117

AN ECONOMIC AND ACTUARIAL EVALUATION AND COMPARATIVE STUDY OF ALL-RISK CROP INSURANCE PROGRAMS IN MANITOBA

By

Yongsheng Ye

A Dissertation Submitted to the Faculty of Graduate Studies of the UNIVERSITY OF MANITOBA in Partial Fulfillment of the Requirements of the Degree of

DOCTOR OF PHILOSOPHY

Department of Agricultural Economics and Farm Management

Winnipeg Canada

ISBN 0-612-13588-8

AN ECONOMIC AND ACTUARIAL EVALUATION AND COMPARATIVE STUDY OF ALL-RISK CROP INSURANCE PROGRAMS IN MANITOBA

BY

YONGSHENG YE

A Thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

I hereby declare that I am the sole author of this dissertation.

I authorize the **University of Manitoba** to lend this dissertation to other institutions or individuals for the purpose of scholarly research.

I further authorize the **University of Manitoba** to reproduce this dissertation by photocopying or by other means, in total or in part, at the request of other institutions or individuals for the purpose of scholarly research.

Yongsheng Ye

Contents

List of Tables

0.1 Acknowledgements

I gratefully acknowledge the assistance and support provided by many institutions and individuals during the period of my graduate study and the course of conducting this research. There are too many to mention all individually, but I would like to express especially my thanks to the following:

I wish to extend my sincere thanks to Dr. Martin H. Yeh, my major advisor, for his numerous help. Dr. Yeh has provided a lot of time, patience, and guidance. Without his constant encouragement, the completion of this research would not have been possible.

I also wish to acknowledge with thanks to Dr. J. C. Gilson of the University of Manitoba, Dr. Neil Hamilton of the Manitoba Crop Insurance Corporation, and Dr. S. R. Jeffrey of the University of Alberta, for their valuable comments and suggestions on earlier proposals of this project. Dr. Gilson and Dr. Hamilton provided a thorough and extremely constructive review in initiating and shaping this research.

Special thanks are due to Professor Rea Josephson and Dr. Brian Owen, my advisory and thesis committee members, for their careful reading and thoughtful suggestions towards thesis improvements. Professor Josephson is also acknowledged for his kind supervision when my major advisor was on leave during 1991.

I would like to express my gratitude to the Canadian International Development Agency for its generous financial support for my graduate program. I am also grateful to the University of Manitoba, the Manitoba Crop Insurance Corporation, and Huazhong Agricultural University (China) for their enormous assistance. Many individuals associated with these institutions have contributed to the completion of my graduate studies, my thesis research, and my thesis writing. Dr. J. I. Elliot, Dr. J. A. MacMillan, Mr. Edmumd Mupondwa, Mr. Paul Gamba, and Mr. Ernest Mwape of the University of Manitoba; Professor Dazun Shen, Professor Jizhong Sun, and Professor Xinsheng He of Huazhong Agricultural University; and Mr. Grant Forsman of the Manitoba Crop Insurance Corporation, to just name a few.

Finally, I wish to extend my deepest appreciation to my dear wife, Zhihong Zheng, our lovely son, Peng, and my parents, for their understanding, tolerance, patience, and love. They all have sacrificed much to support my studies overseas and to cope with my long period of absence and anxiety. I am deeply indebted to them.

Any errors and mistakes which have remained in this thesis are solely the responsibility of the author.

0.2 Abstract

This thesis conducts a comprehensive evaluation and a comparative study of crop insurance programs in Manitoba. The main purpose of the study is to investigate the most effective crop insurance program structures for both the government crop insurance corporation and the insured farmers. The major objective is to provide theoretical and empirical insights into some important issues with which a voluntary all-risk crop insurance is often confronted, and to seek some plausible solutions in two distinct ways: (1) to construct improved program designs including better actuarial structures, better premium ratemaking methodologies, and better coverage frameworks, and (2) to induce stronger program demand.

The thesis contains three parts. An extensive economic literature review of general insurance and crop insurance is first conducted. The fundamentals of the actuarial aspect of insurance are outlined. The basic framework of insurance economics is introduced and discussed within the context of demand for and supply of insurance. A general public all-risk crop insurance model is then developed. The resource distortion effects of crop insurance program parameters are isolated and the resource neutrality conditions are derived. In the first part, adverse selection and moral hazard problems inherited in various all-risk crop insurance programs are also emphasized. Theory suggests that a purely or partially individualized farm-level crop insurance program has no sound foundation due to actuarial inconsistencies. Some approximation has to be sought in order to establish an actuarially sound and financially viable program structure in practice. Currently implemented or proposed programs such as homogeneous risk area based individual productivity index (IPI) and the area-yield crop insurance approach are some examples of such approximations. Theoretically, purely homogeneous risk area program structure with some special adjustment mechanisms

could be the best option. The individualization of current program is not recommended. Manitoba's program should be reformed within the present framework with an emphasis on coverage adjustment methodology.

Parts two and three present some empirical results. In part two (Chapter 3), three crop insurance programs are evaluated in terms of their effectiveness of yield risk reductions for more than 450 Manitoba farms. The examination is first conducted with a proposed index method, where the relative yield risk reduction magnitude is calculated and compared for each farm under each program. The generalized stochastic dominance (GSD) methodology is also used to provide an alternative analytical framework in analyzing producers' relative preferences among those alternatives by comparing the net yield distributions generated by each program for each farm. The results suggest that, given an actuarially sound basis, the fully individualized crop insurance (FI) program is the most favorable choice for risk-averse producers. The area coverage and individual indemnity program (IA) is generally the second best option. The full area crop insurance program (FA) is least preferred by risk-averse farmers. The index approach and the GSD results also clearly indicate producers will be less sensitive to the alternative programs if coverage level is increased. Although the fully individualized program is generally preferred over the area coverage and individual indemnity (IA) program, the dominance can only be made marginally. In some cases, the IA program may be more attractive than the FI program.

In part three (Chapter 4), theoretical and analytical issues in pure premium ratemaking for an all-risk crop insurance program are evaluated. Explicit actuarially fair rate formulas for normal yield distribution and beta yield distribution are presented. Some informal and formal ratemaking frameworks, using the Bayesian methodology, are discussed. A deductible-shifted insurance structure using a truncated indemnity schedule is also proposed and evaluated, and its ratemaking formula presented. The results suggest that, given an actuarially sound basis, the ratemaking formula based on the beta yield probability density function produces unbiased and consistent estimates for expected yield losses. The normal rate formula generates biased rate estimates. Given positively skewed wheat yield distributions for most years, as suggested by the normality tests, the normal pure premium rate methodology tends to underestimate expected losses. The Bayesian framework gives rise to some median rates and it is a very useful premium revision technique. It is also found that the current rate formula used by the Manitoba Crop Insurance Corporation is only partly justified and it could be improved by utilizing a formal Bayesian methodology. The chapter also demonstrates that an indemnity-truncated or deductible-shifted program needs to be considered by the insurance corporation. It is attractive for both the insurer and the insured in the sense that an equivalent or higher yield protection could be obtained by the insured and the program management cost savings could be very significant to the insurance agency.

Chapter 1

Introduction

1.1 Problem Statement

Canadian Federal-Provincial Crop insurance programs have been implemented across the country for more than 30 years since the Crop Insurance Act was enacted in 1959. These programs have continued to play an important role in protecting farm production from uncontrollable natural hazards and in stabilizing farmers' net income flow. The programs have been widely recognized by governments and producers as one of the most effective ways to stabilize farmers' net income from production fluctuations due to adverse production conditions. It is this function that makes the programs one of the most important policy instruments in the Canadian agricultural policy framework (Gilson, 1987; Agriculture Canada, 1989).

The Canadian crop insurance experience has generally been favorable, as compared to similar programs in other parts of the World. Canada's programs, however, are by no means free of problems. Many issues and problems have been recognized as the program experience is accumulated. Comprehensive program reviews have been done in a few provinces, including the Alberta Crop and Hail Insurance Corporation(Gilson, 1987), the Manitoba Crop Insurance Corporation (Manitoba Crop Insurance Review Committee, 1992) and an overall Federal-Provincial program review conducted by the Agriculture Canada (1989). Both theoretical and practical issues confronting the programs were discussed and evaluated. Some fundamental program changes were proposed and further systematic studies on some issues were demanded. For example, after evaluating the general program performance at the national level, the Federal-Provincial review called for implementing "an individual coverage approach where feasible". This may have some fundamental implications for the future program reforms, particularly for those provinces like Manitoba where individual coverage approach has never been tried on a large scale.

Few other agricultural programs are as complex as crop insurance. Economic rationale, financial aspects, and statistical foundation and actuarial principles constitute the basic dimensions of the program. In terms of operational aspects, the basic elements include the determination of long term average yields and coverage levels, the premium setting methodology, and indemnity determination. Each of these elements can be very complicated and controversial, particularly the individual coverage approach versus the area coverage approach.[1]

Today, more and more insured producers strongly demand the implementation of an individualized insurance program. They want a move from the current homogeneous risk area approach toward an individual coverage approach because the risk homogeneity hypothesis is widely doubted by this group of people (Manitoba Crop Insurance Review Committee, 1992)[2]. Such a change is practically possible if the farm yield data base is well established. This may be the case for some insured farmers

[1]A common practice is that the area average yields are used to establish the amount of protection for an individual insured farmer when individual yield data are not available (this is the case when a program starts or a farmer is a new insured). The area may be based on regions of similar or uniform risk (*homogeneous risk area*) or in some cases administrative areas such as counties or townships. In many provinces, area average yields are used only as bench marks for newly insured producers, and individual yield data are used to replace the area yield data as experience is gained.

[2]An individual productivity indexing (IPI) system was introduced in 1992 (about the same time as the Crop Insurance Review was done). Now that producers understand how IPI works there is no substantial demand for individual coverage (Hamilton, 1994).

since more than 30 years' experience has been accumulated. The issue, however, may be of a more fundamental nature than it appears, particularly when we consider the *fundamental principles* underlying any insurance program and relevant individualized program experiences. For example, the poor actuarial performance of the individual coverage approach (due to adverse selection and moral hazard problems, Halcrow, 1978; Skees and Reed, 1986; Miranda, 1991) and its failure to attract producer participation has led to dramatic dissatisfaction with the U. S. Federal Crop Insurance Program, including calls for the elimination of the program and replacing it with a standing disaster assistance program by the Bush administration (Chite, 1992). An alternative area yield based crop insurance is now receiving more and more attention and discussion (Halcrow, 1949, 1978; Miranda, 1991), and a new program, Group Risk Plan (GRP), has been experimented since 1992 (Skees, 1994).

Back to Canada, the individual coverage approach vs. the area coverage approach is also quite controversial. This can be easily verified by the fact that different practices are in place across the country. The key issue to be studied is whether an individualized program structure will provide the same or more insurance protection to the insured farmers in terms of the program objective as the current program does. Equally important, what will be the implications and impacts on the insurers?

Crop insurance programs have been offered in Manitoba since 1960. The programs are implemented by the Manitoba Crop Insurance Corporation (MCIC). Actually, the concern with some alternative coverage approaches has been popular among farmers as well as within the Crown Corporation (MacFadden, 1981). Since the homogeneous risk area approach has been adopted for more than thirty years within the province, some new versions of the program may be logically experimented and extended based upon actuarially sound principles. It is, however, not easy to judge whether the individual approach is superior to the area approach or vice versa without some comprehensive comparative study, even though the individualized approach may have been rendered impractical elsewhere. For example, the failure of the individual coverage approach in the United States cannot be used to justify the MCIC's area based approach. This is actually a two-sided theoretical and empirical question.

Some critical questions can be raised logically from the previous discussion: Has the program objective been reached? Is it inevitable to reform the existing program structures? Is there any theoretical foundation for each of alternative approaches? Empirically, does current homogeneous risk area based IPI approach fail or succeed? What may be the best option? These questions will be evaluated and discussed by this study both theoretically and empirically in the context of the Manitoba crop insurance programs.

1.2 Objectives of the Study

The major objectives of the study are:

(1) To discuss and evaluate the theoretical framework of crop insurance, including the economics of crop insurance, actuarial aspects, operational elements as well as the integration of these components.

(2) To provide theoretical and empirical insights into some important issues with which a voluntary crop insurance is confronted.

(3) To make a comprehensive comparative study of the individual coverage approach versus the area coverage approach in order to determine whether or not the individualized crop insurance has a theoretical foundation, and whether or not it could be practically feasible and superior over the homogeneous risk area approach.

(4) To develop a new Bayesian premium-rate methodology.

(5) To propose and discuss some better program designs in order to increase participation and to reduce program costs.

1.3 Assumptions of the Study

The following assumptions are adopted throughout the study in order to simplify the theoretical framework and empirical models:

(1) The objective of the crop insurance program is to provide insurance protection to insured farmers on an actuarially sound basis against crop yield losses caused by natural hazards that cannot be reasonably controlled by the insured. This yield based insurance characteristic is the program foundation and provides a guide for any program change in the future. It also distinguishes crop insurance programs from other government programs such as price support or direct income insurance.

(2) An all-risk crop insurance plan may not be replaced by any other program. The criticisms with crop insurance should be clarified under the program objective. Other government support programs only serve as complementary instruments for the overall policy objective.

(3) Farmers are expected utility (or profit) maximizers with risk aversion attitudes. The Crown Corporation agency is assumed to be risk neutral with zero profit maximization. Actuarial and financial objectives will dictate the Corporation's action.

1.4 Hypotheses of the Study

To accomplish the research objectives, the following hypotheses will be tested and evaluated consistently throughout this study.

Null hypothesis 1: There is no theoretically sound and consistent foundation for individualized crop insurance. The statistical, actuarial and economic aspects cannot be integrated consistently within this crop insurance program.

Null hypothesis 2: The homogeneous risk area hypothesis cannot be rejected. The current homogeneous risk area based IPI approach is an effective approximation to the theoretically sound insurance. The area coverage program can provide the same or larger yield protection than the individual coverage program.

Null hypothesis 3: Actuarially, an individual coverage approach is not superior to area coverage approach. The individualized insurance structure does not necessarily induce stronger demand for the crop insurance.

Null hypothesis 4: Adverse selection is inherent in any crop insurance program as long as (1) a voluntary program is offered, or (2) the asymmetric information problem is present.

Null hypothesis 5: The current program could be improved either by some new program structures or by some better premium setting methodologies.

Chapter 2

Theoretical Dimensions of Crop Insurance Program

2.1 Fundamental Principles of General Insurance

2.1.1 Risk, Uncertainty and Insurance

When risk is involved, there is a case for insurance. **J. C. Gilson**

If, in meeting hazard, average loss is substituted for actual loss, the result is insurance. **C. A. Kulp**

From an academic context, it may be unwise to attempt to define what insurance is. It is however very logical to discuss the definition and measurement of risk, since the insurance industry and its principles of operation are rooted in the nature of risk and uncertainty. The statements quoted above indicate that the insurance industry would not exist at all if risk did not exist. To understand the fundamentals of insurance, one must understand how people define risk, and in particular, how the insurance industry defines and measures risk.

Economists, statisticians, decision theorists, and insurance theorists have long discussed the concepts of risk and uncertainty in an attempt to arrive at a definition of risk that might be useful for analysis in each field of investigation (Vaughan and

Elliot, 1978). Unfortunately and naturally, they have not been able to agree on a definition that can be used in each field; nor does it appear likely that they will do so in the near future.

In insurance the word "risk" has a long tradition, and has been used in many different senses. It is generally believed that risk was first defined as a mathematical concept by Tetens (1786) in a work on life annuities. Tetens defined risk in terms of what we would presently describe as "one half of the mean deviation" (Borch, 1990). If we were to survey the best-known insurance textbooks, we would find many risk definitions. The following is a list of some examples:

Risk is the chance of loss

Risk is the possibility of loss

Risk is uncertainty

Risk is the probability of any outcome different from the one expected

Risk is the dispersion of actual from expected results

The lack of agreement on the definition of risk in insurance could be attributed to many reasons. One of major reasons for this is that insurance theorists have never tried to distinguish uncertainty from risk. They have attempted to borrow the definitions of risk used in other fields such as statistics or mathematics (Rejda, 1982). It is clear that many definitions of risk are mixed up with tools with which well-defined classes of decision makers measure and order risky choice (Robison and Barry, 1987).

Economists will generally recall Knight's (1921) sharp distinction between the concepts of "risk" and "uncertainty". In Knight's terminology, risk is present in a situation where an action can lead to several different, mutually exclusive outcomes, each with known probability. If these probabilities are unknown, only uncertainty is involved. "The fact is that while a single situation involving a known risk may be regarded as 'uncertainty', this uncertainty is easily converted into effective certainty; for in a considerable number of such cases the results become predictable in accordance with the law of chance and the error in such prediction approaches zero as the number of cases is increased " (Knight, 1921). Knight's distinction between risk and uncertainty is based on the empirical information available for generating probabilities and he is merely stating the Law of Large Numbers (LLN). The problem is that decision makers must make probability judgements even with little or no empirical information. The application of the Bayesian approach to statistics and decision theory seems to have made Knight's distinction not essential. As a result, few economists would like to maintain the distinction imposed by Knight, but use uncertainty and risk interchangeably (Borch, 1990; Robison and Barry, 1987).

Freifelder (1976) proposed that " uncertainty is the lack of certainty; doubt as to the actual outcome of an event or trial of an experiment". In this definition, uncertainty is characterized by the inability to predict the outcome without error. Freifelder defined risk as " the individual evaluation of the uncertainty surrounding the choice of a course of action or the outcome of an event". According to Freifelder, the distinction between risk and uncertainty by reference to the existence or non-existence of probabilities is not important. What is critical is the distinction between the three types of probabilities: *a priori, relative frequency, and subjective.* Consequently, risk has both subjective and objective aspects and is potentially present in all uncertain situations, whether characterized as a priori probabilities (gambling), relative frequency probabilities (insurance), or subjective probabilities (decision making).

The concept of risk becomes more relevant through the development of modern economic theory. The concept of utility developed in the last decade of the nineteenth century was widely believed to be the cornerstone of economic theory. However, many economists found it difficult to accept the utility concept since it was impossible to measure. It was therefore considered as a key break-through when Pareto showed that one could do without utility, and derived all the results of classical economics from the theory of indifference curve.

However, classical theory was not very successful when it dealt with the uncertain elements in economics. When the first real break-through, modern expected utility theory, was made by von Neumann and Morgenstern (1947), it appeared that utility was indispensable. It is within this context that Robison and Barry (1987) defined risk as those uncertain events whose outcomes affect the decision maker's utility or well-being. This definition seems broader than the popular concept of risk as it involves both possible losses and gains.

Robison and Barry's definition of risk may not be a good one from the insurance point of view, since insurance deals with "pure risk" which is used to designate those situations which involve only the possibility of loss or no loss. If a risk involves a possibility of loss as well as gain, the risk could be defined as speculative risk (Mowbray and Blanchard, 1961). Speculative risk is important in some non-insurance industries. Gambling and the futures market provide some good examples of speculative risk. The distinction between pure and speculative risks is crucial in the insurance industry because normally only pure risks are actuarially insurable.

In a gambling situation, risk is deliberately created in the hope of gain, while insurance is a technique for dealing with an already existing pure risk. Gambling is socially unproductive since the winner's gain comes at the expense of the losser. In contrast, insurance tends to be socially productive since both the insurer and the insured are to be better off. Insurance is also different from hedging. Insurance normally involves the transfer of insurable risks, yet hedging is a technique for handling risks that are typically uninsurable such as risks arising from fluctuating prices.

As risk can be defined several ways, the definition of the degree of risk or risk measurement is also not unique. The insurance literature indicates that many people accept the variance or the standard deviation of the sample mean as the proper measure of risk. This is natural since risk and uncertainty have been traditionally identified with variability. Since the standard deviation or variance of a probability distribution is a measure of variation, it appears reasonable to declare the variance to be a measure of absolute risk, with the coefficient of variation (C.V.) being a relative measure. The validity of a variance measure has already been questioned by some authors. For example, McCall (1971) observed that " the variance is an intuitive measure of riskiness, and for this reason has been frequently used to measure the risk associated with a given random variable. However, in some circumstances the variance lacks internal consistency and, therefore, is not reliable measure of risk." Rothschild and Stiglitz (1970) also demonstrated the variance may not be appropriate for measuring risk, for the concept of increasing risk is not equivalent to that implied by equating the risk of a random variable with the variance of the variable.

Risk is subjective in terms of the decision maker's evaluation of uncertain events and consequences. It follows then that the measurement of risk depends upon the individual and the context within which the decision is made. It is for this reason it has been suggested that the most accurate and reliable method of measuring risk is through utility theory (Freifelder, 1976). When utility theory is followed, risk is not measured by any one specific quantity such as variance. Instead the riskiness of an event is determined by applying the decision maker's utility function to the probability distribution of possible outcomes. Since the utility function summarizes the preferences for various outcomes, it contains all the information about how the decision maker would evaluate this uncertainty. In this case, risk is measured by the risk premium as defined by Pratt (1964). Risk premium can be interpreted as the maximum amount of money (price) an insured, with a given utility function, is willing to pay above the expected outcome (actuarially fair premium) to avoid risk.

2.1.2 Insurable Risks and Insurability

The conditions under which a risk is insurable are generally discussed in any popular insurance text. Within the insurance context, it is normally accepted that an insurance company insures only pure risks. Certain requirements or conditions must be fulfilled before a pure risk can be insured. Among these conditions is the requirement that it must be possible to make reliable estimates of the relevant probabilities from statistical observations. The bottom line is simply that a risk is insurable if and only if the insurer can apply the Law of Large Numbers (LLN), i.e., a risk is insurable only if the fundamental principle of insurance is workable. From the viewpoint of the insurer, the following conditions are basic requirements of an insurable risk:

(1) There must be a large number of *homogeneous* risk exposure units to make the losses reasonably predictable.

(2) The loss produced by the risk must be definite and measurable.

(3) The loss must be random in nature (accidental and unintentional).

(4) The loss must not be catastrophic (independent loss).

These requirements are obviously self-explanatory from the insurer's point of view. Although it seems theoretically possible to insure all possibilities of loss, some are not insurable at a reasonable price. For practical reasons, insurers are not willing to accept all the risks that the insured may wish to transfer to them. The four prerequisites listed above represent the "ideal" elements of an insurable risk.

The essence of the first condition is to enable the insurer to predict (calculate) the probability of loss based upon the LLN. Exposure units or insureds with similar loss-producing characteristics are technically grouped by classes. If a sufficiently large number of exposure units are present within a class, the insurer can accurately predict both the average frequency and average severity of loss because the LLN is workable in this situation. The LLN states that *the observed frequency of a random event more nearly approaches the underlying true probability of the population as the number of trials (observations) approaches infinity.* Therefore, the greater the number of exposure units, the more closely will actual results approach the probable results expected from an infinite number of exposures. The LLN can be best illustrated by the following simple example. Consider a case where a coin is flipped and the possible outcome is assumed to be either a head or a tail. The *a priori* probability of getting a head is obviously 0.5. If the coin is flipped only ten times, a head may appear eight times. Although the observed probability of getting a head in this experiment is 0.8, the true probability is still 0.5. If the coin were flipped one million times, however, the actual number of heads would be approximately 0.5. Thus, as the number of random tosses increases, the actual results approach the expected results. Since the observed frequency of a random variable approaches the true (unknown) probability of the population as the number of trials increases, we can obtain a notion of the underlying probability by observing events that have occurred. After observing the proportion of the time that various outcomes have occurred over a long period of time under essentially the same conditions, an index of the relative frequency of the occurrence of each possible outcome (a probability distribution) can be constructed for use in predicting population parameters.

The requirement of an accidental and unintentional loss is necessary for an insurable risk to reduce moral hazard problem. The loss should be accidental (random) because the LLN is based on the random occurrence of events. A deliberately caused loss is obviously not a random event. The third condition of an insurable risk is straightforward. This requirement enables the insurer to determine if the loss is covered under the insurance policy, and if it is covered, how much the insurance company would pay.

The fourth condition of an ideal insurable risk requires that a large proportion of exposure units should not incur losses simultaneously. The insurance principle is based on a notion of sharing and spreading losses over space and time, and the underlying assumption is that only a small percentage of the exposure units will suffer loss at any time. If most or all of the insured in a certain class simultaneously incur a loss, the sharing or pooling technique is unworkable. This will make insurance no longer an effective means by which losses of the few are spread over the entire risk group.

The requirements or conditions of an insurable risk discussed in the previous paragraphs are perhaps not that important for many economists. Most economists tend to take a pragmatic attitude toward this issue: if two parties agree on an insurance contract, the risk covered is by definition insurable (e.g., Borch, 1990). A risk can be insured when no statistics are available, and even when no theoretical analysis seems possible (Brown, 1973). Brown reported a case where the Lloyd's of London wrote an insurance contract to insure against the capture of the Loch Ness Monster in 1971. The LLN and the requirements of an insurable risk are certainly not applicable to this case. The economics literature seems to suggest that every risk is theoretically insurable, and a risk may be practically uninsurable only when moral hazard and adverse selection come to play. The question of what prevents the emergence of a competitive insurance market is often referred to as the "insurability" problem (e.g., Arrow, 1963; Pauly, 1974; Rothschild and Stiglitz, 1976; Chambers, 1989). The basic answer to this question provided by economists is that a risk could be economically uninsurable if asymmetric information (between the insurer and the insured) about the risk is involved. Whether or not a risk is statistically or actuarially insurable seems not critical.

Although it is not difficult to find instances in which an insurance company writes an insurance contract which insures against loss caused by a unique event, technically speaking, such transactions are not true insurance in an actuarial sense. Although they involve the transfer of risk, there is no reduction of risk from the insurer's point of view. Some insurance companies like Lloyd's of London are able to engage in such practices because they substitute mass underwriting (where a single risk is spread among many insureds) for mass exposures and because the premiums charged for such coverages are heavily loaded, that is, higher than probability requires. These kinds of extreme examples are not conducted by well-defined insurance principles, and therefore they cannot be used to justify the notion that all risks are insurable.

2.1.3 Insurance Mechanism and Principles

Insurance is a device which enables an insured to transfer part of his risk to the insurer through the principles of economies of risk (Robison and Barry, 1987) or a risk pooling mechanism. Insurers can reduce the cost of risk bearing as their number of exposure units (i.e. insureds) increases through the LLN, assuming the losses facing their insureds are not perfectly and positively correlated. The insurance company accepts the risk in return for an insurance premium (risk premium) which exceeds the certainty equivalent of the loss. The insured increases his certainty equivalent by paying an insurance premium that is less than the loss of certainty equivalent income created by the risk. In this way both the insurer and the insured are better off from the exchange.

Insurance is a complicated device, and it is consequently difficult to define. However, in its simplest aspect, it is sufficient to consider an insurance contract as described by two elements:

(1) $\rho(x)$ = the premium paid by the insured.

(2) $I(x)$ = the compensation which the insured receives if specific losses occur when the contract is in force.

x is the risk insured (or loss), a random variable which must be actuarially insurable and must be described by a probability distribution, $F(x)$. Since x is a random variable, the compensation function, $I(x)$, is also a random variable.

In this manner, the insurance contract is defined by a pair $(\rho(x), I(x))$ in which the contract will give the insured the right to claim an amount of money (insurance payment) \$ $I(x)$ from the insurer, if certain events should occur. To be entitled to this right, the insured pays the insurer a premium $\rho(x)$.

The essential objective of the theory of insurance is to describe and determine the relationship between these two elements, i.e., how the premium $\rho(x)$ depends on the properties of the probability distribution of the compensation. The general solution to this question is through the principle of equivalence which states that the expected value of claim payments under a contract should be equal to the expected value of premium received, i.e., pure premiums should be equal to the expected losses over time:

$$\rho(x) = \int_0^\infty I(x)f(x)dx, \tag{2.1}$$

where $f(x) = F'(x)$, the probability density function of loss.

The fundamental principles of insurance mechanism can be illustrated by capturing the essential characteristics of true insurance. Since there is no definition acceptable to everyone, it is useful and sufficient to find a working definition. The definition provided by the Commission on Insurance Terminology of the American Risk and Insurance Association (see Vaughan and Elliot, 1978) may be a good one:

Insurance is the pooling of fortuitous losses by transfer of such risks to insurers who agree to indemnify insureds for such losses, to provide other pecuniary benefits on their occurrence, or to render services connected with the risk.

By investigating this insurance definition, one can summarize the key words or the characteristics of a true insurance as follows:

(1) Pooling of Losses (risk sharing among insureds).

(2) Indemnification (risk transfer from insureds to insurer).

Pooling or sharing of losses is the core of insurance. Pooling of losses is the spreading of risks incurred by the few over the entire risk class or risk group, so that in the process, average loss is substituted for actual loss. In addition, pooling involves the classifying of a large number of homogeneous exposure units so that the LLN can operate to provide a substantially accurate prediction of future losses and insurers can reduce their risks. Therefore, pooling of losses or risks has dual implications: (1) the sharing of risks by the entire group (i.e., shifting risk from one individual to a whole class), and (2) the prediction of future losses with some accuracy based on the LLN. As a result, both the insurer and the insured will have a risk reduction.

The first implication can be illustrated by a simple example. Assume that 1000 farmers in a rural community agree that if any farmer's barn is destroyed by a fire, the other members of the community will indemnify the actual costs of the unlucky farmer who incurred the loss. Assume also that each barn has an equal value, say, \$6000, and on average, the probability that a barn burns each year is 0.001, i.e., one out of one thousand. Without insurance, the maximum financial loss to each farmer is \$6000 if the barn should burn. However, by pooling the losses through mutual insurance, the loss can be spread over the entire community, and if one farmer's barn burns down and thus has an actual loss of \$6000, the maximum amount that each farmer could have to assume is only \$6 = \$6000 x 0.001. Consequently, the pooling function leads to the substitution of an average loss of \$6 (premium) for the actual loss of \$6000. The function of risk reductions for the insured is therefore realized.

The second implication indicates that the risk or the variability of outcomes for an insurer is less than for an insured because of risk pooling function. In other words, the risk pooling technique can result in risk reductions for both the insured and the insurer. Consider n potential insured farmers who have identical yet independent distribution (IID) for crop losses caused by hail. Let x_i be the stochastic loss for farmer i with an average value μ_x and variance σ_x^2. The insurance company will cover a farmer' crop loss for a premium equal to the expected loss across all the insured. Assuming the insurance payments are IID for each insured farmer, the average benefit paid by the insurer is

$$E\{\frac{x_1 + x_2 + ... + x_n}{n}\} = E\{X\} = \mu_x. \tag{2.2}$$

The variance of the average insurance payment is

$$\begin{aligned} Var(X) &= E\{(\tfrac{x_1+x_2+...+x_n}{n} - \mu_x)^2\} \\ &= (\tfrac{1}{n})^2 E\{[(x_1 + x_2 + ... + x_n) - n\mu_x]^2\} \\ &= (\tfrac{1}{n})^2 E\{(x_1 - \mu_x)^2 + ... + (x_n - \mu_x)^2\} \\ &= \sigma_x^2/n < \sigma_x^2. \end{aligned} \tag{2.3}$$

Therefore, the larger the risk pool is, the less the insurer's risk will be. If the number of insured farmers in the risk group approaches infinity, the insurance company can accurately predict the probability of crop loss, since $\sigma_x^2/n \to 0$ when n approaches infinity.

2.2 Economics of Insurance

2.2.1 General Literature Review

The discussion on economic aspects of insurance has a long history. Adam Smith

(1776), Leon Walras (1874), and Alfred Marshall all discussed insurance in their most important works. Among these, Smith provided some deep insight into the essentials of insurance, particularly with respect to premium determination. Marshall came close to developing an economic theory of insurance in his Principles (1890) where he treated insurance premiums as the price one has to pay to get rid of the uncertainty. Marshall seemed also to realize that Bernouli's Principle (the Expected Utility Hypothesis) may be the key to the problem of insurance premiums. However, the true economics of insurance did not emerge until von Neumann and Morgenstern (1947) formally proved the Bernouli Hypothesis as a theorem so that risk and uncertainty can be theoretically brought into economic analysis. The break-through was made by Arrow (1953), Debreu (1953), and Allais (1953). In their papers, the values of contingent claims were determined by market forces that drove the demand for risk-bearing services.

The formal insurance economics was shaped in the early 1960's, with the most important work was done by Arrow (1963, 1965) and Borch (1960, 1961, 1962). As a leader in the development of the economics of uncertainty, information, and communication, Arrow did tremendous work in the field of insurance economics. He was the first author in economics to systematically investigate the major reasons why risk shifting in any market is limited. He identified moral hazard, adverse selection, and transaction costs as the fundamental reasons.

Borch also made significant contributions to insurance economics, particularly with respect to the theory of optimal insurance contract. He developed necessary and sufficient conditions for Pareto optimal exchange in risk pooling arrangements. Borch also showed how risk aversion affects the optimal coverage of participants in the risk pool. Borch's work led to many applications in the insurance literature (see Lemaire (1990) for a survey of these applications). Borch's contributions established some important links between actuarial science and insurance economics which have been neglected by actuaries and economists (Louberge, 1990).

In insurance economics, insurance coverage is generally considered to be a contingency commodity which is bought and sold in the market. The premium is considered as a price which is determined by demand for and supply of insurance in the market. Insurance economics is based upon modern utility theory. The heart of the theory is Bernouli's Principle which formed the base for the Expected Utility Theorem. Over 200 years ago Daniel Bernouli postulated his principle in recognition of the fact that an extra dollar is worth more to a poor man than to a rich man by proposing the "moral value ", or the utility, when he used famous " St. Petersburg Paradox " and other examples to show that individuals did not order choice according to their mathematical expected values, but the "moral expectation" given by

$$\int_0^\infty u(x)dF(x) = \int_0^\infty u(x)f(x)dx, \tag{2.4}$$

where $u(x)$ represents the "moral value" or a concave utility function.

The significance of Bernouli's Principle became popular for some time among mathematicians and philosophers, but it had little impact on economic theory and it went unrecognized until von Neumann and Morgenstern (1947) showed that Bernouli's Principle is a logical deduction from a small number of axioms that are reasonable. With objective probabilities, three basic axioms are necessary to obtain the Expected Utility Theorem: *weak order, continuity, and independence.* Given these three axioms, insurance policy A will be preferred to policy B if and only if $E\{u(A)\} > E\{u(B)\}$. With subjective probabilities, additional axioms must be added in order to obtain a unique subjective probability measure over the set of states and a utility function that is unique up to a positive linear transformation. Formally, the Expected Utility Theorem may be stated as follows (Borch, 1990):

A utility function $u(x)$ exists for a decision maker whose preferences are consistent with the axioms of weak ordering, continuity, and independence; this function associates a single real number with any risky prospect and has the following properties:

(i) if $F_1(x) \prec F_2(x)$, then $\int_{-\infty}^{\infty} u(x)dF_1(x) < \int_{-\infty}^{\infty} u(x)dF_2(x)$, and vice versa.

(ii) The utility of a risky prospect is its expected utility.

(iii) The scale on which utility is defined is arbitrary. In particular, the properties of a utility function that are relevant to decision choice are not changed under a positive linear transformation. That is, the function $u(x)$ and $v(x) = \alpha u(x) + \beta$, where $\alpha > 0$, represent the same preference ordering.

Utility function gives a very convenient description of a preference ordering over a set of probability distributions, and this provides the key which opens the door to the economics of uncertainty. To illustrate how the Expected Utility Theorem can be used to analyze insurance problems, consider the following example (Arrow, 1963; Mossin, 1968; Borch, 1990):

Consider a farmer with preferences which can be summarized by a concave utility function $u(x)$, i.e., $u'(x) > 0, u''(x) < 0$. Assume that the farmer has an initial wealth of \$W and with some probability of exposure to a risk which can cause a loss x, a random variable with the density function $f(x)$. By definition, the farmer's expected utility associated with this uncertain situation is

$$E\{u(W-x)\} = \int_0^{\infty} u(W-x)f(x)dx. \tag{2.5}$$

If the farmer agrees buying an insurance policy by paying an insurance premium $\rho(x)$, his risk would be shifted to the insurer and he will get a guaranteed utility

$u(W - \rho(x))$. If the farmer is economically rational, he will be trying to maximize his expected utility and purchase the insurance contract if

$$u(W - \rho(x)) \geq E\{u(W - x)\} = \int_0^\infty u(W - x)f(x)dx. \tag{2.6}$$

There is clearly an upper limit to the premium (the maximum premium the farmer is willing to pay), say, ρ_{max}, at which the contract is acceptable. For $\rho(x) = \rho_{max}$ the equality sign holds in equation (2.6). For a concave utility function (which implies that the farmer is risk averse), and using Jensen's inequality, it follows that

$$E\{u(x)\} < u(E\{x\}), \tag{2.7}$$

that is

$$\int_0^\infty u(W - x)f(x)dx < u(W - E\{x\}). \tag{2.8}$$

The equality sign will hold for some $\rho_{max} > E\{x\}$. The implication is that the risk-averse farmer is willing to pay more than the expected loss (the actuarially fair premium) to have the risk covered by the insurance. This establishes the risk premium[3], as defined by $\rho(x) - E\{x\}$. Risk premium is the insurer's reward for his risk-bearing services, and it is determined by market forces, i.e., by the demand for and supply of insurance. Pure or actuarially fair premium equals the expected loss, $E\{x\}$. It is determined by actuarial principles.

Suppose now that the insurer is also an expected utility maximizer with a concave utility $u_s(.)$. Let his initial wealth be W_s. The insurer will agree to cover the risk x faced by the insured farmer if a premium $\rho(x)$ is paid by the insured. The necessary condition for the insurer to do this is that his expected utility of covering the risk against the premium be at least greater than his initial utility level, i.e., he is assured

[3]Strictly speaking, the amount of $\rho(x) - E\{x\}$ is the loading of the pure premium $E\{x\}$. If other loading factors (e.g., administrative expenses) are ignored, this difference is risk premium.

that

$$u_s(W_s) \leq E\{u_s(W_s + \rho(x) - x)\} = \int_0^\infty u_s(W_s + \rho(x) - x) f(x) dx. \qquad (2.9)$$

Analytically, if there are some values of $\rho(x)$ which satisfy both condition (2.6) and (2.9), the insured and the insurer can make an exchange which will increase their expected utility. The equilibrium values of ρ which match these necessary conditions are the gross premiums. The gross premium consists of at least three parts: *actuarially fair premium or pure premium* due to pure risk, *risk premium* which is the reward to the insurer for his risk-bearing, and some *loading components.* This relationship has been implied by the preceding discussions and it was noticed by Smith (1776) more than 200 years ago. The determination of actuarially fair premium will be discussed in detail in Chapter 4.

2.2.2 Demand for Insurance

The decision to purchase insurance is like the decision to purchase any other commodity. Therefore, demand for insurance for an insured is determined by the same factors as those that influence demand for general consumer goods, except that a risk attitude is involved in insurance demand function. Insured's income, commodity price (premium), and consumer's preferences (particularly risk attitudes) will determine the quantity demanded and the shape of demand curve.

Mossin (1968) and Smith (1968) presented a simple model of insurance demand in which insurance premiums are exogenously determined by some premium calculation principle. Let the optimal insurance coverage rate α be the insured's decision variable. Suppose that the insured has a total wealth (Y) equal to $W - x$ where W is his nonstochastic initial wealth and x is an insurable loss. Assume that the individual can purchase insurance coverage α $(0 < \alpha < 1)$ for a premium $\alpha\rho$. Assume also that the premium is calculated as $\rho = \lambda E\{x\}$, where $\lambda > 1$ is the premium loading factor which should includes the risk premium. $E\{x\}$ is the expected loss which is identical to the actuarially fair premium. Mossin demonstrated that the optimal insurance coverage is such that $0 < \alpha^* < 1$ for $\rho_{max} \geq \rho^* \geq E\{x\}$ where $\rho^* = \lambda^* E\{x\}$ solves

$$E\{u(Y + \alpha^*(x - \rho^*))\} = E\{u(Y)\}, \tag{2.10}$$

and where $u(.)$ is a von Neumann-Morgenstern utility function with $u'(.) > 0$ and $u''(.) < 0$, where u' and u'' are first and second order conditions. Thus, if the premium loading factor exceeds one but is less than λ^*, partial coverage ($0 < \alpha^* < 1$) is demanded.

To illustrate this conclusion, consider the insured's objective function:

$$Max\ E\{u(.)\} = U(.) = \int_0^\infty u(W - x - \alpha\rho + \alpha x) f(x) dx, \tag{2.11}$$

i.e., the insured's problem is to find the optimal value of α which maximizes his expected utility subject to the premium calculation formula. The first order condition is

$$U'(\alpha) = \int_0^\infty (x - \rho) u'(W - x - \alpha\rho + \alpha x) f(x) d(x) = 0. \tag{2.12}$$

If the insured insures all his loss (i.e., $\alpha = 1$), it leads to

$$U'(1) = u'(W - \rho)(E\{x\} - \rho) = 0. \tag{2.13}$$

By assumption, $u'(.) > 0$, thus the only solution which satisfies the condition will be $\rho = E\{x\}$. However, $\rho = \lambda E\{x\}$, so $\lambda = 1$ must hold. This indicates that the individual will buy the full insurance coverage only if the actuarially fair premium is charged (no risk premium, no any other loading components should be added). However, in competitive insurance markets, premiums must be loaded and buyers are willing to pay more than the actuarially fair premiums to have the risk covered by the insurer as indicated by equation (2.7), i.e., $\rho > E\{x\}$ is reasonable. If $\rho > E\{x\}$, then $U'(1)$ will be negative which contradicts the expected utility hypothesis. This implies that it will never be optimal to buy full insurance for a risk averse insured if the gross premium is loaded (higher than the pure premium).

The maximum premium that a risk-averse individual is willing to pay over and above the actuarially fair premium $E\{x\}$ is the risk premium ρ_π as defined by Pratt (1964). According to Pratt, this risk premium can be calculated by solving

$$u(W - E\{x\} - \rho_\pi) = E\{u(W - x)\}. \tag{2.14}$$

As shown by Pratt, a more risk averse individual will have a higher risk premium ρ_π than a less risk averse person.

Mossin's result is very general. However, as Borch (1990) noted, the conclusion does depend on his special assumption about the premium formula. It is easy to show that full insurance coverage will be optimal in Mossin's model if the premium is calculated as $\rho = \alpha E\{x\} + k$ instead of as $\rho = \lambda E\{x\}$, where k is a fixed loading proportion:

$$U'(\alpha) = \int_0^\infty (x - E\{x\})u'(W - x - \alpha E\{x\} - k + \alpha x)f(x)dx = 0. \tag{2.15}$$

Obviously $U'(1) = 0$ holds, indicating that full insurance coverage will be optimal if $\rho = E\{x\} + k$ is taken as the premium formula.

Another form of partial insurance contract is a policy with a deductible (Arrow, 1963; Mossin, 1968; Gould, 1969). For example, in Arrow's model, an indemnity function $I(x)$ is specified with which the insured will receive the amount of $I(x)$ if loss x obtains. Let the premium be $\rho(I(x))$, and it is determined by

$$\rho(I(x)) = (1 + \lambda)E\{I(x)\} \; with \; \lambda > 0, 0 \leq I(x) \leq x, \tag{2.16}$$

the insured will solve his optimization problem as defined by

$$Max \; E\{u(.)\} = U(.) = \int_0^\infty u(W - x - \rho(I) + I(x))f(x)dx. \tag{2.17}$$

Arrow showed that the solution to this problem will be an insurance contract of the form:

$$\begin{aligned} I(x) &= x - D \;\; if \; x > D \\ &= 0 \quad if \; x \leq D, \end{aligned} \tag{2.18}$$

i.e., under this insurance policy, the insured will assume losses under the deductible D, and the insurer will indemnify him any excess loss above the deductible. From the insured's point of view, the optimal deductible level, D^*, can be found by solving the following problem (Borch, 1975):

$$\begin{aligned} Max \; U(D) &= \int_0^D u(W - \rho(D) - x)f(x)dx + \int_D^\infty u(W - \rho(D) - D)f(x)dx \\ St. \; \rho(D) &= (1+\lambda)\int_D^\infty (x - D)f(x)dx. \end{aligned} \tag{2.19}$$

The first order condition, $U'(D) = 0$, is equivalent to

$$(1+\lambda)\int_0^D u'(W - \rho(D) - x)f(x)dx = -u'(W - \rho(D) - D)[1 - (1-\lambda)\int_D^\infty f(x)dx]. \tag{2.20}$$

This condition is satisfied for $D = 0$ if $\lambda = 0$. That is, the zero deductible will be the optimal solution for the insured if the actuarially fair premium is charged. The insured will demand either a full coverage above the optimal deductible D^* or no insurance at all (equation 2.8). It also can be shown that $dD/d\lambda > 0$ if $u''(.) < 0$, suggesting that the insured tends to insure less as insurance becomes more expensive. Schlesinger (1981) and Karni (1985) proved that a more risk averse insured would prefer a contract with a smaller deductible and higher premium. Mossin (1968) also demonstrated that the larger the initial wealth W the insured has, the higher the deductible level will be, given a decreasing absolute risk aversion.

Since insurance markets could be viewed as markets for contingent goods, it is reasonable to investigate them in a more general framework. Borch (1960, 1962) first proposed general equilibrium models of optimal insurance contracts from both demand and supply sides within a reinsurance market context. He also took a more general approach of deriving the optimal insurance policy form endogenously to characterize a Pareto optimal risk-sharing arrangement in a situation where several risk averters were to bear a stochastic loss. Arrow (1971) applied the same framework to Pareto optimal insurance contracts in two distinct cases: (1) if the insurer is risk averse, the insured prefers a policy that involves coinsurance such that the coverage will be some fraction of the loss, and (2) if the insurer is risk neutral, the full insurance coverage of losses above a deductible will be optimal for the insured.

Raviv (1979) extended these results using the same basic framework as Borch (1960), and Arrow (1971). In Raviv's model, the Pareto optimal insurance policy is identified under general assumptions regarding the risk preferences of both the insured and the insurer, and the necessary and sufficient conditions leading to deductibles and coinsurance were derived. Raviv suggested that the insured's risk neutrality is neither a necessary nor sufficient condition for a policy to have a deductible; insured's risk aversion is not a necessary condition for coinsurance, and an optimal insurance may involve a deductible and a coinsurance.

To illustrate Raviv's model, assume that the buyer faces a risk of loss of x, where x is a random variable with probability density function $f(x)$. Denote insurance payment (indemnity) as $I(x)$, and suppose that the premium is exogenously determined and is denoted by ρ. Administrative and other expenses are explicitly specified as $c(I)$, and $c(0) = a \geq 0$. Let W_b and W_s denote the initial level of wealth for the insured and the insurer respectively. On the insurance demand side, the insured is assumed to maximize his expected utility of wealth. To find the Pareto optimal insurance contract, the risk averse insurer is assumed to find the premium $\rho(.)$ and the compensation function $I(.)$ that maximize the insured's expected utility of final wealth subject to two constraints: (1) the insurer's expected utility is constant, and

(2) the insurance payment is nonnegative and it is never larger than loss itself:

$$\begin{aligned} Max\ E\{u_b(\rho, I(x))\} &= \textstyle\int_0^\infty u_b(W_b - \rho(x) - x + I(x))f(x)dx \\ St.\ E\{u_s(\rho, I(x))\} &= \textstyle\int_0^\infty u_s(W_s + \rho(x) - I(x) - c(I))f(x)dx \geq k \\ & \quad 0 \leq I(x) \leq x,\ k \geq u_s(W_s). \end{aligned} \tag{2.21}$$

Raviv proved that the Pareto optimal insurance policies which solve the above model take one of two possible forms: there is either a deductible D provision coupled with the coinsurance of losses above the deductible, or there is full coverage of losses up to a limit M and coinsurance of losses above that limit:

$$\begin{aligned} I^*(x) &= 0\ if\ x \leq D \\ & \quad 0 < I^*(x) < x\ if\ x > D \\ I^*(x) &= x\ if\ x \leq M \\ & \quad 0 < I^*(x) < x\ if\ x > M. \end{aligned} \tag{2.22}$$

Raviv's model is more general than the Arrow and Borch models. With this model, a necessary and sufficient condition for the Pareto optimal deductible to be zero is shown to be that insurance cost $c(I)$ is constant over all states of indemnity schedules, i.e., $c'(I) = 0$. In Raviv's model, it is clear that Arrow's (1971) deductible result was not a consequence of risk neutrality, rather, it was obtained because of the assumption that insurance cost is proportional to coverage.

For a risk neutral insurer, the model can be modified by replacing the constant expected utility constraint by an actuarial regulatory constraint as defined by

$$\rho(x) = R(E\{I(x)\}) = R(\int_0^\infty I(x)f(x)dx), \tag{2.23}$$

where $R(.)$ represents an actuarially fair premium calculation principle. Raviv's model can be easily extended into multiple loss cases.

2.2.3 Moral Hazard and Adverse Selection

In classical insurance models (e.g., Mossin, Raviv, etc.), one of the important assumptions is that the insurer and the insured have the same knowledge about risk probabilities or loss distributions. This is crucial for an equilibrium to be established between demand and supply (e.g., Raviv). However, insurance programs typically suffer from problems of information asymmetry, i.e., the assumption of symmetric knowledge shared by the insured and the insurer is not true. Typically, an insured will know far more about his insured risks than an insurer does. This situation creates an environment conducive to the twin problems of moral hazard and adverse selection, leading to a situation where some competitive markets can not exist while others are inefficient in terms of Pareto optimality (Arrow, 1971; Rothschild and Stiglitz, 1976; Shavell, 1979).

Moral Hazard

The concept of "moral hazard" has its origin in marine insurance which is against "physical hazard". Dover (1957), for example, stated that "... it is often said that whereas physical hazard can be rated, where there is pronounced moral hazard the risk should be declined by the underwriter. 'Moral hazard' is somewhat difficult to define precisely. It may be said to be some element in the nature of the insurance, either with regard to the assured's interest, or the surrounding conditions, which makes - the happening of a casualty a means of benefit to the policy-holder". In Dover's observation, the nature of moral hazard and the relationship between moral hazard and premiums charged was clearly indicated.

Moral hazard was first studied in the economics literature by Arrow (1963) and Pauly (1968). Moral hazard is defined as " the intangible loss-producing propensities of the individual assured " (Dickerson, 1963). Arrow (1971) made this idea very clear in which he stated that " ... the insurance policy might itself change incentives and therefore the probabilities upon which the insurance company has relied. Thus, a fire insurance policy for more than the value of the premises might be an inducement to arson or at least to careless". Shavell (1979) referred to moral hazard as "the tendency of insurance protection to alter an individual's incentive to prevent loss". Moral hazard is a problem because the insured can affect potential loss distribution after he has bought an insurance contract while the insurer cannot afford to observe and monitor the insured's behavior. Beginning with Arrow (1963) and Pauly (1964), economists have proposed some partial solutions to the problem and a partial coverage with deductibles is a major one. Shavell (1979) used a simple two-state model where the insured faces either a known positive loss or no loss with probabilities that depend on loss prevention effort or care to show that partial insurance coverage is optimal in the presence of moral hazard.

Consider the case in which the occurrence of a loss can be observed by the insured and where neither the insured's actions nor the states of nature are observable to the insurer. Assume that the insured's initial wealth is W, the expenditure he is willing to spend on loss prevention is c. Suppose that he is exposed to a loss x with a probability p and his objective is to maximize his expected utility by finding the optimal expenditure on loss prevention and optimal coverage rate α:

$$Max\ E\{u(c,\alpha)\} = U(.) = (1-p)u(W-\rho-c) + pu(W-\rho-c-x+\alpha), \quad (2.24)$$

where ρ is the premium associated with the coverage α. The first order condition with respect to c is

$$\begin{aligned} U'(c) &= 0 \\ \Longrightarrow &\quad p'[u(W-\rho-c-x+\alpha) - u(W-\rho-c)] \\ &= (1-p)u'(W-\rho-c) + pu'(W-\rho-c-x+\alpha), \end{aligned} \quad (2.25)$$

i.e., the insured will choose an optimal level of care, c^*, with which the marginal benefit of taking care (left-hand side) equals the marginal cost of taking care (right-hand side), given optimal coverage level.

The insurer's problem is finding optimal coverage level α^* to maximize insured's expected utility subject to the break-even condition, $\rho = p\alpha$. The first order condition is then:

$$
\begin{aligned}
U'(\alpha) &= 0 \\
\Longrightarrow &\quad -u'(.)(\rho' + c') - p'u(.) + pu'(.)(\rho' + c') \\
+ &\quad p'u(..) - pu'(..)(\rho' + c' - 1) = 0,
\end{aligned} \tag{2.26}
$$

where $u(.) = u(W - \rho - c)$, and $u(..) = u(W - \rho - c - x + \alpha)$. Shavell proved that the condition of $U'(\alpha) = 0$ holds for only $0 < \alpha^* < x$. That is, the optimal insurance under moral hazard always offers partial coverage if care is not observed by the insurer and the cost of taking care is sufficiently low. Since a positive coverage is always offered in the presence of moral hazard, moral hazard alone cannot eliminate possibilities of insurance, i.e., moral hazard reduces but does not eliminate the benefits of insurance. Shavell also demonstrated that moral hazard problems are reduced if the insured's loss-prevention actions are partially observable. This is consistent with Holmstrom's (1979) finding.

Adverse Selection

The concept of adverse selection was first studied in connection with life insurance. Adverse selection comes to play when the existence of choice by insureds leads to higher-than-average loss levels. Adverse selection occurs because the insurer is not able to completely screen risk groups among insureds but the insureds know much more about their risk probability density functions. Consequently, an insured can accurately compare his degree of risk exposure to the average degree of risk exposure assumed by the insurer when developing the premium rate. Those who perceive that they have a high probability of collecting insurance payments in an amount exceeding premiums paid will tend to purchase insurance while others will not be inclined to purchase insurance contracts. Adverse selection works in the direction of accumulating high risks and forces the insurer to increase premiums in order to reach the actuarial objective. This will in turn compound adverse selection problems and result in an adverse chain effect.

Adverse selection can occur in any kind of insurance, and thus it is deeply studied in the economics literature. Some important work was done by Akerloff (1970), Pauly (1974), and Rothschild and Stiglitz (1976). These authors showed that if insurers have imperfect information about risk categories for prospective insureds, some insurance markets may fail to exist and others may be inefficient. The possible solutions to adverse selection include partial insurance coverage (Rothschild and Stiglitz), good experience rating or bonus system, risk categorization, repeated insurance contracts (Dionne, 1983), and multiple period contracts (Cooper and Hayes, 1987).

Rothschild and Stiglitz (1976) investigated the existence of a competitive equilibrium in an insurance market. They showed that a pooling equilibrium [4] cannot exist if a Nash equilibrium concept[5] is adopted. They also illustrated that a separating Nash equilibrium can exist in which high risk and low risk buyers purchase separate contracts. The separating equilibrium is characterized by zero profits for each contract, by partial insurance coverage for the low risk group, and by full coverage for the high risk group. In analytical terms the Rothschild and Stiglitz model can be described as follows:

Suppose that an insured with an initial wealth W is exposed to a risk which can cause a loss x with a probability p. The individual is entitled to claim a compensation αx ($0 < \alpha < 1$, coverage level) if he pays a premium $\rho\alpha$ provided that the loss x occurs. The insured attempts to find the optimal coverage level α which maximizes his expected utility:

$$Max\ E\{u(.)\} = (1-p)u(W-\alpha\rho) + pu(W-\alpha px - x + \alpha x). \qquad (2.27)$$

[4] i.e., different risk group of insureds buy the same contract.

[5] In a Nash equilibrium, both the insured and the insurer will have no incentive to deviate from the equilibrium.

The insurer is assumed to maximize the insured's expected utility (i.e., taking the insured's optimal utility level as given) subject to the break-even constraint as specified by $\rho = xp$, the point where the zero expected profit is obtained (see also Shavell (1979)). Substituting $\rho = xp$ into the insured's objective function, the first order condition, $U'(\alpha) = 0$, leads to

$$u'(W - \alpha px) = u'(W - \alpha px - x + \alpha x). \tag{2.28}$$

Since $u(.)$ is concave by assumption, the above equation holds only for $\alpha = 1$. Thus the insured will demand a full coverage insurance if the insurer's expected profit is zero and the actuarially fair premium is charged, a well known result (e.g., Mossin, 1968).

Assume that there exist two groups of risks in the insurance market, "high" risk insureds and "low" risk insureds, with claim probabilities of respectively p_h and p_l, with $p_h > p_l$. If the insurer is able to distinguish the two groups of insureds without any costs, then each group will be charged its proper premium such that $\rho_h = p_h x$ and $\rho_l = p_l x$, full coverage insurance will be demanded by both groups. However, if the insurance company is not able to screen each risk group or if the cost of the screening is prohibitive, the company has to charge the same premium (say, the average premium which lies in the interval $p_l x \leq \rho \leq p_h x$) for each group. It can be shown that the high risk group of insureds will demand full coverage insurance ($\alpha_h = 1$), and the low risk group of insureds will purchase partial coverage ($0 < \alpha_l < 1$) where α_l solves the following first order condition:

$$(1 - p_l)\rho u'(W - \alpha_l \rho) = p_l(1 - \rho)u'(W - \alpha_l \rho x - x + cx). \tag{2.29}$$

Rothschild and Stiglitz's result suggests that there is an expected loss of $p_h x - \rho$ on each high risk contract for the insurer, and there is an expected profit of $\alpha(\rho - p_l x)$ on each low risk contract. The zero expected profit condition requires that the condition $\alpha(\rho - p_l x) + \rho - p_h x = 0$ must hold, it then follows

$$\rho = \frac{p_h x + \alpha p_l x}{1+\alpha} = \frac{\rho_h + \alpha \rho_l}{1+\alpha}. \tag{2.30}$$

If the simple average premium $\rho = (\rho_l + \rho_h)/2$ is charged for all risk groups, the full coverage level ($\alpha = 1$) is indicated and this kind of contract is naturally demanded by all high risk insureds. It is also obvious that one or more equilibrium of some kind will exist if there are some meaningful solutions in α and ρ.

To show how adverse selection may make a risk uninsurable, Borch (1990) presented a simple analytical model.

Consider a group of n potential insureds, each is exposed to a risk which can cause a loss equal to x. Suppose that the probability that individual i shall suffer the loss x is $p_i(0 < p_i < 1)$, and also assume that $p_1 > p_2 > ... > p_n$. If the insurer indemnifies all the insured's losses, the total expected insurance payments will be $\sum_{i=1}^{n} p_i x$. If the insurer is not able to distinguish the loss probabilities for each individual, the insurer may have to charge a uniform average premium rate for all insureds, the premium is then $\rho(n) = (1/n)\sum_{i=1}^{n} p_i x$. Suppose that individual k is risk averse with a concave utility, he will purchase the contract only if his expected premium is less than or equal to the average premium, i.e.,

$$\rho_k = p_k x \leq \rho(n). \tag{2.31}$$

The maximum insurance price (ρ_k^m) that individual k is willing to pay is obviously determined by

$$u(W_k - \rho_k^m) = (1 - p_k)u(W_k) + p_k u(W_k - x). \tag{2.32}$$

From Jensen's inequality it follows that $\rho_k^m > p_k x$, indicating that the risk averse individual is willing to pay more than actuarially fair premium to have his risk covered by the insurer.

Among these n insureds, obviously there are some individuals whose actuarially fair premiums are greater than the average premium ρ while there are others whose premiums are lower than average. This suggests that the condition (2.31) cannot hold for all individuals in the risk pool. Suppose that the inequality holds only for m individuals with $k \leq m < n$, the insurance will be purchased by these m higher-than-average insureds. As a result, the insurer will receive a total premium $m\rho(n)$, which is not sufficient to cover expected claims from the m insureds since

$$\begin{aligned} m\rho(n) &= (m/n)\sum_{i=1}^{n} p_i x < \sum_{i=1}^{m} p_i x \\ \rho(n) &= (1/n)\sum_{i=1}^{n} p_i x < (1/m)\sum_{i=1}^{m} p_i x = \rho(m). \end{aligned} \tag{2.33}$$

This indicates that higher premiums must be required if only the m high risk insureds purchase the insurance, in order to achieve the actuarial objective (premiums and expected insurance payments should be dynamically equalized). Yet the higher premiums will lead more potential insureds to drop out of their insurance contracts, and the chain reaction will continue.

2.3 Crop Insurance: Problems Identified

The fundamental principles of general non-life insurance and economics of insurance have been reviewed and discussed in the preceding sections of this chapter. The major objective of such a thorough review is to provide a theoretical framework to guide the systematic discussion on theoretical and practical issues of crop insurance that will be covered in the following chapters.

2.3.1 Crop Losses: Insurable Risks?

One of the major observations regarding agricultural insurance is that competitive all-risk agricultural insurance and competitive all-risk crop insurance markets have not emerged (Chambers, 1989). A primary question is then - what prevents the emergence of commercial, all-risk agricultural insurance? Chambers referred to this as the "insurability" problem. As in the economics literature, the general answer to this question provided by agricultural economists is that moral hazard and adverse selection problems (due to asymmetric information between farmers and insurers) make the agricultural risks uninsurable (Ahsan and Ali and Kurian, 1982; Nelson and Loehman, 1987; Chambers, 1989). Since the information problems of moral hazard and adverse selection prevent Pareto optimality from being attained, some "second-best" solutions to these problems are applied in practice. One obvious choice is public intervention: various government supported all-risk crop insurance programs are thus developed.

The above story never seems to be questioned by agricultural economists. Whether or not a risk is actuarially insurable does not appear to be a critical element for the existence of a competitive insurance market for many economists. Economic conditions of the insurability seem much more important than actuarial and statistical requirements in their mind. Borch (1990), for example, stated that " most modern authors tend to take the pragmatic attitude that if two parties agree on an insurance contract, the risk covered is by definition insurable". This is indeed a reasonable statement, but the question is conditions under which the two parties accept the exchange? The necessary economic conditions as summarized by equation (2.6) and (2.9) are certainly critical, but the actuarial and statistical conditions for an insurable risk are also extremely important.

There is no doubt that asymmetric information problems can lead competitive insurance markets for an actuarially insurable risk to fail to exist. This has been demonstrated in the preceding sections. However, whether or not a risk is actuarially insurable is as important as any other economic aspect from the insurer's point of view. If the requirements of an insurable risk are not fulfilled, a private insurer is definitely not willing to insure the risk even if there are no other economic problems such as moral hazard and adverse selection. This is not hard to understand, since the most important operational requirement for any insurance is its actuarial principles. If these actuarial principles can not be implemented, there is no way to operate an actuarially sound program.

Comparing agricultural risks such as crop failure to the requirements of an insurable risk discussed earlier in this chapter, it is apparent that not all the necessary requirements can be satisfied. Regarding the first requirement, a large number of homogeneous risk exposure units are unlikely in agricultural industry. Actually, individual farms are heterogeneous in nature in their production conditions, management skills, micro-environmental climate factors, and soil productivities. This may be the case even within a very small geographical area. This basic reality indicates that the LLN will hardly work for an insurer if he were to cover only a limited farming area. Another obvious violation of an insurable risk in agricultural industry is that crop losses or risks are often catastrophic in nature. A severe adverse weather condition in an area could have similar effect on all the farms located in the area and could easily result in a catastrophic loss. The incidence of risk is therefore hardly independently distributed over individual farms.

The fact that an actuarially insurable risk is one of the necessary conditions for the existence of a commercial insurance industry was observed by a few economists (e.g., Spence and Zechhauser, 1971). Spence and Zechhauser emphasized that there must be substantial independence in the incidence of random events for insurance contracts to exist. If not, the LLN, on which premium and indemnity calculations are based, breaks down. The incidence of crop loss is apparently not independently distributed among individual farms. Favorable and adverse weather may have similar effects on all farms within and in adjoining areas. This indicates that only large insurance companies covering greatly varying agroclimatic areas can hope to balance such risks. This, in turn, is very unlikely for any private insurance company. The uninsurable nature of agricultural risks, therefore, leads any competitive all-risk agricultural insurance market to fail to emerge. This may be the most fundamental reason for the non-existence of any commercial all-risk crop insurance markets.

2.3.2 Insurance and Agricultural Economics Literature

Agricultural economics is one of the most productive research areas among applied economics. It may, however, not be the case as far as agricultural insurance research is concerned. The research on agricultural insurance and particularly on crop insurance has become popular only in recent decades, although various government subsidized all-risk crop insurance programs have been existing since the 1930's. The theories of agricultural insurance developed by agricultural economists are generally guided by the major economic models developed by leading economists, and it is thus not surprising to find that most results derived from agricultural insurance models are already available from pure economics models. In addition, the discussions on agricultural insurance problems have been focused on economic aspects such as moral hazard and adverse selection. Only a few have emphasized actuarial aspects.

Ahsan, Ali and Kurian (1982) developed a theory of crop insurance, using a two-state model following Rothschid and Stiglitz. They first assumed that a representative farmer faces two states of nature. In one, the bad state, the farmer loses his entire output, as defined by a production function $F(A)$, with a probability p. In the other, the farmer retains the entire output $F(A)$ with probability $(1-p)$. The farmer is assumed to be an expected utility maximizer with a risk averse attitude. The risk neutral insurer is assumed to maximize his expected profits. Ashan et al. then demonstrated that a competitive crop insurance market may not exist at all once the problems of imperfect information are present. They proposed two sub-optimal solutions to the issue: (1) market insurance with the public sector as a source of information gathering and dissemination, and (2) direct public provision of all-risk crop insurance. They also presented a simple public all-risk crop insurance model, assuming that the farmer determines his optimal input utilization taking the insurance contract as given, and the insurer chooses the optimal contract so as to maximize the value of aggregate output taking farmer's input utilization level as given.

There are several results which emerged from the Ahsan et al.'s model. Public subsidies are necessary to make agricultural insurance viable; insurance will increase output relative to a case with no insurance; and factor utilization in risky farming is smaller under public crop insurance than under a competitive insurance.

Following Raviv, Nelson and Loehman (1987) extended Ahsan et al.'s model by using a more general production function with multiple outputs and multiple inputs. They analyzed all-risk crop insurance with symmetric information between farmers and insurers. They concluded that information collection and the application of contract design principles are "second-best" solutions which may achieve the benefits of crop insurance at less cost than the public-subsidized insurance program. Nelson and Loehman also pointed out that increased output generally does not occur as a result of crop insurance. This is because the use of a full coverage insurance increases output when inputs are risk increasing (Ahsan et al.), but this conclusion does not generalize to multiple inputs and multiple outputs case where some of the inputs are risk reducing.

The Nelson and Loehman's model was extended by Chambers (1989) where asymmetric information between farmers and insurers is allowed. The economic insurability of agricultural risks and moral hazard were investigated. Chambers' model differs from Nelson-Loehman model in the way that insurance contracts are defined in a revenue insurance context and the model is specified in terms of "result state" (e.g., yield, revenue) and not the "states of nature" (i.e. agricultural risks). Specific conditions for economic insurability and possible violations of these conditions were isolated and discussed. Although no specific new findings can be drawn from Chambers paper, the "result states" model specification technique is obviously an improvement over the " state of nature " specification.

2.3.3 A Public All-Risk Crop Insurance Model

A public all-risk crop insurance model was first studied in the Ahsan et al.'s paper where an exogenously predetermined minimum income M is guaranteed for the insured by the contract. The insured farmer is assumed to choose optimal input x so as to maximize his expected utility of profits. The insurer is assumed to design an optimal contract, (ρ, given M), to maximize social welfare subject to its break-even constraint. The Ahsan et al. model is, however, subject to some obvious limitations. An apparent and critical one is that the exogenously specified minimum income M is questionable since it contradicts the insurance practice and it is difficult to determine. A general public all-risk crop insurance model will be developed and outlined in this section.

An insurance contract is defined to consist of two elements: a gross premium $\rho(y)$ and an indemnity function $I(y)$, with random crop yield y which is described by a stochastic yield distribution, or a stochastic Just-Pope production function $y = y(x, \theta)$, where θ represents all stochastic factors which will affect crop production, and x is a vector of conventional inputs. Assume that the insured farmer pays only a part of gross premiums, the proportion is α with $0 < \alpha < 1$. Let the yield protection level, which is optional to the insured, be β ($0 < \beta < 1$). The farmer is assumed to be an expected utility (of profit Π) maximizer with a risk averse attitude, i.e., his utility is concave with $u'(\Pi) > 0$ and $u''(\Pi) < 0$. He will participate in the crop insurance program only if his expected utility with insurance is greater than that without insurance. Once he picks the contract, he is going to solve the following optimization problem with respect to his optimal input allocation,[6] given output and input prices, P and w, respectively.

$$Max\ E\{u(\Pi)\} = \int_0^\infty u(Py + PL(y)\beta - P\alpha\rho - wx) f(y|\theta, x) dy, \tag{2.34}$$

where $f(y|\theta, x)$ is the yield probability density function which summarizes yield loss probability distributions. $L(y)$ is the crop yield loss function, as defined by:

$$L(y) = Max\{0, C - y\}, \quad \forall\, y < C, \tag{2.35}$$

where C is the insurance coverage, a percentage of the long-term average yield.

The government insurance agency is assumed to provide farmers with insurance coverage subject to its actuarial and financial constraints. For an actuarially sound and financially viable program, premiums must be loaded in order to cover administrative costs and other loading factors.[7] Let the loading factor be $\lambda(\lambda > 1)$, the gross premium calculation principle is simply defined as:

$$\rho = \lambda E\{L(y)\} = \lambda \int_0^C (C - y) f(y|\theta, x) dy. \tag{2.36}$$

If $\lambda = 1$, premiums equal expected losses,and actuarially fair premiums are obtained. It should be noted, however, that risk premiums defined by Pratt (1964) with public crop insurance, is zero. The government is assumed to provide farmers with insurance protection under some social welfare grounds and the services are therefore free. Some

[6]Moral hazard problem could be highlighted, assuming that the insurance company is not able to observe the farmer's particular resource allocation behavior which will affect output distributions through the production function.

[7]Since the administrative costs are fully subsidized by the government, the administration expenses are actually not paid by insured farmers.

operational constraints such as actuarial principles still apply. The insurer is thus supposed to pick up any contract which would maximize the insured farmers' expected utility, subject to equation (2.36). Under these assumptions, a crop insurance contract which satisfies the following model can be sold in the market:

$$\begin{aligned} Max\ E\{u(\Pi)\} &= \int_0^\infty u(Py(|\theta,x) + PL(y)\beta - P\alpha\rho - wx)f(y|\theta,x)dy \\ Subject\ to\ \rho &= \lambda\int_0^C (C-y)f(y|\theta,x)dy. \end{aligned} \tag{2.37}$$

The first order condition with respect to x_i is

$$\frac{dy}{dx_i}(1+\frac{\alpha\lambda}{E(y)}) + \beta\frac{dL}{dx_i} = \frac{w_i}{P}. \tag{2.38}$$

Without crop insurance program, $\alpha = \beta = 0$, the first order condition reduces to:

$$\frac{dy}{dx_i} = \frac{w_i}{P}. \tag{2.39}$$

Condition (2.38) indicates that the optimal resource allocation depends upon output and input prices, production function properties, as summarized by marginal productivities dy/dx_i, the yield coverage level β, the premium contribution portion α, the premium loading factor, and the farmer's expectation of crop yield $E(y)$ or $\bar{y}$. To see how these factors affect the farmer's input decision making, rewrite equation (2.38) as

$$\frac{dy}{dx_i}\frac{P}{w_i} = \frac{1-\frac{P\beta dL}{w_i dx_i}}{1+\frac{\alpha\lambda}{E(y)}} = k, \tag{2.40}$$

It is intuitively easy to see that, ceteris paribus, the following relationships generally hold.[8]

$$\begin{array}{lll} (1)\ \alpha\uparrow & \Longrightarrow & k\downarrow \\ (2)\ \beta\uparrow & \Longrightarrow & k\downarrow \\ (3)\ \lambda\uparrow & \Longrightarrow & k\downarrow \\ (4)\ \bar{y}\uparrow & \Longrightarrow & k\uparrow \end{array} \tag{2.41}$$

Since $k = 1$ indicates a situation where the farmer allocates his inputs such that his marginal costs and marginal revenues for all inputs are equalized, without participating in crop insurance, the certain optimal input level is thus determined when k takes

[8]Obviously $\frac{dL}{dx_i} < 0$, given a positive marginal productivity.

1. Let this certain optimal input level be x_i^*, it is clear that with crop insurance, the value of k could either be greater than one or less than one, suggesting that some resource allocation distortion under crop insurance is obvious. It is interesting to note that farmer's yield expectation $E(y)$ has some important role to play in determining his optimal input utilization. If k increases as one of the program parameters change, it indicates that the farmer will apply more inputs. The farmer will take the opposite action if k decreases as one of the program parameters change. Therefore, the insured farmer tends to use less inputs if the coverage level is increased, or if the premium subsidy is reduced, or if higher loading is applied. The farmer tends to use more inputs if he has a high expectation of his potential yield. The crop insurance program will not have any input distorting effect if $k = 1$, indicating that the following condition must hold.

$$-\frac{E(y)\beta}{\alpha\lambda}\frac{dL}{dx_i} = \frac{w_i}{P} = \frac{dy}{dx_i}. \tag{2.42}$$

The farmer's responses to insurance program parameters regarding input utilization can be formally derived using the standard comparative statics technique. Rewrite the first order condition (2.38) as

$$y'_x(1 + \frac{\alpha\lambda}{\bar{y}}) + \beta L'_x = \frac{w_i}{P} = K. \tag{2.43}$$

Total differential of the equation with respect to x yields

$$D + L'_x\frac{d\beta}{dx} + \frac{y'_x}{\bar{y}^2}[\bar{y}(\lambda\frac{d\alpha}{dx} + \alpha\frac{d\lambda}{dx}) - \alpha\lambda\frac{d\bar{y}}{dx}] = dK, \tag{2.44}$$

where $D = \beta L''_x + (1 + \frac{\alpha\lambda}{\bar{y}})y''_x$. It can be shown that $D > 0$ if $y''_x \leq -L''_x$, and/or $L''_x > 0$ then $y''_x < 0$. With these results, the following comparative statics are easily derived:

$$\begin{array}{c} \frac{d\alpha}{dx} = \frac{-D\bar{y}}{\lambda y'_x} < 0; \ \frac{d\beta}{dx} = \frac{D}{L'_x} < 0 \\ \frac{d\lambda}{dx} = \frac{-D\bar{y}}{\alpha y'_x} < 0; \ \frac{d\bar{y}}{dx} = \frac{D(\bar{y})^2}{\alpha\lambda y'_x} > 0. \end{array} \tag{2.45}$$

The model developed here is of considerable interest. It allows a detailed investigation of an insured farmer's various possible responses to different program changes.

The model itself and the derived results suggest that a systematic change should be made in order to minimize resource distorting effects which could be caused by changing any one of program parameters.

2.3.4 Adverse Selection: Partially Individualized All-Risk Crop Insurance Approach

Adverse selection may occur if the insurer cannot distinguish the inherent riskiness of different farmers. Adverse selection problems are common for any voluntary insurance programs. It is rational that an individual farmer who is aware of a risk of crop failure is more likely to seek crop insurance, although it is not evident that the farmer must represent a risk higher than the average. It is also not clear that this farmer is so price conscious that he will drop the contract if the premium is increased, and thus start the adverse selection chain effect as described in section **2.2.3.** However, adverse selection has been one of the most serious problems in all-risk crop insurance programs, particularly in an individualized crop insurance program (Halcrow, 1949; Skees and Reed, 1986; Miranda, 1991).

For example, Skees (1994) reported that adverse selection was serious in the experience of the U.S. Federal Multiple Peril Crop Insurance (MPCI) programs. In the case of soybeans produced in the Southern United States, adverse selection is estimated to have accounted for almost 20% of the programs' excess losses (indemnities in excess of premiums) during the 1980's. Adverse selection has led to an attempt to replace the current MPCI by a new area-yield-based program called Group Risk Plan (GRP) (Skees, 1994).

Adverse selection with an individualized yield coverage crop insurance program [9]

[9] Partially individualized, fully individualized, or simply individualized approach will be interchangeably used thereafter in the thesis, and this program structure is compared to purely or perfectly individualized crop insurance where indemnities and premiums are all assessed according to individual farm yields for each farm.

was elegantly discussed in one of the most important crop insurance papers written by Halcrow (1949) forty years ago. The theoretical elements developed by Halcrow served as the foundations for the U. S. federal crop insurance program (Gilson, 1987; Skees, 1994). The theoretical background of the GRP is also rooted in Halcrow's seminal paper (Miranda, 1991; Skees, 1994). The actuarial structures for a voluntary all-risk crop insurance were evaluated in detail.

Halcrow identified various types of adverse selection which may occur in a partially individualized all-risk crop insurance where farm-yield based coverages and area-yield based premiums are constructed. In order for an insurance company to maintain an actuarially sound crop insurance program (i.e., premiums collected and indemnities paid should be balanced over time) within a classified area, the following basic necessary conditions are required: (1) there is no secular yield trend for each insured farm within the area, (2) the average deviation of yields from the mean is the same for all the farmers, i.e., yield risks are identical for all the insured, (3) the mean yield is the same for all the farmers, and (4) insured farmers and the insurer have the same knowledge about yield expectation, and farmers can not make their insurance purchase decisions simply by their yield predictions. If any one of these assumptions is violated, some adverse selection may develop.

The first type of adverse selection (Intertemporal Adverse Selection, or ITAS) occurs if the yield trends are present. As the first assumption is relaxed, the level of crop yields changes on individual farms over time. In this case, farmers with higher-than-average increasing yields would find that the average coverages based on the long-term area average yield would be too low while the premiums tend to be in excess of indemnities. The opposite would hold for farmers with decreasing or lower-than-average increasing yields. As a result, the first group of farmers would have net losses and the second group would receive windfall benefits through the use of insurance, and inevitably, the first group of the insured would likely withdraw from insurance while the second group of farmers would remain. This, in turn, would force the insurer to raise premiums to maintain its actuarial objectives, and thus the adverse selection chain of events would commence.

The ITAS may also happen if the fourth assumption is violated, i.e., if farmers can accurately predict their expected yields and take the expectations into consideration of their contract purchase decision-making. This is because farmers are free to either participate in or exclude themselves from the program. Farmers can insure less or drop the contract in good crop years and insure more or purchase the contract in poor crop years, if they believe that their predictions are accurate enough. Similarly, farmers can also vary their insured crop combinations in order to increase their probability of collecting insurance payments.

Interpersonal (or interspatial) adverse selection (IPAS) will occur if the second condition (equal yield deviations) is not fulfilled. Farmers who have greater yield variations (higher yield risks) would receive higher indemnities than those having smaller yield variations, even though they have the same average yields. This situation would lead the higher-than-average yield risk farmers to purchase insurance and the lower-than-average group to drop insurance.

As Halcrow noted, the violation of the equal mean yields could also lead to interpersonal adverse selection. Generally, the farmers with low average yields would receive indemnities more often and in larger amounts than the farmers having higher average yields, if the actual deviation from the mean yield in bushels is the same for all insured farmers. If, however, the yield variations, in terms of coefficient of variation (C. V.), is the same for all farmers, the farmers with the higher average yields would tend to collect larger indemnities in poor crop years than those with low mean yields.

It should be noted that the classification of the intertemporal and interpersonal adverse selections is not absolute. The real implication of the assumptions (1) through (3) is that the identification of some homogeneous risk group is essential for an actuarially sound crop insurance program. Ideally, an individual farm seems such a natural candidate. However, the difficulty of working out an individual actuarial schedule (both the indemnities and premiums based on individual yield records) for each farm is apparent.[10] Theoretically, the necessary conditions for an insurable risk are not fulfilled, and more importantly, the purely individual farm actuarial structure is not consistent with the fundamental working principles of risk sharing and risk pooling. The risk spreading over time for each farm is also very limited under this program structure. This purely individualized crop insurance program may be practically plausible only if there are sufficient time series data for all insured farms and if farmers are required to remain in the program for an extended period of time.

In summary, an individualized all-risk crop insurance program, where coverages are based on the long-term average yields of individual farms while a uniform premium is charged on an area basis, will be financially and actuarially sound only if the four necessary conditions about farm yields and yield risks (as stated above) are systematically fulfilled. Some adverse selection will be inevitable once one of these assumptions breaks down. Since violations of these ideal situations are natural in the real farming industry, the individualized crop insurance program would definitely suffer from various adverse selection problems and is deemed to fail over most farming areas (particularly in those high risk farming regions). This was clearly pointed out by Halcrow. Unfortunately, this important and critical point had not been brought to the attention of the public crop insurance industries.[11] The seemingly ideal program

[10] Such a crop insurance structure was attempted in the late 1940's in the United States, and it was discarded after a few years due to a total failure. See Halcrow, 1949.

[11] This was particularly true in terms of the U. S. Federal Crop Insurance Programs. The U. S.

structure, a purely individualized crop insurance program in which a homogeneous risk group may be naturally established, however, is not practical and theoretically inconsistent. The partially individualized program does not work towards actuarial objectives simply because premium calculation and indemnity determination are not based on a consistent insurable risk exposure unit. This consistency, however, is essential in order to force premiums and expected losses to be in balance over time. It is therefore not surprising to find the partially individualized programs practically unsuccessful.

In order to drive an all-risk crop insurance program to meet the actuarial requirements, the *" protection would be offered under such a premium-indemnity schedule that all farmers facing similar probabilities for indemnities would be assessed similar premiums"*(Halcrow, 1949). This critical condition can not be established unless homogeneous risk classes or groups can be identified. Therefore, the key to a successful program is to classify and distinguish different risk groups by some criteria. The preceding discussions have indicated that individual farms, seemingly ideal risk units, can not be served as such a basis due to both the theoretical and practical difficulties. This suggests that some approximation has to be sought to establish homogeneous risk groups.

2.3.5 Area-Yield All-Risk Crop Insurance

A natural and an easy approximation to a homogeneous risk group is to define a risk area as a risk group. If we assume that farmers within a certain geographical area are all exposed to similar crop production risks, the area-yield crop insurance plan[12] may be developed. Under this program structure, indemnities and premiums are all

program had been implemented under such partially individualized approach. The program was not challenged until tremendous financial disasters were experienced in the mid-1980's.

[12]This program is also referred to as full area crop insurance program, or FA for short.

based on the area yield experiences, i.e., all insured farmers would receive an identical insurance payment whenever the area average yield drops below the predetermined area coverage, regardless of a farmer's individual production level. In its simplest form, a political boundary like a county could be designated as such an area. As Halcrow has discussed, one of the most important conditions for this program to work for individual farmers is that the individual farm yields and yield risks are highly positively correlated to the area yields and yield risks within the area. Similar to the individual program, the non-trend and the equal yield risk assumptions are critical to the success of an area based program.

There are potentially many advantages of the area-yield insurance program[13] over individualized coverage approach. Adverse selection problems would be significantly reduced[14] because information regarding the area yield distribution is generally available and more reliable than information regarding the distributions of individual yields, thus the insurer could more accurately assess premiums (Miranda, 1991). In addition, because the indemnity that each individual receives is independent of his own yield, the insured can not significantly increase his claim by unilaterally altering his production practices. As such, moral hazard essentially would be eliminated. Finally, administrative and program delivery costs would be substantially reduced. All these advantages emerge from the simple fact that the indemnities and premiums are consistently assessed against an identical risk exposure unit (i.e., the area) with this framework.

[13]The acceptance of this program structure is increasing. Some examples are the experimental U. S. Group Risk Plan (GRP) and the Collective Approach (CA) used in Quebec. In Manitoba, however, the opposite is true where the Crop Insurance Review Committee (1992) called for more individualized coverage and a recent review of Manitoba's Livestock Feed Security Program also found that the area based program was rejected by the majority of producers (Hamilton, 1994, personal communications).

[14]Some intertemporal adverse selection would also be reduced, although it cannot be completely eliminated under this program.

Two major drawbacks associated with the area-yield all-risk crop insurance program can be identified (Halcrow, 1949; Skees, 1994). One is that an insured farmer may not be able to receive protection when he actually suffers from a partial or total crop loss, if the area yield did not fall below the area coverage. The other problem is that the insured may be paid by the insurer even if he did not experience a crop loss. These two problems would not be uncommon if the positive correlation between farm yields and area yields is weak.

Miranda (1991) claimed that area-yield crop insurance can provide more effective yield-loss protection than individualized crop insurance for most farmers. His conclusion, however, was derived from a linear assumption about farm yield risks and area yield risks. Actually, the extent to which an area-yield program can work for individual farmers within a political area (e.g., a county) is essentially an empirical question. Whether or not the potential advantages of an area-yield program can be fully realized simply depends on how the area is defined. In most cases, a simple approximation to homogeneous risk group by an administrative boundary will not be a satisfactory choice. Therefore, the incentives to participate in this kind of program are still questionable.

A readily available example of area yield based program is the newly implemented Group Risk Plan (GRP) in the United States. Under GRP, farmers only receive an insurance payment if county yields drop below a trigger level. The trigger level, coverage level α, is selected by the insured farmer as a percentage of the expected county yield. The crop revenue protection level β is optional and it can be selected by the farmer up to 150% of the expected county revenue from the crop. Indemnities are made based on the *percentage shortfalls* below the trigger level. Indemnity in dollars is calculated as

$$\$I(y) = Max\{0,\ (C - y) \times \frac{\beta}{\alpha}\} \times P, \quad 0 < \alpha < 1;\ 0 < \beta < 1.5 \tag{2.46}$$

where C is the yield coverage in bushels, as determined by $C = \alpha\bar{Y}$, α and β are coverage level and revenue protection level, respectively, P is the insured crop price, y is the actual area yield, and $\bar{Y}$ is the long-term area average yield. For example, if a farmer selects a 90 percent coverage level ($\alpha = 0.9$) in a county where the long-term yield for corn is 100 bushels per acre (i.e., $\bar{Y} = 100$), chooses 150% of expected corn county revenue as the revenue protection level ($\beta = 1.5$), an insured corn price of \$2.00 per bushel ($P = 2$) and a county corn yield is 70 bushels per acre ($y = 70$), the insured farmer would receive

$$\$I(y) = Max\{0,\ (100 \times 0.9 - 70) \times \frac{1.5}{0.9}\} \times 2 = \$66.67. \tag{2.47}$$

It should be noted that GRP is not purely crop insurance since expected crop revenue is taken into consideration through the use of revenue protection level β. Moreover, the protection level can be greater than one in order to attract farmers with yields above the county average. GRP reduces to pure area-yield insurance if the coverage level and the revenue protection level are identical, i.e., if $\alpha = \beta$. In this case, the payout equation (2.46) becomes:

$$\$I(y) = Max\{0,\ (C - y)\}P, \tag{2.48}$$

which is simply a crop insurance indemnity schedule.

Investigating GRP, it can be seen that one additional problem arises. Since the revenue protection level β gets into the indemnity formula, the actuarial structure becomes more completed if $\alpha \neq \beta$. In this case, the indemnity a farmer may receive is also determined by his subjective crop revenue protection level, and this is difficult to be incorporated in an actuarially fair premium rate formula. Another difficulty with GRP is that the intertemporal adverse selection may be serious: producers would buy higher coverage if they predict that the probability of payout is higher. Therefore, the actuarial performance of GRP is somewhat doubtful.

2.3.6 Homogeneous Risk Area Approach

One important problem of the area-yield program is that area boundaries are political. In larger counties it is very unlikely that the area is homogeneous in soils and climate. Production risks may be heterogeneous even in a very small county. Since risks of crop failure are indeed related to geographical, ecological, and climatic conditions, it may be plausible to classify homogeneous risk groups on homogeneous risk area bases.

A geographically homogeneous risk area approach is apparently an improvement over an administrative area approach. If a homogeneous risk area is well defined by some criteria, all the farms located in the area are more likely to be subject to similar risks of crop failure. As a result, insurance contracts as defined by premium-indemnity schedules will be more theoretically consistent, since premiums and indemnities are evaluated against a more homogeneous risk exposure unit. All the potential advantages of an area-yield crop insurance would be carried over to this program whereas the disadvantages inherent in the area based design would be substantially reduced, because farm yields and area yields would be more positively correlated. Consequently, the program would be more attractive to risk averse farmers.

One major problem with defining more homogeneous risk areas is that it may not be easy to determine the areas. There also may have some operational difficulties. For example, it is hard to explain to a farmer why he and his neighbor are not in the same risk area. In addition, theoretical risk area boundaries are not necessarily the same for different crops. Generally, historic production levels, soil characteristics, climate conditions, and production conditions could be major references in zoning risk areas. Determining the size of a risk area may also become very technical. It is necessary and essential to define a risk area such that it is large enough to contain sufficient number of farmers in the risk pool.

2.3.7 Quasi-Homogeneous Risk Area Approach

The fundamental problems that accompany farm-level individualized all-risk crop insurance program have been known since the early days of crop insurance practices. However, these problems and issues did not receive serious consideration in some programs such as the U. S. Federal programs until these programs experienced extremely poor financial performance. Fortunately, Canadian Federal-Provincial all-risk crop insurance programs have performed much better from an actuarial and financial perspective. The reason may be that the problems associated with the individualized crop insurance program were sufficiently recognized when these programs were initiated in the early 1960's. The awareness of the potential problems inherent in individualized farm-level crop insurance made the program designers seek some better alternatives (Gilson, 1994, personal communications). It was under this circumstance that a hybrid of risk area-yield and individual coverage has been developed in Manitoba.

Manitoba Crop Insurance has been in operation since 1960. Like any other all-risk crop insurance programs, the mandate of Manitoba's program is to provide yield insurance against natural hazards which can not be reasonably controlled by the insured farmers.

The premium-indemnity schedules under Manitoba program are generally based upon risk areas which are defined as homogeneous areas with respect to production conditions and soil productivities. These areas were determined in 1960's according to crop yield histories, soil characteristics, and other climatic information (precipitation, frost free days, etc.,). The whole province is divided into fifteen risk areas.

A uniform base premium per acre for each crop is charged for all insured farmers within an area. An experience discount/surcharge system is used to make adjustments for individuals. The determination of the bushel-equivalent coverages for each insured farmer is a three-step process. First, the average probable yield (long-term area expected yield, $\bar{Y}$) for the risk area in which the insured farm is located is established. Then further differentiations are made for soil type differences and individual yield performance relative to average. That is, when the risk area expected yield $\bar{Y}$ is determined, it is then adjusted by all soil types in the risk area. The soil is rated from letter A to letter J with type A representing the highest productivity soil and type J the lowest. The soil type adjustment coefficient is determined by comparing how that particular soil has yielded relative to risk area average over the last 15 years. Multiplying the risk area yield $\bar{Y}$ by soil type adjustment coefficient produces the soil zone probable yield. The final step in determining coverage for an insured is to adjust the base soil zone yield by the Individual's Productivity Index (IPI). The IPI is calculated by comparing the individual's yield to the soil zone average yield. For example, if the individual has outyielded the soil type average yield for red spring wheat by 5% over time, the individual's IPI would be 1.05.[15] The soil type yield multiplied by the IPI yields the individual's probable yield.

Once the individual's probable yield has been determined, the individual's coverage can be assigned. An insurance payment is triggered if the farmer's actual yield drops below the coverage. It is clear that the program ends up with an individualized

[15] In the case of an abnormal yield event, a producer's annual index is limited to a range of 70% to 130% of the producer's previous IPI. In the event of a hail loss, the annual index is calculated as $Harvest\ Yield/(1 - \%\ Hail\ Loss)$.

coverage. The critical difference between this program and the individualized program we have discussed earlier is that the individual's coverage in the latter is simply determined by the farmer's own yield records while the coverage for the individual in Manitoba's program is based on the area yield but adjusted by soil productivities and the individual's yield records.

The quasi-homogeneous risk area approach or area based IPI approach originates from the purely homogeneous risk area approach, and is in fact a hybrid of the area coverage approach and individual coverage approaches. The advantages and disadvantages of this combined program structure could be identified from its mother-structures. Since the backbone of this structure is the purely homogeneous risk area methodology,[16] it is important to understand why the purely homogeneous risk area program is not practical so that some special adjustments have to be made in the basic area coverages and area premiums, and why a more straightforward individual coverage approach is not applied.

The purely homogeneous risk area approach is acceptable from an actuarial point of view only if some crucial assumptions or conditions are simultaneously fulfilled. Of these assumptions, the key is that the crop production risks faced by each individual located in the defined risk area are strictly homogeneous. To achieve homogeneity, the following conditions are fundamental: (1) the individual yield data are randomly and independently distributed; (2) all the farms within the area have the same yield variations; (3) there are no yield cycles or trends; (4) all insured farms are exposed to similar or uniform production conditions such as soil and climatic conditions; (5) there are no management differences among the farmers within the area; and (6) the incidence of natural hazards are randomly distributed over the risk area. If one

[16]One practical reason for this program being implemented was documented as "the lack of sufficient individual farm yield data " when the Gross Revenue Insurance Program (GRIP) was introduced in 1991.

of these conditions is violated, some adverse selection and other actuarial problems would arise. However, these ideal conditions are violated in varying degrees in practice. Consequently, some special adjustments are made to the basic area coverages and area premiums in order to correct for departures or violations from the standard situations. The practical reasons for these adjustments are many: to establish greater equity among the insured individuals; to reduce adverse selection; to compensate for deficiencies in the data used to establish the basic area coverages and premiums.

The difficulties of the purely homogeneous risk area approach and the individualized approach seem to suggest that some kind of combination is plausible and inevitable. Actually, it is not uncommon in many insurance programs to provide for special adjustments in the basic coverages and premium rates. A fire insurance company may provide for a scaled reduction in premium rates according to the number of smoke detectors installed in the building or the distance to the nearest fire hydrant or fire station. A life insurance company may adjust its basic coverage rates or premiums based on such criteria as evidence of health status or occupation or whether a person is a non-smoker. Some sort of adjustments are required simply because there is no way to exactly define and classify homogeneous risk groups in insurance. These adjustments are designed, in principle, to correct for deviations from the norm, that is to compensate for differences between the actual conditions and the standard conditions used as the foundation for the calculation of the basic coverages and premium rates. However, these special adjustments generally have some actuarial and operational consequences, and these must be observed and assessed when they are developed (Gilson, 1987).

In order to establish the long-run actuarial soundness of the crop insurance program or the equitable treatment of participants in the program, the assessment of the consequences of any deviation from the basic principles, in the form of special adjustments, is important. Will the special adjustments add to the overall cost of the program and, if so, who should bear the increased premiums? Will the special adjustments inject a bias into the basic structure of the program, and if so, who will gain? Will the special adjustments create adverse selectivity in the program? If the "symmetric adjustment principle " is followed, i.e., the upward (e.g., premium surcharges) adjustments and the downward (e.g., premium discounts) adjustments are in balance over time, the overall cost of the program will not be loaded and no insured will be treated in a biased way. Ferther to this, the actuarial principles would not be negatively affected.

The current area based IPI approach implemented by the Manitoba Crop Insurance Corporation is a complex procedure. The methodology used to determine individual coverages could be driven to an individual coverage approach in the long run, if the area is fairly homogeneous and the adjustments are fairly made to each individual. The only practical obstacle to prevent the IPI approach from becoming the individual coverage approach is the 70%/130% stabilization rule which defines the maximum and the minimum coverage adjustment.[17] Theoretically, however, there is no reason to set such limits.

Under the MCIC program structure, premium rates (area based) are calculated by crop using a simple 25 year average of loss experience adjusted for loads. Premium loads are applied for the loading factors including crop based uncertainty load (5% - 10%), area based self-sustainability load,[18] reseeding benefit load, net premium discount load, and special loads adjustment as required. The premium paid by an individual producer is adjusted by premium discount/surcharge schedule. The indi-

[17]It is however important to note that the 70/130 stabilization rule does not create an absolute cap on IPI. A producer's IPI can move above 130% or below 70% (Hamilton, 1994).

[18]It is calculated so that there is a 90% probability that premium income will exceed losses in any three year period. The purpose of this self-sustainability load is to provide for a surplus build up over time. The MCIC ratemaking methodology will be detailed in Chapter 4.

vidual adjustment is made based on the producer's years of program participation, loss free years and total loss ratio. The maximum adjustment is ±25%. The net discount bias is calculated by risk area and built into the rate calculation through the net discount load.

Some potential problems with the Manitoba's approach can be observed. The actuarial performance of the program will largely depends upon the justification of special adjustments which are made to both individual coverages and producer's premiums. The consistency between coverage and premium adjustments are essential for the soundness of the program. Ideally, the adjustments on both coverages and premiums should be consistently assessed on the same risk assessment basis. It is, however, not difficult to find that such consistency is not entirely established within the MCIC's framework. The adjustment criteria of years of participation, years of free loss, and total loss ratio are obviously ad hoc and are oversimplified.

The obvious advantages of the current approach over a straight individual coverage approach can be summarized as follows (Hamilton, 1994):

(1) The IPI approach provides a more appropriate level of coverage when an insured farm has different soil types.

(2) It provides a buffer against abnormal yield events (70%/130% stabilization rule).

(3) The IPI methodology allows an insured increased coverage following a generally poor crop year, provided the insured producer outyields the area average by more than his previous IPI.

(4) It allows a rebalancing of coverage between producers in a risk area without increasing the overall level of coverage provided (hail adjustment).

The disadvantages of the program compared to the universal individual coverage approach are also apparent:

(1) The program administration is complicated by the fact that producers are required to provide yields by quarter section in order to maintain the soil zone base.

(2) IPI only works if the methodology used to determine soil zone base yield is accurate. Producers will not be satisfied with adjustments around a base probable yield that they consider to be too low.

(3) It takes longer for an insured farmer who has a productivity level significantly different from area average to reach the correct coverage level. This applies to above-average and below-average producers.

Chapter 3

A Comparative Evaluation of Yield Risk Reductions With Alternative Crop Insurance Programs

The purpose of this chapter is to provide a comprehensive evaluation and assessment for various crop insurance programs in order to determine their relative merits in terms of yield risk reductions.

3.1 Simulation Models

The models for analyzing the various insurance approaches are developed and evaluated in this section, based on the yield risk reduction criteria. In particular, three alternatives are examined for risk reduction in yield-equivalent (Carriker et al.1991) or net yield (Williams et al. 1993, Miranda, 1991) using farm-level yield data from the MCIC research questionnaires. An index which measures relative yield risk reduction for each crop insurance scheme is constructed. The generalized stochastic dominance analysis is also made to provide an alternative evaluation. The programs analyzed include: (1) the individual yield coverage and individual indemnity program, (2) the area coverage and individual indemnity program, and (3) the area yield coverage and area indemnity program.

3.1.1 Individual Yield Coverage and Individual Indemnity Program (Fully Individualized (FI) Program)

Under the FI program, the protection or coverage is determined by the historical yield records of each insured farm. An indemnity is triggered for the farm whenever its actual yield falls below the predetermined coverage in any particular year. The yield-equivalent or net yield measure for each farm under the FI is calculated as:

$$y_f^{net} = y_f + I_f - \rho, \quad with \ \ I_f = Max\{0, C_f - y_f\}. \tag{3.1}$$

Where y_f^{net} is the yield-equivalent per acre in bushels, y_f is the actual farm yield per acre, I_f is the insurance indemnity in bushels, C_f is the farm yield coverage, and ρ represents the bushel-equivalent actuarially fair insurance premium per acre.

The yield risk, as measured by net yield variance for the insured farmer equals

$$Var(y_f^{net}) = \sigma_f^2 + \sigma_{I_f}^2 + \sigma_\rho^2 + 2COV(y_f, I_f) - 2COV(y_f, \rho) - 2COV(I_f, \rho), \tag{3.2}$$

where $Var(y_f) = \sigma_f^2$, $Var(I_f) = \sigma_I^2$, $Var(\rho_f) = \sigma_\rho^2$, and $COV(.)$ denotes covariance between the arguments of (.). Without participating in the program, the farmer has to assume his whole yield risk as σ_f^2. By acquiring the FI insurance, however, the farmer can reduce his yield risk by an amount

$$\begin{aligned} \triangle_f^{Var} &= Var(y_f) - Var(y_f^{net}) \\ &= -(\sigma_{I_f}^2 + \sigma_\rho^2 + 2COV(y_f, I_f) - 2COV(y_f, \rho) - 2COV(I_f, \rho)). \end{aligned} \tag{3.3}$$

It is obvious that the FI program will be yield risk reducing for the insured farmer if and only if $\triangle_f^{Var} > 0$. This is equivalent to saying that the condition

$$\beta_c = \frac{\sigma_{I_f}^2 + \sigma_\rho^2}{2COV(y_f, \rho) + 2COV(I_f, \rho) - 2COV(y_f, I_f)}, \ and \ 0 < \beta_c < 1 \tag{3.4}$$

must hold, where β_c defines some yield risk reducing critical value.

The condition of ($0 < \beta_c < 1$) for a yield-risk reducing farm can be easily justified. Theoretically, a positive relationship is expected between crop yields and premiums charged. A non-negative response between indemnities paid and premiums charged is also obvious. A negative correlation between insured yields and indemnities triggered obviously holds for an actuarially sound and financially viable crop insurance program. Therefore, a positive sign for the denominator in the equation above is established. Since a positive yield risk reduction ($\triangle_f^{net} > 0$) must hold for a yield-risk reducing farm, $\beta_c < 1$ follows immediately.

The index β_c provides a comprehensive measure for the relative magnitude of yield risk reductions for the insurance program. This measure is better than usual yield risk measures such as variance or coefficient of variation (C.V.) in the sense it incorporates all the fundamental dimensions of crop insurance in the formula. This index is constructed such that it automatically combines the base strategy (no insurance) into the formula, thus making the comparison much easier. The comparative analyses for the premium setting methodology, for the indemnity schedule, and for the coverage determination, can be simulated conveniently with this simple index.

Given coverage level, the relative magnitude of yield risk reductions regarding the FI program for an insured farmer can be assessed on the following scale: it is said that the program for the insured farmer is yield risk reducing if his β_c falls into the interval of $(0, 1), i.e., 0 < \beta_c < 1$; the program is said to be yield risk increasing if $\beta_c < 0$ or $\beta_c > 1$; it is yield risk neutral if $\beta_c = 1$, and the larger the β_c is, the less yield risk reducing the program will be.[19]

[19]This claim holds only for different programs at a given coverage level, e.g., this is true for the FI, the IA and the FA program at any, say, 80% coverage level. If the coverage level changes for a certain program, e.g., increased from 80% to 100% for the FI program, then the β_c value will generally increase. This, of course, does not indicate that the higher coverage level for the FI leads to the lower yield risk reducing.

3.1.2 Area Yield Coverage, Individual Indemnity Schedule (IA)

Under this program, the insured farmer receives an indemnity if his actual yield (y_f) falls below the insured yield (i.e., the coverage, C_a). Unlike the FI program, the coverage is determined by the long-term average area yield within a predetermined risk area, rather than by the insured own farm yields. Defining the area indemnity schedule as I_a, the yield-equivalent of the insured farmer, in a particular year, with this plan is then

$$y_f^{net} = y_f + I_a - \rho, \quad with \quad I_a = Max\{0, C_a - y_f\}. \tag{3.5}$$

Applying the same procedure as used for the FI program, the critical condition of $\Delta_f^{Var} > 0$ leads to

$$\beta_c = \frac{\sigma_{I_a}^2 + \sigma_\rho^2}{2COV(y_f, \rho) + 2COV(I_a, \rho) - 2COV(y_f, I_a)}, \; and \; 0 < \beta_c < 1. \tag{3.6}$$

The relative magnitude of yield risk reduction for this program can be analyzed in the same manner as for the FI program.

3.1.3 Area Yield Coverage, Area Indemnity Program (or Full Area Insurance Program (FA))

With this scheme, both coverages and indemnities are based upon area yield experiences within a relatively homogeneous risk area. An indemnity is triggered for each participant whenever the actual average area yield (Y_a) drops below some predetermined critical yield level (the area coverage, C_a), in a particular year, regardless of the farm's observed yield (y_f). Under this program, all the insured farms would receive an identical indemnity as defined by

$$I_a = Max\{0, C_a - Y_a\}, \tag{3.7}$$

and his yield risk, as measured by the variance of the net yield, equals

$$Var(y_f^{net}) = \sigma_f^2 + \sigma_{I_a}^2 + 2COV(y_f, I_a) - 2COV(y_f, \rho) - 2COV(I_a, \rho). \tag{3.8}$$

By participating in this insurance program, the producer can reduce his yield risk by an amount

$$\Delta_f^{Var} = -(\sigma_{I_a}^2 + \sigma_\rho^2 + 2COV(y_f, I_a) - 2COV(y_f, \rho) - 2COV(I_a, \rho)). \tag{3.9}$$

The index β_c for this program is the same as that for the IA program, except for a different indemnity schedule.

The full area or Halcrow Plan is claimed to be superior to individualized insurance in the sense that it reduces adverse selection dramatically and the moral hazard problem is essentially removed (Halcrow, 1949, 1978; Miranda, 1991). It is clear that the Halcrow Plan will be reduced to the IA scheme if the individual yields are identical with the area yields ,i.e., $y_f = Y_a$.

The critical condition which indicates whether or not the farmer is interested in area programs (e.g., FA) largely depends upon the yield relationship between his own farm yields and the established area yields, since β_c is determined by the indemnity variance and the farm-yield / area-indemnity covariance $COV(y_f, I_a)$. Halcrow (1949) has demonstrated that the key to the successful implementation of the FA program is that individual farm's yields have to be highly positively correlated with the area yields. Farmers whose yields are not highly correlated with the area yields may find this program ineffective in reducing yield risk (often indicated by a negative β_c). It is thus important to test the existing relationship between individual yields and area yields empirically. This is done by defining the following:

$$y_f = F(Y_a; \beta, u). \tag{3.10}$$

It is generally expected that some nonlinear relationship between farm yields (y_f) and area yields (Y_a) exists (e.g., Marra and Schurle, 1992). Miranda (1991) suggests a simple model, assuming a linear relationship between the individual yield deviation and the area yield deviation :

$$y_f - E(y_f) = \beta_f(Y_a - E(Y_a)) + u_f, \tag{3.11}$$

where $\beta_f = COV(y_f, Y_a)/Var(Y_a) = \rho_{af}\sqrt{Var(y_f)/Var(Y_a)}$, ρ_{af} is the Pearson coefficient of correlation between farm yields y_f and area yields Y_a, and u_f is a random error term. The coefficient β_f measures the sensitivity of the individual's yields to the systemic factors that affect the area yields. Generally, the higher the β_f, the greater the chance that an area yield insurance will be risk reducing for the insured farm. The program will be risk reducing for the farmer only if β_f is above a critical beta value, β_m. The formula to calculate β_m may be derived as follows:

Since $y_f - E(y_f) = \beta_f[Y_a - E(Y_a)] + u_f$, by multiplying through $[I_a - E(I_a)]$ and taking the mathematical expectation, it immediately follows that:

$$\begin{aligned} COV(y_f, I_a) &= \beta_f COV(Y_a, I_a) \\ \beta_f &= COV(y_f, I_a)/COV(Y_a, I_a), \end{aligned} \tag{3.12}$$

and the farmer will be indifferent with and without the insurance if and only if $\triangle_f^{Var} = 0$, i.e., $COV(y_f, I_a) = \sigma_{I_a}^2/2$. It then follows that the program will be yield risk reducing if and only if

$$\beta_f > \beta_m = \frac{COV(y_f, I_a)}{COV(Y_a, I_a)} = -\frac{\sigma_{I_a}^2}{2COV(Y_a, I_a)}. \tag{3.13}$$

This is also the formula presented by Miranda (1991) and he has proved that the β_m ranges from 0 to $\frac{1}{2}$. It thus follows from the equation (3.13) that the FA or the IA will be risk reducing for any insured producer for whom $\beta_f > \beta_m$, provided that the assumption (3.11) is retained.

The linear relationship assumption of individual yields and area yields is crucial in the previous discussion. Yield or yield risk relationship between farm yields and area yields has been an important issue in agricultural economics (Musser and Fackler, 1992). The issue itself is empirical in nature and raises the following questions: (1) Is there any long-term stable relationship between farm yield risk and area yield risk? (2) Does farm yield or yield risk cause area yield or yield risk, or vice versa? (3) What may be the best approximation to the true relationship, if it exits? A set of econometrics procedures are employed in answering the first question, using farm yield and area yield variables.

The existence of a long run stable relationship between farm yields and area yields are tested by the theory of cointegration (Engle and Granger, 1987). The theory states that two time series should cointegrate in order to have a long run stable relationship between them. A two step procedure is used in testing cointegration.

Step 1: The hypothesis of unit root about two time series of farm yields (y_f) and area yields (Y_a) in the following Augmented Dickey-Fuller (ADF) regression is tested:

$$\triangle x_t = \gamma + \delta t + \alpha x_{t-1} + \Sigma \beta_i \triangle x_{t-i} + u_t. \tag{3.14}$$

where $\triangle x_t = y_{ft} - y_{f_{t-1}}$ and $Y_{at} - Y_{a_{t-1}}$, the first differences of two time series, y_f and Y_a are defined as earlier. The selection of lag structure of $t - i$ is based on Akaike's Information Criterion. The test of $\alpha = 0$ in the above equation is equivalent to testing a unit root (stationary time series process). The Dickey-Fuller (1979) t-statistic is utilized in performing step one test.

Step 2: If the unit root hypothesis is not rejected, the cointegration test is then conducted based on the following equation:

$$y_{ft} = \gamma + \beta Y_{at} + u_t. \tag{3.15}$$

The test of cointegration between y_{ft} and Y_{at} is equivalent to the test of a stationary stochastic process of u_t. A long run stable relationship is established if the cointegration hypothesis cannot be rejected.

The nonrejection of the cointegration hypothesis rules out the possibility of no causality between farm yield and area yield, although they do not indicate the direction of causation. Since the area yields are calculated from the individual yields, it is reasonable to assume that only individual yields cause area yields.

3.1.4 Stochastic Dominance Analysis

Though the values of β_c provide measures of yield variability reductions, they do suffer from some limitations. These indices are useful only if a risk aversion attitude is assumed. More importantly, the same risk attitude for every insured farm has to be assumed. Therefore, a more direct comparison of the net yield distributions is made using generalized stochastic dominance (GSD) criteria.

Stochastic dominance allows an examination of risky choices without imposing any functional restriction on the shape of decision maker's utility. It accounts for the difficulty in quantifying the risk preference with some inexact representation of decision maker's risk attitudes. The basis of stochastic efficiency criterion is the Expected Utility Hypothesis. By using stochastic dominance criteria, a choice set of distributions can be reduced to a relatively smaller subset in such manner which ensures that some member of the subset maximizes expected utility. This subset is referred to as the efficient set and is identified for an admissible class of utility functions. Each of the different stochastic dominance criteria is associated with a different class of admissible utility functions.

The first important concept of stochastic efficiency is First-Degree Stochastic Dominance (FSD). FSD assumes nothing about decision maker's risk attitude except that the decision maker always prefers more (income) to less. This is none other than the assumption of a monotonically increasing utility function, i.e., $U'(y) > 0$.

Suppose there are two choices or strategies to be chosen: strategy F and strategy G. Each strategy is associated with its own probability density function (PDF) for the different outcomes which are denoted as $f(y)$ and $g(f)$, respectively. The corresponding cumulative density functions (CDF) are $F(y)$ and $G(y)$, respectively. Under FSD, strategy F is said to dominate strategy G if and only if

$$\begin{aligned} \textstyle\int_0^y [f(y) - g(y)]dy \ &\leq \ 0 \ for \ \ all \ \ y \\ &< \ 0 \ for \ at \ least \ one \ y. \end{aligned} \tag{3.16}$$

Since $F(y) = \int_{-\infty}^{\infty} f(y)dy$ and $G(y) = \int_{-\infty}^{\infty} g(y)dy$, the above criterion simply states: the strategy F associated with the CDF of f(y) is always preferred to the strategy G which is associated with its CDF of g(y) by all decision makers who prefer more to less if the condition $F(y) \leq G(y)$ holds for all outcomes $y's$ with strict inequality for at least one y. Graphically speaking, FSD implies that the CDF curve for f(y) and the CDF curve for g(y) will never cross for all possible outcomes of $y's$, and the F(y) curve is always to the right of the curve G(y), if F is to dominate G.

Second-Degree Stochastic Dominance (SSD) allows for the predicting of a decision maker's risk choice between pairs of strategies without any knowledge of the decision maker's utility function except that it displays risk aversion. Under SSD, two constraints are placed on the admissible utility functions, $U'(y) > 0$ and $U''(y) < 0$. With SSD, the strategy F will dominate strategy G if and only if

$$\begin{aligned} \textstyle\int_0^y [F(y) - G(y)]dy \ &\leq \ 0 \ for \ \ all \ \ y \\ &< \ 0 \ for \ at \ least \ one \ y. \end{aligned} \tag{3.17}$$

Graphically speaking, the strategy F will dominate the strategy G if the difference in areas between the two CDFs before they cross is greater than the difference in areas between the two CDFs after they cross. For many kinds of PDFs, the corresponding

CDFs will cross somewhere over possible outcomes of $y's$. A normal distribution for y is a typical example (Anderson, Dillon and Hardarker, 1977). It is for this reason that First-Degree Stochastic Dominance is not applicable if a normal distribution is assumed.

The disadvantages associated with FSD and SSD are often the lack of high discriminatory power among different choices and the resulting efficient sets are too large (King and Robison, 1981). Moreover, the criteria are not useful if decision makers display both risk-loving and risk-averse attitudes.

Generalized Stochastic Dominance (Meyer, 1977) is a more flexible criterion, in that alternative restrictions on the admissible utility functions are defined with bounds on the Pratt-Arrow absolute risk aversion function $r(y)$. By definition, $r(y) = -U''(y)/U'(y)$, under GSD, strategy F is said to dominate strategy G if and only if

$$\begin{array}{ccc} \int_{-\infty}^{\infty}[F(y) - G(y)]U'(y)dy & \leq & 0 \ \text{for all } U \\ \text{Subject to} & r_l < r(y) < r_u & \text{for all } y. \end{array} \quad (3.18)$$

It is apparent that FSD and SSD are only special cases of this more general efficiency criterion. To illustrate, the class of decision makers ordered by FSD was assumed to have a monotonically increasing utility function, $U'(y) > 0$. This assumption places no bounds on the absolute risk aversion function, since $U''(y)$ are free to take on any value. Thus the class of risky choices consistent with FSD is defined as:

$$-\infty < r(y) < \infty. \quad (3.19)$$

The SSD set is more discriminating. Besides a positive marginal utility ($U'(y) > 0$), it requires $U''(y) < 0$. These utility restrictions can be equivalently stated as follows:

$$0 < r(y) < \infty, \quad (3.20)$$

that is, the applicable class of decision makers are restricted to the risk-averse class with $r(y) > 0$.

GSD is calculated by identifying the utility function from the admissible class which is least likely to result in the expected utility of F being greater than G. If the expected utility of strategy F is greater than that of G for this utility function, then it is known that the result will hold for the entire class of admissible utility functions.

Unlike FSD and SSD, the use of GSD requires the researcher to explicitly define the class of admissible utility functions. This is done by specifying the preference intervals in terms of lower bounds (r_l) and upper bounds (r_u) on the Pratt-Arrow risk aversion coefficients. The determination of these bounds, however, is often difficult. Robison et al. (1984) identify four general approaches to assessing risk preferences: (1) direct elicitation of single valued utility functions; (2) direct elicitation of risk preference intervals; (3) experimental methods; and (4) inferences from observed economic behavior. To use GSD, an interval approach is certainly convenient.

The GSD programs developed by Cochran and Raskin (1988) are used to test different strategies within GSD framework. FSD and SSD tests can also be easily conducted with their programs.

3.2 Data And Procedures

Data: Historical farm production records of red spring wheat for risk area 2, risk area 6, and risk area 12 are used as the dataset for this study. The database was provided by the Manitoba Crop Insurance Corporation. Each MCIC farm record contains individual field information by year and crop. The information includes acres of each crop seeded, average crop yields harvested in metric tons, fertilizer applied by nutrient in pounds, chemical and herbicides applied by product, soil classification rated from A to J, and the risk area code. The three designated risk areas are chosen based on several criteria. The availability of sufficient data and the representativeness of different production risk categories are major concerns in the choice of those risk areas. Table 3.1 presents descriptive statistics of the red spring wheat for the three risk areas.

Table 3.1. Area Yield Descriptive Statistics
(Bushel/Acre)

AREA 2				
PERIOD	Mean	Std Dev	Minimum	Maximum
1960-70	21.9723	6.1807	8.8440	28.3651
1971-80	25.9852	3.6667	20.4936	30.0687
1981-92	28.7972	8.4635	15.4753	42.0186
1960-92	25.8769	7.0444	8.8440	42.0186
AREA 6				
1963-70	28.2401	2.5767	24.4617	32.7992
1971-80	28.2681	3.3063	24.3064	34.1292
1981-92	31.1619	6.8072	16.9099	40.7084
1963-92	29.3208	4.8201	16.9099	40.7084
AREA 12				
1960-70	20.7921	5.3170	11.2742	26.4915
1971-80	24.0442	5.5824	13.0813	30.6775
1981-92	34.8644	9.7921	13.3412	47.1301
1960-92	27.7477	9.7385	11.2742	47.1301

Risk area 2 has traditionally higher risk of crop failure and lower yields. It has a high evapotranspiration rate due to high temperatures and relatively low levels of precipitation. Drought is the major production risk within this area. Soil type F takes 20% of the total area landbase. Risk area 6 traditionally has a lower risk of crop failure and higher yields. Frost hazard is quite significant and only periodic droughts are encountered in a small part of the area. Soil type D makes up 21% of the landbase. Risk area 12 provides a median and mixed case in terms of production risk. It is a large area and takes in the entire Red River Valley. The temperature regime is excellent with very little frost hazard and normally adequate to above average rainfall. The main hazard in this area is excessive moisture. The major soil types are D and D^+ and which make up 40% of the total land. These three risk areas, therefore, will allow an evaluation of yield risk reductions for each of the alternative insurance programs under different production risk circumstances. Table 3.1 presents descriptive statistics of the red spring wheat yields for these three risk areas.

To simplify the empirical models, the sample set is also restricted in several ways. For each producer, only the production records on some specific soil type are retained. Specifically, the production information on soil type D for the risk area 6 and 12, and on soil type F in the case of the risk area 2 are kept in the sample. Moreover, the sample set is restricted to those producers who have participated in the MCIC program and have grown red spring wheat for at least 20 years during the period 1960 to 1992. Based upon these criteria, more than 500 producers remain in the sample. Of these total farms in the sample set, about 40 are based in risk area 2, about 30 are in risk area 6, and 370 are based in risk area 12.

Procedures: Yield risk reductions for the three alternative crop insurance programs are first analyzed using the proposed β_c approach. To capture the impact of different protection levels on the relevant insurance plans, the coverage levels are set at 80% and 100%, respectively. The actuarially fair premiums are calculated in each case in order to put the comparisons on a common base. Technically, the premiums incurred for each program are calculated such that the total indemnities will be averaged out over a certain period, based upon the empirical premium rate setting formula. Both the variable actuarially fair premium rates (Set I) and the fixed actuarially fair premiums (Set II) are used. The index β_c for each farm is also estimated for the alternative programs by using the detrended yield data (Set III). This is calculated as residuals from regressions of individual farm yields on a linear time trend. The detrending data process is utilized since the yield variabilities are likely overestimated without removing the trend which accounts for technical change. To be consistent with the current program, a ten-year moving average yield series is calculated using historical production records for each farm under each program analyzed. This moving average yield distribution is the basis for estimating relevant insured yield. Since each selected farm in the sample has more than 20 years records, the length of calculated net yield distributions for insured farms ranges from ten to twenty-three years.

The GSD approach is used to provide an alternative assessment to the simulated programs by directly comparing net yield distributions. The GSD results are particularly useful in evaluating the implications for the alternative programs under different coverage levels. This popular method is also used to justify the β_c index method. To alleviate the computation burden in performing GSD analysis, only 75 farms from the risk area 12 and with more than thirty years records are chosen as the dataset.

The Miranda β_m and the cointegration test for farm-area yield relationship is conducted to detect some fundamental problems with various area coverage programs. The average "financial" exposures in terms of indemnities for each farm with each simulated program are presented and examined in the final section of the chapter. The group of models consisting of the index, GSD and some supplementary testing procedures will provide some systematic and consistent empirical insights into the problems being studied.

3.3 Results

3.3.1 Yield Risk Reductions With β_c Approach

The summary statistics for β_c's under different programs with various simulation procedures are presented in Table 3.2, Table 3.3 and Table 3.4.

In these tables, three sets of estimated mean β_c values and associated statistics are reported. Set I provides the summary statistics for index β_c for all insured farms within the given risk area, using variable actuarially fair premiums for each of the programs. Under Set II, a fixed actuarially fair premium rate is calculated for each farm. Set III presents the summary statistics for the index β_c, through the use of the same methodology as for Set II. The yield data, however, are systematically detrended by a simple regression trend model for each farm. The specific statistics in these three tables include the mean β_c's, the variance of the estimated indices, the range of the estimated mean values, the skewness, and the normality statistic (Shapiro-Wilks) for all calculated β_c's. Both 80% and 100% coverage levels are simulated. The 80% coverage level is chosen since this is the major option currently offered by the MCIC. The 100% coverage level is simulated to represent a full insurance coverage option. In this manner, the impact of an increased coverage level on the insurance program evaluated can be assessed.

Table 3.2. Summary Statistics for β_c: Risk Area 12

Set I	MEAN	VAR	MIN	MAX	SKEW	NORMAL
FI80	0.2951	0.0259	0.0833	1.0000	2.4611	0.7572
IA80	0.3209	0.0320	0.0574	1.0000	1.7842	0.8246
FA80	0.3336	0.0234	< 0	> 1	0.3489	0.8743
FI100	0.4031	0.0304	0.1111	1.0000	1.3301	0.8867
IA100	0.4345	0.0387	0.0833	1.0000	0.9097	0.9155
FA100	0.5825	0.1213	< 0	> 1	0.7348	0.8574
Set II						
FI80	0.1768	0.0033	0.0292	0.3426	-0.3754	0.9675
IA80	0.1875	0.0048	0.0000	0.3755	-0.2643	0.9747
FA80	0.1968	0.0311	< 0	> 1	1.4553	0.8730
FI100	0.2365	0.0046	0.0467	0.4502	-0.3824	0.9718
IA100	0.2471	0.0070	0.0000	0.5080	-0.0542	0.9755
FA100	0.2961	0.0675	< 0	> 1	1.7117	0.8131
Set III						
FI80	0.2960	0.0027	0.0763	0.4159	-1.0530	0.9390
IA80	0.2797	0.0088	0.0000	0.4561	-0.9580	0.9215
FA80	0.3469	0.0741	< 0	> 1	1.6121	0.8190
FI100	0.3421	0.0020	0.1512	0.4357	-0.9491	0.9474
IA100	0.3219	0.0077	0.0000	0.5090	-1.1425	0.9203
FA100	0.4244	0.0870	< 0	> 1	1.3306	0.8263

Since risk area 12 contains the largest sample in this study, we will examine its results first. In this risk area, given 80% coverage level, the individual coverage and individual indemnity (FI) program is the most efficient design regarding yield risk reductions, for it has the smallest mean β_c. The area coverage and individual indemnity (IA) program comes next with a median average β_c index value. The area coverage and area indemnity, or the full area program (FA), is least preferred by insured farmers as it gives rise to the largest mean β_c. The FI program and the IA program are found either yield risk reducing or yield risk neutral for all the insured farms. This is the case since no negative β_c is obtained. There is also no β_c greater than one. The full area program, however, will be yield risk increasing for some farms, because negative and large (greater than one) values can be observed. The positive skewnesses for all the simulated programs indicate that these programs are not equally preferred by all group of farmers.

As the coverage level is increased to 100%, the mean β_c estimates are increased accordingly for all the three programs. This, however, does not necessarily mean that the programs will be less yield risk reducing provided that the same methodology is applied (see footnote 19). An insurance with higher coverage will likely be more attractive than lower coverage for producers, provided that other conditions remain unchanged. This is intuitively obvious since more protection is provided with higher coverage level. The systematically higher β_c's with the full coverage level, however, do have some policy implications. They suggest that producers will be less sensitive to the alternative programs, in terms of their decision-making as coverage level is increased.

For simulation Set II of the Risk Area 12 analysis, the estimated mean β_c's for the FI, the IA and the FA programs are all reduced. This is not surprising since the variance of premiums is automatically zero for each farm with a fixed actuarially fair premium rate over the simulation period. With this simulation set, the fully individualized program still ranks first in terms of yield risk reducing magnitude, the area coverage and individual indemnity program comes next and the full area program ranks third, given 80% coverage option. This ranking order also holds for the full coverage option. Similar to Set I, the FA program is not preferred by some farmers, as unfavorable β_c's are observed. Unlike Set I, however, the FI and the IA program are yield risk reducing for all insured producers, indicating that some less variable premium schedule is preferred.

Table 3.3. Summary Statistics for β_c: Risk area 2

Set I	MEAN	VAR	MIN	MAX	SKEW	NORMAL
FI80	0.4422	0.0490	0.1315	0.9074	0.5235	0.9398
IA80	0.5483	0.0547	0.1555	0.9359	0.0742	0.9523
FA80	0.5701	0.2348	< 0	> 1	0.8049	0.8653
FI100	0.5369	0.0340	0.1212	0.9217	0.3071	0.9539
IA100	0.6298	0.0480	0.1265	0.9679	-0.3600	0.9444
FA100	0.8827	0.3015	< 0	> 1	-0.7857	0.8601
Set II						
FI80	0.1719	0.0030	0.0340	0.2661	-0.5059	0.9565
IA80	0.2138	0.0049	0.0584	0.3555	-0.3235	0.9614
FA80	0.2654	0.0612	< 0	> 1	3.6025	0.6289
FI100	0.2515	0.0026	0.0734	0.3399	-0.8522	0.9511
IA100	0.2897	0.0051	0.0968	0.4330	-0.4752	0.9657
FA100	0.3466	0.1243	< 0	> 1	0.4215	0.9304
Set III						
FI80	0.2308	0.0038	0.0689	0.3167	-0.7698	0.9273
IA80	0.2363	0.0111	0.0000	0.4066	-0.4813	0.9495
FA80	0.3072	0.0806	< 0	> 1	1.6167	0.8244
FI100	0.3220	0.0018	0.2186	0.3882	-0.4660	0.9601
IA100	0.3187	0.0096	0.0000	0.4839	-0.9662	0.9490
FA100	0.3997	0.0707	< 0	> 1	0.4091	0.8793

With the trend removed (Set III), the β_c pattern changes slightly. Given 80% coverage level, it is found that the area coverage and individual indemnity (IA) program is the most efficient choice for insured farmers. It generates the lowest mean β_c value among those three alternatives. The FI program is the next best choice and the FA program is still least preferred. With the detrended data, the IA program remains a most favorable program even if the coverage level is switched to the full coverage option. This changed ranking order with the detrended procedure suggests that yield variability plays an important role in estimating β_c values. This is because detrending has a significant impact on the calculation of expected losses (Skees and Reed, 1986). Whether some unbiased estimates for the true yield variability can be developed or not will directly affect the farmers' preference choice for relevant insurance programs.

Looking at Table 3.3 and Table 3.4, it is found that the risk area 2 and the risk area 6 have similar β_c patterns with some minor variations compared to risk area 12. Within risk area 2, the preference ranking order remains the FI, the IA and the FA for simulation Set I and Set II with both coverage options. Similar to the risk area 12, the IA program may also become the most favorable program in the area, this occurs when farm yields are detrended and the 100 percent coverage level is chosen (Set III). In this risk area both the individual coverage / individual indemnity and the area coverage / individual indemnity programs are found yield risk reducing. The full area insurance program is basically yield risk reducing on average, but to some producers it is yield risk increasing. This is verified by negative β_c values and also by some high β_c values.

Risk area 6 has the same β_c pattern as risk area 12 in terms of simulation Set II and Set III. In the case of Set I, however, the full area program is found yield risk increasing on average with the full coverage option. The average β_c value is 1.011. This negative result regarding the program's yield risk reduction function may simply be due to very large estimated β_c values occurring for some farms (note the average variance is as large as 0.58). The very small sample size may be another reason.

Although the fully individualized crop insurance (FI) is generally preferred over the IA and the FA programs, the effectiveness of these programs varies depending on the risk associated with production conditions in a risk area. It also varies with different coverage levels. More importantly, the estimated β_c values associated with the IA program design are very close to those associated with the FI program in all cases. This indicates that the claim that the FI is superior to the IA cannot be made too strongly. This is one significant implication which could be derived from this section.

Table 3.4. Summary Statistics for β_c: Risk area 6

Set I	MEAN	VAR	MIN	MAX	SKEW	NORMAL
FI80	0.4585	0.0579	0.1975	0.9769	0.9774	0.8797
IA80	0.4667	0.0595	0.1428	0.9366	0.5405	0.9247
FA80	0.7219	0.2572	< 0	> 1	-0.2216	0.8694
FI100	0.6973	0.0421	0.3421	0.9735	-0.3555	0.9229
IA100	0.7231	0.0459	0.2500	1.0000	-0.5917	0.9452
FA100	1.0112	0.5799	< 0	> 1	-1.0754	0.6028
Set II						
FI80	0.1886	0.0010	0.1278	0.2643	0.4410	0.9744
IA80	0.1938	0.0038	0.0931	0.2867	-0.1951	0.9409
FA80	0.4217	0.4569	0.1258	> 1	3.1066	0.4861
FI100	0.2836	0.0013	0.2243	0.3846	1.2883	0.8949
IA100	0.2912	0.0030	0.1996	0.3759	-0.2722	0.9296
FA100	0.3607	0.1711	< 0	> 1	1.6561	0.7458
Set III						
FI80	0.2451	0.0024	0.1442	0.3257	-0.3640	0.9577
IA80	0.2275	0.0145	0.0000	0.3937	-0.3379	0.9427
FA80	0.3625	0.2193	< 0	> 1	1.5707	0.7218
FI100	0.3479	0.0012	0.2928	0.4231	0.4100	0.9588
IA100	0.3156	0.0149	0.0000	0.4586	-1.3496	0.8664
FA100	0.4159	0.1738	< 0	> 1	1.3093	0.7898

The empirical results from Table 3.2, Table 3.3 and Table 3.4 provide some general insights into the three simulated programs. The non-zero skewness statistics and non-normality result suggested by the Shapiro-Wilks statistic imply that those programs are not equally preferred by the insured producers. It is therefore, useful to further evaluate farmers' preferences to these programs by analyzing different interest groups among all the insured. This is done by grouping all the calculated β_c values into different categories, through the use of β_c ranges. This leads to some more detailed empirical insights about our previous general results.

Table 3.5 shows the β_c distributions for the three risk areas in the case of simulation Set I. Given 80% coverage option, 80% of insured farmers in risk area 2 find that they will have the β_c's which are lower than 0.6 under the FI program. About 60% of these farms fall into this β_c range for the IA program. For the FA program, 28% insured farms find the program yield risk increasing. This figure increases to 38% for risk area 6. Risk area 6 has similar β_c distributions for the FI and the IA program. For risk area 12, the full area program will be yield risk increasing for only 4% of insured farms. Eighty-five percent of the β_c's fall into the range of (0 - 0.4) with both the FI and IA programs, indicating that most farms will find the FI and IA very yield risk reducing.

Table 3.5. β_c Distributions (%): Set I

AREA 2	FI80	IA80	FA80	FI100	IA100	FA100
< 0	0	0	7	0	0	14
0-0.2	14	7	3	5	7	0
0.2-0.4	34	24	42	26	12	0
0.4-0.6	32	31	17	34	26	7
0.6-0.8	10	21	7	21	26	17
0.8-1.0	10	17	3	14	29	17
> 1.0	0	0	21	0	0	45
AREA 6						
< 0	0	0	14	0	0	27
0-0.2	14	24	0	9	9	0
0.2-0.4	43	33	0	9	9	0
0.4-0.6	24	9	10	19	19	0
0.6-0.8	5	24	52	27	23	0
0.8-1.0	14	10	0	36	36	0
> 1.0	0	0	24	0	5	73
AREA 12						
< 0	0	0	4	0	0	5
0-0.2	25	25	27	10	10	0
0.2-0.4	60	60	64	52	43	17
0.4-0.6	11	11	4	24	29	44
0.6-0.8	2	2	0	8	12	20
0.8-1.0	2	2	0	3	7	5
> 1.0	0	0	0	2	0	9

With 100% coverage level, the β_c distributions in the risk area 2 and area 6 under the FI and the IA programs are similar to that with 80% coverage. However, more farms will find the FA program even less preferred, since the percentage of β_c below 0.4 is increased. The farms located in the risk area 12 have similar prospects for these programs.

Table 3.6. β_c Distributions (%): Set II

AREA 2	FI80	IA80	FA80	FI100	IA100	FA100
< 0	0	0	9	0	0	18
0-0.2	73	46	66	20	14	14
0.2-0.4	27	54	18	80	82	24
0.4-0.6	0	0	5	0	4	27
0.6-0.8	0	0	0	0	0	13
0.8-1.0	0	0	0	0	0	2
> 1.0	0	0	2	0	0	2
AREA 6						
< 0	0	0	8	0	0	12
0-0.2	72	52	68	8	12	4
0.2-0.4	28	48	8	92	88	65
0.4-0.6	0	0	0	0	0	4
0.6-0.8	0	0	4	0	0	4
0.8-1.0	0	0	4	0	0	4
> 1.0	0	0	8	0	0	8
AREA 12						
< 0	0	0	4	0	0	5
0-0.2	68	56	44	31	29	19
0.2-0.4	32	44	44	68	67	57
0.4-0.6	0	0	4	1	3	13
0.6-0.8	0	0	4	0	0	3
0.8-1.0	0	0	0	0	0	1
> 1.0	0	0	0	0	0	3

The simulation results with Set II and Set III are presented in Table 3.6 and Table 3.7, respectively. Table 3.6 reflects the β_c distributions for the three risk areas, using a fixed actuarially fair premium for each farm without detrending yield data. Set III estimates (Table 3.7) are derived by the same methodology used for Set II but with yield data detrended. It can be seen that the calculated β_c's are more centred around some smaller mean values than in the Set I, i.e., the β_c's are less viable. Without removing the trends, 50-70% β_c values fall into the 0-0.2 interval for all risk areas with the 80% coverage level under the FI and the IA program, and no calculated β_c's exceed 0.4. Under full coverage, this is also the case for these two programs. As far as the FA program is concerned, it is seen that the β_c's are distributed more evenly than those of Set I. This suggests that a kind of rate making methodology where a more stable actuarially fair premium schedule could be developed would be more appealing from the producers' perspective.

The implications of detrending on the alternative programs can be clarified from Table 3.7. In this case, the general distribution pattern for the β_c indices is similar to that of simulation Set II. It should be noted that the percentage of β_c's that fall between 0 and 0.2 decreases dramatically with 80% coverage for the FI and IA programs, which is why some higher overall β_c values are obtained in the case of Set III. With this methodology, some farms may be over-detrended while others may be under-detrended, thus leading to some biased estimation of true yield variability.

Table 3.7. β_c Distributions (%): Set III

AREA 2	FI80	IA80	FA80	FI100	IA100	FA100
< 0	0	0	9	0	0	9
0-0.2	28	34	15	4	15	2
0.2-0.4	72	59	54	96	65	61
0.4-0.6	0	7	13	0	20	11
0.6-0.8	0	0	4	0	0	9
0.8-1.0	0	0	2	0	0	4
> 1.0	0	0	2	0	0	4
AREA 6						
< 0	0	0	12	0	0	12
0-0.2	27	43	27	8	20	0
0.2-0.4	73	58	42	88	50	62
0.4-0.6	0	0	0	4	31	8
0.6-0.8	0	0	4	0	0	4
0.8-1.0	0	0	2	0	0	4
> 1.0	0	0	2	0	0	12
AREA 12						
< 0	0	0	5	0	0	5
0-0.2	12	20	13	4	14	5
0.2-0.4	88	76	51	89	70	16
0.4-0.6	0	4	22	7	16	32
0.6-0.8	0	0	3	0	0	5
0.8-1.0	0	0	2	0	0	3
> 1.0	0	0	4	0	0	5

Table 3.8 gives the calculated Miranda mean β_m's and β_f's for all sample farms for 80% and 100% coverage. In Table 3.8, ρ_{af} is the Pearson correlation coefficient between the individual farm yields and the area yields. The estimated Pearson cor-

relation coefficients for the risk area 2, the risk area 6 and risk area 12 are 0.60269, 0.5556 and 0.75577, respectively. The corresponding average β_f's for each risk area are estimated as 0.78308, 0.96160 and 0,92520. These results suggest that the calculated ρ_{af}'s and β_f's may not always be consistent. This occurs simply because the linearity assumption between farm yields and area yields may be questionable.

Table 3.8. Miranda β_f and β_m

	MEAN	VAR	MIN	MAX	SKEW	NORMAL
AREA 2						
80%						
ρ_{af}	0.60269	0.03208	0.08579	0.83340	-0.82952	0.92630
β_f	0.78308	0.09338	0.09548	1.37349	-0.12651	0.98115
β_m	0.18903	0.00085	0.13840	0.24081	-0.04023	0.93889
100%						
ρ_{af}	0.60269	0.03208	0.08579	0.83340	-0.82952	0.92630
β_f	0.78308	0.09338	0.09548	1.37349	-0.12651	0.98115
β_m	0.34926	0.00181	0.27125	0.50199	1.14783	0.93817
AREA 6						
80%						
ρ_{af}	0.55560	0.03648	0.08078	0.80560	-1.07470	0.89587
β_f	0.96160	0.14872	0.10113	1.85527	-0.36415	0.93079
β_m	0.24736	0.00277	0.00309	0.27063	-4.69697	0.34975
100%						
ρ_{af}	0.5556	0.03648	0.08078	0.80560	-1.07470	0.89587
β_f	0.96160	0.14872	0.10113	1.85527	-0.36415	0.93079
β_m	0.39473	0.00102	0.29677	0.44472	-1.07188	0.93335
AREA 12						
80%						
ρ_{af}	0.75577	0.02242	0.01947	0.94645	-1.79849	0.84177
β_f	0.92520	0.06658	0.01705	1.53329	-0.51337	0.97252
β_m	0.26299	0.01660	0.00000	0.46170	-0.58166	0.79969
100%						
ρ_{af}	0.75577	0.02242	0.01947	0.94645	-1.79849	0.84177
β_f	0.92520	0.06658	0.01705	1.53329	-0.51337	0.97252
β_m	0.42754	0.01494	0.15689	0.79402	-0.27753	0.94756

Based on the Miranda β_f values, it can be seen that farm yields are highly sensitive to the area yields within risk area 6 and risk area 12. The estimated critical mean betas (β_m's), are much below the observed betas (β_f's), indicating that the full area crop insurance should work well within these three risk areas, if there is no question about the equation (3.11). This is certainly a strong assumption. Other statistics in the table suggest that ρ_{af}'s, β_f's, and β_m's vary dramatically from farm to farm.

The unit root tests for farm yields and for area yields are conducted using the ADF equation defined in equation (3.14). The results are reported in Table 3.9. T_f denotes the calculated Dickey-Fuller T-statistic for the tested farm yields, T_a is the calculated Dickey-Fuller T-statistic for the tested area yields, and T_c denotes the Dickey-Fuller cointegration T-statistic for farm-area yields relation. The relevant critical statistics are presented in the last column in the table.

Table 3.9. Unit root test and Cointegration test results
Selected farms (75 farms)

Stat	NO	Mean	Std Dev	Minimum	Maximum	10% Crit.
T_f	75	-2.86303	0.98648	-5.37460	-0.57540	-3.24
T_a	75	-3.09322	0.88587	-5.19260	-1.36530	-3.24
T_c	75	-3.05180	0.89426	-4.95410	-1.53360	-3.50

Table 3.9 suggests that the hypotheses of unit root for both farm yields and area yields can not be rejected for this particular group of farms. The estimated mean Dickey -Fuller statistics are greater than the critical values. Actually, 53 out of 75 farms passed the unit root test for farm yields and 60% farms passed the unit root test for the corresponding area yields. This means the cointegration test will be meaningful for most farms.

To determine whether a linear combination of the farm yields and the area yields is stationary, cointegrating regressions are tested using equation (3.15). The average T_c value is -3.05, which is greater than the 10% critical of -3.50. This implies that the hypothesis of no cointegration between farm yields and area yields cannot be rejected on average. That is, there is no reason to believe that a long-term stable relationship between farm yields and area yields exists, given our selected sample.[20] The bottom line is that the basic premise of positive correlation between farm and area average

[20]For 23 out of 75 (31%) farms it was observed that some sort of long-term stable relationship exists from our tests.

yields is not acceptable by the test data.

3.3.2 Yield Risk Reductions With the GSD Approach

The Generalized Stochastic Dominance results are presented in Table 3.10. GSD criteria are used to choose efficient strategies from among a set of alternatives by comparing the distribution of net yields for each.

Table 3.10. GSD results: selected farms

	r(1)			r(2)			r(3)			r(4)		
	1	0	?	1	0	?	1	0	?	1	0	?
FI80	49	49	52	83	65	2	80	64	6	76	41	33
	(33)	(33)	(34)	(55)	(43)	(2)	(53)	(43)	(4)	(51)	(27)	(22)
IA80	46	51	53	78	70	2	79	68	3	62	51	37
	(31)	(34)	(35)	(52)	(47)	(1)	(53)	(45)	(2)	(41)	(34)	(25)
FA80	46	43	61	61	89	0	64	87	1	33	80	37
	(31)	(28)	(41)	(41)	(59)	(0)	(43)	(58)	(.6)	(22)	(53)	(25)
No.Eff												
FI80			36		31		33		39			
			(32)		(42)		(44)		(48)			
IA80			35		25		24		28			
			(31)		(33)		(32)		(35)			
FA80			41		19		18		14			
			(37)		(25)		(24)		(17)			
FI100	28	57	63	73	74	1	73	73	2	62	37	49
	(19)	(39)	(42)	(49)	(50)	(1)	(49)	(49)	(2)	(42)	(25)	(33)
IA100	44	42	62	76	72	0	77	63	8	71	35	42
	(29)	(28)	(43)	(51)	(49)	(0)	(52)	(43)	(5)	(48)	(24)	(28)
FA100	52	24	72	69	79	0	63	77	8	18	78	52
	(35)	(16)	(49)	(47)	(53)	(0)	(43)	(52)	(5)	(12)	(53)	(35)
No.Eff												
FI100		20			27			27			42	
		(19)			(36)			(33)			(37)	
IA100		35			21			28			48	
		(33)			(28)			(35)			(42)	
FA100		51			26			26			24	
		(48)			(35)			(32)			(21)	

In performing the GSD analysis, the risk aversion coefficients r(y) are divided into four groups with r(1)=-5.0-0, r(2)=-0.00001-0.00001, r(3)=0-0.0001, and r(4)=0.0001-5.0, respectively. These intervals are selected based upon some simulation process and the ASSESS procedures developed by Cochran et al. (1990), and are used to approximate risk attitudes of risk preferring, risk neutral, risk averse, and strong risk averse, respectively. There are two parts to Table 3.10. The numbers in the first part of the table indicate the number of times a strategy dominates any other strategy in each risk preference interval. The "0" indicates the strategy is dominated by other alternatives, the "1" indicates the opposite case, and the question mark "?" indicates that no dominance can be made among those strategies. The second part of Table 3.10 shows the number of times for which the program is the efficient choice (No. Eff) for the insured farms within the given risk averse interval. A strategy is an efficient one only if it is not dominated by any other strategies and is restricted to those distributions on the rows with only "1's" or "?'s". The numbers in parentheses are corresponding percentages.

As shown in Table 3.10, based on the 80% coverage option, fully individualized insurance is dominated the least number of times in all risk-averse intervals. So it is for risk-neutral interval. The second least dominated alternative strategy is the area coverage and individual indemnity (IA) program. The full area program is dominated the least number of times for the risk-preference interval. These results are highly consistent with those derived from previous β_c approach. Moreover, it can be seen that, although the FI is the most efficient strategy with 80% coverage, it does not overwhelmingly dominate the IA program, the dominance can only be made marginally.

With the full coverage option, however, the FA crop insurance becomes the least dominated strategy for risk-averse intervals, while the IA program comes next. The FI program, on the other hand, is still most preferred by farmers for the risk-neutral interval. As with 80% coverage, the dominance between the FI and the IA is also marginal for the risk-averse interval. The inconsistency regarding the full coverage option between the GSD and the previous index approach may be due to a sample problem.[21]

[21] With the GSD analysis, only those farms which have grown red spring wheat for at least thirty years are chosen as our sample. This likely covers only highly risk-averse producers. This biased sample may directly affect the GSD result.

It is interesting to note that the discriminating power with the GSD for risk-averse farmers declines as coverage level increases. This result strongly supports our previous finding with the index β_c approach that producers will be less sensitive to those alternative programs as coverage approaches the full coverage level. This clearly suggests that offering higher coverage level is indeed an effective option to stimulate higher participation rate, as long as the program is actuarially sound.

Table 3.11. GSD Results: 80% and 100% Coverage Levels
Number of Efficient sets (74 farms)

r(1)			r(2)			r(3)			SSD		
p1	p2	p3	p1	p2	p3	p1	p2	p3	p1	p2	p3
37	34	44	26	14	2	27	17	11	34	25	15
(22)	(20)	(26)	(35)	(19)	(3)	(21)	(13)	(9)	(17)	(12)	(7)
g1	g2	g3	g1	g2	g3	g1	g2	g3	g1	g2	g3
9	16	32	14	9	9	21	30	22	52	50	27
(5)	(9)	(19)	(19)	(12)	(12)	(16)	(23)	(17)	(26)	(25)	(13)

The final GSD analysis directly compares "six" alternative programs, i.e., each program will be treated as two sub-programs with different coverage levels. For example, the FI program with 80 percent coverage (FI80) and with 100 percent coverage (FI100) are treated as two alternatives in this case. This comparison is less meaningful, but it still has some policy implications. The results are shown in Table 3.11. The second degree stochastic dominance (SSD) results[22]are also presented.

In Table 3.11, the risk aversion coefficient intervals are redefined as: r(1)=-5.0-0, standing for risk-preferring category, r(2)=0-0.0001, a risk-averse interval, and r(3)=0.0001-1.0, a strongly risk-averse category. With risk-preference category, it is identified that the full area crop insurance program with eighty percent coverage level (FA80) is the least dominated strategy. The percentage is up to 26%. The second

[22] Actually, these are only quasi-SSD results. With quasi-SSD, the upper bound is set such that the relative risk aversion coefficient will never exceed 100. However, in most cases no significant differences should be encountered between the true and the quasi-second degree stochastic dominance. p1=FI80, p2=IA80, p3=FA80, g1=FI100, g2=IA100, and g3=FA100.

least dominated strategy for this category is the individual program (FI80). With risk-averse interval r(2) , the individual insurance with 80% coverage (FI80) is found most efficient. The IA program with the full coverage (IA 100), however, is found most favorable for the strongly risk-averse insured. The fully individualized program with the full coverage option (FI100) is most preferred using SSD criteria. SSD criteria generate a larger feasible efficient set than GSD criteria.

3.3.3 Program Outlays

The average farm indemnities, measured as bushels per acre, for each risk area under each program are presented in Table 3.12.

As expected, the alternative that would result in the smallest indemnity per acre, for all the three areas, is the full area insurance program (FA). The area coverage and individual indemnity approach (IA) is the most expensive alternative, although it is only slightly more expensive than the FI program. These results are applicable for both coverage levels. The highest average indemnity per acre for the IA program is simply due to higher viability in its indemnity schedules (see St.D's of the table). This result may likely be explained by the fact that ten-year moving average yields are calculated for both the FI and the IA programs to determine their indemnity schedules.[23] This implies that the FI program would very likely be more expensive than the IA program if a longer period had been used in our simulation for the IA program. Normally, crop yields in a short period may change dramatically. They would be much more stable over a longer period.

Generally, either the FI program or the IA program would result in much higher farm level indemnities than the FA alternative.

[23]The MCIC uses a 10-year moving average yield methodology to the FI program and a 25-year or 15-year moving average yield approach to the IA program. This suggests that the viability in indemnities for the IA program in our simulation case is probably overestimated, given a 10-year moving average approach.

Table 3.12. Farm Indemnity Statistics
(Bushel/Acre)

Area 2	Mean	St.D	Min	Max
FI				
IDM80	1.3171	0.7727	0.0944	3.1859
IDM100	3.0286	1.1537	0.6627	5.6192
IA				
IDM80	2.2116	1.5867	0.0000	5.5892
IDM100	4.6542	2.4544	0.4532	9.8484
FA				
IDM80	0.5648	0.2341	0.0345	1.0233
IDM100	1.8217	0.4414	0.7613	2.6237
Area 6				
FI				
IDM80	1.5959	0.7527	0.3730	3.3678
IDM100	3.6366	1.3250	1.8642	7.7169
IA				
IDM80	1.7641	0.9828	0.2365	3.6564
IDM100	3.8338	1.7601	1.2206	7.0727
FA				
IDM80	0.5616	0.1173	0.0781	0.7010
IDM100	1.9060	0.2322	1.2743	2.3395
Area 12				
FI				
IDM80	1.1169	0.7652	0.0000	3.8687
IDM100	2.2134	1.1305	0.0000	5.9530
IA				
IDM80	1.3045	1.0620	0.0000	6.9855
IDM100	2.5863	1.6998	0.0000	10.6022
FA				
IDM80	0.5513	0.3798	0.0000	1.2001
IDM100	1.2819	0.5746	0.0000	2.2887

3.4 Summary and Conclusions

Three crop insurance programs have been evaluated in terms of the effectiveness of yield risk reductions for more than 450 Manitoba farms. The examination is first conducted with the proposed index method with which the relative yield risk reduction magnitude is calculated and compared for each farm under each alternative program. The evaluation is made with 80% and 100% coverage levels, and other important elements such as different premium setting methodologies and a detrending process are also incorporated in the simulation process. The generlized stochastic dominance methodology is also used to provide an alternative analytical framework based on a producer's relative preferences among those alternatives, by comparing the net yield distributions generated by each program for each farm.

The results suggest that, given an actuarially sound basis, the fully individualized crop insurance (FI) is the most favorable choice for risk-averse producers. The area coverage and individual indemnity (IA) program is generally the second best option. The area coverage and area indemnity, or the full area crop insurance plan (FA), is least preferred by risk-averse farmers. This overall ranking holds for both coverage levels. The index approach and GSD results also clearly indicate producers will be less sensitive to the alternative program designs if the coverage level is increased. This suggests that offering higher coverage level, without negatively affecting the actuarial basis, will be an effective means to induce higher participation.

Although a fully individualized program is generally preferred over the area coverage and individual indemnity program (IA), the dominance is only marginal. This is verified by both methodologies. In some cases, the latter may be more attractive than the former. It is found that the FI program and the IA program are yield risk reducing for almost all the farms, except for a few who may find these two programs to be yield risk neutral.

The study has identified that the full area program is also yield risk reducing for most farms. The yield risk increasing cases, however, can be identified for some insured producers with this program. The reason for this is that the individual farm yields for some producers are not highly positively correlated with the relevant area yields and a long run stable relationship between farm yields and area yields may not hold as suggested by the Dickey-Fuller cointegration test. From an insurer's perspective, the FA program is the cheapest program design and the IA program the most expensive, if only the average indemnities per acre is accounted for. This finding is consistent with *a priori* expectations.

Chapter 4

Pure Premium Ratemaking And Actuarial Structures For An All-Risk Crop Insurance Program

4.1 Premium Rates: Theoretical Foundation

4.1.1 General Principles

Premium ratemaking is the core actuarial dimension of any insurance program. If a risk (e.g., crop yield loss) is described by a random variable, a premium setting principle or formula is then a rule that assigns a real number (the premium) to the given risk.

The fundamental principle used to determine gross premiums for general insurance was observed by Adam Smith (1776) in Wealth of Nations where he stated that the insurance "premium must be sufficient to compensate the common losses, to pay the expenses of management, and to afford such a profit as might have been drawn from an equal capital employed in any common trade". This shows that over 200 years ago Adam Smith had a deep insight into the essentials of insurance.

The observation by Adam Smith implies that an insurance premium consists of three components:

$$\rho_{gross} = L + A + R. \tag{4.1}$$

Where the first term L is expected insurance claim payment or pure premium as determined by actuarial principles. It is convenient to write $L = E\{x\} = \rho$, where x denotes actual loss, a random variable. The second term A represents the administrative expenses of the insurance company. The third term R is usually referred to as the " risk premium ".[24] It represents the reward to the insurer for his service as risk-bearer. R is determined by market forces, i.e., the supply of and demand for insurance in a competitive market determine the equilibrium for this insurer's reward.

In practice, premium rates also need to include some adjustments to account for the cost of additional benefits (e.g., premium discount). This indicates that the right-hand side of above identity often contains the fourth term Ld (loads). This term could be complex since all the major elements of insurance might be involved. Fortunately, Ld is generally very small.

All-risk crop insurance is typically non-competitive and public intervention is generally involved. This is because the necessary and sufficient conditions for a competitive crop insurance market cannot be met (Rothschild and Stiglitz, 1976. Ahsan, Ali, and Kurian, 1982). The insurer is also assumed to be a zero profit maximizer with a risk-neutral attitude. This suggests that the third term R vanishes in Smith's equation for a government-subsidized crop insurance.

Crop insurance programs are often fully subsidized in terms of program administrative costs. It is observed that in practice, the administrative cost is often insignificant where the profit element is not included. The administrative cost of crop insurance can however, be substantial, depending on the program.

[24] The "risk premium" concept is formally defined by Pratt (1964) in which the risk premium R is measured as the *amount of money that makes the decision maker indifferent between the random variable (loss) x and the non-random amount (certainty equivalent) $E\{x\} - R$*. The traditional view in the insurance literature is that *uncertainty* is the lack of certainty, and *risk* is the uncertainty which exposes loss and could be predicted by a probability distribution (Knight, 1921). In the economics literature, however, risk is observable only if the decision maker's *utility* is affected by this probability distribution (Robison and Barry, 1987).

Pure premium ρ is determined by statistical and actuarial principles. Statistically, it is determined by the Law of Large Numbers. Actuarially, the principle of equivalence (PE) is its foundation (Borch, 1990). The latter states that the expected value of insurance claim payments or expected losses under a contract should be equal to the expected value of premiums paid. Defining x as the claim payments, a random variable, its probability density function is denoted by $f(x)$ and the actuarially fair pure premium, by the principle of equivalence, is then calculated as

$$\rho = E\{x\} = \int_0^\infty x f(x) dx. \tag{4.2}$$

Equation (4.2) constitutes the fundamental foundation of insurance theory and pure premium ratemaking methodology.

It should be noted that the PE principle is related to the more fundamental Bernoulli principle. In the early days of the calculus of probability, it was taken for granted that the fair price of a gamble was the mathematical expectation of the gain. If the probability of a gain is f(x), the fair price would be $E\{x\} = \int_0^\infty x f(x) dx$. Applied to insurance, this means that a fair premium for a risk described by the probability distribution $f(x)$ would simply be determined by equation (4.2).

The counter example given by Bernoulli (1738) has become known as the St. Petersburg Paradox. To demonstrate this, consider a game in which a coin is tossed until it shows heads. If the first head appears at the nth toss, a prize of 2^n is paid. It is clear that the expected gain in this gamble would be an infinity, since $E\{x\} = \Sigma(1/2)^n 2^n = \infty$. Bernoulli argued that no rational person would be willing to pay an arbitrarily large amount for the right to participate in this gamble. This Paradox cannot be solved without the expected utility hypothesis, where the physical gain x is replaced by some concave utility function of the gain, $u(x)$. The Bernoulli principle is then stated as:

$$E\{u(x)\} = \int_0^\infty u(x)f(x)dx. \tag{4.3}$$

With $u'(x) > 0$ and $u''(x) < 0$. From Jensen's inequality, it follows that

$$E\{u(x)\} < E\{x\}. \tag{4.4}$$

It indicates that a person will pay less than the mathematical expectation for the right to play a gamble. The implication is that an insurer will demand a "risk premium" besides the expected loss (actuarially fair premium) to cover a risk if he is risk-averse.

The actuarially fair premium or the PE equation is the simplest premium calculation principle. It is widely applied in insurance industry. It is a purely actuarial principle[25] and has nothing to do with economic aspects. However, insurance is also a typical economic activity. In his Principles Marshall (1890) discussed insurance premiums as the price one has to pay to get rid of the "evils of uncertainty". This indicates that it is possible to derive insurance premiums through economics principles. The problem is solved by what is called economics of insurance, where premiums are calculated according to some notion of utility. The so-called zero utility principle provides a general solution.

The zero utility premium calculation principle is based upon the economic definition of risk and it states that the premium ρ for a risk x should be calculated such that the expected utility is (at least) equal to the zero utility, i.e.,

$$E\{u(\rho - x)\} = u(0). \tag{4.5}$$

This principle yields a technical minimum premium in the sense that the risk x should not be accepted at a premium below ρ. Quite often also the initial wealth W is taken

[25]Improvements associated with this principle could be made by including some direct loading mechanisms in the formula. This leads to several popular actuarial principles, e.g., the variance principle, flat loading principle, etc., as summarized by Buhlmann (1970).

into account by writing

$$\begin{aligned} u(W) &= E\{u(W+\rho-x)\} \\ &= \int u(W+\rho-x)f(x)dx. \end{aligned} \tag{4.6}$$

Equation (4.6) is the utility theory premium calculation principle. It is noted that in the special case of $u(x) = x$ the zero utility premium equals the actuarially sound premium.

A more interesting case, however, is the exponential utility

$$u(x) = \frac{1}{\lambda}(1 - e^{-\lambda x}), \tag{4.7}$$

which leads to a premium ρ of

$$\rho = \frac{1}{\lambda} ln \int e^{\lambda x} f(x)dx. \tag{4.8}$$

It is interesting to note that the parameter $\lambda = -u''(x)/u'(x)$, is the Pratt risk aversion coefficient (Pratt, 1964), the greater λ, the greater ρ. For a risk-neutral insurer, $\lambda = 0$, $\rho = E\{x\}$, i.e., we are back to the actuarially fair premium.[26] In the case of a normally distributed risk x the premium is

$$\rho = E\{x\} + \frac{\lambda}{2} Var\{x\}, \tag{4.9}$$

which is identical with the variance principle used by actuaries.

4.1.2 Pure Ratemaking in All-Risk Crop Insurance

With all-risk crop insurance, the expected insurance payments are determined by its indemnity schedule as defined by

$$E\{x\} = E\{Max(0, C - y)\}. \tag{4.10}$$

[26] It is for this reason that the actuarial premium ratemaking is emphasized by this study since risk-neutral behavior is assumed for the Crown crop insurance corporation. However, impacts of risk attitude on premium ratemaking can be easily simulated within this utility theory ratemaking framework.

Where C is the coverage (the insured yield) which is a percentage of the long term average yield, y is the actual yield of insured crop, a random variable. The loss variable is simply described as $x = C - y$. The PE equation is immediately translated into

$$\rho = E\{C - y\} = \int_0^C (C - y) f(y) dy, \quad \forall \;\; y_i < C. \tag{4.11}$$

Where $f(y)$ is the probability density function of crop yields. Clearly pure premium rate ρ is a function of the parameters of $f(y)$, the probability density function of crop yields, and the coverage yield determined by the insurance company. As seen from above theoretical formula, pure premium rates can be calculated as long as the yield probability density function $f(y)$ is assumed and estimated.

A few studies in the agricultural economics literature can be found with reference to premium ratemaking. The classical work is done by Botts and Boles (1958) in which a rate formula with a normal yield distribution is derived. Yeh and Sun (1980) investigate the possibility of using the Pearson Family distributions to approximate actual wheat crop yields in calculating Manitoba all-risk crop insurance pure premium rates. Nelson (1990) also calculates pure premium rates assuming the beta (Pearson Type I) yield distributions by using seven Iowa county average corn yield data. Both researches conclude that classical normality assumption about crop yield losses is questionable and leads to some biased estimation of pure premiums. However, no formal pure premium ratemaking formula based on their yield distributions can be found in both papers.[27] Skees and Reed (1986) proposed ratemaking issues for farm-level crop insurance using trend-adjusted yield data. They found that detrending has important implications for premium ratemaking in a sense that unadjusted (for

[27] A theoretically consistent rate-setting formula for a given yield distribution can be obtained by substituting the probability density function into equation (4.11) and getting the integral over the range of zero and the coverage yield C. If yield variability and mean yield are estimated based on one distribution (e.g., beta) but the rates are calculated from another formula (say, Botts & Boles formula), biased results will be obtained.

trend) yields reduce the expected insurance payments, thus resulting in some biased estimates.

4.1.3 Premium Ratemaking: the MCIC Methodology

The MCIC's premium ratemaking methodology has experienced two stages in its development. In the first stage, a theoretical rate based on normal curve theory (the Botts and Boles formula) is calculated for most programs. This was not changed until 1985. Since 1985, an actual loss cost formula (ALC) has been developed. With this formula, actual annual losses are calculated as a percentage of coverage. The premium charged is an average of annual losses over the last 25 years. The ALC formula is expressed as

$$\rho = \frac{\sum_{i=1}^{25}(C_i - Y_i)/C_i}{25}. \tag{4.12}$$

Where Y_i is the annual average area yield and C_i is the annual average area coverage. This formula is not workable if the length of the data series is less than 25 years or actual loss records are not tracked. Therefore, in practice MCIC uses a hybrid of the theoretical loss cost as determined by the normal rate formula and the ALC formula. That is, the actual rate is determined by actual annual cost, plus a theoretical base rate. This theoretical base rate is an average rate over pre-1985 as calculated by the normal curve theory. The weighting on the base rate is reduced by 4 percent for every year of actual loss experience added. Letting $k = 1$ for 1985, $k = 2$ for 1986, ..., and k=25 for 2010, then it follows that

$$\rho = \frac{1}{k}\sum_{i=1}^{k}\frac{(C_i - Y_i)}{C_i} + \textit{Normal Base Rate} * (1 - k * 0.04). \tag{4.13}$$

It is clear that for the year 2010 and each year thereafter, the simple actual loss cost formula (4.12) will apply completely.

4.2 Rate Formulas for Normal and Beta yield Distributions

The Botts and Boles formula can be directly derived by substituting the normal yield probability density function of $f(y)$ into equation (4.11). If letting $E(y) = \bar{Y} = \mu$, $Var(y) = \sigma^2$, representing the expected yield and the variance of yields, respectively, then the formula is

$$\rho_{nml} = A(C - \mu) + d\sigma. \tag{4.14}$$

Where $A = \frac{1}{\sqrt{2\pi}} \int_0^k e^{-\frac{1}{2}z^2} dz$, the proportion of total acres with yields $y_i < C$ in a particular year (i.e., the probability of collecting indemnity). $d = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}k^2}$, the ordinate at the insured yield C, and $k = (C - \mu)/\sigma$. This is the premium setting formula[28] which has been used by the U.S Federal Crop Insurance Cooperation for at least 36 years (Nelson, 1990). The theoretical pure premium rate formula based on the normality assumption may be rewritten as:

$$\rho_{nml} = \frac{\sigma}{\sqrt{2\pi}} (e^{-(\frac{C-\mu}{\sigma\sqrt{2}})^2} - e^{-(\frac{\mu}{\sigma\sqrt{2}})^2}) + \frac{C - \mu}{2} (erf(\frac{C - \mu}{\sigma\sqrt{2}}) + erf(\frac{\mu}{\sigma\sqrt{2}})), \tag{4.15}$$

where $erf(z) = \int_0^z (2/\sqrt{\pi}) e^{-t^2} dt$, is the error function. This version of the formula is derived using MATHEMATICA (Wolfram, 1988) and it is much easier to use since the error function is built in many statistical packages (e.g., SAS, SHAZAM, MATHEMATICA).

Expected loss can also be calculated from any empirical distribution (such as those that are generated when performing Monte Carlo simulations) if one is unwilling to

[28] In the FCIC formula, σ, the standard error of the yield, is simply set at 25% of the mean, i.e., $\sigma_y = 0.25E(y)$, the reason for assuming this linear relationship between the yield standard error and the mean yield is not documented. The formula may be reduced to a more specific formula for estimating expected loss if a polynomial function for integration of a normal distribution is utilized, i.e., $A = k(a_1T + a_2T^2 + a_3T^3)$, where a_1= 0.4361836, a_2 = -0.1201676, a_3 = 0.937298, and $T = 1/(1 + 0.33267k)$ (Abramowitz and Stegun, 1984).

assume any theoretical yield distribution (say, a normal distribution)[29]:

$$\rho_{emp} = \sum_{i=1}^{n} (C - y_i)/n, \quad \forall \;\; y_i < C. \tag{4.16}$$

Where y_i is the observed farm yield, n is the number of random yields developed, and $\sum(C - y_i)$ is the total crop losses of those yields below the coverage C in the particular year.

The assumption of normality of crop yields is often questionable. The normality will lead to some biased estimates of the expected loss since the normal yield distribution is rarely observed. Therefore, some flexible yield distribution is preferable in rate-making process. The beta distribution, as suggested by most authors (e.g., Day, 1965; Yeh and Sun, 1980; Nelson, 1990) is a good candidate for such flexible distribution. The beta distribution is defined by

$$f(y^*|\alpha, \beta) = \frac{y^{*\alpha-1}(1-y^*)^{\beta-1}}{B(\alpha, \beta)}, \;\; 0 < y^* < 1, \;\; \alpha, \beta > 0. \tag{4.17}$$

This distribution is flexible because it can take on any form of skewness and symmetry. Many distributions could be derived from the beta distribution as the distributional parameters α and β vary (Rothschild and Logothetis, 1986). For example, if the ratio α/β remains constant, but α and β increase, the variance decreases and the distribution tends to the standard normal distribution. The beta distribution becomes the distribution of $X_1^2/(X_1^2+X_2^2)$ where $X_i, i = 1, 2$, are independent random variables distributed as $\chi^2_{v_i}, i = 1, 2$, and where $v_1 = 2\alpha$ and $v_2 = 2\beta$. The arc-sine distribution is obtained from the beta distribution if $\alpha = \beta = 1/2$. When $\alpha = \beta = 1$ the beta distribution becomes the standard continuous uniform distribution. If $\beta = 1$ a power function distribution arises and y^{*-1} has a pareto distribution. A weibull-beta

[29]The impact of the normality or any other theoretical distribution assumption on the premium rate making can be assessed easily by comparing this empirical distribution rate making formula with the formula derived from the relevant theoretical distribution.

distribution is obtained if a random variable Z is such that for any constant k, Z^k has a standard beta distribution. In terms of the Pearson Family distributions, the beta distribution is covered by the Pearson Type I distribution (Kendall and Stuart, 1977).

The premium rate based on the beta yield distribution can be derived as

$$\rho^* = \frac{c^{*1+\alpha} H(1-\beta, \alpha, 1+\alpha, c^*)}{\alpha B(\alpha,\beta)} - \frac{c^{*1+\alpha} H(1-\beta, 1+\alpha, 2+\alpha, c^*)}{(1+\alpha)B(\alpha,\beta)}. \tag{4.18}$$

where H(.) is the hypergeometric function, a simple mathematical function,[30] and y^* and c^* are transformed variables defined as:

$y^* = y/Max\{y_1, ..., y_n\}$, $c^* = C/Max\{y_1, ..., y_n\}$. $B(\alpha,\beta)$ is the simple beta function.[31] The estimated mean yield and the yield variance can be obtained through the following transformations:

$$\begin{aligned} E(y*) &= \alpha/(\alpha+\beta), \;\; V(y*) = (\alpha\beta)/((\alpha+\beta)^2(\alpha+\beta+1)) \\ E(y) &= E(y*)Max\{y_1, ..., y_n\}, \;\; \sigma_y = \sigma_y^* Max\{y_1, ..., y_n\}, \end{aligned} \tag{4.19}$$

and the actual estimated pure premium rate can be calculated as

$$\rho_{beta} = \rho^* Max\{y_1, ..., y_n\}. \tag{4.20}$$

The Beta formula is also derived using the MATHEMATICA program and premium rates can be calculated conveniently with this program.

4.3 A Bayesian Approach to Ratemaking

4.3.1 The Rationale of the Bayesian Statistics

An excellent discussion on the general Bayesian statistics and econometrics can be found in Zellner (1971), and Judge et al. (1988, Ch. 4, Ch. 7). The fundamental

[30] It is defined as: $H(a,b,c,z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a} dt$.

[31] The beta function $B(\alpha,\beta)$ is defined by $\int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt$.

rationale of the approach may be illustrated in the following example. Suppose we are interested in estimating the parameters β in the model:

$$\mathbf{y} = \mathbf{f}(\mathbf{X}|\beta; \mathbf{e}). \tag{4.21}$$

In the context of classical econometrics, the true parameters β (unknown) are assumed to be objective and from a given (unknown) population. Because the true distribution about β is unknown, the distribution has to be assumed and the data on $(\mathbf{y}, \mathbf{X})$ are collected by sampling and then the sampling parameters $\hat{\beta}$ are estimated. Whether those estimated $\hat{\beta}$ of the true β are good or not, is simply judged by certain criteria or by some desirable properties such as the Best Linear Unbiased Estimator (BLUE), highest R^2, etc.. In the Bayesian approach, however, the population which possesses β is random while the sample is assumed to be objective and given. That is, before collecting data $(\mathbf{y}, \mathbf{X})$ and investigating β, one must have some subjective (his own) knowledge about β and this knowledge has to be utilized systematically. This prior distribution $g(\beta)$, reflecting the researcher's beliefs about the parameters in question before looking at the data, is combined with the sample information contained in the likelihood function $\ell(\beta|\mathbf{y}, \mathbf{X})$, via Bayes theorem, to produce the posterior distribution, $g(\beta|\mathbf{y}, \mathbf{X})$, the main output of a Bayesian approach:

$$g(\beta|\mathbf{y}, \mathbf{X}) \sim \ell(\beta|\mathbf{y}, \mathbf{X})g(\beta), \tag{4.22}$$

where $\ell(\beta|\mathbf{y}, \mathbf{X}) \sim$ certain theoretical distribution which could be any kind.

4.3.2 Applications to Rate Revisions

Since the first part of this century it has been recognized in the general insurance industry that the best insurance premiums will be obtained when the pure premium is set somewhere between the actual loss experience of the insured and the overall average loss for all insured. This can be justified since an insured individual or group of insured individuals will rarely provide enough information to accurately estimate their potential for future claims. To illustrate how the Bayesian statistics may be applied to premium rate-making, it is appropriate to introduce the so-called *credibility problem* in the insurance literature. Most casualty insurance is offered on a short-term basis (6-12 months), so that premiums can be revised regularly to reflect the changing cost of insurance. The practical problem faced by actuaries is this: given a volume of old experience, as reflected in the old premium and a volume of new experience, as reflected in the recent claims history of the insured, to what extent should these two experiences be reflected in the revised premium ρ? The traditional solution to this problem provided by the credibility theory is:

$$\rho = z(n) * \rho_{new} + [1 - z(n)] * \rho_{old}, \tag{4.23}$$

where ρ_{old} is the old premium per risk, n is the size of the collective generating the new data, $z(n)$ is the credibility factor, $0 < z(n) < 1$, and ρ_{new} is the new premium per risk as defined by I/n where I is the new claims experience. This equation states that a revised premium is a weighted average of old premium and new premium. The problem with this formula is determining the relative weights or credibility factor to be placed on the specific and overall loss experiences. It will be shown that the Bayesian approach is the best way to solve this problem. Bailey (1950) has shown that the credibility factor $z(n)$ could be obtained by the Bayesian approach.

In Bayesian terminology, old or historical premiums may be thought of as emanating from a *prior distribution.* The experience of a particular collective (e.g., an area loss experience in the crop insurance case) in the most recent time period is the new data *(sample)*, and the new revised premium will be some characteristics of the *posterior distribution* resulting from combining the prior and the sample information via the Bayesian theorem. Such an explicitly Bayesian formulation has been presented by Bailey (1950). Klugman (1992) gives a more detailed discussion under the Hierarchical Bayesian Approach.

With the Bayesian approach, premium rates are not entirely determined by the sample data, the researcher's knowledge would play some crucial role and it has to be incorporated into the rate-setting procedure systematically. For example, suppose we are observing the number of claims (x) (frequency claim rate) from one randomly selected insured from the collection of all insureds. Assume that the number of claims in one year has a Poisson distribution with parameter ρ and that the numbers of claims in different years are independent. If we have t year observations, the likelihood function[32]is:

$$f(x|\rho) = e^{-t\rho}\rho^{\Sigma x_i}/\Pi x_i! \tag{4.24}$$

Further, suppose that we have some prior knowledge about ρ before investigating the data, say, we know that the parameter ρ is distributed in the form of the gamma distribution with the parameter α and β:

$$g(\rho) = \frac{\rho^{\alpha-1}e^{-\rho/\beta}}{\beta^\alpha\Gamma(\alpha)}. \tag{4.25}$$

The posterior distribution is then:

$$g(\rho|x) \sim \frac{e^{-t\rho}\rho^{\Sigma x_i}}{\Pi x_i!}\frac{\rho^{\alpha-1}e^{-\rho/\beta}}{\beta^\alpha\Gamma(\alpha)} \sim \rho^{\alpha+\Sigma x_i-1}e^{-(t+1/\beta)\rho}. \tag{4.26}$$

The denominator of above equation does not depend on ρ and so must be the constant that makes the equation a density function, that is, integrate to one. Clearly this posterior distribution is also a gamma distribution with parameters $\alpha + \Sigma x_i$ and

[32]The probability density function $f(x)$ could be written in the form $\ell(x|\rho)$, which emphasizes that it is the probability density for the x's, given parameter ρ; Alternatively, it could be written as $\ell(\rho|x)$, the likelihood function, which stresses that for given x, it can be regarded as a function of the parameter(s).

$\beta/(1+t\beta)$. The Bayes estimate of ρ is the posterior mean, $(\alpha+\Sigma x_i)\beta/(1+t\beta)$. This can be rewritten as

$$\hat{\rho} = \frac{t\beta}{1+t\beta}\bar{x} + \frac{1}{1+t\beta}\alpha\beta, \tag{4.27}$$

a weighted average of the sample mean $\bar{x}$ and the prior mean $\alpha\beta$.

The precision of this estimate and the predictive density function can also be evaluated conveniently. Equation (4.27) is clearly analogous to equation (4.23) in a manner that ρ_{old} is $\alpha\beta$, ρ_{new} is $\bar{x}$, and $z(n)$ is $1/(1+t\beta)$. One of the advantages with the Bayesian approach is that the choice of credibility factor (weight) is established based on theoretical foundation, rather than on some ad hoc procedures.

The credibility theory and the Bayesian methodology could be used to justify the current MCIC ratemaking methodology. The MCIC formula is partly justified in the sense that the calculated rate is a combination of the old premium (or prior rate) and the new (sample) rate. The old premium rate ρ_{old} is determined by the normal base rate, the new premium ρ_{new} is reflected by the actual annual loss cost. The formula is however questionable in terms of its weight determination. It is also hard to justify why the formula will be finally switched to a single actual loss cost formula.

An empirical difficulty with the general Bayesian approach in crop insurance is that it is not easy to incorporate some prior information about the expected loss itself into the framework. This is because pure premium rate itself is not a parameter to be estimated. It is generally a function of some other estimated parameters (σ, μ, coverage level, etc.). Moreover, there is no relevant computer program which can be used readily in this context. It is for this reason that we may fit the Bayesian approach into some slightly different framework. That is, in applying the Bayesian methodology, we often focus on the parameters which determine the pure premium rates, rather than on the premium rate itself. However, there is no theoretical difficulty in utilizing a true Bayesian methodology. A possible framework of using the true Bayesian methodology may is proposed below.

Suppose premium rate ρ is directly estimated from loss data measured by actual indemnity per risk (I, in bushels per acre). By the actuarial principle, the relationship of $E\{I\} = \rho$ holds. Assuming insurance payments are normally distributed, the probability density function for the payments is:

$$f(I|\rho,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(I_i-\rho)^2}{2\sigma^2}}. \tag{4.28}$$

Suppose the prior distribution about premiums ρ could be specified as a gamma distribution with parameter α and β,

$$g(\rho|\alpha,\beta) = \frac{\rho^{\alpha-1}e^{-\rho/\beta}}{\beta^{\alpha}\Gamma(\alpha)}, \tag{4.29}$$

the posterior density function is then:

$$g(\rho|I,\alpha,\beta,\sigma) = \frac{f(I|\rho,\sigma)g(\rho|\alpha,\beta)}{\int_0^\infty f(I|\rho,\sigma)g(\rho|\alpha,\beta,\sigma)d\rho}, \tag{4.30}$$

which is equivalent to

$$g(\rho|I,\alpha,\beta,\sigma) = \frac{\sqrt{2/\pi}\Gamma(\frac{1+\alpha}{2})A^{\alpha}\rho^{\alpha-1}e^{-\rho/\beta+I\rho/\sigma^2-\rho^2/2\sigma^2}}{\sqrt{2}(\frac{1}{2})^{\frac{\alpha}{2}}\sigma^{\alpha}\Gamma(\alpha)\Gamma(\frac{\alpha}{2})A^{\alpha}H(Z_3)+2\sigma\Gamma(\frac{1+\alpha}{2})AH(Z_2)}. \tag{4.31}$$

Where $A = \frac{I}{\sigma^2} - \frac{1}{\beta}$, $H(Z_2) = H(\frac{1+\alpha}{2}, \frac{3}{2}, \frac{A^2\sigma^2}{2})$, and $H(Z_3) = H(\frac{\alpha}{2}, \frac{1}{2}, \frac{A^2\sigma^2}{2})$. $H(.)$ is Kummer confluent hypergeometric function[33] and $\Gamma(.)$ is standard gamma function.

The Bayesian estimate of ρ can be derived from this posterior distribution. The posterior pure premium mean is

$$\begin{aligned} \rho_{Bayesian} = & \frac{-2^{3/2}\alpha\Gamma(\frac{\alpha}{2})H(Z_1)I}{2\beta\sigma AB\Gamma(\frac{1+\alpha}{2})H(Z_2)+\sqrt{2}\Gamma(\frac{\alpha}{2})H(Z_3)} + \\ & \frac{\sqrt{2}\Gamma(\frac{\alpha}{2})H(Z_1)I^2}{2\sigma^3 AB\Gamma(\frac{1+\alpha}{2})H(Z_2)+\sqrt{2}\Gamma(\frac{\alpha}{2})H(Z_3)} + \\ & \frac{\sqrt{2}\sigma^2\Gamma(\frac{\alpha}{2})H(Z_1)}{2\sigma\beta^2 AB\Gamma(\frac{1+\alpha}{2})H(Z_2)+\sqrt{2}\Gamma(\frac{\alpha}{2})H(Z_3)} + \\ & \frac{2\sqrt{\pi}\sigma B(\frac{\alpha}{2})!L(\frac{\alpha}{2},-\frac{1}{2},-\frac{A^2\sigma^2}{2})e^{-\frac{A^2\sigma^2}{2}}}{2\sigma AB\Gamma(\frac{1+\alpha}{2})H(Z_2)+\sqrt{2}\Gamma(\frac{\alpha}{2})H(Z_3)}. \end{aligned} \tag{4.32}$$

[33] As defined by $H(a,b,c,k) = \frac{\Gamma(b)}{\Gamma(a-b)\Gamma(a)}\int_0^1 e^{kt}t^{a-1}(1-t)^{b-a-1}dt$.

Where $H(Z_1)$ is the Kummer confluent hypergeometric function as defined by $H(1+\alpha/2, 3/2, A^2\sigma^2/2)$, $B = (\sigma^2 - \beta I)/\beta\sigma^2$, and $L(.)$ is generalized Laguerre polynomials.

Equation (4.32) gives true Bayesian estimates for actuarially fair premiums provided that loss experiences are coming from the normal distributions and prior information about premiums is contained in the gamma distribution. It is apparent that the Bayesian premiums are weighted average of actual losses (I) and prior information summarized by parameters α and β.

The Bayesian framework could also be used as a prediction tool. This could be done by formulating a *predictive* density. Here of interest is the value of a future observation whose distribution also depends on the parameters contained in the posterior distribution.[34] The old observations are the insurance payments paid in the past to insured producers and we wish to predict the payments that will be made in the future.

Assume that the density of this new observation is $g(I_{new}|\rho)$, the predictive density is then:

$$f^*(I_{new}|I) = \int g(I_{new}|\rho)\pi^*(\rho|I)d\rho. \tag{4.33}$$

Where $\pi^*(.)$ is the posterior density function. This equation represents all of our knowledge about a future observation. Both point and interval estimates for this value can be constructed as is done for the parameter ρ itself. In our example, the predictive density function will be

$$\begin{aligned} f^*(I_{new}|I) \quad = \quad & \frac{(\frac{1}{4})^{\alpha/2} 2^{2-\alpha} A^\alpha \sigma C^{1+\alpha}\Gamma(\alpha) H(\frac{1+\alpha}{2},\frac{3}{2},\frac{\sigma^2 C^2}{4})}{4(\frac{1}{2})^{\alpha/2} e^{I_{new}^2/2\sigma^2} A^\alpha \sigma^2 C^\alpha \Gamma(\frac{\alpha}{2}) B\Gamma(\frac{1+\alpha}{2}) H(Z_2) + \sqrt{2}\Gamma(\frac{\alpha}{2}) H(Z_3)} + \\ & \frac{(\frac{1}{4})^{\alpha/2} A^\alpha C^\alpha \Gamma(\frac{\alpha}{2}) H(\frac{\alpha}{2},\frac{1}{2},\frac{\sigma^2 C^2}{4})}{2(\frac{1}{2})^{\alpha/2} e^{I_{new}^2/\sigma^2}\sqrt{\pi} A^\alpha \sigma^2 C^\alpha B H(Z_2) + \sqrt{2}\Gamma(\frac{\alpha}{2}) H(Z_1)}, \end{aligned} \tag{4.34}$$

if the normal distribution is assumed for the $g(I_{new}|\rho)$. Where C is $I/\sigma^2 + I_{new}/\sigma^2 - 1/\beta$.

[34]This new observation I_{new} does not necessarily have the same distribution as the old observations $I's$, but it must depend on the same parameter ρ.

The theoretical formula for the pure premium rate indicates that the actuarially sound premiums depend upon the accurate estimation of the mean yield and yield risk, as measured by the yield expectation and yield variability. An important question that arises is: How can the Bayesian methodology be used to improve the accuracy of empirical estimates with the expected yield and yield variability? The Bayesian Inequality Constrained Estimation developed by Gewek (1986) is utilized in answering this question.

Suppose the estimated parameters are $\beta = (\sigma, \mu)$. In the Gewek framework, the prior information about the parameters to be estimated (β) is formulated in a sort of inequality formation. If, for example, we have some prior information about β such that the parameters are confined in the range between β_{lower} and β_{upper}, then the appropriate noninformative prior density function can be formulated as:

$$g(\beta) = \begin{cases} 1 & if\ \beta_{lower} < \beta < \beta_{upper} \\ 0 & otherwise \end{cases} \tag{4.35}$$

This prior density function suggests that only those β values which fall into the range of $(\beta_{lower}, \beta_{upper})$ are feasible, while all the other values of β are assumed to be zero. It also indicates that all values within the feasible category are equally likely. With this justification, the posterior density function is:

$$g(\beta|y) \sim \begin{cases} \ell(\beta|y)g(\beta) & \text{if } \beta_{lower} < \beta < \beta_{upper} \\ 0 & otherwise \end{cases} \tag{4.36}$$

This posterior density function is a truncated normal distribution. It is truncated at point 0.

Given this framework, the practical question is how to determine the feasible range of β_{lower} and β_{upper}. The following procedures are possible options in applying this quasi-Bayesian approach:

(1) The estimated mean yield and yield risk from the homogeneous risk area could

be treated as prior information, the individual farm's actual yield and yield variability will be the sample.

(2) The statistical estimates for yield expectation and yield variability from different yield distributions could serve as the prior. In this study, the yield variances from normal distribution will be treated as the lower bounds for yield risks, and the mean yields estimated from the same yield distribution are treated as upper bounds for yield expectation.

4.4 An Indemnity-Truncated or Deductible-Shifted Program

An all-risk crop insurance, in particular, a farm-level crop insurance program typically suffers from high administrative costs. In the U.S., for example, administrative costs account for almost 40 percent of the net cost of the crop insurance program. These costs escalate in areas that are characterized by a preponderance of small scale producers (Skees, 1994). High management cost is one common feature for many government-subsidized crop insurance programs. As a result, reducing administrative costs is critical in order for the government to establish a financially viable and affordable program. The following proposed program structure is aimed at addressing this particular problem.

Under the current MCIC programs, an insured farmer will receive an insurance payment whenever his actual yield of insured crop drops below the predetermined insured yield, no matter how much his actual claim is. This indicates that the whole management process will apply to him whether he is actually making a claim only worth \$2 or a claim worth as much as \$200. This is because the indemnity scale is determined between zero and actual indemnity payment. If the proportion of those making small claims is significant, the resulting inefficiency in administrative expenses is obvious. Therefore, it may be effective to design a structure in which small claimers will be eliminated from actual insurance payment receivers. A truncated indemnity structure (or a deductible-shifted program) proposed here is one possible choice.

The truncated indemnity schedule works in a way that an explicit truncated deductible level (θ) is specified and the schedule is defined by

$$I = Max\{0, (1-\theta)C - y\} \quad with \;\; 0 < \theta < 1. \tag{4.37}$$

Where C is the yield coverage and y is the actual crop yield. Under this program, an insured producer receives an insurance indemnity payment if his actual yield falls below the coverage and if the shortfall is larger than a certain amount, as measured by θC. The insured farmer will not receive payment if his loss is less than θC bushels even if he suffers from a sort of crop failure. Obviously, the θ value should be small enough not to discourage participation and big enough to effectively reduce administrative costs. When $\theta = 0$, shedule (4.37) reduces to the conventional indemnity schedule.

The indemnity-truncated or deductible-shifted program provides some important advantages over the conventional program. One obvious advantage is that the program delivery costs associated with this program will be significantly lower than that of the basic deductible program due to the elimination of those small claimers whose crop losses are less than θC bushels. The program will be more attractive to farmers because higher yield protection could be obtained without exceeding the maximum coverage to average yield ratio (i.e., the maximum real coverage level).[35] Given a maximum real coverage level c, the coverage could be flexibly provided from $\theta\%$ to $(c + \theta)\%$ so that the real coverage level would remain constant at c whereas the deductible or unindemnified loss range could change. For instance, coverage from 5% to 85%, 10% to 90%, or 15% to 95% would provide an equivalent 80% real coverage

[35]This alternative approach to determining coverage at any coverage level has been proposed by farmers as a means of increasing the protection available from crop insurance (Gilson, 1987).

level. Clearly, the yield protection will actually be increased, since the probability of collecting insurance payments becomes larger. This is obvious given that the probability of losses between any yield values of coverage C and average yield $\bar{Y}$ *or* (μ) is higher than the probability of losses between yields of 0 and C of corresponding range, and that the area between 0 and θC is generally less than that between C and $C + \theta C$ (i.e., $\int_0^C f(y)dy < \int_C^{C+\theta C} f(y)dy$, $f(y)$ is the crop yield probability density function). These relationships will be held for the normal yield distribution and any other positively skewed yield distributions.

With this program design, the theoretical premium rate formula (4.11) is modified as

$$\rho = E\{(1-\theta)C - y\} = \int_0^C [(1-\theta)C - y]f(y)dy, \ \forall \ y_i < C. \tag{4.38}$$

A formal rate formula can be derived if the yield probability density function $f(y)$ is assumed and estimated for crop yields. For example, given the normal yield distribution, the formula is shown to be

$$\rho_{nml} = \frac{\sigma}{\sqrt{2\pi}}(e^{-(\frac{C-\mu}{\sigma\sqrt{2}})^2} - e^{-(\frac{\mu}{\sigma\sqrt{2}})^2}) + \frac{(1-\theta)C-\mu}{2}(erf(\frac{C-\mu}{\sigma\sqrt{2}}) + erf(\frac{\mu}{\sigma\sqrt{2}})). \tag{4.39}$$

The beta formula (4.18) is modified as

$$\rho^* = \frac{(1-\theta)c^{*1+\alpha}H(1-\beta, \alpha, 1+\alpha, c^*)}{\alpha B(\alpha,\beta)} - \frac{c^{*1+\alpha}H(1-\beta, 1+\alpha, 2+\alpha, c^*)}{(1+\alpha)B(\alpha,\beta)}. \tag{4.40}$$

Where ρ^* is specified as earlier so that $\rho = \rho^* Max\{y_1, .., y_n\}$.

4.5 Data and Results

Table 4.1. ML Estimates for Mean Yield μ's (Bushel/Acre)

Year	$\hat{\mu}_{nml}$	t	$\hat{\mu}_{bayes}$	St.E	$\hat{\mu}_{beta}$	$\hat{\alpha}$	t	$\hat{\beta}$	t
60	20.565	84.535	20.551	0.0079	20.5371	6.1857	21.112	17.9100	20.557
61	13.272	73.027	13.265	0.0039	13.2582	3.6007	24.877	18.1260	23.464
62	23.485	87.744	23.388	0.0578	23.2812	3.4136	26.375	8.3164	24.851
63	11.963	62.928	11.985	0.0131	12.0078	2.2074	28.592	12.4990	26.031
64	24.476	184.800	24.454	0.0127	24.4312	7.7885	29.185	17.7150	28.664
65	26.559	130.220	26.516	0.0247	26.4727	5.4966	30.886	11.1140	30.187
66	20.095	105.800	20.063	0.0180	20.0319	3.7680	33.279	11.2800	31.845
67	27.041	160.280	27.018	0.0131	26.9947	6.6660	34.505	13.0890	33.883
68	23.715	114.890	23.704	0.0064	23.6933	3.9184	33.650	9.3120	32.468
69	14.327	65.776	14.339	0.0065	14.3503	2.1100	30.625	9.6528	27.908
70	17.606	73.825	17.581	0.0149	17.5546	3.5986	40.835	12.8010	39.861
71	27.187	125.310	27.176	0.0063	27.1650	7.0757	33.429	13.7620	31.615
72	27.725	171.990	27.733	0.0050	27.7429	8.2798	24.707	15.5960	24.368
73	26.039	123.270	26.035	0.0018	26.0319	7.2590	26.095	15.0490	25.634
74	19.886	86.335	19.874	0.0067	19.8632	4.6384	24.295	14.0430	23.430
75	25.619	104.350	25.591	0.0161	25.5635	5.8475	24.420	12.4520	23.843
76	28.734	114.610	28.710	0.0139	28.6858	6.5713	24.591	11.7550	24.177
77	30.965	125.250	30.950	0.0087	30.9344	8.2125	25.487	13.0260	24.712
78	27.885	102.810	27.855	0.0170	27.8248	7.1963	21.219	13.4940	21.247
79	30.136	100.800	30.131	0.0032	30.1249	7.3574	21.891	12.1810	21.620
80	19.942	56.245	19.959	0.0101	19.9762	2.6023	21.725	7.8193	20.375
81	32.050	102.960	31.949	0.0595	31.8415	5.7561	21.934	8.7058	21.616
82	37.770	134.450	37.758	0.0069	37.7452	10.5210	20.951	11.7780	20.896
83	29.327	117.770	29.272	0.0324	29.2148	7.8132	22.745	13.5820	22.436
84	36.507	115.990	36.322	0.1202	36.0671	4.9746	24.498	6.0595	24.045
85	46.010	147.930	45.939	0.0419	45.8652	8.3298	22.489	6.1994	22.725
86	36.560	148.700	36.464	0.0580	36.3568	8.8453	35.657	10.6180	39.786
87	39.004	157.550	38.945	0.0316	38.8884	10.5480	23.659	11.1510	23.625
88	13.500	61.208	13.614	0.0708	13.7536	2.2634	32.655	10.9020	32.371
89	41.317	245.670	41.284	0.0191	41.2504	12.2720	39.994	11.5280	38.150
90	45.155	275.730	45.092	0.0376	45.0228	11.8620	33.528	9.2153	34.030
91	31.893	87.079	31.693	0.1266	31.4376	5.5338	33.439	8.5482	32.914
92	46.700	283.610	46.623	0.0480	46.5284	6.7853	33.749	4.8812	34.359
Mean	27.928	125.489	27.893	0.0278	27.8526	6.2964	28.260	11.5806	27.664

Farm-level yield data for red spring wheat for risk area 12 are used to calculate pure premiums with different premium setting methodologies. To test the efficiency associated with various formulas, annual average area rates are estimated for each year for the period 1960 - 1992. The sample size ranges from 900 to 3000 since the participation rate fluctuates from year to year. The annual mean yield μ and the yield variance σ^2 are estimated with Maximum Likelihood (ML) technique, and these estimates are utilized in our premium rate formulas. Table 4.1 and 4.2 report mean yield and yield variance estimates.

In Table 4.1, $\hat{\mu}_{nml}$ represents the ML estimates for the annual mean yield under the normal yield distribution assumption, $\hat{\mu}_{beta}$ denotes the ML estimates for mean yield under the beta distribution, and $\hat{\mu}_{bayes}$ represents the Gewek estimates for mean yield. $\hat{\alpha}$ and $\hat{\beta}$ are estimated parameters associated with the beta distribution, t and St.E are t-ratio and standard error, respectively. The notations in Table 4.2 are similar to that in Table 4.1.

Table 4.1 shows that the estimated mean yields fluctuate significantly from year to year. For most years, the normal estimates are higher than the beta estimates. For only five out of thirty-three years the beta estimates are higher than the normal estimates. This indicates that the yield distributions for this area are generally positively skewed and the normality hypothesis can not be accepted (Table 4.3). The Bayesian Inequality or the Gewek estimates fall between the normal and beta estimates. This is because the normal estimates and the beta estimates are used as possible ranges for these Bayesian estimates. As seen in Table 4.1, all the ML estimates are statistically significant.

In terms of yield variances, the normal estimates are lower than the beta estimates (Table 4.2). The normal estimates of yield variances for 31 out of 33 years are underestimated. This suggests that the premiums based on the normality assumption will also be biased and underestimated. The Gewek variance estimates drop between the normal and beta estimates as well. Again, all the estimates are statistically significant, particularly for beta parameters.

Table 4.2. ML Estimates for Yield Risk σ's (Bushel/Acre)

Year	$\hat{\sigma}_{nml}$	t	$\hat{\sigma}_{bayes}$	St.E	$\hat{\sigma}_{beta}$	$\hat{\alpha}$	t	$\hat{\beta}$	t
60	6.7963	45.096	6.8878	0.0551	6.9758	6.1857	21.112	17.9100	20.557
61	6.1308	47.683	6.1859	0.0298	6.2398	3.6007	24.877	18.1260	23.464
62	9.5171	50.260	9.6703	0.1113	10.1848	3.4136	26.375	8.3164	24.851
63	7.1888	53.479	7.2005	0.0054	7.2098	2.2074	28.592	12.4990	26.031
64	6.9191	56.884	7.0079	0.0597	7.1571	7.7885	29.185	17.7150	28.664
65	8.6498	59.942	8.7588	0.0766	8.9702	5.4966	30.886	11.1140	30.187
66	8.3576	74.909	8.4473	0.0636	8.6519	3.7680	33.279	11.2800	31.845
67	8.0781	67.568	8.1633	0.0576	8.3030	6.6660	34.505	13.0890	33.883
68	9.4365	64.653	9.5367	0.0648	9.6824	3.9184	33.650	9.3120	32.468
69	8.8060	57.169	8.7131	0.0605	8.5916	2.1100	30.625	9.6528	27.908
70	7.6605	45.416	7.7715	0.0731	7.9374	3.5986	40.835	12.8010	39.861
71	7.9713	51.961	8.0371	0.0376	8.1070	7.0757	33.429	13.7620	31.615
72	7.6228	48.585	7.6298	0.0022	7.6341	8.2798	24.707	15.5960	24.368
73	7.6367	51.127	7.6976	0.0364	7.7637	7.2590	26.095	15.0490	25.634
74	7.6678	47.053	7.7274	0.0344	7.7905	4.6384	24.295	14.0430	23.430
75	8.2564	47.561	8.3589	0.0627	8.4915	5.8475	24.420	12.4520	23.843
76	8.5262	48.033	8.6195	0.0551	8.7273	6.5713	24.591	11.7550	24.177
77	8.1215	46.544	8.1909	0.0394	8.2615	8.2125	25.487	13.0260	24.712
78	7.8566	44.313	7.9815	0.0821	8.1812	7.1963	21.219	13.4940	21.247
79	8.5443	37.696	8.5489	0.0018	8.5531	7.3574	21.891	12.1810	21.620
80	10.2780	40.992	10.2660	0.0105	10.2460	2.6023	21.725	7.8193	20.375
81	9.4263	42.820	9.5994	0.1212	9.9587	5.7561	21.934	8.7058	21.616
82	8.2146	41.428	8.2462	0.0155	8.2737	10.5210	20.951	11.7780	20.896
83	7.8638	44.651	7.9781	0.0725	8.1394	7.8132	22.745	13.5820	22.436
84	10.6260	47.535	10.8050	0.1315	11.4748	4.9746	24.498	6.0595	24.045
85	9.7515	44.464	9.8779	0.0785	10.0407	8.3298	22.489	6.1994	22.725
86	8.5733	49.315	8.6752	0.0628	8.8057	8.8453	35.657	10.6180	39.786
87	8.1852	46.691	8.2784	0.0564	8.3925	10.5480	23.659	11.1510	23.625
88	7.9299	50.836	7.9753	0.0250	8.0200	2.2634	32.655	10.9020	32.371
89	7.6354	64.409	7.7301	0.0708	8.0283	12.2720	39.994	11.5280	38.150
90	8.3633	72.160	8.4037	0.0231	8.4457	11.8620	33.528	9.2153	34.030
91	16.9390	65.231	16.7360	0.1575	10.0611	5.5338	33.439	8.5482	32.914
92	10.8340	67.296	10.9370	0.0668	11.0884	6.7853	33.749	4.8812	34.359
Mean	8.6171	52.235	8.6862	0.0576	8.6178	6.3424	28.093	11.6412	27.506

The tests of normality for cross-sectional farm yields are reported in Table 4.3. The sample size involved in the test for each year is presented in the first column. The estimated yield variability, skewness, and kurtosis statistics are presented in the fourth, fifth, and sixth column, respectively. The normality test statistic is presented in the last column. The tests are conducted with the SAS Univariate Procedure. With this procedure, the normality test statistic will be computed as the Shapiro-

Wilk's W statistic if sample size is less than 50 (small sample), and a statistic based on Kolomogorov's D statistic will be calculated if the sample size is larger than 50. This statistic ranges between zero and one, and a small value tends to reject the normality hypothesis. The simulated critical values and the methodology can be found in Shapiro & Wilk (1965) and Stephens (1974). For example, given a 5 percent significance level, the critical value is 0.931 if sample size is 33. This critical value will be 0.947 if the sample size becomes 50. The larger the sample size is, the greater the critical value will be. It is clear from Table 4.3 that the normal yield distribution hypothesis for cross-sectional farm yields can not be accepted.

Table 4.3 also clearly shows that farm wheat yields within risk area 12 for most years are skewed to the right. It is also interesting to note that the normality tests for wheat yields using farm-level time series data (the results not reported here) lead to the rejection of the normality hypothesis as well. The rejection of the normality hypothesis and the confirmation of a positively skewed yield distribution indicate that crop yield loss distributions are generally positively skewed. This is consistent with the general perspective that most insurance loss distributions are skewed to the right with an heavy right tail (Klugman, 1984).

Table 4.3. Normality Test (Bushel/Acre)

Year	Sample	Yield	St-D	Skewness	Kurtosis	NORMT
60	976	20.5113	1.2628	0.1998	0.118	0.9694
61	1376	13.2755	1.0114	0.5842	0.201	0.9587
62	1579	23.7581	2.4258	-0.2108	-0.345	0.9687
63	1870	12.2350	1.3645	1.0193	3.168	0.9397
64	2111	24.0891	1.3160	-0.0643	0.187	0.1042
65	2286	26.3539	1.9873	-0.0064	0.000	0.0898
66	2555	19.8477	1.8496	0.2761	-0.011	0.0887
67	2994	26.7914	1.7414	0.1303	0.253	0.0951
68	2599	23.7207	2.4724	0.3133	0.306	0.0771
69	1900	14.3548	2.1730	0.8443	0.561	0.9186
70	1145	17.5198	1.5931	0.4040	-0.053	0.9650
71	1530	27.2075	1.7450	0.2634	0.366	0.9721
72	1374	27.5276	1.6404	0.3979	0.527	0.9650
73	1551	25.8797	1.5509	0.4120	0.795	0.9680
74	1265	19.9035	1.6107	0.3966	0.174	0.9670
75	1278	25.6573	1.8394	0.0749	0.019	0.9736
76	1343	28.8227	1.9293	0.0250	0.259	0.9727
77	1217	30.7421	1.7693	0.1084	0.246	0.9672
78	971	27.8381	1.6878	0.0053	0.618	0.9747
79	914	30.2026	2.0346	0.2954	0.306	0.9719
80	937	20.1501	2.9675	0.8108	1.273	0.9459
81	1005	31.8127	2.5110	-0.4515	0.058	0.9621
82	927	37.7456	1.8361	-0.1469	0.234	0.9671
83	1105	29.3215	1.6838	0.2207	4.898	0.9526
84	1233	36.6761	2.9991	-0.7932	0.859	0.9405
85	1070	45.7994	2.6724	-0.6025	1.937	0.9559
86	1305	36.5117	2.0302	-0.5654	2.218	0.9434
87	1177	38.9387	1.8276	-0.6910	1.092	0.9453
88	1414	13.4909	1.7010	1.4920	5.355	0.9130
89	2284	41.2325	1.5597	-0.7440	2.094	0.1521
90	3003	45.1586	1.9256	-0.9265	1.682	0.1459
91	2386	31.7426	7.3165	24.8689	972.554	0.1489
92	2540	46.5602	3.1829	-0.5863	0.825	0.0497
Mean	1162	27.9205	2.0979	0.8289	30.387	0.9747

The estimated actuarially sound annual premium rates with an 80 percent coverage level for different crop yield distributions are presented in Table 4.4. For simplicity, the coverage yield is determined as an 80 percent actual annual average yield. This allows us to reflect and test sensitivities of coverages to calculated premiums. To simulate effectiveness and implications of different yield distribution assumptions, an identical coverage applies to different rate formulas for each particular year.

ρ_{emp} is an ex ante empirical rate, as calculated by equation (4.16). This ex ante annual empirical rate reflects an exact average crop loss or "expected " loss (bushels per acre). This rate is calculated to provide a comparison basis for different rate formulas in evaluating various ratemaking methodologies. A ratemaking methodology or formula is good only if it produces a premium estimate which is equal or very close to this ex ante empirical rate. The normal rates, ρ_{nml}, are computed by formula (4.15). The beta rates, as denoted by ρ_{beta}, are derived from equation (4.18) and equation (4.20). ρ_{Gewek} represents a quasi-Bayesian rate which is based on Gewek yield variability and yield expectation estimations. ρ_{mcic} is the actual MCIC rate. The MCIC rates are calculated by the Corporation's ratemaking methodology and coverages are determined by the long term average area yield. Therefore, they are not entirely comparable with our simulated ex ante rates. $\rho_{Bayesian}$ presents the premium rates based on the true Bayesian framework calculated from the posterior mean (equation (4.32)). The Bayesian rates are simulated by taking actual losses as the sample and the gamma distribution is assigned to capture our prior knowledge regarding historical premiums. Because the actual losses are not available and the historical MCIC premium rates are influenced by many factors, the hypothetical data generated from the sample used in this study are utilized. This is reasonable since the Bayesian rates are primarily calculated to demonstrate a methodological framework.

Table 4.4 shows that the normal rates ρ_{nml}'s are generally underestimated, as compared to the actual losses. The reason for this is that yield loss risks under normality hypothesis are underestimated, provided that some skewed yield loss distributions are observed. The normal rates cover the actual losses only for ten out of thirty three years. The differences between these normal rates and the actual losses are not insignificant for most years over the simulation period. For the period 1960 - 1992, the average annual crop loss for wheat in risk area 12 is 1.295 bushels per acre, and the average annual normal rate is only 1.226 bushels per acre over the same period, a 5.3% shortfall annually.

Table 4.4. Estimated Pure Premium Rates
80% Coverage: Risk Area 12 (Bushel/Acre)

Year	ρ_{emp}	ρ_{nml}	ρ_{beta}	ρ_{Gewek}	ρ_{mcic}	$\rho_{Bayesian}$
60	1.076	1.115	1.102	1.145	N/A	0.992
61	1.250	1.150	1.251	1.161	N/A	1.139
62	2.100	1.754	2.089	1.815	N/A	1.932
63	1.657	1.226	1.651	1.222	N/A	1.509
64	0.979	0.972	0.979	1.004	N/A	0.913
65	1.431	1.401	1.469	1.444	N/A	1.300
66	1.717	1.551	1.696	1.579	N/A	1.565
67	1.168	1.206	1.219	1.237	1.022	1.068
68	1.826	1.728	1.838	1.755	0.953	1.668
69	2.177	1.478	1.990	1.475	1.007	2.006
70	1.603	1.434	1.589	1.462	0.944	1.459
71	1.089	1.167	1.145	1.189	1.151	1.102
72	0.900	1.037	0.968	1.037	1.070	0.851
73	0.976	1.118	1.086	1.137	1.080	0.911
74	1.402	1.387	1.402	1.405	0.882	1.374
75	1.279	1.326	1.349	1.363	0.944	1.164
76	1.278	1.264	1.278	1.298	0.952	1.163
77	0.930	1.042	1.037	1.064	1.036	0.875
78	1.076	1.101	1.147	1.144	0.939	0.992
79	1.137	1.206	1.154	1.207	0.926	1.042
80	2.312	1.892	2.230	1.886	0.838	2.137
81	1.562	1.385	1.570	1.460	0.911	1.421
82	0.791	0.797	0.803	0.806	1.003	0.774
83	1.023	1.038	1.086	1.083	0.986	0.948
84	1.812	1.544	1.944	1.643	0.983	1.656
85	1.051	0.905	1.074	0.948	1.528	0.971
86	1.088	0.939	1.026	0.985	1.437	1.001
87	0.938	0.746	0.814	0.780	1.528	0.882
88	1.696	1.372	1.741	1.353	1.537	1.545
89	0.742	0.543	0.650	0.569	1.934	0.740
90	0.863	0.598	0.669	0.615	2.866	0.826
91	1.704	3.085	1.680	2.223	1.977	1.554
92	1.393	1.166	1.373	1.209	2.107	1.266
Mean	1.295	1.226	1.297	1.264	1.205	1.278

In contrast to the normal rates, the calculated beta premium rates provide a very good approximation to the actual indemnity payments. For most years, the beta rates are very close to ex ante empirical rates, and some are even identical. From 1960 to 1992, the estimated average beta rate is 1.297 bushels per acre, and the difference between the average beta rate and the average loss is only 0.005 bushels per acre. This result suggests that the beta rate formula (4.18) can provide rather optimal premium rate estimates.

Average annual quasi-Bayesian rate is calculated as 1.264 bushels per acre. For most years, ρ_{Gewek} produces a median rate which is between the normal and the beta rate. This is because the Gewek Inequality Restrictions Estimation provides median yield variability and yield expectation estimates. The reliability and precision of these ρ_{Gewek}'s depend on the quality of specified prior information for yield variability and average yield. Their power and usefulness can be improved significantly if some realistic prior restrictions are incorporated. The key to this procedure is how to formulate prior information. The example discussed here is only a very simple case.

The true Bayesian methodology generates median premium rates which fall between "actual losses" and " historical premiums". It should be noted that a fixed premium rate for "historical" or prior premium is assumed ($\hat{\alpha} = 1.6534$, $\hat{\beta} = 0.7517$ so that $\bar{\rho} = 1.24$, and $\hat{\sigma} = 0.4155$) while annual actual losses denoted by ρ_{emp} are used in the Bayesian simulation. Consequently, $\rho_{Bayesian}$ represents an evaluation of how premiums should be revised as "actual losses" change, given a fixed old premium. The simulation results presented in Table 4.4 apparently suggest that the Bayesian approach gives an average of this prior premium (1.24 bushels per acre) and the actual loss experience. More importantly, it can closely fit the revised (posterior) premium pattern to the changed loss pattern, i.e., the revised premium increases as the loss goes up, and vice versa.

Table 4.5 presents the estimated pure premiums for the truncated indemnity program. These rates are calculated with a 5 percent truncated level (θ=0.05) with different coverage levels. As seen from the table, the premium rates, based on 80% coverage level, as computed either by the normal formula or by the beta formula, are lower than those with a zero truncation (i.e., θ=0, see Table 4.4) as expected. This result is logical because with a (0.05, 0.80) combination for (θ, c), the real coverage level is only 75 percent.

Table 4.5. Simulation of Indemnity-Truncated Program
5 Percent Truncation (θ=0.05, Bushel/Acre)

			$c = 80\%$				$c = 85\%$	
Year	portion	Loss	ρ_{nml}	ρ_{beta}	portion	Loss	ρ_{nml}	ρ_{beta}
60	0.09	0.856	0.892	0.854	0.09	1.119	1.137	1.128
61	0.00	1.038	0.982	1.049	0.09	1.289	1.172	1.278
62	0.09	1.842	1.468	1.744	0.05	2.149	1.788	2.134
63	0.00	1.452	1.072	1.446	0.19	1.695	1.248	1.653
64	0.05	0.795	0.738	0.791	0.40	1.015	0.991	1.002
65	0.01	1.124	1.115	1.149	0.11	1.489	1.429	1.503
66	0.04	1.448	1.303	1.406	0.07	1.769	1.581	1.734
67	0.05	0.901	0.934	1.644	0.19	1.220	1.230	1.247
68	0.00	1.558	1.442	1.510	0.30	1.878	1.762	1.879
69	0.05	1.903	1.294	1.815	0.09	2.229	1.505	1.998
70	0.07	1.375	1.312	1.327	0.00	1.648	1.462	1.624
71	0.03	0.825	0.898	0.854	0.20	1.139	1.199	1.171
72	0.15	0.649	0.778	0.697	0.07	0.954	1.056	0.989
73	0.49	0.707	0.861	0.808	0.13	1.032	1.140	1.110
74	0.00	1.142	1.152	0.970	0.00	1.452	1.414	1.231
75	0.40	1.026	1.052	1.046	0.05	1.339	1.353	1.381
76	0.19	1.024	0.977	0.964	0.04	1.331	1.289	1.307
77	0.03	0.735	0.766	0.741	0.02	0.972	1.059	1.057
78	0.18	0.844	0.834	0.552	0.07	1.126	1.121	1.723
79	0.08	0.899	0.916	0.847	0.44	1.184	1.229	0.850
80	0.00	1.990	1.634	1.908	0.00	2.374	1.927	2.268
81	0.38	1.261	1.067	1.207	0.05	1.630	1.412	1.606
82	0.56	0.587	0.527	0.520	0.07	0.863	0.802	0.810
83	0.06	0.827	0.770	0.789	0.09	1.061	1.056	1.108
84	0.15	1.567	1.185	1.519	0.38	1.859	1.573	1.988
85	0.02	0.799	0.587	0.793	0.16	1.101	0.908	1.086
86	0.15	0.893	0.651	0.887	0.47	1.126	0.950	1.042
87	0.52	0.689	0.480	0.624	0.13	0.991	0.748	0.820
88	0.00	1.460	1.198	1.504	0.04	1.740	1.397	1.761
89	0.18	0.557	0.312	0.488	0.40	0.782	0.533	0.646
90	0.40	0.642	0.344	0.595	0.15	0.912	0.587	0.664
91	0.17	1.402	2.672	1.403	0.16	1.767	3.142	1.719
92	0.16	1.102	0.803	1.077	0.18	1.454	1.179	1.396
Mean	0.14	1.088	1.006	1.077	0.15	1.382	1.288	1.360

Table 4.5 confirms again that the beta formula produces better premium rate estimates in both 80% and 85% coverage cases. It should be noted that in order for an insurer to provide an actual 80% coverage level, the norminal coverage should be

set at 85% with the 5% deductible truncated. It is observable from Table 4.5 that an equivalent 80% coverage under 5% truncation (i.e., θ=0.05, c=0.85) under truncated indemnity schedule generates higher premiums than a non-truncated program (i.e. simple 80% coverage level with zero truncation).

As seen in Table 4.5, an 85 percent coverage level with a five percent truncation program provides an equivalent insurance protection as a simple 80 percent coverage program does, but in the former, insured producers are required to pay some higher premiums per acre. This is because expected indemnity payments for a truncated program are higher than a non-truncated program. For example, with a simple 80% coverage level, the average annual indemnity payment is 1.295 bushels per acre. This figure becomes 1.475 bushels when a 5 percent shifted deductible is offered, given an 85% norminal coverage level. This clearly suggests that the probability for an insured farmer to collect insurance indemnities is increased if he is willing to pay a bit higher premium rate under deductible-shifted program. Therefore, from an insurance participant's perspective, a deductible-shifted insurance package, say, a combination of (0.05, 0.85) for (θ, c), could provide the same or even larger protection than a basic 80% coverage program.

From an insurer's perspective, an increased indemnity cost could be balanced by an actuarially sound premium ratemaking methodology. It also may be offset by a reduction in administrative costs if the indemnity schedule is truncated. This is very likely since the proportion of claimers who make small claims among claimer population is not insignificant in many cases. On the contrary, it is often very significant. The column under *portion* in Table 4.5 displays the percentage of those small claimers in the total indemnity payment receivers for each year under specified deductible and coverage levels. It is obvious that this proportion is significant for most simulation years, and it is increased as coverage level increases. For example, the percentage ranges from zero to 56 for an 80% coverage option. If a real 80% coverage level is offered, the small claim percentage becomes more stable than a 75% coverage, it is often around 30 percent. With a deductible-shifted program, however, all these small claimers are eliminated from actual claimer population. As a result, cost reductions due to this structure is expected to be very significant. Take (θ = 0.05, c=0.85) program as an example. The average indemnity for the past 33 years is 1.475 bushels per acre, which is 14 percent higher than that of the basic program (80% coverage with no truncation). However, the average annual small claim proportion for the deductible-shifted program is 28 percent over the same period for the 0.05, 0.85 combination, and 33 percent for the 0.05, 0.80 combination. This shows that a five percent indemnity-truncated program may lead to some 30 percent reduction in terms of the number of claims to be processed.

Table 4.6 presents the simulation results for the programs with a ten percent deductible truncation. With this level, the indemnity schedule is truncated at 0.1*C bushels where C is the coverage yield, and a 90 percent coverage level will give an actual 80 percent yield protection only. Obviously, the small claim proportion goes up as truncated level θ increases. This indicates that potential program management cost savings associated with higher coverage program are even more significant. For example, given an 80% coverage level, the possible reduction in claims could be about 31 percent, and this number might be 37 percent if the coverage level is increased to 90 percentage of expected yield given our simulation cases. As expected, the average insurance payment under a 90% coverage level with a 10% deductible truncation is slightly increased as compared to the 85% coverage and 5% truncation option. Although these two programs provide an identical 80% yield loss protection, the former may be more attractive to producers than the latter, since the probability for an insured producer to collect insurance payments is higher in the former program design.

Table 4.6. Simulation of Indemnity-Truncated Program 10 Percent Truncation (θ=0.10, Bushel/Acre)

	$c = 80\%$				$c = 90\%$			
Year	portion	Loss	ρ_{nml}	ρ_{beta}	portion	Loss	ρ_{nml}	ρ_{beta}
60	0.45	0.664	0.668	0.665	0.27	1.133	1.079	1.068
61	0.27	0.874	0.813	0.873	0.09	1.303	1.148	1.256
62	0.13	1.606	1.182	1.493	0.30	2.166	1.737	2.091
63	0.10	1.260	0.918	1.171	0.19	1.708	1.233	1.654
64	0.25	0.630	0.503	0.662	0.48	1.027	0.907	0.916
65	0.46	0.929	0.831	0.929	0.15	1.508	1.353	1.429
66	0.26	1.220	1.056	1.213	0.18	1.787	1.539	1.694
67	0.45	0.693	0.662	0.622	0.24	1.237	1.423	1.158
68	0.10	1.316	1.155	1.181	0.32	1.896	1.709	1.824
69	0.05	1.640	1.111	1.417	0.09	2.246	1.488	1.908
70	0.11	1.154	0.994	1.060	0.24	1.663	1.427	1.584
71	0.51	0.613	0.629	0.563	0.10	1.157	1.100	1.188
72	0.64	0.414	0.519	0.425	0.12	0.972	0.958	0.973
73	0.50	0.557	0.603	0.530	0.22	1.051	1.054	1.023
74	0.32	0.952	0.914	0.735	0.09	1.469	1.368	1.458
75	0.43	0.842	0.779	0.743	0.21	1.359	1.277	1.323
76	0.23	0.797	0.690	0.651	0.38	1.349	1.196	1.315
77	0.09	0.550	0.489	0.445	0.04	0.996	0.944	0.994
78	0.22	0.631	0.568	0.556	0.47	1.143	1.025	1.079
79	0.19	0.676	0.627	0.541	0.48	1.200	1.126	1.152
80	0.21	1.715	1.376	1.566	0.04	2.394	1.897	2.263
81	0.40	1.039	0.749	0.843	0.10	1.653	1.308	1.508
82	0.62	0.431	0.256	0.230	0.11	0.886	0.632	0.688
83	0.18	0.654	0.503	0.493	0.49	1.073	0.949	1.011
84	0.09	1.337	0.826	1.094	0.42	1.874	1.452	1.882
85	0.38	0.636	0.269	0.372	0.53	1.118	0.697	0.909
86	0.21	0.724	0.363	0.388	0.51	1.139	0.797	0.988
87	0.52	0.555	0.214	0.234	0.46	1.008	0.568	0.676
88	0.23	1.252	1.024	1.242	0.18	1.755	1.380	1.692
89	0.60	0.400	0.080	0.126	0.48	0.795	0.322	0.446
90	0.48	0.495	0.091	0.121	0.46	0.928	0.357	0.546
91	0.27	1.145	2.259	0.925	0.26	1.788	3.095	1.724
92	0.38	0.874	0.440	0.581	0.38	1.475	0.983	1.219
Mean	0.31	0.893	0.734	0.758	0.27	1.403	1.197	1.295

4.6 Conclusions

This chapter has examined the theoretical and analytical issues with respect to ratemaking framework and actuarial structures for an all-risk crop insurance program. The ratemaking simulations are made with more than 1000 insured red spring wheat producers in risk area 12. The specific actuarially fair or pure premium rate formulas for the normal yield probability density function, and for the beta yield distribution are developed and presented. These formulas are empirically easy to use and premium rates can be calculated by common statistical and mathematical packages such as SAS and MATHEMATICA. A quasi-Bayesian approach based on the Inequality Restriction estimation in calculating premium rates is also developed. A formal Bayesian framework is proposed and discussed within the context of premium revision techniques, and the current MCIC ratemaking methodology is examined within this analytical framework. In terms of program design, a deductible-shifted program by defining a truncated indemnity schedule is proposed and evaluated, and the premium rate formulas associated with this new structure are derived and simulated.

The results of this chapter suggest that, given an actuarially sound basis, the rate formula based on the beta yield distribution produces best unbiased and consistent estimates for yield losses. The normal rate formula tends to underestimate expected losses significantly. The major reason for this is that the normality hypothesis for farm wheat yields can not be accepted in risk area 12, and positively skewed yield distributions (skewed to the right) are common as suggested by the normality tests. The quasi-Bayesian procedure could generate some median rate estimates. The efficiency and the quality of these estimated rates depend upon the reliability of specified prior information. Theoretically, the true Bayesian approach is particularly appealing and useful regarding the premium rate revision methodology, the difficulty is essentially empirical and computational in nature. It is found that the current MCIC methodology is partly justified using the credibility theory and/or the Bayesian methodology. Some formal application of Bayesian approach may give rise to an improvement relative to the current formula.

A deductible-shifted program is found very interesting. It appears attractive to both the insured and the insurer. From an insured perspective, it is attractive because an equivalent or higher loss coverage could be obtained as compared to basic non-deductible-truncated programs. The probability for the insured producer to collect indemnity payments is essentially increased provided the producer is willing to pay slightly higher premium rates. The program is more flexible in terms of coverage determination. The insurance corporation finds the program appealing because some significant administrative cost reductions could be expected. This is the case since small claim insured are discriminated and eliminated from the total claimer population. For a farm-level all-risk crop insurance, the proportion of claimers who make small amount claims is often significant.

Chapter 5

Summary and Conclusions

5.1 General Conclusions

This thesis has provided an examination of the fundamental elements of crop insurance. The evaluation was conducted using theoretical and empirical approaches. To have a clear picture of the overall results which have emereged from this study, it may be useful to recall the hypotheses stated at the beginning of this thesis:

Null hypothesis 1: There is no theoretically sound and consistent foundation for an individualized crop insurance. The statistical, actuarial and economic aspects cannot be integrated consistently in this kind of crop insurance program.

Null hypothesis 2: The homogeneous risk area hypothesis cannot be rejected. The current homogeneous risk area based IPI approach is an effective approximation to the theoretically sound insurance. The area coverage program can provide the same or larger yield protection coverage than the individual coverage program.

Null hypothesis 3: Actuarially, an individual coverage approach may not be superior to area coverage approach. The individualized insurance structure does not necessarily induce stronger demand for the crop insurance. The financial exposure with the individual coverage will be much higher than with the current homogeneous risk area or some other area approaches.

Null hypothesis 4: Adverse selection is inherent in any crop insurance program as long as (1): a voluntary program is offered, or (2): the asymmetric information problem is present.

Null hypothesis 5: The current program could be improved either by some new programs (e.g., an indemnity-truncated program) or by some better premium setting methodologies. The Bayesian methodology could play an important role in the premium rate-making procedure.

The theoretical discussion suggests that the purely and partially individualized all-risk crop insurance programs typically suffer from some critical problems including actuarial inconsistency, adverse selection and moral hazard, and some practical difficulties. These observations indicate that there is no reason to reject hypothesis one, i.e., there is no reason to believe that theoretically sound and consistent foundation could be established within individualized program structures.

Although the purely homogeneous risk area approach has not been implemented in practice, the difficulty is only practical in nature in defining risk homogeneity, and it may provide an effective approximation to the theoretically sound program. Currently implemented homogeneous risk area based individual productivity indexing (IPI) method provides a basic framework toward this sub-optimal program structure. The fourth hypothesis is not rejected as suggested by our theoretical discussions and supported by practical evidence. The empirical results conducted in the study tend to more positively support the non-rejection of hypothesis two, hypothesis three, and hypothesis five.

Farmers' demand for crop insurance program is determined by many factors. These factors can be determined by the standard economic tools with the help of modern utility theory. A risk averse farmer will participate in insurance only if his expected utility with insurance is greater than that without insurance. Since all-risk crop insurance is designed to provide yield protection to the insured, the magnitude of yield risk reductions for a particular program structure will play an important role in determining farmers' demand for that particular program. The empirical study with respect to this aspect is conducted in Chapter Three, and these results are used to partially test hypothesis two and three.

The results suggest that, given an actuarially sound basis, the fully individualized crop insurance (FI) is the most favorable choice for risk-averse producers. The area coverage and individual indemnity (IA) program is generally the second best option. The area coverage and area indemnity, or the full area crop insurance plan (FA), is least preferred by risk-averse farmers. This overall ranking holds for both coverage levels. The index approach and the GSD results also clearly indicate producers will be less sensitive to the alternative programs if coverage level is increased. This suggests that offering higher coverage level, without negatively affecting the actuarial basis, will be an effective means to induce higher participation.

Although the fully individualized program is generally preferred over the area coverage and individual indemnity (IA) program, the dominance can only be made marginally. This is verified by both methodologies. In some cases, the latter may be more attractive than the former. It is found that the FI program and the IA program are yield risk reducing for almost all the farms, except for a few who may find these two programs to be yield risk neutral. The study has identified that the full area program is also yield risk reducing for many farms. The yield risk increasing cases, however, can be identified for some insured producers with this program. The reason for this is that the individual farm yields for some producers are not highly positively correlated with the respective area yields. The hypothesis of a long run stable relationship between farm yields and area yields is not accepted by the Dickey-

Fuller cointegration test. From an insurance company perspective, the FA program is the cheapest program design as expected, and the IA program is most expensive, based solely on indemnity costs.

With respect to the supply of crop insurance, it has been demonstrated that public provision is inevitable due to the uninsurable nature of crop production risk and asymmetric information problems. The determination of ratemaking methodologies and administration of programs are critical for the financially sound performances of any government subsidized all-risk crop insurance program. They are also critical if farmers are going to be encouraged to purchase insurance. These aspects are covered in Chapter Four.

Chapter 4 examined theoretical and analytical issues with respect to ratemaking framework and actuarial structures for an all-risk crop insurance program. Ratemaking simulations are conducted with more than 1000 insured red spring wheat producers in risk area 12. The specific actuarially fair or pure premium rate formulas for the normal yield probability density function, and for the beta yield distribution are developed and presented. These formulas are empirically easy to use and premium rates can be calculated by common statistical and mathematical packages such as SAS and MATHEMATICA. A quasi-Bayesian approach based on the Inequality Restriction estimation in calculating premium rates is also developed. A formal Bayesian framework is proposed and discussed within the context of premium revision techniques.

The results suggest, given an actuarially sound basis, the rate formula based on the beta yield distribution produces best unbiased and consistent estimates for yield expected losses. The normal rate formula tends to underestimate expected losses significantly. The major reason for this is that the normality hypothesis for farm wheat yields cannot be accepted in risk area 12, and positively skewed yield distributions (skewed to the right) are common as suggested by the normality tests. The quasi-Bayesian procedure could generate some median rate estimates. The efficiency and the quality of these estimated rates depend upon the reliability of specified prior information. Theoretically, the true Bayesian approach is particularly appealing and useful regarding the premium rate revision methodology, the difficulty is essentially empirical and computational in nature. It is found that the current MCIC methodology is partly justified using the credibility theory and/or the Bayesian methodology. Some formal application of Bayesian approach may give rise to an improvement relative to the current formula.

A deductible-shifted or indemnity-truncated program is also discussed. It appears attractive to both the insured and the insurer. From an insured perspective, it is attractive because an equivalent or higher loss protection could be obtained without increasing the real coverage level, as compared to conventional programs. The probability for the insured producer to collect indemnity payments is essentially increased provided the producer is willing to pay slightly higher premium rates. The program is more flexible in terms of coverage determination. The insurance corporation finds the program appealing because some significant administrative cost reductions could be expected. This is the case since small claim insured are discriminated and eliminated from the total claimer population. For a farm-level all-risk crop insurance, the proportion of claimers who make small amount claims is often significant.

5.2 Limitations of the Study

Several important assumptions and simplifications were made that place some limitations on the analysis in this research.

The simulations conducted in Chapter Three deal with the relative merits for several program alternatives in terms of their yield risk reductions. The currently implemented IPI program is however not included in the simulations. The models are at best a simple approximation of what is actually in place, and as a result, the comparisons between the model results and programs offered by the Manitoba Crop Insurance Corporation should carefully be made. Nevertheless, the basic analysis is directly applicable to crop insurance issues currently under discussion in both Canada and the United States.

The second obvious limitation is that the same MCIC data was used for all models and this may create biases in the results of alternative program designs. This is because adverse selection and moral hazard effects could be present in the data set.

The fact that yields are based on one soil type rather than the average over the entire farm may have reduced the level of yield variability and could have an effect on the results.

Administrative costs and the costs associated with program abuse are not modeled. As a result, financial comparisons are only made based on program payouts.

5.3 Suggestions For Further Research

Several approaches in terms of program structure have been proposed. Examples include an area plan, individualized insurance structure and weather crop insurance scheme. Among these, area approach and individualized crop insurance plan have received much attention, and have been implemented in different programs. Weather crop insurance program has never been experimented with for some practical reasons. Unfortunately both the area approach and the individual approach have been shown to suffer from critical problems such as moral hazard and adverse selection. Theoretically, these problems cannot be overcome thoroughly as long as the program is based upon a voluntary basis and the current methodologies are maintained. Fundamentally, for an insurance scheme to be financially viable and actuarially sound, the symmetric information condition, the reasonable loss distribution assumption and the homogeneity risk condition have to be met. However, the question is: Is there any system in which all these fundamental principles can be incorporated consistently? Stated differently, can we design a kind of structure in which all relative advantages inherited in each current approach may be reconstructed consistently while without their limitations? We believe that the answer to this question is yes! A fundamental problem with current approaches (either area approach or individual approach) is that the loss information system inherited in these programs (i.e. crop yield and its loss experience) is somewhat *subjective* and not *objective* because the insurer cannot afford to get the true information (true crop yield and yield variation data) due to information collection cost. What is meant by this statement is that a kind of objective system is crucial for a program to be practical. It is on the basis of this reasoning that *a homogeneous risk group approach* is proposed and a detailed study of this non-contiguous risk areas is one obvious entension of this research. In this approach, a homogeneous risk *group* (not *area*) will be defined according to a set of *objective indices* such as some meteorological observations and soil productivity indices. The insured farmers will be divided into different groups according to homogeneity risk criteria. The same coverage and premium rate will be determined within each group. This *nongeographical* conceptual framework will overcome many potential problems and lead to consistency between area approach and individual approach in terms of their advantages. It is conceptually superior to all the current approaches and practically feasible.

Bibliography

[1] Abramowitz, Milton and Irene Stegun, 1984, Handbook of Mathematical Functions, Washington DC: Government Printing Office.

[2] Agriculture Canada, Federal-Provincial Crop Insurance Review: Discussion Paper, Ottawa, May 1989.

[3] Akerlof, G. A., The Market for 'Lemons': Quality Uncertainty and Market Mechanism, Quart. J. of Economics, 84:488-500, 1970.

[4] Allais, M., Généralisation des théories de l'équilibre écononique geénéral et du rendement social au cas du risque, in Econométrie, 81-110, Paris: CNRS, 1953

[5] Anderson J. R., J. L. Dillion and J. B. Hardaker, 1977, Agricultural Decision Analysis, Ames IA: Iowa State University press.

[6] Ahsan, S. M., A. G. Ali and N. John, Toward a Theory of Agricultural Insurance, Amer. J. of Agr. Econ., 64(3):520-529, 1982.

[7] Arow, K. J., Uncertainty and the Welfare Economics of Medical Care, American Economic Review, 53:941-69, 1963.

[8] Arrow, K. J., 1971, Essays In The Theory of Risk Bearing, Chicago: Markham Publishing Co..

[9] Bailey, A. L., Credibility Procedures, Proceedings of the Casuality Actuarial Society, 37:7-45, 1950.

[10] Bardsley, Peter, Arun Abey and Scott Davenport, The Economics of Insuring Crops Against Drought, Australian J. of Agri. Econ., 28(1): 1-14, 1984.

[11] Barnaby, G. A. and Jerry Skees, Public Policy for Catastrophic Yield Risk: An Alternative Crop Insurance Program, Choices, 7-9, second Quarter, 1990.

[12] Bernouli, D., Specimen Theoriae Noviae De Mensura Sortis, Proceedings of the Imperial Academy 5, 175-192, 1738, St. Petersburg. English Translation: Exposotion of a New Theory on the Measurment of Risk, Econometrica, 22:23-36, 1954.

[13] Borch, Karl H., The Safety Loading of Reinsurance Premiums, Skandinavisk Akuarietidskrift, 163-184, 1960.

[14] Borch, K. H., The Utility Concept Applied to the Theory of Insurance, Astin Blletin, 1:245-55, 1961.

[15] Borch, K. H., Equilibrium in a Reinsurance Market, Econoetrica, 30:424-44, 1962.

[16] Borch, K. H., Optimal Insurance Arrangements, The Astin Bulletin, VIII(3): 284-90, 1975.

[17] Borch, Karl H., 1990, Economics of Insurance, Amsterdam: North-Holland.

[18] Brown, A., 1973, Hazard Unlimited. The Story of Lloyd's of London, London: Peter Davies Ltd.

[19] Botts, R. R. and J. N. Boles, Use of Normal-Curve Theory in Crop Insurance Rate making, J. of Farm Econ., XL, 1958.

[20] Buhlmann, Hans, 1970, Mathematical Methods in Risk Theory, New York: Springer-Verlag.

[21] Carriker, G. L.,Jerry R. Williams, G. A. Barnaby, and Jr.,J. Roy Black, Yield and Income Risk Reduction Under Alternative Crop Insurance and Disaster Assistance Designs, Western J. of Agr. Econ., 16(2): 238-250, 1991.

[22] Chambers, R. G., Insurability and Moral Hazard in Agricultural Insurance Markets, A.J.A.E, 71(3): 604-615, 1989.

[23] Chite, R. M, Federal Crop Insurance: Current Issues and Options For Reform, CRS Report for Congress, CRS Report No. 88-739 ENR, March 1992.

[24] Cochran, Mark and Rob Raskin, A user's guide to the Generalized Stochastic Dominance Program for the IBM PC Version GSD 2.1, Dept. of Agri. Econ. and Rural Sociology, Staff Pap. SPO688, University of Arkansas, April 1988.

[25] Cochran, Mark, Peter Zimmel, S. C. Goh, Nicholas D. Stone, Troy W. Toman and Gary L. Helms, An Expert System to Elicit Risks Preferences: The Futility of Utility Revisited, Computers and Electronics in Agriculture, 4(1): 361-375, 1990.

[26] Cooper, R. and B. Hayes, Multi-period Insurance Contracts, International J. of Industrial Organization, 5:211-231, 1987.

[27] Day, Richard, Probability Distributions of Field Crop Yields, J. of Farm Econ., 47: 713-741, 1965.

[28] Debreu, G., Une économie de l'incertain, Miméo, Électricité de France, 1953.

[29] Dickerson, O. D., 1963, Health Insurance, Rev. Ed., Homewood.

[30] Dickey, D. A. and W. A. Fuller, Distribution of the Estimators for Autoregressive Time Series with a Unit Root, J. of the Amer. Statistical Association, 74:427-31, 1979.

[31] Dionne, G., Adverse Selection and Repeated Insurance Contracts, Geneva Papers on Risk and Insurance, 8:316-333, 1983.

[32] Dover, V., 1957, A Handbook to Marine Insurance, Fifth Edition, London: Witherby.

[33] Driscoll, J. L., Changes in Rate Making for Federal Crop Insurance, in Multiple Peril Crop Insurance: A Collection of Empirical Studies, Southern Cooperative Series, Bulletin No. 334, pp 27-47, May 1988.

[34] Freifelder, L., 1976, A Decision-Theoretic Approach to Insurance Rate making, Homewood IL: Richard D. Irwin.

[35] Gewek, J., Exact Inference in the Inequality Constrained Normal Linear Regression Model, J. of Applied Econometrics, 1(2): 127-41, 1986.

[36] Gilson, J. C., Agriculture Instability and Crop Insurance in Alberta, Dept. of Agricultural Economics and Farm Management, the University of Manitoba, Winnipeg, Canada, 1987.

[37] Gould, J. P., The Expected Utility Hypothesis and the Selection of Optimal Deductibles for a Given Insurance Policy, Journal of Business, 42:143-151, 1969.

[38] Halcrow, H. G., Actuarial Structure for Crop Insurance, J. of Farm Econ., 31(3): 418-449, 1949.

[39] Halcrow, H. G., A New Proposal for Federal Crop Insurance, Illinois Agr. Econ., 18(2): 20-29, 1978.

[40] Hamilton, Neil, Determination of Premiums, Coverage Levels and Indemnities in Administering Crop Insurance Programs in Manitoba, Paper presented at the International Symposium on Agricultural Insurance Development, Yannlin, China, April 25-28, 1994.

[41] Hazell, P., C. Pomareda and A Valdes, 1986, Crop Insurance For Agricultural Development, Baltimore MD: Johns Hopkins University Press.

[42] Holmstrom, B., Moral Hazard and Observability, Bell J. of Economics, 10(1): 74-91, 1979.

[43] Judge, G. G, R. C. Hill, W. E. Griffiths, H. Lutkepohl and T-C Lee, 1988, Introduction to the Theory and Practice of Econometrics, New York: John Wiley & Sons, 3rd Eddition.

[44] Just, R. E. and R. Pope, Stochastic Specification of Production Functions and Economic Implications, J. of Econometrics, 7:67-86, 1978.

[45] Karni, E., 1985, Decision Making Under Uncertainty, Cambridge, Mass: Havard University Press.

[46] Kendall, M. and Alan Stuart, 1977, The Advanced Theory of Statistics, Vol. 1-3, 4th Edition, New York: Macmillan Publishing Co..

[47] King, R. P., Adjusting Crop Insurance Premium and Production Guarantee Levels to Reflect Farmer Experience, in Multiple Peril Crop Insurance: A Collection of Empirical Studies, Southern Cooperative Series, Bulletin No. 334, pp 49-58, May 1988.

[48] King, R. P., Presentation to Federal Crop Insurance Four-states research team, San Francisco CA, June 1984.

[49] King, R. and L. Robison, An Interval Approach to Measuring Decision Maker Preferences, Amer. J. of Agr. Econ., 63(2): 510-520, 1981.

[50] Klugman, Stuart A., Loss Distributions: Estimation, Large Sample Theory, and Applications, in Premium Calculation in Insurance, ed. by F. De Vylder, Reidel Publishing Company, pp 263-284, 1984.

[51] Klugman, Stuart A., 1992, Bayesian Statistics in Actuarial Science: With Emphasis on Credibility, Boston/Dordrecht/London: Kluwer Academic Publishers.

[52] Lemaire, J., 1990, Borch's Theorem: A Historical Survey of Applications, in H. Louberge's (ed.) Risk, Information, and Insurance, Kluwer Academic Publishers, 15-17.

[53] Louberge, H. 1990, 'Introduction', in H. Louberge (ed.) Risk, Information, and Insurance, Klumer Academic Publishers, 1-14.

[54] Manitoba Crop Insurance Corporation, Manitoba Crop Insurance Corporation Annual Report, various issues.

[55] Manitoba Crop Insurance Corporation, Manitoba Crop Insurance Program Manual, 1992, 1993.

[56] Manitoba Crop Insurance Review Committee, Report of Manitoba Crop Insurance Review Committee on Canada-Manitoba Crop Insurance, June 1992.

[57] Marra, Michele and B. Schurle: Kansas Wheat Yield Risk Measures and Aggregation: A Meta-Analysis Approach, WAEA Annual Meetings, Colorado Springs, Co., July 1992.

[58] Marshall, A., 1890, Principles of Economics, London.

[59] McCall, J. J., Probabilistic Economics, Bell J. of Econ. and Management Science, 2:403-433, 1971.

[60] McFadden, Don, Manitoba Crop Insurance Program Study: Report to the Board of Directors, Manitoba Crop Insurance Cooperation, March 1981.

[61] Meyer, J., Choice Among Distributions, J. Econ. Theory, 14(1): 326-36, 1977.

[62] Miller, R. B., Actuarial Applications of Bayesian Statistics,in Bayesian Analysis in Econometrics and Statistics, ed. by A. Zellner, PP197-212, Amsterdam: North-Holland Publishing Co.. 1980.

[63] Miranda, Mario J., Area-yield Crop Insurance Reconsidered, Amer. J. of Agri. Econ., 73(2): 233-242, 1991.

[64] Mossin, J., Aspects of Rational Insurance Purchasing, J. of Political Economy, 79:553-68, 1968.

[65] Mowbray, A. H. and R. H. Blanchard, 1961, Insurance, Its Theories and Practice in the United States, New York: McGraw-Hill.

[66] Musser, Wesley N and Paul L. Fackler, Historical Overview of Estimation of Risk Coefficients, Invited paper, WAEA Meeting, Colorado Spring, 1992.

[67] Nelson, Carl H. and Edna T. Loehman, Further Toward a Theory of Agricultural Insurance, A.J.A.E, 69(3): 523-531, 1987.

[68] Nelson, C. H., The Influence of Distributional Assumptions on The Calculation of Crop Insurance Premia, Northern Central J. of Ag. Econ., 12(1): 71-78, 1990.

[69] Pauly, M. V., the Economics of Marol Hazard: Comment, Amer. Econ. Review, 58:531-36, 1968.

[70] Pauly, M. V., Overinsurance and Public Provision of Insurance: The Role of Moral Hazard and Adverse Selection, Quarterly J. of Economics, 88: 44-62, 1974.

[71] Petkau, David Robert, An Assessment of the Livestock Feed Security Program, M.Sc Thesis, Dept. of Agri. Econ. & Farm Management, University of Manitoba, 1992.

[72] Pratt, J. W., Risk Aversion in the Small and Large, Econometrica, 32(1): 122-36, 1964.

[73] Raviv, Arther, The Design of an Optimal Insurance Policy, Amer. Econ. Review, 69:84-96, 1979.

[74] Rejda, G. E., 1982, Principles of Insurance, Scott, Foresman and Company.

[75] Robison, L. J. and P. J. Barry, 1987, The Competitive Firm's Response to Risk. New York: Macmillan.

[76] Robison, L., P. Barry, J. Kliebenstein, and G. Patrick, 1984, Risk Attitudes: Concepts and Measures for Decision Analysis. In P. Barry (Editor), Management in Agriculture. Ames, IA: Iowa State University Press. PP 11-30.

[77] Rothschild, M. and J. E. Stiglitz, Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. Quartly J. of Economics, 30(1): 629-649, 1976.

[78] Rothschild, V. and N. Logothetis, 1986, Probability Distributions, New York: John Willey & Sons.

[79] SAS Institute Inc. SAS User's Guide: Basics, 1985 Edition. Box 8000, Cary, North Carolina, 27511.

[80] Schlesinger, H., the Optimal Level of Deductibility in Insurance Contracts, J. of Risk and Insurance, 48:465-81, 1981.

[81] Shapiro, S. S. and M. Wilk, An Analysis of Variance Test for Normality (complete sample), Biometrika, 52, 591-611, 1965.

[82] Shavell, S., On Moral Hazard and Insurance, Q. J. of Economy, 93:541-62, 1979.

[83] Skees, Jerry and Michael R. Reed, Rate making for Farm-Level Crop Insurance: Implications for Adverse Selection, Amer. J. of Agri. Econ., 68(3): 653-659, 1986.

[84] Skees, Jerry, The Development of the New Group Risk Plan in the United States, Paper presented at the International Symposium on Agricultural Insurance Development, the Northwestern Agricultural University, China, April 25-28, 1994.

[85] Smith, A., 1776, The Wealth of Nations, Edinburgh.

[86] Smith, V., Optimal Insurance Coverage, J. of Political Economy, 79:68-77, 1968.

[87] Spence, M. and R. Zechhauser, Insurance, Information and Individual Action, Amer. Econ. Review, 61:380-87, 1971.

[88] Stephens, M. A., EDF Statistics for Goodness of Fit and Some Comparisons, J. of Amer. Stat. Association, 69, 730-737, 1974.

[89] Tetens, J., 1786, Einleitung Zur Berechnung der Leibrenten und Anwartschaften, Leipzig.

[90] Vankoughnet, R. Brent, An Economic Analysis of A Crop Cost Approach to Crop Insurance, Unpublished M.Sc. Thesis, University of Manitoba, Winnipeg, 1992.

[91] Vaughan, E. J. and Curtis M. Elliot, 1978, Fundamentals of Risk and Insurance, 2nd Edition, John Wiley & Sons.

[92] von Neumann, J. and O. Morgenstern, 1947, Theory of Games and Economic Bahavior, Princeton Univ. Press.

[93] Walras, L., 1874, Eléments d'Economie Politique Pure, Lausanne. English translation (1954): Elements of Pure Economics.

[94] White, K. J., S. D. Wong, D. Whistler and S. A. Haun, 1990, SHAZAM User's Reference Manual, Version 6.2, McGraw-Hill Book Company.

[95] Williams, J. R., Gordon L. Carriker, G. A. Barnaby and Jayson K. Harper, Crop Insurance and Disaster Assistance Designs For Wheat and Grain Sorghum, Amer. J. of Agri. Econ., 75(2):435-447, 1993.

[96] Williams, J. R., J. K. Harper and G. A. Barnaby, Government Program Impacts On the Selection of Crop Insurance in Northeastern Kansas, N. Cent. J. Agr. Econ., 12(1):207-21, 1990.

[97] Wisner, R., Weather Crop Yields in the Prairie Provinces, 1961-1986, Discussion Paper No. 353, Economic Council of Canada, Ottawa, 1988.

[98] Wolfram, Stephon, 1988, *Mathematica*^{TM}: A System for Doing Mathematics by Computer, Addison-Wesley Publishing Company, Inc.

[99] Ye, Yongsheng, A Bayesian Econometric Analysis of Technical Change in China's Agriculture, Contributed Paper, International Agric. Economics Conference held by CAEFMS/WAEA, Edmonton, Canada. July 11-14,1993.

[100] Yeh, Martin H., and Shih-ping Sun, An Evaluation of the Actuarial Structure of Manitoba Crop Insurance Program and Its Implications, Canadian J. of Agri. Econ., 28(1): 54-70, 1980.

[101] Yeh, M. H., and Roland Y. Wu, Premium Rate making in An All Risk Crop Insurance Program, J. of Farm Economics, 48(5):1580-1586, 1966.

[102] Yeh, Martin H., Yongsheng Ye and G. Tuo, Crop Insurance Premium Rates and Discounts Under Alternative Distribution, Management Science, 13(12): 7-13, Canadian Administrative Sciences Association (ASAC), 1992.

[103] Zellner, A., 1971, An Introduction to Bayesian Inference in Econometrics, New York: John Wiley & Sons Inc.

Appendix

In this appendix, we present pure premium rate formulas for various yield distributions. Since crop yield loss distributions are most likely skewed to the right, the distributions with right tails are often used to derive pure premium rate formulas. There are two "natural" distributions (i.e., the normal distribution and the gamma distribution) from which a number of heavy-tailed distributions can be created. Two methods of creating such distributions are by transformation and by mixing. Two common transformations which shift probability to the right are $y = exp(x)$ and $y = x^{1/\theta}$. Mixing occurs when one of the parameters is considered to be random with a specified parametric distribution. The new random variable is the marginal distribution of the original variable. That is, let $f(x; \beta_1, .., \beta_p)$ be the continuous p.d.f. of variable x, and $g(\beta_p; \theta_1, ..., \theta_q)$ be the continuous p.d.f. of β_p, then the new, mixed distribution will have p.d.f.

$h(x; \beta_1, .., \beta_{p-1}, \theta_1, .., \theta_q) = \int f(x; \beta_1, .., \beta_p) g(\beta_p; \theta_1, .., \theta_q) d\beta_p$.

By these methods, various distributions can be obtained.

Pure Premium Rate Formulas: Various Yield Distributions

Name	Yield Distribution	Range	Pure Premium Rate Formula
Lognormal	$\frac{1}{\sqrt{2\pi\sigma^2 y^2}} e^{-(logy-\mu)^2/2\sigma^2}$	$\sigma > 0, 0 < y < \infty$	$\frac{C}{\sigma\sqrt{2\pi} Exp((logx-\mu)^2/2\sigma^2)} + \frac{C\int_0^C \sigma\sqrt{\pi} y^{-1} dy}{\pi\sigma^2\sqrt{2} Exp((logx-\mu)^2/2\sigma^2)}$
Gamma	$\frac{1}{\Gamma(\alpha)\beta^\alpha} y^{\alpha-1} e^{-y/\beta}$	$y > 0, \alpha, \beta > 0$	$\frac{C^\alpha/\beta^{\alpha-1} - \alpha\beta e^{C/\beta}\Gamma(\alpha,0,c/\beta) + C.e^{C/\beta}\Gamma(\alpha,0,C/\beta)}{\beta^{\alpha-1}\Gamma(\alpha)e^{C/\beta}}$
Logistic	$\frac{e^{-(y-\alpha)/\beta}}{\beta(1+e^{-(y-\alpha)/\beta})^2}$	$\beta > 0, -\infty < y < \infty$	$c - \frac{c}{1+e^{\alpha/\beta}} - \beta log(\frac{1+e^{\alpha/\beta}}{1+e^{\alpha-C}})$
Rayleigh	$2\alpha y e^{-\alpha y^2}$	$\alpha > 0, 0 < y < \infty$	$C - \frac{\sqrt{\pi} erf(C\sqrt{\alpha})}{2\sqrt{\alpha}}$
Gumbel	$\frac{e^{-(y-\alpha)/\beta}}{\beta} Exp(e^{-(y-\alpha)/\beta})$	$\beta > 0, -\infty < y < \infty$	$\frac{C}{e^{e^{\alpha/\beta}}} + \beta E(-e^{\alpha/\beta}) - \beta E(e^{(\alpha-C)/\beta})$
Beta II	$\frac{1}{B(\alpha,\beta)} \frac{y^{\alpha-1}}{(1+y)^{\alpha+\beta}}$	$\alpha, \beta > 0, 0 < y < \infty$	$\frac{C^{1+\alpha} H(\alpha+\beta,\alpha,1+\alpha,-C)}{\alpha B(\alpha,\beta)} - \frac{C^{1+\alpha} H(\alpha+\beta,1+\alpha,2+\alpha,-C)}{(1+\alpha)B(\alpha,\beta)}$
Burr	$\alpha\theta\beta^\alpha y^{\theta-1}(\beta + y^\theta)^{-\alpha-1}$	$0 < y < \infty$	$C - \frac{\beta^\alpha C}{(\beta+C^\theta)^\alpha} - \frac{\alpha\theta C^{\theta+1} H(1+\alpha,1+1/\theta,2+1/\theta,-\theta C/\beta)}{\beta(1+\theta)}$
Weibull	$\alpha\beta y^{\beta-1} e^{-\alpha y^\beta}$	$\alpha, \beta > 0, y > 0$	$C - \int_0^C e^{-\alpha y^\beta} dy$
Pareto	$\frac{\alpha k^\alpha}{y^{\alpha+1}}$	$\alpha > 0, y \geq k > 0$	$(k^\alpha/C^{\alpha-1})/(\alpha - 1)$
Exponential	$\lambda e^{-\lambda y}$	$\lambda > 0, y > 0$	$-\frac{1-\lambda C}{\lambda} + \frac{C+(1-\lambda C)/\lambda}{e^{\lambda C}}$

Note: E(z) is the exponential integral, $E(z) = \int_{-z}^{\infty}(e^t/t)dt$, $H(.)$ is the hypergeometric function, and $erf(.)$ is the error function.