Capacity Planning under Fuzzy Environment using Possibilistic Approach

By

Gurmail S. Bassan

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Department of Mechanical and Manufacturing Engineering
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ABSTRACT

Currently, capacity planning is receiving more emphasis in management of operations in Industrial Engineering because insufficient capacity may lead to deteriorating delivery performance and high work-in-process inventories. On the other hand excess capacity may lead to wastage of resources. Even the most modern and sophisticated capacity planning systems may face a great deal of uncertainty, imprecision and vagueness due to uncertain market demand, set up resources, capacity constraints, pessimistic time standards, and subjective beliefs of managers etc., leading to inferior planning decisions. Under such circumstances fuzzy models which explicitly consider these uncertainties, generate more robust, flexible and efficient planning.

The traditional fuzzy logic-based models though are capable of dealing with some complex capacity-planning systems where various uncertain parameters and vagueness are involved, yet they use complex membership functions to calculate the degree of truth that involve complicated, time consuming and tedious mathematical operations. In this thesis, the solution techniques and methods developed are based on possibility theory. These techniques not only eliminate the need of calculation of complex membership functions but also yield crisp answers to fuzzy problems in capacity planning.

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INTRODUCTION

Capacity planning is a critical element of any successful production planning and control system and is receiving increased emphasis in the management of operations. The efficient utilization of capacity results in many financial benefits. Insufficient capacity will lead to deteriorating delivery performance and high work-in-process inventories but on the other hand excess capacity is an undesirable expense that can be reduced. According to Filho and Marcola, (2001), the objective of the production planning phases for long and medium terms is to verify whether the available production capacity is sufficient to produce the projected plan. Wortman (1995), affirmed that, it is not viable to elaborate detailed production and capacity plans for an unknown future in terms of specific orders for final products, inventory levels, specific machine availability and individual operator availability.

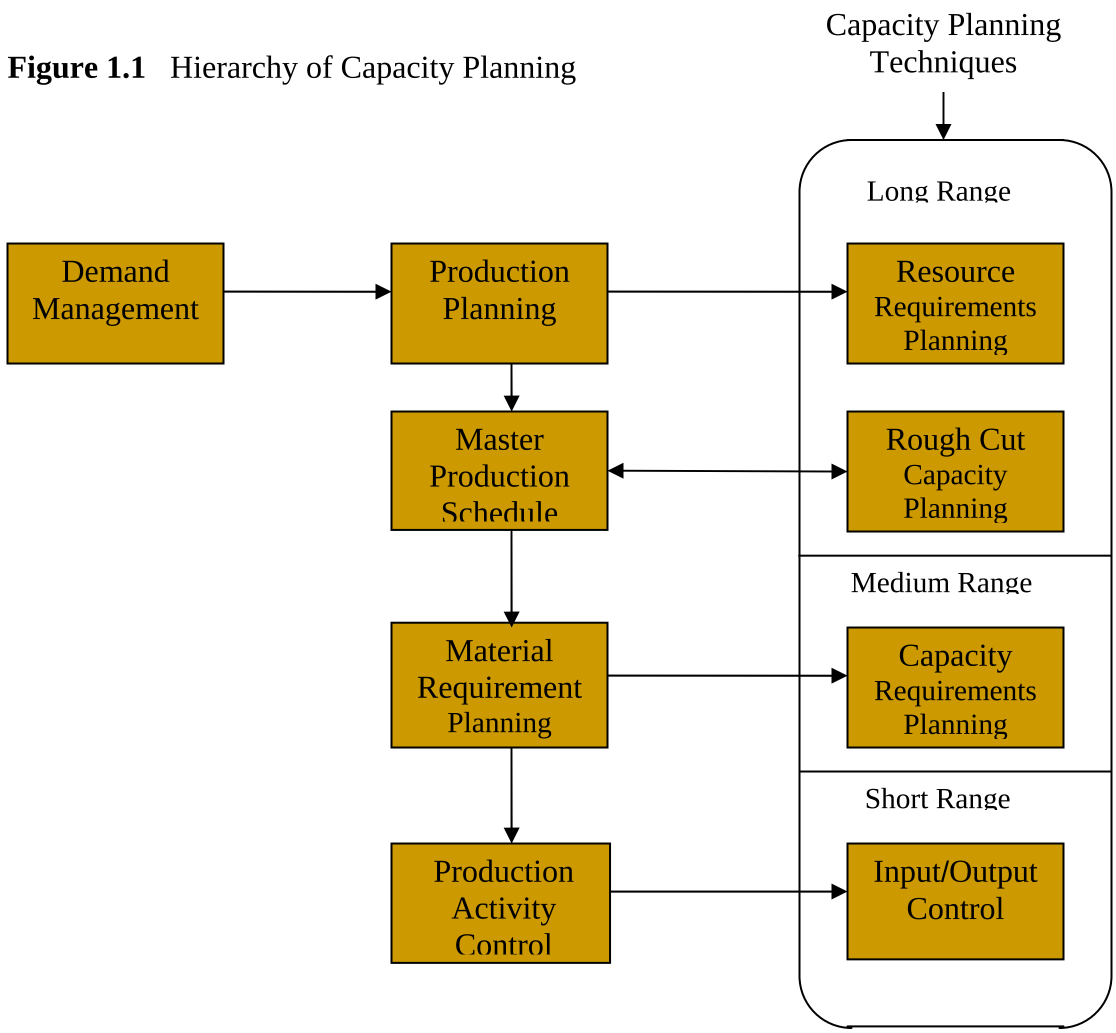
Traditional production control systems based on material requirements planning (MRP); do not sufficiently support a planner in solving capacity problems who ignores capacity constraints. Such systems are capacity insensitive and implicitly assume that sufficient capacity is available to produce a required set of components at the time they are needed. According to Fogarty et al. (1991), a problem commonly encountered in operating these systems is the existence of an overstated master production schedule (MPS). An overstated MPS generates more production than the shop can complete, thus causing certain problems, such as an increase in raw material and work-in-process inventories. More materials are purchased and released to shop than are completed and shipped, thus building queues on a shop floor (Chase et al. 2006).

Many possible forms of uncertainty could be present in the decision environment of a production planning system such as uncertainties in the market demand, set-up resources, and capacity constraints (Pai et al, 2004). According to Fogarty et al. (1991), Russell and Taylor, (2008), most capacity planning techniques use detailed data on time standards for each product at any work center. The time standards for each product have provisions for allowances for rest, to overcome fatigue and other unavoidable delays etc. When a time standard is set, it is usually reliable. However, production processes continually improve and the time standards set become pessimistic. As a result, the use of crisp estimates of time standards becomes less satisfactory and unreliable in capacity management. Apart from the uncertainty in market demand, forecast of uncertainty is determined by the subjective beliefs of managers as well. However, the measurement of manager's judgments is difficult and vague. According to Mula et al. (2008), models for production planning which do not recognize uncertainty can be expected to generate inferior planning decisions compared with models which explicitly take into account uncertainty. *Hence, it is natural to deal with such systems through fuzzy systems.*

Capacity planning is the process of determining the amount of capacity required to produce in future, or in other words how much output is needed from plant facilities and from suppliers. It is a function of establishing, monitoring and adjusting level of capacity in order to execute all manufacturing plans. Capacity planning is usually performed at four different levels depending upon the length of planning horizon as depicted in Figure 1.1 and as follows:

- Resource Requirements Planning (RRP).
- Rough Cut Capacity Planning (RCCP).
- Capacity Requirements Planning (CRP).
- Input / Output Control.

Figure 1.1 Hierarchy of Capacity Planning



Thus, the process of capacity planning starts with an overall plan of resources followed by rough cut capacity planning (RCCP) for validating the master production schedule (MPS). At resource requirement planning level, the information is treated in an aggregated way, and the decisions cover a long range planning horizon, and are taken by the higher level administration. The capacity planning has a distinct role to play at all different levels of production planning and control systems. According to Fogarty et al. (1991), in a MRP system the typical sequence is to create a MPS, use a RCCP to verify that the MPS is feasible, perform the MRP explosion, and send the planned order release data to capacity requirements planning (CRP). Thus, the main emphasis in this thesis would be focused towards the capacity planning techniques of rough cut capacity planning (RCCP) and capacity requirements planning (CRP) under fuzzy environment using fuzzy systems.

1.1 Rough Cut Capacity Planning (RCCP)

Validation of MPS, with respect to capacity is termed as RCCP. It verifies whether a given MPS is feasible or not. An overstated MPS orders more production to be released than the shop can complete, thus causing increased inventories, irregular supplies and wasteful manufacturing. Currently, in industry, there are three most commonly used RCCP methods (Fogarty et al. 1991):

- Capacity planning using overall factors (CPOF)
- Bill of labor approach (BOL).
- Resource profile approach (RP).

All these methods convert the master production schedule from units of end items to be produced into amount of time required on certain key resources. Capacity planning, using overall factors, requires least detailed data and least computational endeavor (Fogarty et al. 1991). Rough cut capacity is calculated by multiplying the time the total plant requires to produce one typical part by the MPS quantity to obtain the total time required in the entire plant to meet the MPS. This time is prorated among the key resources by multiplying the total plant time by the historical proportion of time used at a given work centre.

Another approach followed is the bill of labor (BOL) approach. The BOL approach uses detailed data on the time standards for each product at the key resources or work center and do not consider lead times in manufacturing. It assumes that the final product is made (or assembled) at the same time as that of its components or parts, or in other words the lead times in manufacturing are not considered. It converts the MPS units of end products to be produced into amount of time required at certain work centers or key resources.

Another method is resource profile approach. A resource profile is similar to bill of labor except that the time at each department is associated, reflecting lead time of the part. The other approaches of rough cut capacity analysis like bill of labor approach (BOL) and capacity planning using overall factors (CPOF) do not consider lead time offsets. Both these approaches assume that all components and parts are manufactured at the same time as the end item. Resource profile approach of RCCP considers lead time offsets and time phases the labor requirements.

1.2 Capacity Requirements Planning (CRP)

CRP is the process to determine the machine and labor resources required to attain the production. Open shop orders and planned orders in the MRP system are inputs to the capacity requirements planning, which ‘translates’ these orders into hours of work by work center for any time period. According to Fogarty et al. (1991), CRP is a detailed comparison of the capacity required by the MRP and by the orders currently in progress versus available capacity. It is applied to determine the capacity requirements after RCCP has validated the MPS. Once the MPS is accepted, CRP determines the load that is expected at each work center during each time period. Currently, CRP is a deterministic technique and it uses more information than RCCP.

Depending upon the demand of the situation, the capacity required is calculated by using one of the techniques discussed above. If the capacity available is less than the capacity required then there are certain options available, by which one can enhance capacity to some extent. Buying short-term capacity is usually avoided since it narrows down profit margins. According to Zobolas et al. (2008), whenever RCCP or CRP indicates short-term capacity shortages that render a MPS/MRP infeasible, plant managers have a number of options (Ip et al. 2000).

- Work longer and/or in more shifts (Overtime).
- 'Buy' capacity by subcontracting a number of orders.
- Alternate routing.
- Adding shifts/personnel.
- Cancel orders.
- Delay product delivery.
- Re-arrange the production plan to balance the system (if possible) and produce some orders earlier if capacity and storage space allows.

These options are discussed in brief as follows:

Overtime is probably the popular solution to insufficient capacity because it requires few advance arrangements to be made and do not increase hiring, training or fringe benefit cost while increasing labor capacity. However, direct costs usually increase due to both premium wages and decreasing productivity rates. Buying capacity by subcontracting a number of orders is another way to obtain additional capacity but it requires extensive cost analysis and prior arrangements to find a suitable vendor to deliver quality work. Sometimes subcontracting increases lead times, transportation cost and do not guarantee a quality product. By alternate routing, we temporarily change the routing of specific parts so that work usually performed at a busy work center is temporarily changed to another work center that has little work during a given period but not at the expense of quality. Adding shifts/personnel can enhance capacity if there are no equipment constraints. Adding shifts/personnel can be done either by adding a shift, hiring new personnel to the existing shift or moving personnel from some under utilized work center. Cancelling orders and delaying product delivery affects business reputation and is usually followed when there are no other alternatives. A production plan can be rearranged to balance the system (if possible) and some orders can be produced earlier if capacity and storage space allow. This will allow generation of a new revised MPS. In other words, if the capacity is still inadequate there is no other option but to revise the master production schedule.

When insufficient capacity exists and all the above mentioned techniques have been exhausted, some organizations consider the revision of the master production schedule to be a solution of last resort. According to Fogarty et al. (1991), MPS revision should be the first thing an organization should consider because a number of factors can cause an order to be expedited and in case of inadequate capacity it is impossible to complete all orders in time. Furthermore, the management must take the responsibility to see that

RCCP is performed. If an unavoidable overload exists, management should take the responsibility to revise the job due dates in order to provide a feasible MPS.

1.3 Organization of the Thesis

In the present thesis, we consider, under fuzzy environment, an important Industrial Engineering problem i.e. Capacity Planning Problem, addressed under crisp environment by Fogarty et al. (1991), Pai et al. (2004), Mula et al. (2008)). The bill of labor approach for RCCP, resource profile approach for RCCP and capacity requirement planning approach will be modeled under fuzzy environment using Possibility Theory.

Chapter 1 provides an introduction to the problem considered in the thesis. Chapter 2 will provide a review of the literature of the related work done by other researchers. The capacity analysis under fuzzy environment using possibility theory with bill of labor approach for RCCP is considered in Chapter 3. Chapter 4 deals with capacity analysis under fuzzy environment using possibility theory with resource profile approach for RCCP. In Chapter 5, capacity requirements planning under fuzzy environment using Possibility Theory is discussed. Finally, the conclusions and discussions on the contributions made by the thesis, along with some recommendations for future research, will be given in Chapter 6.

CHAPTER 2

LITERATURE SURVEY

The main objective of this chapter is to provide a survey of the literature pertaining to capacity analysis problem, fuzzy arithmetic and other related concepts used in this thesis. An effort is made to identify the status of the existing literature and review the recent developments in these areas. A summary of literature search is also provided at the end of the chapter.

2.1 Literature Review on Capacity Analysis and Capacity Planning Problems

Currently, several manufacturing companies face numerous problems like global competition, uncertain market environment and shorter product life cycles etc. This has resulted in the use of complex manufacturing systems with equally complex control mechanisms. Production control systems based on the conventional manufacturing resource planning (for example MRP II) framework have often proved unsatisfactory in meeting the above challenges (*Pandey* et al. 2000). MRP II systems operate on the assumption of infinite capacity for the resources, thus leaving the capacity problem unresolved (McCarthy and Barber 1990). To overcome this deficiency, Taal and Wortmann (1997), suggest that the capacity problems be resolved at the material requirement planning (MRP) stage itself through an integrated approach to MRP and finite capacity planning. Traditional production control systems (based on the manufacturing resource planning concept) do not sufficiently support the planner in solving capacity problems, ignore capacity constraints and assume that lead times are fixed. This leads to problems on the shop floor that cannot be resolved in the short term. They integrate finite capacity planning and MRP to avoid capacity problems thus resulting in a planning method for simultaneous capacity and material planning. *Pandey* et al. (2000), present a finite capacity material requirements planning (FCMRP) algorithm which is executed in two stages. Firstly, capacity-based production schedules are generated from the input data. Secondly, the algorithm produces an appropriate material requirements plan to satisfy the schedules obtained from Stage 1.

MRP was the first attempt to link the demand of raw materials and sub-assemblies with the total demand for finished products (Hollier and Cooke 1991), in order to achieve reduction of inventory shortages or overstocks (Muhlemann et al. 1992, Slack et al. 2001). However, MRP fell short of expectations (Darlington and Moar 1996) and it is generally accepted that the culprit for the low MRP performance is its ‘capacity insensitive’ nature (Ho and Chang 2001). According to Zobolas et al. (2008), many approaches have been suggested to tackle this problem, the most important being MRP II with closed loop feedback to the MPS and the MRP in relation to capacity constraints.

The RCCP (Rough Cut Capacity Plan) module of MRP II checks whether a proposed MPS is feasible (Hopp and Spearman 1996). However, its main drawback is that, in order to simplify calculations, it does not perform any offsetting before evaluating resource utilization. Moreover, RCCP does not consider detailed routing information and, usually, calculates the load only on critical resources. Capacity Requirements Planning (CRP), on the other hand, is a more detailed capacity check of a MRP plan after the bill-of-materials (BOM) explosion and takes into account inventory records. However, the CRP routine usually entails extensive calculations. Gunther and Tempelmeier (2005), propose an extension of the original RCCP (ERCCP) routine which includes lead time offset information. Zobolas et al. (2008) developed a decision support system to improve the MPS. The system developed comprises of two main components: an extension of the RCCP loading output routine (ERCCP) that provides accurate resource loading information for the assessment of each solution (MPS with offset production orders), and a metaheuristic algorithm (a Genetic Algorithm) that adjusts MPS based on the ERCCP output. These two methods compare the algorithm with the traditional RCCP approach as well as with a typical ERP system.

According to Pires (1994), it is highly desirable to have flexibility in the production capacity, not only in technological terms (capability) but also in volume (capacity). According to Filho and Marcola, (2001), annualized hours (AH) (Annualized hours is a work time control system that helps in increasing the flexibility of available capacity) system aids in the search of a solution for the capacity flexibility problem. It transfers available capacity from one production work centre to another and increases or decreases the day's work in the period defined as planning horizon. Filho and Marcola, (2001), develop a linear programming (L.P.) model to insert annualized hours as a tool of the RCCP and further develop an algorithm to compare the aggregate production capacity and demand during a certain planning horizon.

Smunt et al. (1996), illustrate the benefits of integrating the learning curve analysis with capacity planning procedures and affirm that more accurate capacity projections are possible through learning curve analysis, allowing capacity management and scheduling decisions to be more accurate and efficient. Guide and Spencer, (1997), describe the modifications required to develop a bill of resources approach for rough-cut capacity planning (RCCP) for remanufacturing environment. The problems of capacity planning in a remanufacturing environment are compounded by highly variable processing times and probabilistic routing files and material recovery rates. The modified bill-of-material approach presented is an effective planning tool in remanufacturing environment.

Ding et al. (2007), address considerations of extending capacity planning to suppliers, present a preliminary design of a program tool, and report the implementation experience of such a tool in a manufacturing firm. The approach proposed by him represents an extension of capacity requirement planning in the context of extended enterprise. To perform supplier capacity analysis, a program tool that contains two parts is proposed to collect capacity-related data from suppliers, and to collect part requirements data from the plant, perform comparison, and display results. The web-based supplier-side program and the plant-side program are considered in a supplier-capacity-analysis tool, separate from ERP. This tool can be used to identify in advance the Tier I suppliers (immediate suppliers) that will have capacity load exceeding the available capacity.

According to Marvel et al. (2005), well known limitations of the MRP approach issues, such as demand uncertainties and production equipment availability, require a more robust method of generating and validating shop floor schedules. They use a discrete event simulation to validate the capacity planning process. The simulation model (Marvel et al. 2005), provides a planning tool that provides not only the ability to determine if the capacity planning process is valid but also provides the ability to schedule the balance of the product line, and identify problems that may cause customer service issues.

2.2 Capacity Planning under Fuzzy Environment

According to Pai et al. (2004), in capacity-planning systems, various sources of uncertainty and imprecision are encountered. In most cases, the uncertainty is determined by the subjective beliefs of managers. However, most of the times a manager's judgment is subjective and vague, and therefore is difficult to measure in terms of crisp values. Therefore, under such circumstances, a fuzzy logic approach coupled with fuzzy systems and fuzzy arithmetic appears to be a natural way to deal with capacity planning problems in the presence of the uncertain demand, set-up resources, and the capacity constraints. Models for production planning which do not recognize uncertainty can be expected to generate inferior planning decisions compared with models which explicitly take into account uncertainty (Mula et al. 2008). Different stochastic modeling techniques based on probability distributions have been successfully applied to production planning problems with randomness. Escudero et al. (1993), analyze different modeling approaches for the production and capacity problem using stochastic programming. However, stochastic programming is only appropriate if parameters are given as random variables. Probability distributions derived from evidence recorded in the past are not always reliable because of market changes that influence demand or backlog costs, or technological innovation, which in turn has an influence on capacity data. As sufficient data are not always available for predicting uncertain parameters, the choice of the fuzzy set theory is more logical and convincing for expression of the uncertainty of expert knowledge (Mula et al. 2008). Various researchers have used fuzzy arithmetic to investigate production-planning problems. Triangular fuzzy numbers (see Appendix 1, A.10) were used to represent the order quantity and demand in the economic ordering quantity (EOQ) problem by Yao et al. (2000). Yao and Lee (1999), employed trapezoidal fuzzy numbers (see Appendix 1, A.11) to deal with the EOQ problems. Pai et al. (2004), used fuzzy if-then rules in modeling uncertain and imprecise capacity data and vaguely defined relations between them. The proposed fuzzy logic-based models, although were capable of dealing with complex capacity-planning systems where various uncertain parameters and incomplete knowledge are involved, used complex membership functions (π-functions) to calculate the degree of truth. Mula et al. (2008), proposed fuzzy linear programming approach to generate an optimal fuzzy solution for a transformed fuzzy linear programming (LP) model with uncertain data for production planning from a deterministic structure.

Verma (2001), proposed capacity analysis under fuzzy environment using complex membership functions to calculate the degree of truth in uncertain data that involved complicated, time consuming and tedious mathematical operations.

2.3 Possibility Theory, Possibilistic Mean Value and other Moment Properties of Fuzzy Numbers

Dubois and Prade (1980), explained the ranking of fuzzy numbers (see Appendix 1, A.7) and formed the basis of possibility theory. Assuming that we have two fuzzy numbers A and B, according to Zadeh's (1996) extension principle, the crisp inequality $x \le y$ can be extended to obtain the truth value of the assertion that A is less than or equal to B, as follows (Bector and Chandra, 2005), (Appadoo, 2006):

$$T(A(\le)B) = \sup_{x \le y} (\min(\mu_A(x), \mu_B(x))).$$

This truth value $T(A(\leq)B)$ is called the degree of possibility of dominance of B over A and is denoted by $Poss\,(A(\leq)B)$.

Similarly, the degree of possibility of dominance that the assertion "A is greater than or equal to B" is true, is given by

$$Poss\,(A(\geq)B) = \sup_{x\geq y}(\min(\mu_A(x), \mu_B(x))),$$

and
$$Poss\,(A(=)B) = \sup_{x}(\min(\mu_A(x), \mu_B(x))).$$

Here it may be noted (Dubois and Prade 1980), that $(A(\leq)B)$ if and only if $Poss(A(\leq)B) \geq Poss(B(\leq)A)$. For example for two TFN's, A = (a_1, a_2, a_3) and B = (b_1, b_2, b_3), if $a_2 \leq b_2$ then Poss $(A(\leq)B)$ = 1 and $Poss(B(\leq)A)$ = Height$\left(A \cap B\right) \leq 1$.

Therefore for the case of TFN's it can be defined that $(A(\leq)B)$ with respect to

Poss $(A(\leq)B)$ if $a_2 \leq b_2$.

2.3.1 Lower Possibilistic Mean, Upper Possibilistic Mean and Possibilistic Mean Value of a Fuzzy Number

Goetschel and Voxman (1986), introduced a method for ranking of fuzzy numbers. For example for two fuzzy numbers A and B $\in F$ and their α- level sets written for A as A (α) = $\left[a_1(\alpha), a_2(\alpha)\right]$ and for B as B (α) = $\left[b_1(\alpha), b_2(\alpha)\right], \alpha \in \left[0,1\right]$,

$$A \leq B \Leftrightarrow \int_0^1 \alpha\left(a_1\left(\alpha\right) + a_2\left(\alpha\right)\right)d\alpha \;\leq\; \int_0^1 \alpha\left(b_1\left(\alpha\right) + b_2\left(\alpha\right)\right)d\alpha \quad . \tag{2.3.1}$$

In Goetschel and Voxman (1986), (2.3.1) is motivated in part by the desire to give less importance to lower levels of fuzzy numbers.

For a fuzzy number $A \in F$, and an α - level set A (α) $= \left[p_1(\alpha), p_2(\alpha) \right], \alpha \in [0,1]$,

Carlsson and Fuller (2001), define the equality

$$\text{Possibility}\left[A \le p_1(\alpha) \right] = \pi\left[\left(-\infty, p_1(\alpha) \right) \right] = \sup_{u \le p_1(\alpha)} A(u) = \alpha$$

Since A is continuous, Carlsson and Fuller (2001), define that the lower possibility-weighed average of the minima of α- level sets, is nothing else but lower possibilistic mean value of A, denoted by $E_L(A)$.

Therefore

$$E_L(A) = \frac{\int_0^1 Poss\left[A \le a_1(\alpha) \right] \min\left[A(\alpha) \right] d\alpha}{\int_0^1 Poss\left[A \le a_1(\alpha) \right] d\alpha}$$ (Where "Poss" denotes possibility).

$$= \frac{\int_0^1 Poss\left[A \le a_1(\alpha) \right] a_1(\alpha) d\alpha}{\int_0^1 Poss\left[A \le a_1(\alpha) \right] d\alpha}$$

$$= \frac{\int_0^1 \alpha a_1(\alpha) d\alpha}{\int_0^1 \alpha d\alpha}$$

$$= \frac{\int_0^1 \alpha a_1(\alpha) d\alpha}{\frac{1}{2}}$$

$$E_L(A) = 2\int_0^1 \alpha a_1(\alpha) d\alpha \tag{2.3.2}$$

Similarly, Carlsson and Fuller (2001), define the equality

$$\text{Possibility}\left[A \geq p_2(\alpha)\right] = \pi\left(\left[p_2(\alpha), \infty\right]\right) = \sup_{u \geq p_2(\alpha)} A(u) = \alpha$$

Carlsson and Fuller (2001), define that the upper possibility-weighed average of the maxima of α- level sets, is nothing else but upper possibilistic mean value of A, denoted by $E_R(A)$.

Therefore,

$$E_R(A) = \frac{\int_0^1 Poss\left[A \geq a_2(\alpha)\right] \max\left[A(\alpha)\right] d\alpha}{\int_0^1 Poss\left[A \geq a_2(\alpha)\right] d\alpha}$$

$$= \frac{\int_0^1 Poss\left[A \geq a_2(\alpha)\right] a_2(\alpha)\, d\alpha}{\int_0^1 Poss\left[A \geq a_2(\alpha)\right] d\alpha}$$

$$= \frac{\int_0^1 \alpha a_2(\alpha) d\alpha}{\int_0^1 \alpha d\alpha} = \frac{\int_0^1 \alpha a_2(\alpha) d\alpha}{\frac{1}{2}}$$

$$E_R(A) = 2\int_0^1 \alpha a_2(\alpha) d\alpha \qquad (2.3.3).$$

Taking the weight of the arithmetic means of $a_1(\alpha)$ and $a_2(\alpha)$ as α, Carlsson and Fuller (2001), define the level weighted average E(A) of the arithmetic mean of the α-level sets of the fuzzy number A as:

$$E(A) = \int_0^1 \alpha\left(a_1(\alpha) + a_2(\alpha)\right) d\alpha$$

$$= \frac{2\int_0^1 \alpha a_1(\alpha)d\alpha + 2\int_0^1 \alpha a_2(\alpha)d\alpha}{2}$$

$$= \frac{1}{2}\left(\frac{\int_0^1 \alpha a_1(\alpha)d\alpha}{\frac{1}{2}} + \frac{\int_0^1 \alpha a_2(\alpha)d\alpha}{\frac{1}{2}} \right)$$

$$= \frac{1}{2}\left(\frac{\int_0^1 \alpha a_1(\alpha)d\alpha}{\int_0^1 \alpha d\alpha} + \frac{\int_0^1 \alpha a_2(\alpha)d\alpha}{\int_0^1 \alpha d\alpha} \right).$$

Thus, one can see that

$$\mathrm{E}\ (\mathrm{A}) = \frac{E_L(A) + E_R(A)}{2} \qquad (2.3.4)$$

Thus, Carlsson and Fuller (2001), define the crisp possibilistic mean value E(A) of fuzzy number A as the arithmetic mean of its lower possibilistic mean value $E_L(A)$ and upper possibilistic mean value $E_R(A)$.

2.4 Summary

Capacity planning is a complex problem and many different aspects involved, make it a challenging research area. Production control systems based on MRP ignore capacity constraints assuming infinite capacity for the resources. The RCCP module of MRP II checks whether a proposed MPS is feasible. Although some of RCCP techniques (for example resource profile approach) consider lead times of manufacturing assuming they are fixed, thus leaving problems on the shop floor. There is a great deal of uncertainty in production planning systems in the form of uncertain market demand, set-up resources, and capacity constraints. Most capacity planning techniques use detailed data on time standards for each operation at any work center. The time standards set become pessimistic with passage of time. Apart from this many sources of uncertainty and imprecision are encountered in capacity-planning systems, thus demanding a more robust method of generating and validating shop floor schedules. Stochastic modeling techniques based on probability distributions have been used in production planning problems with randomness. Probability distributions derived from evidence recorded in the past are not always reliable because of market changes. *When sufficient data are not available for predicting uncertain parameters, the choice of the fuzzy set theory is more logical and convincing for expression of the uncertainty.* Though some researchers have used fuzzy logic based models to address the complex capacity planning problem, complex membership functions to account the degree of truth have been used.

The present work has adopted some of the elements from the work done by Carlsson and Fuller (2001), in terms of possibility theory and possibilistic mean values. The equations and method developed based on possibility theory not only eliminate the need for calculation of complex membership functions, but also generate crisp results for the complex capacity planning problem for which prior information (or prior data) may not be available.

CHAPTER 3

BILL OF LABOR APPROACH FOR POSSIBILISTIC ROUGH CUT CAPACITY PLANNING

Researchers (Pai et al, 2004, Mula et al. 2008) have extended the capacity analysis problem using bill of labor approach with crisp numbers to fuzzy environment by replacing each crisp number by a fuzzy number. They have used complex membership functions to calculate the degree of truth. In this chapter, the possibility theory approach suggested by Carlson & Fuller (2001), is applied to the capacity analysis problem when the data available, instead of being crisp, is in the form of triangular (TFN) or trapezoidal (TrFN) fuzzy numbers.

3.1 Introduction

Validation of master production schedule (MPS) with respect to capacity is called rough cut capacity planning (Fogarty et al.1991). Rough cut capacity planning (RCCP) verifies whether the MPS is feasible or not. Though there are many RCCP techniques, most of them convert the MPS from units of end items to be produced into amount of time required on certain key resources. According to Fogarty et al. (1991), Russell and Taylor, (2008), the bill of labor (BOL) approach for rough cut capacity planning is an important approach applied usually for capacity analysis. It assumes that the final product is made (or assembled) at the same time as that of its components or parts, for example in assembly plants. Thus bill of labor approach of RCCP do not consider the lead times in manufacturing. The bill of labor approach uses detailed data on the time standards for each product at the key resources or work centers.

3.2 Rough Cut Capacity Analysis using Bill of Labor (BOL) Approach when the data available is in the form of Crisp Numbers

Using the approach proposed by Fogarty et al. (1991), assume that the MPS contains p number of products to be manufactured over time buckets t, where $p = 1, 2. . . n$; $t = 1, 2, . . ., m$. Let the work centers (or key resources) whose capacity is to be analyzed be represented by i, where $i = 1, 2. . . s$.

In such a model, as in Fogarty et al. (1991), the following assumptions are made:

- Each bill of labor is time phased.
- All the components and parts of the end product are manufactured at the same time as the end product, i.e. lead times of manufacturing are not considered.

Let,

a_{ip} = Bill of labor amount in time units at work centre i for product p.

b_{pt} = MPS quantity for product p during time t.

c_{it} = Capacity needed in time units at work centre i for period t.

P_p = Product p, where, $p = 1, 2,. . ., n$.

WC_i = Work Center i, where $i = 1, 2,. . ., s$.

T_t = Period t, where $t = 1, 2, . . , m$.

According to Fogarty et al. (1991), the bill of labor and MPS in a crisp case can be represented in a tabulated form as follows:

Table 3.1 Bill of Labor in time units

	P_1	P_2	----	P_n
WC_1	a_{11}	a_{12}	----	a_{1n}
WC_2	a_{21}	a_{22}	----	a_{2n}
----	----	----	----	----
WC_s	a_{s1}	a_{s2}	----	a_{sn}

Table 3.2 MPS in product units

	T_1	T_2	----	T_m
P_1	B_{11}	b_{12}	----	b_{1m}
P_2	B_{21}	b_{22}	----	b_{2m}
----	----	----	----	----
P_n	b_{n1}	b_{n2}	----	b_{nm}

Formulation for Rough Cut Capacity Calculation

Let

c_{it} = Capacity required at work centre i for time period t.

Then,

$$c_{it} = \sum_{p=1}^{n} a_{ip}(.)b_{pt}\ ,\ \ i = 1, 2,. . . , s;\ \ t = 1, 2, . . ., m\ . \qquad (3.2.1)$$

Using equation (3.2.1), the required capacity at any key resource can be calculated. If the numbers a_{ip}'s, b_{pt}'s for all values of i, p and t are crisp, then their product will also be crisp. Using Table (3.1), Table (3.2) and equation (3.2.1), the capacity requirement can be shown in a tabulated format as follows:

Table 3.3 Capacity required in time units

	T_1	T_2	----	T_m
WC_1	c_{11}	c_{12}	----	c_{1m}
WC_2	c_{21}	c_{22}	----	c_{2m}
----	----	----	----	----
WC_s	c_{s1}	c_{s2}	----	c_{sm}

3.3 Rough Cut Capacity Analysis Using Bill of Labor Approach under Fuzzy Environment using Possibility Theory

For the purpose of analysis developed in this section, one can rewrite the equation (3.2.1) as follows;

$$c_{it} = \sum_{p=1}^{n} a_{ip}(.)b_{pt}, \; i = 1, 2, \ldots, s; \; t = 1, 2, \ldots, m. \tag{3.3.1}$$

In equation (3.3.1), if at least one of the a_{ip}'s or b_{pt}'s is assumed to be a fuzzy number, then their product will also be a fuzzy number (Kaufman and Gupta (1988, 1991). Hence, corresponding c_{it}'s, will also be fuzzy numbers (Kaufman and Gupta (1988, 1991). It is pertinent to mention here that in equation (3.3.1), in general, each of the c_{it} may not necessarily be a TFN (Kaufman and Gupta (1988, 1991). In general, the product of two TFN's is a parabolic fuzzy number.

It is now assumed that in (3.3.1) each of the a_{ip} and b_{pt} for $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n$, $t = 1, 2, \ldots, m$, is a TFN of the type:

$$a_{ip} = (a_{ip1}, a_{ip2}, a_{ip3}) \quad \text{and} \quad b_{pt} = (b_{pt1}, b_{pt2}, b_{pt3}). \tag{3.3.2}$$

The values in (3.3.2) can be obtained from expert opinion of individuals who have the same information source but different opinion about the same problem. This means that instead of having only one crisp data/estimate of bill of labor or MPS, one can have three estimates based on the expert opinion yielding a TFN to account for uncertainty.

Hence, each c_{it} in equation (3.3.1), is a fuzzy number given by

$$c_{it} = (c_{it1}, c_{it2}, c_{it3}) \quad \text{for each } i = 1, 2, \ldots, s, \; t = 1, 2, \ldots, m. \tag{3.3.3}$$

Each fuzzy number c_{it} in (3.3.3), can be represented by its α-cut obtained by using the α-cuts of a_{ip} and b_{pt} (see Appendix 1, A.2).

Let

$a_{ip(\alpha)} = \left[a_{ip1}(\alpha), a_{ip2}(\alpha)\right]$ as α -cut for a fuzzy number a_{ip} for $i = 1, 2, \ldots, s$, $p = 1,$

$2, \ldots, n,$

such that

$$a_{ip(\alpha)} = \left[a_{ip1} + \alpha\left(a_{ip2} - a_{ip1}\right),\ a_{ip3} + \alpha\left(a_{ip2} - a_{ip3}\right)\right] \qquad \forall\ \alpha \in [0,1] \qquad (3.3.4)$$

Similarly, let

$b_{pt(\alpha)} = \left[b_{pt1}(\alpha), b_{pt2}(\alpha)\right]$ as α -cut for a fuzzy number b_{pt} for $p = 1, 2, \ldots, n,$ $t = 1,$

$2, \ldots, m,$

such that

$$b_{pt\alpha)} = \left[b_{pt1} + \alpha\left(b_{pt2} - b_{pt1}\right), b_{pt3} + \alpha\left(b_{pt2} - b_{pt3}\right)\right] \qquad \forall\ \alpha \in [0,1] \qquad (3.3.5)$$

As in Carlson and Fuller (2001), using (3.3.4), equations (2.3.2), (2.3.3) and (2.3.4), one can derive an expression, for the lower possibilistic mean $E_L(a_{ip})$, upper possibilistic mean $E_R(a_{ip})$ and possibilistic mean $E(a_{ip})$ of an a_{ip} as follows:

Using (3.3.4) and equation (2.3.2), one can obtain

$$E_L(a_{ip}) = 2\int_0^1 \left\{a_{ip1} + \alpha\left(a_{ip2} - a_{ip1}\right)\right\}\alpha d\alpha$$

$$= 2\int_0^1 a_{ip1}\alpha d\alpha + 2\int_0^1 \alpha\left(a_{ip2} - a_{ip1}\right)\alpha d\alpha$$

$$= 2a_{ip1}\int_0^1 \alpha d\alpha + 2\left(a_{ip2} - a_{ip1}\right)\int_0^1 \alpha^2 d\alpha$$

$$= 2a_{ip1}\left|\frac{\alpha^2}{2}\right|_0^1 + 2\left(a_{ip2} - a_{ip1}\right)\left|\frac{\alpha^3}{3}\right|_0^1 = a_{ip1} + \frac{2}{3}\left(a_{ip2} - a_{ip1}\right)$$

$$= \frac{1}{3}a_{ip1} + \frac{2}{3}a_{ip2} \qquad \text{for } i = 1, 2, \ldots, s\ , p = 1, 2, \ldots, n. \qquad (3.3.6)$$

Similarly, using (3.3.4) and equation (2.3.3), one can obtain

$$E_R(a_{ip}) = 2\int_0^1 \left\{a_{ip3} + \alpha\left(a_{ip2} - a_{ip3}\right)\right\}\alpha d\alpha$$

$$= 2\int_0^1 a_{ip3}\alpha d\alpha \ + \ 2\int_0^1 \alpha\left(a_{ip2} - a_{ip3}\right)\alpha d\alpha$$

$$= 2a_{ip3}\int_0^1 \alpha d\alpha \ + \ 2\left(a_{ip2} - a_{ip3}\right)\int_0^1 \alpha^2 d\alpha$$

$$= 2a_{ip3}\left|\frac{\alpha^2}{2}\right|_0^1 + 2\left(a_{ip2} - a_{ip3}\right)\left|\frac{\alpha^3}{3}\right|_0^1 = a_{ip3} + \frac{2}{3}\left(a_{ip2} - a_{ip3}\right)$$

$$= \frac{1}{3}a_{ip3} + \frac{2}{3}a_{ip2} \qquad \text{for } i = 1, 2, \ldots, s\ , p = 1, 2, \ldots, n. \qquad (3.3.7)$$

Possibilistic mean value $E(a_{ip})$ can now be deduced from equations (3.3.6), (3.3.7) and (2.3.4), as follows:

$$E(a_{ip}) = \frac{\frac{1}{3}a_{ip1} + \frac{2}{3}a_{ip2} + \frac{1}{3}a_{ip3} + \frac{2}{3}a_{ip2}}{2}$$

$$= \frac{1}{6}a_{ip1} + \frac{1}{6}a_{ip3} + \frac{2}{3}a_{ip2} \quad \text{for } i = 1, 2, \ldots, s\ , p = 1, 2, \ldots, n. \qquad (3.3.8)$$

Similarly, using (3.3.4), (3.3.5) and equations (2.3.2), (2.3.3) and (2.3.4), one can derive an expression, for the lower possibilistic mean $E_L(a_{ip}.b_{pt})$, upper possibilistic mean $E_R(a_{ip}.b_{pt})$ and possibilistic mean $E(a_{ip}.b_{pt})$, for each combination of $a_{ip}.b_{pt}$'s, similar to that of equation (3.3.1), as follows:

Using (3.3.4), (3.3.5) and equation (2.3.2), one can obtain

$$\mathrm{E_L}(a_{ip}.b_{pt}) = 2\int_0^1 \left\{a_{ip1} + \alpha\left(a_{ip2} - a_{ip1}\right)\right\}\left\{b_{pt1} + \alpha\left(b_{pt2} - b_{pt1}\right)\right\}\alpha d\alpha$$

$$= \frac{1}{6}a_{ip1}b_{pt1} + \frac{1}{6}a_{ip1}b_{pt2} + \frac{1}{6}a_{ip2}b_{ip1} + \frac{1}{2}a_{ip2}b_{pt2} \ . \tag{3.3.9}$$

Similarly, using (3.3.4), (3.3.5) and equation (2.3.3), one can obtain

$$\mathrm{E_R}(a_{ip}.b_{pt}) = 2\int_0^1 \left\{a_{ip3} + \alpha\left(a_{ip2} - a_{ip3}\right)\right\}\left\{b_{pt3} + \alpha\left(b_{pt2} - b_{pt3}\right)\right\}\alpha d\alpha$$

$$= \frac{1}{6}a_{ip3}b_{pt3} + \frac{1}{6}a_{ip3}b_{ip2} + \frac{1}{6}a_{ip2}b_{pt3} + \frac{1}{2}a_{ip2}b_{pt2} \ . \tag{3.3.10}$$

For all $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n$, $t = 1, 2, \ldots, m$, possibilistic mean value $\mathrm{E}(a_{ip}.b_{pt})$, can now be calculated from equations (3.3.9), (3.3.10) and (2.3.4), as follows:

$\mathrm{E}(a_{ip}.b_{pt}) =$

$$\frac{\frac{1}{6}a_{ip1}b_{pt1} + \frac{1}{6}a_{ip1}b_{pt2} + \frac{1}{6}a_{ip2}b_{pt1} + \frac{1}{2}a_{ip2}b_{pt2} + \frac{1}{6}a_{ip3}b_{pt3} + \frac{1}{6}a_{ip3}b_{pt2} + \frac{1}{6}a_{ip2}b_{pt3} + \frac{1}{2}a_{ip2}b_{pt2}}{2}$$

$$= \frac{1}{12}a_{ip1}b_{pt1} + \frac{1}{12}a_{ip1}b_{pt2} + \frac{1}{12}a_{ip2}b_{pt1} + \frac{1}{12}a_{ip3}b_{pt3} + \frac{1}{12}a_{ip3}b_{pt2} + \frac{1}{12}a_{ip2}b_{pt3} + \frac{1}{2}a_{ip2}b_{pt2} \tag{3.3.11}$$

Using equation (3.3.11), one can calculate, the possibilistic mean values for all the combinations of $a_{ip}.b_{pt}$'s as in equation (3.3.1). Thus, the mean capacity $\mathrm{E}(c_{it})$, for $i = 1, 2, \ldots, s$, $t = 1, 2, \ldots, m$, required at any key resource using bill of labor approach under fuzzy environment for a period is obtained as follows:

$$\mathrm{E}(c_{it}) = \sum_{p=1}^{n} \mathrm{E}\left(a_{ip}.b_{pt}\right), \ i = 1, 2, \ldots, s; \ t = 1, 2, \ldots, m. \tag{3.3.12}$$

It is important to point out here that $E(c_{it})$, calculated above in (3.3.12), is the crisp capacity required under fuzzy environment at work center i for period t using bill of labor approach.

3.4 Numerical Example of Possibilistic Rough Cut Capacity Calculation under Fuzzy Environment using Bill of Labor Approach

In this section, the results are illustrated with the help of a numerical example. Four key resources (work centers) WC_1, WC_2, WC_3, and WC_4; five products P_1, P_2, P_3, P_4 and P_5; and six time periods T_1, T_2, T_3, T_4, T_5 and T_6 are considered. The following TFN's in Table (3.4), represent bill of labor (BOL) in hours, Tables (3.5) and (3.6) represent MPS in number of product units. These TFN's can be generated randomly or using random number tables, but for purpose of comparison and verification of results, the data from Verma, (2001), has been adopted.

Table 3.4 Bill of Labor in hours

	P_1	P_2	P_3	P_4	P_5
WC_1	(.25, .30, .31)	(.12, .14, .15)	(.25, .26, .28)	(.24, .28, .30)	(.12, .14, .16)
WC_2	(.24, .27, .29)	(.18, .20, .23)	(.36, .40, .41)	(.25, .26, .30)	(.30, .32, .35)
WC_3	(.15, .17, .19)	(.30, .32, .33)	(.40, .43, .44)	(.50, .52, .55)	(.36, .40, .41)
WC_4	(.12, .16, .18)	(.35, .37, .40)	(.15, .17, .20)	(.50, .55, .60)	(.40, .43, .46)

Table 3.5 Master Production Schedule in product units for Periods T_1 to T_3

	T_1	T_2	T_3
P_1	(50, 60, 65)	(42, 46, 55)	(40, 45, 55)
P_2	(60, 72, 75)	(75, 78, 80)	(90, 92, 95)
P_3	(75, 80, 90)	(80, 90, 100)	(115, 120, 128)
P_4	(90, 101, 105)	(100, 105, 108)	(107, 109, 110)
P_5	(70, 80, 85)	(80, 90, 95)	(60, 70, 80)

Table 3.6 Master Production Schedule in product units for Periods T_4 to T_6

	T_4	T_5	T_6
P_1	(100, 105, 110)	(70, 75, 78)	(80, 90, 95)
P_2	(32, 40, 42)	(80, 90, 92)	(65, 75, 80)
P_3	(75, 78, 80)	(110, 114, 120)	(80, 95, 100)
P_4	(50, 52, 64)	(72, 78, 84)	(70, 75, 85)
P_5	(100, 105, 110)	(55, 65, 70)	(90, 95, 110)

Using equation (3.3.11), the possibilistic mean values of all the above a_{ip}'s and b_{pt}'s given in the Tables (3.4) to (3.6), respectively, are calculated for all the combinations of $a_{ip}.b_{pt}$'s similar to those of equation (3.3.1), in the following tables. Using equation (3.3.1), tables (3.7) to (3.10) represent the possibilistic mean values of all the combinations of $a_{ip}.b_{pt}$'s, as follows:

Table 3.7 Possibilistic Mean Values for Work Center 1

	T_1	T_2	T_3	T_4	T_5	T_6
P_1	17.396	13.768	13.479	30.825	21.923	26.196
P_2	9.773	10.773	12.756	5.408	12.282	10.279
P_3	21.171	23.575	31.548	20.373	29.930	24.446
P_4	27.660	28.978	30.118	14.880	21.610	21.017
P_5	11.108	12.508	9.833	14.717	9.008	13.567
Total	87.108	89.603	97.734	86.203	94.753	95.504

Table 3.8 Possibilistic Mean Values for Work Center 2

	T_1	T_2	T_3	T_4	T_5	T_6
P_1	15.908	12.593	12.329	28.196	20.053	23.958
P_2	14.248	15.707	18.598	7.885	17.905	14.988
P_3	31.958	35.592	47.623	30.755	45.182	36.913
P_4	26.484	27.753	28.847	14.255	20.695	20.129
P_5	25.496	28.713	22.558	33.796	20.671	31.138
Total	114.094	120.357	129.955	114.887	124.505	127.125

Table 3.9 Possibilistic Mean Values for Work Center 3

	T_1	T_2	T_3	T_4	T_5	T_6
P_1	10.083	7.983	7.817	17.867	12.707	15.183
P_2	22.463	24.783	29.346	12.428	28.242	23.629
P_3	34.513	38.433	51.434	33.218	48.798	39.858
P_4	52.110	54.618	56.781	28.027	40.715	39.592
P_5	31.304	35.254	27.692	41.496	25.379	38.221
Total	150.473	161.072	173.069	133.035	155.841	156.483

Table 3.10 Possibilistic Mean Values for Work Center 4

	T_1	T_2	T_3	T_4	T_5	T_6
P_1	9.308	7.368	7.217	16.475	11.718	14.008
P_2	26.233	28.938	34.266	14.515	32.978	27.596
P_3	13.908	15.492	20.713	13.372	19.648	16.063
P_4	54.971	57.600	59.871	29.575	42.950	41.771
P_5	34.079	38.379	30.150	45.175	27.629	41.617
Total	138.499	147.778	152.217	119.112	134.924	141.054

Using bill of labor approach under fuzzy environment, the possible mean capacity required at any key resource for a period is calculated by using equation (3.3.12). The last row in each of the tables (3.7) to (3.10) represents the summation of the crisp possibilistic mean value of $a_{ip}.b_{pt}$'s, and yields the crisp possibilistic mean value of the corresponding c_{it}'s (E(c_{it})'s), as in equation (3.3.12). The E(c_{it})'s calculated above are the crisp capacity requirements under fuzzy environment at work center i for period t using bill of labor approach. Using equation (3.3.12) and Tables (3.7) to (3.10), the calculation of E(c_{11}), for WC_1, T_1, $p = 1, 2, \ldots, 5$, is demonstrated as follows:

$$\mathrm{E}(c_{11}) = \sum_{p=1}^{5} \mathrm{E}\left(a_{1p}.b_{p1}\right)$$

$$= \mathrm{E}\,(a_{11} \,.\, b_{11}) + \mathrm{E}\,(a_{12} \,.\, b_{21}) + \mathrm{E}\,(a_{13} \,.\, b_{31}) + \mathrm{E}\,(a_{14} \,.\, b_{41}) + \mathrm{E}\,(a_{15} \,.\, b_{51})$$

$$= 17.396 + 9.773 + 21.171 + 27.660 + 11.108$$

$$= 87.108.$$

The other E(c_{it})'s are calculated similarly using Tables (3.7) to (3.10), and are shown in Table (3.11) below.

Table 3.11 Capacities required in hours for Work Center *i* for Period *t*

	T_1	T_2	T_3	T_4	T_5	T_6
WC_1	87.108	89.603	97.734	86.203	94.753	95.504
WC_2	114.094	120.357	129.955	114.887	124.505	127.125
WC_3	150.473	161.072	173.069	133.035	155.841	156.483
WC_4	138.499	147.778	152.217	119.112	134.924	141.054

Capacities calculated above are the crisp possibilistic mean values of the capacities required at different work centers (WC_1 to WC_4) for different time periods (T_1 to T_6). These capacities are compared with the available capacities to foresee whether the MPS is feasible or not? Depending upon the situation, one can take a timely step in either enhancing capacity of the work centers that are short in capacity or can revise the MPS.

3.5 Comparison and Interpretation of Results

In Table (3.11), the fuzzy capacities required in time units for each work center for different time periods are calculated using possibilistic approach. These capacities are the crisp capacities in the form of possibilistic mean values of the fuzzy capacities required at different Work Centers to satisfy the MPS. These results are compared with the fuzzy capacities calculated by using same input data (same TFN's for bill of labor and MPS). The method uses multiplication of their α-cuts and then calculating membership functions for each fuzzy number as shown in tables (3.12) and (3.13):

Table 3.12 Capacities required in hours for Period T_1 to T_3

	T_1	T_2	T_3
WC_1	(68.45, **88.36**, 101.7)	(73.10, **90.12**, 104.65)	(82.43, **97.90**, 112.94)
WC_2	(93.3, **114.46**, 134.25)	(101.38, **120.12**, 141)	(111.95,**129.29**,151.28)
WC_3	(125.7, **152.16**, 169.3)	(139.6, **162.08**, 179.2)	(154.1, **173.37**, 191.42)
WC_4	(111.25, **139.79**, 161.8)	(125.29, **147.97**, 170.4)	(131.05, **151.69**, 176.3)

Table 3.13 Capacities required in hours for Period T_4 to T_6

	T_4	T_5	T_6
WC_1	(71.59, **86.64**, 99.6)	(78.48, **95.68**, 107.98)	(75.4, **96.5**, 112.55)
WC_2	(99.26, **114.67**, 132.06)	(105.3, **124.93**, 142.68)	(104.2, **127.2**, 150.95)
WC_3	(115.6, **133.23**, 150.26)	(134.3, **157.13**, 172.88)	(130.9, **157.15**, 180.3)
WC_4	(99.45, **118.61**, 141.6)	(110.9, **135.53**, 157.44)	(115.35, **140.4**, 170.7)

If one compares the results generated by using possibilistic approach with the results generated by the method using complex membership functions at $\alpha = 1$ (interior point of the TFN), one can easily observe that there is very minor difference between both the results. Furthermore, the method using complex membership functions provides one the capacities in the form of fuzzy numbers. On the other hand, possibilistic approach suggested here generates capacities in the form of crisp possibilistic mean values, which are easy to work with.

The bill of labor approach, considered in this chapter, for capacity planning under fuzzy environment, although does not consider the lead time offsets, it uses detailed data on time standards for each product at key resources. It is relatively easy and predicts better the actual changes required in capacity requirement at key resources or in MPS from period to period.

The approach suggested in this chapter appears to be simple and promising. It is easy to work with their crisp possibilistic mean values instead of calculating the complex membership functions of each and every fuzzy number which is quite time consuming.

CHAPTER 4

CAPACITY PLANNING UNDER FUZZY ENVIRONMENT USING POSSIBILITY THEORY WITH RESOURCE PROFILE APPROACH

In this chapter, possibility theory is used for resource profile approach of rough cut capacity analysis under fuzzy environment. The crisp resource profile approach to find the capacity requirements, at certain key resources, is extended to the fuzzy environment, where MPS quantity and resource profile amount for each product at a Work Center are represented by a triangular fuzzy number (TFN). It is proposed to use possibility theory approach to tackle the computational tediousness of mathematical operations of TFN's. The approach can also be extended further when the MPS quantity and resource profile amount for each product at a Work Center are represented by some other special type of fuzzy numbers (for example, trapezoidal fuzzy numbers).

4.1 Introduction

Resource profile approach considers lead time offsets and time phases the labor requirements. The problem considered in this chapter is different from the one considered in Chapter 3. The bill of labor approach (BOL) considered in Chapter 3 does not consider *lead time offsets,* whereas in this chapter the lead time offsets are considered. However, bill of labor approach of RCCP assumes that all components and parts are manufactured at the same time as the end product. It is important to point out here that certain products have substantially long manufacturing lead times running over several weeks or months. For example many large and complex products such as airplanes, ships, boilers, and machine tools have very lengthy lead times. For such products (having lengthy manufacturing lead times), the resource profile approach of capacity analysis appears to be very useful. According to Fogarty et al. (1991), Chase et al. (2006), the resource profile technique is the most detailed rough cut approach, but is not as detailed as capacity requirements planning.

Resource profile is somewhat similar to a bill of labor except that in this case the time at each department or Work Center reflects the lead time of a part or a component. To prepare a resource profile under such circumstances, the lead time is converted into periods prior to the period in which the delivery is promised. For example, the last operation always occurs immediately prior to delivery, therefore, it is shown as occurring at zero periods prior to delivery (Fogarty et al, 1991).

4.2 Rough Cut Capacity Analysis under Crisp Environment using Resource Profile Approach

Suppose, the MPS contains p number of products to be manufactured over time buckets t, where $p = 1, 2, \ldots, n$; $t = 1, 2, \ldots, m$. Let the work centers i (or key resource), whose capacity to be analyzed, be represented by WC_i i where $i = 1, 2, \ldots, s$. Let d represent the time to due date, where $d = t, t + 1 \ldots m$.

The following assumptions are made under which the model is developed.

- Each resource profile is time phased.

- The manufacturing lead times of all the components and parts at each department or work center are attached with them as periods prior to the delivery of the end product.
- The end product is delivered immediately after the last operation is completed on it.

Let,

$a_{ip(d-t)}$ = Resource profile amount at Work Center i for product p due in period $d - t$ (measured in time units), $t = 1, 2, \ldots, m$, and $d = t, t + 1, \ldots, m$.

b_{pt} = MPS quantity for product p during time t.

c_{it} = Capacity needed in time units at work centre i for period t.

P_p = Product p, where $p = 1, 2, \ldots, n$.

WC_i = Work Center i, where $i = 1, 2, \ldots, s$.

T_t = Period t, where $t = 1, 2, \ldots, m$.

In a crisp case, the resource profile and MPS can be represented in a tabular form as shown below:

Table 4.1 Resource Profile in time units

	$m - 1$	----	1	0
P_1	$a_{i1(m-1)}$	----	a_{i11}	a_{i10}
P_2	$a_{i2(m-1)}$	----	a_{i21}	a_{i20}
----	----	----	----	----
P_n	$a_{in(m-1)}$	----	a_{in1}	a_{in0}

Table 4.2 MPS in product units

	T_1	T_2	----	T_m
P_1	b_{11}	b_{12}	----	b_{1m}
P_2	b_{21}	b_{22}	----	b_{2m}
----	----	----	----	----
P_n	b_{n1}	b_{n2}	----	b_{nm}

Formulation for Capacity Calculation

Let

c_{it} = Capacity required at work centre i for time period t.

Therefore,

$$c_{it} = \sum_{p=1}^{n}\sum_{d=t}^{m} a_{ip(d-t)}(.)b_{pt}\ , \ i = 1, 2, \ldots, s;\ t = 1, 2, \ldots, m. \qquad (4.2.1)$$

Using equation (4.2.1), the required capacity at any key resource or Work Center can be calculated. If the numbers $a_{ip(d-t)}$'s and b_{pt}'s for all values of i, p, d and t are crisp then their product will also be crisp, and hence, each c_{it}, which is summation of these individual products will also be a crisp number. Using Table 4.1, Table 4.2 and equation (4.2.1), one can obtain the following table of the capacity required under crisp environment.

Table 4.3 Capacity required in time units

	T_1	T_2	----	T_m
WC_1	c_{11}	c_{12}	----	c_{1m}
WC_2	c_{21}	c_{22}	----	c_{2m}
----	----	----	----	----
WC_s	c_{s1}	c_{s2}	----	c_{sm}

4.3 Rough Cut Capacity Calculation using Resource Profile Approach under Fuzzy Environment

In this section, for analysis purpose, the equation (4.2.1) is rewritten as follows:

$$c_{it} = \sum_{p=1}^{n}\sum_{d=t}^{m} a_{ip(d-t)}(.)b_{pt}\ ,\ \ i\ = 1, 2,. . . , s;\ \ t = 1, 2, . . ., m, \qquad (4.3.1)$$

In equation (4.3.1), if at least one of the $a_{ip(d-t)}$'s or b_{pt}'s is a fuzzy number then their product would also be a fuzzy number (Kaufman and Gupta, 1988, 1991) and hence corresponding c_{it} will also be a fuzzy number. It is important to mention here that in (4.3.1), although each of the value of c_{it} is a fuzzy number, it may not be necessarily a TFN. (Kaufman and Gupta, 1988, 1991). As explained earlier, in general, the product of two TFN's is a parabolic fuzzy number.

It is now assumed that in equation (4.3.1), each of the $a_{ip(d-t)}$ and b_{pt} for $i = 1, 2, . . ., s,$ $p = 1, 2, . . ., n,\ \ t = 1, 2, . . , m,$ and $d = t, t + 1, \ldots, m,$ is a TFN of the type

$$a_{ip(d-t)} = (a_{ip(d-t)1}, a_{ip(d-t)2}, a_{ip(d-t)3}) \text{ and } b_{pt} = (b_{pt1}, b_{pt2}, b_{pt3}) \qquad (4.3.2)$$

As discussed in Chapter 3, the values in (4.3.2), can be obtained from expert opinion of individuals who have the same information source but different opinion about the same problem. This means that instead of having only one crisp data/estimate of resource profile or MPS, one can have three estimates based on the expert opinion yielding a TFN to account for uncertainty.

Hence, each c_{it} in equation (4.3.1), is a fuzzy number given by

$$c_{it} = (c_{it1}, c_{it2}, c_{it3}) \text{ for each } i = 1, 2, . . ., s,\ \ t = 1, 2, . . , m, \qquad (4.3.3)$$

Each fuzzy number c_{it} in (4.3.3), can be represented by its α-cut obtained by using the α-cuts of a_{ip} and b_{pt} (see Appendix 1, A.2).

Let,

$a_{ip(d-t)}(\alpha) = \left[a_{ip(d-t)1}(\alpha), a_{ip(d-t)2}(\alpha)\right]$ as α-cut for $a_{ip(d-t)}$'s, for $i = 1, 2, . . ., s$, $p = 1,$ $2, . . ., n, d = t, t + 1, \ldots, m,$ such that

$a_{ip(d-t)}(\alpha) =$

$$\left[a_{\text{ip(d - t)1}} + \alpha\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)1}}\right),\ a_{\text{ip(d - t)3}} + \alpha\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)3}}\right)\right] \quad \forall\ \alpha \in [0,1]. \tag{4.3.4}$$

Similarly,

$b_{pt}(\alpha) = \left[b_{pt1}(\alpha), b_{pt2}(\alpha)\right]$ as α -cut for b_{pt}'s, $p = 1, 2, \ldots, n$, $t = 1, 2, \ldots, m$,

such that

$$b_{pt}(\alpha) = \left[b_{pt_1} + \alpha\left(b_{pt2} - b_{pt1}\right), b_{pt3} + \alpha\left(b_{pt2} - b_{pt3}\right)\right] \qquad \forall\ \alpha \in [0,1] \tag{4.3.5}$$

for all $i = 1, 2, \ldots, s, p = 1, 2, \ldots, n, t = 1, 2, \ldots, m.$

As in Carlson and Fuller (2001), one can use (4.3.4) and equations (2.3.2), (2.3.3) and (2.3.4), in deriving the crisp lower possibilistic mean value $E_L(a_{ip(d-t)})$, crisp upper possibilistic mean value $E_R(a_{ip(d-t)})$ and crisp possibilistic mean value $E(a_{ip(d-t)})$, respectively, for a fuzzy number $a_{ip(d-t)}(\alpha) = \left[a_{\text{ip(d - t)1}}(\alpha),\ a_{\text{ip(d - t)2}}(\alpha)\right]$. It is pertinent to mention here that $E_L(a_{ip(d-t)})$, $E_R(a_{ip(d-t)})$ and $E(a_{ip(d-t)})$ are crisp numbers.

Using a similar approach of Carlson and Fuller (2001), from (4.3.4) and equation (2.3.2), for $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n, d = t, t + 1, \ldots, m,$ one can obtain

$$E_L(a_{ip(d-t)}) = 2\int_0^1 \left\{a_{\text{ip(d - t)1}} + \alpha\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)1}}\right)\right\}\alpha d\alpha$$

$$= 2\int_0^1 a_{\text{ip(d - t)1}}\alpha d\alpha + 2\int_0^1 \alpha\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)1}}\right)\alpha d\alpha$$

$$= 2a_{\text{ip(d - t)1}}\int_0^1 \alpha d\alpha + 2\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)1}}\right)\int_0^1 \alpha^2 d\alpha$$

$$= 2a_{\text{ip(d - t)1}}\left|\frac{\alpha^2}{2}\right|_0^1 + 2\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)1}}\right)\left|\frac{\alpha^3}{3}\right|_0^1$$

$$= \; a_{\text{ip(d - t)1}} + \frac{2}{3}\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)1}}\right)$$

$$= \; \frac{1}{3}a_{\text{ip(d - t)1}} + \frac{2}{3}a_{\text{ip(d - t)2}} \tag{4.3.6}$$

Similarly from (4.3.4) and equation (2.3.3), for *i = 1, 2, . . ., s* , *p = 1, 2, . . ., n, d = t, t + 1, …., m,* one can obtain

$$\mathrm{E_R}(a_{ip(d-t)}) = 2\int_0^1 \left\{a_{\text{ip(d - t)3}} + \alpha\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)3}}\right)\right\}\alpha\, d\alpha$$

$$= \; 2\int_0^1 a_{\text{ip(d - t)3}}\alpha\, d\alpha \; + \; 2\int_0^1 \alpha\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)3}}\right)\alpha\, d\alpha$$

$$= \; 2a_{\text{ip(d - t)3}}\int_0^1 \alpha\, d\alpha \; + \; 2\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)3}}\right)\int_0^1 \alpha^2 d\alpha$$

$$= \; 2a_{\text{ip(d - t)3}}\left|\frac{\alpha^2}{2}\right|_0^1 + 2\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)3}}\right)\left|\frac{\alpha^3}{3}\right|_0^1$$

$$= \; a_{\text{ip(d - t)3}} + \frac{2}{3}\left(a_{\text{ip(d - t)2}} - a_{\text{ip(d - t)3}}\right)$$

$$= \; \frac{1}{3}a_{\text{ip(d - t)3}} + \frac{2}{3}a_{\text{ip(d - t)2}} \tag{4.3.7}$$

Possibilistic mean value $\mathrm{E}(a_{ip(d-t)})$ of a fuzzy number $a_{ip(d-t)}$ is deduced from equations (2.3.4), (4.3.6) and (4.3.7), for *i = 1, 2, . . ., s* , *p = 1, 2, . . ., n, d = t, t + 1, …., m,* as follows:

$$\mathrm{E}(a_{ip(d-t)}) \; = \; \frac{\frac{1}{3}a_{\text{ip(d - t)1}} + \frac{2}{3}a_{\text{ip(d - t)2}} + \frac{1}{3}a_{\text{ip(d - t)3}} + \frac{2}{3}a_{\text{ip(d - t)2}}}{2}$$

$$= \; \frac{1}{6}a_{\text{ip(d - t)1}} + \frac{1}{6}a_{\text{ip(d - t)3}} + \frac{2}{3}a_{\text{ip(d - t)2}}\,. \tag{4.3.8}$$

Similarly, using (4.3.4), (4.3.5) and equations (2.3.2), (2.3.3) and (2.3.4), one can derive an expression, for the lower possibilistic mean $E_L(a_{ip(d-t)}.b_{pt})$, upper possibilistic mean $E_R(a_{ip(d-t)}.b_{pt})$ and possibilistic mean $E(a_{ip(d-t)}.b_{pt})$, for each combination of $a_{ip(d-t)}.b_{pt}$'s, similar to those of equation (4.3.1), as shown below:

Using (4.3.4), (4.3.5) and equation (2.3.2), for $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n$, $d = t, t + 1, \ldots, m$, $t = 1, 2, \ldots, m$, one can obtain

$$E_L(a_{ip(d-t)}.b_{pt}) = 2\int_0^1 \left\{a_{ip(d-t)1} + \alpha\left(a_{ip(d-t)2} - a_{ip(d-t)1}\right)\right\}\left\{b_{pt1} + \alpha\left(b_{pt2} - b_{pt1}\right)\right\}\alpha d\alpha$$

$$= \frac{1}{6}a_{ip(d-t)1}b_{pt1} + \frac{1}{6}a_{ip(d-t)1}b_{pt2} + \frac{1}{6}a_{ip(d-t)2}b_{pt1} + \frac{1}{2}a_{ip(d-t)2}b_{pt2} \qquad (4.3.9)$$

Similarly, using (4.3.4), (4.3.5) and equation (2.3.3), for $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n$, $d = t, t + 1, \ldots, m$, $t = 1, 2, \ldots, m$, one can obtain

$$E_R(a_{ip(d-t)}.b_{pt}) = 2\int_0^1 \left\{a_{ip(d-t)3} + \alpha\left(a_{ip(d-t)2} - a_{ip(d-t)3}\right)\right\}\left\{b_{pt3} + \alpha\left(b_{pt2} - b_{pt3}\right)\right\}\alpha d\alpha$$

$$= \frac{1}{6}a_{ip(d-t)3}b_{pt3} + \frac{1}{6}a_{ip(d-t)3}b_{pt2} + \frac{1}{6}a_{ip(d-t)2}b_{pt3} + \frac{1}{2}a_{ip(d-t)2}b_{pt2} \qquad (4.3.10)$$

Possibilistic mean value $E(a_{ip(d-t)}.b_{pt})$, can now be calculated from equations (4.3.9), (4.3.10) and (2.3.4), for $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n$, $t = 1, 2, \ldots, m$, $d = t, t + 1, \ldots, m$, as follows:

$$E(a_{ip(d-t)}.b_{pt}) = \frac{\frac{1}{6}a_{ip(d-t)1}b_{pt1} + \frac{1}{6}a_{ip(d-t)1}b_{pt2} + \frac{1}{6}a_{ip(d-t)2}b_{pt1} + \frac{1}{2}a_{ip(d-t)2}b_{pt2}}{2} + \frac{\frac{1}{6}a_{ip(d-t)3}b_{pt3} + \frac{1}{6}a_{ip(d-t)3}b_{pt2} + \frac{1}{6}a_{ip(d-t)2}b_{pt3} + \frac{1}{2}a_{ip(d-t)2}b_{pt2}}{2}$$

$$E(a_{ip(d-t)}.b_{pt}) = \begin{array}{l} \frac{1}{12}a_{ip(d-t)1}b_{pt1} + \frac{1}{12}a_{ip(d-t)1}b_{pt2} + \frac{1}{12}a_{ip(d-t)2}b_{pt1} + \frac{1}{12}a_{ip(d-t)3}b_{pt3} \\ + \frac{1}{12}a_{ip(d-t)3}b_{pt2} + \frac{1}{12}a_{ip(d-t)2}b_{pt3} + \frac{1}{2}a_{ip(d-t)2}b_{pt2} \end{array} \quad (4.3.11)$$

Using expression (4.3.11), one can calculate, the possibilistic mean values for all the combinations of $a_{ip(d-t)}.b_{pt}$'s, similar to those of (4.3.1). The capacity required at any key resource using resource profile approach under fuzzy environment for a period is calculated by summation of crisp possibilistic mean values of $a_{ip(d-t)}.b_{pt}$'s to find the crisp possibilistic mean values and the corresponding c_{it} (E(c_{it})) as given below:

$$E(c_{it}) = \sum_{p=1}^{n}\sum_{d=t}^{m} E\left(a_{ip(d-t)}.b_{pt}\right) \; i = 1, 2, \ldots, s; \; t = 1, 2, \ldots, m. \quad (4.3.12)$$

The E(c_{it}) calculated above is the crisp capacity requirement under fuzzy environment at work center *i* for period *t* using resource profile approach.

4.4 Numerical Example of Possibilistic Rough Cut Capacity Calculation under Fuzzy Environment using Resource Profile Approach

In this section, the results are illustrated with the help of a numerical example in which four key resources (work centers) WC_1, WC_2, WC_3, and WC_4; five products P_1, P_2, P_3, P_4 and P_5, six time periods T_1, T_2, T_3, T_4, T_5 and T_6 and six months lead time (time to due date) are considered. The following TFN's in Tables (4.4), (4.5), (4.6) and (4.7) represent resource profiles in time units for Work Center 1, 2, 3 and 4 respectively; Tables (4.8) and (4.9) represent MPS in number of product units for periods T_1 to T_3 and T_4 to T_6 respectively. As pointed out earlier, these TFN's could be generated randomly or using random number tables, but for purpose of comparison and the verification of results, they have been adopted from Verma, (2001).

Table 4.4 Resource Profile in time units for Work Center 1

P_p ↓	Time to Due Date					
	5	4	3	2	1	0
P_1	(.2, .3, .7)	(.3, .4, .8)	(.3, .7, .8)	(.4, .6, .8)	(.2, .3, .7)	(.35, .4, .7)
P_2	(.6, .7, .9)	(.4, .5, .9)	(.2, .6, .9)	(.1, .5, .6)	(.15, .2, .5)	(.6, .75, .8)
P_3	(.65, .7, .9)	(.6, ..8, .9 ,)	(.1, .5, .7)	(.4, .8, .9)	(.2, .8, .9)	(.3, .4, .5)
P_4	(.6, .8, .9)	(.12, .15, .2)	(.15, .25, .4)	(.4, .9, .95)	(.3, .6, ..8)	(.4, .6, .9)
P_5	(.6, .7, .9)	(.4, .5, .9)	(.2, .6, .9)	(.1, .5, .6)	(.15, .2, .5)	(.6, .75, .8)

Table 4.5 Resource Profile in time units for Work Center 2

P_p ↓	Time to Due Date					
	5	4	3	2	1	0
P_1	(.25, .3, .31)	(.15, .2, .25)	(.4, .5, .8)	(.2, .3, .7)	(.4, .6, .8)	(.4, .5, .9)
P_2	(.1, .2, .4)	(.2, .4, .9)	(.1, .8, .9)	(.7, .8, .9)	(.6, .9, .95)	(.3, .8, .9)
P_3	(.2, .4, .9)	(.8, .85, .95)	(.1, .3, .7)	(.3, .8, .9)	(.1, .5, .7)	(.2, .7, .9)
P_4	(.1, .6, .9)	(.3, .7, .9)	(.7, .8, .9)	(.6, .9, .95)	(.5, .6, .9)	(.1, .4, .7)
P_5	(.2, .3, .7)	(.3, .4, .8)	(.3, .7, .8)	(.4, .6, .8)	(.2, .3, .7)	(.35, .4, .7)

Table 4.6 Resource Profile in time units for Work Center 3

P_p ↓	Time to Due Date					
	5	4	3	2	1	0
P_1	(.1, .2, .4)	(.2, .4, .7)	(.1, .5, .7)	(.7, .8, .9)	(.6, .8, .9)	(.3, .8, .9)
P_2	(.4, .5, .7)	(.7, .8, .9)	(.4, .5, .7)	(.1, .8, .9)	(.2, .7, .8)	(.7, .8, .9)
P_3	(.1, .6, .9)	(.3, .7, .9)	(.2, .4, .6)	(.3, .5, .8)	(.1, .2, .3)	(.3, .5, .6)
P_4	(.2, .3, .7)	(.3, .4, .8)	(.4, .6, .9)	(.4, .6, .8)	(.2, .3, .6)	(.3, .7, .8)
P_5	(.5, .7, .8)	(.3, .4, .7)	(.6, .8, .9)	(.15, .2, .3)	(.6, .7, .9)	(.1, .3, .5)

Table 4.7 Resource Profile in time units for Work Center 4

P_p ↓	Time to Due Date					
	5	4	3	2	1	0
P_1	(.2, .3, .7)	(.3, .5, .7)	(.3, .7, .8)	(.4, .6, .8)	(.2, .4, .6)	(.1, .2, .5)
P_2	(.2, .4, .9)	(.8, .85, .9)	(.1, .3, .7)	(.1, .5, .7)	(.3, .5, .8)	(.15, .2, .3)
P_3	(.1, .8, .9)	(.1, .2, .8)	(.3, .6, .9)	(.4, .6, .8)	(.7, .8, .9)	(.3, .7, .9)
P_4	(.2, .3, .7)	(.2, .7, .9)	(.6, .7, .9)	(.5, .6, .8)	.5, .7, .8)	(.3, .4, .7)
P_5	(.6, .8, .9)	(.2, .3, .7)	(.6, .7, .8)	(.4, .7, .8)	(.2, .7, .9)	(.1, .4, .6)

Table 4.8 Master Production Schedule in product units for Periods T_1 to T_3

	T_1	T_2	T_3
P_1	(200, 230, 270)	(200, 205, 250)	(170, 240, 250)
P_2	(100, 175, 210)	(230, 240, 260)	(95, 100, 150)
P_3	(295, 299, 309)	(300, 310, 390)	(200, 210, 260)
P_4	(300, 305, 390)	(200, 295, 350)	(210, 220, 280)
P_5	(100, 175, 210)	(230, 240, 260)	(95, 100, 150)

Table 4.9 Master Production Schedule in product units for Periods T_4 to T_6

	T_4	T_5	T_6
P_1	(300, 325, 375)	(100, 150, 160)	(100, 175, 200)
P_2	(105, 190, 195)	(100, 140, 200)	(200, 250, 270)
P_3	(250, 255, 280)	(125, 175, 200)	(100, 170, 175)
P_4	(110, 120, 200)	(100, 130, 200)	(210, 230, 290)
P_5	(105, 190, 195)	(100, 140, 200)	(200, 250, 270)

Using equation (4.3.11), one can calculate the possibilistic mean values of all the above $a_{ip(d-t)}$'s and b_{pt}'s given in the Tables (4.4) to (4.7); (4.8) and (4.9), respectively, for all the combinations of $a_{ip(d-t)}.b_{pt}$'s as in equation (4.3.1). The capacity required at any key resource using resource profile approach under fuzzy environment for a period is calculated by summation of crisp possibilistic mean values of $a_{ip(d-t)}.b_{pt}$'s, to find the crisp possibilistic mean value of the corresponding c_{it} ($E(c_{it})$), as per equation (4.3.12). The $E(c_{it})$'s calculated above are the crisp capacity requirements under fuzzy environment at work center i for period t using resource profile approach. Using equation (4.3.12), and possibilistic mean values calculated as explained above, the calculation of $E(c_{11})$, for WC_1 , T_1 , $p = 1, 2, \ldots, 5$, $t = 1, 2, \ldots, 6$, is demonstrated as follows:

$$E(c_{11}) = \sum_{p=1}^{5}\sum_{d=1}^{6} E\left(a_{1p(d-1)}.b_{p1}\right)$$

$$\begin{aligned} E(c_{11}) = {} & E(a_{110} . b_{11}) + E(a_{111} . b_{12}) + E(a_{112} . b_{13}) + E(a_{113} . b_{14}) + E(a_{114} . b_{15}) \\ & + E(a_{115} . b_{16}) + E(a_{120} . b_{21}) + E(a_{121} . b_{22}) + E(a_{122} . b_{23}) + E(a_{123} . b_{24}) \\ & + E(a_{124} . b_{25}) + E(a_{125} . b_{26}) + E(a_{130} . b_{31}) + E(a_{131} . b_{32}) + E(a_{132} . b_{33}) \\ & + E(a_{133} . b_{34}) + E(a_{134} . b_{35}) + E(a_{135} . b_{36}) + E(a_{140} . b_{41}) + E(a_{141} . b_{42}) \end{aligned}$$

$+ E(a_{142} . b_{43}) + E(a_{143} . b_{44}) + E(a_{144} . b_{45}) + E(a_{145} . b_{46}) + E(a_{150} . b_{51})$

$+ E(a_{151} . b_{52}) + E(a_{152} . b_{53}) + E(a_{153} . b_{154})+ E(a_{154} . b_{55}) + E(a_{155} . b_{56})$

$E(c_{11})$ = 103.375 + 75.292 +139.333 + 215.417 + 65.583 + 60.208 + 124.417 + 58.875 + 49.333 + 105.792 + 81.0 + 176.417 + 120.117 + 232.667 + 163.583 + 121.250 + 134.792 + 116.042 + 198.292 + 171.375 + 189.667 + 35.0 + 21.3 + 186.333 + 124.417 + 58.875 + 49.333 + 105.792 + 81.0 + 176.417

$E(c_{11})$ = 3541.292

It is important to mention here that the $E(c_{11})$ calculated above is the crisp capacity required under fuzzy environment at work center 1 for period 1 (c_{11}) using resource profile approach

Thus,. $E(c_{11})$ = 3541.292

The other $E(c_{it})$'s are calculated similarly and are shown in table (4.10), as given below:

Table 4.10 Possibilistic Mean Values of Capacities required for Periods T_1 to T_6

	T_1	T_2	T_3	T_4	T_5	T_6
WC_1	3541.292	2750.338	1962.833	1567.646	868.083	647.438
WC_2	3423.279	2922.625	2283.917	1779.479	1050.583	586.833
WC_3	3410.729	2867.896	2180.042	1631.208	1019.458	629.542
WC_4	3465.438	2764.250	2084.563	1505.354	947.042	399.458

Capacities calculated above are the possibilistic mean values of the capacities required at different work centers (WC_1 to WC_4) for different time periods (T_1 to T_6).

Possibilistic mean values although are crisp numbers, they better predict the fuzziness involved due to any impreciseness or vagueness.

These capacities are compared with the available capacities to foresee whether the MPS is feasible or not? Hence, a timely step can be taken either in enhancing capacity of the work centers that are short in capacity or to revise the MPS.

4.5 Comparison and Interpretation of Results

The fuzzy capacities required for each work center for different time periods are calculated using resource profile approach of RCCP with possibilistic approach, and are depicted in Table (4.10). These capacities are crisp capacities in the form of possibilistic mean values of the fuzzy capacities required at different work centers for different time periods to satisfy the MPS.

The results computed by using same TFN's for resource profile and MPS, and using same input data but different method using multiplication by α-cuts and then calculating membership functions for each fuzzy number are depicted in tables (4.11) and (4.12), as given below:

Table 4.11 Capacities required in hours for Period T_1 to T_3

	T_1	T_2	T_3
WC_1	(1727, **3527.6**, 5656)	(1375.2, **2735.5**, 4429.5)	(793.5, **1963.5**, 3289)
WC_2	(1582.5, **3362.55**, 5812.1)	(1324.5, **2884**, 4878.75)	(979.25, **2293**, 3717.25)
WC_3	(1655.75, **3411**, 5405.4)	(1504.25, **2865**, 4478)	(1047.5, **2218**, 3300)
WC_4	(1733, **3402.8**, 5738.1)	(1388, **2717**, 4555.5)	(1022.25, **2065.5**, 3390)

Table 4.12 Capacities required in hours for Period T_4 to T_6

	T_4	T_5	T_6
WC_1	(659, **1606**, 2463.5)	(395.5, **844.5**, 1511.5)	(389, **651**, 920.5)
WC_2	(1827.75, **1786**, 2851.5)	(455, **1045.5**, 1773)	(211, **598.5**, 972.5)
WC_3	(688.5, **1683.5**, 2478.5)	(449.5, **1045.5**, 1569.5)	(283, **661**, 895)
WC_4	(656.75, **1509.5**, 2468)	(397.5, **955.5**, 1548.5)	(153, **396.5**, 703.5)

Comparing these with the results generated by possibilistic approach for RCCP using resource profile approach with same input data, one can easily come to a conclusion that the results are almost similar at $\alpha = 1$ (interior point of the TFN). An advantage of possibilistic approach is that it provides the final results in the form of crisp possibilistic mean values, which are easy to manipulate, while the other methods provide the final results in the form of fuzzy numbers.

The Resource Profile approach of RCCP is a detailed rough cut capacity planning technique since it includes the lead time amounts in it. It appears to be very useful for products having substantially long manufacturing lead times. The important benefit of the resource profile approach of RCCP is that it accommodates both product mix variations and manufacturing lead time offsets in preparing the capacity analysis plans.

As pointed out in Chapter 3, the approach suggested in this chapter also appears to be simple and practical. Possibilistic mean values of fuzzy capacities are crisp numbers and are easily workable on any computer software.

CHAPTER 5

CAPACITY REQUIREMENTS PLANNING UNDER FUZZY ENVIRONMENT USING POSSIBILITY THEORY

In the present chapter, capacity requirements planning (CRP) problem under fuzzy environment is considered using possibility theory approach. The problem considered in this chapter is different from the ones considered in Chapters 3 and 4 in the sense that the proposed approach includes the setup times per lot, run times per part, lot size of each part and number of parts to be manufactured/processed at certain specified work centers or key resources for planned order releases, and released orders. The crisp CRP approach to find the capacity requirements at certain key resources is extended to the fuzzy environment, where, the input data is in the form of setup times per lot, run time per piece and lot sizes for planned order releases and released orders. The MPS and inventory records are represented by triangular fuzzy numbers (TFN's). Under these assumptions, possibility theory approach is developed to tackle the computational tediousness of mathematical operations of TFN's. The approach can be extended further when the input data are represented by trapezoidal or some other type of fuzzy numbers.

5.1 Introduction

Capacity requirements planning (CRP) is the process to determine the machine and labor resources required to attain the production. Open shop orders and planned orders are inputs to the capacity requirements planning. CRP is applied to determine the capacity requirements after rough cut capacity planning has validated the MPS. CRP conceptually starts with explosion of MPS via MRP system. Planned orders are taken from the MRP system and used to perform a simulation that uses lead time offsets to determine the time each order passes through each Work Centre. The orders already released are included in the simulation. Based upon this simulation, a machine load report for each Work Centre is prepared and then compared to the capacity available at that Work Centre. Thus CRP verifies that there is sufficient capacity to process all planned orders released by MRP and orders already released within the planning horizon. This verification endorses the final acceptance of MPS, and CRP determines the load of each Work Centre during each period of that planning horizon.

5.2 Capacity Requirements Planning under Crisp Environment

Suppose, p number of parts are to be manufactured/processed over time buckets t, where $p = 1, 2, \ldots, n;\ \ t = 1, 2, \ldots, m$. Let the work centers (or key resources) whose capacity to be analyzed be represented by i where $i = 1, 2, \ldots, s$. It is assumed that Work Centers do not break down and are available throughout the planning horizon.

Let,

a_{ipt} = Setup time per lot at Work Center i for Part p in Period t.

b_{ipt} = Run time per piece at Work Center i for Part p in Period t.

L_{pt} = Lot size for Part p in Period t.

c_{it} = Capacity required at Work Center i for Period t.

P_p = Product p, where $\quad p = 1, 2, \ldots, n,$

WC_i = Work Center i, where $\quad i = 1, 2, \ldots, s,$

T_t = Period t, where $t = 1, 2, \ldots, m,$

In a crisp case, the a_{ipt}'s, b_{ipt}'s and L_{pt}'s can be represented in a tabular form shown below.

Table 5.1 Setup time per Lot in time units

	T_1	T_2	----	T_m
P_1	a_{i11}	a_{i12}	----	a_{i1m}
P_2	a_{i21}	a_{i22}	----	a_{i2m}
----	----	----	----	----
P_n	a_{in1}	a_{in2}	----	a_{nm}

Table 5.2 Run time per piece (.) Lot Size

	T_1	T_2	----	T_m
P_1	$b_{i11}L_{11}$	$b_{i12}L_{12}$	----	$b_{i1m}L_{1m}$
P_2	$b_{i21}L_{21}$	$b_{i22}L_{22}$	----	$b_{i2m}L_{2m}$
----	----	----	----	----
P_n	$b_{in1}L_{n1}$	$b_{in2}L_{n2}$	----	$b_{inm}L_{nm}$

Formulation for Capacity Requirements Calculation

Let

c_{it} = Capacity required at Work Centre i for time period t in time units

Then,

$$c_{it} = \sum_{p=1}^{n} \left(a_{ipt} + b_{ipt}\,(.)\,L_{pt} \right), \; i = 1, 2, \ldots, s; \; t = 1, 2, \ldots, m. \qquad (5.2.1)$$

Using equation (5.2.1), the required capacities at any key resource can be calculated. If the numbers a_{ipt}'s,b_{ipt}'s and L_{pt}'s for all values of i, p and t are crisp, then their product will also be crisp. Using Table (5.1), Table (5.2) and equation (5.2.1), one can obtain the values of all c_{it}'s in the form of following capacity table:

Table 5.3 Capacity required in time units

	T_1	T_2	----	T_m
WC_1	c_{11}	c_{12}	----	c_{1m}
WC_2	c_{21}	c_{22}	----	c_{2m}
----	----	----	----	----
WC_s	c_{s1}	c_{s2}	----	c_{sm}

5.3 Capacity Requirements Planning under Fuzzy Environment using Possibility Theory

To analyze the equation (5.2.1) under fuzzy environment, one can rewrite it as:

$$c_{it} = \sum_{p=1}^{n} \left(a_{ipt} + b_{ipt}\,(.)\,L_{pt} \right), \; i = 1, 2, \ldots, s; \; t = 1, 2, \ldots, m. \tag{5.3.1}$$

In equation (5.3.1), if at least one of the a_{ipt}'s, b_{ipt}'s or L_{pt}'s is a fuzzy number then their product would also be a fuzzy number (Kaufman and Gupta 1988, 1991). Hence, corresponding c_{it}'s will also be fuzzy numbers. It is important to mention here that in equation (5.3.1), though each of the value of c_{it} is a fuzzy number, it may not be necessarily a TFN. (Kaufman and Gupta 1988, 1991). As pointed out earlier, in general, the product of two TFN's is a parabolic fuzzy number.

It is now assumed that in equation (5.3.1), each of the a_{ipt}'s, b_{ipt}'s and L_{pt}'s for $i = 1, 2, \ldots, s,\;\; p = 1, 2, \ldots, n, t = 1, 2, \ldots, m,$ is a TFN of the type

$$a_{ipt} = (a_{ipt1}, a_{ipt2}, a_{ipt3}),\; b_{ipt} = (b_{ipt1}, b_{ipt2}, b_{ipt3}) \text{ and } L_{pt} = (L_{pt1}, L_{pt2}, L_{pt3}) \tag{5.3.2}$$

As in the previous chapters, the values of (5.3.2), can be obtained from expert opinion of individuals who have the same information source but different opinion about the same problem. As pointed out earlier, in place of only one crisp data /estimate for a_{ipt}'s, b_{ipt}'s and L_{pt}'s, one can have three estimates in the form of a TFN to account for uncertainty, based on the expert opinion or intuitiveness.

Hence, each c_{it} given by equation (5.3.1), is a fuzzy number given by

$$c_{it} = (c_{it1}, c_{it2}, c_{it3}) \quad \text{for each } i = 1, 2, \ldots, s, \ t = 1, 2, \ldots, m, \tag{5.3.3}$$

Each fuzzy number c_{it} in (5.3.3), can be represented by its α-cut obtained by using the α-cuts of a_{ipt}'s, b_{ipt}'s and L_{pt}'s (see Appendix 1, A.2).

Let,

$a_{ipt(\alpha)} = \left[a_{i\,pt1}(\alpha), a_{i\,pt2}(\alpha)\right]$ as α-cut for a_{ipt}'s, for $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n$, and $t = 1, 2, \ldots, m$, such that

$$a_{ipt(\alpha)} = \left[a_{ipt1} + \alpha\left(a_{ipt2} - a_{ipt1}\right),\ a_{ipt3} + \alpha\left(a_{ipt2} - a_{ipt3}\right)\right] \qquad \forall\ \alpha \in [0,1] \tag{5.3.4}$$

Similarly,

$b_{ipt(\alpha)} = \left[b_{i\,pt1}(\alpha), p_{i\,pt2}(\alpha)\right]$ as α-cut for b_{ipt}'s, $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n$, $t = 1, 2, \ldots, m$, such that

$$b_{ipt(\alpha)} = \left[b_{ipt1} + \alpha\left(b_{ipt2} - b_{ipt1}\right), b_{ipt3} + \alpha\left(b_{ipt2} - b_{ipt3}\right)\right] \qquad \forall\ \alpha \in [0,1] \tag{5.3.5}$$

and for all $i = 1, 2, \ldots, s, p = 1, 2, \ldots, n, t = 1, 2, \ldots, m,$

and,

$L_{pt(\alpha)} = \left[L_{pt1}(\alpha), L_{pt2}(\alpha)\right]$ as α-cut for L_{pt}'s, $p = 1, 2, \ldots, n$, $t = 1, 2, \ldots, m$, such that

$$L_{pt(\alpha)} = \left[L_{pt_1} + \alpha\left(L_{pt2} - L_{pt1}\right), L_{pt3} + \alpha\left(L_{pt2} - L_{pt3}\right)\right] \qquad \forall\ \alpha \in [0,1] \tag{5.3.6}$$

As in Carlson and Fuller (2001), using (5.3.4) and equations (2.3.2), (2.3.3) and (2.3.4), one can derive an expression for the crisp lower possibilistic mean value $E_L(a_{ipt})$, crisp upper possibilistic mean value $E_R(a_{ipt})$ and crisp possibilistic mean value $E(a_{ipt})$ respectively, for a fuzzy number $a_{ipt}(\alpha) = \left[a_{ipt1}(\alpha), a_{ipt2}(\alpha)\right], \alpha \in [0,1]$. It is important to mention here that $E_L(a_{ipt})$, $E_R(a_{ipt})$ and $E(a_{ipt})$ are crisp numbers.

Similar to that of Carlson and Fuller (2001), using (5.3.4) and equation (2.3.2), for $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n$, and $t = 1, 2, \ldots, m$, one can obtain

$$
\begin{aligned}
E_L(a_{ipt}) &= 2\int_0^1 \left\{a_{ipt1} + \alpha\left(a_{ipt2} - a_{ipt1}\right)\right\}\alpha d\alpha \\
&= 2\int_0^1 a_{ipt1}\alpha d\alpha + 2\int_0^1 \alpha\left(a_{ipt2} - a_{ipt1}\right)\alpha d\alpha \\
&= 2a_{ipt1}\int_0^1 \alpha d\alpha + 2\left(a_{ipt2} - a_{ipt1}\right)\int_0^1 \alpha^2 d\alpha \\
&= 2a_{ipt1}\left|\frac{\alpha^2}{2}\right|_0^1 + 2\left(a_{ipt2} - a_{ipt1}\right)\left|\frac{\alpha^3}{3}\right|_0^1 \\
&= a_{ipt1} + \frac{2}{3}\left(a_{ipt2} - a_{ipt1}\right) \\
&= \frac{1}{3}a_{ipt1} + \frac{2}{3}a_{ipt2}\ .
\end{aligned}
\tag{5.3.7}
$$

Similarly, from (5.3.4) and (2.3.3), for $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n$, and $t = 1, 2, \ldots, m$, one can obtain

$$
\begin{aligned}
E_R(a_{ipt}) &= 2\int_0^1 \left\{a_{ipt3} + \alpha\left(a_{ipt2} - a_{ipt3}\right)\right\}\alpha d\alpha \\
&= 2\int_0^1 a_{ipt3}\alpha d\alpha + 2\int_0^1 \alpha\left(a_{ipt2} - a_{ipt3}\right)\alpha d\alpha
\end{aligned}
$$

$$= 2a_{ipt3}\int_0^1 \alpha d\alpha + 2\left(a_{ipt2} - a_{ipt3}\right)\int_0^1 \alpha^2 d\alpha$$

$$= 2a_{ipt3}\left|\frac{\alpha^2}{2}\right|_0^1 + 2\left(a_{ipt2} - a_{ipt3}\right)\left|\frac{\alpha^3}{3}\right|_0^1$$

$$= a_{ip3} + \frac{2}{3}\left(a_{ip2} - a_{ip3}\right)$$

$$= \frac{1}{3}a_{ipt3} + \frac{2}{3}a_{ipt2}. \qquad (5.3.8)$$

Possibilistic mean value E(a_{ipt}) of a fuzzy number a_{ipt} for $i = 1, 2, \ldots, s,$ $p = 1, 2, \ldots, n,$ and $t = 1, 2, \ldots, m,$ is calculated from (2.3.4), (5.3.7) and (5.3.8) as under:

$$E(a_{ipt}) = \frac{\frac{1}{3}a_{ipt1} + \frac{2}{3}a_{ipt2} + \frac{1}{3}a_{ipt3} + \frac{2}{3}a_{ipt2}}{2}$$

$$E(a_{ipt}) = \frac{1}{6}a_{ipt1} + \frac{1}{6}a_{ipt3} + \frac{2}{3}a_{ipt2} \qquad (5.3.9)$$

Similarly, using (5.3.4), (5.3.5), (5.3.6); and equations (2.3.2), (2.3.3) and (2.3.4), one can derive an expression, for the lower possibilistic mean value $E_L(a_{ipt} + b_{ipt} \,.\, L_{pt})$, upper possibilistic mean value $E_R(a_{ipt} + b_{ipt} \,.\, L_{pt})$ and possibilistic mean value $E(a_{ipt} + b_{ipt} \,.\, L_{pt})$, for each combination of $(a_{ipt} + b_{ipt} \,.\, L_{pt})$'s, similar to those of equation (5.3.1), as follows:

Using (5.3.4), (5.3.5), (5.3.6) and equation (2.3.4), one can obtain

$E_L(a_{ipt} + b_{ipt} \,.\, L_{pt}) =$

$$2\int_0^1 \left[\left\{a_{ipt1} + \alpha\left(a_{ipt2} - a_{ipt1}\right)\right\} + \left\{b_{ipt1} + \alpha\left(b_{ipt2} - b_{ipt1}\right)\right\}\left\{L_{pt_1} + \alpha\left(L_{pt2} - L_{pt1}\right)\right\}\right]\alpha d\alpha$$

$$= \frac{1}{2}b_{ipt2}L_{pt2} + \frac{1}{6}b_{ipt2}L_{pt1} + \frac{1}{6}b_{ipt1}L_{pt2} + \frac{1}{6}b_{ipt1}L_{pt1} + \frac{2}{3}a_{ipt2} + \frac{1}{3}a_{ipt1} \qquad (5.3.10)$$

Similarly, using (5.3.4), (5.3.5), (5.3.6) and (2.3.5), one can obtain

$E_R(a_{ipt} + b_{ipt} \,.\, L_{pt}) =$

$$2\int_0^1 \left[\left\{a_{ipt3} + \alpha\left(a_{ipt2} - a_{ipt3}\right)\right\} + \left\{b_{ipt3} + \alpha\left(b_{ipt2} - b_{ipt3}\right)\right\}\left\{L_{pt3} + \alpha\left(L_{pt2} - L_{pt3}\right)\right\}\right]\alpha d\alpha$$

$$= \frac{1}{2}b_{ipt2}L_{pt2} + \frac{1}{6}b_{ipt2}L_{pt3} + \frac{1}{6}b_{ipt3}L_{pt2} + \frac{1}{6}b_{ipt3}L_{pt3} + \frac{2}{3}a_{ipt2} + \frac{1}{3}a_{ipt3} \qquad (5.3.11)$$

Possibilistic mean value $E(a_{ipt} + b_{ipt} \,.\, L_{pt})$ for $i = 1, 2, \ldots, s$, $p = 1, 2, \ldots, n$, and $t = 1, 2, \ldots, m$,, can now be deduced from (5.3.10), (5.3.11) and (2.3.6), as follows:

$$E(a_{ipt} + b_{ipt} \,.\, L_{pt}) = \frac{\frac{1}{2}b_{ipt2}L_{pt2} + \frac{1}{6}b_{ipt2}L_{pt1} + \frac{1}{6}b_{ipt1}L_{pt2} + \frac{1}{6}b_{ipt1}L_{pt1} + \frac{2}{3}a_{ipt2} + \frac{1}{3}a_{ipt1}}{2} + \frac{\frac{1}{2}b_{ipt2}L_{pt2} + \frac{1}{6}b_{ipt2}L_{pt3} + \frac{1}{6}b_{ipt3}L_{pt2} + \frac{1}{6}b_{ipt3}L_{pt3} + \frac{2}{3}a_{ipt2} + \frac{1}{3}a_{ipt3}}{2}$$

$$E(a_{ipt} + b_{ipt} \,.\, L_{pt}) = \begin{array}{l} \frac{1}{2}b_{ipt2}L_{pt2} + \frac{1}{12}b_{ipt2}L_{pt1} + \frac{1}{12}b_{ipt1}L_{pt2} + \frac{1}{12}b_{ipt1}L_{pt1} + \frac{1}{12}b_{ipt2}L_{pt3} \\ + \frac{1}{12}b_{ipt3}L_{pt2} + \frac{1}{12}b_{ipt3}L_{pt3} + \frac{2}{3}a_{ipt2} + \frac{1}{6}a_{ipt1} + \frac{1}{6}a_{ipt3} \end{array} \qquad (5.3.12)$$

Using equation (5.3.12), one can calculate, the possibilistic mean values for all the combinations of $(a_{ipt} + b_{ipt} \,.\, L_{pt})$'s, similar to that of (5.3.1). The capacity required at any key resource or Work Center under fuzzy environment for a period is calculated by summation of crisp possibilistic mean values of $(a_{ipt} + b_{ipt} \,.\, L_{pt})$'s *to* find the crisp possibilistic mean values of the corresponding $E(c_{it})$'s, as given below:

$$E(c_{it}) = \sum_{p=1}^{n} E(a_{ipt} + b_{ipt} \,.\, L_{pt}),\ i = 1, 2, \ldots, s;\ t = 1, 2, \ldots, m. \qquad (5.3.13)$$

The $E(c_{it})$ calculated above is the crisp capacity requirement at work center *i* for period *t* under fuzzy environment. Thus using equation (5.3.13), one can calculate the capacity requirement at work center *i* for period *t* under fuzzy environment.

5.4 Numerical Example of Capacity Requirements Calculation under Fuzzy Environment using Possibility Theory

In this section, a numerical example is considered to illustrate the approach suggested. Let's consider three key resources (Work Centers) WC_1, WC_2, and WC_3; four parts P_1, P_2, P_3 and P_4 of a product ABC (say), six time periods T_1, T_2, T_3, T_4, T_5 and T_6. It is assumed that the setup time per lot and run time per piece for planned order releases and released orders are measured in minutes and lot size is measured in product units. The following TFN's in Tables (5.4) and (5.5), represent item master record files and routing files data respectively, for ABC. Tables (5.6) and (5.7) represents planned order releases for four parts extracted by exploding MPS by MRP system in product units for periods T_1 to T_3 and T_4 to T_6 respectively. This input data in the form of TFN's (or any other type of fuzzy numbers) can be generated randomly. As before, the data from Verma, (2001) is utilized, for comparison and verification of results, as follows:

Table 5.4 Item Master Record Files for (ABC)

Part	Order Quantity	On Hand	On Order	Due Date	Lead Time
P_1	(190, 200, 205)	(90, 100, 105)	(190, 200, 205)	First week	1 week
P_2	(395, 400, 410)	(395, 400, 410)	(395, 400, 410)	Second week	2 week
P_3	(2395, 2400, 2410)	(1490, 1500,1505)	(2395, 2400, 2410)	Second week	3 week
P_4	(5995, 6000, 6010)	(2490, 2500, 2505)	(5995, 6000, 6010)	Second week	4 week

Table 5.5 Routing Files for (ABC)

Part	Work Center	Setup Time per Lot (minutes)	Run Time per Piece(minutes)
P_1	1	(25, 30, 45)	(2, 2.5, 2.7)
P_2	2	(8, 10, 25)	(.6, .75, .8)
P_2	1	(5, 15, 35)	(.4, .5, .7)
P_3	3	(10, 15, 30)	(.2, .3, .35)
P_3	1	(10, 25, 30)	(.2, .25, .40)
P_3	2	(12, 15, 25)	(.15, .25, .3)
P_4	2	(20, 25, 45)	(.7, .75, .85)
P_4	3	(10, 30, 40)	(.1, .15, .3)
P_4	1	(50, 75, 90)	(.3, .5, .6)
P_4	3	(20, 30, 35)	(.3, .35, .45)

Table 5.6 Planned Order Releases for Week 1 to Week 3

Part	Week 1	Week 2	Week 3
P_1	(190, 200, 205)	(190, 200, 205)	(190, 200, 205)
P_2	(395, 400, 410)	(395, 400, 410)	(395, 400, 410)
P_3	(2395, 2400, 2410)	----	(2395, 2400, 2410)
P_4	(5995, 6000, 6010)	----	----

Table 5.7 Planned Order Releases for Week 4 to Week 6

Part	Week 4	Week 5	Week 6
P_1	(190, 200, 205)	(190, 200, 205)	(190, 200, 205)
P_2	(395, 400, 410)	(395, 400, 410)	(395, 400, 410)
P_3	----	(2395, 2400, 2410)	(2395, 2400, 2410)
P_4	(5995, 6000, 6010)	----	----

The setup time per lot table (Table 5.8) is produced directly for planned order releases of the MRP system.

Table 5.8 Setup Time per lot computation for Planned Order Releases (in minutes).

	Part	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
WC_1	P_1	(25, 30, 45)	(25, 30, 45)	(25, 30, 45)	(25, 30, 45)	(25, 30, 45)	(25, 30, 45)
	P_2	----	(5, 15, 35)	(5, 15, 35)	(5, 15, 35)	(5, 15, 35)	(5, 15, 35)
	P_3	----	(10, 25, 30)	----	(10, 25, 30)	----	(10, 25, 30)
	P_4	----	----	(50, 75, 90)	----	----	(50, 75, 90)
WC_2	P_1	----	----	----	----	----	----
	P_2	(8, 10, 25)	(8, 10, 25)	(8, 10, 25)	(8, 10, 25)	(8, 10, 25)	(8, 10, 25)
	P_3	----	----	(12, 15, 25)	----	(12, 15, 25)	----
	P_4	(20, 25, 45)	----	----	(20, 25, 45)	----	----
WC_3	P_1	----	----	----	----	----	----
	P_2	----	----	----	----	----	----
	P_3	(10, 15, 30)	----	(10, 15, 30)	----	(10, 15, 30)	(10, 15, 30)
	P_4	----	(10, 30, 40)	----	(20, 30, 35)	(10, 30, 40)	----

The run time per piece table (Table 5.9) is also produced directly for planned order releases of the MRP system.

Table 5.9 Run time per piece computation for Planned Order Releases (in minutes)

	Part	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
WC_1	P_1	(2, 2.5, 2.7)	(2, 2.5, 2.7)	(2, 2.5, 2.7)	(2, 2.5, 2.7)	(2, 2.5, 2.7)	(2, 2.5, 2.7)
	P_2	----	(.4, .5, .7)	(.4, .5, .7)	(.4, .5, .7)	(.4, .5, .7)	(.4, .5, .7)
	P_3	----	(.2, .25, .4)	----	(.2, .25, .4)	----	(.2, .25, .4)
	P_4	----	----	(.3, .5, .6)	----	----	(.3, .5, .6)
WC_2	P_1	----	----	----	----	----	----
	P_2	(.6, .75, .8)	(.6, .75, .8)	(.6, .75, .8)	(.6, .75, .8)	(.6, .75, .8)	(.6, .75, .8)
	P_3	----	----	(.15, .25, .3)	----	(.15, .25, .3)	----
	P_4	(.7, .75, .85)	----	----	(.7, .75, .85)	----	----
WC_3	P_1	----	----	----	----	----	----
	P_2	----	----	----	----	----	----
	P_3	(.2, .3, .35)	----	(.2, .3, .35)	----	(.2, .3, .35)	(.2, .3, .35)
	P_4	----	(.1, .15, .3)	----	(.3, .35, .45)	(.1, .15, .3)	----

Using equation (5.3.12), one can calculate the possibilistic mean values of the above mentioned TFN's in the form of a_{ipt}'s, b_{ipt}'s and L_{pt}'s given in the tables (5.8), (5.9) and (5.6) & (5.7) respectively, for all the combinations of (a_{ipt} + b_{ipt} . L_{pt})'s, similar to that of equation (5.3.1), for planned order releases. The possibilistic mean values for all the a_{ipt}'s, b_{ipt}'s and L_{pt}'s given in the tables (5.8), (5.9) and (5.6) & (5.7) respectively, are calculated as explained above and arranged in table (5.10), for planned order releases as follows:

Table 5.10 Possibilistic Mean Values for Planned Order Releases (in minutes)

	Part	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
WC_1	P_1	520.083	520.083	520.083	520.083	520.083	520.083
	P_2	----	223.958	223.958	223.958	223.958	223.958
	P_3	----	663.688	----	663.688	----	663.688
	P_4	----	----	2973.917	----	----	2973.917
	Total	520.083	1407.729	3717.958	1407.729	744.042	4381.646
WC_2	P_1	----	----	----	----	----	----
	P_2	306.229	306.229	306.229	306.229	306.229	306.229
	P_3	----	----	596.458	----	596.458	----
	P_4	4578.229	----	----	4578.229	----	----
	Total	4884.458	306.229	902.688	4884.458	902.688	306.229
WC_3	P_1	----	----	----	----	----	----
	P_2	----	----	----	----	----	----
	P_3	717.000	----	717.000	----	717.000	717.000
	P_4	----	1028.604	----	2179.563	1028.604	----
	Total	717.000	1028.604	717.000	2179.563	1745.604	717.000

The capacity required at any key resource or Work Center under fuzzy environment for a period is calculated by summation of crisp possibilistic mean values of (a_{ipt} + b_{ipt} . L_{pt})'s to find the crisp possibilistic mean values of the corresponding (E(c_{it})'s), as per equation (5.3.13). The row (shown as total for each Work Center in table 5.10) represents the summation of the crisp possibilistic mean value of (a_{ipt} + b_{ipt} . L_{pt})'s and yields the crisp possibilistic mean value of the corresponding (E(c_{it})'s), as in equation (5.3.13), for planned order releases. The E(c_{it})'s calculated above are the crisp capacity requirements under fuzzy environment at each Work Center for a given time period for the planned order releases from the MRP system. Using equation (5.3.13) and table (5.10), the calculation of E(c_{11}), for WC_1 ,T_1, for $p = 1, 2, \ldots, 4$, for planned order releases is demonstrated as follows:

$$E(c_{11}) = \sum_{p=1}^{4}\left(a_{1p1} + b_{1p1}\,(.)\,L_{p1}\right)$$

$$= \left[E\left(a_{111} + b_{111}\,.L_{11}\right) + E\left(a_{121} + b_{121}\,.L_{21}\right) + E\left(a_{131} + b_{131}\,.L_{31}\right) + E\left(a_{141} + b_{141}\,.L_{41}\right)\right]_P$$

$$= 520.083 + 0 + 0 + 0$$

$$= 520.083$$

The E(c_{11}) calculated above is the crisp capacity requirement under fuzzy environment at work center 1 for period 1 for planned order releases c_{11}. The other E(c_{it})'s for planned order releases are calculated on similar lines using table (5.10), and are shown in Table (5.11), given below:

Table 5.11 Possibilistic Mean Values of Fuzzy Capacities required at Work Center *i* for Period *t* for Planned Order Releases

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
WC_1	520.083	1407.729	3717.958	1407.729	744.042	4381.646
WC_2	4884.458	306.229	902.688	4884.458	902.688	306.229
WC_3	717.000	1028.604	717.000	2179.563	1745.604	717.000

Now, one can account for the orders already released to the shop. According to Table (5.4), four orders (190, 200, 205), (395, 400, 410), (2395, 2400, 2410) and (5995, 6000, 6010) have already been released to the shop. These four orders are all on schedule, that is, the number of operations remaining to be completed is equal to the number of weeks remaining until due date. Part P_1 is due on first week and has one operation (the last operation) left to be completed; the other three parts are due on second week and have two operations (the final two) to be completed. The following table, (Table 5.12), depicts the TFN's for the setup time per lot, run time and lot size for the released orders.

Table 5.12 Setup time per Lot, Run time and Lot size for the Released Orders.

Part	WC	Week	Setup time	Run time	Lot size
P_1	1	1	(25, 30, 45)	(2, 2.5, 2.7)	(190, 200, 205)
P_2	2	1	(8, 10, 25)	(.6, .75, .8)	(395, 400, 410)
P_2	1	2	(5, 15, 35)	(.4, .5, .7)	(395, 400, 410)
P_3	1	1	(10, 25, 30)	(.2, .25, .40)	(2395, 2400, 2410)
P_3	2	2	(12, 15, 25)	(.15, .25, .3)	(2395, 2400, 2410)
P_4	1	1	(50, 75, 90)	(.3, .5, .6)	(5995, 6000, 6010)
P_4	3	2	(20, 30, 35)	(.3, .35, .45)	(5995, 6000, 6010)

Using equation (5.3.12), one can calculate the possibilistic mean values of the above mentioned TFN's in the form of a_{ipt}'s, b_{ipt}'s and L_{pt}'s given in Table (5.12), for all the combinations of $(a_{ipt} + b_{ipt} \,.\, L_{pt})$'s, similar to the ones of equation (5.3.1), for released orders. The possibilistic mean values for all the a_{ipt}'s, b_{ipt}'s and L_{pt}'s given in the table

(5.12), are calculated as explained above and are arranged in the last column of table (5.13), for released orders, as follows:

Table 5.13 Possibilistic Mean Values of the Setup time per Lot, Run time and Lot size for the Released Orders.

Part	WC	Week	Setup time	Run time	Lot size	Possibilistic Mean Values
P_1	1	1	(25, 30, 45)	(2, 2.5, 2.7)	(190, 200, 205)	520.083
P_2	2	1	(8, 10, 25)	(.6, .75, .8)	(395, 400, 410)	306.229
P_2	1	2	(5, 15, 35)	(.4, .5, .7)	(395, 400, 410)	223.958
P_3	1	1	(10, 25, 30)	(.2, .25, .40)	(2395, 2400, 2410)	663.688
P_3	2	2	(12, 15, 25)	(.15, .25, .3)	(2395, 2400, 2410)	596.458
P_4	1	1	(50, 75, 90)	(.3, .5, .6)	(5995, 6000, 6010)	2973.917
P_4	3	2	(20, 30, 35)	(.3, .35, .45)	(5995, 6000, 6010)	2179.563

The capacity required at any key resource or Work Center under fuzzy environment for a period is calculated by summation of crisp possibilistic mean values of $(a_{ipt} + b_{ipt} \,.\, L_{pt})$'s to find the crisp possibilistic mean values of the corresponding $E(c_{it})$'s, as per equation (5.3.13), for released orders. The $E(c_{it})$'s calculated above are the crisp capacity requirements under fuzzy environment at each Work Center for a given time period for the released orders from the MRP system. Using equation (5.3.13) and table

(5.13), the calculation of E(c_{11}), for WC_1 and T_1 for $p = 1, 2, \ldots, 4,$ for released orders is demonstrated as follows:

$$E(c_{11}) = \sum_{p=1}^{4}\left(a_{1p1} + b_{1p1}(.)L_{p1}\right)$$

$$=\left[\left(a_{111} + b_{111}(.)L_{11}\right)+\left(a_{121} + b_{121}(.)L_{21}\right)+\left(a_{131} + b_{131}(.)L_{31}\right)+\left(a_{141} + b_{141}(.)L_{41}\right)\right].$$

$$= 520.083 + 0 + 663.688 + 2973.917$$

$$= 4157.688$$

The E(c_{11}) calculated above is the crisp capacity requirement under fuzzy environment at work center 1 for period 1 for released orders. The other E(c_{it})'s for released orders are calculated on similar lines using table (5.13), and are given in Table (5.14), as below:

Table 5.14 Possibilistic Mean Values of Fuzzy Capacities Required at each Work Center for the Released Orders

Work Center	Week 1	Week 2
WC_1	4157.688	223.958
WC_2	306.229	596.458
WC_3	----	2179.563

The fuzzy capacities required by planned order releases as per Table (5.11), are now added to the fuzzy capacities required by the orders already released to the shop as per Table (5.14), to calculate the total fuzzy capacity requirement plan as in Table (5.15).

Table 5.15 Possibilistic Mean Values of Capacity Requirement Plan for Work Center *i* for Period *t*

Work Center	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
WC_1	4677.771	1631.688	3717.958	1407.729	744.042	4381.646
WC_2	5190.688	902.688	902.688	4884.458	902.688	306.229
WC_3	717.000	3208.167	717.000	2179.563	1745.604	717.000

Capacities calculated above using possibilistic approach are the possibilistic mean values of the capacities required at different work centers (WC_1 to WC_3) for different time periods (T_1 to T_6) incorporating planned order releases and released orders. These capacities are compared with the available capacities to permit planning by foreseeing and expanding those resources that may be short in capacity, in a timely fashion. If available capacity is insufficient, either available capacity must be increased or the schedule must be revised until available capacity is sufficient.

5.5 Comparison and Interpretation of Results

The fuzzy capacity requirement plan for planned order releases and released orders for the numerical example considered above using possibilistic approach is depicted in table (5.15). This capacity requirement plan in the form of crisp possibilistic mean values is required for different Work Centers to satisfy the MPS. Possibilistic mean values are crisp numbers, yet they better predict the fuzziness involved due to any vagueness or imprecision.

The fuzzy capacity requirement plan computed by using same TFN's for different parameters considered above that utilizes the same input data but employs different methods involving tedious membership function calculation, is presented below:

Table 5.16 Fuzzy Capacities required for Period T_1 to T_3

	T_1	T_2	T_3
WC_1	(3147.5, **4760**, 5887)	(1220, **1585**, 2236.5)	(2416.5, 3820, 4616.5)
WC_2	(4706.5, **5145**, 5859.5)	(616.25, **925**, 1101)	(616.25, **925**, 1101)
WC_3	(489, **735**, 873.5)	(2428, **3060**, 4582.5)	(489, **735**, 873.5)

Table 5.17 Fuzzy Capacities required for Period T_4 to T_6

	T_4	T_5	T_6
WC_1	(1057, **1370**, 1914.5)	(568, **745**, 920.5)	(2905.3, **4445**, 5610.5)
WC_2	(4461.5, **4835**, 5506.5)	(616.25, **925**, 1101)	(245, **310**, 353)
WC_3	(1818.5, **2130**, 2739.5)	(1098.5, **1665**, 2716.5)	(489, **735**, 873.5)

Comparing the results generated by using possibilistic approach with the results generated by an alternative method adopted by Verma, 2001, that involves complex membership functions, one can easily observe that the results are very similar. Moreover the traditional methods using membership functions generate the results in the form of fuzzy numbers but the possibilistic approach suggested in this chapter, generates the results in the form of crisp possibilistic mean values. As compared to fuzzy numbers it is easy to work with their crisp possibilistic mean values. Calculation of membership functions of each and every fuzzy number is time consuming. Furthermore, possibilistic mean values are crisp numbers and can be easily implemented on any computer software.

The capacity requirements planning (CRP) is more detailed than the RCCP techniques of bill of labor and resource profile approach because RCCP is generally interpreted using average capacity. On the other hand CRP examines cumulative capacity and time buckets for RCCP are generally large and plans often by month or week, whereas, CRP plans by week, day or even hourly. RCCP shows sufficient average capacity but does not contain sufficient information regarding timing of order releases for component items for accurate cumulative capacity requirements. CRP, on the other hand uses exact order release data from the MRP system.

The possibilistic approach suggested in this Chapter not only appears to be simple and practical, but also in comparison to other methods it generates the results in crisp form representing fuzziness, which are easy to interpret and manipulate.

CHAPTER 6

CONTRIBUTION, CONCLUSION AND RECOMMENDATIONS

In the present chapter, the contribution and conclusions made in this thesis are presented briefly. Finally, some recommendations for further research on the problems considered in this dissertation are suggested.

6.1 Contribution and Conclusion

In the present dissertation, an important problem in the area of Capacity Planning in Industrial Engineering addressed by Fogarty et al. (1991), Verma (2001), Pai et al. (2004), Mula et al. (2008), has been re-examined. In this thesis, three problems have been addressed

1. Capacity planning problem under fuzzy environment using bill of labor approach for RCCP.
2. Resource profile approach under fuzzy environment including lead time offsets.
3. Capacity requirements planning under fuzzy environment that includes fuzzy setup times per lot, fuzzy run time per part and fuzzy lot of each part.

The solution technique used to solve the above problems is the possibilistic approach. It is observed that the methods presented in this thesis are computationally simple in analyzing and determining the satisfactory and flexible solution to the capacity planning problem. The results obtained in this thesis using possibility theory, fuzzy sets and fuzzy systems, provide more satisfactory solution and flexibility in the form of range of estimates accounting for uncertainty and vagueness in Capacity Planning problem. The methods suggested can be easily implemented using any simple computer software (for example MS Excel). Furthermore, the approach suggested is simple in dealing with the complex mathematical operations of the fuzzy numbers. As compared to fuzzy numbers it is easy to work with their crisp possibilistic mean values, instead of calculating the complex membership functions of each and every fuzzy number which is not only time consuming but also involves tedious mathematical computation. Possibilistic mean values although are crisp numbers, predict the fuzziness involved due to any imprecision or vagueness more easily.

Furthermore, main contributions of this thesis have been included in Chapters 3 to 5. To summarize, in Chapter 3, the rough cut capacity requirements for various key resources for different time periods under fuzzy environment using bill of labor (BOL) approach has been discussed. Assuming that the data for both BOL and master production schedule (MPS) is known in the form of triangular fuzzy numbers (TFN's), the required capacity in the form of crisp possibilistic mean values has been calculated. The rough cut capacity requirements for various key resources for different time periods under fuzzy environment by including the lead-time dimension has been considered in Chapter 4. Considering that the data for both the resource profile and master production schedule (MPS) is known in the form of triangular fuzzy numbers (TFN's), the required capacities calculated in the form of crisp possibilistic mean values. In Chapter 5, the problem of capacity requirements planning has been discussed for various key resources for different time periods under fuzzy environment by assuming that the data for setup time per lot, run time per piece and lot size of planned order releases and released orders is known in the form of triangular fuzzy numbers (TFN's), and the required capacity is calculated in the form of crisp possibilistic mean values. Furthermore, at the end of each chapter the results obtained by the suggested possibilistic approach are compared with the results obtained by other methods using complex membership functions to prove the utility of the suggested possibilistic approach.

6.2 Recommendations for Future Research

The results developed in this thesis can be applied to a number of problems and a number of extensions are possible to the capacity planning problems. The results of Chapters 3 to 5 can be extended to the case when the capacity input data is in the form of trapezoidal, LR-fuzzy numbers or O(m, n) fuzzy numbers. The results deduced in the form of possibilistic mean values can be further improved in the form of weighted possibilistic mean values.

The approach suggested in this thesis can be used in a number of engineering and financial problems, as follows

- In Group Technology (GT) setup under fuzzy environment, where data for processing and setup times of the parts is in the form of fuzzy numbers, the minimum processing time in the form of fuzzy numbers can be determined. The optimal routing for the parts considered can also be obtained.
- In linear programming (LP) models when uncertain demand and input data, and imprecise setup resources and capacity constraints are present, planning and scheduling can be generated.

- In financial management for call price options as well as determining the master investment schedule, when resources in the form of available funds are imprecise.

It is believed that approach suggested in this thesis will initiate further research in a number of areas (fuzzy statistics and its applications in industry) where input data and information is imprecise.

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Appendix 1

FUZZY LOGIC AND FUZZY SETS THEORY

As explained by Liberatore (2002), fuzzy logic is not fuzzy thinking. Zadeh (1965), introduced fuzzy sets to represent knowledge that is vague or imprecise, that is, “fuzzy.” In a classical set theory, an element either is or is not a member of a set. In contrast to the “crisp” boundaries of classical sets, fuzzy sets allow degree of membership in a set, as expressed by a number between 0 and 1.

In this appendix, some of the basic terminology, notation, definitions and pre-requisites of fuzzy sets theory are introduced. Theory of fuzzy sets is basically a theory of graded concepts (Zimmerman, 1991).

A.1 Fuzzy Set

Let X be a classical set of objects, called the universe, whose generic elements are denoted by X. The membership in a crisp subset A of X is viewed as characteristic function $\mu_A(x)$ from X to $\{0,1\}$ such that

$$\mu_A(x) = \begin{cases} 0 & for \;\; x \notin A \\ 1 & for \;\; x \in A \end{cases}$$

Where $\{0,1\}$ is called a valuation set (Dubois and Prade, (1980), Zadeh, (1996)).

If the valuation set is allowed to be the closed real interval $[0, 1]$, then A is called a fuzzy set as proposed by Zadeh (1996).

$\mu_A(x)$ is the degree of membership of x in A, the closer the value of $\mu_A(x)$ is to 1, the more x belongs to A. Therefore, a fuzzy set A is completely characterized by the set of ordered pairs:

A = $\{(x, \mu_A(x) \;\; x \in X\}$ where $\mu_A(x)$ maps X to the membership space $[0, 1]$.

Elements with zero degree of membership are usually not listed. If $\sup_{x \in A} \mu_A(x) = 1, \forall\, x \in R$, then the fuzzy set A is called a normal fuzzy set in R. A fuzzy set that is not normal is called subnormal fuzzy set.

A.2 α - Cut or α -Level Set

One of the most important properties of fuzzy sets is the concept of α-cut or α-level set. An α - cut denoted by A(α) is the crisp set of elements $x \in R$ whose degree of belonging to the fuzzy set A is at least $\alpha \in [0, 1]$.

This means A(α) $= \{x \in X \mid \mu(x) \geq \alpha, \alpha \in [0, 1]\}$. The α-cut is the crisp set A(α) that contains all elements of the universal set X $\in R$ whose membership grades in A are greater than or equal to the specified value of α, $\alpha \in [0, 1]$.

A.3 Support of a Fuzzy Set

Let A be a fuzzy set in X. Then the support of A, denoted by S(A), is the crisp set given by:

S(A) = $\{x \in X : \mu_A(x) > 0\}$.

If $\mu_A(x)$ is constant over S(A), then A is non-fuzzy.

A.4 Intersection of Fuzzy Sets

Intersection of two fuzzy sets A and B is a fuzzy set C denoted by C = A $\cap$ B, whose membership function is related to those of A and B by:

$$\mu_c(x) = \text{Min}\left[\mu_A(x),\ \mu_B(x)\right]\ \forall\, x \in X.$$

A.5 Algebraic Operations on Fuzzy Sets

In addition to the set theoretical operations, one can also can define a number of combinations of fuzzy sets and relate them to one another. Here some more important operations among them are presented.

1. The algebraic sum of two fuzzy sets A and B is A + B, whose membership function is defined as:

$$\mu_{A+B}(x) = \mu_A(x)\ (+)\ \mu_B(x),\ \forall\, x \in X.$$

$$\text{Provided}\ \ \mu_A(x)\ (+)\ \mu_B(x)\ \leq\ 1,\ \forall\, x \in X.$$

2. The absolute difference $|A - B|$, of A and B is given by

$$\mu_{|A-B|}(x)\ =\ \left|\mu_A(x)(-)\mu_B(x)\right|\quad x \in X.$$

3. The algebraic product of two fuzzy sets A and B, is A (.) B, whose membership function is:

$$\mu_{A(.)B}(x)\ = \mu_A(x)\ (.)\ \mu_B(x),\ \forall\, x \in X.$$

A.6 Convexity of Fuzzy Sets

The notion of convexity can be extended to fuzzy sets in such a way as to preserve many of the properties that it has in case of crisp sets. In what follows, one assumes that

X is the n-dimensional space R^n. The following two definitions of convexity of a fuzzy set can be formulated.

A fuzzy set A, is convex if and only if the sets $A_\alpha = \{x \in X \,|\mu_A(x) \geq \alpha\}$ for all $\alpha \in [0,1]$ is a convex set. The second definition of convexity of a fuzzy set is as follows:

A fuzzy set A is said to be convex set if,

$\mu(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu(x_1), \mu(x_2)), x_1, x_2 \in X \quad \alpha \in [0,1]$, i.e. if $\mu(x)$ is a quasi-concave function on X.

A.7 Fuzzy Number

The definition of the convex fuzzy set allows us the following definitions of a fuzzy number:

A fuzzy number A, is a fuzzy set on the real line R, which possesses the following properties:

1. A is normal convex set on R.
2. The α-cut A_α is a closed interval for every $\alpha \in [0,1]$.
3. The support of A, S(A) $= \{x \in X : \mu_A(x) > 0\}$, is bounded.

A.8 Fuzzy Arithmetic

Fuzzy arithmetic is based on two properties of fuzzy numbers:

- Each fuzzy set and thus, each fuzzy number can be fully and uniquely represented by its α-cuts or α-level sets.
- α- Cuts of each fuzzy number are closed intervals of real numbers for all $\alpha \in [0,1]$.

A fuzzy number can be characterized by an interval of confidence at level α, (Kaufmann and Gupta, 1988, 1991), as follows:

$A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$ which has the property:

$\alpha \le \alpha' \Rightarrow A_{\alpha'} \subset A_\alpha$

These properties enable one to define an arithmetic operation on fuzzy numbers in terms of arithmetic operations on their α-cuts or α-level sets (i.e. arithmetic operation on closed intervals).

A.9 Fuzzy Arithmetic Based on Operations on Closed Intervals

Let $A = [a, b] \in R$ and $B = [c, d] \in R$ be two fuzzy intervals, then according to

Kaufmann and Gupta, (1988, 1991), arithmetic operations on them are as follows:

- **Addition** A + B = $[a + c, b + d]$
- **Subtraction** A – B = $[a - d, b - c]$
- **Multiplication** A × B = $[Min(ac, ad, bc, bd), Max(ac, ad, bc, bd)]$
- **Division** A / B = $\left[Min\left(\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}\right), Max\left(\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}\right)\right]$
- **Inverse** of A $A^{-1} = \left[Min\left(\frac{1}{a}, \frac{1}{b}\right), Max\left(\frac{1}{a}, \frac{1}{b}\right)\right]$

Let A and B be two fuzzy numbers such that $A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$ is the α-cut of A and $B_\alpha = \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right]$ is the α-cut of B. Let * denote any of the arithmetic operations +, -, $\times$, $/$, or inverse on fuzzy numbers. Then we define a fuzzy set A * B in R, by defining its α-cuts $(A * B)_\alpha$ as $(A * B)_\alpha = A_\alpha * B_\alpha$ for any $\alpha \in [0,1]$. Since $(A * B)_\alpha$ is a closed interval for each $\alpha \in [0,1]$ and A and B are fuzzy numbers, A * B is also a fuzzy number.

A.10 Triangular Fuzzy Number

A triangular fuzzy number (TFN), A, can be represented completely by a triplet $A = (a_1, a_2, a_3)$, whose membership function is defined as follows

$$\mu_A(x) = \begin{cases} 0 & x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \le x \le a_3 \\ 0 & x \ge a_3 \end{cases}$$

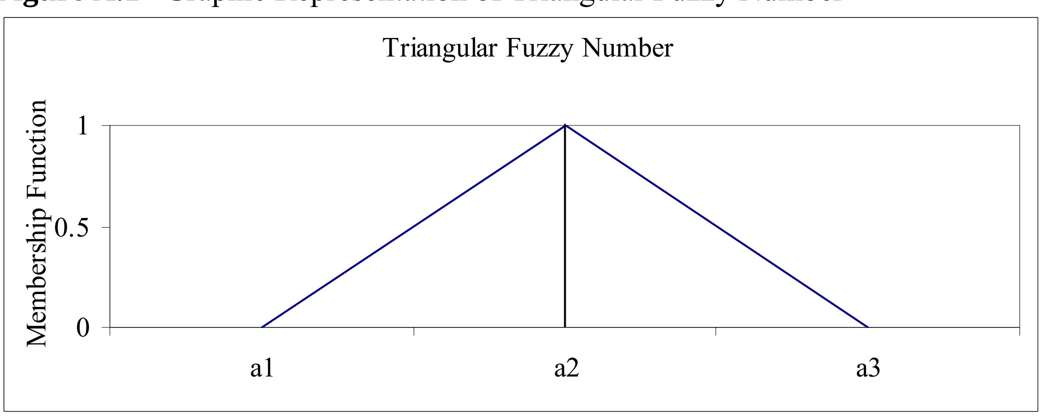
According to (Kaufmann and Gupta, (1988, 1991)), the interval of confidence at level-α as:

$$A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right],$$

We characterize the TFN, $A = (a_1, a_2, a_3)$ as

$$A_\alpha = \left[a_1 + \alpha(a_2 - a_1), a_3 + \alpha(a_2 - a_3)\right] \quad \forall \alpha \in [0,1]$$

Figure A.1 Graphic Representation of Triangular Fuzzy Number



Algebraic operations on TFN's

Let A = (a_1, a_2, a_3) and B = (b_1, b_2, b_3) be two TFN's, Then

- **Addition** $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- **Subtraction** $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

For the following two operations, we assume that a_i and b_i, i = 1, 2, 3, are positive.

- **Multiplication** $A\,(.)\,B = (a_1b_1, a_2b_2, a_3b_3)$
- **Division** $A / B = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$

A.11 Trapezoidal Fuzzy Number

A trapezoidal fuzzy number (TrFN) A, can be represented completely by a quadruplet A = (a_1, a_2, a_3, a_4), whose membership function is written as follows:

$$\mu_A(x) = \begin{cases} 0 & x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \dfrac{x - a_4}{a_3 - a_4} & a_3 \le x \le a_4 \\ 0 & x \ge a_4 \end{cases}$$

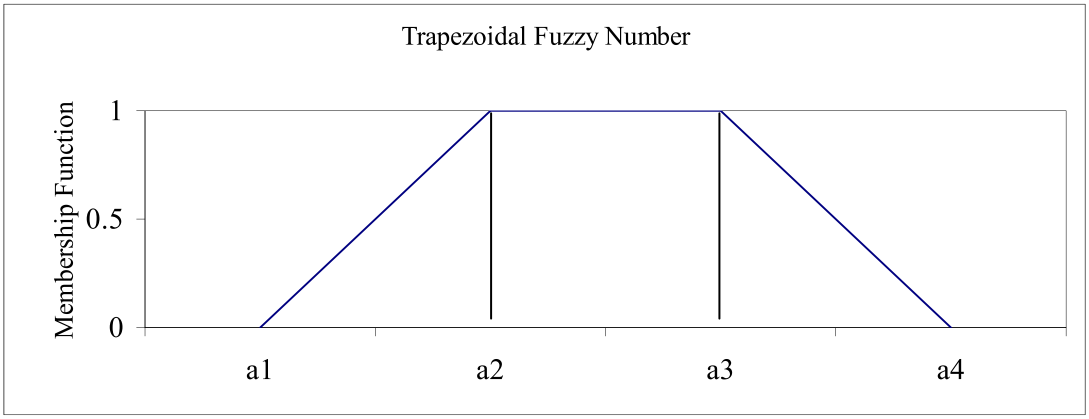
According to Kaufmann and Gupta, (1988, 1991), the interval of confidence at level-α as

$$A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right],$$

We characterize the TrFN, A = (a_1, a_2, a_3, a_4) as

$$A_\alpha = \left[a_1 + \alpha(a_2 - a_1), a_4 + \alpha(a_3 - a_4)\right] \qquad \forall \alpha \in [0, 1]$$

Figure A.2 Graphic Representation of Trapezoidal Fuzzy Number



Algebraic operations on TrFN's

Let A = (a_1, a_2, a_3, a_4) and B = (b_1, b_2, b_3, b_4) be two TrFN's, Then

- **Addition** $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
- **Subtraction** $A - B = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

For the following two operations, we assume that a_i and b_i, i = 1, 2, 3, 4 are positive.

- **Multiplication** $A (.) B = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$
- **Division** $A / B = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$

A.12 Zadeh's Extension Principle

Zadeh's extension principle also known as sup min extension principle in fuzzy literature, allows us to extend any point operations to operations involving fuzzy sets and is stated in terms of notation introduced already, as follows (Bector and Chandra, 2005):

i. $\mu_{f(A)}(y) = \sup\limits_{x \in X,\ f(x)=y} \left(\mu_A(x)\right)$, for all $A \in F(X)$, and

ii. $\mu_{f^{-1}(B)}(x) = \mu_B\left(f(x)\right)$, for all $B \in F(Y)$.

Sometimes the function f maps n-tuple in X to appoint in y i.e. $X = X_1 \times X_2 \times .... \times X_n$ and $f : X \to Y$ given by $y = f\left(x_1, x_1, ...x_n\right)$. Let A_1, A_2 …An be n fuzzy sets in X_1, X_2….X_n respectively. The extension principle of Zadeh allows to extend the crisp function $y = f\left(x_1, x_1, ...x_n\right)$ to act on n fuzzy sub sets of X, namely A_1, A_2, …A_n such that B = $B = f\left(A_1, A_1, ...A_n\right)$

Here the fuzzy set B is defined by

$$B = \left\{\left(y, \mu_B\left(y\right)\right) : y = f\left(x_1, ..., x_n\right), \left(x_1, ..., x_n\right) \in X_1 \times .... \times X_n\right\}$$

And

$$\mu_B(y) = \sup_{x \in X\,,\; y = f(x)} \min\left(\mu_{A_1}\left(x_1\right), ..., \mu_{A_n}\left(x_n\right)\right).$$