Essays on Designing Optimal Spectrum License Auctions

by

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Contents

DEDICATION

To Jacob and Jingbo

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ABSTRACT

Basically, my dissertation focuses on License Auctions. Four chapters of my dissertation are theoretical analysis of license auctions. Broadly speaking, I analyze the effects of different auction rules on revenue, efficiency and social welfare.

The first chapter studies the flaw in the design of the 2000 Turkish GSM (Global System for Mobile Communications) auction. In this auction, the Turkish government wants to raise as much revenue as possible and to increase competition in the cell-phone market by selling two licenses to new firms via a sequential auction, but it ends up with only one license sold. I identify this auction design failure. And I also show that if the auction were designed as a ”simultaneous auction”, the government would sell two licenses and receive more revenue.

In the second chapter, I show that if the cost asymmetry between the bidding firms is large enough, then having fewer firms in the market will surprisingly result in higher social welfare. This result is contrast to the common or general case in which ”social welfare” will be higher if there are more firms competing in the market.

In the first two chapters, I study the optimal auction designs when the firms bid for only one object. From the third chapter of my dissertation, I extend the analysis to the cases in which some bidders want more than one different object (multi-unit auctions). In the third chapter, I characterize the optimal bidding strategies of local and global bidders for two heterogeneous licenses in a multi-unit simultaneous ascending auction with synergies. In such auctions, some bidders want to win the bundle of licenses to enjoy synergies. So they may bid more than their stand-alone valuation of a license. This exposes them to

the risk of losing money since they may win parts of licenses. I determine the optimal bidding strategies in the presence of an exposure problem and show that global bidders may accept a loss even when they win all licenses and moreover, if a "bid-withdrawal" rule is introduced to the auction, the exposure problem disappears, and the simulation results show that revenue will be higher.

In the last chapter, I study the Canadian AWS auction in which 40 percent spectrum are set aside for new firms. I characterize the effect of spectrum set-aside auctions on seller's revenue, consumer surplus and social welfare. In a two-license simultaneous ascending bid auction model, I show that a spectrum set aside may not only encourage new entry and increase competition in the downstream market, but also under some circumstance, decreases the seller's revenue and consumer surplus. But a spectrum set aside results in inefficient allocation, and this inefficient entry further reduces social welfare.

FIGURES

1 Introduction

Countries such as Canada, United States, and Turkey have been using auctions to award spectrum licenses since 1990s. These countries have used different auction designs. In this thesis, I analyze how to improve these spectrum license auction designs so that governments will maximize revenue while allocating spectrum rights to firms that will use them in the best possible way. This is a game theoretical analysis that focuses on government revenue, social welfare implications and the strategic manipulations possible in existing auction designs.

In the first paper, we study the Turkish Global Mobile Telecommunications (GSM) auction held in 2000. In this auction, the government wanted to sell two GSM licenses with a sequential auction. The winning bid in the first auction would be the reserve price for the second auction. There were two incumbent firms in the market; hence, the government's objective was to increase competition in the cell-phone service market by having two more firms and to get as much revenue as possible. However, the government ended up selling only one license; no firm participated in the second auction. We show how and why a sequential auction results in predatory bidding, and hence, less competition in the market.

In the second paper, we continue to study the 2000 Turkish GSM auction in a model in which firms have heterogeneous cost functions. We mainly focus on the government revenue and social welfare implications of the Turkish auction design. First, we find that the seller may receive higher revenue with the Turkish auction. Second, we find a non-monotonic relation between the bidders' cost asymmetry and the seller's revenue. Third, we discuss the social welfare implications.

The first paper is published in Economics Bulletin and the second paper is forthcoming in Applied Economics Research Bulletin as joint papers with H. Gunay.

In the remaining two papers of this thesis, we study the simultaneous ascending bid auctions. This is the auction design used by Industry Canada-government agency responsible from designing spectrum auctions in Canada.

In the third paper, we characterize the optimal bidding strategies of local and global bidders for two heterogeneous licenses in a multi-unit simultaneous ascending auction with synergies. *'Local'* firms are interested in winning only specific licenses in order to serve in local markets and *'global'* firms are interested in winning all licenses in order to serve nationwide. The global firms enjoy synergies if they win all licenses. This gives them an incentive to bid over their stand alone valuations for some licenses. As a result, there is a risk of accepting losses. This is known as the exposure problem. We determine the optimal bidding strategies in the presence of an exposure problem and show that global bidders may accept a loss even when they win all licenses and moreover, if a "bid-withdrawal" rule is introduced to the auction, the exposure problem disappears, and the simulation results show that revenue will be higher.

In the last paper, I study the recent Canadian advanced wireless services (AWS) license auction in which 40 percent megahertz spectrum is set aside for new entrant firms in order to encourage the new entry. Incumbents such as Telus, Rogers, and Bell Mobility, called for an open auction of all wireless spectrum to the highest bidder. They argued that a set-aside auction will result in lower revenue, lower social welfare, and inefficient allocation. Spectrum set-aside auction can guarantee the new entry in the Canadian wireless services market and the price of wireless services will go down due to intense competition. But the effects of a set-aside auction on revenue, consumer surplus, and social welfare are not clear. In this paper, I show that-unlike what Telus, Rogers, and Bell Mobility argued-under some conditions an auction with set asides may result in higher auction revenue and consumer surplus.

I show that spectrum set asides indeed result in inefficient allocation, since a new entrant firm (even though its valuation is lower than incumbents' willingness to pay) will enter the market. Moreover, this inefficient entry further reduces social welfare. But under some circumstance, a spectrum set aside increases the seller's revenue and consumer surplus.

All proofs and simulation codes are included in the respective Appendix.

2 Predatory Bidding in Sequential Auctions

2.1 Introduction

Auctions are increasingly used in allocating spectrum licenses since they provide efficient results. However, if the auctions are not designed carefully, seller might get unwanted results like less revenue. In this chapter, we will study one such auction; the Turkish Global Mobile Telecommunications (GSM) auction held in 2000.

In this auction, the government wanted to sell two GSM licenses with a sequential auction.[1] The winning bid in the first auction would be the reserve price for the second auction. There were two incumbent firms in the market; hence, the government's objective was to increase competition in the cell-phone service market by having two more firms and to get as much revenue as possible. However, the government ended up selling only one license; no firm participated in the second auction.

Klemperer (2002) writes that Turkish GSM auction is biased towards creating monopoly. We model the auction formally and show that it will result in government selling only one license. We also compare this auction with a "no-reserve price" auction, and hence, show that the Turkish government's mistake was to set a reserve price in the second auction that can be manipulated by the firm that wins the first auction.

Ozcan (2004), in his working paper, also studies the same auction but compares it with a second-price auction. In his paper, firms' costs are private information and both licenses can be sold under some cases in the original auction. This chapter is different in a number of respects. First, we model the firms' costs as public information which gives a

[1] Ashenfelter (1989) and Jeitschko (1999) are a few examples that discuss sequential auctions.

very simple and transparent model. Second, we show that only one license will be sold in all possible cases. Third, we compare the original auction with a no-reserve price auction and show that the government would be able to sell both licenses and get more revenue with the latter one. Therefore, we identify the reserve-price that can be manipulated by the firms as the auction design flaw.[2]

McMillan (1994) gives an overview of spectrum rights auctions and mentions the possibility of predatory bidding in sequential auctions. Pitchik and Schotter (1988), in an experimental study, shows that predatory bidding occurs in sequential auctions if bidders are budget constrained and bidders valuations are common knowledge. In this chapter, bidders valuations are common knowledge but predatory bidding occurs due to the design of the auction.

In this paper, my contribution is to build most of the model and complete most of the proofs.

2.2 The Model

The model mimics the real world case of the Turkish GSM auction as much as possible. We consider a cell-phone market with two incumbent firms. The government will sell (lease) two more cell-phone licenses in a sequential auction administered in the same period. Each auction format is a first-price sealed-bid auction. Firms can buy at most one license. The winning bid of the first auction will be the reserve price for the second auction; that is, the firm who wins the second auction will pay at least as much as the first auction winning bid. We define the market inverse demand function as $p = A - bQ$, where p and Q stand

[2] We became aware of this working paper well after starting our paper.

for the market price and the market quantity demanded, respectively; A and b are positive numbers. Incumbent and entrant firms are identical. The firms have zero costs. There are at least 3 bidding (entrant) firms. We assume that firms in the cell-phone market will Cournot- compete at each period $t = 0, 1, ..$ once they get the licenses. Each firm's objective is to maximize its discounted profit (net of the bid they pay) with the discount factor$\delta \in (0, 1)$. All firms will decide how much to bid in the first and/or the second auction.

Assumption 1 If there is a tie, the government will award the license with a lottery that only the highest bidders participate.

In the proposition below, we characterize how the firms will bid in the first auction and prove that no firm will win in the second auction.

Proposition 1 *In the first auction, all firms will bid* $\frac{A^2}{16b(1-\delta)}$. *No firm can win the second auction.*

Proof We denote the discounted profit of a firm with π_j and per period quantity by q_j where the subscript $j = 3, 4$ denotes how many firms are competing in the market. After straightforward "Cournot-oligopoly profit maximization" calculations, equation 1 show q_j and π_j , respectively.

$$q_j = \frac{A}{(j+1)b} \qquad \pi_j = \frac{A^2}{(j+1)^2 b(1-\delta)} \tag{1}$$

If the firms bid high enough in the first auction, then no firm will enter to the second auction. Therefore, each firm will bid in order to prevent the fourth firm to enter to the market until their profits are zero. That is, they will bid $\pi_3 = \frac{A^2}{16b(1-\delta)}$ in the first auction. One firm will be awarded the license via lottery according to our assumption. Then, in the second auction, the remaining firms can bid at most 4 $\pi_4 = \frac{A^2}{25b(1-\delta)}$ but this is less than the reserve price, and no firm will win the second auction.

■

The firms take advantage of the auction design flaw and only one license is sold. Firms are willing to give up immediate money by bidding high since they know that they will make up this in the form of higher profits due to the less competition in the market. Hence, government's increased competition objective is not achieved. Now, we assume that the government designs a sequential auction but does not specify a reserve price for the second auction.[3] We will call this auction a "no reserve price auction."

Proposition 2 *In a no reserve price auction, firms will bid* $\frac{A^2}{25b(1-\delta)}$ *in both auctions. Both licenses will be sold.*

If a firm bids π_3 in the first auction, then it will make a loss since it can only make a (gross) profit of π_4 in the market which is less than its bid. Once the reserve price condition is removed from the auction design, firms can bid at most π_4 since there is no way of preventing entry of the fourth firm; hence, both licenses are sold. Not only that the government can sell two licenses, but it will also increase its revenue according to corollary 3 below. This shows that setting the reserve price that can be manipulated with a bid in the first auction is the flaw in the auction design.

[3]This is actually same as a simultaneous auction in which two licenses are sold.

Corollary 3 *The government will have more revenue with a no reserve price auction compared to the original auction.*

Proof In the no reserve price auction, two licenses will be sold at a lower price compared to the original auction. In the original auction, however, only one license will be sold. The following calculation shows that revenue is higher in the no reserve price auction.

$$\frac{A^2}{16b(1-\delta)} < 2\frac{A^2}{25b(1-\delta)} \Leftrightarrow 12.5 < 16 \tag{2}$$

Hence, revenues are always greater with the no reserve price auction.

■

If the government had not specified a reserve price, it would have been able to sell two licenses (which means more competition and more social welfare) and would have raised more revenue.

2.3 Conclusion and Discussion

We would like to discuss the possibility of one firm being strategic and the others being not strategic. Not strategic in the sense that they will bid π_4 in the first auction. If the strategic firm believes that the other firms are strategic, then only one license will be sold and the results will be the same as the original auction! If the strategic firm knows that the other firms are not strategic, then we will get the results of "no reserve price" auction.[4] We show how and why a sequential auction results in predatory bidding, and hence, less competition

[4]The only change will be that the strategic firm will bid slightly higher (we can say 1 cent more, if we change our continuous bid assumption to discrete bids of multiples of 1 cents) than π_4 in the first auction.

in the market. Turkish government sets a reserve price in the second auction that depends on the first auction's winning bid. This gives the firms the incentive of predatory bidding. We show that in a sequential auction without a reservation price, two licenses would be sold and more revenue would be raised.

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3 The 2000 Turkish Cell-Phone License Auction

3.1 Introduction

Turkish government intended on selling two cell-phone licenses with a sequential auction in 2000. Its design was very different than from those generally used before. Specifically, the winning bid in the first auction would be the reserve price in the second auction. We show that the Turkish auction design gives predatory (preemptive) bidding incentives which results in only one license being sold, which was in fact the case in the 2000 Turkish auction. However, depending on the cost asymmetry of the bidding firms, this may result in higher seller revenue or (even) higher social welfare compared to some other auction designs that result in two licenses being sold.

In 2000, two incumbent firms were already operating in the Turkish cell-phone market. The government announced that two more licenses will be sold with the auction rules as defined above.[5] The incumbent firms were not allowed to participate in the auctions. Six groups/firms participated in the first auction. A Turkish-Italian group, Is-Tim, won the first license by bidding an amount much higher than the analysts' expectations. This "high" bid became the reserve price in the second auction. No firm participated in the second auction since profits (benefit of the second license) would not justify paying the second auction's "high" reserve price (cost of the license). Is-Tim were able to deter the entry of another firm by strategically bidding. Turkish government -fearing law suits or reputation concerns- did not try to auction the second license later.

In our model, one of the bidder firms (an efficient firm) has the same cost function as

[5] A third license was reserved for the state-owned telecommunications company.

the incumbent monopolist.[6] The other bidders are identical high-cost (inefficient) firms. All costs are common knowledge.[7] The value of each license depends on which firms will receive the licenses and the resulting market structure; hence, the license valuations are endogenous.[8] We show that low cost firm successfully deters the entry of another firm by bidding predatorily. We also show that if the government were to use a simultaneous auction (or a sequential auction) with no reserve price, then it would be able to sell two licenses. We compare the Turkish auction with these auctions. First, we find that the seller may receive higher revenue with the Turkish auction. Second, we find a non-monotonic relation between the bidders' cost asymmetry and the seller's revenue. Third, we discuss the social welfare implications. Finally, we show that one can always design an auction, which some of them involves selling one license, that will give better or same results as the Turkish auction.

Krishna (1993, 1999), Rodriguez (2002), and Hoppe, Jehiel, and Moldavanu (2006) are some other papers that analyze the preemptive bidding incentives.[9] These aforementioned papers do not study the Turkish auction design.

Klemperer (2002) writes that the Turkish GSM auction is biased towards creating monopoly without modelling the details. We show that his insight is correct. Moreover,

[6]We make some small variations such as modelling one incumbent firm instead of two; however, the number of incumbent firms will not change the qualitative results as long as there is room for at least one entrant in the downstream market. We also do not model the fact that a third license is reserved for the state-owned company. This dose not affact the qualitative results.

[7]Complete information auction models are used in papers such as Krishna (1993 and 1999), Jehiel and Moldavanu (1996), Grimm, Riedel, and Wolfstetter (2003), Riedel and Wolfstetter (2006), and Hoppe, Jehiel, and Moldavanu (2006). This assumption rules out complexities like winner's curse and enable us to focus on the link of auctions with the revenue and the market structure.

[8]Some of the complete information papers that study endogenous license valuations are Krishna (1993, 1999), Rodriguez (2002), Hoppe, Jehiel and Moldovanu (2006). Some of the incomplete information papers are Rosenthal and Wang (1996) and Ozcan (2004).

[9]The literature uses the term preemptive bidding when the incumbent bids in order to prevent the new entry. Here, we use the term predatory bidding since not the incumbent, but one of the entrants is bidding (predatorily) to prevent the entry of the others.

we compare this auction with the other well-known auctions that will allow government to sell two licenses and show when the Turkish auction will generate more revenue.

There are many papers that study the real-world auctions. For example, Grimm, Riedel, and Wolfstetter (2003), and Hoppe, Jehiel, and Moldavanu (2006) analyze German spectrum auctions. However, -to our best knowledge- there is only one published paper, Gunay and Meng (2007), that study the Turkish auction in the literature. Another exception is Ozcan's (2004) working paper. Unlike these two papers, our modelling of cost function enables us to show the link between the bidders' cost asymmetry and the seller's revenue. We find a non-monotonic relation. We compare the Turkish auction with the other well-known auction designs that result in selling two licenses. We show that the Turkish auction may generate more revenue.[10] Unlike these two papers, we show that the Turkish auction design may generate higher social welfare for some parameter spaces compared to a simultaneous auction that sells two licenses.

In short, our modest aim is to model the 2000 Turkish cell-phone license auction and compare it with the other auction designs. In what follows, we set up our simple model. We discuss whether and when to use the Turkish auction design at the end. In this paper, my contribution is to build most of the model and complete most of the proofs.

3.2 The Model

We assume that the government will sell two cell-phone licenses in the form of a sequential auction administered in the same period. Each auction format is a first-price sealed-bid auction. Each entrant firm can buy at most one license.

[10]In his working paper, Ozcan compares the Turkish auction only with selling monopoly rights. We think that the comparison should be done with the auctions that result in selling two licenses.

There is already one incumbent firm in the market which cannot participate in the auction. The incumbent firm and the successful entrants will Cournot-compete at each period $t = 0, 1, ...$ in the downstream market. And the inverse demand function is a standard function $p(q) = A - bq$, where $p > 0$ and $q \geq 0$ denote the market price and the market quantity demanded, respectively. We assume that the parameters A and b are positive. There are at least 3 bidding firms. One of the bidding firms and the incumbent firm have zero marginal costs; these low cost firms are denoted as firm L. The other bidding firms are identical and has marginal costs, $c > 0$. Each of these firms is denoted as firm H.[11] All firms have zero fixed costs. All firms will decide whether to participate the auction and how much to bid in the first and/or in the second auction, if they participate.

We will look for an equilibrium where symmetric bidders use symmetric strategies and do not use (weakly) dominated strategies. The bids will be multiples of a very small monetary unit ϵ.[12] If there is a tie between n firms, one of these firms wins with some positive probability, $\kappa = \frac{1}{n}$. We denote profit with π_j^i, where $i = L, H$ shows the firm type and $j = LL, LH, LLH, LHH$ shows the number and type of firms competing in the market. For example, π_{LHH}^H denotes the profit of firm H when there is a total of three firm in the market (one L type and two H types).

3.2.1 Turkish auction Auction

In this auction, the winning bid of the first auction will be the reserve price for the second auction; that is, the firm who wins the second auction will pay **strictly more** than the first auction winning bid.

[11]We can assume that these H-firms are not identical. The qualitative results will not change.

[12]For notational simplicity, we assume that all profits will be multiples of ϵ. This monetary unit ϵ is almost equal to zero.

Assumption 2: A firm will not participate in the second auction if it expects a negative payoff (after paying the reservation price.)

First we show that there are many weakly dominated strategies. Let $(s_{i1}, s_{i2}) \in S_i$, where s_{i1} and s_{i2} denote the firm i's,$i = L, H$'s action in the first and second auction, respectively. Note that $s_{i2} \in \{0, \epsilon, 2\epsilon ...\} \cup \{not\ participate\}$ due to assumption 1.

Lemma 4 *: Let there be one L, and two H bidders. Moreover assume that the bidders will not participate in the second auction if they will make negative profits after paying the reserve price. For H-type firms and* $j = LH, LLH$*, the strategy* $(\pi_j^H - \epsilon, s_{H2}) \in S_i$ *weakly dominates* $(\pi_j^H + k\epsilon, s_{H2}) \in S_i$*, where* $k = 0, 1, 2, ...$

This lemma essentially tells that a strategy that involves an action in which a high type firm bids more than its "expected" profits in the first auction is weakly dominated. According to our equilibrium concept, strategies involving such actions cannot be an equilibrium.

In the proposition below, we characterize how the firms will bid in the first auction and prove that no firm will participate in the second auction. Firm L wants less competition in the market, so it will bid predatorily in the first auction to make sure that firm H will not participate in the second auction. Firm L bids slightly higher than firm H's highest possible (not weakly dominated) bid and will win the first auction. Let us emphasize that the bid of L firm depends on the cost asymmetry between the firms.[13]

Lemma 5 *: The Turkish auction will result in the bidders' incentives of predatory bidding*

[13] Note that c is not only the marginal cost of H type firm but also the cost asymmetry between L and H type firm.

in the first auction and deter the entry of others in the second auction.

Proof. See the Appendix.

According to Lemma 5, we know that if there is a reserve price in the second auction, then all the bidders have incentives to bid preemptively in order to deter the entry of other bidders in the second auction. Because in the first auction in order to maximize the expected profits, firm L will decide whether to bid so high to deter the entry of firm H in the second auction, or to wait until the second auction to pay a relative lower reserve price when firm H wins the first auction. However, when the cost asymmetry or firm H's marginal costs are under a reasonable range, that is, $c_H < \frac{A+(n+1)c_L}{n+2}$, it is optimal for firm L to bid so high to deter the entry of firm H in the second auction in the Turkish auction. That means, the Turkish auction will always result in the predatory bidding.

Proposition 6 *(Equilibrium in the Turkish auction)*

A)If $0 < c < \frac{A}{11}$, *then firm L will win the first auction by bidding* $\pi^H_{LH} = \frac{(A-2c)^2}{9b(1-\delta)}$. *No firms will participate in the second auction.*

B)If $\frac{A}{11} \leq c \leq \frac{A}{3}$, *then firm L will win the first auction by bidding* $\pi^H_{LLH} = \frac{(A-3c)^2}{16b(1-\delta)}$. *No firms will participate in the second auction.*

Proof. See the Appendix.

While firms are bidding, they take into account the possible competition (and the resulting profits) in the downstream market. To understand Proposition 6 part B, assume that one of the H-firms wins the first auction by offering the possible maximum (not weakly dominated) bid that gives them (almost)zero expected profits. Moreover, assume that firm L participates in the second auction. Since the marginal cost of firm H is high in this region, firm L can still make a positive payoff after paying the reservation price. But then, in the first auction, when firm H is bidding, it should bid assuming that firm L will also be in the market. This bid will be $\pi^H_{LLH} - \epsilon$ due to competition between the H-firms.[14] But firm L is better off if there is less competition in the market. Hence, firm L will bid π^H_{LLH}, just ϵ higher than H-type firms' bid in the first auction. No firm H can bid more than this (reservation price) without making negative profits, and hence, no firm will participate in the second auction. Firm L's predatory bidding strategy will work.

To understand part A, assume that firm H wins the first auction. Since the cost of firm H is low enough, firm L cannot make enough profits in the downstream market after paying the reservation price in the second auction. That is, firm H deters the entry of firm L. Specifically, firm H will bid $\pi^H_{LH} - \epsilon$ in the first auction as if there will be two firms in the market. Expecting this, firm L will bid slightly more than firm H's bid in the first auction and deter the entry of firm H.

Complete-information endogenous valuations papers such as Gunay and Meng (2007) and Hoppe et. al. (2006) do not allow cost differences among the entrant firms/bidders. However, we show that the cost difference affects the bids (and hence the seller's revenue). Note that as c increases, the bids (hence the seller's revenue) decrease but there is a discontinuity point.

[14]Lemma 1 shows that strategies containing π^H_{LLH} is weakly dominated by strategies containing $\pi^H_{LLH} - \epsilon$.

3.2.2 "No Reserve Price" Simultaneous Auction

Now, we assume that the government designs a simultaneous auction with no reserve price in which two licenses are for sale. Using first or second sealed bid price format will give the same outcome since this is a complete information game. We will call this auction "no reserve price auction". Also, note that this auction's outcome is the same as the outcome of selling two licenses in a sequential auction with no reserve price.

Proposition 7 *(Equilibrium in "no reserve price auction") In a no reserve price auction, two licenses will be sold. Firm L will win by bidding* $\pi^H_{LLH} = \frac{(A-3c)^2}{16b(1-\delta)}$ *and one H firm will win by bidding* $\pi^H_{LLH} - \epsilon$.

Once the reservation price is removed from the auction design, both licenses are sold. This shows that setting the reserve price that can be manipulated with a bid in the first auction causes only one license to be sold. Now, we can compare these two different auctions in terms of seller's revenue and social welfare.

3.2.3 Comparison Of Two Auctions

Since ϵ is very small (almost zero), we assume that the revenue in "no reserve price auction" is equal to $2\pi^H_{LLH}$. We make this assumption for exposition purposes. Qualitative results do not change because of this assumption.

Proposition 8 *(Revenue Comparison) The government will have more revenue with a no reserve price auction compared to the Turkish auction when*

$$c < \frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} \approx \frac{A}{20} \quad or \quad \frac{A}{11} \leq c \leq \frac{A}{3} \tag{3}$$

The government will have more revenue with the "Turkish auction" auction when

$$\frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} < c < \frac{A}{11} \tag{4}$$

Proof. See the Appendix.

This Proposition follows from Proposition 7 and 8 directly and confirms that the government will have more revenue from selling one license through the "Turkish auction" sequential auction when $\frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} < c < \frac{A}{11}$, otherwise, the government will get more revenue from selling two licenses through a no reserve price auction.

As one can see from figure 1, there is not a monotonic relation between the firms' cost difference and the seller's revenue when we compare the two auctions. When $0 < c < \frac{A}{11}$, in a no reserve price auction, government revenue will be $2\pi^H_{LLH}$. In the Turkish auction auction, government revenue will be only π^H_{LLH}. These bids are decreasing functions of the marginal cost (and the cost difference), c. As a result, bids (hence the seller's revenue) decrease in both type of auctions. However, as c increases, government revenue in a no reserve price auction will decrease **faster** than that in the Turkish auction auction. When c is zero (the extreme case), "no reserve price" auction gives a higher revenue. Since bids will decrease faster in the no reserve price auction, as c increases, eventually, revenue becomes higher in the Turkish auction. This happens when $\frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} \leq c \leq \frac{A}{11}$.

At $c = \frac{A}{11}$, there is a discontinuous jump in the Turkish auction case. This is the point in which the market is profitable for three firms rather than two firms. Beyond this point, entry deterrence is much cheaper for the firm L, and it bids π^H_{LLH} in the Turkish auction which is also the seller's revenue. In the no reserve price auction, two licenses are sold for a total of $2\pi^H_{LLH}$. Hence, revenue will be higher with the no reserve price auction.

Proposition 9 *(Social Welfare Comparison)*

A) When $\frac{7A}{69} < c \leq \frac{A}{3}$, there will be higher social welfare with the original auction than that with a no reserve price auction.

B) When $0 < c < \frac{7A}{69}$, there will be higher social welfare with a no reserve price auction.

Proof. See the Appendix.

If the cost difference between the firms is large enough; that is when $\frac{7A}{69} < c \leq \frac{A}{3}$, then having one firm in the market is better than having two firms in terms of social welfare. In other words, less competition results in higher social welfare. The reason for this result is as follows. When the high-cost firm enters the market, the total output, and hence, consumer surplus increases. This effect increases social welfare. However, the low-cost firm produces fewer outputs in the new equilibrium. The high-cost firm produces outputs previously produced by the low cost firm. As a result, total profits in the new equilibrium will be lower. This decreases the social welfare. When the cost asymmetry between the firms is high, the latter effect dominates and the social welfare decreases.

In Lahiri and Ono (1988) and Zhao (2001), if the less efficient firm's marginal cost decreases, then the social welfare decreases. Since the Turkish auction auction prevents the entry of the high cost firm, we find that the Turkish auction gives higher social welfare compared to "no reserve price" auction. However, we caution the reader that this is a complete information model. Hence, if the cost asymmetry were in that region, the government would auction only one license with a no reserve price auction. Because of competition, the seller's revenue would be higher while the social welfare is the same with this auction.

Figure 1 summarizes propositions 8 and 9.

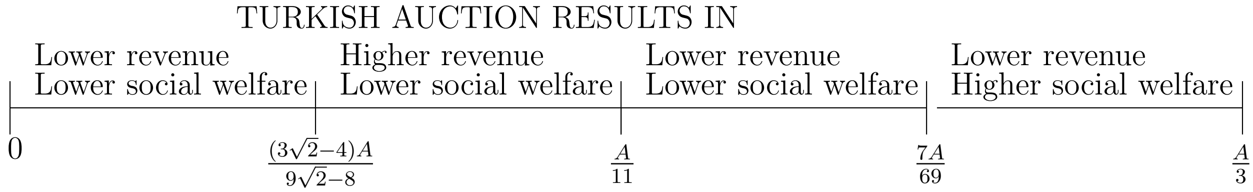

Figure 1: Comparison of Turkish and no reserve price auctions under different cost ranges

3.3 When to use the Turkish auction design?

Should we use the Turkish auction? We will show that a seller who cares about social welfare or revenue can always use a different auction design that gives better or same results than the Turkish auction. We have already discussed that a seller who wants to maximize social welfare can do better (or same) by using other auction designs. What about a seller who wants to maximize revenue? Compared to no reserve price auction that sells two licenses, we have showed that the Turkish auction can generate more revenue for some c. However, if c is in such a region, then the Turkish government could sell one license with a no reserve price auction, then, the seller's revenue would be equal to π^H_{LLH} which is the same as the Turkish auction.[15]

3.4 Conclusion

In this paper, our modest aim was to analyze the 2000 Turkish cell-phone license auction. We showed how this auction design gives predatory (preemptive) bidding incentives to the bidders.

[15] Depending on the government and firm's discount factors, government may even get more revenue by setting a higher exogenous reserve price. Our aim is to show that one can find other auction designs that perform same or better than the Turkish auction design. Since we showed one, we do not model this possibility.

Our main result is that a seller who cares about maximizing social welfare or revenue can always design another auction that gives better or same results as the Turkish auction.

3.5 Appendix

Proof of Lemma 4: We will prove this for the case $j = LH$; that is, the case in which the H firm will not participate in the second auction if it loses the first one by Assumption 1 since it will be making negative profits. The proof for the case $j = LHH$ is similar and omitted.

Let S_i be the strategy space of firm $i = H, L$ and σ_i a strategy. We will show that one H firm's strategy $(\pi^H_{LH} + k\epsilon, x) \in S_i$, where $k = 0, 1, 2, ..$, is weakly dominated by the strategy $(\pi^H_{LH} - \epsilon, x) \in S_i$. Let $\sigma_i = (s_{i1}, s_{i2}) \in S$ be the strategy of the $i = L, H$ firm. The first and second components of the strategy show the action in the first and the second auction, respectively. In short, we have to show that the payoff of the H-type firm $U(.)$ is lower under weakly dominated strategies:

$$U((\pi^H_{LH} + k\epsilon, x), \sigma_H, \sigma_L) \leq U((\pi^H_{LH} - \epsilon, x), \sigma_H, \sigma_L) \qquad (5)$$

The relation should hold with strict inequality at least for one strategy combination. We will show this for different cases.

Case 1: $Max\{s_{H1}, s_{L1}\} \geq \pi^H_{LH} + k\epsilon$.

In this case, both strategies give zero payoff since the H-firm will lose the first auction and will not participate in the second auction according to our assumption. They will not participate since then they will be the third firm and their profits will definitely be lower than the reserve price of the second auction.

Case 2: $\pi^H_{LH} - \epsilon < Max\{s_{H1}, s_{L1}\} \leq \pi^H_{LH} + k\epsilon$

In this case, the left hand side of inequality 5 is non-positive (and negative when $k \neq 0$. The right hand side or inequality 5 is zero since the firm cannot win in the first auction, and hence, does not participate in the second auction.

Case 3: $\pi^H_{LH} - \epsilon = Max\{s_{H1}, s_{L1}\}$

In this case, the left hand side of the inequality is non-positive. The right hand side's expected payoff is $\kappa\epsilon > 0$ assuming that each firm wins the auction with probability κ.

Case 4: $\pi^H_{LH} + k\epsilon \geq \pi^H_{LH} - \epsilon > Max\{s_{H1}, s_{L1}\}$

In this case, $U((\pi^H_{LH} + k\epsilon, x), \sigma_H, \sigma_L) < U((\pi^H_{LH} - \epsilon, x), \sigma_H, \sigma_L)$. In the first auction, the H-firm we consider will win with a lower bid. This gives a higher payoff to H firm regardless of what happens in the second auction.

■

Proof of Lemma 5: We assume that there are n incumbent firms with low costs denoted by c_L, we call firm L, and m entrant firms including one firm with low costs as the incumbents, c_L, and $m-1$ firms with high costs denoted by c_H, and we call firm H.

Now, we consider the two strategies of firm L in the first auction. Strategy 1: firm L enters the first auction and submits the predatory bid to deter the entry of firm H in the second auction. Strategy 2: firm L lets form H win the first auction and waits to enter the second auction with winning the second license at a relative lower reserve price. Moreover, we assume that the market demand function is $p(q) = A - bq$. If firm L chooses Strategy 1, then the expected profit of firm L is denoted by π_L^1; if firm L chooses Strategy 2, then the expected profit of firm L is denoted by π_L^2.

So firm 1's expected profit under Strategy 1 is, $\pi_L^1 = [A - b(nq_L + q_L) - c_L]q_L$.

We take a partial derivative of the expected profit of firm L under Strategy 1 with respect to firm L's quantity, then the profit-maximizing quantity and maximum profits are the following,

$$\frac{\partial \pi_L^1}{\partial q_L} = 0 \Longrightarrow q_L^{1*} = \frac{A-c_L}{b(n+2)} \Longrightarrow \pi_L^{1*} = \frac{(A-c_L)^2}{b(n+2)^2}$$

And firm 1's expected profit under Strategy 2 is determined by the optimal quantities of both firm L and firm H, $\pi_L^2 = [A - b(nq_L + q_H + q_L) - c_L]q_L$, and $\pi_H^2 = [A - b(nq_L + q_H + q_L) - c_H]q_H$.

We take partial derivatives of the expected profits of firm L under Strategy 1 with respect to firm L's quantity and the expected profits of firm H with respect to firm H's quantity, respectively, then the profit-maximizing quantities and maximum profits are the following,

$$\frac{\partial \pi_H^2}{\partial q_H} = 0 \Longrightarrow q_H^* = \frac{A+(n+1)c_L-(n+2)c_H}{b(n+3)}$$

$$\frac{\partial \pi_L^2}{\partial q_L} = 0 \Longrightarrow q_L^{2*} = \frac{A-2c_L+c_H}{b(n+3)} \Longrightarrow \pi_L^{2*} = \frac{(A-2c_L+c_H)^2}{b(n+3)^2}$$

Now we compare π_L^{1*} with π_L^{2*}, and find, if $\pi_L^{1*} < \pi_L^{2*}$, then we have,

$$\frac{(A-c_L)^2}{b(n+2)^2} < \frac{(A-2c_L+c_H)^2}{b(n+3)^2} \Longrightarrow \frac{(A-c_L)}{(n+2)} < \frac{(A-2c_L+c_H)}{(n+3)} \Longrightarrow c_H > \frac{A+(n+1)c_L}{n+2}$$

Otherwise, when $c_H < \frac{A+(n+1)c_L}{n+2}$, $\pi_L^{1*} > \pi_L^{2*}$.

But note that all the quantities should be positive, that means, $q_L^{1*} = \frac{A-c_L}{b(n+2)} > 0 \Longrightarrow c_L < A$, $q_L^{2*} = \frac{A-2c_L+c_H}{b(n+3)} > 0 \Longrightarrow c_L < \frac{A+c_H}{2}$, and $q_H^* = \frac{A+(n+1)c_L-(n+2)c_H}{b(n+3)} > 0 \Longrightarrow c_H < \frac{A+(n+1)c_L}{(n+2)}$.

As a result, we have the following restriction about the firms' costs, $c_L < \frac{A+c_H}{2}$, and $c_H < \frac{A+(n+1)c_L}{(n+2)}$. Thus, in the Turkish auction firm L can not make higher profits by waiting to the second auction. The Turkish auction with a reserve price will result in the predatory bidding.

If we assume that $c_L = 0$ and $c_H = c$. We have the similar results.

$$\frac{\partial \pi_L^1}{\partial q_L} = 0 \Longrightarrow q_L^{1*} = \frac{A}{b(n+2)} \Longrightarrow \pi_L^{1*} = \frac{A^2}{b(n+2)^2}$$

$$\frac{\partial \pi_H^2}{\partial q_H} = 0 \Longrightarrow q_H^* = \frac{A-(n+2)c}{b(n+3)}$$

$$\frac{\partial \pi_L^2}{\partial q_L} = 0 \Longrightarrow q_L^{2*} = \frac{A+c}{b(n+3)} \Longrightarrow \pi_L^{2*} = \frac{(A+c)^2}{b(n+3)^2}$$

Now we compare π_L^{1*} with π_L^{2*}, and find, if $\pi_L^{1*} < \pi_L^{2*}$, then we have,

$$\frac{A^2}{b(n+2)^2} < \frac{(A+c)^2}{b(n+3)^2} \Longrightarrow \frac{A}{(n+2)} < \frac{(A+c)}{(n+3)} \Longrightarrow c > \frac{A}{n+2}$$

Otherwise, when $c_H < \frac{A}{n+2}$, $\pi_L^{1*} > \pi_L^{2*}$.

But note that all the quantities should be positive, that means, $q_L^{1*} = \frac{A}{b(n+2)} > 0 \Longrightarrow A > 0$, $q_L^{2*} = \frac{A+c}{b(n+3)} > 0 \Longrightarrow A + c > 0$, and $q_H^* = \frac{A-(n+2)c}{b(n+3)} > 0 \Longrightarrow c < \frac{A}{(n+2)}$.

As a result, we have the following restriction about the firms' costs, $c < \frac{A}{(n+2)}$. Thus, in the Turkish auction firm L can not make higher profits by waiting to the second auction. The Turkish auction with a reserve price will result in the predatory bidding.

■

Proof of Proposition 6: We denote per period profit with π^i_j where $i = L, H$ shows the firm type and $j = LL, LH, LLH, LHH$ denotes how many and which firms are competing in the market. After straightforward calculations, equation 4, equation 5, and equation 6 show per period profit and the quantities produced by each type of firm when there are two firms and three firms in the market, respectively.

$$q^L_{LL} = \frac{A}{3b} \qquad \pi^L_{LL} = \frac{A^2}{9b} \qquad q^H_{LH} = \frac{A-2c}{3b} \qquad \pi^H_{LH} = \frac{(A-2c)^2}{9b} \tag{6}$$

$$q^L_{LLH} = \frac{A+c}{4b} \qquad \pi^L_{LLH} = \frac{(A+c)^2}{16b} \qquad q^H_{LLH} = \frac{A-3c}{4b} \qquad \pi^H_{LLH} = \frac{(A-3c)^2}{16b} \tag{7}$$

$$q^L_{LHH} = \frac{A+2c}{4b} \qquad \pi^L_{LHH} = \frac{(A+2c)^2}{16b} \qquad q^H_{LHH} = \frac{A-2c}{4b} \qquad \pi^H_{LHH} = \frac{(A-2c)^2}{16b} \tag{8}$$

Part A): We look for an equilibrium that does not involve weakly dominated strategies. In the first auction, Type H firms will bid until their profits are (almost) zero because of the competition between them. But their profits depend on whether L-firm will participate in the second auction or not (assuming that L-firm loses the first auction). Moreover, bidding π^H_{LH} is weakly dominated by $\pi^H_{LH} - \epsilon$.[16] However, firm L can prevent the entry of H firms and maximize its payoff by offering π^H_{LH} in the first auction.

First let us calculate the bid of type H firms. Firm H knows that if its cost is low enough and it wins the first auction with a bid $\pi^H_{LH} - \epsilon = \frac{(A-2c)^2}{9b(1-\delta)} - \epsilon$, then firm L will not

[16]We note that we assume ϵ to be a sufficiently small monetary unit.

enter to the second auction. Because, if firm L enters the second auction, it has to pay the (smallest) reserve price and there will be three firms in the market and hence, its net profit will be negative when $0 < c < \frac{A}{11}$ as shown in equations below:

$$Net\ profit\ of\ Firm\ L = \pi^L_{LLH} - \pi^H_{LH} = \frac{(A+c)^2}{16b(1-\delta)} - \frac{(A-2c)^2}{9b(1-\delta)} < 0 \tag{9}$$

$$\Longleftrightarrow \frac{(A+c)^2}{16} < \frac{(A-2c)^2}{9} \tag{10}$$

$$\Longleftrightarrow \frac{(A+c)}{4} < \frac{(A-2c)}{3} \Longleftrightarrow 0 < c < \frac{A}{11} \tag{11}$$

Hence, type H firm will bid $\frac{(A-2c)^2}{9b(1-\delta)} - \epsilon$ in the first auction. Firm L will bid just $\frac{(A-2c)^2}{9b(1-\delta)}$ and win the first auction. Firm H will make a negative profit if it wins the second auction with a bid higher than the reservation price $\frac{(A-2c)^2}{9b(1-\delta)}$; hence, no firm H will participate in the second auction.

Part B): From Part A, we know that when $c \geq \frac{A}{11}$, firm L will enter the second auction. Hence, any firm H can bid at most $\pi^H_{LLH} - \epsilon = \frac{(A-3c)^2}{16b(1-\delta)} - \epsilon$. Firm L will bid $\frac{(A-3c)^2}{16b(1-\delta)}$ in the first auction. Because of the reservation price, no firm H will enter to the second auction.

■

Proof of Proposition 8: When $\frac{A}{11} \leq c \leq \frac{A}{3}$, the bids in both auctions are the same but government will sell two licenses in the "no-reserve price" auction; Hence, their revenue will be doubled. If $0 < c < \frac{A}{11}$, then in the original auction, government will sell one license but will make $\frac{(A-2c)^2}{9b(1-\delta)}$. In the no-reserve price, government will sell two licenses, each of them at a price of $\frac{(A-3c)^2}{16b(1-\delta)}$. When we compare the revenue:

$$\frac{(A-2c)^2}{9b(1-\delta)} > 2\frac{(A-3c)^2}{16b(1-\delta)} \Longleftrightarrow \frac{(A-2c)^2}{9} > 2\frac{(A-3c)^2}{16} \Longleftrightarrow \frac{(A-2c)}{3} > \sqrt{2}\frac{(A-3c)}{4} \tag{12}$$

$$\Longleftrightarrow 4A - 8c > 3\sqrt{2}A - 9\sqrt{2}c \Longleftrightarrow c > \frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} \tag{13}$$

■

Proof of Proposition 9: We consider social welfare in auctions as the sum of producer surplus, consumer surplus and government revenue from the auctions. We denote social welfare by W^i and government revenue by R^i, $i = 1, 2$. Where W^1 and R^1 denote social welfare and government revenue with the original auction, and W^2 and R^2 denote social welfare and government revenue with a no reserve price auction. According to Proposition 2 and 3, we know that $R^1 = \frac{(A-2c)^2}{9b(1-\delta)}$, when $0 < c < \frac{A}{11}$ or $R^1 = \frac{(A-3c)^2}{16b(1-\delta)}$, when $\frac{A}{11} \leq c \leq \frac{A}{3}$, and $R^2 = 2\frac{(A-3c)^2}{16b(1-\delta)}$. From Proposition 4, we know that the government will have more revenue by selling two licenses when $c < \frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} \approx \frac{A}{20}$ *or* $\frac{A}{11} \leq c \leq \frac{A}{3}$ and more revenue by selling one license when $\frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} < c < \frac{A}{11}$.

When $0 < c \leq \frac{A}{3}$, social welfare and deadweight loss from the original auction will be,

$$W^1 = PS + CS + R^1 = 2 \cdot \frac{A^2}{9b} - R^1 + \frac{2A}{3b} \cdot \frac{1}{2} \cdot \frac{2A}{3} + R^1$$

$$W^1 = \frac{4A^2}{9b} \tag{14}$$

$$W^2 = PS + CS + R^2 = 2 \cdot \frac{(A+c)^2}{16b} + \frac{(A-3c)^2}{16b} - R^2 + \frac{3A-c}{4} \cdot \frac{1}{2} \cdot \frac{3A-c}{4b} + R^2$$

$$W^2 = \frac{15A^2 + 23c^2 - 10Ac}{32b} \tag{15}$$

Since $W^1 - W^2 = \frac{-7A^2 - 207c^2 + 90Ac}{288b} > 0$, when $(A-3c)(7A-69c) < 0$.

Thus, if $\frac{7A}{69} < c \leq \frac{A}{3}$, $W^1 > W^2$ and $R^2 > R^1$. If $0 < c < \frac{7A}{69}$, $W^2 > W^1$ and if $0 < c < \frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8}$ and $\frac{A}{11} \leq c \leq \frac{7A}{69}$, $R^2 > R^1$; if $\frac{(3\sqrt{2}-4)A}{9\sqrt{2}-8} < c < \frac{A}{11}$, $R^1 > R^2$.

Since $CS^1 = \frac{2A}{3b} \cdot \frac{1}{2} \cdot \frac{2A}{3}$ and $CS^2 = \frac{3A-c}{4} \cdot \frac{1}{2} \cdot \frac{3A-c}{4b}$, then $CS^2 - CS^1 = \frac{[9(3A-c)^2 - 64A^2]}{288b} = \frac{(A-3c)(17A-3c)}{288b}$, we always have $CS^2 > CS^1$, when $0 < c \leq \frac{A}{3}$.

■

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4 Exposure Problem in Multi-Unit Auctions

4.1 Introduction

In a typical American or Canadian spectrum license auction, hundreds of (heterogenous) licenses are sold simultaneously. Each of these licences gives the spectrum usage right of a geographical area to the winning firm. Some *'local'* firms are interested in winning only specific licenses in order to serve in local markets, while other *'global'* firms are interested in winning all licenses in order to serve nationwide.[17] The global firms enjoy synergies if they win all licenses. This gives them an incentive to bid over their stand alone valuations for some licenses. As a result, there is a risk of accepting losses. This is known as the exposure problem.[18]

In a model simplifying the American and the recent Canadian spectrum license auctions, we derive the optimal bidding strategies of local and global firms in a simultaneous ascending auction. We mainly focus on the optimal bidding strategies when there is the possibility of an exposure problem. We also study an auction that has a "bid withdrawal" feature. This feature allows bidders to withdraw their highest standing bid later in the auction (without any penalty). We show that the bid withdrawal rule will eliminate the exposure problem. Additionally, our simulation results show that an auction with bid withdrawal will bring higher revenues.

The multi-unit auction literature generally assumes that global bidders have either

[17]In the recent Canadian Advanced Wireless Spectrum auction, firms such as Globalive and Rogers were interested in all licenses whereas firms such as Bragg Communication and Manitoba Telecom Services (MTS) were interested in East Coast and Manitoba licenses, respectively.

[18]As a result, bidders may bid less aggressively, which results in less revenue. Englmaier et. al (2009), Kagel and Levin (2005), and Katok and Roth (2004), among others, discuss the exposure problem in identical object auctions.

equal valuations (Englmaier et. al (2009), Albano et. al. (2001), Kagel and Levin (2005), Katok and Roth (2004), Rosenthal and Wang (1996), and Krishna and Rosenthal (1996)) or very large synergies (Albano et al. (2006)). The spectrum licenses for different geographic areas are not homogenous objects; hence, the equal valuation assumption does not fit for the Canadian and the American spectrum license auction. Moreover, in a heterogeneous license environment, bidders may not drop out from both auctions simultaneously. This enables us to analyze bidding behavior in the remaining auction.

Albano et. al (2006) assume very large synergies, and do not analyze the exposure problem.[19] In contrast, we allow for moderate synergies, and our focus is on the exposure problem and bid withdrawal. In our paper, global bidders will lower their bids because of the exposure problem (when there is no bid-withdrawal); however, their optimal strategy still requires them to bid over their stand alone valuation for at least one license. If they win this license by receiving a potential loss, then they may need to stay in the other license auction to minimize their loss. Therefore, there are cases in which the exposure problem may arise even when the bidder wins all licenses.

Kagel and Levin (2005) and Krishna and Rosenthal (1996) show that bidders bid more aggressively as the number of bidders decreases in multi-unit auctions with synergies. Chow and Yavas (2009) test this experimentally in a simultaneous second-price auction setting. We also find the same result in our simultaneous ascending auction; the global bidders' optimal drop out price increases as the other bidders drop out.

This chapter also shows that bid withdrawal will eliminate the exposure problem. To

[19]Their global bidders cannot end up winning only one license. Our results coincide with theirs when we assume large synergies. Let us note that they also discuss the moderate synergy cases for *identical* objects.

understand this, assume that the global bidder has the highest standing bid in one license, and this bid is greater than his stand alone valuation. Then, he will continue to stay in the other license auction until his (potential) payoff becomes zero. At that price, if he cannot win the license, he will drop out of this license auction and will withdraw his highest standing bid from the other one. This will give him a zero payoff; therefore, there would be no exposure problem. The seller's revenue will be affected with the introduction of bid withdrawal in two ways. On the one hand, the bid withdrawal rule encourages the global bidder to bid even more aggressively without the fear of accepting a loss. This effect tends to increase revenues. On the other hand, the global bidder, after dropping out of one license auction, will bid less aggressively in the remaining one and never allow the exposure problem to happen. This effect tends to decrease revenues. Our simulations show that the first effect dominates, and hence, revenue is higher in an auction with bid withdrawal.

There are no theoretical papers we are aware of that analyze bid withdrawal rules in multi-unit auctions.[20] The only experimental paper on this issue belongs to Porter (1999). His set-up is different from ours. For example, there are no local bidders for each object. Like Porter, we also find that revenue is higher in auctions with a bid withdrawal rule.

This chapter is organized as follows. We first consider a special case of one global and two local bidders. We derive their optimal bidding strategies in an auction with and without bid withdrawal. Then, we compare the revenue and efficiency in these auctions. Finally, we extend our results to a more general case with two global bidders and a finite number of local bidders. All proofs are included in the Appendix.

[20] Harstad and Rothkopf (1995) discuss bid-withdrawal in single unit auctions. Wang (2000) discusses re-negotiation in a single unit procurement auction.

This paper is joint with H. Gunay. My contribution is to participate in building the model and write most of the proofs. I also write all of the simulation codes using MATLAB.

4.2 The Model

There are 2 licenses, license A and B for sale.[21] There are N global bidders who demand both licenses and M_j local bidders who demand only license $j = A, B$. Both local bidders and global bidders have a private stand alone valuation for a single license, v_{ij}, where i and j represent the bidder and the license, respectively. The valuations v_{ij} are drawn from the continuous distribution function $F(v_{ij})$ with support on $[0, 1]$ and probability density function $f(v_{ij})$. The type of bidders, global or local, is publicly known.

Like Albano et al. (2006), we assume that there are homogeneous positive synergies for global bidders, and denote this kind of synergy by $\alpha > 0$ and α is public information. Then, the global bidder i's total valuation, given that it wins two licenses is, $V_i = v_{iA} + v_{iB} + \alpha$. The valuation to a global bidder i who receives only one license j or the valuation of local bidder i who receives license j is $V_i = v_{ij}$.[22]

We consider a situation where the licenses are auctioned off simultaneously through an ascending multi-unit auction. Prices start from zero for all licenses and increase simultaneously and continuously at the same rate (or by a very small pre-determined increment in each round). When only one bidder is left on a given license, the clock stops for that license; hence, he wins the license at the price that the last bidder drops. At the same time, the price on the remaining license will continue to increase, if there are more than

[21] We use two licenses like Albano et. al. (2001 and 2006), Brusco and Lopomo (2002), Chow and Yavas (2009), and Menucicci (2003).

[22] A local bidder who is interested in license j participates only on license j auction.

one bidder.

The dropout is irreversible; once a bidder drops out of bidding for a given license, he cannot bid for this license later.[23] The number of active bidders and the drop-out prices are publicly known. We also assume that there is no budget constraint for the bidders.

4.2.1 A Special Case Without Bid Withdrawal Rule

We first start with a special case in which there is a single global bidder, called Firm 1, and two local bidders, called Firm 2 and Firm 3. The global bidder, Firm 1, is interested in both licenses A and B. Firm 1's total valuation of two licenses is given by $V_1 = v_{1A} + v_{1B} + \alpha$. His stand-alone valuation of license A or B is given by v_{1A} or v_{1B}, and we assume that $v_{1A} > v_{1B}$.[24] Firm 2 is only interested in license A; her private valuation is v_{2A} and Firm 3 is only interested in license B; her private valuation is v_{3B}.[25] The stand-alone valuations are drawn from the uniform distribution function F with support on $[0, 1]$ and probability density function f.

We will derive a symmetric perfect Bayesian equilibrium with the help of lemmas that follow. First, we describe the equilibrium strategy of the local bidder.

Lemma 10 *: Each local bidder has a weakly dominant strategy to stay in the auction until the price reaches his stand alone valuation.*

This is a well-known result so we skip the proof.

[23]In the real-world auctions, there is activity rule. If the bidders do not have enough highest standing bids, then the number of licenses they may bid is decreased (in the next rounds). Hence, when there are two licenses, this translates into an irreversible drop-out.

[24]The justification of this assumption is as follows. The global bidder will value Toronto license (license A) more than the Winnipeg license (license B), for example. We assume that two independent draws are made and the higher amount will be the valuation for license A.

[25]We do not assume that $v_{2A} > v_{3B}$ since local firms are different; hence, their efficiency may differ.

Lemma 11 : *a) The global bidder stays in both license auctions at least until the price reaches the minimum of his/her stand alone valuations.*

b) If his average valuation is greater than 1, his optimal strategy is to stay in until the price reaches his average valuation.[26]

The result comes from comparing the expected profits from dropping before the minimum of the stand alone valuations and dropping out at the minimum stand alone valuation. If a global bidder drops out before the minimum of its stand alone valuation, it loses the possibility of winning both licenses and enjoying the synergy. We skip the proof of this lemma.

When his average valuation which is defined by the half of total valuation for the two licenses including synergies: $\frac{v_{1A}+v_{1B}+\alpha}{2}$ exceeds 1; that is, the synergy is large enough, the global bidder will bid up to his average valuation, $\frac{V_1}{2}$. This will shut out the local bidders since local bidders' stand alone valuation can be at most 1.[27]

To calculate the optimal drop out price, at the beginning of the auction, the global bidder must compare the payoffs for two cases: **Case 1** is the payoff from dropping out of license B auction at price p and optimally continuing on license A auction. **Case 2** is the payoff from winning license B at price p and optimally continuing on license A auction.[28] The price that the global bidder is indifferent between these two cases (if the two local bidders are still active) is the optimal drop out price. Let p_1^* denote this price. We will show the uniqueness of p_1^* by using the monotonicity of payoff fuctions. Note that

[26]Note that this lemma is also valid for the general case when there are two global bidders.
[27]This is, in a way, a large synergy case and coincides with Albano et. al. (2006) paper.
[28]Remember that we assume $v_{1A} > v_{1B}$; hence, the global bidder will drop out from license B first -assuming that he has not won license A yet- .

according to Lemma 11, $p_1^* > v_{1B}$ and the optimal [updated] drop out price for license A -after winning license B at price p- is $v_{1A} + \alpha$.[29]

We denote the expected profit of Firm 1 for Case 1 by $E\Pi_1^1$ and the expected profit for Case 2 by $E\Pi_1^2$, respectively. The superscript represents which case Firm 1 plays and the subscript represents the global bidder, Firm 1. We have:

$$E\Pi_1^1 = Max\{0, \int_{p_1^*}^{v_{1A}} (v_{1A} - v_{2A}) f(v_{2A}|p_1^*) dv_{2A}\} \tag{16}$$

$$E\Pi_1^2 = \int_{p_1^*}^{Min\{v_{1A}+\alpha,1\}} (V_1 - p_1^* - v_{2A}) f(v_{2A}|p_1^*) dv_{2A} + \int_{Min\{v_{1A}+\alpha,1\}}^{1} (v_{1B} - p_1^*) f(v_{2A}|p_1^*) dv_{2A} \tag{17}$$

The explanation of equation 16 is as follows. After the global bidder drops out of the auction for license B at p_1^*, he will continue to stay in the auction for license A until v_{1A}. If he wins, he will pay v_{2A} since the local bidder will drop out at his valuation v_{2A}. In order to calculate his expected profit, he will be using $f(v_{2A}|p_1^*)$ which is the density of the local bidder's valuation for license A given p_1^*.

The first term of $E\Pi_1^2$ is Firm 1's expected profit of winning two licenses (assuming that he wins license B at the price p_1^*). If Firm 2's valuation v_{2A} is less than Firm 1's (updated) willingness to pay, $v_{1A} + \alpha$, then Firm 1 wins license A and pays v_{2A}. Since $v_{2A} < 1$, we use the minimum function in the upper limit of the first integral. The second term of $E\Pi_1^2$ is Firm 1's expected profit of winning only license B which can happen only if

[29] The global bidder's updated price is found by equation these two equations: $v_{1A} + v_{1B} + \alpha - v_{3B} - p = v_{1B} - v_{3B} \Rightarrow p = v_{1A} + \alpha$. The left hand side of the first equation is his payoff from winning license A at price p and B at price v_{3B}. The right hand side is his payoff from dropping out from license A auction after winning license B.

$v_{2A} > v_{1A} + \alpha$. Note that the second term is negative by lemma 11 (which is the exposure problem arising from winning only one license) unless $v_{1A} + \alpha > 1$.

By equating equation 16 and 17 and assuming that $f(.)$ is a uniform distribution, the optimal drop-out prices are given in lemma 12.

Lemma 12

$$p_1^* = \begin{cases} \frac{1}{2}\{v_{1B} + \alpha + 1 - (v_{1B}^2 + 1 - 2v_{1B} - \alpha^2 + 2v_{1B}\alpha + 2\alpha - 4v_{1A}\alpha)^{\frac{1}{2}}\}, \\ \qquad \text{if } 0 < v_{1A} < 1-\alpha \text{ and } 2(1-v_{1A})(v_{1A}-v_{1B}) > \alpha^2; \\ \frac{1}{3}\{v_{1A} + v_{1B} + \alpha + 1 - ((v_{1A} + v_{1B} + \alpha + 1)^2 - 3(v_{1A}+\alpha)^2 - 6v_{1B})^{\frac{1}{2}}\}, \\ \qquad \text{if } 0 < v_{1A} < 1-\alpha \text{ and } 2(1-v_{1A})(v_{1A}-v_{1B}) \leq \alpha^2; \\ \frac{1}{2}\{v_{1B} + \alpha + 1 - \{(v_{1B}+\alpha+1)^2 - 4(v_{1A}+v_{1B}+\alpha) + 2 + 2v_{1A}^2\}^{\frac{1}{2}}\}, \\ \qquad \text{if } 1-\alpha \leq v_{1A} < 1 \text{ and } 1 + v_{1A} > 2(v_{1B}+\alpha); \\ \frac{2(v_{1A}+v_{1B}+\alpha)-1}{3}, \\ \qquad \text{if } 1-\alpha \leq v_{1A} < 1 \text{ and } 1 + v_{1A} \leq 2(v_{1B}+\alpha). \end{cases} \tag{18}$$

The optimal drop-out price is a function that takes a unique value for each case defined in the lemma above. For example, case $0 < v_{1A} < 1-\alpha$ and $2(1-v_{1A})(v_{1A}-v_{1B}) > \alpha^2$ implies that $p_1^* < v_{1A}$. The proof is in the appendix.

In Lemma 13 below, we characterize the global bidder's optimal bids depending on the information available to the players. We have already calculated the optimal drop out price. Different cases can arise depending on whether the global bidder wins license A or license B first.

Lemma 13 *:*

A)Firm 1 will stay in the auctions for license A and license B until the optimal drop-out price p_1^* (as long as any of the local bidders did not drop out).

B)Firm 1 will drop out of the auction for license B at p_1^* and continue to stay in the auction for license A until its stand-alone valuation v_{1A}, if $v_{1A} > p_1^*$. Otherwise, it will also *drop out from license A auction.*

C) If Firm 1 wins license B first (that is, $v_{3B} < v_{2A}$ and $v_{3B} \leq p_1^*$), then it will continue to stay in the auction for license A until $v_{1A} + \alpha$.

D) If Firm 1 wins license A first, (that is, $v_{2A} < v_{3B}$ and $v_{2A} \leq p_1^*$), then it will continue to stay in the auction for license B until $v_{1B} + \alpha$.

Proof. See the Appendix.

In part c of lemma 13, we characterize what will happen if the global bidder wins license B (at v_{3B}). As price p increases, Firm 1 will choose to continue for license A only if $v_{1A} + v_{1B} + \alpha - v_{3B} - p$ is greater than the drop-out payoff which is $v_{1B} - v_{3B}$. In other words, the firm will continue until price p becomes $v_{1A} + \alpha = p$.

We are ready to summarize our Perfect Bayesian equilibrium.

Proposition 14 *(Perfect Bayesian Equilibrium)*

a) Out-of-equilibrium-path beliefs: If the global bidder, Firm 1, drops out of license A before license B then the local bidder, Firm 3, believes that Firm l will act like a local bidder and bid at most 1 on license B.

b) Lemma 10,11,12, and 13 and out of equilibrium path beliefs constitute a Perfect Bayesian Nash Equilibrium.

At the beginning of the game, each firm calculates its optimal drop-out price. For local bidder A and local bidder B, the optimal drop out prices are their valuations. In equilibrium, it is optimal for a global bidder to stay in the auctions for both licenses up to

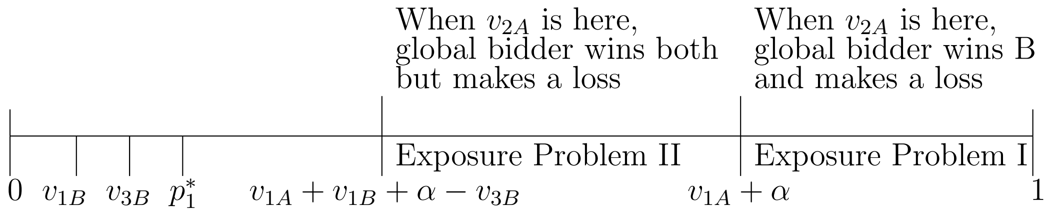

Figure 2: EXPOSURE PROBLEM

his optimal drop-out price calculated in lemma 12 when his average valuation is below 1 or to his average valuation when his average valuation exceeds 1. When the price reaches the minimum of these prices, one firm drops out of license auction. If, for example, Firm 3 dropped out before Firm 1 such that $v_{1B} > v_{3B}$, Firm 1 would continue to stay in the auction for license A until the price reaches $v_{1A} + \alpha$. At this price, it finds that the payoff from winning only license B is more than winning both licenses even though it will enjoy synergy; hence, it drops out.

If the licenses were identical (e.g. Albano et. al. (2001)), the global firm would drop out from both licenses at the same time.

The optimal drop out price calculated at the beginning of the game (or whenever both local bidders are active) will be increasing as the global bidder's stand-alone valuations, v_{1A}, v_{1B} and/or synergy, α, increases, respectively.

Corollary 15 *: Firm 1's drop-out price p_1^* will increase as α, v_{1B} and/or v_{1A} increases.*

This is an expected result. As the total valuation of the global bidder increases, he bids higher.

4.2.2 Exposure Problem

Now we can discuss the exposure problem with the help of figure 2. In the first type of exposure problem, the global bidder may win license B at a price above his stand alone valuation (i.e., $v_{1B} < v_{3B} < p_1^*$) and loses the other license (i.e., $v_{2A} > v_{1A} + \alpha$, since he will continue to bid until $v_{1A} + \alpha$). This is the type of exposure problem Chakraborty (2004) focuses on.

In the second type of exposure problem, the global bidder wins both licenses but makes a loss. This is the case when he wins license B at $v_{1B} < v_{3B} < p_1^*$ and wins license A at $v_{1A} + \alpha > v_{2A} > v_{1A} + \alpha + v_{1B} - v_{3B}$. Note that if he wins license A at the price $v_{1A} + \alpha + v_{1B} - v_{3B}$, his payoff is zero. The global bidder stays in the auction for license A to minimize its loss once the price passes $v_{1A} + \alpha + v_{1B} - v_{3B}$.

If objects were homogenous, second type of exposure problem would not be observed since the bidder would drop out from both license auctions at the same time.

In the next section, we will show that bid-withdrawal eliminates the exposure problem.

4.2.3 A Special Case With Bid Withdrawal Rule

Now we assume that the bidder can withdraw its standing high bid without any penalty. We will assume that if a firm withdraws its bid, the seller will immediately re-auction the license to the -same- bidders. [30]

[30] In the recent Canadian license auction, the auctioneer could start the auction for the re-auctioned license at any price it finds appropriate. So we interpret that the starting price will be zero. We will show that if the global bidder withdraws one license, then this license will be re-auctioned after both license auctions are completed. In such a re-auction, the global bidder will act as a local bidder; hence, the actual selling price of the license will be v_{1B}.

We denote by p_1^{**} the optimal drop-out price for license B of Firm 1 for the auction with bid-withdrawal.

First, we find the optimal drop-out price. At the price, p_1^{**}, the firm has to be indifferent between dropping out from license B auction (**Case 1**), and winning license B. (**Case 2**).

We denote the expected profit of Firm 1 when playing Case 1 by $E\Pi_1^1$ and the expected profit of Firm 1 when playing Case 2 by $E\Pi_1^2$, respectively.

$$E\Pi_1^1 = Max\{0, \int_{p_1^{**}}^{v_{1A}} (v_{1A} - v_{2A}) f(v_{2A}|p_1^{**}) dv_{2A}\} \tag{19}$$

$$E\Pi_1^2 = \int_{p_1^{**}}^{v_{1A}+v_{1B}+\alpha-p_1^{**}} (v_{1A} + v_{1B} + \alpha - p_1^{**} - v_{2A}) f(v_{2A}|p_1^{**}) dv_{2A} \tag{20}$$

After the global bidder drops out of the auction for license B at p_1^{**}, he will continue to stay in the auction for license A until v_{1A} when $v_{1A} > p_1^{**}$. If he wins, he will pay v_{2A}. Thus, $E\Pi_1^1$ in equation 19 is Firm 1's expected payoff from winning license A after dropping out of license B auction. $E\Pi_1^2$ in equation 20 is Firm 1's expected payoff after winning license B at price p_1^{**} and continuing optimally. If Firm 2's valuation v_{2A} is less than Firm 1's [updated] willingness to pay, $v_{1A}+v_{1B}+\alpha-p_1^{**}$, then Firm 1 wins license A at the price v_{2A}. In this case, Firm 1 will win two licenses with positive profits. If $v_{2A} > v_{1A}+v_{1B}+\alpha-p_1^{**}$, then Firm 1 will drop out of license A auction and withdraw its bid from license B auction at the price $v_{1A} + v_{1B} + \alpha - p_1^{**}$. This will give him a zero payoff.

By equating equations 19 and 20, the optimal drop-out prices is given in lemma 16.

Lemma 16

$$p_1^{**} = \begin{cases} v_{1B} + \alpha, & \textit{if } v_{1A} > v_{1B} + \alpha; \\ \frac{v_{1A} + v_{1B} + \alpha}{2} & \textit{if } v_{1A} \leq v_{1B} + \alpha. \end{cases} \tag{21}$$

Proof. See the Appendix.

The maximum value of license B with synergy is $v_{1B}+\alpha$; hence, the bidder will continue to stay in the auction until this price when $v_{1A} > v_{1B} + \alpha$. If the condition does not hold, the bidder will bid, at most, half of his total valuation.

In Lemma 17 below, we characterize the global bidder and the local bidders' optimal drop out prices (i.e., bids). Note that the global bidder's optimal continuation price after winning license B depends on whether it makes profit from license B or not. If it makes a profit (that is $v_{1B} - v_{3B} > 0$), then it will continue to stay in license A auction until $v_{1A} + \alpha$. If it makes a loss, it will stay until $v_{1B} + v_{1A} + \alpha - v_{3B}$. At that price, it should also withdraw its bid on license B guaranteeing a zero profit. We skip its proof.

Lemma 17 *:*

A) Firm 1 will stay in the auctions for license A and license B until the optimal drop-out price p_1^{**}*. The local bidders will stay in the auction until their valuations.*

B) Firm 1 will drop out of the auction for its lower valuation license B at p_1^{**} *and continue to stay in the auction for license A until its stand-alone valuation* v_{1A}*, if* $v_{1A} > p_1^{**}$*; otherwise, it will drop out from license A auction.*

C) i) If Firm 1 wins license B first and $v_{1B} - v_{3B} < 0$*, then it will continue to stay in the auction for license A until valuation* $v_{1A} + v_{1B} + \alpha - v_{3B}$*.*

ii) If Firm 1 wins license B first and $v_{1B} - v_{3B} > 0$, then it will continue to stay in the auction for license A until valuation $v_{1A} + \alpha$.

D) i) If Firm 1 wins license A first and $v_{1A} - v_{2A} < 0$, then it will continue to stay in the auction for license B until valuation $v_{1A} + v_{1B} + \alpha - v_{2A}$.

ii) If Firm 1 wins license A first and $v_{1A} - v_{2A} > 0$, then it will continue to stay in the auction for license B until valuation $v_{1B} + \alpha$.

We would like to analyze when the global firm will use bid withdrawal in this auction.

Proposition 18 *:*

a) If $v_{1B} < v_{3B} < p_1^{**}$ and $v_{1A} + v_{1B} + \alpha - v_{3B} < v_{2A}$, the global bidder will withdraw its bid for license B and drop out from license A auction.

b) If $v_{1A} < v_{2A} < p_1^{**}$ and $v_{1A} + v_{1B} + \alpha - v_{2A} < v_{3B}$, the global bidder will withdraw its bid for license A and drop out from license B auction.

Now, we show that the optimal drop out price with bid withdrawal is higher compared to the optimal drop out price of no bid withdrawal case. If there is no risk of losing money, the bidder may bid more aggressively.

Proposition 19 *:*

The global bidder's optimal drop out price in an auction without bid withdrawal rule is lower than that in an auction with bid withdrawal rule.

Proof. See the Appendix.

Since the synergies between licenses induce the global bidder bid aggressively in order to win both licenses, the global bidder will bid higher than his stand-alone valuation for a single license. With bid-withdrawal, the global bidder will drop-out of license B later compared to an auction without bid withdrawal. This higher drop-out price of license B will both increase the probability of winning license B and the expected winning price of license B. Both effects increase the government's expected revenues. On the other hand, the elimination of exposure problem in the auction may decrease the seller's revenue. The global bidder drops out from the remaining auction at $v_{1A} + v_{1B} + \alpha - v_{3B}$ for license A (instead of $v_{1A} + \alpha$). This decreases the seller's revenue from the auction with bid-withdrawal.

If the global bidder withdraws its bid from one license, then, our rule is that it will be re-auctioned. Say the global bidder withdrew its bid from license B. In the re-auction, the global bidder, since it lost license A, will stay in the auction until v_{1B}, and the local bidder will bid up to his valuation v_{3B}. We also know that since the global bidder withdrew its bid, it must be making a loss on license B; that is, $v_{1B} < v_{3B}$. In this case, the selling price would be v_{1B}.[31]

Unfortunately, the expected revenue functions are too complicated to compare algebraically. Because of that, we run simulations with MATLAB.[32] In figure 3, we find the revenue for different synergy levels. For each synergy level, we draw 40000 observations. From these observations, we only take the values with $v_{1A} > v_{1B}$; hence, approximately, we are left with 20000 observations. Out of these 20000 observations, we only select the

[31]We also could have put an arbitrary rule in which license B is offered to the local bidder at half of his valuation, $\frac{v_{3B}}{2}$, since his valuation can be deduced from his bidding in the previous auction. Our simulations with this rule also gave higher revenue for the auction with bid-withdrawal. Also, let us note that, in such a case, the local bidder does not find it optimal to drop earlier than its valuation with the hope that the global bidder may withdraw its bid later.

[32]This program and the program for the general case in the next section are available on H. Gunay's web-site at http://home.cc.umanitoba.ca/ gunay/ The latter program is very long.

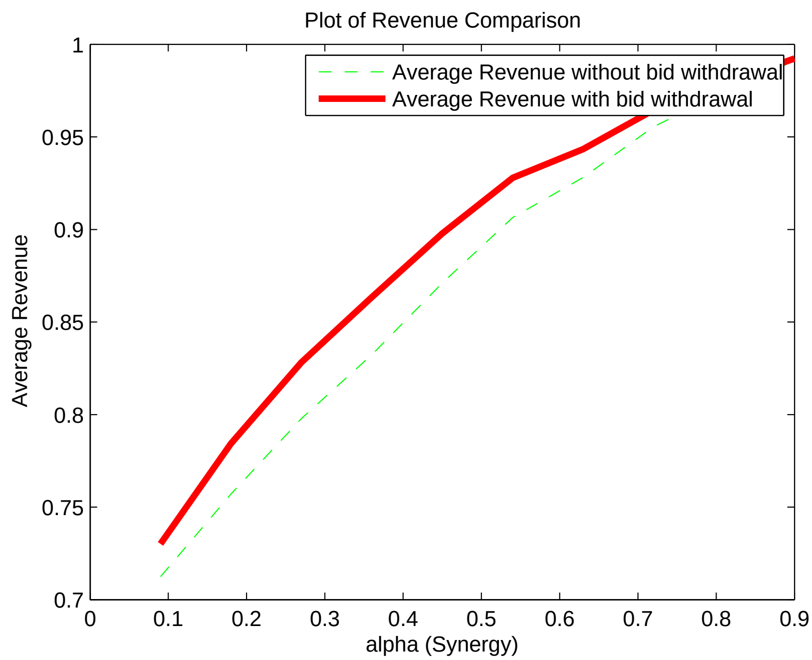

Figure 3: Revenue with bid withdrawal is higher.

moderate and small synergy cases; that is, $v_{1A} + v_{1B} + \alpha < 2$. We calculate the revenue for each qualifying draw and find the average of these revenues. The auction with bid withdrawal rule gives a higher revenue.

4.2.4 Efficiency

We find that an auction with bid withdrawal rule is more efficient than an auction without bid withdrawal rule. The efficient auction is the auction design in which the licenses will be awarded to the bidders who value them most. We discuss this in three cases: **Case 1**, if the local bidders win two licenses respectively or the global bidder wins one or both without a loss, then these two auctions with or without bid withdrawal rule is equivalent in terms of efficiency. **Case 2**, If the global bidder wins both licenses with a loss, an auction without bid withdrawal rule is inefficient. The prices of licenses the global bidder paid are the local bidders' valuations. These prices are higher than the global bidder's valuation. However, this case can not happen in an auction with bid withdrawal rule. Since the global bidder will either win licenses with non-negative profits or withdraw his bid when there is a loss. Thus, the licenses will be allocated to the local bidders who have the highest valuations. **Case 3**, If the global bidder wins one license with a loss, an auction with bid withdrawal rule is more efficient than an auction without bid withdrawal rule. With bid withdrawal rule, the global bidder can withdraw his bid to avoid this loss. Hence, the license will be reallocated to the local bidder who value it most.

4.3 A General Case with 2 Global Bidders and 2M Local Bidders

In this section, we assume that there are two global bidders and $2m$ local bidders. Half of the local bidders are interested in receiving license A and the other half are interested in receiving license B. Their valuations are given as v_{ji} where $j = 3, 4, .., m + 2$ denote the local bidder firms and $i = A, B$ denote the license they are interested in. We will use firm 1 and firm 2 for the global bidders. We still keep the assumption that global bidders value license A more than license B; i.e, $v_{1A} \geq v_{1B}$ and $v_{2A} \geq v_{2B}$. We mainly look for the moderate synergy cases in which $0 < \frac{v_{1A}+v_{1B}+\alpha}{2} < 1$. We assume that the synergies are common knowledge.[33]

4.3.1 A General Case Without Bid Withdrawal Rule

Since the local bidders will not drop out until the price reaches their valuation, we concentrate on finding the global bidders' optimal strategy. We will show how to find Firm 1's drop out price. Firm 2's drop out price can be calculated symmetrically. As in the previous section, we have to compare the expected profit, $E\Pi_1^1$, in Case 1 (dropping out

[33] The case in which $\alpha > 1$ is analyzed by Albano et. al (2006).

without winning license B) and the expected profit, $E\Pi_1^2$, in Case 2 (winning license B). The equations below show these expected profits in an auction without bid withdrawal rule. In the equations, p_A denotes the price of license A, if Firm 1 wins license A. The function $g(p_A|p_1^*)$ denotes the density of p_A given p_1^*, the drop out price of Firm 1.

$$E\Pi_1^1 = Max\{0, \int_{p_1^*}^{v_{1A}} (v_{1A} - p_A) g(p_A|p_1^*) d(p_A)\} \tag{22}$$

$$E\Pi_1^2 = \int_{p_1^*}^{Min\{v_{1A}+\alpha,1\}} (V_1 - p_1^* - p_A) g(p_A|p_1^*) dp_A + \int_{Min\{v_{1A}+\alpha,1\}}^{1} (v_{1B} - p_1^*) g(p_A|p_1^*) dp_A \tag{23}$$

We have $p_A = \max\{B_2^A, v_{3A},...,v_{(2+m)A}\}$, where B_2^A represents Firm 2's (i.e., the other global bidder's) valuation of license A. If firm 2 does not win license B, then $B_2^A = v_{2A}$. If it wins (or has a chance to win) license B, then $B_2^A = v_{2A} + \alpha$.

If $B_2^A = v_{2A}$, the distribution function $G(p_A|p_1^*) = (F(p_A|p_1^*))^{m+1} = (\frac{p_A - p_1^*}{1-p_1^*})^{m+1}$ and the density function $g(p_A|p_1^*) = (m+1)F(p_A|p_1^*)^m f(p_A|p_1^*) = (\frac{m+1}{1-p_1^*})(\frac{p_A-p_1^*}{1-p_1^*})^m$ since v_{2A} is uniformly distributed on $[0,1]$.

If $B_2^A = v_{2A} + \alpha$, then B_2^A has the uniform density functions on the interval $[\alpha, 1+\alpha]$. In this case, the corresponding conditional density function, $g(p_A|p_1^*)$, will be:

$$g(p_A|p_1^*) = \begin{cases} 0, & \text{if } p_1^* \le p_A < \alpha; \\ \frac{m+1}{1-\alpha}(\frac{p_A-\alpha}{1-\alpha})^m, & \text{if } p_1^* \le \alpha \le p_A \le 1; \\ \frac{m+1}{1-p_1^*}(\frac{p_A-p_1^*}{1-p_1^*})^m, & \text{if } \alpha < p_1^* \le p_A < 1; \\ \frac{1}{\alpha}, & \text{if } 1 \le p_A < \alpha+1; \\ 0, & \text{Otherwise.} \end{cases} \tag{24}$$

We calculate the optimal drop-out price from the following fundamental equation.

$$E\Pi_1^1 = E\Pi_1^2 \tag{25}$$

Note that the equation is dynamic in the sense that it changes as the other bidders drop out. Also, the equations will change as the density function $g(.)$ takes different values. Hence, we have to calculate equation 25 for different subcases.

SUBCASE I: If the other global bidder is still active in the license B auction and when $(m+2)(\alpha - v_{1B})(1-\alpha)^{m+1} \geq v_{1A}^{m+2} - (v_{1A}-\alpha)^{m+2}$ holds, equation 25 will be as follows.

$$\int_{\alpha}^{v_{1A}} (v_{1A} - p_A)(\frac{m+1}{1-\alpha})(\frac{p_A-\alpha}{1-\alpha})^m dp_A = \int_{p_1^*}^{Min\{v_{1A}+\alpha,1\}} (V_1 - p_1^* - p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A$$
$$+[\int_{Min\{v_{1A}+\alpha,1\}}^{1} (v_{1B} - p_1^*)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A] \tag{26}$$

In our proofs, we show that our assumption for this subcase implies that $p_1^* \leq \alpha$. We also know that $\alpha < p_A$ should hold since the other global bidder is active in license B auction. In short, for $p_1^* \leq p_A < \alpha$, we have $g(.) = 0$ by equation 24. This is why we start the lower limit of the first integral from α. Hence, we have $p_1^* \leq \alpha \leq p_A$ which implies that $g(p_A|p_1^*) = (\frac{m+1}{1-\alpha})(\frac{p_A-\alpha}{1-\alpha})^m$.

The right hand side of equality assumes Firm 1 wins license B. Hence, we use $g(.)$ when $B_2^A = v_{2A}$.

SUBCASE II: If the other global bidder drops out of license B auction or when $(m+2)(\alpha - v_{1B})(1-\alpha)^{m+1} \leq v_{1A}^{m+2} - (v_{1A}-\alpha)^{m+2}$ and $(m+2)(v_{1A}-v_{1B})(1-v_{1A})^{m+1} \geq \alpha^{m+2}$ (which implies that $\alpha \leq p_1^* \leq p_A$ and $p_1^* \leq v_{1A}$) global bidder 1 will use the following equation to calculate its drop out price from license B,

$$\int_{p_1^*}^{v_{1A}} (v_{1A} - p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A = \int_{p_1^*}^{Min\{v_{1A}+\alpha,1\}} (V_1 - p_1^* - p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A$$
$$+ \int_{Min\{v_{1A}+\alpha,1\}}^{1} (v_{1B} - p_1^*)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A \quad (27)$$

SUBCASE III: When $\alpha \geq v_{1A}$, or when $\alpha \leq v_{1A}$ and $(m+2)(v_{1A}-v_{1B})(1-v_{1A})^{m+1} \leq \alpha^{m+2}$ (the latter condition implies that $p_1^* \geq v_{1A}$ and $(m+2)(\alpha - v_{1B})(1-\alpha)^{m+1} \leq v_{1A}^{m+2} - (v_{1A} - \alpha)^{m+2}$ implies $p_1^* \geq \alpha$), global bidder 1 will use the following equation to calculate its drop out price p_1^*.

$$0 = \int_{p_1^*}^{Min\{v_{1A}+\alpha,1\}} (V_1 - p_1^* - p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A +$$
$$\int_{Min\{v_{1A}+\alpha,1\}}^{1} (v_{1B} - p_1^*)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A \quad (28)$$

There are no analytic solutions for these cases; however, we show that a unique optimal drop out price exists for each case. The propositions below show how the global bidder will behave during the auction.

Proposition 20 *:Assuming that $v_{1A} + \alpha < 1$.*

A) i)If $(m+2)(\alpha - v_{1B})(1-\alpha)^{m+1} \geq v_{1A}^{m+2} - (v_{1A} - \alpha)^{m+2}$ and the other global bidder is active in the license B auction, Firm 1's optimal drop out price will be calculated by using equation 26. The unique optimal drop out price will be such that $p_1^* \leq \alpha \leq v_{1A}$.

ii) If $(m+2)(\alpha - v_{1B})(1-\alpha)^{m+1} \leq v_{1A}^{m+2} - (v_{1A} - \alpha)^{m+2}$ and $(m+2)(v_{1A} - v_{1B})(1 - v_{1A})^{m+1} > \alpha^{m+2}$, or the other global bidder drops out of license B auction, Firm 1's optimal *drop out price will be calculated by using equation 27. The unique optimal drop out price will be such that* $\alpha \leq p_1^* \leq p_A$ *and* $p_1^* \leq v_{1A}$.

B) When

i) $v_{1A} \leq \alpha$ *or*

ii) $v_{1A} \geq \alpha$ *and* $(m+2)(v_{1A} - v_{1B})(1 - v_{1A})^{m+1} \leq \alpha^{m+2}$,

Firm 1's optimal drop out price will be calculated by using equation 28. The unique optimal drop out price will be $p_1^* \geq v_{1A}$ *for case ii.*

The equations above are dynamic in the sense that as the other bidders drop out, the global bidder revises its optimal drop out price -to a higher level-. This seemingly contradictory result (i.e., with less competition, a higher price is bid) is first shown by Krishna and Rosenthal (1996) and later tested with experiments by Chow and Yavas (2009) in a second-price sealed bid environment.

Now we will write the proposition for the case $v_{1A} + \alpha > 1$.

Proposition 21 *:Assuming that* $v_{1A} + \alpha > 1$.

A) i) $(m+2)(1-\alpha)^{m+1}(v_{1A} + v_{1B}) \leq (m+1+\alpha)(1-\alpha)^{m+1} + (v_{1A} - \alpha)^{m+2}$ *and the other global bidder is active in the license B auction, Firm 1's optimal drop out price will be calculated by using equation 26. The unique optimal drop out price will be such that* $p_1^* \leq \alpha \leq v_{1A}$.

ii) $(m+2)(1-\alpha)^{m+1}(v_{1A} + v_{1B}) \geq (m+1+\alpha)(1-\alpha)^{m+1} + (v_{1A} - \alpha)^{m+2}$ *and* $(m+2)(v_{1B} + \alpha) \leq (m+1) + v_{1A}$, *or the other global bidder drops out of license B auction,* *Firm 1's optimal drop out price will be calculated by using equation 27. The unique optimal drop out price will be such that* $\alpha \leq p_1^* \leq p_A$ *and* $p_1^* \leq v_{1A}$.

B) When

i) $v_{1A} \leq \alpha$ *or*

ii)$v_{1A} \geq \alpha$ *and* $(m+2)(v_{1B}+\alpha) \geq (m+1)+v_{1A}$,

Firm 1's optimal drop out price will be calculated by using equation 28. The unique optimal drop out price will be $p_1^* \geq v_{1A}$ *for case ii.*

In the proposition below, we explain how the global bidder will behave after winning license B or after dropping out from license B auction (without winning it).

Proposition 22 *:*

A) (Global bidder drops out of license B) If Firm 1 drops out of the auction for license B, it will continue to stay in the auction for license A until the price reaches v_{1A}*, if* $v_{1A} \geq p_1^*$*; otherwise, it will drop out from license A auction.*

B) (Global bidder wins License B first) If Firm 1 wins license B at the price p_B *(i.e.,* $p_B \leq p_1^*$ *and* $p_B \leq p_A$*) then it will continue to stay in the auction for license A until valuation* $v_{1A}+\alpha$.

C) (Global bidder wins license A first) If Firm 1 wins license A first, (i.e., $p_A \leq p_1^*$ *and* $p_A \leq p_B$*) then it will continue to stay in the auction for license B until valuation,* $v_{1B}+\alpha$

Now, we can state our equilibrium.

Proposition 23 *(Perfect Bayesian Nash Equilibrium)*

A) Out-of-equilibrium-path beliefs: If a global bidder drops out of license A before license B at the price p, all other bidders will believe that this global bidder's valuation of B, v_{1B}, is uniformly distributed on $[0, 1]$.

B) Lemma 10,11, and proposition 20, 21, 22 and the out of equilibrium path beliefs constitute a Perfect Bayesian Nash Equilibrium.

At the beginning of the game, each global firm calculates its optimal drop-out price p_i^*. For local bidders, the optimal drop out price is equal to their valuations. When the price reaches the minimum of these prices, one firm drops out of license B (or A) auction. The global bidders will update their optimal drop out prices p_i^* and will continue to stay in the auction for both licenses until their new p_i^*. Note that a global bidder will drop out of the auction for one license with lower valuation at his optimal drop-out price. In equilibrium, it is optimal for a global bidder to stay in the auctions for both licenses up to his optimal drop-out price when his average valuation is below 1.[34]

4.3.2 A General Case With Bid Withdrawal Rule

Now we consider a bid withdrawal case in which the bidder can withdraw its standing high bid without any penalty. We denote by p_1^{**} the optimal drop out price for license B of Firm 1.

In the proposition below, we describe the optimal strategies. Note that with bid-withdrawal, the global bidder will stay in the license auction up to $v_{1B} + \alpha$ which is the

[34] A global bidder should stay until his average valuation when his average valuation exceeds 1.

maximum possible amount if $v_{1A} > v_{1B} + \alpha$.

Proposition 24 :

A) If $v_{1A} \geq v_{1B} + \alpha$, then $p_1^{**} = v_{1B} + \alpha$.

B) If $\alpha \leq v_{1A} \leq v_{1B} + \alpha$ or $v_{1A} \leq \alpha$, then $p_1^{**} = \frac{v_{1A}+v_{1B}+\alpha}{2}$.

C) (Global bidder drops out of license B) If Firm 1 drops out of the auction for license B, it will continue to stay in the auction for license A until the price reaches v_{1A}, if $v_{1A} \geq p_1^{**}$; otherwise, it will drop out from license A auction.

D) (Global bidder wins License B first) If Firm 1 wins license B at the price p_B such that $v_{1B} \leq p_B$ then it will continue to stay in the auction for license A until valuation $v_{1A} + v_{1B} + \alpha - p_B$; when $p_B \leq v_{1B}$, then it will continue to stay in the auction for license A until valuation $v_{1A} + \alpha$.

E) (Global bidder wins license A first) If Firm 1 wins license A at the price p_A such that $v_{1A} \leq p_A$, then it will continue to stay in the auction for license A until valuation $v_{1A} + v_{1B} + \alpha - p_A$; when $p_A \leq v_{1A}$, then it will continue to stay in the auction for license B until valuation $v_{1B} + \alpha$.

The elimination of exposure problem will increase the global bidders' optimal drop out price; they will bid more aggressively *initially*.

Proposition 25 : *The global bidder's optimal drop out price is lower in an auction without bid withdrawal rule than that in an auction with bid withdrawal rule.*

Proof. See the Appendix. Now we discuss when the global bidder withdraws its bid.

Firm 1 will withdraw his standing high bid on license B if it is making a loss on license B; that is, $p_B > v_{1B}$. It will withdraw its bid, when it drops out of license A auction. The highest price that Firm 1 will pay for license A (after winning license B at price p_B) is $p_A = v_{1A} + v_{1B} + \alpha - p_B$ which makes its profit of winning both licenses 0. When the clock price reaches to this maximum price, the global bidder will drop out and withdraw its bid.

Symmetrically, we can find the condition to withdraw from license A auction.

Proposition 26 *(Bid Withdrawal)*

A) If the global bidder wins License B first at $p_B > v_{1B}$ then the global bidder will withdraw from license B at $v_{1A} + v_{1B} + \alpha - p_B$.

B) If the global bidder wins License A first, $p_A > v_{1A}$ then the global bidder will withdraw from license A at $v_{1A} + v_{1B} + \alpha - p_A$.

What happens when a bid is withdrawn on a license? We will assume that this license will be offered to the second highest bidder at his drop out price as a take it or leave it offer.[35] If he does not accept, it will be offered to the third highest bidder at her drop out price and so on. If nobody accepts, it will be re-auctioned. The second highest bidder, if this is a local bidder, will accept the license at the offered price. Actually, this bidder is indifferent between accepting or rejecting the offer; however, we assume as a tie-breaking rule that it will accept.[36]

The bid-withdrawal and the re-assigning rule we describe will have an effect on the

[35] We can assume that the seller leaves this bidder a very small positive payoff.

[36] By applying backward induction, we can deduce that the second highest "local" bidder will accept the offer especially if it is given a very small payoff. If it does not accept, the third highest local bidder should definitely accept the offer.

seller's revenue. If the global bidder withdraws his high bid and if the second highest bidder is a local bidder, then the revenue will not change. This is because the withdrawn price is the valuation of this local bidder. However, if the second highest bidder on the license is the other global bidder, it may not accept the license, if offered at his drop out price. Remember that the global bidder's drop out price is greater than his stand alone valuation.[37] Hence, the license will be offered to the local bidder (which was the third highest bidder) at her drop out price which is less than the withdrawn price. In this case, allowing bid-withdrawal will decrease seller's revenue.

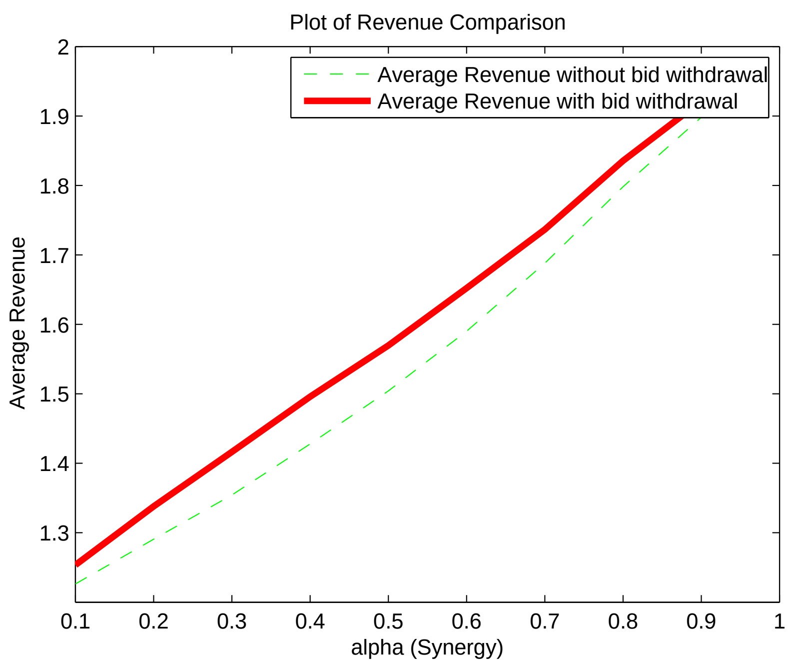

Figure 4: Revenue with bid withdrawal is higher.

Our simulations in figure 4 show that revenue is still higher with an auction with bid-withdrawal. We assumed two global bidders and one local bidder for each license for this simulation. We draw 40000 observations for each α and select draws that satisfies our assumptions (i.e., $v_{iA} > v_{iB}$ for $i = 1, 2$). Moreover, we looked for small/moderate synergy

[37] One case the the global bidder may accept the offer is if it is the one who win license A. Then, suddenly, it finds himself in a position to win both licenses and enjoy synergy although it dropped out from license B auction before.

cases in which $v_{iA} + v_{iB} + \alpha < 2$, for $i = 1, 2$.

4.4 Conclusion and Discussion

Our conclusion is that an auction with bid-withdrawal is superior to an auction without bid-withdrawal given the environment of our paper.

We add to the literature in at least three ways. First, we show the optimal bidding strategies of global bidders when there are moderate synergies and the licenses are heterogeneous. Second, we study bid-withdrawal in multi-unit auctions theoretically for the first time (to our best knowledge). Third, we analyze the exposure problem extensively and show when it will arise even though the bidder wins all licenses. One of our main contribution is to show that the exposure problem is solved when bid-withdrawal is introduced. Elimination of the exposure problem makes the global bidder more aggressively at first; however, it bids less aggressively later in the remaining auction. As the net effect, we show that revenue is still higher.

More studies are needed to determine whether there are other superior bid-withdrawal rules. For example, one may study a bid-withdrawal rule in which the global bidder may pay a penalty. While this will be an interesting study, the analysis will be more complex when there are two global bidders.

4.5 Appendix

4.5.1 Proofs

Proof of Lemma 11: Let p be the drop-out price before the global bidder's lowest stand-alone value v_{1B}, that is, $p < v_{1B}$. When the global bidder drops out of bidding for the single license at p without winning it, then he will keep bidding for license A until the price reaches v_{1A}. The expected profits in this case are given by,

$$E\Pi_1(p) = \int_p^{v_{1A}} (v_{1A} - p_A) g(p_A|p) dp_A$$

When the global bidder keeps bidding and drops out of bidding for the license B until v_{1B}, then the expected profits are given by,

$$E\Pi_1(V_{1B}) = \int_p^{v_{1A}+\alpha} \int_p^{v_{1B}} (V_1 - p_A - p_B) g(p_B|p) g(p_A|p) dp_B dp_A$$

$$+ \int_{v_{1A}+\alpha}^{1} \int_p^{v_{1B}} (v_{1B} - p_B) g(p_B|p) g(p_A|p) dp_B dp_A$$

$$+ \int_p^{v_{1A}} \int_{v_{1B}}^{1} (v_{1A} - p_A) g(p_B|p) g(p_A|p) dp_B dp_A$$

Where p_A and p_B denote the prices of the given license A and B respectively, and are defined by $p_A = \max\{B_2^A, B_3^A\}$, and $p_B = B_2^B$, where B_i^j denote the bidder i's bid for license j. $g(p_A|p)$ denotes the probability density function of the highest bid for license A between global bidder 2 and local bidders given the current price p. Moreover, we assume that B_i^j is independently distributed on $[0, 1]$ with the distribution function $F(B_i^j)$ and the corresponding density function $f(B_i^j)$. $g(p_j|p)$ denotes the density of the highest bid for license A among two global bidders and m local bidders' bids equal to p_j given that the current price is p. Moreover, $g(p_j|p) = (m+1)(F(p_j|p))^m f(p_j|p) = (\frac{m+1}{1-p})(\frac{p_j-p}{1-p})^m$. In particular, let $p_B \leq v_{1B}$ when the global bidder wins license B. Then,

$E\Pi_1(V_{1B}) = \int_p^{v_{1A}+\alpha} \int_p^{v_{1B}} (V_1 - p_A - p_B) g(p_B|p) g(p_A|p) dp_B dp_A$

$+ \int_{v_{1A}+\alpha}^{1} \int_p^{v_{1B}} (v_{1B} - p_B) g(p_B|p) g(p_A|p) dp_B dp_A + \int_p^{v_{1A}} \int_{v_{1B}}^{1} (v_{1A} - p_A) g(p_B|p) g(p_A|p) dp_B dp_A$

$\geq \int_p^{v_{1A}} \int_p^{v_{1B}} (v_{1A} - p_A) g(p_B|p) g(p_A|p) dp_B dp_A + \int_p^{v_{1A}} \int_{v_{1B}}^{1} (v_{1A} - p_A) g(p_B|p) g(p_A|p) dp_B dp_A$

$+ \int_{v_{1A}}^{1} \int_p^{v_{1B}} (v_{1B} - p_B) g(p_B|p) g(p_A|p) dp_B dp_A + \int_p^{v_{1A}} \int_p^{v_{1B}} (v_{1B} - p_B) g(p_B|p) g(p_A|p) dp_B dp_A$

$= \int_p^{v_{1A}} \int_p^{1} (v_{1A} - p_A) g(p_B|p) g(p_A|p) dp_B dp_A + \int_p^{1} \int_p^{v_{1B}} (v_{1B} - p_B) g(p_B|p) g(p_A|p) dp_B dp_A$

$\geq \int_p^{v_{1A}} \int_p^{1} (v_{1A} - p_A) \frac{(m+1)(p_B - p)^m}{(1-p)^{m+1}} g(p_A|p) dp_B dp_A$

$= \int_p^{v_{1A}} (v_{1A} - p_A) g(p_A|p) dp_A = E\Pi_1(p)$

So we can conclude that,

$E\Pi_1(V_{1B}) \geq E\Pi_1(p)$

Thus, it is optimal for the global bidder to stay in both auctions until his lowest stand-alone valuation for a single license. Therefore, the expected profits from continuing to stay in the auction at least up to $\min\{v_{1A}, v_{1B}\}$ are greater than or equal to that from dropping out before $\min\{v_{1A}, v_{1B}\}$. So the global bidder prefers to stay in the auctions until the price reaches $\min\{v_{1A}, v_{1B}\}$.

When $\frac{V_1}{2} > 1$, the global bidder, Firm 1, can stay in the auction until $\frac{V_1}{2} > 1$ to shut out all the local bidders and then competes with the other global bidder only in the following rounds. In this case, Firm 1's behavior is similar to a local bidder's bidding strategy. According to Lemma 1, it is optimal for the global bidder to bid until his average valuation to win both or none.

■

Proof of Lemma 12: We will prove this by solving $E\Pi_1^1 = E\Pi_1^2$. We have four cases.

Case I: We restrict our attention for the case $v_{1A} \leq 1 - \alpha$ so that the lower limit of the second integral in $E\Pi_1^2$ is lower than its upper limit. We also assume that the optimal drop out price is such that $p_1^* < v_{1A}$ holds. Later, we show this holds when $2(1 - v_{1A})(v_{1A} - v_{1B}) > \alpha^2$. The condition $p_1^* < v_{1A}$ implies that $E\Pi_1^1 > 0$.

The equation:

$$E\Pi_1^1 = \int_{p_1^*}^{v_{1A}} (v_{1A} - v_{2A}) f(v_{2A}|p_1^*) dv_{2A}$$

$$= E\Pi_1^2 = \int_{p_1^*}^{v_{1A}+\alpha} (V_1 - p_1^* - v_{2A}) f(v_{2A}|p_1^*) dv_{2A} + \int_{v_{1A}+\alpha}^{1} (v_{1B} - p_1^*) f(v_{2A}|p_1^*) dv_{2A}$$

$$\Rightarrow 2(p_1^*)^2 - 2p_1^*(1 + v_{1B} + \alpha) + \alpha^2 + 2v_{1A}\alpha + 2v_{1B} = 0$$

gives the following two roots:

$$p_1^* = \frac{1}{2}\{v_{1B} + \alpha + 1 - (v_{1B}^2 + 1 - 2v_{1B} - \alpha^2 + 2v_{1B}\alpha + 2\alpha - 4v_{1A}\alpha)^{\frac{1}{2}}\} \tag{29}$$

$$p_+^* = \frac{1}{2}\{v_{1B} + \alpha + 1 + (v_{1B}^2 + 1 - 2v_{1B} - \alpha^2 + 2v_{1B}\alpha + 2\alpha - 4v_{1A}\alpha)^{\frac{1}{2}}\} \tag{30}$$

The optimal drop-out price cannot exceed the global bidder's average valuation. However, in equation 30, we have $p_+^* \geq \frac{V_1}{2}$ since $v_{1B} + \alpha + 1 +$ *some positive constant* is greater than $V_1 = v_{1B} + \alpha + v_{1A}$. Note that v_{1A} can be at most 1. Hence, we rule out this root.

To show that $2(1 - v_{1A})(v_{1A} - v_{1B}) > \alpha^2 \Rightarrow p_1^* < v_{1A}$, we will prove the negation of the latter logical statement implies the negation of the former logical statement.

$p_1^* = \frac{1}{2}\{v_{1B} + \alpha + 1 - (v_{1B}^2 + 1 - 2v_{1B} - \alpha^2 + 2v_{1B}\alpha + 2\alpha - 4v_{1A}\alpha)^{\frac{1}{2}}\} \geq v_{1A}$

$\Rightarrow 2(1 - v_{1A})(v_{1A} - v_{1B}) < \alpha^2$ (We skip the algebra here).

Thus, when $v_{1A} \leq 1 - \alpha$ and $2(1 - v_{1A})(v_{1A} - v_{1B}) > \alpha^2$ the optimal drop out price [38] is $p_{1(1)}^* = \frac{1}{2}\{v_{1B} + \alpha + 1 - (v_{1B}^2 + 1 - 2v_{1B} - \alpha^2 + 2v_{1B}\alpha + 2\alpha - 4v_{1A}\alpha)^{\frac{1}{2}}\}$

Case II: We restrict our attention to the case $v_{1A} \leq 1 - \alpha$. We also assume that $2(1 - v_{1A})(v_{1A} - v_{1B}) \leq \alpha^2$ holds. This implies that $v_{1A} \leq p_1^*$, which in turn implies that $E\Pi_1^1 = 0$. The equation:

$$\begin{aligned} E\Pi_1^1 = 0 = E\Pi_1^2 = & \int_{p_1^*}^{v_{1A}+\alpha} (V_1 - p_1^* - v_{2A}) f(v_{2A}|p_1^*) dv_{2A} \\ & + \int_{v_{1A}+\alpha}^{1} (v_{1B} - p_1^*) f(v_{2A}|p_1^*) dv_{2A} \\ \Rightarrow 3(p_1^*)^2 - 2p_1^*(1 + v_{1A} + v_{1B} + \alpha) + (v_{1A} + \alpha)^2 + 2v_{1B} = 0 \end{aligned}$$

There are two solutions to this equation but the root that is not greater than $\frac{V_1}{2}$ is as follows,

$$p_{1(2)}^* = \frac{1}{3}\{v_{1A} + v_{1B} + \alpha + 1 - ((v_{1A} + v_{1B} + \alpha + 1)^2 - 3(v_{1A} + \alpha)^2 - 6v_{1B})^{\frac{1}{2}}\} \quad (31)$$

Since this is the case in which $p_1^* \geq v_{1A}$, we need the following restriction on the root,

$p_{1(2)}^* = \frac{1}{3}\{v_{1A} + v_{1B} + \alpha + 1 - ((v_{1A} + v_{1B} + \alpha + 1)^2 - 3(v_{1A} + \alpha)^2 - 6v_{1B})^{\frac{1}{2}}\} \geq v_{1A}$

$\Rightarrow 2(1 - v_{1A})(v_{1A} - v_{1B}) \leq \alpha^2$

Thus, when $v_{1A} \leq 1 - \alpha$ and $2(1 - v_{1A})(v_{1A} - v_{1B}) \leq \alpha^2$ the optimal drop out price

[38] Since the negation of the statements; that is, $p_1^* < v_{1A} \Rightarrow 2(1 - v_{1A})(v_{1A} - v_{1B}) \geq \alpha^2$, the condition is necessary and sufficient.

is[39],

$$p^*_{1(2)} = \tfrac{1}{3}\{v_{1A} + v_{1B} + \alpha + 1 - ((v_{1A} + v_{1B} + \alpha + 1)^2 - 3(v_{1A} + \alpha)^2 - 6v_{1B})^{\frac{1}{2}}\}$$

Case III: Assume that $1 - \alpha \leq v_{1A}$ and $1 + v_{1A} > 2(v_{1B} + \alpha)$ hold. The latter one imply that $p^*_1 < v_{1A}$. Therefore, we have $E\Pi^1_1 > 0$. The equation:

$$E\Pi^1_1 = \int_{p^*_1}^{v_{1A}} (v_{1A} - v_{2A}) f(v_{2A}|p^*_1) dv_{2A}$$

$$= E\Pi^2_1 = \int_{p^*_1}^{1} (v_{1A} + v_{1B} + \alpha - p^*_1 - v_{2A}) f(v_{2A}|p^*_1) dv_{2A}$$

$$\Rightarrow 2(p^*_1)^2 - 2p^*_1(1 + v_{1B} + \alpha) + 2(1 + v_{1B} + \alpha) - 1 - v^2_{1A} = 0$$

There are two solutions to this equation but the root that is not greater than $\frac{V_1}{2}$ is as follows,

$$p^*_{1(3)} = \frac{1}{2}\{v_{1B} + \alpha + 1 - \{(v_{1B} + \alpha + 1)^2 - 4(v_{1A} + v_{1B} + \alpha) + 2 + 2v^2_{1A}\}^{\frac{1}{2}}\} \tag{32}$$

If $p^*_{1(3)} < v_{1A}$, then $1 + v_{1A} > 2(v_{1B} + \alpha)$.

Thus, when $1 - \alpha \leq v_{1A}$ and $1 + v_{1A} > 2(v_{1B} + \alpha)$, the optimal drop out price is $p^*_{1(3)} = \frac{1}{2}\{v_{1B} + \alpha + 1 - \{(v_{1B} + \alpha + 1)^2 - 4(v_{1A} + v_{1B} + \alpha) + 2 + 2v^2_{1A}\}^{\frac{1}{2}}\}$

Case IV: When $1 - \alpha \leq v_{1A}$ and $1 + v_{1A} \leq 2(v_{1B} + \alpha)$. The latter one implies that $v_{1A} \leq p^*_1$. Therefore we have $E\Pi^1_1 = 0$. The equation:

[39] This condition is necessary and sufficient because of the same reason given in footnote 38

$$E\Pi_1^1 = 0 = E\Pi_1^2$$

$$= \int_{p_1^*}^{1} (v_{1A} + v_{1B} + \alpha - p_1^* - v_{2A}) f(v_{2A}|p_1^*) dv_{2A}$$

$$\Rightarrow 2(v_{1A} + v_{1B} + \alpha - 2p_1^*) - (1 - p_1^*) = 0$$

A unique solution to this equation is as follows,

$$p_{1(4)}^* = \frac{2(v_{1A} + v_{1B} + \alpha) - 1}{3} \tag{33}$$

If $p_{1(4)}^* \geq v_{1A}$, then $1 + v_{1A} \leq 2(v_{1B} + \alpha)$.

Thus, when $1 - \alpha \leq v_{1A}$ and $1 + v_{1A} \leq 2(v_{1B} + \alpha)$, the optimal drop out price is $p_{1(4)}^* = \frac{2(v_{1A}+v_{1B}+\alpha)-1}{3}$

■

Proof of Corollary 15:

We take partial derivative of p_1^* from equation 29 with respect to α, when $v_{1A} \leq 1 - \alpha$,and $2(1 - v_{1A})(v_{1A} - v_{1B}) \geq \alpha^2$, we have

$\frac{\partial p_1^*}{\partial \alpha} = \frac{1}{2}\{1 + \frac{\alpha+2v_{1A}-1-v_{1B}}{\sqrt{v_{1B}^2+1-2v_{1B}-\alpha^2-4v_{1A}\alpha+2\alpha+2v_{1B}\alpha}}\} > 0$

by eliminating $\frac{1}{2}$ and taking the fraction to the left hand side and multiplying each side with the denominator, we get

$\iff \sqrt{v_{1B}^2 + 1 - 2v_{1B} - \alpha^2 - 4v_{1A}\alpha + 2\alpha + 2v_{1B}\alpha}$

$> 1 + v_{1B} - \alpha - 2v_{1A}$

by squaring both sides and with some algebra that we skip, we get:

$\iff (2v_{1B} - 2v_{1A} - \alpha)(1 - \alpha - v_{1A}) + \alpha(v_{1A} - 1) < 0$

The term above is negative since the first parenthesis is negative by the fact that $v_{1B} < v_{1A}$; the second parenthesis is non-negative by the fact that $v_{1A} \leq 1 - \alpha$, and the third term is negative by the fact that $v_{1A} < 1$.

Thus, $\frac{\partial p_1^*}{\partial \alpha} > 0$ when $v_{1A} \leq 1 - \alpha$.

Next, we show that $\frac{\partial p_1^*}{\partial v_{1B}} > 0$.

$\frac{\partial p_1^*}{\partial v_{1B}} = \frac{1}{2}\{1 + \frac{1-\alpha-v_{1B}}{\sqrt{v_{1B}^2+1-2v_{1B}-\alpha^2-4v_{1A}\alpha+2\alpha+2v_{1B}\alpha}}\} > 0.$

This is positive since the numerator is positive by the assumption that $v_{1B} < v_{1A}$ and $v_{1A} \leq 1 - \alpha$. The denominator is always positive since it is a square root.

Now we show that $\frac{\partial p_1^*}{\partial v_{1A}} = \frac{\alpha}{\sqrt{v_{1B}^2+1-2v_{1B}-\alpha^2-4v_{1A}\alpha+2\alpha+2v_{1B}\alpha}} > 0.$

Then we take partial derivative of p_1^* from equation 31 with respect to v_{1A}, v_{1B}, and α, respectively, when $v_{1A} < 1 - \alpha$ and $2(1 - v_{1A})(v_{1A} - v_{1B}) \leq \alpha^2$, we have,

$\frac{\partial p_1^*}{\partial \alpha} = \frac{\partial p_1^*}{\partial v_{1A}} = \frac{1}{3}\{1 - \frac{1+v_{1B}-2\alpha-2v_{1A}}{\sqrt{(v_{1A}+v_{1B}+\alpha+1)^2-3(v_{1A}+\alpha)^2-6v_{1B}}}\} > 0$

$\iff \sqrt{(v_{1A} + v_{1B} + \alpha_1 + 1)^2 - 3(v_{1A} + \alpha)^2 - 6v_{1B}} > 1 + v_{1B} - 2\alpha - 2v_{1A}$

By squaring both sides and with some algebra that we skip, we get:

$\iff (v_{1A} + \alpha)(1 + v_{1B} - \alpha - v_{1A}) - v_{1B} > 0$

$\iff (1 - v_{1A} - \alpha)(\alpha + v_{1A} - v_{1B}) > 0$ since $v_{1A} \leq 1 - \alpha$ and $v_{1A} > v_{1B}$.

Thus, $\frac{\partial p_1^*}{\partial \alpha} = \frac{\partial p_1^*}{\partial v_{1A}} > 0$.

And $\frac{\partial p_1^*}{\partial v_{1B}} = \frac{1}{3}\{1 + \frac{2-v_{1A}-v_{1B}-\alpha}{\sqrt{(v_{1A}+v_{1B}+\alpha+1)^2-3(v_{1A}+\alpha)^2-6v_{1B}}}\} > 0$ since $2 - v_{1A} - v_{1B} - \alpha > 0$ for the case $v_{1A} < 1-\alpha$ and $2(1-v_{1A})(v_{1A}-v_{1B}) \leq \alpha^2$.

Next, we take partial derivative of p_1^* from equation 32 with respect to v_{1A}, v_{1B}, and α, respectively, when $1-\alpha \leq v_{1A}$ and $1+v_{1A} \geq 2(v_{1B}+\alpha)$, and we have,

$\frac{\partial p_1^*}{\partial \alpha} = \frac{\partial p_1^*}{\partial v_{1B}} = \frac{1}{2}\{1+\frac{1-v_{1B}-\alpha_1}{\sqrt{(v_{1B}+\alpha+1)^2-4(v_{1A}+v_{1B}+\alpha)+2+2v_{1A}^2}}\} > 0$ Since $1-\alpha \leq v_{1A}$ and $1+v_{1A} \geq 2(v_{1B}+\alpha)$, and $2 > 1+v_{1A} \geq 2(v_{1B}+\alpha)$, thus, $(1-v_{1B}-\alpha_1) > 0$.

And $\frac{\partial p_1^*}{\partial v_{1A}} = \frac{1-v_{1A}}{\sqrt{(v_{1B}+\alpha+1)^2-4(v_{1A}+v_{1B}+\alpha)+2+2v_{1A}^2}} > 0$ unless v_{1A} is equal to 1.

Finally, we take partial derivative of p_1^* from equation 33 with respect to v_{1A}, v_{1B}, and α, respectively, when $1-\alpha \leq v_{1A} < 1$, and we have,

$\frac{\partial p_1^*}{\partial \alpha} = \frac{\partial p_1^*}{\partial v_{1A}} = \frac{\partial p_1^*}{\partial v_{1B}} = \frac{2}{3} > 0$

■

Proof of Lemma 16:

We will prove this by solving $E\Pi_1^1 = E\Pi_1^2$. We have two cases.

Case I: When $v_{1B}+\alpha < v_{1A}$, we will show that $v_{1A} > p_1^{**}$ holds. This implies that, in equation 19, $E\Pi_1^1 > 0$.

The equation:

$$E\Pi_1^1 = \int_{p_1^{**}}^{v_{1A}} (v_{1A}-v_{2A}) f(v_{2A}|p_1^{**}) dv_{2A}$$
$$= E\Pi_1^2 = \int_{p_1^{**}}^{v_{1A}+v_{1B}+\alpha-p_1^{**}} (v_{1A}+v_{1B}+\alpha-p_1^{**}-v_{2A}) f(v_{2A}|p_1^{**}) dv_{2A}$$
$$\Rightarrow (v_{1B}+\alpha-p_1^{**})(v_{1B}+\alpha+2v_{1A}-3p_1^{**}) = 0 \ \ (we \ \ skip \ \ algebra)$$

gives the following two roots:

$$p_1^{**} = v_{1B} + \alpha \tag{34}$$

$$p_+^{**} = \frac{v_{1B} + \alpha + 2v_{1A}}{3} \tag{35}$$

We will show that the second root cannot be the answer. Our assumption for this case is $v_{1A} > v_{1B} + \alpha$. The optimal drop-out price cannot exceed the global bidder's updated valuation of license A (after winning license B). However, in equation 35, we have $p_+^{**} = \frac{v_{1B}+\alpha+2v_{1A}}{3} < v_{1A} + v_{1B} + \alpha - p_1^{**} = \frac{v_{1A}+2v_{1B}+2\alpha}{3}$. This implies $v_{1A} < v_{1B} + \alpha$. This is a contradiction. Hence, we rule out this root.

We assume that $v_{1A} > v_{1B} + \alpha$ so we have:

$p_1^{**} = v_{1B} + \alpha < v_{1A}$

Thus, when $v_{1A} > v_{1B} + \alpha$ the optimal drop out price is $p_1^{**} = v_{1B} + \alpha$

Case II: When $v_{1A} \leq v_{1B} + \alpha$, we have $v_{1A} \leq p_1^{**}$. This implies that $E\Pi_1^1 = 0$. The equation:

$$\begin{aligned} E\Pi_1^1 = 0 = E\Pi_1^2 \\ = \int_{p_1^{**}}^{v_{1A}+v_{1B}+\alpha-p_1^{**}} (v_{1A} + v_{1B} + \alpha - p_1^{**} - v_{2A}) f(v_{2A}|p_1^{**}) dv_{2A} \\ \Rightarrow (v_{1A} + v_{1B} + \alpha - 2p_1^{**})^2 = 0 \end{aligned}$$

gives the following unique root:

$$p_1^{**} = \frac{v_{1A} + v_{1B} + \alpha}{2} \tag{36}$$

We assume $v_{1A} \leq v_{1B} + \alpha$. If we add v_{1A} and divide by 2 both sides of the inequality, we get

$p_1^{**} = \frac{v_{1A}+v_{1B}+\alpha}{2} \geq v_{1A}$.

Thus, when $v_{1A} \leq v_{1B} + \alpha$ the optimal drop out price is $p_1^{**} = \frac{v_{1A}+v_{1B}+\alpha}{2}$

■

Proof of Lemma 17:

Now we consider the optimal strategy of local bidder, Firm 3 with valuation of license B, v_{3B}. We also describe two strategies for Firm 3: **Case 1**: at $p^{'} \leq v_{3B}$, Firm 3 will stay in the auction for license B until v_{3B}. **Case 2**: at $p^{'} \leq v_{3B}$, Firm 3 will drop out of the auction for license B before his valuation, v_{3B} and hope that the global bidder will withdraw license B at $p^{'}$ ultimately and win license B with positive profits.

We denote the expected profit of Firm 3 when playing Case 1 by $E\Pi_3^1$ and the expected profit of Firm 1 when playing Case 2 by $E\Pi_3^2$, respectively. By making the local bidder, Firm 3, indifferent between playing Case 1 and Case 2, we can find the optimal drop-out price p'.

$$E\Pi_3^1 = \int_{p^{'}}^{v_{3B}} (v_{3B} - p_1^{**}) f(p_1^{**}|p^{'}) dp_1^{**} \tag{37}$$

$$E\Pi_3^2 = P(v_{1B} < p^{'})(v_{3B} - p^{'})P(v_{1A} + v_{1B} + \alpha - p^{'} < v_{2A}) \tag{38}$$

Where $P(v_{1B} < p^{'})$ denotes the probability of the local bidder, Firm 3's drop out price, $p^{'}$, is greater than Firm 1's stand-alone valuation of license B, v_{1B}, and $P(v_{1A}+v_{1B}+\alpha-p^{'} < v_{2A})$ denotes the probability of the other local bidder, Firm 2's valuation, v_{2A} is greater than the global bidder's new valuation of license A, $v_{1A}+v_{1B}+\alpha-p^{'}$. By solving $E\Pi_3^1 = E\Pi_3^2$, we have $\frac{(v_{3B}-p^{'})^2}{2(1-p^{'})} = p^{'}(v_{3B} - p^{'})P(v_{1A} + v_{1B} + \alpha - p^{'} < v_{2A})$. So when $p^{'} = v_{3B}$, then $E\Pi_3^1 = E\Pi_3^2$, which means that the local bidder, Firm 3's optimal drop out price is v_{3B}, in other words, Firm 3 will stay in the auction for license B up to v_{3B}. Moreover, as $p^{'} < v_{3B}$ decreases, both $p^{'}$ and $P(v_{1A}+v_{1B}+\alpha-p^{'} < v_{2A})$ will decrease, and thus $E\Pi_3^2$ will decrease. Since the partial derivative of $\frac{(v_{3B}-p^{'})}{2(1-p^{'})}$ with respect to $p^{'}$ is negative, $\frac{\partial[\frac{(v_{3B}-p^{'})}{2(1-p^{'})}]}{\partial p^{'}} = \frac{v_{3B}-1}{(1-p^{'})^2} < 0$, when $v_{3B} < 1$. Thus, as $p^{'} < v_{3B}$ decreases, then $E\Pi_3^1$ will increase. As a result, when $p^{'} < v_{3B}$, $E\Pi_3^1 > E\Pi_3^2$; when $p^{'} = v_{3B}$, then $E\Pi_3^1 = E\Pi_3^2$. So it is optimal for Firm 3 to stay in the auction for license B up to v_{3B}.

■

Proof of Proposition 19 :

We show that the optimal drop out price in an auction without bid withdrawal rule is lower than that in an auction with bid withdrawal rule. We should prove this in four different cases, since p_1^* takes different values depending on the case (Check lemma 12). We will give the proof for the first case, the others are similar.

Case I:

When $v_{1A} \leq 1-\alpha$ and $2(1-v_{1A})(v_{1A}-v_{1B}) \geq \alpha^2$ the optimal drop out price in an auction without bid withdrawal rule is,

$p^*_{1(1)} = \frac{1}{2}\{v_{1B}+\alpha+1-(v^2_{1B}+1-2v_{1B}-\alpha^2+2v_{1B}\alpha+2\alpha-4v_{1A}\alpha)^{\frac{1}{2}}\}$, and the optimal drop out price in an auction with bid withdrawal rule is $p^{**}_1 = v_{1B}+\alpha$ when $v_{1A} > v_{1B}+\alpha$, and $p^{**}_1 = \frac{v_{1A}+v_{1B}+\alpha}{2}$ when $v_{1A} \leq v_{1B}+\alpha$.

First, when $v_{1A} > v_{1B}+\alpha$, we will show that $p^*_{1(1)} < p^{**}_1$.

Since $\frac{1}{2}\{v_{1B}+\alpha+1-(v^2_{1B}+1-2v_{1B}-\alpha^2+2v_{1B}\alpha+2\alpha-4v_{1A}\alpha)^{\frac{1}{2}}\} < v_{1B}+\alpha$

$\Leftrightarrow (1-v_{1B}-\alpha)^2 < (v^2_{1B}+1-2v_{1B}-\alpha^2+2v_{1B}\alpha+2\alpha-4v_{1A}\alpha)$

$\Leftrightarrow (v_{1A}+\alpha-1)+(v_{1A}-1) < 0$ since $v_{1A} \leq 1-\alpha$, we have $p^*_{1(1)} < p^{**}_1$.

Second, when $v_{1A} \leq v_{1B}+\alpha$, $p^*_{1(1)} < p^{**}_1$.

Since $\frac{1}{2}\{v_{1B}+\alpha+1-(v^2_{1B}+1-2v_{1B}-\alpha^2+2v_{1B}\alpha+2\alpha-4v_{1A}\alpha)^{\frac{1}{2}}\} < \frac{v_{1A}+v_{1B}+\alpha}{2}$

$\Rightarrow (1-v_{1A})^2 < (v^2_{1B}+1-2v_{1B}-\alpha^2+2v_{1B}\alpha+2\alpha-4v_{1A}\alpha)$

$\Rightarrow v^2_{1B}-v^2_{1A}-2v_{1B}+2v_{1A}-\alpha^2+2v_{1B}\alpha+2\alpha-4v_{1A}\alpha > 0$

$\Rightarrow (v_{1A}-v_{1B})[2-(v_{1B}+\alpha)-(v_{1A}+\alpha)]+\alpha[(1-v_{1A})+(1-v_{1A}-\alpha)] > 0$

Since $v_{1A} \leq 1-\alpha$ and $v_{1A} > v_{1B}$, we have $p^*_{1(1)} < p^{**}_1$.

■

Proof of Proposition 20:

We know that when $\alpha \leq p^*_1 \leq v_{1A}$, or if the other global bidder drops out of the license B auction, global bidder 1's optimal drop-out price will be calculated by using equation 27.

And when $p_1^* \leq \alpha \leq v_{1A}$, if the other global bidder stays in the license B auction, global bidder 1's optimal drop-out price will be calculated by using equation 26.

We will prove that there is a unique optimal drop out price by solving $E\Pi_1^1 = E\Pi_1^2$ first. And then we prove that this optimal drop out price will increase as the number of active firms in license A auction decreases. We have three cases.

Case I: In this case, we have $\alpha \leq v_{1A} < 1-\alpha$. We will show that this implies $E\Pi_1^1 > 0$. We will also show that our assumption $(m+2)(\alpha - v_{1B})(1-\alpha)^{m+1} \geq v_{1A}^{m+2} - (v_{1A} - \alpha)^{m+2}$ implies $\alpha \geq p_1^*$,

Taking the integrals in the equation 26:

$$E\Pi_1^1 = \int_{\alpha}^{v_{1A}} (v_{1A} - p_A)(\frac{m+1}{1-\alpha})(\frac{p_A - \alpha}{1-\alpha})^m dp_A = E\Pi_1^2$$

$$= \int_{p_1^*}^{v_{1A}+\alpha} (V_1 - p_1^* - p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A + \int_{v_{1A}+\alpha}^{1} (v_{1B} - p_1^*)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A$$

gives us the following:

$$\frac{(v_{1A} - \alpha)^{m+2}}{(1-\alpha)^{m+1}(m+2)} = \frac{(v_{1A} + \alpha - p_1^*)^{m+2}}{(1-p_1^*)^{m+1}(m+2)} + (v_{1B} - p_1^*) \qquad (39)$$

First, we show that there exists a unique solution to equation 39 which is the optimal drop out price p_1^*. We define a new function, $F(p, m) = E\Pi_1^1 - E\Pi_1^2$. To prove uniqueness, we will show that this function is monotonically increasing and it is negative when $v_{1B} = p$ and is positive when $p = \alpha$. Hence, there must be a unique root at the interval $v_{1B} < p < \alpha$.

$F(p, m) = \frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}(m+2)} - \frac{(v_{1A}+\alpha-p)^{m+2}}{(1-p)^{m+1}(m+2)} - (v_{1B} - p)$.

We take partial derivative of $F(p,m)$ with respect to p, we have,

$\frac{\partial F(p,m)}{\partial p} = 0 - (\frac{v_{1A}+\alpha-p}{1-p})^{m+1}(-1 + \frac{m+1}{m+2}\frac{v_{1A}+\alpha-p}{1-p}) + 1$

$\Rightarrow (\frac{v_{1A}+\alpha-p}{1-p})^{m+1} + 1 - \frac{m+1}{m+2}(\frac{v_{1A}+\alpha-p}{1-p})^{m+2} > 0$

since $v_{1A} + \alpha > p$ and $\frac{m+1}{m+2}(\frac{v_{1A}+\alpha-p}{1-p})^{m+2} < 1$ (by the fact that $v_{1A} + \alpha < 1$).

Thus, $F(p,m)$ is monotonically increasing function of p, when $v_{1B} \leq p < \alpha$.

If $p = v_{1B}$, then $F(v_{1B}) = \frac{(v_{1A}-\alpha)^{m+2}}{(m+2)(1-\alpha)^{m+1}} - \frac{(v_{1A}+\alpha-v_{1B})^{m+2}}{(m+2)(1-v_{1B})^{m+1}} < 0$.

$\Leftrightarrow (v_{1A} - \alpha)^{m+2}(1 - v_{1B})^{m+1} < (v_{1A} + \alpha - v_{1B})^{m+2}(1 - \alpha)^{m+1}$

$\Leftrightarrow (v_{1A} - \alpha)\{(v_{1A} - \alpha)(1 - v_{1B})\}^{m+1} < (v_{1A} + \alpha - v_{1B})\{(v_{1A} + \alpha - v_{1B})(1 - \alpha)\}^{m+1}$

$\Leftrightarrow (v_{1A} - \alpha)\{v_{1A} - \alpha - v_{1A}v_{1B} + v_{1B}\alpha\}^{m+1} < (v_{1A} + \alpha - v_{1B})\{v_{1A} - \alpha - v_{1A}v_{1B} + v_{1B}\alpha + \alpha(1 - \alpha) + (1 - v_{1A})(\alpha - v_{1B})\}^{m+1}$

since $(v_{1A} - \alpha) < (v_{1A} + \alpha - v_{1B})$ and $(v_{1A} - \alpha - v_{1A}v_{1B} + v_{1B}\alpha) < v_{1A} - \alpha - v_{1A}v_{1B} + v_{1B}\alpha + \alpha(1 - \alpha) + (1 - v_{1A})(\alpha - v_{1B})$.

Our assumption $(m + 2)(\alpha - v_{1B})(1 - \alpha)^{m+1} \geq v_{1A}^{m+2} - (v_{1A} - \alpha)^{m+2}$ implies that $F(p = \alpha, m) = \frac{(v_{1A}-\alpha)^{m+2}}{(m+2)(1-\alpha)^{m+1}} - \frac{v_{1A}^{m+2}}{(m+2)(1-\alpha)^{m+1}} - (v_{1B} - \alpha) > 0$. Hence, there is a unique root to the equation 39 in the interval $v_{1B} < p < \alpha$.

Next, we show that as the number of active firms in license A auction decreases, the optimal drop out price will increase. We will use the implicit function theorem for this:

$\Leftrightarrow \frac{dp_1^*}{dm} = -\frac{\frac{\partial F(p_1^*,m)}{\partial m}}{\frac{\partial F(p_1^*,m)}{\partial p_1^*}} < 0$.

We have already shown that $\frac{\partial F(p_1^*,m)}{\partial p_1^*} > 0$.

We take partial derivative of $F(p_1^*, m)$ with respect to m, when $v_{1A} < 1-\alpha$, and $(m+2)(\alpha - v_{1B})(1-\alpha)^{m+1} \geq v_{1A}^{m+2} - (v_{1A}-\alpha)^{m+2}$. We have,

$$\frac{\partial F(p_1^*,m)}{\partial m} = (\tfrac{v_{1A}-\alpha}{1-\alpha})^{m+1} \ln(\tfrac{v_{1A}-\alpha}{1-\alpha})\tfrac{v_{1A}-\alpha}{m+2} + (\tfrac{v_{1A}-\alpha}{1-\alpha})^{m+1}\tfrac{v_{1A}-\alpha}{(m+2)^2}$$

$$-(\tfrac{v_{1A}+\alpha-p_1^*}{1-p_1^*})^{m+1} \ln(\tfrac{v_{1A}+\alpha-p_1^*}{1-p_1^*})\tfrac{v_{1A}+\alpha-p_1^*}{m+2} + (\tfrac{v_{1A}+\alpha-p_1^*}{1-p_1^*})^{m+1}\tfrac{v_{1A}+\alpha-p_1^*}{(m+2)^2} > 0$$

$$\Leftrightarrow \tfrac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}}|\ln(\tfrac{v_{1A}-\alpha}{1-\alpha}) - \tfrac{1}{m+2}| < \tfrac{(v_{1A}+\alpha-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}}|\ln(\tfrac{v_{1A}+\alpha-p_1^*}{1-p_1^*}) - \tfrac{1}{m+2}|$$

$$\Leftrightarrow \frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}}(\ln(\frac{1-\alpha}{v_{1A}-\alpha}) + \frac{1}{m+2}) < \frac{(v_{1A}+\alpha-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}}(\ln(\frac{1-p_1^*}{v_{1A}+\alpha-p_1^*}) + \frac{1}{m+2}) \tag{40}$$

We will show that these inequalities hold by showing that the right hand side of the inequality 40 is increasing and the left hand side is decreasing in α.

Now let $H(\alpha) = \frac{(v_{1A}+\alpha-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}}(\ln(\frac{1-p_1^*}{v_{1A}+\alpha-p_1^*}) + \frac{1}{m+2})$, and we take partial derivative of $H(\alpha)$ with respect to α, when $0 \leq \alpha < 1$ and we have,

$$\frac{\partial H(\alpha)}{\partial \alpha} = \tfrac{(m+2)(v_{1A}+\alpha-p_1^*)^{m+1}}{(1-p_1^*)^{m+1}}(\ln(\tfrac{1-p_1^*}{v_{1A}+\alpha-p_1^*}) + \tfrac{1}{m+2}) - \tfrac{(v_{1A}+\alpha-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}}\tfrac{v_{1A}+\alpha-p_1^*}{1-p_1^*}\tfrac{1-p_1^*}{(v_{1A}+\alpha-p_1^*)^2}$$

$$\Leftrightarrow (m+2)\ln(\tfrac{1-p_1^*}{v_{1A}+\alpha-p_1^*}) > 0$$

Since $H(.)$ is increasing, for $\alpha > 0$, we have:

$$H(0) = \frac{(v_{1A}-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}}(\ln(\frac{1-p_1^*}{v_{1A}-p_1^*})+\frac{1}{m+2}) < H(\alpha)\frac{(v_{1A}+\alpha-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}}(\ln(\frac{1-p_1^*}{v_{1A}+\alpha-p_1^*})+\frac{1}{m+2})$$

Now, we will show that the left hand side of the inequality is decreasing in α.

Now let $K(\alpha) = \frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}}(\ln(\frac{1-\alpha}{v_{1A}-\alpha}) + \frac{1}{m+2})$, and we take partial derivative of $K(\alpha)$

with respect to α:

$\frac{\partial K(\alpha)}{\partial \alpha} = \{(\ln(\frac{1-\alpha}{v_{1A}-\alpha}) + \frac{1}{m+2})[(m+1)(v_{1A}-1) + (\alpha - 1)] + (1 - v_{1A})\}\frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}} < 0$

$\Leftrightarrow (\ln(\frac{1-\alpha}{v_{1A}-\alpha}) + \frac{1}{m+2})[(m+1)(1-v_{1A}) + (1-\alpha)] > (1 - v_{1A})$

$\Leftrightarrow (\ln(\frac{1-\alpha}{v_{1A}-\alpha}) + \frac{1}{m+2})[(m+1)(1-v_{1A}) + (1-\alpha)] > (\ln(\frac{1-\alpha}{v_{1A}-\alpha}) + \frac{1}{m+2})[(m+1)(1 - v_{1A}) + (1 - v_{1A})] > (1 - v_{1A})$

$\Leftrightarrow (m+2)\ln(\frac{1-\alpha}{v_{1A}-\alpha}) > 0.$

Thus, we conclude that $K(\alpha)$ is a decreasing function. That is, when $\alpha > p_1^*$, then $K(\alpha) < K(p_1^*)$. We have:

$$K(\alpha) = \frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}}(\ln(\frac{1-\alpha}{v_{1A}-\alpha}) + \frac{1}{m+2}) \quad (41)$$

$$< K(p_1^*) = H(0) = \frac{(v_{1A}-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}}(\ln(\frac{1-p_1^*}{v_{1A}-p_1^*}) + \frac{1}{m+2}) \quad (42)$$

$$< H(\alpha) = \frac{(v_{1A}+\alpha-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}}(\ln(\frac{1-p_1^*}{v_{1A}+\alpha-p_1^*}) + \frac{1}{m+2}) \quad (43)$$

.

Note that inequality 40 is proved; hence, we prove that $\frac{\partial F(p_1^*, m)}{\partial m} > 0$.

By the implicit function theorem, we show that the optimal drop out price increases as the number of firm, m, decreases.

$\Leftrightarrow \frac{dp_1^*}{dm} = -\frac{\frac{\partial F(p_1^*,m)}{\partial m}}{\frac{\partial F(p_1^*,m)}{\partial p_1^*}} < 0$

Case II: We have $\alpha \leq v_{1A} < 1 - \alpha$. We will show that our assumption $(m+2)(\alpha - v_{1B})(1-\alpha)^{m+1} \leq v_{1A}^{m+2} - (v_{1A}-\alpha)^{m+2}$ implies $\alpha \leq p_1^*$ and the other assumption $(m+2)(v_{1A}-v_{1B})(1-v_{1A})^{m+1} > \alpha^{m+2}$ implies $p_1^* < v_{1A}$. The latter condition implies that

$E\Pi_1^1 > 0$.

The equation 27:

$$E\Pi_1^1 = \int_{p_1^*}^{v_{1A}} (v_{1A} - p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A = E\Pi_1^2$$
$$= \int_{p_1^*}^{v_{1A}+\alpha} (V_1 - p_1^* - p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A + \int_{v_{1A}+\alpha}^{1} (v_{1B} - p_1^*)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A$$

gives the unique root.

By taking the integral and rearranging the equation, we find that p_1^* is the unique solution to the following equation.

$$\frac{(v_{1A} - p_1^*)^{m+2}}{(1-p_1^*)^{m+1}(m+2)} = \frac{(v_{1A} + \alpha - p_1^*)^{m+2}}{(1-p_1^*)^{m+1}(m+2)} + (v_{1B} - p_1^*) \tag{44}$$

As in Case I, we define a new function, $F(p_1^*, m) = E\Pi_1^1 - E\Pi_1^2$

$\Leftrightarrow F(p,m) = \frac{(v_{1A}-p)^{m+2}}{(1-p)^{m+1}(m+2)} - \frac{(v_{1A}+\alpha-p)^{m+2}}{(1-p)^{m+1}(m+2)} - (v_{1B} - p)$.

We take partial derivative of $F(p,m)$ with respect to p, when $v_{1A} < 1 - \alpha$, and $v_{1B} \leq p < v_{1A}$. We have,

$\frac{\partial F(p,m)}{\partial p} = (\frac{v_{1A}-p}{1-p})^{m+1}(-1 + \frac{m+1}{m+2}\frac{v_{1A}-p}{1-p})$

$-(\frac{v_{1A}+\alpha-p}{1-p})^{m+1}(-1 + \frac{m+1}{m+2}\frac{v_{1A}+\alpha-p}{1-p}) + 1$

$= [(\frac{v_{1A}-p}{1-p})^{m+1} - (\frac{v_{1A}+\alpha-p}{1-p})^{m+1}](-1 + \frac{m+1}{m+2}\frac{v_{1A}-p}{1-p}) + 1 - (\frac{v_{1A}+\alpha-p}{1-p})^{m+1}\frac{m+1}{m+2}\frac{\alpha}{1-p}$

$\Leftrightarrow \frac{\partial F(p,m)}{\partial p} > 0$. We conclude that this is positive since $(-1 + \frac{m+1}{m+2}\frac{v_{1A}-p}{1-p}) < 0$ and $[(\frac{v_{1A}-p}{1-p})^{m+1} - (\frac{v_{1A}+\alpha-p}{1-p})^{m+1}] < 0$. Also we need $1 - (\frac{v_{1A}+\alpha-p}{1-p})^{m+1}\frac{m+1}{m+2}\frac{\alpha}{1-p} > 0$. This is true since $p < v_{1A}$ (which we will prove that this is true) and $\alpha < 1 - v_{1A} < 1 - p$. The last condition shows that the last fraction $\frac{\alpha}{1-p}$ is also less than 1.

Thus, $F(p, m)$ is monotonically increasing function of p, when $\alpha \leq p < v_{1A}$.

Our assumption $(m+2)(\alpha - v_{1B})(1-\alpha)^{m+1} \leq v_{1A}^{m+2} - (v_{1A} - \alpha)^{m+2}$ implies that $F(\alpha) = \frac{(v_{1A}-\alpha)^{m+2}}{(m+2)(1-\alpha)^{m+1}} - \frac{(v_{1A}+\alpha-\alpha)^{m+2}}{(m+2)(1-\alpha)^{m+1}} - (v_{1B} - \alpha) < 0$. Our other assumption $(m+2)(v_{1A} - v_{1B})(1 - v_{1A})^{m+1} > \alpha^{m+2}$ implies that $F(v_{1A}) = 0 - \frac{\alpha^{m+2}}{(m+2)(1-v_{1A})^{m+1}} - (v_{1B} - v_{1A}) > 0$. If that is true, there is a unique solution to equation 44 which is p_1^*. Hence, $\alpha < p_1^* < v_{1A}$.

Next, we show that when the number of active firms in license A auction decreases, this optimal drop out price will increase.

$$F(p_1^*, m) = \frac{(v_{1A}-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}(m+2)} - \frac{(v_{1A}+\alpha-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}(m+2)} - (v_{1B} - p_1^*) = 0$$

We take partial derivative of $F(p_1^*, m)$ with respect to m, when $v_{1A} < 1 - \alpha$, and $(m+2)(v_{1A} - v_{1B})(1 - v_{1A})^{m+1} > \alpha^{m+2}$. We have proven this in the proof of Case I, that is, $\frac{\partial F(p_1^*,m)}{\partial m} > 0$.

$$\frac{dp_1^*}{dm} = -\frac{\frac{\partial F(p_1^*,m)}{\partial m}}{\frac{\partial F(p_1^*,m)}{\partial p_1^*}} < 0$$

Case III: We have $v_{1A} < 1 - \alpha$ and $\alpha \geq v_{1A}$ or $v_{1A} < 1 - \alpha$, $\alpha \leq v_{1A}$, and we will show that $(m+2)(v_{1A} - v_{1B})(1 - v_{1A})^{m+1} \leq \alpha^{m+2}$ implies $v_{1A} \leq p_1^*$. The latter condition implies that $E\Pi_1^1 = 0$.

The equation 28:

$$E\Pi_1^1 = 0 = E\Pi_1^2 = \int_{p_1^*}^{v_{1A}+\alpha} (V_1 - p_1^* - p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A + \int_{v_{1A}+\alpha}^{1} (v_{1B} - p_1^*)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A$$

gives the unique root which is the optimal drop out price.

By taking the integral and rearranging the equation, we find the following equation.

$$0 = \frac{(v_{1A} + \alpha - p_1^*)^{m+2}}{(1 - p_1^*)^{m+1}(m+2)} + (v_{1B} - p_1^*) \tag{45}$$

Now let $F(p, m) = E\Pi_1^1 - E\Pi_1^2$

$\Leftrightarrow F(p, m) = 0 - \frac{(v_{1A}+\alpha-p)^{m+2}}{(1-p)^{m+1}(m+2)} - (v_{1B} - p)$.

We take partial derivative of $F(p, m)$ with respect to p, when $v_{1A} < 1-\alpha$, and $v_{1A} \leq p$. We have,

$\frac{\partial F(p,m)}{\partial p} = -(\frac{v_{1A}+\alpha-p}{1-p})^{m+1}(-1 + \frac{m+1}{m+2}\frac{v_{1A}+\alpha-p}{1-p}) + 1 > 0$

since $-1 + \frac{m+1}{m+2}\frac{v_{1A}+\alpha-p}{1-p} < 0$ (by the fact that $v_{1A} + \alpha < 1$).

Thus, $F(p, m)$ is monotonically increasing function of p, when $v_{1A} \leq p$.

Our assumption $(m+2)(v_{1A} - v_{1B})(1 - v_{1A})^{m+1} \leq \alpha^{m+2}$ implies that $F(p = v_{1A}, m) = -\frac{\alpha^{m+2}}{(m+2)(1-v_{1A})^{m+1}} - (v_{1B} - v_{1A}) \leq 0$. If $p = v_{1A} + \alpha$, then $F(v_{1A} + \alpha) = 0 - 0 - (v_{1B} - v_{1A} - \alpha) > 0$. Thus, there is a unique solution, p_1^*, to equation 45 in the interval $(v_{1A}, v_{1A} + \alpha)$.

Next, we show that when the number of active firms in license A auction decreases, this optimal drop out price will increase.

$F(p_1^*, m) = -\frac{(v_{1A}+\alpha-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}(m+2)} - (v_{1B} - p_1^*) = 0$

We take partial derivative of $F(p_1^*, m)$ with respect to m, when $v_{1A} < 1 - \alpha$, and $(m+2)(v_{1A} - v_{1B})(1 - v_{1A})^{m+1} \leq \alpha^{m+2}$. We have,

$\frac{\partial F(p_1^*,m)}{\partial m} = -(\frac{v_{1A}+\alpha-p_1^*}{1-p_1^*})^{m+1} \ln(\frac{v_{1A}+\alpha-p_1^*}{1-p_1^*})\frac{v_{1A}+\alpha-p_1^*}{m+2} + (\frac{v_{1A}+\alpha-p_1^*}{1-p_1^*})^{m+1}\frac{v_{1A}+\alpha-p_1^*}{(m+2)^2} > 0$

$\Leftrightarrow -\ln(\frac{v_{1A}+\alpha-p_1^*}{1-p_1^*}) + \frac{1}{m+2} > 0$

Since $\frac{v_{1A}+\alpha-p_1^*}{1-p_1^*} < 1$, then $\ln(\frac{v_{1A}+\alpha-p_1^*}{1-p_1^*}) < 0$. Thus, $\frac{\partial F(p_1^*,m)}{\partial m} > 0$.

Since $\frac{\partial F(p_1^*,m)}{\partial p_1^*} > 0$ and $\frac{\partial F(p_1^*,m)}{\partial m} > 0$, we have,

$\frac{dp_1^*}{dm} = -\frac{\frac{\partial F(p_1^*,m)}{\partial m}}{\frac{\partial F(p_1^*,m)}{\partial p_1^*}} < 0$

■

Proof of Proposition 21:

We will explain this in three cases that we call case IV, V, and VI.

Case IV: We have $\alpha \leq v_{1A}$, $1-\alpha \leq v_{1A} \leq 1$, and we will show that $(m+2)(1-\alpha)^{m+1}(v_{1A}+v_{1B}) \leq (m+1+\alpha)(1-\alpha)^{m+1} + (v_{1A}-\alpha)^{m+2}$ implies $\alpha \geq p_1^*$. The condition $\alpha \leq v_{1A}$ implies that $E\Pi_1^1 > 0$.

The equation 26:

$$E\Pi_1^1 = \int_{\alpha}^{v_{1A}} (v_{1A}-p_A)(\frac{m+1}{1-\alpha})(\frac{p_A-\alpha}{1-\alpha})^m dp_A$$
$$= E\Pi_1^2 = \int_{p_1^*}^{1} (v_{1A}+v_{1B}+\alpha-p_1^*-p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A-p_1^*}{1-p_1^*})^m dp_A$$

gives the unique root.

By taking the integral and rearranging the equation, we have:

$$\frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}(m+2)} = (v_{1A}+v_{1B}+\alpha-2p_1^*) - \frac{(m+1)(1-p_1^*)}{m+2} \qquad (46)$$

The solution to this equation is:

$$p_1^* = \frac{(m+2)(1-\alpha)^{m+1}(v_{1A}+v_{1B}+\alpha) - (m+1)(1-\alpha)^{m+1} - (v_{1A}-\alpha)^{m+2}}{(m+3)(1-\alpha)^{m+1}} \tag{47}$$

The necessary and sufficient condition to have $\alpha \geq p_1^*$ is $(m+2)(1-\alpha)^{m+1}(v_{1A}+v_{1B}) \leq (m+1+\alpha)(1-\alpha)^{m+1} + (v_{1A}-\alpha)^{m+2}$. We skip the algebraic explanation of this.

Now let $F(p_1^*, m) = E\Pi_1^1 - E\Pi_1^2$

$\Leftrightarrow F(p_1^*, m) = \frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}(m+2)} - (v_{1A}+v_{1B}+\alpha-2p_1^*) + \frac{(m+1)(1-p_1^*)}{m+2}$.

We take partial derivative of $F(p_1^*, m)$ with respect to p_1^*, and have,

$\frac{\partial F(p_1^*,m)}{\partial p_1^*} = 2 - \frac{m+1}{m+2} > 0$.

Thus, $F(p_1^*, m)$ is monotonically increasing function of p_1^*, when $p_1^* < \alpha$.

Next, we show that when the number of active firms in license A auction decreases, this optimal drop out price will increase.

We take partial derivative of $F(p_1^*, m)$ with respect to m and have,

$\frac{\partial F(p_1^*,m)}{\partial m} = (\frac{v_{1A}-\alpha}{1-\alpha})^{m+1} \ln(\frac{v_{1A}-\alpha}{1-\alpha}) \frac{v_{1A}-\alpha}{m+2} - (\frac{v_{1A}-\alpha}{1-\alpha})^{m+1} \frac{v_{1A}-\alpha}{(m+2)^2}$

$+ \frac{(m+2)(1-p_1^*)-(m+1)(1-p_1^*)}{(m+2)^2} > 0$

$\Leftrightarrow \frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}} (\ln(\frac{1-\alpha}{v_{1A}-\alpha}) + \frac{1}{m+2}) < \frac{1-p_1^*}{m+2}$

$\Leftrightarrow \ln(\frac{1-\alpha}{v_{1A}-\alpha})^{m+2} < \frac{(1-\alpha)^{m+2}(1-p_1^*)}{(v_{1A}-\alpha)^{m+2}} - 1$

If we replace $(1-p_1^*)$ in the numerator of the right hand side of the inequality with $(1-\alpha)$, we get $\frac{(1-\alpha)^{m+1}(1-p_1^*)}{(v_{1A}-\alpha)^{m+2}} - 1 > (\frac{1-\alpha}{v_{1A}-\alpha})^{m+2} - 1$ since $p_1^* < \alpha$.

Now, we will show that $\ln(\frac{1-\alpha}{v_{1A}-\alpha})^{m+2} < (\frac{1-\alpha}{v_{1A}-\alpha})^{m+2} - 1$

which implies the desired result $\ln(\frac{1-\alpha}{v_{1A}-\alpha})^{m+2} < \frac{(1-\alpha)^{m+1}(1-p_1^*)}{(v_{1A}-\alpha)^{m+2}} - 1$.

Let $(\frac{1-\alpha}{v_{1A}-\alpha})^{m+2} = x > 1$, then,

$\Rightarrow \ln x < x - 1$

$\Rightarrow x < e^{x-1}$

Since $\frac{\partial e^{x-1}}{\partial x} > \frac{\partial x}{\partial x} = 1$, when $x > 1$ and when $x = 1$, $x = e^{x-1}$. So $x < e^{x-1}$, when $x > 1$. And thus, $\frac{\partial F(p_1^*,m)}{\partial m} > 0$.

Since $\frac{\partial F(p_1^*,m)}{\partial p_1^*} > 0$ and $\frac{\partial F(p_1^*,m)}{\partial m} > 0$, we have,

$\frac{dp_1^*}{dm} = -\frac{\frac{\partial F(p_1^*,m)}{\partial m}}{\frac{\partial F(p_1^*,m)}{\partial p_1^*}} < 0$

Case V: We have $\alpha \leq v_{1A}$, $1-\alpha \leq v_{1A} \leq 1$, and we will show that one assumption $(m+2)(1-\alpha)^{m+1}(v_{1A}+v_{1B}) \geq (m+1+\alpha)(1-\alpha)^{m+1} + (v_{1A}-\alpha)^{m+2}$ implies $\alpha \leq p_1^*$ and the other assumption $(m+2)(v_{1B}+\alpha) \leq (m+1) + v_{1A}$ implies $p_1^* \leq v_{1A}$. The condition $\alpha \leq v_{1A}$ implies that $E\Pi_1^1 > 0$.

The equation 27:

$$E\Pi_1^1 = \int_{p_1^*}^{v_{1A}} (v_{1A} - p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A$$
$$= E\Pi_1^2 = \int_{p_1^*}^{1} (v_{1A} + v_{1B} + \alpha - p_1^* - p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A - p_1^*}{1-p_1^*})^m dp_A$$

gives the unique root which is the optimal drop out price. By taking the integrals, we find:

$$\frac{(v_{1A} - p_1^*)^{m+2}}{(1-p_1^*)^{m+1}(m+2)} = (v_{1A} + v_{1B} + \alpha - 2p_1^*) - \frac{(m+1)(1-p_1^*)}{m+2} \tag{48}$$

Now let $F(p_1^*, m) = E\Pi_1^1 - E\Pi_1^2$

$\Leftrightarrow F(p, m) = \frac{(v_{1A}-p)^{m+2}}{(1-p)^{m+1}(m+2)} - (v_{1A} + v_{1B} + \alpha - 2p) + \frac{(m+1)(1-p)}{m+2}$.

We take partial derivative of $F(p, m)$ with respect to p, when $1 - \alpha \leq v_{1A} \leq 1$, and $p < v_{1A}$. we have,

$\frac{\partial F(p,m)}{\partial p} = (\frac{v_{1A}-p}{1-p})^{m+1}(-1 + \frac{m+1}{m+2}\frac{v_{1A}-p}{1-p}) + 2 - \frac{m+1}{m+2} > 0$ by a and be below:

a) $(-1 + \frac{m+1}{m+2}\frac{v_{1A}-p}{1-p}) \in (-1, 0)$ since $\frac{m+1}{m+2} < 1$ and $\frac{v_{1A}-p}{1-p} < 1$ by the fact that $v_{1A} < 1$.

b) and $1 - \frac{m+1}{m+2} > 0$.

Thus, $F(p, m)$ is monotonically increasing function of p, when $p < v_{1A}$.

Our assumption $(m+2)(1-\alpha)^{m+1}(v_{1A}+v_{1B}) \geq (m+1+\alpha)(1-\alpha)^{m+1} + (v_{1A}-\alpha)^{m+2}$ implies that $F(\alpha) = \frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}(m+2)} - (v_{1A} + v_{1B} - \alpha) + \frac{(m+1)(1-\alpha)}{m+2} < 0$, and the other assumption $(m+2)(v_{1B}+\alpha) < (m+1) + v_{1A}$ implies that $F(v_{1A}) = 0 - (v_{1B} + \alpha - v_{1A}) + \frac{(m+1)(1-v_{1A})}{m+2} > 0$. Thus, when $1 - \alpha \leq v_{1A} \leq 1$ and $\alpha \leq v_{1A}$, there is a unique solution, p_1^*, to equation 48 in the interval $\alpha \leq p \leq v_{1A}$.

Next, we show that when the number of active firms in license A auction decreases, this optimal drop out price will increase.

$F(p_1^*, m) = \frac{(v_{1A}-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}(m+2)} - (v_{1A} + v_{1B} + \alpha - 2p_1^*) + \frac{(m+1)(1-p_1^*)}{m+2}$

We take partial derivative of $F(p_1^*, m)$ with respect to m, when $1 - \alpha \leq v_{1A} \leq 1$, and $(m+2)(v_{1B}+\alpha) < (m+1) + v_{1A}$. We have,

$\frac{\partial F(p_1^*,m)}{\partial m} = (\frac{v_{1A}-p_1^*}{1-p_1^*})^{m+1}\ln(\frac{v_{1A}-p_1^*}{1-p_1^*})\frac{v_{1A}-p_1^*}{m+2} - (\frac{v_{1A}-p_1^*}{1-p_1^*})^{m+1}\frac{v_{1A}-p_1^*}{(m+2)^2}$

$+\frac{(m+2)(1-p_1^*)-(m+1)(1-p_1^*)}{(m+2)^2} > 0$

$\Leftrightarrow \frac{(v_{1A}-p_1^*)^{m+2}}{(1-p_1^*)^{m+1}}(\ln(\frac{1-p_1^*}{v_{1A}-p_1^*}) + \frac{1}{m+2}) < \frac{1-p_1^*}{m+2}$

$\Leftrightarrow \ln(\frac{1-p_1^*}{v_{1A}-p_1^*})^{m+2} < (\frac{1-p_1^*}{v_{1A}-p_1^*})^{m+2} - 1$

We have shown that the last inequality holds in the proof of Case IV.

And thus, $\frac{\partial F(p_1^*,m)}{\partial m} > 0$.

Since $\frac{\partial F(p_1^*,m)}{\partial p_1^*} > 0$ and $\frac{\partial F(p_1^*,m)}{\partial m} > 0$, we have,

$\frac{dp_1^*}{dm} = -\frac{\frac{\partial F(p_1^*,m)}{\partial m}}{\frac{\partial F(p_1^*,m)}{\partial p_1^*}} < 0$

Case VI: We have $1-\alpha \leq v_{1A} \leq 1$, and $\alpha \geq v_{1A}$ or $\alpha \leq v_{1A}$, $1-\alpha \leq v_{1A} \leq 1$, and we will show that $(m+2)(v_{1B}+\alpha) \geq (m+1)+v_{1A}$ implies $v_{1A} \leq p_1^*$. The latter condition implies that $E\Pi_1^1 = 0$.

The equation 28:

$$E\Pi_1^1 = 0 = E\Pi_1^2 = \int_{p_1^*}^1 (v_{1A}+v_{1B}+\alpha-p_1^*-p_A)(\frac{m+1}{1-p_1^*})(\frac{p_A-p_1^*}{1-p_1^*})^m dp_A$$

gives the unique root.

By taking the integral and rearranging the equation, we find the unique solution, p_1^*:

$$p_1^* = \frac{(m+2)(v_{1A}+v_{1B}+\alpha)-(m+1)}{m+3} \tag{49}$$

Our assumption $(m+2)(v_{1B}+\alpha) \geq (m+1)+v_{1A}$ implies that $v_{1A} \leq p_1^* = \frac{(m+2)(v_{1A}+v_{1B}+\alpha)-(m+1)}{m+3}$. Thus, there is a unique solution to equation 49.

Next, we show that when the number of active firms in license A auction decreases, this optimal drop out price will increase.

We take partial derivative of p_1^* with respect to m, when $1-\alpha \leq v_{1A} \leq 1$ and $(m+2)(v_{1B}+\alpha) \geq (m+1)+v_{1A}$. We have,

$\frac{\partial p_1^*}{\partial m} = \frac{(v_{1A}+v_{1B}+\alpha-1)(m+3)-(m+2)(v_{1A}+v_{1B}+\alpha)+(m+1)}{(m+3)^2}$

$\Rightarrow \frac{\partial p_1^*}{\partial m} = \frac{(v_{1A}+v_{1B}+\alpha)-2}{(m+3)^2} < 0$

Since $v_{1A}+v_{1B}+\alpha < 2$, so we have $\frac{\partial p_1^*}{\partial m} < 0$.

■

Proof of Proposition 24:

We will prove this by solving $E\Pi_1^1 = E\Pi_1^2$ and using the proper density function $g(.)$. We have three cases.

We show in Case I that the global bidder's optimal drop out price cannot be lower than the synergy, α. And then we calculate the global bidder's optimal drop out price when it is lower or greater than v_{1A}.

Case I: If $v_{1A} \geq \alpha \geq p_1^{**}$, then we have $E\Pi_1^1 > 0$. The equation:

$$E\Pi_1^1 = \int_{\alpha}^{v_{1A}} (v_{1A}-p_A)(\frac{m+1}{1-\alpha})(\frac{p_A-\alpha}{1-\alpha})^m dp_A$$
$$= E\Pi_1^2 = \int_{p_1^{**}}^{v_{1A}+v_{1B}+\alpha-p_1^{**}} (V_1-p_1^{**}-p_A)(\frac{m+1}{1-p_1^{**}})(\frac{p_A-p_1^{**}}{1-p_1^{**}})^m dp_A$$

gives the following equation,

$$\frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}} = \frac{(v_{1A}+v_{1B}+\alpha-2p_1^{**})^{m+2}}{(1-p_1^{**})^{m+1}} \tag{50}$$

Now, we show that there is no solution to equation 50 in the interval $v_{1B} \leq p_1^{**} \leq \alpha$. We define, $F(p_1^{**}, m) = \frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}} - \frac{(v_{1A}+v_{1B}+\alpha-2p_1^{**})^{m+2}}{(1-p_1^{**})^{m+1}}$. And we take partial derivative of $F(p_1^{**}, m)$ with respect to p_1^{**}, we have,

$$\frac{\partial F(p_1^{**},m)}{\partial p_1^{**}} = 0 - (\frac{v_{1A}+v_{1B}+\alpha-2p_1^{**}}{1-p_1^{**}})^{m+1}\frac{2(m+2)(1-p_1^{**})-(m+1)(v_{1A}+v_{1B}+\alpha-2p_1^{**})}{1-p_1^{**}} > 0$$

since $v_{1A}+v_{1B}+\alpha < 2$ and $1-p_1^{**} > 0$.

Thus, $F(p_1^{**}, m)$ is monotonically increasing function of p_1^{**}, when $v_{1B} \leq p_1^{**} \leq \alpha$.

If $p_1^{**} = v_{1B}$, then $F(v_{1B}) = \frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}} - \frac{(v_{1A}+\alpha-v_{1B})^{m+2}}{(1-v_{1B})^{m+1}} < 0$. The proof is the same as that for Case I of Proposition 20.

If $p_1^{**} = \alpha$, then $F(\alpha) = \frac{(v_{1A}-\alpha)^{m+2}}{(1-\alpha)^{m+1}} - \frac{(v_{1A}+v_{1B}-\alpha)^{m+2}}{(1-\alpha)^{m+1}} < 0$. Since $(v_{1A}+v_{1B}-\alpha) > (v_{1A}-\alpha)$.

That is, when $p_1^{**} < \alpha$, there is no solution to equation 50.

Case II: We have $v_{1A} \geq \alpha$ and show that $v_{1A} \geq v_{1B}+\alpha$ implies $v_{1A} \geq p_1^{**}$ and in turn implies $E\Pi_1^1 > 0$.

$$E\Pi_1^1 = \int_{p_1^{**}}^{v_{1A}} (v_{1A}-p_A)(\frac{m+1}{1-p_1^{**}})(\frac{p_A-p_1^{**}}{1-p_1^{**}})^m dp_A$$

$$= E\Pi_1^2 = \int_{p_1^{**}}^{v_{1A}+v_{1B}+\alpha-p_1^{**}} (V_1-p_1^{**}-p_A)(\frac{m+1}{1-p_1^{**}})(\frac{p_A-p_1^{**}}{1-p_1^{**}})^m dp_A$$

$$\Rightarrow (v_{1A}-p_1^{**})^{m+2} = (v_{1A}+v_{1B}+\alpha-2p_1^{**})^{m+2}$$

Or when the other global bidder withdraws license B at the price p_1^{**}, then global bidder 1 will accept license B at p_1^{**}.

$$E\Pi_1^1 = \int_{p_1^{**}}^{v_{1A}} (V_1 - p_1^{**} - p_A)(\frac{m+1}{1-p_1^{**}})(\frac{p_A - p_1^{**}}{1-p_1^{**}})^m dp_A$$
$$= E\Pi_1^2 = \int_{p_1^{**}}^{v_{1A}+v_{1B}+\alpha-p_1^{**}} (V_1 - p_1^{**} - p_A)(\frac{m+1}{1-p_1^{**}})(\frac{p_A - p_1^{**}}{1-p_1^{**}})^m dp_A$$
$$\Rightarrow v_{1A} - p_1^{**} = v_{1A} + v_{1B} + \alpha - 2p_1^{**}$$

give the following unique root:

$$p_1^{**} = v_{1B} + \alpha \tag{51}$$

Hence, our assumption $v_{1A} \geq v_{1B} + \alpha$ implies $p_1^{**} \leq v_{1A}$.

Case III: We have $v_{1A} \leq \alpha$, or $v_{1A} \geq \alpha$ and $v_{1A} \leq v_{1B} + \alpha$ implies $v_{1A} \leq p_1^{**}$ and in turn implies $E\Pi_1^1 = 0$.

$$E\Pi_1^1 = E\Pi_1^2$$
$$0 = \int_{p_1^{**}}^{v_{1A}+v_{1B}+\alpha-p_1^{**}} (V_1 - p_1^{**} - p_A)(p_A - p_1^{**})^m dp_A$$
$$0 = (v_{1A} + v_{1B} + \alpha - 2p_1^{**})^{m+2}$$

gives the following unique root:

$$p_1^{**} = \frac{v_{1A} + v_{1B} + \alpha}{2} \tag{52}$$

Our assumption $v_{1A} \leq v_{1B} + \alpha$ implies that $p_1^{**} \geq v_{1A}$.

■

Proof of Proposition 25:

Next, we show that the optimal drop out price in an auction without bid withdrawal rule in six cases is lower than that in an auction with bid withdrawal rule in a general case with two global bidders and $2m$ local bidders.

Case I:

When $v_{1A} < 1 - \alpha$, the optimal drop out price in an auction without bid withdrawal rule is derived by equating the following equations,

$Max\{0, \int_{p_1^*}^{v_{1A}} (v_{1A} - p_A) g(p_A|p_1^*) d(p_A)\} = \int_{p_1^*}^{v_{1A}+\alpha} (V_1 - p_1^* - p_A) g(p_A|p_1^*) dp_A + \int_{v_{1A}+\alpha}^{1} (v_{1B} - p_1^*) g(p_A|p_1^*) dp_A$

and the optimal drop out price in an auction with bid withdrawal rule is derived by equating the following equations,

$Max\{0, \int_{p_1^{**}}^{v_{1A}} (v_{1A} - p_A) g(p_A|p_1^{**}) d(p_A)\} = \int_{p_1^{**}}^{v_{1A}+v_{1B}+\alpha-p_1^{**}} (V_1 - p_1^{**} - p_A) g(p_A|p_1^{**}) dp_A$

Let $F(x) = Max\{0, \int_{x}^{v_{1A}} (v_{1A} - p_A) g(p_A|x) d(p_A)\} - \int_{x}^{v_{1A}+\alpha} (V_1 - x - p_A) g(p_A|x) dp_A - \int_{v_{1A}+\alpha}^{1} (v_{1B} - x) g(p_A|x) dp_A$

$F(x) = Max\{0, \int_{x}^{v_{1A}} (v_{1A} - p_A) g(p_A|x) d(p_A)\} - \int_{x}^{v_{1A}+v_{1B}+\alpha-x} (V_1 - x - p_A) g(p_A|x) dp_A - \int_{v_{1A}+v_{1B}+\alpha-x}^{v_{1A}+\alpha} (V_1 - x - p_A) g(p_A|x) dp_A - \int_{v_{1A}+\alpha}^{1} (v_{1B} - x) g(p_A|x) dp_A$

And let $G(x) = Max\{0, \int_{x}^{v_{1A}} (v_{1A} - p_A) g(p_A|x) d(p_A)\} - \int_{x}^{v_{1A}+v_{1B}+\alpha-x} (V_1 - x - p_A) g(p_A|x) dp_A$

If $G(p_1^{**}) = 0$, then $F(p_1^{**}) = 0 - \int_{v_{1A}+v_{1B}+\alpha-p_1^{**}}^{v_{1A}+\alpha} (V_1 - p_1^{**} - p_A) g(p_A|p_1^{**}) dp_A - \int_{v_{1A}+\alpha}^{1} (v_{1B} - p_1^{**}) g(p_A|p_1^{**}) dp_A > 0$. It is positive since the first integral in the expression is non-positive by the fact that the density function is conditional on $p_A \geq p_1^{**}$ and the lower limit of the integral suggests that we look for cases where $p_A \geq v_{1A} + v_{1B} + \alpha - p_1^{**}$. Note that this is -in a way- the exposure problem in which the bidder wins both licenses and makes a loss. The second integral is negative since we know that $p_1^{**} > v_{1B}$. In a way, this is the exposure problem of winning one license. This shows that the expression is positive.

Since we have shown that $F(x)$ is monotonically increasing function of x. If $F(p_1^*) = 0$, then we have, $p_1^* < p_1^{**}$, since $F(p_1^{**}) > F(p_1^*) = 0$.

Case II:

When $1-\alpha \leq v_{1A} \leq 1$, the optimal drop out price in an auction without bid withdrawal rule is derived by equating the following equations,

$$Max\{0, \int_{p_1^*}^{v_{1A}} (v_{1A} - p_A) g(p_A|p_1^*) d(p_A)\} = \int_{p_1^*}^{1} (V_1 - p_1^* - p_A) g(p_A|p_1^*) dp_A$$

and the optimal drop out price in an auction with bid withdrawal rule is derived by equating the following equations,

$$Max\{0, \int_{p_1^{**}}^{v_{1A}} (v_{1A} - p_A) g(p_A|p_1^{**}) d(p_A)\} = \int_{p_1^{**}}^{v_{1A}+v_{1B}+\alpha-p_1^{**}} (V_1 - p_1^{**} - p_A) g(p_A|p_1^{**}) dp_A$$

Let $F(x) = Max\{0, \int_{x}^{v_{1A}} (v_{1A} - p_A) g(p_A|x) d(p_A)\} - \int_{x}^{1} (V_1 - x - p_A) g(p_A|x) dp_A$

$F(x) = Max\{0, \int_{x}^{v_{1A}} (v_{1A} - p_A) g(p_A|x) d(p_A)\} - \int_{x}^{v_{1A}+v_{1B}+\alpha-x} (V_1 - x - p_A) g(p_A|x) dp_A - \int_{v_{1A}+v_{1B}+\alpha-x}^{1} (V_1 - x - p_A) g(p_A|x) dp_A$

And let Let $G(x) = Max\{0, \int_{x}^{v_{1A}} (v_{1A} - p_A) g(p_A|x) d(p_A)\} - \int_{x}^{v_{1A}+v_{1B}+\alpha-x} (V_1 - x - p_A) g(p_A|x) dp_A$

If $G(p_1^{**}) = 0$, then $F(p_1^{**}) = 0 - \int_{v_{1A}+v_{1B}+\alpha-x}^{1} (V_1 - x - p_A) g(p_A|x) dp_A > 0$. It is positive by the same logic of case I. Since we have shown that $F(x)$ is monotonically increasing function of x. If $F(p_1^*) = 0$, then we have, $p_1^* < p_1^{**}$, since $F(p_1^{**}) > F(p_1^*) = 0$.

■

4.5.2 Simulation Code

Because the simulation code for a general case is too long, it is available at http://home.cc.umanitoba.ca/ nay/. The following code is for special case only.

$fork = 1 : 10$

$n = 200$

$c = 0.09 * k * ones(n)$

$a = rand(n)$

$b = rand(n)$

$x = a > b$

$x = x == 1$

$v = a + b + c < 2$

$v = v == 1$

$z = x + v == 2$

$z = z == 1$

$a = a(z)$

$b = b(z)$

$c = c(z)$

$I = zeros(length(a), 1)$

$J = zeros(length(a), 1)$

$K = zeros(length(a), 1)$

$L = zeros(length(a), 1)$

$M = zeros(length(a), 1)$

$N = zeros(length(a), 1)$

$O = zeros(length(a), 1)$

$fori = 1 : length(a)$

$forj = 1 : 1$

$ifa(i, j) <= 1 - c(i, j)anda(i, j) >= b(i, j) + c(i, j)and2 * (1 - a(i, j)) * (a(i, j) - b(i, j)) >= c(i, j)^2$

$I(i, j) = 1$

$elseifa(i, j) <= 1 - c(i, j)anda(i, j) <= b(i, j) + c(i, j)and2 * (1 - a(i, j)) * (a(i, j) - b(i, j)) >= c(i, j)^2$

$J(i, j) = 1$

$elseifa(i, j) <= 1 - c(i, j)anda(i, j) <= b(i, j) + c(i, j)and2 * (1 - a(i, j)) * (a(i, j) -$

$b(i,j)) <= c(i,j)^2$

$K(i,j) = 1$

$elseifa(i,j) >= 1 - c(i,j)anda(i,j) >= b(i,j) + c(i,j)and1 + a(i,j) >= 2 * (b(i,j) + c(i,j))$

$L(i,j) = 1$ $elseifa(i,j) >= 1 - c(i,j)anda(i,j) <= b(i,j) + c(i,j)and1 + a(i,j) >= 2 * (b(i,j) + c(i,j))$

$M(i,j) = 1$ $elseifa(i,j) >= 1 - c(i,j)anda(i,j) <= b(i,j) + c(i,j)and1 + a(i,j) <= 2 * (b(i,j) + c(i,j))$

$N(i,j) = 1$ $else$ $O(i,j) = 1$

end

end

end

$I = I == 1$

$J = J == 1$

$K = K == 1$

$L = L == 1$

$M = M == 1$

$N = N == 1$

$O = O == 1$

$A = a(I)$

$B = b(I)$

$C = c(I)$

$d = rand(length(A), 1)$ $e = rand(length(A), 1)$

$p1 = 1/2 * (B + C + 1 - sqrt(1 + B.^2 - 2 * B - C.^2 + 2 * B. * C + C. * 2 - A. * C. * 4))$

$p2 = B + C$

$fori = 1 : length(A)$

$if(e(i) <= p1(i)andd(i) <= A(i) + C(i))||(d(i) <= p1(i)andp1(i) <= e(i)ande(i) <= B(i) + C(i))$

$f1(i) = d(i) + e(i)$

$elseif(e(i) <= p1(i)andd(i) >= A(i) + C(i))$

$f1(i) = e(i) + A(i) + C(i)$

$elseif(d(i) <= p1(i)ande(i) >= B(i) + C(i))$

$f1(i) = d(i) + B(i) + C(i)$

$elseif(e(i) >= p1(i)andp1(i) <= d(i)andd(i) <= A(i))$

$f1(i) = p1(i) + d(i)$

$elseif(e(i) >= p1(i)andd(i) >= A(i))$

$f1(i) = p1(i) + A(i)$

$else$

$f1(i) = 0$

$end\, if(e(i) <= B(i) and d(i) <= A(i)+C(i)) || (B(i) <= e(i) and e(i) <= p2(i) and d(i) <= A(i) + B(i) + C(i) - e(i)) || (d(i) <= p2(i) and p2(i) <= e(i) and e(i) <= B(i) + C(i))$

$f2(i) = d(i) + e(i)$

$elseif(B(i) <= e(i) and e(i) <= p2(i) and d(i) >= A(i) + B(i) + C(i) - e(i))$

$f2(i) = B(i) + A(i) + B(i) + C(i) - e(i)$

$elseif(e(i) <= B(i) and d(i) >= A(i) + C(i))$

$f2(i) = A(i) + C(i) + e(i)$

$elseif(d(i) <= p2(i) and e(i) >= B(i) + C(i))$

$f2(i) = d(i) + B(i) + C(i)$

$elseif(e(i) >= p2(i) and p2(i) <= d(i) and d(i) <= A(i))$

$f2(i) = p2(i) + d(i)$

$elseif(e(i) >= p2(i) and d(i) >= A(i))$

$f2(i) = p2(i) + A(i)$

$else$

$f2(i) = 0$

end

end

$A1 = a(J)$

$B1 = b(J)$

$C1 = c(J)$

$d1 = rand(length(A1), 1)$

$e1 = rand(length(A1), 1)$

$p1p = 1/2*(B1+C1+1-sqrt(1+B1.^2-2*B1-C1.^2+2*B1.*C1+C1.*2-A1.*C1.*4))$

$p2p = 1/2*(A1 + B1 + C1)$

$fori = 1 : length(A1)$

$if(e1(i) <= p1p(i)andd1(i) <= A1(i)+C1(i))||(d1(i) <= p1p(i)andp1p(i) <= e1(i)ande1(i) <= B1(i) + C1(i))$

$h1(i) = d1(i) + e1(i)$

$elseif(e1(i) <= p1p(i)andd1(i) >= A1(i) + C1(i))$

$h1(i) = e1(i) + A1(i) + C1(i)$

$elseif(d1(i) <= p1p(i)ande1(i) >= B1(i) + C1(i))$

$h1(i) = d1(i) + B1(i) + C1(i)$

$elseif(e1(i) >= p1p(i)andp1p(i) <= d1(i)andd1(i) <= A1(i))$

$h1(i) = p1p(i) + d1(i)$

$elseif(e1(i) >= p1p(i) and d1(i) >= A1(i))$

$h1(i) = p1p(i) + A1(i)$

$else$

$h1(i) = 0$

end

$if(e1(i) <= B1(i) and d1(i) <= A1(i)+C1(i))||(e1(i) >= B1(i) and e1(i) <= p2p(i) and d1(i) <= A1(i) + B1(i) + C1(i) - e1(i))||(p2p(i) <= e1(i) and e1(i) <= B1(i) + C1(i) and d1(i) <= A1(i))||(p2p(i) <= e1(i) and e1(i) <= A1(i)+B1(i)+C1(i)-d1(i) and d1(i) >= A1(i) and d1(i) <= p2p(i))$

$h2(i) = d1(i) + e1(i)$

$elseif(e1(i) >= B1(i) and e1(i) <= p2p(i) and d1(i) >= A1(i) + B1(i) + C1(i) - e1(i))$

$h2(i) = B1(i) + A1(i) + B1(i) + C1(i) - e1(i)$

$elseif(d1(i) >= A1(i) and d1(i) <= p2p(i) and e1(i) >= A1(i) + B1(i) + C1(i) - d1(i))$

$h2(i) = A1(i) + A1(i) + B1(i) + C1(i) - d1(i)$

$elseif(e1(i) >= p2p(i) and d1(i) >= p2p(i))$

$h2(i) = p2p(i) + p2p(i)$

$elseif(e1(i) <= B1(i) and d1(i) >= A1(i) + C1(i))$

$h2(i) = e1(i) + A1(i) + C1(i)$

$elseif(e1(i) >= B1(i) + C1(i)andd1(i) <= A1(i))$

$h2(i) = d1(i) + B1(i) + C1(i)$

$else$

$h2(i) = 0$

end

end

$A2 = a(K)$

$B2 = b(K)$

$C2 = c(K)$

$d2 = rand(length(A2), 1)$

$e2 = rand(length(A2), 1)$

$p1p1 = 1/3*(A2+B2+C2+1-sqrt((A2+B2+C2+1).^2-3*(A2+C2).^2-6*B2))$

$p2p2 = 1/2 * (A2 + B2 + C2)$

$fori = 1 : length(A2)$

$if(e2(i) <= p1p1(i)andd2(i) <= A2(i) + C2(i))||(d2(i) <= p1p1(i)andp1p1(i) <= e2(i)ande2(i) <= B2(i) + C2(i))$

$j1(i) = d2(i) + e2(i)$

$elseif(e2(i) <= p1p1(i) and d2(i) >= A2(i) + C2(i))$

$j1(i) = A2(i) + C2(i) + e2(i)$

$elseif(e2(i) >= B2(i) + C2(i) and d2(i) <= p1p1(i))$

$j1(i) = d2(i) + B2(i) + C2(i)$

$elseif(e2(i) >= p1p1(i) and d2(i) >= p1p1(i))$

$j1(i) = p1p1(i) + p1p1(i)$

$else$

$j1(i) = 0$

end

$if(e2(i) <= B2(i) and d2(i) <= A2(i)+C2(i))||(e2(i) >= B2(i) and e2(i) <= p2p2(i) and d2(i) <= A2(i) + B2(i) + C2(i) - e2(i))||(p2p2(i) <= e2(i) and e2(i) <= B2(i) + C2(i) and d2(i) <= A2(i))||(p2p2(i) <= e2(i) and e2(i) <= A2(i)+B2(i)+C2(i)-d2(i) and d2(i) >= A2(i) and d2(i) <= p2p2(i))$

$j2(i) = d2(i) + e2(i)$

$elseif(e2(i) >= B2(i) and e2(i) <= p2p2(i) and d2(i) >= A2(i) + B2(i) + C2(i) - e2(i))$

$j2(i) = B2(i) + A2(i) + B2(i) + C2(i) - e2(i)$

$elseif(d2(i) >= A2(i) and d2(i) <= p2p2(i) and e2(i) >= A2(i) + B2(i) + C2(i) - d2(i))$

$j2(i) = A2(i) + A2(i) + B2(i) + C2(i) - d2(i)$

$elseif(e2(i) >= p2p2(i) and d2(i) >= p2p2(i))$

$j2(i) = p2p2(i) + p2p2(i)$

$elseif(e2(i) <= B2(i) and d2(i) >= A2(i) + C2(i))$

$j2(i) = e2(i) + A2(i) + C2(i)$

$elseif(e2(i) >= B2(i) + C2(i) and d2(i) <= A2(i))$

$j2(i) = d2(i) + B2(i) + C2(i)$

$else$

$j2(i) = 0$

end

end

$A3 = a(L)$

$B3 = b(L)$

$C3 = c(L)$

$d3 = rand(length(A3), 1)$

$e3 = rand(length(A3), 1)$

$p1p1p = 1/2 * (B3 + C3 + 1 - sqrt((1 + B3 + C3).^2 - 4 * (A3 + B3 + C3) + 2 + 2 * A3.^2))$

$p2p2p = B3 + C3$

$for i = 1 : length(A3)$

$if(e3(i) <= p1p1p(i))||(p1p1p(i) <= e3(i)and e3(i) <= B3(i) + C3(i)and d3(i) <= p1p1p(i))$

$l1(i) = d3(i) + e3(i)$

$elseif(e3(i) >= B3(i) + C3(i)and d3(i) <= p1p1p(i))$

$l1(i) = d3(i) + B3(i) + C3(i)$

$elseif(e3(i) >= p1p1p(i)and p1p1p(i) <= d3(i)and d3(i) <= A3(i))$

$l1(i) = p1p1p(i) + d3(i)$

$elseif(e3(i) >= p1p1p(i)and d3(i) >= A3(i))$

$l1(i) = p1p1p(i) + A3(i)$

$else$

$l1(i) = 0$

end

$if(e3(i) <= B3(i))||(p2p2p(i) >= e3(i)and e3(i) >= B3(i)and d3(i) <= A3(i) + B3(i)+C3(i)-e3(i))||(e3(i) >= p2p2p(i)and e3(i) <= B3(i)+C3(i)and d3(i) <= p2p2p(i))$

$l2(i) = d3(i) + e3(i)$

$elseif(e3(i) >= B3(i)and e3(i) <= p2p2p(i)and d3(i) >= A3(i)+B3(i)+C3(i)-e3(i))$

$l2(i) = B3(i) + A3(i) + B3(i) + C3(i) - e3(i)$

$elseif(e3(i) >= p2p2p(i)and p2p2p(i) <= d3(i)and d3(i) <= A3(i))$

$l2(i) = p2p2p(i) + d3(i)$

$elseif(e3(i) >= p2p2p(i) and d3(i) >= A3(i))$

$l2(i) = p2p2p(i) + A3(i)$

$elseif(e3(i) >= B3(i) + C3(i) and d3(i) <= p2p2p(i))$

$l2(i) = d3(i) + B3(i) + C3(i)$

$else$

$l2(i) = 0$

end

end

$A4 = a(M)$

$B4 = b(M)$

$C4 = c(M)$

$d4 = rand(length(A4), 1)$

$e4 = rand(length(A4), 1)$

$p1p1p1 = 1/2*(B4+C4+1-sqrt((1+B4+C4).^2-4*(A4+B4+C4)+2+2*A4.^2))$

$p2p2p2 = 1/2*(A4+B4+C4)$

$fori = 1 : length(A4)$

$if(e4(i) <= p1p1p1(i))||(p1p1p1(i) <= e4(i) and e4(i) <= B4(i) + C4(i) and d4(i) <=$

$p1p1p1(i))$

$m1(i) = d4(i) + e4(i)$

$elseif(e4(i) >= B4(i) + C4(i) and d4(i) <= p1p1p1(i))$

$m1(i) = d4(i) + B4(i) + C4(i)$

$elseif(e4(i) >= p1p1p1(i) and p1p1p1(i) <= d4(i) and d4(i) <= A4(i))$

$m1(i) = p1p1p1(i) + d4(i)$

$elseif(e4(i) >= p1p1p1(i) and d4(i) >= A4(i))$

$m1(i) = p1p1p1(i) + A4(i)$

$else$

$m1(i) = 0$

end

$if(e4(i) <= B4(i))||(p2p2p2(i) >= e4(i) and e4(i) >= B4(i) and d4(i) <= A4(i) + B4(i) + C4(i) - e4(i))||(e4(i) >= p2p2p2(i) and e4(i) <= B4(i) + C4(i) and d4(i) <= A4(i))||(e4(i) >= p2p2p2(i) and e4(i) <= A4(i) + B4(i) + C4(i) - d4(i) and d4(i) >= A4(i) and d4(i) <= p2p2p2(i))$

$m2(i) = d4(i) + e4(i)$

$elseif(e4(i) <= p2p2p2(i) and e4(i) >= B4(i) and d4(i) >= A4(i) + B4(i) + C4(i) - e4(i))$

$m2(i) = B4(i) + A4(i) + B4(i) + C4(i) - e4(i)$

$elseif(d4(i) <= p2p2p2(i) and d4(i) >= A4(i) and e4(i) >= A4(i) + B4(i) + C4(i) - d4(i))$

$m2(i) = A4(i) + A4(i) + B4(i) + C4(i) - d4(i)$

$elseif(e4(i) >= p2p2p2(i) and d4(i) >= p2p2p2(i))$

$m2(i) = p2p2p2(i) + p2p2p2(i)$

$elseif(e4(i) >= B4(i) + C4(i) and d4(i) <= A4(i))$

$m2(i) = d4(i) + B4(i) + C4(i)$

$else$

$m2(i) = 0$

end

end

$A5 = a(N)$

$B5 = b(N)$

$C5 = c(N)$

$d5 = rand(length(A5), 1)$

$e5 = rand(length(A5), 1)$

$p1p1p1p = 1/3 * (2 * (A5 + B5 + C5) - 1)$

$p2p2p2p = 1/2 * (A5 + B5 + C5)$

$fori = 1 : length(A5)$

$if(e5(i) <= p1p1p1p(i))||(d5(i) <= p1p1p1p(i)andp1p1p1p(i) <= e5(i)ande5(i) <= B5(i) + C5(i))$

$n1(i) = d5(i) + e5(i)$

$elseif(e5(i) >= B5(i) + C5(i)andp1p1p1p(i) >= d5(i))$

$n1(i) = d5(i) + B5(i) + C5(i)$

$elseif(e5(i) >= p1p1p1p(i)andd5(i) >= p1p1p1p(i))$

$n1(i) = p1p1p1p(i) + p1p1p1p(i)$

$else$

$n1(i) = 0$

end

$if(e5(i) <= B5(i))||(p2p2p2p(i) >= e5(i)ande5(i) >= B5(i)andd5(i) <= A5(i) + B5(i) + C5(i) - e5(i))||(e5(i) >= p2p2p2p(i)ande5(i) <= B5(i) + C5(i)andd5(i) <= A5(i))||(e5(i) >= p2p2p2p(i)ande5(i) <= A5(i) + B5(i) + C5(i) - d5(i)andd5(i) >= A5(i)andd5(i) <= p2p2p2p(i))$

$n2(i) = d5(i) + e5(i)$

$elseif(e5(i) <= p2p2p2p(i)ande5(i) >= B5(i)andd5(i) >= A5(i) + B5(i) + C5(i) - e5(i))$

$n2(i) = B5(i) + A5(i) + B5(i) + C5(i) - e5(i)$

$elseif(d5(i) <= p2p2p2p(i) and d5(i) >= A5(i) and e5(i) >= A5(i) + B5(i) + C5(i) - d5(i))$

$n2(i) = A5(i) + A5(i) + B5(i) + C5(i) - d5(i)$

$elseif(e5(i) >= p2p2p2p(i) and d5(i) >= p2p2p2p(i))$

$n2(i) = p2p2p2p(i) + p2p2p2p(i)$

$elseif(e5(i) >= B5(i) + C5(i) and d5(i) <= A5(i))$

$n2(i) = d5(i) + B5(i) + C5(i)$

$else$

$n2(i) = 0$

end

end

$f1 = sum(f1)$

$h1 = sum(h1)$

$j1 = sum(j1)$

$l1 = sum(l1)$

$m1 = sum(m1)$

$n1 = sum(n1)$

$f2 = sum(f2)$

$h2 = sum(h2)$

$j2 = sum(j2)$

$l2 = sum(l2)$

$m2 = sum(m2)$

$n2 = sum(n2)$

$ff1 = length(A)$

$hh1 = length(A1)$

$jj1 = length(A2)$

$ll1 = length(A3)$

$mm1 = length(A4)$

$nn1 = length(A5)$

$q = ff1 + hh1 + jj1 + ll1 + mm1 + nn1$

$s(k) = (f1 + h1 + j1 + l1 + m1 + n1)/q$

$t(k) = (f2 + h2 + j2 + l2 + m2 + n2)/q$

$cb(k) = 0.09 * k$

end

$x = [cb]$

$y = [s]$

$plot(x, y,' g--')$

$z = [t]$

$holdon$

$plot(x, z,' r-',' linewidth', 3)$

$xlabel('alpha(Synergy)')$

$ylabel('AverageRevenue')$

$title('PlotofRevenueComparison')$

$legend('AverageRevenuewithoutbidwithdrawal',' AverageRevenuewithbidwithdrawal')$

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5 The Effect of Set-Aside Auctions

5.1 Introduction

In 2008, Industry Canada conducted the advanced wireless services (AWS) spectrum auction in which 40 percent megahertz spectrum is set aside for new entrant firms in order to encourage the new entry. Incumbents such as Telus, Rogers, and Bell Mobility, called for an open auction of all wireless spectrum to the highest bidder.[40] They argued that a set-aside auction will result in inefficient allocation, reduce auction revenue and social welfare. Moreover, an open auction will promote market competition efficiently and make consumers better off. A spectrum set-aside auction can guarantee the new entry in the Canadian wireless services market and the price of wireless services will go down due to the intense competition. But the effects of a set-aside auction on revenue, consumer surplus, and social welfare are not clear. In this paper, I show that-unlike what Telus, Rogers, and Bell Mobility argued-under some conditions an auction with set asides may result in higher auction revenue and higher consumer surplus.

There are a few papers in set-aside auctions in the literature. Ayres and Cramton (1996) and Cramton et. al. (2008) study the incumbents bidding strategies in the FCC (Federal Communications Commission) set-aside auctions. They show that an incumbent firm took advantage of the set aside program and won the licenses originally reserved for small firms by creating a bidding front (a new small firm) to participate in the auction with set asides. In Canadian AWS auction, new entrants are defined as "*an entity, including affiliates and associated entities, which holds less than 10 percent of the national wireless*

[40] "*Bell believes strongly that the Canadian wireless industry is highly competitive and such measures (set asides) are not in Canadians' best interest.*"

market based on revenue". Therefore, three biggest national incumbents, Telus, Rogers, and Bell Mobility, cannot bid on the set aside spectrum licenses.It is impossible to be dominant in the set aside auction for national incumbents.

Crandall et. al. (2007) argue that set-aside spectrum for Canadian AWS auction will result in inefficient allocation and reduce economic welfare of consumers. Milgrom (2004) illustrates that an auction with set asides generates higher revenue than an auction without set asides by using an example of four bidders with different exogenous valuation on licenses.

In contrast, I assume that bidders' valuation will be determined endogenously by the profits from the downstream market. Moreover, I show that expected revenue and consumer surplus from a set-aside auction depends on the cost asymmetry among bidders. That is, the expected revenue and consumer surplus from a set-aside auction may be lower than from a non set-aside auction. Hoppe et. al. (2006) illustrate the results with an example that set asides can lower social welfare due to inefficient market entry. They show that revenue is higher from a set-aside. In this paper, I find that a set-aside auction may decrease the seller's revenue, but social welfare is always lower in a non set-aside auction. Moreover, I show that as the market size increases, in other words, the cost asymmetry among bidders decreases, then the seller's revenue, consumer surplus, and social welfare for each auction will increase.

To summarize, in a model simplifying the recent Canadian advanced wireless services spectrum license auction, I show that if the market size or cost asymmetry is moderate, both revenue and consumer surplus from an auction with a set-aside are higher than that from an auction without a set-aside. If the market size is big enough or the cost asymmetry is low enough, revenue will be lower, but consumer surplus will be higher with a set-aside auction. Social welfare is lower in all cases with a set-aside auction.

The chapter is organized as follows. Section 4.2 characterizes a simple model with two incumbent firms and three potential entrant firms. Section 4.3 simulates a two-license model with three incumbent firms and three potential entrant firms. All proofs are in the Appendix.

5.2 The Model

There are 2 licenses, license A and B for sale via a simultaneous ascending bid auction. The auction proceeds in rounds. Prices start from zero for two licenses and increase simultaneously and continuously until only one bidder is left on a given license. On that license auction, the last bidder wins that license at the stopping price. At the same time, the price on the other license will continue to increase, if there are more than one bidder.

The dropout is irreversible; once a bidder drops out of bidding for a given license, he cannot bid for this license again in the next round. The number of active bidders and the drop-out prices are publicly known.

There are 2 homogenous incumbent firms, denoted by firm 1 and firm 2 who are interested in new spectrum licenses to provide advanced wireless services, and 3 homogenous entrant firms, denoted by firm 3, firm 4, and firm 5, who want to win a license to enter the downstream cell-phone service market. Both incumbent and entrant firms' valuation for a single license is determined by the downstream market structure. We assume that if an incumbent firm wins a new license, her marginal cost will be C_L, otherwise, her marginal cost is C_H. We also assume that if an entrant firm wins a license, then her marginal cost will be C_H[41]. There is no fixed costs.

Moreover, we assume that the market demand function is $p(Q) = A - Q$, where A is the market size, and firms will Cournot compete with each other. We consider two types of auctions, auction without set-aside spectrum license and auction with set-aside spectrum license.

In a non set-aside auction, two licenses are open to two incumbents and three entrants. When incumbents' willingness to pay for a new license is higher than entrants' valuation, then two incumbents will win both licenses because they are identical, and pay the third highest valuation, that is, one of entrants' expected profit from the market.

In a set-aside auction, one of licenses will be set aside for three entrants and the other one will be open to both incumbents and entrants. Then, one of entrant firms will win one license and one of the incumbents wins the other license. Because spectrum set-aside guarantees the new entry and three entrants are homogeneous, one of entrants wins the license with expected zero profit. On the other license auction, incumbents will compete for only one new license. So one of incumbents wins the license with her willingness to pay when taking new entry into account, because incumbents are identical.

The following Lemma 27 shows that all the quantities are positive and incumbent firms' valuation is higher than entrants' valuation. Otherwise, two of three entrants will win both licenses in both auctions, there is no need to set aside for entrants.

[41] After winning a new license, incumbents' marginal cost is lower than entrants' because of their existing, well established customer base in previous generation wireless services.

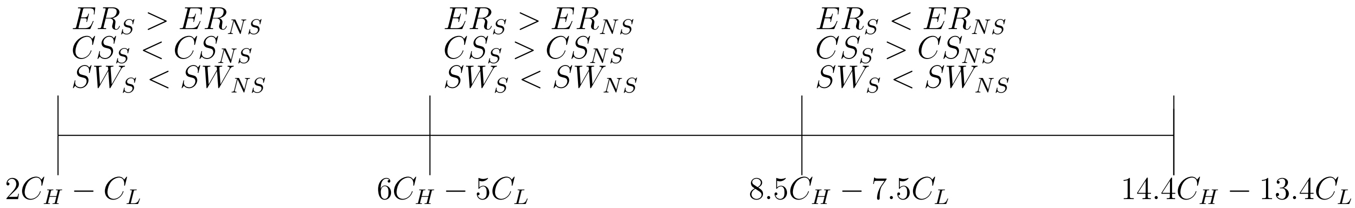

Figure 5: Revenue,CS and SW Comparison

Lemma 27 *: When $2C_H - C_L < A < 14.4C_H - 13.4C_L$, incumbent firms' valuation is higher than entrants' valuation.*

Proof. See the Appendix.

When we compare a non set-aside auction with a set-aside auction in forms of seller's expected revenue, consumer's surplus, and social welfare, we find the following results.

Proposition 28 *:*

a) When $2C_H - C_L < A < 6C_H - 5C_L$, then $ER_S > ER_{NS}$, $CS_{NS} > CS_S$;

b) When $6C_H - 5C_L < A < 8.5C_H - 7.5C_L$, then $ER_S > ER_{NS}$, $CS_S > CS_{NS}$;

c) When $8.5C_H - 7.5C_L < A < 14.4C_H - 13.4C_L$, then $ER_S < ER_{NS}$, $CS_S > CS_{NS}$;

d) When $2C_H - C_L < A < 14.4C_H - 13.4C_L$, $SW_{NS} > SW_S$.

Proof. See the Appendix.

The seller's expected revenue functions in both a set-aside and a no set aside auctions are decreasing functions in cost asymmetry. When the cost asymmetry is high, the seller's expected revenue in a no set-aside auction decreases faster than that in a set aside auction. Thus, the expected revenue is higher in a set aside auction when the cost asymmetry is high enough.

In general, more firms in the market will benefit all consumers because more competition will result in lower prices. In this model, consumer surplus will depend on the relationship between the size of the market and marginal costs. In a set aside auction, one of the high type firms will enter the market. And this new entry will affect consumer surplus in two ways. On the one hand, if high type firm enters the market, then low type firm will produce fewer outputs because of competition. Therefore, there is a negative effect on total output and thus consumer surplus. On the other hand, if a high type firm enters the market, then the high type firm will produce outputs which are previously produced by the low type firm. Therefore, there is a positive effect on total output and hence consumer surplus. Thus, when the size of the market is big enough or the cost asymmetry is not significant, that is, a new entrant firm is relatively efficient, then the positive effect dominates the negative effect, total output increases and thus consumer surplus is higher in set-aside case. When the size of the market is small or the cost asymmetry is significant, that is, a new entrant firm is relatively inefficient, then the negative effect dominates the positive effect, total output decreases and thus consumer surplus is lower in set-aside case.

Social welfare is defined the sum of consumer surplus and producer surplus. Lahiri and Ono (1988) argue that "*by eliminating or impairing minor firms a government can actually increase welfare. This happens primarily because elimination of a less efficient firm, shifts production from it to more efficient ones and thus increases total profits. This increase in profits dominates the loss in consumer's surplus caused by the fall in output which the elimination entails.*"

A spectrum set aside has two different effects on social welfare. On the one hand, there is a positive effect on social welfare due to increased competition in the downstream market which further results in lower prices and higher consumer surplus. On the other hand, there is a negative effect on producer surplus. A spectrum set aside guarantees an inefficient entrant firm will win a license and enter the market. This will result in each firm's profits in the downstream market to decrease as the number of active firms increases. A spectrum set aside indeed lowers social welfare which means that the positive effect is dominated by the negative effect. Thus, this social welfare reducing effect of set asides is derived from the inefficient entry.

5.3 Results of Simulation

Now, if each bidder's marginal costs are different. We assume that there are two licenses for sale via a simultaneous ascending bid auction. There are three incumbents with different marginal costs denoted by, $a1 < a2 < a3$, random drawn on [0.5,1] if they win new licenses, then their marginal costs be lower and denoted by, $b1 < b2 < b3$, random drawn on [0,0.5]. There are three entrants with marginal costs denoted by, $c1 < c2 < c3$ random drawn on [0.5,1] if they win a license. We run simulations with MATLAB.[42] In figures, we find each revenue, consumer surplus and social welfare for a different market size. For each market size, we draw 200 observations, calculate revenue, consumer surplus and social welfare for each case and find the average of these results. From the figures, we find that revenue from a set-aside auction will be higher when the market size is lower than 2, consumer surplus from a set-aside auction will be higher when the market size is lower than 4, social welfare is always lower in set-aside case because of inefficient entry.

Form the figures, I conclude the following observations:

[42] The program is available in the Appendix.

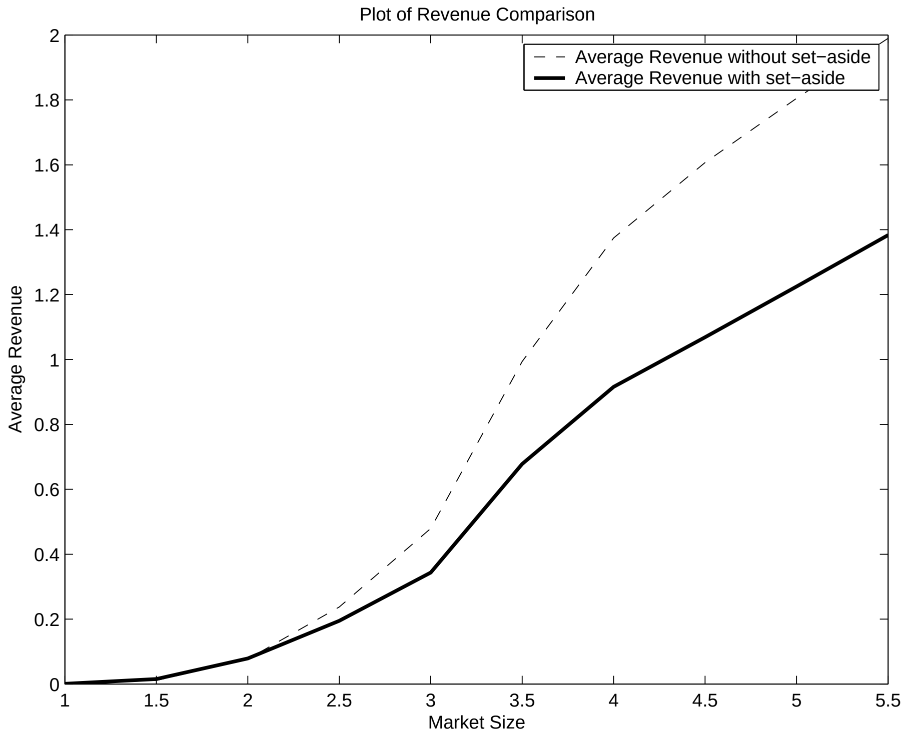

Figure 6: Revenue Comparison.

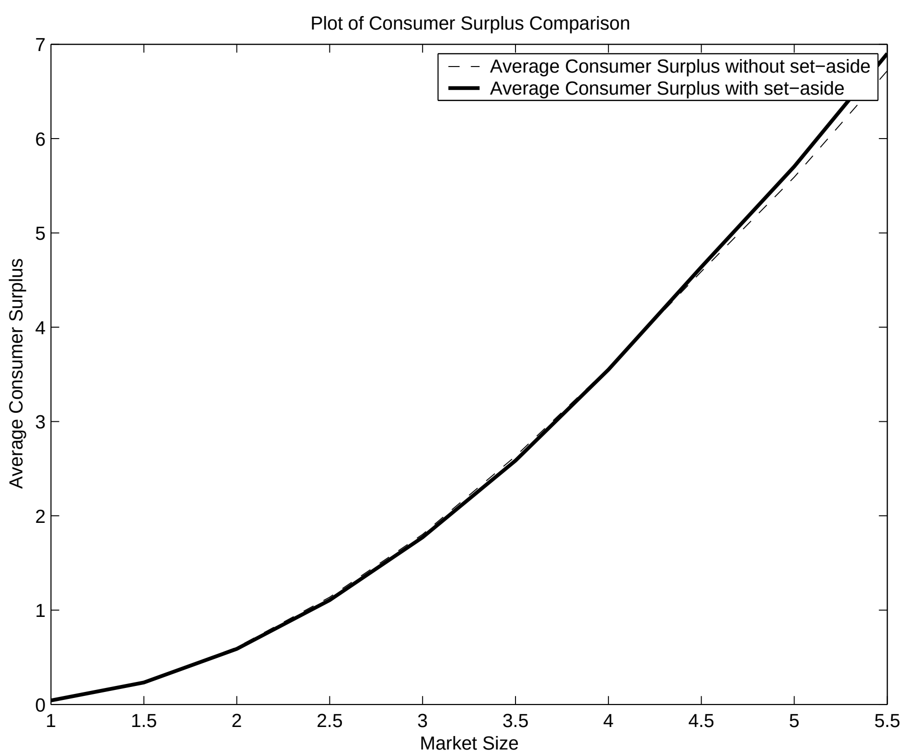

Figure 7: Consumer Surplus Comparison.

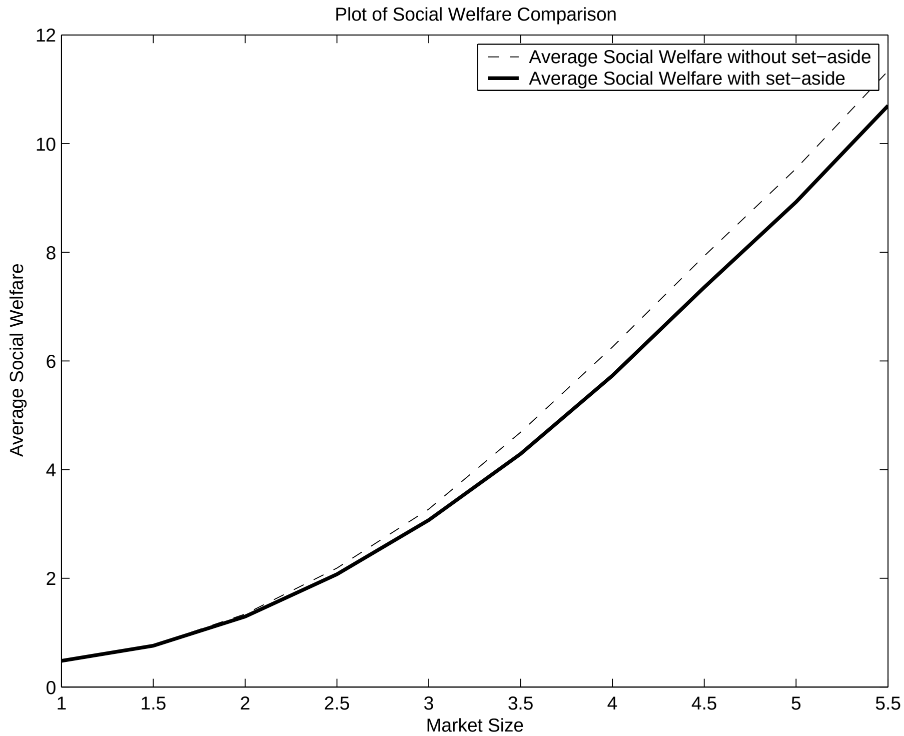

Figure 8: Social Welfare Comparison.

Observation 1 *:*

a) When $A < 2$*, then* $AR_S > AR_{NS}$*,* $CS_S < CS_{NS}$*.*

b) When $2 \leq A \leq 4$*, then* $AR_S < AR_{NS}$*,* $CS_S < CS_{NS}$*.*

c) When $A > 4$*, then* $AR_S < AR_{NS}$*,* $CS_S > CS_{NS}$*.*

d) When $1 < A < 5.5$*,* $SW_S < SW_{NS}$ *and as the market size increases, revenue, consumer surplus and social welfare will increase.*

5.4 Conclusion and Discussion

A spectrum set aside indeed results in inefficient allocation, since a new entrant firm (even though its valuation is lower than incumbents' willingness to pay)will enter the market. Moreover, this inefficient entry further reduces social welfare. But under some circumstances, a spectrum set aside increases the seller's revenue and consumer surplus, even though the allocation is inefficient.

5.5 Appendix

5.5.1 Proofs

Proof of Lemma 27:

In a non set-aside auction, the expected profit of firm i is denoted by $E\Pi^i_{NS}$ and in a set-aside auction, the expected profit of firm i is denoted by $E\Pi^i_S$, where $i = 1, 2, 3, 4, 5$.

In a non set-aside auction, 2 licenses are open to all the incumbent and entrant firms. 2 incumbent firms will win licenses and pay an entrant's valuation. In a set-aside auction, one of two licenses will be set aside for entrant firms and the other license will be open to both incumbent and entrant firms. Then, one entrant firm, F_i, where $i = 3, 4, 5$, will win license A at a price equal to F_i's expected profit where $i = 3, 4, 5$, and one of incumbents will win the other license at a price equal to F_i's willingness to pay, where $i = 1, 2,$.

In a non set-aside auction, 2 licenses are open to all the incumbent and entrant firms. If incumbent firms F_i, $i = 1, 2$ win these two licenses, then its profit is, $\Pi^{iW}_{NS} = [A - C_L - (Q_L + Q_L)]Q_L$. If an incumbent firm F_i does win a license, then two entrants win and its profit is, $\Pi^{iNW}_{NS} = [A - C_H - (2Q_H + 2Q_H)]Q_H$[43]. If an entrant firm does win a license, then two entrants win and each profit is, $\Pi^{iW}_{NS} = \frac{2}{3}[A - C_H - (2Q_H + 2Q_H)]Q_H$.

[43]Because two incumbents are identical. Their strategies are symmetric. If one incumbent chooses to free-ride, then the other one cannot keep bidding. If I assume that incumbents' valuation of a new license is defined by $V_1 = V_2 = \Pi^W_2 - p\Pi^{NW}_3 - (1-p)\Pi^{NW}_4$, where p denotes the probability of free-ride and entrants' valuation is denoted by $V_3 = V_4 = V_5$, then there are three cases: Case 1. two incumbents win both licenses when $V_1 = V_2 > V_3 = V_4 = V_5$; Case 2. two entrants win both licenses when $V_1 = V_2 < V_3 = V_4 = V_5$; Case 3. each of both incumbents and entrants has an equal probability to win a license when $V_1 = V_2 = V_3 = V_4 = V_5$. We consider the first two cases only, either two incumbents win or lose the licenses.

We take a partial derivative of the profit of Π^i_{NS} with respect to F_i's quantity, Q_L or Q_H, then the profit-maximizing quantity and profits are as follows,

$\frac{\partial \Pi^{iW}_{NS}}{\partial Q_L} = 0 \Longrightarrow Q^{iW*}_L = \frac{A+C_L+C_L-3C_L}{3}$

$\Longrightarrow \Pi^{iW*}_{NS} = [\frac{A+C_L+C_L-3C_L}{3}]^2$

Similarly, we have, $\frac{\partial \Pi^{iNW}_{NS}}{\partial Q_H} = 0 \Longrightarrow Q^{iNW*}_H = \frac{A-C_H}{5}$ where $i = 1, 2$

$\Longrightarrow Q^{iW*}_H = \frac{A-C_H}{5}$ where $i = 3, 4, 5$

$\Longrightarrow \Pi^{iNW*}_{NS} = [\frac{A-C_H}{5}]^2$ where $i = 1, 2$

$\Longrightarrow \Pi^{iW*}_{NS} = \frac{2}{3}[\frac{A-C_H}{5}]^2$ where $i = 3, 4, 5$

One assumption we need that all the quantities are positive. So we assume,

$A - C_H > 0.$

We will show that incumbent firms' willingness to pay is higher than entrant firms' valuation. If an incumbent firm, Firm 1, wins one license and one entrant firm, for example, Firm 3, wins the other license, then entrant firm's profit is, $\Pi^{3*}_{NS} = \frac{2}{3}[\frac{A-C_H}{5}]^2$.

So we need, $\Pi^{2W*}_{NS} - \Pi^{2NW*}_{NS} > \Pi^{3*}_{NS} \Leftrightarrow [\frac{A+C_L+C_L-3C_L}{3}]^2 - [\frac{A-C_H}{5}]^2 > \frac{2}{3}[\frac{A-C_H}{5}]^2$

$\Leftrightarrow 5(A - C_L)^2 > 3(A - C_H)^2$

$\Leftrightarrow \sqrt{5}(A - C_L) > \sqrt{3}(A - C_H)$

Since,$C_L < C_H$, we have $\Pi^{2W*}_{NS} - \Pi^{2NW*}_{NS} > \Pi^{3*}_{NS}$.

In a set-aside auction, one license is open to all the incumbent and entrant firms and the other one is set aside only for the entrants. If an incumbent firm F_1, wins the license, then its profit is, $\Pi_S^{1W} = \frac{1}{2}[A - C_L - (Q_L + 2Q_H)]Q_L$. If an incumbent firm F_1 does win a license, then the other incumbent will win it and its profit is, $\Pi_S^{1NW} = \frac{1}{2}[A - C_H - (Q_L + 2Q_H)]Q_H$. If an entrant firm F_3 does win a license, then its profit is, $\Pi_S^3 = \frac{1}{3}[A - C_H - (Q_L + Q_H + Q_H)]Q_H$.

We take the partial derivative of these profits with respect to F_i's quantity, then the profit-maximizing quantity and profits are as follows,

$\frac{\partial \Pi_S^{1W}}{\partial Q_L} = 0 \Longrightarrow Q_L^{1W*} = \frac{A + C_L + 2C_H - 4C_L}{4}$

$\Longrightarrow \Pi_S^{1W*} = \frac{1}{2}[\frac{A + C_L + 2C_H - 4C_L}{4}]^2$

Similarly, we have, $\frac{\partial \Pi_S^{1NW}}{\partial Q_H} = 0 \Longrightarrow Q_H^{1NW*} = \frac{A + C_L + 2C_H - 4C_H}{4}$ where $i = 1, 2$

$\Longrightarrow \Pi_S^{1NW*} = \frac{1}{2}[\frac{A + C_L + 2C_H - 4C_H}{4}]^2$

And $\frac{\partial \Pi_S^3}{\partial Q_H} = 0 \Longrightarrow Q_H^{4*} = \frac{A + C_L + 2C_H - 4C_H}{4}$

$\Longrightarrow \Pi_S^{3*} = \frac{1}{3}[\frac{A + C_L + 2C_H - 4C_H}{4}]^2$

Similarly, one assumption we need that all the quantities are positive. So we assume,

$A + C_L - 2C_H > 0.$

The other assumption we require is that incumbent firms' willingness to pay is higher than entrant firms' valuation. If an incumbent firm, Firm 1, wins one license and one entrant firm, for example, Firm 4, wins the other license in an open auction, then entrant firm's profit under Auction 2 is, $\Pi_S^{4*} = \frac{1}{2}[\frac{A + 4C_H - 5C_H}{5}]^2$.

So we need, $\Pi_S^{1W*} - \Pi_S^{1NW*} > \Pi_S^{4*} \Leftrightarrow \frac{1}{2}[\frac{A + C_L + 2C_H - 4C_L}{4}]^2 - \frac{1}{2}[\frac{A + C_L + 2C_H - 4C_H}{4}]^2 > \frac{1}{2}[\frac{A + 4C_H - 5C_H}{5}]^2$

$\Leftrightarrow 2A^2 + 2C_H^2 - 25C_L^2 + 25AC_L - 29AC_H - 25C_LC_H < 0$

$\Leftrightarrow A < 14.4C_H - 13.4C_L$

Combined with these assumption, we conclude that $2C_H - C_L < A < 14.4C_H - 13.4C_L$.

■

Proof of Proposition 28:

Now we compare revenue from a non set-aside auction with revenue from a set-aside auction. Since 2 incumbents win both licenses and pay entrants'valuation in a non set-aside auction, $\frac{2}{3}[\frac{A-C_H}{5}]^2$, then total revenue from a non set-aside auction is determined by, $R_{NS} = 2\{\frac{2}{3}[\frac{A-C_H}{5}]^2\}$; And one of incumbents wins one license and pays incumbents' valuation, $\frac{1}{2}[\frac{A+C_L+2C_H-4C_L}{4}]^2 - \frac{1}{2}[\frac{A+C_L+2C_H-4C_H}{4}]^2$ and one of entrants wins the other license and pays entrants' valuation, $\frac{1}{3}[\frac{A+C_L+2C_H-4C_H}{4}]^2$, then total revenue from auction with a set-aside is determined by, $R_S = \frac{1}{2}[\frac{A+C_L+2C_H-4C_L}{4}]^2 - \frac{1}{2}[\frac{A+C_L+2C_H-4C_H}{4}]^2 + \frac{1}{3}[\frac{A+C_L+2C_H-4C_H}{4}]^2$.

If $R_{NS} > R_S$, we have,

$\Leftrightarrow 8[\frac{A-C_H}{5}]^2 > 3[\frac{A+C_L+2C_H-4C_L}{4}]^2 - [\frac{A+C_L+2C_H-4C_H}{4}]^2$

$\Leftrightarrow 39A^2 - 36C_H^2 - 325C_L^2 - 328AC_H + 250AC_L + 400C_HC_L > 0$

$\Leftrightarrow A > 8.5C_H - 7.5C_L or A < C_L - 0.1C_H + 0.1C_L$

Since, $C_L < C_H \Leftrightarrow A < C_L - 0.1C_H + 0.1C_L < 2C_H - C_L$, thus, we have, when $A > 8.5C_H - 7.5C_L$, $R_{NS} > R_S$ and when $2C_H - C_L < A < 8.5C_H - 7.5C_L$, $R_{NS} < R_S$.

Next, we compare consumer surplus and social welfare CS_{NS}, SW_{NS} under a non set-aside auction and CS_S, SW_S under a set-aside auction. Since the market demand is given by $p = A - Q$, then consumer surplus is determined by $CS = \frac{1}{2}Q(A-p) = \frac{1}{2}Q^2$. Social welfare is defined by the sum of consumer surplus and producer surplus.

In a non set-aside auction, 2 incumbent firms win licenses, total quantity $Q_{NS} = 2Q_L^* + Q_H^* = \frac{2A-2C_L}{3}$, thus,

$CS_{NS} = \frac{1}{2}(\frac{2A-2C_L}{3})^2$.

$PS_{NS} = \Pi_{NS}^{1*} + \Pi_{NS}^{2*}$

$= 2[\frac{A+2C_L-3C_L}{3}]^2$

$SW_{NS} = CS_{NS} + PS_{NS} = \frac{1}{2}(\frac{2A-2C_L}{3})^2 + 2[\frac{A+2C_L-3C_L}{3}]^2$

In a set-aside auction, the entrant firm F_3 wins one license and the incumbent firm F_1, wins the other license, respectively, so there are 3 firms in the market. Total quantity is determined by $Q_S = Q_L^* + 2Q_H^* = \frac{3A-C_L-2C_H}{4}$, thus,

$CS_S = \frac{1}{2}(\frac{3A-C_L-2C_H}{4})^2$.

$PS_S = \Pi_S^{1*} + \Pi_S^{2*} + \Pi_S^{3*}$

$= [\frac{A+C_L+2C_H-4C_L}{4}]^2 + 2[\frac{A+C_L+2C_H-4C_H}{4}]^2$

$SW_S = CS_S + PS_S = \frac{1}{2}(\frac{3A-C_L-2C_H}{4})^2 + [\frac{A+C_L+2C_H-4C_L}{4}]^2 + 2[\frac{A+C_L+2C_H-4C_H}{4}]^2$

When we compare CS_{NS} with CS_S, we have,

If $CS_{NS} > CS_S$, then,

$\Leftrightarrow \frac{1}{2}(\frac{2A-2C_L}{3})^2 > \frac{1}{2}(\frac{3A-C_L-2C_H}{4})^2$

$\Leftrightarrow \frac{2A-2C_L}{3} > \frac{3A-C_L-2C_H}{4}$

$\Leftrightarrow A < 6C_H - 5C_L$

Thus, when $2C_H - C_L < A < 6C_H - 5C_L$, $CS_{NS} > CS_S$ and when $6C_H - 5C_L \leq A < 14.4C_H - 13.4C_L$, $CS_{NS} < CS_S$.

Finally, we compare SW_{NS} with SW_S and find that

When $SW_{NS} > SW_S$, we have,

$\Leftrightarrow \frac{1}{2}(\frac{2A-2C_L}{3})^2 + 2[\frac{A+2C_L-3C_L}{3}]^2$

$> \frac{1}{2}(\frac{3A-C_L-2C_H}{4})^2 + [\frac{A+C_L+2C_H-4C_L}{4}]^2 + 2[\frac{A+C_L+2C_H-4C_H}{4}]^2$

$\Leftrightarrow 7A^2 + 79C_L^2 + 252C_H^2 + 166AC_L - 180AC_H - 324C_HC_L < 0$

$\Leftrightarrow 1.5C_H - 0.5C_L < A < 24.2C_H - 23.2C_L$

Since $C_H > C_L \Leftrightarrow 2C_H - C_L > 1.5C_H - 0.5C_L$, and $24.2C_H - 23.2C_L > 14.4C_H - 13.4C_L$, when $2C_H - C_L < A < 14.4C_H - 13.4C_L$, we always have $SW_{NS} > SW_S$.

■

5.5.2 Simulation Code

$n = 200$

$a = rand(3, n)$

$forj = 1 : 3$

$fork = 1 : n$

$ifa(j, k) < 0.5$

$a(j,k) = a(j,k) + 0.5$

end

end

end

a

$u = sort(a)$

$a = u$

$b = rand(3,n)$

$b = b * 0.5$

$w = sort(b)$

$b = w$

$c = rand(3,n)$

$forj = 1 : 3$

$fork = 1 : n$

$ifc(j,k) < 0.5$

$c(j,k) = a(j,k) + 0.5$

end

end

end

c

$v = sort(c)$

$c = v$

$for i = 1 : 10$

$for k = 1 : n$

$A(i, k) = i * 0.5 + 0.5$

$if A(i, k) + b(1, k) + a(2, k) + a(3, k) + c(2, k) - 5 * a(2, k) > 0 and A(i, k) + b(1, k) + a(2, k) + a(3, k) + c(2, k) - 5 * c(2, k) > 0$

$v55(i, k) = ((A(i, k) + a(1, k) + a(2, k) + a(3, k) + c(1, k) + c(2, k) - 6 * c(1, k))/6)^2$

$v15(i, k) = ((A(i, k) + b(1, k) + a(2, k) + a(3, k) + c(1, k) - 5 * b(1, k))/5)^2 - ((A(i, k) + a(1, k) + a(2, k) + a(3, k) + c(1, k) + c(2, k) - 6 * a(1, k))/6)^2$

$v44(i, k) = ((A(i, k) + b(1, k) + a(2, k) + a(3, k) + c(1, k) - 5 * c(1, k))/5)^2$

$v54(i, k) = ((A(i, k) + b(1, k) + a(2, k) + a(3, k) + c(2, k) - 5 * c(2, k))/5)^2$

$v14(i, k) = ((A(i, k) + b(1, k) + a(2, k) + a(3, k) + c(1, k) - 5 * b(1, k))/5)^2 - ((A(i, k) + a(1, k) + b(2, k) + a(3, k) + c(1, k) - 5 * a(1, k))/5)^2$

$v24(i, k) = ((A(i, k) + a(1, k) + b(2, k) + a(3, k) + c(1, k) - 5 * b(2, k))/5)^2 - ((A(i, k) + b(1, k) + a(2, k) + a(3, k) + c(1, k) - 5 * a(2, k))/5)^2$

$v34(i, k) = ((A(i, k) + a(1, k) + a(2, k) + b(3, k) + c(1, k) - 5 * b(3, k))/5)^2 - ((A(i, k) +$

$b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(3,k))/5)^2$

$v13(i,k) = ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4 * b(1,k))/4)^2 - ((A(i,k) + a(1,k) + b(2,k) + b(3,k) - 4 * a(1,k))/4)^2$

$v23(i,k) = ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4 * b(2,k))/4)^2 - ((A(i,k) + b(1,k) + a(2,k) + b(3,k) - 4 * a(2,k))/4)^2$

$v33(i,k) = ((A(i,k) + b(1,k) + a(2,k) + b(3,k) - 4 * b(3,k))/4)^2 - ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4 * a(3,k))/4)^2$

$else$

$v55(i,k) = 0$

$v15(i,k) = 0$

$v44(i,k) = 0$

$v54(i,k) = 0$

$v14(i,k) = 0$

$v24(i,k) = 0$

$v34(i,k) = 0$

$v13(i,k) = 0$

$v23(i,k) = 0$

$v33(i,k) = 0$

end

$ifv54(i,k) > 0andv55(i,k) >= v15(i,k)$

$R1(i,k) = v15(i,k) + v15(i,k)$

$R2(i,k) = v55(i,k) + v15(i,k)$

$CS1(i,k) = 1/2 * ((5 * A(i,k) - a(1,k) - a(2,k) - a(3,k) - c(1,k) - c(2,k))/6)^2$

$CS2(i,k) = 1/2 * ((5 * A(i,k) - a(1,k) - a(2,k) - a(3,k) - c(1,k) - c(2,k))/6)^2$

$SW1(i,k) = 1/2 * ((5 * A(i,k) - a(1,k) - a(2,k) - a(3,k) - c(1,k) - c(2,k))/6)^2 + ((A(i,k) + a(1,k) + a(2,k) + a(3,k) + c(1,k) + c(2,k) - 6 * a(1,k))/6)^2 + ((A(i,k) + a(1,k) + a(2,k) + a(3,k) + c(1,k) + c(2,k) - 6 * a(2,k))/6)^2 + ((A(i,k) + a(1,k) + a(2,k) + a(3,k) + c(1,k) + c(2,k) - 6 * a(3,k))/6)^2 + ((A(i,k) + a(1,k) + a(2,k) + a(3,k) + c(1,k) + c(2,k) - 6 * c(1,k))/6)^2 + ((A(i,k) + a(1,k) + a(2,k) + a(3,k) + c(1,k) + c(2,k) - 6 * c(2,k))/6)^2$

$SW2(i,k) = SW1(i,k)$

$elseifv54(i,k) > 0andv44(i,k) >= v14(i,k)andv14(i,k) >= v54(i,k)andv54(i,k) >= v24(i,k)$

$R1(i,k) = v54(i,k) + v54(i,k)$

$R2(i,k) = v54(i,k) + v54(i,k)$

$CS1(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$CS2(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$SW1(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2 + ((A(i,k) +$

$b(1,k)+a(2,k)+a(3,k)+c(1,k)-5*b(1,k))/5)^2+((A(i,k)+b(1,k)+a(2,k)+a(3,k)+c(1,k)-5*a(2,k))/5)^2+((A(i,k)+b(1,k)+a(2,k)+a(3,k)+c(1,k)-5*a(3,k))/5)^2+((A(i,k)+b(1,k)+a(2,k)+a(3,k)+c(1,k)-5*c(1,k))/5)^2$

$SW2(i,k)=SW1(i,k)$

$elseif v54(i,k)>0 and v44(i,k)>=v14(i,k) and v14(i,k)>=v24(i,k) and v24(i,k)>=v54(i,k) and v54(i,k)>=v34(i,k)$

$R1(i,k)=v24(i,k)+v24(i,k)$

$R2(i,k)=v54(i,k)+v24(i,k)$

$CS1(i,k)=1/2*((4*A(i,k)-b(1,k)-a(2,k)-a(3,k)-c(1,k))/5)^2$

$CS2(i,k)=1/2*((4*A(i,k)-b(1,k)-a(2,k)-a(3,k)-c(1,k))/5)^2$

$SW1(i,k)=1/2*((4*A(i,k)-b(1,k)-a(2,k)-a(3,k)-c(1,k))/5)^2+((A(i,k)+b(1,k)+a(2,k)+a(3,k)+c(1,k)-5*b(1,k))/5)^2+((A(i,k)+b(1,k)+a(2,k)+a(3,k)+c(1,k)-5*a(2,k))/5)^2+((A(i,k)+b(1,k)+a(2,k)+a(3,k)+c(1,k)-5*a(3,k))/5)^2+((A(i,k)+b(1,k)+a(2,k)+a(3,k)+c(1,k)-5*c(1,k))/5)^2$

$SW2(i,k)=SW1(i,k)$

$elseif v44(i,k)>=v14(i,k) and v14(i,k)>=v24(i,k) and v24(i,k)>=v34(i,k) and v34(i,k)>=v54(i,k)$

$R1(i,k)=v24(i,k)+v24(i,k)$

$R2(i,k)=v54(i,k)+v24(i,k)$

$CS1(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$CS2(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$SW1(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * b(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(2,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(3,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * c(1,k))/5)^2$

$SW2(i,k) = SW1(i,k)$

$elseif v54(i,k) > 0 and v14(i,k) >= v44(i,k) and v44(i,k) >= v54(i,k) and v54(i,k) >= v24(i,k) and v24(i,k) >= v34(i,k)$

$R1(i,k) = v54(i,k) + v54(i,k)$

$R2(i,k) = v54(i,k) + v54(i,k)$

$CS1(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$CS2(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$SW1(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * b(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(2,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(3,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * c(1,k))/5)^2$

$SW2(i,k) = SW1(i,k)$

$elseif v54(i,k) > 0 and v14(i,k) >= v44(i,k) and v44(i,k) >= v24(i,k) and v24(i,k) >= v54(i,k) and v54(i,k) >= v34(i,k)$

$R1(i,k) = v24(i,k) + v24(i,k)$

$R2(i,k) = v54(i,k) + v24(i,k)$

$CS1(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$CS2(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$SW1(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * b(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(2,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(3,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * c(1,k))/5)^2$

$SW2(i,k) = SW1(i,k)$

$elseif v54(i,k) > 0 and v14(i,k) >= v44(i,k) and v44(i,k) >= v24(i,k) and v24(i,k) >= v34(i,k) and v34(i,k) >= v54(i,k)$

$R1(i,k) = v24(i,k) + v24(i,k)$

$R2(i,k) = v54(i,k) + v24(i,k)$

$CS1(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$CS2(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$SW1(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * b(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(2,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(3,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * c(1,k))/5)^2$

$SW2(i,k) = SW1(i,k)$

$elseif v54(i,k) > 0 and v23(i,k) >= v44(i,k) and v54(i,k) >= v33(i,k) and v24(i,k) >= v44(i,k) and v54(i,k) >= v34(i,k)$

$R1(i,k) = v44(i,k) + v44(i,k)$

$R2(i,k) = v54(i,k) + v24(i,k)$

$CS1(i,k) = 1/2 * ((3 * A(i,k) - b(1,k) - b(2,k) - a(3,k))/4)^2$

$CS2(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$SW1(i,k) = 1/2 * ((3 * A(i,k) - b(1,k) - b(2,k) - a(3,k))/4)^2 + ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4 * b(1,k))/4)^2 + ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4 * b(2,k))/4)^2 + ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4 * a(3,k))/4)^2$

$SW2(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * b(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(2,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * a(3,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5 * c(1,k))/5)^2$

$elseif v54(i,k) > 0 and v33(i,k) >= v44(i,k) and v34(i,k) >= v44(i,k)$

$R1(i,k) = v33(i,k) + v33(i,k)$

$R2(i,k) = v54(i,k) + v24(i,k)$

$CS1(i,k) = 1/2 * ((3 * A(i,k) - b(1,k) - b(2,k) - a(3,k))/4)^2$

$CS2(i,k) = 1/2 * ((4 * A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$SW1(i,k) = 1/2*((3*A(i,k) - b(1,k) - b(2,k) - a(3,k))/4)^2 + ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4*b(1,k))/4)^2 + ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4*b(2,k))/4)^2 + ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4*a(3,k))/4)^2$

$SW2(i,k) = 1/2*((4*A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5*b(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5*a(2,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5*a(3,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5*c(1,k))/5)^2$

$elseif v54(i,k) > 0 and v23(i,k) >= v44(i,k) and v44(i,k) >= v33(i,k) and v33(i,k) >= v54(i,k) and v24(i,k) >= v44(i,k) and v44(i,k) >= v34(i,k) and v34(i,k) >= v54(i,k)$

$R1(i,k) = v44(i,k) + v44(i,k)$

$R2(i,k) = v54(i,k) + v24(i,k)$

$CS1(i,k) = 1/2*((3*A(i,k) - b(1,k) - b(2,k) - a(3,k))/4)^2$

$CS2(i,k) = 1/2*((4*A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2$

$SW1(i,k) = 1/2*((3*A(i,k) - b(1,k) - b(2,k) - a(3,k))/4)^2 + ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4*b(1,k))/4)^2 + ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4*b(2,k))/4)^2 + ((A(i,k) + b(1,k) + b(2,k) + a(3,k) - 4*a(3,k))/4)^2$

$SW2(i,k) = 1/2*((4*A(i,k) - b(1,k) - a(2,k) - a(3,k) - c(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5*b(1,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5*a(2,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5*a(3,k))/5)^2 + ((A(i,k) + b(1,k) + a(2,k) + a(3,k) + c(1,k) - 5*c(1,k))/5)^2$

$else$

$R1(i,k) = 0$

$R2(i,k) = 0$

$CS1(i,k) = 0$

$CS2(i,k) = 0$

$SW1(i,k) = 0$

$SW2(i,k) = 0$

end

$CB(i) = i * 0.5 + 0.5$

end

end

$RR1 = permute(R1, [2,1])$

$RR2 = permute(R2, [2,1])$

$TR1 = sum(RR1)$

$TR2 = sum(RR2)$

$AR1 = TR1/n$

$AR2 = TR2/n$

$CCS1 = permute(CS1, [2,1])$

$CCS2 = permute(CS2, [2, 1])$

$TCS1 = sum(CCS1)$

$TCS2 = sum(CCS2)$

$ACS1 = TCS1/n$

$ACS2 = TCS2/n$

$SSW1 = permute(SW1, [2, 1])$

$SSW2 = permute(SW2, [2, 1])$

$TSW1 = sum(SSW1)$

$TSW2 = sum(SSW2)$

$ASW1 = TSW1/n$

$ASW2 = TSW2/n$

$x = [CB]$

$y = double(AR1)$

$plot(x, y,' b - -')$

$z = double(AR2)$

$holdon$

$plot(x, z,' r-',' linewidth', 2)$

$xlabel('MarketSize')$

$ylabel('AverageRevenue')$

$title('PlotofRevenueComparison')$

$legend('AverageRevenuewithoutset-aside','AverageRevenuewithset-aside')$

$x = [CB]$

$y = double(ACS1)$

$plot(x, y,'b--')$

$z = double(ACS2)$

$holdon$

$plot(x, z,'r-','linewidth', 2)$

$xlabel('MarketSize')$

$ylabel('AverageConsumerSurplus')$

$title('PlotofConsumerSurplusComparison')$

$legend('AverageConsumerSurpluswithoutset-aside','AverageConsumerSurpluswithset-aside')$

$x = [CB]$

$y = double(ASW1)$

$plot(x, y,'b--')$

$z = double(ASW2)$

$holdon$

$plot(x, z,' r-',' linewidth', 2)$

$xlabel('MarketSize')$

$ylabel('AverageSocialWelfare')$

$title('PlotofSocialWelfareComparison')$

$legend('AverageSocialWelfarewithoutset-aside',' AverageSocialWelfarewithset-aside')$

5.6 References

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6 Conclusion

This thesis mainly focuses on multi-unit spectrum license auctions. I analyze the effects of different auction rules on revenue, efficiency, and social welfare, and show how to improve the auction designs.

In the second chapter, "Predatory Bidding in Sequential Auctions," we show that setting the winning bid of the first auction as the reserve price of the second auction is a design failure; hence, the Turkish government could not sell two licenses. Setting such endogenous reserve price gives the firms the incentive of predatory bidding. We show that if the auction were designed as a simultaneous auction, the government would sell two licenses and receive more revenue.

In the third chapter, "The 2000 Turkish Cell-Phone License Auction," we show that a seller who cares about social welfare or revenue can always use a different auction design which gives better or same results than the Turkish auction. We also find that if the cost asymmetry between the bidding firms is large enough, then having fewer firms in the market may result in higher social welfare.

In the fourth chapter, "Exposure Problem in Multi-Unit Auctions," our conclusion is that an auction with bid withdrawal is superior to an auction without bid withdrawal in terms of revenue and allocation efficiency. We add to the literature in at least three ways. First, we show the optimal bidding strategies of global bidders when there are moderate synergies and the licenses are heterogeneous. Second, we study bid-withdrawal in multi-unit auctions theoretically for the first time (to our best knowledge). Third, we analyze the exposure problem extensively and show when it will arise even though the bidder wins all licenses. One of our main contributions is to show that the exposure problem is solved when bid-withdrawal is introduced. Elimination of the exposure problem makes the global bidder bid more aggressively at first; however, it bids less aggressively later in the remaining auction. As the net effect, we show that revenue is still higher.

In the fifth chapter, "The Effect of Set-Aside Auctions," I study the Canadian AWS auction in which 40 percent spectrum are set aside for new firms. I show that a spectrum set aside indeed results in inefficient allocation, since a new entrant firm will enter the market. This inefficient entry further reduces social welfare. But under some circumstance, a spectrum set aside increases the seller's revenue and consumer surplus.