A Fundamental Approach using Theories of Elasticity to Study
Texture-Related Mechanical Properties of Foods

BY
Alok Anand

A Thesis
Submitted to the Faculty of Graduate Studies
In Partial Fulfillment of the Requirements
for the Degree of

MASTER OF SCIENCE

Department of Food Science
University of Manitoba
Winnipeg, Manitoba

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A Fundamental Approach using Theories of Elasticity to Study Texture-Related Mechanical Properties of Foods

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Alok Anand

A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University of Manitoba in partial fulfillment of the requirements of the degree of Master of Science

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A       Cross-sectional area of indenter or specimen (m²)
β       Dundur’s coefficient (Dimensionless)
b       Radius of the indenter (m)
C       Constant (N)
δ       Original length in transverse direction (m)
d       Deformation at the point of pressure application (m)
D       Flexural rigidity of the plate (Nm)
Δδ      Change in length in transverse direction (m)
ΔL      Change in length in the direction of force (m)
E       Young’s modulus of elasticity (Nm⁻²)
f       Factor (Dimensionless)
F       Applied load (N)
F_{critical}  Maximum load at which the specimen fails (N)
F_{indent}   Load at which a layered composite system fails by indentation (N)
G       Shear modulus (Nm⁻²)
h       Thickness of the plate (m)
k       A constant known as constraint factor, k = 2 (Wen et al. 1998)
K       Modulus of subgrade reaction (Nm⁻³)
K_c      Compression coefficient of the specimen (Nm⁻²)
ker', kei'  First derivatives of the real and imaginary parts of Bessel functions
K_s      Shear Coefficient (Nm⁻¹)
\[ l \] Characteristic length (m)

\[ L \] Original length of the specimen (m)

\[ v \] Poisson's ratio (Dimensionless)

\[ P \] Perimeter of indenter (m)

\[ p \] Pressure on the foundation at a given point (Nm\(^{-2}\))

\[ p(r) \] Pressure at a distance \( r \) from the origin (Nm\(^{-2}\)), \( r \leq b \)

\[ q \] Pressure at any point (Nm\(^{-2}\))

\[ r \] Distance of a point from the center of loading (m); \( 0 \leq r < b \)

\[ \sigma \] Intensity of uniformly distributed load \( F \) (Nm\(^{-2}\))

\[ \sigma_r \] Compressive strength of the foundation material (Nm\(^{-2}\))

\[ \sigma_s \] Strength in the small deflection plate theory (Nm\(^{-2}\))

\[ \sigma_y \] Yield stress (Nm\(^{-2}\))

\[ \sigma_z \] Stress along the Z-axis at a depth \( z \) from the surface (Nm\(^{-2}\))

\[ t \] Thickness of top layer (m)

\[ \tau_{top} \] Shear strength of the top layer (Nm\(^{-2}\))

\[ y \] Distance of any point along the Y-axis from the surface (m)
ABSTRACT

A fundamental approach was taken to evaluate the mechanical properties of foods, which are often associated with the perceived texture of foods. Indentation test, which is a widely used empirical test in the modern food industry, was studied with a fundamental perspective so that the results could be obtained in fundamental units instead of empirical units. Indentation test was used to measure the Young’s modulus of elasticity and the yield stress of a model food system. Slabs of 1% agar gel, prepared in three sample dimensions were indented with seven flat cylindrical indenters. Results obtained from the indentation test were compared with those obtained by a uniaxial compression test. It was postulated that the indentation test could be used to measure the fundamental mechanical properties of foods. Effects of sample dimensions were observed and explained on the basis of known physical principles. Results were then extended to a model food system, which could be regarded as a layered composite structure, in a similar way to many examples in the discipline of engineering, e.g., highways, runways, ice structures, sandwich panels etc. Scientists and engineers involved in these areas have used theories of elasticity to predict the stresses and resulting displacements in layered composite structures. In the second part of this research, stiffness and strength parameters of a layered composite food were analyzed by the theories of elasticity and it was found that these theories were also applicable to a layered food system.

The fundamental approach taken in this research was based on known physical principles and it made possible to apply a rigorous analysis of mechanical attributes of food texture which was otherwise, not possible with conventional and
empirical approach of food texture evaluation. A rigorous analysis based on known physical principles will enable food scientists to predict the texture of both, homogeneous and composite foods, while they undergo different processing steps. It can also enable food process engineers to design and evaluate a process to produce foods with desired textural attributes.
1. INTRODUCTION

For most foods, texture is considered to be the most important quality attribute among the four major food quality attributes, namely the physical appearance, flavor, nutritional value, and texture (Bourne 1982). Many food processing operations are designed to change the textural properties of foods (Bourne 1982) so that the food becomes more palatable and more acceptable to consumers. For example, a crispier potato chip is preferred to a less crispy one and consumers usually are willing to pay a premium price for crispier chips. Another example is an easily spreadable margarine which can sell for a higher price than that for a hard margarine. Therefore optimizing the textural properties of foods is directly associated with higher financial returns.

Food texture evaluation is required at various stages of processing to control the quality, to meet the desired quality standards and also to ensure consistent quality over different batches. Texture evaluation is a necessary component of new product development and is also required for process control and design (Finney 1969). If food texture can be evaluated accurately, it can also be changed to meet the expectations for an acceptable food quality. However, there are a number of concerns that must be addressed first before food texture can be correctly evaluated. One such concern is the lack of officially accepted definitions of the terms that are often used to describe textural characteristics of foods, e.g. toughness, chewy, hardness, grittiness (Finney 1969; Bourne 1982). There exists a general worldwide disagreement on the terms that are often used to describe texture (Bourne 1982). As a result, it is difficult to ensure that the same textural characteristic is measured under the same name and same conditions by different researchers at different geographical locations (Finney 1969).
Some other concerns include the empirical nature of methods that are used to evaluate food texture (Mohsenin 1970), lack of comparability between results (Mohsenin 1986; Segars and Kapsalis 1987; Bourne 1994), lack of understanding of the mechanisms responsible for texture development in foods (Segars and Kapsalis 1987), etc.

Two major approaches can be taken in evaluation of food texture (Mohsenin 1986). It can be evaluated either on the basis of scores given by the members of a sensory panel or on the basis of physical and mechanical properties of foods themselves. The first approach is known as sensory evaluation of food texture. Although it does provide satisfactory results, influence of human factors cannot be ruled out (Mohsenin 1986; Kilcast 1999). In addition, it is time consuming and labor intensive (Cardello and Maller 1987). The second approach is widely known as instrumental evaluation of food texture. This method is more consistent and is generally free from human factors (Mohsenin 1986). Instrumental methods of texture evaluation are further divided into two major categories of tests: empirical and fundamental tests. Currently, the empirical tests are more common in the food industry for the simplicity and economy that they offer. They are also suited for foods because of the complex (non-homogeneous, anisotropic, and viscoelastic) nature of many foods (Segars and Kapsalis 1987). However, empirical methods do not produce results in fundamental units of measurement and therefore comparison of results is difficult. For example, they do not allow the reduction of forces to understandable stresses primarily because the areas over which the forces are applied are not easily defined (Segars and Kapsalis 1987). Therefore, in order to develop optimum predictive measurements of food texture, one should consider taking the fundamental approach. This approach
heavily relies upon the true mechanical properties of foods such as the modulus of
elasticity, failure stress, fracture toughness etc., which are known to be related to the
perceived texture of foods (Mohsenin 1986). Therefore, good knowledge of mechanical
properties of foods can provide a more complete knowledge of the texture of foods.
Since mechanical properties are measured by physical experiments, a careful choice of
experimental conditions such as the rate of force application, magnitude of strain,
sample dimensions etc. is critical to develop theoretical models which can later be used
to evaluate texture of foods on the basis of their mechanical properties (Segars and
Kapsalis 1987).

Indentation is one of the most widely used empirical tests in the food industry
primarily because it offers rapid test results and is easy to use (Timbers et al. 1965).
Therefore the first part of this research was dedicated to studying the use of indentation
as a fundamental approach instead of an empirical approach. The objective was to see
if an indentation test can be used to characterize the true mechanical properties of a
model food system. The geometrical dimensions of the test specimens also affect the
tests of fundamental nature (Brinton and Bourne 1972; Peleg 1977; Peleg 1987). For
foods that contain large amounts of water, effect of sample dimensions is mostly due to
the hydrostatic pressure build up in the samples (Peleg 1987). Therefore, another
objective was to better understand the effects of sample dimensions on characterization
of mechanical properties using an indentation test.

There are numerous examples of foods that are made of more than one
component and each of these components have their own mechanical properties, which
can be substantially different from one another. A set of layered food systems such as
layered cakes, pies, sandwich cookies, fruits and vegetables with skin etc., lies within the domain of multi-component food systems. These layered foods can be conveniently regarded as layered composite structures, where each layer has unique mechanical properties. For example, a pie crust has entirely different properties from its inside filling. When a layered composite food is consumed, the perception of texture is a combined effect of the mechanical properties of each of the layers (Roy et al. 1989; Scanlon and Ross 2000). Therefore one objective of the second part of this research was to characterize the mechanical behavior of a model composite food system (see section 2.3).

Many examples such as highways, runways, foundations, sandwich panels, ice structures are found in the field of engineering, which are also classified as layered composite structures. Researchers have used the classical theories of elasticity to compute the relationship between load and deformations in these structures with the goal of predicting and improving the overall performance of these structures under designed and accidental loads (Westergaard 1926; Burmister 1945; Wyman 1950; Nevel 1970; Selvadurai 1979). The overall performance of these structures under loading is a function of the mechanical properties of individual layers. In a number of ways, a layered food composite can be considered reasonably similar to the engineering structures as mentioned above. In foods, the mastication load produces deformation in food being consumed and thus produces a perception of texture during consumption of food (Mohsenin 1986). Also when foods are handled or transported, external loads such as the transit loads may produce deformations and may even cause the foods to lose their structural integrity. Theories of elasticity have provided classical treatments
for the mechanical behavior of layered composite structures (Timoshenko and Goodier 1951) and solutions are readily available in many of the engineering fields (Westergaard 1926; Burmister 1945; Wyman 1950; Nevel 1970; Selvadurai 1979). Therefore another objective of the second part of this research was to demonstrate the applicability of theories of elasticity in studies pertaining to the mechanical behavior of layered composite foods.

For the convenience of readers, objectives of the entire research are listed below:

1. To see if an indentation test can characterize the mechanical properties of a model food system.
2. To study the effects of sample dimensions on characterization of mechanical properties of a model food system.
3. To characterize the mechanical behavior of a layered model composite food system.
4. To demonstrate the applicability of theories of elasticity in studying the mechanical behavior of a layered model composite food system.
CHAPTER 2

2.0 Literature Review
2.1 FOOD TEXTURE AS A QUALITY INDEX

Much food processing is designed to change the textural characteristics of foods (Bourne 1982). For example wheat grain as a whole is hard and therefore unsuitable to eat, but when converted into soft breads, cakes, or cookies, it makes a nice meal (Bourne 1994). The very first decision on whether or not to consume the bread can be made without even unwrapping the bread on the basis of perceived “softness” upon squeezing the unwrapped bread, regardless of how good the taste and flavor is.

Food texture is a sensory attribute and is one of the three major quality characteristics (appearance, flavor, and texture) for the consumer acceptance of foods (Bourne 1990). It is a combined effect of structural characteristics of foods and their interactions with sensory organs (Szczesniak 1963). Food texture is a multi-parameter attribute which covers a number of different textural properties or sensory feelings such as hardness, gumminess, chewiness, crispiness, tenderness etc. and therefore “textural properties” is a more versatile term instead of texture (Bourne 1990). Szczesniak (1963) divided the textural properties of foods into three major classes: mechanical characteristics, geometrical characteristics, and other characteristics. The mechanical characteristics were considered to be the most important in evaluation of textural properties of foods and were further divided into five basic classes (Szczesniak 1963).

i. Hardness – It is defined as the force required to attain a given deformation.

ii. Cohesiveness – It is defined as the strength of the internal bonds which holds the physical structure together.

iii. Viscosity – It is the rate of flow of matter under the action of an applied force.
iv. Elasticity – it is a material property by virtue of which the deformed material regains its original shape as soon as the force is removed.

v. Adhesiveness – It is defined as the energy required to overcome the forces of attraction between the food and other materials.

The geometrical characteristics are mostly related to the physical appearance of foods and are sensed visually. The other characteristics include some of the mouthfeel factors mostly affected by fat and moisture contents and they cannot be easily explained on the basis of mechanical and geometrical properties of foods. The above classification of textural properties of foods indicates that a better understanding of food texture requires the need to understand the mechanical properties of foods (Szczesniak 1963).

2.1.1 Evaluation of Food Texture

Food texture is not easy to define and therefore is not easy to evaluate (Mohsenin 1970). However, a number of methods have been developed to evaluate food texture, which can be broadly classified into two major categories, subjective and objective methods (Mohsenin 1970). Subjective methods of texture evaluation involve sensory analyses, where the judgements about the textural quality are made on the basis of actual human perception. Mohsenin (1986) recognized the fact that the textural characteristics of finished food products must be well correlated with human senses, which are the ultimate judges of food quality (Mohsenin 1986). Generally speaking, in modern food processing plants, sensory results for textural quality supersede the results
obtained by other methods. On the other hand, sensory methods are expensive and time consuming (Bourne 1994). Often they are affected by physiological and psychological factors and therefore more information on individual preferences must be gathered before the results of sensory evaluation can be reasonably interpreted (Mohsenin 1986). Objective methods of texture evaluation primarily aim to exclude the physiological and psychological factors and thereby evaluate the textural properties solely on the basis of mechanical properties of foods. Objective methods are further divided into two categories of tests, fundamental and empirical tests (Mohsenin 1986). Each of these methods has its own advantages and disadvantages, which will be discussed in subsequent sections. Regardless of the type of method used in evaluation of food texture, several points must be considered such as identifying factors which may affect the test results, defining what exactly is to be measured, selecting a statistically representative sample, and precisely describing the experimental conditions (Kamel and deMan 1977).

2.1.1.1 Empirical tests

Empirical tests are most widely used (Bourne 1982; Mohsenin 1970) in food industry because they are developed from practical experiences as a rapid way to measure textural characteristics (Bourne 1994). A number of devices have been developed for performing empirical tests on foods which include the shear press, consistometer, penetrometer, tenderometer, etc (Mohsenin 1986). Empirical tests are inexpensive, quick, and often reliable for general use as they correlate well with sensory methods (Bourne 1994). They can measure more than one mechanical property at a
given time and can produce an average of results if a statistically large number of samples (Bourne 1994) is used. For these reasons, empirical tests can be suitable for routine texture evaluation and quality control. Major disadvantages of empirical tests include their poor definition and arbitrary nature (Mohsenin 1970). The measured quantities are not rigorously defined by mathematical equations. Units of measurement are undefined and therefore the test data cannot be converted into another system of measurements (Bourne 1994). Even so, in certain sectors of the food industry, empirical tests are still used as a standard for textural quality, e.g. the Adam’s consistometer is used as a standard device to measure the consistency of apple puree and creamed corn (Rosenthal 1999). Another disadvantage of empirical tests is that these tests are specific to a particular food product or a narrow range of products. An empirical test measures a specific characteristics of a given food in an arbitrary way and therefore the results are often not comparable and cannot be predictive (Rosenthal 1999). In situations when a process development or a process evaluation on the basis of textural qualities must be carried out in order to develop a new product or to improve an existing product, empirical tests may not find extensive usage regardless of them being rapid and inexpensive.

2.1.1.2 Fundamental tests

Fundamental tests facilitate texture evaluation on the basis of the mechanical properties of foods (Rosenthal 1999) such as the modulus of elasticity, failure stress, Poisson’s ratio etc. These tests have been developed by engineers and scientists on the basis of testing construction engineering materials (Bourne 1994).
However the objectives of performing these tests for foods and for engineering materials are entirely different. For foods the prime concern generally is to measure the "weakness" of foods in order to attain easy crushing during mastication and produce a pleasant mouthfeel during consumption. For construction engineering materials, prime concern is to measure strength so that the material can sustain excessive loading and environmental abuse for a longer period of time (Bourne 1994).

Advantages of fundamental tests include the fact that they are rigorously defined and produce results in known fundamental units (Mohsenin 1970; Bourne 1994). As a matter of fact, the very first step in fundamental testing is to identify the specific mechanical property to be measured (Mohsenin 1986). Then the test data is analyzed according to the fundamental equations in mechanics to obtain the specific mechanical property in the correct units of measurement. Care is taken so that experimental conditions and geometry of the samples do not affect the measurements (Mohsenin 1986). Therefore, fundamental tests facilitate the comparison of results easily and precisely, a reason why they are the tests of choice for engineering materials (Bourne 1994). On the other hand, they are not as rapid, easy, and inexpensive as empirical tests (Mohsenin 1986) for routine use in food processing plants. Also, fundamental tests often fail to correlate well with sensory tests (Finney 1971; Finney and Abbott 1972; Mohsenin 1986; Bourne 1994).
2.1.2 Need to Correlate Empirical and Fundamental Tests

It is interesting to note the dilemma which Bourne (1994) has expressed: "As scientists, we prefer the fundamental tests because they are rigorously defined and are described by an equation. As food technologists, we do not use them because they have a poor record in predicting the consumer's opinion of textural quality. On the other hand, as scientists, we despise the empirical tests because they are arbitrary, poorly defined, and have no impressive equation to describe the test, but as food technologists, we use them all the time because they are successful in predicting the consumer's opinion of textural quality".

Bourne's dilemma clearly indicates the need for a standard test method which, on one hand is rigorously defined by equations, and on the other hand can successfully predict the consumer's opinion of textural quality. Therefore a novel idea for food scientists would be to develop standard testing conditions which have the advantages of both empirical and fundamental tests and can easily lend themselves for routine use in food industry. One way this can be done is by seeing how well the results of empirical tests can be explained in terms of fundamental mechanical properties of foods. Or in other words, by using empirical tests, try to obtain the mechanical properties of food that are related to the texture of foods.

2.1.3 Texture Related Fundamental Mechanical Properties

2.1.3.1 Modulus of elasticity

Modulus of elasticity or the Young's modulus (E) of a specimen is determined by the ratio of stress to the corresponding strain within the elastic range of the specimen.
(Mohsenin 1986). It is calculated by the slope of the linear portion (i.e. within the elastic range) of the stress-strain plot. It is a measure of stiffness and rigidity of the specimen or in other words, a measure of how easily a specimen can be deformed (Dobraszczyk and Vincent 1999). When Robert Hooke proposed his law of proportionality between force and deformation in 1676 that “as is the extension, so is the force”, he missed the fact that the proportionality was also a function of sample geometry (Dobraszczyk and Vincent 1999). After more than a century, Thomas Young proposed that expressing this proportionality in terms of stress and strain instead of force and deformation could exclude the effects of specimen’s geometry (Dobraszczyk and Vincent 1999). Therefore this material property was named as Young’s modulus of elasticity. Mathematically it is expressed as:

\[ E = \frac{\text{stress}}{\text{strain}} = \frac{(F/A)}{(\Delta L/L)} \]  

(2.1)

Where, \( E \) = Young’s modulus of elasticity (Nm\(^{-2}\))

\( F \) = Applied load (N)

\( A \) = Area of cross section (m\(^2\))

\( \Delta L \) = Change in length in the direction of force (m)

\( L \) = Original length of the specimen (m)

Since no material is known to be perfectly elastic, and most biological materials including foods are only elastic up to a limited extent (Mohsenin 1986), this definition of modulus of elasticity holds for small deformations only. Modulus of elasticity was
shown to describe the firmness of fruits and vegetables (Finney 1969) and tenderness of cooked meats (Mohsenin 1970).

2.1.3.2 Failure stress

Failure stress (σ_f) is an important mechanical property that can be used to describe textural characteristics of foods. It is the stress at which a specimen fails under the action of applied loading (Segars and Kapsalis 1987). It is calculated as the maximum load on the force deformation curve divided by the cross sectional area through which the load is applied, and is mathematically expressed as:

\[ \sigma_f = \frac{F_{\text{critical}}}{A} \]  

Where, \( F_{\text{critical}} = \) Maximum load at which the specimen fails (N)

The unit for failure stress is the same as that for modulus of elasticity, i.e., Nm\(^{-2}\) (Segars and Kapsalis 1987). There is a little bit of confusion between the terms failure stress and failure stress. Failure stress is related to the strength of the materials (Mohsenin 1970; Nussinovitch et al. 1990). The failure point marks the end of the elastic limit of the specimen and therefore, beyond this point, a deformed specimen cannot return to its original dimensions upon removal of force. The deformation beyond the failure point is permanent, which is also known as plastic deformation (Dobraszczyk and Vincent 1999). Therefore the failure stress is commonly used as a measure of the usable strength of the material (Byras and Snyder 1975).
Nussinovitch et al. (1990) found that yield stress (or failure stress) was a useful parameter to compare the strength of different gels. They also found that it correlated well with the sensory attributes of gel texture. Dupont et al. (1992) showed a correlation between sensory crispiness and mechanical strength of the french fries.

2.1.3.3 Poisson’s ratio

When a specimen is subjected to an external loading, the deformation occurs not only in the direction of the force (or longitudinal deformation), but also in the direction, perpendicular to the force (or transverse deformation). The ratio of the two strains is known as Poisson’s ratio (\(\nu\)) and is mathematically expressed as:

\[
\nu = -\frac{(\Delta \delta / \delta)}{(\Delta L / L)}
\]

Where, \(\Delta \delta = \text{Change in length in transverse direction (m)}\)

\(\delta = \text{Original length in transverse direction (m)}\)

The negative sign is added because the two deformations are of opposite nature i.e., if one is an extension then the other one will be a contraction or vice versa. Poisson’s ratio is a dimensionless quantity. Poisson’s ratio only varies from 0.0 to 0.5 for most materials (Mohsenin 1986; Peleg 1987). Typically for steel, the value is about 0.3 (Bowes et al. 1984), for gels it is 0.3 to 0.5 (Yano et al. 1987), and for apple, potato, and water, it is 0.23, 0.49, and 0.50 respectively (Mohsenin 1970).
2.1.3 Methods for Computing Mechanical Properties

2.1.4.1 Compression test

There is a large number of reported studies using compression tests in foods and other biological materials such as gels, mostly appearing in Journal of Texture Studies, Journal of Food Engineering etc. The uniaxial compression test is one of the most popular tests for the determination of deformation properties of foods (Dobraszczyk and Vincent 1999). The test does not require gripping of specimens in the apparatus (Luyten et al. 1992) and therefore is applicable to a large array of specimen sizes and shapes. In this test, usually a cylindrical specimen is compressed between parallel plates (or compressive platens) on a suitable testing machine and the force is measured in relation to the observed displacement of the parallel plates. Although simple, a compression test might give highly erroneous results if not done properly (Luyten et al. 1992).

The complications arise from the presence of friction between the test specimen and the compressive platens. During the test, if the specimen is restrained and is therefore unable to slide along the platens as the test proceeds, the static frictional forces have to be overcome. As a result, a large part of the applied forces will be used up overcoming this friction (Felbeck and Atkins 1984). Since the value of the dynamic frictional coefficient is less than that of the static frictional coefficient, it is better that the test specimen is free to slide along the platens. Frictional forces may also induce shear effects and thereby may generate a complex stress field. For these reasons, proper lubrication of platens is critical in uniaxial compression tests.
Another problem with the uniaxial compression test is barreling of the specimen. When the ends are restrained at the platens due to friction, the central part of the specimen barrels out (Dobraszczyk and Vincent 1999). In this case, the deformation is non-homogeneous and the strain patterns vary along the height of the specimen.

2.1.4.2 Indentation test

Indentation or penetration testing is one of the simplest and most widely used methods for objective measurement of textural characteristics (Timbers et al. 1965). The test measures the force required to push an indenter into a food (Bourne 1982). Use of indentation tests for evaluation of mechanical properties of foods has been commonly cited (deMan 1969; Mohsenin 1970; Bourne 1994). In this test, a rigid indenter (usually made of high carbon steel and also known as probe, punch or die) is pressed against the surface of the test specimen. The data thus obtained are used to generate a force-deformation curve and sections of this curve are analyzed to obtain various mechanical properties (Mohsenin 1986). Advantages of the indentation test include its simplicity, rapidity, and lesser dependence on the geometry of the specimen. It is an ideal test for texture evaluation in localized regions of foods (Bohler et al. 1987). Although the indentation test for the evaluation of food texture is primarily classified as an empirical test (Bourne 1982), it can be a suitable test for adopting a fundamental approach to texture evaluation. The test has been the subject of much research in the areas of food science and engineering, materials science, civil, geotechnical and mechanical engineering, and biomechanics. Researchers have defined the many aspects of this test pertaining to their specific interests and a lot of information is readily available in the
literature (Bourne 1966; Nevel 1970; Lanir et al. 1990; Dempsey et al. 1991; Briscoe et al. 1998; Wen et al. 1998; Ross and Scanlon 1999). The test is almost completely defined in terms of fundamental units and therefore, its use facilitates an easy transition from an arbitrary empirical test to a more rigorously defined fundamental test. Recently the test has caught the attention of researchers in the areas of polymer science, where micro and nano-indentation techniques are used to determine the strength of tiny crystals and polymer coatings (Lanir et al. 1990; Briscoe et al. 1998).

The pressure distribution under a rigid circular indenter indenting an elastic material is given by (Mohsenin 1986):

\[
p(r) = \frac{F}{2\pi b \sqrt{b^2 - r^2}} \tag{2.4}
\]

Where, \( p(r) \) = Pressure at a distance \( r \) from the origin (\( \text{Nm}^{-2} \)), \( r \leq b \)

\( b \) = Radius of indenter (m)

\( F \) = Load on the indenter (N)

The above equation is also known as the Boussinesq equation and will be further discussed later in section 2.4.2. The equation shows that there is a large variation in pressure under the indenter. Directly underneath the indenter (i.e. at the origin, \( r=0 \)) the pressure has a finite value whereas at the edges of the indenter (\( r=b \)), it is infinite. Bourne (1966) explained the indentation test by separating the total indentation load into its shear and compression components. The compressive load is determined by the area under the indenter (or the cross section area of the indenter).
whereas the shear load is determined by the perimeter of the indenter. He proposed the following expression for the force $F$ needed to push an indenter through a specimen:

$$F_{\text{critical}} = K_cA + K_sP + C$$  \hspace{1cm} (2.5)

Where, $K_c$ = Compression coefficient of the specimen (Nm$^{-2}$)

$K_s$ = Shear coefficient of the specimen (Nm$^{-1}$)

$A$ = Cross section area of indenter (m$^2$)

$P$ = Perimeter of the indenter (m)

$C$ = Constant (N)

The two coefficients in the above equation are area and perimeter dependent quantities and the constant $C$ was found to be zero in most cases within the limits of experimental error (Bourne 1966). Several studies using the above relationship (Equation 2.5) can be found in the literature (DeMan 1969; Bourne 1975; Ross and Scanlon 1999). The relationship shown in equation 2.5 can be a good starting point to understand the indentation process as it depicts a relationship between the force at failure, $F_{\text{critical}}$, and the indenter size. Boussinesq's equation also depicts such a relationship but is applicable when the strains are low and the force is not accompanied by the failure of the specimen. For flat cylindrical indenters of radius $b$, a square term of the radius $b$ ($A = \pi b^2$) is also involved in equation 2.5 and therefore, a nonlinear relationship between failure force and indenter size can well be expected.
2.1.4.3 Tensile test

The tensile test is opposite in nature to the uniaxial compression test in that the test specimen is stretched between the jaws of the apparatus. In a tensile test, fracture occurs at the outer face of the specimen whereas in a compression testing it occurs inside the specimen (Luyten et al. 1992). For studies aimed at fracture of food materials, tensile testing has an advantage in a way that it allows the initiation and propagation of fracture to be observed easily (Luyten et al. 1992). By making notches on the test specimen in a tensile test, notch sensitivity can easily be observed during the test (Van Vliet 1999). Another advantage of the tensile test is that the problem of friction can be avoided. However tensile tests are not very popular for determination of mechanical properties of foods for a number of reasons (Mohsenin 1986). A major disadvantage in using tensile testing for foods is the need to grip the specimens firmly between the jaws of the apparatus. Shape, size, moisture content and the softness of foods represent problems in gripping, alignment, and attaining uniform stress distribution throughout the test specimen (Mohsenin 1986). Gluing of specimens may be a possible solution, but gluing may not necessarily work. Regardless of these factors, researchers have still used tensile tests in their studies.

Gillet et al. (1978) used a tensile test on processed meats to measure the adhesion between the meat pieces. Schoorl and Holt (1983) measured the strength of apple tissues by means of tensile testing. Stading and Hermansson (1991) performed a tensile test on their β-lactoglobulin gels. They used glue as the means of gripping their gel specimens to the jaws of the apparatus. Scanlon and Ross (2000) used superglue to
stick the french fry crust onto the jaws of the apparatus and performed tensile test to study the fracture behavior of the french fry crust.

2.1.4.4 Three point bending test

Bending is a combination of compression, tension and shear (Vincent 1990). The test specimen for a three point bending test is usually much longer than its thickness and width (Van Vliet 1999) and therefore can be considered as a beam. The two points of contact at the two ends act as simple supports and the third contact point is at the center of the beam at the point of load application. Span (distance between the two supporting ends) to depth ratio in bending tests is important because for a lower ratio, shear stresses become very significant (Vincent 1990).

Mechanical properties of certain foods such as vegetable stalks, strips of pasta, rectangular bars of cheese and butter can easily be determined by testing them as simply supported beams (Mohsenin 1986). Fracture mostly starts on the outside of the specimen on its tension side and therefore can be observed easily. Bending tests are often used in materials testing because they are easy to perform (Van Vliet 1999) and there is no need to fix the specimen to an apparatus. However, use of bending test for foods is limited. Obtaining a homogeneous specimen in the shape of a beam is a problem for many foods. Bending of a test specimen under the action of its own load is also more likely to happen with food samples which is a major problem in using bending tests for foods. Therefore, the modulus of elasticity of the food should be substantially high (Van Vliet 1999) which turns out to be a major limitation for many foods. Andersson (1973) used a three point bending test to study the fracture properties
of crisp bread. Kapsalis et al. (1972) used the bending test to study the deformation behavior of foods.
2.2 MODEL FOOD SYSTEMS

Determination of mechanical properties and application of any of the above mentioned methods assume certain characteristics of the test specimen. The test specimen should ideally be isotropic and homogeneous (Mohsenin 1986). Specimen geometry should preferably be simple so that the specimen can conveniently be used in the apparatus. Most materials from plant and animal sources including foods are cellular composite structures formed by the aggregation of fibers and/or plate like components (Atkins and Mai 1985) and therefore are inherently non-homogeneous and anisotropic. In many cases foods do not possess a self supported structure, e.g., any type of semisolid or liquid food. Geometry is complex in many cases (Mohsenin 1986), e.g. pears, apples, potatoes, pepper, bagels, pastries etc. In addition, processed foods are also likely to undergo structural changes at every stage of processing. For example cooking, baking, frying and cooling cause redistribution of tissue water (Ross and Porter 1966), freezing and thawing cause collapse of tissues and microstructural changes (Edwards 1999), and slicing is usually accompanied by significant loss of moisture.

Owing to all these problems of non-homogenity, anisotropicity, and large changes in properties, determining the fundamental properties of foods from the above techniques is fraught with difficulties. As a consequence, model food systems are required for studies related to food texture. A model food system is considered to be reasonably homogeneous and isotropic. A test specimen made from a model food system is considered to have simple geometry and be free of variations in mechanical properties throughout the specimen.
2.2.1 Agar Gel

Agar is the most ancient phytocolloid extracted from red seaweed (Klose and Glicksman 1968). It was discovered in Japan in 1658. The major sources of agar are the species of *gelidium* and *cartilagineum*, although many species of *rhodophyceae* are also used to produce agar (Klose and Glicksman 1968). The weeds of commercial importance grow in marine habitat from the tide line out to depths of 120 feet which are harvested, cleaned, and washed with fresh water. Weeds are then spread in thin layers and dried for 4 to 20 days, pressed into bales and shipped to the processor (Klose and Glicksman 1968).

Agar has been used in foods for about 300 years in the far east and later was introduced to Europe and North America about 100 years ago. The Food and Drug Administration (FDA) has granted agar GRAS (generally recognized as safe) status and the results of FDA-recommended toxicological, teratological, and mutagenic evaluations of agar have been highly satisfactory (Armisen 1997). Because of its unique gel forming properties, agar has wide commercial applications in the areas of food, microbiology, biotechnology, medicine and dentistry (Klose and Glicksman 1968). Agar is graded into food, bacteriological, medicinal, and dental grades.

2.2.1.1 Chemical structure

Agar is a polysaccharide and is assigned a linear galactan structure (Armisen 1997). The structure is chemically composed of 3, 6-anhydro-L-galactose and D-galactopyranose residues in varying proportions, and with a small amount of ester sulphate (Klose and Glicksman 1968). The amount of sulphate is the basis for
distinguishing between the different polysaccharides which are extracted from the red seaweed, such as agar (low sulphate), carrageenan (high sulphate), and furcellaran (substantially high in sulphate) (Moirano 1977).

Agar is a mixture of two polysaccharides agarose, which is the major gelling component, and agarpectin, which is a non-gelling or a weak gelling component (Clarke and Ross-Murphy 1987). Agarose is a neutral non-sulfated linear molecule and is the dominant component in agar (Nijenhuis 1997). Agarose has a high commercial value and most of its mechanical properties are similar to those of agar (Watase and Arakawa 1968). Molecular weight of agarose is in the order of $10^5$ (Clarke and Ross-Murphy 1987).

2.2.1.2 Physical properties

Agar is insoluble in cold water but soluble in boiling water (Klose and Glicksman 1968). The aqueous solution is clear and forms a firm resistant gel on cooling to about 32-39°C. Upon heating the gel does not melt below 85°C (Klose and Glicksman 1968). The lag between gelling and melting of agar is known as gel hysteresis (Klose and Glicksman 1968). Hysteresis is an important property of agar gel and is the basis for many of its applications in food and biotechnology (Klose and Glicksman 1968). The reason for this hysteresis lag is the presence of a wide range of strand thicknesses in the gel network of agar due to strands of different thicknesses having different temperature stability (Brigham et al. 1994). Brigham et al. (1994) studied the ultrastructure of an agar gel and reported that all the strands were less than 10 nm thick and most of them were less than 5 nm thick. Hydrogen bonds and ionic
bonds are considered to play a major role in gelation (Watase and Arakawa 1968; Clarke and Ross-Murphy 1987). Activation energies associated with crosslink breakage are reported to be 21 kJ/mol for agar gel (Watase and Arakawa 1968) which is the order of magnitude for breaking hydrogen bonds. During gelation, hydrogen bonds are formed causing the gel to stiffen (Nijenhuis 1997). Hydrogen bonds formed after cooling during the gelation process can be disrupted on heating and vice versa (Watase 1983). Additives which can disrupt hydrogen bonding, e.g., urea, have deleterious effects on the gelation process (Nijenhuis 1997). Network formation in agar gel is classified as a disorder to order phenomenon which results in the formation of crosslinks or “junction zones” (Clarke and Ross-Murphy 1987).

Agar solutions are less viscous than the solutions of other seaweed extracts and the viscosity depends on the temperature and pH. Properties such as solution viscosity, gelling temperature, gel strength, degree of syneresis, and gel clarity may vary with the source of agar (Renn 1984). Even bacteriological grade agar exhibits appreciable variations in properties from one lot to another. Therefore, agarose, which is the more pure form, is often recommended for use in precise applications (Renn 1984).

2.2.1.3 Syneresis

Gels contain large amounts of water entrapped in a high molecular weight network, made of protein or polysaccharide (Clark 1990). The network shrinks with time and thus has less capacity to hold water inside. This unintentional release of water is known as syneresis or bleeding or weeping or product separation (Glicksman 1969). When gels are subjected to mechanical tests for studying their viscoelastic properties,
syneresis is often a problem (Nijenhuis 1997) because it changes the mechanical properties during the test. Control of syneresis is important in order to maintain the desired textural quality of finished food products where the gel has been used as an ingredient.

2.2.1.4 Agar in mechanical tests

Gels are viscoelastic materials (Mitchell 1980; Van Vliet et al. 1990). They are highly homogeneous as compared to most foods and they can be easily formed into a number of shapes suitable for mechanical testing (Nussinovitch and Peleg 1990). Their response to external loading is linear in a strain regime as large as 15% (Arakawa 1961 as cited by Mitchell 1976). Therefore rheological experiments on gels and data interpretation afterwards are easy to carry out (Mitchell 1980). For these reasons, gels are frequently used to model a food or biological system (Nussinovitch and Peleg 1990). Gels can also be mechanically considered as 'soft solids' (Clark 1990) and therefore their mechanical behavior is affected by both the solid and the liquid phases. A correlation between the rheological properties and the state of water in the gel has been shown by NMR studies (Aizawa et al. 1973 as cited by Mitchell 1976). Among the gels from red seaweed origin (agar, carrageenan, and furcellaran), agar has been the subject of most rheological studies, primarily because of its greatest gelling capacity (Mitchell 1976).

Nussinovitch et al. (1990) studied the effect of gum and ion concentration on the mechanical properties of agar gel in the concentration range of 1 to 5%. Cylindrical samples of 1.5 cm x 1.5 cm were subjected to uniaxial compression at a cross head
speed of 6mm/min. They reported almost a linear relationship between the gum concentration and the gel's mechanical properties. However it was interesting to note that attaining the maximum strength was not necessarily accompanied by the maximum of other mechanical properties. Nussinovitch et al. (1990) also reported that the mechanical properties of the agar gel were responding to calcium ions. Calcium increased the strength of the gel network by inducing additional cross linking in the network. Clark (1990) reported that the calcium ions could affect the average diameter of the gel’s network strands. The presence of additional crosslinks makes a gel network more stiff and therefore the elastic modulus of the gel will increase (Mitchell 1980).

Kaletunc et al. (1991) attempted to quantify the degree of elasticity of agar, alginate, and carrageenan gels under large strain compression-decompression cycles. The cross head speed in their investigation was 10mm/min on an Instron universal testing machine. They measured strength, failure strain, and deformability modulus of these gels at 1 and 2.5% concentration levels. The researchers reported that these mechanical properties were independent of each other.

Mitchell (1980) also found in his extensive literature review on rheology of gels that the rupture strength of a gel was not necessarily related to its elastic modulus, and therefore experiments which involve deformations large enough to rupture the gel would not necessarily rank a series of gels in the same order as experiments which involve small deformations, which did not cause the rupture of the gel. Experiments conducted by Wood (1979) may bring another illustration on this topic. Wood (1979) prepared a series of seven gels from agar, xanthan gum plus locust bean and carrageenan plus locust bean gum and measured the peak force (large deformation),
initial slope (small deformation) and the bloom number (small deformation). He also obtained the sensory results for shakeability (small deformation), finger hardness (small deformation), gel strength in mouth (large deformation), and gel toughness in mouth (large deformation). The ranking orders obtained for the seven gels using large and small deformation tests were not the same. One of the plausible reasons for this discrepancy could be the fact that the elastic modulus (small deformation) and rupture strength (large deformation) depend in different ways on the primary molecular weight of the polymer from which the gel is made (Mitchell 1980). Rupture strength increases with increasing molecular weight while the elastic modulus becomes independent of molecular weight above a certain limiting value (Mitchell 1980).

Dependence of elastic modulus of agar on its concentration has been investigated by Watase and Arakawa (1967). They found that below a certain concentration, the elastic modulus was proportional to the fourth power of concentration, and past that certain value of concentration, elastic modulus was proportional to the square of the concentration.

### 2.2.1.5 Food usage of agar

Excellent gelling properties of agar and its ability to withstand high temperatures have made agar a suitable candidate for use in foods. Agar is broadly used in bakery products, confectionery, dairy products, and canned meat and fish (Glicksman 1969). In baking applications agar is used as a stabilizer in pies, pie fillings, icings, toppings, cookies, cream shells etc. It is used as an anti-staling agent in bread and cakes in the concentration range of 0.1 to 1%. To prevent transit damage of canned meat, fish
and poultry, agar is used in the concentration range of 0.5 to 2%. In the dairy industry agar is used as stabilizer in sherbet, cheese, and yogurt often in combination with other food gums. In the beverage industry agar is used as a refining agent in wine, juice, and vinegar in the concentration range of 0.05 to 0.15%. Agar is not metabolized in the human body and therefore does not add calories to the foods (Armisen 1997). This is a major advantage to the producers of low calorie diets and health conscious consumers. According to the FDA the acceptable daily intake of agar for humans is 50 mg per kg of body weight (Armisen 1997).

2.2.2 Pasta

Pasta is an important food commodity, which has been known to be in consumption for thousands of years (Dick and Matsuo 1988). It is an extruded product which is usually made in thin brittle sheets. Durum semolina, durum flour, and hard wheat flour are major ingredients in pasta production (Hahn 1990). Often other ingredients such as eggs, spinach, and tomatoes are also added for specific taste and flavor. The surface of freshly extruded pasta is a continuous protein film while the inner portion of pasta is a compact structure of starch granules embedded in an amorphous protein matrix (Hahn 1990).

In traditional pasta consuming countries such as Italy, textural characteristics are considered most important by the pasta manufacturers among all other quality attributes such as color, taste, and nutrition (D’Egidio and Nardi 1996). Texture of dry and cooked pasta is affected by a number of variables such as wheat variety, semolina and
water quality, use of salt and other ingredients, water to pasta ratio, cooking time and temperature, draining method, etc.

Mechanical properties of dry pasta are important to ensure proper packaging, handling, and transportation (Dick and Matsuo 1988). Smewing (1997) performed instrumental analyses on both dry and cooked pasta. She measured the breaking strength and flexure for dry pasta, and firmness, stickiness, and tensile strength for cooked pasta. Breaking strength of dry pasta determines how well the product tolerates transportation and it also indicates how well the pasta holds together during cooking (Hahn 1990). Akiyama and Hayakawa (1994) studied crack formation in pasta products as a result of tensile stress created by hygro-shrinkage. They measured the tensile fracture stress of dry pasta at temperatures between 20 and 70°C and reported a value of 3.3 MNm⁻² which was independent of temperature. Liu et al. (1997) investigated crack development in pasta upon drying. They determined modulus of elasticity, yield stress, tensile fracture stress, and failure strain of pasta at various moisture contents commonly encountered in conventional pasta drying process. They reported a value of 1.9 GNm⁻² (10% moisture content) for the Young's modulus and 9.2 MNm⁻² (8.4% moisture content) for yield stress of their pasta samples.
2.3 COMPOSITE MATERIALS

A composite material is made of two or more materials combined on a macroscopic scale to form a more useful material which usually exhibits the superior qualities of its constituent materials and some unique qualities which neither of the constituents possess (Jones 1975). The three common types of composites are fibrous composites, particulate composites, and laminated composites (Jones 1975). Food examples of these types of composites would be fruit pulp, plain biscuits and cookies, and layered cakes respectively.

The scope of this study is limited to laminated composites only, which are made of layers of different materials and each of these materials has its unique mechanical properties. The overall mechanical properties of a layered composite become a complex function of the mechanical properties of each of the layers, their dimensions, and the type of interaction between the layers. For example if the two layers are tightly bonded, then the overall mechanical behavior of the composite would be different than if they were not bonded. The complex function may be defined by the structure, orientation, dimensions, and relative proportions of individual layers (Jones 1975). Examples of layered composites can be found in foundation engineering (Terzaghi 1955; Selvadurai 1979), runway design (Burmister 1945), highway construction (Westergaard 1926), cold region engineering (Nevel 1970), and railroad construction (Li and Selig 1998a; 1998b). In all these examples, layers of different mechanical properties are utilized to sustain excessively large loads and heavy impacts.

For engineering materials, some of the properties that can be improved by fabricating composites are strength, stiffness, physical appearance, weight, wear
resistance, corrosion resistance, conductivity etc. (Jones 1975). For foods, the properties that can be improved may include texture, flavor, taste, nutritional value, transportability etc.

2.3.1 Industrial Composites

There are numerous examples of industrial composites being used, researched, and developed for various uses. These composite materials are of primary importance in structural applications where high strength to weight and stiffness to weight ratio is critical (Jones 1975). Industrial composites offer a number of advantages such as higher strength, lighter weight, corrosion resistance etc. over traditional construction materials (Wen et al. 1998). Composites in the form of sandwich panels (Olsson and McManus 1996), are widely sought in the aerospace industry where the aircraft construction material must sustain a heavy impact but at the same time be light in weight (Jones 1975). Fiber reinforced plastic is the desired construction material in armor applications because it is light in weight and is capable of sustaining high velocity impacts (Wen et al. 1998).

2.3.2 Biological and Food Composites

Most animal and plant materials are cellular composite structures formed of fibrous or plate-like components aggregated together (Atkins and Mai 1985). A system of human skin and subcutaneous tissues is an example of a biological composite (Zheng and Mak 1997). Cream filled candies, crispy french fries, sandwich cookies, pies, layered cakes, fruits and vegetables with skin etc. are examples of food composites. To
a certain extent, soft bread in a wrapper can also be considered as an example of a food composite. The mechanical properties and thickness of the wrapper affect the squeeze test of wrapped bread. In all of these examples, the two principal phases, e.g., pie filling and pie crust in pies or outer skin and inner flesh in fruits, have appreciably different mechanical properties. One phase is mechanically "softer" than the other is. The overall textural characteristics of the food as an independent entity will be a function of the properties of both phases.

Khan and Vincent (1991) have evaluated the role played by apple skin in preventing bruising and splitting of apples under compression. Scanlon and Ross (2000) have used agar gel and fried potato crust to model the french fry as being composed of two phases: the outer crust (made of fried potato crust) and the inner mealy material (made of agar gel). They studied the composite behavior of the simulated french fry when the two phases were glued together. The behavior of a composite food under mechanical tests can be explained using the theories of elasticity. Doing so brings the advantage of much research done in other fields where a similar situation has been treated and analytical solutions to the problems have been developed.
2.4 THEORY OF ELASTICITY

The theory of elasticity permits the analysis of stresses and displacements in elastic bodies. It has numerous applications in solving practical engineering problems (Timoshenko and Goodier 1951). A theory of stresses and displacements in a multilayer system has been developed in accordance with the theory of elasticity (Burmister 1945). The following are the basic assumptions to be made before using the theories of elasticity (Timoshenko and Goodier 1982):

i. Materials possess elastic properties i.e. within a certain limit, the deformation disappears as soon as the external force causing deformation is removed.

ii. The material is homogeneous and continuously distributed over its volume so that the smallest element cut from the material possesses the same specific physical properties as the material itself.

iii. Elastic properties of the material are the same in all directions, i.e., the material is isotropic.

2.4.1 Anisotropic Elasticity

The general assumption of isotropy is only a convenient approximation as no material is truly isotropic (Felbeck and Atkins 1984). The modulus of elasticity of any material varies with the direction of measurement (Timoshenko and Goodier 1982). In the case of metals, anisotropy may occur as a result of various metallurgical treatment such as rolling, forging, extruding etc. It may also be due to difference in grain sizes at various locations as a result of different cooling rates (Felbeck and Atkins 1984). For biological materials including foods, anisotropy is an inherent natural property (Atkins
and Mai 1985; Mohsenin 1986) and is further affected by processing steps. Layered food composites exhibit anisotropy for the simple reason that any transverse section will contain two or more different materials of different mechanical properties. On the other hand, a longitudinal section may contain just one phase of pure material with unique mechanical properties.

Anisotropy can be useful in situations where loading is to be performed in a given direction only (Felbeck and Atkins 1984). Klintworth and Stronge (1990) considered the indentation problem of an anisotropic half space and found that the stress field was distorted by the effects of material orientation and the relative magnitudes of the elastic moduli.

However, most engineering materials can still be considered isotropic particularly when the dimensions of the test material is very large compared to the dimensions of an elementary particulate from the same material (Timoshenko and Goodier 1982). Similarly a model food system made of an agar gel can reasonably be considered isotropic although as is apparent from 2.2.1.2, this assumption is not valid in the nm range.

2.4.2 Boussinesq Problem of a Rigid Indenter

Rigid indenters are often used to study force-deformation behavior of foods for evaluation of the mechanical properties of foods (Mohsenin 1986). This approach of evaluating mechanical properties is an approximation of the Boussinesq problem which involves the contact between a rigid flat cylindrical indenter and a plane boundary of a semi-infinite elastic material. The semi-infinite elastic material is also
known as an “elastic half space” and is defined by a region which has infinite dimensions in the plane perpendicular to the direction of force and has finite dimension in a plane parallel to the direction of force (Mohsenin 1970). The elastic half space is also known as a foundation and in this study, the term foundation will also be used liberally. The elastic half space is an assumption that enables transformation from three dimensional elasticity problem to a more convenient two dimensional elasticity problem. Some of the assumptions that are made in the Boussinesq problem (Mohsenin 1986) are following:

i. Bodies in contact are homogeneous.

ii. External load on the indenter is static.

iii. Hooke's law holds.

iv. There is no contact stress at the opposite end of the semi-infinite elastic body as a result of the indentation load.

v. Surfaces of the bodies in contact are smooth so that the traction due to shear stresses are eliminated.

Timoshenko and Goodier (1951) gave the relationship between the applied force and the resultant deformation in the Boussinesq problem:

\[
\frac{F}{d} = \frac{2bE}{1 - \nu^2}
\]  

(2.6)

Where, \( d \) = Deformation (m)
The above relationship facilitates the computation of the modulus of elasticity 'E' for any given indenter radius 'b'. The term on the left hand side 'F/d' can be calculated from the initial slope of force-deformation plots.

2.4.3 Elastic Layer Lying on an Elastic Half Space

Problems involving mechanical behavior of a multi-layer composite system can be modeled as plates on elastic foundations. A tremendous amount of literature is available on the problems of elastic plates lying on elastic foundations (or elastic half spaces) where the prime interest has been to compute the contact stresses and the displacement as a result of loading (Gladwell 1980). The plate is defined as a relatively flat three-dimensional object for which two dimensions (length and width) are large compared to the third dimension (thickness). The plate is also known as a layer and in this study the two terms will be used interchangeably. The major purpose of a plate or layer overlying an elastic foundation is to increase the apparent strength of the foundation and its performance when subjected to excessive loading (Chen and Engel 1972). Numerous examples can be found in the areas of railroad track design (Li and Selig 1998a; 1998b), highway design (Westergaard 1926), aircraft runway (Burmister 1945) and foundation engineering (Selvadurai 1979) etc. Some examples involving biological materials can be found in the works of Zheng and Mak (1997) and Scanlon and Ross (2000) who worked on human skin and gels, respectively.

In all of the above cases the plate or the layer distributes the load over a wider area and thereby reduces the stress that the foundation is actually subjected to (Chen and Engel 1972). However, problems of this type need the foundation to be a
well-defined elastic object for which the theories of elasticity can be readily applicable. Researchers have proposed a number of different models for the foundations primarily on the basis of continuity of the elastic properties of the foundations.

2.4.3.1 The Winkler model

The Winkler model represents one of the simplest approximations for defining the properties of the foundation in the problem of plates on elastic foundations (Dempsey et al. 1991). It is characterized by a system of independent spring elements, where each spring in the system is independent of the loading and the resulting displacement of adjacent spring elements in that system (Gladwell 1980). In the Winkler model, deformation only occurs at a point directly underneath the applied load. Outside this loading region, there is no deformation regardless of the magnitude of load (Selvadurai 1979).

According to the above description of a Winkler model, it is clear that a Winkler model does not describe the foundation as an elastic continuum (Vallabhan et al. 1991). However, this model, by its nature of being a system of independent spring elements, does not treat the foundation in a proper way. Firstly, it calls for a need to determine the modulus of subgrade reaction (to be discussed in the following section), the true value of which can be hard to determine (Terzaghi 1955; Selvadurai 1979; Bowles 1982; Vallabhan et al. 1991). Secondly, in reality, a load applied at one point would also cause deformations at other points located distantly to the point of application of the load (Vallabhan et al. 1991). Therefore the foundation should ideally be considered as an elastic continuum instead. For these reasons, the use of Winkler
model is at times, questionable (Bowles 1982). Still for the simplicity that a Winkler model offers, it is continuously being used by researchers.

**2.4.3.1.1 Modulus of subgrade reaction.** The Winkler model is characterized by the modulus of subgrade reaction which is defined as the ratio between the pressure on a foundation and the resultant deformation at a given point (Bowles 1982):

\[
K = \frac{q}{d}
\]

(2.7)

Where, \( K = \) Modulus of subgrade reaction (Nm\(^{-3}\))

\( q = \) Pressure on the foundation at a given point (Nm\(^{-2}\))

\( d = \) Deformation at the point of applied pressure (m)

\( K \) is also known as the modulus of foundation or the subgrade modulus (Teng 1962). Since the Winkler model is a system of closely placed spring elements, the modulus of subgrade reaction is analogous to the spring constant of a spring. The value of \( K \) is assumed to be independent of the magnitude of pressure and the location of the point of pressure application (Selvadurai 1979). Plate bearing tests have been described by researchers (Terzaghi 1955; Selvadurai 1979) for in situ measurement of the value of \( K \). Accuracy is always a concern in determining an exact value of \( K \) (Westergaard 1926; Terzaghi 1955; Selvadurai 1979). However it may not pose a great concern in the calculation of stresses in the plate foundation system for the reason that
the fourth root of \( K \) enters into the analysis (Westergaard 1926) which will be shown in following section.

2.4.3.1.2 Characteristic length and flexural rigidity. The plate overlying an elastic foundation bends or flexes under the action of applied load. The flexural behavior of a plate on an elastic foundation is described by a parameter called the characteristic length of the plate (Sodhi et al. 1982). The characteristic length is a measure of stiffness of the plate relative to that of the foundation (Westergaard 1926) and it describes the interaction between the foundation and the overlying layer (Selvadurai 1979). The stiffer the plate and the less stiff the foundation, the greater the characteristic length is. The characteristic length can be calculated by the following relationship between the modulus of subgrade reaction of the foundation and the flexural rigidity of the plate:

\[
I = \left( \frac{D}{K} \right)^{0.25}
\]  

(2.8)

Where, \( I \) = Characteristic length (m)

\( D \) = Flexural rigidity (Nm)

Flexural rigidity or the bending stiffness (Shield 1991) describes the flexural behavior of a plate, or in particular, the resistance of the plate to shear stresses. For an elastic plate, flexural rigidity is a function of the plate’s thickness and its elastic
properties (Timoshenko and Woinowsky-Krieger 1959). Flexural rigidity is mathematically expressed as:

\[ D = \frac{Eh^3}{12(1-\nu^2)} \]  

(2.9)

Where,  
- \( h \) = Thickness of the plate (m)
- \( E \) = Modulus of elasticity of the plate (Nm²)
- \( \nu \) = Poisson’s ratio of the plate (dimensionless)

Based on the extent of deformation, the problem of plate on elastic foundation is further classified as small deflection plate theory, larger deflection plate theory, and membrane theory (Olsson and McManus 1996). The problem is treated differently in each of the three classifications. Small deflection plate theory is applied in situations where the deformation is less than half the thickness of the plate (Olsson and McManus 1996). In this case the middle plane of the plate remains unstrained and the normal stresses in the direction transverse to the plate can be neglected (Timoshenko and Woinowsky-Krieger 1959). Large deformation plate theory is applicable when the deformation is not small as compared to the plate thickness but is still small as compared to the other dimensions of the plate (Timoshenko and Woinowsky-Krieger 1959). In this case the middle plane of the plate is strained and a non-linear component of deformation is introduced. As the deformation exceeds the thickness of the plate, the load deformation relationship becomes non-linear due to increased magnitude of membrane stresses (Olsson and McManus 1996). Membrane
stresses are defined as stresses in a plate, for which the deformations at the center of the plate are very large in comparison to the thickness of the plate (Timoshenko and Woinowsky-Krieger 1959). In this case, the plate offers negligible resistance to bending.
CHAPTER 3

Dimensional Effects on the Prediction of Texture-Related Mechanical Properties of Model Foods by Indentation
3.1 ABSTRACT

Indentation is a convenient method for evaluating food texture. However, results obtained are empirical and are influenced by various testing conditions as well as specimen dimensions. The present work is aimed at studying the variability in the values of fundamental mechanical properties (the Young's modulus and failure stress) obtained by indentation of specimens of different dimensions. To establish a valid comparison, values of Young's modulus and failure stress were also determined by compression. A model food system was created by using 1% agar gel fashioned into specimens of three different dimensions. Flat cylindrical indenters of seven different radii were used for indenting the specimens. To make use of the equation for frictionless elastic indentation, Poisson's ratio was also determined independently under compression. Mechanical properties for different experimental conditions were compared. Differences were seen depending on the ratio of depth of the specimen and indenter radius. Under half-space conditions (for specimen dimensions and indenter radius) values obtained by indentation for Young's modulus (12.46 kN/m²) was 34% higher than the value obtained by compression (9.29 kN/m²). The overestimation of 34% was attributed to the various problems that are often encountered in indentation testing, such as the elastic mismatch between indenters and test specimens. Values for indentation failure stress were substantially higher than compressive ones. It was concluded that experimental conditions can be chosen so that the results for Young's modulus obtained by indentation can be relied upon when evaluating food texture.
3.2 INTRODUCTION

Evaluation of food texture is employed in the development of new food products, improvement of existing food products, process control, and evaluation of the textural quality of finished food products (Finney 1969). However, evaluation of food texture is not easy to define because of the complex and anisotropic structure of foods (Mohsenin 1970) and the viscoelastic nature of many foods (Finney 1969). Therefore, a fairly large vocabulary is used to refer to different textural attributes (Bourne 1982). Sensory evaluation of food texture is often influenced by various psychological factors (Mohsenin 1970). Objective or instrumental evaluation of food texture provides information on the mechanical properties of foods and is free from psychological factors (Mohsenin 1970). It has been studied by many researchers, for example, Bourne (1982) and Mohsenin (1970).

Objective measurements are further divided into three major categories: fundamental, empirical, and imitative tests (Mohsenin 1970). Fundamental tests first define the specific mechanical property to be measured and provide the results in fundamental units which are easy to compare with one another. The parallel plate compression test is an example of a fundamental test. Empirical tests, even though they are one of the most commonly used tests in the food industry (deMan 1969), provide the results in empirical units which are not easy to compare. The indentation test is an example of such a test, where the force/deformation required to push an indenter into a food product is measured. The potential for empirical tests to provide results in fundamental units has been studied (Adams et al. 1996; Ross and Scanlon 1999). The objectives of this research were the following:
i. To study the effects of sample dimensions in indentation measurement of mechanical properties such as Young’s modulus of elasticity and failure stress.

ii. To compare the values of Young’s modulus obtained by indentation to those obtained by compression in order to see how well indentation derived mechanical parameters are predicted.
3.3 THEORETICAL APPROACH

The Boussinesq problem is applicable in studying the pressure distribution under an indenter as well as the evaluation of the elastic modulus (Mohsenin 1970). The basic assumptions in applying the Boussinesq solutions are the following:

i. Materials of the contacting bodies are homogeneous.

ii. Loads applied are static.

iii. Hooke's law holds.

iv. Contacting stresses vanish at the opposite ends of the body.

v. Radii of curvature of the contacting bodies are very large when compared to the radius of contact surface.

vi. The surfaces of contacting bodies are sufficiently smooth so that the tangential forces are eliminated.

The relationship between the deformation $d$ caused by a rigid indenter under the action of an applied force $F$ has been given by the following equation (Timoshenko and Goodier 1951):

\[
\frac{F}{d} = \frac{2 b E}{1 - \nu^2}
\]  

(3.1)

Where:  
$F$ = Applied load (N)  
$d$ = Deformation (m)  
$b$ = Radius of the indenter (m)  
$E$ = Modulus of elasticity (Nm$^{-2}$)  
$\nu$ = Poisson’s ratio of the material (Dimensionless)
The left hand side of the above equation is the slope of the force-deformation curve that can be obtained experimentally. By substituting the values for the Poisson's ratio of the food and the radius of the indenter, modulus of elasticity can be determined from indentation.
3.4 MATERIALS AND METHODS

For the purpose of modeling a food system, agar gel of 1% (Weight/Volume) concentration was used. Microbiological grade granulated agar (Difco, Detroit, MI) was dissolved in distilled water and was brought to the boil with continuous stirring (Pappas et al. 1987). The solution was boiled until it was clear and small air bubbles started to form. The solution was left on the hot plate for a few minutes till the bubbles stopped forming and then it was carefully poured into two glass pans which were large enough to provide 15 specimens each. The pan was left overnight at room temperature for equilibration. After 24 h (Nussinovitch and Peleg 1990; Pappas et al. 1987), the gel from the pan was transferred on to a large tray and was marked so that specimens of appropriate sizes could be cut. To avoid dehydration, specimens were kept covered until they were tested between 24-36 h after creation of the gel.

To determine the Poisson’s ratio, 4 cm sided cubes were compressed to 4% strain as described by Ross and Scanlon (1999). For compression testing, cylindrical specimens (Finney 1973) of 1.5 cm height and 1.5 cm diameter (Kaletunc et al. 1991) were prepared using a cork borer (Nussinovitch et al. 1990). For indentation testing, specimens of 6 cm length, 6 cm width, and three different depths of 4, 2 or 1 cm were cut from the slabs of agar gel. Seven flat cylindrical indenters of radii 1.5, 2.0, 2.5, 3.0, 3.5, 4.0 and 4.5 mm were used to indent specimens. On any given day, only one of the three specimen dimensions was indented using all seven indenters. Six replicates for each combination of indenter and the specimen dimension were analyzed in order to eliminate natural textural variations (Pollak and Peleg 1980). The entire experiment was completely randomized over a period of 6 days. On any given day, 21 specimens of
one single dimension were indented. All testing was performed at ambient temperature (23 ± 2°C). Both compression and indentation testing were carried out on a Lloyd L1000 universal testing machine (Lloyd Instruments Limited, Fareham, UK), equipped with a 20 N load cell and a 386 analog computer (Hewitt Rand Corporation) through Lloyd Rcontrol 2.21 software (Lloyd Instruments Limited, Fareham, UK). The Lloyd instrument was used primarily because it was capable of producing a cross head speed as low as 1 mm.min⁻¹ which could be considered appropriate for quasi-static loading condition (Ross and Scanlon 1999). At a low rate of loading and with a low strain regime, foods tend to exhibit elastic characteristics (Mohsenin 1970). The compression platen was coated with Teflon and lubricated with oil (Kaletunc et al. 1991) in order to reduce friction (Nussinovitch et al. 1994) and thereby shear effects during deformation. Specimens were compressed or indented until they failed, with the force-deformation curve being recorded by the computer for further analysis.

The initial part of the force-deformation curve, lying within 0.75 to 4% of strain regime was used to compute the slope (Mohsenin 1970; Ross and Scanlon 1999). Values for failure stress (σ_f) were obtained by dividing the peak force by the cross-sectional area of indenters (for indentation testing) and by the cross-sectional area of the cylindrical specimens (for compression testing).
3.5 RESULTS AND DISCUSSION

3.5.1 Poisson’s Ratio

On the basis of 30 replications, a value of 0.5 was established for the Poisson’s ratio of the 1% agar gel and thus it was considered to be incompressible. Oakenfull et al. (1989) and Yano et al. (1987) have considered gels to be generally incompressible. Ross and Scanlon (1999) determined the Poisson’s ratio of a 3% agar gel to be 0.32, but the higher agar concentration was thought to inhibit release of air bubbles, and so lower the Poisson’s ratio for the gel (Yano et al. 1987). The 1% gel used in the present study was low in concentration and thus more fluid upon cooling.

3.5.2 Young’s Modulus of Elasticity

The compression test yielded a value of \(9.29 \pm 0.90 \text{ kN} \cdot \text{m}^{-2}\) for the modulus of elasticity. Experimental values of the slopes (Table 3.1) obtained from the initial linear region (0.75 to 4% strain regime) of the indentation load-deformation curves were substituted into Equation 3.1 along with the experimentally determined value for Poisson’s ratio (0.5). Values of Young’s modulus were then determined for each indenter. Theoretical values of slopes for each indenter size were determined by equation 3.1 by substituting values of Young’s modulus (9.29 kN\cdot m^{-2}) and Poisson’s ratio (0.5). These values are presented in Table 3.2.
Table 3.1. Theoretical and experimental slopes (and standard deviations) for the three dimensions of 1% agar gel specimens when indented by seven indenters

<table>
<thead>
<tr>
<th>b (mm)</th>
<th>Theoretical slope (Nm⁻¹)</th>
<th>Experimental slope (Nm⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4 cm depth</td>
</tr>
<tr>
<td>1.5</td>
<td>36</td>
<td>44 ± 4.0</td>
</tr>
<tr>
<td>2.0</td>
<td>48</td>
<td>71 ± 4.1</td>
</tr>
<tr>
<td>2.5</td>
<td>60</td>
<td>88 ± 2.9</td>
</tr>
<tr>
<td>3.0</td>
<td>72</td>
<td>101 ± 6.9</td>
</tr>
<tr>
<td>3.5</td>
<td>84</td>
<td>127 ± 6.6</td>
</tr>
<tr>
<td>4.0</td>
<td>96</td>
<td>136 ± 9.1</td>
</tr>
<tr>
<td>4.5</td>
<td>108</td>
<td>164 ± 10.3</td>
</tr>
</tbody>
</table>

Table 3.2. Means and standard deviations for the Young’s modulus (E) for all specimen dimensions when indented by seven indenters

<table>
<thead>
<tr>
<th>b (mm)</th>
<th>4 cm depth</th>
<th>2 cm depth</th>
<th>1 cm depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>11 ± 1.0</td>
<td>11 ± 1.0</td>
<td>12 ± 1.2</td>
</tr>
<tr>
<td>2.0</td>
<td>13 ± 0.8</td>
<td>13 ± 0.5</td>
<td>13 ± 1.0</td>
</tr>
<tr>
<td>2.5</td>
<td>13 ± 0.4</td>
<td>11 ± 1.0</td>
<td>13 ± 0.4</td>
</tr>
<tr>
<td>3.0</td>
<td>13 ± 0.9</td>
<td>11 ± 1.0</td>
<td>13 ± 1.8</td>
</tr>
<tr>
<td>3.5</td>
<td>14 ± 0.7</td>
<td>13 ± 0.7</td>
<td>15 ± 2.5</td>
</tr>
<tr>
<td>4.0</td>
<td>13 ± 0.9</td>
<td>14 ± 1.0</td>
<td>16 ± 1.0</td>
</tr>
<tr>
<td>4.5</td>
<td>14 ± 0.9</td>
<td>14 ± 1.0</td>
<td>18 ± 1.6</td>
</tr>
</tbody>
</table>
The average value for Young’s modulus from Table 3.2 (excluding eight observations to be discussed later in this section) was found to be 12.46 kN/m². Therefore, indentation produced a value for Young’s modulus which was an overestimate of 34% compared to the value obtained by parallel plate compression.

Figure 3.1 represents a plot between indenter radius and experimentally determined slopes. Ideally the plot should yield straight lines of zero intercept. Theoretical slope values can be obtained for each indenter size by substituting a Young’s modulus value of 9 kN/m² (as obtained by compression) and a Poisson’s ratio of 0.5 into Equation 3.1. These theoretical slopes are included in Table 3.1 and have been plotted in Figure 3.1.

![Figure 3.1](image)

**Figure 3.1.** Plot of theoretical and experimental slopes against the indenter radius for the three specimen dimensions (dashed line represents the line of fit for theoretical slope values using E = 9 kN/m² and v = 0.50 in Equation 1)
The slopes, and therefore the Young's modulus predicted by the experimental model was higher than that observed by the theoretical model (Equation 3.1). High R-squared values (0.990, 0.988, and 0.964 for 4, 2, and 1 cm depths respectively) in all cases validated the experimental approach. Values of the intercepts in Figure 3.1 were found to be -9.00, -19.91, and -43.14 for 4 cm, 2 cm and 1 cm depths, respectively. A greater deviation from zero of the intercept for the 1 cm depth specimen was an indication of departure from linearity as would be predicted by Equation 3.1. Such a result was expected since the 1 cm depth specimen was knowingly prepared to violate the requirements for half space conditions at larger indenter sizes.

A reason for the discrepancy between the results obtained by compression and indentation may be due to the elastic mismatch between the two elastically dissimilar bodies in contact, i.e., a rigid steel indenter and a highly compliant gel. In such a case the more compliant body (the gel) would deform more, and a modified stress field will be generated (Warren and Hills 1994). Warren and Hills quantified the influence of this mismatch by a single dimensionless quantity for axisymmetric contact problems as:

\[ \beta = \frac{(1-2\nu_1) - (1-2\nu_2)}{G_1 G_2} \]  
\[ \beta = \frac{2\alpha}{(1-2\nu_1) G_1 + (1-2\nu_2) G_2} \]  

Where: \( \beta = \) Dundur's coefficient (Dimensionless)
\[ G = \text{Shear modulus (Nm}^{-2}\text{)} \]

Subscripts 1 and 2 represent the two elastically dissimilar materials which are steel and gel in this case. Considering the values of shear modulus and Poisson's ratio for steel as 77 GNm\(^{-2}\) and 0.3 respectively (Bowes et al. 1984), and those for 1% agar gel used as 3 kNm\(^{-2}\) (one third of the Young's modulus) and 0.5 respectively, a Dundur's coefficient of \(-1.56 \times 10^{-5}\) was obtained. A low value of this coefficient implied that stick existed for the entire contact area (Warren and Hills 1994) and thus violated assumption vi given above. However, incorporating Dundur's coefficient in the Boussinesq analysis does not allow for more than 10% discrepancy between the compression and indentation results, so that other mechanisms must be responsible for the enhanced stiffness upon indentation (Warren and Hills 1994).

The Young's modulus determined from all combinations of specimen dimensions and indenter radii were comparable except for the combinations of 1 and 2 cm specimen depth with larger indenters (Table 3.2). The values predicted in these cases were higher than the majority of values. This can be explained by assumption (iv) in using Boussinesq's equation, which has been violated in these cases. Contact stresses must vanish at the opposite end of the bodies under loading, i.e., the bottom face of the material under testing should be free from any stresses and therefore the base plate effects should not be "seen". To calculate the stresses at the opposite end, Timoshenko and Goodier (1951) proposed the following relationship:
\[ \sigma_y = \sigma f \quad (3.4) \]

where

\[ f = \frac{y^3}{(b^2 + y^2)^{\frac{3}{2}}} - 1 \quad (3.5) \]

Where: \( \sigma_y \) = Stress along the Y-axis at a depth \( y \) from the surface (Nm\(^{-2}\))

\( \sigma \) = Intensity of uniformly distributed load \( F \) (Nm\(^{-2}\))

\( y \) = Distance of any point along the Y-axis from the surface (m)

\( f \) = Factor (Dimensionless)

The Y-axis is parallel to the longitudinal axis of the indenter. Table 3.3 lists the values of the factor ‘\( f \)’ for all combinations of specimen depths and indenters. The factor ‘\( f \)’ is independent of the material properties of the specimen but dependent on the depth \( y \) and the indenter radius \( b \). Within the limits of experimental error, it can be seen that the factor retains a non-zero value for some indenters when used on 1 and 2 cm depths of specimens. The reaction force due to non-zero stresses gave rise to the base plate effect and the gel appeared to be stiffer.
Table 3.3. Values of the factor ‘f’ for the stresses at the opposite end of the gel specimen

<table>
<thead>
<tr>
<th>b (mm)</th>
<th>4 cm depth</th>
<th>2 cm depth</th>
<th>1 cm depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.1*</td>
</tr>
<tr>
<td>2.5</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.1*</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.1*</td>
</tr>
<tr>
<td>3.5</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.2*</td>
</tr>
<tr>
<td>4.0</td>
<td>0.0</td>
<td>-0.1*</td>
<td>-0.2*</td>
</tr>
<tr>
<td>4.5</td>
<td>0.0</td>
<td>-0.1*</td>
<td>-0.2*</td>
</tr>
</tbody>
</table>

*values indicative of base plate effects

3.5.3 Failure Stress

The value of the failure stress (stress at failure) by compression was found to be 8 ± 2 kNm². Results for the failure stresses as obtained by indentation are listed in Table 3.4. No clear trend in change in failure stress could be noticed on the basis of the depth of the specimens. It was concluded that the depth of specimen had no measurable effects on the specimen’s failure behavior within experimental error. Failure stress decreased as the size of indenter increased. Maximum loads at failure are listed in Table 3.4, along with the failure stresses; they increased with an increase in indenter size. Smaller indenters brought about failure at small loads.
Table 3.4. Means and standard deviations for maximum load (F) and failure stresses ($\sigma_y$) for all specimen dimensions, when indented by seven indenters

<table>
<thead>
<tr>
<th>b (mm)</th>
<th>4 cm depth</th>
<th>2 cm depth</th>
<th>1 cm depth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F (N)</td>
<td>$\sigma_y$ (kNm$^{-2}$)</td>
<td>F (N)</td>
</tr>
<tr>
<td>1.5</td>
<td>0.60 ± 0.03</td>
<td>85 ± 4.52</td>
<td>0.63 ± 0.10</td>
</tr>
<tr>
<td>2.0</td>
<td>0.87 ± 0.14</td>
<td>69 ± 10.90</td>
<td>0.84 ± 0.13</td>
</tr>
<tr>
<td>2.5</td>
<td>1.05 ± 0.10</td>
<td>54 ± 5.12</td>
<td>0.99 ± 0.27</td>
</tr>
<tr>
<td>3.0</td>
<td>0.99 ± 0.36</td>
<td>35 ± 12.78</td>
<td>1.04 ± 0.28</td>
</tr>
<tr>
<td>3.5</td>
<td>1.28 ± 0.15</td>
<td>33 ± 3.94</td>
<td>1.08 ± 0.38</td>
</tr>
<tr>
<td>4.0</td>
<td>1.21 ± 0.45</td>
<td>24 ± 8.86</td>
<td>1.45 ± 0.21</td>
</tr>
<tr>
<td>4.5</td>
<td>1.72 ± 0.35</td>
<td>27 ± 5.54</td>
<td>1.58 ± 0.30</td>
</tr>
</tbody>
</table>

Timoshenko and Goodier (1951) proposed the following relationship to describe the pressure distribution $p(r)$ under a rigid indenter, which was based on the approach taken by Boussinesq:

$$p(r) = \frac{F}{2\pi b \sqrt{b^2 - r^2}}$$

(3.6)

Where: $p(r) =$ Pressure distribution under the indenter (Nm$^{-2}$)

$r =$ Distance of a point from the center of loading (m); $0 \leq r < b$

Pressure at the center of loading ($r = 0$) will be the minimum and it will be half the value of the reported failure stress. The pressure around the perimeter would be infinite and theoretically the gel will fail at infinitesimally low loads. Therefore, the failure of a solid material by a rigid indenter will constitute two components: shear and
compression. The former is a perimeter dependent quantity while the later is an area dependent quantity. Depending on the indenter area and perimeter, the contribution of these components towards failure will be different (Ross and Scanlon 1999). Since the perimeter is directly proportional to the indenter radius whereas the area is proportional to the square of the radius, a somewhat complex relationship exists between these two parameters when using flat cylindrical indenters (Bourne 1966). Contributions of shear and compression in failure of the gel has been given by Bourne (1975) as:

\[ F_{\text{critical}} = K_c A + K_s P + C \]  

(3.7)

Where:

- \( K_c \) = Compression coefficient \((\text{Nm}^2)\)
- \( A \) = Cross-sectional area of indenter \((\text{m}^2)\)
- \( K_s \) = Shear Coefficient \((\text{Nm}^{-1})\)
- \( P \) = Perimeter of indenter \((\text{m})\)
- \( C \) = Constant \((\text{N})\)

The value of the constant \( C \) has been reported in most cases as zero (Bourne 1966). Subsequently dividing the above equation by \( A \) and \( P \), compression and shear coefficients can be calculated (Bourne 1975). These coefficients are listed in Table 3.5 on the basis of both \( F_{\text{critical}} / A \) and \( F_{\text{critical}} / P \) plots. Representative \( F_{\text{critical}} / A \) and \( F_{\text{critical}} / P \) plots are shown in Figures 3.2 and 3.3 for the 4 cm depth of specimens. Good agreement was found between the values obtained for 2 and 4 cm depths. A small difference was found for the values obtained for 1 cm depth, where the half space conditions were slightly transgressed in the case of larger indenters.
Table 3.5. Values of compression and shear coefficients ($K_c$ and $K_s$) as obtained from $F_{\text{critical}}/A$ and $F_{\text{critical}}/P$ plots

<table>
<thead>
<tr>
<th></th>
<th>4 cm depth</th>
<th>2 cm depth</th>
<th>1 cm depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_c$ (kNm$^{-2}$)</td>
<td>$K_s$ (Nm$^{-1}$)</td>
<td>$K_c$ (kNm$^{-2}$)</td>
<td>$K_s$ (Nm$^{-1}$)</td>
</tr>
<tr>
<td>$F_{\text{critical}}/A$</td>
<td>-8.13</td>
<td>72</td>
<td>-11</td>
</tr>
<tr>
<td>$F_{\text{critical}}/P$</td>
<td>-8.47</td>
<td>73</td>
<td>-9.3</td>
</tr>
</tbody>
</table>

Figure 3.2. Representative $F_{\text{critical}}/A$ plot for the 4 cm depth of specimen

Figure 3.3. Representative $F_{\text{critical}}/P$ plot for the 4 cm depth of specimen
3.6 CONCLUSIONS

The dimensions of the specimens affected the Young’s modulus when the requirements for elastic half spaces were slightly transgressed. Within half space conditions, reasonable agreement was found between the theoretical predictions and the experimental values. The indentation model overestimated the value of Young’s modulus of 1% agar gel by 34%. For 1 and 2 cm depth of specimens, elastic half space assumptions were invalid, particularly for larger indenters so that on average Young’s modulus was 56% greater than that obtained from compression. Owing to differences in the mechanical properties of test specimens and steel indenters, elastic mismatch between the specimens and indenters occurred. Specimen dimensions had no measurable effect on failure stresses within the limits of experimental error. It was therefore concluded that indentation has potential for use as a fundamental test for characterization of texture-related mechanical properties of model foods, provided the experimental conditions are carefully chosen.
CHAPTER 4

Indentation Analysis of a Layered Food Composite using the Classical Theories of Elasticity
4.1 ABSTRACT

Numerous foods, both processed and unprocessed can be structurally classified as layered composite structures. Mechanical properties of each layer can be different from those of other layers in a composite food and therefore texture of a layered composite food would be a complex function of the mechanical properties and dimensions of each of the layers. Unfortunately not many studies in the areas of food texture have been dedicated to studying the relationship between texture and mechanical properties of layered food composites. The present study is aimed at studying the interaction between the two layers and their effects on the overall mechanical properties of a layered composite food. Classical theories of elasticity were used to predict two mechanical properties, stiffness and strength, of a model composite food system. It was found that the stiffness parameter was reasonably explained by a classic 'elastic plate on an elastic foundation' model whereas the strength parameter could be reasonably explained by a model for failure theory for a sandwich panel. The study demonstrated that the theories of elasticity can be successfully used to characterize the behavior of a model composite food system in a similar way that they are used to characterize the behavior of engineering structures. This enables food scientists to gather valuable information about the texture of a layered food composite on the basis of the mechanical properties of individual layers and the type of interaction between the layers.
4.2 INTRODUCTION

Numerous examples of foods, both processed and natural, can be structurally regarded as layered composites (Roy et al. 1989), e.g., sandwich cookies, cream filled candies, layered cakes, french fries, fruits and vegetables with skin. Freshly baked wrapped bread is another type of example that can also be considered as a layered food composite. A layered composite food consists of two or more layers of foods and mechanical properties of different layers are different from one another. For example, in sandwich cookies, a creamy smooth filling is sandwiched between two crispy cookies. Mechanical properties of the creamy smooth filling are different from those of the crispy cookies.

Mechanical properties of foods have been shown to greatly influence the perceived texture of foods (Finney 1969; Bourne 1975; Mohsenin 1986; Peleg 1977; Peleg 1987). Texture of a layered food composite, which depends on the texture of the individual layers, is affected by the mechanical properties of individual layers. Overall mechanical behavior of a layered food composite is not only important for its texture but also for safe processing, handling, transportation, and storage of foods. Yet, not many researchers have studied the mechanical behavior of a layered composite food system (Roy et al. 1989). Langley et al. (1990) have addressed textural issues of a particulate composite food system which is different from a layered composite food system. Roy et al. (1989) have proposed a method to predict the compressive behavior of a multi-layer food system. With certain limitations, their method predicted a reasonably good correlation between observed and predicted compressive behavior of
layered food composites. Scanlon and Ross (2000) have studied the texture of french fries as a composite material.

A number of research studies have been done in the areas of highway and runway construction (Westergaard 1926; Burmister 1945), railroad construction (Li and Selig 1998a, 1998b), soil foundation (Selvadurai 1979), cold regions sciences and ice structures (Wyman 1950; Nevel 1970), and sandwich panels (Olsson and McManus 1996), where the major objective has been to study the stresses and deformations in a layered composite system. Classical theories of elasticity (Westergaard 1926; Burmister 1945; Timoshenko and Goodier 1951; Gladwell 1980; Dempsey et al. 1991) have been used for mathematical formulation of the stresses and the deformations. A layered composite food is structurally similar to the composite structures usually encountered in the above mentioned areas. In the previous chapter of this research, it has been shown that the elastic behavior of foods can be reasonably characterized by the theories of elasticity. Indentation, although classified as an empirical test (Timbers et al. 1965), has been established as a suitable method to study the load-deformation relationship in model food systems (Anand and Scanlon 1999; Ross and Scanlon 1999). Therefore, the objectives of this study were the following:

(i) To model the strength and stiffness behavior of a layered composite food system under indentation loads, and

(ii) To demonstrate the applicability of the classical theories of elasticity, that were originally developed for engineering materials and were applicable in engineering problems, to the areas of food texture.
4.3 THEORETICAL APPROACH

In most engineering situations, the major objective of a multi-layer system is to improve the overall mechanical response of the system to large loads. Another purpose is to determine the safe thickness of the top layer so that it can sustain predicted large loads by evenly distributing the loads to the underlying layers over a wider area. A plate or an elastic layer lying on an elastic foundation is a classical treatment available within the theories of elasticity to deal with the stresses and deformations in layered composite structures (Westergaard 1926; Burmister 1945; Timoshenko and Woinowsky-Krieger 1959; Dempsey et al. 1991). Recently Scanlon and Ross (2000) have modeled a french fry system as a plate on an elastic foundation. The crispy fry crust was modeled as plate whereas the mealy interior was considered a foundation. A plate, as shown in Figure 4.1(a), is defined as a three dimensional elastic object, one dimension of which (thickness) is small compared to the other two dimensions (length and width). The elastic foundation is also regarded as an infinite plane and it is defined as an elastic object for which the dimensions in the direction of force (usually the thickness) is finite but the dimensions perpendicular to the direction of force (length and width) are infinite. When all dimensions are infinite, the foundation is called an elastic half space, as indicated in Figure 4.1(b).
Figure 4.1. (a) An infinite elastic plate or layer; (b) Elastic half space
In geotechnical and soil engineering, subgrade is a more common term for an elastic foundation (Westergaard 1926; Terzaghi 1955; Selvadurai 1979). The Winkler model (Westergaard 1926; Terzaghi 1955; Selvadurai 1979; Nevel 1970; Dempsey et al. 1991; Scanlon and Ross 2000) is one of the most popular and simpler models for describing the properties of elastic foundations. Even though a Winkler model only approximates the actual behavior of an elastic foundation (Bowles 1982; Vallabhan et al. 1991), the use of the Winkler model can still be justified for the simplicity that it offers (Westergaard 1926; Vallabhan et al. 1991). A Winkler foundation is characterized by the 'modulus of subgrade reaction' (K) also known as 'stiffness modulus' or 'foundation modulus' and is defined as the ratio of applied pressure at any point on the foundation and the resultant deformation at that point. The elastic plate lying on an elastic foundation is characterized by the 'flexural rigidity' (D), (also known as 'rigidity modulus' or 'bending stiffness') which describes the resistance of the plate to bending. The interaction between the plate and the foundation is characterized by a parameter called a 'characteristic length' or 'radius of relative stiffness' (l). The following expressions define and relate each of the above mentioned parameters (modulus of subgrade reaction, flexural rigidity, characteristic length):

\[ K = \frac{q}{d} \]  \hspace{1cm} (4.1)

\[ D = \frac{Eh^3}{12(1-\nu^2)} \]  \hspace{1cm} (4.2)
\[ I = \sqrt[3]{\frac{D}{K}} = \frac{\sqrt[3]{Eh^3}}{12(1-\nu^2)K} \]  \hspace{1cm} (4.3)

Where, 

- \( K \) = Modulus of subgrade reaction (Nm\(^{-1}\))
- \( q \) = Pressure at any point (Nm\(^{-2}\))
- \( d \) = Deformation at the point of pressure application (m)
- \( D \) = Flexural rigidity of the plate (Nm)
- \( E \) = Modulus of elasticity of the plate (Nm\(^{-2}\))
- \( h \) = Thickness of the plate (m)
- \( \nu \) = Poisson’s ratio of the plate (dimensionless)
- \( l \) = Characteristic length parameter (m)

When a flat cylindrical indenter of radius 'b' is applied against a system of 'plate on an elastic foundation', the entire system experiences deformation. Based on the magnitude of deformation and the state of the middle plane of the plate, the problem of a plate on an elastic foundation is further classified according to small deflection plate theory, larger deflection plate theory, and membrane theory (Olsson and McManus 1996). Small deflection plate theory is applied in situations where the deformation is less than half the thickness of the plate (Olsson and McManus 1996). In this case the middle plane of the plate remains unstrained and the normal stresses in the direction transverse to the plate can be neglected (Timoshenko and Woinowsky-Krieger 1959). Nevel (1970) modeled the problem of a floating ice sheet as an elastic plate lying on an elastic foundation. He treated the specific weight of water (Nm\(^{-3}\)) as the
modulus of subgrade reaction. The safe bearing thickness of ice was thus determined in this case using small deflection plate theory. Nevel (1970) used the following expressions for the stiffness and strength of a floating ice sheet.

\[
Stiffness = \frac{F}{d} = \frac{\pi K b^2}{1 + \left(\frac{b}{l}\right) \text{ker}' \left(\frac{b}{l}\right)}
\]

\[
Strength = \sigma_s = \frac{3 F (1 + \nu) \text{kei}' \left(\frac{b}{l}\right)}{\pi h^2 \left(\frac{b}{l}\right)}
\]

Where, 
- \( F \) = Applied load (N)
- \( b \) = Indenter radius (m)
- \( \sigma_s \) = Strength in the small deflection plate theory (Nm\(^{-2}\))
- \( \text{ker}' \), \( \text{kei}' \) = First derivatives of the real and imaginary parts of Bessel functions

The following assumptions are made when dealing with problems of stresses and deformations in a layered system (Burmister 1945):

(i) Each of the two layers are assumed to be homogeneous, isotropic, and elastic so that Hooke's law is valid.

(ii) The top layer is infinite in the horizontal direction but of a finite thickness 'h'. The underlying layer is of infinite dimensions in both horizontal and vertical dimensions and therefore can be considered a half space.
(iii) Shear stresses are zero and normal stresses are limited to the area of loading only. Stresses and displacements are zero at an infinite depth of the underlying layer.

(iv) The two layers remain in contact at all times. The type of contact could be either frictional or frictionless.

Stresses and displacements in a layered composite system have also been analytically treated by a different approach (Wen et al. 1998). This approach is based on how a layered composite system fails under the action of indentation loads. When a composite structure is indented using rigid indenters, the entire structure tends to bend while the top layer experiences more bending in the vicinity of indenter. As soon as the shear strength of the top layer is exceeded, failure occurs which is usually accompanied by some local crushing of the foundation underneath the indenter (Wen et al. 1998). Therefore the total failure load should consist of two components: the load that causes failure of the top layer as soon as the shear strength of top layer is reached, and the load that causes foundation crushing when the compressive strength of the foundation is reached. The total failure load as given by Wen et al. (1998) is:

$$F_{\text{indent}} = 2 \pi b h \tau_{\text{top}} + \pi b^2 k \sigma_f$$  \hspace{1cm} (4.6)

Where, $F_{\text{indent}} =$ Load at which a layered composite system fails by indentation (N)

$\tau_{\text{top}} =$ Shear strength of the top layer (Nm$^{-2}$)

$\sigma_f =$ Compressive strength of the foundation material (Nm$^{-2}$)
\[ k = \text{Constraint factor; } k = 2 \text{ (Wen et al. 1998)} \]

The advantage of the approach taken by Wen et al. (1998) is that it yields a simple model that can predict the failure loads and energies in a layered composite system. A pure analytical treatment of the problem is a major advantage especially for food scientists. In the present study, the approach taken by Wen et al. (1998) will be referred to as the ‘indentation failure model’.
4.4 MATERIALS AND METHODS

4.4.1 Preparation of 'Pasta on Gel' Composites

Microbiological grade granulated agar (Difco Laboratories, MI) in a concentration of 3% (w/v) was dissolved into distilled water and boiled with continuous stirring, until the formation of air bubbles stopped. To inhibit microbial growth, 0.2% (W/V) sodium azide was added to the agar gel solution. Hot 3% agar gel solution was poured into a pan (24 cm x 18 cm x 4 cm) to different depths (0.4, 1, 2, and 4 cm) and was left overnight at room temperature for gelation. Nussinovitch and Peleg (1990) reported that the changes in mechanical properties of an agar gel were minimal after 18 to 48 h of gel preparation and therefore, gels were used in the experiments after 24 h of preparation till the experiment was finished, on any given day. An hour prior to experimentation, the 3% gel was transferred to a larger tray and cut into desired dimensions using a sharp blade. Four sample dimensions were thus prepared; all with a cross sectional area of 4 cm x 4 cm and of four different depths, 4 cm, 2 cm, 1 cm, and 4 mm.

Dry pasta was bought from the local supermarket and was cut into 4 cm x 4 cm flat pieces, using a hand held cutting tool fitted with a cut off wheel no. 409 (Dremel, Racine, WI). The average thickness of the pasta, as measured by a digital caliper, was 1.10 ± 0.10 mm. A few minutes prior to experimentation, the pasta layer was glued onto the 3% gel specimens using superglue (LEPAGE 8, Brampton, ON). A thin layer of superglue was scrapped off the gel using a sharp blade and the thickness was measured using a vernier caliper. The average thickness of the superglue layer was 0.03 ± 0.01 mm. The modulus of elasticity of superglue has been reported as 1.7 GNm⁻²
(Yokoyama and Shimizu 1998) which is comparable to the modulus of elasticity of dry pasta (2.9 GNm\(^{-2}\)), determined in an auxiliary experiment.

4.4.2 Preparation of ‘Gel on Gel’ Composites

In all ‘gel on gel’ composites, the foundation was made of 1% gel and the top layer was made of 3% gel. Following the procedure described in 4.4.1, 1% agar gel blocks were prepared. However, the cross sectional area of the ‘gel on gel’ composite specimen was 6 cm x 6 cm instead of 4 cm x 4 cm as mentioned in 4.4.1. For the top layer, 3% gel was prepared by the same method as the 1% gel, but the hot 3% gel was poured into a beaker (900 ml) instead, and was left overnight at room temperature for gelation. An hour prior to start of mechanical testing, the 3% gel was taken out of the beaker and was sliced into layers of 2 and 4 mm thickness using a Hobart meat slicer (Model 410, Troy, OH). Half an hour prior to experimentation, the 3% gel layer was glued to the 1% gel foundation using a freshly prepared 3% gel. Using a freshly prepared 3% gel as a gluing medium eliminated the use of any other synthetic glue, which would likely have very different mechanical properties than the agar gel. Hanging edges of layers of 3% gel were trimmed using a sharp pathology blade.

4.4.3 Indentation Testing

Indentation testing was carried out using a universal testing machine (Lloyd L 1000, Lloyd Instruments Limited, Fareham, UK) complete with a computerized data recording facility. All experiments were performed at room temperature under quasi-static conditions with a crosshead speed of 1 mm.min\(^{-1}\). To reduce friction induced by
shear effects, lubrication was provided between the specimens and the bottom platen (used to hold samples) of the universal testing machine (Kaletunc et al. 1991). Besides lubrication, the sample holding platen was also coated with teflon. Flat cylindrical indenters made of stainless steel were used for indentation of each of the ‘pasta on gel’ composites and ‘gel on gel’ composites. Indenters of diameters 1, 3, and 5 mm were used for ‘pasta on gel’ composites whereas indenters of 3, 5, and 7 mm were used for all ‘gel on gel’ composites.

4.4.4 Mechanical Properties of Gels and Dry Pasta

The mechanical properties of the individual parts of the composites, 1% gel, 3% gel, and dry pasta were determined in auxiliary experiments. Uniaxial compression tests were applied on cylindrical specimens of 1 and 3% gels as described in section 3.4. Cubes (4 cm) of 1% agar gel were compressed between lubricated parallel plates on an Ottawa Texture Measuring System (Canners Machinery, Simcoe, ON) equipped with a 110 N load cell to measure Poisson’s ratio. Following the same method, Ross and Scanlon (1999) have previously reported a value of 0.32 for the Poisson’s ratio of an agar gel. Therefore, their value of 0.32 was used for the Poisson’s ratio of 3% agar gel, in this study. Strips of dry pasta were characterized by tensile testing (ASTM D828). Shear strength of 3% gel and dry pasta were calculated by dividing the measured failure stress values by a factor of 2.33 (Hill 1950). Modulus of subgrade reaction (K) was first obtained for each indenter size by dividing the initial slopes of the force-deformation curves of 1 and 3% agar gel specimens by the cross section area of the indenter. Then values of ‘K’ were characterized for each foundation depth by the following method.
(Scanlon and Ross 2000). For each depth, values of the modulus of subgrade reaction (K) were plotted against the indenter radius (b). A regression equation of the form \( K = m.b^n \) was obtained, where m and n were the regression coefficients, values of which were known. A representative plot for 1% gel has been shown in Figure 4.2 where the foundation depth was 4 mm. By substituting a hypothetical value for ‘b’ so that the indenter size became the same as the size of the specimen, a value of ‘K’ was obtained (Scanlon and Ross 2000). For example, the value of ‘b’ to be used for a specimen of cross sectional dimensions 6 cm x 6 cm would be 3 cm. The value of K thus obtained was used as the value of the modulus of subgrade reaction for a given foundation depth (Scanlon and Ross 2000).

![Figure 4.2. Representative plot for calculation of the modulus of subgrade reaction (4 mm foundation depth of 1% agar gel)](image-url)
4.5 RESULTS AND DISCUSSION

4.5.1 Characterization of Materials

The fundamental mechanical properties of 1 and 3% gel, and dry pasta are given in Table 4.1. Representative plots for the model layered composites are shown in Figure 4.3.

<table>
<thead>
<tr>
<th></th>
<th>1% gel</th>
<th>3% gel</th>
<th>Dry pasta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus (E, Nm⁻²)</td>
<td>12.16±2.10 x 10³</td>
<td>74.09±18.43 x 10³</td>
<td>2.88±0.40 x 10⁹</td>
</tr>
<tr>
<td>Failure stress (σ, Nm⁻²)</td>
<td>9.27±0.90 x 10³</td>
<td>57.44±5.45 x 10³</td>
<td>4.13±0.97 x 10⁶</td>
</tr>
<tr>
<td>Poisson's ratio (ν)</td>
<td>0.50±0.12</td>
<td>0.32</td>
<td>0.25</td>
</tr>
<tr>
<td>Shear strength (τ, Nm⁻²)</td>
<td>3.98 x 10³</td>
<td>24.65 x 10³</td>
<td>1.77 x 10⁹</td>
</tr>
<tr>
<td>Modulus of subgrade reaction (K, Nm⁻³)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 mm depth</td>
<td>0.30 x 10⁶</td>
<td>6.21 x 10⁶</td>
<td></td>
</tr>
<tr>
<td>1 cm depth</td>
<td>0.88 x 10⁶</td>
<td>10.02 x 10⁶</td>
<td></td>
</tr>
<tr>
<td>2 cm depth</td>
<td>0.56 x 10⁶</td>
<td>8.45 x 10⁶</td>
<td></td>
</tr>
<tr>
<td>4 cm depth</td>
<td>0.49 x 10⁶</td>
<td>10.17 x 10⁶</td>
<td></td>
</tr>
</tbody>
</table>

a. Ross and Scanlon (1999)

b. Scanlon (2000)

4.5.2 Stiffness

4.5.2.1 'Pasta on gel' composites

Following the approach taken by Nevel (1970) and as shown in equation 4.4, theoretical values of stiffness were calculated. Experimental values for stiffness were obtained from the initial slopes (where the deformation was less than half the thickness.
Figure 4.3. Representative plots for the three types of composites when indented by an indenter of radius 1.5 mm (a) ‘Pasta on gel’ - pasta layer on 4 cm foundation depth, (b) ‘Gel on gel’ - 2 mm thick top layer on 4 cm foundation depth, (c) ‘Gel on gel’ - 4 mm thick top layer on 4 cm foundation depth
of the layer) of the force-deformation curves (Figure 4.4). Both theoretical and experimental values of stiffness are listed in Table 4.2. Values for the characteristic length for 'pasta on gel' composites were 15.30, 13.58, 14.17, and 13.53 mm for 4 mm, 1, 2, and 4 cm foundation depths, respectively.

![Figure 4.4. A typical load-deformation curve obtained by indentation of model layered food composites; h is the thickness of the top layer](image)

### Table 4.2. Mean and standard deviations of stiffness (kNm⁻¹) for all 'pasta on gel' composites as obtained theoretically and experimentally

<table>
<thead>
<tr>
<th></th>
<th>0.5 mm indenter</th>
<th>1.5 mm indenter</th>
<th>2.5 mm indenter</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 mm depth</td>
<td>7.75</td>
<td>37.86 ± 3.08</td>
<td>7.79</td>
</tr>
<tr>
<td>1 cm depth</td>
<td>6.10</td>
<td>14.19 ± 0.06</td>
<td>6.14</td>
</tr>
<tr>
<td>2 cm depth</td>
<td>6.64</td>
<td>7.92 ± 1.61</td>
<td>6.68</td>
</tr>
<tr>
<td>4 cm depth</td>
<td>6.06</td>
<td>4.54 ± 0.61</td>
<td>6.09</td>
</tr>
</tbody>
</table>
The model (Nevel 1970) underpredicts the stiffness values for shallow foundations. As the foundation depth increases, the stiffness values predicted by the model start to agree with the experimental stiffness values. For the purpose of normalization, experimental stiffness values were divided by the theoretical stiffness values. Normalization therefore, allows a plot of both experimental and theoretical values simultaneously on a single plot. In this case, normalization also makes the quantities dimensionless and therefore general trends in the plot can be easily observed and compared.

Experimental stiffness values normalized by predicted stiffness values are plotted against indenter radius in Figure 4.5. Theoretically all points in this plot should lie on the $Y=1$ line if the model was to perfectly fit the experimental data, i.e., if the experimental observations are the same as predicted by the theoretical model. For larger foundation depths (2 and 4 cm depths) the points are somewhat close to the line $Y=1$ showing that the model fits reasonably well for these foundation depths. For all four foundation depths the plots are almost parallel to the line $Y=1$ showing that indenter size has essentially no effect on the stiffness nor on the validity of the model. Experimental stiffness values for the 4 mm and the 1 cm depths are approximately 5 and 2 times larger than the predicted values, respectively. Effect of foundation depth can be explained on the basis of the assumption made in the plates on elastic foundation model that the foundation must behave as an elastic half space (Burmister 1945). Therefore as the depth of the foundation increases, base plate effects (Anand and Scanlon 1999) become less pronounced and as a result, the model becomes more valid. The foundation depth effect is consistent as it can be observed that the four foundation depths are ordered in Figure 4.5, according to foundation depths. The plot for the 4 cm
foundation depth is closest to the line $Y=1$, whereas the plot for the 4 mm foundation depth is the farthest.

Figure 4.5. Normalized stiffness of 'pasta on gel' composites

- □ 4 mm depth
- ◆ 1 cm depth
- ◆ 2 cm depth
- ▲ 4 cm depth
4.5.2.2 'Gel on gel' composites

Means and standard deviations for theoretical (Equation 4.4) and experimental values of stiffness are listed in Table 4.3 below.

<table>
<thead>
<tr>
<th></th>
<th>2 mm layer</th>
<th>4 mm layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theoretical</td>
<td>Experimental</td>
</tr>
<tr>
<td><strong>4 mm depth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5 mm indenter</td>
<td>35</td>
<td>185 ± 29</td>
</tr>
<tr>
<td>2.5 mm indenter</td>
<td>37</td>
<td>323 ± 40</td>
</tr>
<tr>
<td>3.5 mm indenter</td>
<td>41</td>
<td>504 ± 107</td>
</tr>
<tr>
<td><strong>1 cm depth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5 mm indenter</td>
<td>61</td>
<td>176 ± 34</td>
</tr>
<tr>
<td>2.5 mm indenter</td>
<td>69</td>
<td>185 ± 4</td>
</tr>
<tr>
<td>3.5 mm indenter</td>
<td>79</td>
<td>283 ± 63</td>
</tr>
<tr>
<td><strong>2 cm depth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5 mm indenter</td>
<td>48</td>
<td>150 ± 9</td>
</tr>
<tr>
<td>2.5 mm indenter</td>
<td>53</td>
<td>175 ± 68</td>
</tr>
<tr>
<td>3.5 mm indenter</td>
<td>60</td>
<td>247 ± 15</td>
</tr>
<tr>
<td><strong>4 cm depth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5 mm indenter</td>
<td>45</td>
<td>148 ± 11</td>
</tr>
<tr>
<td>2.5 mm indenter</td>
<td>49</td>
<td>195 ± 15</td>
</tr>
<tr>
<td>3.5 mm indenter</td>
<td>55</td>
<td>222 ± 10</td>
</tr>
</tbody>
</table>

Normalized stiffness values are plotted against indenter radius for each of the foundation depths in Figures 4.6 through 4.9. Indenter size effects are quite obvious.
Figure 4.6. Normalized stiffness for gel on gel composites (4 mm thick foundation)

\[ \text{Normalized stiffness} \]

\[ \text{Indenter radius, } b(\text{m}) \]

\[ \text{2 mm top layer} \quad \text{4 mm top layer} \]

Figure 4.7. Normalized stiffness for gel on gel composites (1 cm thick foundation)

\[ \text{Normalized stiffness} \]

\[ \text{Indenter radius, } b(\text{m}) \]

\[ \text{2 mm top layer} \quad \text{4 mm top layer} \]
Figure 4.8. Normalized stiffness for gel on gel composites (2 cm thick foundation)

Figure 4.9. Normalized stiffness for gel on gel composites (4 cm thick foundation)
Small indenters tend to give a better fit of the model than do the larger indenters. This trend is consistent for all four foundation depths although the effect of indenter size gradually diminished with an increase in foundation depth (plots became less steep for larger foundation depths, 1, 2, and 4 cm). The effect of layer thickness can also be noticed from each of these plots. A 4 mm top layer leads to a better fit of the model as compared to a 2 mm thick top layer which is a result of better load distribution caused by a thicker layer (Terzaghi 1955). Load distribution efficiency of top layers is characterized by a physical parameter called ‘range of influence’ (R), which is related to the characteristic length (l) (Equation 4.3) by the following expression (Terzaghi 1955):

\[ R = 2.5 l \]  

(4.7)

Where, \( R \) = Range of influence (m)

The range of influence is an imaginary radius of load distribution when an indenter of radius \( b \) is applied against a layered medium. Because of the interaction between the layer and the underlying foundation (as characterized by the parameter \( l \)), the layer redistributes the load onto the foundation over a wider area given by \( \pi R^2 \) instead of an area equal to \( \pi b^2 \) (Terzaghi 1955). Therefore the foundation experiences an apparently lower stress than the layer itself (\( R > b \)). This is how providing a top layer (e.g. a thin layer of concrete) improves the loading performance of a layered medium such as a highway or a runway (Burmister 1945), even in case when the foundation itself is weak and cannot sustain large loads (e.g., a sandy subgrade). Values of characteristic lengths are shown for all ‘gel on gel’ composites (Table 4.4).
Table 4.4. Values of characteristic length ($l$) and range of influence ($R$) for all ‘gel on gel’ composites

<table>
<thead>
<tr>
<th>Foundation Depth</th>
<th>2 mm thick top layer</th>
<th>4 mm thick top layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$l$ (mm)</td>
<td>$R$ (mm)</td>
</tr>
<tr>
<td>4 mm</td>
<td>3.68</td>
<td>9.20</td>
</tr>
<tr>
<td>1 cm</td>
<td>2.81</td>
<td>7.03</td>
</tr>
<tr>
<td>2 cm</td>
<td>3.15</td>
<td>7.87</td>
</tr>
<tr>
<td>4 cm</td>
<td>3.26</td>
<td>8.15</td>
</tr>
</tbody>
</table>

Values of $l$ are higher for a 4 mm thick top layer than for a 2 mm thick top layer. Therefore, the range of influence ($R = 2.5l$) would be larger for the 4 mm thick top layer than for the 2 mm thick top layer (Terzaghi 1955). Larger values of range of influence would result in a low apparent stress, which the underlying foundation will be subjected to. An empirical parameter $(R/b)^2$ can be treated as an index for the load distribution efficiency of top layers. Because of the square term, this empirical index for load distribution efficiency would change faster than a change in layer thickness causing the relationship between layer thickness and its efficacy in distributing the load to a wider area to be non-linear.

Also the model fits better for larger foundation depths as the data points in Figures 4.7, 4.8, and 4.9 are closer and more parallel to the line $Y=1$. Decreased base plate effects (Anand and Scanlon 1999) as mentioned for ‘pasta on gel’ composites can again account for the better fit of the model.

4.5.3 Strength

For analysis of strength, the model used by Nevel (1970) needs the value of peak load at failure (Equation 4.5) to be used. Associated with the peak loads at failure, the
average deformations for 'pasta on gel' composites were 0.8 mm, 1.40 mm, 2.38 mm, and 4.04 mm for foundation depths of 4 mm, 1, 2, and 4 cm respectively. Average thickness of dry pasta used as the top layer was only 1.10 mm. For the 4 mm top layer of all 'gel on gel' composites, the average deformation at failure was 3.46 mm. For the 2 mm top layer of all 'gel on gel' composites, the average deformation was 3.09 mm. Therefore, the deformations associated with the peak loads in the present experiments are no longer small as compared to the thickness of the top layer. In fact the deformations are in excess of half of the layer thickness for all composites. On this ground the validity of small deflection plate theory can be questioned (Olsson and McManus 1996) and therefore, the strength analysis was carried out using the indentation failure model proposed by Wen et al. (1998).

4.5.3.1 'Pasta on gel' composites

Strength values for 'pasta on gel' composites were calculated by dividing the failure loads obtained theoretically (as shown in Equation 4.6) and experimentally (Figure 4.4) by the cross sectional area of the indenters. Both experimental and theoretical values of strength are listed in Table 4.5.

<table>
<thead>
<tr>
<th></th>
<th>0.5 mm indenter</th>
<th>1.5 mm indenter</th>
<th>3.5 mm indenter</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 mm depth</td>
<td>7.91</td>
<td>36.01 ± 0.93</td>
<td>2.71</td>
</tr>
<tr>
<td>1 cm depth</td>
<td>7.91</td>
<td>30.39 ± 5.75</td>
<td>2.71</td>
</tr>
<tr>
<td>2 cm depth</td>
<td>7.91</td>
<td>28.24 ± 2.59</td>
<td>2.71</td>
</tr>
<tr>
<td>4 cm depth</td>
<td>7.91</td>
<td>27.98 ± 4.04</td>
<td>2.71</td>
</tr>
</tbody>
</table>
The indentation failure model underpredicts strength values in most cases. However, if the 4 mm foundation is excluded, good agreement can be seen particularly for larger indenters. Indentation results for 'pasta on gel' composites are plotted in Figure 4.10 in which failure loads normalized by foundation depths are plotted against indenter sizes.

![Figure 4.10. Plot of $F_{\text{indent}}$/foundation depth vs. $b$ for 'pasta on gel' composites (lines are theoretical)](image)

For each foundation depth, failure load is proportional to indenter size. Failure loads are however affected by foundation depths. If there was no effect due to foundation depths, there would have been just one plot for all four foundation depths (Wen et al.)
Instead there are four distinct plots for the four foundation depths (Figure 4.10). The 4 mm foundation depth shows a substantially different behavior than the other three foundation depths. Indenter effects are much more pronounced for this foundation depth (as evidenced by the increased slope). The other three depths of foundations (1, 2, and 4 cm) show a behavior not very different from each other. Lack of base plate effects (Anand and Scanlon 1999) could account for improved validity of the model at larger foundation depths. Smaller foundation depths would experience large base plate effects (Anand and Scanlon 1999) and as a result, high reaction forces to the indentation process. Therefore, smaller foundation depths would apparently exhibit larger strength than would be predicted by the model and thus causing the failure loads to be higher than expected (Mohsenin 1986, Pp. 348-382; Anand and Scanlon 1999). Plots for 1, 2, and 4 cm foundation depths are parallel and close to each other showing that with an increase in foundation depth, the effect of foundation depth on failure load has gradually decreased.

4.5.3.2 ‘Gel on gel’ composites

Experimental and theoretical values of strength for all ‘gel on gel’ composites are shown in Table 4.6 and good agreement between experimental and predicted values can be seen for both layers and all indenters.
Table 4.6. Means and standard deviations for theoretical and experimental strengths (kN/m²) for all ‘gel on gel’ composites

<table>
<thead>
<tr>
<th></th>
<th>2 mm layer</th>
<th>4 mm layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theoretical</td>
<td>Experimental</td>
</tr>
<tr>
<td><strong>4 mm depth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5 mm indenter</td>
<td>84.24</td>
<td>97.14 ± 15.13</td>
</tr>
<tr>
<td>2.5 mm indenter</td>
<td>57.95</td>
<td>47.87 ± 7.91</td>
</tr>
<tr>
<td>3.5 mm indenter</td>
<td>46.69</td>
<td>40.19 ± 2.87</td>
</tr>
<tr>
<td><strong>1 cm depth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5 mm indenter</td>
<td>84.24</td>
<td>55.17 ± 22.64</td>
</tr>
<tr>
<td>2.5 mm indenter</td>
<td>57.95</td>
<td>37.35 ± 4.15</td>
</tr>
<tr>
<td>3.5 mm indenter</td>
<td>46.69</td>
<td>33.35 ± 4.68</td>
</tr>
<tr>
<td><strong>2 cm depth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5 mm indenter</td>
<td>84.24</td>
<td>63.19 ± 8.53</td>
</tr>
<tr>
<td>2.5 mm indenter</td>
<td>57.95</td>
<td>36.16 ± 6.18</td>
</tr>
<tr>
<td>3.5 mm indenter</td>
<td>46.69</td>
<td>29.54 ± 6.75</td>
</tr>
<tr>
<td><strong>4 cm depth</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5 mm indenter</td>
<td>84.24</td>
<td>58.00 ± 16.32</td>
</tr>
<tr>
<td>2.5 mm indenter</td>
<td>57.95</td>
<td>35.99 ± 1.28</td>
</tr>
<tr>
<td>3.5 mm indenter</td>
<td>46.69</td>
<td>28.06 ± 4.29</td>
</tr>
</tbody>
</table>

In Figures 4.11 through 4.14, experimental and theoretical values of strength normalized by the thickness of the top layer are plotted against indenter size for each of the four foundation depths. The indentation failure model seems to fit reasonably well for all ‘gel on gel’ composites as can be seen in Figures 4.11 through 4.14. Failure load is proportional to the size of the indenter, which is also evident from equation 4.6. For all ‘gel on gel’ composites, failure loads are also affected by the thickness of the top layer which is also expected (Equation 4.6).
Figure 4.11. Plot of $F_{\text{indent}}/t$ vs. $b$ for 4 mm gel on gel composites (lines are theoretical)

- ■ 4 mm top layer
- ▲ 2 mm top layer
- -- 4 mm top layer
- --- 2 mm top layer

Figure 4.12. Plot of $F_{\text{indent}}/t$ vs. $b$ for 1 cm gel on gel composites (lines are theoretical)

- ■ 4 mm top layer
- ▲ 2 mm top layer
- -- 4 mm top layer
- --- 2 mm top layer
Figure 4.13. Plot of $F_{\text{indent}}/t$ vs. $b$ for 2 cm gel on gel composites (lines are theoretical)

---

Figure 4.14. Plot of $F_{\text{indent}}/t$ vs. $b$ for 4 cm gel on gel composites (lines are theoretical)
4.5.3.3 Comparison with Bourne’s equation

The indentation failure model accounts for the contributions from both the foundation and the top layer when predicting failure loads (Equation 4.6). A dimensional similarity can be found between equation 4.6 and the following equation 4.8 given by Bourne (1966).

\[ F_{\text{critical}} = K_p P + K_c A + C \]  

(4.8)

Where,  
- \( F_{\text{critical}} \) = Maximum load at which a test specimen fails (N)
- \( K_p \) = Compression coefficient (Nm\(^{-2}\))
- \( A \) = Cross section area of the indenter (m\(^2\))
- \( K_s \) = Shear coefficient (Nm\(^{-1}\))
- \( P \) = Perimeter of the indenter (m)
- \( C \) = A constant (N), value of which is usual taken to be zero (Bourne 1966)

Bourne’s equation accounts for the coefficients for shear and compression contributions, \( K_s \) and \( K_c \) respectively, which are related to the perimeter and cross sectional area of a flat cylindrical indenter, respectively. Other researchers (Anand and Scanlon 1999; Ross and Scanlon 1999) have also used equation 4.8, to calculate the shear and compression coefficients in an indentation process. Following this approach, another check on the validity of the model given by equation 4.6 was carried out.

The first term on the right hand side of the equation 4.6 is a product of the perimeter of a flat cylindrical indenter of radius \( b \) (2\( \pi b \)), the thickness of the top layer (h), and the shear strength of the top layer (\( \tau_{\text{top}} \)). A product of ‘h’ and ‘\( \tau_{\text{top}} \)’ is
dimensionally similar to the shear coefficient ($K_s$) in Bourne's equation. The second term on the right hand side of the equation 4.6 is a product of the cross sectional area of the indenter ($\pi b^3$), the constraint factor ($k=2$), and the compressive strength of the foundation ($\sigma_f$). Units of the compressive strength of the foundation are same as those of the compression coefficient ($K_c$) in the equation 4.8. By subsequently dividing both sides of equation 4.6 by the area and the perimeter of the indenter, relationships of the form $y = m \cdot x + c$ can be obtained as shown below in equations 4.9 and 4.10.

\[
\frac{F_{indent}}{2\pi b} = (b)\sigma_f + h\tau_{top}
\]  

\[
\frac{F_{indent}}{\pi b^2} = \left(\frac{2}{b}\right)h\tau_{top} + 2\sigma_f
\]

It should be noted that the constraint factor ($k$) has been assumed a numerical value of 2 (Wen et al. 1998). By plotting the indentation load/indenter area against $2/b$, and the indentation load/indenter perimeter against $b$, values of the shear strength of the top layer (slope in equation 4.9 and intercept in equation 4.10, both divided by ‘h’) and the values of the compressive strength of the foundation (slope in equation 4.10 and intercept in equation 4.9 divided by 2) can be calculated. This is analogous to the approach taken by Bourne and co-workers (Bourne 1966; Bourne 1975). Representative plots for equations 4.9 and 4.10 are shown in Figures 4.15 and 4.16, for gel on gel
composites of 1 cm foundation depth. Values of $\tau_{\text{top}}$ and $\sigma_{\text{f}}$, calculated from the Bourne plots are shown in Table 4.7.

Figure 4.15. Load/area plot for 'gel on gel' composites of 1 cm foundation depth

\[\triangle 2 \text{ mm top layer}\quad \square 4 \text{ mm top layer}\]
Figure 4.16. Load/perimeter plot for 'gel on gel' composites of 1 cm foundation depth

△ 2 mm top layer  □ 4 mm top layer
Table 4.7. Values of the shear strength of the top layer ($\tau_{\text{top}}$) and the compressive strength of the foundation ($\sigma_f$) as obtained by equations 4.8 and 4.9 (values shown in the bracket are R-square values for the plots)

<table>
<thead>
<tr>
<th></th>
<th>$\tau_{\text{top}}$ (Nm$^{-2}$)</th>
<th></th>
<th>$\sigma_f$ (Nm$^{-2}$)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Load/area</td>
<td>Load/perimeter</td>
<td>Load/area</td>
<td>Load/perimeter</td>
</tr>
<tr>
<td>4 mm foundation depth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 mm layer</td>
<td>$77.88 \times 10^3$ (0.879)</td>
<td>$35.42 \times 10^3$ (0.034)</td>
<td>$-4.24 \times 10^3$ (0.879)</td>
<td>$-1.26 \times 10^3$ (0.034)</td>
</tr>
<tr>
<td>4 mm layer</td>
<td>$49.26 \times 10^3$ (0.587)</td>
<td>$19.50 \times 10^3$ (0.114)</td>
<td>$-1.60 \times 10^3$ (0.587)</td>
<td>$7.25 \times 10^3$ (0.114)</td>
</tr>
<tr>
<td>1 cm foundation depth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 mm layer</td>
<td>$14.75 \times 10^3$ (0.985)</td>
<td>$13.79 \times 10^3$ (0.955)</td>
<td>$7.68 \times 10^3$ (0.985)</td>
<td>$8.49 \times 10^3$ (0.955)</td>
</tr>
<tr>
<td>4 mm layer</td>
<td>$20.60 \times 10^3$ (0.997)</td>
<td>$21.23 \times 10^3$ (0.656)</td>
<td>$-2.28 \times 10^3$ (0.997)</td>
<td>$-3.33 \times 10^3$ (0.656)</td>
</tr>
<tr>
<td>2 cm foundation depth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 mm layer</td>
<td>$22.66 \times 10^3$ (0.988)</td>
<td>$21.36 \times 10^3$ (0.423)</td>
<td>$1.05 \times 10^3$ (0.988)</td>
<td>$2.15 \times 10^3$ (0.423)</td>
</tr>
<tr>
<td>4 mm layer</td>
<td>$16.54 \times 10^3$ (0.996)</td>
<td>$17.08 \times 10^3$ (0.001)</td>
<td>$0.97 \times 10^3$ (0.996)</td>
<td>$0.051 \times 10^3$ (0.001)</td>
</tr>
<tr>
<td>4 cm foundation depth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 mm layer</td>
<td>$19.82 \times 10^3$ (0.999)</td>
<td>$19.43 \times 10^3$ (0.931)</td>
<td>$2.47 \times 10^3$ (0.999)</td>
<td>$2.80 \times 10^3$ (0.931)</td>
</tr>
<tr>
<td>4 mm layer</td>
<td>$20.57 \times 10^3$ (0.998)</td>
<td>$20.12 \times 10^3$ (0.035)</td>
<td>$-1.09 \times 10^3$ (0.998)</td>
<td>$-0.33 \times 10^3$ (0.035)</td>
</tr>
<tr>
<td>'Pasta on gel' composites</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 mm deep foundation</td>
<td>$9.94 \times 10^6$ (0.987)</td>
<td>$8.65 \times 10^6$ (0.756)</td>
<td>$4.04 \times 10^6$ (0.987)</td>
<td>$-2.93 \times 10^6$ (0.756)</td>
</tr>
<tr>
<td>1 cm deep foundation</td>
<td>$8.52 \times 10^6$ (0.947)</td>
<td>$7.41 \times 10^6$ (0.724)</td>
<td>$3.70 \times 10^6$ (0.947)</td>
<td>$-2.75 \times 10^6$ (0.724)</td>
</tr>
<tr>
<td>2 cm deep foundation</td>
<td>$7.79 \times 10^6$ (0.984)</td>
<td>$7.12 \times 10^6$ (0.869)</td>
<td>$3.12 \times 10^6$ (0.985)</td>
<td>$-2.54 \times 10^6$ (0.869)</td>
</tr>
<tr>
<td>4 cm deep foundation</td>
<td>$7.79 \times 10^6$ (0.967)</td>
<td>$6.94 \times 10^6$ (0.803)</td>
<td>$3.27 \times 10^6$ (0.967)</td>
<td>$-2.54 \times 10^6$ (0.803)</td>
</tr>
</tbody>
</table>
Table 4.7 shows that the values of the shear strength of the top layer ($\tau_{\text{top}}$) as obtained from the two plots are in good agreement with the exception of 'gel on gel' composites of 4 mm foundation depth. The 4 mm depth of foundation behaves differently as the values of $\tau_{\text{top}}$ as obtained by indentation load/indenter area plot are more than double the values obtained by indentation load/indentation perimeter plot. Values of $\tau_{\text{top}}$ for the other three ‘gel on gel’ composites are in reasonably good agreement with the actual value which is 24.65 kNm$^{-2}$ for a 3% gel. Calculated values of $\tau_{\text{top}}$ for ‘pasta on gel’ composites are however, not in agreement with the actual value which is 1.77 GNm$^{-2}$ for dry pasta and a plausible explanation for this could not be given at this time. Calculated values of compressive strength ($\sigma_f$) from the two plots are generally in good agreement with one another for all composites. However the values of $\sigma_f$ are not in good agreement with actual values which is 9.27 kNm$^{-2}$ for a 1% gel and 57.44 kNm$^{-2}$ for a 3% gel. However more information on stresses, displacements, and failure characteristics of the composite system could help in explaining this part of the discussion.
4.6 CONCLUSIONS

The mechanical behavior of a model layered composite food system was found to be similar to that of an industrial composite. Stiffness and strength parameters for layered food composites of very different properties were modeled by 'plate on elastic foundation model' and 'failure of a sandwich panel model', respectively. The experimental observations were found to be in reasonable agreement with the theoretical predictions and therefore it was concluded that a model composite food system could be analyzed by using theories of elasticity. Further research in this area is desirable to fully develop models for mechanical behavior of real composite food systems.
5. OVERALL DISCUSSION AND CONCLUSION

A fundamental approach can be successfully used to measure those mechanical properties of foods that affect the texture of foods. It was shown that the indentation test which is regarded as an empirical test in the food industry, can reasonably measure the mechanical properties such as the Young’s modulus of elasticity \( E \) and the failure stress \( \sigma_f \) of a model one-component food system. Values obtained for these two mechanical properties \( (E \) and \( \sigma_f) \) were compared with the values obtained by a uniaxial compression test which falls under the category of fundamental tests. Within the limits of experimental error, these values compared reasonably well. Discrepancies could be explained on the basis of experimental conditions and simplifying assumptions. Results obtained in the initial part of the research further confirmed the results on gels reported by Ross and Scanlon (1999). In the second part of this study, theories of elasticity were found to be reasonably applicable to model composite food systems (Jackman and Stanley 1992; Ross and Scanlon 1999; Scanlon and Ross 2000), with the overall mechanical behavior of a model composite food system being reasonably explained by the theories used for stresses and deformations in engineering structures. The stiffness parameter was modeled by the small deflection plate theory whereas the strength parameter was modeled by theories developed for sandwich panels.

Base plate effects due to the effect of sample dimensions, as discussed in section 3.5, have a major role in load-deformation behavior of both homogeneous (chapter 3) and composite (chapter 4) systems. Base plate effects were used to explain the effects of sample depth (or thickness) in both homogeneous and composite systems. Some of the samples, e.g. the 1 cm depth in homogenous system and the 4 mm depth in
composite system, were knowingly made to violate the requirements of an 'elastic half space' at larger indenter radii. The failure loads at larger indenter sizes as experienced by these samples were indeed sizably higher than those experienced by the rest of the samples. Witnessing base plate effects was a result of violation of the 'elastic half space' assumption and therefore was expected.

Indenter size effects were also consistent for both homogenous and composite systems. Smaller indenters showed a better agreement between the experimental data and the theoretical predictions, for each sample depth. For composite systems, it was attributed to the quantitative differences in the values of ratios of 'range of influence' and the indenter radius. The parameter \((R/b)^2\) as mentioned in the section 4.5.2.2 has higher values for smaller indenters for any given combination of top layer thickness and foundation depth. Therefore, smaller indenters would be associated with better efficiencies of load distribution for any given composite dimension. Jackman and Stanley (1992) stated about the presence of a 'zone of influence' around the indenter when fruits and vegetables with skin were subjected to indentation process. The zone of influence \((R)\), as discussed in the field of geotechnical engineering by Terzaghi (1955), was a result of interaction between the top layer and the underlying foundation. Terzaghi (1955) further proposed that the zone of influence could be quantified and numerically was equal to 2.5 times the value of characteristic length \((l)\). Apart from compressive stresses directly underneath the indenter, the zone of influence also carries stresses such as the lateral tensile stresses and shear stresses acting upon the surface of the sample (Jackman and Stanley 1992). If the lateral tensile stresses could be neglected, the force required for indentation can be represented by Bourne's equation,
which accounts for both compressive and shear forces imposed by the indenter. For homogeneous systems, the values of compressive and shear coefficients as obtained by load/area and load/perimeter plots agreed for all sample depths. This shows that Bourne's equation was valid for homogeneous systems. For composite systems, Bourne's equation was slightly modified and presented in a form, dimensionally similar to that of indentation failure model (Equation 6 in chapter 4) proposed by Wen et al. (1998). Compression and shear contributions of the indentation load came from the mechanical properties of the underlying foundation (compressive strength of the foundation) and the overlying top layer (shear strength of the layer) respectively. Again in this case, load/area and load/perimeter plots were made for the purpose of quantification of compression and shear contributions (Table 4.7). These contributions generally agreed with each other for both 'pasta on gel' and 'gel on gel' composites. However, in some cases, the experimentally determined values of compressive strength of foundation and shear strength of top layer did not agree with the theoretical values and reasonable explanations accounting for this could not be postulated at this time. Apart from this discrepancy, the Bourne equation was again shown to have potential validity for the composite systems.

Therefore, a fundamental approach can be adopted while measuring the texture related mechanical properties of foods provided the requirements for certain basic assumptions are fulfilled or corrected for. It was further postulated that despite their non-homogeneous and anisotropic nature, foods could also be treated with the theories that are usually applied to engineering materials.
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