CERTAIN MODELS FOR FACILITY LOCATION AND PRODUCTION PLANNING UNDER FUZZY ENVIRONMENT

BY

NADEEN AL-BADER

A Thesis
Submitted to the Faculty of Graduate Studies
In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE

Department of Mechanical and Industrial Engineering
University of Manitoba
Winnipeg, Manitoba

THE UNIVERSITY OF MANITOBA

FACULTY OF GRADUATE STUDIES

*****

COPYRIGHT PERMISSION

CERTAIN MODELS FOR FACILITY LOCATION AND PRODUCTION PLANNING UNDER FUZZY ENVIRONMENT

BY

NADEEN AL-BADER

A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University of Manitoba in partial fulfillment of the requirement of the degree

Of

MASTER OF SCIENCE

DEDICATED TO MY FAMILY,

HUSBAND WALEED

AND

DAUGHTERS REEMA & LEENA

ABSTRACT

For a production engineer, there are three main issues that should be taken in consideration. First is to select the best and the right place to locate the new or existing facility; next, to select the type of production planning systems, either single stage or multistage structure; finally, to cope with the changes in the production planning, e.g. increase and decrease in production rate limits, change in work force level (hiring and laying off workers) and so on. This thesis deals with these three issues under a fuzzy environment, since fuzzy set theory has proved to be very efficient to handle certain types of uncertainties encountered in a variety of areas. Fuzzy algebra provides an excellent mathematical framework for explicitly incorporating imprecision and vagueness into the decision making models, especially when the system involves human subjectivity.

In the present thesis, we provide an introduction to the concepts of decision making and presents introduction, prerequisites and the needs for a methodology for analyzing certain problems under fuzzy environment, and deal with the literature review of the related work done by other researchers relevant to this research. Furthermore, we deal with rating models under fuzzy environment with an application to a facility location selection problem, formulate and analyze the multistage production planning problem using linear programming approach under both crisp and fuzzy environment, and present a production planning model with production rate change, under crisp and fuzzy environment using linear programming approach. Finally, the conclusion and the contributions made in the thesis, along with some recommendations for further research, are provided.

ACKNOWLEDGEMENTS

I would like to express my appreciation to my advisor, Professor C. R. Bector, for his encouragement and academic support throughout the course of my thesis. I also wish to thank other members of my examining committee; Professor O. Hawaleshka and Professor G. Wang, who provided extremely helpful comments, thoughtful suggestions, and a careful review of the thesis.

I would like to thank my husband Waleed for his enormous support and patience during the course of my work.

A special thanks to my lovely daughters Reema and Leena, their innocent love and enduring patience made it all possible.

LIST OF FIGURES

Figure		Page
1.1	Graphic representation of a Triangular Fuzzy Number	9
1.2	Facilities in series (a serial system).	22
1.3	Facilities in parallel, common inventories.	23
1.4	Facilities in parallel, separate inventories between stages.	23
4.1	Two-stage model of the problem.	64

LIST OF TABLES

Table		Page
3.1	Crisp Ratings for Each Location Criterion	43
3.2	Crisp Weights for each Location Criterion	44
3.3	Computation of Crisp Rates for Location 1, 2, 3 and 4	45
3.4	Fuzzy Rating for Location1 (L1)	46
3.5	Fuzzy Weights for Each Location Criterion	46
3.6	Computation of Fuzzy Rates for Location1 (L1)	47
3.7	Error analysis for Location1 (L1)	49
3.8	Fuzzy Rating for Location2 (L2)	49
3.9	Computation of Fuzzy Rates for Location2 (L2)	50
3.10	Error analysis for Total Fuzzy Rates Location2 (L2)	52
3.11	Fuzzy Rating for Location3 (L3)	52
3.12	Computation of Fuzzy Rates for Location3 (L3)	53
3.13	Error analysis for Location3 (L3)	55
3.14	Fuzzy Rating for Location4 (L4)	55
3.15	Computation of Fuzzy Rates for Location4 (L4)	56
3.16	Error analysis for Location4 (L4)	58
3.17	Ranking Method 1: Rating for Decision Criteria and Decision Alternative	59

3.18 Ranking Method 2: Rating for Decision Criteria and Decision Alternative 59

3.19 Ranking Method 3: Rating for Decision Criteria and Decision Alternative 60

3.20 Ranking Method 4: Rating for Decision Criteria and Decision Alternative 61

4.1 Results of crisp linear programming problem. 67

4.2 Optimal Utilization of production capacity 67

4.3 Results of fuzzy linear programming problem. 78

4.4 Optimal Utilization of production capacity 79

4.5 Value of λ corresponding to capacity tolerance and total cost tolerance 79

4.6 Total cost corresponding to capacity tolerance and total cost tolerance 80

5.1 Production Planning Problem Data 85

5.2 Results of crisp linear programming problem. 88

5.3 Results of fuzzy linear program problem 99

5.4 Values of λ for production increase/decrease limits change tolerance=10% 100

5.5 Values of λ for production increase/decrease limits change tolerance=15% 101

5.6 Values of λ for production increase/decrease limits change tolerance=20% 101

5.7 Values of λ for production increase/decrease limits change tolerance=25% 101

5.8 Values of λ for production increase/decrease limits change tolerance=30% 101

5.9 Total cost corresponding to Demand tolerance and total cost tolerance when the production rate tolerance = 10 % 102

5.10 Total cost corresponding to Demand tolerance and total cost tolerance when the production rate tolerance = 15 % 102

5.11 Total cost corresponding to Demand tolerance and total cost tolerance when the production rate tolerance = 20 % 102

5.12 Total cost corresponding to Demand tolerance and total cost tolerance when the production rate tolerance = 25 % 103

5.13 Total cost corresponding to Demand tolerance and total cost tolerance when the production rate tolerance = 30 % 103

CONTENTS

ABSTRACT		**I**
ACKNOWLEDGEMENTS		**II**
LIST OF FIGURES		**III**
LIST OF TABLES		**IV**

CHAPTER			**Page**
1	INTRODUCTION		1
	1.1	Fuzzy Set Theory	3
		1.1.1 Fuzz Set	3
		1.1.2 α – Cut or α – Level Set	4
		1.1.3 Support of a fuzzy Set	4
		1.1.4 Intersection of Fuzzy Sets	4
	1.2	Algebraic Operation on Fuzzy Sets	5
	1.3	Convexity of Fuzzy Sets	5
	1.4	Fuzzy Arithmetic	6
		1.4.1 Fuzzy Number	6
		1.4.2 Fuzzy Arithmetic Based on Operations on Closed Intervals.	7
		1.4.3 Triangular Fuzzy Number	8
	1.5	Kaufmann and Gupta Error Analysis with T.F.N. Approximation	10
	1.6	Ranking of Fuzzy Numbers.	11
	1.7	Rating Models	13

		1.7.1 Weighted Rating Model under Crisp Environment	13
	1.8	Linear Programming	14
		1.8.1 The Generalized Linear Programming Model	15
	1.9	Fuzzy Linear Programming	16
	1.10	Zimmerman's Approach – Symmetric Model	17
	1.11	Multistage Planning Problems	21
	1.12	Organization of the Thesis	24
2	LITRATURE SURVEY		27
	2.1	Fuzzy Weighted Rating Models	27
	2.2	Multistage Production System	30
	2.3	Summary of the Thesis	30
3	FUZZY RATING MODEL AND FACILITY LOCATION PROBLM		33
	3.1	Introduction	33
	3.2	Fuzzy Rating Model	33
	3.3	The Approximated Fuzzy Rating Model	36
	3.4	Error Analysis of T.F.N	37
	3.5	Kaufmann and Gupta Error Analysis with T.F.N. approximation.	39
	3.6	Left and Right Divergence of the Parabolic Fuzzy Rating Number	40
	3.7	Facility Location Problem: A Crisp Rating Model	42
	3.8	Facility Location Problem: A Fuzzy Rating Model	45
	3.9	Facility Location Selection	58

3.10 Conclusion 61

4 MULTISTAGE PLANNING PROBLEM UNDER CRISP AND FUZZY ENVIRONMENTS 62

4.1 Introduction 62

4.2 Linear Programming Formulation under Crisp Environment 63

4.2.1 Assumptions 63

4.2.2 Notation 63

4.2.3 Objective 63

4.2.4 General Formulation 63

4.2.5 Numerical Example under Crisp Environment 64

4.2.6 Results 67

4.2.7 Interpretation of the results 67

4.3 Formulation under Fuzzy Environment 68

4.3.1 Additional Assumptions 69

4.3.2 Objective 69

4.3.3 Additional Notation 69

4.3.4 Formulation of Multistage Planning Problem under Fuzzy Environment 70

4.3.4.1 Membership functions 71

4.3.5 Numerical Example under Fuzzy Environments 74

4.3.6 Results 78

4.3.7 Interpretation of the results 79

4.3.8 Discussion of the solution in view of Table 4.5 & 4.6 80

5 A LINEAR PROGRAMMING APPROCH TO PRODUCTION PLANNING MODEL UNDER CRISP AND FUZZY ENVIRONMENT 82

5.1 Introduction 82

5.2 Linear Programming Formulation under Crisp Environment 83

5.2.1 Assumptions 83

5.2.2 Notation 83

5.2.3 Objective 84

5.2.4 General Formulation 84

5.2.5 Numerical Example under Crisp Environment 85

5.2.6 Results 88

5.2.7 Interpretation of the results 89

5.3 Formulations under Fuzzy Environment 89

5.3.1 Additional Assumptions 90

5.3.2 Objective 90

5.3.3 Additional Notation 90

5.3.4 Formulation of Production Problem under Fuzzy Environments 91

5.3.5 Numerical Example under Fuzzy Environments 95

5.3.6 Results 99

5.3.7 Interpretation of the results 100

5.3.8 Discussion of the solution in view of Table 5.4-Table 5.13 103

6 CONCLUTION, CONTRIBUTION AND RECOMMENDATIONS 105

6.1 Conclusion and Contribution 105

6.2 Recommendation for Future Research 107

REFERENCES 109

APPENDEX 1 114

APPENDEX 2 123

CHAPTER 1

INTRODUCTION

Uncertainty is one of the main and most important issues that have to be addressed by modern management and engineering systems. It is present in most decision making problems due to unknown future events. Most subjects of modern analysis are characterized by a number of general features that make them particularly difficult for existing methods. These features are: complexity, dynamics, and uncertainty. In certain cases the presence of uncertainty makes the traditional approaches insufficient [37].

When the time to make a sound decision on an uncertain problem comes, it is important for the decision makers to consider and evaluate the uncertainty involved in it and its surroundings. Uncertainty may result from a multiple of sources. For example,

- Imprecise/vague knowledge regarding future conditions,
- Inaccurate data,
- Forecasting errors,
- Subjective influence,
- Existence of external uncontrollable disturbances.

For decision making under uncertainty, one should normally develop an active approach rather than ignore it.

Probability theory is the traditional theory describing and measuring the phenomenon of uncertainty. In most situations it is assumed that the probability theory can be used to deal with uncertainty of every type. However, the monopoly position of probability theory to treat every kind of uncertainty has become highly questionable. As a consequence, the scientific literature of the past four decades has offered some new models and techniques to describe and transfer vagueness, imprecision and uncertainty in a useful manner. Undoubtedly, one of those is the theory of fuzzy sets that was introduced in 1965 by the papers of Lotfi A. Zadeh [44, 45].

Fuzzy set theory is a relatively new mathematical tool to deal with vagueness, imprecision uncertainty [22, 23, 47] and is highly suitable for application areas where an expert's subjective judgment is involved. Used to represent uncertain reasoning, it mimics the human ability to take rational decisions in an ambiguous and inexact environment.

We now give a brief introduction to fuzzy set theory and fuzzy linear programming. We shall use such linear programming models in the following chapters. An important underlying assumption in most of the models (linear programming and others) is that "demand is deterministically known". However, demand is always forecasted, and most of the times the forecasts do not turn out to be precisely correct. Furthermore, in practice most of the companies are limited by budget restrictions. Setting targets or goals on cost figures is a very common practice in the industrial and business world, and in some situations these restrictions have some elasticity/ambiguity. This suggests the possibility of applying fuzzy logic to some industrial problems.

In some cases, the decision-maker might not really want to actually maximize or minimize the objective function, but rather may want to reach some "aspiration level" which might not even be crisply defined. In real world problems, this can happen because sometimes it is simply not possible to obtain precise data, or the cost of obtaining precise data may be too high. This imprecision in data arises because of the complex nature of real world problems. So imprecise data will be used in modeling the problem . We will analyze our problems by means of fuzzy logic when some sort of ambiguity in available budget and demand is involved. Fuzzy set theory is a tool that gives reasonable analysis of complex systems without making the process of analysis too complex.

In the following sections, a brief introduction to fuzzy set theory and linear programming will be given.

1.1 Fuzzy Set Theory

Theory of fuzzy sets is basically a theory of graded concepts (Zimmerman, [46]). A central concept of fuzzy set theory is that it is permissible for an element to belong partly to a fuzzy set.

1.1.1 Fuzzy Set

Let X be a classical set of objects, called the universe, whose generic elements are denoted by x. The membership in a crisp subset of X is viewed as characteristic function $\mu_A(x)$ from X to $\{0, 1\}$ such that:

$$\mu_A(x) = \begin{cases} 0 & \text{for } x \notin A \\ 1 & \text{for } x \in A \end{cases}$$

where $\{0,1\}$ is called a valuation set [7, 22, 23, 29, 45, 47].

if the valuation set is allowed to be the closed real interval [0, 1], A is called a fuzzy set as proposed by Zadah [44], with $\mu_A(x)$ as the degree of membership (degree of belonging) of x in A. The closer the value of $\mu_A(x)$ is to 1, the more x belongs to A. Therefore, A is completely characterized by the set of ordered pairs:

$$A = \{(x, \mu_A(x)/ x \in X\}$$

where $\mu_A(x)$ maps X to the membership space [0,1]. Elements with zero degree of membership are usually not listed.

1.1.2 α – Cut or α – Level Set

One of the most important concepts of fuzzy sets is the concept of an α-cut or α-level set. An α-cut denoted by A_α is the crisp set of elements x in R whose degree of belongings to the fuzzy set A is at least $\alpha \in [0, 1]$. This means

$$A_\alpha = \{ x \in R \mid \mu_A(x) \geq \alpha, \alpha \in [0, 1]\}.$$

That is, the α-cut or α-level set of fuzzy set is the crisp set A_α that contains all elements of the universal set $X \in R$ whose membership grades in A are greater than or equal to the specified value of α, $\alpha \in [0, 1]$.

1.1.3 Support of a fuzzy Set

The support of a fuzzy set A is a set S(A) such that $x \in S(A) \Leftrightarrow \mu_A(x) > 0$. If $\mu_A(x)$ is constant over S(A), then A is non-fuzzy.

1.1.4 Intersection of Fuzzy Sets

Intersection of two fuzzy sets A and B is a fuzzy set C denoted by $C = A \cap B$, whose membership function is related to those of A and B by

$$\mu_C(x) = \min [\mu_A(x), \mu_B(x)], \ \forall x \in X$$

1.2 Algebraic Operations on Fuzzy Sets

In addition to the set theoretic operations, we can also define a number of combinations fuzzy sets and relate them to one another. Here we present some more important operations among those:

1. Algebraic product of two fuzzy sets A and B, is A (.) B whose membership function is

$$\mu_{A(.)B}(x) = \mu_A(x)\ (.)\ \mu_B(x), \quad \forall\ x \in X$$

2. Algebraic Sum of A and B is A + B whose membership function is defined as

$$\mu_{(A+B)}(x) = \mu_A(x)\ (+)\ \mu_B(x), \quad \forall\ x \in X$$

provided $0 \le \mu_A(x)\ (+)\ \mu_B(x) \le 1$

1.3 Convexity of Fuzzy Sets

The notation of convexity can be extended to fuzzy sets in such a way as to preserve many of the properties that it has in case of crisp sets. In what follows, we assume that X is the n-dimensional space R^n. We now have the following two equivalent definitions of convexity of a fuzzy set.

Convex Fuzzy Set

A fuzzy set A is convex if and only if the sets $A_\alpha = \{x \in X \mid \mu_A(x) \ge \alpha\}$ for all $\alpha \in [0, 1]$ is a convex set.

The second definition of convexity of a fuzzy set is as follows:

A fuzzy set A is said to be a convex set if

$$\mu(\lambda x_1 + (1-\lambda)x_2) \ge \text{Min}(\mu(x_1), \mu(x_2)), \quad x_1, x_2 \in X, \quad \lambda \in [0, 1].$$

Normal Fuzzy Set

A fuzzy set A is normal if and only if

$$\forall x \in A: \qquad \vee_{x} \mu_A(x) = 1.$$

This means that the maximum value of $\mu_A(x)$ on A is equal to 1.

1.4 Fuzzy Arithmetic

The first definition of a fuzzy set allows us to extend various properties of crisp sets and operations on crisp sets to their fuzzy counterparts.

The definition of a fuzzy set leads us to the definition of a fuzzy number given below.

1.4.1 Fuzzy Number

A fuzzy Number A is a fuzzy set on the real line R, which possesses the following properties:

- A is a normal, convex fuzzy set on R,
- The α-level set A_α is a closed interval for every $\alpha \in [0, 1]$, and
- The support of A, $S(A) = \{x \mid \mu_A(x) > 0\}$, is bounded.

Fuzzy arithmetic is based on two properties of fuzzy numbers:

1. Each fuzzy set and thus, each fuzzy number can be fully and uniquely represented by its α-level sets.
2. α-level sets of each fuzzy number are closed intervals of real numbers for all $\alpha \in [0, 1]$.

These properties enable us to define an arithmetic operation on fuzzy numbers in terms of arithmetic operations on their α–level sets.

1.4.2 Fuzzy Arithmetic Based on Operations on Closed Intervals.

A fuzzy number can be characterized by an interval of confidence at level α, [22, 23], as follows.

$$A_\alpha = \left[a_1^{(\alpha)},\ a_2^{(\alpha)}\right]$$

which has the property

$$\alpha \le \alpha' \quad \Rightarrow A_{\alpha'} \subset A_\alpha$$

Let A = [a, b] ∈ R and B = [c, d] ∈ R be two fuzzy numbers then we define the arithmetic operations on them as follows [22, 23]:

Addition $A + B = [\, a + c, b + d \,]$

Subtraction $A - B = [\, a - d, b - c \,]$

Multiplication $AB = [\ \mathrm{Min}\,(ac, ad, bc, bd),\ \ \mathrm{Max}\,(ac, ad, bc, bd)]$

If a fuzzy set is defined over R^+ the formula can be simplified to: $AB = [ac, bd]$

Inverse of A $A^{-1} = [\ \mathrm{Min}\,(\frac{1}{a}, \frac{1}{b}),\ \mathrm{Max}\,(\frac{1}{a}, \frac{1}{b})\,]$

Division $\frac{A}{B}$ $\frac{A}{B} = [\mathrm{Min}\,(\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}), \mathrm{Max}\,(\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d})]$

Minimum (∧) $A \wedge B = [a \wedge c, b \wedge d]$

Maximum (∨) $A \vee B = [a \vee c, b \vee d]$

Let A and B be two fuzzy numbers such that $A_\alpha = \left[a_1^{(\alpha)},\ a_2^{(\alpha)}\right]$ be the α–level set of A and $B_\alpha = \left[b_1^{(\alpha)},\ b_2^{(\alpha)}\right]$ is the α –level set of B.

Let * denote any of the arithmetic operations +, − , , ., /, $\wedge$ and $\vee$ on fuzzy numbers.

Then, we define a fuzzy set A * B in R by defining its α –level sets $(A * B)_\alpha$ as

$$(A * B)_\alpha = A_\alpha * B_\alpha \text{ for any } \alpha \in [0, 1].$$

Since $(A * B)_\alpha$ is a closed interval for each $\alpha \in [0, 1]$ and A and B are fuzzy numbers, A * B is also a fuzzy number.

The multiplication of fuzzy number $A \subset R$ by an ordinary number $k \in R^+$ is defined as

$$(k * A_\alpha) = k\,(.)\,A_\alpha = \left[ka_1^{(\alpha)},\ ka_2^{(\alpha)}\right]$$

or equivalently, $\mu_{K.A}(x) = \mu_A\left(\frac{x}{k}\right) \quad \forall\, x \in R$

1.4.3 Triangular Fuzzy Number

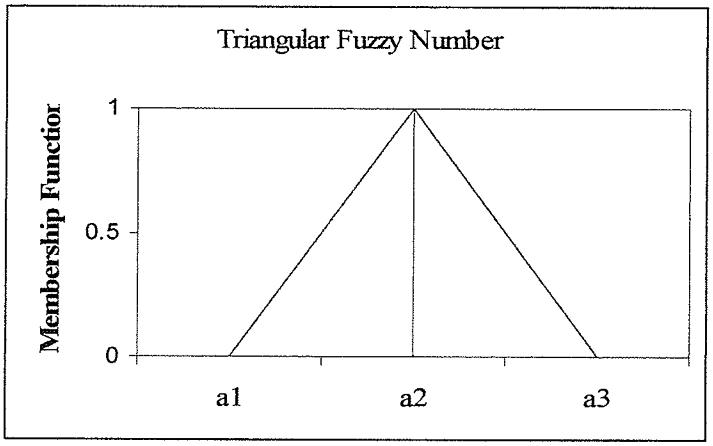
A triplet (a, b, c) is defined as a triangular fuzzy number (T.F.N.) if its membership function is defined as

$$\mu_A(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{x-c}{b-c} & b \le x \le c \\ 0 & x > c \end{cases}$$

Alternatively [23, pp.26, 27], in terms of confidence at level α we characterize the T.F.N. (a_1, a_2, a_3) as,

$$A_\alpha = \left[a_1^{(\alpha)},\ a_2^{(\alpha)}\right] = [(a_2 - a_1)\,\alpha + a_1,\ -(a_3 - a_2)\,\alpha + a_3] \qquad \forall\ \alpha \in [0,1]$$

Figure 1.1 Graphic representation of a Triangular Fuzzy Number



Algebraic operations on T.F.N.

Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be two T.F.Ns then ,

- Addition $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- Subtraction $A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

For the following two operations, we assume that a_i and b_i, $i = 1, 2, 3$ are positive.

- Multiplication $A(.)B = (a_1 b_1, a_2 b_2, a_3 b_3)$
- Division $A(:)B = \left\langle \frac{a_1}{b_3} \frac{a_2}{b_2} \frac{a_3}{b_1} \right\rangle$

Associated Ordinary Number

Let $A_i = (a_{i1}, a_{i2}, a_{i3})$ be a triangular fuzzy number. Then, an associated ordinary number (AON) corresponding to A is a crisp number and is defined as follows.

$$(AON)_i = \frac{a_{i1} + 2a_{i2} + a_{i3}}{4}$$

1.5 Kaufmann and Gupta Error Analysis with T.F.N. Approximation.

Using Kaufmann and Gupta [23] notation, suppose we have a fuzzy number A whose α – cut is given explicitly by

$A(\alpha) = [\ A(\alpha_L), A(\alpha_R)\ ]$

where $A(\alpha_L)$ = represents the α – cut of left segment of the exact fuzzy number A.

$A(\alpha_R)$ = represents the α – cut of right segment of the exact fuzzy number A.

Suppose P, a T.F.N., is obtained as an approximation of the fuzzy number A, such that the α – cut of P is given by

$P(\alpha) = [\ P(\alpha_L), P(\alpha_R)\ ]$

where $P(\alpha_L)$ = represents the α – cut of left segment of the approximate fuzzy number P.

$P(\alpha_R)$ = represents the α – cut of right segment of the approximate fuzzy number P.

The left divergence $\varepsilon_{l\alpha}$ is given by [23],

$$\varepsilon_{l\alpha} = A(\alpha_L) - P(\alpha_L) \tag{1.5.1}$$

The right divergence $\varepsilon_{r\alpha}$ is given by [23],

$$\varepsilon_{r\alpha} = A(\alpha_R) - P(\alpha_R) \tag{1.5.2}$$

According to Kaufmann and Gupta [23], If each of the right and left divergence is small (say, less than 3%), then P can be considered as a good approximation of the fuzzy number A, and one can use P in place of A to obtain certain interesting results. Kaufmann and Gupta [23] show that if the fuzzy number A is obtained as a result of multiplication of two fuzzy numbers, then the maximum value of both $\varepsilon_{l\alpha}$ and $\varepsilon_{r\alpha}$ occurs at $\alpha = \frac{1}{2}$.

1.6 Ranking of Fuzzy Numbers.

Fuzzy numbers are convenient for representing imprecise numerical quantities in a vague environment, and for purpose of application their comparison or ranking is very important. Also, ranking fuzzy numbers is a fundamental problem of fuzzy decision making. Since fuzzy numbers do not form a natural linear order (for example, real numbers form a natural linear order), different comparison methods have been developed by various researchers [7, 10, 23, 28]. Each of these methods has its own advantages and disadvantages and it would be a hard task to decide which one of them is the best. Despite of many methods suggested in the literature [7, 10, 23, 28] there is no single measure which is, in case of fuzzy numbers, universally applicable to a wide variety of situations. Below we give briefly, few methods of ranking triangular fuzzy numbers.

Brief Description of the ranking and ordering indices for triangular fuzzy numbers.

Let $A_i = (a_{i1}, a_{i2}, a_{i3})$ for i = 1, 2, 3, …, n, represent the triangular fuzzy numbers to be ranked. Researchers in [7, 10, 23, 28] developed a real number as $(\text{index})_i$ for each A_i as the ordering value or ranking value of A_i. The $(\text{index})_i$ is treated as a fuzzy measure of A_i. Then, the fuzzy numbers are ranked according to the descending (or ascending) order of their corresponding indices, and the fuzzy number with the largest index is considered as the most preferred fuzzy number. Below we give some of the indices for triangular fuzzy numbers developed by various researchers.

1. Chang's Ranking Index (Komolananij [28])

$$(Index)_i = \frac{(a_{i3} - a_{i1})(a_{i1} + a_{i2} + a_{i3})}{6} \qquad \text{for } i = 1, 2, \ldots, n.$$

2. Chiu and Park Ranking Index (Chiu and Park [10])

Let w_{i1} and w_{i2} be the weights associated with the fuzzy number A_i whose ranking index has to be computed. Then,

$$Index_i = \frac{(a_{i1} + a_{i2} + a_{i3})w_{i1}}{3} + w_{i2}a_{i2} \qquad \text{for } i = 1, 2, \ldots, n.$$

Chiu and Park [10] suggest that we let $w_{i1} = 1$ and w_{i2} be between [0.1 and 0.3].

3. Kaufmann and Gupta Ranking Index (Kaufmann and Gupta [23])

The Kaufmann and Gupta Ranking Method is a hierarchical test for which

$$(Index)_i = \frac{a_{i1} + 2a_{i2} + a_{i3}}{4} \qquad \text{for } i = 1, 2, \ldots, n.$$

Kaufmann and Gupta [23] suggest that if we have certain fuzzy numbers having the same indices, then the fuzzy number having the largest [a_{i2}] is the best alternative. If an index and [a_{i2}] are the same for a set of fuzzy numbers, then examine the range of [$a_{i1} - a_{i3}$]. The fuzzy number with the largest [$a_{i1} - a_{i3}$] is chosen as the best alternative.

4. Fuzzy Weighted Methods (Bortolan [7])

Let w_{i1} and w_{i2} represent the weights associated with the fuzzy number whose ranking index has to be computed. Then,

$$(Index)_i = w_{i1}\left[\frac{(a_{i1} + a_{i3})}{2}\right] + w_{i2}a_{i2} \qquad \text{for } i = 1, 2, \ldots, n.$$

According to bortolan [7] w_{i1} and w_{i2} can be chosen as $w_{i2} = 1$, $w_{i1} = 0.5$.

1.7 Rating Models

Project selection models, first introduced by Keeney and Raiffa [24, 25], are based on rating or ranking attempt to broaden the base on which the selection of various decision alternatives takes place. This is not to say that they ignore questions of cost and profit, but rather they seek to include other factors so as to get a wider perspective on the merits of a decision alternative. These other factors are often chosen so as to reflect company strategies or goals. Rating models [24, 25], are based on analytical tool and often help to highlight the level of compatibility that a project might have with the firms overall strategic aims. The analytical tool used enables one to explicitly rank tangible and intangible factors, having certain degree of importance, against each other for the purpose of resolving conflict or setting priorities. It is a multi-attribute modeling methodology that has helped various researchers to investigate successfully problems of multi-decision modeling [3, 19, 24, 25, 34, 43].

1.7.1 Weighted Rating Model under Crisp Environment

The weighted rating model under crisp environment, as stated by Keeney and Raiffa [24, 25], is stated as follows.

Suppose we have n decision alternatives and m type of different criteria. Let

w_i = the weight associated with the i-th criterion, $i = 1, 2, \ldots, m$

r_{ij} = the rate associated with the i-th criterion and j-th decision alternative, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$

R_j = the total rate associated with the j-th decision alternative, $j = 1, 2, \ldots, n$

Then according to [24, 25] the total rate Rj, in the rating model is obtained as follows:

$$R_j = \sum_{i=1}^{n} w_i r_{ij} \quad \text{for } j=1,2,\ldots,n. \tag{1.7.1.1}$$

This decision alternative selection model uses a rating system [22, 42] in which the designers of the model are required to give i-th criterion a weight w_i. The weights, w_i may be generated by various techniques. One effective and widely used procedure for this purpose is the Delphi technique [22, 42], however, any weighting which is considered to reflect accurately the organization's priorities can be used. This weighting is chosen to reflect the importance of that particular criterion from the point of view of management, or the importance of the criterion with respect to achieving the company's strategic goals, and is usually allocated a value belonging to the interval [1, 100] depending upon the importance of the criterion. Often, for convenience, the interval [1, 100] is also replaced by [1, 10].

1.8 Linear Programming

Linear Programming is a mathematical method of allocating scarce resources to achieve an objective, such as maximizing profit [30] or minimizing cost. Linear programming approach is a mathematical representation of a real world decision situation that consists of a linear objective function and linear resource constrains. Once the problem has been identified, the goals of management established, and the applicability of the linear programming determined, the next step in solving an unstructured real world problem is the formulation of a mathematical model. This entails three major steps:

- Identification of decision variables (the quantity of the activity in question).

- The development of an objective function that is a linear relationship of the solution variables, and
- The determination of system constraints, which are also linear relationships of the decision variables, which reflect the limited resources of the problem.

1.8.1 The Generalized Linear Programming Model

Decision variables

In each problem, decision variables, which denote a level of activity or quantity produced, are defined. For a general model, n decision variables are defined as

x_j = quantity of activity j, where j = 1, 2, …, n.

Objective Function

The objective function represents the sum total of the contribution of each decision variable in the model towards an objective. It is represented as

Maximize or Minimize $Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_jx_j + \ldots + c_nx_n$

where

Z = the total value of the objective function

c_j = the contribution per unit of activity j (j = 1, 2, …, n)

System Constraints

The constraints of a linear programming model represent the limited availability of resources in the problem. Let the amount of each of m resources available be defined as b_i (for i = 1, 2, ..., m). We also define a_{ij} as the amount of resource i consumed per unit of activity j (j = 1, 2, ..., n). Thus, the constraint equations can be defined as

$a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ij}x_j + \ldots + a_{inj}x_n \ (\leq, =, \geq) \ b_i \quad i = 1, 2, \ldots, m$

Non - negativity

$x_1, x_2, \ldots, x_n \geq 0.$

1.9 Fuzzy Linear Programming

Most of the time, due to incomplete or forecasted information the input data for c_j's, b_i's and a_{ij}'s, and/or the objective function and/or inequalities are imprecise. With these fuzzy/imprecise data the above problem is called fuzzy linear programming problem. Thus, a fuzzy linear programming problem is not uniquely defined. The fuzzy problem depends upon the type of fuzziness present and specified by the decision-maker. Fuzzy linear programming problem can be broadly classified as:

- Linear programming problem with fuzzy resources or fuzzy inequalities and crisp objective function.
- Linear programming problem with fuzzy resources or fuzzy inequalities and fuzzy objective function.
- Linear programming problem with fuzzy resources and fuzzy coefficients.

Two major fuzzy linear programming models as given in Zimmermann [46] are:

1. Symmetric
2. Non-symmetric.

The symmetric models are based on the definition of fuzzy decision proposed by Bellman and Zadeh [4]. It is assumed [4] that the objective function and constraints are imprecise and can be represented by fuzzy sets and the decision is the confluence of the fuzzy objective function and fuzzy constraint.

The non-symmetric models [4] are based on the following two approaches:

1. The determination of the fuzzy set decision.
2. The determination of a crisp maximizing decision by aggregating the objective function, after appropriate transformations with the constraints.

Thus, in a general format, a fuzzy linear problem (FLPP) can be written as:

$$
\text{(FLPP)} \qquad \text{maximize } z = f_0(x)
$$

subject to

$$
\begin{aligned}
f_i(x) &\precsim d_i && i = 1, 2, \ldots, k \\
g_i(x) &\leq b_i && i = k+1, k+2, \ldots, m \\
x &\geq 0
\end{aligned}
$$

where " $\precsim$ " is called the " fuzzy less than or equal to ", or " essentially less than or equal to ", f_0 , f_i and g_i, i = 1, 2, …, m are linear functions and $x \in R^n$.

1.10 Zimmerman's Approach – Symmetric Model

In this approach, on the lines of Zimmermann [46], the goals and the constraints are represented by fuzzy sets and we assume that the decision maker can establish an aspiration level z for the value of the objective function he/she wants to achieve.

Therefore, as proposed by Zimmermann [46], we consider the following format of the symmetric fuzzy linear programming problem (SFLP)

(SFLP) Find x such that

$$f_0(x) \gtrsim z_0$$

$$f_i(x) \lesssim d_i \qquad i = 1, 2, \ldots, k$$

$$g_i(x) \leq b_i \qquad i = k+1, k+2, \ldots, m$$

$$x_j \geq 0 \qquad j = 1, 2, \ldots, n$$

where f_0 , f_i , $i = 1, 2, \ldots, k$, and g_i, $i = 1, 2, \ldots, m$ are linear functions, z_0 , d_i, and b_i are vaguely specified values.

Also, $\gtrsim$ is the fuzzified version of $\geq$ and represents "essentially greater than or equal to" and $\lesssim$ represents "essentially less than or equal to".

Then the problem is interpreted as:

- Make a decision $x \geq 0$ such that at x:
- The value of the objective function $f_0(x)$ "essentially greater than or equal to" the predetermined aspiration level z, and
- The constraints $f_i(x) \lesssim d_i$, $i = 1, 2, \ldots, k$ are satisfied in fuzzy sense, and the constraints $g_i(x) \leq b_i$ $i = k+1, k+2, \ldots, m$ are crisply satisfied.

Then the problem (SFLP) is equivalent to

(EFLP) Find x such that

$$-f_0(x) \lesssim -z_0$$

$$f_i(x) \lesssim d_i \qquad i = 1, 2, \ldots, k$$

$$g_i(x) \leq b_i \qquad i = k+1, k+2, \ldots, m$$

$$x_j \geq 0 \qquad j = 1, 2, \ldots, n$$

where each of the fuzzy constraints, $-f_0(x) \lesssim -z$, and $f_i(x) \lesssim d_i$, $i = 1, 2, \ldots, k$

represents a fuzzy set whose membership function is $\mu_i(x)$, $i = 0, 1, 2, \ldots, k$, is interpreted as the degree to which x satisfies the fuzzy constraints $-f_0(x) \lesssim -z_0$, and $f_i(x) \lesssim d_i$, $i = 1, 2, \ldots, k$. Then, following Zimmermann [46], we write a symmetric fuzzy programming problem as follows:

(EFLP-1) Find x that satisfies

$$f_0(x) \lesssim z_0 \tag{1.10.1}$$

$$f_i(x) \lesssim d_i \qquad i = 1, 2, \ldots, k \tag{1.10.2}$$

$$g_i(x) \leq b_i \qquad i = k+1, k+2, \ldots, m \tag{1.10.3}$$

$$x_j \geq 0 \qquad j = 1, 2, \ldots, n \tag{1.10.4}$$

z_0 is called the aspiration level of $f_0(x)$ and is given some pre-assigned value. Let, $q_0 > 0$, and $q_i > 0$, ($i = 1, 2, \ldots, k$), be subjectively chosen constants of admissible violations such that q_0 is associated with (1.10.1), and q_i ($i = 1, 2, \ldots, k$) are associated with i-th linear constraint of (1.10.2). We assume that the membership functions of $\mu_I(x)$, $i = 0, 1, 2, \ldots, k$, are linearly decreasing over the "tolerance level" q_i.

Now, on the lines of Zimmerman [46], we define the membership function corresponding to (1.10.1) and (1.10.2), as follows.

Corresponding to $f_0(x)$ membership function $\mu_i(x)$ for objective function is written as

$$\mu_0(x) = \begin{cases} 1 & \text{if} \quad f_0(x) \leq z_0 \\ 1 - \dfrac{f_0(x) - z_0}{q_0} & \text{if} \quad z_0 \leq f_0(x) \leq z_0 + q_0 \\ 0 & \text{if} \quad z_0 + q_0 \leq f_0(x) \end{cases} \tag{1.10.5}$$

Corresponding to $i = 1, 2, \ldots, k$, the membership function is

$$\mu_i(x) = \begin{cases} 1 & \textit{if} \quad f_i(x) \le d_i \\ 1 - \dfrac{[f_i(x) - d_i]}{q_i} & \textit{if} \quad d_i < f_i(x) \le d_i + q_i \\ 0 & \textit{if} \quad d_i + q_i \le f_i(x) \end{cases} \tag{1.10.6}$$

Once the membership functions are known, then a solution that belongs to the intersection of the fuzzy sets of objective function (1.10.1) and the constraints (1.10.2) and satisfied the crisp constraints (1.10.3) and (1.10.4) is a solution to (EFLP-1). Suppose that $\mu_D(x)$ is the membership function of the fuzzy set "decision" of the model. Then,

$$\mu_D(x) = \text{Min } (\mu_0(x), \mu_1(x), \mu_2(x), \mu_3(x), \ldots, \ldots, \mu_k(x))$$

Since, we are interested in a large value of $\mu_D(x)$, therefore, following Zimmermann [46], we want to obtain the maximum value of $\mu_D(x)$. Thus, our interest is to:

Maximize $\mu_D(x) = \text{Min } [\mu_0(x), \mu_1(x), \mu_2(x), \mu_3(x), \ldots, \ldots, \mu_k(x)]$

Subject to the constraints of (1.10.3) and (1.10.4).

Now, along the lines of Zimmermann [46], replacing $\mu_D(x)$ by λ, and using (1.10.5) and (1.10.6) respectively for $\mu_0(x)$, $\mu_i(x)$, $i = 1, 2, \ldots, k$, we have the following problem;

(EFLP-2) Max λ

subject to

$$f_0(x) + q_0 \lambda \le z_0 + q_0$$

$$f_i(x) + q_i \lambda \le d_i + q_i \qquad i = 1, 2, \ldots, k$$

$$g_i(x) \le b_i \qquad i = k+1, k+2, \ldots, m$$

$$0 \le \lambda \le 1$$

$$x_j \ge 0 \qquad j = 1, 2, \ldots, n$$

It is observed that (EFLP-2) is a crisp optimization problem whose optimal solution, if it exists, provides a solution to (SFLP).

Remark 1.10.1. If in (SFLP), we replace

$$f_0(x) \precsim z_0 \quad \text{by} \quad f_0(x) \mathrel{\tilde{\leq}} z_0$$

that is, if we replace the requirement " essentially less than or equal to" denoted by " $\precsim$ ", by the requirement " desired to be less than or equal to" denoted by " $\tilde{\leq}$ ", then, we take the membership function $\mu_0(x)$ corresponding to $f_0(x)$ as follows:

$$\mu_0(x) = \begin{cases} 1 & \text{if} \quad f_0(x) \leq z_0 - q_0 \\ 1 - \dfrac{f_0(x) - (z_0 - q_0)}{q_0} & \text{if} \quad z_0 - q_0 \leq f_0(x) \leq z_0 \\ 0 & \text{if} \quad z_0 \leq f_0(x) \end{cases}$$

In this case corresponding to (EFLP-1) we have the following (EFLP-2)

(EFLP-2) Max λ

subject to

$$\begin{aligned} f_0(x) + q_0\lambda &\leq z_0 & & \\ f_i(x) + q_i\lambda &\leq d_i + q_i & & i = 1, 2, \ldots, k \\ g_i(x) &\leq b_i & & i = k+1, k+2, \ldots, m \\ 0 \leq \lambda &\leq 1 & & \\ x_j &\geq 0 & & j = 1, 2, \ldots, n. \end{aligned}$$

1.11 Multistage Planning Problems

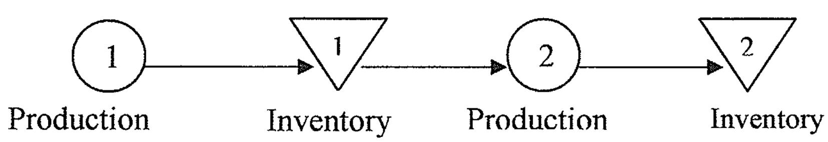
Production planning is the activity of establishing production goals over some future time period, called the *planning horizon*. The objective of production planning is to plan the optimal use of resources to meet stated production requirements or to take advantage of potential sales opportunities [21].

The production systems consist of either single stage, or multistage. Multiple facilities arranged in a multistage structure so that the output from one stage becomes the input to a following stage [21]. The facilities can be operated on different schedules, with coordination required only because some facilities obtain parts or semi finished products from one or more other facilities. By allowing flexibility in scheduling, reduced production costs may be realized; however, this usually is at the expense of additional inventory holding costs resulting from increased inventories between stages. The between-stage inventories act as a buffer to absorb the effect of imbalances between the production rates of successive stages. The larger is the inventory, the more independence between the stages.

The key to modeling a multistage system are the decisions about what groupings of production operations constitute the stages in the system, whether or not each stage is to have multiple facilities operating in parallel, and how many inventory points between stages are to be defined.

The following diagrams represent a two-stage production system:

Figure 1.2 Facilities in series (a serial system).



In Figure 1.2, the system is viewed as two facilities in series. The term *facility* is used here to mean a subsystem that is to be scheduled as a whole, so that in particular, it can be modeled without specifically accounting for inventory accumulations between production centers within the facility. A facility could be a single machine, a department, a production line, an entire plant, or a grouping of plants, depending on the nature of the planning decisions [21].

When a stage consists of parallel sequences of production centers, where each sequence produces the product of that stage, it may be desirable to model the stage as parallel facilities and define separate decision variables for each facility. This would be nearly always be true in cases where a stage involves multiple plants. Both figures 1.3 and 1.4 represent a system where the first stage consists of two parallel facilities, each producing the same type of product; the second stage, which uses the product of the first stage, consists of three parallel facilities.

Figure 1.3 Facilities in parallel, common inventories.

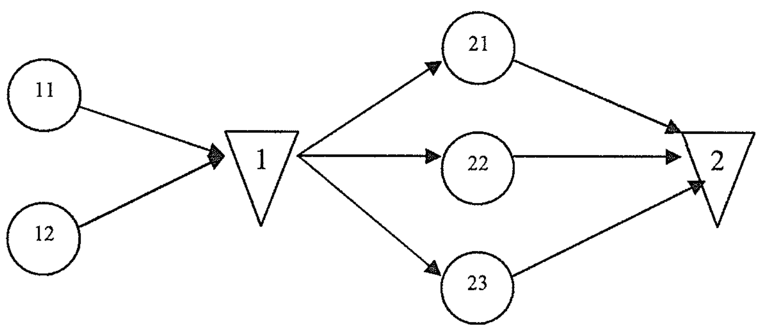
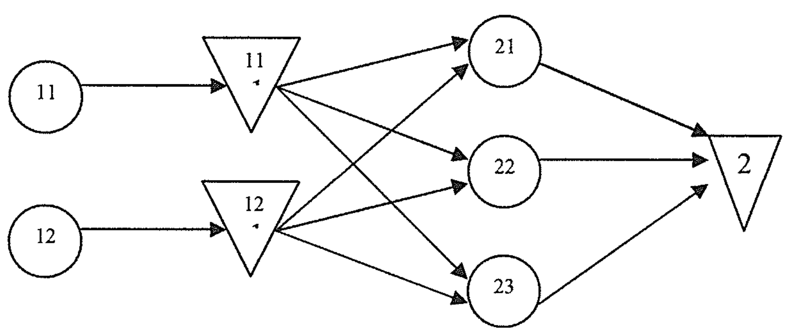


Figure 1.4 Facilities in parallel, separate inventories between stages.



In Figure 1.3, the inventory between stages is not identified by facility, while in Figure 1.4, it is. The former model would be used when the inventory is, in fact, stored at a common location, or when it is not important to distinguish between locations. The latter model may be necessary when transportation costs depend heavily on plant locations, when there are storage limitations at separate inventory locations, or when it desired to obtain the optimal shipping pattern between stages, as well as the production program at each stage, as direct output from solving the model.

Mathematical programming models, particularly linear programming models, are commonly used to analyze multistage systems.

1.12 Organization of the thesis

In the present thesis, a number of related problems from a variety of areas are model under fuzzy environment. Also, we discuss the methods to obtain their solutions and interpretation to the solutions.

To be involved in production area, the following three issues are important:

1. First, select the best and the right place to locate the new or existing facility, then
2. select the type of production planning systems; either single stage or multistage structure, and then
3. cope up with the changes in the production planning, e.g. increase and decrease in production limits, change in work force level (hiring and laying off workers), and cost of overtime, etc.

The present thesis deals with all these three issues under fuzzy environment.

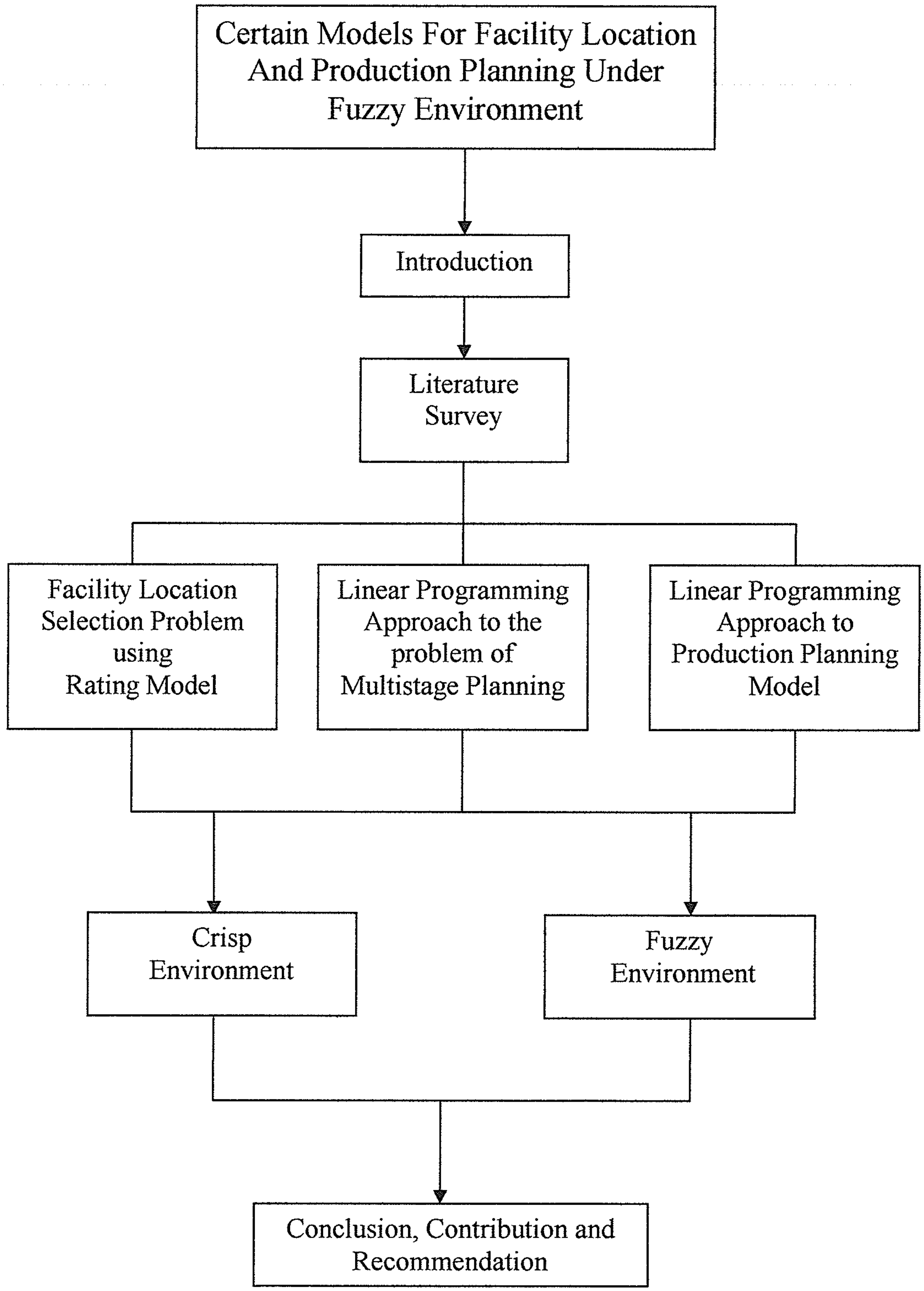
Chapter 1 provides an introduction to the concepts and problems considered in this thesis.

Chapter 2 deals with the literature review of the related work done by other researchers.

Chapter 3 deals with the weighted rating model under fuzzy environment with an application to a facility location selection problem. In Chapter 4, a linear programming approach to the problem of multistage planning is presented under both crisp and fuzzy environment. Chapter 5 presents a production planning model under crisp and fuzzy environment using linear programming approach. Finally, the conclusion and the contributions made by the thesis, along with some recommendations for further research, are given in Chapter 6.

The flow chart of the organization of the thesis is shown in the next page.

Organization of the Thesis



CHAPTER 2

LITERATURE SURVEY

The main objective of this chapter is to provide a survey of the literature dealing with production planning problems, Multistage Planning and the Weighted Rating Model and other concepts considered in this thesis.

2.1 Fuzzy Weighted Rating Models

The weighted rating model is a multi-objective decision making tool which was introduced by Keeney and Raiffa [24, 25] as a selection technique process to overcome the common inconsistency in human judgment. It is considered as a powerful decision making tool in many disciplines, and is a promising and important field of study. A classical crisp weighted rating model, as introduced by Keeney and Raiffa [24, 25] is stated as follows.

Suppose we have n decision alternatives and m criteria such that the m criteria are used on each of the n projects.

We take

r_{ij} = the rate associated with the i-th criterion and j-th decision alternative, i = 1, 2, …, m; j = 1, 2, …, n, and

w_i = the weight of the i-th criterion, i = 1, 2, …, m,

such that R_j, the total weighted rate of the j-th decision alternative, j = 1, 2, …, n, is given by:

$$R_j = \sum_{i=1}^{m} w_i r_{ij} \quad j = 1, 2, \ldots, n.$$

Criterion	Weight	Dec. Alt. 1	Dec. Alt. 2 …	Dec. Alt. n
1	w_1	r_{11}	r_{12}	r_{1n}
2	w_2	r_{21}	r_{22}	R_{2n}
..	….	….	….	….
..	….	….	….	….
m	w_m	r_{m1}	r_{m2}	r_{mn}
Total Rate R_j		$R_1 = \sum_{i=1}^{m} w_i r_{i1}$	$R_2 = \sum_{i=1}^{m} w_i r_{i2}$	$R_n = \sum_{i=1}^{m} w_i r_{in}$

The decision alternative with the highest (lowest) rate is the optimal solution when the objective is to maximize (minimize, respectively) the total weighted rate.

The process of deriving weights is fundamental to the effectiveness of the model's application. As already stated in Section 1.7, the weights w_i may be generated by various techniques. One effective and widely used procedure for this purpose is the Delphi technique [23, 42]. However, any weighting which is considered to reflect accurately the organization's priorities can be used. A weighted rating model allows the relative importance of the criteria to be modeled. The degree to which a criterion satisfies the relevant goal is modeled and the rates are then summed. More preferred decision alternatives rate higher on the scale, and less preferred decision alternatives rate lower. In practice, scales extending from 1 to 100 are often used, where 1 represents a real or hypothetical least preferred decision alternatives, and 100 is associated with a real or hypothetical most preferred decision alternatives. Often, for convenience, scales belonging to the interval [1,10] are also used. The most common way to combine rates on criteria, and relevant weights between criteria, is to calculate a simple weighted average of rates. Use of such weighted averages depends on the assumption of mutual independence of preferences. This means that the judged strength of preference for a decision alternative on one criterion will be independent of its judged strength of preference on another. Constraints can also be modeled into this procedure to give the constrained weighted rating model. Using a variety of techniques, certain types of fuzzy decision making problems have been studied by Kaufmann and Gupta [22, 23], Zadeh [45], and Zimmermann [46, 47].

Lucas and Moore [32] presented a decision procedure for the selection systems projects. The procedure includes a steering committee which develops project evaluation criteria. The committee specifies weights for each criteria several alternatives for each computer application are developed and a scoring is used to evaluate each alternative. The procedure allowed for inclusion of both objective and subjective criteria and result in consistent project selection decision.

Dubois and Prade [13] proposed a survey of decision-analysis-oriented methods based on the concept of a fuzzy number. Fuzzy numbers are useful to perform sensitivity analysis on utility-based model or scoring methods, when probability or utility values, weights of attributes cannot be precisely estimated but are obtained through verbal statements. Algorithms for computing fuzzy expectations of utility of fuzzy global ratings are provided.

Ince et al. [20] presented the application of fuzzy multiple-criteria decision-making to the problem of project selection. The category of multiple-criteria techniques known as fuzzy ranking methods are used to rank projects under conditions where the criteria of selection are imprecisely defined and the project scores for the criteria are subjectively determined.

Appadoo [2] developed the scoring models under fuzzy environment and applied them to assess the best decision alternative from multicriteria decision problem like Bond Selection Problem.

2.2 Multistage Production System

The production systems consist of either single stage or multistage. Multiple facilities arranged in a multistage structure so that the output from one stage becomes the input to a following stage.

Mathematical programming models, particularly linear programming models, are commonly used to analyze multistage systems.

Gabbay [15] presented an analytic framework for a hierarchical treatment of a multistage linear programming production and inventory problem. The problem addressed is multi-item, capacitated and linear.

Teny and Kochhar [40] developed a formal mathematical approach to aggregate production planning for a multi-product, multi-cell and multi-stage manufacturing system. The model, based upon a vector space approach, includes all the important variables relating to the demand for individual items, inventory levels, the availability of machines taking into account any breakdowns, subcontracting of orders and overtime / under time working and increase / decrease in the number of orders subcontracted are presented.

Pizzolato and Goyal [35] modeled a multi-stage multi-system production process. A serial production process is defined in which the production output of each stage serves as an input for the next one while intermediate stage inventories absorb productive imbalances between successive stages.

Tamura [38, 39] formulated a generalized model of the production planning problem as a mixed- integer programming problem, and is approximated as a linear programming problem by applying the Dantzig–Wolfe decomposition principle [12]. An approximate algorithm is developed in detail for a two- stage production system.

Gunasekaran et. al [17] developed a mathematical model for determining the optimum lot-sizes for a set of products and the capacity required to produce them in multi-stage production system to minimize the total system cost per unit time.

Wang [41] developed a fuzzy multi-objective linear programming model for solving the multi-product aggregate production planning decision problem in a fuzzy environment. The proposed model attempted to minimize total production costs, carrying and backordering cost and rates of changes in labor levels considering inventory level, labor levels, capacity, warehouse space and the time value of money.

2.3 Summary of the Thesis

The results and methods conducted in this thesis are contained in Chapter 3, Chapter 4, and Chapter 5. We summarize them as follows:

Chapter 3 Fuzzy Weighted Rating Models

In this chapter, we consider the weighted rating model of Keeney and Raiffa [24, 25] under fuzzy environment and apply it to a facility location problem.

Chapter 4 Linear Programming Approach to Multistage Planning Problem.

The purpose of this chapter is to extend the result proved by Johnson [21] further by solving the multistage planning problem under fuzzy conditions. The advantage of using fuzzy mathematics is that it gives decision–maker flexibility and quantifies the certain type of uncertainty involved in the multi-stage production planning problem.

Chapter 5 Linear Programming Approach to Production Planning Model.

In this chapter, the effect of the changes in production planning problem {e.g. increase and decrease in production limits}, has been solved under fuzzy conditions. A numerical example has considered and compared the results obtained in the crisp environment and observed that the results obtained using fuzzy assumptions give much more flexible results.

Chapter 6 Conclusion, Contribution and Recommendations.

In this chapter, the contributions made in thesis are presented, conclusion along with some recommendations for further research on the problems considered in the thesis.

CHAPTER 3

FUZZY RATING MODEL AND FACILITY LOCATION PROBLEM

The purpose of this chapter is to develop a rating model under fuzzy environment and apply it to a facility location selection problem. We call such models as Fuzzy Rating Models (FRM).

3.1 Introduction

In modeling real world problems we often encounter multicriteria decision making problems with independent objectives. A rating model introduced by Keeney and Raiffa [24, 25] is a relatively quick and easy way to identify the best decision alternatives in such problems [1]. In this chapter we consider a rating model under fuzzy environment and show how such a model can assist in analyzing a multi criteria decision making problem when the information available is vague, imprecise and subjective. Numerical examples, with both crisp and fuzzy cases, are provided at the end of the chapter to demonstrate the application of the model in analyzing a facility location selection problem. It is believed that FRM developed here will find its applications in a wide variety of areas like fuzzy part family formation in a production system and fuzzy job selection problems etc.

3.2 Fuzzy Rating Model

Assume that

r_{ij} = the rating associated with the criterion i and decision alternative j,

$i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$ is a T.F.N. such that,

$r_{ij} = (r_{ij1}, r_{ij2}, r_{ij3})$ for $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$

whose α–cut is given by:

$$r_{ij}(\alpha) = [r_{ij1} + (r_{ij2} - r_{ij1})\alpha, r_{ij3} - (r_{ij3} - r_{ij2})\alpha] \quad (3.2.1)$$

for $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$ and $0 \leq \alpha \leq 1$

Let:

w_i = the weight of the criterion i for $i = 1, 2, \ldots, m$

be T.F.N. such that

$w_i = (w_{i1}, w_{i2}, w_{i3})$ for $i = 1, 2, \ldots, m$

whose α–cut is given by:

$$w_i(\alpha) = [w_{i1} + (w_{i2} - w_{i1})\alpha, w_{i3} - (w_{i3} - w_{i2})\alpha] \quad (3.2.2)$$

for $i = 1, 2, \ldots, m$ and $0 \leq \alpha \leq 1$

$w_i = (w_{i1}, w_{i2}, w_{i3})$ for $i = 1, 2, \ldots, m$, where 1 represents a real or hypothetical least preferred decision alternatives, and 100 is associated with a real or hypothetical most preferred decision alternatives. Often, for convenience, scales belonging to the interval [1, 10] are also used.

We now let

R_j = fuzzy Rate for decision alternative j, $j = 1, 2, \ldots, n$

In view of (3.2.1) and (3.2.2), we have

$$R_j(\alpha) = \sum_{i=1}^{m} w_i(\alpha) \otimes r_{ij}(\alpha) \quad (3.2.3)$$

$$= \sum_{i=1}^{m} [w_{i1} + (w_{i2} - w_{i1})\alpha, w_{i3} - (w_{i3} - w_{i2})\alpha] \otimes [r_{ij1} + (r_{ij2} - r_{ij1})\alpha, r_{ij3} - (r_{ij3} - r_{ij2})\alpha] \quad (3.2.4)$$

Where $\otimes$ denotes the fuzzy multiplication.

$$R_j(\alpha) = (\sum_{i=1}^{m} w_{i1}r_{ij1} + [\sum_{i=1}^{m} w_{i1}r_{ij2} - 2\sum_{i=1}^{m} w_{i1}r_{ij1} + \sum_{i=1}^{m} w_{i2}r_{ij1}]\alpha$$

$$+$$

$$[\sum_{i=1}^{m} w_{i2}r_{ij2} - \sum_{i=1}^{m} w_{i2}r_{ij1} - \sum_{i=1}^{m} w_{i1}r_{ij2} + \sum_{i=1}^{m} w_{i1}r_{ij1}]\alpha^2,$$

$$\sum_{i=1}^{m} w_{i3}r_{ij3} + [\sum_{i=1}^{m} w_{i3}r_{ij2} - 2\sum_{i=1}^{m} w_{i3}r_{ij3} + \sum_{i=1}^{m} w_{i2}r_{ij3}]\alpha$$

$$+$$

$$[\sum_{i=1}^{m} w_{i3}r_{ij3} - \sum_{i=1}^{m} w_{i3}r_{ij2} - \sum_{i=1}^{m} w_{i2}r_{ij3} + \sum_{i=1}^{m} w_{i2}r_{ij2}]\alpha^2) \qquad (3.2.5)$$

The fuzzy number R_j corresponding to R_j (α) is obtained by setting $\alpha = 0$ for obtaining end values, and $\alpha = 1$,

for obtaining the interior value. Therefore,

$$R_j = (\sum_{i=1}^{m} w_{i1}r_{ij1}, \sum_{i=1}^{m} w_{i2}r_{ij2}, \sum_{i=1}^{m} w_{i3}r_{ij3}), \ j=1,2,\dots,n$$

for which the membership function μ_{sj} (x) for j = 1 , 2, …, n, is obtained by setting

$$(\sum_{i=1}^{m} w_{i1}r_{ij1} + [\sum_{i=1}^{m} w_{i1}r_{ij2} - 2\sum_{i=1}^{m} w_{i1}r_{ij1} + \sum_{i=1}^{m} w_{i2}r_{ij1}]\alpha +$$
$$[\sum_{i=1}^{m} w_{i2}r_{ij2} - \sum_{i=1}^{m} w_{i2}r_{ij1} - \sum_{i=1}^{m} w_{i1}r_{ij2} + \sum_{i=1}^{m} w_{i1}r_{ij1}]\alpha^2 = x$$

and

$$\sum_{i=1}^{m} w_{i3}r_{ij3} + [\sum_{i=1}^{m} w_{i3}r_{ij2} - 2\sum_{i=1}^{m} w_{i3}r_{ij3} + \sum_{i=1}^{m} w_{i2}r_{ij3}]\alpha +$$
$$[\sum_{i=1}^{m} w_{i3}r_{ij3} - \sum_{i=1}^{m} w_{i3}r_{ij2} - \sum_{i=1}^{m} w_{i2}r_{ij3} + \sum_{i=1}^{m} w_{i2}r_{ij2}]\alpha^2 = x$$

respectively, and solving each of them for α in terms of x. Thus, we have the membership function as given on the next page.

$$\mu_s(x)=\begin{cases} 0 & \text{if } x \le \sum\limits_{i=1}^{m} w_{i1}r_{ij1} \\[2ex] \dfrac{-\left[\sum\limits_{i=1}^{m} w_{i1}r_{ij2} - 2\sum\limits_{i=1}^{m} w_{i1}r_{ij1} + \sum\limits_{i=1}^{m} w_{i2}r_{ij1}\right] + \sqrt{\begin{array}{l} 4\left[\left[\sum\limits_{i=1}^{m} w_{i2}r_{ij2} - \sum\limits_{i=1}^{m} w_{i2}r_{ij1} - \sum\limits_{i=1}^{m} w_{i1}r_{ij2} + \sum\limits_{i=1}^{m} w_{i1}r_{ij1}\right]\right]^2 x \\ \quad + \left[\sum\limits_{i=1}^{m} w_{i1}r_{ij2} - 2\sum\limits_{i=1}^{m} w_{i1}r_{ij1} + \sum\limits_{i=1}^{m} w_{i2}r_{ij1}\right]^2 \\ -4\left[\left[\sum\limits_{i=1}^{m} w_{i2}r_{ij2} - \sum\limits_{i=1}^{m} w_{i2}r_{ij1} - \sum\limits_{i=1}^{m} w_{i1}r_{ij2} + \sum\limits_{i=1}^{m} w_{i1}r_{ij1}\right]\right]^2 \left[\sum\limits_{i=1}^{m} w_{i1}r_{ij1}\right] \end{array}}}{2\left[\left[\sum\limits_{i=1}^{m} w_{i2}r_{ij2} - \sum\limits_{i=1}^{m} w_{i2}r_{ij1} - \sum\limits_{i=1}^{m} w_{i1}r_{ij2} + \sum\limits_{i=1}^{m} w_{i1}r_{ij1}\right]\right]} & \text{if } \sum\limits_{i=1}^{m} w_{i1}r_{ij1} \le x \le \sum\limits_{i=1}^{m} w_{i2}r_{ij2} \\[2ex] \dfrac{-\left[\sum\limits_{i=1}^{m} w_{i3}r_{ij2} - 2\sum\limits_{i=1}^{m} w_{i3}r_{ij3} + \sum\limits_{i=1}^{m} w_{i2}r_{ij3}\right] + \sqrt{\begin{array}{l} 4\left[\left[\sum\limits_{i=1}^{m} w_{i3}r_{ij3} - \sum\limits_{i=1}^{m} w_{i3}r_{ij2} - \sum\limits_{i=1}^{m} w_{i2}r_{ij3} + \sum\limits_{i=1}^{m} w_{i2}r_{ij2}\right]\right]^2 x \\ \quad + \left[\sum\limits_{i=1}^{m} w_{i3}r_{ij2} - 2\sum\limits_{i=1}^{m} w_{i3}r_{ij3} + \sum\limits_{i=1}^{m} w_{i2}r_{ij3}\right]^2 \\ -4\left[\left[\sum\limits_{i=1}^{m} w_{i3}r_{ij3} - \sum\limits_{i=1}^{m} w_{i3}r_{ij2} - \sum\limits_{i=1}^{m} w_{i2}r_{ij3} + \sum\limits_{i=1}^{m} w_{i2}r_{ij2}\right]\right]^2 \left[\sum\limits_{i=1}^{m} w_{i3}R_{ij3}\right] \end{array}}}{2\left[\left[\sum\limits_{i=1}^{m} w_{i3}r_{ij3} - \sum\limits_{i=1}^{m} w_{i3}r_{ij2} - \sum\limits_{i=1}^{m} w_{i2}r_{ij3} + \sum\limits_{i=1}^{m} w_{i2}r_{ij2}\right]\right]} & \text{if } \sum\limits_{i=1}^{m} w_{i2}r_{ij2} \le x \le \sum\limits_{i=1}^{m} w_{i3}r_{ij3} \\[2ex] 0 & \text{if } x \ge \sum\limits_{i=1}^{m} w_{i3}r_{ij3} \end{cases}$$

3.3 The Approximated Fuzzy Rating Model

If we approximate the above fuzzy number by a T.F.N., and denote it by AR_j, then AR_j is given by

$$AR_j = \left(\sum_{i=1}^{m} w_{i1}r_{ij1}, \sum_{i=1}^{m} w_{i2}r_{ij2}, \sum_{i=1}^{m} w_{i3}r_{ij3}\right),\ j=1,2,\ldots,n \tag{3.3.1}$$

and we call it the approximate fuzzy rate (AFR). The corresponding confidence interval at α – cut level of the approximated T.F.N. $AR_j(\alpha)$ is given by

$AR_j(\alpha) =$

$$\left(\sum_{i=1}^{m} w_{i1}r_{ij1} + [\sum_{i=1}^{m} w_{i2}r_{ij2} - \sum_{i-1}^{m} w_{i1}r_{ij1}]\alpha, \quad \sum_{i=1}^{m} w_{i3}r_{ij3} - [\sum_{i=1}^{m} w_{i3}r_{ij3} - \sum_{i-1}^{m} w_{i2}r_{ij2}]\alpha\right) \tag{3.3.2}$$

The approximate membership function of the AFR model is as given below.

$$\mu_{ARj}(\mathrm{x}) = \begin{cases} 0 & \text{if } \mathrm{x} \le \sum\limits_{i=1}^{m} w_{i1}r_{ij1} \\ \dfrac{x - \sum\limits_{i=1}^{m} w_{i1}r_{ij1}}{\sum\limits_{i=1}^{m} w_{i2}r_{ij2} - \sum\limits_{i=1}^{m} w_{i1}r_{ij1}} & \text{if } \sum\limits_{i=1}^{m} w_{i1}r_{ij1} \le x \le \sum\limits_{i=1}^{m} w_{i2}r_{ij2} \\ \dfrac{\sum\limits_{i=1}^{m} w_{i3}r_{ij3} - x}{\sum\limits_{i=1}^{m} w_{i3}r_{ij3} - \sum\limits_{i=1}^{m} w_{i2}r_{ij2}} & \text{if } \sum\limits_{i=1}^{m} w_{i2}r_{ij2} \le x \le \sum\limits_{i=1}^{m} w_{i3}r_{ij3} \\ 0 & \text{if } \mathrm{x} \ge \sum\limits_{i=1}^{m} w_{i3}r_{ij3} \end{cases}$$

where "x" represent the rating value

3.4 Error Analysis of T.F.N

Unfortunately the triangular fuzzy numbers are not closed under multiplication and division and it has been pointed out by Kaufmann and Gupta [22, 23] that the result of these operators is a polynomial membership function, and the triangular shape only approximates the actual result. It is a common mistake to blindly assume that the error introduced by the approximations is small and acceptable. According to Kaufmann and Gupta [22, 23], the linear approximation can be quite poor and can lead to incorrect results when used in a variety of applications. Thus, in this case, it is important to perform error analysis to check the accuracy of the estimate. Error of approximation between the approximated membership function and the actual membership function can be sufficiently large to produce erroneous results. Each triangular or polynomial fuzzy number can be separated into a left and right segment.

Using the notation of Kaufmann and Gupta [22, 23] the actual fuzzy number for the value x, at a given α is defined as T_L for the left segment and T_R for the right segment. The standard approximation for value x at a given α is defined as P_L and P_R for the left and right segments respectively. This allows us to separately analyze the left and right portions of the membership curve [22, 23].

The left segment error, ε_L, is

$$\varepsilon_L = P_L - T_L \tag{3.4.1}$$

and the right segment error, ε_R, is

$$\varepsilon_R = P_R - T_R \tag{3.4.2}$$

A more meaningful measure of the error can be obtained by taking the absolute percent error. The absolute percent error with respect to the actual value is defined for the left segment as

$$\%\ \varepsilon_L = \left|\frac{P_L - T_L}{T_L}\right| \cdot 100 \tag{3.4.3}$$

and the right segment error is defined as

$$\%\ \varepsilon_R = \left|\frac{P_R - T_R}{T_R}\right| \cdot 100 \tag{3.4.4}$$

These expressions require knowledge of both the approximation and the actual value at every α–cut to have any utility.

If the above errors are small that less than 3%, we can take the approximate triangular fuzzy number as a legitimate approximation (Kaufmann and Gupta [23]).

3.5 Kaufmann and Gupta Error Analysis with T.F.N. approximation.

Using Kaufmann and Gupta [23] notation, let the fuzzy number A be obtained as the multiplication of the two T.F.N.'s such that the α–cut of A is given explicitly by

$A(\alpha) = [A(\alpha_L), A(\alpha_R)]$

Where $A(\alpha_L)$ = represents the α–cut of left segment of the exact fuzzy number A.

$A(\alpha_R)$ = represents the α–cut of right segment of the exact fuzzy number A.

Suppose P, a T.F.N., is obtained as an approximation of the fuzzy number A, such that the α–cut of P is given by

$P(\alpha) = [P(\alpha_L), P(\alpha_R)]$

Where $P(\alpha_L)$ = represents the α–cut of left segment of the approximate fuzzy number P.

$P(\alpha_R)$ = represents the α–cut of right segment of the approximate fuzzy number P.

The left divergence $\in_{L\alpha}$ is given by [23],

$$\in_{L\alpha} = A(\alpha_L) - P(\alpha_L) \qquad (3.5.1)$$

The right divergence $\in_{R\alpha}$ is given by

$$\in_{R\alpha} = A(\alpha_R) - P(\alpha_R) \qquad (3.5.2)$$

According to Kaufmann and Gupta [22, 23], The maximum left and right divergences occur at $\alpha = \frac{1}{2}$

If each of the right and left divergence is small, then P can be considered as a good approximation of the product of the two fuzzy numbers.

3.6 Left and Right Divergence of the Parabolic Fuzzy Rating Number.

From (3.2.5), we have

$$R_j(\alpha) = (\sum_{i=1}^{m} w_{i1}r_{ij1} + [\sum_{i=1}^{m} w_{i1}r_{ij2} - 2\sum_{i=1}^{m} w_{i1}r_{ij1} + \sum_{i=1}^{m} w_{i2}r_{ij1}]\alpha$$

$$+$$

$$[\sum_{i=1}^{m} w_{i2}r_{ij2} - \sum_{i=1}^{m} w_{i2}r_{ij1} - \sum_{i=1}^{m} w_{i1}r_{ij2} + \sum_{i=1}^{m} w_{i1}r_{ij1}]\alpha^2,$$

$$\sum_{i=1}^{m} w_{i3}r_{ij3} + [\sum_{i=1}^{m} w_{i3}r_{ij2} - 2\sum_{i=1}^{m} w_{i3}r_{ij3} + \sum_{i=1}^{m} w_{i2}r_{ij3}]\alpha$$

$$+$$

$$[\sum_{i=1}^{m} w_{i3}r_{ij3} - \sum_{i=1}^{m} w_{i3}r_{ij2} - \sum_{i=1}^{m} w_{i2}r_{ij3} + \sum_{i=1}^{m} w_{i2}r_{ij2}]\alpha^2) \qquad (3.2.5)$$

and from (3.3.2) we have

$AR_j(\alpha) =$

$$(\sum_{i=1}^{m} w_{i1}r_{ij1} + [\sum_{i=1}^{m} w_{i2}r_{ij2} - \sum_{i-1}^{m} w_{i1}r_{ij1}]\alpha,$$

$$\sum_{i=1}^{m} w_{i3}r_{ij3} - [\sum_{i=1}^{m} w_{i3}r_{ij3} - \sum_{i-1}^{m} w_{i2}r_{ij2}]\alpha) \qquad (3.3.2)$$

According to Kaufmann and Gupta [23], the left Divergence for j = 1, 2, . . ., n is given by using equation (3.6.1)

$$\varepsilon_{Lj} = \left[\frac{\begin{array}{l}(\sum_{i=1}^{m} w_{i1}r_{ij1} + [\sum_{i=1}^{m} w_{i1}r_{ij2} - 2\sum_{i=1}^{m} w_{i1}r_{ij1} + \sum_{i=1}^{m} w_{i2}r_{ij1}]\alpha + \\ [\sum_{i=1}^{m} w_{i2}r_{ij2} - \sum_{i=1}^{m} w_{i2}r_{ij1} - \sum_{i=1}^{m} w_{i1}r_{ij2} + \sum_{i=1}^{m} w_{i1}r_{ij1}]\alpha^2,\end{array}}{\sum_{i=1}^{m} w_{i1}r_{ij1} + [\sum_{i=1}^{m} w_{i2}r_{ij2} - \sum_{i-1}^{m} w_{i1}r_{ij1}]\alpha}\right] \tag{3.6.1}$$

The maximum left divergences occurs at $\alpha = \frac{1}{2}$, therefore, substitute $\alpha = \frac{1}{2}$ in (3.6.1).

The maximum left divergence for $j = 1, 2, \ldots, n$ is given by

$$\text{Max}\_\varepsilon_{Lj} = \frac{\sum_{i=1}^{m} w_{i1}r_{ij2} + \sum_{i=1}^{m} w_{i2}r_{ij1} + \sum_{i=1}^{m} w_{i2}r_{ij2} + \sum_{i=1}^{m} w_{i1}r_{ij1}}{4} \tag{3.6.2}$$

Similarly, the right divergence is given by (3.6.3).

$$\varepsilon_{Rj} = \left[\frac{\begin{array}{l}\sum_{i=1}^{m} w_{i3}r_{ij3} + [\sum_{i=1}^{m} w_{i3}r_{ij2} - 2\sum_{i=1}^{m} w_{i3}r_{ij3} + \sum_{i=1}^{m} w_{i2}r_{ij3}]\alpha + \\ [\sum_{i=1}^{m} w_{i3}r_{ij3} - \sum_{i=1}^{m} w_{i3}r_{ij2} - \sum_{i=1}^{m} w_{i2}r_{ij3} + \sum_{i=1}^{m} w_{i2}r_{ij2}]\alpha^2\end{array}}{\sum_{i=1}^{m} w_{i3}r_{ij3} + [\sum_{i=1}^{m} w_{i3}r_{ij3} - \sum_{i-1}^{m} w_{i2}r_{ij2}]\alpha)}\right] \tag{3.6.3}$$

The maximum right divergences occurs at $\alpha = \frac{1}{2}$, therefore, substitute $\alpha = \frac{1}{2}$ in (3.6.3).

The maximum right divergence for $j = 1, 2, \ldots, n$ is given by

$$\text{Max}\_\varepsilon_{Rj} = \frac{\sum_{i=1}^{m} w_{i3} r_{ij1} + \sum_{i=1}^{m} w_{i2} r_{ij3} + \sum_{i=1}^{m} w_{i2} r_{ij2} + \sum_{i=1}^{m} w_{i3} r_{ij3}}{4} \qquad (3.6.4)$$

We will be using Max $\_\varepsilon_{Lj}$ and Max $\_\varepsilon_{Rj}$ for j = 1, 2, …, n, in the example below.

3.7 Facility Location Problem: A Crisp Rating Model

For the purpose of comparing the crisp approach with the fuzzy approach we first consider a numerical example under crisp environment.

Assuming that an industrial engineer is faced with the decision of location of a new facility, and the choices is to be made from the following locations:

- Location 1 (L1)
- Location 2 (L2)
- Location 3 (L3)
- Location 4 (L4)

For the sake of completion, we first demonstrate the crisp model [24, 25] given in Section 1.7.1.

A summary of the ratings assigned to the different criteria and decision alternatives is provided in Table 3.1, and a summary of the weights assigned to different criteria are provided in the Table 3.2.

There are 8 criteria that related to the facility location problem; these criteria have been taken from [14].

- **Travel distance:** Depends on location (starting and ending points of trips).

- **Travel cost:** This must be optimized
- **Political decision.**
- **Convenience of access:** For each location, there is a distance between the location and the other facility, so the closer the distance to Li the more convenient will be the access of Li to that distance.
- **Material handing cost** is very important in multifacility location problems, the longer a material handling transport, the more it can cost.
- **Working condition** consists of the size, comfort, car parking, …, etc.
- **Cost of renting and maintenance.**
- **Other characteristics**: There are some characteristics that may effect the decision of the new facility location; such characteristics are termed as `other characteristics'.

Table 3.1. Crisp Ratings for Each Location Criterion

Criterion	L1	L2	L3	L4
Travel distance	9	8	9	7
Travel cost	8	7	7	8
Political decision	6	7	5	6
Convenience of access	5	6	6	6
Material handling cost	5	5	5	5
Working conditions	4	5	4	4
Cost of renting& maintenance	5	4	5	6
Other characteristics	5	5	5	5

Before assigning the weights to different criteria we should first have a standard scale of the level of importance which is provided in the following table:

Level of importance	**Scale**
Extremely important	8
Very important	7
Important	6
Moderately important	5
Average important	4
Somewhat important	3
Somewhat unimportant	2
Unimportant	1

Table 3.2. Crisp Weights for each Location Criterion

Criterion	**Level of importance**	**Weight**
Travel distance	Extremely important	8
Travel cost	Extremely important	8
Political decision	Somewhat important	3
Convenience of access	Average important	4
Material handling cost	Somewhat unimportant	2
Working conditions	Average important	4
Cost of renting& maintenance	Somewhat important	3
Other characteristics	Average important	4

Table 3.3 below, provides a summary of the rating value for each of the sub-criterion for Location 1, Location 2, Location 3 and Location 4.

Table 3.3 Computation of Crisp Rates for Location 1, 2, 3 and 4

Criterion	Weight	L (1)	L (2)	L (3)	L (4)
Travel distance	8	9	8	9	7
Travel cost	8	8	7	7	8
Political decision	3	6	7	5	6
Convenience of access	4	5	6	6	6
Material handling cost	2	5	5	5	5
Working conditions	4	4	5	4	4
Cost of R&M	3	5	4	5	6
Other characteristics	4	5	5	5	5
Total rate	-	235	227	228	226

Comparing the results in Table 3.3, we see that the highest crisp rate of 235 corresponds to the Location1. Thus, under crisp environment, Location1 is the recommended decision alternative.

3.8 Facility Location Problem: A Fuzzy Rating Model

We now consider the model in which each of the Locations has their weights and ratings given in terms of T.F.N.'s in place of crisp value. We consider each of them one by one. Various ranking procedures [7, 10, 23, 28] are used to rank the different alternatives in fuzzy environment.

- **Case 1: Location 1 (L1).**

We assume that the fuzzy ratings and the fuzzy weights for the L1 are given in the form of T.F.N.'s in Tables 3.4 and 3.5 respectively, along with their α cuts.

Table 3.4. Fuzzy Rating for Location 1 (L1)

Criterion	Rating	Rating (α)
Travel distance(α)	(8 , 9, 10)	(8 + α , 10 – α)
Travel cost (α)	(7, 8 , 9)	(7 + α , 9 – α)
Political decision (α)	(5, 6, 7)	(5 + α , 7 – α)
Convenience of access (α)	(4, 5 ,6)	(4 + α , 6 – α)
Material handling cost (α)	(4, 5 ,6)	(4 + α , 6 – α)
Working conditions (α)	(3, 4 ,5)	(3 + α , 5 – α)
Cost of R&M (α)	(4, 5 ,6)	(4 + α , 6 – α)
Other characteristics (α)	(4, 5 ,6)	(4 + α , 6 – α)

Table 3.5. Fuzzy Weights for Each Location Criterion

Criterion	Level of importance	Weight	Weight (α)
Travel distance	Extremely important	(7, 8 , 9)	(7+ α , 9– α)
Travel cost	Extremely important	(7, 8 , 9)	(7+ α , 9– α)
Political decision	Somewhat important	(2, 3 ,4)	(2+ α , 4– α)
Convenience of access	Average important	(3, 4 ,5)	(3+ α , 5– α)
Material handling cost	Somewhat unimportant	(1, 2 ,3)	(1+ α , 3– α)
Working conditions	Average important	(3, 4 ,5)	(3+ α , 5– α)
Cost of renting& maintenance	Somewhat important	(2, 3 ,4)	(2+ α , 4– α)
Other characteristics	Average important	(3, 4 ,5)	(3+ α , 5– α)

If we plug in values of the rating (Table 3.4) and weight (Table 3.5) in the formula of multiplication we end up with values in Table 3.6

Table 3.6. Computation of Fuzzy Rates for Location1 (L1)

Criterion	Weight (α)	Rating (α)	Score (α)= Weight(α)* Rating(α)
Travel distance	$(7+\alpha, 9-\alpha)$	$(8+\alpha, 10-\alpha)$	$(56+15\alpha+\alpha^2, 90-19\alpha+\alpha^2)$
Travel cost	$(7+\alpha, 9-\alpha)$	$(7+\alpha, 9-\alpha)$	$(49+14\alpha+\alpha^2, 81-18\alpha+\alpha^2)$
Political decision	$(2+\alpha, 4-\alpha)$	$(5+\alpha, 7-\alpha)$	$(10+7\alpha+\alpha^2, 28-11\alpha+\alpha^2)$
Convenience of access	$(3+\alpha, 5-\alpha)$	$(4+\alpha, 6-\alpha)$	$(12+7\alpha+\alpha^2, 30-11\alpha+\alpha^2)$
Material handling cost	$(1+\alpha, 3-\alpha)$	$(4+\alpha, 6-\alpha)$	$(4+5\alpha+\alpha^2, 18-9\alpha+\alpha^2)$
Working conditions	$(3+\alpha, 5-\alpha)$	$(3+\alpha, 5-\alpha)$	$(9+6\alpha+\alpha^2, 25-10\alpha+\alpha^2)$
Cost of R&M	$(2+\alpha, 4-\alpha)$	$(4+\alpha, 6-\alpha)$	$(8+6\alpha+\alpha^2, 24-10\alpha+\alpha^2)$
Other characteristics	$(3+\alpha, 5-\alpha)$	$(4+\alpha, 6-\alpha)$	$(12+7\alpha+\alpha^2, 30-11\alpha+\alpha^2)$
Total fuzzy rate (α)=	***Weight (α)***	***Rating (α)***	$(160+67\alpha+8\alpha^2, 326-99\alpha+8\alpha^2)$

Defining the interval of confidence at level α, the total fuzzy rate is a parabolic fuzzy number as follows.

Total Fuzzy Rate$_{L1}$ $(\alpha) = (160+67\alpha+8\alpha^2, 326-99\alpha+8\alpha^2)\ \forall\alpha \in [0,1]$ (3.8.1)

The membership function for the Location 1 with parabolic curvature is given as shown below.

$$\mu_{L1(rate)} = \begin{cases} 0 & \text{if} \quad \text{rate} \leq 160 \\ \dfrac{-67+\sqrt{32\ rate-631}}{16} & \text{if} \quad 160 \leq \text{rate} \leq 235 \\ \dfrac{99-\sqrt{32\ rate-631}}{16} & \text{if} \quad 235 \leq \text{rate} \leq 326 \\ 0 & \text{if} \quad \text{rate} \geq 326 \end{cases}$$

Now if we let $\alpha = 0$ and $\alpha = 1$, in the equation (3.8.1) we get in terms of a fuzzy number the total score for the L1.

Total Fuzzy Rate$_{L1}$ = (160 , 235, 326)

Approximating the **Total Fuzzy Rate**$_{L1}$ by a T.F.N., we obtain the following α -cut.

Total Fuzzy Rate$_{L1}$ $(\alpha) = (160 + 75\,\alpha,\ 326 - 91\,\alpha) \qquad \forall \alpha \in [0,1]$ (3.8.2)

The membership function for the approximated Total Fuzzy rate$_{L1}$ is given as below.

$$\mu_{L1(rate)} = \begin{cases} 0 & if \quad rate \le 160 \\ \dfrac{rate - 160}{75} & if \quad 160 \le rate \le 235 \\ \dfrac{326 - rate}{91} & if \quad 235 \le rate \le 326 \\ 0 & if \quad rate \ge 326 \end{cases}$$

The error analysis for Location 1 (L1) is provided in Table 3.7, this table consists of 8 columns, and each column represents the following:

Column 1 = the left part of Equation (3.8.1) for $\forall \alpha \in [0,1]$

Column 2 = the right part of Equation (3.8.1) for $\forall \alpha \in [0,1]$

Column 3 = the left part of Equation (3.8.2) for $\forall \alpha \in [0,1]$ which is the approximated T.F.N.

Column 4 = the right part of Equation (3.8.2) for $\forall \alpha \in [0,1]$ which is the approximated T.F.N.

Column 5 = the difference between Column 3 and Column 1.

Column 6 = the difference between Column 4 and Column 2.

Column 7 = computation of right percentage error % $\varepsilon_R = \left| \dfrac{column4 - column2}{column2} \right| \times 100$

Column 8 = computation of left percentage error % $\varepsilon_L = \left| \dfrac{column3 - column1}{column1} \right| \times 100$

Table 3.7. Error analysis for Location 1 (L1)

-	1	2	3	4	5	6	7	8
α	Left Rating	Right Rating	Left Rating	Right Rating	Error	Error	% Error	% Error
-	Curvature	Curvature	T.F.N	T.F.N	(3-1)	(4-2)	-	-
0	160	326	160	326	0	0	0	0
0.1	166.78	316.18	167.5	316.9	0.72	0.72	0.2	0.4
0.2	173.72	306.52	175	307.8	1.28	1.28	0.4	0.7
0.3	180.82	297.02	182.5	298.7	1.68	1.68	0.6	0.9
0.4	188.08	287.68	190	289.6	1.92	1.92	0.7	1.0
0.5	195.5	278.5	197.5	280.5	2	2	0.7	1.0
0.6	203.08	269.48	205	271.4	1.92	1.92	0.7	0.9
0.7	210.82	260.62	212.5	262.3	1.68	1.68	0.6	0.8
0.8	218.72	251.92	220	253.2	1.28	1.28	0.5	0.6
0.9	226.78	243.38	227.5	244.1	0.72	0.72	0.3	0.3
1	235	235	235	235	0	0	0.0	0.0

In this case we see that the left and right maximum percent error is small. Therefore, using the T.F.N. approximation in this case is useful. Graph 3.7 in Appendix 1 shows the Total Fuzzy Rate$_{L1}$ and the approximated T.F.N. We observe that the segments of the multiplication corresponding to the Total Fuzzy Rate$_{L1}$ are not straight lines.

Case 2: Location 2 (L2).

We assume that the fuzzy ratings and the fuzzy weights for the L2 are given in the form of T.F.N.'s in Table 3.8 and Table 3.5 respectively, along with their α cuts.

Table 3.8: Fuzzy Ratings for Location 2 (L2)

Criterion	Rating	Rating (α)
Travel distance(α)	(7, 8, 9)	$(7+\alpha, 9-\alpha)$
Travel cost (α)	(6, 7, 8)	$(6+\alpha, 8-\alpha)$
Political decision (α)	(6, 7, 8)	$(6+\alpha, 8-\alpha)$
Convenience of access (α)	(5, 6, 7)	$(5+\alpha, 7-\alpha)$
Material handling cost (α)	(4, 5 ,6)	$(4+\alpha, 6-\alpha)$
Working conditions (α)	(4, 5 ,6)	$(4+\alpha, 6-\alpha)$
Cost of R&M (α)	(3, 4, 5)	$(3+\alpha, 5-\alpha)$
Other characteristics (α)	(4, 5, 6)	$(4+\alpha, 6-\alpha)$

If we plug in values of the ratings (Table 3.8) and weight (Table 3.5) in the formula of multiplication we end up with values in Table 3.9.

Table 3.9: Computation of Fuzzy Rates for Location 2 (L2)

Criterion	Weight (α)	Rating (α)	Rate(α)=Weight(α)*Rating(α)
Travel distance	$(7+\alpha, 9-\alpha)$	$(7+\alpha, 9-\alpha)$	$(49+14\alpha+\alpha^2, 81-18\alpha+\alpha^2)$
Travel cost	$(7+\alpha, 9-\alpha)$	$(6+\alpha, 8-\alpha)$	$(42+13\alpha+\alpha^2, 72-17\alpha+\alpha^2)$
Political decision	$(2+\alpha, 4-\alpha)$	$(6+\alpha, 8-\alpha)$	$(12+8\alpha+\alpha^2, 32-12\alpha+\alpha^2)$
Convenience of access	$(3+\alpha, 5-\alpha)$	$(5+\alpha, 7-\alpha)$	$(15+8\alpha+\alpha^2, 35-12\alpha+\alpha^2)$
Material handling cost	$(1+\alpha, 3-\alpha)$	$(4+\alpha, 6-\alpha)$	$(4+5\alpha+\alpha^2, 18-9\alpha+\alpha^2)$
Working conditions	$(3+\alpha, 5-\alpha)$	$(4+\alpha, 6-\alpha)$	$(12+7\alpha+\alpha^2, 30-11\alpha+\alpha^2)$
Cost of R&M	$(2+\alpha, 4-\alpha)$	$(3+\alpha, 5-\alpha)$	$(6+5\alpha+\alpha^2, 20-9\alpha+\alpha^2)$
Other characteristics	$(3+\alpha, 5-\alpha)$	$(4+\alpha, 6-\alpha)$	$(12+7\alpha+\alpha^2, 30-11\alpha+\alpha^2)$
Total fuzzy rate (α)=	***Weight (α)***	***Rating (α)***	**$(152+67\alpha+8\alpha^2, 318-99\alpha+8\alpha^2)$**

Defining the interval of confidence at level α, the total fuzzy rate is a parabolic fuzzy number as follows.

$$\textbf{Total Fuzzy Rate}_{L2}\,(\alpha) = (152+67\alpha+8\alpha^2,\ 318-99\alpha+8\alpha^2)\quad \forall\alpha\in[0,1] \qquad (3.8.3)$$

The membership function for the Location 2 with parabolic curvature is given as shown below.

$$\mu_{L2(rate)} = \begin{cases} 0 & \text{if} \quad \text{rate} \le 152 \\ \dfrac{-67+\sqrt{32rate-375}}{16} & \text{if} \quad 152 \le \text{rate} \le 227 \\ \dfrac{99-\sqrt{32rate-375}}{16} & \text{if} \quad 227 \le \text{rate} \le 318 \\ 0 & \text{if} \quad \text{rate} \ge 318 \end{cases}$$

Now if we let $\alpha = 0$ and $\alpha = 1$, in the equation (3.8.3) we get in terms of a fuzzy number the total rate for the L2.

Total Fuzzy Rate$_{L2}$= (152, 227, 318)

Approximating the **Total Fuzzy Rate**$_{L2}$ by a T.F.N., we obtain the following α -cut.

Total Fuzzy Rate$_{L2}$ (α) = (152+75 α, 318 – 91 α) $\forall\alpha \in [0,1]$ (3.8.4)

The memberships function for the approximated Total Fuzzy Rate$_{L2}$ is given as below.

$$\mu_{L2(rate)} = \begin{cases} 0 & if \quad \text{rate} \le 152 \\ \dfrac{rate-152}{75} & if \quad 152 \le \text{rate} \le 227 \\ \dfrac{318-\text{rate}}{91} & if \quad 227 \le \text{rate} \le 318 \\ 0 & if \quad \text{rate} \ge 318 \end{cases}$$

The error analysis for location 2 (L2) are provided in table 3.10, this table consists of 8 columns, each Column represents the following:

Column 1 = the left part of Equation (3.8.3) for $\forall\alpha \in [0,1]$

Column 2 = the right part of Equation (3.8.3) for $\forall\alpha \in [0,1]$

Column 3 = the left part of Equation (3.8.4) for $\forall\alpha \in [0,1]$ which is the approximated T.F.N.

Column 4 = the right part of Equation (3.8.4) for $\forall\alpha \in [0,1]$ which is the approximated T.F.N.

Column 5 = the difference between Column 3 and column 1.

Column 6 = the difference between Column 4 and column 2.

Column 7 = computation of right percentage error % $\varepsilon_R = \left|\dfrac{column4 - column2}{column2}\right| \times 100$

Column 8 = computation of left percentage error % $\varepsilon_L = \left|\dfrac{column3 - column1}{column1}\right| \times 100$

Table 3.10. Error analysis for Total Fuzzy Scores Location 2 (L2)

-	1	2	3	4	5	6	7	8
α	Left Rating	Right Rating	Left Rating	Right Rating	Error	Error	% Error	% Error
-	Curvature	Curvature	T.F.N	T.F.N	(3-1)	(4-2)	-	-
0	152	318	152	318	0	0	0	0
0.1	158.78	308.18	159.5	308.9	0.72	0.72	0.2	0.5
0.2	165.72	298.52	167	299.8	1.28	1.28	0.4	0.8
0.3	172.82	289.02	174.5	290.7	1.68	1.68	0.6	1.0
0.4	180.08	279.68	182	281.6	1.92	1.92	0.7	1.1
0.5	187.5	270.5	189.5	272.5	2	2	0.7	1.1
0.6	195.08	261.48	197	263.4	1.92	1.92	0.7	1.0
0.7	202.82	252.62	204.5	254.3	1.68	1.68	0.7	0.8
0.8	210.72	243.92	212	245.2	1.28	1.28	0.5	0.6
0.9	218.78	235.38	219.5	236.1	0.72	0.72	0.3	0.3
1	227	227	227	227	0	0	0.0	0.0

In this case we see that the left and right maximum percent error is small. Therefore, using the T.F.N. approximation in this case is useful. Graph 3.10 in Appendix 1 shows the Total Fuzzy Rate$_{L2}$ and the approximated T.F.N. We observe that the segments of the multiplication corresponding to the Total Fuzzy Rate$_{L2}$ are not straight lines.

Case 3: Location 3 (L3):

We assume that the fuzzy ratings and the fuzzy weights for the L3 are given in the form of T.F.N.'s in Tables 3.11 and 3.5 respectively, along with their α cuts.

Table 3.11. Fuzzy Ratings for Location 3 (L3)

Criterion	**Rating**	**Rating (α)**
Travel distance(α)	(8 , 9, 10)	(8+ α , 10 – α)
Travel cost (α)	(6, 7 , 8)	(6 + α , 8 – α)
Political decision (α)	(4, 5 ,6)	(4+ α , 6 – α)
Convenience of access (α)	(5, 6, 7)	(5+ α , 7 – α)
Material handling cost (α)	(4, 5 ,6)	(4+ α , 6 – α)
Working conditions (α)	(3, 4 ,5)	(3+ α , 5 – α)
Cost of R&M (α)	(4, 5 ,6)	(4+ α , 6 – α)
Other characteristics (α)	(4, 5 ,6)	(4+ α , 6 – α)

If we plug in values of the ratings (Table 3.11) and weights (Table 3.5) in the formula of multiplication we end up with values in Table 3.12 below.

Table 3.12. Computation of Fuzzy Rates for Location 3 (L3)

Criterion	Weight (α)	Rating (α)	Score (α)=Weight(α)*Rating(α)
Travel distance	$(7+\alpha, 9-\alpha)$	$(8+\alpha, 10-\alpha)$	$(56+15\alpha+\alpha^2, 90-19\alpha+\alpha^2)$
Travel cost	$(7+\alpha, 9-\alpha)$	$(6+\alpha, 8-\alpha)$	$(42+13\alpha+\alpha^2, 72-17\alpha+\alpha^2)$
Political decision	$(2+\alpha, 4-\alpha)$	$(4+\alpha, 6-\alpha)$	$(8+6\alpha+\alpha^2, 24-10\alpha+\alpha^2)$
Convenience of access	$(3+\alpha, 5-\alpha)$	$(5+\alpha, 7-\alpha)$	$(15+8\alpha+\alpha^2, 35-12\alpha+\alpha^2)$
Material handling cost	$(1+\alpha, 3-\alpha)$	$(4+\alpha, 6-\alpha)$	$(4+5\alpha+\alpha^2, 18-9\alpha+\alpha^2)$
Working conditions	$(3+\alpha, 5-\alpha)$	$(3+\alpha, 5-\alpha)$	$(9+6\alpha+\alpha^2, 25-10\alpha+\alpha^2)$
Cost of R&M	$(2+\alpha, 4-\alpha)$	$(4+\alpha, 6-\alpha)$	$(8+6\alpha+\alpha^2, 24-10\alpha+\alpha^2)$
Other characteristics	$(3+\alpha, 5-\alpha)$	$(4+\alpha, 6-\alpha)$	$(12+7\alpha+\alpha^2, 30-11\alpha+\alpha^2)$
Total fuzzy rate (α)=	***Weight (α)***	***Rating (α)***	$(154+66\alpha+8\alpha^2, 318-98\alpha+8\alpha^2)$

Defining the interval of confidence at level α, the total fuzzy rate is a parabolic fuzzy number as follows.

Total Fuzzy Rate$_{L3}$ $(\alpha) = (154+66\alpha+8\alpha^2, 318-98\alpha+8\alpha^2)\ \ \forall \alpha \in [0,1]$ (3.8.5)

The membership function for the Location 3 with parabolic curvature is given as shown below.

$$\mu_{L3(\text{rate})} = \begin{cases} 0 & if \quad \text{rate} \le 154 \\ \dfrac{-33+\sqrt{8\text{rate}-143}}{8} & if \quad 154 \le \text{rate} \le 228 \\ \dfrac{49-\sqrt{8\text{rate}-143}}{8} & if \quad 228 \le \text{rate} \le 318 \\ 0 & if \quad \text{rate} \ge 318 \end{cases}$$

Now if we let $\alpha = 0$ and $\alpha = 1$, in the equation (3.8.5) we get in terms of a fuzzy number the total rate for the L3.

Total Fuzzy Rate$_{L3}$ = (154, 228, 318)

Approximating the **Total Fuzzy Rate**$_{L3}$ by a T.F.N., we obtain the following α -cut.

Total Fuzzy Rate$_{L3}$ (α) = $(154+74\,\alpha,\ 318-90\,\alpha)\ \forall\alpha\in[0,1]$ (3.8.6)

The memberships function for the approximated Total Fuzzy Rate$_{L3}$ is given as below.

$$\mu_{L3(\text{rate})}=\begin{cases}0 & \textit{if} \quad \text{rate}\le 154\\ \dfrac{\text{rate}-154}{74} & \textit{if} \quad 154\le\text{rate}\le 228\\ \dfrac{318-\text{rate}}{90} & \textit{if} \quad 228\le\text{rate}\le 318\\ 0 & \textit{if} \quad \text{rate}\ge 318\end{cases}$$

The error analyses for Location 3 (L3) are provided in table 3.13, this table consists of 8 columns, and each column represents the following:

Column 1 = the left part of Equation (3.8.5) for $\forall\alpha\in[0,1]$

Column 2 = the right part of Equation (3.8.5) for $\forall\alpha\in[0,1]$

Column 3 = the left part of Equation (3.8.6) for $\forall\alpha\in[0,1]$ which is the approximated T.F.N.

Column 4 = the right part of Equation (3.8.6) for $\forall\alpha\in[0,1]$ which is the approximated T.F.N.

Column 5 = the difference between Column 3 and Column 1.

Column 6 = the difference between Column 4 and Column 2.

Column 7 = computation of right percentage error % $\varepsilon_R=\left|\dfrac{column4-column2}{column2}\right|\times 100$

Column 8 = computation of left percentage error % $\varepsilon_L=\left|\dfrac{column3-column1}{column1}\right|\times 100$

Table 3.13. Error analysis for Location 3 (L3)

	1	2	3	4	5	6	7	8
α	Left Rating Curvature	Right Rating Curvature	Left Rating T.F.N	Right Rating T.F.N	Error (3-1)	Error (4-2)	% Error -	% Error -
0	154	318	154	318	0	0	0	0
0.1	160.68	308.28	161.4	309	0.72	0.72	0.2	0.4
0.2	167.52	298.72	168.8	300	1.28	1.28	0.4	0.8
0.3	174.52	289.32	176.2	291	1.68	1.68	0.6	1.0
0.4	181.68	280.08	183.6	282	1.92	1.92	0.7	1.1
0.5	189	271	191	273	2	2	0.7	1.1
0.6	196.48	262.08	198.4	264	1.92	1.92	0.7	1.0
0.7	204.12	253.32	205.8	255	1.68	1.68	0.7	0.8
0.8	211.92	244.72	213.2	246	1.28	1.28	0.5	0.6
0.9	219.88	236.28	220.6	237	0.72	0.72	0.3	0.3
1	228	228	228	228	0	0	0.0	0.0

In this case we see that the left and right maximum percent error is small. Therefore, using the T.F.N. approximation in this case is useful. Graph 3.13 in Appendix 1 shows the Total Fuzzy Rate$_{L3}$ and the approximated T.F.N. We observe that the segments of the multiplication corresponding to the Total Fuzzy Rate$_{L3}$ are not straight lines.

Case 4: Location 4 (L4).

We assume that the fuzzy ratings and the fuzzy weights for the L4 are given in the form of T.F.N.'s in Tables 3.14 and 3.5 respectively, along with their α cuts.

Table 3.14. Fuzzy Ratings for Location 4 (L4)

Criterion	Rating	Rating (α)
Travel distance(α)	(6, 7, 8)	(6+ α , 8 – α)
Travel cost (α)	(7, 8 , 9)	(7+ α , 9 – α)
Political decision (α)	(5, 6, 7)	(5+ α , 7 – α)
Convenience of access (α)	(5, 6, 7)	(5+ α , 7 – α)
Material handling cost (α)	(4, 5 ,6)	(4+ α , 6 – α)
Working conditions (α)	(3, 4 ,5)	(3+ α , 5 – α)
Cost of R&M (α)	(5, 6, 7)	(5+ α , 7 – α)
Other characteristics (α)	(4, 5 ,6)	(4+ α , 6 – α)

If we plug in values of the ratings (Table 3.14) and weight (Table 3.5) in the formula of multiplication we end up with values in Table 3.15.

Table 3.15. Computation of Fuzzy Ratings for Location 4 (L4)

Criterion	Weight (α)	Rating (α)	Score(α)=Weight(α)*Rating(α)
Travel distance	$(7+\alpha, 9-\alpha)$	$(6+\alpha, 8-\alpha)$	$(42+13\alpha+\alpha^2, 72-17\alpha+\alpha^2)$
Travel cost	$(7+\alpha, 9-\alpha)$	$(7+\alpha, -\alpha)$	$(49+14\alpha+\alpha^2, 81-18\alpha+\alpha^2)$
Political decision	$(2+\alpha, 4-\alpha)$	$(5+\alpha, 7-\alpha)$	$(10+7\alpha+\alpha^2, 28-11\alpha+\alpha^2)$
Convenience of access	$(3+\alpha, 5-\alpha)$	$(5+\alpha, 7-\alpha)$	$(15+8\alpha+\alpha^2, 35-12\alpha+\alpha^2)$
Material handling cost	$(1+\alpha, 3-\alpha)$	$(4+\alpha, 6-\alpha)$	$(4+5\alpha+\alpha^2, 18-9\alpha+\alpha^2)$
Working conditions	$(3+\alpha, 5-\alpha)$	$(3+\alpha, 5-\alpha)$	$(9+6\alpha+\alpha^2, 25-10\alpha+\alpha^2)$
Cost of R&M	$(2+\alpha, 4-\alpha)$	$(5+\alpha, 7-\alpha)$	$(10+7\alpha+\alpha^2, 28-11\alpha+\alpha^2)$
Other characteristics	$(3+\alpha, 5-\alpha)$	$(4+\alpha, 6-\alpha)$	$(12+7\alpha+\alpha^2, 30-11\alpha+\alpha^2)$
Total fuzzy rate (α)=	***Weight (α)***	***Rating (α)***	$(151+67\alpha+8\alpha^2, 317-99\alpha+8\alpha^2)$

Defining the interval of confidence at level α, the total fuzzy rate is a parabolic fuzzy number as follows.

Total Fuzzy Rate$_{L4}$ $(\alpha) = (151+67\alpha+8\alpha^2, 317-99\alpha+8\alpha^2) \quad \forall \alpha \in [0,1]$ (3.8.7)

The membership function for the Location 4 with parabolic curvature is given as shown below.

$$\mu_{L4(rate)} = \begin{cases} 0 & \text{if} \quad rate \le 151 \\ \dfrac{-67+\sqrt{32rate-343}}{16} & \text{if} \quad 151 \le rate \le 226 \\ \dfrac{99-\sqrt{32rate-343}}{16} & \text{if} \quad 226 \le rate \le 317 \\ 0 & \text{if} \quad rate \ge 317 \end{cases}$$

Now if we let $\alpha = 0$ and $\alpha = 1$, in the equation (3.8.7) we get in terms of a fuzzy number the total rate for the L4.

Total Fuzzy Rate$_{L4}$= (151,226,317)

Approximating the **Total Fuzzy Rate**$_{L4}$ by a T.F.N., we obtain the following α -cut.

Total Fuzzy Rate$_{L4}$ $(\alpha) = (151+75\,\alpha,\ 317 - 91\,\alpha)\ \forall \alpha \in [0,1]$ (3.8.8)

The memberships function for the approximated Total Fuzzy Rate$_{L4}$ is given as below.

$$\mu_{L4(rate)} = \begin{cases} 0 & if \quad rate \le 151 \\ \dfrac{rate-151}{75} & if \quad 151 \le rate \le 226 \\ \dfrac{317-rate}{19} & if \quad 226 \le rate \le 317 \\ 0 & if \quad rate \ge 317 \end{cases}$$

The error analysis for location 4 (L4) are provided in table 3.16, this table consists of 8 columns, each column represent the following:

Column 1 = the left part of Equation (3.8.7) for $\forall \alpha \in [0,1]$

Column 2 = the right part of Equation (3.8.7) for $\forall \alpha \in [0,1]$

Column 3 = the left part of Equation (3.8.8) for $\forall \alpha \in [0,1]$ which is the approximated T.F.N.

Column 4 = the right part of Equation (3.8.8) for $\forall \alpha \in [0,1]$ which is the approximated T.F.N.

Column 5 = the difference between Column 3 and Column 1.

Column 6 = the difference between Column 4 and Column 2.

Column 7 = computation of right percentage error % $\varepsilon_R = \left|\dfrac{column4 - column2}{column2}\right| \times 100$

Column 8 = computation of left percentage error % $\varepsilon_L = \left|\dfrac{column3 - column1}{column1}\right| \times 100$

Table 3.16. Error analysis for Location 4 (L4)

	1	2	3	4	5	6	7	8
α	Left Rating	Right Rating	Left Rating	Right Rating	Error	Error	% Error	% Error
	Curvature	Curvature	T.F.N	T.F.N	(3-1)	(4-2)	-	-
0	151	317	151	317	0	0	0	0
0.1	157.78	307.18	158.5	307.9	0.72	0.72	0.2	0.5
0.2	164.72	297.52	166	298.8	1.28	1.28	0.4	0.8
0.3	171.82	288.02	173.5	289.7	1.68	1.68	0.6	1.0
0.4	179.08	278.68	181	280.6	1.92	1.92	0.7	1.1
0.5	186.5	269.5	188.5	271.5	2	2	0.7	1.1
0.6	194.08	260.48	196	262.4	1.92	1.92	0.7	1.0
0.7	201.82	251.62	203.5	253.3	1.68	1.68	0.7	0.8
0.8	209.72	242.92	211	244.2	1.28	1.28	0.5	0.6
0.9	217.78	234.38	218.5	235.1	0.72	0.72	0.3	0.3
1	226	226	226	226	0	0	0.0	0.0

In this case we see that the left and right maximum percent error is small. Therefore, using the T.F.N. approximation in this case is useful. Graph 3.16 in Appendix 1 shows the Total Fuzzy Rate$_{L4}$ and the approximated T.F.N. We observe that the segments of the multiplication corresponding to the Total Fuzzy Rate$_{L4}$ are not straight lines.

3.9 Facility Location Selection

Since the error analysis clearly shows that the maximum values of errors and the percentage errors between the parabolic fuzzy numbers and the approximated fuzzy numbers are small, therefore, for selecting the best location, we rank the approximated T.F.N.'s using various ranking techniques given in Chapter 1.

Ranking Method 1. Kaufmann & Gupta [22 23]:

$$(AON)_i = \frac{a_{i1} + 2a_{i2} + a_{i3}}{4} \quad \text{for } i = L1, L2, L3, L4$$

Table 3.17: Rating for Decision Criteria and Decision Alternative.

Criterion	Fuzzy Number	AON
Location1 (L1)	(160, 235, 326)	239
Location2 (L2)	(152, 227, 318)	231
Location3 (L3)	(154, 228, 318)	232
Location4 (L4)	(151, 226, 317)	230

We observe that in this case the L1 is the Location with the highest index 239.

Ranking Method 2. Chang's Ranking Index (Komolanij [28]):

$$Index_i = \frac{(a_{i3} - a_{i1})(a_{i1} + a_{i2} + a_{i3})}{6} \quad \text{for } i = \text{L1 , L2, L3, L4}$$

Table 3.18: Rating for Decision Criteria and Decision Alternative.

Criterion	Fuzzy Number	$\mathbf{Index_i}$
Location1 (L1)	(160, 235, 326)	19948
Location2 (L2)	(152, 227, 318)	19284
Location3 (L3)	(154, 228, 318)	19133
Location4 (L4)	(151, 226, 317)	19201

We observe that in this case the L1 is the location with the highest index 19948.

Ranking Method 3: Chiu and Park Ranking Index (Chiu and Park [7])

Let w_1 and w_2 represents the weights associated with the fuzzy number whose ranking index has to be computed. Then,

$$Index_i = \frac{(a_{i1} + a_{i2} + a_{i3})w_{i1}}{3} + w_{i2}a_{i2} \quad \text{for } i = L1, L2, L3, L4$$

w_{i1} and w_{i2} are the weights.

Chui and Park believe that the weight w_{i2} should be taken as 1 and the weight w_{i1} should range between 0 and 1 and the most likely possible value is 0.5. $Index_i$ can be rewritten as:

$$Index_i = 0.5\left[\frac{(a_{i1} + a_{i2} + a_{i3})}{3}\right] + a_{i2}$$

Table 3.19: Rating for Decision Criteria and Decision Alternative.

Criterion	Fuzzy Number	$Index_i$
Location1 (L1)	(160, 235, 326)	355
Location2 (L2)	(152, 227, 318)	343
Location3 (L3)	(154, 228, 318)	345
Location4 (L4)	(151, 226, 317)	342

We observe that in this case the L1 is the location with the highest index 355.

Ranking Method 4: Chui and Park weighted Ranking Index 2 [10]:

$$Index_i = w_{i1}\left[\frac{(a_{i1} + a_{i3})}{2}\right] + w_{i2}a_{i2} \quad \text{for } i = L1, L2, L3, L4$$

where w_{i1} represents the weight associated with the values of a_{i1} and a_{i3}, and w_{i2} represents the weight associated with a_{i2}. Chui and Park believe that the weight w_{i2} should be taken as 1 and the weight w_{i1} should range between 0 and 1 and the most likely possible value is 0.5. In this case, $Index_i$ can be rewritten as:

$$\text{Index}_i = 0.5\left[\frac{(a_{i1}+a_{i3})}{2}\right]+a_{i2}$$

Table 3.20: Rating for Decision Criteria and Decision Alternative.

Criterion	Fuzzy Number	Index_i
Location1 (L1)	(160, 235, 326)	356.5
Location2 (L2)	(152, 227, 318)	344.5
Location3 (L3)	(154, 228, 318)	346
Location4 (L4)	(151, 226, 317)	343

We observe that in this case the L1 is the location with the highest index 356.5.

Thus, keeping in view the fact that according to the ranking criteria, the T.F.N. with the highest rank is the best alternative, we observe that all the four methods used above yield that the location1 are the most attractive locations.

3.10 Conclusion

Most rating problems deal with future and often uncertain and imprecise data. To cope this uncertainty, fuzzy sets theory has been applied to the traditional deterministic approach for rating models by dealing with uncertain data using triangular fuzzy numbers. The main contribution of this chapter is the application of fuzzy set theory to the rating problem and its application in the selection of Facility Location Problem. A numerical example of a facility location selection problem with fuzzy data is considered using the fuzzy rating model.

CHAPTER 4

MULTISTAGE PLANNING PROBLEM UNDER CRISP AND FUZZY ENVIRONMENTS

Multistage planning problem under crisp environment has been considered by several researchers [11, 15, 17, 21, 35, 38, 39, 40, 41]. In the present chapter, we investigate, formulate and solve the problem under fuzzy environment and compare the results obtained with the results obtained under crisp environment. The multistage planning problem in this chapter is formulated as a linear programming problem under both crisp and fuzzy environment. First we obtain the solution of linear programming problem under crisp environment. Then, we formulate and solve the problem in a fuzzy environment, and compare the results obtained under both crisp and fuzzy environment.

4.1 Introduction

As already stated in Section 1.11, the production systems consist of either single stage, or multistage. Multiple facilities are arranged in a multistage structure so that the output from one stage becomes the input to a following stage [21]. The key to modeling a multistage system are the decisions about what groupings of production operations constitute the stages in the system, whether or not each stage is to have multiple facilities operating in parallel or series, and how many inventory points between stages are to be defined. Mathematical programming models, particularly linear programming models, are commonly used to analyze multistage systems.

The main contribution of the present chapter is the formulation of a multistage production problem under fuzzy environment as a fuzzy linear program and compares the results obtained with the one formulated under crisp environment. We demonstrate the use of models by using the numerical example given in Johnson [21, p. 141].

4.2 Linear Programming Formulation under Crisp Environment

4.2.1 Assumption

For this model, it assumed that all the data are known with certainty.

4.2.2 Notation

Let,

X_{jk} = the number of units of product at stage j produced by process k, (k=1,2,…, N), j = 1, 2, …, M

c_{jk} = unit variable production cost at stage j , if process k is used

d_{jl} = amount of resource l, available at stage j, l= 1 , 2, …, L

h_{jkl} = amount of resource l used to produce one unit at stage j by process k.

Z = total production cost for the period under consideration.

4.2.3 Objective

The objective in this problem is to minimize the total cost of production for the period.

4.2.4 General Formulation

Thus, under the crisp environment, we have the following problem (CLP), [21].

(CLP) $$\text{Min } Z = \sum_{j=1}^{M} \sum_{k=1}^{k_j} c_{jk} X_{jk} \tag{4.2.4.1}$$

Subject to the following constraints:

(1) Resource limitations at each stage:

$$\sum_{k=1}^{k_j} h_{ikl} X_{jk} \le d_{jl} \qquad (j = 1, 2, \ldots, M;\ l = 1, 2, \ldots, L) \tag{4.2.4.2}$$

(2) Inventory balance between stages:

$$\sum_{k=1}^{k_j} X_{jk} = \sum_{k=1}^{k_{j+1}} X_{j+1,k}, \qquad (j = 1, 2, \ldots, M-1) \tag{4.2.4.3}$$

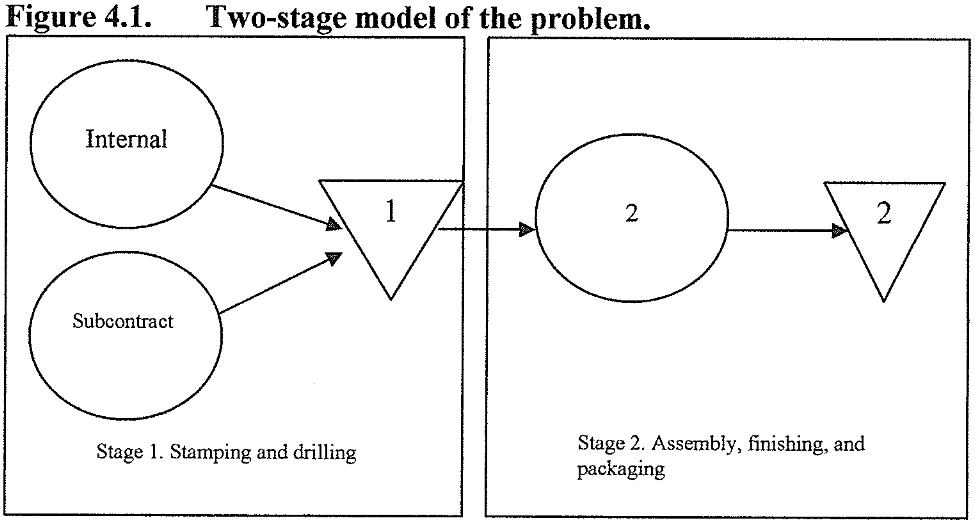
Non-negatively constraints:

$$X_{jk} \ge 0$$

4.2.5 Numerical Example under crisp Environment

We now illustrate the model (CLP) through the numerical example given by Johnson. [21, p.141].

Figure 4.1. Two-stage model of the problem.



The first stage consists of the production centers performing stamping and drilling, while the second consists of the production centers performing assembly, finishing, and packaging. The first stage is divided into two parallel facilities:

1. Internal stamping and drilling and
2. Subcontracted stamping and drilling.

The second stage is treated as a single facility.

Notations

Let,

W_i = the amount of product i stamped and drilled in the plant,

X_i = the amount of product i stamped and drilled by the subcontractor,

Y_{ij} = the amount of product i produced by process j in assembly, finishing, and packaging.

Process 1 (j = 1) = involves only regular time production,

Process 2 (j = 2) = requires that finishing be done on overtime.

a_i = the cost of stamping and drilling a unit of product i internally, including material cost,

b_i = the cost of obtaining a stamped and drilled unit of product i from the subcontractor,

c_{ij} = the unit cost of processing a unit of product i through stage 2 using process j.

The problem under crisp environment can be formulated as follows:

(CLP) $\text{Min } Z = \sum_{i=1}^{4}(a_iW_i + b_iX_i + c_{i1}Y_{i1} + c_{i2}Y_{i2}) =$

$6W_1 + 15\ W_2 + 11\ W_3 + 14\ W_4 + 7.2\ X_1 + 18\ X_2 + 13.2\ X_3 + 16.8\ X_4 + 6.2\ Y_{11} + 15.4\ Y_{21}$

$+11.2\ Y_{31} + 14.3\ Y_{41} + 7.4\ Y_{12} + 18.4\ Y_{22} + 13.4\ Y_{32} + 17.1\ Y_{42}$

Subject to the following constraints:

(1) Stage 1 Capacity:

Stamping: $0.03\ W_1 + 0.15\ W_2 + .05\ W_3 + 0.1\ W_4 \leq 400$

Drilling: $0.06\ W_1 + 0.12\ W_2 + 0.10\ W_4 \leq 400$

Metal: $2.0\ W_2 + 1.2\ W_4 \leq 2000$

(2) Stage 2 Capacity:

Assembly: $\sum_{j=1}^{2} (0.05Y_{1j} + 0.10Y_{2j} + .05Y_{3j} + 0.12Y_{4j}) \leq 500$

Finishing: $0.04Y_{11} + 0.20\ Y_{21} + 0.03\ Y_{31} + 0.12\ Y_{41} \leq 450$

$0.04Y_{12} + 0.20\ Y_{22} + 0.03\ Y_{32} + 0.12\ Y_{42} \leq 100$

Packaging: $\sum_{j=1}^{2} (0.02Y_{1j} + 0.06Y_{2j} + .02Y_{3j} + 0.05Y_{4j})) \leq 400$

(3) Inventory Balance Equations:

Stage 1: $W_1 + X_1 = Y_{11} + Y_{12}$

$W_2 + X_2 = Y_{21} + Y_{22}$

$W_3 + X_3 = Y_{31} + Y_{32}$

$W_4 + X_4 = Y_{41} + Y_{42}$

Stage 2: $Y_{11} + Y_{12} = 3000$

$Y_{21} + Y_{22} = 500$

$$Y_{31} + Y_{32} = 1000$$

$$Y_{41} + Y_{42} = 2000$$

Non-negativity constraints:

$$W_i \geq 0, \quad X_i \geq 0, \quad Y_{ij} \geq 0 \qquad i, j = 1, 2, 3, 4.$$

4.2.6 Results

Solving the above problem using Microsoft excel, we obtain the following Table 4.1.

Table 4.1 Results of crisp linear programming problem.

Variable	Value	Variable	Value	Variable	Value
W_1	3000	X_2	500	Y_{21}	500
W_3	1000	X_4	333	Y_{31}	1000
W_4	1667	Y_{11}	3000	Y_{41}	2000

The utilization of production capacity is shown in Table 4.2 below.

Table 4.2 Optimal Utilization of production capacity

	Regular time Hours			Overtime Hours		
Department	**Available**	**Scheduled**	**Unused**	**Available**	**Scheduled**	**Unused**
Stamping	400	306.7	93.3	0	0	0
Drilling	400	346.7	53.3	0	0	0
Assembly	500	490.0	10.0	0	0	0
Finishing	450	400.0	50.0	100	0	100
Packaging	400	210.0	190.0	0	0	0

4.2.7 Interpretation of the Results

As in [21], results given in the above table summarize the solution of the crisp problem. All 2000 square feet of sheet metal are used in producing product 4. This causes all of product 2 and some of product 4 to be produced from the subcontractor.

The values of the dual variables corresponding to the regular time finishing capacity constraint and the sheet metal constraint are 2.00 and 2.33, respectively. This means that an extra hour of regular time finishing capacity would permit $2 reduction in total cost and that an extra square foot of sheet metal would result in a $2.33 reduction in total cost.

4.3 Formulation under Fuzzy Environment

In general, most of the time, due to incomplete or forecasted information, the data are imprecise. Problems of imprecise capacity, demand or data can be handled effectively by taking advantage of fuzzy set theory [4, 44, 46].

The characteristics that require it to be formulated in fuzzy environment are:

1. **Imprecise total cost limit levels.** The management provides an upper bound of the estimation of the total cost represented by objective function z_o over the entire planning horizon. The actual costs desired to stay below this upper bound. A tolerance that defines the dispersion of the total cost may be given in the form of a fraction of z_o.

2. **Imprecise capacity and production requirements.** The management can provide a tolerance level in the form of a fraction of imprecisely known capacity and production requirements that provides a range above and below the forecasted capacity and production requirements in which the actual capacity and production requirements are likely to occur.

We now formulate the problem under the following additional assumptions.

4.3.1 Additional Assumptions

(i) The total cost (value of objective function) stays possibly below an imprecisely given limit.

(ii) The demand and production requirements for all products are known imprecisely.

4.3.2 Objective

The objective of the model is to stay below an imprecisely stated upper bound on the total cost keeping in view the imprecise data for production requirements

4.3.3 Additional Notation

Let,

z_o = imprecisely known total cost limit specified by the management,

q_o = tolerance level associated with the imprecisely known total cost z_o.

q_j = tolerance level associated with the imprecisely known production requirements h_{ik}. for all stages.

μ_{of} = membership function associated with imprecisely known total cost z_o.

μ_{ikL} = membership function corresponding to lower side of the constraint associated with imprecisely known production requirement in each stage.

μ_{ikU} = membership function corresponding to upper side of the constraint associated with imprecisely known production requirement in each stage.

All other variables and symbols have the same meaning as in crisp formulation.

4.3.4 Formulation of Multistage Planning Problem under Fuzzy Environment

Using Zimmerman's notation Zimmermann [46], in a fuzzy environment, the following crisp constraints given by (4.2.4.3) is replaced as follows.

Fuzzy Inventory balance between stages:

$$\sum_{k=1}^{k_j} X_{jk} \cong \sum_{k=1}^{k_{j+1}} X_{j+1,k}, \qquad (j = 1, 2, \ldots, M\text{-}1)$$

which are further replaced by

$$\sum_{k=1}^{k_j} X_{jk} \gtrsim \sum_{k=1}^{k_{j+1}} X_{j+1,k}, \qquad (j = 1, 2, \ldots, M\text{-}1)$$

with μ_{ikL} as the corresponding membership function, and

$$\sum_{k=1}^{k_j} X_{jk} \lesssim \sum_{k=1}^{k_{j+1}} X_{j+1,k}, \qquad (j = 1, 2, \ldots, M\text{-}1)$$

with μ_{ikU} as the corresponding membership function, where $\gtrsim X_{j+1,k,}$ (or $\lesssim X_{j+1,k,}$ respectively) means that the corresponding fuzzy constraint is 'essentially $\geq X_{j+1,k,}$' (or essentially $\leq X_{j+1,k,}$ respectively), for all j & k .

Then, following Zimmerman [46] under fuzzy environment, our linear programming problem becomes the following problem, denoted by (FLP).

(FLP) Find X_{jk}'s, j = 1, 2, …, M ; k = 1,2,…, N that satisfy the following.

For the fuzzy objective

$$\sum_{j=1}^{M}\sum_{k=1}^{k_j} c_{jk}X_{jk} \ \tilde{\leq}\ z_o \qquad (4.3.4.1)$$

and for the fuzzy constraints with corresponding membership functions μ_{ikL} and μ_{ikU} are,

$$\sum_{k=1}^{k_j} X_{jk} \ \gtrsim\ \sum_{k=1}^{k_{j+1}} X_{j+1,k}, \qquad (j = 1, 2, \ldots, M-1) \qquad (4.3.4.2)$$

$$\sum_{k=1}^{k_j} X_{jk} \ \lesssim\ \sum_{k=1}^{k_{j+1}} X_{j+1,k}, \qquad (j = 1, 2, \ldots, M-1) \qquad (4.3.4.3)$$

$$\sum_{k=1}^{k_j} h_{ikl}X_{jk} \ \lesssim\ d_{jl} \qquad (j = 1, 2, \ldots, M;\ l = 1, 2, \ldots, L) \qquad (4.3.4.4)$$

and the crisp constraints are written as

$$X_{ij} \geq 0 \qquad (4.3.4.5)$$

4.3.4.1 Membership Functions

Following Zimmermann [46], below we define the membership functions for the fuzzy objective and fuzzy constraints.

Objective Function

As in (EFLP) in Section 1.10, Let us denote our objective function by $f_o(x)$.

Then, if

$f_o(x)$ is desired to be possibly lower than z_o , the membership function μ_{of} for objective function is written as:

$$\mu_{of} = \begin{cases} 1 & \text{if} \quad f_0(x) \le z_0 - q_0 \\ 1 - \dfrac{[f_0(x) - (z_0 - q_0)}{q_0} & \text{if} \quad z_0 - q_0 < f_0(x) \le z_0 \\ 0 & \text{if} \quad z_0 < f_0(x) \end{cases}$$

Similarly the membership functions for fuzzy constraints are obtained as below.

We denote our fuzzy constraint functions by $g_j(x)$, j =1, 2, …, N.

Then following Zimmermann [46],

μ_{ikL}, membership function for the lower side of the fuzzy region of the fuzzy constraints are taken as

$$\mu_{jkL} = \begin{cases} 1 & \text{if} \quad g_j(x) \ge d_j \\ 1 - \dfrac{[d_j - g_j(x)]}{q_j} & \text{if} \quad d_j - q_j < g_j(x) \le d_j \\ 0 & \text{if} \quad d_j - q_j > g_j(x) \end{cases}$$

and μ_{ikU} membership function for the upper of the fuzzy region of the fuzzy constraints are taken as

$$\mu_{jku} = \begin{cases} 1 & \text{if} \quad g_j(x) \le d_j \\ 1 - \dfrac{[g_j(x) - d_j]}{q_j} & \text{if} \quad d_j < g_j(x) \le d_j + q_j \\ 0 & \text{if} \quad d_j + q_j > g_j(x) \end{cases}$$

Once the membership functions are obtained, we get a solution to (FLP) by finding the intersection of the fuzzy sets given by (4.3.4.1), (4.3.4.2) and (4.3.4.3) is to be found out to get a decision. Then $\mu_D(x)$ the membership function of the fuzzy set 'decision' of the model that satisfying (4.3.4.1), (4.3.4.2) and (4.3.4.3) is

$$\mu_D(x) = \min(\mu_{of}, \mu_{11L}, \mu_{21L}, \ldots, \mu_{NML}, \mu_{11U}, \mu_{21U}, \ldots, \mu_{NMU})$$

Since we are interested in large value of $\mu_D(x)$ over (4.3.4.4) and (4.3.4.5), therefore, following Zimmermann [46], we obtain

Maximize $\mu_D(x) = \min(\mu_{of}, \mu_{11L}, \mu_{21L}, \ldots, \mu_{NML}, \mu_{11U}, \mu_{21U}, \ldots, \mu_{NMU})$

subject to the constraints (4.3.4.4) and (4.3.4.5).

Replacing $\mu_D(x)$ by λ, we have the following problem (LP) along the lines of Zimmermann [46]:

(LP) max λ

subject to

$$\mu_{of} \geq \lambda$$

$$\mu_{jkL} \geq \lambda \qquad j=1,2,\ldots,M;\ k=1,2,\ldots,N$$

$$\mu_{jkU} \geq \lambda \qquad j=1,2,\ldots,M;\ k=1,2,\ldots,N$$

and crisp constraints (4) and (5)

It is observed that (LP) is a crisp linear program whose optimal solution provides a solution to (FLP).

In view of the membership functions μ_{of}, μ_{jkL}, and μ_{jkU}, $j=1,2,\ldots,M$; $k=1,2,\ldots,N$; the (LP) can be restated as:

max λ

subject to

$$f_0 + \lambda\, q_0 \leq z_o$$

$$f_{jk} - \lambda\, q_{jk} \geq d_{jk} - q_{jk} \qquad j=1,2,\ldots,M,\ k=1,2,\ldots,N$$

$$f_{jk} + \lambda\, q_{jk} \leq d_{jk} + q_{jk} \qquad j=1,2,\ldots,M,\ k=1,2,\ldots,N$$

$$0 \leq \lambda \leq 1$$

and the crisp constraints.

Thus, we see that we obtain a solution to (FLP) by solving (LP) which is a crisp linear programming problem.

4.3.5 Numerical Example under Fuzzy Environments

Below we write a fuzzified format of (CLP). In this example we assume a tolerance level of approximately 30% in production requirements and 0.20% in total cost, and obtain z_0 as \$133,033 and q_0 as \$266.066. For the capacity constraints the tolerances are q_1= 120, q_2= 120, q_3= 600, q_4= 150, q_5= 135, q_6= 30, q_7= 120, q_8= 900, q_9= 150, q_{10}= 300, q_{11}= 600, where as the rest of the data is same as in crisp problem. In view of (FLP) following is fuzzy version of above problem.

(NP-1) $6W_1 + 15\,W_2 + 11\,W_3 + 14\,W_4 + 7.2\,X_1 + 18\,X_2 + 13.2\,X_3 + 16.8\,X_4 + 6.2\,Y_{11} + 15.4\,Y_{21} + 11.2\,Y_{31} + 14.3\,Y_{41} + 7.4\,Y_{12} + 18.4\,Y_{22} + 13.4\,Y_{32} + 17.1\,Y_{42} \lesssim 133{,}033.3$

Stage 1 Capacity:

Stamping: $0.03\,W_1 + 0.15\,W_2 + .05\,W_3 + 0.1\,W_4 \lesssim 400$

Drilling: $0.06\,W_1 + 0.12\,W_2 + 0.10\,W_4 \lesssim 400$

Metal: $2.0\,W_2 + 1.2\,W_4 \lesssim 2000$

Stage 2 Capacity:

Assembly: $\sum_{j=1}^{2} (0.05Y_{1j} + 0.10Y_{2j} + .05Y_{3j} + 0.12Y_{4j}) \lesssim 500$

Finishing: $0.04Y_{11} + 0.20\,Y_{21} + 0.03\,Y_{31} + 0.12\,Y_{41} \lesssim 450$

$0.04Y_{12} + 0.20\,Y_{22} + 0.03\,Y_{32} + 0.12\,Y_{42} \lesssim 100$

Packaging: $\sum_{j=1}^{2} (0.02Y_{1j} + 0.06Y_{2j} + .02Y_{3j} + 0.05Y_{4j}) \lesssim 400$

Inventory Balance Equations:

Stage 1:

$$W_1 + X_1 = Y_{11} + Y_{12}$$

$$W_2 + X_2 = Y_{21} + Y_{22}$$

$$W_3 + X_3 = Y_{31} + Y_{32}$$

$$W_4 + X_4 = Y_{41} + Y_{42}$$

Stage 2:

$$Y_{11} + Y_{12} \cong 3000$$

$$Y_{21} + Y_{22} \cong 500$$

$$Y_{31} + Y_{32} \cong 1000$$

$$Y_{41} + Y_{42} \cong 2000$$

Non-negativity constraints:

$$W_i \geq 0, \quad X_i \geq 0, \quad Y_{ij} \geq 0 \qquad i, j = 1,2,3,4.$$

Replacing each fuzzy equality with two fuzzy inequalities, we obtain:

(NP-2) $6W_1 + 15\, W_2 + 11\, W_3 + 14\, W_4 + 7.2\, X_1 + 18\, X_2 + 13.2\, X_3 + 16.8\, X_4 + 6.2\, Y_{11}$

$+15.4\, Y_{21} + 11.2\, Y_{31} + 14.3\, Y_{41} + 7.4\, Y_{12} + 18.4\, Y_{22} + 13.4\, Y_{32} + 17.1\, Y_{42} \lesssim 133{,}033.3$

Stage 1 Capacity:

Stamping: $0.03\, W_1 + 0.15\, W_2 + .05\, W_3 + 0.1\, W_4 \lesssim 400$

Drilling: $0.06\, W_1 + 0.12\, W_2 + 0.10\, W_4 \lesssim 400$

Metal: $2.0\, W_2 + 1.2\, W_4 \lesssim 2000$

Stage 2 Capacity:

Assembly: $$\sum_{j=1}^{2}(0.05Y_{1j}+0.10Y_{2j}+.05Y_{3j}+0.12Y_{4j}) \lesssim 500$$

Finishing: $$0.04Y_{11}+0.20\,Y_{21}+0.03\,Y_{31}+0.12\,Y_{41} \lesssim 450$$

$$0.04Y_{12}+0.20\,Y_{22}+0.03\,Y_{32}+0.12\,Y_{42} \lesssim 100$$

Packaging: $$\sum_{j=1}^{2}(0.02Y_{1j}+0.06Y_{2j}+.02Y_{3j}+0.05Y_{4j}) \lesssim 400$$

Inventory balance equations:

Stage 1:

$$W_1 + X_1 = Y_{11} + Y_{12}$$

$$W_2 + X_2 = Y_{21} + Y_{22}$$

$$W_3 + X_3 = Y_{31} + Y_{32}$$

$$W_4 + X_4 = Y_{41} + Y_{42}$$

Stage 2:

$$Y_{11} + Y_{12} \lesssim 3000$$

$$Y_{11} + Y_{12} \gtrsim 3000$$

$$Y_{21} + Y_{22} \lesssim 500$$

$$Y_{21} + Y_{22} \gtrsim 500$$

$$Y_{31} + Y_{32} \lesssim 1000$$

$$Y_{31} + Y_{32} \gtrsim 1000$$

$$Y_{41} + Y_{42} \lesssim 2000$$

$$Y_{41} + Y_{42} \gtrsim 2000$$

Non-negativity constraints:

$$W_i \geq 0, \quad X_i \geq 0, \quad Y_{ij} \geq 0 \qquad\qquad i, j = 1,2,3,4.$$

Then the crisp equivalent of this problem can be written as

(NP-3) Maximize λ

Subject to the following constraints:

$$6W_1 + 15 W_2 + 11 W_3 + 14 W_4 + 7.2 X_1 + 18 X_2 + 13.2 X_3 + 16.8 X_4 + 6.2 Y_{11} + 15.4 Y_{21}$$
$$+ 11.2 Y_{31} + 14.3 Y_{41} + 7.4 Y_{12} + 18.4 Y_{22} + 13.4 Y_{32} + 17.1 Y_{42} + 266.066 \lambda \leq 133{,}033.3$$

Stage 1 Capacity:

Stamping: $0.03 W_1 + 0.15 W_2 + .05 W_3 + 0.1 W_4 + 120 \lambda \leq 520$

Drilling: $0.06 W_1 + 0.12 W_2 + 0.10 W_4 + 120 \lambda \leq 520$

Metal: $2.0 W_2 + 1.2 W_4 + 600 \lambda \leq 2600$

Stage 2 Capacity:

Assembly: $0.05Y_{11} + 0.10 Y_{21} + 0.05 Y_{31} + 0.12 Y_{41} + 0.05Y_{12} + 0.10 Y_{22} + 0.05 Y_{32} + 0.12 Y_{42} + 150 \lambda \leq 650$

Finishing: $0.04Y_{11} + 0.20 Y_{21} + 0.03 Y_{31} + 0.12 Y_{41} + 135 \lambda \leq 585$

$0.04Y_{12} + 0.20 Y_{22} + 0.03 Y_{32} + 0.12 Y_{42} + 30 \lambda \leq 130$

Packaging: $0.02Y_{11} + 0.06 Y_{21} + 0.02 Y_{31} + 0.05 Y_{41} + 0.02Y_{12} + 0.06 Y_{22} + 0.02 Y_{32} + 0.05Y_{42} + 120 \lambda \leq 520$

Inventory Balance Equations:

Stage 1: $W_1 + X_1 = Y_{11} + Y_{12}$

$$W_2 + X_2 = Y_{21} + Y_{22}$$

$$W_3 + X_3 = Y_{31} + Y_{32}$$

$$W_4 + X_4 = Y_{41} + Y_{42}$$

Stage 2:

$$Y_{11} + Y_{12} + 900\,\lambda \le 3900$$

$$Y_{11} + Y_{12} - 900\,\lambda \ge 2100$$

$$Y_{21} + Y_{22} + 150\,\lambda \le 650$$

$$Y_{21} + Y_{22} - 150\,\lambda \ge 350$$

$$Y_{31} + Y_{32} + 300\,\lambda \le 1300$$

$$Y_{31} + Y_{32} - 300\,\lambda \ge 700$$

$$Y_{41} + Y_{42} + 600\,\lambda \le 2600$$

$$Y_{41} + Y_{42} - 600\,\lambda \ge 1400$$

$$0 \le \lambda \le 1$$

Non-negativity constraints:

$$W_i \ge 0\,,\quad X_i \ge 0,\quad Y_{ij} \ge 0 \qquad\qquad i, j = 1,2,3,4.$$

4.3.6 Results.

The optimal solution to (NP-3) is as described in the following table.

Table 4.3. Results of fuzzy linear programming problem.

Variable	Value	Variable	Value	Variable	Value
W_1	2994	X_2	499	Y_{21}	499
W_3	998	X_4	327	Y_{31}	998
W_4	1670	Y_{11}	2994	Y_{41}	1996
λ	.9938				

The utilization of production capacity is shown in table 4.4

Table 4.4. Optimal Utilization of production capacity

	Regular time Hours			Overtime Hours		
Department	**Available**	**Scheduled**	**Unused**	**Available**	**Scheduled**	**Unused**
Stamping	520	425.97	94.03	0	0	0
Drilling	520	465.9	54.1	0	0	0
Assembly	650	638.16	11.84	0	0	0
Finishing	585	533.4	51.6	130	29.81	100.19
Packaging	520	328.87	191.13	0	0	0

4.3.7 Interpretation of the Results

Results given in the above table summarize the solution of the linear version of the fuzzy problem. The minimum value of the objective function, which represents the minimization of the total cost, is $132,768.58 and level of satisfaction λ, is .9938.

Table 4.5 and 4.6 show the behavior of the value of λ corresponding to changes in tolerance levels, of 10%, 20%, 30%, 40% and 50% for imprecisely known demand and capacity, and of 0.20%, 0.25% , 0.5% , 1%,2% , 3% , 4% and 5% tolerance level for imprecisely known total cost.

Table 4.5. Value of λ corresponding to capacity tolerance and total cost tolerance

Capacity	Total cost tolerance						
Tolerance	**0.20%**	**0.25%**	**0.5%**	**1%**	**2%**	**3%**	**4%**
10%	0.9816	0.9771	0.9553	0.9145	0.8425	0.7810	0.7279
15%	0.9877	0.9846	0.9698	0.9413	0.8892	0.8425	0.8005
20%	0.9907	0.9884	0.9772	0.9554	0.9145	0.8771	0.8425
25%	0.9926	0.9907	0.9816	0.9640	0.9304	0.8992	0.8699
30%	0.9938	0.9923	0.9847	0.9698	0.9413	0.9145	0.8892

Table 4.6. Total cost corresponding to capacity tolerance and total cost tolerance

Capacity	Total cost tolerance						
Tolerance	**0.20%**	**0.25%**	**0.5%**	**1%**	**2%**	**3%**	**4%**
10%	\$132,772	\$132,708	\$132,398	\$131,816	\$130,791	\$129,916	\$129,160
15%	\$132,770	\$132,706	\$132,388	\$131,781	\$130,667	\$129,671	\$128,773
20%	\$132,769	\$132,704	\$132,383	\$131,762	\$130,600	\$129,533	\$128,550
25%	\$132,769	\$132,704	\$132,380	\$131,751	\$130,558	\$129,444	\$128,404
30%	\$132,769	\$132,703	\$132,378	\$131,743	\$130,528	\$129,383	\$128,301

4.3.8 Discussion of the solution in view of Table 4.5 and Table 4.6

Table 4.5 shows different values of λ for various tolerance levels for the imprecisely known total cost and imprecisely known capacity and demand. Also, Table 4.6 shows different values of total cost for various tolerance levels for the imprecisely known total cost and imprecisely known capacity and demand. Note that in this formulation the membership function λ is used to express the degree of certainty of (trust in) the solution with respect to fuzzy parameters, total cost and imprecisely known capacity [46]. From Table 4.5, it is observed that with the increase in the value of tolerance level for imprecisely known total cost, the value of λ decreases. This shows that the smaller the value of membership grade λ, the smaller is the support for the solution and hence, lower the degree of certainty of solution. On the other hand, it is observed that with increase in tolerance limits for imprecisely known capacity and demand, the value of λ increases. This shows that the larger the value of membership grade λ, the larger is the support for the solution. It can therefore, be concluded that fuzzy programming does not provide just another crisp solution; instead it produces the optimum solution corresponding to the pre-specified tolerance levels of constraints with an associated degree of one's belief in the solution. Graphs 4.5 and 4.6 in Appendix 2 reinforce the above observation.

Another advantage of fuzzy programming is that it admits imprecise data. This feature is particularly useful for the situation when the management in an organization is not able to specify precisely the total cost limit, but is rather able to provide lower and upper bounds, with a pre-specified tolerance interval above or below these bounds, taken as representing imprecision in setting of such bounds. As already stated, fuzzy set theory permits the partial belonging of an element to a fuzzy set characterized by a membership function that takes values in the interval [0, 1]. Thus, fuzzy programming produces most satisfactory solution within a pre specified interval, whereas a conventional crisp set theory constraint only permits an element either to belong (membership grade 1) or not to belong (membership grade 0) to the set [0, 1].

CHAPTER 5

A LINEAR PROGRAMMING APPROCH TO PRODUCTION PLANNING MODEL UNDER CRISP AND FUZZY ENVIRONMENT

As mention before (Section 1.12, p. 24), to be involved in the production area there are three main points that should be taken into consideration. In the present chapter we consider the third point, that is, to cope up with the changes in the production planning (for examples, increase and decrease in production rates limits, change in work force level, hiring and laying off workers, and cost of overtime…etc).

In this chapter, we consider the production planning problem, under the assumption that there exists production rate change and there is a penalty associated with such change. We formulate such a problem as a linear programming problem, under both crisp and fuzzy environment, and compare the results obtained by solving these models.

5.1 Introduction

In this chapter, we are concerned with planning production over a future interval of time, called the planning horizon, during which the rate of demand for the product varies. We assume this time interval is divided into periods and that the planning problem is to establish a production rate for each period in the planning horizon. The demand rate in each period is assumed to be known. An important contribution of this model is that under fuzzy environment it includes penalties for increasing or decreasing production quantity from one period to the next one.

The following formulation is for a single product and single production source in each period.

5.2 Linear Programming Formulation under Crisp Environment

5.2.1 Assumption

For this model, the following assumptions are made:

1. The model involves a single product and a single production stage.
2. The unit costs of all products in each operation are assumed to be known.
3. The demand varies from one time period to another and is assumed to be known.
4. The costs of backorders are not taken into consideration.

5.2.2 Notation

Let,

X_t = quantity produced in period t, $(t = 1, 2, \ldots, T)$

I_t = inventory at the beginning of period t.

S_t = increase in production quantity in period t,

d_t = decrease in production quantity in period t,

c_t = unit variable production cost in period t,

h_t = unit inventory carrying cost held from period t to $t+1$

π_t = unit cost of increase production quantity by period t.

ω_t = unit cost of decrease production quantity in period t.

P_t = maximum production in period t,

D_t = demand in period t.

L_t = maximum increase production change in period t,

K_t = maximum decrease production change in period t,

Z = total production cost for the period.

5.2.3 Objective

The problem is to choose $X_1, X_2, \ldots, X_T$ to minimize the sum of production costs, inventory costs, and production change costs over the planning horizon. Thus the objective in this problem is to minimize the total costs of production for the period.

5.2.4 General Formulation

Thus, under the crisp environment, we have the following problem:

$$\text{Min } Z = \sum_{t=1}^{T} (c_t X_t + h_t I_t + \pi_t S_t + \omega_t d_t)$$

Subject to, for $t = 1, 2, \ldots, T$,

1. **Inventory balance constraints.**

$$I_t + X_t - I_{t+1} = D_t$$

2. **Production capacity constraints.**

$$X_t \leq P_t$$

3. **Change in production constraints.**

$$X_t - X_{t-1} + d_t - S_t = 0$$

4. **Production increase limits.**

$S_t \leq L_t$

5. Production decrease limits.

$d_t \leq k_t$

6. Non negativity constraints

$X_t \geq 0$

$I_t \geq 0$

$S_t \geq 0$

$d_t \geq 0$

5.2.5 Numerical Example under Crisp Environment

The following table shows the data for production planning problem

Table 5.1. Production Planning problem Data

	Month 1	Month 2	Month 3	Month 4	Month 5
Demand (tons)	22	18	20	25	22
Production Cost ($/ton)	30	50	40	30	25
Inv. Cost($/ton/month)	5	10	7	5	8

If the production level is increased from one month to the next, then the company incurs a cost of $3, $5, $2, $3, $7 respectively per ton of increased production to cover the additional labor and /or over time. Each ton of decreased production incurs a cost of $5, $3, $3, $4, $6 respectively to cover the benefits of unused employees. The production during the previous month was 25 tons, and the beginning inventory is 6 tons. Inventory at the end of the fifth month must be at least 12 tons to cover anticipated demand. Production capacity cannot exceed 40 tons in any one month. The increase in production quantity cannot exceed 6 tons in the first month, 8 tons in the second month, 6 tons in the third month and 5 tons in both the forth and fifth month. The decrease in production quantity cannot exceed 6 tons in each month.

5.2.5.1 Notations.

Decision variables are defined as follows:

Let,

X_t = the number of tons to produce during Month t. $t = 1, 2, \ldots, 5$

I_t = inventory in tons at the beginning of Month t. $t = 1, 2, \ldots, 5$

S_t = number of tons of increased production in Month t. $t = 1, 2, \ldots, 5$.

d_t = number of tons of decreased production in Month t. $t = 1, 2, \ldots, 5$

The problem under crisp environment can be formulated as follows:

(CLP) Min $Z = (30 X_1 + 50 X_2 + 40 X_3 + 30 X_4 + 25 X_5 + 5 I_1 + 10 I_2 + 7 I_3 + 5I_4 + 8I_5$
$+ 3S_1 + 5S_2 + 2S_3 + 3S_4 + 7S_5 + 5d_1 + 3d_2 + 3d_3 + 4d_4 + 6d_5)$.

Subjects to the following constraints.

1. **Boundary Conditions**

   Initial and final Inventory constraints:

   Initial Inventory $I_1 = 6$

   Ending Inventory $I_6 \geq 12$

2. **Production Capacity Constraints**

   $X_1 \leq 40$

   $X_2 \leq 40$

$X_3 \leq 40$

$X_4 \leq 40$

$X_5 \leq 40$

3. Inventory Balance Constraints.

$I_1 + X_1 - I_2 = 22$

$I_2 + X_2 - I_3 = 18$

$I_3 + X_3 - I_4 = 20$

$I_4 + X_4 - I_5 = 25$

$I_5 + X_5 - I_6 = 22$

4. Change in production constraints.

X_0 = production during the previous month

= 25

(Month 1) $X_1 - X_0 + d_1 - S_1 = 0$

(Month 2) $X_2 - X_1 + d_2 - S_2 = 0$

(Month 3) $X_3 - X_2 + d_3 - S_3 = 0$

(Month 4) $X_4 - X_3 + d_4 - S_4 = 0$

(Month 5) $X_5 - X_4 + d_5 - S_5 = 0$

5. Production increase/decrease limits constraints

$S_1 \leq 6$

$S_2 \leq 8$

$S_3 \leq 6$

$S_4 \leq 5$

$S_5 \leq 5$

$d_1 \leq 6$

$d_2 \leq 6$

$d_3 \leq 6$

$d_4 \leq 6$

$d_5 \leq 6$

6. Non negativity Constraints.

All variables are nonnegative.

$X_t \geq 0 \qquad (t = 1, 2, \ldots, 5)$

$I_t \geq 0 \qquad (t = 1, 2, \ldots, 5)$

$S_t \geq 0 \qquad (t = 1, 2, \ldots, 5)$

$d_t \geq 0 \qquad (t = 1, 2, \ldots, 5)$

5.2.6 Results

Solving the above problem using Microsoft excel, we obtain the result in the following Table 5.2

Table 5.2 Results of crisp linear programming problem.

Variable	Value	Variable	Value	Variable	Value
X_1	19	I_1	6	S_1	0
X_2	15.25	I_2	3	S_2	0
X_3	21.25	I_3	0.25	S_3	6
X_4	26.25	I_4	1.5	S_4	5
X_5	31.25	I_5	2.75	S_5	5
d_1	6	d_4	0	X_0	25
d_2	03.75	d_5	0	I_6	12
d_3	0				

5.2.7 Interpretation of the Results

Results given in the above table summarize the solution of the crisp problem

The optimal production plan can be summarized as follows.

	Month 1	Month 2	Month 3	Month 4	Month 5
Beginning Inventory	6	3	0.25	1.5	2.75
Amount Produced	19	15.25	21.25	26.25	31.25
Ending Inventory	3	0.25	1.5	2.75	12

OFV = $3,945.75

5.3 Formulation under Fuzzy Environment

Problems of imprecise demand or data can be handled effectively by taking advantage of fuzzy set theory (Zadeh [44] Zimmermann [46])

The characteristics that require it to be formulated in fuzzy environment are:

1. **Imprecise total cost limit levels.** The management provides an upper bound of the estimation of the total cost represented by objective function z_o over the entire planning horizon. The actual costs would be likely to stay below this upper bound. A tolerance that defines the dispersion of the total cost may be given in the form of fraction of z_o.

2. **Imprecise demand.** Since demand is always forecasted and forecasts are rarely accurate to the exact number of units, the management can provide a tolerance level in form of a fraction of imprecisely known demand, that provides arrange above and below the forecasted demand in which the actual demand are likely to occur.

3. **Imprecise production limits change**. The management provides an upper bound of the estimation of the production limits change which is a tolerance level in form of a fraction of the production limits change.

We now formulate the problem under the following additional assumptions.

5.3.1 Additional Assumptions

1. Total cost over the entire planning horizon of T Periods stays possibly below a given limit.
2. Demand that varies from period to period is known imprecisely.
3. Production change limits (increase/decrease) stay below a given limit.

5.3.2 Objective

The objective of the model is to keep the total cost below an imprecisely stated upper bound keeping in view the imprecise demand and the production change for a finite number of periods.

5.3.3 Additional Notation

Let,

z_o = imprecisely known total cost limit specified by the management,

q_o = tolerance level associated with the imprecisely known total cost z_o.

q_t = tolerance level associated with the imprecisely known demand D_t, for all t.

q_j = tolerance level associated with the imprecisely known production change S_t, d_t, for all t.

μ_{of} = membership function associated with imprecisely known total cost z_o.

μ_{iL} = membership function corresponding to lower side of the constraint associated with imprecisely known demand D_t, for all t.

μ_{iU} = membership function corresponding to upper side of the constraint associated with imprecisely known demand D_t, for all t.

μ_{jL} = membership function corresponding to lower side of the constraint associated with imprecisely known production change S_t, d_t, for all t.

μ_{jU} = membership function corresponding to upper side of the constraint associated with imprecisely known production change S_t, d_t, for all t.

All other variables and symbols have the same meaning as in crisp formulation.

5.3.4 Formulation of Production Problem under Fuzzy Environment

Using Zimmerman's notation (Zimmermann, [46]), the following constraints

$$I_t + X_t - I_{t+1} = D_t \qquad t = 1, 2, \ldots, T$$

can be rewritten in fuzzy environment as

$$I_t + X_t - I_{t+1} \lesssim D_t \qquad t = 1, 2, \ldots, T$$

with μ_{iL} as the corresponding membership function, and

$$I_t + X_t - I_{t+1} \gtrsim D_t \qquad t = 1, 2, \ldots, T$$

with μ_{iU} as the corresponding membership function, where $\lesssim d_t$ (or $\gtrsim d_t$, respectively) means that the corresponding fuzzy constraint is 'essentially $\lesssim d_t$ '(or essentially $\gtrsim d_t$ respectively), for all t.

$$S_t \leq L_t \qquad t = 1, 2, \ldots, T$$

$$d_t \leq k_t \qquad t = 1, 2, \ldots, T$$

can be rewritten in fuzzy environment as

$S_t \lesssim L_t \qquad t = 1, 2, \ldots, T$

$d_t \lesssim k_t \qquad t = 1, 2, \ldots, T$

Using Zimmerman's approach, in a fuzzy environment, the objective function, which is the total cost of production, can be written as

$$\sum_{t=1}^{T} (c_t X_t + h_t I_t + \pi_t S_t + \omega_t d_t) \lesssim z_0$$

with μ_0 as the corresponding membership function for the objective function, where $\lesssim$ z_0 means that the corresponding membership function is ' desired to be less than or equal to z_0'.

Then, under fuzzy environments, our crisp linear programming problem (CLP) becomes the following fuzzy linear programming problem, denoted by (FLP)

(FLP) Find X_t, I_t, S_t, d_t, $\quad t = 1, 2, \ldots, T$, that satisfy the following:

For the objective function we have,

$$\sum_{t=1}^{T} (c_t X_t + h_t I_t + \pi_t S_t + \omega_t d_t) \lesssim z_0 \qquad (1)$$

and for the fuzzy constraints with corresponding membership functions μ_{iL} and μ_{iU} are,

$I_t + X_t - I_{t+1} \lesssim D_t \qquad t = 1, 2, \ldots, T \qquad (2)$

$I_t + X_t - I_{t+1} \gtrsim D_t \qquad t = 1, 2, \ldots, T \qquad (3)$

$S_t \lesssim h_t \qquad t = 1, 2, \ldots, T \qquad (4)$

$d_t \lesssim k_t \qquad t = 1, 2, \ldots, T \qquad (5)$

and the crisp constraints are written as

$X_t \le P_t \qquad (6)$

$$X_t - X_{t-1} + d_t - S_t = 0 \qquad (7)$$

$$X_t \geq 0 \qquad (8)$$

$$I_t \geq 0 \qquad (9)$$

$$S_t \geq 0 \qquad (10)$$

$$d_t \geq 0 \qquad (11)$$

$$0 \leq \lambda \leq 1 \qquad (12)$$

Membership Function

The membership functions for the fuzzy objective and fuzzy constraints are defined on similar lines as in Section 4.3.4.1 of Chapter 4.

Once the membership functions are known, then the intersection of these fuzzy sets is to be found out to get a decision. Let $\mu_D(x)$ be the membership function of the fuzzy set 'decision' of the model. Thus $\mu_D(x)$

$$\mu_D(x) = \min(\mu_{of}, \mu_{1L}, \mu_{2L}, \ldots, \mu_{NL}, \mu_{1U}, \mu_{2U}, \ldots, \mu_{NU})$$

Since we are interested in large value of $\mu_D(x)$ over (6)-(12), therefore, following Zimmermann [46], we obtain

Maximize $\mu_D(x) = \min(\mu_{of}, \mu_{1L}, \mu_{2L}, \ldots, \mu_{NL}, \mu_{1U}, \mu_{2U}, \ldots, \mu_{NU})$

Subject to the constraints (6)- (12).

Replacing $\mu_D(x)$ by λ, we have the following problem (LP) along the lines of Zimmermann [46]:

(LP) Max λ

Subject to

$$\mu_{of} \geq \lambda$$

$$\mu_{jL} \geq \lambda \qquad j = 1, 2, \ldots, N$$

$$\mu_{jU} \geq \lambda \qquad j = 1, 2, \ldots, N$$

and crisp constraints (6)-(12)

It is observed that (LP) is a crisp linear program whose optimal solution provides a solution to (FLP).

In view of (LP) we write (FLP) as

(LP-1) max λ

Subject to

$$\sum_{t=1}^{T} (c_t X_t + h_t I_t + \pi_t S_t + \omega_t d_t) + \lambda\, q_0 \leq z_o$$

$$I_t + X_t - I_{t+1} + \lambda\, q_t \leq D_t + q_t \qquad t = 1, 2, \ldots, T$$

$$I_t + X_t - I_{t+1} - \lambda\, q_t \geq D_t - q_t \qquad t = 1, 2, \ldots, T$$

$$S_t + \lambda\, q_j \leq h_t + q_j \qquad t = 1, 2, \ldots, T$$

$$d_t + \lambda\, q_j \leq k_t + q_j \qquad t = 1, 2, \ldots, T$$

and

$$X_t \leq P_t \qquad t = 1, 2, \ldots, T$$

$$X_t - X_{t-1} + d_t - S_t = 0 \qquad t = 1, 2, \ldots, T$$

$$X_t \geq 0 \qquad t = 1, 2, \ldots, T$$

$$I_t \geq 0 \qquad t = 1, 2, \ldots, T$$

$$S_t \geq 0 \qquad t = 1, 2, \ldots, T$$

$$d_t \geq 0 \qquad t = 1, 2, \ldots, T$$

$$0 \leq \lambda \leq 1$$

Thus, we see that we obtain a solution to (FLP) by solving (LP-1) which is a crisp linear programming problem.

5.3.5 Numerical Example under Fuzzy Environments

Below we write a fuzzified format (CLP). In this example we assume a tolerance level of approximately 30% in demand, 0.25% in total cost and 30% in production increase/decrease limits change. Therefore z_0 is \$ 3945.75 and $q_0 = 9.8644$. For the demand constraints, the tolerances are $q_1 = 6.6$, $q_2 = 5.4$, $q_3 = 6$, $q_4 = 7.5$, $q_5 = 6.6$. For the production increase limits change constrains, the tolerances are $q_6 = 1.8$, $q_7 = 2.4$, $q_8 = 1.8$, $q_9 = 1.5$, $q_{10} = 1.5$, For the production decrease limits change constrains, the tolerances are $q_{11} = 1.8$, where as the rest of the data is the same as in the crisp problem (3C).

In view of (FLP) following is fuzzy version of (LP-1) problem.

(NP-1) Maximize λ

Subject to the following constraints

$$30 X_1 + 50 X_2 + 40 X_3 + 30 X_4 + 25 X_5 + 5 I_1 + 10 I_2 + 7 I_3 + 5I_4 + 8I_5 + 3S_1 + 5S_2 + 2S_3 + 3S_4 + 7S_5 + 5d_1 + 3d_2 + 3d_3 + 4d_4 + 6d_5 \lesssim 3{,}945.75$$

$$I_1 + X_1 - I_2 \lesssim 22$$

$$I_1 + X_1 - I_2 \gtrsim 22$$

$$I_2 + X_2 - I_3 \lesssim 18$$

$$I_2 + X_2 - I_3 \gtrsim 18$$

$$I_3 + X_3 - I_4 \lesssim 20$$

$$I_3 + X_3 - I_4 \gtrsim 20$$

$$I_4 + X_4 - I_5 \lesssim 25$$

$$I_4 + X_4 - I_5 \gtrsim 25$$

$$I_5 + X_5 - I_6 \lesssim 22$$

$$I_5 + X_5 - I_6 \gtrsim 22$$

$$S_1 \lesssim 6$$

$$S_2 \lesssim 8$$

$$S_3 \lesssim 6$$

$$S_4 \lesssim 5$$

$$S_5 \lesssim 5$$

$$d_1 \lesssim 6$$

$$d_2 \lesssim 6$$

$$d_3 \lesssim 6$$

$$d_4 \lesssim 6$$

$$d_5 \lesssim 6$$

and crisp constraints

$$X_1 - X_0 + d_1 - S_1 = 0$$

$$X_2 - X_1 + d_2 - S_2 = 0$$

$$X_3 - X_2 + d_3 - S_3 = 0$$

$$X_4 - X_3 + d_4 - S_4 = 0$$

$$X_5 - X_4 + d_5 - S_5 = 0$$

Initial Inventory $I_1 = 6$

Ending Inventory $I_6 \geq 12$

$X_1 \leq 40$

$X_2 \leq 40$

$X_3 \leq 40$

$X_4 \leq 40$

$X_5 \leq 40$

$X_0 = 25$

All variables are nonnegative.

$X_t \geq 0 \quad (t = 1, 2, \ldots, 5)$

$I_t \geq 0 \quad (t = 1, 2, \ldots, 5)$

$S_t \geq 0 \quad (t = 1, 2, \ldots, 5)$

$d_t \geq 0 \quad (t = 1, 2, \ldots, 5)$

$0 \leq \lambda \leq 1$

Then the crisp equivalent of this problem can be written as

(NP-2) Maximize λ

Subject to the following constraints:

$30 X_1 + 50 X_2 + 40 X_3 + 30 X_4 + 25 X_5 + 5 I_1 + 10 I_2 + 7 I_3 + 5 I_4 + 8 I_5 + 3S_1 + 5S_2 + 2S_3 + 3S_4$

$+ 7S_5 + 5d_1 + 3d_2 + 3d_3 + 4d_4 + 6d_5 + 9.8644 \lambda \leq 3{,}945.75$

$I_1 + X_1 - I_2 + 6.6 \lambda \leq 28.6$

$I_1 + X_1 - I_2 - 6.6 \lambda \geq 15.4$

$I_2 + X_2 - I_3 + 5.4 \lambda \leq 23.4$

$I_2 + X_2 - I_3 - 5.4 \lambda \geq 12.6$

$$I_3 + X_3 - I_4 + 6\lambda \leq 26$$

$$I_3 + X_3 - I_4 - 6\lambda \geq 14$$

$$I_4 + X_4 - I_5 + 7.5\lambda \leq 32.5$$

$$I_4 + X_4 - I_5 - 7.5\lambda \geq 17.5$$

$$I_5 + X_5 - I_6 + 6.6\lambda \leq 28.6$$

$$I_5 + X_5 - I_6 - 6.6\lambda \geq 15.4$$

$$S_1 + 1.8\lambda \leq 7.8$$

$$S_2 + 2.4\lambda \leq 10.4$$

$$S_3 + 1.8\lambda \leq 7.8$$

$$S_4 + 1.5\lambda \leq 6.5$$

$$S_5 + 1.5\lambda \leq 6.5$$

$$d_1 + 1.8\lambda \leq 7.8$$

$$d_2 + 1.8\lambda \leq 7.8$$

$$d_3 + 1.8\lambda \leq 7.8$$

$$d_4 + 1.8\lambda \leq 7.8$$

$$d_5 + 1.8\lambda \leq 7.8$$

and crisp constraints

$$X_1 - X_0 + d_1 - S_1 = 0$$

$$X_2 - X_1 + d_2 - S_2 = 0$$

$$X_3 - X_2 + d_3 - S_3 = 0$$

$$X_4 - X_3 + d_4 - S_4 = 0$$

$$X_5 - X_4 + d_5 - S_5 = 0$$

Initial Inventory $\quad I_1 = 6$

Ending Inventory $I_6 \geq 12$

$X_1 \leq 40$

$X_2 \leq 40$

$X_3 \leq 40$

$X_4 \leq 40$

$X_5 \leq 40$

$X_0 = 25$

All variables are nonnegative.

$X_t \geq 0 \quad (t = 1, 2, \ldots, 5)$

$I_t \geq 0 \quad (t = 1, 2, \ldots, 5)$

$S_t \geq 0 \quad (t = 1, 2, \ldots, 5)$

$d_t \geq 0 \quad (t = 1, 2, \ldots, 5)$

$0 \leq \lambda \leq 1$

5.3.6 Results

The optimal solution to (NP-2) is as described in the following table.

Table 5.3. Results of fuzzy linear program problem

Variable	Value	Variable	Value	Variable	Value
X_1	18.98	I_1	6	S_1	0
X_2	15.16	I_2	3.04	S_2	0
X_3	21.18	I_3	0.25	S_3	6.02
X_4	26.19	I_4	1.48	S_4	5.01
X_5	31.20	I_5	2.74	S_5	5.01
d_1	6.02	d_4	0	X_0	25
d_2	03.82	d_5	0	I_6	12
d_3	0	λ	0.9914		

5.3.7 Interpretation of the Result

Results given in the above table summarize the solution of the linear version of the fuzzy problem. The minimum value of the objective function, which represents the minimization of the total cost, is \$3,935.97 and level of satisfaction λ, is 0.9914

The optimal production plan can be summarized as follows.

	Month 1	Month 2	Month 3	Month 4	Month 5
Beginning Inventory	6	3.04	0.25	1.48	2.74
Amount Produced	18.98	15.16	21.18	26.19	31.20
Ending Inventory	3.04	0.25	1.48	2.74	12

Table 5.4 to table 5.13 show the behavior of the value of λ and the objective function, corresponding to changes in tolerance levels, of 10%, 15% , 20%, 25%, and 30% for imprecisely known demand, and of 0.25% , 0.5% , 1%,2% , 3% , 4% and 5% tolerance level for imprecisely known total cost. And 10%, 15%, 20%, 25%, and 30% for imprecisely known production increase/decrease limits change.

Tables 5.4 to Table 5.8 provide value of λ corresponding to variations in production increase/decrease limits change tolerance, demand tolerance, and total cost tolerance.

Table 5.4. Values of λ for production increase/decrease limits change tolerance=10%

Demand tolerance	**Total cost tolerance**						
	0.25%	**0.50%**	**1%**	**2%**	**3%**	**4%**	**5%**
10%	0.9747	0.9507	0.9059	0.8281	0.7625	0.7066	0.6583
15%	0.9824	0.9654	0.9332	0.8747	0.8231	0.7773	0.7363
20%	0.9865	0.9734	0.9482	0.9014	0.8591	0.8205	0.7853
25%	0.9891	0.9784	0.9577	0.9187	0.8829	0.8497	0.8189
30%	0.9908	0.9818	0.9642	0.9309	0.8998	0.8707	0.8435

Table 5.5. Values of λ for production increase/decrease limits change tolerance=15%

Demand tolerance	Total cost tolerance						
	0.25%	0.50%	1%	2%	3%	4%	5%
10%	0.9759	0.9529	0.9101	0.835	0.7713	0.7167	0.6693
15%	0.983	0.9665	0.9353	0.8784	0.8281	0.7832	0.7429
20%	0.9869	0.9741	0.9494	0.9037	0.8622	0.8244	0.7897
25%	0.9893	0.9788	0.9585	0.9203	0.8851	0.8524	0.8221
30%	0.991	0.9821	0.9648	0.932	0.9014	0.8727	0.8458

Table5.6. Values of λ for production increase/decrease limits change tolerance=20%

Demand tolerance	Total cost tolerance						
	0.25%	0.50%	1%	2%	3%	4%	5%
10%	0.9770	0.9550	0.9139	0.8414	0.7789	0.7237	0.6758
15%	0.9835	0.9676	0.9372	0.8819	0.8327	0.7888	0.7492
20%	0.9872	0.9747	0.9507	0.9059	0.8653	0.8281	0.7939
25%	0.9895	0.9792	0.9593	0.9218	0.8872	0.8550	0.8251
30%	0.9911	0.09824	0.9654	0.9332	0.9030	0.8747	0.8481

Table 5.7. Values of λ for production increase/decrease limits change tolerance=25%

Demand tolerance	Total cost tolerance						
	0.25%	0.50%	1%	2%	3%	4%	5%
10%	0.9780	0.9569	0.9173	0.8461	0.7826	0.7280	0.6805
15%	0.9841	0.9686	0.9391	0.8852	0.8372	0.7941	0.7543
20%	0.9875	0.9753	0.9518	0.9081	0.8681	0.8316	0.7980
25%	0.9897	0.9797	0.9601	0.9233	0.8892	0.8575	0.8281
30%	0.9913	0.9827	0.9660	0.9342	0.9045	0.8766	0.8503

Table 5.8. Values of λ for production increase/decrease limits change tolerance=30%

Demand tolerance	Total cost tolerance						
	0.25%	0.50%	1%	2%	3%	4%	5%
10%	0.9789	0.9586	0.9205	0.8489	0.7862	0.7322	0.6851
15%	0.9845	0.9695	0.9409	0.8883	0.8404	0.7965	0.7570
20%	0.9878	0.9759	0.9529	0.9101	0.8709	0.8350	0.8019
25%	0.9899	0.9801	0.9609	0.9247	0.8912	0.8600	0.8309
30%	0.9914	0.9830	0.9666	0.9353	0.9059	0.8784	0.8525

Tables 5.9 to Table 5.13 provide value of total cost corresponding to variations in production increase/decrease limits change tolerance, demand tolerance, and total cost tolerance.

Table 5.9. Total cost corresponding to Demand tolerance and total cost tolerance when the production rate tolerance = 10 %

Demand tolerance	Total cost tolerance						
	0.25%	0.50%	1%	2%	3%	4%	5%
10%	3936.14	3926.99	3910	3880.4	3855.49	3834.23	3815.88
15%	3936.06	3926.07	3908.93	3876.72	3848.32	3823.07	3800.49
20%	3936.02	3926.55	3908.34	3874.61	3844.06	3816.25	3790.82
25%	3935.99	3926.45	3907.96	3873.25	3841.24	3811.64	3784.18
30%	3935.98	3926.38	3907.7	3872.29	3839.24	3808.32	3779.34

Table 5.10 Total cost corresponding to Demand tolerance and total cost tolerance when the production rate tolerance = 15%

Demand tolerance	Total cost tolerance						
	0.25%	0.50%	1%	2%	3%	4%	5%
10%	3936.12	3926.95	3909.84	3879.86	3854.44	3832.63	3813.7
15%	3936.05	3926.68	3908.85	3876.43	3847.73	3822.14	3799.18
20%	3936.02	3926.53	3908.29	3874.43	3843.69	3815.64	3789.95
25%	3935.99	3926.44	3907.93	3873.12	3840.98	3811.21	3783.56
30%	3935.98	3926.37	3907.68	3872.2	3839.05	3808.01	3778.88

Table 5.11 Total cost corresponding to Demand tolerance and total cost tolerance when the production rate tolerance = 20%

Demand tolerance	Total cost tolerance						
	0.25%	0.50%	1%	2%	3%	4%	5%
10%	3936.11	3926.91	3909.69	3879.35	3853.55	3831.53	3812.43
15%	3936.05	3926.66	3908.77	3876.15	3847.18	3821.26	3797.94
20%	3936.01	3926.52	3908.24	3874.26	3843.33	3815.06	3789.12
25%	3935.99	3926.43	3907.90	3873.0	3840.73	3810.8	3782.96
30%	3935.97	3926.37	3907.66	3872.11	3838.86	3807.7	3778.43

Table 5.12 Total cost corresponding to Demand tolerance and total cost tolerance when the production rate tolerance = 25%

Demand tolerance	Total cost tolerance						
	0.25%	0.50%	1%	2%	3%	4%	5%
10%	3936.10	3926.87	3909.55	3878.98	3853.11	3830.85	3811.5
15%	3936.04	3926.64	3908.69	3875.89	3846.65	3820.42	3796.94
20%	3936.01	3926.51	3908.19	3874.09	3842.99	3814.50	3788.32
25%	3935.99	3926.42	3907.87	3872.89	3840.49	3810.40	3782.38
30%	3935.97	3926.36	3907.63	3872.03	3838.68	3807.40	3777.99

Table 5.13 Total cost corresponding to Demand tolerance and total cost tolerance when the production rate tolerance = 30%

Demand tolerance	Total cost tolerance						
	0.25%	0.50%	1%	2%	3%	4%	5%
10%	3936.09	3926.84	3909.43	3878.76	3852.68	3830.19	3810.59
15%	3936.04	3926.62	3908.63	3875.65	3846.27	3820.03	3796.40
20%	3936.01	3926.50	3908.15	3873.93	3842.66	3813.96	3787.54
25%	3935.99	3926.41	3907.84	3872.78	3840.26	3810.02	3781.82
30%	3935.97	3926.36	3907.61	3871.94	3838.51	3807.11	3777.56

5.3.8 Discussion of the Solution in View of Table 5.4 – 5.13

Tables 5.4 to Table 5.8 show different values of λ for various tolerance levels for the imprecisely known total cost and imprecisely known demand. In each table we fixed the value of the tolerance level of production increase/decrease limits change. Note that in this formulation the membership function λ is used to express the degree of certainty of the solution with respect to fuzzy parameters, total cost, imprecisely known demand and imprecisely know production increase/decrease limits. From Table 5.9 to Table 5.13, it is observed that with increase in the value of the tolerance levels for imprecisely known production limits, keeping the demand tolerance and total cost tolerance fixed, the value of λ decreases. This shows that the smaller the value of λ, the smaller is the support for the solution and hence, lower the degree of certainty of solution. On the other hand, it is observed that with increase in the value of the tolerance limits for imprecisely known demand, the value of λ increases. This yields that the larger the value of λ, the larger is the support for (trust in) the solution. It can therefore be concluded that fuzzy programming does not provide just another crisp solution; also, it produces the optimum solution corresponding to the pre-specified tolerance of constraints corresponding to vaguely stated problems.

CHAPTER 6

CONCLUSION, CONTRIBUTION AND RECOMMENDATIONS

In the present chapter, contributions and conclusions of this thesis are provided. Finally, recommendations for further research on the problems considered in this thesis will be given.

6.1 Conclusion and Contribution

Probabilistic models and the models using fuzzy sets and fuzzy logic describe different aspects of uncertainty. Probabilistic models primarily describe random variability in parameters. In contrast, models using fuzzy sets and fuzzy logic incorporate vagueness, imprecision, and subjective judgment. The data sets of forecasted values rarely turn out to be crisply correct, therefore, the models based on precise knowledge of forecasted values have restricted practical applications. This thesis deals with a few such problems through fuzzy set theory approach. Based on the analysis presented in this thesis, we conclude that the fuzzy sets theory approach offers added advantage of flexibility of dealing with uncertainty involved in the weighted rating models, the multistage planning models, and the production planning models. We propose various fuzzy techniques to the weighted rating models, multistage planning problem, and the production planning models and arrive at more flexible solutions than normally obtained in their counterparts under crisp environment. It appears that the fuzzy ranking method and fuzzy linear programming are significantly more flexible than the non-fuzzy based prediction methods. Thus, the major contribution of the present thesis is that the results obtained here, using the theory of fuzzy sets and fuzzy logic, provide more flexible and satisfactory solutions versus the results obtained using their crisp counterparts.

In this thesis, we develop a fuzzified version of the weighted rating model formula. Based on the observations, we conclude that the fuzzy approach offers more flexibility and usefulness for the decision making process than the crisp approach formulated by Keeney and Raiffa [24, 25]. We observe that when there is imprecise and vague information available, a fuzzy weighted rating model provides a better and more flexible decision in certain multi-criteria decision making problems like facility location selection problem.

The multistage planning problem with imprecise values under both crisp and fuzzy environments is considered in this thesis under crisp environments; we formulate the problem as a linear programming problem. However, we deal with such a problem through fuzzy logic approach. Under fuzzy environments the problem is formulated as fuzzy linear program. Fuzzy programming does not provide just another solution; instead it produces the optimum solution corresponding to the pre-specified tolerance levels of constraints. Another advantage of fuzzy programming is that it admits imprecise data, especially the data that are not statistically defined.

In this thesis we develop a production planning problem with variable demand rate and with limits on production increase and decrease (and penalty on them) under both crisp and fuzzy environment with a finite planning horizon. Under crisp environments, we formulate the problem as linear programming. However, one underlying assumption in the above model and most of the models in the literature is that demand is deterministically known. But demand is forecasted and forecasts rarely-if-never turn out to be crisply correct. Therefore, the models based on precise knowledge of demand have little practical applications. We deal with such a problem using fuzzy approach. Under fuzzy environments, the problem is formulated as a linear program.

It is suggested that the methods presented in this dissertation are computationally effective and useful for determining the optimal solution to the problems discussed in Chapter 3, Chapter 4 and Chapter 5.

6.2 Recommendation for Future Research

A number of extensions are possible to the problems addressed in Chapter 4 and Chapter 5. In Chapter 4, we discuss and solve the multistage planning problem using linear programming approach as a symmetric case. However, the non-symmetric problem can be solved utilizing the same approach with appropriate modifications. Similarly, the results of Chapter 5 can be extended to the case when backorders is considered and the cost of backorders is taken in consideration. Also, the production planning problem discussed and solved in Chapter 5 using the linear programming approach for a symmetric case can be formulated and solved for a non-symmetric problem utilizing the same approach with appropriate modification.

In brief, in certain type of industries, the basic goal is to identify the most cost effective or profitable way of getting the right product to the right place at the right time, given a host of working and business constraints and parameters. Therefore, for an organization with multiple locations, production processes, product, models and customers, where instinct and experience are not able to cope up with the size and complexity of the operation, the use of fuzzy logic and fuzzy sets is an aid to be examined.

References

[1] Anderson, David L., Sweeney, Dennis J., Williams, Thomas A., *Introduction to Management Science Quantitative Approaches to Decision Making*, South-western College Publishing, U.S.A., 2000.

[2] Appadoo, S., *Operation Research Approach for Certain Financial Decision Making Problems Under Fuzzy Environment*, M.Sc. Thesis, Department of Business Administration, University of Manitoba, 2001.

[3] Bell, D. E., Raiffa, H., and Tversky, A., *Making Decisions*, Cambridge University Press, 1988, pp. 113-146.

[4] Bellman, R., and Zadeh, L. A., "Decision Making in a Fuzzy Environment", Management Science, 17, 1970, pp. 141-164.

[5] Blog, B., Hoek, Van Der G., Rinnoy, Kan. A. H. G., and Timmer, G. T., " The Optimal Selection of Small Portfolios", Management Science, Vol. 29, 1983, pp. 792-798.

[6] Bojadziev, George and Bojadziev, Maria, *Fuzzy logic for Business, Finance and Management*, World Scientific, New Jersey, U.S.A., 1997.

[7] Bortolan, G., and Degani, R., "A Review of Some Methods for Ranking Fuzzy Subsets", Fuzzy Sets and Systems, Vol. 15, 1985, pp. 1-19.

[8] Carlsson, C., and Korhonen, P., "A Parametric Approach to linear Programming", Fuzzy Sets and Systems, Vol. 20, 1986, pp. 17-30.

[9] Chen, S., "Ranking Fuzzy Numbers with Maximizing Set and Minimizing Set", Fuzzy Sets and systems, Vol. 17, 1985, pp.113-129.

[10] Chiu, Yu., and Park, C., "Fuzzy Cash Flow Analysis Using Present Worth Criterion", The Engineering Economist, 39(2), 1994, pp. 113-138.

[11] Crowston, W. B., Wagner, M., and Williams, J. F., "Economic Lot Size Determination in Multi-stage Assembly Systems", Management Science, 19 (5), 1973, pp. 517-527.

[12] Dantzig, George B., *Linear Programming and Extensions*, The Rand Corporation and University of California, Berkeley, Princeton, New Jersey, 1963,

[13] Dubois, D., Prade, H., "Use of Fuzzy Numbers in Decision Analysis.", Fuzzy Inf. and Decision Processes, 1982, pp. 309-321.

[14] Francis, R. L., and White, J. A., *Facility Layout and Location: an Analytical Approach*, Prentice-Hall Inc., Second Edition, 1974.

[15] Gabbay, H., "Multi-Stage Production Planning", Management Science, Vol. 25, n 11, 1979, pp. 1138-1148.

[16] Gonzales A., "A Study of the Ranking Function Approach Through Mean Values", Fuzzy Sets and Systems, Vol. 35, 1991, pp. 29-41.

[17] Gunasekaran, A., Goyal, S. K., and Martikainen, T., "Production Capacity planning and Control in multi-stage manufacturing.", Journal of the Operational Research Society, Vol. 49, n 6, 1998, pp.625-634.

[18] Hamacher, H., Liberling, H., and Zimmermann, H.-J., "Sensitivity Analysis in Fuzzy Linear Programming", Fuzzy sets and systems, 1 (1978), pp. 269-281.

[19] Hammond, j, S., Keeney, R, L., and Raiffa, H., *Smart Choices: A Practical Guide to Making Better Decisions*, Harvard Business School Press, U.S.A., 1999.

[20] Ince, E., Mutmansky, J. M., and Albert, E. K., "Fuzzy multiple-criteria decision-making. Application to AML project selection.", International Journal of Surface Mining & Reclamation, Vol. 5, n 4, 1991, pp. 167-176.

[21] Johnson, L. A., *Operations Research in Production Planning, Scheduling, and Inventory Control*, John Wiley & Sons, New York, 1974.

[22] Kaufmann, A., and Gupta, M. M., *Mathematical Fuzzy Models in Engineering and Management Science*, North Holland, New York, U.S.A., 1988.

[23] Kaufmann, A., and Gupta, M. M., *Introduction to Fuzzy Arithmetic Theory and Applications*, Von Nostrand Reinhold Company, New York, 1985.

[24] Keeney, R. L., and Raiffa, H., *Decisions with Multiple Objectives*, Wiley Press, New York, U.S.A., 1976.

[25] Keeney, R. L., *Value-Focused Thinking: a Path to Creative Decision Making*, Harvard University Press, Cambridge, Massachusetts, U.S.A., 1992.

[26] Kim K., and Park, C., "Ranking Fuzzy Numbers with Index of Optimism", Fuzzy Sets and Systems, Vol. 35, 1990, pp. 143-150.

[27] Klir, G. J., *Fuzzy set theory: Foundations and Applications*, Prentice-Hall, Inc., 1997.

[28] Komolananij S. *Capital Budgeting in a Fuzzy World*, Department of Industrial Engineering, Ph.D. thesis, University of Texas at Arlington, U.S.A., 1995.

[29] Lai, Joung-Jou, and Hwang, Ching Lai, *Fuzzy Mathematical Programming*, Springer Verlag, Berlin. 1992.

[30] Lee Sang M., Moore Laurence J., and Taylor Bernard W. III, *Management Science*, Wm. C. Brown Publishers, Second Edition, 1981.

[31] Liou, T., and Wang, M., "Ranking Fuzzy Numbers with Integral Value", Fuzzy Sets and Systems, Vol. 50, 1992, pp. 247-255.

[32] Lucas, H. C. Jr., Moore, J. R. Jr., "Multiple-Criterion Scoring Approach to Information System Project Selection.", INFOR Journal (Canadian Journal of Operational Research and Information Processing), Vol. 14, n 1, 1976, pp. 1-12.

[33] Nahmias, S., *Production and Operations Analysis*, Irwin McGraw Hill Company, New York, U.S.A., 1997.

[34] Olson, D., *Decision Aids for Selection Problems*, Springer-Verlag, New York, U.S.A., 1995.

[35] Pizzolato, N. D., Goyal, S. K., "Production Planning in A Multi-Stage Multi-System Production Process.", Journal of the Institution of Engineers (India), Part PE: Production Engineering Division, Vol. 66, 1986, pp. 117-124.

[36] Ross, Timoth J., *Fuzzy Logic with Engineering Applications*, McGraw-Hill, Inc., U.S.A., 1995.

[37] Ruszczynski, Andrzij, et. al. "Optimization Under Uncertainty", IIASA-OPT-Project Description, http://www.ac.at/Research/OPT/docs/OPT_Description.html

[38] Tamura, T., "Solution Procedure for The Multi-Item and Multi-Stage Production Planning Problem", Transactions of the Japan Society of Mechanical Engineers, Part C, Vol. 54, n 504, 1988, pp. 1974-1982.

[39] Tamura, T., " Procedure for A Multi-Item and Multi-Stage Production Planning Problem", JSME International Journal, Series 3: Vibration, Control Engineering, Engineering for Industry, Vol. 32, n 1, 1989, pp. 150-157.

[40] Teny, R. M., Kochhar, A. K., "Vector Space Model Approach to Production Planning for Multi-Product, Multi-Cel and Multi-Stage Manufacturing System.", International Journal of Systems Science, Vol. 15, n 11, 1984, pp. 1169-1187.

[41] Wang, R., Liang, T., "Application of Fuzzy Multi-objective Linear Programming to Aggregate Production Planning", Computers and Industrial Engineering, Vol. 46, n 1, 2004, pp. 17-41.

[42] Ward, T. L., "Fuzzy Discounted Cash Flow Analysis", in G. W. Evans., W. Karwowski., and M. R. Wilhelm., eds., *Applications of Fuzzy Set Methodologies in Industrial Engineering,* Elsevier, Amsterdam, pp.91-102, 1989.

[43] Yoon, K. P., and Hwang, C. L., *Multiple Attribute Decision Making*, Sage Publications, Beverly Hills, U.S.A., 1995.

[44] Zadeh, Lotfi A., *fuzzy sets*, Information and control, Vol. 8, 1965, pp.338-353.

[45] Zadeh, Lotfi A., Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems; Selected papers by Lotfi A. Zadeh, Edited and with a preface by George J. Klir and Bo Yuan. *Advances in Fuzzy Systems---Applications and theory*, 6. World Scientific Publishing Co., Inc., River Edge, NJ, 1996.

[46] Zimmermann, H. J., *Fuzzy Set Theory and its Applications*, Kluwer-Nijhoff Publishing, Second Edition, 1991.

[47] Zimmermann, H. J., *Fuzzy Set Theory and its Applications*, With a foreword by L. A. Zadeh. Second edition. Kluwer Academic Publishers, Boston, MA, 1992.

APPENDIX 1

Rating Model

Output for Fuzzy Weighted Rating Model

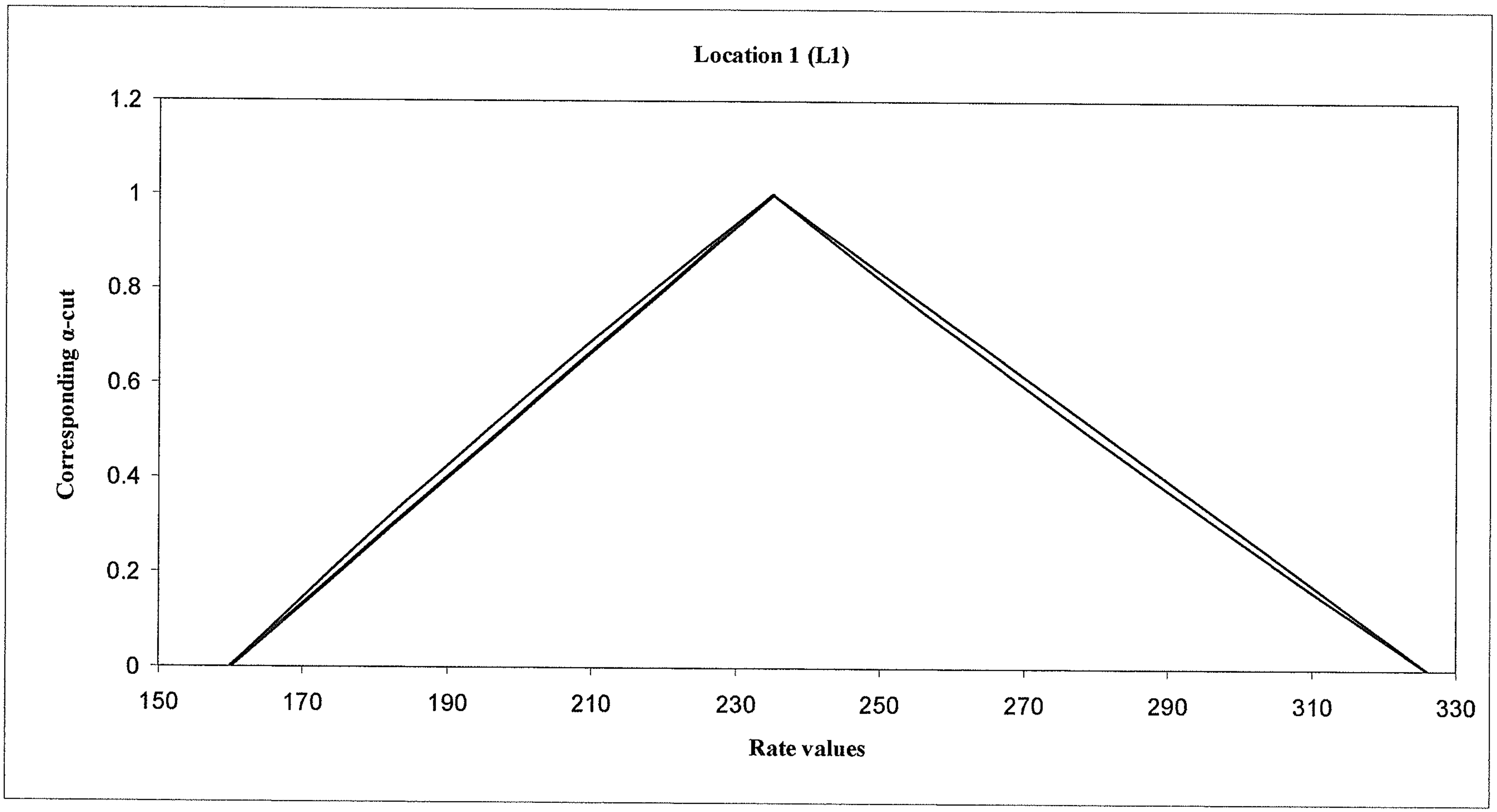
(Page 114 – 122)

Graph 3.7

Depicting

Error analysis for Location 1 (L1)

(As per Table 3.7 p. 49)

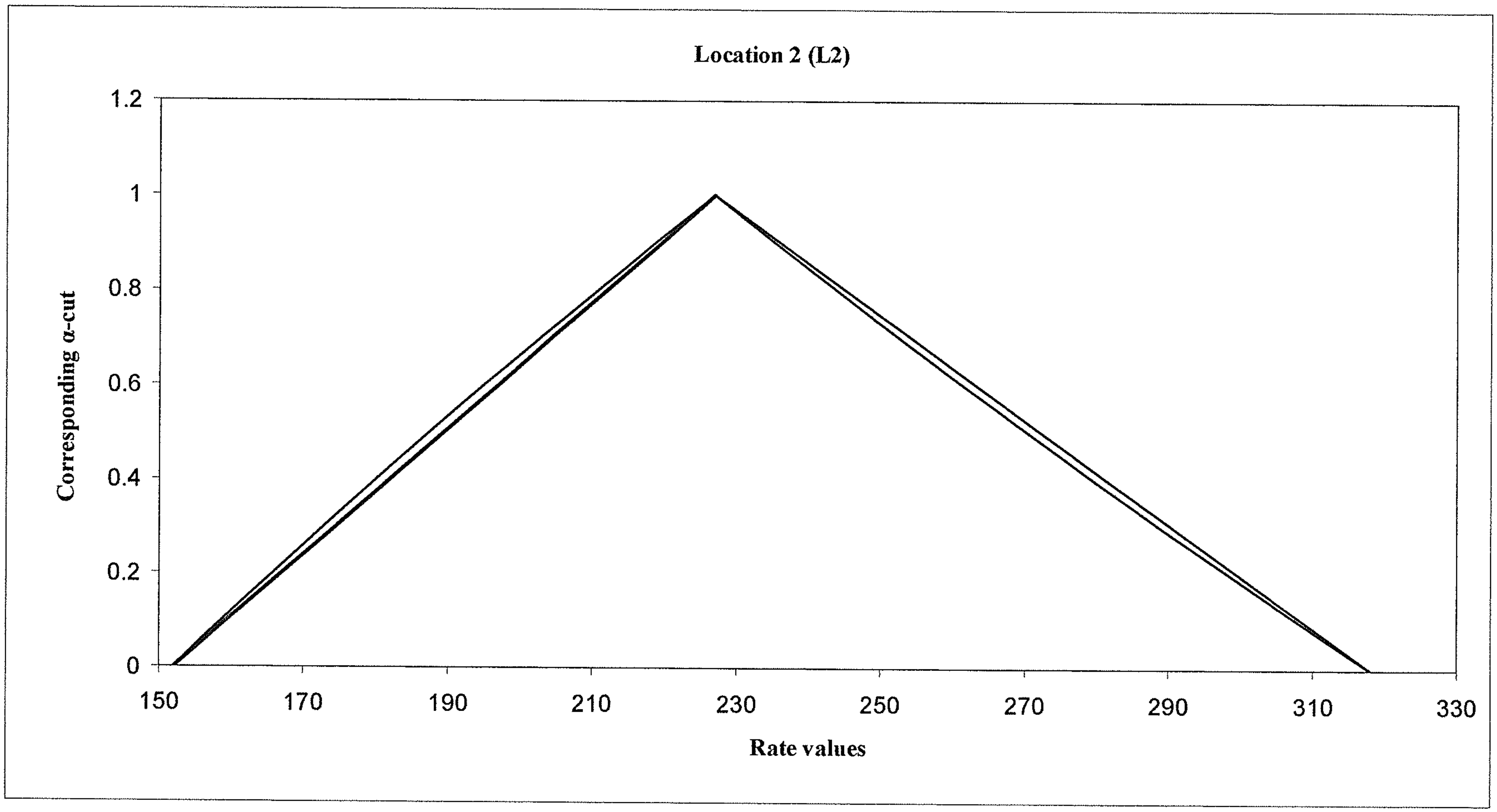

Location 1 (L1)
Corresponding α-cut
1.2
1
0.8
0.6
0.4
0.2
0
150
170
190
210
230
250
270
290
310
330
Rate values

Graph 3.10

Depicting

Error analysis for Location 2 (L2)

(As per Table 3.10 p. 52)

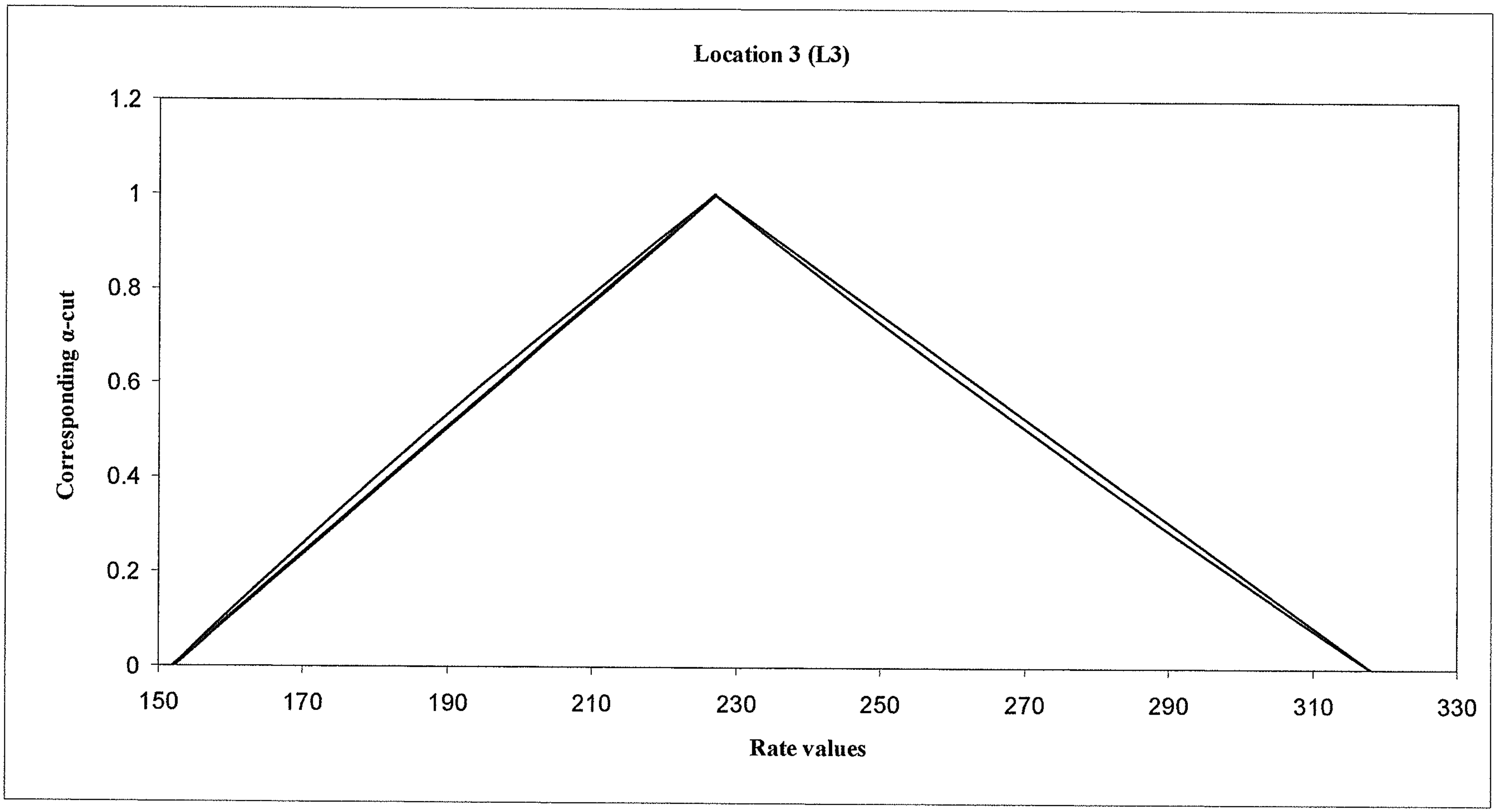

Location 2 (L2)
Corresponding α-cut
1.2
1
0.8
0.6
0.4
0.2
0
150
170
190
210
230
250
270
290
310
330
Rate values

Graph 3.13

Depicting

Error analysis for Location 3 (L3)

(As per Table 3.13 p. 55)

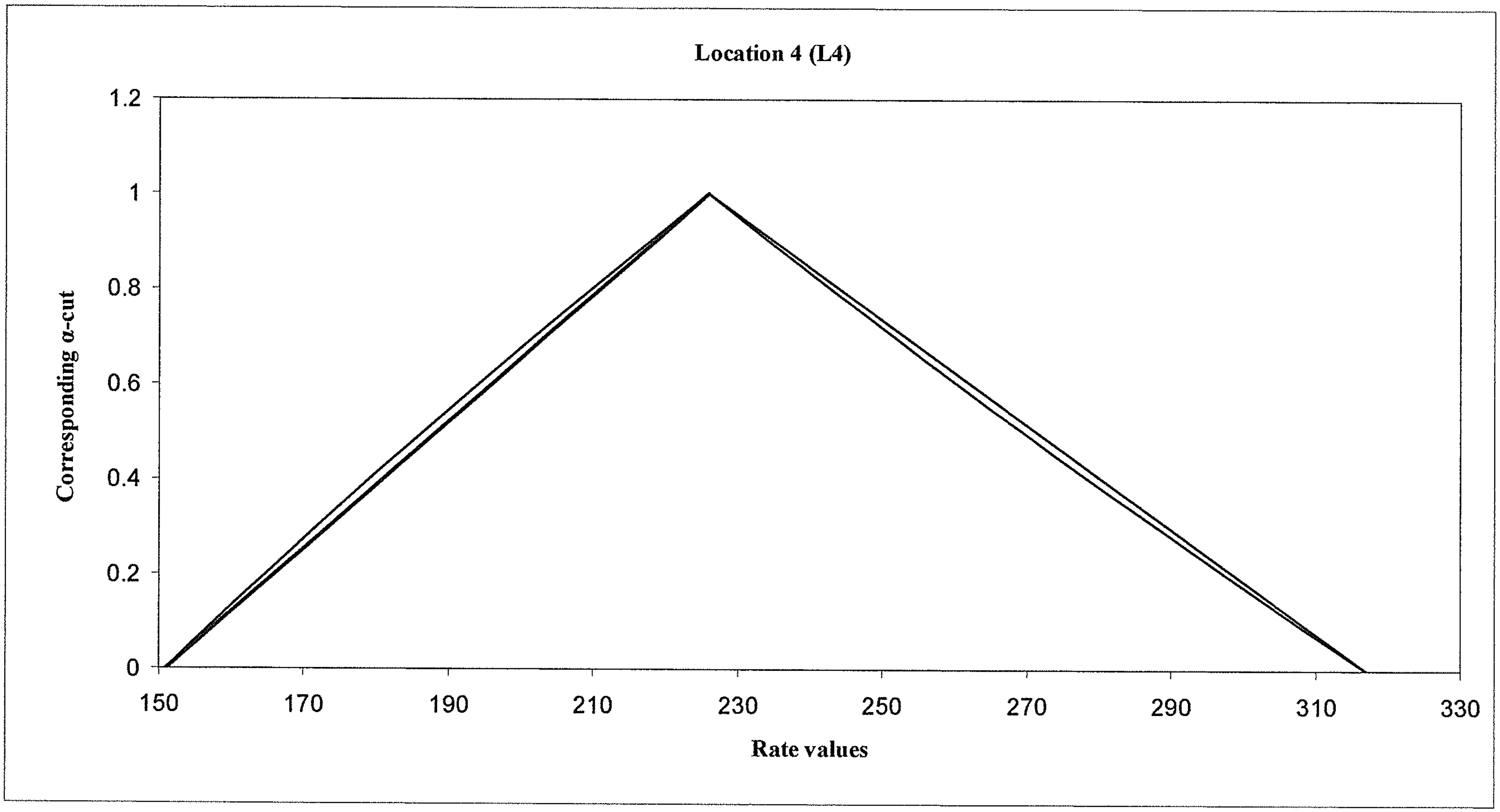

Location 3 (L3)
Corresponding α-cut
1.2
1
0.8
0.6
0.4
0.2
0
150
170
190
210
230
250
270
290
310
330
Rate values

Graph 3.16

Depicting

Error analysis for Location 4 (L4)

(As per Table 3.16 p. 58)


Location 4 (L4)
Corresponding α-cut
1.2
1
0.8
0.6
0.4
0.2
0
150
170
190
210
230
250
270
290
310
330
Rate values

APPENDIX 2

(Page 123 – 131)

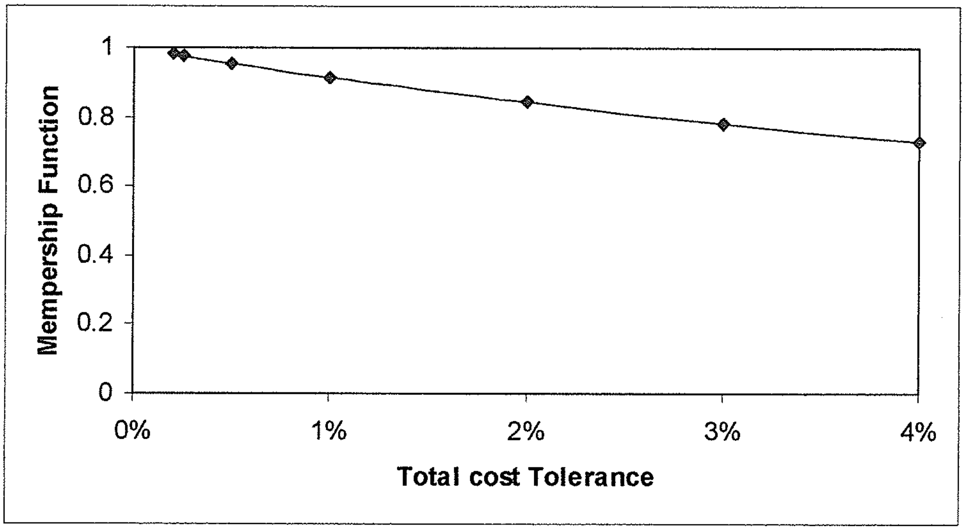
Graphs 4.5

Depicting

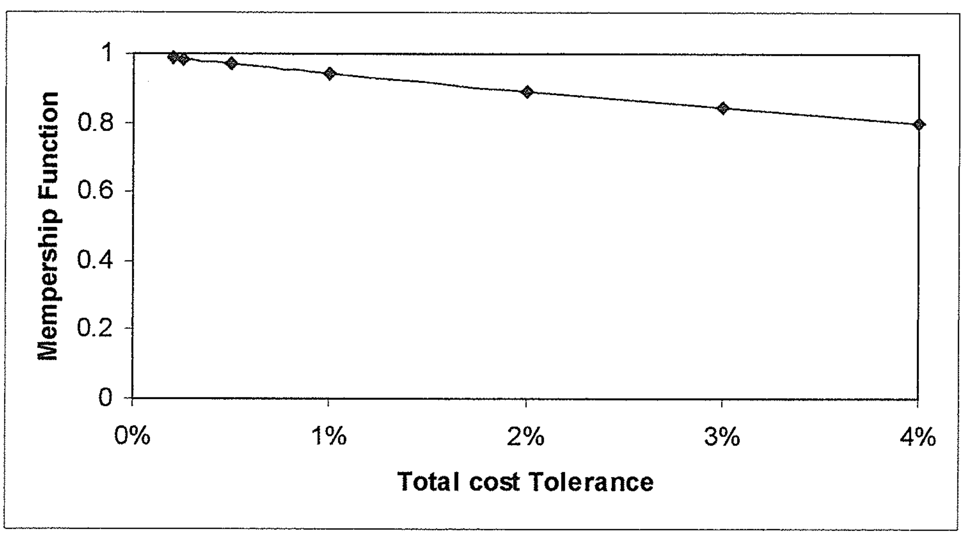
Value of λ Corresponding to capacity tolerance and total cost Tolerance

(As per Table 4.5 p.79)

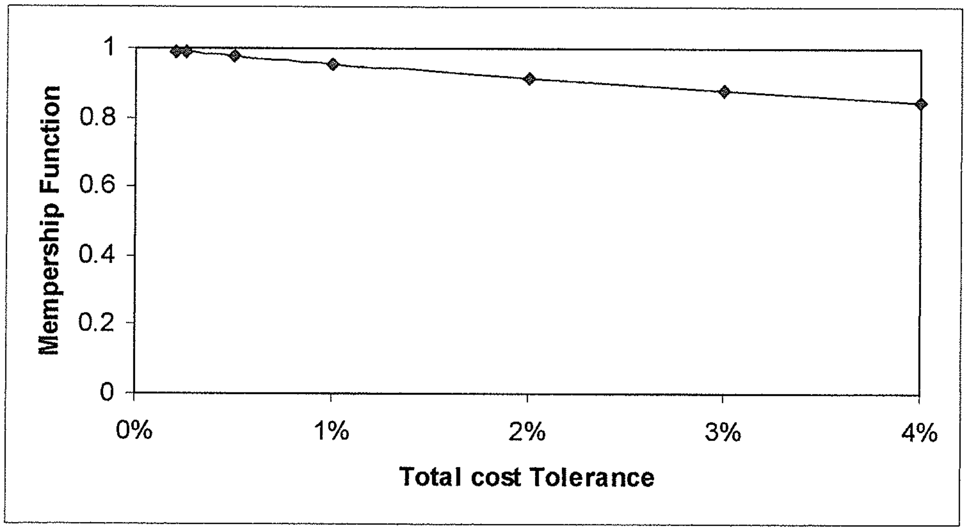
Constraint Tolerance 10 %



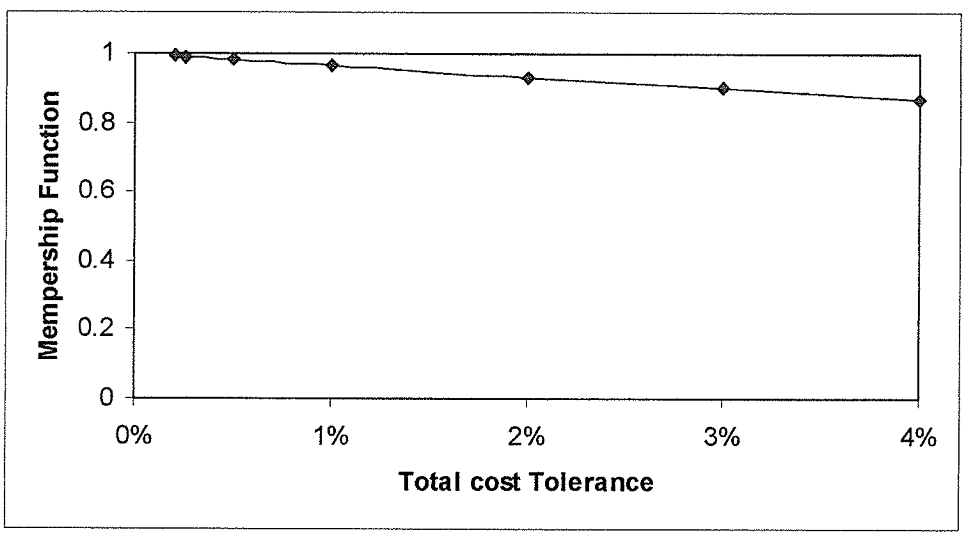
Constraint Tolerance 15 %



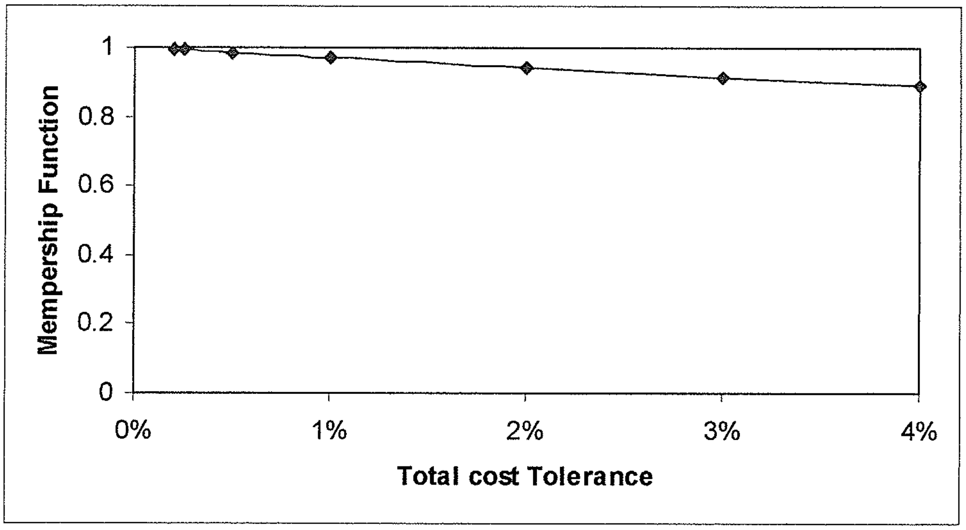
Constraint Tolerance 20 %



Constraint Tolerance 25 %



Constraint Tolerance 30 %



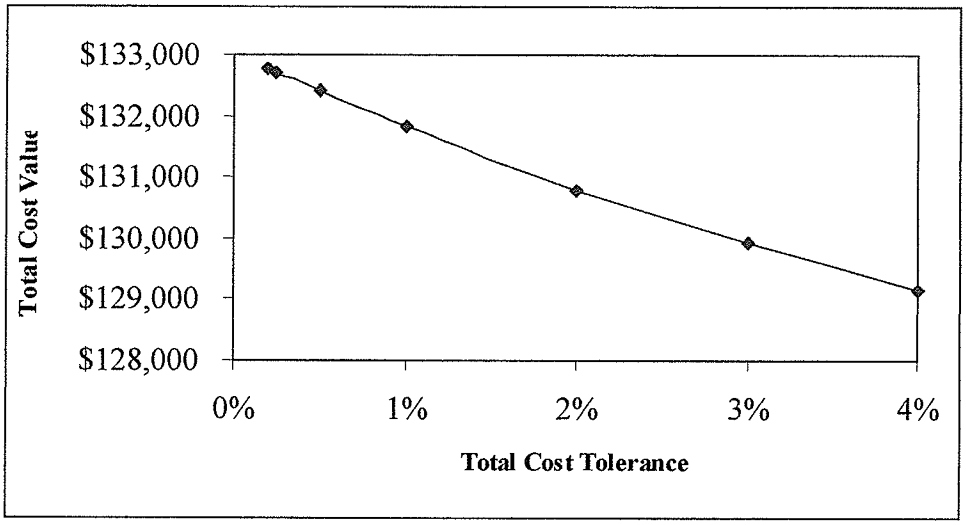
Graphs 4.6

Depicting

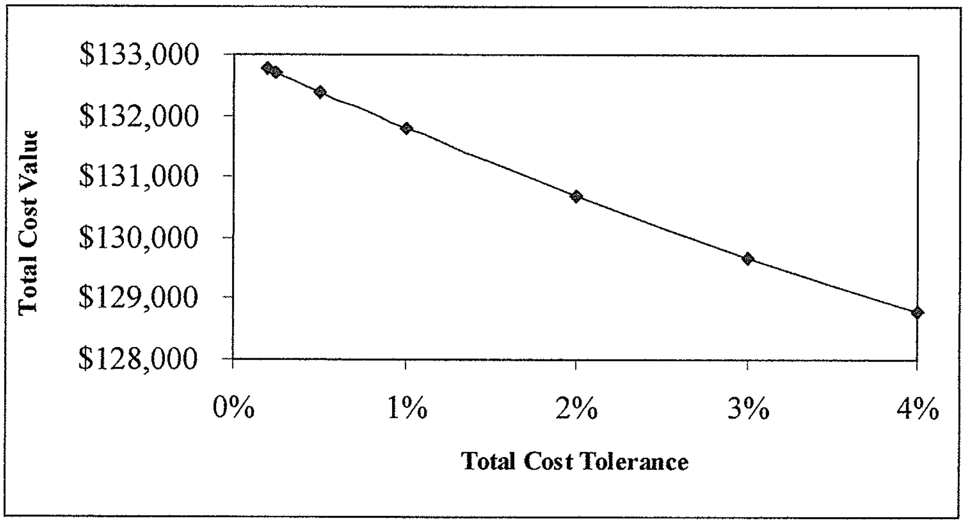
Value of Total Cost Corresponding to Constraints and Total Cost Tolerance

(as per Table 4.6 p. 80)

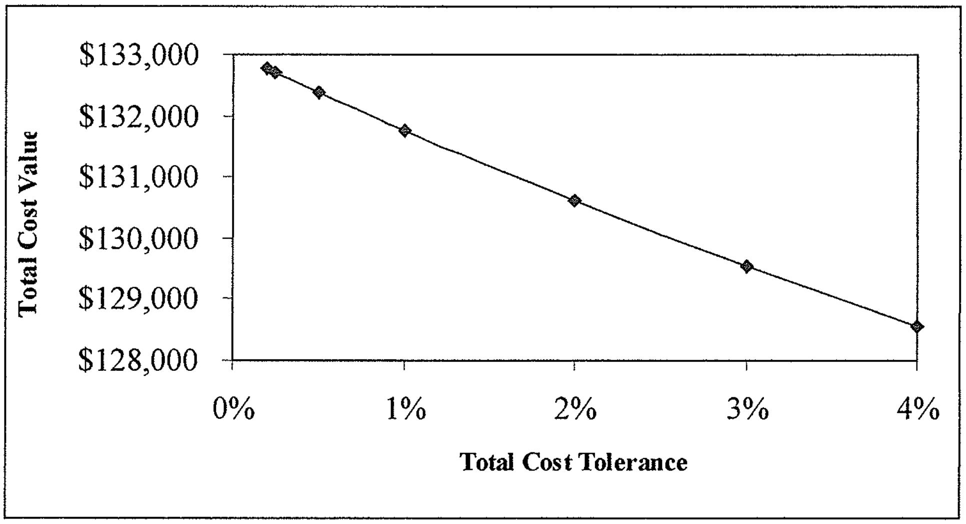
Constraint Tolerance 10 %



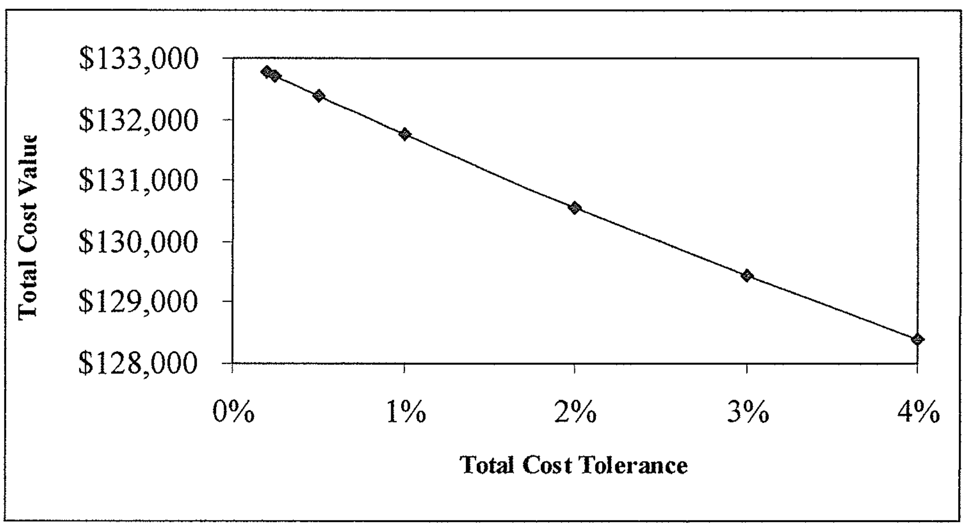
Constraint Tolerance 15 %



Constraint Tolerance 20 %



Constraint Tolerance 25 %



Constraint Tolerance 30%