Sequence Design for CDMA System

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of

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Abstract

Wireless Communication is witnessing rapid advances in volume and range of services. A major thrust for wireless communication technology is to improve spectrum efficiency, with the goal being to support greater user density and higher secure data rates for the allocated frequency spectrum. One promising approach for increased spectrum efficiency in digital wireless technology is the use of Spread Spectrum Communications. It is a system built to resist external interference, to operate at low spectral energy, to provide multiple access capability, and to provide a secure channel inaccessible to the outside listeners. Of the many potential applications of Spread Spectrum Communications, Code Division Multiple Access (CDMA) appears to be the most popular. CDMA has become widely accepted as one of the most promising solutions to increasing demands for high capacity wireless networks. One concern in a CDMA system is to improve the capacity. Several approaches to increase the capacity are investigated here: (1) the use of Welch bound Equality(WBE) sequences, (2) a CDMA/TDMA approach for assigning signals to users, and (3) use of a hybrid of orthogonal sequences. For each different approach, the codes are constructed and the performances are analyzed theoretically or simulated by a special iteration receiver. Comparison of the above schemes shows that the signal assignment technique, where a hybrid of two sets of orthogonal sequences or the WBE sequences are used, is desirable.

Acknowledgements

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Chapter 1

Introduction

The demand for high capacity flexible wireless services is evergrowing. Code Division Multiple Access (CDMA) is an access scheme that shows promise to meet this demand. CDMA has been implemented in second generation wireless systems that form the IS-95 standard. In third-generation(3G) mobile radio networks, wideband code division multiple access (WCDMA) has emerged as the mainstream air interface solution. Two wide band CDMA schemes are being standardized as W-CDMA(by Europe, Japan and Korea) and CDMA-2000 (by the United States and Korea)[3].

This thesis focuses on improving the capacity of the CDMA system. In the first chapter an overview of CDMA is given. Included topics are the comparison of multiple access schemes such as FDMA(Frequency Division Multiple Access), TDMA(Time Division Multiple Access), and CDMA(Code Division Multiple Access), the different types of CDMA, and the spreading codes used in CDMA.

1.1 Multiple access schemes: FDMA, TDMA and CDMA

The basis for any air interface design is how the common transmission medium is shared between users, i.e., the multiple access scheme. There are three basic

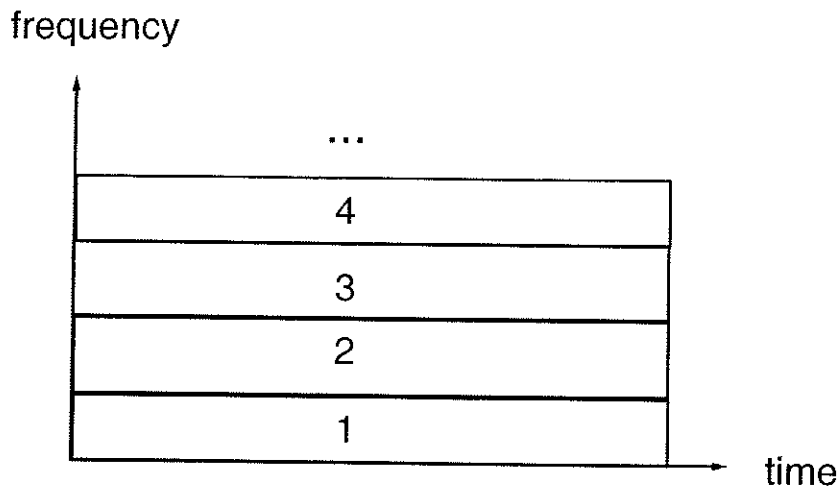

Figure 1.1: *FDMA*

multiple access schemes: FDMA, TDMA and CDMA.

FDMA (figure (1.1)), divides the radio channel into a range of radio frequencies and is used in the traditional analog cellular system. In FDMA, the total frequency bandwidth is divided into frequency channels that are allocated to the users, with only one user assigned to a channel at a time. Other users can access this channel only after the subscriber's call has terminated or is handed off to a different channel by the system. FDMA cellular standards include AMPS (Advanced Mobile Phone Service) and TACS (Total Access Communications System).

TDMA (figure (1.2)) is a common multiple access technique employed in digital cellular systems. It divides conventional radio channels into time slots to obtain higher capacity. Standards based on TDMA are North American Digital Cellular, Global System for GSM (Mobile Communications), and PDC (Personal Digital Cellular). As with FDMA, no other users can access an occupied TDMA channel until the channel is vacated.

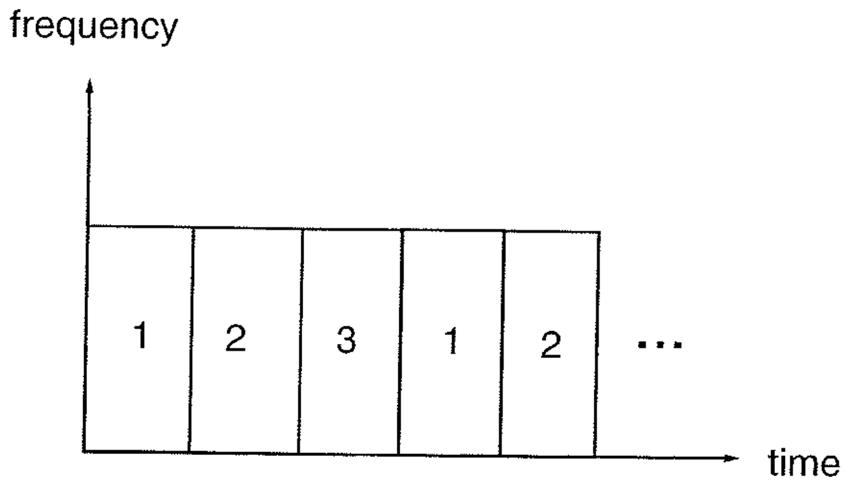

Figure 1.2: *TDMA*

CDMA (figure (1.3) is an advanced digital wireless transmission technique. Instead of using different frequencies or time slots, as the traditional technologies do, CDMA uses "pseudo-noise sequences" to transmit and distinguish between multiple wireless channels. Its bandwidth is much wider than that required for simple point-to-point communications at the same data rate because it uses noise-like carrier wave forms to spread the information symbol of interest over a much greater bandwidth. However, because the channels are distinguished by digital codes, many users can share the same bandwidth simultaneously. The advanced methods used in commercial CDMA technology improve capacity, coverage and voice quality, leading to a new generation of wireless networks. Older radio receivers separate stations and channels by filtering in the frequency domain. CDMA receivers, conversely, separate communication channels by a pseudo-random modulation that is applied and removed in the digital domain. Multiple users can therefore occupy the same frequency band. This universal frequency reuse is crucial to CDMA achieving a high spectral efficiency. CDMA has gained international acceptance by cellular radio system operators as an upgrade because of its universal frequency reuse and noise-like characteristics.

The IS-95 CDMA standard has been adopted by the TIA (Telecommunications Industry Association) and in 1992 became a digital cellular standard. The J-STD-008 standard for personal communications services was also accepted by ANSI. CDMA is the first digital technology which meets the exacting standards of the CTIA (Cellular Telecommunications Industry Association). Depending on the level of mobility of the system, it provides 10 to 20 times the capacity of AMPS, and 4 to 7 times the capacity of TDMA[1][2]. Of the three technologies CDMA is the only one that can efficiently utilize spectrum allocation and offer service to many subscribers without requiring extensive frequency planning. All CDMA users can share the same frequency channel because their messages are distinguished only by digital code, while TDMA operators have to coordinate the allocation of channels in each cell in order to avoid interfering with adjacent channels. The average transmitted power required by CDMA is much lower than what is required by analog, FDMA and TDMA technologies.

The main advantages of CDMA are as follows[4]:

Increased capacity.

Improved voice quality, eliminating the audible effects of multipath fading .

Enhanced privacy and security.

Improved coverage characteristics which reduce the number of cell sites.

Simplified system planning reduces deployment and operating costs.

Reduced average transmitted power, thus increasing talk time for portable devices.

Reduced interference to other electronic devices .

Reduction in the number of calls dropped due to handoff failures.

Development of a reliable transport mechanism for wireless data communications.

Coexistence with previous technologies, due to CDMA and analog being able to operate in the same spectrum range with relatively little interference.

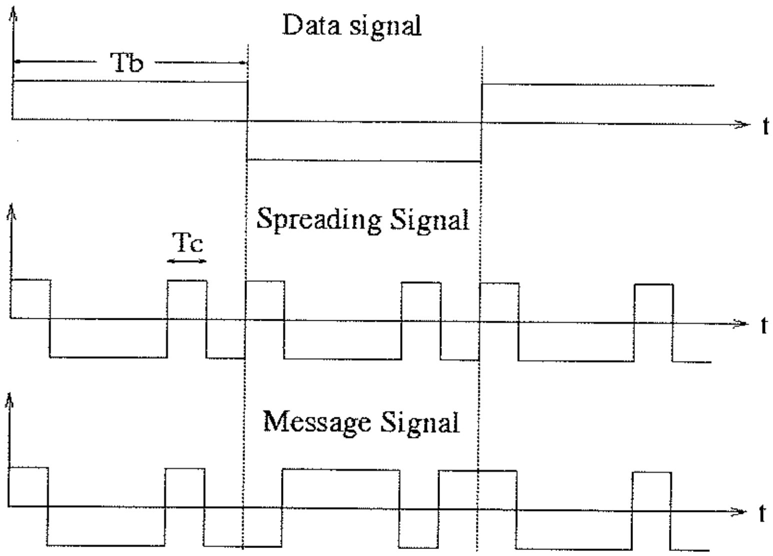

Figure 1.3: *CDMA*

1.2 Classification of CDMA

There are three types of CDMA based on the modulation method used to obtain the wideband signals: direct sequence(DS), frequency hopping(FH) and time hopping(TH). In DS-CDMA, spectrum is spread by multiplying the information signal with a pseudo-noise sequence, resulting in a wideband signal. In the FH-CDMA, a pseudo-noise sequence defines the instantaneous transmission frequency. The bandwidth at each moment is small, but the total bandwidth over, for example, a symbol period, is large. In TH-CDMA, a pseudo-noise sequence defines the transmission moment. This thesis is concerned with DS-CDMA.

1.3 Spreading codes of DS-CDMA system

Orthogonal sequences

There are two kinds of sequences used in CDMA technology. One spreading spectrum scheme known as orthogonal CDMA, or synchronized CDMA spectrum typically uses Walsh functions as spreading codes. Under the conditions of ideal synchronization and power control, there is no inter-user interference. The available number of sequences is equal to the spreading factor N. This is the limit of orthogonal CDMA(O-CDMA) system capacity. In the TIA/EIA/IS-95-A standard, Walsh functions are constructed from 64*64 Hadamard matrix which results in 64 channels.

Pseudo-noise Sequences

Another spreading spectrum scheme uses pseudo-noise codes. One of their features is that they are quite effective in overcoming narrow band interference. In order to do this, the PN codes need to behave like noise. A PN code ideally meets the following constraints:

- The sequences are built from only 2 different levels.
- The codes should have a sharp (1-chip wide) autocorrelation peak to enable code-synchronization.
- The codes should have a low cross-correlation value, the lower this cross-correlation, the more users one can allow in the system. This holds for both full-code correlation and partial-code correlation. The latter because in most situations there will not be a full-period correlation of two codes; it is more likely that codes will only correlate partially (due to the random-access nature).
- The codes should be "balanced": the difference between the number of ones and zeros in the code may only be 1. This last requirement results in good spectral density properties (equally spreading the energy over the whole frequency-band).

Usually the PN sequences are generated by linear feedback shift registers. The number of PN sequences that can be used is limited by the multiple access interference. Commonly used PN codes are M-sequences, or maximum-length sequences. Such sequences can be created by a single shift-register with a number of specially selected feedback-taps. If the shift-register size is k, then the length of the code is equal to $2^k - 1$. Each state of the shift register, i.e., different content of shift register cells, produces a different M-sequence. The generator produces $2^k - 1$ codes.

1.4 Capacity consideration and structure of the thesis

The number of available sequences is always limited by the number of chips or by the dimension of each sequence in a DS-CDMA system. In order to increase the capacity, or increase the number of sequences, one must sacrifice somewhere(such as the balance constraints). Typically one would like to try to minimize the correlation between any pair of codes since in an additive white Gaussian noise channel this determines the error performance.

Essentially, DS-CDMA scheme can be generalized as a signal assignment problem: given a signal space of dimension m, and n users, what's the best signal assignment based on the criteria that the multiple access interference(MAI) is minimized? When n is less than m, orthogonal sequences are the solution. Because all the sequences are orthogonal, under perfect power control and synchronization, there is no MAI. When the number of users is greater than the dimension of the signal space, one has MAI. The presence of MAI and its minimization leads to the construction of WBE(Welch Bound Equality) sequences. These sequences can be demodulated by a two stage receiver(discussed in a later chapter) to accommodate more users than the signal dimension m without appreciable degradation of error performance.

Though various multiple access schemes are present, an ongoing issue is improvement of the spectrum efficiency and reduction of the MAI. This is also the objective of this thesis.

As a preview, the thesis is arranged in the following order: Chapter 2 discusses the criteria of the signal assignment scheme. The problem in assigning signals is illustrated by a toy example of a two dimensional space. In Chapter 3, starting from the paper [5] by Sari et al of a hybrid CDMA/TDMA signal assignment scheme, a simple variation on it is proposed. The performance is analyzed thoroughly. When implementation is taken into consideration, we see that in the hybrid CDMA/TDMA schemes, the TDMA chips concentrate energy into a few chips, a disadvantage. A natural question is: do other methods exist which spread the energy over all the chips? The solution to this problem is discussed in Chapter 4 where a hybrid of two sets of orthogonal codes is introduced. In Chapter 5, an optimum solution for the signal assignment scheme is discussed based on the work of Massey and Mittelholzer [8]: WBE sequences generated from linear codes. Such WBE sequences have some limitations due to the constraint between the number of sequences and the dimension of signal space. Thus, in Chapter 6 the author proposes another type of WBE sequence, one which can be constructed from Hadamard matrices. Although WBE sequences have very good performance, their compatibility with present CDMA used in IS-95 remains a problem.

Finally a comparison of the different approaches: WBE sequences, CDMA/TDMA, hybrid of two sets of CDMA in terms of performance, compatibility, complexity, capacity and implementation, is made.

Chapter 2

Signal Assignment in Two Dimensional Space

2.1 Introduction

In a CDMA system, each spreading code is essentially a vector in a signal space. An orthogonal CDMA system, a special case, is one where all the vectors are mutually perpendicular with each other. From linear algebra, the number of linearly independent vectors in an $n-dimensional$ space is n, i.e., the maximum number of perpendicular vectors is n. If more than n signals need to be assigned in the n-dimensional space with an objective of keeping all signals as orthogonal as possible, then the signals have to be carefully chosen.

In general, assume there are m users and an n-dimensional signal space, with $m > n$. The signal assigned to each user can be expressed as a linear combination of an orthonormal basis set $\{\varphi_1(t), \varphi_2(t), ..., \varphi_n(t)\}$, i.e.

$$s_i(t) = \sum_{j=1}^{n} s_{ij}\varphi_j(t) \quad i = 1, 2, ...m \tag{2.1}$$

where $s_i(t)$ is assumed to be of unit energy.

In the synchronous case with perfect power control, when the additive white Gaussian noise(AWGN) channel is considered, then at the receiver, the received signal can be written as:

$$r(t) = \sum_{i=1}^{m} a_i \sqrt{E_b} s_i(t) + n(t) \tag{2.2}$$

where $a_i = \pm 1$ is the binary transmitted information bit of user i, E_b represents transmitted energy per bit and n(t) is AWGN.

A set of sufficient statistics for the detection of a_i is obtained by correlating the received signal with the users' transmitted signals, i.e,

$$r_i = \int_0^{T_s} r(t) s_i(t) dt = \sqrt{E_b} a_i + \sqrt{E_b} \sum_{j=1, j \neq i}^{m} a_j s_{ij} + n_i \quad i = 1, 2 ... m \tag{2.3}$$

where s_{ij} is the cross correlation between signals $s_i(t)$ and $s_j(t)$:

$$s_{ij} = \int_0^{T_s} s_i(t) s_j(t) dt \tag{2.4}$$

and n_i is a zero mean Gaussian random variable with variance $N_0/2$.

Depending on how the sufficient statistics are used, different receivers are possible. Each receiver typically represents a tradeoff between complexity and error performance. The one considered primarily in this thesis is the correlation receiver. In terms of complexity it is the simplest. A correlation receiver is one where the each user ignores the inter-user interference, represented by the middle term in equation (2.3), commonly called multiuser access interference and also ignores the statistical dependence between the noise samples, n_i. It simply bases its decision on r_i as follows:

$$\begin{cases} \tilde{a}_i = +1 & r_i >= 0 \\ \tilde{a}_i = -1 & r_i < 0 \end{cases}$$

Consider the inter-user interference term experienced by user i:

$$\xi^{(i)} = \sqrt{E_b} \sum_{j=1, j \neq i}^{M} a_j s_{ij}$$

where, the a_j are statistically independent. If the central limit theorem is invoked, then a potential model for $\xi^{(i)}$ is that it is Gaussian with zero mean and variance(assuming E_b=1):

$$\sigma(i)^2 = \sum_{j=1, j \neq i}^{M} \mid s_{ij} \mid^2 \tag{2.5}$$

The total inter-user interference variance is the sum of all the individual variances:

$$\sigma^2_{(TOT)} = \sum_{i=1}^{M} \sum_{j=1, j \neq i}^{M} \mid s_{ij} \mid^2 \tag{2.6}$$

The righthand side of equation 2.6 is also known as the total squared cross correlation(TSC) of the signal set.

When the variance of inter-user interference experienced by each user is the same, then

$$\sigma(i)^2 = \sigma^2_{(TOT)}/M \tag{2.7}$$

or the channel is the same to all the users.

Since the error performance, under the Gaussian assumption, depends on the total noise power(or variance), $\sigma^2_{TOT} + N_0/2$, equation (2.7) gives a signal assignment criteria: minimize the total squared cross correlation which in turn will minimize $\sigma(i)^2$.

As a toy example, consider a two dimensional signal space, where it is desired to assign three users, i.e., to choose three signal points in this space. The

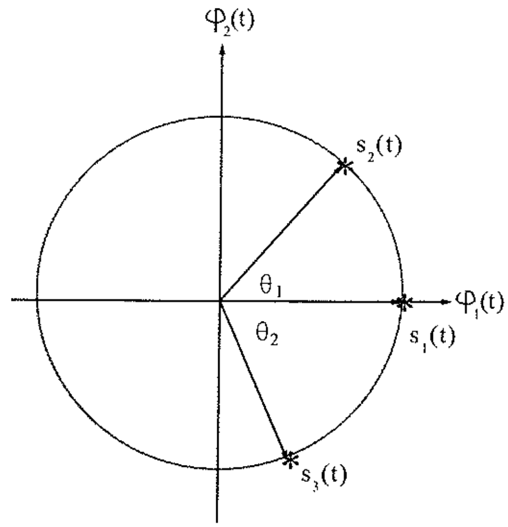

Figure 2.1: *Signal assignment in two dimensional signal space*

example shall serve as motivation for the succeeding work, illustrate the different receivers possible and highlight the potential problems in choosing a signal set.

2.2 Signal assignment in two dimensional space

Consider the simplest problem of three independent users each of whom are to be assigned a signal that lies in a two dimensional signal space. From the discussion in section 2.1, the assignment of the signal should be such as to minimize the total squared cross correlation. Without loss of of generality, let the user signals, $s_i(t)$, have unit energy, i.e., normalized. Each user uses antipodal modulation, with binary information symbols represented as $\pm\sqrt{E_b}$, (E_b therefore is the transmitted energy per bit). The signal space is shown in figure (2.1).

The three signals are represented by linear combinations of the bases $\varphi_1(t)$ and $\varphi_2(t)$:

$$s_1(t) = s_{11}\varphi_1(t) + s_{12}\varphi_2(t) \rightarrow (s_{11}, s_{12})$$

$$s_2(t) = s_{21}\varphi_1(t) + s_{22}\varphi_2(t) \rightarrow (s_{21}, s_{22})$$

$$s_3(t) = s_{31}\varphi_1(t) + s_{32}\varphi_2(t) \rightarrow (s_{31}, s_{32})$$

with $0 \leq t \leq T_s$, where T_s is the signal interval.

Without loss of generality, assign $(s_{11}, s_{12}) = (1, 0)$, $(s_{21}, s_{22}) = (\cos\theta_1, \sin\theta_1)$, $(s_{31}, s_{32}) = (\cos\theta_2, \sin\theta_2)$, The cross correlation between each pair of signals is as follows:

$$s_{12} = s_{21} = \cos\theta_1$$

$$s_{13} = s_{31} = \cos\theta_2$$

$$s_{23} = s_{32} = \cos\theta_1 cos\theta_2 + \sin\theta_1 \sin\theta_2$$

where s_{ij} satisfy the unit energy condition, i.e, $s_{i1}^2 + s_{i2}^2 = 1$.

The total squared error correlation is given by

$$TSC = 2[(\cos\theta_1)^2 + (\cos\theta_2)^2 + (\cos\theta_1 cos\theta_s + \sin\theta_1 \sin\theta_2)^2] \tag{2.8}$$

Minimizing equation (2.8) results in the following values for the two angles:

$$\theta_1 = \pi/3$$

$$\theta_2 = -\pi/3$$

The resultant signal set is shown in Figure (2.2). It is important to note that Figure (2.2) represents the transmitted signals and not the received signal.

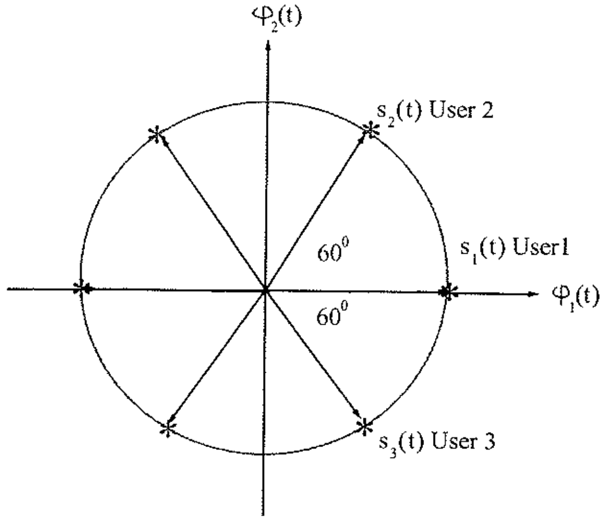

Figure 2.2: $\pi/3$ *signal assignment scheme*

2.3 Error performance analysis

The previous section presented the optimum signal constellation where the signals lie in a 2 dimensional space for the case of three users. The optimality criterion used was that of minimizing the total squared correlation. However, the basic criterion of interest in any communication system is symbol error probability. Though the total squared correlation is closely and directly related to error performance in an additive Gaussian noise channel model that is not the situation here. This is because an individual user experiences interference from the other users; an interference that, in general, is not particularly well modelled as Gaussian.

Further the error performance depends on the type of multiuser receiver that is postulated. The performance of the signal assignment developed in the last section is investigated for four different receivers. These receivers are the most popular ones, at least in the research literature. They are:

1. Correlation receiver

2. Decorrelation receiver

3. Jointly optimum receiver

4. Individually optimum receiver

For each receiver, it is assumed there is perfect power control and that the receiver is synchronized. The received signal is therefore

$$r(t) = \sqrt{E_b} a_1 s_1(t) + \sqrt{E_b} a_2 s_2(t) + \sqrt{E_b} a_3 s_3(t) + n(t) \tag{2.9}$$

where $n(t)$ is additive, white Gaussian noise and $a_i = \pm 1$. The four different receivers and their error performance for the given signal assignment are described in the following sections.

2.3.1 Correlation receiver

A block diagram of the correlation receiver is shown as figure (2.3). Very simply each user correlates the received signal with his/her transmitted signal, ignores the interuser interference, and thresholds the output of the correlator(or matched filter) to produce the decision. Note that in the absence of interuser interference this receiver would be optimum. However, the actual outputs of the matched filter here are:

$$r_1 = \int_0^{T_s} r(t) * s_1(t)\, dt = \sqrt{E_b} a_1 + (1/2)\sqrt{E_b}(a_2 + a_3) + n_1$$

$$r_2 = \int_0^{T_s} r(t) * s_2(t)\, dt = \sqrt{E_b} a_2 + (1/2)\sqrt{E_b}(a_1 - a_3) + n_2$$

$$r_3 = \int_0^{T_s} r(t) * s_3(t)\, dt = \sqrt{E_b} a_3 + (1/2)\sqrt{E_b}(a_1 - a_2) + n_3 \tag{2.10}$$

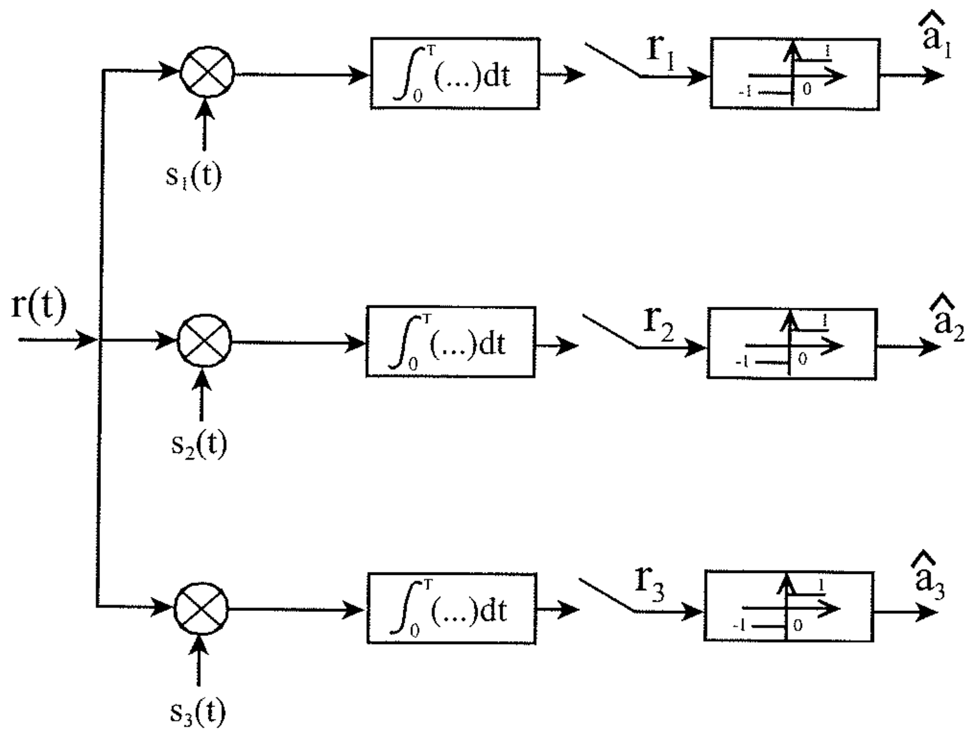

Figure 2.3: *Block diagram of correlation receiver*

The second term of (2.10) represents the interuser interference from the other user while the the third term due to the additive, white Gaussian noise is Gaussian, zero mean, variance $N_0/2$. Since each user sees identical interference, the error performance of each user is the same for all three users. The decision rule for the i^{th} user is:

choose 1 if $r_i \geq 0$ otherwise choose -1.

It is straightforward to show that the bit error probability is:

$$P_b = 0.25 \int_{-\infty}^{-2\sqrt{E_b}} f(n_1)\, dn_1 + 0.5 \int_{-\infty}^{-\sqrt{E_b}} f(n_1)\, dn_1 + 0.25 \int_{-\infty}^{0} f(n_1)\, dn_1 \quad (2.11)$$

In terms of the Q-function, defined as $Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-x^2/2} dx$, equation 2.11 has the form(as plotted in figure (2.4)) :

$$P_b = 0.25Q(2\sqrt{2E_b/N_0}) + 0.5Q(\sqrt{2E_b/N_0}) + 0.125 \quad (2.12)$$

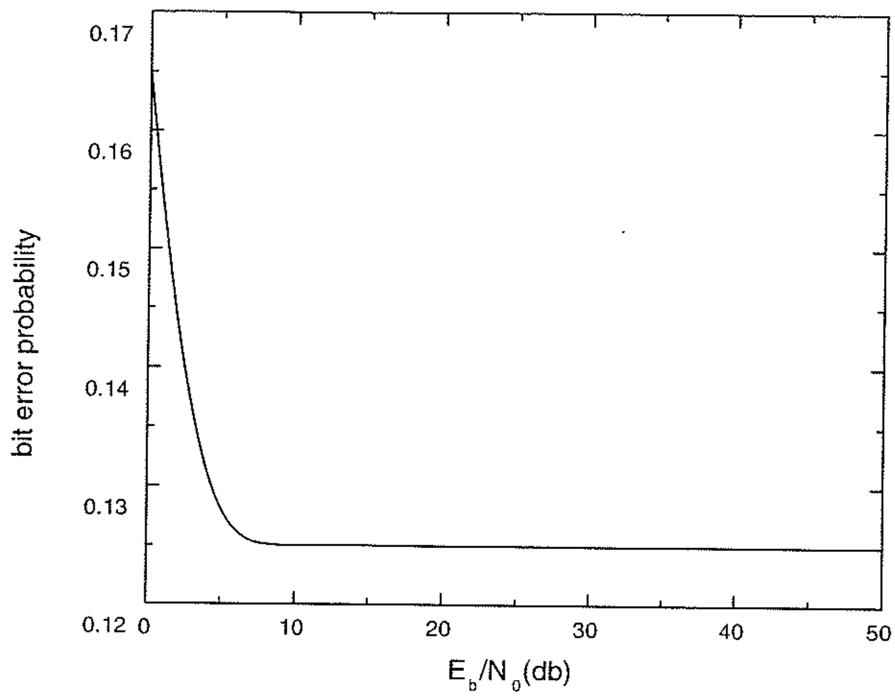

Figure 2.4: *Error performance of $\pi/3$ signal assignment scheme*

Note that regardless of the signal to noise ratio, defined as E_b/N_0, that there is an irreducible error probability of 0.125. This reflects the fact that the interference is not Gaussian.

2.3.2 Decorrelation receiver

The decorrelation receiver decorrelates the cross correlation between different users. Then the decision is made on the decorrelated sufficient statistics. The sufficient statistics of equations (2.10) can be written in matrix format:

$$\begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} = R \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \tag{2.13}$$

where R is 3×3 signal correlation matrix:

$$R = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 1 \end{pmatrix} \tag{2.14}$$

If the matrix R has an inverse R^{-1}, one multiplies equation (2.13) with R^{-1} to produce a new set of sufficient statistics:

$$\begin{pmatrix} r_1^{'} \\ r_2^{'} \\ r_3^{'} \end{pmatrix} == R^{-1}R \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + R^{-1} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} n_1^{'} \\ n_2^{'} \\ n_3^{'} \end{pmatrix} \tag{2.15}$$

Note that the multiuser interference has been eliminated. However, this is at the expense of increased Gaussian noise. The decision rule is:

$$\left\{ \begin{array}{ccc} \hat{a}_1 & = & sgn(r_1^{'}) \\ \hat{a}_2 & = & sgn(r_2^{'}) \\ \hat{a}_3 & = & sgn(r_3^{'}) \end{array} \right\} \tag{2.16}$$

Unfortunately, in the $\pi/3$ signal assignment scheme, the matrix R has no inverse. So the decorrelation receiver is unavailable in this special case.

2.3.3 Jointly optimum receiver

The jointly optimum receiver detects the entire transmitted bit sequence, i.e., (a_1,a_2.a_3) in our case. As a criteria it minimizes the probability of sequence(or symbol) error. For the $\pi/3$ signal assignment scheme considered here the signal space plot is as shown in figure (2.5). Note that sequence (-1,1,1) and (1,-1,-1) result in the same received signal point which lies at the origin(this in essence is why the error floor was present for the correlation receiver) . This implies that an error shall be incurred regardless of the signal to (additive white Gaussian) noise ratio.

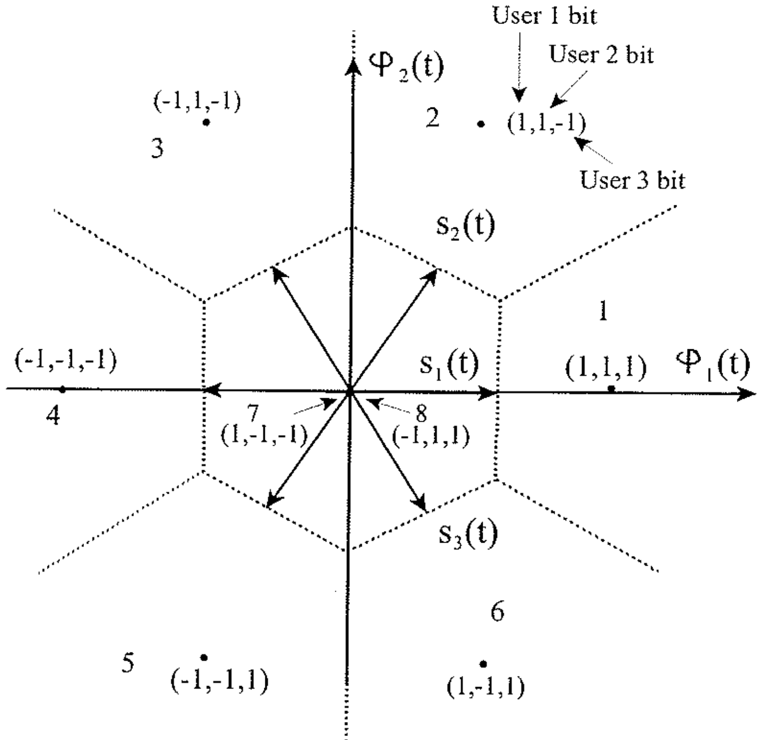

Figure 2.5: $\pi/3$ *Signal points and decision regions for joint optimum receiver.* $s_i(t)$ *represent the transmitted signals. The received signal points are represented by a dot along with transmitted bit pattern when channel noise is ignored.*

The receiver bases its decision on sufficient statistics that are obtained somewhat differently from that of the correlation receivers. The received signal is projected onto the two basis functions of the signal space to obtain the sufficient statistics, i.e.,

$$r_1 = \int_0^T r(t)\varphi_1(t)dt \tag{2.17}$$

$$r_2 = \int_0^T r(t)\varphi_2(t)dt \tag{2.18}$$

The signal space is partitioned as shown in figure (2.5). The partition results in the minimum sequence error probability and is based simply on the minimum distance rule. The jointly optimum receiver is quite efficient at the base station, because three users information bits can be detected simultaneously, but at the mobile station, only one bit out of three bits is of interest to any user. For example, when user one wants to detect his/her information, if the signal in regions of 1,2 6 is detected, then bit "1" is chosen; if the signal in regions of 3,4 or 5 is detected, then "-1" is chosen; if the signal in the hexagon region of 7 or 8 is detected, then either "1" or "-1" can be chosen randomly. Therefore the decision rule and the boundary of user 1 can be expressed as:

$r_1 > 0$, say "1"; otherwise say "-1"

Similarly, one can develop the decision rules for the other two users.

Compared with the correlation receiver, the decision rules in the $\pi/3$ signal assignment case are completely the same. So the error performance of jointly optimum receiver for this case is the same as that of the correlation receiver.

2.3.4 Individually optimum receiver

The individually optimum receiver chooses the individual user information bit a_i to maximize the probability $Pr(a_i \mid r_1, r_2)$, where r_1 and r_2 are sufficient statistics as defined in equation 2.17 and 2.18. The decision rule of individual optimum receiver for user i is therefore:

$$Pr(a_i = +1 \mid r_1, r_2) > Pr(a_i = -1 \mid r_1, r_2) \quad a_i = 1 \tag{2.19}$$

$$Pr(a_i = +1 \mid r_1, r_2) < Pr(a_i = -1 \mid r_1, r_2) \quad a_i = -1 \tag{2.20}$$

The decision rule essentially selects the a_i that maximizes the likelihood function: $f(r_1, r_2 \mid a_i)$, i.e., if

$$f(r_1, r_2 \mid a_i = +1) \geq f(r_1, r_2 \mid a_i = -1), \quad choose \quad a_i = 1 \tag{2.21}$$

else

$$f(r_1, r_2 \mid a_i = +1) < f(r_1, r_2 \mid a_i = -1), \quad choose \quad a_i = -1 \tag{2.22}$$

For symbol a_1, the likelihood function $f(r_1, r_2 \mid a_1 = +1)$ has four terms:

$$f(r_1, r_2 \mid a_1 = +1) =$$

$$f(r_1, r_2 \mid a_1 = +1, a_2 = +1, a_3 = +1)Pr(a_2 = +1, a_3 = +1)+$$

$$f(r_1, r_2 \mid a_1 = +1, a_2 = -1, a_3 = +1)Pr(a_2 = -1, a_3 = +1)+$$

$$f(r_1, r_2 \mid a_1 = +1, a_2 = +1, a_3 = -1)Pr(a_2 = +1, a_3 = -1)+$$

$$f(r_1, r_2 \mid a_i = +1, a_2 = -1, a_3 = -1)Pr(a_2 = -1, a_3 = -1) \tag{2.23}$$

Since the a_i are assumed to be equally probable and statistically independent, the probabilities $Pr(a_2 = +1, a_3 = +1)$, $Pr(a_2 = -1, a_3 = +1)$, $Pr(a_2 = +1, a_3 = -1)$, $Pr(a_2 = -1, a_3 = -1)$ are all equal to 0.25.

Consider user 1 in the $\pi/3$ signal assignment scheme, the first term $f(r_1, r_2 \mid a_1 = +1)$ in equation (2.23) is calculated as follows:

$$f(r_1, r_2 \mid a_1 = +1, a_2 = +1, a_3 = +1) =$$

$$= \frac{1}{\sqrt{2\pi}\sigma} exp\{-\frac{1}{2\sigma^2} \int_0^T [r(t) - s_1(t) - s_2(t) - s_3(t)]^2 dt\} \tag{2.24}$$

where $\sigma^2 = N_0/2$.

Since $s_1(t) = \varphi_1(t)$, $s_2(t) = \frac{1}{2}\varphi_1(t) + \frac{\sqrt{3}}{2}\varphi_2(t)$, $s_3(t) = \frac{1}{2}\varphi_1(t) - \frac{\sqrt{3}}{2}\varphi_2(t)$, equation (2.24) has the form:

$$f(r_1, r_2 \mid a_1 = +1, a_2 = +1, a_3 = +1) = C * exp\{\frac{2r_1 - 2}{\sigma^2}\} \tag{2.25}$$

where

$$C = \frac{1}{\sqrt{2\pi}\sigma} exp\{-\frac{1}{2\sigma^2}\int_0^T r(t)^2 dt\}$$

The other terms in equation (2.23) are calculated similarly:

$$f(r_1, r_2 \mid a_1 = +1, a_2 = -1, a_3 = +1) = C * exp\{\frac{r_1 - \sqrt{3}r_2 - 2}{\sigma^2}\} \tag{2.26}$$

$$f(r_1, r_2 \mid a_1 = +1, a_2 = +1, a_3 = -1) = C * exp\{\frac{r_1 + \sqrt{3}r_2 - 2}{\sigma^2}\} \tag{2.27}$$

$$f(r_1, r_2 \mid a_1 = +1, a_2 = -1, a_3 = -1) = C \tag{2.28}$$

The likelihood function $f(r_1, r_2 \mid a_1 = -1)$ also has four terms:

$$f(r_1, r_2 \mid a_1 = -1) =$$

$$f(r_1, r_2 \mid a_1 = -1, a_2 = +1, a_3 = +1) Pr(a_2 = +1, a_3 = +1) +$$

$$f(r_1, r_2 \mid a_1 = -1, a_2 = -1, a_3 = +1) Pr(a_2 = -1, a_3 = +1) +$$

$$f(r_1, r_2 \mid a_1 = -1, a_2 = +1, a_3 = -1) Pr(a_2 = +1, a_3 = -1) +$$

$$f(r_1, r_2 \mid a_i = +1, a_2 = -1, a_3 = -1) Pr(a_2 = -1, a_3 = -1) \tag{2.29}$$

The four terms in equation (2.29) have the form:

$$f(r_1, r_2 \mid a_1 = -1, a_2 = +1, a_3 = +1) = C \tag{2.30}$$

$$f(r_1, r_2 \mid a_1 = -1, a_2 =_1, a_3 = +1) = C * exp\{-\frac{r_1 - \sqrt{3}r_2 + 2}{\sigma^2}\} \tag{2.31}$$

$$f(r_1, r_2 \mid a_1 = -1, a_2 = +1, a_3 = -1) = C * exp\{-\frac{r_1 - \sqrt{3}r_2 + 2}{\sigma^2}\} \tag{2.32}$$

$$f(r_1, r_2 \mid a_1 = -1, a_2 = -1, a_3 = -1) = C * exp\{-\frac{r_2 + 2}{\sigma^2}\} \tag{2.33}$$

The boundary of the decision rule is determined by considering equality in equation 2.21. After simple manipulation the following is obtained:

$$sinh(\frac{2r_1}{\sigma^2}) + 2 * sinh(\frac{r_1}{\sigma^2}) * cosh(\frac{\sqrt{3}r_2}{\sigma^2}) = 0 \tag{2.34}$$

The solution of equation (2.34) is $r_1 = 0$. So the decision rule is precisely the same as that of the correlation receiver.

The above analysis of error performance shows that it is unsatisfactory regardless of which receiver is considered. Unsatisfactory in the sense that an irreducible error is present for three receivers and a decorrelation receiver does not even exist.

The next two sections explore two other approaches to the signal assignment for three users in a two dimensional signal space. The first approach is an intuitive one while the second is based directly on the fundamental criteria of interest, that of minimizing overall error probability.

2.4 Intuitive approach

In general, the approach is based on assigning a set of orthogonal signals with the cardinality of this set equal to the signal space dimensionality. The remaining signals are assigned with the criteria the correlation is minimized.

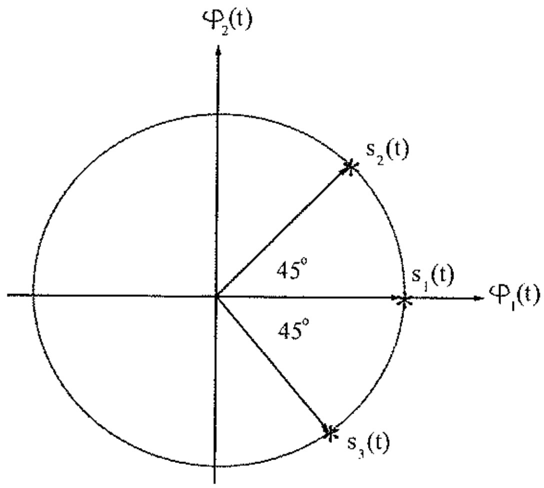

Figure 2.6: $\pi/4$ *signal assignment scheme*

For the toy example of a two dimension space with three users the signal assignment is easily seen to be as illustrated in figure (2.6).

Specifically

$$s_1(t) = \varphi_1(t) \leftrightarrow (1, 0)$$

$$s_2(t) = \sqrt{2}/2(\varphi_1(t) + \varphi_2(t)) \leftrightarrow (\sqrt{2}/2, \sqrt{2}/2)$$

$$s_3(t) = \sqrt{2}/2(\varphi_1(t) - \varphi_2(t)) \leftrightarrow (\sqrt{2}/2, -\sqrt{2}/2)$$

Consider again the performance of the four different receivers. The procedure is identical to that of the last section and the details are shown in Appendix A. The error performance of the four receivers are:

2.4.1 Correlation receiver

The bit error probability of user 1 is

$$P_{b(1)} = 0.25Q[(1+\sqrt{2})\sqrt{2E_b/N_0}] + 0.5Q[\sqrt{2E_b/N_0}] + 0.25Q[(1-\sqrt{2})\sqrt{2E_b/N_0}] \tag{2.35}$$

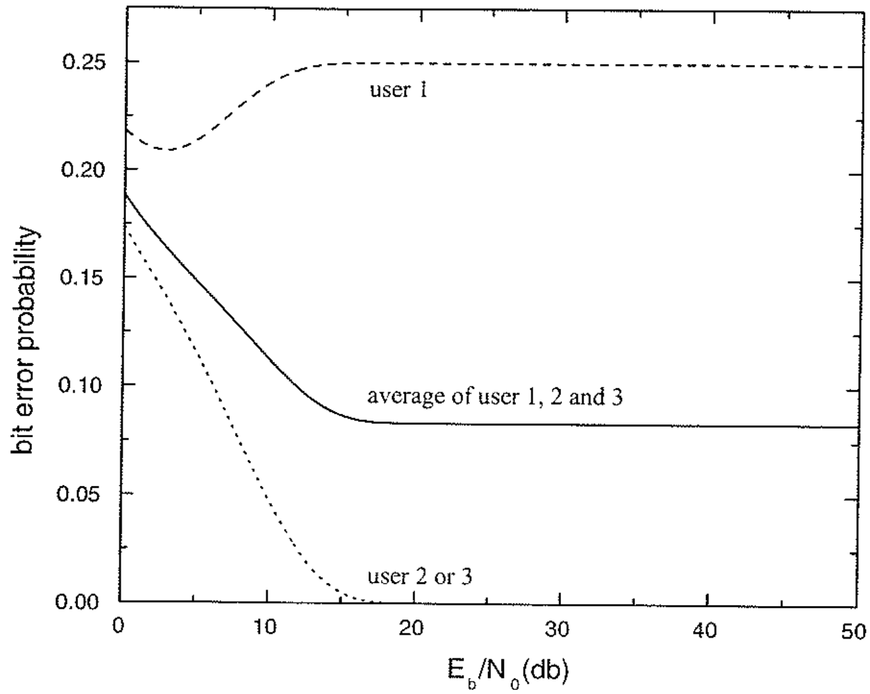

Figure 2.7: *error probability of $\pi/4$ signal assignment scheme*

The bit error probability of both user 2 and user 3 are the same and given by:

$$P_{b(2,3)} = 0.5Q[(1+\sqrt{2}/2)\sqrt{2E_b/N_0}] + 0.5Q[(1-\sqrt{2}/2)\sqrt{2E_b/N_0}] \qquad (2.36)$$

Figure (2.7) shows the error performance of user 1, user 2 and user 3. It can be seen user 2 and 3 have much smaller error probability than user 1. The overall average error probability is also shown .

2.4.2 Decorrelation receiver

The cross correlation matrix R defined in equation (2.14) in this case is

$$R = \begin{pmatrix} 1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 1 \end{pmatrix} \qquad (2.37)$$

The inverse of R

$$R = 10^3 * \begin{pmatrix} 3.3113 & -2.3411 & -2.3411 \\ -2.3411 & 1.6561 & 1.6551 \\ -2.3411 & 1.6551 & 1.6561 \end{pmatrix} \tag{2.38}$$

The sufficient statistics are

$$\begin{pmatrix} r_1' \\ r_2' \\ r_3' \end{pmatrix} == \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} n_1' \\ n_2' \\ n_3' \end{pmatrix} \tag{2.39}$$

Where the new random variables n_1', n_2', n_3' are

$$\begin{pmatrix} n_1' \\ n_2' \\ n_3' \end{pmatrix} = R^{-1} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = 10^3 * \begin{pmatrix} 3.3113n_1 - 2.3411n_2 - 2.3411n_3 \\ -2.3411n_1 + 1.6561n_2 + 1.6551n_3 \\ -2.3411n_1 + 1.6551n_2 + 1.6561n_3 \end{pmatrix} \tag{2.40}$$

Because n_1, n_2 and n_3 are normal, Gaussian random variables with variance $\sigma^2 = N_0/2$, so n_1', n_2' and n_3' are also normal,Gaussian random variables with variances σ_1^2, σ_2^2 and σ_3^2 as follows:

$$\sigma_1^2 = 2.19 \times 10^7 (N_0/2)$$

$$\sigma_2^2 = 1.10 \times 10^7 (N_0/2)$$

$$\sigma_3^2 = 1.10 \times 10^7 (N_0/2) \tag{2.41}$$

The overall average symbol error probability for the three users is

$$Pr = \frac{1}{3} Q(3.0 * 10^{-4} \sqrt{E_b/N_0}) + \frac{2}{3} Q(1.3 * 10^{-3} \sqrt{E_b/N_0}) \tag{2.42}$$

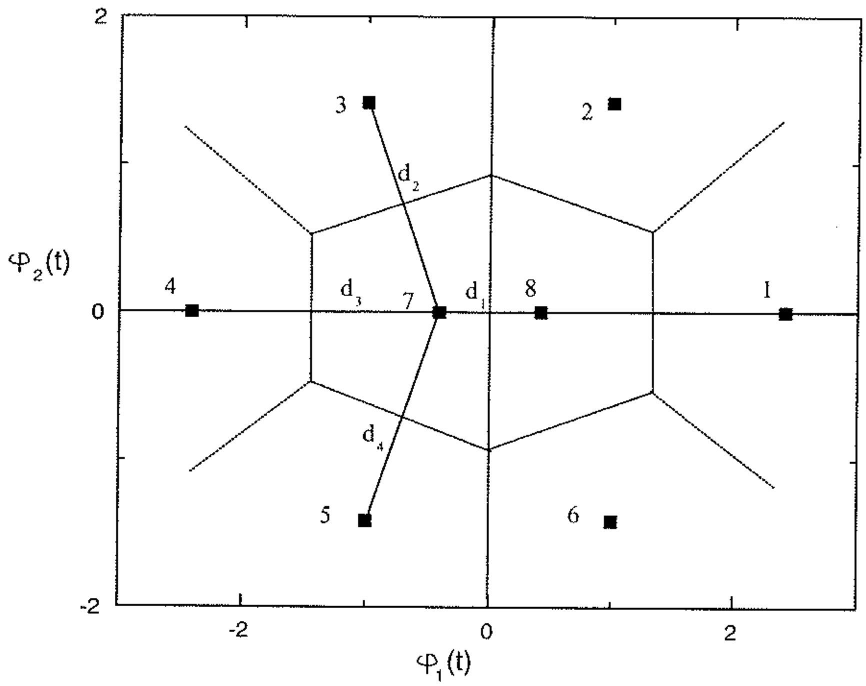

Figure 2.8: *Signal points and decision region of $\pi/4$ signal assignment scheme*

2.4.3 Jointly optimum receiver

When AWGN is not present, the possible signal points are shown in figure (2.8). For user 1, if the signal in regions of 1, 2, 6 or 7 is detected, then bit "1" is chosen; if the signal in regions of 3, 4, 5 or 8 is detected, then "-1" is chosen. It is very hard to obtain the precise symbol error probability, but the worst case lower bound and upper bound can show the approximate error probability. For joint detection, the region 7 and 8 obviously have the biggest error probability. Choose region 7 (with information bit (1,-1,-1)) as an example. One can see point 7 has 4 nearest neighbors 3,4,5 and 8, with "distances" of: $d_1 = 2(\sqrt{2}-1)\sqrt{E_b}$, $d_2 = d_4 = 2\sqrt{(2-\sqrt{2})}\sqrt{E_b}$, $d_3 = 2\sqrt{E_b}$, So the symbol error probability is bounded by:

$$Q[\frac{d_1}{\sqrt{2N_0}}] < Pr(error|symbol) < Q[\frac{d_3}{\sqrt{2N_0}}] + 2Q[\frac{d_2}{\sqrt{2N_0}}] + Q[\frac{d_1}{\sqrt{2N_0}}] \quad (2.43)$$

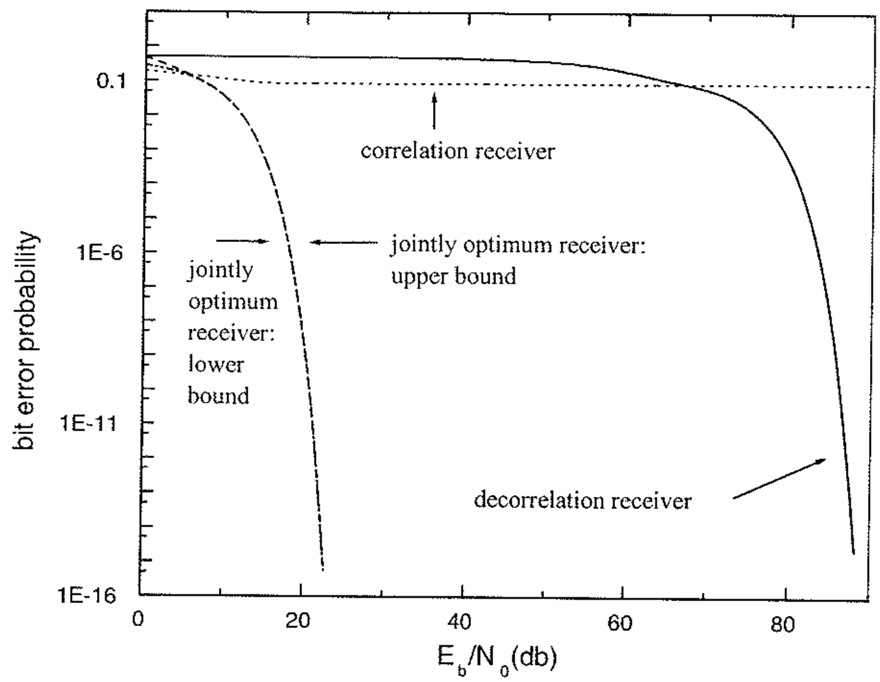

Figure 2.9: *Comparison of error performance of different receivers for $\pi/4$ signal assignment scheme*

i.e.,

$$Q[(2-\sqrt{2})\sqrt{\frac{E_b}{N_0}}] < Pr(error|symbol) < Q[\sqrt{\frac{2E_b}{N_0}}]+2Q[(\sqrt{2\sqrt{2}-4})\sqrt{\frac{E_b}{N_0}}]+Q[(2-\sqrt{2})\sqrt{\frac{E_b}{N_0}}] \tag{2.44}$$

As a comparison, the error performance of correlation receiver, decorrelation receiver and jointly optimum receiver is plotted in figure (2.9). It can be seen from figure (2.9) that the performance of jointly optimum receiver has the best performance, while the correlation receiver is the worst.

2.4.4 Individually optimum receiver

Similar to the case of the $\pi/3$ signal assignment scheme, the boundary of the decision region between $a_1 = +1$ and $a_1 = -1$ is decided by the following equation(See

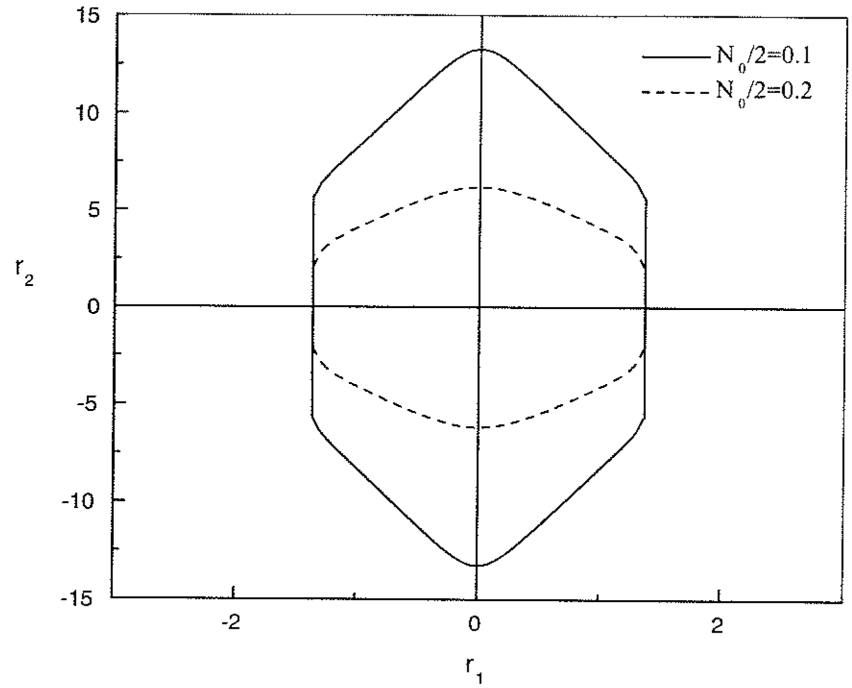

Figure 2.10: *Decision region of the individual optimum receiver with a $\pi/4$ signal assignment scheme*

appendix B):

$$cosh\frac{\sqrt{2}r_2}{\sigma^2} = \frac{cosh\frac{\sqrt{2}r_1+\sqrt{2}}{\sigma^2} - exp(\frac{2r_1}{\sigma^2})cosh\frac{\sqrt{2}r_1-\sqrt{2}}{\sigma^2})}{exp(\frac{2r_1}{\sigma^2}) - 1} \tag{2.45}$$

The solution of equation (2.45) is shown in figure (2.10) when $\sigma^2 = 0.1$ and $\sigma^2 = 0.2$. Obviously the decision region is a function of signal to noise ratio. When the channel becomes noisy, equation 2.45 has no solution. The error probability and even the bound of the error probability are very difficult to determine and beyond the scope of the thesis.

2.5 Signal assignment based on the criteria of minimization of Symbol Error Probability

A fundamental criteria for any communication system is to minimize the symbol error probability. This is considered here for the toy example for the simplest receiver, the correlation receiver. The received signal for three users in two dimensional space can be written as:

$$r(t) = \sqrt{E_b}s_1(t) + \sqrt{E_b}s_2(t) + \sqrt{E_b}s_3(t) + n(t)$$

where $s_1(t) = \varphi_1(t)$, $s_2(t) = cos\theta_1\varphi_1(t) + sin\theta_1\varphi_2(t)$, $s_3(t) = cos\theta_2\varphi_1(t) + sin(\theta_2)\varphi_2(t)$.

For a correlation receiver, the sufficient statistics are:

$$r_1 = \int_0^{T_s} r(t) * s_1(t)\,dt = a_1 + a_2cos\theta_1 + a_3cos\theta_2 + n_1$$

$$r_2 = \int_0^{T_s} r(t) * s_2(t)\,dt = a_1cos\theta_1 + a_2 + a_3(cos\theta_1cos\theta_2 + sin\theta_1sin\theta_2) + n_2$$

$$r_3 = \int_0^{T_s} r(t) * s_3(t)\,dt = a_1cos\theta_2 + a_2(cos\theta_1cos\theta_2 + sin\theta_1sin\theta_2) + a_3 + n_3 \quad (2.46)$$

Again, the receiver is based on the decision rule: if $r_i \geq 0$, choose $a_i = 1$, otherwise choose $a_i = -1$. Based on this decision rule and using symmetry it follows that for user i the symbol error probability is

$P_{b(i)}$=Pr[symbol error$|a_i$=+1].

The symbol error probability for each of the three users is

$$P_{b(1)} = 0.25Q[\sqrt{\frac{2E_b}{N_0}}(1 + cos\theta_1 + cos\theta_2)] + 0.25Q[\sqrt{\frac{2E_b}{N_0}}(1 - cos\theta_1 + cos\theta_2)]$$

$$+0.25Q[\sqrt{\frac{2E_b}{N_0}}(1+cos\theta_1-cos\theta_2)]+0.25Q[\sqrt{\frac{2E_b}{N_0}}(1-cos\theta_1-cos\theta_2)] \quad (2.47)$$

$$P_{b(2)}=0.25Q[\sqrt{\frac{2E_b}{N_0}}(1+cos\theta_1+cos(\theta_1-\theta_2)]+0.25Q[\sqrt{\frac{2E_b}{N_0}}(1-cos\theta_1+cos(\theta_1-\theta_2)]$$

$$+0.25Q[\sqrt{\frac{2E_b}{N_0}}(1+cos\theta_1-cos(\theta_1-\theta_2)]+0.25Q[\sqrt{\frac{2E_b}{N_0}}(1-cos\theta_1-cos(\theta_1-\theta_2)] \quad (2.48)$$

$$P_{b(3)}=0.25Q[\sqrt{\frac{2E_b}{N_0}}(1+cos\theta_2+cos(\theta_1-\theta_2)]+0.25Q[\sqrt{\frac{2E_b}{N_0}}(1-cos\theta_2+cos(\theta_1-\theta_2)]$$

$$+0.25Q[\sqrt{\frac{2E_b}{N_0}}(1+cos\theta_2-cos(\theta_1-\theta_2)]+0.25Q[\sqrt{\frac{2E_b}{N_0}}(1-cos\theta_2-cos(\theta_1-\theta_2)] \quad (2.49)$$

The overall average symbol error probability for the three users is given simply by:

$$\bar{P}_r(\theta_1,\theta_2,E_b/N_0)=[P_{b(1)}+P_{b(2)}+P_{b(3)}]/3 \quad (2.50)$$

One now needs to choose θ_1 and θ_2 for a given E_b/N_0 to minimize equation (2.50).

Numerical solution of equation (2.50) for a given SNR is unique. Table 2.1 shows all the best values of the angle θ_1 and θ_2 which minimize the symbol error probability with SNR ranging from 2 db to 26 db. It should be pointed out that the angles shown in the table are confined in $0 \leq \theta_1, \theta_2 \leq 180^o$ because signal $s_i(t)$ is equivalent to $-s_i(t)$ when binary(± 1) data is considered. From table (2.1), it can be seen that when SNR is less than 12db, the $\pi/3$ signal assignment is the best choice, which is the same as that shown in the previous section where the criteria is to the total cross correlation. However, when the SNR becomes bigger and

Table 2.1: Signal assignments for different signal to noise ratio

SNR(db)	Bit Error Probability	θ_1(degree)	θ_2
2	0.154738	60.03	120.06
3	0.150218	60.03	120.06
4	0.146096	60.03	120.06
5	0.142382	60.03	120.06
6	0.139083	60.03	120.06
7	0.136198	60.03	120.06
8	0.133723	60.03	120.06
9	0.131641	60.03	120.06
10	0.12993	60.03	120.06
11	0.12856	60.03	120.06
12	0.127493	60.03	120.06
13	0.126559	55.8	111.7
14	0.125411	53.9	107.7
15	0.124064	52.5	105
16	0.122539	51.4	102.9
17	0.120851	50.6	101.2
18	0.119012	49.9	99.8
19	0.117033	49.2	98.6
20	0.11493	48.8	97.5
21	0.112722	48.2	96.6
22	0.11043	47.9	95.8
23	0.10808	47.5	95.0
24	0.105699	47.2	94.4
25	0.10332	46.9	93.9
26	0.100974	46.7	93.4

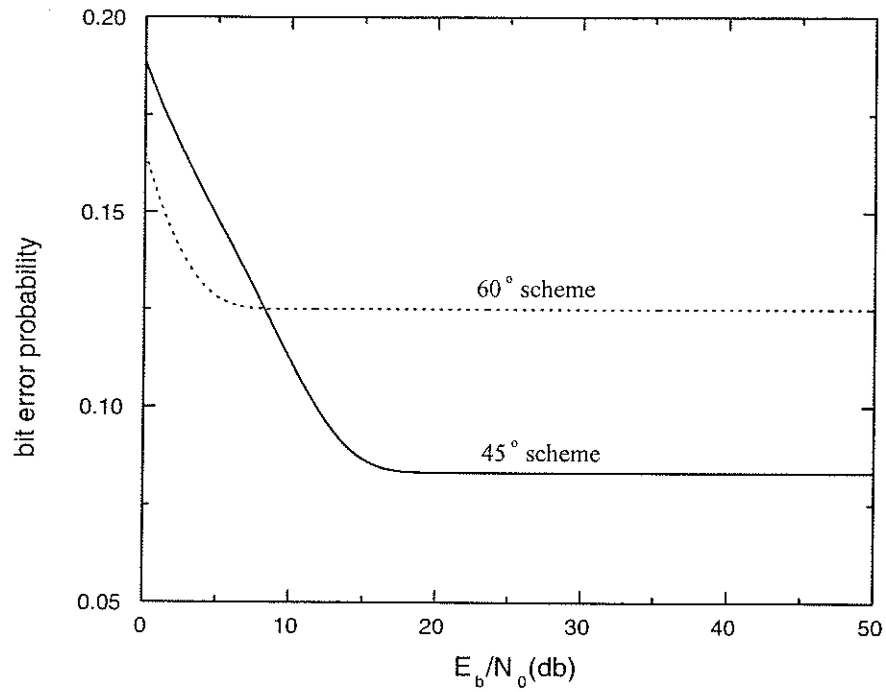

Figure 2.11: *Comparison of error performance between $\pi/3$ and $\pi/4$ signal assignment scheme*

bigger, the signal assignment has to be adjusted to minimize the error probability based on the correlation receiver.

2.6 Discussion

From the above analysis, it is seen that the signal assignment scheme depends on the adopted receiver. When the correlation receiver is used to detect the transmitted information, the $\pi/3$ scheme is better than the intuitive signal assignment(figure (2.11)) at the region of small signal to noise ratio, which means that the criteria of minimizing the TSC is consistent with the criteria of minimizing the bit error probability . However, when other kinds of receivers are considered, this is not the case. For example, the jointly optimum receiver detects the $\pi/4$ signal assignment scheme better than $\pi/3$. When the dimension of signal space becomes bigger, the jointly optimum receiver becomes more and more complicated(2^N signal points in N dimensional space) and thus impractical if not intractable. So the actual receiver is always a trade-off between the performance and the complexity. In the following chapters, only the correlation receiver is considered to explore the signal assignment scheme in higher dimensional space, and an iteration receiver will be introduced to improve the performance.

Chapter 3

Iteration Receiver and the Hybrid of CDMA and TDMA Signal Assignment

3.1 Introduction

This chapter covers the iteration receiver and an intuitive signal assignment scheme called CDMA plus TDMA. In the previous chapter, the correlation receiver, de-correlation receiver, jointly optimum receiver and individually optimum receiver were discussed for signal assignment schemes in a two dimensional signal space. No matter which receiver or signal assignment was used, the performance was not satisfactory. The reason is because relatively the number of users is significantly greater than the dimensionality of the signal space. With this in mind, this chapter considers the situation where the number of users is in a relative sense only slightly more than the dimensionality of the signal space.

In a high dimensional signal space, among all the receivers, only the correlation receiver is simple, while the other three become impractical. But correlation receiver performance degrades significantly when the MAI in the system is large. This is the case when the number of users is much larger than the dimensionality of the signal space or the signal set is not chosen appropriately. From the point-of-view of both performance and implementation, the iteration receiver, an extension of of the correlation receiver, is introduced in this chapter and applied to the signal assignment scheme of CDMA plus TDMA.

The iteration receiver is based on the intuitive notion that if the sequence set is carefully chosen so that the MAI is small or at least "reasonable" then the estimated symbol is likely to be correct. Then the MAI is known and can be subtracted out which results in the classical known signal in additive white Gaussian noise problem. One should observe that the approach is very analogous to decision feedback equalization [6].

3.2 Iteration receiver

The general idea of the iteration receiver is: first use the correlation receiver to obtain the estimated symbols of all users; then for each individual user estimate the possible interference caused by all other users and subtract this interference from the original received signal; lastly apply the correlation receiver again. After this procedure is applied to all the remaining users, the zeroth iteration is finished. The next iteration is similar except the estimated interference is obtained from the decoded symbols during the previous iteration.

Suppose there are M users in an L dimension space($M > L$). The sequence set $s_1(t), s_2(t), ..., s_M(t)$ is modulated by the user symbols $a_1, a_2, ..., a_M$(binary: ± 1). Under the assumption that the channel is AWGN and all the carriers are perfectly synchronized, the received signal r(t) is:

$$r(t) = a_1 s_1(t) + a_2 s_2(t) + ... + a_M s_M(t) + n(t) \tag{3.1}$$

where n(t) is the noise signal. If a correlation receiver is used, the users' symbols are decoded as: $\hat{a}_1, \hat{a}_2, ..., \hat{a}_M$, as seen in figure (3.1). Obviously, if the sequences $s_1(t), s_2(t), ..., s_M(t)$ are not orthogonal, then each user will experience interference. However, if the sequence set is carefully chosen to make the MAI as small as possible, chances are that the decoded symbols are likely to be correct, which immediately means that part of the MAI is known. The iteration receiver makes use of the result of the correlation receiver, makes an estimate of the MAI, then subtracts the estimate from the received signal, and applies the correlation receiver again. The procedure can continue until a performance bound is reached. The procedure of the iteration receiver is shown in figure (3.1). Initially, for a specific user, say user i, the MAI caused by the other M-1 users is estimated as follows:

$$MAI_i = \hat{a}_1 s_1(t) + \hat{a}_2 s_2(t) + ... + \hat{a}_{i-1} s_{i-1}(t) + \hat{a}_{i+1} s_{i+1} + ... + \hat{a}_M s_M(t) \tag{3.2}$$

The estimated received signal $\hat{r}_i(t)$ of user i can be written as:

$$\hat{r}_i(t) = r(t) - MAI_i = a_i s_i(t) + (a_1 - \hat{a}_1) s_1(t) +$$

$$+(a_2 - \hat{a}_2) s_2(t) + ... + (a_{i-1} - \hat{a}_{i-1}) s_{i-1}(t) +$$

$$+(a_{i+1} - \hat{a}_{i+1} i) s_{i+1} + ... + (a_M - \hat{a}_M) s_M(t) + n(t) \tag{3.3}$$

If the sequence $s_1(t), s_2(t), ..., s_M(t)$ is properly chosen, then most substraction terms in equation (3.3) will be zero, and even if there are still some terms which are not equal to zero, if the cross correlation between $s_i(t)$ and $s_j(t)$(j=1,2,..,i-1,i+1,..,M) is small, the estimated received signal will be close to the single user received signal $a_i s_i(t) + n(t)$. The decison rule of the first iteration receiver is based on the sufficient statistic:

$$\hat{r}_i = \int_0^T \hat{r}_i(t) s_i(t) dt \tag{3.4}$$

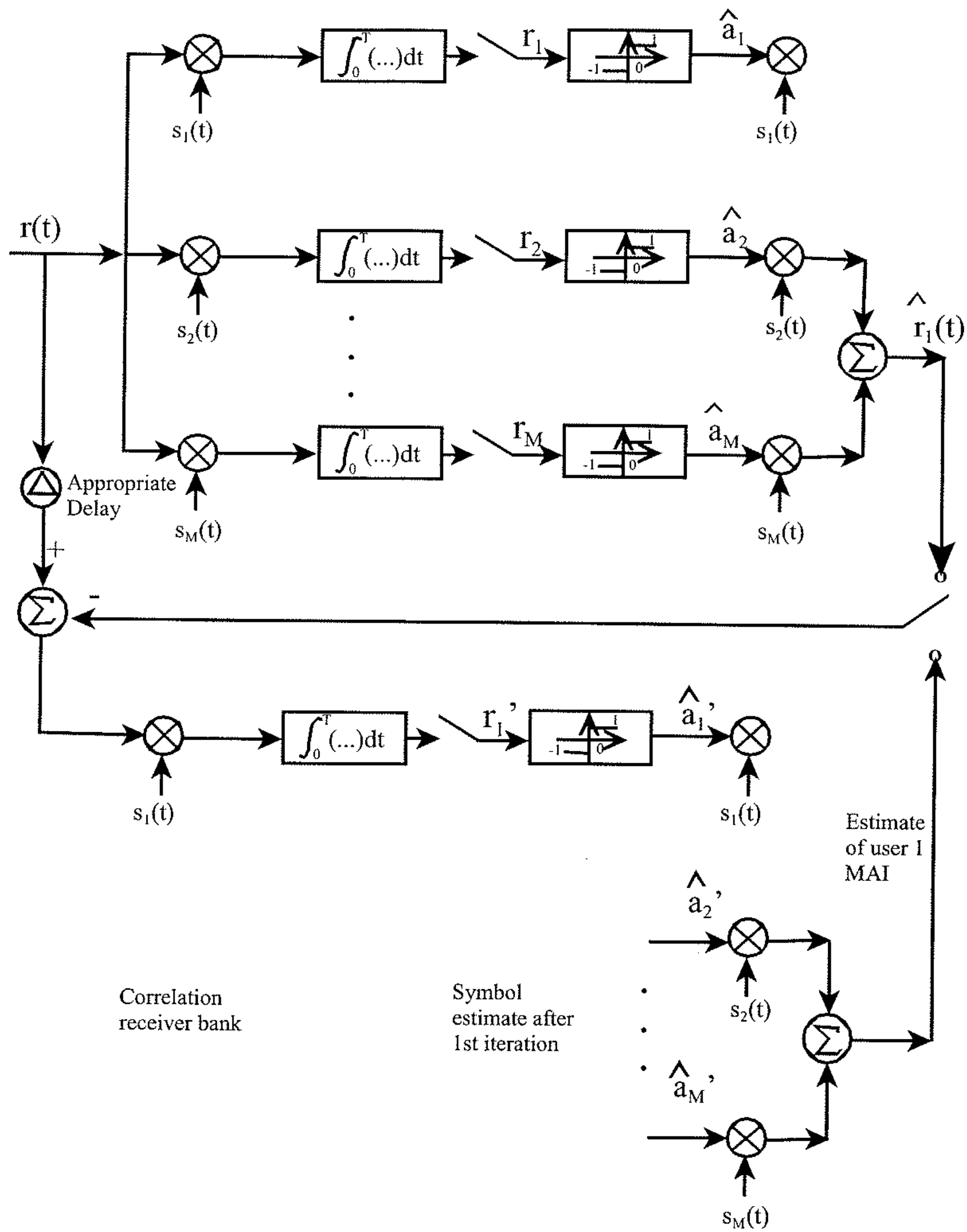

Figure 3.1: *Decoding procedure of iteration receiver*

When $\hat{r}_i > 0$, symbol 1 is detected, otherwise -1 is detected. The iteration can repeat. The problem becoms, how to choose signal set $s_1(t), s_2(t), ..., s_M(t)$ to fully use the property of the iteration receiver? The following section discusses an intuitive signal assignment scheme: CDMA plus TDMA.

3.3 Augmenting CDMA with TDMA

3.3.1 Signal assignment

The signal space defined in IS-95, which is the standard of interest in this thesis, has a dimension of 64, where Walsh functions are used as the signal sets. A direct motivation for this thesis is to increase the capacity of the IS-95 cellular system. This translates the problem to one of how should additional users be assigned in a signal space of dimension 64?

One intuitive solution is to augment the CDMA signals with TDMA signals, first proposed by Hikmet[5]. This signal assignment scheme is called CDMA/TDMA. It is discussed in some detail below:

The ith(i=1,2,...64) Walsh function can be expressed as linear combinations of the basis shown in figure (3.2) as $s_i(t) = \sum_{j=1}^{64} s_{ij}\phi_j(t)$, $i = 1, 2, ..., 64$, where $\int_0^{T_s} s_i(t)s_j(t)\,dt = \delta_{ij}$, T_s is the duration of Walsh sequence, T_c is the duration of one chip of Walsh sequence, and $T_s = 64T_c$.

Now suppose there are m more users than the 64 orthogonal signals that the signal space can support where m is reasonably less than "64". Assign signals to these users as follows:

$$x_i(t) = C * \phi_i(t) \qquad i = 1, 2, ..., m$$

where C a constant that determines the energy of $x_i(t)$. In another word, the m additional users occupy different time slots to avoid mutual interference within

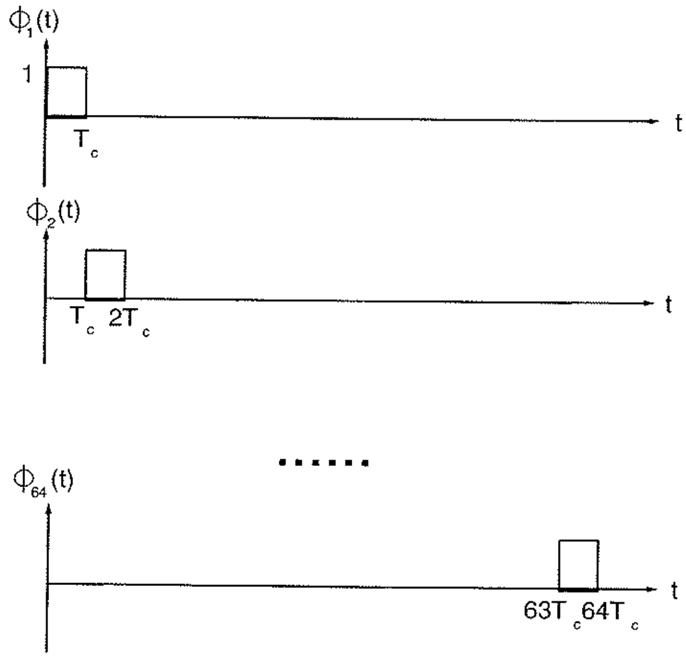

Figure 3.2: *Bases of 64 dimensional space*

themselves, essentially they are TDMA users.

Without loss of generality, suppose the signals have unit energy which means $C = 1$. Note that signals $\{s_i(t)\}$ and $\{x_i(t)\}$ are orthogonal within themselves but there is correlation(or MAI) between signals between the two sets, call this

$$\alpha_{ij} = \int_0^{T_s} x_i(t) s_j(t)\, dt \tag{3.5}$$

The following sections discuss the performance of the CDMA/TDMA signal assignment scheme when the iteration receiver introduced at the beginning of this chapter is used . Actually the complete signal set is divided into two subsets, the correlation receiver is first applied to the CDMA users during an iteration, then to complete an iteration the TDMA symbols are estimated by making use of the estimated CDMA symbols. The iteration can continue until one reach a point of diminishing returns in terms of error performance.

3.3.2 Performance of CDMA user

Under the assumption of perfect synchronization and power control in AWGN, the received signal is:

$$r(t) = \sum_{i=1}^{64} a_i s_i(t) + \sum_{j=1}^{m} b_j x_j(t) + n(t) \tag{3.6}$$

When the correlation receiver is used, the sufficient statistic for the CDMA user is:

$$r_i = \int_0^{T_s} r(t) s_i(t)\, dt = a_i + \sum_{j=1}^{m} b_j \alpha_{ij} + n_i \tag{3.7}$$

where a_i is the ith CDMA user information symbol. The second term in equation (3.7) is the interference caused by TDMA users: a discrete random variable with zero mean value. Call this term r_i^I:

$$r_i^I = \sum_{j=1}^{m} b_j \alpha_{ij} = \sum_{j=1}^{m} b_j s_{ij} \tag{3.8}$$

since

$$\alpha_{ij} = \int_0^{T_s} \sum_{k=1}^{64} [s_{ik}\phi_k(t)]\phi_j(t)\, dt = \sum_{k=1}^{64} s_{ik}\delta_{kj} = s_{ij}$$

Note that since each Walsh signal is normalized to unit energy, s_{ij} is either $+1/8$ or $-1/8$.

Assume the transmitted energy for each data symbol is E_b. Then $b_j = \pm\sqrt{E_b}$ with equal probability. Therefore r_i^I is a random variable with possible values $\sqrt{E_b}/8(-m+2i)$, i=0,1,2,...,m, and a binomial distribution, i.e., there are

$(m+1)$ possible values with the probabilities:

$$Pr[r_i^I = (-m+2*l)\sqrt{E_b}/8|l] = \frac{\begin{pmatrix} m \\ l \end{pmatrix}}{2^m} \qquad l = 0,1,2,...,m \tag{3.9}$$

where l represents the interference sequence.

The third term n_i in equation (3.7) is a Gaussian random variable with zero mean and variance of $N_0/2$, whose probability density is given by

$$f(n_i) = \frac{1}{\sqrt{\pi N_0}} exp(\frac{-n_i^2}{N_0}) \tag{3.10}$$

The decision rule of the correlation receiver is:

$r_i >= 0, \quad \tilde{a_i} = \sqrt{E_b}$

$r_i < 0, \quad \tilde{a_i} = -\sqrt{E_b}$

The bit error probability can be calculated as:

$$P_b(CDMA) = \sum_{l=0}^{m} Pr[r_i^I] \int_{-\infty}^{-\sqrt{E_b}+(m-2l)\sqrt{E_b}/8} f(n_i)\, dn_i \tag{3.11}$$

For convenience, define the probability:

$$P_{b,l} = \int_{-\infty}^{-\sqrt{E_b}+(m-2l)\sqrt{E_b}/8} f(n_i)\, dn_i \tag{3.12}$$

which is the bit error probability when $b_i = \sqrt{E_b}$ is the transmitted symbol.

The plots of equation (3.11) is shown in figure (3.3) for various m=1,3,5,7,9,11. It can be seen that the performance of the CDMA users degrades rapidly when the number of TDMA users increases.

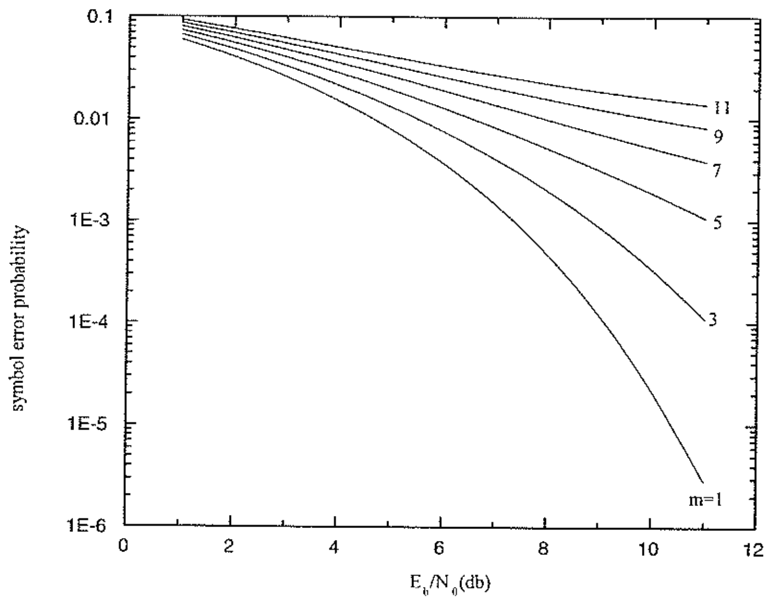

Figure 3.3: *The CDMA users' symbol error probability of the zeroth iteration receiver with m TDMA users (m=1, 3, 5, 7, 9,11)*

3.3.3 Gaussian approximation of CDMA user performance

The performance can be also analyzed as follows([5]). In equation (3.7):

$$r_i = a_i + \sum_{j=1}^{m} b_j \alpha_{ij} + n_i = a_i + r_i^I + n_i$$

even if there are only a few TDMA users, the random variable r_i^I has many possible discrete values. So according to the central limit theorem, r_i^I can be approximately treated as a continuous r.v.(random variable) with variance σ_T^2:

$$\sigma_T^2 = m * E_b/N = m * E_b/64 \tag{3.13}$$

where m is the number of TDMA users, N is the CDMA users or the spreading factor which is 64 for our interest. CDMA user i sees a total interference variance σ_{CDMA}^2 caused by AWGN and TDMA users of:

$$\sigma_{CDMA}^2 = N_0/2 + E_b * m/64 \tag{3.14}$$

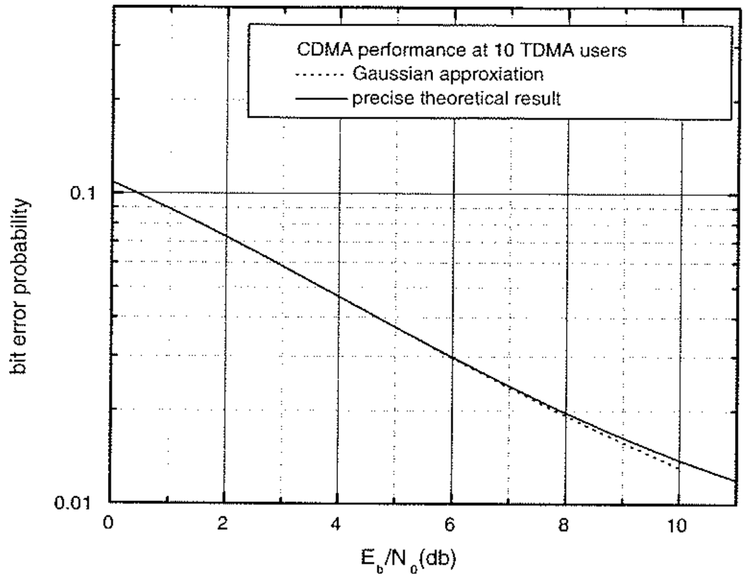

Figure 3.4: *Comparison between Gaussian approximation and the theoretical calculation*

Thus the symbol error probability of CDMA users is:

$$P_b(CDMA) = Q\left(\sqrt{\frac{E_b}{N_0/2 + E_b * m/64}}\right) \tag{3.15}$$

When $m = 10$, the performance of this approximation is shown in figure (3.4) The Gaussian approximation of CDMA performance is consistent with the theoretical calculation.

3.3.4 Performance of TDMA users

Consider now the probability of error experienced by the TDMA users. When the estimated MAI due to the CDMA users is subtracted off, the resultant TDMA signal becomes:

$$\tilde{r}_{TDMA}(t) = \sum_{i=1}^{64}(a_i - \tilde{a}_i)s_i(t) + \sum_{j=1}^{m} b_j x_j(t) + n(t) \tag{3.16}$$

The output of the correlation receiver is:

$$\tilde{r}_{j(TDMA)} = \int_0^{T_s} \tilde{r}_{TDMA}(t) x_j(t)\, dt$$

$$= b_j + \sum_{i=1}^{64} (a_i - \tilde{a}_i) s_{ij} + n_j, j = 1, 2, ...m$$

or

$$\tilde{r}_{j(TDMA)} = b_j + r_j^I + n_j \tag{3.17}$$

where

$$r_j^I = \sum_{i=1}^{64} (a_i - \tilde{a}_i) s_{ij}$$

Each term $(a_i - \tilde{a}_i)$ in r_j^I is one of three values (2,0,-2) with probabilities $\{(0.5P_b(CDMA),$ $1 - P_b(CDMA),\ 0.5P_b(CDMA)\}$ respectively.

For convenience, call the bit error probability of the CDMA users $P_{b,C}$. Note that, s_{ij} is deterministic, and can take on only two values: either $\sqrt{E_b}/8$ or $-\sqrt{E_b}/8$. Therefore r_j^I has values of -128+2n, n=1,2,...,128(scaled by $\sqrt{E_b}/8$).

The probabilities of their values are:

$$P_r(r_j^I = -128) = (\frac{1}{2} P_{b,C})^{64}$$

$$P_r(r_j^I = -126) = \begin{pmatrix} 64 \\ 1 \end{pmatrix} (1 - P_{b,C})(\frac{1}{2} P_{b,C})^{63}$$

$$P_r(r_j^I = -124) = \begin{pmatrix} 64 \\ 1 \end{pmatrix} (1 - P_{b,C})^1 (\frac{1}{2} P_{b,C})^{63} + \begin{pmatrix} 64 \\ 2 \end{pmatrix} (1 - P_{b,C})^2 (\frac{1}{2} P_{b,C})^{62}$$

......

and in general, when $n \leq 64$, it is

$$P_r(r_j^I = -128 + 2n) = \sum_{k=0}^{(int)(n/2)} \begin{pmatrix} 64 \\ n-k \end{pmatrix} (1 - P_{b,C})^{n-2k} (\frac{1}{2} P_{b,C})^{64-(n-2k)} \quad (3.18)$$

and

$$P_r(r_j^I = -128+2n) = \sum_{k=0}^{(int)(128-n)/2} \begin{pmatrix} 64 \\ 128-n-k \end{pmatrix} (1-P_{b,C})^{128-n-2k} (\frac{1}{2} P_{b,C})^{64-(128-n-2k)} \quad (3.19)$$

for $n > 64$, where $(int)x$ means get the integer value of variable x. Again the decision rule is:

$$\begin{array}{ll} \tilde{r}_j(TDMA) >= 0, & \tilde{b}_j = \sqrt{E_b} \\ \tilde{r}_j(TDMA) < 0, & \tilde{b}_j = -\sqrt{E_b} \end{array}$$

Under the condition that $\sqrt{E_b}$ was transmitted, an error is made if $\sqrt{E_b} + (128 + 2n)\sqrt{E_b}/8 + n_j < 0$, i.e.

$$n_j < -\sqrt{E_b} + (64 - n) - \sqrt{E_b}/4, n = 0, 1, ..., 128$$

The bit error probability of a TDMA user is therefore:

$$P_{b,T} = \sum_{n=0}^{128} P_r(r_j^I(n)) \int_0^{-\sqrt{E_b}+(64-n)\sqrt{E_b}/4} f(n_j)\, dn_j \quad (3.20)$$

$f(n_j)$ is defined in equation (3.10), and $P_r(r_j^I(n)$ is defined in equation (3.18) and equation (3.19).

3.3.5 First iteration

Usually the above error performance of CDMA users (equation (3.11)) is not good enough because of the multiple user interference caused by TDMA users. To improve the performance one can iterate. The following presents an analysis of the performance after the first iteration.

Consider first the CDMA users. The estimated TDMA signals from the zeroth iteration are:

$$\tilde{r}_T(t) = \sum_{j=1}^{m} \tilde{b}_j x_j(t) \tag{3.21}$$

Subtracting this from the received signal gives:

$$\tilde{r}_C(t) = r(t) - \tilde{r}_T(t) = \sum_{i=1}^{64} a_i s_i(t) + \sum_{j=1}^{m} (b_j - \tilde{b}_j) x_j(t) + n(t)$$

The output of the iteration receiver for ith CDMA user is:

$$r_{i,C} = \int_0^{T_S} \tilde{r}_C(t) s_i(t)\, dt = a_i + \sum_{j=1}^{m} (b_j - \tilde{b}_j) s_{ij} + n_i$$

where $\sum_{j=1}^{m} (b_j - \tilde{b}_j) s_{ij}$ can take on values of $(-2m + 2k)\sqrt{E_b}/8$, k=0, 1, 2,..., 2m, and as before $(b_j - \tilde{b}_j) \in \{-2, 0, 2\}$ with the probability of $\{0.5P_{b,T}, 1 - P_{b,T}, 0.5P_{b,T}\}$.

The decision rule is again : if $r_{i,C} >= 0$, $a_j = 1$; otherwise $a_j = -1$.

An error occurs if $\sqrt{E_b} + \sum_{j=1}^{m} (b_j - \tilde{b}_j) s_{ij} + n_i < 0$

or $\quad n_i < -\sqrt{E_b} + (m-k)\sqrt{E_b}/4$

The bit error probability of the CDMA user during the first iteration is:

$$P_{b,C}^1 = \sum_{0}^{2m} P_r(r_i^I(k)) \int_0^{-\sqrt{E_b}+(m-k)\sqrt{E_b}/4} f(n_i)\, dn_i \tag{3.22}$$

$$P_r(r_i^I(k)) = \begin{cases} \sum_{l=0}^{(int)k/2} \begin{pmatrix} m \\ k-l \end{pmatrix} (1 - P_{b,T})^{k-2l} (\frac{1}{2} P_{b,T})^{m-(k-2l)} & \text{if } 0 \le k \le m \\ \sum_{k=0}^{(int)(2m-k)/2} \begin{pmatrix} m \\ 2m-k-l \end{pmatrix} (1 - P_{b,C})^{2m-k-2l} (\frac{1}{2} P_{b,C})^{m-(2m-k-2l)} & \text{if } k > m \end{cases}$$

The estimated CDMA user data symbols are now used to detect the TDMA users. Subtracting out the estimated CDMA symbols from $s(t)$ gives:

$$\tilde{r}^1_{TDMA}(t) = \sum_{i=1}^{64}(a_i - \tilde{a}^1_i)s_i(t) + \sum_{j=1}^{m} b_j x_j(t) + n(t) \tag{3.23}$$

Again a correlation receiver results in the following set of equations

$$\tilde{r}^1_{j(TDMA)} = \int_0^{T_s} \tilde{r}_{TDMA}(t)x_j(t)\,dt = b_j + \sum_{i=1}^{64}(a_i - \tilde{a}_i)s_{ij} + n_j, \quad j = 1, 2, ..., m$$

or

$$\tilde{r}_{j(TDMA)} = b_j + r^I_{j,1} + n_j \tag{3.24}$$

where $r^I_{j,1} = \sum_{i=1}^{64}(a_i - \tilde{a}^1_i)s_{ij}$ and $\tilde{a}^1_i$ is the output of the ith CDMA user.

Once again the term $(a_i - \tilde{a}^1_i)$ can take three possible values {2,0,-2} with probabilities $\{0.5P^1_{b,C}, 1 - P^1_{b,C}, 0.5P^1_{b,C}\}$. After a similar derivation, the symbol error probability of TDMA users $P^1_{b,T}$ can be calculated as:

$$P^1_{b,T} = \sum_{0}^{128} P_r(r^I_{j,1}(n)) \int_0^{-\sqrt{E_b}+(64-n)\sqrt{E_b}/4} f(n_j)\,dn_j \tag{3.25}$$

$$P_r(r^I_{j,1}(n)) = \begin{cases} \sum_{k=0}^{(int)n/2} \begin{pmatrix} 64 \\ n-k \end{pmatrix} (1 - P^1_{b,C})^{n-2k}(\frac{1}{2}P^1_{b,C})^{64-(n-2k)} & \text{if } 0 \le n \le 64 \\ \sum_{k=0}^{(int)(128-n)/2} \begin{pmatrix} 64 \\ 128-n-k \end{pmatrix} (1 - P^1_{b,C})^{128-n-2k}(\frac{1}{2}P^1_{b,C})^{64-(128-n-2k)} & \text{if } n > 64 \end{cases}$$

3.3.6 Alternative approach

Another approach to the iteration receiver is: detect the TDMA user first, then use the estimated TDMA data symbol to detect the CDMA user, i.e., the procedure is reversed. Still considering the case of 64 CDMA users and an additional m($m \ll 64$) TDMA users, the sufficient statistics for the TDMA users for the zeroth iteration is:

$$r_j = b_j + \sum_{i=1}^{64} a_i s_{ij} + n_j, \quad j = 1, 2, ..., m \tag{3.26}$$

where $\sum_{i=1}^{64} a_i s_{ij}$ can have 65 possible values with probability $Pr[(-64 + 2n) * \frac{1}{8}\sqrt{E_b}] = \frac{1}{2^{64}} * \begin{pmatrix} 64 \\ n \end{pmatrix}$, n=0,1,2,...,64.

The bit error probability is:

$$P_{b,TT} = \sum_{n=0}^{64} \frac{1}{2^{64}} * \begin{pmatrix} 64 \\ n \end{pmatrix} \int_{-\infty}^{-\sqrt{E_b}+(32-n)\sqrt{E_b}/4} f(n_j)\, dn_j \tag{3.27}$$

As described before, the estimated CDMA received signal is calculated by subtracting the estimated TDMA received signals from the total received signal. The estimated CDMA signal is:

$$\tilde{r}_C(t) = r(t) - \tilde{r}_T(t) = \sum_{i=1}^{64} a_i s_i(t) + \sum_{j=1}^{m} (b_j - \tilde{b}_j) x_j(t) + n(t)$$

The error performance of the CDMA users is calculated simply as before:

$$P_{b,CT} = \sum_{0}^{2m} P_{rT}(r_i^I(k)) \int_{0}^{-\sqrt{E_b}+(m-k)\sqrt{E_b}/4} f(n_i)\, dn_i \tag{3.28}$$

$$P_{rT}(r_i^I(k)) = \begin{cases} \sum_{l=0}^{(int)k/2} \begin{pmatrix} m \\ k-l \end{pmatrix} (1-P_{b,TT})^{k-2l}(\frac{1}{2}P_{b,TT})^{m-(k-2l)} \\ \qquad \text{if } 0 \le k \le m \\ \sum_{k=0}^{(int)(2m-k)/2} \begin{pmatrix} m \\ 2m-k-l \end{pmatrix} (1-P_{b,TT})^{2m-k-2l}(\frac{1}{2}P_{b,TT})^{m-(2m-k-2l)} \\ \qquad \text{if } k > m \end{cases}$$

where the subscript "T" represents the detection sequence of $TDMA \rightarrow CDMA \rightarrow TDMA \rightarrow CDMA....$

After the zeroth iteration the bit error probability of the TDMA users and CDMA users are respectively:

$$P_{b,TT}^1 = \sum_0^{128} P_{rT}^1(r_j^I(n)) \int_0^{-\sqrt{E_b}+(64-n)\sqrt{E_b}/4} f(n_j)\, dn_j \tag{3.29}$$

$$P_{rT}^1(r_j^I(n)) = \begin{cases} \sum_{k=0}^{(int)n/2} \begin{pmatrix} 64 \\ n-k \end{pmatrix} (1-P_{b,CT})^{n-2k}(\frac{1}{2}P_{b,CT})^{64-(n-2k)} \\ \qquad \text{if } 0 \le n \le 64 \\ \sum_{k=0}^{(int)(128-n)/2} \begin{pmatrix} 64 \\ 128-n-k \end{pmatrix} (1-P_{b,CT})^{128-n-2k}(\frac{1}{2}P_{b,CT})^{64-(128-n-2k)} \\ \qquad \text{if } n > 64 \end{cases}$$

and

$$P_{b,CT}^1 = \sum_0^{2m} P_{rT}^2(r_i^I(k)) \int_0^{-\sqrt{E_b}+(m-k)\sqrt{E_b}/4} f(n_i)\, dn_i \tag{3.30}$$

$$P_{rT}^2(r_i^I(k)) = \begin{cases} \sum_{l=0}^{(int)k/2} \begin{pmatrix} m \\ k-l \end{pmatrix} (1-P_{b,TT}^1)^{k-2l}(\frac{1}{2}P_{b,TT}^1)^{m-(k-2l)} \\ \qquad \text{if } 0 \le k \le m \\ \sum_{k=0}^{(int)(2m-k)/2} \begin{pmatrix} m \\ 2m-k-l \end{pmatrix} (1-P_{b,TT}^1)^{2m-k-2l}(\frac{1}{2}P_{b,TT}^1)^{m-(2m-k-2l)} \\ \qquad \text{if } k > m \end{cases}$$

3.4 Simulation result between two demodulation methods

The theoretical analysis of the error performance is very complicated. A better approach is to evaluate the performance by computer simulation. In figure (3.5), the CDMA performance is shown, while figure (3.6) shows the TDMA performance. From these simulation results, the following conclusions are obtained:

(1). CDMA users have better performance with $CDMA \rightarrow TDMA$ than $TDMA \rightarrow CDMA$ for the first and second iteration, but the opposite for the zeroth iteration. Simply CDMA users see less MAI than the TDMA users. The TDMA performance is weakly dependent of SNR, since for a TDMA user, the interference comes almost exclusively from CDMA users.

(2). The performance improvement of second iteration over that of the first iteration is marginal.

(3). In the $CDMA \rightarrow TDMA$ iteration procedure, TDMA users' performance is similar to CDMA users', but much worse than CDMA users' performance in the $TDMA \rightarrow CDMA$ iteration procedure.

3.5 Augmenting TDMA systems with CDMA sequences

Rather than augmenting the CDMA signals with TDMA signals, one can reverse the assignment. In continuing with the IS-95 specification again, 64 users are assigned to individual time slots or chips, $x_i(t) = \phi_i(t)$, i.e., i=1,2...64, where $\phi_i(t)$ are the bases discussed before. Additional users are assigned Walsh signals:

$$s_j(t) = \sum_{k=1}^{64} s_{jk}\phi_k(t) = \frac{1}{8}w_j(t), \qquad j = 1, 2, ..., m << 64$$

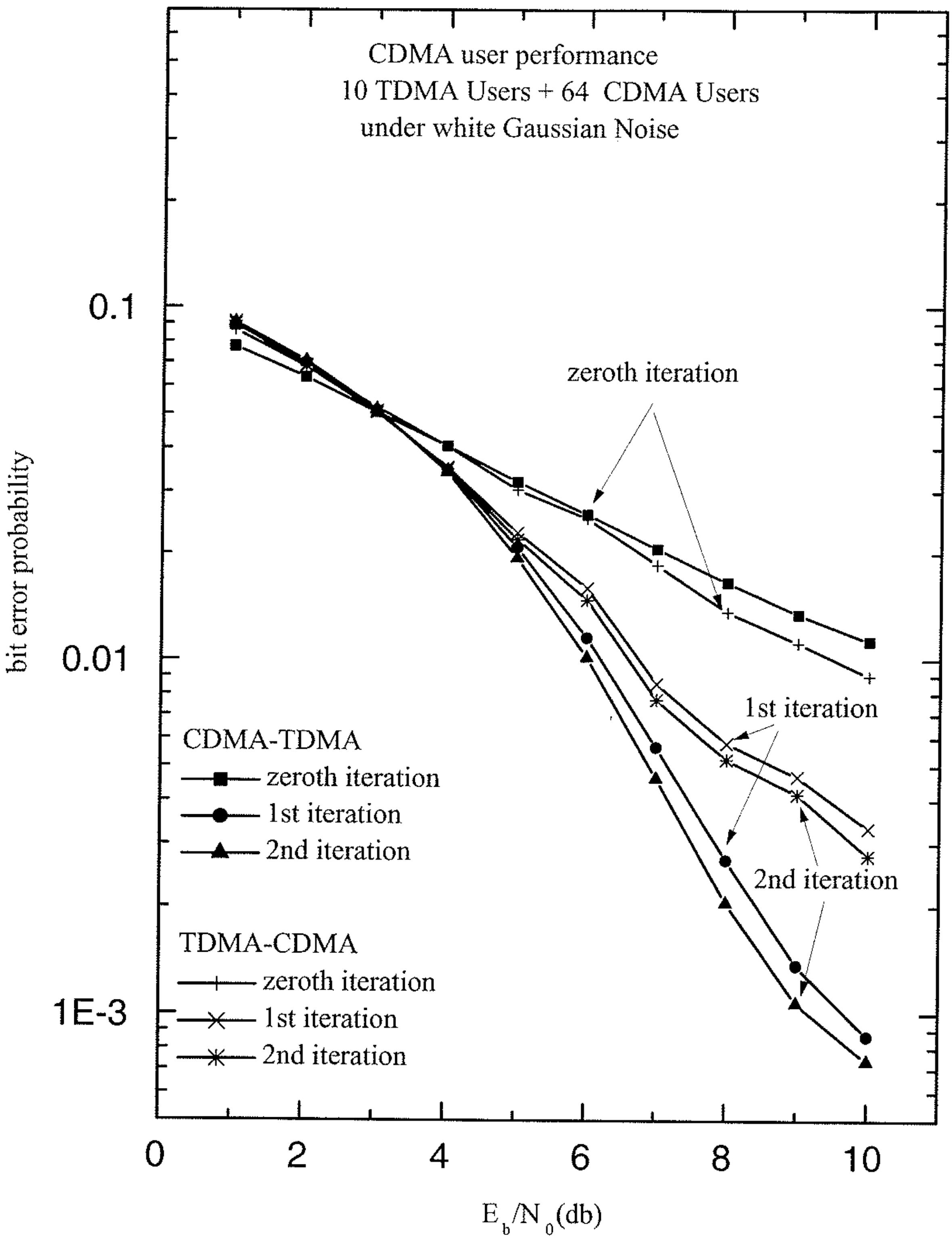

Figure 3.5: *CDMA user performance under two demodulation schemes*

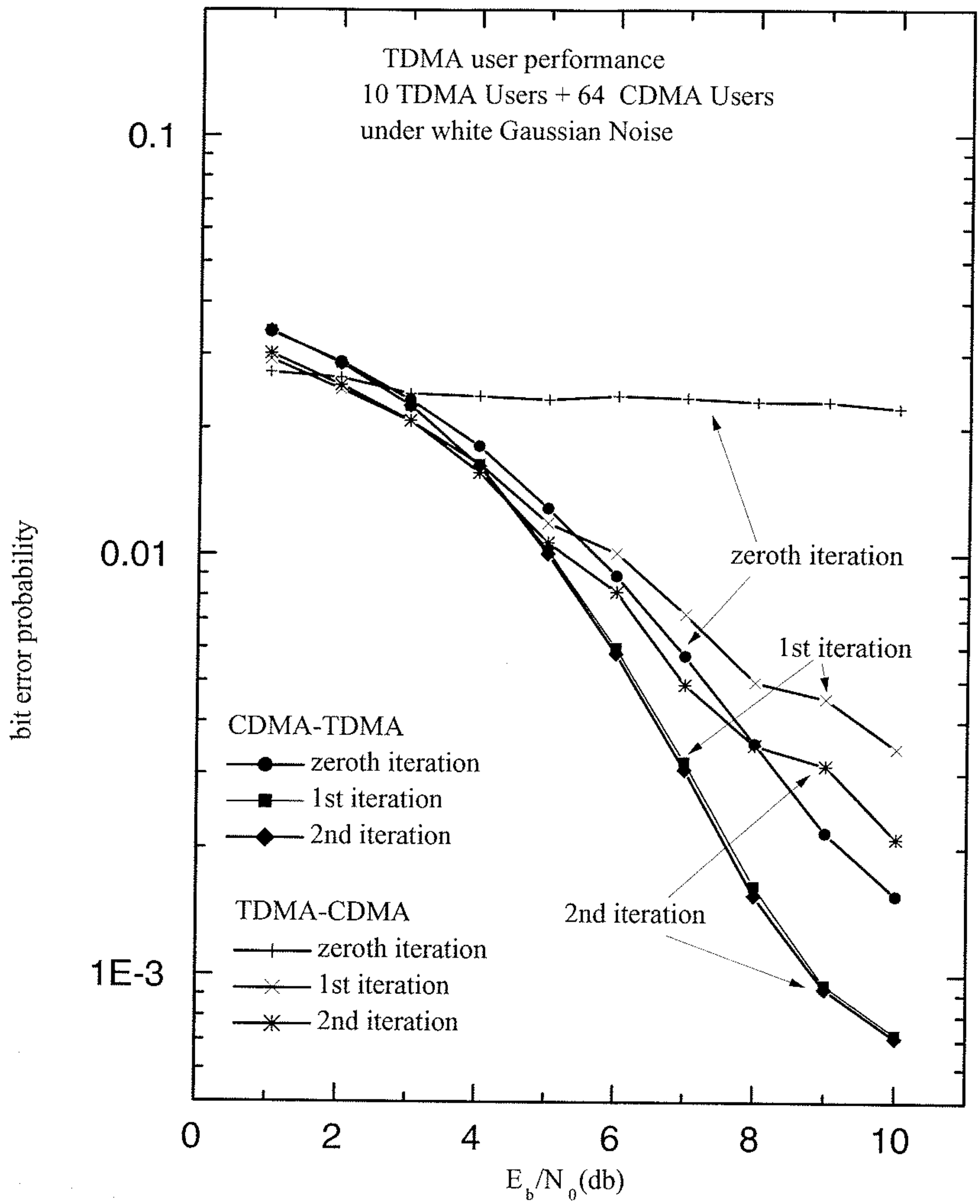

Figure 3.6: *TDMA user performance under two demodulation methods*

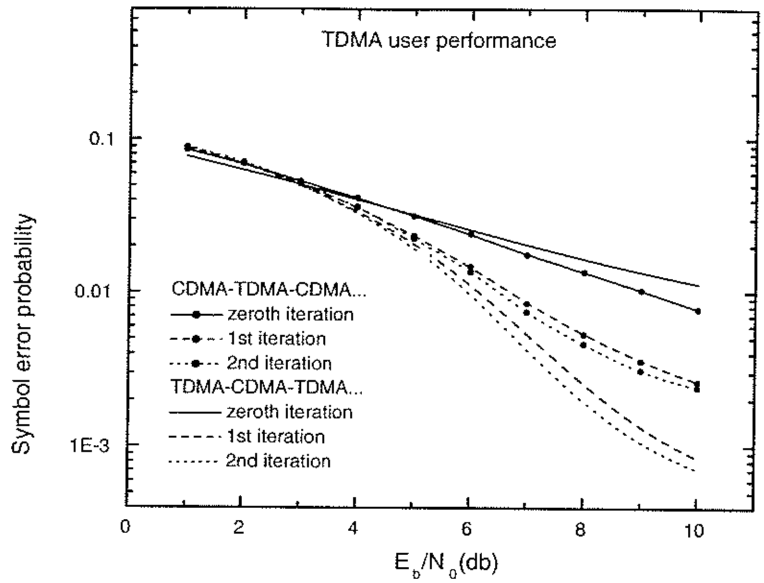

Figure 3.7: *TDMA user performance under two demodulation schemes*

To determine the performance of this signal assignment scheme simulation was used for the iteration receivers as described in the previous section.

The results of simulation are shown in figure (3.7) and (3.8).

Performance in the first iteration:

(1). TDMA users have better performance with $TDMA \to CDMA$ than $CDMA \to TDMA$ for first and second iteration, but the opposite for the zeroth iteration. because TDMA users see less MAI than the CDMA users.

(2). The performance improvement of second iteration after the first iteration is marginal.

(3). In the $TDMA \to CDMA$ iteration procedure, CDMA users' performance is similar to TDMA users', but much worse than TDMA users' performance in the $CDMA \to TDMA$ iteration procedure.

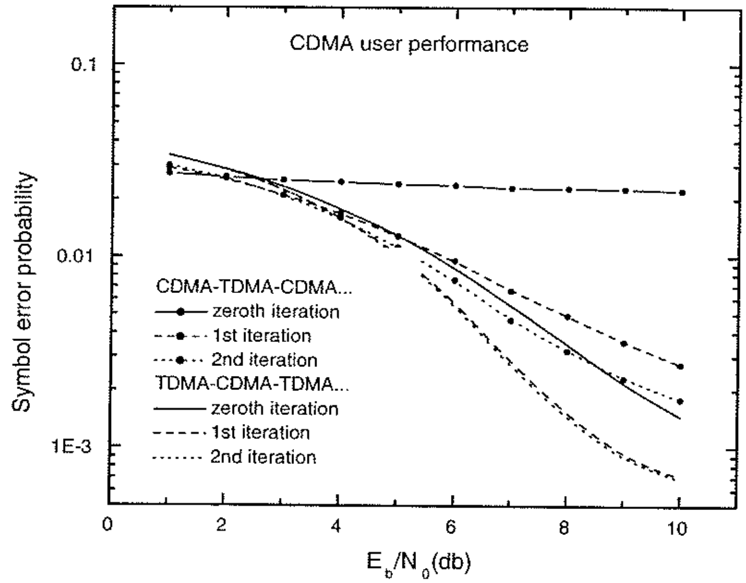

Figure 3.8: *CDMA user performance under two demodulation schemes*

3.6 Comparison between full CDMA + partial TDMA and full TDMA + partial CDMA signal assignment

The following conclusions are obtained by comparing the two signal assignment schemes of full CDMA + partial TDMA and full TDMA + partial CDMA:

(1). The order of detection affects the performance. When 64 CDMA + m TDMA signal assignment scheme is used, the performance is better when the CDMA users are detected first. On the other hand, if 64 TDMA + m TDMA signal assignment scheme is used, then it is better to detect the TDMA users first. If the correct order is used, the second iteration is not even necessary since the gain is marginal. Figure (3.9) compares the performance of the second receiver between 64CDMA+10TDMA and 64TDMA+10CDMA. A more general statement about the detection order of the iteration receiver can be made here: in a system with $nCDMA + mTDMA$ user, if $n > m$, CDMA users should be detected first; on the contrary if $n < m$, TDMA users should be detected first.

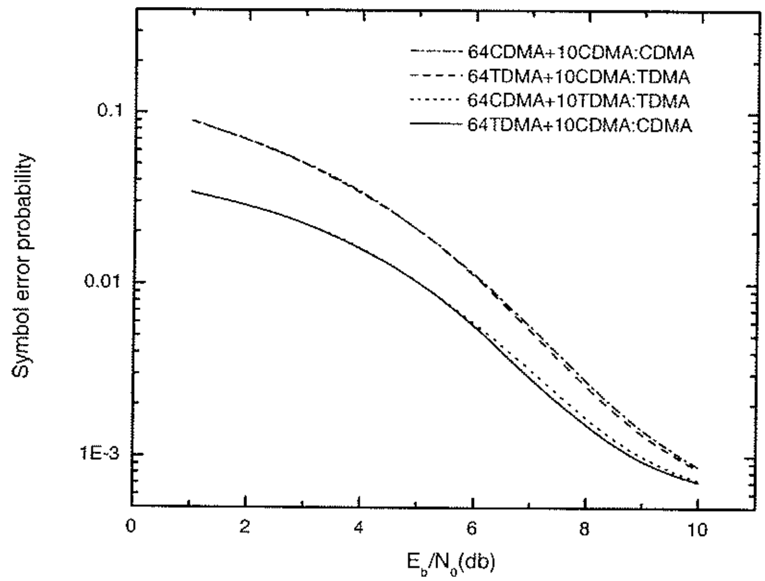

Figure 3.9: *CDMA and TDMA user performance under two different signal assignment*

(2). 64 CDMA + m TDMA is preferred. From figure (3.9), the performance of the two signal assignment schemes are essentially the same in the AWGN channel. But because CDMA has increased resistance to multi-path distortion [10], and is more secure, the $64CDMA + mTDMA$ is preferred to $64TDMA + mCDMA$.

Chapter 4

Hybrid of Two Sets of Orthogonal Codes

4.1 Introduction

The CDMA/TDMA signal assignment scheme discussed in the last chapter has some disadvantages typical of TDMA, such as weak resistance to multi-path distortion, and less security. Moreover, the peak to average power of TDMA is higher than CDMA signals which could be a serious problem in implementation when the linearity range of the amplifier is limited.

In order to overcome these disadvantages, this chapter introduces a new signal assignment scheme based on a hybrid of two sets of orthogonal codes. The two sets are chosen to overcome the disadvantages of CDMA/TDMA. The way to construct such codes is discussed and the performance is analyzed.

4.2 Hybrid of two sets of orthogonal codes and orthogonal transformation

The problem is: Given a complete Walsh sequence set of dimension n, then how does one accommodate m additional users($m << n$) with a set of sequences whose levels are the same as in the Walsh sequence set and simultaneously keeping the MAI to a minimum? Essentially, this question includes the following three constraints:

- The first constraint is to minimize the MAI between the m additional sequence set and the original n Walsh sequence set. This directly results in an assignment of two sets of orthogonal codes. This is explained as follows. In n dimensional space, the Walsh sequence set can be used as a set of bases. The sum of the square of the cross correlation between each basis vector and a vector of fixed length is the same, regardless of which direction the vector is oriented. So, the MAI for the first additional user can't be minimized. If more additional users are added, then the MAI is kept at a minimum if the additional users are mutually orthogonal. This leads to the result of two sets of orthogonal codes.

- The second constraint is to make the projection of each additional signal on each original basis vector equal. This also results in each individual MAI being equal or each signal point unique.

- All the additional vectors should have the same levels, i.e. the levels are either 1 or -1.

The following sections introduce two methods to construct the sequence set which satisfies the above constraints.

4.2.1 Orthogonal transformation

The first approach is a "trial and error" method based on the use of orthogonal matrices. Specifically, one uses Hadamard matrices to construct a new sequence set. Because Hadamard matrices (which is a kind of orthogonal matrix with entries 1 or -1) are known up to a certain order(at least up to the order of 300 [12]), the multiplication of the original matrix with Hadamard matrices gives a new orthogonal matrix. If this new matrix satisfies the uniform correlation constraint, then it is a desired matrix.

Mathematically, For an n dimensional Walsh sequence set: $\vec{x}_1, \vec{x}_2, ..., \vec{x}_n$ expressed in matrix form as:

$$\mathbf{H_1} = \begin{pmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \cdots \\ \vec{x}_n \end{pmatrix} \tag{4.1}$$

choose a Hadamard matrix T, such that $H_2 = T * H_1$ has all the entries 1 or -1.

The following gives an example for four dimensional space. Consider an orthogonal matrix H_1 in the 4-dimensional space,

$$H_1 = \begin{pmatrix} -1 & -1 & -1 & -1 \\ -1 & +1 & -1 & +1 \\ -1 & -1 & +1 & +1 \\ -1 & +1 & +1 & -1 \end{pmatrix} \tag{4.2}$$

It is easy to find an orthogonal matrix T_1:

$$T_1 = \frac{1}{2} \begin{pmatrix} -1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \end{pmatrix} \tag{4.3}$$

The product of matrix $H_1 * T_1$ is

$$H_2 = T_1 * H_1 = \begin{pmatrix} -1 & +1 & +1 & +1 \\ -1 & -1 & +1 & -1 \\ -1 & +1 & -1 & -1 \\ -1 & -1 & -1 & -1 \end{pmatrix} \tag{4.4}$$

Then check the sum of cross correlation of each row of matrix H_2 with all the rows of matrix H_1:

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2 = 16$$

Therefore, T_1 is the desired transformation.

However, if the orthogonal transform matrix $T_2 = \frac{1}{2}H_1^T$ was chosen, then the product of T_2 with H_1 is

$$H_3 = T_2 * H_1 = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \tag{4.5}$$

Though the sum of cross correlation of each row of matrix H_3 with all rows of matrix H_1:

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2 = 16$$

are equal because the elements of matrix H_3 has different levels from 1 and -1, the row sequences of H_3 are not the desired ones. They are typical TDMA signals.

4.2.2 Construct two sets of orthogonal codes by computer search

The above "trial and error" way can't guarantee that one can find the proper transformation matrix. A brute force approach is by computer exhaustive search. In order to speed up the search, one can use all possible constraints to minimize the search region. If there are only m additional users, one just needs to find m sequences instead of n.

The following constraints can be used in the search: If the original bases are $\vec{x}_1$, $\vec{x}_2$, ,...,$\vec{x}_n$, one need to find the sequences $\vec{y}_1$, $\vec{y}_2$, ,...,$\vec{y}_m$, where $\vec{y}_i$ is represented by

$$\vec{y}_i = \frac{1}{\sqrt{n}}(\pm\vec{x}_1 \pm \vec{x}_2 \pm ... \pm \vec{x}_n), 1 \le i \le m \tag{4.6}$$

where the coefficients are chosen to normalize the"energy" to n, i.e. the inner product:

$$< \vec{y}_i, \vec{y}_i >= n \tag{4.7}$$

The correlation between any base $\vec{x}_k$ with $\vec{y}_i$:

$$< \vec{x}_k, \vec{y}_i >= \pm \frac{n}{\sqrt{n}} = \pm\sqrt{n} \tag{4.8}$$

so the construction equation (4.6), (4.7) and (4.8) will guarantee that the correlation between the n Walsh functions and $\vec{x}_i's$ is uniform under the condition that all $\vec{x}_i$ are orthogonal within themselves. The following gives an example of computer search result in 16 dimensional space. One starts with the 16×16 Hadamard matrix $H_1(16 * 16)$:

$$H_1 = \begin{bmatrix}
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\
-1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 \\
-1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 \\
-1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 \\
-1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 \\
-1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 \\
-1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\
-1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\
-1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\
-1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\
-1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\
-1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\
-1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\
-1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1
\end{bmatrix}$$

By computer search many sets of sequences can be found which satisfy the construction equations, each set consists of 16 orthogonal sequences. Only two such sets H_2 and H_3 are given here as examples:

$$H_2 = \begin{bmatrix} -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 \\ +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 \\ -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 \\ -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & +1 & +1 \\ -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 \\ -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} +1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 & +1 & -1 \\ +1 & +1 & -1 & +1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & +1 & +1 \\ +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 \\ -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 \\ -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & +1 & +1 \\ -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 \\ -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 \end{bmatrix}$$

If there are m more users in addition to the original 16 users, then m Walsh sequences chosen from one of the above orthogonal matrices(not all other orthogonal matrices are listed here) would serve as the additional orthogonal set.

With the increase of the dimension of the signal space, the complexity of

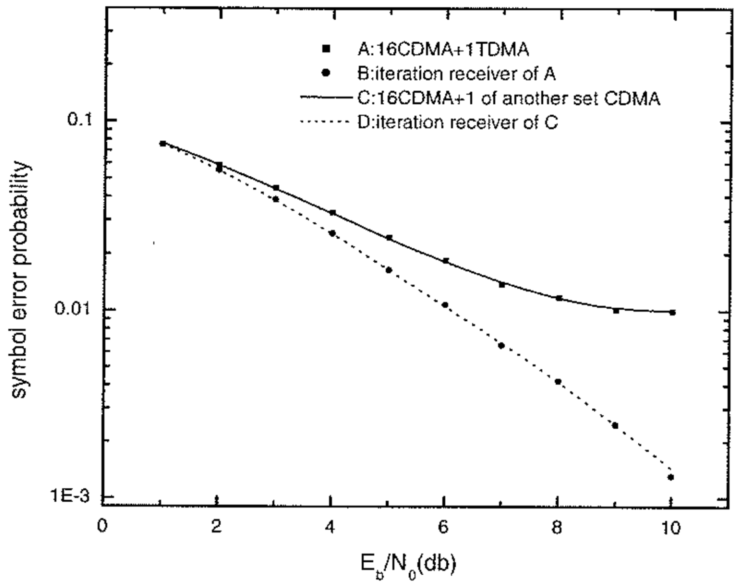

Figure 4.1: *Performance of two sets of orthogonal sequences(17 users)*

the computer search goes up exponentially. For example, when n=64, one has to search 2^{64} possible combinations of the bases; this is a tremendous task.

4.3 Performance analysis

Consider a system with n users which are accommodated by n Walsh sequences, and m additional users which are accommodated by the sequences $\vec{y_1}, \vec{y_1}, \vec{y_2}, ..., \vec{y_m}$. The sum of squared correlation between one Walsh sequence with all the additional sequences is $m * n$. Considering all the Walsh sequences, the sum becomes $n * m * n = mn^2$. At the same time the additional sequences will also see interference of the same level, so the total correlation square of such a system is $2mn^2$, which means it has the same performance as n CDMA + m TDMA system. As an example, in 16 dimensional space, considering the case that there are 18 users, 16 users can use Walsh functions, the remaining additional users are accommodated by another set of orthogonal sequences of 2-level:

$$\begin{bmatrix} -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 \\ +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 \end{bmatrix}$$

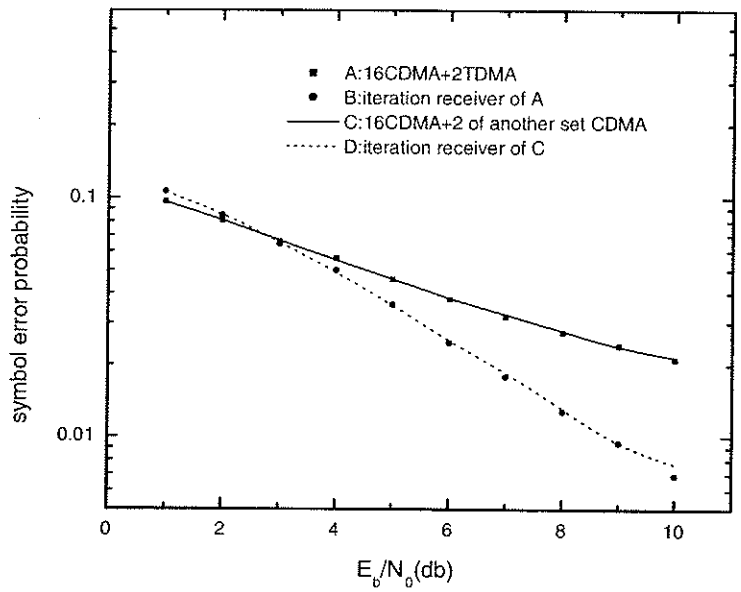

Figure 4.2: *Performance of two sets of orthogonal sequences(18 users)*

For CDMA+TDMA scheme, 2 sequences are chosen as follows:

$$\begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure (4.1) and Figure (4.2) show the performance for the case of 17 and 18 users respectively. In each case, the performance of the CDMA+TDMA scheme is identical to the two level scheme. This is also true for the first iteration receiver, which is intrinsically dominated by the multiple access interference that each user experiences. However, the two level system in which any sequence has equal projection on all the bases is preferred over the CDMA/TDMA system for the reasons mentioned at the beginning of this chapter.

Chapter 5

Signal Assignment in Multiple Dimensional Space: WBE Sequence

5.1 Motivation

This chapter discusses a signal assignment scheme based on the criteria to minimize the multiple access interference. In chapter 3 and 4, intuitive solutions were proposed to improve the capacity of CDMA system. The more general question is: in a CDMA systems with M users, and a certain number of chips, L, i.e, signal space dimensionality, how does one construct a set of sequences to get the best performance?

When $M < L$, one can always use orthogonal sequences, for example, Walsh sequences. But when $M > L$, the sequences cannot be orthogonal, and any user shall experience inter-user interference due to the other users. For the correlation receivers, which is the one considered in the rest of this thesis, the appropriate criteria is that of minimizing the sum of squares of the cross correlations between all pairs of sequences. The design of the sequences that accomplish this is discussed next.

Again, as in Chapter 2, it is assumed that all the M users have the same energy, and that the sequences are binary, i.e, either 1, or -1. Before presenting optimum sequences and their construction, a general formulation of the problem is given.

5.2 Synchronous CDMA systems

Assume the vector representation of the M sequences are: $\vec{s}^{(1)}, \vec{s}^{(2)}, ..., \vec{s}^{(M)}$, where each vector represents a user and has L binary($\pm$) components:

$$\vec{s}^{(i)} = [s_1^i, s_2^i, ..., s_L^i]$$

Note that each user has the same energy, i.e.

$$< \vec{s}^{(i)}, \vec{s}^{(i)} >= L$$

In the synchronous case under additive white Gaussian noise channel, the received signal is represented as:

$$\vec{r} = \sum_{i=1}^{M} b^i \vec{s}^{(i)} + \vec{n}$$

where b^i represent the data bit for i-th user. If a matched filter receiver is used, then the sufficient statistics of the decision rule are given by:

$$r_i =< \vec{r}, \vec{s}^{(i)} >=< (\sum_{j=1}^{M} b^j \vec{s}^{(j)} + \vec{n}), \vec{s}^{(i)} >, i = 1, 2, ...M$$

Under the equal energy condition, the above equation can be written as:

$$r_i = b^i L + \sum_{j=1, j\neq i}^{M} b^j < \vec{s}^{(j)}, \vec{s}^{(i)} > + < \vec{n}, \vec{s}^{(i)} > \tag{5.1}$$

The third term $< \vec{n}, \vec{s}^{(i)} >$ in above equation is a Gaussian random variable with zero mean, variance $L * N_0/2$. For simplicity, this term is defined as:

$$\eta^{(i)} \equiv < \vec{n}, \vec{s}^{(i)} >$$

The inter-user interference term experienced by user i is

$$\xi^{(i)} = \sum_{j=1, j \neq i}^{M} b^j < \vec{s}^{(j)}, \vec{s}^{(i)} >$$

Assuming the information symbols b^j are statistically independent, then $\xi^{(i)}$ is approximately a Gaussian random variable with zero mean and variance:

$$\sigma(i)^2 = \sum_{j=1, j \neq i}^{M} |< \vec{s}^{(j)}, \vec{s}^{(i)} >|^2 \tag{5.2}$$

Equation (5.2) can be written as:

$$\sigma(i)^2 = \sum_{j=1}^{M} |< \vec{s}^{(j)}, \vec{s}^{(i)} >|^2 - L^2 \tag{5.3}$$

The total inter-user interference variance is then given by:

$$\sigma^2_{(TOT)} = \sum_{i=1}^{M} \sum_{j=1}^{M} |< \vec{s}^{(j)}, \vec{s}^{(i)} >|^2 - ML^2 \tag{5.4}$$

The problem is thus: choose the binary(± 1 valued) sequences $\vec{s}^{(i)} = [s_1^i, s_2^i, ..., s_L^i]$ of length L to minimize σ^2_{TOT}.

A further constraint is: if possible, make the variance of the inter-user interference experienced by each user the same,i.e.

$$\sigma(i)^2 = \sigma^2_{(ind)} = \sigma^2_{(TOT)}/M \tag{5.5}$$

Table 5.1: $M \times L$ array contain the row vectors and the column vectors.

$M \times L$	$\vec{y}^{(1)}$	$\vec{y}^{(2)}$	...	$\vec{y}^{(L)}$
$\vec{s}^{(1)}$	s_{11}	s_{12}	...	s_{1L}
$\vec{s}^{(2)}$	s_{21}	s_{22}	...	s_{2L}
...	...	...	...	...
$\vec{s}^{(M)}$	s_{M1}	s_{M2}	...	s_{ML}

5.3 Welch Bound Equality relation

Since $\sigma^2_{(TOT)}$ is obviously greater or equal to zero, it follows from equation (5.4) that the sum of the squared inner products is bounded below by what is known as Welch's bound[8]. Stated formally it is:

Welch's Bound: if $\vec{s}^{(1)}, \vec{s}^{(2)}..., \vec{s}^{(M)}$ are vectors in the complex field C, and all have the same energy L,i.e., $\| \vec{s}^{(i)} \|^2 = < \vec{s}^{(i)}, \vec{s}^{(i)} > = L$, then,

$$\sum_{i=1}^{M} \sum_{j=1}^{M} |< \vec{s}^{(j)}, \vec{s}^{(i)} >|^2 \geq M^2 L \tag{5.6}$$

with equality if and only if the columns $\vec{y}^{(1)}, \vec{y}^{(2)}..., \vec{s}^{(L)}$ of the $M \times L$ array whose rows are $\vec{s}^{(1)}, \vec{s}^{(2)}..., \vec{s}^{(M)}$ are orthogonal and all columns have the same energy. i.e. $\| \vec{y}^{(k)} \|^2 = M$, for $1 \leq k \leq L$.(see table 5.1)

Therefore

$$\sigma^2_{TOT} = \sum_{i=1}^{M} \sum_{j=1}^{M} |< \vec{s}^{(j)}, \vec{s}^{(i)} >|^2 - ML^2 \geq M^2 L - ML^2 = ML(M - L)$$

and

$$\sigma^2_{ind} \geq L(M - L) \tag{5.7}$$

If $\vec{s}^{(1)}, \vec{s}^{(2)}, ..., \vec{s}^{(M)}$ are sequences in C^L, such that $| s_{ij} | = 1$, for $1 \leq i \leq M$

and $1 \leq j \leq L$, then Welch's bound becomes

$$\sum_{i=1}^{M}\sum_{j=1}^{M} |< \vec{s}^{(i)}, \vec{s}^{(j)} |^2 \geq M^2 L \tag{5.8}$$

with equality if and only if the columns $\vec{y}^{(1)}, \vec{y}^{(2)}, ..., \vec{y}^{(L)}$ of the $M \times L$ array whose rows are $\vec{s}^{(1)}, \vec{s}^{(2)}, ..., \vec{s}^{(M)}$ are orthogonal. More over, if equality holds, then:

$$\sum_{j=1}^{M} |< \vec{s}^{(i)}, \vec{s}^{(j)} >|^2 = ML$$

5.4 Construction of Welch Bound Equality sequences from linear codes

When a sequence is such that it meets the lower bound it is called a Welch Bound Equality(WBE) sequence. Construction of these sequences are of interest and importance. In this section a construction technique based on the linear block codes is discussed. The material in this section is taken from reference [8] (see it for proofs).

The structure of linear codes can be used to calculate the correlation between WBE sequences. In order to do this, WBE sequences are represented by linear codes by the following mapping: $-1 \rightarrow 0, 1 \rightarrow 1$. If after such a mapping, the WBE sequence $\vec{s} = [s_1, s_2, ...s_L]$ is represented by a linear code $\vec{b}$, then it's easier to calculate the correlation between two WBE sequences through their corresponding linear codes as follows:

Theorem: Assume two sequences $\vec{s}, \vec{s'}$ (± 1 valued with length L) correspond to two linear codes $\vec{b}, \vec{b'}$. Then the inner product

$$< \vec{s}, \vec{s'} >= L - 2d(\vec{b}, \vec{b'})$$

where $d(\vec{b}, \vec{b'})$ is the Hamming distance between $\vec{b}$ and $\vec{b'}$. From the above theorem, the rule to construct WBE sequences can be obtained:

Construction rule: The binary (± 1) sequence set corresponding to the linear code V is a WBE set if and only if the dual code $V^{\perp}$ contains no codewords of Hamming weight 2.

Corollary: The binary (± 1) sequence set corresponding to the linear code V is a WBE set if the minimum distance $d^{\perp}$ of the dual code $V^{\perp}$ is at least 3.

Unipolar sequence set: The binary sequences used in a CDMA system spread the user data bit by either -1 or 1. So if the corresponding linear code contains the all-one codeword, then according to a property of linear codes, the complementary codeword of every codeword is also in the set. The correlation between a sequence and its complementary sequence has the maximum possible value: obviously not a satisfactory sequence. So, only linear codes which do not contain the all-one codeword are of interest. The sequence set satisfying this condition is called a unipolar sequence set.

5.5 Construction of WBE sequences from cyclic code

Cyclic codes form an important subset of linear codes. Of particular importance in the construction of WBE sequences are cyclic Hamming codes which have a d_{min} of 3. Therefore from the corollary above its dual code can serve as the basis to construct a WBE sequence set. A Hamming code has the following characteristics for every $m \geq 3$[[13]] :

code length: $n = 2^m - 1$

Number of information symbols: $k = 2^m - m - 1$

Number of parity check symbol: $n - k = m$

Minimum distance: $d_{min} = 3$

Table 5.2: Minimal polynomial of $x^{15}+1$

conjugate roots	minimal polynomial
1	x+1
$\alpha, \alpha^2, \alpha^4, \alpha^8$	x^4+x+1
$\alpha, \alpha^3, \alpha^6, \alpha^9, \alpha^{12}$	$x^4+x^3+x^2+x+1$
α^5, α^{10}	x^2+x+1
$\alpha^7, \alpha^{11}, \alpha^{13}, \alpha^{14}$	x^4+x^3+1

As an example consider m=4, n=15, k=11, i.e. a (15,11) Hamming code with a (15,9) dual code, If the dual code is a unipolar code(which it is) then it corresponds to a WBE sequence set.

The generator polynomial $g(x)$, for the cyclic (15,11) Hamming code is readily determined, since in GF(16), all elements are roots of the polynomial $x^{15}-1$. It is given by

$$g(x) = x^4 + x + 1$$

From this the generator polynomial of the dual code is given by

$$g_{\perp}(x) = (x^{15}+1)/(x^4+x+1) = x^{11}+x^8+x^7+x^5+x^3+x^2+x+1 \quad (5.9)$$

The codebook corresponding to this dual code is shown in the following table.

Since M=16 and L=15, then according to equation (5.4), the total variance of this sequence set is:

$$\sigma^2_{TOT} = ML(M-L) = 16*15*(16-15) = 240$$

In order to evaluate the performance of WBE sequence set, the following section will compare the WBE sequence set with a signal assignment scheme of CDMA plus TDMA, proposed by Hikmet[5] and which has been discussed in greater detail in Chapter 3.

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1
\end{bmatrix}
$$

To obtain m=16 signals this scheme takes 15 orthogonal Walsh sequences, shown below, and augments it with a single TDMA sequence which occupies one chip period.

$$
\begin{bmatrix}
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\
-1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 \\
-1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 \\
-1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 \\
-1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 \\
-1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 \\
-1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\
-1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\
-1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\
-1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\
-1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\
-1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\
-1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\
-1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1
\end{bmatrix}
$$

TDMA sequence: $4, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$

The total variance of this spreading scheme is:

$$\sigma^2_{TOT} = 240 + 240 = 480$$

twice that of the WBE sequence. Further, the WBE sequence set has a uniform single user MAI (multiple access interference) variance:

$$\sigma_i^2(WBE) = L(M - L) = 15, 1 \leq i \leq 16$$

whereas in the 15 CDMA + 1 TDMA scheme the CDMA user experiences a MAI variance of $\sigma_i^2(CDMA) = 16, 1 \leq i \leq 15$ while the TDMA user sees a MAI variance of $\sigma^2(TDMA) = 15 * 16 = 240$.

Another benefit of using the WBE sequence instead of CDMA+TDMA is that the sequence has only two levels(± 1 valued instead of $\pm 1, +4$).

Performance simulation Under additive white Gaussian noise channel, an iteration receiver(explained also in Chapter 3) is used to detect the user's information symbol. The zeroth iteration receiver is just a single user matched filter receiver as shown in equation (5.1):

$$r_i^{(1)} = Lb^i + \xi^i + n^i \tag{5.10}$$

where the decision rule is:

if $r_i^{(1)} \geq 0, \hat{b}_i^{(1)} = 1$;

otherwise $\hat{b}_i^{(1)} = -1$.

where "1" represents the first stage receiver.

After all the user's data symbols are detected, we can construct the first iteration receiver. The sufficient statistics of first iteration receiver are

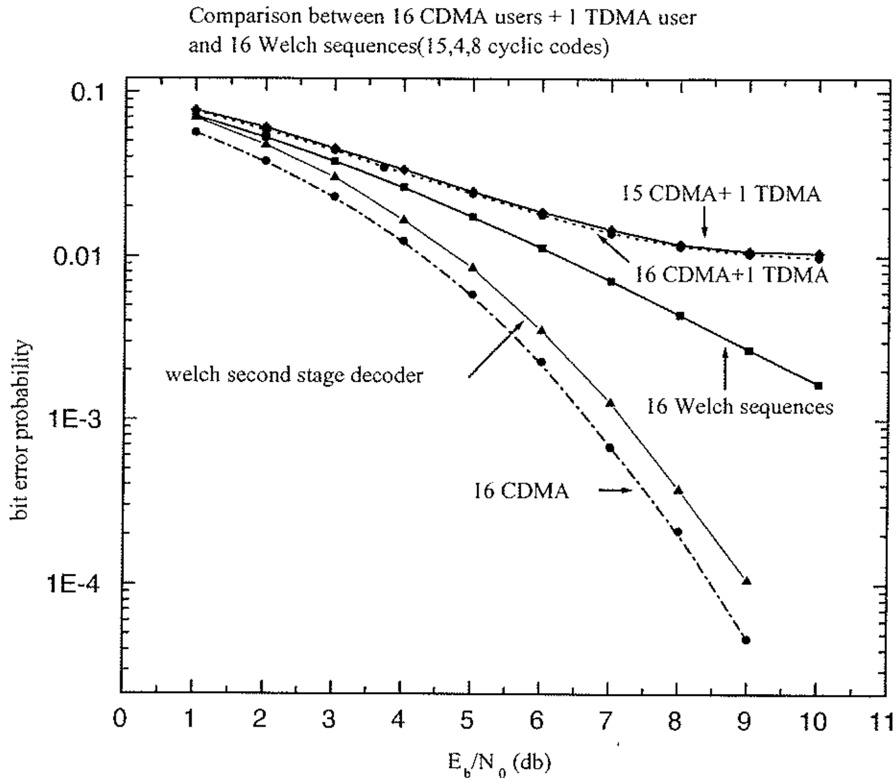

Figure 5.1: *Performance simulation of WBE and CDMA+TDMA*

$$r_i^{(2)} = r_i^{(1)} - \sum_{j=1, j\neq i}^{15} \hat{b}_j^1 < \vec{s}^i, \vec{s}^j >$$

and decision rule of the first iteration receiver is:

if $r_i^{(2)} \geq 0, \hat{b}_i^{(2)} = 1$;

otherwise $\hat{b}_i^{(2)} = -1$.

The symbol error probability of 15 CDMA + 1 TDMA, WBE and pure CDMA are shown in figure (5.1). As can be seen:

(1). The zeroth iteration performance of WBE sequences is much better than CDMA+TDMA scheme,especially when the signal-to-noise ratio is high. The poor performance of the latter is caused by the big variance of multiuser interference term which is relatively smaller for WBE sequences.

(2). The first iteration receiver performed similarly for both schemes. Both approach the pure CDMA spreading scheme.

(3). There some limitations in constructing WBE sequences from the linear cyclic Hamming codes, because the length of the codeword and information symbol must satisfy the relation $n = 2^m - 1$, where n is the length of the sequence, m is an integer.

Chapter 6

Construction of WBE Sequences from Hadamard Matrices and Performance Evaluation

6.1 Motivation

In the previous chapter, WBE sequences were constructed from binary linear codes, in particular, from binary linear cyclic codes. The disadvantage of this construction is that one can't construct the desired length and desired number of WBE sequences, there are always certain constraints. For (n, k) cyclic codes, the number of codewords is 2^k, $k = 2^m - m - 1$, and the length of each codeword is $n = 2^m - 1$. Therefore the number of sequences can only be increased in exponential steps which results in a huge increase in inter-user interference.

A natural problem to ask is: If there are M users, and the length of each sequence is L, where $M \geq L$, do WBE sequences exist for any M and L? If so, how can one construct the best WBE sequences with reasonable capacity to accommodate the M users? Of particular interest is the situation where M is not that "much greater" than L. Construction of WBE sequences based on Hadamard matrices and Walsh functions is discussed in this chapter. WBE sequence performance is compared with the CDMA+TDMA signal assignment. Again this is done for the case if $M = 64$ user.

6.2 Hadamard matix

A Hadamard matrix H is an n x n matrix with entries of -1 and 1 which satisfies the condition: $H*H^T = n*identity\ matrix[n]$ [16], where n is the order of the matrix, H^T is the transpose of H. A very important property of Hadamard matrix is that all rows as well as all columns are pairwise orthogonal. Historically, the Hadamard matrix originated from solving the following problem: given a $n \times n$ matrix with entries of +1, -1 , what is the maximum possible value of its determinant? The solution is the Hadamard matrix which has the determinant [17][18]:

$$det \mid H \mid = n^{1/2}$$

6.3 Construction of WBE Sequences from the Hadamard matrix

6.3.1 The relation between WBE sequences and Walsh functions

From the discussion on WBE sequences in Chapter 5 , the row vector of an $M \times L$ matrix with ± 1 elements are WBE sequences if and only if the column vectors are mutually orthogonal. Obviously, then Walsh functions which are the row vectors of the Hadamard matrix are WBE sequences. But in this case, the number of row vectors is the same as the column vectors, i.e., the number of users is equal to the length of the sequence. What is desired is "shortened" Walsh functions, ones where $M > L$. They are easily obtained from a $M \times M$ Hadamard matrix by deleting any $M - L$ columns of the matrix. The remaining columns are still mutually orthogonal. Though the rows are not orthogonal any more, they are the desired WBE sequences.

The total variance of inter-user interference, which is an indication of performance, of such WBE sequences is readily obtained using equation 5.7 and is given by:

$$\sigma^2_{TOT}(WBE) = ML(M-L) \tag{6.1}$$

In comparison with the $CDMA+TDMA$ scheme where it is assumed there are a total of M users, L of whom use Walsh sequences, while the other $(M-L)$ users use TDMA pulses, then:

variance of each CDMA user: $(M-L)*L$

variance of all CDMA users: $L*(M-L)*L$

variance of each TDMA user: $(M-L)*L$

variance of all TDMA users: $(M-L)*L*L$

Note in the above the Walsh sequences have an amplitude of ± 1, while the TDMA sequences have a pulse amplitude of $\sqrt{L}$.

The total variance of CDMA+TDMA is

$$\sigma^2_{TOT}(CDMA+TDMA) = (M-L)*L^2 + (M-L)L^2 = 2*(M-L)L^2$$

and therefore one gets the following ratio:

$$\eta = \frac{\sigma^2_{TOT}(WBE)}{\sigma^2_{TOT}(CDMA+TDMA)} = \frac{M}{2L}$$

As long as the number of users is less than two times the sequence length, the ratio η is always less than one, which indicates that WBE sequences are better than the CDMA+TDMA scheme. Practically M is considerably less than 2L. As an example considers the IS-95 standard where L=64. Let $M = 74$, i.e., one is interested in an extra 10 users. Then $\eta = 0.58$ which means that a system using WBE sequences shall experience much less MAI than one based on CDMA+TDMA sequences.

To explore further the construction of WBE sequences from Hadamard matrices, consider the simple case where $L = 16$, but $M = 20$ or 24. One can construct the set of WBE sequences from Hadamard matrices of order 20 and 24 respectively. The two matrices are given in appendix D.

Consider first $M = 20$. Then 20 WBE sequences of length 16 are obtained from the 20×20 Hadamard matrix by simply deleting the last 4 columns. The resulting set is(W_{20}):

$$
(W_{20}) = \begin{bmatrix}
+1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
+1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 \\
+1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 \\
+1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 \\
+1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 \\
+1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 \\
+1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 \\
+1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 \\
+1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 \\
+1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 \\
+1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 \\
+1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 \\
+1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 \\
+1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 \\
+1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 \\
+1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 \\
+1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 \\
+1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 \\
+1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 \\
+1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & +1
\end{bmatrix}
$$

It is easy to calculate the total variance of the WBE sequence set from equation(6.1). For convenience, the average individual variance, normalized to the energy of each sequence, is calculated in terms of db.

The choice of the WBE sequence subset from a complete set remains another issue. The detailed calculation shows that the average individual variance differs

Table 6.1: The average individual variance at different number of users(from20 $\times$ 20 Hadamard matrix)

number of users	WBE: $\sigma_{IND}(db)$	CDMA+TDMA: $\sigma_{IND}(db)$
17	2.63	1.37
18	2.88	2.88
19	3.13	3.76
20	3.36	4.38

only slightly between two subsets. The average individual variance of the subset where the first M sequences are chosen from matrix W_{20} is shown in table 6.1. Note that when the number of users is less than 18, one should use CDMA + TDMA, but when it is larger than 18, WBE sequences perform better. If there are more than 20 users, say 24, then WBE sequences can be constructed from the 24 $\times$ 24 Hadamard matrix by deleting the last 8 columns. The final WBE sequence set W_1 is shown as below.

However, since for any given order, the Hadamard matrix is not unique, one can use a different 24*24 Hadamard matrix to obtain the 16 WBE sequences, this results in set W_2.

For a particular M($16 < M \leq 24$), the average individual variance is calculated for the three sequence sets, based on W_1, based on W_2 CDMA+TDMA. The results are shown in table 6.2.

Table 6.2: Total variance at different number of users(from 24×24 Hadamard matrix)

number of users	WBE1: $\sigma_{IND}(db)$	WBE2: $\sigma_{IND}(db)$	CDMA+TDMA: $\sigma_{IND}(db)$
17	4.38	4.18	1.37
18	4.51	4.32	2.75
19	4.39	4.25	3.51
20	4.31	4.31	4.02
21	4.26	4.31	4.40
22	4.34	4.41	4.70
23	4.41	4.41	4.94
24	4.52	4.52	5.14

$$W_1 = \begin{bmatrix}
+1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\
-1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & -1 & +1 \\
+1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 \\
-1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 \\
+1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & +1 & -1 \\
-1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 \\
+1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 \\
-1 & -1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 \\
+1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 \\
-1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 \\
+1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 \\
-1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 \\
+1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & -1 \\
-1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\
+1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\
-1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 \\
+1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\
-1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 \\
+1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\
-1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 \\
+1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & +1 \\
+1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & +1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1
\end{bmatrix}$$

$$
W_2 = \begin{bmatrix}
+1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\
+1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\
+1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 \\
+1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
+1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 \\
+1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & +1 \\
+1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 \\
+1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 \\
+1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 \\
+1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 \\
+1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 \\
+1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 \\
+1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 \\
+1 & -1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 \\
+1 & -1 & +1 & -1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 \\
+1 & -1 & +1 & -1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 \\
+1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 \\
+1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 \\
+1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\
+1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 \\
+1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 \\
+1 & -1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & -1 \\
+1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & -1 & +1 & -1 & -1 & -1 \\
+1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 & +1
\end{bmatrix}
$$

In figure (6.1) the individual variance for three WBE sequence sets and that of CDMA+TDMA are shown as a function of M. Three conclusions can be made from the figure:

(1). In almost all cases(except M=17), the performance of WBE sequences is better than CDMA + TDMA as long as the proper sequences are chosen based on the number of users. For example, if the number of users is less than 20 but greater than 16, the WBE sequences should be constructed from the 20×20 Hadamard matrix; if the number of users is greater than 20 but less than 24, the WBE sequence set has to be constructed from Hadamard matrix of order 24×24.

(2). The variance of WBE sequences from the 24×24 Hadamard matrix is much bigger than that from 20×20. For example, in the case that there are

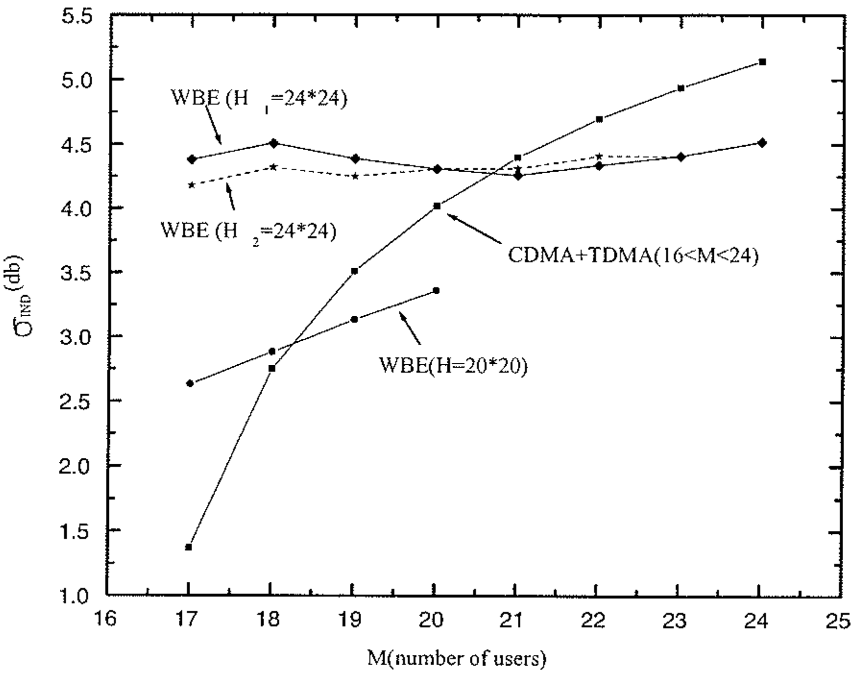

Figure 6.1: *The individual variance changes with the number of users when L=16*

20 users, table 6.1 gives a standard deviation(σ_{IND}) of 3.36 db , while table 6.2 gives a standard deviation of 4.31 db. This discrepancy strongly suggests that the number of users should not be greater than the length of the sequences by 4, otherwise, the users have to suffer an increase in interference.

(3). A WBE sequence set constructed from one Hadamard matrix differs only slightly in variance from a WBE sequence set that is constructed from another Hadamard matrix when the number of users doesn't reach the full capacity(not all the WBE sequences are used in a system). This is also true within one WBE sequence set. The variance of one subset is marginally different from that of another subset.

The above observations can be used to choose WBE sequences of longer length.

6.3.2 WBE sequences of arbitrary length

The availability of WBE sequences depends exclusively on the available Hadamard matrices. Each Hadamard matrix yields a set of WBE sequences. The dimension of Hadamard matrix is believed to be given by Hadamard conjecture, still unproven,but true for all sizes of practical interest. The conjecture states that Hadamard matrices exist of dimension n = 1, n = 2, and $n \bmod 4 = 0$. To accommodate M users, the dimensionality of a Hadamard matrix should be greater than or equal to M.

Consider now the design of signal sets for the case where $M > 64$, i.e., an extension of the IS-95 standard to accommodate more users. Two design approaches, one based on WBE sequences as obtained from Hadamard matrices, the other on the CDMA+TDMA approach are compared.

Suppose there are M($M > 64$) users in a CDMA system, where the length of each sequence is limited to L=64. There are many choices to construct WBE sequence set. Any Hadamard matrices with order K($K > M$) can be used to construct the WBE sequence set. For example, in the case that there are 65 users, the Hadamard matrix, 68×68, 72×72, 76×76..., etc, can be used to construct the WBE sequence set.

What is the best WBE sequence set? The criteria is that the proper WBE sequence set must minimize the inter-user interference while at the same time accommodating the number of users in the system. The average individual variance in the system is actually bounded by $M * L * (M - L)/M * L = (M - L)$ (normalized to the energy of the sequence set). Specifically, each standard deviation is calculated as follows :

When $64 \leq M \leq 68$, the Hadamard matrix of order 68 is the best one to use to construct WBE sequence set. The upper bound of each individual standard

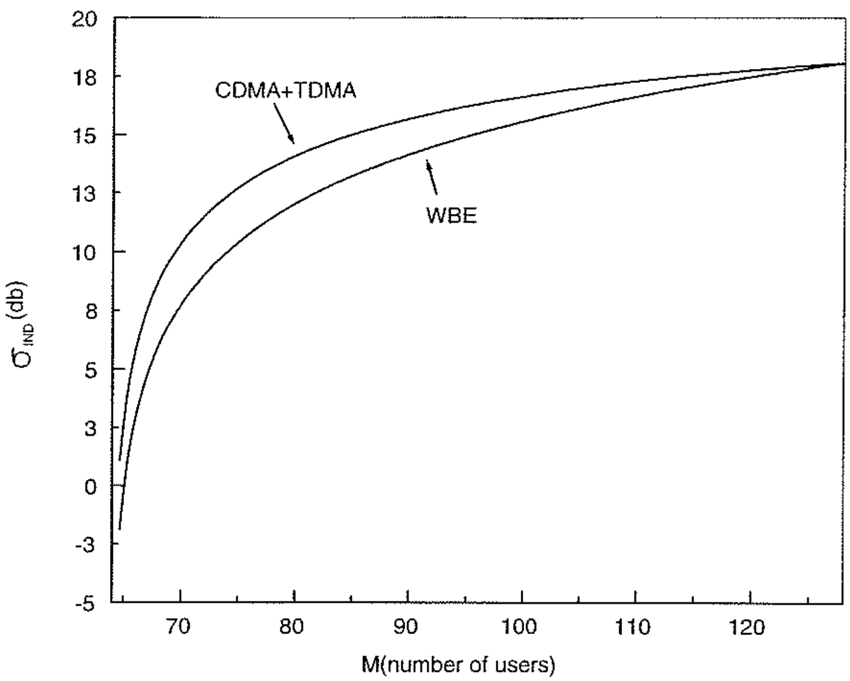

Figure 6.2: *The average individual variance of WBE sequence set and CDMA+TDMA set when L=64*

deviation is: $\sigma_{IND} \leq 10 * log(\sqrt{(64*68*4/64*68)}) = 3.01db$.

When $68 \leq M \leq 72$, Hadamard matrix with order 72 is best. The upper bound is: $\sigma_{IND} \leq 10 * log(\sqrt{(64*72*8/64*72)}) = 4.52db$.

When $72 \leq M \leq 76$, Hadamard matrix with order 76 is best. The upper bound is: $\sigma_{IND} \leq 10 * log(\sqrt{(64*76*12/64*76)}) = 5.40db$.

By choosing the proper WBE sequences based on the number of users, it is always possible to make the variance of each individual user smaller than that of the CDMA+TDMA system, as can be seen in figure(6.2), where the individual variance is drawn as a function of the number of users. As a comparison, two curves overlap at M=64 and 128. In between the two points, the average individual variance of WBE is less than that of CDMA+TDMA which indicates that WBE sequence set is better than CDMA+TDMA.

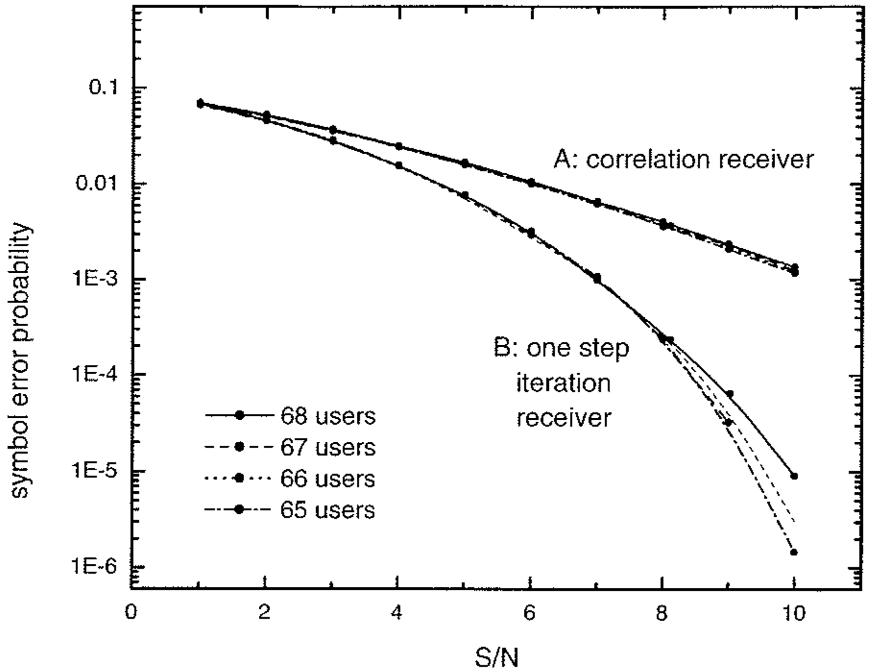

Figure 6.3: *Performance of WBE from 68*68 Hadamard matrix*

6.3.3 Performance simulation and comparison with that of CDMA + TDMA

The performance of the correlation receiver and the one step iteration receiver is simulated for both WBE sequence and CDMA+TDMA in this section.

1. The performance of WBE sequence set

Figure (6.3) shows the performance of a subset of 68 WBE sequences from 68×68 Hadamard matrix corresponding to M=65, 66, 67, 68. For both the correlation receiver and the one step iteration receiver the curves indicate that the performance is not sensitive to M. This property is due to the fact that the interference of each individual user is very close to the average value, which changes little when M changes. This is determined by an exhaustive computer search with the results shown in Table (6.3) and (6.4).

Table 6.3: Average individual user's variance at different number of users(from 68×68 Hadamard matrix)

number of users	WBE $\sigma_i(db)$	CDMA $\sigma_i(db)$
64	2.87	1.05
65	2.94	3.01
67	2.97	3.89
68	3.01	4.51

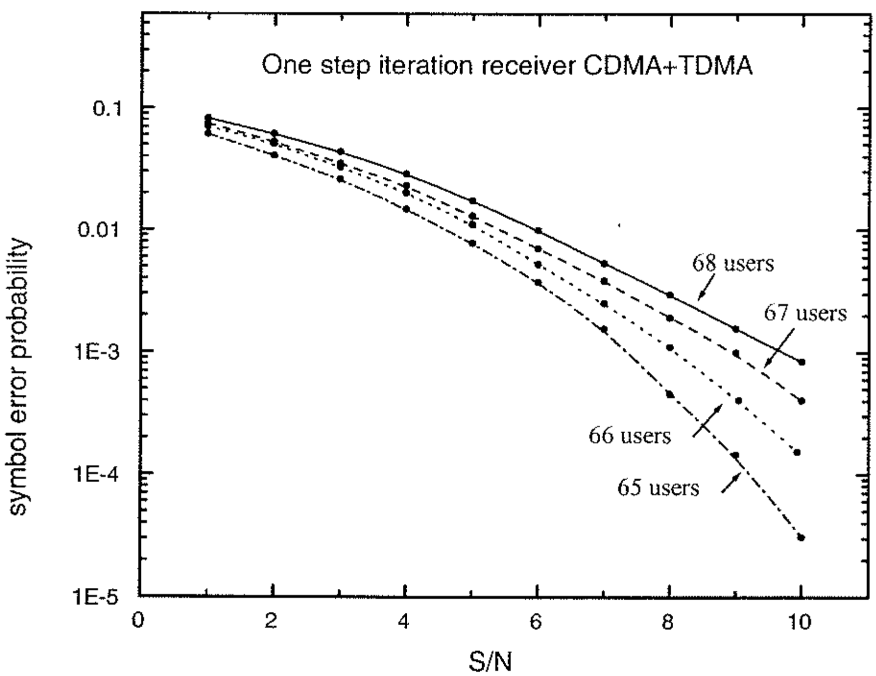

Figure 6.4: *Performance of 64 CDMA + x TDMA*

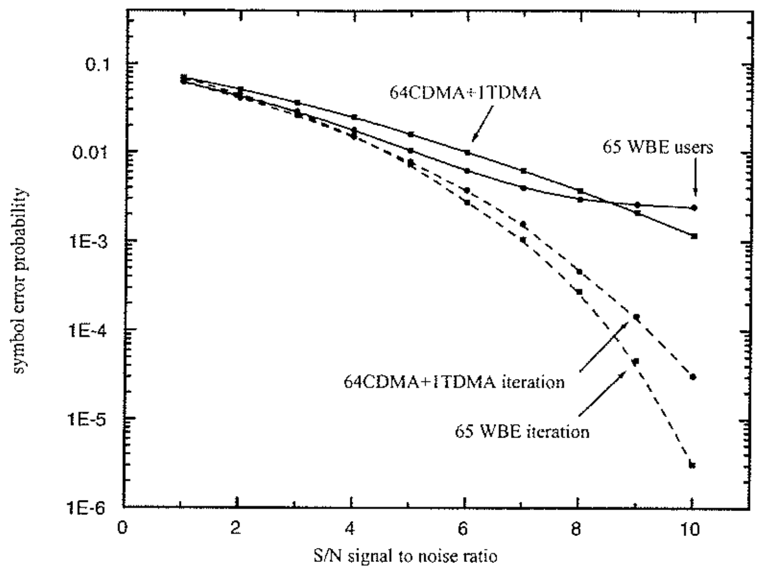

Figure 6.5: *Performance of 65 WBE(from* $68 * 68$ *Hadamard matrix) users and 64CDMA + 1 TDMA*

2. The performance of CDMA + TDMA system

Figure (6.4) shows the performance of the correlation receiver for 64 CDMA users with an additional TDMA users numbering 1 to 4, so the total number of users M=65, 66, 67, 68. For different M, the performance differs a lot even when M changes only by 1. This is due to the great increment of interference caused by the addition of a TDMA user, as can be seen from table (6.4) where the individual variance changes greatly with M.

3. Performance comparison of WBE sequence set and CDMA + TDMA sequence set

Figure (6.5), Figure (6.6), Figure (6.7) and Figure (6.8) compare the performance of WBE sequence set with CDMA+TDMA sequence set in the case M=65, 66, 67, 68.

When $M = 65$(Figure (6.5)), the error probability of WBE crosses that of CDMA+TDMA at 9 db for the correlation receiver; even though, the first step iteration receiver of WBE sequence is still better that of 64 CDMA + 1 TDMA.

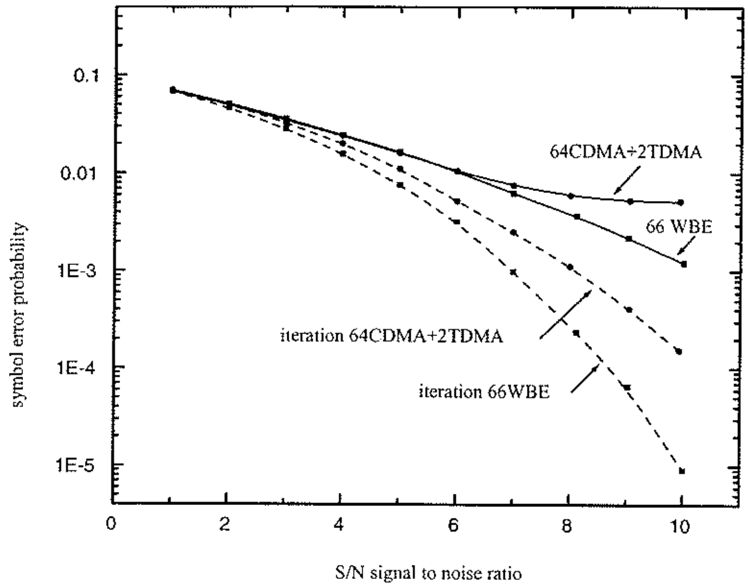

Figure 6.6: *Performance of 66 WBE(from* 68 ∗ 68 *Hadamard matrix) users and 64CDMA + 2 TDMA*

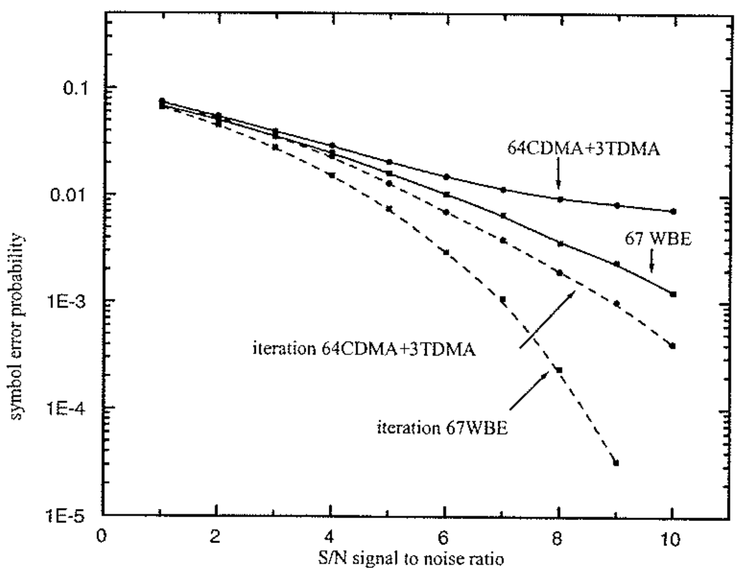

Figure 6.7: *Performance of 67 WBE(from* 68 ∗ 68 *Hadamard matrix) users and 64CDMA + 3 TDMA*

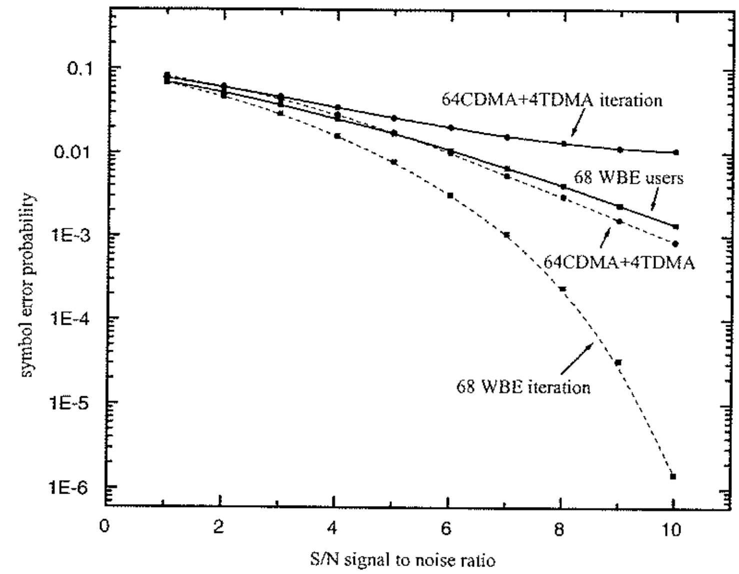

Figure 6.8: *Performance of 68 WBE(from* $68 * 68$ *Hadamard matrix) users and 64CDMA + 4 TDMA*

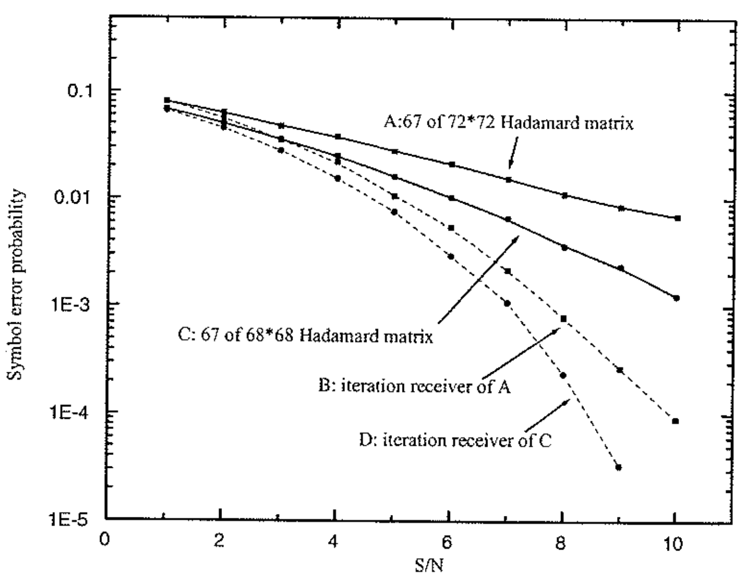

Figure 6.9: *Performance of 67 users from the Hadamard matrix of order 68 and 72*

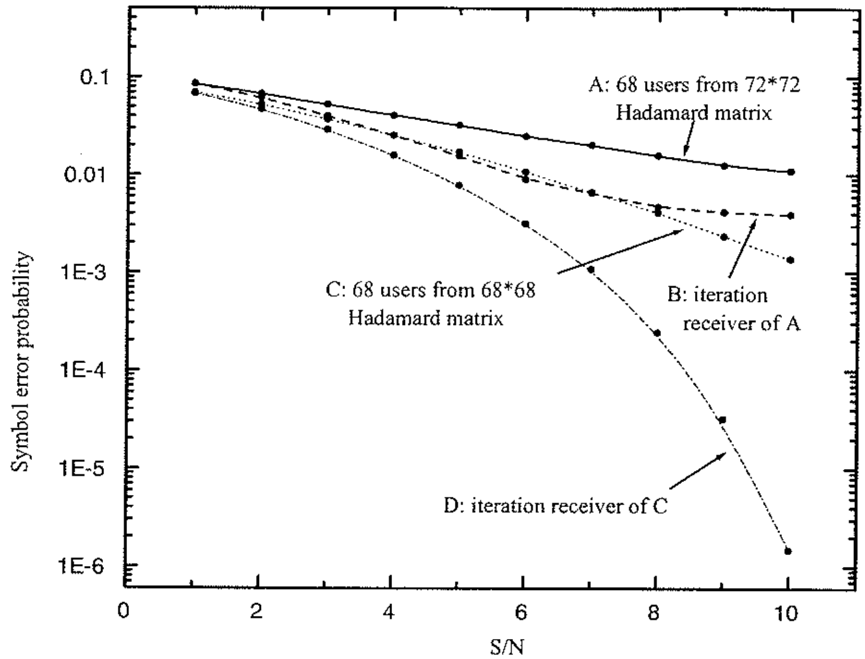

Figure 6.10: *Performance of 68 users from the Hadamard matrix of order 68 and 72*

when M = 66, 67, 68, both the correlation receiver and the first step iteration receiver of WBE sequence have better performance than the CDMA+TDMA system, So in all cases of $65 \leq M \leq 68$, the iteration receiver of the WBE sequence set has a better performance than the CDMA+TDMA sequence set.

4. Performance of WBE sequence set when $M > 68$

When there is a requirement to accommodate more than 68 users with fixed $L = 64$, the solution is to construct the WBE sequence set from a higher order Hadamard matrix. Without any doubt, the performance will deteriorate due to the increased MAI. There is another potential problem: when the users in a cell keep registering and un-registering dynamically, if the capacity of each cell is designed to accommodate more than 72 users, but actually there are less than 72 users, then the users have to suffer bigger MAI. To analyze such case, figure (6.10) shows the performance of 68 users by using the WBE sequence set from Hadamard matrix of order 68 and the sequences Hadamard matrix of order 72. Figure (6.9) shows the similar case when $M = 67$.

Chapter 7

Conclusion

7.1 Conclusions

Three different approaches to increase the capacity of CDMA sytems have been discussed up till now: CDMA+TDMA scheme, the hybrid of two sets of orthogonal sequences and the WBE sequence set. The three spreading schemes are compared in the table (7.1) in terms of performance, compatibility, complexity of construction, capacity:

Performance: CDMA+TDMA and two sets of orthogonal codes have the same performance, but they are worse than WBE sequences. Further, all the users of WBE sequences have identical performance, while in CDMA+TDMA, the TDMA users have worse performance than the CDMA users. In two orthogonal sets, the set which doesn't reach the full capacity has a poor performance. When the

Table 7.1: Comparison of different schemes to improve the capacity of CDMA systems

	TDMA/CDMA	CDMA/TDMA	two Orthogonal sets	WBE sequences
performance	not good	not good	not good	good
compatibility	good	good	good	not good
complication	good	simple	hard	hard
capacity	flexible	flexible	flexible	flexible
spreading	bad	bad	good	good

number of users changes slightly, the performance of WBE sequences remains relatively constant, while the performance in the other two schemes changes greatly when the number of users changes.

Compatibility: CDMA+TDMA and two sets of orthogonal codes have good compatibility with the existing system: when the number of users are less than the dimensionality of the system, the system is just OCDMA(orthogonal CDMA); if there are additional users more than the OCDMA can accommodate, they can be easily expanded to be accommodated by TDMA sequences. Since WBE is completely different from OCDMA, it can't coexist with the OCDMA system.

Complexity: In CDMA+TDMA system, TDMA users can be easily put in different time slots, while the other two schemes need a special buffer to store the codes. This, however, is not a major concern.

Capacity: All three schemes have good ability to accomodate additional users. For nCDMA/mTDMA, m can be as big as n to accommodate m additional users, though this is not suggested because this leads to large MAI; For two orthogonal sets of sequences, from one set of n orthogonal codes, another set with the same number can be generated by linear transformation to accomodate as many as n additional users; While WBE sequences can make use of a Hadamard matrix to accommodate enough users.

Spreading: In CDMA+TDMA, the TDMA users are in fact not spread, so the scheme is less desirable than the other two which have a spreading factor equal to the length of each sequence.

7.2 Limitations?

The schemes to improve the capacity of CDMA systems discussed in this thesis are good and practical for uplink. It's easy to implement the iteration receiver in base station. While it could be a problem in the mobile station depending on what kind of sequence set the mobile station uses. For the scheme of two orthogonal signal sets, if the original capacity is n, and the capacity is increased to $n+m$, then the original mobile station needs a bank of m correlation receivers while the additional mobile stations need n correlation receivers. For the scheme of WBE sequences, each mobile station always needs n correlation receivers built in. The additional complexity creates a practical problem.

Bibliography

[1] Rifaat D. Dayem, Prentice Hall, “PCS & digital cellular technologies: accessing your options”,1997.

[2] Vijay K. Garg and Joseph E. Wilkes, Prentice Hall, “Wireless communication systems”, 1996.

[3] Tero Ojianperä, Ramjee Prasad, “Wideband CDMA for third generation mobile communications”.

[4] Glisic, Savo G. , Leppanen, Pentti A., “Code Division Multiple Access Communications”,1995.

[5] Sari, H. and Vanhaverbeke, F. and Moeneclaey, M. “Increasing the capacity of CDMA using hybrid spreading sequences and iterative multistage detection”. Proc. VTC 1999- Fall. Amsterdam, the Netherlands. Sept. 19-22 1999. 1160-1164.

[6] D. Williamson; R.A. Kennedy; G.W. Pulford, “Block Decision Feedback Equalization”, IEEE Trans. Commun., vol. 40, no. 2, pp. 255–264, February 1992.

[7] Harmuth, H. F. “Applications of walsh functions in communications.” IEEE Spectrum 6, 82-91, 1969.

[8] James L. Massey and Thomas Mittelholzer, Welch bound and sequence sets for code-division multiple access systems. InR. Capocelli, A. De Santis, and U. Vaccaro, editors, Sequences II: Methods in communications, security and computer science. Springer-Verlag, 1991. 63-78.

[9] Robinson, Enders A. Statistical communication and detection; with special reference to digital data processing of radar and seismic signal.

[10] http://www.iec.org/online/tutorials/tdma/topic06.html.

[11] Goldstein, H. "Orthogonal Transformations" , pp132-137,1980.

[12] http://www.research.att.com/ njas/hadamard/ .

[13] Lin and Costello. " Error control coding: Fundamental and Application", Prentice Hall, Englewood Cliffs, 1983. pp 51.

[14] Paley, R. E. A. C. "On orthogonal matrices." J. Math. Phys. 12, 311-320, 1933.

[15] S. S. Agaian "Lecture notes in mathematics" J. Math. Phys. 12, 311-320, 1980.

[16] J. Hadamard "Résolution d'une question relative aux déterminants" Bull. Des Sciences, Math., 17:240-246,1893.

[17] G . Barb. "Intorno al teroema di Hadamard sui determinanti a valore massimo." Giorn. Mat. battaglini, 71:70-86,1993.

[18] U. Scarpis. "On determinants of maximum value." Rend. Inst. Lombardo di Sci. Lett., 31:1441-1446. 1898.

[19] TIA/EIA Interim Standard, TIA/EIA/IS-95, pp 6-17

[20] J. Williamson. Hadamard's determinant theorem and the sum of four squares. Duke Math. J., 1165-81,1994.

[21] L. D. Baumert, S. W. Golomb, and M. Hall Jr. "Discovery of an Hadamard matrix of order 92." Bull. Amer. math. Soc., 68:237-238,1962.

[22] R. J. Turyn. "Character sums of and difference sets". Pacific J. Math. Sinica, 5:411-423, 1955.

[23] R. J. Turyn. "Sequences with small correlation." In. H. B. Mann, editor, Error-correcting codes" pages 195-228. Wiley, New York, 1968.

[24] R. J. Turyn. "An infinite class of Williamson matrices". J. Combinatorial structures and their applications, pages 435-437, New York, 1970.

[25] J. Seberry Wallis and Albert Leon Whiteman. "Some classes of Hadamard matrices with constant diagonal." Bull. Austral. Math. Soc. 7:233-249,1972.

[26] http://mathworld.wolfram.com/PaleyConstruction.html.

Appendix A

Correlation Receiver Derivation

The received signal is:

$$r(t) = a_1 s_1(t) + a_2 s_2(t) + a_3 s_3(t) + n(t)$$

Sufficient statistics of the receiver are obtained by:

$$r_1 = \int_0^{T_s} r(t) * s_1(t)\, dt = a_1 + (\sqrt{2}/2)(a_2 + a_3) + n_1$$

$$r_2 = \int_0^{T_s} r(t) * s_2(t)\, dt = a_2 + (\sqrt{2}/2)a_1 + n_2$$

$$r_3 = \int_0^{T_s} r(t) * s_3(t)\, dt = a_3 + (\sqrt{2}/2)a_1 + n_3$$

The first term is just the information bit, the second term is a discrete random variable with possible value 1 (Pr=0.25), 0(Pr=0.5), -1(Pr=0.25).

$$Pr[a_1 = 1] = 0.25 \int_{-\infty}^{-(1+\sqrt{2})\sqrt{E_b}} f(n_1)\, dn_1$$

$$+0.5 \int_{-\infty}^{-\sqrt{E_b}} f(n_1)\, dn_1 + 0.25 \int_{-\infty}^{(-1+\sqrt{2})\sqrt{E_b}} f(n_1)\, dn_1 \qquad \text{(A.1)}$$

The decision rules for user s_2 and s_3 are similar with user s_1, the bit error probability:

$$Pr[a_2 = 1] = 0.5 \int_{-\infty}^{(-1-\sqrt{2}/2)\sqrt{E_b}} f(n_2)\, dn_2 + 0.5 \int_{-\infty}^{(-1+\sqrt{2}/2)\sqrt{E_b}} f(n_2)\, dn_2 \quad \text{(A.2)}$$

$f(n_1)$ and $f(n_2)$ are normal, zero mean Gaussian densities with variance $N_0/2$. This scheme is equivalent to two orthogonal signals with one additional signal, the number of dimensions are essentially the number of chips in the CDMA spreading scheme.

Appendix B

Individual Optimum Receiver Derivation

The individually optimum receiver chooses the individual user information bit a_i to maximize the probability $Pr(a_i \mid r_1, r_2)$, where r_1 and r_2 are sufficient statistics as defined in equation 2.17 and 2.18. The decision rule of the individual optimum receiver for user i is therefore:

$$Pr(a_i = +1 \mid r_1, r_2) > Pr(a_i = -1 \mid r_1, r_2) \quad a_i = 1 \tag{B.1}$$

$$Pr(a_i = +1 \mid r_1, r_2) < Pr(a_i = -1 \mid r_1, r_2) \quad a_i = -1 \tag{B.2}$$

The decision rule essentially selects the a_i that maximizes the likelihood function: $f(r_1, r_2 \mid a_i)$, i.e.,

if $f(r_1, r_2 \mid a_i = +1) > f(r_1, r_2 \mid a_i = -1)$, choose $a_i = 1$

else

$$f(r_1, r_2 \mid a_i = +1) < f(r_1, r_2 \mid a_i = -1), \quad \textit{choose} \quad a_i = -1 \tag{B.3}$$

The likelihood function $f(r_1, r_2 \mid a_i = +1)$ has four terms:

$$f(r_1, r_2 \mid a_i = +1) = f(r_1, r_2 \mid a_i = +1)Pr(a_2 = +1, a_3 = +1) +$$

$$f(r_1, r_2 \mid a_i = +1)Pr(a_2 = -1, a_3 = +1) +$$

$$f(r_1, r_2 \mid a_i = +1)Pr(a_2 = +1, a_3 = -1) +$$

$$f(r_1, r_2 \mid a_i = +1)Pr(a_2 = -1, a_3 = -1) \tag{B.4}$$

Since the a_i are assumed to be equally probable and statistically independent, the probabilities $Pr(a_2 = +1, a_3 = +1)$, $Pr(a_2 = -1, a_3 = +1)$, $Pr(a_2 = +1, a_3 = -1)$, $Pr(a_2 = -1, a_3 = -1)$ are all equal to 0.25.

Consider user 1 in the $\pi/3$ signal assignment scheme, the first term $f(r_1, r_2 \mid a_1 = +1)$ in equation (B.4) can be calculated as follows:

$$f(r_1, r_2 \mid a_1 = +1, a_2 = +1, a_3 = +1) =$$

$$= \frac{1}{\sqrt{2\pi}\sigma} exp\{-\frac{1}{2\sigma^2}\int_0^T [r(t) - s_1(t) - s_2(t) - s_3(t)]^2 dt\} \tag{B.5}$$

where $\sigma^2 = N_0/2$.

Since $s_1(t) = \varphi_1(t)$, $s_2(t) = \frac{\sqrt{2}}{2}\varphi_1(t) + \frac{\sqrt{2}}{2}\varphi_2(t)$, $s_3(t) = \frac{\sqrt{2}}{2}\varphi_1(t) - \frac{\sqrt{2}}{2}\varphi_2(t)$, equation (B.5) has the form:

$$f(r_1, r_2 \mid a_1 = +1, a_2 = +1, a_3 = +1) = C * exp\{-\frac{3 + 2\sqrt{2} - 2(1 + \sqrt{2})r_1}{2\sigma^2}\} \tag{B.6}$$

where

$$C = \frac{1}{\sqrt{2\pi}\sigma} exp\{-\frac{1}{2\sigma^2}\int_0^T r(t)^2 dt\}$$

Similarly, other terms in equation (B.4) can be calculated as follows:

$$f(r_1, r_2 \mid a_1 = +1, a_2 = -1, a_3 = +1) = C * exp\{-\frac{3 - 2r_1 - 2\sqrt{2}r_2}{2\sigma^2}\} \qquad \text{(B.7)}$$

$$f(r_1, r_2 \mid a_1 = +1, a_2 = +1, a_3 = -1) = C * exp\{-\frac{3 - 2r_1 + 2\sqrt{2}r_2}{2\sigma^2}\} \qquad \text{(B.8)}$$

$$f(r_1, r_2 \mid a_1 = +1, a_2 = -1, a_3 = -1) = C * exp\{-\frac{3 - 2\sqrt{2} - 2(1 - \sqrt{2})r_1}{2\sigma^2}\} \qquad \text{(B.9)}$$

The likelihood function $f(r_1, r_2 \mid a_1 = -1)$ also has four terms:

$$\begin{aligned} f(r_1, r_2 \mid a_1 = -1) = {} & f(r_1, r_2 \mid a_i = -1)Pr(a_2 = +1, a_3 = +1) + \\ & f(r_1, r_2 \mid a_1 = -1)Pr(a_2 = -1, a_3 = +1) + \\ & f(r_1, r_2 \mid a_i = -1)Pr(a_2 = +1, a_3 = -1) + \\ & f(r_1, r_2 \mid a_i = +1)Pr(a_2 = -1, a_3 = -1) \end{aligned} \qquad \text{(B.10)}$$

The four terms in equation (B.10) have the form:

$$f(r_1, r_2 \mid a_1 = -1, a_2 = +1, a_3 = +1) = C * exp\{-\frac{3 - 2\sqrt{2} + 2(1 - \sqrt{2})r_1}{2\sigma^2}\} \qquad \text{(B.11)}$$

$$f(r_1, r_2 \mid a_1 = -1, a_2 = -1, a_3 = +1) = C * exp\{-\frac{3 + 2r_1 - 2\sqrt{2}r_2}{2\sigma^2}\} \qquad \text{(B.12)}$$

$$f(r_1, r_2 \mid a_1 = -1, a_2 = +1, a_3 = -1) = C * exp\{-\frac{3 + 2r_1 + 2\sqrt{2}r_2}{2\sigma^2}\} \qquad \text{(B.13)}$$

$$f(r_1, r_2 \mid a_1 = -1, a_2 = -1, a_3 = -1) = C * exp\{-\frac{3 + 2\sqrt{2} + 2(1 + \sqrt{2})r_1}{2\sigma^2}\} \qquad \text{(B.14)}$$

The boundary of the decision rule is by considering equality in equation B.3. After simple manipulation the following is obtained:

$$exp(\frac{r_1}{\sigma^2})(cosh\frac{\sqrt{2}r_1 - \sqrt{2}}{\sigma^2} + cosh\frac{\sqrt{2}r_2}{\sigma^2}) = exp(-\frac{r_1}{\sigma^2})(cosh\frac{\sqrt{2}r_1 + \sqrt{2}}{\sigma^2} + cosh\frac{\sqrt{2}r_2}{\sigma^2}) \tag{B.15}$$

After separating the variables of r_1 and r_2, the following formula can be obtained:

$$cosh\frac{\sqrt{2}r_2}{\sigma^2} = \frac{cosh\frac{\sqrt{2}r_1+\sqrt{2}}{\sigma^2} - exp(\frac{2r_1}{\sigma^2})cosh\frac{\sqrt{2}r_1-\sqrt{2}}{\sigma^2})}{exp(\frac{2r_1}{\sigma^2}) - 1} \tag{B.16}$$

Appendix C

Construction of Hadamard matrix

C.1 Introduction

In 1933 Paley [15] proved that the order of Hadamard a matrix is divisible by 4, but the reverse statement is true or not is unknown: i.e., does there exist Hadamard matrices of order n for all n divisible by 4. This is the so called Hadamard conjecture. When n increases, the possible combinations goes to 2^n, which explodes exponentially. Even when n=28, it is difficult to calculate a brutal-force search. There are no general construction methods. However, with the help of the computer, the order of about 4000 Hadamard matrices have been constructed with the exception of order 268 [15].

For a certain order n, the number of distinct Hadamard matrices are usually not unique. For $n \leq 28$, The numbers of orders 0, 4, 8, 12, 16, 20, 24, 28, are respectively 1, 1, 1, 1, 5, 3, 60, 487.

Several main methods to construct Hadanmard matrix are introduced in the following sections.

C.2 Recursive method

This method can construct a Hadamard matrix of order 2^n by a recursive procedure[19].

$$\mathbf{H_1} = \mathbf{0}$$

$$\mathbf{H_2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{C.1}$$

$$\mathbf{H_4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \tag{C.2}$$

$$\mathbf{H_{2n}} = \begin{pmatrix} H_n & H_n \\ H_n & \bar{H}_n \end{pmatrix} \tag{C.3}$$

where n is the power of 2 and $\bar{H}_n$ denotes the binary complement of H_n.

C.3 Cronecker Product construction

If (H_{ij}) is any $n \times n$ Hadamard matrix, and $B_1, B_2, ...B_n$ are any $m \times m$ Hadamard matrix, then the matrix obtained from the Kronecker product:

$$H \otimes [B_1, B_2, ...B_n] = \begin{pmatrix} h_{11}B_1 & h_{12}B_1 & ... & h_{1n}B_1 \\ h_{21}B_2 & h_{22}B_2 & ... & h_{2n}B_2 \\ ... & ... & ... & ... \\ H_{n1}B_n & h_{n2}B_n & ... & h_{nn}B_n \end{pmatrix} \tag{C.4}$$

is an $nm \times nm$ Hadamard matrix.

A special case is: if $B_1 = B_2 = B_3 = ... = B_n$, the Kronecker product is:

$$H \times [B_1, B_2, ...B_n] = H \times B$$

C.4 Paley construction

PALEY'S THEOREM If q is an odd prime or $q = 0$ and n is any positive integer, then there is an Hadamard matrix of order $m = 2^e(q^n + 1)$, where e is any positive integer such that $Mod[m, 4] = 0$ [14][26].

Hadamard matrices H^n can be constructed by using the finite field $GF(p^m)$ when $p = 4l - 1$ and m is odd. Pick a representation r relatively prime to p, then by setting to -1 $\lfloor (p-1)/2 \rfloor$ (where $\lfloor x \rfloor$ is the floor function)distinct equally spaced residues mod p $(r^0, r, r^2, ..., r^0, r^2, r^4, ..., etc)$ in addition to 0, a Hadamard matrix is obtained if the powers of r (mod p) run through $< \lfloor (p-1)/2 \rfloor$. For example,

$$n = 12 = 11^1 + 1 = 2(5+1) = 2^2(2+1)$$

is of the form with $p = 11 = 4 \times 3 - 1$ and $m = 1$. Since $m = 1$, we are dealing with GF(11). So pick r=2 and compute its residue(mod 11), which are:

$$\begin{aligned} r^0 &\equiv 1 \\ r^1 &\equiv 2 \\ r^2 &\equiv 4 \\ r^3 &\equiv 8 \\ r^4 &\equiv 16 \equiv 5 \\ r^5 &\equiv 10 \\ r^6 &\equiv 20 \equiv 9 \end{aligned}$$

$$\begin{aligned} r^7 &\equiv 18 \equiv 7 \\ r^8 &\equiv 14 \equiv 3 \\ r^9 &\equiv 6 \\ r^{10} &\equiv 12 \equiv 1. \end{aligned}$$

Picking the first $\lfloor 11/2 \rfloor = 5$ residues and adding 0 gives: 0, 1, 2, 4, 5, 8, which should then be set to 1 in the matrix obtained by writing out the residues increasing to the left and up along the border(0 through $P-1$, followed by ∞), then adding horizontal and vertical coordinates to get the residue to place in each square.

$$\begin{bmatrix}
\infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
10 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \infty \\
9 & 10 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \infty \\
8 & 9 & 10 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \infty \\
7 & 8 & 9 & 10 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & \infty \\
6 & 7 & 8 & 9 & 10 & 0 & 1 & 2 & 3 & 4 & 5 & \infty \\
5 & 6 & 7 & 8 & 9 & 10 & 0 & 1 & 2 & 3 & 4 & \infty \\
4 & 5 & 6 & 7 & 8 & 9 & 10 & 0 & 1 & 2 & 3 & \infty \\
3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 0 & 1 & 2 & \infty \\
2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 0 & 1 & \infty \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 0 & \infty \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & \infty
\end{bmatrix}$$

This gives rise to the following matrix when "0,1,2,4,5,8" are replaced by

"1", and all other entries by "-1":

$$\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 \\ -1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 \\ -1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 \\ +1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & -1 \\ -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 \\ -1 & -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & +1 & -1 \\ +1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 \\ -1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 \\ +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 \\ +1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & +1 & -1 \\ +1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 & -1 & -1 \end{bmatrix}$$

H_{16} can be trivially constructed from $\mathsf{H}_4 \otimes \mathsf{H}_4$. H_{20} can't be built up from smaller matrices, so $n = 20 = 19 + 1 = 2(3^2 + 1) = 2^2(2^2 + 1)$ can be used. Only the first form can be used, with $p = 19 = 4 \times 5 - 1$ and $m = 1$. Therefore using GF(19), and set 9 residues plus 0 to 1. H_{24} can be constructed from $\mathsf{H}_2 \otimes \mathsf{H}_{12}$.

C.5 Williamson construction

C.5.1 Circulant matrix:

A circulant matrix is one obtained by circulating its first row[20]:

$$\begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$$

C.5.2 Williamson theorem:

If A, B, C, D are symmetric, circulant $n \times n$ matrices satisfying: $A^2 + B^2 + C^2 + D^2 = 4nI$, then the matrix

$$W = \begin{pmatrix} A & -B & -C & -D \\ B & A & -D & C \\ C & D & A & -B \\ D & -C & -B & -A \end{pmatrix} \tag{C.5}$$

is an $nm \times nm$ Hadamard matrix. The matrices A,B,C,D are called Williamson matrices.

Williamson produced Williamson matrices of odd orders n=3,...,21,25,37, and 43. There are now known Williamson matrices for all odd $n < 100$ except n=35, 47, 51, 53, 65, 67, 71, 73, 77, 81, 83, 93, 95.

C.6 Baumert and M. Hall construction

Baumert and M. Hall[21] produced Williamson matrices of order 23 in 1962, therefore demonstrating for the first time the existence of an Hadamard matrix of order $4 \times 23 = 92$, the smallest unsolved case at that time. The work was aided by computer.

Another idea is ariese for the Williamson array A,B,C,D. With the following arrangement, one can get another kind of Hadamard matrix:

$$\begin{bmatrix}
A & A & A & B & B & B & C & C & C & D & D & D \\
A & -A & -B & A & -B & B & D & D & D & -C & -C & -C \\
A & B & -A & -A & C & D & -B & D & -D & -B & C & -C \\
B & -A & A & -A & C & D & -D & -B & D & -C & -B & -C \\
B & -C & -D & -D & D & -A & C & -C & A & A & B & -B \\
B & -D & D & -D & -C & -A & A & B & -A & -C & C & B \\
C & B & -C & -C & -A & -D & -D & D & B & B & -A & A \\
C & -D & C & D & D & -C & -A & B & -A & A & -B & -B \\
C & -D & -D & C & -D & C & -B & -A & -B & B & A & A \\
D & C & -B & B & A & -B & C & -A & -C & -D & -A & D \\
D & C & B & B & -B & -A & -A & -C & C & -A & D & -D \\
D & B & C & -C & -A & C & B & -A & -B & D & -D & A
\end{bmatrix}$$

From this Williamson array of order q, the Hadamard matrix of order 12q can be obtained.

C.7 Turyn construction

The circular Hadamard matrix:

$$\begin{bmatrix}
+1 & -1 & +1 & +1 \\
+1 & +1 & -1 & +1 \\
+1 & +1 & +1 & -1 \\
-1 & +1 & +1 & +1
\end{bmatrix}$$

Turyn proved that the order of circular Hadamard matrix must have the form $4p^2$, where $p > 55$ and p is odd, but not a prime power[22][23][24].

C.8 Goethals and Seidel construction

As an improvement on the Williamson construction, Goethals and Seidel altered the Williamson matrices so that the circulant matrices A, B, C, and D need not be symmetric. They must satisfy:

$$AA^t + BB^t + CC^t + DD^t = 4nI$$

rather than $A^2 + B^2 + C^2 + D^2 = 4nI$. Under these conditions the matrice

$$\begin{bmatrix} A & -BR & -CR & -DR \\ BR & A & -DR & CR \\ CR & DR & A & -BR \\ DR & -CR & BR & A \end{bmatrix}$$

is a Hadamard matrix, where R is a certain $back - circulant$ matrix, now known as the $Goethals - Seidel\ array$[25].

Appendix D

Hadamard matrices of order 20 and 24

Some examples of Hadamard matrices of order 20 and 24 are listed here. (for brevity, "+" represent "+1", "-" represent "-1".) Note that not all Hadamard matrices of order 20 and 24 are listed.

D.1 Hadamard matrices of order 20

$$H_1 = \left[\begin{array}{cccccccccccccccccccc}
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\end{array}\right]$$

$$H_2 = \begin{bmatrix}
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\end{bmatrix}$$

$$H_3 = \begin{bmatrix}
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\end{bmatrix}$$

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H_4 = \begin{bmatrix}
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\end{bmatrix}
$$

D.2 Hadamard matrices of order 24

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H_1 = \begin{bmatrix}
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\end{array}\right]
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$$H_3 = \begin{bmatrix}
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