THE UNIVERSITY OF MANITOBA

A STUDY OF THE ABILITY OF A GROUP OF GRADE XII STUDENTS TO SOLVE REAL WORLD MATHEMATICAL PROBLEMS

by
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A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
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ABSTRACT

The purpose of this study was to analyse the ability of a group of grade XII students to recognize, approach, and solve real world mathematical problems. For this purpose two parallel forms of a mathematical test were developed by the investigator; a real world mathematical problem form and a textbook mathematical problem form. The real world mathematical problems were designed in a real world mathematical problem format, whereas the textbook mathematical problems were written in the usual textbook problem format. There were ten questions in each form, for every problem in the real world mathematical problem form there was an equivalent problem in the textbook mathematical problem form with an identical mathematical skill for the solution of the problem.

The study was conducted in four public high schools in the City of Winnipeg. A pilot testing in two of the four high schools was conducted for the purpose of establishing content validity, readability, and students' understanding of the wording of the mathematical problems in the test. The final testing was administered in the remaining two high schools. A sample of 47 grades XII and XI subjects from Grant Park High School, divided into two equal subgroups and randomly assigned to the two forms of the test, wrote the two parallel forms of the test in two sessions on April 7, and April 19, 1983. In the second session, April 19, 1983, the subgroup that wrote the textbook mathematical problem form of the test wrote the real world mathematical problem form of the test and vice versa. In St. John's High School 26 subjects from two grade XII classes wrote the two forms of the test in two sessions on April 22nd, and April 27, 1983. The two grade XII classes of St.

John's High School were not randomly assigned. Instead the two intact classes were assigned to the two forms of the test. In the second session, April 17, 1983, the two classes exchanged the two forms of the test.

The study showed that students generally performed better on the textbook mathematical problems than on the real world mathematical problems with an identical mathematical skill requirement. Good textbook mathematical problem solvers had a better recognition of the real world mathematical problem than less-able textbook mathematical problem solvers. However, good and less-able textbook problem solvers alike had trouble with real world problems that had extraneous data as well as with those real world problems that had missing information/data to be generated by the problem solver.

These results led to the conclusion that the usual textbook mathematical problem solving experience is not adequate to prepare the student to function "mathematically" with real world problems.

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CHAPTER I

Nature And Purpose Of The Investigation

Introduction

The literature of mathematics education in the area of problem solving frequently indicates that there is a difference between real world mathematical problems and the usual textbook mathematical problems. But this author was unable to find any detailed examination of the difference in the literature. Most writers seem to assume that the difference lies in a worded presentation as opposed to a situation in a natural context which is interpreted as implying some mathematical problem. It is arguable at what point of intellectual abstraction this natural configuration becomes a mathematical problem. Certainly the real world is not, in and of itself, a mathematical problem. A human mind must attempt to impose some structure on it before any mathematical problem exists.

The difficulty, then, in defining "real world" problems is in deciding how much verbalization or imposition of structure has occured by the time a "problem" exists.

Meiring (1980) reported that textbook problems do not fully reflect the circumstances of real world problems. He observed that:

> Most textbooks organize problems to provide practice with specific kind of content or process. Students may so realize that an entire set of word problems will involve a particular operation or application. Therefore, problems become thinly disguised exercises in which most students merely select numerical data for some standard operation. (Meiring, 1980, page 45).
> Give children problems for which they must collect or generate some of the information or data. Textbook problems provide all the necessary facts and only the essential information. Every-day experiences in problem solving require selection

> of relevant information or seeking out data that must be used. (Meiring, 1980, page 54).

This author views the real world mathematical problem as analogous to raw material, like iron ore, that must undergo continued processing. Textbook mathematical problems are often processed so as to call directly for the direct application of a particular mathematical skill. In other words, most textbooks problems do not give the solver the experience or the satisfaction of solving problems at least closer to real world situations.

Blosser (1981), commenting on real problems and the solver's first task in tackling them, said:

> Real problems are usually cluttered with irrelevant information or even may be missing some necessary data. One of the solver's first task is to distinguish between the pertinent and non-pertinent information and to decide whether he/she has enough information to generate a solution. (Blosser, 1981, Eric Bulletin).

The Purpose Of The Study

The purpose of this study is to examine the ability of a group of grade XII students to solve worded mathematical problems which approach, so closely as is possible, the complexity of the real world.

The Questions Of The Study:

1. Do good standard (textbook) problem solvers and less-able standard (textbook) problem solvers differ in the recognition, approach, and solution of more real-world-like mathematical problems?
2. Do female and male students differ in the recognition, approach, and solution of real-world-like mathematical problems?

Significance Of The Study

It is evident that a main goal of mathematics educators is to produce good problem solvers.

Potential mathematical problems are literally everywhere around every student. Yet most students seem to have difficulty finding mathematical structure in situations even when their attention is called to some particular situation where they certainly are given a broad hint that they are intended to find such structure.

In this study, we consider at least a first step towards embedding problems in a real-world context. The results provide some information on:

1) The degree to which grade XII students recognize the application of a mathematical skill learned through solving a usual textbook mathematical problem to an analogous more deeply embedded situations that require the same mathematical skill; and
2) any cautions that should be taken by textbook writers in attempting to link high school students with their real world through more deeply embedded problems.

From this point on, such more deeply-embedded problems will be called "real world problems", even though they still appear in print and are only part-way towards the ultimate objective.

Assumptions Of The Study

It is assumed that if textbook mathematical problems are presented closer to real world format high school students will find the study of mathematics more relevant and directly useful and applicable.

This author believes that textbook authors have tended to overprotect students by polishing problems, especially in senior grades (X-XII). And, therefore, students have too often failed to see the intended relationship between the textbook problems and the mathematical problems they may run into in the real world. Although teachers could generate supplementary real world problems, the task is often too difficult and too time-consuming for teachers in senior grades (X-XII).

Statement Of Research Hypotheses

A study by Moskol (1980) led her to conclude that students seemed to have trouble transferring mathematical concepts, learned in an academic environment, to real-world situations (Moskol, 1980, page 158). Arter and Clington (1974) found that children solve fewer problems under extraneous data condition (Arter and Clington, 1974, page 30). These results furnish the ground for the following hypotheses:

(1) Generally students, regardless of sex and ability, will score significantly lower on a real world mathematical problem test than on a textbook mathematical problem test involving exactly the same mathematical tasks at the $\alpha \leq .05$ level of significance.

(2) Sex will have no significant effect on students' performance on either the textbook problem test (Test T) or the real world problem test (Test R) at the $\alpha \leq .05$ level of significance.

(3) Good textbook problem solvers (males and females) as opposed to less-able textbook problem solvers (males and females) will perform better on both tests at the $\alpha \leq .05$ level of significance.

(4) Good textbook problem solvers, regardless of sex, will have better recognition of real world mathematical problem in comparison to less-able textbook problem solvers (males and females) at the $\alpha \leq .05$ level of significance.

Definitions Of Terms

Mathematical Skill: is the recognition of the formula, theorem, principle or rule necessary for the solution of a mathematical problem.

Recognition of the real world mathematical problem: is the identification of the mathematical skill necessary for the solution of the real world mathematical problem.

A good textbook mathematical problem solver: is any student who scores high (B or better) on past classroom mathematics tests or a student who is stated to be at that level by his/her mathematics teacher.

A less-able textbook mathematical problem solver: is any student who scores lower (B^- or lower) on past classroom mathematics tests or a student who is stated to be at that level by his/her mathematics teacher.

A real world mathematical problem: is a problem that is more deeply embedded in irrelevant data and has less direct indications of the skill to be called for. Such a problem may contain non-pertinent, but not frivolous, information. The following simple examples illustrate the definition.

Example (1):

How many dollars do five pupils have altogether if each has 4 dollars?

Example (2):

How many legs do 5 cows have, if each has two horns?

Example (1) is a textbook model of a simple problem where the number of the sets and the size of each set are explicitly stated. Example (2) is a more deeply embedded problem that calls for the same mathematical skill needed to solve the problem of example (1). In example (2) the size of the set must be generated by the problem solver. The non-pertinent, but not frivolous, information is the expression "each has two horns."

Limitations And Delimitations

The study was limited to four public high schools in the city of Winnipeg. Both tests were limited to ten mathematical problems. The problems in the real world mathematical problem test are mathematical formulations of situations that might be reasonably supposed to be within the range of experience of these students. The problems in the textbook mathematical test are the same problems reduced to provide direct application of the mathematical skills needed to solve the real world mathematical problem test.

The Design Of The Study

How To Determine The Answers To The Questions Of The Study?

Student's solutions of the real world mathematical problems must meet the following criteria in order for proposed solutions to be considered acceptable.

1) The criterion for the students' recognition of the real world mathematical problem will be the identification of the mathematical skill underlying the problem. This is only possible if the students attempt each problem and show all work in the answer booklet.
2) The criterion for students' approach is the identification of the pertinent from the non-pertinent data given in the problem.
3) The criterion for the complete solution of the problem will be the correct manipulation of the data and equations leading to the correct answer.

Students' abilities to recognize, approach, and solve the real world mathematical problem test are graded accordingly.

Question 1) of the study will be answered by the statistical analysis of the performance of good textbook problem solvers versus less-able textbook problem solvers.

Question 2) of the study will be answered by the statistical analysis of male versus female students performance.

As for the experimental design of the study, the subjects will be randomly assigned to the two forms of the test (real world and textbook) then their performance will be measured. Each candidate will write the two forms in two separate sessions with a time interval long enough to expect that there will be no carry over from the first session to the second session.

Summary

In this chapter different aspects of the nature and the purpose of the investigation were outlined. This included the significance and the assumptions of the study, definition of terms, limitations and delimitations, and the design of the study. It should be noticed that it was the investigator's personal experience rather than the review of the related literature that spurred the investigation. However, the review of the literature, an account of which follows, shed light on some aspects of the problem, such as extraneous data conditions and missing information.

CHAPTER II

The Review Of The Literature

A review of some of the findings on the teaching of problem solving in schools along with detailed descriptions of real world and textbook problems are presented in this chapter. This is followed by a summary of the effects of extraneous information on problems solving achievement and a number of other references related to solving real world mathematical problems. This review is limited to reports published during the last twenty years.

Teaching Heuristics And Real World Problem Solving

A number of investigations and studies have been conducted on the possibility of teaching the skills of problem solving through the teaching of heuristics. Although a majority of the authors of these reviews concluded that teaching heuristics improved problem solving ability a contrary conclusion was expressed by Schoenfeld (1979). He concluded that instruction in mathematical problem solving by means of general strategies had resulted in only marginal success. The following three reasons were given by Schoenfeld:

1. the strategies have yet to be described in sufficient detail,
2. they are descriptive, rather than prescriptive, and
3. there are too many potentially useful strategies. (Schoenfeld, 1979, page 3).

Schoenfeld supported his argument by reference to the results of a study which he conducted with 19 college students in an instructional period of three and one half weeks which he refers to as "the model of expert problem solving". This model consisted of reformulating the problem in a useful analytic way, properly construed design, exploring the problem, and verification. However, the results were that the students showed little of the general strategic abilities of the expert.

Greenes (1981) somewhat agreed with Schoenfeld in her finding that gifted children tend to use a variety of non-standard approaches and strategies for solving problems (Greenes, 1981, page 21).

On the other hand, Mendoza (1980) found, in a study of the effects of teaching heuristics on the ability of grade X students to solve novel mathematical problems, that students could be taught to apply at least one heuristic to novel algebraic and geometric problems. Mendoza indicated that it appeared more effective to teach heuristics alone than to teach them in combination with specific content.

Allen and Ross (1977) in a study addressed to the "recognition that an idea is indispensably relevant to the solution of the problem", with eight graders, supported the proposition that skills in applying mathematical ideas could be improved by learning procedures that were rich in opportunities for such application at appropriate levels of complexity for each student.

Swafford and Kepner (1980) studied the extent to which an application approach helps in solving real life problems. They reported that the experimental group, which received the application oriented first-year algebra program, showed a less favorable view of the need for mathematics in jobs outside science and engineering than the control group. They also found that both groups improved in their applied problem solving skills in the course of the year.

Another author who showed a great deal of concern about applied mathematical problem solving was Lesh (1981). Lesh believed that there must not be any separation of problem solving and concept formation and that the emphasis should be on the dynamic interaction between basic mathematical concepts and applied problem solving processes. He added that "concrete models not only serve as a bridge from the real world into the world of mathematics, they can also serve as a bridge to help students apply their ideas back in the real world." (Lesh, 1981, page 247).

Polya said that "if other means fail, we should try to imagine an analogous problem". (Polya, 1957, page 182). Indeed, Branda (1979) found that making analogies, among other heuristics, was used by the five highest scoring students on a creativity factors test. Branda argued that the search for a meaning or relevance in the data, the statement that some data that would be needed are not provided, and the use of analogies should be three of the heuristics taught for solving real world mathematical problems.

Real World Mathematical Problems

What is a real world mathematical problem? There is no clear consensus among authors as to what is a "real world mathematical problem". To Lomon; Beck; and Arbetter (1975) a real problem "connotes a practical, immediate impediment to good, safe or pleasurable living." (Lomon, Beck, and Arbetter, 1975, page 54). To Shan, and others (1975) a real problem must embody some valid aspect of school community life. (Shan; and others, 1975, page 20).

Lesh (n.d.) views real problems as mathematical problems in real situations.

Casey and Sowel (1982) argue that real world mathematical problems are out-of-doors mathematical activities which provide strong motivational advantages for the students.

Authors such as Meiring (1980), Lesh (1981), and Blosser (1981), observe that real world problem situations contain too much information, irrelevant data, and, sometimes, missing information that has to be generated by the problem solver.

Real problems often occur as "ouches" rather than as well defined questions with clearly specified goals. (Lesh, 1981, page 251)

Lester (1978) is of the opinion that the term "real-world" is difficult to define since, according to Lester, a real-world or real-life problem for one person may not be a real-life problem for another. (Lester, 1978, page 63).

Kerr and Maki (1979) argued that the model starts with a real situation and ends in testing the usefulness of the model back in a real situation again. That is to say, the real situation and the model constitute a continuum.

Kerr and Maki suggest that real world mathematical problem solving should start with the identification of a real world problem or area of study. Then the problem is described in written form: "a real model". The next step is to replace the words and concepts of the real model with mathematical symbols and expressions: "a mathematical model". Mathematical tools and techniques are next used to arrive at conclusions based on the model. These conclusions are then tested in the real world to determine the usefulness of the model. (Kerr and Maki, 1979, page 2).

Kerr and Maki point out that problems in the real world are often too complicated to deal with mathematically. In order to make the problem manageable, a decision must be made as to which aspects of a problem can be ignored when a real model is drawn from the real world problem. (Kerr and Maki, 1979, page 2).

According to Kerr and Maki, we encounter difficulties when we read of problems in newspapers or in textbooks, because the authors almost always simplify real-world problems in the process of writing about them. The teacher will often want to modify the setting and further simplify the problem into a "classroom" model in order to make it manageable for students. (Kerr and Maki, 1979, page 4).

Other authors who emphasize the role of modeling in solving real world problems are Lomon; Beck; and Arbetter (1975). They say that dealing with problems involves observations, modeling, discussion, value judgements, decision making, and communicating. (Lomon; Beck; and Arbetter, 1975, page 54).

Problem formulation is critical in many real problem situations; the goal may be to understand a problem, not to solve it. (Lesh, 1981, page 251).

"Interest" and "motivation" are two other factors in solving real world problems. Students should be involved in identifying problems which are real, important, and interesting to them. (Shan; and others, 1975; Lester, 1978; Casey and Sowel, 1982).

In this review of the literature, the first genuine attempt to study the processes students use to solve real world mathematical problems was by Moskol (1980).

Moskol studied the processes a sample of 30 undergraduate mathematics majors used to solve real-world problems in which mathematics could be applied. The 30 subjects participated in a clinical interview in which they thought out loud while solving problems. The problems covered estimation, non-standard word problems, model construction, and open-ended questions. It was found that students had trouble estimating physical quantities and determing appropriate data in the presence of extraneous data. Moskol concluded that these students seemed to have trouble transferring mathematical concepts, learned in an academic environment, to real-world problems. (Moskol, 1980, page 158).

Mathematics-Textbook Problems

The role mathematics-textbook problems play in promoting real world applications of mathematics is always an issue among authors and researchers. Lesh (1981) stated that:

> In most mathematics textbook application problems, the student is given an idea and then asked to use it in a variety of word problems that refer to real objects. Presumably, those problems increase the meaningfulness and usefulness of the underlying ideas by encouraging students to relate ideas to real world situations. Most problems, however, require the student to begin with a real world situation and look for relevant mathematical ideas. (Lesh, 1981, page 252)

Some authors believe that mathematics textbooks do not exactly reflect real world circumstances, or that mathematics-textbook problems are actually little more than exercises designed for practicing the use of a formula or algorithm. (Lester, 1978; Meiring, 1980; Lesh, 1981).

Keuhls (1976) studied the effect of interspersed questions on learning from mathematical texts. The study was conducted to determine if there was a significant difference in learning, as measured by an achievement test, between students who read mathematical material with questions inserted after each paragraph or two and students who read the same material with no questions inserted. Kuehls found that the two treatments were different for students at a high level of achievement and ability. However, Kuehls concluded that the efficiency of a particular mathematics text depends on the ability and achievement of the mathematics student. He argued that students at high levels of achievement and ability, when taught by a text that has questions inserted periodically, should do as least as well as when taught by a conventional text.

Kuehls pointed out that more than one text should be selected in order to meet the needs of the students at different levels of achievement and ability. (Kuehls, 1976, page 76).

The following example by Robinson (1977) demonstrated that mathematical processes and mathematical algorithms do not have universal application. For example, explained Robinson:

> Do 2 and 3 always "make 5"? If x lives 2 blocks from y, for instance and y lives 3 blocks from z, is it necessarily true that x lives 5 blocks from z? In other words under what conditions should one add 2 blocks and 3 blocks to determine the total distance? (Robinson, 1977, page 23).

Robinson suggested that there is a need to know when some bit of mathematics fits and when it does not. Robinson also concluded that the importance of problem solving as an aim of mathematics instruction is not just that the students be able to solve the problems in the book, the ultimate aim is that they be able to solve problems whenever the need arises. (Robinson, 1977, page 26).

Authors such as Moskol (1980) observed that textbooks often give students applications of mathematics by giving them questions which describe physical quantities. Seldom, if ever, are students encouraged to question the authority of the constants or the relationships described in such questions. (Moskol, 1980, page 154).

Greenes (1981) recommended non-textbook materials at the intermediate and junior high levels as a major media for problem solving. According to Greenes good story problems may be found in textbooks but context often determines the method for their solution.

The Effect Of Extraneous Data Condition On Problem Solving

Arter and Clington (1974) conducted a study about the consequences of irrelevant data and question placement in arithmetic word problems with fourth graders. Ages ranged from eight years, ten months to nine years, eleven months. Arter and Clington found that children solve fewer problems under the extraneous data condition. However, Arter and Clington concluded that (1) irrelevant data required additional processing steps and/or (2) irrelevant data reduce the efficiency of the processing system. (Arter and Clington, 1974, page 30).

Another study of extraneous data conditions was by Blankenship and Lovitt (1976). The authors indicated that learning-disabled children who were of average intelligence were troubled by problems containing extraneous information. The author's finding with regular public school children was that students' ability to solve problems containing extraneous information increased with grade level. (Blankenship and Lovitt, 1976, page 296). Blankenship and Lovitt suggested that if students are unable to solve problems containing extraneous information because of the "rote computational habit" theory, then a technique that stresses careful reading in teaching students to discriminate between relevant and irrelevant informations must be developed. (Blankenship and Lovitt, 1976, page 297).

The above mentioned "rote computational habit" theory which Blankenship and Lovitt referred to was first observed by Earp (1970). This theory states that "frequently children left to their own procedures read only until they encounter a number, then they work with that number and any other they see without actually reading the entire problem". (Earp, 1970, page 577).

Meiring (1980), Lesh (1981), and Blosser (1981) stress that it is vital for the growth of a real world mathematical problem solver to solve problems where some missing information/data must be generated.

In mathematics textbooks, problems with "not enough information" are usually identical to other short word problems, except that some relevant bit of information is missing, and the student is expected to conclude that no solution is possible. (Lesh, 1981, page 17). Applied problems typically require generating and testing hypotheses to determine the goodness of fit of a proposed model which (a) accounts for the given information, (b) fills in the gaps created by missing information, and (c) suggests additional information which may be obtained. (Lesh, 1981, pages 17, 18).

Further References

Criticism of the public secondary schools' mathematics program and its apparent failure to help graduates of public secondary schools function effectively in the "real world" has been expressed elsewhere. Milson (1979) noticed that individuals and groups from divergent segments of society were currently expressing the concern that students in the secondary schools could not read, write or use mathematics. (Milson, 1979, page 695). It was charged that in many cases graduates of our public secondary school could not function effectively in the "real world". (Milson, 1979, page 695).

Milson reported a study in which a questionnaire was administered to geometry classes in a medium-size secondary school. Among the items were "Do you think geometry is a valuable and important subject?" Responses included:

"Geometry is good for thinking, but has no real use in my life.";
"I can not see any application for geometry.";
"Geometry seems to be subject where all we do is argue over picky points. What does one really learn from it." (Milson, 1979, page 695).

Milson summarized the responses to the questionnaire by concluding that in nearly all cases the lack of an awareness of practical everyday applications of geometry is evident. He suggested the development of a practical application resource module. According to Milson, the module would correct the students' negative perception of the secondary school geometry.

The results of a study by Milson, in which a module was used to develop an awareness of the practical side of geometry, indicated that the experimental group had positive changes of perception of geometry. (Milson, 1979, page 697).

Janvier (1981) argued that science and mathematics essentially deal with situations stripped of many factors or with schematical situations. Janvier added that to hasten, and thereby to create artifical contexts, was the main pitfall which we must avoid. (Janvier, 1981, page 120). A central difficulty for many appeared, argued Janvier, to be referring back to the original situations in order to enrich the solution process. (Janvier, 1981, page 121).

Janvier reported a research project in which the interpretation of Cartesian graphs was undertaken. Janvier found that pupils tended to stay at the linguistic or graphical level and experienced trouble in getting out of it. He wondered if the ability to enrich the solution process is a major component of what is traditionally called intelligence or if it is more situation related. (Janvier, 1981, page 121).

Janvier stated that the results of his research project led him gradually to the conviction that the use of situations is not a panacea which guarantees"concretisation" of abstract notions. (Janvier, 1981, page 121).

On problem formulation Kochen, Brade, and Brade (1976) said that formulating a mathematical "story problem" is more difficult than solving one already formulated mathematically. The difficulty, according to Kocher, Brade, and Brade, is that formulation involves recognizing a problem that needs formulation and which arises naturally in a person's daily activities. (Kochen, Brade, and Brade, 1976, page 115).

Kochen, Brade, and Brade remarked that from kindergarten to graduate school we train people primarily to solve well-defined problems that are posed and presented to them. Scientists and engineers are taught in school to solve such problems. Only from their professional experience do they learn to recognize and formulate real problems on their own. (Kochen, Brade, and Brade, 1976, page 116).

Kochen, Brade, and Brade think that training in the recognition and formulation of real problems should be a matter of top priority in formal education. They said:

> because of the immensity of problems that we face but have not been recognized or properly formulated, it is imperative that formal education does not wait until the "scientist" becomes a "professional" in problem-solving before he gets his training in the recognition and formulation of problems. To accomplish such a shift in training priorities, we need a better understanding of how one can and does recognize and formulate a real problem. (Kochen; Brade; and Brade, 1976, page 116).

Bell (1974) wrote an article addressed to the need to use mathematics in real life. Bell's opinion was that any citizen unable to use mathematics readily is already handicapped in understanding many important decisions affecting public policy and his own person life.

Bell pointed out that the trend for every man to need more mathematical understanding to reponsibly cope with the actual world he lives in is nearly certain to continue. (Bell, 1974, page 196).

Bell blamed the school mathematics program for what he called a "clear failure" for at least a majority of people to competently use even quite simple mathematics tools. (Bell, 1974, page 196).

Kulm and Days (1979) conducted a study on the transfer of information acquired in solving four categories of related problems onto a target problem. Kulm and Days found that solving an equivalent problem resulted in transfer for puzzle problems but not for algebraic problems. (Kulm and Days, 1979, page 101).

Another study on solving mathematically related problems was conducted by Silver (1981). In Silver's study students from grade 7 were asked to form groups of problems that were mathematically related and to explain their basis for categorizing them. The 16 problems consisted of four sets of structurally related problems and four sets of problems related in cover story, or problem context. Silver found that good problem solvers tended to form groups of related problems on the basis of common problem structure, whereas poor problem solvers tended to form groups on the basis of common problem details. (Silver, 1981, page 58).

Ogilvy (1962) viewed "practical" problems as problems that could be related in some direct way to everyday life or problems that could be categorized as applied mathematics. (Ogilvy, 1962, page 17). Ogilvy also categorized "the most interesting problems" as the ones that are relatively easy to state, have wide application either in mathematics or elsewhere, and possess a certain intrinsic charm of their own. (Ogilvy, 1962, page 3).

Cohen (1979) studied the relationship between scientific interest and verbal problem solving. Cohen did not arrive at any conclusive answer as to whether there existed a relationship between outdoor, computational, or scientific interest and achievement on a verbal problem-solving test.

Summary

It is clear, from the articles and the research studies covered in this review of the literature, that the area of real world mathematical problem solving is still behind other areas of mathematics education in terms of the amount of the literature dedicated to the area. It is also apparent that the points of disagreement among the authors and the researchers on issues such as "Teaching heuristics: does it improve the ability to solve real world mathematical problems?" and the definition of the real world mathematical problem are still wider.

However, in the articles and the research studies reviewed in this paper, it is unanimously felt that there exist real world mathematical problems, not of the kind we encounter in the usual mathematics textbooks.

Almost all the authors expressed their dissatisfaction with the usual mathematics program, from kindergarten to graduate school, for its apparent failure "real-worldwise".

The literature supports the proposition that problems presented with extraneous data conditions present difficulties for junior graders and has little or no effect with older students. But that proposition is coupled with the researchers' stress that it is vital for the growth of good real world problem solvers to solve problems where some missing information/data must be generated.

Last but not least the authors discussed miscellaneous topics on real world mathematical problem solving. Among the issues were the importance of real world problem solving, students' perception of mathematics units that have little real world applications, the public concern about the inability of today's students to use mathematics in real life, and the formulation of real world mathematical problems.

CHAPTER III

The Process Of Data Collection

Introduction

In order to test the research hypotheses, two forms of mathematical test were developed by this investigator. Form one (Test R) was composed of ten mathematical problems written in the real world mathematical problem format, whereas form two (Test T) was composed of ten mathematical problems in the usual textbook mathematical problem format. It is assumed that the solution of problems in Test T and Test R require identical underlying mathematical skills.

This chapter contains a full desciption of the procedure of a pilot study, the sample, the instrumentation, the procedure of the final study, and the statistical procedure.

The Procedure Of The Pilot Study

A pilot study involving both the T and R forms of the test was conducted in two high schools in the City of Winnipeg. The purpose of the pilot study was to establish readability, content validity, and students' understanding of the wording of the problems of the two forms of the test. The first study involved 30 grade XII students at St. John's Ravenscourt Collegiate who were divided into two subgroups each of which wrote one form of the test on February 8, 1983.

In the pilot study it appeared that 60 minutes was adequate time for all candidates in the two subgroups to complete the test.

The second school was Fort Richmond Collegiate. Thirty students wrote the two forms of the test on February 16, 1983. Again the students were divided, by their classroom mathematics teacher, into two equal subgroups. The two forms of the test were administered during the first two periods of a school day. However, ten subjects were 5 to 10 minutes late for the test. This led to the consideration of only 26 subjects, 13 in each subgroup. Also it should be noted that the subjects did not have enough time to finish writing the test. Added to that was some apathy demonstrated by the subjects.

The results and the outcome of the pilot study (see appendix B) did not suggest any major changes with respect to instrumentation or test administration to be considered for the final study.

The Sample

For the final investigation two high schools, Grant Park and St. John's high schools in the Metropolitan City of Winnipeg, were chosen on the grounds of easy accessibility and the readiness of the school principals, teachers, and the students to cooperate with the investigator. St. John's High School is located in the North End of the City in a working class neighborhood, whereas Grant Park High School is a relatively new high school with new facilities located in a middle class neighborhood in the south of the city of Winnipeg.

In Grant Park High School the 56 students (49 from grade XII and 7 from grade XI) who wrote the test were enrolled in a Mathematics 300 class. The seven grade XI students (3 good textbook problem solvers and 4 less-able textbook problem solvers, 3 males and 4 females) were included to make the total number 60. Fifty six showed up for the first session and 49 for the second session. Only the 47 subjects who attended both sessions were considered for the final analysis.

The grade XII subjects were two separate classes taught by two male mathematics teachers. The subjects were divided into two equal subgroups such that the subgroups could be balanced in terms of the student ability, as good or less-able textbook problem solver, and sex. The subgroups were then randomly assigned to the two forms of the test with each subgroup writing one form of the test in the first session and the other form in the second session.

In St. John's High School the subjects were students from two grade XII classes taught by two male mathematics teachers. The subjects were enrolled in a Mathematics 300 class. The subjects were not randomly assigned; instead the two intact classes were each assigned to either of the two forms of the test with 10 candidates in one class and 17 in the other in the first session; 10 and 16, respectively, in the second session.

Table I illustrates the sample from Grant Park High School in detail.

Table I

Sample From Grant Park High School

	Subgroup RT		Subgroup TR	
	R First Session	T Second Session	T First Session	R Second Session
Females	13	11	9	7
Males	17	15	17	16
Good problem solvers	14	13	14	11
Less-able problem solvers	16	13	12	12
TOTAL	30	26	26	23

Table II illustrates the sample from St. John's High School in detail.

Table II

Sample From St. John's High School

	Subgroup RT		Subgroup TR	
	R First Session	T Second Session	T First Session	R Second Session
Females	6	6	7	7
Males	4	4	10	9
Good problem solvers	5	5	10	10
Less-able problem solvers	5	5	7	6
TOTAL	10	10	17	16

These sample groups and the subgroups there-in were identified and used on the test groups for the assessment of problem solving ability.

Instrumentation

A testing instrument having two parallel forms was designed by the investigator. A real world mathematical problem test (Test R) and the usual textbook mathematical problem test (Test T). Each of the two forms was composed of ten mathematical questions (see appendix A) such that for every question in Test T there was an equivalent question in Test R with the same mathematical skill requirement for the solution of the question. One difference between the two forms was that some Test R questions require the problem solver to generate more "common sense" information or data. This difference may be noticed in questions 9 and 10. In question 9 the radius of the circle is not provided and in question 10 the straight line is not extended to include all the G.p.A. (Grade point Average) points along the axis (see appendix A, Test R Problem #10). The missing data, in both question 9 and question 10, are crucial for the solution of the questions. The other aspect of non-similarity between the two forms was that non-pertinent or extraneous data in Test R questions 3, 4, and 7 must be detected and ignored by the problem solver. In question 3 the distance of racer (2) from the centre of the circles does not matter in the solution of the question. In question 4 the thickness of the walls is a non-pertinent datum, and in question 7 the sizes of the populations of the provinces of Manitoba and Ontario are provided as extraneous data condition.

The ten questions in Test R were classified, in the interest of further analysis, into three categories: -

(1) Questions with extraneous data conditions or questions with non-pertinent data. These were questions 3, 4 and 7.

(2) Questions with missing information or data. These were questions 9 and 10.

(3) Questions that emphasize real life or concrete applications, questions 1, 2, 5, 6 and 8.

The following table illustrates that for every question in Test R there was an equivalent question in Test T with similar mathematical skill requirement for a solution.

Table III

Item Equivalents In Test Forms R And T

TEST R	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
TEST T	#7	#2	#3	#6	#5	#4	#8	#1	#9	#10

The Procedure Of The Final Study

Data Collection: On April 7, 1983 fifty six students at Grant Park High School in the City of Winnipeg, wrote both Test R and Test T. The two forms of the test (Test R and Test T) were written simultaneously by dividing the 56 students into two groups. One group (30 students) wrote Test R and the other group (26 students) wrote Test T.

The students' two classroom mathematics teachers, with a little help from the investigator, supervised the administration of the test. Each student was provided with an answer booklet together with the question sheet. The students were asked to attempt each problem and show all work in the answer booklet. They were also asked to turn in the question sheet together with an answer booklet when they finished writing the test. The use of calculators was allowed, but no mathematics textbooks, notes, or other extraneous materials were allowed.

The maximum time for writing the test was set to be 80 minutes. A 50-minute time was set as the minimum time for students to turn in the answer booklet and leave the classroom. This arrangement was to avoid the likelihood of fast students encouraging slow students to leave the classroom before attempting all the problems in the test.

The test was administered for a second time on April 19, 1983. This session the group that wrote Test R now wrote Test T and vice versa. Twenty-three out of 26 students (in the first session) showed up to write Test R and 26 out of 30 students (in the first session) showed up to write Test T making a total of 49 students in the second session in comparison to 56 in the first session.

The two classroom mathematics teachers in Grant Park High School provided this investigator with a list indicating each student's name, sex, and teacher rating of the student's ability as a good textbook problem solver or a less-able textbook problem solver.

On April 22, 1983 two intact grade XII classes at St. John's High

School in the City of Winnipeg wrote Test T and Test R with 17 students in the first class and 10 in the other respectively. The two classes wrote the two forms of the test simultaneously in one classroom. Test procedures and test administration that were followed in Grant Park High School were exactly repeated with St. John's High School.

On April 27, 1983 the same two forms of the test were administered for a second session to the same two St. John's High School grade XII classes, except that the class that wrote Test R in the first session now wrote Test T and vice versa. Sixteen students out of 17 (in the first session wrote Test R and 10 out of 10 wrote Test T making a total of 26 in comparison with 27 in the first session.

The Statistical Procedure

First, a t-test was used to establish the kind of effect test sequence would have on students' performance. The t-test was to compare the scores of those students who wrote textbook problem test (Test T) first and real world problem test (Test R) second with the scores of those who wrote real world problem test first and textbook problem test second.

Second, 3-Factorial Analysis of Variance with repeated measures was used to determine the interaction of type of question (real world or textbook) with sex, ability, and test results. The application of 3-Factorial ANOVA was contingent upon a finding of no significant difference in testing sequence.

Summary

In the preceding chapter the process of data collection was described, including pilot testing, the instrumentation used in the study, and the sample. This chapter also includes a brief reference of the statistical procedure that was used for the analysis of data. An account of the organization and the statistical analysis of data follows.

CHAPTER IV

Organization And Analysis Of Data

Introduction

Tables IV and V provide the results of the t-tests on testing sequence in both St. John's and Grant Park High Schools respectively. Table VI provides the results of the 3-Factorial Analysis of Variance for sex (S), ability (A) and problem type (P), with repeated measures on the last factor. Table VII provides the results of a 3-Factorial Analysis of Variance for the interaction of the recognition of the real world mathematical problem with sex and ability. Tables VI and VII have been subdivided into a, b, and c, for convenience. Each table is followed by a brief description of the statistical significant of the results.

Tables And Results

Testing Sequence: Tables IV and V provide the results of the t-tests between the results of the students in each subgroup for each of the tests administered at each of the high schools involved in the final study. Subgroup (TR) wrote the textbook form of the test first and the real world form of the test second, whereas, subgroup (RT) wrote the real world form of the test first and the textbook form of the test second. The t-tests test for any sequence effect.

TABLE IV

St. John's High School, Means and Standard Deviations

VARIABLE	NUMBER OF CASES	MEAN	STANDARD DEVIATION	T VALUE	DEGREES OF FREEDOM	2-TAIL PROB.
TEXT						
Subgroup (TR)	16	66.38	15.513			
Subgroup (RT)	10	83.20	12.155	-3.08	22.60	0.005*
REAL						
Subgroup (TR)	16	53.63	16.382			
Subgroup (RT)	10	69.10	16.603	-2.32	19.05	0.031*

*Significant at $\alpha \leq .05$ level of significance

For $\alpha \leq .05$ level of signficance and ($p = 0.005$), it is shown that subgroup (RT) significantly out scored subgroup (TR) on the textbook test. On the real world test ($p = 0.031$), it is also clear that subgroup (RT) signficantly out scored subgroup (TR) at $\alpha \leq .05$ level of significance.

TABLE V

Grant Park High School, Means and Standard Deviations

VARIABLE	NUMBER OF CASES	MEAN	STANDARD DEVIATION	T VALUE	DEGREES OF FREEDOM	2-TAIL PROB.
TEXT						
Subgroup (TR)	21	61.43	16.765			
Subgroup (RT)	26	70.54	18.520	-1.77	44.37	0.084
REAL						
Subgroup (TR)	21	49.95	18.691			
Subgroup (RT)	26	57.92	20.038	-1.41	44.02	0.166

Testing sequence did not have any significant effect on the performances of subgroup (TR) and subgroup (RT) on the textbook form of the test ($p = 0.084$). Also on the real world test the two subgroups' performance was not signficantly affected by testing sequence at $\alpha \leq .05$ level of significance ($p = 0.166$).

Table VI illustrates a 3-Factorial Analysis of Variance test results for the interaction of type of problem (Real world or textbook) with sex and ability (good textbook problem solver or less-able textbook problem solver) in Grant Park High School. The 3-Factorial ANOVA test was contingent upon a finding of no significant difference in testing sequence.

Type of Problem: The Interaction With Sex And Ability

TABLE VI(a)

CELL means for 1-st dependent variable - TEXT, REAL

	Able girls	Less-able Girls	Able Boys	Less-able Boys
TEXT	72.45	47.83	79.38	59.29
REAL	65.45	37.17	59.15	49.59
MARGINAL	68.95	42.50	69.27	56.44
COUNT	11	6	13	17

TABLE VI(b)

STANDARD DEVIATIONS for 1-st Dependent Variable - TEXT, REAL

	Able girls	Less-able Girls	Able Boys	Less-able Boys
TEXT	14.77	12.58	11.45	17.84
REAL	17.56	11.37	18.34	19.66
COUNT	11	6	13	17

TABLE VI(c)

ANALYSIS OF VARIANCE for SEX, ABILITY, AND PROBLEM TYPE

SOURCE	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	F	TAIL PROB.
SEX	763.78	1	763.78	1.67	0.2025
ABILITY	8665.94	1	8665.94	19.00	0.0001*
SA	687.35	1	687.35	1.51	0.2262
(1) ERROR	19607.95	43	456.00		
P	2880.69	1	2880.69	31.13	0.0000*
PS	191.39	1	191.39	2.07	0.1577
PA	59.79	1	59.79	0.65	0.4259
PSA	256.02	1	256.29	2.77	0.1035
(2) ERROR	3979.59	43	95.55		

*Significant at $\alpha \leq .05$ level of significance

From Table VI(c), no significant main effect was found due to sex on type of problem at $\alpha \leq .05$ level of significance. As indicated in Table VI(c), no significant interactions at $\alpha \leq .05$ level of significance were established due to type of problem with sex and ability (PSA).

A significant main effect on type of problem due to ability could be noticed in Table VI(c) where good textbook problem solvers ($\bar{x}$ = 69.13) score significantly higher on both textbook and real world than less-able textbook problem solvers ($\bar{x}$ = 51.33), at $\alpha \leq .05$ level of significance (p = 0.0001). Another significant main effect shows that everyone does significantly better on text ($\bar{x}$ = 66.47), than on real ($\bar{x}$ = 54.36) at $\alpha \leq .05$ level of significance (p = 0.0000).

Table VII illustrates a 3-Factorial Analysis of Variance on the recognition of the real world mathematical problem and its implication with sex and ability. The results were obtained from data gathered in Grant Park High School.

Real World Problem Recognition

TABLE VII(a)

CELL MEANS for 1-st Dependent Variable - Ratio

	Able Girls	Less-able Girls	Able Boys	Less-able Boys
RATIO*	0.81	0.58	0.79	0.70
COUNT	11	6	13	17

TABLE VII(b)

STANDARD DEVIATIONS for 1-st Dependent Variable - Ratio

	Able Girls	Less-able Girls	Able Boys	Less-able Boys
RATIO	0.14	0.15	0.17	0.22
COUNT	11	6	13	17

TABLE VII(c)

ANALYSIS OF VARIANCE for 1-st Dependent Variable - Ratio

SOURCE	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	F	TAIL PROB.
SEX	1.03	1	0.03	0.79	0.3794
ABILITY	0.27	1	0.27	8.40	0.0059[b]
SA	0.05	1	0.05	1.65	0.2064
ERROR	1.39	43	0.03		

*RATIO: Is the ratio of the real world mathematical problems recognized to the real world mathematical problems attempted in the real world mathematical problem form of the test

[b]: Significant at $\alpha \leq .05$ level of significance.

From Table VII(c), no significant main effect was found due to sex on the recognition of the real world mathematical problem at $\alpha \leq .05$ level of significance ($p = 0.3794$). Also there was no significant main effect on students' recognition of the real world mathematic problem as a result of sex-ability (SA) interaction at $\alpha \leq .05$ level of significance ($p = 0.2064$).

The only significant main effect on student's recognition of the real world mathematical problem was due to ability at $\alpha \leq .05$ level of significance ($p = 0.0059$).

Summary

The preceding chapter was concerned with the presentation of tables and results, the description of statistically significant and non-significant main effects on students' performances on textbook and real world mathematical problem forms of the test due to sex and ability of the student as good or less-able textbook problem solver. In the following chapter the educational implications of the results are discussed.

CHAPTER V

Interpretations And Implications Of The Results

Introduction

This chapter is concerned with the interpretation of the statistical significance of the data as well as with the implication of the results within the framework of problem solving in mathematics education. Although broader terms have been used in this chapter, this does not mean that the implications of the results are assumed to extend beyond the actual context in which the study was conducted.

Discussion

The result of a t-test was used to assess whether or not the testing sequence (writing the real world mathematical problem test first and the textbook mathematical problem test latter or vice versa) might have influenced the results. The testing sequence did not have a statistically significant effect on the performance of the two subgroups of students who wrote the two forms of the test in Grant Park High School. However, there was a statistically significant difference between the performances of the two subgroups of students who wrote the two forms of the test in St. John's High School. The disparity in the results of the two schools could be attributed to the fact that the sample of Grant Park High School was divided into two equally balanced groups in terms of ability and sex, and the two subgroups were then randomly assigned to the two forms of the test, whereas the sample of St. John's High School consisted of two intact grade XII classes taught by two classroom mathematics teachers. Therefore it was not surprising that the original differences between the two classes were manifested in the results of their performances with one of the two classes always outscoring the other class irrespective of testing sequence.

The fact that testing sequence did not have a significant impact on students performance in Grant Park High School (this school becomes the focus of subsequent analysis) can be used to support the premise that the real world mathematical problem is not only rich in mathematical application, but it is also, perhaps, the best model of a mathematical problem that may help its solver to solve other types of mathematical problems.

The interaction of type of problem (real world or textbook) with the sex of the student was not statistically significant. And, perhaps, it will stay like that as long as the real world and the mathematical relations that link parts of it to one another remain the same for men and women of this country. The result might have been otherwise in a typical African setting where the real world and its mathematical structure usually experienced by male hunters in the wilderness are by no means similar to the domestic real world usually experienced by the female.

Good textbook problem solvers performed significantly better than less-able textbook problem solvers on both the textbook and the real world mathematical problems. It was to be expected that good textbook problem solvers would excell over less-able textbook problem solvers in solving textbook mathematical problems. But the same argument should not be taken for granted when it comes to the real world mathematical problem test, for the simple reason that being a good textbook problem solver does not necessarily mean being also good at solving real world mathematical problems. For a particular student to be a good real world mathematical problem solver, he/she needs more than the necessary mathematical skills of the usual mathematics textbook problem solving excercise. This argument is supported by the result that everyone, good or less-able textbook problem solvers, did significantly worse on the real world problem test than on the textbook problem test.

The recognition of real world mathematical problems as defined in this paper is the identification of the mathematical formula, theorem, or principle underlying the solution of the problem. Problem recognition is crucial for the solution of real world mathematical problems. If the real world mathematical problem is one intellectual step superior to the usual textbook mathematical problem, then problem recognition becomes an essential tool to reduce the real world mathematical problem to the solution level of the usual textbook mathematical problem.

The results indicate that problem recognition does not significantly vary as a function of the student's sex as it does as a function of the student's ability. However, good textbook problem solvers did recognize a significantly greater proportion of the real world mathematical problems than less-able textbook problem solvers. This finding revives the argument that unless good textbook problem solvers have an intellectual advantage over less-able textbook problem solvers other than just being good in solving textbook problems, it will be difficult to conclude that good textbook problem solvers have relatively better recognition of real world mathematical problems.

It is worth noting that good and less-able textbook problem solvers alike had trouble with the problems that contained extraneous data and the problems that had missing information/data (problems # 3, 4, 7, 9, and

10) in the real world mathematical problem test (see appendix C for a sample of students' solutions of the real world mathematical problem test).

Summary Of The Study

The primary concern of this study was to analyze the recognition, approach, and the solution of real world mathematical problems by a group of grade XII students who acquired mathematical skills through twelve years of studying school mathematics. To accomplish this objective, a 10-question real world problem test was developed by this investigator. Another 10-question textbook problem test was also designed for the purpose of measuring and comparing the performances of the students with respect to the two forms of the test. The two forms of the test were identical in terms of the mathematical skills needed.

The analyses of the recognition, approach, and the solution of the real world mathematical problem would be difficult without taking into consideration the ability of the student (good or less-able textbook problem solver) which should indicate the degree of the students' mastery of the mathematical skills learned from studying mathematics in a classroom setting. Ability and sex were the two independent variables along which subgroup sampling was polarized in Grant Park High School, but it was not possible to do likewise with the sample of St. John's High School nor was it possible to assure ideal testing conditions because intact classes were used.

The research hypotheses were primarily inspired by the review of the related literature and by this investigator's personal experience. The five research hypotheses were all set in the "null hypotheses" form for two reasons: -

(1) to remove any kind of vagueness or ambiguity, and

(2) null hypotheses lend themselves to the statistical procedure used in this study.

Conclusion And Recommendation

It is fair to say that this study is not a replication of any known study in the literature and did accomplish its objectives. The four statistical null hypotheses (research hypotheses) were supported by the data at $\alpha \leq .05$ statistical significance.

The research questions were answered as follows: -

1. Do good textbook problem solvers and less-able textbook problem solvers differ in the recognition, approach, and solution of the real world mathematical problem?

Yes they do differ. Data do suggest that good textbook problem solvers perform better on a real world mathematical problem test than do less-able textbook problem solvers. Also, good textbook problem solvers recognize a greater proportion of real world mathematical problems than do less-able textbook problem solvers. However, good and less-able alike have trouble with real world mathematical problems than with textbook mathematical problems.

2. Do female and male students differ in the recognition, approach, and solution of the real world mathematical problem?

There is no evidence in the data to suggest that sex is a factor in students capability to recognize, approach, and solve the real world mathematical problem.

Solving a related textbook mathematical problem does not seem to help students to recognize and hence solve real world problems. This is supported by the fact that testing sequence does not have a significant effect on performance.

The data also suggest that good and less-able textbook problem solvers alike experience trouble with those real world mathematical problems that were fitted with non-pertinent data as well as with those problems that had missing information/data. This finding confirms the finding of Arter and Clington (1974) about the consequences of irrelevant data and question placement in arithmetic word problems with fourth graders. But it obviously contradicted Blankenship and Lovitt's (1974) finding with regular public school children that student's ability to solve problems containing extraneous information increased with grade level.

If the findings of this study have any significance at all, it should be the suggestion that no matter how good a student is in solving usual textbook mathematical problems, he/she needs to be exposed to some sort of problem solving experience beyond the traditional textbook problem solving in order to function "mathematically" in the real world. That "some sort" of problem solving experience could be by encouraging students to observe and examine carefully and critically those forms of mathematics and mathematical relations they encounter at home, in the store, or in the street. Literally, have them search for mathematics and mathematical relations anywhere in their real world. For younger pupils, real world mathematical problem solving experience could be established by drawing their attention to geometric figures on the traffic signs. Older students could be encouraged to examine critically the mathematical principles used in designing playgrounds, racetracks, spider webs, and many other natural and man-made objects. Out-door-education is another discipline that could be integrated with the real world problem solving experience.

Real world mathematical problem solving experience would, probably, assure the student that the mathematics he/she learns from mathematics textbooks in the classroom is not a meaningless isolated bit of knowledge but is part of the over all dynamic action that is responsible for the equilibrium of the universe at large.

Recommendation For Further Research

The Instrument: -

If the investigator in a further research should develop a set of real world mathematical problems for testing purposes, then it would be advisable to formulate the problems from real events, or real "mathematical situation" in the real world of the student. It would be meaningless if the real events, that are to be mathematically formulated, are of no interest to the student. It must be admitted that formulating a real world mathematical problem is not easy and caution must be taken when formulating real world mathematical problems in the categories of missing information/data, or extraneous data condition problems. This is because withholding critical information/data would turn the problem into a riddle, and inserting extraneous frivolous data would be an affront to the intelligence of the student.

Methodology: -

It would be more clinical and experimental in a further research if the investigator could expose an experimental group to a three-week real world mathematical problem solving exercise where the group will be solving real world mathematical problems while the control group solve just textbook mathematical problems.

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APPENDIX A

(i) A letter to the student soliciting his/her cooperation.

(ii) Ten questions of the real world mathematical problem test (Test R).

(iii) Ten questions of the textbook mathematical problem test (Test T).

E UNIVERSITY OF MANITOBA

FACULTY OF EDUCATION
Department of Curriculum:
Mathematics and Natural Sciences

Winnipeg, Manitoba
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Dear Student:

The mathematical problems you are about to try to solve are by no means intended to test "how much mathematics do you know" as they are intended to help explore new frontiers of scientific inquiry in mathematics education. Therefore, your cooperation is fully appreciated for the benefit of all. Wishing you the best of luck in your studies.

Sincerely,

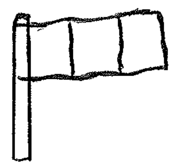
Researcher: O. Geil

MATHEMATICAL PROBLEM SOLVING

TEST R

Candidates will attempt each problem and show all work in the answer booklet.

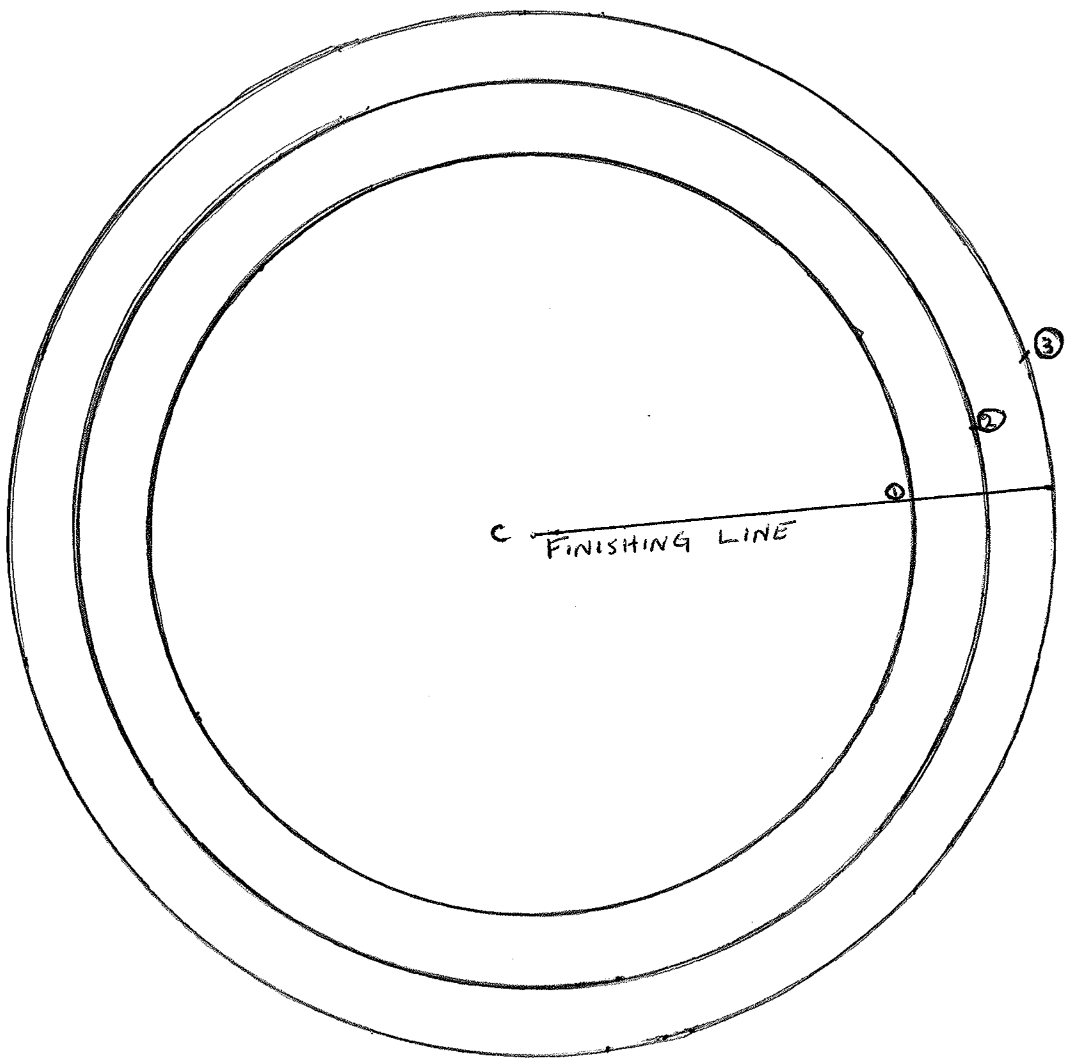
1. The Manitoba Association of School Trustees requested that each school division should have its own three-color rectangular flag in the upcoming Manitoba Schools Parade. Each flag will be shaped as follows:



The Department of Education has approved only five colors (green, red, white, yellow and black) from which each school division must choose any three different colors which will be placed in the flag in any order. How many of Manitoba's 80 school divisions could have a unique flag?

2. You might have heard of the incident in the Mediterranean Sea during September 1981, when the Libyan war jets attacked two maneuvering U.S. war jets for an alleged invasion of the Libyan territorial waters by the U.S. jets. However, international law determines the Libyan Territorial waters as the path of a point that moves such that three times its distance from Greenwich added to two times its distance from the Equator is equal to 4502 km. According to a source based upon Satellite intelligence, the U.S. jets were spotted maneuvering at a position 500 km from Greenwich and 1500 km from the Equator. At that position two Libyan war jets fired at the U.S. planes. From the given mathematical data, tell whether or not the U.S. jets did, indeed, invade the Libyan territorial waters.

3.



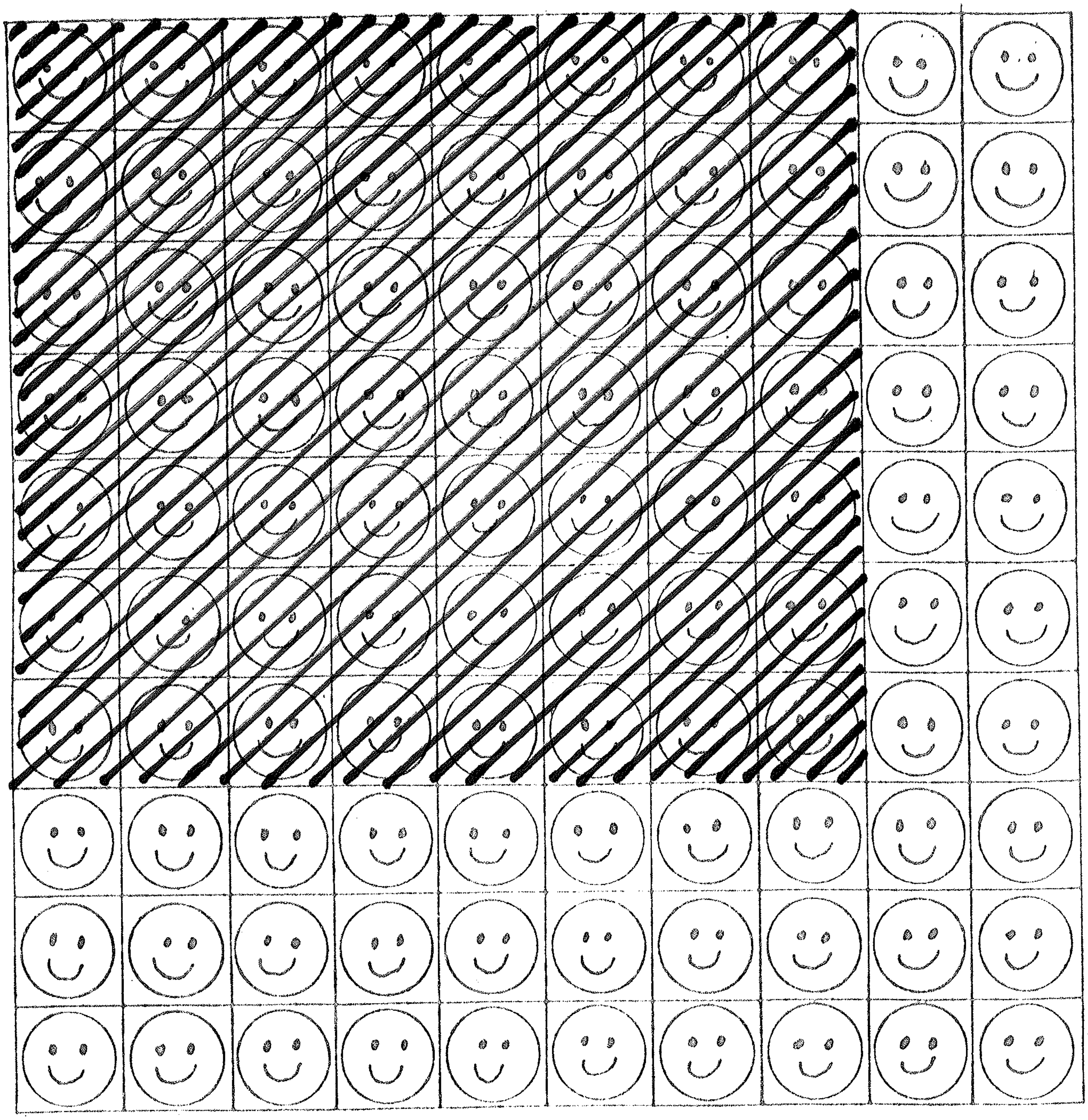
On this circular running track, three racers are to start a race. Racer 1's starting point is right on the finishing line (see the figure) but the other two racers' starting points are at various distances from the finishing line along their respective circles. Racer 1 is 100 m from the central point C of the three circles, racer 2 102 m, and racer 3 104 m from the same point C. How many meters from the finishing line should you set the starting point of racer 3 in order for him/her to complete evenly with racers 1 and 2?

4. Your classroom is 8 m long, 6 m wide, and the walls (0.5 m thick) are $\sqrt{21}$ m high. If you extend a string from one of the corners at the floor to the opposite corner at the ceiling, how long would the string be?

5. Statistics Canada reports that the population of Canada increased by 100,000 persons every year over the preceding year since 1980. Canada's population was 20,000,000 on January 1, 1981. What is your projection of Canada's population at the end of 2001 if the increase continues in the same pattern?

6. There are two flocks of birds. One flock is on the ground and the other flock in four trees. If one bird from the ground joins the other flock in the trees, the two flocks become equal in number. But, instead, if one bird from the flock in the trees joins the flock on the ground, the flock on the ground becomes twice as big in number as the flock in the trees. Find how many birds were originally in each of the two flocks.

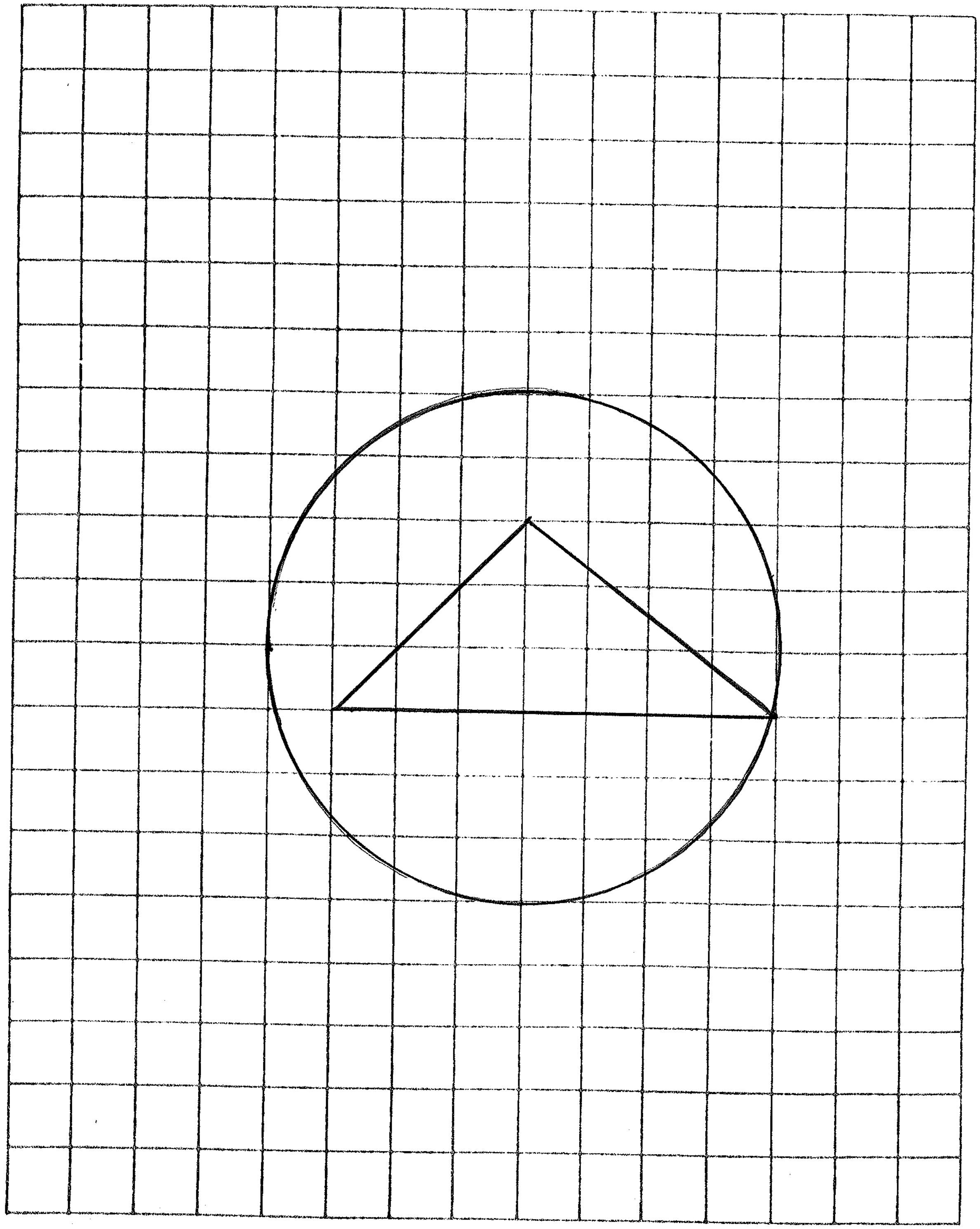
7. It has been suggested that the Canadian Schools' Students Camping should take place in Niagara Falls in the summer of 1983. The Province of Ontario, population 8 million, will be represented by 500 students out of its 2 million school students. If the provinces are to be represented proportionally to the student populations, and Manitoba has one million people of whom 200,000 are students, how many students will represent Manitoba?

8. 

Let $X = 10$ heads (see the above figure). There is a total of 100 heads in the above figure, and that is to say, there are X^2 heads in the figure. In terms of X and X^2 express the number of heads in the shaded area.

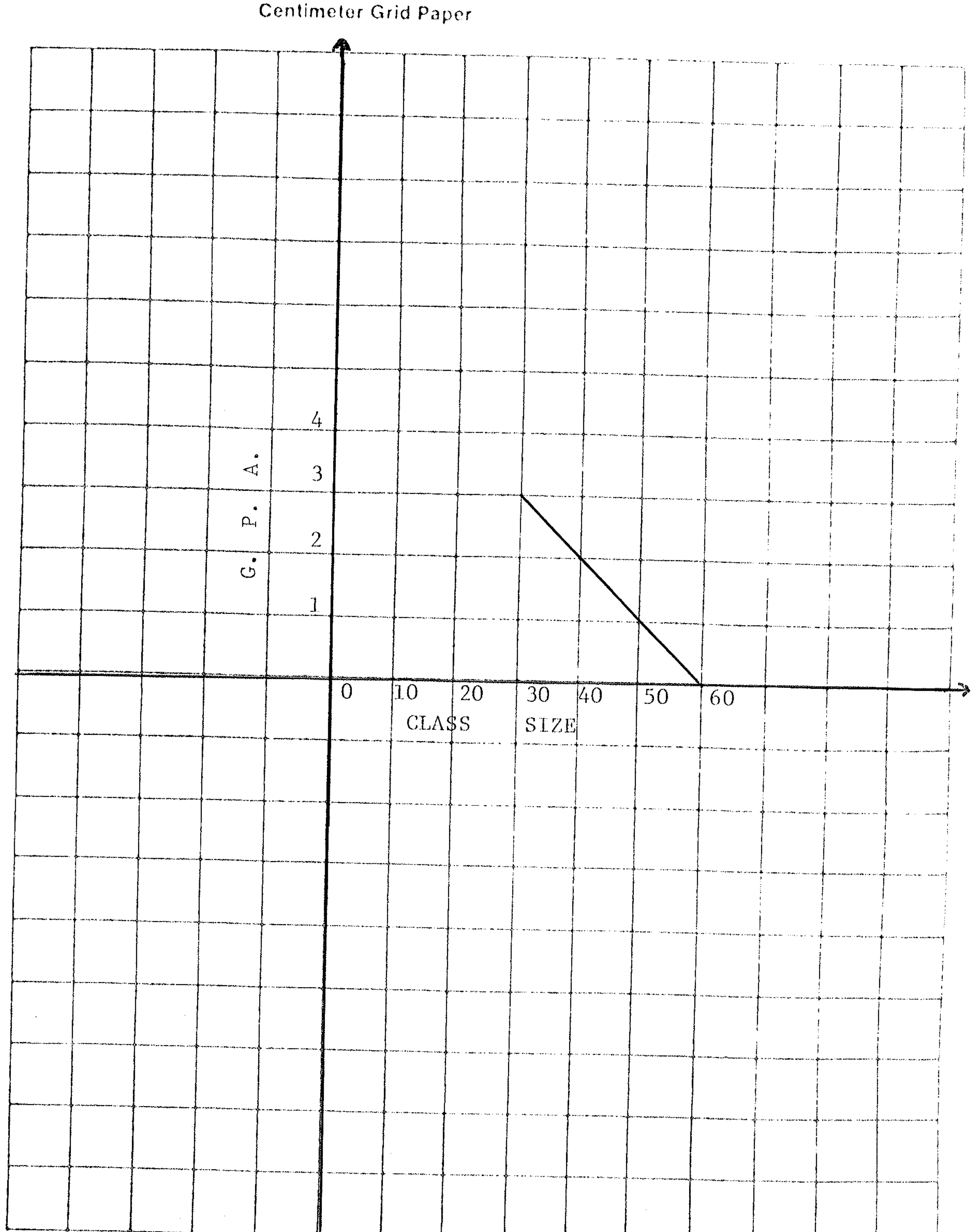
9.

Centimeter Grid Paper



Find both the area of the circle and the area of the triangle in the above figure. Remember each square in the grid is 1 sq. cm.

10.



In a study, the quality of education in Canadian Schools (elementary-senior high) measured by Grade Point Average (GPA) from 0 to 4 was plotted against the average size of the class (number of students per class). The graph yielded a straight line (see the figure). From your reading of the graph (1) express in your words the type of the linear relationship between the quality of education and the size of the class; (2) How large should the average size of the class be in order to have the best possible quality of education?

Researcher: O. Geil

MATHEMATICAL PROBLEM SOLVING

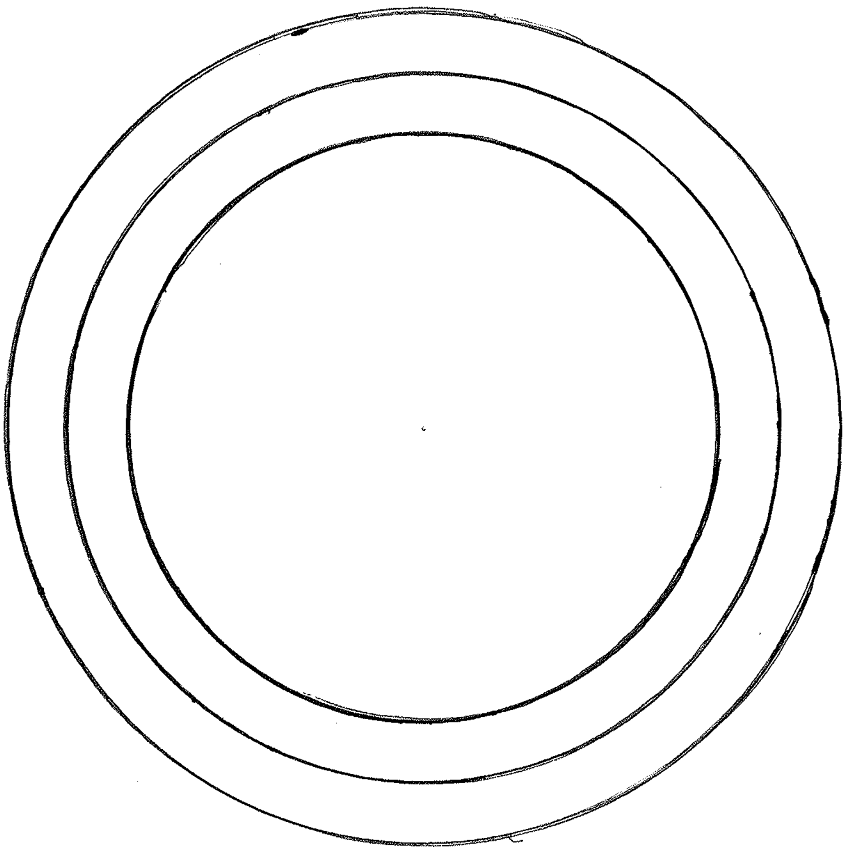
TEST T

Candidates will attempt each problem and show all work in answer book.

1. Find the area of a rectangle that is (X-2) cm long and (X-3) cm wide.

2. If 4X-3Y=12 is the equation of a line and (4,5) is a point in the X-Y plane, show whether the point (4,5) is on the line 4X-3Y=12, above the line, or below the line given by the equation 4X-3Y=12.

3.



The above three concentric circles have radii of 5, 7, and 10 cm. Find the difference between the circumferences of the smaller (radius 5 cm) and the larger (radius 10 cm) circles.

4. Suppose you have X dollars and your friend Y dollars and the following two statements are true:

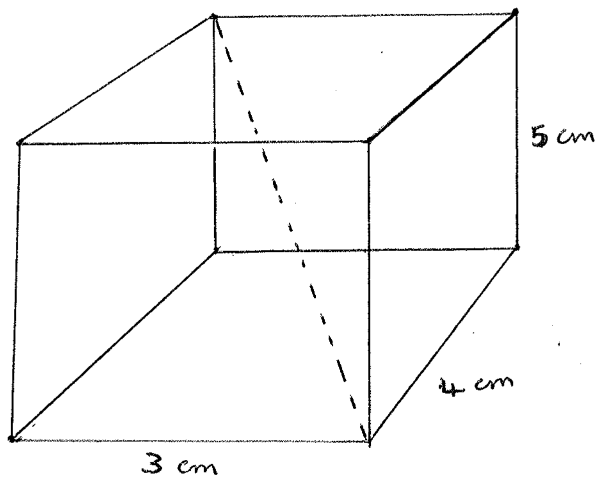
$$X - Y = 2$$
$$-X + Y = 2(Y-1)$$

Find the values of X and Y.

5. 7, 9, 11, 13, ...

The above series starts with 7 and increases by 2. Calculate the 300th number in the series.

6.



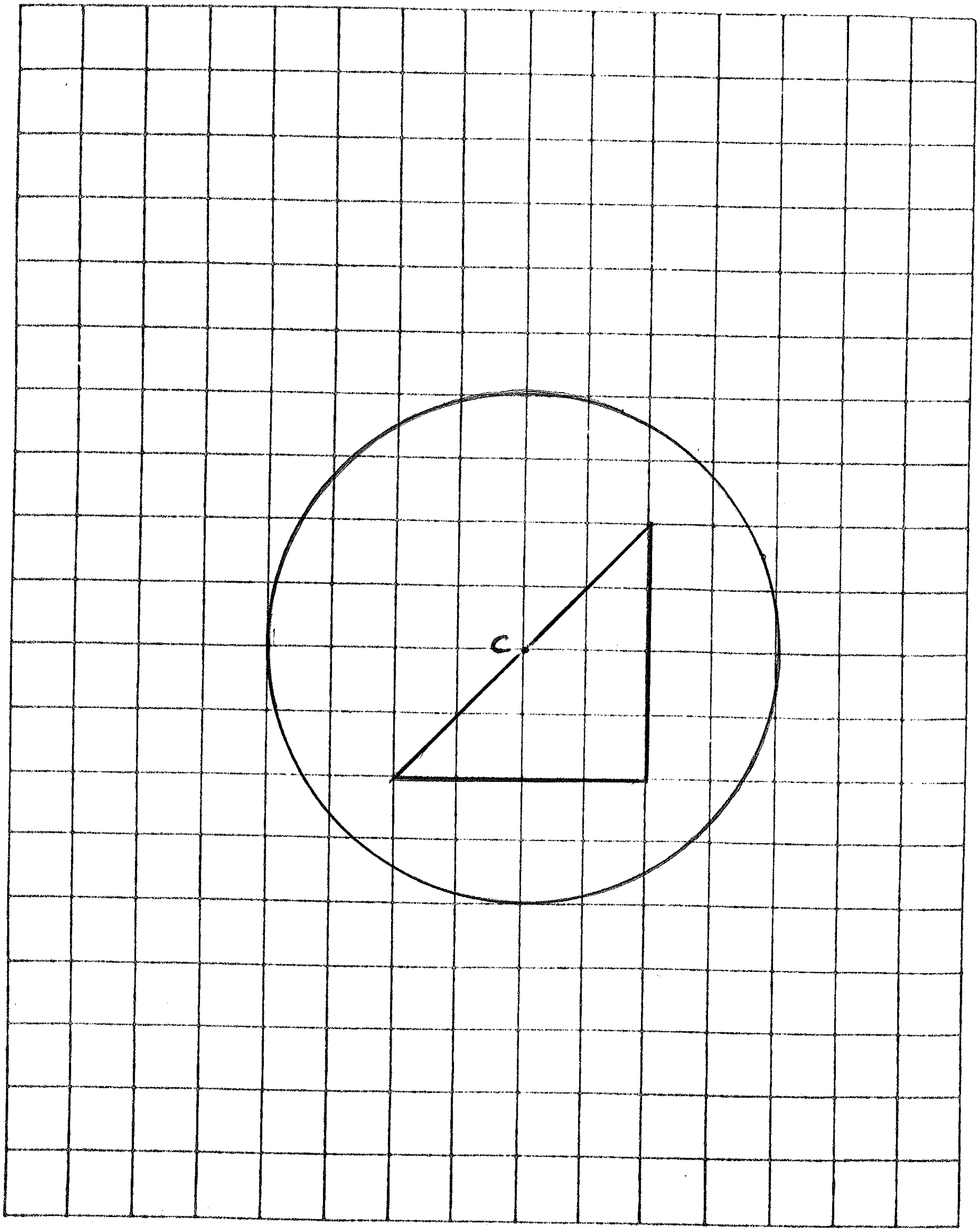
Find the length of the broken line (the diagonal of the cube). The dimensions of the rectangular solid (box) are 3, 4, and 5 cm as shown in the figure.

7. How many three-digit figures can you get out of the set of the numbers (1, 2, 3, 4, 5) if no digit may be used twice. For example, 534 is a three-digit figure from the set.

8. If in every single stick of the chewing-gum you chew 0.5 gm of sugar, suppose you and your playmates need 150 gm of sugar to keep you active in a football game. How many sticks of chewing-gum will you need?

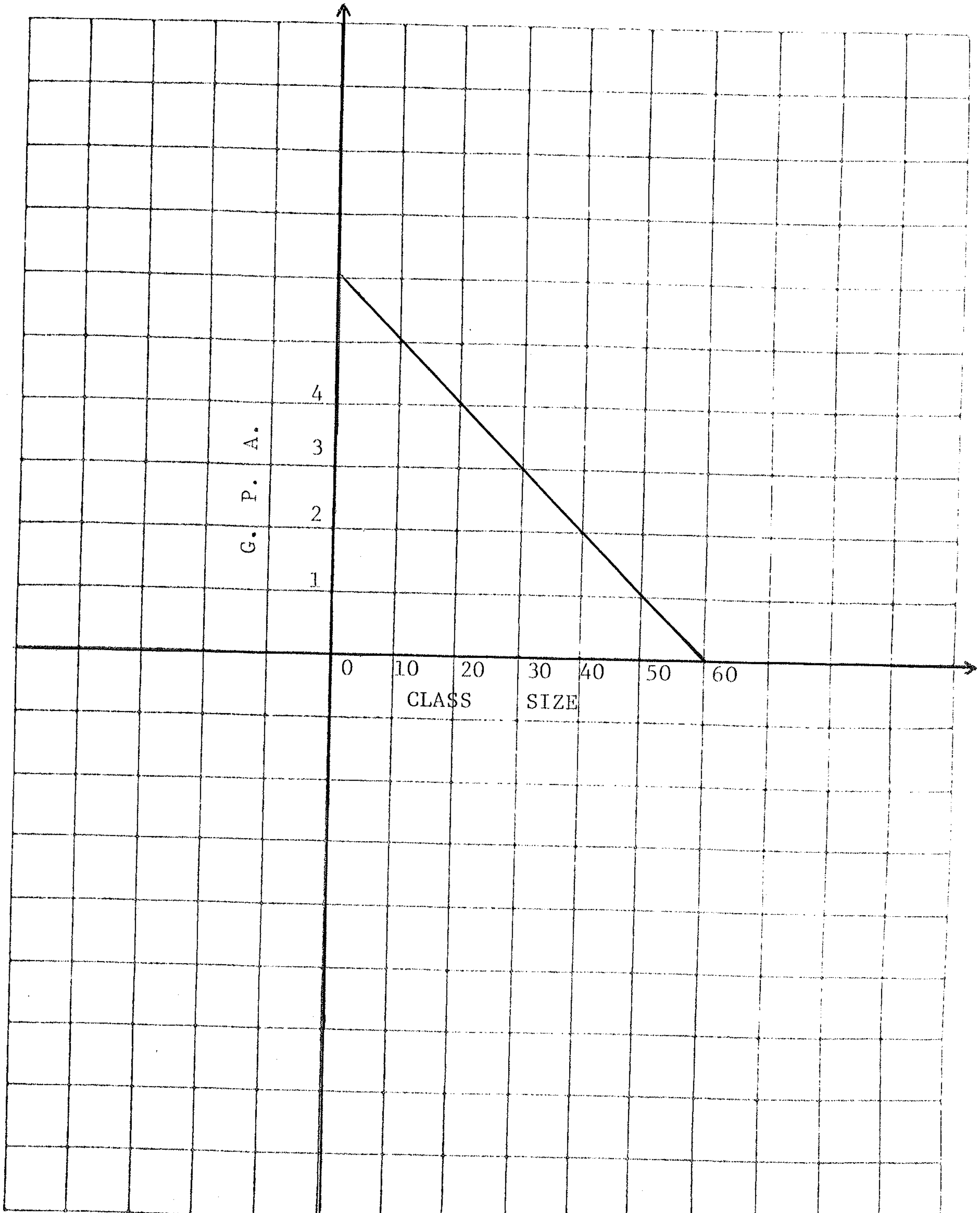
9.

Centimeter Grid Paper



Find both the area of the circle and the area of the triangle in the above figure. Remember each square in the grid is 1 sq. cm. and c is the centre of the circle.

10. Centimeter Grid Paper



In a study, the quality of education in Canadian Schools (elementary-senior high) measured by Grade Point Average (GPA) from 0 to 4 was plotted against the average size of the class (number of students per class). The graph yielded a straight line (see the figure). From your reading of the graph (1) express in your words the type of the linear relationship between the quality of education and the size of the class; (2) How large should the average size of the class be in order to have the best possible quality of education?

APPENDIX B

Tables providing the results of the pilot study in St. John's Ravenscourt and Fort Richmond Collegiates.

ST. JOHN'S RAVENCOURT COLLEGIATE, 15 Candidates

Table (1)

Scores and means of the candidates on the two forms of the test.

REAL WORLD	SCORE	85	74	71	68	65	63	62	61	59	52	49	43	41	41	39	$\bar{x} = 58.2$
TEXTBOOK	SCORE	97	88	88	85	80	78	76	75	75	73	70	65	60	60	58	$\bar{x} = 75.2$

Table (2)

The number of candidates who scored correct answers on each question of the real world problem test.

Question #	9	1	2	4	7	10	3	5	6	8
# of Candidates with correct answers	14	12	11	11	11	11	10	9	9	4

FORT RICHMOND COLLEGIATE, 26 Candidates

Table (3)

TEST T TEXTBOOK PROBLEMS			TEST R REAL WORLD PROBLEMS		
SEX	Teacher rating of the candidates	SCORE	SEX	Teacher rating of the candidates	SCORE
M	AVERAGE	90	M	GOOD	79
M	AVERAGE	75	M	AVERAGE	78
F	GOOD	73	F	AVERAGE	76
F	POOR	67	M	GOOD	74
M	AVERAGE	67	F	GOOD	71
F	AVERAGE	66	F	POOR	64
F	AVERAGE	66	F	GOOD	64
F	AVERAGE	59	M	AVERAGE	62
F	AVERAGE	58	F	GOOD	54
M	POOR	50	M	AVERAGE	52
M	GOOD	47	M	POOR	28
M	GOOD	46	F	AVERAGE	22
F	AVERAGE	44	M	AVERAGE	21

TEST T	TEST R
$\bar{x} = 62.15$	$\bar{x} = 57.31$
FEMALES $\bar{x} = 64.83$	FEMALES $\bar{x} = 58.50$
MALES $\bar{x} = 62.50$	MALES $\bar{x} = 62.17$
GOOD $\bar{x} = 55.3$	GOOD $\bar{x} = 68.4$
AVERAGE $\bar{x} = 65.6$	AVERAGE $\bar{x} = 58.0$

APPENDIX C

Two samples of the students' solutions of the real world mathematical problem test (Test R) questions. Sample (1) represents full recognition, approach, and solution of the real world mathematical problem on Test R by a student who was either a good textbook problem solver (G) or a less-able textbook problem solver (L.A.) as indicated. Sample (2) represents students' solutions which did not fully meet the criteria for full recognition, approach, and solution of the real world mathematical problem on Test R.

SAMPLE (1)

1/ There are 5 possible colours and each flag uses 3 colours.

∴ no of flags combinations possible

$= P(5,3)$

$= \frac{5!}{2!} = \frac{120}{2} = 60$ ✓ 10 G

∴ 60 of the 80 ~~sho~~ school divisions could have a unique flag.

~~2/ Let x be the point whose path determines the Libyan Territorial waters.~~

3

2/ 3 (distance from Greenwich) + 2(distance from Equator) = 4502 km

distances given:

distance from Greenwich = 500 km

" " Equator = 1500 km

$3(500\,km) + 2(1500\,km)$

$= 1500\,km + 3000\,km$

$= 4500\,km$

10 G

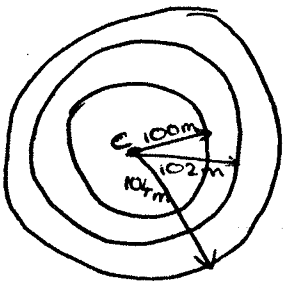
~~Since~~ the border of the Libyan Territorial Waters satisfies

3(distance from Greenwich) + 2(distance from Equator) = 4502 km

$4500\,km < 4502\,km$

∴ U.S. jets __did__ invade ~~L~~ Libyan Territorial Waters

3/



Racer ① will run a distance ~~equ~~ equal to the circumference of the innermost circle

$C = 2\pi r$

$= 2\pi(100m)$

$= 200\pi\,m$ $(\doteq 628m)$

G

In order for Racer ③ to compete evenly he must also run a distance of $200\pi\,m$.

$C_{\text{outer circle}} = 2\pi(104) = 208\pi\,m$

10

$208\pi\,m$

$-\ 200\pi\,m$

$8\pi\,m$ ⟹ Racer ③ must start $8\pi\,m$ $(\doteq 25m)$ from the finishing line.

4/ ~~8~~ classroom → 8m long

6m wide

$\sqrt{21}$ high

G

Since measurements are taken from inside, the wall's thickness doesn't affect the measurements.

Length of a diagonal across the floor = ~~?~~ x = ?

$$(\text{length})^2 + (\text{width})^2 = x^2$$

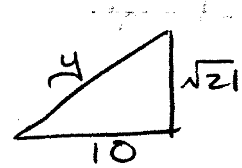
$$64 + 36 = x^2$$

$$100 = x^2$$

$$\underline{x = 10}$$ ✓

10

~~A stri~~ The extended string will form the hypotenuse of a right Δ:



$$y^2 = (10)^2 + (\sqrt{21})^2$$

$$y^2 = 100 + 21$$

$$y^2 = 121$$

$$\underline{\underline{y = 11}}$$ ✓

∴ length of string = 11

5) $t_1 = 20{,}000{,}000$ $t_n = ?$ $n = 20$ $d = 100{,}000$

10

$$t_{20} = t_1 + (n-1)d$$

$$= 20{,}000{,}000 + 19(100{,}000)$$

$$= 20{,}000{,}000 + 1900{,}000$$

$$= 21{,}900000 \text{ people.}$$ ✓

G

6) $x \rightarrow$ flock on the ground
$y \rightarrow$ flock in the trees

① $x - 1 = y + 1$
② $2(y-1) = x + 1$
$2y - 2 = x + 1$

① $x - y = 2$
② $-x + 2y = 3$

G

Using ①
$x = y + 2$

10

Subst in ②
$-(y+2) + 2y = 3$
$-y - 2 + 2y = 3$
$y = 3 + 2$
$y = 5$

Subst in ①
$x = 5 + 2$
$= 7$

There are 7 birds in the flock on the ground and there are 5 birds in the flock in the trees.

10

7.) $\dfrac{500}{2,000,000} = \dfrac{x}{2,000,00}$

$\therefore$ $2,000,000\,x = 100,000,000$
$x = 50$ students

G

8) $X = 10$ heads

X^2 heads in the figure

heads in shaded area 10

$= (X-3)(X-2)$

$= X^2 - 2X - 3X + 6$

$= X^2 - 5X + 6$

G

9.) area of triangle $= \frac{1}{2} bh$

$= \frac{1}{2}(7)(3)$

$= \frac{21}{2}$

10 $= 10.5$ sq. cm

Area of circle $= \pi r^2 = 3.14(4)^2$

$= 3.14(16)$

$= 50.24$ sq. cm.

---

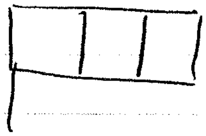
10) 1) The Grade point Average is inversely proportional to the class size.

2) 20 students per class to obtain the highest G.P.A.

10

L.A.

SAMPLE (2)

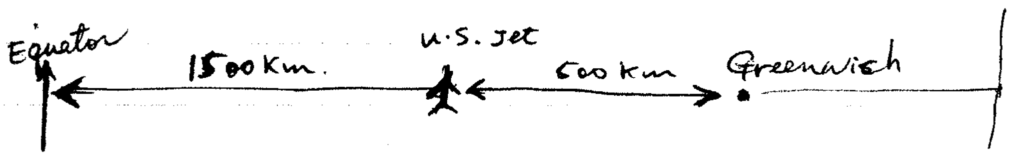
1)  only five colors

there are 54 ways to place three different colors in the flag. ✗

4 G

There are 80 school

$\therefore$ there are $80 \times 54 = 4320$ flag in three different colors. ✗

2) 

If the Jet is is at a distance less than 4502 Km

$3x + 2y = 4502$

$500 + x = 1500 + y$ ~~1500 - y~~ $y - 1500 = x + 500$.

$3x + 2y = 4502$

$y - 1500 = x + 500$ G

~~$x = y$~~

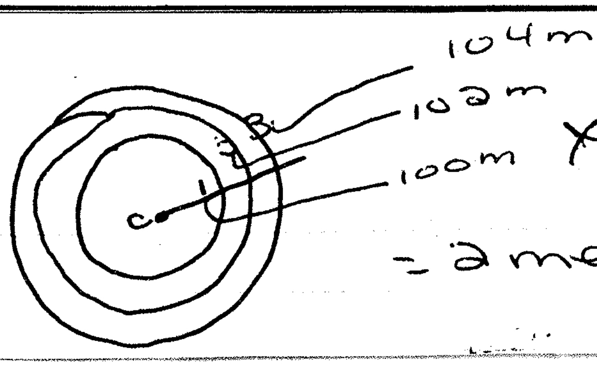
$2x - 2y = -4000.$

$3x + 2y = 4502.$

$5x = 502$

$x = 100.4$ Km. ⟶ the jets is 100.4 Km away from Greenwich.

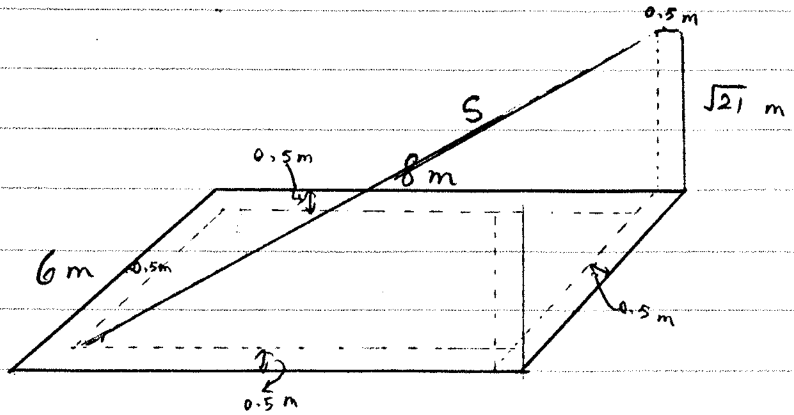
$3 \times 100.4 = 301.2$ .Km. ✗

3) 

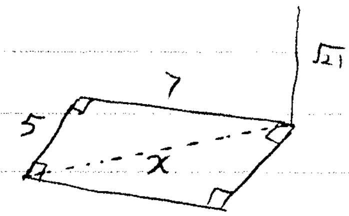
✗ / 0 L.A.

= 2 meters.

4)



G



actual length of classroom $= 8 - 0.5 - 0.5$

$= 7$ m

actual wide of classroom $= 6 - 0.5 - 0.5$

$= 5$ m

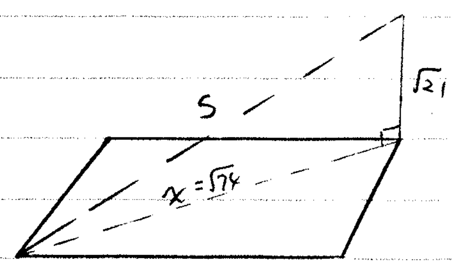
$x^2 = 5^2 + 7^2$

$x^2 = 25 + 49$

$x^2 = 74$

$x = \sqrt{74}$ m

4



let S be the length of the string

$S^2 = (\sqrt{74})^2 + (\sqrt{21})^2$

$S^2 = 74 + 21$

$S^2 = 95$

$S = \sqrt{95}$ m

Ans: The string would be $\underline{\underline{\sqrt{95} \text{ m}}}$.

5.) 100,000 persons have increased every year since 1980.

Population of Canada in 1981 is 20,000,000.

$2001 - 1981 = 20$ yrs (L.A.)

$20(100{,}000) = 2{,}000{,}000$

$(20{,}000{,}000)(2{,}000{,}000)$

4

$= 4 \times 10^{13}$

The population of Canada will be 4×10^{13} in the year 2001

6) 2 flocks of birds

1 flock in four trees

1 " on the ground.

let n = flock in four trees

s = flock on ground

① $s-1 = n+1$ ✓

② $n-1 = 2s$

L.A.

6

① $s-1 = n+1$

$s = n+2$

Subst.

$s = -3+2$

$s = -1$

subst.

$n-1 = 2s$

$n-1 = 2(n+2)$

$n-1 = 2n+2$

$-3 = n$

∴ the flock of birds in the four trees is 1 & the flock of birds on the ground is 3. ✗

7) percentage of students of population in Ontario

$= \frac{2}{8} \times 100\%$

G

$= 25\%$

percentage of students of population in Manitoba

$= \frac{200,000}{1,000,000} \times 100\%$

$= 20\%$

Let x be the students to represent Manitoba

$$\frac{500}{25} = \frac{x}{20}$$

4

$$x = \frac{500 \cdot 20}{25}$$

$$= 400$$

Ans: 400 students

8) 7×8

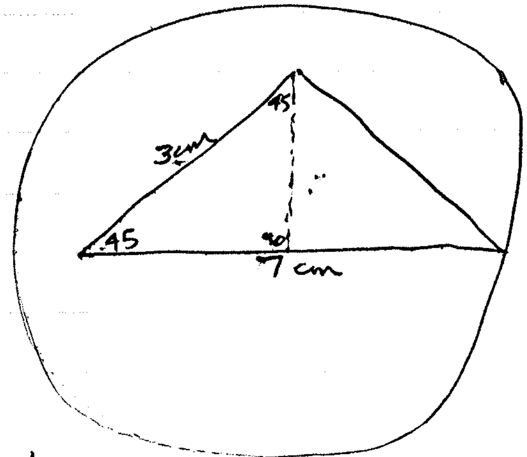
Let $\boxed{x = 7 \text{ heads}}$

4 G

there is a total of 56 heads.

That is to say, there are $\boxed{x^2 + x}$ heads in the shaded area.

9)



area of circle

$= \frac{1}{2}\pi r^2$

$\frac{1}{2}\cdot\pi\cdot 4^2$

Area Circle 25.13 sq. cm.

area $\Delta = \frac{1}{2} b \cdot h$

$7.42 = \frac{1}{2}\cdot(7)\cdot 2.12$

area of triangle

4

$h = \sin 45 = \frac{O}{H}$ $\sin 45 = \frac{O}{3} = 2.12$

G

10) ① As the class sizes increase from 30 students per class beyond this the grade point average goes constant down by 1 ~~point~~ per every 10 extra people

G

② The average class should be 30 people to fulfill ~~most~~ highest education level.