# Resource Allocation and Performance Evaluation in Relay-Enhanced Cellular Networks

by

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# Abstract

The focus of this thesis is on end-to-end (e2e) queueing performance evaluation and resource allocation in order to improve the performance of the relay-enhanced cellular networks. It is crucial to study both the performance of the data link layer and the physical layer issues. Therefore, we first consider end-to-end queueing performance evaluation and after that to consider physical layer issues, we present power allocation schemes, relay load balancing and relay assignment. First, we presented a framework for the link-level end-to-end queueing performance evaluation. Our system model consists of a base station, a relay, and multiple users. The e2e system is modeled as a probabilistic tandem of two finite queues. Using the decomposed model, radio link-level performance measures such as e2e packet loss rate, e2e delay and throughput are obtained analytically and compared with simulation results. A framework for power allocation for downlink transmissions in decode-and-forward relay networks is investigated. We consider a system with a single base station communicating with multiple users assisted by multiple relays. The relays have limited power which must be divided among the users they support in order to maximize the data rate of the whole network. Based on knapsack problem, the optimal power allocation is proposed. To consider fairness, weighted-based scheme is presented. Moreover, to utilize the power wisely, an efficient power reallocation scheme is proposed. Simulation results demonstrate the efficacy of the proposed schemes. By applying the relay selection scheme, it may happen that some relays have more users connected to them than other relays, which results in having unbalanced load among the relays. In order to address this issue, a game theoretic approach is presented. Coalition formation game is proposed based on merge-and-split rule to form the optimal structure. The simulation results demonstrate the effect of applying game in proposed problem. Finally, the relay assignment procedure is studied. The optimal solution is found using Lagrangian Relaxation. Then, a lighter algorithm is proposed to efficiently carry out the relay assignment. Simulation results show that the proposed algorithm can achieve near optimal data rate, while it decreases the processing time significantly.

**Keywords**: Queueing, power allocation, relay load balancing, relay assignment, coalitional game theory.

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# List of Abbreviations

| | |
|---|---|
| E2E | End-to-End |
| ARQ | Automatic Repeat Request |
| TDM | Time-Division Multiplexing |
| QoS | Quality of Service |
| 3GPP | 3rd Generation Partnership Project |
| LTE Avanced | Long Term Evolution Advanced |
| OFDMA | Orthogonal Frequency Division Multiple Access |
| WiMAX | Worldwide Interoperability for Microwave Access |
| MIMO | Multiple-Input Multiple-Output |
| STC | Space-Time Coding |
| BS | Base Station |
| TSG j | Technical Specification Group j |
| UMB | Ultra Mobile Broadband |
| AF | Amplify-and-Forward |
| DF | Decode-and-Forward |
| CF | Compress-and-Forward |
| RF | Regenerate-and-Forward |
| BER | Bit Error rate |
| RC | Repetition Coding |

| | |
|---|---|
| UC | Unconstrained Coding |
| PF | Proportional Fairness |
| NE | Nash Equilibrium |
| AMC | Adaptive Modulation and Coding |
| WRR | Weighted Round Robin |
| MRC | Maximum Ratio Combining |
| SINR | Signal to Interference plus Noise Ratio |
| NACK | Negative Acknowledgment |
| DTMC | Discrete Time Markov Chain |
| PER | Packet Error Rate |
| HOL | Head of Line |
| GAP | Generalized Assignment Problem |
| FSMC | Finite State Markov chain |

# Chapter 1

# Introduction

## 1.1 Background and Motivation

Wireless communication networks have remarkably developed in recent years. Although separated by only a few years, each new generation of wireless devices has brought significant improvements in terms of link communication speed, device size, battery life, applications, etc. Increasing number of users demanding wireless access and growing number of wireless applications require a higher link data rate and certain quality of service (QoS) requirements such as lower end-to-end (e2e) delay. With a rapid growth of the number of users, and the scarcity of frequency spectrum, cellular systems are facing difficulty in providing satisfactory requirements to users, especially to those at the cell edge.

In wireless communication systems, the received signal varies as a result of the destructive and constructive interference of the multipath signals. Destructive interference results dramatic effects on the overall system performance. Wireless communications suffer from great challenges due to detrimental fading effects of wireless channels [1].

Rapid development of wireless communication technologies and systems during the last decade has provided ubiquitous high data communication to mobile users by also developing new hardware and standards. Wireless communication networks such as 3rd Generation Partnership Project (3GPP) long term evolution advanced (LTE-Advanced) are expected to provide high data rate coverage in the most cost-effective manner. To achieve this objective for future communication systems, and overcome capacity degradation in a shadowing area or a cell edge, high spectral efficiency schemes are required as compared to current wireless communication systems to ensure efficient use of scarce resources such as power and bandwidth.

One solution to these problems is to increase the density of base stations significantly, which results in considerably higher deployment costs, and would only be feasible if the number of users would also increase at the same rate [2]. This issue is caused not only because of increase in the number of wireless communication users, but also because of the fact that the information which has to be transported has also grown significantly.

Classical cellular-like network architectures are unable to efficiently overcome these challenges and cope with the stringent quality of service requirements of emerging services. Due to the random quality of the wireless channels such as scattering, reflection and diffraction of the transmitted energy caused by obstacles such as buildings, trees, etc., multiple versions of a signal transmitted from a source may arrive at the destination via multiple paths with different attenuations, phase shifts and delays. The overall received signal could be a constructive or destructive superposition of these versions [3], [4]. In fact, severe destructive combinations may occasionally happen, causing a severe drop of the channel gain and a temporarily failure or discontinuity of the service.

In order to combat fading effects and boost system performance of wireless communications, some techniques known as diversity techniques have been proposed and widely adopted in practice. Depending on the characteristics of the channels as well as the transceiver structures, various diversity techniques in time, frequency, and space domains have been studied extensively in the literature and applied in practice [5]. By applying diversity techniques, multiple replicas of the transmitted signal experience independently faded channels. The probability that all of them go into deep fade decreases. Time diversity can be achieved by channel coding and interleaving [5]. However in delay-sensitive applications, time diversity is not applicable due to the delay constraints. Frequency diversity can be achieved in frequency-selective wideband systems [6].

Orthogonal frequency division multiple access (OFDMA) is an important multiple access technique for high data rate wireless communication systems, such as 3GPP long term evolution advanced and IEEE 802.16 Worldwide Interoperability for Microwave Access (WiMAX). It is not only because of its flexibility in resource allocation, but also because of its ability to exploit multiuser diversity. This technology is becoming a popular choice since the frequency selectivity effect can be eliminated by transmitting the wideband signal on multiple narrowband signals [7–10].

Wireless cellular networks have to be designed and deployed with unavoidable constraints on the limited radio resources such as bandwidth and transmit power [11]. The solutions should improve the capacity and utilization of the radio resources [12]. Spatial diversity using multiple antennas at the transmitter and/or the receiver enhances the signal quality while not degrading the system performance in terms of delay and bandwidth efficiency [3]. Various space-time codes have been proposed to provide spatial diversity [13], [14]. Multiple-input multiple-output (MIMO) is a well known technique which can increase spectral efficiency and link reliability in wireless communication, without additional bandwidth or transmit power [15], [16]. In the MIMO technique diversity relies on uncorrelated channels, and is achieved by employing multiple antennas at the terminal nodes, and by sufficiently separating the multiple antennas [17–19]. MIMO communication through space-time coding (STC) has been accepted as an effective means to increase the quality of service of point-to-point communications in terms of combating fading effects, increasing error performance, and boosting throughput by using multiple collocated antennas at the transmitter and/or the receiver [20], [3].

However, incorporation of multiple antennas at mobile nodes may not be desirable due to increase in size of mobile devices [21]. The spacing between antenna elements needs to be larger than half a wavelength to avoid fading correlation and antenna coupling. In many practical wireless applications, wireless devices are so miniaturized that such spacing between multiple antennas cannot be employed. Due to size, cost, and hardware complexity limitations, the implementation of multiple antennas might not be feasible in many applications [22], [12].

Cooperative communications is a technique which provides an alternative to MIMO systems to achieve spatial diversity gain [23]. Cooperative diversity or relay-assisted communication has been proposed as an alternative solution where several distributed terminals cooperate to transmit/receive their intended signals [24–28]. Cooperative diversity exploits the broadcast nature of the wireless medium. In cooperative communication, by use of fixed simple and small terminals, called relays, messages are transferred between the base station and users [2,29–38]. Cooperative diversity avoids the cost, size, and hardware complexity limitations and achieves spatial diversity in a distributed fashion. Therefore, it has been widely accepted as one of the most promising techniques in wireless communication networks.

In the next section, we give a brief overview of relays and their applications in wireless cellular networks.

## 1.2 Relays

The information theoretical properties of the relay channel have already been studied in the 1970s by Cover [39]. However the integration of relays into a cellular system has only gained attention around the year 2000. The research in [31] motivated to study the effect of relay deployment in cellular networks.

Deploying relays in cellular networks can help to improve the performance of the users efficiently and has the potential to solve the coverage problem for high data rates in wide areas [33]. Relays increase the total coverage area of the cell, especially at the cell edge [40], [127]. In a wireless communication system, relays with less cost and functionality than the base station ($BS$) [22–24, 42], can offer significant benefits in the throughput enhancement, transmission reliability and range extension [2,23,26,32,43–51].

Relays, with less functionality than a $BS$, can forward high data rates to remote areas of the cell and thus overcome the high path losses while maintaining low infrastructure cost [2]. By deploying relays in cellular networks, the cell radius increases. As a result, the number of base stations would decrease or even eliminated in some infrastructure-less deployments. Since the total cost of relays is less than base stations, the infrastructure cost would reduce to a great extent. Relaying for wireless networks has received considerable interest, as it provides coverage extension and reduces power consumption without incurring the high costs of additional base station deployment. Relays can be also used to increase the capacity of the cell.

By applying relays in the network, users require less resources from the BS due to the signal quality improvement achieved by help of the relays. Thus, the same resources can be shared among other users resulting in overall capacity improvement.

Relaying technologies are demonstrated to enhance the coverage of wireless systems as well as to provide considerable throughput gains [52], [53] and [54]. In the emerging OFDMA-based standards such as 3GPP long term evolution [55], [56] and IEEE 802.16j [57], [58] the multihop relay concept has been introduced to provide ubiquitous high-data-rate coverage. Coverage extension through multi-hop relays is considered in [49, 59–61]. The authors in [63] showed through simulation-based results, that relays provide range extension and spectral efficiency enhancement when deployed in different location in the cell. Multiple relay deployment in cellular networks is also discussed in [62]. Simulation results showed that deploying multiple relays per cell can significantly improve system capacity and coverage. The increase in coverage radius of the cell depends upon the placement of relays in the cell. There is a need for optimal relay placement to achieve maximum extension of the coverage radius of the cell. Researchers so far have addressed the issue of optimal placement of cellular relays. The authors in [64] and [65] analyze relay placement for wireless sensor networks, where the objective is to achieve maximum connectivity between pairs of relay nodes. In [66] and [67], the relay placement problem is analyzed from the perspective of increasing system capacity rather than coverage radius extension. [68] considers a dual relay architecture with cooperative relay pairs and proposes an algorithm to select the two best relay locations from a predefined set of candidate positions. In [69], an iterative relay placement algorithm is proposed which divides all points in the cell into good and bad coverage points and places relays at the good points whose neighbors have bad coverage.

Relays as part of infrastructure based networks have been standardised in the Technical Specification Group j (TSG j) of IEEE802.16j [58], [57]. 3GPP and 4G mobile systems considered relay technologies in the standardization process as well [44, 70–72]. Different relay classification will be discussed in next subsection.

## 1.3 Relays Classifications

Relays not only can be used in fixed infrastructures, but also can provide coverage on mobile vehicle, and temporary coverage for emergency and disaster recovery. Fixed relays can be deployed strategically and cost effectively in cellular networks to extend coverage, reduce total transmission power, enhance the capacity of a specific region with high traffic demands, and improve signal reception. Fixed relays have low-cost and fixed radio infrastructures without wired backhaul connections. They store the data received from the base station and forward to the users, and vice versa. Fixed relay, with possible mesh extensions, is also a very strong candidate technology for future wireless networks.

Similar to fixed relays, mobile relays can enlarge the coverage area, reduce the overall transmit power, and increase the capacity at cell edges. Mobile relays differ from fixed relays in the sense that the relays are mobile and are not deployed as the infrastructure of a network. Mobile relays are therefore more flexible in accommodating varying traffic patterns and adapting to different propagation environments. However, mobile relays are less reliable than fixed relays since the network topology is highly dynamic and unstable. Two types of mobile relay systems can be distinguished as moving networks and mobile user relays. The moving network employs dedicated relays on moving vehicles (e.g., trains) to receive data from the $BS$ and forwards to the mobile users onboard, and vice versa. The purpose of the moving network is to improve the coverage on the vehicle. The mobile user relay enables distributed mobile users to self-organize themselves into a wireless ad hoc network, which complements the cellular network infrastructure using multihop transmissions. In this thesis, we adopt fixed relay scenario due to its practicality.

A variety of different classifications have been used to categorize relay nodes in the LTE-A standard. In one category, relays may be distinguished based on the functionality [73] as

. *Repeater*: This type of relays is the simplest in terms of implementation and functionality. The relay simply receives the signals from the base station, amplifies it and then forwards it to the user.

. *Decoder/Encoder*: This relay is able to decode the received signals and re-code the transmit signals in order to achieve higher link quality. The advantage of achieving higher link quality comes at the expense of higher cost and complexity of the relay and also adds delay to the communication link.

. *Base station*: This type of relay has the functionality of a base station like mobility management, session set-up, and handover. Such functionality adds more complexity to the implementation of this relay and the delay budget is further increased.

A different classification is used in 3GPP standardization where two types of relays have been defined, Type I and Type II in [70], or non-transparency and transparency in [74].

. *Type I (or non-transparent)*: This relay type can help a remote mobile station, which is located far away from a base station, to access the base station. So a Type I relay needs to transmit the common reference signal and the control information for the base station, and its main objective is to extend signal and service coverage. Type I relays can mainly make some contributions to the overall system capacity by enabling communication services and data transmissions for remote mobile stations.

. *Type II (or transparent)*: the relays can help a local mobile station, which is located within or outside the coverage of a base station and has a direct communication link with the base station, to improve its service quality and link capacity. So a Type II relay does not transmit the common reference signal or the control information, and its main objective is to increase the overall system capacity by achieving diversity and transmission gains for local mobile stations.

It has been demonstrated that by deploying one or multiple relays that can perform cooperative transmission in a wireless network [2,26,52–54,75,76], a significant performance improvement can be witnessed in terms of throughput, bit error rate, capacity, or other metrics. Adding relays to a cellular network can potentially provide multiple transmission paths from source to destination. Availability of alternative paths not only increases diversity, but also increases reliability [77], [26]. Cooperative communication details will be discussed in the next section.

## 1.4 Cooperative Communication

In wireless communications, users experience various impediments such as interference, fluctuations in power due to reflections and attenuation, and randomly-varying channel conditions caused by mobility and changing environment. Cooperative communication have attracted interest as an emerging technology for wireless networks that can mitigate these degradations [78]. Cooperative communications take advantage of the broadcasting nature of wireless networks. Cooperative communication is formed by deploying group of relays, each with independent channel condition, which combat impairments caused by shadowing and path loss [79]. The basic idea is that, geographically distributed relays provide diversity that can significantly improve system performance [23, 80, 81], while resolving the difficulties of installing multiple antennas on small communication terminals.

The basic idea of cooperative communications can be traced back to the 1970s to van der Meulen [39, 82–84], in which a basic three-terminal communication model was first introduced. Consider a scenario which consists of a user, relay and $BS$. $BS$ tries to communicate with the user with the help of a relay. Due to the broadcasting nature of wireless transmission, user can receive the transmitted signal from $BS$ and then relay tries to assist user by sending some version of its received signal to the user. Because the two versions of the source signal experience independent fading paths, spatial diversity can be obtained in such a system. Depending on the propagation conditions, a direct link between a source and the destination may be useful or maybe not. Without the direct link, only propagation attenuation can be reduced. In contrast, with the direct link, the diversity benefit can also be obtained. Beyond the diversity capability to mitigate the fading effects, relaying transmission can also reduce the propagation attenuation to increase the capacity and coverage of the networks.

Starting from the early 2000s, cooperative communication techniques have drawn extensive interests in both academic research and industrial applications. Various cooperation schemes have been designed for enhancing the performance of wireless communication networks [85–89]. Cooperative communications have been studied in different aspects of information theory [26, 27, 91, 92, 133], the channel effect [39, 93, 110], relay selection mechanisms [75, 95–101], differential modulation in cooperative systems [102–111]. Diversity gains of cooperative transmission techniques have been studied in [24, 26, 112].

Research suggests large benefits by cooperative communication, which is required to meet the ever growing demands in cellular networks [24, 42, 113, 114]. The deployment of relay nodes, dedicated for cooperative communications, is a key challenge in next generation networks such as 3GPP's long term evolution advanced [115], [116] or IEEE 802.16j WiMAX standard [117]. For instance, in [118], the authors study the capacity gains and the resource utilization in a multi-hop LTE network in the presence of relays. Further, the performance of different relaying strategies in an LTE-Advanced network is studied in [127]. Furthermore, the authors in [119] study the possibility of coverage extension in an LTE-Advanced system, through the use of relaying. In [120], the communication possibilities between the relays and the base station are studied and a need-basis algorithm for associating the relays to their serving $BS$ is proposed for LTE-Advanced networks. The possibilities for handover in an LTE network in the presence of relays are analyzed in [121]. Other aspects of relay deployment in next generation networks are also considered in [122–126].

Most wireless systems, such as ultra mobile broadband (UMB), Long Term Evolution (LTE), and IEEE802.16e (WiMAX) promise very high data rates per user over high bandwidth channels.

In order to become more familiar with cooperative communication, different kind of cooperative relaying protocols will be studied in next subsection.

## 1.5 Cooperative Relaying Protocols

Many cooperative relaying protocols, which control the exchange of information between terminals on the network, have been proposed to establish a two-hop communication between a base station and user through a relay [73, 74, 127, 128]. Well known cooperation techniques are amplify-and-forward (AF), decode-and-forward (DF), compress-and-forward (CF) [28], and regenerate-and-forward (RF) [129].

In AF relay-assisted protocol, the relay simply amplifies the incoming signal and forwards it to the destination without doing any decoding. It is also called non-regenerative relaying. In this scheme, received signal is neither detected, decoded, nor compressed before retransmission [23]. Amplify-and-forward relays are low complexity and easy to implement. The main drawback of this strategy is that the relay terminal amplifies the received noise at the same time. Applying this strategy to cooperative communication leads to a lower bit error rate (BER) than direct transmission [130–132].

In DF relay-assisted protocol, relay decodes the incoming signal and and re-encodes it before forwarding a copy [26], [114]. Due to decoding, the noise in the received signal is cleaned out [26]. Depending on the type of symbols retransmitted, the strategy at the relay is repetition coding (RC) or unconstrained coding (UC). In RC, the relay retransmits the same symbols previously estimated, while in UC the symbols transmitted are not the same as the received ones, but are related to the same information sent by the source. Hence, this protocol is also called regenerative relaying. The DF can effectively avoid error propagation through the relay, but has the processing delay.

For the CF protocol, each relay first maps its received signal into another signal in a reduced signal space, then encodes and forwards the compressed signal as a new codeword by taking the signal received at the destination as side information. In CF scheme relay exploits the statistical dependency between the message received at the relay and destination, and compresses the received signal prior to retransmission [133]. In RF, relay retransmits a regenerated version of the detected signal to the destination [129].

Depending on the network topology and the quality of the backhaul link between the source and the relay, one protocol may outperform the other in terms of system capacity or diversity. Generally speaking, for systems with good backhaul links, DF based cooperation schemes are more favorable, while for systems with relative poor backhaul links, AF or CF based cooperation schemes are more advantageous. The AF and DF protocols are the most popular ones due to their simplicity and intuitive designs. Compared with amplify-and-forward relaying [134], DF has significant advantages on noise propagation avoidance. Since both AF and DF protocols provide various performance enhancements, they are included in the 4th generation OFDMA system implementation [73] and [136]. For application of relays to cellular systems, the IEEE 802.16m WiMAX standard [37], [38] has given preference to decode-and-forward over amplify-and-forward. DF has received more attention in the standardization community and technical reports for implementation. We concentrate on decode-and-forward relays in this thesis.

## 1.6 Power Allocation Related Works

OFDMA combined with relaying techniques offer a promising technology to provide ubiquitous high data rate coverage [137] and improve the system performance by taking advantage of both techniques. To fully exploit the benefits of relaying in an OFDMA system, advanced power allocation schemes are crucial for the future OFDMA-based relay-enhanced cellular networks and employment of conventional schemes will be highly inefficient [138].

The literature shows that power allocation algorithms have been widely studied for

OFDMA-based networks without relays, surveyed in [139]. After that, most of the research in OFDMA relaying either do not consider the power allocation problem since they mostly consider relay selection scheme, or make unrealistic assumptions to simplify the problem. As an example, the relay selection and subcarrier assignment is solved in [140] while the power allocation problem is not considered. After that, there have been many research focused on improving the system throughput by resource allocation in OFDMA-based relay networks.

Resource allocation for relay channel has been addressed in several studies [78, 141–145]. In [146], the joint subcarrier assignment and relay selection is solved with the assumption of equal power allocation, and the power allocation subproblem is solved by an iterative method. In [147], the authors use the Lagrange dual-decomposition method to propose a modified water-filling algorithm for power allocation solution. In [148], [66] and [149], the optimal relays locations are studied when the network topology, traffic distribution and transmission power are determinate. However, in OFDMA cellular networks with fixed relays, it is costly to re-install relays at different locations when the traffic distribution changes. Subcarrier assignment and power allocation are proposed in [150], where each node may act as a source/destination or relay simultaneously and the work in [151], aims at maximizing the downlink capacity with minimal rate requirements from users.

There are many problems that need to be considered in designing a relay network such as power allocation. Most of the research that consider the power allocation in OFDMA-based relay networks, such as [147, 152–154] and [155], aim to maximize the system throughput. However, fairness among multiple users mostly has not been considered. Therefore, users with bad channel conditions are starved since all resources are assigned to users with good channel conditions.

Considering fairness, authors in [156] propose a resource allocation algorithm with joint considerations on fairness among users and efficient subcarrier utilization. A sum-rate optimization problem with minimal rate requirements is solved by a subgradient method in [151]. Authors in [157] developed an optimization framework to solve the problem of joint selection and power allocation. The optimization problem uses the achievable sum rate and max-min user rate. In [158] and [159], optimal resource allocation for max-min fairness are proposed. In [160] and [161], relay power allocation for maximizing the minimum rate and the weighted sum rate was investigated for AF scheme.

Since max-min fairness is limited by rates of users in poor channel state and proportional fairness (PF) can achieve the tradeoff between system throughput and fairness [162], the PF seems more attractive in wireless networks. PF maximizes the summation of logarithmic function of users' throughput.

In conventional cellular networks PF is widely adopted [163–166]. However, in cooperative relay networks DF, PF resource allocation has not been extensively proposed due to the complexity. In [167], PF based subcarrier allocation is discussed in the network with only one relay and using equal power allocation. In [168], efficient greedy algorithms are proposed to maximize the total capacity of a single-cell OFDMA based relaying network while giving users proportional fairness. To reduce the complexity, in [151, 167–169], the authors assumed that there is no direct transmission between the $BS$ and users and the direct path is ignored.

In [77], the authors have studied throughput maximization schemes for AF, in the absence of direct path and in presence of frequency selective fading. The authors by analyzing the system under both individual and total power constraints, have shown that maximum throughput achievable is higher in case of total power constraint. A selective relaying scheme is proposed in [170], where the system chooses to relay only if an increase in throughput can be achieved by relaying. Otherwise, only the direct path is used for transmission. In [171], the authors have proposed power allocation and relay selection under the objectives of minimizing total transmit power and maximizing user rates. A multi-level relaying system with fixed subscriber stations is dealt in [172], and an algorithm is proposed to determine the minimum number of relays to achieve the minimum rate requirement [77, 170–172]. In [173], joint bandwidth and power allocation strategies for a Gaussian relay network are investigated. AF and DF schemes are analyzed for joint bandwidth and power allocation. The main objective of joint bandwidth and power allocation is to maximize the signal-to-noise ratio at the receiver using AF and DF schemes. The study in [174] proposes a centralized framework that selects multiple relays for transmission in a two-hop network. The aim of the multiple relay selection is to maximize the SNR at the destination using binary power allocation at the relays. An optimal relay assignment and power allocation in a cooperative cellular network is discussed in [175]. Using the sum-rate maximization as a design metric, the authors proposed a convex optimization problem that provides an upper bound on performance. A heuristic water-filling algorithm is also suggested to find a near-optimal relay assignment and power allocation. In [176], a linear-marking mechanism is investigated for relay assignment in a multi-hop network with multiple source destination pairs. The aim of the proposed linear-marking mechanism is to maximize the worst user capacity.

A distributed nearest neighbor relay selection protocol and its outage analysis are presented in [177]. For the relay assignment in a multiuser communication system, decentralized protocols are discussed in [178] and [179]. The decentralized framework in [178] uses DF scheme and assigns relays without considering power allocation. In [179], decentralized AF scheme is used for joint relay assignment and power allocation.

## 1.7 Game Theory

Game theory is a formal analytical framework with a set of mathematical tools to study the complex interactions among players. Throughout the past decades, game theory has made a revolutionary impact on a wide number of disciplines ranging from economics, politics, philosophy, or even psychology [180]. Games may generally be categorized as noncooperative and cooperative games. A noncooperative game is concerned with the analysis of strategic choices, and it explicitly models the decision making process of rational but selfish players to maximize their individual payoffs in a self interested manner without being concerned with the impact of their strategies on the other players. A typical solution to a noncooperative game is the Nash equilibrium (NE). NE is a status or a combination of strategies of the players where no player can increase his/her payoff by changing his/her strategy unilaterally. Unfortunately, the NE has been proven to be not always socially optimal since individually rational strategies always lead to worse results than the theoretical possibility of an enforceable agreement among rational players.

Unlike in noncooperative games, the players in a cooperative game are grouped together and establish an enforceable agreement in their group. Cooperative games emphasize collective rationality and social optimality. Cooperative game theory is dedicated to the study of cooperation among a number of players. Cooperative game theory mainly includes two branches of Nash bargaining and coalition formation game. In this thesis, we restrict our attention to the latter, although the former can also be quite useful in different scenarios. In this context, coalitional games prove to be a very powerful tool for designing fair, robust, practical, and efficient cooperation strategies in communication networks.

One of the major parts of coalition formation game involves the formation of a bunch of cooperative players, called coalitions [180]. A coalition, can be formed by players to gain a higher payoff, and the worth of this coalition is called the coalitional value. There are no general rules for coalition formation. The coalitional game can be implemented through two main rules for forming or breaking coalitions referred to as merge and split relying on the interaction among players [181]. The basic idea behind merge-and-split rule is that, given a set of players, any collection of disjoint coalitions can merge into a single coalition, if this new coalition is preferred over the previous state. Similarly, a coalition splits into smaller coalitions if the resulting collection is preferred. The coalition function is attained by determining the payoff and cost function. Coalitional game is formed by applying the pareto order as the comparison relation, and merge-and-split rule. A stable solution for a coalition formation game ensures that the outcome is immune to deviations by groups of players, meaning that no player has an incentive to move from its current coalition to another coalition.

## 1.8 Data Link Layer

The high requirements of future wireless communication systems motivate researchers to enhance existing technologies. It should be noted that, many works on cooperative diversity scheme mainly focused on the physical layer aspects of the network [24]. However, it is essential to study the performance of the data link layer as well.

The continually increasing number of users and the demand for multimedia services in cellular wireless networks require a lower end-to-end delay. Authors in [182] obtained the packet delay in a multiple-source single-destination network with the help of a relay selection algorithm. The analysis for wireless systems with adaptive modulation and coding (AMC) at the physical layer and finite buffer queuing at the link layer was presented in [183]. The radio link-level delay statistics in a wireless network using AMC, weighted round robin (WRR) scheduling, and automatic repeat request (ARQ)-based error control is analyzed in [184]. In [185], the operation of a node relaying packets from multi users to a destination node is examined. Analytical expressions for the average length of the queue as functions of the probabilities of transmissions and the outage probabilities of the links are obtained. The analysis in [186] is developed based on a vacation queueing model in a multi-rate wireless network where the exact statistics of queue length and delay are obtained under both saturated and non-saturated buffer scenarios. The unified tandem queue framework in [187] considers the multi-rate transmission in the physical layer and ARQ in the link layer for a multi-hop wireless network. The performance measures, derived through the queueing model, are then used to address the problem of quality of service routing.

The e2e performance evaluation of a system consists of a $BS$, one typical relay and multiple users leads to a tandem queueing problem. In each time-slot, only one user can be in tandem with relays buffer. Hence, depending on the probability of selection, each user can either be in connection or no-connection state. The probability of connection is not necessarily the same for all users. Evaluation of the radio link-level performance measures such as e2e packet loss rate, e2e delay and throughput can be carried out to develop a queueing analytical framework.

## 1.9 Thesis Contributions

This thesis consists of two main parts. In the first part, an analytical framework for the link-level end-to-end queueing performance evaluation in a multiuser wireless relay network with ARQ-based error control is studied. In the second part, the physical layer issues, mainly the power allocation, relay load balancing and relay assignment are considered to gain the potential capacity and improvements of multihop relaying.

In Chapter 2, the e2e performance evaluation of a two-hop relay assisted communication system in a relay-enhanced cellular network is considered. We assume a system consisting of the $BS$, one typical relay and multiple users. A cell is divided into multiple sectors, each serviced by a fixed relay, and the base station is located at the cell centre. The e2e network of $BS$, relay and users is modeled as a tandem of two finite buffer queues for relay and each user. The wireless scheduler deployed at the base station schedules the transmissions corresponding to the different users in a time-division multiplexing fashion, such that, in each time-slot only one user is in tandem with relay's buffer.

The relay only stores packets, which experience failure in the transmission to the $BS$. The e2e evaluation of this system leads to a probabilistic tandem queueing problem. To make the analysis tractable, the finite buffer of the relay is decomposed into smaller non-overlapping portions, each corresponding to an individual user's packets (per-user queueing). Using the decomposed model, radio link-level performance measures such as buffer overflow probability, e2e packet loss rate, e2e delay and throughput are obtained analytically and compared with simulation results.

To the best of our knowledge, none of the previous work addressed the e2e performance evaluation of a multi-user relay network with probabilistic tandem queues. The probability of connection is not necessarily the same for all users in our model.

As an application of this model, a method of obtaining optimum values for selection probabilities, based on exhaustive search, is presented such that the e2e aggregate throughput is maximized subject to the users individual delay constraints.

After that, since advanced radio resource management schemes are crucial for the future OFDMA-based decode-and-forward cellular relay networks, the physical layer issues, mainly the power allocation, relay load balancing and relay assignment are considered in the second part of the thesis.

In Chapter 3, we consider a system with a single base station communicating to multiple users being assisted by multiple relays. Each relay forms a coalition with the users who use this relay. The relays have limited power which must be divided among the users they support in order to maximize the data rate of the whole network.

Our work is different from all other works, since we first obtain the optimal power requirement of each user and based on that we propose the optimal power allocation which maximizes the data rate as an *upper bound*. Moreover, other power allocation scheme is proposed which consider power allocation and fairness jointly. We also propose an efficient power reallocation scheme to utilize the power wisely and improve the data rate of the network by efficiently reallocating the power of relays. In the proposed schemes, the direct path is not ignored and both direct path and relay path are considered, DF is applied along with the maximum ratio combining (MRC). The simulation results of our proposed scheme show the average amount of improvement of our proposed scheme compared to traditional scheme.

Besides power allocation, relaying brings in issues such as efficient relay selection and relay load balancing which should be considered as well. Most of the studies focused on maximizing the system throughput without considering load balancing among the relays. However, in this thesis, we consider the joint power allocation and load balancing to avoid relay overloading and efficiently improve the performance.

In Chapter 3, we assumed that the network is formed by applying the traditional relay selection scheme based on channel gains. Then, a power allocation scheme was proposed based on the knapsack problem to maximize the total data rate of the network. However, in Chapter 4, we do not apply the traditional relay selection scheme. It is because the users are distributed randomly and by applying the traditional relay selection scheme, it may happen that some relays have more users connected to them than other relays, which results in having unbalanced load among the relays. By applying relay load balancing, users who can connect to uncongested relays join them as opposed to connecting to congested relays, so users are evenly distributed among the relays. In Chapter 4, by considering load balancing among relays, an optimization problem is formalized in order to maximize the total throughput of the network.

For solving the problem, with the need for tools that allow to study the behavior and interactions of the nodes, cooperative game theory scheme as a suitable analytical method is proposed to achieve load balancing among relays while considering the total data rate of the network and fairness issue among users as well.

To distribute the power of relays among the users wisely, the efficient power allocation scheme obtained in Chapter 3, is used. By defining the coalition value, cost function and using merge-and-split rule, the coalitional formation game is run. After the termination of the merge-and-split rule, the coalition with the highest coalition value among the formed coalitions is selected. The same procedure continues for the all relays as well, until coalitions are formed around all relays of the network. Simulation results show that, the proposed game-based power allocation scheme can improve the average sum-spectral efficiency approximately 20% compared to the traditional scheme. Moreover, by applying the game, users are evenly distributed among the relays and load balancing is obtained.

In Chapter 5, the relay assignment procedure is studied to improve the aforementioned common relay selection method and maximize the data rate of the network. It is taken into account that each relay has a limited power, which should be distributed among the users they support, and each user has to be assigned to a single relay. First, the optimization problem is formulated and the optimal solution is found using Lagrangian Relaxation. Then, a lighter algorithm is proposed to efficiently and quickly carry out the relay assignment with a close-to-optimal performance. Simulation results show that the proposed algorithm can achieve near optimal data rate in the network, while it decreases the processing time significantly. All simulation results are obtained by using Matlab.

The rest of this research is organized as follows. End-to-end queueing performance evaluation is considered in Chapter 2. Section 2.1 describes the system model and assumptions, followed by a discussion on problem formulation in Section 2.2. End-to-end QoS measures are derived in Section 2.3 and validated by numerical results and simulations in Section 2.4. Chapter 3 discusses power allocation in OFDMA-based decode-and-forward cellular relay networks. The system model and the exact problem definition are described in Section 3.1. In Section 3.2, first we find the optimal power requirement of each user and based on that the *optimal power allocation* which maximizes the data rate is achieved by following the definition of *knapsack problem*. However, this solution does not take into account any fairness issue. Considering the fairness issue, other *power allocation scheme* is proposed which is based on weights. In order to wisely utilize the power which is scarce and expensive and improve the data rate of the network, an efficient *power reallocation scheme* is proposed in Section 3.3. In Section 3.4, simulation results are shown. Chapter 4 presents game theoretic relay load balancing approach in decode-and-forward cellular relay networks. The system model and the exact problem definition are described in Section 4.1. Coalitional formation scheme is discussed in Section 4.2. In Section 4.3, relay power allocation scheme and coalitional game procedure are presented followed by simulation results in Section 4.4. Relay assignment scheme is considered in Chapter 5. Section 5.1 describes the system model and problem formulations. Lagrangian relaxation problem is investigated in Section 5.2. The optimal relay assignment scheme is considered in Section 5.3 followed by lagrangian bounds in Section 5.4. In Section 5.5, relay assignment algorithm is discussed. Simulation results are shown in Section 5.6. Finally, conclusions and future work are discussed in Chapter 6.

# Chapter 2

# End-to-End Queueing Performance Evaluation in Relay-Enhanced Cellular Networks

The analytical framework for radio link level end-to-end queueing performance evaluation in a multiuser wireless relay network under scheduling and ARQ-based error control is presented in this chapter. First, system model and assumptions are described. Then, problem formulation and analysis are presented, followed by end-to-end QoS measures. After that, numerical and simulation results are illustrated. An application of our queueing analysis conclude this chapter.

## 2.1 System Model and Assumptions

Most of the works on the cooperative diversity schemes mainly focus on the physical layer of the network [39], [24]. However, it is helpful to study the performance of the data link layer as well as the physical layer as pointed out in [188]. End-to-end performance evaluation of a two-hop relay assisted communication system in a multi-user multi-relay cellular network is considered in this chapter.

In this work, we consider the uplink transmission in a single cell network. However, using per-user queueing, similar system model can be applied to the downlink scenario as well. Each cell is divided into several regions, each serviced by a fixed relay. The $BS$ assigns each user to a specific relay depending on location, network traffic, link gain, etc. For simplicity and without loss of generality, we only consider the network consisting of $BS$, one typical relay and the users assigned to it in the relay selection phase. The relay only stores the packets in its buffer that their transmission to the destination experience failure. Therefore, the packets that are successfully transmitted to $BS$ by the user are not stored in the relay.

The e2e network of $BS$, relay and users is modeled as a tandem of two finite buffer queues for relay and each user. The wireless scheduler deployed at the $BS$ schedules the transmissions corresponding to the different users in a TDM fashion, such that, in each time-slot only one user is in tandem with relay's buffer. This means that in each time-slot only one user is sending its information to the relay in uplink scenario. In packet level view, only one user's buffer is in tandem with relay's buffer in each time-slot.

Selective repeating automatic repeat request at the link layer is considered as a very efficient technique to eliminate the residual error and to avoid the costly use of a strong error correction code at the physical layer [189–191].

The e2e evaluation of this system leads to a tandem queuing problem, which is the focus of this chapter. The tandem of buffers corresponding to each user and relay is approximated by decomposed tandems to make the problem tractable. In this case, a specific and separate part of relay's finite buffer is allocated to each user depending on the probability of transmission from that user, user's queue length and different priorities or traffic types of users. The impact of channel condition is modeled as a maximum number of packets that can be transmitted depending on the signal to interference plus noise ratio (SINR) on each link. We assume that traffic arrives at the relay node buffer according to a batch Bernoulli arrival process. The receiver decodes the received packets and sends negative acknowledgments (NACKs) to the transmitter asking for retransmission of the erroneous packets

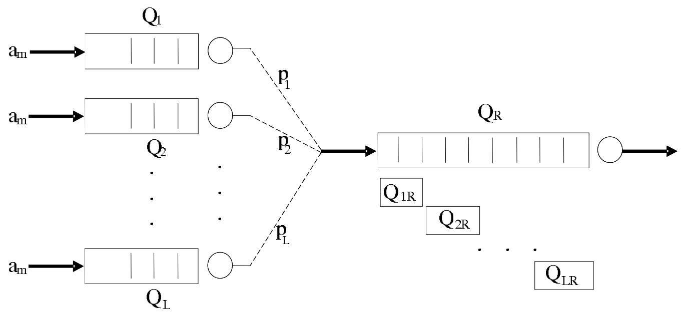

**Figure 2.1:** The e2e tandem queue modeling of an $L$-user system.

(if any). In this work, an error-free and instantaneous feedback channel is assumed; therefore, the transmitter knows exactly if there is any transmission error at the end of each time frame. Using this model, radio link level performance measures such as buffer overflow probability, e2e packet loss rate and e2e delay can be obtained.

In our model, each allocated time-slot of transmission has two phases and the main policy of the users is as follows. The transmission from the user to $BS$ and relay occurs in the first phase and the transmission from relay to $BS$ happens in the second phase (half-duplex transmission). The packets that the users successfully transmit to $BS$ are not stored in relay. The relay does not have packets of its own and only forwards the packets that have been received from the users. We assume $L$ users in the sector, to each one a separate portion of the relays common buffer, $\mathrm{Q}_{1R}, \mathrm{Q}_{2R}, ..., \mathrm{Q}_{LR}$ is allocated in our decomposed model.

It should be noted that in each time-slot we have a tandem of two finite queues, either $(\mathrm{Q}_1, \mathrm{Q}_{1R})$, or $(\mathrm{Q}_2, \mathrm{Q}_{2R})$,..., or $(\mathrm{Q}_L, \mathrm{Q}_{LR})$, as shown in Fig. 2.1. In the long run, it is assumed that user $v$ is granted a time-slot for transmission with the probability $p_v$, i.e., with probability $p_v$, $\mathrm{Q}_v$ and $\mathrm{Q}_{vR}$ are connected. For all $L$ users we have

$$\sum_{v=1}^{L} p_v = 1. \tag{2.1}$$

The queue size for the $v$th user, the $v$th user's allocated portion in the relay's buffer, and the relay is $Q_v$, $Q_{vR}$ and $Q_R$ respectively. The impact of channel condition is modeled as

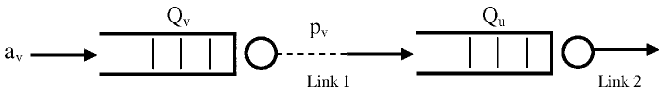

**Figure 2.2:** The decomposed tandem queue modeling of an arbitrary user.

the maximum number of packets that can be transmitted on each link, denoted by $N$.

## 2.2 Problem Formulation and Analysis

In this section, due to similar behaviour of all users in the decomposed model, we do the e2e analysis of an arbitrary user and its counterpart in the relay. Let us consider the $v$th user ($\mathrm{Q}_v$) which is in tandem (connected) with the $v$th portion in the relay ($\mathrm{Q}_{vR}$) with probability $p_v$. For simplicity, we call this $v$th portion $\mathrm{Q}_u$ henceforth, where $u = 1, 2, ..., L$.

We assume that traffic arrives at $\mathrm{Q}_v$ according to a batch Bernoulli process, where $b$ packets arrive in one time-slot with probability $a_b$ $(b = 0, 1, 2, ..., M)$. The arrival process is assumed similar for all users without loss of generality. The decomposed tandem model of $v$th user is shown in Fig. 2.2.

Let $q_v^n$ be the number of packets in $\mathrm{Q}_v$ at time-slot $n$. Note that $\mathrm{R}_n = \{q_v^n, q_u^n\}$ forms a discrete time Markov chain (DTMC). When these two buffers are connected, the number of packets in $\mathrm{Q}_v$ and $\mathrm{Q}_u$ can decrease or increase as they serve or receive each packet. However, the number of packets in $\mathrm{Q}_u$ cannot change when it is not connected to $\mathrm{Q}_v$ (no arrival to $\mathrm{Q}_u$, no service from $\mathrm{Q}_u$). On the other hand, the number of packets in $\mathrm{Q}_v$ can increase even if it is not in tandem, since it can receive new arrivals with the rate $a_v$. Therefore, in each time-slot each user can either be in *connection* state or in *no-connection* state.

Let $(x_n, y_n)$ be the generic system state, $q_v^n = x_n$, $q_u^n = y_n$ and $(x_n, y_n) \rightarrow (x_{n+1}, y_{n+1})$ be the system transition from state $(x_n, y_n)$ to $(x_{n+1}, y_{n+1})$. Let $b$ be the number of packets in a particular time-slot that arrive to $\mathrm{Q}_v$, and the maximum transmission capability on link 1 (from user to relay) and link 2 (from relay to $BS$) be $N$ packets. We assume a packet error rate (PER) on each link and employ ARQ to compensate for it. The whole transition probability matrix can be written as

$$\mathbf{A}_{x_n,x_{n+1}}(y_n, y_{n+1}) = \text{Pr}\{(x_n, y_n) \to (x_{n+1}, y_{n+1})\}. \tag{2.2}$$

The order of block matrix $\mathbf{A}_{x_n,x_{n+1}}$ is $2(Q_u+1) \times 2(Q_u+1)$, since for certain values of $x_n$ and $x_{n+1}$, there exists $(Q_u + 1)$ and $(Q_u + 1)$ possible states for $y_n$ and $y_{n+1}$ respectively, each of them can be in the two possible inner states of $(c)$ or $(nc)$. The number of packets in Q$_v$ after accepting new arrivals at time $n + 1$ is min$\{x_n + b, Q_v\}$. When Q$_v$ is in tandem with Q$_u$, the number of packets transmitted on link 1 is min$\{x_n, N\}$. Among these transmitted packets, $i$ packets are correctly received at the receiving end and these $i$ successfully transmitted packets will enter Q$_u$, that is $x_{n+1} = \text{min}\{x_n + b, Q_v\} - i$. Similarly for Q$_u$, we have $y_{n+1} = \text{min}\{y_n + i, Q_u\} - j$, where $j$ packets are correctly received at $BS$. If the average PER on link 1 is equal to $\alpha$, when $m$ packets are transmitted from the user, the probability that only $k$ of them will be successfully received at the relay, can be calculated as

$$\omega_\alpha(m, k) = \binom{m}{k} \alpha^{m-k}(1 - \alpha)^k. \tag{2.3}$$

We assume that $j$ packets among min$\{y_n, N\}$ transmitted packets are correctly received at the receiving end of link 2, i.e., $BS$, and average PER of link 2 is $\beta$. Now, the total state transition probability can be written as

$$\begin{aligned}&\text{Pr}\{(x_n, y_n) \to (x_{n+1}, y_{n+1})\} = \\ &\sum_{b,i,j} p_v a_b \omega_\alpha(\text{min}\{x_n, N\}, i)\omega_\beta(\text{min}\{y_n, N\}, j),\end{aligned} \tag{2.4}$$

where all possible cases such that $x_{n+1} = \text{min}\{x_n + b, Q_v\} - i$ and $y_{n+1} = \text{min}\{y_n + i, Q_u\} - j$ are included in the sum. Using $\mathbf{A}_{x_n,x_{n+1}}(y_n, y_{n+1})$ as mentioned in (2.4), the total transition matrix can be formulated as matrix $\mathbf{P}$ given in (2.5). It is enough to construct an $2(Q_u + 1) \times 2(Q_u + 1)$ block matrix and two of its circularly-shifted versions to obtain all sub-matrices in (2.5) by simple scalar to matrix multiplications.

To become more familiar with the structure of the transition matrix $\mathbf{P}$ and the corresponding transition probability block matrices, an example is provided in *Appendix A*. In the example, the user and the relay have equal buffer size of 5 and the packet arrival rate at the users' buffer is according to a batch Bernoulli process $a_b$ $(b = 0, 1, 2)$ with $N = 2$.

$$\mathbf{P} = \begin{bmatrix} \mathbf{A}_{0,0} & \mathbf{A}_{0,1} & \cdots & \mathbf{A}_{0,M} & & & & & \\ \mathbf{A}_{1,0} & \mathbf{A}_{1,1} & \mathbf{A}_{1,2} & \cdots & \mathbf{A}_{1,M+1} & & & & \\ \vdots & & & & & & & & \\ \mathbf{A}_{N,0} & \mathbf{A}_{N,1} & \mathbf{A}_{N,2} & \mathbf{A}_{N,3} & \cdots & \mathbf{A}_{N,N+M} & & & \\ & & & \ddots & \ddots & \ddots & & & \\ & & & \mathbf{A}_{Q_v-1,Q_v-N-1} & \mathbf{A}_{Q_v-1,Q_v-N} & \mathbf{A}_{Q_v-1,Q_v-N+1} & \mathbf{A}_{Q_v-1,Q_v-N+2} & \cdots & \mathbf{A}_{Q_v-1,Q_v} \\ & & & & \mathbf{A}_{Q_v,Q_v-N} & \mathbf{A}_{Q_v,Q_v-N+1} & \mathbf{A}_{Q_v,Q_v-N+2} & \cdots & \mathbf{A}_{Q_v,Q_v} \end{bmatrix} \tag{2.5}$$

Now that we derived the total transition matrix in (2.5), the steady state probability vector $\boldsymbol{\pi}$ can be obtained from

$$\boldsymbol{\pi}\mathbf{P} = \boldsymbol{\pi}\ , \ \boldsymbol{\pi}\mathbf{1} = \mathbf{1}, \tag{2.6}$$

where $\mathbf{1}$ is a column vector of all ones with the same dimension as $\boldsymbol{\pi}$. We can expand $\boldsymbol{\pi}$ as $\boldsymbol{\pi} = [\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \cdots, \boldsymbol{\pi}_{Q_v}]$, where $\boldsymbol{\pi}_i$ is a row vector of dimension $2(Q_u + 1)$ corresponding to the probability of having $i$ packets in Q$_v$, and it can be further expanded as $\boldsymbol{\pi}_i = [\pi_{i,0}^{(c)}, \pi_{i,0}^{(nc)}, \pi_{i,1}^{(c)}, \pi_{i,1}^{(nc)}, \cdots, \pi_{i,Q_u}^{(c)}, \pi_{i,Q_u}^{(nc)}]$, where $\pi_{i,j}^{(c)}$ is the probability of having $i$ and $j$ packets in Q$_v$ and Q$_u$ respectively, while being in $(c)$ state. Given the steady state probability vector $\boldsymbol{\pi}$ from (2.6), we can now derive the e2e queueing performance measures.

## 2.3 End-to-End QoS Measures

The e2e performance measures such as the buffer overflow probability, e2e loss rate, throughput, and delay are studied in this section. In tandem queues, packets can be lost due to the overflow of the buffers. The buffer overflow probability for Q$_v$ can be calculated as a ratio between the average number of dropped packets due to overflow at Q$_v$, denoted as $\bar{O}_v$, and the average number of packets arriving at Q$_v$ in one time-slot, which is $\bar{A}_v = \sum_{b=1}^{M} b a_b$.

Therefore, the buffer overflow probability for Q$_v$ can be developed as

$$P_{bo}^{(v)} = \frac{\bar{O}_v}{\bar{A}_v}. \tag{2.7}$$

We define $y_k = \boldsymbol{\pi}_k \mathbf{1}_{2(Q_u+1)}$ as the marginal probability that there are $k$ packets in Q$_v$. In order to calculate the average number of dropped packets due to the overflow at Q$_v$ we have

$$\bar{O}_v = \sum_{b=1}^{M} \sum_{k=Q_v-M}^{Q_v} a_b y_k \times \max\{0, b+k-Q_v\}. \tag{2.8}$$

Note that $\max\{0, b+k-Q_v\}$ is the number of dropped packets in Q$_v$, given that there are $k$ packets in Q$_v$ and $b$ arriving packets. For Q$_u$, the probability of arriving $i$ packets due to successful transmissions from Q$_v$ can be formulated as $f_i = \sum_{k=0}^{Q_v} y_k \omega_\alpha(\min\{k, N\}, i)$. Note that we defined $\omega_\alpha(m, k)$ before, in (2.3). Moreover, the average arrival rate to Q$_u$ can be calculated as $\bar{A}_u = \sum_{i=1}^{N} i f_i$. We then define $z_l$ as $z_l = \sum_{k=0}^{Q_v} \pi_{k,l}$ which is the marginal probability that there are $l$ packets in Q$_u$ regardless of the tandem being in ($c$) or ($nc$) state. In other words, $\pi_{k,l} = \pi_{k,l}^{(c)} + \pi_{k,l}^{(nc)}$. The average number of dropped packets due to overflow at Q$_u$ can be calculated as

$$\bar{O}_u = \sum_{i=1}^{N} \sum_{k=Q_u-N}^{Q_u} f_i z_k \times \max\{0, i+k-Q_u\}. \tag{2.9}$$

Assuming that the propagation delay over the wireless channel is negligible, the e2e delay is caused by waiting time in the buffers along with the delay due to retransmissions of the ARQ protocol. With the use of Little's law, the e2e average delay for the tandem of $v$th user (Q$_v$ and Q$_u$) can be written as

$$D_{e2e}^{(v)} = \frac{\sum_{k=1}^{Q_v} k y_k}{\bar{A}_v(1 - P_{bo}^{(v)})} + \frac{\sum_{k=1}^{Q_u} k z_k}{\bar{A}_u(1 - P_{bo}^{(u)})}, \tag{2.10}$$

where the denominator is the average arrival rate considering packet loss due to overflow and the numerator of each term is the average length of each queue. The total e2e delay of

$L$ users can be obtained as $D_{e2e} = \sum_{v=1}^{L} D_{e2e}^{(v)}$.

The e2e loss rate can be obtained analytically with the help of transition matrix $\mathbf{P}$. A packet drop occurs in the relay's buffer in *connected* states corresponding to situations where there is no or not enough buffer space for the arriving packets. To obtain e2e loss rate, the probability of being in these states ($q_u^n = Q_u$ or $Q_u - 1$), obtained from (2.6), are multiplied by the number of dropped packets in each state and finally summed together. As there is no closed-form expression for this calculation, we give an example to clarify the process. Assuming $M = 2$ and $Q_u = 5$ in (2.5), $A_{0b}, b = 0, 1, 2$ do not introduce any packet losses as the $Q_v$ is empty and there is no packet transmission to $Q_u$. However, for $A_{1b}, b = 0, 1, 2$ we can have a maximum of one packet loss when $Q_u = 5$ and the tandem is *connected*. All other states producing packet loss can be obtained using the same approach. The total e2e loss rate of $L$ users can be obtained by summation over all users.

All the *connected* states in which relay serves a packet contribute to the e2e throughput. Therefore, to obtain throughput, the probability of being in these states are multiplied by the number of served packets in each state and finally summed together. As there is no closed-form expression for this calculation, we give an example to clarify the process. Assuming $M = 2$ and $Q_v = Q_u = 3$, the matrix elements contributing to throughput are $A_{00}(3, 1), A_{00}(5, 1)$, and the portion in $A_{00}(5, 3)$ which are contributing with one, two and one packets respectively. These probabilities are multiplied by $\pi_{0,1}^{(c)}$, $\pi_{0,2}^{(c)}$ and $\pi_{0,2}^{(c)}$ respectively and summed together. All other states producing throughput can be obtained using the same approach. The total e2e throughput of $L$ users can be obtained by summation over all users. These analytical results are shown and validated by simulation in the next section.

## 2.4 Numerical and Simulation Results

In order to validate our queueing model, some numerical and simulation results of the performance measures for typical values of $a_b, \alpha, \beta, Q, N$, and $M$ are presented in order to validate our queueing model. The numerical results are based on our analysis of decomposed modeling queues in the previous section, and simulations are based on the exact model. A case of two users is considered first but the results can be extended to the $L$-user case. Each user and the relay have queue length of $Q_1, Q_2$ and $Q_R$ respectively. The maximum achievable capacity for each link is $N = 2$ and for the batch Bernoulli arrival process we have $M = 2$. For each realization, a total of 100 warm-up packets followed by the total of 1000 packets are transmitted.

Figs. 2.3 and 2.4 show the average e2e delay and loss rate for different PER parameters $(\alpha, \beta)$ on each link. It can be inferred from the figures that, with increasing the average arrival rate, the e2e delay increases since a newly arrived packet sees more head of line (HOL) packets each time. Since all buffers are finite, more HOL packets leads to more losses. As for the loss rate, since the decomposed queues are shorter in length, they become full more quickly which results in loss rate degradation. Having decomposed queues, however, causes less HOL packets and each user only see its own HOLs in the relay's buffer. Therefore, the results for delay obtained from analysis are less than those obtained from simulations. The numerical results and simulations are carried out for three different probabilities of error in each link and as expected, higher probabilities of error causes more e2e delay and loss rate.

In Fig. 2.5, the e2e throughput of a 3-user system with different queue sizes is depicted in which with increasing average arrival rate, the e2e throughput increases up to a specific point which gives the maximum achievable throughput. It should be noted that as long as we have more packets than $N$ in the user's and relay's buffer, the e2e throughput can not increase further and the links become saturated. Consequently, the throughput results obtained from the analysis and simulation are quite the same. Moreover, with the increase in number of users, the analytical results do not deviate from simulation.

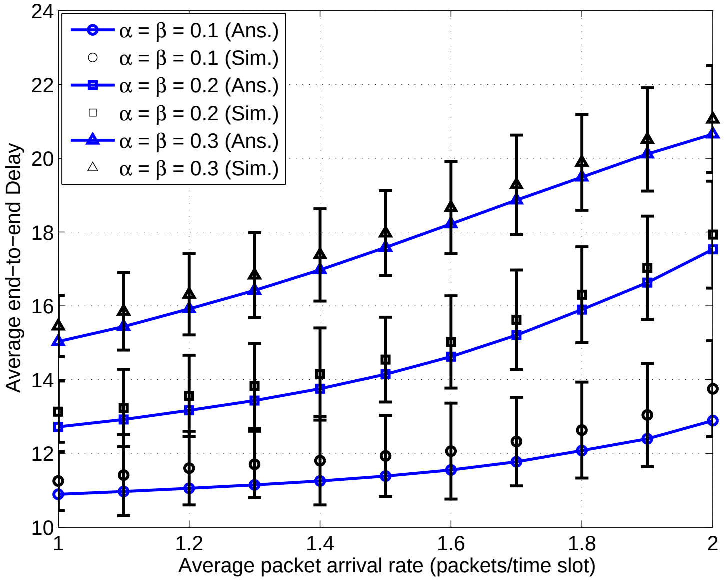

**Figure 2.3:** The average e2e delay versus the average packet arrival rate ($r$) for $Q_1 = 5$, $Q_2 = 10$, $Q_R = 15$, $p_1 = p_2 = 0.5$, and $\alpha = \beta = 0.1, 0.2, 0.3$.

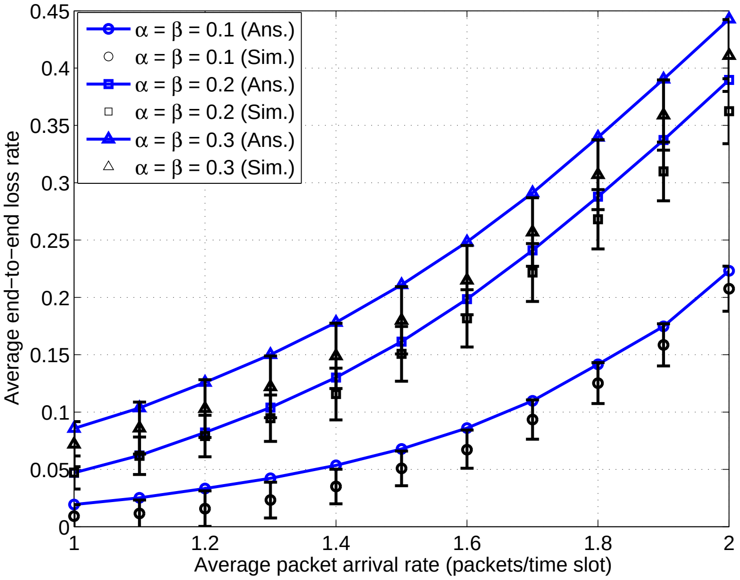

**Figure 2.4:** The average e2e loss rate versus the average packet arrival rate ($r$) for $Q_1 = 5$, $Q_2 = 10$, $Q_R = 15$, $p_1 = p_2 = 0.5$, and $\alpha = \beta = 0.1, 0.2, 0.3$.

### 2.4.1 Application of the Queueing Analysis

As an application to our queueing analysis, we consider the problem of allocating the user selection probabilities (i.e., scheduling probabilities) to maximize the aggregate throughput

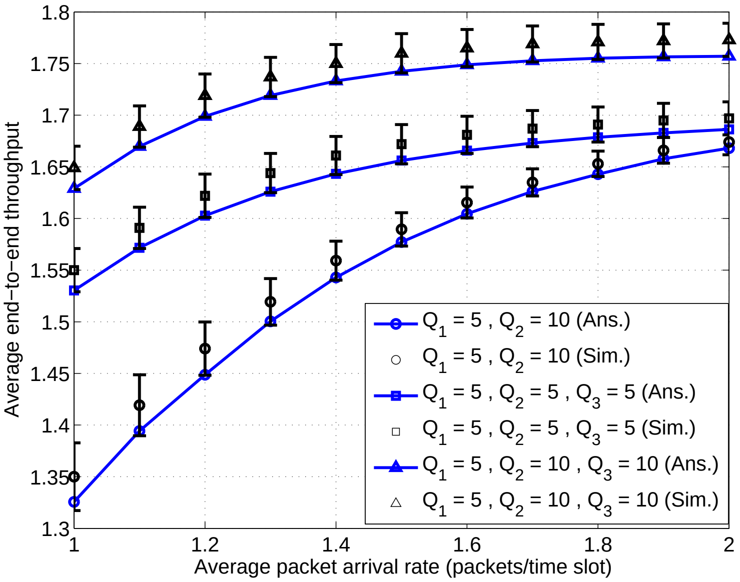

**Figure 2.5:** The average e2e throughput versus the average packet arrival rate ($r$) for a 3-user system, $p_1 = p_2 = p_3 = 1/3$, and $\alpha = \beta = 0.1$.

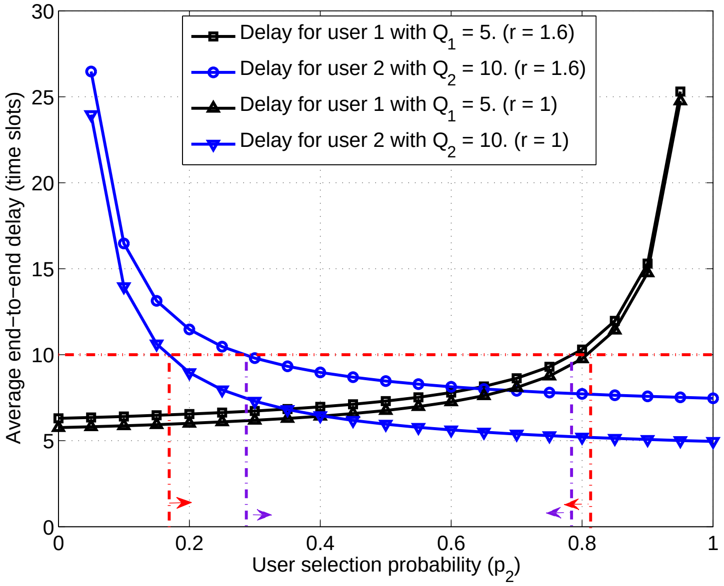

**Figure 2.6:** The average e2e delay versus the user selection probability ($p_2$) for $\alpha = \beta = 0.1$, different average packet arrival rates.

of the system subject to users individual delay constraints. The procedure is described for a scenario with two users, but can be generalized to multiple users as well. This access probability allocation can be performed at the relay and it is considered as a long-run opti-

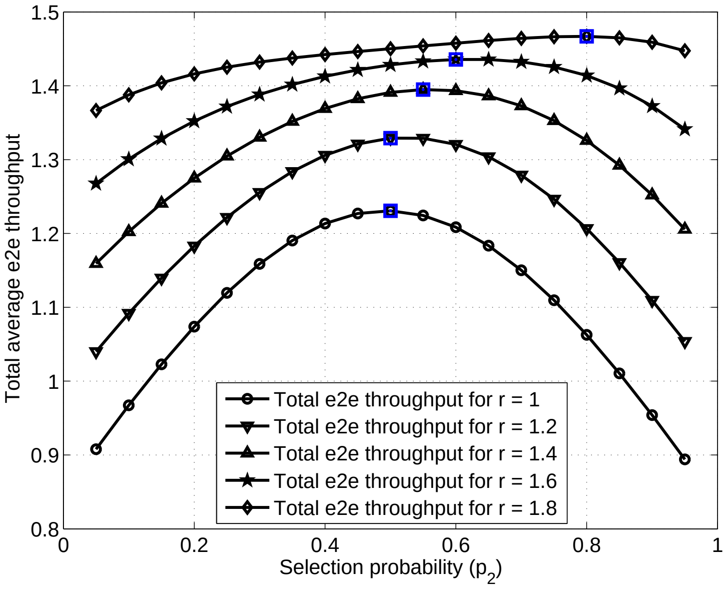

**Figure 2.7:** The total average e2e throughput versus the user selection probability ($p_2$) for $\alpha = \beta = 0.1$, different average packet arrival rates.

mization.

To describe the procedure we consider the case of a two-user system with buffer sizes $Q_1 = 5$ which is selected with probability $p_1$ and $Q_2 = 10$ with probability $p_2$. Individual delays of each user and aggregate throughput of the system versus $p_2$ are depicted in Fig. 2.6 and 2.7 respectively for different average arrival rates. Using (2.10), the average e2e delay for each user versus its corresponding selection probability is obtained as in Fig. 2.6. Since, $p_1 = 1 - p_2$ in the considered example, both curves are shown on a similar figure. It is obvious that depending on the average arrival rate ($r$), different graphs are obtained. For all feasible values of $p_1$ and $p_2$ from 0 to 1, the range of possible values for these probabilities to satisfy the delay constraint is obtained. The delay threshold of both users is assumed to be $\gamma = 10$. The feasible range of $p_2$ is between 0.17 and 0.81 for average arrival rate $r = 1$ (i.e., $p_1 = 1 - p_2$ is between 0.19 and 0.83), and for average arrival rate $r = 1.6$ is between 0.29 and 0.78 (i.e., $p_1 = 1 - p_2$ is between 0.22 and 0.71), each marked by two arrows in Fig. 2.6. Once the possible ranges are obtained for each user, relay goes to the aggregate throughput graph and chooses the combination of $(p_1, p_2)$ that maximizes the total throughput. In other words, for the limited number of points in the interval of $p_2$, i.e., $p_2 = 0.3, 0.35, 0.4, \cdots, 0.75$ for $r = 1$, relay should undertake an exhaustive search for all combinations of these points with the point in the selected interval of $p_1$ to find the pair which leads to the maximum throughput in Fig. 2.7. For our understudy example, for the case of $r = 1$, $p_1 = p_2 = 0.5$, and for the case of $r = 1.6$, $p_1 = 0.4$ and $p_2 = 0.6$ give the maximum throughput. The optimum points corresponding to $p_2$ are marked with square markers in Fig. 2.7 for other average arrival rates, but only two possible arrival rates are considered in Fig. 2.6 to make it more readable.

Similar to the case of two users, for the case of multiple users, after the initial phase of obtaining the possible selection probability ranges which satisfy the delay constraints, relay has to examine all possible combinations of the discrete points in these ranges of probabilities through an exhaustive search to find the optimum combination that maximizes the throughput, as in [192]. It should be noted that the value of each users threshold depends on the application, however, it is assumed that the thresholds are selected such that there exists a possible range of probabilities to maximize the total throughput; otherwise, the aggregate throughput cannot be optimized.

# Chapter 3

# Power Allocation Framework in OFDMA-Based Decode-and-Forward Relay Networks

A framework for power allocation of downlink transmissions in OFDMA-based decode-and-forward cellular relay networks is investigated in this chapter. First, system model and problem definition are described. Then, power allocation scheme is proposed followed by optimal power allocation and fairness issues. After that, power reallocation scheme is presented. Finally, simulation results conclude this chapter.

## 3.1 System Model and Problem Definition

In this chapter, we consider the downlink scenario of a OFDMA-based relay-enhanced cellular network. In the cell, a $BS$ is located at cell center and $K$ fixed relays are located uniformly. There are $N$ users distributed randomly over the cell. Each user is equipped with a single antenna. Relays are assumed to be fixed, the cell radius is $R$, and the distance from $BS$ to each relay is $\tau R$, where $0 < \tau < 1$. The channels between stations are frequency selective and OFDMA is employed to convert the channel into orthogonal subcarriers with flat fading. We assume the half-duplex operation of relays. By pre-subcarrier allocation, the best unallocated subcarrier is assigned to each user and the power of $BS$ is equally allocated among all users. For simplicity, we assume that each relay uses the same subcarrier to relay information it received. There is no more than one relay assisting each user. The overall bandwidth $B$ is divided equally among the $N$ users.

In our model, DF cooperative scheme discussed in [26] is used, each time slot has two phases and the policy is as follow. In the first phase, the $BS$ transmits and the rest of the nodes receive. In the second phase, the relays transmit what they have received in the first phase to users. The user combines the directly received signal from $BS$ and the relayed signal from relay on subcarrier together using the maximal ratio combining (MRC). The achievable data rate between $BS$ and user $i$ by help of relay $j$ is as follow [26],

$$DR^{j}_{(BS,i)} = \frac{1}{2}min\{\frac{B}{N}log_2(1+\frac{P_{BSj}|h_{BS,j}|^2}{BN_0/N}), \frac{B}{N}log_2(1+\frac{P_{BSi}|h_{BS,i}|^2}{BN_0/N}+\frac{P_{ji}|h_{j,i}|^2}{BN_0/N})\}, \tag{3.1}$$

where $h_{BS,j}$ ($h_{BS,i}$) denotes the channel gain between $BS$ and relay $j$ (user $i$), $P_{BSj}$ ($P_{BSi}$) is the transmit power from $BS$ to relay $j$ (user $i$) and $P_{ji}$ is the transmit power from relay $j$ to user $i$. $N_0$ is the power spectral density of additive white Gaussian noise. The first term in Equation (3.1) represents the maximum rate at which the relay $j$ can reliably decode the $BS$ message, while the second term in Equation (3.1) represents the maximum rate at which the user $i$ can reliably decode the $BS$ message given repeated transmissions from the $BS$ and relay $j$. In the first phase, the $BS$ transmits and the rest of the nodes receive, so in Equation (3.1), $P_{BSj}$ is equal to $P_{BSi}$. As a result, the Equation (3.1) can be written as

$$DR^{j}_{(BS,i)} = \frac{1}{2}min\{\frac{B}{N}log_2(1+\frac{P_{BSi}|h_{BS,j}|^2}{BN_0/N}), \frac{B}{N}log_2(1+\frac{P_{BSi}|h_{BS,i}|^2}{BN_0/N}+\frac{P_{ji}|h_{j,i}|^2}{BN_0/N})\}. \tag{3.2}$$

The total transmission power of each relay which must be divided among users who use this relay is limited. In our model, for users who are strong enough (with good channel gain) a direct link is used and for other users a relay should be selected. Let us define the direct achievable rate between user $i$ and $BS$, $I_{BSi}$, as

$$I_{BSi} = log_2(1 + \frac{P_{BSi}|h_{BS,i}|^2}{BN_0/N}). \tag{3.3}$$

The direct link between the $BS$ and user $i$ is used if the signal is strong enough that can be decoded by the user, i.e., $I_{BSi} > \xi$. Where $\xi$ is the *spectral efficiency* determined by designer depending on the application. Otherwise, $BS$ should transmit the data to user $i$ by help of a relay. If $\xi$ is chosen to be a larger number, more users use relay.

The objective of this work is to distribute the transmit power of each relay $j$ among its users, in order to maximize the total data rate of the network. Therefore, the optimization problem is defined as

*Problem $P_A$:*

$$\max_{x,P} \sum_{j=1}^{K} \sum_{i=1}^{N} x_{ij} DR^{j}_{(BS,i)} \tag{3.4}$$

$$s.t. \quad \sum_{j=1}^{K} x_{ij} \leq 1 \qquad \forall i \tag{3.5}$$

$$\sum_{i=1}^{N} x_{ij} P_{ji} \leq P_j^{max} \qquad P_{ji} > 0, \forall i, j \tag{3.6}$$

where $x_{ij} \in \{0, 1\}$ is a binary variable that indicates whether or not user $i$ is assigned to relay $j$ and it is defined as follow

$$x_{ij} = \begin{cases} 1 & \text{user } i \text{ is assigned to relay } j \\ 0 & \text{otherwise.} \end{cases} \tag{3.7}$$

There are $K$ relays and $N$ users in the network, $x_{ij}$ and $P_{ji}$ are the decision variables. Constraint (3.5) implies that each user at most can be assigned to one relay. As we explained earlier, for users who are strong enough ($I_{BSi} > \xi$), direct link is used. The total transmission power of each relay $j$ is limited to $P_j^{max}$ as shown in constraint (3.6).

The above mentioned ***Problem*** $P_A$ is a mixed nonlinear integer programming problem since it contains not only the continuous variable $P_{ji}$, but also the binary variable $x_{ij}$. It is difficult to find the optimal solution to this kind of a problem since there is no general algorithm for solving such a mixed nonlinear integer programming problem efficiently [193]. Therefore, we exploit the nature of the problem and find an approach to achieve a reasonable performance. In order to simplify the problem all $x_{ij}$ satisfying constraint (3.5) are determined by relay selection scheme as follow. As we explained earlier, $BS$ should transmit the data to user $i$, whose $I_{BSi}$ is less than $\xi$, by help of a relay. In order to select a relay, we apply a common relay selection method in the literature, where for each user a relay which has the highest channel gain among all available relays is selected [194]. As a result, each relay $j$ will form a coalition with the users who use this relay. Let $\mathcal{N}_j$ be the set of users assigned to relay $j$ $(j = 1, 2, ..., K)$. We let $\mathcal{N}_0$ be the set of users using direct link (without relay). As a result,

$$\mathcal{N}_0 \cup \mathcal{N}_1 \cup \mathcal{N}_2 \cup ... \cup \mathcal{N}_K = \mathcal{N}, \tag{3.8}$$

where $\mathcal{N}$ is the set of all users and $N = |\mathcal{N}|$.

These $K$ relays are independent of each other and each of them has a specific amount of power. Moreover, relay selection procedure is done before applying any power allocation scheme. Thus, the ***Problem*** $P_A$, is converted to the following problem for each relay $j$.

*Problem* $P_B$*:*

$$\underset{P}{max} \sum_{i=1}^{N_j} DR^j_{(BS,i)} \tag{3.9}$$

$$s.t. \quad \sum_{i=1}^{N_j} P_{ji} \leq P_j^{max} \qquad P_{ji} > 0, \forall i, j \tag{3.10}$$

where $N_j = |\mathcal{N}_j|$ and $P_{ji}$ is the decision variable.

Taking into account the formed network, ***Problem*** $P_B$ implies that, to maximize the total data rate of the network, the data rate of users on each relay should be maximized.

Since $P_{ji}$ is not known a priori in Equation (3.2), distributing the transmit power of each relay $j$ among its users is a challenging issue.

In case of using the relay, the second term of (3.2) should meet $\xi$, which requires $P_{ji}$ to satisfy a threshold, called $P_{ji}^{th}$. To obtain $P_{ji}^{th}$, we let the second term of Equation (3.2) equal to $\xi$,

$$log_2(1 + \frac{P_{BSi}|h_{BS,i}|^2}{BN_0/N} + \frac{P_{ji}|h_{j,i}|^2}{BN_0/N}) = \xi. \tag{3.11}$$

The corresponding $P_{ji}$ which lets this equality take place, called $P_{ji}^{th}$, can be calculated as

$$P_{ji}^{th} = \frac{(BN_0/N)(2^{\xi} - 1) - P_{BSi}|h_{BS,i}|^2}{|h_{j,i}|^2}. \tag{3.12}$$

By this, if $P_{ji}^{th}$ has a negative value, we let it be zero. The power allocation scheme will be discussed in the next section.

## 3.2 Power Allocation Scheme

In this section, we find how to distribute the transmit power of each relay $j$ among all users who use it. In order to find that, we rewrite the Equation (3.2) as

$$DR^{j}_{(BS,i)} = \frac{1}{2}min[\frac{B}{N}log_2(a_i), \frac{B}{N}log_2(b_i + (c_i * P_{ji}))] \tag{3.13}$$

$$a_i = (1 + \frac{P_{BSi}|h_{BS,j}|^2}{BN_0/N}) \tag{3.14}$$

$$b_i = (1 + \frac{P_{BSi}|h_{BS,i}|^2}{BN_0/N}) \tag{3.15}$$

$$c_i = \frac{|h_{j,i}|^2}{BN_0/N}. \tag{3.16}$$

To determine the appropriate power distribution of each relay $j$ we should consider all possible states. As shown in Equation (3.13), the data rate of each user $i$ is the minimum between the first and second term. In Equation (3.13), where all parameters except $P_{ji}$ are known , if $a_i < b_i$ the minimum function will select the first term. Moreover, the first term will be selected if $a_i > b_i$ and $(c_i * P_{ji}) > a_i - b_i$. Otherwise, if $a_i > b_i$ and $(c * P_{ji}) < a_i - b_i$, the minimum function will select the second term as illustrated in (3.17).

$$DR^{j}_{(BS,i)} = \begin{cases} \frac{1}{2}\frac{B}{N}log_2(a_i) & a_i < b_i \quad \text{or} \\ & a_i > b_i, \\ & (c_i * P_{ji}) > a_i - b_i \\ \frac{1}{2}\frac{B}{N}log_2(b_i + (c_i * P_{ji})) & a_i > b_i, \\ & (c * P_{ji}) < a_i - b_i. \end{cases} \tag{3.17}$$

The case where $a_i < b_i$ means that the channel gain between the $BS$ and relay $j$ is worse than the channel gain between the $BS$ and user $i$. Therefore, in this case the direct link is selected and there is no need to use a relay.

In the case where $a_i > b_i$, if we make $(c_i * P_{ji}) > a_i - b_i$, the minimum function will select the first term and the extra given power of the relay $j$ allocated to user $i$ will be wasted. Moreover, if we make $(c * P_{ji}) < a_i - b_i$, the minimum function will select the second term, while it would be possible to select the first term with greater value and maximize the minimum value. As a result, the best strategy for power distribution of the relay is to let the first term and second term of the Equation (3.13) become equal. The corresponding $P_{ji}$ which let this equality take place, called $P^{*}_{ji}$, is the optimal power requirement of user $i$

$$P^{*}_{ji} = P_{BSi}\frac{|h_{BS,j}|^2 - |h_{BS,i}|^2}{|h_{j,i}|^2}. \tag{3.18}$$

Therefore, the appropriate power distribution for the relay $j$ is to assign $P^{*}_{ji}$ to each user $i$ who use relay $j$. It should be mentioned that, achievable rate of the DF cooperative scheme can be limited by the capacity of the link between the $BS$ and relay. As a result, in case of *equal power allocation* among all users of a relay, it may happen that for some users the link between the relay and $BS$ becomes a bottleneck and a part of the power allocated to that specific user cannot be fully utilized and will be wasted. However, our scheme by finding the optimal power requirement of each user $i$ ($P^*_{ji}$), avoids such kind of problems. Note that, the transmit power of each relay $j$ is limited as shown in constraint (3.10). Therefore, it may happen that the transmit power of the relay $j$ cannot support all the $P^*_{ji}$ of the users who use relay $j$. In this case, we should find the optimal solution to maximize the data rate. The optimal solution will be discussed in next subsection.

### 3.2.1 Optimal Power Allocation

The optimal solution which maximizes the data rate can be setup as a *knapsack problem*. In *knapsack problem* [195], with a set of $n$ items, each item $i$ with a weight of $w_i$ and value $v_i$, the aim is to maximize the total value while the total weight does not exceed a given limit. The number of each item is denoted by $x_i$.

$$max \sum_{i=1}^{n} v_i x_i \tag{3.19}$$

$$s.t. \quad \sum_{i=1}^{n} w_i x_i \leq W. \tag{3.20}$$

In our model, each user $i$ (referred as an item) has optimal required amount of power to reach, called $P^*_{ji}$, (*as a bound*) and a fraction of power can be assigned to each user (*as a continuous variable*). The total amount of knapsack is the maximum power of the relay $j$ and this power should be distributed among the users who use relay $j$. Therefore, power allocation of relay $j$ among its users can be modeled as a *bounded continuous knapsack problem*. A *greedy algorithm* can find the optimal solution to maximize the total value (data rate) as follow.

The greedy algorithm starts with the user that can have the maximum data rate (value) per unit of power (weight), who is the user with the highest channel gain. Equation (3.2) shows the relation between the channel gain and data rate. The proposed user is selected and as much power as possible is given to this user until the bound ($P^*_{ji}$) is reached. If the power is still available, the algorithm will continue to give the power to the second user with the highest channel gain and so on (based on decreasing order of their channel gain), until the relay $j$ runs out of power.

This solution sets the *upper bound* for maximum achievable data rate and there is no other way to reach more data rate than this scheme. It is due to the fact that this scheme gives each unit of power to the user which can result in highest data rate. However, this solution does not take into account any fairness. We should consider the fairness issue as well which will be explained in next subsection.

### 3.2.2 Power Allocation Scheme with Fairness

With limited available radio resources, increasing system throughput and maintaining fairness are usually conflicting with each other [196], leading to a tradeoff between these two performance measures. In particular, balancing system throughput and fairness is necessary, depending on different application and specific scenarios [197].

Conventional power allocation algorithms have some weak points that degrade the system performance. As an example, *equal power allocation algorithm* does not take the channel state information of each user into consideration. To overcome such weak points and consider the throughput and fairness jointly, we developed a power allocation scheme, which is explained in **Algorithm 3.2.1**.

The values of $P^*_{ji}$ and $P^{th}_{ji}$ of each user $i$ are calculated as explained earlier, (3.12) and (3.18). In the case that the relay $j$ can support the total $P^*$ of all of its users, the relay $j$ allocates to each user $i$ its corresponding $P^*_{ji}$. However, if the power of the relay $j$ is less than the total $P^*$ of all aforementioned users, the $P_{th}$*-check scheme* will be run, shown in **Algorithm 3.2.2**.

As it is explained in $P_{th}$*-check scheme*, relay $j$ allocates $P^{th}_{ji}$ to each corresponding user $i$, if its power can support the total $P_{th}$ of all users who use relay $j$. User satisfaction and fairness can be achieved by this scheme, since the minimum data rate requirement of all users can be met. Moreover, the remaining amount of power denoted as $P_{rem}$ in

**Algorithm 3.2.1** POWER ALLOCATION SCHEME

```
Initialization:
    • For each user $i$, $P^*_{ji}$ and $P^{th}_{ji}$ is calculated.
if $\sum_{i=1}^{N_j} P^*_{ji} \le P^{max}_j$ then
    • Relay allocates $P^*_{ji}$ to each corresponding user $i$.
else if $\sum_{i=1}^{N_j} P^*_{ji} > P^{max}_j$ then
    • Run $P_{th}$-CHECK SCHEME.
end if
```

**Algorithm 3.2.2** $P_{th}$-CHECK SCHEME

```
if $\sum_{i=1}^{N_j} P^{th}_{ji} = P^{max}_j$ then
    • Relay allocates $P^{th}_{ji}$ to each corresponding user $i$.
else if $\sum_{i=1}^{N_j} P^{th}_{ji} > P^{max}_j$ then
    • Run WEIGHTED-BASED SCHEME followed by POWER REALLOCATION SCHEME.
else if $\sum_{i=1}^{N_j} P^{th}_{ji} < P^{max}_j$ then
    • Relay allocates $P^{th}_{ji}$ to each corresponding user $i$
    • Run WEIGHTED-BASED SCHEME to allocate the remaining amount of power ($P^{max}_j - \sum_{i=1}^{N_j} P^{th}_{ji}$).
end if
```

Equation (3.21), is distributed among the users by the *weighted-based scheme* as shown in **Algorithm 3.2.3**.

$$P_{rem} = P^{max}_j - \sum_{i=1}^{N_j} P^{th}_{ji}. \tag{3.21}$$

The *weighted-based scheme* combines fairness and preferential weighting. It assigns a weight to each user and different users have different resource shares based on their pre-assigned weights. If $N$ users are available with weights of $w_1, w_2, w_3, ..., w_N$ user $i$ will achieve $Share_i$ of the whole resource amount as follow

$$Share_i = \frac{w_i}{(w_1 + w_2 + w_3 + ... + w_N)}. \tag{3.22}$$

This scheme can prevent users with bad channel gain (poor users) from overwhelming the

**Algorithm 3.2.3** WEIGHTED-BASED SCHEME

```
1: Initialization:
     • Having the channel gain ($h_{j,i}$) and $P^*_{ji}$ of each user $i$.
2: Calculate the corresponding weight of each user $i$ based on $P^*_{ji}h^2_{j,i}$
3: Allocate the fraction $\frac{P^*_{jl}h^2_{j,l}}{\sum_{i=1}^{N_j} P^*_{ji}h^2_{j,i}}$ of the total relay power, $P_j^{max}$, to each user $l$.
```

resources of the network. In our scenario, it may happen that the power of the relay cannot support the total $P^*$ of all the users who select the relay $j$ as their relay. Therefore, in order to find how to distribute the power of the relay $j$ among these users, we should define a weight for each user $i$.

The weight of each user is defined by channel gain and the optimal power requirement of each user $i$ ($P^*_{ji}$). A user with higher channel gain and power request should be given more power. The data rate which should be maximized is related to power and channel gain according to (3.2), so the weight for each user $i$ is defined as follow

$$w_i = P^*_{ji}h^2_{j,i}. \tag{3.23}$$

As a result, the relay $j$ allocates to each user $l$ the fraction $p_l$ of the total relay power ($P_j^{max}$) as follow

$$p_l = \frac{P^*_{jl}h^2_{j,l}}{\sum_{i=1}^{N_j} P^*_{ji}h^2_{j,i}}. \tag{3.24}$$

After the above case, which the relay $j$ can support the total $P_{th}$ of all its users, now we should explain the case which relay $j$ cannot support the total $P_{th}$ of all its users. In this case since the power of relay $j$ is less than the total $P_{th}$ of all its users, relay $j$ does not allocate $P_{th}$ to users. The relay $j$ allocates the power to its users by *weighted-based scheme* followed by *power reallocation scheme*, which will be explained in the next section.

## 3.3 Power Reallocation Scheme

By *power allocation scheme*, the assigned power of the relay $j$ to each of its users is denoted by $P_i$. It may happen that some users using the relay $j$ have allocated power which is less than the predefined threshold ($P_{th}$). This allocated power does not satisfy $\xi$ and the received signal cannot be decoded, so the power is wasted. In order to wisely utilize the power which is scarce and expensive and improve the data rate of the network, we propose an efficient *power reallocation scheme* which is shown in **Algorithm 3.3.1**.

This algorithm first finds the users whose allocated power is less than their threshold ($P_{ji}^{th}$). The larger the difference of the allocated $P_i$ and $P_{ji}^{th}$, the less chance for user $i$ to reach $P_{ji}^{th}$. Therefore, this algorithm sorts the users whose allocated power are less than their $P_{th}$ based on descending order of difference between the allocated $P_i$ and $P_{ji}^{th}$. *Power reallocation scheme* starts from distributing the power of the user $k$ which has the largest difference between the allocated power $P_k$ and $P_{jk}^{th}$, equally among the other users which their power are less than their $P_{th}$. Then, the algorithm continues the same procedure for the other users in a descending order.

As a result, users whose allocated power is less than their threshold ($P_{ji}^{th}$), have the chance to gain the distributed power and compensate their power to the level which is greater than their threshold power. This scheme provides a power pooling approach, since the extra and useless power of users which could have wasted, is distributed among other users. In order to evaluate our proposed schemes, simulation results are presented in the next section. As explained earlier, our proposed optimal solution maximizes the data rate. However, this solution does not take into account any fairness.

Increasing system throughput and maintaining fairness are usually leading to a tradeoff between these two parameters. Generally, policies for sharing resources that are characterized by low level of fairness provide high average throughput but it may result in more unhappy users. Therefore, it is important to provide schemes that allocate resources fairly and efficiently. To consider the throughput and fairness jointly, our proposed power allo-

**Algorithm 3.3.1** POWER REALLOCATION SCHEME

```
Initialization:
    • Having the $P_{ji}^{th}$ and allocated power $P_i$ of each user $i$ resulted from POWER ALLOCATION SCHEME.
Find users on the relay $j$ which have $P_i < P_{ji}^{th}$ and make the set $\mathcal{R}_j^* \subset \mathcal{R}_j$. ($\mathcal{R}_j$ is the set of the users that use relay $j$.)
Sort the users, $i \in \mathcal{R}_j^*$, based on descending order of $P_{ji}^{th} - P_i$ and make the set $\mathcal{R}_j^{**} \subseteq \mathcal{R}_j^*$.
for $i \in \mathcal{R}_j^{**}$ do
    if $P_i < P_{ji}^{th}$ then
        • Distribute $P_i$ equally among users in $\mathcal{R}_j^{**}$ excluding users who could not reach $P_{ji}^{th}$ in the previous round.
        • $P_i = 0$
    end if
end for
```

cation scheme, *weighted-based scheme* followed by $P_{th}$-*check scheme*, combines fairness and preferential weighting.

In the next section, simulation results are presented. In order to evaluate our proposed schemes, we compare our schemes with *traditional* and *weighted max-min* power allocation schemes. *Traditional scheme* does not take the channel state information of each user into consideration, each relay simply divides its transmit power *equally* among all its users. Therefore, some users may not be able to utilize their share of the resources.

*Max-min fairness* is a classical sharing principle which assigns power to users according to their demands. The simple *max-min fair allocation* is not capable of providing various shares to users based on variations in their weight. As a result, adjusted max-min fair allocation called *weighted max-min scheme* is considered. In *weighted max-min scheme*, a new parameter called weight is assigned to each user. It should be noted that for *weighted max-min scheme*, demand and weight vector should be defined. In our work, demand and the weight vector are set to be the corresponding $P^*$ and channel gain of users, respectively.

## 3.4 Simulation Results

In this section, we evaluate the performance of the proposed *power (re)allocation schemes*. We consider a wireless DF cellular network with $1Km$ radius. The $BS$ is located at the cell center, fixed relays are located uniformly at $2/3$ of the cell radius and users are distributed randomly. For the channel gain, the fading coefficients are i.i.d. Rayleigh random variables and the standard deviation of zero mean lognormal random variable of shadowing is 4 $dB$. The parameters in the simulations for path loss exponent, noise power, and $\xi$ are equal to 3, -50 $dBm$, and 0.9, respectively. The power of $BS$ is 0.1 $W$ and the total bandwidth is 1 $MHz$. The results are obtained using 100 simulation runs. Simulation results illustrate the comparison among the *optimal scheme* resulting from knapsack problem which is an *upper bound*, our proposed *power allocation scheme with and without power reallocation*, *weighted max-min power allocation*, and *traditional scheme*.

Fig. 3.1 shows the sum-spectral efficiency of the network versus the maximum power of each relay. The sum-spectral efficiency is the total normalized data rate. The number of relays and users in the network are 4 and 50 respectively, and the maximum power of each relay is increases from 0.01 $W$ to 0.08 $W$. The maximum power of each relay is limited. As shown in Fig. 3.1, when the maximum power of each relay increases, the amount of power which can be allocated to each user increases, and the sum-spectral efficiency of the network increases.

It is illustrated that, the *weighted max-min scheme* gives the lowest sum-spectral efficiency. It is due to the fact that, by definition, *weighted max-min scheme* helps users with bad channel gain, by maximizing the minimum rate. The sum-spectral efficiency of our *proposed scheme* with *power reallocation* is 12% more than that of *traditional scheme*. The result of proposed *power allocation scheme with power reallocation* is very close to *optimal solution* (*upper bound*). Note that the sum-spectral efficiency of the proposed scheme increases slowly for higher values of the maximum power of each relay. It is because, when the power of each relay increases, every relay can support the total $P^*$ of more

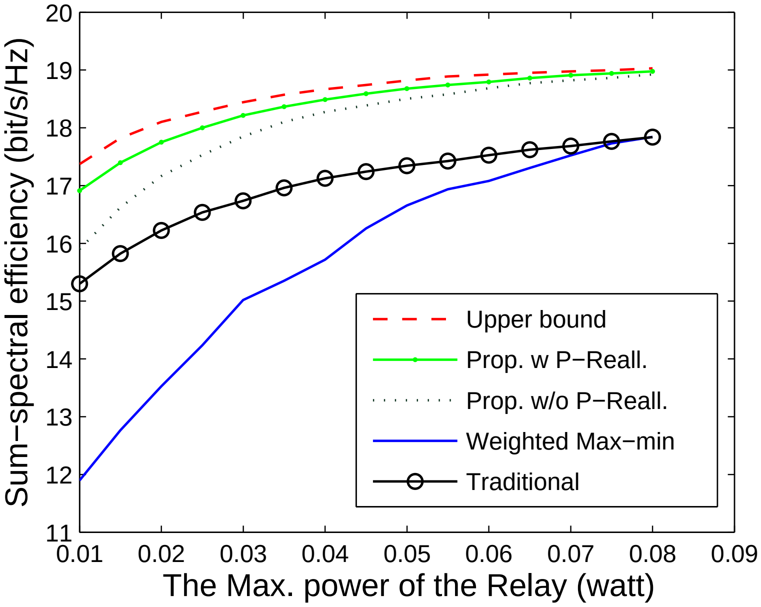

**Figure 3.1:** The sum-spectral efficiency of the network versus the max. power of each relay (number of relays and users are 4 and 50 respectively, and the max. power of each relay is 0.01-0.09).

users connected to them, up to the point that all users are given $P^*$ and do not need extra power.

Fig. 3.2 shows the sum-spectral efficiency of the network versus the number of users using relay. The number of users in the network is increases from 10 to 50 and there are 3 relays in the network with maximum power of 0.05 $W$ for each relay. As shown in the figure, when the number of users increases, more users select each relay and the sum-spectral efficiency increases. The *optimal solution* resulting from knapsack problem, outperforms the other schemes since it allocates power to users more efficiently. The difference between the *proposed power allocation scheme with and without power reallocation* shows the effect of proposed *power reallocation scheme*. *Power reallocation scheme* tries to compensate more users to their threshold power ($P_{th}$). As Fig. 3.2 illustrates, the performance of our proposed schemes are significantly better for larger number of users, compared to weighted max-min scheme. It is because, when the number of users using relay increases, the weighted max-min scheme helps more poor users (users with bad channel gains).

Fig. 3.3 depicts the impact of increasing the number of relays. As the number of

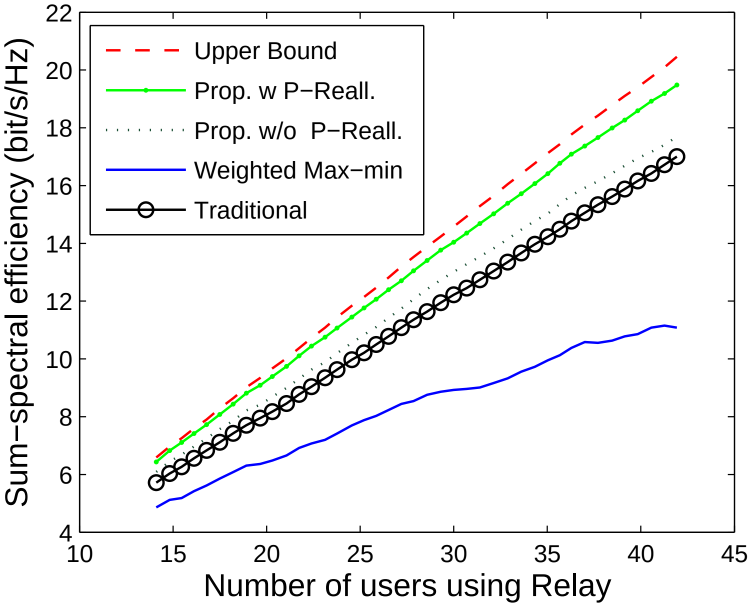

**Figure 3.2:** The sum-spectral efficiency of the network versus the number of users using relay (number of relay is 3, the maximum power of each relay is 0.05, and number of users in the network is 10-50).

relays increases from 3 to 6, less users select each relay and the probability for each relay to be able to support $P^*$ of all its users increases. Therefore, as Fig. 3.3 shows when the number of relays increases to 6, the results of all schemes merge to the same point of sum-spectral efficiency. Moreover, the result of proposed *power allocation scheme* with *power reallocation* is very close to upper bound. Fig. 3.3 illustrates that the sum-spectral efficiency of the traditional scheme is 12% less than that of *proposed scheme*.

Fig. 3.4 depicts the average share of each relay from the sum-spectral efficiency of the network. In each relay, the first bar indicates the *optimal scheme* resulting from knapsack problem, the second bar shows our *proposed power allocation scheme with power reallocation*, the third bar illustrates the *traditional scheme*, and the forth bar shows the *weighted max-min power allocation with power reallocation scheme*. It can be seen that the *optimal scheme* and *weighted max-min power allocation with power reallocation scheme* resulted in the highest and lowest spectral efficiencies in each relay, respectively. As Fig. 3.4 shows, the amount of spectral efficiency of different relays resulting from the same power allocation scheme (e.g. the first bar of each relay) are different, since different number of users

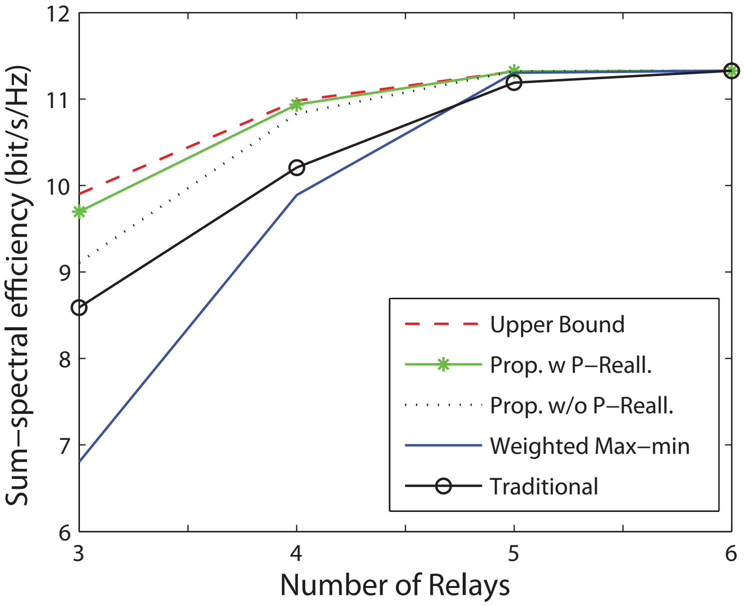

**Figure 3.3:** The average sum-spectral efficiency of the network versus number of relays (number of users is 50, the maximum power of each relay is 0.05 and the number of relays is 3-6).

are connected to each relay, i.e., the most users are connected to relay 3, followed by relay 1 and relay 2. The figure illustrates that when more users are connected to a relay, the *proposed power allocation scheme with power reallocation* can significantly outperform the *traditional scheme*, while it maintains its superiority, when minimum users are connected to the relay. Moreover, the performance of the *proposed power allocation scheme with power reallocation* can be very close to the the *optimal scheme* for all relays with different number of connected users.

Fig. 3.5 illustrates the comparison between our *proposed power allocation* and *weighted max-min power allocation scheme* with and without *power reallocation scheme*. By applying power reallocation scheme (*Algorithm 3.3.1*), the extra power of users which could have been wasted, is distributed among other poor users whose power are less than the threshold power ($P_{th}$) leading to improvement in the spectral efficiency. As Fig. 3.5 shows, the power reallocation scheme is more effective for larger number of users (for both *proposed* and *weighted max-min* power allocation schemes). It is because, when the number of users using relay increases, both (*Algorithm 3.3.1*) and the weighted max-min scheme can help

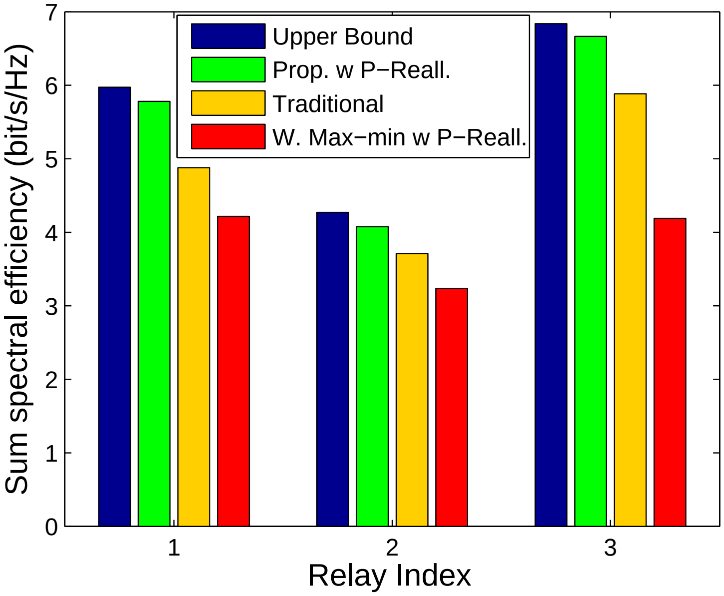

**Figure 3.4:** The sum-spectral efficiency of each relay versus the relay index (number of relay is 3, the maximum power of each relay is 0.05, and number of users in the network is 50).

more poor users reach $P_{th}$.

Fig. 3.6 depicts the effect of applying the *power reallocation scheme* on spectral efficiency improvement. As Fig. 3.6 shows, *power reallocation scheme* is significantly more effective on the *proposed scheme* compared to *weighted max-min scheme*. It is due to the fact that, by definition, *weighted max-min scheme* itself helps poor users, before applying *Algorithm 3.3.1*, and tries to maximize the minimum rate. Therefore, less number of users need to obtain extra power to reach $P_{th}$. However, the *proposed scheme* focuses on the other users to increase spectral efficiency of the network. As a result, the *power reallocation scheme* can be complement and help the remaining users, poor users, to reach $P_{th}$ and contribute to the total spectral efficiency of the network.

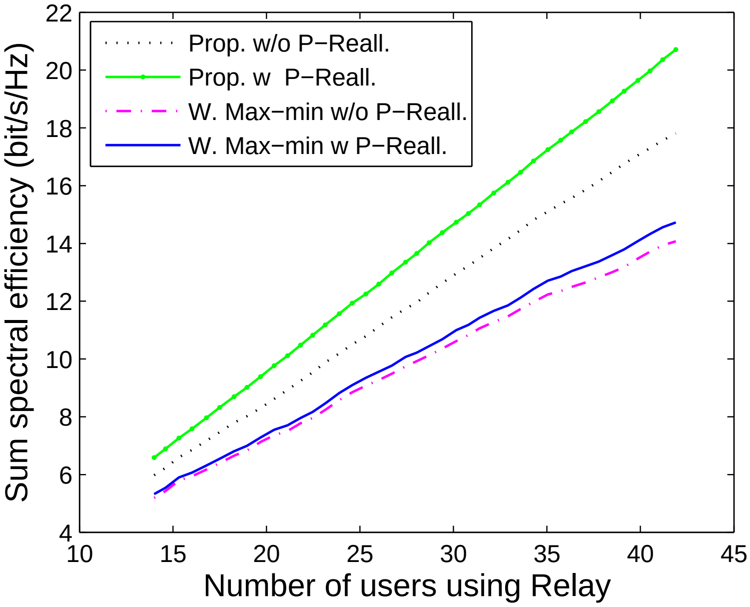

**Figure 3.5:** The sum-spectral efficiency of the network versus the number of users using relay (number of relay is 3, the maximum power of each relay is 0.05, and number of users in the network is 10-50).

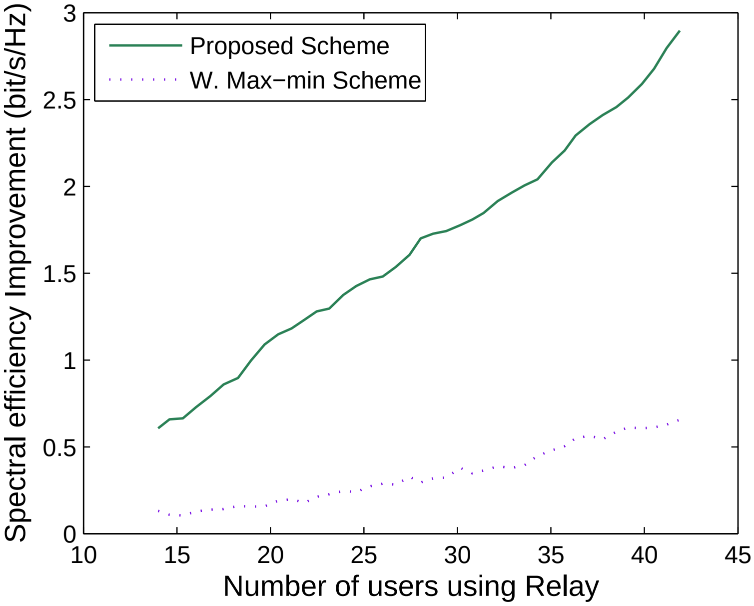

**Figure 3.6:** The improvement of sum-spectral efficiency of the network versus the number of users using relay (number of relay is 3, the maximum power of each relay is 0.05, and number of users in the network is 10-50).

# Chapter 4

# A Coalitional Game-Based Relay Load Balancing and Power Allocation in Decode-and-Forward Relay Networks

After considering power allocation in OFDMA-based decode-and-forward cellular relay networks in previous chapter, a game theoretic relay load balancing scheme is proposed for OFDMA-based DF cellular relay networks in this chapter. First, system model and problem definition are described. Then, coalitional formation scheme is proposed followed by joint power allocation and relay load balancing scheme and coalitional game description. After that, simulation results conclude this chapter.

## 4.1 System Model and Problem Definition

In this chapter, the same downlink scenario of DF cellular relay network as previous chapter is considered. In the cell, a $BS$ is located at cell center and $K$ fixed relays are located uniformly. There are $N$ users distributed randomly over the cell. The cell radius is $R$, and the distance from $BS$ to each relay is $\tau R$, where $0 < \tau < 1$. The channels between stations are frequency selective and OFDMA is employed to convert the channel into orthogonal subcarriers with flat fading. By pre-subcarrier allocation, the best unallocated subcarrier is assigned to each user and the power of $BS$ is equally allocated among all users. For simplicity, we assume that each relay uses the same subcarrier to relay information it received. There is no more than one relay assisting each user. The overall bandwidth $B$ is divided equally among the $N$ users.

As explained in previous chapter, DF cooperative scheme is used and the user combines the directly received signal from $BS$ and the relayed signal from relay together using MRC. As a result, the achievable data rate between $BS$ and user $i$ by help of relay $j$ is as follow.

$$DR^{j}_{(BS,i)} = \frac{1}{2}min\{\frac{B}{N}log_2(1+\frac{P_{BSi}|h_{BS,j}|^2}{BN_0/N}), \frac{B}{N}log_2(1+\frac{P_{BSi}|h_{BS,i}|^2}{BN_0/N}+\frac{P_{ji}|h_{j,i}|^2}{BN_0/N})\}. \tag{4.1}$$

where $h_{BS,i}$ ($h_{BS,j}$) denotes the channel gain between $BS$ and user $i$ (relay $j$), $P_{BSi}$ ($P_{BSj}$) is the transmit power from $BS$ to user $i$ (relay $j$) and $P_{ji}$ is the transmit power from relay $j$ to user $i$. $N_0$ is the power spectral density of additive white Gaussian noise.

Similar to Chapter 3, $BS$ transmits the data to user $i$ either directly or via a relay, depending on the quality of the channel between the $BS$ and user $i$. Let us define $I_{BSi}$ as

$$I_{BSi} = log_2(1+\frac{P_{BSi}|h_{BS,i}|^2}{BN_0/N}). \tag{4.2}$$

The direct link between the $BS$ and user $i$ is used if the signal is strong enough that can be decoded by the user, i.e., $I_{BSi} > \xi$. Where $\xi$ is the *spectral efficiency* determined by designer depending on the application. Otherwise, $BS$ should transmit the data to user $i$ by help of a relay.

In Chapter 3, we found how to distribute the transmit power of each relay $j$ among users who use this relay, in order to maximize the total data rate of the network with and without fairness issues. In previous chapter, we assumed that the network is formed by applying the traditional relay selection scheme, where for each user a relay which has the highest channel gain among all available relays is selected [194]. As a result, each relay $j$ formed a coalition with the users who use this relay.

However, in this chapter, we do not apply the traditional relay selection scheme. It is because the users are distributed randomly and by applying the traditional relay selection scheme, it may happen that some relays have more users connected to them than other relays. As a result, the load is unbalanced among the relays which leads to the decrease in the assigned power to users that connected to relays with high load. Therefore, the major contribution of this chapter is relay load balancing, which is an important issue that aims to utilize resources effectively and avoid overload of any relays. By applying relay load balancing, users who can connect to uncongested relays join them as opposed to connecting to congested relays, so users are evenly distributed among the relays.

In this chapter, we want to maximize the total data rate of the network while having load balancing among the relays. Therefore, the optimization problem is defined as

*Problem $P_C$:*

$$\max_{x,P} \sum_{j=1}^{K}\sum_{i=1}^{N} x_{ij} DR^{j}_{(BS,i)} \tag{4.3}$$

$$s.t. \quad \sum_{j=1}^{K} x_{ij} \leq 1 \qquad \forall i \tag{4.4}$$

$$N_{min} < \sum_{i=1}^{N} x_{ij} < N_{max} \qquad \forall j \tag{4.5}$$

$$\sum_{i=1}^{N} x_{ij} P_{ji} \leq P_j^{max} \qquad P_{ji} > 0, \forall i, j \tag{4.6}$$

where $x_{ij} \in \{0, 1\}$ is a binary variable that indicates whether or not user $i$ is assigned to relay $j$ and it is defined as follow.

$$x_{ij} = \begin{cases} 1 & \text{user } i \text{ is assigned to relay } j \\ 0 & \text{otherwise.} \end{cases} \tag{4.7}$$

Constraint (4.4) implies that each user at most can be assigned to one relay. As we explained earlier, for users who are strong enough ($I_{BSi} > \xi$) direct link is used. In constraint (4.5), $N_{max}$ and $N_{min}$ define the maximum and minimum number of users which can be assigned to each relay $j$. Constraint (4.5) imposes the load balancing requirements. Moreover, the total transmission power of each relay $j$ is limited to $P_j^{max}$ as shown in constraint (4.6).

It is difficult to find the optimal solution to the *Problem* $P_C$, since *Problem* $P_C$ is more complicated than generalized assignment problem ($GAP$), which is a NP-hard combinatorial optimization problem [198]. Compared to $GAP$, assigning users to relays with limited power should be done while constraints (4.5) should be met and power allocation should be applied as well. Therefore, we exploit the nature of the problem and find an approach to achieve a reasonable performance.

The aim of this chapter is to achieve load balancing among relays while considering the total data rate of the network and fairness issue among users as well. In order to achieve relay load balancing, we define $N_{max}$ and $N_{min}$, in constraint (4.5), as the maximum and minimum number of users which can be assigned to each relay $j$, respectively.

In order to solve the problem, first $N_{max}$ should be determined. The maximum number of users that a relay $j$ can support, $N_{max}$, is a system parameter which can be calculated as

$$N_{max} = \lfloor \frac{P_j^{max}}{\bar{p}_k} \rfloor, \tag{4.8}$$

where $\bar{p}_k$ is the average power that should be allocated to an arbitrary user $k$ in order to satisfying the spectral efficiency requirement. To obtain $\bar{p}_k$, we let the second term of Equation (4.1) become equal to $\xi$.

$$log_2(1 + \frac{P_{BSk}|h_{BS,k}|^2}{BN_0/N} + \frac{P_{jk}|h_{j,k}|^2}{BN_0/N}) = \xi. \tag{4.9}$$

The corresponding $P_{jk}$ which lets this equality take place, called $\bar{p}_k$, is as follow

$$\bar{p}_k = \frac{(BN_0/N)(2^{\xi} - 1) - P_{BSk}|h_{BS,k}|^2}{|h_{j,k}|^2}, \tag{4.10}$$

where $h_{BS,k}$ and $h_{j,k}$ are the average channel gains of an arbitrary user $k$ in the network by considering pathloss. The minimum number of users connected to relay, i.e., $N_{min}$, is a system parameter that can be determined by the network designer. Having the $N_{min}$ and $N_{max}$, we should solve the problem.

Note that, although having more users connected to a relay increases the total data rate of that relay, but if too many users are connected to that relay the power which relay can assign to each user decreases. As a result, in order to maximize the data rate and satisfy the constraint (4.5) as well, cooperative game theory provides analytical tools to study the behavior of users and achieve balance between the assigned power and total data rate. By applying the game, users join uncongested relays, instead of congested relays. Thus users are evenly distributed among the relays and load balancing can be obtained.

With the emergence of cooperation as a new communication paradigm, it has become imperative to seek suitable game theoretical tools that allow to study the behavior and interactions of the nodes. The main branch of cooperative games describes the formation of cooperating groups of players, referred to as coalitions [180]. Forming a coalition brings gains to its members, but the gains are limited by a cost for forming the coalition. Our proposed problem can be modeled as a coalitional game by forming coalitions around each relay.

A centralized approach can be used in order to find the optimal coalition structure that maximizes the data rate subject to existing constraints. However, as shown in [199], finding the optimal coalition structure in a centralized manner leads to an optimization problem which is NP-complete. This is mainly due to the fact that the number of possible coalition structures, given by the Bell number, grows exponentially with the number of users $N$ [200]. Moreover, the complexity increases further as the amount of power dedicated to each user should be calculated for each possible coalition structure. Therefore, in order to derive a distributed solution and having lower complexity, our proposed problem can be modeled as a coalitional game. In the next section, our proposed coalitional formation schemes will be discussed after a brief introduction to coalitional game theory.

## 4.2 Coalitional Formation Scheme

Coalitional game theory aims at finding an optimal structure of players to optimize the worth of each coalition. The game can be described by a pair $(\mathcal{N}, v)$, where $\mathcal{N}$ is the set of players and $v$ denotes the coalition function, which designates each coalition a number, to reflect the value of the corresponding coalition [180]. In addition, a *comparison relation* $\triangleright$ is defined to compare two collections of coalitions. Consider two sets of coalitions $\mathcal{T} = \{\mathcal{T}_1, \cdots, \mathcal{T}_s\}$ and $\mathcal{R} = \{\mathcal{R}_1, \cdots, \mathcal{R}_t\}$ which are formed by the same set of the players (i.e., $\bigcup_{a=1}^{s} \mathcal{T}_a = \bigcup_{b=1}^{t} \mathcal{R}_b$), and the number of disjoint coalitions in them are $s$ and $t$, respectively. $\mathcal{T} \triangleright \mathcal{R}$ implies that the way $\mathcal{T}$ partitions the set of the players is preferred to the way $\mathcal{R}$ partitions the set of the players. Various well known orders can be used as comparison relations [181]. In the cooperation game, the *Pareto order* is highly appealing as a comparison relation for the merge-and-split rules. The comparison relation, called *Pareto order*, can be defined as

$$\mathcal{T} \triangleright \mathcal{R} \Leftrightarrow \{\phi_n(\mathcal{T}) \geq \phi_n(\mathcal{R}), \forall n \in \mathcal{T}, \mathcal{R}\} \tag{4.11}$$

with at least one strict inequality ($>$) for a player. Note that, $\phi_n(\mathcal{T})$ and $\phi_n(\mathcal{R})$ denote the payoff of the same player $n$ in two different collections of coalitions, (i.e., $\mathcal{T}$, $\mathcal{R}$). The *Pareto order* implies that a collection $\mathcal{T}$ is preferred over $\mathcal{R}$, if at least one player is able to improve its payoff when the coalition structure changes from $\mathcal{R}$ to $\mathcal{T}$ without decreasing other players' payoffs.

Coalition formation attracts high interest in game theory [199], [201] and [181]. The approaches used for distributed coalition formation are quite varied and range from heuristic approaches [199] to set theory based methods [181] as well as approaches that use bargaining theory or other negotiation techniques from economics [202].

There are no general rules for distributed coalition formation. The coalitional game can be implemented through two main rules for forming or breaking coalitions referred to as merge and split relying on the interaction among players [181]. Consider a collection of coalitions, e.g., $\mathcal{C} = \{\mathcal{C}_1, \cdots, \mathcal{C}_m\}$, and a coalition formed by all players in $\mathcal{C}$, i.e., $\{\bigcup_{i=1}^{m} \mathcal{C}_i\}$. The merge and split rules are defined as

- Merge rule: If $\{\bigcup_{i=1}^{m} \mathcal{C}_i\} \rhd \{\mathcal{C}_1, \cdots, \mathcal{C}_m\}$, the $m$ coalitions in $\mathcal{C}$ merge together to form one coalition $\{\bigcup_{i=1}^{m} \mathcal{C}_i\}$, i.e., $\{\mathcal{C}_1, \cdots, \mathcal{C}_m\} \rightarrow \{\bigcup_{i=1}^{m} \mathcal{C}_i\}$;
- Split rule: If $\{\mathcal{C}_1, \cdots, \mathcal{C}_m\} \rhd \{\bigcup_{i=1}^{m} \mathcal{C}_i\}$, the players in coalition $\{\bigcup_{i=1}^{m} \mathcal{C}_i\}$ split into $m$ disjoint coalitions $\{\mathcal{C}_1, \cdots, \mathcal{C}_m\}$, i.e., $\{\bigcup_{i=1}^{m} \mathcal{C}_i\} \rightarrow \{\mathcal{C}_1, \cdots, \mathcal{C}_m\}$.

The basic idea behind merge-and-split rule is that, given a set of players, any collection of disjoint coalitions can merge into a single coalition, if this new coalition is preferred over the previous state. Similarly, a coalition splits into smaller coalitions if the resulting collection is preferred.

It has been proved in [181] that any arbitrary sequence of these two rules (merge-and-split) converges to a final partition, and the optimal structure is formed with the feature that each player has no incentive to leave its coalition, called $\mathbb{D}_{hp}$-stable.

Our proposed problem is modeled as a coalitional game and in the initialization phase each user pre-selects the relay with the highest channel gain. The set of all users that pre-select relay $j$ as their relay is $\mathcal{M}_j$. After that, coalitional game by applying merge-and-split rule over users who pre-select relay $j$, forms the optimal structure and select the coalition with the highest coalitional value for relay $j$. Note that $P_{ji}$ which is needed to calculate the data rate should be obtained from the proposed power allocation scheme. As introduced earlier, to form a coalitional game, the player set and the coalition function should be defined properly. In our system, each user can be regarded as a player. Hence, the key issue is to find a suitable coalition function for the game on relay $j$ as $v_j(S)$, with $S \subseteq \mathcal{M}_j$ being a coalition of users of $\mathcal{M}_j$.

According to (4.3), our aim is to maximize the total data rate satisfying all the constraints (4.4)-(4.6). As mentioned earlier, constraint (4.5) is applied in order to have load balancing among the relays in the network. Although having more users connected to a relay increases the total data rate of that relay, but if too much users are connected to that relay the power which relay can assign to users decreases. The value $v_j(S)$ of a coalition $S$ must capture the trade off between increasing the total data rate and having load balancing. Therefore, $v_j(S)$ should be an increasing function of data rate and decreasing function of the cost. Thus, the coalition value $v_j(S)$ can be consider as

$$v_j(S) = TD_j(S) - C_j(S) \tag{4.12}$$

where $TD_j(S)$ and $C_j(S)$ are the payoff and the cost function of coalition $S$ on relay $j$, respectively. $TD_j(S)$, is the total normalized data rate of the coalition $S$. Note that, to calculate the data rate, $P_{ji}$ is obtained by the proposed power allocation scheme, which is described in Chapter 3.

One of the most important contributions of this paper is to achieve relay load balancing. In order to determine $C_j(S)$, the cost function should reflect the constraint (4.5), where $N_{max}$ and $N_{min}$ define the maximum and minimum number of users which can be assigned to each relay $j$. To do so, we first define two cost functions, i.e., $C_j^{dec}(S)$ and $C_j^{inc}(S)$ such that the requirements for load balancing (constraint (4.5)) are satisfied.

By applying the traditional relay selection scheme for randomly distributed users, it may happen that some relays with small number of connected users are being underutilized. The cost function $C_j^{dec}(S)$ is defined to alleviate underutilization of relays, while satisfying $N_{min} < \sum_{i=1}^{N} x_{ij}$. Then, $C_j^{dec}(S)$ should tend to $\infty$, when the constraint $N_{min}$ is violated, i.e., $N_{min} > \sum_{i=1}^{N} x_{ij}$, and decrease, while more users connect to relay $j$. As a result, a well suited cost function can be derived as

$$C_j^{dec}(S) = \begin{cases} \frac{1}{N_S - N_{min}}, & N_s > N_{min} \\ \infty, & \text{otherwise}, \end{cases} \tag{4.13}$$

where $N_S$ is the number of users in the coalition $S$.

On contrary, if too many users connect to a relay, the power, which can be assigned to each user by the relay, decreases significantly and the relay is being overutilized. Thus,

$C_j^{inc}(S)$ is defined to mitigate this issue and satisfy $\sum_{i=1}^{N} x_{ij} < N_{max}$. Then, $C_j^{inc}(S)$ should tend to $\infty$, when $\sum_{i=1}^{N} x_{ij} > N_{max}$, while it should increase with increase in the number of connected users. A proper function for $C_j^{inc}(S)$ can be defined as

$$C_j^{inc}(S) = \begin{cases} \frac{1}{N_{max} - N_S}, & N_s < N_{max} \\ \infty, & \text{otherwise.} \end{cases} \tag{4.14}$$

Finally, the cost function in (4.12) can be determined as

$$C_j(S) = C_j^{dec}(S) + C_j^{inc}(S). \tag{4.15}$$

After obtaining the coalition value, the detailed procedure of the game will be discussed in the next section.

## 4.3 Scheme Description and Power Allocation

The amount of transmit power of the relay dedicated to each user who selected this relay is not known a priori. Therefore, before discussion on the game procedure, our power allocation scheme is discussed shortly, obtained from previous chapter. After that, in order to maximize the total data rate in the network and having the load balancing among the relays, the coalitional game procedure will be discussed.

### 4.3.1 Relay Power Allocation Scheme

The appropriate power distribution for the relay $j$ is to assign $P_{ji}^*$ to each user $i$ who use relay $j$. However, if the power of the relay $j$ is less than the total $P^*$ of all aforementioned users, the *$P_{th}$-check scheme Algorithm* will be run. By *$P_{th}$-check scheme Algorithm*, relay $j$ allocates $P_{ji}^{th}$ to each corresponding user $i$ if its power can support the total $P_{th}$ of all users who use relay $j$. The remaining amount of power, is distributed among the users by the *weighted-based scheme Algorithm*. Moreover, in case which relay $j$ cannot support the total $P_{th}$ of all users, the relay $j$ allocates the power to users by *weighted-based scheme algorithm*. The details of relay power allocation scheme, $P_{th}$*-check scheme Algorithm* and *weighted-based scheme algorithm* can be found in Chapter 3.

### 4.3.2 Coalitional Game Procedure

In the initialization phase of the game, each user pre-selects the relay with the highest channel gain, explained in **Algorithm 4.3.1**. Based on *Relay Pre-Selection Algorithm*, the direct link from $BS$ to user $i$ will be used if the corresponding $I_{BSi}$, obtained from (4.2), is greater than $\xi$. Otherwise $BS$ should transmit the data to user $i$ by help of a relay.

The game starts with the relay who has the most pre-selected users and continues for the other relays with less pre-selected users until it forms coalition around all relays of the network. As explained earlier, the coalition value of a coalition $S$, $(v_j(S))$, must capture the trade off between increasing the data rate and load balancing. The power assigned to each user is determined by power allocation scheme discussed in previous subsection and the data rate of each user is calculated by (4.1). Moreover, the cost of each coalition is obtained from (4.15).

Coalitional game is formed by applying the pareto order [180] as the comparison relation, and merge-and-split rule is run over users who pre-select relay $j$. After the termination of the merge-and-split rule, the optimal structure is obtained and several coalitions might be formed around relay $j$. The coalition $S_j^*$ with the highest coalition value among the formed coalitions, $\mathscr{S}_j$, will be selected for relay $j$, i.e,

$$S_j^* = \underset{S \in \mathscr{S}_j}{\text{argmin}} v_j(S). \tag{4.16}$$

This coalition is the best one among all possible coalitions since no user has incentive to leave its coalition to avoid coalition value reduction. The operation details of merge-and-split rule can be found in [204]. Note that, users who pre-selected relay $j$ but are not in a final coalition of relay $j$, should select the other relay with the next highest channel

**Algorithm 4.3.1** RELAY PRE-SELECTION

```
Initialization:
    • Each user $i$ calculates $I_{BSi}$ and compares it to $\xi$.
if $I_{BSi} > \xi$ then
    direct link is selected.
else
    if $I_{BSj} > \xi$ then
        calculate channel gains between user $i$ and relays
        sort channel gains based on descending order.
        pre-select the relay $j$ with the highest channel gain.
    end if
end if
```

gain. The same procedure continues for the other relays as well, until coalitions have been formed around all relays of the network. The coalition formation scheme is explained in **Algorithm 4.3.2**.

The complexity of the game lies in the complexity of the merge-and-split operations. In the merge operation, consider the number of coalition formation proposals sent by each of the $N$ nodes. The most complex and worst case for the merge occurs when all the proposals are rejected. In this case, if the first node submits $N-1$ proposals and the second one submits $N-2$ proposals and so on, then the total number of proposals is $N(N-1)/2$. In practice, the process is far less complex and the number of proposals is much lower than $N(N-1)/2$. It is because once a group of users merges into a larger coalition, the number of merging possibilities for the remaining users will decrease. Thus, in the worst case, the complexity is of the order $\mathcal{O}(N^2)$. As for split operation, a coalition is not required to search all the split forms. As soon as a coalition finds a split form verifying the Pareto order, the users in this coalition will split, and the search for further split forms is not required.

In order to evaluate the proposed game, simulation results are presented in the next section.

**Algorithm 4.3.2** COALITION FORMATION OF THE RELAY $j$

```
Initialization:
    • Find the number of users who select the relay $j$, $N_{min}$ and $N_{max}$.
Run the coalition formation scheme (merge-and-split rule)
Among formed coalitions, select the coalition with the maximum coalition value.
Find the users, who pre-selected the relay $j$ and are not in the final formed coalition.
For each above mentioned user $n$, sort channel gain based on descending order.
if user $n$ has not selected the relay $m$ previously then
    select the relay $m$ with the highest channel gain.
end if
```

## 4.4 Simulation Results

In this section, we evaluate the performance of the proposed *game theoretic power allocation schemes*. We consider a wireless DF cellular network with $1Km$ radius. The $BS$ is located at the cell center, fixed relays are located uniformly at $\frac{2}{3}R$ of the cell radius and users are distributed randomly. For the channel gain, the fading coefficients are i.i.d. Rayleigh random variables and the standard deviation of zero mean lognormal random variable of shadowing is 4 $dB$. The parameters in the simulations for path loss exponent, noise power, $N_{min}$ and $\xi$ are equal to 3, -50 $dBm$, 1 and 0.9, respectively. The power of $BS$ is 0.1 $W$ and the total bandwidth is 1 $MHz$. The results are obtained using 100 simulation runs. In Chapter 3, each user is connected to the relay with the highest channel gain but in this chapter the relay selection procedure is taken place by explained game theoretic approach. Simulation results illustrate the comparison among game theoretic approach with applying the proposed power allocation scheme, the proposed power allocation scheme without game, and traditional scheme. Traditional scheme does not take the channel state information of each user into consideration, each relay simply divides its transmit power equally among all its users.

Fig. 4.1 shows the average sum-spectral efficiency of the network versus number of users using relay. The average sum-spectral efficiency is the total normalized data rate. The number of users in the network is increasing from 20 to 60 and the number of relays is 3. As the number of user in the network increases, the number of users using relay increases and the average sum-spectral efficiency increases. In addition, when the number of users increases, game-based scheme is more effective as users are distributed among relays and the power of the relays is not wasted. By using game-based scheme along with the proposed power allocation scheme, the average sum-spectral efficiency improves furthermore. The proposed game-based power allocation scheme can improve the average sum-spectral efficiency approximately 20% compared to the traditional scheme. In traditional scheme, each relay simply divides its transmit power equally among all its users.

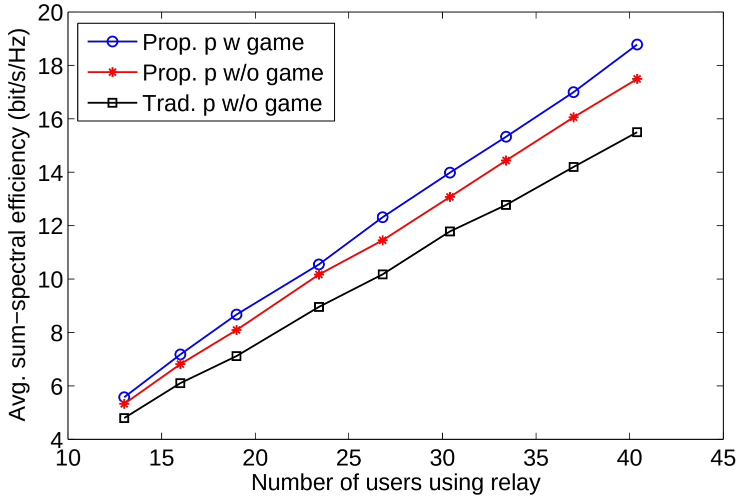

**Figure 4.1:** The average sum-spectral efficiency of the network versus the number of users using relay (number of relays and users are 3 and 60 respectively, and the max. power of each relay is 0.05).

Fig. 4.2 shows the number of users connected to each relay. It shows the effect of applying game on load balancing of the relays. Note that, the first bar of each relay indicates our proposed power allocation scheme with game, the second and third bar of each relay indicate our proposed power allocation scheme without game and traditional scheme, respectively. The figure indicates that, when the game is not applied, it may happen that some relays have more users than other relays which results in having unbalanced load among the relays. By applying the game, users who can connect to uncongested relays join them as opposed to connecting to congested relays. Thus users are evenly distributed among the relays and load balancing is obtained.

Fig. 4.3 depicts the average share of each relay from the sum-spectral efficiency of

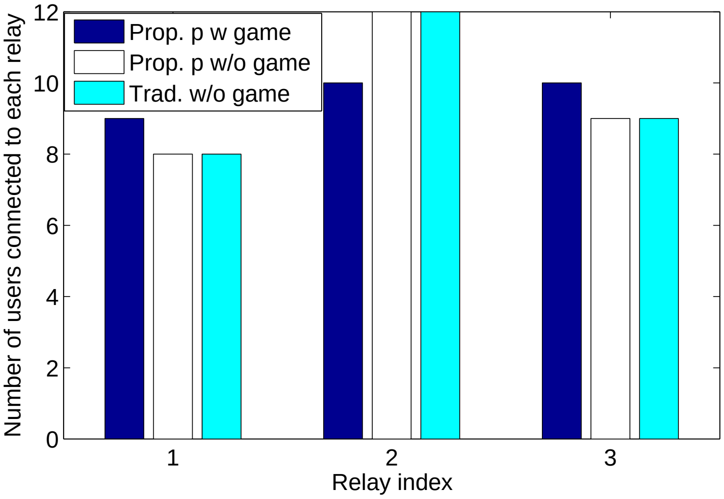

**Figure 4.2:** The number of users connected to each relay versus the relay index (number of relays and users are 3 and 50 respectively, and the max. power of each relay is 0.05).

the network. In each relay, the proposed power allocation with game scheme and traditional scheme obtain the highest and lowest sum-spectral efficiency, respectively. The sum-spectral efficiency is different in different relay since different number of users are connected to each relay.

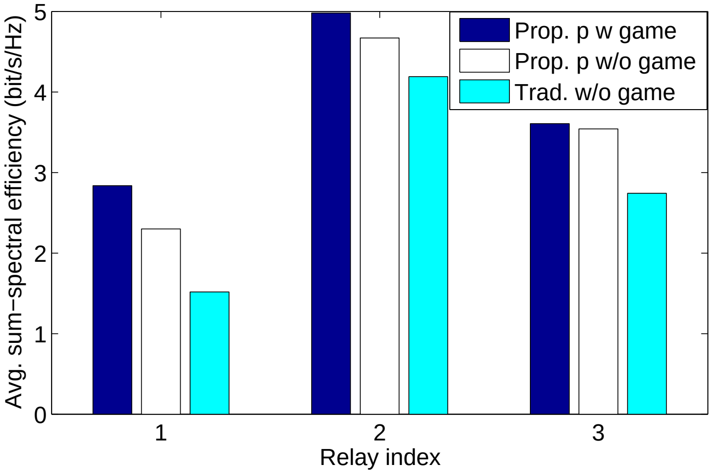

**Figure 4.3:** The average sum-spectral efficiency of each relay versus the relay index (number of relays and users are 3 and 50 respectively, and the max. power of each relay is 0.05).

Fig. 4.4 shows the average spectral efficiency of a user in each relay. The results are obtained by dividing the total sum-spectral efficiency of each relay by its corresponding number of connected users. Since the relay's power is distributed among the connected users more efficiently by applying the power allocation scheme, average sum-spectral efficiency is improved compare to the traditional scheme. In addition, game-based scheme can even more improve the user's average sum-spectral efficiency as less congested coalitions formed for each potentially congested relay and each user is given more share of relay's power. The sum-spectral efficiency is different in each relay since different number of users are connected to each relay.

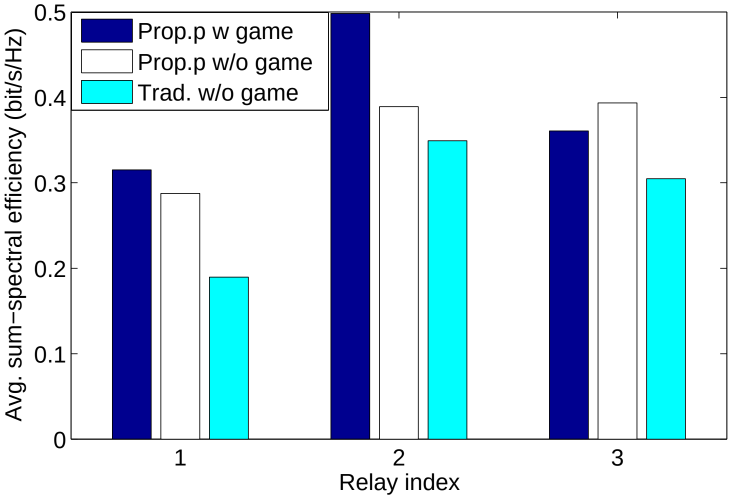

**Figure 4.4:** The average sum-spectral efficiency of each user versus the relay index (number of relays and users are 3 and 50 respectively, and the max. power of each relay is 0.05).

Fig. 4.5 illustrates the average sum-spectral efficiency of the network versus the number of users using relay considering the effect of increasing the relay power. The maximum power of each relay is limited. As the maximum power of each relay increases, the amount of power which can be allocated to each user increases, and the sum-spectral efficiency of the network increases. Fig. 4.5 indicates that instead of increasing the power of each relay, the proposed game-based power allocation scheme can be used to achieve higher average sum-spectral efficiency. This improves the energy efficiency of the network as the resources are scarce and expensive.

Fig. 4.6 shows the effect of applying the proposed power allocation scheme. In this figure by applying game approach, two different power allocation schemes, proposed power allocation scheme and equal power allocation among the users connected to the relay, are

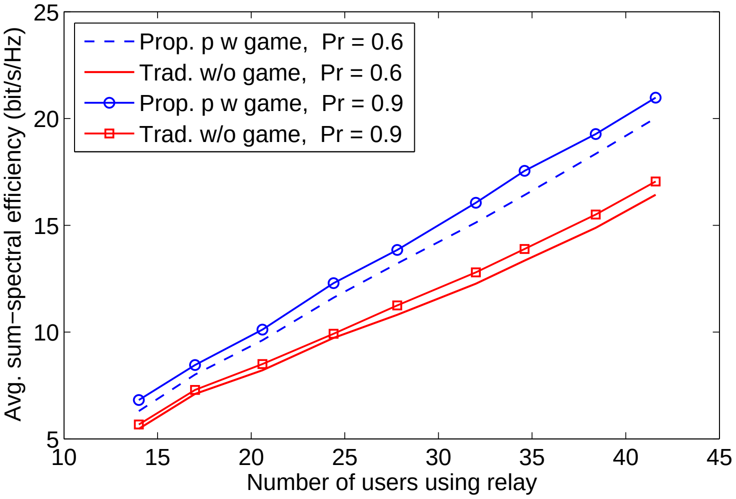

**Figure 4.5:** The average sum-spectral efficiency of the network versus the max. power of each relay (number of relays and users are 3 and 60 respectively, and the max. power of each relay is 0.06, 0.09).

illustrated. As Fig. 4.6 depicts the effect of proposed power allocation is more considerable than equal power allocation.

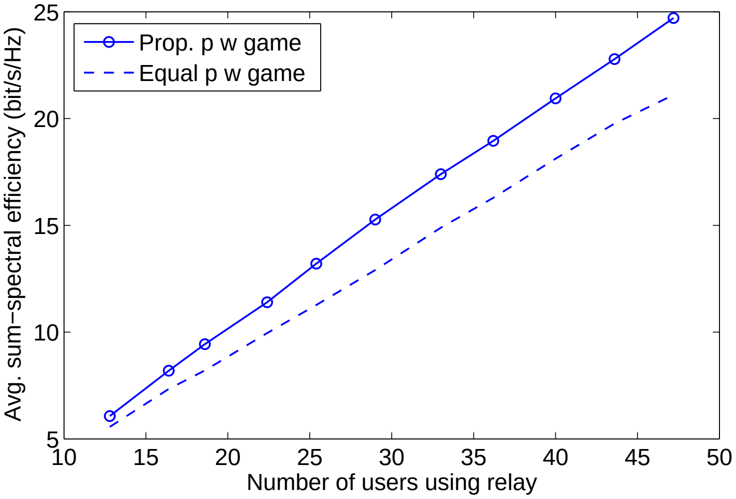

**Figure 4.6:** The average sum-spectral efficiency of the network versus the number of users using relay (number of relays and users are 3 and 70 respectively, and the max. power of each relay is 0.05).

In order to evaluate the effect of the proposed power allocation scheme, in Fig. 4.7, the average sum-spectral efficiency of the network versus the maximum power of each relay is shown for the proposed power allocation scheme, weighted max-min power allocation and traditional scheme, without applying coalitional game. It should be noted that for weighted max-min scheme, demand and weight vector of users should be defined, which are set to be the corresponding $P^*$ and the channel gains, respectively. Moreover, in traditional scheme, each relay divides its power equally among all its users.

As shown in Fig. 4.7, when the maximum power of each relay increases, the amount of power which can be allocated to each user increases, and the sum-spectral efficiency of the network increases.

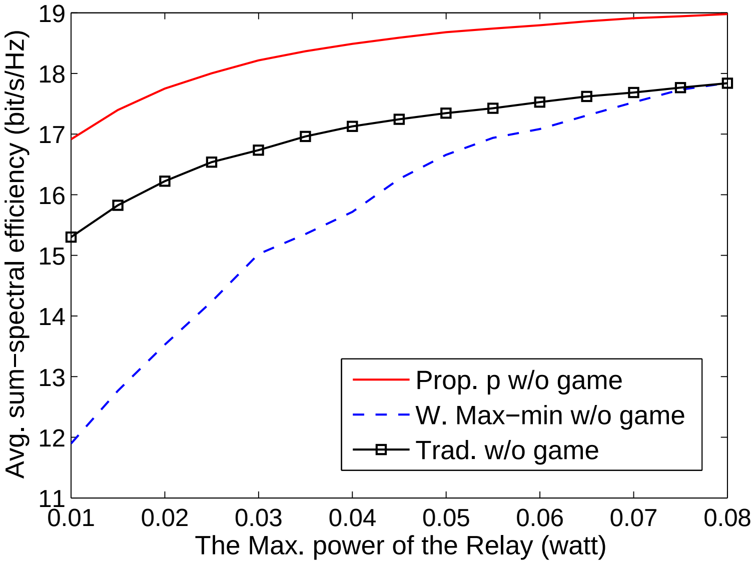

**Figure 4.7:** The average sum-spectral efficiency of the network versus the max. power of each relay (number of relays and users are 4 and 50 respectively, and the max. power of each relay is 0.01-0.08).

As Fig. 4.7 depicts, the *weighted max-min scheme* has the lowest sum-spectral efficiency. It is because, per definition, it helps users with bad channel gain, by maximizing the minimum rate. In addition, the sum-spectral efficiency of the *proposed scheme* is 11% more than that of *traditional scheme*. Note that the sum-spectral efficiency of the *proposed scheme* increases slowly for higher values of the maximum power of each relay. The reason is that, by increasing the maximum power of each relay, every relay can support the total $P^*$ of more users connected to them, up to the point that all users are given $P^*$ and do not need extra power.

Fig. 4.8 shows the average sum-spectral efficiency of the network versus the number of users using relay for the proposed power allocation scheme, weighted max-min power allocation, and traditional scheme, without applying coalitional game. As shown in the figure, the average sum-spectral efficiency increases with increase of the number of users for all schemes. The figure also shows that the *proposed scheme* outperforms the other two, since it allocates power to users more efficiently. In addition, the performance of the *proposed scheme* is significantly better for larger number of users, compared to *weighted max-min scheme*. It is due to the fact that when the number of users using relay increases, the weighted max-min scheme helps more poor users (users with bad channel gains). For example, the difference between the average sum-spectral efficiency of the *proposed scheme* and *weighted max-min scheme* increase from 32% to 71%, when the number of users using relays changes from 15 to 40.

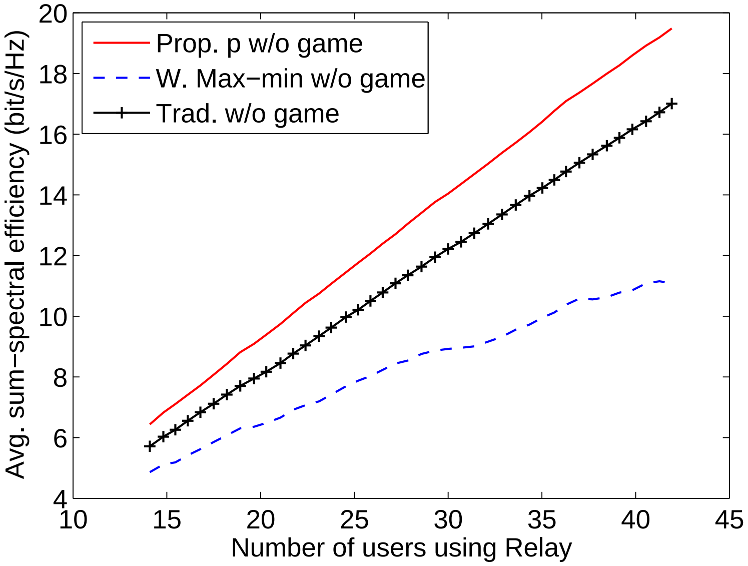

**Figure 4.8:** The average sum-spectral efficiency of the network versus the number of users using relay (number of relay is 3, the maximum power of each relay is 0.05, and number of users in the network is 10-50).

# Chapter 5

# Relay Assignment Scheme in Decode-and-Forward Relay Networks

In Chapter 3, the optimization problem was defined to maximize the total data rate of the network. Since the mentioned *Problem* $P_A$ was a mixed nonlinear integer programming problem, it was difficult to find the optimal solution. Therefore, we exploited the nature of the problem and found an approach to achieve a reasonable performance. In order to simplify the problem, we applied a common relay selection method, where for each user a relay which has the highest channel gain among all available relays is selected. As a result, taking into account the formed network, the problem was solved by applying power (re)allocation scheme, $P_{th}$-check and weighted-based schemes.

In Chapter 4, in order to achieve load balancing among the relays, the optimization *Problem* $P_C$ was defined and solved by applying game theory.

In this chapter, the relay assignment procedure is studied to improve the aforementioned common relay selection method and maximize the data rate of the network. It is taken into account that each relay has a limited power, which should be distributed among the users they support, and each user has to be assigned to a single relay. First, the optimization problem is formulated and the optimal solution is found using Lagrangian Relaxation. Then, a lighter algorithm is proposed to efficiently and quickly carry out the relay assignment with a close-to-optimal performance. Simulation results show that the proposed algorithm can achieve near optimal data rate in the network, while it decreases the processing time significantly.

## 5.1 System Model and Problem Formulation

In this chapter, the same downlink scenario of DF cellular relay network as previous chapter is considered. In the cell, a $BS$ is located at cell center and $K$ fixed relays are located uniformly. There are $N$ users distributed randomly over the cell. The channels between stations are frequency selective and OFDMA is employed to convert the channel into orthogonal subcarriers with flat fading. There is no more than one relay assisting each user. The overall bandwidth $B$ is divided equally among the $N$ users.

As explained in Chapter 3, DF cooperative scheme is used and the user combines the directly received signal from $BS$ and the relayed signal from relay together using MRC. The achievable data rate between $BS$ and user $i$ by help of relay $j$ is as follow

$$DR^{j}_{(BS,i)} = \frac{1}{2} min\{\frac{B}{N} log_2(1 + \frac{P_{BSj}|h_{BS,j}|^2}{BN_0/N}), \frac{B}{N} log_2(1 + \frac{P_{BSi}|h_{BS,i}|^2}{BN_0/N} + \frac{P_{ji}|h_{j,i}|^2}{BN_0/N})\}, \tag{5.1}$$

which can be rewrite as

$$DR^{j}_{(BS,i)} = \frac{1}{2} min[\frac{B}{N} log_2(a_i), \frac{B}{N} log_2(b_i + (c_i * P_{ji}))] \tag{5.2}$$

$$a_i = (1 + \frac{P_{BSi}|h_{BS,j}|^2}{BN_0/N}) \tag{5.3}$$

$$b_i = (1 + \frac{P_{BSi}|h_{BS,i}|^2}{BN_0/N}) \tag{5.4}$$

$$c_i = \frac{|h_{j,i}|^2}{BN_0/N}. \tag{5.5}$$

The data rate of each user $i$ is the minimum between the first and second term. As discussed in Chapter 3, the case where $a_i < b_i$ means that the channel gain between the $BS$ and relay $j$ is worse than the channel gain between the $BS$ and user $i$. Therefore, in this case the direct link is selected and there is no need to use a relay.

In the case where $a_i > b_i$, if we make $(c_i * P_{ji}) > a_i - b_i$, the minimum function will select the first term and the extra given power of the relay $j$ allocated to user $i$ will be wasted. Moreover, if we make $(c * P_{ji}) < a_i - b_i$, the minimum function will select the second term, while it would be possible to select the first term with greater value and maximize the minimum value.

As a result, the best strategy for power distribution of the relay is to let the first term and second term of the Equation (5.2) become equal. The corresponding $P_{ji}$ which let this equality take place, called $P^*_{ji}$. In this case, we can rewrite the Equation (5.1) as

$$DR^j_{(BS,i)} = \frac{B}{N} log_2(1 + \frac{P_{BSi}|h_{BS,i}|^2}{BN_0/N} + \frac{P^*_{ji}|h_{j,i}|^2}{BN_0/N})\}. \tag{5.6}$$

Note that, each relay has a limited power which should be distributed among the users they support. In order to consider how users should be assigned to relays to maximize the data rate of the whole network, the optimization problem can be formalized as

*Problem $P_D$:*

$$\max_x \sum_{j=1}^{K} \sum_{i=1}^{N} x_{ij} DR^j_{(BS,i)} \tag{5.7}$$

$$s.t. \quad \sum_{j=1}^{K} x_{ij} \leq 1 \qquad \forall i \tag{5.8}$$

$$\sum_{i=1}^{N} x_{ij} P^*_{ji} \leq P^{max}_j \qquad P_{ji} > 0, \forall i, j \tag{5.9}$$

where $x_{ij} \in \{0, 1\}$ is a binary variable that indicates whether or not user $i$ is assigned to relay $j$ and it is defined as follow

$$x_{ij} = \begin{cases} 1 & \text{user } i \text{ is assigned to relay } j \\ 0 & \text{otherwise.} \end{cases} \tag{5.10}$$

Relay assignment scheme which determines which users should be connected to each relay, in order to maximize the data rate of the whole network, will be discussed in this chapter. By applying the Lagrangian Relaxation method, we will solve the optimization problem.

## 5.2 Lagrangian Relaxation Problem Formulation

There are $K$ relays and $N$ users in the network. The above mentioned optimization problem can be formulated as

$$Z = max\ cx \tag{5.11}$$

$$s.t. \quad Ax \leq b \tag{5.12}$$

$$Bx \leq d \tag{5.13}$$

$$x_{ij} = \ 0\ or\ 1, \forall ij \tag{5.14}$$

where $x_{ij} \in [0, 1]$ is a binary variable that indicates whether or not user $i$ is assigned to relay $j$. In order to solve the problem, by dualizing the second constraint, the lagrangian relaxation approach can be formulated as

$$Z_D(u) = max\ [cx + u(d - Bx)] \tag{5.15}$$

$$s.t. \quad Ax \leq b \tag{5.16}$$

$$x_{ij} = \ 0\ or\ 1, \forall ij \tag{5.17}$$

where $u$, which is a vector of dual variables can be written as

$$u = [u_1, u_2, ..., u_N]^T, \tag{5.18}$$

and

$$u_i = [u_{i,1}, u_{i,2}, ..., u_{i,K}] \qquad\qquad i = 1, 2, ..., N \tag{5.19}$$

The objective function $z$ is the data rate. The data rate of each relay can be written as

$$DR^j = [DR_1^j, DR_2^j, ..., DR_N^j] \qquad\qquad j = 1, 2, ..., K \tag{5.20}$$

The data rate of all the relays can be shown as

$$c = [DR^1, DR^2, ..., DR^K]. \tag{5.21}$$

$P_j$ is a diagonal matrix which indicates the amount of power that relay $j$ assigns to each user, $e(K)$ is a $K$ column vector of ones and $I(N)$ ia an identity matrix of order $N$ which are shown as

$$\mathbf{P_j} = \begin{bmatrix} P_{j1} & & & & \\ & P_{j2} & & 0 & \\ & & P_{j3} & & \\ & 0 & & \ddots & \\ & & & & P_{jN} \end{bmatrix}, \tag{5.22}$$

$$\mathbf{e(K)} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{N*1}, \tag{5.23}$$

and

$$\mathbf{I(N)} = \begin{bmatrix} 1 & & & \\ & 1 & 0 & \\ & & 1 & \\ & 0 & & \ddots & \\ & & & & 1 \end{bmatrix}_{N*N} \tag{5.24}$$

We can formulate $x_i$, which indicates whether user $i$ is connected to relay $j$ or not as

$$x_i = [x_{i,1}, x_{i,2}, ..., x_{i,K}] \qquad i = 1, 2, ..., N \tag{5.25}$$

$x$ is defined as

$$x = [x_1, x_2, ..., x_N]^T. \tag{5.26}$$

We can rewrite

$$A = e(K)^T \otimes I(N), \tag{5.27}$$

where operator $\otimes$ is the kronecker product. The power of relays, $B$, is

$$B = [P_1, P_2, ..., P_K]. \tag{5.28}$$

We can write $b$ and $d$ as

$$b = e(N), \tag{5.29}$$

and

$$d = [P_1^{max}, P_2^{max}, ..., P_K^{max}]^T. \tag{5.30}$$

In order to make the formulations more clear, an example will be considered in next subsection.

### 5.2.1 Example

In this example, we consider a setup consists of 3 relays and 4 users. To simplify the notations, we can rewrite Equation (5.6) as Equation (5.31), since all $b_i$ and $c_{ij}$ are known parameters.

$$DR^j_{(BS,i)} = \frac{B}{2N} log_2(b_i + c_{ij} P^*_{ji}), \tag{5.31}$$

As a result, Equation (5.7) can be written as

$$\max_{x} \sum_{j=1}^{3} \sum_{i=1}^{4} x_{ij}(log_2(b_i + c_{ij}P^*_{ji})) \tag{5.32}$$

$$s.t. \quad \sum_{j=1}^{3} x_{ij} \leq 1 \qquad \forall i \tag{5.33}$$

$$\sum_{i=1}^{4} x_{ij}P^*_{ji} \leq P_j^{max} \qquad P^*_{ji} > 0, \forall i, j \tag{5.34}$$

so we have

$$\begin{aligned}
max[&x_{11}(log_2(b_1 + c_{11}P^*_{11})) + x_{21}(log_2(b_2 + c_{12}P^*_{12})) \\
&+x_{31}(log_2(b_3 + c_{13}P^*_{13})) + x_{41}(log_2(b_4 + c_{14}P^*_{14})) \\
&+x_{12}(log_2(b_1 + c_{21}P^*_{21})) + x_{22}(log_2(b_2 + c_{22}P^*_{22})) \\
&+x_{32}(log_2(b_3 + c_{23}P^*_{23})) + x_{42}(log_2(b_4 + c_{24}P^*_{24})) \\
&+x_{13}(log_2(b_1 + c_{31}P^*_{31})) + x_{23}(log_2(b_2 + c_{32}P^*_{32})) \\
&+x_{33}(log_2(b_3 + c_{33}P^*_{33})) + x_{43}(log_2(b_4 + c_{34}P^*_{34}))]
\end{aligned} \tag{5.35}$$

and then

$$x_{11} + x_{12} + x_{13} \leq 1 \tag{5.36}$$

$$x_{21} + x_{22} + x_{23} \leq 1 \tag{5.37}$$

$$x_{31} + x_{32} + x_{33} \leq 1 \tag{5.38}$$

$$x_{41} + x_{42} + x_{43} \leq 1 \tag{5.39}$$

which means that, each user at most can be connected to one relay. For the second constraint, the matrices $P_1$, $P_2$, $P_3$, $x$ and $d$ can be written as

$$\mathbf{P_1} = \begin{bmatrix} P^*_{1,1} & 0 & 0 & 0 \\ 0 & P^*_{1,2} & 0 & 0 \\ 0 & 0 & P^*_{1,3} & 0 \\ 0 & 0 & 0 & P^*_{1,4} \end{bmatrix}, \tag{5.40}$$

$$\mathbf{P_2} = \begin{bmatrix} P^*_{2,1} & 0 & 0 & 0 \\ 0 & P^*_{2,2} & 0 & 0 \\ 0 & 0 & P^*_{2,3} & 0 \\ 0 & 0 & 0 & P^*_{2,4} \end{bmatrix}, \tag{5.41}$$

$$\mathbf{P_3} = \begin{bmatrix} P^*_{3,1} & 0 & 0 & 0 \\ 0 & P^*_{3,2} & 0 & 0 \\ 0 & 0 & P^*_{3,3} & 0 \\ 0 & 0 & 0 & P^*_{3,4} \end{bmatrix}, \tag{5.42}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \tag{5.43}$$

and

$$\mathbf{d} = \begin{bmatrix} P_1^{max} \\ P_2^{max} \\ P_3^{max} \end{bmatrix}. \tag{5.44}$$

As a result, $Bx \leq d$ can be formulated as

$$\left[ \begin{bmatrix} P^*_{1,1} & 0 & 0 & 0 \\ 0 & P^*_{1,2} & 0 & 0 \\ 0 & 0 & P^*_{1,3} & 0 \\ 0 & 0 & 0 & P^*_{1,4} \end{bmatrix}, \begin{bmatrix} P^*_{2,1} & 0 & 0 & 0 \\ 0 & P^*_{2,1} & 0 & 0 \\ 0 & 0 & P^*_{2,3} & 0 \\ 0 & 0 & 0 & P^*_{2,4} \end{bmatrix}, \begin{bmatrix} P^*_{3,1} & 0 & 0 & 0 \\ 0 & P^*_{3,2} & 0 & 0 \\ 0 & 0 & P^*_{3,3} & 0 \\ 0 & 0 & 0 & P^*_{3,4} \end{bmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \leq \begin{bmatrix} P_1^{max} \\ P_2^{max} \\ P_3^{max} \end{bmatrix} \tag{5.45}$$

By use of this example, the formulations became more clear. In the next section, in order to solve the optimization problem, the Lagrangian Relaxation approach will be discussed.

## 5.3 Optimal Relay Assignment Scheme

In this section, we analyze the optimal solution for the aforementioned *Problem* $P_D$. As recall from Section 5.1, *Problem* $P_D$ can be written as

*Problem $P_D$:*

$$Z = \max_{x} \sum_{j=1}^{K} \sum_{i=1}^{N} x_{ij} DR^{j}_{(BS,i)} \tag{5.46}$$

$$s.t. \quad \sum_{j=1}^{K} x_{ij} \leq 1 \qquad\qquad \forall i \tag{5.47}$$

$$\sum_{i=1}^{N} x_{ij} P^{*}_{ji} \leq P^{max}_{j} \qquad P_{ji} > 0, \forall i, j \tag{5.48}$$

By applying Lagrangian Relaxation approach and dualizing the constraint (5.48), *Problem $P_D$* is reformulated to the following optimization problem

*Problem $P_E$:*

$$Z_D(u) = max \sum_{j=1}^{K} \sum_{i=1}^{N} x_{ij} DR^{j}_{(BS,i)} + \sum_{j=1}^{K} u_j (\sum_{i=1}^{N} P^{max}_{j} - x_{ij} P^{*}_{ji}) \tag{5.49}$$

$$s.t. \quad \sum_{j=1}^{K} x_{ij} \leq 1 \qquad\qquad \forall i \tag{5.50}$$

where $u_j$ is the Lagrangian multiplier. *Problem $P_E$* can be simplified to

$$Z_D(u) = max \sum_{j=1}^{K} \sum_{i=1}^{N} x_{ij} (DR^{j}_{(BS,i)} - u_j P^{*}_{ji}) + \sum_{j=1}^{K} u_j P^{max}_{j} \tag{5.51}$$

$$s.t. \quad \sum_{j=1}^{K} x_{ij} \leq 1 \qquad\qquad \forall i \tag{5.52}$$

In order to find the solution, the Lagrangian bounds should be obtained.

## 5.4 Lagrangian Bounds

In order to obtain Lagrangian bounds, we need to calculate Lagrangian upper bounds and Lagrangian lower bounds.

### 5.4.1 Lagrangian Upper Bounds

A feasible solution of the *Problem* $P_E$ sets an upper bound for the problem. The known feasible solution of the *Problem* $P_E$ can be defined as $Z^*$, which can be obtained as follows.

The allocated power of relays to users are dedicated such that the data rate is maximized, regardless of the constraint (5.47). Therefore, for each relay $j$, users with highest $p^*_{ji}$ are assigned such that the constraint (5.48) is satisfied. After that, the users assigned to more than one relay are assigned to the relay with the highest $p^*_{ji}$ to fulfil the constraint (5.47).

### 5.4.2 Lagrangian Lower Bounds

By duallizing the constraint (5.48), the lagrangian *Problem* $P_D$ is reformulated as (5.49). The optimal value of the lagrangian problem is a lower bound on the optimal value of the original problem. The Lagrangian multiplier, $u$, can be obtained by the optimal solution to the dual problem

$$Z_D = \min_u Z_D(u) \tag{5.53}$$

Gradients are substituted with subgradients, since $Z_D(u)$ is non-differentiable. A series of $u$ which is the result of subgradient method is calculated by [205]

$$u^{k+1} = \text{max}\{u^k + S_k \Delta u, 0\} \tag{5.54}$$

where $u^0$ is the initial value, $\Delta u = \{\Delta u_j\}$, $\Delta u_j = \sum_{i=1}^{N} P_j^{max} - x_{ij}P^*_{ji}$, $x_{ij}$ is an optimal solution to the lagrangian problem and $S_k$ is a scalar step size.

Per [205], the step size can be practically selected as

$$S_k = \frac{\lambda_k(Z^* - Z_D(u^k))}{\parallel \Delta u \parallel^2} \tag{5.55}$$

where $\{0 < \lambda_k \leq 2 | \lambda_k \in \mathbb{N}\}$. The series of $\lambda_k$ is initially set as $\lambda_k = 2$ and for a specific number of iterations, if $Z_D(u^k)$ does not increase, it is halved.

To obtain the optimal solution, a branch-and-bound algorithm is applied by considering upper and lower bounds. Consider $\hat{\mathscr{X}}$ as the feasible solution. We set $\hat{x}_{ij} = 1 - x_{ij}$ and compute the corresponding lower bound by lagrangian relaxation. Set $x_{ij} = \hat{x}_{ij}$, if the corresponding lower bound is greater than upper bound. During the procedure, if a better feasible solution is found, the upper bound is updated. Then, for each node, a lower bound of the corresponding subproblem is computed. In case the upper bound is greater than lower bound, the node is branched into two nodes by $x_{ij} = 0$ and $x_{ij} = 1$. Otherwise, the node is fathomed.

## 5.5 Relay Assignment Algorithm

In this section, we propose a new relay assignment algorithm as a faster and less complex alternative mechanism to solve the main problem, ***Problem** $P_D$*.

In the following, the procedure of the proposed relay selection algorithm, **Algorithm 5.5.1**, is explained. The first phase is initialization, which is carried out prior to the main algorithm. At the beginning, all users are unassigned to any relay, i.e., $\mathscr{X} = (0)_{N \times K}$, and the $p^*_{ji}$ for each user is a priori known, i.e., $\mathscr{P}^* = (p^*_{ji})_{N \times K}$. Additionally, two sets are defined to show the users that have their final assignments and users that should still be considered for further assignments or reassignments. The prior set is called the set of closed assignments and is denoted as $\mathcal{S}_{\mathcal{A}}$, and the latter, called the set of open assignments, is denoted as $\mathcal{S}_{\mathcal{B}}$. We set $\mathcal{S}_{\mathcal{A}} = \varnothing$ and $\mathcal{S}_{\mathcal{B}} = \{i, \forall i \in \{1, ..., N\}\}$, which means that no assignment is final and all users should be taken into account for the assignment by the algorithm.

The main phase of the algorithm begins with no closed assignments, $\mathcal{S}_{\mathcal{A}} = \varnothing$, and runs until all assignments are final, $|\mathcal{S}_{\mathcal{A}}| = N$. First, the users in $\mathcal{S}_{B}$ are sorted on each relay based on the ascending order of the $p^*_{ji} h^2_{j,i}$, $p^*_{ji} \in \mathscr{P}^*$. For each relay $j$, the users are assigned, $x_{ij} = 1$, based on their orders, while the relay has enough power to allocate to the users, i.e., $\sum_{i \in \mathcal{S}_B} p^*_{ji} x_{ij} \leq P^{max}_j - \sum_{n \in \mathcal{S}_A} p^*_{jn} x_{nj}$. Note that $\sum_{n \in \mathcal{S}_A} p^*_{jn} x_{nj}$ is the total power allocated by relay $j$ to the users in the closed assignment set.

We define the set of over-assigned users as $\mathcal{S}_O$, which are assigned to more than one relay leading to deviation from the constraint (5.47). If $\mathcal{S}_O = \varnothing$, the current assignment meets all constraints and is the final solution. Therefore, the open assignments should change to the closed assignments, i.e, $\mathcal{S}_A \longleftarrow \mathcal{S}_B$. Otherwise, for each $i \in \mathcal{S}_O$, the user with maximum number of assigned relays, $\sum_{j=1}^{K} x_{ij} > \sum_{j=1}^{K} x_{nj}, n \in \{\mathcal{S}_O - i\}$, should release its extra assignments to satisfy the constraint (5.47) and open up more opportunities for other users. If the users have the same number of extra assignments, the user $i$ with the highest $p_{ji}^{*} h_{j,i}^{2}$ among the users in $\mathcal{S}_O$ is selected. Then, the corresponding relay to the maximum $p_{ji}^{*} h_{j,i}^{2}$ is selected, other assignments are dropped, and the user $i$ is added to the set of closed assignments, $\mathcal{S}_A \longleftarrow \{i\}$, and excluded from the reassignment. In other words, $x_{iJ} = 1, J = \underset{j \in \{1,...,K\}}{\text{argmax}}\, p_{ji}^{*}$ and $x_{ij} = 0, j \in \{\{1, ..., K\} - J\}$. The algorithm runs again without considering user $i$ until no open assignment has been left.

**Algorithm 5.5.1** Relay Assignment

```
Initialization:
    • $\mathscr{P}^* = (p^*_{ji})_{N\times K}$.
    • $\mathscr{X} = (0)_{N\times K}$.
    • $\mathcal{S}_{\mathcal{A}} = \varnothing$.
    • $\mathcal{S}_{\mathcal{B}} = \{i, \forall i \in \{1, ..., N\}\}$.
while $|\mathcal{S}_{\mathcal{A}}| < N$ do
    for $j = 1$ to $K$ do
        Sort $\mathcal{S}_B$ based on the ascending order of $p^*_{ji}h^2_{j,i}$, $p^*_{ji} \in \mathscr{P}^*$
        while $\sum_{i\in\mathcal{S}_B} p^*_{ji}x_{ij} \le P^{max}_j - \sum_{n\in\mathcal{S}_A} p^*_{jn}x_{nj}$ do
            $x_{ij} = 1$
        end while
    end for
    for $i \in \mathcal{S}_B$ do
        if $\sum_{j=1}^{K} x_{ij} > 1$ then
            $\mathcal{S}_O \longleftarrow \{i\}$
        end if
    end for
    if $\mathcal{S}_O = \varnothing$ then
        $\mathcal{S}_A \longleftarrow \mathcal{S}_B$
    else
        for $i \in \mathcal{S}_O$ do
            if $\sum_{j=1}^{K} x_{ij} > \sum_{j=1}^{K} x_{nj}, n \in \{\mathcal{S}_O - i\}$ or
            user $i$ has the highest $p^*_{ji}h^2_{j,i}$ among the users in $\mathcal{S}_O$ then
            $x_{iJ} = 1, J = \underset{j\in\{1,...,K\}}{\text{argmax}}\, p^*_{ji}h^2_{j,i}$
            $x_{ij} = 0, j \in \{\{1, ..., K\} - J\}$
            $\mathcal{S}_A \longleftarrow \{i\}$
            end if
        end for
    end if
end while
```

## 5.6 Simulation Results

In this section, we evaluate the performance of the optimal solution which results from Lagrangian Relaxation approach and the proposed relay assignment approach. We consider a wireless DF cellular network with $1Km$ radius. The $BS$ is located at the cell center, fixed relays are located uniformly at $2/3$ of the cell radius and users are distributed randomly. For the channel gain, the fading coefficients are i.i.d. Rayleigh random variables and the standard deviation of zero mean lognormal random variable of shadowing is 4 $dB$. The parameters in the simulations for path loss exponent, noise power, and $\xi$ are equal to 3, -50 $dBm$, and 0.9, respectively. The power of $BS$ is 0.1 $W$ and the total bandwidth is 1 $MHz$. The results are obtained using 100 simulation runs.

In Fig. 5.1, we compare the total spectral efficiency of the network for the proposed relay assignment algorithm and optimal approach. As it is shown in the figure, the total spectral efficiency increases when the number of users increases. The performance of the proposed relay assignment algorithm is close to the optimal approach, since in the proposed relay assignment algorithm, the users are sorted and gain power based on the ascending order of $p^*_{ji}h^2_{j,i}$ and over-assigned users, release its extra assignments wisely.

Optimal approach has a slightly better total spectral efficiency compared to the relay assignment approach, which shows that, the relay assignment approach can be considered without a significant compromise on the total spectral efficiency.

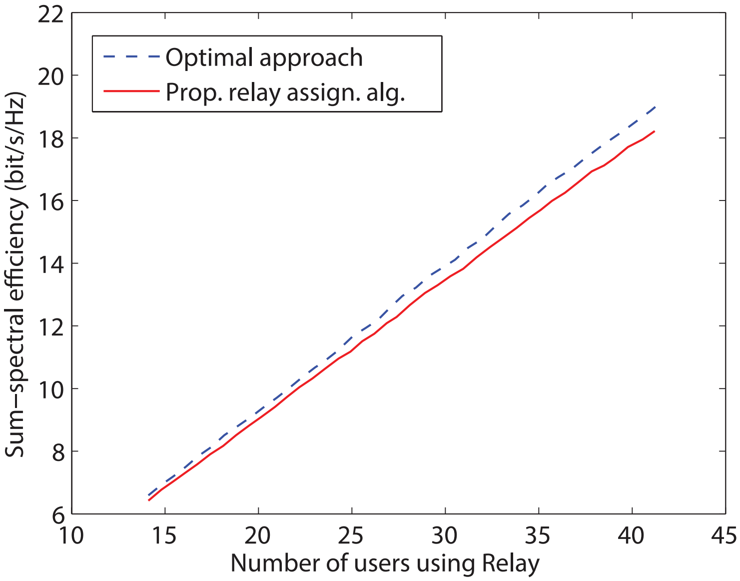

**Figure 5.1:** The sum-spectral efficiency versus the number of users using relay

In Fig. 5.2, the processing time of the optimal approach and the proposed relay assignment algorithm is illustrated. As it shows in the figure, the processing time for the proposed relay assignment algorithm is negligible compared to the optimal approach. The optimal approach takes a considerable time. As a result, the proposed relay assignment algorithm outperforms the optimal solution in terms of computational complexity.

Furthermore, it should be noted that, the proposed relay assignment algorithm can achieve close to optimal performance in total spectral efficiency. As a result, the proposed relay assignment algorithm is a wise choice for applications with limited time.

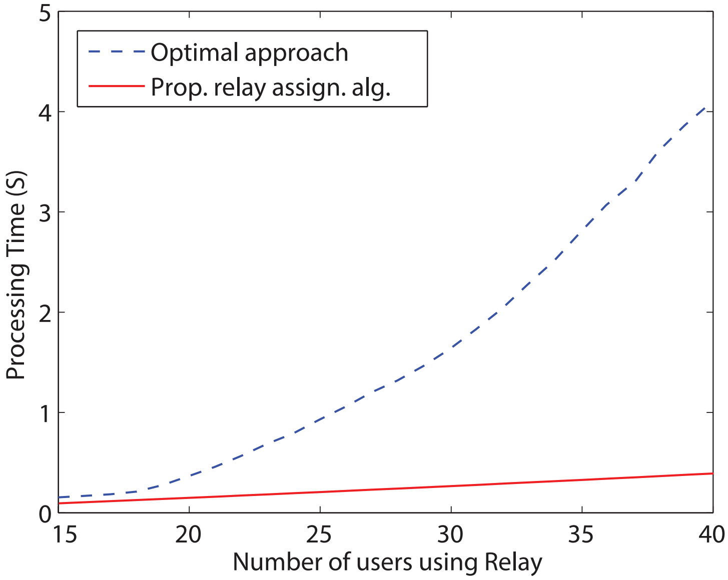

**Figure 5.2:** The processing time versus the number of users using relay

# Chapter 6

# Conclusions and Future Work

In this research, in order to improve the performance of the relay-enhanced cellular networks, an analytical framework for the link-level end-to-end queueing evaluation with ARQ-based error control is studied. After that the physical layer issues, mainly the power allocation, relay load balancing and relay assignment are considered.

First, we have developed a queueing analytical framework for radio link-level performance evaluation of a multi-user wireless relay network. We assume a system consisting of a BS, one typical relay and multiple users in its corresponding sector. The relay only stores packets, which experience failure in the transmission to the BS. The e2e evaluation of this system leads to a tandem queueing problem. To make the analysis tractable, the finite buffer of the relay is decomposed into smaller non-overlapping portions, each corresponding to an individual user's packets (per-user queueing). In each time-slot, only one user is in tandem with the relays buffer with a certain probability. Using the decomposed model, performance measures such as buffer overflow probability, e2e packet loss rate, throughput, and delay have been obtained. The results obtained from the analytical model closely match those from simulations.

After that, since advanced radio resource management schemes are crucial for the future relay-enhanced cellular networks, the power (re)allocation problem is studied. We consider a system with a single base station communicating to multiple users being assisted by multiple relays. Each relay forms a coalition with the users who use this relay. The relays have limited power which must be divided among the users they support in order to maximize the data rate of the whole network. By finding the optimal power requirement of each user, the appropriate amount of power to allocate is obtained. The upper bound solution is proposed based on the *optimal power allocation scheme*, which does not take into account the fairness issue. However, fairness issue is addressed in our proposed *power allocation scheme* by assigning a weight to each user. In order to avoid wasting the power and improving the performance of our proposed scheme, *power reallocation scheme* is applied. By this scheme, some kind of power pooling takes place among users since the extra and unused power of users which could have wasted, is distributed among other users. The simulation results of our proposed scheme show the average amount of improvement of our proposed scheme compared to traditional scheme is more than 10%.

Since users are distributed randomly over the cell, by applying the relay selection scheme, it may happen that some relays have more users than other relays which results in having unbalanced load among the relays. Therefore, to achieve load balancing among relays while considering the total data rate of the network and fairness issue among users as well and with the need for decentralized networks and tools that allow to study the behavior and interactions of the nodes, a game theoretic relay load balancing approach in relay-enhanced cellular networks is considered. A coalition formation game is proposed based on merge-and-split rule to form the optimal structure. The basic idea behind merge-and-split rule is that, given a set of players, any collection of disjoint coalitions can merge into a single coalition, if this new coalition is preferred over the previous state. Similarly, a coalition splits into smaller coalitions if the resulting collection is preferred. The coalition function is attained by determining the payoff and cost function. Relay power allocation scheme is obtained, and coalitional game is formed by applying the pareto order as the comparison relation, and merge-and-split rule. After the termination of the merge-and-split rule, the coalition with the highest coalition value among the formed coalitions is selected. The simulation results demonstrate the effect of applying game theory in proposed problem.

Finally, the relay assignment procedure is studied to improve the aforementioned common relay selection method and maximize the data rate of the network. It is taken into account that each relay has a limited power, which should be distributed among the users they support, and each user has to be assigned to a single relay. First, the optimization problem is formulated and the optimal solution is found using Lagrangian Relaxation. Then, a lighter algorithm is proposed to efficiently and quickly carry out the relay assignment with a close-to-optimal performance. Simulation results show that the proposed algorithm can achieve near optimal data rate in the network, while it decreases the processing time significantly. The following issues and extensions can be considered for the future.

In Chapter 2, the tandem of buffers corresponding to each user and relay is approximated by decomposed tandems to make the problem tractable. It should be noted that, choosing the appropriate buffer allocation ratio for each user is an open problem, which can be investigated in future work. Moreover, the system model can be extended to multi-hop scenarios, multi-rate systems, using adaptive modulation and coding technique increases the transmission rate. By using of finite-state Markov chain (FSMC), more exact channel model which can capture the channel dynamics, can be obtained. In [206], a scheduling scheme is carried out to assign different priorities to the packets in the queue to guarantee the better system throughput. To achieve differentiated service guarantees, authors in [207] assigned different priority values accordingly. In [208–210], the complex interactions between multiple stages are studied. But in such queuing networks, the steady state of each stage is analyzed without considering the impact of congestion at any stage. However, [211] establishes a general multi-stage queuing network model with feedback flow to analyze the behavior of other stages.

In this work we did not consider interference, since considering power allocation, relay load balancing and relay assignment all together with considering interference make the problem too complicated. However, since we proposed power allocation schemes and relay assignment in detail in this work, we try to take the interference into account as well in our future work.

Increasing number of users with various demands and growing number of wireless applications and multimedia services require dynamic adequate bandwidth allocation. Since users have different kind of traffic such as video and voice, different amount of bandwidth is required for each user. This scenario rises an interesting issue in resource allocation, bandwidth allocation, which can be considered in future work. The system can be modeled as a generalized assignment problem in order to assign users to relay while assigning them their requested bandwidth to maximize the data rate of the whole network.

# Appendix A

# The Transition Matrix $\mathrm{P}$ and The Corresponding Transition Probability Block Matrices

The user and the relay have equal buffer size of 5. The maximum achievable capacity for each link is $N = 2$ and the packet arrival rate at the users' buffer is according to a batch Bernoulli process, where $b$ packets arrive in one time frame with probability $a_b$ $(b = 0, 1, 2)$.

$$\mathbf{P} = \begin{bmatrix} A_{0,0} & A_{0,1} & A_{0,2} & & & \\ A_{1,0} & A_{1,1} & A_{1,2} & A_{1,3} & & \\ A_{2,0} & A_{2,1} & A_{2,2} & A_{2,3} & A_{2,4} & \\ & A_{3,1} & A_{3,2} & A_{3,3} & A_{3,4} & A_{3,5} \\ & & A_{4,2} & A_{4,3} & A_{4,4} & A_{4,5} \\ & & & A_{5,3} & A_{5,4} & A_{5,5} \end{bmatrix}, \tag{A.1}$$

where each corresponding transition probability block matrices is as follows

$$\mathbf{A_{0,0}} = \begin{bmatrix} a_0 & & & & & \\ a_0\bar{\beta} & a_0\beta & & & & \\ a_0\bar{\beta}^2 & 2a_0\beta\bar{\beta} & a_0\beta^2 & & & \\ & a_0\bar{\beta}^2 & 2a_0\beta\bar{\beta} & a_0\beta^2 & & \\ & & a_0\bar{\beta}^2 & 2a_0\beta\bar{\beta} & a_0\beta^2 & \\ & & & a_0\bar{\beta}^2 & 2a_0\beta\bar{\beta} & a_0\beta^2 \end{bmatrix}, \tag{A.2}$$

$$\mathbf{A_{0,1}} = \begin{bmatrix} a_1 & & & & & \\ a_1\bar{\beta} & a_1\beta & & & & \\ a_1\bar{\beta}^2 & 2a_1\beta\bar{\beta} & a_1\beta^2 & & & \\ & a_1\bar{\beta}^2 & 2a_1\beta\bar{\beta} & a_1\beta^2 & & \\ & & a_1\bar{\beta}^2 & 2a_1\beta\bar{\beta} & a_1\beta^2 & \\ & & & a_1\bar{\beta}^2 & 2a_1\beta\bar{\beta} & a_1\beta^2 \end{bmatrix}, \tag{A.3}$$

$$\mathbf{A_{0,2}} = \begin{bmatrix} a_2 & & & & & \\ a_2\bar{\beta} & a_2\beta & & & & \\ a_2\bar{\beta}^2 & 2a_2\beta\bar{\beta} & a_2\beta^2 & & & \\ & a_2\bar{\beta}^2 & 2a_2\beta\bar{\beta} & a_2\beta^2 & & \\ & & a_2\bar{\beta}^2 & 2a_2\beta\bar{\beta} & a_2\beta^2 & \\ & & & a_2\bar{\beta}^2 & 2a_2\beta\bar{\beta} & a_2\beta^2 \end{bmatrix}, \tag{A.4}$$

$$\mathbf{A_{1,0}} = \begin{bmatrix} 0 & a_0\bar{\alpha} & & & & \\ 0 & a_0\bar{\alpha}\bar{\beta} & a_0\bar{\alpha}\beta & & & \\ 0 & a_0\bar{\alpha}\bar{\beta}^2 & 2a_0\bar{\alpha}\beta\bar{\beta} & a_0\bar{\alpha}\beta^2 & & \\ 0 & & a_0\bar{\alpha}\bar{\beta}^2 & 2a_0\bar{\alpha}\beta\bar{\beta} & a_0\bar{\alpha}\beta^2 & \\ 0 & & & a_0\bar{\alpha}\bar{\beta}^2 & 2a_0\bar{\alpha}\beta\bar{\beta} & a_0\bar{\alpha}\beta^2 \\ 0 & & & & a_0\bar{\alpha}\bar{\beta}^2 & 2a_0\bar{\alpha}\beta\bar{\beta}+ a_0\bar{\alpha}\beta^2 \end{bmatrix}, \tag{A.5}$$

$$\mathbf{A_{1,1}} = \begin{bmatrix} a_0\alpha & & & & & \\ a_0\alpha\bar{\beta} & a_0\alpha\beta & & & & \\ a_0\alpha\bar{\beta}^2 & 2a_0\alpha\beta\bar{\beta} & a_0\alpha\beta^2 & & & \\ & a_0\alpha\bar{\beta}^2 & 2a_0\alpha\beta\bar{\beta} & a_0\alpha\beta^2 & & \\ & & a_0\alpha\bar{\beta}^2 & 2a_0\alpha\beta\bar{\beta} & a_0\alpha\beta^2 & \\ & & & a_0\alpha\bar{\beta}^2 & 2a_0\alpha\beta\bar{\beta} & a_0\alpha\beta^2 \end{bmatrix} \tag{A.6}$$

$$+ \begin{bmatrix} 0 & a_1\bar{\alpha} & & & & \\ 0 & a_1\bar{\alpha}\bar{\beta} & a_1\bar{\alpha}\beta & & & \\ 0 & a_1\bar{\alpha}\bar{\beta}^2 & 2a_1\bar{\alpha}\beta\bar{\beta} & a_1\bar{\alpha}\beta^2 & & \\ 0 & & a_1\bar{\alpha}\bar{\beta}^2 & 2a_1\bar{\alpha}\beta\bar{\beta} & a_1\bar{\alpha}\beta^2 & \\ 0 & & & a_1\bar{\alpha}\bar{\beta}^2 & 2a_1\bar{\alpha}\beta\bar{\beta} & a_1\bar{\alpha}\beta^2 \\ 0 & & & & a_1\bar{\alpha}\bar{\beta}^2 & 2a_1\bar{\alpha}\beta\bar{\beta}+ a_1\bar{\alpha}\beta^2 \end{bmatrix}, \tag{A.7}$$

$$\mathbf{A_{1,2}} = \begin{bmatrix} a_1\alpha & & & & & \\ a_1\alpha\bar{\beta} & a_1\alpha\beta & & & & \\ a_1\alpha\bar{\beta}^2 & 2a_1\alpha\beta\bar{\beta} & a_1\alpha\beta^2 & & & \\ & a_1\alpha\bar{\beta}^2 & 2a_1\alpha\beta\bar{\beta} & a_1\alpha\beta^2 & & \\ & & a_1\alpha\bar{\beta}^2 & 2a_1\alpha\beta\bar{\beta} & a_1\alpha\beta^2 & \\ & & & a_1\alpha\bar{\beta}^2 & 2a_1\alpha\beta\bar{\beta} & a_1\alpha\beta^2 \end{bmatrix}, \tag{A.8}$$

$$+ \begin{bmatrix} 0 & a_2\bar{\alpha} & & & & \\ 0 & a_2\bar{\alpha}\bar{\beta} & a_2\bar{\alpha}\beta & & & \\ 0 & a_2\bar{\alpha}\bar{\beta}^2 & 2a_2\bar{\alpha}\beta\bar{\beta} & a_2\bar{\alpha}\beta^2 & & \\ 0 & & a_2\bar{\alpha}\bar{\beta}^2 & 2a_2\bar{\alpha}\beta\bar{\beta} & a_2\bar{\alpha}\beta^2 & \\ 0 & & & a_2\bar{\alpha}\bar{\beta}^2 & 2a_2\bar{\alpha}\beta\bar{\beta} & a_2\bar{\alpha}\beta^2 \\ 0 & & & & a_2\bar{\alpha}\bar{\beta}^2 & 2a_2\bar{\alpha}\beta\bar{\beta}+ a_2\bar{\alpha}\beta^2 \end{bmatrix}, \tag{A.9}$$

$$\mathbf{A_{1,3}} = \begin{bmatrix} a_2\alpha & & & & & \\ a_2\alpha\bar{\beta} & a_2\alpha\beta & & & & \\ a_2\alpha\bar{\beta}^2 & 2a_2\alpha\beta\bar{\beta} & a_2\alpha\beta^2 & & & \\ & a_2\alpha\bar{\beta}^2 & 2a_2\alpha\beta\bar{\beta} & a_2\alpha\beta^2 & & \\ & & a_2\alpha\bar{\beta}^2 & 2a_2\alpha\beta\bar{\beta} & a_2\alpha\beta^2 & \\ & & & a_2\alpha\bar{\beta}^2 & 2a_2\alpha\beta\bar{\beta} & a_2\alpha\beta^2 \end{bmatrix}, \tag{A.10}$$

$$\mathbf{A_{2,0}} = \begin{bmatrix} 0 & 0 & a_0\bar{\alpha}^2 & & & & \\ 0 & 0 & a_0\bar{\alpha}^2\bar{\beta} & a_0\bar{\alpha}^2\beta & & & \\ 0 & 0 & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta} & a_0\bar{\alpha}^2\beta^2 & & \\ 0 & 0 & & & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta} & a_0\bar{\alpha}^2\beta^2 \\ 0 & 0 & & & & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta}+ a_0\bar{\alpha}^2\beta\bar{\beta}^2 \\ 0 & 0 & & & & & a_0\bar{\alpha}^2\bar{\beta}^2+ 2a_0\bar{\alpha}^2\beta\bar{\beta}+ a_0\bar{\alpha}^2\beta^2 \end{bmatrix}, \tag{A.11}$$

$$\mathbf{A_{2,1}} = \begin{bmatrix} 0 & 2a_0\alpha\bar{\alpha} & & & & \\ 0 & 2a_0\alpha\bar{\alpha}\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta & & & \\ 0 & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 & & \\ 0 & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 & \\ 0 & & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 \\ 0 & & & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta}+2\, a_0\alpha\bar{\alpha}\beta^2 \end{bmatrix} \tag{A.12}$$

$$+ \begin{bmatrix} 0 & 0 & a_1\bar{\alpha}^2 & & & & \\ 0 & 0 & a_1\bar{\alpha}^2\bar{\beta} & a_1\bar{\alpha}^2\beta & & & \\ 0 & 0 & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta} & a_1\bar{\alpha}^2\beta^2 & & \\ 0 & 0 & & & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta} & a_1\bar{\alpha}^2\beta^2 \\ 0 & 0 & & & & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta}+ a_1\bar{\alpha}^2\beta\bar{\beta}^2 \\ 0 & 0 & & & & & a_1\bar{\alpha}^2\bar{\beta}^2+ 2a_1\bar{\alpha}^2\beta\bar{\beta}+ a_1\bar{\alpha}^2\beta^2 \end{bmatrix}, \tag{A.13}$$

$$\mathbf{A_{2,2}} = \begin{bmatrix} a_0\alpha^2 & & & & & \\ a_0\alpha^2\bar{\beta} & a_0\alpha^2\beta & & & & \\ a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & & & \\ & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & & \\ & & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & \\ & & & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 \end{bmatrix} \tag{A.14}$$

$$+ \begin{bmatrix} 0 & 2a_1\alpha\bar{\alpha} & & & & \\ 0 & 2a_1\alpha\bar{\alpha}\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta & & & \\ 0 & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 & & \\ 0 & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 & \\ 0 & & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 \\ 0 & & & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta}+2\, a_1\alpha\bar{\alpha}\beta^2 \end{bmatrix} \tag{A.15}$$

$$+ \begin{bmatrix} 0 & 0 & a_2\bar{\alpha}^2 & & & & \\ 0 & 0 & a_2\bar{\alpha}^2\bar{\beta} & a_2\bar{\alpha}^2\beta & & & \\ 0 & 0 & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta} & a_2\bar{\alpha}^2\beta^2 & & \\ 0 & 0 & & & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta} & a_2\bar{\alpha}^2\beta^2 \\ 0 & 0 & & & & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta}+ a_2\bar{\alpha}^2\beta\bar{\beta}^2 \\ 0 & 0 & & & & & a_2\bar{\alpha}^2\bar{\beta}^2+ 2a_2\bar{\alpha}^2\beta\bar{\beta}+ a_2\bar{\alpha}^2\beta^2 \end{bmatrix}, \tag{A.16}$$

$$\mathbf{A_{2,3}} = \begin{bmatrix} a_1\alpha^2 & & & & & \\ a_1\alpha^2\bar{\beta} & a_1\alpha^2\beta & & & & \\ a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & & & \\ & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & & \\ & & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & \\ & & & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 \end{bmatrix} \tag{A.17}$$

$$+ \begin{bmatrix} 0 & 2a_2\alpha\bar{\alpha} & & & & \\ 0 & 2a_2\alpha\bar{\alpha}\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta & & & \\ 0 & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 & & \\ 0 & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 & \\ 0 & & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 \\ 0 & & & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta}+2\, a_2\alpha\bar{\alpha}\beta^2 \end{bmatrix} \tag{A.18}$$

$$\mathbf{A_{2,4}} = \begin{bmatrix} a_2\alpha^2 & & & & & \\ a_2\alpha^2\bar{\beta} & a_2\alpha^2\beta & & & & \\ a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & & & \\ & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & & \\ & & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & \\ & & & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 \end{bmatrix} \tag{A.19}$$

$$\mathbf{A_{3,1}} = \begin{bmatrix} 0 & 0 & a_0\bar{\alpha}^2 & & & & \\ 0 & 0 & a_0\bar{\alpha}^2\bar{\beta} & a_0\bar{\alpha}^2\beta & & & \\ 0 & 0 & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta} & a_0\bar{\alpha}^2\beta^2 & & \\ 0 & 0 & & & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta} & a_0\bar{\alpha}^2\beta^2 \\ 0 & 0 & & & & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta}+ a_0\bar{\alpha}^2\beta\bar{\beta}^2 \\ 0 & 0 & & & & & a_0\bar{\alpha}^2\bar{\beta}^2+ 2a_0\bar{\alpha}^2\beta\bar{\beta}+ a_0\bar{\alpha}^2\beta^2 \end{bmatrix}, \tag{A.20}$$

$$\mathbf{A_{3,2}} = \begin{bmatrix} 0 & 2a_0\alpha\bar{\alpha} & & & & \\ 0 & 2a_0\alpha\bar{\alpha}\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta & & & \\ 0 & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 & & \\ 0 & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 & \\ 0 & & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 \\ 0 & & & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta}+2\, a_0\alpha\bar{\alpha}\beta^2 \end{bmatrix} \tag{A.21}$$

$$+ \begin{bmatrix} 0 & 0 & a_1\bar{\alpha}^2 & & & & \\ 0 & 0 & a_1\bar{\alpha}^2\bar{\beta} & a_1\bar{\alpha}^2\beta & & & \\ 0 & 0 & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta} & a_1\bar{\alpha}^2\beta^2 & & \\ 0 & 0 & & & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta} & a_1\bar{\alpha}^2\beta^2 \\ 0 & 0 & & & & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta}+ a_1\bar{\alpha}^2\beta\bar{\beta}^2 \\ 0 & 0 & & & & & a_1\bar{\alpha}^2\bar{\beta}^2+ 2a_1\bar{\alpha}^2\beta\bar{\beta}+ a_1\bar{\alpha}^2\beta^2 \end{bmatrix}, \tag{A.22}$$

$$\mathbf{A_{3,3}} = \begin{bmatrix} a_0\alpha^2 & & & & & \\ a_0\alpha^2\bar{\beta} & a_0\alpha^2\beta & & & & \\ a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & & & \\ & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & & \\ & & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & \\ & & & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 \end{bmatrix} \tag{A.23}$$

$$
+\begin{bmatrix}
0 & 2a_1\alpha\bar{\alpha} & & & & \\
0 & 2a_1\alpha\bar{\alpha}\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta & & & \\
0 & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 & & \\
0 & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 & \\
0 & & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 \\
0 & & & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta}+2\ a_1\alpha\bar{\alpha}\beta^2
\end{bmatrix} \tag{A.24}
$$

$$
+\begin{bmatrix}
0 & 0 & a_2\bar{\alpha}^2 & & & & \\
0 & 0 & a_2\bar{\alpha}^2\bar{\beta} & a_2\bar{\alpha}^2\beta & & & \\
0 & 0 & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta} & a_2\bar{\alpha}^2\beta^2 & & \\
0 & 0 & & & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta} & a_2\bar{\alpha}^2\beta^2 \\
0 & 0 & & & & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta}+\ a_2\bar{\alpha}^2\beta\bar{\beta}^2 \\
0 & 0 & & & & & a_2\bar{\alpha}^2\bar{\beta}^2+\ 2a_2\bar{\alpha}^2\beta\bar{\beta}+\ a_2\bar{\alpha}^2\beta^2
\end{bmatrix}, \tag{A.25}
$$

$$
\mathbf{A_{3,4}} = \begin{bmatrix}
a_1\alpha^2 & & & & & \\
a_1\alpha^2\bar{\beta} & a_1\alpha^2\beta & & & & \\
a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & & & \\
 & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & & \\
 & & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & \\
 & & & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2
\end{bmatrix} \tag{A.26}
$$

$$
+\begin{bmatrix}
0 & 2a_2\alpha\bar{\alpha} & & & & \\
0 & 2a_2\alpha\bar{\alpha}\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta & & & \\
0 & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 & & \\
0 & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 & \\
0 & & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 \\
0 & & & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta}+2\ a_2\alpha\bar{\alpha}\beta^2
\end{bmatrix} \tag{A.27}
$$

$$
\mathbf{A_{3,5}} = \begin{bmatrix}
a_2\alpha^2 & & & & & \\
a_2\alpha^2\bar{\beta} & a_2\alpha^2\beta & & & & \\
a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & & & \\
 & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & & \\
 & & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & \\
 & & & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2
\end{bmatrix} \tag{A.28}
$$

$$
\mathbf{A_{4,2}} = \begin{bmatrix}
0 & 0 & a_0\bar{\alpha}^2 & & & & \\
0 & 0 & a_0\bar{\alpha}^2\bar{\beta} & a_0\bar{\alpha}^2\beta & & & \\
0 & 0 & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta} & a_0\bar{\alpha}^2\beta^2 & & \\
0 & 0 & & & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta} & a_0\bar{\alpha}^2\beta^2 \\
0 & 0 & & & & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta}+\ a_0\bar{\alpha}^2\beta\bar{\beta}^2 \\
0 & 0 & & & & & a_0\bar{\alpha}^2\bar{\beta}^2+\ 2a_0\bar{\alpha}^2\beta\bar{\beta}+\ a_0\bar{\alpha}^2\beta^2
\end{bmatrix}, \tag{A.29}
$$

$$
\mathbf{A_{4,3}} = \begin{bmatrix}
0 & 2a_0\alpha\bar{\alpha} & & & & \\
0 & 2a_0\alpha\bar{\alpha}\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta & & & \\
0 & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 & & \\
0 & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 & \\
0 & & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 \\
0 & & & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta}+2\ a_0\alpha\bar{\alpha}\beta^2
\end{bmatrix} \tag{A.30}
$$

$$+\begin{bmatrix} 0 & 0 & a_1\bar{\alpha}^2 & & & & \\ 0 & 0 & a_1\bar{\alpha}^2\bar{\beta} & a_1\bar{\alpha}^2\beta & & & \\ 0 & 0 & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta} & a_1\bar{\alpha}^2\beta^2 & & \\ 0 & 0 & & & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta} & a_1\bar{\alpha}^2\beta^2 \\ 0 & 0 & & & & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta}+ a_1\bar{\alpha}^2\beta\bar{\beta}^2 \\ 0 & 0 & & & & & a_1\bar{\alpha}^2\bar{\beta}^2+ 2a_1\bar{\alpha}^2\beta\bar{\beta}+ a_1\bar{\alpha}^2\beta^2 \end{bmatrix}, \tag{A.31}$$

$$\mathbf{A_{4,4}} = \begin{bmatrix} a_0\alpha^2 & & & & & \\ a_0\alpha^2\bar{\beta} & a_0\alpha^2\beta & & & & \\ a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & & & \\ & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & & \\ & & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & \\ & & & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 \end{bmatrix} \tag{A.32}$$

$$+\begin{bmatrix} 0 & 2a_1\alpha\bar{\alpha} & & & & \\ 0 & 2a_1\alpha\bar{\alpha}\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta & & & \\ 0 & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 & & \\ 0 & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 & \\ 0 & & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 \\ 0 & & & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta}+2\ a_1\alpha\bar{\alpha}\beta^2 \end{bmatrix} \tag{A.33}$$

$$+\begin{bmatrix} 0 & 0 & a_2\bar{\alpha}^2 & & & & \\ 0 & 0 & a_2\bar{\alpha}^2\bar{\beta} & a_2\bar{\alpha}^2\beta & & & \\ 0 & 0 & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta} & a_2\bar{\alpha}^2\beta^2 & & \\ 0 & 0 & & & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta} & a_2\bar{\alpha}^2\beta^2 \\ 0 & 0 & & & & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta}+ a_2\bar{\alpha}^2\beta\bar{\beta}^2 \\ 0 & 0 & & & & & a_2\bar{\alpha}^2\bar{\beta}^2+ 2a_2\bar{\alpha}^2\beta\bar{\beta}+ a_2\bar{\alpha}^2\beta^2 \end{bmatrix}, \tag{A.34}$$

$$\mathbf{A_{4,5}} = \begin{bmatrix} a_1\alpha^2 & & & & & \\ a_1\alpha^2\bar{\beta} & a_1\alpha^2\beta & & & & \\ a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & & & \\ & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & & \\ & & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & \\ & & & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 \end{bmatrix} \tag{A.35}$$

$$+\begin{bmatrix} 0 & 2a_2\alpha\bar{\alpha} & & & & \\ 0 & 2a_2\alpha\bar{\alpha}\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta & & & \\ 0 & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 & & \\ 0 & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 & \\ 0 & & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 \\ 0 & & & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta}+2\ a_2\alpha\bar{\alpha}\beta^2 \end{bmatrix} \tag{A.36}$$

$$+\begin{bmatrix} a_2\alpha^2 & & & & & \\ a_2\alpha^2\bar{\beta} & a_2\alpha^2\beta & & & & \\ a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & & & \\ & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & & \\ & & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & \\ & & & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 \end{bmatrix} \tag{A.37}$$

$$\mathbf{A_{5,3}} = \begin{bmatrix} 0 & 0 & a_0\bar{\alpha}^2 & & & & \\ 0 & 0 & a_0\bar{\alpha}^2\bar{\beta} & a_0\bar{\alpha}^2\beta & & & \\ 0 & 0 & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta} & a_0\bar{\alpha}^2\beta^2 & & \\ 0 & 0 & & & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta} & a_0\bar{\alpha}^2\beta^2 \\ 0 & 0 & & & & a_0\bar{\alpha}^2\bar{\beta}^2 & 2a_0\bar{\alpha}^2\beta\bar{\beta}+ a_0\bar{\alpha}^2\beta\bar{\beta}^2 \\ 0 & 0 & & & & & a_0\bar{\alpha}^2\bar{\beta}^2+ 2a_0\bar{\alpha}^2\beta\bar{\beta}+ a_0\bar{\alpha}^2\beta^2 \end{bmatrix}, \tag{A.38}$$

$$\mathbf{A_{5,4}} = \begin{bmatrix} 0 & 2a_0\alpha\bar{\alpha} & & & & \\ 0 & 2a_0\alpha\bar{\alpha}\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta & & & \\ 0 & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 & & \\ 0 & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 & \\ 0 & & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta} & 2a_0\alpha\bar{\alpha}\beta^2 \\ 0 & & & & 2a_0\alpha\bar{\alpha}\bar{\beta}^2 & 4a_0\alpha\bar{\alpha}\beta\bar{\beta}+2\ a_0\alpha\bar{\alpha}\beta^2 \end{bmatrix} \tag{A.39}$$

$$+ \begin{bmatrix} 0 & 0 & a_1\bar{\alpha}^2 & & & & \\ 0 & 0 & a_1\bar{\alpha}^2\bar{\beta} & a_1\bar{\alpha}^2\beta & & & \\ 0 & 0 & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta} & a_1\bar{\alpha}^2\beta^2 & & \\ 0 & 0 & & & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta} & a_1\bar{\alpha}^2\beta^2 \\ 0 & 0 & & & & a_1\bar{\alpha}^2\bar{\beta}^2 & 2a_1\bar{\alpha}^2\beta\bar{\beta}+ a_1\bar{\alpha}^2\beta\bar{\beta}^2 \\ 0 & 0 & & & & & a_1\bar{\alpha}^2\bar{\beta}^2+ 2a_1\bar{\alpha}^2\beta\bar{\beta}+ a_1\bar{\alpha}^2\beta^2 \end{bmatrix}, \tag{A.40}$$

$$\mathbf{A_{5,5}} = \begin{bmatrix} a_0\alpha^2 & & & & & \\ a_0\alpha^2\bar{\beta} & a_0\alpha^2\beta & & & & \\ a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & & & \\ & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & & \\ & & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 & \\ & & & a_0\alpha^2\bar{\beta}^2 & 2a_0\alpha^2\beta\bar{\beta} & a_0\alpha^2\beta^2 \end{bmatrix} \tag{A.41}$$

$$+ \begin{bmatrix} 0 & 2a_1\alpha\bar{\alpha} & & & & \\ 0 & 2a_1\alpha\bar{\alpha}\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta & & & \\ 0 & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 & & \\ 0 & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 & \\ 0 & & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta} & 2a_1\alpha\bar{\alpha}\beta^2 \\ 0 & & & & 2a_1\alpha\bar{\alpha}\bar{\beta}^2 & 4a_1\alpha\bar{\alpha}\beta\bar{\beta}+2\ a_1\alpha\bar{\alpha}\beta^2 \end{bmatrix} \tag{A.42}$$

$$+ \begin{bmatrix} 0 & 0 & a_2\bar{\alpha}^2 & & & & \\ 0 & 0 & a_2\bar{\alpha}^2\bar{\beta} & a_2\bar{\alpha}^2\beta & & & \\ 0 & 0 & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta} & a_2\bar{\alpha}^2\beta^2 & & \\ 0 & 0 & & & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta} & a_2\bar{\alpha}^2\beta^2 \\ 0 & 0 & & & & a_2\bar{\alpha}^2\bar{\beta}^2 & 2a_2\bar{\alpha}^2\beta\bar{\beta}+ a_2\bar{\alpha}^2\beta\bar{\beta}^2 \\ 0 & 0 & & & & & a_2\bar{\alpha}^2\bar{\beta}^2+ 2a_2\bar{\alpha}^2\beta\bar{\beta}+ a_2\bar{\alpha}^2\beta^2 \end{bmatrix}, \tag{A.43}$$

$$+ \begin{bmatrix} a_1\alpha^2 & & & & & \\ a_1\alpha^2\bar{\beta} & a_1\alpha^2\beta & & & & \\ a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & & & \\ & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & & \\ & & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 & \\ & & & a_1\alpha^2\bar{\beta}^2 & 2a_1\alpha^2\beta\bar{\beta} & a_1\alpha^2\beta^2 \end{bmatrix} \tag{A.44}$$

$$
+\begin{bmatrix}
a_2\alpha^2 & & & & & \\
a_2\alpha^2\bar{\beta} & a_2\alpha^2\beta & & & & \\
a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & & & \\
 & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & & \\
 & & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2 & \\
 & & & a_2\alpha^2\bar{\beta}^2 & 2a_2\alpha^2\beta\bar{\beta} & a_2\alpha^2\beta^2
\end{bmatrix} \tag{A.45}
$$

$$
+\begin{bmatrix}
0 & 2a_2\alpha\bar{\alpha} & & & & & \\
0 & 2a_2\alpha\bar{\alpha}\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta & & & & \\
0 & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 & & & \\
0 & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 & & \\
0 & & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta} & 2a_2\alpha\bar{\alpha}\beta^2 & \\
0 & & & & 2a_2\alpha\bar{\alpha}\bar{\beta}^2 & 4a_2\alpha\bar{\alpha}\beta\bar{\beta}+2 & a_2\alpha\bar{\alpha}\beta^2
\end{bmatrix} \tag{A.46}
$$

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# Publications

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[2] **Y. Farazmand**, A.S. Alfa, “A Coalitional Game-Based Relay Load Balancing and Power Allocation in Decode-and-Forward Cellular Relay Networks”, *Wireless Communications and Mobile Computing (WCM)*, Accepted, 2014.

[3] **Y. Farazmand**, A.S. Alfa, “A game theoretic power allocation and relay load balancing in OFDMA-based DF cellular relay networks”, *IEEE Vehicular Technology Conference (VTC)*, pp. 1-6, Las Vegas, USA, Sep. 2013.

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