A MULTIOBJECTIVE APPROACH TO THE CONSIDERATION OF RELIABILITY IN URBAN WATER DISTRIBUTION NETWORKS

by

Anthony John Kettler

A thesis
presented to the University of Manitoba
in partial fulfillment of the requirements
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in

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ABSTRACT

A multiobjective linear programming model which considers the tradeoff between cost and reliability has been developed for the design of looped water distribution networks. The technique, which generates a set of alternative systems, is similar to the constraint method, where a set of reliability constraints based on pipe failure data are varied. Pipe failure data obtained from the City of Winnipeg showed a decreasing relationship between pipe diameter, which is the linear programming decision variable, and failure incidence. Using this relationship, a reliability constraint limiting the expected number of failures per kilometre per year is applied to each link in the system. The reliability constraints were varied in a stepwise fashion to improve the reliability of the path having the lowest Poisson based probability of having zero failures in a given year. Since a path is typically made up of several links, dual variables are used to identify the link whose reliability can be improved at least cost to the system.

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CHAPTER I

INTRODUCTION

One of the most efficient optimization/operations research approaches developed for water distribution network design is based upon the well known linear programming formulation. These linear programming water distribution network design models have progressed from simple branched network designers (Karmeli et al. (1968)) to more sophisticated looped system models capable of optimal layout and sizing of components for various loading conditions (Morgan (1983)). However, none of these models consider that performance will also be governed by such factors as soil type, system operation, pipe bedding and cover, soil shifting, traffic loads, joint type, temperature, and pipe characteristics. That is, no 'optimal' design procedure explicitly considers reliability of some or all system components.

Some earlier simulation based models have, however, addressed reliability in some form. Damelin et al. (1972) developed a simulation model where reliability of a water supply system is related to the amount of water the system fails to supply in a given time period. Shamir and Howard (1981) refined this concept by developing a screening model to be used prior to simulation of the system. These procedures stop far short of explicity considering reliability in the actual least cost design of the network.

The purpose of this thesis is to develop a linear programming optimization model for the explicit consideration of reliability in the least cost design of looped water distribution networks, where reliability of a network is related to records of failure incidence.

Research into the failure of water mains has been going on for many years. As early as the 1920's Smith and Shipley (1921), and Smith (1921) were carrying out studies related to the corrosion of cast iron in difficult soils. Over the years a great deal of knowledge has been gained with regard to pipe characteristics and environment, and how these factors relate to failure incidence in a general sense. With the advent of computerized record systems intricate leak records can be kept and utilized in sophisticated pipe failure analyses, theoretically enabling pipe and environmental variables to be related to failure incidence.

The design model developed here will take failure incidence into account, where layout and flow pattern are given, and system component (pipe) size is to be optimized with respect to cost under certain reliability constraints. Networks considered will be single source with network layout and pressure distribution assumed to be optimal.

CHAPTER II

LITERATURE REVIEW

2.1 Linear Programming Models in Water Distribution

In one of the early uses of optimization models for water distribution system design Karmeli et al. (1968) developed a linear programming model for least cost design of simple branched water distribution networks. In this formulation the objective function minimizes total system cost, defined by the sum of pipe and pumping costs. Minimum heads are specified at demand points such that adequate service pressures are provided for the given outflows. Because outflow for each demand point is known, flow along each link in the network is known. Hence, for a given pipe diameter and roughness coefficient, unit length head losses for all possible pipe sizes in all specified links may be calculated by the Hazen-Williams equation and used as coefficients for the head loss constraints. Length constraints are specified to ensure that the sum of the lengths of pipe selected for a given link is sufficient to cover the distance between the nodes at each end of the link.

This technique is well-suited to systems where supply pressure is given. However, it does not consider alternate supply or layout, storage, looping, or multiple demand patterns.

Bhave (1979) developed a technique to decrease the size of the linear programming model for branched water distribution network designers, such as that put forth by Karmeli et al. (1968). The method allows the designer to reduce the set of candidate diameters by using a critical path approach.

Schaake and Lai (1969) addressed the problem of system design capacity expansion through the use of linear and dynamic programming. The linear programming method is an approximation of a non-linear formulation to find optimum pipe sizes where the objective function and node continuity constraints are approximations of non-linear expressions. The authors justify this transformation by pointing out that a reasonably accurate piece-wise linearization may be used for the objective function. For this approach pressure distribution for looped systems must be prespecified, while pressure distribution for branched networks is optimized.

The objective function of the Schaake and Lai (1969) model consists of capital costs for pipes and power costs for pumping, where a continuous cost function is used for pipe costs. The continuity constraint at each node ensures that flow into and out of a node balances. The Hazen-Williams equation is used here to account for prespecified head losses between nodes, and where the continuous decision variable is directly related to pipe diameter. Constraints for multiple demand patterns are also developed.

Kally (1972) finds the least cost branched or looped network by using linear programming and a combination of linear programming and Hardy-Cross respectively. Following each solution of the linear programming the actual pressure at each node is examined to determine whether there is an excess or deficiency of head at that point. Knowing that the pressure at a given node is a function of head at the source, and of the upstream pipe diameters, pipe diameters upstream are then modified to correct the situation in the linear programming. This formulation is valid for branched systems only, and must be used as an approximation for looped systems, where Hardy-Cross is used after each linear programming iteration to restore hydraulic consistency. If minimum pipe sizes are not specified in the looped system design approach, the system will become branched, as this is the ultimate least cost system.

The objective function of the Kally (1972) formulation is unusual, in that the cost is associated with the length of pipe changed from one diameter to another. Pressure constraints for nodes specify a maximum difference between existing and required head at a junction where change in head is related to the length of pipe of changed diameter in a path to the junction in question.

Alperovits and Shamir (1977) suggested a linear programming design method for looped systems in which the basic framework of the formulation is identical to that of Karmeli et al. (1968). This approach, however, is also capable of considering looping, improving flow distribution, reservoirs, and multiple loading together with valves. Looping was handled by the simple addition of a zero head loss constraint for each loop in the system. In this constraint the algebraic sum of head loss around a loop must be equal to zero, thus ensuring hydraulic consistency.

Alperovits and Shamir addressed the problem of optimizing flow distribution with a method known as linear programming gradient, where a gradient vector is used to indicate which path has the greatest point reduction in cost with respect to change in flow. Due to the looped nature of the system, and the interdependance of flow paths, this technique was not entirely correct. To overcome this shortcoming Quindry et al. (1979) modified the linear programming gradient equation.

The Alperovits and Shamir formulation meets hydraulic requirements without the use of Hardy-Cross, and provides a local optimum for a given layout. However, explicit consideration of reliability remains a concern, since the linear programming approach tends to drive the system towards a branched network unless minimum pipe sizes are specified.

Quindry et al. (1981) improved the method developed by Schaake and Lai (1969) by adding a gradient search technique. This technique allowed them to alter pressures at junctions to reduce system cost while meeting flow requirements at demand points. The outflows at nodes were altered, then the heads were modified to restore required flow to the nodes. The linear programming model chooses optimum pipes to meet these requirements, as in the method of Schaake and Lai.

Morgan and Goulter (1982) developed a two-stage linear programming technique for layout and design of water distribution networks. The model consists of two linear programs; a layout and flow distribution model given pressure distribution, and a pressure and pipe size model, given flow distribution. Redundancy is provided by ensuring that any given node in the layout and flow distribution model must be connected to at least two links. In most cases this will provide a looped system by iterating back and forth between two linear programs until further improvement is negligible.

Morgan (1983) designed a layout model for water distribution networks utilizing Hardy-Cross and linear programming. In this model a least cost layout and component system is designed to meet hydraulic requirements under a range of fire flows, given that any single pipe may be out of service.

2.2 General Watermain Break Analysis

Dolson (1955) recognized the need for analysis of failures in existing pipelines, where the information gained is useful in determining what the shortcomings of a water system might be. Dolson also noted that a standard system for recording break statistics was necessary. Possible problem areas in distribution systems were given as selection of materials, handling, installation procedure, trench conditions, defective joints, corrosion, disturbances by adjacent construction, variation of temperature, water hammer, overly large taps, poor service connections, and inadequate thrust blocking.

Baracos et al. (1955) monitored vertical movement, strain, soil and water temperature, and tested soil at three test installations of small diameter cast iron pipe. From the test results and break data available from the City of Winnipeg it was evident that the corrosive nature and physical characteristics of local soils are major factors in watermain failures. In particular, swelling and shrinking of local clay soil with seasonal moisture content fluctuations was noted to be a significant contributing factor, especially where pipe was already weakened by corrosion.

Arnold (1960) concluded that main breaks in Philadelphia usually are a result of some external physical force acting upon a weakened section of pipe. This weakening is typically a result of corrosion in cast iron pipe. Arnold recommends preconstruction surveys of the pipe environment and strict adherance to proper construction practice.

In a similar study for New York City Clark (1960) cites casting flaws, excess internal pressure, corrosion, electrolysis, freezing, excessive loading of backfill, poor thrust blocking, differential settlement, adjacent structures, and blasting vibration as causes of cast iron main breaks. The main causes in New York are stated to be excess backfill loads, poor thrust blocks, differential settlement, adjacent structures, and blasting vibrations. Recommendations for improvements include rigorous inspection, improved construction techniques, and a more thorough consideration of redundancy in network design.

Remus (1960) considered reliability of service, as well as maintenance cost, to be of major importance to water costs. In this study it was noted that causes of failure are related to one another, and as such, the relative importance of each is difficult to determine. Pipe wall thickness, temperature change of soil and water, construction procedure and materials, water hammer, traffic loading, sewer washouts, rigid joint, and unsuitable soil conditions were suggested as the prime factors affecting performance ofcast iron water-mains in Detroit.

Niemeyer (1960) states that a saving in initial system cost may prove to be a false economy, as it may result in high failure rates, with the associated property damage, increased maintenance cost, disruption of customer service, and loss of fire protection. Niemeyer provided break statistics for Indianapolis, including such details as pipe type and size, length of pipe, number of breaks, type of break, number of joints, type of joint, and seasonal failure rates. Based on this data, it was recommended that thicker cast iron pipe walls, shorter pipe lengths, and "more ductile iron"[1] could be possible areas of improvement.

Morris (1967) analyzed the major factors involved in watermain failures and recommended remedies. This fairly comprehensive paper suggests that, if armed with suitable knowledge of pipe environment, pipe characteristics, previous problems, etcetera, designers may reasonably safeguard a distribution system against excessive breakage. Morris states that accurately kept break records are necessary tools for a successful break reduction program.

Fitzgerald (1968) examined the major cause of failure of cast iron water distribution systems, namely corrosion. It is shown that if all other parameters remain constant, the rate of corrosion related failures will increase yearly, while non-corrosion types of failure should remain constant,

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[1] Niemeyer, H.W., "Experience with Main Breaks in Four Large Cities - Indianapolis", Journal of the American Water Works Association. pp. 1051-1058, August, 1960.

all things remaining equal. Fitzgerald also notes that increased pipe wall thickness, along with several other methods of corrosion protection may be used as preventative measures against corrosion related failures. This observation agrees with the results of other studies which noted an increased failure incidence with smaller pipes whose wall thickness are smaller, and are hence prone to a reduced time to failure by corrosion (Morris (1967), O'Day et al. (1980), O'Day (1982, 1983), Brcic (1983), Ciottoni (1983), Kettler and Goulter (1983)). The reduced wall thicknesses associated with smaller diameter pipes also causes the smaller pipes to be structurally weaker than the larger diameter pipes, thus makng them more prone to non-corrosion types of failures. The specification or selection of larger diameter pipes will, therefore, reduce the rate of failures. Like Morris (1967), Fitzgerald, however, also notes that a unique approach may have to be developed for each locale to ensure idenfication of a site specific cost effective solution. Fitzgerald also noted that effective corrosion control not only results in maintenance cost savings, but also results in increased customer satisfaction and reduced traffic delays.

O'Day et al. (1980) and O'Day (1982, 1983) recognized that useful life, pipe age, and other 'rule of thumb' criteria are inappropriate for use in replacement studies of water distribution networks. It is suggested that a more scientific approach be taken where decisions are made according to appropriate criteria. It is also suggested that a computerized leak record system, coupled with trend analysis and predictive models can result in financial savings. Local factors to be considered in records and analysis should include temperature , soil type, frost penetration, shrinking and swelling of soil, external loading, construction practice, and characteristics of the pipe itself, such as size and material. However, it must be kept in mind that these studies are only valid for certain locales where pipe and environment are relatively uniform.

Chambers (1982, 1983) reported that problems with cast iron pipe in the City of Winnipeg are mainly due to corrosion and stresses exerted on watermains by moisture sensitive clays. The City of Winnipeg uses a computerized record of break data, which has been found useful in the prediction of future maintenance through the study of the break history of the pipe system. This type of prediction allows the city to budget for anticipated future maintenance and halt further worsening of system failure rates. It should be noted that the 1980-1982 Winnipeg leak rate of 1.1 leaks per kilometre per year was found to be more than twice as high as those for nine other Canadian cities.

Brcic (1983) describes the "watermain priority rating system"[2] devised for the City of St. Catharines. The replacement criteria are, in descending order of importance, break history, concurrent municipal works, existing main data, consumers-development, system improvement, existing road condition, customer complaints, distribution of works, disruption factor, and others. This hierarchical type of approach allows the engineer to make rational decisions with regard to elimination of deficiencies and or upgrading, with a time scale and budgeting in mind. Brcic also notes a higher incidence of failure in smaller pipe sizes and in cast iron pipe, while age was not found to have a strong correlation with incidence of failure.

Ciottoni (1983) presented computerized water main break records and subsequent analysis of the data for the City of Philadelphia. The Philadelphia data base enabled the author to draw conclusions about factors resulting in main breaks, where it allows the engineer to implement a water main replacement program based on a sound decision-making process. More specifically, pipe was analyzed with regard to age, size, location, break type, time of break, thickness of pipe wall and electrolysis effects. Age of pipe was not found to be a significant factor in break rates, while pipe size showed a strong correlation. Sullivan (1982) also found that

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[2]Brcic, C., "Criteria for Replacement of Watermains", <u>1983 AWWA/OMWA Joint Annual Conference</u>, Toronto.

smaller diameter pipe in Boston exhibited an increased rate of failure.

Kettler and Goulter (1984) investigated rates of pipe breakage with increasing pipe diameter and time. They concluded that failure rates of a relatively constant population of cast iron pipe increased with time and with decreased diameter. Failure rates of asbestos cement watermain were found to increase with time as well. The predominant modes of failure were also investigated.

Shamir and Howard (1979) utilized historical pipe failure data and engineering economics to predict optimal replacement time of watermains. In this study it was emphasized that for the predictions to be valid pipe must be of the same type and in a homogeneous environment under similar operating conditions. If new pipe being installed is of the same type as old then a similar failure history can be expected. Kettler and Goulter (1983) make a similar assumption.

Clark et al. (1982) extend the work of Shamir and Howard (1979) by developing two equations relating leak events to specific pipe parameters. Parameters for the time to first repair equation are pipe diameter, internal pressure, percent of pipe overlaid by industrial or residential development, length of pipe in corrosive soil, and pipe type. The accumulated number of repair equation parameters are pipe type, pressure differential, age of pipe from first break, and percent of land over pipe of low and moderately corrosive soil. No mention is made of pipe bedding, depth of cover, susceptability of soils to differential movement or shrink swell, or of temperature effects. Furthermore, the correlation of the equations to actual failure data is not particularly strong.

CHAPTER III

ANALYSIS OF PIPE FAILURE DATA

3.1 Overview

The objective of this portion of the study was to determine whether the size of pressurized cast iron water pipe could be related to the average number of failures per kilometre per year experienced over a given time period. In other words, it was hoped to identify whether larger pipes experienced fewer failures than smaller pipes. It was recognized that if such a relationship does indeed exist, it could be used in a linear programming formulation to design looped water distribution networks, with the explicit consideration of reliability. A constraint set limiting the expected number of breaks per year permitted in each link of a network could then be formulated.

Quantitative estimates of failure incidence for each pipe size were not calculated according to age of pipe, narrowing the scope of the study to the investigation of the nature of the general relationship between failure incidence and pipe size. The analysis is therefore directed towards proving whether the relationship is suitable for use in the linear programming model rather than providing exact operational estimates of values to be used in the formulation.

3.2 Raw Data

The data used in the analysis was provided by the City of Winnipeg for District 4, one of six City of Winnipeg Works and Operations Division Operations Department Districts.

District 4 is located in the northeast of Winnipeg, as shown in Figure 1, and consists of 27 local neighbourhoods.

The computer based data on watermain breakage for District 4 in the City of Winnipeg was obtained for the six year period 1975 through 1980, and included details of date of failure, failure address, leak type, year installed, pipe size, pipe material, joint type, and failure mode.

Due to the need for a sample of known length, further data was required on the length of each of the cast iron pipe sizes in District 4. Since this information was not available from the computer printouts, it was obtained from a separate study conducted by the City of Winnipeg, where lengths of each type and diameter of pipe were measured for the local neighbourhoods within District 4. Table 1 gives the lengths of each type of pipe without regard to diameter for the local neighbourhoods of District 4, while Table 2 shows the breakdown of lengths according to pipe type and diameter for 7 of the 27 local neighbourhoods.

The pipe length data given in Tables 1 and 2 are in feet. They are represented in Imperial units because this was the form in which the data was obtained. Subsequent related calculations, however, were done in SI units, where a hard conversion for pipe length and a soft conversion for nominal pipe diameter were utilized. The conversion takes place in Tables 3 and 4.

FIGURE 1

LOCATION OF DISTRICT 4
IN THE CITY OF WINNIPEG

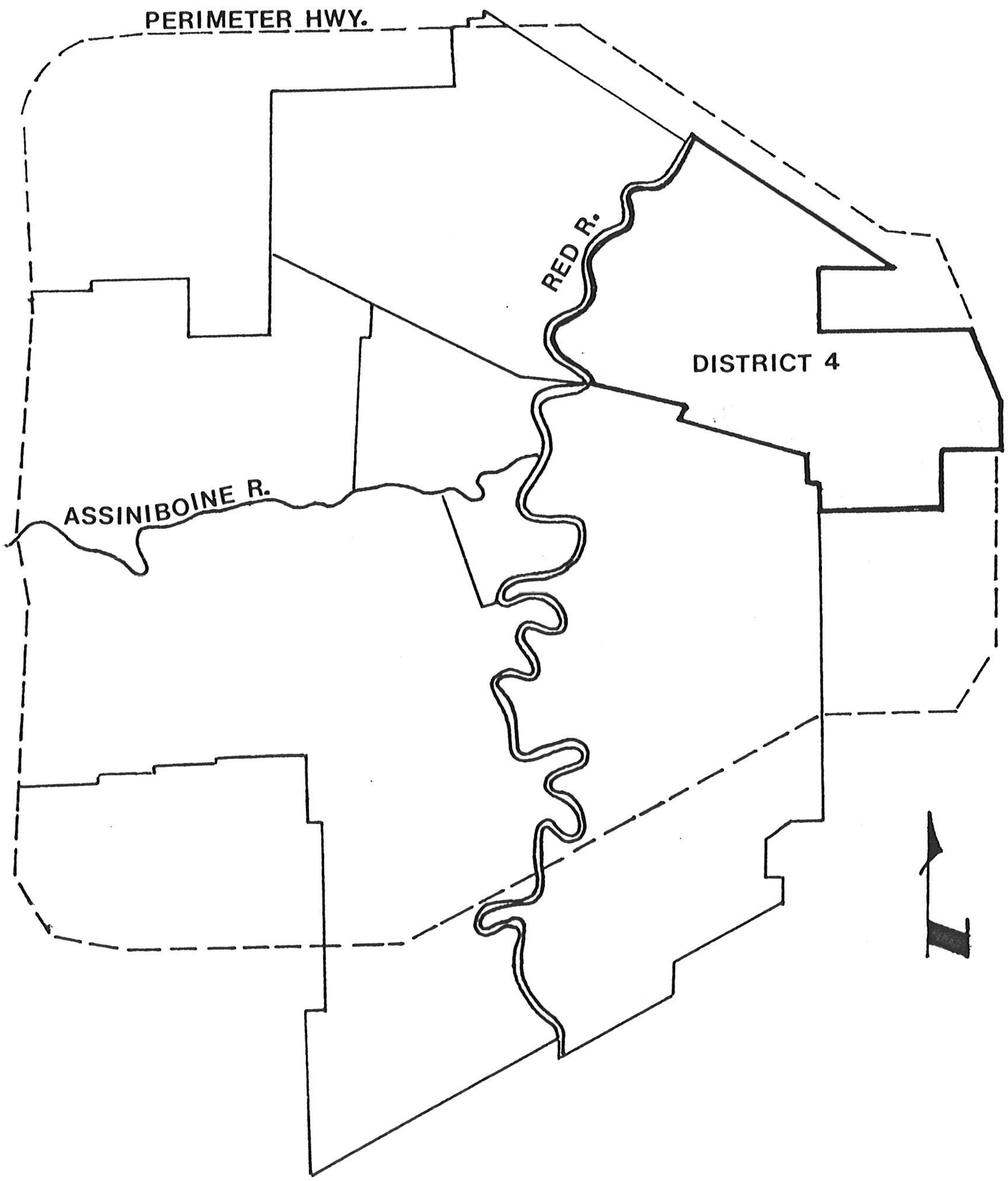

TABLE 1

PIPE LENGTH BY TYPE FOR DISTRICT 4

Local Neighbourhood	Pipe Quantities (feet) Cast Iron	Asbestos Cement	Other
Valhalla		3,650	
Kildonan Drive	69,050	10,450	3,100
Rossmere	148,500	27,500	1,300
Munroe West	47,050	6,500	
West Elmwood	27,310	4,750	700
Chalmers	67,640	5,920	5,670
Tyne Tees		19,110	
East Elmwood	13,450	16,100	2,300
Talbot Grey	24,940	1,550	
Munroe East	43,900	30,810	10,200
Valley Gardens	1,800	11,280	3,500
McLeod Industrial	1,900	14,950	11,450
Peguis		8,950	
River East	7,000	100,100	3,200
Springfield South			
Springfield North			

TABLE 1 (Continued)

PIPE LENGTH BY TYPE FOR DISTRICT 4

Local Neighbourhood	Pipe Quantities (feet) Cast Iron	Asbestos Cement	Other
Regent	1,250	28,850	4,850
Mission Gardens	5,200	34,350	
Transcona Yards	18,260	6,340	
Melrose	27,830	1,350	
Victoria West	37,540	3,040	
Kern Park	23,450		
Canterbury Park	300	33,000	
Kildare Redonda	50,950	15,930	
Radisson	42,950	2,000	
Lakeside Meadows		38,400	3,600
Transcona South	3,600	14,850	

TABLE 2

PIPE LENGTH BY TYPE AND DIAMETER FOR DISTRICT 4

Local Neighbourhood	Diameter (inches)	Pipe Quantities (feet) Cast Iron	Asbestos Cement	Other
Rossmere	6	115,750	14,400	
	8	26,050	3,400	
	10	6,700	7,800	
	12		2,200	
	24			1,300
		148,500	27,800	1,300
Munroe West	4	300	600	
	6	27,000		
	8	11,400		
	10	4,800	3,300	
	12	3,550		
	16		2,600	
		47,050	6,500	
West Elmwood	6	12,100		
	8	5,590		
	12	5,900	1,350	
	14			700
	16	3,720		
	20		3,400	
		27,310	4,750	700
Chalmers	4	1,000		
	6			
	8			
		1,000		

TABLE 2 (Continued)

PIPE LENGTH BY TYPE AND DIAMETER FOR DISTRICT 4

Local Neighbourhood	Diameter (inches)	Pipe Quantities (feet) Cast Iron	Asbestos Cement	Other
River East	2		300	
	6	7,000	37,200	
	8		27,760	
	10		21,000	
	12		19,900	
	30			3,200
		7,000	106,160	3,200
Valhalla	6		2,750	
	12		900	
			3,650	
Kildonan Drive	2	550		
	4	600		
	6	38,550	6,850	
	8	8,350	600	
	10	3,000	2,200	
	12	9,550	800	
	16	8,450		
	24			3,100
		69,050	10,450	3,100

3.3 Data Reduction

Cast iron pipes were the only type considered because they comprised the bulk of the data. It was also felt that the sample should be chosen such that it showed relatively uniform pipe and environmental characteristics. Since the reliability of a certain type of pipe is assumed to be based on pipe and environmental characteristics for that given pipe and environment type, it would not be reasonable to base reliability considerations on data from other types of pipe or environment, or a combination thereof. The pipe type in the sample chosen is constant. As data on pipe environment was not available at the time of the study, the pipe environment is assumed to be consistent throughout.

The first step in the reduction process was to check for completeness of the cast iron pipe length data in Tables 1 and 2. The lengths given for each diameter for a given neighbourhood (See Table 2) were added up and checked against the total length of cast iron pipe for each neighbourhood as shown in Table 1. It was found that data for Kildonan Drive, Rossmere, Munroe West, River East, West Elmwood and Valhalla were complete. Since data for Chalmers were incomplete they were eliminated from the study. Valhalla was eliminated because it contained no cast iron pipe. Even though the data for River East was complete it was eliminated due to the fact that all pipe here was installed from 1970 to 1980. A relatively constant population could not be assumed for River

East, where the study period is from 1975 through 1980. However, this dismissal would not have a large effect on the outcome of the study, since River East represents a very small proportion of the data, as shown in Table 3. It should be noted that the remaining 20 neighbourhoods could not be considered because the cast iron pipe lengths in these areas were not broken down with respect to diameter.

During the data reduction it was noticed that the proportion of 150 mm pipe in Rossmere was much greater than that for the other neighbourhoods (See Table 3). However, it could not be decided whether this was an isolated case which would be detrimental to the analysis, or whether it was the prevalent case. Furthermore, 40% of pipe in the Rossmere area was installed from 1970 to 1980, possibly including some cast iron pipe. Because of these uncertainties with Rossmere, the data was split into 2 groups for analysis, one including and one excluding Rossmere; analysis A and analysis B respectively. The installation date was important because a basic assumption was that the length and type of pipe is the same from 1975 through 1980, giving roughly the same proportion of breaks to pipe length in each of the 6 years. Over a given time period the proportion of breaks to pipe length is expected to increase, ie, more failures in older pipe. (See Kettler and Goulter (1984)).

TABLE 3

CAST IRON PIPE LENGTH AND PERCENT OF TOTAL BY DIAMETER

Pipe Diameter (mm)	NEIGHBOURHOOD River East	Valhalla	Kildonan Drive	Rossmere	Munroe West	West Elmwood	Chalmers
50	Pipe Length (Feet) / % of Total		550 / 0.80				
100			600 / 0.87		300 / 0.64		1000 / 100
150	1000 / 100		38550 / 55.83	115750 / 77.95	27000 / 57.39	12100 / 44.31	
200			8350 / 12.09	26050 / 17.54	11400 / 24.23	5590 / 20.47	
250			3000 / 4.34	6700 / 4.51	4800 / 10.20		
300			9550 / 13.83		3550 / 7.55	5900 / 21.60	
350							
400			8450 / 12.24			3720 / 13.62	
Total	7000 / 100		69050 / 100	148500 / 100	47050 / 100	27310 / 100	1000 / 100

The distribution of pipe diameters by pipe length and percent for each neighbourhood is shown in Table 3. Given the partial data on pipe lengths for each diameter, the approximate distribution for the whole of District 4 was calculated. The length of each pipe size was estimated by multiplying the estimated percentage by the total cast iron pipe quantity known to exist in District 4 to 1980. This distribution is shown in Table 4. The percent distribution for each neighbourhood and for the district as a whole is shown in Figure 2, which clearly illustrates that the distribution is reasonably uniform from neighbourhood to neighbourhood and from analysis A to B.

When Rossmere was included for analysis A, the total cast iron pipe length sampled was 291,910 feet, or 44% of the total amount of cast iron pipe in District 4, which was 663,870 feet. When Rossmere was excluded for analysis B, the sample length was 143,410 feet, or 22% of the total existing.

Given that the length of each cast iron pipe diameter in District 4 was found by extrapolation, and that failures for each year from 1975 through 1980 by diameter were known, it was then possible to calculate the number of failures per kilometre per year for each pipe diameter. The number of failures for each pipe size for each of the 6 years of computer data were counted, as shown in Table 5. To obtain the number of failures per kilometre per year, these figures

TABLE 4

ESTIMATED PIPE LENGTH FOR DISTRICT 4 BY DIAMETER

Analysis A Including Rossmere				Analysis B Excluding Rossmere			
Pipe Diameter (mm)	Sample Length (ft)	% of Total	Estimated Length (ft/km)	Pipe Diameter (mm)	Sample Length (ft)	% of Total	Estimated Length (ft/km)
50	550	0.19	1261 / 0.38	50	550	0.38	2523 / 0.77
100	900	0.31	2058 / 0.63	100	900	0.63	4182 / 1.27
150	193400	66.25	43984 / 134.06	150	77650	54.15	359486 / 109.57
200	51390	17.60	116841 / 35.61	200	25340	17.67	117306 / 35.75
250	14500	4.97	32994 / 10.06	250	7800	5.44	36115 / 11.01
300	19000	6.51	43218 / 13.17	300	19000	13.25	87963 / 26.81
400	12170	4.17	27683 / 8.44	400	12170	8.49	56363 / 17.18
Total	291910	100	663869 / 202.35	Total	143410	100	663937 / 202.36

FIGURE 2
PIPE SIZE DISTRIBUTION IN PERCENT OF TOTAL PIPE LENGTH FOR NEIGHBOURHOODS AND DISTRICT

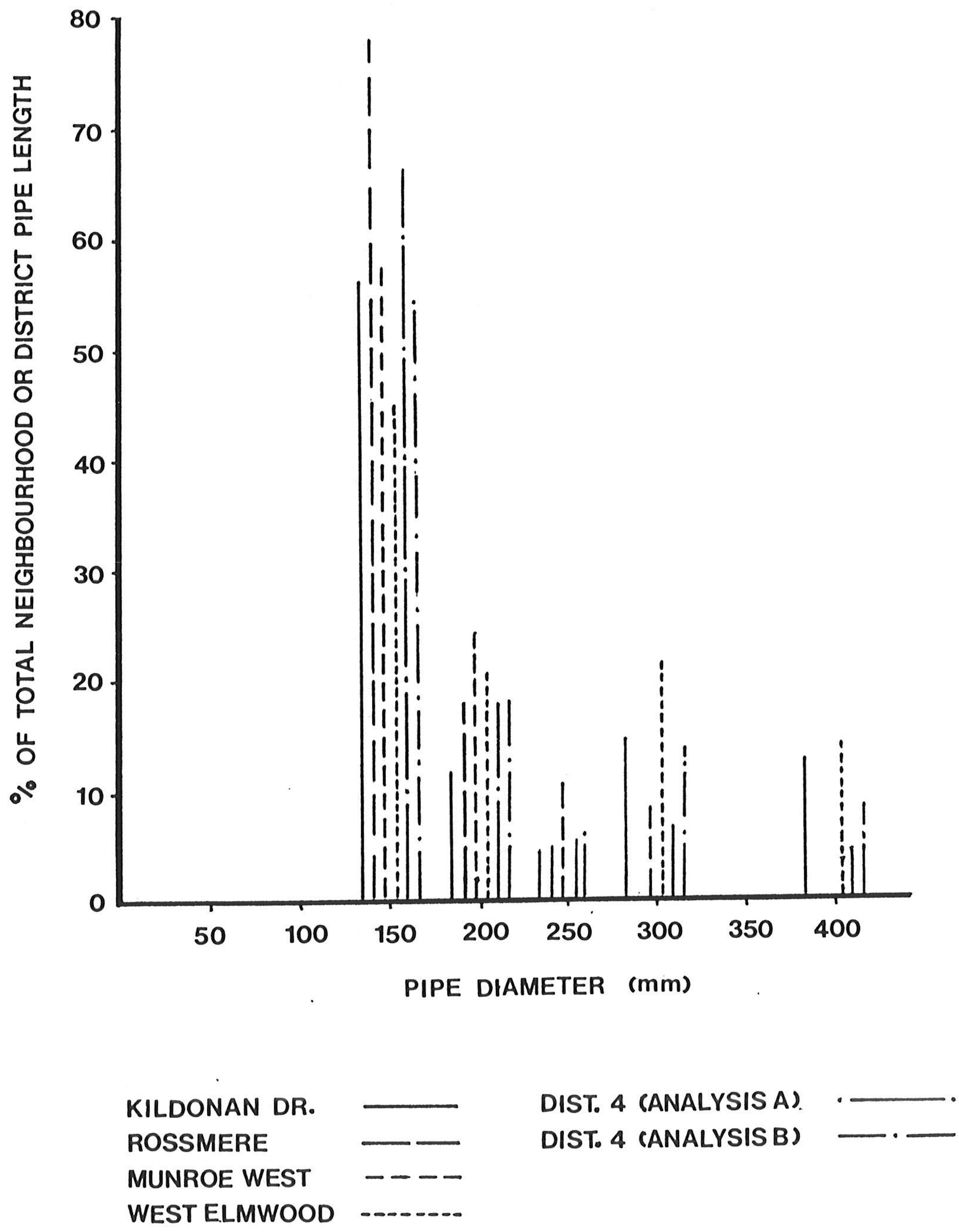

TABLE 5

NUMBER OF PIPE FAILURES IN DISTRICT 4
BY YEAR AND DIAMETER

Year	Number of Pipe Failures							
	Pipe Diameter (mm)							
	50	100	150	200	250	300	350	400
1975	0	0	119	15	1	2	0	0
1976	0	1	112	26	2	4	0	0
1977	0	3	160	27	10	2	0	0
1978	0	2	107	25	3	1	0	0
1979	0	1	103	28	7	1	0	0
1980	0	1	98	41	3	1	0	0
Total	0	8	699	162	26	11	0	0

were divided by the estimated pipe lengths from Table 4. The results are shown in Table 6, for both analysis A and analysis B. It should be noted that the 50, 350, and 400 mm pipe sizes were not used in this part of the study. The removal of these pipe sizes was due to the lack of breaks recorded for any of these pipe sizes, as shown in Table 5, and that the 50 and 350 mm pipes make up only 0.19 or 0.38% and 0% of the population respectively, as shown in Table 4.

An overall weighted mean by pipe length was calculated, and is shown in Table 6. The values were 0.75 and 0.76 for analysis A and B respectively, where the incidence of failure per kilometre per year for all pipe in the City of Winnipeg was calculated to be 0.83 by the Waterworks Waste and Disposal Division in 1979 (Yeung (1980)). Given the fact that this combines all pipes and the calculations here involve extrapolation, the figures are reasonable. In 1983, Chambers stated that the average for the years 1980 through 1982 was 1.1 failures per kilometre per year, where the failure rate increased dramatically with time.

3.4 Sample Correlation Coefficients

Sample correlation coefficients considering both the sets of yearly data and 6 year averaged data were calculated for analyses A and B.

The calculated r values, where r is measuring the "degree

TABLE 6

NUMBER OF PIPE FAILURES PER KILOMETRE IN DISTRICT 4
BY YEAR AND DIAMETER

Analysis A Including Rossmere						Analysis B Excluding Rossmere					
Year	Pipe Diameter (mm)					Year	Pipe Diameter (mm)				
	100	150	200	250	300		100	150	200	250	300
1975	0.00	0.89	0.42	0.10	0.15	1975	0.00	1.09	0.42	0.09	0.07
1976	1.61	0.84	0.73	0.20	0.31	1976	0.79	1.03	0.73	0.18	0.15
1977	4.84	1.20	0.76	1.00	0.15	1977	2.36	1.47	0.76	0.91	0.07
1978	3.23	0.80	0.71	0.30	0.08	1978	1.57	0.98	0.70	0.27	0.04
1979	1.61	0.77	0.79	0.70	0.08	1979	0.79	0.95	0.79	0.64	0.04
1980	1.61	0.74	1.16	0.30	0.08	1980	0.79	0.90	1.15	0.27	0.04
Six Year Average	2.15	0.87	0.76	0.43	0.14	Six Year Average	1.05	1.07	0.76	0.39	0.07
Overall Average = 0.75 Breaks/km/Year						Overall Average = 0.76 Breaks/km/Year					

of linear relationship among variables"[3], indicate that the yearly data had only a moderately strong value. The values for analyses A and B are -0.638 and -0.684, where negative values indicate a decrease in the number of failures per kilometre per year as pipe size is increased. This relationship is clearly illustrated in Figures 3 and 4, where yearly data is plotted.

When the data for each pipe size is averaged over the 6 year study period, the large and negative r values indicate a strong decreasing relationship. The r values for analyses A and B are -0.915 and -0.963 respectively. The 6 year averaged data is plotted in Figures 5 and 6. It should be noted that these are the more important failure estimates, for the information desired is long term, and not yearly, as indicated in the introductory remarks. The primary parameter of interest is the long term failure characteristics of pipes and how they do or do not differ between pipes of varying diameters. Here, the indication is clearly a reduction in failures with increasing pipe size. It should again be stressed that time has not been taken into account.

3.5 Linear Regression Analysis

The simple linear regression for analysis A is illustrated in Figures 3 and 5, where it is shown for both

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[3] Devore, Jay L., Probability and Statistics for Engineering and the Sciences, Brooks/Cole, Monterey, California, 1982.

FIGURE 3

SIMPLE LINEAR REGRESSION ANALYSIS A

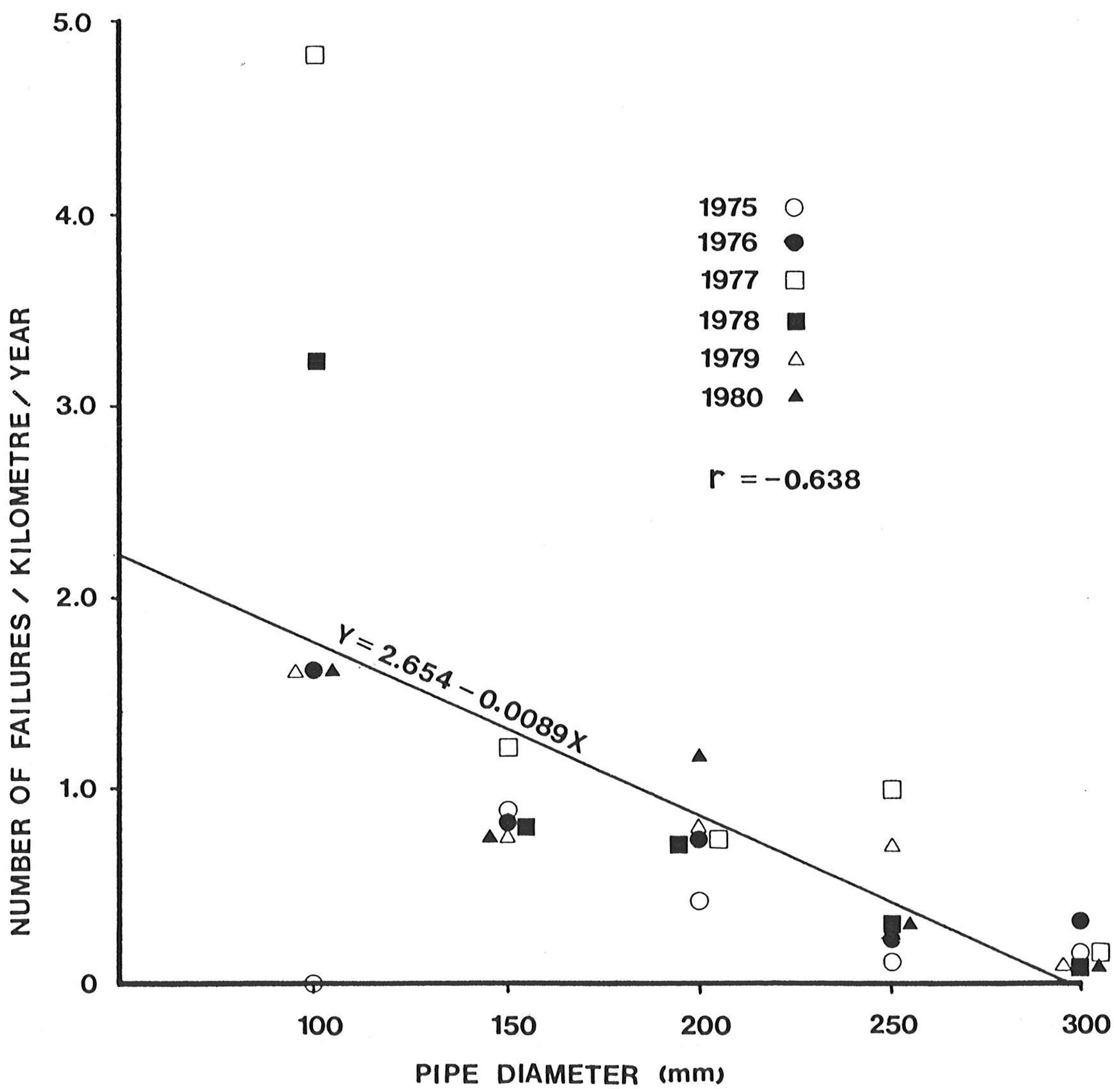


FIGURE 4

SIMPLE LINEAR REGRESSION ANALYSIS B

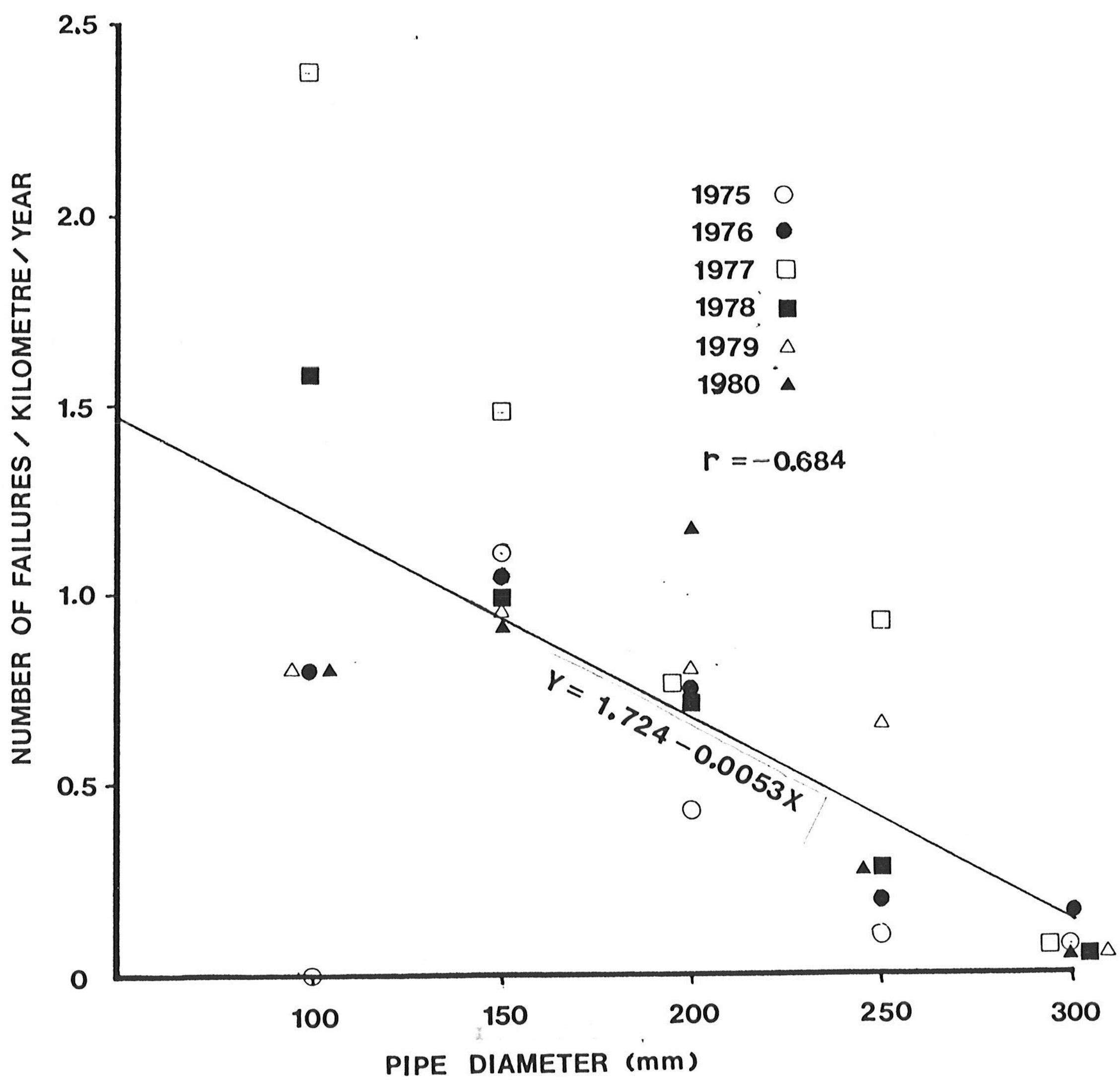


FIGURE 5

SIMPLE LINEAR REGRESSION ANALYSIS A

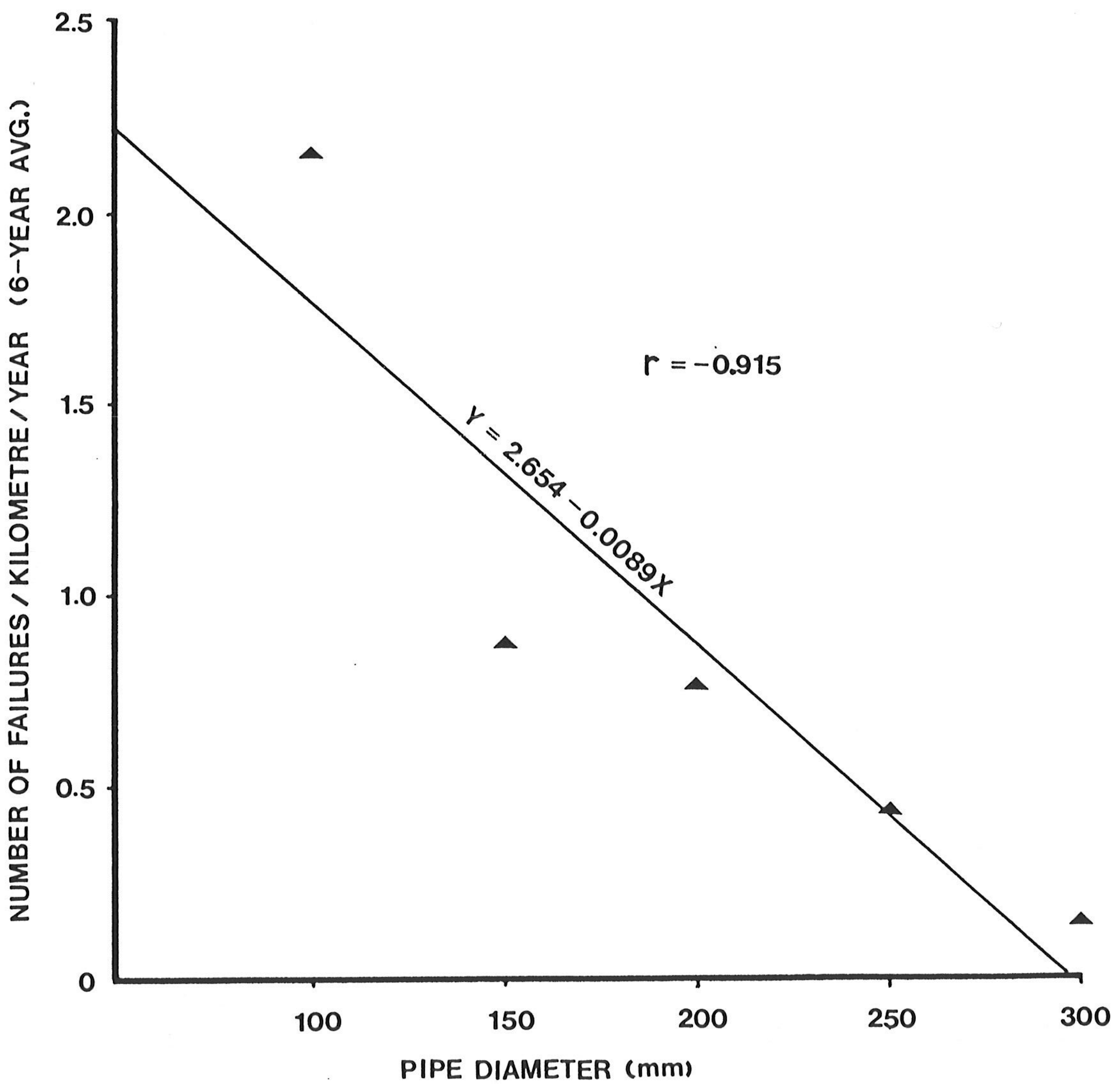
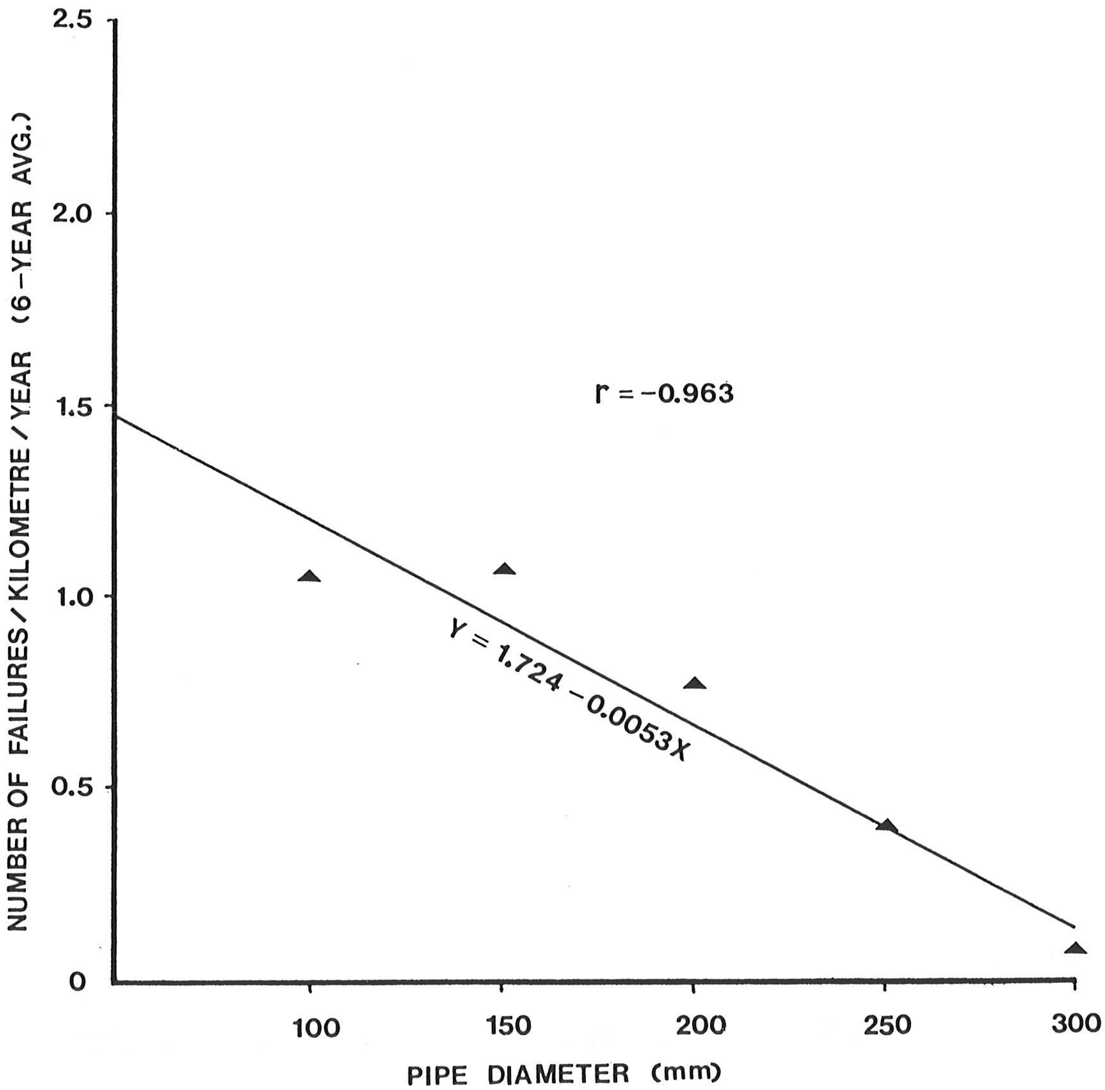


FIGURE 6

SIMPLE LINEAR REGRESSION ANALYSIS B



the 6 year averaged and yearly data. It should be noted that in Figure 3 there is a great deal of scatter for the 100 mm pipe relative to other sizes. Recall from Table 4 that the 100 mm pipe accounts for only 0.31% of the sample lengths, while for example the 150 mm pipe, showing much less scatter, accounts for 66.25% of the sample length.

The simple linear regression for analysis B is illustrated in Figures 4 and 6, where once again it is shown for both the 6 year averaged and yearly data. It should also be noted from Figure 4 that there is a great deal of scatter for the 100 mm pipe, where it accounts for only 0.63% of the sample, as stated in Table 4. A pipe such as the 150 mm, with much less scatter, accounts for a much greater proportion of the sample, namely, 54.15%.

From the above discussion it is evident that dependability of the data varies from pipe size to pipe size, where the 100 mm pipe data is relatively non-dependable when compared to the other sizes, which show much less variation. In order to get a more representative relationship it was decided to use a weighted regression analysis by variation (same as regression weighted by variance), where more weight was given to pipes with less variation in the yearly failure data. These linear regression lines are shown in Figures 7 and 8 for analyses A and B respectively, where the most weight was given to the 300 and 150 mm pipes, and the least

FIGURE 7

WEIGHTED LINEAR REGRESSION
ANALYSIS A BY VARIATION

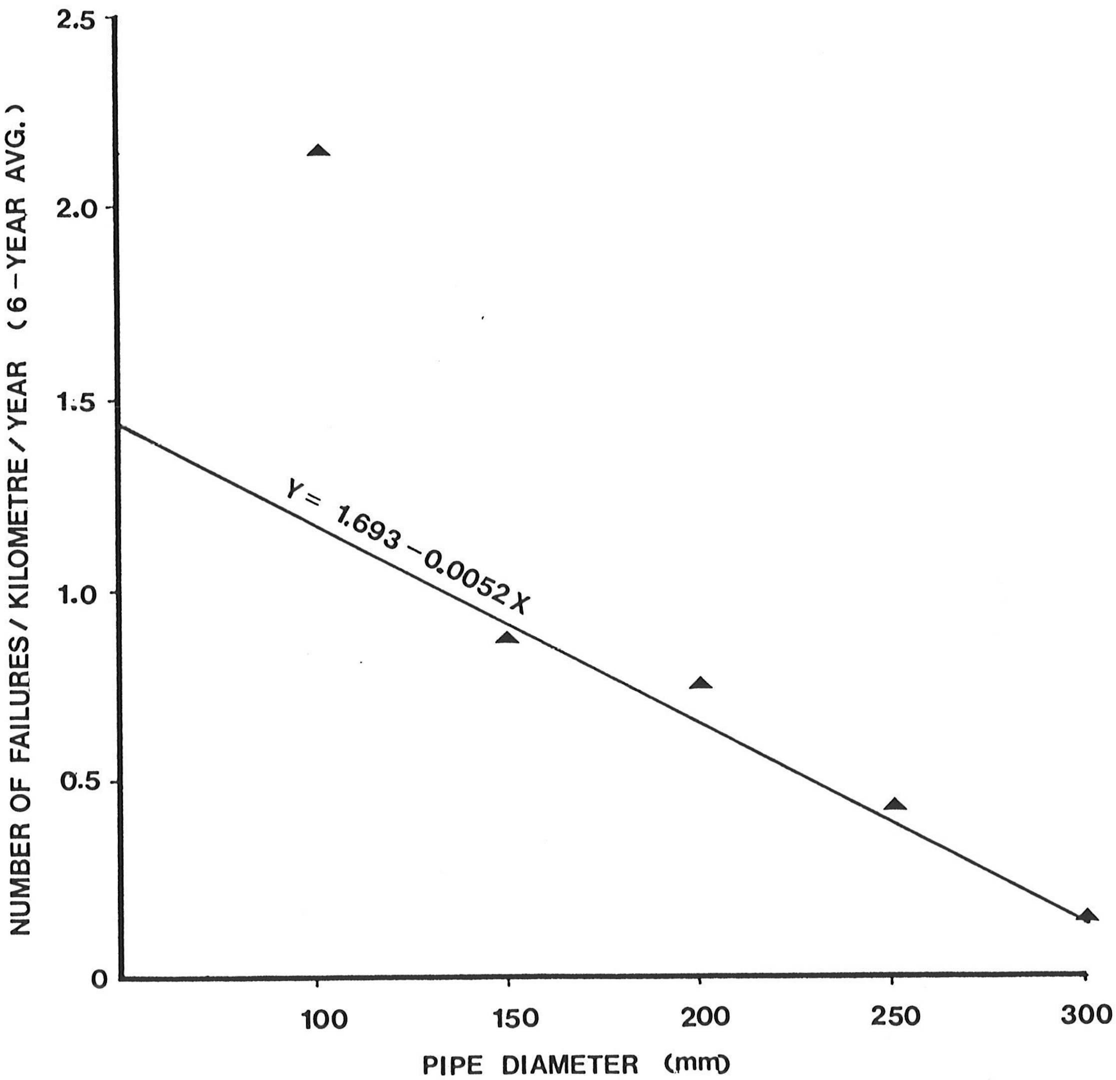
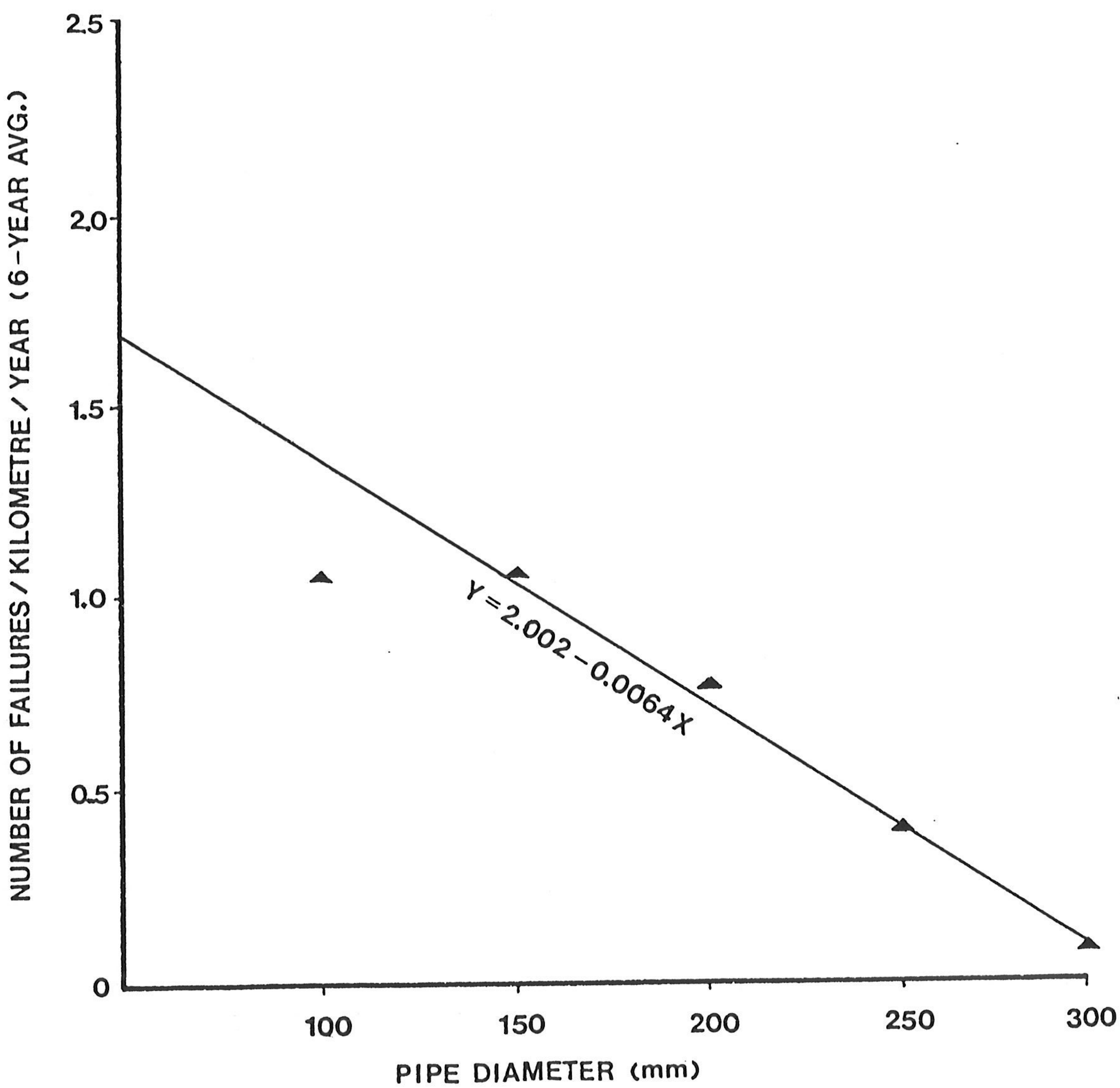


FIGURE 8

WEIGHTED LINEAR REGRESSION
ANALYSIS B BY VARIATION



to the 100 mm pipe. It seemed surprising that the greatest weight was given to the 300 mm pipe if one considered that it represented only 4.17 or 8.49% of the sample length, while the 150 and 200 mm pipes represented 66.25 or 54.15% and 17.60 or 17.67% respectively. However, it should also be realized that the 300 mm yearly pipe data is grouped near the x-axis. The reason for this grouping is that the failure rate cannot beome negative as failure incidence decreases, consequently causing a large proportion of the data points to cluster around the zero failures point, thereby reducing the variation.

Although not shown in this study, weighted linear regression analyses by population were also carried out. This analysis gave very similar relationships to those found using variation as a weight. This was not surprising, for one can expect less variation with a larger, and therefore more dependable sample. The exception here was the 300 mm pipe, which because its values were found to be crowded near to the zero point, is very dependable with a small population.

It should be noted that if larger pipe were included in the study, the relationship would not have been linear. As the x-variable, ie, pipe diameter, increases, it is expected that the "best fit" line through the points would become horizontal or nearly so. Furthermore, if pipe size were decreased (even though it may be impractical to consider much smaller pipes) the relationship would probably not have continued in a linear fashion. More data is necessary to determine the exact relationship outside of the range studied here.

3.6 Test for Linearity

Tests for linearity were carried out on the four derived linear regression lines, as a determination of whether or not the assumption of linearity (for the pipe sizes chosen) was reasonable. In the test for linearity, it was necessary to determine whether or not the calculated F-statistic was significantly larger than the tabled F-value at a given confidence level. If the F-statistic was significantly larger than the F-value, then the relationship was assumed to be non-linear.

The F-statistic of 1.293 for simple linear regression analysis A, shown in Table 7, was found to be lower than the F-value of 2.99 at $\alpha = 0.05$. Therefore, the regression was accepted as being linear. The tendency of simple linear regression analysis B to be linear is even stronger, with an F-statistic value of 0.087. This is understandable, since the 100 mm pipe average number of failures is much nearer to the regression line in B, resulting in a lower value for the lack of fit sum of squares (SSLF).

The values of the F-statistic for the linear regression lines weighted by variation were expected to be larger, since little weight was given to the 100 mm pipe, resulting in an increase in the lack of fit sum of squares (SSLF). Values for the 150, 200, 250 and 300 mm pipes affecting SSLF increased by a lesser degree or remained nearly the same, while the

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TABLE 7

TEST FOR LINEARITY

Sample Calculation for "Analysis A"
Simple Linear Regression

Pipe Diameter (mm)	$\bar{y}_{i.}$	$\hat{y}_i$	$(y_{ij} - \bar{y}_{i.})^2$
100	2.15	1.76	13.889
150	0.87	1.32	0.142
200	0.76	0.87	0.280
250	0.43	0.42	0.593
300	0.14	0.00	0.040
			Σ 14.944

$$SSLF = \sum n_i (\bar{y}_{i.} - \hat{y}_i)^2 = 2.318$$

$$MSLF = SSLF/I-2 \qquad = 2.318/3 = 0.773$$

$$SSPE = \sum\sum(y_{ij} - \bar{y}_{i.})^2 = 14.944$$

$$MSPE = SSPE/\sum n_i - I = 14.944/25 = 0.598$$

$$F = \frac{MSLF}{MSPE} = 1.293 \qquad F_{0.05,3,25} = 2.99 \qquad F_{0.01,3,25} = 4.68$$

Regression Type	Analysis A F-Statistics (F)	Regression Type	Analysis B F-Statistics (F)
Simple Linear	1.293	Simple Linear	0.087
Weighted By Variation	3.266	Weighted By Variation	0.193

100 mm pipe value increased dramatically, resulting in a large overall increase in SSLF, and hence MSLF. The pure error sum of squares (SSPE) remained the same from the simple linear to the weighted regression.

The F-statistic for analysis A weighted by variation was 3.266, greater than the F-value of 2.99 at $\alpha = 0.05$. However, at $\alpha = 0.01$ the F-value is 4.68. Therefore, at $\alpha = 0.05$ non-linearity is somewhat significant, and is rejected at $\alpha = 0.01$, where it is reasonable to accept the linear relationship. The F-statistic for analysis B, from Table 7, was found to be 0.193, giving strong evidence for the existence of a linear relationship.

Given the above results, it appeared reasonable to accept the assumption of linearity. The existence of a linear relationship is useful in that it eases interpretation of the nature of the relationship between pipe failure incidence and pipe size. However, it must be accepted with caution for weighted regression analysis A, where linearity was rejected at $\alpha = 0.05$, primarily due to wide variation between the mean and estimated failure value for the 100 mm pipe.

3.7 Confidence Intervals and Bands

Confidence intervals were constructed for each of the 5 pipe sizes to determine just how predictable failures were on a year to year basis, ie, within what range is it possible to predict next year's failures?

The details of the calculations for and illustrations of the confidence intervals are shown for analyses A and B in Figures 9 and 10 respectively. The α value was taken to be 0.05, with n-1 = 5 degrees of freedom since there are n=6 data points for each pipe size.

Overall, the predictability ranges from very poor to fairly reasonable with the 100 mm pipe being totally unpredictable for all practical purposes. The 150, 200 and 250 mm pipes are much better, even if not very useful, while the confidence interval for the 300 mm pipe is fairly good, allowing a greater degree of predictability. It should be realized that since the main objective of the study was not yearly prediction, but rather failure incidence values over a long time period, these confidence intervals are not particularly significant. However, they do serve some purpose in clearly indicating that relatively little confidence can be placed on the calculated failure values for the 100 mm pipe.

Analysis B has tighter confidence intervals than analysis A, especially for the 100 mm pipe. These tighter confidence intervals cannot be taken as an indication that analysis B data is better, or that it more closely resembles the real world situation. It simply means that manipulation of the data has forced the 100 mm failure values closer to the x-axis in analysis B, thereby reducing scatter and the confidence interval.

FIGURE 9

90% CONFIDENCE LIMITS BY PIPE SIZE
FOR ANALYSIS A

Pipe Diameter (mm)	S	μ	$t_{0.05/5}$	$\frac{S}{\sqrt{n}}\ t$	% Data
100	1.67	2.15	2.015	1.37	0.31
150	0.17	0.87	2.015	0.14	66.25
200	0.24	0.76	2.015	0.20	17.60
250	0.34	0.43	2.015	0.28	4.97
300	0.09	0.14	2.015	0.07	6.51

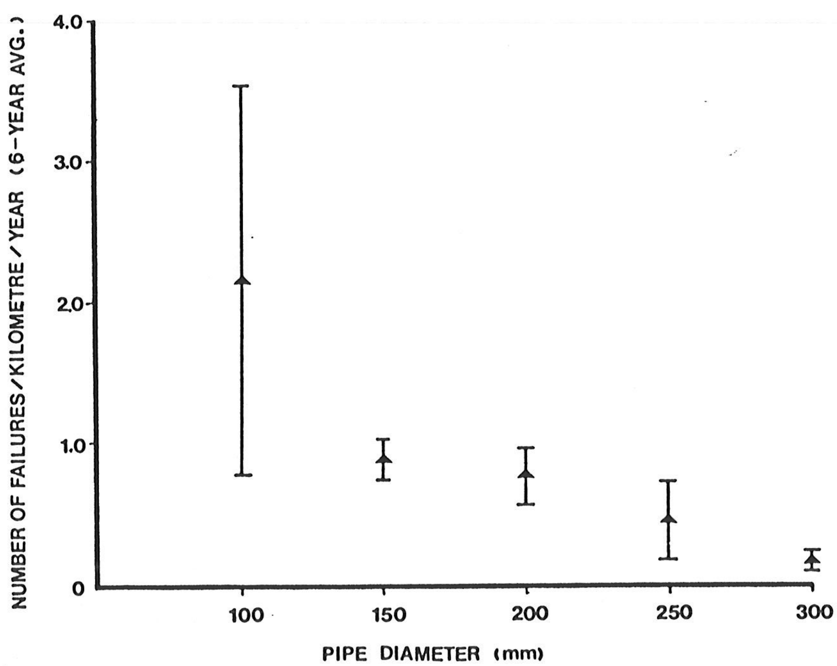
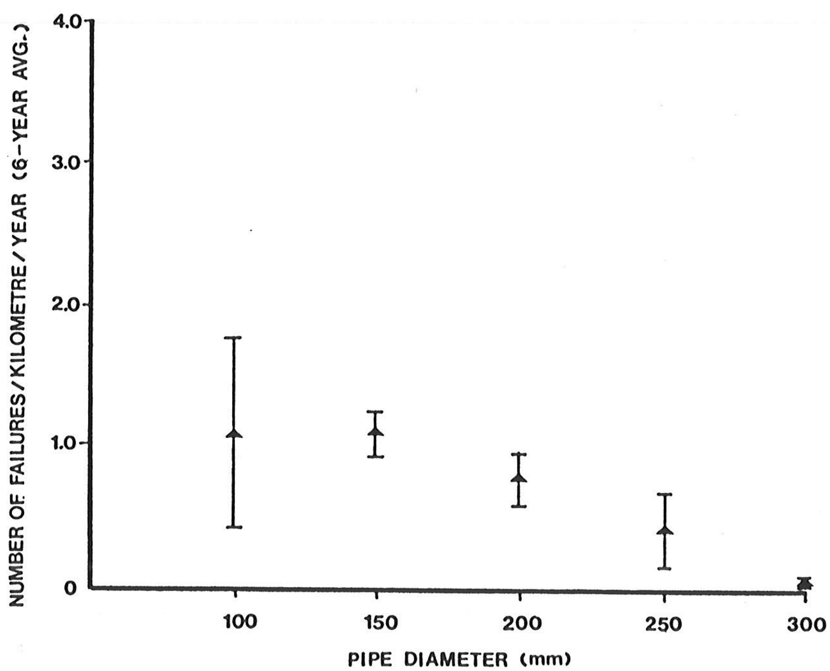


FIGURE 10

90% CONFIDENCE LIMITS BY PIPE SIZE
FOR ANALYSIS B

Pipe Diameter (mm)	S	μ	$t_{0.05/5}$	$\frac{S}{\sqrt{n}}\ t$	% Data
100	0.81	1.05	2.015	0.67	0.63
150	0.21	1.06	2.015	0.17	54.15
200	0.23	0.76	2.015	0.19	17.67
250	0.31	0.39	2.015	0.26	5.44
300	0.04	0.07	2.015	0.03	13.25



Confidence bands about regression lines weighted by variation were constructed to assess the value of the regression line as a predictor of the mean failure values, and to determine whether the relationship is decreasing if kept within the confines of the confidence bands.

A sample calculation for analysis A is shown in Table 8. When all 5 pipe sizes were used, the resulting confidence bands for $\alpha = 0.05$, with n=30, were poor, as shown in Figure 11. This was mainly due to the fact that the 100 mm pipe was responsible for a very large value for MSE. This was understandable, given its large variation. It was decided that all the data could not be judged as poor, since the variation seemed to be caused by only one pipe size. Therefore, confidence bands were constructed utilizing all sizes except the 100 mm, as illustrated in Figure 11 for analysis A. These confidence bands are much narrower than those for the 5 pipe sizes, indicating a greater confidence in prediction of mean values.

Much tighter confidence bands were constructed for analysis B in Figure 12, where the 5 size bands are nearly as good as the 4 size bands of analysis A. The 4 size bands for analysis B showed further improvement. Again, it must be stressed that this does not mean analysis B is representative of better data. It is merely representative of better test results in the analysis, where one can be fairly confident of a decrease in failure incidence with increased pipe sizes.

TABLE 8

SAMPLE CALCULATION FOR REGRESSION ANALYSIS A WEIGHTED BY VARIATION OF α = 0.05 CONFIDENCE BANDS FOR $E(Y_n)$

x Pipe Diameter (mm)	$\hat{y}$	$\sum_{i=1}^{6}(y_{ij}-y)^2$	$(x_h-\bar{x})^2$	$\sum_{i=1}^{6}(x_i-\bar{x})^2$
100	1.17	19.66	10,000	60,000
150	0.91	0.150	2,500	15,000
200	0.65	0.355	0	0
250	0.39	0.605	2,500	15,000
300	0.13	0.041	10,000	60,000
		Σ 20.811		Σ150,000

$$MSE = \frac{SSE}{n-2} = \frac{20.811}{28} = -.743$$

$$s^2(y_x) = MSE[1/n \quad \frac{(x_h-\bar{x})^2}{\sum_{i=1}^{6}(x_i-\bar{x})^2}] \qquad t_{0.05/28} = 1.701$$

$$s^2_{(\hat{y}100)} = 0.743[1/30 + \frac{10,000}{150,000}] = 0.074 \qquad S = 0.273 \qquad tS=0.464$$

$$s^2_{(\hat{y}150)} = 0.743[1/30 + \frac{2,500}{150,000}] = 0.037 \qquad S = 0.193 \qquad tS=0.328$$

$$s^2_{(\hat{y}200)} = 0.743[1/30 + 0 \quad] = 0.033 \qquad S = 0.183 \qquad tS=0.311$$

$$s^2_{(\hat{y}250)} = \qquad = 0.037 \qquad S = 0.193 \qquad tS=0.328$$

$$s^2_{(\hat{y}300)} = \qquad = 0.074 \qquad S = 0.273 \qquad tS=0.464$$

FIGURE 11

90% CONFIDENCE BANDS BY PIPE SIZE FOR ANALYSIS A

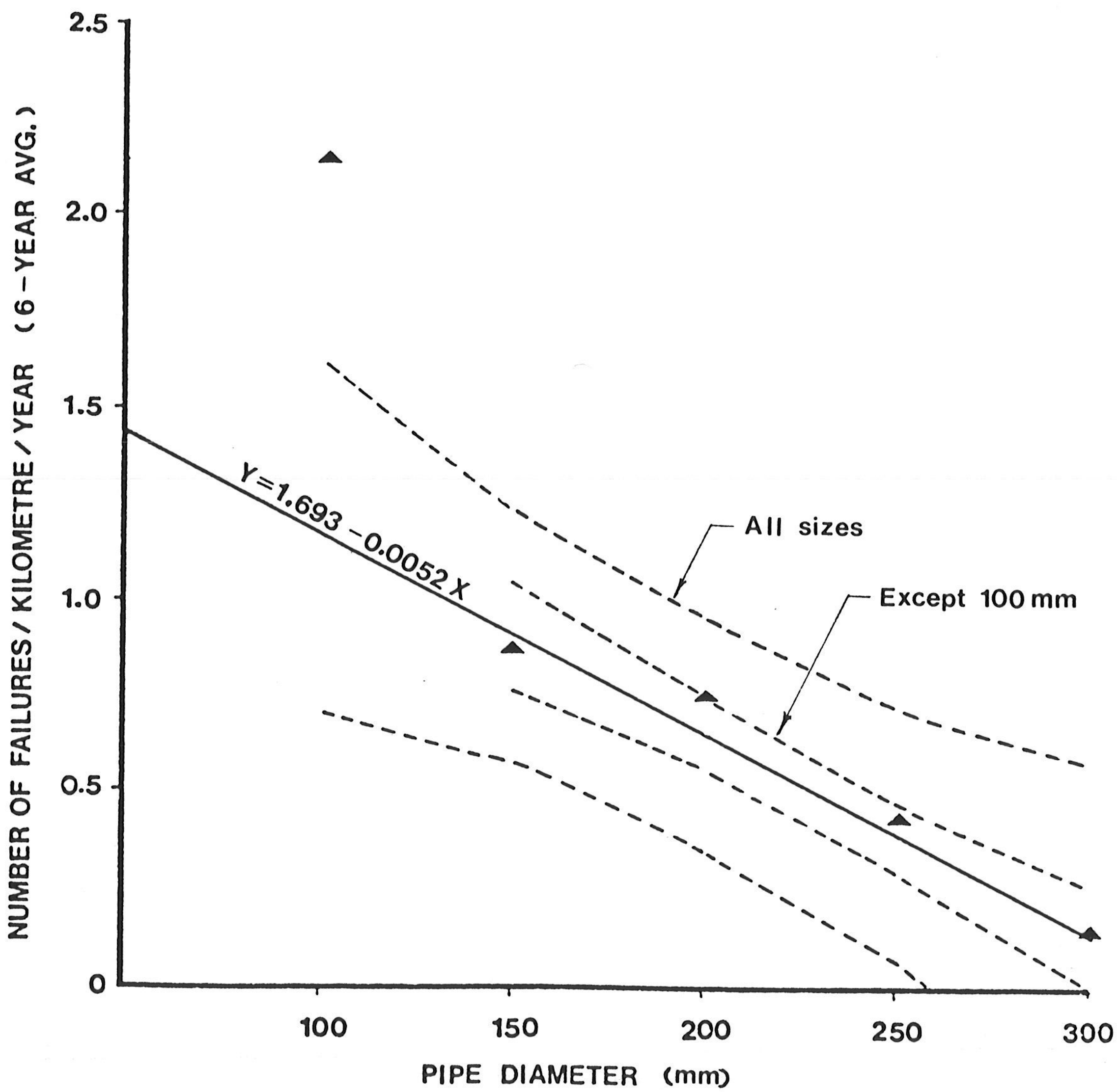


FIGURE 12

90% CONFIDENCE BANDS BY PIPE SIZE FOR ANALYSIS B

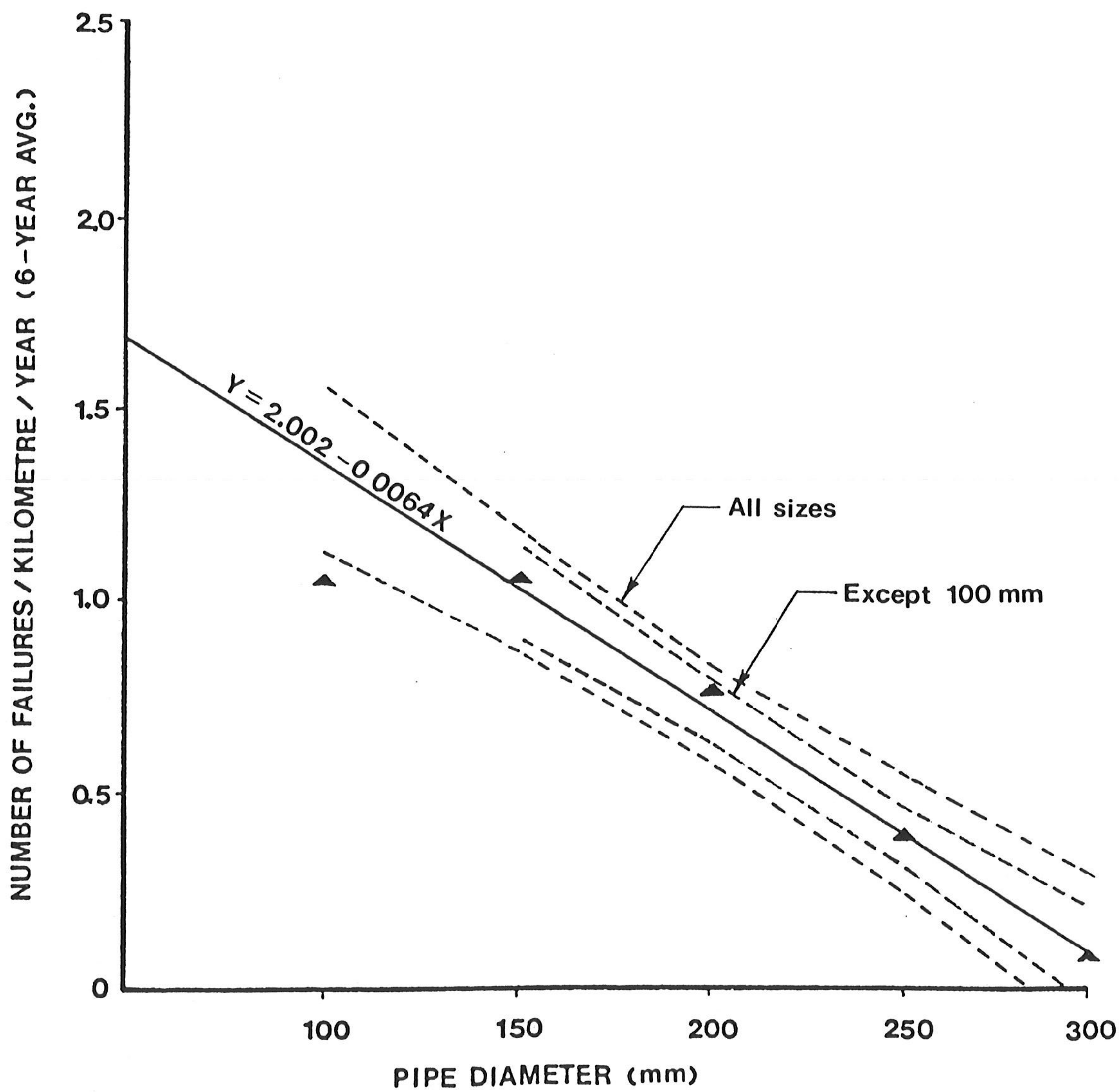


TABLE 9

NUMBER OF FAILURES BY MODE OF FAILURE 1975 - 1980

Pipe Diameter (mm)	Mode of Failure						
	J	O	L	X	S	C	K
50	0	0	0	0	0	0	0
100	6	1	0	1	0	0	0
150	428	129	28	80	12	1	1
200	79	52	8	11	6	1	1
250	11	9	4	2	0	0	0
300	4	3	1	3	0	0	0
350	0	0	0	0	0	0	0
400	0	0	0	0	0	0	0

J - Joint
O - Hole
L - Longitudinal Split
X - Circular Crack
S - Sleeve
C - Corporation Stop
K - Old Clamp Leaking

TABLE 10

NUMBER OF FAILURES / KILOMETRE / YEAR
BY MODE OF FAILURE 1975 - 1980

Pipe Diameter (mm)	Mode of Failures for Analysis A						
	J	O	L	X	S	C	K
100	1.61	0.27	0	0.27	0	0	0
150	0.54	0.16	0.04	0.10	0.02	0	0
200	0.37	0.24	0.04	0.05	0.03	0	0
250	0.18	0.15	0.07	0.03	0	0	0
300	0.05	0.04	0.01	0.04	0	0	0
Pipe Diameter (mm)	**Mode of Failures for Analysis B**						
	J	O	L	X	S	C	K
100	0.79	0.13	0	0.13	0	0	0
150	0.65	0.20	0.04	0.12	0.02	0	0
200	0.37	0.24	0.04	0.05	0.03	0	0
250	0.17	0.14	0.06	0.03	0	0	0
300	0.03	0.02	0.01	0.02	0	0	0

TABLE 11

SINGLE FACTOR ANALYSIS OF VARIANCE FOR PIPE DIAMETER (ANALYSIS A)

Null Hypothesis - There is no difference in the number of failures/ kilometre / year between the various pipe sizes.

Pipe Diameter	$x_{i.}$	$\bar{x}_{i.}$	$(x_{i.} - x_{..})^2$	$\frac{1}{n-1}\sum(x_i - \bar{x}_{i.})^2 = s^2 x_i$
100	2.15	0.54	4.45	0.527
150	0.84	0.21	11.70	0.051
200	0.70	0.18	12.67	0.025
250	0.43	0.11	14.69	0.005
[illegible]	0.14	0.04	16.89	0.0004
	Σ 4.26		Σ 60.38	Σ 0.608

$$s^2\bar{x}_{i.} = \frac{1}{n-1}(60.38) = \frac{1}{19}(60.38) = 3.18$$

$$\sigma^2_B = 4(3.18) = 12.72$$

$$\sigma^2_W = 0.608$$

$$\sigma^2_B / \sigma^2_W = 20.92 \quad > \quad F_{0.05,4,19} = 2.90$$

Therefore reject null hypothesis

TABLE 12

SINGLE FACTOR ANALYSIS OF VARIANCE FOR PIPE DIAMETER (ANALYSIS B)

Null Hypothesis - There is no difference in the number of failures/ kilometre / year between the various pipe sizes.

Pipe Diameter	$x_{i.}$	$\bar{x}_{i.}$	$(x_{i.} - x_{..})^2$	$\frac{1}{n-1} \sum (x_i - \bar{x}_{i.})^2 = s^2 x_i$
100	1.05	0.26	4.80	0.127
150	1.01	0.25	4.97	0.075
200	0.70	0.18	6.45	0.025
250	0.40	0.10	8.07	0.004
300	0.08	0.02	9.99	0.0002
	$\sum$ 3.24		$\sum$ 34.28	$\sum$ 0.456

$$s^2 \bar{x}_{i.} = \frac{1}{n-1}(34.28) = \frac{1}{19}(34.28) = 1.804$$

$$\sigma_B^2 = 4(1.804) = 7.217$$

$$\sigma_w^2 = 0.456$$

$$\sigma_B^2 / \sigma_w^2 = 15.83 \quad > \quad F_{0.05,4,19} = 2.90$$

Therefore reject null hypothesis

3.8 Single Factor Analysis of Variance

It was decided to carry out single factor analysis of variance to test statistically whether there is a decrease in failure incidence with larger pipe sizes. The data are shown in Table 9 as number of failures by mode of failure for each pipe size. These data were further broken down, for anaylses A and B, to failures per kilometre per year, as shown in Table 10.

The null hypothesis stated that there is no difference in the number of failures per kilometre per year between the various pipe sizes. The null hypothesis was rejected at α = 0.05 and α = 0.01 for analyses A and B, as shown in Tables 11 and 12.

3.9 Conclusions From the Statistical Analysis

1) Data averaged over a 6 year period shows that there is a decrease in the incidence of failures per kilometre per year with increased cast iron pipe size.

2) Differences in variation of yearly data from pipe size to pipe size makes simple linear regression a poor choice in illustrating the nature of the relationship between failure incidence and pipe size. For example, a pipe size with highly variable yearly data will be given the same weight as a pipe size with less variable data in a simple linear regression analysis.

3) Regression analysis weighted by variation represents the data trend better than simple linear models, as it puts less weight on less dependable data. That is, less weight will be applied to pipe sizes with more variable yearly data.

4) The assumption of linearity is reasonable if it is weighted properly and it is realized that it can be used only for a certain range of pipe sizes. The test for linearity validates this assumption for the range of pipe sizes chosen.

5) Confidence intervals for each pipe size show an overall poor predictability for yearly failure incidence.

6) Confidence bands for prediction of mean failure values, given the weighted regression line, are poor for analysis A if the 100 mm pipe is considered. They are much improved if the 100 mm is not considered, giving a greater degree of confidence in prediction of mean failure value over a long time period by the regression line. Confidence bands for anaylsis B are satisfactory. If kept within the confidence bands a decrease in failure incidence with increased pipe size is clearly indicated.

7) Analysis of variance shows that larger pipes are more reliable, ie, less failures per kilometre per year than smaller pipes.

8) The study has a major weakness in that age of pipe has not been taken into consideration. It would have been better to analyze pipe failures where the pipes were all nearly the same age, or where several analyses were carried out on several age groups, where different age groups may represent different pipe characteristics, as suggested by Ciottoni (1983).

CHAPTER IV

FORMULATION OF THE LINEAR PROGRAMMING MODEL

4.1 Overview

The model presented here is similar to that of Alperovits and Shamir (1977). It uses the same type of objective function, head loss constraints, and length constraints. A set of reliability constraints based on the analysis of pipe failure data in Chapter III is added to provide a measure of reliability. The linear programming gradient vector approach of Alperovits and Shamir is not used to modify the flows to find a "more optimal" flow distribution.

Layout, flow, and pressure head distribution are held constant while reliability of the system is improved in a cost effective iterative fashion. The least costly link to improve in the least reliable path is modified by decreasing the allowable number of failures per year, ie, the right hand side of the corresponding reliability constraints, in each iteration. The least reliable path is identified by the Poisson probability of zero failures in a given year, while the least costly link in this worst path is found using dual variables. Each link bound by a reliability constraint has a corresponding dual variable which gives the change in system cost for a unit adjustment in reliability. This technique, similar to the constraint method, allows the user to generate a set of alternative networks with successive improvements in reliability at increased cost.

4.2 General Formulation

The objective function minimizes total system cost, where pipe costs are a constant for a unit length of each pipe candidate. It is typical for the set of candidate pipes in a link to be of different diameters. However, it is possible to have two or more pipes of the same diameter in a link, given that the pipes of equal or near equal diameter have a combination of different flow, cost, and reliability characteristics. The objective function can be written mathematically as;

$$\text{Minimize} \quad \sum_{i=1}^{N} \sum_{J=1}^{n(i)} C_{ij} X_{ij} \qquad (1)$$

where N = number of links in the network

$n(i)$ = number of candidate pipes in link i

C_{ij} = unit cost of pipe candiate j in link i

X_{ij} = length of pipe candidate j in link i (decision variable)

Length constraints shown by Equation 2 are necessary to ensure that the lengths of pipe chosen for a link add up to the length of that link, and are written mathematically as;

$$\sum_{j=1}^{n(i)} X_{ij} = L_i \quad \forall \text{ links } i \qquad (2)$$

where L_i = length of link i

Head loss constraints shown in Inequality 3 are necessary to ensure that the head loss along a pre-specified path is less than or equal to the difference in head between the extreme upstream and downstream nodes of that path. Since required pressure heads at demand nodes are known, the right hand side of a head loss constraint is derived by simple subtraction of the head at the downstream node from that of the upstream node, as shown in Equation 4. The required head of any node along a unidirectional flow path cannot exceed the hydraulic gradient defined by the first and last nodes in that path, or its pressure requirements will not be met. To ensure hydraulic consistency for loops, the allowable head loss for a path defined as a loop is set equal to zero, where Inequality 3 becomes an equation. The hydraulic gradient is defined by the Hazen-Williams equation, shown as Equation 5. Expressions 3 through 5 are mathematically written as;

$$\sum_{i \varepsilon p'(b)} \sum_{j=1}^{n(i)} J_{ij} X_{ij} \leq B_p \quad \forall \text{ paths } p \qquad (3)$$

where $p'(b)$ = the set of links in the path associated with lead loss B_p

J_{ij} = hydraulic gradient for pipe candidate j in link i

B_p = maximum allowable head loss along path p

from $$B_p = H_o - H_m \tag{4}$$

where H_o = head at source node

H_m = head requirement at node m

and $$J_{ij} = \frac{4.3586(10)^{18}\ F_{ij}^{\ 1.852}}{C_j^{\ 1.852}\ D_j^{\ 4.87}} \tag{5}$$

where F_{ij} = flow in pipe j of link i (m^3/s)

C_j = pipe friction coefficient for pipe candidate j

D_j = diameter of pipe candidate j (mm)

Non-negativity constraints described in Inequality 6 are included to prevent the choice of negative lengths of pipe, and written mathematically as;

$$X_{ij} \geq 0 \qquad \forall\ i,j \tag{6}$$

4.3 Reliability Constraints

In Chapter III a relationship was developed between pipe size and failure incidence. It was found that larger pipes are more reliable, being associated with fewer failures per kilometre per year. Reliability constraints have been formulated for each link based on this relationship. These constraints, described in Inequality 7, allow the user to increase the level of reliability (reduce the expected number of failures per year) in a link. The reliability constraints are mathematically written as;

$$\sum_{j=1}^{n(i)} a_j X_{ij} \leq F_i \quad \forall \quad \text{Links } i \qquad (7)$$

where a_j = expected number of failures per kilometre per year for pipe candidate j.

F_i = maximum allowable number of failures per year in link i.

4.4 Poisson Path Reliability

The reliability of a particular path in the network is defined as the probability of having zero failures per year in that path. This probability is obtained using a Poisson distribution described in Equation 8. This probability enables the user to identify paths requiring improvement, ie, paths with low probability of zero expected failures per year. Poisson path reliability is expressed mathematically as;

$$PF(o)_p = 1 - \sum_{x=1}^{\infty} \frac{e^{-M_p} \; M_p^{\,x}}{x!} \qquad (8)$$

where $PF(o)_p$ = probability of zero failures in a given year for path p

M_p = expected number of failures per year in path p

from $$M_p = \sum_{k \varepsilon p} \sum_{j=1}^{n(i)} a_j X_{kj}$$

where k = all links in path p

4.5 Method of Application

The method of application is an iterative procedure consisting of the decision-maker's input and the linear programming (LP) algorithm. The LP algorithm contains the objective function, length constraints, head loss constraints, non-negativity constraints, and reliability constraints, while the user's input is based on Poisson path reliabilities and dual variables associated with the reliability constraints.

The procedure is initiated by running the LP model to obtain an initial design, where the right hand side of the reliability constraints are set at artificially high values so as to be non-binding. This ensures a least cost system solution based solely upon length and hydraulic requirements. Now that the process has been initiated, the user may proceed iteratively.

Since pipe size is related to failure incidence, the lengths of various pipe sizes chosen by the LP are the pieces of information necessary for the user to estimate the expected number of failures in each link. Using this information, the Poisson probability of zero failures for each path in the network can be calculated. The user is then able to decide which path in the network requires improvement, ie, identify the path with the lowest probability of zero failures per year.

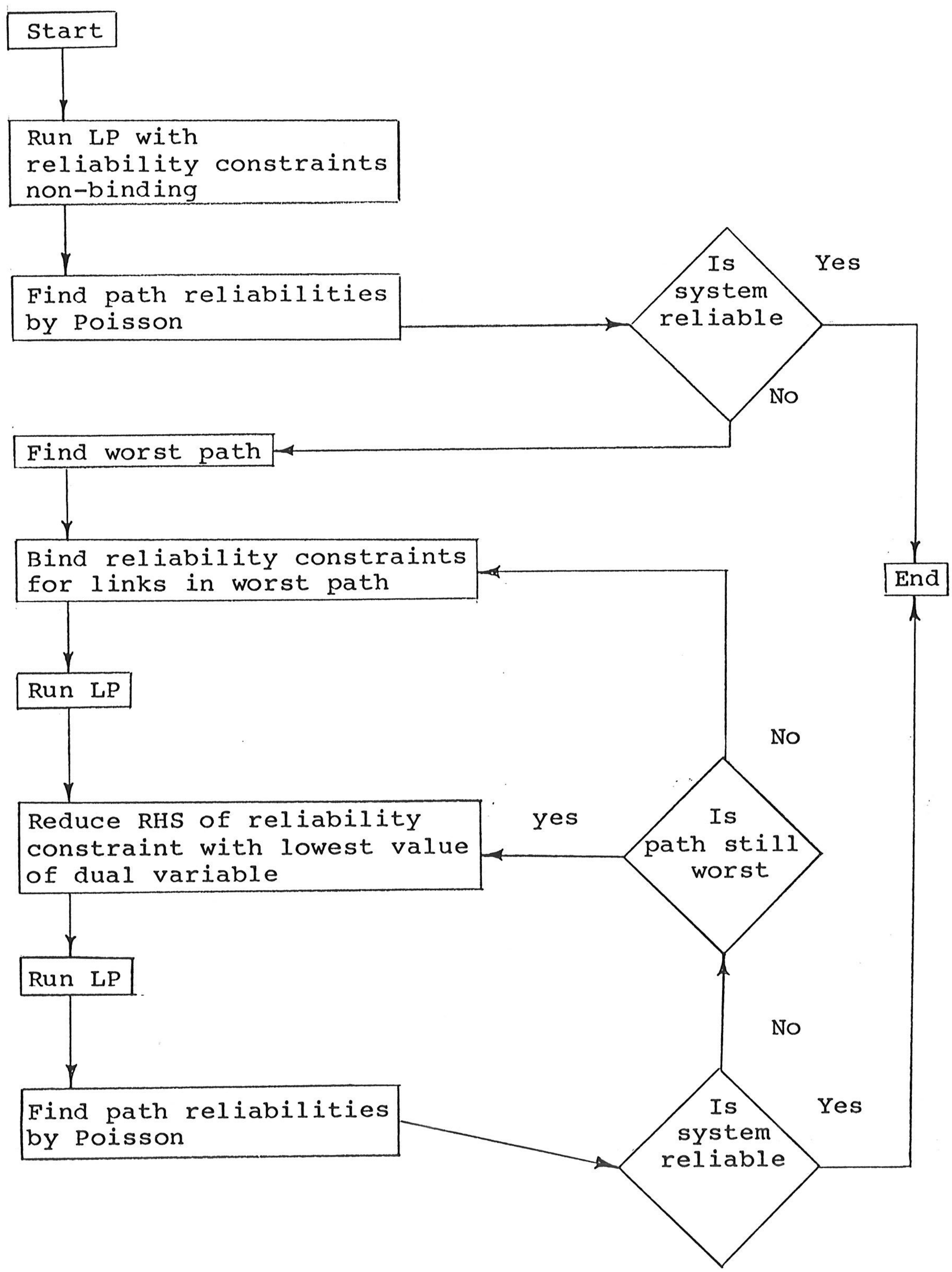
The decision-maker now reduces the right hand side of the reliability constraints of the links in the path with lowest reliability until they just bind. By doing so and running the LP, the user will obtain dual variables associated with the corresponding reliability constraints. These dual variables give the cost per unit of reliability improvement in a given link, ie, the overall system cost change per unit of reduction in the right hand side of the reliability constraints. The right hand side of the reliability constraint corresponding to least cost improvement of a link in the worst path is reduced and the LP is run. This results in an increase in the value of the Poisson probability of zero failures for the path considered, and possibly for other paths which happen to contain the link for which the reliability constraint was modified. The Poisson probabilities are again calculated to redetermine the "worst" path. If the previous worst path is more reliable than one or more of the others, the user will move on to the new least reliable path. If not, the dual variables for the previously considered path will be re-examined to reduce the number of expected failures in the least costly link.

This procedure continues as shown in Figure 13 until some limit has been reached with regard to system cost or reliability.

It should be noted that difficulties will be encountered if several reliability constraints in one or more paths are left to bind simultaneously. As well, pipes of non-adjacent sizes may be chosen in links where reliability constraints are binding. These difficulties will be discussed in conjunction with the sample application.

FIGURE 13

FLOW CHART FOR METHOD OF APPLICATION



CHAPTER V

SAMPLE PROBLEM

5.1 Overview

The sample exercise undertaken here illustrates how a more reliable water distribution network may be developed utilizing the pipe failure data of Chapter III and the linear programming model of Chapter IV. It also demonstrates some of the computational difficulties encountered with this linear programming based procedure.

5.2 Sample Network

The network chosen for analysis consists of a single source node, 19 outflow nodes, and 24 links, each 1 kilometre in length, as shown in Figure 14. The 7 paths and 5 loops of the system are illustrated in Figure 15.

The inflow or outflow at each node, amount and direction of flow in the links, and minimum pressure head requirements for each path and loop remain constant throughout the analysis. Node inflows and outflows, and links flows are given in Tables 13 and 14 respectively. C_j=120 for all pipes.

5.3 Implementation

The process was initiated by finding optimal pipe sizes for the given flows with reliability constraints non-binding. Pipe size alternatives used and their corresponding expected

FIGURE 14

NETWORK LAYOUT

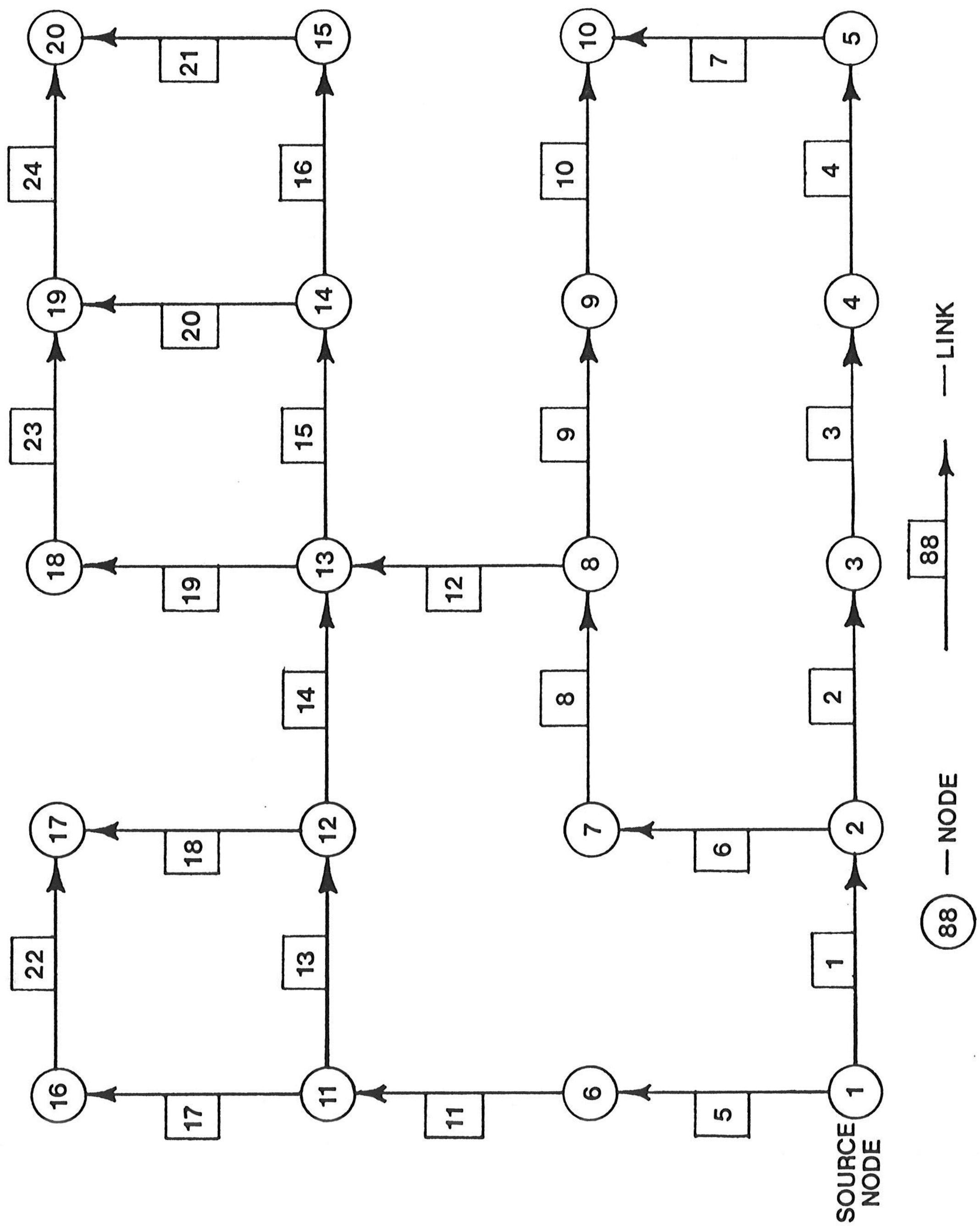


FIGURE 15

NETWORK PATHS AND LOOPS

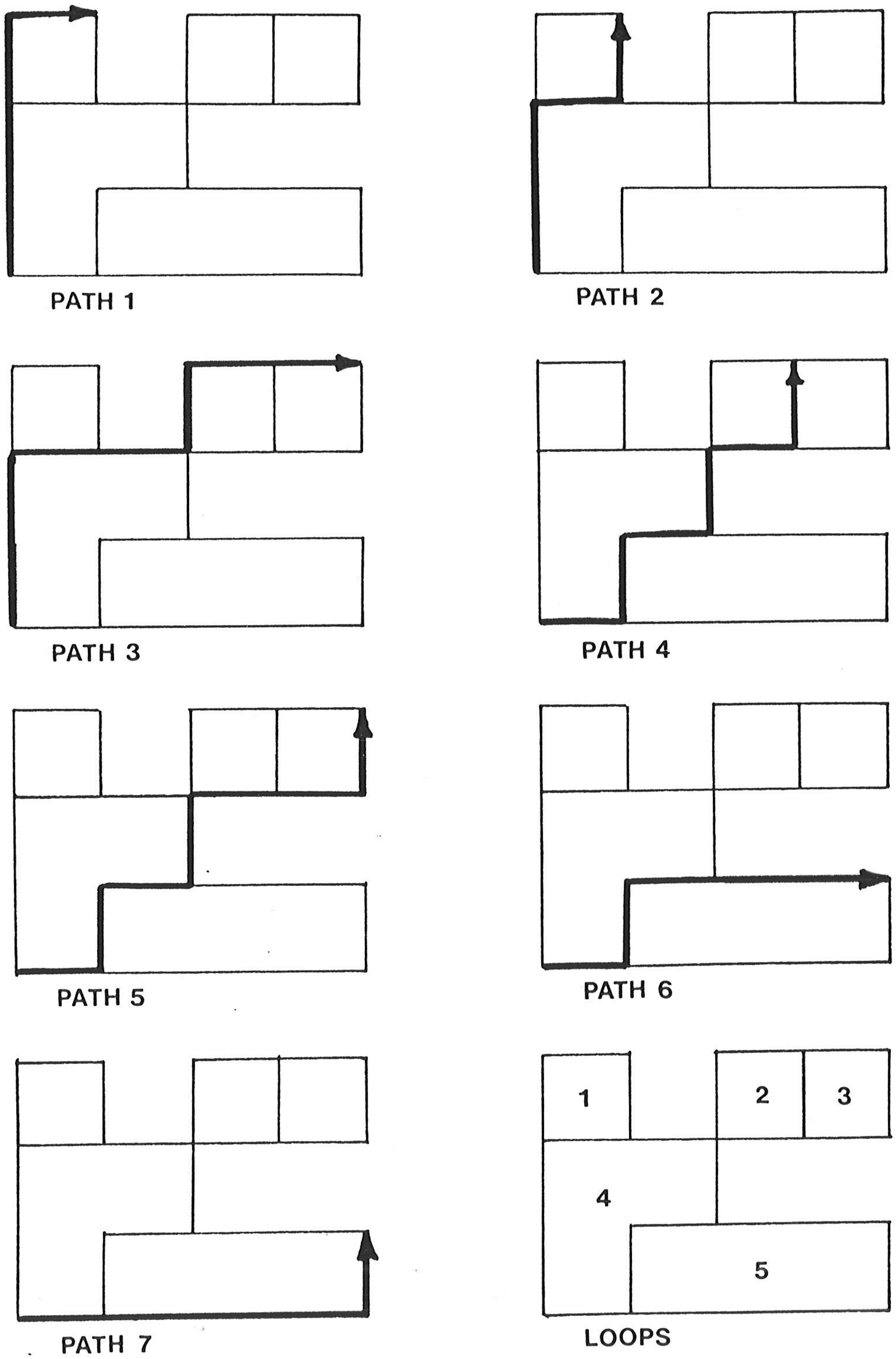


TABLE 13

NODE FLOW REQUIREMENTS

Node Number	Flow (m^3/s)
1 (Source Node)	-0.200
2	0.018
3	0.006
4	0.009
5	0.009
6	0.009
7	0.009
8	0.012
9	0.012
10	0.012
11	0.006
12	0.009
13	0.006
14	0.030
15	0.006
16	0.030
17	0.018
18	0.011
19	0.006
20	0.006

Note: 1) positive flows are outflows
2) negative flow is inflow

TABLE 14

LINK FLOWS

Link Number	Flow (m^3/s)
1	0.095
2	0.029
3	0.023
4	0.014
5	0.105
6	0.048
7	0.005
8	0.042
9	0.019
10	0.007
11	0.096
12	0.011
13	0.076
14	0.054
15	0.043
16	0.007
17	0.014
18	0.013
19	0.016
20	0.006
21	0.001
22	0.005
23	0.005
24	0.005

number of failures per kilometre per year and cost per metre are shown in Table 15. Estimated pipe failure values were taken dirctly from the regression analysis B weighted by variation, shown in Figure 8. In a design situation the actual values derived for individual pipe sizes should be used.

The maximum pipe size considered was 350 mm. The expected failure incidence for the 350 mm pipe had to be estimated since data was unavailable from the Chapter III pipe failure analysis. In some cases this maximum pipe diameter, ie, 350 mm, was chosen for certain links. This condition implies a suboptimal solution in that larger pipe sizes may have been chosen in upstream links if they were available. It was assumed that the 350 mm pipe was the maximum size available for this problem.

The pipe sizes and lengths chosen in the initial solution are denoted as Run 1 in Table 16. The corresponding total system cost of $775,511.19 and reliability of 18.15 expected failures per year are shown in Table 17.

In this demonstration of the model paths with Poisson probabilities less than the arbitrary value of 0.05 were considered unsatisfactory. Therefore 4 paths, namely 3, 4, 5 and 7, required reliablity improvements, with path 5 being the most unsatisfactory. Poisson probabilities for all paths and runs are given in Table 18.

TABLE 15

PIPE FAILURE AND COST DATA

Pipe Diameter (mm)	Expected Number of Failures per Kilometre per Year	Pipe Cost ($/m)
100	1.36	14.30
150	1.04	16.90
200	0.71	24.10
250	0.39	43.20
300	0.07	69.20
350	0.05	98.20

TABLE 16

LP RESULTS - LENGTHS OF PIPE CHOSEN BY LP (m)

(BRACKETED NUMBERS ARE DUAL VARIABLES)

16	17	18	19	20	21	22	23
00	1000	1000	1000	1000	1000	1000	1000
00	1000	1000	1000	1000	1000	1000	1000
84(0.71)	309(0.71)	309(0.71)	309(0.71)	309(0.71)	309(0.71)	-- (0.53)	-- (0.53)
92	149	149	149	149	149	758	758
24	542	542	542	542	542	242	242
- (0.41)	-- (0.41)	-- (0.41)	-- (0.41)	-- (0.41)	-- (0.41)	31(0.62)	31(0.62)
-	424	424	424	424	424	969	969
00	576	576	576	576	576	--	--
00	1000	1000	1000	1000	1000	1000	1000
00	1000	1000	1000	1000	1000	1000	1000
31(0.60)	31(0.60)	31(0.60)	31(0.60)	31(0.66)	31(0.60)	31(0.60)	31(0.60)
69	969	969	969	969	969	969	969
-	--	--	--	--	--	--	--
00	1000	1000	1000	1000	1000	1000	1000
-	--	--	--	--	--	--	--
31	231	231	231	231	231	893	893
69	769	769	769	769	769	107	107
00	1000	1000	1000	1000	1000	1000	1000
00	1000	1000	1000	1000	1000	1000	1000
15(0.80)	218(0.80)	215(0.80)	215(0.80)	215(0.80)	215(0.80)	215(0.80)	215(0.80)
-	--	--	--	--	--	--	--
85	785	785	785	785	785	785	785
00	1000	1000	1000	1000	1000	1000	1000
91	891	891	891	891	891	891	891
09	109	109	109	109	109	109	109
00	1000	1000	1000	1000	1000	1000	1000
- 0.71)	-- (0.71)	-- (0.71)	-- (0.71)	191(0.86)	328(1.43)	328(1.43)	--
58	158	161	161	546	--	--	1000
.8	718	713	713	--	672	672	--
24	124	126	126	263	--	--	--
-	--	--	--	--	--	--	--
33	933	933	933	933	933	933	933
67	67	67	67	67	67	67	67
47	847	847	847	847	847	847	847
53	153	153	153	153	153	153	153
40	740	740	740	740	775	775	781
60	260	260	260	260	225	225	219
-	--	--	--	--	--	--	589
-	--	--	--	--	1000	1000	411
00	1000	1000	1000	1000	--	--	--
-	--	--	--	--	--	--	--
56(0.60)	656(0.60)	969(0.60)	1000	1000	1000	1000	1000
44	344	31	--	--	--	--	--
-	--	--	--	--	--	--	--
-	--	--	--	--	--	--	--
00	1000	1000	1000	1000	1000	1000	1000
00	1000	1000	1000	1000	1000	1000	1000
-	--	--	--	--	178	178	1000
00	1000	1000	1000	1000	822	822	--
-	--	--	--	--	--	--	--

7	8	9	10	11	12	13	14	15
	1000	1000	1000	1000	1000	1000	1000	1000
	1000	1000	1000	1000	1000	1000	1000	1000
	--	--	--	--	5(0.71)	5(0.71)	38(0.71)	38(0.70)
	754	754	754	754	748	748	682	682
	246	246	246	246	247	247	280	280
	--	--	--	--	-- (0.41)	-- (0.41)	-- (0.41)	-- (0.41)
	--	--	--	--	--	--	--	--
	1000	1000	1000	1000	1000	1000	1000	1000
	1000	1000	1000	1000	1000	1000	1000	1000
	1000	1000	1000	1000	1000	1000	1000	1000
	--	--	--	--	-- (0.25)	-- (0.25)	-- (0.25)	-- (0.25)
	--	--	--	--	--	--	424	424
	1000	1000	1000	1000	1000	1000	576	576
	1000	1000	1000	1000	1000	1000	1000	1000
	--	--	--	--	--	--	--	--
	231	231	231	231	231	231	231	231
	769	769	769	769	769	769	769	769
	1000	1000	1000	1000	1000	1000	1000	1000
	1000	1000	1000	1000	1000	1000	1000	1000
(0.80)	215(0.80)	215(0.80)	215(0.80)	215(0.80)	215(0.80)	215(0.80)	215(0.80)	215(0.80)
	--	--	--	--	--	--	--	--
	785	785	785	785	785	785	785	785
	1000	1000	1000	1000	1000	1000	1000	1000
	891	891	891	891	891	891	891	891
	109	109	109	109	109	109	109	109
	1000	1000	1000	1000	1000	1000	1000	1000
(0.24)	-- (0.24)	-- (0.24)	-- (0.24)	-- (0.62)	-- (0.62)	-- (0.71)	-- (0.71)	-- (0.71)
	--	--	--	--	--	154	154	158
	47	48	531	350	350	728	728	718
	905	903	359	493	493	118	118	124
	48	49	110	157	157	--	--	--
	933	933	933	933	933	933	933	933
	67	67	67	67	67	67	67	67
	847	847	847	847	847	847	847	847
	153	153	153	153	153	153	153	153
	577	577	577	577	577	740	740	740
	423	423	423	423	423	260	260	260
	--	--	--	--	--	--	--	--
	--	--	--	--	--	--	--	--
	--	--	--	--	--	1000	1000	1000
	1000	1000	1000	1000	1000	--	--	--
(0.09)	-- (0.22)	-- (0.22)	31(0.60)	31(0.60)	31(0.60)	31(0.60)	31(0.60)	656(0.60)
	121	424	969	969	969	969	969	344
	879	576	--	--	--	--	--	--
	--	--	--	--	--	--	--	--
	1000	1000	1000	1000	1000	1000	1000	1000
	1000	1000	1000	1000	1000	1000	1000	1000
	--	--	--	--	--	--	--	--
	--	--	--	--	1000	1000	1000	1000
	1000	1000	1000	1000	1000	1000	--	--

Link	Pipe Diameter (mm)	RUN1	2	3	4	5	6	
1	300	1000	1000	1000	1000	1000	1000	10
2	200	1000	1000	1000	1000	1000	1000	10
3	250	--	--	--	--	--	--	-
	200	754	754	754	754	754	754	7
	150	246	246	246	246	246	246	2
4	250	--	--	--	--	--	--	-
	200	--	--	--	--	--	--	-
	150	1000	1000	1000	1000	1000	1000	10
5	350	1000	1000	1000	1000	1000	1000	10
6	250	1000	1000	1000	1000	1000	1000	10
7	250	--	--	--	--	--	--	-
	200	--	--	--	--	--	--	-
	150	1000	1000	1000	1000	1000	1000	10
8	250	943	943	923	876	759	1000	10
	200	57	57	77	124	241	--	-
9	200	284	284	303	347	457	231	2
	150	716	716	697	653	543	769	7
10	150	1000	1000	1000	1000	1000	1000	10
11	300	1000	1000	1000	1000	1000	1000	10
12	250	--	--	-- (0.07)	-- (0.07)	-- (0.07)	215 (0.80)	2
	200	--	--	--	121	424	--	-
	150	1000	1000	1000	879	576	785	7
13	300	1000	1000	1000	1000	1000	1000	10
14	300	561	561	518	518	518	891	8
	250	439	439	482	482	482	109	1
15	250	1000	1000	1000	1000	1000	1000	10
16	300	--	--	-- (0.22)	-- (0.22)	-- (0.22)	-- (0.24)	-
	250	--	--	--	--	--	--	-
	200	--	--	--	--	--	16	
	150	985	985	1000	1000	1000	967	9
	100	15	15	--	--	--	17	
17	150	933	933	933	933	933	933	9
	100	67	67	67	67	67	67	
18	150	847	847	847	847	847	847	8
	100	153	153	153	153	153	153	1
19	200	577	577	603	603	603	577	5
	150	423	423	397	397	397	423	4
20	300	--	--	--	--	--	--	-
	250	--	--	--	--	--	--	-
	200	--	--	162	162	162	--	-
	150	1000	1000	838	838	838	1000	10
21	250	--	--	-- (0.08)	-- (0.08)	-- (0.08)	-- (0.09)	-
	200	--	--	--	--	--	--	-
	150	--	--	--	--	--	--	5
	100	1000	1000	1000	1000	1000	1000	5
22	100	1000	1000	1000	1000	1000	1000	10
23	150	1000	1000	1000	1000	1000	1000	10
24	250	--	--	--	--	--	--	-
	200	--	--	--	--	--	--	-
	150	1000	1000	1000	1000	1000	1000	10

TABLE 17

LP RESULTS - SYSTEM COST AND RELIABILITY

Run	System Cost ($)	System Reliability (expected failures/year)
1	775,511.19	18.1510
2	775,511.94	18.1510
3	775,546.37	18.0981
4	775,842.50	18.0590
5	776,582.25	17.9598
6	790,580.31	17.9003
7	791,947.75	17.7422
8	794,190.12	17.5401
9	796,376.94	17.4400
10	804,439.44	17.1002
11	813,252.62	16.9002
12	813,341.50	16.8990
13	825,843.75	16.1849
14	829,302.44	16.0451
15	841,298.81	15.8448
16	846,596.37	15.6457
17	852,372.75	15.5057
18	858,371.06	15.4053
19	858,971.19	15.3954
20	873,937.25	15.1950
21	894,348.12	14.8068
22	900,115.69	14.3883
23	935,488.25	13.8246

TABLE 18

LP RESULTS - PATH POISSON PROBABILITIES OF ZERO FAILURES PER YEAR

Run	Path 1	2	3	4	5	6	7
1	0.079	0.278	0.036	0.036	0.018	0.058	0.027
2	-	-	-	-	-	-	-
3	-	-	-	0.038	-	-	-
4	-	-	-	0.039	-	-	-
5	-	-	-	0.041	-	-	-
6	-	-	0.040	0.042	-	-	-
7	-	-	-	-	-	-	-
8	-	-	-	-	0.019	-	-
9	-	-	-	-	0.020	-	-
10	-	-	-	-	0.030	-	-
11	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-
13	-	-	0.059	0.058	-	-	-
14	-	-	-	-	-	-	0.031
15	-	-	-	-	0.036	-	-
16	-	-	-	-	-	-	0.037
17	-	-	-	-	-	-	0.042
18	-	-	-	-	0.040	-	-
19	-	-	-	-	-	-	-
20	-	-	-	-	0.049	-	-
21	-	-	0.063	0.080	-	-	-
22	-	-	-	-	-	0.072	0.052
23	0.079	0.278	0.082	0.096	0.054	0.072	0.052

Certain links in path 5 were bound by reliability constraints. This allowed dual variables associated with the reliability constraints for those links to give an indication of which link in this worst path was least costly to improve with respect to reliability. The dual variables for Run 3, as illustrated in Table 16, indicated that link 12 was the most cost effective to change with respect to reliability improvement. It was found, however, that the reliability constraints on a given path could not be forced to bind simultaneously. This phenomenon resulted in complications. Choices of smaller pipes had to be eliminated from non-binding links before the first improvements were made to link 12. Due to the cost (pipe size) minimizing objective of linear programming an increase in size of pipe in a given link by the tightening of reliability constraints may result in a decrease in length of larger pipe sizes in other upstream or downstream links. This will produce a least cost alternative given governing hydraulic constraints. If all links in a path could be bound by reliability constraints, this problem would be eliminated for that path. If the steady decrease in length of 250 mm pipe and increase in length of 200 mm pipe for link 8 of path 5 is traced in Table 16, it becomes readily apparent that the minimum pipe size candidate must be increased in the absence of universal reliability constraints.

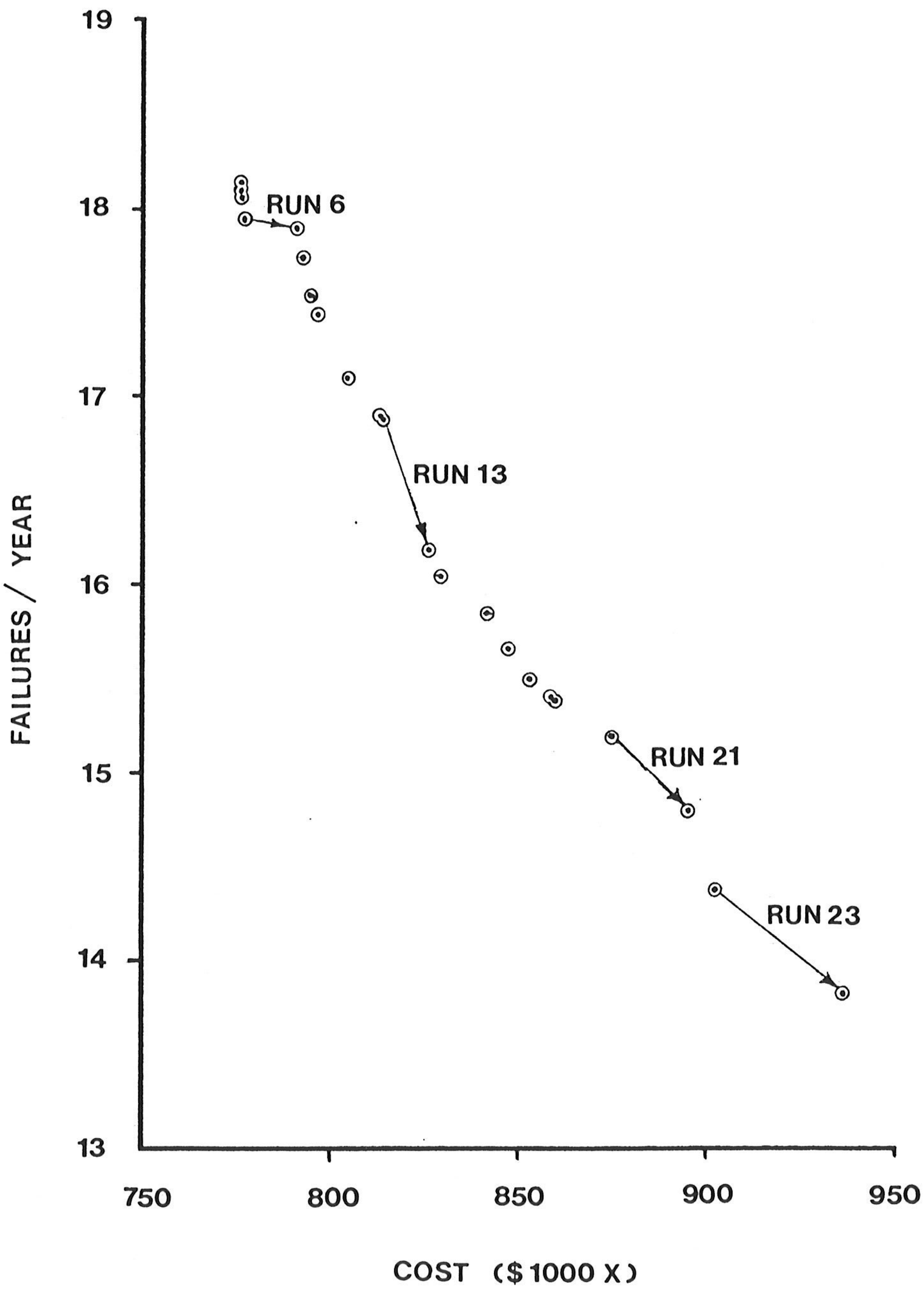
The 200 mm pipe was eliminated from candidacy in link 8 for Run 6. This prevented undermining of the increase in reliability for path 5. However, this upsets what was expected to be a steadily decreasing rate of decrease in failure incidence with increased cost. This is shown in Figure 16. Relatively large breaks in the curve occur at runs where a minimum pipe size candidate for a link was increased in diameter. Breaks for runs 6, 13, 21, and 23 are due to elimination of candidate pipes.

In Run 6 links 12 and 16 exhibit unusual pipe size choices. The 200 mm pipe has been skipped in link 12 and three pipe sizes have been chosen for link 16. In hydraulically constrained linear programming models no more than two adjacent pipe sizes are chosen per link.

These unusual pipe choices are made where loop constraints (Equation 3) must be satisfied. Increases in reliability of a certain link in a loop represent increases in length of larger diameter pipe. Increasing pipe size in one link of a loop requires an offsetting change of pipe size in another link of that loop in order to balance the loop, ie, bring the total head losses around the loop to zero. This change can be achieved by either decreasing the length of larger diameter pipes carrying flow in the same direction as the link with the modified reliability constraint or by increasing the length of larger diameter pipe carrying flow

FIGURE 16

EXPECTED SYSTEM FAILURES
PER YEAR VERSUS
SYSTEM COST



in the opposite direction to the modified link. Reliability constraints are therefore satisfied at the highest head loss possible, thereby minimizing cost and eliminating or reducing size increases in links with opposite flow direction.

The choice of non-adjacent pipe sizes for link 12 occurred when the 200 mm pipe was eliminated from candidacy in link 8. This indicates that increasing head losses at link 12 was least costly with respect to maintaining the loop constraint for loop 4. Given that link 12 is assumed to be least costly to modify, and all other things remain equal, the choice of pipe sizes in link 12 can be examined using simple linear equations. To keep from increasing pipe sizes in the opposite flow direction of loop 4 the required minimum head loss in link 12 must be 2.94 m, obtained by multiplying the length of pipe choices for Run 6 in Table 16 by the corresponding unit head losses for the flow in link 12 from Table 14. The minimum reliability for link 12 was set at 0.90 failures per kilometre per year, ie, the right hand side of the corresponding reliability constraint. If the combinations of candidate pipes are assumed to be limited to pairs, then certain candidate pairs can be eliminated. The 300/250, 300/200, 250/200 pairs greatly exceed reliability requirements and provide too little head loss at high cost, while the 150/100 pair is unreliable. The complete range of pipe candidates for link 12 and their pertinent characteristics are shown in Table 19.

TABLE 19

PIPE CANDIDATES AND CHARACTERISTICS FOR LINK 12

Pipe Diameter (mm)	Expected Number of Failures per Kilometre per Year	Pipe Cost ($/m)	Head Loss per Kilometre (m)
300	0.07	69.20	0.125
250	0.39	43.20	0.304
200	0.71	24.10	0.902
150	1.04	16.90	3.663

The least costly pair of pipes will be that which produces the maximum possible head loss. These pairs can be found by combining equations 9 and 10 to give equation 11.

$$J_a X_a + J_b X_b = 2.94 \qquad (9)$$

where J_a, J_b = unit head losses for pipe candidates a and b

X_a, X_b = lengths of pipe candidates a and b

$$X_b = 1 - X_a \qquad (10)$$

where each link is 1 kilometre long

$$X_a = \frac{J_b - 2.94}{J_b - J_a} \qquad (11)$$

These combinations of pipe sizes and their associated reliabilities and head losses are outlined in Table 20. Equation 11 can be modified to find combinations of lengths for given reliabilities instead of head losses. These combinations are also given in Table 20, where the 250/150 pair is the best choice, meeting head loss and reliability criteria at least cost.

Improvements to path 5 were continued until its Poisson probability exceeded that of path 7, in Run 10. However, the links of path 7 were not bound until run 12. The minimum diameter pipe candidate of link 16 was allowed to increase in length while the 150 mm pipe size disappeared. This serves to further illustrate the occurrence of the choice of non-adjacent pipe sizes. The minimum allowable pipe size for link 16 was raised to 150 mm in Run 13, resulting in an

TABLE 20

HEAD LOSS AND RELIABILITY PAIRS FOR LINK 12

Pipe Pair	Length of Pipe by Head Loss (m)	Cost per Metre ($)	Associated Reliability (failures per year)
300/150	0.204/0.796	27.57	0.84
250/150	0.215/0.785	22.55	0.90
200/150	0.262/0.738	18.79	0.95

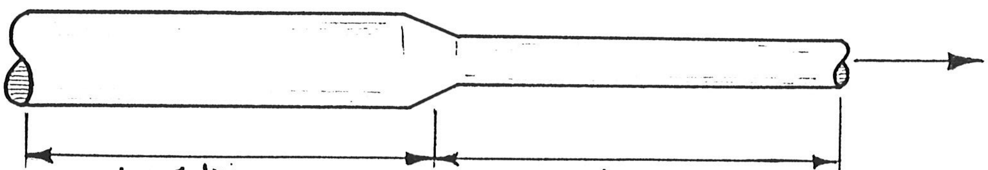
Pipe Pair	Length of Pipe by Reliability (m)	Cost per Metre ($)	Associated Head Loss per Kilometre (m)
300/150	0.144/0.856	24.43	3.15
250/150	0.215/0.785	22.55	2.94
200/150	0.424/0.576	19.95	2.49

increase in pipe sizes of opposite flow direction links in loops 2 and 3.

The system reliability improvements were continued until all path Poisson probabilities exceeded 0.05, giving a total system cost of $935,488.25 at 13.82 expected failures per year. This final answer, although it provides a more reliable system, is not completely satisfactory from the standpoint of hydraulic efficiency. Since flows in upstream links are greater than in downstream links head loss will be at a minimum if larger pipes are located upstream. This point is illustrated by the sample problem of Figure 17.

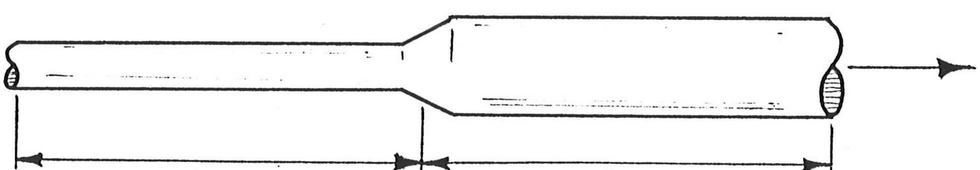
FIGURE 17

LOCATING LARGER DIAMETER PIPES
UPSTREAM FOR GREATER HYDRAULIC EFFICIENCY
(REDUCED HEAD LOSS)



L = 1 km	L = 1 km
F = 0.1 m^3/s	F = 0.05 m^3/s
C = 100	C = 100
D = 300 mm	D = 150 mm
J = 10.5 m	J = 84.8 m

TOTAL HEAD LOSS = 95.3 m



L = 1 km	L = 1 km
F = 0.1 m^3/s	F = 0.05 m^3/s
C = 100	C = 100
D = 150 mm	D = 300 mm
J = 306.0 m	J = 2.9 m

TOTAL HEAD LOSS = 308.9 m

CHAPTER VI

CONCLUSIONS

1. Statistical analysis of pipe failure data shows that there is a decrease in the incidence of failures per kilometre per year with increased cast iron pipe size for District 4 in the City of Winnipeg.
2. Present day 'optimal' design techniques do not explicitly consider reliability of pipes in the system, while the linear programming method presented here directly incorporates results of a statistical analysis for reliability of cast iron pipe.
3. The linear programming model developed here, coupled with pipe failure data, provides a means for assessing and modifying reliability of links and paths in a hydraulically constrained, cost optimized water distribution network.
4. Reliability improvement by iterative tightening of link reliability constraints according to dual variables gives a set of alternative systems with increasing reliability and cost. However, this technique results in reduced hydraulic efficiency and unusual pipe choices about loops.

CHAPTER VII

RECOMMENDATIONS

1. A more detailed investigation of pipe failure data with respect to environmental conditions and pipe characteristics is required. Pipe installed in different environmental conditions may display different rates of failure. Factors such as soil type and moisture, bedding conditions, traffic and backfill loading, pressure head and surges, proximity to structures, and frost penetration should be assessed with respect to their individual and combined effects on watermain failure incidence.

   Pipe itself is variable in nature due to the different types available, and different strength classes available within a certain size group of a particular type of pipe. Further, appurtenances such as couplings, clamps, service connections, fittings, valves, air release valves, fire hydrants, and thrust blocks should be investigated with respect to failure incidence. For example, copper service connections associated with smaller pipe sizes promote rapid corrosion of cast iron pipe, perhaps biasing the relationship between pipe size and failure incidence.

2. The linear programming model requires modifications in the area of reliability improvement, since the resulting hydraulic configuration is not entirely satisfactory.

The reliability of supply to each node should instead be assessed. Reliability of supply to a node is a function of reliability of the paths supplying the node, where path reliability will be based upon the expected number of failures in the path. Modifications to the pressure head at a given node will result in direct reliability improvement to paths supplying that node because pressure head is related to pipe size for given flow conditions. It should also be investigated whether upgrading pipe strength class is more cost effective than a size increase.

Further refinements to the model should include optimization of flow distribution and multiple loadings.

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