Modeling Inflation in String theory

by

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Abstract

The standard big bang theory (BB) is, at present, the best model one has to describe the evolution of our universe. Most of the predictions of the model have been verified to extraordinary precision. However, BB has some extreme fine-tuning problems. The inflationary theory was proposed as a way to resolve these problems. Inflation can be described as a period of very rapid expansion in the very early universe dominated mostly by some field(s) (known as inflaton field) potential energy V. For any successful inflationary model, the scalar potential has to be almost flat in some direction to drive inflation and so has to obey the slow-roll conditions. These conditions require that the slow-roll parameters be much less than unity.

In this thesis, we consider an approach to derive V for a specific toy model in the context of string theory. We study inflation in type IIB compactified on Calabi-Yau manifold since it comes with many classically flat moduli fields. Stabilization of these moduli fields is vital for any inflationary models in this context. Our starting point is to assume parts of the complex structure moduli and axio-dilation are stabilized by some classical mechanism such as flux compactification. The remaining unstabilized field(s) could then leave some almost flat directions after considering nonperturbative correction that could drive inflation. Thus, the resulting potential for these field(s) is derived. We show that the derived scalar potential in a toy model does indeed satisfy the slow-roll conditions, and so is a suitable candidate for inflation.

We also look at models of moduli stabilization in a region close to conifold singularity for different Kahler potentials. The masses and the warp factor for the complex structure moduli and axio-dilaton were calculated. We find evidence for runaway directions in the resulting scalar potential.

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Dedication

To my parents and my aunt.

Table of Contents

List of Figures

Chapter 1

Why we need Inflation

The modern era of cosmology began with the advent of Einstein's theory of gravity, which laid out the theoretical understanding of the universe, combined with the experimental observations, such as Slipher's and Edwin Hubble's discovery of the expansion of the universe [1] [2]. After the discovery of the expanding universe, Lemaitre began to speculate about the origin of the universe. As the universe expands, one can show that the density of matter and radiation decrease continuously as a^{-3} and a^{-4} respectively. Here a is the scale factor and it relates the physical coordinate r to comoving coordinate x so $r = ax$. Rewinding time backwards, Lemaitre postulated that the present universe must have emanated from a tiny, infinitely dense volume billions of years ago, expanding and cooling ever since, a postulate now known as the Standard Big Bang model.

The categories of evidence supporting the standard big bang model are the expanding universe, the existence of the cosmic microwave background (CMB) [3][1] as measured by the COBE (Cosmic Background Explorer) in 1989 [4] and then with WMAP in 2001 [5] and Planck in 2013 [6]. However, the model does not take into account a number of initial value problems such as uniformity (the Horizon Problem) and the flatness (the Flatness Problem).

The horizon problem is related to the communication between different regions of the universe. The universe is extremely homogeneous on a large scale. Given the finite age of the universe, light could only travel a finite distance at any time. However, looking at the cosmic microwave background, it can be seen that the temperature is very uniform over the entire sky, to an accuracy of 1 part in 10^5 [4]. At the time of last-scattering in the early universe, the photons were coupled to the matter, and the farthest neighbouring photons that could swap energy with each other at the speed of light is about $2°$ in the sky today [7]. So how can the parts of the sky that are very far apart (say $180°$) have reached thermal equilibrium?

[1] The CMB is the leftover radiation formed when the universe cooled enough so that the opaque plasma neutralized into a transparent gas.

Now to understand the flatness problem let's first define the density parameter $\Omega(t)$, which is a dimensionless quantity

$$\Omega(t) = \frac{\rho}{\rho_c}, \tag{1.1}$$

where ρ is the total energy density and ρ_c is the critical density. The critical density ρ_c is the amount of matter required for the universe to be spatially flat and is a function of time. The critical density at the present is defined as

$$\rho_c = \frac{3H^2}{8\pi G}, \tag{1.2}$$

where H is the Hubble constant and G is the Newton's Gravitational constant. Note that the ρ in eq. (1.1) is the total energy density of the universe. Therefore $\Omega(t)$ is the sum of normal matter Ω_B, dark matter Ω_D and dark energy Ω_Λ as suggested by recent observations:

$$\Omega(t) = \Omega_B + \Omega_D + \Omega_\Lambda. \tag{1.3}$$

Now from the following relation, which comes from the Friedmann equation (defined in chapter 2), we can draw some conclusions about the values of the density parameter at different times in the history of the universe

$$|\Omega(t) - 1| = \frac{|k|}{a(t)^2 H(t)^2}, \tag{1.4}$$

where $k = \pm 1/R^2$ sets the curvature radius and $H(t) \equiv \dot{a}(t)/a(t)$ is the Hubble parameter. From eq. (1.4), we see $k = 0$ is a special case as $k \to 0$ leads $\Omega \to 1$ and thus, it maintains 1 all the time. The recent observational evidence [8] suggests that the value of Ω currently is equal to $(1.012^{+0.018}_{-0.022})$ meaning that the present universe is spatially flat (or at least very close to being flat). However, this presents a problem, because if there is any deviation from it in the past, this difference would have grown over time. In particular, the difference between $\Omega(t)$ and 1 for matter and radiation is seen to grow over time as

$$\text{Matter} \begin{cases} a \propto t^{2/3} \\ \dot{a} \sim t^{-1/3} \end{cases} \Rightarrow |\Omega(t) - 1| \sim t^{2/3}, \tag{1.5}$$

$$\text{Radiation} \begin{cases} a \propto t^{1/2} \\ \dot{a} \sim t^{-1/2} \end{cases} \Rightarrow |\Omega(t) - 1| \sim t, \tag{1.6}$$

where the definition of the Hubble parameter is used to get the final relations. Eqs. (1.5, 1.6) tells us how the scalar factor evolution during the matter and radiation epoch. Let us calculate using eq. (1.4) how close to 1 the density parameter must have been at various early times, based on the constraint today. It turns out for Ω to be as close to one today as measured, corresponds to $|\Omega(10^{-44}s) - 1| = 10^{-60}$ at the Planck epoch and a value of $|\Omega(1s) - 1| = 10^{-16}$ at the time of neutrino decoupling. Neutrino decoupling refers to a period when the interaction rate between neutrinos and the other form of matter in the very early universe is smaller than the expansion rate of the universe. These values imply that the value of $(\Omega - 1)$ at early times have to be fine-tuned to values very close to zero, but without being exactly zero. Such an astonishing fine-tuning appears implausible in the big bang model.

In order to solve these problems, inflation was introduced by Alan Guth in the 80's [9]. Several models have been proposed since its first discovery. The characteristic feature of all the models is that the very early universe undergoes through a period of rapid expansion known as inflation. The mechanism for inflation is that the very early universe is permeated by a scalar field, known as the inflaton field. The energy density of the inflaton field is mostly dominated by its potential energy V that remains nearly constant. This causes the universe to expand at an exponential rate. As the field reaches the minimum of the potential, the rate of the expansion slows down, eventually ending the inflationary epoch. At the end of inflation, the energy stored in the inflaton field decays producing radiation and matter due to the interaction with other matter fields. This is known as reheating. The universe from this point onwards is radiation dominated and therefore follows the standard big bang paradigm. The nature of the inflaton field and the mechanism perturbing it from its initial condition is unknown at present.

This simple hypothesis resolves the above problems as follows. In the horizon problem, uniformity is reached initially on microscopic scales by normal thermal equilibrium processes before inflation. Then the exponential expansion stretches the regions of microscopic uniformity to become large enough to encompass the observed universe and more. As the new regions outside our cosmological horizon come into view during normal expansion, these are precisely the same regions from the original small patch of space before inflation, and so they are at nearly the uniform temperature. Moreover, it resolves the flatness problem because the universe expands at such an extraordinary rate. The scale factor in eq. (1.4) increases by a considerable amount due to exponential expansion during inflation, while H is $\sim$ constant and thus the RHS approaches zero driving Ω close to one.

Including the role of quantum mechanics, inflation stretches the tiny quantum fluctuations to the macroscopic scale, thus producing the perturbations we see in the CMB, which later resulted into formation of large-scale structure.

Although inflation does solve the problems initially stated, there remain some unanswered questions. There is no explanation for the origin of the effective scalar potential and it is an open question as to how to embed inflation in a theory of fundamental physics. Due to the incompleteness of a theory, a standard practice has been a phenomenological approach where an effective scalar potential $V(\phi)$ is postulated. A complete inflationary model should explain the origin of the potential.

One way to approach the problem is to embed inflationary models in higher dimensional models such as string theory and derive the form of the potential. String theory provides clear motivation for considering inflationary models that have many scalar fields. Our aim in this thesis is to consider this approach to explicitly derive the potential $V(\phi)$ for a specific model of inflation in string theory. Inflation requires a sufficiently flat potential, as measured by the smallness of the two slow-roll parameters. Thus not all potentials lead to inflation. In considering this approach, we also aim to answer questions such as how often do we get flat potentials and thus the possibility of a scalar field(s) reaching these areas.

A brief outline of this thesis is as follows:

Chapter 2: In this chapter, we review the necessary mathematical foundations of modern cosmology. Furthermore, we derive some conditions that a successful inflationary model has to satisfy.

Chapter 3: This chapter is a short introduction to string cosmology.

Chapter 4 and 5: Here, we present an explicit derivation of $V(\phi)$ for a specific toy model and show that it satisfies the conditions derived in chapter 2. In chapter 5, we look at some more realistic models of moduli stabilization for type IIB in a region close to conifold singularity.

Chapter 2

Overview of Inflation

We review the nature of inflation and a number of conditions that successful inflationary models should obey. This includes deriving these slow-roll conditions and associated slow-roll parameters for multiple fields.

The universe is isotropic and homogeneous on a large scale. This is known as *the cosmological principle* and is described by the Friedmann-Robertson-Walker (FRW) metric, eq. (2.1). The evolution of the universe can be described by the Friedman eqs.(2.2, 2.3):

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right], \tag{2.1}$$

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \tag{2.2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{2.3}$$

The scale factor, a describes the scale of the universe at a given time, ρ the energy density of the universe, Λ the cosmological constant, $d\Omega^2$ the two sphere metric, and k the curvature radius. The evolution of the energy density ρ of the universe is described by the continuity equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0. \tag{2.4}$$

To use the above equation, we also need to specify a relationship between pressure p and the energy density ρ. These are related by the equation of state

$$p = w\rho, \tag{2.5}$$

where w is a constant.

Let's now quantify the idea of inflation mathematically. Consider a slowly-rolling real homogenous scalar field ϕ evolving in a potential $V(\phi)$ as described by the following Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \tag{2.6}$$

The canonical energy momentum tensor $T_{\mu\nu}$ is given by

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}L = \partial^\mu\phi\partial^\nu\phi - \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right). \tag{2.7}$$

Comparing the above to $T^{\mu\nu}$ for a perfect fluid, namely

$$T^{\mu\nu} = diag(\rho, p, p, p), \tag{2.8}$$

we get the following energy density and pressure of the scalar field

$$\rho(\phi) = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p(\phi) = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{2.9}$$

where the first term is just the kinetic energy density in the above two expressions, and the second term is the potential energy density. Using these equations one can restate the Friedman equations in terms of the scalar field

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{2.10}$$

$$\ddot{a} = -4\pi G a\left(\frac{1}{3} + w\right)\rho. \tag{2.11}$$

Here we define the reduced Planck mass $M_p = 1/\sqrt{8\pi G}$ and henceforth will set $M_p = 1$ for simplicity unless otherwise stated. Also, the equations of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{2.12}$$

for the scalar field can be derived from the action, where $V'(\phi) = dV/d\phi$. As can be noted eq. (2.12) is similar in form to the equation of motion of damped harmonic oscillator with a friction term $3H\dot{\phi}$, resulting from the expansion. This equation is a form of the Klein-Gordon equation.

For a successful inflation one requires

$$\dot{\phi}^2 < V(\phi), \quad |\ddot{\phi}| < |V'(\phi)| \tag{2.13}$$

i.e for inflation to take place in the very early universe, the first condition is that the potential energy density should dominate the kinetic energy density. That gives some amount of inflation. The second condition ensures that the inflation can lasts long enough to solve various cosmological problems, as discussed in chapter 1. Note that the above conditions are possible as long as the potential is flat enough to allow the inflaton field to roll slowly towards the minima, where inflation ceases. The conditions shown above are called the *slow-roll conditions or slow-roll approximations*, these provide natural conditions for any inflationary model.

In this thesis, we work with multiple scalar fields; thus, we have a scalar field space. These scalar fields have indices associated with it. That means we need a scalar field metric K_{ab} to raise and lower indices. To this end, it is important to note that we are dealing with two types of metrics – a spacetime metric $g_{\mu\nu}$, where the indices run over the usual four spacetime indices and a scalar field metric K_{ab}, where the indices run over the scalar fields. All of the above equations need to include this metric K_{ab}, which appears in the following action in the kinetic energy term in the scalar field Lagrangian:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}g^{\mu\nu}K_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V\right), \tag{2.14}$$

where R is the Ricci scalar. With this metric, we now have the following energy density and the pressure

$$\rho(\phi_a) = \frac{1}{2}K_{ab}\dot{\phi}^a\dot{\phi}^b + V(\phi_a), \quad \text{and} \quad p(\phi_a) = \frac{1}{2}K_{ab}\dot{\phi}^a\dot{\phi}^b - V(\phi_a). \tag{2.15}$$

Plugging this into the Friedman equation as before, we get the Hubble parameter for multiple fields

$$H^2 = \frac{1}{3}\left(\frac{K_{ab}\dot{\phi}^a\dot{\phi}^b}{2} + V(\phi_a)\right). \tag{2.16}$$

The slow roll conditions shown earlier are conveniently handled by introducing the *slow-roll parameters* for multiple scalar fields. These can be expressed in terms of the scalar potential as

$$\epsilon = \frac{1}{2}\frac{K^{ab}\partial_a V\partial_b V}{V^2}, \quad \text{and} \quad N^a_b = \frac{K^{ac}(\partial_c\partial_b V - \Gamma^d_{cb}\partial_d V)}{V}. \tag{2.17}$$

In particular, N^a_b is a matrix and η is defined as the most negative eigenvalue of this matrix. Successful inflation is possible if and only if the potential is flat and has minimal curvature. What this means for the slow-roll parameters is that inflation occurs as long as $\epsilon, |\eta| < 1$, as a consequence of slow-roll conditions.

Let's now derive these equations for ϵ and η from the given conditions. If we use the slow roll conditions given in eq. (2.13) with the Friedman eq. (2.16) and KG equation eq. (2.12), we are left with the following terms

$$H^2 \simeq \frac{V}{3}, \quad \text{and} \quad 3H\dot{\phi}^b \simeq -\partial_a V K^{ab}. \tag{2.18}$$

Squaring the second equation and replacing H using the first equation, we get

$$\dot{\phi}^b \dot{\phi}_b \simeq \frac{\partial_a V \partial^a V}{3V}. \tag{2.19}$$

Now we can define the ϵ as a ratio of kinetic to potential energy but up to a factor 3. Then using eq. (2.19), we get the desired result

$$\epsilon = 3\left(\frac{K_{ab}\dot{\phi}^a\dot{\phi}^b}{2V}\right) = \frac{1}{2}\frac{K^{ab}\partial_a V \partial_b V}{V^2}. \tag{2.20}$$

This approximation must eventually fail for inflation to end.

To derive the second slow-roll parameter η, we use the KG equation analogus to geodesic equation A.9 derived in appendix (A)

$$\ddot{\phi}^e + \dot{\phi}^a\dot{\phi}^b\Gamma^e_{ab} + 3H\dot{\phi}^e + \partial_a V K^{ae} = 0. \tag{2.21}$$

Based on the slow-roll conditions, we are making an approximation that eq. (2.21) simplifies to just last two terms. So we can now estimate the size of the first two terms

$$\ddot{\phi}^a + \Gamma^a_{bc}\dot{\phi}^c\dot{\phi}^b, \tag{2.22}$$

to see if they are small. Now the connection Γ^a_{bc} can be replaced with its usual definition in field space. Expanding out the terms and swapping some indices leads to

$$\ddot{\phi}^a + K^{ad}\partial_c K_{db}\dot{\phi}^c\dot{\phi}^b - \frac{1}{2}K^{ad}\partial_d K_{bc}\dot{\phi}^b\dot{\phi}^c. \tag{2.23}$$

Now multiplying the above expression by $-3H$, then adding and subtracting $1/2K^{ad}\partial_b K_{dc}\dot{\phi}^b\dot{\phi}^c$ gives

$$-3H\left[\ddot{\phi}^a + K^{ad}\partial_c K_{db}\dot{\phi}^c\dot{\phi}^b - \frac{1}{2}K^{ad}\partial_d K_{bc}\dot{\phi}^b\dot{\phi}^c + \frac{1}{2}K^{ad}\partial_b K_{dc}\dot{\phi}^b\dot{\phi}^c - \frac{1}{2}K^{ad}\partial_b K_{dc}\dot{\phi}^b\dot{\phi}^c\right]. \tag{2.24}$$

Swapping the indices on the last term $c \leftrightarrow b$ we get

$$-3H\left[\ddot{\phi}^a + K^{ad}\partial_c K_{db}\dot{\phi}^c\dot{\phi}^b\right] + 3H\left[\frac{1}{2}K^{de}K^{ac}(\partial_c K_{be} + \partial_b K_{ce} - \partial_e K_{bc})K_{df}\dot{\phi}^f\dot{\phi}^b\right]. \tag{2.25}$$

Here we see that the first term is simply the time derivative of the KG equation eq. (2.18), given H is approximately constant. Notice that the second terms just gives the Christoffel symbol, so we have

$$K^{ac}\partial_c\partial_b V\dot{\phi}^b + 3HK_{df}\dot{\phi}^f\dot{\phi}^b\Gamma^d_{cb}. \tag{2.26}$$

This quantity is most negative when $\dot{\phi}^b$ is the eigenvector with most negative eigenvalue. So the eigenvalue tells us how quickly the field rolls away from a particular spot on V. We can compare this to eq. (2.18), which tells us how quickly the universe is inflating at that instance. This gives the desired result

$$N^a_b = \frac{K^{ac}(\partial_c\partial_b V - \Gamma^d_{cb}\partial_d V)}{V}. \tag{2.27}$$

Why are slow-roll parameters so vital for inflation? They are vital because they restrict the many kinds of potentials that give rise to inflation. During inflation the slow-roll parameters can be considered to be approximately constant since the potential is flat. The parameter ϵ measures the slope of the potential. Meanwhile, η measures the curvature which indicates how long the potential stays flat enough for inflation to last. Violation of either of these slow-roll parameters means either no inflation or the end of inflation.

In addition to resolving the horizon and flatness problems, inflation also provides an explanation to the observed spectrum of the density perturbations in the CMB Planck results [10]. The density perturbations are responsible for the formation of large scale structures. Associated with this is an important observational parameter, the tilt $n_s - 1$ of the spectrum of the perturbation [10]. The tilt is related to the slow-roll parameters as $n_s = 1 - 6\epsilon + 2\eta$ [11]. The observational value of the tilt from the recent Planck result gives $n_s = 0.9649 \pm 0.0042$ [10]. We immediately see that this is possible if and only if $\epsilon < 1$ and η is small. So in general, inflation requires ϵ to be less than unity, which the recent Planck analysis constrains to $\epsilon < 0.0097$ [10], and η to be small. Note that ϵ is positive by definition, whilst η can have either sign. We will look for $\epsilon <\ 0.01$ as a rough estimate of meeting the CMB constraint.

To summarize, slow-roll conditions were used to derive two slow-roll parameters ϵ eq. (2.20) and η eq. (2.27), which the potential has to satisfy. These equations will be used in the later chapters for calculations.

Chapter 3

String Inflation

String theory is a theory of ten spacetime dimensions. However, we only have three spatial dimensions plus time. The other six of these dimensions are assumed to be curled up into tiny bundles, i.e. compactified. The Calabi-Yau (CY) manifolds $\mathcal{M}_6$ are the most commonly studied compactifications because of their simplicity. CY manifolds are complex manifolds with an even number of dimensions and vanishing Ricci tensor. Some examples of CY manifolds are the complex plane and the 2-torus in two (real) dimensions. In six (real) dimensions, there are many CY manifolds known, and there is no way of knowing at present which of these manifolds might lead us to our 4D world. The metric of these 6D compact manifolds contains moduli which are scalar fields in low-energy 4D effective theory, among which an inflaton field might exist.

The moduli are the parameters characterizing the shape and size of the extra dimensions, and the low-energy 4D effective theories depend on the vacuum expectation values (VEVs) of these moduli fields. An example is the expectation value of the dilaton field Φ describing the string interaction strength [12]. However, as string theory is compactified to lower dimensions, it turns out that these moduli fields (which are now scalar fields) are massless and their VEVs cannot be determined because the potential is flat in the effective four-dimensional theory. This is the famous moduli stabilization problem. This problem seems to have been resolved in the context of flux compactifications of type IIB string theory[1] [13,14]. Here we summarize the basic elements of flux compactification. For more details see [15,16].

If we want to find a viable inflationary model in string theory, the first task is to identify a suitable inflaton candidate among the moduli in a particular direction. It turns out compact CY three-folds, in general, come with many classically flat directions [17] (useful for inflation models as the potential needs to be almost flat) in the scalar potential. These classically flat directions correspond to the moduli fields of the 4D scalar fields that parameterize the

[1]The fluxes are the higher dimensional generalization of EM fields and are pointed only along the extra dimension. Flux compactification is simply introducing fluxes in the compactification.

shape and size of the compact dimensions.

The 4D low-energy effective description can be written as a 4D supergravity theory, which is a locally supersymmetric theory incorporating gravity. The 4D supergravity then describes the dynamics of these moduli fields, which are charaterized by the superpotential W and the Kahler potential K. Specifying these, the dynamics of the low-energy scalars is given by the Lagrangian density

$$\mathcal{L} = -K_{i\bar{\jmath}}\partial_\mu \phi_i \partial^\mu \overline{\phi}_{\bar{\jmath}} - V(\phi, \overline{\phi}), \tag{3.1}$$

where indices $i, \bar{\jmath}$ run over all moduli fields and their complex conjugates. The total scalar potential V for type IIB compactified on CY three-fold consists of sum of F and D-terms:

$$V = V_F + V_D. \tag{3.2}$$

The first term comes from the standard N= 1 supergravity formula for the F-term potential [12] and is given as

$$V_F = e^K \left(K^{i\bar{\jmath}} D_i W \overline{D_{\bar{\jmath}} W} - 3|W|^2 \right). \tag{3.3}$$

Here K is the Kahler potential, whose second derivative with respect to the fields, $K_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} K$, defines the kinetic terms of the scalars in $\mathcal{L}$. $K_{i\bar{\jmath}}$ is the Kahler metric on the moduli space with its inverse $K^{i\bar{\jmath}}$, W is the superpotential. Moreover D_i is the covariant derivative defined as, $D_i = \partial_i + \partial_i K$. The resulting scalar potential depends on the indepedent real fields given by the real and imaginary parts of the moduli fields.

The second part of the scalar potential, coined the D-term, can either be induced by the tension of the anti-D3 branes or various other sources [18, 20, 25]. Dp-branes in type II string theory are dynamical $(p+1)$-dimensional objects (with p spatial dimensions) on which the ends of the open strings can be confined. An Anti-Dp-branes are simply the opposite of the Dp-branes in nature. This term is given as follows for the anti-D3 case

$$V_D = \frac{D}{(\rho - \bar{\rho})^2}, \tag{3.4}$$

where ρ is the Kahler modulus to be described below, and D is some positive constant that depends on the number of anti-D3-branes and potentially warp factor discussed in chapter 5. We assume that the value of D is fine-tuned to give a small positive cosmological constant. Also we should note that the potential contribution for the D-branes cancels but adds for the anti-branes. The exponent on the denominator can either be 2 (as considered in [18]) or 3 as in [25]. This depends on the position of the anti-D3 brane in the CY manifold.

All moduli may be divided into three major groups- the complex structure moduli t, the Kahler moduli ρ responsible for the sizes of the CY, and the axio-dilaton τ. The stabilization of these massless moduli fields and compactification are essential for any inflationary models in string theory. Early attempts to derive cosmological inflation from string theory assume that all the moduli were fixed by some unknown mechanism except the dilaton and the volume modulus. These were considered as inflaton fields. However, it was later shown that the models were problematic as the nonperturbative potentials considered were not flat enough in order to satisfy slow-roll parameters. An alternative solution was proposed, D-brane inflation [21–23], which instead considers a separation between two branes and the lowest mode of the open string connecting the two branes is considered as an inflaton field. However, such models required extraordinary fine-tuning of the parameters of the model. Some of these problems were resolved in yet another exciting proposal where the scenario instead consists of a brane and an anti-brane (to be more specific a D3/anti-D3 branes system located at a specific point of a CY three-fold or in a warped geometry) as the inflaton candidates, including moduli stabilization and warp factor from flux compactifications [24–27].

So how does one stabilize these fields? The fluxes, which are the magnetic fields in the compact space, in general stabilize the axio-dilaton τ and complex structure moduli, but leave the Kahler/volume moduli unstabilized. The existence of flux induces the superpotential for the complex structure moduli and axio-dilaton given by [14]

$$W = \int_{\mathcal{M}_6} G_3 \wedge \Omega. \tag{3.5}$$

The complex 3-form field strength $G_3 \equiv F_3 - \tau H_3$ and Ω denotes the holomorphic three-form on the CY three-fold that only depends on the complex coordinates. The $F_3 \equiv dC_2$ is the 2-form RR field strength and $H_3 \equiv dB_2$ is the NS-NS 2-form/Kalb-Ramond field. Finally the τ in G_3 is the axio-dilaton defined as $\tau \equiv C_0 + ie^{-\Phi}$. Turning on the G_3-flux induces V_F that fixes the complex structure moduli and the dilaton.

The Kahler moduli are stabilized by introducing a stack of D7 branes in flux compactification as shown by KKLT [18]. These branes, in turn, give rise to the non-perturbative effects with an exponential that depends on ρ, leading to the overall superpotential in type IIB flux compactification as follows

$$W = W_0(t, \tau) + f e^{ia\rho}, \tag{3.6}$$

where W_0 is the tree level contribution from the fluxes, the coefficient a in the exponential is a constant that depends on the sources of the nonperturbative effects, and f is an unknown complex function of t and τ. Here we consider only one exponential correction to W, but there can be multiple exponentials in general [18, 19]. The corrections to the superpotential considered above are what stabilize the Kahler moduli.

We can now define the Kahler potential associated with these moduli fields. The Kahler potential we consider here goes into the supergravity theory. The Kahler potential can be computed from the dimensional reduction of the 10D type IIB action. Doing this we get the total Kahler potential of the form

$$K = K(\rho) + K(\tau) + K(t), \tag{3.7}$$

where each of the terms corresponds to

$$K = -3\log\left[-i(\rho-\bar{\rho})\right] - \log\left[-i(\tau-\bar{\tau})\right] - \log\left[-i\int_M \Omega\wedge\bar{\Omega}\right]. \tag{3.8}$$

It is a function of the fields and their conjugates. In general, one also needs to take into account the corrections to the Kahler potential (see for instance [28]). However, they can be ignored in the large-volume limit as it is done in this thesis. Interestingly, the perturbative corrections to the Kahler potential and the superpotential stabilize the remaining Kahler moduli of type IIB flux compactification as shown in [29, 30].

For the low-energy moduli obtained from the flux compactification, the superpotential is independent of the compactification moduli so the Kahler potential has the important property that

$$K^{\rho\bar{\rho}}\partial_\rho K\partial_{\bar{\rho}}K = 3. \tag{3.9}$$

This is referred to as no-scale property of a Kahler potential. Note we consider only one Kahler modulus ρ in this thesis. So eq. (3.5) together with the no-scale property implies that

$$\left(K^{\rho\bar{\rho}}D_\rho W\overline{D_{\bar{\rho}}W} - 3|W|^2\right) = 0$$

So, the resulting V_F potential takes the form

$$V_F = e^K\left(K^{i\bar{j}}D_iW\overline{D_{\bar{j}}W}\right), \tag{3.10}$$

where $i, \bar{j}$ sum over all the moduli but ρ. This potential is manifestly positive and minimized for $D_iW = 0\ \forall i$; hence the Kahler moduli are not fixed by the fluxes themselves. This means $V_F = 0$ at the minimum. As a result, V_F is often known as no-scale type.

Few things to note about the scalar potential. Firstly, the potential has a nontrivial minimum for these fields, and these occur at large values of the moduli provided that W_0 is small enough. We want W_0 to be small enough so that it is comparable with the nonperturbative contributions to W near the minimum. The scalar potential V_F at the minimum corresponds to anti-de Sitter (AdS) spacetime, which has negative cosmological constant. This contradicts with the observation of an expanding universe, which would instead require a small positive cosmological constant. In order to prevent this situation, KKLT introduced an additional $\overline{D3}$ brane (hence the additional D-term we defined earlier) whose effects add a positive contribution to the overall scalar potential through the value of D. This uplifts the AdS minimum to a deSitter (dS) minimum with a positive cosmological constant.

Our primary concern in this thesis is to derive inflation from string theory and the approach considered here is that the inflaton field may be one or more of the moduli fields mentioned above that may have been left unstabilized. For typical choices of G_3, all the complex structure moduli and axio-dilaton are stabilized by fluxes. In particular, we are looking for exceptional cases where part of the complex structure and axio-dilaton are not stabilized by flux. The potential resulting from the F and D-term in a context of slow-roll is examined very carefully, to ensure that the unstabilized fields play the role of the inflaton.

Chapter 4

Scalar Potential for toy models

4.1 Deriving the Scalar Potential

In the preceding chapter, we defined the Kahler potential and superpotential in general. We will now discuss possible inflationary potentials in a simplified toy model where the flux does not stabilize all the complex structure moduli. The starting point for the toy model is a factorized 6D torus with the Kahler moduli and complex structure moduli set equal. The field t relates to torus shape, ρ describes the torus volume, and τ controls the string interaction strength. The Kahler potential from chapter 3 is shown here again, but now the last term is replaced with the term specific to our case

$$K = -3\Big(\log[-i(t-\bar{t})] + \log[-i(\rho-\bar{\rho})]\Big) - \log[-i(\tau-\bar{\tau})]. \tag{4.1}$$

The W_0 in the superpotential in eq. (3.6) is replaced with $t^3 - \tau$

$$W = t^3 - \tau + f(t;\tau)e^{ia\rho}. \tag{4.2}$$

Note that this is the case for a single component each of F_3 and H_3 so that the superpotential doesn't stabilize all real and imaginary parts of t and τ. Having defined the Kahler potential and superpotential, we are now in a position to calculate the scalar potential explicitly. The scalar potential from chapter 3 is presented here again

$$V_F = e^K \left[K^{i\bar{j}} D_i W \overline{D_{\bar{j}} W} - 3|W|^2\right]. \tag{4.3}$$

The exponential factor is simply given by

$$e^K = \frac{-i}{(\tau-\bar{\tau})(t-\bar{t})^3(\rho-\bar{\rho})^3}. \tag{4.4}$$

With the terms inside the bracket of eq. (4.3), the Kahler metric with respect to the fields are calculated as

$$K^{t\bar{t}} = -\frac{1}{3}(t-\bar{t})^2, \quad K^{\tau\bar{\tau}} = -(\tau-\bar{\tau})^2, \quad K^{\rho\bar{\rho}} = -\frac{1}{3}(\rho-\bar{\rho})^2. \tag{4.5}$$

The superpotential term is

$$-3|W|^2 = -3\left(\overline{f}e^{-ia\overline{\rho}} + \overline{t^3} - \overline{\tau}\right)\left(fe^{ia\rho} + t^3 - \tau\right). \tag{4.6}$$

The covariant derivatives are given by

$$\begin{aligned} D_tW &= \partial_t f e^{ia\rho} - \frac{3}{t-\bar{t}}W + 3t^2, \\ D_\tau W &= \partial_\tau f e^{ia\rho} - \frac{W}{\tau-\overline{\tau}} - 1, \\ D_\rho W &= -\frac{3}{\rho-\overline{\rho}}W + iafe^{ia\rho}. \end{aligned} \tag{4.7}$$

Calculating the associated complex part and putting together everything gives the potential of the form

$$\begin{aligned} V_F(t,\tau,\rho,f) = &\frac{-i}{(\rho-\overline{\rho})^3(t-\bar{t})^3(\tau-\overline{\tau})}\Bigg[-\frac{1}{3}(\rho-\overline{\rho})^2\left|iafe^{ia\rho} - \frac{3\left(fe^{ia\rho}+t^3-\tau\right)}{\rho-\overline{\rho}}\right|^2 \\ &- (\tau-\overline{\tau})^2\left|-\frac{fe^{ia\rho}+t^3-\tau}{\tau-\overline{\tau}} + e^{ia\rho}\partial_\tau f - 1\right|^2 \\ &- 3(t-\bar{t})^2\left|-\frac{fe^{ia\rho}+t^3-\tau}{t-\bar{t}} + e^{ia\rho}\partial_t f + t^2\right|^2\Bigg]. \end{aligned} \tag{4.8}$$

Here we see that the potential is a function of complex and conjugate fields. So the next task is to rewrite the potential as a function of real variables. To do so we write the fields in terms of their real and imaginary parts

$$t = \beta + ie^{-\alpha}, \quad \tau = c + ie^{-\phi}, \quad \rho = b + ie^{-u}. \tag{4.9}$$

where $\beta, \alpha, c, \phi, b$ and u are all real variables. The imaginary parts are all positive because e^{-u} describes the volume of the extra dimenstion, $e^{-\phi}$ controls the string interacting strength and $e^{-\alpha}$ gives the ratio of the two sides of a two dimensional torus. We set $f = 0$ to examine how the leading flux term stabilizes τ and complex structure moduli. This gives

$$V_F = \frac{1}{32}e^{3(\alpha+u)+\phi}\left[\left(c-\beta^3\right)^2 + 3e^{-2\alpha}\beta^4 + 3e^{-4\alpha}\beta^2 + \left(e^{-3\alpha} - e^{-\phi}\right)^2\right]. \tag{4.10}$$

Since we want to minimize the potential, eq. (4.10) indicates that the variables should satisfy the following conditions

$$\beta = c = 0, \quad \phi = 3\alpha. \tag{4.11}$$

However, away from the minimum, the potential is too steep for inflation. The scalar potential does not stabilize all moduli at its minimum. In particular α is not stabilized. We turn on the nonperturbative terms as it can stabilize the remaining moduli, and we examine how steep it is. Substituting t, τ and ρ defined earlier into eq. (4.8), we get the following simplified potential:

$$\begin{aligned} V_F(\alpha, u, b, f) \;=\; & \frac{1}{128}e^{6\alpha+3u}\left(\frac{4}{3}e^{-2u}\left|iae^{ia(b+ie^{-u})}f + \frac{3ie^{u}e^{ia(b+ie^{-u})}f}{2} + 3e^{u}e^{-3\alpha}\right|^2 + \frac{4}{3}e^{-2\alpha}\right. \\ & \left|\partial_t f e^{ia(b+ie^{-u})} + \frac{3ie^{\alpha}fe^{ia(b+ie^{-u})}}{2}\right|^2 - 3\left|2ie^{-3\alpha} + fe^{ia(b+ie^{-u})}\right|^2 \\ & \left. +4e^{-6\alpha}\left|\partial_\tau f e^{ia(b+ie^{-u})} + \frac{ie^{3\alpha}fe^{ia(b+ie^{-u})}}{2}\right|^2\right). \end{aligned} \tag{4.12}$$

The function f implicity depends on α through the complex struture moduli and axio-dilaton. We can compute the derivatives using $\alpha = -ln\,[(t-\bar{t})/2i]$ and $\beta = [(t+\bar{t})/2]$. Let $f(t,\tau) = g(\alpha)$, we can get the derivatives by simply inverting t and $\bar{t}$. So using chain rule we have

$$\begin{aligned} \partial_t f &= (\partial_t\beta)\partial_\beta f + (\partial_t\alpha)\partial_\alpha f = \frac{1}{2}\partial_\beta f - \frac{1}{(t-t^*)}\partial_\alpha f = \frac{1}{2}\partial_\beta f + \frac{i}{2}e^{\alpha}\partial_\alpha f = \frac{1}{2}\Big(h(\alpha) + ie^{\alpha}g_1'(\alpha)\Big), \\ \partial_\tau f &= (\partial_\tau c)\partial_c f + (\partial_t\phi)\partial_\phi f = \frac{1}{2}\partial_c f + \frac{i}{2}e^{\phi}\partial_\phi f = \frac{1}{2}\left(q(\alpha) + \frac{i}{3}e^{3\alpha}g_2'(\alpha)\right). \end{aligned} \tag{4.13}$$

The last equalities are the definition of h, g_1', q, g_2'. The idea behind why we define it this way is that the relations between ϕ and α are set by the higher energy physics (as well as $\beta = c = 0$), so we now effectively have one remaining variable at low energies. The idea now is to set the derivatives to separate functions and then we pick functions to test robustness of inflation in this type of inflation. Replacing the derivatives with those in eq. (4.12) with eq. (4.13) gives

$$\begin{aligned} V \quad = \quad & \frac{1}{128}e^{3u+6\alpha}\left[\frac{4}{3}|g|^2 e^{-2ae^{-u}}\left(a^2e^{-2u}+3ae^{-u}+3\right)+e^{-2ae^{-u}}\left(\frac{1}{3}|h|^2e^{-2\alpha}+|q|^2e^{-6\alpha}\right)\right. \\ & -4ae^{-ae^{-u}-u-3\alpha}\left(ige^{iab}-i\bar{g}e^{-iab}\right)+ie^{-2ae^{-u}-\alpha}\left(\bar{h}\left(g+\frac{g_1'}{3}\right)-\left(\bar{g}+\frac{\bar{g'}_1}{3}\right)h\right) \\ & +ie^{-2ae^{-u}-3\alpha}\left(\bar{q}\left(g+\frac{g_2'}{3}\right)-\left(\bar{g}+\frac{\bar{g'}_2}{3}\right)q\right)+\frac{1}{9}e^{-2ae^{-u}}\left(3|g_1'|^2+|g_2'|^2\right) \\ & \left.+e^{-2ae^{-u}}\left(g\left(\bar{g'}_1+\frac{\bar{g'}_2}{3}\right)+\bar{g}\left(g_1'+\frac{g_2'}{3}\right)\right)\right]-\frac{D}{4e^{-u}}. \end{aligned} \tag{4.14}$$

The last term is the D-term.

Having successfully derived the scalar potential explicitly, now we can find the scalar field metric in terms of real variables, as this will be used in the slow-roll parameters. To do so, we consider the following action for the multiple scalar fields

$$S=\int d^4\sqrt{-g}[K_{i\bar{j}}\partial_\mu\phi^i\partial^\mu\overline{\phi^{\bar{j}}}-V]. \tag{4.15}$$

Here the indices run over all the moduli. To summarize the journey so far, we initially considered complex fields (t,τ,ρ). The scalar potential was then found in terms of these complex fields. However, we then defined the fields in terms of real and imaginary parts and the scalar potential was then derived in terms of these variables. We then simplified the potential by minimizing it in steps. At each step, we reduced the number of variables in the model.

Now we can substitute t, τ and ρ defined earlier into this action and impose eq. (4.11). Doing this, we get the following action in terms of the three light real variables.

$$S(\alpha,u,b)=\int d^4x\sqrt{-g}\left[K_{t\bar{t}}e^{-2\alpha}\partial_\mu\alpha\partial^\mu\alpha+9K_{\tau\bar{\tau}}e^{-6\alpha}\partial_\mu\alpha\partial^\mu\alpha+K_{\rho\bar{\rho}}\left(\partial_\mu b\partial^\mu b+e^{-2u}\partial_\mu u\partial^\mu u\right)\right]. \tag{4.16}$$

To find the scalar field metric, we simply multiply the correspoinding terms in the action with eq. (4.5). The metric turns out to be a diagonal 3×3 metric given below

$$K_{AB}=\begin{pmatrix} 3 & 0 & 0 \\ 0 & \frac{3}{4} & 0 \\ 0 & 0 & \frac{3}{4}e^{2u}\end{pmatrix}, \tag{4.17}$$

where indices $A,B\in\{\alpha,u,b\}$. We will use this metric to calculate both the slow-roll

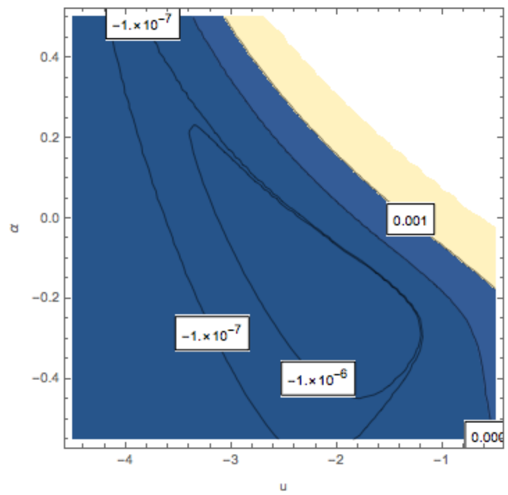


Figure 4.1: Plot of the potential with $f(t,\tau) = f_0\tau t^2$ as a function of two variables (u, α). The value of the potential and the size depend on the values used for a and f_0. The figure displays a minimum at which the potential is negative leading to an AdS vacuum.

parameters.

4.2 Scalar potential with $f(t,\tau) = f_0\tau t^2$

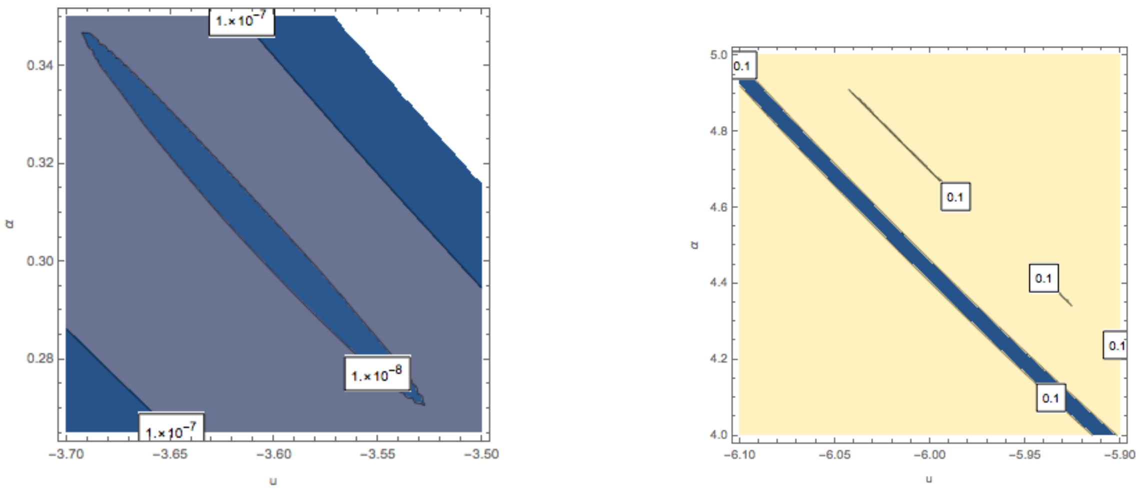
To start with, we consider a simple form of $f(t,\tau) \equiv g(\alpha) = f_0\tau t^2$ and investigate the scalar potential. Computing the derivatives of f with respect to the fields and simplying gives

$$\begin{aligned}
f(t,\tau) &\equiv g(\alpha) = f_0\tau t^2 = -if_0 e^{-5\alpha},\\
\partial_\beta f &\equiv h(\alpha) = -2f_0 e^{-4\alpha},\\
\partial_\alpha f &\equiv g_1'(\alpha) = 2if_0 e^{-5\alpha},\\
\partial_c f &\equiv q(\alpha) = -f_0 e^{-2\alpha},\\
\partial_\phi f &\equiv g_2'(\alpha) = if_0 e^{-5\alpha}.
\end{aligned} \tag{4.18}$$

and their conjugate respectively. Putting everything together into eq. (4.14), we have the following simple potential:

$$\begin{aligned}
V_F(\alpha, u, f_0 = 1, a = 1/10, b) \;=\; & \frac{e^{4\alpha+u-2ae^{-u}}}{1152}\Big[12a^2 e^{12\alpha} - 72a\cos(ab)e^{ae^{-u}+4\alpha+u} + 36ae^{12\alpha+u} \\
& + \left(-24e^{6\alpha} + 163e^{12\alpha} + 9\right)e^{2u}\Big].
\end{aligned} \tag{4.19}$$

The f_0 is defined as a constant, expected to be of order 1, and the value of a is chosen as



(a) Contour plot of the potential including a V_D term with $D = 0.00369152$ which uplifts the AdS minimum to a dS-minima.

(b) Contour plot of ϵ away from the dS-minimum of the scalar potential.

Figure 4.2: Plots of potential with $f(t, \tau) = f_0 \tau t^2$ and the slow-roll parameter ϵ as a function of u.

in [18]. The value of ab is set to 0 at the minimum as this minimizes the cosine term. The scalar potential is manifestly real but now has the property that it depends only on two varibles (α, u).

Now we plot the potential as a function of u and α to locate the minima. From fig.4.1, we see that there is a minimum at negative energy corresponding to an anti-deSitter (AdS) universe. The value of the potential at the minimum is

$$V_{AdSMin} = -9.56554 \times 10^{-6}. \tag{4.20}$$

Now we consider the contribution of $V_D = D/(\rho - \bar{\rho})^2$; it gives a small positive energy to the potential. For suitable choice of D the AdS minimum is uplifted to a dS minimum as shown in fig.4.2a. This term uplifts the potential without changing the shape too much around the minimum. The value of D is choosen so that the new minimum is small but positive. With the value of $D = 0.00369152$, we get the uplifted minimum at

$$V_{dSMin} = 1.00 \times 10^{-8}. \tag{4.21}$$

We now look at the slow-roll parameters to examine the dynamics of the fields as it rolls towards the minima. Particularly, we can calculate the slow-roll parameters, for which the scalar energy density is dominated by potential energy rather than its kinetic energy. The first slow-roll parameter ϵ can be calculated by using eq. (2.20) from chapter 2. A necessary condition for any inflationary toy models is to ensure that the slow-roll parameters are sufficiently small. The contour plot of ϵ is shown in fig.(4.2b). We find $\epsilon \sim 0.1$ along a linear region extending with $u \in [-5, -6]$, $\alpha \in [4, 5]$ and at $ab = 0$. While this value of ϵ leads to inflation, it is too large to be consistent with the CMB results.

The eigenvalues of the matrix N^a_b in the region where ϵ is small turn out to be

$$N^a_b = \left(-7.27268 \times 10^{-7}, -7.91109 \times 10^{-9}, 4.81198 \times 10^{-10}\right). \tag{4.22}$$

The only value we care about is the most negative eigenvalue of this matrix. With the particular toy model we are considering here, and with the choice of g, the η turns out to be

$$\eta = -7.91109 \times 10^{-9}. \tag{4.23}$$

Such smallness of η means that it is compatible with the CMB measurements. The derived scalar potential with the particular function considered is not realistic for our universe, since it does not satisfy the constraint on ϵ. However, we consider other possible values for this function.

Having derived a generalized scalar potential, the goal of the thesis is to now determine if this is robust, we will now test different functions for g, g', h, q in eq. (4.14), even if they are not consistent with inflationary behaviour.

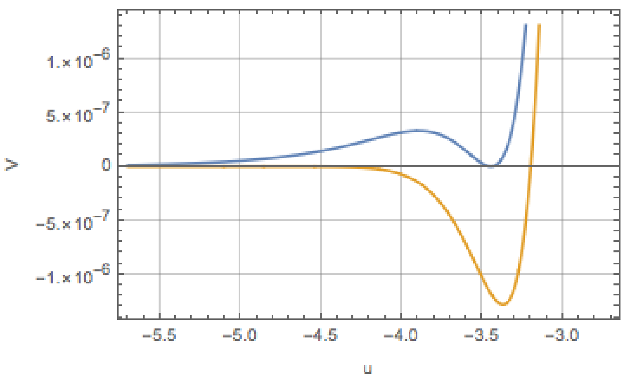
4.3 Scalar potential with $g = \alpha$

To this end, we set the parameter $g = \alpha$ with all other functions set to zeros. This gives the potential

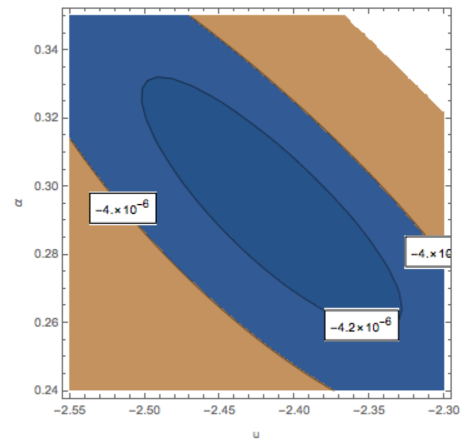
$$\begin{aligned} V_F(\alpha, u, a = 1/10) \;\; = \;\; & \frac{1}{384} e^{-2ae^{-u}+u+3\alpha} \left(12\alpha^2 e^{u+3\alpha} + 12\alpha^2 e^{2u+3\alpha} + 0.4 e^{3\alpha}\alpha^2 \right. \\ & \left. -24 e^{u+ae^{-u}} \alpha \sin(ab) + 8\alpha e^{2u+3\alpha} + \frac{4}{3} e^{2u+3\alpha}\right). \end{aligned} \tag{4.24}$$

As before, we plot the scalar potential as a function of u. The value of $ab = 5\pi$ at the minimum as this minimizes the sine term. In fig.4.3a the orange line corresponds to an anti-deSitter minimum and blue corresponds to uplifted potential with a deSitter minimum with $\alpha = 0.8$. Fig.4.3b is the associated contour with the value of the potential vs u and α at the AdS-minimum, which is

$$V_{AdSMin} = -4.2708 \times 10^{-6}. \tag{4.25}$$



(a) The figure displays a minimum at which potential is negative (orange line), which is then uplifted to positive value (blue line) with $\alpha = 0.8$.



(b) Contour of potential in the vicinity of the AdS-minimum with $V = -4.2708 * 10^{-6}$.

Figure 4.3: Plots of potential with $g = \alpha$ as a function of u.

Demanding that the value of the potential at this minimum be at zero or some positive value, we get this value of $D = 0.00465242$. Including this in the potential, now the value of the minimum of the potential lies at, see fig.4.4a,

$$V_{dSmin} = 1.00859 \times 10^{-11}. \tag{4.26}$$

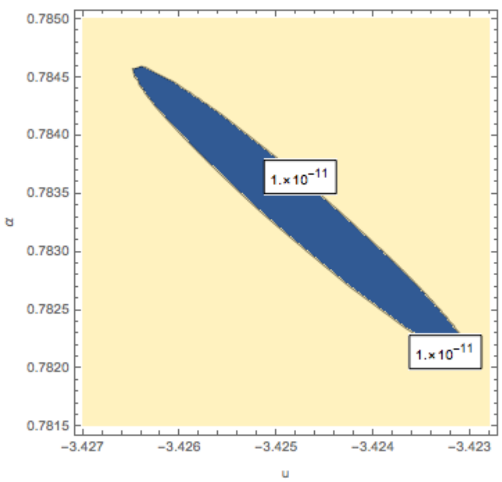
We next ask when this potential admits a slow roll. As before a contour is useful to find the minimum value. And as can be seen from fig.4.4b, we find a region where $\epsilon \sim 0.01 < 1$. Having satisfied the epsilon slow-roll condition, we can now proceed to check the second slow-roll condition η. And the most negative eigenvalue of the second slow-roll parameter η is

$$\eta = -1.15332 \times 10^{-6}. \tag{4.27}$$

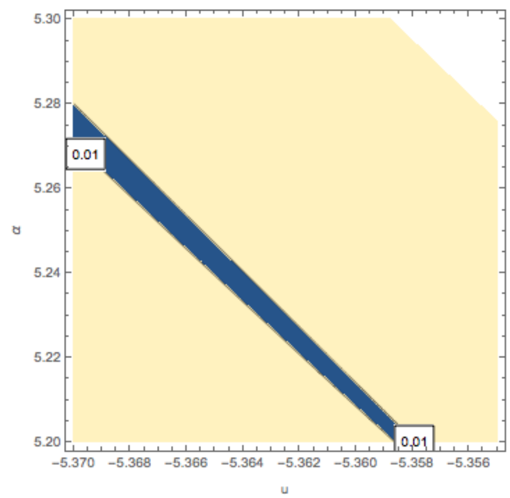
in the region where ϵ is small. The derived potential and the form of f considered here seem to satisfy both of the slow-roll conditions. What this means is that the scalar potential and the particular g considered here indeed have the potential to drive inflation and are roughly consistent with CMB.

4.4 Generalizing the form of g

In this section, we are going to consider a form of $g = e^{-n\alpha^n}$. We consider two cases where $n = 1$ and $n = 2$.

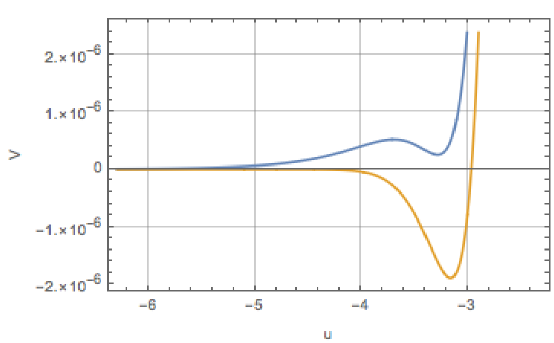


(a) The figure displays a minimum at which the potential is positive leading to a deSitter vacuum.

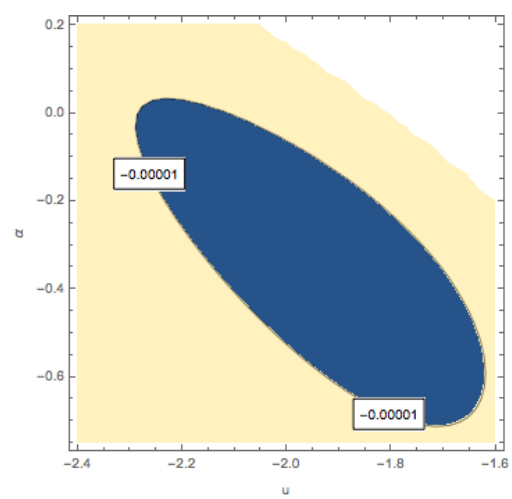


(b) Contour plot of the measure ϵ in dS space.

Figure 4.4: Plots of potential with $g = \alpha$ and slow-roll paramter ϵ.

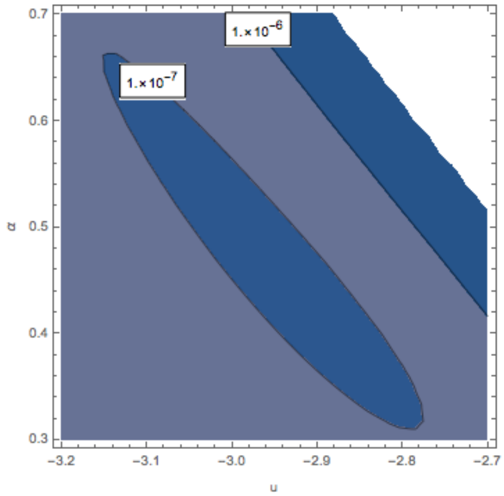


(a) Plot of V with (blue line) and without (orange line) the contribution of the D-term for the case of exponential function $g = e^{-\alpha}$ in the vicinity of the minima with $\alpha = 0.8$.

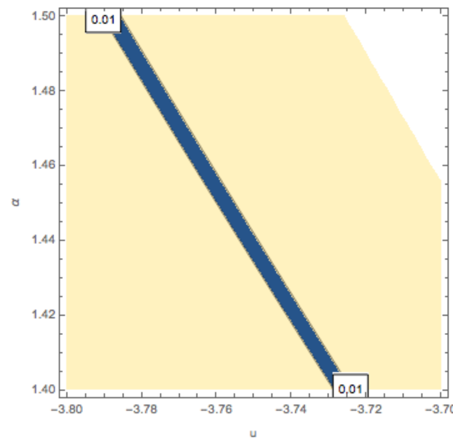


(b) Contour plot of the effective potential in the vicinity of the AdS-minimum.

Figure 4.5: Plots of potential with $g = e^{-\alpha}$ as a function of u.



(a) Contour plot of the potential in the vicinity of the dS-minima, including a D-term with $D = 0.00497031$.



(b) Contour plot of the ϵ in the vicinity of the dS-minima.

Figure 4.6: Plots of potential with $g = e^{-\alpha}$ and slow-roll parameter ϵ.

4.4.1 With $n = 1$

Consider a simple case where $n = 1$ and so $g = e^{-\alpha}$ and other functions are set to zero. Most of the terms in the potential don't change much. Following the same procedure, the scalar potential is

$$V_F(\alpha, u) = \frac{1}{384} e^{-0.2e^{-u}+u+3\alpha} \Big(-2.4e^{0.1e^{-u}+u-\alpha} + 1.2e^{u+\alpha} + 12e^{2u-\alpha} + 8e^{2u+2\alpha} + \frac{4}{3}e^{2u+3\alpha} + 0.04e^{\alpha} \Big). \tag{4.28}$$

The plot of the potential can be seen in fig.4.5a and fig.4.5b for different values of α and u. The minimum now lies at

$$V_{AdSMin} = -1.05353 \times 10^{-5}. \tag{4.29}$$

With $D = 0.00497031$ the AdS is uplifiting to dS with and the minimum from fig.4.6a is at

$$V_{dSMin} = 1.00 \times 10^{-7}. \tag{4.30}$$

From fig.4.6b, it can be seen that there is a region with $\epsilon \sim 0.01$ is small enough to satisfy the slow-roll condition. The η slow-roll condition is also satisfied in the same region, with

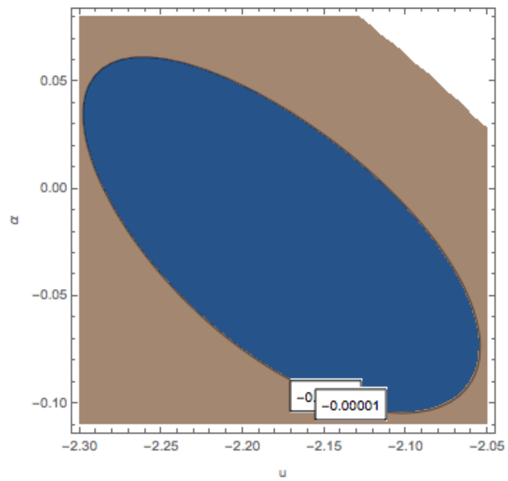


Figure 4.7: Contour plot of the effective potential with $g = e^{-2\alpha^2}$ as a function of u and α in the vicinity of the AdS-minimum.

the most negative eigenvalue being

$$\eta = -3.12593x10^{-8}. \tag{4.31}$$

4.4.2 With $n = 2$

Let's consider a final case where $n = 2$. As usual most of the terms in the potential don't change much including the V_D term. The potential including the V_D term is

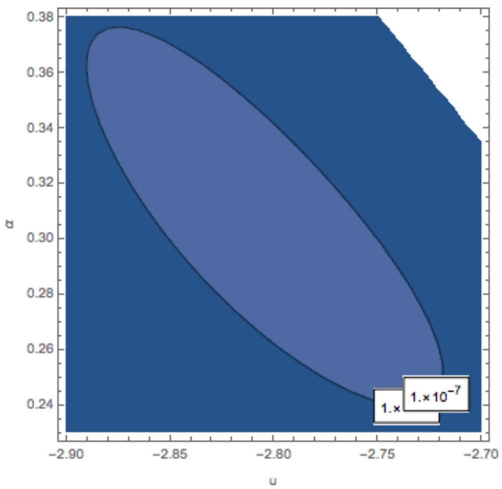
$$\begin{aligned} V_F(\alpha, u) &= \frac{1}{384} e^{3\alpha - 0.2e^{-u} + u} \Big(1.2e^{-4\alpha^2 + 3\alpha + u} - 2.4e^{-2\alpha^2 + 0.1e^{-u} + u} + 8e^{-2\alpha^2 + 3\alpha + 2u} + \\ & \quad 0.04e^{3\alpha - 4\alpha^2} + 12e^{2u - \alpha} + \frac{4}{3} e^{3\alpha + 2u} \Big) \end{aligned} \tag{4.32}$$

and the minimum (from fig.4.7) lies at

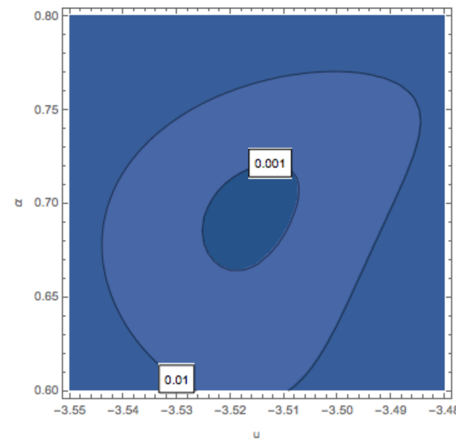
$$V_{AdSMin} = -1.05353 \times 10^{-5}. \tag{4.33}$$

With the value of $D = 0.00527921$, the deSitter minimum from fig.4.8a is

$$V_{dSMin} = 1.00 \times 10^{-7}. \tag{4.34}$$



(a) Contour plot of the effective potential in the vicinity of the dS-minima.



(b) Contour plot of the ϵ in the vicinity of the dS-minima.

Figure 4.8: Plots of potential with $g = e^{-2\alpha^2}$ and slow-roll parameter ϵ.

Both the slow-roll conditions were satisfied in some region, with $\epsilon \sim 0.001$ and the eigenvalue of the η

$$\eta = -5.28077 \times 10^{-6}. \tag{4.35}$$

The plot of the ϵ slow-roll parameter is shown in fig.4.8b. The cases we considered are robust because we consistenly get inflation.

4.5 Discussion

It is important to note that the fine-tuning of dS minimum does not affect whatever is going on in the inflationary region. This is because the potential at dS minimum is much less than inflationary region, so changing the minimum a little does not change inflationary potential appreciably.

A question one might ask is how large or small does the potential have to be? From the calculations, we see that the values of the potential for our particular toy models were in the order of $10^{-5} - 10^{-6}$ in Planck units in AdS space. Uplifting to dS-minima, the potentials were now on the order of $10^{-8} - 10^{-11}$ in Planck units. How does this compare to the present value of the cosmological constant? The present value of the cosmological constant $\Lambda \sim 10^{-120}$ in units of Planck density. Even if we fine-tune the dS-minimum the inflationary region does not change much. It stays at high potential V compared to Λ. The fine-tuning that sets the value of Λ to such smallness does not change what is going on in the inflationary region. Our ability to accomplish this relies on the freedom to tune the variables of the model.

To summarize this chapter, we considered a simple toy model where flux does not stabilize all the complex structure moduli. We then defined these fields in terms of real variables (β, α) since parts of t and τ were stabilized. The values of the variables were found by calculating the scalar potential by considering the nonperturbative superpotential to be small so that it can be neglected. We then examined one sample function for the nonperturbative term, then considered some simplified models since this is a toy model. This checks if this class of potentials robustly gives inflation. The potentials were shown to obey the slow-roll conditions and so were good candidates for causing inflation.

Chapter 5

Moduli stabilization in conifold throat

5.1 Singular and Deformed Conifold

In this chapter, we look at more realistic models of moduli stabilization for type IIB on the CY threefold in a region close to a conifold singularity. CY manifolds can have singular points [12, 32]. One particular type is the conifold singularity which can be described locally by a noncompact CY three-fold. Near the singularity, the spacetime approaches this noncompact version. The conifold is a 6D noncompact CY space defined by four complex coordinates w_i constrained by the condition

$$\sum_{i=1}^{4} w_i^2 = 0, \quad w_i \in \mathbb{C}^4. \tag{5.1}$$

Eq.(5.1) describes a manifold that is smooth except at $w_i = 0$ and looks similar to a cone. The conifold singularity is at the tip where all the $w_i = 0$. The base of the cone is the space $T^{1,1}$ which has the topology $S^2 \times S^3$, a 5D manifold. It is possible to smooth out the singularity at the tip of the cone. This can be done either by deformation or resolution. The focus here will be on the deformed conifold. This is done by replacing eq.(5.1) by [36]

$$\sum_{i=1}^{4} w_i^2 = Z, \tag{5.2}$$

where Z is the complex structure modulus[1]. As one approaches the tip of the conifold, S^2 starts shrinking to a point, and we are left with S^3. With eq.(5.2) where $Z \neq 0$, the tip of the deformed conifold becomes a smooth point and so S^3 is no longer vanishing but has a size determined by Z. This means that there is no longer a spacetime singularity for finite

[1]Note on notation- in this chapter, we are using Z for the complex structure modulus and S for the axio-dilaton.

Z, but rather the conifold tip now looks like a deformed conifold [31].

The Kahler potential of the complex structure moduli space is entirely determined by the integral of the holomorphic 3-form. So the Kahler potential near the conifold for an unwarped case can be calculated using the complex structure part of eq.(3.8) and it takes the logarithm form [31]

$$K \sim \log(|Z|^2 \log|Z|^2). \tag{5.3}$$

Note that as $Z \to 0$, the moduli space metric vanishes and K as defined above diverges.

5.2 Warped Throat

If background nonvanishing fluxes F_3 and H_3 defined in chapter 3 backreact on the geometry, it produces a warped background (also referred to as the warped throat) because of the nonvanishing stress-energy tensors. This backreaction introduces a warping of the 10D geometry, and so for general CY, the metric is given by [13]

$$ds^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn} dy^m dy^n. \tag{5.4}$$

In this expression, $g_{\mu\nu}$ denotes the noncompact 4D metric whereas the metric of the 6D CY is described by g_{mn}, and both are modified by the warp factor. The exponents are the warp factor which are determined by F_3 and H_3 field strength. If the warp factor scales nontrivially, it has significant effects on the scalar potential and the masses of the moduli fields.

The best-understood example of such a warped background geometry is given by Klebanov-Strassler (KS). The CY manifold normally has a conifold point on it, but with the addition of 3-form fluxes F_3 and H_3, the geometry near the conifold is deformed and looks like KS, see [36] for details. The value of the warp factor at the deformed conifold point is known in KS geometry.

Also, note that the form of the Kahler potential in terms of the moduli space depends on the warp factor. In other words, eq.(5.3) may be modified in the presence of a warp factor. However, no complete calculations of K with warped factor dependence is known at present. Close to the tip, the warp factor can be related to the deformation parameter Z of the conifold via $e^{A_{conifold}} \sim |Z|^{1/3}$ [32]. In the calculations to follow, we give ad hoc prescriptions for including the warp factor dependence in K.

5.3 Mass of complex structure modulus and axio-dilaton

This chapter looks at the moduli stabilization close to conifold singularity in the limit of small Z approximation, for different Kahler potentials. Since the warp factor influences the form of the K, the idea here is to consider a reasonable approximation for K. By doing this, we check if it has the right behaviour to model dependence warping.

<u>*Case A:*</u>

First, we reproduce the results of [34] in a slightly different manner. Let us start by calculating the mass hierarchies. So we consider complex struture part of Kahler potential extracted from [34]

$$K = -\log\left(A + \frac{|Z|^2\log(|Z|^2)}{2\pi}\right) - \log(S+\overline{S}) - \log[-i(\rho-\overline{\rho})], \tag{5.5}$$

where A depends on the CY manifold (we pick $A = 0.10440$ as in [34]) and $S = -i\tau$ axio-dilaton. The Kahler moduli is defined as $\rho \to \mu + ie^{-\nu}$ and the axio-dilaton as $S \to s + it$. As we turn on the three-form fluxes, we freeze some of the complex structure moduli and thus we induce the superpotential of the form

$$W = f\left(\frac{iZ\log(Z)}{2\pi} + B_0 + DZ\right) + ihSZ, \tag{5.6}$$

where f, h are flux quanta and D, B_0 are the geometric quantities of the particular CY considered in [34]. Note that eq.(5.6) comes from the flux superpotential defined in chapter 3. The covariant derivative with respect to the moduli fields to leading order in Z are

$$D_Z W = Df + \frac{if\log(Z)}{2\pi} + \frac{if}{2\pi} + ihS, \tag{5.7}$$

$$D_S W = ihZ - \frac{B_0 f + DfZ + \frac{ifZ\log(Z)}{2\pi} + ihSZ}{\bar{S}+S}, \tag{5.8}$$

and the corresponding conjugates. Here we do not worry about the ρ part of V_F since $|D_\rho W|^2$ cancels with the $-3|W|^2$ in the scalar potential. Moreover, the $(\partial_Z K)$ is smaller than the terms shown in eq.(5.7) when Z is small so we ignore the Kahler potential contribution to $D_Z W$. To minimize the scalar potential we can set the above covariant derivatives to zero when considering only the superpotential from the flux. First, we define Z as follows

$$Z = re^{iT}. \tag{5.9}$$

Now the size of the S^3 at the tip of the conifold gets stabilized due to presence of the flux, and so by solving eq.(5.7) the complex struture moduli takes the value at leading order

$$Z \sim Ce^{-\frac{2\pi h}{f}S}, \tag{5.10}$$

with $C = e^{-1+2i\pi D}$. What this means is that Z is exponentially small at the minimum. Similarly, from eq.(5.8), we get

$$Z = exp\left[\frac{2\pi h}{f}(s-it) + 2\pi iD + \frac{2\pi iB_0}{Z}\right]. \tag{5.11}$$

We can compare eq.(5.11) with eq.(5.10). Doing this we get

$$1 = exp\left(-1 - \frac{4\pi hs}{f} - \frac{2\pi iB_0}{Z}.\right) \tag{5.12}$$

This equation determines the value of s and therefore r if $B_0 = 0$, which means that s is stabilized. However, it tell us nothing about t and T so when B_0 is zero, these are not stabilized. If $B_0 \neq 0$, we get an transcendental equation that can determine t and T.

Given the definition of Z we can also compute the masses of the fields. The complex struture modulus is fixed via $D_Z W = 0$ and $D_S W = 0$. These can be found by first computing the derivative of the scalar potential w.r.t Z, $\overline{Z}$, S, $\overline{S}$ and then multiplying by the corresponding Kahler metric

$$V_{Z\bar{Z}} = e^K\left(K^{Z\bar{Z}}\partial_Z(D_Z W)\partial_{\bar{Z}}(\overline{D_{\bar{Z}}W}) + K^{S\overline{S}}\partial_Z(D_S W)\partial_{\bar{Z}}(\overline{D_{\bar{S}}W})\right). \tag{5.13}$$

Because of the cancellations in the scalar potential as mentioned earlier, the derivatives do not affect other factors in eq.(5.13). The Kahler metric at leading order for Z and S comes out to be

$$K^{Z\bar{Z}} = -\frac{2\pi A}{\log(|Z|^2)}; \quad K^{S\bar{S}} = (\bar{S}+S)^2, \tag{5.14}$$

so we get

$$V_{Z\bar{Z}} = \frac{1}{\left(A + \frac{|Z|^2\log(|Z|^2)}{2\pi}\right)}\left((\bar{S}+S)\left|ih - \frac{Df + \frac{if\log(Z)}{2\pi} + \frac{if}{2\pi} + ihS}{\bar{S}+S}\right|^2 - \frac{Af^2}{2\pi|Z|^2(\bar{S}+S)\log(|Z|^2)}\right). \tag{5.15}$$

The mass of the complex structure moduli is given by the ratio of energies. This is because Z is not canonically normalized so to convert it to correct mass we have to take the ratio of potential to kinetic terms

$$M_Z^2 = \frac{1}{2}K^{Z\bar{Z}}V_{Z\bar{Z}} = -\frac{A\pi}{\log(|Z|^2)}V_{Z\bar{Z}}. \tag{5.16}$$

We are mainly interested in small Z, so the mass approximates to

$$M_Z^2 \sim \frac{Af^2}{4\pi Re(S)|Z|^2\log^2(|Z|^2)}. \tag{5.17}$$

Similarly, we can calculate the mass of the axio-dilaton. Using the previously calculated Kahler metric for S, we get $V_{S\bar{S}}$ to be

$$V_{S\bar{S}} = \frac{(\bar{S}+S)}{\left(A + \frac{|Z|^2\log(|Z|^2)}{2\pi}\right)}\left(h^2 + \left|\frac{B_0 f + DfZ + \frac{ifZ\log(Z)}{2\pi} + ihSZ}{(\bar{S}+S)^2} - \frac{ihZ}{\bar{S}+S}\right|^2\right), \tag{5.18}$$

so that

$$M_S^2 = \frac{1}{2}K^{S\bar{S}}V_{S\bar{S}} = \frac{(\bar{S}+S)^2}{2}V_{S\bar{S}}, \tag{5.19}$$

and for small Z, this approximates to

$$M_S^2 \sim \frac{2\pi h^2(\bar{S}+S)^3}{4\pi A}. \tag{5.20}$$

The terms corresponding to B_0 are considered to be small. Moreover, only h^2 term in 5.18 survives as Z is small and given $s \sim \log(Z)$.

Case B:

Having reproduced the results of [34], the idea is to see if changing the Kahler potential in a certain way affects the masses of the fields, in a way that we would expect. The warp factor is a function of a position and so if in CY space we are in a region where the warp factor is small such as in conifold throat; this results in a small masses of the fields. We say the masses of the fields get redshifted by a power of the warp factor. From our 4D perspective, this happens if the Kahler potential changes. Thus we are looking at how changing K in a region of conifold throat scales down the masses and whether it gives the right behaviour that we expect.

Now we compare what happens if we change the Kahler potential in a way to try to mimic what the warp factor is doing. We expect the mass to be proportional to e^A. Consider the following potential from [37] [38].

$$K = -\log\left(\frac{Aa}{|Z^c|^2} + \frac{a|Z^{1-c}|^2\log(|Z|^2)}{2\pi}\right) - \log(S+\overline{S}) - \log[-i(\rho-\overline{\rho})]. \tag{5.21}$$

In computing the masses of the fields as before all that happens is that the prefactor changes since the derivatives do not acts on the prefactor. So we expect the second derivative of the scalar potential and the masses to have a similar behaviour. The prefactor for both the complex struture moduli and axio-dilaton simply scales as

$$M^2 = \frac{|Z^c|^2}{a}, \tag{5.22}$$

with constant a defined in chapter 3. The warp factor scales as

$$e^{2A}_{conifold} \sim |Z^c|^2. \tag{5.23}$$

We know that the warp factor scales as $e^{A_{conifold}} \sim |Z|^{1/3}$ [32] close to the tip. We can compare it to eq.(5.23) and notice that we need $c = 1/3$. So here we see that the mass is proportional to the warp factor as we expect.

We can now summarize what we have done so far. First, the fluxes stabilized the complex structure modulus Z and the axio-dilaton. This is essential for any realistic string model. Second, warping introduces interesting possibilities for generating mass hierarchies. It is attainable to deform the conifold so it's not singular. This is done by setting the sum to complex structure moduli and is known as a deformed singularity. Finally, we have shown how warp factors relate to Z.

5.4 Deriving the scalar potential

Now we look at the scalar potential. Starting with the Kahler potential from *case A*, we add the nonperturbative term defined in chapter 3 to the superpotential with W_0 defined as eq.(5.6)

$$W = W_0 + P(S,Z)e^{ia\rho}. \tag{5.24}$$

Following a similar procedure as in chapter 4, we calculate the scalar potential without the nonperturbative term (assuming its small). With the fields defined as $Z \to re^{iT}$, $S \to s+it$,

$\rho \to \mu + ie^{-\nu}$ and the corresponding conjugates, the scalar potential is

$$\begin{aligned} V_F &= \frac{e^{3\nu}}{16As}\left(-\frac{Af^2\left(-2\pi D+\frac{2\pi ht}{f}+T\right)^2}{2\pi\log\left(r^2\right)}-\frac{Af^2\left(\frac{2\pi hs}{f}+\log(r)+1\right)^2}{4\pi\log(r)}\right. \\ &\quad +\frac{f^2r^2\left(\log(r)-\frac{2\pi hs}{f}\right)^2}{4\pi^2}+\frac{B_0f^2r\cos(T)\left(-2\pi D+\frac{2\pi ht}{f}+T\right)}{\pi} \\ &\quad \left.+\frac{B_0f^2r\sin(T)\left(\log(r)-\frac{2\pi hs}{f}\right)}{\pi}+\frac{fr^2\left(-2\pi D+\frac{2\pi ht}{f}+T\right)^2}{4\pi^2}+f^2B_0^2\right). \end{aligned} \tag{5.25}$$

We keep only leading terms in small Z approximation. These approximations will apply in all the calulations to follow. To further simplify the potential we can set

$$T+\frac{2\pi ht}{f}=\theta, \quad log(r)-\frac{2\pi hs}{f}=\alpha, \quad log(r)+\frac{2\pi hs}{f}=\beta, \tag{5.26}$$

so the simplified potential is

$$\begin{aligned} V_F &= \frac{e^{3\nu}}{16As}\left[-\frac{Af^2(\theta-2\pi D)^2}{2\pi\log\left(r^2\right)}-\frac{A(\beta+1)^2f^2}{4\pi\log(r)}+f\left(B_0f\cos(T)+\frac{r(\theta-2\pi D)}{2\pi}\right)^2\right. \\ &\quad \left.+f^2\left(B_0\sin(T)+\frac{\alpha r}{2\pi}\right)^2\right]. \end{aligned} \tag{5.27}$$

Furthermore we can get an expression for r and s in terms of real variables α and β from eqs.(5.26). Solving the second equation for r and then substituting it into the third equation gives $s \to f(\beta-\alpha)/4\pi h$. This can then be subsituted back into the second equation to get $r \to e^{(\alpha+\beta)/2}$. These can be plugged into the above eq.(5.27) so we have a potential as a function of real variables

$$\begin{aligned} V_F &= \frac{\pi h e^{3\nu}}{4A(\beta-\alpha)}\left[-\frac{A\left[(\beta+1)^2+(\theta-2\pi D)^2\right]}{2\pi(\alpha+\beta)}+\left(B_0\cos(T)+\frac{e^{\frac{\alpha+\beta}{2}}(\theta-2\pi D)}{2\pi}\right)^2\right. \\ &\quad \left.+\left(\frac{\alpha e^{\frac{\alpha+\beta}{2}}}{2\pi}+B_0\sin(T)\right)^2\right]. \end{aligned} \tag{5.28}$$

Note that we set $f=1$ to simplify the potential. If B_0 is set to zero, we see that there is no potential for T so it may be stabilized at a lower scale. The mass term for α has r in front

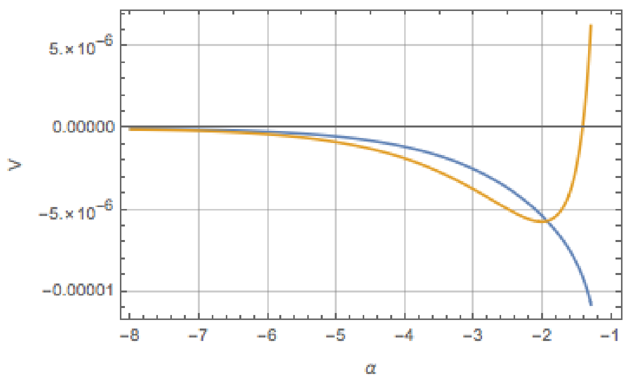


Figure 5.1: Plot of the scalar potential eq.(5.30) as a function of α with $A = 0.104404$ [34], $a\mu = \pi/2$, and $B_0 = 0$. The orange line corresponds to $\nu = -4$, and blue line to $\nu = -4.2$.

which is small, so the mass for α is small.

The above potential indicates that the parameters defined in eqs.(5.26) should be set to $\theta = 2\pi D$ and $\beta = -1$ as this minimizes the potential. Doing so we get the full scalar potential as

$$V_F = \frac{\pi h e^{3\nu}}{4A(-1-\alpha)}\left[\left(\frac{e^{\frac{1}{2}(\alpha-1)}\alpha}{2\pi} + B_0\sin(T)\right)^2 + B_0^2\cos^2(T)\right]. \tag{5.29}$$

Considering the contribution of the nonperturbative term we can calculate the terms associated with it.

We choose $P = Z$ as an illustrative possible example. The potential is simplified using eqs.(5.26) and is optimized using values of θ and β defined earlier. The scalar potential works out to be as follows

$$\begin{aligned} V_F \;=\; & \frac{\pi h e^{3\nu}}{4Af(\beta-\alpha)}\Bigg(\frac{4\left(\alpha^2+1\right)a^2 e^{-2ae^{-\nu}+\alpha-2\nu-1}}{3(\alpha-1)^2} - \frac{8\alpha a^2 e^{-2ae^{-\nu}+\alpha-2\nu-1}}{3(\alpha-1)^2} - \frac{8\pi A e^{-2ae^{-\nu}}}{(\alpha-1)^2} \\ & + \frac{e^{\alpha-1}\alpha^2 f^2}{4\pi^2} + 4ae^{-2ae^{-\nu}+\alpha-\nu-1} + e^{-2ae^{-\nu}+\alpha-1} - \frac{2ae^{-ae^{-\nu}+\alpha-\nu-1}\sin(a\mu)}{\pi(\alpha-1)^2} \\ & + \frac{2\left(2\alpha-\alpha^2\right)afe^{-ae^{-\nu}+\alpha-\nu-1}\sin(a\mu)}{\pi(\alpha-1)^2} + \frac{fe^{-ae^{-\nu}+\alpha-1}\sin(a\mu)}{\pi} + B_0^2 f^2 \\ & + \frac{B_0\sin(T)e^{\frac{\alpha-1}{2}}\left(-3f^2-\alpha f^2-3\right)}{\pi} + 2B_0 f cos\left(T+a\mu\right)\left(7+2ae^{-\nu}\right)e^{\frac{1}{2}\left(-2ae^{-\nu}+\alpha-1\right)}\Bigg). \end{aligned} \tag{5.30}$$

We start by considering B_0 to be small so T is stabilized only by nonperturbative potential. The allowed possible values for α has to be less than -1 since s by definition is positive and also $\beta = -1$. However, we see that eq. (5.12) determines $s \sim (-f/h)$ with $B_0 = 0$. The only way s can be positive is if f/h is negative, which causes the terms in the bracket of eq. (5.30) to be negative. But to minimize the potential we want the bracket terms to be positive otherwise; the potential drops off to $-\infty$ so it is unbounded below. Now the possible values for ν have to be negative because the supergravity approximation breaks down due to the fact that the corrections from string theory become important. Plotting the potential eq. (5.30) in fig.5.1 we see the potential does have minimum (orange line) as $\nu \to 0$. But the minimum vanishes as $\nu \to -\infty$, this corresponds to blue line in fig.5.1. It turns out that there is always an unbounded below potential as $\nu \to -\infty$ since $(e^{\alpha-1}\alpha^2 f^2)/(4\pi^2)$ dominates in this range and is manifestly positive. The scalar potential without and with the B_0 as with the other Kahler potential were calculated and all exhibit similar behaviour.

However, there is still work to be done. We assumed some terms in the e^K and $(\partial_Z K)W$ in the covariant derivatives were small due to small Z approximation. Also, the values B_0 and A could be different from the particular case we are considering here. It is important to note that runaway behaviour is not affected by our choice of the "standard" Kahler potential vs the other guesses. It is possible that accounting for the extra terms may resolve the runaway behaviour. Moreover, the KS geometry is a very important part of model building in string cosmology, so confirmation of this runaway behavior would be significant.

Chapter 6

Conclusion

In this thesis, we investigated a mechanism for deriving inflation in type IIB string theory. In particular, we derived the scalar potential that drives inflation. In order for the potential to drive inflation, it has to obey the slow-roll conditions. These conditions for the general multiple fields were derived in chapter 2. In chapter 3, we review the relevant construction of type IIB string theory. We considered type IIB string theory as it consists of three types of moduli fields i.e. complex structure moduli t, axio-dilaton τ and Kahler moduli ρ. These fields are massless in the low-energy effective theory. However, fields τ and t were shown to be stabilized by turning on the higher dimensional generalization of magnetic fluxes. Moreover, the Kahler moduli are stabilized via nonperturbative corrections to the superpotential. We considered a scenario where possibly one or more of these complex structure moduli fields may not be stabilized by the fluxes. The same nonperturbative corrections perhaps stabilize these moduli as the Kahler moduli. These moduli then correspond to almost flat directions of the scalar potential and can play the role of an inflaton.

The toy models associated with the moduli were considered, and the corresponding scalar potentials in this context were derived. A particular form of Kahler potential and a superpotential associated with the field(s) was considered in chapter 4 and the scalar potential was then derived in terms of real variables and was minimized in steps. It was shown in [31] that the potential without the nonperturbative correction (i.e. $f(t;\tau)e^{ia\rho}$ was considered small so it can be neglected) to the superpotential was much steeper, and thus inflation was not possible. The full scalar potential with the nonperturbative corrections was then derived for different functions f. This may work for inflation because steep part of the potential is already minimized.

Moreover, the corresponding slow-roll parameters were recalculated. Plots of the slow-roll parameters showed that the minimum value that these parameters could take was less than unity and small enough to fit observations. This shows that the scalar potentials we derived with the nonperturbative correction can drive inflation in the early universe.

Furthermore, in chapter 5, we explored the moduli stabilization close to conifold singularity in the approximation that Z is small. Besides, the mass hierarchies of complex structure and axio-dilaton were computed, and we found Kahler potentials that give a redshifted mass as expected. As with chapter 4, the scalar potential was derived; however, the potential had no minima leading to a runaway type behaviour. This will be verified with further work.

To conclude if we want to describe the nature of inflation in the very early stage of the universe, we need to take into account all the possible corrections to the scalar potential. We have seen that inflation consistent with cosmological constraints can arise in a string-inspired model when some complex structure moduli are stabilized only by the nonperturbative superpotential. We have also studied models based on the warped KS geometry; our results indicate that the potential is possibly unbounded below, which would have a major impact on string model building. We intend to pursue these aspects further in future work.

Appendix A

Derivation of generalized geodesic equation

Now let's derive the generalized Klein-Gordon equation starting with the modification of the action for scalar field space. The action is given as

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}g^{\mu\nu}K_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V\right). \tag{A.1}$$

Here, $g_{\mu\nu}$ is the symmetric spacetime metric, K_{ab} is the symmetric scalar field metric function of the fields, R is the Ricci scalar. The first term in the action is the familiar Einstein-Hilbert action part of the action whereas the last two terms are related to the matter part of the action. Varying the action w.r.t the fields leads to the equation of motion or multi-field KG equation

$$\begin{aligned}\delta S &= \int d^4x\delta\left(\sqrt{-g}\right)\left(\frac{R}{2} - \frac{1}{2}g^{\mu\nu}K_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V\right) + \int d^4x\sqrt{-g}\left(\frac{1}{2}\delta R - \frac{1}{2}(\delta g^{\mu\nu})K_{ab}\right.\\ &\quad \partial_\mu\phi^a\partial_\nu\phi^b - \frac{1}{2}g^{\mu\nu}\partial_c K_{ab}\partial_\mu\phi^a\partial_\nu\phi^b(\delta\phi^c) - \frac{1}{2}g^{\mu\nu}K_{ab}\partial_\mu(\delta\phi^a)\partial_\nu\phi^b - \frac{1}{2}g^{\mu\nu}K_{ab}\partial_\mu\phi^a\partial_\nu(\delta\phi^b)\\ &\quad \left.-\partial_a V\delta\phi^a\right) = 0.\end{aligned}$$

Integrating by parts and throwing away the boundary terms since the variations vanish at boundaries, we get

$$\begin{aligned}\delta S = \int d^4x \frac{\sqrt{-g}}{2}\delta g^{\mu\nu}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{2}g^{\alpha\beta}g_{\mu\nu}K_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V - K_{ab}\partial_\mu\phi^a\partial_\nu\phi^b\right)\\ -\delta\phi^c\left[-\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_c K_{ab}\partial_\mu\phi^a\partial_\nu\phi^b + \partial_c V\right) - \partial_\mu\left(g^{\mu\nu}\sqrt{-g}K_{ab}\partial_\nu\phi^b\right)\right] = 0.\end{aligned} \tag{A.2}$$

From here we get a variation with respect to the metric corresponding to the Einstein equation. The stress-energy tensor, $T_{\mu\nu}$ is

$$T_{\mu\nu} = K_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - g_{\mu\nu}\left(\frac{1}{2}K_{ab}\partial^\alpha\phi^a\partial_\alpha\phi^b + V\right), \tag{A.3}$$

and also a variation with respect to the fields corresponding to EoM/KG equation

$$g^{\mu\nu}\partial_c K_{ab}\partial_\mu\phi^a\partial_\nu\phi^b + 2\partial_c V - \frac{2}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}K_{ab}\partial^\mu\phi^b\right) = 0. \tag{A.4}$$

Let's now use A.4 to derive a generalized Klein-Gordon equation which will be useful in deriving the multifield slow-roll parameters. Using the spacetime metric signature for the FLRW metric, we have

$$g^{00}\partial_c K_{ab}\dot{\phi}^a\dot{\phi}^b + 2\partial_c V - \frac{2}{a^3(t)}\partial_0\left(g^{00}a^3(t)K_{ab}\dot{\phi}^b\right) = 0. \tag{A.5}$$

Computing the time derivative of the second term and simplifying further leads to

$$2K_{cb}\ddot{\phi}^b + 2\dot{\phi}^b\dot{\phi}^a\partial_a K_{cb} - \partial_c K_{ab}\dot{\phi}^a\dot{\phi}^b + 6HK_{ab}\dot{\phi}^b + 2\partial_a V = 0. \tag{A.6}$$

Swapping the dummy indices c and a on the second term and factoring back out $\dot{\phi}^a\dot{\phi}^b$ from the second and third term gives

$$2K_{cb}\ddot{\phi}^b + \dot{\phi}^a\dot{\phi}^b(\partial_a K_{cb} + \partial_a K_{cb} - \partial_c K_{ab}) + 6HK_{ab}\dot{\phi}^b + 2\partial_a V = 0. \tag{A.7}$$

If we swap the dummy indices a and b on the second term in the bracket, we see that this gives us the Christoffel symbol in field space. Dividing through by 2 gives us the following expression

$$K_{cb}\ddot{\phi}^b + \dot{\phi}^a\dot{\phi}^b\Gamma^d_{ab}K_{dc} + 3HK_{ab}\dot{\phi}^b + \partial_a V = 0. \tag{A.8}$$

Contracting by K^{ce} gives

$$\ddot{\phi}^e + \dot{\phi}^a\dot{\phi}^b\Gamma^e_{ab} + 3H\dot{\phi}^e + \partial_a V K^{ce} = 0. \tag{A.9}$$

This is Klein-Gordon equation analogus to the geodesic equation in field space which we will incorporate in our slow-roll derivation.

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