LEARNING PLACE VALUE: INSTRUCTION USING A COMPARISON OF PHYSICAL AND COMPUTER GENERATED BLOCKS

BY

ALAN T. YOSHINO

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Department of Education
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Name ALAN YOSHINO

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ABSTRACT

Understanding place value has been a problem for many years and the proper use of concrete materials has been advocated to help rectify the problem. This study was a comparison of two methods of teaching place value to two comparable groups, each with twenty-two students. One group was taught with actual base-ten blocks and the other was taught using a computer program, *Blocks Microworld.* The general questions posed for the study involved improving creativity through less constraints and enhancing the understanding of place value through continual reinforcement of written numerical notation. Specific questions involving the comparison of the two methods were studied in detail. The evaluation was based upon pretest/posttest score analysis, classroom observations and student interviews. Certain subgroups of students seemed to benefit more than others depending on the mode of teaching. The overall results indicated that there were some limited advantages in using the computer program for place value understanding, but also indicated that it was very difficult to alter students' methods of solving problems once an algorithmic approach had been learned regardless which methodology was used.

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Chapter I. Introduction

Importance of the Study

The understanding of place value and numeration has been a concern of mathematics educators for many years. Payne and Rathmell (1975) indicate "that children have difficulty learning about numeration has been well documented by informal observations of teachers as well as by more careful evaluations of researchers" (p. 149). More recently Fuson and Briars (1990) cited several reports which indicate:

> that on digit correspondence tasks most second graders and many third and fourth graders receiving traditional instruction show no understanding that the tens digit means ten things (these children show one chip - rather than ten chips - to demonstrate what the 1 in 16 means), or they are misled by nonten groupings and show only a grouping face-value meaning (for 13 objects arranged as three groups of four objects and one left-over object, they say that the 3 means the three groups and the 1 means the one left-over object). (p. 195-196)

In addition to the difficulties that children experience in understanding place value, recommendations from the National Council of Teachers of Mathematics, through the publication *Curriculum and Evaluation Standards for School Mathematics* (1989), calls for an increased emphasis on meaningful experiences in school mathematics and a decreased emphasis on rote memorization of computational algorithms. This recommendation addresses the concern expressed by Kamii (1989), that "the ability to produce correct answers in double-column addition by following the algorithm does not mean that children understand place value" (p. 19).

A study by Thompson incorporated the computer program, *Blocks Microworld*, to determine if it would enhance curiosity and hence understanding in children as they learn about place value. It is suspected that in many classrooms, connections fail to be made from the manipulations of physical base-ten materials to symbolic notation. This problem seems to have been addressed in *Blocks Microworld* since there is a continual automatic reference to symbolic notation in the program as one manipulates the blocks. Hiebert and Wearne (1992) point out that "instruction that focuses on helping students construct connections provides one form of teaching for understanding" (p. 99). *Blocks Microworld* also provides an opportunity for children to construct their own ideas as to the meaning of arithmetic notation and operations.

Thompson's Study

Thompson (1991) had developed a software package, *Blocks Microworld*, which was designed to enhance the understanding of place value. It is a mouse-driven, computerized microworld that presents base-ten blocks and numeration as linked systems. Thompson (1992) elaborates on the program indicating that:

> the microworld was designed with one restriction not available in physical base-ten blocks. Students could perform representation-transforming actions on the

> blocks (sometimes called trades) only by acting on digits in a quantity's numeral representation. *Blocks Microworld* is an example of a mathematical microworld that employs multiple, linked notational systems. . . . The design of *Blocks Microworld* is intended to support students' continual making of meaning for their notational actions and interpretation of notation. This support springs from their being oriented continually toward notation even when their intention is to manipulate blocks. (p. 127-128)

Thompson conducted a study which he describes as follows:

> Twenty fourth grade children were matched according to performance on a whole number calculation and concepts pretest and assigned at random to one of two groups: wooden base-ten blocks and computerized microworld. Instruction in each group was designed to orient students toward relationships between notation and meaning. Instruction given the two groups was based upon a single script that extended whole number numeration to decimal numeration, and emphasized solving problems in concrete settings while inventing notational schemes to represent steps in solutions. (p. 123)

Thompson (1992) states that a primary focus of the study was "to investigate what features of students' engagement in tasks involving base-ten blocks contribute to students' construction of meaning for decimal numeration and their construction of notational methods for determining the results of operations involving decimal numbers" (p. 125). The study investigates the understanding of the base-ten numeration system and in particular, pursues the question of transfer of concept understanding in concrete and pictorial representation to symbolic representation. It also examines the use of a dynamic versus a static medium with special attention offered to constraints and supports in those environments. Briefly, Thompson's findings showed that,

> Neither group changed in regard to whole-number notational methods. Blocks children understood decimal numbers as if they were funny whole numbers; Microworld children attempted to give meaning to decimal notational methods, but were largely in a state of disequilibrium at the end of the study. (p. 123)

The state of "disequilibrium", as explained by Thompson, is a possible indication of students "trying to make sense" of problem situations. Thompson further concludes that:

> We must take care not to conclude from this study that uses of wooden base-ten blocks, or physical materials in general, are ineffectual in producing understanding of notational methods. . . . Rather, this study would suggest that we reexamine previous studies, asking to what degree students have internalized procedures as prescriptions, as distinct from having internalized them as conventions. (p. 145-146)

This Study

The intent of the study reported upon here was to determine if the findings would support the results which were suggested by Thompson. The general questions posed for the study were:

1. Would less restrictive constraints (ie. freedom from memorized algorithms) placed upon students improve their creativity in approaching problem-solving in regard to place value?
2. Would continual reinforcement of written numerical notation with real or simulated base-ten blocks enhance the students' understandings of place value?
3. How would the students using the computer program, *Blocks Microworld* compare to the students using real blocks?

The study was a replication of the study done by Thompson which was an experimental learning experience comparing subgroups of students learning with physical Dienes blocks and the computer program, *Blocks Microworld.*

Forty-four students were divided into two groups of comparable proficiency according to their performance on a test of whole numbers, fractions and decimal fractions. One group received instruction using base-ten blocks and the other received instruction using the computer program, *Blocks Microworld.* Both groups received instruction following similar scripts. The instruction focused on the relationships and meaning which developed between block manipulations and notation. The content area extended from calculations with whole numbers into addition and subtraction of decimal numbers. There was an emphasis on having students understand the concept of place value and also an attempt to dispel any belief that there is only one correct way to add and subtract.

Limitations

An increased amount of time for instruction might have been beneficial to the study since it would have been possible to collect more data through observations and interviews and to use a more comprehensive script for the lessons. But the time for the classroom instruction for this study was kept to approximately the same as in Thompson's study. This decision was made so that the study would be as close a replication as possible, and also because the teachers and students in the study could not afford more time from their regular curriculum.

It may have been better to have selected the students randomly from a larger population, but the mechanics involved in accomplishing this would be nearly impossible. There may have been a benefit in using larger numbers of students, but the size of classes of the selected teachers was twenty-two students and it would have been very difficult to manage more than the two classes in this study. But the fact that the two groups had come from a common pool of students from the previous year and the random method by which the two groups were selected helped to ensure appropriate grouping.

The two classes had to proceed simultaneously because of timetabling considerations, which meant that data from observations by the researcher was collected in half the desired time.

Chapter II. Literature Review

There is an abundance of literature on the topic of teaching and learning the base-ten numeration system.

First, literature is cited which supports the notion that there is a lack of understanding of the base-ten numeration system. This is followed by a review of literature on manipulatives which describes the attempts to rectify the lack of understanding of base-ten concepts. Thirdly, literature which supports the importance of making necessary connections for concept transfer is examined. This portion of the literature review attempts to demonstrate the value of connections from concrete to symbolic representation for effective understanding of place value concepts. The idea of connections is further explored in terms of notation systems. Fourth, aids to the understanding of the base-ten numeration system are identified through literature which elaborates on constraints and supports offered to students in learning mathematical concepts. Some of the literature directly identifies *Blocks Microworld* as a favorable example of the good use of constraints and supports as well as an example of learning through a dynamic medium as opposed to a static medium.

Lack of Understanding of Base-ten Concepts

The understanding of the concepts of base-ten numeration and place value is very basic to the development of further mathematical concepts. Nevertheless, it is one of the areas in early mathematical development that seems to be done without understanding in many cases. As pointed out by Ross (1986), "It has been repeatedly documented, however, that children throughout their elementary school years generally understand place value poorly" (p. 1). There are many studies that reflect the fact that children have difficulty in understanding the meaning of the symbolic notation system. Engelhardt, Ashlock and Wiebe (1984) document a number of reports in support of the foregoing statement; Bednarz and Janvier (1979) found only 10 percent of 8-and 9-year olds displayed a good understanding of numeration (place value); Smith (1973) found that third graders had difficulty renaming numerals; Engelhardt and Wiebe (1979) analyzed calculation errors made by middle class 4th grade students on problems involving whole numbers and decimal fractions. Of those, 23 percent were classified as numeration errors. In a subsequent test of numeration understanding, pupils performed more poorly than anticipated and poorer performance seemed to be associated with numeration errors in the earlier computation problems. More recently, Labinowicz (1985) noted that children had difficulty in interpreting place-value notation. In a study done by Baroody, Berent, Gannon and Ginsberg (1983), it was found that younger children often wrote larger, unfamiliar numbers as they were spoken. That is, they were usually correct in writing one digit numerals, but tended to write teen numerals in reverse (51 for "fifteen"). Fuson and Briars (1990) state that "many children who carry out the algorithms correctly do so procedurally and do not understand reasons for crucial aspects of the procedure or cannot give the values of the trades they are writing down" (p. 181). As attested to by these educators, there is strong evidence that there exists a lack of understanding in the base-ten numeration system.

Miura and Okamoto (1989), in a study examining the differences in mathematical performances of students in Japan and the United States, indicate that the "cognitive representation of number may be differentially influenced by the structural characteristics of the particular numerical languages and that these differences in number representation may contribute to variation in mathematics performance between the two groups" (p. 113). This may or may not be a factor in children learning base-ten numeration, but in North America we must look elsewhere to further promote understanding of place value concepts. This is reiterated by Fuson (1990):

> English-speaking U.S. children may need considerable and extended support in the classroom for constructing multiunit meanings based on ten, because the English language and the U.S. culture provide relatively little support for such meanings. English does not explicitly name the tens in 2-digit numbers, in contrast to several Asian languages (Chinese, Japanese, Korean) that do name the tens (12 is 'ten two' and 58 is said 'five ten eight'). (p. 278)

Use of Concrete Materials

In order to alleviate a general lack of understanding of mathematical concepts, educators have been promoting the use of concrete materials (manipulative materials) in the teaching of mathematics. Manipulative materials have been advocated for the instruction of early and middle years mathematics for at least the past decade. The importance of using manipulatives for conceptual understanding of mathematics has been stressed by many educators (Ball, 1988; Curcio et al, 1987; Madell, 1985; Sweetland, 1984; Van de Walle & Thompson, 1984). Sowell (1989), through the use of meta-analysis, compiled the results of 60 studies to determine the effectiveness of mathematics instruction with manipulative materials and states, "Results showed that mathematics achievement is increased through the long-term use of concrete instructional materials and that students' attitudes toward mathematics are improved when they have instruction with concrete materials provided by teachers knowledgeable about their use" (p. 498). Throughout the *Curriculum and Evaluation Standards for School Mathematics*, there are numerous references to teaching with manipulative materials. The message is that:

> Children come to understand number meanings gradually. To encourage these understandings, teachers can offer classroom experiences in which students first manipulate physical objects and then use their own language to explain their thinking. This active involvement in, and expression of, physical manipulations encourages children to reflect on their actions and to construct their own number meanings. In all situations, work with number symbols should be meaningfully linked to concrete materials. (p. 38)

However, even with the advocation of concrete materials, there are studies which still indicate many shortcomings with respect to student understanding of mathematical concepts. The literature on the effectiveness of concrete materials has ranged from no effect to significant change (Fennema, 1972; Labinowicz, 1985; Resnick & Omanson, 1987; Sowell, 1989; Wearne & Hiebert, 1988).

There are many reasons for the lack of understanding with the notation system for numeration. One factor that cannot be ignored is the strong possibility of the limited use of concrete manipulative materials. There are numerous studies and surveys which indicate a limited use of manipulatives, especially at upper elementary and higher grades. (Matas, 1992; McBride & Lamb, 1986; Scott, 1983; Suydam, 1984). Because of this lack of manipulative usage, Caully (1988) points out, "traditional instruction often overlooks children's own construction of the base-ten system, they are forced to learn conventional rules by rote. Students are left with an incomprehensible list of rules that are often wrong" (p. 205).

Moreover, there seems to be a reluctance on the part of teachers to spend the time and effort to promote the learning of concepts when it appears their students can "perform well" on a paper-and-pencil test. Swart (1985) although addressing the "back to the basics" movement, emphasizes the preceding point:

> It appears, to our dismay, that many people think of the basics as being little more than the ability to compute with paper and pencil to achieve 80 percent correct on tests of computational ability. This superficial view of the basics has two problems. Not only does it lead to superficial treatment, or no treatment at all, of topics that are basic to mathematical literacy, such as geometry, statistics, and probability, but it leads to superficial

> treatment of the operations themselves. This lack of attention will affect students' later learning. (p. 36)

Caully (1988) also reinforces the foregoing when noting the failings of teachers to realize "that procedural knowledge does not guarantee conceptual understanding" (p. 205).

The lack of manipulative usage is still a barrier to overcome even when the materials are advocated by educators such as Fuson and Briars (1990). They state that "second graders using the base-ten blocks showed performance considerably above that ordinarily shown by second graders receiving traditional instruction" (p. 196).

Concept Transfer

On the assumption that one is able to hurdle the first steps in the teaching of numeration in the way of using concrete materials; this is no guarantee that numeration will be better understood. There is still the important step of linking concepts from the concrete to the symbolic form. The improper or incomplete use of the materials is cited as another possibility for the lack of understanding of base-ten numeration. An example of this may be the use of manipulatives without the connections made to the symbolic representations in numeration. Hynes (1986) echoes the views of Fennema (1972) in stating that the purpose of using manipulatives is to assist students in bridging the gap from their own concrete environment to the more abstract level of mathematics. Fuson (1990) states, "the blocks must be linked very tightly and clearly to the words and the base-ten marks" (p. 277). Fuson and Briars (1990) reiterate the same, "In order to use and understand English words and base-ten written marks and add and subtract multi-digit numbers, children need to link the words and the written marks to each other and need to give meaning to both the words and the marks" (p. 181). Similar references to the importance of the linking of physical and symbolic representations are made by many educators (Engelhardt, Ashlock and Wiebe, 1984; Madell, 1985; Caully, 1988). Thompson (1992) also theorizes that the seeming mediocrity in results in studies that compare manipulative usage can be attributed to the "nature of students' engagement with concrete materials and their orientation toward materials in relation to notation and numerical value" (p. 124).

The notion of connections is a primary focus of the computer program devised by Thompson. As he indicates, multi-base arithmetic blocks were invented to support childrens' meaningful learning of base-ten numeration and numeration-based algorithms. Unfortunately, children often develop ways to solve arithmetic problems using the blocks that have few meaningful connections with symbolic methods. Ross (1989) suggests that children can learn procedures by rote with manipulatives as easily as they can with written symbols. Childrens' understandings of base-ten blocks and their understandings of the symbols that represent the blocks are often in isolation.

There are recognized levels of learning as identified by Jerome Bruner which include concrete and formal operations. Bruner (1977) professes, "What is most important for teaching basic concepts is that the child be helped to pass progressively from concrete thinking to the utilization of more conceptually adequate modes of thought" (p. 38). *Blocks Microworld* concentrates on the connections between the pictorial and the symbolic. It is at this stage that it may be necessary to go back and forth between pictorial and symbolic representations. *Blocks Microworld* accommodates this connection very readily in order to reinforce the vital connections between forms of representations of numbers.

The blocks learning systems (both physical and computer medium) include translations between two systems, i.e. the blocks system and the numeration system. In both the physical and computer blocks learning systems, it is very necessary to connect the actions with the blocks with their related symbols. In reference to using the physical blocks, Thompson and Van de Walle (1984) state "Using the numeral cards with the counters will help children connect the symbols to the place-value concepts represented. They can 'see' how the 2 in 23 represents two 10's" (p. 8). Herein lies one of the advantages claimed by the computer program devised by Thompson. With the physical materials the connections between the two systems can only be pointed to, whereas in the computer medium, the actions are executed simultaneously. In each system the student is constrained to act on one notation system in order to achieve a target goal in the other system, but in the case of the computer medium, there is continual feedback from the numeric representation.

As indicated, there is strong support for the use of concrete materials for the teaching of base-ten concepts. But, the foregoing is evidence that the use of concrete materials such as Dienes blocks does not necessarily ensure the understanding of the underlying concepts. One reason may very well lie within the notion of meaningful transfer of the concept to the symbolic stage. There are numerous other studies that emphasize the importance of connecting concrete manipulations with the equivalent symbolic representation (Beattie, 1986; Bright, 1986; Clements & Battista, 1990; Driscoll, 1986). Learning with concrete materials linked to the symbolic notations is an important area for the understanding of place-value concepts and this is addressed through Thompson's *Blocks Microworld.*

Notation Systems

One aspect of base-ten comprehension which is addressed by Thompson's study focuses on the understanding of notation systems. The term "notation systems" in this paper refers to a very broad category. Notation systems could include the base-ten numeration system as well as the notation system specially constructed by educators using Dienes blocks. This system and other similar systems amount to particular organizations of physical materials based on certain rules that determine the allowable objects and the allowable transformations. Therefore, as expressed by Kaput (1992), we actually "define a notation system to be a system of rules (i) for identifying or creating characters, (ii) for operating on them, and (iii) for determining relations among them" (p. 523).

It is notation systems, concrete, pictorial, and symbolic representations, which require connections as indicated in the previous section. Much of school mathematics in arithmetic and algebra amounts to transformations within a particular notation system, where the transformations are primarily the application of the rules of the notation system being used. Kaput (1992) identifies four different kinds of mathematical activities in school mathematics, one of which is constrained transformations within a particular notation system, without reference to any external meanings. He is alluding to the vital connections which were expounded upon in the previous section. He maintains that this type of activity, "in the form of manipulation of symbols on paper without reference to external meanings, strongly dominates school mathematics" (p. 525). This is also very much the case with physical base-ten materials and the symbolic notations. Most true mathematical activity involves the coordination of and translations between different notation systems. According to Kaput (1992):

> To make sense of the interactions between processes involving mental structures and processes involving physical ones, we need a language that, in the linguist's sense, has separate 'registers' for each, as well as a register for the interactions. After all, the process of making sense of our experience, in this case experience with new media and notations, requires a language for its expression, and we must adopt or define such here. Hence we posit two worlds, a world of mental operations, which is always hypothetical, and a world of physical operations which is frequently observable. . . . these two worlds interact in opposite directions, although in a subtle sense, each can be presumed to be cyclical. (p. 522)

Constraints and Supports

There is another aspect which is often lacking in the understanding of place value even when base-ten blocks are used conscientiously. As pointed out by Ross (1990), "even extensive experience with embodiments like base-ten blocks and other place-value manipulatives does not appear to necessarily facilitate understanding of place value as measured by the digit-correspondence tasks"
(p. 14). Ross (1990) explains further:

> Careful instruction with place-value manipulatives can facilitate the acquisition of the procedural knowledge required for facility with computational algorithms. . . . We must not mislead ourselves, however, that as a result of such instruction children construct understanding of either the complex place-value numeration system or the algorithm. . . . If understanding is the goal, it probably doesn't matter if you show children procedures with paper and pencil, beans and cups, or base-ten blocks. When you show, children may not find it necessary to think; they simply follow directions.
>
> Understanding comes only from thinking; children must be provoked into constructing their own knowledge of numbers and the relations between them. They need to engage in problem-solving tasks that challenge them to think about useful ways to partition and compose numbers. (p. 15)

Ross indicates that a method of promoting the foregoing is to encourage students to find sums and differences in their own ways, not to have them constrain themselves to a set algorithm.

The idea of constraints in this paper is not necessarily a negative aspect of a system. Too many constraints may inhibit creativity, but a system with no constraints may not accomplish what one wishes to attain. For example, if there were no constraints, there would be a lack of convention which would in turn affect communication. One must introduce some constraints and supports as guidelines in order to develop convention for communication.

Thompson addresses the ideas of constraints and supports in his study by promoting free expression of methods of addition and subtraction. Kaput (1992) cites an aspect of Thompson's program as,

> Another feature of the system is the lack of a close tie between its CS (constraint-support) structure and the traditional algorithms for computing in the standard numeration system. This reflects the designer's view that the operations on quantities are different from the usual procedures for putting the results of those operations in canonical form. (p. 528)

If a statement is written on a blank piece of paper, the paper does not provide much in the way of support or constraint to the writing. In the case of lined or graph paper, some support and constraints are provided by the lines themselves. According to Resnick and Omanson (1987), students' active participation in a prescribed activity may have little effect if they think that they are following a prescription. If students follow a prescribed procedure, they are not allowed the opportunity to construct personal constraints in their quest for understanding. Constraints are met only because the students are obliged to adhere to prescription regardless of whether or not concrete materials are a part of those prescriptions. The point is further emphasized by Madell (1985):

> Young children can do mathematics. Without explicit instruction in the use of algorithms, they can develop quite sophisticated procedures. Although not all these procedures are as efficient as the usual ones, their creation and use builds a deep understanding of the arithmetic operations. (p. 22)

Kamii (1989) solidifies the point in stating that "the system of tens has to be constructed by children, in their heads, on *their* system of ones, through constructive abstraction" (p. 30). This is acknowledged by Smith (1985) and again by Labinowicz (1985), in stating that "children need to construct their own place value relations based on systematic groupings of tens and to express their own ideas for representing these groups. If not, we impose on children an already-constructed and sophisticated system that eluded scholars for centuries" (p. 281). Dienes (1964) also suggests that construction should always precede analysis. He suggests that a child must be allowed to develop concepts in a global intuitive manner through their own experiences. Thompson (1992) reinforces the idea that if we place the use of concrete materials within the context of "their development of an orientation toward using notation to express their reasoning as it occurs in concrete settings, . . . we must be sensitive to students' images of their activities" (p. 124).

An aspect of Thompson's program that seems to distinguish it from the concrete base-ten materials is its use of constraints and supports which are built into the program. There are constraints within the program, but there is also the freedom for students to explore and develop their own ideas which can then be reinforced by program and notation constraints. *Blocks Microworld* attempts to address many of the concerns noted previously. According to Thompson (1991), its design is meant to "facilitate students' creation of meaningful, and perhaps idiosyncratic, methods - but methods grounded on deep understanding of big, important mathematical ideas" (p. iv). *Blocks Microworld* is a computer program that was designed to assist children in overcoming difficulties when making meaningful connections between their actions with base-ten blocks and their actions with numerical symbols. A main feature of the program is that, when supported, it can help the children to make important connections between concrete representations of numerical value and traditional symbolic arithmetic.

As pointed out by Kaput (1992), in reference to Dienes and the structure of his blocks, "while his imposition of the structure on the objects of the system is strong, his constraints on actions, imposed via externally-provided written statements guiding activity-structure, are necessarily weak" (p. 527). *Blocks Microworld* attempts to extend the strengths of the constraints on the physical materials through an interactive medium. *Blocks Microworld* provides a structural environment within which students act upon things presented by the program. Their actions are constrained by built-in principles which orient students in productive directions.

Blocks Microworld has built-in constraints on actions which differ from the actions on concrete materials. When using concrete base-ten materials, constraints must be provided externally. For example, as one manipulates the materials, one must also change any written notation as another operation, whereas in *Blocks Microworld*, a change in one results in simultaneous change in the other. This can help to establish an equivalence among the objects and, as Thompson theorizes, "the successful students internalize the CS (constraint-support) action structure (in the sense of Piaget) to build their own knowledge structures" (p. 527).

Dynamic versus Static Medium

A key aspect that distinguishes Thompson's approach to the teaching of numeration from that of traditional concrete methods incorporates the dynamic interactivity provided by the computer as opposed to the static environment of manipulative materials. *Blocks Microworld* provides simultaneous changes to notation when changes are made to the materials and visa versa.

According to Kaput (1992), "technologies based on dynamic interactive electronic media", in this case an interactive computer program, "embody fundamental attributes that distinguish them from traditional static media in ways likely to have tremendous long term impact on mathematics education" (p. 525).

Kaput offers a very simple explanation which distinguishes dynamic and static medium:

> In static media, the states of notational objects cannot change as a function of time, whereas in dynamic media they can. Hence, time can become an information-carrying dimension. . . . Dynamic media inherently make variation easier to achieve. In static media, one must

> resort to such compensatory strategies . . . (such as) providing multiple instances organized spatially rather than temporally. . . . Physical materials such as Dienes blocks provide an example of media which are somewhat dynamic in the following sense: While one physically moves the elements of the system to produce a particular state, once produced, the state remains static until changed to a new one by direct action of the user. (p. 525-526)

The dynamic aspect as opposed to the static has been studied by Mayton (1991). Although the content area of the study was in reference to the human heart, the results of the study showed that the use of animation in computer-based instruction to teach a dynamic process can be beneficial.

O'Brien (1983) offers an interesting comparison of dynamic versus static knowledge. He states, "dynamic knowledge - a fabric of enabling ideas rather than a storehouse of inert associations" (p. 111). Kaput (1992) in reference to Thompson's program, applauds the dynamic, interactive media as opposed to the static, inert media in stating, "In the former as opposed to the latter, one has much more freedom to create and link new notations and create variations within and across them" (p. 525). The dynamic aspect of Thompson's program seems to accomplish this feat by connecting direct actions on the blocks with the corresponding notations as opposed to reference to "inert associations" which are the best that can be accomplished with the physical Dienes blocks.

In summary, a final comment on *Blocks Microworld,* as reiterated by Kaput (1992),

> We have seen that it is possible to impose much stronger CS structure using the interactive computer medium than using inert physical media, especially at the level of coordinating between notation systems, because one can transfer to the computer the mechanics of the translation process. This frees the student to focus on the connections between actions on the two systems, actions which otherwise have a tendency to consume all of the student's cognitive resources even before translation can be carried out, let alone be monitored. (p. 529)

In summation, the literature cited indicated a concern for the lack of understanding of base-ten concepts as well as giving support for the use of manipulatives to aid in rectifying the problem. The literature revealed that one area which contributed to the lack of understanding is the connections between stages of number representations or notation systems. Finally, commentaries which promote constraints and supports, as well as compare dynamic and static mediums lead into the design and purpose of the computer program, *Blocks Microworld* and this study.

Chapter III. Thompson's Study

As previously indicated, this study is a replication of an investigation by Thompson (1992). Following are some of the particulars of his study.

Statement of the Problem

Thompson's study (1992) was designed "to investigate what features of students' engagement in tasks involving base-ten blocks contribute to students' construction of meaning for decimal numeration and their construction of notational methods for determining the results of operations involving decimal numbers" (p. 125).

Thompson's instruction emphasized freedom of method for solving addition and subtraction problems and also emphasized the freedom of notational expression provided that the expression accurately reflected the meaning and method. His instruction also emphasized the relationships between the actions on blocks and the corresponding results to notation.

Thompson hypothesized that students who have experienced typical mathematical instruction would not be easily convinced that they have the freedom to use notation creatively. He further hypothesized that continual exposure to the relationship between the actions on blocks and the corresponding changes in notation would be more effective than a setting that did not orient the students to that continual relationship.

Methodology

The study was conducted over nine days, one day each for the pretest and posttest and seven days for instruction.

Twenty fourth-grade students were assigned to one of two treatments: ten to microworld instruction (microworld), the other ten to wooden block instruction (blocks). The students were assigned to their respective groups using pretest scores. Consecutive pairs of students were taken from a list of scores in descending order and students from each pair were assigned to either the microworld or blocks group. The rankings were tested for robustness through correlation to their total pretest scores and also the item scores were analyzed by factor analysis. There were 17 pretest items covering whole number computation, place value, and fractions.

The microworld teacher used a Macintosh connected to a large-screen projector during class discussions. The blocks teacher used an overhead projector and plastic blocks during class discussions. Students in the microworld group used computers for in-class activities and students in the blocks group used wooden blocks for their in-class activities. In both groups, classes were arranged with two students per station.

Each teacher worked from a highly detailed script written for the microworld group. The blocks teacher modified segments that were appropriate only for the microworld group so that they were appropriate for the blocks group.

Students were given the freedom to approach problems without constraints and were not forced to solve arithmetic problems using conventional methods, such as beginning the addition with the units digit. Instruction was designed so that students had as much freedom as possible to develop their own methods for solving addition and subtraction problems and to develop their own methods of notation. The instruction was designed with the intention that students reason naturally about solving problems and that they reflect their reasoning in notation. Care was taken to have the students realize that along with the freedom mentioned, that: (i) whenever they performed a concrete action with blocks, they represented it in notation, (ii) whatever they wrote expressed a performed action, and (iii) their cumulative record expressed all their actions.

During the instructional periods, base-ten numeration was not addressed directly. It was addressed thematically by encouraging students to refer to the types of blocks by their number-name (e.g. hundred instead of flat).

The posttest consisted of the pretest together with additional items on ordering decimals, decimal representations, appropriateness of method, and decimal computation. Items were scored for correctness of result and validity of method.

Eight students were interviewed following the posttest: four students scoring highest on the pretest and the four students scoring lowest on the pretest. The intent of the interviews was to assess the students' abilities to relate their written work to some mathematical principle or concrete model.

Posttest results were analyzed in two parts. The first part of the analysis examined the changes in students' performances from pretest to posttest. The second part of the posttest examined student responses to questions dealing with content: decimal numeration and calculation with decimal numerals. Performance and method were part of the analysis of the posttest. Analysis of performance focused on the correctness of the students' answers and the analysis of method focused on whether and in what ways students' use of notational methods were influenced by instruction.

Results and Conclusions

The results of the various parts of the study are summarized as follows:

For both microworld and blocks students, there was no substantial improvement on any items on the pretest and posttest, although there was a slight improvement in both the microworld and blocks students' understanding of the ratio nature of decimal fractions (see Table 1).

On the remainder of the posttest, the blocks students were generally more accurate on the decimal computation items than the microworld students (see Table 2), but they were less successful than the microworld students on two of the three conceptual items involving order and equivalence (see Table 3).

A good portion of the instruction time was spent with students discussing the methods for representing the actions that they took while solving addition and subtraction problems in concrete settings.

Table 1

Number of Correct Responses on Pretest and Posttest Items for Blocks and Microworld Groups

(Thompson's Study Results)

Item	Blocks Pre	Blocks Post	Microworld Pre	Microworld Post
3004 - 286	5	6	5	5
7814 +2648	9	10	9	9
5002 - 493	5	5	4	5
Shade 2/10 of a 10x10 grid	6	6	7	7
Shade 3/5 of a 10x10 grid	3	2	2	2
If a flat stands for one, then what does a long stand for?	3	5	2	5
If a flat stands for one, then what does a cube stand for?	5	5	3	5
If a flat stands for one, then what does a single stand for?	3	5	3	5

Note: n = 10 for each group.

Each blocks student expressed the same whole-number subtraction notation method on the posttest as on the pretest as did eight of the microworld students. Two of the microworld students changed to a novel method.

Results on whole number instruction suggests that students are reluctant to modify their already-automatized notational schemes.

For instance, blocks students were more likely to have assimilated decimal addition to whole-number notational schemes (see Table 4).

Blocks students were more likely than microworld students to use standard addition on decimal addition problems.

Table 2

Number of Correct Responses for Decimal Computation Items on the Posttest for Blocks and Microworld Groups (Thompson's Study Results)

Item	Blocks	Microworld
12.27 + 5.84	10	6
7.31 - 6.4 = ____	8	5
8.03 - 2.9	9	9
14.8 + 7.23 = ____	4	2

Note: n = 10 for each group.

One item was given to assess the extent to which students felt instruction on alternative methods conflicted with prior conceptions of what constitutes school mathematics. Blocks students tended to feel that the standard addition was the right way, even if other ways gave the correct answer, and most microworld students disagreed with the statement (see Table 5).

Table 3

Number of Specific Responses to Decimal Fraction Equivalence and Order Items on the Posttest

(Thompson's Study Results)

Item		Response	Blocks	Microworld
4.325	$4+\frac{32}{100}+\frac{5}{1000}$	Same	3	7
		Different	4	2
		Can't tell	3	1
4.325	$4\ \frac{325}{1000}$	Same	6	4
		Different	3	4
		Can't tell	1	2
7.89 is smaller than 7.9		Yes	3	6
		No	6	3
		Don't know	1	1

Note: n = 10 for each group.

Table 4

Decimal Addition Notational Methods on Posttest (Thompson's Study Results)

Item	Method	Blocks	Microworld
12.27 + 5.84	Standard Addition	7	2
	Add within Columns	3	8
14. 8 + 7.23 = ___	Standard Addition	5	0
	Add within Columns	2	5
	Align right	3	1
	Novel	0	1
	No work	0	3

Note: n = 10 for each group.

Table 5

Children's Attitudes About Correctness of Unprescribed Methods (Thompson's Study Results)

Statement		Response	Blocks	Microworld
This is the RIGHT way to add 8276 and 4185. Other ways might give the same answer, but they are not the right way:	$\begin{array}{r} 1\ 1 \\ 8\ 2\ 7\ 6 \\ +\ 4\ 1\ 8\ 5 \\ \hline 1\ 2\ 4\ 6\ 1 \end{array}$	Yes	5	1
		No	2	7
		Don't know	3	2

Note: n = 10 for each group.

The interviews focused on students' reasoning as expressed on the posttest. The students in the low blocks group gave explanations that were largely procedural, without reference to actions on blocks or to constraints imposed by the decimal numeration system. The students in the low microworld group showed little understanding of decimal numeration and had even lost the little facility they previously had with written computation. This result could be attributed to the fact that these students were absent for two of the three days of instruction involving decimal numeration.

For the high blocks students, there was no evidence that either of the two had formed connections among decimal numeration, base-ten blocks, and notational conventions. It was felt that these students had evidently modified their whole number procedures for processing numerals to accommodate the presence of a decimal point. The two high microworld students showed competencies with decimal notation and appeared to have made sense of sources of conventions.

Through the classroom observations, microworld students appeared to use quite creative methods during instruction of whole number operations. Whereas, during blocks instruction, when nonstandard use of notation was employed, Thompson expresses the feeling that block students were missing the point of the instruction.

At some points, blocks students showed little evidence of feeling constrained to write something that actually represented what they did with blocks. They appeared to consider actions on the blocks and writing on paper as separate activities. In contrast, microworld students made repeated references to actions on symbols as referring to actions on blocks. The relationship between notation and the blocks appeared to be prominent in their experience.

Another observation was that block students showed resistance throughout the instruction to considering alternative methods of solving problems. The microworld group, on the other hand, although resisting alternative methods at the outset, had that resistance diminish over the seven days of instruction.

Thompson (1992) concludes that "before students can make productive use of concrete materials, they must first be committed to making sense of their activities and be committed to expressing their sense in meaningful ways" (p. 146). He further concludes that for "concrete embodiments of a mathematical concept to be used effectively in relation to learning some notational method, students must come to see each as a reflection of the other" (p. 146).

Chapter IV. Methodology for the Study

School Setting

The setting for the study was an elementary school with a student population of 450-500 students. The school, located in Winnipeg, Manitoba, Canada draws from a wide economic and cultural range.

School Facilities

The school in which the study took place has three grade five classes, two of which participated in this study. Each of the classes involved had three Macintosh computers in the classroom area. There was also a Macintosh computer lab equipped with thirteen computers, a system server, an overhead projector and a computer projection unit. The computer room was organized into two long rows of computer tables back to back with six computers down each side. The system server, a Macintosh SE and the computer projection unit are situated among the two rows of computers.

Materials

There were twenty-two students in each of the groups and within each group the students worked in pairs. In the blocks group, each pair of students had a set of base-ten blocks and a place-value mat. The teacher used an overhead set of base-ten blocks with an overhead projector. The microworld group had each pair of students using a Macintosh computer with the program, "Blocks Microworld" installed (site licence was purchased). The teacher used the Mac SE computer with an overhead data projection unit. The scripts for the teachers were similar to the scripts provided for the microworld group in the Thompson study (see Appendix A). The block's script was edited to better suit the terminology used when using actual base-ten blocks, as opposed to the computer program (see Appendix B). The pretest/posttest were similar to those used in the Thompson study. There were 48 test items covering the concepts of common fractions, decimal fractions, relationships between fractions and decimals, and addition and subtraction of whole numbers and decimal numbers (see Appendix C). Permission to use the foregoing was obtained from Patrick Thompson (see Appendix D).

Deviations from Thompson's Study

The intended deviations from Thompson's study were those suggested and/or supported by Thompson (1993). These included:

1. The sizes of the two groups for this study were larger, two classes, each with 22 students, from the same elementary school as opposed to a total of twenty students in Thompson's study.
2. The students were from fifth grade classes as opposed to the fourth grade. It was felt that the fifth grade students would be more familiar with fractions and therefore would be more familiar with items on the pretest.
3. The pretest and posttest instruments for this study were identical. Thompson omitted several items on decimals from the pretest in his study because the students were not familiar with decimals and would not be able to cope with those particular questions. He felt that the grade level change would warrant the pretest instrument

being the same as the posttest instrument because the students should be somewhat familiar with decimals (Thompson, 1993).

Design and Procedure

The classes chosen to be involved in the study were selected prior to December, 1993. The actual work with the students took place in early January, 1994. This seemed to be the most appropriate time for students to be involved in "extra-curricular" mathematics and a time which created the least amount of interruption to their regular school activities.

The selection of the teachers and their classes for this study was very deliberate in that both teachers were active on divisional mathematics teams and were strong proponents for improving the teaching of mathematics. One of the teachers was a member of the Elementary Mathematics Leadership Team and the other was a member of the Middle Years Mathematics Inservicing Team.

Since both classes were formed from a common pool of students from the previous year, the majority of students had had similar experiences in their previous year's mathematics classes. Most students had not used base-ten blocks extensively in the past.

Approval for the study from the Faculty of Education Research and Ethics Committee was obtained and permission to approach the teachers was granted from the principal of the school. This was followed by permission of the Superintendent of the school division to proceed with the study. Formal permission from the principal was obtained after the teachers were asked and consented to participate.

Consent from the parents of the students was obtained through a parent letter (See Appendix E).

The teachers involved were given the scripts and met with the researcher on several occasions prior to the beginning of the student lessons to clarify questions or concerns.

The students were assigned to their respective groups using raw pretest scores. Scores were placed in descending order and students from each of the consecutive pairs were assigned to either the blocks or microworld groups by a toss of a coin.

The students' schedules had the flexibility to accommodate the restructuring for mathematics lessons, so that the students selected for the blocks and microworld groups were in their respective groups for their particular mathematics instruction. The teachers used a script similar to that which was prepared for the Thompson study. The blocks teacher received a modified script which was more appropriate for students using physical blocks.

The regular teachers taught each of the assigned groups. This arrangement allowed the researcher to better observe the students and document reactions to different segments of the instruction in each of the classes. The blocks group used physical base-ten blocks with the teacher using an overhead projector with a transparent set of base-ten blocks. The microworld students used Macintosh computers with the program *Blocks Microworld* installed, while the teacher used a Macintosh connected to a overhead projection unit.

The teachers involved were made thoroughly familiar with the study, the script and the intent of the lessons. The actual lessons including the pretest and posttest were extended over eleven class periods. Each class period was approximately sixty minutes in duration. The students in both groups were working in pairs with one pair of students per station. The actual pairing of students within each group was determined according to the teachers' perceptions as to partner compatibility.

The general questions posed for the study were:

1. Would less restrictive constraints (ie. freedom from memorized algorithms) placed upon students improve their creativity in approaching problem-solving in regard to place value?
2. Would continual reinforcement of written numerical notation with real or simulated base-ten blocks enhance the students' understandings of place value?
3. How would the students using the computer program, *Blocks Microworld* compare to the students using real blocks?

The researcher made classroom observations in both classes during their class presentations and discussions. The types of questions and reactions that came from the students were documented through hand-scribed notes. The researcher observed the students for indications of:

1. Novel approaches to solving problems.
2. Understanding of place-value concepts.

These were two of the questions of the study. This data along with the data collected from the posttests and interviews gave some indication of students' understanding of place-value concepts.

Interviews with twelve students followed the posttest: six students from the blocks group and six students from the microworld group. The students from each group were selected according to the following criteria: four students scoring the lowest on the pretest, four students scoring closest to the median of the pretest and the four students scoring the highest on the pretest. The questions asked during the interview were based upon the students' answers on the posttest. The objective was to have students explain their answers in terms of how and why they answered as they did. The researcher specifically attempted to determine which students had:

1. An understanding of the place-value concepts.
2. A comfort with using non-standard algorithms and novel approaches to solving problems.

The interviews were audio-taped so that exact responses were documented.

The test items on pretest/posttest were categorized into four subtests. The subtests are identified as i) fraction concepts ii) decimal concepts and place value iii) relationships between fraction and decimal iv) operations with decimals and whole numbers. The posttest was analyzed in two ways:

1. An examination of the changes in students' overall performances from pretest to posttest.
2. An examination of student responses to questions dealing with specific content in the subtests.

The analysis included the comparison of specific groups of students on the specific subtests. The students were grouped as high microworld, low microworld, high blocks and low blocks. The foregoing groups were established from the pretest scores using the median as the dividing line for the groups.

The following null hypotheses were tested:

1. There is no difference in mean scores within each group from pretest to posttest.
2. There is no difference in mean scores between the groups for the posttest.
3. There are no differences in mean increases from pretest to posttest between the two groups for the entire test and subtests.
4. There are no differences in the sum of the ranked increases from pretest to posttest between the high microworld and the high blocks groups for the entire test and subtests.
5. There are no differences in the sum of the ranked increases from pretest to posttest between the low microworld and the low blocks groups for the entire test and subtests.
6. There are no differences in the sum of the ranked increases from pretest to posttest between the high microworld and the low microworld groups for the entire test and subtests.
7. There are no differences in the sum of the ranked increases from pretest to posttest between the high blocks and the low blocks groups for the entire test and subtests.

The statistical analysis done on the data from the tests were as follows:

For hypotheses 1-3, parametric t-tests were performed on the means and mean differences in pretest and posttest scores whenever there was a comparison of the entire microworld group to the entire blocks group. The comparisons made were on the entire test and on each of the four subtests.

For hypotheses 4-7, non-parametric tests (Mann-Whitney) were performed on the rankings of subgroups with respect to the differences from pretest to posttest. The Mann-Whitney tests the distribution of a combined ranking of the two groups being analyzed. This test was done whenever there was a comparison of any subgroups. The subgroups compared were high microworld versus high blocks, low microworld versus low blocks, high microworld versus low microworld, and finally high blocks versus low blocks. Comparisons were made on the entire test and on each of the four subtests.

Finally the overall findings of the study were compared to the findings of Thompson's study. This was done through the construction of tables similar to the tables in which Thompson reported his results.

Chapter V. Results

Comparison of Mean Scores

The mean scores for the students in both the microworld and blocks groups improved significantly within their groups from the pretest to the posttest. Significance was established using parametric t-tests (see Table 6). Null hypothesis (1) is thus rejected. The difference in mean scores on the posttests between groups was not statistically significant (see Table 6). Null hypothesis (2) is therefore not rejected.

Table 6

Mean Scores of Students on the Pretest and Posttest

	Blocks	Microworld	*t*
Pretest	13.95	13.82	
Posttest	17.36	20.09	1.13
t	2.03*	2.44*	

*$p < .05$.

Note: n = 22 for each group.

Comparison of Mean Increase in Scores - Between Groups

When the entire microworld group was compared to the entire blocks group with respect to their mean increase in scores from the pretest to posttest, the microworld group scored higher, not only on the test as a whole but on all the subtests (see Table 7). A significant difference was found in the whole test comparison. It was also found that the microworld group had significantly increased their scores on the subtest dealing with the concepts of common fractions as compared to the blocks group. Statistical significance was established using parametric t-tests. Thus null hypothesis (3) is rejected for the whole test and the common fractions subtest.

Table 7

Mean Increase in Scores from Pretest to Posttest Between Groups

		Subtest			
	Whole Test	Common Fractions	Decimals/ Place Value	Relationships Decimals and Fractions	Operations with Decimals and Whole Numbers
Blocks	3.41	0.55	0.77	0.68	1.41
Microworld	6.27	1.36	1.09	1.59	2.23
t	2.19**	1.91*	.62	1.33	1.09

*$p < .05$. **$p < .025$.

Note: n = 22 for each group.

Comparison of Increases in Scores - Differential Learning Groups

When considering the differential learning groups, the mean increase in scores of the high microworld group was higher than that of the high blocks group for all categories of comparison (see Table 8 and Table 9). There were significant differences for the whole test as well as the subtests dealing with fraction concepts and with the relationships between decimal fractions and common fractions using the non-parametric Mann-Whitney test on their rankings. Null hypothesis (4) is thus rejected.

The mean increase in score of the low microworld group was also higher than that of the low blocks group in all subtests except in the subtest dealing with relationships between decimal fractions and common fractions. The differences were not significant using the Mann-Whitney test. Therefore hypothesis (5) is not rejected.

Table 8

Mean Increase in Scores from Pretest to Posttest for Differential Learning Groups

		Subtest			
	Whole Test	Common Fractions	Decimals/ Place Value	Relationships Decimals and Fractions	Operations with Decimals and Whole Numbers
High Microworld	7.45	1.55	1.36	2.00	2.55
Low Microworld	5.09	1.18	0.82	1.18	1.91
High Blocks	2.45	0.00	1.09	- 0.36	1.73
Low Blocks	4.36	1.09	0.45	1.73	1.09

Note: n = 11 for each group.

In the comparison of the high microworld group with the low microworld group, and the high blocks group with the low blocks group in all subtests, there were no significant differences in the sum of the ranked increases in scores. Therefore null hypotheses (6) and (7) are not rejected. However the low blocks group did improve more than the high blocks students on the entire test as well as on two of the subtests dealing with fraction and decimal concepts.

The increase in scores of the high blocks group was the least of all groups and this included a decrease in mean score on the subtest dealing with relationships between decimal fractions and common fractions.

Table 9

Summary of Comparisons of Ranked Increases in Scores from Pretest to Posttest for Differential Learning Groups

High Microworld vs. Low Microworld

- increases in scores were greater for high microworld in all categories of tests.
- differences were not significant.

High Microworld vs. High Blocks

- increases in scores were greater for high microworld in all categories of tests.
- significance was established for the whole test and subtests on fractions and relationships between decimals and fractions.

High Blocks vs. Low Blocks

- increases in scores were greater for low blocks on the whole test and subtests on common fractions and relationships between decimals and fractions.
- differences were not significant.

Low Microworld vs. Low Blocks

- increases in scores were greater for low microworld in all categories except on subtest on relationships between decimals and fractions.
- differences were not significant.

Comparison of Findings in the Studies

Thompson found that there was no substantial improvement in the performance of either group with respect test results and in particular to certain test items (see Table 1, p. 28). Similar results were found in this study when comparing the same items (see Table 10). When looking at the test in its entirety, both groups in this study did improve. The microworld students showed better understanding of the ratio nature of decimal fractions as illustrated through the last four items in Table 10. Thompson found similar results when examining these items.

Table 10

Number of Correct Responses on Pretest and Posttest Items for Blocks and Microworld Groups

Item	Blocks Pre	Blocks Post	Microworld Pre	Microworld Post
3004 - 286	13	9	14	11
7914 + 2648	10	15	13	20
5002 - 493	12	9	14	14
Shade 3/10 of a 10x10 grid	14	20	16	20
Shade 2/5 of a 10x10 grid	0	1	3	6
If a cube stands for one, then what does a supercube stand for?	8	9	5	11
If a cube stands for one, then what does a long stand for?	0	3	1	6
If a cube stands for one, then what does a single stand for?	0	3	1	6
If a cube stands for one, then what does a flat stand for?	0	3	1	5

Note: n = 22 for each group.

On decimal computations in Thompson's posttest, blocks students were generally more accurate than the microworld students (see Table 2, p. 29). In this study, the scores and also the amount of improvement of the microworld students were slightly better than that of the blocks group (see Table 11).

Table 11

Number of Correct Responses for Decimal Computation Items on the Posttest for Blocks and Microworld Groups

Item	Blocks Pre	Blocks Post	Microworld Pre	Microworld Post
12.27 +5.84	13	15	16	19
7.31 - 6.4 = _____	5	7	2	12
8.03 -2.9	9	13	12	15
14.8 + 7.23 = _____	6	9	3	9

Note: n = 22 for each group.

Thompson found that the blocks students were slightly less successful than the microworld students on two of the three items involving order and equivalence (see Table 3, p. 29). The findings in this study do not concur with those of Thompson's (see Table 12). According to Thompson's results, blocks students tended to feel that standard addition was the right way to do addition. Even if other methods gave the right answer, they would not be considered as the right way (see Table 5, p. 30). Again, the results of this study do not concur with the results of Thompson's study (see Table 13).

Table 12

Number of Specific Responses to Decimal Fraction Equivalence and Order Items on the Posttest

Item		Response	Blocks Pre	Blocks Post	Microworld Pre	Microworld Post
4.325	$4+\frac{32}{100}+\frac{5}{1000}$	Same	13	9	6	7
		Different	3	11	8	11
		Can't tell	5	1	5	2
4.325	$4\ \frac{325}{1000}$	Same	11	9	7	11
		Different	5	8	4	4
		Can't tell	5	4	8	5
7.89 is smaller than 7.9		Yes	9	8	5	3
		No	12	13	14	13
		Don't know	0	1	1	2

Note: n = 22 for each group.

Table 13

Children's AttitudesAbout Correctness of Unprescribed Methods

Statement		Response	Blocks Pre	Blocks Post	Microworld Pre	Microworld Post
This is the RIGHT way to add 8276 and 4185. Other ways might give the same answer, but they are not the right way:	$\begin{array}{r} {\scriptstyle 1\,1} \\ 8276 \\ +\underline{4185} \\ 12461 \end{array}$	Yes	14	10	13	16
		No	1	3	2	4
		Don't know	6	9	5	1

Note: n = 22 for each group.

Observations

Approximately half of both classes went unobserved. This was the result of the timetabling and the limited amount of time that the students were to spend on this study. It was necessary for classes to run simultaneously and this required the researcher to move from one class to the other as the lessons were progressing.

During the first day, there was a sense of abnormal enthusiasm and attentiveness in both classes because of the novelty of using base-ten blocks and the computers for mathematics. Both groups seemed to be quite comfortable with the relationship of exchanging cubes for flats and the concept of "tens of". Both teachers expressed some nervousness at the beginning, but both were fine once they had begun the lesson.

During the second day, students seemed to understand the subtraction of whole numbers example. At this point, the students of each class were allowed to work on their homework in an extra class.

The third day showed little difference in either group with regard to students' understanding of the work covered to this point. The lessons for the first three days had dealt with block values and calculations with whole numbers. What became evident was that the microworld group was able to move through the lessons more quickly. This was primarily because the base-ten blocks themselves were much more cumbersome to manipulate when doing the actual exchanging. The base-ten simulations of the computer screen allowed for the exchanges to occur more quickly and efficiently.

The script for the fourth day caused some difficulty because it appeared to be moving too quickly in the presentation of the concepts. The idea of allowing a particular block piece to stand for "one" was confusing to many of the students. Both groups used an extra period for the fourth day activities. The continual feedback connecting the symbolic representations with the simulated blocks was more easily made for the microworld students than for the blocks group. The microworld students found it easier to respond to the continual feedback between the connection of the simulated blocks and their symbolic representations than the blocks students. The blocks students were not privy to that type of continual reinforcement and spent much of their time writing out the actions which were performed on the blocks.

The fifth day of the study still found a number of students confused with the notion that any piece could be assigned the value of "one". Both teachers commented on the fact that more time to develop this concept would have been beneficial.

The sixth day seemed to have students a little more comfortable about the different pieces being used to stand for "one". There was a little confusion with respect to the amount of recording that was expected in each of the problems.

There appeared to be some students who had a good understanding of the decimal place value concepts by the seventh day. There were also a number of students who were reluctant to offer any solutions to the homework. There were definitely a few students who did understand the concepts well. This was the case for both groups. There was no real evidence that novel methods for solving problems were being attempted by either group.

Interviews

A main focus of the interviews was to determine the reasoning behind students' work in the posttests. In particular, the interviews were to help determine if place-value concepts were understood and if freedom from constraints would allow students to create novel ways to solve problems.

When students were specifically asked questions about place-value concepts, it was only the students from the high microworld group and the high blocks group that illustrated a solid understanding of place-value. A partial dialogue with a high microworld student illustrates the level of understanding:

R: (Referring to question 7) Marianne lets the cube stand for 'one', what do you think the supercube stands for?

A: Ten.

R: Ten, OK, why?

A: Because there's ten cubes in the supercube.

R: OK, looking at the same thing, if the cube still stands for 'one', tell me what a long stands for?

A: One-one hundredth.

R: OK, why do you say that?

A: Because there's one hundred longs in a cube.

R: Good, and what about a single, what does that stand for?

A: One-one thousandth, because there's a thousand singles in a cube.

R: Good, and finally, what does a flat stand for?

A: One-tenth, because there's ten flats in a cube.

R: (referring to question 8) Tom and Sally each let one of *(illustration of each of the base ten pieces)* stand for the number 'one'. Their friend, James, put out some blocks. Tom said that James' blocks made 4.025. Sally said James' blocks made 402.5. Let me ask you this, for Tom, which one of these did he let stand for 'one'?

A: A cube.

R: Why do you say that?

A: Because there's only four different kinds of blocks, and there's four different place values in there, so, and the top one is that *(pointing to the cube)* and that is the largest one in all these four so that has to be 'one'.

R: So that has to be 'one', good OK, how about Sally?

A: It will have to be the long because it's the third number or the third largest one of the blocks.

R: Good for you, so now when they ask this question as to who required more singles to make 'one', and if each used the blocks that you said as their 'ones', who had more singles in their 'one'?

A: Tom.

R: Sure.

A: Ha, then I got that question right!

There was very little evidence that the other students had achieved understanding of place-value concepts. A partial dialogue with a low blocks student which typifies the lack of understanding is as follows:

R: (referring to question 7) I have a new piece here, called a supercube. It has 10,000 singles in it. This one has a thousand singles in it. How many of these cubes do you think are in the supercube?

B: Ten?

R: Ten! Right! That's exactly right. Now, remember in your lessons, you could let any one of the pieces stand for 'one'? So, let's say that Marianne lets the cube stand for 'one', what do you think the supercube stands for?

B: Ten?

R: Ten! That's right. Now if the cube is still 'one', what do you think the flat stands for?

B: Ten?

R: Let's look at numbers going the other way. *(wrote a number 4.567)* If these are 'ones' *(pointing to the 4)*, and I have a decimal point here, what do these become? *(pointing to the 5)* The one right beside the decimal place, the other way.

B: Umm. . . *(shaking her head, indicating not knowing)*

R: Do you remember the word 'tenths'?

B: Oh yeah.

R: OK, so these are tenths. What do the think the next ones are? What do the longs stand for?

B: Ones?

When students were asked specifically about alternative methods for solving certain problems, students in the high microworld group as well as the high blocks group stated that alternative methods for doing questions were still correct. A partial dialogue with a high blocks student indicating flexibility follows:

R: (referring to question 12) This is the right way to add 8276 and 4185. *(Showing the question done in the standard way)* Other ways might give the same answer, but they are not the right way. Do you think that this is true?

C: I don't think so.

R: You don't think so? In other words you think that you can do it in another way and still get the right answer and still be correct?

C: Uh huh.

R: Sure, of course, Why do you say that?

C: Well, because in the math class we did that.

R: (referring to question 13) Tom and Sally each solved 45.32-32.37. Tom's work is shown on the left and Sally's work is shown on the right. Sally has subtracted starting from the right and Tom has subtracted starting from the left. Both got the same answer, do you think that both are OK?

C: Uh huh, just as long as you don't break any rules, you can do it in any way you want.

All other students except one from the low blocks and one from the low microworld indicated that there was only one correct method to do the addition and subtraction questions. A partial dialogue with a low microworld student indicating entrenchment in procedures follows:

R: (referring to question 12, as in previous interview) This is . . ., but they are not the right way. Do you agree with the statement?

D: Uh huh.

R: You mean you agree that any other way is wrong?

D: Uh huh. *(Nodding in agreement)*

R: (referring to question 13, as in previous interview) Tom and Sally each solved . . . . Do you think both are OK?

D: No, I don't think so.

R: Why?

D: You can get the wrong answer.

R: You think you can get the wrong answer by starting from the left?

D: Yes, I think she should start at the right hand side.

The preceding extracts from the student interviews indicate that high microworld students as well as high block students had improved in two important areas as opposed to other groups of students:

1. understanding of place-value concepts.
2. acceptance of alternative methods as correct solutions to problems.

In the summation of the findings, the mean scores and the mean increase in scores of the microworld group was greater than that of the blocks group from pretest to posttest. This included the scores of the whole test as well as the scores on all the subtests. Microworld subgroups had also scored higher than their comparative subgroups in the blocks students in all aspects with only one exception where the low blocks group had a greater mean increase in one subtest than the low microworld group.

In comparison to Thompson's findings, the findings of this study were similar in showing the microworld students performed slightly better than the blocks students. Any differences of results in the studies generally seemed to indicate a better understanding on the behalf of the microworld students in this study.

In general, the observations of the classes revealed small differences between the groups with minimal evidence of attempts to use novel methods for solving problems in either group. The interviews generally showed that it was the high microworld and high blocks students who showed understanding of place value and felt that alternative methods to solve problems were also correct solutions.

Chapter VI. Discussion and Conclusion

First, general comments with respect to the findings related to the general research questions will be stated followed by a more detailed discussion of the findings. The general questions posed for the study were:

1. Would less restrictive constraints (ie. freedom from memorized algorithms) placed upon students improve their creativity in approaching problem-solving in regard to place value?
2. Would continual reinforcement of written numerical notation with real or simulated base-ten blocks enhance the students' understandings of place value?
3. How would the students using the computer program, *Blocks Microworld* compare to the students using real blocks?

Since the scripts of both groups emphasized decreased constraints, it is possible that the improvement of the test scores of both groups could substantiate that less restrictive constraints placed upon students improve their creativity in approaching problems. The fact that the microworld students seemed to have more time to explore novel methods may have been one reason for the greater increase in test scores for the microworld group. The interviews seem to indicate more specifically that it was the high microworld and high blocks students who benefitted in this regard. The observations revealed very little evidence with respect to novel approaches to solutions of problems.

The continual reinforcement of written number notation with the illustrations to which the microworld students were subjected did enhance the understanding of place value according to the test results. Interviews seem to isolate this improvement more with the high microworld and high blocks students. Once again, the observations revealed little evidence in this regard.

The improvement of the posttest scores for both groups seems to speak favorably for the use of blocks, real or simulated in teaching base-ten concepts. This concurs with the many studies that indicate the necessity of manipulative usage (Ball, 1988; Curcio et al, 1987; Madell, 1985; Sweetland, 1984; Van de Walle & Thompson, 1984).

That the scores of the microworld students had improved more than those of the blocks group may be partly attributed to the fact that students in the microworld seemed to move more quickly through their lessons primarily because of the ease by which the blocks could be moved on the screen as opposed to the actual blocks. Hence the microworld students were afforded more time to explore different strategies when problem solving. In addition, the students had more exposure to the continual reinforcement of the reciprocal representations of the blocks and numerals. Whatever the reason for the difference in improvement of the microworld group over the blocks group, it speaks favorably for the microworld computer program.

It was noted that the low microworld group did not improve as much as the high microworld group. This may be attributed to at least three factors. Firstly, the students in the low microworld group may have been fully occupied with the computer and the workings of the program and could not devote their full concentration to the mathematical concepts that were being addressed. Secondly, the fact that the microworld students were using only pictorial representations of the blocks as opposed to the real materials may have had more effect on the low microworld students than on the high microworld students. The low microworld students might have benefitted from more exposure and familiarity with the actual base-ten blocks since very few of the students in the study had a great deal of experience with actual base-ten blocks in their previous years. This could be verification that certain students benefit from the actual manipulation of the real blocks and are in need of the concrete representation of the numbers as indicated by the research of Sowell (1989). This may, in part, account for the difference in scores of the low microworld group on the posttest as opposed to the high microworld students. Thirdly, as indicated by Thompson (1992) "before students can make productive use of concrete materials, they must first be committed to making sense of their activities and be committed to expressing their sense in meaningful ways" (p. 146).

The low blocks group showed substantial gains on their posttests from their pretests and their average increase over the tests was more than the high blocks group. This again may be verification of the research of Sowell (1989) with respect to the necessity of concrete materials.

Another factor which may have contributed to some differences in the test results with respect to the two groups was the fact that the tests were paper-and-pencil tests without the use of actual manipulatives. These tests made use of pictorial and symbolic representations similar to which the microworld group had used throughout the lessons. The blocks group encountered these representations only in their homework questions and had actual blocks available as a resource for homework. This change in aids may have contributed to some of the differences in the test results of the groups.

The effect of the continual exposure of the symbolic representation along with the blocks representation seemed to benefit the students in the microworld group according to the test results. They had the constant reminder of the notational representation of the blocks displayed on the same screen and any change in one was simultaneously altered in the other. The blocks students had to record these changes if they wished to see the continually changing representation. Since it was very time consuming to do an abundance of the recording, these students were not able to complete as much work as the microworld students. Through the classroom observations, the students in neither group showed that the continual exposure to the dual representation had any significant effect on their understanding of base-ten relationships. Again, the time constraints may have been the reason that this difference in environments did not produce any evident effects. If the study had proceeded over a longer period of time, then possibly, there might have been some differences in understanding of place value in the two groups. However, in the interviews, when students were asked about questions requiring understanding of place value, the high blocks and the high microworld students showed good understanding of the place-value concepts.

An objective of the study was to measure change in students' creativity in their attempts to solve problems. Teachers in both the microworld and blocks groups used scripts that encouraged students to try different methods of solving problems and attempted to have students realize that unconventional methods may be as correct as the conventional method. It may have been easier for the microworld students to try other methods since their simulated blocks were easier to manipulate. The ease of manipulating blocks gave them some extra time to explore. In either case, it appears that the accomplishment of that particular objective was met with some success with particular groups of students. One reason for this seeming lack of creativity may be attributed to the length of the study and the nature of the scripts. Although the teachers were attempting to encourage freedom of method in their students, the scripts themselves contained constraints which did not allow for an abundance of free exploration. The constraints were the pressure of time with the presence of an underlying message of completing certain tasks in the lesson plans within a designated time frame. More time for lessons may have made a difference in both groups. Both teachers expressed the concern that they did not feel the students were given enough time for certain parts of the lessons to reach a level of comfort in understanding.

The concern that it is very difficult to have students use unique problem solving procedures for standard calculations when previously learned procedures were memorized had been expressed by Thompson (1992). Through the observations of both classes, there was little evidence of students using alternative methods of solving problems. Again, this may be attributed to the fact that there was limited time for exploration. It appears that a longer period of time would be required to have students set aside previously learned procedures for new methods. It must also be stated again that only portions of the classes were observed due to the fact that classes were running simultaneously. However, there was some evidence of creativity shown during the interviews. When students were asked specifically about alternative methods for solving certain problems, students in the high blocks as well as the high microworld stated that alternative methods for doing questions were still correct. The analysis of the test items dealing with alternative methods also suggests that most students who showed acceptance to alternative solutions were from either the high microworld or the high blocks groups. The minimal effect on either group with respect to changing methods is consistent with past studies (Labinowicz, 1985; Resnick & Omanson, 1987; Wearne & Hiebert, 1988).

In summary, the results of this study seem to suggest several important points. Firstly, it seems that regardless of using either real blocks or the computer program, there is benefit to the students when learning place-value concepts. There seems to be some advantage in using *Blocks Microworld* as opposed to the actual base-ten blocks. This must be qualified by saying that under certain conditions it appears that it is advantageous to use the program as opposed to the actual base-ten blocks. In particular, the continual reinforcement of symbolic representations with the manipulations of the blocks provided by the computer program seems to aid certain types of students to understand place value concepts.

Secondly, there is reason to believe that using the program or using the actual blocks could aid in developing some independence from rigid algorithmic procedures. In particular, it seems to depend largely on the teacher to continually reinforce and encourage students to feel free to explore with their own methods as opposed to always using pre-determined algorithms. It must be noted that it seems to be a long term agenda to have students readily use novel methods when previous learning of the same or similar concepts had already taken place.

The foregoing suggests thoughts for further investigations in this area. One may wish to determine which stage in a student's understanding of numeration would benefit most from the use of actual blocks as opposed to a computer simulation.

For further research, it also seems appropriate to recommend that the long term effects of blocks and the computer program be studied. This may enable one to determine the effects of the continual exposure of the dual representation of the numbers. The longer time span for the study may also make clearer the time necessary for students to accept alternative methods to solving problems as correct procedures.

A final recommendation is that a control group be included, so that benefits of either treatment could be clearly determined. This may better differentiate any improvements attributable to the treatments.

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Appendix A

Script for Microworld Students

This is the script used for the students in the group who were taught the lessons using the program Blocks Microworld. This script was obtained from Dr. Patrick Thompson and was used in this study with his permission. Any duplication of the script in whole or in part must be authorized by Dr. Thompson.

Day 1, 2

Class demonstration of computer and computer program (9 minutes)

Students are at computers in pairs. You are at your computer at front of classroom, projected image of your Mac display is shown on projection screen.

Explain purpose of activity (1 minute)

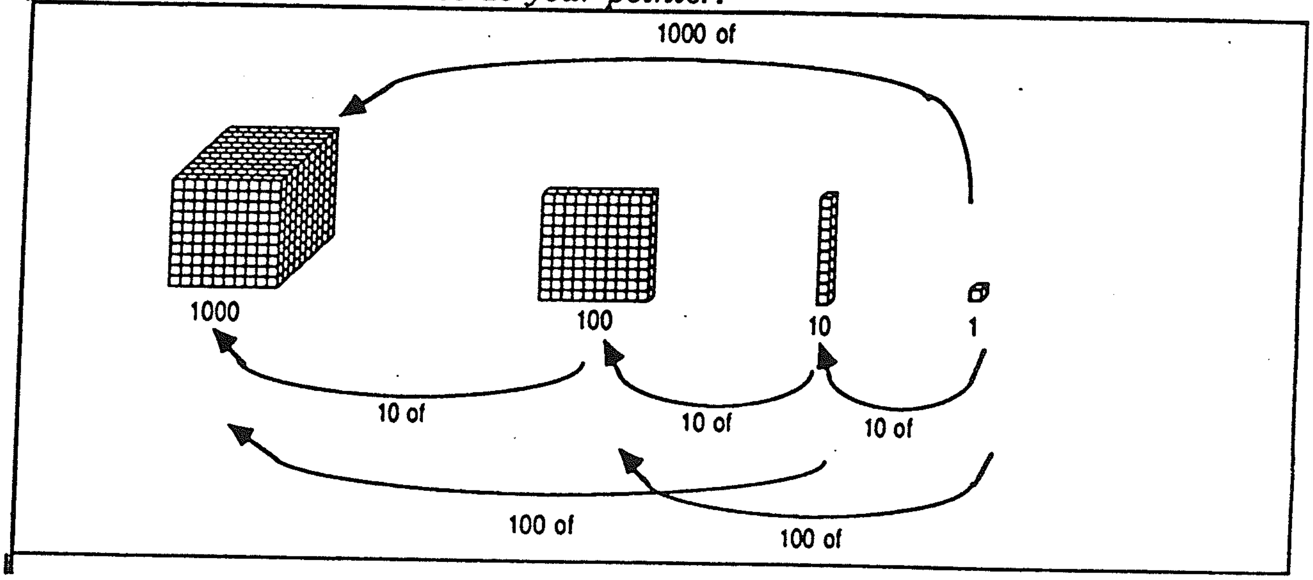
That when we work with numbers, what we do should have meaning and make sense in terms of what the ~~numbers stand~~ for. We are going to do some review of adding and subtracting whole numbers before doing some more work with decimal numbers.

The mouse (1 minute)

Be sure to mention picking up the mouse and moving it when out of room.

Making numbers (5 minutes)

- Discuss "ten of" relationship among the blocks. *Use projected screen as your "blackboard." Use mouse as your pointer.*



- Write **2413** on blackboard. Ask students what blocks we would use to show 2413.

Do not ask students how to make 2413 on the mats. Rather lead them to saying that 2413 would be made of 2 cubes (thousands), 4 flat(hundred), 1 long (tens), and 3 singles (ones).

- *"This is how we make numbers with the computer."* Make 2413 by dragging blocks. Have students mimic your actions on their computers.

Point out the dual representational systems (blocks and digits) as you drag blocks from the "reservoir."

The digit display (1 minute; skip this activity with the "real block" group*)*

- This activity is for them to understand that the digit display always gives reliable information, while the "blocks" display does not.

Tell students to put their heads down and cover their eyes; you are going to do "something tricky."

Move a flat so that it covers all the singles.

Say "How many longs do I have now?"
"How many singles do I have now?"

Point out the digit display, and the fact that *it* says that you still have 3 singles. Uncover the singles.

Dragging blocks (1 minute; skip this activity with the "real blocks" group*)*

- Demonstrate how to move blocks around.

Say that to move a block, they must put the *tip* (pointy end) of the mouse-pointer on a block.

Have students move several blocks from one region to the other. Point out how the digit displays always correspond to what is on the screen.

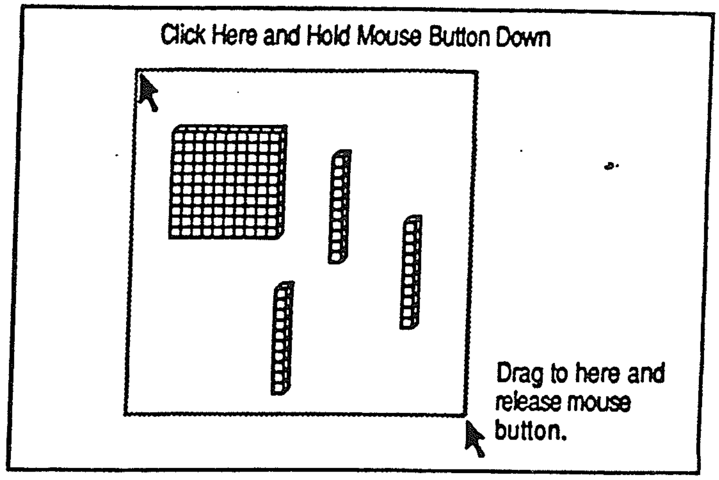
- Tell students to move all blocks back to the *left* region.

Point out to students that they can move more than one block at a time.

Have them click and drag so that they draw a rectangle around all the blocks in the right region. When they release the mouse button, all the blocks intersecting the interior of the rectangle will be selected (blackened).

Or, students can hold down a digit key and BLOCKS MICROWORLD *will automatically select that many blocks of the kind they clicked.*

Moving any one of the selected blocks will move all the selected blocks.



After moving all blocks to the left region, tell students to click Clean Up.

Carrying/Borrowing Demonstration (10 minutes)

You lead the discussion with your computer. Students will mimic your actions when you tell them to.

- You borrow from the cubes.

Click on the "cubes" digit, then click Borrow.

Refer to the blocks as "thousands," "hundreds," "tens," and "ones." *Explain to students that they have to click on a digit before they click Borrow, for otherwise the computer will not know which kind of block to unglue.*

Ask whether the total number of singles has changed or has remained the same.

Make sure that they understand that you are referring to the singles contained in the longs, flats, and cubes and not just to "loose" singles.)

- Point out the "digit" display at the top of the number region and how it corresponds with the new configuration of blocks.

Say *"Before we borrowed one cube, the display said '4-flat' After we borrowed, it said '14 flats.' Why did it change?"*

- Tell students, "Borrow one thousand on your computers."

Representing changes in blocks by changes in numeral

Discuss how one reflects changes in the blocks with corresponding changes in the numeral, such as by:

$$\overset{1}{\cancel{2}}\,\overset{14}{\cancel{4}}\,1\,3$$

Have students read numeral as "one cube, fourteen flats, one long, and three singles." Also have them read the numeral as "one thousand, fourteen hundreds, one ten, and three ones."

Stress that the number of singles represented by this numeral is still the same, just as the number of singles in the "blocks" diagram is still the same.

Say: Suppose we ask a farmer to deliver 2413 apples. He brings the apples in 1 box of 1000, 14 boxes of 100, 1 box of 10, and 3 loose apples. Did we get the correct number of apples?

The moral of this example is that that even though the numeral is "funny looking," it names the same number as does the "better" looking numeral 2413.

- **Click "cube" digit. Place arrow over *Borrow* button. *DO NOT CLICK THE BORROW BUTTON.* Tell students you are going to borrow a cube again.**

Ask them to explain what the computer will do to the blocks.

Ask them to explain what the computer will display in the "digits" area of the screen.

- Click *Borrow*.

- Ask students to explain what it meant to "borrow" a block.

Make sure they explain in terms of actions on the blocks. *Do not allow them to give their explanations only in terms of clicking a digit and the "borrow" button.*

- Ask them to read the digit display; point out the correspondence between the digit display and the block representation of the number.

- Represent the result as:

0 24
~~1~~ ~~14~~
~~2~~ ~~4~~ 1 3

- Tell students to borrow from the cubes.

Have students read numeral as"no cubes, twenty-four flats, one long, and three singles" Also have them read the numeral as "twenty-four hundreds, one ten, and three ones."

Stress that the number of singles represented by this numeral is still the same, just as the number of singles in the "blocks" diagram is still the same.

- **Click the "flat" digit. Place the arrow over the *Borrow* button. *DO NOT CLICK THE BORROW BUTTON.* Tell students you are going to borrow a flat.**

  - Ask them to predict what the computer will display in the "digits" area of the screen.
  - Ask them to predict what will happen to the blocks.
  - Borrow a flat.

- Ask them to read the digit display.

  - Represent the result as:

23
0 ~~24~~
~~1~~ ~~14~~ 11
~~2~~ ~~4~~ ~~1~~ 3

Have students read numeral as "no cubes, twenty-three flats, eleven longs, and three singles". Also have them read the numeral as "twenty-three hundreds, one ten, and three ones."

Stress that the number of singles represented by this numeral is still the same, just as the number of singles in the "blocks" diagram is still the same.

Transition

Four problems:

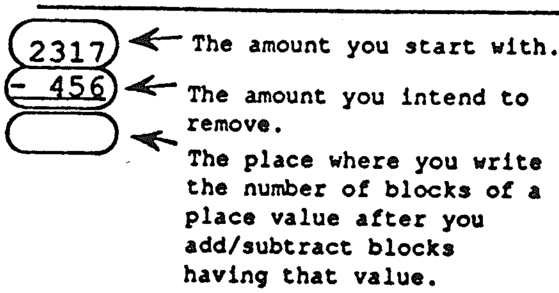
301 - 134 = ___; Demonstration & Discussion (no recording)

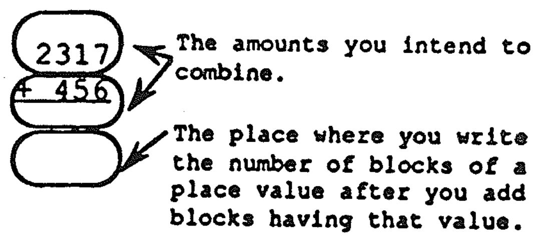
403 - 125 = ___; Students do with help (no recording)

Recording One's Actions and Results of Actions

Note to reader: One can adopt any set of conventions for recording actions performed on blocks-as-representations-of-quantities. We did not leave it entirely to students to create their own conventions, but we did emphasize that conventions aid and simplify communication, and that conventions also aid us in keeping track what we have done and what remains to be done.

We proposed a set of global conventions about how we write an initial statement of an arithmetic problem, and what it means to write something in a particular area of the initial setup. The global conventions we proposed are shown below. They were our attempt to give students freedom in creating their own schemes for recording actions performed on blocks, and yet were consistent with their prior experience with standard algorithms. Some students adopted these conventions straightforwardly; some made substantial modifications to them in creating their own recording schemes (see examples for Problem 1 of first homework).





Always record the effects of carrying or borrowing in the areas containing the digits that represent the blocks affected by the action.

- **Mention that we have "solved" these problems, but that it would be better if after solving a problem we could show anyone else how we did it without actually having to do it over with the blocks.**

There are two kinds of actions one can perform–actions that change a quantity's *representation* and actions that change a quantity's *value*. Borrowing and carrying change a quantity's representation but do not change its value. Adding and subtracting change a quantity's value. Be sensitive to students' remarks that might indicate they have confounded these two very different notions of "change" in the display.

1432 - 445 = ___; Demonstration &Discussion (with recording)

Do it twice—once conventionally, once unconventionally—if time permits. *(Note to reader: SeeHomeowork Problem 1 for an example of what it means to record actions.)*

2014 - 1132 = ___; Students do with help (with recording)

- Stress that when two people solve problems differently, both of their solutions can be correct even though their solution methods might be very different. Solutions are correct or incorrect; solution *methods* are appropriate or inappropriate.

Explain assignment.

Say "Some children solved these four problems with blocks, but they did not record how they solved them. Your job is to do the recording for these children."

- Discuss first homework problem. Ask "What did Frank do?" for each of the first few steps. Record succesive changes in the display.

Stress to students that the task is for them to record *what these children did with the blocks*. Students may think (when they sit down at home to do their homework) that *they* are supposed to do the indicated subtractions using their already-known standard algorithms.

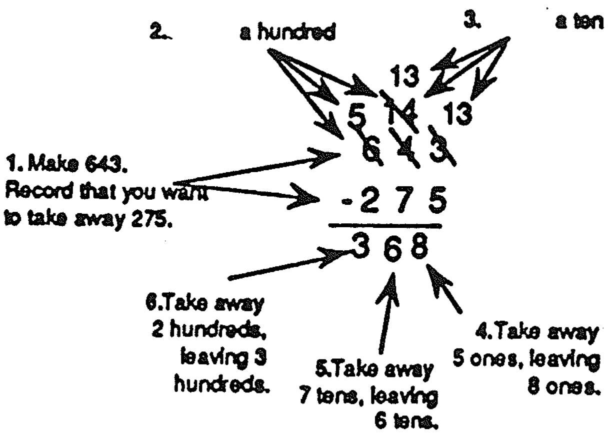
Homework 1

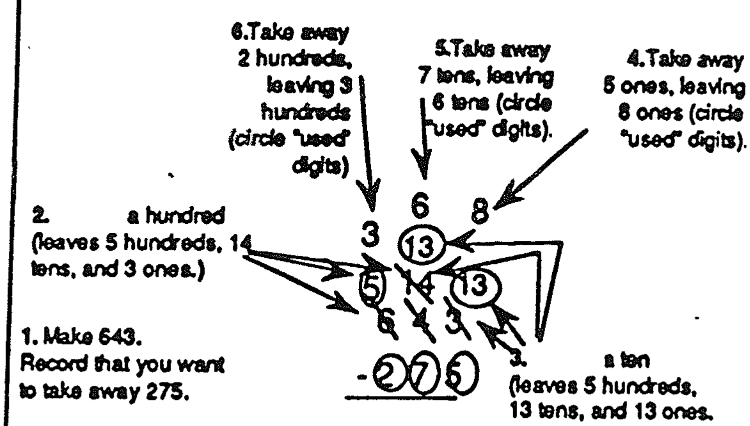
Frank solved the problem "643 - 275 = ___" using blocks, but he did not record his steps. What he did with the blocks is shown on the next page.

Use the setup below to record the steps in what Frank did with the blocks.

(Note to reader: Two ways in which students recorded Frank's "blocks" solution are shown below. These records reflect Frank's actions; the point made to students was that it is okay to do anything that makes sense when solving a subtraction problem with blocks. The main requirement is that one must record each and every action taken, whether it be a change of representation or a change of quantity.

$$\begin{array}{r} 643 \\ \underline{-\,275} \end{array}$$



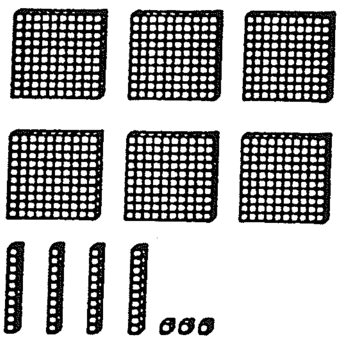


Problem 1

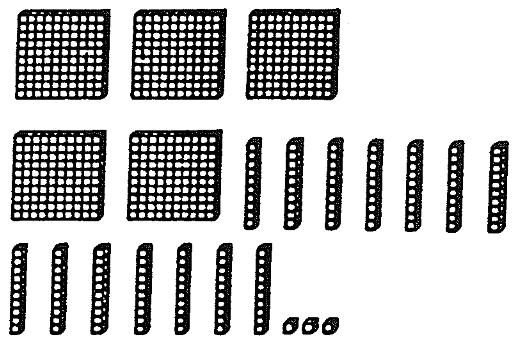
Frank solved the problem "643 - 275 = ___" using blocks, but he did not record his steps. What he did with the blocks is shown below. Use the setup at the right to record the steps in what Frank did with the blocks.

$$\begin{array}{r} 643 \\ -\ 275 \\ \hline \end{array}$$

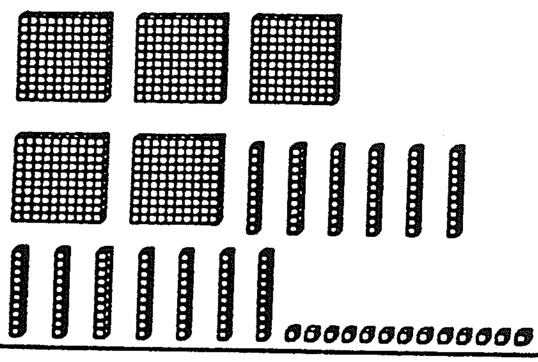
Step 1 of Frank's solution.



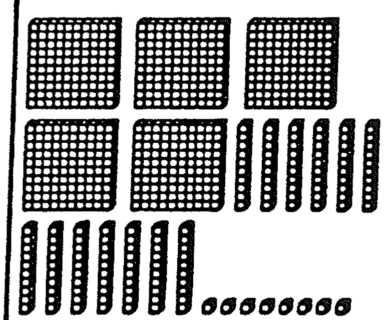
Step 2 of Frank's solution.



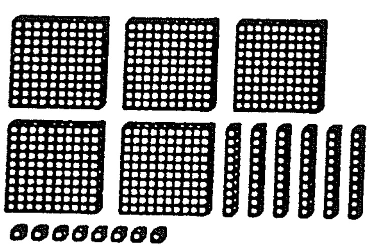
Step 3 of Frank's solution.



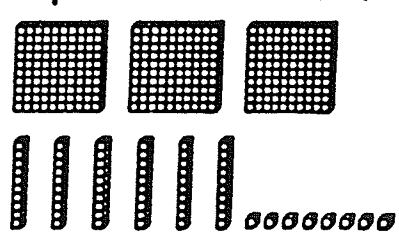
Step 4 of Frank's solution



Step 5 of Frank's solution



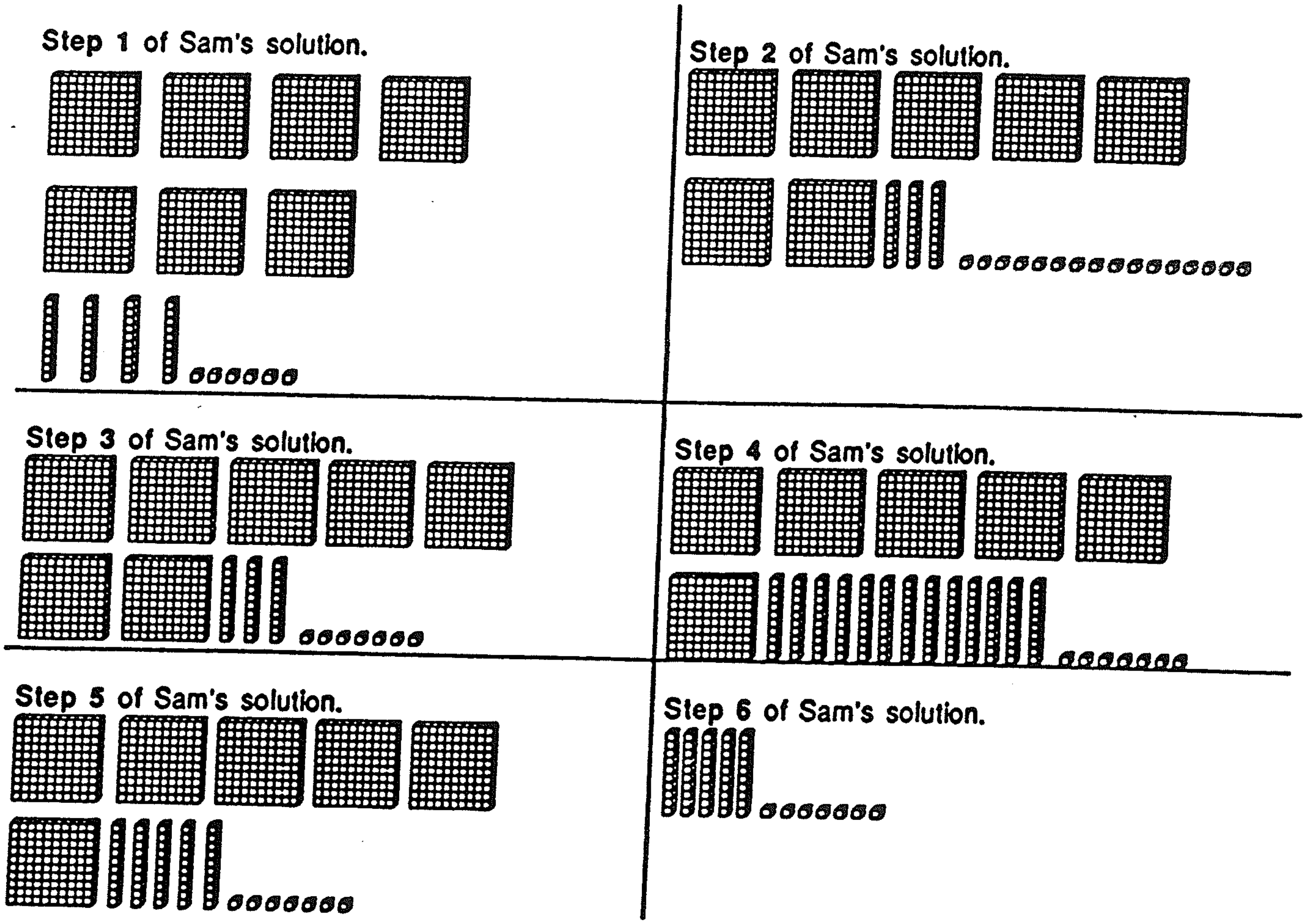
Step 6 of Frank's solution.



Problem 2

Samantha solved the problem "746 - 689 = ___" using blocks. What she did with the blocks is shown below. Use the setup at the right to record the steps in what Samantha did with the blocks.

$$\begin{array}{r} 746 \\ -\ 689 \\ \hline \end{array}$$

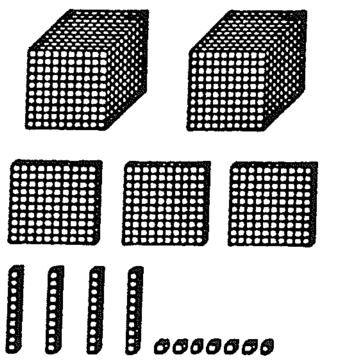


Problem 3

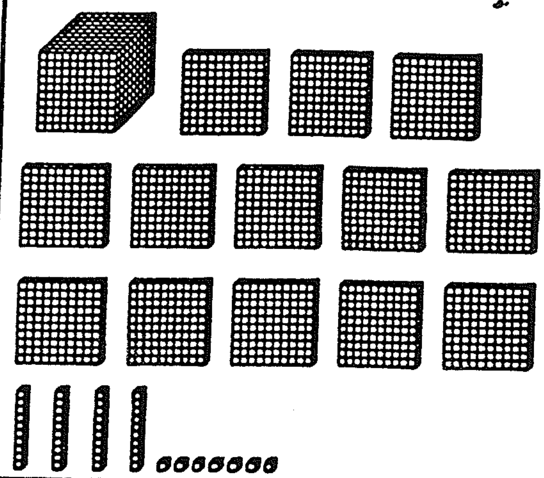
Jimmy solved the problem "2347 - 958 = ___" using blocks. What he did with the blocks is shown below. Use the setup at the right to record the steps in what Jimmy did with the blocks.

$$\begin{array}{r} 2347 \\ -\ 958 \\ \hline \end{array}$$

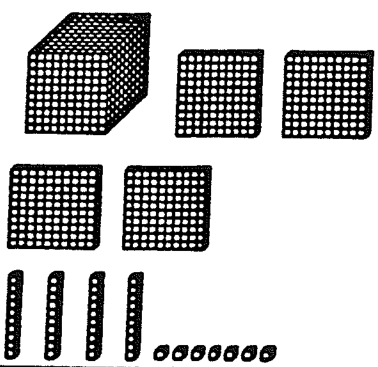
Step 1 of Jimmy's solution



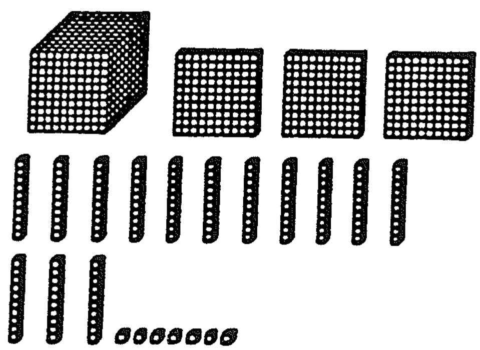
Step 2 of Jimmy's solution



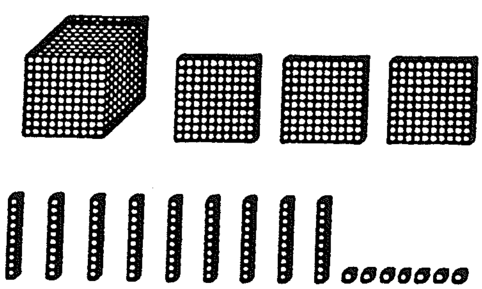
Step 3 of Jimmy's solution



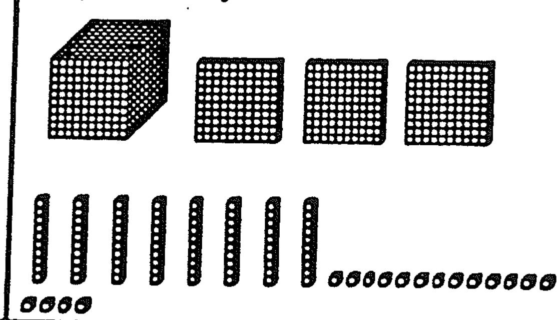
Step 4 of Jimmy's solution



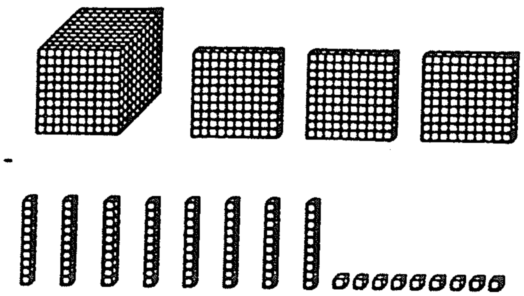
Step 5 of Jimmy's solution



Step 6 of Jimmy's solution



Step 7 of Jimmy's solution

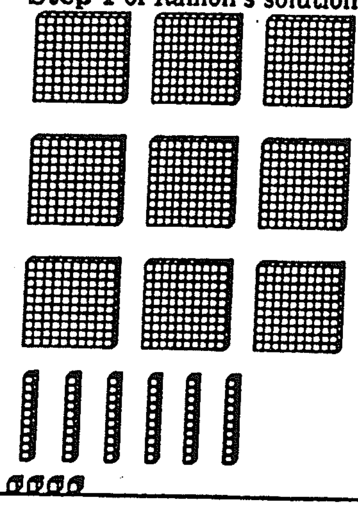


Problem 4

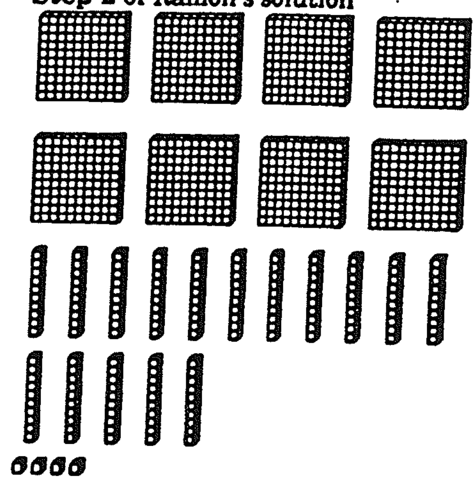
Ramon solved the problem "964 - 475 = ___" using blocks. What he did with the blocks is shown below. Use the setup on the right to record the steps in what Ramon did with the blocks.

```
  964
- 475
```

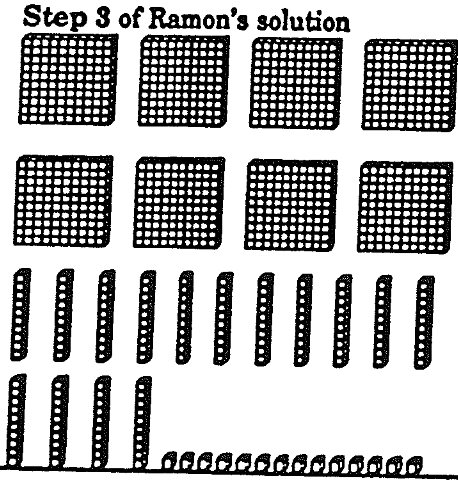
Step 1 of Ramon's solution



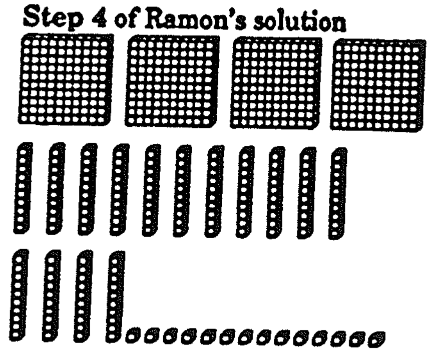
Step 2 of Ramon's solution



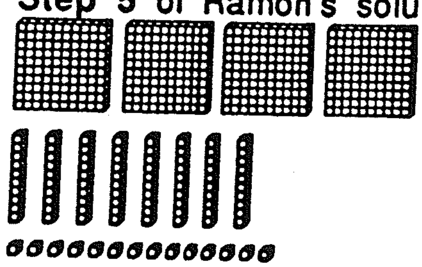
Step 3 of Ramon's solution



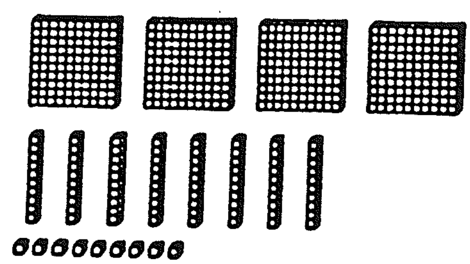
Step 4 of Ramon's solution



Step 5 of Ramon's solution



Step 6 of Ramon's solution



Problem 5

Use Frank's method to solve 2358 - 1429 = ____

Use Samantha's method to solve 2358 - 1429 = ____

Use Jimmy's method to solve 2358 - 1429 = ____

Use Ramon's method to solve 2358 - 1429 = ____

Day 3

Go over previous day's homework.

- Have students use red pencils while going over homework. *Collect homework.*

Be attentive for this confusion: Students may have thought that *they* were supposed to do the subtraction, and may have completely ignored the "fictitious" children's solution methods. If anyone did this, have him or her do the problems over again (as homework), in addition to the homework assigned for today. *GIVE THEM A FRESH HOMEWORK SHEET. KEEP THEIR ORIGINAL HOMEWORK.*

Discuss what it means to add

- - Make 132 in left region.
  - Make 61 in right region.
  - Say that to add the numbers represented by these blocks means to combine the blocks.

  - Click ***Combine*** (at the top of the screen).

Draw students' attention to the digit display. Emphasize that we can combine two groups of blocks by simply thinking of them as going together to make one group. When we combine two groups of blocks, we get a number of ones, a number of tens, and a number of hundreds in the new group.

The main idea to get across is that addition means to *combine*. The next example will show that the idea of "carrying" comes up *after one has already combined two numbers of things (i.e., after one has combined groups of blocks).*

Point out that an alternative to combining the two quantities all at once is to combine them a little at a time. For example, you could have first taken the ones from the right region to the left region, then have taken the tens from the right region to the left region. This is the conventional method.

- Click ***Separate*** (to prepare for the next example).

Click ***ClearAll*** to clear the screen.

Exemplify addition with Blocks

- Make 864 in the left region.
  Make 137 in the right region.

- Tell students that you are going to click ***Combine*** (but don't do it yet).

> ☞ Ask them to *predict*:
> - What the computer will do (it will erase the line that separates the two regions).
>
> - What the digit display will have in it (it will display the total number of flats, the total number of longs, and the total number of singles).

Say Are we done adding? (Yes. But, the children will probably say no because the digits aren't "right." If they don't say this, then provoke the point by pretending that you yourself aren't sure that you are done adding.)

Ask whether the total number of singles is the correct total number of singles that you should have after adding. (Yes, it is. So you must be done adding.)

[Here is an analogy: Suppose we are going to order 1001 apples from a farmer. If we ask for 9 boxes with 100 apples each, 9 boxes with 10 apples each, and 11 loose apples, we would have the 1001 apples that we desired.]

But we do not have the total number of singles represented in the *conventional* way that other people expect total numbers to be represented. (Which is to have all digits being between 0 and 9 inclusive.)

Thus we need to fix the representation.

- Click the singles digit in the digit display (it should have a box around it, like this: $\boxed{11}$.

  Click ***Carry*** (middle left side of the set of action buttons).

> ☞ Draw students' attention to the actions that the computer performed. It selected 10 singles and then glued them together to make a ten.
>
> Then, draw students' attention to the digits display. Emphasize that after the computer glued 10 singles to make 1 ten, it changed the ditit display to reflect the actual number of each kind of block.

Ask whether you are done (i.e., done putting the sum of 864 and 137 into a "conventional" format; you aren't done—there now are 10 tens).

Say that you want to "carry" the tens so that there are fewer than ten of them.

Ask How do I carry the tens?
(This is the procedure: Click on the "longs" digit so that it looks like $\boxed{10}$, and then click ***Carry***.)

☞ Get the procedure settled before going on. *Don't do the procedure yet!*

Ask students to __predict__ what the computer will do after you click the digit and click ***Carry***.

Ask students to __predict__ what the digit display will look like after you click ***Carry***.

(It will say 10 Flats 0 Longs 1 Single.)

Do the procedure. Draw students' attention to the actions the computer performed and to the correspondence between the digit display and the current blocks representation of the number.

- Repeat the last step with the flats. We're not done yet, since there are 10 flats. Have students reiterate the procedure for "conventionalizing" the display [click the digit, then click ***Carry***], and have them predict the computer's actions and to predict what will be in the digits display.

Students solve problem on computer

- Have students solve 496 + 505 = ___.

☞ Point out that they are done "adding" as soon as they click ***Combine***. But, they are not done representing the solution in a "conventional" format until they have finished making all digits less than 10.

Relate actions with blocks to algorithms with numerals

This next example is to make connections between solving the problem with the computer and how we record solutions on a "set up" representation of an addition problem using numerals.

- Click ***ClearAll*** and ***Separate*** (to prepare for next problem).
- Write this problem on the board:

$$\begin{array}{r} 6\ 4\ 5 \\ +\ \underline{1\ 6\ 6} \end{array}$$

Say that you are going to do this problem on the computer, but that you are also going to record your actions on the "setup" numerals. This way, we can explain to someone else how we did this addition without having actually to show them on the computer.

- On the computer: Put 645 in one region; put 166 in the other. Click ***Combine***.
- Record the digits display. The result should look like this:

$$\begin{array}{r} 6\ 4\ 5 \\ +\ \underline{1\ 6\ 6} \\ 7\ {}^{1}0\ {}^{1}1 \end{array}$$

Read this result as "645 plus 166 is 7 hundreds, 10 tens, and 11 ones."

Point out that we normally write a small "tens" digit in a column so that no one gets confused as to what we are representing. Were we to write all digits the same size in a column, it could become confusing, like this:

$$\begin{array}{r} 6\ 4\ 5 \\ +\ \underline{1\ 6\ 6} \\ 71011 \end{array}$$

- Go through successive steps of "conventionalizing" the sum. Record the display each time you carry.

Students solve and record solution

- Hand out "463 + 648 = ____" problem page.
  Have students solve and record 463 + 648 = ________.

Assign Homework

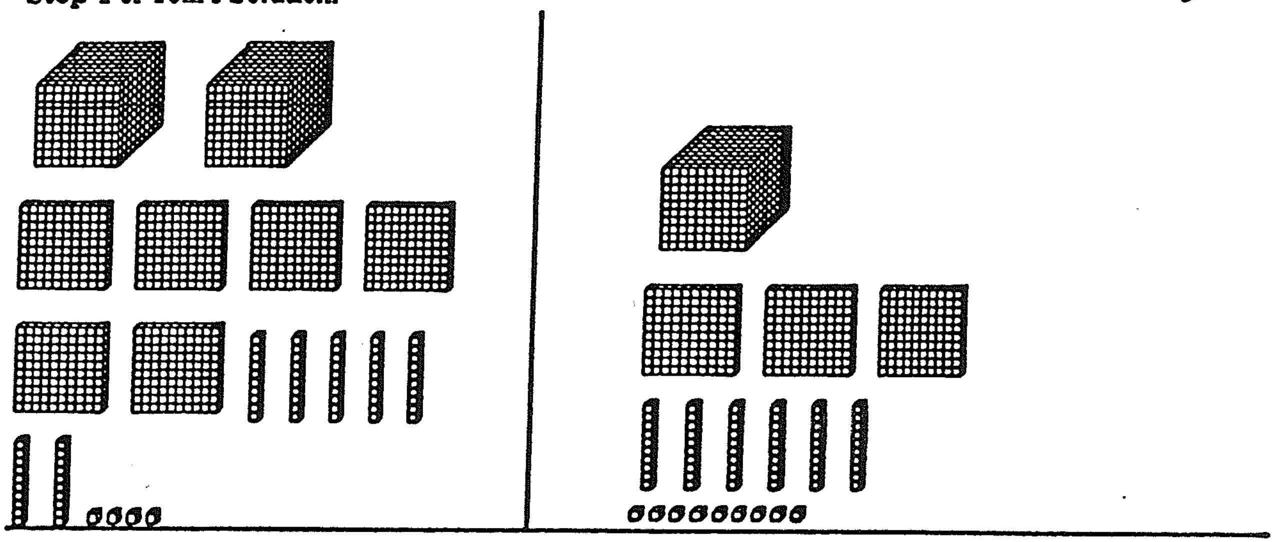
$$
\begin{array}{r}
463 \\
+\underline{648}
\end{array}
$$

Tom's solution to 2674 + 1369 = ____

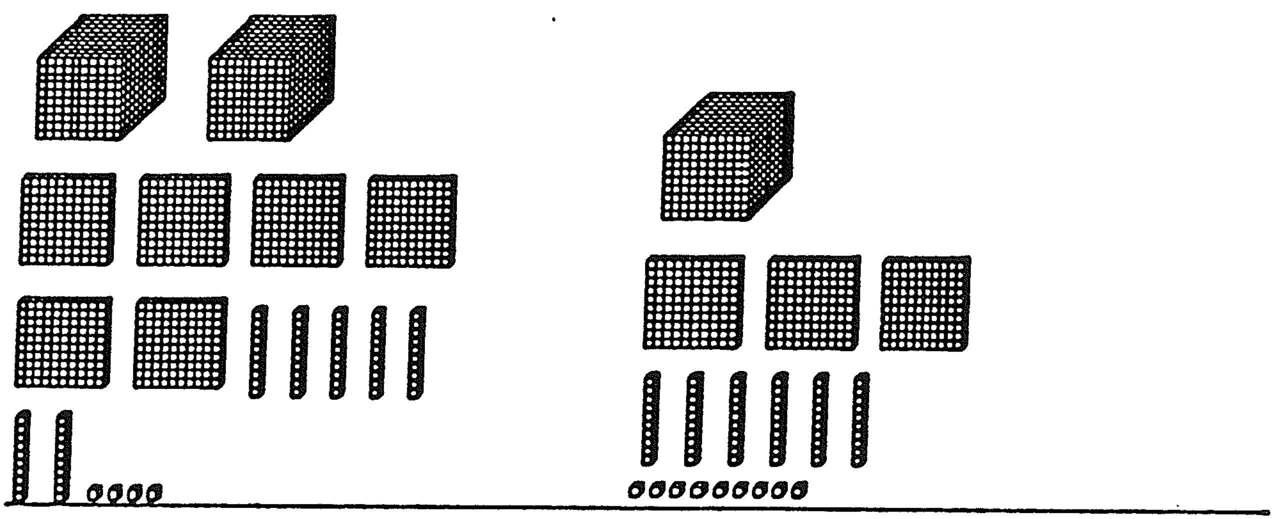
Tom solved the problem "2674 + 1369 = ____" using blocks. What he did with the blocks is shown below and on the next page. Use the setup at the right to record the steps in what Tom did with the blocks.

$$\begin{array}{r} 2674 \\ +\ 1369 \\ \hline \end{array}$$

Step 1 of Tom's Solution.



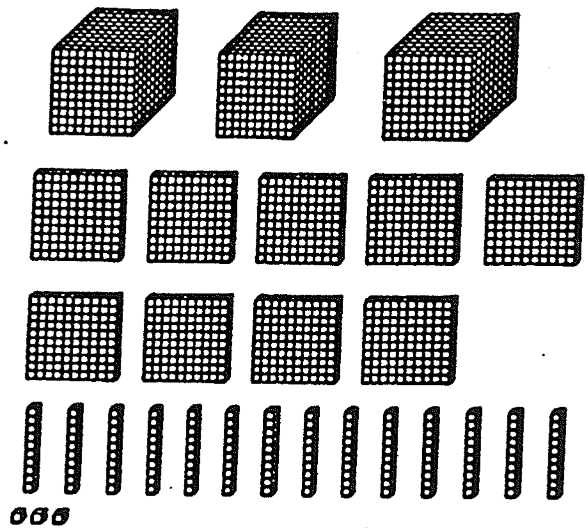
Step 2 of Tom's Solution.



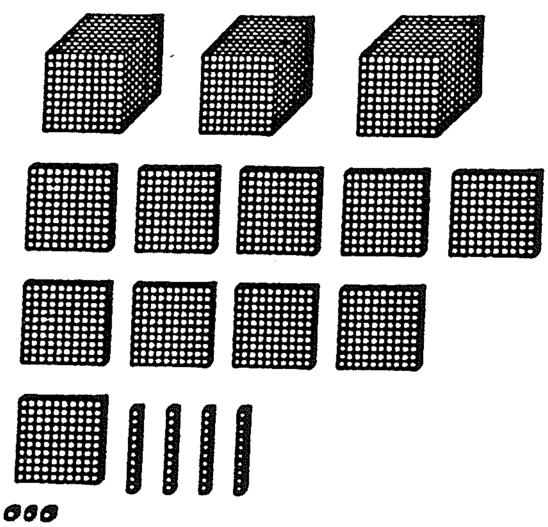
(Steps continued on next page)

Tom's solution to 2674 + 1369 = ____ (continued)

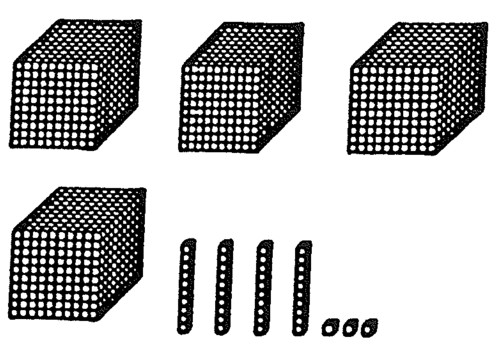
Step 3 of Tom's Solution.



Step 4 of Tom's Solution.



Step 5 of Tom's Solution.

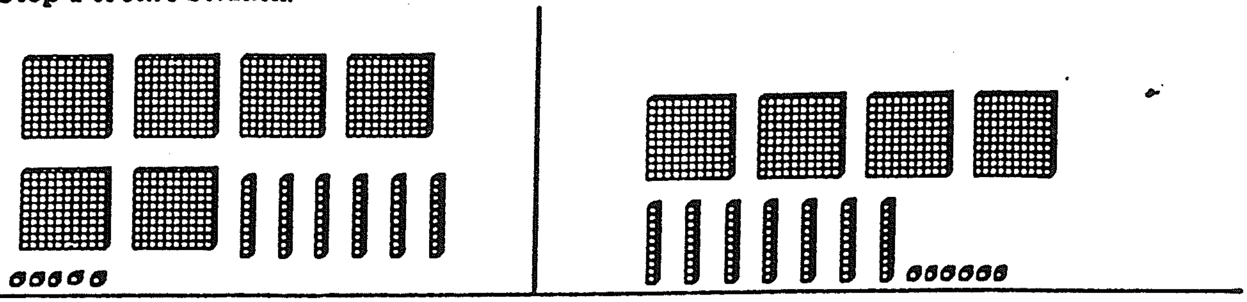


Jill's Solution to "665 + 476 = ____"

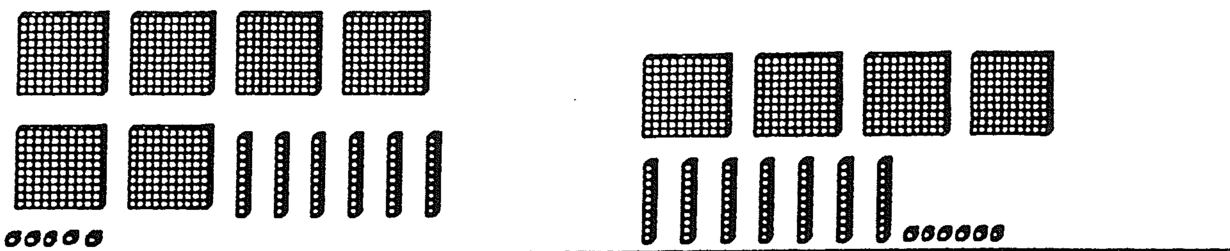
Jill solved the problem "665 + 476 = ____" using blocks. What she did with the blocks is shown below. Use the setup at the right to record the steps in what Jill did with the blocks.

$$\begin{array}{r} 665 \\ +\ \underline{476} \end{array}$$

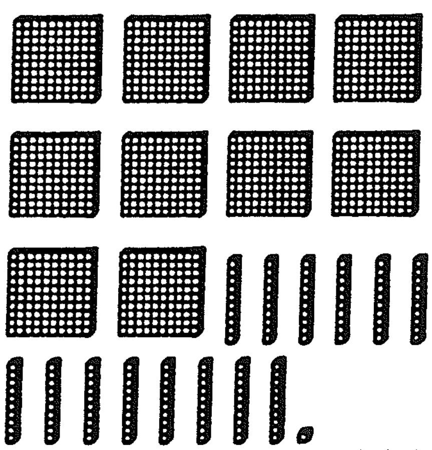
Step 1 of Jill's Solution.



Step 2 of Jill's Solution.

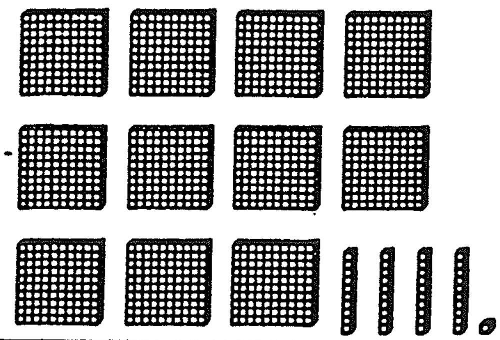


Step 3 of Jill's Solution.

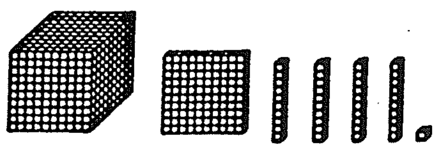


(Steps continued on next page)

Step 4 of Jill's Solution.



Step 5 of Jill's Solution.



Day 4

Go over previous day's homework.

- Have students use red pencils while going over homework. *Collect homework.*

Introduction to base-ten decimal numeration

Ask: How many flats are in a cube? *(10)*
How many hundreds are in one thousand? *(10)*

How many longs are in a flat? *(10)*
How many tens are in one hundred? *(10)*

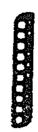
How many longs are in a cube? *(100)*

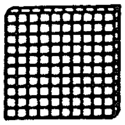
If this is problematic for students, ask "How many flats are in a cube? *(10)* How many longs are in each flat? *(10)* How many longs in a cube? *(100)*

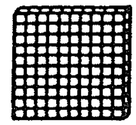
How many tens are in one thousand? *(100)*

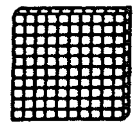
- Remind students of Marianne on the test that they took. Marianne liked to think of different blocks as being one instead of a single as being one.

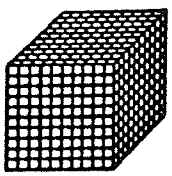
Turn contrast on projector (big knob) to "white out" the display. Put transparencies of following figure on the projector. Discuss one at a time.

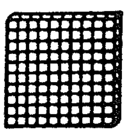
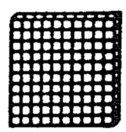
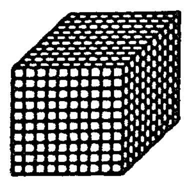
If  is one, then  is $\frac{1}{10}$, since there are 10 of  in one .

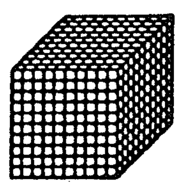
If  is one, then

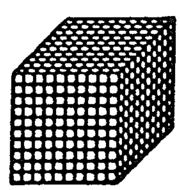
 is $\frac{1}{10}$, since there are ten of  in one .

and  is $\frac{1}{100}$, since there are 100 of  in one .

If  is one, then

 is $\frac{1}{10}$, since there are ten of  in one .

 is $\frac{1}{100}$, since there are 100 of  in one .

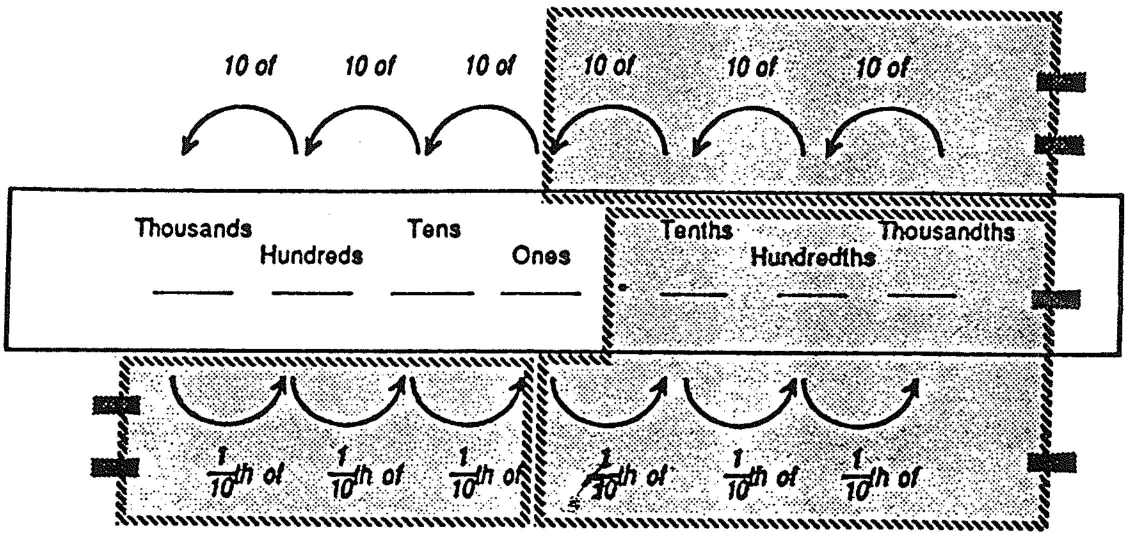
and  is $\frac{1}{1000}$, since there are 1000 of  in one .

Decimal place value

- Introduce students to decimal numeration notation. Say that the way we normally write numbers contains an idea that we can use to write what we have just said about fractions.

Put transparency of following figure on the projector

All flaps should be down.



- Things to point out about this scheme:

— A *digit* in a place tells us how many of those "value groups" we have.

— Each *place* is worth ten times as much as the one to its right.

— The *number* represented by a digit in a place need not be 10 times greater than the number represented by the digit to its right. (In "234", 2 hundreds is not ten times greater than 3 tens. But a hundred is 10 times greater than a ten.) ← *Omit if running long on time.*

Ask How many ones in one ten? — *Point at arrow after they have answered. Say that this diagram shows what they just said.*

How many tens are in one hundred? — *Point at arrow after they have answered. Say that this diagram shows what they just said.*

How many hundreds are in one thousand? — *Point at arrow after they have answered. Say that this diagram shows what they just said.*

Lift bottom left flap.

— Each *place* is worth $\frac{1}{10}$th as much as the one to its left.

Say There are ten hundreds in one thousand. So one hundred is $\frac{1}{10}$th of one thousand.

Point at arrow. Say that this diagram shows what you just said.

Ask How many tens in one hundred?

One ten is what part of one hundred? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

How many ones in one ten?

One one is what part of ten? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

Lift bottom right flap.

Say We can keep going to the right of the one's place. But, we need to make a mark to show where the one's place is, because the right-most place in a numeral need not be the one's place when we use places to the right of one. This dot is called a decimal point. It separates the ones place from the tenths place.

Ask Since each place is worth one-tenth of the place to its left, what value should the place to the right of one have? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

Note that they have seen a decimal point in money before. It separates the "one dollar" place fromthe "tenth of a dollar" place.

[Continue with hundredths and thousandths.]

Lift top right flap.

Ask How many thousandths in one hundredth? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

Say It does not matter where a digit is. Its place value is 10 times the place value of the digit to its right and its place value is $\frac{1}{10}$th the place value of the digit to its left. (Point at arrows while saying this.)

Representing numbers in decimal notation

Have students select the flat as their unit in the computer program. To do this:

Put the pointer over the **Unit** menu.
Press the mouse button and hold it down.
Slide the pointer down till *Flat is One* is highlighted.
Release the mouse button.

Draw students' attention to the fact that it now says **A Flat is One** below the master blocks, whereas it used to say **A Single is One**. This is to remind them that the *flat* now stands for "one."

- Tell students to make "ten and 11 hundredths."

Tell them that if they are clever, they will need to use only three blocks (viz., a cube, a long, and a single). Let them think about that.

- Hand out activity sheet (they will do these at their computers). Tell students that when they make these numbers, they will need to decide what block represents one. Different problems may require different blocks to stand for one.

Assign Homework

Computer Group

Do this to choose a block to represent one:

- Put the pointer over the **Unit** menu.
- Press the mouse button and hold it down.
- Slide the pointer so that your choice is highlighted.
- Release the mouse button.

Make these numbers *Fill in the blanks after you make a number*

4.30 _ Cubes _ Flats _ Longs _ Singles *(Put a decimal point between the ones and the tenths.)*

6.03 _ Cubes _ Flats _ Longs _ Singles *(Put a decimal point between the ones and the tenths.)*

12.1 _ Cubes _ Flats _ Longs _ Singles *(Put a decimal point between the ones and the tenths.)*

3.01 _ Cubes _ Flats _ Longs _ Singles *(Put a decimal point between the ones and the tenths.)*

1.032 _ Cubes _ Flats _ Longs _ Singles *(Put a decimal point between the ones and the tenths.)*

Tear the last two pages off. They have pictures of blocks that you will use to do your homework. Cut pictures of blocks from those pages when you need them.

Show 3.41 with blocks in two ways. Use different units.

Pick a block that will be the unit in your representation of 3.41. Paste a picture of that block into a "unit box". Then paste pictures of blocks to show 3.41.

Unit Box Put a unit here is the unit. 3.41 *First way*	*Unit Box* Put a unit here is the unit. 3.41 *Second Way*

Show 12.7 with blocks in two ways. Use different units.

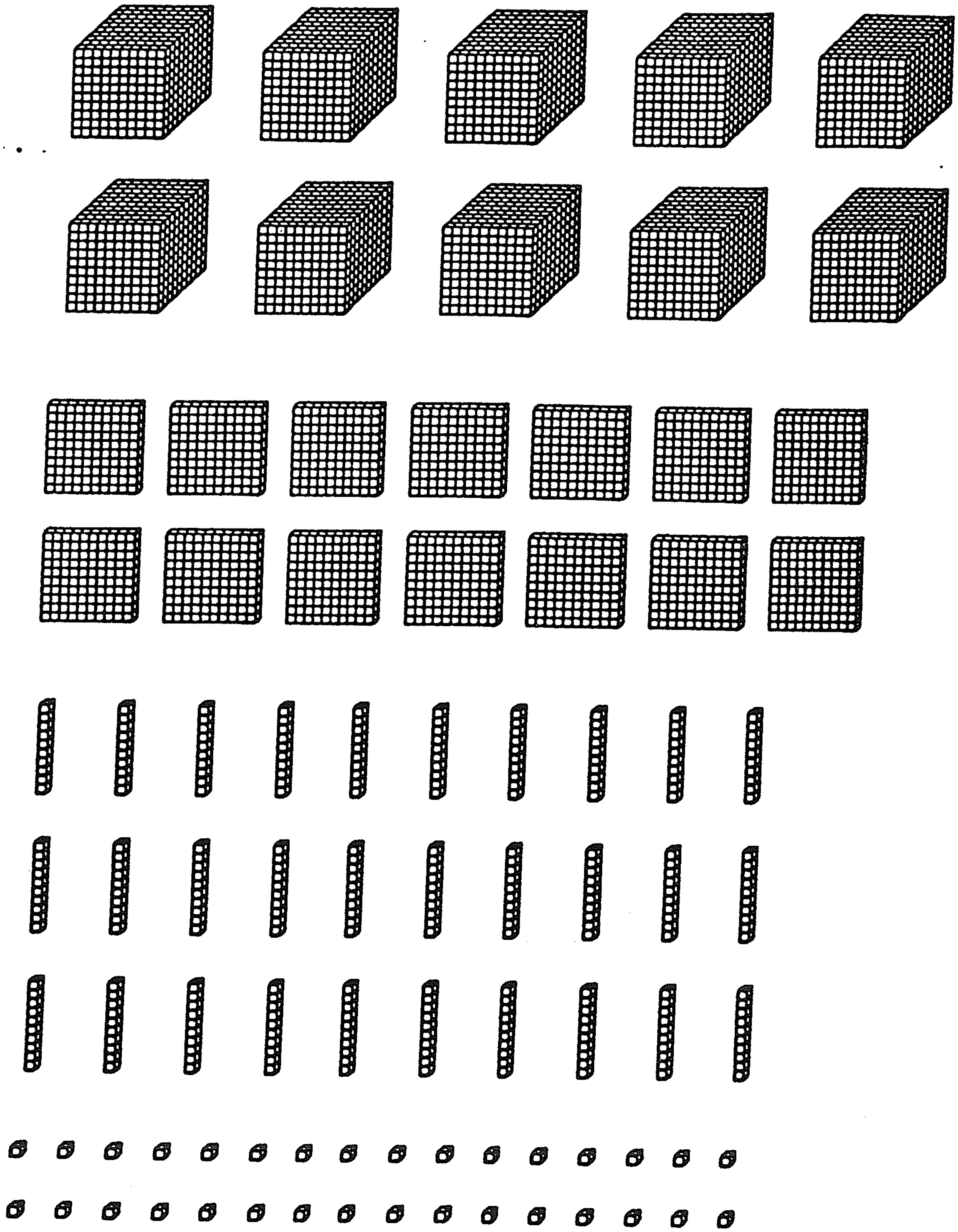
Pick a block that will be the unit in your representation of 12.7. Paste a picture of that block into a "unit box". Then paste pictures of blocks to show 12.7.

Unit Box	*Unit Box*
Put a unit here — **is the unit.**	Put a unit here — **is the unit.**
12.7	12.7
First way	*Second Way*

Show 11.32 with blocks in two ways. Use different units.

Pick a block that will be the unit in your representation of 11.32. Paste a picture of that block into a "unit box". Then paste pictures of blocks to show 11.32.

Unit Box	*Unit Box*
Put a unit here — is the unit.	Put a unit here — is the unit.
11.32	11.32
First way	*Second Way*



Day 5

Go over homework from previous day

Hand out red pencils.

- Be brief in going over homework. Give emphasis to the selection of a unit and, and especially give emphasis to *the relationship between choice of unit and what number the blocks represent.*

Once a unit is chosen, values of all other blocks are determined by the relationships "ten of" and "$\frac{1}{10}$th of."

Changing representations without changing the number

Have students go to computers. They should have pencils and papers with them.

- Tell students to select either the flat or the cube as their unit (it is their choice).

Write 4.32 on the blackboard.

Say Make "four and thirty-two hundredths" on your computers.

(After they are done)

Write four and thirty-two hundredths on a sheet of paper.

We are going to do some borrowing, and your job will be to record on paper what you do with the blocks.make "four and thirty-two hundredths" on their computers.

Tell students to *Borrow* a tenth.

Be sure to use the language of "tenths," "hundredths," and "thousandths." Let students translate tenths, hundredths, and thousandths into the language of blocks.

- Have students read the digit display in ones, tenths, and hundredths.

Stress that we want to know how many ones, tenths, etc.: 4 ones, 2 tenths, and 12 hundredths. Emphasize that the number that they originally represented has not changed:

Say Record on your paper the change you made in the blocks.

Write $4.\overset{2}{\cancel{3}}\,\overset{12}{\cancel{2}}$ (after students have finished writing on their papers).

- Tell students to *Borrow* a one.
- Have students read the digit display in ones, tenths, and hundredths: 3 ones, 12 tenths, and 12 hundredths.

Say Record on your paper the change you made in the blocks.

Write $\overset{3}{\cancel{4}}.\overset{12}{\cancel{3}}\,\overset{12}{\cancel{2}}$ (after students have finished writing on their papers).

Say This shows that 4 ones, 3 tenths, and 2 hundredths represents the same number as is represented by 3 ones, 12 tenths, and 12 hundredths.

Point out similarities between decimal numbers and whole numbers

Say the following (don't make it a matter of lengthy discussion):

Think about what stands for what,

Borrow whenever you do not have enough of what you want to take away from,

Record on the numeral any changes in the blocks.

Subtraction of decimal numbers

Tell students to select the flat as their unit.

- Write this on the board: **4.02 - .5 = ____**
  Have students solve this problem *without recording.*

After they have solved it, ask students what was hard about this problem. (I expect that the hard part will be deciding and keeping in mind what stands for what.)

- Tell students to *ClearAll.*

Tell students to write this on a piece of paper:
(write it just like it appears here)

```
  10.4
- __.62
```

Ask students if it would make any difference if we wrote it like this:

```
 10.4
-_.62
```

(It wouldn't matter if we paid close attention to place-value of digits; it would matter if we did not pay close attention.) Bring out the point that we align digits that have the same place value merely as a matter of convenience—so that when we subtracting digits, we know that we are subtracting tens from tens, ones from ones, tenths from tenths, and so on. *We align the decimal points to ensure that digits with the same place value are in the same columns.*

- Have students solve "10.4 - .62 = ____" *(with recording).*

Discuss solution briefly when they are finished.

Speak about actions on blocks (using the language of tens, ones, tenths, and hundreds) and record actions on numerals. *But don't make any reference to the numeral.*

Hand out activity sheet (next page)

Tell students to do these problems any way they wish. They can do them with or without using blocks. If they do them without using blocks, then they should imagine that they *are* using blocks. In either case, tell them to record on the numerals changes they make (or imagine making) in the blocks.

Assign homework

Solve these problems.

You will need to decide what block is your unit in each problem.

You can do these problems with or without using blocks. *It is your choice.*

In either case, record on the numbers whatever you do (or imagine doing) with the blocks.

$$\begin{array}{r} 2.31 \\ -\underline{.43} \end{array}$$

$$\begin{array}{r} 1.01 \\ -\underline{.002} \end{array}$$

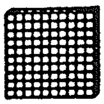
Problem 1

James solved the problem "42.41 - 17.56 = _____" using blocks. What he did with the blocks is shown on the next two pages.

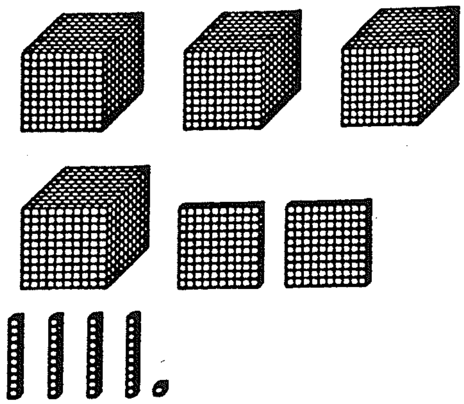
Use the setup below to record the steps in what James did with the blocks.

$$\begin{array}{r} 42.41 \\ -\ 17.56 \\ \hline \end{array}$$

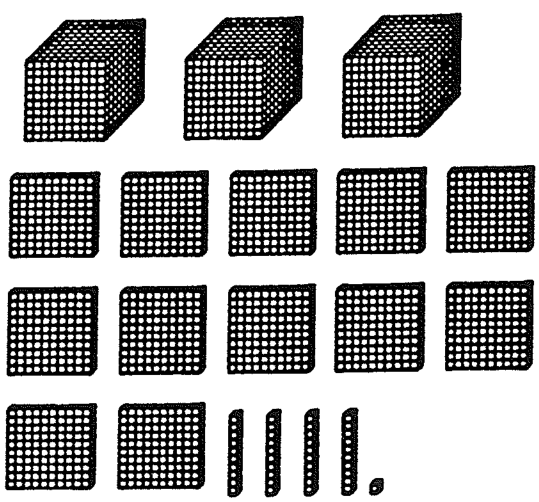
James' Solution to "42.41 - 17.56 = ____"

 is the unit.

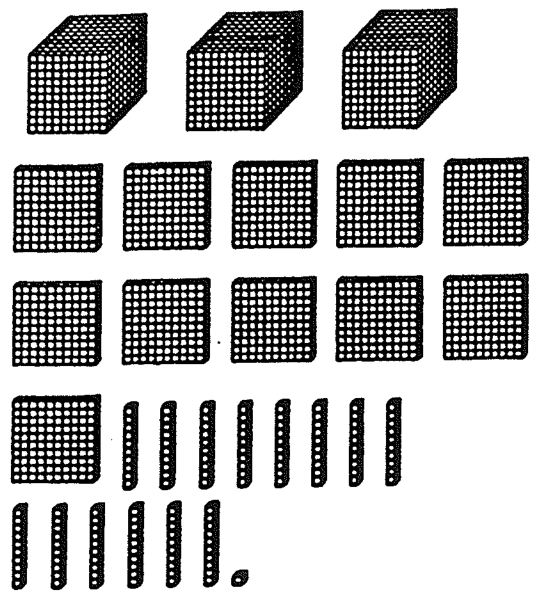
Step 1 of James' Solution.



Step 2 of James' Solution

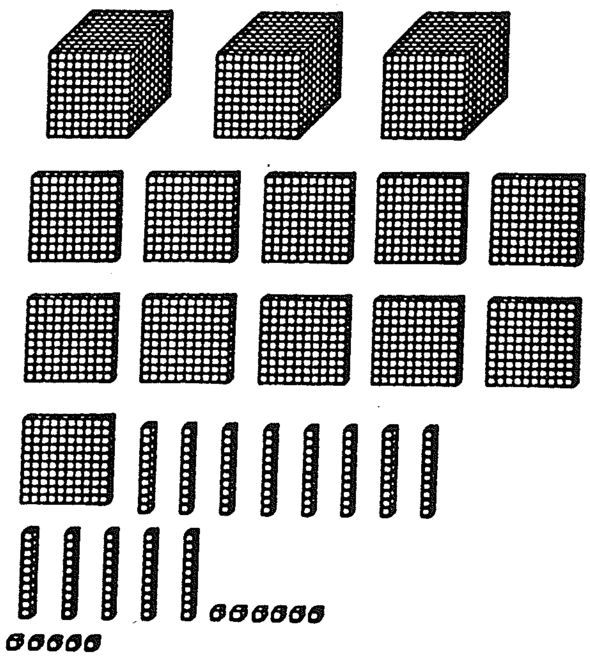


Step 3 of James' Solution.

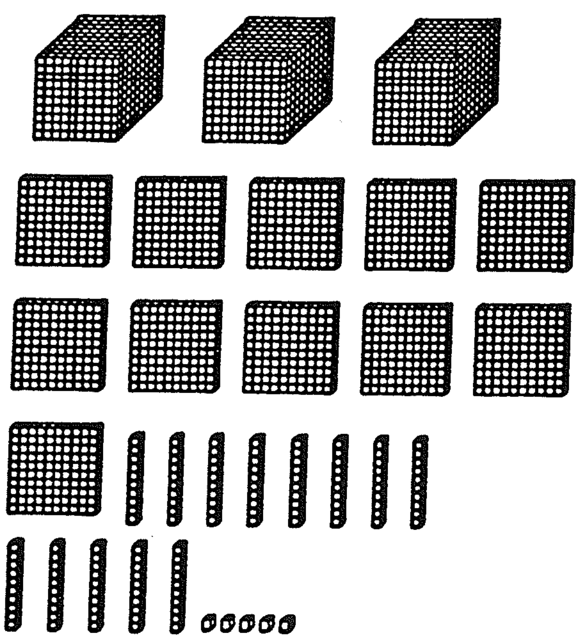


James' Solution to "42.41 - 17.56 = ____"

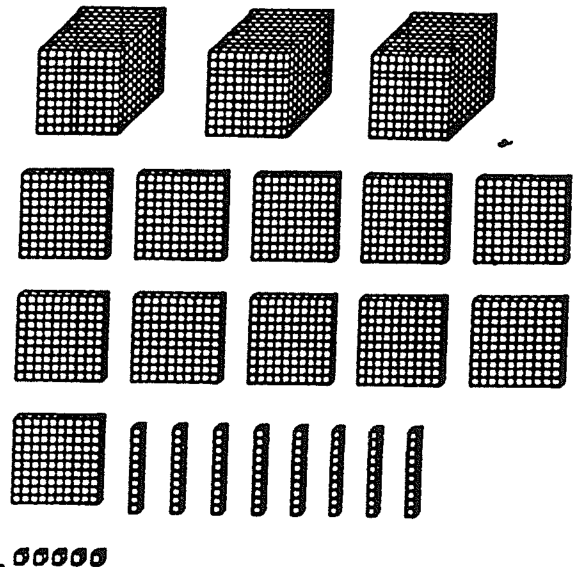
Step 4 of James' Solution.



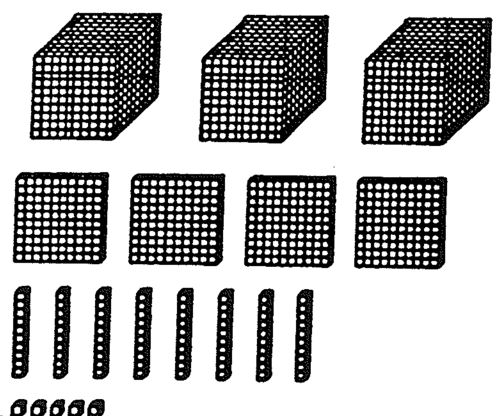
Step 5 of James' Solution.



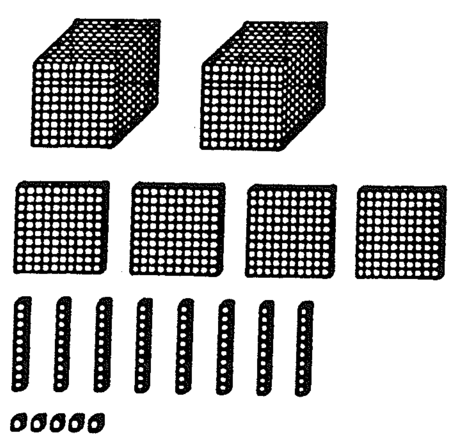
Step 6 of James' Solution



Step 7 of James' Solution



Step 8 of James' Solution



Problem 2

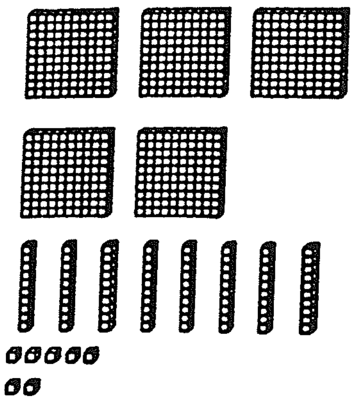
Sally solved the problem "58.7 - 49.8 = ______" using blocks. What she did with the blocks is shown on the next pages.

Use the setup below to record the steps in what Sally did with the blocks.

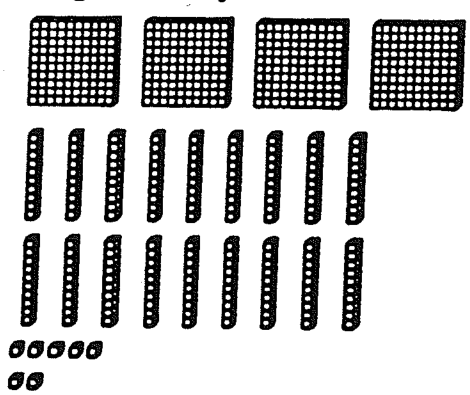
$$\begin{array}{r} 58.7 \\ -\ \underline{49.8} \end{array}$$

 is the unit.

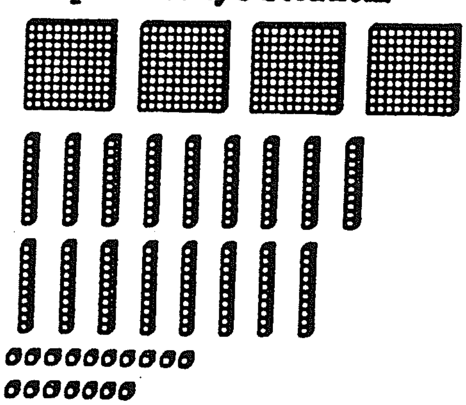
Step 1 of Sally's Solution



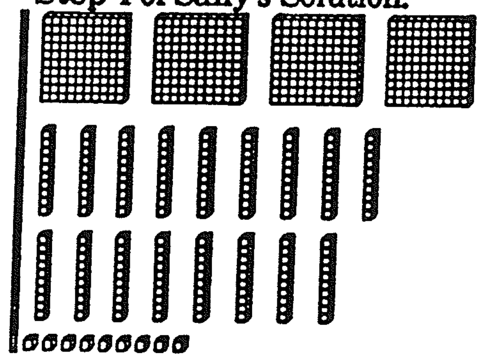
Step 2 of Sally's Solution.



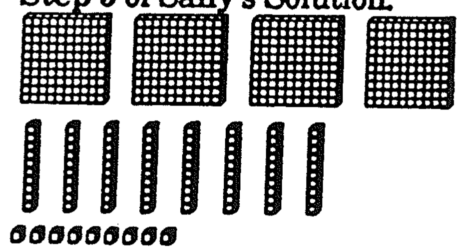
Step 3 of Sally's Solution.



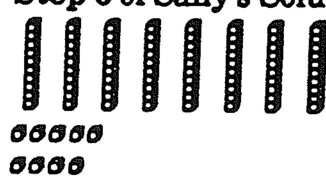
Step 4 of Sally's Solution.



Step 5 of Sally's Solution.



Step 6 of Sally's Solution.



Day 6

Go over previous day's homework briefly.

At the end, ask students if there is one particular method that these children used that they like best. That they liked least. *(The point of this discussion is that they reflect on the methods; the point is not to settle on a prescribed method.)*

Tell them that if they like a method, they can feel free to use it themselves.

Addition of decimal numbers

- Remind them that to *add* means to *combine.*
- Remind them that when we work with a numeral, it is to *record* what we do with the numbers that blocks stand for.
- Give students a copy of the next sheet. Tell them to solve these problems with blocks, and to record their solutions on the numerals.

They will need to pick an appropriate unit for each problem. This is part of the problem. Remind them how to change from one unit to another:

- Put the pointer over the Unit menu.
- Press the mouse button and hold it down.
- Slide the pointer so that your choice is highlighted.
- Release the mouse button.

- When students are finished, have each pair pick *one* problem to write on the board. Discuss differences among recording methods.

Differences among recording methods should be reflective of different methods of solving problems with blocks. Make sure that this comes out in the discussion.

Assign homework

Do these addition problems with blocks. Record on this paper what you do with the blocks.

You will need to decide what block represents one in each problem.

$$\begin{array}{r} 11.37 \\ +\ \underline{8.64} \end{array}$$

$$\begin{array}{r} 32.6 \\ +\ \underline{28.51} \end{array}$$

$$\begin{array}{r} 6.4 \\ +\ \underline{4.812} \end{array}$$

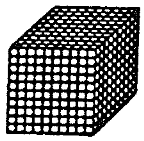
$$\begin{array}{r} 1.111 \\ +\ \underline{9.99} \end{array}$$

Problem 1

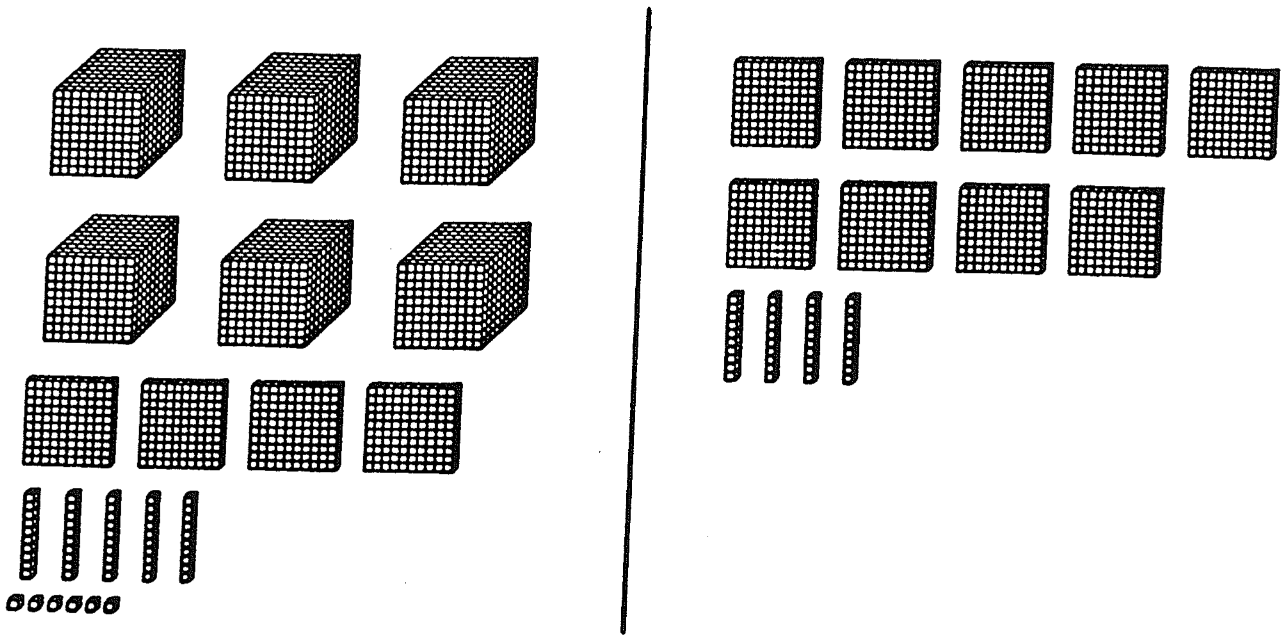
Jamie solved the problem "6.456 + .94 = ____" using blocks. What she did with the blocks is shown on the next two pages..

Use this setup to record the steps in what Jamie did with the blocks.

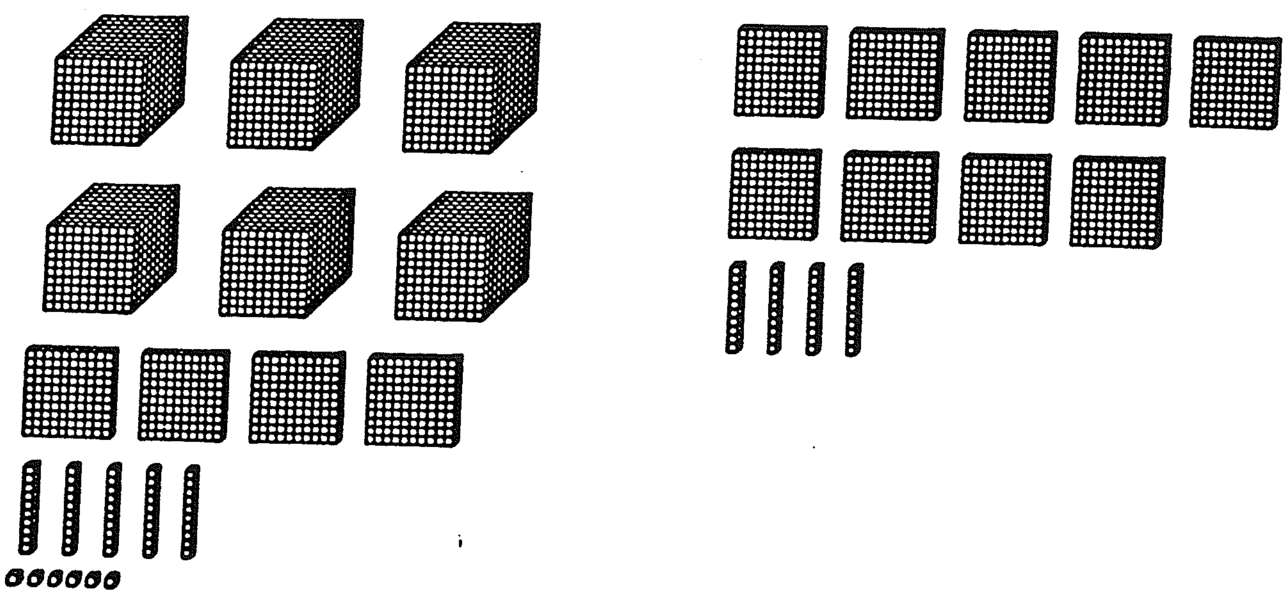
$$\begin{array}{r} 6.456 \\ \square \quad .94 \\ \hline \end{array}$$

 is the unit.

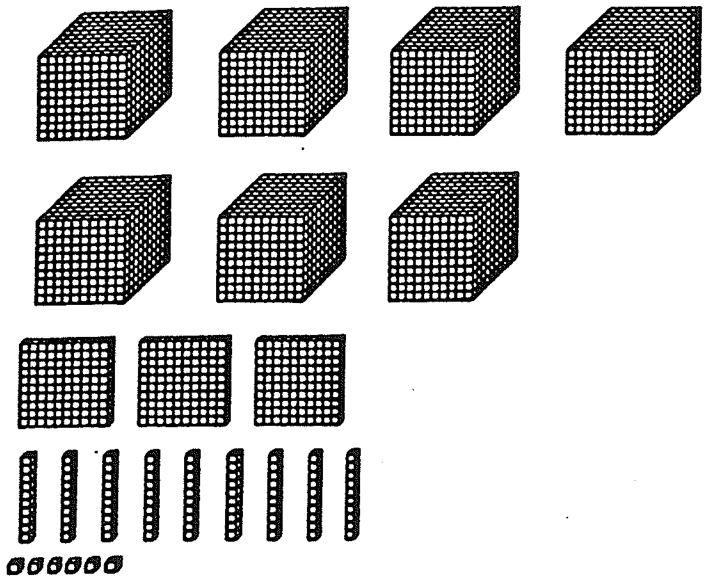
Step 1 of Jamie's Solution.



Step 2 of Jamies Solution.



Step 3 of Jamie's Solution.

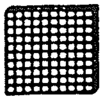


Problem 2

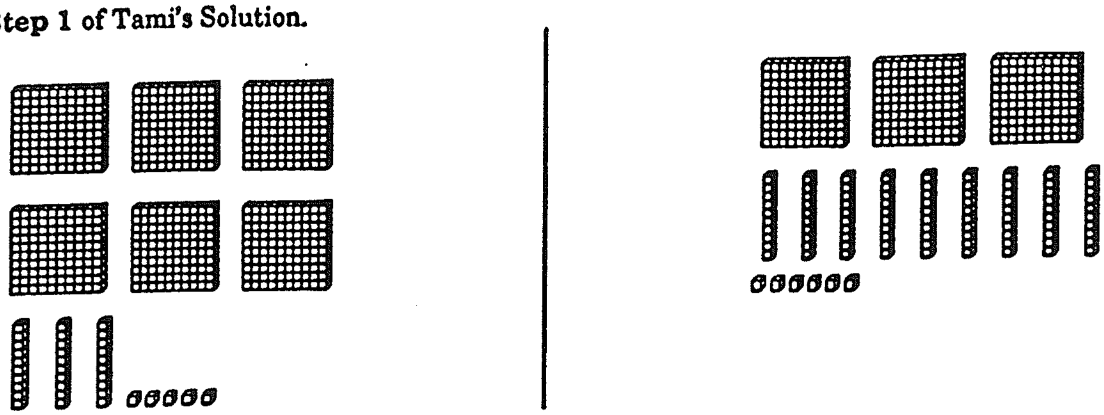
Tami solved the problem "6.35 + 3.96 = ____" using blocks. What she did with the blocks is shown on the next two pages.

Use this setup to record the steps in what Tami did with the blocks.

$$\begin{array}{r} 6.35 \\ \square\ \underline{3.96} \end{array}$$

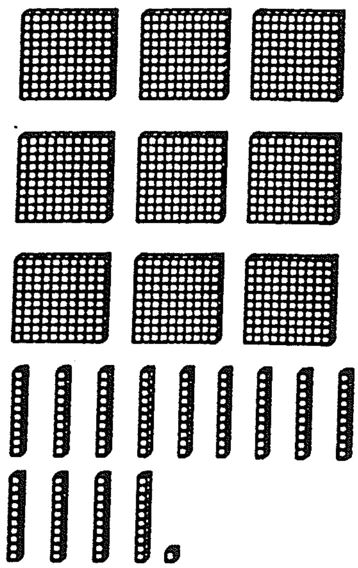
 is the unit.

Step 1 of Tami's Solution.

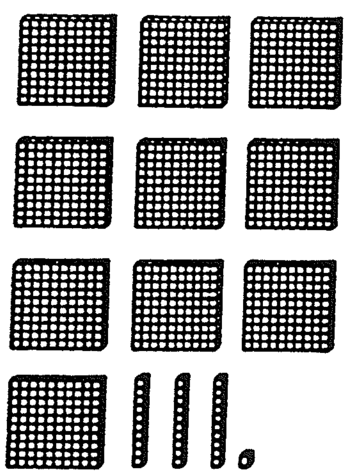


Step 2 of Tami's Solution.

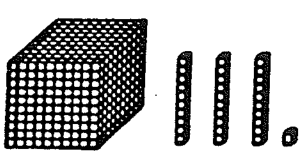
Step 3 of Tami's Solution.



Step 4 of Tami's Solution.



Step 5 of Tami's Solution.



Day 7

Go over previous day's homework briefly

Assign activity sheet for mixed practice (next page).

Mixed Practice

Try to do these problems without blocks.

If you become confused while doing a problem, then think about what you would do with the blocks.

If thinking about the blocks doesn't help, then use real blocks.

Show your work on this paper.

$$\begin{array}{r} 15.4 \\ +\underline{16.81} \end{array}$$

$1748 + 2253 =$ ______

$6.02 - 5.4 =$ ______

$$\begin{array}{r} 5.001 \\ -\underline{1.82} \end{array}$$

$$\begin{array}{r} 5.08 \\ -\underline{2.93} \end{array}$$

Appendix B

Script for Blocks Students

This is the script used for the students in the group who were taught the lessons using the base-ten blocks. This script is a modified version of the script used for the microworld group.

The original script was obtained from Dr. Patrick Thompson and was used in this study with his permission. Any duplication of this script in whole or in part must be authorized by Dr. Thompson.

BLOCKS

Day 1, 2

Class demonstration (9 minutes)

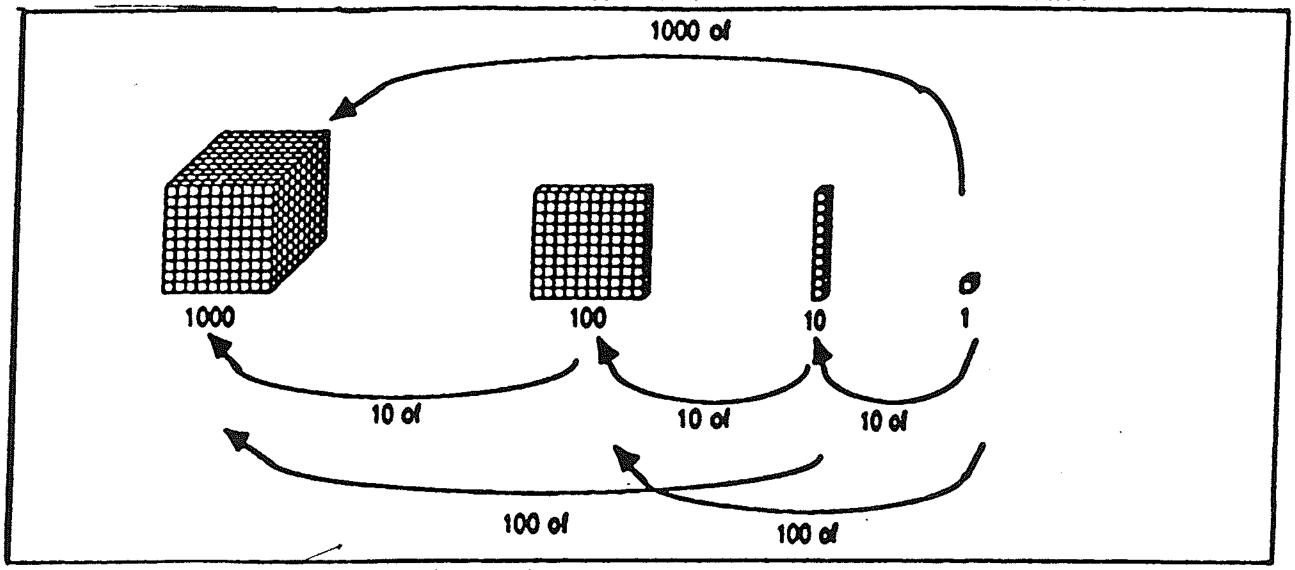
Students are in pairs. You are at the front of classroom, projected image of your mat is shown on projection screen.

Explain purpose of activity (1 minute)

That when we work with numbers, what we do should have meaning and make sense in terms of what the numbers stand for. We are going to do some review of adding and subtracting whole numbers before doing some more work with decimal numbers.

Making numbers (5 minutes)

- Discuss "ten of" relationship among the blocks. Do on overhead.



- Write **2413** on blackboard. Ask students what blocks we would use to show 2413.

Do not ask students how to make 2413 on the mats. Rather lead them to saying that 2413 would be made of 2 cubes (thousands), 4 flat(hundred), 1 long (tens), and 3 singles (ones).

- *"This is how we make numbers with the blocks."* Make 2413 by using blocks. Have students mimic your actions on their mats.

Point out the dual representational systems (blocks and digits) as you place blocks on the mat.

Carrying/Borrowing Demonstration (10 minutes)

You lead the discussion with your display. Students will mimic your actions when you tell them to.

- You borrow from the cubes.

Exchange "Thousands" for "Hundreds", "Hundreds for "Tens", and "Tens' for "Ones".

Refer to the blocks as "thousands", "hundreds", "tens" and "ones"

Ask whether the total number of singles has changed or has remained the same.

Make sure that they understand that you are referring to the singles contained in the longs, flats, and cubes and not just to "loose" singles.

- Write out the "digit" display at the top of the number region of the mat and show how it corresponds with the new configuration of blocks.

Say *"Before we exchanged one cube, the display read '4 flats' After we exchanged, it should read '14 flats'. Why should it change?"*

- Tell students, "Exchange one thousand on your mats."

Representing changes in blocks by changes in numeral

Discuss how one reflects changes in the blocks with corresponding changes in the numeral, such as by:

$$\overset{1}{\cancel{2}}\,\overset{14}{\cancel{4}}\,1\,3$$

Have students read numeral as "one cube, fourteen flats, one long, and three singles." Also have them read the numeral as "one thousand, fourteen hundreds, one ten, and three ones."

Stress that the number of singles represented by this numeral is still the same, just as the number of singles in the "blocks" diagram is still the same.

Say: Suppose we ask a farmer to deliver 2413 apples. He brings the apples in 1 box of 1000, 14 boxes of 100, 1 box of 10, and 3 loose apples. Did we get the correct number of apples?

The moral of this example is that that even though the numeral is "funny looking," it names the same number as does the "better" looking numeral 2413.

- Point out a "cube" digit. Do not exchange. Tell students you are going to exchange a cube.

Ask them to explain what you will do to the blocks.

Ask them to explain what the display in the "digits" area of the mat will be.

Do the exchange; write the new "digit display"

- Ask students to explain what it meant to exchange a block.

Make sure they explain in terms of actions on the blocks. Do not allow them to give their explanations only in terms of exchanging pieces, but also in terms of digits.

- Ask them to read the digit display; point out the correspondence between the digit display and the block representation of the number.

- Represent the result as:

0 24
~~1~~ ~~14~~
~~2~~ ~~4~~ 1 3

- Tell students to borrow from the cubes.

Have students read numeral as"no cubes, twenty-four flats, one long, and three singles" Also have them read the numeral as "twenty-four hundreds, one ten, and three ones."

Stress that the number of singles represented by this numeral is still the same, just as the number of singles in the "blocks" diagram is still the same.

- Point out a "flat" digit. Do not exchange. Tell students you are going to exchange a flat.

    - Ask them to predict what you will display in the "digits" area of the mat.

    - Ask them to predict what will happen to the blocks.

    - Exchange a flat and write the new "digit display"

- Ask them to read the digit display.

    - Represent the result as:

23
0 ~~24~~
~~1~~ ~~14~~ 11
~~2~~ ~~4~~ ~~1~~ 3

Have students read numeral as "no cubes, twenty-three flats, eleven longs, and three singles". Also have them read the numeral as "twenty-three hundreds, one ten, and three ones."

Stress that the number of singles represented by this numeral is still the same, just as the number of singles in the "blocks" diagram is still the same.

Transition

Four problems:

301-134 = ; Demonstration & Discussion (no recording)

403-125 = ; Students do with help (no recording)

Recording One's Actions and Results of Actions

Note to reader: One can adopt any set of conventions for recording actions performed on blocks-as-representations-of-quantities. We did not leave it entirely to students to create their own conventions, but we did emphasize that conventions aid and simplify communication, and that conventions also aid us in keeping track what we have done and what remains to be done.

We proposed a set of global conventions about how we write an initial statement of an arithmetic problem, and what it means to write something in a particular area of the initial setup. The global conventions we proposed are shown below. They were our attempt to give students freedom in creating their own schemes for recording actions performed on blocks, and yet were consistent with their prior experience with standard algorithms. Some students adopted these conventions straightforwardly; some made substantial modifications to them in creating their own recording schemes (see examples for Problem 1 of first homework).

2317 ← The amount you start with.

− 456 ← The amount you intend to remove.

[ ] ← The place where you write the number of blocks of a place value after you add/subtract blocks having that value.

2317, + 456 ← The amounts you intend to combine.

[ ] ← The place where you write the number of blocks of a place value after you add blocks having that value.

Always record the effects of carrying or borrowing in the areas containing the digits that represent the blocks affected by the action.

- Mention that we have "solved" these problems, but that it would be better if after solving a problem we could show anyone else how we did it without actually having to do it over with the blocks.

There are two kinds of actions one can perform–actions that change a quantity's *representation* and actions that change a quantity's *value*. Borrowing and carrying change a quantity's representation but do not change its value. Adding and subtracting change a quantity's value. Be sensitive to students' remarks that might indicate they have confounded these two very different notions of "change" in the display.

1432 - 445 = ___; Demonstration &Discussion (with recording)

Do it twice—once conventionally, once unconventionally—if time permits. *(Note to reader: SeeHomeowork Problem 1 for an example of what it means to record actions.)*

2014 - 1132 = ___; Students do with help (with recording)

- Stress that when two people solve problems differently, both of their solutions can be correct even though their solution methods might be very different. Solutions are correct or incorrect; solution *methods* are appropriate or inappropriate.

Explain assignment.

Say "Some children solved these four problems with blocks, but they did not record how they solved them. Your job is to do the recording for these children."

- Discuss first homework problem. Ask "What did Frank do?" for each of the first few steps. Record succesive changes in the display.

Stress to students that the task is for them to record *what these children did with the blocks*. Students may think (when they sit down at home to do their homework) that *they* are supposed to do the indicated subtractions using their already-known standard algorithms.

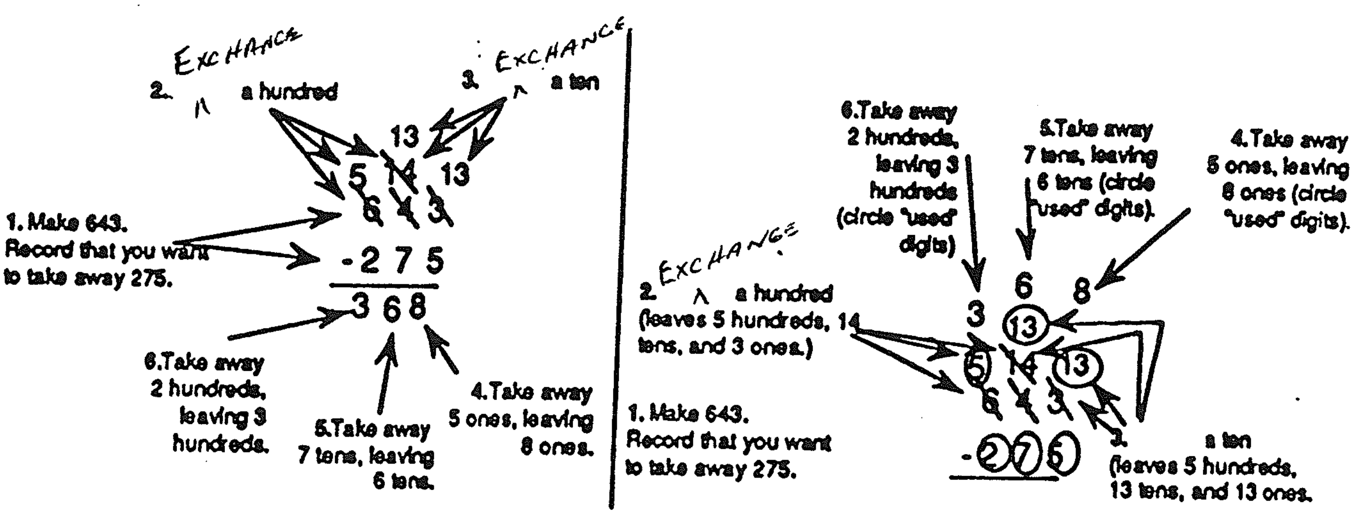
Homework 1

Frank solved the problem "643 - 275 = ___" using blocks, but he did not record his steps. What he did with the blocks is shown on the next page.

Use the setup below to record the steps in what Frank did with the blocks.

(Note to reader: Two ways in which students recorded Frank's "blocks" solution are shown below. These records reflect Frank's actions; the point made to students was that it is okay to do anything that makes sense when solving a subtraction problem with blocks. The main requirement is that one must record <u>each and every action taken</u>, whether it be a change of representation or a change of quantity.)

$$\begin{array}{r} 643 \\ \underline{-\ 275} \end{array}$$

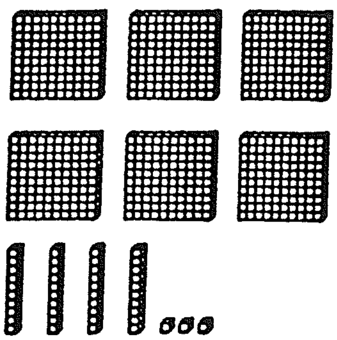


Problem 1

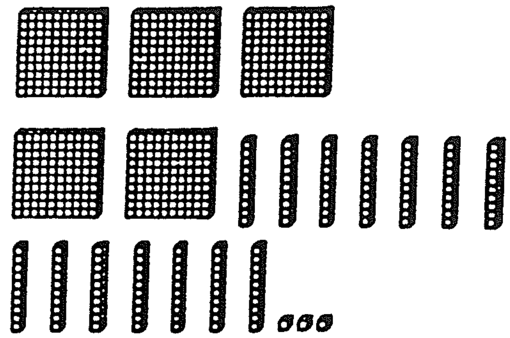
Frank solved the problem "643 - 275 = ___" using blocks, but he did not record his steps. What he did with the blocks is shown below. Use the setup at the right to record the steps in what Frank did with the blocks.

$$\begin{array}{r} 643 \\ -\ 275 \\ \hline \end{array}$$

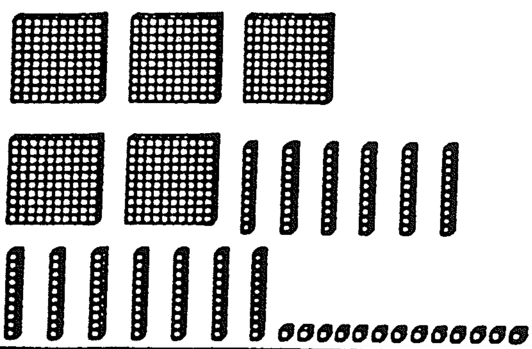
Step 1 of Frank's solution.



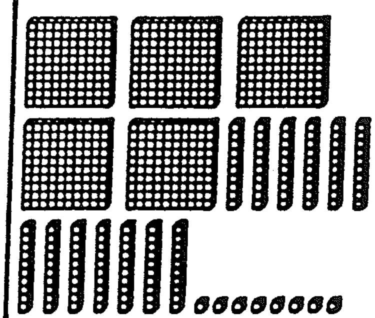
Step 2 of Frank's solution.



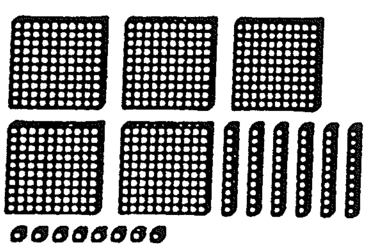
Step 3 of Frank's solution.



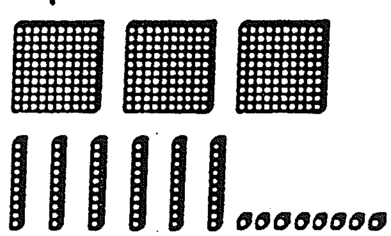
Step 4 of Frank's solution



Step 5 of Frank's solution



Step 6 of Frank's solution.

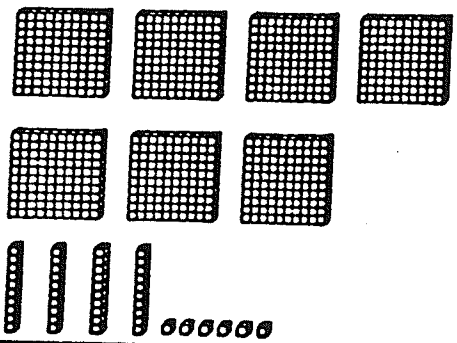


Problem 2

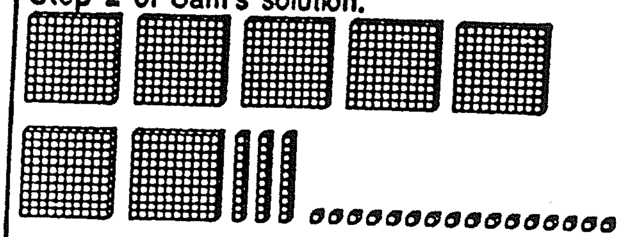
Samantha solved the problem "746 - 689 = ___" using blocks. What she did with the blocks is shown below. Use the setup at the right to record the steps in what Samantha did with the blocks.

$$\begin{array}{r} 746 \\ -\ 689 \\ \hline \end{array}$$

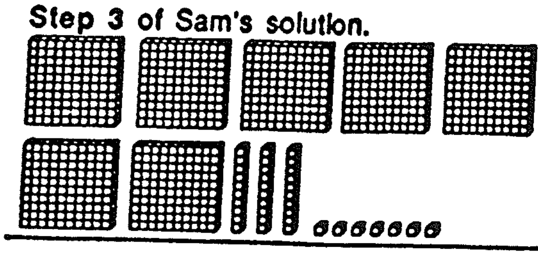
Step 1 of Sam's solution.



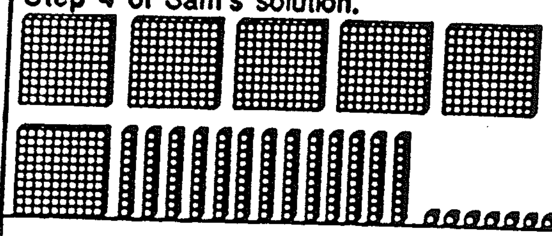
Step 2 of Sam's solution.



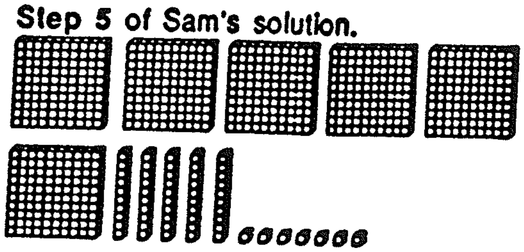
Step 3 of Sam's solution.



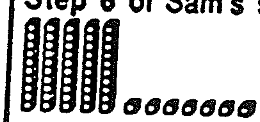
Step 4 of Sam's solution.



Step 5 of Sam's solution.



Step 6 of Sam's solution.

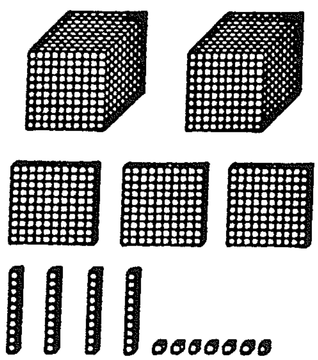


Problem 3

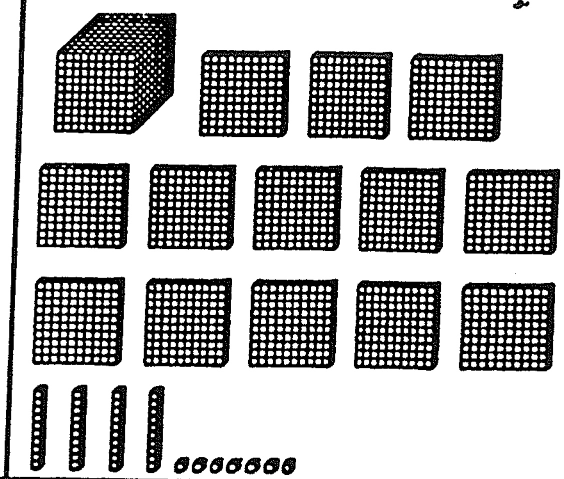
Jimmy solved the problem "2347 - 958 = ___" using blocks. What he did with the blocks is shown below. Use the setup at the right to record the steps in what Jimmy did with the blocks.

$$\begin{array}{r} 2347 \\ -\ \ 958 \\ \hline \end{array}$$

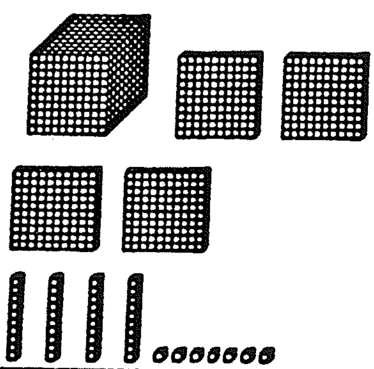
Step 1 of Jimmy's solution



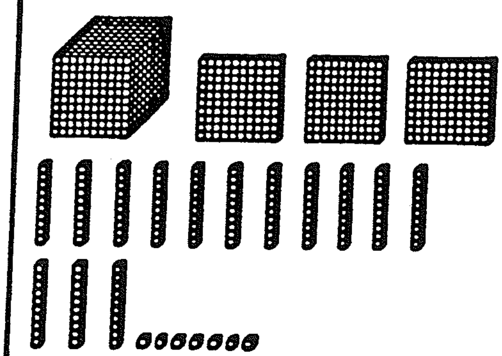
Step 2 of Jimmy's solution



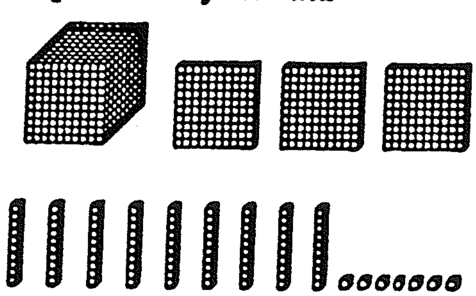
Step 3 of Jimmy's solution



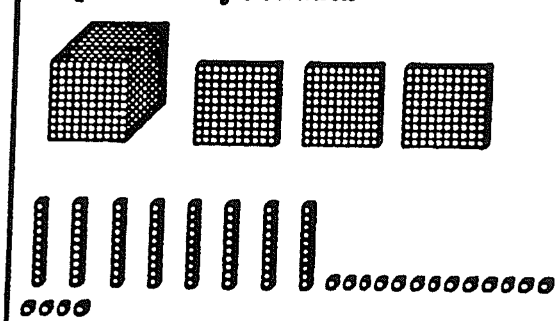
Step 4 of Jimmy's solution



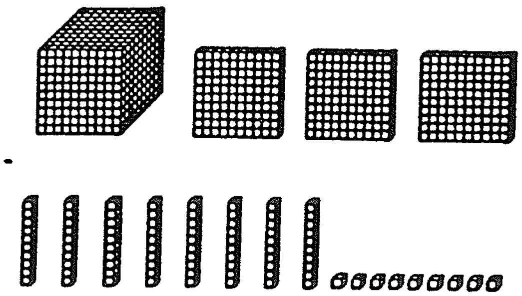
Step 5 of Jimmy's solution



Step 6 of Jimmy's solution



Step 7 of Jimmy's solution

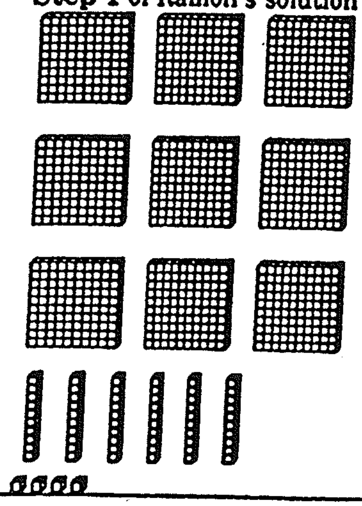


Problem 4

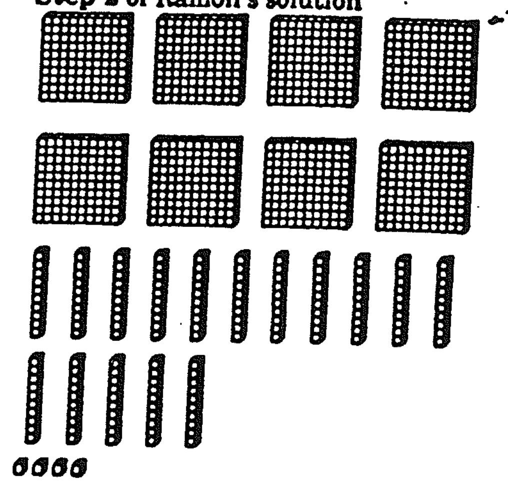
Ramon solved the problem "964 - 475 = ___" using blocks. What he did with the blocks is shown below. Use the setup on the right to record the steps in what Ramon did with the blocks.

$$\begin{array}{r} 964 \\ -\ \underline{475} \end{array}$$

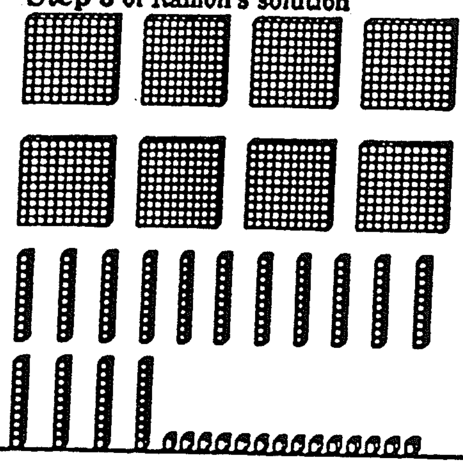
Step 1 of Ramon's solution



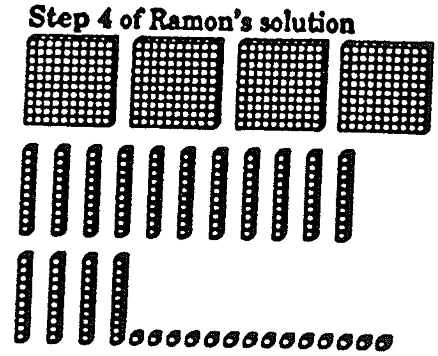
Step 2 of Ramon's solution



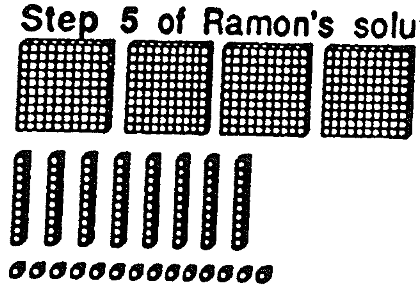
Step 3 of Ramon's solution



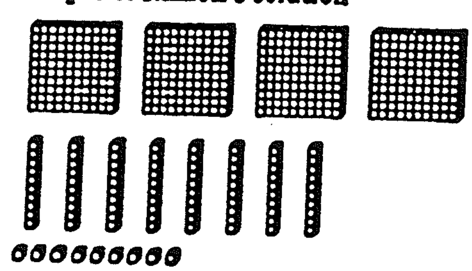
Step 4 of Ramon's solution



Step 5 of Ramon's solution



Step 6 of Ramon's solution



Problem 5

Use Frank's method to solve 2358 - 1429 = ____

Use Samantha's method to solve 2358 - 1429 = ____

Use Jimmy's method to solve 2358 - 1429 = ____

Use Ramon's method to solve 2358 - 1429 = ____

Day 3

Go over previous day's homework.

Have students use red pencils while going over homework. *Collect homework.*

> *Be attentive for this confusion: Students may have thought that they were supposed to do the subtraction, and may have completely ignored the "fictitious" children's solution methods. If anyone did this, have him or her do the problems over again (as homework), in addition to the homework assigned for today. Give them a fresh homework sheet. Keep their original homework.*

Discuss what it means to add

-Make 132 in left region of the mat; record the number.
-Make 61 in right region of the mat; record the number.
-Say that to add the numbers represented by these blocks means to combine the blocks.

> *Emphasize that we can combine two groups of blocks by simply thinking of them as going together to make one group. When we combine two groups of blocks, we get a number of ones, a number of tens, and a number of hundreds in the new group.*
>
> *Record the number.*
>
> *The main idea to get across is that addition means to combine. The next example will show that the idea of "carrying" comes up <u>after one has already combined two numbers of things (i.e. after one has combined groups of blocks).</u>*
>
> *Point out that an alternative to combining the two quantities all at once is to combine them a little at a time. For example, you could have first taken the ones from the right region to the left region, then have taken the tens from the right region to the left region. This is the conventional method.*

Exemplify addition with Blocks

Make 864 in the left region. Record the number.
Make 137 in the right region. Record the number.

Tell students that you are going to combine (but don't do it yet).

Ask them to predict:
What the digit display should have in it (it will display the total number of flats, the total number of longs, and the total number of singles).

Record the total # of flats; total # of longs; total # of singles.

Say: Are we done adding? (Yes. But, the children will probably say no because the digits aren't "right." If they don't say this, then provoke the point by pretending that you yourself aren't sure that you are done adding.)

Ask: whether the total number of singles is the correct total number of singles that you should have after adding. (Yes, it is. So you must be done adding).

[Here is an analogy: Suppose we are going to order 1001 apples from a farmer. If we ask for 9 boxes with 100 apples each, 9 boxes with 10 apples each, and 11 loose apples, we would have the 1001 apples that we desired.]

But we do not have the total number of singles represented in the conventional way that other people expect total numbers to be represented. (Which is to have all digits being between 0 and 9 inclusive.)

Thus we need to fix the representation. Therefore, we need to trade (or exchange).

-Exchange "singles"

Ask whether you are done (i.e., done putting the sum of 864 and 137 into a "conventional" format; you aren't done - there now are 10 tens.)

Say that you want to "carry" the tens so that there are fewer than ten of them.

Ask How do I carry the tens?

Show how to exchange ten "Longs"(tens) for a "Flat" (Hundred).

Record # of flats; # of longs; # of singles, (It should say 10 Flats 0 Longs 1 Single.)

Repeat for "Flats". (Exchange ten flats for a cube)

Now record.

Students solve problem on their mats.

-Have students solve 496 + 505 =

Point out that they are done "adding" as soon as they combine. But, they are not done representing the solution in a "conventional" format until they have finished making all digits less than 10.

Relate actions with blocks to algorithms with numerals.

This next example is to make connections between solving the problem with blocks and how we record solutions on a "set up" representation of an addition problem using numerals.

- Write this problem on the board:

$$\begin{array}{r} 6\ 4\ 5 \\ +\ \underline{1\ 6\ 6} \end{array}$$

Say that you are going to do this problem with blocks, but that you are also going to record your actions on the "setup" numerals. This way, we can explain to someone else how we did this addition without having actually to show them with blocks.

- On the mat: Put 645 in one region; put 166 in the other. Then add (combine).

- Record the digits display. The result should look like this:

$$\begin{array}{r} 6\ \ 4\ \ 5 \\ +\ \underline{1\ \ 6\ \ 6} \\ 7\ 10\ 11 \end{array}$$

Read this result as "645" plus "166 " is 7 hundreds, 10 tens, and 11 ones."

Point out that we normally write a small "tens" digit in a column so that no one gets confused as to what we are representing. Were we to write all digits the same size in a column, it could become confusing, like this:

$$\begin{array}{r} 6\ 4\ 5 \\ +\ \underline{1\ 6\ 6} \\ 71011 \end{array}$$

- Go through successive steps of "conventionalizing" the sum. Record the display each time you carry.

Students solve and record solution

- Hand out "463 + 648 = ____" problem page.
  Have students solve and record 463 + 648 = _____.

Assign Homework.

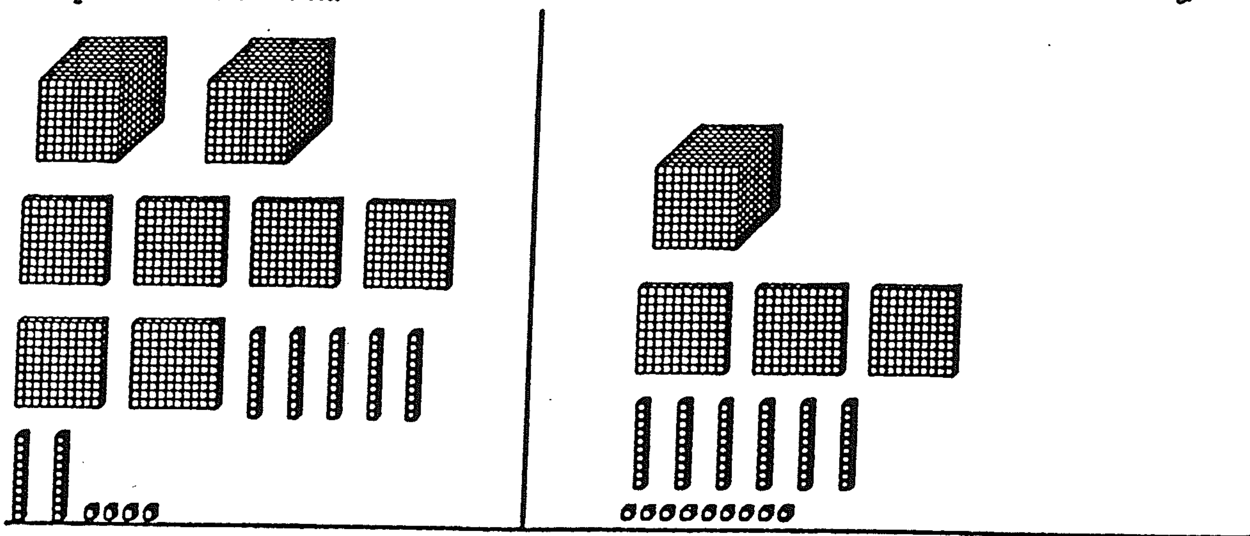
$$\begin{array}{r} 463 \\ +\underline{648} \end{array}$$

Tom's solution to 2674 + 1369 = ____

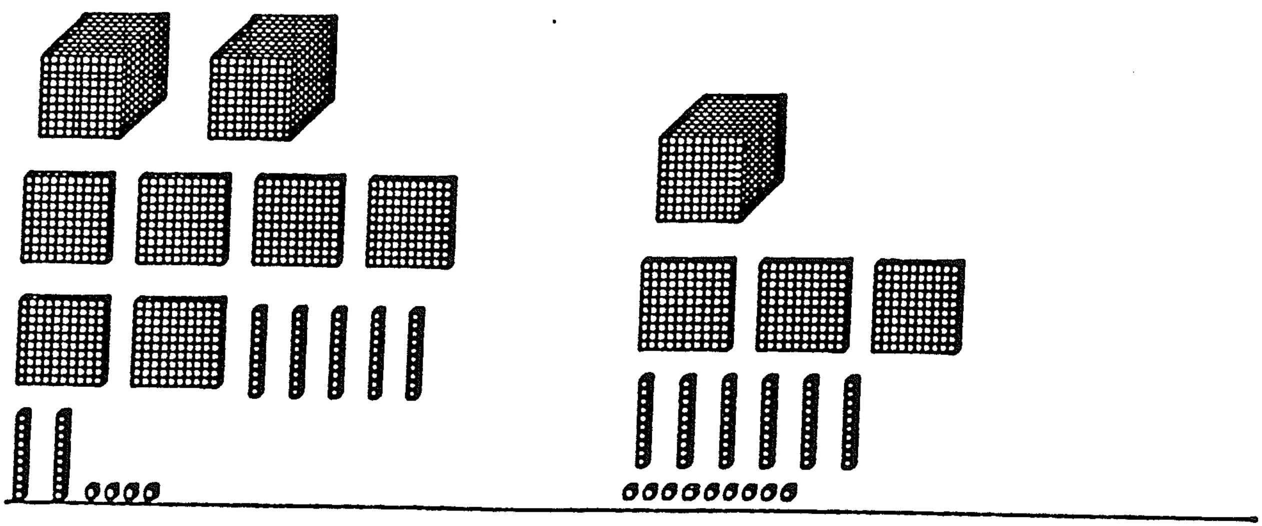
Tom solved the problem "2674 + 1369 = ____" using blocks. What he did with the blocks is shown below and on the next page. Use the setup at the right to record the steps in what Tom did with the blocks.

$$\begin{array}{r} 2674 \\ +\ 1369 \\ \hline \end{array}$$

Step 1 of Tom's Solution.



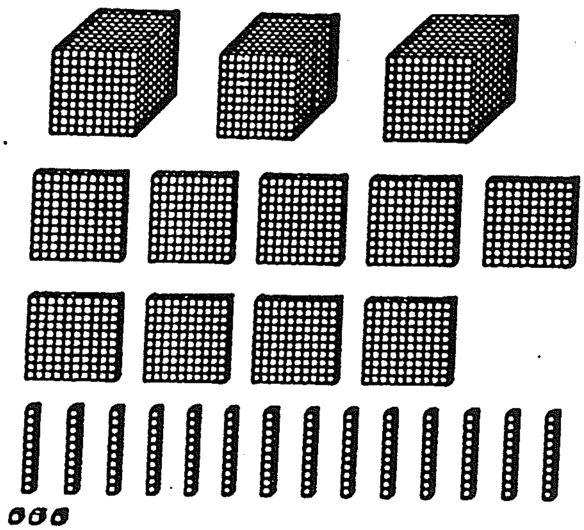
Step 2 of Tom's Solution.



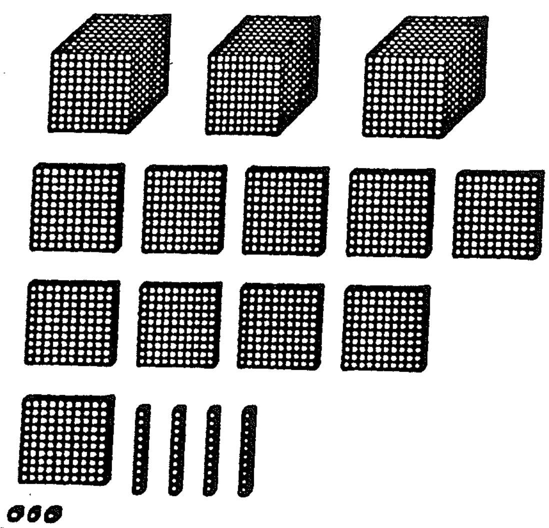
(Steps continued on next page)

Tom's solution to 2674 + 1369 = ____ (continued)

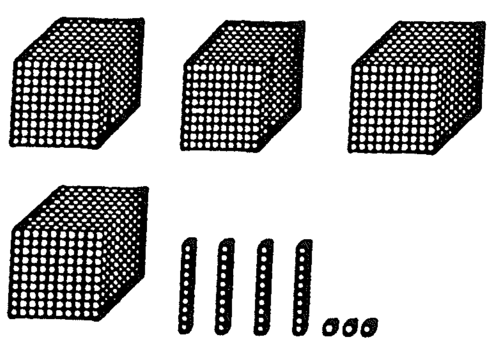
Step 3 of Tom's Solution.



Step 4 of Tom's Solution.



Step 5 of Tom's Solution.

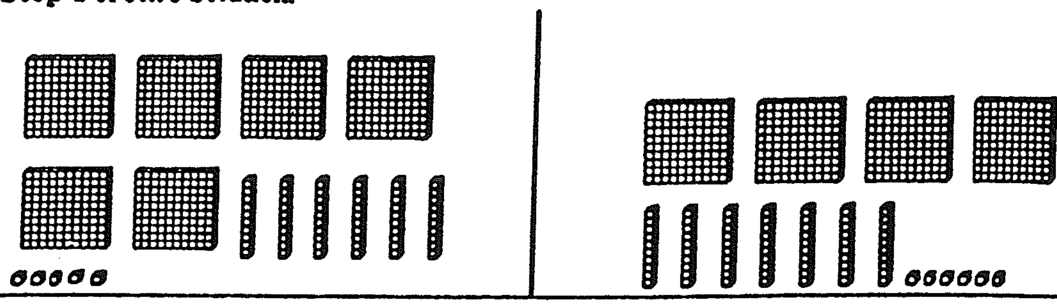


Jill's Solution to "665 + 476 = ____"

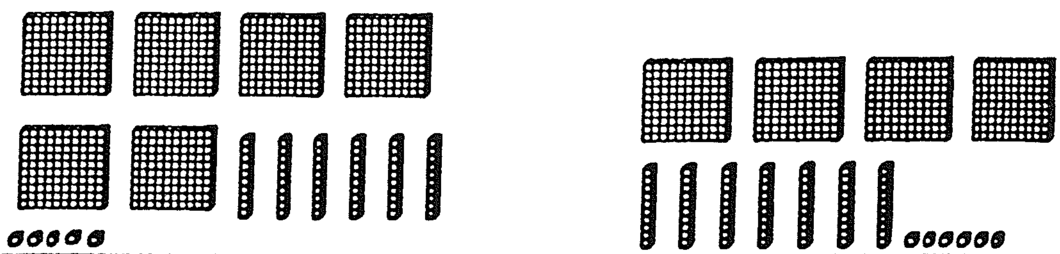
Jill solved the problem "665 + 476 = ____" using blocks. What she did with the blocks is shown below. Use the setup at the right to record the steps in what Jill did with the blocks.

$$\begin{array}{r} 665 \\ +\underline{\;476} \end{array}$$

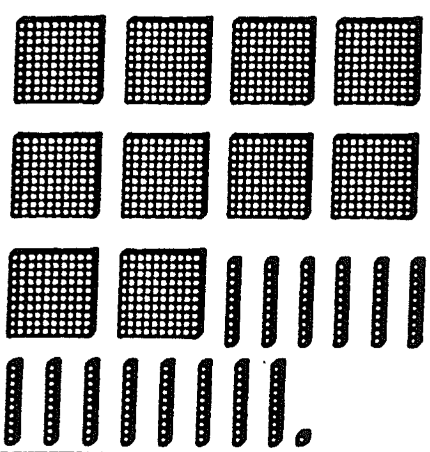
Step 1 of Jill's Solution.



Step 2 of Jill's Solution.

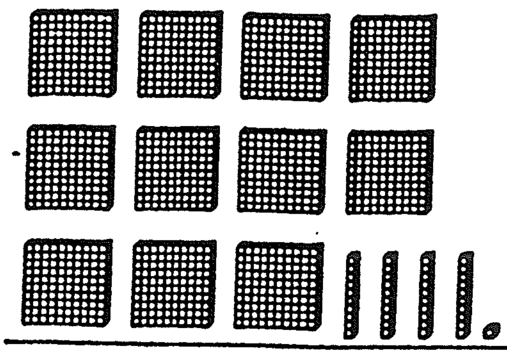


Step 3 of Jill's Solution.

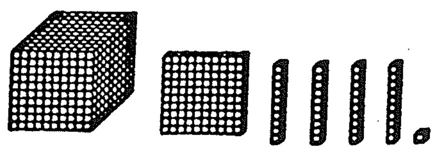


(Steps continued on next page)

Step 4 of Jill's Solution.



Step 5 of Jill's Solution.



Day 4

Go over previous day's homework.

- Have students use red pencils while going over homework. Collect homework.

Introduction to base-ten decimal numeration

Ask: How many flats are in a cube? (10)
How many hundreds are in one thousand? (10)

How many longs are in a flat? (10)
How many tens are in one hundred? (10)

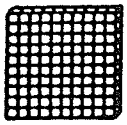
How many longs are in a cube? (100)

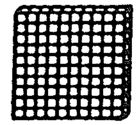
How many tens are in one thousand? (100)

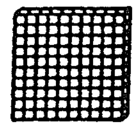
- Remind students of Marianne on the test that they took. Marianne liked to think of different blocks as being one instead of a single as being one.

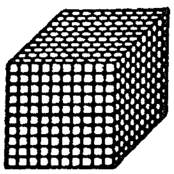
Put transparencies of the following figure on the projector. Discuss one at a time.

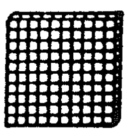
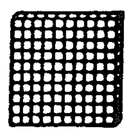
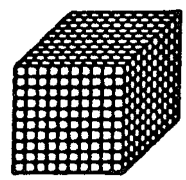
If  **is one, then**  **is $\frac{1}{10}$, since there are 10 of**  **in one**  **.**

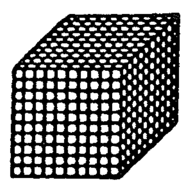
If  is one, then

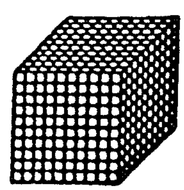
 is $\frac{1}{10}$, since there are ten of  in one .

and  is $\frac{1}{100}$, since there are 100 of  in one .

If  is one, then

 is $\frac{1}{10}$, since there are ten of  in one .

 is $\frac{1}{100}$, since there are 100 of  in one .

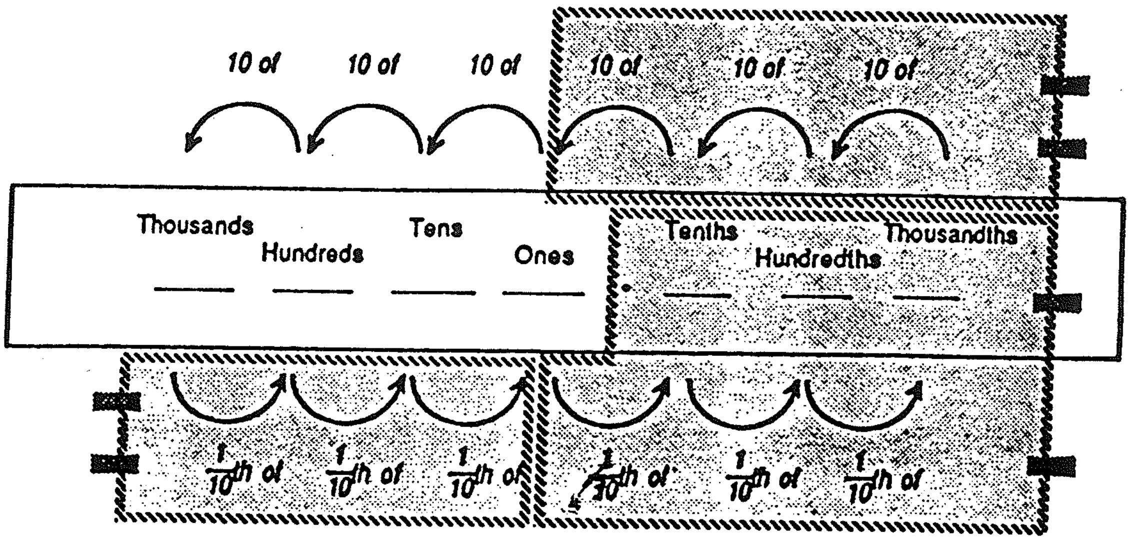
and  is $\frac{1}{1000}$, since there are 1000 of  in one .

Decimal place value

- Introduce students to decimal numeration notation. Say that the way we normally write numbers contains an idea that we can use to write what we have just said about fractions.

Put transparency of following figure on the projector

All flaps should be down.



- Things to point out about this scheme:

  — A *digit* in a place tells us how many of those "value groups" we have.

  — Each *place* is worth ten times as much as the one to its right.

  — The *number* represented by a digit in a place need not be 10 times greater than the number represented by the digit to its right. (In "234", 2 hundreds is not ten times greater than 3 tens. But a hundred is 10 times greater than a ten.) ← *Omit if running long on time.*

Ask How many ones in one ten? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

How many tens are in one hundred? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

How many hundreds are in one thousand? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

Lift bottom left flap.

— Each *place* is worth $\frac{1}{10}$th as much as the one to its left.

Say There are ten hundreds in one thousand. So one hundred is $\frac{1}{10}$th of one thousand.

Point at arrow. Say that this diagram shows what you just said.

Ask How many tens in one hundred?

One ten is what part of one hundred? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

How many ones in one ten?

One one is what part of ten? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

Lift bottom right flap.

Say We can keep going to the right of the one's place. But, we need to make a mark to show where the one's place is, because the right-most place in a numeral need not be the one's place when we use places to the right of one. This dot is called a decimal point. It separates the ones place from the tenths place.

Ask Since each place is worth one-tenth of the place to its left, what value should the place to the right of one have? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

Note that they have seen a decimal point in money before. It separates the "one dollar" place fromthe "tenth of a dollar" place.

[Continue with hundredths and thousandths.]

Lift top right flap.

Ask How many thousandths in one hundredth? *Point at arrow after they have answered. Say that this diagram shows what they just said.*

Say It does not matter where a digit is. Its place value is 10 times the place value of the digit to its right and its place value is $\frac{1}{10}$th the place value of the digit to its left. (Point at arrows while saying this.)

- Tell students to make "ten and 11 hundredths."

Tell them that if they are clever, they will need to use only three blocks (viz., a cube, a long, and a single). Let them think about that.

- Hand out activity sheet (they will do these at their mats). Tell students that when they make these numbers, they will need to decide what block represents one. Different problems may require different blocks to stand for one.

Assign Homework.

Non-computer Group

Make these numbers *Fill in the blanks after you make a number*

4.30 __ Cubes __ Flats __ Longs __ Singles *(Put a decimal point between the ones and the tenths.)*

6.03 __ Cubes __ Flats __ Longs __ Singles *(Put a decimal point between the ones and the tenths.)*

12.1 __ Cubes __ Flats __ Longs __ Singles *(Put a decimal point between the ones and the tenths.)*

3.01 __ Cubes __ Flats __ Longs __ Singles *(Put a decimal point between the ones and the tenths.)*

1.032 __ Cubes __ Flats __ Longs __ Singles *(Put a decimal point between the ones and the tenths.)*

Tear the last two pages off. They have pictures of blocks that you will use to do your homework. Cut pictures of blocks from those pages when you need them.

Show 3.41 with blocks in two ways. Use different units.

Pick a block that will be the unit in your representation of 3.41. Paste a picture of that block into a "unit box". Then paste pictures of blocks to show 3.41.

Unit Box	*Unit Box*
Put a unit here — is the unit.	Put a unit here — is the unit.
3.41	3.41
First way	*Second Way*

Show 12.7 with blocks in two ways. Use different units.

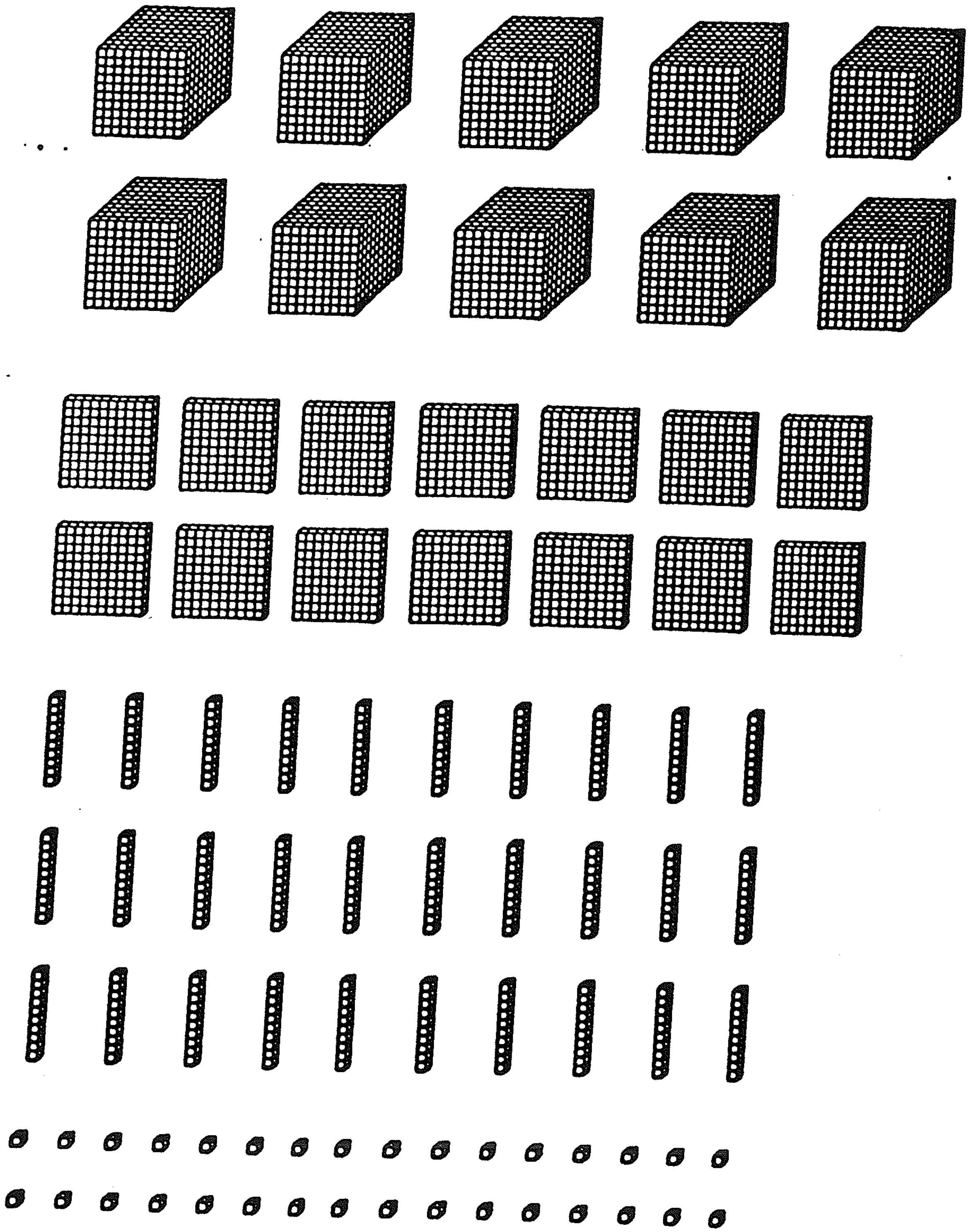
Pick a block that will be the unit in your representation of 12.7. Paste a picture of that block into a "unit box". Then paste pictures of blocks to show 12.7.

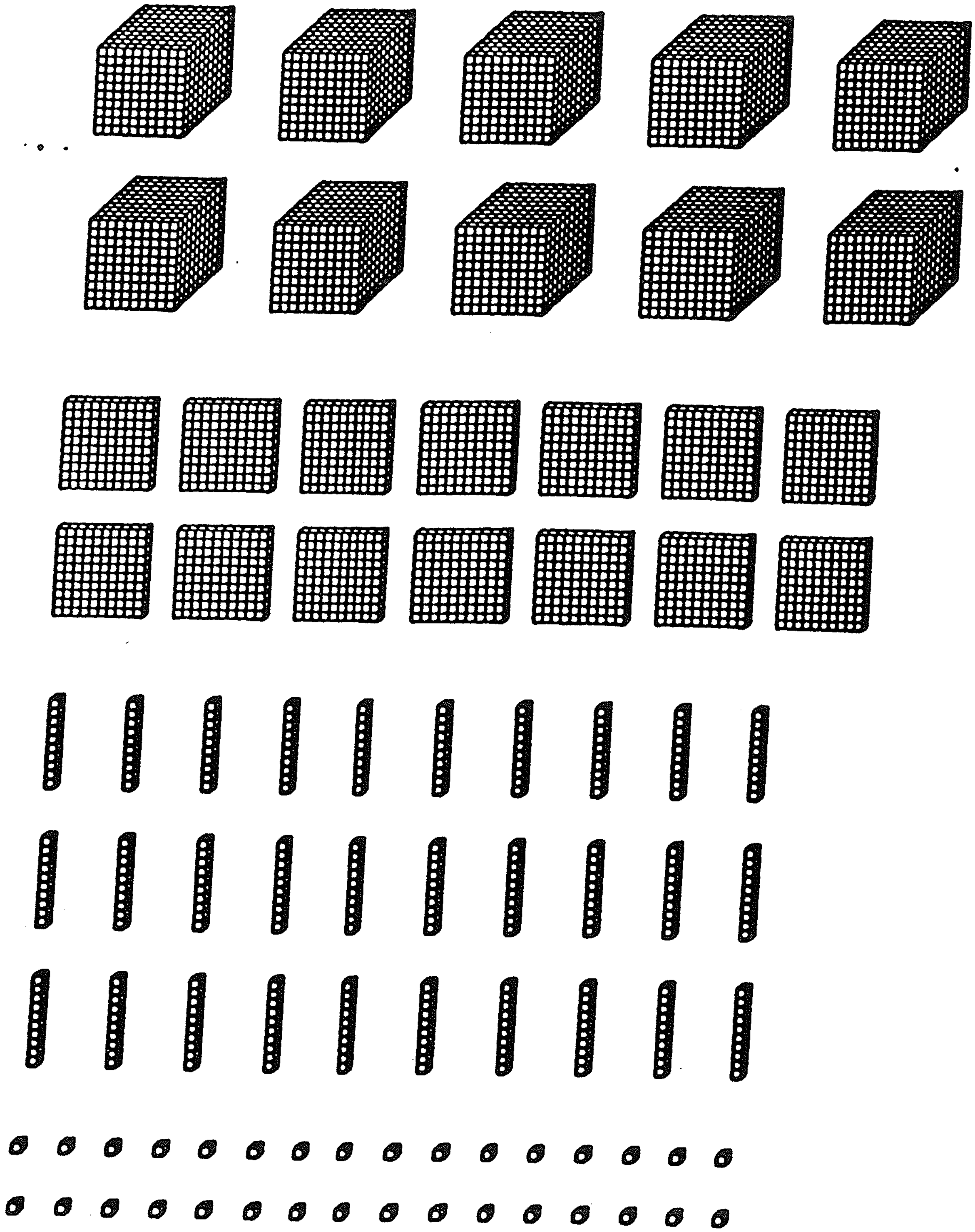
First way	Second Way
Unit Box Put a unit here is the unit. 12.7	*Unit Box* Put a unit here is the unit. 12.7

Show 11.32 with blocks in two ways. Use different units.

Pick a block that will be the unit in your representation of 11.32. Paste a picture of that block into a "unit box". Then paste pictures of blocks to show 11.32.

Unit Box	*Unit Box*
Put a unit here — **is the unit.**	**Put a unit here** — **is the unit.**
11.32	11.32
First way	*Second Way*





Day 5

Go over homework from previous day.

Hand out red pencils.

- Be brief in going over homework. Give emphasis to the selection of a unit and, and especially give emphasis to the relationship between choice of unit and what number the blocks represent.

Once a unit is chosen, values of all other blocks are determined by the relationships "ten of" and "1/10 th of."

Changing representations without changing the number

Have students use their mats. They should have pencils and papers with them.

- Tell students to select either the flat or the cube as their unit (it is their choice).

Write **4.32** on the blackboard.

Say Make "four and thirty-two hundredths" on your mats.

(After they are done)

Write four and thirty-two hundredths on a sheet of paper.

We are going to do some exchanging, and your job will be to record on paper what you do with the blocks; make "four and thirty-two hundredths" on their mats.

Tell students to exchange a tenth.

Be sure to use the language of "tenths", "hundredths", and "thousandths". Let students translate tenths, hundredths, and thousandths into the language of blocks.

- Have students read the digit display in ones, tenths, and hundredths.

Stress that we want to know how many ones, tenths, etc.: 4 ones, 2 tenths, and 12 hundredths. Emphasize that the number that they originally represented has not changed.

Say Record on your paper the change you made in the blocks.

Write $4.\overset{2}{\cancel{3}}\overset{12}{\cancel{2}}$ (after students have finished writing on their papers).

- Tell students to EXCHANGE [redacted] a one.
- Have students read the digit display in ones, tenths, and hundredths: 3 ones, 12 tenths, and 12 hundredths.

Say Record on your paper the change you made in the blocks.

Write $\overset{3}{\cancel{4}}.\overset{12}{\cancel{3}}\ \overset{12}{\cancel{2}}$ (after students have finished writing on their papers).

Say This shows that 4 ones, 3 tenths, and 2 hundredths represents the same number as is represented by 3 ones, 12 tenths, and 12 hundredths.

Point out similarities between decimal numbers and whole numbers

Say the following (don't make it a matter of lengthy discussion):

Think about what stands for what.

Borrow whenever you do not have enough of what you want to take away from.

Record on the numeral any changes in the blocks.

Subtraction of decimal numbers

Tell students to select the flat as their unit.

- Write this on the board: **4.02 - .5 = ____**
  Have students solve this problem *without recording.*

After they have solved it, ask students what was hard about this problem. (I expect that the hard part will be deciding and keeping in mind what stands for what.)

- Tell students to [redacted] CLEAR THEIR MATS

Tell students to write this on a piece of paper:
(write it just like it appears here)

$$\begin{array}{r} 10.4 \\ -\ \underline{.62} \end{array}$$

☞ Ask students if it would make any difference if we wrote it like this:

$$\begin{array}{l} 10.4 \\ -\ \underline{.62} \end{array}$$

(It wouldn't matter if we paid close attention to place-value of digits; it would matter if we did not pay close attention.) Bring out the point that we align digits that have the same place value merely as a matter of convenience—so that when we subtracting digits, we know that we are subtracting tens from tens, ones from ones, tenths from tenths, and so on. *We align the decimal points to ensure that digits with the same place value are in the same columns.*

- Have students solve "10.4 - .62 = ____" *(with recording).*

Discuss solution briefly when they are finished.

☞ Speak about actions on blocks (using the language of tens, ones, tenths, and hundreds) and record actions on numerals. *But don't make any reference to the numeral.*

Hand out activity sheet (next page)

Tell students to do these problems any way they wish. They can do them with or without using blocks. If they do them without using blocks, then they should imagine that they *are* using blocks. In either case, tell them to record on the numerals changes they make (or imagine making) in the blocks.

Assign homework

Solve these problems.

You will need to decide what block is your unit in each problem.

You can do these problems with or without using blocks. *It is your choice.*

In either case, record on the numbers whatever you do (or imagine doing) with the blocks.

$$\begin{array}{r} 2.31 \\ -\underline{.43} \end{array}$$

$$\begin{array}{r} 1.01 \\ -\underline{.002} \end{array}$$

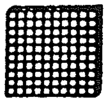
Problem 1

James solved the problem "42.41 - 17.56 = _____" using blocks. What he did with the blocks is shown on the next two pages.

Use the setup below to record the steps in what James did with the blocks.

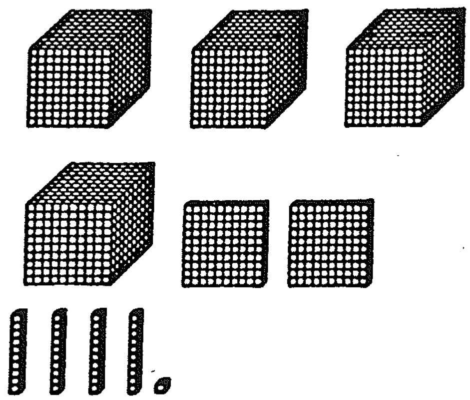
$$\begin{array}{r} 42.41 \\ -\ 17.56 \\ \hline \end{array}$$

James' Solution to "42.41 - 17.56 = ___"

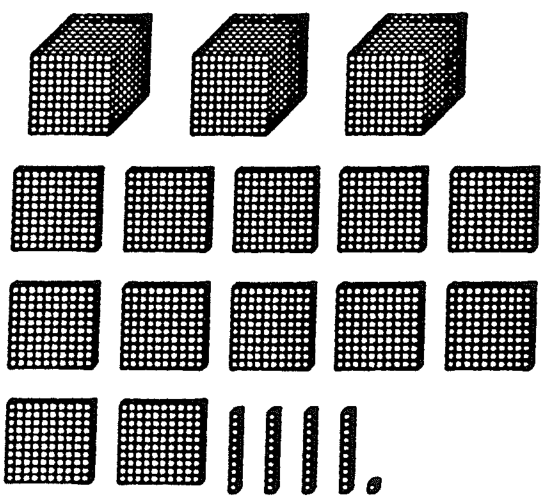


Is the unit.

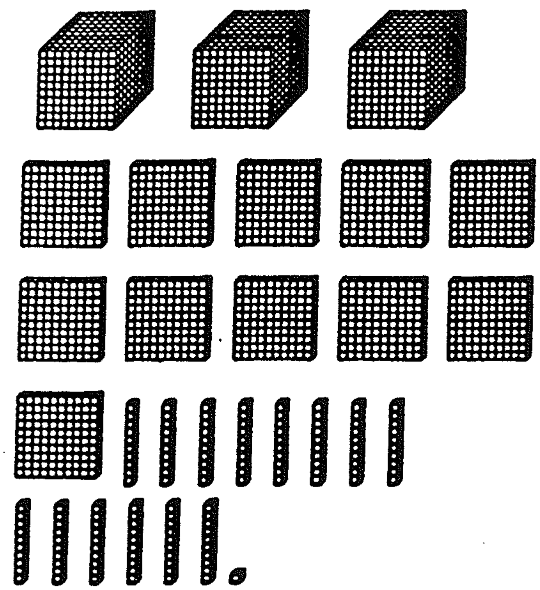
Step 1 of James' Solution.



Step 2 of James' Solution

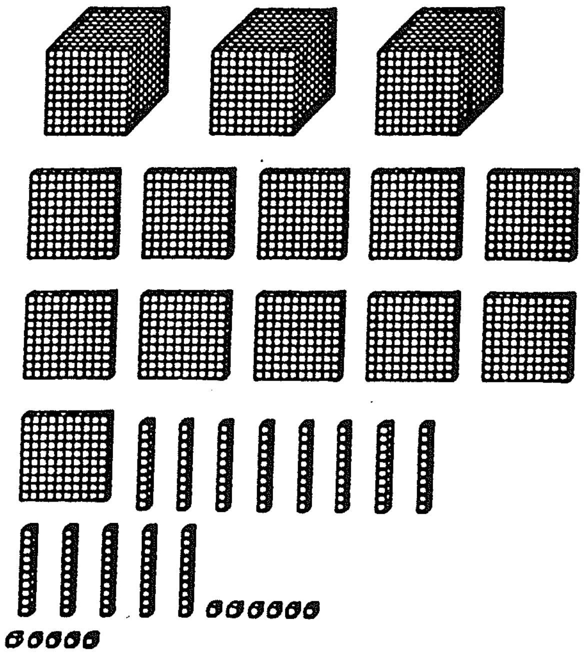


Step 3 of James' Solution.

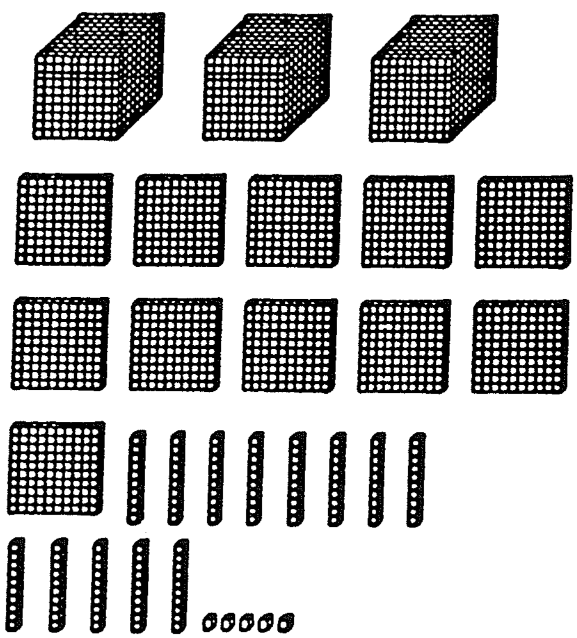


James' Solution to "42.41 - 17.56 = ____"

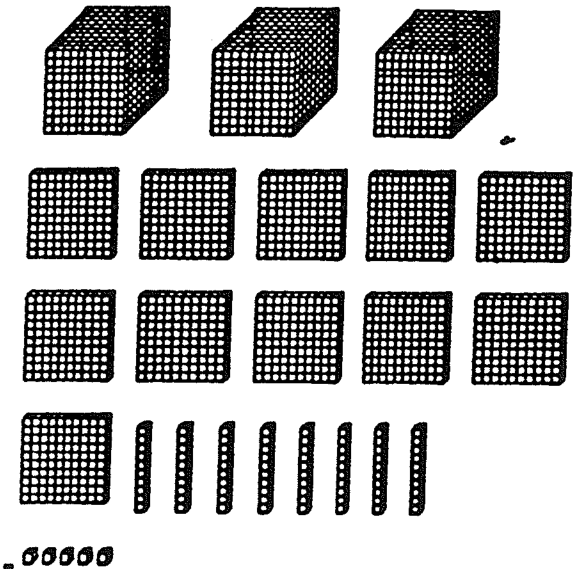
Step 4 of James' Solution.



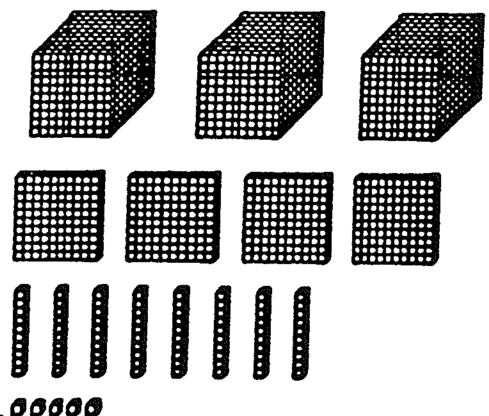
Step 5 of James' Solution.



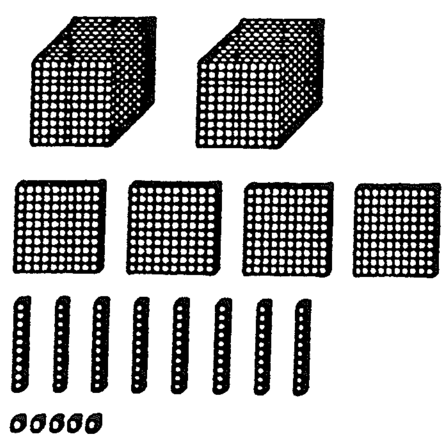
Step 6 of James' Solution



Step 7 of James' Solution



Step 8 of James' Solution



Problem 2

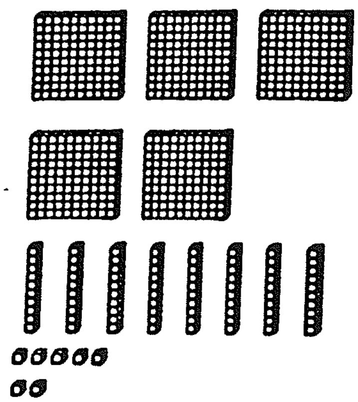
Sally solved the problem "58.7 - 49.8 = ______" using blocks. What she did with the blocks is shown on the next pages.

Use the setup below to record the steps in what Sally did with the blocks.

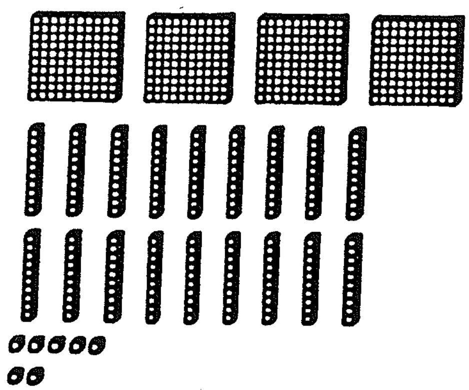
$$\begin{array}{r} 58.7 \\ -\ \underline{49.8} \end{array}$$

 is the unit.

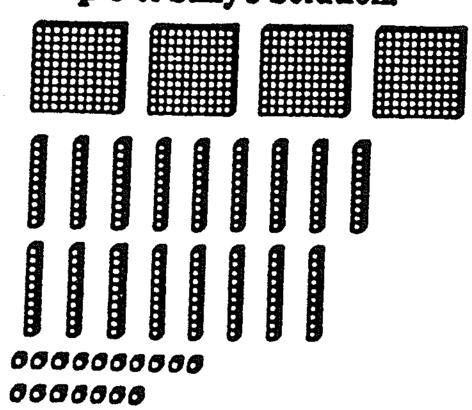
Step 1 of Sally's Solution



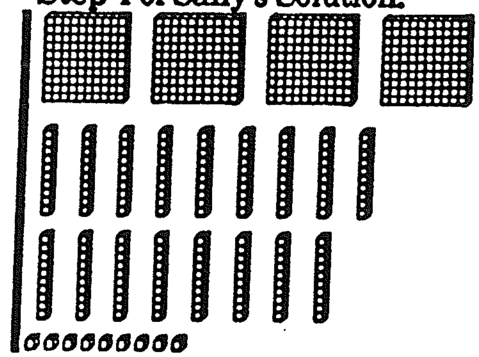
Step 2 of Sally's Solution.



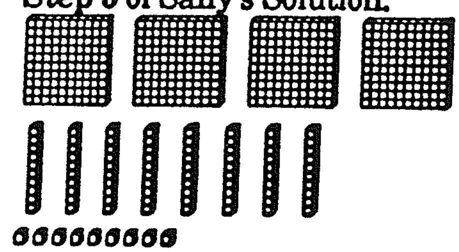
Step 3 of Sally's Solution.



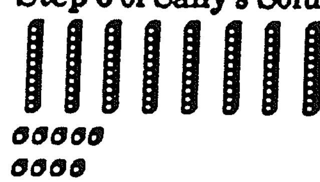
Step 4 of Sally's Solution.



Step 5 of Sally's Solution.



Step 6 of Sally's Solution.



Day 6

Go over previous day's homework briefly.

At the end, ask students if there is one particular method that these children used that they like best. That they liked least. (the point of this discussion is that they reflect on the methods; the point is not to settle on a prescribed method.)

Tell them that if they like a method, they can feel free to use it themselves..

Addition of decimal numbers.

- Remind them that to *add* means to *combine*.

- Remind them that when we work with a numeral, it is to *record* what we do with the numbers that blocks stand for.

- Give students a copy of the next sheet. Tell them to solve these problems with blocks, and to record their solutions on the numerals

They will need to pick an appropriate unit for each problem. This is part of the problem.

- When students are finished, have each pair pick *one* problem to write on the board. Discuss differences among the recording methods.

Differences among recording methods should be reflective of different methods of solving problems with blocks. Make sure that this comes out in the discussion.

Assign homework.

Do these addition problems with blocks. Record on this paper what you do with the blocks.

You will need to decide what block represents one in each problem.

$$\begin{array}{r} 11.37 \\ +\underline{\ \ 8.64} \end{array}$$

$$\begin{array}{r} 32.6 \\ +\underline{\ 28.51} \end{array}$$

$$\begin{array}{l} \ 6.4 \\ +\underline{\ 4.812} \end{array}$$

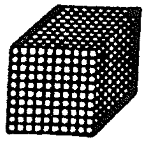
$$\begin{array}{l} \ 1.111 \\ +\underline{\ 9.99} \end{array}$$

Problem 1

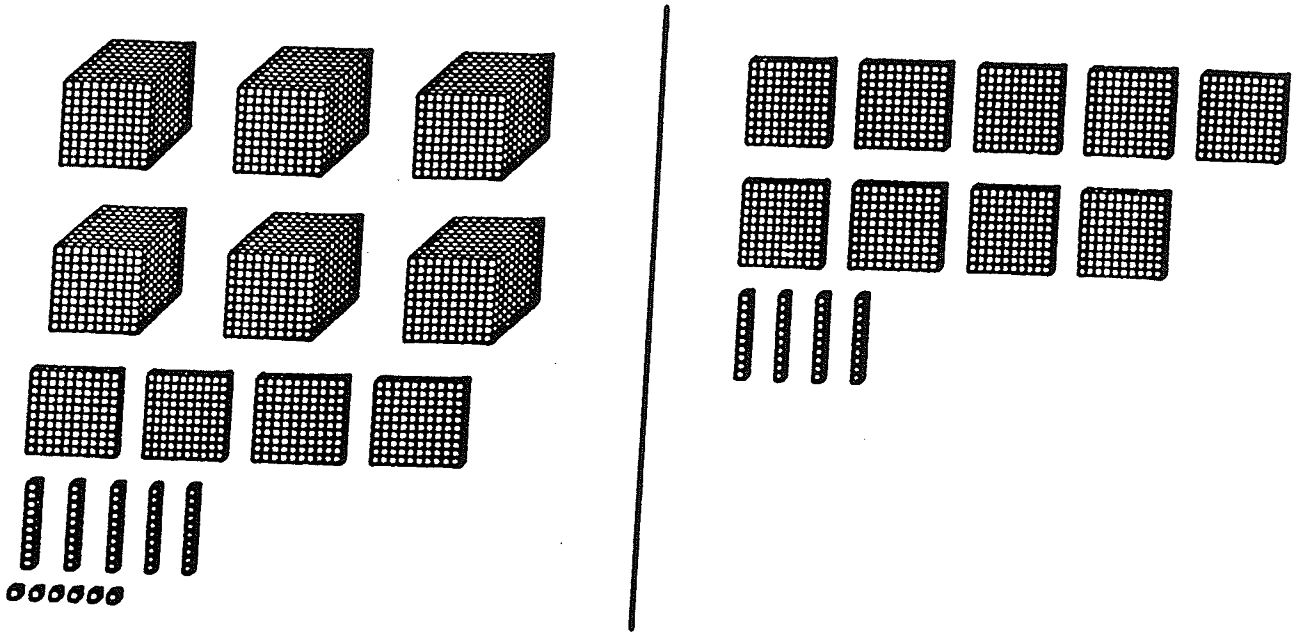
Jamie solved the problem "6.456 + .94 = ____" using blocks. What she did with the blocks is shown on the next two pages.

Use this setup to record the steps in what Jamie did with the blocks.

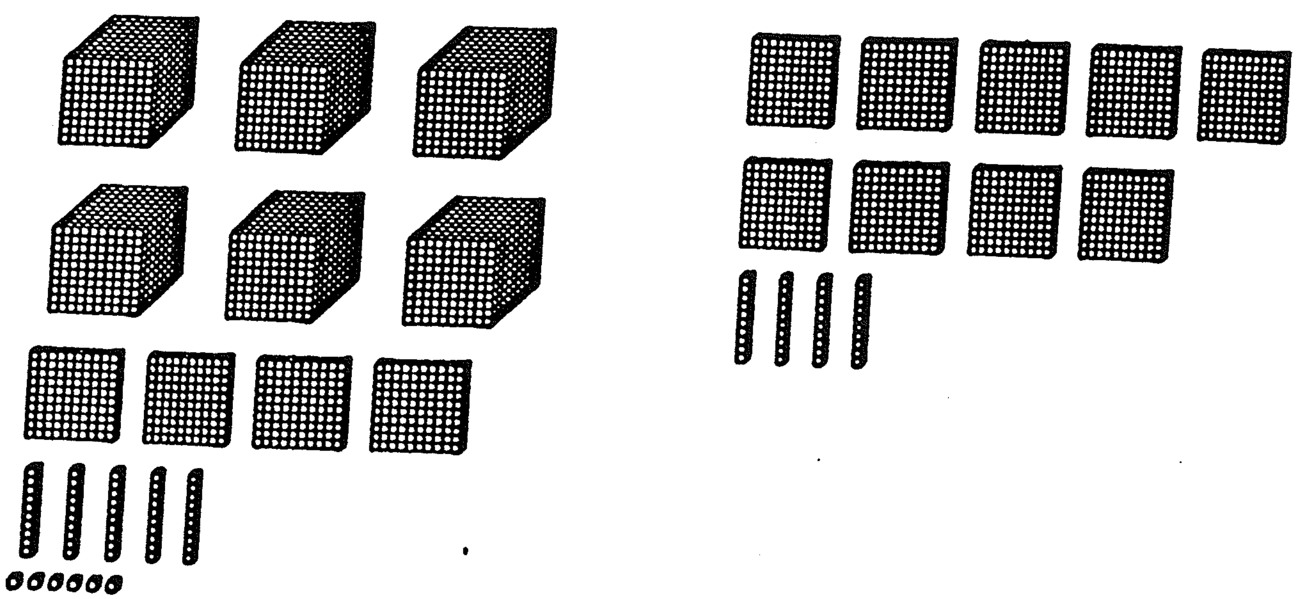
$$\begin{array}{r} 6.456 \\ \square \quad .94 \\ \hline \end{array}$$

 is the unit.

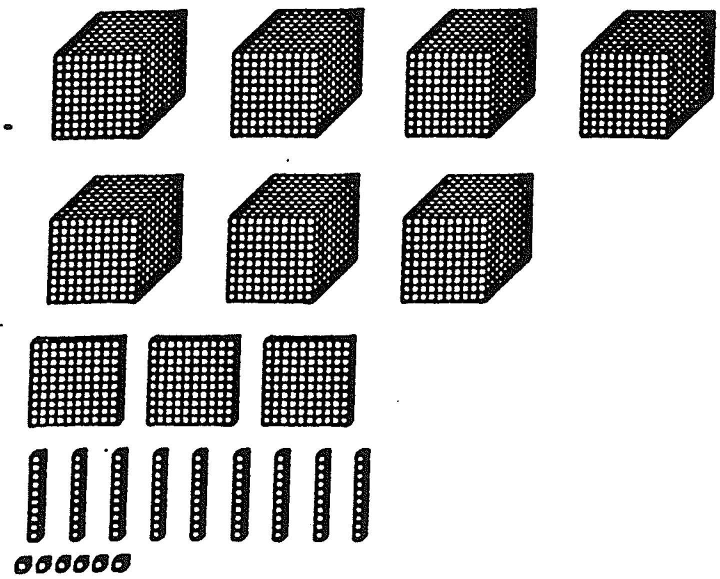
Step 1 of Jamie's Solution.



Step 2 of Jamies Solution.



Step 3 of Jamie's Solution.

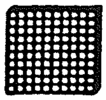


Problem 2

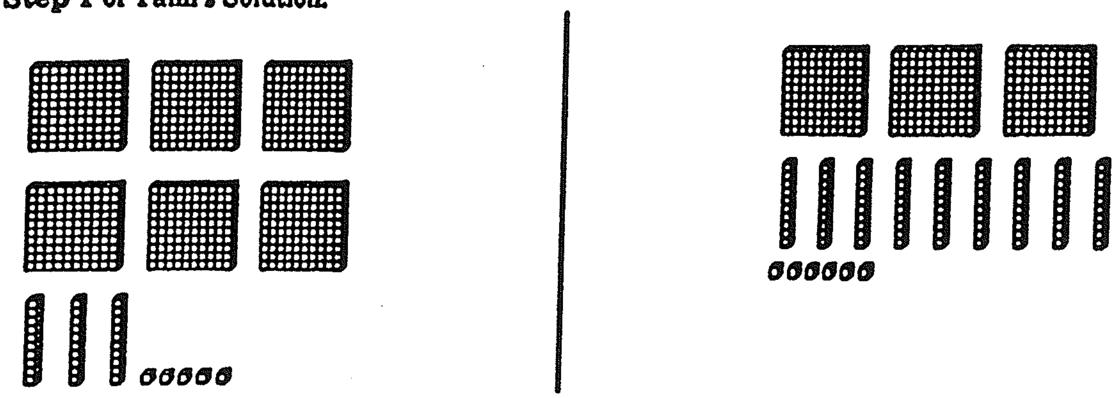
Tami solved the problem "6.35 + 3.96 = ____" using blocks. What she did with the blocks is shown on the next two pages.

Use this setup to record the steps in what Tami did with the blocks.

$$\begin{array}{r} 6.35 \\ \square\ \underline{3.96} \end{array}$$

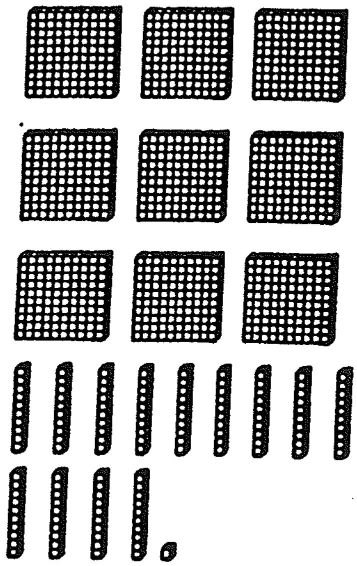
 is the unit.

Step 1 of Tami's Solution.

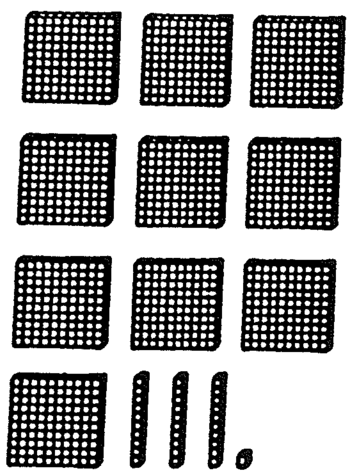


Step 2 of Tami's Solution.

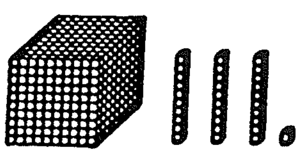
Step 3 of Tami's Solution.



Step 4 of Tami's Solution.



Step 5 of Tami's Solution.



Day 7

Go over previous day's homework briefly

Assign activity sheet for mixed practice (next page).

Mixed Practice

Try to do these problems without blocks.

If you become confused while doing a problem, then think about what you would do with the blocks.

If thinking about the blocks doesn't help, then use real blocks.

Show your work on this paper.

$$\begin{array}{r} 15.4 \\ +\underline{16.81} \end{array}$$

$1748 + 2253 =$ ______

$6.02 - 5.4 =$ ______

$$\begin{array}{r} 5.001 \\ -\underline{\ \ 1.82} \end{array}$$

$$\begin{array}{r} 5.08 \\ -\underline{\ 2.93} \end{array}$$

Appendix C

Pretest/Posttest

This is the pretest/posttest used for the study. It is similar to the test used by Dr. Patrick Thompson in his study. All items on the test were obtained from Dr. Patrick Thompson and were used in this study with his permission.

Question #8 was modified slightly at the request of the Research and Ethics Committee.

Any duplication of the test in whole or in part must be authorized by Dr. Thompson.

Decimal Number Project

Your Name: ____

This set of questions is designed to help us understand what you learned about whole numbers and decimal fractions.

Please read each question carefully before you begin it.

Do all your work on these pages. *Do not erase your scratch work.* Instead, cross out your mistaken scratch work when you need to correct an error.

Wait for your teacher to tell you to begin.

1. Each fraction in the left column is the same as one decimal number in the right column.

   Draw a line between each fraction and the decimal number that it is the same as. *Some decimal numbers might end up not having any lines drawn to them. Some might have more than one line drawn to them.*

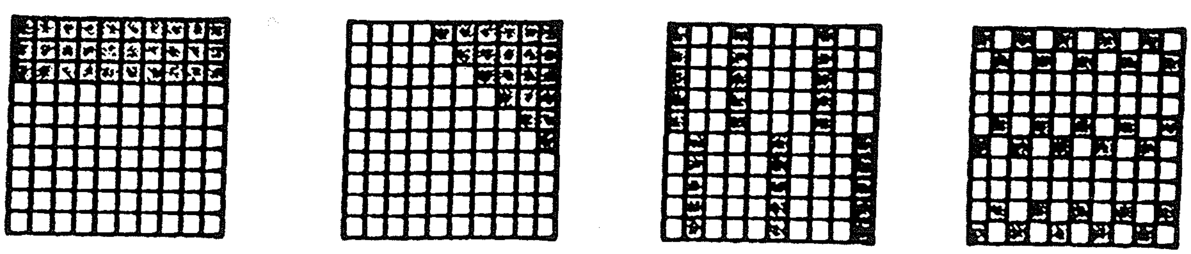
$\frac{1}{2}$	.10
$\frac{1}{4}$	.21
$\frac{3}{4}$	.25
$\frac{5}{10}$	.30
$\frac{7}{10}$	.40
$\frac{21}{30}$	.50
$\frac{25}{100}$	.70
	.75
	.80
	1.0

2. A group of 2000 people has broken into subgroups, with 20 people in every subgroup.

The people in each subgroup make up ____ of the total group. *Circle your choice to fill the blank.*

$\frac{1}{10}$ $\frac{1}{20}$ $\frac{1}{100}$ $\frac{1}{200}$ $\frac{1}{1000}$ $\frac{1}{2000}$

3. Put a circle around each picture that has $\frac{3}{10}$ of its area shaded.



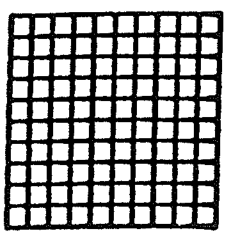
4. Each line has three numbers. Circle the largest number in each line.

(a) $\frac{15}{75}$ .123 .0947

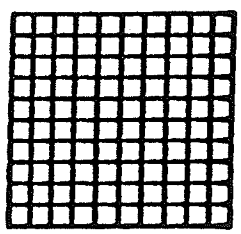
(b) $3\frac{1}{4}$ 3.7198 3.72

(c) 8.35 .972 $8\frac{1}{2}$

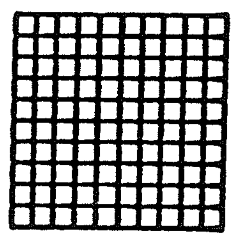
5. The grids shown below are each 10 squares by 10 squares. Shade each grid so that the shaded part goes with the fraction below the grid.



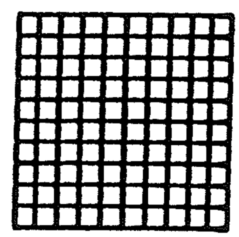
$\frac{3}{10}$



$\frac{2}{5}$



$\frac{5}{20}$



$\frac{41}{100}$

6. Calculate the answers to each of these arithmetic problems:

3004 - 286 = ____

10.02 - 1.034 = ____

7914 +2648 = ____

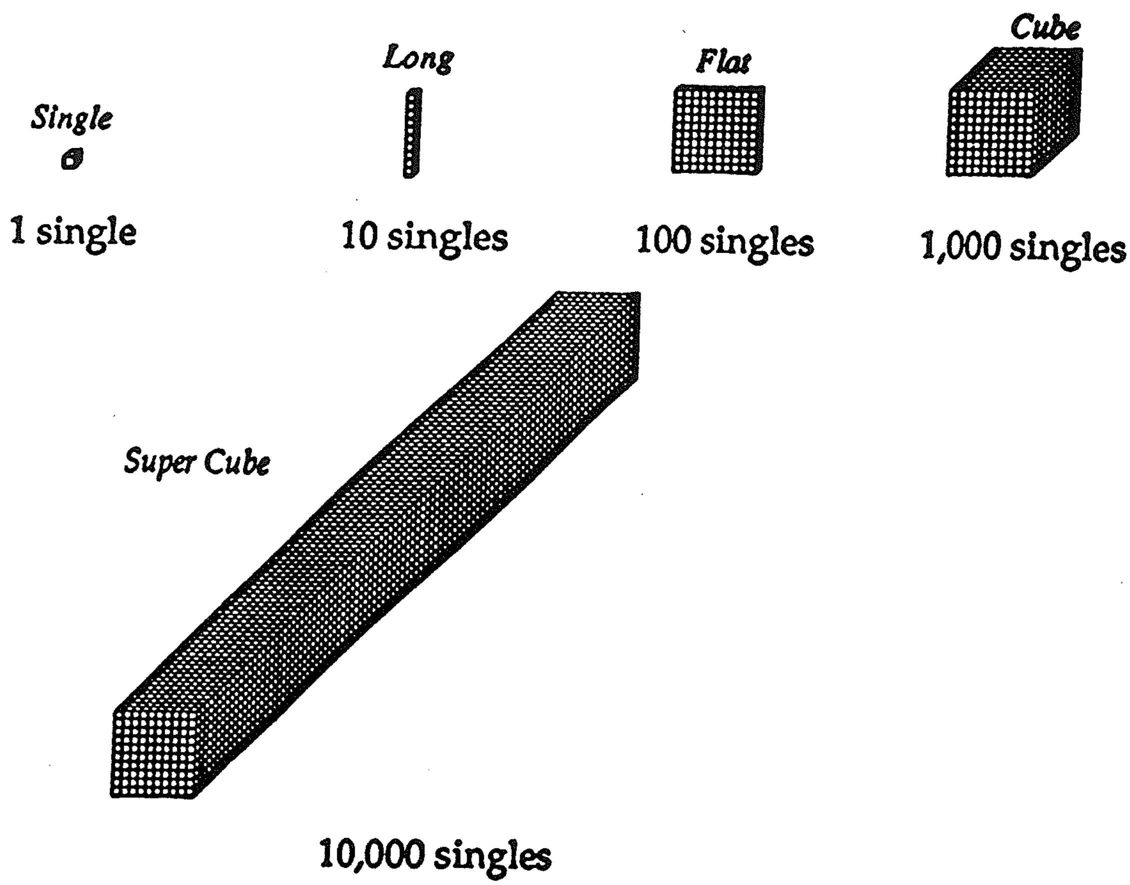
.642 + .89 = ____

5002 - 493 = ____

1.020 - .35 = ____

7. The pictures shown below are of blocks. Larger blocks are made of "single" blocks glued together.

Underneath each block is a number. The numbers show how many "singles" are inside the blocks.



Marianne did not like to count singles. She liked to count the cubes (1,000 singles). In the way Marianne thinks of the blocks, a cube stands for the number 1.

Pretend that you are Marianne, and that the cube stands for the number 1.

a. Since a cube stands for 1, what does a super cube stand for?

Answer: ______________________

(Question 5, continued from previous page)

b. Since a cube stands for 1, what does a long stand for?

Answer: ______________________

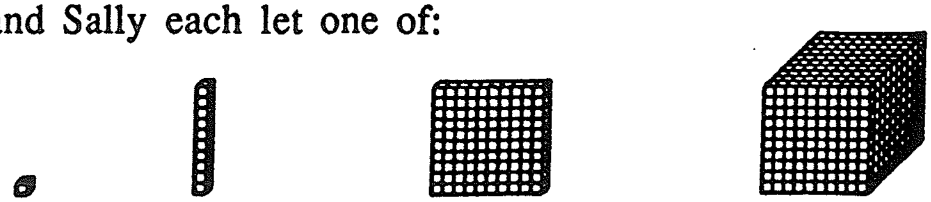
c. Since a cube stands for 1, what does a single stand for?

Answer: ______________________

d. Since a cube stands for 1, what does a flat stand for?

Answer: ______________________

8. Tom and Sally each let one of:

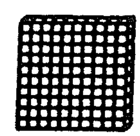


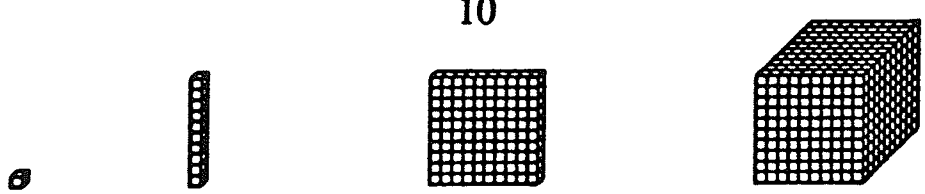
stand for the number 1. Their friend, James, put out some blocks. Tom said that James' blocks made 4.025. Sally said James' blocks made 402.5

a. Who required more singles to make 1 (one)?

Tom Sally (Circle one)

Explain your answer:

b. If Sally's choice for the number 1 was a  , then what would Tom say is $\frac{1}{10}$? (Circle your choice)



Explain your answer:

9.a Do this subtraction problem:

$$\begin{array}{r} 2.000000001 \\ -\underline{1.97989} \end{array}$$

.b Do this subtraction problem:

$$\begin{array}{r} 2.001 \\ -\underline{1.97989} \end{array}$$

10. Circle "Same" if an item describes a number that is the same as 4.325.
Circle "Different" if an item describes a number that is different from 4.325.
Circle "Can't Tell" if you cannot tell if the number is the same or different.

$4 + \frac{32}{100} + \frac{5}{1000}$	*Same*	*Different*	*Can't Tell*
Four ones, three tens, two hundreds and five thousands	*Same*	*Different*	*Can't Tell*
$4\frac{325}{1000}$	*Same*	*Different*	*Can't Tell*
Four ones, three tenths, two hundredths, and five thousandths	*Same*	*Different*	*Can't Tell*
$3.^{1}31^{1}5$	*Same*	*Different*	*Can't Tell*

11. Calculate the answers to each of these arithmetic problems:

$$\begin{array}{r} 12.27 \\ +\underline{\ 5.84} \end{array} \qquad\qquad \begin{array}{r} 8.03 \\ -\underline{\ 2.9\ \ } \end{array}$$

$7.31 - 6.4 =$ ______ $\qquad$ $14.8 + 7.23 =$ ______

12. Circle "Yes" if you think a statement is true.
Circle "No" if you think a statement is not true.
Circle "Don't Know" if you cannot decide if a statement is true or false.

7.89 is smaller than 7.9	*Yes*	*No*	*Don't Know*
12.30 is the same as 12.03	*Yes*	*No*	*Don't Know*
This is the RIGHT way to add 8276 and 4185. Other ways might give the same answer, but they are not the right way:	*Yes*	*No*	*Don't Know*

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 8276 \\ +\ 4185 \\ \hline 12461 \end{array}$$

13. Tom and Sally each solved 45.32 - 32.37. Tom's work is shown on the left. Sally's work is shown on the right.

Put a circle around whomever's work is correct.

Put an X through whomever's work is incorrect.

Tom	*Sally*
$$\begin{array}{r} 12 \\ 45.3\cancel{2} \\ -32.37 \\ \hline 1\cancel{3}.\cancel{0}5 \\ 2\ \ \cancel{10} \\ 9 \end{array}$$	$$\begin{array}{r} 14\ \ 12 \\ 3\ \cancel{15}\ \cancel{13}12 \\ \cancel{4}\cancel{5}.\cancel{3}\cancel{2} \\ -32.37 \\ \hline \overset{1}{\cancel{2}}.95 \\ 12 \end{array}$$

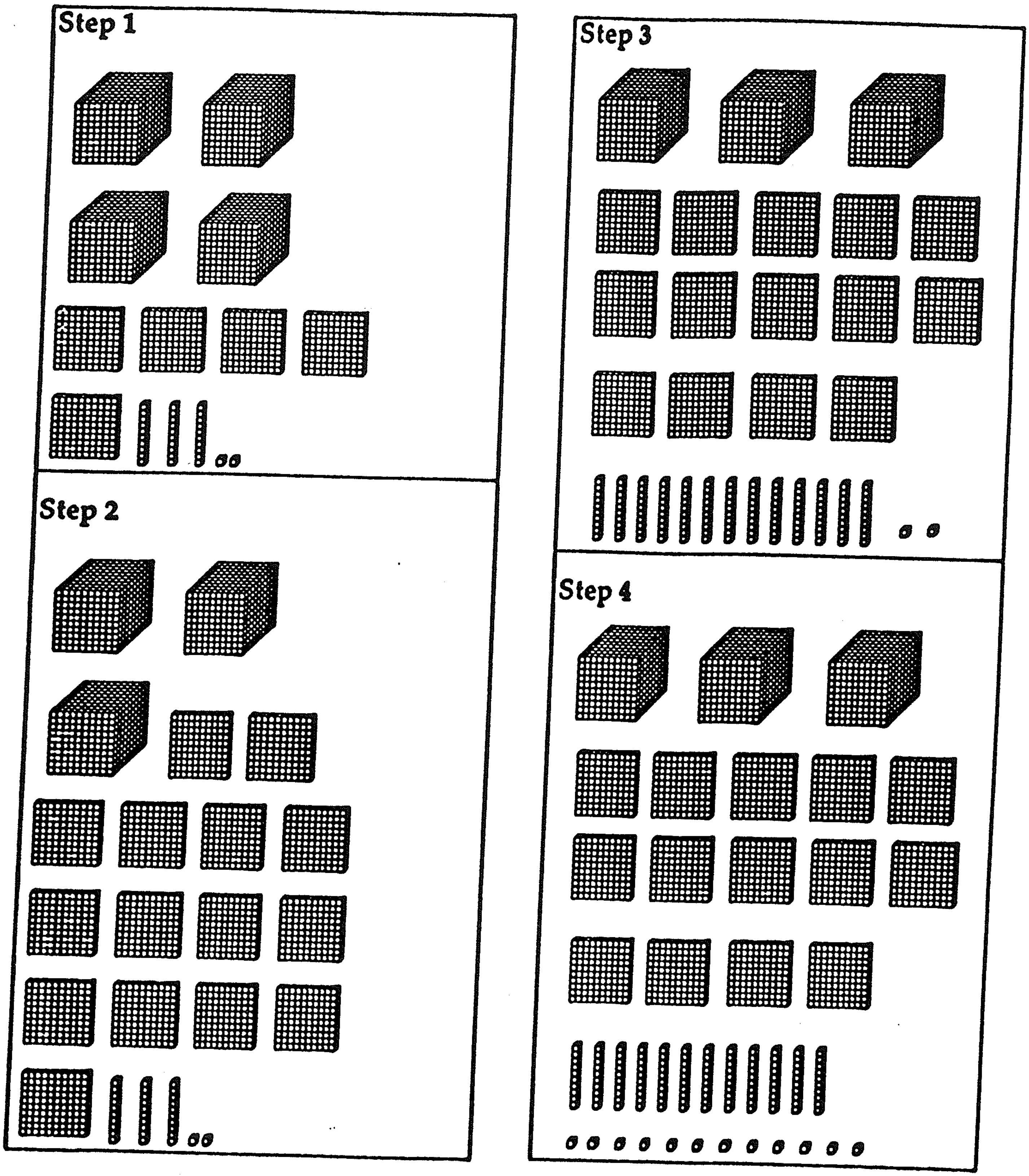
b. Tom and Sally actually solved 45.32 - 32.37 using blocks. Their work, as shown above, was done as they recorded their steps while using blocks.

The blocks on the next page show the first four steps in Tom's or Sally's solution.

With whose solution do these steps belong?

Tom's Sally's *(Circle one)*

Explain your decision.


Step 1
Step 2
Step 3
Step 4

Appendix D

Letter of Consent from Dr. P. Thompson

San Diego State University

Center for Research in Mathematics and Science Education
6475 Alvarado Road #206
San Diego, CA 92120
Tel: 619-594-2363 Fax: 619-594-1581
E-mail: PTHOMPSON@SCIENCES.SDSU.EDU

Quantitative Reasoning Project

August 23, 1993

Al Yoshino
Dept. of Curriculum
River East School Division No. 9
589 Roch Street
Winnipeg, Manitoba
CANADA R2K 2P7

Dear Al,

Thanks for you letter of 8/17/93 and for your kind words regarding Blocks Microworld. As for using the materials I've already sent you to in your thesis, please be my guest. If you would like to talk about changes that might be incorporated into your study I'd be glad to chat with you on the phone. By the way, did I send you a copy of the article in Journal for Research in Mathematics Education that reported this study? If not, then let me know and I'll send you a copy.

Best wishes,

Patrick W. Thompson
Professor
Mathematics

Appendix E

Letter for Parental Consent

River East School Division No. 9

589 ROCH STREET, WINNIPEG, MANITOBA, CANADA R2K 2P7 (204) 667-7130

November 25, 1993

Dear Parents/Guardians,

This letter is to inform you about a study entitled "Microworld: A Computer Program for Better Understanding of Place Value" which is planned to be done in January, 1994, involving students in Grade 5 classes at Bertrun E. Glavin School. The study is being conducted as part of my Master of Education requirements at the Faculty of Education, University of Manitoba.

The intent of the study is to determine if interaction with a particular computer program has advantages over the physical manipulations of base-ten blocks. The study will focus on students' understanding of place value acquired through the use of base-ten blocks as compared to a computer simulation of the blocks.

The proposed study will involve 2 teachers and their classes for a period of 9 - 1/2 hour periods. All students will be given a pretest. Two groups of students will be formed by alternately placing students in each of the groups according to a ranking of pretest scores. One group will be given 7 lessons using physical base-ten blocks and the other group will be given similar instruction using simulated base-ten blocks via a computer program. Both methods of instruction are recognized forms of sound teaching practice.

The content of the lessons will be concentrated on the understanding of place value in decimal fractions which is part of the regular Grade 5 mathematics program. At the conclusion of the lessons, all students will be given a posttest.

Data will be collected from the pretest, posttest, classroom observations, and brief interviews with selected students. Twelve students will be selected for interviews from their pretest rankings: four from each of the pretest score categories of low, middle, and high. Students will be audio-taped during the interviews to ensure accuracy in recording data and information. All data and records collected will be kept confidential and will be disposed of at the completion of the study.

Your child's scores and data will be omitted at your request at anytime without penalty. A summary of the results will be made available to you at the completion of the study. The results of the study will be used as information for the school to determine the extent to which the program may be used in the future.

If you have concerns or questions, please contact me at the above address or phone number.

Thank you

Yours sincerely.

Al Yoshino
Math/Science Consultant

Parent/Guardian Consent Form

I do / do not (circle one) consent to have ________________________
(Student's Name)

participate in the study "Microworld: A Computer Program for Better Understanding of Place Value" with Al Yoshino.

Signature: ________________________________
(Parent/Guardian)

Date: __________________________