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Brushless Doubly-fed AC Field Machine

By

Arjang Maapaar

Advisor: Prof. R.W. Menzies

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Arjang Maapaar

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Master of Science

Abstract

The intermittent nature of the wind has made it more unique and yet challenging in comparison with most of other sources of energy. Wind speed might have not been a matter of concern in the old days when stone windmills were used for grinding wheat or other agricultural crops. But as far as producing electricity is concerned, fixed frequency and stable output voltage are the two major grid requirements and can neither be deviated from nor compromised. The most common approach to bridge between a variable speed source and fixed frequency/voltage in wind energy systems is a doubly-fed induction generator (DFIG), which has the disadvantage of using brushes and slip rings, and hence, elevating the maintenance costs.

The thesis studies an alternative to the above design; a "Brushless Doubly-fed AC Field Machine" based on a patent. This brushless configuration is studied from different perspectives including analysis, simulation, and operating zones.

It is concluded that the machine is capable of being operated as a variable speed generator if set up in certain configurations and as long as proper controls are provided. The brushless generator is then compared to the conventional DFIG to demonstrate the advantages and disadvantages.

Acknowledgements

"The journey of discovery consists not in seeking new landscapes, but in having new eyes." ***Marcel Proust***

It shouldn't be hard to confess that during the course of completing this thesis, I learnt much more about life, people and humanity than I did about science, technology and engineering. It was quite an honor for me to work under advisory of Professor R.W. Menzies and I am extremely grateful to him for his knowledge, meticulosity and inspiring manners, needless to say that without his generous financial support this research would not be viable.

I wish to adduce my accolades and kudos to "Power group" staff and graduate students who lent me a hand whenever needed.

I would also like to dedicate this work to my Father and Mother, though far way but their paces were always heard along side me in this journey.

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Nomenclature

ν_{s_M} : Main machine stator field speed wrt stator (synch.speed) (rad/sec)

ω_s : Main stator(Network) frequency(rad/sec)

P_M : Number of the main machine Poles

ν : Mechanical(shaft) speed(rad/sec)

ν_{r_M} : Main machine rotor field speed wrt rotor (slip speed)(rad/sec)

ω_{r_M} : Main machine rotor field frequency (rad/sec)

ω_{r_A} : Auxiliary machine rotor field frequency (rad/sec)

ν_{r_A} : Auxiliary machine rotor field speed wrt rotor (Slip Speed)(rad/sec)

P_A : Number of the auxiliary machine poles

ν_{s_A} : Main machine stator field speed wrt stator (synch. speed) (rad/sec)

ω_I : Auxiliary stator(Inverter) frequency(rad/sec)

$F_{(\theta)}$: Magnetomotive Force

N_s : Number of turns in each phase

θ : an angle wrt stator ref

γ : an angle wrt rotor ref

β : rotor angle wrt stator ref

$\beta_\circ$: rotor angle at $t = 0$

$F_{m\theta}$: Total (magnetizing) magneto motive force

H_θ : Magnetic field intensity

g' : Effective length of each air gap

B_θ : Magnetic flux density

λ_{turn} : Flux linkage of a single turn

N_s : Number of turns of phase "a" of stator

n_a : Conductor density for phase "a" of stator

λ_{ma} : Flux linkage of phase "a" of stator

N_r : Number of turns of phase "A" of rotor

n_A : Conductor density for phase "A" of rotor

λ_{mA} : Flux linkage of phase "A" of rotor

e_{ma} : Induced voltage in phase "a" of stator

e_{mA} : Induced voltage in phase "A" of rotor

E_s : Stator voltage

E_r : Rotor voltage

RsM: Main machine per-phase stator winding resistance

XsM: Main machine per-phase stator leakage inductance

RrM: Main machine per-phase rotor winding resistance

XrM: Main machine per-phase rotor leakage inductance

XmM: Main machine per-phase magnetizing inductance

RsA: Auxiliary machine per-phase stator winding resistance

XsA: Auxiliary machine per-phase stator leakage inductance

RrA: Auxiliary machine per-phase rotor winding resistance

XrA: Auxiliary machine per-phase rotor leakage inductance

XmA: Auxiliary machine per-phase magnetizing inductance

S_M : Main machine slip

S_A: Auxiliary machine slip

V_{s_M} : Main machine Terminal Voltage

V_{s_A} : Auxiliary machine Terminal Voltage

P_{s_M} : Main machine stator power

P_{s_A} : Auxiliary machine stator power

$v_{M_{pu}}$: Mechanical speed of the main machine shaft in per unit

fi : Inverter frequency

Chapter 1: Introduction

1.1 Purpose

The main objective of this thesis is to study the feasibility of a novel variable frequency generator that would be suitable for use in wind turbines.

The generator is a cascading configuration of two electrical machines wound on the same stator and rotor. It can provide fixed frequency ac power to the utility system while operating over a range of shaft speeds.

The idea of this machine is based on a United States Patent named "Brushless AC Field System for Stable Frequency Variable Speed Alternators" by Vithayathil et al. [1].

As the statement "Variable Speed Alternators" suggests, the idea of possible application of this machine for wind power came to the mind of the author of this thesis. What follows in Chapter 1 is a summary of existing generators that are being used in wind turbines and a review of the Patent.

1.2 Thesis Outline

Chapter 2, in a step by step fashion, strives to develop a "Brushless Doubly-fed AC Field Machine" based on the standard existing machines, taking into consideration different winding arrangements. Next the equations are calculated and equivalent circuit is shown.

Based by this theoretical background a simulation case is put together in Chapter 3 to both validate the materials of Chapter 2 and also perform a behavioral study of the machine at different speeds. The limits of operation are also confirmed in this chapter.

Chapter 4 presents a basic cost analysis of this generator in comparison with the commonly used doubly-fed induction generator.

Chapter 5 presents the conclusions and recommendations for further study.

1.3 Review of Conventional Wind Generators

A wind turbine can basically be equipped with any kind of generator, but due to the intermittent, variable nature of the wind some generators suit the wind power application more than the other ones. It has become a common practice to utilize DC generators for small wind energy systems (i.e. less than 10 kW) and for large wind turbines *"Induction"* (also known as "Asynchronous") and *"Synchronous"* generators are used either in their original design or with some modifications and/or along with other power electronic devices such as converters and soft starters.

1.3.1 Induction (Asynchronous) Generators

1.3.1.1 Squirrel-cage Induction Generator

Squirrel-cage induction machines are the simplest type of a generator. These machines are the most common generators used in large scale grid connected wind turbines. Once the stator winding is connected to the network a magnetic field starts to rotate around the rotor, which induces a current in the windings of the rotor and consequently another rotating magnetic field generated by the rotor starts to rotate in rotor as well. The interaction between these two magnetic fields results in the torque which acts on the rotor.

As far as speed is concerned in AC generators, the frequency of the generated AC is fixed by the rotational speed and the number of poles for which the generator is designed. This is depicted by the following well-known equation:

$$v_s = \frac{2\omega_s}{P}$$ **Eq. 1-1**

ω_s = Frequency of the stator/network (rad/sec)

v_s = Speed of the stator field wrt stator = Synchronous speed of the shaft (rad/sec)

P = Number of poles

Rewriting the above equation yields:

$$\omega_s = \frac{v_s P}{2}$$ **Eq. 1-2**

Therefore, it is essential to maintain the speed constant at the exact value determined by the number of poles of the machine to ensure a constant frequency output, however constant speed operation is not desired in a lot of applications such as wind mills.

Mechanical speed of the shaft v (rad/sec) is not the same as synchronous speed of the shaft v_s (rad/sec). This speed difference $(v_s - v)$ is referred to as slip speed and is usually expressed in terms of a percentage or per unit of v_s:

$$v_s - v = s v_s$$ **Eq. 1-3**

This percentage is called "Slip" and is then defined as:

$$s = \frac{v_s - v}{v_s}$$ **Eq. 1-4**

In Squirrel-cage induction generators slip is normally limited to a small range. (0.01 to 0.05)

In terms of frequency, Equation 1-3, can be written as:

$$\omega_s - \frac{vP}{2} = s\omega_s \qquad \textbf{Eq. 1-5}$$

where:

ω_s= Frequency of the stator/network (rad/sec)

The term $s\omega_s$ is equal to the frequency in the rotor circuit, ω_r (rad/sec), thus:

$$\omega_s - \frac{vP}{2} = \omega_r \qquad \textbf{Eq. 1-6}$$

Note: The use of *"pole changing"* squirrel-cage induction generators is also a common practice to achieve two different synchronous speeds, which makes it possible to run the generator with higher number of poles in lower wind speeds and vise versa. The following figure shows a Nordex N60 1300kw turbine which utilizes a "Pole changing (6/4-poles) Squirrel-cage Induction Generator" **[2]**:

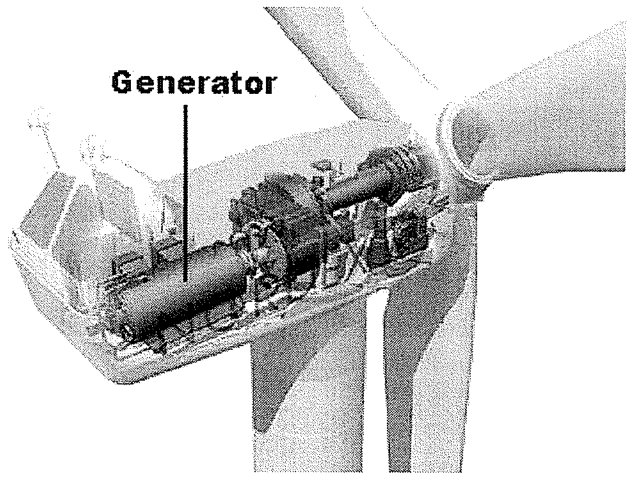

Figure 1-1: Nordex N60 1300kw turbine with a "Squirrel-cage Induction Generator".

Courtesy of Nordex AG Germany.

Operation Speeds:

From the above discussion, it is obvious that the rotor speed can only vary over a very limited range, thus this configuration is regarded as a **"Constant Speed"** Generator.

Advantages:

- Mechanical simplicity which leads to: an inexpensive design and with low maintenance.
- Smooth operation around the rated power (At the rated power, when a wind gust hits the wind turbine rotor, slip enables the generator speed to increase a little in response to the gust without causing a corresponding increase in the generated power output.)

Disadvantages:

- Unable to operate at variable speeds
- Limited power quality control
- Mechanical stress on drive train
- Low efficiency

1.3.1.2 Wound Rotor Induction Generator with Variable Ext. Resistors

As discussed previously the slip of a "Squirrel-cage Induction Generator" is confined to a limited range, therefore it only allows a small deviation from the synchronous speed. The following equation shows that the slip is a function of rotor resistance, therefore by connecting the rotor of an induction generator to external variable resistors by means of brushes and slip rings, the slip value can be increased, which results in a wider spectrum of operational speeds. From the Thevenin equivalent circuit of an induction machine we know that:

$$\bar{I}'_r \approx \frac{\bar{V}_{th}}{(R_{th} + \frac{R'_r}{s}) + jX_{th}} \approx \bar{V}_{th} \frac{s}{R'_r} \qquad (A) \qquad \textbf{Eq. 1-7}$$

We also know that the mechanical torque of a 3-phase induction machine is:

$$T_{mech} = \frac{3}{V_s} \left|\bar{I}'_r\right|^2 \frac{R'_r}{s} \qquad (N.m) \qquad \textbf{Eq. 1-8}$$

Thus if we substitute for $\bar{I}_r'$ in the above equation, we have:

$$T_{mech} \approx \frac{3\overline{V}_{th}}{V_s}\frac{s}{R_r'} \qquad (N.m) \qquad \textbf{Eq 1-9}$$

This is illustrated in the following figure, the torque speed characteristics of an induction machine with variable external resistors:

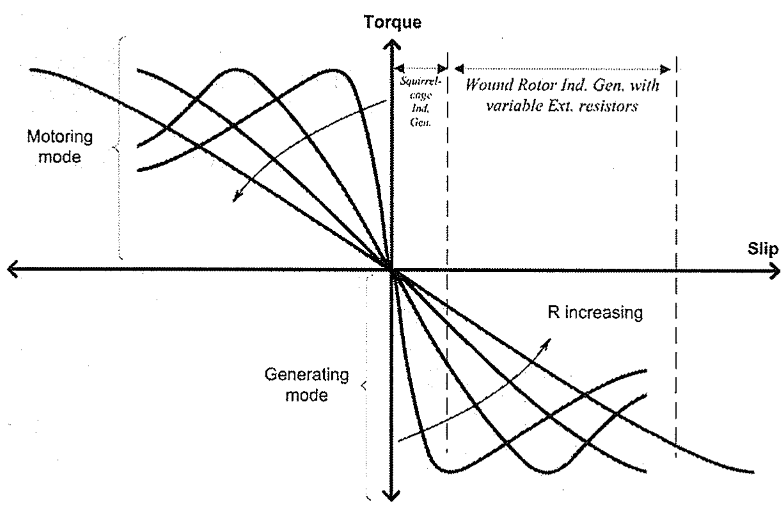

Figure 1-2: Torque speed characteristics of an induction generator with external variable resistors

Adding resistors to the rotor circuit can increase the slip value up to 0.1, which means the capability of speed deviation from the synchronous speed by 10%. Such a configuration is commercially used in wind generators. This is illustrated in the following figure according to reference **[3]**:

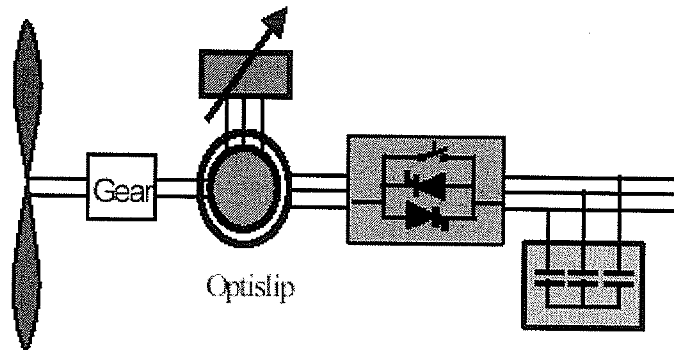

Figure 1-3: Wound Rotor Induction Generator with variable external resistors.

Vestas Wind Systems has integrated the external resistors to the rotor, along with the rotor current control (RCC) unit to obviate the need for brushes. The following figure shows a Vestas V47/660kW turbine which utilizes a "Wound Rotor Induction Generator with variable internal resistors". Vestas calls this design OptiSlip **[4]**:

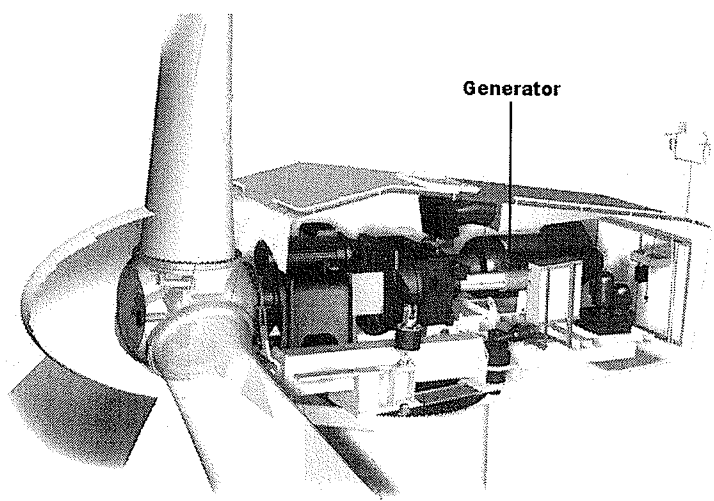

Figure 1-4: Vestas 47/660kW turbine with a "Wound Rotor Induction Generator with variable internal resistors". Courtesy of Vestas Wind Systems A/S.

Operation Speeds:

From the above discussion, it is obvious that the rotor speed can vary over a wider range than the *"Squirrel-Cage Induction Generator"* (i.e., up to 10% of the synchronous speed), which is still not an adequately wide range taking into the account the broad spectrum of the wind speeds, thus this configuration is regarded as a **"semi-variable speed"** generator.

Advantages:

- Some variable speed capability
- Lower stress on the drive train

Disadvantages:

- Limited range of variable speed operation
- Brush Maintenance (except for the Optislip design)
- Lower efficiency at higher slips

1.3.1.3 Doubly-fed Wound Rotor Induction Generator (DFIG)

As explained previously, the rotor circuit can be accessed by means of brushes and slip rings. Therefore the rotor can be connected to a converter and fed by AC variable-frequency current. The stator of this generator is usually connected directly to the power network; this design is thus named *"Doubly-fed Wound Rotor Induction Generator"* or also known by its short form of (DFIG). A schematic diagram of such a configuration is illustrated here:

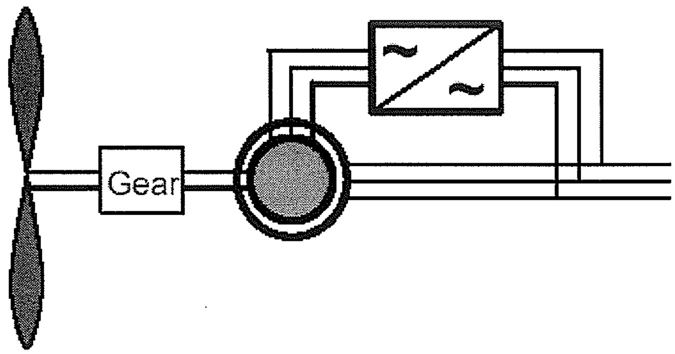

Figure 1-5: Doubly fed Wound Rotor Induction Generator (DFIG). [3]

"TACKE (later Enron, now overtaken by GE) a German Company introduced this concept for the first time and was followed by other major wind energy companies such as, Zond (Enron), DeWind, Südwind, Jacobs, Fuhrländer, Windtec, and Vestas." [5]

As stated formerly, the speed relations of a "*Squirrel-cage Induction Generator*" is depicted by this equation:

$$\omega_s - \frac{vP}{2} = \omega_r \qquad \textbf{Eq. 1-10}$$

Obviously the rotor of a DFIG system is not short circuited but is fed by the converter current; therefore its speed equation can be rewritten in the following form:

$$\omega_s - \frac{vP}{2} = \omega_I \qquad \textbf{Eq. 1-11}$$

where ω_r is replaced by ω_I (rad/sec) which is the frequency of the converter. The converter does not need to be a full-scale converter and usually a converter with the capacity of 25% of the nominal power suffices. The following figure shows a Vestas V90/2MW turbine which utilizes a "*Doubly fed Wound Rotor Induction Generator*":

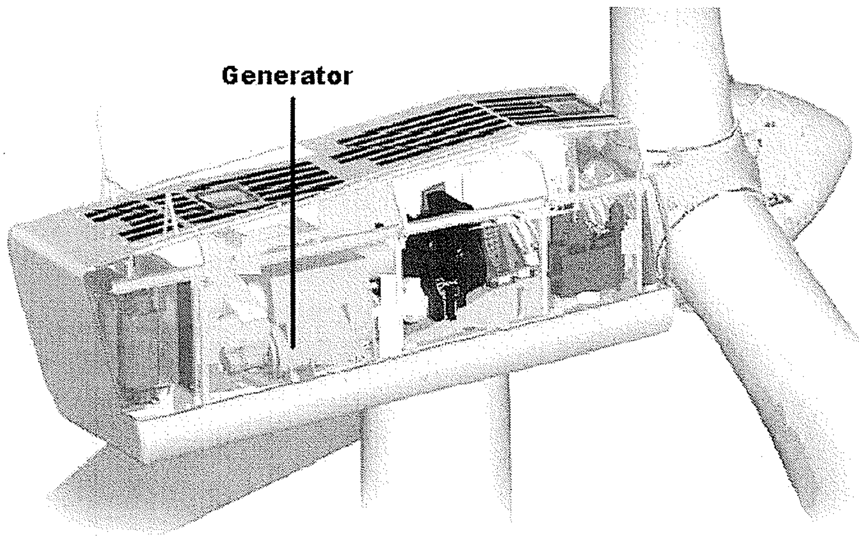

Figure 1-6: Vestas V90/2MW turbine with a "Doubly fed Wound Rotor Induction Generator". Courtesy of Vestas Wind Systems A/S. [4]

Operation Speeds:

Typically 70% of the speed range will be utilized by this generator [3], therefore this generator falls under the **variable speed** category.

Advantages:

- Full variable speed operation
- Less stress on drive train

Disadvantages:

- Brush and slip ring maintenance
- Losses in converter

1.3.1.4 Squirrel-cage Induction Generator with a full-Scale Converter

The stator of a squirrel-cage induction generator can be directly connected to the network through converter in order to facilitate a variable speed operation. The converter must be rated the same as the generator. A schematic diagram of such a configuration is illustrated here:

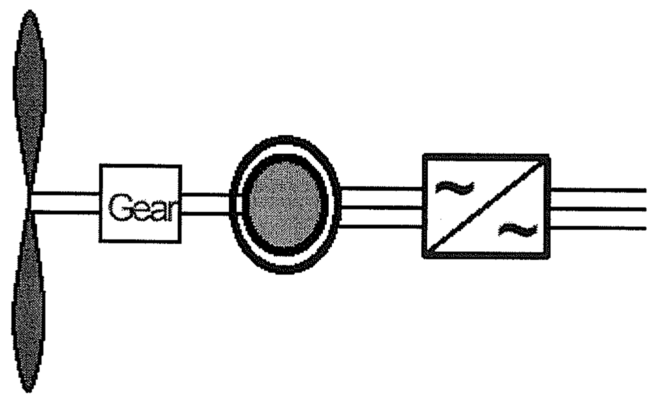

Figure 1-7: Squirrel-cage Induction Generator with a full-Scale Converter [3].

Operation Speeds:

As the stator of the generator is directly connected to the frequency converter, the range of variable speed operation theoretically is from 0 to 100%. Therefore this generator falls under **full variable speed** category.

Advantages:

- Full variable speed operation
- Less stress on drive train

Disadvantages:

- Cost of full scale frequency converter

1.3.2 Synchronous Generators

1.3.2.1 Synchronous Generator with Brushed Excitation System

This is the conventional synchronous generator, in which the DC field winding is placed on the rotor and by means of brushes and slip rings is connected to a DC source. The DC source also known as *"the excitation system"* can be a DC generator itself on the same shaft. Another option would be rectifying the AC power of the synchronous generator and feeding it back to its field winding through the above mentioned brushes and slip rings. The use of brushes and slip rings does not make this concept commercially viable for a wind turbine application where the effort is made to design generators of lower maintenance requirements. Other options to utilize synchronous generators in wind power industry are explained below.

1.3.2.2 Synchronous Generator with Brushless Excitation System

In order to eliminate the maintenance drawback of "Synchronous Generator with Brushed Excitation" another alternative came to existence called "Brushless Synchronous Machines". This machine uses a wound rotor AC machine for its excitation. The excitation system is coupled with the main machine and both are placed in the same casing. The stator of the exciter gets its power from the exciter of the synchronous machine and the AC current induced in its rotor is rectified and fed to the rotor of the synchronous machine obviating the need for brushes and slip rings. This concept is widely used in fossil fuel power plant generators and has not been exercised in wind turbines.

1.3.2.3 Permanent Magnet Synchronous Machines

Another option to eliminate the brushes and slip rings would be the use of a permanent magnet rotor field. If such a generator is designed with higher number of poles it can operate at a lower shaft speed while still producing a fast rotating field in its stator fulfilling the frequency demanded by the network. This innovation totally makes it possible to do without a gearbox and directly connect the generator to the turbine low speed shaft. This configuration is referred to as "*Direct Drive Multi Pole Permanent Magnet Synchronous Machine*". The stator is then connected to a full-scale converter, making it possible to operate as a variable speed generator. A schematic diagram of such a configuration is illustrated here:

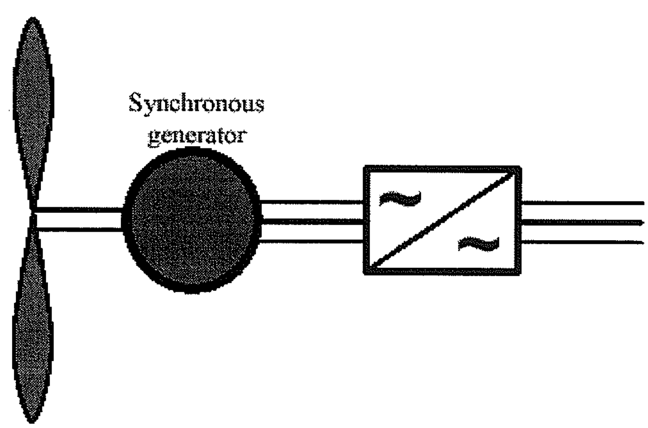

Figure 1-8: Direct Drive Multi Pole Permanent Magnet Brushless Synch. Machine [3]

Due to the large number of poles (e.g.128 poles) the structure of these generators is considerably bigger than conventional generators of the same capacity; hence they are also referred to as *"ring generators"*.

The following figure shows an Enercon E66/1.5MW turbine which utilizes a "*Direct Drive Multi Pole Permanent Magnet Synchronous Machine*" **[6]**:

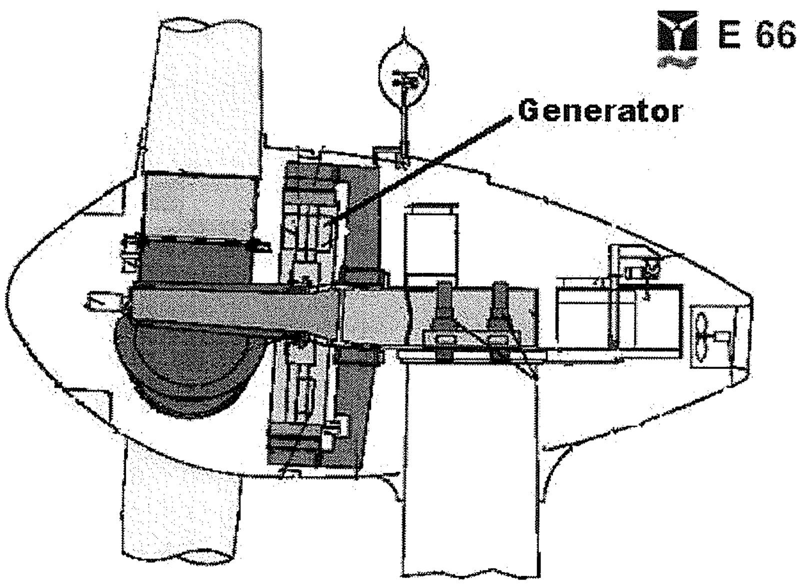

Figure 1-9: Enercon E66/1.5MW turbine with a "Direct Drive Multi Pole Permanent Magnet Synchronous Machine". Courtesy of Enercon GmbH.

Operation Speeds:

Similar to a "Squirrel-cage Induction Generator with a Full-Scale Converter" as the stator of the generator is directly connected to the frequency converter, the range of variable speed operation theoretically is from 0 to 100%. Therefore this generator falls under **full variable speed** category.

Advantages:

- Full variable speed operation
- Low wear due to slow rotation of turbine parts
- Less stress on drive train
- Gearless

Disadvantages:

- Cost of a fully rated frequency converter

All the above discussed configurations are then combined with different wind turbine blade power regulation systems such as passive (classic) stall, active stall or pitch (electrical or hydraulic) to form a complete variable speed system.

1.4 Brushless Doubly-fed Induction Generator

As discussed earlier, it might not be possible to keep the shaft speed at a constant value which is the case in wind power turbines. To overcome this problem the Patent 188,204 B1 [1] suggests a cascade configuration of two machines, referred to as a" Doubly-fed AC Field Machine". The following figure illustrates the general idea of such a design:

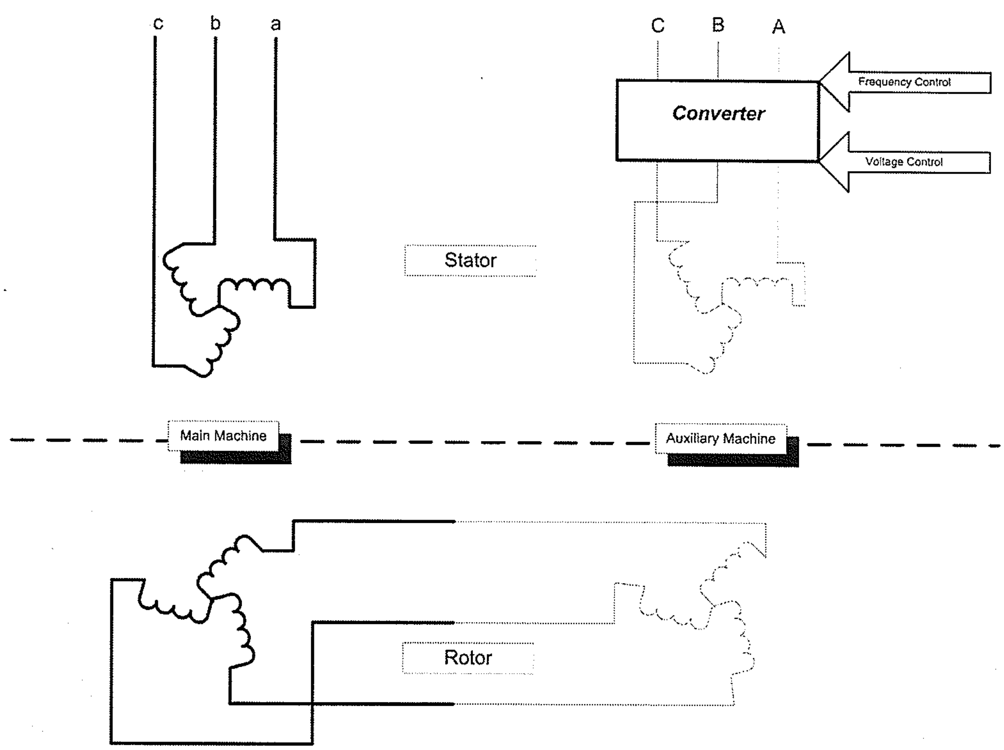

Figure 1-10: Doubly-fed AC Field Machine

1.4.1 Overview

In this figure the circuit blocks shown above the broken line are located on the stator and those below it on the rotor. As it is clear the scheme basically consists of 2 wound-rotor induction generators with their rotors connected to each other both mechanically and electrically. One machine considered to be the power generating unit and referred to as **"Main Machine"** and the other playing the role of an AC exciter, of a considerably smaller size and referred to as the **"Auxiliary Machine"**. The stator of the main machine is connected to the AC network and the stator of the auxiliary machine is connected to the output of a static converter. The input of the converter can be powered by the same AC network.

1.4.2 Description of the Machine Operation

The machine is claimed to operate in variable speeds according to the following description provided by this Patent [1]:

> *"In contrast to the earlier schemes, the field system of the present study, besides being a brushless scheme, totally eliminates the need for a coupled exciter machine. Using the new scheme it is possible to use a speed which is different from that of a corresponding DC excited machine. The new scheme enables stabilization of the frequency at the predetermined fixed value*

during speed variations. Independent control of the field is possible for adjustment of the voltage and the power factor as in conventional DC excited alternators. Contrary to conventional DC field machines, the new scheme employs an AC field winding, which is excited by an AC source. The new scheme is applicable with single phase and poly phase excitation. For the purposes of this description we will assume that the AC is poly phase. The poly phase AC source itself is located on the rotor in a separate winding and therefore there is no need for contact brushes or a separate exciter machine. We may designate this AC winding as the auxiliary rotor winding. For creating a poly phase voltage in the auxiliary rotor winding the new scheme employs an auxiliary stator winding which is separate from the main winding of the generator. The voltage and frequency of the AC voltage induced in the auxiliary rotor winding are determined by the voltage and frequency of the input to the auxiliary winding on the stator and the speed of rotation of the machine. Since the input to the poly phase main field winding is from an AC source, the field created by this winding will have a rotational speed relative to the rotor. Since the AC input to the main field winding is from the auxiliary rotor winding, this rotational speed will be determined by the frequency of the induced AC, which itself is determined by the *frequency of the AC input to the auxiliary stator winding and the speed of rotation. The stator auxiliary winding will be fed through an inverter whose frequency will be adjustable. Therefore the speed of rotation of the main field with respect to the rotor will be adjustable at any value of the speed of rotation of the rotor by adjusting the frequency of the inverter feeding the auxiliary stator winding. The speed of rotation of the main field in space is the algebraic sum of the rotational speed of the rotor and the rotational speed of the field relative to the rotor. Therefore the rotational speed of the main field relative to the stator is adjustable by adjustment of the inverter frequency. Therefore this invention makes it possible to maintain the output frequency of the generator by the adjustment of the output frequency of the inverter feeding the auxiliary stator winding. The magnitude of the induced voltage is adjustable by adjustment of the magnitude of the AC output from the inverter."* [1]

1.4.3 Points to be Noted:

1) Static converter, through which power is provided to the auxiliary stator circuit, typically may consist of a rectifier, which converts the input to DC followed by an inverter. It could also be a direct frequency converter without an intermediate DC link. The input to the static converter may be from the same AC bus as the one to which the terminals of the main winding are connected or it could be a separate one. A transformer may be included in the static converter block if needed for the purpose of voltage matching. The output frequency of the static converter may be adjustable. This may be implemented by a control signal applied to its switching control circuit. The output voltage magnitude of the static converter also may be adjustable. A voltage control signal may be used for voltage adjustment.

2) In a DC excited conventional alternator the power transferred between the prime mover and the AC source is by the process of mechanical rotation. In the present case there will be power transfer also by transformer action between the field winding and the stator winding. The ratings of the auxiliary winding and the main field winding must be adequate to handle this power.

3) Magnetic coupling between the main machine windings and the auxiliary windings can be eliminated by choosing different number of poles, as explained in Chapter 2.

In conclusion, the main advantage of the machine over the existing brushless systems is claimed in the Patent to be its capability of operating at variable speeds.

It is also worth mentioning that due to some marketing restrictions in North America on "Doubly-fed Induction Generators" (DFIG) another advantage of this concept is to achieve variable speed operation in wind generators without conflicting with other patented designs.

1.5 `Variable Speed Pitch Regulated Operation

Although stall regulation can be used in a variable speed wind turbine system, pitch regulation seems to be a more precise aerodynamic control over higher range of wind speeds. Therefore, several major wind turbine manufactures have favored this method more than stall regulation techniques. The following figure illustrates how a pitch regulated system acts in the different operating regions of a wind turbine together with generator torque control.

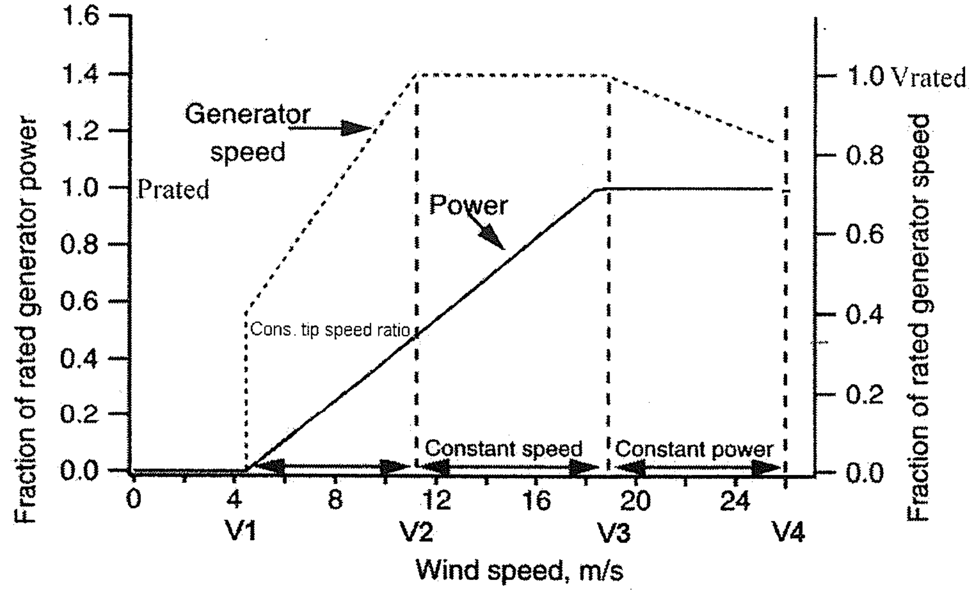

Figure 1-11: Variable speed pitch regulated operation [7]

In the above figure, V_1 and V_2 are cut-in and rated wind speeds respectively. The region between these two speeds is called the constant tip speed ratio region as the blade pitch is kept constant, the turbine operates at variable speed and generator torque is used to maintain the optimal tip speed ratio. It should also be noted that V_2 is also when the generator reaches its rated speed (V_{rated}). Once the wind speed passes V_2 the turbine enters the "Constant Generator speed" region. In this region pitch acts to keep the generator speed constant while the power increases with wind speed. As the wind speed reaches V_3, the turbine enters the "Constant Power" region where the generator produces its rated power (P_{rated}). Within this region pitch acts to keep the speed within the acceptable limits and generator torque is used to control its output power till the wind speed reaches at V_4 which is the cut-out wind speed of the turbine.

We know that the power in the wind changes with the cube of wind speed where as the first operating region of the above figure is suggesting a linear relation between wind speed and output power. This is simply due to the power coefficient curve of the turbine as shown in the following picture:

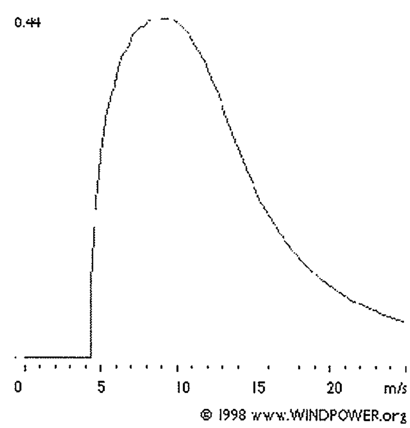

Figure 1-12: Power Coefficient vs. Wind Speed [8]

Combining the above picture and the fact that power in the wind varies with the cube of wind speed yields a nearly linear relationship between the wind speed and output power of the turbine in the first operating region. In Chapters 2 and 3 we will see how a "Brushless Doubly-fed AC Field Machine" can fit within the above operating zones.

1.6 Chapter Summary

In this chapter the importance of studying wind energy was discussed followed by an introduction to various types of generators used in wind turbines.

From the generator point of view, wind energy systems fall under the two major categories of induction (asynchronous) and synchronous generators. Different configurations were taken into consideration ranging from fixed speed wind turbines to full variable-speed ones utilizing these two types of generators. It was realized that due to the intermittent, variable nature of the wind some of them are more suitable to wind power generation than other designs; in other words variable speed generators are preferred over fixed speed designs. On the other hand, when the options are narrowed down to variable speed configurations, some disadvantages appear to be challenging to tackle, for instance the high cost of full scale converters, maintenance of brushes and slip rings, and often times the weight and cost of the huge gear boxes. In order to overcome some of the above problems, a "Brushless Doubly-fed AC Field Machine" is introduced based on a patent to be discussed in the following chapters in more detail. The variable speed pitch regulated system was briefly explained and will be used in Chapter 3 as the aerodynamic control option for a "Brushless Doubly-fed AC Field Machine".

Chapter 2: Mathematical Model of the Machine

2.1 Main Machine:

The following well-known equation shows the speed and frequency relation in general AC generators:

$$v_{s_M} = \frac{2\omega_s}{P} \ {rad}/{\sec} \qquad \textbf{Eq. 2-1}$$

In the main machine P_M is the number of poles, thus:

$$v_{s_M} = \frac{2\omega_s}{P_M} \ {rad}/{\sec} \qquad \textbf{Eq. 2-2}$$

Where:

v_{s_M} = Main machine stator field speed wrt stator (synch.speed) $({rad}/{\sec})$

ω_s = Main stator(Network) frequency$({rad}/{\sec})$

P_M = Number of the main machine poles

The summation of the mechanical speed of the shaft and the slip speed is equal to the synchronous speed:

$$v_{s_M} = v + v_{r_M} \qquad \textbf{Eq. 2-3}$$

where:

ν : Mechanical(shaft) speed(rad/sec)

ν_{r_M} : Main machine rotor field speed wrt rotor (slip speed)(rad/sec)

Thus by substituting ν_{s_M} in the above equation we can write:

$$\nu + \nu_{r_M} = \frac{2\omega_s}{P_M} \qquad \textbf{Eq. 2-4}$$

For the rotor of the main machine it is also obvious that:

$$\nu_{r_M} = \frac{2\omega_{r_M}}{P_M} \qquad \textbf{Eq. 2-5}$$

where:

ω_{r_M} = Main machine rotor field frequency (rad/sec)

Again by substituting ν_{r_M} in equation $\nu + \nu_{r_M} = \frac{2\omega_s}{P_M}$ **Eq. 2-4** we can write:

$$\nu + \frac{2\omega_{r_M}}{P_M} = \frac{2\omega_s}{P_M} \qquad \textbf{Eq. 2-6}$$

Rewriting the above equation yields:

$$\omega_s = \frac{P_M \nu}{2} + \omega_{r_M} \qquad \textbf{Eq. 2-7}$$

As the rotors of both machines are mechanically and electrically connected to each other, the rotor frequencies of both machines are the same ($\omega_{r_M} = \omega_{r_A}$) and referred to as ω_r, therefore:

$$\omega_s = \frac{P_M \nu}{2} + \omega_r \qquad \textbf{Eq. 2-8}$$

2.2 Auxiliary Machine:

Similar to what was discussed for the main machine, except that we start from the rotor, for the auxiliary machine the following equation also applies:

$$v_{r_A} = \frac{2\omega_r}{P_A} \qquad \textbf{Eq. 2-9}$$

where:

v_{r_A} = Auxiliary machine rotor field speed wrt rotor (Slip Speed)(rad/sec)
P_A = Number of the auxiliary machine poles

For the stator of the auxiliary machine it is also obvious that:

$$v_{s_A} = \frac{2\omega_I}{P_A} \qquad \textbf{Eq. 2-10}$$

where:

v_{s_A} = Main machine stator field speed wrt stator (synch. speed) (rad/sec)
ω_I = Auxiliary stator(Inverter) frequency(rad/sec)

We know that "auxiliary stator field synchronous speed" v_{s_A} is equal to the summation of "auxiliary machine slip speed" v_{r_A} and "mechanical (shaft) speed v :

$$v_{s_A} = v_{r_A} + v \qquad \textbf{Eq. 2-11}$$

By substituting v_{s_A} and v_{r_A} in the above equation we can write:

$$\frac{2\omega_I}{P_A} = \frac{2\omega_r}{P_A} + v \qquad \textbf{Eq. 2-12}$$

Rewriting the above equation yields:

$$\omega_r = \omega_I - \frac{P_A v}{2} \qquad \textbf{Eq. 2-13}$$

The last step in finding the speed relations of the "Brushless Doubly-fed AC Field Machine" would be substituting ω_r in equation 2.8:

$$\omega_s = \frac{P_M v}{2} + \omega_I - \frac{P_A v}{2} \qquad \textbf{Eq. 2-14}$$

Rewriting the above equation yields:

$$\omega_s = \frac{P_M v}{2} - \frac{P_A v}{2} + \omega_I \qquad \textbf{Eq. 2-15}$$

$$\omega_s = \frac{(P_M - P_A)v}{2} + \omega_I \qquad \textbf{Eq. 2-16}$$

Assuming a 4-pole main machine and a 2-pole auxiliary machine, the above equation is reduced to a shorter form:

$$\omega_s = v + \omega_I \qquad \textbf{Eq. 2-17}$$

The following table shows the machine speed and frequency parameters at a glance:

Table 2-1: Main and auxiliary machines speed and frequency parameters in rad/sec

ν = Mechanical (shaft) speed (rad/sec)	
Main Machine	P_M: Number of poles
	ω_s: Stator/network Frequency (rad/sec)
	ν_{sM} : Stator field speed wrt stator (synch. speed) (rad/sec)
	ν_{rM} : Rotor field speed wrt rotor (slip speed) (rad/sec)
	ω_{rM} : Rotor field frequency (rad/sec)
Aux. Machine	P_A: Number of poles
	ω_{rA}: Rotor field frequency (rad/sec)
	ν_{rA} :Rotor field speed wrt rotor (slip speed) (rad/sec)
	ν_{sA} : Stator field speed wrt stator (synch. speed) (rad/sec)
	$\omega_{sA} = \omega_I$:Stator/Inverter Frequency (rad/sec)

2.3 Verification of Network and Inverter Frequency Relation (Stator of the main machine and stator of the auxiliary machine)

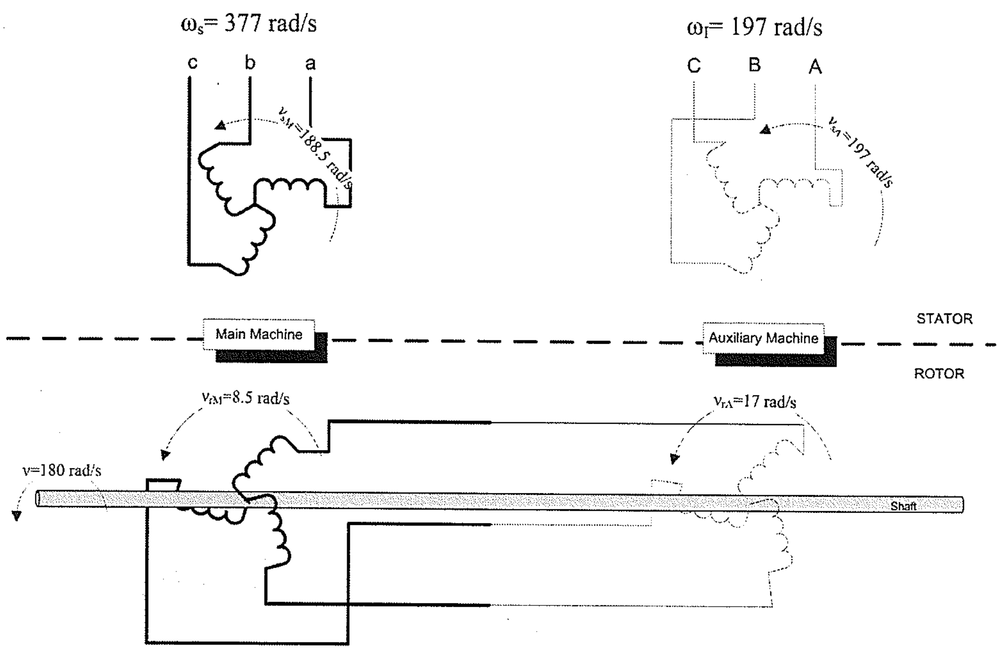

Figure 2-1: Representation of main and auxiliary machines speed and frequency parameters

Starting from stator of main machine we calculate the angular velocity of the magnetic field in the aux. machine stator (ω_I). Provided that stator of the main machine is connected to a 60 Hz network and mechanical speed of the shaft (v) is 180 rad/sec:

$$\omega_S = 377 \ {}^{rad}\!/_{s}$$

$$4 \ pole \quad main \ stator \rightarrow v_{sM} = \frac{2\omega_s}{p} = \frac{2 \times 377}{4} = 188.5$$

$$
\begin{aligned}
v_{sM} &= v + v_{rM} \\
v_{rM} &= v_{sM} - v \\
&= 188.5 - 180 \\
&= 8.5\, {rad}/{sec}
\end{aligned}
$$

$$4 \text{ pole main rotor} \rightarrow v_{rM} = \frac{2\omega_{rM}}{p} \rightarrow \omega_{rM} = 2\times v_{rM} = 2\times 8.5 = 17\, {rad}/{sec}$$

Knowing that the rotors of both main and auxiliary machines are connected to each other, $\omega_{rA}=\omega_{rM}=17 rad/sec$

$$2 \text{ pole aux. rotor} \rightarrow v_{rA} = \frac{2\omega_{rA}}{p} \rightarrow \omega_{rA} = v_{rA} = 17\, {rad}/{sec}$$

$$
\begin{aligned}
v_{sA} &= v + v_{rA} \\
&= 180 + 17 \\
&= 197\, {rad}/{sec}
\end{aligned}
$$

$$\left.\begin{array}{r} \left.\begin{array}{r} 2 \text{ pole aux. stator} \rightarrow v_{sA} = \dfrac{2\omega_{sA}}{p} \rightarrow \omega_{sA} = v_{sA} \\ v_{sA} = 197\, {rad}/{sec} \end{array}\right\} \Rightarrow \omega_{sA} = 197\, {rad}/{sec} \\ \omega_{sA} = \omega_I \end{array}\right\} \Rightarrow$$

$$\omega_I = 197\, {rad}/{sec}$$

Verification by the equation:

$$
\begin{aligned}
\omega_S &= \omega_I - v \rightarrow \\
\omega_I &= \omega_S + v \\
&= 377 - 180 \\
&= 197\ {rad}/{sec}
\end{aligned}
$$

This shows the authenticity of the equation.

2.4 Operation at the Synchronous Speed

In the numerical example we assumed that the stator of the main machine is connected to a 60 Hz network: $\omega_s = 377 \ {rad}/{s}$

And the mechanical speed of the shaft is:

$\nu = 180 \ {rad}/{s}$

Therefore the inverter frequency is:

$\omega_l = 197 \ {rad}/{s}$

But if the mechanical speed of the shaft continues increasing and reaches the shaft synchronous speed:

$$\nu = 188.5 \ {rad}/{s} \qquad \textbf{Eq. 2-18}$$

And considering that $\omega_s = \nu + \omega_l$, it would be concluded that:

$$\omega_l = 188.5 \ {rad}/{s} \qquad \textbf{Eq. 2-19}$$

This implies that at the synchronous speed of the main generator the rotor frequency must be zero which means the auxiliary machine must induce a DC current into the rotor which is not possible.

Therefore the machine can appropriately work under or over synchronous speed but is not capable of working exactly at the synchronous speed of the main machine. This is examined in Chapter 3.

2.5 Mutual Magnetic Coupling of a 4-pole and a 2-pole Winding

From Appendix D.3 we know that the flux density of a four pole machine is equal to:

$$B_{\theta 4} = \hat{B}Cos(\omega_s t + \alpha_m - 2\theta)$$

We also know that the flux linkage of a two pole machine could be calculated by the following integration:

$$\lambda_{turn_2} = \int_{\theta}^{\theta+\pi} B_{\theta 4} \times l \times r \times d\theta \qquad \textbf{Eq. 2-20}$$

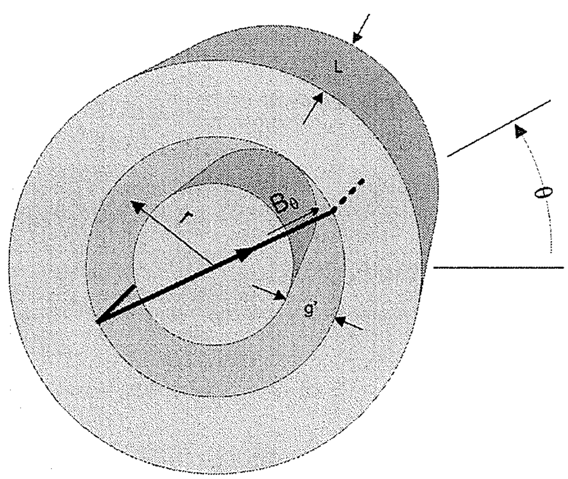

Figure 2-2: Flux linkage of a 2 pole machine [11]

Therefore, to investigate the "mutual magnetic coupling" of a 4-pole winding with a 2-pole winding we can calculate the flux linkage of a 2-pole winding by substituting the $B_{\theta 4}$ in the above equation:

$$\lambda_{turn} = \int_{\theta}^{\theta+\pi} \hat{B}Cos(\omega_s t + \alpha_m - 2\theta) \times l \times r \times d\theta$$

$$= \hat{B}lr \int_{\theta}^{\theta+\pi} Cos(\omega_s t + \alpha_m - 2\theta) d\theta$$

$$= -\frac{1}{2}\hat{B}lrSin(\omega_s t + \alpha_m - 2\theta)\Big|_{\theta}^{\theta+\pi}$$

$$= -\frac{1}{2}\hat{B}lr\left[Sin(\omega_s t + \alpha_m - 2\theta - 2\pi) - Sin(\omega_s t + \alpha_m - 2\theta)\right]$$

$$= -\frac{1}{2}\hat{B}lr\left[Sin(\omega_s t + \alpha_m - 2\theta) - Sin(\omega_s t + \alpha_m - 2\theta)\right]$$

$$\lambda_{turn} = 0 \qquad \textbf{Eq. 2-21}$$

This implies that there will not be any "mutual magnetic coupling" between a 4-pole winding and a 2-pole winding which gives us the leeway of winding both main and auxiliary machines in the same housing as an integrated winding if desired. This has the advantage of a more compact design.

2.6 Equivalent Circuit of the Machine

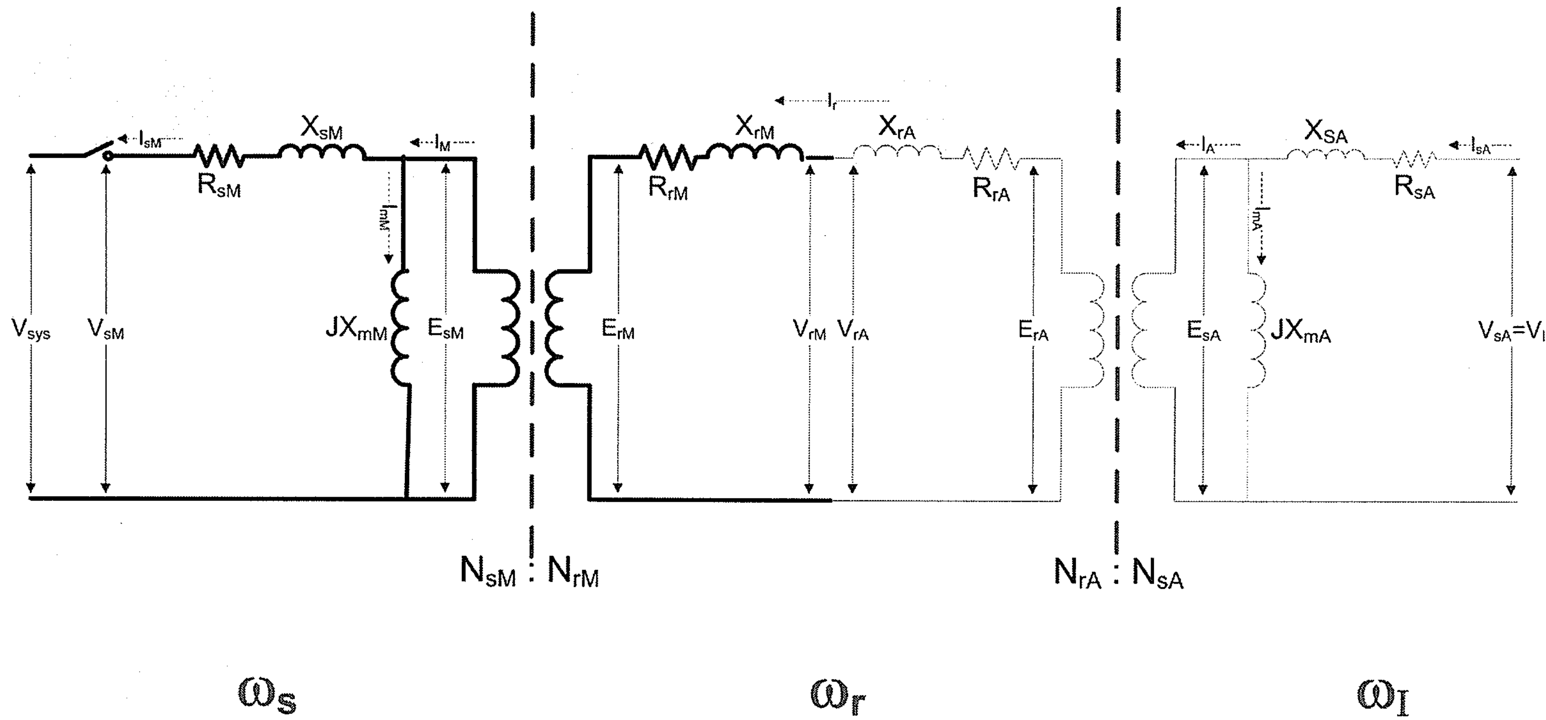

Figure 2-3: Equivalent circuit of the machine

Table 2-2: Main Machine Parameters

Stator	R_{sM}	Main machine per-phase stator winding resistance
	X_{sM}	Main machine per-phase stator leakage inductance
Rotor	R_{rM}	Main machine per-phase rotor winding resistance
	X_{rM}	Main machine per-phase rotor leakage inductance
X_{mM}		Main machine per-phase magnetizing inductance

Table 2-3: Auxiliary Machine Parameters

Stator	R_{sA}	Auxiliary machine per-phase stator winding resistance
	X_{sA}	Auxiliary machine per-phase stator leakage inductance
Rotor	R_{rA}	Auxiliary machine per-phase rotor winding resistance
	X_{rA}	Auxiliary machine per-phase rotor leakage inductance
X_{mA}		Auxiliary machine per-phase magnetizing inductance

2.7 Phasor Calculations

2.7.1 Calculation of Inverter Voltage (VI) in Terms of System Values

Open-Circuit Operation

Assuming that the machine is stationary (considered as a transformer) and at steady state, we know that:

$$I_{s_M} = 0$$

thus: $\overline{E}_{s_M} = \overline{V}_{s_M}$

$$\overline{E}_{s_M} = jX_{m_M}\overline{I}_M \quad \rightarrow \quad \overline{I}_M = \frac{\overline{E}_{s_M}}{jX_{m_M}} = \frac{\overline{V}_{s_M}}{jX_{m_M}} \qquad \textbf{Eq. 2-22}$$

$$\frac{\overline{I}_M}{\overline{I}_r} = \frac{N_{r_M}}{N_{s_M}} \quad \rightarrow \quad \overline{I}_r = \frac{N_{s_M}}{N_{r_M}} \times \overline{I}_M = \frac{N_{s_M}}{N_{r_M}} \times \frac{\overline{V}_{s_M}}{jX_{m_M}} \qquad \textbf{Eq. 2-23}$$

$$\left.\begin{array}{l} \dfrac{\overline{E}_{s_M}}{E_{rM}} = \dfrac{N_{s_M}}{N_{r_M}} \quad \rightarrow \quad E_{rM} = \dfrac{N_{r_M}}{N_{s_M}} \times \overline{E}_{s_M} = \dfrac{N_{r_M} \times \overline{V}_{s_M}}{N_{s_M}} \\ \overline{E}_{r_A} = \overline{E}_{r_M} + \left[(R_{r_M} + R_{r_A}) + j(X_{r_m} + X_{r_A})\right]\overline{I}_r \end{array}\right\} \rightarrow$$

$$\overline{E}_{r_A} = \frac{N_{r_M} \times \overline{V}_{s_M}}{N_{s_M}} + \left[(R_{r_M} + R_{r_A}) + j(X_{r_m} + X_{r_A})\right] \times \left[\frac{N_{s_M}}{N_{r_M}} \times \frac{\overline{V}_{s_M}}{jX_{m_M}}\right] \qquad \textbf{Eq. 2-24}$$

$$\frac{\overline{E}_{r_A}}{\overline{E}_{s_A}} = \frac{N_{r_A}}{N_{s_A}}$$

thus:

$$\overline{E}_{s_A}=\frac{N_{s_A}}{N_{r_A}}\times\overline{E}_{r_A}=\frac{N_{s_A}}{N_{r_A}}\left[\frac{N_{r_M}\times\overline{V}_{s_M}}{N_{s_M}}+\left[\left(R_{r_M}+R_{r_A}\right)+j(X_{r_m}+X_{r_A})\right]\times\left[\frac{N_{s_M}}{N_{r_M}}\times\frac{\overline{V}_{s_M}}{jX_{m_M}}\right]\right]$$

Eq. 2-25

For $\overline{I}_{s_A}$ we have

$$\left.\begin{array}{r}\overline{I}_{m_A}=\dfrac{jX_{m_A}}{\overline{E}_{s_A}}\\ \\ \dfrac{\overline{I}_r}{\overline{I}_A}=\dfrac{N_{s_A}}{N_{r_A}}\quad\rightarrow\quad\overline{I}_A=\dfrac{N_{r_A}}{N_{s_A}}\overline{I}_r\end{array}\right\}\rightarrow\ \overline{I}_{s_A}=\overline{I}_{m_A}+\overline{I}_A=\frac{jX_{m_A}}{\overline{E}_{s_A}}+\frac{N_{r_A}}{N_{s_A}}\overline{I}_r \qquad \textbf{Eq. 2-26}$$

$$\left.\begin{array}{r}\overline{I}_{s_A}=\dfrac{jX_{m_A}}{\overline{E}_{s_A}}+\dfrac{N_{r_A}}{N_{s_A}}\overline{I}_r\\ \text{From before}:\overline{I}_r=\dfrac{N_{s_M}}{N_{r_M}}\times\dfrac{\overline{V}_{s_M}}{jX_{m_M}}\end{array}\right\}$$

Hence we have

$$\overline{I}_{s_A}=\frac{jX_{m_A}}{\overline{E}_{s_A}}+\frac{N_{r_A}\times\dfrac{N_{s_M}}{N_{r_M}}\times\dfrac{\overline{V}_{s_M}}{jX_{m_M}}}{N_{s_A}}$$

which can be rewritten as

$$\overline{I}_{s_A}=\frac{jX_{m_A}}{\overline{E}_{s_A}}+\frac{N_{r_A}\times N_{s_M}\times\overline{V}_{s_M}}{N_{r_M}N_{s_A}jX_{m_M}} \qquad \textbf{Eq. 2-27}$$

$\overline{V}_I$ is calculated as

$$\overline{V}_I=\overline{E}_{s_A}+\left(R_{s_A}+jX_{s_A}\right)\overline{I}_{s_A} \qquad \textbf{Eq. 2-28}$$

Substituting for $\overline{I}_{s_A}$ in the above equation yields

$$\overline{V}_I = \overline{E}_{s_A} + \left(R_{s_A} + jX_{s_A}\right)\left(\frac{jX_{m_A}}{\overline{E}_{s_A}} + \frac{N_{r_A} \times N_{s_M} \times \overline{V}_{s_M}}{N_{r_M} N_{s_A} jX_{m_M}}\right) \quad \textbf{Eq. 2-29}$$

And eventually substituting for $\overline{E}_{s_A}$ in the above equation yields the inverter voltage:

$$\overline{V}_I = \frac{N_{s_A}}{N_{r_A}}\left[\frac{N_{r_M} \times \overline{V}_{s_M}}{N_{s_M}} + \left[\left(R_{r_M} + R_{r_A}\right) + j(X_{r_m} + X_{r_A})\right] \times \left[\frac{N_{s_M}}{N_{r_M}} \times \frac{\overline{V}_{s_M}}{jX_{m_M}}\right]\right]$$
$$+ \left(R_{S_A} + jX_{S_A}\right) \times$$
$$\left[\frac{jX_{m_A}}{\frac{N_{s_A}}{N_{r_A}}\left[\frac{N_{r_M} \times \overline{V}_{s_M}}{N_{s_M}} + \left[\left(R_{r_M} + R_{r_A}\right) + j(X_{r_m} + X_{r_A})\right] \times \left[\frac{N_{s_M}}{N_{r_M}} \times \frac{\overline{V}_{s_M}}{jX_{m_M}}\right]\right]} + \frac{N_{r_A} \times N_{s_M} \times \overline{V}_{s_M}}{N_{r_M} N_{s_A} jX_{m_M}}\right]$$

Eq. 2-30

2.8 Ratio of Different Quantities of the Machine

For a deeper understanding of the "Brushless Doubly-fed AC Field Machine", equations of a basic electric machine are presented in Appendix D: Equations based on which the ratio of different machine quantities are summarized as follows.

2.8.1 Standstill

Table 2-4: Ratio of Different Quantities of a Standstill Machine

Quantity	Stator	Rotor
VOLTAGE	1	$\frac{N_r}{N_s}\angle-\beta$
CURRENT	1	$\frac{N_r}{N_s}\angle-\beta$
IMPEDANCE	1	$\left(\frac{N_r}{N_s}\right)^2$
POWER	1	1

The ratios of stator and rotor voltages, currents, impedances and powers imply that a *"standstill"* machine acts as an ideal transformer.

2.8.2 Rotating

Table 2-5: Ratio of Different Quantities of a Rotating Machine

Quantity	Stator	Rotor
VOLTAGE	1	$\frac{N_r}{N_s} \times S \angle -\beta_\circ$
CURRENT	1	$\frac{N_r}{N_s} \angle -\beta_\circ$
IMPEDENCE	1	$\left(\frac{N_r}{N_s}\right)^2 \times S$
POWER	1	S
FREQUENCY	ω_s	$(v_s - v) = v_r$

The rotor induced voltage, impedance and power imply that these quantities are proportional to the speed of rotation in a *"rotary"* machine. As manifested in the above tables, it is worth mentioning that the current ratio of the machine is independent of the rotor speed. Considering the fact that both machines are identical to each other (except for the number of poles and their rated MVA), the above relations are valid for both machines.

2.9 Calculation of Slips (Main and Auxiliary Machines)

Knowing that the slip is defined by the following equation

$$S = \frac{v_s - v}{v_s} \qquad \textbf{Eq. 2-31}$$

According to section 2.3 slip of both main and auxiliary machines can be calculated as

$$S_M = \frac{v_{sM} - v}{v_{sM}} = \frac{188.5 - 180}{188.5} = \frac{8.5}{188.5} = 0.045$$

$$S_A = \frac{v_{sA} - v}{v_{sA}} = \frac{197 - 180}{197} = \frac{17}{197} = 0.086$$

It is once more noted that for the main machine,

$$v_{sM} = \frac{\omega_s}{2}$$

and for the auxiliary machine,

$$v_{sA} = \omega_I$$

The following figure shows the inverter frequency (ωI) rad/sec vs. mech. speed of the shaft (v) rad/sec

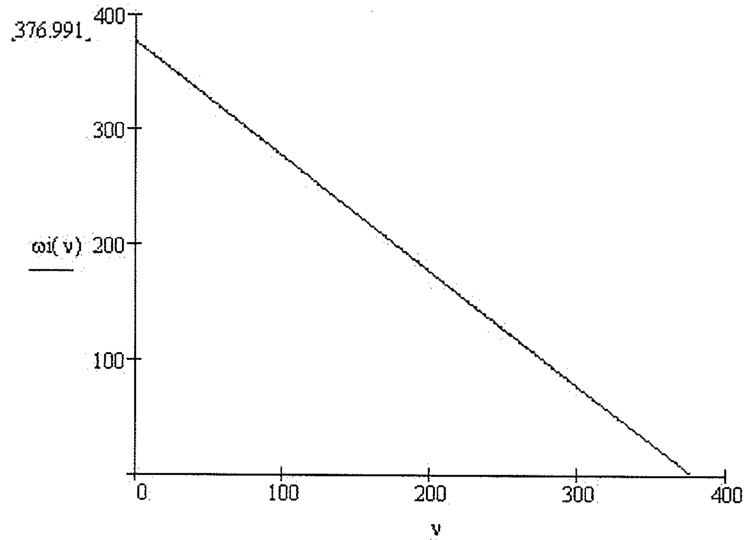

Figure 2-4: Inverter frequency (ωI) rad/sec vs. mech. speed of the shaft (v) rad/sec

The phase relationship between the stator voltage and the network voltage prior to synchronization can be controlled by varying the phase of the inverter voltage. Once synchronized, the inverter phase angle, δ_i , can control the phase or load angle δ_s of the main machine. In a similar fashion control of the inverter phase angle will control the active power output of the main machine. If we increase the output voltage of inverter(E_i), it pushes more current (I_i) to the auxiliary machine and creates more current in rotor of main machine which creates more current in the stator of the main machine and injects "reactive power" to the network. In the same fashion by decreasing the output voltage of inverter(E_i), the reactive power can be absorbed from the network.

2.10 Terminal Voltage Relation of the Main and Auxiliary Machines

Knowing that the stator and rotor voltage ratio of the main machine (provided that $\frac{N_{r_M}}{N_{s_M}} = 1$) is

$$1 : S_M ,$$

the voltage of the rotor can be found:

$$E_{r_M} = S_M \times E_{s_M} \qquad \textbf{Eq. 2-32}$$

and also before closing the breaker (i.e., connecting the stator of the main machine to the network):

$$E_{s_M} = V_{s_M}$$

thus

$$\left.\begin{matrix} E_{r_M} = S_M \times V_{s_M} \\ E_{r_M} \cong V_{r_M} \end{matrix}\right\} \rightarrow V_{r_M} = S_M \times V_{s_M}$$

$$V_{s_M} = \frac{V_{r_M}}{S_M} \qquad \textbf{Eq. 2-33}$$

and in the same fashion for the auxiliary machine the stator and rotor voltage ratio of the main machine (provided that $\frac{N_{r_A}}{N_{s_A}} = 1$) is

$$1 : S_A$$

$$\left.\left.\begin{array}{l}E_{r_A} = S_A \times E_{s_A} \xrightarrow{E_{s_A} \cong V_{s_A}} E_{r_A} = S_A \times V_{s_A} \\ \qquad\qquad\qquad\qquad\quad E_{rA} \cong V_{r_A}\end{array}\right\} \rightarrow V_{r_A} = S_A \times V_{s_A}\right\} \rightarrow$$

The rotors of both machines are connected together, as a result: $V_{r_A} \cong V_{r_M}$

$$V_{r_M} = S_A \times V_{s_A}$$

$$V_{s_M} = \frac{S_A}{S_M} \times V_{s_A} \qquad \textbf{Eq. 2-34}$$

2.11 Power Relation of the Main and Auxiliary Machines

Knowing that the power transfer ratio from stator of the main machine to its rotor is

$$1 : S_M \, ,$$

power transferred to the rotor can be found

$$P_{r_M} = S_M \times P_{s_M} \qquad \textbf{Eq. 2-35}$$

and in the same fashion the power transfer ratio from stator of the auxiliary machine to its rotor is

$$1 : S_A$$

$$P_{r_A} = S_A \times P_{s_A}, P_{s_A} = \frac{1}{S_A} \times \underbrace{P_{r_A}}$$

$$\xrightarrow{P_{r_A} = P_{r_M}} P_{s_A} = \frac{1}{S_A} \times S_M \times P_{s_M}$$

$$P_{s_A} = \frac{S_M}{S_A} \times P_{s_M} \qquad \textbf{Eq. 2-36}$$

2.12 Direct Connection of the Rotor Circuits

It was shown that the power relation of the main and auxiliary machines is depicted by the following equation:

$$P_{s_A} = \frac{S_M}{S_A} \times P_{s_M}$$

This shows that the rating of the auxiliary machine (and consequently the inverter) is directly related to the proportion of main machine's slip over auxiliary machine's slip (i.e. $\frac{S_M}{S_A}$), to investigate this more meticulously the $\frac{S_M}{S_A}$ is calculated with the shaft speed ranging from 0.4 p.u. to 2.5 p.u. . (It is assumed that the gearbox ratio of this turbine is 63.2:1 as in a Vestas V47 660kw turbine.) Table 2-1 indicates that the auxiliary machine is consuming a large portion of the power produced by the main machine over much of the speed range. It is also noted that as the shaft speed increases the rating of the inverter decreases which might make the direct sequence connection of the rotors a proper option where the wind mill is operating at higher speeds. This is discussed more in the following sections.

Table 2-6: Inverter Power at different Shaft Speeds (Direct connection of the rotor circuits)

Low speed shaft (rpm)	High speed shaft (rpm)	V (rad/sec)	V_{s_M} (rad/sec)	$V_{M_{pu}}$ (p.u.)	V_{s_A} (rad/sec)	fi (Hz)	S_M	S_A	$\frac{P_{s_A}}{P_{s_M}} = \frac{S_M}{S_A}$
11.3927	720.0171	75.4	188.5	**0.4**	301.6	**48**	0.6	0.75	**0.8**
14.2408	900.0214	94.25	188.5	**0.5**	282.75	**45**	0.5	0.67	**0.75**
17.089	1080.026	113.1	188.5	**0.6**	263.9	**42**	0.4	0.57	**0.7**
19.9372	1260.03	131.95	188.5	**0.7**	245.05	**39**	0.3	0.46	**0.65**
22.7854	1440.034	150.8	188.5	**0.8**	226.2	**36**	0.2	0.33	**0.6**
25.6335	1620.039	169.65	188.5	**0.9**	207.35	**33**	0.1	0.18	**0.55**
28.4817	1800.043	188.5	188.5	**1**	188.5	**30**	0	0	**-**
31.3299	1980.047	207.35	188.5	**1.1**	169.65	**27**	-0.1	-0.2	**0.45**
34.178	2160.051	226.2	188.5	**1.2**	150.8	**24**	-0.2	-0.5	**0.4**
37.0262	2340.056	245.05	188.5	**1.3**	131.95	**21**	-0.3	-0.9	**0.35**
39.8744	2520.06	263.9	188.5	**1.4**	113.1	**18**	-0.4	-1.3	**0.3**
42.7225	2700.064	282.75	188.5	**1.5**	94.25	**15**	-0.5	-2	**0.25**
45.5707	2880.068	301.6	188.5	**1.6**	75.4	**12**	-0.6	-3	**0.2**
48.4189	3060.073	320.45	188.5	**1.7**	56.55	**9**	-0.7	-4.7	**0.15**
51.267	3240.077	339.3	188.5	**1.8**	37.7	**6**	-0.8	-8	**0.1**
54.1152	3420.081	358.15	188.5	**1.9**	18.85	**3**	-0.9	-18	**0.05**
56.9634	3600.086	377	188.5	**2**	0	**0**	-1	-	**-**
59.8115	3780.09	395.85	188.5	**2.1**	-18.85	**-3**	-1.1	22	**-0.05**
62.6597	3960.094	414.7	188.5	**2.2**	-37.7	**-6**	-1.2	12	**-0.1**
65.5079	4140.098	433.55	188.5	**2.3**	-56.55	**-9**	-1.3	8.67	**-0.15**
68.3561	4320.103	452.4	188.5	**2.4**	-75.4	**-12**	-1.4	7	**-0.2**
71.2042	4500.107	471.25	188.5	**2.5**	-94.25	**-15**	-1.5	6	**-0.25**

2.13 Reverse Connection of the Rotor Circuits

As the "Direct Connection of the Rotor Circuits" does not seem to be capable of operating over a continuous speed range (running at synchronous speed is not possible), therefore the "Reverse Connection of the Rotor Circuits" is studied as another alternative to this concept as shown in the following figure:

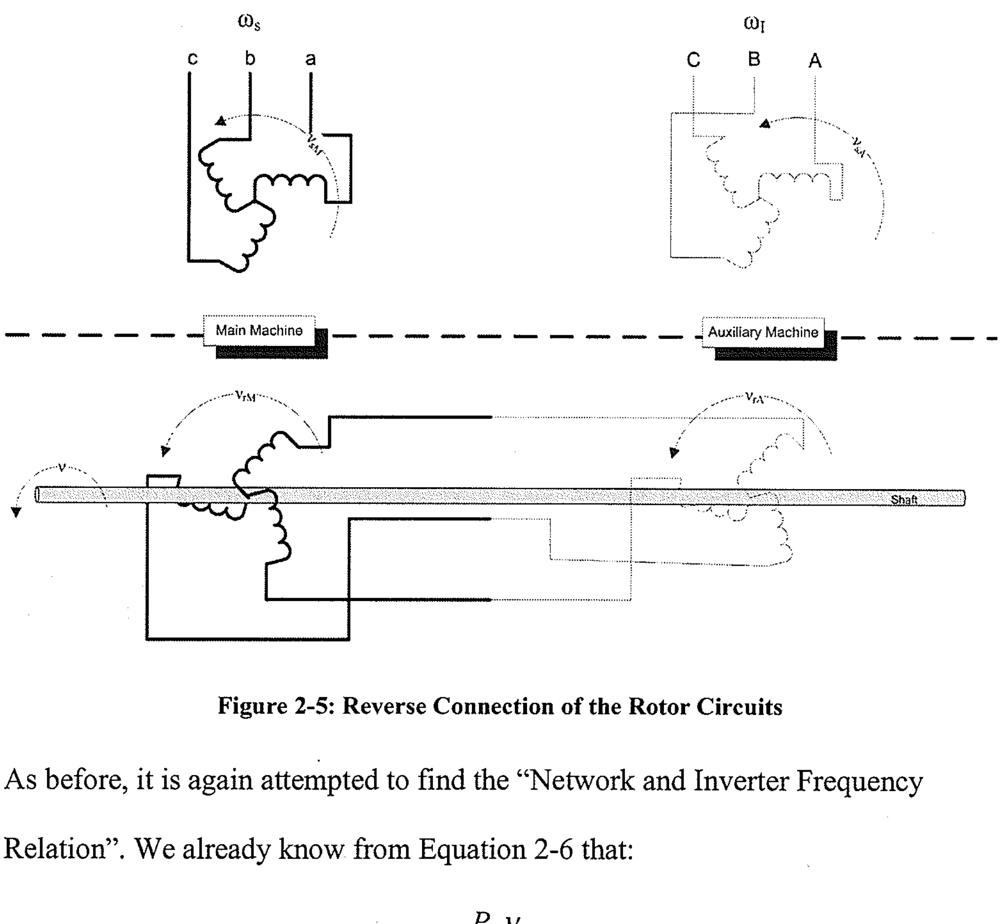

Figure 2-5: Reverse Connection of the Rotor Circuits

As before, it is again attempted to find the "Network and Inverter Frequency Relation". We already know from Equation 2-6 that:

$$\omega_s = \frac{P_M v}{2} + \omega_{r_M}$$

It is obvious that due to the "Reverse connection of the rotor circuits" the above equation can be rewritten as:

$$\omega_s = \frac{P_M v}{2} - \omega_{r_M} \qquad \textbf{Eq. 2-37}$$

Also equation 2-12 suggests that:

$$\omega_{r_M} = \omega_I - \frac{P_A v}{2}$$

By substituting the above equation in equation 2-75 we have

$$\omega_s = \frac{P_M v}{2} - \omega_I + \frac{P_A v}{2}$$

Rewriting the above equation yields:

$$\omega_s = \frac{(P_M + P_A)v}{2} - \omega_I \qquad \textbf{Eq. 2-38}$$

Assuming a 4-pole main machine and a 2-pole auxiliary machine, the above equation is reduced to

$$\omega_s = 3v - \omega_I \qquad \textbf{Eq. 2-39}$$

Repeating the same example of the "Direct connection" discussed in Section 2.3 where the stator of the main machine is connected to a 60 Hz network and mechanical speed of the shaft (v) is 180 rad/sec the above equation yields the inverter frequency:

$$\omega_I = 3v - \omega_s$$
$$\omega_I = 3 \times 180 - 377$$
$$\omega_I = 163 \ {}^{rad}\!/_{\sec}$$

This is also validated by taking a glimpse at the following figure:

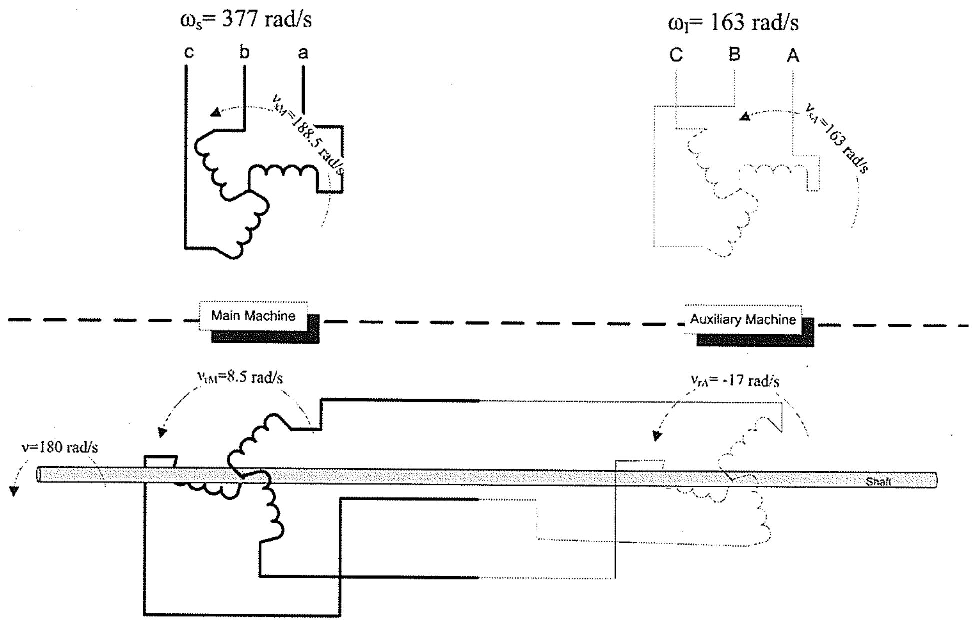

Figure 2-6: Representation of main and auxiliary machines speed and frequency parameters for the "Reverse connection of the rotor circuits"

Note that v_{r_A} (field speed of the auxiliary machine rotor wrt rotor (slip speed)) is negative now which means that it is rotating at the opposite direction with respect to the rotation direction of the shaft. As before different parameters as well as $\frac{S_M}{S_A}$ are calculated with the shaft speed ranging from 0.4 p.u. to 2.5 p.u. to investigate the required inverter rating for this connection. The following table indicates that the "reverse connection of rotor circuits" enables the machine to operate in other speed ranges too. This is visualized more clearly in the following section in comparison with the case where rotors are connected in "Direct sequence".

Table 2-7: Inverter Power at different Shaft Speeds (Reverse connection of the rotor circuits)

Low speed shaft (rpm)	High speed shaft (rpm)	V (rad/sec)	V_{s_M} (rad/sec)	$V_{M_{pu}}$ (p.u.)	V_{s_A} (rad/sec)	fi (Hz)	S_M	S_A	$\frac{P_{s_A}}{P_{s_M}} = \frac{S_M}{S_A}$
11.39268	720.0171	75.4	188.5	**0.4**	-150.8	**-24**	0.6	1.5	**0.4**
14.24084	900.0214	94.25	188.5	**0.5**	-94.25	**-15**	0.5	2	**0.25**
17.08901	1080.026	113.1	188.5	**0.6**	-37.7	**-6**	0.4	4	**0.1**
19.93718	1260.03	131.95	188.5	**0.7**	18.85	**3**	0.3	-6	**-0.05**
22.78535	1440.034	150.8	188.5	**0.8**	75.4	**12**	0.2	-1	**-0.2**
25.63352	1620.039	169.65	188.5	**0.9**	131.95	**21**	0.1	-0.3	**-0.35**
28.48169	1800.043	188.5	188.5	**1**	188.5	**30**	0	0	**-**
31.32986	1980.047	207.35	188.5	**1.1**	245.05	**39**	-0.1	0.15	**-0.65**
34.17803	2160.051	226.2	188.5	**1.2**	301.6	**48**	-0.2	0.25	**-0.8**
37.0262	2340.056	245.05	188.5	**1.3**	358.15	**57**	-0.3	0.32	**-0.95**
39.87437	2520.06	263.9	188.5	**1.4**	414.7	**66**	-0.4	0.36	**-1.1**
42.72253	2700.064	282.75	188.5	**1.5**	471.25	**75**	-0.5	0.4	**-1.25**
45.5707	2880.068	301.6	188.5	**1.6**	527.8	**84**	-0.6	0.43	**-1.4**
48.41887	3060.073	320.45	188.5	**1.7**	584.35	**93**	-0.7	0.45	**-1.55**
51.26704	3240.077	339.3	188.5	**1.8**	640.9	**102**	-0.8	0.47	**-1.7**
54.11521	3420.081	358.15	188.5	**1.9**	697.45	**111**	-0.9	0.49	**-1.85**
56.96338	3600.086	377	188.5	**2**	754	**120**	-1	0.5	**-2**
59.81155	3780.09	395.85	188.5	**2.1**	810.55	**129**	-1.1	0.51	**-2.15**
62.65972	3960.094	414.7	188.5	**2.2**	867.1	**138**	-1.2	0.52	**-2.3**
65.50789	4140.098	433.55	188.5	**2.3**	923.65	**147**	-1.3	0.53	**-2.45**
68.35605	4320.103	452.4	188.5	**2.4**	980.2	**156**	-1.4	0.54	**-2.6**
71.20422	4500.107	471.25	188.5	**2.5**	1036.75	**165**	-1.5	0.55	**-2.75**

2.14 Operation Regions of the Machine

Figure 2-7 illustrates possible operation regions of a brushless doubly fed AC field machine for both direct and reverse connection of the rotor circuits as far as the inverter power is concerned (Note that the active power of the main machine is kept at **1p.u**):

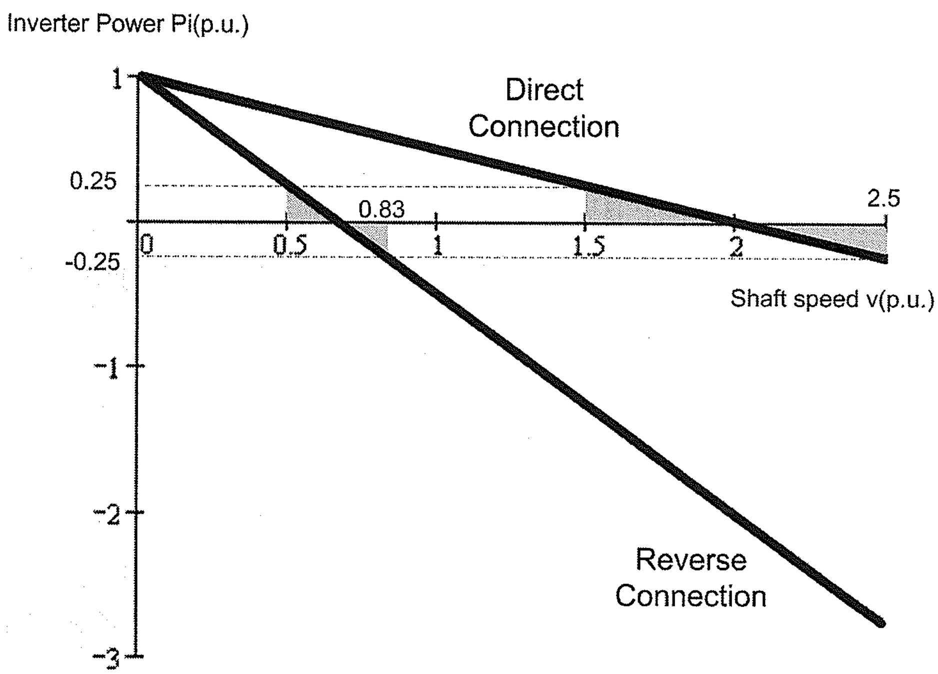

Figure 2-7: Operation regions of a brushless doubly fed AC field machine

If it is assumed that an inverter rated at 25% of the main machine power is economically viable for this concept, it is then concluded that machine is capable of working in two different speed regions:

1- Reverse connection: from 0.5 p.u. to 0.83 p.u. (Continuous operation)

2- Direct connection: from 1.5 pu. to 2.5 p.u. (Discontinuous operation at 1 p.u.)

In direct connection the machine goes through synchronous speed (i.e. 1 p.u), and as the machine is not capable of operating at this speed, it is impossible to achieve a "Continuous Operation" with the "direct connection of the rotor circuits".

The more detailed discussion and PSCAD simulation regarding the operation regions of the machine will follow in Chapter 3.

2.15 Chapter Summary

In this chapter, the proposed "Brushless Doubly-fed AC Field Machine" was studied in detail starting from developing its speed equations. It was shown that the main stator (network) frequency, ω_S is directly related to auxiliary stator/inverter frequency, ω_I as stated in Chapter 1. The speed equation was then validated by an example.

In order to eliminate the magnetic interference between two machines, the main and auxiliary machines were assumed to be 4- and 2-pole machines respectively, then the winding configurations (including the sinusoidally distributed windings) of 4- and 2-pole machines were studied at a glance to make it easier to derive the machine equations. "Total Magnetomotive Force" of the main machine was calculated based on "Rotating Magnetic Field" of its stator and rotor, and then field intensity H_θ and flux density B_θ were derived. The flux linkages of stator and rotor of the main machine were also derived consequently. As the last step the stator and rotor voltages of the main machine were found for both standstill and rotating modes.

The equivalent circuit of a "Brushless Doubly-fed AC Field Machine" was developed and the operating characteristics of the machine were provided for both standstill and rotating modes. Power and terminal voltage relations of the main and auxiliary machines were also obtained to be used in Chapter 3 where the machine behavior is simulated in PSCAD.

Reverse connection of the rotor circuits was also taken into consideration and was compared to the former connection (direct connection) from the speed and inverter power points of view. Reverse connection provided lower shaft speed operation, and avoided operation at synchronous speed.

Chapter 3: Simulation

3.1 Introduction

In the previous Chapters, ideal equations were developed. In this Chapter practical machines will be simulated using EMTDC/PSCAD simulation program. As this program did not contain a model for the "Brushless doubly-fed AC Field Machine", two separate machines were modeled; one for the main machine rated at 1MVA and one for the auxiliary machine rated at 0.25 MVA.

These models require parameters to be entered as per unit (p.u.) values. The base values for the complex power, voltage and speed for the main 4-pole machine are given in Table 3-1, followed by typical per unit values for a machine of this size. Table 3-2 lists these values for the auxiliary 2-pole machine.

3.2 Main & Auxiliary Machines Specifications

3.2.1 Main Machine:

Table 3-1: Main Machine Data

Parameter	Assigned Value
Base Power	1 MVA
Base Voltage (L-L)	1kV
Base Speed	188.5 rad/s
Base Angular Frequency	377 rad/s
Stator/Rotor Turns Ratio	1
Angular Moment of Inertia (J=2H)	0.3 p.u.
Mechanical Damping	0.05 p.u.
Stator Resistance	0.005 p.u.
Wound Rotor resistance	0.005 p.u.
Magnetizing Inductance	4 p.u.
Stator Leakage Inductance	0.0613 p.u.
Wound Rotor Leakage Inductance	0.0613 p.u.

3.2.2 Auxiliary Machine:

Table 3-2: Auxiliary Machine Data

Parameter	Assigned Value
Base Power	0.25 MVA
Base Voltage (L-L)	1kV
Base Speed	377 rad/s
Base Angular Frequency	377 rad/s
Stator/Rotor Turns Ratio	1
Angular Moment of Inertia (J=2H)	0.3 p.u.
Mechanical Damping	0.05 p.u.
Stator Resistance	0.008 p.u.
Wound Rotor resistance	0.008 p.u.
Magnetizing Inductance	4 p.u.
Stator Leakage Inductance	0.0613 p.u.
Wound Rotor Leakage Inductance	0.0613 p.u.

3.3 Speed Control

It was shown in Chapter 2 that the frequency relation of the two machines is as follows:

$$\omega_s = v + \omega_I$$

As the wind speed changes, the speed of the rotor varies, in order to keep ω_s constant at all times ω_I needs to vary based on the above equation:

$$\omega_I = \omega_s - v \qquad \textbf{Eq. 3-1}$$

and in per unit form:

$$\omega_{s_{pu}} = \frac{\omega_s}{\omega_{b_M}} \text{ and } \omega_{I_{pu}} = \frac{\omega_I}{\omega_{b_A}}$$

$$v_{M_{pu}} = \frac{v}{v_{b_M}} \text{ and } v_{A_{pu}} = \frac{v}{v_{b_A}}$$

Hence

$$v_{A_{pu}} = 0.5 v_{M_{pu}} \qquad \textbf{Eq. 3-2}$$

Thus, in the simulation the following block is used to implement the effect of the different number of poles:

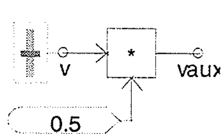

Figure 3-1: Implementation of Different Number of Poles Operation

The inverter model requires the frequency input in Hz, the inverter input frequency is:

$$\omega_{I_{pu}} = \frac{\omega_I}{\omega_{b_A}} = \frac{2\pi f_I}{2\pi f_b} = \frac{f_I}{f_b}$$

$$f_I = \omega_{I_{pu}} \times f_b$$

$$f_I = \omega_{I_{pu}} \times 60$$

The following block illustrates this procedure:

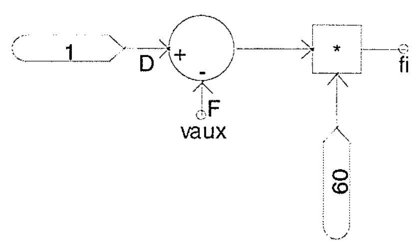

Figure 3-2: Frequency Control of the Inverter

3.4 Voltage Control

As a general requirement for every generator, the output voltage of a wind turbine generator must comply with the voltage level of the grid at the point of common coupling and reach this voltage before it is connected to the network. The commercial wind turbine generators are usually rated at 690 volts (3 phase) and with the advent of larger wind turbines this voltage has increased to 1000 volts (as in a Vestas V90-3MW) or even 6000v (as in a Vestas V120-4.5MW).

It was concluded in Chapter 2 that the output voltage of a "Brushless Doubly-fed AC Field Machine" is a function of both main and auxiliary machines' slips and also the voltage of auxiliary machine as depicted by the following equation:

$$V_{s_M} = \frac{S_A}{S_M} \times V_{s_A} \qquad \textbf{Eq. 3-3}$$

$$V_{s_A} = \frac{S_M}{S_A} \times V_{s_M} \qquad \textbf{Eq. 3-4}$$

Therefore to be able to generate 1p.u. at the terminals of the main machine, the inverter voltage (connected to the stator of the auxiliary machine) is required to be set at the following value for each different shaft speed:

$$V_{s_A} = \frac{S_M}{S_A} \times 1 \qquad \textbf{Eq. 3-5}$$

In doing so, the slips of both machines are calculated for each single shaft speed using the following blocks in PSCAD:

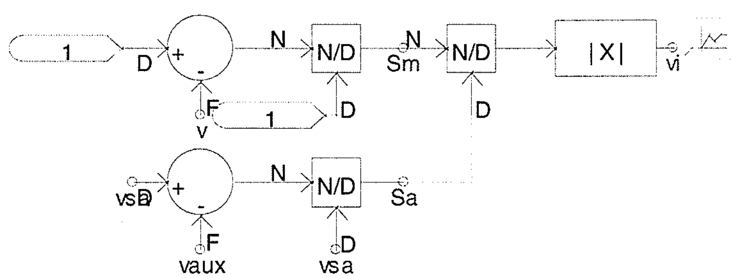

Figure 3-3: Voltage control of the inverter

The inverter is then set at Vi to make sure that the output voltage of the wind turbine (the main machine) is continuously kept at 1p.u.

3.5 Simulation in PSCAD

In order to validate the theoretical discussions and calculations in Chapter 2, a PSCAD case is set up and the machine behavior is studied in different operation modes. It is a common practice to operate electrical machines in a "Torque Control" mode, but as in our case the torque provided by the wind turbine can not be proportionally split into two separate torques (to be fed to each machine), thus the machines are run in a "Speed Control" mode. The following figure shows the simulation model (not showing the controls):

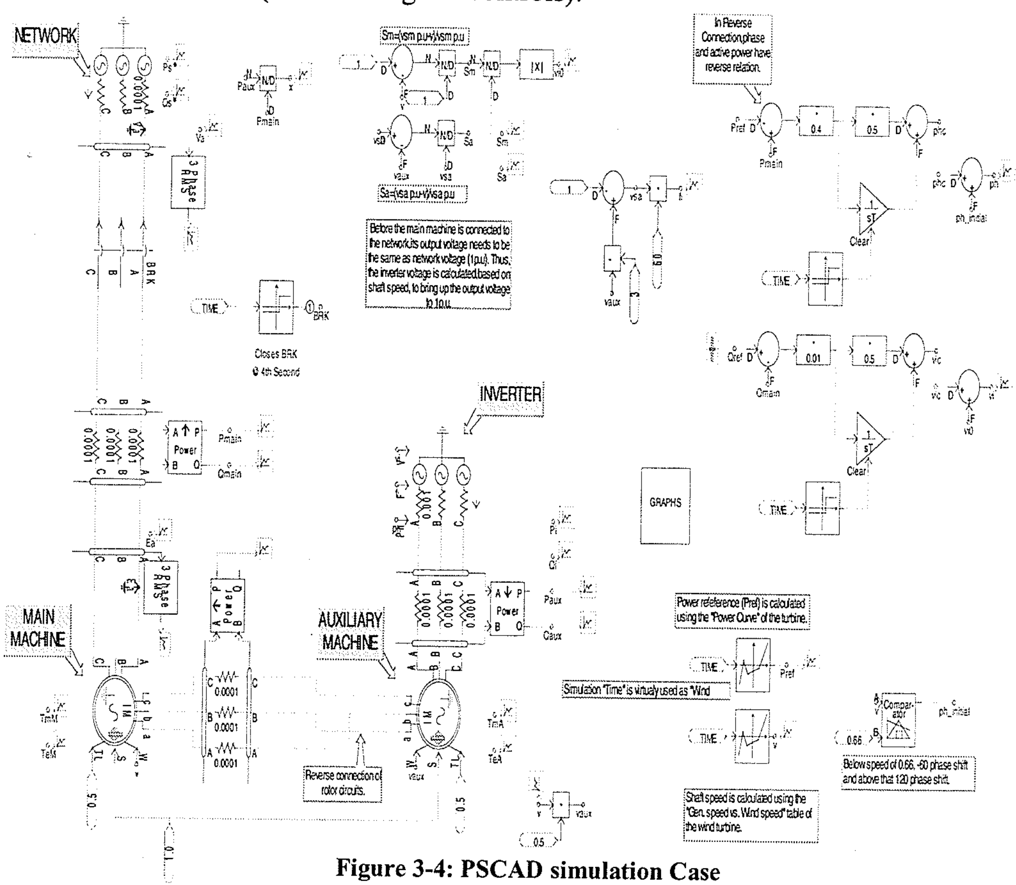

Figure 3-4: PSCAD simulation Case

3.6 Operation at the Synchronous Speed

As discussed in Chapter 2, the machine is not capable of operating at the synchronous speed (i.e. $\omega_s = \nu + \omega_I$, $\omega_I = 188.5 \, {rad}/{s}$). A simulation is carried out for 15 seconds and the behavior of the machine is investigated at and around this speed to verify this. The following figure shows the resulting generated power as the machine is synchronized and the speed is varied. The Inverter voltage (Vi) was maintained at 0.86 p.u.

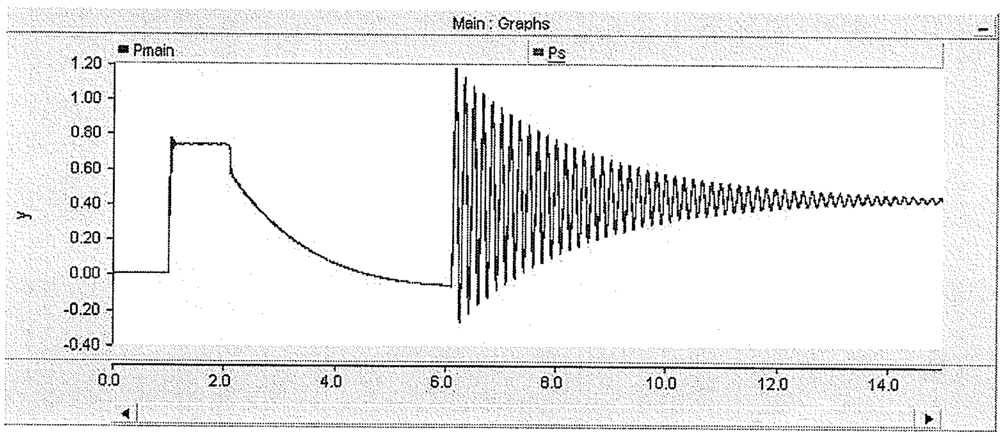

Figure 3-5: Generator power at and around the synchronous speed

1st second: Breaker is closed with the shaft rotating at 1.1p.u.

2nd second: Shaft speed is decreased to 1 p.u. (i.e. *synchronous speed*)

6th second: Shaft speed is decreased to 0.9 p.u.

It is shown that the machine is producing power both in lower and higher speeds than synchronous speed where as its not producing any power at synchronous speed.

3.7 Variable Speed Operation

In keep with the changing nature of the wind, often times it is desirable for the wind turbines to be capable of operating at variable speeds. The "Brushless Doubly-fed AC Field Machine" is expected to serve this purpose as discussed in Chapter 1. To show the variable speed operation ability of the machine, a simulation is run for 40 second at different speeds (Direct connection of the rotor circuits). As before the inverter voltage is set to (Vi): 0.86 p.u. The following figure illustrates the power generated by the main machine at various speeds.

1st second: Breaker is closed with the shaft rotating at 1.3p.u.
3rd second: Shaft speed is decreased to 1.1 p.u. (i.e. *synchronous speed*)
5th second: 1.02 p.u.
10th second: 1(i.e. *synchronous speed*)
17th second: 0.98 p.u.
27th second: 0.9 p.u.
30th second: 0.8 p.u.
32nd second:0.7 p.u.
34th second: 0.6 p.u.
36th second: 0.5 p.u.
38th second: 0.4 p.u.

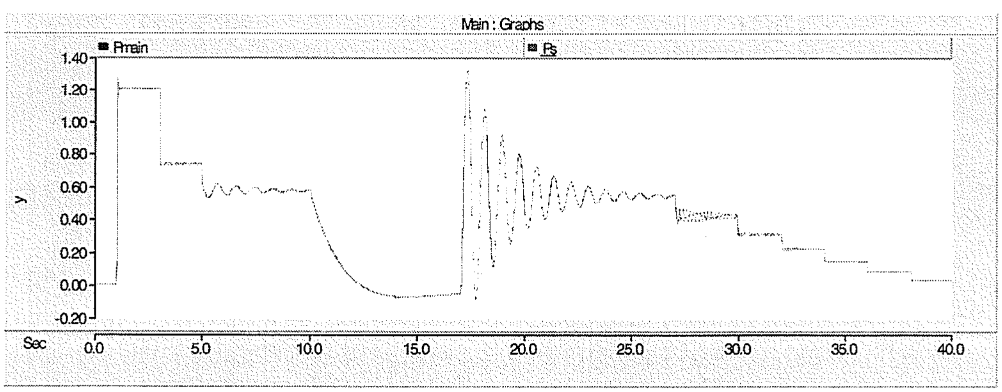

Figure 3-6: Variable speed operation of the machine

Note that it is also possible to achieve variable speed operation with reverse connection of the rotor circuits.

3.8 Active Power

As discussed in Chapter 2, it appears that the "Reverse connection of the rotor circuits" makes it possible to operate at lower speeds (0 to 0.83 p.u.) while keeping the inverter power below 0.25 p.u. compared to the direct connection. Therefore from this point forward, all the simulations are carried out with the "Reverse connection of the rotor circuits".

In order to validate the equation ($\frac{P_{s_A}}{P_{s_M}} = \frac{S_M}{S_A}$) presented in Chapter 2, a simulation case is run over 0.1 to 0.83 p.u. speed range. As the main power is kept at 1p.u. therefore the above equation can be rewritten as $P_{s_A} = \frac{S_M}{S_A}$. It is shown that inverter active power P_{s_A} calculated here complies with the theoretical values driven in Chapter 2 (i.e. $\frac{S_M}{S_A}$). It should be noted that the slight difference between simulation values and those of Chapter 2 is due to neglecting the losses in deriving the above equation in Chapter 2.

At first glance it might be implied from the graph that at lower shaft speeds inverter power is well over 0.25 p.u., but this is only when the main power is set at 1p.u. which is not the case for a wind turbine at low wind speeds. The power curve presented in Chapter 1 clearly indicates that the lower the wind speed (or shaft speed) the lower the turbine active power (i.e. main generator active power) will be. Later in this chapter we will see how the power reference to the main machine is gradually increased (based on the power curve) with the shaft speed increasing resulting in much lower inverter power demands.

Figure 3-7 shows simulation results for a speed variation of 0.1 to 0.83 p.u., while maintaining the output power at 1 p.u. and unity power factor.

3.9 Reactive Power

To study the machine from the reactive power point of view and its capability of meeting the network reactive power demands, the "Brushless Doubly-fed AC Field Machine" is run at 0.8 leading power factor, again over the shaft speed range of 0.1 to 0.83 p.u.. The simulation results are shown in the Figure 3-8 including the main machine active and reactive power and auxiliary machine reactive power.

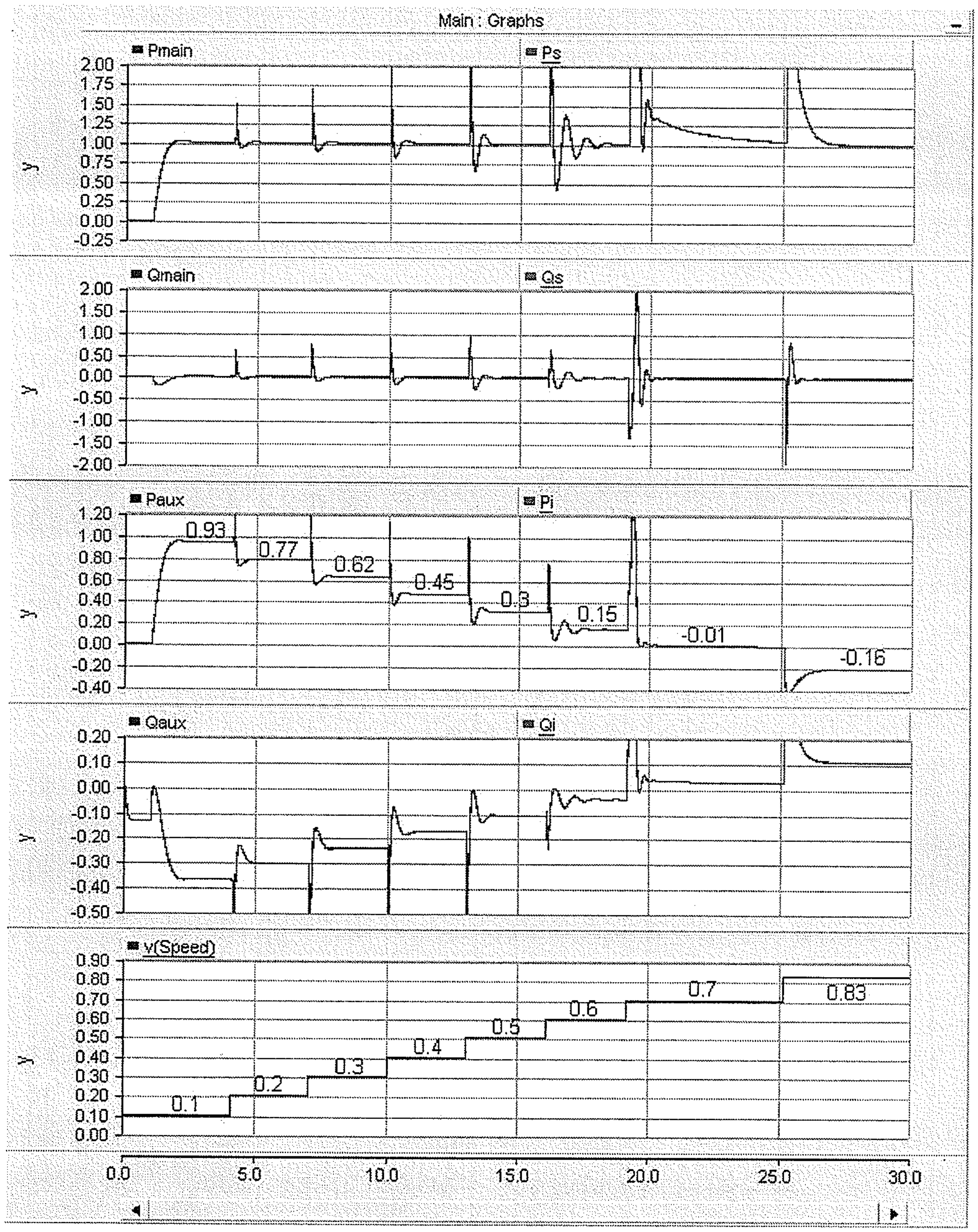

Figure 3-7 : Auxiliary machine (Inverter) active power vs. shaft speed (p.u.)

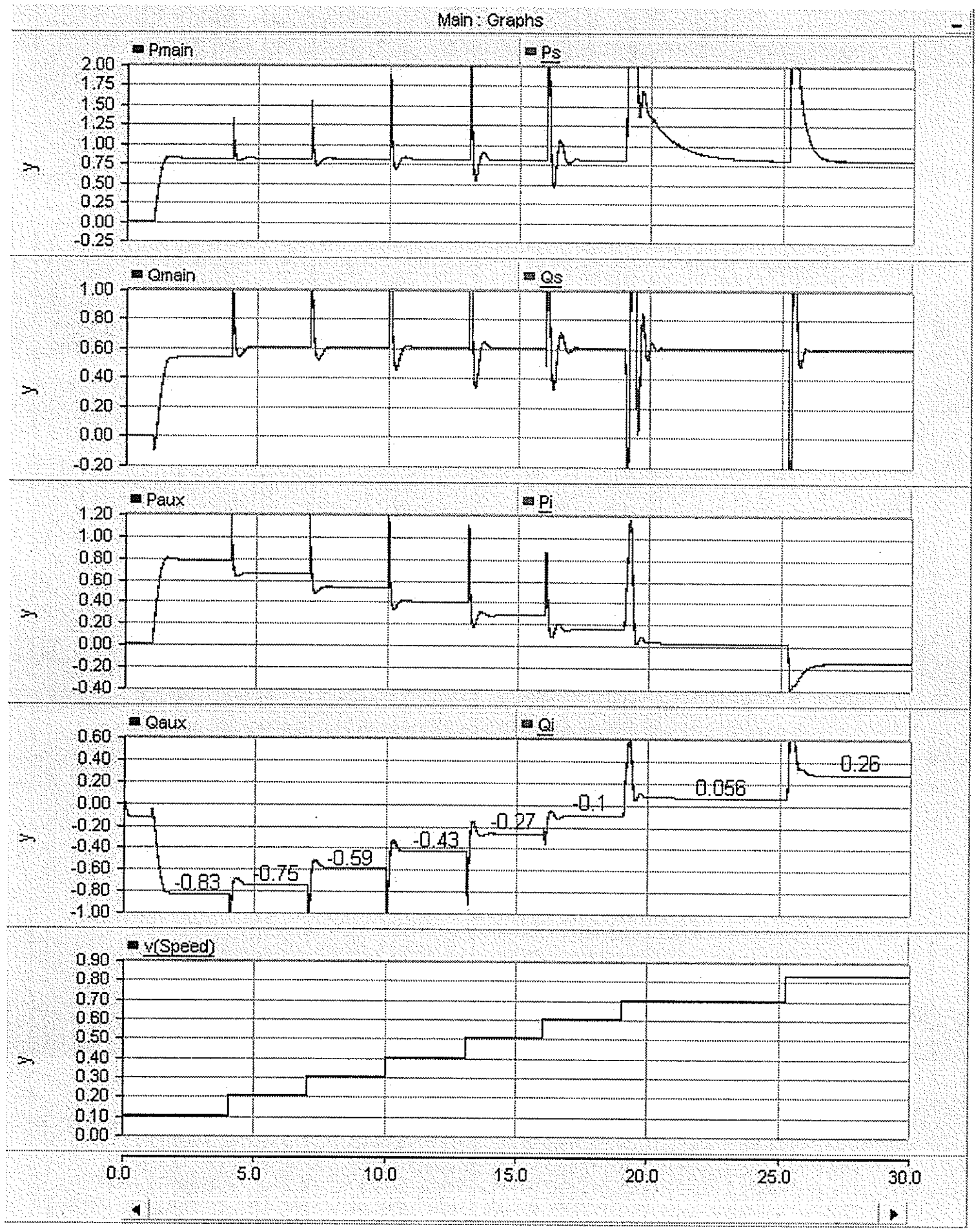

Figure 3-8: Auxiliary machine (Inverter) reactive power vs. shaft speed (p.u.)

3.10 Power Curve

Figure 1-11 illustrated how a variable speed pitch regulated wind turbine operates in different wind speed conditions. To investigate whether a "Brushless Doubly-fed AC Field Machine" can fit within such criteria, a simulation is run where the power and generator speed curves in Figure 1-11 are used to provide the power and generator speed references respectively.

The breaker is closed when the wind speed reaches cut-in wind speed of 4 m/s (corresponding to the 4^{th} second of the simulation). The generator reaches its rated speed (0.83 p.u.) at wind speed of 10 m/s. At 18 m/s the turbine reaches its rated power which is kept constant thereafter by decreasing the generator speed to 0.73 p.u. (using the pitch mechanism) while the wind speed creeps up. Eventually the generator is disconnected from the grid as soon as the wind speed reaches the cut-out speed of 25 m/s by means of turbine controls including feathering the blades off the wind.

The above operation is illustrated in Figure 3-9. The first graph (main machine power) manifests that the "Brushless Doubly-fed AC Field Machine" has successfully traced the power reference dictated by the turbine over the different operation conditions. As observed before, the machine is experiencing some transients at 0.66 p.u shaft speed due to the change of inverter frequency sign from negative to positive.

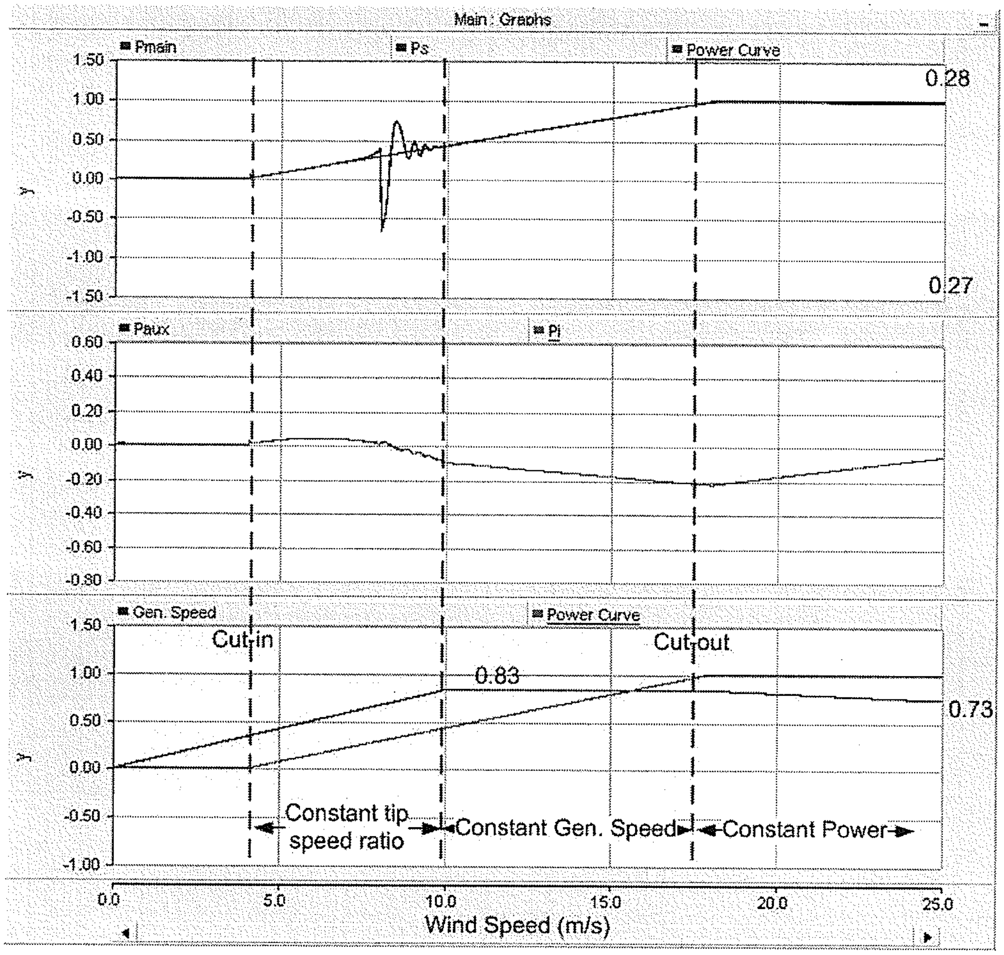

Figure 3-9: Brushless Doubly-fed Induction Generator Operation in a Variable Speed Pitch Regulated Wind Turbine

A deeper look into the inverter power graph in the above figure, reveals the interesting fact that the inverter active power (i.e. auxiliary active power) is following a curved pattern in "Constant tip speed ratio" region whereas it is changing in a linear fashion in the next other two regions (i.e. Constant generator speed and Constant power). As described earlier in Section 3.8, the inverter is not absorbing/injecting high amounts of active power at lower speeds compared to Figure 2-7 because the main power reference is not at 1 p.u but it is changed with respect to speed fluctuations.

3.11 Brushless Doubly-fed AC field Machine vs. DFIG

A brushless doubly-fed AC field machine can be compared to a "DFIG" from several different aspects, but as far as this study is concerned a simulation is run to investigate the "inverter power demands" of the above mentioned designs. This is shown in the following graph:

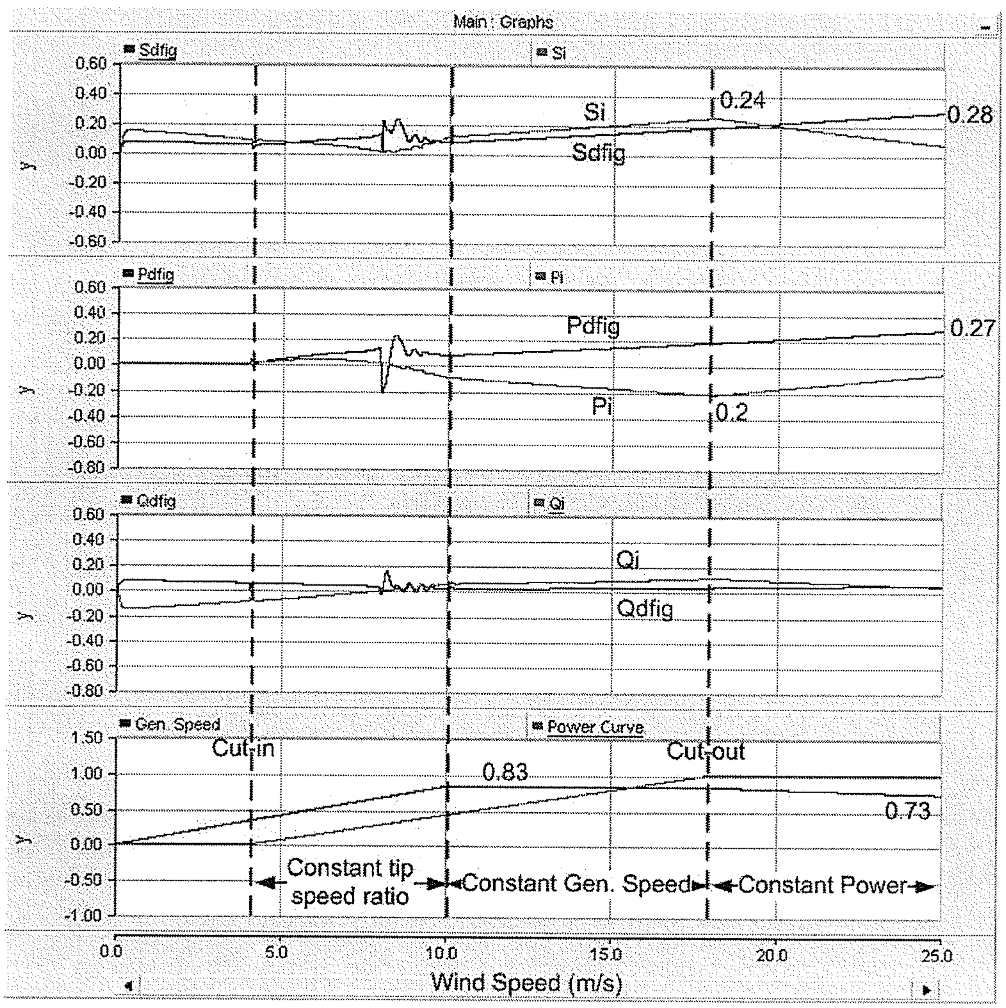

Figure 3-10: "Brushless Doubly-fed AC Field Machine" vs. DFIG

It is interesting to note that while the inverter power of a brushless doubly-fed AC field machine is injecting active power (i.e. auxiliary machine in generation mode) back into the network over most of its operation range, the inverter of a "DFIG" is absorbing power from the network under the same conditions. As far as the rating of the two inverters is concerned, the brushless design requires lower rating (0.24 p.u) whereas inverter rating of a DFIG needs to be at least 0.28 p.u.

3.12 Chapter Summary

The objective of this chapter was to validate the calculations, equations and behavioral study of the machine. Based on per unit values of the auxiliary and main machine a simulation case was put together to validate the theory introduced in Chapter 2. In doing so different control blocks such as speed and voltage control were added to ensure a smooth operation of the machine over different operation zones. While the variable speed operation of the machine was studied it was realized that operation at the synchronous speed of the main machine is not feasible as expected in Chapter 2. The machine was then studied from active and reactive power points of view as well as its capability to operate in the different operating zones of a real wind turbine as discussed in Chapter 1. The machine was eventually compared to a DFIG in terms of its inverter power demand.

Chapter 4: Cost Study

4.1 Introduction

It was investigated throughout the thesis that the design of a "Brushless Doubly-fed AC Field Machine" is theoretically feasible. It might also be worthwhile to consider the economical viability of such a design briefly. In doing so a general cost break down of a typical wind turbine is provided followed by a more detailed explanation based on a comprehensive report carried out by National Renewable Energy Laboratory [9].

4.2 Cost Breakdown of a Wind Turbine

The general cost breakdown of a typical large wind turbine is presented in the following table based on reference [10]:

Table 4-1: Wind turbine component cost breakdown

Component	% Cost
Rotor (blades, hub and pitch drive)	28%
Nacelle and machinery , w/o drive train and generator	21.7%
Gearbox and drive train	17.3%
Generator	**7%**
Converter	**7%**
Tower and foundation	19%
Total	100%

The generator stated above is a "Doubly-fed Induction Generator" which has been is widely used in commercial variable speed wind turbines. The converter is a back to back IGBT base converter rated usually at 25% to 33% of the generator rating. It should be noted that in case of a "Direct Drive Multi Pole Permanent Magnet Synchronous Machine" the cost of generator increases significantly which is offset greatly by the absence of the gearbox cost. Contrary to the fact that such a design requires a full-scale converter as described in Chapter 1, the cost of converter does not vary much due to a cheaper thyristor design rather than utilizing an IGBT converter.

A "Brushless Doubly-fed AC Field Machine" has an extra generator (i.e. auxiliary generator) which introduces extra costs in comparison with other designs discussed in Chapter 1. Although the above table gives useful information in regards to cost estimation of a "Brushless Doubly-fed Induction AC Field Machine", a more thorough study is done considering other factors that affect the cost of a wind turbine.

4.3 Cost of Energy (COE) Analysis

This section describes an economic assessment method used to estimate the cost of a "Brushless doubly-fed AC Field Machine". What follows is based on a report by "National Renewable Energy Laboratory", NREL entitled "Alternative Design Study Report: WindPACT Advanced Wind Turbine Drive Train Designs Study", written by R. Poore, T. Lettenmaier in August 2003 no.SR-500-33196.[9]

This report studies different wind turbine drive train configurations including low/high speed shafts, brakes, gearbox and generator and then based on information provided by a number of wind turbine manufacturers calculates the COE of each design. It should be noted that the conventional DFIG (which is of interest in comparison with a "Brushless Doubly-fed AC Field Machine) is referred to as "Baseline" design in the above report.

Economic comparisons of different drive trains in this report are made using COE estimate defined as:

$$COE = \frac{(ICC \times FCR) + O\,\&\,M + LRC}{AEP} \qquad \textbf{Eq. 4-1}$$

Where

COE : Cost of Energy (dollars/kilowatt - hours)
ICC : Initial Capital Cost (dollars)
FCR : Fixed Charge Rate (percentage/year)
O & M : Operations and Maintenance costs (dollars/year)
LRC : Levelized Replacement Cost (dollars/year)
AEP : Annual Nate Energy Production (kilowatt - hours/year)

The report then calculates and summarizes the COE for different drive train designs in the following abbreviated table [9]:

Table 4-2: Summary of COEs for Various Drive Trains

	Baseline	Integrated Baseline	Direct Drive	Single PM	Multi-PM
Transmission system	155,000	120,000	NA	90,000	58,000
Support structure	34,000	21,000	55,000	20,000	19,000
External cooling system	2,400	3,000	3,400	4,400	5,300
Brake	1,400	1,300	12,400	3,200	5,600
Coupling	2,400	2,100	NA	NA	NA
Nacelle cover	17,000	9,000	14,000	8,200	7,000
Generator	60,000	60,000	304,000	54,000	78,000
Power electronics	62,000	62,000	53,000	53,000	53,000
Substation VAR control	NA	NA	12,000	12,000	12,000
Transformer	23,000	23,000	26,000	26,000	26,000
Cable	18,000	18,000	16,000	16,000	16,000
Switchgear	12,000	12,000	10,000	10,000	13,000
Other subsystems	25,000	25,000	25,000	25,000	25,000
Drive train assembly and test	8,000	4,900	9,400	5,500	7,900
Drive train component cost total	420,000	361,000	540,000	327,000	325,000
Percentage of baseline drive train cost	100	86	129	78	77
Annual net energy production (AEP; in kilowatt-hours)	4.841E+06	4.841E+06	4.990E+06	5.001E+06	4.978E+06
Percentage of baseline AEP	100	100	103	103	103
Replacement costs—LRC ($/yr)	5100	5100	5600	4800	4500
O&M ($/yr)	24600	23500	23700	21200	23400
O&M ($/kWh)	0.0051	0.0049	0.0047	0.0042	0.0047
COE ($/kWh)	0.0358	0.0339	0.0378	0.0313	0.0317
Percentage of baseline COE	100	95	106	87	89

As it was discussed earlier, the auxiliary generator only requires to be rated at 24% of the main generator, which obviously means at most 24% extra cost. According to the above table the generator in a "baseline design" (i.e. a DFIG) costs $60000, which should increase by about 24% or to $74400 for a "Brushless Doubly-fed AC Field Machine". The power electronics including converter would be 9% cheaper, 33% for the DFIG versus 24% for the brushless design, which reduces the cost from $60000 to $54600. According to the report the generator shares 5% of total operation and maintenance (O&M) of a wind turbine (4%

scheduled and 6% unscheduled maintenance), 3% of which could be assumed to go towards maintenance and replacements of brushes and slip rings which for a brushless design brings the O&M costs down to about $23862 from $24600. The auxiliary generator might also add 1% to the "Support structure" due to its weight, increasing the cost by $340. Considering the above modifications the total capital cost (ICC) of the machine comes to $428602 which is 2% more expensive than the baseline (DFIG) design

Substituting the above values in the COE equation yields a COE of **0.0351** for a "Brushless Doubly-fed AC Field Machine" which shows that this design is slightly cheaper than the baseline design and also can compete with other designs considering the fact that it could also be used in areas where a DFIG design is patented.

Chapter 5: Conclusions & Recommendations

5.1 Conclusions

The main objective of this thesis was to carry out a feasibility study on a novel variable frequency generator that would be suitable for variable speed wind turbine applications. The generator was based on a patent by Vithayathil et al called “Brushless AC Field System for Stable Frequency Variable Speed Alternators” [1].

The first aspect investigated was the variable speed capability of the claimed configuration, which was confirmed through theoretical discussion as well as simulation results, leading to the fact that the machine is incapable of operating at “synchronous speed” with the ”direct connection of the rotor circuits”. This major drawback led to the idea of the “reverse connection of the rotor circuits”. The reverse connection appeared to be a proper alternative as it reduced the active power demand of the inverter compared to the direct connection, as well as obviating the need for operation at synchronous speed. Having had in mind that the inverter of a conventional DFIG is rated at about 0.25 p.u the reasonable generator speed range for the reverse connection to keep the inverter power within a similar range turned out to be from 0.5 to 0.83 p.u. It was later realized that operating the machine within the standard power curve in different wind turbine operating regions (i.e. constant tip speed, constant speed and constant power), could decrease the inverter power demand even more. In conclusion, compared to a DFIG design, the "Brushless Doubly-fed AC Field Machine" proves itself to be more advantageous for it makes it possible to achieve a brushless operation with less inverter power demands than a DFIG.

As far as the construction of the machine is concerned, the study showed that both machines (auxiliary and main) could be wound together and fit in a single housing, as long as proper number of poles is chosen to eliminate the "mutual magnetic coupling" of the two machines. This obviously makes this design more complex compared to a DFIG design which results in a larger and heavier generator.

The maintenance requirement of the machine and overhaul downtime of the turbine, as a key factor in designing wind turbines would also improve due to the brushless design.

5.2 Recommendations

Further research could include the economical viability of such a machine in terms of its single housing design costs, maintenance breaks and inverter price. More study is suggested to improve and optimize controls in order to achieve a smoother operation at and around the critical speeds discussed in this thesis.

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Appendices

Appendix A: Equations in (Rev/min)

For the main machine:

$$\left.\begin{array}{l} N_{s_M} = \dfrac{120F}{p}\ {}^{rev}\!/_{\min} \\ p = P_M \end{array}\right\} \rightarrow \left.\begin{array}{r} N_{s_M} = \dfrac{120F}{P_M} \\ N_{s_M} = N + N_{r_M} \end{array}\right\} \rightarrow$$

N_{s_M} : Main machine stator field speed wrt stator (synch.speed) $({}^{\mathrm{rev}}\!/_{\mathrm{min}})$

F_s : Main stator/network Frequency (Hz)

P_M : Number of the main Machine Poles

N : Mechanical(Shaft) Speed$({}^{\mathrm{rev}}\!/_{\mathrm{min}})$

N_{r_M} : Main machine rotor field speed wrt rotor (Slip Speed) (rev/min)

$$\left.\begin{array}{r} \rightarrow N + N_{r_M} = \dfrac{120F_s}{P_M} \\ N_{r_M} = \dfrac{120F_{r_M}}{P_M} \end{array}\right\} \rightarrow N + \frac{120F_{r_M}}{P_M} = \frac{120F_s}{P_M} \rightarrow F_s = \frac{P_M N}{120} + F_{rM}$$

F_{r_M} = Main machine rotor field frequency (Hz)

Note: As the rotors of both machines are mechanically and electrically connected to each other, the rotor frequency of both machines is the same ($F_{r_M} = F_{r_A}$) and referred to as F_r:

$$F_s = \frac{P_M N}{120} + F_r \,(1)$$

For the Auxiliary Machine:

$$\left.\begin{array}{l} N_{r_A} = \dfrac{120F_r}{P} \\ P = P_A \end{array}\right\} \rightarrow \quad N_{r_A} = \dfrac{120F_r}{P_A}$$

$$\left.\begin{array}{l} N_{s_A} = \dfrac{120F_I}{P} \\ P = P_A \end{array}\right\} \rightarrow \quad N_{s_A} = \dfrac{120F_I}{P_A}$$

$\rightarrow$

N_{r_A} : Auxiliary machine rotor field speed wrt rotor (Slip Speed) ($^{rev}/_{min}$)

P_A : Number of the auxiliary machine poles

N_{s_A} : Main machine stator field speed wrt stator (synch. speed) ($^{rev}/_{min}$)

F_I : Auxiliary stator(Inverter) frequency(Hz)

We know that "auxiliary stator field synchronous speed" N_{s_A} is equal to the summation of "auxiliary machine slip speed" N_{r_A} and "mechanical (shaft) speed N:

$$N_{s_A} = N_{r_A} + N$$

$$\rightarrow \frac{120F_I}{P_A} = \frac{120F_r}{P_A} + N \longrightarrow F_r = F_I - \frac{P_A N}{120} \quad (2)$$

$$(2) \text{ in } (1) \rightarrow F = \frac{P_M N}{120} + F_I - \frac{P_A N}{120}$$

$$F_s = \frac{P_M N}{120} - \frac{P_A N}{120} + F_I \rightarrow$$

$$F_s = \frac{(P_M - P_A)N}{120} + F_I$$

$$N : (^{rev}/_{min})$$

$$F_s : Hz$$

In Chapter 1 equations were driven in rad/sec, this can also be done here using the conversion factors:

$$F_s = \frac{(P_M - P_A)N}{120} \times \underline{60} + F_I = \underbrace{\frac{(P_M - P_A)N}{2}}_{\frac{rev}{sec}} + F_I$$

$$\xrightarrow{\times 2\pi} 2\pi \times F_I = 2\pi \times \frac{(P_M - P_A)N}{2} + 2\pi \times F_I \rightarrow$$

$$\omega_s = \frac{(P_M - P_A)\nu}{2} + \omega_I$$

$$\omega_s, \omega_I, \nu : (rad/_{\text{sec}})$$

Assuming a 4-pole Main machine and a 2-pole auxiliary machine, the above equation is reduced to a shorter form:

$$F_s = \frac{N}{60} + F_I \text{ or } \omega_s = \nu + \omega_I$$

The following table shows the speed and frequency parameters at a glance:

Table A-1Main and auxiliary machines speed and frequency parameters in rev/min

N= Mechanical (shaft) speed (rev/min)	
Main Machine	P_M: Number of poles
	F_s: Stator/network Frequency (Hz)
	N_{sM} : Stator field speed wrt stator (synch. speed)(rev/min)
	N_{rM} : Rotor field speed wrt rotor (slip speed) (rev/min)
	F_{rM} : Rotor field frequency (Hz)
Aux. Machine	P_A: Number of poles
	F_{rA}: Rotor field frequency (Hz)
	N_{rA} :Rotor field speed wrt rotor (slip speed) (rev/min)
	N_{sA} : Stator field speed wrt stator (synch. speed)(rev/min)
	F_I :Stator/Inverter Frequency (Hz)

Appendix B: Simulation Parameters

fi : Inverter frequency (Hz)

ph: Inverter phase angle

v_i : Inverter voltage (p.u)

P_i : Inverter active power (p.u)

Q_i : Inverter reactive power (p.u)

P_{main} : Main machine active power (p.u)

P_{aux} : Auxiliary machine active power (p.u)

P_s : Network active power (p.u)

Q_{main} : Main machine reactive power (p.u)

Q_{aux} : Auxiliary machine reactive power (p.u)

Q_s : Network reactive power (p.u)

P_{DFIG} : "Doubly-fed Induction Generator" Inverter active power (p.u)

P_{ref} : Active power reference (p.u)

Q_{ref} : Reactive power reference (p.u)

v : Main machine mechanical(shaft) speed(p.u)

v_{aux} : Auxiliary machine mechanical(shaft) speed(p.u)

S_m : Main machine slip

S_a : Auxiliary machine slip

T_{mM} : Main machine mechanical torque (p.u)

T_{eM} : Main machine electrical torque (p.u)

T_{eA} : Auxiliary machine electrical torque (p.u)

T_{mA} : Auxiliary machine mechanical torque (p.u)

Appendix C: Understanding 2- and 4-Pole Winding Configurations

It was mentioned in Chapter 1 that the auxiliary machine is chosen to be a 2-pole machine, and a 4-pole one to be used as the main machine. In order to calculate the magnetomotive forces of these machines, an appropriate understanding of the 2 and 4-pole windings is helpful. What follows is a brief description of the above matter.

2-Pole Machine

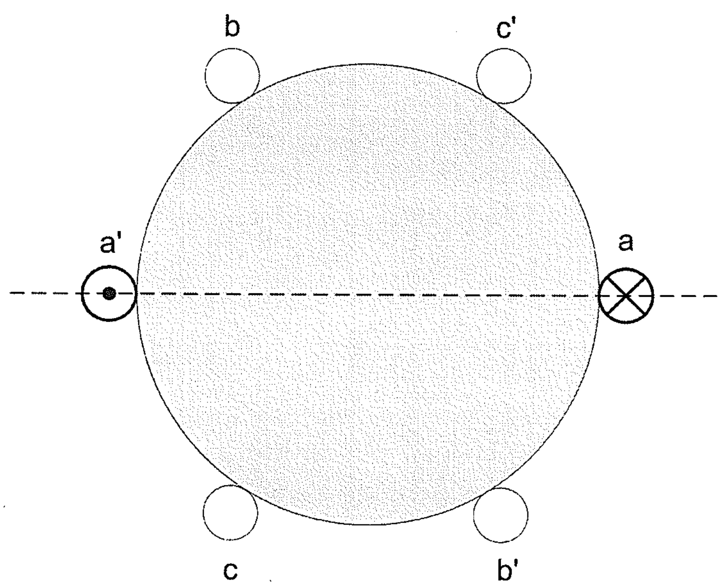

Figure C-1: Stator of a 2-pole concentrated wound machine

In the above picture [11], the dot presents a current leaving the plane of the page and perpendicular to it. The cross represents a current entering the plane of the page and perpendicular to it. This is better shown in the following figure [12]:

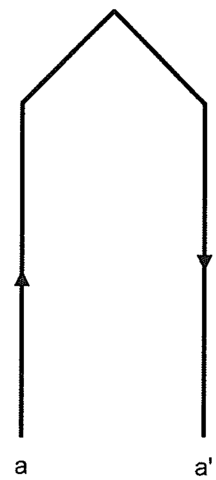

Figure C-2: A winding diagram for the phase "a" of the stator of a 2-pole machine

In a 2-pole concentrated wound machine, the Fourier series of the mutual inductance contains terms in $\cos\theta, \cos 3\theta, \cos 5\theta ...$ .It is desirable to eliminate the higher harmonic terms of this series, as they produce only oscillatory torque terms. This is accomplished by distributing the windings. As might be expected, a sinusoidally varying mutual inductance can be achieved if the turns of the windings are distributed so that the density of turns varies sinusoidally, as suggested by the following figure. In a practical machine, various approximations to this ideal sinusoidal distribution are used. [11-page 339]

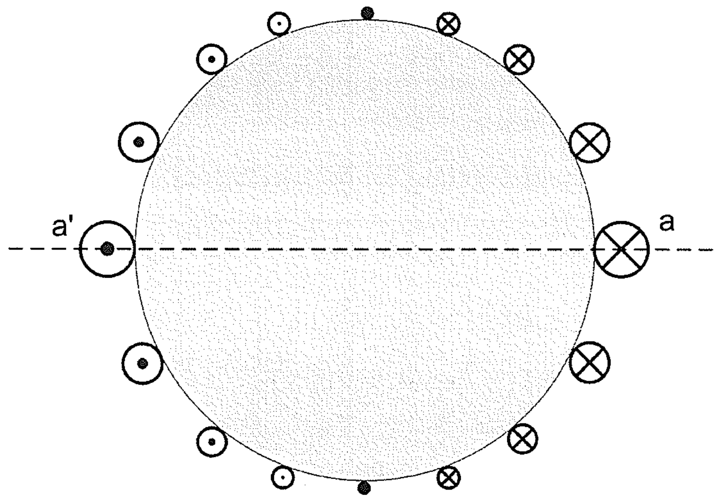

Figure C-3: Stator of a 2-pole sinusoidally wound machine (Showing phase a) [11]

4-Pole Machine

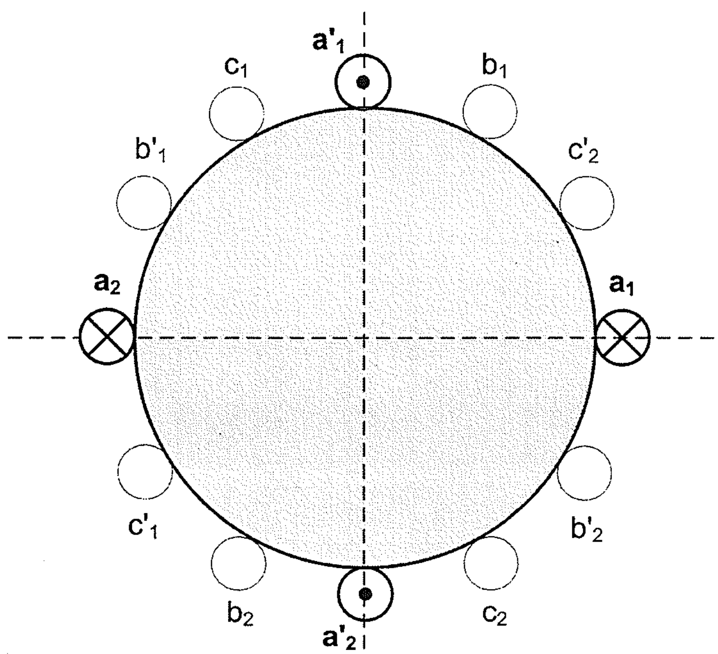

Figure C-4: Stator of a 4-pole concentrated wound machine

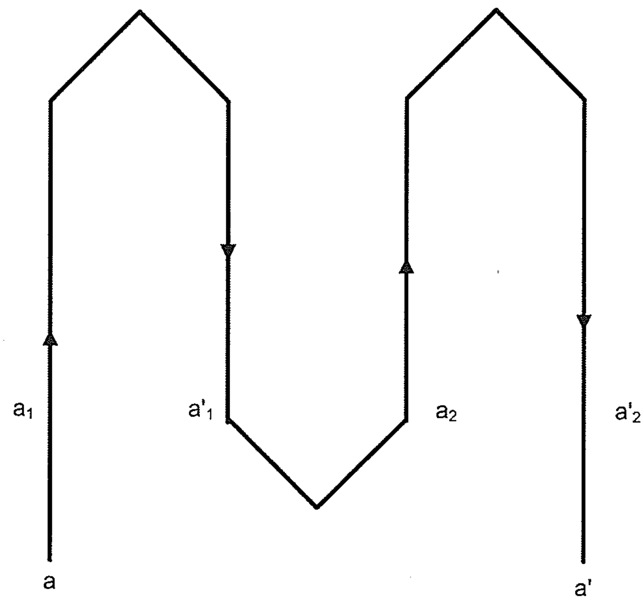

Figure C-5: A winding diagram for the phase "a" of the stator of a 4-pole machine [12]

And as before, to eliminate the harmonics, a sinusoidally distributed winding is also used for a 4-pole machine:

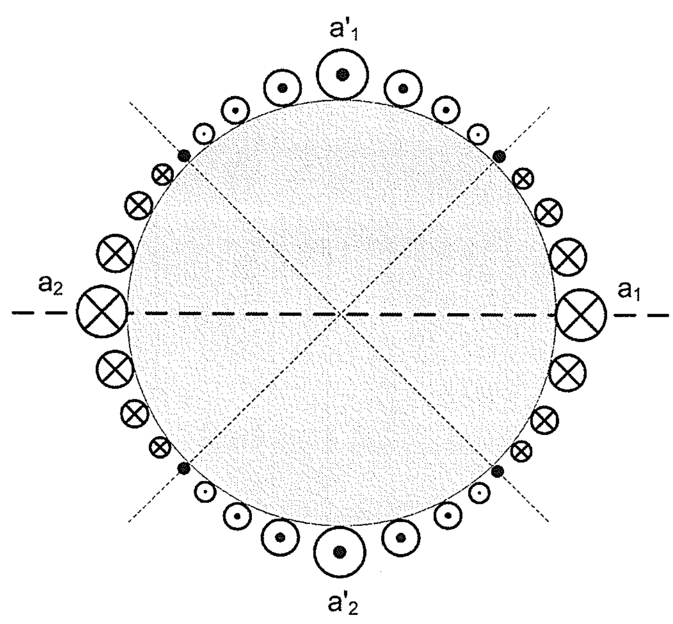

Figure C-6: Stator of a 4-pole sinusoidally wound machine (Showing phase a) [11]

It is worth mentioning that the rotors for both 2- and 4-pole machines are wound in the same fashion as the stator.

Practical windings

It should be noted that due to limited number of slots in the practical manufacturing of the machine, a pure sinusoidal winding is not achieved and the winding arrangements will be approximately sinusoidal. The following figure shows the magnetomotive force for both Ideal (Sinusoidal) and Practical (Actual) winding cases [11]:

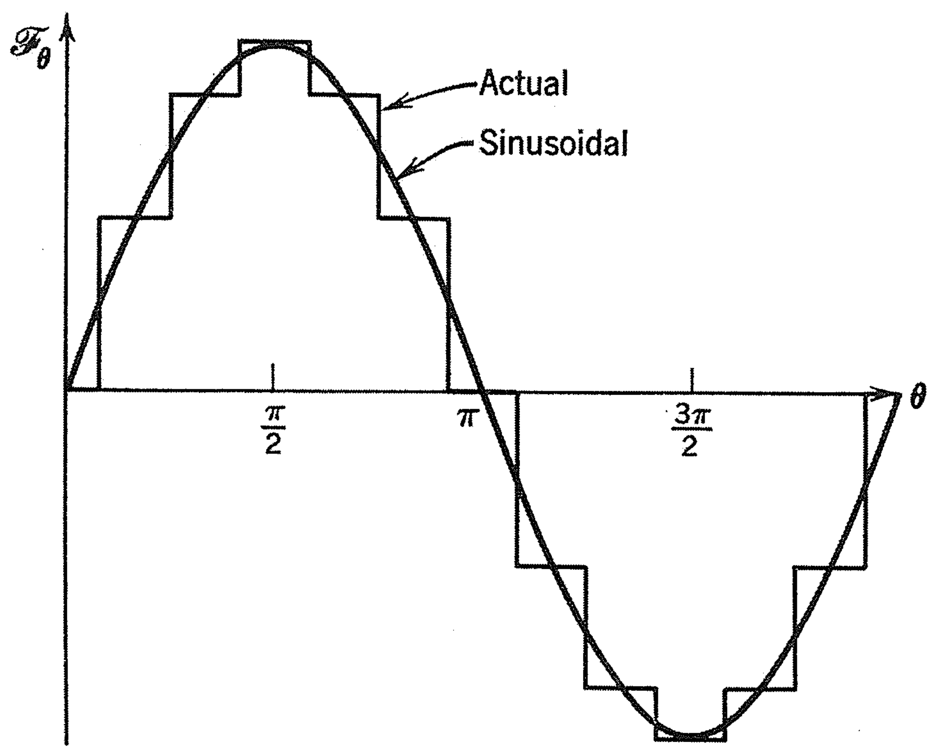

Figure C-7: Magnetomotive force distribution for practical and ideal winding arrangements

Appendix D: Equations

D.1 Rotating Magnetic Field

D.1.1 Stator mmf

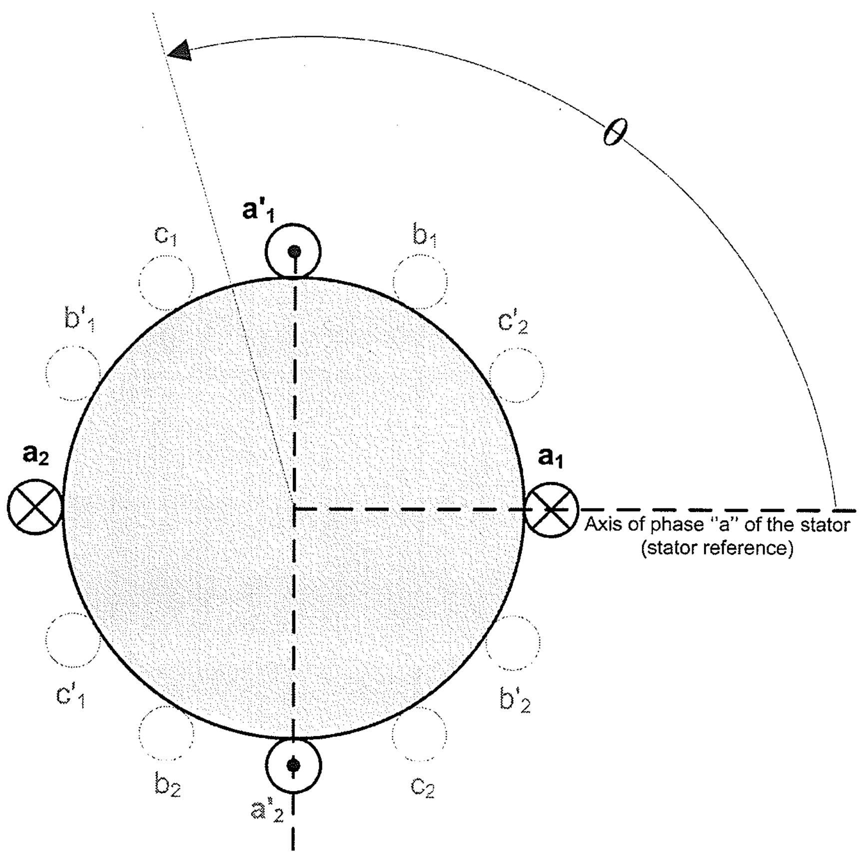

Figure D-1: Definition of θ in stator [13]

The above figure shows how θ is defined in stator. The mmf along θ is:

$$F_{(\theta)} = F_a(\theta) + F_b(\theta) + F_c(\theta) \qquad \textbf{Eq. D-1}$$

For a 2-pole machine, the mmf produced by each phase can be described as:

$$\begin{aligned} F_a(\theta) &= Ni_a Cos\theta \\ F_b(\theta) &= Ni_b Cos(\theta - 120) \\ F_c(\theta) &= Ni_c Cos(\theta - 240) \end{aligned} \qquad \textbf{Eq. D-2}$$

The above equations can be re-written for a 4-pole machine (main machine) as follows:

$$\begin{aligned} F_a(\theta) &= N_s i_a Cos 2\theta \\ F_b(\theta) &= N_s i_b Cos(2\theta - 120) \\ F_c(\theta) &= N_s i_c Cos(2\theta + 120) \end{aligned} \qquad \textbf{Eq. D-3}$$

N_s: Number of turns in each phase

Therefore at t = 0:

$$F(\theta, t) = N_S i_a Cos 2\theta + N_S i_b Cos(2\theta - 120) + N_s i_c Cos(2\theta + 120) \qquad \textbf{Eq. D-4}$$

Balanced three phases currents flowing through the three-phase windings of the main machine stator:

$$\begin{aligned} i_a &= I_{m_s} Cos(\omega_s t + \alpha_s) \\ i_b &= I_{m_s} Cos(\omega_s t + \alpha_s - 120) \\ i_c &= I_{m_s} Cos(\omega_s t + \alpha_s + 120) \end{aligned} \qquad \textbf{Eq. D-5}$$

If we substitute for these currents in the above equation:

$$\begin{aligned} F(\theta) = {} & N_s I_{m_s} Cos(\omega_s t + \alpha_s) Cos 2\theta + \\ & N_s I_{m_s} Cos(\omega_s t + \alpha_s - 120) Cos(2\theta - 120) + \\ & N_s I_{m_s} Cos(\omega_s t + \alpha_s + 120) Cos(2\theta + 120) \end{aligned}$$

We know from trigonometric equation that:

$$CosACosB = \frac{1}{2}[Cos(A+B) + Cos(A-B)] \qquad \textbf{Eq. D-6}$$

$$\begin{aligned}
F(\theta,t) = {} & \frac{1}{2}N_s I_{m_s}[Cos(\omega_s t+\alpha_s+2\theta)+Cos(\omega_s t+\alpha_s-2\theta)]+ \\
& \frac{1}{2}N_s I_{m_s}[Cos(\omega_s t+\alpha_s-120+2\theta-120)+Cos(\omega_s t+\alpha_s-120-2\theta+120)]+ \\
& \frac{1}{2}N_s I_{m_s}[Cos(\omega_s t+\alpha_s+120+2\theta+120)+Cos(\omega_s t+\alpha_s+120-2\theta-120)] \\
= {} & \frac{1}{2}N_s I_{m_s}Cos(\omega_s t+\alpha_s-2\theta)+\frac{1}{2}N_s I_{m_s}Cos(\omega_s t+\alpha_s+2\theta)+ \\
& \frac{1}{2}N_s I_{m_s}Cos(\omega_s t+\alpha_s-2\theta)+\frac{1}{2}N_s I_{m_s}Cos(\omega_s t+\alpha_s+2\theta-240)+ \\
& \underbrace{\frac{1}{2}N_s I_{m_s}Cos(\omega_s t+\alpha_s-2\theta)}_{\text{Forward Rotating Components}}+\underbrace{\frac{1}{2}N_s I_{m_s}Cos(\omega_s t+\alpha_s+2\theta+240)}_{\text{Reverse Rotating Components}}
\end{aligned}$$

Eq. D-7

Reverse rotating components add up to zero and "forward rotating components" result in the following equation:

$$F_s(t,\theta) = \frac{3}{2}N_s I_{m_s}Cos(\omega_s t+\alpha_s-2\theta) \qquad \textbf{Eq. D-8}$$

$$\hat{F}_{m_s} = \frac{3N_s\hat{I}_{m_s}}{2} \qquad \textbf{Eq. D-9}$$

therefore:

$$F_s(t,\theta) = \hat{F}_{m_s}Cos(\omega_s t+\alpha_s-2\theta) \qquad \textbf{Eq. D-10}$$

Note: if $\omega_s t+\alpha_s-2\theta=0 \rightarrow Cos(\omega_s t+\alpha_s-\theta)=1$ thus the maximum value of magnetomotive force occurs at:

$$2\theta = \omega_s t+\alpha_s$$

Thus:

$$\theta = \frac{\omega_s t + \alpha_s}{2} \qquad \textbf{Eq. D-11}$$

D.1.2 Rotor mmf

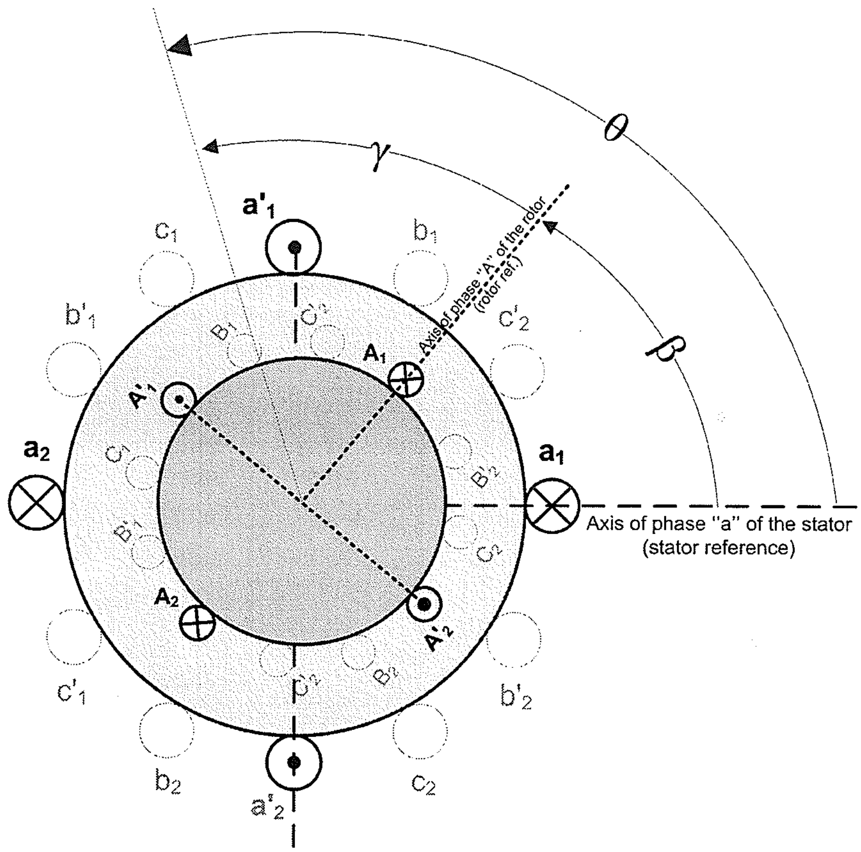

Figure D-2: Definition of β and γ [11]

$$\begin{cases} \theta : \text{an angle wrt stator ref} \\ \gamma : \text{an angle wrt rotor ref} \\ \beta : \text{rotor angle wrt stator ref} \\ \beta_\circ : \text{rotor angle at } t = 0 \\ \theta = \beta + \gamma \\ \beta = vt + \beta_\circ (\text{if rotor is rotating}) \\ \beta = \beta_\circ (\text{if rotor is standstill}) \end{cases}$$

It was shown that the mmf produced by stator of a 4-pole machine is given as:

$$F(\theta,t) = \frac{3}{2} N_s I_{m_s} Cos(\omega_s t + \alpha_s - 2\theta)$$

Similarly according to the above definition of angles for the rotor we can write:

$$F(\gamma,t) = \frac{3}{2} N_r I_{m_r} Cos(\omega_r t + \alpha_s - 2\gamma)$$

Angles are related to each other as follows:

$$\left.\begin{matrix} \theta = \beta + \gamma \\ \beta = vt + \beta_\circ \end{matrix}\right\} \rightarrow \theta = vt + \beta_\circ + \gamma \rightarrow \gamma = \theta - vt - \beta_\circ$$

Substituting for γ yields the following equation:

$$\begin{aligned} F_r(\theta,t) &= \frac{3}{2} N_r I_{m_r} Cos[\omega_r t + \alpha_r - 2(\theta - vt - \beta_\circ] \\ &= \frac{3}{2} N_r I_{m_r} Cos[\omega_r t + \alpha_r - 2\theta + 2vt + 2\beta_\circ] \end{aligned}$$

$$F_r(\theta,t) = \frac{3}{2} N_r I_{m_r} Cos[(\omega_r + 2v)t + \alpha_r + 2\beta_\circ - 2\theta]^1 \quad \textbf{Eq. D-12}$$

Now with comparing the mmfs of stator and rotor and considering the fact that they should be equal and opposite, the following limits are derived:

$$\omega_s = \omega_r + 2v$$

$$v = \frac{\omega_s - \omega_r}{2} \quad \textbf{Eq. D-13}$$

and

$$\alpha_r + 2\beta_\circ = \alpha_s \quad \textbf{Eq. D-14}$$

also:

$$\frac{3}{2} N_s I_{m_s} = \frac{3}{2} N_r I_{m_r}$$

$$\frac{I_{m_r}}{I_{m_s}} = \frac{N_s}{N_r} \quad \textbf{Eq. D-15}$$

The current ratio of stator and rotor based on no. of turns is expressed by the above equation.

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[1] This implies that this magnetic field is rotating forward at an angular velocity of $\frac{\omega_r}{2} + v$ as seen by a stationary observer.

D.2 Total Magnetomotive Force

Therefore the total magnetomotive force at any instant of time can be found by adding stator and rotor mmfs in a vector form (at t=0):

$$F_{m\theta} = \hat{F}_m Cos(\omega_s t + \alpha_m - 2\theta) \quad \text{(A)} \qquad \textbf{Eq. D-16}$$

where:

$F_{m\theta}$: Total (magnetizing) magnetomotive force

$$\alpha_m = \alpha_s + \alpha_r + \beta_\circ$$

This magnetizing force can be considered as the result of a magnetizing component of the stator current. In phase "a" this current component would be:

$$i_{ma} = \hat{I}_{ms} Sin(\omega_s t + \alpha_m) \quad \text{(A)} \qquad \textbf{Eq. D-17}$$

Also from before we know that

$$\hat{F}_{m_s} = \frac{3N_s \hat{I}_{ms}}{2} \quad \text{(A)}$$

therefore

$$\hat{I}_{ms} = \frac{2\hat{F}_m}{3N_s} \quad \text{(A)} \qquad \textbf{Eq. D-18}$$

D.3 Field Intensity, Flux Density and Flux Linkage

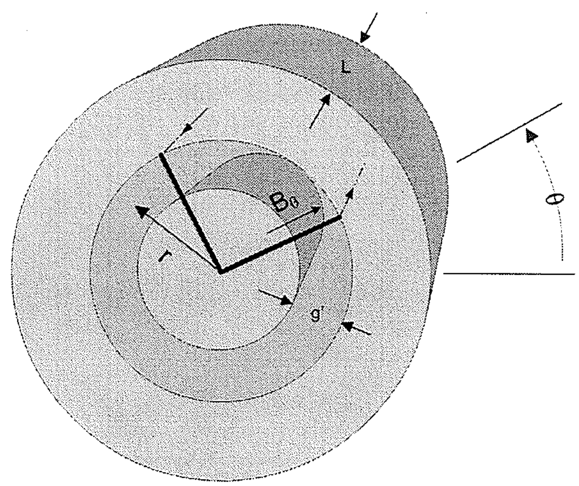

Figure D-3: Flux linkage of a 4-pole machine

Magnetic field intensity directed outward across the air gap at angle θ is:

$$H_\theta = \frac{F_{m\theta}}{2g'} \quad \text{(A/m)} \qquad \textbf{Eq. D-19}$$

g' : Effective length of each air gap

Thus the corresponding magnetic flux density in the air gap is

$$B_\theta = \mu_\circ H_\theta = \mu_\circ \frac{F_{m\theta}}{2g'} = \frac{\mu_\circ \hat{F}_{m\theta}}{2g'} Cos(\omega_s t + \alpha_m - 2\theta) \quad \text{(T)} \qquad \textbf{Eq. D-20}$$

Therefore we can write B_θ as

$$B_\theta = \hat{B} Cos(\omega_s t + \alpha_m - 2\theta) \quad \text{(T)} \qquad \textbf{Eq. D-21}$$

$$\hat{B} = \frac{\mu_\circ \hat{F}m}{2g'} \quad (\text{T}) \qquad \textbf{Eq. D-22}$$

D.3.1 Stator

Therefore considering a single turn, the flux linkage this air gap flux density produces in stator is:

$$\lambda_{turn} = \int_{\theta}^{\theta+\frac{\pi}{2}} B_\theta \times l \times r \times d\theta \qquad \textbf{Eq. D-23}$$

If B_θ is substituted in the above equation we would have

$$\lambda_{turn} = \int_{\theta}^{\theta+\frac{\pi}{2}} \hat{B}Cos(\omega_s t + \alpha_m - 2\theta) \times l \times r \times d\theta = \hat{B}lr \int_{\theta}^{\theta+\frac{\pi}{2}} Cos(\omega_s t + \alpha_m - 2\theta) d\theta$$

$$= \frac{-1}{2}\hat{B}lrSin(\omega_s t + \alpha_m - 2\theta)\Big|_{\theta}^{\theta+\frac{\pi}{2}}$$

$$= \frac{-1}{2}\hat{B}lr[Sin(\omega_s t + \alpha_m - 2\theta - \pi) - Sin(\omega_s t + \alpha_m - 2\theta)]$$

$$= \frac{-1}{2}\hat{B}lr[-Sin(\omega_s t + \alpha_m - 2\theta) - Sin(\omega_s t + \alpha_m - 2\theta)]$$

$$\lambda_{turn} = \hat{B}lrSin(\omega_s t + \alpha_m - 2\theta) \qquad \textbf{Eq. D-24}$$

We know that conductor density for phase "a" of the stator is:

$$n_a = \frac{N_s}{2} Cos2\theta \qquad conductors/rad \qquad \textbf{Eq. D-25}$$

Integration over the band of conductors in phase "a" gives its flux linkage due to the air gap flux density as:

$$\lambda_{ma} = \int_0^{\frac{\pi}{2}} n_a \times \lambda_{turn}\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{N_s}{2} Cos 2\theta \times Blr\, Sin(\omega_s t + \alpha_m - 2\theta) d\theta$$

$$= \frac{N_s}{2} \times B \times l \times r \int_0^{\frac{\pi}{2}} Cos 2\theta\, Sin(\omega_s t + \alpha_m - 2\theta) d\theta$$

$$\lambda_{ma} = -\frac{\pi}{4} N_s \times B \times l \times r\, Sin(\omega_s t + \alpha_m) \quad (V.s) \qquad \textbf{Eq. D-26}$$

D.3.2 Rotor

The flux linkage of the rotor windings is similarly found by first considering a single turn at the angle of γ:

$$\lambda_{turn} = \int_{\gamma}^{\gamma+\frac{\pi}{2}} b_{\gamma} lr\, d\theta$$

$$\lambda_{turn} = Blr\, Sin(\omega_s t + \alpha_m - 2\theta) \qquad \textbf{Eq. D-27}$$

Knowing that $\theta = \beta + \gamma$, λ_{turn} would be:

$$\lambda_{turn} = Blr\, Sin(\omega_s t + \alpha_m - 2\beta - 2\gamma) \quad (V.s) \qquad \textbf{Eq. D-28}$$

Similarly for the phase A of the rotor, the conductor density is:

$$n_A = \frac{N_r}{2} Cos(2\gamma) \qquad conductors/_{rad} \qquad \textbf{Eq. D-29}$$

thus

$$\lambda_{mA} = \int_{0}^{\frac{\pi}{2}} n_A \, \lambda_{turn} \, d\gamma \qquad \textbf{Eq. D-30}$$

$$\lambda_{mA} = \int_{0}^{\frac{\pi}{2}} \frac{N_r}{2} Cos(2\gamma) \times BlrSin(\omega_s t + \alpha_m - 2\beta - 2\gamma) \, d\gamma$$

$$= \frac{N_r}{2} \times B \times l \times r \int_{0}^{\frac{\pi}{2}} Cos(2\gamma) Sin(\omega_s t + \alpha_m - 2\beta - 2\gamma) \, d\gamma$$

$$\lambda_{mA} = -\frac{\pi}{4} N_r \hat{B} lr Sin(\omega_s t + \alpha_m - \beta) \qquad \textbf{Eq. D-31}$$

D.4 Standstill Operation

At standstill β is a constant $\beta_\circ$ (rotor angle at t =0);

$\beta = vt + \beta_\circ \xrightarrow{v=0} \beta = \beta_\circ$

The induced voltage in winding "a" of the stator is:

$$e_{ma} = -\frac{d\lambda_{ma}}{dt}$$

$$= \omega_s \times \underbrace{N_s \frac{\pi}{4}}_{E_s} BlrCos(\omega_s t + \alpha_m) \qquad Volts \qquad \textbf{Eq. D-32}$$

And similarly corresponding induced voltage in winding "A" of the rotor is:

$$e_{mA} = \frac{-d\lambda_{mA}}{dt}$$

$$= \omega_s \times \underbrace{N_r \frac{\pi}{4}}_{E_r} BlrCos(\omega_s t + \alpha_m - \beta) \quad Volts \qquad \textbf{Eq. D-33}$$

Comparison of e_{ma} and e_{mA} shows that the induced voltages in the rotor and stator are related by the following expression:

$$E_r = \frac{N_r}{N_s} \times E_s \angle -\beta \qquad \textbf{Eq. D-34}$$

D.5 Rotating Operation

Rotation of an electric round rotor machine does not change its inductance and resistance parameters, but it does change the relationship between the stator and rotor voltages and frequencies.

The induced voltage in winding "a" of the stator remains the same as was calculated previously:

$$e_{ma} = -\frac{d\lambda_{ma}}{dt} = \omega_s \times \underbrace{N_s \frac{\pi}{4}}_{E_s} BlrCos(\omega_s t + \alpha_m) \quad Volts \qquad \textbf{Eq. D-35}$$

The corresponding induced voltage in winding "A" of the rotor can be found by substituting for $\beta = vt + \beta_\circ$ (which manifests that rotor is no longer at standstill) in rotor flux linkage λ_{mA}:

$$\lambda_{mA} = -\frac{\pi}{4} N_r \hat{B} lrSin(\omega_s t + \alpha_m - \beta)$$
$$= -\frac{\pi}{4} N_r \hat{B} lrSin(\omega_s t + \alpha_m - vt - \beta_\circ)$$

$$\lambda_{mA} = -\frac{\pi}{4} N_r \hat{B} lr Sin\left[(\omega_s - v)t + \alpha_m t - \beta_\circ\right] \qquad \textbf{Eq. D-36}$$

And similarly corresponding induced voltage in winding "A" of the rotor is:

$$e_{mA} = \frac{-d\lambda_{mA}}{dt}$$

$$e_{mA} = \left(\omega_s - v\right) \times \underbrace{N_r \frac{\pi}{4} \hat{B} lr}_{E_r} Cos\left[(\omega_s - v)t + \alpha_m - \beta_\circ\right] \quad Volts \qquad \textbf{Eq. D-37}$$

Comparison of e_{ma} and e_{mA} for a "rotating machine" shows that the induced voltages in the rotor and stator are related by the following expression:

$$E_{m_r} = \frac{N_r}{N_s} \times \frac{(\omega_s - v)}{\omega_s} \times E_{m_s} \angle -\beta_\circ$$

$$E_r = \frac{N_r}{N_s} \times S \times E_s \angle -\beta_\circ \qquad \textbf{Eq. D-38}$$