Routing Procedures for Multiple Reservoirs with Backwater-Affected Outlet Structures

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Robert Alan Graham

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ROUTING PROCEDURES FOR MULTIPLE RESERVOIRS

WITH BACKWATER-AFFECTED OUTLET STRUCTURES

BY

ROBERT ALAN GRAHAM

A thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements of the degree of

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in

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Abstract

The purpose of this thesis research is to develop a robust and computationally efficient computer model to perform level-pool reservoir routing calculations on multiple reservoirs with backwater-affected outlet structures. Two methods are developed and demonstrated with specific test cases. The first method, named the matrix method, involves the linearization of the discharge-storage relationship using a first-order Taylor-Series approximation. A set of simultaneous equations is then solved to determine the outflow from the reservoirs. The second method, referred to as the delta factor method, is an iterative search technique which converges upon the solution reliably and efficiently.

Test cases are presented with three reservoirs in series to illustrate how the two methods function under three operating conditions: a typical rising and falling inflow hydrograph, rapidly changing discharge conditions and, finally, reservoir drawdown conditions. Stage-discharge and stage-storage relationships are entered into the computer in tabular form. Results are presented in figures and discussed. The test cases are also used to evaluate the computer time requirements for both methods.

Both the matrix and the delta factor methods provide solutions for the test cases without modification to the original input data. The matrix method requires more computer time than the delta factor method but, during times of rapidly changing flow conditions, the matrix method provides more reliable results by automatically shortening the analysis time step.

Acknowledgments

I want to thank the many people who have assisted me throughout the development of the concepts contained in this project. The constant encouragement of my friends and colleagues provided me with the enthusiasm to continue looking for solutions after encountering numerous obstacles in the search for new ideas.

The support of the engineering staff at Ducks Unlimited Canada Inc. was invaluable. First and foremost, I want to thank the Chief Engineer, Mr. Ron Coley P. Eng., for initially identifying the need for research in the area of reservoir routing and for having the persistence to ensure that the project produced practical results. I thank the Manitoba Provincial Engineer, Mr. Larry Leavans P. Eng., for his patience as he would often wait for the last little problem to be worked out. This project would be years from completion without the preferential treatment I always got from the Mr. Dieter Bonas of the Computer Services Department at DU Canada. Dieter's encouragement and energy lead to not only the creation of this thesis but also a cherished friendship.

I especially wish to thank my advisor, Professor C. Booy, for the time and energy he has put into this project. His ideas doubled the complexity and scope of this thesis making it more than I ever dreamed it would be. Without his insights, the matrix method would never have been developed. His assistance with the writing and presentation of this thesis is greatly appreciated.

Finally, I want to thank my wife, Romana Villa, for waiting patiently while "I am almost finished!". There were many times when we gave up our time together so that this thesis would be finished sooner and not later. Thank you, Romana, for your constant love and support.

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1. Definitions and Abbreviations

Average-volume induced oscillations: oscillations in discharges and elevations that result when the outflow from a reservoir does not change linearly over the time step selected.

Discharge error: CEOP minus EEOP outflow.

SOP: (start-of-period) start of the reservoir routing time step.

EOP: (end-of-period) end of the reservoir routing time step.

Time step (ΔT): increment in time (seconds) from the SOP to the EOP.

CEOP outflow: EOP outflow calculated from rating curves using the EOP elevations.

EEOP outflow: EOP outflow estimated using one of the proposed methods.

ds: property of the reservoir or outlet immediately downstream of the reservoir being referenced.

us: property of the reservoir or outlet reservoir being referenced.

CPU time: time required by the central processing unit of the IBM 3090 mainframe computer to perform the requested operations.

2. Introduction

Reservoir routing is the procedure for determining the changes in water levels on lakes and reservoirs in response to environmental events such a rainfall and snow melt. Reservoirs are constructed for many purposes such as hydroelectric power generation, flood water retention, irrigation water management and for creation of recreational facilities. Generally, these reservoirs raise the natural water levels a great deal to create large water storage areas and high-head hydroelectric power plants. The outlet structures which control the outflow from these reservoirs are, therefore, rarely affected by the natural downstream water levels far below and are termed "free-flow outlets". The methods for calculating the water levels and outflow from these types of reservoirs have been developed and are commonly used by water resource engineers.

Ducks Unlimited Canada Ltd. constructs and manages hundreds of small reservoirs to create wet land habitat for the preservation of numerous wild life populations. These reservoirs generally raise the water level less than ten feet. Since many are in close proximity to each other and are connected by small waterways, the water level of a downstream reservoir, therefore, often affects the water level and outflow of the adjoining upstream reservoir. This type of outlet structure is termed "backwater-affected". The outlet structures for these reservoirs must be made large enough to ensure that rainfall events do not significantly alter the water levels during critical seasons while, at the same time, the cost of these structures must be kept to a minimum.

Reservoir routing for backwater-affected outlet structures and reservoirs has not been extensively studied. Two models are used in Canada but both models simplify the condition back to the free-flow outlet condition. Since Ducks Unlimited Canada designs and operates hundreds of these reservoirs, the economies of scale have required that reliable and efficient procedures be developed for analyzing reservoirs with backwater-affected outlet structures. The development of these procedures and their incorporation into a computer program is the focus of this thesis.

Two methods for reservoir routing with backwater-affected outlets are developed and demonstrated with specific test cases. The first method involves the linearization of the discharge-storage relationship using a first-order Taylor-Series approximation. A set of simultaneous equations is then solved to determine the outflow from the reservoirs. The second method is an iterative search technique which converges upon the solution reliably and efficiently.

The methods are discussed in this thesis using a simple example which consists of three reservoirs in series as shown in Figure 1. The inflow for the first reservoir is set by a user-defined inflow hydrograph and the outflow from this reservoir is the inflow to the second reservoir. The outflow from the second reservoir flows into the third reservoir and the outflow form the third reservoir exits the system. The outlet structure for Reservoir 1 is backwater-affected by Reservoir 2 and similarly Reservoir 2 is backwater-affected by Reservoir 3. Although only three reservoirs are used in the demonstrations, the methods derived for the analysis of this system can be readily extended to more reservoirs.

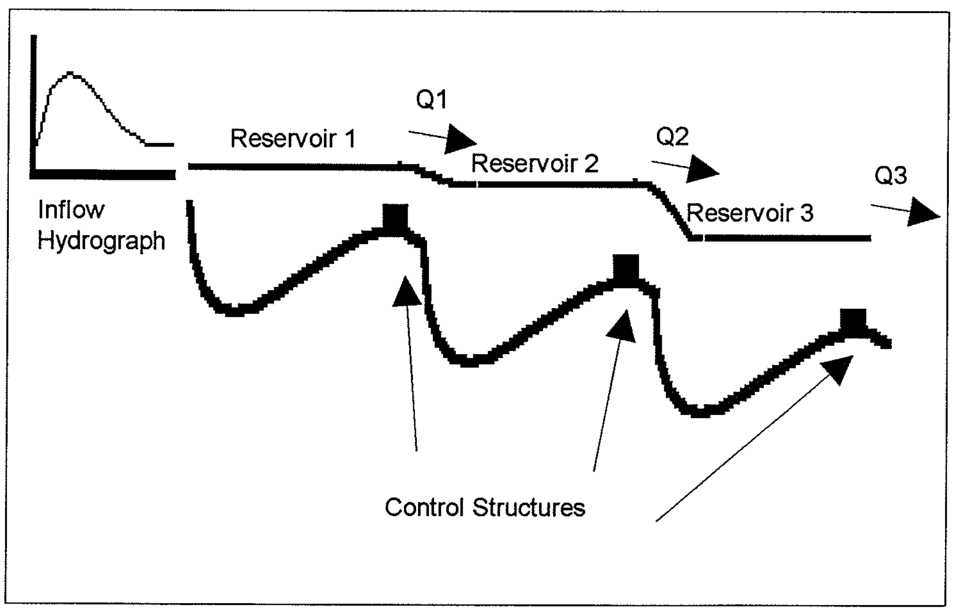

Figure 1 - Three Reservoirs in Series

The two reservoir routing methods presented are assessed by the following criteria.

1) The methods must not assume a constant downstream water level to calculate the reservoir outflow. If assumptions are made in the analysis, a verification of the assumptions and subsequent corrections must be included in the procedure.

2) The computational procedure should directly use the tabular form of the discharge rating curves and stage-storage curves as they are commonly compiled in practice today so that the method is an efficient tool for the water resource engineer.

3) The method should be computationally efficient. This is evaluated using the computational time and the number of iterations required by each method for the test cases.

4) The method must remain robust enough to avoid non-convergence. The method developed must be efficient for the water resource engineer to use even at the cost of additional computational time for the digital computer.

3. Current Methods

In 1986, an extensive search was conducted for literature or existing computer programs in the area of reservoir routing procedures for multiple reservoirs with backwater-affected outlet structures. Only the SIMPAK and the ACRES models were found (to be discussed later). Numerous computer models were found to perform reservoir routing on multiple reservoirs but they all employ the Modified Puls technique which assumes unregulated free flow at the outlet of each reservoir.

In 1995, Haestad Methods, an engineering software distribution company specializing in hydrological computer models, was contacted and, once again, it was confirmed that no additional commercial software had been developed. The United States Bureau of Reclamation was also contacted and personnel reported that they were unaware of any developmental work taking place in this field. Finally, no additional literature was found in a search through the engineering periodicals.

The two reservoir routing models that were found are the SIMPAK model developed by the Water Planning and Management Branch of Environment Canada and the reservoir routing model developed by Acres International for in-house use. Both of these models perform the reservoir routing calculations by assuming that the downstream elevation remains constant at the SOP elevation. The time step must be kept short to keep the error involved in this assumption from affecting the results.

Some conditions require very short time steps. For example, the discharge from an outlet operating under submerged conditions may change significantly even with small downstream fluctuations. This problem is compounded when a small reservoir is located downstream of the reservoir. Small reservoirs rise and fall rapidly with small changes in inflow. Finally, with steep hydrographs, the inflow to the reservoirs changes rapidly. It can then be expected that the reservoir elevations also change rapidly.

Neither the SIMPAK or the Acres International routing models provide a verification check to ensure that the change in downstream elevation, although small, does not significantly affect the discharge. The discharges and elevations of the reservoirs begin to oscillate when the time step is chosen too large. If such oscillations are observed in the results, then the user must shorten the time step and repeat the calculations.

Oscillations observed in the results may be caused by backwater-affected structures or they may be the result of a more basic assumption that is used in both free-flow and backwater-affected reservoir routing. This assumption is discussed in the following section.

3.1. The Requirement for Short Time Steps in All Reservoir Routing

All of the reservoir routing procedures discussed or developed in this thesis use the assumption that the inflow or outflow volumes used in the water balance equation (see equation [1]) may be approximated using the arithmetic average of the SOP and EOP discharges (see equation [2]). This includes the modified Puls technique used for unregulated free-flow outlets.

$$\text{change in storage} = (\text{inflow volume} - \text{outflow volume}) \qquad [1]$$

For both inflow and outflow.

$$\text{Volume}\,(\text{ft}^3) = \frac{(Q_s + Q_e)}{2} \times \Delta t \qquad [2]$$

where

Q_s	=	start-of-period flow (cfs)
Q_e	=	end-of-period flow (cfs)
Δt	=	time step (seconds)

Regardless of the inflow condition, the time step must be kept short for conditions where the outflow from a reservoir changes non-linearly. A short time step ensures that the non-linear changes in outflow are adequately approximated by a straight line. Failure to use a short enough time step results in oscillating discharges. This type of oscillation is referred to as "average discharge-induced oscillation". An example is used to explain its occurrence.

The solid line in Figure 2 shows an outflow hydrograph representing a situation where the gate of a control structure is opened, resulting in a sudden outflow that diminishes quickly with time. If this curved line were approximated by a straight-line segment connecting the SOP to the EOP outflow, the total approximated outflow volume would be much greater than the actual outflow volume. The greater outflow volume would result in a lower EOP elevation and a lower EOP outflow being calculated from the elevation-discharge curves. Therefore, if the outflow is to be approximated by a straight line, the EOP outflow must be less than the actual EOP outflow indicated by the solid line. The dashed line on Figure 2 indicates the linear changing outflow where the EOP discharge equals the discharge calculated from the elevation-discharge curves.

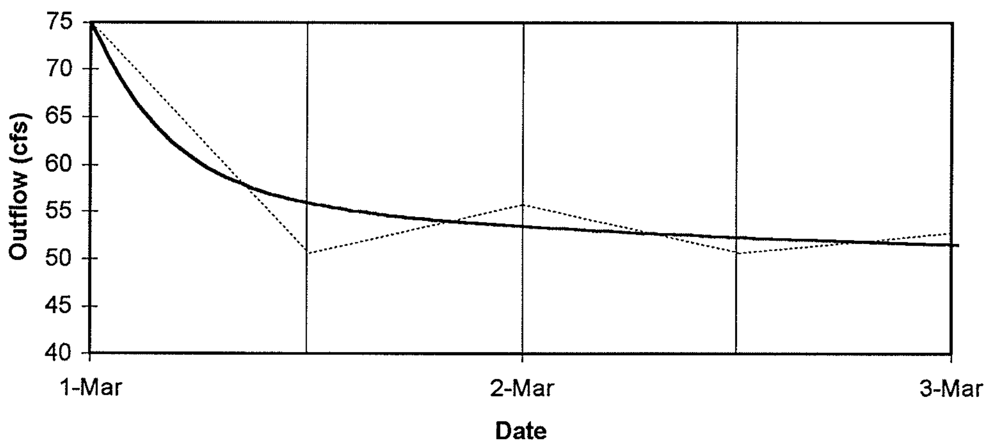

Figure 2 - Averaged Outflow Hydrograph

Using the same rationale, the EOP discharge would have to be overestimated at the end of the second day but usually to a lesser extent. This oscillation generally decreases with time but the oscillation in the discharge may cause a convergence to a solution that satisfies the equations but is physically unacceptable. For example, if the outflow volume is overestimated, it is possible for the water level to drop below the crest of the outlet structure. This problem is discussed in the test case section, "Results: Delta Test Case 1 - Run 3: (DF-TC1-R3) Delta Factor+Linear+Zero Inflow+24hr", page 53.

Although the use of equation [2] to calculate the outflow volume may cause the oscillation problems discussed above it is still commonly accepted in the formulation of the basic reservoir routing equation.

3.2. The Reservoir Routing Equation

Using the assumption that mean flows during a time interval equal the average between flows at the start and end of the time interval, the water balance equation can be written as equation [3]. This is the basic reservoir routing equation used in the modified Puls technique and in both procedures discussed for this thesis.

$$\text{change in storage } (\text{ft}^3) = (S_e - S_s) = \left[\left(\frac{I_s + I_e}{2}\right) - \left(\frac{Q_s + Q_e}{2}\right)\right] \times \Delta t \qquad [3]$$

where

S_s	=	start-of-period storage (ft^3)
S_e	=	end-of-period storage (ft^3)
I_s	=	start-of-period inflow (cfs)
I_e	=	end-of-period inflow (cfs)
Q_s	=	start-of-period outflow (cfs)
Q_e	=	end-of-period outflow (cfs)
Δt	=	time period (sec)

For an unregulated reservoir, only the end-of-period storage and the end-of-period outflow are unknown. If the outlet structure has no downstream control, then there is a simple solution for calculating the reservoir outflow.

3.3. Modified Puls Technique

When the reservoir routing equation is applied to a reservoir with a non-regulated free-flow (without downstream control) outlet structure and a given inflow hydrograph, the only unknowns are the end-of-period outflow and storage. The technique proposed by Puls was to rearrange equation [3] so that the unknown variables were grouped together on the left, as shown in equation [4]. Using the fact that the outflow is uniquely defined by the storage, a graph or table is made for each reservoir relating the left-hand term of the equation to the outflow. When the relationship is given in graphical form, the curve is called the storage indication curve.

$$\frac{2S_e}{\Delta t} + Q_e = \frac{2S_s}{\Delta t} - Q_s + I_s + I_e \qquad [4]$$

The steps in Puls' method are to calculate the value of the right-hand side of equation [4] and then to look this value up on the curve and to read the outflow. This curve lookup step is easily performed from a table with a digital computer.

When the outlet structure is backwater-affected, the outflow is a function of both the reservoir storage and the downstream reservoir storage (see equation [5]). It is this interdependence between the reservoirs that makes calculating the outflow from backwater-affected reservoirs more complex.

$$\text{Outflow} = f(\text{reservoir storage, ds reservoir storage}) \qquad [5]$$

3.4. Constant Downstream Water Level Assumption

A simple solution to the problem of the backwater-affected flood routing is to select a very short time step to ensure that the downstream water levels do not change significantly over the time step. Given the SOP downstream elevation, the storage indication curve used in the Puls technique can be recreated for each time step and each reservoir may be treated as if its outlet were operating under free-flow conditions. However, if the time step is selected too large, oscillations in the discharge may commence. In the following example, the oscillation problem is described for a pair of reservoirs in series with a rising inflow hydrograph. Reservoir 1 is the upstream reservoir and Reservoir 2 is downstream.

It is assumed that, at the start, the two reservoirs are operating in a steady-state condition with inflows equal to outflows. Reservoir 1 now begins to rise over the first time step because of an increase in inflow. The EOP outflow can then be calculated using the modified Puls technique with the SOP elevation from Reservoir 2. This outflow, however, is overestimated because the EOP elevation for Reservoir 2 should have been increased and this would have reduced the discharge. The overestimated EOP outflow from Reservoir 1 is used for the inflow to Reservoir 2. This overestimated inflow causes Reservoir 2 to appear to rise more rapidly than it should.

Returning to Reservoir 1 for the second time step, the SOP outflow was overestimated. Now the EOP outflow is calculated using a storage indication curve developed for a high SOP downstream elevation that is too high. Both the overestimated SOP outflow and the overestimated downstream elevation result in an EOP outflow for Reservoir 1 that is too low. The cycle of overestimating outflow and then underestimating it may then cause the model to become unstable.

Shortening the time step can often solve this problem but it may make the computer calculations inefficient. There is no verification check for the acceptability of the constant downstream elevation assumption, but the presence of oscillations is used as an indicator that the time step is too long. An advantage of this method is that, when the time steps are kept short, the problem of "average discharge-induced oscillation" mentioned previously is also minimized.

4. The Proposed Methods

This thesis develops and investigates two methods for calculating reservoir outflow. The first is the matrix method which is a linearization technique and the second is the delta factor method which is an incremental search technique.

4.1. General Overview of the Procedures

In both methods presented, the EOP outflow is first determined. This outflow is called the estimated end-of-period (EEOP) outflow. It is entered into the water balance equation to determine the end-of-period storage. The end-of-period storage is then used to calculate the EOP elevation. The elevation is then used to determine the calculated end-of-period (CEOP) outflow from the rating curves. The difference between the EEOP and CEOP outflow is then determined. The absolute value of this difference is referred to as the discharge error. If the discharge error is less than the acceptable tolerance, the EEOP outflow is accepted as correct and the model proceeds to the next time step. If the discharge error is not within the acceptable tolerance, the outflow is re-estimated and the procedure is repeated. These steps are similar to a reservoir routing technique for uncontrolled free-flow reservoirs described in "*USBR Design of Small Dams Method*" (p 349, Second Edition, 1973). The difference in the techniques lies in the determination of the EEOP outflow.

4.1.1. Linearization of the Stage-versus-Storage Relationships

The first method for determining the outflow uses a first-order Taylor-Series approximation to represent the relationship between outflow and storage. This method assumes that the relationship between outflow and storage remains linear over the time increment chosen. If it does not closely approximate linearity, the EEOP outflow does not agree with the CEOP outflow and the iteration is said to fail. The time step is then shortened, which is done automatically in the computer procedure, and the procedure is repeated until the EEOP outflow agrees with the CEOP outflow.

4.2. Incremental Search (Delta Factor) Method

The second method for estimating the outflow is an incremental search named the delta factor method. An initial EEOP outflow (equal to the SOP outflow) is used in the water balance equation to produce an EOP storage. This storage is used to calculate the EOP elevation and the CEOP outflow. If the discharge error is not within the acceptable tolerance, the iteration is said to fail. The EEOP outflow is then changed by the discharge error divided by the delta factor (see equation [6]).

$$\text{Change in EEOP outflow} = \frac{(\text{CEOP outflow} - \text{EEOP outflow})}{\text{Delta Factor}} \qquad [6]$$

This method guides the incremental changes in outflow to avoid slow convergence and non-convergence by altering the value of the delta factor for each reservoir. Thus, determining the delta factor is critical to the success of this technique. When the EEOP outflow is allowed to change too quickly the model becomes unstable and the method does not produce convergence. Very small increments in the EEOP outflow yield a very robust model that reliably converges but it is not efficient. The initial value of the delta factor is selected as 2.0. In typical calculations, it is found to range from this minimum of 2.0 to a maximum near 10.0.

4.3. The Matrix Method for Determining EEOP Outflows

The matrix method uses a first-order Taylor-Series approximation to represent the relationship between the change in outflow and the change in storage (for both the reservoir under investigation and the downstream reservoir) that may affect the outflow. The use of only the first two terms of a Taylor-Series approximation is based on an assumption that the relationship between outflow and storage remains sufficiently linear over the time step. A brief introduction to the formulation of the method follows.

The reservoir farthest downstream in a system of reservoirs is assumed to operate with either a free-flow outlet or a known downstream elevation. This reservoir has, therefore, only two unknowns: the end-of-period (EOP) outflow and the EOP storage. To solve for these unknowns, two equations are available: the water balance equation (see equation [10]) and the first-order Taylor-Series approximation (see equation [8]). The EEOP outflow and the EOP storage can thus be determined.

Adding a second reservoir upstream that has a backwater-affected outlet structure adds two more unknown values to the system: the EOP outflow and the storage of that reservoir. At the same time, another water balance equation and a Taylor-Series approximation are also added to the system. This results in four unknowns and four equations. The system of equations can be solved by substitution and matrix algebra. Additional reservoirs similarly add equal numbers of unknowns and equations.

4.3.1. Derivation of the Matrix Equations

The relationship between outflow and storage is shown in equation [7].

$$Q_{n,e} = f(S_{n,e}, S_{n+1,e}) \qquad [7]$$

where

n	=	reservoir designation number 1,2 or 3 ordered from upstream to downstream
$Q_{n,e}$	=	EOP outflow for Reservoir n
$S_{n,e}$	=	EOP storage for Reservoir n

Representing equation [7] using only the first two terms of the Taylor-Series approximation yields equation [8].

$$\delta Q_n = \frac{\delta Q_n}{\delta S_n}\,\delta S_n + \frac{\delta Q_n}{\delta S_{n+1}}\,\delta S_{n+1} \qquad [8]$$

where

n	=	reservoir designation number 1, 2 or 3 ordered from upstream to downstream
δQ_n	=	change in outflow for Reservoir n over a time period δt
δS_n	=	change in storage for Reservoir n over a time period δt

Neglecting the higher-order terms in the Taylor-series approximation implies that the relationship between outflow and the storage remains practically linear over the time increment chosen. The ratios of the outflow to storage are represented by the variables A_n and B_n as follows in equation [9].

$$A_n = \frac{\delta Q_n}{\delta S_n}, \quad B_n = \frac{\delta Q_n}{\delta S_{n+1}} \qquad [9]$$

The A-ratio in equation [9] represents the change in discharge from a change in storage and the B-ratio in equation [9] represents the change in discharge from a change in downstream storage. Substituting these ratios into equation [8] yields equation [10].

$$\delta Q = A_n \, \delta S_n + B_n \, \delta S_{n+1} \qquad [10]$$

The water balance equation can be written as:

$$\delta S_n = (Q_{n-1} - Q_n)\,\delta t \qquad [11]$$

where

Q_{n-1} = outflow from the upstream reservoir
Q_n = inflow to the current reservoir

Substitution into equation [10] results in equation [12]

$$\delta Q_n = A(Q_{n-1} - Q_n)\delta t + B(Q_n - Q_{n+1})\delta t \qquad [12]$$

The assumption that the mean outflow during the time period can be represented by the average between outflows at the start and at the end of the period is expressed as:

$$Q_{n,a} = \frac{(Q_{n,s} + Q_{n,e})}{2}$$ [13]

where

$Q_{n,a}$ = average outflow over period for Reservoir n
$Q_{n,s}$ = SOP outflow for Reservoir n
$Q_{n,e}$ = EOP outflow for Reservoir n

Substituting equation [13] into equation [12] and rearranging the terms with the unknown values on the left and the known values on the right yields equation [14].

$$(\frac{2}{\Delta t} + A_n - B_n)\ Q_{n,e} - A_n\ Q_{n-1,e} + B_n\ Q_{n+1,e} =$$
$$(\frac{2}{\Delta t} + B_n - A_n)\ Q_{n,s} + A_n\ Q_{n-1,s} - B_n\ Q_{n+1,s}$$ [14]

where

Δt = the reservoir routing time step

Equation [14] is used for each reservoir in the system that has a backwater-affected outlet with the exception of the reservoir farthest upstream. The reservoir farthest upstream (n=1) is represented by an equation with one less unknown, because the EOP inflow is a given value. The equation for the reservoir farthest upstream becomes equation [15].

$$(\frac{2}{\Delta t} + A_n - B_n)\ Q_{1,e} + B_n\ Q_{2,e} =$$
$$(\frac{2}{\Delta t} + B_n - A_n)\ Q_{1,s} + A_n\ Q_{0,s} - B_n\ Q_{2,s} + A_n\ Q_{n-1,e}$$ [15]

where

$Q_{0,s}$ = known SOP inflow
$Q_{0,e}$ = known EOP inflow

Reservoirs, including the reservoir farthest downstream, that do not have a backwater-affected outlet structure have a simplified form of equations [14] or [15], as the value for the B-ratio is zero.

Equation [16] is the matrix formulation using equation [15] for Reservoir 1 and equation [14] for Reservoirs 2 and 3. The first line of the matrix is Reservoir 1 and the last line is Reservoir 3. Note that, for Reservoir 3, there are no terms containing the B-ratio.

$$\begin{bmatrix} \frac{2}{\Delta t} + A_1 - B_1 & +B_1 & 0 \\ -A_2 & \frac{2}{\Delta t} + A_2 - B_2 & +B_2 \\ 0 & -A_3 & \frac{2}{\Delta t} + A_3 \end{bmatrix} \begin{Bmatrix} Q_{1,e} \\ Q_{2,e} \\ Q_{3,e} \end{Bmatrix} = \begin{bmatrix} +A_1 Q_{0,s} + (\frac{2}{\Delta t} + B_1 - A_1) Q_{1,s} - B_1 Q_{2,s} \\ A_2 Q_{1,s} + (\frac{2}{\Delta t} + B_2 - A_2) Q_{2,s} - B_2 Q_{3,s} \\ A_3 Q_{2,s} + (\frac{2}{\Delta t} - A_3) Q_{3,s} \end{bmatrix} \quad [16]$$

With this matrix formulation, the estimated end-of-period (EEOP) outflow can be calculated directly.

4.3.2. Linearization of the Stage-Discharge Curves

The stage-discharge relationship for a backwater-affected outlet structure can be represented by a set of curves for a range of downstream elevations. Each curve can be approximated by straight-line segments. A sample curve is shown in Figure 13, page 37, for a five-foot-long stop-log weir. The slope of these line segments affects the A-ratio defined in equation [9] and used in equation [10]. A significant change in the slope may invalidate the linearity assumption of the Taylor-Series approximation. When the elevations at the beginning and end of the time step span either side of one or more slope break points, then the slope used in the estimating calculations does not equal the actual slope of the discharge curve over the time step and a significant error in the EEOP outflow may result (see Figure 3).

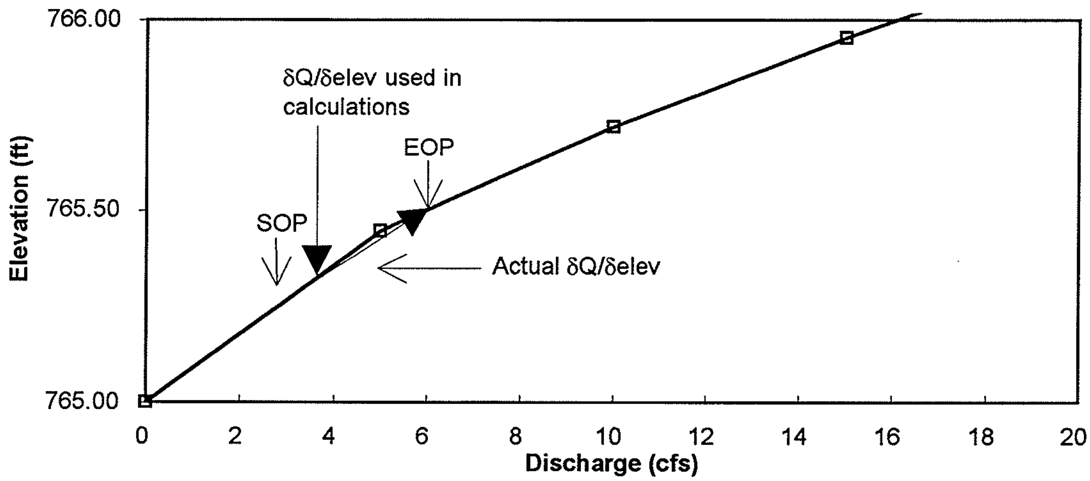

Figure 3 - Non-linear Discharge Curve

Shortening the time step brings the EOP elevation closer to the break point and, therefore, helps to reduce error in the outflow estimate. However, if several reservoirs all have discharge rating curves with significant changes in slope, it is likely that the elevations for not all of the reservoirs move to the break points at the same time. The best solution is to have several short-line segments so that the change in slope in not as severe. For the weir structure shown in Figure 13, it is only the free-flow curve, elevation 765 ft, at the lower upstream elevations, that needs several short-line segments.

Under submerged conditions with large backwater effects, the stage-discharge curve has a very small slope, yielding extremely large values of the A-ratio (see Figure 13 downstream elevation = 775 ft, page 37). At less than 50 cfs, the curve is almost horizontal. As the discharge increases, the curve begins to rise, the slope increases and the A-ratio drops extremely quickly. Again, to avoid rapid changes in the A-ratio, a single slope should be used over the largest range of discharge possible. The stage-discharge curve is very difficult to define accurately in this region and, therefore, little accuracy is lost by using a straight-line segment.

4.3.3. Determining the A-ratio at Break Points on the Stage-Discharge Curves

Break points in the stage-discharge curves as described in the section "Linearization of the Stage-Discharge Curves" produce a second problem. In the initial formulation of the matrix method, the slope of the rating curve was calculated in the direction of increasing elevation (see Figure 4). If the SOP elevation is very close to the break point on the curve and the water level is decreasing, then using the calculated slope of the rating curve does not produce an accurate estimate of the EOP discharge. The solution to this problem is to evaluate the slope twice. The slope calculated from the increasing elevation is averaged with the slope calculated by decreasing the elevation.

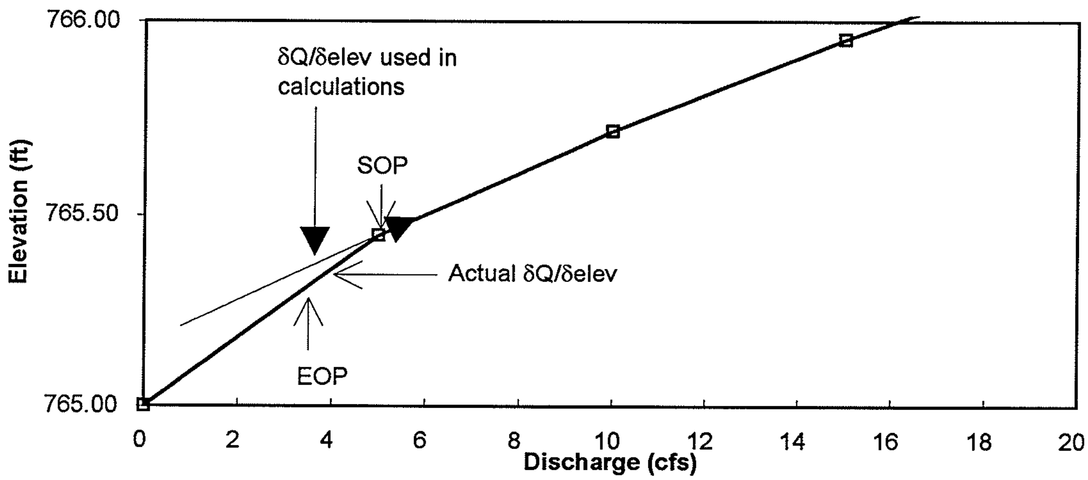

Figure 4 - Effect of Direction with Non-linear Stage-Discharge Curves

4.3.4. Non-linearity of the Stage-Storage Curves

The stage-storage curve is also represented by a series of straight-line segments. As with the stage-discharge curves, the slope of these segments also affects the A- and B-ratios in equations [9] and [10]). The slope of these line segments tends to be fairly constant as the slope of the stage-storage curve is the reciprocal of the lake area. However, for reservoirs with large marsh lands, there are sudden changes in slope when the marsh lands flood or become dry. This can produce iteration problems.

4.3.5. Rapidly Changing Conditions and Starting Elevations

Modeling problems may arise with steep inflow hydrographs. The rapid change in inflow may result in greater non-linearity in the δdischarge/δstorage ratios. The model must then shorten the time step to allow for this as discussed in the section, "Linearization of the Stage-Discharge Curves", page 16.

A second cause of rapidly changing conditions is specifying starting elevations that are significantly different from the steady-state flow. Again, rapidly changing storage levels result as the reservoirs adjust.

It has been assumed that the outflow changes linearly over the time step. This is rarely the case for rapidly changing outflows. As discussed in the section, "The Requirement for Short Time Steps in All Reservoir Routing", page 6, the model then may produce oscillating outflows. Fortunately, the shortening of the time step to solve the non-linearity problem also helps ensure that this does not occur.

4.4. The Delta Factor Method for Determining EEOP Outflows

The second method for estimating the outflows is an incremental search procedure. The EEOP outflow is changed by dividing the discharge error by a systematically selected factor, referred to as the delta factor (see equation [6], page 12). The value of the delta factor is different for each reservoir and it is assessed and changed if needed at every failed iteration. The process of assessing and updating the delta factor is crucial to the efficiency of the method and its ability to converge steadily to a solution.

4.4.1. Determining Values for the Delta Factor

The selection of the delta factor is the critical component of the technique. If the delta factor is selected as low as 1.0, the EEOP outflows changes very rapidly and the model often becomes unstable and does not converge. Conversely, with the delta factor of about 10.0, the EEOP outflows change very slowly. The model then converges consistently but at the cost of large amounts of computer time.

All reservoirs in the system must converge together. This became apparent during the initial development of the delta factor method when the outflow from the free-flow reservoir farthest downstream was determined using the modified Puls technique for each iteration. This was done because it seemed rather illogical to use an incremental step technique when a direct solution for the one reservoir could be used. However, it soon became apparent that, as the inflows to that reservoir changed from one iteration to the next, the modified Puls technique produced rapidly changing outflows and often caused the model to become unstable.

As mentioned previously, the delta factor is determined for each reservoir separately because the effect of one reservoir on another is different for each reservoir and this effect changes with the water level. It is the most difficult to calculate the outflow for reservoirs that have large outlet structures and small storage volumes. Referring back to the terminology described in matrix method, this type of reservoir has a large A-ratio. Also, reservoirs with backwater-affected outlets are difficult to route when the backwater effect is large. This coincides with a large B-ratio for the matrix method. When the downstream reservoir causing the backwater effect has a small surface area, its elevation tends to change rapidly. This may cause the problems with the routing to become even more difficult (a larger B-ratio).

Reservoirs that have large values of the A- or B-ratios require larger values of the delta factor. A starting delta factor value of 2.0 is used in the model. From there, it is either increased or decreased depending on relative values of the EEOP and CEOP. The procedure for changing the delta factor has a significant effect on the efficiency of the method and ensuring that the model leads to convergence. The method for changing the delta factor that has proved most efficient while remaining robust to consistently lead to convergence was developed through experience while working on numerous project for Ducks Unlimited Canada. In principle, the following rules were developed for the computer calculations.

If the EEOP outflow is continuously greater than or continuously less than the CEOP outflow for five consecutive iterations, this is taken as an indication that the EEOP outflow should be changing more rapidly. The delta factor is reduced by 0.25 to accomplish this and it is not reduced below 2.0. The delta factor must be decreased rather slowly to avoid convergence problems.

If, on iteration i, the EEOP outflow is greater than the CEOP outflow and, on iteration i-1, the EEOP outflow is less than the CEOP outflow, this is taken as an indication that the EEOP outflow is changing too rapidly and that the value of the delta factor should be increased. The delta factor is then increased immediately by 0.5 to prevent non-convergence. There is no maximum placed on the delta factor values.

4.4.2. Determining EEOP Outflows

For each time step, the first estimate of the EEOP outflow is equal to the start-of-period (SOP) discharge. This assumption would be correct for a steady-state flow condition where inflow equals outflow. After the water volumes are routed through the reservoirs (EOP storages are calculated), EOP elevations and CEOP outflows may be determined. The EEOP outflow is subtracted from the CEOP outflow to produce the discharge error. If the absolute value of the discharge error is not within a specified tolerance, then the iteration is said to fail and the EEOP outflows must be re-determined. The new EEOP outflow value is calculated by adding the change indicated in equation [6] to the old EEOP outflow.

4.5. The Verification Procedure

Both the matrix and the delta factor method incorporate a verification step to ensure that the assumptions made are verified before continuing to the next time step. The steps in the procedure follow.

1) Determine the EEOP outflows.

   The matrix method constructs the matrix equations and solves for the EOP outflows. The delta factor method estimates the EOP outflows.

2) Perform the water balance (reservoir routing) calculations.

   The outflows from step 1 are used to calculate the water volumes to be transferred between reservoirs. The EOP storages and elevations are now calculated.

3) Verify estimated outflows.

   The calculated end-of-period (CEOP) outflows are determined by entering the elevations calculated in step 2 into the outlet structure rating curves. The EEOP outflow is subtracted from the CEOP outflow to produce the discharge error. If the absolute value of the discharge error is not within a specified tolerance, then the iteration is said to fail and the EOP outflows must be re-determined.

Failure by the matrix method is assumed to be a violation of the linearity assumptions and, therefore, the time step is shortened by one half the previous value. The procedure is then to repeat from step 1 to recalculate the EEOP outflow. An iteration failure for the delta factor method results in a requirement to return to step 1 where the EEOP outflow is re-estimated using the delta factor equation (see equation [6]).

4) Moving to the next time step.

The EEOP outflow is the value used in the reservoir routing calculations. Therefore, it is the value which is carried on to become the SOP outflow for the next routing period and which is printed in the table of results. It also becomes the SOP inflow for the downstream reservoir.

4.5.1. Transferring EEOP or CEOP Outflow to the Next Time Step

In developing the methods, it was found to be essential that the EEOP outflows, and not the CEOP outflows, be transferred to become the SOP outflows for the next time period (see step 4 above). In a first attempt at developing the methods, the CEOP outflows were the values transferred to the next time step as the SOP outflows. Non-convergence problems were often encountered. The initial reasoning for transferring the CEOP outflows was that these discharges were derived from the final elevations calculated for the time period and, thus, they would be the most accurate results. However, recalling step 2 above, it is the EEOP outflow that is used in the water balance equations to determine the EOP storage volume. If the CEOP outflow were transferred to the SOP outflow to be used in the reservoir routing calculations, there would be a sudden change in the discharge hydrograph from the reservoir. The maximum value of the "sudden change" is the tolerance used in the calculations.

5. General Description of the Test Cases

Both methods are applied to test cases to assess their performance against the criteria outlined in the section, "Introduction", page 2. The first test case consists of three small reservoirs (lake area of 25 acres) in series and all having linear stage-storage curves and linear stage-discharge curves. With these linear relationships, the matrix method should operate quite efficiently since the linearity assumptions required by the first-order Taylor-Series approximation are not be violated. The second test case is similar to the first except that actual stage-discharge curves for five-foot-long stop-log weirs are used as outlet structures for the reservoirs. The stage-storage curves are taken from a small lake with an area of about 29 acres. Small reservoirs were chosen with relatively large outlet structures because it is under these conditions that the reservoir routing calculations are the most difficult.

Each test case is run for three different discharge and starting water level conditions. The first run is a typical rising and falling hydrograph over a one-month period with a one-day time step for analysis. The starting reservoir level for Reservoir 1 is set 0.5 ft lower than the steady-state elevation so that the model starts in an unsteady state with rapidly changing discharges. The second run has the same conditions except that it is analyzed on a half-hour time step. This test case shows how the oscillations that result from the outflow changing non-linearly over the time step can be eliminated with the shortened time step. The third run is a zero inflow case where the reservoirs are drawn down to the outlet crest elevations. These test runs are summarized in Table 1. The delta factor method has one additional run to show a solution to problems that occur during the rapid drawdown of the reservoir.

Table 1 - Test Cases

Method	Case	Run	Identification Label	Stage-Discharge and Stage-Storage Curves	Inflow Hydrographs	Time Step Length
Matrix	1	1	M-TC1-R1	Linear	Rise and Fall	24 hour
		2	M-TC1-R2	Linear	Rise and Fall	1/2 hour
		3	M-TC1-R3	Linear	Zero Inflow	24 hour
Matrix	2	1	M-TC2-R1	Non-linear	Rise and Fall	24 hour
		2	M-TC2-R2	Not Run	Rise and Fall	1/2 hour
		3	M-TC2-R3	Non-linear	Zero Inflow	24 hour
Delta Factor	1	1	DF-TC1-R1	Linear	Rise and Fall	24 hour
		2	DF-TC1-R2	Linear	Rise and Fall	1/2 hour
		3	DF-TC1-R3	Linear	Zero Inflow	24 hour
		4	DF-TC1-R4	Linear	Zero Inflow	6 hour
Delta Factor	2	1	DF-TC2-R1	Non-linear	Rise and Fall	24 hour
		2	DF-TC2-R2	Not Run	Rise and Fall	1/2 hour
		3	DF-TC2-R3	Non-linear	Zero Inflow	24 hour

5.1. Reservoir Description and Notation

Three reservoirs in series, as shown in Figure 1, are analyzed. The two upstream reservoirs have backwater-affected outlets and the reservoir farthest downstream has a free-flow outlet structure. The reservoirs are numbered, with Reservoir 1 being the reservoir farthest upstream and Reservoir 3 being the reservoir farthest downstream.

5.2. EOP Inflows

For the reservoir farthest upstream (Reservoir 1), the EOP inflow hydrograph is entered with daily inflow values. For the middle and downstream reservoirs (Reservoirs 2 and 3), the inflows are the EEOP outflows from the reservoirs immediately upstream.

5.3. Selection of the Outflow Tolerance

As stated before, the discharge error is defined as the absolute value of the CEOP outflow minus the EEOP outflow. The maximum allowable error (tolerance) is set by the user of the program to a value that defines acceptable accuracy. For the test cases in this thesis, the discharges are in the 0 to 250 cfs range and a maximum error of 0.05 cfs has been set (0.02%). This is admittedly an extremely low tolerance value. It was chosen to fine-tune the calculation procedures of the two methods.

5.4. Choosing the Length of the Routing Period

The length of the time step chosen for the model depends initially upon the detail which the user desires in the calculations. If the inflow hydrograph were changing appreciably on an hourly basis and the peak discharge were to be accurately described, a time step of one hour at most would be required. For the test cases in this thesis, the inflow data are entered on a daily basis and, therefore, time steps of one day are selected. If there were a rapid change in the inflow or outflow conditions, a shorter time step should be considered as discussed in the section, "Rapidly Changing Conditions and Starting Elevations", page 18.

The second criterion for the selection of the time step is the efficiency of the computer program. The value of the time step Δt magnifies any difference between the EEOP outflow and the CEOP outflow. The matrix and the delta factor methods both determine an EEOP outflow which, after being multiplied by Δt, affects the EOP storage value (see equation [3]). This EOP storage value is then in turn used to calculate the CEOP outflow. The longer the time step the more likely that the discharge error would be greater than the discharge tolerance and the more likely that the calculations for this period would have to be repeated. With a very short time step, very few iterations are needed. If almost every time period requires several iterations, then the model becomes more efficient with a smaller time step.

6. Matrix Method Test Cases

6.1. Matrix Method: Linear Model - Test Case 1

A hypothetical test case with linear stage-discharge and stage-storage curves was created to investigate the operation of the matrix method. Most problems with the method are expected to be a result of the non-linearity of the stage-versus-discharge and the stage-versus-storage curves. Therefore, with the non-linearity problem removed, each time step was expected to converge with only one iteration.

The δstage-δdischarge and δstage-δstorage values are constant at the values indicated in Table 2 until either the upstream or the downstream water level drops below the crest elevation. When the upstream water level drops below the crest elevation, the A-ratio becomes 0.0 and, when the downstream elevation drops below the crest elevation, the B-ratio becomes 0.0. As the stage-discharge curve is linear, it does not represent any real-life structure (culvert, weir, etc.). The one exception is the proportional discharge weir which is designed to have a linear stage discharge curve.

Table 2 - Reservoir and Discharge-Storage Ratios - Test Case 1 - All Runs

Variable	Value	Units
$\frac{\delta discharge}{\delta el\ (us)}$	50.0	$\frac{cfs}{ft}$
$\frac{\delta discharge}{\delta el\ (ds)}$	-50.0	$\frac{cfs}{ft}$
$\frac{\delta storage}{\delta el}$	25.0	$\frac{acre\text{-}ft}{ft}$
A-Ratio $\frac{\delta discharge}{\delta storage\ (us)}$	2.0	$\frac{cfs}{acre\text{-}ft}$
B-Ratio $\frac{\delta discharge}{\delta storage\ (ds)}$	2.0	$\frac{cfs}{acre\text{-}ft}$

While linear stage-discharge and stage-storage curves seem rather unrealistic, it should be remembered that the method assumes linearity of the discharge-storage relationship. The latter relationship tends to be far more linear than the former.

6.1.1. Description: Matrix Test Case 1 - Run 1: (M-TC1-R1) Matrix+Linear+Rise and Fall+24hr

The inflow hydrograph for Reservoir 1 (the farthest upstream) is given in Table 3 and the crest elevations of the outlet structures and the starting water level for each reservoir are given in Table 4. The starting water level is selected so that the reservoirs are in an unsteady state (inflow not equal to outflow). Reservoir 1 is 0.5 feet below the steady-state elevation and Reservoir 3 is 0.5 feet above the steady-state elevation. These starting water levels result in rapidly changing conditions on all three reservoirs.

Table 3 - Inflow Hydrograph to Reservoir 1- Test Case 1 - Runs 1 and 2

Date	Inflow (cfs)	Date	Inflow (cfs)
March 1	50.0	March 17	80.0
March 2	50.0	March 18	70.0
March 3	50.0	March 19	60.0
March 4	50.0	March 20	50.0
March 5	50.0	March 21	50.0
March 6	65.0	March 22	50.0
March 7	80.0	March 23	50.0
March 8	100.0	March 24	50.0
March 9	130.0	March 25	50.0
March 10	145.0	March 26	50.0
March 11	150.0	March 27	50.0
March 12	135.0	March 28	50.0
March 13	125.0	March 29	50.0
March 14	110.0	March 30	50.0
March 15	100.0	March 31	50.0
March 16	90.0		

Table 4 - Outlet Structure Crest Elevations and Starting Water Levels - Test Cases 1 and 2 - Runs 1 and 2

Reservoir Number	Location	Crest Elevation (ft)	Starting Water Level (ft)
1	Upstream	766.00	768.00
2	Middle	765.00	767.50
3	Downstream	765.00	766.50

6.1.2. Results: Matrix Test Case 1 - Run 1: (M-TC1-R1) Matrix+Linear+Rise and Fall+24hr

The discharges calculated using the matrix method are displayed in Figure 5. The left vertical axis displays the discharge in cfs and the right vertical axis indicates the number of iterations required for each time step which, in this case, is always 1. The discharge for Reservoir 1 rises very quickly over the first day and the discharge from Reservoir 3 drops off very quickly. The oscillation in the discharge that takes place results from the changes in outflow not being linear over the 24-hour time step. These oscillations do not take place in the next test case where the time step is chosen as one half hour.

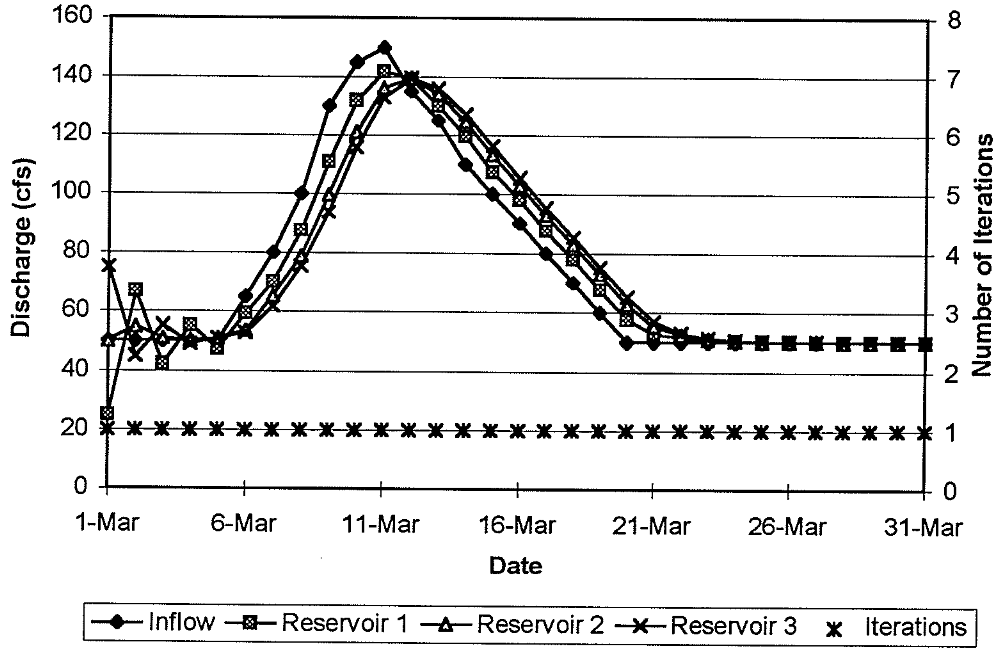

Figure 5 - Reservoir Discharge and Number of Iterations versus Time - Matrix Method: Test Case 1 - Run 1

The elevation results from this run are shown in Figure 6 with the elevation on the vertical axis and the date on the horizontal axis.

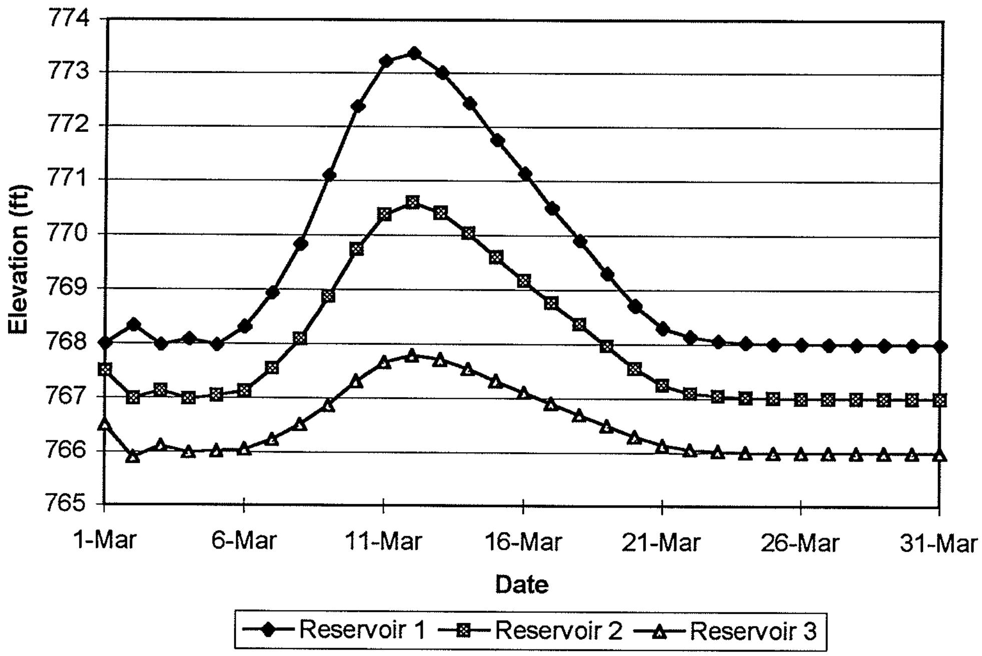

Figure 6 - Reservoir Elevation versus Time - Matrix Method: Test Case 1 - Run 1

6.1.3. Description: Matrix Test Case 1 - Run 2: (M-TC1-R2) Matrix+Linear+Rise and Fall+½hr

Run 2 is identical to Run 1 except the time step has been changed to one half hour instead of 24 hours. The purpose of this test run is to show the elimination of the oscillation in discharges and to show how the discharge error remains unchanged with a shortened time step.

6.1.4. Results: Matrix Test Case 1 - Run 2: (M-TC1-R2) Matrix+Linear+Rise and Fall+½hr

The discharges for Run 2 are shown in Figure 7 for the first seven days of the run. The outflow from Reservoir 1 rapidly increases from 25 cfs to just above 50 cfs in only half a day. During the second half of the day, the outflow is fairly constant. The oscillations observed in Test Case 1 were caused by this bilinear shape in the outflow hydrograph. This outflow hydrograph would not be well represented by a linear approximation.

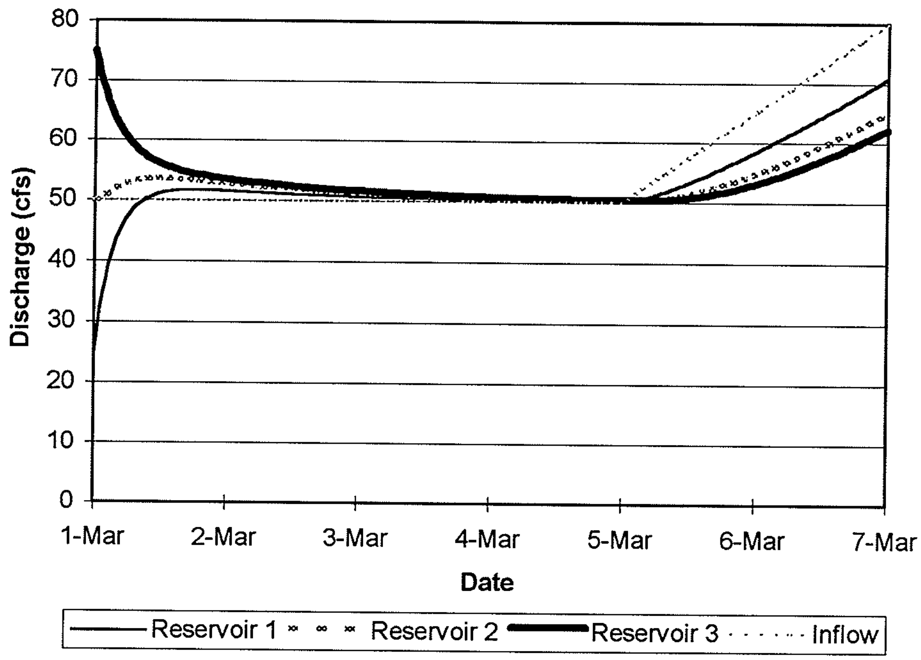

Figure 7 - Reservoir Discharge versus Time: Matrix Method: Test Case 1 - Run 2

6.1.5. Observed Discharge Errors

The discharge error for both Test Case 1 - Run 1 and Test Case 1 - Run 2 was plotted in Figure 8 for the first four days of the run. The absolute value of the discharge error is less than 0.02 cfs, with outflows ranging from 25 to 150 cfs. This accuracy is considered excellent (0.04% of a 50 cfs discharge).

There is a lower limit to the tolerance obtainable by the matrix method. It may be noted that there was no decrease in discharge error with the shortening of the time step from 24 hours to one half hour. If a tolerance of less than 0.02 cfs had been requested for Test Case 1 Run 1, then shortening the time step would not have produced acceptable discharge errors and the procedure would never proceed to the next time step.

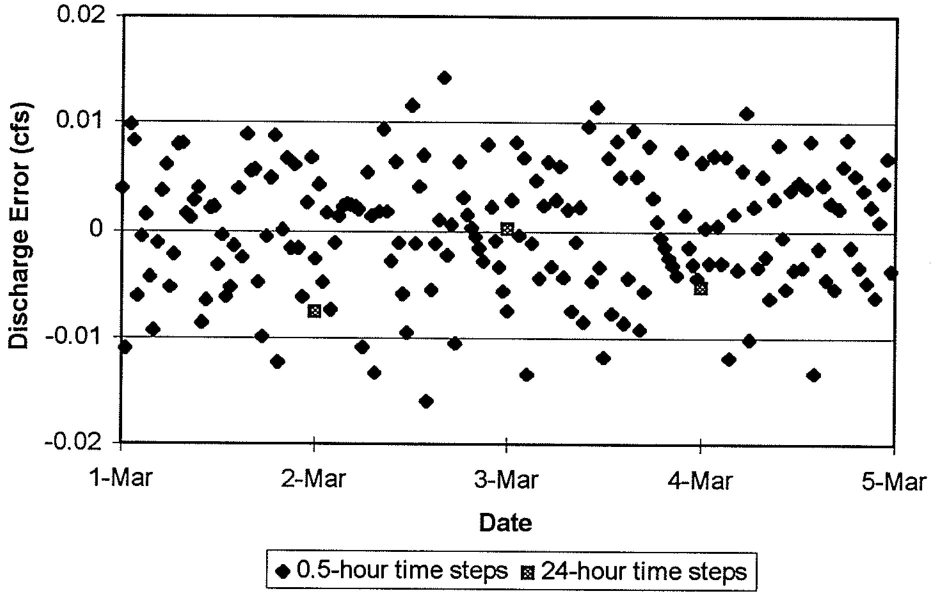

Figure 8 - Discharge Error and the Time Step: Test Case 1 - Runs 1 and 2

6.1.6. Description: Matrix Test Case 1 - Run 3: (M-TC1-R3) Matrix+Linear+Zero Inflow+24hr

Test Case 1 - Run 3 was selected to observe the performance of the method as the reservoir outlets change successively from backwater-affected to free-flow to zero-flow, as the water levels change. Run 3 uses the same A- and B-ratios for the outlet structures but the crest elevations for Reservoirs 2 and 3 have been lowered by one foot so that the change from backwater-affected to free-flow and then to the zero-flow state can be readily observed on Reservoir 1. These crest elevations and the starting water levels are summarized in Table 5. There is no inflow to Reservoir 1 so the discharge and elevations are expected to drop quickly.

Table 5 - Test Case 1 - Run 3 - Crest Elevations and Starting Water Levels

Reservoir Number	Location	Crest Elevation (ft)	Starting Water Level (ft)
1	Upstream	766.00	768.50
2	Middle	764.00	766.10
3	Downstream	764.00	765.10

6.1.7. Results: Matrix Test Case 1 - Run 3: (M-TC1-R3) Matrix+Linear+Zero Inflow+24hr

The discharge results are displayed in Figure 9 and the elevation results in Figure 10. The outlet structure for Reservoir 1 changes from backwater-affected to free-flow during the first day. During the second day, the water level drops to the crest elevation and there is zero discharge.

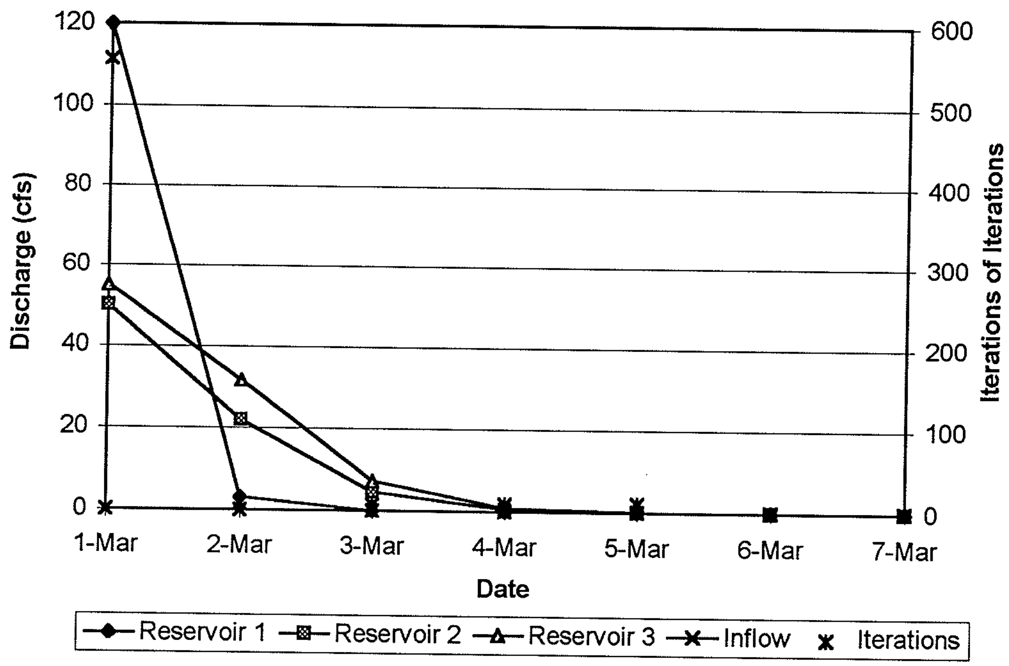

Figure 9 - Reservoir Discharge and Number of Iterations versus Time - Matrix Method: Test Case 1 - Run 3

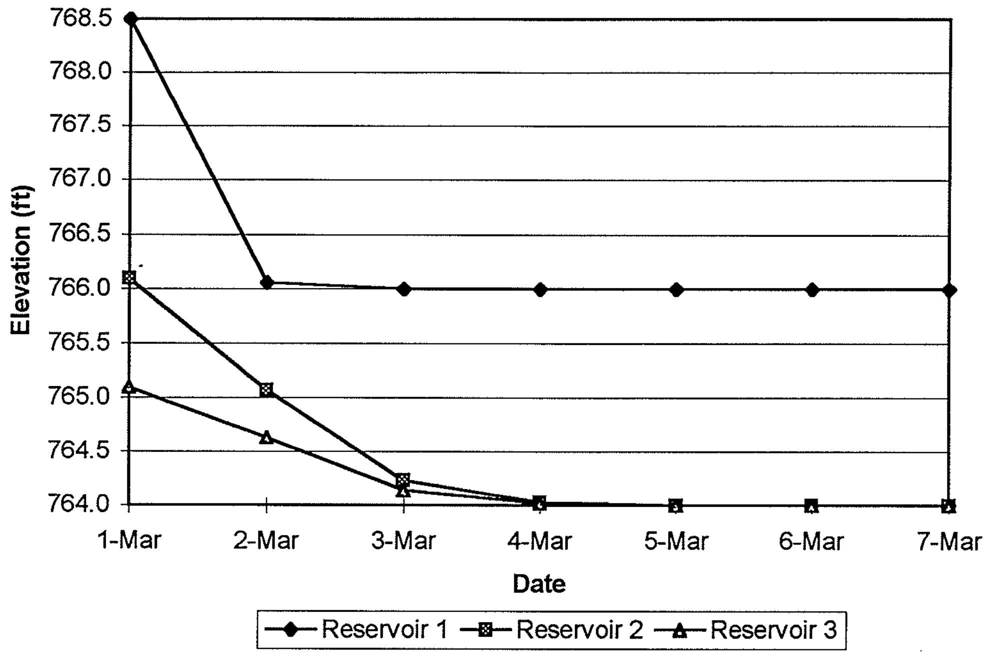

Figure 10 - Reservoir Elevation versus Time - Matrix Method: Test Case 1 - Run 3

6.1.8. Boundary Conditions on the Stage-Discharge Curves

There are two boundary conditions in the stage-discharge relationship that may cause problems with determining the EEOP outflows. The first case occurs when the upstream water level drops below the crest elevation and the A-ratio becomes zero. The second occurs when the downstream elevation drops low enough that the outlet no longer is submerged and the value of the B-ratio equals zero. The SOP A- and B-ratios are calculated every time the procedure moves to the next time step and these values are used to determine the EEOP outflow. If the iteration fails, it is usually because the values of the A- and B-ratios at the EOP have changed significantly.

On the first day of this test case (M-TC1-R3) the outlet for Reservoir 1 changes from backwater-affected to free-flow as the downstream water level drops below the crest elevation. To illustrate this effect on the number of iterations, the B-ratios were calculated and plotted on Figure 11. It can be seen that the values change from the initial value of -2.0 to its free-flow value of zero. The B-ratio is indicated on the left vertical axis and the downstream elevation is indicated on the right vertical axis. The horizontal axis indicates the iteration counter. This counter begins with a zero value at the SOP and increases with each iteration whether successful or unsuccessful.

At the start of the first day, the iteration number is equal to zero, the time step is 24 hours (1440 minutes), the downstream elevation equals 766.1 ft and the SOP B-ratio is calculated as -2.0 cfs/acre-ft. These values are plotted in Figure 11 at Day 1 iteration number value of zero. The EEOP outflow is determined, the EOP elevations and storages and the CEOP outflow are calculated, the discharge error is too great and the iteration fails. To investigate why the iteration fails, the resulting B-ratio at the EOP elevation is calculated. Both the EOP B-ratio and the EOP downstream elevation are plotted with the iteration counter equal to one. The downstream elevation is less than the crest elevation of 766.0 ft and, therefore, the EOP B-ratio equals 0.0 cfs/acre-ft.

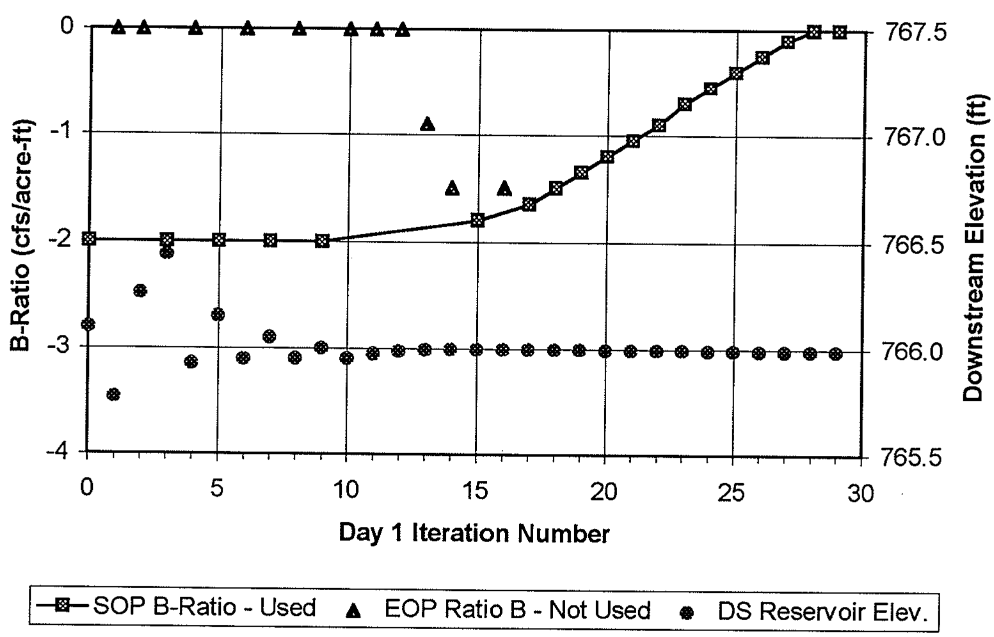

Figure 11 - Reservoir 1, B-Ratio from Backwater-Affected to Free-Flow
Matrix Method: Test Case 1 - Run 3

On the second iteration, the EOP B-ratio is zero not because the downstream elevation is below the crest but because the downstream elevation was greater than the upstream elevation. Again, the iteration fails.

On the third iteration, the EOP downstream elevation is greater than the crest elevation, the EOP B-ratio is still -2.0 cfs/acre-ft and the iteration succeeds. The EOP B-ratio now becomes the SOP B-ratio and is plotted on the SOP B-ratio line.

Until Iteration 13, each time the downstream water level is less than the crest elevation, the EOP B-ratio equals zero and the iteration fails. If the downstream elevation is above the crest and the B-ratio equals -2.0 cfs/acre-ft, the iterations succeed. At Iteration 13, the downstream water level is less than 0.01 feet below the crest. As the A- and B-ratios are calculated by varying the elevation by 0.01 feet, the resulting B-ratio begins to be an combination of the backwater-affected value of -20.0 and the free-flow value of 0.0 cfs/acre-ft. At Iteration 15, the value of the B-ratio is close enough to the SOP value (Iteration 9) that the iteration succeeds. Iteration 16 fails again but, from Iteration 17 onward, each iteration succeeds, with the B-ratio slowly changing towards a zero value.

At Iteration 29, the B-ratio value has reached a value of 0.0 and it remains at this value for the rest of the routing periods. With each failed iteration, the time step is shortened and, at Iteration 16, the time step is approximately 1 min.

During the second day of routing, the water level on Reservoir 1 drops to the crest elevation. Figure 12 illustrates the change in the A-ratio as the water level decreases with each successful iteration. Note that the A-ratio never reaches a zero value. Even when the water level is at the crest elevation, the A-ratio remains at a value of 2.0 because the ratio is calculated in the direction of increasing elevation. At Iteration 2, the EOP elevation is estimated below the crest but the iteration fails and the A-ratio of less than 2.0 is never used. At Iteration 4, the ratio is not used because, although the discharges for station 250 are within the allowable tolerance, the time step has to be shortened because another of the reservoirs fails the tolerance test.

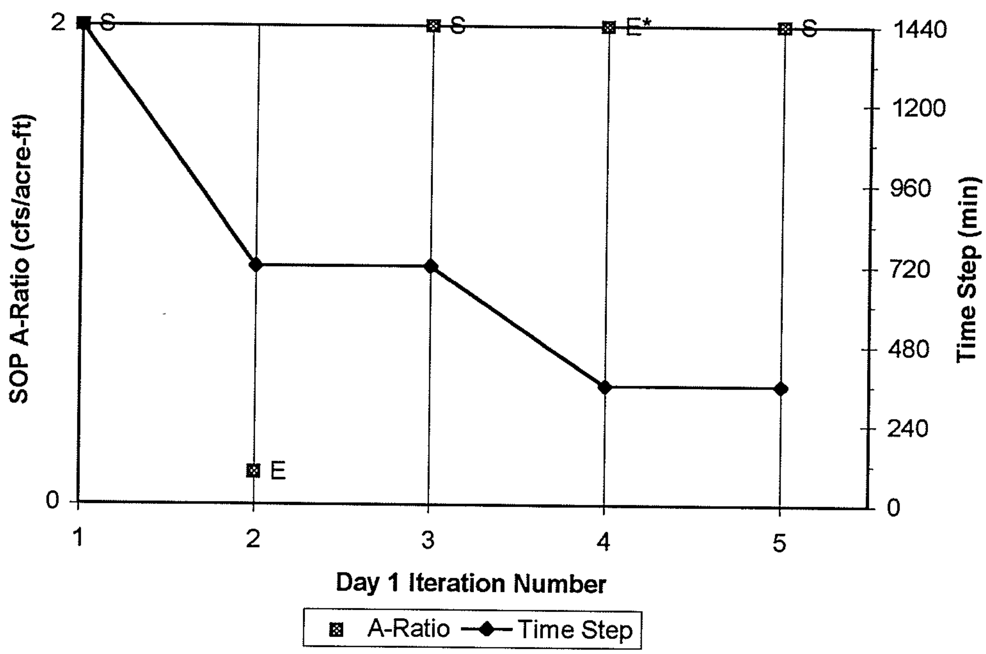

Figure 12 - A-Ratio and the Time Step - Matrix Method: Test Case 1 - Run 3

6.2. Matrix Method: Non-linear Example - Test Case 2.

The second test case includes three very small reservoirs with areas of approximately 29 acres each. Five-foot-wide stop-log weirs are used as outlet structures for each reservoir. The crest elevations of the weirs and starting water levels were given previously in Table 4. The stage-discharge and stage-storage curves for Reservoir 2 (middle reservoir) are shown in Figure 13 and Figure 14. The stage-discharge curve and stage-storage curve for Reservoir 1 are similar except that they are shifted vertically up one foot. The stage-storage curve for Reservoir 3 is the same as Reservoir 2 and the stage-discharge curve is the same as for Reservoir 2 with a downstream elevation of 765 ft.

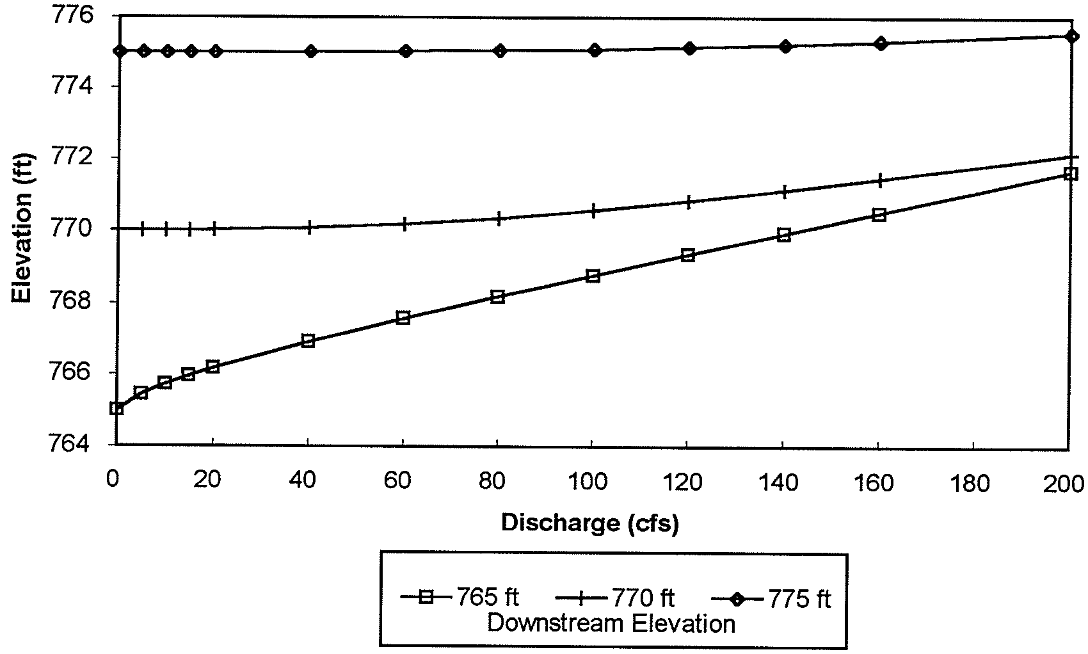

Figure 13 - Reservoir Elevation versus Discharge (Reservoir 2) - Matrix Method: Test Case 2

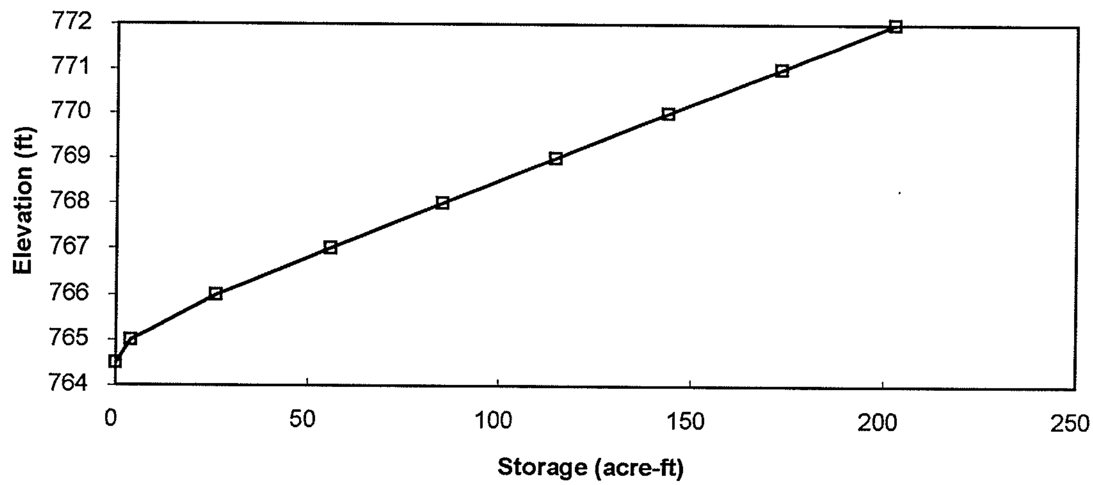

Figure 14 - Stage versus Storage (Reservoirs 2 and 3) - Matrix Method: Test Case 2

The use of a Taylor-Series approximation to represent the discharge-storage relationship requires that the δdischarge/δstorage ratio (for both upstream and downstream) remain nearly constant over the time step. Figure 15 illustrates how the δdischarge/δu/s storage ratio (A-ratio) changes with elevation for Reservoir 2 for two different downstream water conditions. The curve to the left for a downstream water elevation of 765 ft represents downstream water levels less than the weir crest elevation of 765 ft (free-flow conditions). The A-ratio remains almost constant at a value of approximately 1.1 for elevations greater than 768 ft. Referring back to figures Figure 13 and Figure 14 , it can be seen that the slopes of the discharge and storage curves become constant at elevations above 768 ft, while the slopes of both curves are changing below this elevation.

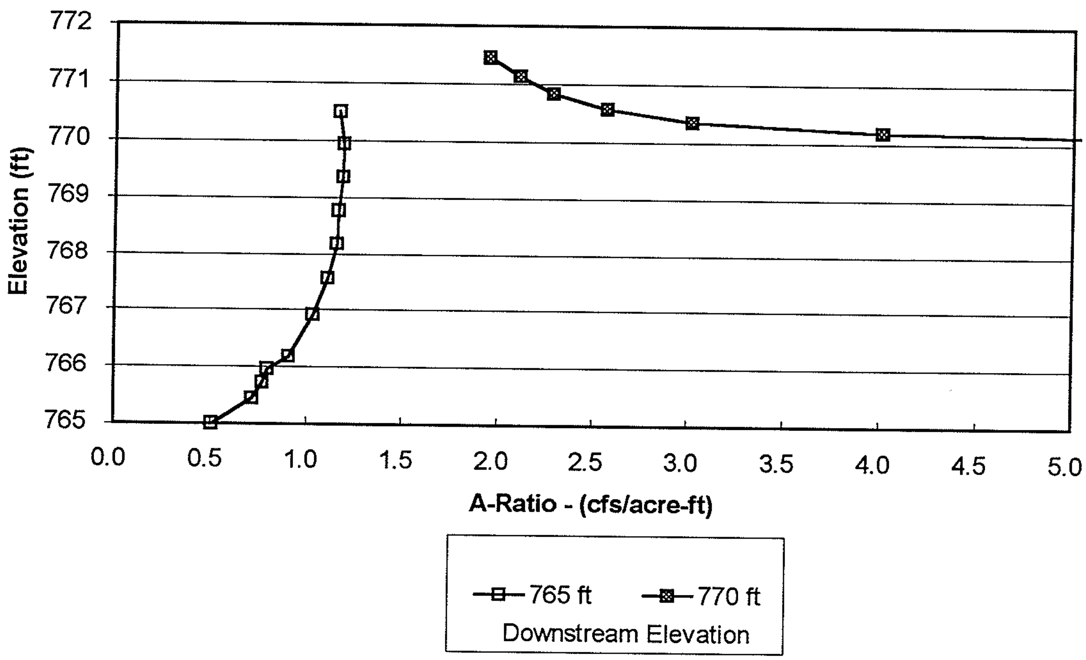

Figure 15 - Stage versus A-Ratio (Reservoir 2) - Matrix Method: Test Case 2

The curve to the right (downstream elevation 770 ft) in Figure 15 illustrates how the A-ratio changes with elevation for submerged conditions. When the upstream water level is only slightly higher than the downstream level, the A-ratio is very large (170 cfs/acre-ft, which is far beyond the limits of Figure 15) and then reduces rapidly to values near 2 cfs/acre-ft. This region with such a rapidly varying ratio will require very short time steps in order to obtain a constant ratio.

Figure 16 illustrates how the δdischarge/δds storage ratio (B-ratio) varies with elevation for different downstream water conditions. The value of the B-ratio is negative because an increase in water level results in a decrease in discharge. The curve to the right for a downstream elevation of 765 ft represents the downstream water condition moving from a free-flow to a backwater-affected condition. This curve is almost vertical, indicating that the B-ratio does not vary significantly as the downstream water levels change. An unchanging value for this B-ratio allows fairly large time steps to be used in the routing calculations.

The curve to the left for a downstream elevation of 770 ft represents a backwater control condition downstream. As with the A-ratio, the B-ratio changes rapidly when the upstream and downstream water levels are almost the same. For higher upstream water levels, a change in downstream storage has a far smaller effect on the discharge, as indicated by the decrease value of the B-ratio.

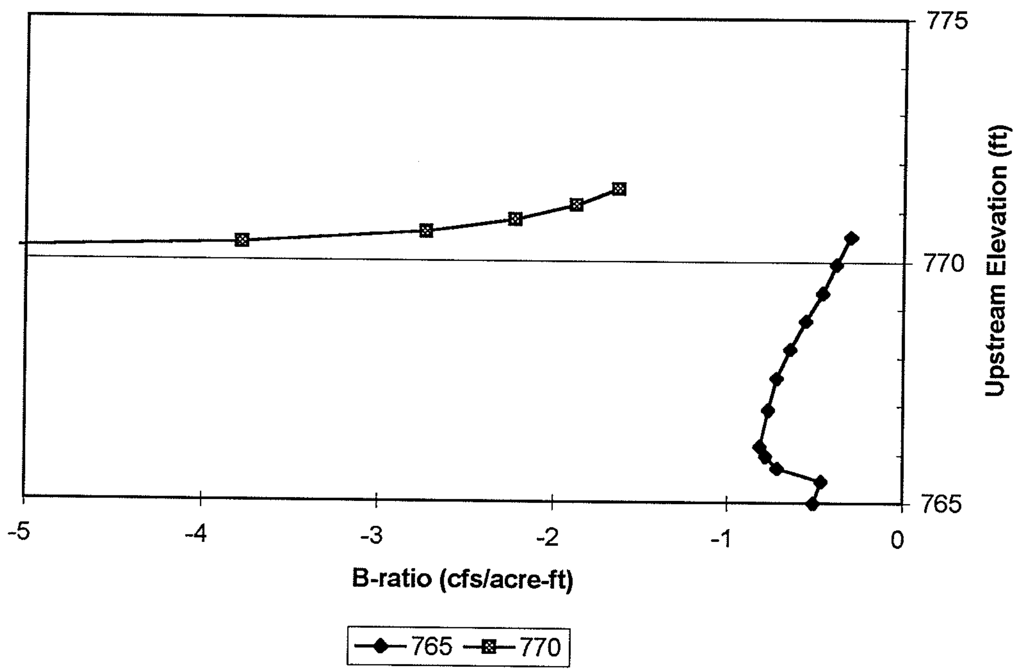

Figure 16 - Stage versus B-ratio (Reservoir 2) - Matrix Method: Test Case 2

6.2.1. Description: Matrix Test Case 2 - Run 1: (M-TC2-R1) Matrix+Non-linear+Rise and Fall+24hr

Test case 2 - Run 1 has the same inflow hydrograph and starting elevations as Test Case 1 - Run 1. However, the stage-discharge curves are from stop-log weir structures with a five-foot crest length and the stage-storage curves are from a typical small reservoir with an area of approximately 29 acres.

6.2.2. Results: Matrix Test Case 2 - Run 1: (M-TC2-R1) Matrix+Non-linear+Rise and Fall+24hr

The discharge results and number of iterations from this run are plotted in Figure 17 and the elevations are plotted in Figure 18. When a similar run was made in the linear model, each time period required only one iteration to complete. With this "real-life" non-linear case, up to 36 iterations per day were required.

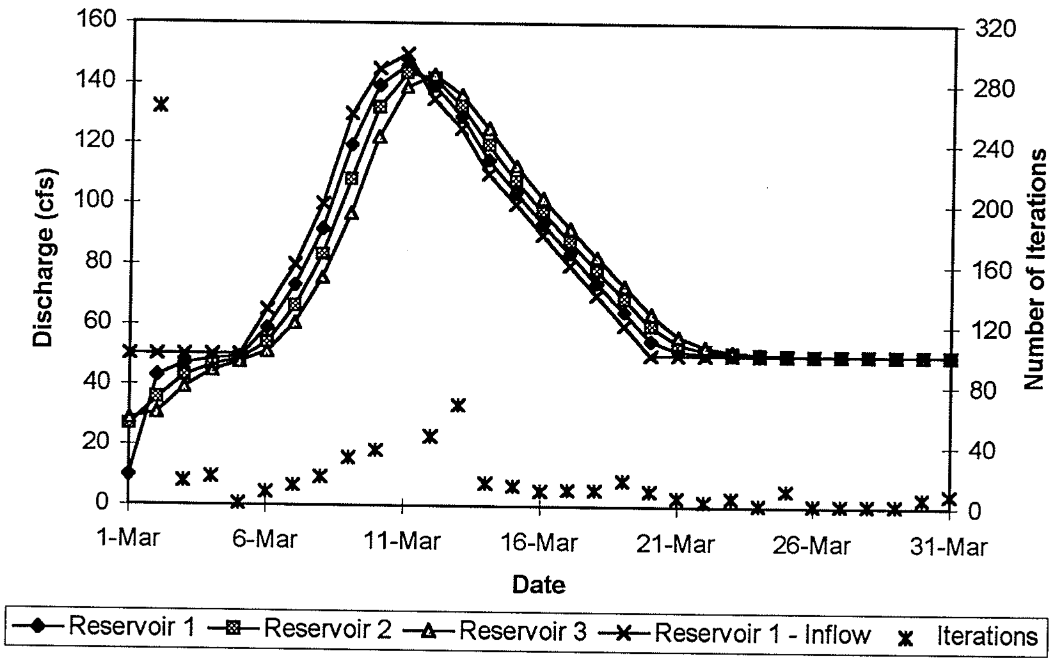

Figure 17 - Reservoir Discharge and Number of Iterations versus Time - Matrix Method: Test Case 2 - Run 1

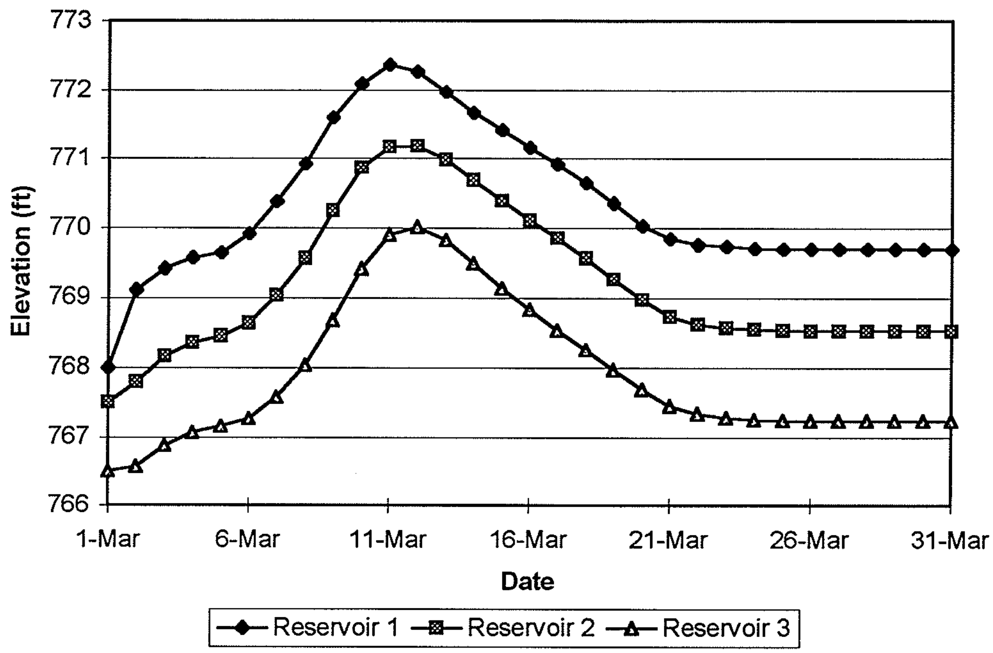

Figure 18 - Reservoir Elevation versus Time - Matrix Method: Test Case 2 - Run 1

Figure 19 and Figure 20 display how the A- and B-ratios change during the second day of the routing. The reservoir elevation is plotted with the A-ratio and the downstream reservoir elevation is plotted with the B-ratio to illustrate how the ratios are changing with elevation. For each successful iteration, the SOP ratio that was used in the matrix calculations is plotted. If the iteration fails, the EOP ratios are calculated and plotted to investigate why the iteration failed. At Iterations 2 and 3, the large changes between the SOP and EOP ratios cause the iteration failures. Note how there is a significant change in both the reservoir elevation and the downstream reservoir elevation. Iteration 3 is successful because the EOP ratios are much closer to the SOP ratios and the time step is much shorter. With a short time step, the EOP storage changes less and, thus, the CEOP discharge changes less from the SOP to the EOP. Since the discharge is changing less, there is less opportunity for the discharge error to become larger than the acceptable tolerance. An example of this phenomenon is Iteration 8 which fails, yet Iteration 9 is successful when the ratios change almost twice as much. Referring to Table 6, the length of the time step for Iteration 9 is 1.5 hours while Iteration 8 has a time step of 3.0 hours.

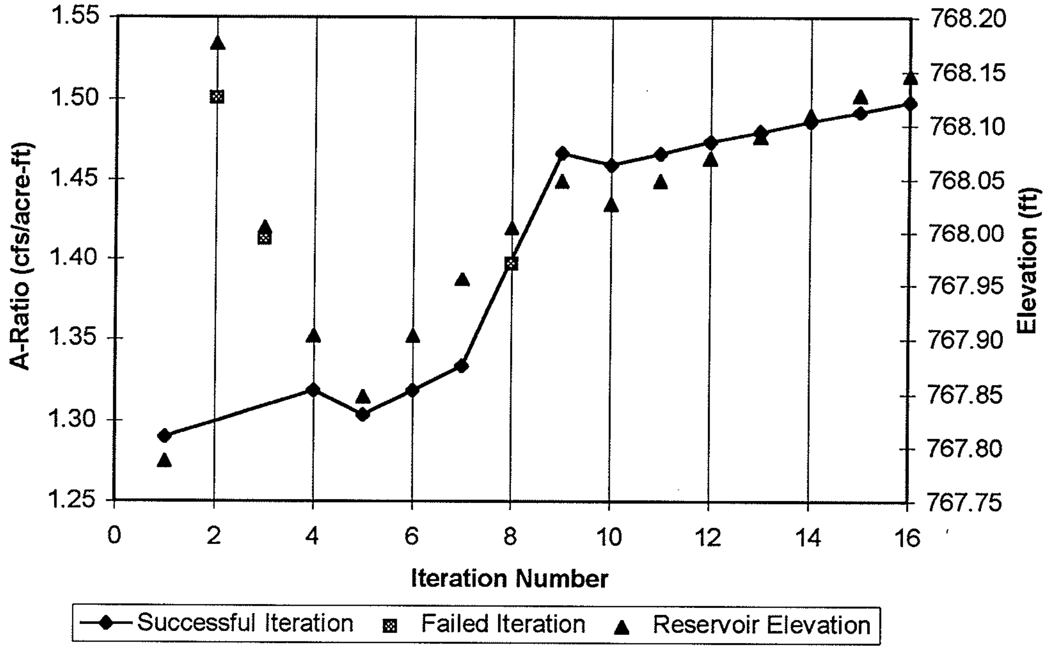

Figure 19 - Reservoir 1 - A-Ratio versus Iteration Number - Matrix Method: Test Case 2 - Run 1

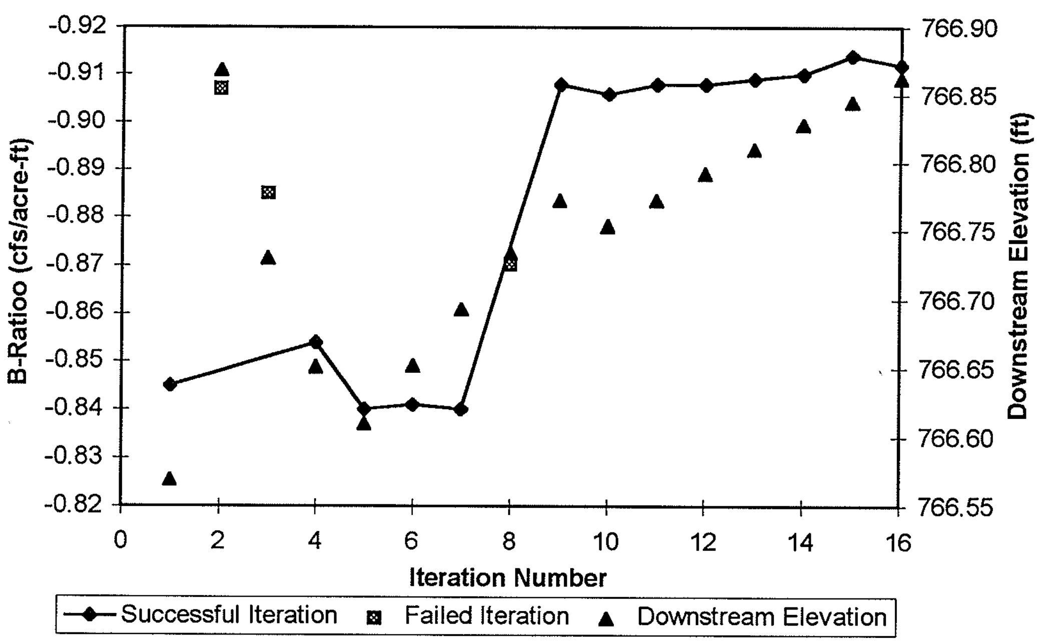

Figure 20 - Reservoir 1 - B-ratio versus Iteration Number - Matrix Method: Test Case 2 - Run 1

Table 6 - Matrix Method - Test Case 2 - Run 1 - Iteration Number and Time Step Duration

Iteration Number	Time Step (hrs)	Iteration Number	Time Step (hrs)
1	24	9	1.5
2	12	10	1.5
3	6	11	1.5
4	3	12	1.5
5	3	13	1.5
6	3	14	1.5
7	3	15	1.5
8	1.5	16	1.5

6.2.3. Description: Matrix Test Case 2 - Run 2: (M-TC2-R2) Matrix+Non-linear+Rise and Fall+½hr

This test case is not repeated for the non-linear case because the objective of illustrating the effect of a shortened time step are accomplished with M-TC1-R2. This reference is included so that the run labeling system will remain synchronized between test cases and method.

6.2.4. Description: Matrix Test Case 2 - Run 3: (M-TC2-R3) Matrix+Non-linear+Zero Inflow+24hr

The inflow and starting water level conditions for Test Case 1 - Run 3 are selected to observe the performance of the method as the reservoir outlets change from backwater-affected to free-flow to zero-flow. Again, the crest elevations for Reservoirs 2 and 3 have been lowered by one foot so that the change from backwater-affected to free-flow and then to the zero-flow condition can be readily observed on Reservoir 1. These crest elevations and the starting water levels were summarized previously in Table 5, page 32.

6.2.5. Results: Matrix Test Case 2 - Run 3: (M-TC2-R3) Matrix+Non-linear+Zero Inflow+24hr

The discharge and number of iterations for each day are displayed in Figure 21 and the reservoir elevations are displayed in Figure 22. Approximately 120 iterations are required to complete the routing for March 1 and 2 because of the rapid changes in outflow from Reservoir 1. After March 7, when the reservoirs approach their crest elevations and the system approaches a steady-state condition, only one iteration is required per time step.

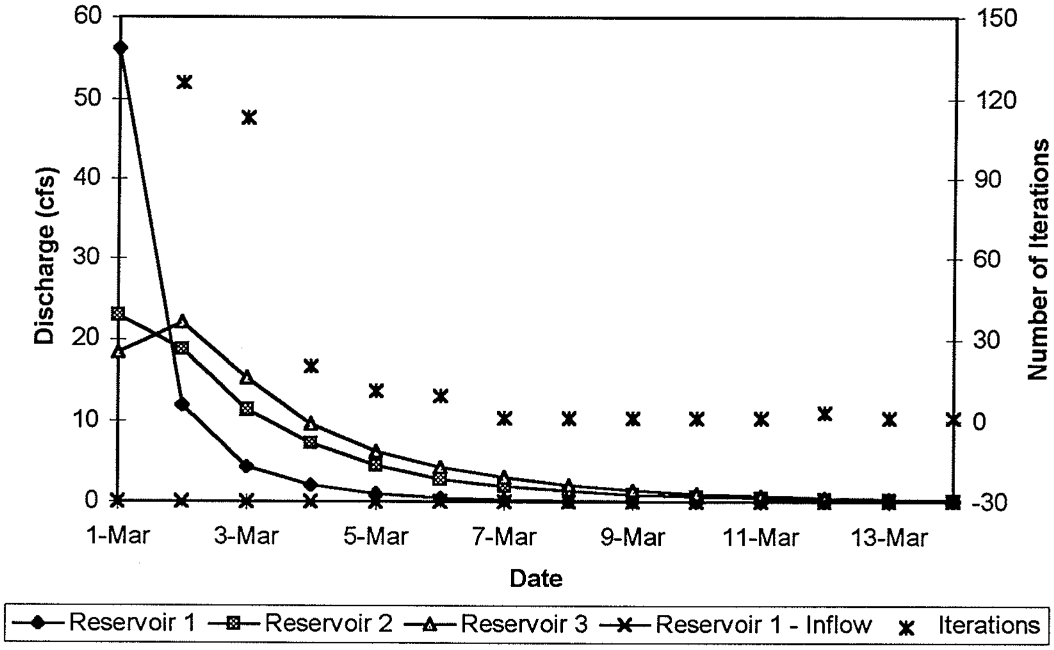

Figure 21 - Reservoir Discharge and Number of Iterations versus Time - Matrix Method: Test Case 2 - Run 3

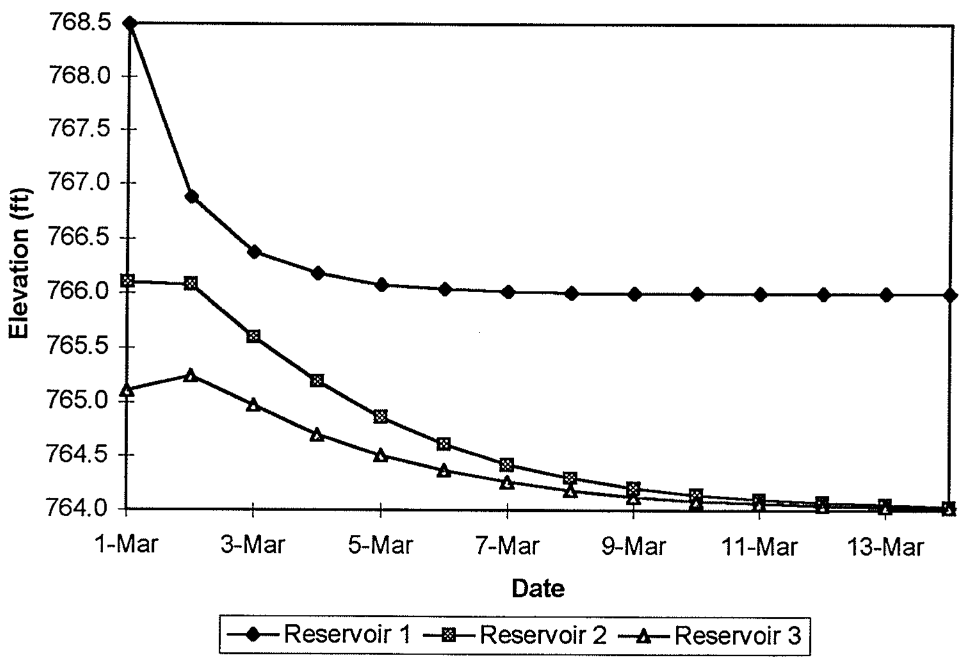

Figure 22 - Reservoir Elevation versus Time - Matrix Method: Test Case 2 - Run 3

7. Delta Factor Test Cases

The test cases and runs used to display the characteristics of the matrix method are used again for the delta factor method. The test case and run numbers are kept the same for easy comparison between the two methods. Most of the discussion of the method is included in the Test Case 1 results. The Test Case 2 runs are included primarily for comparison against the matrix method runs. The delta factor method operates equally as efficiently in the linear Test Case 1 as in the non-linear Test Case 2.

7.1. Delta Factor: Linear Model - Test Case 1

As described previously in the section, "Matrix Method: Linear Model - Test Case 1", page 26, the stage-discharge and stage-storage relationships are linear. Linear relationships were chosen to show the operation of the matrix method but they also provide good working examples for the delta factor method.

7.1.1. Description: Delta Test Case 1 - Run 1: (DF-TC1-R1) Delta+Linear+Rise and Fall+24hr

The inflow to the upstream reservoir ranges from 50 to 150 cfs (see Figure 23) and a time step of one day is used for this run. The starting reservoir levels and crest elevations were listed previously in Table 4, page 27. The run begins at an unsteady-state with a constant 50 cfs inflow and then the hydrograph rises and falls back to a 50 cfs inflow.

7.1.2. Results: Delta Test Case 1 - Run 1: (DF-TC1-R1) Delta+Linear+Rise and Fall+24hr

The discharge results are illustrated in Figure 23 and the elevation results in Figure 24. As expected, these results are almost identical to the results from the matrix method (see Figure 5 and Figure 6, page 28). The oscillating discharges observed until March 6 are the result of choosing the one-day time step while the outflow is changing rapidly (see the section ,"The Requirement for Short Time Steps in All Reservoir Routing", page 6, for a discussion of this type of oscillating outflows). The number of iterations required for each time step is indicated on the vertical axis on the right of Figure 23. Each time step requires about 20 iterations.

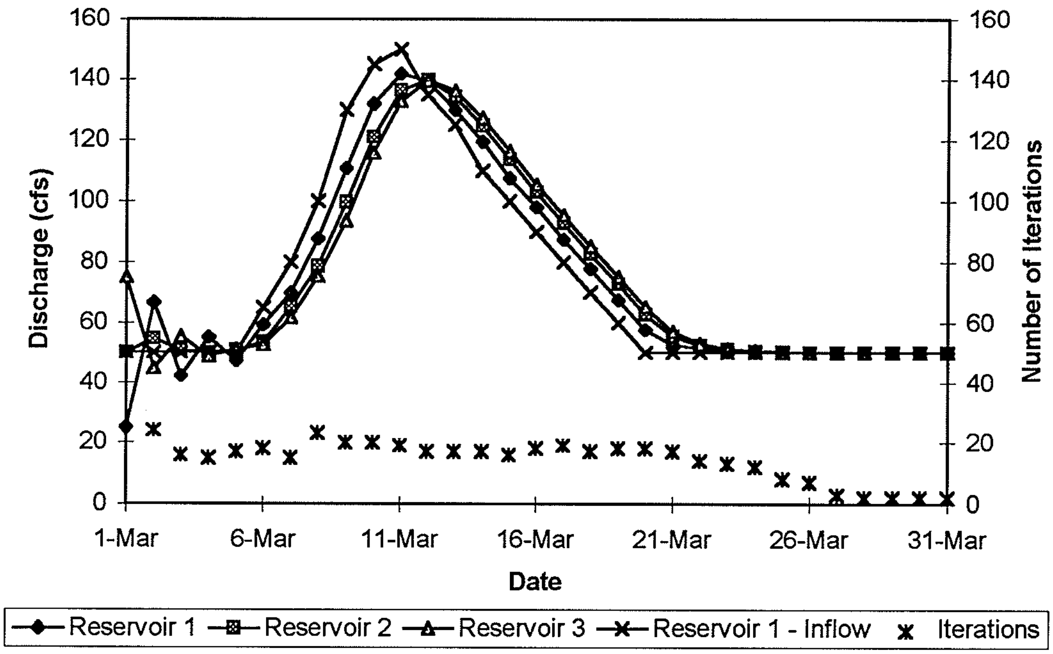

Figure 23 - Reservoir Discharge and Number of Iterations versus Time - Delta Factor: Test Case 1 Run 1

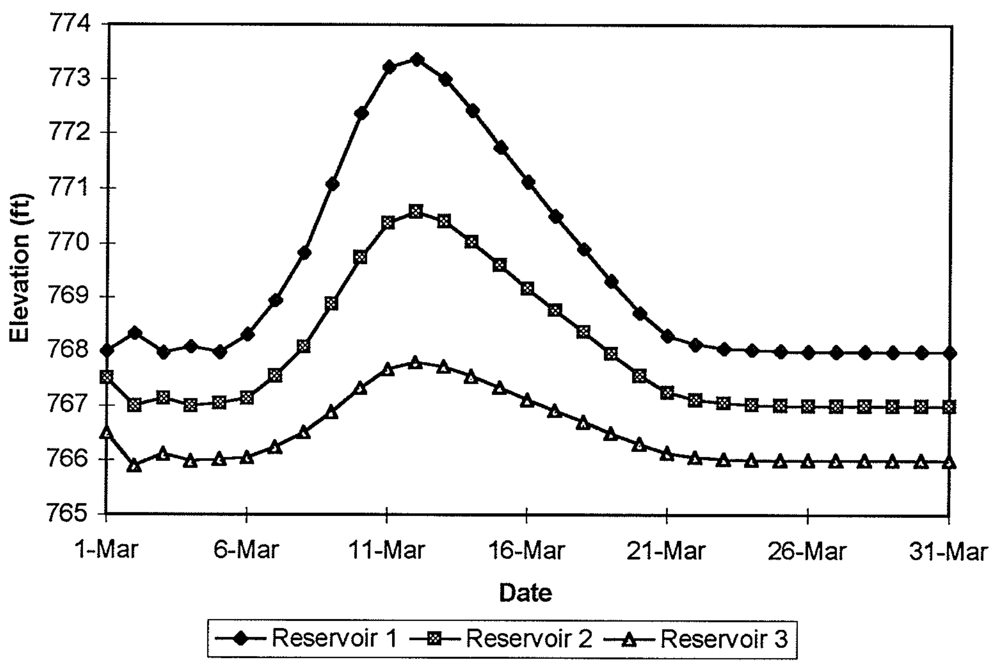

Figure 24 - Reservoir Elevation versus Time - Delta Factor: Test Case 1 - Run 1

7.1.3. Convergence between CEOP and EEOP Discharges

Figure 25 illustrates the convergence between EEOP and CEOP outflows for Reservoir 2. Each increment between the graphic symbols on the horizontal axis is one iteration. The first EEOP value for each time step equals the SOP outflow for the reservoir. The initial assumption that the outflow does not change often is not correct and results in a dramatically different value for the CEOP outflow. This frequently leads to oscillations for the first two or three iterations before a more consistent convergence develops. This initial assumption is used because the reservoir routing models are commonly used for annual reservoir operations. Throughout a typical year, there are long periods when the outflow does not change and the assumption is correct.

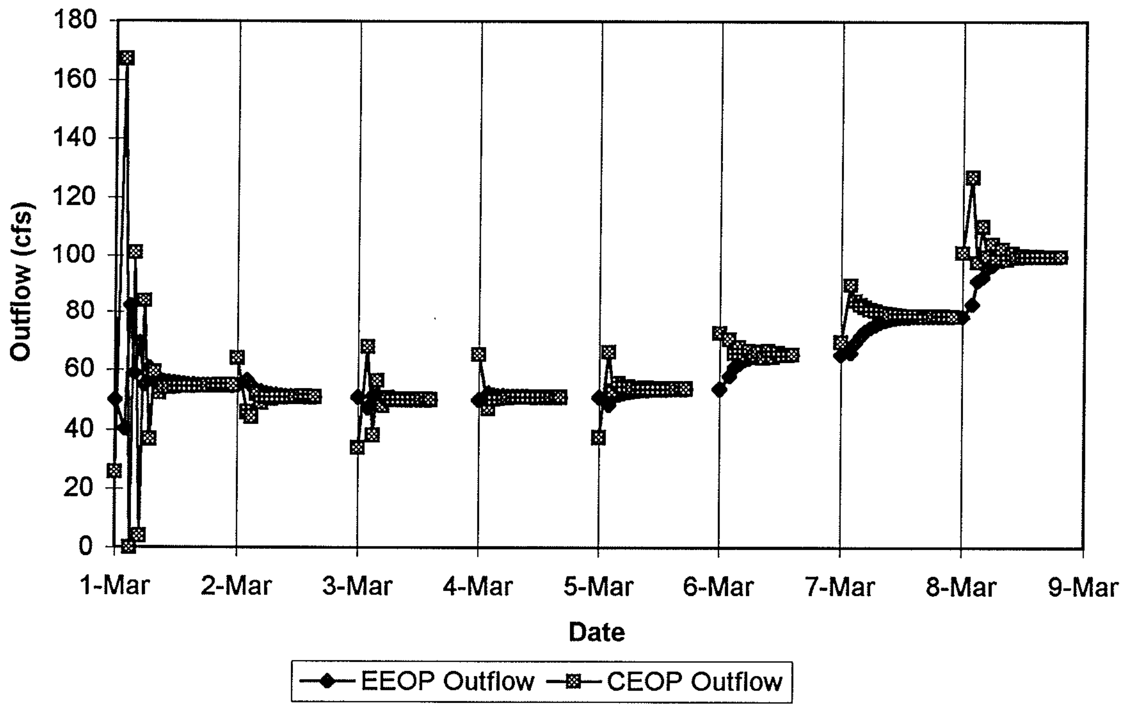

Figure 25 - Reservoir 2 EEOP and CEOP Outflow Convergence - Delta Factor: Test Case 1 - Run 1

Figure 26 illustrates the convergence between the EEOP outflow and the CEOP outflow for both Reservoir 1 and Reservoir 2 during a typical routing period (March 8). It is important that the two reservoirs converge simultaneously because the outflow from Reservoir 1 is the inflow for Reservoir 2. The horizontal axis represents the outflow from Reservoir 1 and the vertical axis represents the outflow from Reservoir 2. The figure shows both the EEOP and CEOP values. The EEOP outflow is indicated as a diamond and the CEOP outflow is indicated with a square. For the first iteration, both are labeled with the numerals "1". Considering the second iteration labeled with the numerals "2", Reservoir 1 is converging while Reservoir 2 is diverging. The convergence of the outflows is not a consistent trend and small changes in the EEOP discharge can result in much larger changes in the CEOP value. The convergence paths yield different shapes depending upon the backwater relationships upstream and downstream of the reservoir and the operating conditions of other reservoirs in the system.

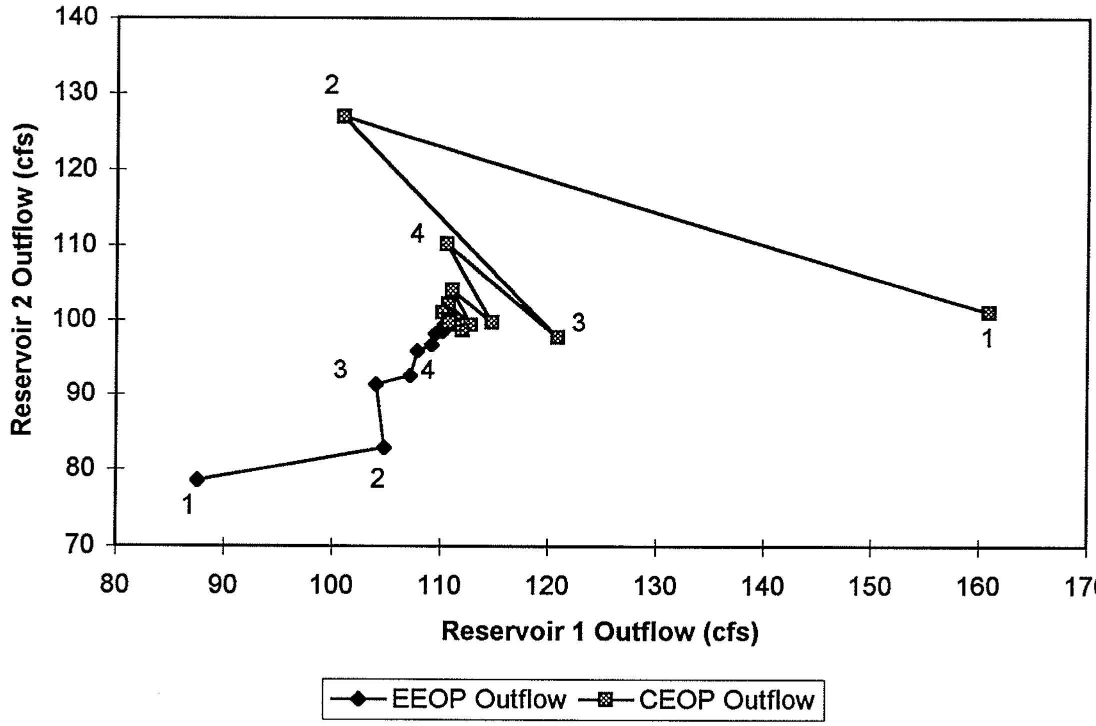

Figure 26 - Convergence of EEOP and CEOP Outflows for Reservoirs 1 and 2. (March 8).

The area between Points 4 on Figure 26 has been expanded in Figure 27 to show the convergence just before the successful iteration. Again, it can be observed in this figure that small changes in the EEOP discharges result in larger changes in the CEOP discharges.

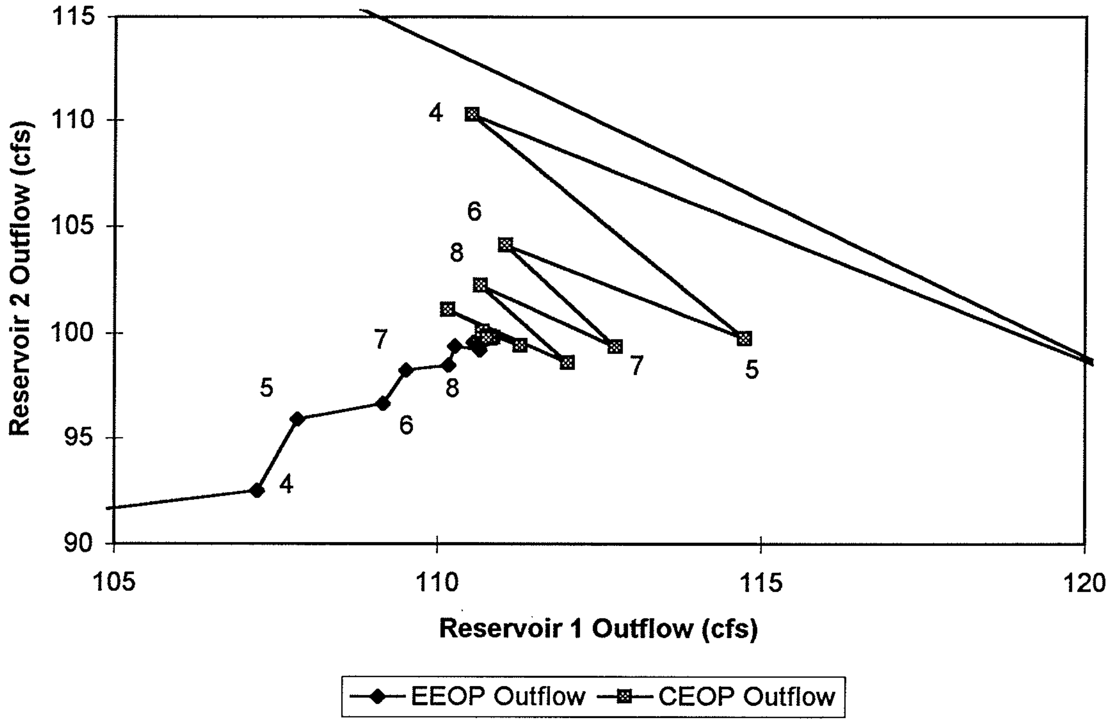

Figure 27 - Convergence of EEOP and CEOP Outflows for Reservoirs 1 and 2 (March 8 - expanded).

7.1.4. Changing the Delta Factor

As discussed previously in the section, "Determining Values for the Delta Factor", page 19, the delta factor is continuously changed throughout the iteration process. The method of changing the delta factor was derived through experience with many reservoir routing examples. Figure 28 illustrates how the delta factor changes during each time step. The interval between each data point on the horizontal axis is one iteration. The pattern exhibited from March 1 to March 7 is the most typical pattern for the changes in the delta factor. First, the delta factor increases to prevent the oscillation between EEOP and CEOP outflows. Once the oscillation stops, the EEOP and the CEOP outflows merge more consistently and the delta factor decreases. Finally, as the EEOP and CEOP outflows merge more closely together, the delta factor increases, yielding small changes in the EEOP outflow. Figure 29 illustrates the delta factor and discharges changing together.

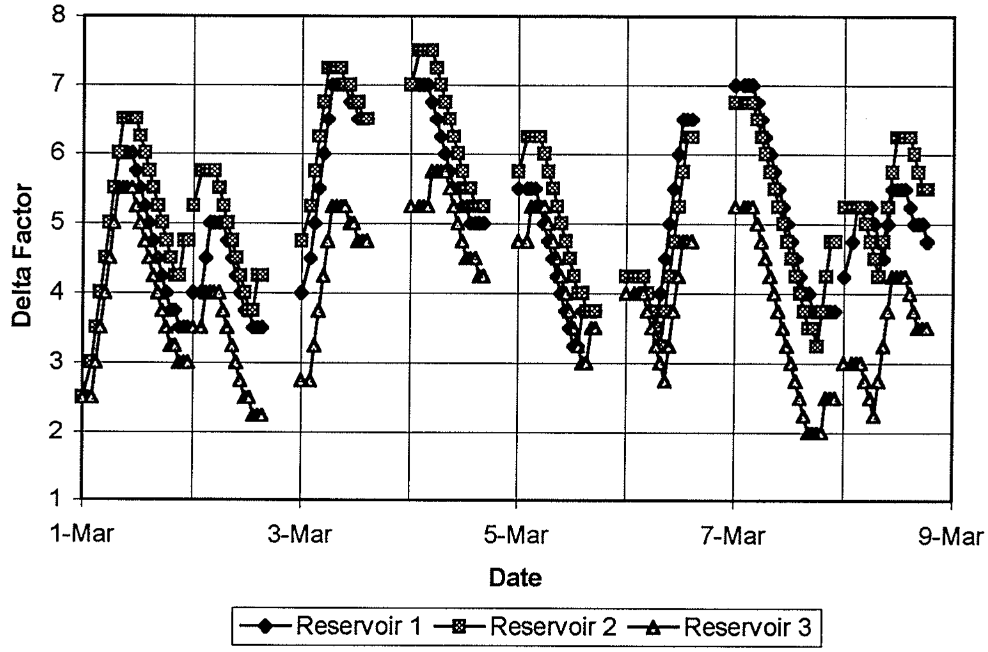

Figure 28 - Delta Factor versus Time - Delta Factor: Test Case 1 - Run 1

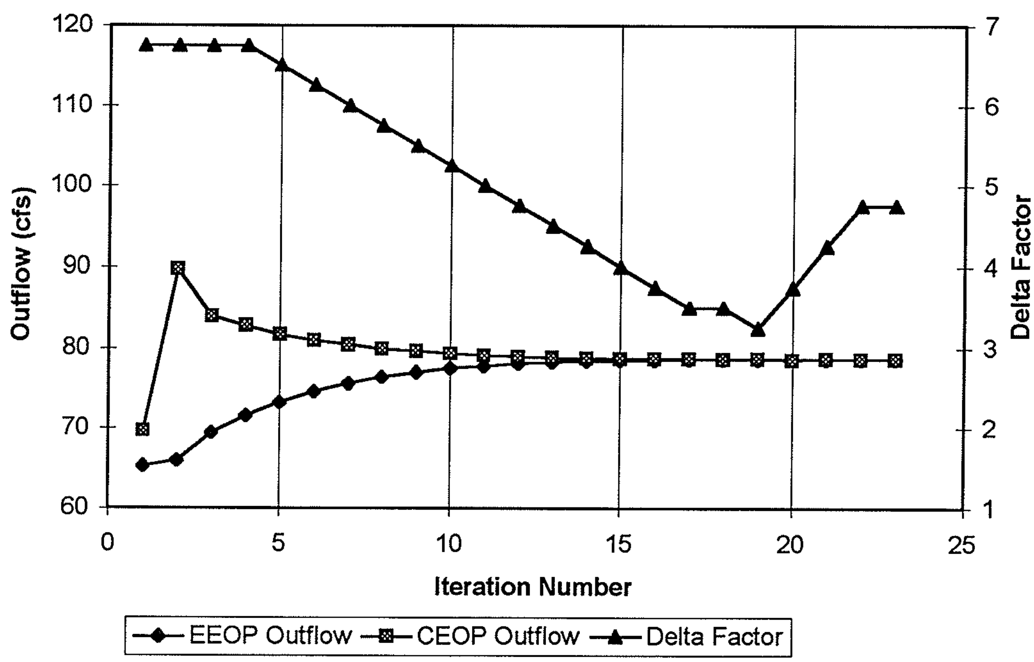

Figure 29 - EEOP, CEOP and Delta Factor versus Iteration Number (March 7) - Delta Factor: Test Case 1 - Run 1

Since all three reservoirs are connected via backwater-affected structures, it is common for the delta factor to change similarly for all reservoirs. In general, the delta factor values for Reservoir 2 should remain higher than for the other two reservoirs because the inflow and outflow constantly change throughout the time period. Reservoir 1 should have the next higher delta factor values because it has a backwater controlled outlet but the inflow is steady throughout the time period (user-defined inflow hydrograph). Lastly, Reservoir 3 has the lowest delta factor values because it has a free-flow outlet and, therefore, it is the most stable. The exceptions to this, again, are on March 1 and 2 when Reservoir 1 is in the unsteady state and, therefore, the delta factor value for this reservoir is the highest.

7.1.5. Description: Delta Test Case 1 - Run 2: (DF-TC1-R2) Delta+Linear+Rise and Fall+½hr

Run 2 is identical to Run 1 except that the time step has been changed to one half hour instead of 24 hours. The purpose of this test case is to illustrate the effect of the choice of time steps on the number of iterations required. Secondly, this case shows again that the oscillation in discharges in the first five days is eliminated with the use of a shorter time step.

7.1.6. Results: Delta Test Case 1 - Run 2: (DF-TC1-R2) Delta+Linear+Rise and Fall+½hr

The discharge results are illustrated in Figure 30 and the elevation results in Figure 31. As expected, these results are almost identical to the results from the matrix method (see Figure 7, page 3). The number of iterations required for each time step is indicated on the vertical axis on the right of Figure 30. Each time step requires about 10 iterations, except for the period from March 2 to 4 where the reservoir system is in a steady state and only one iteration is required. Referring back to the matrix method Test Case 1 - Run 1, 20 iterations are required for each time step. Therefore, the total number of iterations required for Run 2 with half hour time steps is more than required for Run 1 with 24 time steps.

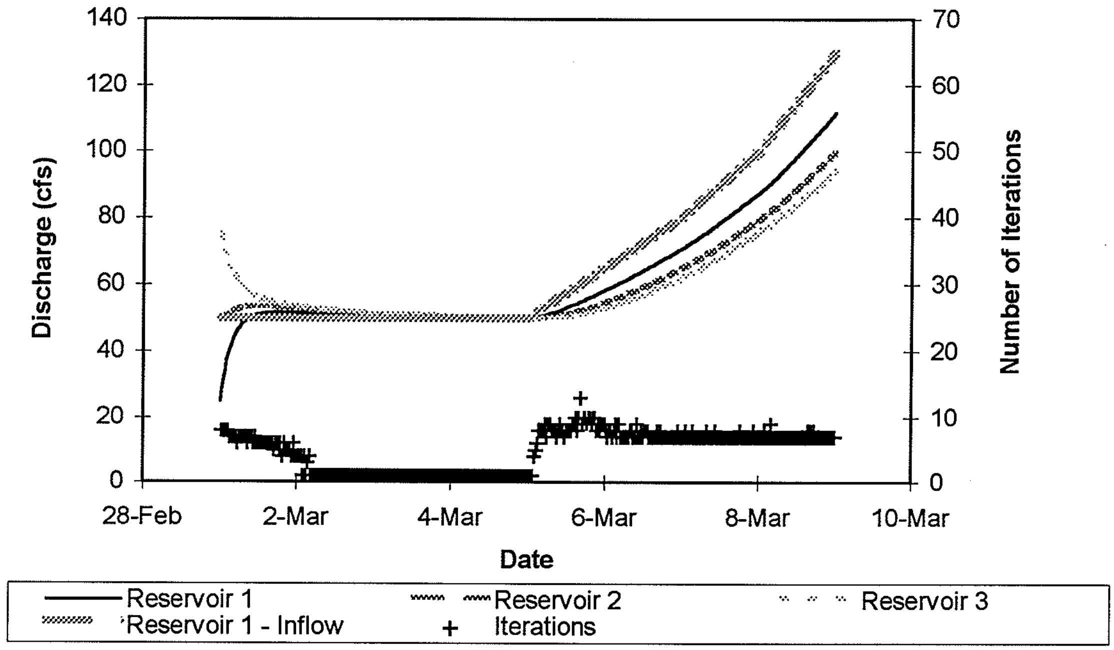

Figure 30 - Reservoir Discharge and Number of Iterations versus Time - Delta Factor: Test Case 1 - Run 2

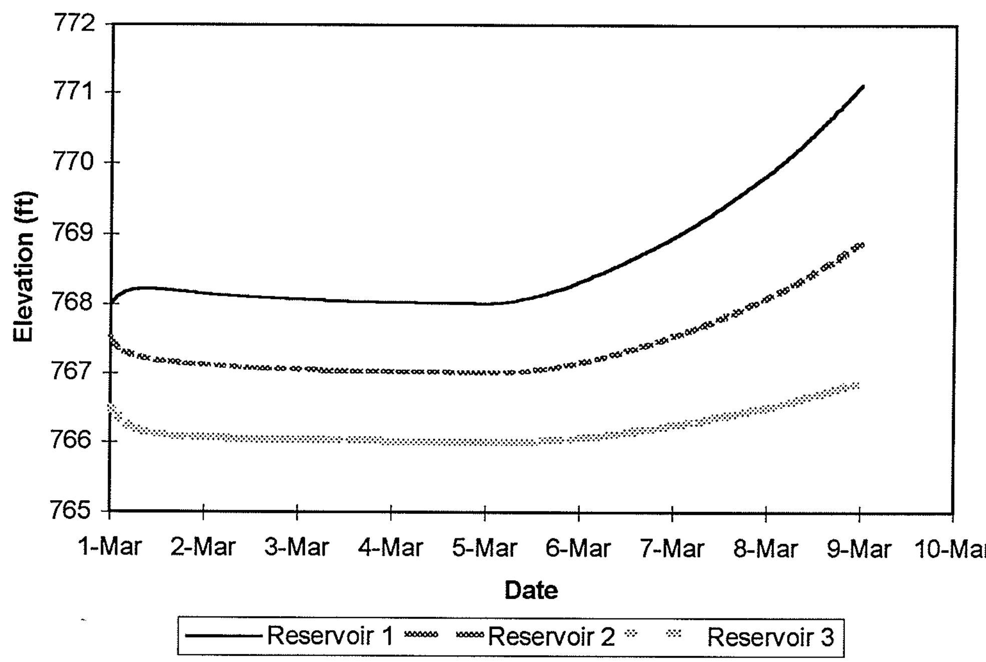

Figure 31 - Reservoir Elevation versus Time - Delta Factor: Test Case 1 - Run 2

7.1.7. Description: Delta Test Case 1 - Run 3: (DF-TC1-R3) Delta Factor+Linear+Zero Inflow+24hr

As with the matrix method, Test Case 1 - Run 3 was selected to observe the performance of the method as the reservoir outlets change from backwater-affected to free-flow to zero-flow. The crest elevations for Reservoirs 2 and 3 are one foot lower than Test Case 1 - Run 2 so that the change from backwater-affected to free-flow and then to the zero-flow state can be readily observed on Reservoir 1. These crest elevations and the starting water levels are summarized in Table 5, page 32. There is no inflow to Reservoir 1 so the discharge and elevations are expected to drop quickly.

7.1.8. Results: Delta Test Case 1 - Run 3: (DF-TC1-R3) Delta Factor+Linear+Zero Inflow+24hr

The discharge results are displayed in Figure 32 and the elevation results in Figure 33. The discharge results illustrated in Figure 32 are not the same as the results from the matrix method presented in Figure 9, page 32. As mentioned previously in the section, "The Requirement for Short Time Steps in All Reservoir Routing", page 6, the selection of a time step that is too long may cause convergence upon an unacceptable solution when reservoir elevations reach the crest level. Referring to Figure 33, note that the water level for Reservoir 1 has dropped below the crest elevation of 766 ft by almost one foot. This discrepancy occurs for the following reasons.

The solution obtained using the matrix method reaches the end of the day by reducing the time step to approximately 1 minute while the delta factor method maintains the one-day time step. Using the delta factor method, Reservoir 1 converges to a zero outflow at the end of the routing period. With zero inflow and a SOP outflow of 120 cfs, the decrease in storage is 60 cfs-days or 116 acre-ft. This decrease in storage results in the water level for Reservoir 1 becoming lower than the crest elevation. However, when this elevation is used to determine the CEOP outflow, the result is still zero and the iteration is successful. This large outflow volume becomes the inflow volume for the downstream reservoirs and, thus, they drop in elevation only after discharging this extra volume of water. The solution to this problem is to select a shorter time step so that the discharge volumes are not so large as to cause the water level to drop below the crest elevation. Neither of the methods presented provides an automatic correction for these conditions and selecting the shorter time step must be done by the user of the program. The test case in the following section, "Description: Delta Test Case 1 - Run 4: (DF-TC1-R4) Delta Factor+Linear+Zero Inflow+6hr" uses a 6-hour time step for the same conditions as this run.

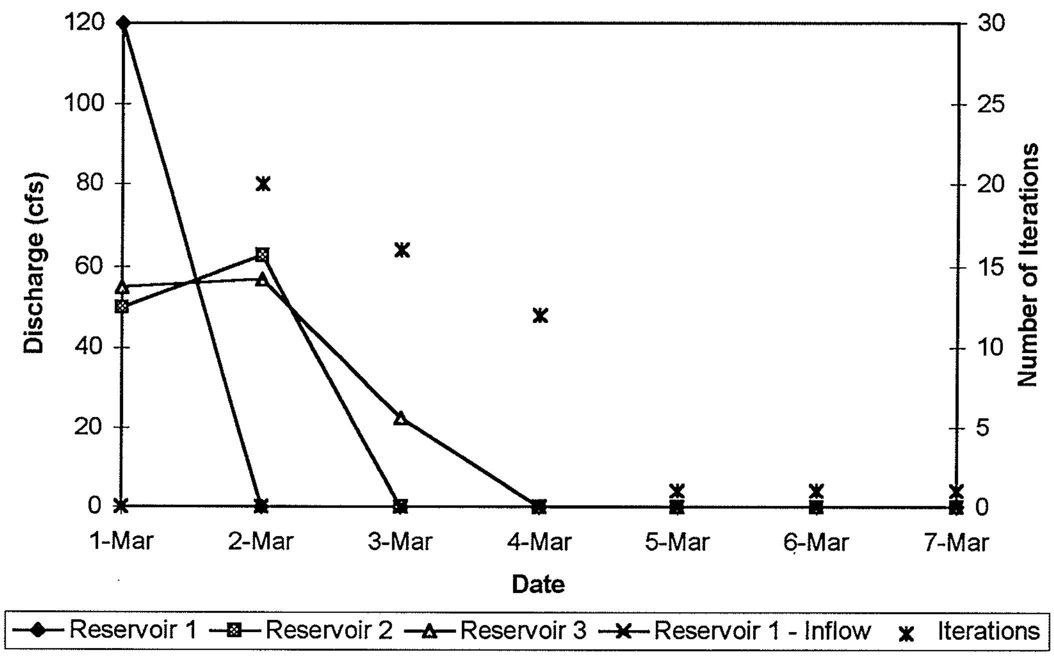

Figure 32 - Reservoir Discharge and Number of Iterations versus Time - Delta Factor: Test Case 1 - Run 3

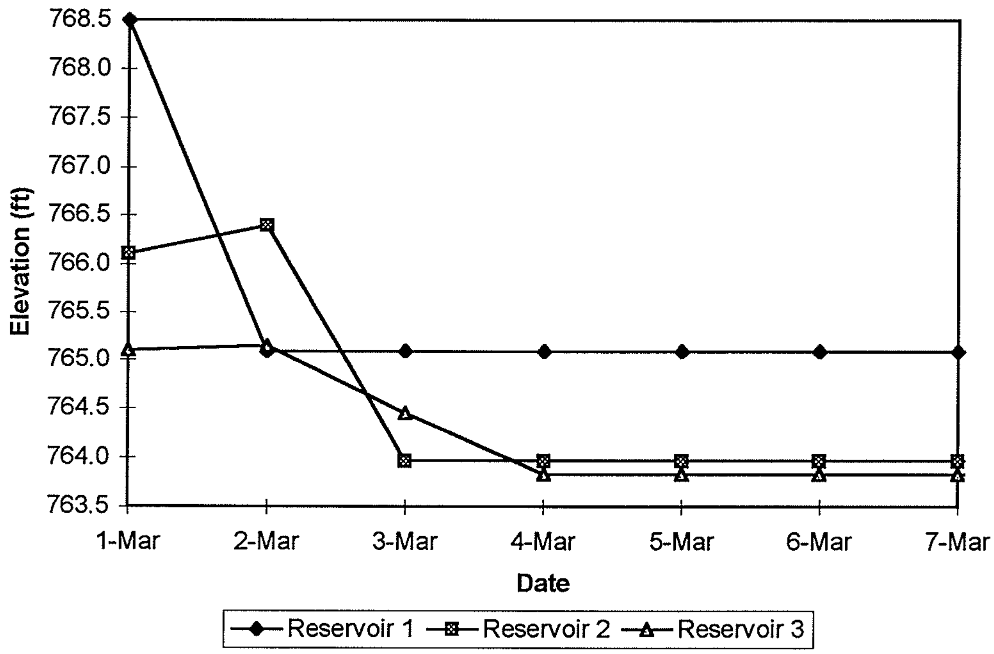

Figure 33 - Reservoir Elevation versus Time - Delta Factor: Test Case 1 - Run 3

7.1.9. Description: Delta Test Case 1 - Run 4: (DF-TC1-R4) Delta Factor+Linear+Zero Inflow+6hr

This test case is identical to Delta Factor+Linear+Zero Inflow+24hr - Test Case 1 - Run 3 except that the time step is shortened to 6 hours. The shorter time step is expected to reduce the oscillations in the discharges and to prevent the water levels from dropping below the crest elevations.

7.1.10. Results: Delta Test Case 1 - Run 4: (DF-TC1-R4) Delta Factor+Linear+Zero Inflow+6hr

The discharge results presented in Figure 34 compare much closer to the results from the matrix method (see Figure 9, page 32) than to the results obtained in Delta Factor Test Case 1 - Run 3 (see Figure 32). Figure 34 and Figure 35 indicate only a slight oscillation in outflow and elevation for Reservoir 2 at the first time step on March 1. If the time step is shortened even more, this oscillation can be eliminated. Although there is a slight oscillation in the discharges, it should be noted that, by March 2 (3 time steps ahead), the discharges from this case agree very closely with those from the matrix method which uses a one-minute time step. Equally as important, the elevations presented in Figure 35 show that the water levels do not drop below the crest elevations.

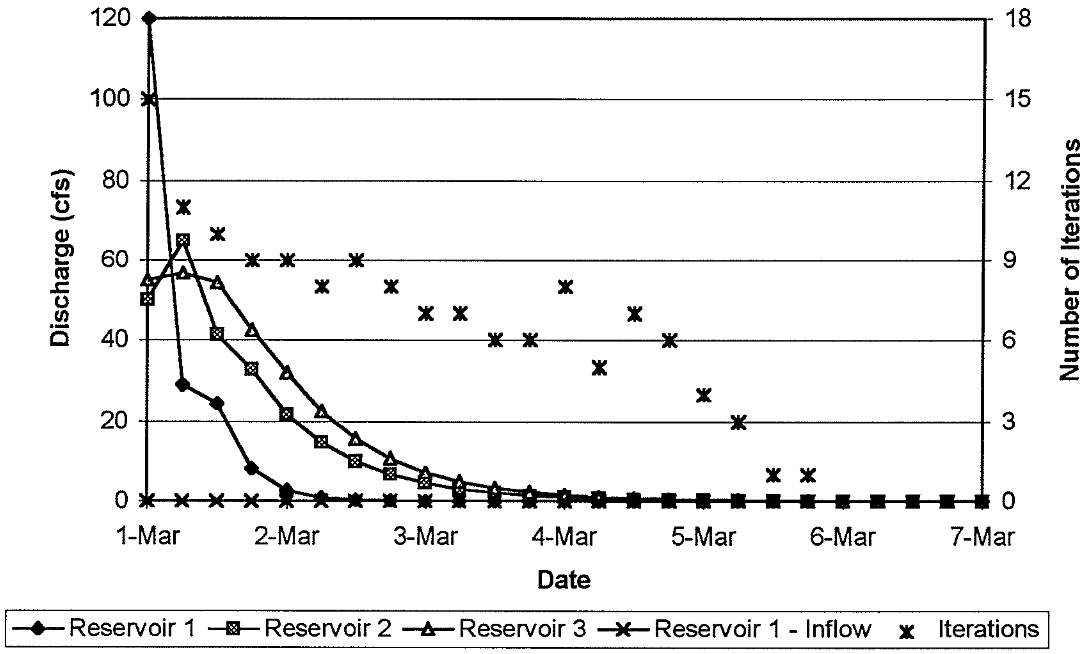

Figure 34 - Reservoir Discharge and Number of Iterations versus Time - Delta Factor: Test Case 1 - Run 4

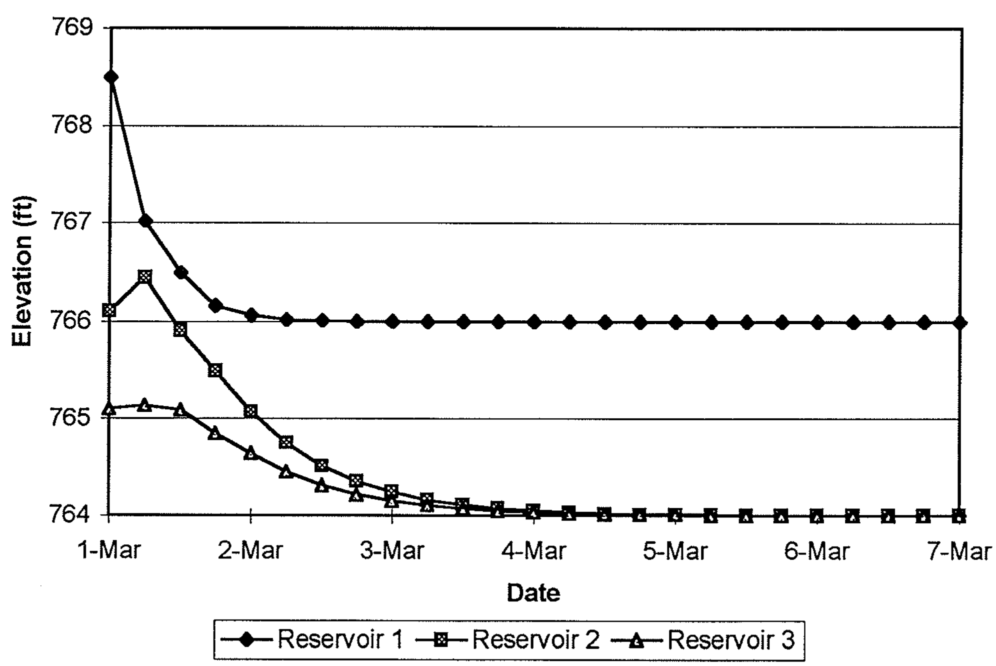

Figure 35 - Reservoir Elevation versus Time - Delta Factor: Test Case 1 - Run 4

7.2. Delta Factor: Non-linear Model (Test Case 2)

The stage-discharge and stage-storage curves used in Test Case 2 for the delta factor method are the same as the curves used for the matrix method Test Case 2. The use of non-linear curves has little effect on the operation of the delta factor method and Test Case 2 has been included in this thesis for comparison against the matrix method.

7.2.1. Description: Delta Test Case 2 - Run 1: (DF-TC2-R1) Delta Factor+Non-linear+Rise and Fall+24hr

This run uses the same inflow hydrograph to Reservoir 1 as Test Case 1 - Run 1 (see Figure 36). The starting elevations have been changed to illustrate how the method operates under different conditions. The starting elevations are all below the steady-state elevations. Thus, the water level in the reservoirs immediately starts to rise. A 24-hour time step and a 0.05 cfs discharge tolerance are used in the calculations.

7.2.2. Results: Delta Test Case 2 - Run 1: (DF-TC2-R1) Delta Factor+Non-linear+Rise and Fall+24hr

The outflow results from the delta factor method are given in Figure 36 and the elevation results in Figure 37. The discharges oscillate for the first five days of the test case because the starting elevations are approximately one foot below the steady-state conditions. Approximately 10 iterations are required throughout the entire test case until the model reaches a steady-state condition on about March 22 after which only one iteration per day is required.

These results compare very closely with those from the matrix method except for the oscillations in discharge that occur from March 1 to March 6. These oscillations do not occur in the matrix method results because the matrix method reduces the length of the time step to less than one minute for the March 1 period and 1.5 hours for the March 2 period.

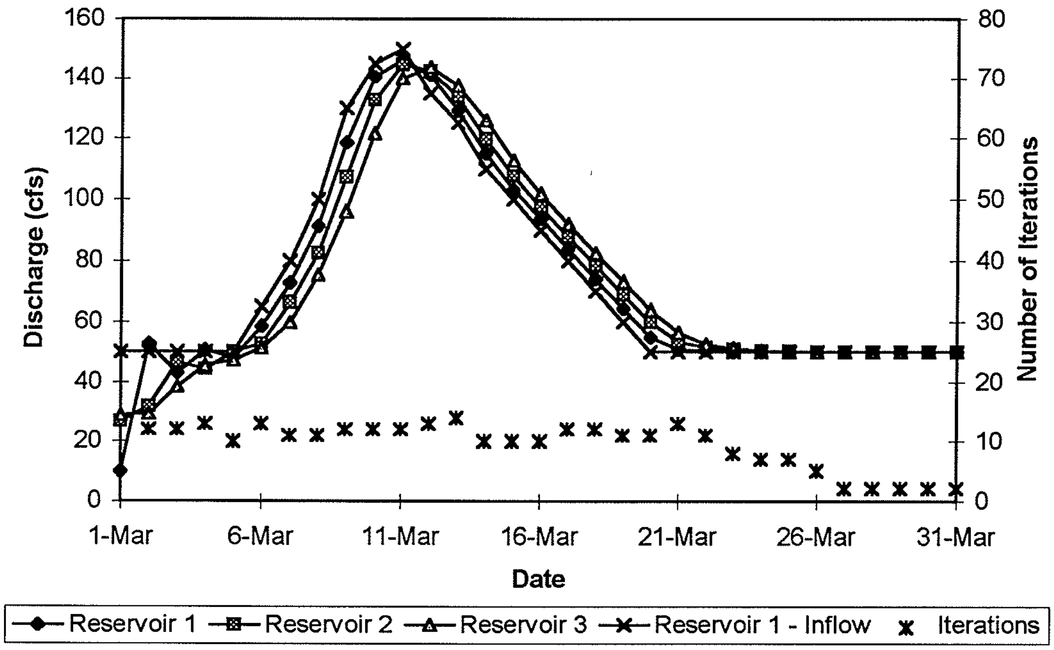

Figure 36 - Reservoir Discharge and Number of Iterations versus Time - Delta Factor: Test Case 2 - Run 1

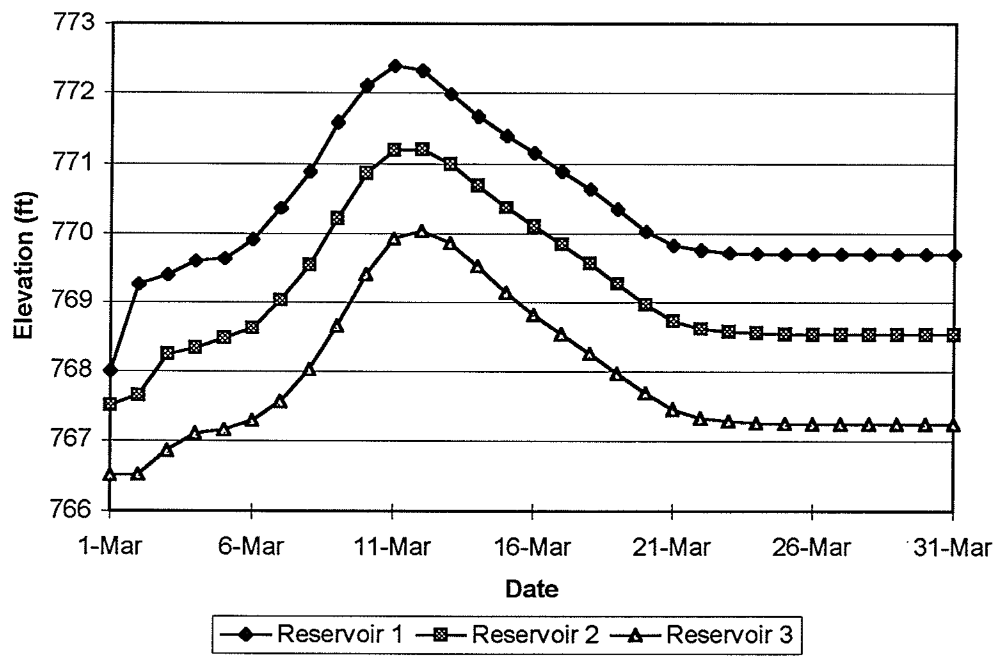

Figure 37 - Reservoir Elevation versus Time - Delta Factor: Test Case 2 - Run 1

7.2.3. Description: Delta Test Case 2 - Run 2: (DF-TC2-R2) Delta Factor+Non-linear+Zero Inflow+½hr

This test case is not repeated for the non-linear case because the objective of illustrating the effects of a shortened time step are accomplished with M-TC1-R2. This reference is included so that the run labeling system will remain synchronized between test cases and method.

7.2.4. Description: Delta Test Case 2 - Run 3: (DF-TC2-R3) Delta Factor+Non-linear+Zero Inflow+24hr

This test case has zero inflow to Reservoir 1 to demonstrate the operation of the delta factor as the elevation of a reservoir lowers to the crest of the outlet structure. The starting water levels and crest elevations are given in Table 5, page 32.

7.2.5. Results: Delta Test Case 2 - Run 3: (DF-TC2-R3) Delta Factor+Non-linear+Zero Inflow+24hr

The reservoir hydrographs and the number of iterations required for each time step are illustrated in Figure 38. Because the outflow from Reservoir 1 is changing rapidly, 14 iterations are required on March 2. After March 2 the discharges are not changing as quickly and only 5 to 10 iterations per day are required. This oscillation is an indication that the outflow from these reservoirs is not changing linearly over the time period and, to eliminate this oscillation, the time step must be shortened. Referring back to the results from the matrix method presented previously in Figure 21, page 44, the outflow from reservoirs 1 and 2 steadily decreases over this period with no oscillations. For the period ending March 2, the matrix method uses 126 iterations with the time step shortened to approximately 20 minutes.

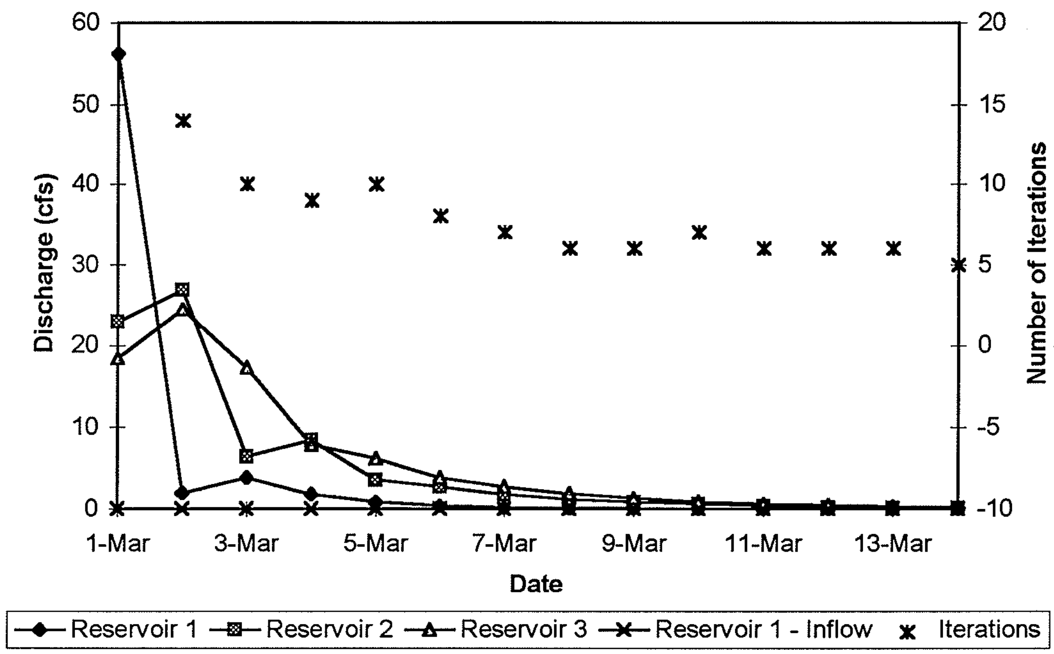

Figure 38 - Reservoir Discharge and Number of Iterations versus Time -
Delta Factor: Test Case 2 - Run 3

The reservoir elevations for this test case are shown in Figure 39 and the delta factor values in Figure 40. On March 1 and 2, the outlet from Reservoir 1 is backwater-affected by Reservoir 2. On March 3, the elevation of Reservoir 2 drops below 766.0 ft and the outlet for Reservoir 1 operates under free-flow conditions. Referring to Figure 40, there is no significant change in the delta factor values. After March 7, the water level for Reservoir 1 is at the crest elevation and the outflow is zero. Under these conditions, constant outflow (zero), this reservoir requires only one iteration per time step. The delta factor method sets the first EEOP outflow value for each time step equal to the SOP outflow. Because they are both zero, a successful iteration for this reservoir always occurs. The value for the delta factor remains constant for the rest of the routing period because it is changed only during an unsuccessful iteration for the reservoir.

The delta factors for Reservoirs 2 and 3 steadily rise from one time step to the next from March 7 onwards.

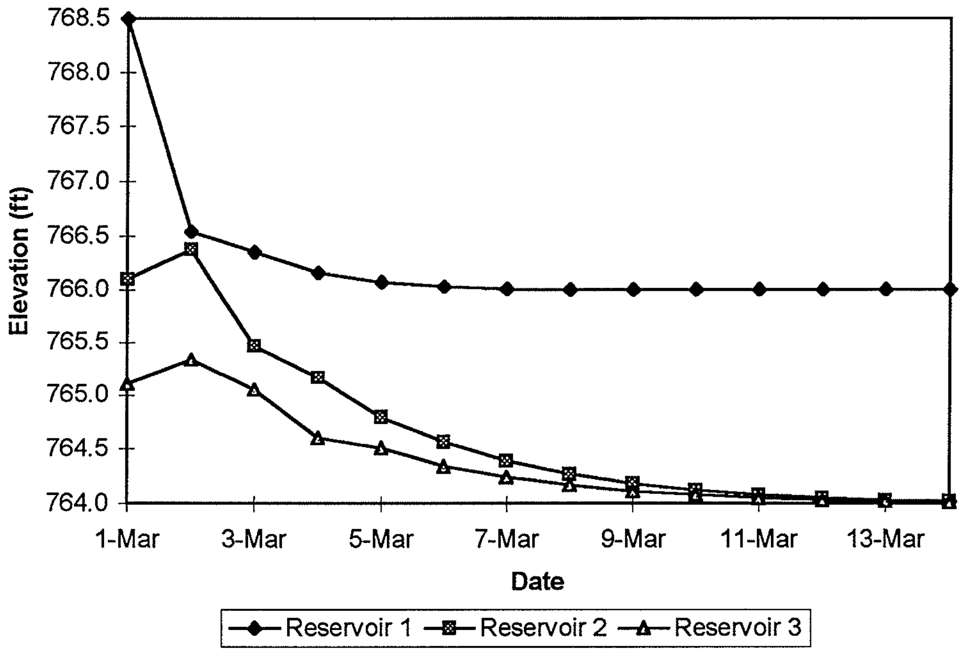

Figure 39 - Reservoir Elevation versus Time - Delta Factor: Test Case 2 - Run 3

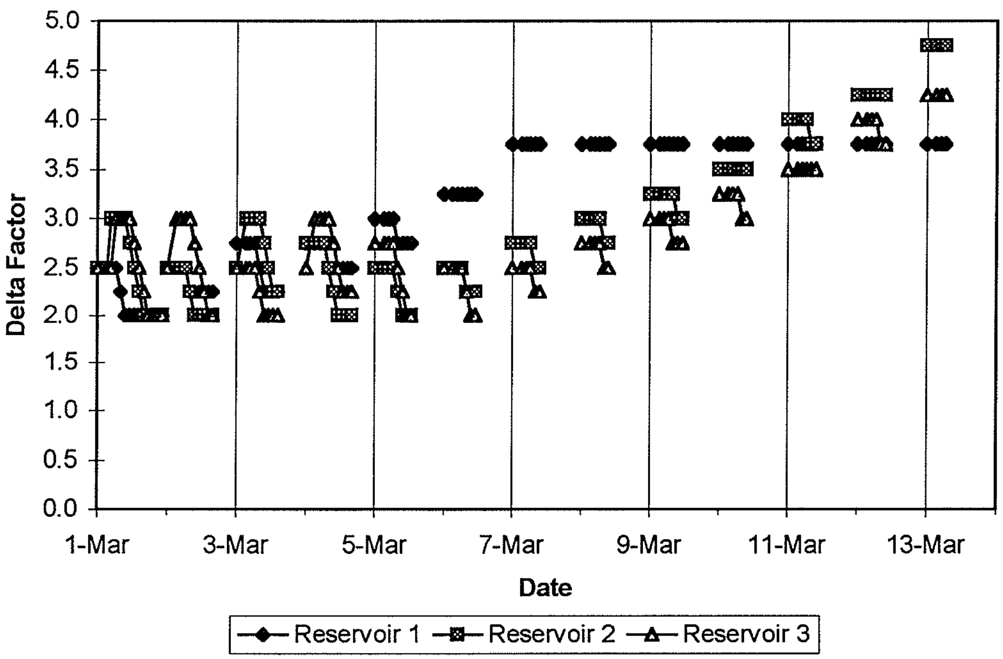

Figure 40 - Delta Factor Values versus Time - Delta Factor: Test Case 2 - Run 3

8. Computation Time and Iteration Comparisons

Statistics on the number of iterations and total central processing unit (CPU) time are presented for four of the test cases in Table 7 and Table 8. These data have been gathered to compare the efficiency of the two methods when applied to the different test cases. Column 1 of Table 7 contains the test case label while Column 2 contains the total number of iterations used for the period from March 1 to March 31. Column 3 contains the total time required by the computer for the entire run. This time includes reading the data files, performing the calculations and printing the results. The printout was kept to a minimum so that it would have little effect on the CPU time. Column 4 contains the average time (in milliseconds) to complete one iteration. It is calculated by dividing the total CPU time in Column 3 by the total number of iterations in Column 2. Column 5 contains the average number of iterations per day (Column 1 divided by 30 days). Each column also contains the average of the four test cases run for each method. These averages will provide an overall evaluation of the efficiency for each method.

The first observation to be noted from Column 4 of Table 7 is that the time required to perform an iteration using the matrix method is approximately 12 milliseconds whereas the delta factor requires only 6 milliseconds. This 2:1 time ratio should be kept in mind when comparing the number of iterations required by each method. The most computationally intensive steps in the matrix method are the inversion of the matrix and determining the A- and B-ratios. To determine the B-ratio of the reservoirs for one iteration of the matrix method, three discharges must be calculated using the backwater stage-discharge curves. In comparison, the delta factor requires only one discharge to be calculated. If the computer program subroutines could be further optimized, the performance of the matrix method would be greatly enhanced.

The matrix method performed extremely well on TC1-R1 (linear model with a typical hydrograph) with 30 iterations for the month (one iteration per day). This is expected, as the matrix method should perform extremely well with constant stage-storage ratios. In comparison, the delta factor method required an average of 14 iterations per day. The use of linear relationships has no benefits to the efficiency of the delta factor method.

Table 7 - Matrix Method and Delta Factor Central Processing Unit (CPU) Time and Iteration Comparison

Test Case Label	Total Number of Iterations	Total CPU Time (seconds)	Average Iteration Time (milliseconds)	Average Number of Daily Iterations
Matrix Method				
M-TC1-R1	30	0.53	18	1
M-TC2-R1	843	9.45	11	28
M-TC1-R3	1133	9.07	8	38
M-TC2-R3	323	3.24	10	11
Average	582	5.57	12	19
Delta Factor Method				
DF-TC1-R1	426	1.97	5	14
DF-TC2-R1	282	1.52	5	9
DF-TC1-R3	75	0.60	8	3
DF-TC2-R3	120	0.76	6	4
Average	226	1.21	6	8

Table 8 - Matrix Method and Delta Factor Iteration Comparison for Specific Conditions

Test Case Label	Number of Iterations March 2	Average Number of Iterations March 6-31
Matrix Method		
M-TC1-R1	1	1
M-TC2-R1	264	21
M-TC1-R3	1100	1
M-TC2-R3	126	2
Average	373	6
Delta Factor Method		
DF-TC1-R1	24	13
DF-TC2-R1	12	9
DF-TC1-R3	20	1
DF-TC2-R3	14	3
Average	18	7

The first five days of the period are characterized by rapidly changing outflow conditions followed by the typical rise and fall of an inflow hydrograph. Column 2 of Table 8 contains the number of iterations for the first day of the rapidly changing flows. Column 3 contains the average number of iterations per day for the last 26 days of the month. Note that the matrix method requires only one iteration for March 1 even though the discharges are changing quite rapidly.

Test Case 2 - Run 1 uses the same inflow hydrographs as Test Case 1-Run 1 but the discharge-storage relationships are no longer linear. The average number of iterations per day rises from 1 to 28 for the matrix method whereas the number of iterations for the delta factor method remains approximately the same (14 reduced to 9). The rise in the number of iterations required for the matrix method is a direct result of the non-linear stage-discharge and stage-storage curves. Referring to Table 8, the matrix method now required 264 iterations rather than one iteration to reach March 2.

Test Case 1-Run 3 and Test Case 2-Run 3 are the reservoir drawdown simulations where there is no inflow into the system and where the water levels lower to the crest elevations. The drawdown is completed within approximately five days after which there is no inflow or outflow from the reservoirs. Referring to Table 8, it is evident that the number of iterations required during the first day of the drawdown (Column 2) is substantially higher than the average number of iterations required after the drawdown is complete (Column 3). The matrix method for Test Case 1-Run 3 required 1100 iterations to reach March 2 even though the discharge-storage ratios were linear. The difficulty arises in the method when the reservoirs changed from backwater-affected to free-flow. The A-ratio suddenly changes from the constant value of 2.0 to 0.0 cfs/acre-ft.

Referring again to Table 8, conclusions can be made about the overall efficiency of the two methods operating under either rapidly changing flow conditions (March 1-2) or under more typical operating conditions (March 6-31). The matrix method required an average of 373 iterations per day during the rapidly changing flow conditions whereas the delta factor required only 18. However, under the more typical operating conditions, the matrix method and delta factor methods both require approximately the same number of iterations (6 and 7, respectively).

9. Conclusions

Both the matrix and the delta factor methods are able to produce acceptable results for the test cases analyzed. The four criteria by which the methods are to be evaluated are repeated below from the introduction.

1) The methods must not assume a constant downstream water level to calculate the reservoir outflow. If assumptions are made in the analysis, a verification of the assumptions and subsequent corrections must be included in the procedure.

2) The computational procedure should directly use the tabular form of the stage-discharge curves and stage-storage curves as they are commonly compiled in practice today so that the method is an efficient tool for the water resource engineer.

3) The method should be computationally efficient. This will be evaluated using the computational time and the number of iterations required by each method for the test cases.

4) The method must remain robust enough to avoid non-convergence. The method developed must be efficient for the water resource engineer to use even at the cost of additional computational time for the digital computer.

Criterion 1 specifies that assumptions made in the model must be verified. The assumption contained in the formation of the matrix method is that the discharge-versus-storage relationships must remain linear over the time step selected. The validity of this assumption is checked by calculating the EOP discharge from the resulting elevations. Similarly, the delta factor method estimates the EOP outflows and, again, the estimate is verified against the EOP outflow calculated from the stage-discharge curves. In this study, the outflows determined by both methods always agreed with the calculated outflow value within a tolerance of 0.05 cfs.

To satisfy Criterion 2, the stage-versus-discharge and stage-versus-storage curves were inputted in this study in tabular form for both methods. The curves were checked to ensure they are constantly increasing in value for elevations, discharges and storages.

Criterion 3 specifies that the method must be computationally efficient. In this study, the matrix method required more computation time than the delta factor method for the real-life test case (Test Case 2). However, any improvement in efficiency for the function which calculates the discharges from the backwater curves would greatly reduce the central processing unit (CPU) time requirements for this method. The delta factor method required far fewer iterations during periods of rapidly changing flow while, during more typical routing conditions, both methods required approximately the same number of iterations. A significant advantage of the matrix method was that, during the rapidly changing flow conditions, it automatically shortened the analysis time step producing more reliable results.

Finally, Criterion 4 states that the procedure must be robust. In this study, all of the test cases were run on the model without any modifications to the initial data input. The test cases selected included very small reservoirs with relatively large outlet structures. These are the most difficult cases for a reservoir routing procedure and, therefore, it may be concluded that the procedure is sufficiently robust.

One disadvantage to the matrix method is that the solution of the set of equations using the inversion of the matrices is not a process easily understood by most practicing engineers. If problems occur in the procedure, it is difficult for an end user to discover why the process is not proceeding correctly. The additional complexities at this stage of development do not produce significant tangible benefits.

Conversely, the delta factor method is based on extremely simple concepts. By observing the changes in the outflows during the iteration procedure, it is fairly simple to determine the source of any iteration problems.