conventional maximum power transfer theorem, only 50% power transmission efficiency is possible with above resultant parameter values. This is due to fact that the same amount of power delivered to load is dissipated at source side impedance. Hence maximum power transfer and maximum transmission efficiency are two contradictory requirements in power circuits. A compromised solution for both the power transfer and transmission efficiency can be achieved through a multiple objectives optimization problem as explained in [17]. This is done by introducing weighing factors to the individual objective functions, namely power transfer or transmission efficiency, according to their relative importance. In this case, Let f be the multiple objective function:

$$f = \gamma P_{load} - (1-\gamma) P_{loss} \quad (4.14)$$

where, P_{load} – power delivered to load

P_{loss} – power loss in source side ($P_{loss} = |I|^2 R_S$)

γ – weighing factor

The above multiple objectives function was optimized with a fixed step size in the search direction using the sensitivities derived from adjoint network approach. The code developed in Mathcad software for optimization process is attached to Appendix 2. The results were tabulated as shown below for different values of γ.

Table 4-2. Results comparison for different γ values

γ	P_{load}	P_{loss}	R_L	X_L	Power to load (%)	Transmission Efficiency (%)
1.0	50.0	50.0	0.50	-0.040	100.0	50.0
0.8	48.3	33.3	0.72	-0.041	96.6	59.2
0.6	44.8	22.8	0.98	-0.044	86.6	66.9
0.5	42.9	19.5	1.10	-0.044	85.8	68.8
0.4	41.2	16.9	1.22	-0.045	82.4	70.9

The above result reveals that the compromise between power transfer to load and transmission efficiency can be achieved through optimization process which uses the sensitivities derived from adjoint network approach.

4.2 Buck-boost DC-DC Converter

The buck boost converter is a controllable DC-DC converter in which the output voltage is either larger or less than the input voltage depending on the duty cycle of the switching element. Hence it is inherently a switching circuit and therefore it periodically changes its configuration with the switching operation. The result is a non-linear dynamical system. Hence the application of adjoint network approach or incremental network approach in sensitivity analysis becomes complicated and time consuming. Such an application of adjoint network approach is explained in [19] with piecewise linear approximation of the DC-DC converter.

In this case study, it is investigated the applicability of those sensitivity analysis methods in average model of the converter instead of the original circuit to cut off the computational effort significantly. For the simplicity, the index-0 average model of DC-DC converter developed under generalized averaging method in section 3.3 is considered.

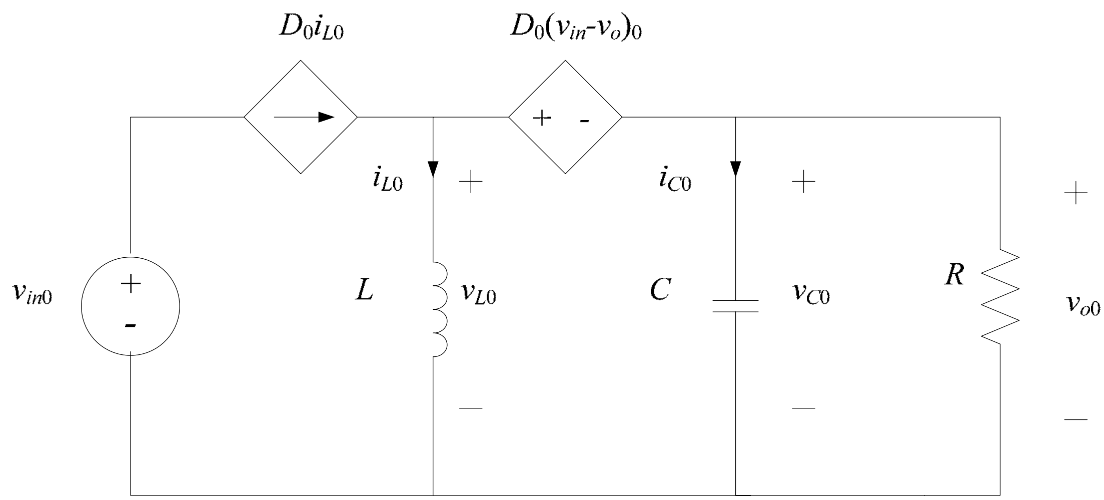
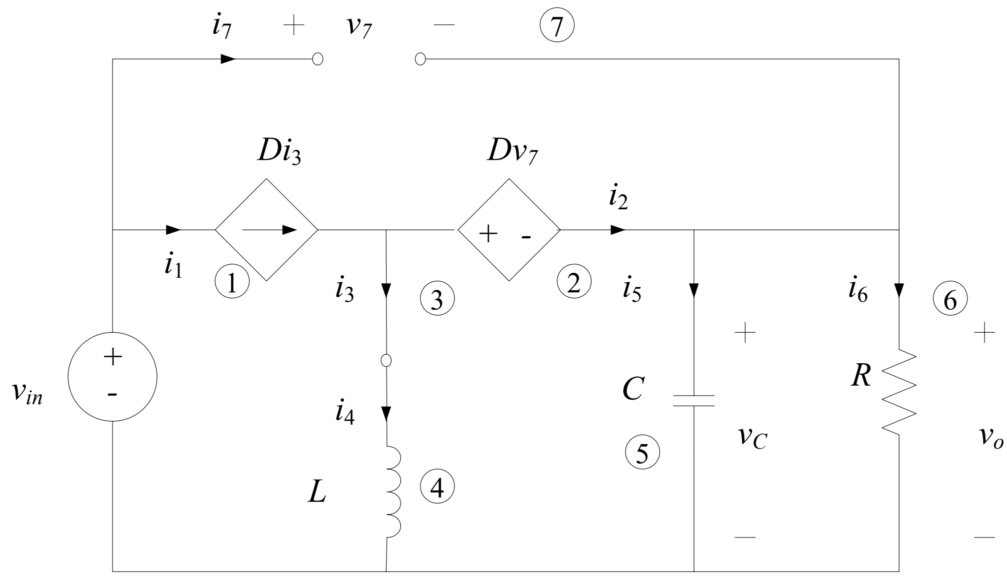


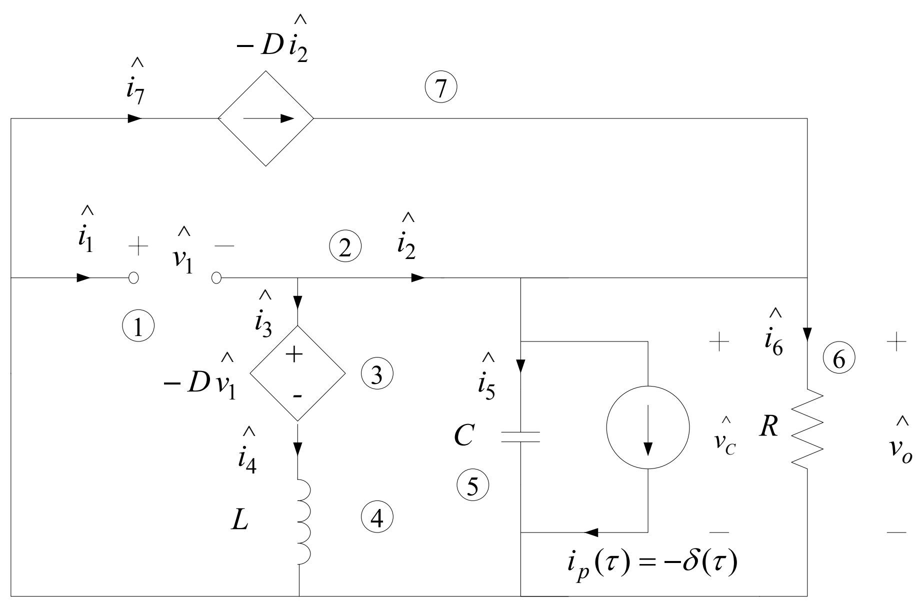
Fig. 4-4. Index-0 average circuit of buck-boost converter

4.2.1 Adjoint Network Sensitivity Analysis of Buck-boost Converter

The adjoint network sensitivity analysis in time domain is used here to analyse the transient behaviour of the circuit. Suppose that the transient sensitivity of out-put voltage, $v_o(t)$ of the converter is the measure of interest. Since the voltage across the capacitor, $v_C(t)$ equals to the output voltage, $v_o(t)$ the analysis of capacitor voltage is adequate. The index-0 averaged circuit was reconfigured as shown in Fig. 4-5(a) for the sensitivity analysis and the corresponding adjoint network developed is as shown in Fig. 4-5(b).



(a) Index-0 average network



(b) Adjoint network of index-0 average network

Fig. 4-5. Sensitivity analysis using adjoint network approach

By solving the index-0 average circuit, the transient solution for $v_C(t)$ is obtained as follows:

$$v_C(t) = \frac{-Dv_{in}}{(1 - D)}\left[1 - e^{-t/2RC}(\cos(\omega t) + \frac{1}{2RC\omega}\sin(\omega t))\right] \tag{4.15}$$

where, $\omega = \left(\frac{-1}{4R^2C^2} - \frac{(1 - D)^2}{LC}\right)^{\frac{1}{2}}$

Then, the time derivative of $v_C(t)$,

$$\dot{v}_C(t) = \frac{-Dv_{in}}{(1 - D)}\left[e^{-t/2RC}(\omega + \frac{1}{4R^2C^2\omega})\sin(\omega t)\right] \tag{4.16}$$

By solving the adjoint circuit, the transient solution for $\hat{v}_C(\tau)$,

$$\hat{v}_C(\tau) = \frac{1}{C}e^{-\tau/2RC}\left[\cos(\omega\tau) - \frac{1}{2RC\omega}\sin(\omega\tau)\right] \tag{4.17}$$

The time domain sensitivity of output voltage against capacitor value is equal to that of capacitor voltage against the capacitor value. Hence from (2.51),

$$\Delta v_o(t_f) = \Delta v_C(t_f) = -\left\{\int_0^{t_f}[\hat{v}_C(\tau)\,\dot{v}_C(t)]_{\tau=t_f-t}\,dt\right\}\Delta C \tag{4.18}$$

Substituting values,

$$\Delta v_o(t_f) = \left(\frac{Dv_{in}}{(1 - D)C}\cdot(\omega + \frac{1}{4R^2C^2\omega})\cdot e^{-t_f/2RC}\right)$$
$$\times\left[\left(\frac{1}{2}\sin(\omega t_f) + \frac{1}{4RC\omega}\cos(\omega t)\right)t_f - \frac{1}{4RC\omega^2}\sin(\omega t_f)\right]\Delta C$$

Suppose that the change in capacitor value is sufficiently small to take $\frac{\Delta v_o}{\Delta C}(t_f)$ as the time domain partial derivative $\frac{\partial v_o}{\partial C}(t_f)$ which leads to next formula:

$$\frac{\partial v_o}{\partial C}(t_f) = \left(\frac{Dv_{in}}{(1 - D)C} \cdot (\omega + \frac{1}{4R^2C^2\omega}) \cdot e^{-t_f/2RC} \right) \times \left[\left(\frac{1}{2}\sin(\omega t_f) + \frac{1}{4RC\omega}\cos(\omega t) \right) t_f - \frac{1}{4RC\omega^2}\sin(\omega t_f) \right] \tag{4.19}$$

The partial derivative $\frac{\partial v_o}{\partial C}(t_f)$ is the time domain sensitivity of output voltage against capacitor value.

Similarly, the time domain sensitivity of output voltage against inductor value can be derived and the derivative is as follows.

$$\frac{\partial v_o}{\partial L}(t_f) = \left(\frac{Dv_{in}}{(1 - D)RLC\omega} \cdot e^{-t_f/2RC} \right) \times \left[\left(\frac{(1 - D)^2 R}{2L}\sin(\omega t_f) - \frac{\mathrm{A}}{2}\cos(\omega t) \right) t_f + \frac{\mathrm{A}}{2\omega}\sin(\omega t_f) \right] \tag{4.20}$$

where, $\mathrm{A} = \left(\omega + \frac{1}{4R^2C^2\omega} - \frac{(1 - D)^2 R}{2LC\omega} \right)$

For the demonstration purpose, the average model of converter and its adjoint network are solved analytically here. But in real case the power electronic circuits are complex and hence getting analytical solution is a time consuming process. In such cases, the average model and its adjoint network are solved using simulation software and the resulting branch information for both networks as in (4.15) and (4.17) are stored. Then the sensitivity analysis can be carried out applying those data as shown in (4.18).

4.2.2 Results Comparison

To verify the accuracy of above sensitivities derived using adjoint network approach, those are compared with the same sensitivities calculated in brute-force manner. The simulation-based numerical method explained in section 2.1.1 is used as the brute-force method to calculate the sensitivities of index-0 average circuit as well as the original buck-boost converter. To calculate the sensitivity of output voltage v_o against capacitor value C in brute-force manner, two simulations were carried out for capacitor value C and incremented capacitor value C_1. Let v_{o1} and v_{o2} are the corresponding output voltages for the capacitor values C and C_1 respectively.

Recalling equation (2.8),

$$\frac{\partial v_o}{\partial C} \approx \frac{v_{o2} - v_{o1}}{C_1 - C} = \frac{v_{o2} - v_{o1}}{\Delta C} \tag{4.21}$$

Similarly, sensitivity against inductor value, L

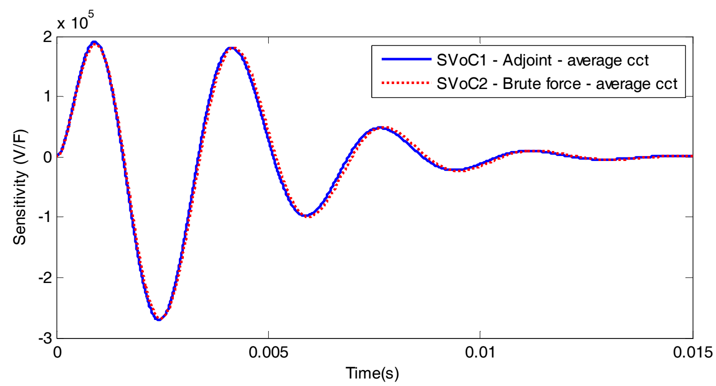
$$\frac{\partial v_o}{\partial L} \approx \frac{v_{o2} - v_{o1}}{L_1 - L} = \frac{v_{o2} - v_{o1}}{\Delta L} \tag{4.22}$$

According to (4.21), for calculation of sensitivities in brute-force manner, an incremental value for the parameters (ΔC or ΔL) is to be introduced. The sensitivities of index-0 average circuit derived from adjoint network approach and those of index-0 average circuit and original buck-boost converter calculated in brute-force manner were graphed and compared as follows.

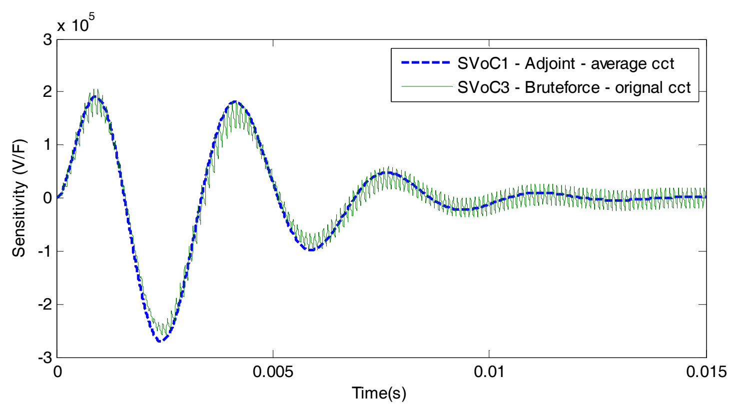
For the calculations in brute-force manner, the change in capacitor value, ΔC is taken as 5% of the capacitor value first. i.e. C = 100 μF and C_1 = 105 μF

Change in capacitance, Δ*C* = 5% of capacitor value

The waveforms of the time domain sensitivities mentioned above are as follows. For index-0 average circuit, the sensitivity of output voltage against capacitor value derived using adjoint network approach is denoted by SVoC1. For the same circuit, the same sensitivity calculated in brute-force manner is denoted by SVoC2. For the original circuit, SVoC3 denotes the sensitivity of output voltage against capacitor value derived in brute-force manner.



(a)

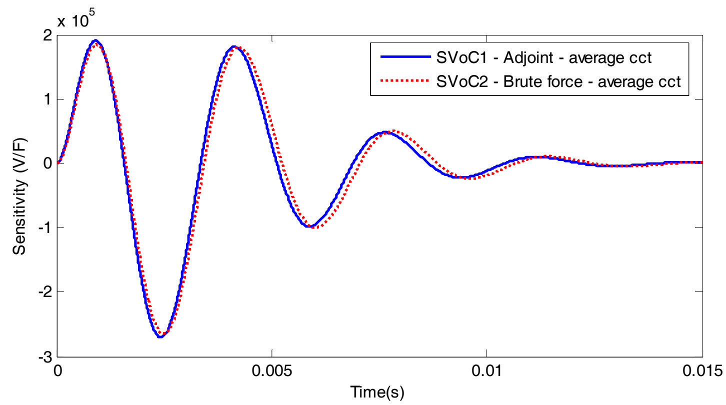


(b)

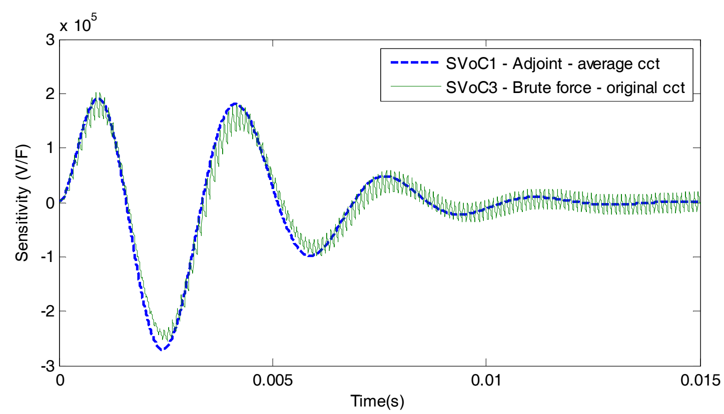
Fig. 4-6. Comparison of time domain sensitivities (Δ*C* = 5%)

The results shows that the time domain sensitivities derived by the adjoint network approach combined with the circuit averaging technique are similar to those of original network calculated brute-force manner. Several ΔC values, i.e. 10%, 20% and -10%, were tried out for the brute-force method to calculate sensitivities and the comparison of those with adjoint sensitivities is as follows.

Change in capacitance, ΔC = 10% of capacitor value



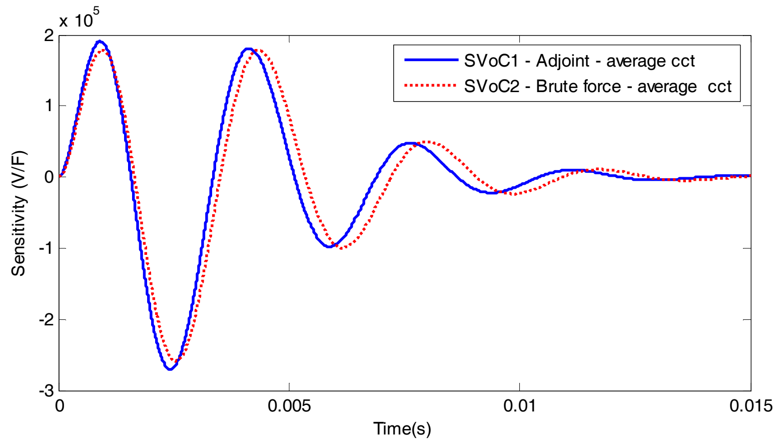
(a)



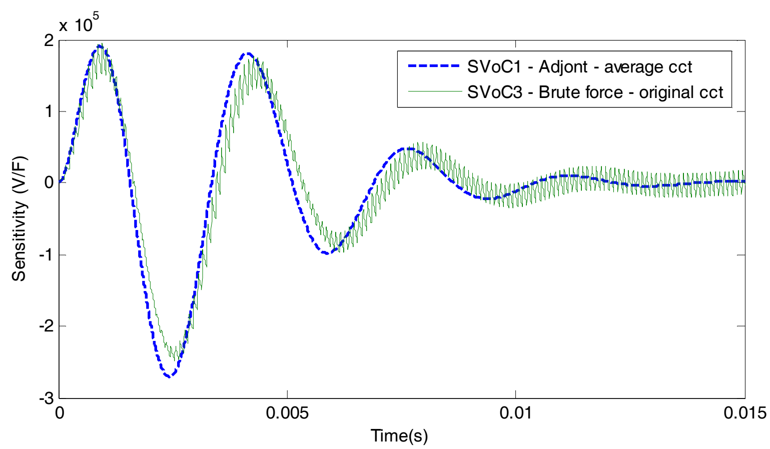
(b)

Fig. 4-7. Comparison of time domain sensitivities (ΔC = 10%)

Change in capacitance, ΔC = 20% of capacitor value



(a)



(b)

Fig. 4-8. Comparison of time domain sensitivities (ΔC = 20%)

Change in capacitance, Δ*C* = -10% of capacitor value

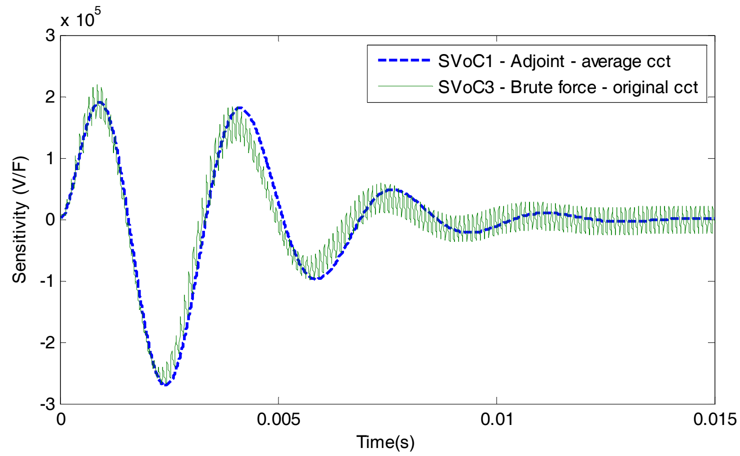
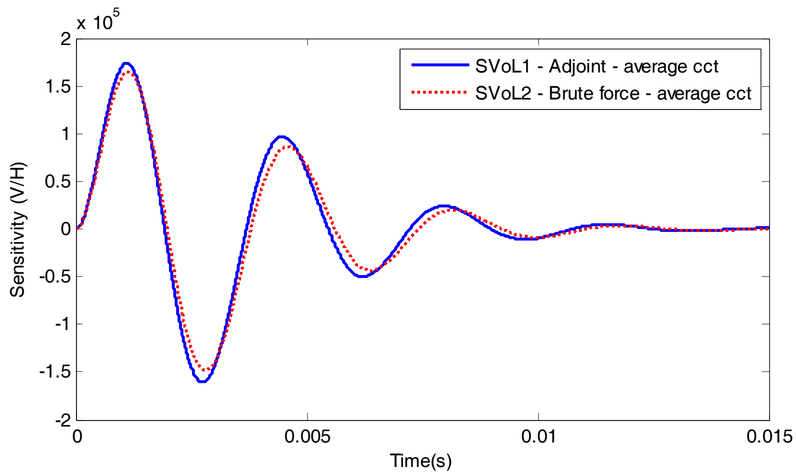


Fig. 4-9. Comparison of time domain sensitivities (Δ*C* = -10%)

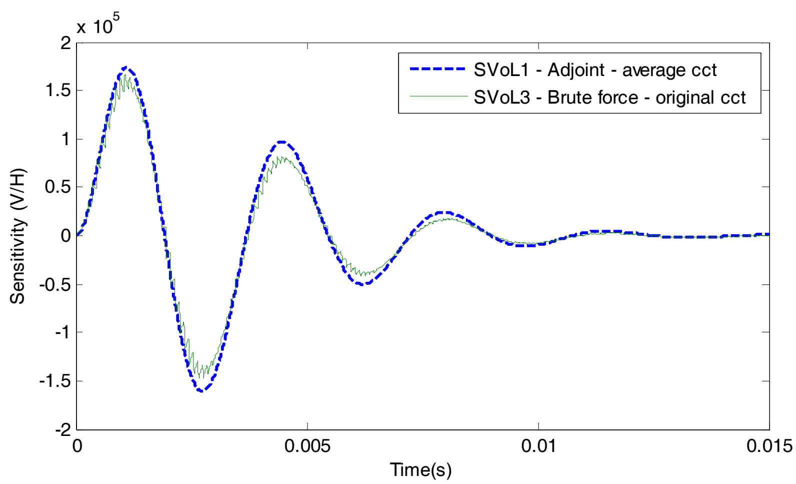
It was noticed that as the Δ*C* value is increased there is a phase shift in sensitivities calculated by brute-force manner with respect to those of adjoint network approach. This is happened due to the introduction of Δ*C* value which differs the natural frequency of the circuit in brute-force method. But in case of adjoint network approach, it takes originally introduced parameter values for the sensitivity calculation. A negative Δ*C* value causes a phase shift in reverse direction as shown in Fig. 4-9.

The same comparison explained above was carried out for the sensitivity of output voltage, v_o against inductor value, *L*. The comparison is as presented in the following graphs.

Change in inductance, ΔL = 5% of inductor value



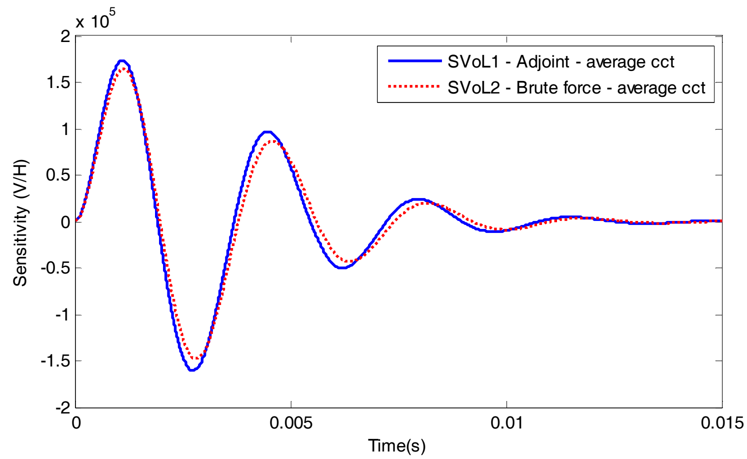
(a)



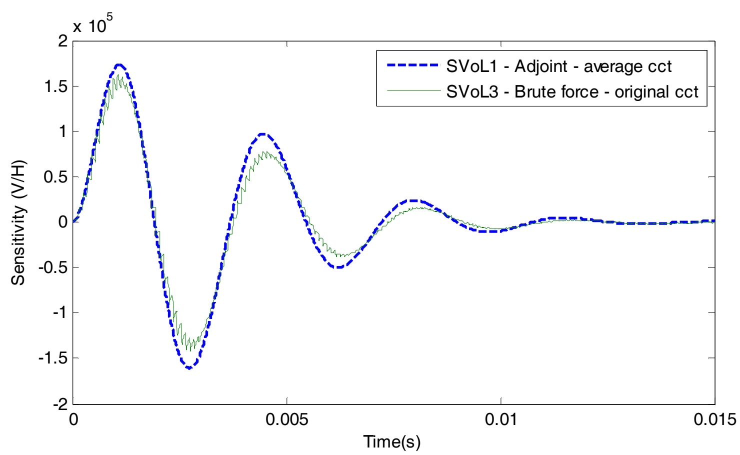
(b)

Fig. 4-10. Comparison of time domain sensitivities (ΔL = 5%)

Change in inductance, ΔL = 10% of inductor value



(a)



(b)

Fig. 4-11. Comparison of time domain sensitivities (ΔL = 10%)

The above results confirm that the time domain sensitivities derived by the adjoint network approach combined with the circuit averaging technique are similar to those of original network calculated in brute-force manner. Hence, in sensitivity analysis of switching circuits, the salient features of adjoint network approach can be achieved by analyzing the average model instead of its original network with preserving reasonable accuracy.

One of the applications of above derived time domain sensitivities using adjoint network approach combined with circuit averaging techniques may be the optimization of circuit parameters to reduce the transient overshoot of a measured circuit variable. For example, the transient overshoot of the output voltage of buck-boost converter can be reduced by optimising capacitor value, C and inductor value, L with the sensitivities derived in the above manner.

4.2.3 Sensitivity in Optimization

With given parameter values of the buck-boost converter, the output voltage of index-0 average circuit and its reference value are as shown in Fig. 4-12.

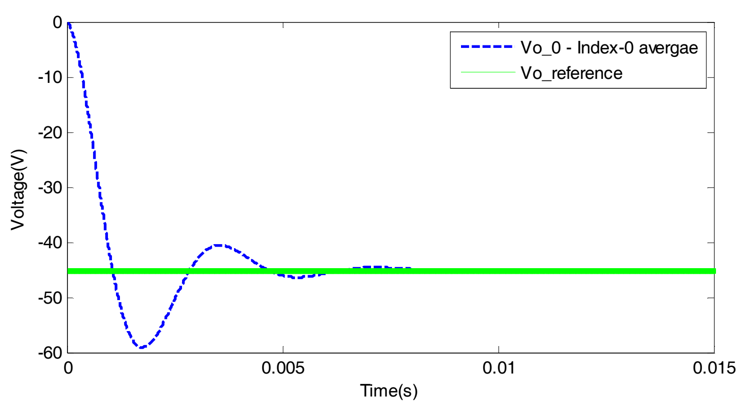


Fig. 4-12. Overshoot of output voltage of index-0 average circuit

An objective function f_o, is defined to reduce overshoot of output voltage in index-0 average circuit as given by (4.23).

$$f_o = \int_{t_0}^{t_1} (v_o(t) - v_{o,ref})^2 \, dt \tag{4.23}$$

where, t_0 and t_1 are to be decided upon output voltage pattern

The sensitivity of objective function against the circuit parameters can be defined using Leibniz Integral Rule as follows.

$$\frac{\partial f_o}{\partial C} = \int_{t_0}^{t_1} 2(v_o(t) - v_{o,ref}) \frac{\partial v_o(t)}{\partial C} dt \tag{4.24}$$

$$\frac{\partial f_o}{\partial L} = \int_{t_0}^{t_1} 2(v_o(t) - v_{o,ref}) \frac{\partial v_o(t)}{\partial L} dt \tag{4.25}$$

The quantities $\frac{\partial v_o(t)}{\partial C}$ and $\frac{\partial v_o(t)}{\partial L}$ are the time domain sensitivities and can be replaced with those derived in previous section.

Once the sensitivities of objective function are calculated from (4.24) and (4.25), the parameters C and L can be optimized as follows.

$$C_{new} = C_{old} - \alpha_C \left(\frac{\partial f_o}{\partial C} \right)_{old} \tag{4.26}$$

$$L_{new} = L_{old} - \alpha_L \left(\frac{\partial f_o}{\partial L} \right)_{old} \tag{4.27}$$

where, α_C and α_L are the step lengths in the search direction.

A code developed in Mathcad software for the optimization of circuit parameters to reduce overshoot of the output voltage of the index-0 average circuit is shown in Appendix 3. It was revealed that the optimization process continues to reduce overshoot drastically as shown in Fig. 4-13. The objective function does not have a local minimum as increasingly larger inductors and increasingly smaller capacitors will continue to lower this objective function. Moderate values for parameters were selected through the optimization as given below.

Table 4-3. Optimized circuit parameters

Parameter	Before optimization	After optimization
Inductance, *L*	180 μH	194 μH
Capacitance, *C*	100 μF	27 μF

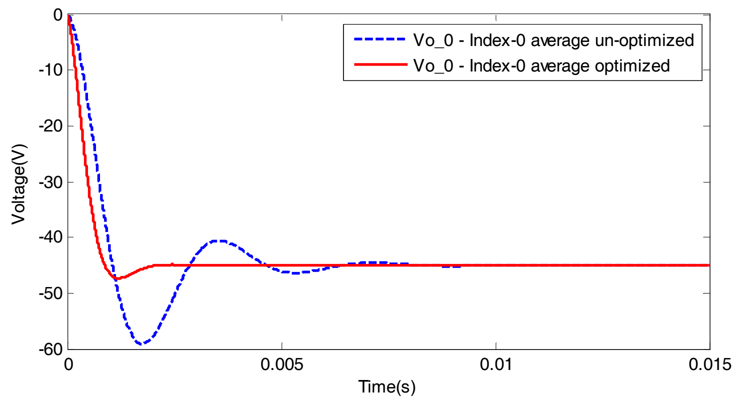
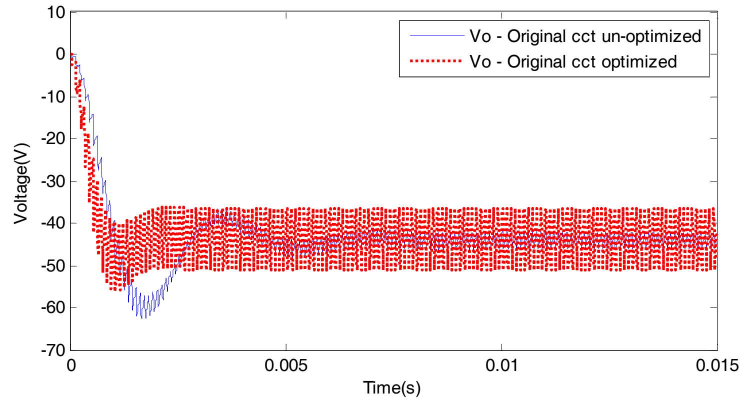
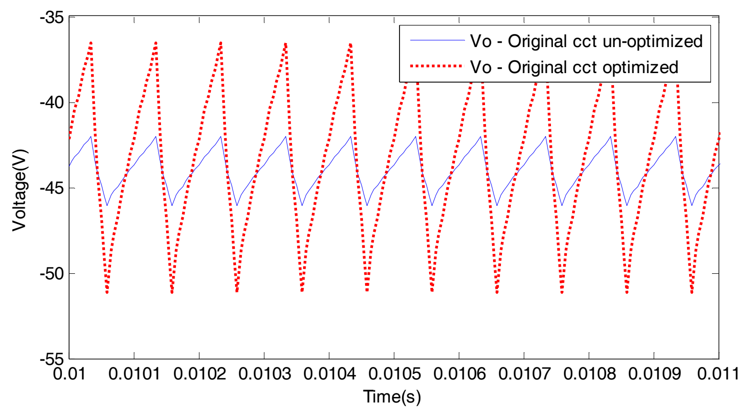


Fig. 4-13. Output voltage of index-0 average circuit after and before optimization

Once the optimized parameters are substituted in original buck boost converter, it revealed a practical limitation of using only the index-0 average circuit to represent original circuit. The index-0 average circuit does not take in to account the ripple in the output voltage. Hence the optimization of transient overshoot results in amplified ripple at steady state as shown in Fig. 4-14. This problem can be overcome simply by including the index-1 average circuit which provides an approximation to the ripple waveform. Then the optimization problem can be taken as a multiple objective functions problem to reduce both the transient overshoot and ripple waveform.



(a) Voltage overshoot and ripple voltage



(b) Zoomed view of ripple voltage

Fig. 4-14. Output voltage of original circuit before and after optimization

4.2.4 Sensitivity with Higher Index Average Circuit

Since the objective of including the index-1 average circuit is to minimize steady state voltage ripple, a frequency domain sensitivity analysis method would be sufficient to study the sensitivities of the circuit. Either the adjoint network approach or incremental network approach in frequency domain can be used to derive the sensitivities. The incremental network approach is used here to demonstrate the use of combination of sensitivities derived from different analysis methods in optimization.

The index-1 average circuit derived in section 3.2 was simplified using convolution theorem as shown in the following figure.

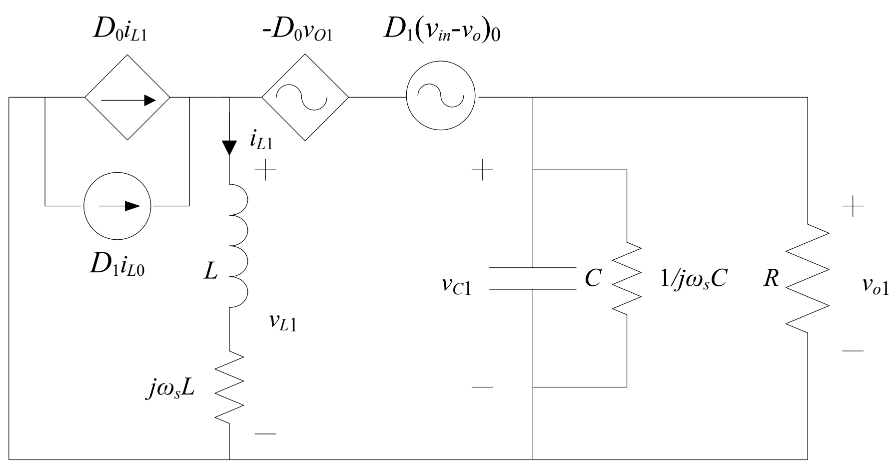


Fig. 4-15. Index-1 average model of buck-boost converter

The steady state output voltage of the combination of the index-0 and index-1 average circuits is as shown in following Fig. 4-16.

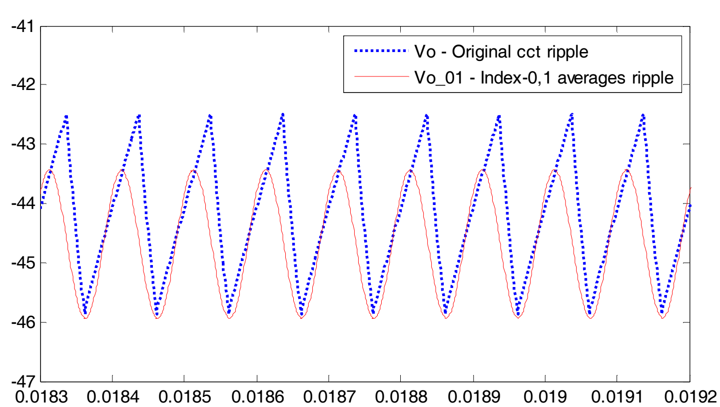


Fig. 4-16. Steady state output voltage

The corresponding incremental network of index-1 average circuit is developed as shown in Fig. 4-17.

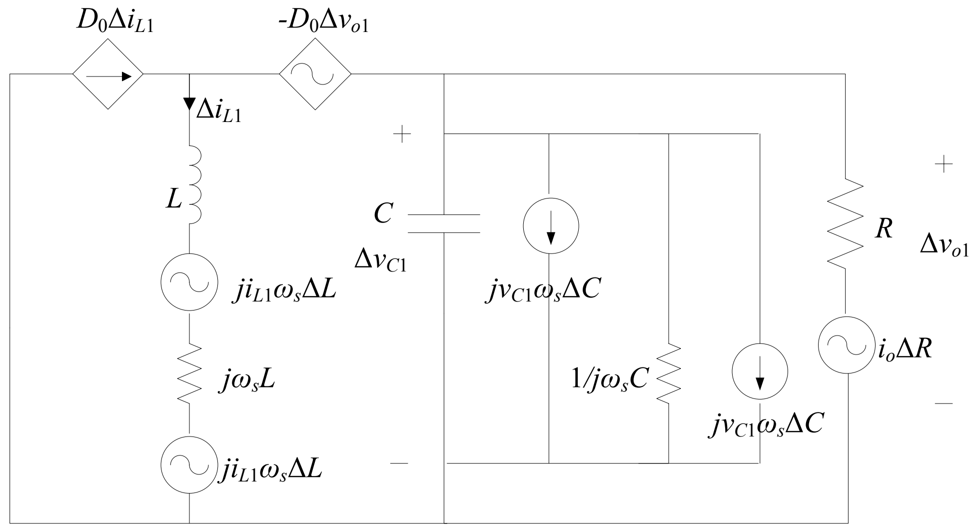


Fig. 4-17. Incremental network of index-1 average circuit

By solving the incremental circuit, the change in output voltage, Δv_{o1} or Δv_{C1} (both are same) can be expressed as follows.

$$\Delta v_{o1} = \Delta v_{C1} = \frac{4v_{C1}\omega_s \Delta C + j\left(2(1-D)i_{L1}/L\right)\Delta L + j\,(2i_{o1}/R)\,\Delta R}{\sqrt{\left(\left((1-D)^2/\omega_s L\right) - 4\omega_s C\right)^2 + \left(2/R\right)^2}} \tag{4.28}$$

Hence, the sensitivities of output voltage can be approximated as follows.

$$\frac{\partial v_{o1}}{\partial C} \approx \frac{\Delta v_{o1}}{\Delta C} = \frac{4v_{C1}\omega_s}{\sqrt{\left(\left((1-D)^2/\omega_s L\right) - 4\omega_s C\right)^2 + \left(2/R\right)^2}} \tag{4.29}$$

$$\frac{\partial v_{o1}}{\partial L} \approx \frac{\Delta v_{o1}}{\Delta L} = \frac{2(1-D)i_{L1}/L}{\sqrt{\left(\left((1-D)^2/\omega_s L\right) - 4\omega_s C\right)^2 + \left(2/R\right)^2}} \tag{4.30}$$

To reduce the voltage ripple, let the objective function, f_1 is to be the output voltage of the index-1 circuit.

$$f_1 = v_{o1} \quad \text{then,} \quad \frac{\partial f_1}{\partial C} = \frac{\partial v_{o1}}{\partial C} \qquad \frac{\partial f_1}{\partial L} = \frac{\partial v_{o1}}{\partial L} \tag{4.31}$$

With the combination of objective functions to reduce the transient overshoot and voltage ripple, the multiple objectives function, f can be defined as;

$$f = \gamma_0 f_0 + \gamma_1 f_1 \tag{4.32}$$

Where, γ_0 and γ_1 are weighing factors

Then, $$\frac{\partial f}{\partial C} = \gamma_0 \frac{\partial f_0}{\partial C} + \gamma_1 \frac{\partial f_1}{\partial C} \quad \text{and} \quad \frac{\partial f}{\partial L} = \gamma_0 \frac{\partial f_0}{\partial L} + \gamma_1 \frac{\partial f_1}{\partial L} \tag{4.33}$$

From (4.24), (4.25) and (4.33),

$$\frac{\partial f}{\partial C} = \int_{t_0}^{t_1} 2(v_o(t) - v_{ref}) \frac{\partial v_o(t)}{\partial C} dt + \gamma \frac{\partial v_{o1}}{\partial C}$$

$$\frac{\partial f}{\partial L} = \int_{t_0}^{t_1} 2(v_o(t) - v_{ref}) \frac{\partial v_o(t)}{\partial L} dt + \gamma \frac{\partial v_{o1}}{\partial L} \tag{4.34}$$

The optimized parameter values given by the following equations where α is the step length in the search direction.

$$C_{new} = C_{old} - \alpha \left(\frac{\partial f_0}{\partial C} + \gamma \frac{\partial f_1}{\partial C} \right)_{old}$$

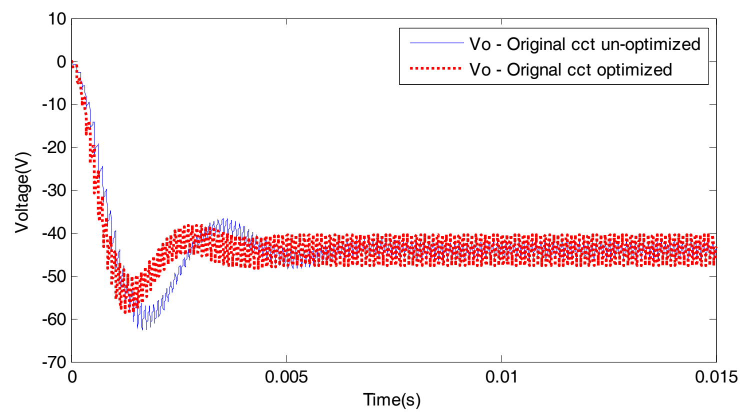
$$L_{new} = L_{old} - \alpha \left(\frac{\partial f_0}{\partial L} + \gamma \frac{\partial f_1}{\partial L} \right)_{old} \tag{4.35}$$

An optimization code was written in Mathcad as shown in Appendix 4 and the resulting parameter values after optimization were as follows.

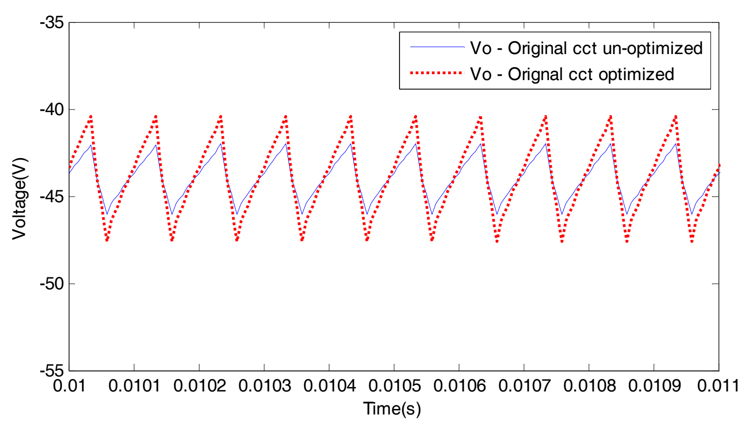
Table 4-4. Optimized circuit parameters (multiple objectives)

Parameter	Before optimization	After optimization
Inductance, *L*	180 μH	187 μH
Capacitance, *C*	100 μF	56 μF

The output voltage waveforms of the original buck-boost converter circuit before and after optimization were graphed as shown below. Note that the optimization of this multi-objective function yields a controlled compromise between the importance of the two competing objective functions. The designer can select a suitable compromise level depending on design specifications.



(a) Output voltages



(b) Zoomed view of steady state ripple

Fig. 4-18. Output voltage waveform before and after optimization

Hence, it can be summarized the steps involved in sensitivity analysis of power electronic circuits as follows.

1) Developing the index-0 average model of the given circuit
2) Augmenting higher order average circuits depending on requirement
3) Synthesizing the adjoint network of developed average model
4) Selecting suitable excitation of the circuits depending on the desired performance index
5) Performing circuit simulation on average model and its adjoint network
6) Deriving sensitivities of interested performance index

Chapter 5

Conclusions, Contributions and Recommendations

5.1 Conclusions and Contributions

The main objective of this research was to investigate the applicability of computationally efficient sensitivity analysis method to complex power electronic circuits by deriving linearized average model. The following highlights are the concluding remarks of the research work carried out.

1. The existing simulations based sensitivity analysis methods were thoroughly studied to identify their capabilities and limitations in application to power electronic circuits. The studies revealed that the adjoint network approach has salient features over the others but is limited in application to linear circuits only.

2. The circuit averaging techniques available in literature were investigated for their applicability to a variety of circuits. The average model developed using generalized circuit averaging techniques provided better representation of the original circuit compared to those developed by other method. The average model was validated by comparing its response to that of original circuit.

3. A computationally efficient sensitivity analysis method was introduced for power electronic switching circuits by combining adjoint network approach and equivalent average model of underlying circuit. The resulting sensitivities were verified with those derived in brute-force manner and it was demonstrated that the results are acceptable.

4. The practical limitation of above proposed method with only index-0 average circuit was identified since it does not represent the ripple behaviour of performance index such as in power electronic converters. This problem was eliminated by embedding index-1 average model to the analysis.

5. The practical usage of above derived sensitivities was demonstrated by applying them successfully in an optimization process.
6. In the context of this thesis, sensitivity analysis is the process by which sensitivity of a given performance index is assessed when certain circuit parameters vary. This is a valid and important aspect of a design, which is done after design (parameter selection) is completed. However a by-product of the sensitivity analysis

is generation of partial derivatives, which can be useful in the optimization and parameter selection phase as well.

5.2 Recommendations

Based on the sensitivity analysis method introduced in this research, the following are proposed.

1. Developing a method for sensitivity analysis of power electronic circuits embedded with a control system. The control system introduces non-linearity (such as control limits) to circuit in many cases and may not be linearized through circuit averaging process itself.

2. Developing an automated sensitivity analysis scheme for power electronic circuits in circuit simulation software such as PSCAD/EMTDC. Such a scheme may involve following steps.

    i. Modeling the given circuit in software tool

    ii. Deriving average model of the given circuit

        - Using generalized averaging technique, the average model of the given circuit can be developed by replacing all the branch elements with their average models

    iii. Constructing adjoint network corresponding to average model

        - By replacing the branch elements of average circuit with corresponding adjoint elements, the adjoint network can be developed.

iv. Performing circuit simulations and storing data

- Both the average model and its adjoint circuit can be solved by simulating the circuits and all the branch voltages and currents can be stored.

v. Deriving sensitivities based on stored data

- The adjoint network approach theory can be used to derive the sensitivities using stored branch data.

Bibliography

[1] G.P. Kurpis and C.J. Booth, *IEEE Standard Dictionary of Electrical and Electronics Terms*, 5th ed., New York, Jan. 1993, pp. 1192.

[2] L.O. Chua and P. Lin, *Computer-Aided Analysis of Electronic Circuits*, Prentice-Hall Inc., Englewood Cliffs: New Jersey, 1975, ch. 15.

[3] S.W. Director and R. A. Rohrer, "The generalized Adjoint Network and Network Sensitivities," *IEEE Trans. Circuit Theory*, vol. CT-16, no. 3, pp. 318-323, Aug. 1969.

[4] C.A. Desoer, "Teaching adjoint network to juniors," *IEEE Trans. Education*, vol. E-16, no. 1, pp. 10-14, Feb. 1973.

[5] L.M. Vallese, "Incremental Versus Adjoint Models for Network Sensitivity Analysis," *IEEE Trans. Circuits and Systems*, vol. CAS-21, no. 1, pp. 46-49, Jan. 1974.

[6] G.W. Roberts and A.S. Sedra, "Adjoint networks revisited," in *Proc. IEEE Int. Symp. Circuits and Systems*, 1990, vol. 1, pp. 540-544

[7] F. Yuan and A. Opal, "Adjoint network of periodically switched linear circuits with applications to noise analysis," *IEEE Trans. Circuits and Systems*, vol. 48, no. 2, pp. 139-151, Feb. 2001.

[8] T.V. Nguyen, A. Devgan and O.J. Nastov, "Adjoint transient sensitivity computation in piecewise linear simulation," in *Proc. Design Automation Conference,* 1998, pp. 477- 482.

[9] M.F.N. Tajuddin, N.A. Rahim, I. Daut, B. Ismail and M.F. Mohammed, "State space averaging technique of power converter with digital PID controller," *TENCON 2009 - 2009 IEEE Region 10 Conference*, pp. 1-6, Jan. 2009.

[10] M. Plesnik, "Use of the State-Space Averaging Technique in Fast Steady-State Simulation Algorithms for Switching Power Converters," in *Proc. Canadian Conference on Electrical and Computer Engineering,* 2006*,* pp. 2224-2227, May 2006.

[11] J.M. Noworolski and S.R. Sanders, "Generalized in-place circuit averaging," in *Proc. Applied Power Electronics Conference and Exposition,* 1991, pp. 445-451, Mar. 1991.

[12] S.R. Sanders, J.M. Noworolski, X.Z. Liu and G.C. Verghese, "Generalized averaging method for power conversion circuits," *Power Electronics Specialists Conference,* 1990, *21st Annual IEEE*, pp.333-340, Jun. 1990.

[13] S.R. Sanders, J.M. Noworolski, X.Z. Liu and G.C. Verghese, "Generalized averaging method for power conversion circuits," *IEEE Trans. Power Electronics*, vol. 6, no. 2, pp.251-259, Apr. 1991.

[14] A. Yazdani and R. Iravani, "A generalized state-space averaged model of the three-level NPC converter for systematic DC-voltage-balancer and current-controller design," *IEEE Trans. Power Delivery,* vol. 20, no. 2, pp. 1105- 1114, Apr. 2005.

[15] A. Merdassi, L. Gerbaud and S. Bacha, "A new automatic average modelling tool for power electronics systems," *IEEE Power Electronics Specialists Conference,* 2008, pp. 3425-3431, Jun. 2008.

[16] P.T. Krein, J. Bentsman, R.M. Bass and B.L. Lesieutre, "On the use of averaging for the analysis of power electronic systems," *IEEE Trans. Power Electronics*, vol. 5, no. 2, pp. 182-190, Apr. 1990.

[17] C.S. Kong, "A general maximum power transfer theorem," *IEEE Trans. Education,* vol. 38, no. 3, pp. 296-298, Aug. 1995.

[18] P. Feldmann, T.V. Nguyen, S.W. Director and R.A. Rohrer, "Sensitivity computation in piecewise approximate circuit simulation," *IEEE Trans. Computer-Aided Design of Integrated Circuits and Systems,* vol. 10, no. 2, pp. 171-183, Feb. 1991.

[19] Y. Elcherif and P. Lin, "Transient analysis and sensitivity computation in piecewise-linear circuits," *IEEE Trans. Circuits and Systems,* vol. 38, no. 12, pp. 1525-1533, Dec. 1991.

[20] L. Quan and Y. Fei, "Time-domain response and sensitivity of periodically switched nonlinear circuits," *IEEE Trans. Circuits and Systems I: Fundamental Theory and Applications,* vol. 50, no. 11, pp. 1436- 1446, Nov. 2003.

[21] S. Parker, "Sensitivity Analysis and Models of Nonlinear Circuits," *IEEE Trans. Circuit Theory,* vol. 16, no. 4, pp. 443- 447, Nov. 1969.

[22] A. Khaligh, A.M. Rahimi, M. Khaligh and A. Emadi, “Sensitivity Analyses of Pulse Adjustment Control Technique of a Buck-Boost Converter Operating in Discontinuous Conduction Mode and Driving Constant Power Loads,” *IEEE Vehicle Power and Propulsion Conference,* 2006, pp. 1-5, Sep. 2006.

[23] S. Barmada, A. Musolino, M. Raugi and M. Tucci, ”Time domain sensitivity of non linear circuits via wavelet transform,” *Microwave Symposium Digest, IEEE MTT-S International*, 2008, pp. 1393-1396, Jun. 2008.

Appendices

Appendix 1: Mathcad code for the optimization of load impedance for maximum power transfer

Data: $Rs := 0.5$ $Ls := 0.000106$ $Vs := 10$

$w := 2\cdot\pi\cdot 60$

Assuming initial values for load impedance,

$Rl := 0.3$ $Ll := -0.000120$

where, $Xs := w\cdot Ls$

$Xl := w\cdot Ll$

Optimization of power transfer

```
Opti1(n, alpha) := | for k ∈ 1.. n
                   |   I ← Vs / ((Rs + Rl) + iw·(Ls + Ll))
                   |   I1 ← 1 / ((Rs + Rl) + iw·(Ls + Ll))
                   |   OF ← −1(|I|)^2·Rl
                   |   I1_Rl ← −Vs·(|I1|)^2
                   |   I1_Ll ← −Vs(|I1|)^2·w
                   |   I2 ← | (−|I|)  if |Ll| > |Ls|
                   |        | |I|     otherwise
                   |   OF_Rl ← −[(|I|)^2 + 2Rl·|I|·I1_Rl]
                   |   OF_Ll ← −(2Rl·I2·I1_Ll)
                   |   Rl ← Rl − 10alpha·OF_Rl
                   |   Ll ← Ll − 0.000000alpha·OF_Ll
                   | return (OF, OF_Rl, OF_Ll, Rl, w·Ll)
```

Optimized values:

$$\mathrm{Opti1}(500, 0.0001) = \begin{pmatrix} -50 \\ -3.02454\times 10^{-7} \\ -3.76991\times 10^{4} \\ 0.5 \\ -0.03993 \end{pmatrix}$$

Appendix 2: Mathcad code for the optimization of load impedance for power transfer and transmission efficiency

```
Data:   Rs := 0.5          Ls := 0.000106        Vs := 10
        w := 2·π·60        gamma := 1.0

Assuming initial values for load impedance,              where,  Xs := w·Ls
        Rl := 0.3       Ll := −0.000120                          Xl := w·Ll

Optimization of power transfer and transmission efficiency

Optil(n, alpha) := | for k ∈ 1.. n
                   |   | I ← Vs / ((Rs + Rl) + iw·(Ls + Ll))
                   |   | I1 ← 1 / ((Rs + Rl) + iw·(Ls + Ll))
                   |   | OF1 ← −1(|I|)^2·Rl
                   |   | OF2 ← −1(|I|)^2·Rs
                   |   | OF ← gamma·OF1 − (1 − gamma)·OF2
                   |   | I1_Rl ← −Vs(|I1|)^2
                   |   | I1_Ll ← −Vs(|I1|)^2·w
                   |   | I2 ← | (−|I|)  if |Ll| > |Ls|
                   |   |      | |I|   otherwise
                   |   | OF_Rl ← −gamma·[(|I|)^2 + 2Rl·|I|·I1_Rl] + (1 − gamma)2Rs·|I|·I1_Rl
                   |   | OF_Ll ← −[gamma·2Rl·I2·I1_Ll − (1 − gamma)·2Rs·I2·I1_Ll]
                   |   | Rl ← Rl − 10alpha·OF_Rl
                   |   | Ll ← Ll − 0.0000001alpha·OF_Ll
                   |   return (OF1, OF2, OF, Rl, w·Ll)

Optimized values:   Optil(500, 0.00001) = ( −49.99997
                                            −50.07691
                                            −49.99997
                                              0.49924
                                             −0.03995 )
```

Appendix 3: Mathcad code for the optimization of transient voltage over-shoot by inductor and capacitor values

Data: Vin := 15 D := 0.75 D1 := 0.75 R1 := 9 C1 := 0.0001 L1 := 0.00018(

$$\mathrm{Vref} := \frac{-\mathrm{D1}\cdot\mathrm{Vin}}{1-\mathrm{D1}} \qquad w := \left[\frac{-1}{4\cdot\mathrm{R1}^2\,\mathrm{C1}^2} + \frac{(1-\mathrm{D})^2}{\mathrm{L1}\cdot\mathrm{C1}}\right]^{0.5}$$

Optimization of inductor and capacitor values

$$\mathrm{Opti}(n,\mathrm{alpha}) := \text{for } k \in 1..n$$

$$w \leftarrow \left[\frac{-1}{4\cdot\mathrm{R1}^2\,\mathrm{C1}^2} + \frac{(1-\mathrm{D})^2}{\mathrm{L1}\cdot\mathrm{C1}}\right]^{0.5}$$

$$\mathrm{Vo}(t) \leftarrow \frac{-\mathrm{D1}\cdot\mathrm{Vin}}{1-\mathrm{D1}}\left[1-\exp\left(\frac{-t}{2\cdot\mathrm{R1}\cdot\mathrm{C1}}\right)\left(\cos(wt)+\frac{1}{2\cdot\mathrm{R1}\cdot\mathrm{C1}\cdot w}\sin(wt)\right)\right]$$

$$\mathrm{SVo\_C}(t) \leftarrow \frac{\mathrm{D}\cdot\mathrm{Vin}}{(1-\mathrm{D})\cdot\mathrm{C1}}\cdot\left(w+\frac{1}{4\cdot\mathrm{R1}^2\cdot\mathrm{C1}^2\cdot w}\right)\cdot\exp\left(\frac{-t}{2\cdot\mathrm{R1}\cdot\mathrm{C1}}\right)\cdot\left[\left(0.5\sin(w\cdot t)+\frac{\cos(w\cdot t)}{4\cdot\mathrm{R1}\cdot\mathrm{C1}\cdot w}\right)\cdot t-\frac{\sin(w\cdot t)}{4\cdot\mathrm{R1}\cdot\mathrm{C1}\cdot w^2}\right]$$

$$\mathrm{SVo\_L}(t) \leftarrow \frac{-\mathrm{D}\cdot\mathrm{Vin}}{(1-\mathrm{D})\cdot\mathrm{R1}\cdot\mathrm{L1}\cdot\mathrm{C1}\cdot w}\cdot\exp\left(\frac{-t}{2\cdot\mathrm{R1}\cdot\mathrm{C1}}\right)\cdot\left[\left[\frac{(1-\mathrm{D})^2\cdot\mathrm{R1}}{2\cdot\mathrm{L1}}\cdot\sin(w\cdot t)-\left[w+\frac{1}{4\cdot\mathrm{R1}^2\cdot\mathrm{C1}^2\cdot w}-\frac{(1-\mathrm{D})^2}{2\cdot\mathrm{L1}\cdot\mathrm{C1}\cdot w}\right]\frac{\cos(w\cdot t)}{2}\right]\cdot t+\left[w+\frac{1}{4\cdot\mathrm{R1}^2\cdot\mathrm{C1}^2\cdot w}-\frac{(1-\mathrm{D})^2}{2\cdot\mathrm{L1}\cdot\mathrm{C1}\cdot w}\right]\frac{\sin(w\cdot t)}{2\cdot w}\right]$$

$$\mathrm{OF} \leftarrow \int_{0.001}^{0.015}(\mathrm{Vo}(t)-\mathrm{Vref})^2\,dt$$

$$\mathrm{OF\_C} \leftarrow \int_{0}^{0.015}[2(\mathrm{Vo}(t)-\mathrm{Vref})\cdot\mathrm{SVo\_C}(t)]\,dt$$

$$\mathrm{OF\_L} \leftarrow \int_{0}^{0.015}[2(\mathrm{Vo}(t)-\mathrm{Vref})\cdot\mathrm{SVo\_L}(t)]\,dt$$

$$\mathrm{C1} \leftarrow \mathrm{C1}-\mathrm{alpha}\cdot\mathrm{OF\_C}$$

$$\mathrm{L1} \leftarrow \mathrm{L1}-\mathrm{alpha}\cdot\mathrm{OF\_L}$$

$$\text{return} \begin{pmatrix}\mathrm{OF}\\ \mathrm{OF\_C}\\ \mathrm{OF\_L}\\ \mathrm{C1}\\ \mathrm{L1}\end{pmatrix}$$

Optimized values:

$$\mathrm{Opti}(800, 0.0000000001) = \begin{pmatrix}8.206\times 10^{-3}\\ 9.113\times 10^{3}\\ -1.8\times 10^{3}\\ 2.71\times 10^{-5}\\ 1.944\times 10^{-4}\end{pmatrix}$$

Appendix 4: Mathcad code for the optimization of transient voltage overshoot and voltage ripple by inductor and capacitor values

Data: $Vin := 15 \quad D := 0.75 \quad D1 := 0.75 \quad R1 := 9 \quad C1 := 0.0001 \quad L1 := 0.000180$

$$Vref := \frac{-D1 \cdot Vin}{1 - D1} \qquad w1 := 2 \cdot \pi \cdot 10000 \qquad w := \left[\frac{-1}{4 \cdot R1^2 C1^2} + \frac{(1-D)^2}{L1 \cdot C1}\right]^{0.5}$$

$\text{Opti}(n, alpha) :=$ for $k \in 1..n$

$$w \leftarrow \left[\frac{-1}{4 \cdot R1^2 C1^2} + \frac{(1-D)^2}{L1 \cdot C1}\right]^{0.5}$$

$$Vo(t) \leftarrow \frac{-D1 \cdot Vin}{1 - D1}\left[1 - \exp\left(\frac{-t}{2 \cdot R1 \cdot C1}\right)\left(\cos(w\,t) + \frac{1}{2 \cdot R1 \cdot C1 \cdot w}\sin(w\,t)\right)\right]$$

$$SVo\_C(t) \leftarrow \frac{D \cdot Vin}{(1-D) \cdot C1} \cdot \left(w + \frac{1}{4 \cdot R1^2 \cdot C1^2 \cdot w}\right) \cdot \exp\left(\frac{-t}{2 \cdot R1 \cdot C1}\right) \cdot \left[\left(0.5 \cdot \sin(w \cdot t) + \frac{\cos(w \cdot t)}{4 \cdot R1 \cdot C1 \cdot w}\right) \cdot t - \frac{\sin(w \cdot t)}{4 \cdot R1 \cdot C1 \cdot w^2}\right]$$

$$SVo\_L(t) \leftarrow \frac{-D \cdot Vin}{(1-D) \cdot R1 \cdot L1 \cdot C1 \cdot w} \cdot \exp\left(\frac{-t}{2 \cdot R1 \cdot C1}\right) \cdot \left[\left[\frac{(1-D)^2 \cdot R1}{2 \cdot L1} \cdot \sin(w \cdot t) - \left[w + \frac{1}{4 \cdot R1^2 \cdot C1^2 \cdot w} - \frac{(1-D)^2}{2 \cdot L1 \cdot C1 \cdot w}\right]\frac{\cos(w \cdot t)}{2}\right] \cdot t + \left[w + \frac{1}{4 \cdot R1^2 \cdot C1^2 \cdot w} - \frac{(1-D)^2}{2 \cdot L1 \cdot C1 \cdot w}\right]\frac{\sin(w \cdot t)}{2 \cdot w}\right]$$

$$X \leftarrow \sqrt{\left[\frac{(1-D)^2}{w1 \cdot L1} - 4 \cdot w1 \cdot C1\right]^2 + \left(\frac{2}{R1}\right)^2}$$

$$Vo1 \leftarrow 9 \cdot \frac{\sqrt{\left[1 - \frac{3 \cdot (1-D)}{w1 \cdot L1}\right]^2 + 1}}{X}$$

$$Il1 \leftarrow \frac{\sqrt{\left[9(1-D) + 27 \cdot w1 \cdot C1 + \frac{1}{R1}\right]^2 + (27 \cdot w1 \cdot C1)^2}}{w1 \cdot L1 \cdot X}$$

$$SVo1\_C \leftarrow \frac{4 \cdot Vo1 \cdot w1}{X}$$

$$SVo1\_L \leftarrow \frac{2 \cdot (1-D) \cdot Il1}{L1 \cdot X}$$

$$OF1 \leftarrow \int_{0.001}^{0.015} (Vo(t) - Vref)^2 \, dt$$

$$OF1\_C \leftarrow \int_{0}^{0.015} [2(Vo(t) - Vref) \cdot SVo\_C(t)] \, dt$$

$$OF1\_L \leftarrow \int_{0}^{0.015} [2(Vo(t) - Vref) \cdot SVo\_L(t)] \, dt$$

$$OF2 \leftarrow \int_{0.001}^{0.015} Vo1^2 \, dt$$

$$OF2\_C \leftarrow \int_{0.001}^{0.015} 2 \cdot Vo1 \cdot SVo1\_C \, dt$$

$$OF2\_L \leftarrow \int_{0.001}^{0.015} 2 \cdot Vo1 \cdot SVo1\_L \, dt$$

$$OF \leftarrow OF1 + 10OF2$$

$$C1 \leftarrow C1 - alpha \cdot (OF1\_C + 10OF2\_C)$$

$$L1 \leftarrow L1 - alpha \cdot (OF1\_L + 10OF2\_L)$$

$$\text{return} \begin{pmatrix} OF \\ OF1 \\ OF1\_C \\ OF1\_L \\ OF2 \\ OF2\_C \\ OF2\_L \\ C1 \\ L1 \end{pmatrix}$$

Optimized values:

$$\mathrm{Opti}\,(405\,,0.00000000001\,) = \begin{pmatrix} 0.19363482 \\ 0.08926257 \\ 9.1125\times 10^{3} \\ -1.8\times 10^{3} \\ 0.01043722 \\ 367.42037054 \\ 3.71690835 \\ 5.66999304\times 10^{-5} \\ 1.8720404\times 10^{-4} \end{pmatrix}$$