A MODEL CLARIFICATION STUDY
OF MOTIVATORS AND ANTI-MOTIVATORS
IN MATHEMATICS EDUCATION

A Dissertation

Submitted to the Faculty of Graduate Studies

by Lloyd Lawrence

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A Thesis/Practicum submitted to the Faculty of Graduate Studies of The University of Manitoba in partial fulfillment of the requirements of the degree

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Abstract

Motivation in school mathematics is problematic partly because of the diverse literature on the subject and consequent incompatibilities in conclusions. There is no clear conceptualisation of motivation in mathematics that can serve both the researcher and the practitioner and no comprehensive view of those factors that influence whether or not a learner wants to know and understand mathematics. Such influences are the motivators and anti-motivators in mathematics education.

This study proposes a model for motivators and anti-motivators in mathematics education that can help to clarify the referents used in discourse and research. The primary material used in the model's construction was a combination of preliminary categories and reflections in conjunction with prior literature and the results of a pilot study. Part of this material's use was to reframe in terms of motivation some elements of research on beliefs, abilities, and constructivist orientations towards pedagogy.

The construction of the model was based on David Willer's (1967) analysis and prescription for a scientific approach to model and theory building. Using Willer's classifications, the proposed model is conceptual, immediately relevant to theory, and represents phenomena symbolically. The model was tested against three criteria:

1. It must be able to explain the natural language of non-specialists when they discuss motivation.

2. It must be able to explain direct and indirect references to motivation in the literature.

3. It must be capable of facilitating scholarly discourse on motivation.

The details of the test of the model against those criteria are provided in the thesis.

Chapter 1: Rationale

Mathematical expertise is no longer an essential component of many occupations (Keitel, 1988) and it is now arguable how much mathematics, if any, ought to be mandated for which students. That issue, however, is not the burden of this thesis. For whatever mathematics is taught, we must still deal with the observation that students' performances in mathematics (or lack thereof) can be associated with their perceptions of a broad range of apparent needs (compare with Maslow, 1970): survival, comfort, interpersonal relations and self-esteem, and possibly a thirst for knowledge, understanding, order and beauty. And each of those informal categories allows a plethora of subdivisions.

Discourse and research about motivation have ranged over that broad spectrum of referents and will likely continue to do so. At least part of the lack of precision and inconsistent observations in the current literature can be attributed to that diversity and the usefulness of further discourse and research will depend, not on establishing a single "correct" definition, but on a recognition of all the ways in which the word "motivation" is used and on success in giving each of those meanings clear referents.

As Hidi & Harackiewicz (2000) assert: "Only by dealing with the multidimensional nature of motivational forces will we be able to help our academically unmotivated children" (p. 151). These authors state that their intention is to "find ways in which the academic motivation of children who are 'turned off' could be increased" (p. 152). The present study considers this multidimensional nature of academic motivation and claims that the categories suggested for the proposed model reflect the nature of the above diversity.

The term "motivation" is used throughout the educational literature with the tacit expectation that its meaning is understood. Probably the best attempt to date at a comprehensive definition is Csikszentmihalyi & Nakamura's (1989) discussion of the relationship between effort, energy, and activity in which motivation is characterised by intentionality and commitment. Those qualities can be called "motivators" and since all such influences can be positive or negative, it is more accurate to call them "motivators" and "anti-motivators."

But Csikszentmihalyi & Nakamura's definition does not begin to address the diverse ways in which the word is commonly used. Given that diversity, no single definition could possibly subsume all of those uses.

The purpose of this study is not to propose a "correct" set of motivators and anti-motivators, nor is it intended to report on further exploration of a question about any one kind of "motivation." It is rather a model clarification study; an attempt to propose categories and language that can allow us to clarify discussions of motivation and anti-motivation in mathematics in an unequivocal way.

And it is not sufficient merely to propose a model. Shortly after Thomas Kuhn (1962) called attention to the central place of models (calling them "paradigms") in all scientific discourse, and David Willer (1967) analysed their role in the social sciences and provided good advice as to how to construct them, a plethora of proposed models soon revealed that anyone proposing a model must go beyond providing categories and language. No matter how thorough the writer's analysis might be, it must be demonstrated that his or her categories and language have clear and reasonably unequivocal referents and general utility within the domain for which they are proposed.

For that reason, the model generated in this study is tested against three criteria:

1. Can it make sense of the natural language of non-specialists who are asked to discuss their own and others' motivation and non-motivation to learn mathematics?

2. Can it make sense of what is said and found in both direct and implicit references to motivation in the literature?

3. Having been introduced to the model, can experienced researchers from outside mathematics education use the categories and language of the model to discuss possible researchable questions entailing students' motivators or anti-motivators?

The degree to which the model proposed in chapter four satisfies those criteria is addressed in chapter five.

Steps to a model

The categories of the model proposed in chapter four evolved through three stages, guided throughout by Willer's (1967) prescription for the construction of models. The details of that prescription are provided in chapter four.

Stage 1.

The author's attention to the meanings of "motivation" began with MacPherson's (1991) reflections on the variety of mechanisms available to teachers to "motivate" students. That reading was followed by conversations, further reading, the accumulation of "street" language, and reflection. It soon became evident that there is more to "motivation" than was suggested in MacPherson's paper. His categories were amended and several tentative new categories were appended.

Stage 2.

There followed a more formal pilot study, in which as broad as possible a spectrum of students and adults were asked to reflect on what had and had not motivated them (in one direction or the other) as they encountered mathematics in school. Their attention was called to each of the tentative categories established at stage 1, and those categories were modified, subdivided, and extended as the interviews continued. The details of the pilot study are provided in chapter three.

Stage 3.

The literature on motivation was then searched but not reviewed in the conventional way, since this study is not directed to a further exploration of any particular facet of the domain. Instead, two objectives concerning the domain as a whole guided the literature search.

The first intent was to establish the variety of ways the word has been used and the lack of consistency in those uses. For example, studies that explore the relationship between interests, arousal, and control (Middleton, 1995) are difficult to reconcile with theoretical accounts of attitudes in terms of beliefs, emotions, and behaviour (Hart, 1989). There is no reason to believe that any such writers have deliberately avoided language and concepts that could make it possible to co-ordinate their observations. It is more likely that the historical development of the area from heterogeneous sources using diverse language has led to irreconcilable statements and results. Chapter two provides sufficient examples of that diversity of language and results to demonstrate the need for this study.

The second intent was to survey those prior studies and positions, whether or not addressed to motivation per se, that seemed to have promise as foundations for the construction of the model presented in chapter four. That literature is examined in chapter three.

The references provided in chapters two and three may be useful to someone intending to pursue specific questions about motivation raised in those studies in light of the clarifications recommended in this study.

Central themes identified in stages 1 and 2

Among the many facets of motivation exposed in the assembly of natural language and in the pilot study, a few themes recurred so frequently and so obviously demanded close attention in the growing model that they warrant attention at this point. They do not necessarily parallel components of the model proposed in chapter four, but certainly point to a corpus of general language that must be accommodated within it.

Grades and qualifications.

On occasion, the use of external rewards — in particular, grades — to motivate the learning of mathematics is conjectured to also lead students to value mathematics. That concatenation of two quite different phenomena is common in discussions of motivation and is likely most often attributable to indifference to the precision of the model being used, but in this context they must be distinguished. As was found in the pilot study, the impact that grades can have in generating student affect towards mathematics depends on the grades students receive. Whether or not valuing mathematics is a resultant or residual effect is uncertain. Assuredly, the association of mathematics with low grades can have quite the opposite of the intended effect.

Further, it was pointed out by several non-specialists that even when grades motivate acceptable performances, the external reward is unrelated to mathematics per se, and the motivation so aroused can easily be transferred to other means of obtaining the same rewards, perhaps for less effort.

Overall, then, school grades currently have a high societal profile. It is commonly maintained that grades are a prime motivator and anti-motivator of mathematics. Any model for discourse about motivation must accommodate that belief.

In-school competition.

Education systems continue to express the goals of schooling much as they did at the onset of mass education a hundred and fifty years ago. Essentially, schools exist to prepare children for productive lives within a competitive society. Some of the means for achieving that goal have changed, but not the practice of fostering competition between students. Competition in the adult world has become competitiveness in school — an obsession with being the first, or with being the best. Despite the anti-elitist message of the National Council of Teachers of Mathematics standards (NCTM, 1989), which exhorts schools and teachers not to give their attention only to the talented, in-school competition still is, as several interviewees in the pilot study pointed out, a strong determiner of who will be, and who will not be, motivated to pursue mathematics.

As they attempt to satisfy their most cogent needs, students face potential success or failure. Those who succeed reap the approval and praise of parents, teachers, and peers (perhaps along with some jealousy). For those who fail, there is more often pity or scorn. The pilot study indicated that a system of awards and preferential treatment encourages school achievers to learn and perform, while others are discouraged by their lack of achievement. Several comments supported the notion that if a student is historically predisposed to failure in mathematics, s/he will have reason to avoid the subject. In particular, criterion-referenced grading provides students with indicators of success or failure. Quite obviously then, competition is not likely to motivate those who fall below the mandated standard of performance.

On the other hand, there are many students whose need to compete is so much a part of their innate or learned temperament that they would be unable to perform without attempting to satisfy it. For them, the competition justifies the effort, whether they are attracted to mathematics or not. The removal of such competition from the school environment would be as detrimental to these students as competition is to those discouraged students mentioned above. As the pilot study found, school culture reflects its societal context and interacts with the paradigms and conventions that govern its value system.

A model for discourse about motivation in mathematics must therefore accommodate references to both the positive and negative aspects of competition.

Meaningfulness.

In contrast to what can easily be the ephemeral benefits of competition, meaningful learning is more likely to survive long after schooling is over:

> [The meaning we take from what we learn] asks us to ask ourselves what significance we can attach to our own presence in the world. This is a sobering assignment, and quite probably an unpleasant one, too. Yet, for all its unpleasantness, it is a task we seem unable to turn from. (Morris, 1966)

Meaningfulness has been a recurring theme in twentieth century education. Dewey (1916) was scathing in denouncing meaningless rote recitals, as were others, such as Brownell (1935). They regarded meaningfulness as an implicit quality that encouraged students' attention to the material being learned, and asserted its importance as a foundation for further learning. Bruner (1962) attacked "the mechanical process of manipulating numbers without any intuitive sense of what it is all about … the meaning [and] the significance" (p. 102). It is certain that any model for motivation in mathematics must accommodate the possible impact of meaningfulness.

Family and social issues.

The pilot study found evidence suggesting that parents and surrogates, as they respond to the child's first gropings and questions, usually have a strong initial influence on a child's inclinations, encouraging and discouraging interest and stimulating and curbing arousal. A parent's beliefs and feelings about mathematics and its artifacts are the original stimuli for a child's acquisition of mathematical concepts. It is not possible to test the hypothesis that there are innate mathematical inclinations that are strong enough to withstand negative influences from parents. There is no doubt, however, that parents can influence how a child feels about "doing mathematics" (e.g. Goldberg, 1990; Onslow, 1992; Mendoza, 1996). Likely everyone in a developing child's circle of intimate contacts has, to some degree, the potential to activate a bias one way or the other. Commentators of all kinds hold that it is important to acknowledge the contribution of family and social influences, and commonly recommend interventions to enhance their positive effects.

Summary

The work of this dissertation is to propose a model that can enable clear and unequivocal discourse about motivation. This first chapter identifies some of the most cogent channels of influences that must be accommodated. Chapter two reviews the pertinent literature that must be accommodated but which is characterized by inconsistent usage of definitions and gaps in referents. Chapter three identifies those areas of prior theory and research that can make more useful contributions to the content and substance of a model. Chapter four describes the construction of a model intended to accommodate the data from the pilot study and the analysis of the literature. And chapter five reports on tests of the usefulness of the model by addressing the three criteria given at the beginning of this chapter.

Chapter 2: Problematics in the use of language and sanctions in the use of model construction theory

This and the following chapter review the literature on education and motivation in two ways. First, the need for this study is established by examining some parts of the literature that demonstrate that the uses of the term "motivation" are often inconsistent and non-comprehensive. The following chapter considers some thrusts that seem to hold promise as a basis for a well-defined, consistent and comprehensive model for discourse about motivation.

Stating the question

As was noted above, one of the first difficulties encountered in the literature is that of finding well-defined and consistent definitions. Concepts are appropriated to accommodate applications and motivation is taken to be anything that entails arousal, inclination, interest, attitude, belief, control, perception, anxiety, and related psychological constructs. In other words, "motivation" is "an umbrella term having a wide variety of connotations and denotations" (Ball, 1982, p.1256). Further, what is meant by any one of these labels varies. Consequently, the results and conclusions of studies are frequently uncertain, and their accounts of motivation are often incomplete, and fail to specify the meaning of mathematical "attitudes" and "proclivity."

Kulm (1980) asserts that in mathematics education an attitude is "a general predisposition leading to a set of intentions to behave in a certain fashion" (p. 372; also see Fischbein & Ajzen, 1975). Hart (1989) defines an attitude as "a predisposition to respond in a favourable or unfavourable way with respect to a given object" (p. 39) and speaks of three components (see also Rajecki, 1982): (a) an affective or (equivalent) emotional reaction to the object, (b) behaviour towards the object, and (c) beliefs about the object. She claims that mathematics educators generally do not recognise emotional reactions as belonging to attitude. But McLeod (1992) provides the contrary proposition that "attitudes develop out of emotional responses" (p. 582). As usual, there is no consensus.

Most research is classroom based; however, Goldberg, 1990, Onslow, 1992 and Mendoza, 1996 hold that parental attitudes significantly influence students' mathematical motivation. Many other experiences beyond the classroom can influence the development of positive or negative associations with mathematics and there may be no way to separate these contexts. Cobb, Yackel, & Wood (1989) note that "children's beliefs, their emotional acts, and the network of obligations and expectations that constitute the social context within which they do mathematics are all intimately related" (p. 140). Clearly, motivators and anti-motivators beyond the classroom cannot be ignored.

Stipek (1998) draws on psychological relations between motivation and other (non-mathematical) phenomena when she indicates that mathematics anxiety is an unresolved problem. Stipek's response identifies coping mechanisms aimed at minimising the negative effects of anxiety rather than addressing the causes (e.g. pp. 197-201). Because she has stressed the qualifier "achievement" in achievement motivation, she addresses her comments to the reduction of achievement anxiety in her remarks on mathematics anxiety. While this might remedy a performance problem, it is not really oriented to the object of the performance, which is mathematics. In other words, she does not address mathematics anxiety as an anti-motivator.

Ball (1982) points out that motivation is only one of a set of elements that determine behaviour and that it involves many processes (see below). The example above accomplishes two goals at once: first, it shows how the literature commonly conflates motivation with other contexts; and second, it further demonstrates the diversity of meanings of "motivation."

The reform movement led by the NCTM (1989) has set goals and priorities that imply attention to motivation, advocating the following: active thinking and meaningful learning in mathematics; bringing the content to <u>all</u> students, not solely to those whose personal qualities equip them to succeed; and valuing mathematics instead of merely learning it.

Ball's (1982) cautionary notes are worth repeating here (see also Ball, 1977). Each of these observations has inferences which command attention in a model for discourse about motivation:

1. Motivation is a hypothetical construct that we infer from a person's behaviour in a particular environment.

2. The concept of motivation should not be overused as an explanatory device, even though it may be useful in helping to make predictions about future behaviour.

3. Motivation is only one of many constructs with the potential for affecting a person's behaviour.

4. Motivation concerns many processes that are perhaps related. Extensive research exists on curiosity (Berlyne, 1963), locus of control (DeCharms, 1976), achievement motivation (Fyans, 1980), anxiety (Tobias, 1979), self-esteem (Coopersmith, 1967), and attribution (Weiner, 1979), as well as other topics. But an

attempt to explore that vast literature so as to achieve a full synthesis of motivation-related concepts would be very difficult (Ball says, "impossible") and largely irrelevant to the intent of this study.

5. Reflecting on motivation in education inevitably leads us to questions about values. For example, social motives based on respect for authority, interpersonal motivation based on competition, and motives involving personal autonomy and self-reliance potentially affect students in school and can lead to changes in the wider society.

In short, a way of looking at motivation in mathematics has yet to be proposed that can claim to be both consistent and comprehensive. As contexts vary, language is often modified and meanings changed. Concepts have remained defined by their contexts and attempts to use what seem to be parallel concepts have led to incompatible conclusions (for example, Hart's (1989) and McLeod's (1992) analyses cited earlier in this chapter).

Education theorists have not been entirely consistent in their uses of the characterisations and componential analyses offered by cognitive and affective psychologists (see Hart, 1989). The area is too disorganised to make it possible to carry out much consistent discourse about motivation. A model and set of categories are required that are capable of coordinating the concepts and literature sufficiently for more productive, generalisable investigations in mathematics education.

Historical analysis

Initially, the concept of the individual's autonomous will, which could transform desire into action and maintain the pursuit, was posited to explain all human activity. But later psychologists disputed this notion (see Csikszentmihalyi & Nakamura, 1989;

Cannon, 1932; Hull, 1943). With the adoption of Drive Theories, metaphors for the homeostatic maintenance of physiological health, psychologists discounted volition in psychological motivation, at least to the extent that it was regarded as less of a determining force than the involuntary need to reduce psychological discomfort. Subsequently, to accommodate within Drive Theory the individual's deliberate search for intellectual stimulation, psychologists hypothesised a state of optimal arousal. In this way, theories no longer treated motivation as a one-dimensional phenomenon. Its sources could be both external and internal to the individual, leading some analysts to an extrinsic/intrinsic partitioning of motivation.

Difficulties arise, however, out of this partitioning of motivation. Under one definition an external reward is the criterion for extrinsic motivation; another definition views the decision-making that leads to a reward as intrinsic motivation (see Csikszentmihalyi & Nakamura, 1989). Furthermore, if the rewards in extrinsically motivated activites are understood as subgoals towards more intrinsic goals (Maehr, 1984) — such as the satisfaction of inner needs — then all activity is perforce intrinsically motivated!

Middleton's (1995) decision-based "model of academic intrinsic motivation," constructed in terms of interests, arousal, and control, is not explicit about what makes a motivational process intrinsic rather than extrinsic. Further, Middleton declares that the terms interests, arousal, and control are "general categories" and provides little clarification of the words or concepts being used. Hidi & Harackiewicz (2000) note that "many researchers use the terms interest and intrinsic motivation [italics in original] almost interchangeably," while "individual interest is viewed as a pre-condition of intrinsic motivation" (p. 158). They also advocate "the importance of considering the separate effects of both extrinsic and intrinsic motivation in education" (p. 159) and refer to the extrinsic/intrinsic dichotomy as "problematic" (p. 160).

Concepts and language

Alternative definitions of motivation, motivators, and anti-motivators rely on cognitive and affective conditions such as attitude and emotion. This approach dispenses with the sphere-of-origin categorisation of motivation and focuses instead on an individual's state of consciousness. The literature adopting this stance discusses mainly the positive orientation of motivation — the "spring of action" towards an activity (Ames & Ames, 1984), or the conscious effort to invest "psychic energy" (Csikszentmihalyi & Nakamura, 1989).

"Attitudes" and "emotions" have at least the virtue of including both positive and negative characteristics; hence these concepts provide a means for incorporating both motivators and anti-motivators into theories. Doing so depends on coherent descriptions of attitudes and emotions. However, while researchers agree that attitude is a predisposition to act in a certain manner given a set of circumstances or an object (Kulm, 1980; Hart, 1989), they do not agree on whether and how this constitutes a relationship between attitude and emotion (Rajecki, 1982; McLeod, 1992). What is more, the conceptualised relationships between attitudes and beliefs on the one hand, and emotions and beliefs on the other, are frequently in conflict. Such problems raise questions about the cognitive foundations for belief systems; underlying these ambiguities is the conceptualisation of an inherent cognitive/affective duality in states of consciousness (James, 1890; Rokeach, 1960). It is not possible to analyse these motivational phenomena through their components, since it is difficult to find agreement as to what those components are.

All of these constructs are presumably antecedent to the behaviours from which "motivation" can be inferred, and since none of these model builders have been specific about what behaviours to expect or, in particular, what sorts of "motivation" are to be inferred from them, these analyses must be regarded, at least for the time being, as peripheral to the focus of this study.

Studies in the literature concerning models

Since the present study proposes to clarify the language of discourse about motivation, there is no need for a more extensive review of the above literature. However, there is a range of literature concerning models in education that could be useful to classify models of motivation in mathematics education. Willer's (1967) work on scientific models is a longstanding source of authority for model construction in the social sciences and compares well with the ideas of Black (1962), Churchman, Ackoff, & Arnoff (1957), Hutten (1954), and Toulmin (1953). Willer considers model classifications in terms of its components (<u>concepts</u> or <u>data</u>), its scope (<u>general</u> or <u>theoretical</u>), and its depiction (<u>analogue</u>, <u>iconic</u>, or <u>symbolic</u>). In particular, Willer (p. 24) defends the framing of models in terms of <u>concepts</u> rather than <u>data</u> by quoting Toulmin (p. 42):

> … it is not that our theoretical statements [in models] ought to be entailed by the data, but fail to be, and so assert things the data do not warrant: they neither could be nor need to be entailed by them, being neither generalisations from them, nor other logical constructs out of them, but rather principles in accordance with

which we can make inferences about phenomena.

Willer's definition of a model identifies "a group of phenomena constructed by means of a rationale" (p. 15), and draws a line between a "model" and a "theory": "The theory can be deduced from the model …" (p. 18). Thus, when Moss & Case (1999) speak of a "model" in their study on children's understanding of the rational numbers, they are referring to a theoretical framework with a conceptual structure that generates relationships and simulates reality. Similarly, Jones, Langrall, Thornton, & Mogill (1999) study students' probabilistic thinking using a framework previously constructed over a two-year period and combined with a synthesis of prior research in probabilistic thinking.

Willer (1967, pp. 15-16) describes a model as <u>theoretical</u> if it can immediately generate formal systems and is therefore immediately relevant to theory. Otherwise, the model is <u>general</u> and may be used in the construction of theoretical models. A general model may "refer to a wider range of data or to more inclusive data" (p. 15) than a theoretical model. The mental schemes that Christou & Philippou (1998) explore in their "identification of hierarchical levels among additive and multiplicative word problems" seem to be examples of general models. However, the symbolic and verbal precedence models used by Nathan & Koedinger (2000) for their work on the development of reasoning are characterised by a specificity and a formality that suggest theoretical structure; this clearly fits Willer's description of a theoretical model.

As stated in chapter one, the proposed model construction follows Willer's prescription for the construction of theoretical models. In the remaining choice of <u>analogue</u>, <u>iconic</u>, or <u>symbolic</u> depictions, Willer provides the following distinctions (p.

29): "The first consists of borrowing a mechanism from another application, the second apprehends a mechanism from the data, and the third creates the mechanism for the model by relating the concepts themselves." As will be shown in chapter four, the proposed model is a symbolic depiction of motivators and anti-motivators in mathematics education.

Summary

Motivation research is fettered by a lack of consensus concerning its nature. Propositions about motivation therefore do not have firm foundations. There are too many conflicting conceptualisations between different research perspectives. With the theory in this condition, any study of motivators and anti-motivators is likely to be confounded by diffuse language and incompatible models. It is necessary first to "clarify" the models — that is, to develop a way of clearly identifying the various characterisations given to the informal uses of the word "motivation." Chapter four addresses that task, presenting a model which accommodates the theoretical background and resolves its apparent contradictions. Chapter five will report on a test of the model's clarity and referents.

Chapter 3: Foundational material

This chapter lays the groundwork for the proposed model of categories for motivators and anti-motivators in mathematics. A generic view of motivation is adopted and current applications are outlined. Persuasive arguments about epistemology and learning from the past three decades are weighed and those elements that clearly refer to motivation are emphasised.

It is not the intent of this study to consider in what ways the philosophies and world-views held by teachers affect how students learn mathematics (see Ernest, 1990). These are implicit in the discussion of motivators and anti-motivators, but it would be unnecessary in this study to view motivation ideologically or to place it in such a context and an ideological analysis such as Ernest's seems not to be relevant to the present study.

Part One: The literature

Each of the following analytical accounts has ramifications, often subtle, for the construction of a model for discourse about motivation. Chapter four examines the process of construction of that model, but it would be quite impossible to mark each and every possible linkage between the inferences that can be drawn here and the categories and relationships of the model.

Useful standpoints on motivation.

To date there are no specific definitions of motivation in mathematics education. As suggested in chapter one, Csikszentmihalyi & Nakamura (1989) have probably done as well as can be done: "Whenever we encounter human activity that requires concentrated investment of psychic energy, we assume that this event is not random but the product of conscious effort. Motivation is what makes such effort possible" (p. 51). While this definition is so general as to be of little operational use, the connection it establishes between an activity and its origins relates action and motivation causally, and at least distinguishes motivation from a passive interest.

In Middleton's (1995) study, mentioned in chapter two, on the effects of personal constructs on motivation mentioned in chapter two, he asserts that if the belief systems of teacher and student coincide, so will their conceptions of instructional tasks (p. 257). He also speculates that differences in their belief systems can lead to an eventual aversion to mathematics. These comments on beliefs (where beliefs are identified as components of attitudes) lead to some serviceable hypotheses, such as that (dis)similar beliefs can generate (dis)similar attitudes, and that the social influence of a teacher's preferences can be an (anti-)motivator for learning mathematics.

In passing, it might be noted that this relationship between students' (intrinsic) belief systems and teachers' (extrinsic to the student) belief systems is one of several reasons why "self-esteem" and "social drive" are closely linked categories in the model of chapter four.

National standards and motivation.

The National Council of Teachers of Mathematics (NCTM) has proposed the following general goals for students (1989, p. 5): that they (a) learn to value mathematics, (b) become confident in their ability to do mathematics, (c) become mathematical problem solvers, (d) learn to communicate mathematically, and (e) learn to reason mathematically. It seems to be implicit in these goals that a student will only value mathematics if s/he is motivated to do so. All of the goals can be associated with motivators in that way. A number of conditions may be precursive to the achievement of these goals, but mathematical power and mathematical literacy seem to be critical.

According to the NCTM standards, mathematical power must be developed in all children and includes "… the ability to explore, conjecture, and reason logically; to solve non-routine problems; to communicate about and through mathematics; to connect ideas within mathematics and between mathematics and other intellectual activity. It also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions. Students' flexibility, perseverance, interest, curiosity, and inventiveness also affect the realisation of mathematical power" (1991, p. 1). While the word "motivation" is not used, it is clear that this spectrum of traits is assumed to be motivating, and all of them must be addressed in a comprehensive model.

Mathematical power is thus conceived of as a mixed bag of creativity, inclinations, and effort combined with a set of abilities. From this point of view, then, effective pedagogy is not so much concerned with students' acquiring a liking for mathematics, as with their becoming motivated by acquiring the ability to use mathematics effectively. The NCTM implies that high motivation is not always necessary for a successful pedagogy. However, there would probably be fewer devotees of academic mathematics if the sole result of mathematics education were successful performance. Motivation beyond the utilities of ability, talent, and skill seems to be essential if a student is to fully realise the NCTM goals and find mathematical power enticing.

Ability, performance, and motivation.

Whether or not mathematical ability is taken to be innate, its use and development has been most fully studied and described by Krutetskii (1976). His study on abilities points to some fundamental characteristics also found in the mathematically motivated. He presents the notion that a "mathematical cast of mind," evident in three modes (analytic, geometric, and harmonic), characterises creative mathematicians. MacPherson (1991) makes use of Krutetskii's analysis in his discussion of "levers that teachers can pull that can enhance performance in mathematics." These "levers" are clearly linked to various motivators and reflect an attempt at their classification. MacPherson speculates on seven factors that can stimulate students to know mathematics:

1. The elegance of mathematics.
2. An affinity with an individual associated with mathematics.
3. The peremptory demands of society, family, and friends.
4. Play as a means of practicing skills in a non-threatening context.
5. The evident usefulness in one's life of mathematical knowledge.
6. The power experienced in producing results.
7. The misrepresentation of mathematics in order to enhance its desirability.

Each of these factors speaks to a variety of needs and capacities within an individual, drawing on values that have developed through social conditioning, personal experiences, and thoughtful reflection. Thus, these factors can motivate learning.

Constructivism, pedagogy, and motivation.

Most constructivists are, to some degree, as concerned with social context as with individual development. Piagetian theory includes social interaction by referring to the "collective intellect" as a "social equilibrium" (Fosnot 1996, p. 18). But it is Vygotsky's interpretation of the dialectic between the individual and society that has attracted the largest number of writers on social and cultural constructivism (Cobb, 1996, p. 34; Vygotsky 1979).

Vygotsky's perspective hints at social motivation in classroom pedagogy and interactive learning situations. Pedagogy of this kind becomes a form of socialisation and perhaps a more "natural" process, in the sense that we make the most use of those meanings which meet the least social resistance. This is another way of saying that the desire for social approval motivates us to find communicable meanings; social context provides the motivation for learning.

The task of reworking constructivist philosophy as a theory of learning and thence of pedagogy is incomplete. In part this arises from unresolvable philosophical standpoints (Fosnot, 1996). However, when we think about pedagogical principles in relation to constructivist theories of learning we find that each is either a motivator or a precursor to motivation for studying mathematics. A list of such principles follows (summarised and adapted from Fosnot, 1996, pp. 29-30).

1. Curiosity will be motivating if validated by the teacher: Since learning is a process of development requiring invention and self-organisation on the part of the learner, teachers must be prepared for students to raise questions, generate hypotheses and models, and test them for viability.

2. A non-threatening environment motivates attempts at discovery: Since disequilibration facilitates learning, "errors" need not be avoided and opportunities must exist for students to engage in meaningful, challenging, exploratory investigations, leaving no contradiction undisturbed.

3. Communication can motivate intellectual synthesis: Since reflective abstraction is fundamental to learning, all forms of expression need to be available for organising meaning, including writing, discussion, personal reflection, and multi-symbolic representation.

4. The negotiation of meanings among members of the classroom will motivate a sense-making approach to mathematics: Since dialogue nurtures thoughtfulness the classroom must be a community of discourse, a forum for debating ideas, communicating them, and elevating them to the level of "taken as shared."

5. New knowledge can motivate rethinking prior mathematical views: Since learning consists of the development of (cognitive) structures through the generalisation of experiences, there will be ongoing progressive shifts in perspective which construct and reconstruct organising principles and which reorganise earlier conceptions.

Part Two: The pilot study

A pilot set of interviews involving 12 individuals (Lawrence, 2000) provided the source material for a tentative categorisation of motivators and anti-motivators. An expansion and modification of MacPherson's (1991) "levers" mentioned in chapter one was used to sort participants' responses into the following categories: aesthetics, social sense, necessity, curiosity, practicality, efficacy, and security. At this point the discussion on motivators and anti-motivators clearly extended beyond the classroom.

Data collection.

Each of the 11 interviews (one comprising two students) was audiotaped and transcribed. Participants were selected to represent a variety of backgrounds and experience. Initial categories for the selection of participants were parents, teachers, student-teachers, school students, school administrators, mathematical and non-mathematical academics, the general public. As anticipated, participants occasionally fell into more than one category. Since the two school students were interviewed together, there was a total of 12 interviewees.

It would have been difficult to obtain a more diverse sample with this number of interviews. Gender balance was not achieved, all participants (with the exception of the two school students) were university educated, and none of them had experienced a serious failure in mathematics. A significant task for the interviewer, therefore, was to identify differences in participants' experiences and backgrounds that may have influenced mathematics learning.

Each interview lasted approximately 45 minutes. It was guided by a set of leading questions and supported by an optional set of prompts (see Appendix A). Interview questions were "projective" in nature, so as to encourage participants to respond from their own experiences. Participants were to express their interpretations of their experiences of mathematics, in keeping with the qualitative nature of the study. This technique also maximised the potential for the discovery of further components for the developing model. Interviewees were encouraged to expand on personal histories, anecdotes, speculations, and characterisations, to the extent that doing so added depth and substance to their responses.

Results.

A set of reactions typical of responses in the interviews appears in Appendix B. On questions about mathematics rather than about social influences within the context of mathematics, interviewees commented not so much on the aesthetics of the subject as on its relative benefits, such as higher grades and a sense of achievement. This was especially true of those participants who had experienced little or no difficulty with mathematics at school or later. Failure to develop a working relationship with mathematics was attributed by some participants to an impenetrable or at least confusing period of instruction, particularly associated with algebra or calculus.

When asked to identify characteristics of the subject itself, participants commonly referred to their opinions about whether or not mathematics "made sense" during their schooling. As broad and subjective as this assessment was, it appeared to have had significance for all interviewees in their developmental years. To the extent that they were able to recognise how the mathematics they learned could fit into their understanding of the way the world operated, they considered the subject clear and complete; they were confident that everything would follow logically. If this conceptual integration was missing or if they had been unable to apprehend it, then mathematics was blocked from them: irrelevant and meaningless at best, obscure, confusing, and anxiety-provoking at worst.

An interesting pivot for these disparate perceptions was the abstract quality of mathematics. Interviewees made references to "the rules of the game," a game played with symbols. A connection was acknowledged between a sense of concreteness in mathematics and the everyday activities that make use of computation, such as financial transactions, scoring in games, and the communication of all kinds of information. It was expected that those who had difficulties with the subject would comment that mathematics was too abstract for them, or that problem solving was unrealistically presented. That complaint was heard in some interviews, although not as frequently nor as strongly as anticipated. For the mathematics enthusiasts, on the other hand, the subject's abstract character was cited as one of the primary reasons for its appeal.

Several participants had a school history in which they excelled or at least succeeded in all other subjects, even if they had problems with mathematics. Where interviewees credited their successful mathematical performance in large part to "a good memory," they tended to report that this faculty enabled them to perform well on rote calculations. Such prowess in memorisation, however, did not afford the interviewees a sense of pride in their mathematical achievement, since memorisation was seen as functioning without comprehension and as such did not prepare them for later mathematics.

These accounts highlight the unfortunate role that memorisation was perceived to play in the learning of mathematics. For those who were drawn to mathematics, the reasons for its appeal went far beyond the attainment of mechanical proficiency. These individuals emphasised such qualities as mathematics' internal logic, its comprehensive usefulness, its challenge to the intellect, and the satisfying feeling of applying it effectively. More than one participant spoke in terms of its beauty. It should be clear how this range of views influenced the growing model.

Predictably, the people most commonly seen to be major influences on the learning of mathematics were teachers. When a person's first response cited family members or friends, they were usually identified as practising teachers at the time. Although this disclosure is somewhat clichéd, it does tend to support a view that focuses on the relationships between teachers and learners as a primary influence. On the other hand, the evidence of these interviews suggested that classroom activity alone cannot account for the effects on learners.

Summary

Among other things, the current literature has explored the direct relationship between motivation and the thought processes of teachers and students. Furthermore, meaningfulness in instruction and content is the touchstone of the NCTM recommendations for students' interest in mathematics. A tentative classification of motivators in the guise of performance enhancers has existed for over ten years (MacPherson, 1991), while constructivist theory, which explains the effect of interactions between the learner and his or her environment, alludes to suitable categories that can enhance a model. In addition, a pilot study (Lawrence, 2000) has delivered personal experiences and opinions which added to the background and offered further provisional categories.

On this foundation, it is now possible to consider the proposed model. Chapter four will describe a list of categories chosen as a well-defined, consistent, and comprehensive set of motivators and anti-motivators that can accommodate, in some manner, every direct influence and nuance that can be drawn from the accumulation of natural language, the pilot study, and the literature. Chapter five examines the ability of the model to satisfy the criteria listed in chapter one.

Chapter 4: The model

Background for building a model

Willer (1967) begins by reiterating the commonplace, "Models are unprovable but essential" (p. 21). He adds, "... it is neither common sense nor convention which is the basis for judging a model, but its ability to represent the abstract features of the phenomena isomorphically while providing an adequate formal system for them" (p. 24). He asserts that a model should be built by blending facts — "phenomena in their most primitive form" (p. 28) — with instinctual and logical analyses. He then provides guidance (see below) for constructing a model for discourse. The model suggested in this chapter was constructed using Willer's (1967) protocol.

The development of the present model rests on three foundations. The first of these is the assembly of natural language mentioned in chapter one and the creation of tentative catagories. The second is the considerable literature reviewed in chapters two and three — chapter two demonstrating the diffuse ways of referring to "motivation" in mathematics, and chapter three the major thrusts that seem to have some relevance and that must be accommodated. The third foundation lies in the results of a pilot study by this researcher (Lawrence, 2000). Responses gathered from interviews conducted in that study led to further refinements of the incipient model and grounding for the conceptual definitions of the ultimate model.

Following Willer's process of <u>systematic abduction</u> (an orderly development from facts towards theory), concepts are "applied" to the facts — tentatively at first, making adjustments to ensure consistency, but firmly at its conclusion with the help of a rationale. There are three major steps to systematic abduction (Willer, 1967, pp. 28-29, 52-53):

1. Initially, "imaginative guessing" suggests hypotheses about known data and entails working concepts which have been applied to that data and may imply one or more points of view toward the data. Willer asserts that while the overall aim of this step is to discover hypotheses for eventual theory construction, these initial hypotheses will often be theoretically unmanageable. He holds that if the construction of a symbolic model begins with the concepts, their meaningful interconnection will constitute a rationale.

2. "Mental experiments" performed on the concepts are intended to "find their limitations and make manifest their implicit points of view" (p. 29), while searching for the simplest hypotheses. This is the stage of "meaningful interconnection of concepts" at a theoretical level, and in the symbolic model may require explicit "rationally consistent assumptions" for completion (Willer indicates that there are probably no completed symbolic models in the social sciences).

3. The primary characteristic of systematic abduction is an "internally consistent model" achieved through the use of a rationale which may describe any mechanism involved. Since in a symbolic model the rationale "consists of allowing a set of connected concepts to symbolise a set of phenomena," the process cited above of interconnecting concepts' meanings constitutes the development of the rationale.

The rationale explains the nature of the studied phenomena, creates nominal definitions for the concepts, and seeks to establish meaningful interconnections between the proposed categories. Willer classifies the kind of model proposed in this study as a symbolic model because its rationale consists of using a set of connected concepts to symbolise a set of phenomena.

There are two interrelated stages to the construction of a symbolic model (pp. 52-57):

1. Concept connection must develop within the meaning of the model by way of definition of concepts, rationally consistent assumptions, or both.

2. Concept definition may precede, follow, or alternate with the connections of concepts to concepts.

Willer states that so long as the completed model is conceptually consistent, the starting point of the process is unimportant. Also, a statement of the rationale may be unnecessary if the concepts are established first. However, the significance to the model of a rationale is not in doubt:

> Ultimately the usefulness of a theoretical model may be traced back to its rationale, the point of view toward the phenomena. This point of view is in no sense given in the data but is a consequence of the imaginative thinking of the theorist when confronted with the data.... Model construction, like any process of scientific discovery, cannot be fully systematised and explained. (p. 64)

In the pilot study a broad range of non-specialists were invited to expose the sources and natures of their reactions to the subject by providing largely unstructured accounts of their reactions to learning mathematics. Those reactions were tentatively sorted into categories (see Appendices A and B) and, as necessary, combined with categories derived from the literature (as noted in chapter three prior to the description of the pilot study). Systematic abduction was then used to consolidate that combination as a set of nominal definitions for the final categories. These categories formed the basis of the model. No attempt was made, nor should it be, to restrict these categories to those phenomena that are currently operationally defined as measurable. Instead, the goal was to create a full set of categories for the discussion of motivators and anti-motivators in mathematics.

Although motivators and anti-motivators may be active under many circumstances in one's life, the literature focuses on their presence in schools. There are, however, other contexts and circumstances, such as in the home and community, and when working with mathematical ideas, materials, or instruments. All of these must be accommodated in this model.

As Willer suggests, the data from the assembly of natural language and the pilot study were continually resorted and reclassified as the model grew, with further categories considered when necessary. When the model was judged to be complete and consistent it became possible to create the more precise definitions and relationships given below.

Discussion during systematic abduction from pilot data

The commentary under each heading below is intended to identify the range of ways that discourse about motivation may be addressed to the model (Figure 1). Each commentary is meant to be useful to those using the model to examine testable hypotheses concerning motivation. References to Maslow's (1970) hierarchy of needs relate the descriptions below to the list of general needs mentioned at the beginning of chapter one.

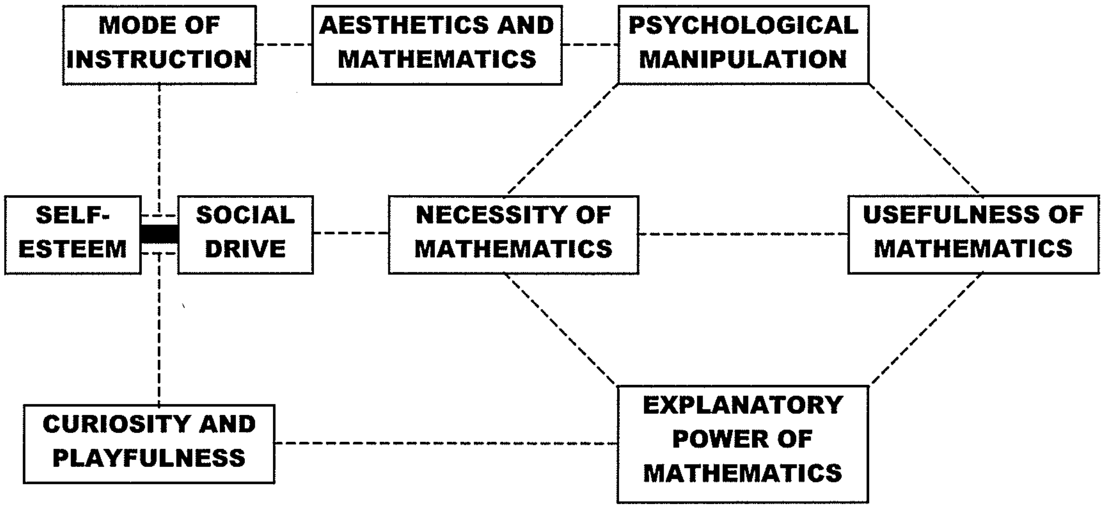

Figure 1. Symbolic model for motivators and anti-motivators in mathematics

Mode of Instruction.

A variety of modes intended to engage students in class can be motivating or anti-motivating. For example: worksheets may be incentives to the competitive or tedious to the pensive; a lecturing style can stimulate a student's imagination or add to a growing confusion; practical applications might inspire concrete thinkers yet undermine those who feel mechanically inept. While a teacher's mode of instruction has little to do with the physical, self, or social needs described in Maslow's hierarchy, the higher mental functions associated with knowledge and understanding — "being" needs (Maslow, 1970) — are fully represented.

It is within this category that a teacher's effect on learning is most obvious, since delivery of the lesson is the teacher's principal function. Mode of instruction may be cited as a direct positive or negative influence on a student's understanding and interest; it may also indirectly induce the social factors arising out of the interactions between student and teacher. In this way the teacher's mode of instruction may be a prime source of other (anti-)motivators.

In considering mode of instruction, attention should be paid to the continuum ranging from a strongly language-based style of instruction to one which primarily emphasises concrete experience. Language can, of course, be a powerful channel of communication, but for some students language is an obstacle to learning. For such students, language-based concepts may be expected to cause difficulties. If the use of language in learning mathematics is inherently problematic to the learner, then the use of that language is an anti-motivator. Visual representations of mathematical concepts, whether diagrammatic, iconic, or physical, may circumvent some difficulties for some students. Geometrical images, figuratively representing the complexities of mathematical ideas, can make for better pedagogy for those students than qualified definitions and annotated formulations. In summary, mode of instruction can be motivating or anti-motivating.

Self-esteem and Social Drive.

For the purposes of this model, self-esteem and social drive are conjoined as companion categories, although they are distinct "deficiency" needs (Maslow, 1970). It can be maintained that variations in self-esteem dominate social influences, but it can also be argued that social influences determine self-esteem. The issues involved in these contesting perspectives parallel the ongoing unresolved debate between the Piagetians and the Vygotskians — the individualistic, self-organising, endogenic view of knowledge construction versus the social, interactional, exogenic explanation (see: Steffe & Thompson, 2000; Lerman, 2000).

Clearly it would be unwise in a study such as this to increase dissonance by the questionable attempt to either fully separate or unite these categories. It might be held that self-esteem and social drive are interdependent sources of motivation, with some uses of the word having a stronger connection to one than to the other. In order to accommodate this ambivalence, the categories are placed in tandem, with the provision that some of those who use the model will want to separate them.

It is reasonable, in discussing self-esteem, to commence with intrapersonal factors. Psychologists have begun to challenge the self-esteem movement's insistence on "success" as the crucial guarantor of a child's self-esteem. Recognition of the inefficacy of attempts to enhance self-esteem directly has led to the realisation that self-esteem may best be promoted by experiences that afford a child opportunities to develop mastery. As Seligman (1995) notes, "Feelings of self-esteem ... develop as side-effects of mastering challenges" (p. 33). Thus, a teacher's emphasis on providing students with opportunities to develop mastery, rather than on ensuring their "success," can significantly influence self-esteem. This example also serves as a good illustration of the close interrelationship between self-esteem and social drive.

How we see ourselves depends on — among other things — what we expect of ourselves and what others expect and reflect back to us. We are willing to learn and perform such skills as will further our attempts to satisfy our needs for self-esteem and a sense of belonging. If we are frustrated in those attempts, whether by human agency or not, we are likely to avoid the type of painful experience that has led to our frustration. In terms of our self-esteem, then, there is then no "cash value" in our investment in the activity — there is too little to be gained and too much to lose. Experiences of failure in learning mathematics and the attendant loss of self-esteem may result in our withdrawing from further engagement in mathematics — and, potentially, from other challenging situations as well. Thus, feelings generated by experiences of mathematics can be an important determinant of the kind of person one becomes.

As we analyse motivation in terms of self-esteem, we find that those who expect that they can deal with the challenges they are offered — in other words, those with high self-efficacy (Bandura, 1982) — often enjoy rising to those challenges. As Bandura notes, "People avoid activities that they believe exceed their coping capabilities, but they undertake and perform assuredly those that they judge themselves capable of managing" (p. 123). Some consider mathematics a desirable pursuit precisely because it is challenging. Attraction to the formidable characteristics of mathematics also implies a connection with the explanatory power of mathematics discussed below. Some individuals evince pride in their accomplishments, in their making sense of mathematical subtleties and in their use of mathematical complexities in applications. Mathematical activity therefore contributes to their self-esteem. Conversely, there are others who find mathematical demands tedious or distressing, contradicting their expectations about their abilities or confirming their worst fears. As a result, they shun contact with mathematics.

An analysis of motivation in terms of social drive shows that learners commonly attribute some part of their success in mathematics to the support or example of parents and other relatives, friends, teachers, and colleagues. To greater or lesser extents, many learners view these influences as significant in their development; often, a particular teacher will be associated with the rewarding learning experiences. However, the same sources can produce the opposite effect. Repeated failure to meet teachers' expectations, especially when combined with public humiliation, can destroy learners' confidence in their abilities. What is more, labels such as "remedial" may perpetuate teachers' negative perceptions of a child's abilities from year to year. This in turn may convince the child of his or her inability to succeed at mathematics, precluding any redress of past failures for the duration of grade school.

The social dynamics of the (mathematics) classroom are related to the tensions between the institutional power of the teacher and the intended empowerment of the student. A student who needs to work autonomously in order to feel empowered by mathematics will likely benefit from a teacher's mellow approach to classroom management. A student who prefers instructional guidance to detailed instructions may not respond well to a heavy-handed approach. Technology in the classroom can also have an impact on social dynamics. For example, the use of computers may vitiate social interaction and isolate students or, as has been found, it may encourage a sharing of techniques and discoveries. These effects may undermine competency and confidence for some while facilitating learning for others.

Self-esteem and social drive are complex factors which function in many ways both as motivators and anti-motivators.

Aesthetics and Mathematics.

A learner's aesthetic needs are, for Maslow (1970), penultimate in the process of self-realisation; they may coincide with learning experiences in mathematics at several points. Cumulatively, these events can support and strengthen whatever inclination an individual has towards learning mathematics. Similarly, cumulative conflicts between aesthetic sensibilities and experiences in mathematics can discourage a learner. Thus, the aesthetics of "doing mathematics" can have a substantial influence on the learner's motivation.

Schooling can focus a learner's attention on mathematics in an abstract academic fashion (as opposed to an everyday one). Some students may feel freed by this abstract approach. They are satisfied by their participation in creative acts, appreciating the order and beauty, the elegance, and the sheer pleasure of "doing mathematics." On the other hand, this emphasis on the abstract and impersonal can lead some students to feel that mathematics is disconnected from "life" — that it is meaningless, insubstantial, and impractical. Therefore, there are motivating aesthetics as well as anti-motivating aesthetics in learning mathematics.

Curiosity and Playfulness.

Curiosity and playfulness are vital factors in learning. Through these, human beings explore and learn to control their environment. White (1959, pp. 317-318) argues that the intrinsic need to deal with the environment is a primary motivator. He proposes "competence" as a motivational concept, encompassing all behaviors that promote an effective interaction with the environment. Play is clearly a prime vehicle for a child's exploration of his/her world. Seligman (1995, pp. 281-282), using the term "mastery" to denote the behavioral control of outcomes, speaks of the profound motivational potential of exploration as an aspect of play: exploration creates a "natural biofeedback loop," in which a child's exploration of his/her environment leads to success, which leads to a greater sense of mastery and more exploration.

There may be a connection between being playful with mathematical ideas and being motivated to learn mathematics. Perhaps the former can encourage the latter, whether or not the learner has a prior inclination towards mathematical activity. However, a student who has a propensity for abstraction or analysis is likely to develop an affinity for mathematical play. This propensity can also be stifled (for example: "Quit playing and get to work!"). Several "deficiency" needs (Maslow, 1970) are involved in these interactions: certain levels of comfort and security are necessary for play to be enjoyable; self-esteem and interpersonal social factors may also be involved (see earlier discussion of mastery under Self-esteem and Social Drive).

At various times in the history of education, educators such as Pestalozzi, Montessori, Dewey, and Bruner have proposed that children make sense of the world by reaching out to it. Curiosity can be thought of as guiding a child's explorations and development of a world-view, and pedagogies based on that notion treat this craving as a resource. In order that curiosity be encouraged, it would seem desirable that teachers provide non-threatening opportunities for such explorations. There are many ways of stimulating a child's imagination and creating a "need" to know, but it is just as easily possible to create fear of failure by having too much depend on results.

Events, circumstances, or conditions can contrive to mystify mathematics for a developing child, making the exploration of mathematics an unpleasant experience that leads to avoidance. As a student's understanding declines, s/he may reject mathematical activity and treat it, and especially those exercises relying on curiosity, with contempt. On the other hand, individuals who have learned mathematics through a series of welcome discoveries can come to be motivated by their appreciation of its complex nature. Clearly, context will determine whether or not curiosity and playfulness is motivating or anti-motivating.

Explanatory Power of Mathematics.

It is common to speak of the power of mathematics: its power to deliver an answer and its power to penetrate the mysteries of physical and abstract phenomena and deliver meaningful information. It is their own knowledge and understanding of mathematics that these people are praising, of course, and Maslow (1970) would suggest that they are therefore expressing their need for such knowledge and understanding.

One of the most impressive examples of the power of mathematics is the use of mathematical systems to represent physical situations, in which theoretical modelling can predict the results that can be expected in scientific investigations. Differential and integral calculus explain the constantly changing states of nature. Statistical analysis can demonstrate overall regularity in random events. Observing the co-ordination of information through combinations of mathematical tools, even the most basic ones, can be a powerfully motivating experience. On the other hand, exposure to what purports to be a means for decoding all of the universe's secrets can be intimidating and discouraging if the mathematical explanations are incomprehensible.

Modern mathematics developed from the logic of Euclidean methodology and the symbolism of Arabic science. When students refer to mathematics as being "rule-based," they are perceiving its structure — the relationships and properties that define its consistency. Reasoned arguments justify results; methodological proofs document algorithmic calculations. Mystery is removed from numerical procedures by analysing their mathematical foundations. There is an organising quality to mathematics that can demystify seemingly mystical results. But some teachers are less capable than others of conveying this sense of order and harmony, and some students are unable to grasp the logic of mathematics in the form that the teacher presents it. Thus, again, the power of mathematics can be a motivator or an anti-motivator.

Necessity of Mathematics.

Everyone has a fundamental physiological need for survival (Maslow, 1970) and among those pursuits which ensure survival mathematics has a privileged place. Although few would claim that mathematics is unnecessary — or worthless — there are a variety of perspectives on its necessity. Those views are often the peremptory reasons for learning and achieving in school mathematics. For example, mathematical accreditation may be an inescapable prerequisite for further education and employment. Because of this and other factors, grades and qualifications drive many students' performances, as noted in chapter one. We here distinguish the belief in the necessity of mathematics from the notion that knowledge of it may be useful in the future. This is an explicit distinction in this model.

School systems generally require mathematical performance until school leaving age, and thus mathematics is essential to graduation. However, some students are not convinced that mathematics is otherwise essential. While many quickly acknowledge that learning mathematics is worthwhile given the applications that arise in everyday activities, this sometimes means that for them mathematics is a tool, required for work at school and possibly later, but not a significant part of everyday life. The easy availability of cheap, hand-held calculators has made it possible for them to carry out almost all of the arithmetic of daily life without having to know how to perform calculations.

Those who think of mathematics as essential are not always those who think of it as worthwhile. Both, however, conceive of it as central to much scientific problem-solving. This sense of the relevance of mathematics does not necessarily translate into performance, but it still falls within the category of motivators. Correspondingly for anti-motivators: if the learner views mathematics as irrelevant to his or her objectives, s/he will likely be disinclined to learn it. Such differences in beliefs about the relevance of mathematics can shape the way individuals' perspectives develop. Negative beliefs about mathematics may transform into negative attitudes towards applications perceived to be related to mathematics; as, for instance, when a student avoids science courses because of a persuasion that mathematics is "irrelevant." In summary, necessity can be a motivator or an anti-motivator.

Usefulness of Mathematics.

Related to physiological survival but emphasising the need for useful applications of knowledge and understanding, mathematics has a history of success. Almost everyone acknowledges that mathematics can be useful. Up to a point in their schooling, around grade 7 or 8, most of what they have learned about mathematics can be directly applied to situations encountered daily. Logical thinking and mathematics are synonymous and desirable for those whose skills develop analytically within the natural sciences. On the other hand, those who rely on their creative, artistic, or instinctual insights sometimes find the restrictions of mathematical thinking to be a hindrance. In this sense, the mathematical practicality that is motivating for some is anti-motivating for others.

Problem-solving is that quality of mathematical activity most often considered pre-eminent. Games and puzzles attract people indiscriminately, without regard to their academic preferences, and the experience of logically working through alternatives brings to their minds the processes involved in "doing mathematics." They will attribute their ability to manipulate data and reasoning to their training in mathematical thinking: an orderly approach, a practical outlook, and a systematic use of information. As a result, they may be motivated to learn more mathematical techniques.

It is undeniable that mathematics can be an important component of our lives, however we may feel about the way that function operates in our lives. Certainly, it is a tool, useful to some, irrelevant to others. Most would agree that mathematics is used in many other activities and that, after school has become a memory, there are still occasions when old algorithmic practices must be resurrected. And those who feel that they have no need of school mathematics after their school days are over may be unaware of their use of it. It is therefore the perception of the usefulness of mathematics that can determine whether or not someone becomes motivated.

Psychological Manipulation.

Mathematics educators know that they do not always persuade students to study mathematics by honest means; whether or not it is recognised by their students, manipulation is sometimes used. Any of the needs discussed above can become a source of manipulation. Parents, teachers, and other dispensers of authoritative guidance may, independently of the facts about what the needs for mathematics are, manufacture propaganda as an expedient means to encourage performance. Whether that propaganda consists of half-truths about the benefits of a mathematics course or claims about the dire consequences of failing it, the tactic has been convincing to some students who see their qualifications in mathematics as vital to their futures. However, deceit can provoke the opposite of what it intended; instead of acquiring a constructive view of mathematics, students may come to associate it with misleading advertising.

A common method of promoting mathematics is by linking it to the computer industry — the "information economy." In fact, it is usually not necessary to have much understanding of mathematics to be able to work with a computer or use sophisticated software. The reality is quite often the opposite, since the intent of many technological advances has been to have computer processing replace slower and less accurate human activity. However, for the study of mathematics, this piece of propaganda is fairly often successful and unquestioned. It supports the traditional view of mathematics as a fundamental building block of knowledge.

Psychological manipulation, therefore, is a double-edged sword which can work either as a motivator or as an anti-motivator.

Supplementary notes on tacit elements of the model

At first glance a number of antecedent influences might appear to have been omitted from the model in Figure 1, such as parents, teachers, peers, and social milieu. However, each of these sources of motivation and anti-motivation is a factor in at least one category of the model. For example, parents and teachers have important roles. They are not identified with any category in particular, but they can be important elements of every category. As mentioned above, a teacher's influence is most apparent in terms of Mode of instruction. But a teacher is capable of being a channel for the effects of all other motivators and anti-motivators. A parent, though less involved in the instruction of mathematics than a teacher, can also be a channel for the effects of all other motivators and anti-motivators. Other influences that are external to the student may be subsumed under the defined categories by similar statements.

Whether or not an influence is motivating or anti-motivating depends on the learner's perception of it and within the parameters of Psychological Manipulation the learner is "falsely influenced." It may or may not be that s/he attributes motivation to factors intended by the manipulation. More important, the learner is not likely to describe "psychological manipulation" as a motivating influence; s/he is not likely to be both motivated and aware of being manipulated to be motivated. Necessarily, therefore, this category can be related to most of the others.

Nominal definitions

The points of view that emerged towards the data during the initial stages of systematic abduction suggested the following definitions:

Mode of Instruction is the result of a teacher's choices in presenting and organising content: mathematical content for students to understand and learn; and editorial content to draw students' attentions. Bruner (1962) distinguishes between "expository" and "hypothetical" teaching methods as promoting, respectively, passive and active learning; he refers to the hypothetical mode as that which "characterises the teaching that encourages discovery" (p. 83).

Self-esteem is determined by an individual's self-concept as a result of the way others express their opinions of the individual, or despite those events, or as a dialectical synthesis of internal and external responses. In particular, Woolfolk (1993, p.76) observes that mathematics is one of the three major divisions of a student's self-concept. And as Ball (1982) says, "It has long been a cliché of educational theory that a student's behaviour is a function of the student's self-concept" (p. 1259).

Social drive covers all active relationships with family, friends, acquaintances, peers, colleagues and others with existential significance in one's life. During schooling parental influences are of primary importance, for better or worse, along with those of teachers and other authority figures. Self-image is closely connected to social drive through the effects of family dynamics, friendships, intimate relationships, and relative isolation.

Aesthetics refers to an individual's beliefs in and loyalties to ideal patterns and rhythms in his or her world. If an aesthetical congruence exists between the learner and the mathematics s/he is learning, then that will also facilitate learning. A level of comfort with the material seems to be fundamental to the experience.

Curiosity and playfulness encompass those modes of activity which allow speculation and discovery with the least commitment to one's ability to apply what is learned. Both can function as catalysts for exploring environments and ideas. As educators, curiosity is "one of the oldest [motivational constructs] in our repertoire" (Ball, 1982, p. 1259). Curiosity and playfulness allow spontaneous activity to be productively directed to mathematical learning.

Explanatory power can be described in terms of "unreasonable effectiveness" — mathematics' inherent capacity to organise, categorise, reconstruct, and simplify our worldly experiences: "Perhaps it is because we are so often impotent that we take pleasure in once in a while being able to control something that precisely" (MacPherson, 1991, p. 53). Explanatory power and control are features not only of mathematics itself, but also characterise its many applications.

Necessity refers to those conditions which the learner believes must be fulfilled without question. These may include the stipulations for advancement in school and the requirements for admission to university or for the attainment of career goals. In sum, necessity is the final argument for perseverance in mathematical studies.

Usefulness refers to the practical value of studying mathematics, its relevance to daily life as well as to sciences and business. It is not skill with symbols and calculations per se that determines usefulness, but the way that these skills can be applied. With every learner, motivation depends on the perception of the extent of the potential usefulness of mathematics.

Psychological manipulation may be as blatant as lying about the facts, or as covert as subtle misrepresentation. It is a duplicitous act, however well-meant, but in education it may sometimes be a necessary evil.

Attribution, achievement, and performance

Weiner's (1986) attributional theory of motivation expands on the causal relations of Rotter's (1954) external or internal "locus of control" and includes the stability and controllability of factors. For the student, a teacher's use of materials and class participation can vary stability, controllability, and locus of control, altering student perspectives or attributions, and thus influencing motivation to learn mathematics. In other words, the student experiences a teacher's mode of instruction as a stable, uncontrollable, externally situated factor in learning, which can be motivating or anti-motivating.

Ball (1982, p. 1258) reports that studies of achievement motivation, which have a history of almost fifty years, include factors such as the motive to succeed, the motive to avoid failure, the perceived probability of success, and the incentive value of success. Each of these factors may be considered in terms of self-esteem or social drive. He also points out that Weiner's (1972) explanation in terms of attribution theory is well supported by research. Both views of achievement motivation offer advice to the practitioner, the first in terms of needs and the second in terms of causal attributions.

Bandura's (1986) social cognitive theory of learning is a departure from the traditional behaviourist view since it rejects a direct causal connection between learning and performance. Bandura considers learning factors that are both external and internal to the individual as interactional and describes the process as "reciprocal determinism"

(Woolfolk, 1993, p. 220). A key component of Bandura's theories is the concept of self-efficacy — a person's sense of his or her ability to perform particular tasks. This is a causal relation: "Bandura's self-efficacy concept implies that educational and child-rearing methods highlighting the person's abilities or successes will lead to greater success than methods highlighting inabilities or failures" (Gray, 1994, p. 589). Clearly, self-efficacy relies on self-confidence and self-esteem.

The blending of external and internal factors indicate some commonalities between Bandura's and Weiner's theories. It is of passing interest here that current work on extrinsic and intrinsic motivation has also rejected extreme polar views: "Most observers of human behaviour ... would agree that both external and internal factors influence individuals' motivation and learning" (Hidi & Harackiewicz, 2000, p. 159).

Relationships creating meaningful interconnections

After carrying the proposed model across the data from the pilot study and the contexts identified in chapter three, it seemed that the categories of the proposed model are well-defined, complete, consistent, and comprehensive. It is also evident that the links between the categories are "soft." In some models, it is possible to identify necessary linkages between categories, even when there may be alternative routes through the model. Such is not the case with the present model. In this model, it is possible to suggest how virtually every category can be linked to every other one, and none of those linkages seem imperative. Nevertheless, some linkages turned up significantly more often than others in the assembly of natural language, the pilot study, and the literature. The stronger ones are shown in Figure 1 above.

Teachers most likely have the greatest influence through their modes of instruction in mathematics lessons, but teaching has social elements that may well be significant in learning. As role-models and authority-figures, teachers may have effects which are antecedent to the appreciation of beauty or usefulness in mathematics, or indeed of any of the qualities that motivate learning. In other words, the effect of the teacher in the model is a major factor, both directly and indirectly.

The student who is driven to "get good marks" in mathematics may be responding to social (and/or internal) pressures to meet performance demands, or to a convincing (and perhaps manipulative) argument that mathematics is an inevitable sine qua non, or to the necessity of mathematical qualifications (and knowledge) for employment or admission to further education. It is likely that all three motivators may be operative in various degrees. In all three cases the study of mathematics is a by-product of concerns other than the mathematics itself; the third motivator seems to be of greatest interest to practitioners. Nevertheless, the influences of all three motivators cannot be ignored and we must acknowledge their potential linkages.

Similarly, mathematical usefulness suggests mathematical necessity, and both are often found as motivators in a single individual. As a consequence, students working through the mathematics of practical applications can find that the explanatory power of mathematics also stimulates their liking for the subject. The combined influence of motivators can thus be a significant determinant in learning mathematics.

From the reverse point of view, however, it is not always possible to separate the influences as distinct motivators (or anti-motivators), even when the influences themselves appear to be distinct. "Self-esteem" and "social drive" may well be relatively distinct characteristics of personality, but as motivators they are difficult to disentangle. As was seen in chapter two in referring to intrinsic and extrinsic motivation, internal and external influences may seem to overlap, and some approaches even interchange them.

Figure 1, then, does not imply any strong linkages between categories. To this point they are best seen as potential relationships open to further research, and will quite possibly be found to be idiosyncratic to individual learners. However, the list of connections below completes the process of systematic abduction. Each motivator is matched with others as they arose in the natural language and in the pilot interviews.

1. Mode of instruction: self-esteem, social drive, aesthetics.
2. Aesthetics: mode of instruction, psychological manipulation, curiosity.
3. Self-esteem: curiosity and playfulness, mode of instruction, social drive.
4. Social drive: self-esteem, mode of instruction, necessity, curiosity and playfulness.
5. Curiosity and playfulness: social drive, self-esteem, explanatory power, aesthetics.
6. Explanatory power: curiosity and playfulness, necessity, usefulness.
7. Usefulness: explanatory power, psychological manipulation, necessity.
8. Necessity: usefulness, social drive, explanatory power, psychological manipulation.
9. Psychological manipulation: necessity, usefulness, aesthetics.

Testing the model

Of course, as observed in chapter one, the potential usefulness of a model of this kind cannot be determined by the above kind of subjective judgement alone. It must be demonstrated in less potentially prejudiced ways. This model, then, was submitted to evaluation in terms of the following three questions:

1. Can it be used, reasonably unequivocally, to make sense of "street" language about motivation and anti-motivation in mathematics?

2. Can it comfortably make sense of the various informal "definitions" of motivation found in the literature of chapters two and three?

3. Can it be used comfortably by non-mathematical experienced researchers to discuss how motivation and anti-motivation might be incorporated or controlled in research studies?

Those questions and their answers are addressed in chapter five.

Chapter 5: Testing the model

The three criteria that this or any model must satisfy were given in chapter one.

1. Can it make sense of the natural language of non-specialists who are asked to discuss their own and others' motivation and non-motivation to learn mathematics?

2. Can it make sense of what is said and found in both direct and implicit references to motivation in the literature?

3. Having been introduced to the model, can experienced researchers from outside mathematics education use the categories and language of the model to discuss possible researchable questions entailing students' motivators or anti-motivators? The reason for selecting researchers from outside mathematics education is given in the analysis below.

In order to consider the first and third criteria, two sets of interviews were conducted. Each set contained five interviewees, solicited individually and personally according to the sample specifications. For the first test criterion the sample from members of the public was partially stratified using such categories as "blue collar" worker, "white collar" worker, and unemployed, with an adequate gender balance. For the third criterion the sample consisted of non-mathematical education researchers. Gender was skewed in this sample, since the participants' non-mathematical orientation was the determining specification factor in their selection.

Each interview lasted approximately 45 minutes for members of the first set and approximately 90 minutes for members of the second set. Protocols for the interviews are given in Appendix C and Appendix D respectively. Members of the public were encouraged to elaborate on their accounts as in the pilot interviews, while academic researchers were offered a copy of the interview protocol as a guide to the discussion. A categorised set of comments from interviews with members of the public in both the pilot study and this study is given in Appendix E.

Analysis of natural language

The first tentative set of categories was inferred from an assembly of street language. These categories were subsequently modified and extended as the results of the pilot study were analysed, in accordance with Willer's description of the abduction process. This style of analysis follows the tradition of interpretive research in consonance with Wilhelm Dilthey's "aim of discovering and communicating the meaning-perspectives of the people studied" (Erickson, 1986, p. 123), since:

> [t]hese are questions of basic significance in the study of pedagogy. They put mind [italics in original] back in the picture, in the central place it now occupies in cognitive psychology. The mental life of teachers and learners has again become crucially significant for the study of teaching (Shulman, 1981, and Shulman, 1986), and from an interpretive point of view mind is present not merely as a set of "mediating variables" between the major independent and dependent variables of teaching — the inputs and outputs. Sense-making is the heart of the matter, the medium of teaching and learning that is also the message. (p. 127)

Members of the public expressed themselves in the interviews according to how they perceived mathematics in school. Their most common observations about motivators and anti-motivators are summarised below.

Self-esteem.

Some of the perceived motivators seem to have been seeded in childhood and nurtured by school experiences. These could most often be referred to the twinned categories of Self-esteem and Social Drive in the model. Self-esteem was clearly seen as a major factor; it was aroused when encouraged and ignited when challenged. Here, the "cash value" to the subjects was praise, self-justification, or both.

Usefulness.

The usefulness of mathematics in dealing with money, physical measurements, and timekeeping all motivated subjects to achieve a minimum level of competency. An interest in technology stimulated an interest in the requisite mathematics, although this did not necessarily translate into the pursuit of further mathematics. These examples of the impetus for learning mathematics are addressed by the category Usefulness of Mathematics in the model.

Necessity.

For two participants, learning mathematics was seen to be a necessity for their chosen school programs. They did not question the requirement at the time and they could not say how much mathematics they would otherwise have decided to study. It was clear from their comments that the achievement of acceptable grades signified the fulfillment of this need; however, both saw their motivation to do mathematics as deriving from a more primary objective. As the Necessity of Mathematics category of the model represents it, the peremptory reasons for learning mathematics are sometimes sufficient motivation at school.

Mode of instruction.

Three participants cited both their classroom environments and the way teachers presented mathematical material as impediments to motivation to learn mathematics. Their examples of memorable anti-motivators included a class size too large for all questions, a teacher's impatience and inadequate explanations, and teachers' peremptory demands. Such examples of institutional inadequacies and ill-humored interactions clearly belong with the model's category of Mode of Instruction. It is curious that those discouraged by a teacher's mode of instruction tended to mention it, while mode of instruction was largely transparent or considered of minor importance where it served as encouragement.

Aesthetics.

Mathematics as a means of calculation struck some participants as unexciting, boring, confusing, over-complicated, and unintelligible: "a different language," as one interviewee put it. Two participants referred to an academic interest in the mathematics learned at school, which seemed to be attributable to the aesthetics of the subject. The others rejected academic pursuits as a whole, in particular those that seemed to them useless — particularly "abstract," "pointless," and "meaningless" exercises in mathematics. When specific attributes of style or content could not be named, these anti-motivated participants lamented a general negative sense of the experience of studying mathematics. These conflicts of sensibilities and their consequent erosions of mathematical interest may well be related to the category Aesthetics and Mathematics.

Distribution of motivators and anti-motivators in natural language.

The phenomenon that practical and everyday mathematics was a motivator for some but not for others is related to the Usefulness of Mathematics. Those in this sample who felt that Mode of Instruction, Necessity of Mathematics, Self-esteem, and Social Drive had been hindrances or deterrents to their learning of mathematics and viewed such unpalatable experiences as anti-motivating influences, seemed to outnumber those who felt otherwise. But two interviewees reported that the elegance and simplicity of mathematical formulae and calculations were substantial motivators. Some participants in the pilot study found the Aesthetics of mathematics motivating to about the same extent as others found it anti-motivating.

Overall, these interviewees addressed all of the categories of the model and provided no comments that could not be accounted for within it. References to the scope, impact, and variety of mathematics were addressed to the Explanatory Power category. Predictably the intellectual appeal of mathematics drew the curious-minded under Curiosity and Playfulness. In each case the influences described were positive. The only mention of a peremptory influence came from one student who spoke of being forced to learn mathematics — and it was distinctly negative. This example may serve to illustrate the anti-motivating side of the Necessity of Mathematics.

It cannot be considered remarkable that the model is able to account for all of the references to motivation in the natural language and in the pilot study. The model was refined and extended as that information and the results of interviews were analysed, evolving in response to them; naturally, therefore, it grew to accommodate both.

Analysis of references in the literature

For the same reason cited above, it is to be expected that the model can accommodate the diverse references to motivation found in the literature. Using the language of the model, it was possible to restate the opinions expressed and the conclusions reached in those studies in a consistent way and to help resolve the ambiguities found therein. As suggested in chapter two (Ball, 1982) a complete review of the literature is forbidding. Nevertheless, the following examples cogently demonstrate the model's range.

Middleton's (1995) triad of interests, arousal, and control can be associated with Aesthetics and Mathematics as well as being indicators of Self-esteem. Middleton's tacit identification of interest with intrinsic motivation is understandable here, since both would be consistently interpreted within those categories of the model. As Deci (1992) points out, "… it is hard to talk about 'intrinsically motivated' activity without describing people being interested in the activity" (p. 49).

Kulm's (1980) and Hart's (1989) definitions of attitude in terms of "predispositions" are easily identified with Aesthetics and Mathematics, Curiosity and Playfulness, and Usefulness of Mathematics. Each of these categories would find a place for each of the three components of attitude cited by Hart. The same categories, along with Self-Esteem and Social Drive, would accommodate McLeod's (1992) view of an emotionally based origin for attitudes. Placed in this context, there is no need to determine attitudes or assess emotional factors and the formerly unresolved perspectives are reconciled.

The problems with the extrinsic/intrinsic dichotomy of motivation and sphere-of-origin are not at issue so far as as this model is concerned, nor is the putative duality of cognitive and affective impulses, the complications of componential analysis or the distinctions between beliefs, emotions, and attitudes.

That motivation may be external or internal is not relevant to the structure of this model, since goal-based reasoning can be viewed either way and the model accommodates either view. Decision-based reasoning is taken to entail an interplay of motivators and anti-motivators; and meaning as a source of motivation is individualised for the learner within the categories of the model. Attitudes, emotions, and beliefs are accommodated without the necessity of explaining their natures. Any or all of those constructs are certainly open to further study. Moreover, since this model accommodates all of them, it is likely that its categories could be used to clarify such further research and discussion. But at this point the model need not address them directly.

Concluding analytical test

The final set of interviews was conducted after the model was established, and may therefore be considered a critical test of the usefulness of the model.

Several mathematics educators had participated in early discussions as the model was being framed and developed. Since these were more likely than other scholars to interpret the model in terms specific to their own discipline; and since the model, to be generally useful, must be understandable and applicable by persons without that specialised background, it was decided to conduct the concluding test of the model with well-qualified scholars who were not specialists in mathematics education.

None of the academic researchers interviewed were specialists in mathematics or mathematics education. They were chosen for their methodological and analytical proficiencies in education research. This choice did raise the risk that these participants might introduce some broader questions of education not relevant to this study. A few consequences of having taken that risk can be found in the following summary, particularly in the first interview.

Why must all students learn mathematics?

The first participant made points of contention of some facets of his personal stance that were peripheral to the orientation of this study. He took issue with the status of mathematics (which he referred to as the "New Latin") in the school curriculum and disputed the assumptions which he inferred lay behind the model. Discussions about motivation in mathematics were recast as polemics against using school mathematics as a measure of ability and worth. Nevertheless, the interview was highly beneficial in demonstrating the need to clarify the nature of this study for further interviewees. In the remaining interviews with academic researchers the introductory explanation included such clarification and led to discussions that focused more clearly on the role and acquisition of motivation in school mathematics.

Two other participants also raised, in a minor way, the question of mathematics as an appropriate choice in the school curriculum. They were less concerned with mathematics than with its influences on learners' decision-making. They argued that for learners to whom achievement comes easily and for whom mathematics is only one of many options, decisions about whether or not to study mathematics may depend in part on who is influencing them. One participant observed that for the less capable or the less self-confident, mathematics may be seen as an option for bright learners, but not for themselves. Another, apparently either unaware of or not convinced by recent attempts to make mathematics attractive to females, suggested that gender-discrimination in mathematics in favour of males could lead to the alienation of female learners, discouraging them from further contact with the subject.

Using the model, participants easily placed issues raised within one or more categories and quickly determined which would be the most pertinent. Self-esteem and Social Drive were prominent in many suggested researchable questions and were believed to be cogent factors in underachievement and gender issues. It was apparent that the participants were comfortable in applying the model to their suggested research questions.

Appraising the model.

Each interview closed with a discussion of the part the model could play in discussing motivation research in school mathematics. Consistently, the model helped define a broad view of the reasons for the choices made at school. Those discussions were commonly grounded, especially for those academics whose family members' experiences were a source for thoughts on suitable research questions. Potential relationships between the categories of the model were ignored by all the participants.

One interviewee proposed cultural comparisons as a possible source of research, suggesting an ethnographic perspective on motivational categories for learning mathematics. He suggested as a possible avenue of inquiry an investigation into whether or not mathematics phobia is greater among Canadians than among Japanese or Taiwanese. It was finally agreed, however, that such a question would not likely call for anything beyond the categories of the model, such as Mode of Instruction or Self-esteem and Social Drive.

Participants were invited to suggest categories that they thought might be added to the model, but it was found that all the suggestions could readily be subsumed within the existing model. One interviewee suggested Time On Task as a possible candidate, but it was finally excluded. Mention of this topic as a motivator or anti-motivator did not occur in the interviews with members of the public, nor does the literature mention it.

Conclusions and recommendations

Overall, then, the model seems to satisfy the criteria that it accommodate natural language and prior literature, as well as demonstrate its general utility and clarity. This foundation can enable ensuing studies to make a greater contribution to knowledge about mathematics motivation. It is important that those who use the model continually monitor its ability to account for research studies and facilitate discourse about motivation.

In particular, Ball's (1982) fourth point cited in chapter two must be given close consideration: "… it is impossible to explore all the vast literature or to achieve a full synthesis of motivation-related concepts." It is therefore unlikely that any theory of motivation can provide a complete integration of topics. Nevertheless, theoretical models must conform to the principles of scientific theory, whose purpose is "to explain, predict, and guide new research" (Willer & Willer, 1973, p. 3). The proposed model is intended to serve as a clarifying agent in discourse and scientific investigations of the relationships between motivation and learning mathematics.

The clear goal of this study is to clarify the language of discourse for commentary and research on motivation in mathematics education. That goal can be fulfilled by the use of this model in at least two directions. First, where reports and commentaries on motivation originate in diverse areas, using contextual languages and theories, they will continue to be in need of clarification. Researchers and commentators will be able to consider them under the aegis of this model so as to resolve any apparent dissonance.

Second, the model will be of direct value to classroom teachers since it accommodates most past and current motivation studies and commentary. Familiarity with the model will encourage teachers to consider the motivational aspects of their classroom practices. By examining the full range of motivators and anti-motivators, their attention will likely be called to some of which they were not previously aware.

Appendix A

Interview protocol for pilot study

Primary questions:	Why did you drop/choose mathematics?
	What do you think of mathematics?
	How do you feel about learning mathematics?
	When do you use mathematics?
Prompting questions:	Is there one person who you would associate most with your learning of mathematics?
	What image comes to mind when you hear the word "mathematics"?
	Is there any other subject you like more/less than mathematics?

Appendix B

A Set of Representative Reactions to the Pilot Study Interview

Positive	Negative
Why Take Mathematics?	Why Drop Mathematics?
Needed in the present or future • required knowledge • qualification for further education • useful for other activities (sport, music)	Not found to be useful • impractical, insubstantial • pointless, meaningless • irrelevant to activities
Made sense • logical, rule-based • strong in subject • enjoyed	Difficult • declining understanding • no 3-D thought • hated
Good experiences & associations • abstract thought • successfully studied • felt challenged • subject least dependent on language • successful at high school contests • facility with the subject	Bad experiences & associations • too abstract • no confidence • no autonomy • too scientific • compulsory at high school • anxiety ("fog"); labelled remedial
When Did Mathematics First Become Desirable? • grades 7 & 8 • grade 9 gifted & talented program • grade 12 & university • during 15^{th} & 16th years of teaching elementary • by tutoring mathematics	When Did Mathematics First Become Detestable? • grade 7 • grade 10 • grade 11 • grade 12 • on reaction to the New Math pedagogy • on acquiring mechanics without concepts

Appendix B continued

A Set of Representative Reactions to the Pilot Study Interview

Positive	Negative
Who Most Influenced Towards Mathematics? • parents in grade 1 or 2 • grade 7 teacher • grade 9 teacher (promoting Pascal tests) • grade 11 & 12 teacher (calculus), as well as mother & father • grade 12 teacher • a friend who also was a mathematics teacher • teaching grades 1 to 3 & looking for innovative material	Who Most Influenced Against Mathematics? • parents • older sister • grade 7 teacher • grade 8 teacher & grade 11 teacher • high school teacher
Positive Feelings About Learning Mathematics • discovery, openness, willing • worthwhile • an art, a creative act • essential, relevant, a language • a means to empowerment (technology) • responsibility to requite a child's interest • democratic classroom; community	Negative Feelings About Learning Mathematics • imposed • only as needed as a tool • resenting non-practical basis • bewildering • a means for disempowerment (technology) • disconnected from humanities • competitive
How Was A Positive Attitude Acquired? • ability to memorise patterns • taking a B.Ed. course in elementary methods • the mathematics of the magic results	How Was A Negative Attitude Acquired? • feeling like a "runt" in grade 3 • being humiliated in grade 4 • getting only limited teacher response

Appendix B continued

A Set of Representative Reactions to the Pilot Study Interview

Positive	Negative
Mathematics is... • logical, organising, sensemaking, orderly • beautiful, elegant, pleasurable • curiosity • a problem-solving way of thinking • useful, practical • practice, a tool • a challenge, a game • fascinating, satisfying • an answer to everything • accomplishment, pride, ability • a function of our lives, connected, interactive, musical (emotional spectrum)	Mathematics is... • memorisation • intrinsically uninteresting • fear • mechanical • useless, irrelevant • abstract, impersonal • painful • functionally unempowering • right-wrong

Appendix C

Test interview protocol for members of the general public

1. Explain that the study is concerned with a person's motivation for or against learning mathematics, without elaborating on this initial statement.

2. Ask the following questions, supplemented as necessary to elicit and clarify responses for later comparisons with the model:

   (a) What do you think about being motivated in mathematics?

   (b) What has motivated you (or someone you know) to learn or do mathematics?

   (c) What has turned you off mathematics?

3. Allow the natural language of the interviewee to dominate the conversation. The terms used to describe motivators and anti-motivators should determine the responding language of the interviewer.

Appendix D

Test interview protocol for academic researchers

1. Explain the terms motivator and anti-motivator, i.e. that they are influences on learners' desires to know, or to avoid, mathematics. Provide generic examples, such as the charismatic teacher whose enthusiasm is a stimulant, or the authoritative teacher whose intimidation is a deterrent. Supply a sufficient variety of examples for an appreciation of the scope of the terms.

2. Examine briefly the diagram of the model, pointing out that the categories of motivators and anti-motivators are related to each other only in the most general way, and that these relationships may vary with individual learners.

3. Propose general questions for consideration of how much each of the categories provides motivation towards or away from mathematics:
   - mode of instruction — what is the vehicle for understanding the performance of mathematics? (continuum: language...physical)
   - self-esteem and social drive — what part is played by sense of self and its relationship to others?
   - aesthetics and mathematics — how does mathematics fit into one's sense of balance in the world?
   - curiosity and playfulness — to what extent is mathematics an invitation to experiment?
   - explanatory power — how much does mathematics make sense of the world?
   - necessity — what are the peremptory reasons for studying mathematics?
   - usefulness — is mathematics a pragmatic fact in one's life?
   - psychological manipulation — a convincing argument for the study of mathematics?

4. Ask for suggested hypotheses about motivation in mathematics that the researcher would want to test using the model. For example, suppose a First Nations student after a lifetime of living on a reserve wants to be educated in the city's secondary education system. Focusing on mathematics education in particular, which of the above factors will be present and which will be absent? Alternatively, there may be hypotheses that require motivation factors to be controlled while other studies are performed.

5. Using the model, discuss each hypothesis until understandings of its parameters have been achieved that are satisfactory and mutual to both interviewer and interviewee.

Appendix E

Categorised samples of comments from members of the public

Mode of instruction: "Motivation is … the ability to be able to understand something complex, and having someone who can teach it in a way I can understand it."

Self-esteem: "And I remember being shamed by my grade four teacher, and made fun of in class because I stayed home, because my mum didn't seem to think that attendance and punctuality meant much."

Self-esteem: "Math is not something that comes easily to me, so I feel better about being able to do it than other things."

Self-esteem: "It's almost like a fog descends, descending over me. And it probably dates back to an experience in my junior high — grade seven — when I did very poorly on an assessment test …"

Social drive: "Overall the math teacher I had was mean and hard to cope with …"

Aesthetics: "Mathematics — it was beautiful; it was elegant; it was — mind working on pure mind. Something about that, something about that I always liked — you know, in the areas of tests that had to do with abstract reasoning, I always scored very highly, and I enjoyed those things; I enjoyed just the mental exercise."

Aesthetics: "I don't see excitement from numbers — no meaning. It's just numbers and manipulating numbers."

Curiosity and playfulness: "… at the same time some of the things presented were interesting; it tied into my natural curiosity, made me want to solve problems …"

Explanatory power: "Something like quantum mechanics has such a great scope and effect, and something like math — that I thought of as so basic — could actually have such a great impact; that would motivate me to learn it, if I had time."

Necessity: "If I want to take physics I have to pass math …"

Usefulness: "In some cases it's the practical use and in others it's fascination …"

Usefulness: "Yeh, a lot of stuff it just seems like there's no — it doesn't have a use, or it won't have a use in our future."

Psychological manipulation: "Everyone says that you can't get a good job without knowing grade 12 math …"

Bibliography

Ames, R. E. & Ames, C. (1984). Introduction to R. E. Ames & C. Ames (Eds.), Research on motivation in education, vol. 1. Orlando, FA: Academic Press.

Ball, S. (1977). Motivation in Education. New York: Academic Press.

Ball, S. (1982). Motivation. In H. E. Mitzel, J. H. Best, W. Rabinowitz (Eds.), Encyclopedia of Educational Research (5th Ed.), New York: Macmillan Publishing Co.

Bandura, A. (1982). Self-efficacy mechanism in human agency. American Psychologist (37).

Bandura, A. (1986). Social foundations of thought and action. Englewood Cliffs, NJ: Prentice-Hall Inc.

Berlyne, D. E. (1963). Motivational problems raised by exploratory and epistemic behaviour. In S. Koch (Ed.), Psychology: A Study of a Science (Vol. 5). New York: McGraw-Hill.

Black, Max (1962). Models and Metaphors: Studies in Language and Philosophy. Ithica: Cornell University Press.

Bruner, Jerome (1962). On Knowing — Essays for the Left Hand. Cambridge, Mass.: Belknap Press of Havard University Press.

Brownell, W. A. (1935). Psychological considerations in the learning and teaching of arithmetic. The teaching of arithmetic, the tenth yearbook of the National Council of Teachers of Mathematics. New York: Teachers College, Columbia University.

Cannon, Walter (1932). The Wisdom of the Body. New York: Norton.

Christou, C. & Philippou, G. (1998). The Develpmental Nature of Ability to Solve One-Step Word Problems. Journal for Research in Mathematics Education 29 (4), pp. 436-442.

Churchman, C. W., Ackoff, R. L., and Arnoff, E. L. (1957). Introduction to Operations Research. New York: John Wiley & Sons, Inc.

Cobb, Paul (1996). Where Is the Mind? A Coordination of Sociocultural and Cognitive Constructivist Perspectives. In Catherine Twoney Fosnot (Ed.), (1996), Constructivism: theory, perspectives, and practice, pp. 34-52. New York: Teachers College, Columbia University. (Original work published 1994)

Cobb, P., Yackel, E., Wood, T. (1989). Young children's emotional acts while engaged in mathematical problem solving. In D. B. McLeod & V. M. Adams (Eds.), Affect and Problem Solving. NY: Springer-Verlag.

Coopersmith, S. (1967). The Antecedents of Self-esteem. San Francisco: Freeman.

Csikszentmihalyi, M. & Nakamura, J. (1989). Dynamics of intrinsic motivation. In R. E. Ames & C. Ames (Eds.), Research on motivation in education, vol. 3. Orlando, FA: Academic Press.

DeCharms, R. (1976). Enhancing Motivation: A Change Project in the Classroom. New York: Wiley.

Deci, E. L. (1992). The Relation of Interest to the Motivation of Behaviour: A Self-Determination Theory Perspective. In K. A. Renninger, S. Hidi, A. Krapp, The Role of Interest in Learning and Development. Hillsdale, NJ: Lawrence Erlbaum Associates.

Dewey, John (1916). Democracy and Education. New York: Collier-Macmillan.

Erickson, Frederick (1986). Qualitative Methods in Research on Teaching. In M. Wittrock, Handbook on Reasearch on Teaching , 3rd Ed, A Project of the American Educational Research Association. New York: MacMillan Publishing Company.

Ernest, Paul (1990). The Philosophy of Mathematics Education. UK: Falmer Press.

Fennema, E. (1989). The study of affect and mathematics: A proposed generic model for research. In D. B. McLeod & V. M. Adams (Eds.), Affect and problem solving. New York: Springer-Verlag.

Fischbein, M. & Ajzen, I. (1975). Belief, Attitude, Intention, and Behaviour: An Introduction to Theory and Research. Reading, Ma: Addison-Wesley.

Fosnot, Catherine Twoney (1996). Constructivism: A Psychological Theory of Learning. In Catherine Twoney Fosnot (Ed.), Constructivism: Theory, Perspectives, and Practice, pp. 8-33. NY: Teachers College, Columbia University.

Fyans, L. J. (1980). Achievement Motivation. New York: Plenum.

Goldberg, S. (1990). Developing and implementing a parental awareness program to enhance children's mathematics performance and attitude. Ed. D. practicum, Nova University (ERIC Document Reproduction Services No. ED 327383).

Gray, P. (1994). Psychology (2nd Ed.). New York: Worth Publishers, Inc.

Hart, L. E. (1989). Describing the affective domain: Saying what we mean. In D. B. McLeod & V. M. Adams (Eds.), Affect and Problem Solving. NY: Springer-Verlag.

Hidi, Suzanne & Harackiewicz, Judith M. (2000). Motivating the academically unmotivated: A critical issue for the 21st century. Review of Educational Research 70 (2), pp. 151-179.

Hull, Clark (1943). Principles of Behaviour. New York: Appleton.

Hutten, E. H. (1954). The Role of Models in Physics. The British Journal for the Philosophy of Science 4, p. 285.

James, William (1890). The Principles of Psychology, Vol.1. NY: Holt.

Jones, G. A., Langrall, C. W., Thornton, C. A., & Mogill, A. T. (1999). Students' Probabilistic Thinking in Instruction. Journal for Research in Mathematics Education 30 (5), pp. 487-519.

Keitel, Christine (1988). Mathematics Education and Technology. In L. Pereira-Mendoza (Ed.), Proceedings of the Canadian Mathematics Education Study Group, 1988 (ERIC Document Reproduction Service No. ED 306259).

Krutetskii, V. A. (1976). The psychology of mathematical abilities in schoolchildren (Joan Teller, Trans.; Jeremy Kilpatrick & Izaak Wirszup, Eds.). Chicago: University Press.

Kuhn, Thomas (1962). The Structure of Scientific Revolutions. Chicago, U.S.: University Press.

Kulm, G. (1980). Research on mathematics attitude. In Richard J. Shumway (Ed.), Research in Mathematics Education. Reston, Va: NCTM.

Lawrence, Lloyd H. (2000). Reactions to mathematics and mathematics education. Unpublished manuscript.

Lerman, Stephen (1989). Constructivism, Mathematics, and Mathematics Education. Educational Studies in Mathematics 20, pp. 211-223.

Lerman, Stephen (1996). Intersubjectivity in Mathematics Learning: A Challenge to the Radical Constructivist Paradigm? Journal for Research in Mathematics Education 27 (2), pp. 133-150.

Lerman, Stephen (2000). A Case of Interpretations of Social: A Response to Steffe and Thompson. Journal for Research in Mathematics Education 31 (2), pp. 210-227.

MacPherson, E. D. (1991). Motivation and mathematics education. Saskatchewan Mathematics Teachers Society Journal 27(3), 51-55.

Maehr, M. L. (1984). Meaning and motivation: Toward a thory of personal investment. In R. E. Ames & C. Ames (Eds.), Research on Motivation in Education, vol. 1. Orlando, FA: Academic Press.

Maslow, A. H. (1970). Motivation and Personality (2nd Ed.). New York: Harper & Row.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualisation. In D. A. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning. Reston, Va: NCTM.

Mendoza, Y. (1996). Developing and implementing a parental awareness program to increase parental involvement and enhance mathematics performance and attitude of at-risk seventh grade students. Masters final report, Nova Southeastern University (ERIC Document Reproduction Services No. ED 400971).

Middleton, J. A. (1995). A study of intrinsic motivation in the mathematics classroom: A personal constructs approach. Journal of Research in Mathematics Education 26(3), pp. 254-279.

Morris, Van Cleve (1966). Existentialism in Education. New York, U.S.A.: Harper and Row.

Moss, J. & Case, R. (1999). Developing Children's Understanding of the Rational Numbers: A New Model and an Experimental Curriculum. Journal for Research in Mathematics Education 30 (2), pp. 122-147.

Nathan, M. J. & Koedinger, K. R. (2000). Teachers' and Researchers' Beliefs About the Development of Algebraic Reasoning. Journal for Research in Mathematics Education 31 (2), pp. 168-190.

National Council of Teachers of Mathematics (NCTM) (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, Va: Author.

National Council of Teachers of Mathematics (NCTM), (1991). Professional Standards for Teaching Mathematics. Reston,Va:Author

Onslow, B. (1992). Improving the attitudes of students and parents through family involvement in mathematics. Mathematics Education Research Journal 4(3), 24-31.

Piaget, J. (1970). Structuralism. New York: Basic Books.

Piaget, J. (1971). Genetic Episemology. New York: Norton.

Rajecki, D. W. (1982). Attitudes: Themes and advances. Sunderland, MA: Sinauer Associates.

Rokeach, Milton (1960). The Open and Closed Mind. NY: Basic Books.

Rotter, J. (1954). Social learning and clinical psychology. Englewood Cliffs, NJ: Prentice Hall.

Seligman, E. P. (1995). The Optimistic Child. New York: Houghton Mifflin Company.

Shulman, L. S. (1981). Recent developments in the study of teaching. In B. Tabachnik, T. S. Popkewitz, & P. B. Szekely (Eds.), Studying teaching and learning. New York: Praeger.

Shulman, L. S. (1986). Paradigms and Research Programs in the Study of Teaching: A Contemporary Perspective. In M. Wittrock, Handbook on Reasearch on Teaching, 3rd Ed, A Project of the American Educational Research Association. New York: MacMillan Publishing Company

Steffe, L. (1993, April). Interaction and Children's Mathematics. Paper presented at the American Educational Research Association, Atlanta, Georgia.

Steffe, Leslie P. & Thompson, Patrick W. (2000). Interaction or Intersubjectivity? A Reply to Lerman. Journal for Research in Mathematics Education 31 (2), pp. 191-209.

Stipek, D. (1998). Motivation to learn: From theory to practice (3rd Ed.). Needham Heights, MA: Allyn & Bacon.

Tobias, S. (1979). Anxiety research in educational psychology. Journal of Educational Psychology 71, pp. 573-582.

Toulmin, S. (1953). The Philosophy of Science. London: Hutchinson's University Library.

Von Glasersfeld, Ernst (1990). An Exposition of Constructivism: Why Some Like it Radical. Journal for Research in Mathematics Education, monograph series no. 4, pp. 19-29. Reston, VA: National Council of Teachers of Mathematics.

Vygotsky, L. S. (1960). Razvitie Vysshikh Psikhicheskikh Funktsii [The Development of the Higher Mental Functions]. Moscow: Akad. Ped. Nauk. RSFSR.

Vygotsky, L. S. (1979). Consciousness as a Problem in the Psychology of Behaviour. Soviet Psychology 17(4), pp. 3-35.

Weiner, B. (1972). Attribution theory, achievement motivation, and the educational process. Review of Educational Research 42, pp. 203-216.

Weiner, B. (1979). A theory of motivation for some classroom experiences. Journal of Educational Psychology 71, pp. 3-25.

Weiner, B. (1986). An attributional theory of motivation and emotion. New York: Springer-Verlag.

White, R. W. (1959). Motivation Reconsidered: The Concept of Competence. Psychological Review (66).

Willer, David (1967). Scientific Sociology: Theory and Method. Englewood Cliffs, NJ: Prentice-Hall Inc.

Willer, D. & Willer, J. (1973). Systematic empiricism: critique of a pseudoscience. Englewood Cliffs, NJ: Prentice-Hall Inc.

Woolfolk, A. E. (1993). Educational Psychology (5th Ed.). Needham Heights, Ma: Allyn & Bacon.