Perceiving

Mathematics Teaching

Through the Eyes of Children

by

M. Patricia Barnson

A Thesis Submitted to the Faculty of Graduate Studies in Partial

Fulfillment of the Requirements for the Degree of

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Abstract

In this study I adopted a hermeneutic research stance to inquire into the nature of learning that occurs when children communicate their mathematical understandings.

I paired learners from grades 3 and 6 who had been identified as being in need of remedial help in mathematics with learners two years their junior, in hopes that communicating about mathematical topics might provide them learning benefits. I provided them with activities to investigate, in hopes that the activities would encourage their communication about mathematics. I collected audio- and video-tape recordings, as well as reflections written by the learners and myself, which I used in my reflection and analysis.

I tell two versions of the story of what occurred. The first telling describes the course of events as they unfolded chronologically. In the second telling, I draw together events that point towards four emergent themes, evident as the study progressed. The themes detailed are titled:

- What does it mean to play the role of mathematics teacher?
- Attitudes towards mistakes;
- What is mathematics?
- Story of the researcher – a description of my thesis journey.

In the concluding chapter, I discuss the potential value of this research within two realms. First, I consider the applicability of the research findings as well as the potential use of this research stance for other practitioners in their own context. Finally, I indicate the relevance this research has in the context within my own teaching practice today

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Table of Contents

List of Figures

Chapter One - Introduction

Knowing where to begin to relate the story of my thesis journey was a hurdle for me. I decided to take the advise of Amy Schwartz - that is - to 'Begin at the Beginning' (1983). Thus, I shall begin by telling of the occasion when I first began to consciously think about the possibilities for learning that surround the communicating of mathematics.

In 1997, I had the pleasure of participating in the Canadian Mathematics Education Study Group conference. Several powerful ideas became apparent from this experience. They inspired the framework upon which this thesis is based.

At the conference, I entered into discussion with a working group that was considering 'Communicating Mathematics'. We began our activities with an invitation to "concentrate on the role of representations - whatever they might be - in the communication of mathematics" (Pirie & Franks, 1997, p.54).

The working group brainstormed for various modes of communicating mathematics. We came up with over fifty. Then the list was divided up. Working in pairs, we were given a mathematical topic on which to base a lesson. We were to present the lesson using only those modes of communication that we were given. This required the taking apart of what I already understood about the topic in order to reorganize it so that it better suited the modes of communication with which we had to work.

It was my observation that those presenting their 'micro' lessons to the group were often gaining more insight or understanding of their topic than were those who were at the receiving end of the communication. It occurred to me that the act of communicating required reorganization of thoughts and ideas in order to make them clear to another. This translation of thought from one mode of representation to another required active thinking on the part of the communicator.

Later, while looking over my notes from the conference, I noticed my working group partner had written: 'write to generate; not to demonstrate - to express understanding or to develop it? -to show it or to grow it?' I wondered if I could set up a situation that would benefit elementary school mathematics learners to grow in understanding through communicating mathematical ideas.

The following pages are a description of my attempt to do just that. I begin by reviewing how other researchers have addressed similar interests. Then I tell of how I paired together learners in need of remedial help with learners two years their junior, in hopes that communicating about mathematical topics might provide them learning benefits.

I tell two versions of the story of what occurred. The first telling describes the course of events as they unfolded chronologically. In the second telling, I draw together events that point towards various emergent themes, evident as the study progressed. These retellings are the result of an interpretive hermeneutic stance -- the framework upon which this research is based.

The final chapter provides examples of possible benefits of the study, as well as limitations to its applicability.

Chapter Two - Review of the Literature

Overview

Recently, there has been much discussion on the role communication plays in learning. The National Council of Teachers of Mathematics (1991) devotes much attention to this topic, as is evident in a detailed discussion concerning classroom discourse. Topics such as 'Teacher's Role in Discourse', 'Students' Role in Discourse', and 'Tools for Enhancing Discourse' are itemized and elaborated.

Discussions of the benefits of various modes of communication frequently look at how learning is enhanced for the recipient of the communication. Such a perspective may describe only half of the story. As Smith puts it,

> I regret the tendency, especially widespread in education, to regard language primarily as ... a vehicle for transmitting information from one person to another.... The perspective completely ignores the possibility that language can be the means of creating worlds and of exploring ideas, our own as well as other people's, that language creates as well as communicates. (1982, p. 67)

It seems to me that learning is enhanced for the person actively engaged in communicating, probably more so than the recipient of the communication. Articulating an idea may require the speaker to reformulate their understanding so that it may be expressed clearly. This seems evident when considering tutorial programs such as the one described by Linder, Leonard-McIntyre, Marshall, & Nchodu (unpublished manuscript). They are researchers, currently working in South Africa, facing the overwhelming task of dealing with huge increases in student enrollment, following the end to apartheid. Many of the new students coming to the university were educated up until now under less than ideal conditions, and require much support so that they might now meet with success in their studies. The researchers set up a tutorial program whereby third year

physics students give tutorials to first year physics students. The tutors are given additional support in this through meetings with the researchers where they reflect on their experience using an extension of Schon's reflective practice.[1] The researchers found that the tutors developed a far richer understanding of the nature of physics through their participation in this program.

Bargh and Schul cite several published studies in which "the tutor showed even greater achievement gains than did the tutee as a result of the program", namely: Allen & Feldman, 1973; Cloward, 1967; Johnson, Sulzer-Azaroff, & Maass, 1977; Morgan & Toy, 1970; and Richer, 1973. (1980, p. 593).

Pimm describes a common experience reported by teachers, where a learner comes to understand an idea for him/herself merely by articulating to the teacher the difficulty that he/she is experiencing. He concludes that "the act of attempting to express their thoughts aloud in words has helped pupils to clarify and organize the thoughts themselves" (1987, p. 23).

In light of all this, I will review the literature surrounding learning benefits experienced by the communicator while communicating mathematics. I will begin with a discussion of the common paradigm underlying research in this area. Then I will turn my discussion to research that describes how language use (speaking and writing) seems to affect the development of understanding. I will consider the role collaboration plays in enhancing understanding, and discuss arguments made for expert versus peer collaboration. I will look at the possibility of how modes of communication other than language may also serve to enhance learning for the communicator. Then I will turn to a discussion of practical implications of this research for the classroom. Finally, I will consider the extent to which the literature addresses the value of communication as a process for learning mathematics.

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[1] Schon, Donald A. (1983). *The Reflective Practitioner: How Professionals Think in Action*. New York: Basic Books.

Underlying Paradigm

The prevailing paradigm behind the research is constructivism (Fullerton, 1993; Holens, 1996; Leikin & Zaslavsky, 1997; Wood, Cobb & Yackel, 1989), with various subtle differences in emphasis. The learner is deemed as the "meaning maker"(Bishop, 1985; Wells, 1986), and learning is viewed as "a process of theory refinement"(Stoessinger & Edmunds, 1990; Smith, 1982). Cazden(1988) draws upon the work of Leont'ev, Piaget, and Perret-Clermont to describe the active construction on the part of the learner, together with the role social interaction plays in provoking cognitive conflict. This in turn stimulates cognitive elaboration. In Clarke's study, the recommended instructional approaches for the program also, "were derived from 'social constructivist' notions about the learning process" (Clarke, 1997, p. 282). Likely acting on this paradigm, an interrelationship between language and thinking forms the basis of many theories (Halliday, 1978; Lindfors, 1991; Smith, 1982; Tough, 1976; Wells, 1986).

Speaking, Writing and Learning

Bargh and Schul cite several studies which demonstrate that overt verbalization can enhance performance in memory as well as problem solving tasks (so long as time constraints are not an issue in the latter)(1980, p.595). Leikin and Zaslavsky point out: "Glasser(1991) suggests that the quality of students' elaboration and self-explanations as they work through a problem play a critical role in their acquisition of knowledge"(1997, p.334). In this study the mentors were encouraged to express their processes within mathematical tasks.

Vygotsky studied "egocentric speech"(speech for oneself - not relying on a listener), and found it to occur when "an impediment or disturbance in an automatic activity makes the actor aware of that activity"(1962, p.16). For Vygotsky, egocentric speech "serves mental orientation,

conscious understanding; it helps in overcoming difficulties; it is speech for oneself, intimately and usefully connected with the child's thinking"(p.133).

Although Vygotsky writes that egocentric speech disappears by school age, Fullerton(1993) often observed it in students from grades four through six when encountering difficulties in independent activities. Pimm makes the point that when he is engaged in performing an intricate computation, he tends to talk himself through it. He notes that "social and school conventions which militate against such vocalizations (eg. working silently so as not to disturb a neighbour)" may stifle this natural and very useful tool in self-monitoring of mental activity (1987, pp.24-25). This argument makes me wonder what an examination room should sound like.

Several researchers describe the benefit that language has for externalizing thought, thus making it accessible to the learner for reflection (Cazden, 1988; Pimm, 1987; Waters & Montgomery, 1990; Wells, 1986). It is at this point where my discussion perhaps crosses the path of discussions related to meta-learning and its benefits (Preece, 1995).

Smith describes how reflection furthers learning:

> Language permits thought to fold back on itself; the product of thought itself becomes an object that thought can operate upon, ... and thereby provide a basis for new or modified ideas. We can contemplate a statement we have ourselves made ... and go forward to make further statements that would not have been possible if the original statement had not been contemplated (1982, p.65).

Lindfors also acknowledges this aspect of language, suggesting it allows the learner to act on ideas as one would shape and reshape clay(1991, p.9).

Fullerton, in her discussion of Britton's research (1982) on the force which he entitles 'Shaping at the point of utterance', explains: "His contention is that this 'force' or energy virtually enables learners to learn while speaking. In other words, we give 'shape' to and deepen our understandings as we give voice to our thoughts"(1993, p.16). This idea of Britton's is similar to

what Waters and Montgomery(1990) term the "generative nature of writing", and is related to the supposition that in order to use language (spoken or written), a synthesis of ideas must be done (Bruner, 1968; Small, 1990).

Cazden compares the work of several researchers on the generative nature of spoken language as follows: "Speaking 'without the answers fully intact' is what British educator Douglas Barnes calls 'exploratory talk', in contrast to a 'final draft'(Duckworth,1981,51; Barnes,1976)" (1988, p.133). A composite of the ideas expressed on this issue by researchers MacGregor and Pimm would explain the issue by saying, "Concepts that were previously partly formed at a non-verbal level"(MacGregor, 1990), once expressed, "however haltingly and incompletely" (Pimm, 1987), can be explored, extended, and refined for greater precision and clarity.

Much discussion has evolved surrounding the relative merits and shortcomings of written versus oral language. Written language provides a more permanent, and therefore more tangible record of thinking for later reflection (McIntosh, 1991; Pimm, 1987). It may force deeper contemplation and more accurate expression of ideas (Buerk, 1985; Pimm, 1987). It allows for more active participation in construction of knowledge for all learners simultaneously (McIntosh, 1991; Mett, 1989). However, many researchers have noted that part of the power of mathematics is in its abbreviated symbolic system. This system allows a single symbol to represent a complex idea, so that one might more easily build upon these complex ideas without having to elaborate the details of the basic elements of the argument. Pimm puts it this way:

> when caught up in a rush of ideas, having to record can act as a serious brake .... it should be borne in mind that considerable mental effort is involved in writing. It is possible to lose sight of what was trying to be expressed because the problems of writing itself takes up all of conscious attention. Expression through the written medium can be both time-consuming and arduous. (1987, pp. 117-118)

In her research with journal writing in high school mathematics classes, Holens noted that responses to mathematics writing activities were generally more positive in the higher level classes. She conjectured:

> students with greater cognitive processing capabilities find journal writing to be both easier to handle and of more benefit to them as learners. Putting one's thoughts into words requires a great deal of cognitive effort, and may thus be very frustrating to students operating from a lower cognitive processing level. (1996, p.21)

Yet Pengelly found great success with writing in mathematics class with younger students. This may be because students only recorded what they chose as being useful to them. "Very little recording was expected during the initial stage....In order to keep track they [the learners, themselves] found it useful to keep records"(1990, p.17). Here we find learners naturally being drawn to a mode of communication that acts as a tool to help organize and enables them to reflect on their own thinking.

Collaboration and the Role of Audience

When I first started thinking about this topic, my ideas were not well formed and were poorly articulated. As I discussed the topic with family, friends, and colleagues, even in cases where the person I was talking to did not speak or add to my ideas, I became aware that they were developing more fully in the course of discussion. As I worked at putting my ideas on paper, I found that I was further reorganizing and even elaborating on my thoughts. Other researchers have described this evolution of their own thought processes in the course of discussion and writing(Clarke, 1997; Cobb, Boufi, McClain & Whitenack, 1997; Smith, 1982). For instance, in Clarke's case-study research considering two teachers' use of innovative mathematics materials at the grade six level, he notes that "it seemed that both teachers valued the opportunity to reflect on

classroom events in conversation with a third party ... and that providing a "sounding board" was an important form of support to them"(p.299). It seems obvious that this form of support would benefit all learners, students included.

The subject of mathematics seems particularly suited to this type of "collaborative building up of knowledge". Lakatos speaks of axioms and definitions being "proof generated"(1976, p.144). Lampert explains this notion. She says that

> [t]he mathematician figures out what he or she is talking about in the process of trying to talk about it, not beforehand by some magical intuition. Setting out to prove something, the practitioner sees that the original terms of the arguments were unclear and usually even changes what it was that was being asserted in the first place....
> Once a conjecture is made, the practitioner sets out to prove it and in doing so becomes clearer about the assumptions that led to the conjecture in the first place (1992, p.306 & p.308).

Notice here the role taken by the rest of the mathematical community - as an audience that must be convinced of the validity of the argument. Indeed, "Nothing encourages one to talk so much as the presence of an interested listener"(Lindfors, 1991).

Collaborative discourse also provides the possibility for learners to "pool their ideas"(Lindfors, 1991), "hitch-hike on what others have said"(Booth & Thornley-Hall, 1991), or "borrow ideas and language from each other"(Brissenden, 1988). Collaborative discourse at its best demands that students learn to "consider and respond to alternative perspectives"(Fullerton, 1993, p.139), and compare one's own ideas with those of others(Lindfors, 1991). Vygotsky argues: "our schema of development ... contrasts both with traditional behaviorist schema...and with Piaget's.... In our conception, the true direction of the development of thinking is not from the individual to the socialized, but from the social to the individual" (1962, pp.19-20). This study did not adopt this one-direction conception of learning through communication.

Fullerton cites the work of Hoyles(1985), who "conjectures that it is in the 'silences' of discussion that real learning takes place"(1993, p.23-24). Pirie also points out the need for "quiet thoughtful moments", citing "Ericsson's(1975) findings that 'think aloud' subjects tend to plan their moves or think ahead less than the silent control groups"(1991, p.159). Furthermore, Pirie notes that in mathematics,

> [t]he power of the subject lies in the ability to move forward without recourse to explanations and to automate intermediate stages of process. In the data processing model proposed by Ericsson and Simon(1980), 'verbal explanation of automated activities' are seen as 'cumbersome and would change the course of the processing from a largely perceptual to a more cognitive one'.
> ...Thus pupils verbalizing mathematical thinking may be found apparently to form their possibly erroneous mathematical beliefs by the force of their own explanatory words"(pp.159-160).

Here, once again it seems, a balance is called for. Collaborative discussion of ideas may be useful at times, but should not be done to the exclusion of everything else. Quiet time for individual reflection of ideas is also a necessary part of learning.

The Teacher-Expert in Peer Communication

"Because there is a limited amount of whole class 'floor time' in any school day, teachers need to devise other means to encourage all children to talk"(Fullerton, 1995, p.16). For some, this is a call for peer collaboration in small group work(Webb, 1991). Others are not so sure. Pimm contends that "merely increasing the amount of pupil discourse in mathematics classrooms may not, by itself, prove beneficial"(1987, p.42). Fullerton(1993) illustrates how learners can, in fact, lead each other to false conclusions if left unchecked. Partly for this reason, many writers recommend dialogue be with an "expert"(Reeves, 1990) or "another more knowledgeable person"(Wells, 1986). Furthermore, Cambourne points out that a person whose motive for

listening to the learner is to support the learner's growth in understanding may be able to identify weaknesses and lead interactions towards:

- focussing - on a gap which the learner needs;
- extending - challenging or raising the ante;
- refocussing - encouraging clarification;
- redirecting - offering new information if there is a mismatch between the learner's intent and the message or in the teacher's expectations of the capacity of the learner. (1988, p.117)

This sort of guided learning, building and extending on what the learner knows, is termed "scaffolding"(Bruner, 1968; Bickmore-Brand & Gauned, 1990; Duckworth, 1981; Cazden, 1988). Wells cites the work of Vygotsky, who

> refers to this sort of collaborative exploration at the limits of the child's ability as working in 'the zone of proximal development'.... What the child is able to do today in conversation with a supportive adult, he or she will tomorrow be able to manage alone"(1986, p.112).

Alternate Modes of Communication

As mentioned earlier, little has been written on the benefits of communicating through modes other than language, when it comes to the research on the pedagogy of mathematics. Some mention is made of "manipulative materials as communication devices(Lowenthal)"(Clement, 1991; Fullerton, 1993). Perhaps we are just so used to using language, we have overlooked the many other possible modes for communication that are available to learners. My conjecture is that every time the same idea is 'translated' to a different mode, understanding of that idea is given greater depth. This phenomenon is suggested by what we have already discussed concerning translation from writing to speaking, and vice versa. Bargh and Schul explain the cognitive benefits of teaching this way:

> Perhaps more important, the teacher must explain the troublesome content in a different manner from its initial presentation. This allows the teacher to see the issue from new perspectives, enabling him or her to see previously unthought of relationships between the discrete elements. It may be this building of new relationships that facilitates a better fundamental grasp of the material.(1980, p.595)

In retrospect, these considerations became especially relevant for this study with those participants who were reluctant to verbalize their cognition.

Implications for the Classroom

Several recommendations based on the ideas elaborated above seem relevant for classrooms. First, there is "a considerable imbalance" when comparing the amount of time the teacher speaks to that of the learners in a typical classroom. For example, Wells found "three teacher utterances on average for every one by the child"(1986, p.87). It seems clear that teachers need to encourage students to take the lead in discussion, providing "scaffolding-type" support only as needed. Furthermore, the teacher needs to establish in the classroom an atmosphere of mutual trust "where both teacher and student are free to wonder out loud"(Berk, 1985, p.69; see also Lampert, 1992; Waters & Montgomery, 1990; Yakel, Cobb, Wood, Wheatley & Merkel, 1989). Teachers should model "conversation repair" by asking clarifying questions when the meaning of a statement is unclear, and encourage the students to do the same(Bickmore-Brand & Gawned, 1990, p.53). According to Wells(1986), large group discussion may be less than adequate and one-to-one or small group situations with the teacher should be planned as often as possible.

One of the teachers in Clarke's study mused that he used to try to make lessons as straightforward as possible, by anticipating potential difficulties, then either removing them or offering procedures to overcome the difficulties before the learners began the task. This he comes to view as "short-circuiting" their thinking(1997 p.287). In fact, errors can be a wonderful

springboard for learning. Fullerton cites the work of Stigler (1988), and Stigler, Lee & Stevenson (1987) to describe how errors may be used to enable teaching and learning.

> A method used regularly by Japanese teachers was to have students who had made errors explain those mistakes to their peers through blackboard use. Class discussion and correction of the errors followed. In contrast, American teachers concentrated on correct responses simply asking students who had correct responses to raise their hands.... As Stigler points out, more discussion can be generated around errors than around correct responses. (1993, p.33)

A teacher who models desired behaviors is more successful in influencing a learner's behavior. It would seem reasonable, then for the teacher to become a "fellow player" in the development of the "mathematical discourse community" of the classroom,(Clarke, 1997, p.280). Thus, when the teacher asks questions, "it is in order to be further informed, not to check that the child's answer is in conformity with her knowledge about the topic"(Wells, 1986, p.113). At the same time, the teacher cannot usurp the responsibility that comes with his/her expertise. Rather, he/she must "initiate shifts in the discourse by capitalizing on the students' mathematical contributions"(Cobb, Boufi, McClain, & Whitenack, 1997).

Teachers need to become active listeners, to try to minimize the number of "lost opportunities", a term use by Fullerton to describe when teachers miss occasions to expand children's understanding of concepts. "Sometimes it was a matter of the teacher not responding to a child's question. Sometimes it was a matter of the teacher having a specific goal in mind and being unwilling to pursue different avenues suggested by the children's responses and conversations"(1993, p.66).

Davis points out the importance of the role of the listener in communication with the following:

> Just as a sounding board in a musical instrument is intended to resonate with, echo and amplify the sounds generated by other parts of the instrument - that is, the sounding board

> participates in the music generated - so the listener who acts as a sounding board participates in the emerging conversation (1996, p.38).

Types of Listening

Davis identifies three different types of listening that a teacher might engage in, depending on the paradigm he/she is acting from, be it conscious or subconscious. "Evaluative listening" involves listening for a preconceived response, rather that listening to the speaker. For the evaluative listener,

> communication is about speaking....Listening is thus understood uncritically as a largely passive attending to the verbal utterances of others. It is the sort of listening that tends to forget its own responsibility in interactions, often blaming miscommunication on the speaker. (1997, p.360)

"Interpretive listening" arises from constructivist notions. It involves endeavoring to make sense of the sense being made by the speaker, but still involves looking for particular responses (Davis, 1997, p.363).

"Hermeneutic Listening [is] a title intended to reflect the negotiated and participatory nature of this manner of interacting with learners"(Davis, 1997, p.369). It is by this type of listening whereby a teacher might genuinely enter into conversation with a learner, and mutually advance their understandings of the topic. For Davis, this orientation to listening arises from an enactivist framework,

> starting with the evolutionary metaphors of Darwin rather than the analytic model of Descartes - a shift that might be characterized in terms of embracing the complexity of tangled relationships and problemetizing the divisive and reductionist tendancies of modern thought. This focus is thus on the dynamic interdependence of agent and setting, thought and action, knowledge and knower, self and other, individual and collective, rather than on autonomous constitution or construction. (1997, p.370)

Davis' conceptions of listening provided more of a research stance than a method to be followed in this study.

Reflecting on the Literature

After this review of the literature, I am further convinced that the act of communicating mathematical ideas may prove to have powerful implications for learning. Yet, it is frequently overlooked by researchers. For example, when I came across the writing of Reeves, I was delighted to read:

> talk plays two vital roles.... The second is to generate creative thinking in individuals, to encourage different ideas and problem solving skills for intellectual development and personal satisfaction. It is this aspect that prepares children for the abstract nature of the discipline (1990, p.94).

I believed this to be directly referring to my concerns, until I read further and found that Reeves was describing teacher talk, not learner talk. Later, I was gratified to read the results of Webb's study:

> Even controlling for ability, giving content-related explanations was positively related to achievement....receiving content-related explanations did not seem to be beneficial to achievement.... Receiving help other than content-related explanations was either negatively or not related to achievement. (1991, pp.372-374)

Unfortunately, when it came to the "Discussion and Conclusions" portion of her paper, Webb all but ignored the significance of her findings concerning enhanced learning on the part of the explainer. The bulk of this section proposed ideas for training students to give "better explanations" to their peers; in other words, to be better able to transmit their own knowledge to others. From my perspective, this analysis seems to have completely ignored the learning benefits associated with the act of explaining, which her study had indicated so well.

Given all that I have read, it is clear that two themes at least emerge that have relevance for my study. The first is that in the learning of mathematics, children need to talk. The second is that teachers of mathematics and mathematics researchers need to listen to what children say as they make sense of ideas in mathematics. So, then, this gives rise to a critical question that guides the remainder of this study and my intellectual journey. **What is the nature of learning that occurs when children communicate their mathematical understandings?**

Chapter Three - The Research Process

Research Methodology

In my search for a legitimate methodology for my research, I have found my own greatest growth. I needed to critically analyze my own implicit assumptions, and develop a personal philosophical framework. This framework would act as a foundation upon which I could build a research methodology that is not only in line with and seen as legitimate by the research community, but is also in line with my own espoused values and beliefs. The personal story of how I made this journey is delivered in Chapter five. The results of that journey, together with reference to the literature that would indicate support for this methodology, are what follows here.

This study is based on a hermeneutic research stance. Davis explains:

> Hermeneutic listening demands the willingness to interrogate the taken for granted and the prejudices that frame our perceptions and actions. Further, drawing from the traditions of hermeneutics - a field of inquiry that seeks both to expose our enacted assumptions and to investigate the social and historical conditions that make those assumptions possible - the notion of hermeneutic listening is intended to imply an attentiveness to the historical and contextual situations of one's actions and interactions.
> (1997, p.370)

A hermeneutic methodology then, would require a thoughtful and rigorous interpretive understanding of any text, human action or human experience which is mindful of the researcher's own role in how those events unfolded. The subjectivity of the researcher's understanding is not to be denied, but considered as an integral part of that which is being studied. For Davis,

> the very "object" of our inquiry - changes ... in part because of our efforts to understand it.... Hans-Georg Gadamer has provided us with a provocative exploration of this issue, arguing that the relationship between the research question and the phenomenon under study is not uni-directional, but reciprocal. Briefly, his suggestion is that the topic of investigation, at least in part, reveals the manner in which it should be investigated.... In the hermeneutic investigation, therefore, the questions are never fixed. Indeed, the

> hermeneutic question might better be thought of as an issue or a topic of wonder. It is an entry point for excavation, not an arrow for answer seeking (1996, pp.22-25).

This research stance seems to be particularly well suited to someone in the position of researcher/teacher. Wong(1995) speaks of a "conflict in purpose and conduct", when faced with, what he considers to be, the dual role of researcher/teacher. I found myself having to contend with similar problems at the outset of the study, as I struggled with the issue of objective versus subjective knowledge. Wilson(1995), offers a resolution to the apparent conflict in roles in her rejoinder to Wong's article. She notes the necessary and appropriate place of the researcher, within the context of the learning process under study.

In keeping with this philosophical foundation, I present my research in narrative form, attempting to honour all the complexity of context which accompanies the events as they unfolded. By the stories therein, it is my intention not so much to present some objective truth but, in the words of van Manen, to invite you "to insert [y]our own experiences in a reflective dialogue. That is, [I] invite you, the reader, to collaborate in the construction of [an] analysis" (1979, p.12).

Intervention Design - An Overview

The following is a description of my initial research design. That is, I describe the plan I developed prior to commencing the study, which reflects the results of negotiation between the school, the Ethics Review Committee, and my own early understandings of the possibilities associated with communicating mathematics.

I decided to set up a study which would look at how the communication of mathematical ideas might enhance a child's own understanding about mathematics. The study would investigate what happens when an older child - a "mentor" - engages a younger child - a "buddy" - in mathematical activity.

Students from grades three and six, who were identified by their teachers as being in need of remedial help in mathematics, were invited to act as mentors for younger students. Willing mentors were paired with buddies two grades younger, who were identified by their teachers as being willing learners. Three mentor-buddy pairs were established at each grade level. My intention was to discover evidence that would indicate that the mentors experienced a learning gain by their interactions with their buddies.

I provided the mentors with activities to investigate with their buddies. These activities were intended to encourage communication in mathematics between mentor and buddy.

Mentor-buddy pairs met immediately following the lunch hour on Mondays, Wednesdays and Fridays for approximately twenty minutes. Mentors remained for an additional ten minutes following these sessions to do some reflective writing, and to get direction on what they would do in the following session.

The mentor buddy pairs met with each other eleven times during the course of the study, which lasted from mid-March through May, 1998. In addition to the 'debriefing period' following mentor-buddy sessions, the mentors also met with me on separate occasions for further direction and planning of what they would do with their buddies. The grade six mentors did this four times; the grade three mentors did this twice. Please refer to Table 1 for a chronological listing of participants and activities.

Notice that not all mentor-buddy pairs met every session, and that some of the activities involved only one group. For example, 'Smath was played only by the grade 6/4 pairs. The names in the chart indicate which mentors attended a session, and an asterisk (*) beside their name indicates that their buddy attended as well. The column on the right indicates the activity they engaged in during the course of the session.

Figure 1: TABLE 1

session #	Date	Gr 6 Mentors *=with buddy	Gr 3 Mentors *=with buddy	Activity
1	Fri, Mar 13	Bill Lori Mary		Introduction to Study; & 'Smath
2	Mon, Mar 16	Bill* Lori* Mary*		'Smath
3	Wed, Mar 18	Bill* Lori* Mary*		'Smath
4	Fri, Mar 20	Bill* Lori* Mary*		'Smath
5	Mon, Mar 23	Bill* Lori* Mary*		'Smath
6	Wed, Apr 8	Bill Lori Mary	Anne Debbie Jenny	Intro to study (Gr 3); Boxes(Gr 3&6)
7	Wed, Apr 15	Bill* Lori* Mary*	Anne* Debbie* Jenny*	Boxes
8	Fri, Apr 17	Bill Mary	Anne Debbie Jenny	Boxes
9	Mon, Apr 20	Bill* Mary*	Anne* Debbie* Jenny*	Boxes
10	Wed, Apr 22	Bill* Mary*	Anne* Debbie* Jenny*	Boxes
11	Fri, Apr 24	Lori*		Boxes - begin
		Bill* Mary*	Anne* Debbie* Jenny*	Boxes
12	Mon, Apr 27		Anne* Debbie* Jenny*	Boxes
13	Fri, May 1	Bill* Lori* Mary*	Anne* Debbie* Jenny*	Boxes

session #	Date	Gr 6 Mentors *=with buddy	Gr 3 Mentors *=with buddy	Activity
14	Mon, May 4	Mary*	Anne* Debbie* Jenny*	Boxes
15	Fri, May 8		Anne* Debbie* Jenny*	Snap It
		Lori*		Boxes
		Bill Lori Mary		Patterns
16	Mon, May 11	Bill* Lori* Mary*		Patterns
17	Wed, May 13	Bill* Lori* Mary*		Patterns
			Anne* Debbie* Jenny*	String Bean
18	Wed, May 20		Anne**	intro Snap It & String Bean
			Debbie* Jenny*	cont Snap It & String Bean
19	Fri, May 22		Anne* Debbie* Jenny*	Problem-Solving Activities
20	Wed, May 27		Anne Debbie Jenny	Boxes

Data Collection

I needed to keep a record to help support my documentation of events as they occurred over the course of the study. I hoped to use this record for reflection and analysis.

I audio taped my initial sessions with the mentors (sessions 1 & 6). I carried a video camera to record interactions between mentors and their buddies during sessions 2, 3, 4, and 5. I put the video recorder on a tripod so that I might also interact with the learners during session 7. I decided to stop video recordings of sessions after session 7.

I carried an audio tape recorder with me, recording my interactions with mentors and buddies from session 7 on. A sampling of conversations between mentors and their buddies also was captured with a second audio tape recorder.

Reflective writings by the mentors following each session with their buddies and my own reflective writings following each session supplement the taped conversations.

Intervention Design - The Five Activities

I gathered together a set of mathematical activities which I believed had the potential of encouraging communication based on each participant's perceptions or understandings of mathematical ideas.

In the following section, I will describe these activities as they were planned in the Intervention Design. As you read, imagine a grade 6 mentor leading a grade four buddy, or a grade three mentor leading a grade one buddy through the activity. Consider the potential opportunities for communicating of mathematical ideas between mentor and buddy, and the possibility of a joint elaboration of ideas.

I plan to describe the activities as they were originally intended here, then detail how events actually unfolded in the next section.

It may be helpful to bookmark this section, as these activities will be referred to, but not necessarily detailed again, at various points throughout the document.

'Smath

'Smath (a commercially produced game by Irwin) is a scrabble-type game that requires players to build mathematical equations in a crossword-style fashion on a game board. Two examples generated by children in the study are "$2 \times 1 = 0 + 2$" and "$5 + 4 - 1 = 8$" The game

begins by each player randomly drawing ten playing pieces from a bag. Each playing piece has either a single digit numeral, or an operation symbol to add, subtract, multiply or divide. Equal signs and brackets are put to one side, and may be used freely as required. The first player to play uses as many of the game pieces as is possible to build an equation that passes through the center square of the playing board. At the end of each turn, the equation as well as the score for that equation are recorded,(see Appendix page 76 for a sample score sheet) and the player draws new playing pieces to replace the ones that were used. Then, each player on any given turn might choose to build a new equation passing through and using a piece from one of the equations that is already on the board. Otherwise, the player might add to an equation already there so that the equation is more elaborate, but continues to be balanced on either side of the equals sign.

I anticipated that this game would elicit conversation or discussion concerning the meaning of the various operations, and/or learners' operational understandings of the commutative, associative, or distributive properties or the special/identity properties associated with 1 and zero.

Boxes

Boxes (Mason, 1997) involves learners using small wooden uniform-size cubes to build rectangular prisms of any dimension of their choosing. They represent their 3-dimensional creation on paper, so that another learner might reproduce it. They analyze the particular characteristics of their own creation, and compare/contrast that to boxes created by the other learners.

Mentors were to introduce their buddies to the idea of 'box', by guiding them in building a box of dimension 2×2×2, and then analyzing and documenting the various ways this box might be described to someone: considering the number of blocks that went in to making it; describing what each face looks like; making stamp prints or rubbings of its faces. They were to have their buddy colour each face of the box a different colour, then allow them to take the box apart and try to

reconstruct it. Buddies would then be allowed to take the box home with them at the end of the introductory session.

In the following session, each mentor and buddy pair would work as a team in creating and describing a box of dimensions of their own choosing. They would draw face views or corner views of the box using grid paper or dot paper, then conceal these creations in an ice cream pail, and leave them with a paper that would describe the box to others. (See Appendix page 77 for a sample of a box record sheet.)

Finally, each mentor and buddy pair would work together to build the boxes created by other teams, and then reflect on those attributes of boxes which are common to all boxes, and those attributes which would be specific to a particular box.

I intended that these activities would encourage conversation or discussion concerning: what to name the various attributes of the boxes so that common understandings for communication purposes could be achieved; what attributes are common to all boxes; how does change in one attribute affect another; what does one need to know about a box in order to have enough information to build it; how does one represent 3-dimensional objects in a 2-dimensional form?

Snap It and String Bean

Snap It and **String Bean** (Barone & Taylor, 1996) are activities which focus on sums of 10.

In Snap It, the mentor holds ten linked connecting cubes up for the buddy to see, then hides them behind his/her back. The buddy says "snap it", whereupon the mentor breaks the string of cubes apart and holds one part up for the buddy to see. "How many cubes are in my hand?"

Then, "how many cubes are behind my back?" Together, they record a number sentence that describes what they did, then change roles and play again.

In String Bean, a string is laid out straight on the playing surface. Ten beans are dropped close to the string. The number of beans on each side of the string is counted. Then a number sentence to describe the situation is recorded.

I expected that these activities would provoke conversation or discussion concerning the relationship between addition and subtraction.

Patterns

Patterns are activities in which learners set up patterns, and recognize or continue patterns that have been set up by others.

In Patterns I, the mentor sets up a pattern in a string of objects, then challenges the buddy to continue that pattern. They record the pattern, and then they repeat the activity with roles reversed.

In Patterns II, mentor-buddy pairs work as a team to discover, describe and extend patterns they find in various puzzle activity sheets. (See appendix pages 78 - 80 for sample activity sheets.)

I hoped that these activities would encourage conversation or discussion concerning the nature of pattern, the possibility of more than one evolving pattern, and ways in which objects or number might be patterned.

Problem Solving Activities

Problem Solving with 3 Addends and **Problem-solving Puzzles** (Barone & Taylor, 1996) are puzzle-type activities.

In Problem solving with 3 addends, mentor-buddy pairs work together to arrange the numbers 1 through 6 on a triangle, so that the sum of the numbers on each side of the triangle add up to 9. This activity could be repeated to solve for sums of 10, 11, and 12 (see Appendix page 81 for a sample game board).

In Problem-solving puzzles, mentor-buddy pairs work together to connect the boxes on the game board so that all the dots are divided equally into groups of 2, 3, 4, or 5. (See Appendix page 81 for sample game boards.)

I assumed that these activities would suggest conversation or discussion concerning strategies for solving the puzzles.

Chapter Four - What Took Place

Stories from the Study

I find it difficult to write the story of what took place during the course of this study in a linear fashion. The unfolding of events was more like the weaving of a tapestry.

The purpose of this section is two-fold. Firstly, I plan to narrate the events as they unfolded chronologically during the course of the study. In essence, this will be an expanded version of Table 1. Think of this as the vertical threads in the fabric. Secondly, I wish to call attention to a series of themes that developed as the study progressed. These themes were not intended, nor anticipated at the outset, but emerged as the study progressed. Think of this as being the horizontal threads in the fabric - those threads which give the fabric vibrancy, pattern, and colour. These themes will be flagged in this section, and then expanded upon later.

I have separated the narration into sections, centering around the five activities: 'Smath; Boxes; Snap It and String Bean; Patterns; and Problem Solving Activities; provided by me for the mentors to engage in with their buddies

I begin each section with a general description of what took place at each session. Then, through the use of particular examples, I point to themes that begin to emerge. As you read, you might consider some of the possible interpretations for the events as described. In a later section, I will revisit the ideas and describe my own interpretaion and analysis of the themes.

Introduction to the Study / 'Smath

The following is a description of how I introduced the grade six mentors to the study, and introduced them to the activity 'Smath. Then I describe how they led their buddies in playing the game of 'Smath.

I began by introducing the grade six mentors to the study. I told them when and how often we would be meeting. I drew attention to the use of the tape recorder and video recorder as a way for me to collect data. I described the reflective writing I would like them to do following each session they had with their buddies. I suggested a variety of ways they could approach me if they had any questions or concerns that they wanted to share with me. We determined which buddy would work with each mentor. I described broadly the types of things they would be doing with their buddies. I asked them to suggest what types of things they might do to help their buddies understand or learn. I introduced the activity 'Smath to them, and we practiced playing a round of 'Smath. I addressed any questions or concerns as they arose during the course of the session. Finally, I asked them to do some reflective writing on how they were feeling about what I was asking them to do for the study.

It was interesting to note the individual perceptions each mentor brought with them to the table during our discussions about what types of things they might do and not do in order to help their buddies learn or understand. One mentor, Lori, suggested that she would "teach". When I asked her what she meant by 'teach', she elaborated: "Show them how to do stuff". Another mentor, Bill, added "give instructions" to the list, and Lori continued with "examples" and "pictures". When I asked the mentors if they learn better by watching the teacher do something, or by doing it themselves, they unanimously agreed it is better "if you get a chance to do it [yourself]". When I asked them if there is anything a teacher does while the learner is trying something, the conversation continued as follows:

Bill: Well, if you make a mistake, she might help you?

Me: Okay. How might the teacher help you?

Bill: If you're doing a Math problem, and you make a mistake, after you're done she will, uh he or she will point out the mistake.

Me: Point out a mistake, Okay.

Lori: Or when you're doing long division or something like that, she'll say 'Can anyone help them figure out their mistake?'

Later, I directed them with: "The idea is to get the buddy to talk as much as possible and you talk as much as possible. Try to talk about the Math part of whatever you're doing. Try to connect things that they've learned before to what you're doing now." Lori's immediate response to these instructions was "Do we get a paper that we could fill out what they're having trouble on while we're doing that?" 'What does it mean to play the role of a teacher' became a recurring theme in the study.

During the introduction to the game of SMATH, when it came to his turn, Bill placed the playing pieces down: 0-9=9. When Lori questioned: "Zero minus nine is nine?", he responded "It's nine". She suggested: "Nine minus zero . . ." and he responded: "Whatever, it's the same". When I asked him to think about it again, he quickly rearranged the playing pieces in his equation, and was able to give an explanation of why the revised equation made more sense when prompted to do so using counting blocks. It is interesting to note that Bill persisted in his initial response when a peer questioned his equation, and only 'reconsidered' the equation after I had voiced a need to think again. Furthermore, I'm now not convinced that his parroting of why it would make more sense really made 'more sense' to him. This type of error arose a few times during the study, along with an explanation by me to 'clear everything up'. I will henceforth refer to these incidents as 'authority declarations to correct mistakes', be they declared by myself or parroted by a learner when prompted to do so.

Another thread that weaves itself to this one began to emerge at this point - 'the use of manipulatives in teaching and learning' also becomes a recurring theme in the study.

Finally, one other theme emerged during the course of this session. Lori in particular expressed concern that she might make a mistake in front of her buddy. We spent some time discussing this issue. It was interesting to note the various ways in which the mentors thought about the possibility of making a mistake, as well as how they might handle the situation. When I asked what positive effects making a mistake would have on the buddy, the conversation developed as follows:

Bill: It doesn't make the student think that the person, the teacher is, like, perfect?

Me: Yah, everyone makes mistakes, and it actually makes it easier on a student if they know that it's safe to make mistakes.

Bill: Make a mistake on purpose.

Me: I don't think it's necessary to make mistakes on purpose ....

Lori: Okay, um, like, sorta like if I say 'I'm gonna make a couple of mistakes. You tell me what the mistakes are.'

In the following session, the grade six mentors played SMATH with their buddies. I videotaped this session. They sat opposite one another, and played the games with very little communicative interaction. It became evident that the competitive nature of the game impeded rather than promoted communication between the players. I attempted to remedy this situation when I met with the mentors following this session. I directed the mentors to henceforth play the game cooperatively with their buddy. I suggested they could do this by working as a team to try to best their own past performance, or perhaps achieve, as a team, the highest score when later compared to the other mentor/buddy pairs.

'Authority declarations to correct mistakes' and 'the use of manipulatives in teaching /learning' themes came into play again during this session. In my reflection following the session, I had written:

> I noticed two of the groups misinterpret the ÷ sign, saying 'divided into', and therefore switching the position of divisor and dividend. I explained the symbol actually is read 'divided by' at the end of the session, individually to each group, including the group that had not made the mistake - in case there was any confusion. I brought out counters to show how the first number might be divided into piles (second number) and the number in each pile would represent the answer.
>
> As I looked over the score sheets, I see several other arithmetic errors. I'm going to try to encourage the mentors to work at justifying their equations using manipulatives. Perhaps that will help them to identify and correct their own errors, rather than needing me to point the errors out to them.

At this point, I began to question the use of the video camera as an effective tool for gathering data. I also began to question the extent to which I should interact with mentor/buddy pairs. I will expand on my reflections about these issues in a later section.

The Grade six/four mentor-buddy pairs played the modified version of 'Smath for the following three sessions. I decided to focus most of my attention on a different pair each session.

In addition to their own unprompted reflections about the session, I gave the mentors some guided questions to answer during their reflective writing following the first of these sessions:

1. Was it different playing the game this way? If so, how?
2. How much talking did you do with your buddy?

3. Does talking together help your thinking?

4. When you're not talking, do you do anything else to communicate with your buddy?

I left the mentors to write their own reflections, unguided by me, for sessions 4 and 5. It is interesting to note my quietest mentor, Mary, wrote after session 4: "Right now was fun because it was quite(sic) in the room. So we can concentrate". Perhaps group work and conversation interferes with some students' thinking.[2]

Following session 4, I met with the mentors and opened up a discussion with: "Sometimes what teachers do is they will get together and share ideas and plan on how to make themselves more effective teachers and help their students to learn; and so become more aware of what they are learning themselves. Now, is there anything you've been noticing as you've been teaching your buddy?" The discussion that ensued dealt with what they perceived to be the strengths and weaknesses their buddies had in doing math. I tried to redirect with: "Have you noticed yourself learning anything as you do this?" Bill offered "Basically, everything I know, 'cause, I know my times tables pretty good...", and then proceeded on to explain where his buddy needed extra work. 'What does it mean to 'know' math' is also a recurring theme.

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[2] One issue which Fullerton raises is that of the silent child: "While we know that some children can and do develop alternative strategies to compensate partially for lack of speaking opportunities"(1993, p.139), there remains concern that even these are not working to their potential. Webb cites research evidence that the most able member of the group may give the most explanations, even if his or her ability level is not high in the class as a whole. "Thus, assigning introverted students to groups in which they have relatively high ability may increase their levels of participation, particularly giving explanations"(Webb, 1991, p.383).

> Recently researchers have begun to label the reluctance to speak as communication apprehension(Hittleman, 1988; McCroskey & Richmond, 1991). The research findings around classroom communication apprehension (CA) are enlightening. Allen, O'Mara & Lang(1987) and Stockard & Wood(1984) report that females experience significantly more CA than males and that children who have communication apprehension learn less and suffer low self-esteem as well. In general, both groups found that academic achievement is significantly and negatively related to communication apprehension.
>
> (Fullerton, 1993, p.136)

The theme of 'what does it mean to be a teacher' came up again. Lori asked if she could give her buddy flash cards and keep a checklist of her buddy's progress. Bill explained "When I talk I help her understand" and "I point to what she wants or she just takes a longer look at her equation". Mary explained that to help her buddy understand, "I was talking with her a little, but I know she understands"(hence further communication is unnecessary? This dyad had very little communicative interaction.)

In looking over my reflection notes after session 4, I saw that 'authority declarations to correct mistakes' and 'the use of manipulatives in teaching /learning' began to be problematic for me. I wrote:

> The division question came up again today - this time with Bill. Obviously my 'explanation' the first day did little to clear the issue up for him. Nevertheless, I felt compelled to repeat my mini-lecture, pulling out my counters 'to make everything clear' ... I wonder ...
>
> I encouraged mentors once again to use the counters to justify their equations' validity. So far, I've been the only one to use them.

Those two themes were evident once again in my reflection notes following session 5:

> The ÷ problem came up again today. Lori seemed to hear me for the first time. I do believe she heard me though.
>
> Lori faithfully used counters to confirm the equations she and her buddy had made. However, she used counters only to substitute for the numbers → she used counters as symbols. Thus, the equation
>
> 5 ÷ 10 = 2 was totally justifiable because:

Figure 2: Lori's Representation of Division

Boxes

We had two weeks break at this point (Spring Break vacation). When we re-convened, I met with grade six and grade three mentors to introduce the grade three mentors to the study, assign them their buddies, and introduce both sets of mentors to the Boxes activities.

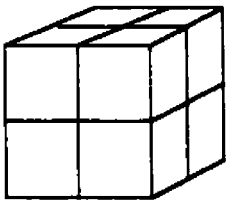

Figure 3: Initial Box

I directed the mentors in building a box that was 2x2x2, using sticky-tack to hold the blocks together. I conversed with them about the various characteristics that could be used to describe the box (counting the number of building blocks; counting the number of faces; describing the shape of each face). When a grade three mentor, Anne, thought there were 20 blocks used to construct the box, I suggested that she build another box exactly like the first, and then count the number of blocks she had used. She was surprised to find only 8 blocks were used. The mentors could not agree on the number of faces that the box had. I had them use crayons to colour each face of the box a different colour. I also directed them to make rubbings of each face in a corresponding colour. With this as evidence, they were able to count the number of faces and

agree. Finally, they took the boxes apart, removing all the sticky-tack, and the mentors played at trying to rebuild their boxes. They were surprised to find it not a straight-forward task. I suggested that they take their boxes home with them to play with and get comfortable with.

In my reflections following this session I had written:

> Bill [a grade six mentor] seemed to find the activities trivial. However I do believe he was very surprised that he couldn't re-combine his blocks to make the box within the class period. Then he quit trying - perhaps because he didn't want anyone else to see he had not already done it???

This thread winds itself with the 'teacher as expert' and 'reaction to mistakes' threads, weaving themselves throughout the study.

A week later, the mentor buddy pairs commenced work on the Boxes activities. I noticed that the mentors were busy working on building their own boxes, with very little interaction with their buddies. The buddies either tried to do what their mentor was doing, or sat watching the mentor build a box. When I noticed this, along with the absence of 'box-related conversation' going on, I directed them with: "mentors, could you ask your buddies things about their boxes? Can you ask them how many blocks there are? You can talk to them about faces..." I captured the conversation a grade-3 mentor, Anne, had with her buddy, following these directions:

Anne: What shape?

Kyle: A square.

Anne: Good! How much, what, how much edges do they have?

Kyle: Four.

Anne: Mmm-hmm. They have faces, too. I forget which. But they do have faces. Tell me, what does it look like? Is it a window sometimes, or is it a picture, like the calendar?

Kyle: It looks like a box xxx

Anne: Yah. A coloured box. Hmm. And you get to take that home. xxx And you can make a castle!

Following this session, I tried to remind mentors of the need to interact with their buddies and involve their buddies in thinking about boxes. As I tried to give them a quick reminder of all that is involved with the boxes activities, it became apparent that they needed at least another full session to gain better familiarity with the activities. The activities were so novel to them that they needed more time to play with the ideas on their own before they would be ready to share the ideas with another learner, and take a 'back-seat' letting the buddies control the manipulation of the blocks.

Session 8 was spent with mentors alone for further direction on Boxes. Grade six mentor Lori was unable to attend at this point for a week - three sessions - due to medical problems.

Mentors worked on depicting boxes with face views, and building boxes based on face view drawings. Grade six mentors tried drawing a corner view of their box using isometric dot paper.

In the next session, mentors led their buddies through the process of building various boxes in order to agree upon what makes a box a box, and what characteristics might differ between boxes. Then they chose one box to put together more permanently using sticky tack. Finally, they began to document its attributes using the box record sheet as shown in the Appendix on page 77.

I asked the mentors to consider three questions in their reflective writing following this session: (1) Does your buddy seem to understand the idea of boxes? (2) Does your buddy seem to

understand what you would need to draw in two dimensions in order to show what a three-dimensional box looks like? (3) What are you doing to get your buddy to understand something, and what seems to work, and what doesn't seem to work? The grade three mentors wrote yes/no responses to all three of the questions. The grade six mentors both gave a similar response to question 3: "First I show her what I did last day. Then she understands"; "I get them to understand by showing an example". It is interesting that the latter mentor, Bill, does not address the question "what doesn't seem to work", particularly because he indicates in questions 1 & 2 that he doesn't think his buddy understands the ideas he is presenting. In his response to question 1, he writes: "not really, I don't think she's really listening", and for question 2 he writes: "not really, we didn't get to that". Thus, any lack of success is attributed to a problem outside of his power, and reflection as to the adequacy of his singular method for promoting understanding in his buddy seems to him to be a non-issue.

Session 10 was a continuation of the activities from the previous day's work. I noticed Anne and Kyle seemed to have several answers on their box record sheet that did not correspond to their box.

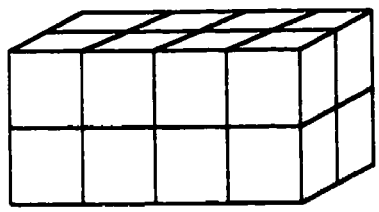

Figure 4: Anne and Kyle's Box

When I asked if their work was complete, they happily reported that they'd answered all the questions. When I made further inquiries about the correspondence between the answers on their

record sheet and the actual characteristics of their own box, an interesting attitude on the part of the mentor came up. For her, 'almost' seemed to be good enough. For example, in counting the number of blocks their box required, they initially thought it to be 12. I probed: "If you wanted to build this again, how many of these would you have to use?" "Eight!" declared Anne. "Eight, uh, four?" suggested Kyle. I suggested they try to build a clone of their first box, using the number of blocks they think they would need. They counted out the blocks, putting them in place as they drew each one. When they got to eight, I asked "Does that look like that?" "Almost" declared Anne. "Almost?" I asked. "Does it look exactly like that yet?" "Only the front face," Anne asserted. They continued to pull blocks and count them until they had reproduced the original box using 16 blocks.

Then we turned to the questions concerning faces. I asked Anne if the picture they had drawn looked the same as a face of the box they were working with. She said, "Almost". I explained that when they were drawing a face, they were to draw exactly what it looks like. I then asked her, "So if you look at the top, how would you describe what this top looks like?" "Almost like, the windows?" she suggested.

Then I decided to draw the faces of a mystery box for them to build, hoping that this might demonstrate to them the need for 'exactly' rather than 'almost' in their drawings. After they had built it and compared their building to the hidden box, they exclaimed "We did it!" Then I pointed out "That's right, and it doesn't look just a little bit like or kinda like; it is exactly like, isn't it?" "Yup" agreed Anne. I pointed out "So, that's what you're trying to do, is make it look exactly like. So when you look at this [referring to their drawings and box] and say does this look like that?" Kyle reflected "Mmm, no." Then they turned to the task of revising their drawings of the faces of their box.

I left the tape recorder on their table as I went to organize the groups to end this session and return to their classrooms. It was interesting to note the mentor's reaction to my intervention in the work she had been doing with her buddy. I believe her attitude is reflected in her final conversation with her buddy:

Anne: ...See? We made a big mistake. I thought we did it, but everybody makes mistakes, right Kyle? ... Right?

Kyle: Mm, right.

Anne: Yah. It's kind of hard when you do a block like that.

It is interesting to note how this attitude towards making mistakes compares and contrasts with that of the other mentors.

Lori, who had been away for a week due to illness, returned for session 11. She, with her buddy, began the boxes activities, and I left a tape recorder on their table. For some reason, the machine malfunctioned, and did not record their conversation. Lori said her buddy did very well and progressed quickly through the initial activity.

The rest of the mentors carried on from where they had left off in the previous session, documenting the box which they would leave hidden for others to reproduce using the information they recorded.

Debbie, a grade three mentor, sought clarity on her role with her buddy. I had been encouraging the mentors to allow their buddies to actually build the box, rather than leaving the buddy to watch as the mentors did all the activity. "But can I help her?" she asked. "Of course", I replied. "You're there to help her, but think of how you might help her without doing it for her". Debbie's face lit up. "Oh, I'll do like [Teacher's name], saying, 'You're doing good', and when

she gets it right: 'That's excellent!'". To this mentor, a significant role of the teacher is to encourage and praise. It is interesting to note how this perspective contrasts with other mentors.

There were several students from grades four and six absent from school on April 27, so I worked with only the grade three/one mentor-buddy pairs in session 12. They carried on from where they had left off in the previous session, documenting their box.

I approached Jenny(mentor) and Angie(buddy) whose documentation did not seem to match their box:

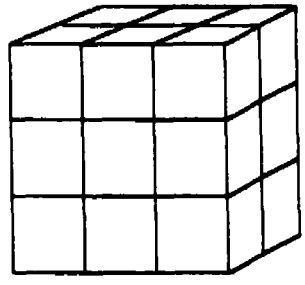

Figure 5: Jenny and Angie's Box

In the conversation that took place, the pair began to re-think their answer to the question: 'How many blocks?' Angie continually referred to a particular face to answer the question, as can be seen in this excerpt of the transcript.

Me: So Angie, is this the same as that?

Angie: Nnno.

Me: Nnno. So, you must have a different number of blocks in here.

Angie: The top?

Me: There's six on top, but how many in all?

Angie: Umm. The whole thing?

Me: The whole thing. That's what this question is asking.

Jenny: (giggle) I did it wrong too.

Later, in conversation with me, another mentor-buddy pair discovered and corrected a mistake in their counting of the total number of blocks in their box as follows:

Debbie(the mentor) & Lynne(the buddy): ... 78, 79, (pause)

Debbie: 80.

Lynne: (giggle) 80.

Debbie: It's 80.

Me: 80. Okay.

Debbie: It's close to 90.

Lynne: I got ahead of myself.

The theme of how the various individuals react to mistakes is represented here, once again.

Session 13 involved all six mentor-buddy pairs once again working on building and documenting a box. Some of the Grade three/one mentor-buddy pairs worked on a second box. Even with this second box, many of the mentors continued to do the work, leaving their buddy to observe their activity.

During the course of the session, I had left my tape recorder on the table of a grade six/four mentor-buddy pair, and had captured their conversation as Bill had attempted to teach Tracy how to draw a corner view of her box using isometric dot paper. Unfortunately, when his attempts at communicating with her proved inadequate for her to be successful in the activity, his communication with her reduced itself to insults, as is evidenced in this exerpt of transcript:

Bill: Do the Y-shape thing, you know, the corner?

Tracy: Like this? [see Figure 6]

Bill: iiii. Is yours spread enough?

Tracy: I dunno.

Bill: Okay, look at the

Tracy: Oh.

Bill: How many times does it go down?

Tracy: Three.

Bill: Right. How many times does it go across?

Tracy: Umm. Two.

Bill: Two. So then you've got five and three. ... Hang on. Wait for a minute, let me see that. Yah, that's ... xxxxxx ... there we go, on top. ... Now does that look like a box?

Tracy: giggles.

Bill: It has to go on the dotted lines! ... Oh, that's the problem, see? ... Look at your box. Does it look like that? Does that look like it has squares? ...

Tracy: There!

Bill: You're a nasty drawer, do you know that? Go. Be gone. Get out of here.

Tracy: giggle.

Bill: Get out of here.

Me: (coming on the scene) If you're done, you can go. Does that look right to you Bill?

Bill: No, it doesn't.

Me: If you're done, you may go.[announcing to all groups] Does that look right to you Bill? What looks wrong?

Tracy: See, I told you it was my worst enemy.

Bill: It's only your worst enemy because you don't listen on how to use it. Get out of here.

Tracy: It's my box.

Bill: What did you do?

Tracy: giggle

Bill: You know, you're a bad drawer; I'm gonna have to erase this entire side?

Tracy: Ohh (giggle)

Bill: How many times does it go down? Does it go down four times?

Tracy: It goes down two times. giggle.

Bill: Not four. There.

It is evident here that when I became involved, the situation deteriorated even further. Bill took over the task, and blamed Tracy for her inability to follow his directions. Consider this reaction to mistakes as compared with that of the other mentors.

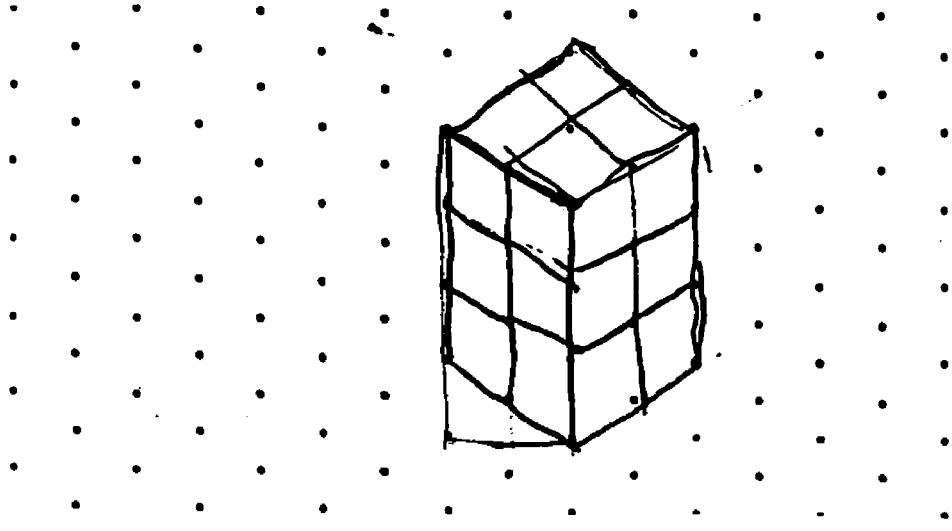

Figure 6: Photocopy of Bill and Tracy's Corner View Drawing

I noticed that grade three mentor, Jenny, and her buddy, Angie, had still not completed their record sheet for their first box, but had various other drawings on their box record sheet [see Figure 7], and was curious as to how they had arisen during the course of the conversation about boxes. Following this session, I worked with Jenny on her own, leading her through the activity of drawing face views for their box. Then I asked Jenny what she had been doing with her buddy, the following conversation took place:

Jenny: I tell her about this box.

Me: Okay.

Jenny: And then I tell her about what I drawn here and the thing.

Me: Okay, what were you telling her about it? Why did you draw this? [referring to the octagon] I'm just curious. What was this? What were you talking about with this?

Jenny: I was just saying to her did that look like a circle? Then she said no.

Me: What did it look like?

Jenny: Like this. [referring to the stop sign]

This was not the only instance where a mentor altered or extended an activity. I began to consider possible reasons why a mentor might change or extend an activity.

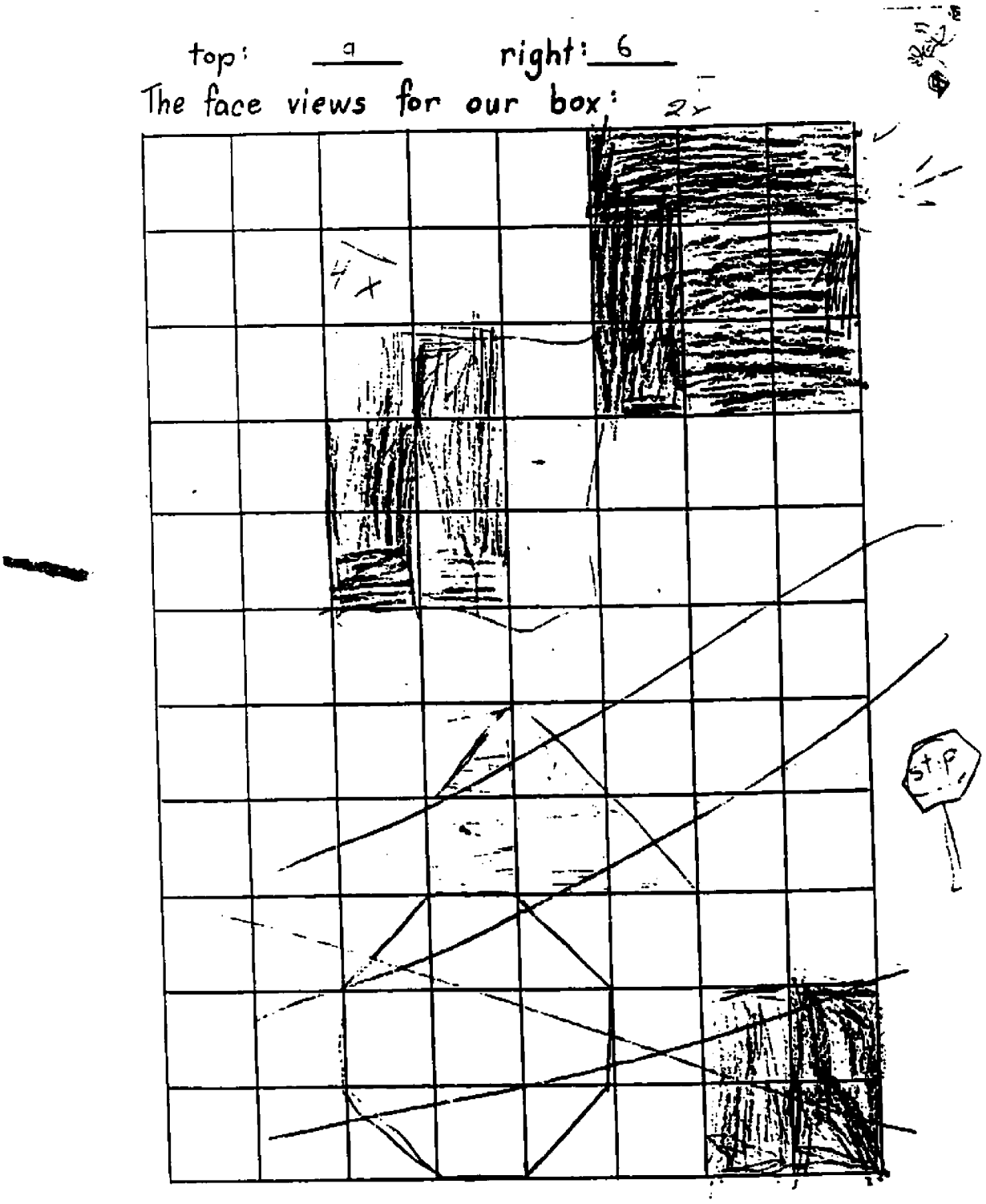

Figure 7: Photocopy of Jenny and Angie's Box Record Sheet

Two Grade six/four mentor-buddy pairs could not join us for session 14 due to illness. The remaining dyads built each other's boxes, and reflected on what generalizations about boxes they might be able to postulate.

Once again, the buddies were put in a passive role, observing as the mentors worked at building the boxes. For example, when I asked Anne if Kyle was helping to build the box, she confidently replied: "Yup. He's passing me the blocks". The transcript is a litany of my pleas for communicative interaction between mentors and buddies as I traveled from group to group. The mentors seemed too caught up in their own activity to concern themselves with how their buddy was able to attempt the reconstructions. Either that, or they thought that silent demonstration of correct procedure was, in itself, the best way to teach their buddy how to reconstruct a box. The following excerpt of transcript is an illustration of this point:

Me: So how are you figuring these out ladies?

Angie(grade one buddy): It's fun.

Me: It's fun. How do you figure it out? How do you know to build it like that?

Angie: I dunno.

Me: What did you do? Did you just follow what Jenny had done?

Angie: Mm-hmm.

I recommended to various dyads that they let the buddy do the building, with the mentor overseeing the operation and offering advice. This led to another difficulty. I came upon Kyle building the various faces of the box separately, as Anne silently looked on. I intervened and was offered an explanation by Anne of what she considered to be her renewed role as follows:

Me: Are you trying to build this face? But remember this face is on two of the sides, and this face is on four of the sides, right? ... This is all one box, right? This is all one box. That's what it looks like from two of the sides, and that's what it looks like from four sides, right? You're building two boxes.

Anne: I told him if he needs help I can give just a *little* help (emphasis Anne's)

It seems my instructions had left Anne out of the picture unless called upon by her buddy. Curious as to what I had said to prompt this outlook, I reviewed the transcript to uncover what my instructions to her had been:

Me: ... Are you having Kyle make it, and then you can help him if he's having trouble?

Anne: Okay.

Me: Okay? Maybe rather than you doing it, see what Kyle's doing, and see if you can help him understand how to do it. Ask Kyle what he's doing. How is he going to build it up?

My initial instructions were being followed to the letter, with the buddy's request for help being the chosen indicator of 'if he's having trouble'. This left Anne in a passive role, in that she did not need to attend to what Kyle was doing unless he saw the need to request her assistance. My further elaboration of possible dialogue between mentor and buddy may have been too vague, or possibly the point of this dialogue was not obvious to Anne, and hence dismissed as unimportant to her role.

The dyads were asked to reflect on and write about anything that they learned about boxes or anything that they thought might be true for all boxes. The problem of generalization seemed to be a difficult thing for mentors to understand. This is evident in their written responses as well as their conversations with me. Consider the following exerpt from the transcript:

Me: ... Are you noticing anything interesting about boxes?

Debbie(grade three mentor): Yes.

Me: What can you say about boxes?

Debbie: They all have sixteen.

Me: For this one they all have sixteen. Do all boxes have sixteen on every face?

Debbie: N-O no.

Me: No. THIS box had sixteen on every face.

Later, this dyad's written reflection about boxes was simply: "some othere blocks have difriet shaps".

The written reflection of Jenny and Angie boiled down to "It was fun". Mary and Cindy (the grade six/four dyad) referred only to the characteristics of a particular box which they had built. Anne and Kyle offered the reflection shown in Figure 6.

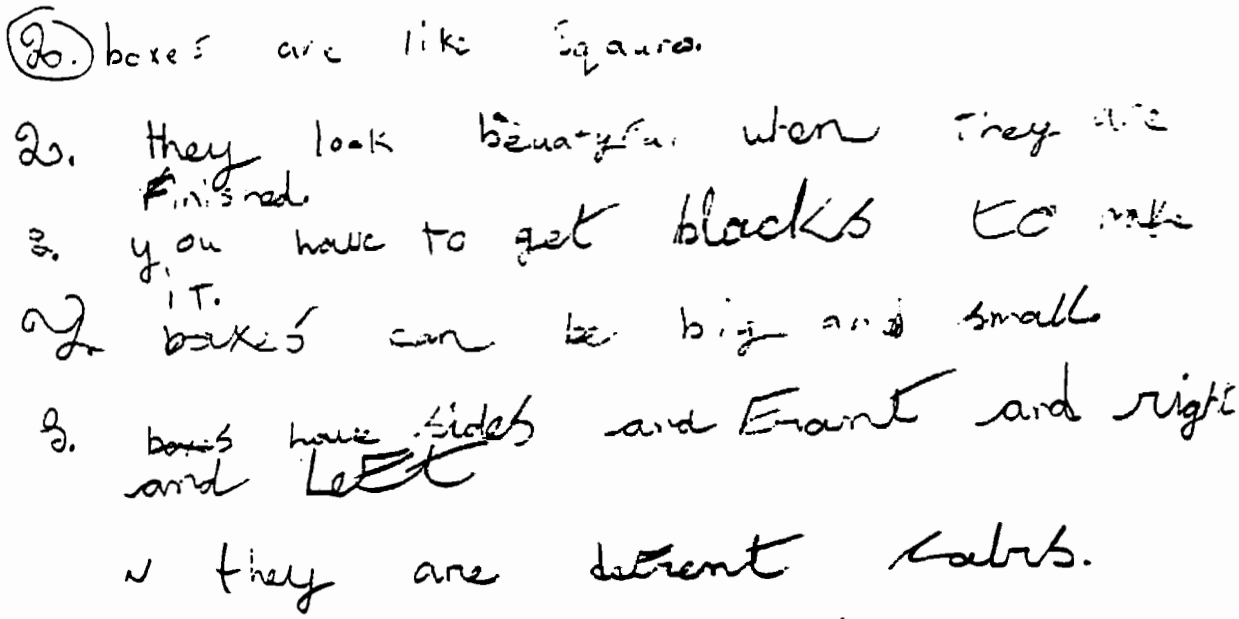

Figure 8: Photocopy of Anne & Kyle's Reflection About Boxes

The difficulty in ensuring that the dyads attended to and communicated about what I had perceived as being worthy of discussion is apparent here and throughout the study.

Lori, a grade six mentor and her buddy, Jill, unable to attend on Monday, carried out the activities of the previous session, during session 15, building the boxes which had been created by the other dyads and reflecting on what generalizations about boxes they might be able to postulate. At one point, I heard Lori saying that the box they were about to build was impossible. I asked Jill if she too thought the box was impossible, and she said no. I suggested that Jill try building it, and Lori commenced also to try building it, separately, with the words "I don't know if it will work". When I walked away to another group, I left my tape recorder on their table. Later in transcribing, I was surprised to find the next comment made by Lori to her buddy: "Oh, you might as well do it. You're going to win anyways". It hadn't occurred to me that this activity would provoke a competitive element. It seems that because they had differing initial hypotheses as to whether or not the box was possible, they were then in competition and the motive for building the box, at least for Lori, was to discover which of them had been correct. Correct was analagous to winning, and mistaken was analagous to losing. I think it is not surprising that Lori is the mentor who was very apprehensive from the start of the study, worried that she might make a mistake in front of her buddy.

Patterns

All three grade six mentors met with me at the end of session 15 to prepare the activities in Patterns which they would do in the following session with their buddies. The grade six/four mentor-buddy pairs worked on the pattern activities in sessions 16 and 17. Once again, there seemed to be very little conversation happening between mentors and buddies, and the transcript in

again a litany of my pleas for communicative interaction between mentors and buddies as I travelled from group to group.

One of the mentors changed the initial activity, describing her change to me as follows:

Lori: I tried this. Umm, what I did was I said I have twenty blue, and thirty yellows. Can you make my pattern? And what I did was I had a pattern already made, and I was going like this, [she indicates that she was setting up the pattern under cover so that her buddy couldn't see it] and she was counting how many blocks; how many yellows and how many blues I was taking in. And then she guessed, umm, xxx she guessed 2, 6, 4, 6, 5, 5, 5, 6,

Me: Ohh. That would be difficult, wouldn't it?

Lori: But it was 2, 6, 3, 6, 4, 6, 5, 6, 6, 6,

Me: Oh, I see. Why don't you actually put a pattern out? Just put any pattern out, but put it right in front of her where she can see it, okay?

I left this pair for awhile. When I returned, Lori told me of a different adaptation she had made to the pattern activity: "I think I've got an idea for her to help her learn patterns. What I do is I give her some blocks and then, I have a pattern underneath the paper. I give her a quick glimpse, and xxx". She had turned the activity into a memory game this time.

This was not the only instance where a mentor altered or extended an activity. In considering why Lori chose to change the activity, I saw that this thread weaves itself in around the theme of what it means to play the role of a teacher. I believe that Lori saw the need to change the activity so that it better suited what she believed to be her role in relation to her buddy.

Session 17 was the final session for the grade six/four mentor-buddy pairs.

Snap It, String Bean, & Problem-Solving Activities

In session 15, the grade three/one mentor-buddy pairs played 'Snap It'. Towards the end of the session, Anne approached me, elated, announcing:

Anne: Mrs. Barnson, I fooled him. Do you know how?

Kyle: She took away zero!

Anne: (shows me an empty hand, then brings the full stack of blocks forward) Ten!

It is interesting to note that this mentor, as well as her buddy, felt that what had been done was of significance. I agreed, seeing the significance of introducing the special qualities associated with zero in addition/subtraction. However, I'm not convinced that it was this aspect of what was done that impressed the pair. They seemed to be mostly impressed by the fact that it had 'fooled' the buddy. Making something more difficult or challenging seems, in their eyes, to be an important aspect of the role of a mathematics teacher.

In session 17, the grade three/one mentor-buddy pairs played 'String Bean'. In session 18, Anne worked with a new buddy, teaching him how to play 'Snap It' and 'String Bean'. I left a tape recorder on their table, recording their conversation for the entire session. The other two dyads played their choice of 'Snap It' and/or 'String Bean'.

I noticed that Jenny and Angie, who had decided to play 'Snap It', had written both '10-2=7' and '10-3=7' on their record sheet. When I asked Jenny, the mentor, if both of those statements were possible, she assured me that, indeed, both had occurred when they played the game. I told her that I was amazed that this could happen, and I asked her to try to reproduce the occurrence of 10-2=7. She gamely tried, snapping the string of 10 behind her back, bringing forward the 2 cubes in one hand, then bringing forward and counting the remaining cubes in the other hand. She persisted over several trials, trying to reproduce her original results. Finally, her

eyes lit up as she had an idea. She snapped apart the string of 10 behind her back, brought forward 2 in one hand, then brought forward a string of 7 in the other and triumphantly counted them out. In mock amazement, I repeated her count, but then pointed out "But hey, if we put these together again, there are only 9. There's one missing". She smiled and stood up to reveal the missing cube that she had hidden. Upon further probing, she couldn't tell me why she couldn't reproduce the 10-2=7 results without hiding a cube, and I'm not convinced that she didn't still believe that that statement was a real possibility.

It was interesting to track the progress of Anne who was introducing the new buddy to both games. This exerpt of transcript revealed some note-worthy underlying assumptions or attitudes on the part of Anne:

Anne: Okay. This is what you do. You get the beans in your hands;

Joe: Yes . . .

Anne: You shake it; (she dropped the beans on the table, then tapped the table beside each bean on one side of the string (3X); then tapped the table beside each bean on the other side of the string (6X).) Okay, 3 here; and how many there?

Joe: 15.

Anne: No. 1, 2, 3, 4, and 1, 2, 3, 4, 5, 6. So, We're gonna do minus or subtracting. Six plus four equals ten; six MINUS four equals ... two. Your turn. You wanna try? D'you get it though? D'you get it though?

Joe: You put the beans in your hand; then you shake it; then you ... then you do like that (he drops the beans on the table).

Anne: Now where they land, right?

Joe: Ya?

Anne: And then you count them, add them, and minus them. Do you get it now?

Joe: I think so.

Anne: Okay, you try.

Joe: (He drops the beans on the table)

Anne: Whoa. (Tapping the table beside each bean on one side of the string, she counts) 1, 2, 3, 4, 5.

Anne with Joe: (Tapping the table beside each bean on the other side of the string) 1, 2, 3, 4, 5.

Anne: Oh! That's easy! You know what you do?

Joe: xxxxx Five take away five equals zero.

Anne: Um-hmm. You got it. So, we're gonna do string bean. Uh, no, no. Snap it. Okay? Snap it, right? You have to put it behind your back. Don't let no one see it. You say: "Snap it". I snap it;

Joe: Ya,

Anne: and how much is here? Count by tens. Four right?

Joe: Five?

Anne: No, like, uh, like for example: 10, 20, 30, 40, 50, 60, 70. For 10's like that xxxx

Joe: 6?

Anne: (sighs) You have to say 60, or you can say 5 - 50, 70...xxx

Joe: 'kay.

Anne: Okay. What do you say?

Joe: 60?

Anne: Mm-hmm ... Or you can count by 5's ... Give up?

Joe: Yah.

Anne: (sigh) I'm going to do it. You see, if you add them all together in 10's or 5's, it will equal a hundred. So...you try.

...

Anne: Okay, put this behind your back and do the same thing.

Joe: I got it.

Anne: Snap it. ... Is there a hundred?

Joe: Yes.

Anne: Always at the end. So do you get that?

Joe: Ya.

Anne: Okay. Let's try one more time.

Joe: Snap it. ... a hundred.

Anne: Good. See, it always adds a hundred, right?

Joe: Ya

Anne: So, you get this one and you get this one, don't you? Do you get both of them? Do you know them now?

Joe: Ya.

At this point, the mentor came to me and announced that they had finished.

I decided to deliver that rather lengthy excerpt from transcript for a few reasons. First, I wanted to draw attention to what the mentor did to ensure 'understanding' in her buddy. I believe that she was acting towards this end, because she frequently stopped to inquire of her buddy "D'you get it?" In String Bean, she demonstrated the procedure to play once, then had the buddy

play one round following her guided questions. When he correctly answered a question, she pronounced "You got it", and moved on to instruction in how to play Snap It.

She decided to alter the game of Snap It, possibly to make it more challenging for Joe, seeing as how he had been so quick to 'get it' in the game of String Bean. Or perhaps she wanted to increase the challenge for herself, since she had already played the game with a different buddy in session 15. She demands that he count by tens. This time, rather than demonstrating a complete round of play first, she involves him in the first round of play, asking him guided questions. When he doesn't immediately catch on, she sighs, then says she'll demonstrate. Then she provides him with what she believes to be the punch line: "it [always] will equal a hundred". Once he snaps the links behind his back and says "a hundred", the mentor decides he's 'got it' and they're finished.

It seems that to this mentor, 'understanding in mathematics' is synonymous with correctly carrying out a procedure, and correctly answering a question. Furthermore, it seems that her understanding of the 'answer' to a math question is the number you will find to the right of the equals sign in an equation. As a result, the point of the two games String Bean and Snap It would seem trivial to her, as the 'answer' is always the same: 10 (or in her adapted game, 100).

Anne later decides to name this phenomenon saying to Joe, "So this is kind of like a reverse. Do you know what's reverse?" "No." "Reverse is something that repeats again and again and again. Like say we snapped it and it's again a hundred, a hundred, a hundred".

In session 19, the grade three/one mentor-buddy pairs worked together at Problem Solving with Three Addends, and Problem-Solving Puzzles.

In session 20, Dr. Mason met with the grade three mentors, to revisit and extend the work they had done with boxes.

Chapter Five - Discussion and Analysis

Analysis

For various reasons, the factors I had anticipated studying and describing in this study never really occurred. In retrospect, I realize that a crucial element in the communicating of mathematical ideas done at the Canadian Mathematics Education Study Group conference, and missing from the experiences of the mentors, was in the actual planning of the activities to be done by the buddy. The mentors were provided with the activities in which they were to lead their buddy. They became committed towards leading their buddy through the activities, but did not necessarily have a conscious commitment towards communication of mathematical ideas. At times, it seems they were even unaware of the mathematical ideas that I had hoped would be the basis of their communications with their buddies. Furthermore, it is possible that they had different priorities -- they may not have been as interested in whether or not their mathematical meanings were clear to their buddy or to themselves.

Occasionally, a mentor would alter one of the activities provided. However their purpose in altering the activity was not so much to further mathematical understandings in their buddy, as much as it was to put the activity more in line with what they saw they needed to do in their role as a teacher of mathematics.

Furthermore, I am not sure I could tell if a mentor were to have a learning benefit by their communication with a buddy. It seems clear to me now that self-reported improvement is unlikely. The fine line between not knowing and knowing is, in most cases, not even evident to the learner. For example, when I referred to my son's preschool teacher as a 'teacher', my son said, "She's not a teacher! She doesn't teach me things!" I pointed out a rhyme that he had learned. He said, "She didn't teach me that". When I asked him how he knew it then, he explained "I knew it when I was born". Similarly with the mentors, they could not think of anything that they had learned, but were

quick to point out areas where they believed their buddies had learned or where their buddies needed additional work.

This all may sound like it leads towards a conclusion describing a failed attempt. Case closed. Go back to the drawing board. However, over the course of the study, other interesting themes emerged and attracted my attention. As events unfolded, my perspective shifted. I became aware of other dynamics involved in a teaching-learning situation to which formerly I had not attended. I discovered issues that I had not given much thought to previously. This chapter details some of my newly-formed awareness of the teaching-learning dynamic which this study provoked

Weaving Together the Threads

In Chapter four, I narrated events as they unfolded chronologically over the course of the study. As I did so, I hinted at themes which emerged as the study progressed. In this chapter, I retell the story of the study, but this time I have separated the narration into sections, centering around four themes.

At the beginning of chapter four, I likened the study to a tapestry: the stories from chapter four like the vertical threads of the fabric; the themes you are about to read like the horizontal threads, weaving themselves through the stories previously told. The metaphor is not meant to imply that I am the weaver. Rather, the tapestry is the context in all its complexity. In separating out the separate threads for analysis, we risk losing sight of the design. Furthermore, the horizontal threads weave themselves around one another, and their separation means a loss of this inter-connectedness. It is hoped that in this retelling, I might keep track of the placement of each thread, so as not to unravel the threads, losing sight of the rich beauty of the tapestry as a whole.

What Does It Mean To Play the Role of Mathematics Teacher?

It became apparent that the mentors, rather than focusing on what it would mean to help their buddies to learn, became immersed in role-playing the part of the teacher. They concentrated on what it meant for them to be teachers.

When I say 'role playing', I mean that the mentors got caught up in a form of imaginative play. They became totally immersed in playing teacher, enacting their perceptions of what the role of teacher would entail. When I think of this type of imaginative play, I think of my two sons in their role-playing. When Carl or Eric decides to be a dragon, they become that dragon. Their entire countenance emanates those features they have come to understand as being dragon. They no longer answer to their names because they are, in fact, something else now. By watching them, I can see what they perceive to be important characteristics of dragon-ness, even if they aren't consciously aware of, or able to articulate these characteristics themselves. The nostrils flare; the fingers hook; the back curves.

It seems then, that by closely examining the manner of the mentors, we might catch a glimpse of what it means to them to be a teacher. Furthermore, by considering the role they relegate to their buddy, we might have some idea of what they believe to be the role of student. Upon doing this, many different themes emerge, and each mentor brought different characteristics to the fore. It would be interesting to consider to what extent and how well each of these mentors is imitating their lived reality. Are they portraying caricatures of teachers they have known, or are their actions close representations of the mentor's experience of what it means to act as teacher and what it means to be a student?

For Bill, a mathematics teacher's job is to give brief instructions, then sit back in judgement and silently point out mistakes. (See pages 28 and 33) I found myself continually directing him to talk with his buddy about what they were doing. On the days that I left the audio

tape recorder on their table, only the occasional mono-syllable was recorded. Watching his manner on the video recorder, he sat back, across the table from Tracy, (he must have moved the chair around, as I set up the chairs so that each mentor would be beside or kitty-corner to his/her buddy) with arms folded across his chest. Occasionally he would lean forward and point to something Tracy had done. Tracy would offer a hesitant giggle and change the thing to which he had pointed.

When it came time for Tracy to learn how to draw a corner view of her box using isometric dot paper, Bill's technique of watching and pointing out Tracy's mistakes failed. (See page 42.) Tracy did not understand enough to even begin the task. The amount of talking he did increased at this point, however the majority of the talk took the form of verbal abuse, rather than guiding Tracy through the process of drawing her box. For Bill, it was the student's responsibility to learn. If she did not learn, that must mean she was not trying hard enough. (See page 37.)

In contrast to this, Lori made a point of letting me know when she believed her buddy was doing well. (See page 39.) I believe she saw the achievement of her buddy as evidence of her own achievement as a teacher. However, Lori also felt the need to have a paper to fill out, recording topics she thought her buddy was having trouble on, in her role as teacher. (See page 29.) When Jill seemed to be succeeding in every activity she posed for her, she decided to made the activity more difficult - a guessing game - in order to be able to note a mistake in her buddy's performance. (See page 50.) It seems that she, too, saw her role as identifying mistakes, and had to ensure that mistakes would arise in order to legitimize her own role.

Furthermore, Lori seemed to adopt a competitive role with her buddy, even in the boxes activity, which I had not considered to be of a competitive nature. (See page 49.) I wonder if this set up a conflict for Lori, trying to act with opposing purposes - one being, to try to 'win', and the other being to help her buddy succeed. Or possibly, she did not see the latter as being part of her purpose, despite the role she had taken up as 'teacher'.

Many of the mentors, particularly at the grade three level, chose to use the strategy of demonstration to the exclusion of any other. (See pages 35, 36, 39, 41 and 46.) It is unclear as to whether this was a conscious choice in teaching strategy, or whether it was an unconscious result of the mentor engaging in the activity herself while forgetting about the buddy. It would seem, however, that Anne considered Kyle's passive presence at the table to be indicative of his participation in the activity, even if his part merely consisted of handing her the blocks. (See page 46.)

The lengthy transcript from session 18 paints an interesting picture of Anne's perspective of what it means to be a teacher. (See page 52.) The 'lesson' is fast-paced, moving on to a new idea as soon as the buddy articulates a correct answer to her question. When he could not immediately or correctly answer her question, she supplied him with the answer, then asked him to repeat the process (or repeat her answer). As soon as he does this, they continue on to the next thing 'he needs to learn'.

Previous to this, when working with her first buddy, Anne found it a significant event, worthy of reporting to me, when she managed to 'trick' her buddy in the game of Snap It. (See page 51.) Her view of the role of teacher seemed to require that she get a learner to give a wrong answer.

It was a grade 3 mentor - Debbie - who identified encouragement and praise to be a crucial part of a teacher's role. (See page 39.) Indeed, the grade 3 mentors paid much attention to this aspect of a teacher's role, and often encouraged their buddy with "good", "That's right", or "You got it", more so that the grade 6 mentors.

There were similarities in the mentors' perspective towards the role of teacher. Most strikingly, it occurred to me that virtually all the mentors took on the stance of what Davis would

label 'an evaluative listener'. in their interactions with their buddies. Davis describes 'evaluative listening' in this way:

> Within the mathematics classroom, this manner of listening is manifested in the detached, evaluative stance of the teacher who deviates little from intended plans, in whose classroom student contributions are judged as either right or wrong (and thus have little impact on lesson trajectories) and for whom listening is primarily the responsibility of the learner (1996, p.52).

This perspective is evident in the types of questions which the mentors asked their buddies. They tended to ask their buddies questions which had only one correct answer. Furthermore, they limited the questions to those which they were already confident in their own knowledge of the answer. Unfortunately, under this posture, I believe it would be impossible for them to enter into the type of genuine conversation about mathematical ideas which I had envisioned at the outset of this project.

Attitudes Towards Mistakes:

Interwoven with their understanding of what it means to be a teacher, as well as their understandings of the nature of the subject of mathematics, we find the mentors' attitudes towards mistakes. Although this is not as transparent as how they understand the role of teacher, it is possible to discern their attitude towards mistakes, in the way that they react when they make a mistake, as well as how they act in terms of mistakes made by their buddies.

Both Lori and Bill struggled with the idea of the possibility that they might make mistakes, and how that might affect their status as teacher. Bill tended to gloss over his mistakes, and pretend that they were situated in trivial matters unworthy of his careful concern. (See pages 29 and 35.) Lori openly declared her fear of making a mistake in the presence of her buddy. When it was suggested that making mistakes might alleviate a buddy's fear of making mistakes him/herself,

both jumped at the idea that they might gloss over their mistakes by pretending that they were made on purpose, to see if the buddy could identify the mistakes they made. (See page 30.)

Although their reaction to making mistakes is not as obvious, the grade three mentors also exhibited signs of awkwardness or discomfort when they realized they had made a mistake in the work they were doing with their buddy. Jenny giggled "I did it wrong too". (See page 41.) Anne asked for support from her buddy: "See? We made a big mistake. I thought we did it, but everybody makes mistakes, right Kyle? ... Right?" (See page 39.)

This issue of 'teacher as expert' is not isolated to the experience of these mentors. In fact, many professional teachers wrestle with the issue, deliberating over how their status in the eyes of their students might change if they were to make visible their own uncertainty. Davis cites Bruner's work, saying "the world that the teachers were presenting to their students was a far more settled, far less hypothetical, far less negotiatory world than the one they were offering their colleagues" (1996, p.160). Thus, there is a tendancy to present mathematics as a totally integrated set of universal truths, which one can master completely - a *Pi in the Sky*(Barrow, 1993) sort of vision, and so the myth is perpetuated.

The issue of the learner making mistakes, in contrast, was seen as not only acceptable, but necessary. For at least some of the mentors, their role in the teaching /learning dynamic required that the learner make a mistake, so that they might then be able to point out the mistake to the learner. For example, when this did not seem to be happening, Lori changed the activity to make the event of a mistake on the part of her buddy more likely to occur. (See page 50.)

What is Mathematics?

This thread weaves itself with the first thread, 'What does it mean to play the role of mathematics teacher'. By attending to subtleties in action of the mentors as they enact their role as mathematics teacher, we may catch a glimpse of what they perceive to be the essence or the nature of mathematics.

When Tracy was unable to draw a corner view of her box, the only way Bill saw that he could 'fix' the problem was to draw the corner view himself. In so doing, he eliminated the learner from the situation. As if that were not enough, he repeatedly banished Tracy with his words: "Go. Be gone. Get out of here". (See page 42.) It is interesting that he felt a need to correctly complete the task, regardless as to whether or not his buddy was involved. It points to a belief that the 'math' is in the finished product, rather than in the process.

Anne, too, seemed to have a similar perspective on the nature of math. For Anne, the ability to produce a correct answer indicated that Joe 'got it', even if she had fed him that correct answer moments before. (See page 52.) Furthermore, notice how in an activity that was intended to be about the addends that add up to 10, Anne prompts her buddy to discover that the 'answer' would always be the same. To Anne, the 'answer' to any math problem is the number one finds 'on the right hand side of the equal sign'.

Lori frequently requested that I provide her with flash cards so that she might drill her buddy on multiplication facts, and check lists to keep track of which facts her buddy knew. (See page 33.) Lori saw mathematics as being a collection of disconnected facts that the learner needed to memorize. Even counters, which I had thought might serve as a model to help demonstrate the process associated with division, were used as symbolic representations by Lori (see page33), keeping the process of division like a black box (i.e.: When you put two numbers into the black

box we call division, you get a different number. By memorizing the math facts, you can know what that number will be).

Indeed, all the mentors seemed to equate proficiency in math facts and not making mistakes with 'being good at math'. Furthermore, once this was achieved, further activity in math was seen as unnecessary. Bill pointed out that he wasn't learning anything new because, "Basically, everything I know, 'cause, I know my times tables pretty good...". (See page 32.) I believe that it was important to Bill, in enacting his role as teacher, to consider himself to already 'know it all'. I believe it was a power issue for him. Davis would agree, noting that if a teacher openly takes on the stance of a learner, this "prompts a leveling of the hierarchial structure of the conventional classroom.... Power might be the privilege of not having to listen, ... mathematics learners, in being compelled to listen, are the powerless ones in the classroom" (1996, p.140). Thus, if Tracy is the only one learning, she is the one who needs to listen or attend carefully in order to understand; Bill need only engage in evaluative listening, to identify and point out Tracy's mistakes.

Story of the Researcher - a description of my thesis journey

I find this thread to be the most difficult section to write, but also the most compelling. It is in the act of writing, and not in the finished product, where the value of the exercise is found. I am not the same person who began this research project. Indeed, as I sit now and attempt to describe to an audience the subtle shifts in my perspective, I am again changed. Davis tells of how Gallagher captures this idea in: "The 'self' ... is not a totalized, self-identical essence, but a 'self-narrative', a self-process which never stops being a process in play" (1996, pp.191-192).

I will frame this retelling, once again, in stories from my journey. I hope that by these stories, you will come to see the process of my growth rather than the finished product, for it is in the process that I see the merit of this work.

It is interesting to think about the role my video recorder played in my research. Without stopping to really think about it, I assumed that if I could just capture all that happened on tape. I would then be able to analyze the events at my leisure.

Looking back, because of the presence of the camera, I assumed the role of 'fly-on-the-wall' observer ... a role I would not intentionally or consciously consider appropriate for this situation. A 'fly on the wall' has no interest in relationship to the learners, and is unlikely to influence them in a positive way. Perhaps the only influence would be to cause a distraction. Furthermore, the researcher is not likely to learn as much from the learners, as there is no opportunity for 'conversation' as it is understood in hermeneutic listening (Davis, 1996).

Then, something happened that made me keenly aware of the problematic nature of my hiding behind the video recorder. Now, as I review the video tape, the scenario as it occurred seems really quite comical.

Picture this: The camera is focused on Lori, a grade six mentor, and her buddy, Jill, as they play 'Smath. Jill puts some playing pieces on the gameboard. The video camera zooms in on her equation which reads: $4 \div 1 = 4$. The camera once more focuses on the mentor buddy pair, capturing Lori explaining to her buddy that "4 can't go into 1". She changes the tiles around, and Jill smiles and shrugs her shoulders, looking embarrassed by her 'mistake'. At that point, there is an interruption in the video tape as I turn off the camera to intervene. When the picture comes back on, Lori and Jill are once again concentrating on the gameboard. A close-up of the gameboard reveals the tiles, once again, back in the position where Jill had originally placed them.

I began to realize that the camera was not a good replacement for my presence. It may be a useful tool to help me to re-member the situation, but that would require that I be a member of the situation in the first place. I recognized that I needed to put down the camera, in order to avoid the distance I had been establishing between myself and those I was observing. I needed to recognize and respect my own integral role in the research context.

Once I had accepted the fact that I was not, and could not be an objective detached observer, that I was, in fact, an integral part of what I was studying, everything seemed to be connected. Williamson's song took on new significance for me. She writes:

> When you open up your life to the living,
> all things come spilling in on you
> And you're flowing like a river,
> the Changer and the Changed
> (1976, p.4)

So it was, that I recognized 'the living' to be, not isolated observations, but the complete stories of these mentors in all the complexity of their context; a rich narrative that considers not only their dynamic present, but also their past and their future as well. The mere thought of writing such a multi-dimensional thesis paralyzed me for some time. I knew not where to even begin. Then, as I was reading my son one of the books we had picked up from the library, I came upon *Begin at the Beginning*. Schwartz (1983) seemed to be speaking directly to me. In the story, a little girl wants to draw the 'best picture ever', but as she begins to imagine all that would have to be involved in such a picture (the universe itself), she is overwhelmed by the enormity of the task, and spends her time in endless procrastination activities such as sharpening her pencils. Finally, her mother offers the little girl some sage advice: to begin at the beginning. The little girl looks out her window, and decides that the tree outside her window might be considered the beginning of her universe. She begins by drawing that.

I decided that I, too, should follow this advice. I began to write the story of my research, beginning with stories of the interesting things that I noticed from the seemingly endless pages of transcript from the study. As I did this, themes seemed to emerge, providing me with a framework upon which to situate my story. I am keenly aware that these themes did not arise from some pre-ordained truth, outside myself. Rather, these themes are necessarily connected with my own subjectivity. The 'truths' I discovered and relate to you here come, not only from what I observe outside myself, but also from within me - a self that comes with its own unique set of experiences; a self with its own unique perspective; a self that selects and reflects on what I determine to be worthy of my observation, based upon those experiences and resulting perspective.

I am also a social animal, and therefore find value in sharing what I have discovered with others. As I do so, the 'other' may choose to share their experience of my discoveries. When there seems to be a mesh between these experiences, or a common sharing of an experience, we rejoice, not because this is indicative of an ultimate 'truth' we have discovered, but rather, because it represents a point of our inter-connectedness. It helps us know that we are not alone.

So it is, that I have invited the reader of this document to consider my story, and reflect upon what they might consider to be noteworthy points of interest - that is - to determine at which points they might find a commonality or relationship with me.

Chapter Six - Conclusion

As I contemplate how I might conclude this thesis, and capture those points of significance which I deem to be of most importance, I am, once again, humbled by the feelings of my own inadequacy. I wish that I could do this with a 'mathematical elegance', that is, a simple concise statement that captures the very essence of the study. However, the complexity of my story, inter-woven with those of the mentors, make this a challenging task.

In my desire to conclude this work, I recognize that I have again lost sight of the importance of context. 'The' conclusion becomes multiple conclusions, each based on one's particular vantage point or perspective. Hence, in concluding this project, I will suggest how a particular perspective of this research might point in the direction of possible conclusions.

As I look back now on the study, with the understandings I have developed by doing the research, I cannot help but wonder aloud the words 'what if?' How might things have been different had I approached the study at the outset with the understandings I have today? Would it have made a difference if I had not imposed the terms 'mentor' and 'buddy' on the learners? In so doing, did I set up an implied demarcation of roles, which called for particular and separate behaviors in keeping with the newly assigned roles? What would have happened if I had simply paired learners together, and provided them with mathematical activities? Would their communication with each other as fellow-learners been different, more closely resembling the communication I had originally hoped to find? Connelly and Clandinin note that part of the power of narrative is that it "recognizes that people say and do different things in different circumstances and, conversely, that different circumstances bring forward different aspects of their experience to bear on the situation" (1988, p.25-26).

In so far as these questions of 'what if?' seem reasonable extensions to this research, their converse points to an area where one might draw conclusions on the research done. The complement to 'what might have been' in the study, is the issue of 'what was'. If 'what might have been' circles around not encouraging the mentors to take on a role of teacher, then the power of what did occur might be found when attention is directed towards what happened when the mentors assumed that role.

From this perspective emerges possibilities with the potential to inform practice. For instance, in examining the imaginative play of the mentors in their role as teacher, what might it reveal about these children's perceptions of teachers of mathematics? What does it reveal about their perceptions of what it means to be students of mathematics? What does it reveal about their perceptions on the nature of the subject of mathematics? What are the implications for mathematics pedagogy?

For the learners that contributed to the stories of this study, I have already described implications of my observations. Connections among elements of the observation have been woven into explicit statements as threads of the overall tapestry in the previous chapter. Conclusions must preserve the context from which they are derived to maintain the integrity of their meaning.

Furthermore, I believe the answers to these specific questions within this particular context to be secondary in importance to what I consider as the power in this research – that through an interpretive hermeneutic stance, I became aware of the need to consider these questions at all.

The conclusions within this context are tied to the specific learners of this particular context. However, they also point to possible implications to other contexts. For example, teachers may assume that their enactment of the role of teacher portrays one thing, (eg: providing drill and practice of multiplication facts to approach automaticity so that further mathematics learning is made possible) but in the lived experience of the students, the students may perceive the

teaching role very differently (eg: providing drill and practice of multiplication facts because mathematics consists of a collection of disjointed and meaningless facts which one must memorize to master). By taking on a hermeneutic stance a teacher might be better able to listen to learners, to understand their perceptions, to consider the possible implications to learning, and to negotiate with them, common understandings that might best serve their learning of mathematics.

So too, other researchers might consider the benefits which an interpretive hermeneutic stance might offer them in their work. This stance, particularly well suited to someone in a position such as teacher/researcher, is mindful of the necessary and appropriate place of the researcher within the context of that which is under study. Furthermore, this attitude allows an attentiveness to the nuances within a situation which might otherwise be overlooked.

This research influences the choices I make in my own teaching practice. I have been involved with, and am an advocate for the provincial assessment for the Manitoba 40S Consumer Mathematics. This assessment is unique in that it centers itself in listening to students as they present what they have learned in the course. It is a form of evaluation which focuses – not on the mistakes a learner makes – but on the successes and accomplishments they demonstrate to the examiners. The possibility for the examiner to listen in a hermeneutic way, and enter into genuine conversation with the learner means that he/she can draw out and capitalize on the strengths of the learner, discussing the "big ideas" of the course, and providing a much more positive experience for the learner.

A further example of my own application of this research would be found in my work with adult learners in a work-place 'essential skills' mathematics program. I have explicitly attempted to negotiate and reconstruct the roles of teacher and student, and what that means for each student.

These new contexts come with their own set of threads, comparable but not identical to the threads of the context described by this research. The threads weave themselves in differing ways, illuminating different patterns, but that is another story.

I have told the story of this research from a personal standpoint. I hope the readers might enter into the story with me, and let the threads of our interwoven contexts create a richer and fuller tapestry.

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Appendix

'smath

SCORE SHEET AND REGISTRATION

6 MOVES ONLY

Name: _____

Address: _____ Age: _____

_____ Birthdate: _____

Phone: _____

move	EQUATION	score	move	EQUATION	score
1			15		
2			16		
3			17		
4			18		
5			19		
6			20		
7			21		
8			22		
9			23		
10			24		
11			25		
12			26		
13			27		
14			28		
	Total			Total	

OUR BOX

date: _____

our names: _____ _____

How many blocks? _____

How many <u>different</u> faces? _____

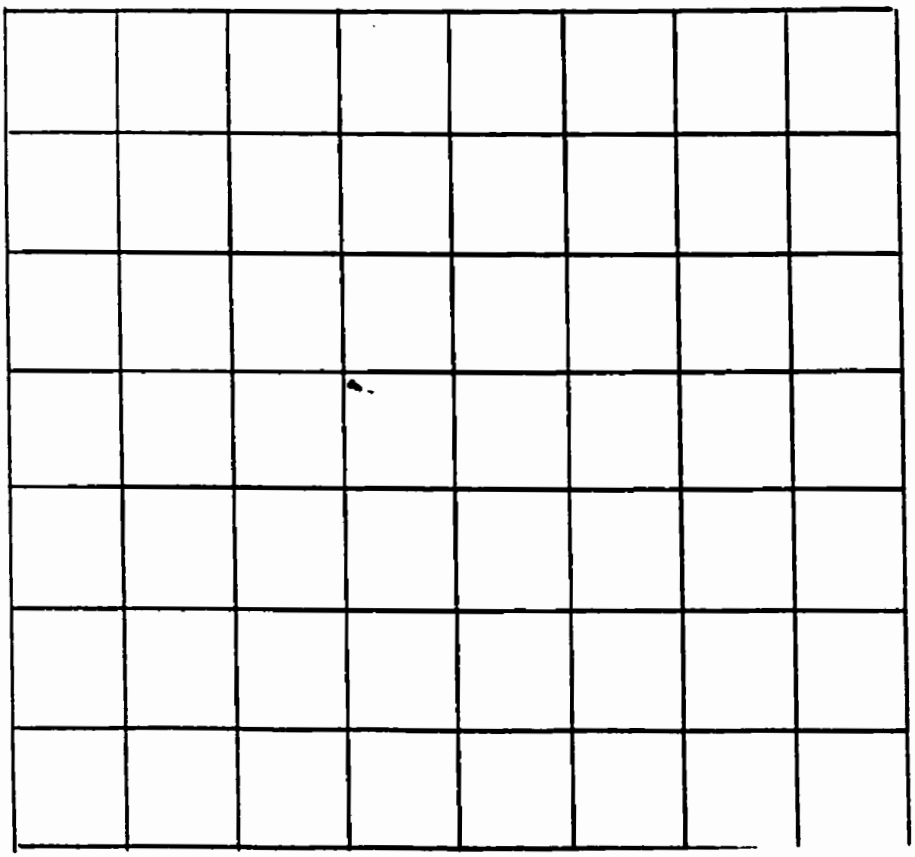
How many squares in each face?

bottom: _____ back: _____

front: _____ left: _____

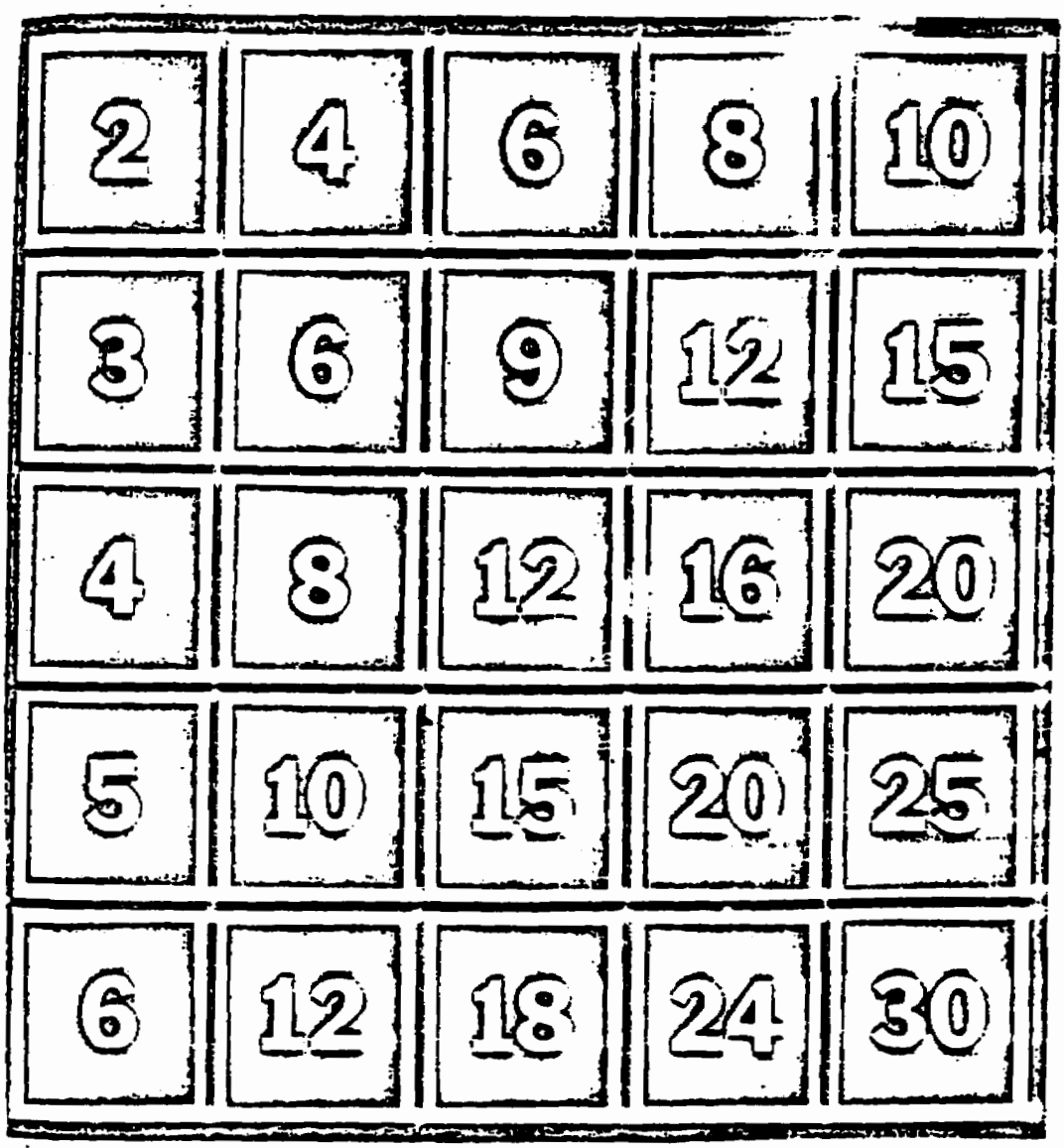
top: _____ right: _____

The face views for our box:



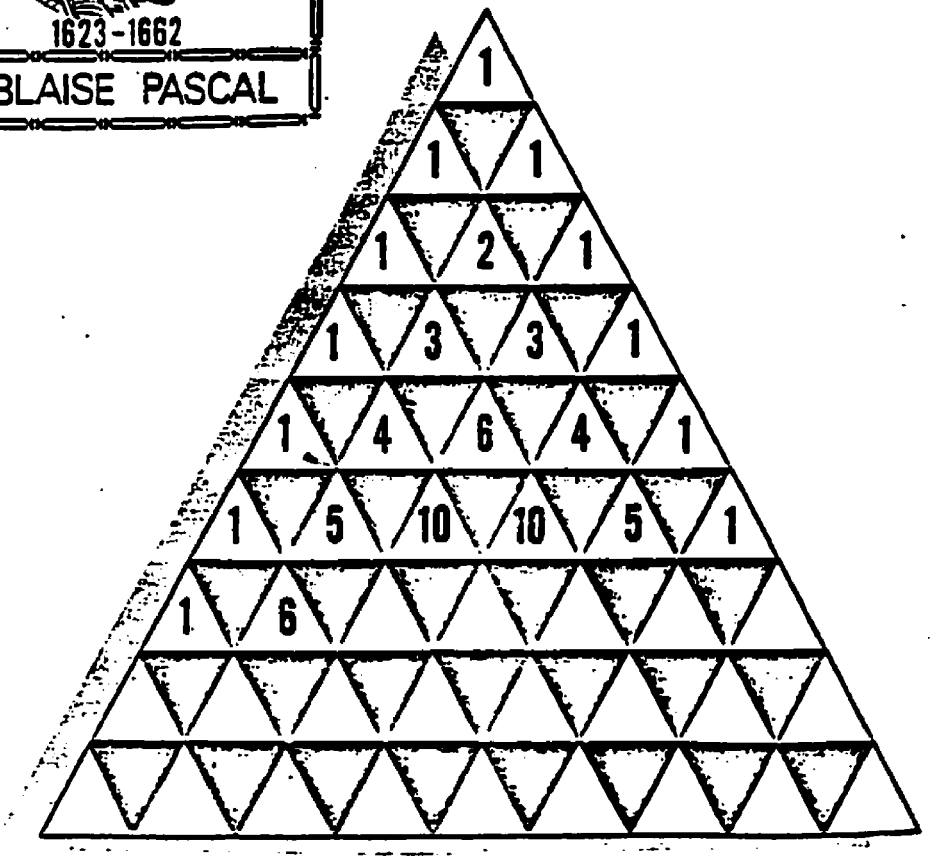
PATTERNS GALORE

MATHEMATICS HAS OFTEN BEEN DEFINED AS THE "STUDY OF PATTERNS". THE OBSERVANCE OF A PATTERN HAS BEEN THE "KEY TO THE SOLUTION" OF MANY MATHEMATICAL DISCOVERIES. THE ARRAY OF NUMBERS BELOW CONTAINS MANY PATTERNS. HOW MANY CAN YOU FIND?



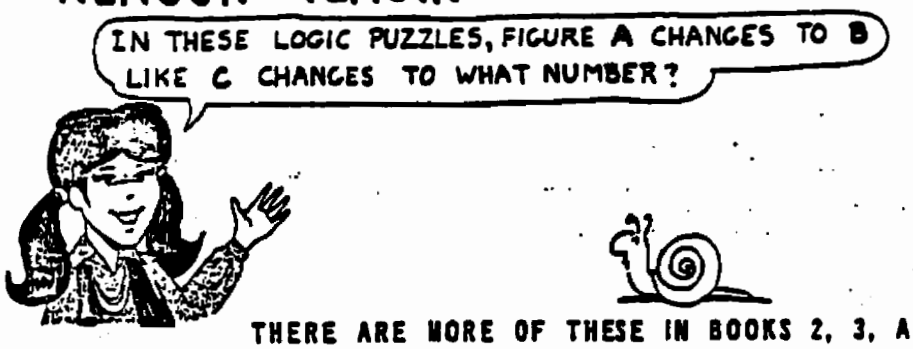
PASCAL'S TRIANGLE

THE TRIANGULAR ARRAY OF NUMBERS BELOW IS NAMED FOR AN EIGHTEENTH CENTURY MATHEMATICIAN, BLAISE PASCAL. THE PATTERN CONTINUES WITHOUT AN END. CAN YOU FIND THE PATTERN AND FILL IN THE BLANKS BELOW?



LOGIC LURE

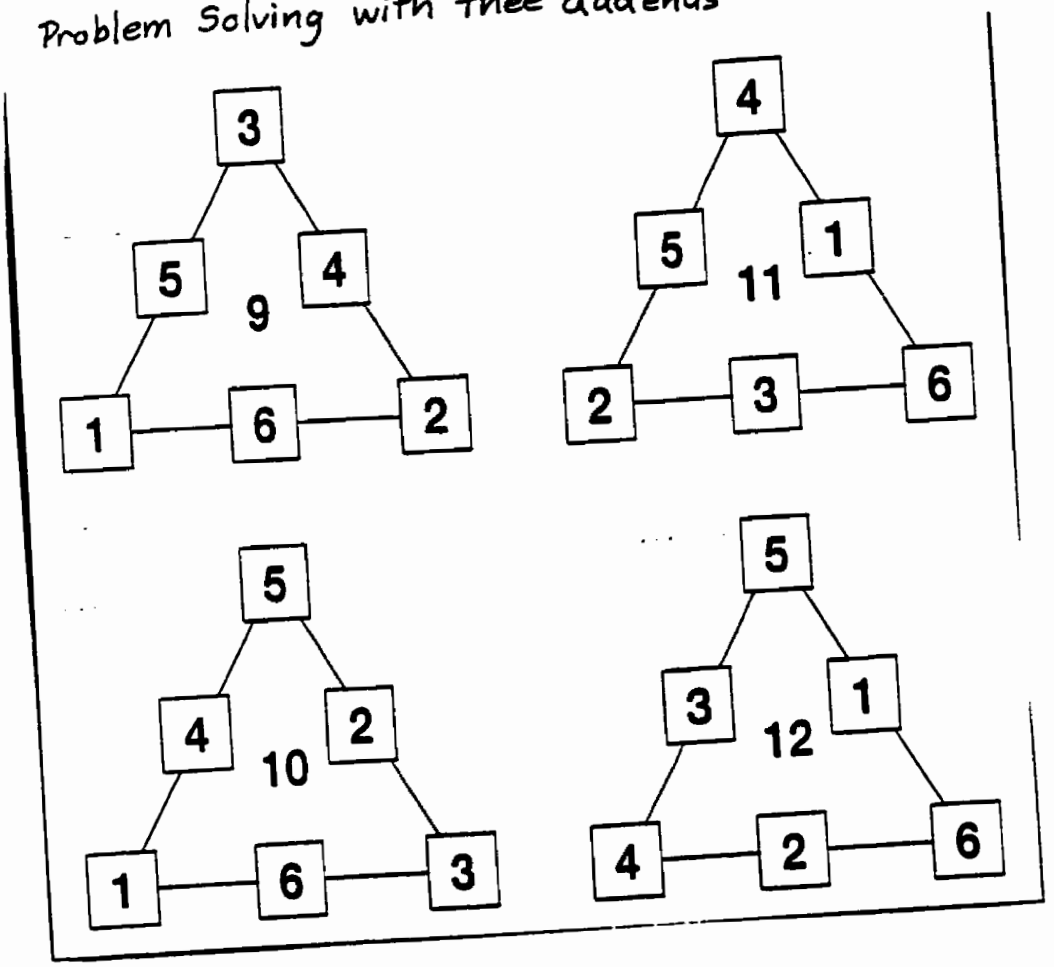
REASON TEASIN'



THERE ARE MORE OF THESE IN BOOKS 2, 3, AND 4.

1	A	B	C	1	2	3	4	5
2	A	B	C	1	2	3	4	5
3	A	B	C	1	2	3	4	5
4	A	B	C	1	2	3	4	5
5	A	B	C	1	2	3	4	5
6	A	B	C	1	2	3	4	5
7	A	B	C	1	2	3	4	5

Problem Solving with thee addends



Problem Solving Puzzles