

LATERAL PRESSURES IN DEEP BULK FERTILIZER STORAGE BINS

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ABSTRACT

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LATERAL PRESSURES IN DEEP BULK FERTILIZER STORAGE BINS
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An investigation was undertaken to determine lateral pressures exerted by commercial bulk fertilizer against deep bin walls. Ammonium phosphate fertilizer was placed in a full scale plywood bin and a model plywood bin in order to determine the magnitude and distribution of the lateral pressures. Manitou wheat was placed in the same bins to serve as a check on the methods and equipment employed.

The coefficient of friction, (granular material on plywood), angle of repose, density and moisture content of wheat, ammonium phosphate and five other commercial fertilizers were determined. These material properties were used to calculate theoretical pressures, predicted by Janssen's equation. Predicted pressures were compared with experimental results.

Pressures were determined in the full scale bin using wall mounted metal diaphragm transducers designed to measure normal loads only. Model bin pressures were determined by subtracting the weight of material resting on the moveable bin floor from the total weight of material in the bin, to get the

total vertical wall load.

Wall rigidity was studied with respect to its effect on lateral pressures. The results were inconclusive.

A number of replications of each test condition and material were made. Throughout the tests, checks were made to detect changes in material properties due to mechanical damage.

Lateral pressures, due to bulk fertilizer in deep bins, were found to be accurately predicted by the Janssen equation. Safe design constants for use in the above mentioned equation were determined. The assumptions that the ratio of lateral to vertical pressure is constant and that vertical pressure is constant across a horizontal section of a bin were not confirmed.

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LIST OF SYMBOLS

A	Cross sectional area in a bin.
A_v	Shear area on a transducer diaphragm.
D	Equivalent diameter of a square bin.
E	Modulus of elasticity.
F	Friction force along a surface.
H	Height of bin wall.
K	Ratio of lateral to vertical pressure.
$K_{ave.}$	Average ratio of lateral to vertical pressure.
K_o	Ratio of lateral to vertical pressure for an at rest state of stress.
L	Lateral pressure in a bin.
N	Normal force.
P	Total lateral load.
P_a	Resultant force due to active pressure, over full height of wall.
p_p	Resultant force due to passive pressure, over full height of wall.
R	Hydraulic radius of a round bin.
R'	Resultant force.
S	Vertical measurement of angle of repose equipment.
T	Vertical measurement on angle of repose equipment.
U	Perimeter, or circumference, of a bin.
V	Vertical pressure in a bin.
W	Total weight of material.
X	Transducer output in microinches.
Y	Pressure against transducer diaphragm in inches of mercury.
a	Radius of diaphragm to inside of transducer wall.

d	Lateral bin dimension.
h	Depth of granular material above the point under consideration.
p	Pressure against transducer diaphragm in p.s.i.
p_a	Active pressure against a wall.
p_p	Passive pressure against a wall.
r	Correlation coefficient.
t	Diaphragm thickness.
w	Density of granular material.
x	Angle of rupture.
α	Level of statistical significance.
γ	Angle of stress obliquity.
Δ_{\max}	Maximum deflection.
ϵ_{\max}	Maximum strain at the center of the diaphragm.
θ	Angle of impending slip.
μ	Coefficient of friction, granular material on itself.
μ'	Coefficient of friction, granular material on bin surface
μ^*	Poisson's ratio.
$\sigma_{r_{\max}}$	Maximum radial stress in a diaphragm.
$\sigma_{t_{\max}}$	Maximum tangential stress in a diaphragm.
$\sigma_{x_{\max}}$	Maximum stress in the X direction.
$\sigma_{y_{\max}}$	Maximum stress in the Y direction.
τ	Maximum shear stress.
ϕ	Emptying angle of repose for a granular material.
ϕ'	Angle of internal friction for a granular material.

CHAPTER I

INTRODUCTION

1.1 Increase in Fertilizer Use

In recent years there has been a general increase in the use of chemical agricultural fertilizer in Canada. The Dominion Bureau of Statistics (1969) indicates that 2,292,728 tons of fertilizer were sold for use in Canada during 1968. This compares with a total use of 1,454,332 tons in 1964. Fig. 1 shows the growth of fertilizer consumption from 1964 to 1968 inclusive.

Faced with an increased quantity of material which must be handled, it is only natural that farmers and dealers alike are considering ways to mechanize their operations. One method which has gained in popularity is the use of bulk storage facilities and mechanical handling equipment similar to that employed in the grain industry.

1.2 Storage Alternatives

The farmer who requires storage for moderate quantities of bulk fertilizer can now choose from a number of commercial bins at costs ranging from ten to thirty dollars per ton of capacity (Alberta Department of Agriculture 1968). These bins are built with attention to the preservation of quality in the stored material. Structural design of the storage bins is of

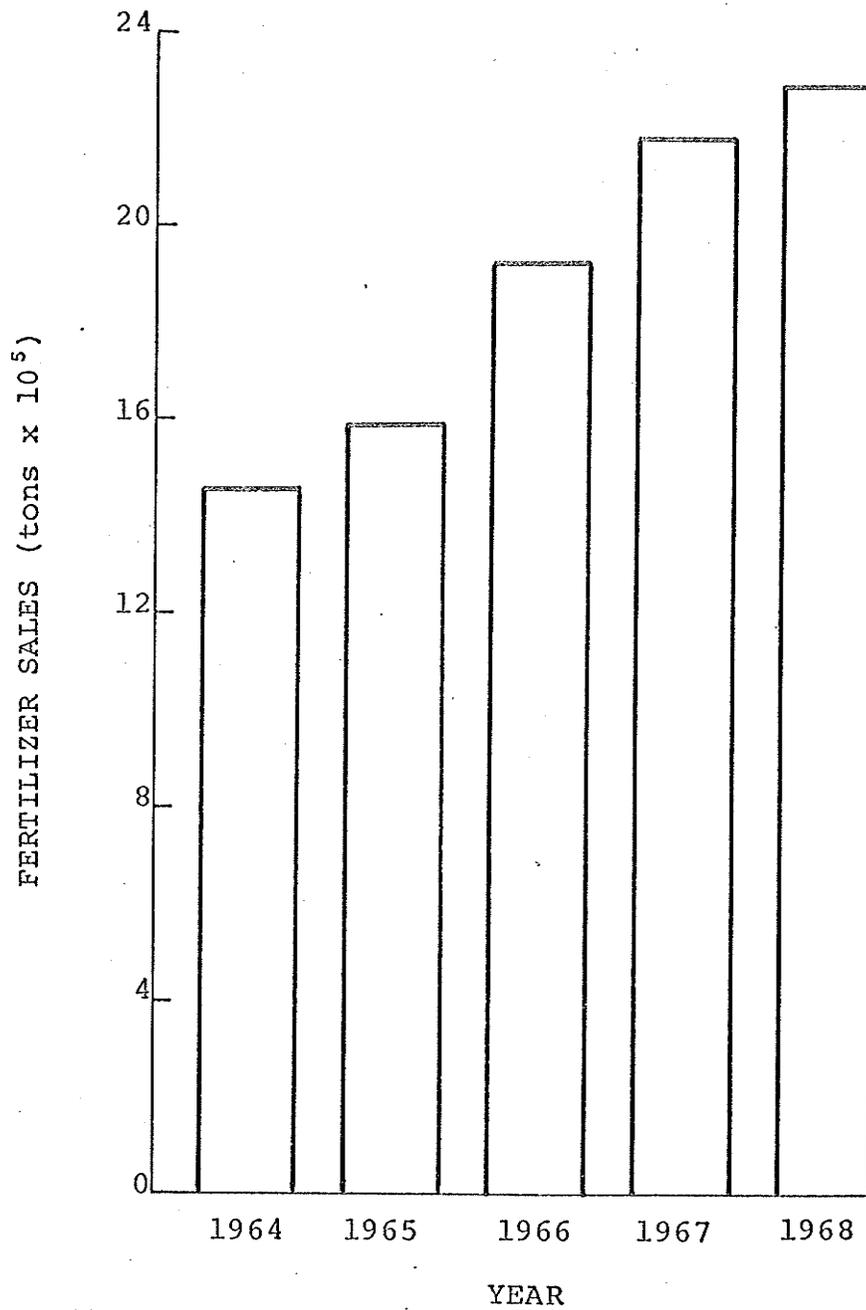


Figure 1. Yearly fertilizer sales in Canada 1964-1968.

minor concern because overdesign will have little effect on the cost of the unit.

The fertilizer dealer and large farmer are faced with a more serious problem. In storage units which range in size from 70 tons (approximate capacity of one bulk delivery car) upwards, substantial savings can be realized by proper design.

There are two alternate types of large bulk storage facilities. The simplest, and presently the most popular, is the horizontal plant. This type of facility employs shallow bin storage (i.e. bins in which the depth of stored material does not exceed the width of the bin) and is usually mechanized by the use of an overhead delivery belt system and a tractor equipped with a front end loader. Construction costs are low and land requirements are relatively high.

The second alternative is a vertical plant. This type of storage is similar in concept to the typical prairie grain elevator. From an engineering point of view, it is considered to be a deep bin (i.e. the depth of the stored material exceeds the width of the bin). This type of storage is mechanized by means of an elevator leg and gravity outflow from hopper bottom bins. Construction costs are high and must be offset by the savings realized in land required for a given quantity of storage.

1.3 Design Information

There is little fertilizer bulk storage design information available which is consistent with performance. Walker

and Grubeler (1965) published design values for density, coefficient of friction and K (the ratio of lateral to vertical pressure) which were used by the American Plywood Association. Using these values, Britton (1967) found wide variations between theoretical loads and the theoretical strengths of existing bins, particularly deep bins. Johnson (1966) stated that "Until there is an extensive test program encompassing the range of materials in their various states including variations in moisture content and length of storage, bulk fertilizer building design will have to be classed as an engineering art rather than an engineering science."

This lack of design information may be a contributing factor to the present popularity of the shallow bin storage unit. An engineer faced with the problem of designing an economical storage unit, would probably tend to recommend the system which required the fewest unknowns. Since a shallow bin seldom exceeds twelve to fourteen feet in depth, the design pressures are not excessive and a design can be produced with relative confidence.

The deep bin, on the other hand, presents a more difficult problem. Since pressures increase with depth, the magnitude of the pressure at the base of a 40 foot high storage unit is much greater than at the base of a twelve foot bin. The parameters (constants for a given material) which contribute to the pressure are not well known so the accuracy of the design pressure is in question. This fact, together with the consequences of estimating low, force the engineer to be conservative

in his design, affecting the economy of the structure.

1.4 Purpose of the Research

The purpose of this research project was to provide some basic information for use in the design of deep bin bulk fertilizer storage structures. The following questions formed the basis of the investigation.

1. Does lateral pressure due to bulk fertilizer of the type studied follow the Janssen distribution when stored in a deep bin?

2. Is wall flexibility a factor in determining the lateral pressure due to bulk fertilizer stored in deep bins?

3. What are the angle of repose, density and coefficient of friction of several commercially available chemical fertilizers?

4. Can model bins be used in bulk fertilizer pressures studies or must full scale storage bins be employed?

CHAPTER II

BIN PRESSURE RESEARCH

2.1 Review of Past Work

Two classical methods of calculating lateral pressures due to earth backfill have been used for over one hundred years. The theories were proposed by Coulomb in 1773 and by Rankine in 1857 (Taylor 1948). Although there has been discussion regarding the reliability of these theories (Baker 1881), they are generally accepted.

Consideration of pressures due to granular material in bins began with the publication of results obtained in experiments carried out by Issac Roberts (1884). These investigations were prompted by failures which had occurred in grain storage buildings. Initial studies were carried out on model bins of square and hexagonal cross section. Wheat and peas were used as fill material. Because of the unexpected results obtained from the model bins, a second series of tests were undertaken on a specially constructed 6 foot 9 inch by 6 foot by 52 foot 2 inch deep bin. Pressures were measured by means of a system of levers and a balance arm, all of which was attached to a square panel placed in an opening in the bin. Roberts presented two theories based on his findings:

1. the lateral pressure in deep bins is independent of depth,

2. the floor load in a bin does not increase after the depth of fill exceeds twice the diameter of an inscribed circle within the bin.

In 1895, H.A. Janssen (Ketchum 1919) reported the results of tests conducted on model bins. The bin walls were supported on screw jacks and the bin floor was allowed to rest, independent of the walls, on a scale. Weighed quantities of dry sand, wheat, corn and other grains were placed in the bins. The proportion of the load resting on the floor was then measured. By use of his formula for bin pressures, lateral and vertical loads were calculated to compare with the experimental results.

In 1896, Prante (Ketchum 1919) reported results of tests undertaken on circular iron bins. Lateral pressures, for the static case, were found to be slightly less than those calculated by use of the Janssen equation. Loads due to moving grain were as much as four times greater than those predicted by Janssen. In the past, this work has generally been disregarded because of the pressure measuring equipment employed.

Toltz (1903) undertook a series of tests in 1897 on a 14 foot square by 65 foot deep wooden bin in order to substantiate design methods for an elevator. Pressures were determined by measuring the deflection of steel plates which were later calibrated. As a result of the tests, Toltz agreed with the theories advanced by Prante and employed these theories in his design work.

In a paper "The Pressure of Grain", Airy (1897) proposed his theory on grain pressures (this theory will be discussed in detail later). As well, values obtained for angle of repose and coefficient of friction between grain and walls were reported for several grains.

During 1902 and 1903, Ketchum and Varnes (Ketchum 1919) conducted tests on a model wooden bin, one foot square by eight feet six inches high. Vertical loads were measured by weighing the moveable bottom and the lateral pressures were measured by means of a series of levers running from a twelve inch square diaphragm to a platform scale. Using wheat, measurements were taken during filling, emptying and while the material was static (no differences in pressure were detected while the wheat was static or in motion). The ratio of lateral to vertical pressure, K , was determined to be equal to 0.4.

Jamieson (1903, 1904) reported on two sets of experiments; first on a full size timber crib bin filled with wheat and then on a series of model bins made of cribbed boards, flat steel plate and trough plate (the corrugations running horizontally). Water filled rubber diaphragm faced pressure cells connected to a mercury column, were used to measure pressures. Good correlation with Janssen's formula was obtained. Dynamic pressures were measured and were found to average ten percent greater than static pressures. Tie-bars through the bins were found to have no affect on lateral pressure.

As a check on Jamieson's results, Bovey (1903, 1904) conducted tests on grain elevators in Montreal and Quebec. Diaphragms similar to those used by Jamieson were employed. A slight decrease in lateral pressure at the bottom of the bin was detected.

Tufft (1904) conducted tests on two circular concrete bins filled with wheat and then with corn. Pressures were measured by means of a rubber membrane placed over a glycerine filled cell attached to a mercury column. A decrease in lateral pressure with time was found.

From 1902-1905, Pleissner (Ketchum 1919) undertook a series of deep bin tests on bins ranging from 1.5 meters square by 18 meters deep to 2.5 meters by 3.15 meters by 17.24 meters deep. Pressures due to wheat and rye were studied. Vertical loads were either weighed or determined by deflection measurements. Lateral pressures were determined by measuring the deflection of calibrated plank pressure surfaces. In static tests, Pleissner's results agree very closely with those of Janssen. The ratio of lateral to vertical pressures, K , was found to decrease with increasing depth.

Ketchum and Williams (Ketchum 1919) in 1909, collaborated on a series of experiments using wheat in a model wooden bin. Vertical pressures were measured using a balance arm and platform scale and the lateral pressures were observed by measuring the electric current flowing through carbon plates. Considering the static case, they concluded that

Janssen's equation was accurate. The ratio, K , was found to increase with the depth of grain.

In his book, Ketchum (1919) summarized all the experimental work done up to 1909. He concluded that:

1. grain behaves as a semi-fluid.
2. Lateral pressure of grain on bin walls is less than vertical pressure and increases little beyond a depth of $2\frac{1}{2}$ to 3 times the bin width or diameter.
3. The ratio of lateral to vertical pressure is not constant but varies with depth.
4. Pressures from moving grain are about ten percent higher than pressures from static grain.
5. Maximum lateral pressures occur immediately after filling.
6. Either Janssen's or Airy's theories are good estimates of actual pressures.

Kaiser and Foster (1921) were the next authors to consider the subject of deep bin pressures when they reviewed past research work. The conclusions reported in their paper were drawn from Ketchum (1919). Long (1932) reviewed the design procedure for grain storage and suggested that values of 0.417 for the coefficient of friction, 0.6 for K , and 50 pounds per cubic foot for density, when used in the Janssen equation, would give a safe design for most grain storage structures.

McCalmont and Ashby (1934, 1938) undertook studies of pressures in corn cribs. On the assumption that large pressure

areas would give less erratic results than small areas, a removeable panel mounted on a calibrated bar was developed. By measuring the horizontal and vertical deflection of the bar, both vertical and lateral wall loads were determined.

Kelly (1940) summarized the state of knowledge in the field of bin pressures. He concluded that there had been little done in the field from the time Ketchum (1919) reviewed the situation.

Kramer (1944) undertook studies to examine the factors which influence pressures in rough rice bulk bins. The ratio K was found to increase rapidly, at shallow depths, but soon became a constant. Jaky (1948), working with various types of grains, also discovered an increase in K with depth, to a maximum value, after which it became constant. It was shown that K was dependent on the geometry of the bin, notably the width, as well as the properties of the material. Jaky found that wall friction increased rapidly at shallow depths and became a constant as depth increased. An approximation for K was suggested using the formula $K = K_0 = 1 - \sin \phi^1$ where K_0 represented an at rest state and ϕ^1 was the internal angle of friction of the grain.

Coughey, Toolles and Scheer (1951) reported on work carried out on a deep circular concrete model bin to determine lateral and vertical pressures caused by granular materials. The pressures due to wheat, shelled corn, soy beans, cement, sand and pea gravel were studied. Lateral pressures were measured using strain gauges mounted on calibrated steel bands

which were placed around the bin and were connected to pressure panels in the bin walls. Vertical pressures were measured by weighing the load on the floor of the bin. Pressures exerted by shelled corn, soy beans, sand and pea gravel were found to be less than those suggested by the Janssen theory. Emptying angle of repose was the only consistent measure of angle of repose. The method used to detect lateral pressures was found to be practical and accurate. As well, values for the ratio of lateral to vertical pressure and the coefficient of friction, for each of the materials studied were listed.

Shallow bin tests conducted by Saul (1953) led to the conclusion that pressure was affected by the method of filling as well as the rigidity of the wall. Lateral pressure against the test wall increased as its rigidity increased.

Dale and Robinson (1954) studied lateral pressures due to corn in deep bins. Pressures were detected using metal diaphragm pressure cells. Their main conclusion was that a change in moisture content makes pressures estimated by Janssen's formula inaccurate.

Writing in a special Materials Handling Issue of Agricultural Engineering, Barre (1958) observed that grain being spouted into a bin tends to segregate by particle size, causing a variation in density through the bin. This variation would cause differences in the physical properties of the grain and probably lead to differences in pressure.

Prompted by a mathematical error in Jaky's (1948) presentation, Jakobson (1958) published an analysis of the

stress conditions within a bulk bin. The basis for the analysis is the assumption that every particle in a bin settles vertically as load increases. By writing a differential equation to describe the above assumption and imposing suitable boundary conditions, a formula was developed which correlated very closely with Janssen's equation.

Lenczner (1963) studied the action of sand in a model bin. The ratio K was found to vary linearly with depth until a depth of four bin diameters was reached, after which it became hyperbolic. The hyperbolic curve reached a maximum at a depth equal to six diameters and then remained constant. According to these studies, the commonly recommended K values for most materials lead to an overestimate of the lateral pressure and an underestimate of the floor pressure. Lenczner's conclusion that lateral pressures in sand are less than those predicted by Janssen's equation agreed with the findings of Coughy et al (1951). Lenczner also took exception to Airy's (1897) theory that the floor load in a bin is equal to the weight of a cone of the filling material having a base equal to the floor area and sides extending up at an angle, equal to the angle of repose. This assumption was found to result in loads which are too low.

Collins (1963) used an aluminum cylinder 3.8 feet in diameter and 12 feet deep to study pressures due to kiln dried sand. The cylinder was instrumented with strain gauges to detect pressures. A "bulge" in the pressure curve was found near the bottom of the tank approximately one half

radius up from the base. No explanation of this "bulge" was attempted.

In a study of lateral pressures using wheat and model circular metal deep bins, Hamilton (1964) found compressive hoop stresses near the top of the bins. As well, the tests showed definite evidence that wall stiffness affected the lateral pressure. This work extended the shallow bin findings of Saul (1953) with regard to the interaction of wall stiffness and lateral pressure.

In the same year, Weiland (1964) studied the causes of grain storage failures. While there was no experimental work undertaken, the conclusions are of very real interest to canadian engineers. Weiland stated that steel undergoes a transition to a brittle state at temperatures of 15 degrees Fahrenheit and below. This transition was thought to be the cause of failures in steel storage structures, and not an increase in pressure as had been suggested by other persons.

Walker and Griebeler (1965) reviewed the present practise in fertilizer storage construction, pointing out trends which were developing. Design values used by the American Plywood Association for substitution into the Janssen equation were listed. The values suggested were based on experience, not experiment, and were felt to result in conservative estimates of pressures.

Janssen's equation was explained and suggested to be accurate by Naberhaus (1965). The use of this formula in bin design for granular materials was recommended. At the

same time, Isaacson and Boyd (1965) presented a mathematical relationship for lateral pressures in deep bins. An equation was developed on which many different boundary conditions could be imposed. If static conditions were chosen, the equation essentially reduced to the Janssen equation.

Brubaker and Pos (1965) conducted tests to determine the static coefficient of friction of grains on various materials. The coefficient increased with an increase in moisture content of either the granular material (except the teflon balls) or the surface. A slip-stick pattern of force was observed on test samples. As a result, it was concluded that the static coefficient of friction was the value obtained after macro-slip had occurred.

Pressures due to moist grain were studied by Cassie and Wood (1966). They adapted the Janssen equation by suggesting values for density, coefficient of friction and K which resulted in a safe design.

As a result of design office difficulties, the writer (Britton 1967) undertook an analysis of some actual structures and compared their theoretical structural strength to the theoretical pressures calculated using Janssen's equation. Some assumptions made regarding the shallow bins do not, in fact, appear to be valid. The writer's analysis of the deep bins was accurate and valid. The deep bins were found to be underdesigned by as much as 200 percent. Since that time, unconfirmed reports of failures have been received.

Studies on the subject of active and passive wall

pressures in shallow bins were reported by Stewart (1967). The Coulomb wedge theory was found to be accurate for predicting active lateral pressures. Contrary to the findings of Saul (1953), wall stiffness did not affect active lateral pressures, as long as the deflection of the wall was great enough to produce active pressures. Wall deflections less than the grain size gave pressures up to 25 percent greater than active pressures. These may be the pressures reported by Saul.

Bickert and Bakker-Arkema (1968) reported on the inconsistencies among friction data for grains. Experimental work showed that surface condition was the single most important factor in determining coefficients of friction. Until more is understood about the effects of the surface on the coefficient, a wide range of values will continue to be reported.

Jenkie and Johanson (1968) advanced a new mathematical theory of bin loading based on two different loading conditions; initial and flow. The switch from initial conditions to flow conditions within the bin was suggested as the cause of the scatter of results reported by many investigators. Design criteria was developed for one type of hopper but work was not completed for all conditions at the time of writing. For vertical bin walls, in the initial state of loading, pressure distribution, as calculated from the proposed theory, closely approximated that predicted by the Janssen equation.

CHAPTER III

THEORIES REGARDING BIN PRESSURE

3.1 Material Properties

Properties of granular materials which affect the pressure are density, coefficient of friction, angle of internal friction, angle of repose and ratio of lateral to vertical pressure. Before discussing theories on pressure, the material properties will be defined.

Density is the weight per cubic foot of the granular material.

The resultant of a normal force (N) and a friction force (F) exists at an angle θ with the normal force. In other words,

$$\frac{F}{N} = \tan \theta \quad (3.1)$$

The function $\tan \theta$ is defined as the coefficient of static friction when θ reaches its maximum value. It is independent of area and constant for given materials. In granular materials, two different coefficients are considered, the coefficient of the material on the bin wall and the coefficient of the material on itself.

The internal friction angle is defined as the angle whose tangent is equal to the coefficient of friction of the material on itself. It is affected by interlocking of the grains, any cohesion that is present and the average conditions

of all the grains within the pile.

Angle of repose is the friction angle of the granular material under zero pressure (Taylor 1948). It tends to be smaller than the internal friction angle since it is dependent on the equilibrium of the least stable grains, has little strength due to interlocking and tends to represent the friction angle of material which is less dense than the average.

Angle of repose has been used in the past as in indication of the internal friction angle. This has led to considerable confusion of the two terms. Taylor (1948) said the angle of repose is a crude approximation of internal friction angle. Stewart (1967) found that the use of this approximation in the Coulomb theory led to an underestimate of pressures.

The ratio of lateral to vertical pressures, or K value, was assumed by Janssen (Ketchum 1919), and Airy (1897) to be a constant, independent of depth. Pleissner (Ketchum 1919), Ketchum (1919), Kramer (1944), Jaky (1948) and Leczner (1963) all found that K varied with depth.

Since K is an experimental value, various approximations have been advanced. Hall (1961) and Canadian Farm Building Standards (National Research Council 1965) suggest the use of the Rankine coefficient of active earth pressures

$$K \approx \frac{1 - \sin \phi}{1 + \sin \phi} \quad (3.2)$$

with the angle of repose substituted for the angle of internal friction. Jaky (1948) suggested that $K = K_0 = 1 - \sin \phi^1$ was more accurate. The two equations represent different stress

conditions. The Rankine coefficient represents the limiting value of K for minimum pressure conditions. Jaky's K_0 represents an at rest condition. General practice in agricultural engineering favors the use of equation (3.2).

3.2 Pressures in Granular Materials, the General Case

3.2.1 Introduction

There are two classic theories on lateral pressures in granular materials; Rankine's theory and Coulomb's theory (Taylor 1948). Both consider a semi infinite mass, cohesionless material and active and passive pressure as the minimum and maximum conditions. Active pressures are defined (Hough 1957) as the conditions which exist when the wall moves away from the mass, allowing a horizontal expansion of the material. Passive pressures result when the wall moves towards the mass, forcing compression of the material.

Each pressure theory will be reviewed considering the case of no surcharge.

3.2.2 Rankine theory

Rankine assumed a relationship existed between the vertical and horizontal pressures in the mass and this relationship was not affected by the presence of a wall. Pressure at any point in the mass was assumed to be proportional to the depth below the surface. This led to the active pressure equation, for no surcharge

$$p_a = wh \frac{1 - \sin \phi'}{1 + \sin \phi'} \quad (3.3)$$

and the passive pressure equation

$$p_p = wh \frac{1 + \sin \phi'}{1 - \sin \phi'} \quad (3.4)$$

in which:

w = the density of the granular material

h = the depth of granular material above the point under consideration

ϕ' = the angle of internal friction

The resultant of these pressures for a unit width over a total wall height H, is given by:

$$P_a = \frac{wH^2}{2} \frac{1 - \sin \phi'}{1 + \sin \phi'} \quad (3.5)$$

and

$$P_p = \frac{wH^2}{2} \frac{1 + \sin \phi'}{1 - \sin \phi'} \quad (3.6)$$

This represented a hydrostatic distribution and since only horizontal strains were considered to exist, this led to the conclusions that:

1. the resultant force always acts in a horizontal plane.

2. the resultant force always acts at a point $\frac{2H}{3}$ below the surface of the granular material.

The basic weakness of this theory is the assumption that the presence of the wall does not alter the relationship of lateral to vertical pressure in the mass. It is reasonable to expect changes in shearing stress at the wall since conditions of friction and cohesion differ from those within the mass.

3.2.3 Coulomb or sliding wedge theory

Coulomb's theory was based on two assumptions:

1. a surface of failure exists which can be considered to be a plane originating at the base of the wall.

2. The thrust on the wall acts in a known direction.

The first assumption implied a wedge bounded by the wall, the free surface and the failure plane (see fig. 2). This wedge, of weight W , was assumed to be maintained in equilibrium by the force P , exerted by the wall and the resultant R , of the stresses on the failure plane.

By analyzing the forces acting on the wedge, the following formulae were developed for the case of no surcharge:

$$P_a = \frac{wH^2}{2} \frac{\cos \phi'}{(1 + \sqrt{2} \sin \phi')^2} \quad (3.7)$$

and

$$P_p = \frac{wH^2}{2} \frac{\cos \phi'}{\sqrt{\cos \gamma} - \sqrt{\sin (\phi' + \gamma)} \sin \phi'} \quad (3.8)$$

where γ = the angle of stress obliquity.

A sliding wedge analysis allows consideration of the effect of wall friction. Results obtained from this method do not differ greatly in magnitude from those obtained using Rankine's equation for the case of no surcharge. The important difference between the two theories is the direction of action of the resultant force, particularly where no surcharge exists. No direct consideration of the point of action of the resultant pressure is made in the analysis. Generally, for wall design, P_a and P_p are assumed to act at $\frac{2H}{3}$ below the surface as in the Rankine analysis.

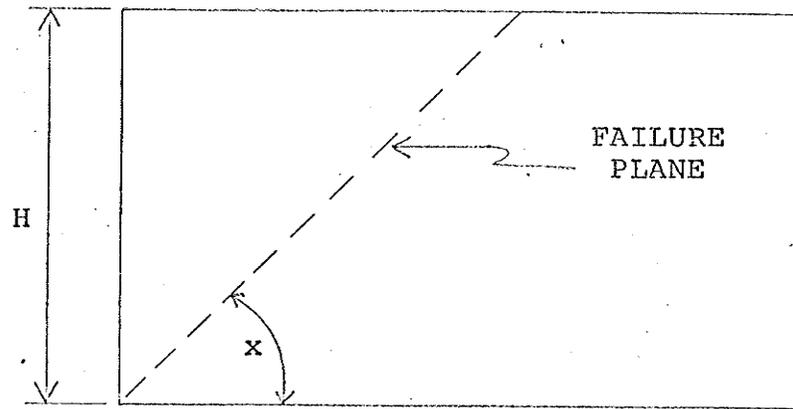


Figure 2. Sliding wedge assumption.

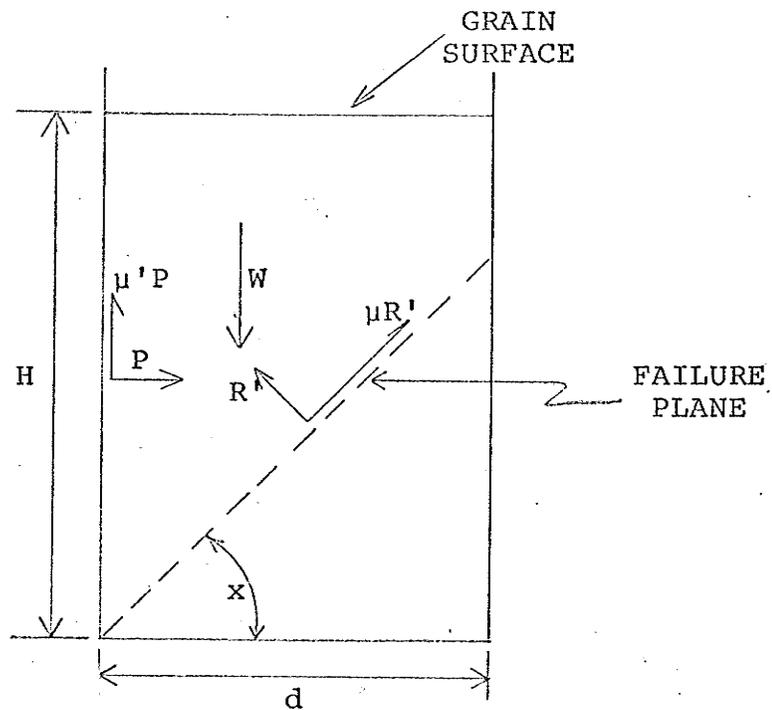


Figure 3. Assumed stresses, Airy's theory.

3.2.4 Discussion

Both theories discussed above assumed a semi-infinite mass. Since in a deep bin, the failure plane of the material intersects the opposite wall before it intersects the surface, the basic assumption is not valid. The deep bin condition precludes the use of the Rankine and Coulomb theories and necessitates special consideration.

3.3 Pressure in Deep Bins

3.3.1 Introduction

There are two classic theories on lateral pressure in deep bins which are generally accepted as predicting safe design pressures, Airy's (1897) theory proposed in 1897 and Janssen's theory (Ketchum 1919) proposed in 1895. The Janssen theory has become the more widely used in view of its relative simplicity. Such authors as Barre and Sammett (1950), Hall (1961) and the Farm Building Standards (National Research Council 1965) quote Janssen's formula for determining lateral pressures in deep bins.

3.3.2 Airy's theory

Airy's theory is basically an expansion of the sliding wedge theory discussed previously. In the deep bin case, the plane of failure intersects the opposite wall before reaching the surface of the grain (see fig. 3). By applying the law of statics to the forces acting parallel to and perpendicular to the failure plane, two equations can be determined. The equations are solved for P , differentiated with respect to x and maximized (i.e. $\frac{dP}{dx}$ set equal to 0). Further mathematical

manipulation leads to the equation

$$P = \frac{wH^2}{2} \left(\frac{1}{\sqrt{\mu(\mu + \mu')} + \sqrt{1 + \mu^2}} \right)^2 \quad (3.9)$$

and by differentiating P with respect to H for any depth $H = h$

$$L = wh \left(\frac{1}{\sqrt{\mu(\mu + \mu')} + \sqrt{1 + \mu^2}} \right)^2 \quad (3.10)$$

The mathematics involved in this derivation is not complicated but is laborious.

3.3.3 Janssen's theory

Janssen's theory is based on the assumption that:

1. a bin of uniform area and constant circumference is under consideration.
2. The ratio of lateral to vertical pressure is constant at all depths.
3. The vertical pressure is uniform across any horizontal plane.

Consider fig. 4. Summing vertical forces

$$VA + Awdy = (V + dV)A + LU\mu'dy$$

this reduces to

$$dV = wdy - \frac{LU\mu'}{A} dy$$

substituting $L = KV$

$$dV = wdy - \frac{KVU\mu'}{A} dy$$

let R , (hydraulic radius) = $\frac{A}{U}$

$$\therefore dV = wdy - \frac{KV}{R} \mu' dy$$

$\frac{KV}{R} \mu'$ will be constant for any particular case

$$\therefore \text{let } \frac{KV}{R} \mu' = n$$

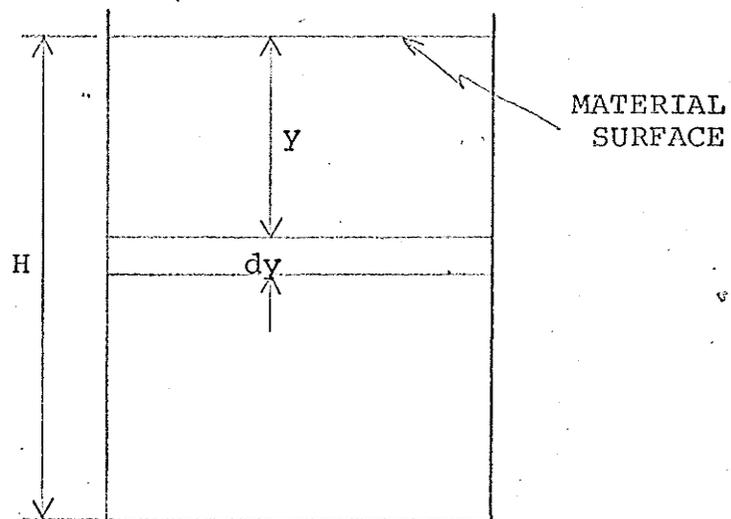


Figure 4a. Location of section considered in Janssen's theory.

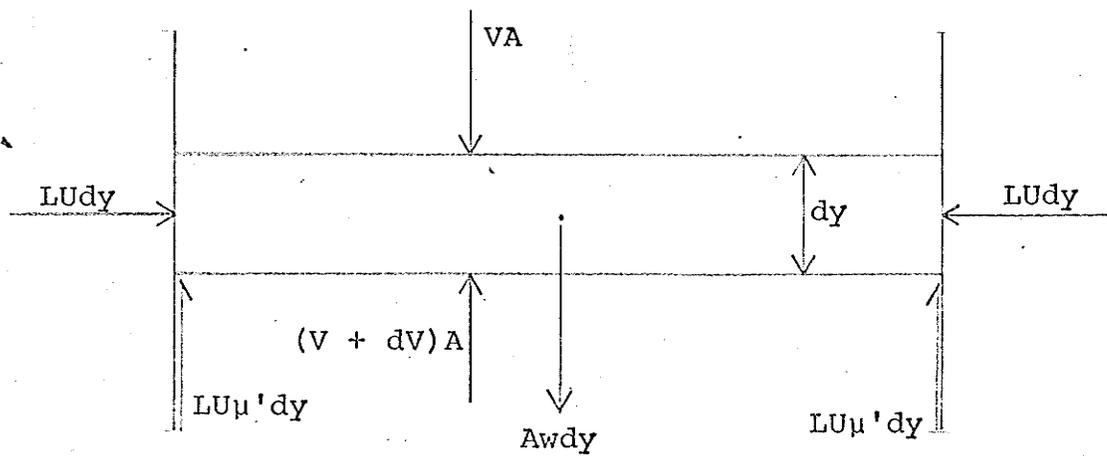


Figure 4b. Assumed stresses on a section, Janssen's theory.

Substitute n and rearrange the terms,

$$\frac{dV}{w - nV} = dy$$

Integrate

$$\log(w - nV) = ny + C$$

for

$$y = 0 \text{ \& } V = 0$$

$$\log w = C$$

$$\therefore \log \left(\frac{w - nV}{w} \right) = -ny$$

and

$$\frac{w - nV}{w} = \frac{1}{e^{ny}} = e^{-ny}$$

solving for V

$$V = \frac{w}{n} (1 - e^{-ny})$$

Substituting

$$n = \frac{K\mu'}{R}$$

gives

$$V = \frac{wR}{K\mu'} \left(1 - e^{-\frac{K\mu' y}{R}} \right)$$

If h is assumed to be the depth below the surface

$$V = \frac{wR}{K\mu'} \left(1 - e^{-\frac{K\mu' h}{R}} \right) \quad (3.11)$$

and since

$$L = KV$$

$$L = \frac{wR}{\mu'} \left(1 - e^{-\frac{K\mu' h}{R}} \right) \quad (3.12)$$

The formula is often written

$$L = \frac{wD}{4\mu'} \left(1 - e^{-\frac{4K\mu'h}{D}} \right) \quad (3.13)$$

where

$$D = \frac{4A}{U} \quad (3.14)$$

This solution is dependent on K and μ' , two factors which must be determined experimentally for each type of material concerned. Both values have been subjected to considerable debate and study with widely varying results.

3.3.4 Discussion

Janssen's equation has been criticized because neither the angle of repose nor the angle of internal friction of the grain were considered. This argument is not entirely true in that K was originally estimated from the relationship $\frac{1 - \sin \phi'}{1 + \sin \phi'}$ where ϕ' was the internal angle of friction of the material. This equation has been altered by the substitution of the angle of repose, ϕ for the angle of internal friction, ϕ' .

Both Janssen and Airy assumed the relationship $L = KV$ existed and was constant throughout the bin. Neither theory considered the effect of a surcharge, a regular occurrence in agricultural storage structures.

While Airy's equation is theoretically more correct, Janssen's equation is much easier to handle. Both result in similar pressure predictions and as a result, Janssen's equation has become more widely used. This study considered the Janssen equation.

CHAPTER IV

EXPERIMENTAL EQUIPMENT

4.1 Introduction

Two separate experiments were conducted; one on a full scale bin and the other on a model bin. The equipment used in each experiment will be discussed separately.

4.2 Full Scale Bin

4.2.1 Bin and handling equipment

A plywood storage bin (see fig. 5) was built using MacMillan Bloedel Limited Plan Number 634. Overall bin dimensions were approximately eight feet square by eight feet high. Two cross walls divided the bin into four cells with inside dimensions of 45.38 inches by 45.38 inches by 94 inches. Each cell had two 0.62 inch plywood walls in common with two adjoining cells. The fir plywood in the walls was placed with the face grain running horizontally.

Test material was elevated into the bins with a vertical cup and belt elevator leg. A spout allowed delivery to the selected cell. Trap doors were installed at the outside edge of each cell floor, to allow the granular material to be shovelled to one of two four inch augers. The augers returned the material to the four foot by eight foot by twelve foot holding bin, completing the handling cycle.

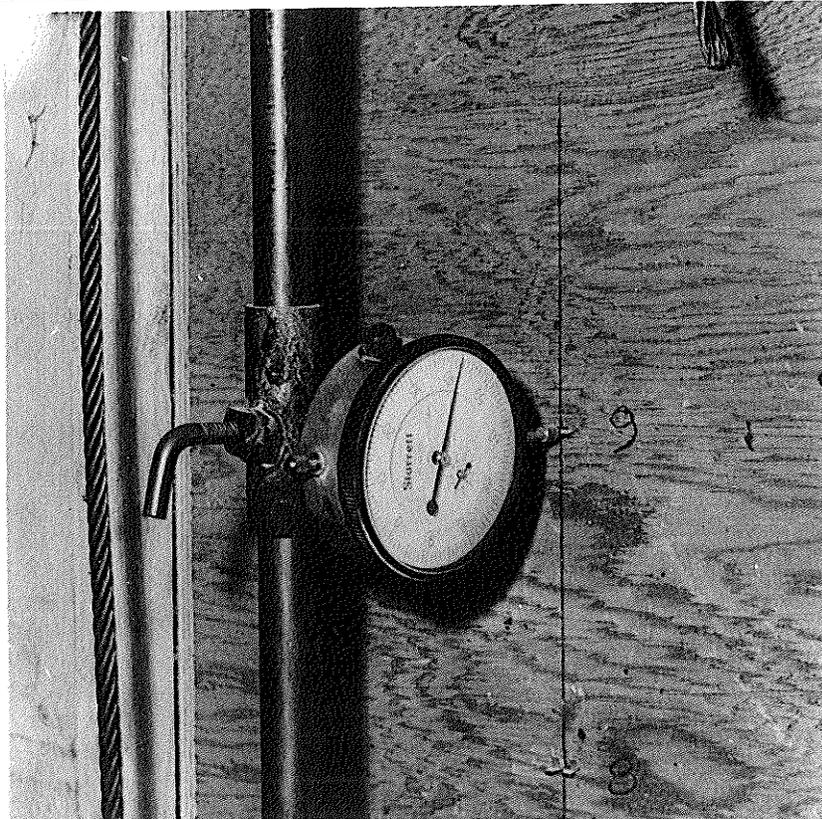
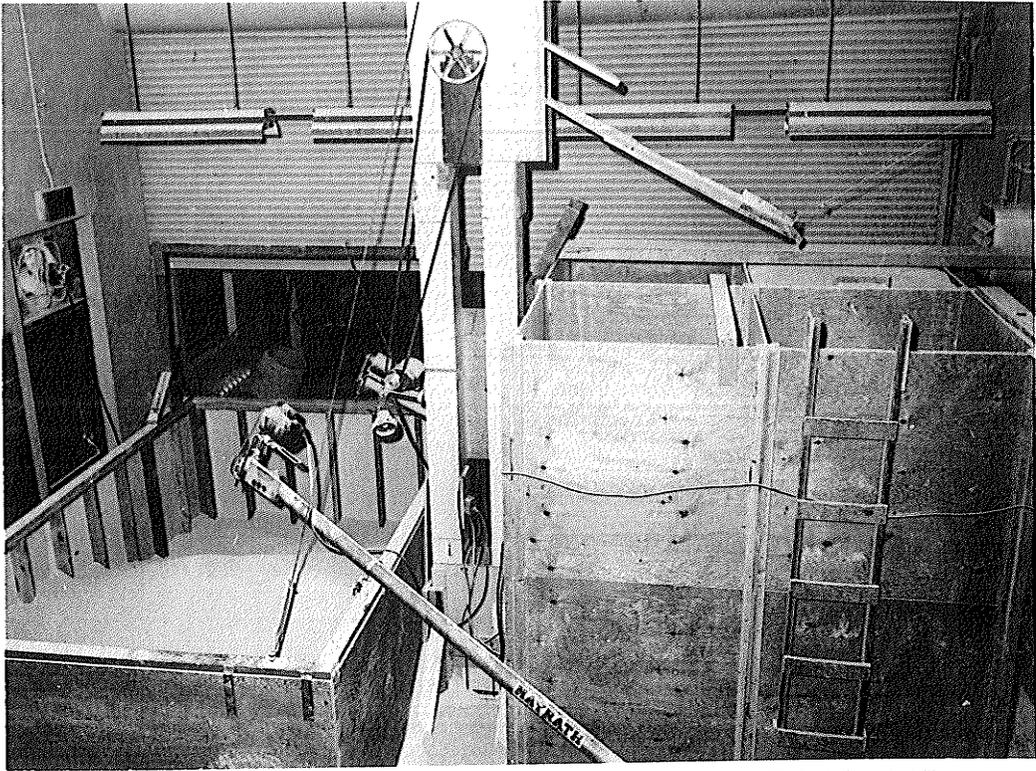


Figure 5. Full scale test bins.

Figure 6. Deflection gauge showing moveable sleeve on support rod.

4.2.2 Deflection measurement

Lateral deflections of the vertical centerline of one bin wall were measured using a Starrett Number 655-441/5 dial gauge (see fig. 6).

A 0.62 inch rod was held rigidly at the top and bottom of the bin wall. The gauge was mounted on a moving sleeve which allowed it to be set opposite any of the sixteen gauging points. Fig. 7 shows the location of the gauging points which were marked on the bin wall with glazing tacks.

4.2.3 Pressure measurement

A pressure measurement system was mounted in one of the common walls of one cell. The system was required to:

1. measure pressure directly;
2. measure normal pressures only, thereby eliminating the vertical wall load component;
3. mount flush with the inside surface of the bin;
4. be independent of wall flexibility;
5. have the same fertilizer to material coefficient of friction as the rest of the wall;
6. function when fertilizer was placed on both sides of the wall.

A review of literature suggested several alternative systems. Roberts (1882, 1884) and Ketchum and Varnes (Ketchum 1919) used a panel linked by a system of levers to a platform scale. Jamieson (1903, 1904), Bovey (1903, 1904) and Lufft (1904) all used liquid filled rubber diaphragms. Toltz (1903) measured deflections of a calibrated panel.

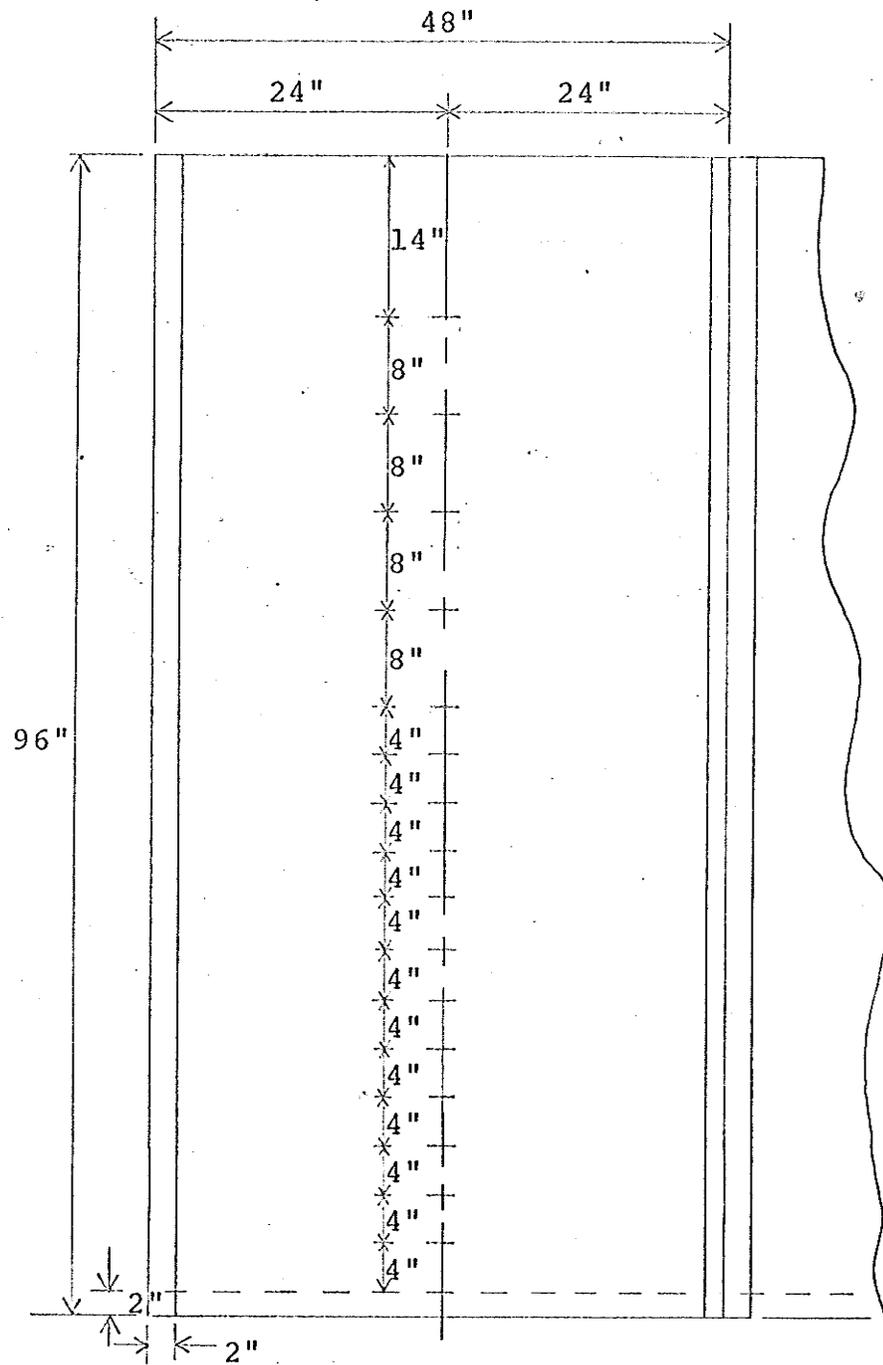


Figure 7. Location of gauge points, full scale bin.

McCalmont (1938), Saul (1953), Stewart (1967) and Willson (1968) used panels supported on calibrated support bars. Strain gauges were mounted directly on the bin wall by Collins (1963) and Hamilton (1964). Dale and Robinson (1954), Pandey (1966) and Nickel (1967) used metal diaphragms mounted with electrical resistance strain gauges.

A thin metal diaphragm mounted with electrical resistance strain gauges was selected. For ease of fabrication, a circular shape was chosen. The problem of coefficient of friction was overcome by bonding a thin veneer of plywood to the diaphragm and machining the transducer to the exact required thickness (see fig. 8).

Beckwith and Buck (1961) report that a thin metal diaphragm pressure transducer generally has a linear output if maximum deflection does not exceed 30 percent of the diaphragm thickness. Dale and Robinson (1954) and Pandey (1966) found that accuracy was greatest when deflection was kept to a minimum.

A diameter of 4.500 inches was chosen. Cell walls were made from 0.250 inch thick mild steel pipe. The diaphragm was made from aluminum as the low modulus of elasticity (10×10^6 p.s.i.) would result in a high unit strain per unit of stress.

Calculations were carried out (see Appendix A) to aid in selecting diaphragm thickness and the type of strain gauge. Due to the unknown stiffness supplied by the plywood and the less than perfect fixity at the diaphragm edge, these calculations

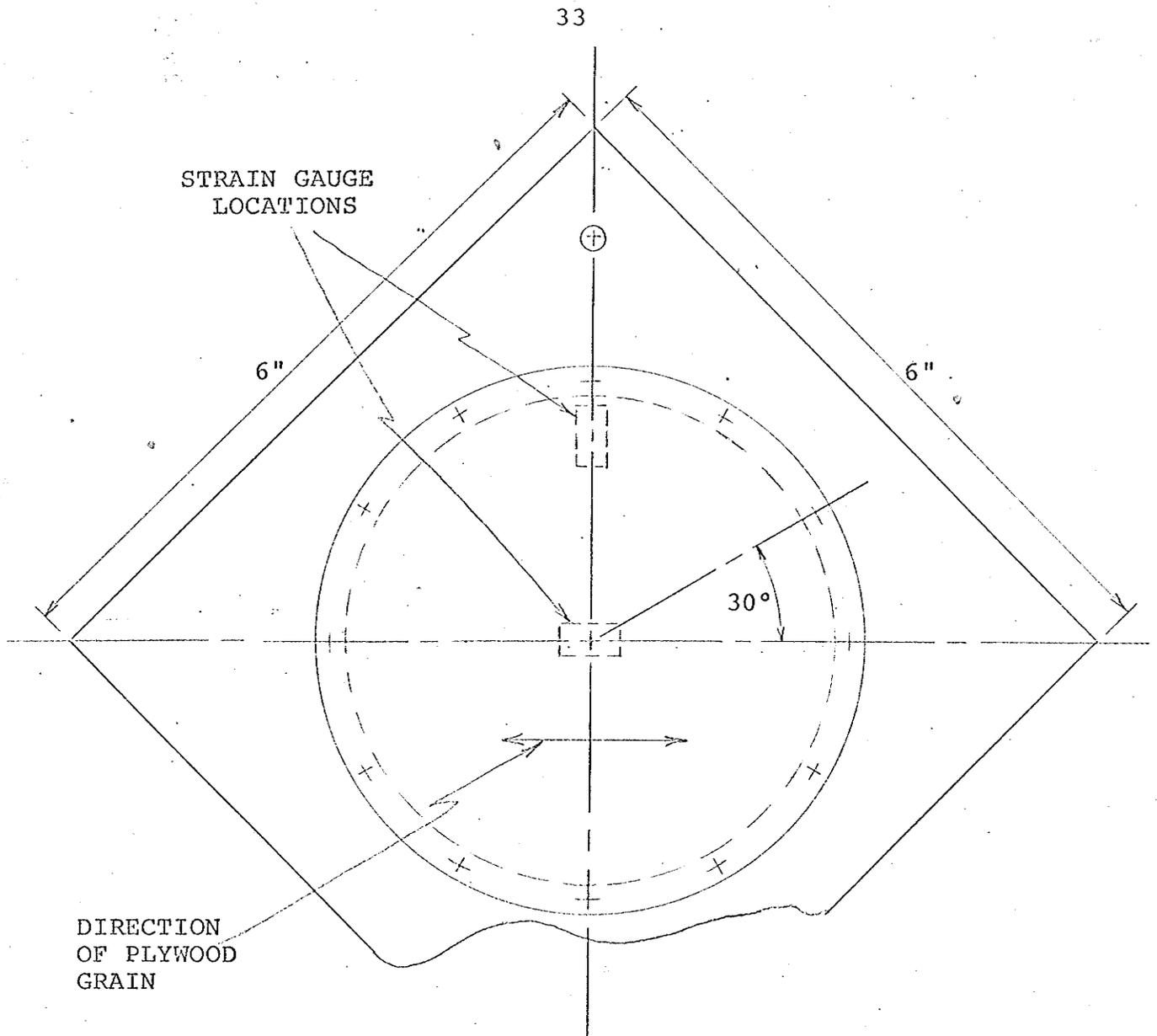


Figure 8a. Plan view of pressure transducer.

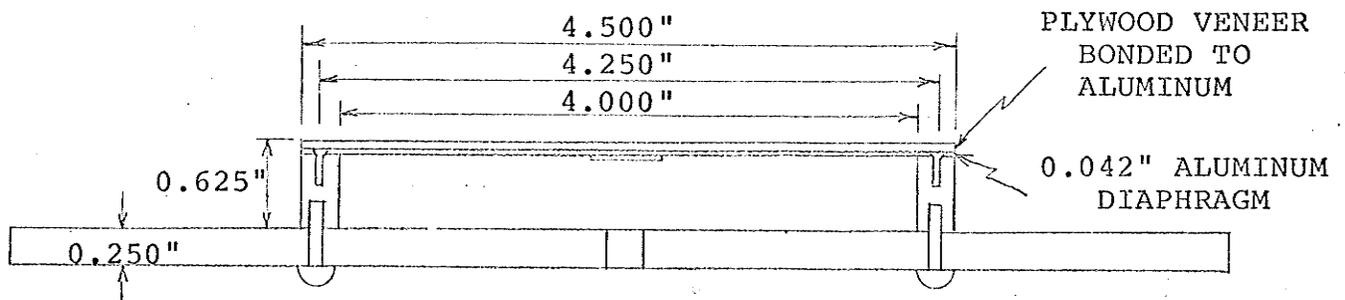


Figure 8b. Sectioned view of pressure transducer.

could do no more than supply a guide for selection. An 0.042 inch thick diaphragm and EA-13-500BH120 foil strain gauges with a gauge factor of $2.09 \pm 0.5\%$ were chosen.

Each transducer was wired into a BLH Model 225, 10 Channel Switch and Balance Unit. The switch and balance unit was wired to a Strainsert Model HW1-D Transducer/Strain Indicator. Theoretical sensitivity was calculated to be 0.013 p.s.i. per micro inch. Actual sensitivity ranged from 0.04 to 0.06 p.s.i. per micro inch of output. The large difference was attributed to the added stiffness of the plywood.

Nine transducers were fabricated, seven were mounted in the bin wall and two on the bin floor. Wall mounted units were placed in holes 4.750 inches in diameter located on the center line of the test wall and centered 6, 18, 30, 42, 54, 66 and 78 inches above the floor (see fig. 9). Two screw nails were placed through the 0.250 inch thick steel back plate to hold each transducer in place. To avoid possible affects of wall deflection, the screw nails were placed on the bin wall centerline. A cover was placed over the backs of the transducers.

Two transducers were placed on the floor, directly below the line of wall mounted units. One transducer was centered six inches from the wall, the other was located at the geometric center of the floor area.(see fig. 10).

4.3 Model Bin

A 16 inch by 16 inch by 47.38 inch model bin was built using fir plywood. Three walls were made from 0.62 inch

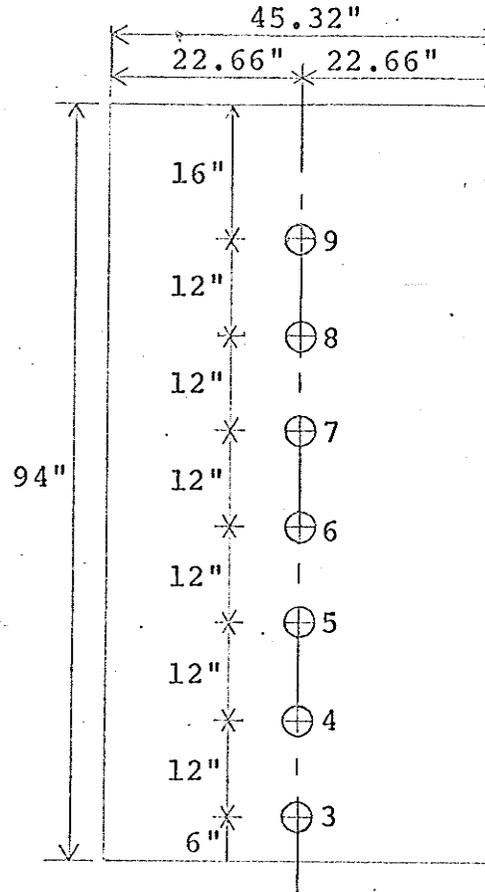


Figure 9. Wall locations of pressure transducers.

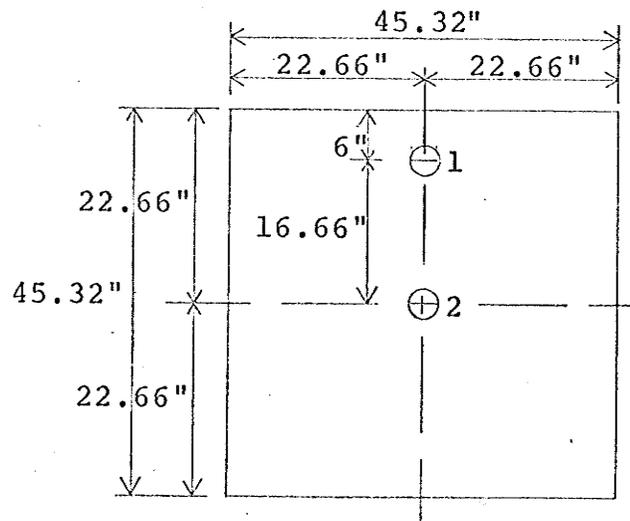


Figure 10. Floor locations of pressure transducers.

select sheathing grade fir plywood, the fourth side was 0.31 inch select sheathing. A 2 inch by 4 inch lumber frame supported the walls one foot above the floor (see fig. 11).

The bin floor was independent of the walls. A Fairbanks standard Number 10 platform scale was used to support the floor and to weigh the load supported on the floor when the bin was full. To allow free floor movement within the walls, a plastic "skirt" was attached to the walls just above the floor. By placing the skirt over the crack between floor and wall, entry of the test material was prevented.

Deflections were recorded on the 0.31 inch thick wall. A bar and dial gauge system similar to that employed on the full scale bin was used (see fig. 6). Gauging points were placed along the vertical centerline of the wall at four inch intervals above the floor. No points were located higher than 36 inches above the floor.

Weights of material being placed into the bin were obtained using a calibrated weight transducer (see fig. 12). Strain in the vertical member of the transducer was measured by two BLH A5-S6-SR4 bonded wire strain gauges, one in tension and one in compression. Output was measured with a Strainsert Model HW1-D Transducer/Strain Indicator. Sensitivity of the transducer, for the range in which it was used, was 78 micro inches per pound (see Appendix D).

Material was placed in the bin by hand. For emptying, a cable and lever system was used to raise the bin floor off the scale. After removing the scale, the floor of the

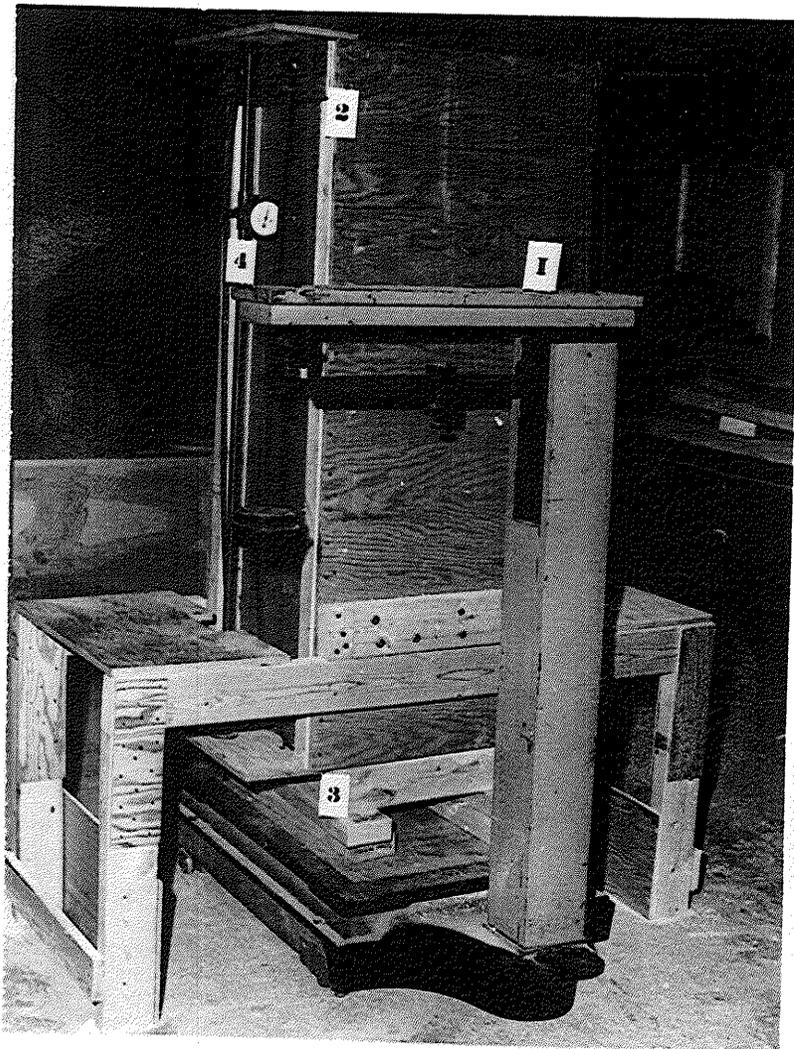


Figure 11. Model test bin showing:
1. platform scale,
2. test bin,
3. moveable floor and,
4. deflection gauge.

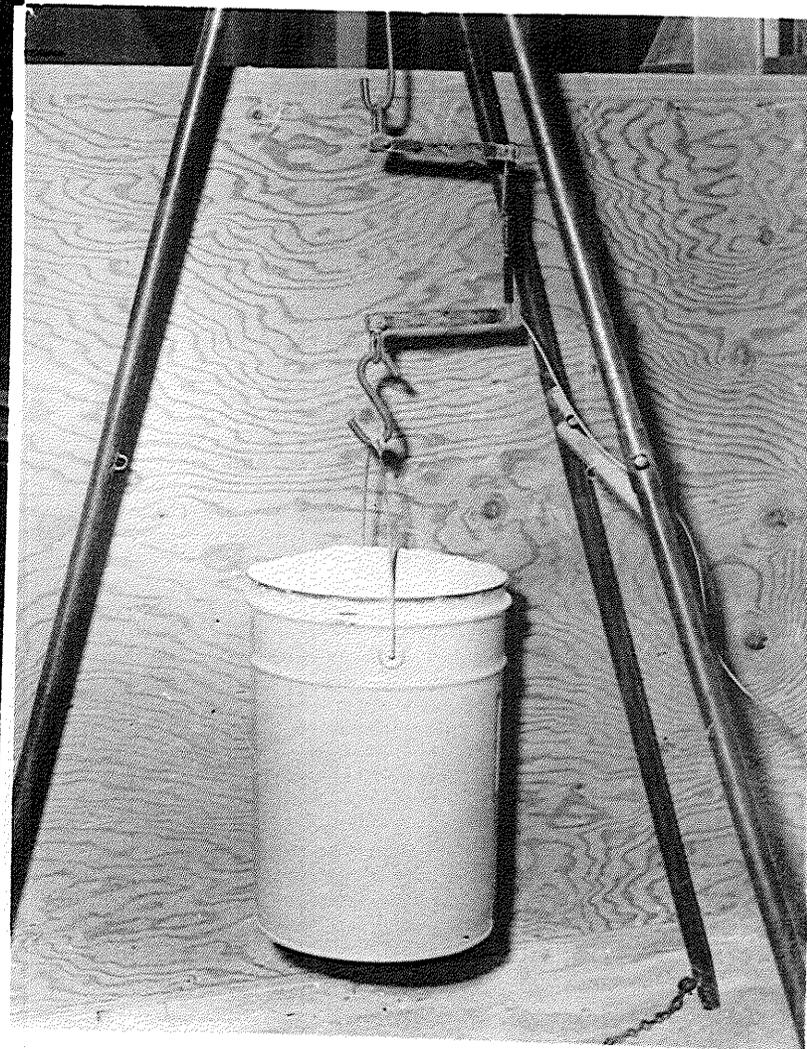


Figure 12. Weight transducer.

bin was lowered and the fill material released.

CHAPTER V

PROCEDURES

5.1 Choice of Fertilizer

Experimentation with all formulations of commercial fertilizer was desirable but limitations on time and facilities prevented this. A decision as to which formulation to use was made based on the following test conditions which would exist:

1. The fertilizer would be repeatedly subjected to mechanical handling equipment.
2. The tests would be carried out over an extended period of time.
3. Precise control of temperature and humidity conditions would not be possible.

Tests conducted in Alberta by Western Cooperative Fertilizers Limited (Harapiak 1968) and the Alberta Department of Agriculture (1968) indicate that ammonium phosphate (11-48-0) offered the least problems in storage due to moisture and caking. Wiens (1969) found that ammonium phosphate did not break down significantly with augering.

Ammonium nitrate (34-0-0, 33.5-0-0) was found to be hygroscopic and subject to breakdown when augered. Urea (46-0-0) is not recommended for long term storage since it tends to "set up" throughout the entire bin (Harapiak 1968).

After consideration of the above, ammonium phosphate (11-48-0) was chosen as the fertilizer to be used in the test bins.

5.2 Consistency of Material Properties

In spite of Wiens' (1969) findings, some reservations were still held regarding the grinding of the test sample by mechanical handling equipment. Checks were made on random samples of the fertilizer at selected intervals to establish that the properties of the fill material had not changed.

Samples of fertilizer were drawn before runs 1, 4, 6, 7, and 10 of the full scale bin tests. Each sample was subjected to a sieve analysis to detect particle grinding. The sieves used were U.S. Bureau of Standards numbers 10, 20, 40, 100, 200 and 270.

Four samples of approximately 1000 grams each were sieved from each major sample. Weights retained on the various sieves were recorded and then averaged for each sieve and each run. The means were subjected to a randomized block analysis of variance with the runs considered as the treatments and the different weights retained treated as blocks. This method of analysis was chosen to remove the obvious variation between weights retained on different sieves and detect differences in the fertilizer due to grinding.

Each sample was subjected to tests of density, angle of repose and coefficient of friction (fertilizer to plywood). Values obtained in each of these tests were analyzed by a completely random analysis of variance, again considering each

pre-run sample as a treatment. The purpose of the tests was to detect differences in mean values of the property under consideration as a result of grinding of the fertilizer.

5.3 Control Material

To check the reliability of the equipment and the techniques employed, wheat was used as a control material for all tests. Manitou wheat was chosen as the control and was subjected to all the tests conducted on the fertilizer.

5.4 Material Properties

5.4.1 Introduction

Although all commercial fertilizers could not be subjected to a complete series of tests, it was decided that the material properties necessary for calculation of pressures by Janssen's formula should be determined for a range of materials.

The following formulations were studied:

1. 11-48-0, Ammonium phosphate, United Grain Growers Limited.
2. 11-55-0, Ammonium phosphate, Sherritt Gordon Mines Limited.
3. 34-0-0, Ammonium nitrate, Simplot Chemical Company Limited.
4. 21-0-0, Ammonium sulphate, Sherritt Gordon Mines Limited.
5. 23-23-0, Ammonium nitrate-phosphate, United Grain Growers Limited.

6. 46-0-0, Urea, United Grain Growers Limited.

5.4.2 Coefficient of friction

A tilting top drafting table was modified for the purpose of these tests (see fig. 13). Select sheathing grade fir plywood similar to that used in the construction of the bins was fastened to the table top. Angle of slope was measured with a large vertical protractor fastened at one end of the table. A plumb bob was fastened at the center point of the protractor and the scale of the protractor adjusted so that the zero line fell directly behind the plumb bob string when the table surface was level. As the table was raised, the angle of slope was read directly.

To conduct a test, a thin layer of granular material was spread over the surface of the plywood. A twelve inch by twelve inch frame was then placed on the test bed and filled with granular material. A 36 inch long bar clamp was used to raise the back edge of the table. When the sample started to move, the angle of slope was recorded.

Twenty tests were conducted on each type of fertilizer, each selected sample of 11-48-0 and both wheat samples. The 11-48-0 tests were analyzed as outlined in Section 5.2. Results were averaged for each set of tests and averages were used in all calculations.

5.4.3 Angle of repose

A one cubic foot plywood box with one removable side was used for these tests. Two 1 inch by 2 inch boards were



Figure 13. Test apparatus for coefficient of friction showing:
 1. fertilizer sample in frame,
 2. protractor and plumb bob,
 3. sample of plywood fastened to table top and,
 4. bar clamp for raising table top.



Figure 14. Test apparatus for angle of repose showing:
 1. fertilizer sample,
 2. test box and,
 3. combination square placed to measure S distance.

fastened across the open top of the box a fixed distance apart. Bar clamps were used to hold the removable side while the box was filled level with the test sample. The side of the box was pulled away and the test sample allowed to flow out (see fig. 14).

A twelve inch combination square was used to measure distances S and T as shown in fig. 15. The tangent of the angle of repose was calculated from the relationship

$$\frac{S - T}{Z} = \tan \phi$$

Tests were repeated ten times on the wheat and each 11-48-0 sample. Fifteen replications were made on the other formulations of fertilizer. Each set of tests was averaged, and the 11-48-0 samples were analyzed as outlined in Section 5.2.

5.4.4 Density

A 250 milliliter graduated glass cylinder (corrected to 248 milliliters) was placed on a Mettler K4 scale and the scale reading was adjusted to read zero. Granular material was poured into the cylinder through a funnel, making certain that free flow was permitted. When the material was level at the 250 milliliter mark, the weight was recorded. Twenty tests were run on each sample of material. The 11-48-0 samples were analyzed as outlined in Section 5.2. Each sample was averaged to obtain the mean density.

During these tests it was observed that the method used to place the material in the graduate affected the

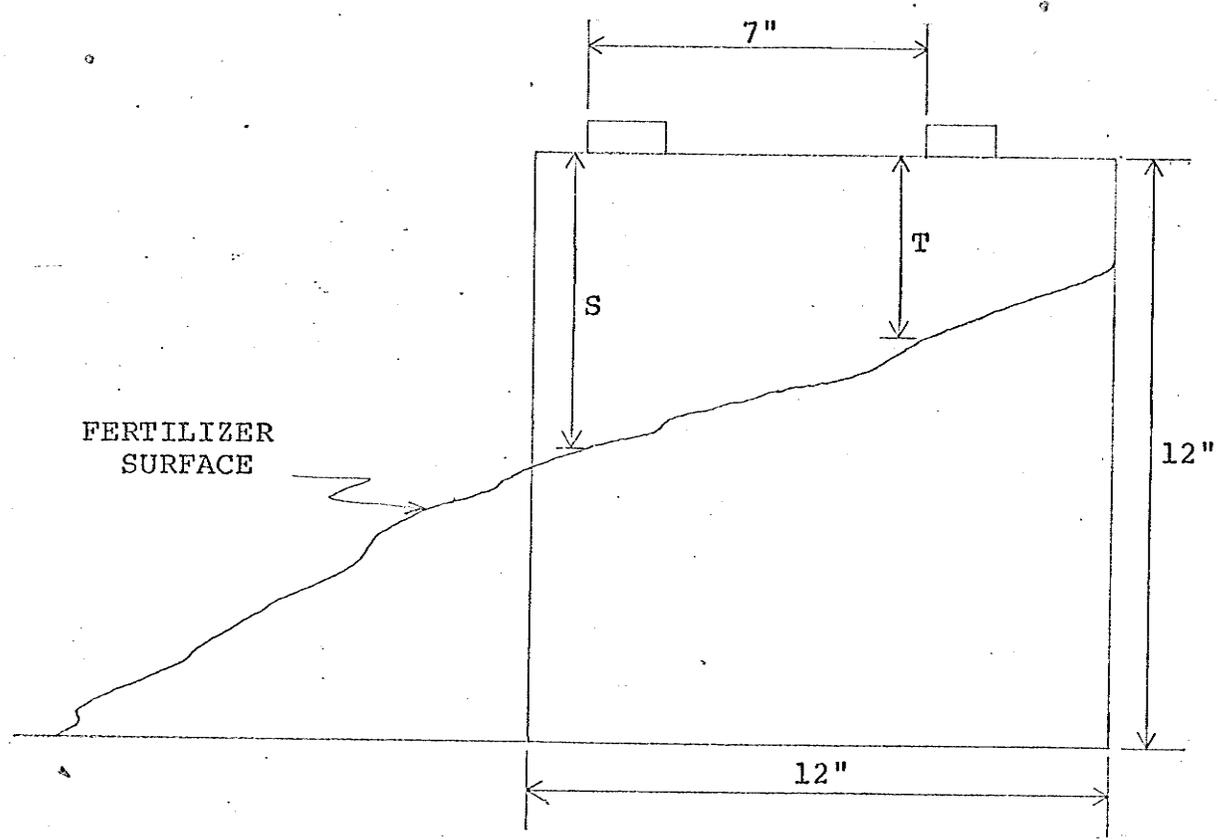


Figure 15. Illustration of measurement locations for angle of repose determination.

density. In order to determine if the height of fall was critical, a short experiment was conducted. Fertilizer was dropped into a two-quart (0.0805 cubic foot) measure through average heights of 0.25, 0.75, 1.75, 2.75, 3.75 and 4.75 feet. The sample was struck off level and weighed to determine the density. Five tests were made at each height and a completely random design analysis of variance was conducted to determine if the mean densities varied. Differences of means were determined by the Student-Newman-Keul multiple range test.

5.4.5 Moisture content

Five samples of each fertilizer formulation were taken to determine the moisture content. Samples were individually weighed, placed in a drying oven for twenty-four hours at 105°C and then reweighed. Moisture contents, on a wet weight basis, were calculated and averaged for each type of fertilizer. Wet weight basis was chosen for direct comparison with wheat.

Five samples of each batch of wheat were taken and moisture contents determined with a Model No. 919 Halross Moisture Meter. The results were averaged for each batch.

5.5 Transducer Calibration

Each transducer was calibrated individually and the data was subjected to a linear regression analysis. The regression equations, together with their correlation coefficients, are listed in Appendix B. Fig. 16 shows a typical strain versus applied normal pressure curve.

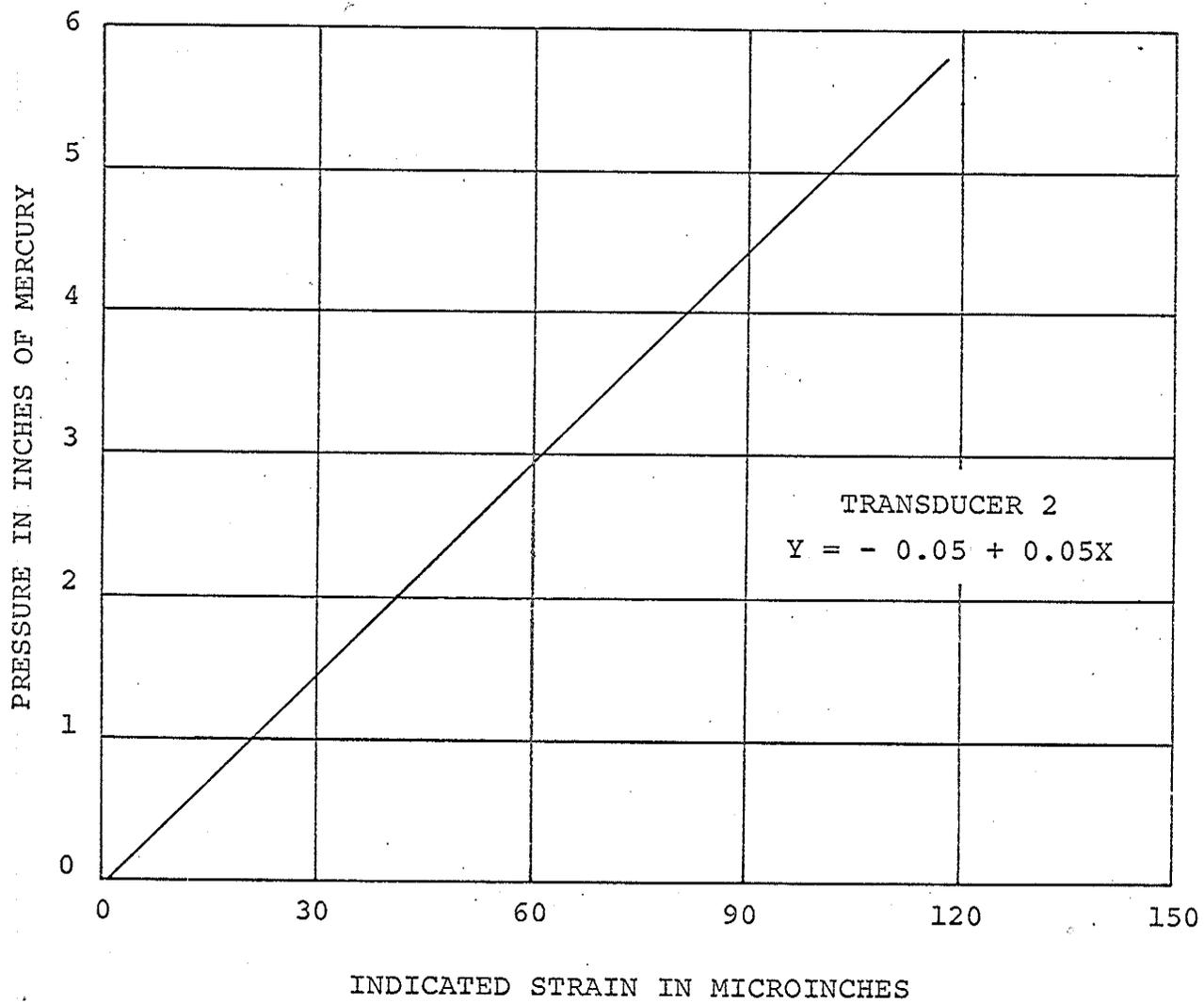


Figure 16. Typical transducer calibration curve.

Fig. 17 shows the calibration system. Air was released from a compressor into a hood which was fastened over a transducer. A seal was obtained by the use of an O-ring between the hood and the transducer's back plate. Pressure in the system was measured with a mercury manometer. The manometer gave almost immediate response to pressure changes, both positive and negative.

Before calibration began, the air system was bled and the strain indicator zeroed. As pressure was increased or decreased, the strain in the diaphragm (X) and the head of mercury (Y) were recorded. The data was then analyzed as discussed previously.

The regression equations yielded pressure in inches of mercury (Y), when the strain, in micro inches, was substituted for X. To make the values more meaningful, the pressure was multiplied by 0.490, the density of mercury in pounds per cubic inch at 70°F, to obtain pressure in pounds per square inch. Since the calibration was carried out at temperatures ranging from 70°F to 73°F, no density correction was applied.

To check that only normal loads were recorded, a layer of masking tape was bonded to the transducer face and connected to a calibrated spring scale by means of a short lumber header and a cord. Normal load was applied with a concrete test cylinder set on several folded hand cloths. The purpose of the hand cloths was to remove the effect of irregularities in the concrete surface.

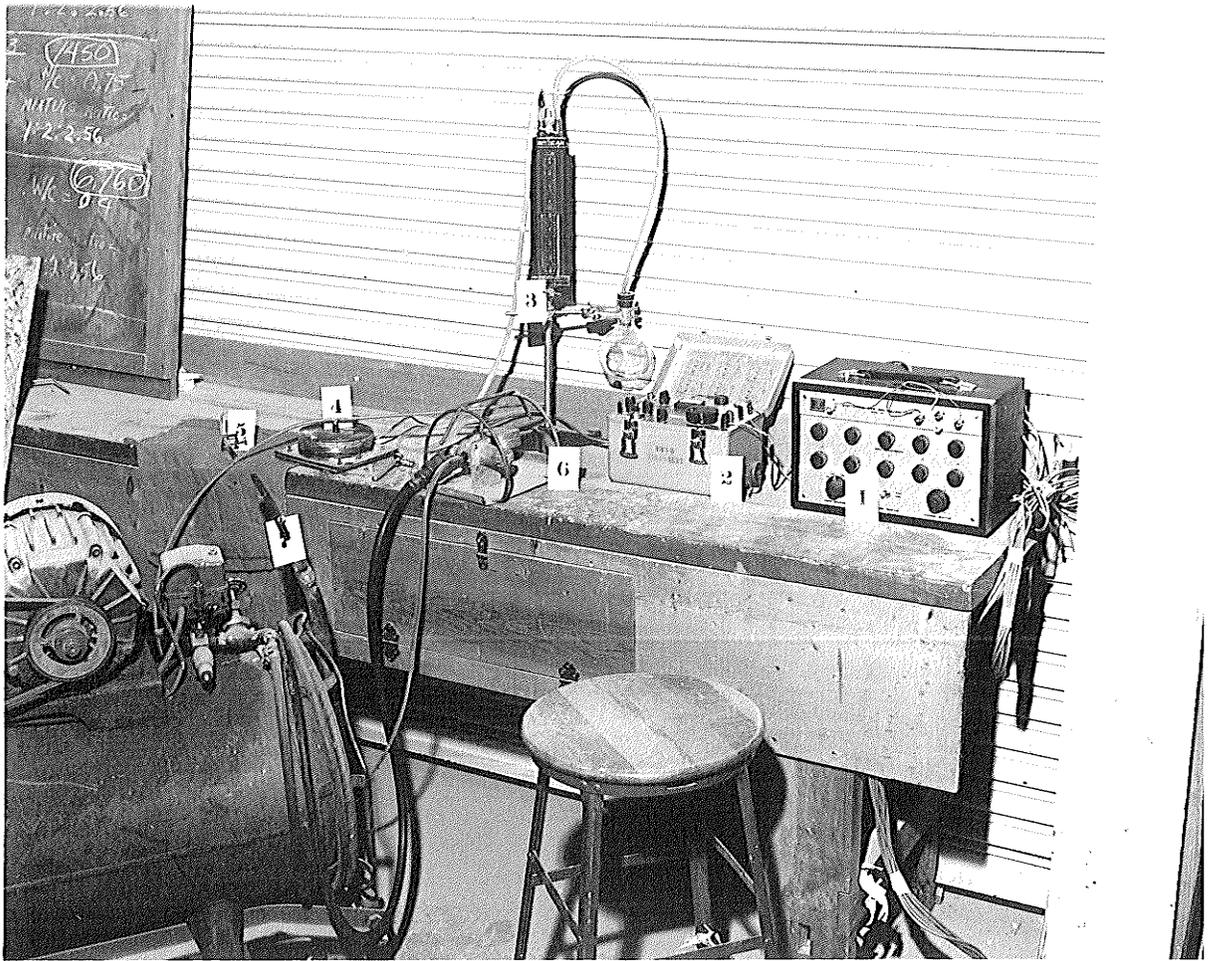


Figure 17. Calibration system for transducers showing:

1. switch and balance unit,
2. strain indicator,
3. mercury manometer,
4. pressure hood mounted over a transducer,
5. pressure relief clamp,
6. air supply valve,
7. compressor.

To conduct a test, the strain indicator was turned on and zeroed, before applying the normal load and recording the indicated strain. Load parallel to the transducer face was then applied manually through the spring scale. If the horizontal load was not truly parallel to the diaphragm, deviations in indicated strain were observed. Loads up to twenty-five pounds, equal in magnitude to the normal load, were applied with no apparent effect on the indicated strain as long as the direction of action was truly horizontal.

After the testing was completed, the transducers were removed from the bin and recalibrated to detect any changes which may have developed.

5.6 Full Scale Bin Tests

5.6.1 Introduction

Sixteen test runs were conducted on the full scale bin, ten using ammonium phosphate fertilizer and six using Number 2 Northern Manitou wheat as the granular material. The fertilizer tests were further divided into six in which the transducer mounted wall was fully flexible and four in which the test wall was held rigid. Six rigid wall tests had been proposed but repeated mechanical handling caused a breakdown of the fertilizer such that material flow was restricted and tests had to be abandoned. The wheat tests were conducted on a fully flexible wall.

The sequence of events was similar for all tests. Before filling, all the transducers were zeroed with the aid of the switch and balance box. Filling was accomplished in a

continuous operation until the test bin was leveled at the maximum depth of 94 inches. Average fill time was 35 minutes for fertilizer and 30 minutes for wheat. Indicated strain was recorded for each transducer immediately after filling and twenty minutes after filling. The two values were averaged.

After pressures were recorded, the test bin was emptied and the material returned to the holding bin. Recovery of the transducers was checked and any zero drift was noted.

5.6.2 Wall flexibility

Two conditions of wall flexibility were investigated, fully flexible and completely rigid. The moment of inertia of the 0.62 inch thick plywood wall (0.194 in^4 per foot width) (Plywood Manufacturers of B.C. 1966) was used as a measure of flexibility.

To make the test wall rigid, the bin adjacent to the test bin was filled with fertilizer. Since the two bins were not filled simultaneously, complete rigidity was not obtained. It was felt that any deflections which did occur would be very small and would have minor effect on the pressures. This accomplished two things: (1) established a condition in which the instrumented wall would have equal pressure from both sides when under test, thereby minimizing lateral deflection; (2) provided storage of fertilizer which was beginning to break down due to mechanical handling.

5.6.3 Deflection readings

During test runs 7 to 10 inclusive, deflections of a flexible wall were recorded. The dial gauge was set at each gauging point and the initial reading recorded before filling commenced. After filling and recording the initial transducer outputs, each gauging point was checked again and the readings for the deflected wall were recorded. Differences between initial and final readings represented the deflections of the points on the wall. The gauge points had to be checked before each filling as recovery of the wall was neither predictable nor uniform.

5.6.4 Analysis

Pressures recorded were averaged for each transducer over each of the test conditions, fertilizer with a rigid test wall, fertilizer with a flexible test wall and wheat with a flexible test wall. Standard deviations were calculated for the data in each test.

Unpaired t-tests were conducted on mean pressures to compare flexible and rigid wall conditions. Lateral pressures were compared to predicted pressures by means of a t-test. Vertical pressures from transducers 1 and 2 were compared using a t-test to detect differences across the bin.

Deflection readings at each gauging point were averaged over the four runs.

5.7 Model Bin

5.7.1 Introduction

Short term and long term tests were conducted, using

Manitou wheat and ammonium phosphate (11-48-0) as the fill materials in both cases. Both granular materials used were subjected to tests for coefficient of friction, angle of repose, and density. Moisture content tests were run on the wheat sample.

5.7.2 Short term tests

Twelve test runs were made, six with ammonium phosphate and six with wheat. Granular material was placed in the bin in lifts of approximately six inches. Each lift was weighed before being placed in the bin and the floor load weighed after the lift was added. Wall deflection readings were taken after the fill reached two, three and four foot depths.

Random block analysis of variance was conducted on lift weights for the tests on each fill material. Test runs were considered as treatments and lift weights at successive depths as blocks.

Increasing floor loads for all tests were considered in a random block analysis of variance to detect differences between test runs. Differences detected in the analysis of variance tests were analyzed by the Student-Newman-Keul multiple range test.

Average material density and the average weight of each lift was determined. Total vertical wall loads were calculated by subtracting the recorded floor load from the cumulative weight of material in the bin.

The first six inch lift was placed in the bin and its

effect was determined. The effect of two lifts was determined in total when 12 inches of fill was placed. This was assumed to be the sum of the effect of the first lift, or top lift, determined above, plus the effect of the second lift. Applying this reasoning, the effect of any lift could be determined by subtracting the total effect of fill material, excluding the lift in question, from the total effect including that lift.

Deflection readings were averaged for each gauge point and both materials. Separate averages were calculated for each fill level at which readings were taken.

5.7.3 Long term tests

After one short term test, the bin was not emptied but allowed to stand overnight. A decrease in floor load was noticed the next morning. Before testing was started again, some short time later, a marked increase in floor load was noticed. These observations led to a series of five tests.

The model bin was filled with a known weight of material, the floor load and wall deflections recorded, and the bin allowed to sit undisturbed for periods of twenty-three to fifty-two hours. Floor load and wall deflection were recorded at intervals during the test.

Two tests were run using Manitou wheat as a fill material. Both tests ran for twenty-three hours and readings were taken at one, four, twelve and twenty-three hours.

Ammonium phosphate was used for three tests. The first

test lasted twenty-three hours and readings were taken at intervals similar to those for the wheat tests. A thirty-six hour test was run with readings as before and at the conclusion of the test. The final test was run for fifty-two hours. Readings were taken every three hours for the first twelve hours of test, from the twenty-fourth to the thirty-sixth hour and again at the conclusion of the test. Maximum and minimum loads were tabulated for each test.

CHAPTER VI

RESULTS AND DISCUSSION

6.1 Consistency of Test Fertilizer

6.1.1 Gradation

A sieve analysis of the 11-48-0 pre-run samples was conducted to detect grinding of the fertilizer. The data is summarized in Appendix C. A random block analysis of variance showed no change in particle size distribution at a level of statistical significance $\alpha = 0.05$.

6.1.2 Coefficient of friction

A completely random analysis of variance of the 11-48-0 samples, as outlined in Section 5.2, showed no difference in the mean coefficient of friction at $\alpha = 0.05$.

6.1.3 Density

A completely random analysis of variance of the 11-48-0 samples, as outlined in Section 5.2, showed no difference in mean density at $\alpha = 0.05$.

6.1.4 Angle of repose

A completely random analysis of variance of the 11-48-0 samples, as outlined in Section 5.2, showed a significant difference in some mean angles. A student-Newman-Keul's multiple range test showed that the mean angle of the sample drawn previous to run number six was greater than the other

means at $\alpha = 0.05$. This was considered to indicate a change in the physical properties of the test fertilizer and the data from number six was neglected in the analysis of the results. Subsequent runs were conducted with a new fertilizer sample.

6.2 Material Properties

Average values obtained for coefficient of friction, angle of repose, K , density and moisture content for the materials tested are given in Table I. Recommended design values, as listed in Walker et al (1965) and Farm Building Standards (National Research Council 1965) are included for comparison purposes.

Coefficients of friction for wheat on plywood are within the range of values predicted in A.S.A.E. D242.1 (Agricultural Engineers Yearbook 1969). Values for fertilizer on plywood are within the range of values suggested by Walker et al (1965). Standard deviations of friction angle data range from a high of 1.04 for the Manitou wheat used in the full scale bin, to a low of 0.55 for the 23-23-0 fertilizer. The values reported are reliable for the materials used on-unsanded fir plywood, perpendicular to the face grain.

Angle of repose for wheat was found to be 22 degrees, considerably less (approximately 21 per cent) than the 28 degrees suggested for hard red spring wheat, by the Farm Building Standards (National Research Council 1965).and A.S.A.E. D240 (Agricultural Engineers Yearbook 1969). An explanation of this difference probably lies in the fact that Manitou is

TABLE I
SUMMARY OF MATERIAL PROPERTIES

Material	Coefficient of Friction	Angle of Repose in Degrees	K ¹	K _O ²	Density lb/cu ft	Moisture Content in per cent
11-48-0	0.566	25.8	0.394	0.573	59.0	0.74
11-55-0	0.549	24.7	0.410	0.582	66.5	1.66
23-23-0	0.465	21.4	0.465	0.635	61.9	0.88
21-0-0	0.535	27.7	0.365	0.535	70.3	0.14
34-0-0	0.422	20.0	0.490	0.658	61.1	0.24
46-0-0	0.508	22.1	0.453	0.624	50.2	0.45
Fertilizer, Suggested Design Values	0.47 ³ to 0.57	30 ³	0.500 ³ or 0.333	0.500	70.0 ³	----
Manitou Wheat Model Bin	0.450	21.8	0.458	0.629	50.0	12.6
Manitou Wheat Full Scale Bin	0.492	22.2	0.452	0.622	51.1	14.3
Wheat, Suggested Design Values	0.382 ⁴	28.0 ⁴	0.361	0.530	48.0 ⁴	13.0 ⁴

¹Estimate from $K = \frac{1 - \sin\phi}{1 + \sin\phi}$

²Estimate from $K_O = 1 - \sin\phi$

³Walker et al (1965)

⁴Farm Building Standards (National Research Council 1965)

a small kernel variety. A smaller ϕ has the effect of increasing the estimate of K when the equation $K = \frac{1 - \sin \phi}{1 + \sin \phi}$ is used. The same discrepancy was found for fertilizer, where values ranged from 20.0 degrees for 34-0-0 to 27.7 degrees for 21-0-0. Walker et al (1965) suggested an angle of repose of 30 degrees. If the value $\phi = 30$ degrees is substituted into the formula $K = \frac{1 - \sin \phi}{1 + \sin \phi}$, K would become 0.333, which is less than the estimated values for K based on experimental data. A value of $K = 0.5$ was suggested (Walker et al 1965) as a safe workable value for fertilizer. This is greater than any of the experimental estimates. Discussion of the effect of the K value will be continued after consideration of experimental results in the test bins.

Densities were found to exist within the expected ranges based on recommendations of companies. The generally accepted practice of using cubic foot samples for determining density was not used due to the size and number of samples required. The results obtained confirm the reliability of the test method employed.

6.3 Density Relative to Height of Fall

Figures 18 and 19 show the relationship of height of fall to average density for ammonium phosphate fertilizer and wheat. An analysis of variance test showed that the mean densities were not equal for either material. Mean densities resulting from different heights of fall were found to be different when subjected to a Student-Newman-Keul multiple range test. From observation, it would appear that the density

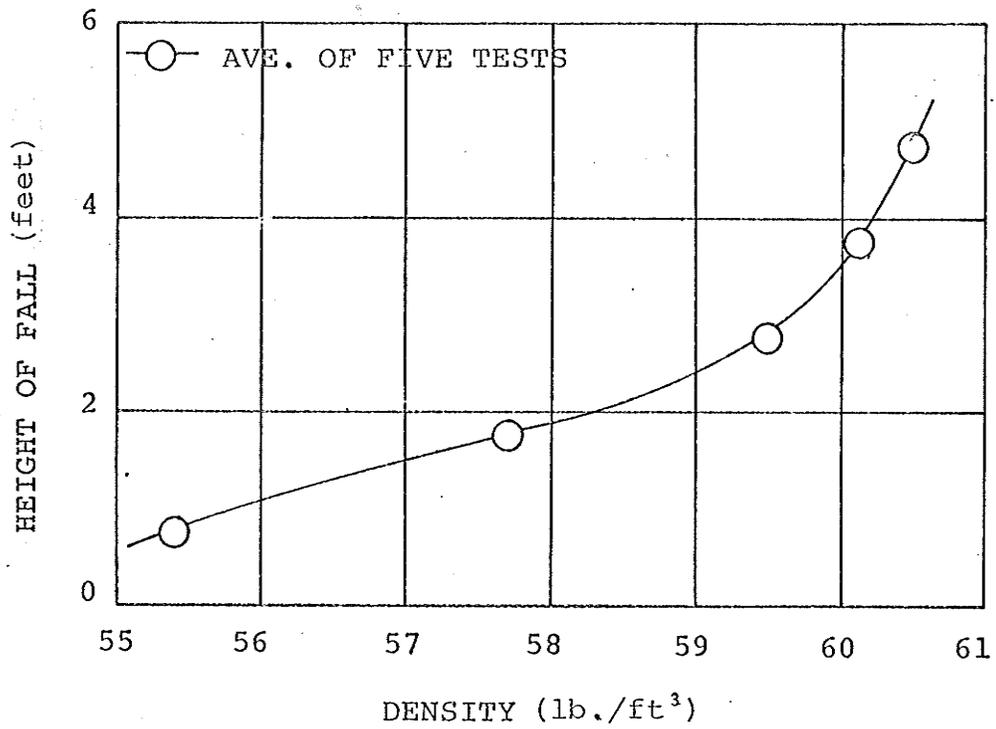


Figure 18. Density versus height of fall curve, ammonium phosphate fertilizer.

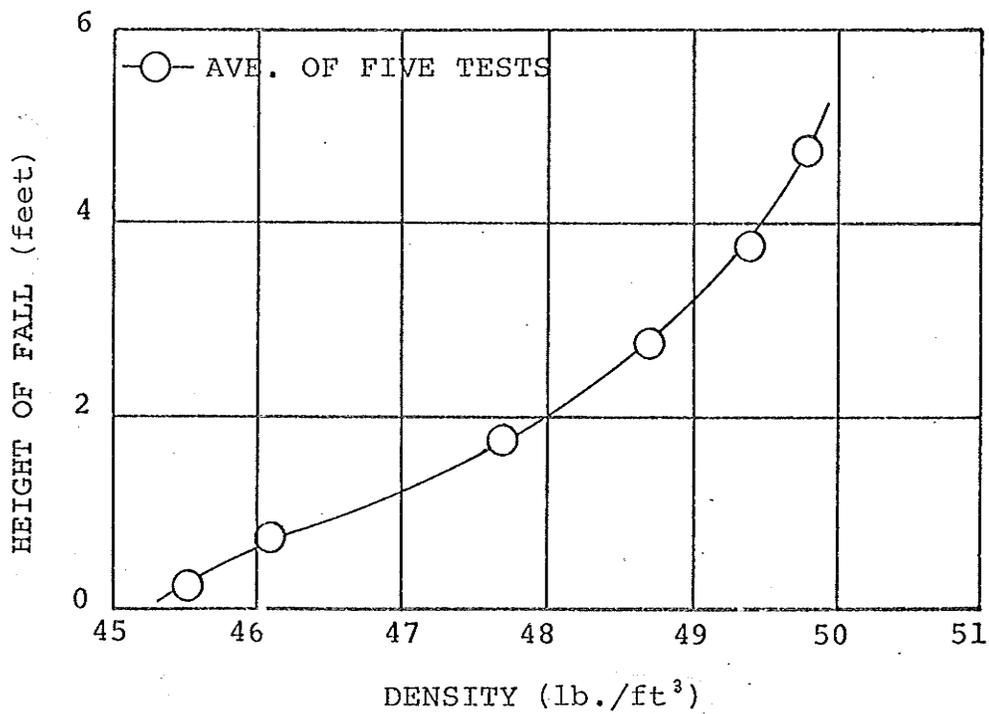


Figure 19. Density versus height of fall curve, Manitou wheat.

would become constant as the height of fall increased beyond some minimum height.

6.4 Lateral Pressures

6.4.1 Full scale bin

Fig. 20 is a plot of lateral pressures versus depth of ammonium phosphate (11-48-0) fertilizer. Curves are shown for theoretical pressures, predicted by Janssen's equation, and experimental results for flexible and rigid wall conditions.

Statistical analysis of the data for the two wall conditions at $\alpha = 0.05$ showed that with the exception of transducer number six, located 52 inches below the material surface, mean lateral pressure did not vary due to different wall flexibilities. This is contrary to previous findings. In spite of the statistical analysis, there was a consistently higher pressure recorded at every level when the wall was rigid.

The shape of the deflection curve (fig. 22) is typical of a wall supported laterally at the base. Below the 64 inch level, deflections decreased due to the lateral restraint introduced by the bin construction. Since an increase in deflection causes a decrease in lateral pressure, it was expected that pressures against the flexible wall would be less than those against the rigid wall at points where flexible wall deflections were greatest. Consideration of figures 20 and 22 shows this to be the case. However, in view of the statistical analysis, the results cannot be considered to be

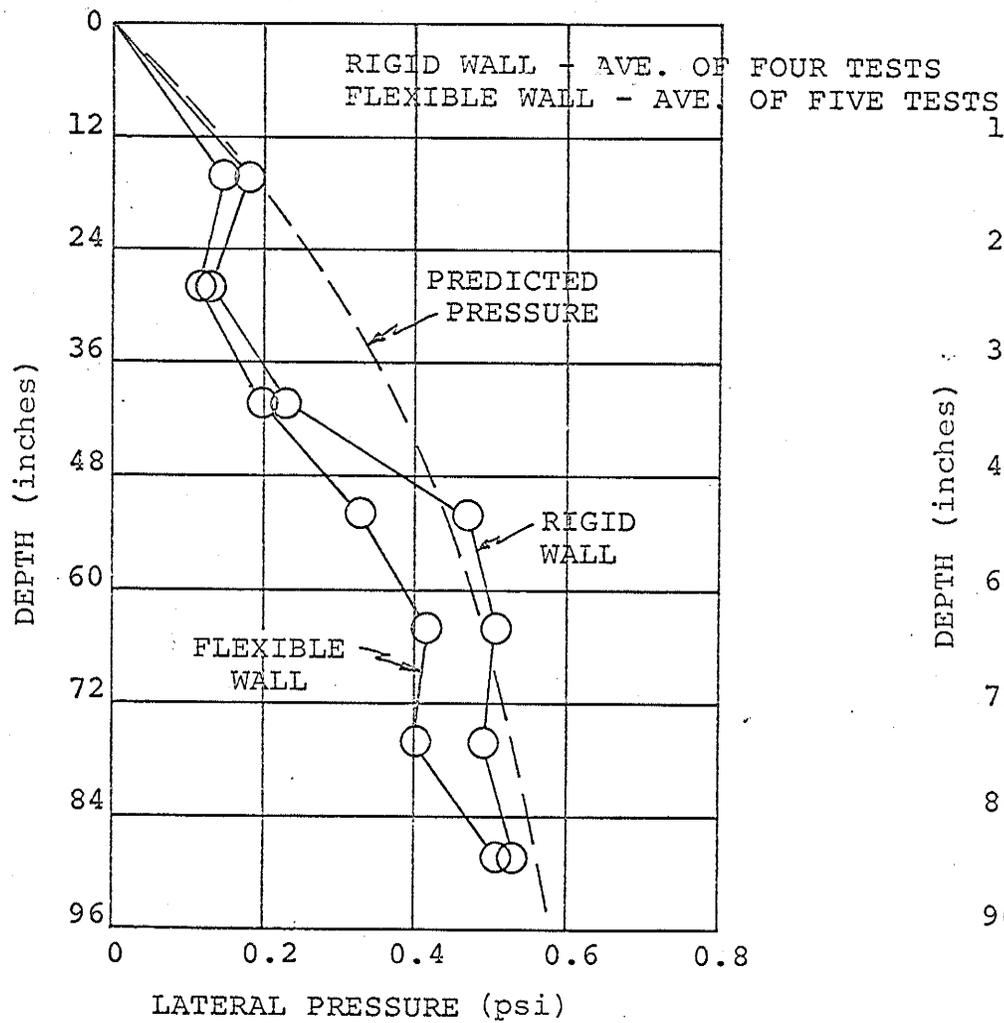


Figure 20. Lateral pressure versus depth curves, full scale bin, ammonium phosphate fertilizer.

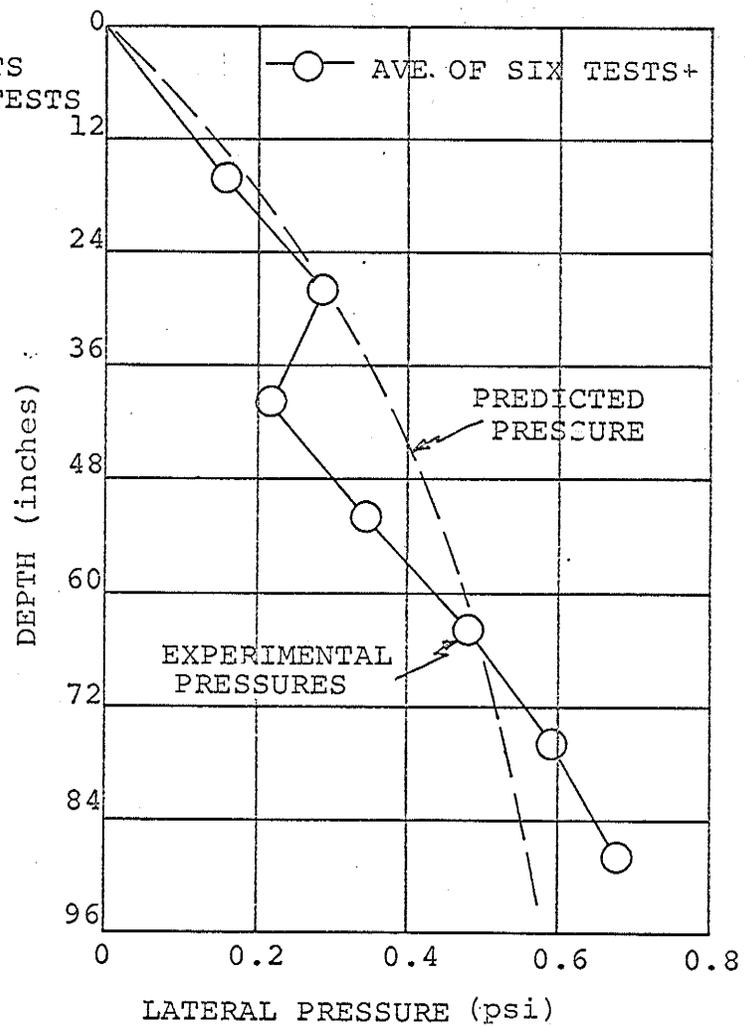


Figure 21. Lateral pressure versus depth curves, full scale bin, Manitou wheat.

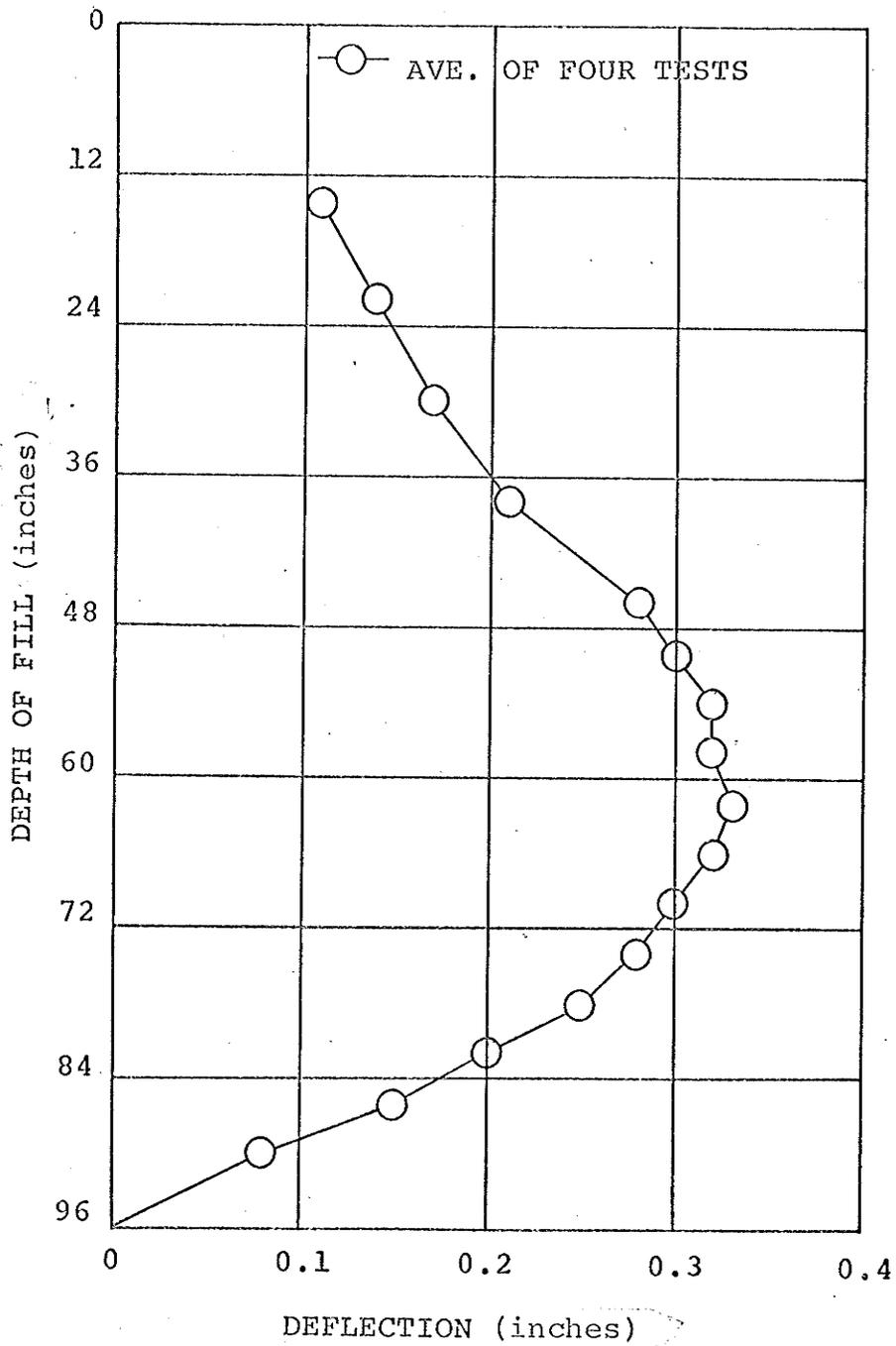


Figure 22. Lateral wall deflection versus depth curve, full scale bin, ammonium phosphate fertilizer.

conclusive.

Average lateral pressures for all tests were computed to theoretical pressures predicted by Janssen's equation. Statistical comparisons showed that experimental values at 28 and 40 inches were less than the theoretical values at $\alpha = 0.05$. All other test values were not significantly different from the theoretical at $\alpha = 0.05$.

Fig. 21 is a plot of lateral pressure versus depth for wheat. Experimental values and theoretical values are shown. A statistical comparison at $\alpha = 0.05$ of the curves showed that experimental values at 40 and 52 inch depths were less than those predicted using Janssen's equation. Test results at 76 and 88 inch depths were greater than theoretical values.

An explanation of the variation in the experimental pressure curves could be in consideration of the wall as a plate held rigid on three sides and free on the fourth side. The wall is subjected to compression, along its long axis, due to the vertical load and is subjected to bending, through its thickness, due to the lateral load. Since the wall is thin relative to its other dimensions, it is possible that an unstable condition could develop. If instability did occur, it would be accompanied by some degree of buckling, causing the wall to move laterally and altering the stress conditions in the fill material.

The deflection measurements would not detect instability in the test wall since they were taken on an adjacent wall. As well, the test wall was instrumented by placing

transducers in seven 4.75 inch diameter cutouts. The cutouts would alter the structural characteristics of the test wall.

Repeatability under each test condition was good, with standard deviations ranging from 0.02 to 0.08. The comparison of experimental values obtained for ammonium phosphate and wheat indicated that the pressure measurement system was consistent. Measured pressures and predictions based on Janssen's equation showed a reasonable correlation.

6.4.2 Model Bin

During the fertilizer tests, the test bin twisted into a rhombic shape, once during run number 2 and again during run number 3. Statistical analysis at $\alpha = 0.05$. indicated that the mean floor loads for these two runs varied from the means of the other runs. Data from these two runs were neglected.

The final lift weight in the wheat tests was found to differ from the weights of the other seven lifts. As a result, only 42 inches of depth were considered.

Table II shows a summary of the vertical wall loads in the model bin, for both ammonium phosphate fertilizer and wheat. The loads shown are the combined effects of gravitation and arching. The maximum degree of arching could not be determined with the equipment employed. The values are not necessarily maximum or minimum and probably represent some state between the two extremes because of the variation of loads with time.

TABLE II

SUMMARY OF VERTICAL LOADS IN MODEL BIN

Depth of Fill (in)	Total Material Weight (lbs)	Floor Load (lbs)	Incremental Increase In Wall Load (lbs)	Cumulative Wall Load (lbs)
Ammonium Phosphate (average of four tests)				
6	49.0	40.8	8.2	8.2
12	98.1	70.1	19.8	28.0
18	147.2	89.5	29.7	57.7
24	196.5	107.5	31.3	89.0
30	246.2	122.5	34.7	123.7
36	296.0	135.5	36.8	160.3
42	344.8	148.8	35.5	196.0
48	393.4	163.0	34.4	230.4
Manitou Wheat (average of six tests)				
6	43.4	35.3	8.1	8.1
12	86.8	62.0	16.7	24.8
18	130.3	80.5	25.0	49.8
24	173.5	92.2	31.5	81.3
30	216.7	103.2	32.0	113.5
36	259.9	111.0	35.4	148.9
42	303.0	118.2	35.9	184.8

Both ammonium phosphate and wheat show a rapid increase in vertical load at shallow depths. The increase in load tends to become small and almost constant at depths greater than 24 inches. Since it is known that lateral load is some function of the vertical load, it can be assumed that the lateral load will show the same tendencies as the vertical load. In this case, the pattern of increasing vertical load corresponds to that which would be expected of a material which followed the Janssen distribution. Actual lateral pressures could not be calculated from the data available.

Figures 23 and 24 show average deflection versus depth curves for ammonium phosphate and wheat. If pressure and deflection are considered to be inter-related, the similarity of the curves suggests that the pressures due to each of the two materials must follow a similar distribution pattern.

6.5 Long Term Pressure Tests

Table III contains the results of long term tests conducted in the model bin. Floor loads decreased with time, resulting in an increase in vertical wall load. Maximum increases in the vertical wall loads were 11.5 per cent for wheat and 14 per cent for fertilizer.

Floor loads were observed to decrease, then increase and decrease again. The time required to reach a minimum value ranged from one hour to 52 hours. The 52 hour period was over a weekend when technicians were not working in the laboratory and disturbance was at a minimum.

After test runs were complete, it was observed that

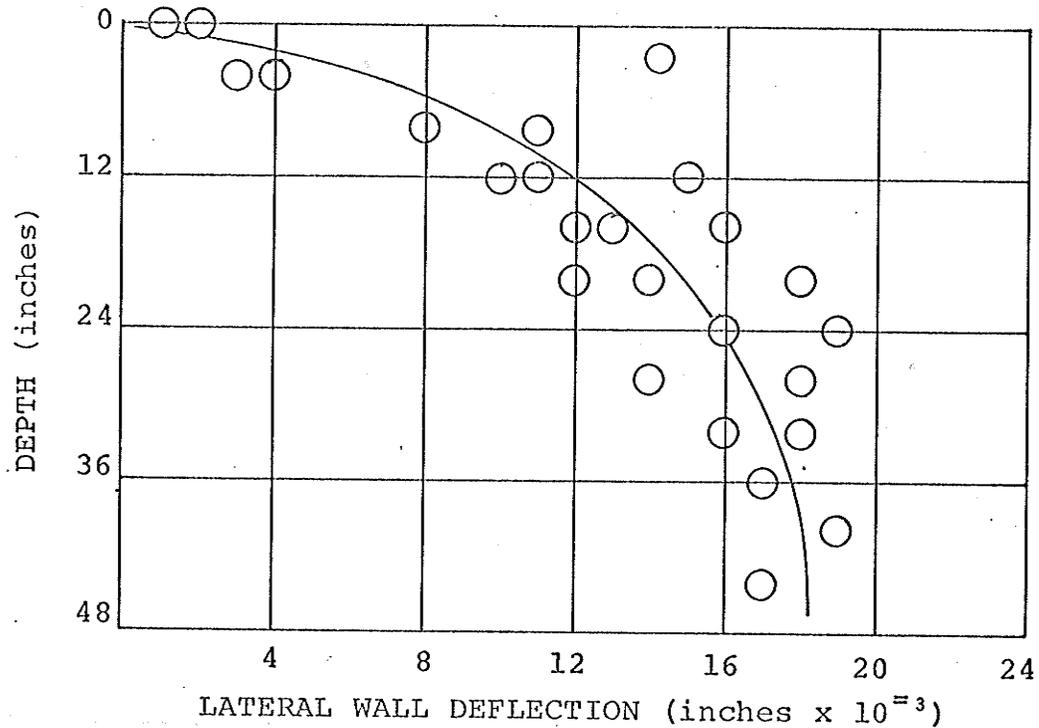


Figure 23. Lateral wall deflection versus depth curve, model bin, ammonium phosphate fertilizer.

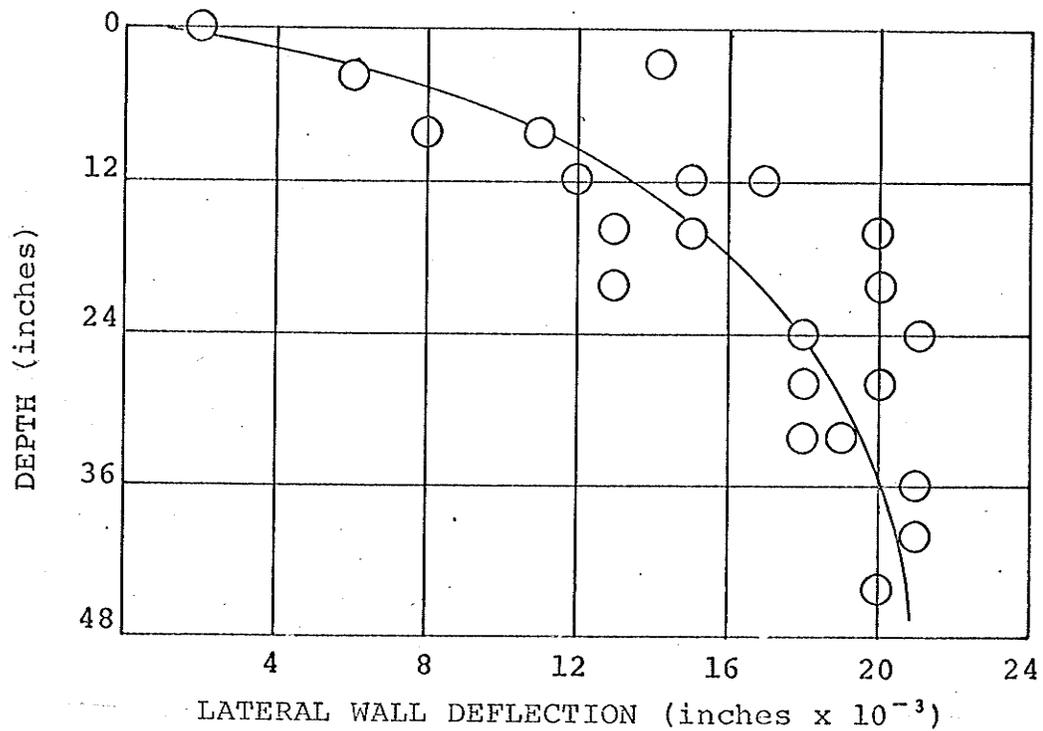


Figure 24. Lateral wall deflection versus depth curve, model bin, Manitou wheat.

TABLE III
SUMMARY OF LONG TERM LOAD RESULTS - MODEL BIN

Test No.	Total Fill Weight (lbs)	Floor Load (lbs)		Vertical Wall Load (lbs)		Time Between Extremes (hrs)
		Max.	Min.	Min.	Max.	
11-48-0 Fill						
13	391.4	157.0	130.0	234.4	260.4	12
16	388.0	154.0	120.0	234.0	268.0	12
17	387.5	130.0	100.0	257.5	287.5	52
Wheat Fill						
14	341.0	124.5	99.5	216.5	241.5	12
15	339.4	125.0	114.0	214.4	225.4	1

floor loads could be increased substantially by hitting the side of the bin once.

The observations outlined led to the conclusion that the test bin was small enough to encourage arching of the granular material over a period of time. Floor load decreased as the arching developed and increased again when the arching collapsed. Variation in rate of weight transfer and effect of vibrations or impact on the wall supports the conclusion that arching did exist in the bin.

6.6 Theoretical Pressures

Fig. 25 shows theoretical lateral pressure versus depth curves calculated using Janssen's equation. The variables used in the equation are equal to those listed in Table I. Bin geometry was similar to the full scale bin. Maximum pressures are exerted by ammonium nitrate (34-0-0). At the sixteen foot depth, the theoretical pressures due to 34-0-0 are almost 40 per cent greater than pressures due to 11-48-0. Ammonium nitrate should be considered the critical design load and used for the design of all deep fertilizer storage bins.

6.7 Floor Loads

6.7.1 Full scale bin

Table IV lists experimental pressures obtained from the floor mounted transducers. Statistical analysis at $\alpha = 0.05$ indicated that pressures varied across the floor, being higher in the center than near the wall. There was not

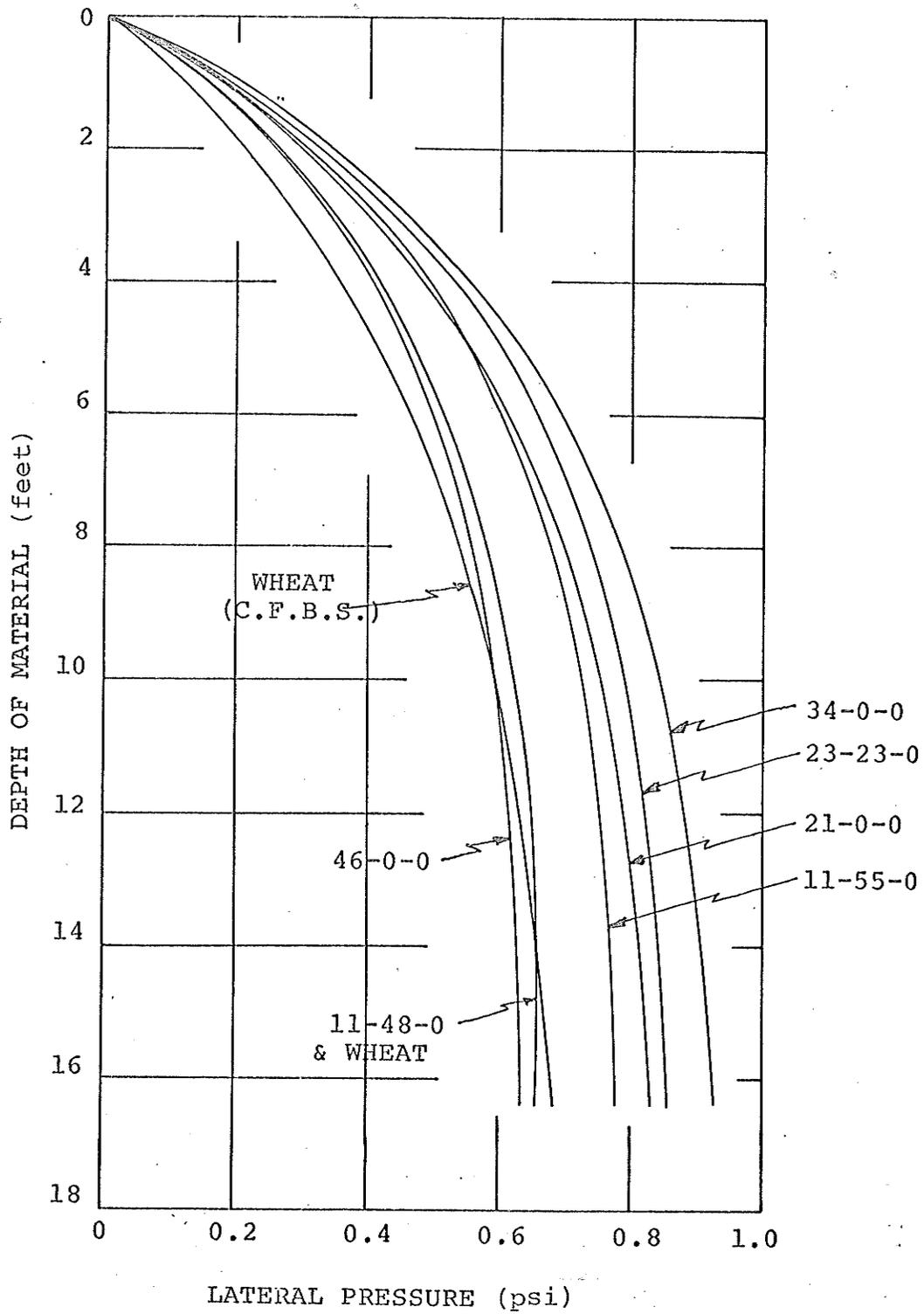


Figure 25. Theoretical lateral pressure versus depth curves, full scale bin.

TABLE IV
SUMMARY OF VERTICAL LOADS, FULL SCALE BIN

	Pressure in p.s.i.			Lateral Pressure at Floor	K, Using Max. V	K, Using Min. V	K, Using Ave. V
	Transducer Number 1	Transducer Number 2	Average of 1&2				
11-48-0, Flexible Wall	1.52	1.89	1.71	0.51	0.270	0.336	0.298
11-48-0 Fill, Rigid Wall	1.17	1.82	1.50	0.53	0.291	0.452	0.353
11-48-0 Weighted Average at Transducer	-----	1.86	-----	0.52	0.280	-----	-----
Wheat Fill	1.46	1.81	1.64	0.68	0.320	0.466	0.415

enough data to determine the rate of change of pressure with distance, therefore the two readings were averaged to obtain an overall floor pressure value.

Mean vertical pressures were found to vary at transducer 1, (six inches from the wall) depending on wall flexibility. Pressure was less when the wall was rigid than when it was flexible. This indicated a variation in distribution of vertical pressure due to arching. The degree of arching would appear to be dependent on wall flexibility. Statistical analysis at $\alpha = 0.05$ indicated that wall flexibility had no significant effect on vertical pressures at transducer number 2 located at the center of the floor.

6.8 The Ratio of Lateral to Vertical Pressure

6.8.1 Full scale bin

Table IV lists a series of K values which were determined from the full scale bin data. Three values are given for each case, considering the maximum measured value, the minimum measured value and the average value of vertical pressure. Since K was based originally on Janssen's assumption that the vertical pressure was constant across any horizontal section in a bin, the K determined from the average vertical pressure probably has the most meaning.

Using the formula $K = \frac{1 - \sin \phi}{1 + \sin \phi}$ as recommended by the National Building Code (1965), K was estimated to be 0.394 for ammonium phosphate and 0.452 for wheat utilized in the full scale bin. K_{ave} was determined to be 0.298 and 0.353 for flexible and rigid walls with fertilizer and 0.415

for wheat. All experimental values are less than the estimated values.

6.8.2 Effect of K on the predicted pressures

K was defined by Janssen as the ratio of lateral to vertical pressure and was assumed to be a constant. Since that time, Ketchum (1919), Jaky (1948), Caughey et al (1951) and others have shown K to be a variable. Despite variability, an average value, when substituted into Janssen's equation, yields a lateral pressure value which is close to measured values.

One of the major uses of K is in the prediction of vertical loads. Farm Building Standards (National Research Council 1965) recommended that vertical loads be estimated from the formula $L = KV$. As a result, a high K value will predict a relatively low vertical load, as indicated by Lenczner (1963). K values determined in this study have, in general, been less than recommended values (Walker et al 1965) for fertilizer. Their use will result in more accurate predictions of actual load conditions.

CHAPTER VII

CONCLUSIONS

1. Fertilizer design constants used in the Janssen equation were density = 62 lb/ft^3 , $K = 0.49$ and μ' (perpendicular to plywood grain) = 0.42 .

2. For the test conditions used, pressures due to ammonium phosphate fertilizer were in agreement with those predicted by the use of the Janssen equation.

3. Wall flexibility appeared to affect lateral pressures but statistical analysis at $\alpha = 0.05$ showed no significant difference between the flexible and rigid wall conditions.

4. Arching of fertilizer occurs to a marked degree in model bins.

5. Vertical pressures were not constant across the bin cross section.

CHAPTER VIII

RECOMMENDATIONS FOR FURTHER STUDY

1. The effect of wall flexibility on deep bin pressures should be studied in more detail.
2. Further investigation of the relationship of material density and height of fall should be undertaken.
3. Further work is required to confirm and expand the data on fertilizer properties as they apply to pressure in bins.
4. The meaning and definition of the K value should be re-examined in light of present knowledge.
5. Plate action of thin bin walls should be investigated to establish design criterion.
6. High capacity, direct ventilation systems should be installed in any laboratory undertaking fertilizer bin tests.
7. Future bin research should approach the problem on a more basic level. This study, as well as many others, had as one of its purposes, the justification of the use of the Janssen equation. In light of the fact that two of Janssen's three basic assumptions are known to be wrong, it would seem reasonable to assume that a more exact solution to the stress conditions in deep bins can be derived.
8. The theories of similitude should be applied more vigorously to future model bin studies.

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APPENDIX A

TRANSDUCER DESIGN CALCULATIONS

TRANSDUCER DESIGN CALCULATIONS

Known:

1. outside diameter = 4.50 in.
2. inside diameter = 4.20 in.

Assumed:

1. allowable bending stress for aluminum = 9000 psi
2. Modulus of Elasticity for aluminum = 10×10^6 psi
3. Poisson's ratio for aluminum = 0.33
4. maximum pressure = 5 psi

Reference:

Beckwith and Buck (1961)

Consider maximum radial stress at $a = \text{maximum}$

$$\sigma_{r_{\max}} = \frac{3 p a^2}{4 t^2} \quad (\text{A1.1})$$

let $\sigma_{r_{\max}} = 9000$ psi and solve for t

$$t = \sqrt{\frac{3 \times 5 \times 2^2}{4 \times 9000}} = 0.0408 \text{ in.}$$

\therefore assume $t = 0.042$ in.

Check maximum tangential stress at diaphragm center.

$$\sigma_{t_{\max}} = \frac{3}{8} \frac{a^2 p (1 + \mu^*)}{t^2} \quad (\text{A1.2})$$

$$= \frac{3 \times 2^2 \times 5 (1 + 0.33)}{8 \times 0.042^2} = 5655 \text{ psi}$$

\therefore t o.k.

Check deflection

$$\begin{aligned}\Delta_{\max} &= -\frac{3}{16} \frac{p a^4 (1 - \mu^2)}{E t^3} & (A1.3) \\ &= -\frac{3 \times 5 \times 2^4 (1 - 0.33^2)}{16 \times 10 \times 10^6 \times 0.042^3} \\ &= 0.0018 \text{ in.}\end{aligned}$$

$$30 \text{ per cent of } t = 0.042 \times 0.30$$

$$= 0.0126 \text{ in.}$$

since $\Delta_{\max} < 30 \text{ per cent of } t$, the transducer should maintain a linear output.

Check shear at $a = 2 \text{ in.}$

$$A_V = 2\pi at = 2\pi \times 2 \times 0.042 = 0.132 \text{ in}^2 \quad (A1.4)$$

$$\text{Load} = \pi a^2 p = \pi \times 2^2 \times 5 = 62.9 \text{ pounds} \quad (A1.5)$$

$$\tau = \frac{62.9}{0.132} = 475.8 \text{ psi}$$

check maximum strain at center of diaphragm

$$\begin{aligned}\epsilon_{\max} &= \frac{\sigma_{x\max}}{E} - \frac{\mu^* \sigma_{y\max}}{E} \\ \sigma_{x\max} &= \sigma_{y\max} = \sigma_{t\max} & (A1.6) \\ &= \frac{5655}{10 \times 10^6} - \frac{0.33 \times 5655}{10 \times 10^6} \\ &= 378.9 \text{ microinches/inch}\end{aligned}$$

This is well within the range of an EA-13-500 BH-120 foil strain gauge.

Theoretical sensitivity of transducer

$$\text{maximum pressure} = 5 \text{ psi}$$

$$\text{maximum strain} = 378.9 \text{ microinches/inch}$$

gauge length = 0.50 in.

$$\begin{aligned}\text{sensitivity} &= \frac{5 \text{ psi}}{3.78.9 \text{ m. in/in} \times 0.50 \text{ in.}} \\ &= 0.026 \text{ psi/microinch}\end{aligned}$$

for a single active gauge.

Using two gauges, one in compression and one in tension, the sensitivity will increase by about two times to approximately 0.013 psi/microinch.

APPENDIX B

REGRESSION EQUATIONS
AND
CORRELATION COEFFICIENTS
FOR
PRESSURE TRANSDUCERS

Transducer Number 1

$$Y = - 0.14 + 0.04X$$

$$r = 1.00$$

Transducer Number 5

$$Y = - 0.12 + 0.06X$$

$$r = 1.00$$

Transducer Number 2

$$Y = - 0.05 + 0.05X$$

$$r = 1.00$$

Transducer Number 6

$$Y = - 0.14 + 0.04X$$

$$r = 1.00$$

Transducer Number 3

$$Y = 0.10 + 0.05X$$

$$r = 1.00$$

Transducer Number 7

$$Y = - 0.21 + 0.04X$$

$$r = 1.00$$

Transducer Number 4

$$Y = - 0.14 + 0.05X$$

$$r = 1.00$$

Transducer Number 8

$$Y = - 0.32 + 0.05X$$

$$r = 1.00$$

Transducer Number 9

$$Y = - 0.17 + 0.04X$$

$$r = 1.00$$

APPENDIX C

SUMMARY
SIEVE ANALYSIS
OF
AMMONIUM PHOSPHATE
FERTILIZER

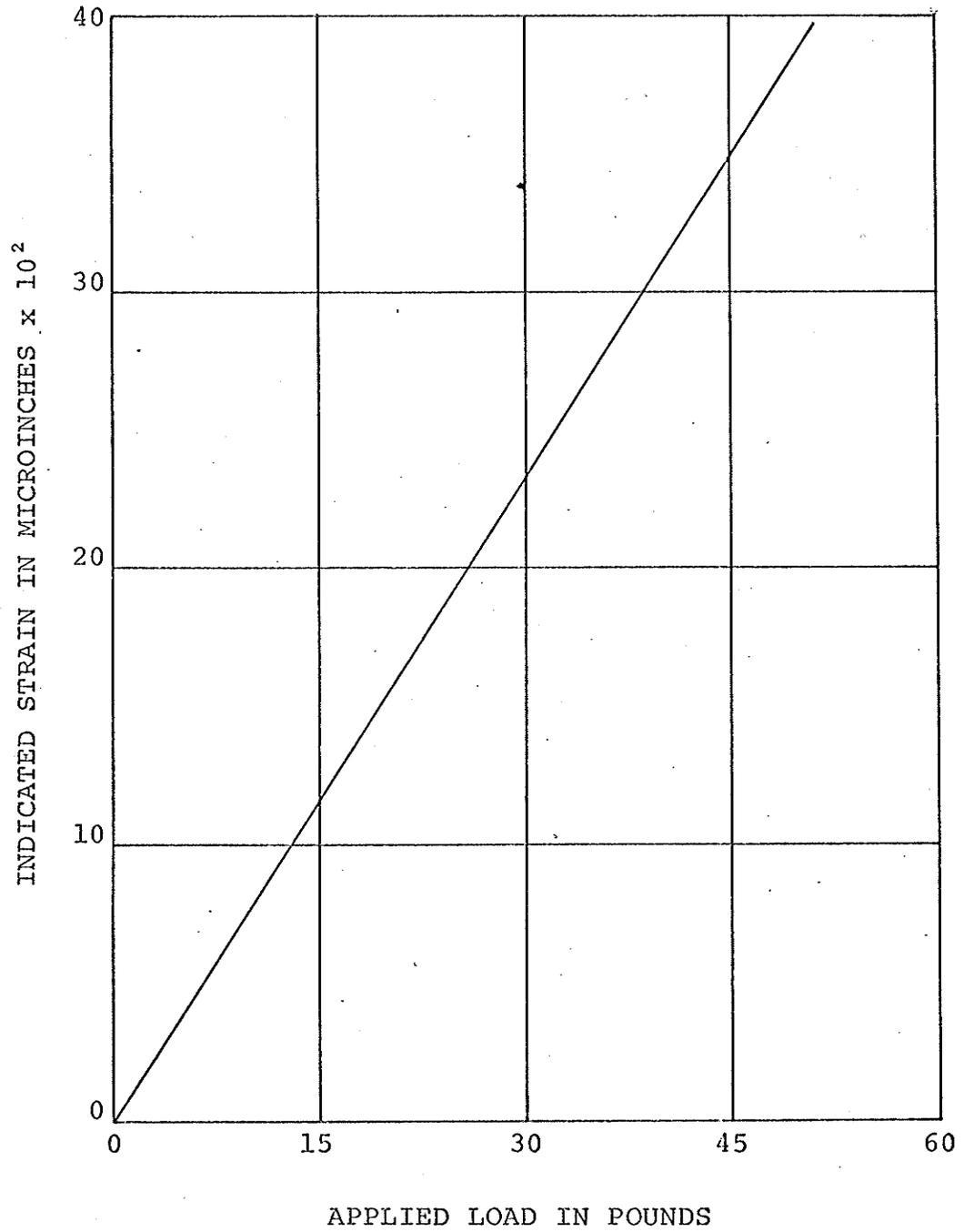
APPENDIX C

SUMMARY OF SIEVE ANALYSIS

Sieve Number		Pre Run 1	Pre Run 4	Pre Run 6	Pre Run 7	Pre Run 10
	Total Wt (gms)	3998.2	4004.2	3998.1	3996.9	3998.5
10	Weight retained gms	1395.1	1583.3	840.5	1556.5	1621.8
	% retained	34.89	39.54	21.02	38.94	40.56
20	Weight retained gms	2478.8	2327.9	2920.5	2333.8	2338.8
	% retained	62.00	58.14	73.05	58.39	58.49
40	Weight retained gms	65.3	66.9	138.2	78.8	27.2
	% retained	1.63	1.67	3.46	1.97	0.68
100	Weight retained gms	41.5	19.8	73	0.5	5.2
	% retained	1.04	0.49	1.82	0.51	0.13
200	Weight retained gms	13.0	4.5	20.2	5.0	3.7
	% retained	0.33	0.11	0.50	0.12	0.09
270	Weight retained gms	3.3	0.5	4.6	0.9	1.2
	% retained	0.08	0.01	0.12	0.02	0.03
Pan	Weight retained gms	1.2	0.5	1.1	0.0	0.6
	% retained	0.03	0.01	0.03	0.00	0.02

APPENDIX D

CALIBRATION CURVE
FOR
WEIGHT TRANSDUCER



Appendix D. Calibration curve for weight transducer.