Routing Vehicles for Minimizing Carbon Dioxide Emissions

by

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Abstract

In this thesis, a new Green Vehicle Routing Problem (GVRP) as well as a novel Pollution-routing Problem (PRP) are formulated and corresponding algorithms to solve these problems are designed. The GVRP is a problem of routing green vehicles and the PRP is a problem of routing vehicles to minimize pollution. Both problems reflect the intensive need for eliminating the negative environmental effects of using conventional vehicles. On the one hand, the routing plan of green vehicles needs to take into account the refueling issue, so more new constraints need to be considered. On the other hand, the routing plan should not only seek to minimize the economic costs (e.g. travel distance, travel time, etc.), but also the environmental cost (e.g. GHG emissions). To solve these problems, two heuristics and two meta-heuristics have been designed. Meanwhile, extensive numerical experiments are conducted to illustrate the efficiency and validity of our proposed algorithms. Based on the analysis of numerical results, some managerial insights are concluded.

Keywords: Vehicle routing; Electric vehicle operations; Capacitated vehicle; Greenhouse gas emissions

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<center>**Chapter 1**</center>

<center>**Introduction**</center>

According to the report of International Energy Agency (IEA), the concentration of carbon dioxide (CO_2) in 2014 was about 40% higher than the concentration of carbon dioxide in the mid-1800s (IEA, 2015). The main reason for the increase in greenhouse gases (GHG) emissions is the rise of the energy consumption of human activities. The pollution from the transportation sector represents 23% of global CO_2 emissions and, in 2013 nearly 75% of the emissions in transportation sector were attributed to the road sector (IEA 2014). These figures indicate an emergent need for reducing the CO_2 emissions of the road sector. The Green Logistics field can address such need to attract more attention from researchers, governments, and industries.

Green Logistics emphasizes on the sustainable issues arising in logistics operations and, thus, requires the reconsideration of current models and, ultimately creation of new business models. Currently, some efforts have been made by scholars to diminish the pollution in the road transportation sector. In terms of the types of vehicle used, green vehicles consuming clean energies such as electric and hydrogen are gradually adopted in the distribution networks to decrease the pollution. However, due to the immaturity of the technology, the driving range of green vehicles is short. In addition, the accessibility of refueling or recharging stations for green vehicles is limited. The problem regarding the design of the routing scheme for green vehicles is called Green Vehicle Routing Problem (GVRP). When solving the GVPR, the above mentioned two barriers need to be dealt with first.

Using of green vehicles in distribution process can reduce carbon dioxide emissions. However, the most effective routing plan, which can significantly reduce the carbon dioxide

<center>1</center>

emissions, needs to have a new objective other than the classic objective emphasizing on minimizing the economic costs. The Pollution-Routing Problem (PRP) aims to design a comprehensive emissions model which seeks to reduce the energy requirements of vehicle routing. The objective of PRP model is to consider environmental costs, such as the cost of greenhouse gas (GHG) emissions along with the operational costs.

In the following subsections, the general description of Vehicle Rouging Problem (VRP) and the introduction of GVRP and PRP problems will be provided.

1.1 Vehicle Routing Problem

The Vehicle Routing Problem (VRP) is a practical-oriented operations research problem. It also is a mathematical problem which is generalized from the distribution process in real life. The VRP can be treated as "a generalization of traveling salesman problem", because it considers two conditions: a). carrier cannot serve the delivery point on one trip, and b). every delivery point has to be served (Dantzig & Ramser, 1959). The studies on VRP are gradually growing and have been an important topic in operations research area. Over the past several decades, various variants of VRP have been proposed, such as Capacitated Vehicle Routing Problem (CVRP), Vehicle Routing Problem with Time Window (VRPTW), etc.

The classic VRP problem is depicted as the following graph-theoretic problem. Let $G = (V, A)$, where $V = \{0, 1, \ldots, n\}$ is the set of nodes and A is the set of arcs. With the excepting for the depot node ($i = 0$), every other node ($i = 1, 2, 3, \ldots, n$) represents the client who has the delivery demand ($D_i > 0$) and can be visited by vehicle only once. The route from node k to node j is represented as the arc between these two nodes and each route has a weight $C_{kj} > 0$, which represents the cost (could be distance, time, etc.) of delivering cargos through the route

between node *k* and *j* (Laporte & Osman, 1995). Figure 1 shows a classical vehicle routing problem which has only one depot and three routes.

The VRP studies can be categorized into two main groups. One category of VRP studies focus on proposing new models of VRP variants by considering more constraints the other category explores how to solve the existing VRP problems. From the perspective of computation complexity, the VRP problems are np-hard, which means, as the size of the problem grows, it cannot be solved by using exact methods in a certain time. Therefore, designing other solution methods such as heuristics and meta-heuristics for solving it is worthy of being studied.

Figure 1 Classical Vehicle Routing Problem
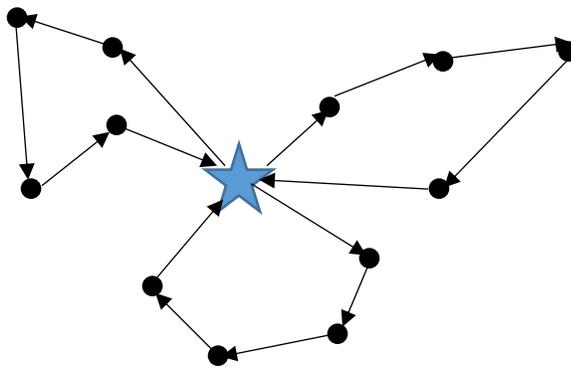
1.2 Green Vehicle Routing Problem

The classic VRP model is focused on the selection of the optimal route to minimize the total transportation costs. The GVRP emphasizes not only on the optimal economic costs of delivery but also the environmental sustainablity issues. To achieve sustainability, the vehicles used in classic VRP are replaced by Alternative Fuel-powered Vehicles (AFVs), which rely on the greener fuel sources such as electricity, natural gas, hydrogen *etc*. (Erdoğan & Miller-Hooks, 2012). Although the use of AFV reduces GHG emissions significantly, it faces two barriers. The first is the limitation of current technology, and the second is the limited availability of

Alternative Fuel Stations (AFSs). The capacity of a fuel tank or battery cannot support AFVs to travel over a long distance. For example, the driving range of an electric truck is about 300 to 400 kilometers on one fuel charge. The availability of AFS is limited because of the low demand for alternative fuels. To overcome these barriers, new models and solution methods are required. This process is difficult, because even small changes in the VRP formulation can initiate huge changes in the design of the algorithms to solve this problem. In addition, the coding of the algorithms is complex.

In this thesis, we address a problem of selecting the delivery routes for AFVs considering a limited loading capacity, limited fuel tank capacity, and scarce availability of AFSs. The objective is to minimize the total route length of all AFVs in the distribution process. We call this problem the Capacitated Green Vehicle Routing Problem (CGVRP). Figure 2 illustrates a simple example of the CGVRP with different participators in a real-world delivery activity. Table 1 shows the distances between different customers. In this example, AFVs have a fuel tank capacity of 50 gallons and a loading capacity of 2.5 tons. It is also assumed that the fuel consumption and the traveling distance have a linear relationship, with a fuel consumption rate of 0.2. In addition, there is one depot (denoted as D_0,), eight customers (denoted as $C_1 - C_8$, individually) and two AFSs (denoted as F_1 and F_2,). The vehicle stars from the depot, visits a set of customers and, finally, returns to the depot. The solution for the VRP shows that the sequence of the delivery process can be expressed as a vector $R = (D_0 - C_6 - C_3 - C_1 - C_8 - C_5 - C_2 - C_4 - C_7 - D_0)$, with a route length of 286.64 miles. The total fuel consumption is about 57.33 gallons, while the fuel tank capacity is 50 gallons. Therefore, this VRP route becomes infeasible for the CGVRP. In order to make the route feasible, AFSs are inserted. In Figure 1, one feasible route of the CGVRP is shown. This route can be represented as $R' = (D_0 - C_6 - C_3 - C_1 -$

$C_8 - C_5 - C_2 - F_2 - C_4 - C_7 - D_0$), with a route length of 315.25 miles. The increase in route length of the CGVRP is 28.61 miles, compared with the route length of the CVRP. Although the low tank capacity of AFV may lead to the increase in the route length, it can be offset by the reduction of GHG emissions.

It is widely known that the VRP is an NP-hard problem. The CGVRP model is a special variant of the CVRP model which extends it by considering the limited capacity of the fuel tank. Therefore, it is also an NP-hard problem. It is hard to obtain the optimal solution of a real-world problem with a large number of customers. In this thesis, the heuristics are proposed to find the solutions of large size problem instances quickly. However, the quality of heuristic solutions is not high enough. So the meta-heuristic algorithms are also proposed to solve the problems.
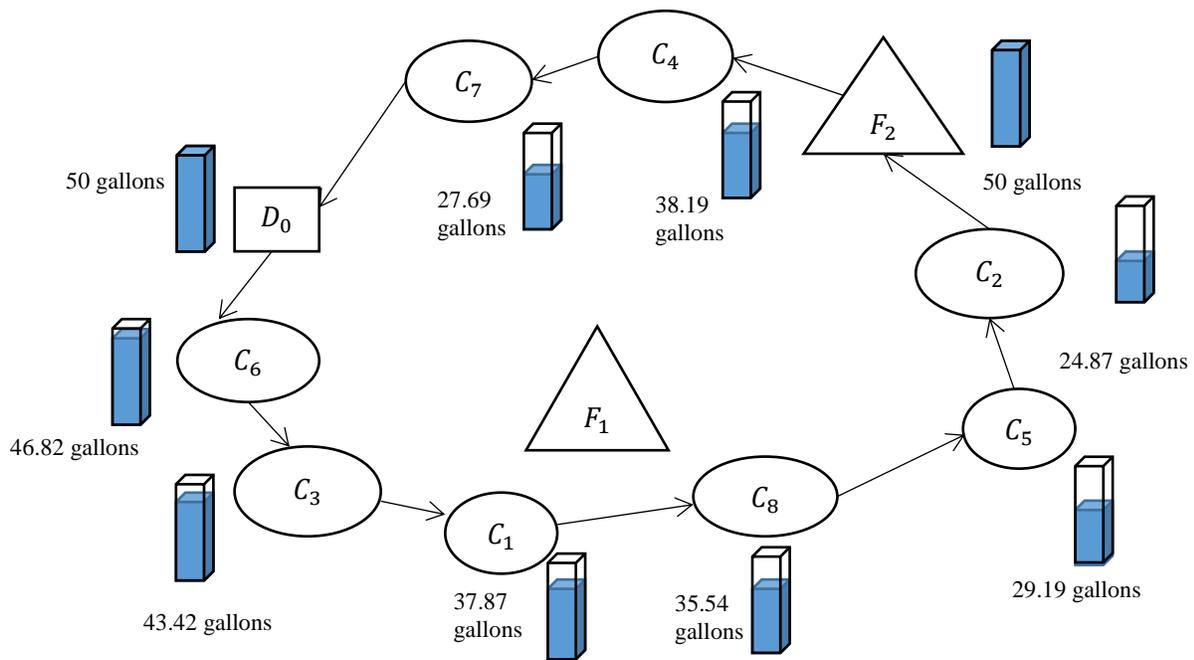
Figure 2 An example of the CGVRP

Table 1 Distance table

	D_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	F_1	F_2
D_0	0	42.72	98.23	31.95	96.74	76.66	15.29	50.63	46.27	36.40	55.90

C_1	-	0	55.90	10.77	59.46	34.37	27.73	47.42	11.66	36.06	14.14
C_2	-	-	0	66.49	75.43	21.59	82.97	90.52	52.92	85.89	45
C_3	-	-	-	0	75.19	44.91	17	44.78	17.08	31.24	24.41
C_4	-	-	-	-	0	69.08	87.46	52.50	79.03	62.01	59.03
C_5	-	-	-	-	-	0	61.40	72.50	31.76	65.73	24.52
C_6	-	-	-	-	-	-	0	47.52	31.06	32.29	41.34
C_7	-	-	-	-	-	-	-	0	58.90	15.13	48.88
C_8	-	-	-	-	-	-	-	-	0	46.86	20.40
F_1	-	-	-	-	-	-	-	-	-	0	41.23
F_2	-	-	-	-	-	-	-	-	-	-	0

1.3 Pollution-Routing Problem

Global warming is becoming an anxious concern of human beings, because it has been proved to have negative impacts on nature, economics and society and will threat the life of future generation (Revesz et al., 2014). In December 2015, the first international climate agreement extending mitigation obligations to all countries, the Paris Agreement on climate change is negotiated at the 21st Conference of the Parties (COP21). The Agreement aims to undertake rapid reductions of carbon dioxide emissions.

In fact, the use of fossil energy represents far the largest source of carbon dioxide emissions (IEA, 2016), so non-fossil energy gradually catches the attentions of people. The innovation of green vehicles, such as hydrogen vehicles and electric vehicles, is a good example of people's attempt to utilize non-fossil energy. However, the implement of electric vehicles in distribution system is hard. Firstly, Due to technology bottleneck, green vehicles usually have short covering range which usually is 100-150 miles (Feng & Figliozzi, 2013), and have to visit refueling or recharging stations during service. Furthermore, the number of refueling or recharging stations is limited. For an instance, there are only 1626 recharging points across Canada, the second largest county in the world (ChargeMap, 2017). Thus, the routing schemes for traditional vehicles which

consume fossil energy are not suitable for routing green vehicles. Moreover, the objective of traditional routing plan seeks to minimize economic cost, such as traveling distance, traveling time, etc. To properly design the routing plan for green vehicles and to mitigate the carbon dioxide emissions of such vehicles, Pollution Routing Problem (PRP) appears in Operations Research literature. Unlike most variants of Vehicle Routing Problem (VRP), the PRP seeks to minimize carbon dioxide emissions instead of economic cost. In addition, the carbon dioxide emissions of vehicles are hard to be measured, because it does not only have relationship with route length, but also is affected by other factors, such as speeds, loads and travelling time. Consequently, the measurement of carbon dioxide emissions requires sufficient consideration of all concerning factors. The solution can bring managerial insights to the practitioners, as long as carbon dioxide emissions are correctly measured in model.

In this chapter, a Pollution Routing Problem with Electric Vehicles is considered, and the corresponding model with the objective which seeks to minimize carbon dioxide emissions is formulated. As solution methods, a saving-algorithm based heuristic with 3-opt interchange and an ant colony (AC) algorithm based meta-heuristics are proposed. To the authors' best knowledge, there are no models aiming to minimize the carbon dioxide emissions of EVs in literature. The motivation of this research is to fill this gap. The reliability and feasibility of the proposed algorithms are proved through extensive numerical experiments on newly designed instances.

Main contributions of this thesis are three-fold. Firstly, two new formulations of GVRP and PRP are proposed. Secondly, new solution methods, i.e. two heuristics and two meta-heuristics, are designed to solve these two problems. Thirdly, some managerial insights are provided through the extensive numerical experiments conducted in the thesis.

This thesis is structured as follows. The second chapter provides a literature review on the two problems. The study on the GVRP published at *Annals of Operation Research*, is discussed in the Chapter 3. The PRP model and its solution methods are addressed in Chapter 4. The final conclusion can be found in Chapter 5.

Chapter 2

Literature Review

The studies on GVRP and PRP have emerged over recent 5 years. The GVRP studies seek to overcome the barriers of routing green vehicles. The PRP studies consider the routing problem from the perspective of minimizing environmental costs. Both problems are new variants of Vehicle Routing Problem (VRP) and only a handful of studies have addressed them which are discussed in this chapter.

2.1 Green Vehicle Routing Problem

The research on the GVRP mainly focuses on replacing conventional vehicles with green vehicles, such as Alternative Fuel-powered Vehicles and Electric Vehicles, in the distribution process. The motivation of the GVRP studies is to overcome the barriers of routing green vehicles, such as limited energy storage (e.g. low battery capacity of electric vehicles) and limited availability of refueling or recharging stations.

The GVRP is developed from early studies on the refueling problem of fossil energy based vehicles (Mehrez and Stern, 1985 and Mehrez et al., 1983). Other relevant studies of the refueling problem are on the multi-depot VRP with satellite stations (e.g. Bard et al. (1998), Crevier et al, (2007), Tarantilis et al, (2008) and Kek et al. (2008)).

Erdoğan and Miller-Hooks (2012) considered a GVRP model to overcome the difficulties brought by the limited number of refueling infrastructures and the short driving range of vehicles. They proposed two solution methods, a Clarke and Wright savings heuristic and a Density-Based Clustering Algorithm, to solve the problem. Based on this study, Felipe, Ortuño, Righini, and

Tirado (2014) focused on a situation where electric vehicles were used for delivery. In their model, the recharging station and the recharging time are decided simultaneously. Schneider, Stenger, and Goeke (2014) considered the customer time windows as well as the limited vehicle capacity in their Electric Vehicle-Routing Problem, given the possibility of recharging at any available station. A hybrid heuristic combing tabu search with nearest neighbour search algorithm was proposed as a solution method. Montoya et al. (2016) developed a simple but effective two-phase heuristic to solve the GVRP and performed extensive experiments on 52 instances from the literature to test their heuristic.

The models in the GVRP literature commonly assume a linear relationship between energy consumption and travelling distance of vehicles. However, energy consumption of vehicle is also affected by other factors such as speed, load, and travelling distance. In this thesis, a GVRP model developed from Erdoğan and Miller-Hooks (2012)'s model is proposed. One of the limitations of their model is the assumption of AFVs' unlimited loading capacity. We extend their model by incorporating a limited loading capacity. Thus, based on the current research, we take two important constraints into consideration: (1) the limited loading capacity and (2) the limited fuel tank capacity. To our best knowledge, this problem has not been addressed in the extant literature. We refer to this problem as the Capacitated Green Vehicle Routing Problem (CGVRP).

2.2 Pollution-Routing Problem

The Pollution-Routing Problem (PRP) aims to design a comprehensive emissions model which seeks to reduce the energy requirements of vehicle routing. The objective of such model has to consider environmental costs, such as the cost of greenhouse gas (GHG) emissions along with the operational costs.

The studies on measuring carbon dioxide emissions are abundant. Based on simulation, the Comprehensive Modal Emission Model and the Parametric Analytical Model of Vehicle Energy Consumption have been proposed to predict the consumption of fuel and the rate of carbon dioxide emissions (Barth & Boriboonsomsin, 2009; Simpson, 2005). However, in terms of transportation planning, the studies which address the pollution issues are rare in quantity. In his PhD thesis, Palmer (2007) illustrated an integrated routing and emissions model, calculated the travelling time as well as carbon dioxide emissions, and manifested that the vehicle speed affects carbon dioxide emissions under different time windows constraints and congestion scenarios. Maden, Eglese, and Black (2010) formulated a vehicle routing and scheduling problem with time window. One interesting characteristic in their model is that the vehicle speed changes with the travelling time which influences the carbon dioxide emissions of vehicles.

Bektaş and Laporte (2011) were the first to introduce the Pollution-Routing Problem and develop a comprehensive objective which accounts for carbon dioxide emissions, fuel and travel times. By using the PRP models and several variants, they revealed the tradeoffs between several performance measures of vehicle routing, such as costs, emissions, distance and load. Demir, Bektaş, and Laporte (2012) further proposed an adaptive large neighborhood search for the PRP to decrease the computing time in the large scale PRP. Franceschetti, Honhon, Van Woensel, Bektaş, and Laporte (2013) introduced the Time-Dependent Pollution-Routing Problem (TDPRP) which extended the PRP by taking traffic congestions into consideration. Assuming a heterogeneous vehicle fleet in research, Koç, Bektaş, Jabali, and Laporte (2014) examined the impact of different fleet sizes on the results of mix pollution-routing problem. Kramer, Maculan, Subramanian, and Vidal (2015) proposed a new algorithm to optimize the speed and departure time of vehicles in the PRP in order to reduce the operational costs. Kramer, Subramanian, Vidal,

and Lucídio dos Anjos (2015) proposed an Iterated Local Search (ILS) with a Set Partitioning (SP) procedure and designed a Speed Optimization Algorithm (SOA) to solve the PRP.

However, there are barely any addressing the PRP model for routing green vehicles, which use clean energy and already have been implemented in the real-life business. This thesis proposes a Pollution-Routing Problem with Electric Vehicles (PRPEV) model which aims to route EVs with an objective of minimizing carbon dioxide emissions. In this model, we consider two aspects: one is the issue of recharging and the other is the consideration of energy consumption. The issue of recharging is brought by low battery capacity of EVs and requires EVs to visit recharging stations. However, in real-life cases, the number of recharging stations is limited. The scarce availability of the recharging stations makes the recharging issue more serious. In this case, the routing plans for EVs have to decide for vehicles which recharging station should be visited and when. As for the second aspect, the reduction of vehicle energy consumption decreases carbon dioxide emissions as well as operational costs. Last but not the least, the calculation of vehicle's energy consumption considering a handful of factors such as curb weight, cargo load, speed and friction of road, is incorporated in our model, whereas current literature only considers a linear relationship between energy consumption and distance.

In addition, the proposed PRPEV model does integrate the PRP and the GVRP to route electric vehicles from the perspective. The route plan generated by this model ensures that EVs can serve all customers and that the carbon dioxide emissions emitted by EVs are optimally low.

2.3 Ant Colony Algorithm

Ant colony (AC) algorithm is inspired by the ants' food searching behavior (Dorigo, Maniezzo, and Colorni 1996). The AC algorithm was initially designed to solve the VRP by

Bullnheimer, Hartl, and Strauss (1999). After this work, the developed AC algorithms have been widely used to solve different variants of the VRP problem.

Barán and Schaerer (2003) proposed a multi-objective ant colony system algorithm for the Vehicle Routing Problem with time window. Mazzeo and Loiseau (2004) further designed an ant colony system algorithm to solve the Capacitated Vehicle Routing Problem (CVRP). To solve the Vehicle Routing Problem with simultaneous delivery and pickup conditions, Gajpal and Abad (2009) proposed an ant colony system algorithm. Yu, Yang, and Yao (2009) proposed an improved ant colony optimization (IACO) which took a new strategy to update the increased pheromone and a mutation operation to solve the VRP. In the work of Yu and Yang (2011), an improved ant colony optimization (IACO) was designed to solve the period vehicle routing problem with time windows (PVRPTW), in which the planning period was extended to several days and each customer was served within a specified time window. Abdulkader, Gajpal, and ElMekkawy (2015) proposed a hybridized algorithm which combined local search with an existent ant colony algorithm to solve the Multi Compartment Vehicle Routing Problem. Schyns (2015) also proposed an ant colony algorithm for the Dynamic Capacitated Vehicle Routing Problems with time windows, (partial) Split Delivery and Heterogeneous fleets.

These studies illustrate the effectiveness of using modified ant colony algorithms to solve different variants of VRP problem.

Chapter 3

Capacitated Green Vehicle Routing Problem

In this chapter, the Capacitated Green Vehicle Routing Problem (CGVRP) is considered as a new variant of the Vehicle Routing Problem (VRP). In this problem, Alternative Fuel-Powered Vehicles (AFVs) are used for distributing products. AFVs are assumed to have low fuel tank capacity. Therefore, during their distribution process, AFVs are required to visit Alternative Fuel Stations (AFSs) for refueling. The design of the vehicle routes for AFVs becomes difficult due to the limited loading capacity, the low fuel tank capacity and the scarce availability of AFSs. Two solution methods, two-phase heuristic algorithms and meta-heuristics based on Ant Colony System (ACS) are proposed to solve the problem. The numerical experiments are then performed on the randomly generated problem instances to evaluate the performance of the proposed algorithms.

3.1 Problem Definition

The CGVRP can be defined as follows. Let $G = (V, E)$ be a complete and directed graph, in which V is a set of vertices and E is a set of edges between different vertices. The vertex set V contains three subsets: customer set $C = \{c_1, c_2, c_3, \dots c_N\}$, AFS set $S = \{c_{N+1}, c_{N+2}, \dots, c_{N+Ns}\}$ and depot set $D = \{c_0\}$, so that $V = C \cup S \cup D$ and $|V| = N + Ns + 1$. It is assumed that the depot is capable of refueling the vehicles, at the time of loading the freights. When a vehicle reaches either the AFSs or the depot, it is refueled to the tank capacity T. The edge set $E = \{(c_i, c_j): c_i, c_j \in V, i \neq j\}$ stands for the edges connecting different vertices of V. Every element of E is associated with the distance between two vertices d_{ij} and the fuel consumption f_{ij} for

traveling this distance. The CGVRP model assumes that the vehicles travel at a constant speed and that the fuel consumption rate is fixed.

In the CGVRP, the vehicles start from the depot, visit a set of customers and finally return to the depot. If there is a need to refuel during the service process, the vehicles visit an AFS. It is assumed that the number of AFSs visited by a vehicle in one tour can be greater than one. Also, a particular AFS can be visited more than once by any vehicle. If the carried products were delivered completely, the vehicle should come back to the depot. To ensure the efficiency of delivery, every customer can only be visited once and the corresponding demand needs to be satisfied after this visit. The visiting frequency of AFSs is not limited. To permit multiple visits to AFSs, we can augment graph G to $G' = (V', E')$ with a dummy vertex set $\theta = \{c_{N+NS+1}, c_{N+NS+2}, \ldots, c_{N+NS+S'}\}$. Each element in θ represents a possible visit to an AFS or the depot. $V' = V \cup \theta$. For every AFS, $c_f \in S$, n_f $(f = 0, \ldots, N + NS)$ is the number of dummy vertices for that AFV. In this way, the number of visiting times for every AFS is recorded by n_f.

Other notations used in modeling the CGVRP formulation are defined as follows:

$C_0 = \{c_0\} \cup C$ is the set that contains the depot and the customer vertices

$S_0 = \{c_0\} \cup S'$ is the set of the depot and the AFSs vertices, $S' = S \cup \theta$

r Fuel consumption rate (gallons per mile)

Q Loading capacity of the vehicle

T Tank capacity of the vehicle (gallons)

e_i The demand of vertex i

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d_{ij} The distance between vertices i and j

l The maximum number of vehicles used

Decision variables

x_{ij} A binary variable, taking value 1 if a vehicle travels from vertex i to j; otherwise, 0

f_j The remaining fuel level after visiting vertex j

m_i The remaining cargo in the vehicle after visiting vertex i

Given the above variables and parameters, the CGVRP can be formulated as follows:

$$\min \sum_{\substack{i,j \in V' \\ i \neq j}} d_{ij} \, x_{ij} \tag{3.1}$$

Subject to

$$\sum_{\substack{j \in V' \\ i \neq j}} x_{ij} = 1, \qquad \forall \, i \in C \tag{3.2}$$

$$\sum_{\substack{j \in V' \\ i \neq j}} x_{ij} \leq 1, \qquad \forall \, i \in S_0 \tag{3.3}$$

$$\sum_{\substack{j \in V' \\ i \neq j}} x_{ji} - \sum_{\substack{j \in V' \\ i \neq j}} x_{ij} = 0, \qquad \forall \, i \in V' \tag{3.4}$$

$$\sum_{i \in V' \backslash \{0\}} x_{0i} \leq l \tag{3.5}$$

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$$\sum_{i \in V' \backslash \{0\}} x_{i0} \le l \tag{3.6}$$

$$f_j \le f_i - r \cdot d_{ij} x_{ij} + T(1 - x_{ij}), \quad \forall\, j \in C, i \in V', i \ne j \tag{3.7}$$

$$f_j \le T - (r \cdot d_{ij}) x_{ij}, \quad \forall\, j \in V', i \in S_0, i \ne j \tag{3.8}$$

$$m_j \le m_i - e_i x_{ij} + Q(1 - x_{ij}), \quad \forall\, j \in V', i \in V', i \ne j \tag{3.9}$$

$$0 \le m_{c_0} \le Q \tag{3.10}$$

$$x_{ij} \in \{0,1\}, \quad \forall\, i, j \in V', i \ne j \tag{3.11}$$

Equation (3.1) minimizes the route length of AFV fleets. Constraint (3.2) ensures that each customer vertex has only one successor. This successor can be the depot, the AFS or a customer. Constraint (3.3) implies that, for each AFS (and the corresponding dummy vertices), at most, one successor (a depot, an AFS and a customer) exists. No visit to some AFSs is also possible. Constraint (3.4) ensures the flow conservation by requiring that the number of vehicles departing from a vertex be equal to the number of vehicles arriving at that vertex. Constraints (3.5) and (3.6) limit the number of vehicles leaving from depot or arriving at depot to, at most m. The remaining fuel of the vehicle is tracked by constraint (3.7), based on the vertex's type and sequence. If the vertex i is a customer, the vertex followed by a vertex j and $x_{ij} = 1$: the first term in constraint (3.7) reduces the corresponding fuel consumption for the route between i and j, based on the distance between two vertices and the fuel consumption rate of the vehicle. Constraint (3.8) ensures that the vehicle has enough fuel to return to the depot directly or via an AFS and that the remaining fuel never falls below 0. Constraints (3.9) and (3.10) guarantee that the quantity of the carried products cannot exceed the vehicle capacity Q. The last constraint

shows a binary integer variable x_{ij} in the mathematical model. Constraints (3.7) and (3.8) distinguish the CGVRP from the classic CVRP. These constraints also eliminate the sub tours.

The CGVRP formulation discussed in this section is adapted from the GVRP formulation proposed by Erdoğan and Miller-Hooks (2012) . The difference between the model in this chapter and their model is the loading capacity constraint. However, our model does not consider the maximum time constraint for the vehicle trip to serve the customers. The maximum tour length constraint is mainly applicable for the delivery of perishable products when the delivery demand is negligible. In real world, the loading capacity is one of the main constraints of the VRP. Therefore, we added the loading capacity constraint in our model (constraints (3.9) and (3.10)).

3.2 Solution Methods

This chapter proposes two solution methods for the CGVRP. The first solution method is the two-phase heuristic, and the second one is the Ant Colony System (ACS) algorithm. These two algorithms are described in the following two subsections.

3.2.1 Two-phase Heuristic for the CGVRP

We first use the two-phase heuristic to seek the solutions of the CGVRP. In the first phase, a Traveling Salesman Problem (TSP) is solved by using the Nearest Neighbour Criteria (NNC) to find the delivery route. The advantage of this criterion lies in its easiness and quick executions for solving the problems with a large number of customers. In the second phase, according to the fuel consumption level and the remaining loaded products, AFSs and the depot are inserted in the TSP route to make a feasible solution for the CGVRP.

1. Phase one: find the TSP routes

In phase 1, the NNC is used to find the TSP route. The NNC is usually used to determine a solution of the TSP (Gutin et al. 2002). In the NNC, a route starts from one customer and then the route is built by adding the nearest customer of the current customer. A whole route is finally built to find the TSP route. Basic steps of this algorithm are described in Figure 3.

Initialize the route set as $R = \emptyset$ and set the depot vertex as current customer z.

Find the nearest unvisited customer of current customer z, say customer y.

Add customer y in the route set R

Mark customer y as a visited customer and set customer y as current customer z.

Are all customers visited?

No

Yes

Return to depot

End
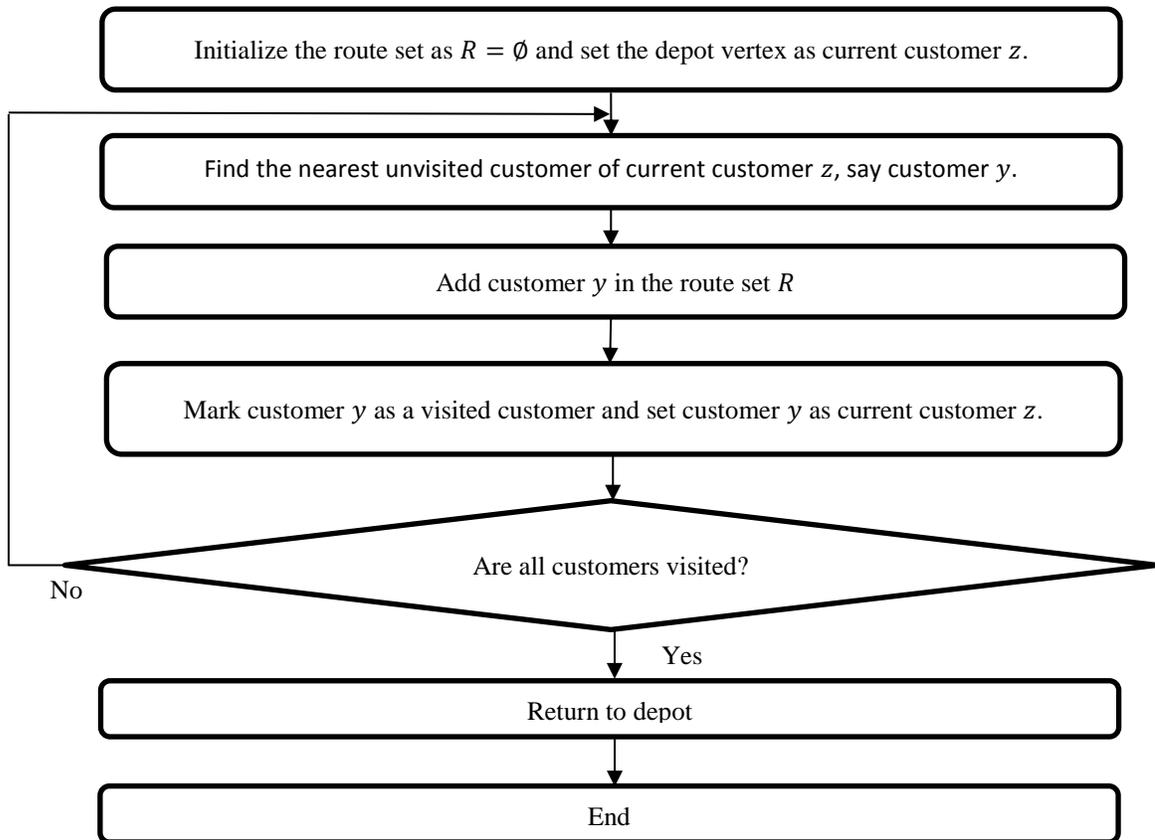
Figure 3. Flow chart diagram of phase one

2. Phase two: generate the CGVRP solutions

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                    ┌──────────────────────────────────────────────┐
                    │  Insert the first customer of TSP route (depot) to the CGVRP  │
                    │           route and mark it as current customer.              │
                    └──────────────────────────────────────────────┘

                              ◇ Current customer is the      Yes
                                last customer in TSP

                                        No

                              ◇ The products carried by
                     No         vehicle meet the demand
                                of the next customer.

   ┌────────────────────────┐
   │ Insert depot as next customer  │        Yes
   │ in the CGVRP route and mark    │
   │ it as the current customer.    │       ◇ The vehicle can return
   └────────────────────────┘          to depot via the next
                                            customer?
 ┌──────────────────────┐
 │ Insert the nearest AFS as  │    No
 │ the next customer and      │
 │ mark it as the current     │
 └──────────────────────┘

                   ◇ The vehicle can           ┌──────────────────────┐
                     reach the nearest          │ Insert the next customer   │
                                                │ from the TSP route to      │
        Yes                                     │ the CGVRP route and        │
                                                │ mark it as the current     │
                                                └──────────────────────┘
   ┌──────────────────────┐
   │ Change the current customer to │
   │ the previous one in the CGVRP  │          ┌──────────────────┐
   │ route and do the same in the   │          │       End        │
   │ TSP route.                     │          └──────────────────┘
   └──────────────────────┘
```

Figure 4. Flow chart diagram of phase two

The TSP solutions found in phase one are used to build the solutions of the CGVRP. When building the CGVRP route, the remaining fuel and products in the vehicle are calculated while inserting a customer. The depot is inserted when the remaining products are not able to satisfy

the demand of next customer on the TSP route. The AFS is inserted when the remaining fuel is not enough to allow the vehicle to reach the next customer from the TSP route or to return to the depot. While building the CGVRP route, a situation arises when visiting the AFS or the depot becomes impossible. In this situation, an AFS is inserted in the already built route to make the solution feasible. The details of the procedure used to create the CGVRP routes are described in Figure 4. Phase two generates the solutions of the CGVRP.

3.2.2 The Ant Colony System (ACS) Algorithm for the CGVRP

Colorni et al. (1991) first proposed the Ant Colony System (ACS) algorithm by observing the food seeking processes of ants. They noticed that the ants could find the shortest route while travelling from their home to the food sources. The mystery is the pheromone released by the ants. When ants travel, they release a pheromone which can stay on the path for a long time. The density of pheromone left on a route decides the route selection of the ants. During one certain period, the density of pheromone becomes higher for the shorter route, because more ants traveled through this route. The increasing amount of pheromone attracts more ants to follow this high density pheromone path. In this way, the pheromone of the shortest path keeps on increasing, while the pheromone of the longest path keeps on decreasing. Finally, most of the ants start following the shortest route.

The ACS algorithm, as a meta-heuristic, is widely used in solving the VRP and its variants (e.g. Barán and Schaerer (2003), Bell and McMullen (2004), Gajpal and Abad (2009a), Gajpal and Abad (2009b) and Abdulkader et al. (2015) etc.). In the ACS algorithm, artificial ants are created using the trail intensity. Detailed explanations and descriptions of the application of the ACS can be found in Stützle and Hoos (2000). Fundamental procedures of the ACS algorithm are listed in follows:

Step 1: Initialize the trail intensity matrix, create m number of artificial ants.

Step 2: Do the following steps, while the termination condition is not fulfilled.

- By using the trail intensity, generate a solution for each ant.

- Carry out the local search to improve the solutions produced by each ant.

- Update the elitist ants.

- Based on the elitist ant solutions, update the trail intensity matrix.

Step 3: Record the best solution generated.

1. Initialization of trail intensity

In this solution method, the trail intensity captures the information of a visit between two customers. The trail intensity τ_{ij} is defined as the intensity of visiting the customer j from the customer i. At the beginning, every element τ_{ij} in the trail intensity matrix is equal, because there is no solution at the first iteration. In this chapter, the initial trail intensity is set as 0.01.

2. Generation of ant solution

In this phase, the problem is first simplified as a TSP problem. Later, based on the TSP solutions, the solutions of the CGVRP are generated as described in section 4.1.2. In every iteration, n ants create n TSP solutions. The principle to select the next customer in the TSP route is mainly influenced by two factors: the saving value and the trail intensity.

- The saving value S_{ij} represents the travelling distance saved by using one vehicle to serve the customer i and j, instead of using two vehicles. let d_{ij} denote the distance between customer i and j, then the function to calculate the saving value S_{ij} can be presented as:

$$S_{ij} = d_{0i} + d_{j0} - d_{ij}$$

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- The trail intensity τ_{ij} represents the information of the solutions from the previous iterations.

The attractiveness value is used to decide which customer is selected as the next visit of vehicle. The attractiveness value is calculated as follows:

$$\xi_{ij} = [S_{ij}]^{\alpha}[\tau_{ij}]^{\beta}$$

In this equation, α is the saving value bias and β is the trial intensity bias. These two parameters are decided by the model and are fixed at the beginning of algorithm execution.

In addition, based on the attractiveness value, the next customer is selected from Ω_q. Here Ω_q represents the set of q unvisited customers. The probability of choosing an unvisited customer as the next customer from set Ω_q is calculated by the following function:

$$P_{xy_i} = \frac{\xi_{xy_i}}{\sum_{j=1}^{q} \xi_{xy_j}}, 1 \leq i \leq q, \qquad y_j \text{ is the } i^{th} element \text{ of } \Omega_q$$

Based on the probability calculation function, the next customer is selected for constructing the TSP route. When the last unvisited customer is inserted, the construction process stops. In the final step of this phase, the TSP route is used to construct the CGVRP route. The procedure to construct the CGVRP route from the TSP route has been described in phase 2 of the heuristic approach discussed in section 4.1.2.

3. Local search

In each iteration, n CGVRP solutions are generated. Nevertheless, the quality of these solutions is not high. Consequently, a local search is required to improve the solution quality. We

used the insertion-based local search schemes to improve the solutions. A description of the local search technique is given below with a numerical example.

Assume that there is a CGVRP problem with one depot (denoted as 0), thirteen customers (denoted as $1, 2, ...\ 13$) and two AFSs (denoted as 14 and 15). A solution to the CGVRP is:

$$R = (0 - 4 - 2 - 10 - 5 - 11 - 6 - 14 - 9 - 12 - 1 - 3 - 15 - 8 - 13 - 7 - 0)$$

The local search can be conducted as follows:

Step 1: Find the first customer (say Customer 1) in the solution and remove it from current position. Then insert it in all other customers' positions to generate a new route. At the end, there will be 12 new routes as below.

$$R_1 = (0 - 1 - 4 - 2 - 10 - 5 - 11 - 6 - 14 - 9 - 12 - 3 - 15 - 8 - 13 - 7 - 0)$$

$$R_2 = (0 - 4 - 1 - 2 - 10 - 5 - 11 - 6 - 14 - 9 - 12 - 3 - 15 - 8 - 13 - 7 - 0)$$

$$R_3 = (0 - 4 - 2 - 1 - 10 - 5 - 11 - 6 - 14 - 9 - 12 - 3 - 15 - 8 - 13 - 7 - 0)$$

... ...

$$R_{11} = (0 - 4 - 2 - 10 - 5 - 11 - 6 - 14 - 9 - 12 - 3 - 15 - 8 - 13 - 1 - 7 - 0)$$

$$R_{12} = (0 - 4 - 2 - 10 - 5 - 11 - 6 - 14 - 9 - 12 - 3 - 15 - 8 - 13 - 7 - 1 - 0)$$

After the generation of the new routes, the feasibility of each route is checked. The route with lowest objective function value among all feasible solutions is reserved for further considerations. In this case, for instance, assume that the feasible route with the lowest objective function value is $R_{10} = (0 - 4 - 2 - 10 - 5 - 11 - 6 - 14 - 9 - 12 - 3 -$

$$15 - 8 - 1 - 13 - 7 - 0).$$

Step 2: Use the solution found in the first step, i.e. $R_{10} = (0 - 4 - 2 - 10 - 5 - 11 - 6 -$
$14 - 9 - 12 - 3 - 15 - 8 - 1 - 13 - 7 - 0)$ and repeat the procedures in the first step
by inserting the next customer (e.g. customer 2) sequentially.

Step 3: Repeat Step 2 for the remaining customers.

The consideration of all customers for the possible insertions is called one cycle of local search. A complete cycle of the local search is repeated till the solution keeps on improving.

4. Update of the elitist ants

The elitist ant set contains the best λ routes found until the current iteration. The main purpose of creating the elitist ant is to change the trail intensities using the best λ solutions. The elitist ants are updated by comparing them with the current ant solutions. If the current ant solutions are better than any elitist ant solutions, then the current solution becomes a part of the elitist ant solution and the worst solution of the elitist ant solution is removed from the list.

5. Update of the trail intensity

The trail intensity is changed by using the updated elitist ant solution set. The function to update the trail intensity τ_{ij} of the edge between customers i and j is:

$$\tau_{ij}{}^{new} = \tau_{ij}{}^{old} \times \varphi + \sum_{\theta=1}^{\lambda} \tau_{ij}{}^{\theta}, \quad i \neq j \text{ and } i, j = 1, 2, \dots, n$$

In this equation, the first term stands for the old trail intensity storing the previous iteration information and φ is the trial persistence which is between 0 and 1. The second term, represents the increase in pheromone brought by the elitist ant θ. The value of $\tau_{ij}{}^{\theta}$ is determined by:

$$\tau_{ij}{}^{\theta} = \begin{cases} 0, & if\ the\ edge\ between\ customer\ i\ and\ j\ is\ not\ in\ the\ elitist\ ant\ route. \\ \dfrac{1}{l^{\theta}}, & otherwise. \end{cases}$$

Here l^{θ} represents the route length of the θ^{th} elitist ant solution. In this chapter, φ is set as 0.95, representing that the pheromone density decreases slowly. The value of φ is set on the basis of the value used in the previous literature.

6. The parameter settings of the ACS

The solution performance and computing time of the ACS algorithm are mainly affected by the number of iterations and the quantity of ants. Due to the complexity of the CGVRP, the number of iterations is set as 500. In each iteration, n new ants are created to generate the feasible solutions, and every ant solution is improved though the local search. The parameters α, β, λ and φ are set as: $\alpha = 5$, $\beta = 5$, $\lambda = 10$ and $\varphi = 0.95$. The detailed information of parameter settings can be found in the study of Dorigo and Stützle (2004).

3.3 Numerical Experiments and Analysis

Numerical experiments were conducted to evaluate the performance of the proposed solution methods. We also used A Mathematical Programming Language (AMPL) with CPLEX solver to obtain the optimal solutions. Based on the results of the experiments, some analyses are shown to illustrate the advantages and shortcomings of our algorithms. The constructions of the proposed algorithms are coded in C programming and implemented on AMD Opteron 2.3 GHz with 16 GB of RAM while AMPL runs on a desktop with Core i5-4590, 3.3 GHz with 8 GM of RAM.

3.3.1 Benchmark Problem Instances

The benchmark problem instances in this chapter are generated randomly. The parameters are set similarly to the parameters used by Erdoğan and Miller-Hooks (2012). The fuel consumption rate of vehicles is defined as 0.2 gallons/mile (Fraer et al, 2005) and the fuel tank capacity is 60 gallons. The vehicle speed is considered to be constant, which is 40 miles/hour. In addition, the loading capacity of a vehicle is set as 300 units of products. The demand of each customer in the instances is generated randomly between 15 and 25 units of products. The depot is located in the center of a 300 by 200 mile grid. The customers are randomly scattered within the grid and the AFSs are randomly situated.

The instances are categorized into two groups, the small instances and the large instances. In small instances, the number of customer ranges from 15 to 24, and the number of AFSs is set to 2. Ten small instances are generated and are labeled as ZGAS1 to ZGAS10. Twenty big problem instances considering 25, 50, 75, 100 and 150 customers and 2, 4, 6 and 8 AFSs, respectively, are generated and denoted as ZGA1 to ZGA20.

3.3.2 Result Analysis

In this section, the results of the proposed two-phase heuristic and ACS algorithm are reported in Table 2. In the table, N is the number of customers, NS is the number of AFSs, and RPD is the algorithm performance evaluation parameter called Relative Percentage Deviation. The formula to calculate RPD_{ij} of the i^{th} algorithm for the j^{th} problem instance is:

$$RPD_{ij} = \frac{(H_j^i - B_j)}{B_j} \times 100\%$$

Here, H_j^i represents the i^{th} algorithm solution in j^{th} problem instance and B_j represents the best algorithm solution in the j^{th} problem instances.

1. Experiments on small instances

The results of the two-phase heuristics, the ACS algorithm, and the exact method for small instances are reported in Table 2. For each problem instance, the average route length and the percentage gap (PG) from the optimal solution are reported.

Table 2. Experiment results of small instances

Instance	N	NS	CPLEX	Two-phase heuristic		Ant colony algorithm	
			Optimal solution (miles)	Route length (miles)	PG (%)	Route length (miles)	PG (%)
ZGAS1	15	2	847.96	1326.17	72.63	850.87	0.34
ZGAS2	16	2	874.79	1647.95	70.47	874.79	0.00
ZGAS3	17	2	875.29	1270.82	86.80	875.29	0.00
ZGAS4	18	2	876.08	1566.06	67.96	876.09	0.00
ZGAS5	19	2	931.82	1617.25	34.41	931.82	0.00
ZGAS6	20	2	950.17	1115.65	40.73	962.02	1.25
ZGAS7	21	2	950.72	1418.15	31.72	962.58	1.25
ZGAS8	22	2	951.33	1430.65	36.34	986.78	3.73
ZGAS9	23	2	963.75	1735.90	22.52	982.78	1.97
ZGAS10	24	2	967.66	1743.39	23.47	1011.63	4.54
Average			918.96	1487.20	61.95	931.46	1.31

From Table 2, it can be observed that the results achieved by the ACS are quite close to the optimal. The average result achieved by using the ACS is just 1.31% away from the optimal solution. However, the computing time taken by AMPL with CPLEX solver is longer than the computing time taken by the ACS algorithm. For example, the computing time for ZGAS10 is about two hours while the problem is solved in seconds by the ACS algorithm. This observation

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justifies the use of ACS for solving the CGVRP. In terms of the two-phase algorithm, the average solution is 61.95% away from the average optimal solution. The large percentage gap shows the performance of the proposed heuristic is not very good: even though, it can solve the instances within seconds.

2. Experiments on large instances

The experiments on large instances are conducted to illustrate the ability of the proposed algorithms for solving large problem instances in a reasonable time. The results are shown in Table 3.

Table 3. Numerical experiment results of large instances

Instance	N	NS	Two-phase Heuristic			Ant Colony Algorithm		
			Route Length (miles)	CPU Time (seconds)	RPD (%)	Route Length (miles)	CPU Time (seconds)	RPD (%)
ZGA1	25	2	1774.76	<1	72.63	1028.06	7	0
ZGA2	50	2	3124.81	<1	70.47	1833.08	95	0
ZGA3	75	2	4240.97	<1	86.80	2270.33	428	0
ZGA4	100	2	4719.21	<1	67.96	2809.75	1311	0
ZGA5	150	2	5147.96	<1	34.41	3829.94	6626	0
ZGA6	25	4	1369.82	<1	40.73	973.38	6	0
ZGA7	50	4	2049.95	<1	31.72	1556.26	70	0
ZGA8	75	4	2867.86	<1	36.34	2103.45	269	0
ZGA9	100	4	3143.08	<1	22.52	2565.43	920	0
ZGA10	150	4	4514.66	<1	23.47	3656.41	3934	0
ZGA11	25	6	1264.71	<1	29.61	975.75	6	0
ZGA12	50	6	1955.81	<1	27.53	1533.61	67	0
ZGA13	75	6	2821.04	<1	33.36	2115.40	256	0
ZGA14	100	6	3132.26	<1	25.34	2499.07	880	0
ZGA15	150	6	4413.43	<1	24.82	3535.75	3960	0
ZGA16	25	8	1252.76	<1	32.43	946.01	6	0
ZGA17	50	8	1958.35	<1	28.74	1521.21	67	0
ZGA18	75	8	2797.89	<1	32.49	2111.80	263	0
ZGA19	100	8	3078.08	<1	23.88	2484.76	884	0
ZGA20	150	8	4278.82	<1	20.26	3558.01	3861	0

	Average	2995.31	<1	38.27	2195.37	1195.80	0

It can be observed from Table 3 that the proposed ACS algorithm is superior to the two-phase heuristic algorithm. The average route length achieved by the ACS is 2195.37, while the average route length achieved by the two-phase heuristic is 2995.31. The RPD of the ACS is 0 for all instances, implying that the ACS produces better solutions in all instances than the proposed heuristics. The average RPD of the two-phase heuristic is 38.27 indicating that the ACS improves the solutions of two-phase heuristic by 38.27% on average. The RPD of the two-phase algorithm varies from 20.26% to 86.80%. The minimum RPD indicates that, even in worst case, the ACS can improve the solution of the two-phase heuristic by 20.26%.

In Table 4, the average results of instances which have the same number of customers and different numbers of AFSs are reported. It is noted that the average RPD of the two-phase heuristic decreases as the number of customers increases. The smallest problem set which contains 25 customers have an average RPD of 43.85%, while the largest problem set with 150 customers have an average RPD of 25.74%. The reason is that, as a meta-heuristic, the ACS algorithm can generate more feasible solutions than the two-phase algorithm. For small problem instances, the quantity of the feasible solutions generated by the ACS is large enough to capture the optimal solution. However, the ACS algorithm may not generate a sufficient number of feasible solutions for large problem instances.

In terms of CPU time, the two-phase heuristic is better than the ACS algorithm. As presented in Table 4, the average computation time of the two-phase heuristic is less than 1 second and that of the ACS algorithm is 1195.80 seconds. In general, when the time constraint is fully required and the concern of the solution accuracy is not important, the proposed two-phase

algorithm can be adopted. On the contrary, if there is a need for accurate solutions and minimizing computation time is not the top priority, the proposed ACS algorithm can be adopted.

Table 4. Impact of the number of customers

N	Two-phase Heuristic		Ant Colony Algorithm	
	Average Route Length (miles)	Average RPD (%)	Average Route Length (miles)	Average RPD (%)
25	1415.51	43.85	980.80	0
50	2272.23	39.61	1611.04	0
75	3181.94	47.25	2150.25	0
100	3518.16	34.92	2589.75	0
150	4588.72	25.74	3645.03	0

3. Analysis of the increase in the number of customers

An analysis on the impact brought by the increase in the customer quantity over the total route length is conducted. In Figure 5, the line graph shows the impact.

In this line graph, there are 4 lines which represent the instances with 2 AFSs, 4 AFSs, 6 AFSs and 8 AFSs. It can be observed that the total route length increases as the number of customers grows. This holds true for all types of AFSs. It is noted that the phenomenon of the scale of economy exists because the route length is not doubled if the number of customers is doubled. For an example, consider the problem instance with 2 AFSs; if the number of customers grows from 50 to 100, the route length increases from 1833.08 miles to 2809.75 miles, less than 3666.16 (1833.08 times by 2) miles. The practical implication is that the strategy to adopt AFVs in delivery process brings more benefits for the company with large number of customers, compared to the company with small number of customers.

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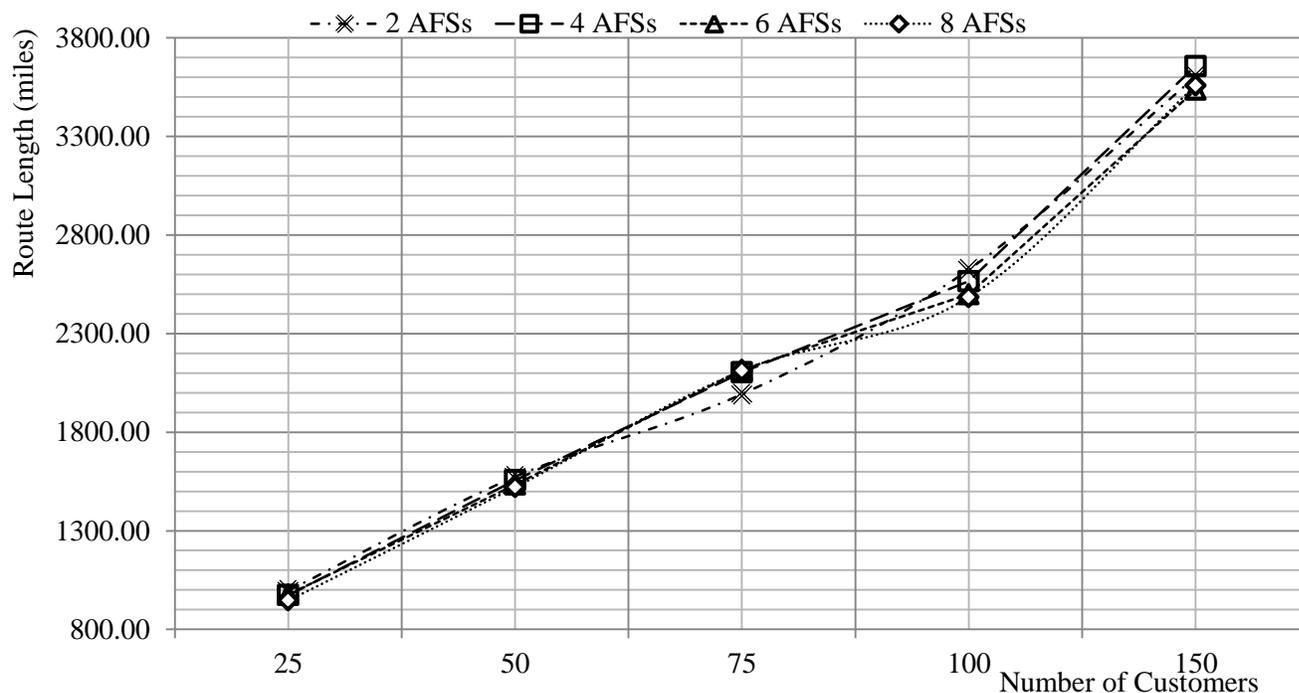
Figure 5. Results of the increase in the number of customers

4. Analysis of the increase in the number of AFSs

We analyze the impact of the total route length over the different number of AFSs in this subsection. The total route length with different number of AFSs is shown in Figure 6.

For most of problem instances, if the number of AFSs increases, the total route length decreases. For example, the route length of the instance with 100 customers and 2 AFSs is 2809.75 miles. As the number of AFSs increases, the route length decreases and when the number of AFSs reaches 8, the route length comes to 2484.76 miles, with a 324.99 decrease. However, an interesting finding is that when the number of AFSs grows, the decrease in route length gets smaller. The line with 100 customers is a good example. The route length for the problem instances with 2 AFSs is 2809.75 miles. When two more AFSs are set up, the route length is reduced to 2565.43 miles, with a 244.32 mile decrease. If the number of AFSs changes

to 6, the route length reduces by 66.36. This trend shows that the benefits (route length decrease) brought by adding one AFS decreases as the number of AFSs increases. If we borrow the principle used in Economics, the most beneficial number of AFSs is decided by the equation:

Marginal Costs Saving Brought by Building up One AFS

$$\geq Marginal\ Costs\ of\ Building\ up\ One\ AFS$$
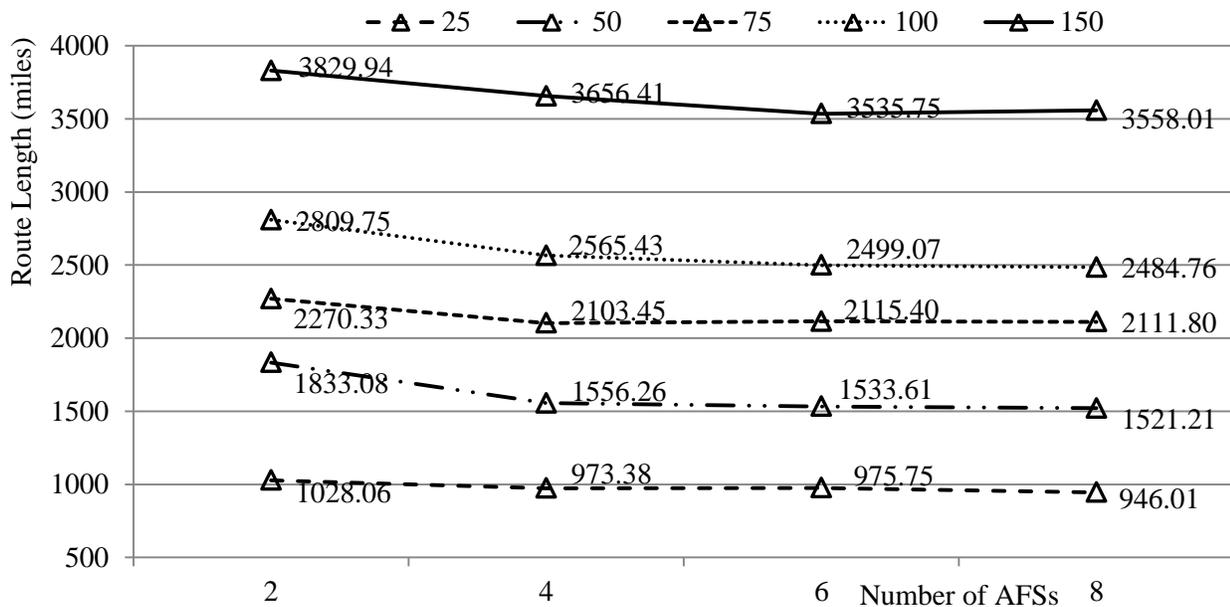
Figure 6. Analysis of the increase in the number of AFSs

5. Comparison between AFVs and conventional vehicles.

Although AFVs generate less GHG emissions than normal vehicles, one of the limitations of AFVs is its short service range. Thus, when companies employ AFVs in their business, the refueling issue appears at the first place. This section explores the impacts on the total route length, when AFVs are used to replace conventional vehicles.

The unlimited fuel tank capacity represents the use of conventional vehicles. When the fuel tank capacity is unlimited, conventional vehicles are refueled at the depot. This assumption is

reasonable and close to the real life cases. The ACS algorithm results with the limited and unlimited fuel tank capacity are shown in Table 5.

The average route length of 20 problem instances with the limited fuel tank capacity is about 300 miles greater than that of 20 problem instances without the fuel tank capacity constraint. It can be concluded that if a company wants to replace normal vehicles with AFVs, the company must prepare for the increase in the route length, which probably leads to an increase in cost. However, it should be noted that the environmental benefits brought by adopting AFVs are obvious. This contradiction between the economic costs and the environmental benefits happens frequently in the pursuit of sustainability. Hence, we are calling for more research on the trade-off decision-making about the economic and environmental benefits.

Table 5. The ACS algorithm results comparison

Instance	N	NS	Route Length with the Limited Fuel Tank Capacity (miles)	Route Length with the Unlimited Fuel Tank Capacity (miles)
ZGA1	25	2	851.37	1028.06
ZGA2	50	2	1393.63	1833.08
ZGA3	75	2	1879.69	2270.33
ZGA4	100	2	2227.67	2809.75
ZGA5	150	2	3160.64	3829.94
ZGA6	25	4	899.18	973.38
ZGA7	50	4	1389.51	1556.26
ZGA8	75	4	1865.89	2103.45
ZGA9	100	4	2243.28	2565.43
ZGA10	150	4	3055.54	3656.41
ZGA11	25	6	925.47	975.75
ZGA12	50	6	1394.74	1533.61
ZGA13	75	6	1866.39	2115.40
ZGA14	100	6	2222.71	2499.07
ZGA15	150	6	3061.26	3535.75
ZGA16	25	8	925.47	946.01
ZGA17	50	8	1394.74	1521.21

ZGA18	75	8	1866.39	2111.80
ZGA19	100	8	2222.71	2484.76
ZGA20	150	8	3061.26	3558.01
Average			1895.38	2195.37

3.4 Discussion

This chapter considers the Capacitated Green Vehicle Routing Problem and provides the corresponding formulation. In this problem, Alternative Fuel-powered Vehicles are employed in the delivery process. It is assumed that these vehicles have the limited tank capacity and loading capacity. In addition, the number of Alternative Fuel Stations is small. The two-phase heuristic and the Ant Colony System algorithm are proposed to solve the problem. The result of the numerical experiment shows that the ACS algorithm is superior to the two-phase heuristic in obtaining the high-quality solutions, but more computation time is required to use the ACS algorithm. On average, the performance of the ACS is 38.27% better than the performance of the two-phase heuristics.

Based on the results, further analyses were conducted. Firstly, it was found that the scale of economy exists in this problem. Secondly, we noticed that the marginal benefit brought by an additional AFS decreases when the number of AFSs increases. Finally, from the view of the firms employing AFVs, there are economic sacrifices for environmental benefits. The managerial implications of these findings can be concluded as follows. The first implication is that large firms are more likely to achieve benefits than small firms. The second implication is that when considering building up a new AFS, the companies need to calculate the marginal benefits brought by an additional AFS and the cost of building it. The last one is that the companies have no economic incentives to adopt AFVs, because the environmental benefits are

hard to measure. Therefore, more policies must be made to encourage firms to do business in a sustainable way.

The limitation of this research is that several parameters used in the proposed model are constant, such as the traveling speed and fuel consumption rate. In fact, these parameters are affected by many factors. Further improvements on setting these parameters can be done to make the model more realistic.

Chapter 4

A Pollution Routing Problem with Electric Vehicles

Driven by the growing concerns about greenhouse gases (GHG) emissions, carriers are commencing to adopt electric vehicles in distribution service. However, because of limited battery capacity, the electric vehicles have to visit the recharging stations en-rout. The traditional vehicle routing schemes for traditional vehicles are not suitable for routing electric vehicles. In this chapter, the Pollution-Routing Problem with Electric Vehicles (PRPEV) is introduced and the corresponding mathematical model is formulated. The PRPEV seeks to minimize the carbon dioxide emissions generated by Electric Vehicles visiting a set of customers and recharging stations. Most of people consider EVs are totally pollution free. However, if the electricity generation is considered, EVs generates pollutants indirectly while they are driving. The reason is that, in most parts of world, the generation of electricity mainly relies on burning fossil energy resources such as coal and nature gas. From this perspective, we project the carbon dioxide emissions of EVs by converting electricity consumptions of EVs to the amount of energy required to generate such amount of electricity. Furthermore, we consider the limited freight capacity of vehicles. As the solution methods of the PRPEV, a saving algorithm based heuristic method in conjunction with 3-opt interchange, and an ant colony system (ACS) based meta-heuristics are checked. The reliability and feasibility of the proposed algorithms are proved through extensive numerical experiments on newly designed instances. In the end, the benefits brought by adopting EVs in distribution are shown in the experiments.

4.1 Problem Formulation

To formulate the PRPEV model, the calculation of EV's energy consumption is firstly required to be described. The energy consumption of EV is affected by the distance traveled as well as other factors such as vehicle weight, speed, engine efficiency and so on. Bektaş and Laporte (2011) showed how to measure the total tractive power demand of a vehicle. The function is as follows:

$$P_{ij} \approx P(d_{ij}/v_{ij}) = \frac{P_{ij}d_{ij}}{v_{ij}}$$

$$= \alpha_{ij}(w + f_{ij})d_{ij} + \beta v_{ij}^2 d_{ij}$$

Where, $\alpha_{ij} = a + gsin\theta + gC_r cos\theta$ is an arc specific coefficient and $\beta = 0.5C_d A\rho$ is a vehicle specific coefficient. The result of this function is in joules ($J = \text{kg m}^2/s^2$) and needs to be translated into Kilowatt hour (kW · h). In addition, due to the waste of energy, total efficiency of utilizing the energy should be taken into consideration. Total efficiency including engine efficiency and charging efficiency of electric vehicle can be found in Davis and Figliozzi (2013):

$$eff_{tot} = eff_{eng} \times eff_{chg}$$
$$eff_{chg} = 0.8$$
$$eff_{eng} = 0.8$$
$$eff_{tot} = 0.8 \times 0.8 = 0.64$$

The rate at which energy consumptions are converted to carbon dioxide emissions released by generating such amount of energy, is given by Feng and Figliozzi (2013):

$$CO_2 \ Emission = 0.69 \ kg/kW \cdot h$$

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This formulation can only be used to the area where the electricity is generated by burning fossil energy.

The PRPEV can be defined as following. Let $G = (V, E)$ be a complete and directed graph, in which V is a set of vertices and E is a set of edges between different vertices. The vertex set V contains three subsets: customer set $C = \{1, 2, 3, \dots, N\}$, AFS set $S = \{N + 2, \ N + 3, \dots N + Ns + 2\}$, and depot set $D = \{0, N + 1\}$, so that $V = C \cup S \cup D$ and $|V| = N + Ns + 2$. It is assumed that the depot can recharge EVs when they are loading. When EVs reach either a recharging station or a depot, they are recharged to the battery capacity T. The edge set $E = \{(c_i, c_j): c_i, c_j \in V, i \neq j\}$ stands for the edges connecting different vertices of V. Every element of E is associated with the distance d_{ij} between two vertices i and j, the energy consumption f_{ij} and the speed s_{ij} for traveling this arch. The PRPEV model assumes that vehicles travel each arc at different speed and that energy consumptions are relevant to the speed, load and traveling distance of vehicles.

In the PRPEV, EVs start from a depot, visit a set of customers and finally return to depot. If it is a necessary to get recharge during service time, EVs have to visit a recharging station. It is assumed that the number of recharging stations visited by a vehicle in one tour can be greater than one. Also, any recharging station can be visited more than once by any vehicle. If the carried products were delivered completely, the vehicle would have to return to depot. To ensure the efficiency of delivery, every customer can be visited only once and the demand of each customer needs to be satisfied after this visit. On contrary, EVs can visit any recharging station more than once. To permit multiple visits to some recharging stations, we can augment graph G by creating graph $G' = (V', E')$ with dummy vertices set $S' = S \cup \{N + Ns + 3, N + Ns +$

$4, \dots, N + Ns + 2 + n_{N+2}, \dots \dots, N + Ns + 2 + \sum_{i=N+2}^{N+Ns+1} n_f, \dots, N + Ns + 2 + \sum_{i=N+2}^{N+Ns+2} n_f\}$.

Each element in θ represents a possible visit to a recharging station or a depot. $V' = V \cup \theta$. n_i is

a non-negative integer is the number of dummy vertices for every recharging station $i \in S$. In

this way, the number of visits for every recharging station is recorded by n_f.

Other notations used in the PRPEV formulation are defined in Table 6.

Table 6 The parameter and variables definition of the PRPEV model

$0, N + 1$	Depot Instances
S'	Set of visits to recharging, including the dummy vertices of recharging stations set S
S_0'	Set of visits to depot instance 0 and recharging stations vertices
C_0	Set of depot instance 0 and customer vertices
V'	Set of visits to customers and recharging stations: $V' = C \cup S'$
V_0'	Set of visits to customers, recharging stations and depot instance 0: $V_0' = V' \cup \{0\}$
V_{N+1}'	Set of visits to customers, recharging stations and depot instance $N + 1$: $$V_{N+1}' = V' \cup \{N + 1\}$$
$V_{0,N+1}'$	Set of visits to customers, recharging stations, depot instance 0 and $N + 1$: $$V_{0,N+1}' = V' \cup \{0\} \cup \{N + 1\}$$
d_{ij}	Distance between vertices i and j
e_i	The demand of vertex i. If $i \in S_0'$, $e_i = 0$
Q	Vehicle capacity
T	Battery capacity
w	Vehicle curb weight
l	The maximum number of vehicles used
α_{ij}	An arc specific constant, $\alpha_{ij} = a + g\sin\theta_{ij} + gC_r\cos\theta_{ij}$
β	$\beta = 0.5 C_d A \rho$, a vehicle specific constant
f_j	Decision variable specifying the remaining energy level after visiting vertex j
m_{ij}	Decision variable specifying the remaining cargo on vehicle while travelling arc (i,j)
x_{ij}	Binary decision variable indicating whether vehicle travel arc (i,j)

The mathematical model of PRP can be formulated as a mixed-integer program as follows:

$$\min \quad \sum_{\substack{i,j \in V' \\ i \neq j}} \alpha_{ij} d_{ij} w \, x_{ij} + \sum_{\substack{i,j \in V' \\ i \neq j}} \alpha_{ij} m_{ij} d_{ij} + \sum_{\substack{i,j \in V' \\ i \neq j}} d_{ij} \beta s_{ij}^2 \qquad (4.1)$$

Subject to

$$\sum_{\substack{j \in V' \\ i \neq j}} x_{ij} = 1, \qquad \forall\, i \in C \tag{4.2}$$

$$\sum_{\substack{j \in V' \\ i \neq j}} x_{ij} \leq 1, \qquad \forall\, i \in S' \tag{4.3}$$

$$\sum_{\substack{j \in V'_{N+1} \\ i \neq j}} x_{ji} - \sum_{\substack{i \in V'_0 \\ i \neq j}} x_{ij} = 0, \qquad \forall\, i \in V' \tag{4.4}$$

$$\sum_{i \in V'_0 \backslash \{0\}} x_{0i} \leq l \tag{4.5}$$

$$\sum_{i \in V'_{N+1} \backslash \{N+1\}} x_{i(N+1)} \leq l \tag{4.6}$$

$$f_j \leq f_i - \left[\alpha_{ij}\left(w + m_{ij}\right)d_{ij} + \beta s_{ij}^2 d_{ij}\right] \cdot x_{ij} + T\left(1 - x_{ij}\right), \quad \forall\, j \in V'_{N+1}, i \in C_0, i \neq j \tag{4.7}$$

$$f_j \leq T - \left[\alpha_{ij}\left(w + m_{ij}\right)d_{ij} + \beta s_{ij}^2 d_{ij}\right] \cdot x_{ij}, \qquad \forall\, j \in V'_{N+1}, \ i \in S_0{}' \tag{4.8}$$

$$\sum_{\substack{j \in V'_0 \\ i \neq j}} m_{ji} - \sum_{\substack{j \in V'_{N+1} \\ i \neq j}} m_{ij} = e_i \qquad \forall\, i \in V' \tag{4.9}$$

$$e_j x_{ij} \leq m_{ij} \leq (Q - e_i)x_{ij}, \qquad \forall\, i, j \in j \in V'_0 \in V'_{N+1}, i \tag{4.10}$$

$$x_{ij} \in \{0,1\}, \qquad \forall\, i, j \in C_0 \tag{4.11}$$

The objective function in this model seeks to minimize the total energy consumption of vehicles in (4.1). Constraints (4.2) guarantee every customer is visited only once. Constraints

(4.3) indicate that, every recharging station (and corresponding dummy vertex) has at most one successor. The flow conservation is established in constraints (4.4) which make the number of incoming vehicle is equal to the number of outgoing vehicle at each vertex. Constraints (4.5) and (4.6) enforce that the number of vehicles used is at most l. Based on the type and sequence of vertex, the remaining energy level of vehicle is tracked by constraints (4.7). Constraints (4.8) ensure the energy level of vehicle never falls below 0. The balance of cargo flow is shown in constraints (4.9) which regulate that the demand of every customer is satisfied when customer is visited. Constraints (4.10) restrict the total cargo on vehicle by vehicle capacity Q. Constraints (4.7) – (4.10) eliminate the possibility of generating sub-tours.

It is noted that the optimal solution for the proposed model is the energy consumption of vehicle in joules. To get the carbon dioxide emissions emitted for generating this energy, one more step of calculation needs to be conducted as follows:

$$Total\ Carbon\ Dioxide\ Emissions = Optimal\ Solution * \frac{0.69}{(3.6 \times 10^6)eff_{tot}}$$

4.2 Solution Method

As the solution methods to the PRPEV, a saving value based two-phase heuristic with 3-opt interchange and an ant colony system based meta-heuristic are proposed.

4.2.1 Two-phrase Heuristic

A two-phase heuristic is proposed to seek the solution of PRPEV. In the first phase, VRP is solved by using the Clarks and Wright saving algorithm with 3-opt interchange. In the second phase, according to the energy consumption level, the recharging stations are inserted in the VRP

route to make a feasible solution to the PRPEV. Finally, carbon dioxide emissions can be estimated by calculating energy consumption.

4.2.1.1 Phase One

In phase one, the Clarke and Wright saving algorithm (Clarke and Wright, 1964) with 3-opt interchange is used to find the VRP route. The Clarke and Wright algorithm is one of the most widely known heuristics for the VRP. The algorithm relies on the notion of savings. For two routes $(0, i, ..., 0)$ and $(0, ..., j, 0)$, the saving value of distance is calculated as $s_{ij} = c_{i0} + c_{0j} - c_{ij}$. The algorithm works as follows:

Step 1. Compute the saving value s_{ij} for $i, j = 1, ..., n$ and $i \neq j$. Sort the saving values in a descending order. Create n routes $(0, i, 0)$ for $i = 1, ..., n$.

Step 2. Start from the top of saving values list and, for any one of the saving values s_{ij}, decide if two routes: $(0, i, ..., 0)$ and $(0, ..., j, 0)$, can be merged feasibly (i.e. if the total customer demands of these two routes less than the vehicle capacity). If so, merge these two routes to generate a new route $(0, ..., j, i, ..., 0)$ and simultaneously delete the two original routes. Otherwise, stop this procedure when merge is not possible. Each of the remaining routes is one tour of the VRP route.

The VRP routes generated by using saving algorithm are developed through executing the 3-opt algorithm, called 3-opt, on all tours of VRP route. 3-opt was proposed by Lin, S. (1965) to solve Travelling Salesman Problem (TSP) and has been proved to be one of the best solution techniques for solving the VRP (Laporte and Semet, 2001). However, 3-opt was rarely used in literature because of coding complexity. In 3-opt, 3 arcs are deleted in one TSP route, and then

reconnect these route fragments in all possible ways. Finally find the optimal one. All 8 possible ways to connect 3 fragments into a tour are shown in Figure 7.
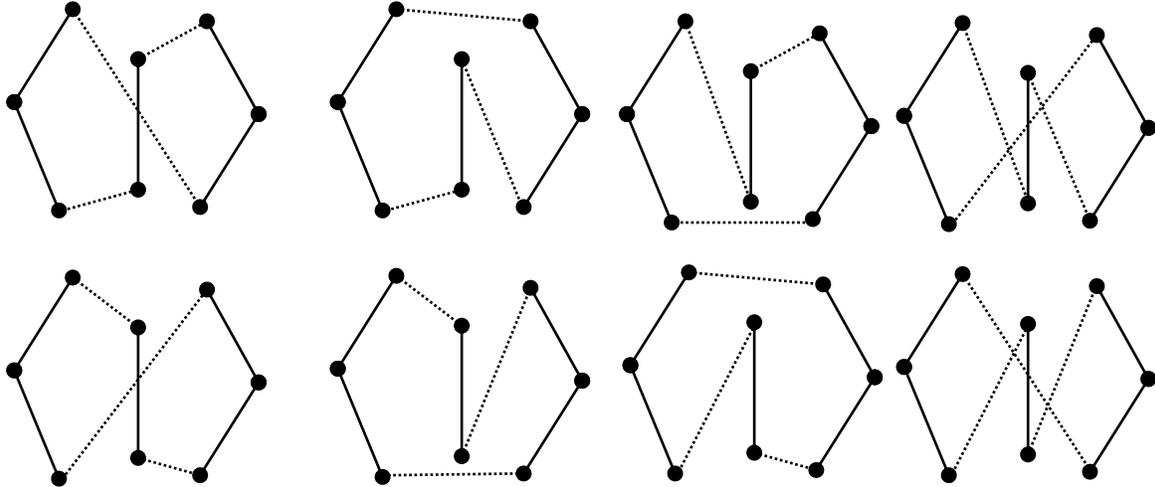

Figure 7 Eight ways to connect 3 fragments into a tour

After using 3-opt, each TSP route is improved from the perspective of objective value. Use the depot as a conjunction to link all TSP routes together. Eventually so the complete VRP route is generated.

4.2.1.2 Phase Two

In phase two, the VRP route found in the first phase is used to build the PRPEV route. The recharging stations are inserted in the VRP route when the remaining energy of vehicle is not enough to support the vehicle to reach next customer or to return to depot. However, the insertion operation is complex. In some cases, a visit to a recharging station or a depot becomes impossible and thus a recharging station needs to be visited before current customer.

When the PRPEV route is built, the energy consumption of vehicle for traveling this route can be calculated. Figure 8 presents the basic steps of our two-phase heuristic.

Input:
 d_{ij} Distance between each vertex i, j (customers, charging stations and depot).
 e_i Demand of vertex i
 w Curb weight of vehicle
 Q Loading capacity of vehicle
 T Battery capacity of vehicle
 N Number of customers
 Ns Number of charging stations
 α_{ij} An arc specific constant
 β A vehicle specific constant
 n Number of feasible routes
Output:
 Route plan R of PRPEV
Pseudo-codes:
1: Create N saving routes $(0, C_n, 0)$ and let the feasibility of each route be 1
2: While number of feasible routes> 1 do
    If new combination exists (the total demands of two routes are less than the vehicle
    capacity)
    Find all possible combinations;
    Select the combination which has the best saving value;
    According to the best combination, create a new route which merges the best two
    fragments together and set the feasibility of this merged route to be 1;
    Make the feasibilities of the merged two routes be 1 and $n = n - 1$
    End if
End while
3. Execute 3-opt on all routes which have feasibility 1 and find the optimal VRP route.
4. Create the PRPEV route by inserting charging stations in the VRP route.
5. Calculate energy consumptions and carbon dioxide emissions, and find the minimum
emissions route

Figure 8 Basic steps of two-phase heuristic

4.2.2 Ant Colony Algorithm Based Meta-heuristic

Ant colony (AC) algorithm is inspired by the ants' food searching behavior (Dorigo,

Maniezzo, and Colorni 1996). Since Bullnheimer, Hartl, and Strauss (1999) designed an AC

algorithm to solve the VRP, the developed AC algorithms have been widely used to solve

different variants of the VRP problem effectively (e.g., Gajpal and Abad, 2009; Yu, Yang, and

Yao, 2009; Yu and Yang, 2011; Abdulkader, Gajpal, and ElMekkawy, 2015; Schyns, 2015).

In the AC algorithm, artificial ants are created through trail density at the beginning of each iteration. Then the artificial ants start from the depot and visit all customers one by one. By conforming to some specific rules of selecting unvisited customers, the artificial ants construct the routes, which are the solution for VRP. At the end of each iteration, some best ants which generate good solutions are recorded as the elitist ants. The trail intensity is updated by using the elitist ant solutions. In this way, the features of good solutions can be recorded in the trail intensity. By setting some specific termination conditions, the final solutions can be achieved.

In the following subsections, the detailed procedures of the proposed AC algorithm based meta-heuristic for solving the PRPEV are performed.

1. Initialization of trail intensity

In the AC algorithm based meta-heuristic, trail intensity τ_{ij} captures the information of the visit form customer i to customer j. Before implementing the meta-heuristic, initial trail intensities of all possible visits are same. In this chapter, the initial trail intensity is set as 0.01.

2. Generation of ant solutions

When generating ant solutions for PRPEV, we first construct m number of ant solutions for the VRP, i.e. battery capacity constraint is not considered. The m VRP solutions are achieved by fulfilling following 2 steps.

1) Construct m solutions for the TSP.

Saving value and trail intensity are two factors which affect the probability of one unvisited customer as the next customer through attractiveness value, which is calculated as follows: $\xi_{ij} = [S_{ij}]^{\alpha}[\tau_{ij}]^{\beta}$. In this equation, α is saving value bias and β is trial intensity bias. These two parameters are decided by the model and are fixed at the

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beginning of algorithm execution.

Based on attractiveness value, the next customer in TSP solution is selected from Ω_q which represents the set of q unvisited customers. The probability of selecting customer y_j in Ω_q is calculated by: $P_{xy_i} = \frac{\xi_{xy_i}}{\sum_{j=1}^{q} \xi_{xy_j}}$, in which $1 \leq i \leq q$ and x is current customer.

2) Construct m solutions for the VRP.

In this step, the depot is inserted in the TSP solution to construct the VRP solution. If the remaining products carried by vehicle are less than the demand of next customer in the TSP solution, a depot needs to be inserted (visited) in the TSP solution. When all customers are visited, a depot needs to be inserted as a destination so that the complete VRP solution is constructed.

3. Execution of 3-opt

To improve the solution quality, the mentioned 3-opt is executed on each tour of the VRP solution achieved in previous step (for instance, an VRP solution, $0 - 1 - 4 - 2 - 10 - 5 - 0 - 11 - 6 - 14 - 9 - 12 - 3 - 0 - 15 - 8 - 13 - 7 - 0$, has 3 tours, i.e. $0 - 1 - 4 - 2 - 10 - 5 - 0, 0 - 11 - 6 - 14 - 9 - 12 - 3 - 0$ and $0 - 15 - 8 - 13 - 7 - 0$).

4. Update elitist ants

The set of elitist ant contains the best λ routes found until the current iteration. The main purpose of using the elitist ants is to change trail intensity. The elitist ants are updated by comparing them with current ant solutions. If one current ant solution is better than one elitist ant solution, then this solution becomes a part of elitist ant solutions and the worst solution of the elitist ant solutions is removed from the list.

5. Update trail intensity

Trail intensity is changed through the elitist ant solutions. The equation to update the trail intensity τ_{ij} of the arc connecting customers i and j is:

$$\tau_{ij}{}^n = \tau_{ij}{}^o \times \varphi + \sum_{m=1}^{\lambda} \tau_{ij}{}^m, \quad i \neq j \text{ and } i,j = 1,2,\dots,n$$

The first term of the right part of this equation stands for the old trail intensity of previous iteration information. φ is the trial persistence which is between 0 and 1. The second term shows the increase of trail intensity brought by the m^{th} elitist ant. The value of $\tau_{ij}{}^m$ is calculated by using the following equation:

$$\tau_{ij}{}^m = \begin{cases} 0, & \text{if the arc between customer } i \text{ and } j \text{ is not in the elitist ant route.} \\ \dfrac{1}{l_m}, & \text{otherwise.} \end{cases}$$

Here l_m is the route length of the m^{th} elitist ant solution. In this chapter, φ is set as 0.95, indicating that pheromone density decreases slowly in every iteration.

6. Parameter settings

The number of iterations and the quantity of ants are two main factors which affect the solution performance and computing time of the AC algorithm. Considering the complexity of the CGVRP, we determine the number of iterations to be 5000. In every iteration, n ants are generated to find feasible solutions. The feasible solutions are improved through 3-opt. In this chapter, $\alpha = 5$, $\beta = 5$ and $\lambda = 10$.Tthe value of φ is set as 0.9, which is recommended by Reed, Yiannakou, and Evering (2014).

4.3 Numerical Experiments and Analysis

In this section, the extensive numerical experiments are presented to evaluate the performance of our heuristic and meta-heuristic. Because the benchmark problem instances of the PRPEV are not available in current literature, the new benchmark problem instances are generated in this chapter. In the first experiments, the performance of our AC algorithm based meta-heuristic is evaluated. To assess the quality of solution achieved by the meta-heuristics, some small instances which can be solved by using the commercial solver CPLEX are designed. In the second experiment, we designed the medium-size PRPEV instances and use our heuristic and meta-heuristic to solve them. The comparison between the solutions of our heuristic and meta-heuristic is performed. The merit performance concerning solution quality and computation time of our based meta-heuristics on the newly designed instances is validated. Finally, we also compare the solution achieved by using different objective functions to illustrate the importance of using new objective functions in routing electric vehicle to lower the environmental costs of distribution.

4.3.1 Parameter Setting and Experimental Environment

The parameters used in the benchmark problem instances are described in this subsection. It is assumed that the electric vehicle used has a battery capacity which is 110 kWh. On each arch between two customers, the vehicle is assumed to run at constant speed, which is randomly selected from $[30, 40, 60, 80] km/h$. In addition, the loading capacity of vehicle is set as 3 tons of products. The proposed solution methods are coded in C programming language and are implemented on AMD Opteron 2.3 GHz with 16 GB of RAM. *A Mathematical Programming Language* (AMPL) with CPLEX solver is used to obtain optimal solutions for small instances. AMPL runs on a desktop with Core i5-4590, 3.3 GHz with 8 GM of RAM.

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4.3.2 Generation of PRPEV Benchmark Instances

10 instances which consider 10-19 customers and 2 recharging stations are designed as small benchmark instances. 40 instances which consider 25, 50, 75, 100 and 150 customers (C) and 2, 4, 6 and 8 recharging stations (R) are designed as medium-size instances. In addition, there are two instances (for example, C25R2-1 and C25R2-2) having same number of customers and recharging stations. The locations of customer in the two instances are same, but the locations of recharging stations are different. In total, 50 problem instances are generated. All instances have only one depot which is located in the center of one 200 by 200 miles grid. The customers are randomly scattered within the grid and the AFSs are randomly situated. The demand of each customer is generated randomly between 0.05 and 0.15 tons of products.

4.3.3 Results Analysis

In numerical experiments, we first evaluate the performance of our AC based meta-heuristic on small problem instances. Then, the proposed heuristic and meta-heuristic are used to solve medium-size instances to show their performance.

4.3.3.1 Experiments on Small Problem Instances

In this subsection, the meta-heuristic solutions for small instances are compared with the optimal solutions found by using CPLEX solver as shown in Table 7. To measure the solution gap between the two solution methods, the percentage gap from optimal solution is used.

Table 7 Numerical results for small problem instances

Instance	Carbon dioxide emissions		Percentage Gap
	Meta-heuristic	Optimal Solution	
C10R2	44.893	43.464	3.29
C11R2	44.923	44.070	1.93
C12R2	42.476	42.164	0.74

C13R2	47.404	46.019	3.01
C14R2	50.753	48.156	5.39
C15R2	51.330	48.702	5.40
C16R2	54.019	53.264	1.42
C17R2	55.285	53.758	2.84
C18R2	56.712	55.491	2.20
C19R2	63.778	59.069	7.97
C20R2	62.047	58.402	6.24
C21R2	65.500	60.805	7.72
C22R2	68.358	63.872	7.02
C23R2	69.655	64.257	8.40
C24R2	69.340	65.048	6.60
Average	56.432	53.769	4.68

From Table 7, it can be seen that the percentage gap between meta-heuristic solution and optimal solution is small. In terms of solutions quality, the solutions achieved by our meta-heuristic are good, even though they are not optimal ones for small problem instances. It is notable that all problem instances can be solved by both solution methods in one hour, showing that both methods are acceptable in computation time. The complete solutions to the small problem instances are reported in the Appendix.

4.3.3.2 Experiments on Medium Problem Instances

Even though they can solve the small problem instances optimally, the commercial solvers such as CPLEX are not able to solve the medium problem instances in a reasonable time or cannot solve the instances due to the great number of variables. However, the proposed heuristic and meta-heuristic can provide good quality solutions for medium problem instances. The numerical results of experiments on medium problem instances and the CPU time are reported in Table 8. To solve the instances, the iterations of executing the proposed meta-heuristic are set as 5000.

It shows in the table that all instances can be solved by the meta-heuristic in or around 30 minutes and demonstrates that our meta-heuristic can be used to solve medium problem instances efficiently. Even though the computation time for using the proposed heuristic is extremely shorter than using meta-heuristic, the solution quality of heuristic is worse. In general, when the time constraint is fully required and the concern of the solution accuracy is not important, the proposed heuristic algorithm can be adopted. On the contrary, if there is a need for accurate solutions and minimizing computation time is not the top priority, the proposed AC based algorithm can be adopted.

From the results of instances with same number of customer, it is clear that more recharging stations are located; more savings on energy consumption can be achieved. The managerial insight of this finding is that, more recharging stations have to be built to satisfy the short driving range limitation of electric vehicles as well as to reduce the carbon emissions. However, the marginal decrease in carbon dioxide emissions brought by one extra recharging station is dropping when the number of recharging stations increases. This phenomenon is worthy of studying and can bring some managerial insights to us. In real life cases, to build one recharging facilitates can induce more cost to the firms who employ electric vehicles. The optimal number of recharging stations is needed to be decided when company designs the recharging network. For the future research, more analysis can be conducted on how to find optimal number of recharging stations located.

Table 8 Numerical results for medium problem instances

Instance	Heuristic			Meta-heuristic		
	Carbon dioxide emissions	Time (s)	RPD	Carbon dioxide emissions	Time (s)	RPD
C25R2-1	92.605	<1	30.33	71.056	35	0

C25R2-2	109.369	<1	36.98	79.846	35	0
C25R4-1	88.857	<1	18.62	74.906	36	0
C25R4-1	78.171	<1	0.75	77.589	35	0
C25R6-1	80.471	<1	8.26	74.333	36	0
C25R6-2	93.850	<1	22.79	76.430	35	0
C25R8-1	80.860	<1	22.72	65.890	36	0
C25R8-2	108.894	<1	30.71	83.312	36	0
C50R2-1	208.983	<1	103.65	102.619	155	0
C50R2-2	177.964	<1	74.58	101.936	157	0
C50R4-1	157.948	<1	41.83	111.360	155	0
C50R4-2	160.349	<1	41.38	113.421	155	0
C50R6-1	151.848	<1	45.98	104.022	156	0
C50R6-2	176.552	<1	53.05	115.355	152	0
C50R8-1	148.816	<1	42.46	104.461	154	0
C50R8-2	177.347	<1	68.93	104.982	158	0
C75R2-1	291.895	<1	108.19	140.209	396	0
C75R2-2	237.255	<1	51.37	156.740	394	0
C75R4-1	263.084	<1	78.12	147.697	385	0
C75R4-2	233.463	<1	61.59	144.483	389	0
C75R6-1	227.799	<1	54.58	147.371	397	0
C75R6-2	227.886	<1	59.46	142.910	399	0
C75R8-1	215.741	<1	59.85	134.968	385	0
C75R8-2	226.163	<1	47.68	153.140	400	0
C100R2-1	417.420	<1	111.46	197.396	1744	0
C100R2-2	356.397	<1	88.15	189.418	791	0
C100R4-1	338.442	<1	98.37	170.614	784	0
C100R4-2	322.312	<1	92.65	167.309	784	0
C100R6-1	302.812	<1	81.13	167.176	777	0
C100R6-2	295.962	<1	78.20	166.087	794	0
C100R8-1	309.689	<1	98.36	156.122	781	0
C100R8-2	378.472	<1	104.95	184.661	798	0
C150R2-1	454.344	<1	96.07	231.720	1824	0
C150R2-2	461.412	<1	80.22	256.033	1846	0
C150R4-1	461.060	<1	105.97	223.844	1840	0
C150R4-2	459.184	<1	93.32	237.524	1835	0
C150R6-1	441.523	<1	100.76	219.926	1824	0
C150R6-2	464.132	<1	104.75	226.679	1841	0
C150R8-1	430.629	<1	92.40	223.822	1828	0
C150R8-2	475.996	<1	110.48	226.152	1832	0
Average	259.649	<1	67.528	146.838	409.436	0

4.3.3.3 Minimizing Carbon Dioxide Emission V.S. Minimizing Route Length

To further explore the potential impacts a carbon dioxide emissions-minimizing objective function can have when routing electric vehicles, we replace the carbon dioxide emissions-minimizing objective with the distance-minimizing objective in our meta-heuristic and use the modified meta-heuristics to solve all medium problem instances. The corresponding results are shown in Table 9.

As shown in Table 9, on average, it is apparent that all solutions achieved by using different objective functions are not identical and the percentage gap is significant. In 42 out of 50 problem instances, the solution achieved by using emissions-minimizing objective has lower carbon emissions than the solution obtained by minimizing route length, even though longer route is traveled. In the other 8 instances, the reasons why the solution of using emissions-minimizing objective is not able to get lower emissions are the disadvantage of meta-heuristic and the complexity of emission-minimizing problem.

This experiment clearly demonstrates the importance of using carbon dioxide emissions-minimizing objective function rather than route length-minimizing objective when we try to achieve our goals to mitigate the side effects on environment in routing electric vehicles.

Table 9 Numerical results with different objective functions

Instance	Minimizing route length	Minimizing carbon dioxide emissions	PG
	Carbon dioxide emissions	Carbon dioxide emissions	
C25R2-1	71.531	71.056	0.669
C25R2-2	90.708	79.846	13.604
C25R4-1	78.971	74.906	5.427
C25R4-1	85.904	77.589	10.717
C25R6-1	78.557	74.333	5.682
C25R6-2	81.442	76.430	6.557
C25R8-1	75.161	65.890	14.070

C25R8-2	86.757	83.312	4.134
C50R2-1	103.927	102.619	1.274
C50R2-2	115.106	101.936	12.920
C50R4-1	123.567	111.360	10.961
C50R4-2	124.910	113.421	10.130
C50R6-1	108.098	104.022	3.918
C50R6-2	117.623	115.355	1.966
C50R8-1	112.186	104.461	7.395
C50R8-2	120.539	104.982	14.818
C75R2-1	156.212	140.209	11.414
C75R2-2	147.257	156.740	-6.050
C75R4-1	172.300	147.697	16.658
C75R4-2	149.865	144.483	3.725
C75R6-1	149.862	147.371	1.690
C75R6-2	140.066	142.910	-1.990
C75R8-1	147.206	134.968	9.067
C75R8-2	160.997	153.140	5.131
C100R2-1	211.117	197.396	6.951
C100R2-2	175.133	189.418	-7.541
C100R4-1	179.140	170.614	4.997
C100R4-2	183.259	167.309	9.534
C100R6-1	172.344	167.176	3.091
C100R6-2	172.254	166.087	3.713
C100R8-1	175.122	156.122	12.170
C100R8-2	184.422	184.661	-0.130
C150R2-1	222.127	231.720	-4.140
C150R2-2	230.884	256.033	-9.822
C150R4-1	223.231	223.844	-0.274
C150R4-2	229.657	237.524	-3.312
C150R6-1	224.879	219.926	2.252
C150R6-2	237.061	226.679	4.580
C150R8-1	233.559	223.822	4.350
C150R8-2	234.872	226.152	3.856
Average	152.195	146.838	4.854

4.4 Discussion

A new problem, the Pollution Routing Problem with Electric Vehicles is proposed. Unlike

the PRP and G-VRP, the PRPEV aims to finding a routing plan for electric vehicles to minimize

the carbon dioxide emissions of vehicles. Several real-life constraints regarding to vehicle capacity, battery capacity, recharging operation and energy consumption are considered. As a new variant of the VRP, NP-hardness brings difficulties to solve this problem. One heuristic integrating Clark and Write algorithm and 3-opt interchanges and one ant colony based meta-heuristics are proposed. Extensive numerical experiments on different size problem instances are conducted to examine the validity and efficiency of the proposed algorithms.

We also compared the results achieved by using different objective functions, i.e. the carbon dioxide emission-minimizing objective function and the route length-minimizing objective function in our model. The comparison illustrates the importance of using the new objective function in routing electric vehicles. Because the traditional objective functions to lower economic cost cannot guarantee that the environmental cost is lowered at the same time. The managerial insights brought by this finding is that when companies intend to design environment friendly routing plan for EVs, they need to adopt new objective function rather than the classic one to achieve their goals.

In future study, more constraints, such as time window constraints and heterogeneous vehicles constraints are called on. In addition, the recycle issue of battery of electric vehicles needs to be addressed because it also creates big environmental problem. Meanwhile, new meta-heuristics for solving this complex problem are also worthy of proposing.

Chapter 5

Conclusion

In this thesis, we discussed two different routing problems and designed the routing plans aiming at minimizing carbon dioxide emissions. In the first problem, AFVs are used in the distribution system of some companies. Due to the limited fuel tank capacity, AFVs need refueling at AFSs while they are in the service time. In the second problem, a more complex problem is discussed. Unlike the fuel consumption rate which is fixed in the first problem, the energy consumption in the second problem is affected by different factors such as vehicle speed, vehicle weight, travelling distance and so on. This difference makes the second problem much more complex than the first one.

To solve the proposed two problems, different solution methods were designed. For the first problem, the two-phase heuristic and the Ant Colony System algorithm were proposed as solution methods. The result of the numerical experiment showed that the ACS algorithm is superior to the two-phase heuristic in obtaining the high-quality solutions, but more computation time is required to use the ACS algorithm. On average, the performance of the ACS is 38.27% better than the performance of the two-phase heuristics. For the second problem, one heuristic integrating Clark and Write algorithm and 3-opt interchanges and one ant colony based meta-heuristics are proposed. Extensive numerical experiments on different size problem instances are conducted to examine the validity and efficiency of the proposed algorithms.

From the different numerical experiments in this thesis, we can find the complexity of the proposed two problems. Because the constraint that energy consumption is relative to only travelling distance is removed, the second problem is much more complex than the first, thus making the problem hard to be solved. We also compared the results achieved by using different

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objective functions, i.e. the carbon dioxide emission-minimizing objective function in the second problem and the route length-minimizing objective function in the first problem. The comparison illustrates the importance of using the new objective function in routing electric vehicles. Because the traditional objective functions to lower economic cost cannot guarantee that the environmental cost is lowered at the same time.

One limitation of this thesis is that, in both proposed models, other constraints like time window constraints which are usually common to be seen in VRP studies have not been considered. In future studies, more constraints need to be considered to make the model more realistic.

Research Output

This thesis is developed from the research events listed below:

Journal paper:

Zhang, S., Gajpal, Y., & Appadoo, S. S. (2017). A meta-heuristic for capacitated green vehicle routing problem. *Annals of Operations Research, 1-19.*

Working paper:

Zhang, S., Gajpal, Y., & Appadoo, S. S. (2017). The pollution routing problem with electric vehicles. Under submission at *Production Economics.*

Conference presentation:

Zhang S. and Gajpal Y., (2016) Two-phase Heuristic Approach for Capacitated Green Vehicle Routing Problem, "*2016 Administrative Sciences Association of Canada (ASAC) Conference, Edmonton, Canada, June 4-7, 2016*"

Zhang S. and Gajpal Y., (2017) The Pollution Routing Problem with Electric Vehicles, "*2017 Administrative Sciences Association of Canada (ASAC) Conference, Montreal, Canada, May 29-June 1, 2017*"

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Appendix

Complete Solutions of Small Problem Instances

Instance	Complete solutions	
	Meta-heuristic	Optimal Solution
C10R2	0-1-5-10-4-2-14-9-7-8-3-6-0	0-6-3-8-7-9-11-2-4-10-5-1-0
C11R2	0-1-5-10-4-2-12-9-7-8-3-11-6-0	0-6-11-3-8-7-9-12-2-4-10-5-1-0
C12R2	0-1-12-9-13-5-10-4-2-7-8-3-11-6-0	0-6-11-3-8-7-2-4-10-5-13-9-12-1-0
C13R2	0-1-12-9-14-10-5-13-4-2-7-8-3-11-6-0	0-6-11-3-8-7-2-4-13-14-9-12-1-10-0
C14R2	0-10-5-2-4-13-9-12-1-0 0-14-11-6-3-8-7-0	0-1-12-9-13-4-2-5-10-15-7-8-3-6-11-14-0
C15R2	0-14-11-6-3-8-7-2-4-13-16-5-10-15-17-9-12-1-0	0-1-12-9-15-10-5-2-4-13-16-7-8-3-6-11-14-0
C16R2	0-1-12-9-19-15-10-5-13-4-2-7-8-3-6-16-11-14-0	0-14-11-16-6-3-8-7-2-4-13-5-10-15-18-9-12-1-0
C17R2	0-1-17-12-9-20-15-10-5-13-4-2-7-8-3-6-16-11-14-0	0-14-11-16-6-3-8-7-2-4-13-5-10-15-19-9-12-17-1-0
C18R2	0-1-17-12-9-21-15-10-5-13-4-2-7-8-18-3-6-16-11-14-0	0-14-11-16-6-3-18-8-7-2-4-13-5-10-15-20-9-12-17-1-0
C19R2	0-1-17-12-9-24-19-22-15-13-4-2-5-10-0	0-7-8-18-3-6-16-11-14-12-1-17-15-9-21-19-5-2-13-4-10-0

	0-14-11-16-6-3-18-8-7-0	
C20R2	0-1-17-12-9-23-19-25-15-10-5-2-13-4-20-7-8-18-3-6-16-11-14-0	0-14-11-16-6-3-18-8-7-20-4-13-2-5-10-15-22-19-9-12-17-1-0
C21R2	0-1-17-12-9-24-19-26-15-10-5-2-13-4-20-7-8-18-3-11-16-6-21-14-0	0-14-21-6 -16-11-3-18-8-7-20-4-13-2-5-10-22-15-9-19-17-12-1-0
C22R2	0-1-12-17-15-27-19-25-9-22-5-10-26-2-13-4-20-7-8-18-3-11-16-6-21-14-0	0-21-6-16-11-3-18-8-7-20-4-13-2-5-10-15-24-19-9-22-17-1-12-14-0
C23R2	0-10-5-22-17-15-28-19-26-9-12-1-0 0-14-21-6-16-11-3-18-8-7-20-4-13-2-23-0	0-14-11-16-6-21-3-18-8-7-20-4-13-2-23-24-10-5-22-15-9-19-17-12-1-0
C24R2	0-10-5-22-17-15-27-19-29-9-12-1-0 0-14-21-6-11-16-24-3-18-8-7-20-4-13-2-23-0	0-14-21-6-11-16-24-3-18-8-7-20-4-13-2-23-25-10-5-22-15-9-19-17-12-1-0