# A STUDY OF PROCESSING TIME AND STORAGE REQUIREMENTS FOR DIGITAL PROCESSING OF SEISMIC DATA

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TITLE: A STUDY OF PROCESSING TIME AND STORAGE REQUIREMENTS FOR DIGITAL PROCESSING OF SEISMIC DATA

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A survey is made of existing software for a specific time series application, viz. seismic refraction data processing. New programs for digital filtering and convolution have been implemented, where necessary, and existing ones modified. Comparisons of processing times by different convolution methods are made.

The related problem of optimum parameters in least squares (Wiener) digital filter design is investigated using a linear prediction filter for refraction data enhancement. Preliminary prediction filtering of actual data emphasizes the need for careful analysis of autocorrelation estimates and mean square error terms before large scale application of least squares digital filters.

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# GLOSSARY OF TERMS USED IN GEOPHYSICAL EXPLORATION

R. E. SHERRIFF Geophysics, Vol. 33, no. 1, February, 1968.

<u>Ambient noise:</u> Undesired signal due to microseismic, atmospheric and other effects.

<u>Convolution:</u> The change of wave shape as a result of passing a signal through a linear filter (See Appendix C).

<u>Convolver:</u> An auxiliary computer under the control of the main computer which performs large-volume, high speed multiplications and additions.

<u>Decibel (db):</u> A unit used in expressing power or intensity ratios. 20 $\log_{10}$ of the amplitude ratio or 10 $\log_{10}$ of the power ratio.

<u>Dip:</u> The angle which a reflector or refractor makes with the horizontal.

<u>Deconvolution:</u> The reverse process of convolution, more specifically removing the filtering action of the earth's subsurface.

<u>Predictive deconvolution:</u> A special form of convolution which equalizes all frequency components within a bandpass in order to shorten the seismic pulse length.

<u>Field tape:</u> The original tape record obtained during actual field recordings, as opposed to a processed tape made from information which originates on another tape.

<u>Filter:</u> That part of a system which discriminates against some of the information entering it.

Frequency modulation. (FM): Modulation in which the instantaneous frequency of the modulated wave differs from the carrier frequency by an amount proportional to the instantaneous value of the modulating wave.

Mohorovicic discontinuity: Seismic discontinuity which separates the earth's crust and mantle. Situated at an average 35 kilometers below the continents and 10 kilometers below the oceans. Characterized by an increase of P-wave velocity to about 8 km/sec.

Multichannel processing: Processing wherein the characteristics are partially based on the characteristics of other channels.

Playback: Producing a new form of a seismic record from magnetic tapes or other reproducible forms of reading.

P wave: A compressional or longitudinal sound wave travelling in the subsurface.

Reflection: The energy or wave from a shot or other seismic energy source which has been returned or reflected from an elastic impedance contrast or series of contrasts within the earth.

Refraction: Also known as headwave or Mintrop wave; a conical wave. Wave travel from a point source obliquely downward to and along a relatively high velocity formation and thence obliquely upward (See Figure (3.1)).

Refractor: An extensive, relatively high, velocity layer, underlying lower velocity layers, which transmits refraction wave nearly horizontally.

Seismograms: A seismic record.

Seismometer: Geophone, usually used in groups or arrays.

Shotpoint: The location where an explosive charge is detonated to create a seismic energy source.

Signal-to-noise ratio (S/N): The energy of a desired event divided by all the remaining energy (noise) at that time.

Time-gate: A specific portion, or time interval, of a time series.

Trace: A record of one seismic channel --usually used in conjunction with galvanometer or cathode ray tube displays or records.

# CHAPTER I

# INTRODUCTION

## 1.1 Time Series Analysis.

The use of digital computers for the analysis of time series, i.e. data which is recorded sequentially in time, leads to many applications in a wide variety of fields of study, including such diverse topics as share prices on stock exchanges, electrocardiographs, meteorological forecasting and many others. The practical problems which arise when large amounts of such data have to be processed will, therefore, effect many computer users and, though this paper deals with a specific type of time series data, viz. multichannel refraction seismograms, the problems and their solutions are, in fact, quite general and will find application whenever digital time series are to be processed on a general purpose computer.

## 1.2 Seismic Data Processing.

In the last twelve months seismic data, recorded by the University of Manitoba Geophysics department on frequency modulated magnetic tape, has been converted to IBM digital format. There is, therefore, a potentially large amount of digital data processing which could be carried out and the development of effective software for this purpose is of prime importance. The main objective of this thesis is to survey and investigate the techniques available for efficient and practical digital filtering.

A considerable body of literature concerning the theory of seismic data processing now exists, however, most of the programs and subroutines, so far published, are not practical for immediate implementation. This is particularly true when extra-long multichannel time series, such as the seismic records considered here, are encountered.

The two main problems concern storage and processing times. Storage difficulties arise in the design and computation of multichannel filter coefficients while processing times can become prohibitive when these filters are applied to long multichannel time series.

A related problem is the choice of suitable parameters in filter design with regard to computational effort involved and effectiveness of the filter in signal detection and enhancement. For the purpose of investigating the optimization of these filter parameters, a multichannel linear prediction filter process was devised and tested on recent seismic data. In addition to having immediate practical value to seismologists, this filtering process also serves as the basis of the more sophisticated technique of predictive deconvolution.

## 1.3 Review of Original Research.

The basic theory and principles of digital seismic data processing were developed at the Massachusetts Institute of Technology by the Geophysical Analysis group in the period from 1952 to 1957, under the directorships of Enders A. Robinson and Stephen M. Simpson. The task of this group was to attempt the realization of Norbert Wiener's time series concepts on the Whirlwind I computer for the solution of problems in seismic exploration for oil [1].

In 1960, S. M. Simpson and E. A. Robinson became associated with VELA UNIFORM, an Advanced Research Agency project at M.I.T., similar to the Geophysical Analysis group, but this time concerned with the problems of detection of underground nuclear explosions [2].

In addition to the many theoretical papers published as a result of the work of these two groups, two sets of computer programs have also been published [3,4]. The first, by S. M. Simpson in FORTRAN II and FAP, is largely specialised to IBM system 709, 7090 and 7094 computers. Of the 267 programs, 90 only are written in FORTRAN, while the remainder, including the convolution subroutine, are written in FAP.

A second suite of programs, by E. A. Robinson [4], is notable for the development of multichannel routines. However, most of the subroutines are written for demonstration purposes rather than efficiency and the author suggests modification of the programs if significant amounts of data are to be processed.

# CHAPTER II

# BASIC CONCEPTS

## 2.1 General Seismic Principles.

The seismic surveying method is based on the reflection and refraction of longitudinal compression waves in the earth's subsurface. A seismic record obtained by a controlled impulse device at the surface, as opposed to an earthquake seismogram which records waves initiated by naturally occurring phenomena, permits depth and dip measurements on the various velocity discontinuities at which reflections and refractions have occurred. To achieve this, the time interval between the energy impulse and energy arrival is measured; the actual energy path and the velocities encountered by the returning waves also have to be known or assumed.

Theoretical seismic energy arrivals in a simple two-velocity layer geologic model are shown in Figure 2.1. The conventional energy source is a charge of dynamite at a designated shot point. Energy returning to the surface can only do so if it has encountered rocks of higher density, and hence higher velocity, and has been reflected, or, in the case of refraction, if it has travelled along the interface at the higher refractor velocity, as shown in Figure 2.1(a). There is a certain critical distance from the shot point, beyond which refracted energy will arrive before energy travelling directly along, or near to, the surface.

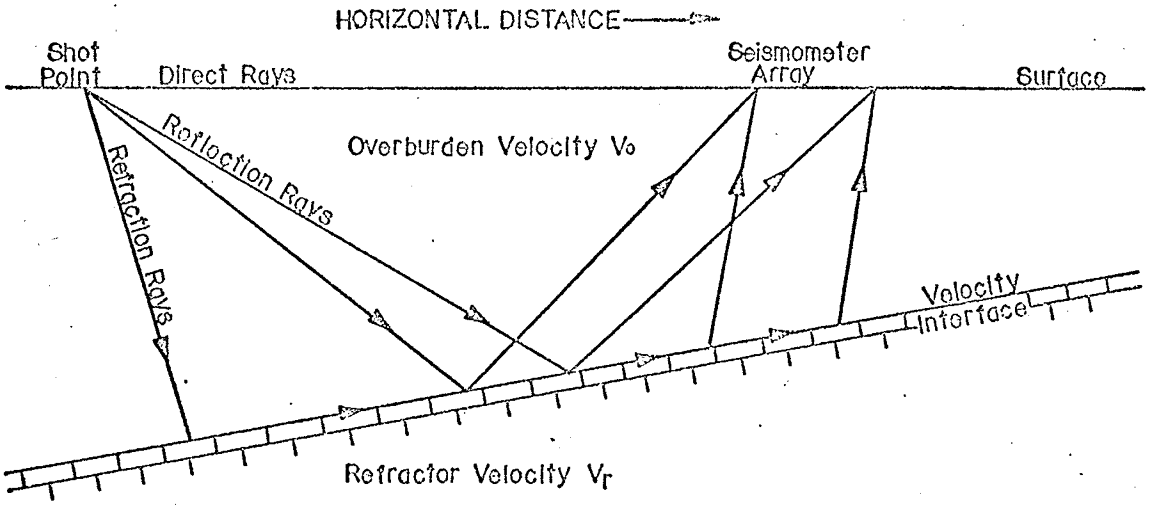



Fig. 2.1(a)

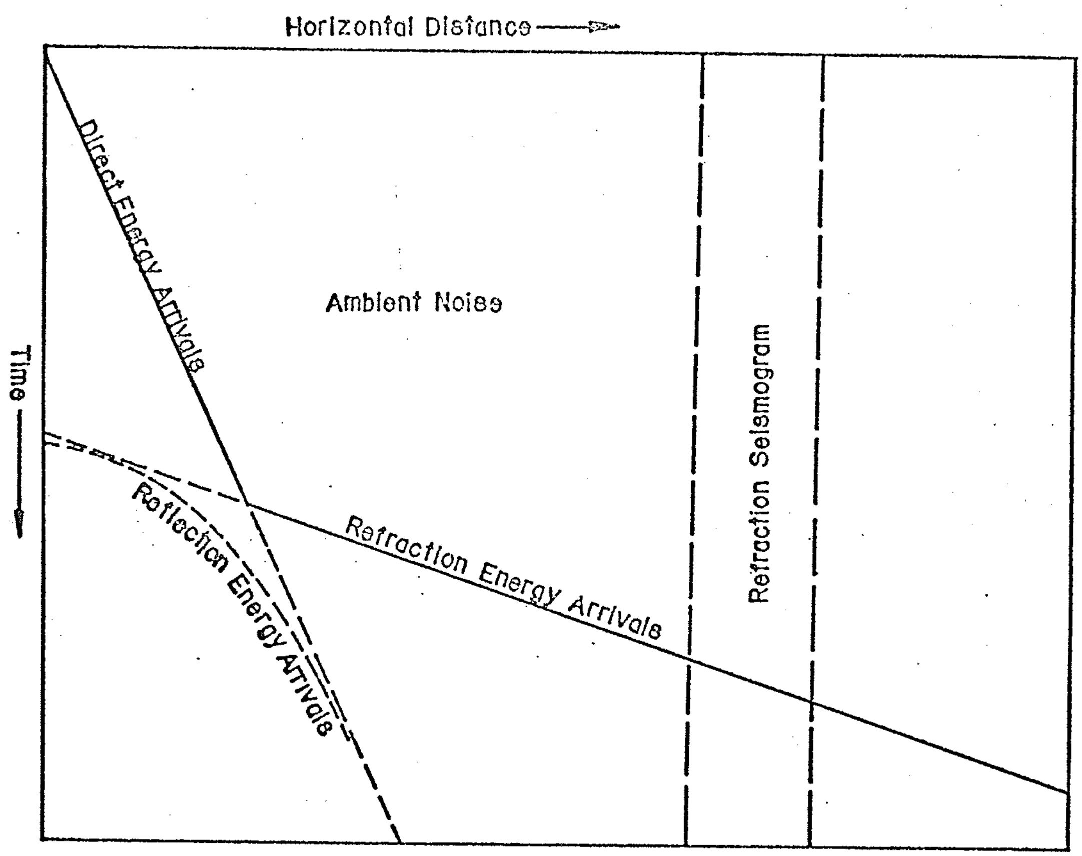



Fig. 2.1(b)

Figure 2.1 Theoretical Seismic Energy Arrivals in a Simple Two Velocity Layer Geologic Model

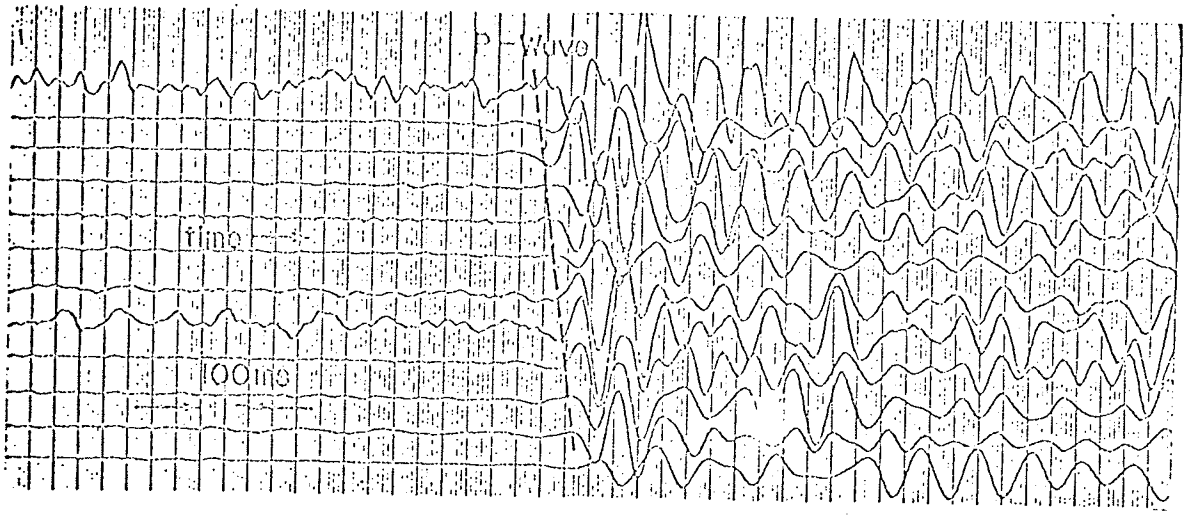
Energy arrivals for all offsets from the shot point are shown in Figure 2.1(b). Refraction arrivals have an apparent velocity which depends on the angle of dip of the velocity interface and the ratio of the overburden and refractor velocities $(V_o/V_r)$. Reflected energy is usually recorded at, or near, the shot point, while refraction records are obtained, as shown, by positioning the seismometer array at a suitable offset from the shot point.

## 2.2 Refraction Records for Crustal Studies.

Seismic refraction records taken by the Geophysics department of the University of Manitoba are designed for studies of the earth's crust. The primary objective is the study of refractions from the massive velocity discontinuities at the base of the earth's mantle (the Mohorovicic discontinuity). The implementation of suitable digital data processing techniques to this type of refraction seismogram is the main topic of this paper.

The data is simultaneously recorded on photographic paper and on frequency modulated magnetic tape. Basically the record comprises the output of 12 seismometer groups with a radio transmitted standard time signal to determine the actual shot instant. A standard timing device gives time calibration (timing lines) on the photographic records to within approximately 2 milliseconds. Details of the field recording system are given in Appendix A.

After the data has been recorded on magnetic tape, playbacks at different frequency and amplitude settings can be made to improve the data quality. Three such playbacks are shown in Figure 2.2. In

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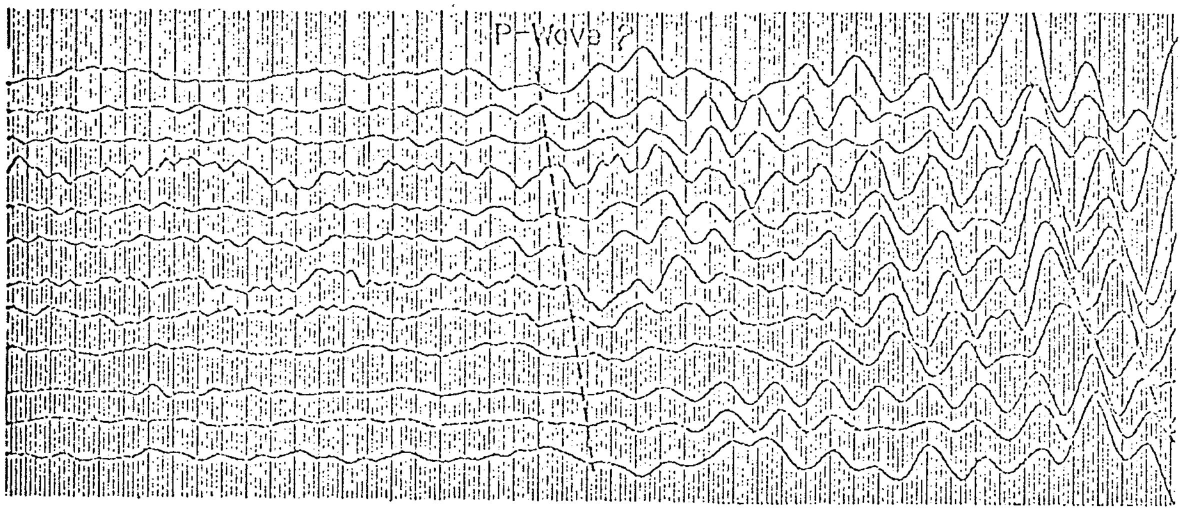
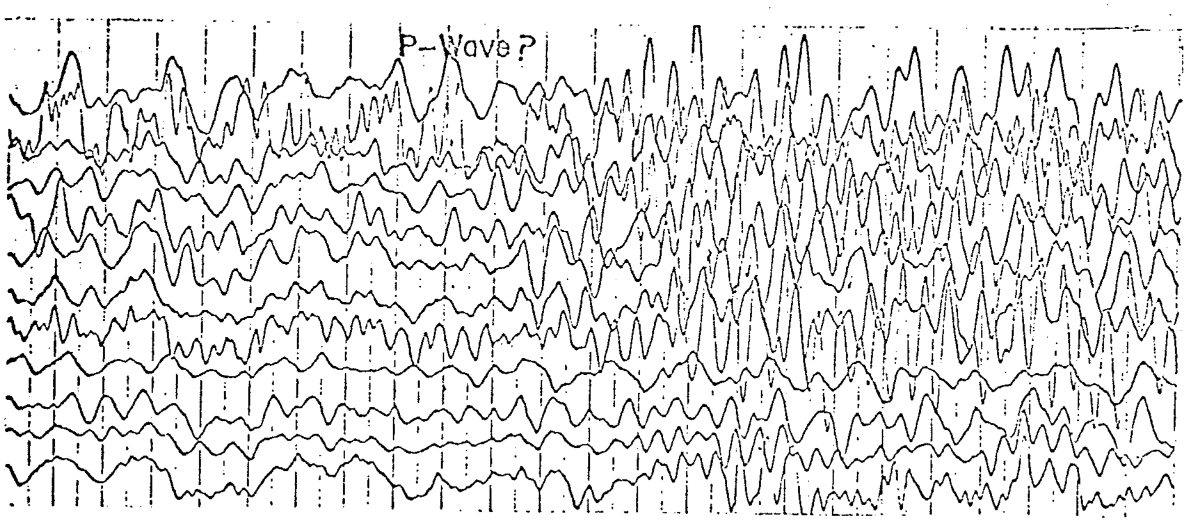



RECORD RB-3 Attn 48-54 db Freq 0-32 cps




RECORD R-49 Attn 30-42db Freq 0-66cps

Figure 2.2 Examples of Analog Seismic Records each case the output of the 12 seismometer groups is shown with the 10 millisecond interval timing lines. The trace for the radio transmitted time signal is not displayed.

Record R-50 shows well defined energy arrivals from a deep refractor, while records R-B3 and R-49 show successively weaker energy onsets. Operational considerations often preclude the recording of perfect refraction arrivals; for example, the seismometer array for record R-B3 was at a distance of some 700 kilometers from the shot point, while record R-49 was an experimental shot taken with a much smaller charge of dynamite than that normally used. As can be seen from the playbacks, though the energy increases to the right of the record (i.e. with increase of time), the actual time of onset of the first energy arrivals (P-waves) is difficult to determine because of their low energy in comparison to the ambient micro-seismic noise. The different appearances of the three playbacks is in some measure due to the different instrument settings.

After conversion of these field tapes to digital format, following the system in Appendix A, digital data processing techniques can be applied to records such as RB-3 and R-49 to improve the signal to noise ratios and hence the accuracy in timing of the refracted energy arrivals.

## 2.3 Principles of Seismic Digital Filters.

The theory and concepts of digital filters are given by E. A. Robinson [5], but the successful application of these techniques cannot be achieved without a full understanding of the theory. An outline of these principles is given below.

2.3.1 Least Squares Digital Filters.

The basic problem in the Wiener theory of signal enhancement is the determination of the numerical values of the filter coefficients, $(f_o, f_1, f_2, ..)$ which, when convolved with the input time series $(x_o, x_1, x_2, .., x_t, .)$, produce an output time series $(y_t)$, which is an approximation to a desired output series $(z_t)$. The form and specifications of the desired output constitute the various types and special cases of the different digital filters.

The solution of this problem, as applied to seismic data processing, depends on the assumption that the input and desired output time series are stationary statistical processes, i.e. their statistical properties are time invariant.

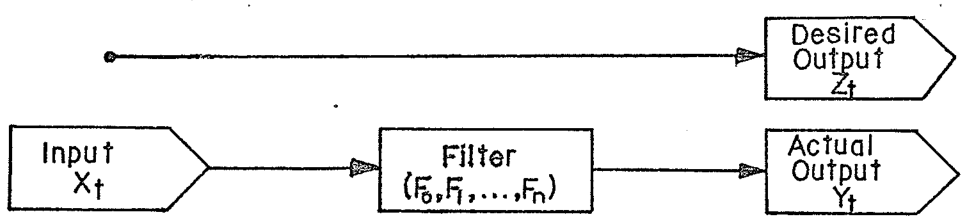



Figure 2.3 Linear Filter Model

The approximation criterion is taken to be the minimization of the mean square error between the desired output $(z_t)$ and the actual output $(y_t)$. The resulting filter is considered to be time invariant over the range of its applicability.

Other approximation criteria have been proposed and tried [6], but the classical least squares approximation appears to be the best criterion, particularly as a fast recursive method for the solution of the normal equations has been developed by N. Levinson [10] and extended to multichannel filters by R. Wiggins and E. Robinson [11].

The stationarity of seismic noise has been investigated by J. N. Galbraith [7] and R. A. Haubrich [8], both of whom concluded that micro-seismic noise could be considered stationary over most records investigated and nearly stationary over all records. Moreover, the widespread success of methods based on this assumption further substantiates its validity.

2.3.2 Optimum Finite Filters.

The determination of the numerical values of a specific filter's coefficients is carried out by the solution of the normal equations. These equations are derived under the assumption that, whereas infinite sums from $-\infty$ to $+\infty$ cannot be realized in practice, the finite sum, over a sufficiently long time interval of discrete data samples, will be a close enough approximation. The optimization criterion is taken to be the minimization of the mean square error between the desired output time series and the actual output series; this is the classical least squares method. The full mathematical derivation of these equations is given in Appendix B.

As can be seen from equations (B.8) and (B.9) in Appendix B, the filter coefficients can be expressed in terms of the autocorrelation of the input series and the cross correlation of the input series with the desired output series. Theoretically, the mean square error can be reduced to any desired small value by taking a sufficiently long filter. In practice, increase of filter length beyond a certain point produces no reduction in the mean square error, as is shown in

Chapter IV. Any method, of solution of the normal equations which allows monitoring of the error term as increasingly larger filters are computed is useful in that it provides a means of avoiding unnecessary computation.

The direct method of solution of equation (B.9) by inversion of the autocorrelation matrix is not used in practice. The autocorrelation matrix can be shown to be a Toeplitz[1] matrix [ 9 ].

2.3.3 Recursive Method for Filter Operator Determination.

Filter operator coefficients are determined numerically by the solution of the normal equations, see paragraphs 2.3.1 and 2.3.2. A fast recursive method for this was originally devised by N. Levinson [10] and extended to multichannel case by R. A. Wiggins and E. A. Robinson [11,12]. The full mathematical theory can be found in the original paper [11]. Briefly it consists in constructing auxiliary sequences (orthonormal polynomials) for each autocorrelation matrix, from which a recursive filter polynomial can be derived.

The recursion (which is actually an interation in the FORTRAN program) can be stopped when any of the following three conditions occurs

i) The filter reaches some maximum pre-set length,

ii) the normalised mean square error reaches some preset minimum,

iii) the normalised mean square error shows no appreciable decrease after successive iterations.

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1. A Toeplitz matrix is positive definite hermitian, with elements constant along any diagonal, so that if the matrix is $n \times n$ there are only $n$ independent elements rather than $n^2$.

The main advantage of the technique is that the computation time is proportional to the square of the filter length rather than the cube as in conventional methods.

2.3.4 Prediction Error Filters.

Using the principles of least squares filtering, a sample of noise on several traces, prior to the first energy arrivals, is chosen and a multichannel filter is computed whose desired output is specified as the actual value of the sampled traces at a later time. This then is a prediction filter and it is the basis of the prediction error filter. Under the assumption that the noise characteristics remain stationary for a time outside the sample interval (or time gate), the multichannel prediction operator is convolved with the input time series to produce a prediction of the noise at a future time, equal in duration to the length (in time units) of the prediction span.

The predicted noise is then subtracted from the actual noise present at that later time: the resultant being the prediction error series. If the prediction filter is reasonably successful in predicting the noise, the amplitude of the error series can be expected to be small until the incidence of a signal, in the form of the first energy refraction arrivals. The signal cannot of course be predicted from the statistical properties of the ambient seismic noise and a large increase in amplitude in the prediction error series indicates the signal arrival.

An important factor in this type of filtering is that signal distortion occurs only after a time delay equal to the prediction span. Thus, for the time represented by the prediction span, the prediction error series is, in fact, the required signal, at a much reduced background noise level.

## 2.4 Digital Filtering of Data (Convolution).

The application of a specific filter to an input time series produces a filtered output series. The output represents a linear transformation of the input data, the actual form of the transformation depends on the design criteria of the filter and the process by which it is carried out is known as convolution. A fuller explanation of convolution is given in Appendix C. Briefly, it consists of a moving summation of the input series data elements which have been weighted by an amount equivalent to the corresponding filter coefficient.

In both single and multichannel filtering processes, therefore, it can be considered as a sequence of multiplication and addition. operations.

Alternatively, convolution can be shown to be mathematically equivalent to polynomial multiplication, where the data elements and filter weights are the polynomial coefficients of the z-transform. In the multichannel case the polynomial coefficients are matrices, or, as Robinson [13] has shown, the multichannel case can be represented by lambda matrix[1] multiplication.

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[1] A lambda matrix is a matrix with polynomial elements.

# CHAPTER III

# SEISMIC DATA PROCESSING TECHNIQUES

## 3.1 Introduction.

The practical implementation of the filters as described in the previous chapter raises serious problems from the point of view of storage requirements and processing times. Excessive storage requirements occur when multichannel filters of lengths greater than 100 have to be computed. Also, it is impractical to store a complete channel in core, let alone a complete 12 channel seismic record.

With regard to processing times, these become prohibitively large for even simple filtering operations, when direct multiply and add operations are employed, even when only a specific portion of a trace or channel is processed.

This chapter deals with the methods of overcoming these problems and compares the performances of the programs developed with previously published results.

The starting point in the implementation of data processing routines for seismic data processing is the two sets of subroutines published by S. M. Simpson [3] and E. A. Robinson [4]. Several of these subroutines were taken and modified, and new subroutines were implemented when the existing programs were found to be inadequate.

Two other program sources which were checked and to which reference will be made are the IBM Scientific Subroutine Package and a suite of programs by D. W. McCowan [14].

A full index of all subroutines referred to, together with their functions, authors and comments, is given in Appendix E.

## 3.2 Storage Requirements.

### 3.2.1 Channel Segmentation - Sectioning.

In general the refraction records to be processed consist of up to $4.0 \times 10^6$ bytes of information, comprising 12 channels of approximately 5 minutes of signal output, sampled at 1.7 millisecond intervals. Though the maximum processing effort would normally not be applied to the whole seismogram, the best method of dealing with such quantities of data is to process it in piece-meal fashion, thereby considerably reducing core storage requirements. Breaking the multi-channel time series into contiguous disjoint segments on input is usually termed trace segmentation or sectioning. Apart from the storage economy thus achieved, two additional factors have to be considered.

Firstly, as is shown below in paragraph 3.3.4.1., there are theoretically optimum section lengths which should be chosen when convolution is carried out by the Fourier transform product method.

Secondly, when an input data series is processed by a linear filter, the resulting output section is greater in length than the input section by an amount $q-1$, where $q$ is the length of the filter. This end-effect has to be taken into account when the input data is sectioned, though none of the published subroutines do so. Fortunately, the necessary modification is fairly simple, as convolution is both associative and commutative, and all that is required is temporary storage of the extra elements for subsequent addition to the following

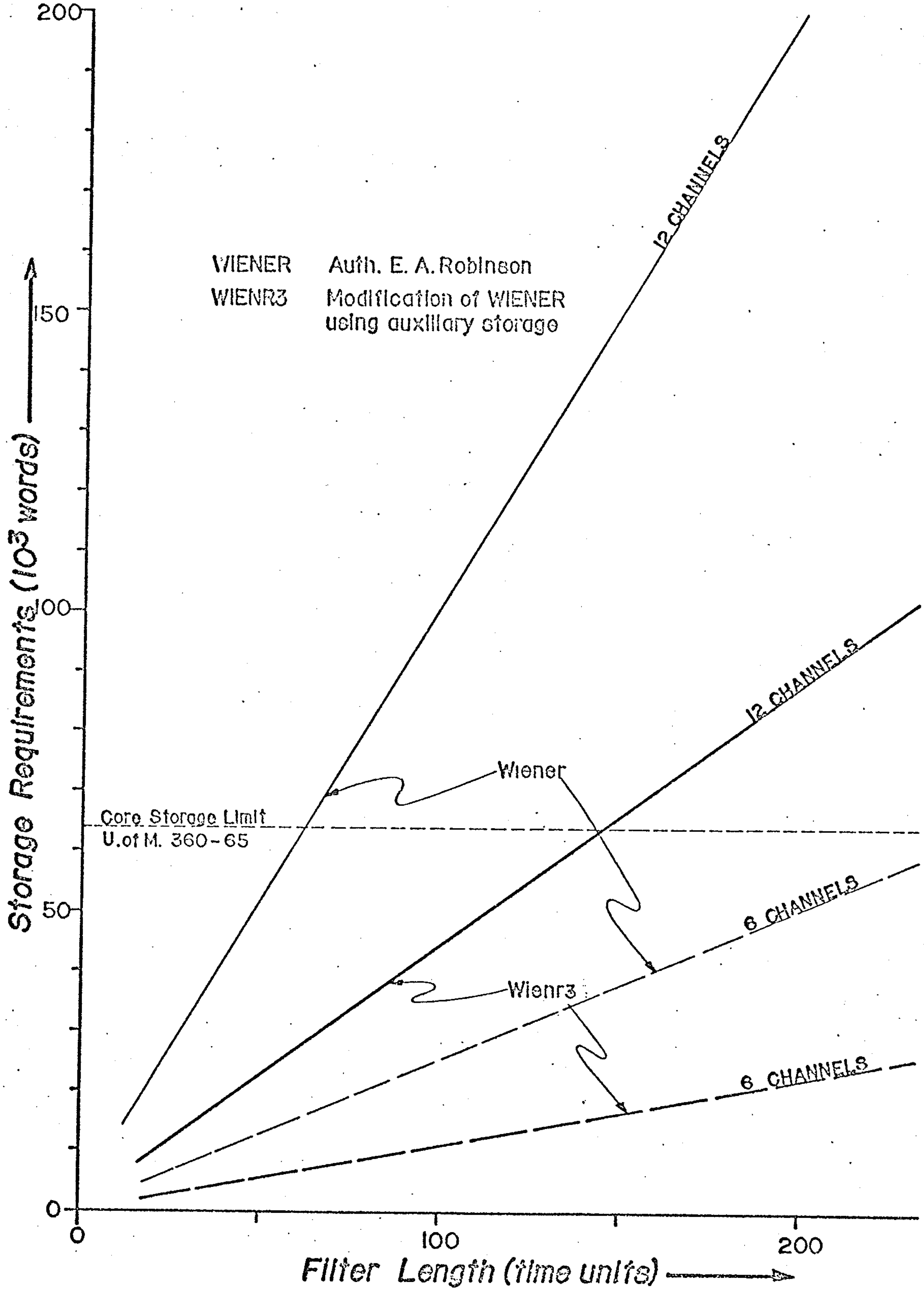



Figure 3.1 Storage Requirements for Least Squares Filter Determination

convolved section.

All the programs, whose implementation is described below, have been modified or designed for repeated convolution of sectioned data series.

3.2.2 Computation of Multichannel Filters using Auxiliary Storage.

The algorithm of Wiggins and Robinson [11] for the computation of the generalized multichannel Wiener filter has been programmed in FORTRAN IV by E. A. Robinson [4], (subroutine WIENER). However, practical application of this collection of subroutines was found to be severely limited with regard to filter length and the number of channels used because of rapid increase of core storage requirements with increase of filter lengths and number of channels. In fact, with an effective upper storage limit of approximately 64,000 single precision words (256k bytes), on the University of Manitoba IBM 360-65 computer, the maximum realizable length of a 12 channel filter was found to be approximately 60.

This program was, therefore, modified to include temporary storage, on a direct access disk file, of the three dimensional correlation matrices and auxiliary polynomials. The saving, in words, attained by using this modified program (WIENR3) is given by:

$$n \times m \times (4q + 1),$$

where $n$ is the number of input channels,

$m$ is the number of output channels and

$q$ is the length of the filter.

For a filter of length 100, which is to produce 12 output channels from 12 input channels, the reduction in core storage amounts to 57,744 words or 226k bytes, see Figure 3.1.

## 3.3 Processing Times.

### 3.3.1 Convolution Methods.

The convolution of discrete data series to produce correlation matrices and the convolution of time series with digital linear filters to produce filtered output are two of the basic processes in the reduction of digitized time series. The convolution process is also prohibitively time consuming if large filters or long time series are employed and direct (polynomial multiplication) methods are used.

Many commercial companies, who deal with seismic data processing, solve this problem with special purpose hardware and "convolvers", e.g. Digital Consultants' CFE-1 and Texas Instruments' TIAC 870.

When special purpose hardware is not available, two methods can be used. The first is to devise a convolution program which groups multipliers so as to replace multiplication by addition, which is then carried out by a linear program. One such program is subroutine PROCOR, developed by S. M. Simpson [3] and used by the Geophysical Analysis Group at the Massachusetts Institute of Technology. Unfortunately, the program is published in FAP and in this form can only be implemented on IBM 7094 computers.

The second method takes advantage of the fact that the convolution of two time series can be shown to be equivalent to the product of their respective Fourier transforms. Since the introduction of the fast Fourier transform technique, in 1965, by J. W. Cooley and J. W. Tukey [15], this method has had a very wide and far reaching impact on methods of numerical convolution.

Specialized subroutines such as PROCOR are largely machine dependent, and because comparisons of processing times show that indirect convolution using fast Fourier transform products is, in fact, the faster method for extra long time series (see paragraph 3.3.5), this latter technique is the one dealt with in this paper.

3.3.2 Convolution Using Fourier Transform Products.

The numerical convolution of two discrete time series functions $x(t)$ and $y(t)$ both of length $n$, is defined to be:

$$c(r) = \sum_{t=0}^{n-1} x(r)y(t-r), \quad r = 1, \ldots, 2n-1$$

where $r$ is the lag and the convolution is said to be formed by lagged products. If the Fourier transforms of the two series $x$ and $y$ are found, then, using the convolution theorem, this same numerical convolution can be expressed as the inverse transform of the product of the Fourier transforms of $x$ and $y$,

$$C(k) = X(k) \times Y(k), \quad k=1,\ldots,2n-1,$$

where $C(k)$, $X(k)$ and $Y(k)$ are the Fourier transforms of $c(r)$, $x(r)$ and $y(r)$ respectively. Then if the inverse transform of $C(k)$ is obtained the result is the convolved output $c(r)$.

In the direct computation of the numerical convolution there are $n(n+1)/2$ multiplications and additions, plus associated indexing operations. The number of operations required to obtain the Fourier transform of a series of length $n$, using fast Fourier techniques, is proportional to $n(\log_2 n)$. In this case $n$ must be a power of 2, i.e. $n=2^m$, where $m$ is a positive integer. The reduction in computation time achieved by use of Fourier transform products to compute the convolution depends on the lengths of the series $x$ and $y$, and on the program details. R. C. Singleton [16] reports tests which show a 16:1 time advantage for the transform product method over the direct method, for $n = 256$, while T. G. Stockham [17] states that speed-up factors of 50 have been realised with $n = 1000$.

3.3.3 Fast Fourier Transform Methods.

The fast Fourier transform technique is an algorithm to find an approximation to the Fourier transform of a function by computing the finite discrete Fourier transform. Given $n$ data points, $x_o, x_1, \ldots, x_{n-1}$, the finite Fourier transform is given by $y_o, y_1, \ldots, y_{n-1}$,

$$y_i = \sum_{k=0}^{n-1} x_k \, e^{-2\pi ijk/n}, \; j = 0, 1, \ldots, n-1 \qquad (3.3.3.1)$$

The given data points can be recovered by the inverse transform:

$$x_m = 1/n \sum_{j=0}^{n-1} y_i \, e^{2\pi ijm/n}, \; m = 0, 1, \ldots, n-1 \qquad (3.3.3.2)$$

Equations (3.3.3.1) and (3.3.3.2) can be rewritten in matrix notation as

$$Y = XW, \tag{3.3.3.3}$$

and $$X = YW^{-1}, \tag{3.3.3.4}$$

where $W$ is the $n \times n$ matrix $\{e^{2\pi ikj/q}\}$ $k,j = 0,1,\ldots,n-1.$

The fast Fourier transform algorithm is based on the factorization of the matrix $W$ into $n$ sparse matrices and a permutation matrix with the requirement that $n$ be a power of 2. By direct computation this process would require $n^2$ multiply and add operations, while the fast Fourier algorithm requires only $n \times n \log_2(n)$ operations.

Several versions of the original algorithm have been published in program form and are listed in the subroutine index, viz. COOL, HARM, FFT4 and NLOGN. The particular subroutine used in this paper is NLOGN (auth. E. A. Robinson), which itself is a later version of subroutine COOL. Subroutine COOL is machine dependent and will not work on IBM system FORTRAN machines, whereas NLOGN is suitable for any FORTRAN system.

3.3.3.1. Fast Fourier Transform for Real Input Series.

Most fast Fourier transform programs are designed for complex input series. A further reduction in computation time can be achieved for seismic time series, which are always in real form. Both real series, which are to be Fourier transformed, are combined, element by element, into the real and imaginary parts of a series of complex numbers, which resultant complex series is then used as the input series to the transform subroutine. This reduces, by half, the number of transforms required. The Fourier transforms of the two real series can be recovered from the joint transform by making use of the

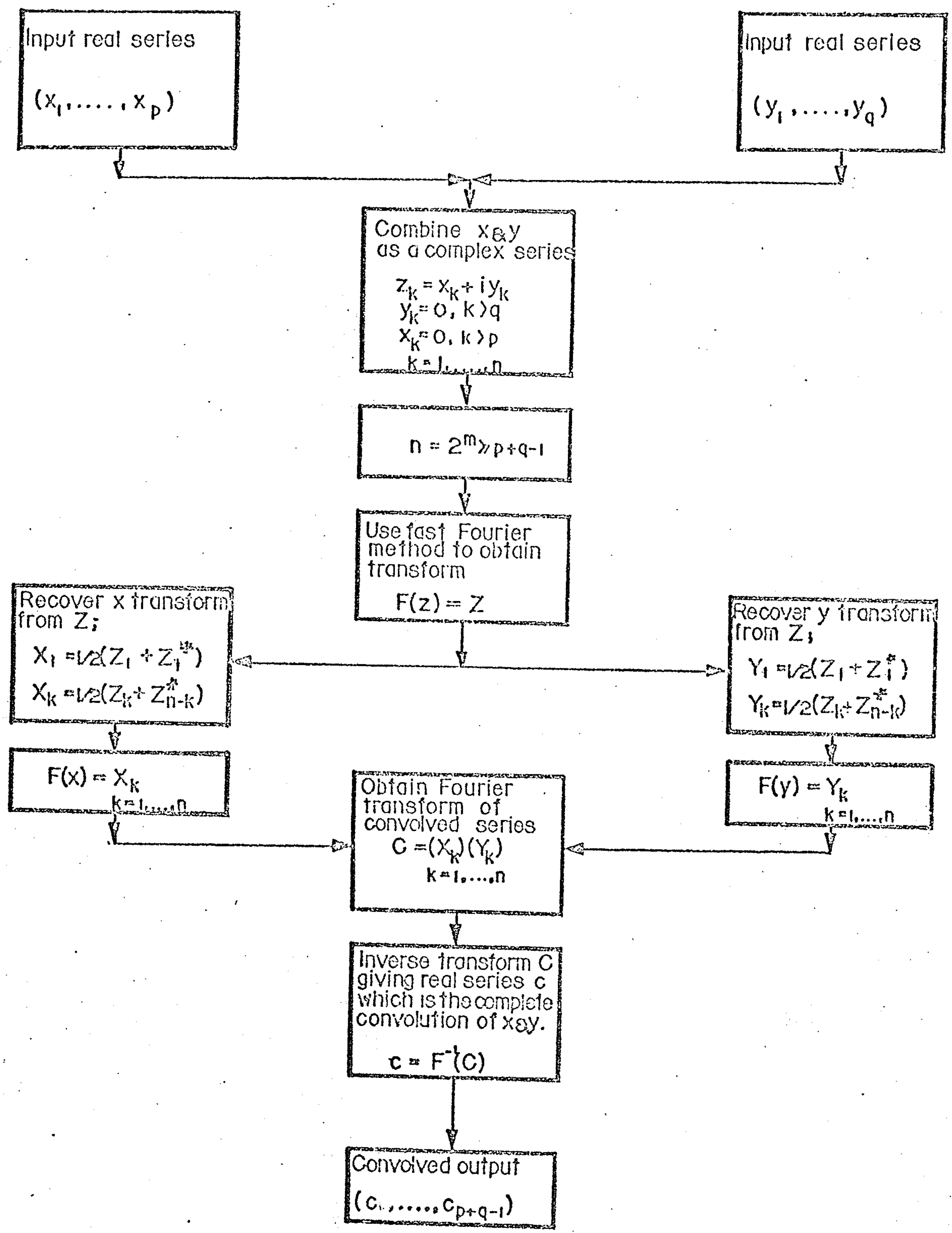



Figure 3.2 Computation Scheme for the Numerical Convolution of Two Real Series

latter's symmetry properties. An outline of such a computation scheme is shown in Figure 3.2.

3.3.4 Convolution of Segmented Time Series Using Transform Products.

3.3.4.1. Optimum Section Length.

When a linear digital filter is applied to a seismic trace the basic process is one of numerical convolution of the filter coefficients with the input time-series. Typical single channel parameters would be a filter of length 50 and a seismic trace of length 36,000: this being equivalent to approximately 60 seconds of record time on a refraction record digitized at a 1.7 millisecond sample rate. Obviously, in terms of storage, it would be extremely wasteful to augment both the input trace and the 50 point filter vector to equally dimensioned vectors of length 65,536 ($=2^{16}$) for convolution by the Fourier transform product method. Instead, the input time series, which is to be filtered, is partitioned into sections or segments.

Both Stockham [17] and Gentlemen [18] deal with sectioning. Stockham gives, for the time required for sectioned convolution using fast Fourier transform techniques, the relationship:

$$T = k_{ct} \times (2K + 1) \times n \times \log_2(n) + k_{aux} \times k \times n$$

where $T$ is the time in seconds for the convolution of a filter of length $q$ with a series of length $p$, partitioned or sectioned into $k$ sections, each of length $p'$.

The constants $k_{ct}$ and $k_{aux}$ are machine constants, which for Stockham's implementation on an IBM 7094, were measured as 60 and

300 microseconds respectively. The variable $n$ is the length of the convolved section and must be a power of 2, i.e.

$$n = 2^m \quad (m \text{ is an integer}),$$

$$\text{and} \quad n \geq p' + q - 1.$$

The second term on the right hand side of equation (3.3.4.1) is the time required to complete the auxiliary processes.

For the purpose of finding the optimum section length, i.e., the value of $p'$ for which the computation time $T$ is a minimum, Gentlemen and Sande [18] ignore the auxiliary processes and the time for the transformation of the filter series and use $T'$ as the time to be minimized, where $T'$ is now:

$$T' = p/(n - q) \times C \times n \times \log(n). \qquad (3.3.4.2)$$

where $C$ is a constant.

Then, under the assumptions that the filter length is very much smaller in magnitude than the length of the input data series and that $n$ is continuous, they show that partial differentiation of equation (3.3.4.2), with respect to $n$, yields the following expression:

$$q = n/(1 + \log(n)) \qquad (3.3.4.3)$$

In the example given, an optimum section length of 300 is suggested for a 50 point filter. In fact, of course, the convolution of a 50 point filter with a section of 300 data points would produce an output segment of length 349. Now the constraints on n viz. it must be an integral power of 2 and must be greater or equal to $(p' + q - 1)$, make it possible to increase this section length to 462 without any increase in computation time. Because n must be an integral power of 2 for the fast Fourier transform algorithm and is, therefore, not continuous, equation (3.3.4.3) is not of great practical use.

Stockham suggests trial evaluations of equation (3.3.4.1) to find the appropriate optimum section length. He also notes that memory (core storage) allocation has to be considered when n becomes very large.

Empirical measurements of convolution times for different filter and section lengths are discussed next.

3.3.4.2. Empirical Measurements for Optimum Section Lengths.

Figure 3.3 shows empirical processing times for different filter and section lengths, measured on the University of Manitoba IBM 360-65. The subroutine used was FTCONV and the timing method is described in Appendix D. Subroutine FTCONV computes the convolution of a linear filter with a sectioned or segmented input series by the Fourier transform product method. The section lengths in each case were chosen so that the convolved output section length was a power of 2. This represents the most efficient use of the transform product method. Intermediate section lengths would be augmented to the next highest integral power of 2 and the processing times would, in fact, follow a step function as is shown for the curve for a filter length of

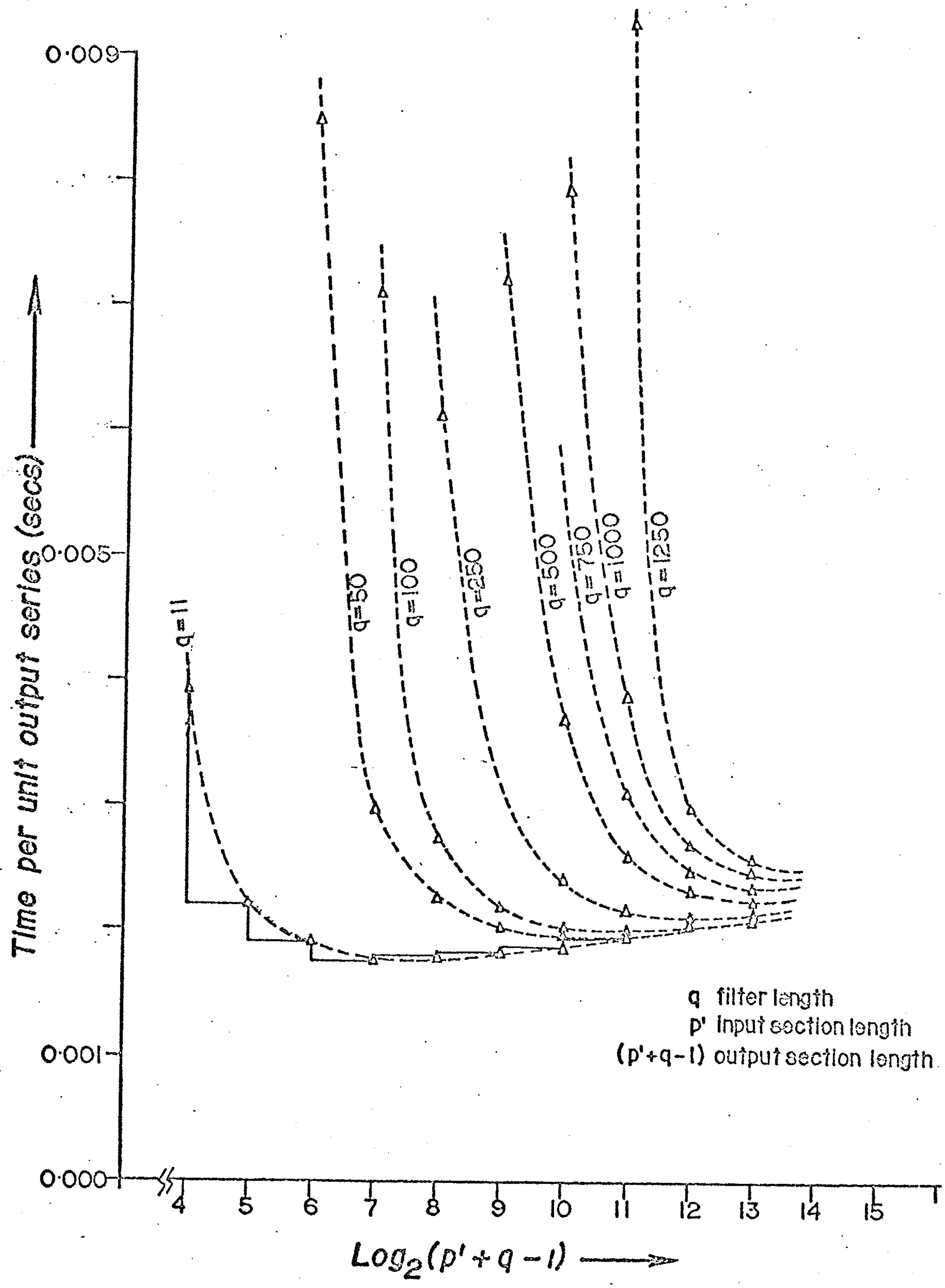



Figure 3.3 Emperical Time Curves for Optimum Section Lengths

11. For representational purposes, however, the other times are connected by a broken curve for each specific filter length.

The general configuration of these empirical time curves shows that, as the section length increases in value away from that of the filter, the processing time per unit output series, (i.e. times are normalized with respect to the total output length) decreases very rapidly to a minimum value, after which, it increases much more gradually as the section length is increased.

As the filter length is increased so also does the optimum section length, until, for filters of lengths of 500 or more, it approaches a value similar in magnitude to the unsectioned input data series. Thus, for filters of these lengths, no actual minimum was obtained, though the curves do show that a dramatic reduction in computation time can be achieved by using an output section length of 8192 ($=2^{13}$). Increasing the output section length to 16,384 ($=2^{14}$) and a doubling of the core storage requirements for FTCONV would not substantially reduce computation times.

3.3.5 Comparison of Processing Times for Different Convolution Methods.

In general, for filters of lengths normally employed in digital data processing, the computation time for convolution can be greatly reduced if the Fourier transform products method is used and an optimum, or near optimum, section length is chosen.

Figure 3.4 shows a comparison of computation times for a direct method subroutine CONCNV and a fast Fourier transform product subroutine FTCONV. Optimum, or near optimum, section lengths were chosen for the transform product computations. Except for small filter

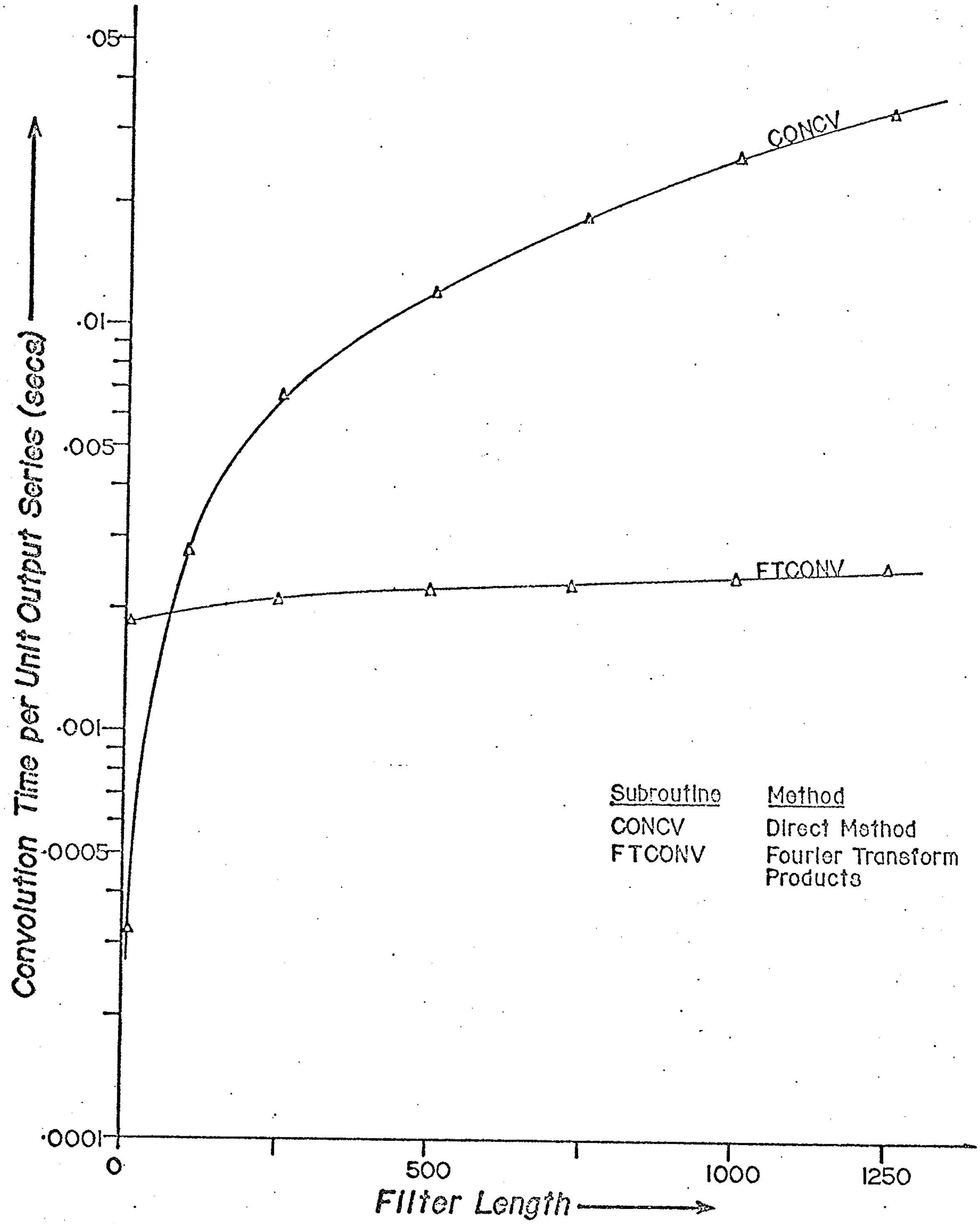



Figure 3.4 Convolution Times for Direct and Transform Product Methods

lengths, where the direct method is the faster of the two, the transform product method shows dramatic reduction in processing times, particularly as the computational effort increases.

Another comparison of computation times is shown in Figure 3.5. Here the computation is for the autocorrelation of a specific data series. This process simply involves the convolution of a time series with one comprising the identical series in reverse order. The data chosen was the digitized ambient noise signal from a refraction seismic record; this would be the first major computation in the determination of a least squares (Wiener) filter.

The subroutines used for the measurements were HEAT (auth. E. A. Robinson), which employs direct multiplication and addition, and FTCORR, which performs the same convolution but uses Fourier transform products. The same reduction in processing times, as in Figure 3.4, are evident.

Also displayed in Figure 3.5, are the empirical times published by S. M. Simpson [3] for a similar range of measurements using subroutines QACORR and FORAC. Subroutine QACORR calls the fast convolution program PROCOR, which does not employ the transform product method but replaces multiplication by summation by grouping of the multipliers, (see paragraph 3.3.1.). FORAC is a direct method subroutine which computes the Tukey approximation to the autocorrelation function.

Though the times for QACORR and FORAC were taken from an IBM 7094 implementation, the comparison in Figure 3.5, between direct methods and indirect methods such as those employed by FTCORR and

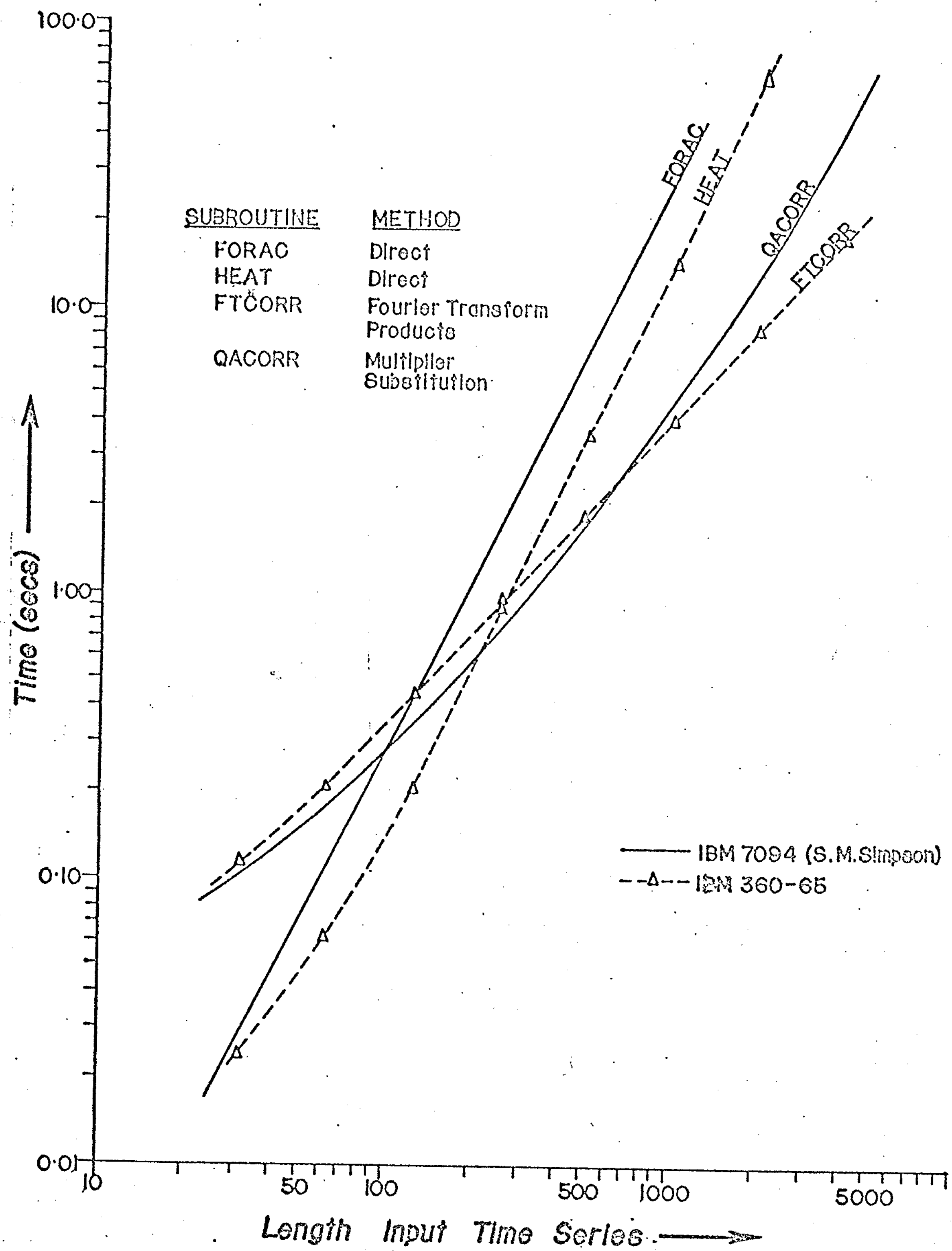



Figure 3.5 Comparison of Process Times for Autocorrelation Subroutines

QACORR, shows the same general relationship. In addition, for data series of lengths greater than 1000, the fast Fourier transform product method shows an increasing smaller increment in computation time for increasing input lengths, compared to that of QACORR.

3.3.6 Observed Error in Fast Fourier Transformation.

The most widely used method for measuring the error introduced during fast Fourier transformation processes is, to first transform a series and, then, to recover the original series from the finite Fourier transform by inverse transformation. Gentlemen and Sande [18] have derived the theoretical bound of the ratio of the root mean square (RMS) of the error and the original data for this method. Their expression is given by:

$$||R|| < 2 \times 1.06 \times \sqrt{N} \times (2 \times n)^{3/2} \times 2^{-b} \qquad (3.3.6.1)$$

where, $||R||$ is the bound for the ratio RMS error/RMS data

$N$ is the number of data elements,

$n$ is the radix (in this case 2), and

$b$ is the number of bits in the characteristic (mantissa) of the fixed length floating point.

Results for the above experiment, using an IBM 360-65, subroutine NLOGN and typical seismic data series, are shown in Figure 3.6. As Gentlemen and Sande note the form of the theoretical bound is a fairly good description of the form of the observed errors, though the magnitude of the theoretical bound is a good deal larger than is typical of the empirical results. These results agree well

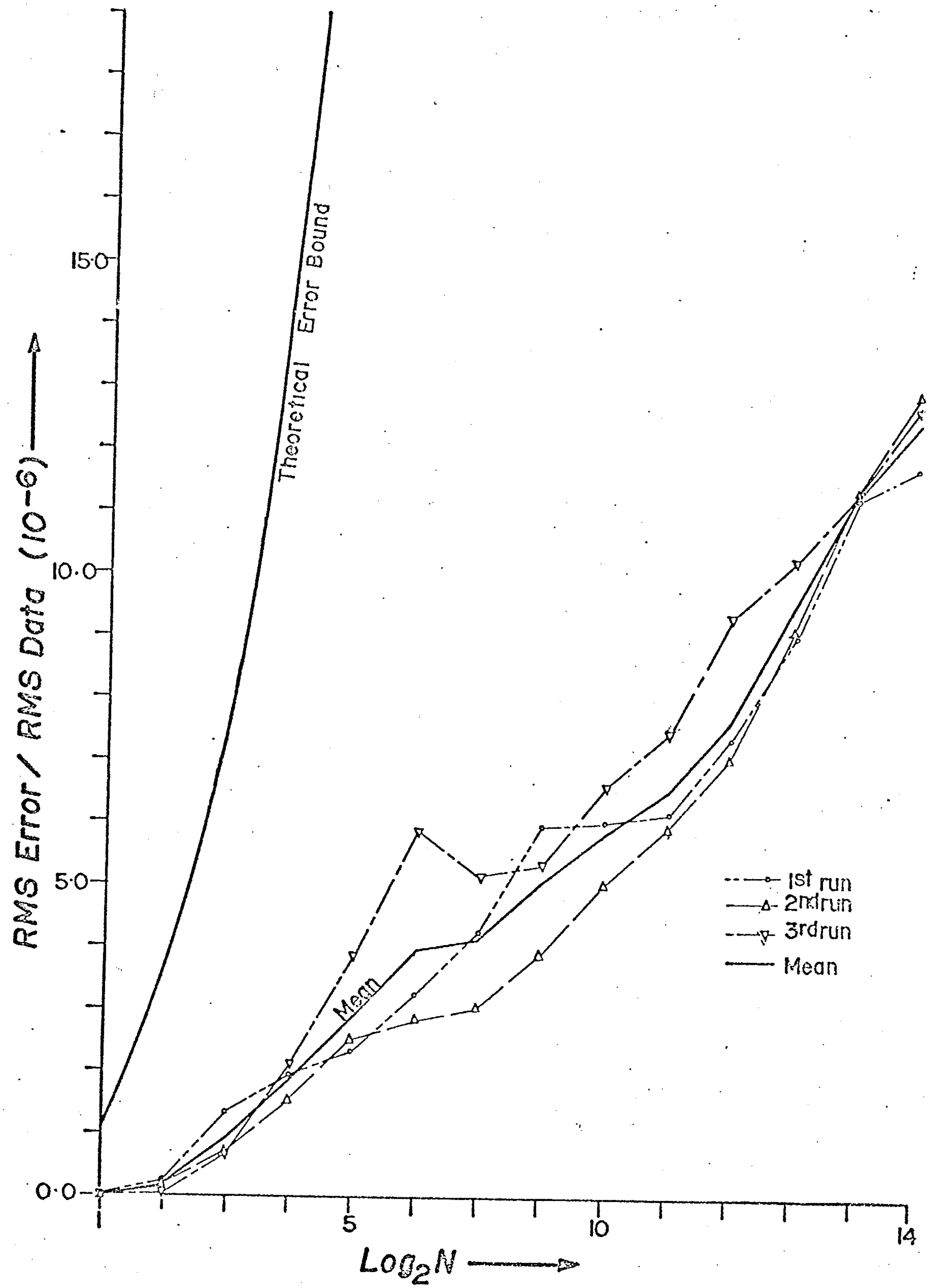



Figure 3.6 Accuracy Measurements for Subroutine NLOGN. Subroutine NLOGN computes the Fast Fourier Transform of a real series, (auth. E. A. Robinson).

with those reported by Gentlemen and Sande [18] and with those of Singleton [16].

When convolution is carried out using Fourier transform products and fast Fourier transformation methods, the resultant, overall error was found to be less than 0.1 and was not considered to be significantly large. Numerical errors such as these, introduced as a result of machine computation, can be considered as equivalent to the instrument noise effects of analog circuits and are usually of insignificant magnitude when compared to the unwanted seismic noise actually recorded as data.

## 3.4 Conclusions.

Conclusions from the results of the comparisons of digital data processing techniques discussed in this chapter can be summarized as follows.

1. Channel segmentation or trace sectioning is a practical method for overcoming core storage problems when dealing with long multi-channel time series data.

2. The storage requirements for the generalized least squares filter computation can be considerably reduced by use of auxiliary storage on direct access disc files.

3. Convolution by Fourier transform products, in conjunction with fast Fourier transformation methods, is far superior, in terms of processing times, to direct convolution methods, if the filter or correlated time series is of length greater than 100 elements. In addition, the transform product method of convolution results in a

reduction in computation times of the same order of magnitude as that afforded by previously published, multiplier replacement methods.

4. Accuracy measurements show that the magnitude of errors, due to the use of fast Fourier transformations, is not significantly large.

5. Empirical time curves for convolution of sectioned input series, using transform products, show the necessity for choosing optimum, or near optimum, section lengths.

# CHAPTER IV

# FILTER DESIGN AND PARAMETER OPTIMIZATION

## 4.1 Introduction.

Related to the question of efficient implementation techniques in digital data processing is the problem of parameter and design criteria for the computation of digital filters. In particular, the use of an unnecessarily long filter operator will result in an inherently inefficient data process even though optimum implementation techniques are used.

The criteria for the design of a specific filter depend largely on the particular application and the underlying assumptions concerning the data. For the purpose of a preliminary investigation into these problems, a linear prediction filter was devised and tested on some seismograms recorded as part of refraction surveys conducted by the University of Manitoba Geophysics department. The objectives of this type of surveying and the general principles of prediction error filtering have been outlined in Chapter II.

The FORTRAN subroutine used to compute the numerical coefficients of the different prediction filters was WIENR3. This is a modification of subroutine WIENR (auth.: E. A. Robinson) which uses the recursion method of Wiggins and Robinson for least squares multichannel filter computation, (see paragraph 2.3.3).

## 4.2 Filter Parameters and Design Criteria.

The known quantities in the normal equations, which define multichannel filter coefficients, are the autocorrelation of the input series, and the cross correlation of the input series and the desired output series. In the case of the linear prediction filter the desired output is a time advanced portion of the input series and the cross correlation function is, therefore, the autocorrelation of the input series for lags advanced in time by an amount equal to the prediction span.

Thus, the first step in the design of a filter is to obtain an estimate of the true autocorrelation function from a finite portion, or sample, of the input series and this requires a decision as to the number of channels and lengths (time gate) of inputs to be included in this sample. This estimate is further improved by smoothing or tapering of the finite autocorrelation function. The type of smoothing operator or spectral window and the amount of smoothing or tapering of the autocorrelation estimate are two additional variables in filter design.

The most important criterion in the choice of a filter, from the point of view of efficient data processing, is of course the length, or number of elements, in the filter operator and this parameter is closely associated with the behaviour of the normalized mean square error curve.

The measurements and discussion of this chapter deal with the effect of variation of the above variables in filter computation on the overall filter performance and the computational effort involved in the application of the filter to the data. These filter parameters can be summarized as follows:

i) Length and number of channels comprising the input data sample.

ii) Type and length of smoothing operator used to improve the estimate of the true autocorrelation function.

iii) The length or number of elements in the filter operator.

The effect of variation of prediction span was not investigated as the required length of undisturbed output following a signal arrival was taken to be a constant 100 time units (0.17 secs.).

## 4.3 The Normalized Mean Square Error.

The recursive method of Wiggins and Robinson, for the computation of the filter coefficients, proceeds by calculating a filter operator of increasing length until either, a maximum length has been achieved, or, the normalized mean square error has reached some previously specified condition. The mean square error between the desired output and actual output is normalized with respect to the first term of the desired output autocorrelation function. It is computed at each step in the filter recursion, that is, for each length of filter operator there is an associated normalized mean square error.

This error term, thus, affords a quantitative measure of the effectiveness of a filter in the range of the given input sample data from which the filter is derived. The overall effectiveness of the filter, when applied to data outside this range, depends on the validity of the basic assumption, that the statistical properties of the time series are time-invariant and remain constant throughout the chosen range of applicability of the filter. Because this assumption is often only approximately true the normalized mean square error cannot serve as a quantitative measure of overall filter effectiveness. It is very useful, however, in demonstrating the effect of parameter variation on the filter characteristics.

A typical normalized mean square error curve for increasing filter lengths is shown in Figure 4.1. The error term decreases fairly rapidly at first and then levels off to an almost stationary value which decreases very little, if at all, with increase in filter length.

## 4.4 Estimation of the Autocorrelation Function.

### 4.4.1. Length of Input Sample.

Most writers recommend that the length of time series, used for the calculation of the estimate of the autocorrelation function, should be as great as possible. The assumption is that, by drawing the input data sample from as large a time span as possible, the effect of variation, with time, of the statistical properties of the ambient seismic noise, will thereby be averaged out.

Jenkins [19], on the other hand, states that the length of the sample and, hence the number of time increments (lags), need only be sufficient to compute the finite autocorrelation function to a length where it has decreased to an insignificantly small magnitude. Wherever practical, therefore, the ambient noise autocorrelation functions for the channels involved, or at least a representative number of them, should be inspected to allow choice of a suitable input sample length.

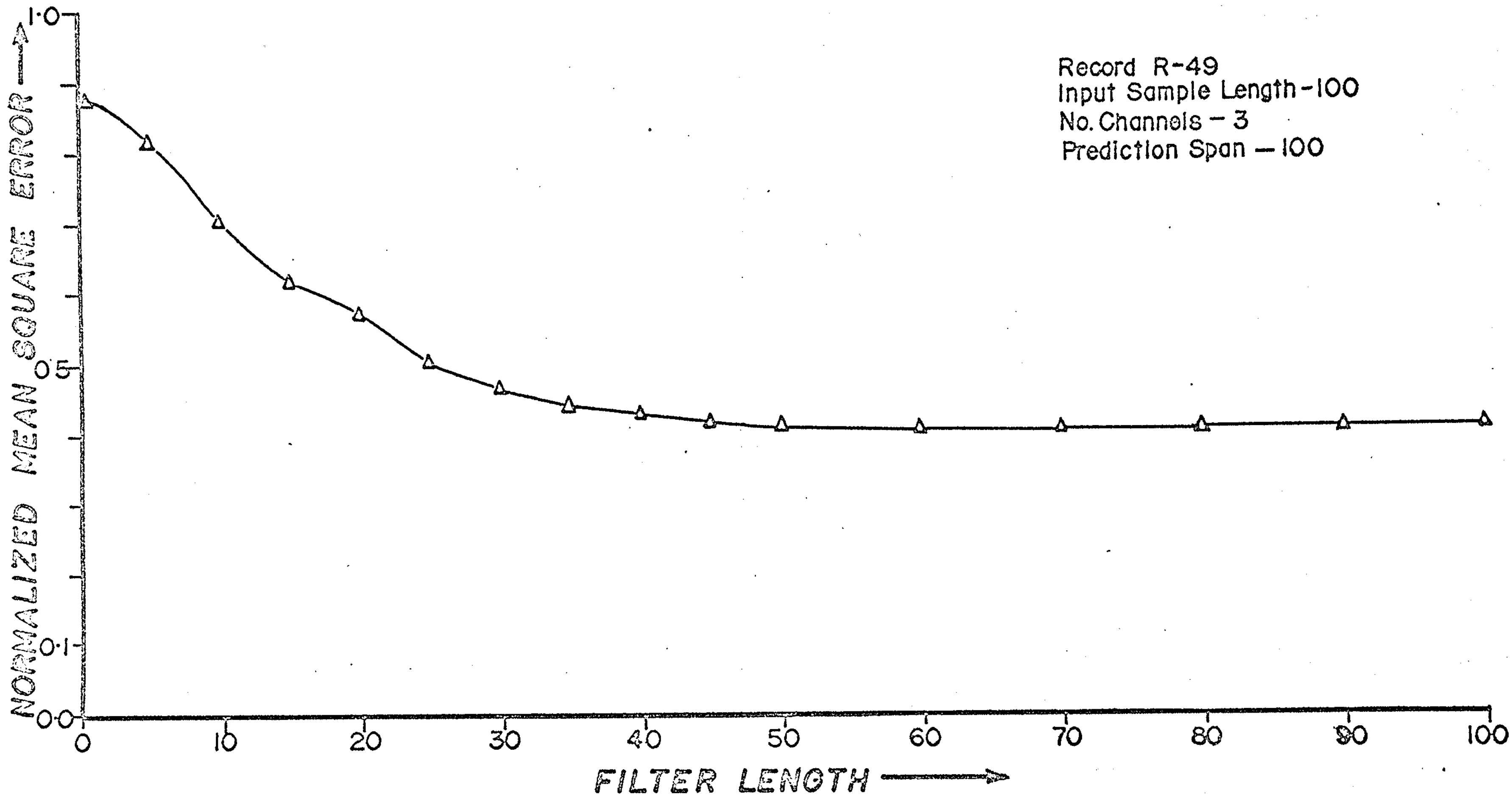



Figure 4.1 Typical Normalized Mean Square Error Term

Figure 4.2(a) shows a typical example of a single channel autocorrelation function while Figure 4.2(b) shows a comparable function for a random series of the same length which has also been frequency filtered with a bandpass of 0-30 cps. The similarity between the two functions supports the assumption that the presignal noise recorded on the seismogram corresponds to random noise in the statistical sense.

In Figure 4.2(a), as the lags increase, the function continues to oscillate at a low but fairly constant rate. Jenkins [20] states that, when an autocorrelation function fails to damp out according to expectation, this is due to correlations between adjacent ordinates. He further states that the magnitude of the residual periodicities will increase as the number of time series terms, used in the estimate, decreases. This indicates that the input sample length should be of sufficient length to avoid augmenting this effect.

In this example the main peak of the autocorrelation is adequately delineated by lags from -100 to +100. That there is sufficient statistical information in this portion of the function is confirmed when it is used to compute a linear prediction filter. It was found that the normalized mean square error showed only slight reduction for filter lengths greater than 60. It must be remembered that in the recursive filter computation the central term (i.e. the maximum value) of the autocorrelation is first used and then successively longer filters are calculated using an increasing number of ordinates of the autocorrelation function.

4.4.2. The Number of Channels in the Input Sample.

The same considerations which apply to single channel auto-

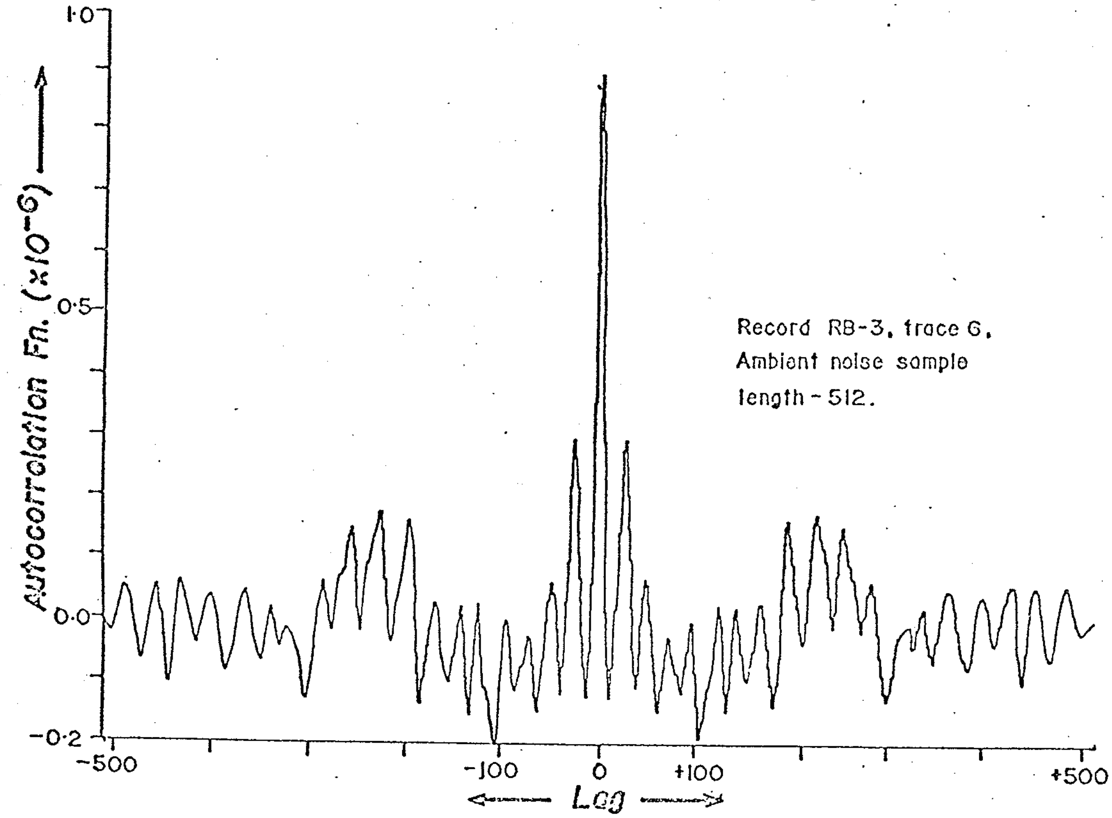



Fig. 4.2(a) Single Channel Autocorrelation Function

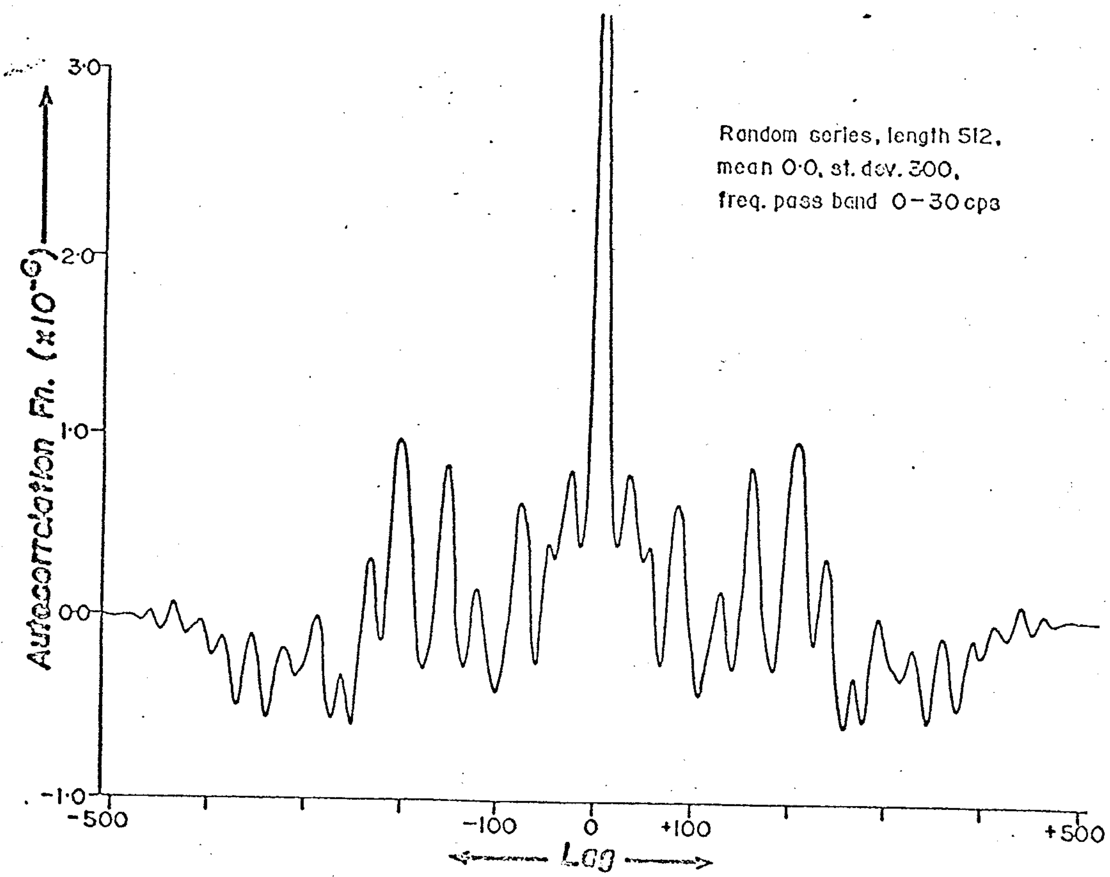



Fig. 4.2(b) Autocorrelation Function of a Random Series

Figure 4.2 Single Channel Autocorrelation Functions correlation estimates apply also to the multichannel case. The number of data points included in each channel input sample must be sufficient to produce an autocorrelation function which does not produce large residual periodicities and which successfully delineates a good approximation to the main peak of the true autocorrelation function.

Otherwise, it is obvious that the multichannel case incorporates samples of the ambient micro-seismic noise from different locations corresponding to the different geophone locations in the field recording layout. Consequently a much more representative sampling of the presignal noise is obtained.

Figure 4.3 shows different error curves for different number of channels used in the filter calculation. For filters of lengths greater than 30 the error term is smaller for increasing number of channels, though it appears that, in this example at least, progressively less advantage is to be gained at each increase in the number of channels. Thus, while the greater computational effort required to compute a 6 channel filter is offset by the smaller length filter required for a specific residual error, this would not necessarily be true for an increase from 6 to 9 channels. The different shape of the 9 channel curve could be due to the inclusion of different noise characteristics from the additional 3 channels which have been sampled.

## 4.5 Type and Length of Smoothing Operator.

The effect of smoothing the finite autocorrelation function with a spectral window is to reduce the variance of the estimate of the

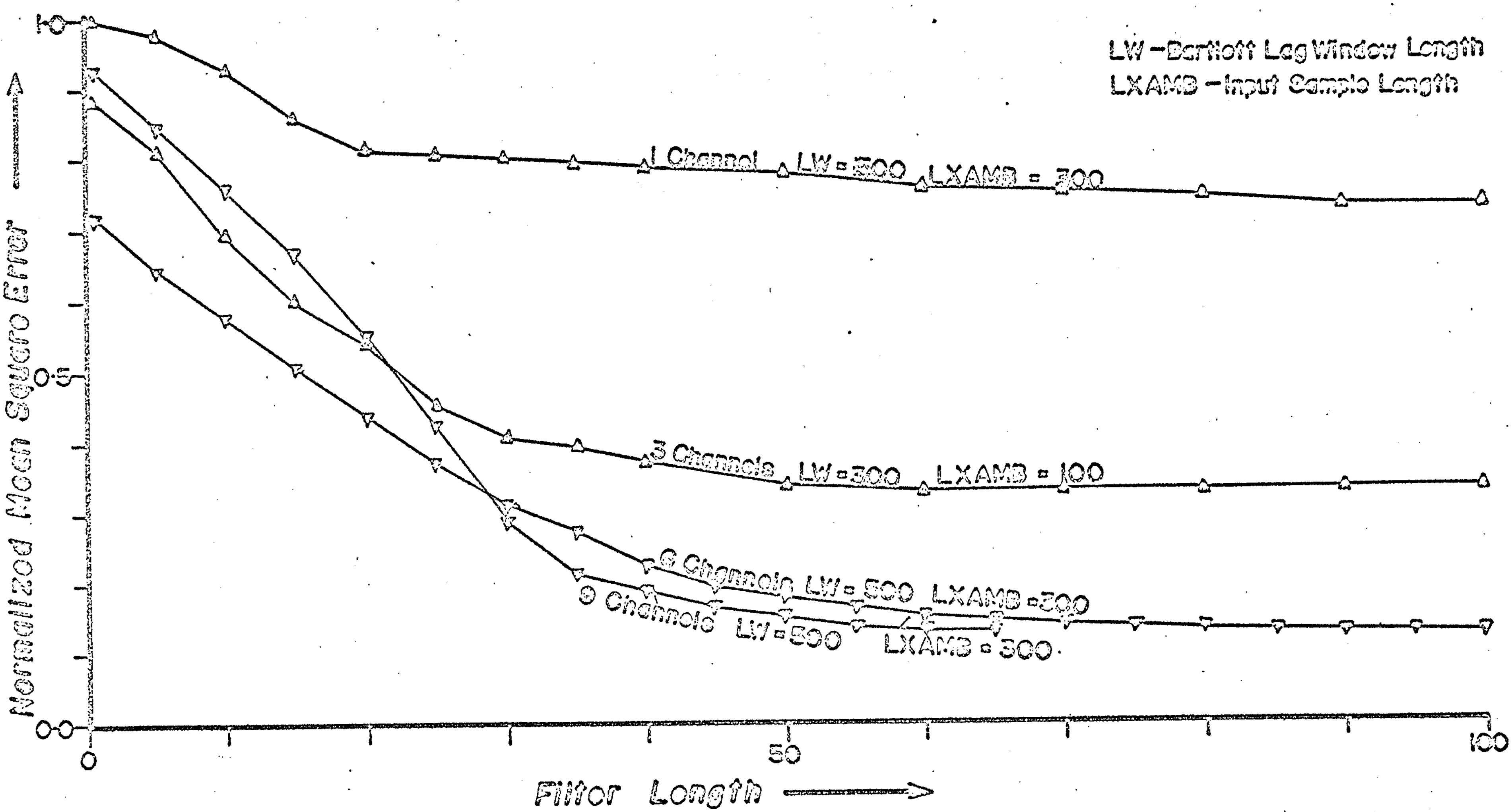



Figure 4.3 Mean Square Error Curves for Varying Number of Input Channels

true autocorrelation function. It can be shown (see Jenkins [20]) that, in the statistical sense, a smoothed autocorrelation function from a short sample series often provoides a better estimate of the true autocorrelation function than an unsmoothed autocorrelation function derived from an input sample of much greater length. It is, therefore, a very practical way of reducing the required computation for a specific filter calculation.

Jenkins [19, 20] has shown that the empirical approach to smoothing is the most practical. He has also shown that the actual form or type of the smoothing operator is of far less importance than the bandwidth, or amount of smoothing, which is carried out. Subroutine WIENR3 uses the Bartlett spectral window, which, in the time domain, is equivalent to multiplying each term of the finite autocorrelation function by the weight $w(t)$ defined by:

$$w(t) = \begin{cases} 1 - \frac{|t|}{LW} & , |t| \leq LW \\ 0 & , |t| > LW \end{cases} ,$$

where $LW$ is the length of the lag window and

$t$ is the time index

In the frequency domain the Bartlett spectral window can be considered as a smoothing operator $W(f)$ of the form:

$$W(f) = LW \times \left( \frac{\sin \pi f LW}{\pi f LW} \right)^2$$

where $f$ is the frequency.

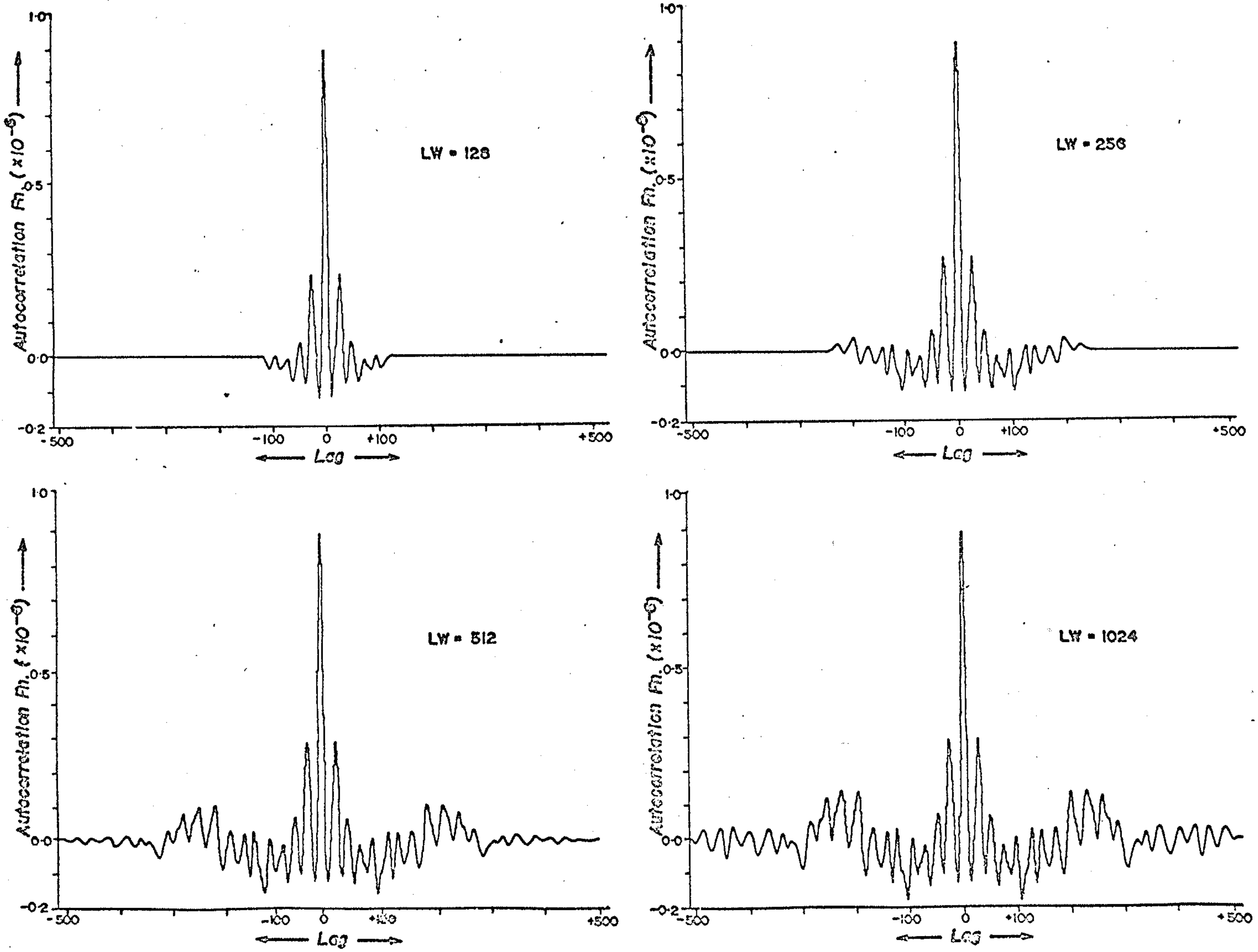



Figure 4.4   Autocorrelation Function with Different Barlett Lag Window Lengths (LW).

(Sample length 512; Record RB-3, Channel No. 6)

Figure 4.4 shows the form of a particular single channel autocorrelation function, in the time domain, for various lengths of lag windows. The severest tapering occurs for small values of lag window lengths (LW). As LW is increased, a point is reached where further increase produces little, if any, change in the autocorrelation function. This point is usually reached when LW is approximately equal to the length of the input sample length. This is further illustrated in Figure 4.5, where the effect on the normalized mean square error curves of differing lag window lengths is shown, for two different 3 channel cases.

As can be seen from a comparison of Figures 4.5(a) and 4.5(b) the most marked effect of variation of lag window length occurs for the smaller input sample lengths.

The actual lag window length which should be used for a given autocorrelation estimate must be chosen to form a compromise between a large value of LW, which produces a relatively small amount of smoothing and a low residual error term, and a small value of LW, which gives a high degree of tapering and a large residual error term. Jenkins [20] shows that this represents a compromise between a high bias and a low variance in the estimate of the true autocorrelation function. He proposes an empirical procedure which consists in starting with a large bandwidth (large LW) which is then decreased until sufficient detail is apparent, in the frequency domain, for the purpose of the estimation. When computing digital filters for seismic data processing, it would probably be more practical to choose that value of lag window length, which, if increased, would not produce any large

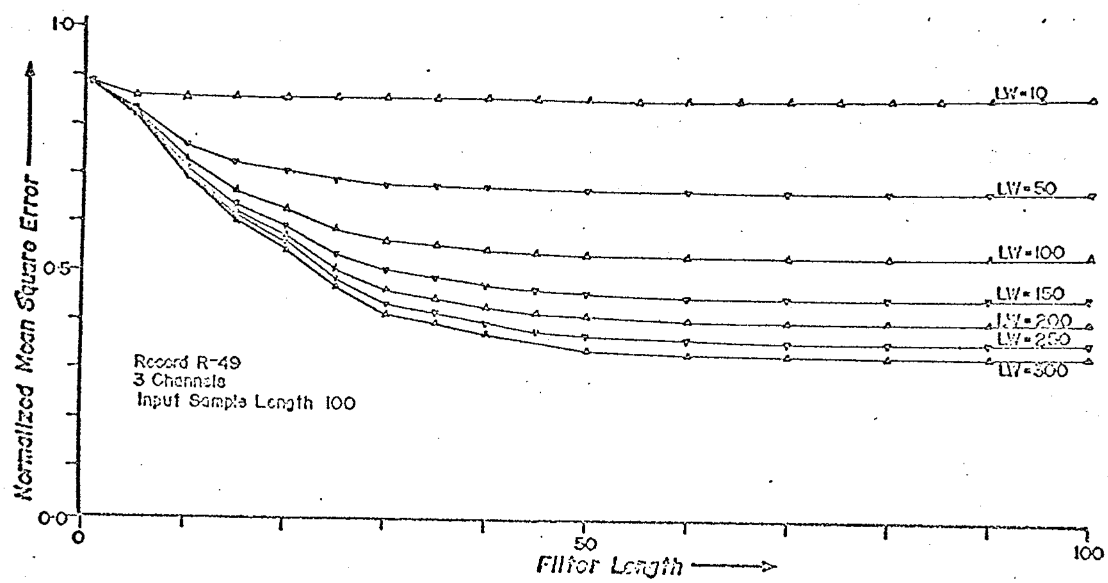



Fig. 4.5(a) Input Sample Length 100

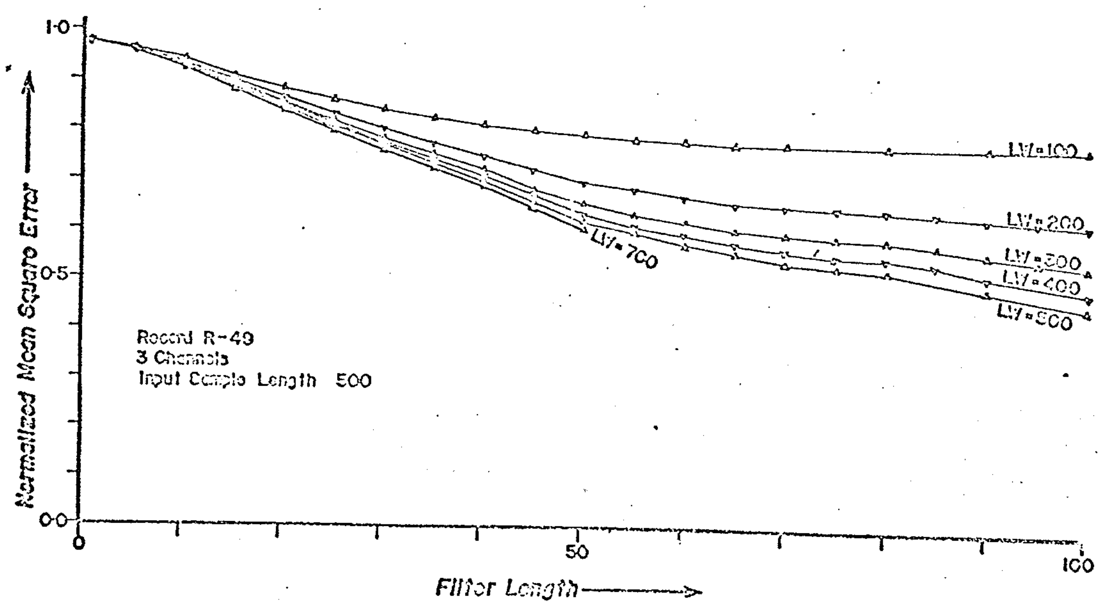



Fig. 4.5(b) Input Sample Length 500

Figure 4.5 Normalized Mean Square Error Curves for Varying Lag Window Lengths (LW) reduction in the error terms for a specific filter length. This value has been shown (see Figure 4.5) to be approximately equal to the length of the input sample.

If it is impractical to examine the autocorrelation function of all the data, i.e. if a standard filter computation is being devised for a large number of seismic records, a fairly good rule of thumb might be to always set LW equal to the length of input time series sample from which the true autocorrelation function is to be estimated.

## 4.6 Filter Operator Length.

Theoretically, the greater the length of filter operator computed the greater the reduction in mean square error between the actual output and the desired output. However, in practice, increasing the filter length beyond a certain point does not necessarily lead to any significant reduction in the error term (see Figure 4.1). Empirical results also indicate that most filter lengths that would be appropriate would be considerably less than 100.

In addition, published results of various least squares filter applications, confirm that filters of greater lengths than 100 are seldom employed.

Unless composite filters are to be used (see paragraph 5.1) these lengths of operators require direct convolution methods of application (see Figure 3.4). It is, therefore, essential for efficient data processing of substantial amounts of data that the minimum acceptable filter length be employed, and this requires inspection or monitoring of the behaviour of the normalized mean square error term.

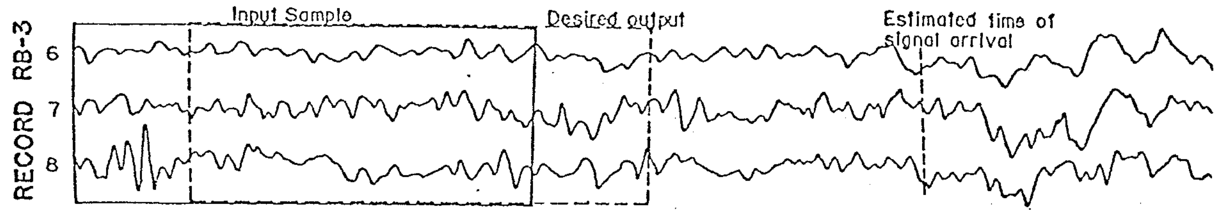
## 4.7 Prediction Filtering of Actual Data.

The mean square error between the actual and desired outputs is a measure of the effectiveness of the filter for that specific portion of the input time series from which it is derived. The filter, however, has to be effective over the whole of that portion of the time series to which it is to be applied. In the case of the linear prediction filter, the filter is applied to that portion of the seismic record immediately preceding and including the first energy arrivals.

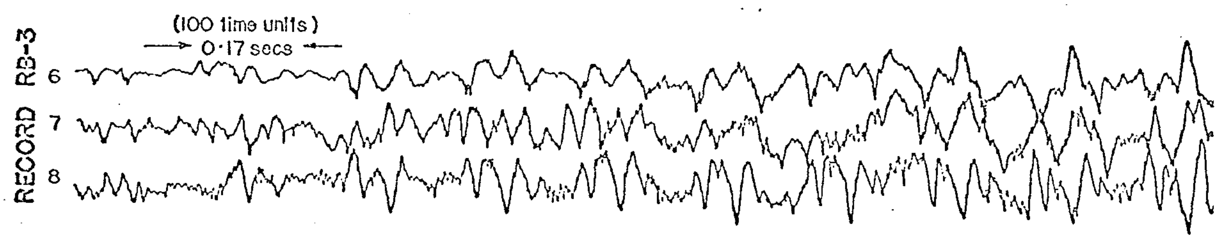
As Peacock and Treitel [21] point out, one measurement of the effectiveness of prediction filtering is the reduction in energy content of the output time series relative to the energy of the input series. Linear prediction filtering has the advantage that no scaling constants are introduced in the filtering process.

Figures 4.6 and 4.7 show the results of the application of two, 3 channel, prediction filters to specific traces on refraction seismograms R-49 and RB-3. In each case the prediction error series have been frequency filtered with a bandpass of 0-30 cps. Figure 4.6 shows the output from each stage in the filtering process while figure 4.7 illustrates the comparison between prediction filtered and simple frequency filtered traces.

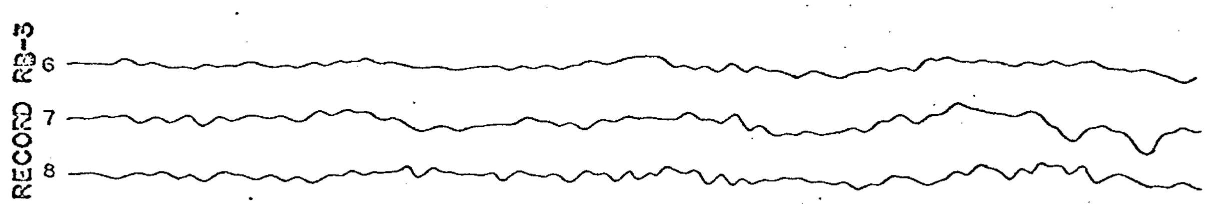
No quantitative measurements of the reduction in noise energy were made as the results are by no means conclusive and the design of the filters is entirely preliminary. However, a definite reduction in noise energy is apparent in the frequency filtered prediction error



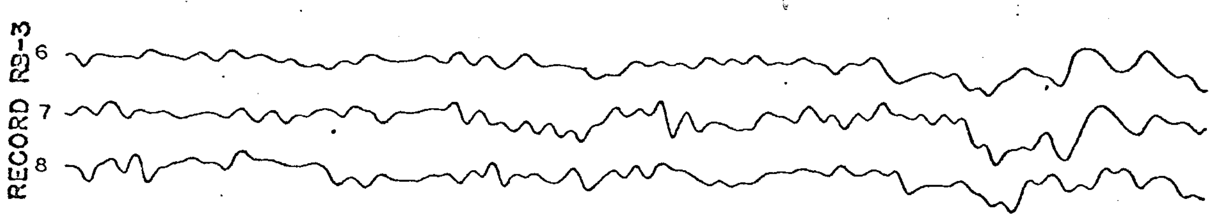

Original data after digital conversion of FM analog field recording




Prediction Error Series : output from application of Linear Prediction Filter.
Filter length: 50; Lag window length: 500.

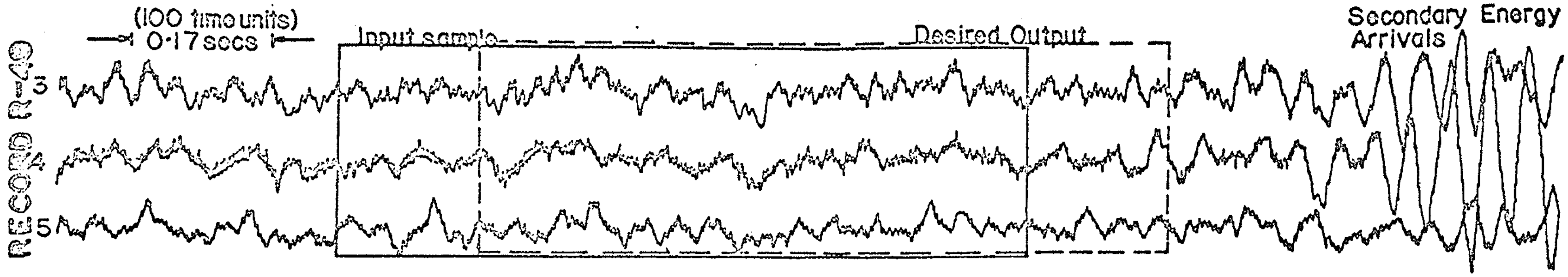



Frequency Filtered Prediction Error Series: bandpass 0-30 cps

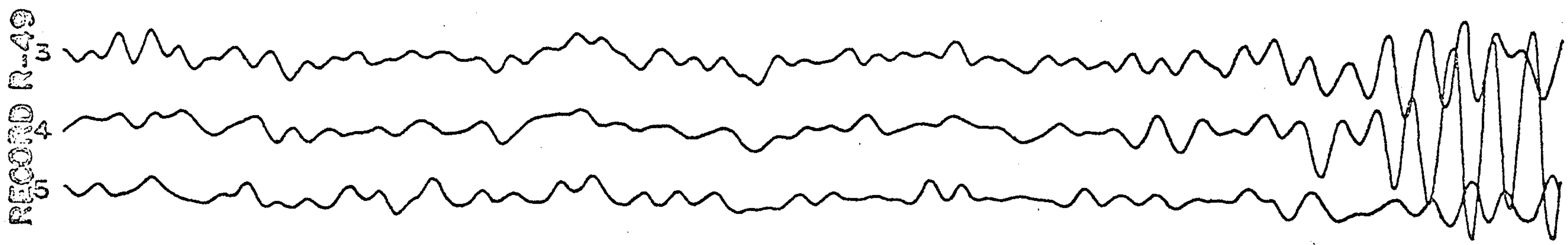



Original data frequency filtered: bandpass 0-30 cps

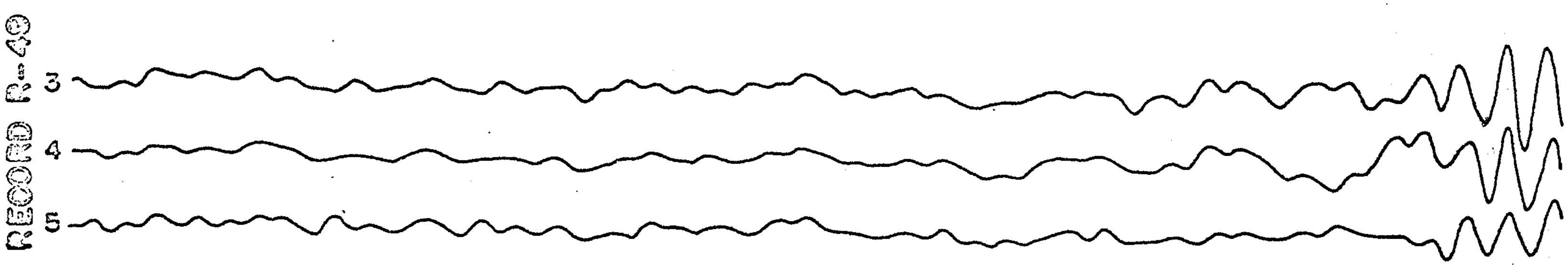
Figure 4.6 Prediction Filtering of Record RB-3




Original data from digital conversion of FM analog field recording.




Original data frequency filtered: bandpass 0-30 cps




Frequency Filtered Prediction Error Series: bandpass 0-30 cps
Linear Prediction Filter length-50, Lag window length-500

Figure 4.7 Prediction Error Filtering of Record R-49 output. The fact that the P-waves, or first energy signals, were not developed, could be due to the field recording, amplitude, settings being too high to include the very weak first arrival energies within the dynamic amplitude range of the recording system. It should be noted, in this context, that very high magnitude energy arrivals occur immediately after the first energy arrivals and, if the initial amplitude settings are too high, overmodulation of the FM recording system might occur. A great deal of experimentation remains to be carried out in this particular field before definite conclusions can be reached.

## 4.8 Conclusions.

The results presented in this chapter are by no means exhaustive and are drawn from a very small sample of the total seismic data on hand. However, they do show how suitable choice of filter length, which itself is affected by both lag window length and input sample length, can substantially reduce processing times for digital filtering.

It has also been shown how the various filter parameters interact and how preliminary examination of the autocorrelation function and normalized mean square error is of utmost importance in the design of an effective filter.

# CHAPTER V

# CONCLUSION

The detailed conclusions and discussion of results on implementation techniques and filter parameter optimization are given in Chapters III and IV. This chapter deals with the overall conclusions and extends the discussion to include further work which might be carried out in this field.

## 5.1 Implementation Techniques.

Use of segmented or sectioned input data and auxiliary disk storage techniques solves the majority of storage problems which arise during seismic refraction data processing. However, storage requirements are still fairly large if filters incorporating more than 6 channels are being computed. This factor may be critical in the choice of the number of channels used if a larger number of channels produces only a marginal improvement in the filtered output.

The Fourier transform product method of convolution, combined with the fast Fourier transform technique, is undoubtedly the best method for convolution of time series with more than 100 elements. At present this method probably finds greatest application in the computation of correlation functions.

The fact that most of the filters, investigated so far, are not of sufficient lengths, by themselves, to warrant the use of the transform product method of convolution, does not preclude its use for filter convolution. Because convolution is a linear process, and because it is both commutative and associative, filters can themselves be convolved to form composite filters prior to application to the data. Thus, if a data process requires several filtering operations, and this is the normal situation, all the filters would be combined together (by convolution) and the resultant composite filter would be convolved with the data on a one-pass system. The length of such composite filters would probably be such that the Fourier transform produce convolution method would be the more efficient. Furthermore, this technique has the added advantage of reduction in input-output operations including tape unit rewinding and auxiliary storage of intermediate results.

## 5.2 Filter Parameters.

Results in Chapter IV show the effect of variation of different filter parameters in the design of least squares (Wiener) filters. Very rarely can specific rules be laid down for the choice of actual parameters. This is particularly true of the estimate of the true autocorrelation function, which plays a major role in the solution of the normal equations. Jenkins [21] has shown that the various optimal criteria approaches are open to criticism and that an empirical approach is the most practical for the choice of the finite autocorrelation function.

Certainly it cannot be emphasized too strongly that haphazard and indiscriminate choice of filter parameters often leads to wasted computer time and to erroneous conclusions as to the effectiveness of a particular least squares filtering process.

It is, therefore, recommended that, until enough data has been processed and sufficient specific examples have been evaluated, both the autocorrelation functions and normalized mean square error curves be carefully examined before a final filter design is chosen for application to the data. When an effective and economical filter has been found, of course, the process can be standardized and the full benefit from the pilot studies can be realized.

## 5.3 Further Research Possibilities.

The possibilities of further research in the design and application of digital filters are many and varied, particularly the problem of deconvolution, for which the linear prediction filter is a basic component. Most of the current literature deals with reflection seismic records and many of the digital processing methods, so far published, are not necessarily appropriate for crustal refraction studies. However, this research belongs more properly to the field of Seismology.

From the point of view of research in the field of Computing Science two topics or areas of research might be considered. The first is the development of a fast multiplication method which could be used to replace direct convolution for filters (or time series) of lengths less than 100. A useful starting point for this work would be the comparison and description of multiplication algorithms by D. E. Knuth [22], A second topic might deal with program organization and the use of peripheral access devices. Preliminary seismic data processes involving standard operations, such as frequency filtering, result in fairly long program times, only a small portion of which can be ascribed to the convolution computation. An investigation into the possibilities of parallel processing and the use of intermediate fast access devices in conjunction with magnetic tape peripherals could lead to a substantial reduction in program run times. This study would, of course, deal with a particular machine implementation and would belong to the field of systems analysis.

## 5.4 Final Conclusion.

One of the major advantages of the implementation of high level programming languages, such as FORTRAN, is that it enables workers in many fields to design digital data processes specifically suited to their own individual needs. By the same token, this involves a responsibility to ensure, as much as possible, that the programs are efficient and not wasteful of storage and computer time. Nowhere is this more apparent than when large amounts of data have to be processed and it is particularly true when time series data, such as seismic records, have to be filtered and no special purpose hardware is available.

Most of the published subroutines are written to demonstrate the principles involved rather than to provide a practical and efficient method of large scale data processing. It is hoped that the material presented in this paper will enable workers in this field to overcome some of these problems or, at least, that it will indicate the various approaches which could lead to a practical solution.

## REFERENCES

1. WADSWORTH, G. P., et al, "Detection of Reflections on Seismic Records by Linear Operators", Geophysics, vol. 27, (Feb. 1962), pp. 552 - 583.

2. CLAERBOUT, J. F., "Detection of P-waves from Weak Sources at Great Distances", Geophysics, vol. 24, no. 2, (April, 1964), pp. 197 - 211.

3. SIMPSON, S. M., "Time Series Computations in FORTRAN and FAP", Addison Wesley, (1966), pp. 12 - 19.

4. ROBINSON, E. A., "Multichannel Time-Series Analysis With Digital Computer Programs", Holden Day, (1967).

5. ROBINSON, E. A., "Statistical Communication and Detection", Griffen, (1967), pp. 284 - 302.

6. RICE, R. B., "Inverse Convolution Filters", Geophysics, vol. 27, No. 1, (Feb. 1962), pp. 7 - 9.

7. GALBRAITH, J. N., "Computer Studies of Microseism Statistics with Applications to Prediction and Detection", M.I.T., Ph.D. thesis, Cambridge, Mass., (1963), pp. 56 - 84.

8. HAUBRICH, R. A., "Earth Noise No. 1", Journal of Geophysical Research, vol. 20, (1965).

9. GRENANDER, U., "Toeplitz Forms and Their Applications", Univ. of California Press, (1958).

10. LEVINSON, N., "The Wiener RMS Error Criterion in Filter Design and Prediction", Journal of Mathematics and Physics, vol. 25, pp. 261 - 278.

11. WIGGINS, R. A., ROBINSON, E. A., "Recursive Solution to the Multichannel Filtering Problem", Journal of Geophysical Research, vol. 70, No. 8, (Feb. 1962), pp. 1885 - 1891.

12. ROBINSON, E. A., "Mathematical Development of Discrete Filters for the Detection of Nuclear Explosions", Journal of Geophysical Research, vol. 63, (1963), pp. 5559 - 5567.

13. ROBINSON, E. A., "Multichannel Z-transforms and Minimum Delay", Geophysics, vol. 31, No. 3, (June 1966), pp. 482 - 500.

14. McCOWAN, D. W., "Finite Fourier Transform Theory", Technical Memorandum, No. 8-66, Earth Science Division, Teledyne Industries, (Dec. 1966).

15. COOLEY, J. W., TUKEY, J. W., "An Algorithm for the Machine Calculation of Complex Fourier Series", Mathematics of Computation, vol. 19, pp. 297 - 301.

16. SINGLETON, R. C., "A Method for Computing the Fast Fourier Transform with Auxiliary Memory and Limited High Speed Storage", IEEE Transactions on Audio and Electroacoustics, vol. AU-15, No. 2, (June 1967), pp. 91 - 98.

17. STOCKHAM, T. G., "High Speed Convolution and Correlation", Proceedings Spring Joint Computer Conference, (1966), pp. 229 - 233.

18. GENTLEMEN, W. M., SANDE, G., "Fast Fourier Transforms", Proceedings Fall Joint Computer Conference, (1966), pp. 569 - 576.

19. JENKINS, G., "Some Examples of and Comments on Spectral Analysis", Proceedings IBM Scientific Computing Symposium on Statistics, No. 10, (1963).

20. JENKINS, G., WATTS, D., "Spectral Analysis and its Applications", Holden Day (1968), pp. 185 - 188; 258 - 294.

21. PEACOCK, K. L., TREITEL, S., "Predictive Deconvolution: Theory and Practice", Geophysics, vol. 14, No. 2, (April 1969), pp. 155 - 169.

22. KNUTH, D., "The Art of Computer Programming: Seminumerical Algorithms", vol. 2, Addison Wesley, (1969), pp. 258 - 279.

23. SINGLETON, R. C., "Algorithm 345: An Algol Convolution Procedure Based on the Fast Fourier Transform", Comm. A.C.M., vol. 12, No. 3, (March 1969), pp. 179 - 187.

# APPENDIX A

# SEISMIC DATA RECORDING SYSTEM

## A.1 Field Recording System.

The data gathering and preliminary processing system for seismic refraction records, used by the Geophysics department of the University of Manitoba, is shown in Figure A.1(a). The signal voltages from each of the 12 geophones, or geophone groups, are recorded both on photographic paper, as galvanometer traces, and on frequency modulated magnetic tape. The photographic display is used primarily for monitoring in the field and for analog playbacks. Variable recording parameters include constant gain amplifiers, with attenuation settings ranging from 0db to 66db, and frequency filters (one filter to two input channels) with settings ranging from 0 cps (open) to 44 cps. A frequency setting of 16 cps would result in attenuation of all frequencies above 16 cps. No low cut filter settings are available, i.e. the pass band is open for low frequencies. The 12 input channels are displayed at two gain settings on the 24 trace camera record, the relative difference being of the order of 12db.

Time calibration is provided on the photographic records by a standard timing device which displays 10 millisecond interval timing lines. Overall time control, for determining the shot instant, is achieved by recording the standard radio time signal (station WWVB), transmitted by the U.S. Bureau of Standards Station at Fort Collins, Colorado. This signal is superimposed on channel 24 of the galvanometer

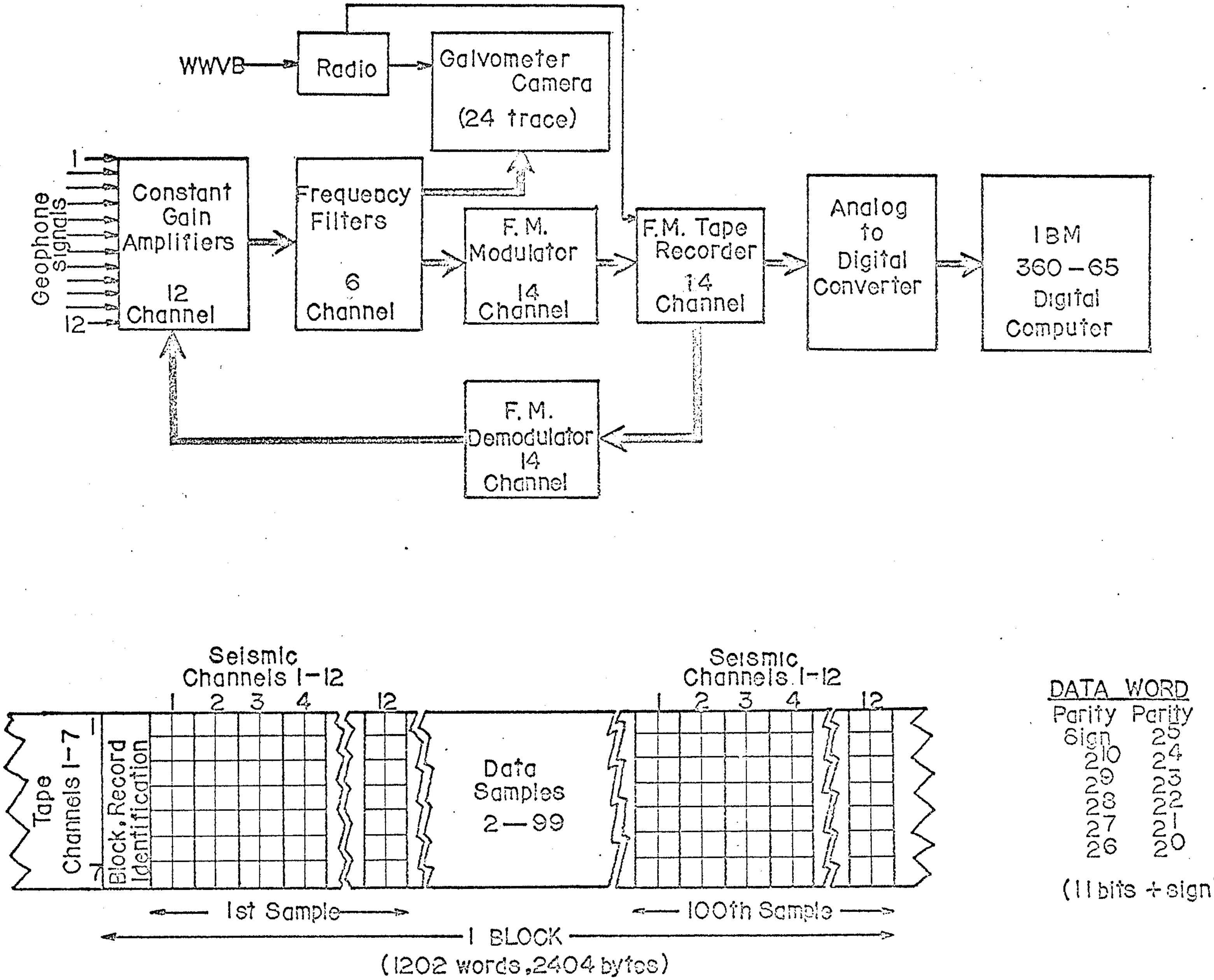



Figure A.1 Seismic Data Recording and Preliminary Processing System

## RECORDING SYSTEM

| | |
|---|---|
| Seismometer .............................. | HALL SEARS (1 cps); Refraction geophones. |
| Amplifiers ............................... | TEX. INSTR. VLF system; 12 channels; Attn: 0-66 db. |
| Filters .................................. | TEX. INSTR. VLF system; 6 channels; High cut: 0-44 cps, Low cut: open |
| Camera ................................... | 24 trace, Line Galvo. |
| FM System ................................ | S.I.E. 14 channel; Carrier Freq. 3375 cps. |
| Tape Recorder ............................ | AMPEX, 14 channel, 1 inch tape. |
| A/D Converter ............................ | RADIATION INC. 16 channel input; 7 channel output. channel sample rate: 0.0017 secs. |

## TYPICAL FIELD RECORDING PARAMETERS

| | |
|---|---|
| Dynamite charge ........................ | 500 - 800 lbs. (in 90 ft. water). |
| Shotpoint to recording location distance .. | 60 - 340 kilometers |
| Frequency bandpass ...................... | open - 16 cps |
| Amplifier gain (attenuation) ............ | 48 db - 24 db. |

FIGURE A.2 SEISMIC INSTRUMENT SPECIFICATIONS camera and is recorded on channel 13 of the FM tape. If radio reception is too poor for communication between the shotpoint and recording positions, the WWVB standard time signal is used to designate a specific time for charge detonation.

The instrument specifications and typical field recording settings are listed in Figure A.2.

A.2 Analog to Digital Conversion.

Conversion of the analog field data to IBM digital format is carried out using the Radiation Inc. A/D converter. This device samples all input channels at a rate of 7000 cps, giving a channel sample rate of 1.71 milliseconds. The data samples are subsequently multiplexed and written on 7 channel, IBM, one inch tape in blocks of 1202, 2 byte, words. The first two data words of a block are reserved for record and block identification and the remaining 1200 data words comprise 100 samples from each of the 12 seismic channels (see Figure A.1(b)). The digitized voltage levels range from -2047 to +2047, corresponding to analog signal voltages with a full scale range of +2 volts to -2 volts.

# APPENDIX B

## DERIVATION OF THE NORMAL EQUATIONS FOR AN OPTIMUM FINITE MULTICHANNEL FILTER

The filter is optimized in the least squares sense, i.e. by the constraint that the average squared difference, between the desired and actual outputs, be a minimum.

The following terms and definitions are used in the derivation of the normal equations:

Input Series, channel $i$, at time $t$ .......... $x_{i,t}$;

Actual output series, channel $j$, time $t$ ...... $y_{j,t}$;

Desired output series ........................ $z_{j,t}$;

Number of input channels ..................... $n$ ;

Number of output channels .................... $m$ ;

Length of input series ....................... $p$ ;

Length of multichannel filter ................ $q$ ;

Filter coefficient at time $t$ ................ $f_{i,j,t}$;

Error squared at time $t$ ..................... $e_t^2$ ;

Mean squared error over all times ............ $\overline{e^2}$ .

The derivation starts by the convolution of a multichannel input series $x_{i,t}$ with a multichannel filter $f_{i,j,t}$ to produce an actual output series $y_{j,t}$. Then the optimum filter coefficients are expressed in terms of the system of normal equations obtained by differentiating, with respect to $f_{i,j,t}$, the mean square error between the actual and desired time series and letting this quantity equal zero.

The particular choice of desired output $z,j,t$ specifies the type of filter being designed. Thus, the derivation is completely general.

If $x_{i,t}$ is the input element on the ith channel at time $t$ and $y_{j,t}$ is the output element on the jth channel, then $y_{j,t}$ can be expressed as the convolution of $x_{i,t}$ with the (so far unknown) filter coefficients, i.e.

$$y_{j,t} = \sum_{i=1}^{n} \sum_{k=1}^{q} f_{i,j,k}\, x_{i,t-k}. \tag{B.1}$$

Now the error square between desired output and actual output is given by:

$$e_t^2 = \sum_{j=1}^{m} (y_{j,t} - z_{j,t})^2,$$

and substituting equation (B.1) for $y_{j,t}$:

$$e_t^2 = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k=1}^{q} (f_{i,j,k}\, x_{i,t-k} - z_{j,t}) \tag{B.2}$$

If the convolution is carried out for the whole length of the finite input time series, i.e. for $t = 1, \ldots, p$, the resulting output series will be of length $p + q - 1$. Also, the mean square error $(\overline{e^2})$ for the jth output channel will be given by:

$$\overline{e^2} = \frac{1}{(p + q - 1)} \sum_{t=1}^{p+q-1} e_t^2 \ ,$$

which, using Equation (B.2), becomes:

$$\overline{e^2} = \frac{1}{(p + q - 1)} \sum_{t=1}^{p+q-1} \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k=1}^{q} (f_{i,j,k}\ x_{i,j-k} - z_{j,t})^2 .$$

The mean square error is minimized by setting its partial derivatives, with respect to the filter coefficients, equal to zero, i.e.;

$$\frac{\partial \overline{e^2}}{\partial f_{u,v,s}} = 0 \ ,$$

where the extra subscripts are introduced to denote the following;

u is the input channel,

v is the output channel, and

s is the specific $m \times n$ filter matrix element.

Now, for any particular filter coefficient $f_{u,v,s}$, the partial derivative of the mean square error is given by:

$$\frac{\partial \overline{e^2}}{\partial f_{u,v,s}} = \frac{1}{(p+q-1)} \sum_{t=1}^{p+q-1} \left[ 2 \sum_{i=1}^{n} \sum_{k=1}^{q} (f_{i,v,k}\ x_{i,t-k} - z_{v,t}) \right] x_{u,t-s} \ ,$$

and, setting this equal to zero, results in the equation:

$$\frac{1}{(p+q-1)} \sum_{t=1}^{p+q-1} \left[ 2 \sum_{i=1}^{n} \sum_{k=1}^{q} (f_{i,v,k}\ x_{i,t-k} - z_{v,t}) \right] x_{u,t-s} = 0 \qquad \text{(B.4)}$$

Rearranging the terms, and discarding the constant gives:

$$\frac{1}{(p+q-1)} \sum_{t=1}^{p+q-1} \sum_{i-1}^{n} \sum_{k-1}^{q} f_{i,v,k}\, x_{i,t-k} = \frac{1}{(p+q-1)} \sum_{t=1}^{p+q-1} z_{v,t}\, x_{u,t-s}.$$

Rearranging the summation:

$$\sum_{i=1}^{n} \sum_{k=1}^{q} f_{i,v,k} \frac{1}{(p+q-1)} \sum_{t=1}^{p+q-1} x_{i,t-k}\, x_{u,t-s} = \frac{1}{(p+q-1)} \sum_{t=1}^{p+q-1} z_{v,t}\, x_{u,t-s} \tag{B.5}$$

Now, $r_{i,u,s-k}$, the autocorrelation function of the input series $x_{i,t}$ is defined as:

$$r_{i,u,s-k} = \lim_{p\to\infty} \frac{1}{(p+q-1)} \sum_{t=1}^{p+q-1} x_{i,t-k}\, x_{u,t-s}\,, \tag{B.6}$$

and $g_{v,u,s}$, the cross correlation function of the input series $x_{i,t}$ with the desired output series $z_{i,t}$, is defined as:

$$g_{v,u,s} = \lim_{p\to\infty} \frac{1}{(p+q-1)} \sum_{t=1}^{p+q-1} z_{v,t}\, x_{u,t-s}. \tag{B.7}$$

Substituting these entities into equation (B.5) gives:

$$\sum_{i=1}^{n} \sum_{k=1}^{q} f_{i,v,k}\, r_{i,u,s-k} = g_{v,u,s}, \tag{B.8}$$

where the subscript ranges are given by:

$$i = 1,\ldots,n \quad ;$$
$$j = 1,\ldots,m \quad ;$$
$$v = [1,\ldots,n] \quad ;$$
$$s,k = 1,\ldots,q \quad .$$

Equation (B.8) can also be written in matrix notation.

Let, $F_k = \begin{bmatrix} f_{1,1,k} & \cdots & f_{1,m,k} \\ \vdots & & \vdots \\ f_{n,1,k} & \cdots & f_{n,m,k} \end{bmatrix}$ , $R_i = \begin{bmatrix} r_{1,1,i} & \cdots & r_{1,n,i} \\ \vdots & & \vdots \\ r_{n,1,i} & \cdots & r_{n,n,i} \end{bmatrix}$

and $G_s = \begin{bmatrix} g_{1,1,s} & \cdots & g_{1,n.s} \\ \vdots & & \vdots \\ g_{m,1,s} & \cdots & g_{m,n,s} \end{bmatrix}$ ,

then equation (B.8) becomes:

$$[F_1^T, F_2^T,\ldots,F_q^T] \begin{bmatrix} R_1 & \cdots & R_m \\ \vdots & & \\ R_{-m} & \cdots & R_1 \end{bmatrix} = [G_1,G_2 \;\cdots\; G_q] \tag{B.9}$$

Equations (B.8) and (B.9) are the normal equations for a finite, least squares, multichannel filter. The autocorrelation function $R_i$ and the cross correlation function $G_s$ are the known quantities and solution of either equation (B.8) or equation (B.9) yields the numerical filter coefficients $F_K(k = 1,\ldots,q)$.

# APPENDIX C

## CONVOLUTION

Convolution is the basic computation for the application of digital filters to time series data. It is also the method for the computation of both cross correlation and autocorrelation functions.

Figure C.1 shows the computation scheme for direct convolution of a single channel filter $(f_1,\ldots,f_q)$ with a discrete time series comprising the data elements $(x_o,\ldots,x_t)$ , where $t$ is the time index. The successive elements of the resultant output series $y_t$ are computed by the moving summation of the input series elements, weighted by the corresponding filter coefficients, which are arranged in reverse order. The length of the output series is $p+q-1$ , where $q$ is the length of the filter and $p$ is the length of the input series which is to be filtered.

This computation can be expressed algebraically as:

$$y_t = \sum_{k=1}^{q} f_k \, x_{t-k} \, , \, x_{t-k+1}$$

or as a FORTRAN program segment by:

```
C FILTER LENGTH LF, INPUT LENGTH LX.

      DO 1 I=1, LX
      DO 1 J=1, LF
      K=I+J-1
    1 Y(K)=Y(K)+F(J)*X(I).
```

The multichannel case is simply an extension from scalar to

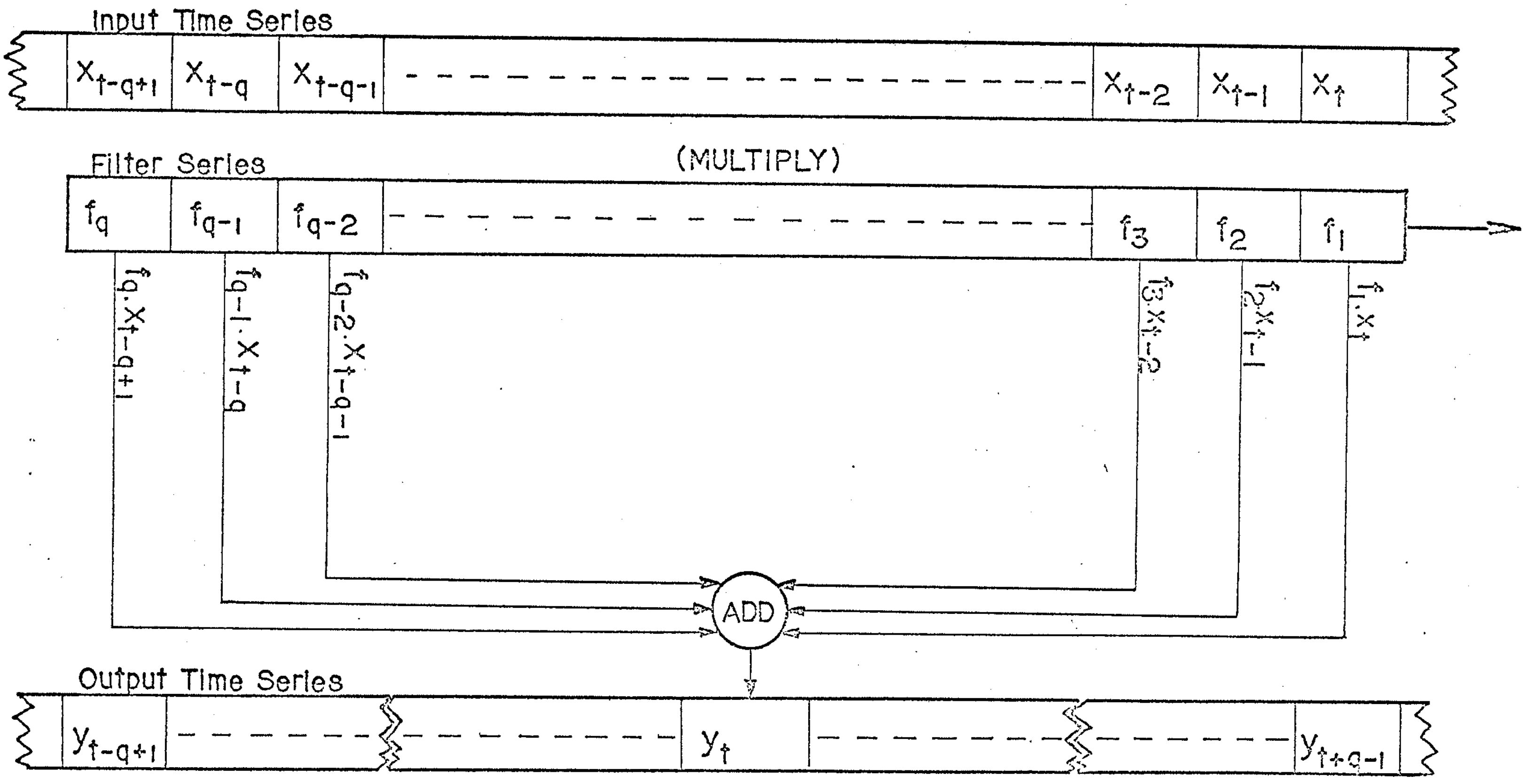



Figure C.1 Numerical Convolution Computation Scheme matrix multiplication where the input and output channels comprise a series of $n \times 1$ and $m \times 1$ vectors and the filter is an $m \times n$ dimensional matrix. Algebraically it can be expressed as:

$$y_{j,t} = \sum_{i=1}^{n} \sum_{k=1}^{q} f_{i,j,k} \, x_{i,t-k+1},$$

where $y_{j,t}$ is the output element at time $t$ on the jth output channel. A FORTRAN program segment which would implement this process would be:

```
      DO 1 I=1,LX
      DO 1 J=1,LF
      IJ=I-J+1
      DO 1 L=1,N
      DO 1 K=1,M
    1 Y(K,IJ)=Y(K,IJ)+F(K,L,J)*X(L,I)
```

In the case of cross correlation (autocorrelation is the special case where the time series is correlated with itself), the identical process is carried out, except that the filter coefficients are now replaced by the data elements of the time series with which correlation is to be carried out. Thus, the algebraic expression for the single channel cross correlation function $r_t$, between two series $x_y$ and $y_t$ is given by:

$$r_t = \sum_{k=1}^{q} x_k \, y_{t-k+1},$$

where $x_t$ and $y_t$ are of length $q$.

Alternatively, as Robinson [11] shows, if the z transforms[1]

[1]The z transform is a polynomial in z whose coefficients are the time series data elements.

```
      SUBROUTINE FTCNV(LF,F,LXX,XX,XFT,ISW,C,XC)
C FTCONV COMPUTES THE CONVOLUTION OF AN OPERATOR F WITH A SINGLE CHANNEL
C TIME-SERIES BY PRODUCT OF THE FOURIER TRANSFORMS, OBTAINED BY USE
C OF NLOGN (AUTH. E.A.ROBINSON). ISW=0 FOR THE FIRST OF POSSIBLY REPEATED
C CALLS. END EFFECTS ARE TO BE HANDLED BY THE CALLING PROGRAM.
C FOR PARAMETER EXPLANATION SEE SUBROUTINE MFTCNV.
      DIMENSION F(1),XX(1),XFT(1)
      COMPLEX C(2),XC(2),A,B
      IF(ISW.NE.0) GO TO 120
      LXFT=LF+LXX-1
      ISW=1
C OBTAIN POWER OF 2 .GE. OUTPUT SECTION LENGTH
      DO 115 I=1,20
      N=2**I
      N2=I
      IF(N.GE.LXFT) GO TO 120
  115 CONTINUE
C LXFT OUT OF RANGE
      ISW=99
      RETURN
C MAKE AUGMENTED COMPLEX ARRAY FROM THE REAL INPUT SERIES
  120 IF=0
      IX=0
      DO 130 I=1,N
      IF=IF+1
      IX=IX+1
      IF(IF.GT.LF) GO TO 125
      CF=F(IF)
      GO TO 126
  125 CF=0.0
  126 IF(IX.GT.LXX) GO TO 127
      XR=XX(IX)
      GO TO 130
  127 XR=0.0
  130 C(I)=CMPLX(CF,XR)
C OBTAIN FOURIER TRANSFORM
      CALL NLOGN(N2,C,-1.0)
C RECOVER INPUT SERIES TRANSFORMS AND MULTIPLY TOGETHER
      AR=REAL(C(1))
      AI=0.0
      A=CMPLX(AR,AI)
      BR=-1.0*AIMAG(C(1))
      BI=0.0
      B=CMPLX(BR,BI)
      XC(1)=A*B
      DO 140 I=2,N
      K=N-I+2
      A=0.5*(C(I)+CONJG(C(K)))
      BR=-0.5*(AIMAG(C(K))+AIMAG(C(I)))
      BI=0.5*(REAL(C(I))-REAL(C(K)))
      B=CMPLX(BR,BI)
  140 XC(I)=A*B
C INVERSE TRANSFORM TO OBTAIN REAL CONVOLVED OUTPUT SERIES.
      CALL NLOGN(N2,XC,1.0)
      DO 150 I=1,LXFT
  150 XFT(I)=-1.0*REAL(XC(I))
      RETURN
      END
```

Figure C.2 Sample FORTRAN Program for Single Channel Convolution of the various filter operators and discrete time series are taken, convolution may be considered as polynomial multiplication. Thus, if $Y(Z)$, $X(Z)$ and $F(Z)$ are the respective z transforms of an output series $y_t$ formed by convolution of an input series $x_t$ with a filter operator $f_t$, the convolution can be shown to be equivalent to:

$$Y(Z) = F(Z)\ X(Z).$$

The z transforms are, in the multichannel case, matrix polynomials i.e. the polynomial coefficients are matrices, or, alternatively can be considered as matrices with polynomial elements (Lambda matrices).

Convolution by Fourier transform products is a direct result of the expression of convolution as z transform polynomial multiplication, for, it can be shown that, when z is restricted to lie on the unit circle, the z transform becomes the Fourier transform, i.e.

$$Z = e^{-2\pi i f}$$

The computation scheme for this method is shown in Figure 3.2 and a sample FORTRAN program for single channel convolution is shown in Figure C.2.

# APPENDIX D

## EXPERIMENTAL METHOD FOR EMPIRICAL TIME MEASUREMENTS

Empirical time measurements were made using IBM system subroutines \$TARTM and \$TOPTM. These two routines permit FORTRAN users to time the execution of a portion of code as is illustrated in the following program segment.

```
C TO TIME THE EXECUTION OF SUBROUTINE FTCONV
      SUBTOT=0.0
      CALL $TARTM(SUBTOT)
      CALL FTCONV(LF,F,LXX,XX,XFT,ISW)
      CALL $TOPTM(SUBTOT)
C SUBTOT NOW CONTAINS TIME INTERVAL BETWEEN THE
C FTCONV CALL AND THE RETURN TO THE MAIN PROGRAM.
```

The time recorded is "wall" time and is accurate to within 16.67 milliseconds.

Times measured in this fashion on the University of Manitoba IBM 360-65 are subject to error due to other programs running concurrently under MVT (Multiprocessing with a Variable number of Tasks). Thus a time interval, measured by the "wall" or "clock" timing mechanism, is guaranteed to equal active CPU task time only if the system is completely free of other programs.

Because of the large number of time measurements needed, for example, to measure the processing time of segmented series convolution, it was found to be impractical to measure active CPU task time from the CPU step time statistics; this being the only current alternative. Also it was deemed unrealistic to exclude all other users to guarantee coincidence of "clock" time and active CPU task time.

Instead, if a time measurement appeared too large, the program would be rerun until a minimum value had been recorded more than once. Typical time variations, due to system interference, were measured by repeated execution of the same piece of coding and were found to be in the region of 12%. This was considered small enough in magnitude to not obscure the overall time relationships which were the main objective of the experiments.

# APPENDIX E

## SUBROUTINE INDEX

| Subroutine Name | Function | Source Reference | Author and Remarks |
|---|---|---|---|
| COOL | Convolution by Fourier Transform Products | [14] | McCowan, D. W., not for IBM FORTRAN systems |
| CONCV | Direct Convolution of segmented single channel series | (*) | Burgess, P.A., Modification of HEAT |
| DHARM | Fast Fourier Transform | (**) | Double precision version of HARM |
| FFT4 | Fast Fourier Transform | [23] | Singleton, R.C., used with SINCNV. |
| FORAC | Direct computation of Tukey Approx. of autocorrelation Fn | [3] | Simpson, S. M., |
| FREQ. | Convolves Multichannel, sectioned input with single channel filter | (*) | Burgess, P.A., uses direct method |
| FTCONV | Single channel convolution by Fourier Transform products | (*) | Burgess, P.A. |
| FTCORR | Single channel correlation | (*) | Burgess, P.A. Uses NLOGN and Transform products |
| HARM | Fast Fourier Transform | (**) | IBM |
| HEAT | Multichannel Convolution | [4] | Robinson, E. A. Direct method |
| MCONV | Multichannel convolution for segmented input | (*) | Burgess, P.A. modification of HEAT |
| MFTCNV | Multichannel convolution for segmented input | (*) | Burgess, P.A., uses and Fourier Transform. Products |

| Subroutine Name | Function | Source Reference | Author and Remarks |
|---|---|---|---|
| NLOGN | Fast Fourier Transform | [4] | Robinson, E.A. A revision and modification of COOL |
| PROCOR | Convolution by multiplier substitution | [3] | Simpson, S.M. Written in FAP; machine dependent for IBM 7094 |
| QACORR | Computes autocorrelation | [3] | Simpson, S.M. Uses PROCOR |
| REVFT4 | Fast Fourier Transform | [23] | Singleton, R.C. used with SINCNV |
| REALTR | Re-orders Reverse binary ordered vector | [23] | Singleton, R.C. used with SINCNV |
| RHARM | Fast Fourier Transform | (**) | IBM. Uses HARM |
| SINCNV | Circular Convolution of two real vectors | [23] | Singleton, R.C. FORTRAN version of uncertified Algol procedure |
| WIENER | Computes general multi-channel least squares (Wiener) filter | [4] | Robinson, E.A. Uses recursion algorithm |
| WIENR3 | (See WIENER) | (*) | Burgess, P.A. Modification of WIENER using auxiliary disk storage. |

* Available from author at Computer Centre, University of Manitoba.

** IBM Scientific Subroutine Package. Available from any IBM branch office.