# Joint Beamforming, Channel and Power Allocation in Multi-User and Multi-Channel Underlay MISO Cognitive Radio Networks

by

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# Abstract

In this thesis, we consider joint beamforming, power, and channel allocation in a multi-user and multi-channel underlay cognitive radio network (CRN). In this system, beamforming is implemented at each SU-TX to minimize the co-channel interference. The formulated joint optimization problem is a non-convex, mixed integer nonlinear programming (MINLP) problem. We propose a solution which consists of two stages. At first, given a channel allocation, a feasible solutions for power and beamforming vectors are derived by converting the problem into a convex form with an introduced optimal auxiliary variable and semidefinite relaxation (SDR) approach. Next, two explicit searching algorithms, i.e., genetic algorithm (GA) and simulated annealing (SA)-based algorithm are proposed to determine optimal channel allocations. Simulation results show that beamforming, power and channel allocation with SA (BPCA-SA) algorithm achieves a close optimal sum-rate with a lower computational complexity compared with beamforming, power and channel allocation with GA (BPCA-GA) algorithm. Furthermore, our proposed allocation scheme shows significant improvement than zero-forcing beamforming (ZFBF).

# Acknowledgements

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# Chapter 1

# Introduction

## 1.1 General Background and Motivations

During last few decades, wireless communication concept has been the prevailing factor in communication industry. In fact, it brought a great deal of benefits over the conventional wired communication. However, wireless communication is confined to a limited number of spectrum bands. Higher frequencies are not advisable to use for the wireless communication because the theory suggests that the median attenuation of the desired signal tends to be more significant [2]. On the other hand, wavelength becomes higher as we decrease the frequency. Hence, lower frequencies are not affordable to the users because the requirement of the antenna size is not practical anymore [1]. Furthermore, the United States frequency allocation chart [3] confirms the above idea and the actual range of spectrum usage in wireless communication is between the stamps of $300$kHz to $300$GHz. Therefore, spectrum is a handful amount of resource, which has to be managed carefully. In fact, the chart further reveals that most of the frequencies have multiple allocations, which actually makes a statement about the current spectrum scarcity. Fixed spectrum allocation to the license users has been the key issue for this problem. Dynamically increasing demand for the spectrum develops the situation even worse with such spectrum allocation. The report [4] published by the Federal Communication Commission (FCC) in 2002 pointed out that $15 \sim 85\%$ of the overall spectrum in most of the time remains underutilized with current spectrum allocation schemes. They believe that the spectrum access problem can be effectively solved by changing and introducing new policies to the existing technologies. Thus, introduction of new technologies are required to manage the spectrum access and spectrum sharing between the users to improve the spectrum utilization and satisfy the user requirements.

In 1998, Joseph Mitola III proposed a novel concept called cognitive radio (CR) for the first time in the history to overcome the aforementioned challenges in wireless communication. According to the descriptions in [5–7], a cognitive radio is a device, which observes the surrounding operating environment, collects information and intelligently makes decisions while adapting its operating parameters such as frequency, transmission power, etc., to establish a connection with another similar device. In a CR network (CRN), there are two kinds of users: users who have the authority to use the spectrum is known as the primary users (PUs) or license users, and unlicensed users are named as cognitive or secondary users (SUs). However, permitting underutilized spectrum to be accessed and shared among the SUs while preserving the requirement of the PUs is a major challenge for the CRNs. In order to solve this problem, SUs' resources (i.e., transmit power, operating frequency, modulation scheme, etc.,) have to be managed accordingly. The authors in [8] identified three different ways (i.e., *interweave*, *overlay* and *underlay*) of dynamic spectrum access strategies in CRNs for the SUs to establish connections while incorporating the PUs' license spectrum. The interweave method [9] has two phases in each time slot. At the first phase, the SUs perform sensing for a certain period of time to identify any vacant PUs' spectrum holes. All the unoccupied spectrum holes detected at the sensing phase are then successfully accessed by each SU during the second phase or data transmission phase. The overlay strategy [10, 11] employs cooperative concurrent transmission among the PUs and SUs. Initially, the SU uses some part of its resources to relay the PU's communication. After that, the remaining resources are utilized for its own transmission. Therefore, both interweave and overlay strategies exhaust some part of SU's resources (i.e., time and transmit power) in order to allow access to the SUs to initiate communications [12]. However, using underlay communication strategy [13, 14], the SUs are allowed to communicate simultaneously with PUs while maintaining a tolerable interference to the PUs without wasting any of their resources. Hence, this dynamic spectrum access strategy has become an attractive blueprint in CRNs.

The main concern with underlay method is the inevitable interference at both primary user receiver (PU-RX) and secondary user receiver (SU-RX) due to the spectrum reuse. Therefore, managing interference has become a critical issue in underlay CRNs. A common solution was to deploy interference constraints at each receiver to protect them from excess disturbances. However, a more technologically advanced method, which controls the direction of the transmitting signal, was preferred to improve the performances of both networks in order to reduce the effect at the unintended receivers. By exploiting multiple-antennas [15], a signal processing technology called *beamforming* [16] has been introduced to CR for directional signal transmission, so as to successfully mitigate the mutual interference and improve the signal-to-interference-plus noise ratio (SINR) [17]. In addition, they afford other advantages such as antenna gain, diversity gain and spatial multiplexing. Joint beamforming and resource allocation in CRNs have been widely studied in literature to enhance the performance of the secondary network. A comprehensive study about different types of beamforming strategies, various multiple antennas systems, and related works in beamforming and resource allocation are presented in Chapter 2.

The zero-forcing beamforming (ZFBF) [18] is a technique, which completely nullifies the interference among the co-channel users. However, application of ZFBF is confined to limited practical scenarios. Furthermore, since it does not consider the potential interference tolerance at SUs, which in turns results in a degradation on overall achievable sum-rate of the secondary network. Recent studies [19–21] showed that both PU and SU receivers can tolerate some amount of interference. As a result, it is not necessary to null the co-channel interference.

The assumption of a single PU channel used in most of the studies [22–24] reduces the degree of freedom available at the secondary base station (SBS) on channel allocation for SUs. As a result, the secondary network offers limited opportunities, which are shared among the SUs. Therefore, joint beamforming and resource allocation with multiple PU channels were considered in this thesis. Beside the benefits given by the multiple PU channels, we address the problem of optimal channel and power allocation with beamforming among the SUs. Moreover, we identified that there are only few studies, which have been focused on joint optimization of three variables, i.e., power, channel allocation and beamforming vectors.

In literature, most of the works were based on the assumption of a cellular architecture for secondary network. Namely, either SU transmitter or receiver is the secondary base station. However, a base station can only provide reduced coverage and serve certain amount of users adopting the limited number of antennas and resources. In fact, device-to-device (D2D) communication based secondary networks offer greater coverage with spatial diversity, higher data rates, and lower energy consumption because of direct and short distance communications. Furthermore, implementation of such network are much easier compared with the conventional cellular system.

## 1.2 Contributions of the Thesis

In this thesis, we consider a joint transmit beamforming and resource (power and channel) allocation problem in an underlay CRNs. Different from traditional works, we consider a CRN with independent D2D communications among multiple PU and SU pairs. Specifically in our work, beamforming is performed by each SU-TX instead of a single SBS, which promotes the spatial diversity and curtails the interference. Our main objective is to maximize the sum-rate of the secondary network while minimizing the co-channel user interference subject to the constraints of total power budget, interference on PUs and signal-to-interference-plus noise ratio (SINR) requirement of each SU. The formulated joint optimization problem is a non-convex, mixed integer nonlinear programming (MINLP) problem [23], which is NP-hard. In order to solve this problem with reasonable computational complexity, a two-stage solution approach is proposed. In the first stage, an iterative algorithm is proposed to determine the feasible beamforming vectors and power allocation for a given channel allocation. After that, two different explicit searching algorithms based on genetic algorithm (GA) [25] and simulated annealing (SA) [26] are proposed to find out the suboptimal channel allocation. The main contributions of this thesis are summarized as follows.

- We consider a multi-channel underlay CR system with the capability of beamforming at each SU-TX to mitigate interference, allow more transmission opportunities, and exploit the benefits of spatial diversity.
- We develop a sum-rate maximization problem to jointly optimize beamforming vectors, power allocation and channel allocation.
- Two different algorithms are proposed to solve the formulated non-convex MINLP problem.
- Simulation results show that the proposed system model outperforms the existing ZFBF model by exploiting interference tolerance capacities, and the proposed algorithms can achieve better performance in terms of sum-rate with lower computation complexity so that they are more suitable for practical applications.

## 1.3 Outline of the Thesis

The organization of this thesis is then filed as follows. Chapter 2 consists of two subsections. During the first subsection, we describe basic fundamental theories behind our research analysis. Following that, other research works similar to our study are listed in the second subsection. An extensive analysis of the proposed optimization problem, i.e., joint beamforming, channel and power allocation in multi-user and multi-channel underlay multiple-input single-output (MISO) CRNs, is presented in Chapter 3. In addition, possible solution approaches, performance comparisons and simulation results are also outlined within the same chapter. Finally, Chapter 4 presents the conclusion and some possible future extensions of this research.

# Chapter 2

# Basic Concepts and Related Works

In this chapter, the fundamental concepts, theories and the related studies behind the wireless communication networks are presented. At the beginning, we explain the developments of the wireless networks, which led to the concept called cognitive radio (CR). The CR capabilities, dynamic spectrum access models and research challenges of the cognitive radio networks (CRNs) are then explained. In addition to that, some preliminary details about multiple antenna systems and beamforming technologies are also pointed out. Last but not least, we provide a comprehensive literature review on related beamforming and resources allocation in CRNs.

## 2.1 Evolution of Wireless Networks

Ever since from the remarkable founding of Guglielmo Marconi, the wireless mobile communication systems changed vastly with outstanding achievements. The *paging system* is identified as the earliest version of the wireless communication system. In fact, it offers one directional communication, called simplex communication between the mobile user (pager) and the public switched telephone network (PSTN). Due to the main drawbacks of high transmit power requirement and no feedback communication, paging system is no longer a favorable communication system. However, most of the shortcomings of the paging system were fulfilled with the innovation of *cordless telephone systems*. The PSTN was connected with a dedicated base station and, which is then connected with the user through a wireless link. This affords two directional (duplex) communication with limited mobility. Later, cordless telephone system were integrated with paging to produce significant performance improvement. However, because of the limited coverage range, such systems were only applicable for small networks. The *cellular telephone system* overcomes many challenges of the previous wireless communication systems. The confined number of frequency spectrums are utilized to accommodate large group of users scattered in vast geographical area. The key feature of such system is that, it partitions the operating environment into small hexagonal geographical areas called *cells* and then reuse the same channel by cell base stations, which are separated with certain distances.

The first time-division-multiple-access (TDMA) based digital cellular telephone system was implemented in 1991 [27]. From then on, research and developments helped the growth of the wireless network systems to be as it today. The analysis in [3, 4] shows that the available spectrum is not enough to satisfy the heterogeneous user requirements. Moreover, the spectrum bands have been underutilized most of the time. Hence, emerging technologies are required to satisfy the increasing spectrum access demand and enhance the utilization of the existing spectrum bands. As a result, a concept called *cognitive radio (CR)* was introduced to the wireless community.

## 2.2 Introduction to Cognitive Radio Networks

In general, the available spectrum bands in wireless communication networks are confined to few usable frequencies [3]. Therefore, once different user-requirements increases, the access possibility of the spectrum bands for each user becomes limited. Furthermore, recent observation about the spectrum utilization revealed that most of the spectrum bands are underutilized. For instance, the FCC report [4] in 2002 mentioned that $15 \sim 85\%$ of the spectrum are underutilized. To overcome these spectrum scarcity problem, Joseph Mitola

III innovated a concept called CR in 1998. The idea was to unfold software defined radio platforms for future mobile radio communication in wireless networks [28]. According to the Haykin's definition in [5],

*"Cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment (i.e., outside world), and uses the methodology of understanding-by-building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters (e.g., transmit-power, carrier-frequency, and modulation strategy) in real-time, with two primary objectives in mind: highly reliable communication whenever and wherever needed; efficient utilization of the radio spectrum."*

Thus, a CRN [29] is defined with the correlation of several CR users. The architectural design and capabilities of CRNs make them unique from the other wireless networks.

### 2.2.1 Cognitive Radio Capabilities and Network Architecture

Basically, the CRN architecture is built upon the combination of two networks [30]: *primary network* and *secondary network (or cognitive network)*. Furthermore, users who belong to the primary network and secondary network are named as primary users (PUs) and secondary users (SUs) (or cognitive users), respectively.

A primary network has a certain amount of licensed spectrum bands, which are assigned to the PUs. Hence, PUs are the legitimate users who have the authority to access the available spectrum bands whenever they needed. Moreover, a primary base station (PBS) is deployed in the primary network to control PUs' activities. On the other hand, the secondary network users, i.e., SUs, have no specific spectrum band for communication. Thus, they are named as unlicensed users. However, SUs are allowed to access the licensed spectrum bands without affecting the PUs' communications. In addition, the required level of quality-of-services (QoS) have to be always guaranteed for each PU. Furthermore, as in the primary network, a secondary base station (SBS) is used to dynamically allocate the resources to SUs in a centralized CRN architecture [23, 31].

Specifically, a CRN has three main characteristics as defined in [32]: *cognitive capability, reconfiguration capability* and *self-organization capability*.

- The cognitive capability: In order to avoid interference to the PUs, the SUs first have to detect the unoccupied spectrum bands (or spectrum holes) of the license spectrum before initiating the transmission. For that, SUs adopt the *spectrum sensing* techniques to timely identify the vacant spectrum band. For example, energy detection based spectrum sensing technique has been the common selection in literature [33] for spectrum hole detection. Furthermore, the available spectrum is shared among the SUs using *spectrum sharing* techniques: *cooperative* or *non-cooperative.* With cooperative spectrum sharing technology, the SBS allocates spectrum to the SUs while associating all sensing information from individual SU. Whereas, in non-cooperative spectrum sharing, each SU accesses the spectrum based on its own sensing information.

- The reconfiguration capability: This describes the ability to alter SUs' operating parameters (i.e., operating frequency, transmit power, modulation scheme, etc.,) based on the observations and adapt to the current operating environment. The operating frequencies of the SUs are dynamically selected to avoid co-channel interference. Furthermore, transmit power of each SU is adjusted appropriately to reduce the effect at the PU and SU receivers. Moreover, the QoS level of each secondary user receiver (SU-RX) is maintained by employing proper coding and modulation schemes.

- The self-organization capability: The optimal setting of the operating parameters are determined by accommodating the self-organized capability of the CRN. It mainly consists of two forms: *radio resource management* and *mobility/connection management*. The former is about managing the resources of the CRN. For example, the detected spectrum holes can be different in space and time. Therefore, we have to manage them among the SUs while considering their required QoS levels and PU

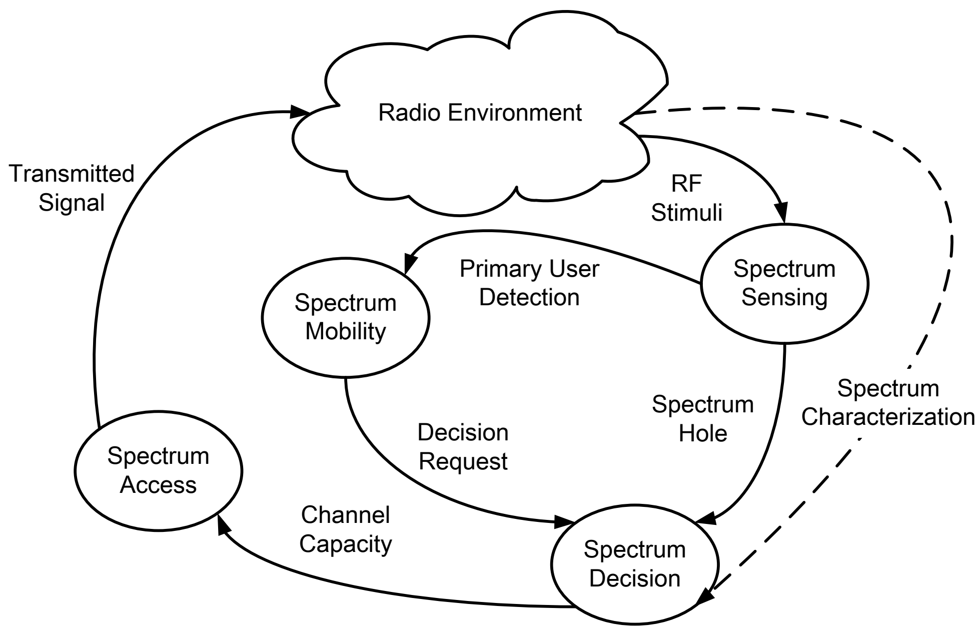


**Figure 2.1:** The cognitive radio cycle.

interference thresholds. However, when a certain PU returns to its frequency band, the SU has to depart from that particular band and has to move to another idle band to continue its transmission. This process is called mobility or connection management.

Using the above three different capabilities of the CRN, we can conclude that each SU is capable of performing four specific functions: *spectrum sensing, spectrum decision, spectrum access* and *spectrum mobility.* The inter-correlation of these functions are depicted in Fig. 2.1 [34]. However, in order to exchange information between the SBS and the SUs, a CRN requires a separate channel. This channel is generally called as a common control channel [35, 36], which is usually employed to perform the initial negotiation between the SBS and SUs before data transmission. Operations such as resource allocation, information exchanging (i.e., channel state information (CSI), sensing information, etc.,) and SU coordination are considered to be completed via this channel.

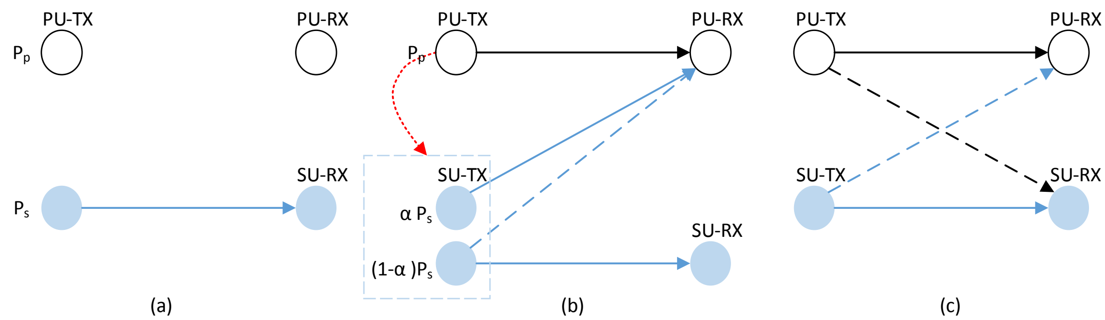


**Figure 2.2:** The three different dynamic spectrum access models, i.e., (a)-interweave dynamis spectrum access model, (b)-overlay dynamis spectrum access model and (c)-underlay dynamis spectrum access model. $P_s$ and $P_p$ are the SU and PU transmit powers, respectively.

### 2.2.2 Cognitive Radio Dynamic Spectrum Access Models

The available spectrum bands are permanently assigned to the PUs. Hence, SUs may not be allowed to use them at their own will. Therefore, they have to find a way to access the license spectrum bands of the PUs. Since, the availability of the spectrum bands are varied in time and space, SUs have to access them dynamically. This spectrum access mechanism is called *dynamic spectrum access* [37]. In fact, its properties help to provide solutions to both excessive spectrum demand requests and spectrum utilization issues in next generation wireless communication networks. Specifically, there are three different dynamic spectrum access models, which are commonly used in CRNs [11]: *interweave, overlay* and *underlay*.

**Interweave dynamic spectrum access model [9]**

This spectrum access model can be employed to avoid interference to the PUs. In other words, each SU uses a fraction of time from its available time slot to detect the occupancy of the PU in that particular spectrum band. Within that time period, they perform sensing, which is a technique (e.g., energy detection) used by SUs to make a decision about the PU's activity (i.e., idle or active) on that specific spectrum band. Thus, the interweave spectrum access model decides access permission of the SUs based on sensing results. For example, a SU admits to a spectrum band if and only if the PU's spectrum is detected as idle and releases it whenever the PU returns. In this way, interference to the PU-RX is avoided. Fig. 2.2-(a) shows the interweave spectrum access when PU is idle. However, due to the imperfect spectrum sensing, there can be two main drawbacks [38], i.e., *false-alarm* and *misdetection*. False-alarm can occur when PU activity is detected as active when it is actually idle. As a result, it will waste possible spectrum access opportunities. Whereas, misdetection causes interference to the PUs since it decides spectrum to be idle even though it is not.

### Overlay dynamic spectrum access model

The overlay dynamic spectrum access model permits SUs to co-exist with the PUs. In fact, each SU associates PU-cooperation to enhance the PU's signal-to-interference-plus-noise ratio (SINR) while appropriately splitting its transmit power between two consecutive transmissions as shown in Fig. 2.2-(b). A fraction of transmit power (i.e., $\alpha \mathrm{P}_s$) is used to transmit the PU's signal. Meanwhile, the remaining part (i.e., $(1-\alpha)\mathrm{P}_s$) is available to transmit SU's own signal. However, $\alpha$ has to be selected appropriately so that overall performance of the PU communication is not degraded. Furthermore, since the PU signal is known at the SU-TX, channel coding [39] can be successfully employed to cancel the interference at the SU-RX.

### Underlay dynamic spectrum access model [13]

Fig. 2.2-(c) depicts the underlay dynamic spectrum access model. This allows PUs and SUs to have concurrent transmissions. Thus, SUs can transmit whenever they wanted apart from being waited for spectrum holes subject to that interference to the PU-RXs is kept under the predefined threshold value. In addition, SU's received signal is also affected by the PU's interference. However, interference tolerance capabilities at the receivers enhance the overall performance of the CRN. Both interweave and overlay schemes use part of their resources (i.e., time and power) on behalf of the PU network for sensing and relaying, but underlay scheme controls its parameters (e.g., power) to reduce interference to the PUs.

### 2.2.3 Research Challenges in Cognitive Radio Networks

In literature, we can find many research, which have been done specifically in CRN to overcome different varieties of challenges. However, still there are some important objectives yet to be solved. In this section, we have listed some of those challenges.

- *Spectrum sensing*: Cooperative sensing can be used to increase the accuracy of the final decision. However, this introduces significant amount of network traffic due to the responses from all the users. Hence, congestion and collisions can be occurred frequently. As a result, the latency of the received information will be higher. Considering those factors, optimal designing of cooperative sensing is still a concern in CRNs. Moreover, energy detection based conventional sensing method is assumed to be synchronized with the PU communication, i.e., time-slotted communication. With asynchronous-sensing, sensing can be performed at whenever and does not want to be waiting for the start of the time slot. However, with energy detection based asynchronous-sensing, PU and SU transmissions cannot be distinguished. Hence, many transmission opportunities can be wasted. Thus, asynchronous sensing with SUs coordination is still a challenge in spectrum sensing.

- *Joint spectrum sensing and access*: Spectrum sensing and spectrum access are two sequential processes which are coupled together. In fact, sensing is performed to detect the PU occupancy in a particular spectrum band. Furthermore, the accuracy of the final decision depends on sensing duration. Whereas, different spectrum access strategies are employed to improve the achievable rates of the secondary network. However, there is a trade-off between the sensing and throughput. Hence, it is still a consideration when designing a system model, which allows random access to the channels while sensing multiple channels at the same time.

- *Distributed resource allocation*: Most of the system models in literature assumed that there exists a central entity (e.g., SBS), which performs all the control operations such as resource allocation. Distributed resource allocation permits each SU to make decisions on its own. However, these decisions may cause interference to the other users. Hence, advanced power control and channel access method have to be developed to support such scenarios.

- *Spectrum mobility management*: Whenever a PU is returned to its operating spectrum band, the existing SU has to stop ongoing transmission and switch to the next available spectrum hole. Thus, to maintain a continuous communication, flexible spectrum handoff and switching delay management is crucial and challenging.

- *Hardware and software architecture*: Conceding the definition of the CR, it is clear that practical development of such device is not simple. Individual development of hardware and software platforms have to be innovative to capture all the theoretical and practical behaviors. Moreover, intercommunication between the hardware and software platforms should be efficient and effective to deliver better outcomes. Thus, robust designing of hardware and software architecture is essential.

## 2.3 Multiple Antenna Systems

The multiple antenna system was proposed to overcome excessive demand for higher data rates and spectrum efficiency, which were unable to achieve with the use of single antenna systems. Furthermore, the known trade-off between the spectrum efficiency (high data rates) and power efficiency (small error rates) [40] for a given bandwidth and transmission power makes it even harder. As a result, the traditional frequency and time domain transmissions are not adequate to support such requirements. Hence, exploiting multiple antenna systems, many challenges have been succeeded in CRNs with the introduction of the new spatial dimension.

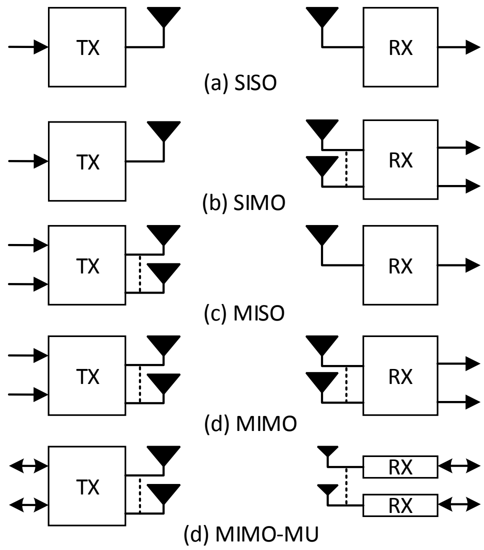


**Figure 2.3:** Antenna cofigurations in wireless systems (Tx:Transmitter, Rx:Receiver).

### 2.3.1 Classification of Multiple Antenna Systems

Fig. 2.3 shows five main types of multiple antenna configurations [15] commonly used in wireless communication networks. The earliest version of antenna configuration is called single-input single-output (SISO) configuration, which consists of single antenna transmitter and receiver. Multiple-input single-output (MISO) configuration has multiple antennas at the transmitter and a single antenna at the receiver. Whereas, the single-input multiple-output (SIMO) has the opposite configuration of MISO. Multiple antennas are deployed both at the transmitter and the receiver in multiple-input multiple-output (MIMO) configuration. Finally, MIMO-Multiuser (MIMO-MU) is used for special scenario where a multiple antenna base station is serving to multiple users with each having one or more antennas.

### 2.3.2 Antenna Array Fundamentals

Each antenna array configuration has specific parameters, which have to be identified properly to understand the performance of the system. Hence, particularly in this subsection, we provide some definitions of antenna parameters.

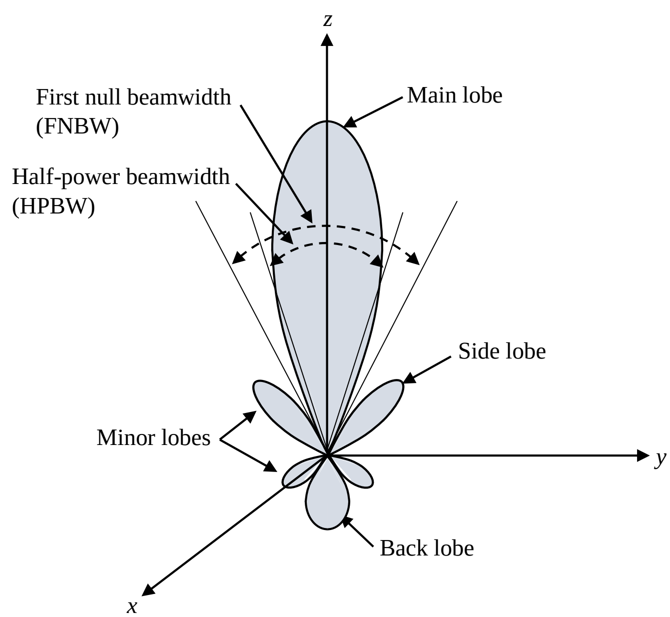


**Figure 2.4:** Characteristics of an antenna radiatio pattern [1].

### Characteristics of an antenna radiation pattern

- An antenna radiation pattern is *"a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates. In most cases, the radiation pattern is determined in the far-field region and is represented as a function of the directional coordinates. Radiation properties include power flux density, radiation intensity, field strength, directivity, phase or polarization."* [41]

A given antenna radiation pattern has some specific elements (i.e., *lobes, main lobe, side lobes, minor lobes* and *nulls*), which are actually used to characterize the radiation pattern. Fig.2.4 [1] illustrates the above mentioned elements more clearly.

- Nulls: The nulls show the almost zero-intensity radiation direction of the antenna radiation pattern.

- Lobe: A lobe is an angular radiation pattern, which has a boundary region defined by the relatively weak radiation intensity, i.e., nulls.

- Main lobe: The lobe with the maximum radiation direction of the antenna radiation pattern.
- Minor lobes: This includes any lobe except the main lobe.
- Side lobes: The unwanted radiation patterns (i.e., close minor lobes), which are in the same hemisphere in the direction of the maximum beam.
- Back lobe: Side lobes, which are in opposite direction to the main lobe.
- Half Power Beamwidth (HPBW) or 3 dB beamwidth: The angular separation of the two points in the main lobe, which has a half-power of the maximum.

**Antenna arrays**

In general, the radiation pattern of an antenna has a wider beamwidth. However, it is not desirable for most of the applications because the effect to the other users is considerably significant. Hence, in order to obtain more directivity in a radiation pattern, we associate multiple antenna elements. Moreover, proper selection of parameters such as geometrical configuration, inter-element spacing and number of antenna elements with synchronized current excitation of the array elements increase the intensity and directivity of the main lobe in the radiation pattern. In addition to that, main lobe can be electronically steered to any desired direction by regulating the phase and/or amplitude of the current excitations. Some examples of antenna array configuration are explained in the following content.

- Linear arrays: Multiple antennas placed in a straight line forms a linear array. However, if all antenna elements are identical and equally spaced, it is then called as a uniform linear array (ULA). The length of the array and the number of antenna elements (i.e., $J$) decide the intensity of the main lobe in a ULA radiation pattern, which is generally at the center and perpendicular to the array. However, it can be steered to an intended direction by controlling the amplitude and phase of each antenna element at the transmitter. Hence, beamforming at the transmitter is called as *transmit beamforming*. The *steering vector* [42] of an antenna array represents the set of phase delays experienced by the signal when it reaches each antenna element. The steering vector for ULA is given by [43]

$$\mathbf{v}(\theta) = [1, \beta, \ldots, \beta^{J-1}]^T \tag{2.1}$$

where $\beta = e^{\iota 2\pi \frac{f_c}{c} d \sin(\theta)}$, $d$ is the spacing between two array elements. $\iota$ is used to represent the imaginary unit and $\theta$ is the direction-of-arrival (DOA) of the desired signal. $f_c$ and $c$ denote the frequency and the wave propagation velocity, respectively.

- Circular arrays: A configuration of identical array elements, which are equally spaced and placed in a circular ring formation, is referred as a uniform circular array (UCA). Unlike ULA, UCA has a major advantage in scanning a beam over a 2D angular direction [44]. Thus, the steering vector of UCA, $\mathbf{v}(\theta, \phi)$ is formulated as [42]

$$\mathbf{v}(\theta, \phi) = [e^{\iota\rho r \sin(\theta)\cos(\phi-\gamma_1)}, \;\; e^{\iota\rho r \sin(\theta)\cos(\phi-\gamma_2)}, \ldots \;\; , e^{\iota\rho r \sin(\theta)\cos(\phi-\gamma_J)}]^T \tag{2.2}$$

where $\gamma_j = 2\pi(j-1)/J, j = 1, \ldots, J$ indicates the angular position of the $j$th array element. The radius of the circular array is given by $r$. $\theta$ and $\phi$ denote the elevation and azimuth angles, respectively, which are used to characterize the DOA of the desired signal. Furthermore, the constant $\rho = (\frac{2\pi f_c}{c})$.

### 2.3.3 Benefits of Multiple Antenna Systems

The benefits given by the multiple antenna systems can be grouped into four main categories [45]: *array gain, spatial diversity, spatial multiplexing* and *interference reduction.*

#### Array gain

In multiple antenna systems, array gain means the average increase in the signal-to-noise ratio (SNR) of a signal at the receiver with respect to the SNR in SISO scenario. In association with the multiple antenna systems, signals can arrive with different phase and amplitudes at the receiver. Thus, taking the coherent combination of such signal helps to improve the SNR at the receiver. In addition, coherent combined effect can be efficiently used at the receiver and/or the transmitter to enhance the performance of the received signal.

**Spatial diversity**

Let's assume we have a SIMO system, which is equipped with one transmit antenna and $J_r$ receive antennas. As a result, at the receiver end, we can have different versions of the same transmitted signal. However, these received copies of the transmitted signal can have variable fluctuations in amplitudes. In other words, these fluctuations of the received signal are generally called as fading. Hence, diversity is determined in terms of number of independent fading branches of the multiple antenna system. For example, a SIMO system has a spatial diversity order of $J_r$. In general, a MIMO with $J_t$ transmit antennas and $J_r$ receive antennas has $J_t J_r$ independent fading links. Thus, its spatial diversity order is determined by $J_t J_r$. In fact, spatial diversity can be used to increase the quality and reliability of the signal reception because combination of different received signal versions help to mitigate fading effect.

**Spatial multiplexing**

Multiple antenna system can facilitate to have different independent data streams without changing the operating bandwidth and the power budget. Hence, it has a linear increase in data rate by a factor given by the minimum of the number of transmit and receive antennas, i.e., min$\{J_t, J_r\}$. Furthermore, spatial multiplexing increases the capacity of the wireless network. However, only the MIMO system can afford spatial multiplexing apart from the MIMO-MU, where a SBS is used to employ independent data streams for each user. In fact, multiple access offered by the later, i.e., MIMO-MU, is known as space-division multiple access (SDMA).

### Interference reduction

The capacity of the wireless network is significantly improved via spectrum reusing. However, inevitable interference to the co-channel users tends to degrade the performance of the received signal. To mitigate such interference, multiple antennas can be effectively employed either at the receiver or transmitter to differentiate desired signal from interference signal while exploiting the new spatial dimension. For example, a signal processing technique called beamforming can be effectively employed with multiple antennas at the transmitter to reduce the disturbance to the co-channel users while directing signal more towards the desired users than others.

## 2.3.4 Overview of Beamforming

The evolution of multiple antenna systems has led to some important signal processing techniques such as *beamforming* [16] to substantially improve the performance of the wireless system. Beamforming exploits the benefits of antenna arrays for directional transmission or reception of a signal, which can be implemented at both receiver and transmitter. Specifically, interference reduction and SINR maximization are some of the key benefits that can be achieved with beamforming [46]. In literature, we can find two main categories of beamforming techniques [47]. They are called *fixed beamforming* and *adaptive beamforming*.

### Fixed beamforming

Fixed beamforming uses a predefined set of beamforming weights and time-delays for each antenna at the transmitter or receiver. The beamforming weights and time-delays are basically calculated while exploiting the location and direction of the desired user. However, there is no correlation between the beamforming weights and the received signal in fixed beamforming. Hence, it can be identified as a multiple fixed beam switching technique.

### Adaptive beamforming

Unlike fixed beamforming, adaptive beamforming technique dynamically computes the beamforming weights based on the properties (i.e., phase, amplitude fading) of the received signal. In other words, this technique tries to determine the optimal beamforming weights in such a way that they increase the quality of the desired received signal and minimize the interference to the other users. For instance, the *least mean square algorithm* [1], *recursive least square algorithm* [48] and *sample matrix inversion* [49] are some of the commonly used adaptive beamforming algorithms in literature.

## 2.4 Beamforming in Cognitive Radio Networks

We already mentioned that there are three dynamic spectrum access models, which are available to the SUs to initiate their communications. However, each dynamic spectrum access model introduces some amount of interference to the PU. Hence, the interference to the PU is inevitable in CRNs and sometimes it can be unintentional. Basically, interweave and overlay dynamic spectrum access models try to minimize the effect up to some extent by employing spectrum sensing and PU-association, respectively. In these kind of situations, SU's resources such as time and power are wasted and hence performance degradation in throughput is also possible. In addition to that, co-channel interference in wireless networks is another drawback due to the reusing of spectrum among the SUs.

In order to overcome those challenges, beamforming technique with multiple antenna system was introduced in CRNs [17]. For example, in hierarchical cellular network, using multiple antennas, SBS performs transmit beamforming to send multiple beams towards different SUs, which are located at various geographical locations. In such way, the energy of the transmitted signals is more concentrated at the intended receivers. As a result interference to the PU and other co-channel SUs are reduced.

Furthermore, *cooperative beamforming* [50] is another type of beamforming method where a particular SU transmitter (i.e., source) uses some SU-relay nodes to send information to a particular SU receiver (i.e., destination). In this case, all users are equipped with single antenna transceivers. However, the SU-relay nodes act as a virtual array of antennas to focus the signal towards the intended SU-destination. Thus, it shows that even without implementation of multiple antennas at each SU, beamforming can be used to mitigate the interference and increase the received signal strength. However, association of the SUs could introduce significant complexity for beamforming in such system configurations.

## 2.5 Related Works on Beamforming and Resources Allocation in CRNs

In this section, we present a general survey about beamforming and resource allocation in CRNs. Specifically, in this survey, we consider resource allocation in CRNs with three different areas in terms of beamforming strategies, number of PU channels and network architectures.

*1) Beamforming in single PU channel CRNs:* In cognitive systems, resource allocation problems, i.e., power and channel allocation, have been widely deployed to increase the achievable sum-rate of the secondary network with simultaneously minimizing interference to the primary network. The authors in [51] considered a joint power and channel allocation problem to maximize the sum-rate of a secondary network with guaranteed protection to primary users. However, the work in [51] tried to control the transmit power of the SUs in order to reduce the interference to the PUs. Different from conventional power control, in [17] beamforming has been successfully adopted in CRNs to enhance SINR at each secondary user receiver (SU-RX) by exploiting the advantages of multiple antenna systems. In literature, joint beamforming and resources allocation have been widely studied for multiple-antenna CRNs. Xie *et al.* in [18] considered a sum-rate maximization problem with beamforming in a single PU channel CRN. In this work, the mutual interference between the SUs are nullified by deploying zero-forcing beamforming (ZFBF). The authors in [52] discussed a joint beamforming and single PU channel assignment problem to maximize the uplink throughput of the CRN while guaranteeing a SINR constraint at each SU-RX and canceling interference at the primary user receiver (PU-RX). However, since ZFBF avoids the potential interference tolerance capabilities at SUs, the overall achievable sum-rate of the secondary network is degraded. The authors in [19, 20] explained that each SU or PU receiver is capable of tolerating some amount of interference, and they further proposed that underlay communication allows SUs to co-exist with PUs as long as the interference to the PU-RX is below the predefined threshold. Therefore, it is not necessary to null the interference all the time. For example, Jiang *et al.* in [22] employed a zero-gradient based iterative approach to determine the local optimal beamforming vectors while maximizing the energy efficiency of the CRN. In [23], beamforming vectors were calculated by using an iterative algorithm based on semidefinite programming to maximize the sum-rate with a total power constraint and co-channel interference constraints at both PU and SU receivers. This work was further extended in [24] by adding an extra quality of service (QoS) constraint. However, the authors in [22–24] only considered a single PU channel CRNs with their problem formulations. As a result, it reduces the degree of freedom available at the SBS on channel allocation for SUs. Therefore, joint beamforming and resource allocation with multiple PU channels were considered.

*2) Beamforming in multiple PU channels CRNs:* In fact, primary user network is not only confined to a single channel. It can employ multiple PU channels to achieve heterogeneous SINR targets at each receiver. Thus, availability of multiple PU channels in secondary network increases the selectivity on channel allocation, while beamforming helps to further reduce the unwanted obstructions to the PUs. Obviously, multiple PU channels access enhances the performance in terms of sum-rate over the single channel CRNs. In [17], a single secondary user transmitter (SU-TX)/SU-RX pair was considered with uniformly distributed primary user transmitters (PU-TXs) and PU-RXs in a circular disc area. Beamforming was implemented by the SU-TX to minimize the interference to the PU-RXs while the received signal strength was maximized at the SU-RX. The authors in [53] considered two beamforming and resource allocation optimization problems, i.e., sum-rate maximization and SINR balancing, in CRNs under the peak power constraints and interference constraints for SUs and PUs, respectively. Authors in [54] presented a framework to minimize the total transmit power of a secondary network subject to some minimum QoS constraints at each SU and interference constraints at each PUs. Gharavol *et al.* in [55] discussed a transmit power minimization problem with a guarantee of SUs' QoS and total power constraints in a multiple channels multiple-input-single-output (MISO) CRN using beamforming. However, in [17, 53–55], the system models were consisted of multiple PUs instead of deploying multiple channels. Thus, the authors in [56] proposed an algorithm based on branch and bound (BnB) method to allocate multiple PU frequency bands with beamforming to serve maximum number of SUs. In addition, in [57], a set of beamforming vectors were determined for each SU to optimally control the power and spectrum bands in order to minimize the interference to the unintended receivers while aiming to maximize the sum-rate of the secondary network.

*3) Network architectures in CRNs:* In literature, we can find different network architectures, which have been proposed for the secondary network. In fact, we can mainly classified them into two categories, i.e., *infrastructure-based CR networks* and *cognitive radio ad-hoc networks (CRAHNs)* [34]. The former consists of a central network entity, which is called as the SBS or access point (AP). A *cellular network CRN* is a common example for the infrastructure-based CR network. On the other hand, the later, i.e., CRAHN, does not have a central unit. Instead, SUs in those networks communicate with each other using the ah-hoc connections. Some works in this area include an adaptive intercell interference cancellation (ICIC) technique for MISO downlink cellular system with channel allocation and beamforming to maximize the weighted sum-rate [58]. Hamdi *et al.* in [59] considered joint beamforming with near-orthogonal user selection method to maximize the downlink throughput of a cellular CRN while subjecting to SINR constraint at each SU, interference constraint at the PU-RX and total power constraint. Authors in [60] presented a joint beamforming with PUs and SUs selection problem to maximize the sum-rate of the entire cellular networks (i.e., both primary and secondary network). In addition, the authors in [22–24] also studied resource allocation and beamforming under the cellular architecture. In [61] the authors discussed a weighted sum-rate maximization problem for ah-hoc networks constrained with per-node (i.e., each SU-TX) transmit power and PU interference. Furthermore, a resource control optimization problem with interference and delay requirements was formulated in [62] with beamforming in an ad-hoc CRN. Different from the traditional architecture, device-to-device (D2D) communication based secondary networks offer many benefits. Improved spectral efficiency, greater coverage with spatial diversity, higher data rates, lower energy consumption and delay are some of the potential advantages due to the direct and short distance communication used in D2D networks. In [63], the authors considered a power allocation problem for CRNs with D2D communications to maximize the data rates of both PU and SU networks without evaluating the effect of interference and beamforming. In [64], joint beamforming and power controlling were studied to minimize the sum power of the primary and D2D networks by subjecting to minimum rate targets both at each PU-RX and SU-RX.

Different from all the existing works, in this thesis, we exploit a joint beamforming, channel and power allocation optimization problem for multi-user multi-channel MISO CRNs. Instead of performing beamforming at a single SBS, we consider beamforming at each SU-TX to mitigate interference and support more transmission opportunities with other benefits. The formulated optimization problem and algorithms for the solution are presented in Chapter 3 with more details.

# Chapter 3

# Problem Formulation - Joint Beamforming, Channel and Power Allocation in Multi-User and Multi-Channel Underlay MISO Cognitive Radio Networks

In this chapter, we consider a joint transmit beamforming and resource allocation problem in an underlay cognitive radio network (CRN). Moreover, different from the traditional system model, beamforming in device-to-device (D2D) based CRN is considered in this work. Furthermore, a sum-rate maximization problem for the secondary network is formulated while minimizing the intra-user interference subject to the constraints of total power budget, interference on PUs and signal-to-interference-plus noise ratio (SINR) requirement of each SU.

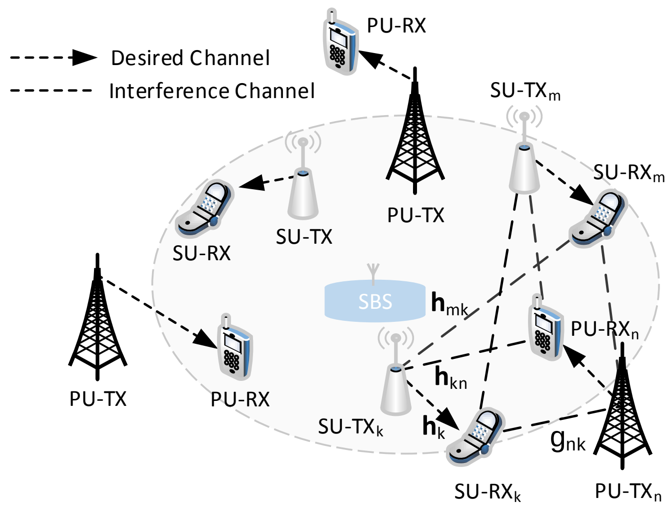


**Figure 3.1:** System model and channel responses.

## 3.1 System model and Problem Formulation

We consider a CRN with $N$ PU transceivers and $K$ SU transceivers randomly distributed in the coverage area of the primary network. Each PU transceiver occupies one separated licensed channel so that there are $N$ PU channels in total. Each SU-TX is equipped with $J$ antennas and its receiver with a single antenna, while each PU (transmitter or receiver) has a single antenna. SU-TXs are allowed to communicate with their corresponding receivers in the underlay manner while satisfying a predefined interference constraint at the corresponding PU-RX. There is a SBS who performs all control functions (e.g., resource allocations) for SUs. An illustration of the system model is shown in Fig. 3.1. We define two sets $\mathcal{S}$ and $\mathcal{P}$ to indicate all possible SU pairs and PU pairs in the network, respectively.

The following notations are used in this chapter. Boldface uppercase and lowercase letters will be used for matrices and vector, respectively. $(\cdot)^{\dagger}, (\cdot)^{T}, E\{\cdot\}$ and $\|\mathbf{x}\|$ denote conjugate transpose, transpose, expectation and Euclidean norm of vector $\mathbf{x}$, respectively. In addition, $\mathrm{Tr}(\mathbf{A})$ indicates the trace operation of a square matrix $\mathbf{A}$. For convenience, Table 3.1 lists some important symbols used in the following sections.

**Table 3.1:** List of Notations

| Symbol | Definition |
|---|---|
| $\mathcal{S}$ | Set of SU pairs |
| $\mathcal{P}$ | Set of PU pairs |
| $\mathcal{S}_n$ | Set of SU pairs using the $n$th PU channel |
| $K$ | Number of SU pairs |
| $N$ | Number of PU channels |
| $s_k/u_n$ | $k$th SU transmit signal / $n$th PU transmit signal |
| $\mathbf{h}_k$ | Channel response between the SU-TX$_k$ and the SU-RX$_k$ |
| $\mathbf{h}_{mk}$ | Channel response between the SU-TX$_m$ and the SU-RX$_k$ |
| $\mathbf{h}_{mn}$ | Channel response between the SU-TX$_m$ and the PU-RX$_n$ |
| $g_{nk}$ | Channel response between the PU-TX$_n$ and the SU-RX$_k$ |
| $g_n$ | Channel response between the PU-TX$_n$ and the PU-RX$_n$ |
| $\eta_k/\eta_n$ | Noise at the SU-RX$_k$/PU-RX$_n$ |
| $\mathcal{Q}_n$ | PU-TX$_n$'s transmit power |
| $\hat{\alpha}_{ki}, \bar{\alpha}_{kn}, \alpha_n, \beta_k$ | Large-scale fading coefficients |
| $\hat{\vartheta}_{ki}, \bar{\vartheta}_{kn}, \vartheta_n, b_k$ | Small-scale fading coefficients |
| $J$ | Number of transmit antennas |
| $\mathbf{\Phi}$ | feasible set of channel allocations |
| $\mathbf{v}_k(\theta, \phi)$ | SU-TX$_k$'s steering vector having a counter-clockwise azimuth angle of $\theta$ and $\phi$ elevation |
| $r_k^n$ | Rate of the $k$th SU-TX on $n$th PU channel |
| $B$ | Transmission bandwidth of each PU channel |
| $I_{th}^n$ | Maximum interference tolerance threshold at the PU-RX$_n$ |
| $\Gamma_k$ | Minimum QoS threshold at the $k$th SU-RX |
| $P_{\max}$ | Power budget of the secondary network |
| $\boldsymbol{\varphi}$ | Auxiliary vector to indicate the intra-user interference thresholds of each SU-RX |
| $\mathbf{w_k}$ | $k$th SU-TX beamforming vector |
| $\mathbf{W_k}$ | $k$th SU-TX positive semidefinite beamforming matrix |
| $x_k^n$ | Binary variable to indicate the $n$th PU channel allocation on $k$th SU-TX |

### 3.1.1 Signal Model

Define transmitted signals from the $k$th SU-TX, i.e., SU-TX$_k$, to its receiver SU-RX$_k$ and from $n$th PU-TX, i.e., PU-TX$_n$, to its receiver PU-RX$_n$ as $s_k$ and $u_n$, respectively. We assume that each modulated transmitted signal has unit energy and is uncorrelated with each other, i.e., $E\{|s_k|^2\} = E\{|u_n|^2\} = 1$, $E\{s_k s_m^\dagger\} = E\{u_n u_d^\dagger\} = 0, \forall k, m \in \mathcal{S}$ and $n, d \in \mathcal{P}$, $k \neq m$ and $d \neq n$. With antenna array, each SU-TX performs transmit beamforming to direct the signal to the intended receiver while at the same time controlling the interference to other users (PUs and SUs). The received signal at the SU-RX$_k$ using the $n$th PU channel can be represented as the aggregation of desired signal, interference from other SU-TXs using the same PU channel for transmission, the interference from the $n$th PU-TX, and noise, i.e.,

$$y_k^n = (\mathbf{w}_k s_k)^\dagger \mathbf{h}_k + \sum_{m \in \mathcal{S}_n, m \neq k} (\mathbf{w}_m s_m)^\dagger \mathbf{h}_{mk} + \sqrt{\mathcal{Q}_n} g_{nk} u_n + \eta_k, \quad k \in \mathcal{S}_n \tag{3.1}$$

where $\mathbf{w}_k$ is the $k$th SU-TX's beamforming vector with a size of $J \times 1$, $\mathcal{Q}_n$ is the transmit power of the $n$th PU-TX, $\mathbf{h}_k$ denotes the $J \times 1$ channel response between the SU-TX$_k$ and SU-RX$_k$, $\mathbf{h}_{mk}$ is the $J \times 1$ channel response between SU-TX$_m$ and SU-RX$_k$, $g_{nk}$ denotes the channel response between PU-TX$_n$ and SU-RX$_k$, and $\mathcal{S}_n$ denotes the set of SU-TXs using the $n$th PU channel. The noise $\eta_k$ is Gaussian distributed with zero mean and variance $\sigma^2$.

Similarly, the received signal at the $n$th PU-RX consists of desired PU signal, co-channel interference from SUs and noise. Thus, it can be formulated as

$$y_n = \sqrt{\mathcal{Q}_n} g_n u_n + \sum_{k \in \mathcal{S}_n} (\mathbf{w}_k s_k)^\dagger \mathbf{h}_{kn} + \eta_n, \quad n = 1, \ldots, N \tag{3.2}$$

where $g_n$ and $\mathbf{h}_{kn}$ are channel responses from the $n$th PU transmitter and the $k$th SU-TX to the $n$th PU-RX, respectively, and $\eta_n$ denotes the noise.

The channel responses $\mathbf{h}_{ki}$, $g_{kn}$ and $g_n$ in (3.1) and (3.2) can be further defined as

$$\mathbf{h}_{ki} = \begin{cases} \mathbf{h}_{ki} = [h_{ki}^1 \ldots h_{ki}^J]^T, \text{if } i \neq k \in \mathcal{S}_n, i \in \mathcal{S}_n \cup \mathcal{P}, & \text{(3.3a)} \\ \beta_k b_k \mathbf{v}_k(\theta, \phi), \text{if } i = k & \text{(3.3b)} \end{cases}$$

$$h_{ki}^l = \hat{\alpha}_{ki} \hat{\vartheta}_{ki}, \; i \neq k \in \mathcal{S}_n, i \in \mathcal{S}_n \cup \mathcal{P} \text{ and } l = 1, \ldots, J \tag{3.4}$$

$$g_{kn} = \alpha_{kn} \vartheta_{kn}, \; k \in \mathcal{S}_n \text{ and } n \in \mathcal{P} \tag{3.5}$$

$$g_n = \bar{\alpha}_n \bar{\vartheta}_n, \; n \in \mathcal{P} \tag{3.6}$$

where $\hat{\alpha}_{ki}, \bar{\alpha}_{kn}, \alpha_n$ and $\beta_k$ denote the large-scale fading, while $\hat{\vartheta}_{ki}, \bar{\vartheta}_{kn}, \vartheta_n$ and $b_k$ represent the small-scale fading which are modeled as Rayleigh distributed random variables.

Consider uniform circular array (UCA) [65] of antenna configuration at each SU-TX. Then, $\mathbf{v}_k(\theta, \phi)$ is the steering vector for the $k$th SU-TX, which indicates the relative responses of the isotropic array elements with respect to the signal impinging at the center of the array from a particular direction. Here, $\phi$ denotes the counter-clockwise azimuth angle measured from X-axis and $\theta$ denotes elevation measured from Z-axis. Thus, the $j$th element of the ideal $J \times 1$ steering vector can be formulated as

$$[\mathbf{v}(\theta, \phi)]_j = e^{\iota r \rho \sin(\theta) \cos(\phi - \gamma_j)}, \quad j = 1, \ldots, J \tag{3.7}$$

where $\gamma_j = 2\pi(j-1)/J$ refers to the angular position of the $j$th array element. The radius of the circular array is given by $r$ and the constant $\rho = (\frac{2\pi f_c}{c})$. In this thesis, we assume that all users are located in the same plane, so that $\theta$ becomes $\frac{\pi}{2}$ and mutual coupling [42] among array elements is not considered.

Similar to [24], we assume that the perfect knowledge of the channel state information (CSI) is available at each SU-TX before transmission.

### 3.1.2 Problem Formulation

Given the $k$th SU transceiver is working on the $n$th PU channel, the SINR at the $k$th SU-RX, denoted as $\mathrm{SINR}_k$, can be represented as

$$\mathrm{SINR}_k = \frac{|\mathbf{w}_k^\dagger \mathbf{h}_k|^2}{\sum\limits_{m \in \mathcal{S}_n, m \neq k} |\mathbf{w}_m^\dagger \mathbf{h}_{mk}|^2 + \mathcal{Q}_n |g_{nk}|^2 + \sigma^2} \tag{3.8}$$

where $|\mathbf{w}_k^\dagger \mathbf{h}_k|^2$ is the desired received signal power at the SU-RX$_k$. $\sum\limits_{m \in \mathcal{S}_n, m \neq k} |\mathbf{w}_m^\dagger \mathbf{h}_{mk}|^2$ denotes the total intra-user interference received at the SU-RX$_k$ from other SU-TXs' who are using the same channel. $\mathcal{Q}_n |g_{nk}|^2$ and $\sigma^2$ are the interference power from PU-TX$_n$ and the noise power at the SU-RX$_k$, respectively.

Let $x_k^n, k = 1, \ldots, K, n = 1, \ldots, N$, denote the channel allocation for SUs. $x_k^n = 1$ means that the $n$th PU channel is assigned to the SU-TX$_k$. Otherwise, $x_k^n = 0$. Then, from

(3.8), the achievable rate of SU pair $k$ on the $n$th PU channel, $r_k^n$, can be calculated as

$$r_k^n = \mathrm{B}\log_2\left[1+\frac{x_k^n|\mathbf{w}_k^{\dagger}\mathbf{h}_k|^2}{\sum\limits_{m\in\mathcal{S}_n, m\neq k}|\mathbf{w}_m^{\dagger}\mathbf{h}_{mk}|^2+\mathcal{Q}_n|g_{nk}|^2+\sigma^2}\right] \tag{3.9}$$

where B is the transmission bandwidth of a single PU channel.

Our major objective is to maximize the sum-rate of the SU network while allowing concurrent transmission with PUs. The optimization problem can be formulated as in *P1*, where $I_{\text{th}}^n$ is the maximum interference tolerance threshold of the $n$th PU-RX. $\Gamma_k$ is the minimum QoS threshold at the $k$th SU-RX. $P_{\max}$ is the available power budget of the secondary network. $\mathbf{w} = [\mathbf{w_1}\ldots\mathbf{w_K}]$ of size $J\times K$ indicates the beamforming weights matrix, where its $k$th column defines the beamforming vector for the SU-TX$_k$. $\mathbf{X}$ is a $K\times N$ channel allocation matrix, which consists of all $x_k^n, k\in\mathcal{S}$ and $n\in\mathcal{P}$.

$$P1: \qquad \max_{\mathbf{w},\mathbf{X}} \sum_{k=1}^{K}\sum_{n=1}^{N}\mathrm{B}\log_2\left[1+\frac{x_k^n|\mathbf{w}_k^{\dagger}\mathbf{h}_k|^2}{\sum\limits_{m\in\mathcal{S}_n, m\neq k}|\mathbf{w}_m^{\dagger}\mathbf{h}_{mk}|^2+\mathcal{Q}_n|g_{nk}|^2+\sigma^2}\right] \tag{3.10}$$

*s.t.*

$$\sum_{k=1}^{K}|\mathbf{w}_k^{\dagger}\mathbf{h}_{kn}|^2x_k^n \leq I_{\text{th}}^n,\ \forall n=1,\ldots,N, \tag{3.11}$$

$$\mathrm{SINR}_k \geq \Gamma_k,\ \forall k=1,\ldots,K, \tag{3.12}$$

$$\sum_{k=1}^{K}\sum_{n=1}^{N}\|\mathbf{w}_k\|^2x_k^n \leq P_{\max} \tag{3.13}$$

$$\sum_{n=1}^{N}x_k^n \leq 1,\ \forall k=1,\ldots,K, \tag{3.14}$$

$$x_k^n \in \{0,1\},\ \forall k\in\mathcal{S},\ \forall n\in\mathcal{P} \tag{3.15}$$

The constraint (3.11) limits the total interference power from SU-TXs to a specific PU-RX below a predefined threshold. Constraint (3.12) guarantees the required SINR level at each SU-RX. Constraint (3.13) keeps the total power consumption under the available power budget. Constraint (3.14) implies that each SU-TX can access at most one PU channel.

Since the power allocation of SU-TX$_k$ is determined by $\|\mathbf{w}_k\|^2$, the problem *P1* in fact integrates beamforming, power allocation and channel allocation.

# Chapter 4

# Solution Approach - Joint Beamforming, Channel and Power Allocation in Multi-User and Multi-Channel Underlay MISO Cognitive Radio Networks

In order to solve the formulated joint beamforming, channel and power allocation optimization NP-hard problem in chapter 3 with reasonable computational complexity, we proposed a two-stage solution approach. In the first stage, the feasible beamforming vectors and power allocations are determined for a given channel allocation. After that, the solution to the channel allocation is determined by proposing two different explicit searching algorithms based on genetic algorithm (GA) [25] and simulated annealing (SA) [26]. Numerical results show that our proposed algorithms can achieve better performance than beamforming in existing system models.

## 4.1 A two-stage solution approach

The problem *P1* consists of continuous and discrete variables, and there are nonlinear terms in both objective function and constraints. It is a non-convex, mixed integer non-linear programming problem, which has been proved to be NP-hard as in [23]. In order to balance performance and computational complexity, in the following a two-stage solution approach is proposed. The idea is to separate the main problem into two sub-problems. In the first sub-problem, the power and beamforming vectors are calculated based on a given channel allocation. After that, the second sub-problem, which determines an optimal channel allocation, will be solved. For the second sub-problem, two algorithms are proposed with different computational complexity.

### 4.1.1 Power and beamforming vector determination based on a given channel allocation

In this section, the beamforming vector and power allocation for each SU-TX will be determined given a channel allocation, $\tilde{\mathbf{X}}$. Given $\tilde{\mathbf{X}}$, constraints (3.11) and (3.13) can be transformed to a summation of quadratic terms and norms so that they become convex. However, the problem is still non-convex because neither the objective function (3.10) nor the constraint (3.12) are convex similar to analysis in [23] and [24], respectively. To overcome this issue, we use semidefinite programming (SDP) approach [66], which allows to express the quadratic terms with some equivalent affine expressions. With SDP, the quadratic terms, $|\mathbf{w}_k^\dagger \mathbf{h}_{kn}|^2$, $|\mathbf{w}_m^\dagger \mathbf{h}_{mk}|^2$ and $\|\mathbf{w}_k\|^2$, can be equivalently represented as

$$|\mathbf{w}_k^\dagger \mathbf{h}_{kn}|^2 = \mathrm{Tr}(\mathbf{w}_k^\dagger \mathbf{h}_{kn} \mathbf{h}_{kn}^\dagger \mathbf{w}_k) = \mathrm{Tr}(\mathbf{W}_k \mathbf{H}_{kn}), \ \ \forall k \in \mathcal{S}, \text{ and } \forall n \in \mathcal{P} \tag{4.1}$$

$$\|\mathbf{w}_k\|^2 = \mathrm{Tr}(\mathbf{W}_k), \ \ \forall k \in \mathcal{S} \tag{4.2}$$

$$|\mathbf{w}_m^\dagger \mathbf{h}_{mk}|^2 = \begin{cases} \mathrm{Tr}(\mathbf{W}_m \mathbf{H}_{mk}), & \forall m \neq k \text{ and } m \in \mathcal{S}_n \\ \mathrm{Tr}(\mathbf{W}_k \mathbf{H}_k), & m = k \end{cases} \tag{4.3}$$

where $\mathbf{W}_k = \mathbf{w}_k\mathbf{w}_k^\dagger$, $\mathbf{H}_{kn} = \mathbf{h}_{kn}\mathbf{h}_{kn}^\dagger$, $\mathbf{H}_k = \mathbf{h}_k\mathbf{h}_k^\dagger$, and $\mathbf{H}_{mk} = \mathbf{h}_{mk}\mathbf{h}_{mk}^\dagger$. From (4.1) - (4.3), we have i) $\mathbf{W}_k$ is a rank one positive semidefinite (PSD) matrix, i.e., $\mathbf{W}_k \succeq 0, \ \forall k$; ii) $\text{Tr}(\mathbf{W}_k\mathbf{H}_{kn}), \text{Tr}(\mathbf{W}_k\mathbf{H}_k)$ and $\text{Tr}(\mathbf{W}_m\mathbf{H}_{mk})$ are all affine expressions with respect to the symmetric PSD matrices $\mathbf{W}_k$ and $\mathbf{W}_m$.

With (4.1), (4.2), (4.3) and the assumption of a known channel allocation, the optimization problem *P1* can be rewritten as

$$\begin{aligned}
\textit{P2}: \qquad & \max_{\mathbf{W}_1,\ldots,\mathbf{W}_K} & & \sum_{k=1}^{K}\sum_{n=1}^{N} r_k^n & (4.4)\\
\textit{s.t.} \quad & \sum_{k=1}^{K} \tilde{x}_k^n \text{Tr}(\mathbf{W}_k\mathbf{H}_{kn}) & \leq \ & I_{\text{th}}^n, \ \forall n = 1,\ldots,N & (4.5)\\
& \text{SINR}_k & \geq \ & \Gamma_k, \ \forall k = 1,\ldots,K & (4.6)\\
& \sum_{k=1}^{K}\sum_{n=1}^{N} \tilde{x}_k^n \text{Tr}(\mathbf{W}_k) & \leq \ & P_{\text{max}}, & (4.7)\\
& \mathbf{W}_k & \succeq \ & 0, \ \forall k = 1,\ldots,K, & (4.8)\\
& \text{Rank}(\mathbf{W}_k) & = \ & 1, \ \forall k = 1,\ldots,K & (4.9)
\end{aligned}$$

where,

$$r_k^n = \text{B}\log_2\left[1 + \frac{\tilde{x}_k^n \text{Tr}(\mathbf{W}_k\mathbf{H}_k)}{\sum_{\substack{m\in\mathcal{S}_n\\ m\neq k}} \text{Tr}(\mathbf{W}_m\mathbf{H}_{mk}) + \mathcal{Q}_n|g_{nk}|^2 + \sigma^2}\right] \tag{4.10}$$

Note that two additional constraints, (4.8) and (4.9), are added in *P2*. Obviously, the former has a convex expression, but the later does not. The constraints (4.5) and (4.7) have been transformed to linear functions without affecting their convexity. The optimization problem *P2* can be further transformed into a convex form by doing the following two steps. First, the non-convex rank constraint of (4.9) can be relaxed by using the SDR approach again [67]. Then, following the similar approach in [23], intra-user interference from other SUs to the SU-RX$_k$, i.e., $\sum_{\substack{m\in\mathcal{S}_n\\ m\neq k}} \text{Tr}(\mathbf{W}_m\mathbf{H}_{mk})$, can be constrained by introducing a new auxiliary variable. After these two steps, the revised optimization problem *P3* can be written as

$$
\begin{aligned}
P3: \quad & \max_{\mathbf{W},\boldsymbol{\varphi}} \quad \sum_{k=1}^{K}\sum_{n=1}^{N} \mathrm{B}\log_2\left[1+\frac{\tilde{x}_k^n \mathrm{Tr}(\mathbf{W}_k\mathbf{H}_k)}{\varphi_k+\mathcal{Q}_n|g_{nk}|^2+\sigma^2}\right] && (4.11)\\
\textit{s.t.} \quad & \text{constraints} \quad (4.5),(4.7),(4.8) \\
& \mathrm{Tr}(\mathbf{W}_k\mathbf{H}_k) \geq \Gamma_k\left(\sum_{\substack{m\in\mathcal{S}_n\\ m\neq k}} \mathrm{Tr}(\mathbf{W}_m\mathbf{H}_{mk})+\mathcal{Q}_n|g_{nk}|^2+\sigma^2\right),\ \forall k\in\mathcal{S}_n && (4.12)\\
\text{and} \quad & \sum_{\substack{m\in\mathcal{S}_n\\ m\neq k}} \mathrm{Tr}(\mathbf{W}_m\mathbf{H}_{mk}) \leq \varphi_k,\ \varphi_k\geq 0,\ \forall k=1,\ldots,K && (4.13)
\end{aligned}
$$

where $\mathbf{W} = [\mathbf{W}_1 \ldots \mathbf{W}_K]$ is a $J \times KJ$ matrix which consists of all the beamforming matrices, and $\boldsymbol{\varphi} = [\varphi_1, \ldots, \varphi_K]^T$ is a $K \times 1$ non-negative auxiliary vector which represents intra-user interference thresholds of all SU-RXs.

Given $\boldsymbol{\varphi}$, the objective function becomes concave since it is just a logarithmic function of an affine expression. In addition, the convexity of the constraint (4.12) can be proven as in [24]. As a result, the problem *P3* turns into a form of convex optimization problem. It is similar to the standard form of SDP, but with a logarithmic rather than a linear objective function. There are many tools available in literature (e.g., CVX [68], a Matlab based modeling software package) to solve such a convex optimization problem. Thus, we only need to determine the optimal value for $\boldsymbol{\varphi}$. Following the similar method in [23], we introduce the following iterative algorithm for *P3*.

(i) *Initialization*: Relax the intra-user interference constraint (4.13) by assigning a feasible non-negative large value for $\boldsymbol{\varphi}$. Then, solve the problem *P3* to find out a feasible value of $\mathbf{W}$, called $\tilde{\mathbf{W}}^{(0)}$. Set the intra-user interference thresholds for each user as $\varphi_k^{(0)} = \sum_{\substack{m\in S_n\\ m\neq k}} \mathrm{Tr}(\tilde{\mathbf{W}}_m^{(0)}\mathbf{H}_{mk}), \forall k \in \mathcal{S}$ and calculate the sum-rate, $R(\tilde{\mathbf{W}}^{(0)}, \boldsymbol{\varphi}^{(0)})$, based on (4.11). Define SU pair index, $k = 1,\ 1 \leq k \leq K$ and the iteration index, $a = 1$.

(ii) *Update*: Update $\boldsymbol{\varphi}^{(a)}$ by $\boldsymbol{\varphi}^{(a)} = \boldsymbol{\varphi}^{(a-1)} - (1-\delta)\varphi_k^{(a-1)} I_k$ where $\delta$ is a fixed step size, $0 < \delta < 1$, and $I_k$ is the $k$th column of an $K \times K$ identity matrix.

(iii) *Iteration results*: Calculate the new value for $R(\tilde{\mathbf{W}}^{(a)}, \boldsymbol{\varphi}^{(a)})$ using $\tilde{\mathbf{W}}^{(a)}$.

(iv) *Check improvements*: Calculate $\Delta R = R(\tilde{\mathbf{W}}^{(a)}, \boldsymbol{\varphi}^{(a)}) - R(\tilde{\mathbf{W}}^{(a-1)}, \boldsymbol{\varphi}^{(a-1)})$. The non-negativity of $\Delta R$ can be proved as in [24]. If $\Delta R$ is greater than a predefined threshold, let $a = a+1$ and repeat steps (ii) and (iii) till it is below a predefined threshold. After that, set $\tilde{\mathbf{W}}^{(a)} = \tilde{\mathbf{W}}^{(a-1)}$, $R(\tilde{\mathbf{W}}^{(a)}, \boldsymbol{\varphi}^{(a)}) = R(\tilde{\mathbf{W}}^{(a-1)}, \boldsymbol{\varphi}^{(a-1)})$ and update the intra-user interference for each user as $\varphi_k^{(a)} = \sum_{\substack{m \in S_n \\ m \neq k}} \mathrm{Tr}(\tilde{\mathbf{W}}_m^{(a)} \mathbf{H}_{mk}),\ \forall k \in \mathcal{S}$.

(v) *Continue iterations and pick the next user*: Set $k = k+1$ and continue steps (ii) - (iv) for the newly selected user.

(vi) *Termination*: If $k > K$, stop the iterations.

At each iteration, the problem *P3* needs to be solved given a channel allocation and $\boldsymbol{\varphi}$. Since most of the convex optimization toolboxes (e.g., CVX) use the interior-point algorithm [69] as a basic solution platform, considering the worst-case scenario as in [67], the computational complexity of a SDR problem *P3* can be expressed as

$$\mathcal{O}(\max\{\xi, \kappa\}^4 \kappa^{(1/2)} \log(1/\epsilon)) \tag{4.14}$$

where $\kappa$ and $\xi$ describe the problem size (i.e., number of PSD matrices) and the number of constraints involved in the optimization problem *P3*, respectively. $\epsilon$ is the given accuracy of the solution defined by the solver. Let $t_T$ denotes the total number of iterations taken by the iterative algorithm to produce a feasible solution for total $K$ SU pairs. Then, the overall complexity to find beamforming and power vectors for a given channel allocation can be derived as

$$\mathcal{O}(\max\{\xi, \kappa\}^4 \kappa^{(1/2)} \log(1/\epsilon)) \times t_T \tag{4.15}$$

Let $\tilde{\mathbf{W}} = [\tilde{\mathbf{W}}_1 \ldots \tilde{\mathbf{W}}_K]$ and $\tilde{\boldsymbol{\varphi}} = [\tilde{\varphi}_1, \ldots, \tilde{\varphi}_K]^T$ be the feasible solutions of the problem *P3*. Then, $\tilde{\mathbf{W}}$ is optimal if and only if the rank of each $\mathbf{W}_k, \forall k \in \mathcal{S}$, is equal to one, i.e., rank($\mathbf{W}_k$) = 1. If such condition is not satisfied, appropriate rank one approximation methods, e.g., eigen-decomposition method [67], can be deployed to get the final solution of $\tilde{\mathbf{W}}$. By using eigen-decomposition method, each beamforming matrix $\mathbf{W}_k$ can be equivalently represented as

$$\mathbf{W}_k = \sum_{j=1}^{J} \lambda_{kj} \mathbf{c}_{kj} \mathbf{c}_{kj}^\dagger \tag{4.16}$$

where $\lambda_{kj}$ and $\mathbf{c}_{kj}$ denote the $j$th eigenvalue and the corresponding eigenvector of the $k$th beamforming matrix $\mathbf{W}_k$, respectively. If $\mathbf{W}_k$ is a rank one matrix, then there exists exactly one non-zero eigenvalue, say $\lambda_{kJ}$. As a result, using (4.16), the $k$th user beamforming matrix $\mathbf{W}_k$ can be written as

$$\begin{aligned} \mathbf{W}_k = \lambda_{kJ} \mathbf{c}_{kJ} \mathbf{c}_{kJ}^\dagger &= (\sqrt{\lambda_{kJ}} \mathbf{c}_{kJ})(\sqrt{\lambda_{kJ}} \mathbf{c}_{kJ})^\dagger \\ &= \mathbf{w}_k \mathbf{w}_k^\dagger \end{aligned} \tag{4.17}$$

Thus, the beamforming vector for the SU-TX$_k$ is,

$$\mathbf{w}_k = \sqrt{\lambda_{kJ}} \mathbf{c}_{kJ} \tag{4.18}$$

### 4.1.2 Finding the optimal channel allocation

In the previous section, we have determined the optimal power and beamforming vectors for a known channel allocation. In order to find the optimal channel allocation, an exhaustive searching algorithm can be used, which needs to compute beamforming vectors, power allocations and sum-rates for all possible channel allocations. With $N$ PU channels and $K$ SU pairs, the searching space size of the exhaustive searching algorithm is $(N+1)^K$, which increases exponentially with $K$. Obviously, this searching method is practically infeasible. Hence, for practical applications, we propose two channel allocation algorithms based on genetic algorithm (GA) [25] and simulated annealing (SA) [70, 71] algorithm to find optimal channel allocations.

### Genetic Algorithm (GA)

GA is a searching algorithm, which can be applied to find out optimal solution to an optimization problem without the knowledge of the objective function's derivatives or any gradient related information. The key idea of GA is to first select a set of feasible values for the decision variables and then design new solutions based on the previous set to improve the objective function [72]. Different from standard GA, in this thesis, we define a $K \times N$ matrix as a chromosome instead of a single string chromosome as in [25], where the $k$th row and $n$th column entry of the chromosome indicates whether the $n$th channel has been allocated to the $k$th SU-TX or not. In fact, a chromosome describes one realization of channel allocation. An example of a chromosome for two PU channels and four SU-TXs can be written as

$$\text{Chromosome} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{4.19}$$

Define $\mathbf{\Phi}$ as the search space, which includes all possible channel allocations, and has a size of $(N+1)^K$. Define the size of each generation as $\nu$. Let $P$ $(0 < P < \nu)$ and $G_{max}$ denote the number of best chromosomes being selected and the maximum number of generations, respectively. The details of GA is as follows.

Initially, $\nu$ chromosomes are randomly selected from $\mathbf{\Phi}$, called the initial generation, $\mathbf{\Phi}^{(0)}$. We define $R^{(i)}(\mathbf{W}, \boldsymbol{\varphi})$, the sum-rate achieved for chromosome $i$, as the fitness function for GA. Then, the chromosomes in $\mathbf{\Phi}^{(0)}$ can be ordered with the descending order of their sum-rates. From the sorted chromosome list ($\mathcal{G}^{(g)}_{sorted}$), the first $P$ chromosomes are selected and put into the set $\mathcal{G}^{(g)}_{best}$, while the last $P$ chromosomes are removed and inserted into the set $\mathcal{G}^{(g)}_{worst}$ at the $g$th generation. The set of remaining chromosomes, $\mathcal{G}^{(g)}_{luckies}$, is named as *luckies*.

Next, a new generation of chromosomes is formed. The new generation consists of

**Algorithm 1** : BPCA-GA

*Initialization*: Given $K, N, \mathbf{\Phi}, \nu, P$ and $\mathrm{G}_{max}$
Start : randomly pick $\nu$ channel realizations from $\mathbf{\Phi}$ to define the initial generation $\mathbf{\Phi}^{(0)}$, $\mathbf{\Phi}^{(0)} \subset \mathbf{\Phi}$
Fitness : find $R^{(i)}(\mathbf{W}, \varphi) \in \mathcal{R}^{(0)}$ for each chromosome $i$ within the set $\mathbf{\Phi}^{(0)}$
Set $g = 0$,
**while** $g < \mathrm{G}_{max}$ **do**
  **if** $g = 0$ **then**
    $\mathcal{G}^{(0)} = \mathbf{\Phi}^{(0)}$, the set of corresponding rates are $\mathcal{R}^{(0)}$
  **else**
    Fitness : find $\mathcal{R}^{(g)} \leftarrow R^{(i)}(\mathbf{W}, \varphi), \forall$chromosome $i \in \mathcal{G}^{(g)}$
  **end if**
  $[\mathcal{R}^{(g)}_{sorted}, \mathcal{G}^{(g)}_{sorted}]$ ←sort($\mathcal{R}^{(g)}, \mathcal{G}^{(g)}$,'Descending')
  $[\mathcal{R}^{(g)}_{best}, \mathcal{G}^{(g)}_{best}]$ ←select($\mathcal{R}^{(g)}_{sorted}, \mathcal{G}^{(g)}_{sorted}, P$,'Best')
  $[\mathcal{R}^{(g)}_{worst}, \mathcal{G}^{(g)}_{worst}]$ ←select($\mathcal{R}^{(g)}_{sorted}, \mathcal{G}^{(g)}_{sorted}, P$,'Worst')
  $[\mathcal{R}^{(g)}_{luckies}] \leftarrow (\mathcal{R}^{(g)}_{best} - \mathcal{R}^{(g)}_{worst})$
  $[\mathcal{G}^{(g)}_{luckies}] \leftarrow (\mathcal{G}^{(g)}_{best} - \mathcal{G}^{(g)}_{worst})$
  **for** $c = 1 : (\nu - P)$ **do**
    $P1 \leftarrow$ select($\mathcal{G}^{(g)}_{best}$, 1,'Random')
    $P2 \leftarrow$ select($\mathcal{G}^{(g)}_{luckies}$, 1,'Random')
    $[TempCH1, TempCH2] \leftarrow$ Crossover($P1, P2$)
    $[CH1, CH2] \leftarrow$ Mutation($TempCH1, TempCH2$)
    $[\mathcal{G}^{(g)}_{child}] \leftarrow [CH1, CH2]$
  **end for**
  $\mathcal{G}^{(g+1)} \leftarrow \{\mathcal{G}^{(g)}_{best} + \mathcal{G}^{(g)}_{child}\}$
  $g \leftarrow g + 1$
**end while**
Output: Optimal channel allocation (or chromosome), optimal beamforming matrices, $\mathbf{W}^{\star}$

the set $\mathcal{G}^{(g)}_{best}$ and $\nu - P$ new chromosomes generated from the two sets $\mathcal{G}^{(g)}_{best}$ and $\mathcal{G}^{(g)}_{luckies}$, through *Children Generation Process (CGP)*. CGP consists of two major operations called *Mutation* and *Cross-over*. The details are explained as follows.

- At first, two chromosomes, $P1$ and $P2$, are randomly selected from the two sets $\mathcal{G}^{(g)}_{best}$ and $\mathcal{G}^{(g)}_{luckies}$, respectively. $P1$ and $P2$ are called parents.

- Next, the cross-over operation is initiated between the two selected parents to generate two new children, called $TempCH1$ and $TempCH2$. Here, we use the *Single Point Cross-over* technique [73] as an example. Specifically, entries of a randomly selected column of the two parents will be swapped to perform the single point cross-over. For example, consider the following two parent channel realizations with four SU pair transceivers and two PU channels (i.e., $K = 4$ and $N = 2$).

$$P1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad P2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Using single point cross-over, the first child (i.e., $TempCH1$) is created by keeping the first column of $P1$ fixed and replacing the second column of its with the second column of $P2$. Similarly, the second child (i.e., $TempCH2$) is also created and can be shown as follows.

$$TempCH1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}, \quad TempCH2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Note that swapping may produce infeasible chromosomes such as multiple channel allocation to a single user. Specifically, in this example both of the PU channels have been assigned to 4th and 3rd users of $TempCH1$ and $TempCH2$, respectively. In this case, except the swapped column, we keep the parent columns unchanged and let only the troublesome entries of the swapped column of both children to be zero to ensure that channel allocation constraint (3.14) is always satisfied. Hence, the new channel realization for the children are determined as follows.

$$TempCH1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad TempCH2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

- At the end, two randomly selected entries, $x_k^{n1}$, $x_k^{n2}$, of an arbitrary selected $k$th row will be swapped, called mutation. Note that, when selecting those two entries, either of them should have a value of one. The importance of mutation is to avoid GA converging to a local optimal value.

The termination condition for GA will be activated when the number of generations produced is beyond the predefined value $\mathrm{G}_{max}$. Eventually, the chromosome in the last generation, which has the maximum fitness, will be the best solution. Furthermore, the corresponding fitness value will be the optimal sum-rate.

At each iteration, the complexity of GA results from three major operations, i.e., sorting, mutation and cross-over, which introduce complexity of $\mathcal{O}(\nu\log(\nu))$, $\mathcal{O}(K)$ and $\mathcal{O}(K)$, respectively. Beside the first generation, $(\nu-P)$ fitness values have to be determined at each generation. Thus, the overall complexity of GA can be computed as

$$\begin{aligned}\mathrm{G}_{max} \times [&\mathcal{O}(\nu\mathrm{log}(\nu)) + \lceil(\nu-P)/2\rceil(\mathcal{O}(K)) \\ &+ (\nu-P) \times t_T \times \mathcal{O}(\mathrm{max}\{\xi,\kappa\}^4\kappa^{(1/2)}\mathrm{log}(1/\epsilon))]\end{aligned} \tag{4.20}$$

The implementation steps of the GA is summarized in **Algorithm 1**. We call the proposed beamforming, power, and channel allocation with GA as BPCA-GA.

### Simulated Annealing (SA)-based algorithm

From (4.20), the complexity of GA mainly depends on the values of $\nu$ and $P$. Since these two parameters also decide the accuracy and the convergence of GA, sufficiently large values of $\nu$ and $P$ have to be set, which causes high computational complexity for GA. In order to further reduce the complexity, a new allocation algorithm based on SA [70] is introduced.

Let $\mathbf{\Phi} = \{\mathbf{X}_1, \ldots, \mathbf{X}_L\}$ denote the feasible set of all channel allocations available with $K$ SU pairs and $N$ PU channels, where a $K \times N$ matrix $\mathbf{X}_l,\ l = 1, \ldots, L = (N+1)^K$, indicates a certain channel allocation with binary entries and a row sum, i.e., $\sum_{n=1}^{N} x_k^n \leq 1$.

Then, the optimization problem for $\mathbf{X}$ can be equivalently expressed as

$$\mathbf{X}^{\star} = \arg \max_{\mathbf{X}_l \in \mathbf{\Phi}} R(\mathbf{X}_l) \tag{4.21}$$

where $R(\mathbf{X}_l)$ indicates the sum-rate associated with $\mathbf{X}_l$. The SA-based algorithm uses neighborhood searching to determine an optimal solution. Specifically, the SA-based algorithm starts with a control parameter and an initial channel allocation that is used to generate new neighbor channel allocation. Then, the new channel allocation is clearly selected if it shows any performance improvement. Otherwise, it may still be accepted with a certain probability, which allows SA-based algorithm to escape from local optimal configurations. The cooling schedule controls the control parameter during the optimization process. The details of the algorithm is as follows.

At the initial state, we set the iteration index $l = 0$ and randomly pick a channel allocation, $\mathbf{X}_0$, from $\mathbf{\Phi}$. Using $\mathbf{X}_0$, a new neighbor channel allocation, i.e., $\hat{\mathbf{X}}_0 \in \mathbf{\Phi}$, is formed by swapping randomly selected row of $\mathbf{X}_0$ with an entry in $\mathcal{A}$, which indicates the set of all feasible combinations of channel allocation to a single user. For example, $\mathbf{X}_0$ with $K = 4$ SU transceiver pairs and $N = 3$ PU channels can be written as

$$\mathbf{X}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{4.22}$$

Thus, we can define $\mathcal{A} \in \{[0\,0\,0], [1\,0\,0], [0\,1\,0], [0\,0\,1]\}, \forall k \in K$ with $N = 3$ PU channels to a certain SU pair $k$. Hence, $\hat{\mathbf{X}}_0$ is generated by replacing a randomly selected row from $\mathbf{X}_0$ with a randomly selected element from the set $\mathcal{A}$. Therefore, an instance of

$$\hat{\mathbf{X}}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{4.23}$$

is generated by replacing the 2nd row of $\mathbf{X}_0$ with the 2nd element in $\mathcal{X}$. Given $\mathbf{X}_0$ and $\hat{\mathbf{X}}_0$, we compute the corresponding sum-rates, which are denoted as $R(\mathbf{X}_0)$ and $R(\hat{\mathbf{X}}_0)$, respectively. The acceptance rule at the $l$th iteration can be formulated as

$$P\{\text{Accept } \hat{\mathbf{X}}_l\} = \begin{cases} 1, & \text{if } R(\hat{\mathbf{X}}_l) \geq R(\mathbf{X}_0) \\ \exp\left(\dfrac{\Delta R}{T_l}\right), & \text{if } R(\hat{\mathbf{X}}_l) < R(\mathbf{X}_0), \end{cases} \tag{4.24}$$

where $\Delta R = R(\hat{\mathbf{X}}_l) - R(\mathbf{X}_0)$. It implies that a candidate neighbor channel allocation, i.e., $\hat{\mathbf{X}}_l$, is accepted with the probability of one if its sum-rate is greater than that of the current channel allocation, or $\exp\left(\dfrac{\Delta R}{T_l}\right)$ if the new neighbor channel allocation has lesser achievable sum-rate than the current allocation. If accepted, we replace $\mathbf{X}_0$ by $\hat{\mathbf{X}}_l$, and update the value of the control parameter for the next iteration as $T_{l+1} =$ cooling-rate $*$ $T_l$, where cooling-rate takes a value in $[0.50; 0.99]$ [74]. Note that $T_0$ can be determined following the similar way as in [75]. We increase $l$ by one and repeat the aforementioned process. When the maximum allowable iterations are reached, the algorithm is terminated and resulting outputs give the optimal channel allocation, sum-rate and the beamforming vectors.

The SA-based algorithm has two major steps, i.e., the generation of a neighbor channel allocation and the determination of sum-rate. A new neighbor channel allocation can be generated from the current channel allocation with a complexity of $\mathcal{O}(N)$, and a sum-rate calculation process involved a complexity of $\mathcal{O}(\max\{\xi, \kappa\}^4 \kappa^{(1/2)} \log(1/\epsilon)) \times t_T$ as in (4.15). If $S_{max}$ is the maximum number of iterations for convergence, the complexity of the algorithm can be computed as

$$S_{max} \times \{\mathcal{O}(N) + \mathcal{O}(\max\{\xi, \kappa\}^4 \kappa^{(1/2)} \log(1/\epsilon)) \times t_T\} \tag{4.25}$$

Compared with the (4.20), the computational complexity of SA-based algorithm is much smaller than the GA. The SA-based algorithm is summarized as in **Algorithm 2**. In this thesis, we call the proposed beamforming, power, and channel allocation with SA-based algorithm as BPCA-SA.

**Algorithm 2** : BPCA-SA

*Initialization*: Given $K, N, \mathbf{\Phi}$, cooling-rate and $\mathrm{S}_{max}$
Set $l = 0$,
Start : Initial channel allocation $\mathbf{X}_0$, $\mathbf{X}_0 \subset \mathbf{\Phi}$ and compute $R(\mathbf{X}_0)$
**while** $l < \mathrm{S}_{max}$ **do**
  Generate new channel allocation, $\hat{\mathbf{X}}_l$ from $\mathbf{X}_0$, $\hat{\mathbf{X}}_l \subset \mathbf{\Phi}$
  Calculate sum-rate, $R(\hat{\mathbf{X}}_l)$
  $\Delta R := R(\hat{\mathbf{X}}_l) - R(\mathbf{X}_0)$
  **if** $l = 0$ **then**
    Compute $T_0$
  **end if**
  **if** $\Delta R \geq 0$ **then**
    $\mathbf{X}_0 \leftarrow \hat{\mathbf{X}}_l$ and $R(\mathbf{X}_0) \leftarrow R(\hat{\mathbf{X}}_l)$
  **else if** $\exp\left(\frac{\Delta R}{T_l}\right) >$ random $[0, 1]$ **then**
    $\mathbf{X}_0 \leftarrow \hat{\mathbf{X}}_l$ and $R(\mathbf{X}_0) \leftarrow R(\hat{\mathbf{X}}_l)$
  **end if**
  Update $T_{l+1} =$ cooling-rate $* T_l$
  $l \leftarrow l + 1$
**end while**
Output: optimal channel allocation and optimal beamforming matrices, i.e., $\mathbf{X}_0$ and $\mathbf{W}^\star$

## 4.2 Simulation Results

In this section, the performance of both BPCA-GA and BPCA-SA algorithms are evaluated using computer simulations.

### 4.2.1 Simulation environment

Consider a CRN with three PU-TX/PU-RX pairs (i.e., $N = 3$). The locations of the PU-TXs are given by the x-y plane coordinates $(300, 0), (-400, 0)$ and $(0, -100)$, and their associated PU-RXs are situated at $(600, 0), (-400, 300)$ and $(0, -400)$, respectively, where all the distances are measured in meters. There are six SU-TX/SU-RX pairs (i.e., $K = 6$) randomly located within a square area of $600m \times 600m$. For each SU-TX, its associated SU-RX is randomly located in a circle centered at the SU-TX with a radius of $100m$. Each SU-TX with $J = 3$ antennas is oriented in a UCA configuration having a radius of $\lambda = \frac{c}{f_c}$, where the carrier frequency $f_c = 900$MHz and the speed of light $c = 3 \times 10^8 ms^{-1}$. The transmit powers, $P_{\text{max}}$, available at the SBS and $\mathcal{Q}_n, \forall n \in \mathcal{P}$ at the PU-TX are set to be $30dBm$ and $23dBm$, respectively. Furthermore, each PU-RX has an interference threshold, $I_{\text{th}}^n, \forall n \in \mathcal{P}$, of $-40dBm$ and each SU-RX's minimum QoS threshold, $\Gamma_k, \forall k \in \mathcal{S}$, is $6dB$. The noise power at any location is considered as $-90dBm$ and the bandwidth of each channel, $B$, is set as 10MHz. The small-scale fading coefficients are modeled as a Circular Symmetric Complex Gaussian (CSCG) random variables with zero mean and unit variance. We consider the free space propagation model to simulate the path-loss, i.e., $PL = \left(\frac{\lambda}{4\pi d}\right)^2$, where $d$ is the distance between the transmitter and the receiver.

### 4.2.2 Convergence of BPCA-GA and BPCA-SA algorithms

Fig. 4.1 shows the convergence of the two proposed algorithms, i.e., BPCA-GA and BPCA-SA. The optimal sum-rate, 42.1765 bps/Hz, is determined by adopting the exhaustive searching method. From the figure, we can see that BPCA-GA approaches to a close optimal sum-rate value of 41.9634 bps/Hz. BPCA-SA can find a suboptimal channel allocation with a sum-rate value of 41.0557 bps/Hz which is only 2.16$\%$ lesser than BPCA-GA. By considering the computational complexities of both BPCA-GA and BPCA-SA, we can conclude that BPCA-SA is more feasible for practical applications. Note that, though BPCA-GA converges much faster than BPCA-SA in terms of number of iterations, each iteration takes more time in BPCA-GA than BPCA-SA because of the high computational complexity involved in GA.

### 4.2.3 The achievable sum-rate with increased number of SU pairs and PU channels

Fig. 4.2 illustrates a comparison of the achievable sum-rate by the BPCA-GA and BPCA-SA algorithms. From the figure, we can see that when the number of SU pairs is small, e.g., $K = 2$, both algorithms show similar performances. This is because the search space

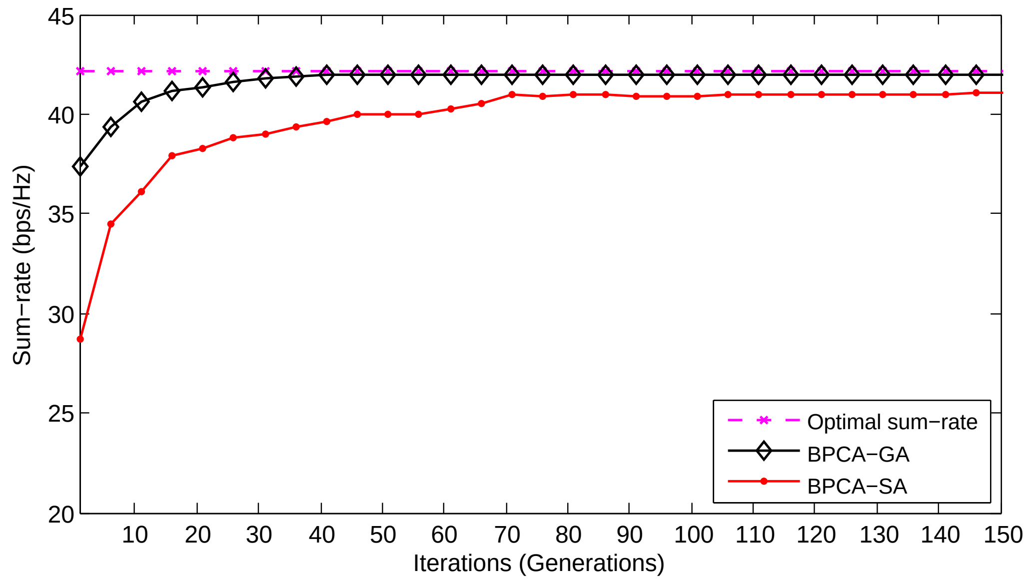


**Figure 4.1:** Convergence of the BPCA-GA and BPCA-SA algorithms with $K = 4, N = 3$, $P_{\text{max}} = 1$W, $J = 3$ and cooling-rate = 0.99.

is small so that both GA and SA-based algorithms have an equal chance to obtain a close optimal value. However, as the number of SU pairs increases, the BPCA-GA can achieve higher sum-rate than BPCA-SA.

Fig. 4.3 depicts performance of BPCA-GA and BPCA-SA with the number of PU channels. From the figure, we can observe a similar behavior as in Fig.4.2. As $N$ increases, achievable sum-rate of both algorithms tend to increase due to the increased degrees of freedom on channel allocation at the SBS. For most of the cases, BPCA-SA is lagged behind BPCA-GA with a small performance gap. By comparing Figs 4.2 and 4.3, we can further observe that the number of SU pairs has more effects on the performance than the number of PU channels.

### 4.2.4 Comparison with Zero-Forcing Beamforming (ZFBF)

We compare the proposed beamforming method with ZFBF in Fig. 4.4. With ZFBF, the beamforming vectors for each SU-TX is determined by confining the interference among

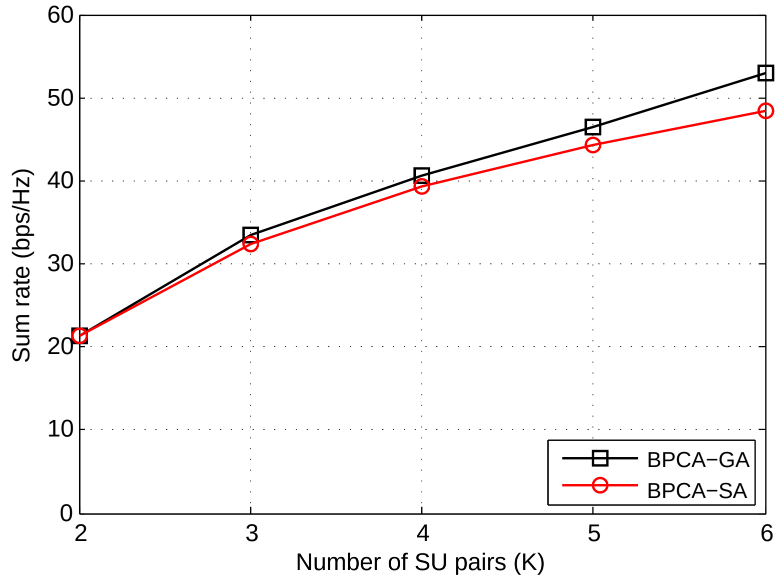


**Figure 4.2:** Sum-rate variation with increased number of SU pairs for fixed $N = 3$ PU channels, $P_{\text{max}} = 1\text{W}$ and $J = 3$.

the SUs to become zero. Mathematically, this condition can be expressed as

$$\sum_{\substack{m \in S_n \\ m \neq k}} Tr(\mathbf{W}_m \mathbf{H}_{mk}) = 0, \ \ \forall k = 1, \ldots, K, \tag{4.26}$$

Hence, the optimization problem, *P3*, can be revised by changing the constraint (4.13) to be (4.26) and solved by using the similar algorithms. We call the zero-forcing beamforming, power and channel allocation with GA and SA-based algorithm as ZFBPCA-GA and ZFBPCA-SA, respectively. From the figure, it can be seen that our proposed model outperforms the ZFBF irrespective of the algorithm used. It is because the degrees of freedom available on channel allocation in our proposed system model are much greater than the ZFBF model. Moreover, after $K > 3$, the curve for ZFBPCA-GA increases gradually while that for ZFBPCA-SA decreases. It is because once $K$ reached the maximum number of channels (i.e., $N = 3$ in our simulation), it is very difficult to find the beamforming vectors with ZFBF due to the increased channel access requirement of the SUs at the SBS. Furthermore, after $K > 3$, the ZFBPCA-SA tends to generate more infeasible channel allocation and hence, eventually converges to a value further from

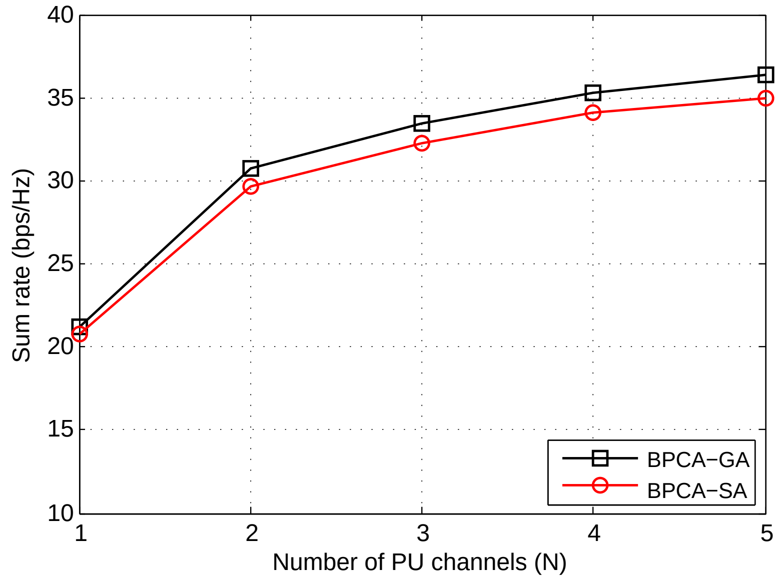


**Figure 4.3:** Sum-rate variation with increased number of PU channels for fixed $K = 3$ SU pairs, $P = 1$W and $J = 3$.

the optimal. However, in our proposed system model, since there is some amount of interference tolerance at SU-RXs and PU-RXs, the sum-rate of the CRN is boosted with the increase of $K$.

### 4.2.5 Comparison of sum-rates with increased number of antennas

The performance of achievable sum-rate with respect to the number of transmit antennas is shown in Fig. 4.5. The figure depicts that the sum-rates of both systems are increased with the number of antennas. It is because once $J$ increase, the transmitter can direct the signal with better intensity to the intended receiver and at the same time can further suppress the interference to the other users who are using the same channel. Similarly, our proposed system model gives much better results than the ZFBF.

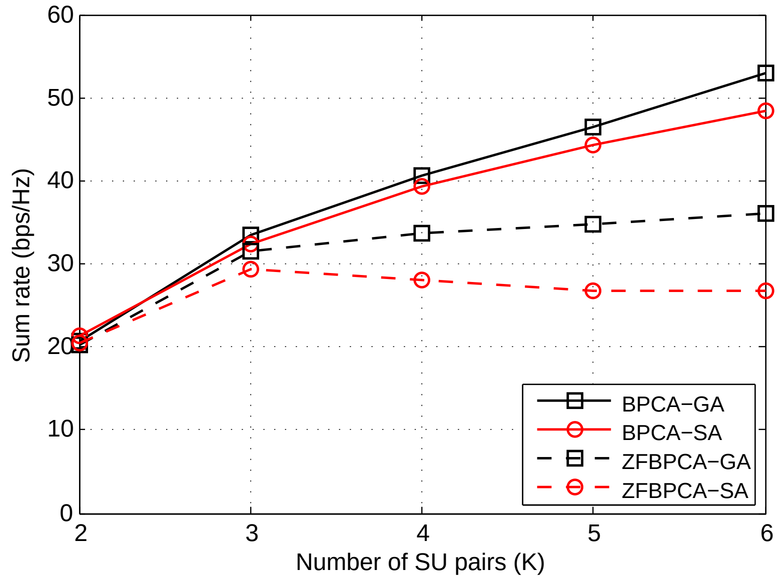


**Figure 4.4:** Comparison of sum-rate variation between ZFBF system model and proposed system model with increased number of SU pairs for fixed $N = 3$ PU channels, $P = 1$W and $J = 3$ antennas.

### 4.2.6 The performance of BPCA-GA and BPCA-SA with different power budget

Fig. 4.6 illustrates the performance of sum-rates by varying power budget, $P_{\text{max}}$. As we increase the power budget, the achievable sum-rate also increases as shown in the figure. In fact, the sum-rate performance gap between the BPCA-GA and BPCA-SA is small and keeps approximately unchanged as we increase $P_{\text{max}}$. It is mainly because the convergence properties of both algorithms do not change with the power. With the increased power budget, the SUs are favored to have more individual power allocation. Hence, they are rewarded with more power as long as the interference constrains are satisfied, which in return increases the sum-rate.

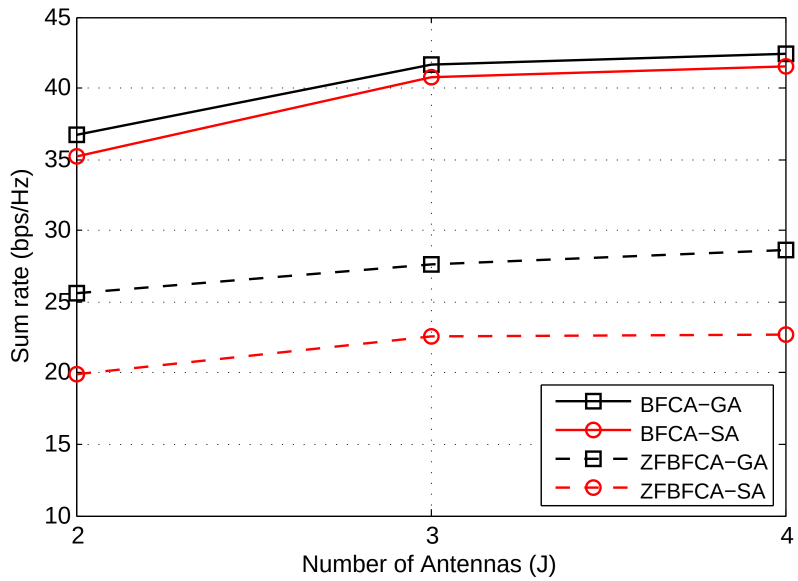


**Figure 4.5:** Comparison of sum-rate variation between ZFBF and proposed system model with increased number of antennas with fixed $N = 2$ PU channels, $K = 4$ SU pairs and $P = 1$W.

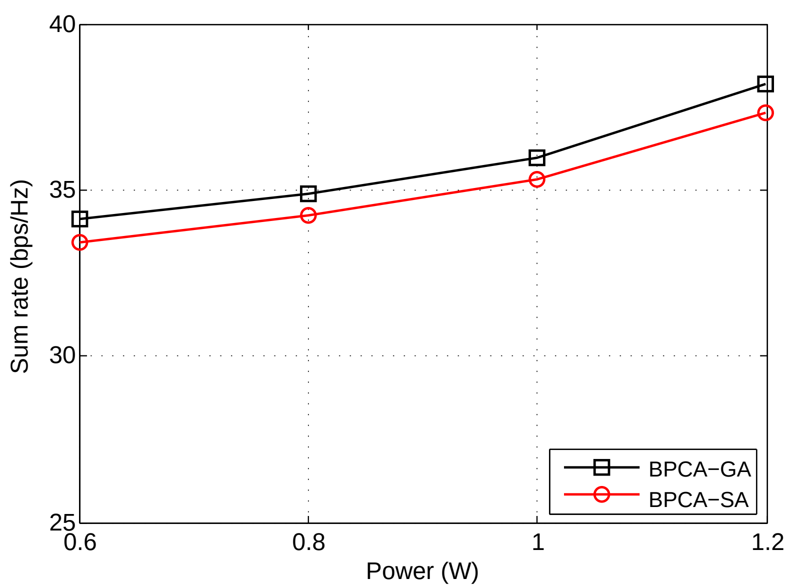


**Figure 4.6:** Comparison of sum-rate variation for BPCA-GA and BPCA-SA with total power budget available for fixed $N = 2$ PU channels, $K = 4$ SU pairs and $J = 3$ antennas.

# Chapter 5

# Conclusions and Extensions

## 5.1 Conclusion

In this thesis, we considered a concept called cognitive radio (CR) in wireless communication networks. Specifically, we focused on resource allocation and beamforming in cognitive radio networks (CRNs) and explained the research challenges in current CR designs. Few related works in literature on these two areas are also listed to identify motivations to this study. In Chapter 2, fundamental knowledge about CRNs and importance of dynamic spectrum access in CR with three common dynamic spectrum access models was addressed. In addition, multiple antenna systems and beamforming theory were studied briefly. At the end of the chapter, a comprehensive literature survey of existing joint resource allocation and beamforming was presented.

In Chapter 3, a problem of joint beamforming, power and channel allocation was considered for multi-user multi-channel underlay CRNs. The problem was formulated as a non-convex mixed integer non-linear programming (MINLP) problem, which was NP-hard. In order to reduce the computational complexity, we decoupled the original problem into two sub-problems. At first, a feasible solution for beamforming vectors and power allocation was obtained from an iterative algorithm for a known channel allocation by using the semi-definite relaxation (SDR) approach with an auxiliary variable. After that, genetic algorithm (GA) and simulated annealing (SA)-based algorithms have been applied to determine the suboptimal channel allocation. Simulation results showed that the performance of beamforming, power, and channel allocation with GA (BPCA-GA) was closer to optimal while having a trade-off in computational complexity. Whereas, beamforming, power, and channel allocation with SA (BPCA-SA) achieved a close performance to GA with lower computational complexity. Moreover, beamforming with interference tolerance capability introduced by our system model achieved better performance than zero-forcing beamforming (ZFBF).

## 5.2 Possible Extensions

In our proposed system model, the secondary user transmitters (SU-TXs) are scattered in a given area while each connecting with a single secondary user receiver (SU-RX). Moreover, each SU-TX performed transmit beamforming to manipulate signal to the intended receiver and minimize interference to both primary user receivers (PU-RXs) and SU-RXs using multiple antenna array elements. However, in order to determine the beamforming vector and channel allocation for each SU-TX, we considered the perfect knowledge of the channel state information (CSI) at secondary base station (SBS). Hence, problem formulation with imperfect CSI is one of the possible extensions to this work. We can use the training sequence channel estimate method [76] to find CSI at the beginning of each time slot. In addition to that, we can use discrete stochastic approximation (DSA)-based channel allocation method [77] to find out the suboptimal channel allocation. Errors in the estimates of the CSI is inevitable due to some sensing limitations. Let $\mathbf{\Phi}$ as in section (4.1.2). Hence, each $\mathbf{X}_l \in \mathbf{\Phi}$ indicates a certain feasible channel allocation in the CRN. Therefore, the corresponding channel gain matrix for $\mathbf{X}_l$ is defined as $\mathcal{C}[\mathbf{X}_l], l = 1, \ldots, L = (N+1)^K$. In general, the optimization problem is expressed as in (4.21). However, due to the estimation errors, the estimated channel gains matrix at the $t$th time slot, denoted as $\hat{\mathcal{C}}[t, \mathbf{X}_l]$, may produce a noisy estimate of $R(\mathbf{X}_l)$, which is defined as $r(\hat{\mathcal{C}}[t, \mathbf{X}_l])$. In

**Algorithm 3** : Discrete Stochastic Approximation

*Initialization*: Given $K, N, \mathbf{\Phi}$
Set $t = 1$,
Randomly select a channel allocation from $\mathbf{\Phi} \leftarrow \mathbf{X}^{(1)}$
Set $\pi[1, \mathbf{X}^{(1)}] \leftarrow 1, \pi[1, \mathbf{X}_l] \leftarrow 0, \forall \mathbf{X}_l \neq \mathbf{X}^{(1)}, \mathbf{X}_l \in \mathbf{\Phi}$ and $\mathbf{D}[1] \leftarrow \mathbf{e}_l$
**for** $t = 1, 2, \ldots$ **do**
    Randomly select a new channel allocation from $\mathbf{\Phi} \leftarrow \tilde{\mathbf{X}}^{(t)}, \forall \tilde{\mathbf{X}}^{(t)} \neq \mathbf{X}^{(t)}$
    $[\mathcal{T}] \leftarrow \{\mathbf{X}^{(t)}, \tilde{\mathbf{X}}^{(t)}\}$
    Compute two rates, $r(\hat{\mathcal{C}}[t, \mathbf{X}^{(t)}]), r(\hat{\mathcal{C}}[t, \tilde{\mathbf{X}}^{(t)}])$
    **if** $\{r(\hat{\mathcal{C}}[t, \mathbf{X}^{(t)}]) \leq r(\hat{\mathcal{C}}[t, \tilde{\mathbf{X}}^{(t)}])\}$ **then**
        $\mathbf{X}^{(t+1)} \leftarrow \tilde{\mathbf{X}}^{(t)}$
    **else**
        $\mathbf{X}^{(t+1)} \leftarrow \mathbf{X}^{(t)}$
    **end if**
    $\mathbf{D}[t+1] \leftarrow \mathbf{e}_{index}$
    $\pi[t+1] = \pi[t] + \epsilon[t](\mathbf{D}[t+1] - \pi[t])$, where $\epsilon[t] = 1/t$
    **if** $\pi[t+1, \mathbf{X}^{(t+1)}] > \pi[t+1, \mathbf{X}^\star]$ **then**
        $\mathbf{X}^\star \leftarrow \mathbf{X}^{(t+1)}$
    **else**
        $\mathbf{X}^\star \leftarrow \mathbf{X}^{(t)}$
    **end if**
    $t \leftarrow t + 1$
**end for**
Output: $\mathbf{X}^\star, \mathbf{W}^\star$

fact, for different iteration times, the value of $r(\hat{\mathcal{C}}[t, \mathbf{X}_l])$ becomes random. Hence, if we assume those estimates are unbiased, then we can have a sequence of independent and identically distributed random variables corresponding to each iteration time $t$. Therefore, the suboptimal channel allocation problem can be approximated by DSA as

$$\mathbf{X}^\star = \arg \max_{\mathbf{X}_l \in \mathbf{\Phi}} E\{r(\hat{\mathcal{C}}[t, \mathbf{X}_l])\} \tag{5.1}$$

At the beginning of the algorithm, we set the iteration index, $t$ to be one. Then, a channel allocation is randomly selected from $\mathbf{\Phi}$, which is denoted as $\mathbf{X}^{(1)}$. Furthermore, a state probability is assigned to each channel allocation in $\mathbf{\Phi}$ including $\mathbf{X}^{(1)}$. Define the set of probabilities for all channel allocations at the $t$th iteration to be $\pi[t] = \pi[t, 1], \pi[t, 2], \ldots, \pi[t, L]$, where $\sum_l \pi[t, l] = 1$. At $t = 1$, we set $\pi[t, \mathbf{X}^{(1)}] = 1$ and $\pi[t, \mathbf{X}] = 0, \forall \mathbf{X} \neq \mathbf{X}^{(1)}$. In addition, $L \times 1$ auxiliary vector, $\mathbf{e}_l$, is defined with all elements equal to zero except the $l$th element, which has a value equals to one. For notation simplicity, the value of $\mathbf{e}_l$ at the $t$th iteration is mapped to a $L \times 1$ column matrix $\mathbf{D}[t]$.

Next, at each iteration a new channel allocation, $\tilde{\mathbf{X}}$, is randomly selected from the set $\mathbf{\Phi}$. Afterward, we compute the corresponding sum-rates for the two selected channel allocation (i.e., $r(\hat{\mathcal{C}}[1, \mathbf{X}^{(1)}])$ and $r(\hat{\mathcal{C}}[1, \tilde{\mathbf{X}}^{(1)}])$. Two sum-rates are then compared to determine the better channel allocation. Finally, the state probabilities of each channel allocation is updated using $\pi[t+1] = \pi[t] + \epsilon[t](\mathbf{D}[t+1] - \pi[t])$ with a step size of $\epsilon[t] = 1/t$. The implementation of the DSA algorithm is summarized as in **Algorithm 3**.

Multiple-input multiple-output (MIMO) communication systems have achieved considerable improvements in capacity over the traditional single-input single-output (SISO) system while using the same transmit power and bandwidth. Spatial diversity and spatial multiplexing strategies (i.e., space division multiple access (SDMA)) are also only possible with MIMO compared with other multiple antenna configurations. Thus, we can suggest a MIMO configuration for the CRN in our system model. Furthermore, instead of transmitting multiple copies of the same signal, we can consider transmitting different signal at each SU-TX.

Furthermore, we assumed that each SU-TX/SU-RX pair is only allowed to utilize at most one PU channel at a time. However, we may allow each SU-TX to use more than one channel to communicate with a single SU-RX. Hence, this could help to significantly improve the performance of the secondary network in term of achievable throughput while simultaneously satisfying the quality-of-service (QoS) requirement of each user. In addition, we can improve the spectrum utilization of both primary and secondary networks with greater extend.

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