ANALYSIS OF LOT SIZE INVENTORY AND EQUIPMENT REPLACEMENT PROBLEMS

by

AVNINDER SINGH (GILL)

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree of

MASTER OF SCIENCE

Department of Mechanical and Industrial Engineering
University of Manitoba
Winnipeg, Manitoba

National Library of Canada

Acquisitions and Bibliographic Services Branch

395 Wellington Street
Ottawa, Ontario
K1A 0N4

Bibliothèque nationale du Canada

Direction des acquisitions et des services bibliographiques

395, rue Wellington
Ottawa (Ontario)
K1A 0N4

Your file Votre référence

Our file Notre référence

ISBN 0-315-77878-4

Canada

ANALYSIS OF LOT SIZE INVENTORY AND EQUIPMENT

REPLACEMENT PROBLEMS

BY

AVNINDER SINGH (GILL)

A Thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

I hereby declare that I am the sole author of this thesis. I authorize the University of Manitoba to lend this thesis to other institutions or individuals for the purpose of scholarly research.

AVNINDER SINGH (GILL)

I further authorize the University of Manitoba to reproduce this thesis by photocopying or by other means, in total or part, at the request of other institutions or individuals for the purpose of scholarly research.

AVNINDER SINGH (GILL)

ABSTRACT

In the present dissertation, two important problems in the field of industrial engineering, viz: a lot size inventory control problem, and an equipment replacement problem, have been analyzed and formulated. A structural similarity between the two formulations presented in the present dissertation has also been identified. We further propose a fuzzy logic approach to deal with the inventory control problem when the data is imprecise. In addition, we suggest a simplified three steps computational algorithm for the equipment replacement problem.

Chapter 1 introduces the two problems, followed by the literature survey in Chapter 2. Chapter 3 is devoted to the lot sizing inventory problem, with variable demand rate, both under crisp and under fuzzy environments. Chapter 4 deals with the equipment replacement problem for which we give a three steps computational method based on dynamic programming, followed by a 0-1 linear programming model for the problem. It is shown in Chapter 5 that equipment replacement and inventory control problems considered in the present dissertation have a common mathematical structure. Finally, conclusions, contributions and recommendations for further research on the afore-mentioned problems are presented in Chapter 6.

ACKNOWLEDGEMENTS

I express my deep sense of gratitude and appreciation to my research advisors, Professor Ostap Hawaleshka and Professor C.R.Bector who, from the beginning to end, filled the role of advisor and co-advisor in an outstanding fashion. My debt of gratitude for what they have done for me at an important stage of my life and carrier, is immense. Sincere thanks are due to the other members of my examining committee, Professor Divakar Rajamani and Professor A.H.Soliman for their comments, valuable discussions and careful review of the thesis. Deep appreciation and thanks are also extended to Mrs. M.K.Bector for her words of wisdom and encouragement.

In particular, I extend my deepest gratitude to my parents and sister who were a constant source of encouragement to me throughout my studies.

I am also thankful to my friends and fellow graduate students Gajendra Adil and Sandeepa Goel for their cooperation and company which has made my stay at Winnipeg pleasant and memorable.

LIST OF FIGURES

LIST OF TABLES

CONTENTS

CHAPTER 1.

DESCRIPTION AND INTRODUCTION OF THE PROBLEMS

The subject of the present research is to investigate and model two important problems in the field of industrial engineering: a lot size inventory control problem (addressed by Wagner and Whitin, 1958) and an equipment replacement problem (addressed by Winston 1991, Waddell 1983, Gupta and Hira 1987). A structural similarity between the two problems has been identified.

In the present chapter, we give an introduction to these problems.

1.1. Basic Inventory Concepts:

1.1.1. Inventory: Inventory can be defined as an idle resource that has economic value. Inventory may also be defined as the stock of goods held to satisfy some eventual demand. The items constituting inventory may be raw materials, purchased or manufactured items, assembled or partially assembled parts or finished products. Inventory is built up when the rate of receipts exceeds the rate of disbursements and is depleted when the reverse is true (Ritzman and Krazewski,1990).

1.1.2. Types of Inventory: In most texts (Silver and Peterson 1985, Ritzman and Krazewski 1990), inventory is classified, as given below, by purpose (or function) of the product.

Functional Classification : Based on the function or purpose of the inventory, five types of the inventory have been identified.

1. Cycle Inventory : When the production or ordering is done in batches instead of one unit at a time, the resulting inventory is called the cycle inventory. Order lot size(Q) varies directly with the time between orders as does cycle inventory. Hence, cycle inventory is that portion of the total inventory that varies directly with lot size. Mathematically,

Cycle Inventory = Q\2 , where Q is the lot size.

The advantages of and reasons for large cycle inventories include economies of scale (because of large setup costs), better customer service, less frequent setups, reduced transportation and purchasing costs (quantity discounts) and to satisfy some technology restrictions (e.g. fixed size of a processing tank in chemical processing, etc.). Cycle inventory is also sometimes called lot sizing inventory or working stock.

2. Safety Stock Inventory: Safety stock is the amount of inventory kept on hand to provide a cushion against the uncertainty in demand, lead time, and supply.

It reduces some customer service problems and the hidden costs of unavailable parts or lost sales. Safety stocks are not needed when the future rate of demand, the lead time and reliability of the vendor are known with certainty. Safety stock is also sometimes called buffer or fluctuation stock.

3. Anticipation Inventories: Anticipation inventory consists of stock accumulated in advance of an expected peak in sales. It is used to cope with uneven rates of demand or supply. For example, peak seasonal demand of air conditioners or lawn mowers occurs only in summer. Such expected peaks in demand may lead the manufacturer to stockpile the items during periods of low demand so that output levels do not have to be increased when the demand is at its peak, because varying output rates and work-force size are normally costly. Anticipation inventories may also occur because of unevenness on the supply side. For example, strawberries ripen during certain months in a year but they are processed into jam which has rather stable demand throughout the year. Anticipation inventories may be accumulated in anticipation of labor strikes, war times, etc. Anticipation inventory is sometimes referred as seasonal or stablisation stock.

4. Pipeline (Transit or Work-In-Process) Inventory: Pipeline inventory is the inventory in transit, i.e. moving from one point to another point within the production facility or within a distribution system. For example, parts move from supplier to the plant, from one work station to the other, from a plant to the warehouse, and from warehouse to the retailer or customer. Pipeline inventory is measured as the average demand during the lead time, i.e. Pipeline inventory = $D_L = d.L$, where D_L is the average demand during lead time, d is average demand per period and L is the number of periods in the lead time between two points.

5. Decoupling stock : Decoupling stock is the stock accumulated between different echelons (stages) of a multi-echelon system to reduce the requirements for complete synchronization between different operations and to permit the independent decision making at different echelons.

1.1.3. Production and Inventory Control: The American Production and Inventory Control Society (APICS) defines these terms as follows (Wallace, 1984):

Inventory Control: The activities and techniques of maintaining the stock of items at desired levels, whether they be raw materials, work-in-process or finished products.

Production Control: The function of directing or regulating the movement of goods through the entire manufacturing cycle from the requisition of raw materials to the delivery of finished products.

In the manufacturing environment, these two terms are frequently intermixed. To some extent, this is justifiable in the production context because the release of a production order will reduce raw materials, increase WIP inventory, and eventually increase finished products. Conversely, a decision to increase the inventory level of items will require a release of a production order.

1.1.3.1. Objectives of Production and Inventory Control:

To discuss the objectives of Production and Inventory Control, we first, briefly discuss some major objectives of manufacturing firms. As given in Plossl (1985), three major objectives of manufacturing firms are:

1. Maximum customer service.
2. Minimum inventory investment.
3. Efficient (low cost) plant operation.

These three objectives are basically in conflict. Maximum level of customer service can be achieved only if inventory levels are very high and the plant is kept flexible by changing production levels and production schedules to meet the customers' changing demands. This is not in agreement with the second and third objectives. Efficient plant operation is normally considered possible if production levels are seldom changed, no overtime is incurred and machines are run for long periods once they are setup. This means efficient plant operation results in large inventory and inflexible customer service which is in disagreement with the first and second objectives. Now, the objective of low inventory can be obtained at the cost of poor customer service and if the plant reacts rapidly to changes in customers' requirements and interruptions in production which violates the first and third objectives. Since, working towards one of these objectives can be done with the exclusion of the others, the inventory control problem really becomes a challenging problem.

The primary objective of inventory control is to reconcile or make consistent these conflicting objectives in a modern company.

1.1.3.2. Economic Importance of Inventory Control:

As reported in Silver and Peterson (1985), Statistics Canada figures say that total inventories owned by Canadian manufacturers are in the neighbourhood of $ 30 billion. Business inventories in the United States exceed $ 500 billion of which half is in manufacturing (Smith, 1989). On the average, 34% of the current assets and 90% of the working capital of a typical company in the United States are invested in inventories. Actual figures for different firms will of course depend on the firm and the type of industry.

Inventories play a major role in the profitability of a firm. Since as quoted above, they constitute a large portion of the assets of a company, inventory control naturally becomes an important function of every firm that produces goods and services. If a good inventory control system can prove to be a key to the success of some companies, a poor inventory control system can be the cause of failure of several others.

There are many views expressed about inventories. Some people say that inventories are the graveyard of American business, but at the same time, the very survival of a company may depend on the inventory. Inventory can act like a double-edged sword. If there is too much inventory, then the firm is not performing optimally and is putting itself in a great risk of obsolescence. On the other hand, if huge reductions in inventory are made without improving manufacturing processes to decrease the lead times, the firm can loose its customers and damage its reputation.

Inventory is also viewed as a necessary evil. Although, it helps to maintain customer service, improves the utilization of resources and reduces set-up costs, each dollar tied up in inventory is unavailable for investments elsewhere (e.g. new products, capacity increases or new technological developments). Therefore, finding an optimal inventory control policy is of paramount importance and crucial for a firm.

1.1.4. Classification of Inventory Models:

There are different ways of classifying the inventory models. Some of the attributes useful in distinguishing between various inventory models are given in this section (Cohen 1988, Silver 1981).

1. Number of Items:
   a) Single Item - This type of model recognizes one product at a time. If the demand rate changes from period to period, then the problem becomes that of a dynamic lot sizing problem.
   b) Multi Item - This type of model considers a number of products simultaneously. These products must have at least one interrelating or binding factor such as a budget or capacity constraint or a common set up.

2. Stocking Points:
   a) Single Echelon Models - Only one stocking location is considered.

b) Multi Echelon Models - More than one interrelated stocking locations are considered.

3. Frequency of Review:

This is the frequency of assessment of the current stock position of the system and implementation of the ordering decision.

a) Periodic - Placement of orders is done at discrete points in time, with a given periodicity.

b) Continuous - Order placement can occur at any time.

4. Order Quantity:

a) Fixed - Order quantity is fixed to the same amount each time.

b) Variable - Order quantity can be variable.

5. Planning Horizon:

a) Finite - Demands are recognised over a limited number of periods.

b) Infinite - Demands are recognised over an unlimited number of periods.

6. Demand:

a) Deterministic - Demands are known with certainty over the planning horizon.

i) Static - Demand rate is constant over every period.

ii) Dynamic - Demand rate is not necessarily constant.

b) Stochastic (Probabilistic) - Demand is unknown, and must be estimated. The demand probability distribution may be known or unknown.

7. Lead Time:

a) Zero - No time elapses between placement and receipt of orders.

b) Non-Zero - Significant time elapses between the placement and receipt of orders. This time may be constant or random.

8. Capacity:

a) Capacitated - There are capacity restrictions on the amount produced or ordered.

b) Uncapacitated - Capacity is assumed to be unlimited.

9. Unsatisfied Demand:

a) Not allowed - In this case, all demand is met and no shortages (no backlogging) are allowed.

b) Allowed - Demand not satisfied in a particular period may be retained and satisfied in a future period (backlogging), partially retained and partially lost or completely lost (no backlogging).

1.1.5. Lot Size Inventory Problem:

Two basic questions to be answered in most of the inventory situations are when to order (the reorder point) and how much to order (the lot size). When the demand rate is constant over time, the associated problem of planning is rather simple because the use of the classical Economic Order Quantity (EOQ) model gives us optimal results. But when the demand rate varies over time i.e. not necessarily constant from one period to another, the associated problem of planning is a bit more challenging and is said to be dynamic in nature, as discussed in the preceeding section. The problem considered for this study is uncapacitated single-item lot sizing problem with dynamic demand. This problem was first addressed by Wagner and Whitin (1958) under the assumption of deterministic demand. The uncapacitated assumption can be justified to some extent in an MRP (Material Requirement Planning) environment on the condition that a good master production schedule exists which takes these capacity restrictions into consideration. This master schedule is aimed at smoothing the production load and can make use of fine tuning devices such as adjusting the lead time, subcontracting, overtime, alternative routings etc. However, in certain situations it may be difficult to ignore the capacity restrictions in actual lot size decisions since this would lead to an infeasible master schedule and subsequently to more frequent replanning. Further, we would add that inventory problems are ubiquitous and complex in nature, so no particular model can represent all the inventory situations.

1.1.5.1. Relevant Costs in the Problem:

There could be many types of relevant costs considered in the problem e.g. the ordering cost, the holding or carrying cost and the total manufacturing cost. There could be other relevant costs depending on the situation. Since, we meet the total demand over the planning horizon and neither do we discuss the case of quantity discounts, so the total manufacturing cost becomes irrelevant to our problem, and we ignore it. Further, we do not discuss the back order case in this study, so we omit the back order or stock out costs as well. Another type of cost which is frequently ignored in the literature is the cost of data acquisition for the model, computational costs and implementation costs including adverse effects of the new model. Such costs are referred to as System Control Costs (Silver, 1981). We are also ignoring this cost since this study deals with the theoretical, not empirical developments of the problem.

1. Holding Cost: It is the cost of keeping items on hand. Holding cost may include taxes, storage, insurance and shrinkage (e.g. pilferage, obsolescence and spoilage) costs. Holding cost increases with the size of the inventory. If r is the dollar amount required to carry one dollar as inventory for one period, and v be the unit cost of the item, then the cost to hold one unit for one period is, $\mathbf{h = v.r}$.

2. Ordering Cost: This type of cost is incurred each time a production or purchase order is placed. This cost increases with the number of orders but is independent of the size of the order. It may include transportation cost, paper work, telephone calls, accounting costs, computer time for record keeping or other receiving costs. In the production context, this type of cost is called set up cost which can include tooling and fixtures, rent on the equipment or if the company owns the equipment, there is a cost of lost opportunity to rent the equipment to some other company during the periods when there is no production. Also, since every production system takes some time before it gets to its full working momentum, the cost associated with that time may be included in the set up cost.

As we can see, there is a cost trade off between the ordering cost and the carrying cost. As the ordering cost increases, the holding cost decreases and vice-versa.

1.1.5.2. Discussion on Solution Approaches:

Wagner and Whitin (1958), suggested a dynamic programming algorithm to deal with the uncapacitated inventory control problem. Though the approach gives optimal results, the complex nature of dynamic programming and the so-called curse of dimensionality makes it difficult to understand by the practitioner and makes the approach practically useless. As reported in Bahl et. al. (1987), an industrial survey conducted by the American Production and

Inventory Control Society found no respondents using the Wagner and Whitin algorithm. There are numerous heuristic methods available in the literature which will be discussed in the literature survey in the next chapter. These heuristics are easy to use but not necessarily optimal. The other efficient approach which immediately comes to mind is linear programming. However, in certain practical situations such as for dedicated production lines, group technology and FMS, it is impossible to ignore set up costs or set up times. Each time a set up is done a cost is incurred. This suggests an integer programming approach with some binary variables representing the set ups. We develop two such 0-1 linear programming models in the third chapter. The main underlying assumption in most of the models is that "demand is deterministically known". But demand is always forecasted and most of the times the forecasts do not turn out to be precisely correct. Precision always demands parameters and structures of a system to be definitely known. In this context Schwartz's quotation (Schwartz,1962) seems appropriate, "An argument which is only convincing if it is precise loses all its force if the assumptions on which it is based are slightly changed, while an argument which is convincing but imprecise may well be stable under small perturbations of its underlying axioms." Those methods which are based on the precise knowledge of data have little practical applications. Furthermore, in practice most of the companies are limited by budget restrictions. Setting targets (or goals) on cost figures is a very common practice in the industrial and business world, and in some situations these restrictions have some elasticity. This suggests the possibility of applying fuzzy logic to some industrial problems.

In some cases, the decision maker might not really want to actually maximize or minimize the objective function, but rather may want to reach some "aspiration level" which might not be even crisply defined. In real world problems, this can happen because sometimes it is simply not possible to obtain the precise data, or the cost associated with obtaining the precise data is too high. This imprecision in data arises because of the complex nature of the real world problems. So the problem becomes that of modelling with imprecise data. We will analyse our problem in Chapter 3 by means of a fuzzy logic approach when some sort of ambiguity in available budget and demands is involved. Zadeh's (1973) principle of incompatibility states that, "In general, complexity and precision bear an inverse relation to one another in the sense that, as the complexity of a problem increases, the possibility of analyzing it in precise terms diminishes. Thus 'fuzzy thinking' may not be deplorable after all, if it makes possible the solution of problems which are too much complex for precise analysis". Fuzzy set theory is a tool which gives reasonable analysis of complex systems without making the process of analysis too complex. In the following lines, we give a brief introduction to fuzzy set theory.

Fuzzy Set Theory:

The theory of fuzzy sets is basically a theory of graded concepts. A central concept of fuzzy set theory is that it is permissible for an element to belong partly to a fuzzy set.

Let X be a space of points or objects, with a generic element of X denoted by x. Thus, $X = \{x\}$.

Fuzzy Set: Let $x \in X$. A fuzzy set A in X is characterized by a membership function (M.F.) $\mu_A(x)$ which associates with each point in X, a real number in the interval [0,1], with the value of $\mu_A(x)$ at x representing the "grade of membership" of x in A. Thus, the nearer the value of $\mu_A(x)$ is to 1, the higher the grade of belongingness of x in A.

In conventional (crisp) set theory, $\mu_A(x)$ takes only two values 1 or 0 depending on whether the element belongs or does not belong to the set A.

Therefore, formally speaking, if $X = \{x\}$ is a collection of objects denoted generically by x, then a fuzzy set A in X is a set of ordered pairs, $A = \{ (x, \mu_A(x)) \ / \ x \in X \}$, where $\mu_A(x)$ maps X to the membership space [0,1].

1.2. Maintenance and Replacement Problems:

Problem situations, concerned with controlling the condition of equipment are termed as maintenance and replacement problems. Models for these problems can be stochastic or deterministic depending upon whether some sort of uncertainty is or is not involved with the timing and/or consequence of the action (Jardine,1978).

1.2.1. Different Problem Areas:

As shown in the following figure, the replacement and maintenance problems can lie in any the following four broad areas (Jardine,1978):

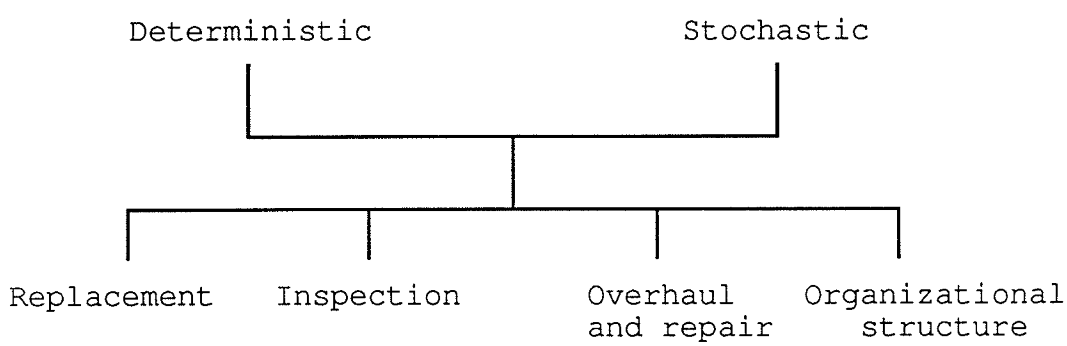

Figure 1.1- Replacement Problem Areas.

1. Replacement: Replacement means to achieve the "as new" condition for the equipment concerned. This is based on the assumption that if a second hand equipment is purchased, it should be considered "as new". Further problems in this area can be Group Replacement, Preventive Replacement etc. Group

replacement problems are those problems in which items are replaced as a group rather than one item at a time because of economies of scale (e.g. quantity discounts if items are purchased from a common supplier). Sometimes failure of one component shuts down the whole production process and it creates the opportunity to replace other parts at the same time to avoid frequent shut downs, this type of replacement situation is called preventive replacement. We will discuss the replacement problem further in the next section.

2. Inspection: The primary function of inspection is to determine the condition of the equipment and the major decisions associated are:
   a) Thoroughness of inspection.
   b) Indicators to be used to describe the equipment's age and condition (bearing wear, product quality etc.).
   c) The timing of inspection.

3. Overhaul and Repair: Overhaul is a restorative maintenance action taken before the equipment has reached a defined failed state, whereas repair is made after the equipment has reached the failed state. Neither action returns the equipment to "as new" state. "Failed state" does not necessarily mean the "broken-down" state, it may be the state in which items produced are outside the specified tolerance limit or there are risks involved with the continued use of equipment. The

major decisions associated in this area are :

a) The interval between overhauls.

b) Degree to which the equipment should be overhauled or repaired, or how close the "as new" condition should be achieved.

4. Organizational Structure: Some major decisions involved in this area are :

a) Determination of what maintenance facilities should be there inside the organisation (e.g. workshops, stores , maintenance staff etc.).

b) How these facilities should be used, taking into account the possible use of outside resources such as contractors. Normally, maintenance work can be performed by the company crew or outside contractor, in the company's workshop or in the contractor's workshop.

Another problem in this area is the determination of crew size with the crew costs and downtime costs identifying the possible trade off. With increases in crew size, cost associated with the crew increases but the down time decreases. These type of problems are normally solved by simulation techniques.

1.2.2. Different Categories of Replacement Problem:

According to Ray (1971), replacement problems normally fall into the following categories depending upon the life pattern of the equipment and its selection.

1. Replacement of items which fail suddenly.
2. Like-for-like replacement.
3. Military aircraft type replacement.
4. Replacement of items which deteriorate and become obsolete with time.

1. Items which fail suddenly : This type of replacement is quite common in the area of electronic components and it has led to the development of an area of study called reliability engineering. Another example of items falling into this category are light bulbs.

2. Like-for-like replacement : Replacement in which an item is replaced by an identical item. This type of replacement is considered to be practically nonexistent. Its main use is a philosophical one; that of an assumed approach in illustrative problems. However, there exist some industrial situations where this type of replacement may be found, e.g. in case of fairly standard machines, a machine tool is replaced by an identical tool. This category is essentially a subset of fourth category.

3. Military aircraft type replacement: In this type of replacement the decision maker is faced with the problems of analysis related to factors other than system which is in service. The problem is that of instant obsolescence, however

the equipment may be functionally downgraded rather than replaced in the literal sense (e.g. it may be used for tasks of lower priority). The value of that equipment for the main line service is lost. This category may overlap a little bit with other categories. For example, if an enemy develops a superior model, the equipment loses its value for main line service. In the business world, electronic computers can be an appropriate example.

4. Replacement of items which deteriorate and become obsolete with time: In this type of replacement, the equipment wears out gradually and becomes obsolete because of cumulative use and new technological developments and starts functioning with decreasing efficiency. Maintenance and operating costs escalate with time and resale values decline. At some stage it becomes necessary to replace the equipment with a new one. Items like industrial vehicles, fork lifts or some other industrial machinery can lie in this category. In this type of replacement, the state-of-the-art of the equipment is dynamic.

1.2.3. Replacement of Deteriorating Items:

To narrow down the topic of our research, in the present dissertation we shall limit our interest to that category of items which deteriorate and become obsolete with time and usage. In the present thesis, we consider the basic replacement problem as deterministic, rather than stochastic in nature. The deterministic problem is addressed in the past by Wagner (1969), Jardine (1978), Gupta and Hira (1987) and Winston (1991). The deterministic assumption is justified in this problem because the increasing probability of breakdowns is reflected by maintenance and operating costs that increase with the time.

In real life all equipment eventually wears out with time due to deterioration and usage, causing it to function with decreasing efficiency. For example, with the passage of time, a milling machine's operating cost and downtime increases and a transportation vehicle such as a car or industrial truck requires increasingly more repair and maintenance. With ever rising repair and maintenance costs, a stage may come when these costs become so high that it is more economical to replace the used piece of equipment by a new one. A natural question then arises about the optimal time of its replacement. If these costs decrease or remain constant with time, the best policy, perhaps, then is never to replace the item. However, in real situations such a condition is hardly met. Technological developments may also force the user to consider the replacement because better designed machinery may result in improved product quality, better efficiency and reduced maintenance and operating costs.

1.2.3.1. Relevant Costs in the Problem:

Generally, those costs which depend upon the choice or age of the equipment are included in the replacements models. Other costs or parameters that effect the cash flows are also normally included in the replacement decisions. Maintenance and operating costs, purchasing costs depend on the choice and age of the equipment, so they become relevant to the problem. Resale values (salvage values) also become a relevant parameter in our problem because besides being dependent on the choice or age, resale values also effect the cash flows.

1.2.3.2. Possible Mathematical Approaches:

Dynamic programming has been suggested, by many researchers, as an approach to modelling and solving the 'equipment replacement problems' (Bellman 1955, Bellman and Dreyfus 1962, Wagner 1969, Stapleton et. al. 1972, Chand and Sethi 1982, Hopp and Nair 1991, Winston 1991 etc.). However, feelings that "high computational burdens due to the so called 'curse of dimensionality' of dynamic programming and the near impossibility of explaining it to the practitioner" or statements like "the complexity of the dynamic programming procedure inhibits its understanding by the layman who, in general, does not possess a high degree of expertise, insight and 'art', acts as an obstacle to its adoption in practice" discourages the users from the use of dynamic programming approach to solve equipment replacement problem (Fordyce and Webster,1984). Recently, the high computational burden argument has diminished somewhat due the developments in computer technology but the other arguments still persist to a large extent. Based on Fordyce and Webster's approach to inventory control, we will give a simple and straight forward dynamic programming algorithm to solve the equipment replacement problem in Chapter 4. In addition, we will also analyse the afore-mentioned problem by means of a pure 0-1 model.

1.2.4. Importance of the Replacement Problem:

Like the inventory control problem, the importance of the replacement problem can also not be underrated. A poor replacement policy can cause unnecessary costs, the continued use of old equipment can cause hazards, industrial accidents or personal injuries. On the other hand, a good replacement policy can be a positive profit contributing factor and help to insure worker safety. An extensive survey conducted by Hsu (1989) clearly indicates that firms have started realizing the importance of equipment replacement and are paying increasing attention to replacement policies. Approximately 89% of the firms surveyed have definite equipment replacement policies. This represents a sizable improvement over his previous survey (Hsu, 1974) which showed that only 52% of the firms surveyed with definite policies regarding equipment replacement. Capital-intensive firms with more expensive and specialized equipment were found to show more concern over equipment replacement than labour-intensive firms (Hsu,1988).

CHAPTER 2.

LITERATURE SURVEY

This chapter provides a survey of literature dealing with inventory control and replacement problems. The purpose of this chapter is to identify the state-of-the-art achieved in the areas mentioned in Chapter 1. However, we will focus our attention on that part of the literature which is relevant to our problems.

2.1. Review of Literature on the Inventory Problem:

Much literature exists on the inventory problems. Some major sources of this literature include the APICS publications (such as Production and Inventory Management Journal, Journal of Operation Management, Conference Proceedings), Operations Research Quarterly, Management Science, Naval Research Logistics, International Journal of Production Research (IJPR), European Journal of Operation Research (EJOR) etc. In the following sections, we will review techniques dealing with single item uncapacitated lot sizing. Expository accounts of some of these techniques are provided in Plossol (1985), Silver and Peterson (1985), Zoller and Robrade (1988), Nydick and Weiss (1989), Hax (1978). Perhaps the first reported work on inventory control was Harris by (1915). Then in 1934, Wilson (1934) gave a statistical approach to find order points. These publications found little practical recognition for at least the following 3-4 decades. This may be due to the fact that the 1930's and 1940's were periods of great depression for the industrial and business world. The question before many companies was that of survival, not optimization. During World War II, different companies were mainly concerned about meeting the war-time needs and a backlog for civilian demands started appearing. This pent-up demand for civilian goods provided a market for every item that could be produced. Once the postwar backlog was satisfied, firms started thinking in terms of optimization, because the problem became that of over-production. Inventory control models received their real boost from operation research techniques developed during World War II. However, the actual bibliography on these models started appearing after 1958 with the well known paper by Wagner and Whitin (1958). Triggered by Wagner and Whitin's work, a number of papers appeared in different research journals. Some of them tried to improve on the Wagner-Whitin algorithm, others gave some heuristics which were basically off-shoots of Wagner-Whitin algorithm with the emphasis on making computation scheme more easy, though may not be optimal. Zoller and Robrade (1988) suggested a convenient classification scheme for the existing literature by categorizing it into following three categories:

1. Optimizing techniques.
2. Stop rules (heuristics) and
3. Heuristic algorithms.

Using their classification scheme, we discuss the literature as follows:

2.1.1. Optimizing Techniques:

2.1.1.1. The EOQ Formula:

The classic EOQ formula was first derived by Ford Harris (1915). This is widely known as the Wilson formula because it was Wilson who popularized this formula in practice. The basic formula for economic order lot sizing is as follows:

$$EOQ=\sqrt{\frac{2.A.D}{v.r}}$$

where

A = fixed cost for the replenishment of an order.

D = demand rate of the item. (normally annual usage rate)

v = unit variable cost.

r = cost of one dollar of item tied up in inventory for a unit time.

Note that D and r should have same unit time basis (i.e.if annual demand is considered then r must be considered for one year, not one month).

2.1.1.2. The Wagner-Whitin Algorithm:

When the demand rate is constant from period to period, then the classical EOQ formula performs in an optimal fashion but, when demand rate varies from period to period, the results from the EOQ formula can be misleading. The only optimal technique which performs optimally in this situation was given by Wagner and Whitin (1958) in their well-known paper. They used dynamic programming to solve this problem, perhaps forced by the recursive nature of the computations. Their work was based on some important theorems established in their paper. These theorems were themselves based on the assumption that initial inventory is zero ($I_0=0$). Before giving their algorithm, we shall briefly state these theorems. Most heuristics given by subsequent researchers are also based on these theorems about the structure of an optimal policy.

Theorem 1: There always exists an optimal policy such that

$$I_{t-1} \cdot X_t = 0 \text{ for } t = 1,2,\ldots,N,$$

where I_{t-1} is the inventory entering period t, X_t is the amount produced in period t and N is the length of planning horizon. This means that replenishment can be made only when the inventory level becomes zero, i.e. having positive inventory and producing at the same time never leads to optimality.

Theorem 2: There exists an optimal policy such that for all t

$$X_t=0 \quad \text{or}$$

$$X_t = \sum_{j=t}^{k} d_j$$

for some k, $t \leq k \leq N$.

where X_t is the amount produced in period t and d_j is the demand in period j. This means that for any given period production is either zero or is the sum of subsequent demands for some number of periods in the future. The dynamic programming approach requires N(N+1)/2 cases to be analyzed.

<u>Theorem 3</u>: There exists an optimal policy such that if demand $d_t{}^*$ in period t^* is satisfied by some amount $X_t{}^{**}$ produced in period t^{**}, $t^{**} < t^*$, then d_t, $t = t^{**}+1, \ldots\ldots, t^*-1$, is also satisfied by $X_t{}^{**}$.

<u>Theorem 4</u>: Given that I=0 for period t, it is optimal to consider periods 1 through t-1 by themselves.

<u>Planning Horizon Theorem</u>: The planning horizon theorem states in part that if it is optimal to incur a setup cost in period t^* when periods 1 through t^* are considered by themselves, then we may let $X_t{}^* > 0$ in the N period model without foregoing optimality. By theorems 1 and 4 it follows further that we adopt an optimal program for periods 1 through t^*-1 considered separately.

<u>The Algorithm</u>: According to Wagner whitin (1958), the algorithm at period t^*, $t^*=1,2,\ldots., N$, may be generally stated as:

1. Consider the policies of ordering at period t^{**}, $t^{**} = 1,2,\ldots, t^*$ and filling demands d_t, $t = t^{**}, t^{**}+1, \ldots, t^*$, by this order.
2. Determine the total cost of these t^* different policies by adding the ordering and holding costs associated with

placing an order at period t^{**}, and the cost of acting optimally for periods 1 through $t^{**}-1$ considered separately. The latter cost is computed previously in computations for periods $t = 1,2,\ldots, t^{*}-1$.

3. From these t^{*} alternatives, select the minimum cost policy for periods 1 through t^{*} considered independently.
4. Proceed to period $t^{*}+1$ (or stop if $t^{*}=N$).

There are however, some potential drawbacks of this method. For example, the computational effort is very great and the complex nature of the algorithm inhibits its understanding by the practitioner (Silver,1985). In the case of rolling horizons, some heuristics outperform the Wagner-Whitin algorithm.

Wagner (1960) further expanded his approach to take into account time-varying manufacturing costs. Eppen et. al.(1969) and Zabel (1964) made extensions to the planning horizon theorem. Zangwill (1969) took backordering costs into consideration and gave a network representation of the problem. Elmaghraby and Baawle (1972) provided an alternative approach to the backorder case by considering batch sizes greater than one, both with and without set up costs. Blackburn and Kunreuther (1974) also consider the backlogging case. Chang (1977) tackles this problem with different demand or production characteristics. Bhaskaran and Sethi (1981) studied the dynamic lot-size model with stochastic demand. Lavis (1981) allows the quantity discount feature in his model.

Fordyce and Webster (1984) further enriched the literature on this problem by presenting the Wagner-Whitin algorithm in a simple and straight forward computational style in tabular form. Fordyce and Webster continued their investigation and the variations of their approach are presented in Fordyce and Webster (1985), where they considered the case of changing manufacturing costs and quantity discounts. Naidu and Singh (1986) gave an algorithm based on incremental cost approach to determine the optimal production policy which they further extended for multi-item case. Jacobs and Khumawala (1987) presented a simple graphic branch and bound optimal procedure which is computationally equivalent to Wagner and Whitin algorithm (1958).

2.1.2. Stop Rules:

Stop rules (Zoller and Robrade,1988) increase the cycle length and stop as soon as some transformation of the controllable cost is reached. Controllable cost C(t), is normally the sum of ordering and holding costs as follows:

$$C(t) = A + H \cdot \sum_{h=1}^{t} (h-1) \cdot d_h$$

where d_h is the demand quantity in period h, H is the holding cost per unit per period and other symbols have the usual meaning.

2.1.2.1. Least Unit Cost Rule (LUC):

This is probably the earliest heuristic, the exact origin of which hasn't been traced out. Gorham (1968) compares the LUC and least total cost (LTC) methods and comes up with the conclusion that the LUC method is erratic. Although it performs well on one set of data, it fairs poorly on another set of data. LUC (Wemmerlov,1981) divides the total cost by the demand quantities to find the cost per unit U(t) as follows:

$$U(t) = \frac{C(t)}{\sum_{h=1}^{t} d_h}$$

and stops as soon as $U(t+1) \geq U(t)$.

2.1.2.2. Part Period Rule:

The Part period rule was developed for IBM's software packages because it is simple to program. It was introduced by DeMatteis (1968) and Mendoza (1968) and is basically the same as the Least Total Cost (LTC) rule (Gorham,1968). The basic criterion in these rules is that requirements for the successive periods can be added to the same lot so long as the cumulative carrying cost does not exceed the ordering cost, i.e.

$$H \, . \sum_{h=1}^{t} (h-1).d_h \leq A$$

and stops stops as soon as :

$$H \, . \sum_{h=1}^{t+1} (h-1).d_h > A$$

2.1.2.3. The Silver and Meal Rule (SMR):

This is perhaps the most famous heuristic method (Silver and Meal,1973). Silver-Meal Rule is identical to Least Unit Cost rule except that the total cost is divided by the number of periods included in the lot instead of by the sum of demand quantities. It computes the cost per period P(t) as follows:

$P(t)=C(t)/t$, and stops as soon as

$P(t+1) > p(t)$.

2.1.2.4. Groff's Rule:

Groff (1979) introduces a policy under which the demand for a period is added to the lot if the marginal savings in ordering cost are greater than the marginal increase in carrying cost.

In mathematical terms,

Marginal savings in ordering cost = $(A/t)-(A/t+1) = A/(t.(t+1))$

Marginal increase in holding cost = $(1/2).H.d_{t+1}$

Groff's rule adds the demand for a period to the lot if

$A/(t.(t+1)) > (1/2).H.d_{t+1}$ and stops as soon as

$(1/2).H.d_{t+1} \geq A/(t.(t+1))$

2.1.2.5. Incremental Order Quantity (IOQ):

Boe and Yilmax (1983), Freeland and Cooley (1982) suggested that cycle length should be increased so long as incremental carrying costs $H.t.d_{t+1}$ does not exceed A and it stops as soon as

$H.t.d_{t+1} \geq A$

2.1.2.6. Period Order Quantity(POQ):

Period Order Quantity is an EOQ based technique. If there are considerable variations in the demand pattern, then the results from simple the EOQ formula can be misleading. Better results can be obtained by adopting a slightly different approach (Brown,1977). The EOQ is calculated from the classical square root formula (EOQ formula), then this EOQ quantity is divided by the average demand during one period to obtain the number of periods whose requirements are to be covered by the lot size (rounded to the nearest positive integer).

T_{POQ} = EOQ/(Average demand during one period)

If D_{avg} is the average demand for one period, then

$$EOQ=\sqrt{\frac{2.A.D_{avg}}{v.r}}$$

$$T_{POQ}=\sqrt{\frac{2.A}{v.r.D_{avg}}}$$

Thus in POQ method,the time between orders remains fixed, but the lot size changes.

2.1.3. Heuristic Algorithms:

In the previous section, we discussed some rules which were basically single pass stop rules. The stop rules terminate when some transformation of controllable cost is reached, while the algorithms further look ahead or back and compare different alternatives to improve the overall decision.

2.1.3.1. IOQ Algorithm:

Trux (1972) proposes to use the IOQ rule to find a safe maximum and then examines if the corresponding lot can be split into two lots. Gaither (1983) determines two subsequent lengths and examines if shifting a demand from first lot to second lot is more profitable or not. In fact, Gaither (1983) is an improved version of his previous algorithm Gaither (1981), after the comments from silver (1983) and Wemmerlov (1983).

2.1.3.2. Part Period Algorithm:

To improve the performance of PPR many attempts have been made. DeMatteis (1968) suggests that the cycle length determined by the PPR should be subjected to a look ahead or look back to determine if the periods of large demand exist. Blackburn and Millen (1980) propose that cycle length determined by PPR could be increased if a closer balance of ordering and carrying costs can be maintained. Karni (1981) proposes that pairs of lots should be combined into a single order through an iterative procedure with a maximum gain in terms of net cost reduction.

2.1.3.3. Silver-Meal Algorithm:

Silver and Meal (1973) made an observation that cost per period is not necessarily convex and may hence have many local minima, however, SMR identifies only the first minima. Blackburn and Millen (1980) suggest that the absolute minima should be found by exhaustive enumeration of C(t) over the entire planning horizon.

2.1.3.4. Other Approaches:

In addition to the approaches discussed above, there are numerous other ones. For example, Bahl and Zionts (1986), formulated the problem as a "fixed charge" problem and made lot-sizing decisions by comparing the minimum savings of having a set up in each period to the maximum savings of having an order in that period. Another technique sometimes used in practice is lot-for-lot (LFL) technique. In LFL, orders are placed in the period in which positive demand exists. While this method minimizes the carrying cost, ordering cost is maximized. If the natural cycle and the range for demand value is small (i.e., ≤ 30) then the LFL is optimal.

Detailed description of these heuristic techniques can be found in Plossl (1985), Silver and Peterson (1985), Zoller and Robrade (1988), Nydick and Weiss (1989). The relative performance of different heuristic methods is compared in Karni (1986), Nydick and Weiss (1989), Zoller and Robrade (1988). Robrade and Zoller (1988) provides an extensive comparative study of different methodologies implemented in commercial software packages. Haddock and Hubicki (1989) conduct a survey on different techniques based on their practical use in industry. Bahl et. al. (1987) give a five point criteria (based on computational effort, Generalizaton, Optimality, Simplicity and Testing) to evaluate the performance of a technique.

This completes our review of the various techniques used in the lot-sizing problem.

2.2. Review of Literature on the Equipment Replacement Problem:

There is no dearth of published papers dealing with replacement and maintenance models. Extensive surveys of these models are provided by Pierskalla and Valker (1976) and Sherif and Smith (1981). The Pierskalla and Valker (1976) survey covers the literature from 1965 to 1976. Sherif and Smith (1981) survey focuses on work since 1976. Work up to 1965 is thoroughly surveyed by McCall (1965). Grant (1950) solved the replacement problem under a set of assumptions such as: no new more efficient equipment available before replacement, the value of money does not change over the useful life of the equipment, and operating costs do not decrease. One of the first significant works on the problem was by Terborgh (1949). Subsequently, Bellman (1955) applied the dynamic programming technique to the replacement problem. Other interesting literature on this topic is by Dean (1951a, 1951b), Churchman et. al. (1957), Wagner (1975), Jardine (1970, 1971, 1978), etc. D'aversa and Shapiro (1978) provide a dynamic programming model for the problem and employ linear programming and enumeration to obtain optimal policies and to test the sensitivity of these optimal policies to various factors. Eilon et.al.(1966) studied the optimum replacement policy of fork lift trucks using two models, the first related to the minimum average cost per truck per year, the second using the discounted cash flow approach. The issue of the effect of technological improvement on the economic life of equipment has also been discussed by different researchers.

For example, Terborgh (1949) model computes the past rate of obsolescence and projects it into the future assuming a constant rate of technological improvement. Subsequent work on this topic is presented in Bellman and Dreyfus (1962), Stapleton et. al. (1972), Chand and Sethi (1982). Stapleton et. al. (1972) contrast the generality of the dynamic programming approach to the problem of optimal asset life determination with the traditional and highly restricted "equal life" solution, and examine the possible effects of different forms of technical change on costs. Chand and Sethi (1982) develop forward algorithm and planning horizon procedures assuming that technological environment is improving over time and that the machine-in-use can be replaced by several different kinds of available machines. Hopp and Nair (1991) also model the problem in an environment of technological change assuming that the costs associated with the presently available technology and future technologies are known, but that the appearance times of future technologies are uncertain. Hopp and Nair claim that their approach requires minimum possible amount of forecasted data.

Most of the above mentioned literature is stochastic in nature. We will not discuss these models any further because our deterministic replacement problem is fundamentally different. The deterministic problem was addressed in the past by Wagner (1969), Gupta and Hira (1987) and Winston (1991). The evidence of practical use of such deterministic models can be found in Waddell (1983) where it is reported that Phillips Petroleum company's fleet managers actually used such a model in making replacement decisions for individual highway tractors and to formulate policies for replacing passanger cars and light trucks.

CHAPTER 3.

A LOT SIZING INVENTORY PROBLEM WITH VARIABLE DEMAND RATE UNDER BOTH CRISP AND FUZZY ENVIRONMENTS

The problem of lot sizing in inventory considers when an order should be placed for a particular product (the reorder point) and how much of it should be replenished in a particular period (lot size) such that the total cost (replenishment cost plus inventory carrying cost) over the planning horizon is minimized while the demand for each period is also satisfied. In the present chapter, we consider single-item uncapacitated lot sizing inventory problem with variable demand rate, both under crisp and fuzzy environments, for a planning horizon of N periods. This problem was first addressed, under crisp environment, by Wagner and Whitin (1958) using dynamic programming. Under crisp environments, we first formulate such a problem as pure 0-1 problem with all the variables restricted to values of zero or one. Then we formulate such a problem as a linear programming problem with exactly N variables restricted to zero-one. Under fuzzy environments such a problem is formulated as a maxmin linear program, once again, with exactly N variables restricted to zero-one. Numerical examples for all the cases are presented.

3.1. Formulation under Crisp Environments:

3.1.1. Assumptions: For the models under crisp environments, we make the following basic assumptions similar to the ones given in Silver and Peterson (1985).

1. The rate of demand varies from one period to the next and is assumed known.
2. The demand pattern terminates at the end of planning horizon or ending inventory must be prespecified.
3. The replenishments are constrained to arrive at the beginings of periods and the entire order quantity is delivered at the same time.
4. The cost factors do not change appreciably with time, except that the ordering (set up) cost for period i is A_i, $i=1,2,...,N$.
5. The replenishment lead-time is known with certainty so that delivery can be timed to occur right at the beginning of a period.
6. The unit variable cost does not depend on the replenishment quantity i.e. no quantity discounts are permitted.
7. The product is treated entirely independently of other products, i.e. benefits from joint replenishment do not exist or are ignored.
8. Carrying cost is only applicable to inventory that is carried over from one period to the next.

<u>Objective</u>: Minimize the total cost over the entire planning horizon of N periods.

3.1.2. Pure 0-1 Formulation:

Now, we give a pure 0-1 formulation for the problem. This formulation will be used again in Chapter 5 to establish a structural similarity between inventory control and equipment replacement problems.

Notation : For this formulation, we base our variable definition on Theorem 1 and Theorem 2 discussed in Chapter 2 (Wagner and Whitin 1958).

Let,

$$x_{ij} = \begin{cases} 1 & \text{if demand of jth period is satisfied from the amount produced in ith period, } i \leq j \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if the replenishment is made in period i} \\ 0 & \text{otherwise} \end{cases}$$

v = unit variable cost in \$ per unit.

r = carrying cost in \$ per \$ per period.

N = length of planning horizon, i.e. the number of periods in the planning horizon.

A_i = ordering cost per replenishment in dollars for period i, i = 1, 2, 3,...., N.

d_j = deterministically (crisply) known demand for period j, j= 1, 2,....,N.

M = any large number $\geq$ N.

Then we have the following model

(C1) Minimize $v \cdot r \sum_{i=1}^{N} \sum_{j=i}^{N} (j-i)\, d_j \,.\, x_{ij} + \sum_{i=1}^{N} A_i . y_i$

subject to

$$\sum_{i=1}^{j} x_{ij} = 1, \quad j = 1, 2, \ldots\ldots, N \qquad (1)$$

$$\sum_{j=i}^{N} x_{ij} \le M\, y_i, \quad i = 1, 2, \ldots., N \qquad (2)$$

$$x_{ij} = 0, 1 \quad i, j = 1, 2, \ldots, N, \text{ and } i \le j \qquad (3)$$

$$y_i = 0, 1 \quad i = 1, 2, \ldots, N, \qquad (4)$$

(i) The objective function expression in the above model minimizes the sum of carrying and ordering costs.

(ii) Constraints (1) along with constraint (3) ensures that demand of a period is met from the amount replenished in either the same period or from only one of the previous periods, which in turn implies that during a particular period either replenishment (set up) cost is incurred or carrying costs is incurred, but both are never incurred simultaneously.

(iii) Constraints (2) in conjunction with constraint (4) and objective function ensure that whenever some amount is replenished in a period, an replenishment or ordering cost is incurred.

(iv) It may be noted that because of the special coefficient structure of this formulation, x_{ij}'s will take values 0 or 1, even if they are not restricted to be 0 or 1.

3.1.2.1. Numerical Example: We illustrate the above formulation through the following numerical example solved on a personal computer using QSB^+ (Chang and Sullivan, 1991). Suppose we have the following data.

$N = 6$, $A_i = \$54$ for $i = 1, 2, \ldots, N$, $v = \$20$ per unit,

$r = \$\ 0.02$ per \$ per month, $v \cdot r = \$0.4$ /unit/month,

Table — 3.1. Showing demand per period

Period	1	2	3	4	5	6
Demand(d_i)	10	62	12	130	154	129

We select

$$M = 10 > N = 6$$

We now have the following linear programming model with all x_{ij}'s and y_i's restricted as zero-one variables.

$$\begin{aligned}\text{Minimize } & 24.8\ x_{12} + 9.6\ x_{13} + 156\ x_{14} + 246.4\ x_{15} + 258\ x_{16} + \\ & 4.8\ x_{23} + 104\ x_{24} + 184.4\ x_{25} + 206.4\ x_{26} + 52\ x_{34} + \\ & 123.2\ x_{35} + 154.8\ x_{36} + 61.6\ x_{45} + 103.2\ x_{46} + 51.6\ x_{56} + \\ & 54\ y_1 + 54\ y_2 + 54\ y_3 + 54\ y_4 + 54\ y_5 + 54\ y_6\end{aligned}$$

subject to

$$x_{11} = 1$$

$$x_{12} + x_{22} = 1$$

$$x_{13} + x_{23} + x_{33} = 1$$

$$x_{14} + x_{24} + x_{34} + x_{44} = 1$$

$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} = 1$$

$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 1$$

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} - 10\, y_1 \leq 0$$

$$x_{22} + x_{23} + x_{24} + x_{25} + x_{26} - 10\, y_2 \leq 0$$

$$x_{33} + x_{34} + x_{35} + x_{36} - 10\, y_3 \leq 0$$

$$x_{44} + x_{45} + x_{46} - 10\, y_4 \leq 0$$

$$x_{55} + x_{56} - 10\, y_5 \leq 0$$

$$x_{66} - 10\, y_6 \leq 0$$

$$x_{ij} = 0,\ 1 \qquad i,j = 1,2,\ldots,6, \quad \text{and} \quad i \leq j$$

$$y_i = 0,\ 1 \qquad i = 1,\ 2,\ldots,6,$$

3.1.2.2. Results: Solving the above problem on a personal computer using QSB$^+$ (Chang and Sullivan, 1991), we obtain the following results given in Table - 3.2.

Table — 3.2. Optimal solution to (C1), pure 0-1 formulation

Variable	Value	Variable	Value	Variable	Value
x_{11}	1	x_{44}	1	y_1	1
x_{12}	1	x_{55}	1	y_4	1
x_{13}	1	x_{56}	1	y_5	1

All other decision variables are equal to 0.

Minimized objective function = 248

Interpretation of Results:

Interpretation of results of problem C1 is simple e.g. $x_{11}=1$, $x_{12}=1$, $x_{13}=1$, $x_{14}=0$, $x_{15}=0$, $x_{16}=0$, $y_1=1$; $x_{22}=0$, $x_{23}=0$, $x_{24}=0$, $x_{25}=0$, $x_{26}=0$, $y_2=0$ and $x_{33}=0$, $x_{34}=0$, $x_{35}=0$, $x_{36}=0$, $y_3=0$ means requirements of the first three periods are satisfied from the amount produced in the first period, and no amount is produced in the second and third periods. Therefore the total amount produced in the first period = $d_1 + d_2 + d_3 = 84$. Similarly, $x_{44}=1$, $x_{45}=0$, $x_{46}=0$, $y_4=1$, implies the demand of the fourth period should be satisfied from the amount replenished (produced) in the fourth period. Hence the amount produced in the fourth period = d_4 =130. Finally, $x_{55}=1$, $x_{56}=1$, $y_5=1$ and $x_{66}=0$, $y_6=0$, suggests that the demands of the fifth and sixth period should be met from the amount produced in the fifth period and no amount should be produced in the sixth period. Hence the amount produced in the fifth period = $d_5 + d_6 = 283$ units. The optimal policy stemming from this interpretation is tabulated below:

Table — 3.3. Showing optimal policy

Period--->	1	2	3	4	5	6	Total
Replenishment -->	84	---	---	130	283	---	497

Total minimized replenishment plus carrying cost = \$248

3.1.3. Formulation with N Variables Restricted as 0-1:

Now, we give a formulation of the problem with exactly N (Length of planning horizon) variables restricted as 0-1. This formulation resembles a transportation type problem.

Notation:

Let,

x_{ij} = Quantity replenished in period i and used in period j, i.e. the quantity carried for (j-i) periods.

$$y_i = \begin{cases} 1 & \text{if the replenishment is made in period i} \\ 0 & \text{otherwise} \end{cases}$$

v = unit variable cost in \$ per unit.

r = carrying cost in \$ per \$ per period.

N = length of planning horizon, i.e. the number of periods in the planning horizon.

A_i = ordering cost per replenishment in dollars for period i, i = 1, 2, 3,...., N.

d_j = deterministically (crisply) known demand for period j, j= 1, 2,....,N

M_i = cumulative demand from period i to period N,

i.e. $d_i + d_{i+1} + \text{-------} + d_N$

Then we have the following model

$$\text{(C2)} \qquad \text{Minimize} \quad v \, . \, r \sum_{i=1}^{N} \sum_{j=i}^{N} (j-i) \; x_{ij} \; + \; \sum_{i=1}^{N} A_i \, . \, y_i$$

subject to

$$\sum_{i=1}^{j} x_{ij} = d_j , \quad j = 1, 2, \ldots\ldots, N \tag{5}$$

$$\sum_{j=i}^{N} x_{ij} \le M_i \; y_i , \quad i = 1, 2, \ldots., N \tag{6}$$

$$x_{ij} \ge 0 \qquad i, j = 1, 2, \ldots, N, \;\; \text{and} \;\; i \le j \tag{7}$$

$$y_i = 0, 1 \qquad i = 1, 2, \ldots, N, \tag{8}$$

(i) The objective function expression in the above model minimizes sum of replenishment and carrying costs.

(ii) Constraints (5) are the demand constraints which ensures that demand in a particular period is met.

(iii) Constraints (6) are the supply constraints, which under certain situations may act as capacity constraints. This constraint set along with constraints (8) and objective function further ensures that whenever some amount is replenished in a particular period, a replenishment cost is incurred.

It is easy to see that (C2) is a linear program in which exactly N variables, y_i, $i = 1, 2, \ldots, N$ are restricted to be zero-one and, therefore, can be easily solved by using available software.

Remarks:

(i). The model (C2) will still be a correct formulation if in the constraints

$$\sum_{j=i}^{N} x_{ij} \leq M_i \, y_i \,, \qquad i = 1, 2, \ldots, N$$

each M_i, $i = 1,2,\ldots,N$ is be replaced by M, where

M = A large number $\geq M_1 = d_1 + d_2 + \text{-------} + d_N$

(ii). The advantage of the formulation (C2) is that by multiplying its objective function by an appropriate multiple we can, trivially, transform it such that all the coefficient in it become integers. Since,the coefficients of all x_{ij}'s in the constraints are already unity, therefore, if the demands d_j's also be discrete (e.g. see Wagner and Whitin, 1958), the formulation will always yield an integer solution without restricting the variables x_{ij}'s as integers.

3.1.3.1. Numerical Example: We illustrate the formulation through the following numerical example that we solved on a personal computer using QSB^+ (Chang and Sullivan, 1991).

Suppose we have the following:

$N = 6$, A_i = \$54 for $i = 1, 2, \ldots, N$, v = \$20 per unit,

r = \$ 0.02 per \$ per month, $v \cdot r$ = \$0.4 /unit/month, and demand is shown in the following table 3.4.

Table - 3.4. Showing demand per period

Period	1	2	3	4	5	6
Demand(d_i)	10	62	12	130	154	129
M_i	497	487	425	413	283	129

We select

$$M = 600 > M_1 = 497$$

and prepare the following Table - 3.5 for convenience.

Table-3.5. x_{ij}=the amount produced in period i and consumed in period j, $i \le j$

		To period						
		1	2	3	4	5	6	Supply(M_i)
	1	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	497
	2		x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	487
From	3			x_{33}	x_{34}	x_{35}	x_{36}	425
Period	4				x_{44}	x_{45}	x_{46}	413
	5					x_{55}	x_{56}	283
	6						x_{66}	129
Demand		10	62	12	130	154	129	

We now have the following linear programming model with y_i's restricted as zero-one variables.

Minimize $0.4\ x_{12} + 0.8\ x_{13} + 1.2x_{14} + 1.6\ x_{15} + 2.0\ x_{16} + 0.4x_{23} + 0.8\ x_{24} + 1.2\ x_{25} + 1.6\ x_{26} + 0.4\ x_{34} + 0.8\ x_{35} + 1.2\ x_{36} + 0.4\ x_{45} + 0.8\ x_{46} + 0.4\ x_{56} + 54\ y_1 + 54\ y_2 + 54\ y_3 + 54\ y_4 + 54\ y_5 + 54\ y_6$

subject to

$x_{11} = 10$

$x_{12} + x_{22} = 62$

$x_{13} + x_{23} + x_{33} = 12$

$x_{14} + x_{24} + x_{34} + x_{44} = 130$

$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} = 154$

$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 129$

$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} - 600\ y_1 \leq 0$

$x_{22} + x_{23} + x_{24} + x_{25} + x_{26} - 600\ y_2 \leq 0$

$x_{33} + x_{34} + x_{35} + x_{36} - 600\ y_3 \leq 0$

$x_{44} + x_{45} + x_{46} - 600\ y_4 \leq 0$

$x_{55} + x_{56} - 600\ y_5 \leq 0$

$x_{66} - 600\ y_6 \leq 0$

$x_{ij} \geq 0 \qquad i,j = 1,2,\ldots,6, \quad \text{and} \quad i \leq j$

$y_i = 0, 1 \qquad i = 1, 2, \ldots, 6,$

3.1.3.2. Results: Solving the above problem on PC using QSB$^+$ (Chang and Sullivan, 1991), we obtain the following results given in Table - 3.6.

Table-3.6. Solution to (C2), the problem with N variables restricted 0-1.

Variable	Value	Variable	Value	Variable	Value
x_{11}	10	x_{44}	130	y_1	1
x_{12}	62	x_{55}	154	y_4	1
x_{13}	12	x_{56}	129	y_5	1

All other decision variables are equal to 0.

Minimum value of the objective function = 248

<u>Interpretation of the results</u>:

Since $x_{ii}+x_{i,i+1}+----+x_{iN}$ where, $i=1,2,...,N$ indicates the quantity replenished in period i and x_{ij} represents the quantity replenished in period i but used in period j, therefore, from above we have that the quantity $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 84$ should be replenished in period 1 out of which 12 units be used in period 1, 62 units in period 2 and 12 units be used in period 3. Similarly, number of units that should be replenished and used in period 4 = 130, number of units replenished in period 5 = 283, out of which 154 units are used in period 5, and 129 units in period 6. Minimum value of the cost is $248.

3.2. Inventory Control under Fuzzy Environments:

In above variable demand inventory problems, the management is forced to provide precise data. However, in practice, management always wants some sort of an elasticity or "leeway". Since the variable demand is forecasted, it is rarely known exactly as the forecasts do not always turn to be crisply correct. This implies that there is always an element of fuzziness in demand. Furthermore, it is a common practice that management specifies a budget limit and asks the production planner to stay 'considerably below' the budget limit. It is this 'considerably below' condition that creates the element of fuzziness in the problem. Under fuzzy environments, when the demand is not a crisp number but is fuzzy, and/or the budget allocated is also not a precise number, the above methods may not work very well to yield an optimal solution. The linear programming formulation for crisp demand (with zero-one variables in it) considered in the present paper, also cannot handle such cases. Therefore, to model such a problem we take advantage of fuzzy set theory (Zadeh 1965, Bellman and Zadeh 1970, Zimmermann, 1988), and by using the idea of symmetric fuzzy linear programming (where both constraints and objective function are fuzzy in nature) we obtain a maxmin linear program with N variables in it restricted as zero-one variables. This is done by creating fuzzy regions around the forecasted demand by using the idea of tolerance as provided by Zimmermann (1988). This formulation is quite simple to follow and always provides a solution to the problem.

Before formulating the problem under a fuzzy environment, we introduce a few concepts of fuzzy set theory that we shall use. For details the reader may refer to Zadeh (1965), Zimmermann (1976, 1988), and Bellman and Zadeh (1970).

3.2.1. Fuzzy Set Theory:

Theory of fuzzy sets is basically a theory of graded concepts. A central concept of fuzzy set theory is that it is permissible for an element to belong partly to a fuzzy set.

Let X be a space of points or objects, with a generic element of X denoted by x. Thus, $X = \{x\}$.

3.2.1.1. Fuzzy Set: Let $x \in X$. A fuzzy set A in X is characterized by a membership function (M.F.) $\mu_A(x)$ which associates with each point in X, a real number in the interval [0,1], with the value of $\mu_A(x)$ at x representing the "grade of membership" of x in A. Thus, the nearer the value of $\mu_A(x)$ is to 1, the higher the grade of belongingness of x in A.

In conventional (crisp) set theory, $\mu_A(x)$ takes only two values 1 or 0 depending on whether the element belongs or does not belong to the set A.

Therefore, formally speaking, if $X = \{x\}$ is a collection of objects denoted generically by x, then a fuzzy set A in X is a set of ordered pairs, $A = \{ (x, \mu_A(x)) / x \in X \}$, where $\mu_A(x)$ maps X to the membership space [0,1].

We shall base our analysis on the following propositions of fuzzy sets (Zadeh 1965, Zimmermann 1988).

3.2.1.2. Union of Fuzzy Sets: The union of two fuzzy sets A and B with respective M.F.'s $\mu_A(x)$ and $\mu_B(x)$ is a fuzzy set C whose M.F. is $\mu_C(x) = \text{Max}\ [\ \mu_A(x),\ \mu_B(x)],\ x \in X$.

3.2.1.3. Intersection of Fuzzy Sets: The intersection of two fuzzy sets A and B with respective M.F.'s $\mu_A(x)$ and $\mu_B(x)$ is a fuzzy set C whose M.F. is $\mu_C(x) = \text{Min}\ [\ \mu_A(x),\ \mu_B(x)],\ x \in X$.

3.2.2. Assumptions: For the model under fuzzy environments, we make the following basic assumptions, similar to Silver and Peterson (1985) with assumption 1 modified as below:

1. <u>The demand rate varies from period to period and is known only imprecisely</u>.
2. The demand pattern terminates at the end of planning horizon or ending inventory must be prespecified.
3. The replenishments are constrained to arrive at the begining of periods and the entire order quantity is delivered at the same time.
4. The cost factors do not change appreciably with time, except that the ordering (set up) cost for period i is A_i, $i=1,2,...,N$.

5. The replenishment lead-time is known with certainty so that delivery can be timed to occur right at the beginning of a period.
6. The unit variable cost does not depend on the replenishment quantity i.e. no quantity discounts are permitted.
7. The product is treated entirely independently of other products, i.e. benefits from joint replenishment do not exist or are ignored.
8. Carrying cost is only applicable to inventory that is carried over from one period to the next.

Objective: The total cost over the entire planning horizon of N periods must stay substantially below a given limit.

3.2.3. Formulation under Fuzzy Environments:

We use the following notation and symbols to model the problem in fuzzy environments.

Notation :

Following Zimmermann (1988), let

B_{OF} = budget limit specified by the management.

P_{OF} = tolerance interval which defines the cost to be considerably below the budget.

μ_{OF} = membership function (M.F.) for the fuzzy objective function.

μ_{jL} = M.F. for lower side of the fuzzy region of fuzzy constraint corresponding to period j.

μ_{jU} = M.F. for the upper side of the fuzzy region of the fuzzy constraint corresponding to period j.

P_j = tolerance interval for the demand data corresponding to period j.

All other symbols and variables have the same meaning as in (C2).

We rewrite the constraints

$$\sum_{i=1}^{j} x_{ij} = d_j \ , \ j = 1, 2, \ldots, N$$

of (C2) in fuzzy environment as follows.

$$\sum_{i=1}^{j} x_{ij} \leq d_j \ , \quad j = 1, 2, \ldots, N \quad \text{with } \mu_{jL} \text{ as the corresponding M.F.} ,$$

and

$$\sum_{i=1}^{j} x_{ij} \geq d_j \ , \quad j = 1, 2, \ldots, N \quad \text{with } \mu_{jU} \text{ as the corresponding M.F.}$$

Then under the fuzzy environments, our linear programming problem (C2) becomes the following problem (F).

(F) Find x_{ij}'s, $i,j = 1, 2, \ldots, N$, $i \leq j$ that satisfy :

<u>The Fuzzy Constraints</u>:

(i). For the objective we have

$$v.r \sum_{i=1}^{N} \sum_{j=i}^{N} (j-i)\ x_{ij} + \sum_{i=1}^{N} A_i.y_i \ \underset{\bullet}{\leq} \ B_{OF} \ , \quad i \leq j$$

with μ_{OF} as the corresponding M.F.

(ii). For the fuzzy demand constraints, we have

$$\sum_{i=1}^{j} x_{ij} \lesssim d_j , \quad j = 1, 2, \ldots, N \quad \text{with } \mu_{jL} \text{ as the corresponding M.F.},$$

and

$$\sum_{i=1}^{j} x_{ij} \gtrsim d_j , \quad j = 1, 2, \ldots, N \quad \text{with } \mu_{jU} \text{ as the corresponding M.F.}$$

The Crisp Constraints :

$$\sum_{j=i}^{N} x_{ij} \le M_i \, y_i , \qquad i = 1, 2, \ldots, N$$

$$x_{ij} \ge 0 \qquad i,j = 1, 2, \ldots, N, \quad \text{and} \quad i \le j$$

$$y_i = 0,1 \qquad i = 1, 2, \ldots, N,$$

where,

(i) M_i in this case is given by

$M_i = d_i + P_i + d_{i+1} + P_{i+1} + \ldots + d_N + P_N$, $i = 1, 2, \ldots, N$

(ii) M is a large number $\ge M_1$, and

(iii) "$\lesssim$"("$\gtrsim$") has the linguistic interpretation "smaller than or equal to with a certain degree lying between 0 and 1" ("greater than or equal to with certain degree lying between 0 and 1" denotes the fuzzified version of "$\le$" ("$\ge$").

In view of Zimmermann (1988), we now define the M.F.'s for the fuzzy objective function and the fuzzy constraints as follows:

For objective function the M.F. is

$$\mu_{OF} = \begin{cases} 1 & \text{if} \quad v.r \sum_{i=1}^{N} \sum_{j=i}^{N} (j-i)\, x_{ij} + \sum_{i=1}^{N} A_i . y_i \le B_{OF} - P_{OF} \\ 0 & \text{if} \quad v.r \sum_{i=1}^{N} \sum_{j=i}^{N} (j-i)\, x_{ij} + \sum_{i=1}^{N} A_i . y_i > B_{OF} \end{cases}$$

and

$$\mu_{OF} = 1 - \frac{v.r \sum_{i=1}^{N} \sum_{j=i}^{N} (j-i)\, x_{ij} + \sum_{i=1}^{N} A_i . y_i - (B_{OF} - P_{OF})}{P_{OF}}$$

when $\quad B_{OF} - P_{OF} < v\,.\,r \sum_{i=1}^{N} \sum_{j=i}^{N} (j-i)\, x_{ij} + \sum_{i=1}^{N} A_i . y_i \le B_{OF}\,, \quad i \le j$

For demand constraints the M.F.'s are

$$\mu_{jL} = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^{j} x_{ij} = d_j\,, \quad j = 1, 2, \ldots, N \\ 0 & \text{if} \quad \sum_{i=1}^{j} x_{ij} < d_j - P_j, \quad j = 1, 2, \ldots, N \end{cases}$$

and

$$\mu_{jL} = 1 + \frac{\sum_{i=1}^{j} x_{ij} - d_j}{P_j}\,, \qquad j = 1, 2, \ldots, N.$$

when $\quad d_j > \sum_{i=1}^{j} x_{ij} \ge d_j - P_j, \qquad j = 1, 2, \ldots, N$

$$\mu_{jU} = \begin{cases} 1 & \text{if } \sum_{i=1}^{j} x_{ij} = d_j, \quad j = 1, 2, \ldots, N \\ 0 & \text{if } \sum_{i=1}^{j} x_{ij} > d_j + P_j, \quad j = 1, 2, \ldots, N \end{cases}$$

and

$$\mu_{jU} = 1 - \frac{\sum_{i=1}^{j} x_{ij} - d_j}{P_j}, \quad j = 1, 2, \ldots, N.$$

when $\quad d_j < \sum_{i=1}^{j} x_{ij} \leq d_j + P_j, \quad j = 1, 2, \ldots, N$

In the above, P_{OF} and P_j's are subjectively chosen constants of admissible violations of the objective function and the constraints respectively (Zimmermann, 1988).

Let, $\mu_D(x)$ be the M.F. of the fuzzy set "decision" of (F). Since, for i= 1,2,...,N, μ_{iL} and μ_{iU} are the membership functions of fuzzy constraints, therefore, the decision space in the fuzzy environment is the intersection of fuzzy sets corresponding to the fuzzy objective function and the fuzzy constraints. Hence,

$$\mu_D(x) = \text{Min}[\mu_{OF}, \mu_{1L}, \mu_{2L}, \ldots, \mu_{NL}, \mu_{1U}, \mu_{2U}, \ldots, \mu_{NU}]$$

Assuming that the management is interested not in a fuzzy set but in a crisp "optimal solution", then on the lines of Zimmermann (1988), we obtain

$$\max \quad \mu_D(x) = \text{Min}[\, \mu_{OF}, \mu_{1L}, \mu_{2L}, \ldots, \mu_{NL}, \mu_{1U}, \mu_{2U}, \ldots, \mu_{NU}]$$

subject to

$$\mu_{OF} \geq \mu_D(x)$$

$$\mu_{iL} \geq \mu_D(x) \qquad i = 1, 2, \ldots, N$$

$$\mu_{iU} \geq \mu_D(x) \qquad i = 1, 2, \ldots, N$$

$$\sum_{j=i}^{N} x_{ij} \leq M_i\, y_i, \qquad i = 1, 2, \ldots, N$$

$$x_{ij} \geq 0 \qquad i, j = 1, 2, \ldots, N, \quad \text{and} \quad i \leq j$$

$$y_i = 0, 1 \qquad i = 1, 2, \ldots, N,$$

Writing λ for $\mu_D(x)$ and using the expressions for the membership functions μ_{OF}, μ_{iL} and μ_{iL} for i = 1, 2,...,N, respectively, we have the following equivalent crisp problem:

(E) Maximize λ

subject to

$$v \,.\, r \sum_{i=1}^{N} \sum_{j=i}^{N} (j - i)\, x_{ij} + \sum_{i=1}^{N} A_i . y_i + P_{OF} . \lambda \leq B_{OF}$$

$$\sum_{i=1}^{j} x_{ij} - P_j \,.\, \lambda \geq d_j - P_j\,, \quad j = 1, 2, \ldots, N.$$

$$\sum_{i=1}^{j} x_{ij} + P_j \,.\, \lambda \leq d_j + P_j\,, \quad j = 1, 2, \ldots, N.$$

$$\sum_{j=i}^{N} x_{ij} \leq M\, y_i\,, \quad i = 1, 2, \ldots, N$$

$$x_{ij} \geq 0 \qquad i, j = 1, 2, \ldots, N, \quad \text{and} \quad i \leq j$$

$$y_i = 0, 1 \qquad i = 1, 2, \ldots, N,$$

It is easy to see that (E) is a linear program in which exactly N variables, y_i, $i = 1, 2, \ldots, N$ are restricted to be zero-one. Also we know (Zimmermann, 1988) that the optimal solution to the problem (E) provides a solution to the problem (F). The problem (E) can be solved by using available software.

3.2.3.1. Numerical Example:

Let us assume that, for the objective function

$B_{OF} = \$300$, $P_{OF} = \$70$,

and for the demand constraints $P_1 = 2$, $P_2 = 4$, $P_3 = 2$, $P_4 = 5$,

$P_5 = 6$, $P_6 = 4$.

the rest of the data is same as in the numerical example under the crisp environment. This simply means that the objective function should be substantially below \$300 with a tolerance of \$70, and the demand is fuzzy with tolerance for the first period being 2, and so on.

Under the given conditions and choosing $M = 600$, our equivalent model is

Maximize λ

subject to

$$0.4\ x_{12} + 0.8\ x_{13} + 1.2\ x_{14} + 1.6\ x_{15} + 2.0\ x_{16} + 0.4\ x_{23} + 0.8\ x_{24} + 1.2\ x_{25} + 1.6\ x_{26} + 0.4\ x_{34} + 0.8\ x_{35} + 1.2\ x_{36} + 0.4\ x_{45} + 0.8\ x_{46} + 0.4\ x_{56} + 54\ y_1 + 54\ y_2 + 54\ y_3 + 54\ y_4 + 54\ y_5 + 54\ y_6 + 70\ \lambda \leq 300$$

$$x_{11} - 2\lambda \geq 8$$
$$x_{11} + 2\lambda \leq 12$$
$$x_{12} + x_{22} - 4\lambda \geq 58$$
$$x_{12} + x_{22} + 4\lambda \leq 66$$
$$x_{13} + x_{23} + x_{33} - 2\lambda \geq 10$$
$$x_{13} + x_{23} + x_{33} + 2\lambda \leq 14$$
$$x_{14} + x_{24} + x_{34} + x_{44} - 5\lambda \geq 125$$
$$x_{14} + x_{24} + x_{34} + x_{44} + 5\lambda \leq 135$$
$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} - 6\lambda \geq 148$$
$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + 6\lambda \leq 160$$
$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} - 4\lambda \geq 125$$
$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} + 4\lambda \leq 133$$
$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} - 600\, y_1 \leq 0$$
$$x_{22} + x_{23} + x_{24} + x_{25} + x_{26} - 600\, y_2 \leq 0$$
$$x_{33} + x_{34} + x_{35} + x_{36} - 600\, y_3 \leq 0$$
$$x_{44} + x_{45} + x_{46} - 600\, y_4 \leq 0$$
$$x_{55} + x_{56} - 600\, y_5 \leq 0$$
$$x_{66} - 600\, y_6 \leq 0$$
$$x_{ij} \geq 0 \qquad i, j = 1, 2, \ldots, 6, \quad \text{and} \quad i \leq j$$
$$y_i = 0, 1 \qquad i = 1, 2, \ldots, 6,$$

3.2.3.2. Results: Solving the above problem on a personal computer using QSB$^+$ (Chang and Sullivan, 1991), we obtain the following results given in Table -3.7.

Table-3.7. Optimal solution to (E), the equivalent problem.

Variable	Value	Variable	Value	Variable	Value
x_{11}	9.5187	x_{44}	128.7967	y_1	1
x_{12}	61.0374	x_{55}	152.5561	y_4	1
x_{13}	11.5187	x_{56}	128.0374	y_5	1

$\lambda = 0.7593$ Cost = \$246.8449

All other decision variables are equal to 0.

Interpretation of Results:

Since $x_{ii}+x_{i,i+1}+----+x_{iN}$ where, $i = 1,2,...,N$ indicates the quantity replenished in period i and x_{ij} represents the quantity replenished in period i but used in period j, therefore, from above we have that the quantity $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 82.0748$ should be replenished in period 1, out of which 9.5187 units are used in period 1, 61.0374 units in period 2, and 11.5187 units should be used in period 3. Similarly, units that should be replenished and used in period 4 = 128.7967, the number of units replenished in period 5 = 280.5935, out of which 152.5561 will be used in period 5, and 128.0374 units in period 6. The corresponding value of the cost is \$246.8449. $\lambda = 0.7593$ indicates the degree of membership of this solution to belong to fuzzy set which corresponds to "Optimal decisions in non-fuzzy environments".

CHAPTER 4.

EQUIPMENT REPLACEMENT PROBLEM

In real life all industrial and military equipment eventually wears out with time and usage. As a result it starts functioning with decreasing efficiency while its operating cost and repair and maintenance costs escalate dramatically. A time will eventually come when these costs become so high that it becomes more economical to replace the used piece of equipment by a new one. A natural question then arises about the optimal time of its replacement. If these costs decrease or remain constant with time, the best policy perhaps then is never to replace the item. However, in real situations such a condition is hardly met. In addition technological developments may also force the user to consider a replacement because better designed machinery may result in improved product quality, better efficiency as well as reduced maintenance and operating costs. Generally, all costs that depend upon the choice or age of the equipment must be taken into account while analyzing the decision of its replacement. In this chapter,we first define the basic equipment replacement problem. Then we propose a simple method (based on dynamic programming) for solving an equipment replacement problem. The algorithm also yields alternative optimal solutions, if any. Then, we give a pure 0-1 linear programming formulation for solving the equipment replacement problem. Numerical examples are provided in each case.

4.1. The Basic Equipment Replacement Problem:

Let some equipment E be purchased by an industry at time i, where, $i = 0, 1, 2, \ldots, n-1$ and n be the length of the planning horizon. Let the origin of the year i be at the time i-1, as shown in the following Figure 4.1.

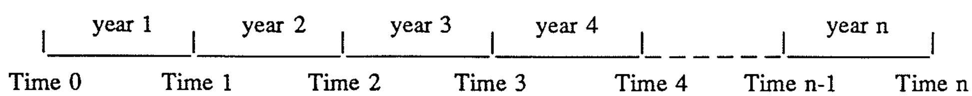

Figure 4.1.- Planning Horizon Figure.

Let

(i) P_i be its acquisition cost of E, m_{ij} be its maintenance cost during the year j and let r_{ij} be its resale value at time j, where $j = 1, 2, \ldots, n$, $(j > i)$.

(ii) m be the maximum allowable age of E and that the first opportunity for its replacement exists not before one year after its purchase. This implies that E may be kept for $p = 1, 2, \ldots,$ or m years, where $m \le n$, and after p ($= 1, 2, \ldots,$ or m) years of use E may be traded in for a new one.

(iii) The resale value of E after time n be zero, i.e. the equipment has no resale value after the planning horizon.

(iv) For $i = 0, 1, 2, \ldots, n-1$; the net cost c_{ij} at time $j = i+1, \ldots, \min(i+m, n)$ is given by

c_{ij} = acquisition cost P_i - resale value r_{ij} + maintenance cost from time i to time j

That is,

$$c_{ij} = P_i - r_{ij} + \sum_{k=i+1}^{j} m_{ik} \qquad \begin{cases} i = 0, 1, 2, \ldots, n-1 \\ j = i+1, i+2, \ldots, \min(i+m, n) \end{cases} \qquad (1)$$

<u>Objective</u>: Given that some new equipment must be purchased at time 0 (at present),we want to find an optimal replacement policy that will minimize its overall cost involved over the time horizon of length n.

4.2. Assumptions:

We made the following assumptions in our model.

1. We assume that acquisition (purchasing) costs, maintenance and operating costs, and resale values of the equipment under consideration are all deterministically known.
2. We assume that at the time of replacement there is only one type of equipment available. There is no loss of generality in making this assumption even if a variety of new equipment is available. We also assume that the age (not the cumulative use) is used to determine the present state of the equipment.
3. It is also assumed that there exists a good second-hand market for used equipment, such that the equipment will have a resale value that may, however, be affected by weather, time, release and/or place of release of new models of the equipment.
4. The structure of the present model is based on the assumption of low initial maintenance and operating costs that increase with age of the equipment while the resale value declines.
5. Due to obsolescence or other reasons (e.g. legal obligations, union contracts, etc.), the equipment age is not allowed to

exceed some specified maximum limit. However, in order to make the model more general, this assumption can be relaxed by giving very high costs and zero resale values for those years which lie beyond this limit.

6. We assume that the planning horizon is finite and that the first opportunity for the equipment replacement appears one year after its purchase.

4.3. Dynamic Programming Formulation:

Following Winston (1991), if

(i) some new equipment be purchased at time i,

(ii) $g_n(i)$ be the minimum net cost incurred from time i (= 0, 1, 2,..., n-1) until time n,then, a dynamic problem formulation of the above problem is given by the following recurrence equation,

$$g_n(i) = \min_j \{c_{ij} + g(j)\} \qquad (i=0,1,2,\ldots,n-1) \qquad (2)$$

where, c_{ij} is as given by (1), j must satisfy the inequalities $i+1 \leq j \leq \min\,(i+m, n)$.

4.3.1. Three Steps Computational Method:

In 1984, Fordyce and Webster (1984) presented the Wagner-Whitin Algorithm (1958) for an inventory control problem with variable demand in a simple and straight forward computational style. In this section we apply the Fordyce and Webster (1984) approach to solve an equipment replacement problem (Winston 1991). The method yields an optimal solution and also identifies alternative optimal solutions, if any.

Along the lines of Fordyce and Webster (1984), we now give a simple and straightforward three steps computational method to solve the equipment replacement problem. The method will be explained through the progressive development of two tables, called the initial tableau and the optimal tableau, and three steps. Since a piece of equipment acquired in any year cannot be used in a prior year and it has a maximum allowable age, therefore, we use only the truncated upper right triangular portion of the tableau . The numbers in the row at the top of each tableau correspond to the states and the numbers at the left of initial tableau correspond to the stages (of the equipment) in a typical dynamic programming problem. The initial tableau contains the initial net costs only, obtained as explained in Step 1 below and the optimal tableau is obtained from the initial tableau as explained in Step 2. Step 3 is for identifying the optimal solution from the optimal tableau. The method also recognizes the alternative optima, if any.

The three steps of our algorithm are as follows:

STEP 1. Obtaining Initial Tableau :

Obtain the initial net costs c_{ij} associated with each state and stage of the equipment by using the formula (1) and prepare the following initial tableau in the form of a truncated upper triangular matrix having n rows (Row 0 to Row (n-1)) and n columns (Column 1 to Column n).

Table - 4.1. Initial tableau for simplified algorithm.

Time →	1	2	3	.... m	m+1	m+2	n−1	n
↓								
Row 0	c_{01}	c_{02}	c_{03}	$\cdot\cdots\cdot c_{0m}$	-	-	-	-
Row 1	-	c_{12}	c_{13}	$\cdot\cdot\cdot\cdot c_{1m}$	$c_{1,m+1}$	-	-	-
Row 2	-	-	c_{23}	$\cdot\cdot\cdot\cdot c_{2m}$	$c_{2,m+1}$	$c_{2,m+2}$	-	-
Row(n−m)	−	−	−	. . . .	. . . .		−	−
Row [n−(m−1)]	−	−	−	. . . .	. . . .	. . . .	. . . . . .	. . . .
Row [n−(m−2)]	. . . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . . . .	. . . .
. . . .	. . . . .	. . . .	. . . .	. . . .	. . . .	. . . .	. . . . . .	. . . .
Row (n−2)	−	−	−	−	−	−	$c_{n-2,n-1}$	$c_{n-2,n}$
Row (n−1)	−	−	−	−	−	−	−	$c_{n-1,n}$

<u>Remark 1</u>: It is important to point out here that if each acquisition cost be independent of the time of acquisition and each of the maintenance costs and salvage values be solely the function of age only then we need to calculate only the first row in the initial tableau. The other rows can be obtained by simply duplicating the first row, shifting to the right by one column each time we move towards the bottom of the table ignoring the entries that lie outside the tableau.

STEP 2. Obtaining the Optimal Tableau:

1. Obtain Row 0 of the optimal tableau by duplicating Row 0 of the initial tableau.

Each subsequent row is obtained as follows.

2. To obtain Row 1 of the optimal tableau :

   (a) Examine Column 1 of the optimal tableau and in this column identify the element with the smallest value v_1 .

   (b) Obtain Row 1 of the optimal tableau by adding v_1 to each element of Row 1 of the initial tableau.

3. To obtain Row 2 of the optimal tableau :

   (a) Examine Column 2 of the optimal tableau and in this column identify the element with the smallest value v_2 .

   (b) Obtain Row 2 of the optimal tableau by adding v_2 to each element of Row 2 of the initial tableau.

In general therefore, we compute Row k, where $k = 1, 2, \ldots, n-1$; of the optimal tableau as follows:

   (a) Examine Column k of the optimal tableau and in this column identify the element with the smallest value v_k .

   (b) Obtain Row k, where $k = 1, 2, \ldots, n-1$, of the optimal tableau by adding v_k to each element of Row k of the initial tableau.

STEP 3. Identifying the Optimal Solution:

1. (i) Examine the Column n of the optimal tableau and identify the minimum value element v_n^* in it. Then,

   $v_n^* = g_n(0)$ is the minimum net cost incurred from time 0 until time n, i.e. during the planning horizon.

   (ii) Suppose the position of v_n^* is in Row s, $s < n$, of the optimal tableau.

Then time s is the trade in time immediately before time n.

2. (i) Examine the Column s of the optimal tableau and identify the minimum value element v_s^* in it. Then, $v_s^* = g_s(0)$ is the minimum net cost incurred from time 0 until time $s < n$

   (ii) Suppose the position of v_s^* is in Row q, $q < s$ of the optimal tableau.

Then time q is the trade in time immediately before time s .

Stopping Rule:

We continue in the same manner until, in the optimal tableau, we identify a minimum value element v_p^*,(say), in Column p, $(p < q)$, such that Row 0 is the row in which v_p^* lies and $v_p^* = g_p(0)$. Once the minimum value element lies in Row 0, the overall optimal policy has been achieved and the process terminates.

The optimal policy is to trade in at time 0, time p,....,time q, time s and the time n (which is the end of the planning horizon) with the overall net optimal cost v_n^* .

Alternative Optimal Solutions:

In Step 3-1(ii), existence of v_n^* (and/or v_s^*in Step 3-2(ii), and so on) in more than one row, implies that the problem has alternative optimal solutions. To obtain an optimal solution we choose any of the rows arbitrarily.

A computer code written in "Think Pascal" for this algorithm is provided in the appendix-B.

4.3.1.1. Numerical Example:

To illustrate our model, we use the example published in Winston(1991) from where we have n = 5, m =3 and the values of all other parameters known deterministically as follows:

$P_0 = P_1 = P_2 = P_3 = P_4 = \$1000 \qquad m_{01} = m_{12} = m_{23} = m_{34} = m_{45} = \$60$

$r_{01} = r_{12} = r_{23} = r_{34} = r_{45} = \$800 \qquad m_{02} = m_{13} = m_{24} = m_{35} = \$80$

$r_{02} = r_{13} = r_{24} = r_{35} = \$600 \qquad m_{03} = m_{14} = m_{25} = \$120$

$r_{03} = r_{14} = r_{25} = \$500$

STEP 1. Initial Tableau:

The entries in each cell are given below :

$$c_{01} = c_{12} = c_{23} = c_{34} = c_{45} = 1000 - 800 + 60 = \$260$$

$$c_{02} = c_{13} = c_{24} = c_{35} = 1000 - 600 + 60 + 80 = \$540$$

$$c_{03} = c_{14} = c_{25} = 1000 - 500 + 60 + 80 + 120 = \$760$$

Note that in this case Remark 1 applies and, therefore, in the initial tableau we calculate the first row only. Other rows are obtained by duplicating the first row.

Thus we have

Table-4.2. Initial tableau for numerical example.

Time → ↓	1	2	3	4	5
Row 0	260	540	760	-	-
Row 1	-	260	540	760	-
Row 2	-	-	260	540	760
Row 3	-	-	-	260	540
Row 4	-	-	-	-	260

STEP 2. Optimal Tableau:

1. Obtain Row 0 of the optimal tableau by duplicating Row 0 of the initial tableau.

Each subsequent row is obtained as follows.

2. To obtain Row 1 of the optimal tableau:

   (a) In Column 1 of the optimal tableau the smallest value v_1 = 260.

   (b) Obtain Row 1 of the optimal tableau by adding $v_1 = 260$ to each element of Row 1 of the initial tableau.

3. To obtain Row 2 of the optimal tableau :

   (a) In Column 2 of the optimal tableau the smallest value v_2 = 520.

   (b) Obtain Row 2 of the optimal tableau by adding $v_2 = 520$ to each element of Row 2 of the initial tableau.

To obtain Row 3 of the optimal tableau :

(a) In Column 3 of the optimal tableau the smallest value v_3 = 760.

(b) Obtain Row 3 of the optimal tableau by adding v_3 = 760 to each element of Row 3 of the initial tableau.

To obtain Row 4 of the optimal tableau :

(a) In Column 4 of the optimal tableau the smallest value v_4 = 1,020.

(b) Obtain Row 4 of the optimal tableau by adding v_4= 1,020 to each element of Row 4 of the initial tableau.

Completed optimal tableau is as follows.

Table - 4.3. Optimal tableau for numerical example.

Time → ↓	1	2	3	4	5
Row 0	260	540	760	-	-
Row 1	-	520	800	1020	-
Row 2	-	-	780	1060	1280
Row 3	-	-	-	1020	1300
Row 4	-	-	-	-	1280

STEP 3. Identifying the optimal solution:

1. (i) In Column 5 of the optimal tableau $v_5^* = 1,280 = g_5(0)$ which is the minimum net cost incurred from time 0 until time 5, the planning horizon.

   (ii) The position of v_5^* is in Row 2 and Row 4. This indicates that there exist alternative optimal solutions. To obtain an optimal solution we choose any of the rows arbitrarily. In this case let us choose Row 4.

Then time 4 is the trade in time immediately before time 5.

2. (i) In Column 4 of the optimal tableau $v_4^* = 1,020 = g_4(0)$. Then, $v_4^* = g_4(0)$ is the minimum net cost incurred from time 0 until time 4.

   (ii) The position of v_4^* is in Row 1 and Row 3. This again indicates that there exist alternative optima. To obtain an optimal solution we choose any of the rows arbitrarily. In this case let us choose Row 3.

3. (i) In Column 3 of the optimal tableau $v_3^* = 760 = g_3(0)$ Then, $v_3^* = g_3(0)$ is the minimum net cost incurred from time 0 until time 3.

   (ii) The position of v_3^* is in Row 0, therefore, the overall optimal policy has been determined and the procedure terminates.

The optimal policy is to trade in at time 0, time 3, time 4 and time 5 (the end of the planning horizon) with the overall net optimal cost $v_5^* = 1280 = g_5(0)$.

Alternative optimal policies with the overall net optimal cost v_5^* $= 1280 = g_5(0)$, are to trade in at

(i) time 0, time 1, time 2 and time 5 (the end of the planning horizon).

(ii) time 0, time 1, time 4 and time 5 (the end of the planning horizon).

4.4. The 0-1 Linear Programming Approach:

In this section, we formulate the deterministic equipment replacement problem (normally solved through the use of dynamic programming) as a 0-1 linear programming problem. The advantage of such a formulation is that it does not require a high degree of expertise, insight and 'art', (i.e.: rather easily understood by practitioners), does not suffer from the so called 'curse of dimensionality' (Budnick et. al., 1988) - a shortcoming of dynamic programming, and can always yield a global optimal solution to fairly large sized problems using available software.

4.4.1. The 0-1 Linear Programming Model:

Let,

$$x_{ij} = \begin{cases} 1 & \text{if equipment purchased at time i is used until time j (j > i)} \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if new equipment is purchased at time i} \\ 0 & \text{otherwise} \end{cases}$$

Since r_{ij} is the resale value of E at time j, for i = 0, 1, 2, ..., n - 1 and j = i+1,....,k, with k = min (i+m, n), and the first opportunity for the equipment replacement exists not before one year after its purchase, therefore $r_{00} = 0$.

Hence, the 0-1 linear programming model of the problem is

$$\text{Minimize } z = \sum_{i=0}^{n-1} \sum_{j=i+1}^{k} \{m_{ij} + (r_{i,j-1} - r_{ij})\} x_{ij} + \sum_{i=0}^{n-1} P_i . y_i \ , \quad k = \min(i+m, n)$$

subject to

$$\sum_{j=i+1}^{k} x_{ij} \leq (k - i) y_i \, , \quad \begin{cases} i = 0,1,2,\text{-----},n-1 \\ k = \min(i+m,n) \end{cases} \qquad (1)$$

$$\sum_{i=0}^{j-1} x_{ij} = 1, \qquad j = 1,\text{-----},m \qquad (2)$$

$$\sum_{i=j-m}^{j-1} x_{ij} = 1, \qquad j = m+1,\text{-----},n \qquad (3)$$

$$x_{ij} \geq x_{i,j+1} \, , \quad \begin{cases} i = 0,1,\text{-----},n-2 \\ j = i+1,\text{-----},k-1 \\ k = \min(i+m,n) \end{cases} \qquad (4)$$

and

$$x_{ij} = 0,1 \ \forall \ i=0,1,2,...,n-1; \ j=i+1,2,...,k \text{ with } k=\min(i+m,n) \qquad (5)$$

$$y_i = 0,1 \ \forall \ i=0,1,2,....,n-1 \qquad (6)$$

In the above 0-1 linear programming model it is interesting to note that:

(i) the total number of variables (x_{ij}'s and y_i's) is always equal to $(1/2)(2n + m)(m + 1) - m^2$, which in turn is also equal to the total number of constraints given by (1) - (4) (for proof see appendix-A).

(ii) Constraints (1) along with binary constraints (5) and (6) ensure that the equipment is replaced not later than after the maximum allowable age of m years, and constraints (2), (3) along with binary constraints (5) ensure that during a particular year only one equipment is in use.

(iii) The function of constraints (4) in conjunction with constraints (5) is to ensure that equipment traded in at a particular time j is not available for use after time j and that the process stops after time n.

4.4.1.1. Numerical Example:

To illustrate our model, we use the example published in Winston (1991) in which we have n = 5, m =3 and the values of all other parameters are known deterministically as follows.

$P_0 = P_1 = P_2 = P_3 = P_4 = \$1000$ $\quad$ $m_{01} = m_{12} = m_{23} = m_{34} = m_{45} = \$60$

$r_{01} = r_{12} = r_{23} = r_{34} = r_{45} = \$800$ $\quad$ $m_{02} = m_{13} = m_{24} = m_{35} = \$80$

$r_{02} = r_{13} = r_{24} = r_{35} = \$600$ $\quad$ $m_{03} = m_{14} = m_{25} = \$120$

$r_{03} = r_{14} = r_{25} = \$500$

Substituting these values in our model, we have the following formulation:

$$\text{Minimize} \quad z = -740x_{01} + 280x_{02} + 220x_{03} - 740x_{12} + 280x_{13} + 220x_{14} - 740x_{23} + 280x_{24} + 220x_{25} - 740x_{34} + 280x_{35} - 740x_{45} + 1000y_0 + 1000y_1 + 1000y_2 + 1000y_3 + 1000y_4$$

subject to

$$x_{01} + x_{02} + x_{03} \leq 3y_0$$
$$x_{12} + x_{13} + x_{14} \leq 3y_1$$
$$x_{23} + x_{24} + x_{25} \leq 3y_2$$
$$x_{34} + x_{35} \leq 2y_3$$
$$x_{45} \leq 1y_4$$
$$x_{01} = 1$$
$$x_{02} + x_{12} = 1$$
$$x_{03} + x_{13} + x_{23} = 1$$
$$x_{14} + x_{24} + x_{34} = 1$$
$$x_{25} + x_{35} + x_{45} = 1$$
$$x_{01} \geq x_{02} \geq x_{03}$$
$$x_{12} \geq x_{13} \geq x_{14}$$
$$x_{23} \geq x_{24} \geq x_{25}$$
$$x_{34} \geq x_{35}$$
$$x_{ij} = 0,1 \ \forall\, i = 0,1,2,3,4; \ j = i+1, 2, \ldots, k \text{ with } k = (i+3, 5)$$
$$y_i = 0,1 \ \forall \ i = 0,1,2,\ldots,4$$

In the above formulation we see that

$$\text{Total number of variables} = (1/2)\ (10 + 3)(3 + 1) - 3^2 = 17$$
$$= \text{Number of constraints obtained using (1) - (4)}$$

Solving the above problem yields the following optimal solution as given in the Table-4.4 below.

Table-4.4 Optimal results of 0-1 model for equipment replacement problem.

Variable	Value	Variable	Value	Variable	Value
x_{01}	1	x_{23}	1	y_0	1
x_{02}	0	x_{24}	1	y_1	1
x_{03}	0	x_{25}	1	y_2	1
x_{12}	1	x_{34}	0	y_3	0
x_{13}	0	x_{35}	0	y_4	0
x_{14}	0	x_{45}	0	-	-

Minimum value of the objective function $z = 1280$

Interpretation of Results:

$y_0 = 1$, $x_{01} = 1$, $x_{02} = 0$, $x_{03} = 0$, $y_1 = 1$ imply that the equipment E purchased at time 0 is used until time 1 and is traded in (a new one is purchased) at time 1.

$y_1 = 1$, $x_{12} = 1$, $x_{13} = 0$, $x_{14} = 0$, $y_2 = 1$ imply that the equipment E purchased at time 1 is used until time 2 and is traded in (a new one is purchased) at time 2.

$y_2 = 1$, $x_{23} = 1$, $x_{24} = 1$, $x_{25} = 1$, $y_3 = 0$ and $x_{34} = 0$, $x_{35} = 0$, $x_{35} = 0$, $y_4 = 0$ imply that the equipment E purchased at time 2 is used until time 5 (is not traded in at time 3 and time 4) and is sold at time 5 (since $n = 5$).

Thus the optimal decision is to purchase at time 0 and to trade in at times 1, 2 and 5. However, since the time horizon $n = 5$, therefore, the equipment is best sold at time 5.

CHAPTER 5.

STRUCTURAL SIMILARITY BETWEEN THE REPLACEMENT PROBLEM AND THE LOT SIZE INVENTORY PROBLEM

In this chapter, we show that the equipment replacement problem discussed in Chapter 4 can be considered as a special type of lot size inventory problem, similar to the one considered in Chapter 3.

5.1. The Lot Size Inventory Problem and the Replacement Problem:

5.1.1. The Lot Size Inventory Problem:

In Chapter 3, under certain assumptions 1-8, (page 39), and notation (page 40), we formulated the following lot size inventory problem, in which the objective is to minimize the total cost over the entire planning horizon of N periods, as the following pure 0-1 linear programming problem.

$$(C_1) \qquad \text{Minimize} \quad \sum_{i=1}^{N}\sum_{j=i}^{N} v \,.\, r\ (j\text{-}i)\ d_j \,.\, x_{ij} \;+\; \sum_{i=1}^{N} A_i\, y_i \qquad (1)$$

subject to

$$\sum_{i=1}^{j} x_{ij} = 1\,, \quad j = 1, 2, \ldots\ldots, N \qquad (2)$$

$$\sum_{j=i}^{N} x_{ij} \leq M\ y_i\,, \quad i = 1, 2, \ldots., N \qquad (3)$$

$$x_{ij} = 0, 1 \qquad i, j = 1, 2, \ldots, N, \ \text{ and } i \leq j \qquad (4)$$

$$y_i = 0, 1 \qquad i = 1, 2, \ldots, N, \qquad (5)$$

where,

$$x_{ij} = \begin{cases} 1 & \text{if demand of period j is satisfied from the amount replenished in the beginning of period i, } i \leq j \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if the replenishment is made in the beginning of period i} \\ 0 & \text{otherwise} \end{cases}$$

v = unit variable (manufacturing or purchase) cost

r = carrying cost, the cost in dollars of carrying one dollar of inventory for one period

N = length of planning horizon, i.e. the number of periods in the planning horizon.

A_i = cost of a replenishment in the beginning of period i ($i = 1, 2, \ldots, N$).

d_j = rate of demand in period j ($j = 1, 2, \ldots, N$).

M = any number $\geq$ N

5.1.2. The Replacement Problem:

Similarly, in Chapter 4, under certain assumptions (page 65) and notation (page 76), we formulated the following replacement problem:

$$(\mathbf{R}_1) \quad \text{Minimize } z = \sum_{i=0}^{n-1} \sum_{j=i+1}^{k} \{m_{ij} + (r_{i,j-1} - r_{ij})\}\, x_{ij} + \sum_{i=0}^{n-1} P_i\, y_i \ , \quad k = \min(i+m, n) \qquad (6)$$

subject to

$$\sum_{j=i+1}^{k} x_{ij} \leq (k-i)y_i, \quad \begin{cases} i = 0, 1, 2, \text{-----}, n-1 \\ k = \min(i+m, n) \end{cases} \tag{7}$$

$$\sum_{i=0}^{j-1} x_{ij} = 1, \qquad j = 1, \text{-----}, m \tag{8}$$

$$\sum_{i=j-m}^{j-1} x_{ij} = 1, \qquad j = m+1, \text{-----}, n \tag{9}$$

$$x_{ij} \geq x_{i,j+1}, \quad \begin{cases} i = 0, 1, \text{-----}, n-2 \\ j = i+1, \text{-----}, k-1 \\ k = \min(i+m, n) \end{cases} \tag{10}$$

and

$$x_{ij} = 0,1 \ \forall \ i=0,1,2,\ldots,n-1;\ j=i+1,2,\ldots,k \text{ with } k=\min(i+m,n) \tag{11}$$

$$y_i = 0,1 \ \forall \ i=0,1,2,\ldots.,n-1 \tag{12}$$

For the above replacement problem (R_1), we have the following.

1. Some equipment E is purchased by an industry at time i, i = 0, 1, 2,....,n-1, n is the length of the planning horizon, and the origin of the period i is at the time i-1, as shown in the following Figure 5.1.

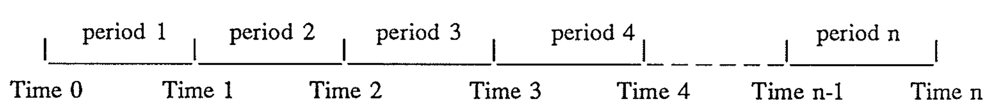

Figure 5.1.-Planning Horizon Figure.

2. P_i is its acquisition cost of E at the beginning of period i,

m_{ij} is its maintenance cost during the period j and let r_{ij} is its resale value at time j, j = 1, 2,..., n, (j > i).

3. m is the maximum allowable age of E and that the first opportunity for its replacement exists not before one period after its purchase. (This implies that E may be kept for p = 1, 2,..,or m periods, where m ≤ n, and after p (= 1,2,...,or m) periods of use E may be traded in for a new one.

4. The resale value of E after time n is zero, i.e. the equipment has no resale value after the planning horizon.

5. For i = 0, 1, 2,..., n-1; the net cost c_{ij} at time j = i+1,...,min (i+m,n) is given by

   c_{ij} = acquisition cost P_i - resale value r_{ij} + maintenance cost from time i to time j.

   That is,

$$c_{ij} = P_i - r_{ij} + \sum_{k=i+1}^{j} m_{ik} \qquad \begin{cases} i = 0, 1, 2, \ldots, n-1 \\ j = i+1, i+2, \ldots, \min(i+m, n) \end{cases}$$

6. Given that some new equipment must be purchased at time 0 (at present),we want to find an optimal replacement policy that will minimize its overall cost involved over the time horizon of length n.

$$x_{ij} = \begin{cases} 1 & \text{if equipment purchased at time i is used until time j } (j > i) \\ 0 & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if new equipment is purchased at time i} \\ 0 & \text{otherwise} \end{cases}$$

5.1.3. Structural Similarity:

In (C_1), setting $c_{ij} \equiv v.r\,(j - i)\,d_j$, and in (R_1), since $c_{ij} \equiv m_{ij} + (r_{i,j-1} - r_{ij})$, setting $P_i \equiv A_i$, and $M > N > k - i$, we obtain the following two models (C_2) and (R_2) respectively

$$(\mathbf{C_2}) \qquad \text{Minimize} \quad \sum_{i=1}^{N}\sum_{j=i}^{N} c_{ij}\, x_{ij} + \sum_{i=1}^{N} A_i\, y_i \tag{13}$$

subject to

$$\sum_{i=1}^{j} x_{ij} = 1, \qquad j = 1, 2, \ldots\ldots, N \tag{14}$$

$$\sum_{j=i}^{N} x_{ij} \le M\, y_i, \qquad i = 1, 2, \ldots, N \tag{15}$$

$$x_{ij} = 0, 1 \qquad i, j = 1, 2, \ldots, N, \text{ and } i \le j \tag{16}$$

$$y_i = 0, 1 \qquad i = 1, 2, \ldots, N, \tag{17}$$

$$(\mathbf{R_2}) \quad \text{Minimize } z = \sum_{i=0}^{n-1}\sum_{j=i+1}^{k} c_{ij}x_{ij} + \sum_{i=0}^{n-1} A_i\, y_i\,, \quad k=\min(i+m, n) \tag{18}$$

subject to

$$\sum_{j=i+1}^{k} x_{ij} \le My_i\,, \quad \begin{cases} i = 0, 1, 2, \text{-----}, n-1 \\ k = \min(i+m, n) \end{cases} \tag{19}$$

$$\sum_{i=0}^{j-1} x_{ij} = 1, \qquad j = 1, \text{-----}, m \tag{20}$$

$$\sum_{i=j-m}^{j-1} x_{ij} = 1, \qquad j = m+1, \text{-----}, n \tag{21}$$

$$x_{ij} \geq x_{i,j+1}\,, \quad \begin{cases} i = 0, 1, \text{-----}, n-2 \\ j = i+1, \text{-----}, k-1 \\ k = \min\,(i+m, n) \end{cases} \tag{22}$$

and

$$x_{ij} = 0,1 \;\forall\; i=0,1,2,\ldots,n-1;\; j=i+1,2,\ldots,k \text{ with } k=\min(i+m,n) \tag{23}$$

$$y_i = 0,1 \;\forall\; i=0,1,2,\ldots,\; n-1 \tag{24}$$

Comparing (C_2) and (R_2), we see that the replacement problem represented by (R_1) can be considered as a special type of the lot size inventory problem, except that it has an additional constraints given by (22) which only indicate that the demand of period j+1 can be satisfied from a lot in period i ($i \leq j$) , only if the demand for period j is satisfied from the same lot.

CHAPTER 6.

CONCLUSIONS, CONTRIBUTIONS AND RECOMMENDATIONS

In the present chapter, we state the conclusions and contributions of this dissertation. Then we discuss the scope of this work under JIT environment. Finally, we give some recommendations for further research on the problems considered in this dissertation.

6.1. Conclusions and Contributions:

In the present dissertation, two important problems in the field of industrial engineering viz: a lot size inventory control problem (addressed by Wagner and Whitin, 1958) and an equipment replacement problem (addressed by Winston 1991, Waddell 1983, Gupta and Hira 1987), have been revisited. A structural similarity between the two formulations presented in the present dissertation has also been identified. Furthermore, we propose a fuzzy logic approach to deal with the inventory control problem when the data is imprecise. In addition, we suggest a simplified three step computational algorithm for the equipment problem.

The inventory problem with variable demand rate under both crisp and fuzzy environments with a planning horizon of N periods, is considered in Chapter 3. Under crisp environments, we formulate the problem as a linear programming problem with exactly N variables restricted to zero-one values. However, one underlying assumption in the above model, and most of the models in the literature is that the demand is deterministically known. But demand is always forecasted and forecasts rarely-if-ever-turn out to be crisply correct. So the models based on the precise knowledge of demand have little practical applications. We deal with such a problem through the fuzzy logic approach. Under fuzzy environments such a problem is formulated as a maxmin linear program, with exactly N variables restricted to zero-one values. Numerical examples are presented for both cases.

Chapter 4 deals with the equipment replacement problem. We formulate a deterministic equipment replacement problem (which is normally solved through the use of dynamic programming) as a 0-1 linear programming problem in which the number of variables is always equal to the number of constraints. The advantage of such a formulation is that it does not require a high degree of expertise, insight and 'art', (i.e. rather easily understood by practitioners), does not suffer from the so called 'curse of dimensionality' (a shortcoming of dynamic programming) and can always yield a global optimal solution to fairly large sized problems using available software. A numerical example is solved on a personal computer and interpretation of the results is provided. In addition to that, we have proposed, along the lines of Fordyce and Webster (1985), a simple method for solving an equipment replacement problem. The algorithm explicitly identifies alternate optimal solutions, if any, as well.

It is shown in Chapter 5 that equipment replacement and inventory control problems considered in the present dissertation have a common mathematical structure. That is, the equipment replacement problem is basically a special type of lot size inventory problem with a constant demand rate of 1 and an additional set of linear constraints peculiar to the replacement problem.

6.2. Scope of this Work under JIT Environment:

The present work is more suitable to MRP type systems. With the increasing number of manufacturers moving towards JIT practices, a discussion on the scope of this research under JIT environment is needed. The main objective of JIT systems is to eliminate wastage of time, material and cost with the goal of zero inventory (lot size ideally equal to 1). Wastage refers to those costs (e.g. inventory costs, setup costs, rework and scrap) which do not add value to the product. In the past, American manufacturers considered setup costs to be unavoidable; but engineering ingenuity can be applied to reduce setup times and costs by designing special jigs and fixtures, simplified dies and machine controls that can switch in little or no time (Hannah,1987). Decreased setup costs result in more frequent set-ups and reduced lot size which in turn reduces holding cost as well. One method of reducing setups could be Group Technology (G.T.). Grouping similar parts results in fewer major set-ups. Even if setup occurs, it is less costly.

A JIT system requires the flow of materials as and when needed, and in exact quantities. But there has to be an advance planning to ensure that material is available when needed. For this purpose a master production schedule in MRP could be moved from monthly or yearly consumption to some hours of usage in a JIT environment.

Although JIT strives for smallest possible lot sizes but there may be valid reasons for acquiring huge inventories of some items. For example, a crop which is harvested only once or twice a year. Other examples could be infrequent and small quantity demand of an item where the cost of delivery of such small quantities would be prohibited (Jordan,1988). For the parts at the entry and exit points of a G.T. cell, standard MRP information is still needed. Final assembly lots can be reduced to 1, but there is still need for a master production schedule; and hence JIT doesn't eliminate the need for a good inventory planning.

Furthermore, constant and effective equipment maintenance and minimal machine breakdown is an important factor in the successful implementation of a JIT system. Therefore, the equipment replacement problem also appears to have potential applications under JIT environment.

6.3. Recommendations for Further Research:

It is believed that a number of extensions are possible to the inventory control and equipment replacement problems. The deterministic and fuzzy models for the inventory control problem, presented in Chapter 3 can be modified to accommodate joint order replenishments in the inventory problem, to take care of capacity restrictions, etc. The joint order model could then be extended even further to incorporate quantity discounts.

It is expected that our 0-1 linear programming approach for the equipment replacement problem should open avenues for its successfully addressing more complicated equipment replacement problems such as group replacement problems. It is believed that the three step method given in Chapter 4 to solve the replacement problem will have many potential applications. It may also be of interest to identify an underlying structure in a set of problems that are currently solved through the use of dynamic programming but could be solved by using the simpler approach, (i) described in the present dissertation for solving a replacement problem, and (ii) described in Fordyce and Webster (1985) for solving the Wagner-Whitin problem (1958) in inventory control. It is highly likely that the above tabular algorithm approach, along with fuzzy dynamic programming will help towards the development of some fuzzy algorithms which, in turn, should become useful in solving the real world problems.

Most of the formulations in this dissertation are assumed to work under non-stochastic conditions. It may be of interest to know, how they work under stochastic conditions.

Though, the present work suits more to determine lot size for a MRP type system, but after initial planning, it could be made applicable to a JIT type environment through some order splitting techniques i.e. obtaining small quantities more frequently.

REFERENCES:

Bahl, H.C., Ritzman, L.P., and Gupta, J.N.D., "Determining Lot Sizes and Resource Requirements: A Riview", Operations Research, Vol. 35, No. 3 (1987), pp.329-345.

Bahl, H.C. and Zionts, S., "Lot Sizing as a Fixed-charge Problem", Production and Inventory Management, Vol.27, No.1 (1986), pp. 1-10.

Bellman, R.E., "Equipment Replacement Policy", SIAM Journal of Applied Mathematics, Vol. 3 (1955), pp. 133-136.

Bellman, R.E. and Dreyfus, S.E., Applied Dynamic Programming, Princeton University Press, Princeton, New Jersey (1962).

Bellman, R. and Zadeh, L.A., "Decision-Making in a Fuzzy Environment", Management Science, 17(1970), 141-164.

Bhaskaran, S. and Sethi, S.P., "The Dynamic Lot Size Model with Stochastic Demands: A Planning Horizon Study", WP, Faculty of Management Studies, University of Toronto, Toronto, Canada (Jan.1981).

Blackburn, J.D. and Kunreuther, H.C., "Planning Horizons for the Dynamic Lot Size Model with Backlogging", Management Science, Vol. 21, No.3 (Nov. 1974), pp.251-255.

Blackburn, J.D., and Millen, R.A., "Heuristic Lot-Sizing Performance in a Rolling Schedule Environment", Decision Science, Vol. 11, (1980), pp.691-701.

Boe, W.J., and Yilmax, C., "The Incremental Order Quantity", Production and Inventory Management, Vol.24, No.2, 2nd Quarter (1983), pp.94-100.

Brown, R.G., Materials Management Systems, Wiley-Interscience, New York, (1977).p.235.

Chand, S. and Sethi, S., "Planning Horizon Procedures for Machine Replacement Models with Several Possible Replacement Alternatives", Naval Research Logistics Quarterly , Vol. 29, No. 3 (1982), pp. 483-493.

Chang, C.H. "A New Dynamic Ordering Rule for Material Requirements Planning", AIIE Systems Engineering Conference, (1977) pp.173-177.

Chang, Yih-Long and Sullivan, Robert S., QSB+ Quantitative Systems for Business Plus, Version 2, Prentice Hall, Englewood Cliffs, NJ (U.S.A.)

Churchman, C.W., Ackoff, R.L. and Arnoff, E.L., Introduction to Operations Research, Wiley, New York (1957).

Cohen, R., Inventory Control at Manitoba Telephone System Phone Centers, M.Sc.Thesis, Dept. of Department of Actuarial & Management Science, University of Manitoba (1983), Winnipeg, MB, Canada.

D'aversa, J.S. and Shapiro, J.F., "Optimal Machine Maintenance and Replacement by Linear Programming and Enumeration", Operational Research Quarterly, Vol. 29, No. 8 (1978), pp. 759-768.

Dean, J., Capital Budgeting, Columbia University Press, New York (1951).

Dean, J., Capital Expenditures, Management and the Replacement of Milk Trucks, Joel Dean Associates, New York (pamphlet).

DeMatteis, J.J., "An Economic Lot-sizing Technique.I: The Part-Period Algorithm", IBM System Journal, Vol. 7, No. 1 (1968), pp. 30-38.

Eilon, S., King, J.R. and Hutchinson, D.E.,"A Study in Equipment Replacement", Operational Research Quarterly, Vol. 17, No. 1 (1966), pp.59-71.

Elmaghraby,S.E., and Bawle,V.Y.,"Optimization of Batch Ordering Under a Deterministic Variable Demand", Management Science ,Vol.18, (1972), pp. 508-517.

Eppen,G.D.,Gould,F.J. and Pashigan,B.D.,"Extensions of the Planning Horizon Theorem in the Dynamic Lot Size Model", Management Science , Vol. 15, No. 5 (January 1969), pp. 268-277.

Fordyce, James. M. and Webster, Francis M., "The Wagner Whitin Algorithm Made Simple", Production and Inventory Management, Second Quarter(1984), pp. 21-30.

Fordyce,James.M. and Webster,Francic.M.,"Non Constant Unit Cost/ Price with Wagner Whitin Algorithm", Production and Inventory Management, First Quarter (1985), pp.71-80.

Freeland, J.R. and Colley, J.L., "A Simple Heuristic Method for Lot Sizing in a Time-phased Reorder System", Production and Inventory Management, Vol. 23, No. 1 (1982), pp. 14-22.

Gaither, N.G.,"A Near-Optimal Lot-Sizing Model for Material Requirements Planning Systems", Production and Inventory Management, Vol. 22, No.4. 4th Quarter (1981), pp. 75-89.

Gaither, N.G.,"An Improved Lot-Sizing Model for MRP Systems," Production and Inventory Management, Vol. 24, No.3. 3rd Quarter (1983), pp.10-20.

Gorham,T.,"Dynamic Order Quantities", Production and Inventory Management, Vol. 9, No. 1(1968), pp. 75-81.

Grant, E., Principles of Engineering Economy, 3rd ed., Ronald Press Company, New York (1950).

Groff,G.,"A Lot-sizing Rule for Time-phased Component Demand", Production and Inventory Management,.Vol.20, No.1(1979),pp. 47-53.

Gupta,P.K. and Hira,D.S.,Operations Research - An Introduction, S.Chand & Company (Pvt.) Ltd. (1987).

Haddock,J., and Hubicki,D.E.,"which Lot-Sizing techniques are Used in Material Requirements Planning?", Production and Inventory Management, Vol.30, No.3.,(1989),pp.53-56.

Hannah,K.H.,"Just-in Time:Meeting the Competitive Challenge" Production and Inventory Management, Vol. 28, No.3. 3rd Quarter (1987), pp.1-3.

Harris,F.W.,Operations and Cost(Factory Management Series), A.W.Shaw Co., Chicago, Chapter 2.(1915).

Hax,A.C., "Aggregate Production Planning", Handbook of Operations Research Models and Applications, edited by Moder, J.J. and Elmaghraby S.E., Van Nostrand Reinhold Co., N.Y. (1978).

Hopp, W.J. and Nair, S.K., "Timing Replacement Decisions Under Discontinuous Technological Change", Naval Research Logistics, Vol. 38,(1991), pp. 203-220.

Hsu, John I.S.,"A Survey of Equipment Replacement practices", Managerial Planning, Mar./Aprl.(1974).

Hsu, John I. S., "Equipment Replacement Policy - A Survey ", Production and Inventory Management Journal, Vol. 29, No. 4 (1988), pp. 23-27.

Jacobs,F.R.and Khumawala,B.M.,"A Simplified Procedure for Optimal Single-Level Lot Sizing" Production and Inventory Management, Vol. 28, No.3. 3rd Quarter (1987), pp.39-43.

Jardine,A.K.S.,"Maintenance and Replacement", Handbook of Operations Research Models and Applications, edited by Moder, J.J. and Elmaghraby,S.E.,Van Nostrand Reinhold Co.,N.Y. (1978).

Jardine,A.K.S.(Ed.)(1970),Operational Research in Maintenance, Manchester University Press and Barnes & Noble (1971).

Jordan, H.H.,"Inventory Management in the JIT Age" Production and Inventory Management, Vol. 29, No.3. 3rd Quarter (1988), pp.57-60.

Karni,R."Maximum Part-Period Gain(MPG)- A Lot Sizing Procedure for Unconstrained and Constrained Requirements Planning Systems", Production and Inventory Management, Vol. 22, No.2. 2nd Quarter (1981), pp. 91-98.

Karni,R."A Study of Lot Sizing Algorithms", IEE Transactions (Dec.1986), pp.356-366.

Krazewski,L.J. and Ritzman,L.P., Operations Management: Strategy and Analysis ,2nd ed., Addison-Wesley Publishing Company,(1990).

Lavis,S.M.,"A Deterministic Inventory Model for a Discrete Demand Schedule With An Available Quantity Discount", WP No. 81-29, Eaton Corp.AIL Division, Deer Park, NY,(April.1981).

Mendoza,A.G.,"An Economic Lot-sizing Technique.II:Mathematical Analysis of the Part-Period Algorithm", IBM System Journal, Vol. 7, No. 1 (1968), pp. 39-46.

McCall,J.J., "Maintenance Policies for Stochastically Failing Equipment: A Survey", Management Science, Vol.11,(1965), pp.493-524.

Naidu,M.M. and Singh,N.,"Lot Sizing for Material Requirement Planning Systems-An Incremental Cost Approach", International Journal of Production Research , Vol.24, No.1(1986), pp.222-240.

Nydick, Robert L,Jr. and Weiss Howard J., "An Evaluation of Variable-demand Lot-sizing Techniques", Production and Inventory Management, First Quarter (1985), pp. 71-80.

Pierskalla, W. and Voelker, J., "A Survey of Maintenance Models: The Control and Surveillance of Deteriorating systems", Naval Research Logistics Quarterly, Vol. 23 (1976) pp. 353-388.

Plossl,G.W. Production and Inventory Control Principles and Techniques , 2nd ed., Prentice-Hall, Englewood Cliffs, (1985).

Ray, Thomas G., A Systems Approach to Replacement , Ph. D. Dissertation , Dept. of Industrial Engineering & Operations Research, Virginia Polytechnic Institute, (August, 1971), Blacksburgh, Virginia, U.S.A.

Reinfeld, N.V., Production and Inventory Control, Reston Publishing Company , (1982).

Robrade,A.,and Zoller,K.,"Dynamische Bestellmengen-und Losgroßen -Planning.", Research Report#87/1. Institut fur Betriebliche Logistik und Organisation, Universitat der Bundeswehr Hamburg,1987.

Schwartz,J.,"The Pernicious Influence of mathematics in Science", Logic Methodology and Philosophy of Science", edited by Nagel,E.,Suppes,P. and Tarski,A.,Stanford,(1962).

Sherif, Y. and Smith, M., "Optimal Maintenance Models for Systems Subject to Failure - A Review", Naval Research Logistics Quarterly , Vol. 28 (1981) pp.47-74.

Silver, E.A.,"Operations Research in Inventory Management:A Review and Critique", Operations Research, Vol.29, No.4. Quarter (1981), pp.628-645.

Silver, E.A.,"Comments on 'A Near-Optimal Lot-Sizing Model", Production and Inventory Management, Vol. 24, No.3. 3rd Quarter (1983), pp.115-116.

Silver.E. A. and Meal, H.C., "A Heuristic for Selecting Lot Sizing Quantities for the Case of Deterministic Time-varying Demand Rate and Discrete Opportunities for Replenishment", Production and Inventory Management, Vol. 14, No. 2 (1973), pp. 64-74.

Silver,E.A. and Peterson, R.,Decision Systems for Inventory Management and Production Planning,2nd ed., Wiley, NY (1985).

Smith,S.B., Computer-Based Production and Inventory Control , Prentice-Hall, Englewood Cliffs (1989).

Stapleton, R.C., Hemmings, D.B. and Scholefield, H.H., "Technical Change and the Optimal Life of Assets", Operations Research Quarterly, Vol. 23,No. 1 (1972), pp. 45-59.

Terborgh,B., Dynamic Equipment Policy, Mc-Graw Hill Book Co., NewYork (1949).

Trux,W.,"Einkauf und Lagerdisposition mit Datenverarbeiting.", 2.aufl.,Munchen: Moderne Industrie, 1972.

Waddell, R., "A Model for Equipment Replacement Decisions and Policies", Interfaces ,Vol. 13 (1983), pp. 1-8.

Wallace, T.F.,ed. APICS Dictionary, 5th edition. Falls Church, Va.: American Production and Inventory Control Society,1984.

Wagner, H.M., "A Postscript to Dynamic Problems in the Theory of the Firm", Naval Research Logistics Quarterly,Vol.7(1960) pp.7-12.

Wagner, H.M., Principles of Operation Research , Prentice-Hall, Englewood Cliffs (1969).

Wagner, H.M., and Whitin T.M., "Dynamic Version of the Economic Lot Size Model", Management Science , Vol. 5, No. 1 (October 1958), pp. 89-96.

Wemmerlov,U., "The Ubiquitous EOQ-its Relation to Discrete Lot Sizing Heuristics", International Journal of Operations and Production Management, Vol. 1, No. 3(1981), pp.161-179.

Wemmerlov,U.,"Comments on 'A Near-Optimal Lot-Sizing Model for Material Requirements Planning Systems'", Production and Inventory Management, Vol. 24, No.3. 3rd Quarter (1983), pp.117-121.

Wilson,R.H."A Scientific Routine for Stock Control", Harvard Business Review,Vol.13, No.1(1934).

Winston W. L. Operations Research, Applications and Algorithms , PWS Publications (1991).

Zabel,E.,"Some Generalizations of an Inventory Planning Horizon Theorem", Management Science , Vol. 10, (1964), pp.465-471.

Zadeh, L.A., "Fuzzy Sets", Information and Control, 8 (1965), 338-353.

Zadeh,L.A.,"Outline of a New Approach to the Analysis of Complex Systems and Decision Processes", IEEE Trans. SMC-2,pp 28-44 (1973).

Zangwill,W.I.,"A Backlogging Model and Multi-Echelon Model of a Dynamic Economic Lot Size Production System-A Network Approach", Management Science , Vol. 15,No.9 (May 1969), pp.506-527.

Zimmermann, H.J.,"Description and Optimization of Fuzzy Systems", Int. Journ. of General Systems, 2(1976), 209-215.

Zimmermann, H.J, Fuzzy Set Theory and Its Application, Kluwer-Nijhoff Publishing, (1988)

Zoller,K.,and Robrade,A.,"Dynamic Lot Sizing Techniques: Survey and Comparison", Journal of Operations Management,Vol. 7, No.4 (December 1988), pp.125-148.

APPENDIX A.

The constraints (1) - (4) (page 76) of Chapter 4 can be written easily by using the following Table A .

Table A.

Time → ↓	1	2	3 . . . . m	m+1	m+2	n-1	n	R.H.S
Row 0	x_{01}	x_{02}	x_{03} . ..... x_{0m}	-	-	-	-	my_0
Row 1		x_{12}	x_{13}	x_{1m}	$x_{1,m+1}$	-	-	$m y_1$
Row 2			x_{23} . . .. x_{2m}	$x_{2,m+1}$	$x_{2,m+2}$	-	-	$m y_2$
.								
Row (n-m)								my_{n-m}
Row (n-(m-1))								$(m-1)y_{n-(m-1)}$
Row (n-(m-2))								$(m-2)y_{n-(m-2)}$
.								
Row (n-2)						$x_{n-2,n-1}$	$x_{n-2,n}$	$2y_{n-2}$
Row (n-1)							$x_{n-1,n}$	$1y_{n-1}$
	1	1	1	.....	1	1	1	

From Table - A, we can now obtain the constraints (2) - (5) as follows:

(i) Constraints (1) are n in number and are obtained from row 0 through row (n-1), obtaining first constraint from row 0 by adding the variables and writing it less than equal to R.H.S., which is my_0. Similarly nth constraint is $x_{n-1,n} \leq 1y_{n-1}$.

(ii) Constraint (2) - (3) are n in number and are obtained from the columns by staring with column 1 and equating the sum of the variables in each column equal to 1 respectively.

(iii) Constraints (4) are (1/2)(2n - m) (m - 1) in number and are obtained by starting by first row which is row 0. Thus from row 0 we have (m-1) constraints $x_{01} \geq x_{02} \geq x_{03} \geq . .... \geq x_{0m}$.

From row 1 we have (m-1) constraints $x_{12} \geq x_{13} \geq x_{14} \geq . .... \geq x_{1,m+1}$

Similarly we have rest of the constraints from other rows up to the row (n-1).

Thus the total number of constraints from the Table A = 2n + (1/2)(2n - m) (m - 1) = (1/2) (2n + m) (m + 1) - m^2 .

Total number of variables are calculated from the Table - A as follows:

Number of variables in row 0 to row (n - m) = m (n-m +1)
Number of variables in row (n - (m + 1)) to row (n - 1) = (m - 1) + (m - 2) + ... + 2 + 1 = (1/2) (m - 1) m
Number of y_i's = n
Therefore, total number of variables = m (n-m + 1) + (1/2) (m - 1) m + n = (1/2) (2n + m) (m + 1) - m^2

APPENDIX B.

```
{*** THIS PROGRAM IS CODED IN THINK PASCAL AND IT ****}
{******* SOLVES EQUIPMENT REPLACEMENT PROBLEM ********}
{********ASSUMPTION:MAINTENANCE COSTS AND SALVAGE VALUES******}
{**************DEPEND SOLELY ON THE AGE*********************}
{*************************************************************}
PROGRAM DYNAMIC_REPLACEMENT (INPUT, OUTPUT);
{*************************************************************}
{***VARIABLES AND VARIABLE TYPE DECLARATION FOLLOWS***********}
{*************************************************************}
  TYPE
     ONE_D_INT = ARRAY[0..99] OF INTEGER;
     ONE_D = ARRAY[1..100] OF REAL;
     ONE_D_LESS = array[0..100] OF REAL;
     TWO_D = ARRAY[0..49, 1..100] OF REAL;
  VAR
     A: REAL;
     MAX, AGEMAX: INTEGER;
     OPTIMAL: ONE_D_LESS;
     SALVAGE, MANTNCE: ONE_D;
     TIME: ONE_D_INT;
     COST: TWO_D;
     CHARACTER: CHAR;
{*************************************************************}
{********************** PART A *******************************}
{***THIS PROCEDURE READS IN MAINTENANCE COSTS AND SALVAGE*****}
{***VALUES AND LOADS THEM TO TWO ONE-DIMENSIONAL ARRAYS*******}
{*************************************************************}
PROCEDURE LOAD_M_S (VAR M, S: ONE_D; AGEMAX: INTEGER);
  VAR
     INDEX: INTEGER;
  BEGIN
     WRITELN;
     WRITELN('DATA ENTRY FOR MAINTENANCE COSTS FOLLOWS : ');
     WRITELN;
```

```
    FOR INDEX := 1 TO AGEMAX DO
      BEGIN
          WRITE('ENTER MAINTENANCE COST DURING PERIOD', INDEX :
                  3,'OF OPERATION: ');
          READLN(M[INDEX]);
      END;  {FOR}
    WRITELN;
    WRITELN('DATA ENTRY FOR SALVAGE VALUES FOLLOWS : ');
    WRITELN;
    FOR INDEX := 1 TO AGEMAX DO
      BEGIN
          WRITE('ENTER SALVAGE VALUE OF',INDEX:3,'PERIOD OLD
                  EQUIPMENT: ');
          READLN(S[INDEX]);
      END;  {FOR}
  END; {LOAD_M_S}
{*************************************************************}
{************************ PART B ****************************}
{*********THIS PROCEDURE COMPUTES THE NET COSTS AND***********}
{*********AND LOADS THEM TO A TWO DIMENSIONAL ARRAY***********}
{*************************************************************}
PROCEDURE LOAD_COST (VAR C: TWO_D; M, S: ONE_D; MAX, AGEMAX:
                      INTEGER);
  VAR
      ROW, COL: INTEGER;
      MAINT: REAL;
  BEGIN
      FOR ROW := 0 TO MAX - 1 DO
        BEGIN
            MAINT := 0
            FOR COL := ROW + 1 TO ROW + AGEMAX DO
                BEGIN
                    IF COL <= MAX THEN
                      BEGIN
                          MAINT := MAINT + M[COL - ROW];
                          C[ROW, COL]:= A + MAINT - S[COL - ROW];
```

```
                    END;{IF-THEN}
                END; {FOR}
            END; {FOR}
   END; {LOAD_COST}

{**********************************************************}
{********************* PART C ****************************}
{***THIS PROCEDURE IS THE KEY PROCEDURE. IT PERFORMS THE***}
{***CALCULATIONS OF DYNAMIC PROGRAMMING ALGORITHM AND******}
{***STORES THE FINAL RESULTS IN ARRAYS OPTIMAL AND TIME****}
{**********************************************************}
PROCEDURE PROCESS (VAR OPTIMAL:ONE_D_LESS; VAR TIME:ONE_D_INT;
                   C: TWO_D;MAX,AGEMAX: INTEGER);
   VAR
      ROW, COL: INTEGER;
      TEMP: REAL;
   BEGIN
      OPTIMAL[MAX] := 0;
      FOR ROW := MAX - 1 DOWNTO 0 DO
         BEGIN
            OPTIMAL[ROW] := C[ROW, ROW + 1] + OPTIMAL[ROW + 1];
            TIME[ROW] := ROW + 1;
            FOR COL := ROW + 2 to ROW + AGEMAX DO
               BEGIN
                  IF COL <= MAX THEN
                     BEGIN
                        TEMP := C[ROW, COL] + OPTIMAL[COL];
                        IF TEMP < OPTIMAL[ROW] THEN
                          BEGIN
                             OPTIMAL[ROW] := TEMP;
                             TIME[ROW] := COL;
                          END; {IF-THEN}
                     END; {IF-THEN}
               END; {FOR}
         END;  {FOR}
   END; {PROCESS}
```

```
{*********************************************************}
{******************** PART D **************************}
{********THIS PROCEDURE  DISPLAYS THE FINAL RESULTS*******}
{*********************************************************}
PROCEDURE RESULTS (OPTIMAL: ONE_D_LESS; TIME: ONE_D_INT;
                    MAX: INTEGER);
  VAR
     HOLD: INTEGER;
  BEGIN
     WRITELN;
     WRITELN('THE RESULTS ARE AS FOLLOWS: ');
     WRITELN;
     WRITELN('THE OPTIMAL POLICY IS TO TRADE IN AT THE FOLLOWING
              TIMES: ');
     HOLD := 0;
     WHILE HOLD < MAX DO
       BEGIN
          WRITELN('TIME ', HOLD : 2);
          HOLD := TIME[HOLD];
       END;  {WHILE}
     WRITELN('TIME ', MAX : 2);
     WRITELN;
     WRITELN('THE MINIMIZED COST FOR THE ABOVE POLICY IS $ ',
              OPTIMAL[0] : 12 : 2);
  END;  {RESULTS}
{*********************************************************}
{****************** MAIN LINE *************************}
{*********************************************************}
  BEGIN    {MAIN LINE}
     CHARACTER := 'Y';
     WHILE (CHARACTER = 'Y') OR (CHARACTER = 'y') DO
       BEGIN
          WRITELN('WELCOME TO EQUIPMENT REPLACEMENT PROBLEM !');
          WRITELN;
```

```
WRITELN('THIS SOFTWARE USES DYNAMIC PROGRAMMING TO
        SOLVE EQUIPMENT REPLACEMENT PROBLEM');
WRITELN('IF OPTIMAL COST IS ≥ 10000000000, THEN IT
        MAY  BE ROUNDED OFF.');
WRITELN;
WRITELN('DATA ENTRY FOLLOWS:');
WRITELN;
WRITE(' ENTER THE LENGTH OF THE PLANNING
       HORIZON (≤100 ): ');
READLN(MAX);
WRITE(' ENTER THE MAXIMUM LIMIT ON THE AGE OF THE
       EQUIPMENT(≤100 ): ');
READLN(AGEMAX);
WRITE('ENTER ACQUISITION COST OF THE NEW EQUIPMENT:');
READLN(A);
LOAD_M_S(MANTNCE, SALVAGE, AGEMAX);
LOAD_COST(COST, MANTNCE, SALVAGE, MAX, AGEMAX);
PROCESS(OPTIMAL, TIME, COST, MAX, AGEMAX);
RESULTS(OPTIMAL, TIME, MAX);
WRITELN('END OF PROCESSING');
WRITELN('THANKS FOR USING THE EQUIPMENT REPLACEMENT
        PROBLEM !');
WRITELN;
WRITELN('FOR A HARD COPY OF THE RESULTS, PRESS
CLOVE-SHIFT-4 KEYS TOGETHER');
WRITE('WANT TO SOLVE ANOTHER PROBLEM (Y/N) ? ');
READLN(CHARACTER);
END;    {WHILE}
END.       {MAIN LINE}
```