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AN INVESTIGATION OF COAXIAL YAGI LOOP ARRAYS

by

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To my parents

ABSTRACT

The electrical characteristics of coaxial circular loop arrays are investigated by a numerical solution of the integral equation for the loop currents. Loop currents are expressed in a Fourier series of the azimuthal coordinate and the resonant property of the loop antenna is utilized to improve the convergence of the series. For arrays with different loop sizes this resonant effect is beneficially used to reduce their electrical length. As a result, with a negligible loss in the accuracy of the computation, the required labour for the investigation of the array characteristics is reduced significantly.

The effect of array parameters on its characteristics such as the input admittance, the directive gain and the radiation pattern are then studied. For a specific wire cross section extensive design data for arrays of three to twelve elements are computed.

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Alireza Shoamanesh

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LIST OF MOST USED SYMBOLS

a_i	radius of wire cross section of the ith-element
b_i	radius of the ith-lement in a circular loop array
C_{gi}	gap capacitance
d_D	director spacing
d_{ij}	the separation distance between the ith and the jth element
d_r	reflector spacing
E_θ	θ component of the electric field
E_ϕ	ϕ component of the electric field
E_{2n}	Weber Lommel function of order 2n
Gd	directive gain in the positive z direction
Gr	directive gain in the negative z direction
H_o^2	zero-order Hankel function of the second kind
h_{n+2m}^2	second kind spherical Hankel function of order n+2m
J_{2n}	Bessel function of the first kind of order 2n
$I_i(\phi_i)$	total current
I_j^n	coefficient of the nth current mode
k	wave number in free space
K_{ij}^n	nth coefficient of Fourier series
R_{ij}	distance between two current components normalized by radius b_i
$U_i(\theta_i)$	a function equal to unity at the driving point and zero elsewhere
V_i	driving voltage of the ith element
$W_{ij}(\phi_i-\phi_j')$	kernel of integral

n	
Z_{ij}	generalized self and mutual impedance
y^n_{ij}	generalized self and mutual admittance
Y_2	input admittance of loop Yagi array
γ	Euler's constant
$\delta\phi_i$	gap width of the driving point of the ith-element
ϵ_o	permittivity of free space
η_o	intrinsic impedance of free space
η_{n+2m}	(n+2m)th order spherical Bessel function of the 2nd kind
μ_o	permeability of free space
λ	wave length in free space
Ω_i	thickness parameter of the ith-element
ω	angular frequency

CHAPTER I

INTRODUCTION

A circular loop antenna, like a dipole, is one of the most common structures and has been the subject of many theoretical studies. Historically, small loop antennas were used as direction finders and flux detectors. As a radiator, the radiation pattern of a small loop antenna shows a figure of eight shape in the vertical plane involving the loop axis. As its size increases in terms of wavelength, the radiation in the axial direction increases and becomes maximum when the circumference is about one wavelength long [1]-[2]. As a result, a highly directive beam in the axial direction may be obtained by an array of coaxial loops of proper dimension.

A coaxial loop array with only one active element is the most common configuration. Its operating principle is similar to that of a Yagi-Uda array, but for a 2-element array a gain difference of about 1.8 dB in favor of the loop array is reported [3]. Unfortunately, published data for loop arrays of more than 2 elements is scarce. It is however reported that the radiation characteristics of loop arrays above the ground plane are less affected by the electrical properties of the soil, when compared with those of dipole arrays [4]. In addition, the use of parasitic loop arrays in place of dipoles has resulted in the elimination of corona problems at high altitudes [3]. These beneficial properties of loop arrays justify further investigation of their characteristics.

There are no difficulties in evaluating the radiation characteristics of a loop which is small with respect to the wavelength of operation. Here a uniform current distribution is assumed, and expressions for the radiated field are easily formulated [5]. For a larger loop, a more realistic current distribution must be found. G. Glinski has assumed a hyperbolic cosine form for the current distribution, similar to that of a lossy parallel-wire transmission line, in order to explain the experimental results for the horizontal radiation pattern of a comparatively large loop. To simplify his analysis, he has retained only the first few terms of an infinite series of the field expressions. This approximation, however, restricts his formula to loops in which the circumference is less than one-half a wavelength [6].

Several other methods have also been applied in the past to the analysis of a thin loop antenna in free space. One of the methods is the electromotive force method, or simply emf, for the determination of antenna impedance, where a current distribution corresponding to that of a wire of zero thickness is assumed. In general, the current distribution on an extremely thin wire antenna can be assumed to approximate a sine wave form very closely. Because of this the self impedance, determined by applying the emf method to an assumed sine wave current distribution, is generally a good practical approximation. Thus, a calculation of the antenna impedance by the emf, gives results, which are quite close to the actual values, except in the region around anti-resonance [7].

Another method which involves a more precise and practical theory, requires the computation of the current distribution from which the impedance is then determined. This technique requires the solution of integral equations by various approximation methods, of which the successive approximation method [1]-[2] and the series expansion method [8]-[11] are representative.

The first general analysis of a circular loop as a transmitting antenna appears to have been carried out by Hallen [8]. He analyzed circular loop antennas driven by an idealized delta function generator and obtained a solution, again in the form of a Fourier series, for the current and impedance. However, owing to the occurance of a singularity or a very large value in the Fourier coefficient, when the index n is close to a certain large number determined by the loop's geometry, Hallen limited its application to loop antennas small compared to the wavelength. He explained that this nonconvergent series might be the result of his assumption of the one-dimensional flow of the current. Moreover, the individual terms were complicated and their evaluation and summation involved a somewhat difficult numerical task.

After Hallen's theory, Storer avoided the contribution from the large term by first replacing the Fourier series by a corresponding integral and then evaluating the integral using the Cauchy principle value. He provided extensive tables and graphs of the impedance, admittance and current distribution

for loops up to a wavelength in circumference and a number of different wire sizes [9]. Storer also examined the validity of the constant current distribution on a small loop and was led to the conclusion that loops larger than $kb = 0.2$ ($k = \frac{2\pi}{\lambda}$) can not be considered small.

T.T. Wu re-examined the problem of evaluating the Fourier series. He pointed out that assuming that the current flows along the center of the conductor, as with Hallen and Storer it does not converge everywhere on the antenna [10]. He explained that the satisfactory values obtained under this assumption were due to a false approximation. He examined the surface current distribution on a perfectly conducting loop and verified that the obtained Fourier series for $I(\phi)$ converges everywhere except at the driving point ($\phi=0$).

Later, King et al. [12] computed the input admittance of a circular loop by taking a partial summation of the infinite series obtained by Wu. They suggested that a Fourier series solution with 20 terms is satisfactory for determining the admittance of thin ($\Omega = 2 \ell n \frac{2\pi b}{a} \geq 10$) and small ($kb \leq 2.5$) loops in air and dissipative media. Here k, a, and b are the free space propagation constant and the wire and loop radii, respectively.

Inagaki et al. by assuming a finite gap at the driving-point gave a theoretical basis for determining the required number of terms in current and admittance computations [11]. They obtained an expression for the driving-point admittance in which the gap capacitance is a lumped representation of the

effect of the gap width. According to their analysis, the driving-point admittance can be treated apart from the gap capacitance, independent of the gap width. It thus is calculated by a partial summation of the Fourier series truncated at a term whose order is equal to the ratio of the loop radius to conductor radius.

On the other hand, Iizuka et al. [13] have analyzed two identical parallel loop antennas. They have decomposed the voltage and current into symmetric and antisymmetric components As a result, the simultaneous integral equations for the distribution of current along the loops have been converted into a single integral equation similar to that for an isolated circular loop antenna which has already been studied. This method, however, is not applicable to antennas with loops of different circumference.

More recently the properties of an array of coaxial circular loop antennas with arbitrary circumferences have been analyzed by means of Fourier series expansions, with emphasis on the existance of finite gaps at the driving-points [14]. The expressions for the current distribution on each loop and the input admittance involve matrix inversion, where the dimension of the matrix is related to the array size. Thus, in investigations of loop arrays the required computer time and storage increase with the array size and for very large arrays become excessive.

This thesis considers coaxial loop arrays and utilizes the resonant property of loops to develop an efficient method for investigation of their characteristics. The current distri-

bution on each loop is expressed by Fourier series around the resonant mode and the Fourier coefficients are found numerically from the integral equation satisfied by the currents. It is shown that for resonant loops a single mode adequately describes the current distribution and a rapid calculation of the antenna parameters is generally feasible. For loops of arbitrary size, other modes are however significant, but a selection of a few modes adjacent to the nearest resonant modes usually provides an accurate representation. Such an approximation further simplifies the problem, if the array consists of loops of different sizes. Here, the resonant modes of loops are different and a given mode may not have a significant excitation in certain loops. As a result, the non-resonant loops can usually be neglected, which effectively reduces the electrical length of the array for such modes. This phenomenon further reduces the computation time and is used whenever possible. Chapter III is devoted to the investigation of these properties.

Chapter IV considers the effects of array parameters on the input admittance, the radiation field and the directivity of the loop Yagi array. It will be seen that the effects of altering the reflector spacing and size are negligible in the forward direction whereas the input impedance of the array and its directive gain Gr in the negative z direction are very much dependent on these alterations. As a result, the reflector size or spacing may be used to vary the back lobe level and the impedance of the array. The driven loop dimensions and the wire cross section have negligible effect on the forward

radiation, but again have considerable effect on the array's input impedance.

It is also shown in Chapter IV that there are two principle ways by which the far field pattern of a Yagi loop array may be controlled. One is to vary the director spacing while holding the element circumference and reflector spacing constant. The second is to vary the director size while holding all other parameters fixed. Either one of these two ways will have a considerable effect on the far-field patterns. Numerical results show that when the exciter size is about a wave length, the director sizes must be less than or equal to the wave length, otherwise the far field patterns will not be useful.

The importance of loop Yagi arrays has already been discussed in [3] and [15]. Unfortunately, there exists very little information on design data for these type of arrays. The prime intention of Chapter V is to present numerical data for rapid design of uniform loop Yagi arrays in which the combination of the exciter and reflector is optimum for the directive gain in the forward direction. The data are for arrays containing between 3 to 12 elements when the director size, kb, changes from 1 to 0.7 with $\Delta(kb) = 0.1$, and the director spacing, d_D, alters from 0.1λ to 0.3λ with $\Delta(d_D) = 0.05\lambda$. From these tables it is seen that for the director circumference $kb = 0.9$ the directive gain Gd has the largest value for any director spacing as long as the array contains more than two directors. The maximum value of directive gain Gd is found

to be 15.2 db, which is for the particular array of 10 directors at a spacing of 0.3λ.

CHAPTER II

FORMULATION OF THE PROBLEM

2.1 Introduction

An array of circular loops composed of N elements with arbitrary circumferences, arranged parallel in a row is analyzed for the case of a finite gap at the driving-point. Integral equations for the current distributions are obtained by expressing the electric field, through the use of Helmholtz integrals, in terms of currents and equating the total electric field to zero along the wire surface. A solution for the integral equations is sought with the aid of Fourier-series expansions of the kernels and the currents. Then the integral equations are reduced to a set of linear simultaneous equations for the Fourier coefficients of the electric currents of the same order. A solution of this set gives the unknown coefficients which are then used to evaluate the required antenna characteristics. In particular, it is shown that the self admittance may be obtained as a sum of two parts: a finite series truncated at a number determined by the ratio of the circular loop to the wire cross section radii and a gap capacitance. The mutal admittance, on the other hand, can be obtained from a finite series only.

2.2 Formulation of the Problem

Consider an array consisting of N circular loops of arbitrary circumferences and arranged parallel to each other with their centers on the z axis, figure (2.1). The radius

of the ith loop, the radius of the conductive wire, and the driving-point voltage are denoted by b_i, a_i, and V_i, respectively.

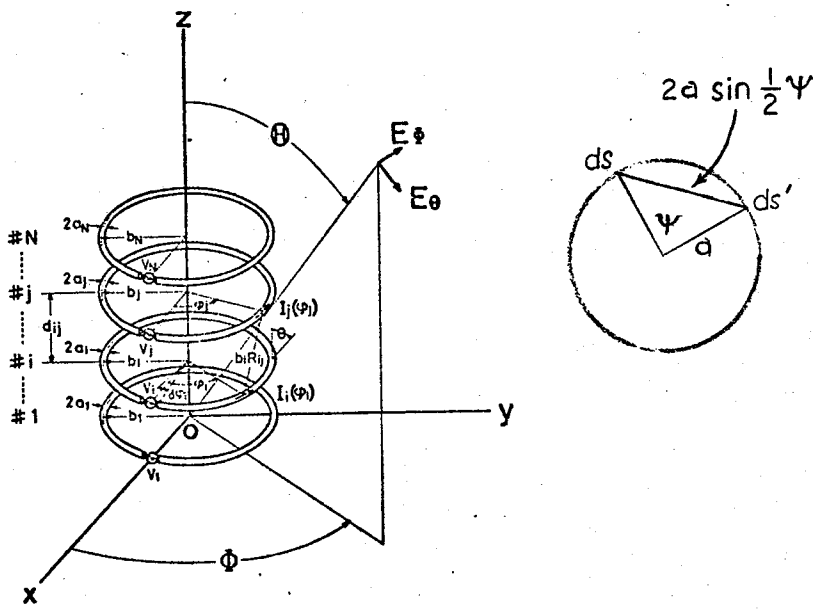

Figure 2.1 Geometry of the array.

It is assumed that

$$\frac{b_i}{a_i} >> 1 \qquad \text{and} \qquad \frac{a_i}{\lambda} << 1 \tag{2.1}$$

where λ is the wave length of the signal in free space.

The array is immersed in an infinite isotropic homogeneous medium, where its complex propagation constant is

$k = \beta - j\alpha$. If the medium is air $\alpha = 0$ and $\beta = \omega\sqrt{\varepsilon_o \mu_o} = 2\pi/\lambda = k$, where ε_o and μ_o are the permittivity and the permeability of the air, respectively. This thesis considers the latter case only, when the array is located in the air.

It is assumed that the driving-point gap width $b_i\ \delta\phi_i$ is finite and that the current $I_i(\phi_i)$ on the ith element flows uniformly in the ψ direction along the conductor surface. Following the procedure in [8]-[10] for a single loop, the integral equation satisfying the boundary conditions of the electric field on the surfaces of N circular loops may be shown to be [14]

$$-\frac{V_i}{b_i \delta\phi_i} U_i(\phi_i) = j\frac{\eta_o}{4\pi b_i} \sum_{j=1}^{N} \int_o^{2\pi} [k\, b_j \cos(\phi_i - \phi_j') + \frac{1}{kb_i} \frac{\partial^2}{\partial \phi_i^2}] W_{ij}(\phi_i - \phi_j')\, I_j(\phi_j') d\phi_j' \qquad (2.2)$$

where $i = 1, 2, 3, \ldots, N$ and

$$W_{ij}(\phi) = \begin{cases} \frac{1}{2\pi} \int_o^{2\pi} \frac{\exp[-jkb_i R_{ii}(\phi,\psi)]}{R_{ii}(\phi,\psi)}\, d\psi, & i=j \qquad (2.3) \\ \frac{\exp[-jkb_i R_{ij}(\phi)]}{R_{ij}(\phi)}, & i\neq j \qquad (2.4) \end{cases}$$

where $I_i(\phi_i)$ is the current on the ith loop, $k = \frac{2\pi}{\lambda}$ is the propagation constant, and $\eta_o = \sqrt{\frac{\mu}{\varepsilon}} = 120\Omega$ is the characteristic impedance of free space. $U_i(\phi_i)$ is equal to 1 within the driving-point gap and is zero elsewhere. The distance R_{ij} between two currents, normalized by the loop radius b_i and for $i = j$ is

$$R_{ii}(\phi,\psi) = [4 \sin^2 \frac{\phi}{2} + 4(\frac{a_i}{b_i})^2 \sin^2 \frac{\psi}{2}]^{\frac{1}{2}} \qquad (2.5)$$

and when $i \neq j$ and assuming $d_{ij} >> a_i$, it is

$$R_{ij}(\phi) = [4(\frac{b_j}{b_i})^2 \sin^2 \frac{\phi}{2} + (\frac{d_{ij}}{b_i})^2 + (\frac{b_j}{b_i} - 1)^2]^{\frac{1}{2}}$$

$$= [1 + (\frac{b_j}{b_i})^2 + (\frac{d_{ij}}{b_i})^2 - 2(\frac{b_j}{b_i}) \cos \phi]^{\frac{1}{2}} \qquad (2.6)$$

The contribution of the magnetic current existing at the driving point is omitted in the right hand side of equation (2.2) but will later be introduced as an adjustment for the gap capacitance.

In order to solve the above simultaneous integral equations the current $I_j(\phi_i)$ and the kernel $W_{ij}(\phi)$ are expanded into Fourier series

$$I_j(\phi_j) = \sum_{n=o}^{\infty} I_j^n \cos n \phi_j \qquad (j= 1, 2, \ldots, N) \qquad (2.7)$$

$$W_{ij}(\phi) = \sum_{n=-\infty}^{\infty} K_{ij}^n e^{-jn\phi} \qquad (i, j=1, 2, \ldots, N) \qquad (2.8)$$

where

$$K_{ij}^{n} = K_{ij}^{-n} = \frac{1}{2\pi}\int_{o}^{2\pi} K_{ij}^{n}(\psi)\, d\psi \qquad (2.9)$$

where for $i = j$

$$K_{ii}^{n}(\psi) = K_{ii}^{-n}(\psi) = \frac{1}{2\pi}\int_{o}^{2\pi} \frac{e^{-jkb_i R_{ii}(\phi,\psi)}}{R_{ii}(\phi,\psi)} \cdot e^{jn\phi} d\phi \qquad (2.10)$$

and for $i \neq j$

$$K_{ij}^{n} = K_{ij}^{-n} = \frac{1}{2\pi}\int_{o}^{2\pi} \frac{e^{-jkb_i R_{ij}(\phi)}}{R_{ij}(\phi)} e^{jn\phi}\, d\phi \qquad (2.11)$$

An introduction of equations (2.7) and (2.8) into the right hand side of equation (2.2), after some manipulation gives [14]

$$\beta_i^n = \frac{\sin n\left(\frac{\delta\phi_i}{2}\right)}{n\,\frac{\delta\phi_i}{2}} V_i = \sum_{j=1}^{N} Z_{ij}^{n} I_j^{n} \qquad (2.12)$$

$$(i = 1, 2, \ldots, N, \quad n \geq o)$$

where

$$Z_{ij}^{n} = \begin{cases} j\pi\eta_o kb_i K_{ij}^{1}, & n = o \\ \frac{j\pi\eta_o}{2}\left(kb_j \frac{K_{ij}^{n+1} + K_{ij}^{n-1}}{2} - \frac{n^2}{kb_i} K_{ij}^{n}\right), & n \geq 1 \end{cases} \qquad (2.13)$$

Equation (2.12) in the matrix form is

$$(Z_{ij}^{n})\,(I_j^{n}) = (\beta_i^{n}), \qquad n > o \qquad (2.14)$$