REGION OF INFLUENCE (ROI) APPROACH TO REGIONAL FLOOD FREQUENCY ANALYSIS

by

Zolt Zrinji

A Thesis
Presented to the University of Manitoba
in partial fulfilment of
the requirements for the degree of
Doctor of Philosophy
in the Department of Civil and Geological Engineering

Winnipeg, Manitoba
June, 1994

National Library of Canada

Bibliothèque nationale du Canada

Acquisitions and Bibliographic Services Branch

Direction des acquisitions et des services bibliographiques

395 Wellington Street
Ottawa, Ontario
K1A 0N4

395, rue Wellington
Ottawa (Ontario)
K1A 0N4

Your file Votre référence

Our file Notre référence

ISBN 0-315-99081-3

Name Zolt Zrinji

Dissertation Abstracts International is arranged by broad, general subject categories. Please select the one subject which most nearly describes the content of your dissertation. Enter the corresponding four-digit code in the spaces provided.

The Sciences and Engineering / Civil
SUBJECT TERM

0543
SUBJECT CODE

U·M·I

Subject Categories

THE HUMANITIES AND SOCIAL SCIENCES

COMMUNICATIONS AND THE ARTS
Architecture 0729
Art History 0377
Cinema 0900
Dance 0378
Fine Arts 0357
Information Science 0723
Journalism 0391
Library Science 0399
Mass Communications 0708
Music 0413
Speech Communication 0459
Theater 0465

EDUCATION
General 0515
Administration 0514
Adult and Continuing 0516
Agricultural 0517
Art 0273
Bilingual and Multicultural 0282
Business 0688
Community College 0275
Curriculum and Instruction 0727
Early Childhood 0518
Elementary 0524
Finance 0277
Guidance and Counseling 0519
Health 0680
Higher 0745
History of 0520
Home Economics 0278
Industrial 0521
Language and Literature 0279
Mathematics 0280
Music 0522
Philosophy of 0998
Physical 0523
Psychology 0525
Reading 0535
Religious 0527
Sciences 0714
Secondary 0533
Social Sciences 0534
Sociology of 0340
Special 0529
Teacher Training 0530
Technology 0710
Tests and Measurements 0288
Vocational 0747

LANGUAGE, LITERATURE AND LINGUISTICS
Language
- General 0679
- Ancient 0289
- Linguistics 0290
- Modern 0291

Literature
- General 0401
- Classical 0294
- Comparative 0295
- Medieval 0297
- Modern 0298
- African 0316
- American 0591
- Asian 0305
- Canadian (English) 0352
- Canadian (French) 0355
- English 0593
- Germanic 0311
- Latin American 0312
- Middle Eastern 0315
- Romance 0313
- Slavic and East European 0314

PHILOSOPHY, RELIGION AND THEOLOGY
Philosophy 0422
Religion
- General 0318
- Biblical Studies 0321
- Clergy 0319
- History of 0320
- Philosophy of 0322

Theology 0469

SOCIAL SCIENCES
American Studies 0323
Anthropology
- Archaeology 0324
- Cultural 0326
- Physical 0327

Business Administration
- General 0310
- Accounting 0272
- Banking 0770
- Management 0454
- Marketing 0338

Canadian Studies 0385
Economics
- General 0501
- Agricultural 0503
- Commerce-Business 0505
- Finance 0508
- History 0509
- Labor 0510
- Theory 0511

Folklore 0358
Geography 0366
Gerontology 0351
History
- General 0578
- Ancient 0579
- Medieval 0581
- Modern 0582
- Black 0328
- African 0331
- Asia, Australia and Oceania 0332
- Canadian 0334
- European 0335
- Latin American 0336
- Middle Eastern 0333
- United States 0337

History of Science 0585
Law 0398
Political Science
- General 0615
- International Law and Relations 0616
- Public Administration 0617

Recreation 0814
Social Work 0452
Sociology
- General 0626
- Criminology and Penology 0627
- Demography 0938
- Ethnic and Racial Studies 0631
- Individual and Family Studies 0628
- Industrial and Labor Relations 0629
- Public and Social Welfare 0630
- Social Structure and Development 0700
- Theory and Methods 0344

Transportation 0709
Urban and Regional Planning 0999
Women's Studies 0453

THE SCIENCES AND ENGINEERING

BIOLOGICAL SCIENCES
Agriculture
- General 0473
- Agronomy 0285
- Animal Culture and Nutrition 0475
- Animal Pathology 0476
- Food Science and Technology 0359
- Forestry and Wildlife 0478
- Plant Culture 0479
- Plant Pathology 0480
- Plant Physiology 0817
- Range Management 0777
- Wood Technology 0746

Biology
- General 0306
- Anatomy 0287
- Biostatistics 0308
- Botany 0309
- Cell 0379
- Ecology 0329
- Entomology 0353
- Genetics 0369
- Limnology 0793
- Microbiology 0410
- Molecular 0307
- Neuroscience 0317
- Oceanography 0416
- Physiology 0433
- Radiation 0821
- Veterinary Science 0778
- Zoology 0472

Biophysics
- General 0786
- Medical 0760

EARTH SCIENCES
Biogeochemistry 0425
Geochemistry 0996
Geodesy 0370
Geology 0372
Geophysics 0373
Hydrology 0388
Mineralogy 0411
Paleobotany 0345
Paleoecology 0426
Paleontology 0418
Paleozoology 0985
Palynology 0427
Physical Geography 0368
Physical Oceanography 0415

HEALTH AND ENVIRONMENTAL SCIENCES
Environmental Sciences 0768
Health Sciences
- General 0566
- Audiology 0300
- Chemotherapy 0992
- Dentistry 0567
- Education 0350
- Hospital Management 0769
- Human Development 0758
- Immunology 0982
- Medicine and Surgery 0564
- Mental Health 0347
- Nursing 0569
- Nutrition 0570
- Obstetrics and Gynecology 0380
- Occupational Health and Therapy 0354
- Ophthalmology 0381
- Pathology 0571
- Pharmacology 0419
- Pharmacy 0572
- Physical Therapy 0382
- Public Health 0573
- Radiology 0574
- Recreation 0575
- Speech Pathology 0460
- Toxicology 0383

Home Economics 0386

PHYSICAL SCIENCES

Pure Sciences
Chemistry
- General 0485
- Agricultural 0749
- Analytical 0486
- Biochemistry 0487
- Inorganic 0488
- Nuclear 0738
- Organic 0490
- Pharmaceutical 0491
- Physical 0494
- Polymer 0495
- Radiation 0754

Mathematics 0405
Physics
- General 0605
- Acoustics 0986
- Astronomy and Astrophysics 0606
- Atmospheric Science 0608
- Atomic 0748
- Electronics and Electricity 0607
- Elementary Particles and High Energy 0798
- Fluid and Plasma 0759
- Molecular 0609
- Nuclear 0610
- Optics 0752
- Radiation 0756
- Solid State 0611

Statistics 0463

Applied Sciences
Applied Mechanics 0346
Computer Science 0984

Engineering
- General 0537
- Aerospace 0538
- Agricultural 0539
- Automotive 0540
- Biomedical 0541
- Chemical 0542
- Civil 0543
- Electronics and Electrical 0544
- Heat and Thermodynamics 0348
- Hydraulic 0545
- Industrial 0546
- Marine 0547
- Materials Science 0794
- Mechanical 0548
- Metallurgy 0743
- Mining 0551
- Nuclear 0552
- Packaging 0549
- Petroleum 0765
- Sanitary and Municipal 0554
- System Science 0790

Geotechnology 0428
Operations Research 0796
Plastics Technology 0795
Textile Technology 0994

PSYCHOLOGY
General 0621
Behavioral 0384
Clinical 0622
Developmental 0620
Experimental 0623
Industrial 0624
Personality 0625
Physiological 0989
Psychobiology 0349
Psychometrics 0632
Social 0451

REGION OF INFLUENCE (ROI) APPROACH

TO REGIONAL FLOOD FREQUENCY ANALYSIS

BY

ZOLT ZRINJI

A Thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

ABSTRACT

Good extreme flow estimation is a necessity for the proper design of river engineering development, flood protection or urban engineering works. Usually, a short record length, or no data record at all, at the site of interest necessitates the use of the regional flood frequency approach. Regional flood frequency analysis employs spatial information in order to enhance the reliability of the temporal data. The region of influence (ROI), employed here, ensures that each site has a region with a potentially unique combination of stations. The regionalization incorporating a homogeneity test ensures that the selected stations have similar extreme flow characteristics. The hierarchical feature is added to the ROI approach in order to further enhance the efficiency of the spatial information transfer. It does this by taking advantage of the different spatial similarity scales that have been observed for different orders of moments for a flood frequency distribution. The incorporation of this concept into the ROI framework is accomplished by allowing for a set of ROIs for a site as opposed to a single ROI.

The regional flood frequency approach presented in this study can be applied to both the case of gauged sites and ungauged sites. The relative merits of the methodology for the ungauged case are demonstrated through an application to extreme flow data for sites in Newfoundland, Canada. The new approach is compared with results obtained from regression analysis and is shown to provide improved estimates of extreme flow quantiles at sites which are considered to be ungauged. The hierarchical ROI approach is evaluated through the use of a Monte Carlo experiment applied to data from another collection of unregulated catchments in midwest Canada. The simulation experiment shows that with this refinement of the new methodology, an improvement in flood quantile estimation is achieved.

ACKNOWLEDGEMENT

I would like to express my thankfulness to all people who provided me with support and understanding while working on this thesis.

I am specially grateful to Professor Donald Burn for his support through this research program and the preparation of the thesis. His guidance, advice, encouragement and thoroughness are greatly appreciated. I am also thankful to Dr. Slobodan Simonovic, Dr. Caspar Booy and Dr. Allan Woodbury who gladly obliged to serve on my advisory committee.

Financial assistance for this research was provided by Natural Sciences and Engineering Research Council of Canada (NSERC) and by University of Manitoba graduate fellowship and they are gratefully acknowledged.

I am grateful to the members of Facility for Intelligent Decision Support (FIDS) environment, and to faculty members at the Department of Civil Engineering, at University of Manitoba, for being sincere friends and for encouraging my work.

Finally, I am deeply appreciate the help and understanding of my wife throughout my studies.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

ACLS	Area Controlled by Lakes and Swamps
AIC	Akaike Information Criteria
BAREA	BAREn Area
CDF	Cumulative Density Function
CV	Coefficient of Variation
DA	Drainage Area
GEV	Generalized Extreme Value
GLS	Generalized Least Squares
K-S	Kolmogorov Smirnov
LAT	LATitude of the catchment centroid
LP3	Log Pearson type 3
MAR	mean annual runoff
MLE	Maximum Likelihood Estimation
MOM	Method Of Moments
MSE	Mean Squared Error
OLS	Ordinary Least Squares
P3	Pearson type 3
PPC	Probability Plot Correlation
PPCC	Probability Plot Correlation Coefficient
PS	Pearson Skewness

PWM	Probability Weighted Moment
RL	Record Length
ROI	Region Of Influence
SHAPE	Catchment SHAPE factor
WLS	Weighted Least Squares

1. INTRODUCTION

The estimation of future flooding potential at a site is frequently required by the design engineer for a variety of urban planning and river engineering works. Obtaining an accurate estimation of the relationship between extreme flows and the associated recurrence interval (the so called Q-T relationship) is more complicated if at the site of concern, the gauging record is shorter than the return period of interest, or there is no flow record at all. To compensate for a short data record, the trade off-between the spatial and the temporal characterization of extreme flow events can be effected through the use of regional flood frequency analysis. Regional flood frequency analysis can facilitate the estimation of an extreme flow value at a location for which limited flow data exist, based on an extreme flow relationship derived using information from basins with similar hydrologic responses.

Regional flood frequency analysis has historically been used in one of two contexts. In the first context, the intent is to improve the accuracy and precision of the extreme flow estimates at a gauged site by pooling at-site information from hydrologically similar gauged sites. In the second context, the task is to estimate extreme flows at a site for which no streamflow data are available. Although there are obvious fundamental differences between the two problem contexts, there is a similarity which can be effectively exploited in the development of a regional flood frequency approach.

Regional flood frequency analysis consists of two major parts. The first part is the development of a dimensionless frequency curve, representing the ratio of the extreme flow of any return period to an index flood, which is generally the mean annual flood. The second part is the estimation of the index flood. In the case of estimating the mean annual flood at an ungauged site, the mean annual flood can be related to the basin's physiographic characteristics, while in the case of gauged site, the estimation of the mean annual flood is generally based on the recorded measurements.

This research is primarily focused on the development of the dimensionless frequency curve, which can be further subdivided into: a) regionalization; b) determination of an appropriate theoretical distribution function; and c) identification of an efficient method for combining all available regional information for predicting the standardized extreme flow.

1.1 RESEARCH OBJECTIVES

The major objective of the research reported in this work is to develop a method to improve the estimation of the extreme flow events for both gauged and ungauged sites. Improvements in the estimation of the extreme flow events are intended to be made by employing the region of influence (ROI) approach in the process of grouping stations into the region. The ROI approach can promote an efficient data transfer through the use of stations that are hydrologically similar to the site of interest. The required degree of hydrologic homogeneity of the regions that are formed can be ensured by the incorporation of a homogeneity test in the regionalization process. Hydrologic homogeneity is ensured by some form of statistical homogeneity test.

Additional improvement of the extreme flow estimation can be obtained by the application of a hierarchical approach to the estimation of distribution parameters with the ROI approach. This is accomplished by taking advantage of the different spatial similarity scales that have been observed for different orders of moments for extreme flow data. The incorporation of the hierarchical concept into the ROI framework is accomplished by defining a set of ROI's for a site as opposed to a single ROI.

The ROI approach, with the incorporation of the above features, can be applied in the analysis of gauged or ungauged sites although the preferred station selection strategy differs for the two cases.

1.2 SCOPE OF WORK

The intent of this research is to improve the estimation of extreme flow events at the site of interest, using regional flood frequency analysis.

A general review of previous research contributions is presented in Chapter 2. This review includes different approaches to the grouping of stations into regions that have previously been used. The chapter continues with the review of the selection of the variables useful in the grouping process used in flood frequency analysis. An overview of homogeneity tests used in regional analysis follows. In addition, a summary of parameter estimation methods is presented. The section concludes with a review of methods dealing with the problem of how the site can be assigned to the one of the existing regions.

In Chapter 3, theoretical considerations that are used in this study are presented. Chapter 3 starts with an overview of L-moments, which are efficient tools for selecting the appropriate theoretical distribution, estimating parameters for the selected distribution function, and calculating the homogeneity of the region based on the measured flows. The regional flood frequency analysis approach presented in this work is based on the index flood method, which is a structured approach that allows each step of the methodology to be considered more or less independently. Different sections are allocated to the issues of selecting an appropriate parent distribution function, developing a technique for grouping stations into a region that incorporates a homogeneity test, and the estimation of at-site extreme flow events.

The application of the proposed regional flood frequency analysis to the case of ungauged sites is presented in Chapter 4. For the evaluation of the methodology, the regional flood frequency analysis with the ROI approach is applied to data from a collection of unregulated Canadian catchments. However, the evaluation of the results is difficult, since the "true" flow quantiles for a selected return period at a site cannot be determined. In order to evaluate the results obtained by the ROI approach, the establishment of an estimate for the true flow quantile is necessary. The true flow quantile estimate is obtained by applying a combination of at-site and regional estimation. Following this, the gauged sites are sequentially treated as ungauged and the estimates of extreme events obtained are compared to the associated assumed true flow quantile. In addition, the results obtained by the ROI approach are also compared with those obtained from an earlier study of the same area.

The application of the hierarchical region of influence approach, where the parameters of the selected distribution are obtained on the hierarchical basis, is presented in Chapter 5. The hierarchical ROI approach is evaluated through the use of a Monte Carlo experiment applied to data derived from a different set of unregulated Canadian catchments. Through a simulation experiment, it is possible to evaluate the refinement of various aspects of the presented methodology and also to quantify the improvements obtained in flood quantile estimation.

Conclusions and recommendations for future research are included in Chapter 6.

2. LITERATURE REVIEW

This chapter summarizes relevant literature for regional flood frequency analysis. These studies deal with the regionalization task, starting with the geographical approach, followed by cluster analysis, a partitioning approach and other statistical approaches such as the region of influence (ROI). A segment which deals with the variables used in the regionalization process follows. Next, a review of studies that describe various measures of regional homogeneity for a generated region are presented. In the following section, an overview of goodness of fit and parameter estimation approaches are described. Finally, a review of studies dealing with the site allocation problem is given.

2.1. REGIONALIZATION APPROACHES

The first task in regional flood frequency analysis is the creation of the region. A region is a group of gauging stations from which it is possible to transfer to the particular location of interest reliable information that is relevant, and typically limited. This information transfer is typically effected through the use of a form of the index flood method (Greis and Wood, 1981). The index flood method involves a normalization of the data and a calculation of a regional distribution through a pooled estimation procedure.

In the early days of regional analysis, the delineation of regions relied on geographic, political, administrative, or physiographic boundaries. Those regions were quite easy to create, but the resulting regions were only assumed to be homogeneous in their hydrologic response.

Such similarity in hydrologic response cannot be guaranteed especially when the neighbouring basins are physically very different. Thus, these methods have been replaced by more sophisticated procedures.

Hydrologists often use a regional regression model to estimate flow characteristics at ungauged sites. Many studies are based on regions created using the residuals from a regression model. These methods use residuals from an overall regression function, for an entire collection of stations, to create subregions based on the positive/negative sign and magnitude of the residuals (Wandle, 1977; Guetzkow, 1977). Deriving subregions using the above method requires a large amount of subjective judgement.

Creating regions on the basis of the correlation of the extreme flow series is another regionalization method. However this is a very tedious task, especially when the number of stations is large. Furthermore, a difficulty arises if the lengths of the flow record at the various sites are different. If the calculation of the correlation involves stations with different lengths of flow record, then the standard error of the estimates would also be different. If a common period of record is used for the calculation, then it is possible to miss some unusual event or events which could affect the correlation matrix (Burn, 1988).

Cluster analysis is a more objective method of creating subregions (Tasker, 1982). The essence of cluster analysis is to identify clusters (groups) of gauging stations such that the stations within a cluster are similar while there is dissimilarity between the clusters. There are many different algorithms available for performing cluster analysis. Since any variable, consisting of physical characteristics or flood statistics, can be used to define the similarity measure required to generate clusters, the variables have to be carefully selected and weighted according to the importance for the actual problem (Burn, 1989). In addition, clusters formed on the basis of physical basin characteristics may reflect the distribution of gaps in the data space instead of any significant hydrological relationships with the basin characteristics.

Another method of defining regions is the multiple partitioning method which involves partitioning the stations by basin characteristics (Wiltshire, 1985; 1986c). This method was shown to be a very effective alternative to geographical regionalization. The basin characteristics used in Wiltshire's (1985) work were basin area, average annual rainfall, and a soil index, which measures a basin's ability to accept winter rainfall. One of the potential drawbacks of this method is that there could be considerable spatial variability in the distinguishing characteristics within a basin which would require the use of an average or some other representative value. This could be the case for rainfall and soil type when the basin is large. Another problem arises from the low correlation between some basin characteristics and the extreme flow events which results in poor hydrologic similarity within a region.

The later studies of Wiltshire (1986c) included additional basin characteristics into the regionalization process, namely one day duration effective rainfall with a five year return period, the mean soil moisture deficit, the stream density, the main stream slope, the fraction of the basin draining through a lake, and the fraction of the basin that is urbanized. It has been shown that pre-selection of the basin characteristics is essential for obtaining effective regionalization with the multiple partition method. Some combination of basin characteristics will generally be found to be effective for defining homogeneous regions, but the subjectivity involved in the process is still high. Thus, the regions formed using the process are not unique, since another set of basin characteristics will lead to a different result.

Other grouping methods use classification by flow statistics (Wiltshire, 1985; 1986c). These studies include the mean annual flood, the coefficient of variation of annual flood series, peaks over threshold flood series, and the variance of flood statistics to locate the optimum division of the data-space of the basin characteristics. The basic idea is that sites with similar statistical measures are expected to have similar extreme flow responses. However, this approach is not feasible for ungauged locations.

A novel regionalization approach first suggested by Acreman and Wiltshire (1987) and Acreman (1987) involves dispensing with fixed regions. This approach allows each site to have a unique set of stations that constitutes the region for that site. Thus, it is possible for two neighbouring stations to have completely different sets of stations that represent the region for each site. This methodology involves the transfer of extreme flow information from similar stations to the site of interest. The subsequent implementation of the method is referred to as the region of influence (ROI) approach by Burn (1990a; b). In this method there is no need for boundaries between regions nor a need for rigid, fixed regions. Furthermore, there is no need that all sites in a particular area use the same number of stations in the extreme flow estimation procedure (Burn 1990a). However even this method is not completely free from subjectivity, since the method requires the choice of a threshold value, which functions as a cut-off point for the dissimilarity measure. All sites which have a dissimilarity measure greater than the threshold value are excluded from the region of influence. The threshold value has a similar effect to the selection of the number of regions when a fixed number of regions are created. A larger threshold value will increase the number of stations included in the ROI, but the homogeneity of the group is expected to decrease in an analogous way to the situation where the number of fixed regions is small. In addition, when the threshold value is small, the number of stations included in the ROI will decrease much as the case when the number of fixed regions is large. An advantage of this method is that dissimilar stations are excluded from the region of influence.

2.2. REGIONALIZATION VARIABLES

The choice of basin characteristics to be used in the regionalization process requires some attention. It is essential that the selected basin attributes are available for each site of interest. One possible way to screen the important characteristics is by plotting basin attribute values versus some measure of extreme flow. The selected basin characteristics are then chosen from those which show a relationship on the resulting graph.

Forming clusters using factor analysis is another possible way to extract the most influential basin characteristics (White, 1975). Factor analysis results in reducing a set of intercorrelated variables to a smaller number of basin characteristics that provides knowledge of the underlying structure of the variables. However, all the attributes involved in the procedure must be tested for normality. If some of them cannot pass the test, some kind of transformation is required. However, this will not diminish the effectiveness of this approach for selecting the influential characteristics (Abrahams, 1972).

The selection of basin characteristics for forming regions can be critical for the determination of the final grouping of the catchments. It is also possible that a catchment characteristic has an important role in one basin (or region), but is not a significant factor in others (Nathan and McMahon, 1990). Thus a purely mechanical approach to the regionalization task should be avoided and every case should be considered separately, with careful judgement employed in the selection of the significant basin characteristics.

If the basin attributes are selected subjectively for regionalization purposes, without any tested relation to the flood creating mechanism, the set of stations obtained by different users may vary substantially due to the large amount of subjectivity involved in the process. This type of approach may be acceptable when the number of sites in the considered area is relatively small. Otherwise, the subjectively chosen attributes may result in a tedious regionalization process due to the large number of possible combinations of attributes and stations.

2.3. HOMOGENEITY TEST

The next step in regional flood frequency analysis is the evaluation of the regional homogeneity for the group of stations. The importance of regional homogeneity has been demonstrated by Hosking et al. (1985), Wiltshire (1986a), and Lettenmaier et al. (1987). Each of the regions should have two basic properties: (1) dissimilarity from other regions; and (2) homogeneity of flood frequency characteristics within a region to allow the definition of an average regional flood frequency curve.

One way to test the homogeneity of the collection of flood series is by the dimensionless coefficient of variation (CV) of each flood series (Wiltshire, 1986c), which is defined as the sample standard deviation divided by the sample mean. The objective is to minimize the variance of the CV within a group and maximize the variance of CV between groups. Differences between groups can be checked by an F statistic which represents the ratio between the variance of CV between groups over the variance of CV within a group. The test for homogeneity within a group can be accomplished using a χ^2 statistic with the hypothesis that there is no statistical difference between the site CV's within a group. The χ^2 test is only valid for large regions with long record lengths. Otherwise, the use of this test will result in the hypothesis being accepted too frequently, since the value of the statistic will be underestimated.

The homogeneity of a region can be also tested by the likelihood ratio test of whether or not the flood frequency characteristics can be represented by a single regional frequency relationship (Acreman and Sinclair, 1986).

Probability plot correlation (PPC) was introduced by Filliben (1975) as a test statistic for normality. Later it was observed that the PPC test can also be applied for the Gumbel, Weibull, and uniform distribution. In regional analysis, the probability plot correlation coefficient (PPCC) test has been used as a graphical aid to test the possibility of fitting a common theoretical distribution function to the gauging stations comprising the region. The PPCC test statistic is defined as the product moment correlation coefficient between the ordered observations and the order statistic means for each assumed distribution function (Vogel and Kroll, 1989). This test has been shown to be efficient when stations are independent. In addition, the test is more efficient when distributions with three or more parameters are tested, since they are more flexible and they tend to have a linear probability plot.

Another way to test the homogeneity of the regions is based on the geometry of the cumulative density function (CDF) of the flood frequency distribution. The hypothesis is based on the fact that the form of the CDF depends on the relative frequency of floods with different magnitudes and also on the parameters of the distribution from which the flood data are assumed to derive (Wiltshire, 1986a).

The formulation of this test is similar to the CV based test and requires a single measure to describe the non-exceedance probability at each site. For example, the point of interest could be the non-exceedance probability of the mean. The non-exceedance probability of the site mean should be around the same value as the non-exceedance probability of the assumed distribution's mean. However, distributions with a high or a low CV could have the same mean, but obviously those are not necessarily similar to the assumed distribution which may have a very different CDF geometry. To overcome this problem, a simple transformation is required (Wiltshire, 1986a). Through the transformation, the difference between high or low CV and between positive or negative skewness can be detected and therefore improve the power of the test. From this point the homogeneity test can proceed as a test of similarity of the exceedance probability of the mean. The hypothesis, that there is no difference in the exceedance probability of the mean, can be checked by a χ^2 statistic.

Hosking and Wallis (1993) suggested a homogeneity test which is based on L-moments. L-moments were introduced by Hosking (1986) and represent a new approach to the estimation of a sample's moments. L-moments have certain characteristics that make them very attractive for use in the identification of the form of the distribution of a random variable. L-moments are also useful to provide information on regional variability and for the estimation of parameters of known theoretical distributions. The estimation of distribution parameters by L-moments is attractive, because estimates of L-moments have been shown to be practically unbiased, even for small sample sizes commonly available in hydrology (Gingras and Adamowski, 1992; Pilon and Adamowski, 1992).

The L-moment based homogeneity test of Hosking and Wallis (1993) proposes calculating five summary statistics of the at site data and comparing the between site variability of these statistics with what would be expected from a homogeneous region. Although the calculated statistics have somewhat arbitrary threshold values, the method has still proven efficient. The main shortcoming of this method is that the homogeneity measure is highly dependent on the number of sites in the region. This means that the method works fine for large regions but has a tendency to give false indications of homogeneity for small regions. In addition, this method can only be used as a significance test of the homogeneity if an assumption is made as to the true regional distribution and if it is assumed that temporal and spatial independence are strictly satisfied. Furthermore, the method is based on the simulation of the homogeneous region with the same number of sites and site record lengths as the sites in the proposed region and it therefore requires a large number of simulations to perform this homogeneity test. This makes the method computationally intensive for cases where the homogeneity test must be applied many times. Applying the test once, as a final homogeneity test is, however, feasible even for a very large region.

Another L-moment based homogeneity test was proposed by Chowdhury et al. (1991). This method provides a statistical judgement in the acceptance of the null hypothesis that the region is homogeneous. The test is based on the difference between at-site sample and the regional value of L-moment ratios. The statistic which is used to test the hypothesis that the region is homogeneous has approximately a standard χ^2 distribution with 2(K-1) degrees of freedom, where K is the number of sites included in the region. The method is efficient only if the overall parent distribution is the Generalized Extreme Value (GEV), since the asymptotic variances and covariances of the unbiased probability weighted moment estimators are easily available only for the GEV distribution.

A recently suggested homogeneity test uses normalized 10-year flood quantile estimators and their sample variances. It has been proved that the normalized 10-year flood estimators have small bias and approximately a normal distribution even in small samples. Therefore the regional homogeneity test is developed on the basis of at-site normalized 10 year flood estimators for sites in the region. Furthermore, this test has the advantage that it focuses on the distribution of the larger events, rather than on the entire empirical CDF as proposed in other tests. The shortcoming of this method is that the assumption of the independent annual floods must be satisfied for a realistic homogeneity test (Lu and Stedinger, 1992).

2.4. GOODNESS OF FIT

Regional flood frequency requires the selection of a distribution function. Example distribution functions include the log-normal, generalized extreme value (GEV), Log Pearson type 3, and Wakeby distribution. The task is to choose a distribution which will adequately reproduce the behaviour of the extreme flows at the upper tail of the distribution. It should be noted that there is no guarantee that the natural events (i.e., extreme flows), will follow any of the existing theoretical distribution functions (Cunnane, 1985). In addition, the lack of sufficient data to identify the correct theoretical distribution required for a reliable goodness of fit test has led some political jurisdictions to specify a single distribution. For example, the U.S. Water Resources Council specified the Log Pearson type 3 (LP3) distribution for the use of estimating flood probabilities, whereas a similar study in the UK (NERC, 1975) proposed the GEV distribution as a standard. In Germany the Pearson type 3 (P3) and Log Pearson type 3 (LP3) distributions were recommended, while in Australia the LP3 distribution was advocated as a regional parent distribution (IEA, 1977).

However, in some situations it is possible to estimate the theoretical distribution from the site measurements. One possible way to do so is the Kolmogorov Smirnov (K-S) goodness of fit test. The advantageous feature of this test is that it gives simultaneous confidence intervals for all observations and thus provides a visual test. The K-S test provides an answer of whether or not the specific distribution can be fit to the sample data, but it gives no preference to any among the distributions which pass the test.

A method based on the calculation of the Akaike Information Criteria (AIC) (Chow and Watt, 1992), gives useful information if a set of possible distributions are selected which can be fit to the sample data. The approach recognizes that considering a distribution with additional parameters does not necessarily lead to the selection of a distribution which will provide a better fit to the sample data. The selection of a distribution which has an additional parameter has to be justified in terms of the improvement in the overall fit to the sample data. This justification is incorporated into the value of the AIC. The distribution with a minimum value of the associated AIC would be the best selection. The approach was originally developed for a selection of a distribution function for a single station. The selection of a regional distribution by this approach is not mentioned, however it does not exclude the possibility of expansion of the approach in that direction. The shortcoming of the approach is that while it has been shown to be efficient with a large sample size ($n>100$), in a small sample size ($n<50$) it is very difficult to show the superiority of the selected distribution to all of the others.

Another method for determining the appropriate distribution function is the procedure based on L-moments. The L-moment ratio diagrams can produce a collection of points, which can be compared with the theoretical curves for known distributions. In addition to the graphical approach, it is possible to calculate a goodness of fit measure based on a simulation process. This gives a numerical value associated with each of the theoretical distributions (Hosking and Wallis, 1993). It is important to mention that the use of the L-moment ratio test cannot assure the identification of the true distribution, but it does focus attention on likely candidates, and removes inappropriate candidates from further consideration.

2.5. PARAMETER ESTIMATION

When a decision has been made about the appropriate distribution, there is an additional decision as to which method for the estimation of parameters for the selected distribution function will be employed. The chosen approach should be an efficient method resulting in a small variance for the estimated parameters, should also yield estimates of the parameters which have a small bias, and perform reasonably well in the case of minor distributional mis-specification.

Commonly used parameter estimation methods are: the method of moments, maximum likelihood, probability weighted moments, ordinary least squares, generalized least squares, and graphical techniques. The method of moments and the graphical method have some typical characteristics which are always true regardless of the nature of the sample data used. The sample estimates of moments are known to be biased and lead to parameter estimates that are less efficient than alternative approaches. Graphical estimation determines the shape of the Q-T relationship. The result is entirely determined by the form of the assumed distribution. Furthermore, the estimated distribution depends on the type of graph paper and the plotting position formula adopted (Cunnane, 1985). For other estimation methods, more complex evaluation is need to determine whether the method provides any inference about the form of the distribution.

In the previous decade, there has been extensive work done on the topic of the benefits versus shortcomings of different parameter estimators. Greis and Wood (1981) have shown the superiority of the probability weighted moment (PWM) method over more conventional methods like method of moments (MOM) and maximum likelihood estimation (MLE) for distributions which can be expressed in inverse form. Many distributions, including Gumbel, Wakeby and Weibull meet this qualification. Parameter estimation by PWM method is efficient. In addition, the combination of PWM parameter estimator with the GEV distribution function has been reported as an efficient combination for regional flood frequency (Potter, 1987).

L-moments can be applied in the process of estimation of a distribution function's parameters because a distribution function whose mean exists is uniquely characterized by its L-moments. The distribution function parameters can be estimated from the sample estimates of the L-moments. L-moments have the advantageous characteristic that they are linear combinations of the data. Therefore, the sample estimates of L-moments tend to be comparatively robust in the presence of outliers and tend to provide accurate and stable parameter estimates even with a small sample size (Hosking, 1990).

Recent work has focused on index flood methods because they have the desirable property of being highly structured. This method assumes that for a group of sites, the scale and shape parameters are identical for all sites in the group, while the location parameter varies from site to site. This idea was used to develop a hierarchical approach to parameter estimation. The assumption inherent in the approach is that, the skewness of the observed variable (e.g. the annual maximum flood) can be considered constant over a larger area than the coefficient of variation which can be considered constant over a subregion. Furthermore, the CV varies less over the space than the location parameter (mean annual flood). The fact that the higher the order of the parameter that is to be estimated, the greater the number of sites that is needed to provide an estimate with a given degree of reliability, also supports the idea that different sizes of regions should be used for the estimation of parameters of different levels (Gabriele and Arnell, 1991).

A hierarchical and empirical Bayes approach presented by Ribeiro-Correa and Rousselle (1993) was based on the same idea as the above hierarchical approach, but differs from the index flood based approach in that the flood series is not standardized. Estimation of the shape parameter of the Pearson type 3 distribution was based on the weighted average of the at site skew coefficients for sites in the region with a long record length (over 35 years). The location parameter is estimated at site with a correction for the regional skew. The scale parameter is estimated using the empirical Bayes approach. This approach to the regional flood frequency showed good results for the Pearson type 3 distribution.

In regional flood frequency analysis, linear regression is often used, either for regionalization, or for the estimation of various extreme flow quantiles. The estimation of regression parameters has received considerable attention. Recent research has focused on the development of methods which combine simplicity of use and effectiveness in providing a set of parameters which will ensure a robust regression function.

Parameters for regression models have traditionally been estimated using ordinary least squares (OLS). However, the assumption that the residual errors associated with the individual observations are homoscedastic and independently distributed may be violated. In the case of regional hydrologic data, variations in the length of the available streamflow records and cross correlations among concurrent flow result in estimates of the T-year return period flood, and other flow statistics, that are cross correlated and of varying precision. Advanced methods like weighted least squares (WLS) and generalized least squares (GLS) were developed to deal with such situations (Stedinger and Tasker, 1985; Stedinger and Tasker, 1986; Tasker et al., 1986).

2.6. ASSIGNMENT OF UNGAUGED SITES TO A REGION

Assigning the site of interest to one of the homogeneous regions is a crucial stage in regional flood frequency analysis. With our present knowledge of the physical environment, it is not possible to model the multitude of processes and interactions which influence the flood frequency characteristics at any site. However, through the regionalization process, it is possible to determine which characteristics have an influential role in any particular situation.

Assigning the ungauged site to one of the regions is trivial if the sites were grouped on the basis of geographic or political criteria. In these situations, borders between regions are clearly defined and the assignment is determined by the simple physical location of the ungauged site.

In the case of regional flood frequency analysis where basin characteristics are used in the formation of the regions, the assignment is not so simple. One approach is to locate the ungauged site in the basin characteristic data space. This is an easy and straightforward process, but it results in potentially abrupt changes in the extreme flow estimates as one crosses a data space partition line. In order to avoid the sudden changes of the flood estimate as one moves from one sector to another sector in the partitioned data space, the weighted mean quantile approach can be employed. The estimated extreme flow can be obtained on the basis of the weighted flood estimates from each surrounding group. Weights applied to the estimates are in inverse proportion to the distance from the ungauged site location to the centroid of each group (Wiltshire, 1986c).

An approach to identify the appropriate region for a site of interest is by discriminant analysis. Discriminant analysis is based on a transformation of basin characteristics, or combination of basin characteristics, to the probability of membership that the site of interest is assigned to the particular region (Wiltshire, 1986b). Discriminant analysis always assigns the site of interest to the existing region with the highest probability of membership. This method always gives an answer, but the answer is based on the relative similarity, which means that the site of interest can be assigned to the most similar region, even if the similarity is quite poor. Thus, this method should be applied with care and critical interpretation of the results.

When regions have been created on the basis of the catchment characteristics, the ungauged site can be assigned to a region based on its location in the catchment characteristic data space, as mentioned earlier. Although this is a simple process, there is no guarantee that similarity in terms of catchment characteristics implies hydrologic similarity. In the case of the gauged site, where some amount of data record is available, obtaining hydrologic similarity has a higher probability.

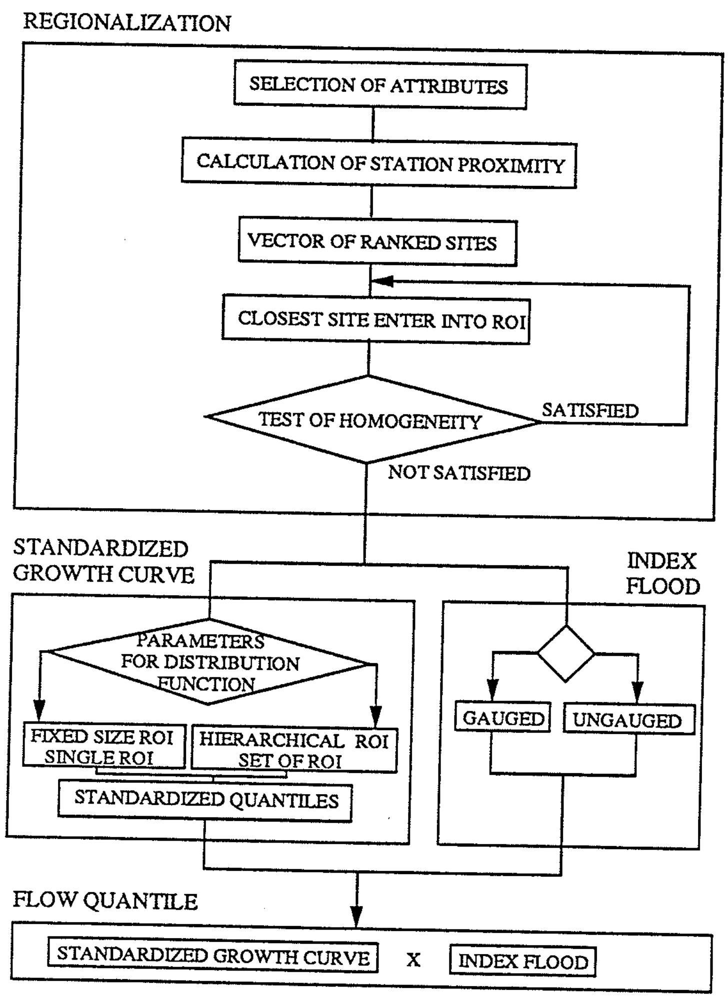
The problem of the assignment of the site of interest to one of the existing regions can be avoided by using the ROI regionalization process, since the ROI approach creates a unique region around the site of interest. This approach is feasible in both the gauged and ungauged cases (Zrinji and Burn, 1994).

3.0 REGIONAL FLOOD FREQUENCY WITH A ROI APPROACH

The theoretical framework for regional flood frequency analysis presented below consists of a regionalization technique which incorporates a homogeneity test, the determination of the standardized growth curve and index flood. These steps are presented in the schematic form in the Figure 3.1.

Since the approach presented in this work involves the use of L-moments, this section will start with a brief overview of the theory of L-moments.

Figure 3.1 Components for flow quantiles estimation with ROI approach



3.1 L-MOMENTS

One of the most widely used techniques for fitting frequency distributions to observed data is known as the method of moments. The method of moments is applied by equating the theoretical moments of a distribution to the sample product moments. Let X be a real-valued random variable, then the first four theoretical moments are:

$$\mu = E[X] \tag{3.1}$$

$$\sigma^2 = Var[X] = E[(X-\mu)^2] \tag{3.2}$$

$$\gamma = \frac{E[(X-\mu)^3]}{\sigma^3} \tag{3.3}$$

$$\kappa = \frac{E[(X-\mu)^4]}{\sigma^4} \tag{3.4}$$

Let X_i be the ith value of a sample of size n, then the first four sample product moments are:

$$m = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{3.5}$$

$$S^2 = \frac{1}{n}\sum_{i=1}^{n} (X_i - m)^2 \tag{3.6}$$

$$G = \frac{1}{S^3}\left[\frac{1}{n}\sum_{i=1}^{n} (X_i - m)^3\right] \tag{3.7}$$

$$k=\frac{1}{S^4}\left[\frac{1}{n}\sum_{i=1}^{n}(X_i-m)^4\right] \tag{3.8}$$

where μ, σ^2, γ, and κ denote the theoretical mean, variance, coefficient of skew, and kurtosis coefficient, respectively, while m, S^2, G, and k denote the sample mean, variance, coefficient of skew, and kurtosis coefficient, respectively. It has been shown that observations from a small sample size ($n \leq 100$) tend to estimate S^2 and G with an extraordinary bias and variance (Wallis et al., 1974). There are several methods which attempt to render the estimate unbiased, but they usually result in increased variance, since the skewness is always downward biased (Vogel and McMartin, 1991).

A good alternative to the method of ordinary moments is the method of L-moments, which was first introduced by Hosking (1986). L-moments are analogous to the conventional moments, but are estimated as a linear combination of order statistics, and thus they are subject to less bias than ordinary product moments. This is because ordinary product moment estimators such as the variance and the skewness, require raising deviations between observations and the mean to the second and third power, which gives a greater weight to observations that deviate from the mean, resulting in a greater bias and variance in parameter estimates.

L-moments can be expressed using probability weighted moment (PWM) estimators. For a random variable X, with a cumulative distribution function F(X), the order r PWM can be defined as (Greenwood et al., 1979):

$$\beta_r=E[X[F(X)]^r] \tag{3.9}$$

Following Hosking (1986), L-moments can be expressed as a linear combination of the PWMs as:

$$\lambda_{r+1}=\sum_{k=0}^{r}\beta_r(-1)^{r-k}\binom{r}{k}\binom{r+k}{k} \tag{3.10}$$

Therefore the first four L moments are:

$$\lambda_1=\beta_0 \tag{3.11}$$

$$\lambda_2=2\beta_1-\beta_0 \tag{3.12}$$

$$\lambda_3=6\beta_2-6\beta_1+\beta_0 \tag{3.13}$$

$$\lambda_4=20\beta_3-30\beta_2+12\beta_1-\beta_0 \tag{3.14}$$

The first L-moment, λ_1, is the arithmetic mean and hence it is a measure of location. The second L-moment, λ_2, is a measure of dispersion, analogous to the standard deviation. In addition, the L-coefficient of variation, L-CV, is defined as:

$$\tau_2=\frac{\lambda_2}{\lambda_1} \tag{3.15}$$

Furthermore, by standardizing the higher order moments through:

$$\tau_r\equiv\frac{\lambda_r}{\lambda_2} \qquad \text{for } r=3,4,... \tag{3.16}$$

they become independent of the units of measurement for X. Analogous to the coefficient of skewness (CS), τ_3 is the L-skewness and reflects the degree of symmetry of a sample. Similarly, τ_4 is a measure of peakedness and is referred to as L-kurtosis. Higher L-moment ratios may be viewed similarly to the conventional moment ratios. τ_5 can be interpreted as a measure of tendency to bimodality, while the τ_r of odd order are generalized skewness measures in so far as symmetric distributions have $\tau_{2r+1}=0$ for all $r\geq1$.

For practical use, the PWMs can be estimated from a finite sample. Let $X_1\leq X_2\leq...\leq X_n$

be the ordered values of a sample of size n. The unbiased estimator of β_r is (Landwehr et al., 1979a):

$$b_r = n^{-1}\sum_{j=1}^{n} \frac{(j-1)(j-2)....(j-r)}{(n-1)(n-2)...(n-r)} x_j \qquad (3.17)$$

Therefore, the unbiased estimators for the λ_r are given by:

$l_1=b_0$ (3.18)

$l_2=2b_1-b_0$ (3.19)

$l_3=6b_2-6b_1+b_0$ (3.20)

$l_4=20b_3-30b_2+12b_1-b_0$ (3.21)

Sample estimates for τ_r are:

$$t_2=\frac{l_2}{l_1} \quad \text{for } r=2 \qquad (3.22)$$

and

$$t_r=\frac{l_r}{l_2} \quad \text{for } r=3,4,... \qquad (3.23)$$

These estimates of t_r are asymptotically unbiased for large values of n.

The ordinary product moment ratios are not bounded. When they are estimated from a finite sample of size n, the sample moment ratios are bounded. CV is limited to $(n-1)^{0.5}$, while the sample CS cannot exceed $(n-2)/(n-1)^{0.5}$ (Kirby, 1974; Wallis et al., 1974).

Hosking (1986) observed that the L-moment ratios are bounded so that $|\tau_r|<1$ for r=3 and 4. For strictly positive random variables, $\tau_2<1$. It has been also shown that for sample size $n\geq4$, the sample L-moment ratios t_3 and t_4, estimated by the unbiased PWM estimator, can take on all feasible values for the population of L-moment ratios. For strictly positive random variables, t_2 can take values between 0 and 1.

3.2 SELECTION OF THE APPROPRIATE THEORETICAL DISTRIBUTION

In order to perform flood frequency analysis, it is generally necessary to select one of the known theoretical distributions as the parent distribution function for a collection of stations, which are forming a homogeneous region. From the available methods for the selection of the parent distribution, the method based on the use of L-moments, recommended by Hosking and Wallis (1993), will be outlined in the following.

As was mentioned in section 3.1, a probability distribution function is fully characterized by its L-moments, and therefore, for a homogeneous region, the weighted average L-moment ratios can be considered to represent the parent distribution. The weighted average L-moment ratios are defined as:

$$\overline{t_r} = \frac{\sum_{k=1}^{K} n_k t_r^{\,k}}{\sum_{k=1}^{K} n_k} \qquad \text{for } r=2,3 \tag{3.24}$$

where n_k is the number of years of record at site k, t_r^k is the rth L-moment ratio for site k, and K is the total number of sites taken into consideration.

Scatter plots of L-moment ratios of the individual sites can be used to determine the sampling variability of the estimates for the average L-moment ratios. For the sake of simplicity, and without losing the power of the method for selecting distributions, it is possible to state that the distribution is mainly characterized by its L-skewness and L-kurtosis, since the location and the scale parameters of the known distributions can be easily matched to the regional mean and L-CV.

One possible way to select the appropriate distribution is by plotting the regional average L-skewness ($\bar{t}_3$) and L-kurtosis ($\bar{t}_4$) values against the L-skewness and L-kurtosis values of a selection of theoretical distributions on a L-skewness - L-kurtosis graph. By simple visual inspection of the graph it would be possible to select the most appropriate distribution if the regional average L-moment ratios exactly match the plots for one of the theoretical distributions.

However, in most circumstances, the regional average L-moment ratios do not exactly correspond to the plots for any of the theoretical distributions. In this situation, it is beneficial to know how far the regional plot is from the plot for each of the theoretical distributions. It is possible to estimate the significance of the difference using the sampling variance of L-kurtosis, $\bar{t}_4$. Let σ_4 be the standard deviation of $\bar{t}_4$, which can be obtained by repeated simulation of a homogeneous region with an assumed parent distribution, where the sites have the same record lengths as the observed data. Then, the goodness of fit measure can be defined as:

$$Z^{DIST} = \frac{\bar{t}_4 - \tau_4^{DIST}}{\sigma_4} \tag{3.25}$$

where τ_4^{DIST} represents the L-kurtosis of the theoretical distribution.

The method described above assumes that the sample L-moment ratio $\bar{t}_4$ is unbiased. In most cases, this assumption is reasonably good. However, when the record lengths are short, (i.e., $n_i \leq 20$), or the population L-skewness is large, (i.e., $\tau_3 \geq 0.4$), the L-kurtosis should not be compared with the $\bar{t}_4$, but with the bias corrected version, $\bar{t}_4 - \beta_4$, where β_4 is the bias in the regional average L-kurtosis for regions with the same number of sites and the same record lengths as the observed data. The bias correction β_4 can be calculated as:

$$\beta_4 = \frac{1}{N_{Sm}} \sum_{m=1}^{N_{Sm}} (\bar{t}_4^{\,m} - \bar{t}_4) \tag{3.26}$$

where $\bar{t}_4^{\,m}$ is the regional average L-kurtosis for the mth simulated region, and N_{Sm} is the number of simulations. Then the standard deviation of $\bar{t}_4$ is:

$$\sigma_4 = \left(\frac{1}{N_{Sm}-1} \left[\sum_{m=1}^{N_{Sm}} (\bar{t}_4^{\,m} - \bar{t}_4)^2 - N_{Sm} \beta_4^{\,2} \right] \right)^{1/2} \tag{3.27}$$

Therefore the bias corrected goodness of fit measure for each distribution can be defined as:

$$Z^{DIST} = \frac{\tau_4^{DIST} - \bar{t}_4 + \beta_4}{\sigma_4} \tag{3.28}$$

Small values of Z^{DIST} indicate that the region is consistent with the associated distribution indicated by DIST. A reasonable criterion for a small value is $|Z^{DIST}| \leq 1.64$. This criterion corresponds to acceptance of the hypothesized distribution at a confidence level of 90%, if the estimated at-site L-kurtosis values have normal distributions. The Z statistic then has the form of a significance test for goodness of fit and has approximately a standard normal distribution.

It should be noted that this method for goodness of fit is not recommended as a formal goodness of fit test (Hosking and Wallis, 1993), since the assumption that the distribution for the Z statistic is normal, is unlikely to be satisfied. In addition, this method cannot confirm the identification of the true distribution, but it does allow the selection of likely candidates and removes inappropriate candidates from further consideration.

A computer program for calculating the above goodness of fit measure is available from Hosking (1991). It should be noted that the computer program incorporates an additional assumption. For the simulation purposes, it assumes that the parent distribution is the four parameter kappa distribution. This assumption is very reasonable if the true parent distribution is from the family of three parameter distributions, since some of the three parameter distributions represent special cases of the kappa distribution. One advantage of this assumption is that the method is not committed in the early stages to any particular theoretical distribution.

3.3 REGIONALIZATION

The basis for regional flood frequency analysis is the region. The region is a set of sites at which observations are assumed to follow the same parent distribution. A homogeneity test should be applied for the examination of the above assumption. Then, if the assumption is satisfied, it is possible to transfer useful information from the surrounding stations to the site of interest to enhance the estimation of the flood frequency characteristics. This idea is followed in this work. Regions are incrementally formed and after each stage of the region forming process, a homogeneity test is applied to check the assumption of homogeneity.

The regionalization method used in this work employs an idea first suggested by Acreman and Wiltshire (1987) and Acreman (1987). They realized that the site of interest cannot always be assigned to one of the previously formed regions. Instead, they employed a fractional membership approach. The fractional membership approach is more efficient if there are a large number of regions, which increases the homogeneity of each region and at the same time decreases the membership of each. They elaborate on this issue even further, to the point that they decreased the size of regions to only one station. In other words they dispensed with regions and formed a group of gauging stations where the site of interest is in the "centre" of a number of similar gauging stations.

Later the method was expanded and implemented and referred to as the region of influence (ROI) approach by Burn (1990a,b). The foundation of the regionalization technique is the identification of a region of influence separately for each site. Thus, regions are created in a flexible way such that there are no rigid boundaries, but rather the regions can overlap. Each site can be considered to have its own region which consists of the collection of stations that comprise the ROI for that site.

The ROI consists of stations that are in close proximity to the site of interest. The proximity is measured by a weighted Euclidean distance in an M-dimensional attribute space, where the attributes are measures applicable for the identification of stations with a similar extreme flow response. The distance metric used is defined as:

$$D_{jk}=\left(\sum_{i=1}^{M} W_i\,(C_{ji}-C_{ki})^2\right)^{1/2} \tag{3.29}$$

where D_{jk} is the weighted Euclidean distance from site j to site k, M is the number of attributes included in the distance measure, W_i is the weight associated with attribute i, and C_{ji} is a standardized value for attribute i for site j.

Standardization of attributes is necessary to remove units and to avoid introduction of bias due to scaling differences of the attributes. While several methods are available for data standardization, in this work the attribute is standardized by dividing each value by the standard deviation of the data calculated for attribute values from the total number of stations. In this way the dimensionality and the differences in the variability of the attributes are considered. The potential drawback of this approach is that any possible useful information on the variability of a particular measure could be lost. However, this difficulty can be overcome by the selection of appropriate weights for the attributes.

Attributes can be basin physiographic measures, such as basin size, stream length, channel slope, stream density, soil type, underlying geology, cultivation, and lake storage. Meteorologic factors, such as type of region (whether humid or arid), storm directions, precipitation intensities, or snowmelt contributions, can also be included. The two types of attributes from above can be combined into a single measure such as, the mean annual runoff, which indicates the available precipitation, and also reflects the runoff inducing parameters of the basin. Flow statistics such as the coefficient of variation, skewness, standardized ten year quantile, or other parameters derived from the extreme flow data, can also be used as indicators of station similarity. All the attributes considered for use in the process of calculating the proximity measure should be easily obtainable from published information. Therefore, there is a compromise between the influential attributes, which have been shown to have a significant correlation with extreme flow events, and those which can be easily derived but are not necessarily as highly correlated with extreme flow events.

Using Equation (3.29), it is possible to assign a numerical value to each pair of gauging stations which indicates the dissimilarity of one gauging station to another, based on the M chosen attributes. This result can be presented in the form of a lower triangular dissimilarity matrix with zeros on the main diagonal. The regionalization process for a given site involves successively adding gauged stations to the ROI for a site, starting with the most similar station and continually adding the next most similar station. The process terminates when adding an additional station would lead to a lack of homogeneity for the collection of stations in the ROI for the site. The homogeneity of the ROI is evaluated after each station addition using a homogeneity test. The set of stations in a site's ROI are thus defined as:

$$I_i = \{K : Stat_R \leq Stat_K\} \tag{3.30}$$

where I_i indicates the set of stations in the ROI for site i, K indicates the number of stations included in the ROI, $Stat_R$ represents the value describing the regional homogeneity, and $Stat_K$ represents the critical value of a homogeneity statistic which varies as a function of the number of stations in the region of influence.

In the process of sequentially adding sites to a region of influence, a station added that results in a heterogenous ROI is deleted and the next most similar station is evaluated. The skipping of one or more stations is implemented to avoid premature cessation of the ROI building process which may occur due to the existence of one or more unusual stations.

There are different strategies that may be used to determine how many stations should be skipped before the regionalization process terminates for a given site. If, in the phase of calculating the dissimilarity matrix, the attributes represent some form of extreme flow statistic, the station skipping process might be completely eliminated. In other words, at the appearance of the first station which results in a heterogeneous ROI, the regionalization process concludes without including this last station in the ROI. However, if the attributes represent a combination of extreme flow statistics and basin characteristics, or they are all basin characteristics, the skipping feature in creating the ROI may be advantageous due to the variable correlation between the attributes and the flood generating mechanisms. The ideal situation would be if the flooding potential for each site could be completely described with the set of basin characteristics. However, this situation will seldom occur and the feature of skipping outlier stations will then lead to an ROI that will consist only of stations with similar flooding characteristics. For a particular site, the skipping of stations could continue until there are no further candidates to enter into the ROI. However, it is desirable to fully investigate the trade-off between a large number of sites in the ROI carrying a large amount of information, but with a lower homogeneity of the ROI, and a limited number of stations with less information, but a higher level of homogeneity (Zrinji and Burn, 1993). This implies that different termination strategies should be investigated to explore this trade-off.

Once the order of stations entering into the ROI for a site is determined, it might be expected that different homogeneity tests, with carefully selected significance level, will perform in the same way, and thus result in the selection of the same set of stations for the ROI. This is not, however, always true. A powerful homogeneity test is essential in order to detect dissimilar stations and prevent their inclusion in the ROI. This becomes more important when the number of stations in the ROI becomes fairly large. It is then possible for the median value for the flow characteristics of the stations in the ROI to deviate from the flow characteristics for the site of interest. This can result in the acceptance of a station into ROI even though the station has substantively different hydrologic characteristics from the site of interest. This is possible because increasing the number of sites included in the ROI results in a larger number of degrees of freedom in the homogeneity test and greater heterogeneity is therefore possible. Thus, it is preferable to perform a homogeneity test with a stringent entering criterion in order to avoid stations entering that will potentially increase the heterogeneity of the region of influence for a site.

A sufficient number of stations are required in order to perform an efficient regional analysis, keeping in mind that the purpose of the regional analysis is to reduce the sampling error and to obtain a reliable estimation of extremes. With just a few stations included in the ROI, the influence of each station will increase allowing some station to have an overwhelmingly large effect on the estimated extreme flow. This is especially problematic when information from each station is weighted by the record length at the station. Jin and Stedinger (1989) suggested the record length not be used for weighting each station's information when a regional value is calculated and some stations have much longer record lengths than others. On the other hand, too many stations in the ROI can have a negative effect as well. If the ROI consists of a large number of stations then the influence of each station has been reduced, but the information pooled from these stations can lead to a false estimate due to the stations in the ROI no longer being representative of the site of interest.

3.4 HOMOGENEITY

The selection of the homogeneity test is essentially independent of the grouping process described above in section 3.3. The selected homogeneity test should be powerful enough to detect any heterogeneity among the stations included in the region of influence. The homogeneity test must be able to separate sites with similar flow characteristics from sites which do not come from the same regional parent distribution.

One of the homogeneity test used herein is based on an idea developed by Chowdhury et al. (1991) and involves the use of sample L-moment ratios to determine if the at site L-moment ratios are similar to the L-moment ratio of the regional parent distribution. This homogeneity test is applicable when the parent distribution is the GEV distribution since the homogeneity test is specific to the GEV distribution. In a case where the parent distribution is not the GEV distribution, the homogeneity test proposed by Hosking and Wallis (1991) could be used. Both homogeneity tests will provide acceptable and similar results, but the homogeneity test by Chowdhury et al. (1991) is preferred due to the substantially lower computational requirements.

3.4.1 HOMOGENEITY TEST 1

The description of the homogeneity test, presented below, follows from Chowdhury et al. (1991).

In order to test regional homogeneity the χ^2_R statistic has been employed. The χ^2_R statistic is defined as:

$$\chi^2_R = \sum_{j=1}^{K} \chi^2_j \tag{3.31}$$

where χ^2_j is the contribution to χ^2_R from station j and K is the total number of stations in the region.

The statistic χ^2_R follows the χ^2 distribution with 2(K-1) degrees of freedom when the observations at each site are independent. The region can be considered as homogeneous if the calculated value of χ^2_R is less than the tabulated critical χ^2 value at a selected level of significance for the appropriate number of degrees of freedom.

The value of χ^2_j is defined as:

$$\chi^2_j = \begin{bmatrix} t_2 - t^R_2 \\ t_3 - t^R_3 \end{bmatrix}^T \begin{bmatrix} Var(t_2) & Cov(t_2,t_3) \\ Cov(t_2,t_3) & Var(t_3) \end{bmatrix}^{-1} \begin{bmatrix} t_2 - t^R_2 \\ t_3 - t^R_3 \end{bmatrix} \tag{3.32}$$

where t_2 and t_3 are the estimates for the L-CV and L-skewness, respectively, at site j, t^R_2 and t^R_3 are weighted regional estimates for L-CV and L-skewness defined by Equation (3.24), and $Var(t_2)$, $Var(t_3)$ and $Cov(t_2,t_3)$ are the asymptotic variance and covariance of the sample L-moment ratios defined as:

$$Var[t_2] = Var[l_2/l_1] \cong \frac{l_2^2}{l_1^4} Var[l_1] + \frac{1}{l_1^2} Var[l_2] - 2\frac{l_2}{l_1^3} Cov[l_1,l_2] \tag{3.33}$$

$$Var[t_3]=Var[l_3/l_2]\cong\frac{l_3^2}{l_2^4}Var[l_2]+\frac{1}{l_2^2}Var[l_3]-2\frac{l_3}{l_2^3}Cov[l_2,l_3] \tag{3.34}$$

$$\begin{aligned}Cov[t_2,t_3]=Cov[l_2/l_1,l_3/l_2]\cong&\frac{l_3}{l_1^2l_2}Cov[l_1,l_2]-\frac{1}{l_1^2}Cov[l_1,l_3]\\&+\frac{1}{l_1l_2}Cov[l_2,l_3]-\frac{l_3}{l_1l_2^2}Var[l_2]\end{aligned} \tag{3.35}$$

The variances and covariances of L-moment estimators l_1, l_2, and l_3 are elements of the matrix $[\Lambda V\Lambda^T]$ where Λ is the coefficient matrix of PWM vectors $[\beta_0, \beta_1, \beta_2]^T$ in Equation (3.11), (3.12) and (3.13) and V is the matrix of asymptotic variances of unbiased PWM estimators b_0, b_1, b_2 in Equation (3.17). The Λ matrix is defined as:

$$\Lambda=\begin{bmatrix}1 & 0 & 0\\ -1 & 2 & 0\\ 1 & -6 & 6\end{bmatrix} \tag{3.36}$$

Elements of the symmetrical matrix V for the first three unbiased PWM estimators are (Hosking, 1986):

$$V_{1,1}=\frac{\alpha^2}{n\kappa^2}[\Gamma(1+2\kappa)-\Gamma^2(1+\kappa)] \tag{3.37}$$

$$V_{2,2}=\frac{2^{-2\kappa}\alpha^2}{n\kappa^2}[\Gamma(1+2\kappa)G(1/2)-\Gamma^2(1+\kappa)] \tag{3.38}$$

$$V_{3,3}=\frac{3^{-2\kappa}\alpha^2}{n\kappa^2}[\Gamma(1+2\kappa)G(2/3)-\Gamma^2(1+\kappa)] \tag{3.39}$$

$$V_{1,2}=V_{2,1}=\frac{\alpha^2}{2n\kappa^2}[2^{-2\kappa}\Gamma(1+2\kappa)+(1-2^{1-\kappa})\Gamma^2(1+\kappa)] \tag{3.40}$$

$$V_{2,3}=V_{3,2}=\frac{\alpha^2}{2n\kappa^2}[3^{-2\kappa}\Gamma(1+2\kappa)G(1/3)+(2^{-2\kappa}-2^{1-\kappa}3^{-\kappa})\Gamma^2(1+\kappa)] \tag{3.41}$$

$$V_{1,3}=V_{3,1}=\frac{\alpha^2}{2n\kappa^2}[3^{-2\kappa}\Gamma(1+2\kappa)-2^{-2\kappa}\Gamma(1+2\kappa)G(1/2)+2(2^{-\kappa}-3^{-\kappa})\Gamma^2(1+\kappa)] \tag{3.42}$$

where α and κ are the scale and shape parameters respectively of the GEV distribution. G denotes the hypergeometric function, and can be calculated as:

$$G(x)=1+\frac{2\kappa^2}{\Gamma(1+2\kappa)}\sum_{m=1}^{\infty}\frac{\Gamma(2\kappa+m)(-x)^m}{(\kappa+m)m!} \tag{3.43}$$

and $\Gamma()$ is the gamma function which is defined for positive arguments as:

$$\Gamma(x)=\int_0^{\infty}e^{-t}t^{x-1}dt \tag{3.44}$$

For a range of [-1,0] the value of the gamma function can be calculated by using the property of a gamma function as:

$$\Gamma(x+1)=x\Gamma(x) \tag{3.45}$$

Equations (3.37) to (3.42) are valid if κ is in the range $[-1/2,\infty)$.

3.4.2 HOMOGENEITY TEST 2

The second homogeneity test used in this study was proposed by Hosking and Wallis (1993) and is based on various orders of sample L-moment ratios. This homogeneity test is based on the idea that in a homogeneous set of stations, all stations have the same population L-moments. This is, however, not true for the sample L-moments, due to the sampling variance. The question arises as to whether any difference is introduced by L-moment ratios from different populations or simply as a result of sampling error. Therefore, the level of the sampling, or random, error has to be determined in order to successfully use this homogeneity test. Simulation can be applied for the establishment of the acceptable variability level.

This homogeneity test has three different levels for testing the homogeneity. The homogeneity is tested by the variation of L-moments of different order, represented by the following three measures:

1) Homogeneity test based on the L-CV only is defined as a weighted standard deviation of L-coefficient of variation (L-CV):

$$V_1 = \frac{\sum_{i=1}^{N_R} n_i (t_{2(i)} - \bar{t}_2)^2}{\sum_{i=1}^{N_R} n_i} \tag{3.46}$$

2) Homogeneity test based on the L-CV and L-skewness is defined as the weighted average distance from the site to the group weighted mean in L-CV L-skewness space:

$$V_2 = \frac{\sum_{i=1}^{N_R} n_i \left[(t_{2(i)} - \bar{t}_2)^2 + (t_{3(i)} - \bar{t}_3)^2 \right]^{1/2}}{\sum_{i=1}^{N_R} n_i} \tag{3.47}$$

3) Homogeneity test based on the L-skewness and L-kurtosis is defined as the weighted average distance from the site to the group weighted mean in L-skewness L-kurtosis space:

$$V_3 = \frac{\sum_{i=1}^{N_R} n_i \left[(t_{3(i)} - \bar{t}_3)^2 + (t_{4(i)} - \bar{t}_4)^2 \right]^{1/2}}{\sum_{i=1}^{N_R} n_i} \tag{3.48}$$

where $\bar{t}_2$, $\bar{t}_3$ and $\bar{t}_4$ are the group mean L-CV, L-skewness and L-kurtosis respectively, $t_{2(i)}$, $t_{3(i)}$ and $t_{4(i)}$ are the L-CV, L-skewness and L-kurtosis respectively at site i, N_R is the number of sites in the region of influence and n_i is the record length at site i.

In order to calculate the homogeneity measure it is necessary to calculate the mean and the standard deviation of V_1, V_2 and V_3. This can be done through simulation and involves the generation of a large number of regions. Regions are generated with the properties that: (i) the data are derived from a kappa world, (ii) there is no cross-correlation or serial correlation in the data and (iii) the sites have the same record lengths as in the case under investigation. For each simulation the values of V_k (k=1,2,3) are calculated. Then the μ_{Vk} and σ_{Vk} are calculated as the mean and the standard deviation for each V_k. The heterogeneity measure is then defined as:

$$H_k = \frac{V_k - \mu_{V_k}}{\sigma_{V_k}} \qquad \text{for } k=1,2,3 \tag{3.49}$$

According to Hosking and Wallis (1993), a region can be declared homogeneous with a corresponding order of L-moment if the H value is less than 1 ($H<1$). The region is possibly homogeneous if the value of H is between 1 and 2 ($1 \leq H < 2$) and the region is definitely declared heterogeneous if the value of H is greater than or equal to 2 ($H \geq 2$) (Hosking and Wallis, 1993).

3.5 ESTIMATION OF THE EXTREME FLOW QUANTILES

Once the ROI for a given site has been determined and an appropriate distribution has been selected, the estimation of the parameters for a given distribution can proceed in order to estimate extreme flow quantiles. Two approaches for parameter estimation are presented below. The first is based on a fixed size ROI, and the second employs the hierarchical approach. By completing the parameter estimation phase, it is possible to estimate the standardized extreme flow quantiles. In order to obtain the magnitude of the extreme flow, the standardized extreme flow is multiplied by the index flood for the site of interest. For the estimation of the index flood at an ungauged site, the method of nonlinear regression is used, as presented at the end of this section.

3.5.1 PARAMETER ESTIMATION

The most frequently used parameter estimators in flood frequency analysis are the maximum likelihood (ML), the probability weighted moment (PWM), and recently the variation of the PWM, the L-moment based parameter estimator.

The asymptotic standard error of ML and the PWM methods are very similar, but for a small sample size (n<50), the standard error of the estimated parameters is less by the PWM than the standard error of the estimated parameters by the method of ML (Greis and Wood, 1981). Therefore, in practice, where only a finite sample is available, the method of PWM is more acceptable. In addition, the PWM method has similarities with the conventional method of moments and thus it is more tractable than the ML method. Furthermore, the PWM method always gives a fast and feasible solution, while the ML in some cases requires a large number of iterations and sometimes the solution is infeasible.

In order to obtain a regional estimation of parameters using the PWM method, two weighting schemes are presented herein. The first method of weighting information from each site included in the ROI is by the record length at each gauged station. Weighting by record length implies that the stations with longer flow records provide greater amounts of information about the regional flood frequency relationship. Scaled PWMs for each site j in the region are calculated as:

$$t_{j1}=\frac{b_{j1}}{b_{j0}} \qquad \forall\ j \tag{3.50}$$

$$t_{j2}=\frac{b_{j2}}{b_{j0}} \qquad \forall\ j \tag{3.51}$$

where b_{j1} and b_{j2} are the unbiased sample estimators for PWMs for site j defined by Equation (3.17). Regional PWMs are then calculated as the weighted average of the scaled at-site PWMs through:

$$T_i = \frac{\sum_{j=1}^{K} t_{ji} n_j}{\sum_{j=1}^{K} n_j} \qquad \text{for i=1,2} \tag{3.52}$$

A second method for weighting information from stations in the region of influence is by the proximity measure defined by Equation (3.29). In this case, the distribution parameters and extreme flows associated with various return periods are estimated at each gauged site individually. The parameter estimation process starts with the calculation of the PWMs, and the distribution parameters can then be estimated. Once the parameters of the distribution have been estimated, standardized extreme flows can be obtained for each station and these standardized extreme flows then weighted to obtain the estimated standardized extreme flow at the site of interest. The weights applied to the standardized extreme flows for a particular station are inversely proportional to the distance defined by Equation (3.29). The weight, w_j, for station j is defined as:

$$w_j = \frac{\frac{1}{D_{0j}}}{\sum_{j=1}^{K} \frac{1}{D_{0j}}} \tag{3.53}$$

where D_{0j} represents the distance (in attribute space) between the site of interest and the regional station j. This weighting scheme implies that stations that exhibit greater similarity with the site of interest are more important for estimating at-site extreme flow quantiles.

For completeness, a set of common distribution functions used in flood frequency analysis is presented in the Appendix A along with the corresponding estimations of parameters via L-moments, taken from Hosking (1990).

3.5.2 HIERARCHICAL APPROACH TO PARAMETER ESTIMATION

Regional flood frequency analysis based on the index flood method is highly structured. The ROI provides an ideal vehicle to capture the benefits of the structured format. A natural extension of the ROI approach is the use of the hierarchical approach for the estimation of parameters of a selected theoretical distribution.

It can be observed that by including more stations in the ROI the variance of the mean annual maximum floods increases more rapidly than does the coefficient of variation of the annual maximum floods. In addition, the increase in the variance of the skewness is less than the increase in variance of the CV as more stations enter the ROI. There is a similar behaviour of the statistics of the observed variable between the large region based on a traditional regionalization approach and the ROI approach, which indicates the possible application of the hierarchical approach to the ROI.

The hierarchical approach was successfully implemented by Gabriele and Arnell (1991), wherein the approach was based on fixed regions. The shape parameter of the distribution was estimated on the basis of an entire set of stations in the region, the scale parameter was estimated based on a subset of stations (i.e., subregions), while the location parameter was estimated on the basis of information obtained at site. Regions and subregions were formed in a manner to obtain an optimum number of regions and subregions, where the criteria of obtaining the optimality was to minimize the mean squared error (MSE) of the parameter to be estimated. Practically, the MSE is a function of time sampling variance and spatial variability of the annual maximum flood.

Applying the hierarchical approach to the ROI, the principle remains the same, namely to include more stations in the ROI for the purpose of estimating the shape parameter than are used for estimating the scale parameter. Although the principle remains the same the implementation of the technique is different. The ROI approach builds the region, starting with the base, which is the site of interest, and sequentially adding more stations to the region, until a homogeneity threshold is reached. By applying a homogeneity test, the size of the ROI can be controlled by the critical χ_K^2 value associated with different significance levels.

In the homogeneity test proposed by Hosking and Wallis (1993), where the homogeneity is based on the variability of different orders of sample L-moments, the concept of a hierarchical approach is also incorporated. The test is based on the variability of different statistics, namely on the sample L-CV, on the sum of L-CV and L-skewness, and on the sum of L-skewness and L-kurtosis. The homogeneity test based on the L-CV statistic results in a ROI with a set of stations which is substantially smaller than the set of stations included in the resulting ROI based on the variability of L-CV and L-skewness. Furthermore, the resulting ROI obtained by the homogeneity test based on the variability of L-CV and L-skewness consists of fewer stations than the resulting ROI obtained by the homogeneity test based on the sum of variability of L-skewness and L-kurtosis. This is a result of higher order moments having less regional variability compared to lower order moments. Although the homogeneity test by Hosking and Wallis readily facilitates the use of the hierarchical approach, the homogeneity test by Chowdhury et al.

(1991) is more practical due to computational aspects.

The incorporation of the hierarchical approach into the ROI framework results in a set of ROIs for the site of interest as opposed to a single ROI. By taking into account the different spatial similarity of different orders of moments for a flood frequency distribution, this approach should lead to improved parameter estimation.

3.5.3 ESTIMATION OF THE MEAN ANNUAL FLOOD

In order to complete the extreme flow estimation at the site of interest, the standardized extreme flows have to be multiplied by the index flood, which is typically the mean annual flood. It is easy to obtain the index flood value if the site of interest is a gauged site with a reasonably lengthy data record. However, if the site of interest is ungauged, or the flow records are short, the estimation of the mean annual flood based on the at-site information is unreliable or simply impossible. In this situation, a regional regression can be applied for the estimation of the mean annual flood.

A nonlinear regression relationship is usually used in hydrology to obtain the required estimate. The regression model has the form of:

$$Q=a_0 C_1^{a_1} C_2^{a_2} \dots C_P^{a_P} \tag{3.54}$$

where Q is the mean annual flood, C_i is the ith basin characteristic, P is the number of basin characteristics included in the regression, and the a_i's are the regression parameters.

The nonlinear form of the regression relationship can be transformed to a linear relationship using:

$$logQ = \log a_0 + a_1 \log C_1 + a_2 \log C_2 + \ldots + a_p \log C_p \tag{3.55}$$

Regression parameters can be estimated using ordinary least squares (OLS) if the residual errors associated with the individual observations are homoscedastic and independently distributed. If this is not the case, a weighted least squares (WLS), or generalized least squares (GLS) parameter estimator should be applied. These estimators incorporate the variances and covariances of the residual errors at each site and thus result in better estimates of the regression parameters, especially when the length of record varies widely from site to site, or when the cross correlation of the flows are greater than 0.6 (Stedinger and Tasker, 1985).

A difficulty with using the WLS or the GLS method arises from the estimation of the variance-covariance matrix of the residual errors. This is addressed in Tasker and Stedinger (1989).

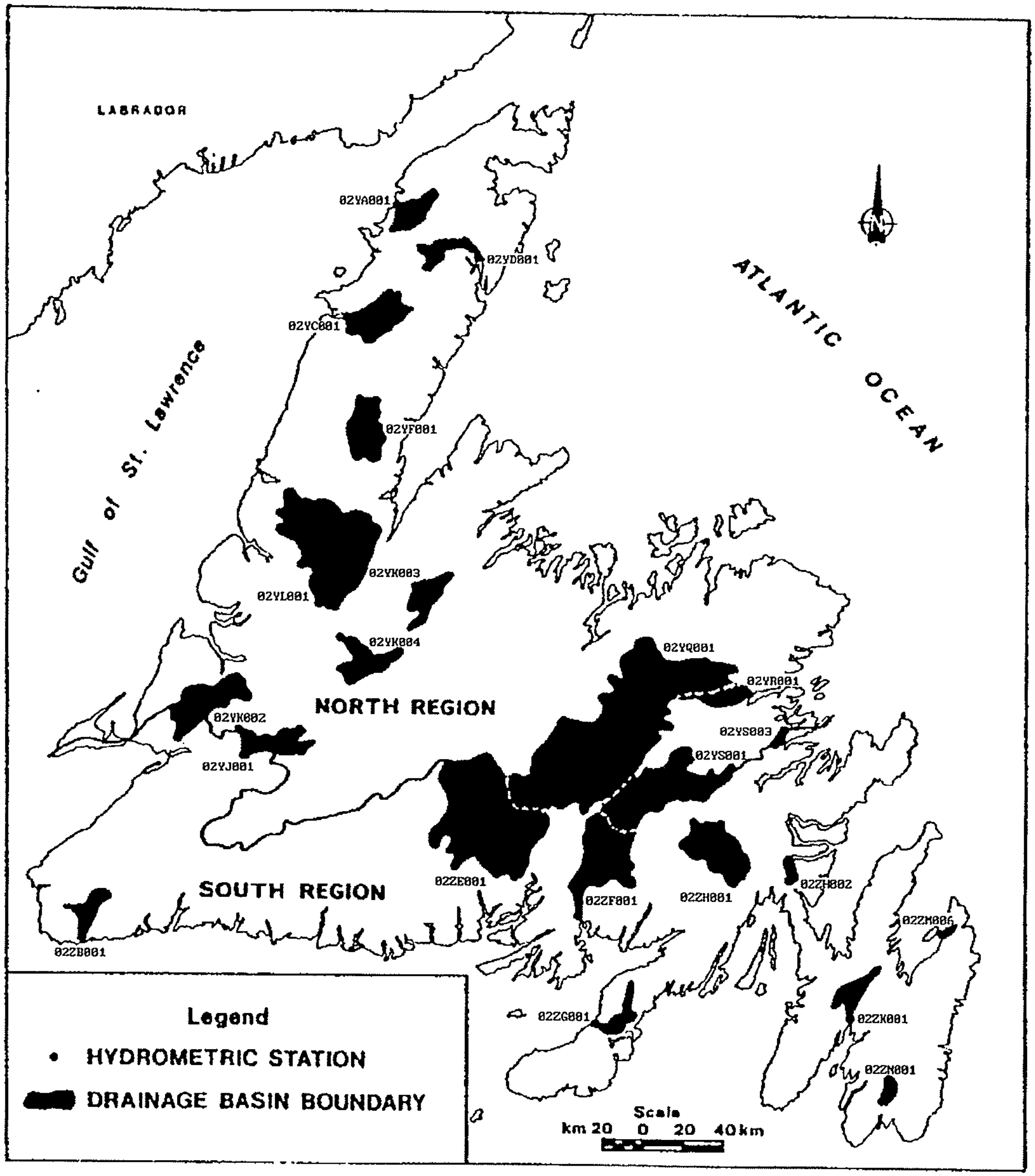
4. EXTREME FLOW ESTIMATION AT UNGAUGED SITES

In this chapter, a new approach to regional flood frequency analysis for ungauged sites is presented. The section starts with a description of the selected case study. In the following sections, the illustrative example is presented with a detailed description of each novel step in the flood frequency analysis. At the end of Chapter 4, the basis for comparison of results is presented along with the presentation and discussion of results.

4.1 DESCRIPTION OF THE CASE STUDY AREA

The study area consists of the entire island of Newfoundland, Canada (see Figure 4.1). The island is roughly triangular in shape and is bounded on the west coast by the Gulf of Saint Lawrence and on the south and northeast by the Atlantic Ocean. Available catchment characteristics for a set of 22 gauging stations include the catchment drainage area (DA) expressed in km^2, a catchment shape factor (SHAPE) which describes the ratio of the drainage area to the drainage perimeter, the percentage of catchment area controlled by lakes and swamps (ACLS), the percentage of barren area (BAREA), the mean annual runoff for the catchment (MAR) expressed in mm, and the latitude of the catchment centroid (LAT) (Panu and Smith, 1988). In addition, annual maximum daily flow data are available for the 22 stations. The length of the flow records varies between 12 and 61 years, with an average record length of 29 years.

Figure 4.1 Location of Gauged Sites and their Watersheds (adapted from Panu and Smith, 1988)



4.2 ILLUSTRATIVE EXAMPLE

The method for extreme flow estimation described in Chapter 3 was applied to the case study described above. Estimates of extreme flows were obtained with each gauging station individually considered to be an ungauged site.

In order to perform a regional flood frequency analysis, the parent distribution function has to be established. In the selection of the parent distribution function, the most commonly used distribution functions were considered, such as generalized extreme value, generalized logistic, generalized normal, lognormal, generalized pareto, kappa, Pearson type III, and Wakeby.

The selection of the appropriate parent distribution function was based on the characteristics of the L-moment ratios obtained at each gauged site. Output from the program by Hosking (1991), presented in the Table 4.1, provides the possible candidates for the parent distribution. A star by the Z statistic in the output of the goodness of fit program (Table 4.1) indicates that the particular distribution could be an acceptable frequency distribution function. The fit of the sample data to a particular distribution is considered to be better as the absolute value of the calculated Z statistic becomes closer to zero.

In addition, the appropriate parent distribution can be selected by visual inspection of the plot of the sample τ_3 versus τ_4 on an L-moment ratio diagram along with the plot of the L-moment ratios for commonly used distributions. This is shown in Figure 4.2. The point corresponding to the mean value of L-skewness and L-kurtosis for the collection of stations is almost coincident with the line representing the GEV distribution. Furthermore, the at-site L-moment ratios are scattered fairly evenly along the line representing the GEV distribution. There are some points that are relatively far from the line representing the GEV distribution, but the main collection of points representing individual site L-skewness and L-kurtosis values is grouped around the GEV distribution function. According to the Z value and the L-moment ratio plot (Figure 4.2) the GEV distribution is an appropriate option as a parent distribution function for the collection of gauging stations.

Table 4.1 Output from the goodness of fit program by Hosking (1991)

***** GOODNESS-OF-FIT MEASURES *****	
GENERALIZED LOGISTIC	Z= 1.33 *
GENERALIZED EXTREME VALUE	Z=-1.29 *
PEARSON TYPE III	Z=-2.26
GENERALIZED PARETO	Z=-6.94

The GEV distribution function is defined as:

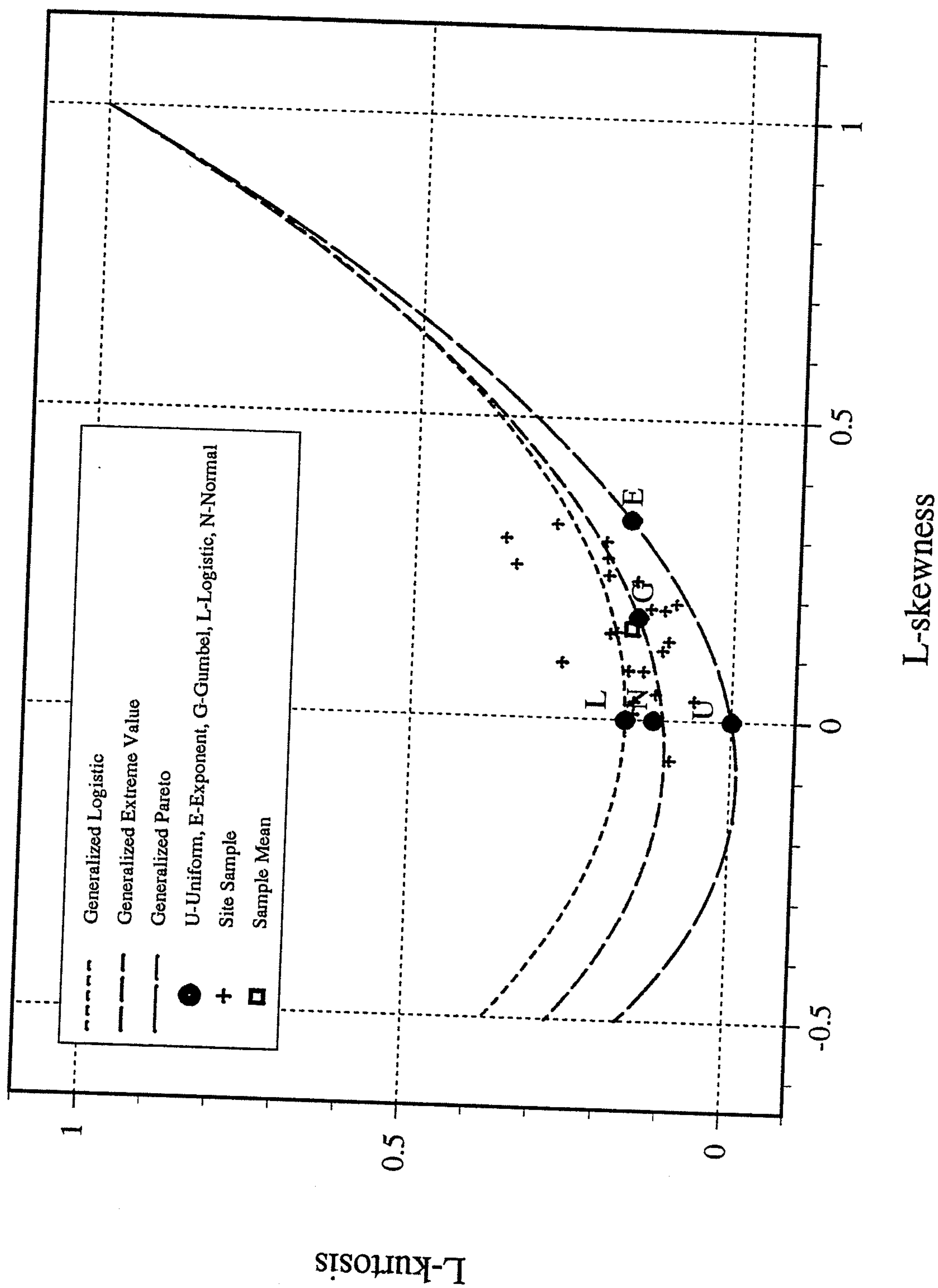
$$F(X)=\exp\left(-\left(1-\frac{k}{\alpha}(X-\xi)\right)^{\frac{1}{k}}\right) \qquad \text{for } k\neq 0 \qquad (4.1)$$

and

$$F(X)=\exp\left(-\exp\left(\frac{1}{\alpha}(x-\xi)\right)\right) \qquad \text{for } k=0 \qquad (4.2)$$

The terms ξ, α, and k are the location, scale and shape parameters respectively. When k=0, the GEV distribution reduces to the Gumbel distribution.

Figure 4.2 L-moment ratio diagram for the case study data



The inverse distribution function is defined as:

$$X_T = \xi + \frac{\alpha}{k}\left(1-\left(-\log\left(1-\frac{1}{T}\right)\right)^k\right) \qquad \text{for } k\neq 0 \qquad (4.3)$$

and

$$X_T = \xi - \alpha\left(\log\left(-\log\left(1-\frac{1}{T}\right)\right)\right) \qquad \text{for } k=0 \qquad (4.4)$$

Parameters for this distribution can be estimated by L-moments, as shown in Appendix A, or using PWMs, through (after Hosking et al., 1985b):

$$c = \frac{2T_1-1}{3T_2-1} - \frac{log2}{log3} \qquad (4.5)$$

$$k = 7.8590c + 2.9554c^2 \qquad (4.6)$$

$$\alpha = \frac{(2T_1-1)k}{\Gamma(1+k)(1-2^{-k})} \qquad (4.7)$$

$$\xi = 1 + \frac{\alpha}{k}\left(\Gamma(1+k)-1\right) \qquad (4.8)$$

Once the selection of the parent distribution is made, the next step is to create homogeneous regions. To form a site specific region of influence, the combination of the six available catchment characteristics were used in Equation (3.29) in order to form a dissimilarity matrix. The catchment characteristics are presented in Table 4.2, along with the corresponding record length (RL), for each station. The selected catchment characteristics are the DA, SHAPE and MAR. This group of basin attributes is comparable to the basin characteristics used by Panu and Smith (1988) to derive a regression relationship between basin characteristics and extreme flows for the entire island. In this study, ACLS is not included in the process of defining the ROI, because ACLS proved to be a poor indicator of a site's extreme flow characteristics. In other words, with the inclusion of ACLS in the regionalization process, the size of the resulting ROI was smaller than the size of the ROI formed without the ACLS, when the same homogeneity level was used. The remaining two catchment characteristics, BAREA and LAT showed a similar behaviour as the ACLS did, namely, that the size of the ROI obtained was smaller when these basin characteristics were included in the calculation of dissimilarity than was obtained without them.

The weights allocated to catchment characteristics represent the contribution of these significant catchment characteristics to the representation of similarity in terms of the extreme flow. Weights are allocated to DA, SHAPE, and MAR as 0.91, 0.03 and 0.06 respectively. This weight distribution clearly indicates that the basin area has an overwhelmingly large influence in the calculated dissimilarity measure. This resulted because the available catchment characteristics cannot fully represent the characteristics of the extreme flow quantiles. Additional catchment characteristics, such as the main basin slope, soil type, vegetation cover, or others could improve the relationship between catchment characteristics and extreme flow quantiles. However, the extension of the catchment characteristic data set is beyond the scope of this study and therefore this study is based on the available data set. Weights were obtained on the basis of trial and error, with an objective to maximize each site's ROI size. The weights presented above represent the best combination of weights providing the most homogeneous ROIs. Weight allocation started with an uniform weight distribution and continued by a trial and error until a satisfactory set of weights were obtained.

In order to construct a site specific ROI, the set of stations are ranked by similarity with the site of interest. Once the rank of the stations is obtained, catchment characteristics no longer have an influence on the composition of the site specific ROI. In the remainder of the process, the flow data at the stations are used to test the homogeneity of the proposed ROI.

Table 4.2 Summary of catchment characteristics

#	**Gauging Station Name**	**ID #**	**DA**	**BAREA**	**SHAPE**	**ACLS**	**MAR**	**LAT**	**RL**
			[km^2]	[%]	[.]	[%]	[mm]	[°]	[yrs]
1	Ste. Genevieve River	02YA001	306.	0.2	1.48	96	982	51.10	20
2	Torrent River	02YC001	624.	49.8	1.45	99	1330	50.62	30
3	Beaver Brook	02YD001	237.	11.2	2.23	73	1197	50.94	20
4	Cat Arm River	02YF001	611.	18.0	1.86	100	1420	50.16	15
5	Harrys River	02YJ001	640.	6.9	1.81	75	1321	48.75	21
6	Lewaseechjeech Brook	02YK002	470.	29.0	2.32	100	1162	48.57	17
7	Sheffield River	02YK003	391.	15.2	1.98	94	856	49.28	12
8	Hinds Brook	02YK004	529.	29.3	1.78	95	984	48.96	24
9	Upper Humber River	02YL001	2110.	14.5	1.56	75	1250	49.53	61
10	Gander River	02YQ001	4400.	6.9	2.08	91	853	48.63	40
11	Middle Brook	02YR001	267.	0.8	1.93	98	788	48.80	30
12	Terra Nova River	02YS001	1290.	14.8	2.35	92	898	48.38	34
13	Southwest Brook	02YS003	36.7	0.5	1.43	100	929	48.58	22
14	Isle Aux Morts River	02ZB001	205.	78.2	2.09	60	2124	47.75	27
15	Salmon River	02ZE001	2640.	49.6	1.75	100	943	48.22	22
16	Bay Du Nord River	02ZF001	1170.	44.1	2.15	96	1082	48.04	39
17	Garnish River	02ZG001	205.	63.4	2.45	96	1332	47.28	31
18	Pipers' Hole River	02ZH001	764.	23.4	1.67	91	1024	48.05	37
19	Come By Chance River	02ZH002	43.3	49.7	1.66	92	1364	47.96	20
20	Rocky River	02ZK001	285.	37.1	2.0	55	1251	47.36	41
21	Northeast Pond River	02ZM006	3.9	3.6	1.24	100	1100	47.65	36
22	Northwest Brook	02ZN001	53.3	78.8	2.06	100	1847	46.91	23

The result of the first part of the ROI forming process is a ranked set of stations for each site. The stations, ranked by proximity in basin attribute space, are presented in Table 4.3. The ordering of the stations for a given site is determined from the dissimilarity matrix, which is a symmetric matrix with zeros on the main diagonal and the remaining elements representing the weighted distance between two sites in the M dimensional attribute space. For example, station 13 is the closest station to site 1 and also the closest station to site 21. Therefore, it is not surprising that stations 1 and 21 are the closest stations to site 13. For any site in Table 4.3, as the station order increases, the corresponding distance between the selected site and the corresponding station, also increases.

Table 4.3 Rank of stations in basin attribute space

#	**Station Order**																				
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**
1	13	8	11	7	21	20	19	2	18	3	5	4	6	17	22	16	14	12	9	15	10
2	5	4	18	8	1	20	6	7	19	3	11	21	13	16	17	22	14	12	9	15	10
3	20	17	6	7	11	19	8	4	1	5	13	22	2	21	18	14	16	12	9	15	10
4	5	2	20	6	8	18	3	7	1	17	19	11	16	22	14	13	21	12	9	15	10
5	4	2	18	8	6	20	7	1	3	17	11	16	19	13	21	22	12	14	9	15	10
6	3	20	17	7	8	5	4	11	18	1	2	19	13	16	22	21	12	14	9	15	10
7	11	8	1	6	20	3	18	5	13	17	4	2	19	21	16	22	12	14	9	15	10
8	7	18	1	5	11	6	20	2	4	3	13	17	19	21	16	12	22	14	9	15	10
9	15	12	16	18	5	2	4	8	6	7	1	20	11	3	17	14	19	13	21	22	10
10	15	9	12	16	18	5	2	4	8	6	7	1	20	11	3	17	14	13	19	22	21
11	7	1	8	3	13	20	6	21	17	19	18	5	4	2	22	16	12	14	9	15	10
12	16	18	5	8	6	4	2	7	9	11	20	3	1	17	13	19	15	21	22	14	10
13	21	1	19	11	7	20	8	3	2	17	6	5	18	4	22	14	16	12	9	15	10
14	22	17	19	4	20	3	5	6	2	21	8	1	7	13	18	11	16	12	9	15	10
15	9	12	16	10	18	5	2	4	8	6	7	1	20	11	3	17	13	19	14	21	22
16	12	18	5	4	8	6	2	7	20	3	11	1	17	9	19	13	21	22	14	15	10
17	3	20	6	19	22	7	4	11	5	8	1	14	13	2	21	18	16	12	9	15	10
18	8	5	2	4	7	1	6	16	20	11	3	12	13	17	19	21	22	14	9	15	10
19	20	21	13	3	1	22	17	11	7	4	8	2	6	5	14	18	16	12	9	15	10
20	3	6	17	19	7	8	4	1	5	11	2	13	21	22	18	14	16	12	9	15	10
21	13	19	1	20	11	7	8	3	2	5	4	17	6	22	18	14	16	12	9	15	10
22	14	19	17	20	3	4	6	5	21	2	1	13	8	7	11	18	16	12	9	15	10

The estimation of extreme flows at the ungauged site starts with the construction of the site specific ROI which entails selecting stations that are in the close proximity to the site of interest. In this problem context, the proximity is calculated in basin attribute space, since basin attributes represent the only information available for an ungauged site. The homogeneity test incorporated into the process examines the similarity of the extreme flow data, pooled from the series of annual maximum daily flows, at the gauged sites. Thus, the ungauged site of interest is not included in the examination of the homogeneity of its own ROI. This is the main reason why it is so important to select those basin characteristics which are highly related to the occurrence of extreme flow events.

In the process of testing the homogeneity of the collection of selected stations, by the homogeneity test proposed by Chowdhury et al. (1991), it is very important to select an appropriate significance level. The significance level indicates the probability of committing an error of rejecting the hypothesis when it is actually true (Hogg and Craig, 1969). For example, for the value of α=0.01, a probability of 1% is associated with the event that the region is considered heterogeneous when in fact the region is homogeneous. With a lower significance level, a region with more stations, but lower homogeneity, will result and vice versa. Since the characteristics of the extreme flow events at the site of interest are not known, it is desirable to have a larger number of gauged sites in the region in order to reduce the influence of each site. However, with the increased number of gauged sites, the centroid of the region can be shifted away, in the basin attribute space, from the position of the site of interest. This could result in an inaccurate estimation of the extreme flow because of the assumption that the combination of basin characteristics are related to the extreme flow events.

Several significance levels were applied in the homogeneity test and the sizes of the resulting ROIs are shown in Table 4.4.

Table 4.4 Size of ROI with various degrees of homogeneity

	Significance level α					
#	**0.005**	**0.01**	**0.025**	**0.05**	**0.10**	**0.25**
1	18	16	12	12	5	5
2	7	7	6	6	6	5
3	3	3	3	3	3	2
4	7	7	7	5	4	4
5	6	5	5	4	4	4
6	18	18	4	3	3	3
7	18	6	5	4	4	4
8	6	5	5	2	2	2
9	11	9	9	8	8	2
10	3	3	3	3	3	3
11	6	6	1	1	1	1
12	7	7	4	4	4	3
13	18	10	6	5	4	4
14	18	18	16	12	7	4
15	2	2	2	2	2	2
16	13	13	8	7	7	5
17	5	5	2	2	2	2
18	8	7	7	6	6	5
19	18	16	15	12	8	8
20	4	4	4	1	1	1
21	18	16	15	12	5	5
22	18	16	6	6	6	6

From Table 4.4, it can be seen that the number of sites included in the ROI decreases as the significance level increases. Upon close analysis of each site's ROI, it is noticed that some stations are seldom included in a site specific ROI, and those site's ROIs consist of a limited number of stations (see Table 4.3 and Table 4.4, stations 9, 10 and 15). These three stations have the largest drainage areas and therefore they are in a distinct position relative to the rest of the stations, indicating that they have a specific combination of basin characteristics. The position of these stations in the basin attribute space makes it difficult to find stations with a similar combination of basin characteristics. On the other hand, some stations (e.g. 7 and 20) are in close proximity to other stations in the basin attribute space, but they have a different extreme flow response, which is detected by the homogeneity test. Observing the site specific ROI sizes that are relatively small in Table 4.4 at the low significance level (α=0.005), and comparing the results in Table 4.3, it can be determined which stations cannot be included in a particular ROI. Stations 7 and 20 are the stations that most frequently terminate the ROI formation process. These stations are outliers in the sense that they have a different relation between basin characteristics and extreme flow events than is observed for the rest of the stations. Outlier stations such as those prohibit the expansion of the ROI, although subsequent stations in the station ranking could include useful information that might improve the extreme flow estimation. To overcome such an early termination of the regionalization, outlier stations were skipped and never again considered as candidates for that site's ROI. In this way, the information entering into the ROI is potentially expanded without violating the specified homogeneity level of the region that is formed.

Table 4.5 summarizes the sizes of the site specific ROI for two cases. One is the case where one station can be skipped (columns labelled a) and the other is the case where an unlimited number of stations may be skipped (columns labelled b). This latter case implies that the regionalization process continues until there are no further stations available. The occurrence of meaningful differences in the size of the ROI for the above two cases (labelled a and b) indicates that outlier stations have been identified. This happens more frequently as the significance level increases. When a strong homogeneity criterion is applied, as is the case with a high significance level, the process of regionalization will detect more stations as outliers than will be the case for a low homogeneity criterion.

Table 4.5 Size of ROI with various skipping schemes and homogeneity levels

	Significance level α											
	0.005		**0.01**		**0.025**		**0.05**		**0.10**		**0.25**	
#	**a**	**b**	**a**	**b**	**a**	**b**	**a**	**b**	**a**	**b**	**a**	**b**
1	19	19	17	18	17	19	17	18	11	17	7	16
2	17	19	17	19	6	18	6	18	6	17	5	15
3	17	19	17	19	5	16	3	15	3	14	2	15
4	17	19	17	19	17	18	6	17	5	15	4	13
5	17	19	5	18	5	17	5	17	4	16	4	15
6	20	20	20	20	17	18	17	19	3	17	3	16
7	20	20	17	19	5	18	5	18	5	17	4	15
8	17	19	17	19	5	18	4	17	4	16	2	8
9	20	20	20	20	10	19	10	19	8	18	7	16
10	9	16	9	16	3	15	3	15	3	13	3	11
11	17	19	17	19	5	17	5	15	4	16	4	14
12	7	19	7	18	7	18	6	17	6	16	3	15
13	19	19	17	19	9	17	9	17	9	17	5	14
14	20	20	19	19	17	19	17	18	15	17	6	16
15	9	16	5	13	3	15	3	12	3	11	3	7
16	20	20	20	20	12	19	12	18	7	18	7	16
17	17	19	17	19	17	18	17	18	4	17	4	14
18	17	19	7	18	7	18	6	17	6	17	5	16
19	19	19	17	19	15	17	15	17	14	16	11	16
20	17	19	17	19	17	19	17	19	15	18	3	17
21	19	19	17	19	15	18	17	18	14	16	5	15
22	19	19	17	18	17	18	17	18	14	16	12	16

To incorporate regional information for the estimation of the extreme flow event both weighting schemes described in section 3.5.1 were employed. The first weighting scheme uses the record length as the factor to weight information from a station, and the second weighting scheme uses basin characteristics as a basis for weighting each site's influence.

The estimation of the mean annual flood was obtained by using a nonlinear regression relationship between a combination of catchment characteristics and the mean annual flood, as described in section 3.5.3. The same combination of catchment characteristics were used as was reported by Panu and Smith (1988), for the application for the entire island. Those catchment characteristics are the catchment area, the catchment shape factor, the percentage of catchment area controlled by lakes and swamps, and the mean annual runoff for the catchment.

4.3 BASIS FOR COMPARISON OF RESULTS

A difficulty in any evaluation of extreme flow estimators using real world data is that we never know the "true" extreme flow for a selected return period at a site. This is especially true when methods for estimating extreme flow events at ungauged sites are considered. Techniques for estimating extreme flows at ungauged sites can be evaluated using a set of stations for which gauging records do exist by sequentially considering each of the gauged sites to be ungauged and conducting the analysis as if no flow record existed for the site considered to be ungauged. However, this only solves part of the evaluation problem in that the at-site flow record provides an imperfect estimate of the true, but unknown, extreme flow relationship at the site. The quality of the available estimates should improve with increases in the record length. A summary of estimated "true" extreme flows of various return periods for the 22 gauged sites, based only on the information from the single site, is given in Table 4.6. For this analysis, the extreme flows were considered to follow the GEV distribution. However, for the record lengths available for the case study considered herein, the at-site extreme flow estimates, based on single station analysis, must be deemed somewhat suspect, particularly for those stations with very short record lengths. As such, the at-site extreme flows estimated from single station analysis are unlikely to provide a reasonable benchmark for comparison of the various ungauged site flood frequency analysis options.

Table 4.6 Summary of "true" extreme flows obtained on the at-site basis in m^3/s

	Return Period [years]					
#	**T=5**	**T=10**	**T=20**	**T=50**	**T=100**	**T=200**
1	38.28	45.99	54.48	67.33	78.55	91.30
2	251.08	297.39	345.42	413.28	468.72	528.19
3	119.81	138.35	154.56	173.46	186.22	197.87
4	312.05	350.19	381.04	414.05	434.51	451.85
5	239.78	278.30	315.89	365.47	403.33	441.67
6	138.03	157.07	176.63	203.95	226.04	249.51
7	83.94	93.91	101.00	107.57	111.10	113.75
8	110.03	122.42	132.61	143.70	150.69	156.71
9	676.45	741.63	797.79	862.27	905.16	943.77
10	719.41	791.83	848.91	908.25	943.99	973.55
11	35.70	39.95	43.60	47.80	50.59	53.10
12	216.08	246.91	275.61	311.52	337.55	362.76
13	12.78	14.63	16.33	18.42	19.91	21.32
14	219.69	263.88	307.42	365.52	410.37	456.22
15	348.59	387.24	418.62	452.30	473.26	491.09
16	217.11	268.66	325.56	411.85	487.36	573.32
17	70.01	84.53	100.07	122.81	142.04	163.30
18	257.16	308.44	358.34	423.99	473.97	524.45
19	29.35	34.06	38.30	43.39	46.93	50.25
20	127.13	157.02	190.89	243.80	291.44	347.04
21	2.67	3.16	3.67	4.41	5.03	5.70
22	33.38	37.90	42.09	47.32	51.10	54.76

An alternative to using single station estimates is to use a regional estimate of the extreme flow at each site incorporating information from both the site of interest and from other similar stations. Although this option is also not ideal, it does reduce the difficulty associated with the short at-site record lengths and provides an assessment of the merit of the ungauged estimation options by comparing regional estimates obtained with and without data at the site of interest.

The regional method used herein also employs the ROI concept but for this case the similarity metric used for defining the ROIs can include flood statistics since in the regional analysis, extreme flow information is available for all sites. The variables used in the similarity metric were the same as those used by Burn (1990b) and included the coefficient of variation

(CV) of the annual flood series, a plotting position estimate of the 10 year flow event (Q10) interpolated from the available annual flood series, and a variation of the Pearson skewness (PS) measure defined as:

$$PS = \frac{\mu - m}{\sigma} \tag{4.9}$$

where μ is the mean, m is the median, and σ is the standard deviation of the annual flood series. As with the case of forming ROIs for ungauged sites, stations were added to the ROI for a site, in the order given by the similarity metric, until adding an additional site results in the collection of stations not passing the homogeneity test. ROIs were created with a significance level of 5%. This significance level was used by other authors, such as in the application of the R-test (Wiltshire, 1986a) and in the application of the q_{10} test (Lu and Stedinger, 1992). The skipping option is not used in this process of creating ROIs for a gauged site since the attributes used in the calculation of the station similarity are based on flow statistics, which were obtained from the series of annual maximum daily flow values. It is expected that the attributes and extreme flows are closely related to each other. In this situation, the problem of outlier stations should be minimized with the selection of this specific set of attributes employed in the construction of the site specific ROI. For the gauged analysis case, streamflow information from the site of interest is also included in the homogeneity test since this information is also available at the site of interest. Parameter estimates and extreme flows for the site of interest were then calculated in accordance with the procedures outlined above, where at-site flow information can now also be incorporated. A summary of the estimated "true" extreme flows based on the regional flood frequency analysis is presented in Table 4.7.

Table 4.7 Summary of "true" extreme flows obtained on the regional basis in m^3/s

	Return Period [years]					
	T=5	**T=10**	**T=20**	**T=50**	**T=100**	**T=200**
1	39.81	46.51	52.93	61.25	67.48	73.69
2	257.06	300.38	342.00	395.94	436.42	476.79
3	115.09	134.65	153.60	178.39	197.16	216.03
4	305.77	357.99	408.58	474.80	524.98	575.46
5	244.80	286.37	326.45	378.62	417.94	457.30
6	146.19	167.19	186.64	210.85	228.30	245.12
7	75.45	88.25	100.63	116.79	129.01	141.27
8	109.05	124.88	139.67	158.28	171.83	185.00
9	713.47	820.62	920.94	1047.30	1139.48	1229.24
10	716.33	821.93	921.35	1047.37	1139.87	1230.45
11	35.82	41.02	45.88	51.97	56.41	60.71
12	220.68	255.41	287.90	328.79	358.60	387.61
13	12.86	14.87	16.76	19.14	20.87	22.56
14	212.04	247.62	281.88	326.46	359.98	393.53
15	344.41	394.38	441.10	499.85	542.64	584.25
16	217.12	252.70	286.54	329.96	362.21	394.09
17	71.01	82.87	94.23	108.91	119.90	130.82
18	246.27	287.45	327.13	378.76	417.66	456.58
19	28.43	33.26	37.93	44.03	48.64	53.28
20	129.93	151.19	171.36	197.16	216.26	235.09
21	2.76	3.23	3.68	4.25	4.69	5.12
22	33.83	38.70	43.23	48.88	52.97	56.92

The estimation of extreme flow quantiles obtained by the ROI approach, presented in this study, were compared to extreme flow quantile estimates obtained by regression analysis, following the work of Panu and Smith (1988). The extreme flow estimation for each site by regression included two different grouping scenarios. Firstly, the entire data set is defined as a group. Secondly, the area was divided into two groups, namely the northern and the southern part of Newfoundland (Panu and Smith, 1988). This station grouping corresponds to geographic regionalization. The dividing line for the two groups follows the topographical highest line in the central part of the island. An extensive analysis (Panu and Smith, 1988) was conducted regarding the influential catchment characteristics in support of the above grouping. It was found that the major flood producing mechanism in the northern part of the island is spring snowmelt aggravated by rainfall and in the south part of the island, rainfall storms and rainfall events contributing to snowmelt in both the winter and the spring season are important. The influential catchment characteristics included in the regression model when the entire island is considered as a group are the catchment area (DA), the mean annual runoff (MAR), the percentage of catchment area controlled by lakes and swamps (ACLS), and the catchment shape factor (SHAPE). For the south group, this same set of catchment characteristics is used. For the northern group, the influential catchment characteristics are the DA, MAR, and the latitude of the catchment centroid (LAT).

For the estimation of the extreme flow, nonlinear regression is used, as shown in Equation (3.54). A separate regression relationship was developed for each desired return period. The study by Panu and Smith (1988) used the annual maximum instantaneous flow. This study, however, is based on the annual maximum daily flow series and the regression parameters were recalculated with respect to the maximum daily flow for each year. Furthermore, for consistency, the values of extreme flows corresponding to the various return periods were estimated, for each gauged site, based on the selected parent probability distribution, which is the GEV distribution, using: (i) single site frequency analysis, and (ii) regional at-site estimates. Finally, to avoid an unfair comparison, separate regression relationships were derived for each site. The extreme flows from the site of interest were not included in the data set used to derive the individual regression relationships. In this way, the regression approach is realistically evaluated as an ungauged site extreme flow quantile estimation approach.

A total of twelve estimation options were considered and are summarized below.

T1 - This is a single station at-site extreme flow estimation option which uses at-site PWMs to estimate the parameters for the GEV distribution.

T2 - This is a regional estimator based on PWMs estimated from extreme flow data for all stations in the ROI defined for each site. PWMs from each of the stations in the ROI are weighted in accordance with the record length at the station to obtain at-site extreme flow estimates. In defining the ROI, the station attributes used were the CV, PS and Q10 with equal weights assigned to each attribute. The 5% level of significance was used in the regional homogeneity test which determines the stopping criterion for adding stations to an ROI.

ROI1 - This is the region of influence approach applied to ungauged sites where catchment characteristics were used as the attributes defining station similarity. The catchment characteristics selected were the DA, SHAPE, and MAR with weights 0.91, 0.03, and 0.06 assigned to the three attributes. PWMs from the stations in the ROI are weighted in accordance with the record length at the station to obtain at-site extreme flow estimates. For the estimation of the mean annual flood, the same basin characteristics were used as in the case of the regression based approach for the entire island. Those are the DA, ACLS, SHAPE, and MAR.

ROI2 - This option is the same as ROI1 except that the extreme flows were calculated for each station and then basin characteristics were used to calculate the weighting factor for the determination of each station's contribution to the extreme flow estimate at the ungauged site.

ROI3 - This option is similar to the ROI1 option, but the regions of influence are created allowing one station to be skipped.

ROI4 - This option is analogous to option ROI2, but the regions of influences were created as in the option ROI3 allowing one station to be skipped.

ROI5 - This option is similar to options ROI1 and ROI3 in the way that information is used for the estimation of the extreme flow quantiles, but in the creation of the regions of influence, an unlimited number of station skips were allowed.

ROI6 - This option is similar to options ROI2 and ROI4, but as in option ROI5, in the creation of the regions of influence, an unlimited number of station skips were allowed.

RG1 - This is a regression based estimator considering all the stations to comprise a single region. The single station at-site extreme flow estimates (Option T1) at 5, 10, 20, 50, 100, and 200 year return periods were used as the "true" flow values to be estimated.

RG2 - This option is the same as RG1 except the set of stations is divided into two regions corresponding to the north and south portions of the study area.

RG3 - This option is equivalent to RG1 except that the "true" at-site estimates were obtained from the Option T2 (regional estimation approach).

RG4 - This option is the same as RG3 except the set of stations is divided into two regions as in option RG2.

The first two estimators, T1 and T2, are two different approximations to the "true" extreme flow values at the sites. The next six methods, from ROI1 to ROI6, are the various ROI ungauged options developed in this study. The last four options, from RG1 to RG4, are regression estimates similar in spirit to the approach of Panu and Smith (1988).

4.4 PRESENTATION AND ANALYSIS OF RESULTS

The first stage in the analysis of the results was a comparison of the two estimates of the "true" extreme flow values, options T1 and T2. One would anticipate that if the regional estimator was a perfect predictor of the true extreme flows that the agreement between options T1 and T2 would improve with an increase in the record length and would also be better for shorter return periods. This argument is valid since as the length of the data record at a site increases, the single station at-site estimates should approach the "true" values. In addition, for a fixed record length, the utility of regional information is greater for longer return periods than it is for shorter return periods and in fact for return periods in excess of the length of record, it can be expected that any regional information is of benefit regardless of the similarity between the new information and the information from the site of interest.

The results from the two "true" flow estimators, presented in Table 4.6 and Table 4.7, were compared at return periods equal to 5, 10, 20, 50, 100 and 200 years. It was noted that for the two sites with the shortest data record length, 12 and 15 years of record at stations 7 and 4 respectively, the agreement between the two options was comparatively poor. The lack of agreement in these results was particularly pronounced for the longer return periods (100 and 200 years). For these two stations, the regional estimates are probably providing more realistic extreme flow estimates. For the site with the longest data record length, 61 years of flow record at station 9, the agreement was also not particularly strong. In this case, there was some indication of a systematic lack of fit. For the remainder of the sites, the agreement was generally quite satisfactory with greater differences in the results obtained noted for the longer return periods, as was expected. There was, however, not as strong a relationship as expected between the agreement in the results and the record length of the site. The results of this comparison indicate that the regional estimator is likely a reasonable approximation to the unknown "true" extreme flow values although the comparisons presented below will consider both Options T1 and T2 as a predictors of the "true" extreme flow values.

The ROI that was defined for a site when the site was assumed to be ungauged tended to contain a similar set of stations to those included when the site was considered to be gauged. The main differences were in the order in which the sites were included in the region of influence and the total number of sites included. For the ungauged case, there were generally fewer stations included in the ROI. The main reason for this behaviour is that when stations are selected based on similarity in terms of the basin characteristics, a collection of stations is formed that contains stations which should not have been added based on their extreme flow characteristics. As a result, the homogeneity test will deem the set of stations to be heterogeneous. The site skipping feature in the regionalization process for the ungauged case reduces the chance of early termination of the regionalization process. However, due to the different basis of ranking stations, the site specific ROIs will still differ for the two sets of attributes used for defining station similarity.

To evaluate and compare the relative merits of the ungauged site estimation options, comparisons were made between the results from: Options ROI1, ROI2, ROI3 ROI4, ROI5 and ROI6 versus both T1 and T2; Option RG1 and RG2 versus T1; and Options RG3 and RG4 versus T2. Note that for the regression options, the comparison is with at-site estimates equivalent to those predicted by the corresponding regression relationship (i.e., either the single station or the regional estimates). Each estimator was evaluated in terms of a measure of the relative mean squared error (MSE) and in terms of the bias of the estimates averaged over return periods of 5, 10, 20, 50, 100 and 200 years. These measures are defined as:

$$MSE=\frac{1}{NS}\frac{1}{J}\sum_{i=1}^{NS}\sum_{j=1}^{J}\frac{(\hat{Q}_j^{\,i}-Q_j^{\,i})^2}{Q_j^{\,i}} \tag{4.10}$$

and

$$BIAS=\frac{1}{NS}\frac{1}{J}\sum_{i=1}^{NS}\sum_{j=1}^{J}\frac{\hat{Q}_j^{\,i}-Q_j^{\,i}}{Q_j^{\,i}} \tag{4.11}$$

where MSE is the relative mean squared error measure, BIAS is the measure of average bias, NS is the number of sites in the data set, J is the number of return periods, Q_j^i is the estimate for the extreme flow for the jth return period at site i, and Q_j^i is the "true" value for the extreme flow for the jth return period at site i.

MSE and BIAS values for various estimators are summarized in Table 4.8.1 and Table 4.8.2. Results for options ROI1 to ROI6 are compared with T1 and T2 for various sizes of region of influence as indicated by the significance level used in the homogeneity test. The MSE and BIAS values that are presented in bold type represent the minimum (preferred) values for a particular option.

It can be seen that for the first two ROI options, (ROI1 and ROI2), the smallest MSE values are obtained for the highest significance level and for the second highest significance level for options T1 and T2 respectively. BIAS values very much follow the pattern of the MSE values for the various options. This is especially apparent for options ROI3, ROI5 and ROI6 where the minimum MSE and BIAS values occur at the same significance level. The BIAS values when the comparison is made with regional "true" values show a slightly lower value than the corresponding values when the comparison is made with at site "true" values.

Table 4.8.1 Summary of MSE values for various ROI options

		Significance level α					
		0.005	**0.01**	**0.025**	**0.05**	**0.10**	**0.25**
ROI1	**T1**	14.63	14.89	15.01	15.59	14.85	**14.21**
	T2	16.33	16.61	16.39	16.71	**15.63**	16.54
ROI2	**T1**	14.07	14.21	14.50	14.73	14.18	**13.92**
	T2	15.74	15.97	16.00	16.12	**15.34**	16.46
ROI3	**T1**	14.93	15.44	15.12	15.11	**14.02**	14.40
	T2	15.35	15.53	16.41	16.92	**15.07**	15.40
ROI4	**T1**	14.54	14.75	14.38	14.29	**13.74**	14.10
	T2	**15.00**	15.03	15.80	16.05	15.14	16.02
ROI5	**T1**	15.38	15.51	15.18	15.09	14.70	**13.46**
	T2	15.43	15.30	15.11	15.87	14.91	**14.72**
ROI6	**T1**	14.82	14.92	14.81	14.54	14.24	**13.30**
	T2	14.89	14.80	14.89	15.01	14.61	**14.71**

Table 4.8.2 Summary of BIAS values for various ROI options

		Significance level α					
		0.005	**0.01**	**0.025**	**0.05**	**0.10**	**0.25**
ROI1	**T1**	0.075	0.070	0.075	0.074	0.072	**0.066**
	T2	0.073	0.068	0.073	0.072	0.069	**0.064**
ROI2	**T1**	0.055	**0.052**	0.058	0.058	0.056	0.053
	T2	0.053	**0.050**	0.056	0.056	0.053	0.050
ROI3	**T1**	0.074	0.073	0.067	0.070	**0.061**	0.067
	T2	0.070	0.069	0.064	0.068	**0.059**	0.065
ROI4	**T1**	0.061	0.058	0.052	0.053	**0.050**	0.055
	T2	0.057	0.055	0.049	0.050	**0.047**	0.052
ROI5	**T1**	0.066	0.063	0.063	0.066	0.061	**0.058**
	T2	0.063	0.059	0.059	0.063	0.057	**0.055**
ROI6	**T1**	0.061	0.060	0.060	0.060	0.056	**0.053**
	T2	0.057	0.056	0.056	0.056	0.052	**0.050**

For options ROI3 and ROI4, in which one station skip is allowed during the formation of the site specific ROI, a tendency can be noticed for the lowest MSE values to be obtained with the second smallest size ROI, with the exception of ROI4 with T2 as the basis of comparison.

Options ROI5 and ROI6 clearly demonstrate a tendency for the best estimates of the extreme flow quantiles to be obtained with the smallest size ROI. In other words, with an ROI of high homogeneity it is possible to have the best estimate of the extreme flow quantiles, regardless of the selected "true" extreme flow option (T1 or T2).

The MSE and BIAS values for the first four ROI options (ROI1, ROI2, ROI3 and ROI4), when compared to both "true" flow options (T1 and T2), reveal that the minimum values occur at various ROI homogeneity levels. These are the ROI options for which one (options ROI1 and ROI2) or two (options ROI3 and ROI4) outlier stations can stop the expansion of the region of influence, preventing the incorporation of useful information in the estimation of extreme flow quantiles. For the last two ROI options (ROI5 and ROI6), when compared to both "true" value options (T1 and T2), the best extreme flow estimation occurs with the employment of a ROI with a strict homogeneity criterion. This illustrates that the best results can be obtained with a region that consists of only those stations which are hydrologically very similar to each other.

It can also be noted that all ROI options have fairly similar MSE values with a note that options ROI2, ROI4 and ROI6 have lower values of MSE than options ROI1, ROI3 and ROI5 respectively. This can be explained by the fact that in options ROI2, ROI4 and ROI6 the effect of weighting extreme flow information from sites in the ROI is more sophisticated than the weighting incorporated in options ROI1, ROI3 and ROI5 in that the latter options involve weighting by record length only. However, the results indicate that the pairs of options, (ROI1 and ROI2, ROI3 and ROI4, ROI5 and ROI6), are very similar in terms of MSE, implying that the approach to weighting extreme flow information from sites in the ROI has less impact on the estimation of the extreme flow quantiles than does the identification of an appropriate set of gauging stations for inclusion in the region of influence for a site.

Table 4.9 combines the best MSE values for the various ROI options and the MSE values for the regression options.

Table 4.9 MSE values for various estimation options

Estimation option	"True" Extreme Flow Quantiles Option	
	T1	**T2**
ROI1	14.21	15.63
ROI2	13.92	15.34
ROI3	14.02	15.07
ROI4	13.74	15.00
ROI5	13.47	14.72
ROI6	13.30	14.71
RG1	17.00	
RG2	32.50	
RG3		15.65
RG4		23.58

The MSE values for the various options summarized in Table 4.9 reveal a meaningful improvement in MSE associated with the ROI options relative to the regression based alternatives. The best estimates of extreme flow quantiles can be obtained with a region of influence approach, and especially with options ROI6 and ROI5.

In contrast to the ROI options, the regression results were noted to provide better estimates when T2 was used to define the "true" extreme flow quantiles as opposed to the case when T1 was used. There is no apparent explanation for this pattern since in both cases the regression equations were derived using the assumed "true" values as the dependent variables.

In the comparison of the RG1 and RG3 versus RG2 and RG4 methods respectively, it can be noted that RG1 and RG3 have a notably lower MSE value. This can be attributed to the fact that in the options RG1 and RG3, the whole island is treated as a region resulting in a larger number of stations included in the regression than for options RG2 and RG4 where the island is divided into two regions. While the regression curves for options RG2 and RG4, may provide a better fit to the data, they obviously cannot handle the attributes of the ungauged site in order to provide a good estimate of the extreme flow value at the corresponding site. Options RG1 and RG3 provide better estimates of the extreme flow at the ungauged site, although the regression curve may have lower R^2 value measuring the overall fit of the model. This may be an indication that statistical measures are not always the best indicators of the appropriateness of the applied method.

A disadvantage of the regression approach arises from the need to formulate separate regression relationships for each return period of interest. This can lead to the definition of inconsistent Q-T relationships at some sites where in unusual cases, the predicted extreme flow quantile for a return period T_1 could be greater than the value for return period T_2 where T_1 is less than T_2. Furthermore, the relationship between the extreme flows and the set of basin characteristics is not necessarily the same for all return periods. An anomalous result like this cannot occur with the ROI based approach in that the entire Q-T relationship for a site is explicitly defined.

The principal shortcoming in any extreme flow estimation which involves regression analysis is that the majority of the error introduced into the estimates is during the application of the regression relationship between various catchment attributes and extreme flow quantiles. This is the case with an ROI approach as well. A better index flood estimate is needed for a major improvement of the extreme flow estimation at ungauged sites. Usually, it is difficult to obtain catchment attributes for the catchments of interest and access to catchment characteristic data that are relevant is even more limited. The big advantage of the index flood approach, as applied in the ROI options, is that the regression is applied only once, to the index flood.

5. HIERARCHICAL ROI APPROACH

It has been shown in many studies that the use of regional versus at-site flood frequency analysis leads to improved extreme flow quantile estimation. The information collected at the sites incorporated in the regional flood frequency analysis is assumed to give a better measure of the flood frequency characteristics at the site of interest. The hierarchical approach to the estimation of parameters of the selected distribution function exploits this fundamental premise of regional flood frequency to extract information from a number of sites, but takes a further step in the employment of the available data. The main idea employed in this approach is that values of higher order statistics vary less in space than do the lower order statistics. A pragmatic approach would then imply that the higher the order of the regional statistic to be estimated, the greater the number of sites that are needed to achieve a desired degree of reliability. The hierarchical approach, in combination with the ROI approach, results in a set of ROI's for a site of interest. Each ROI in this set is created to provide the most efficient estimation of the corresponding parameters of the distribution function.

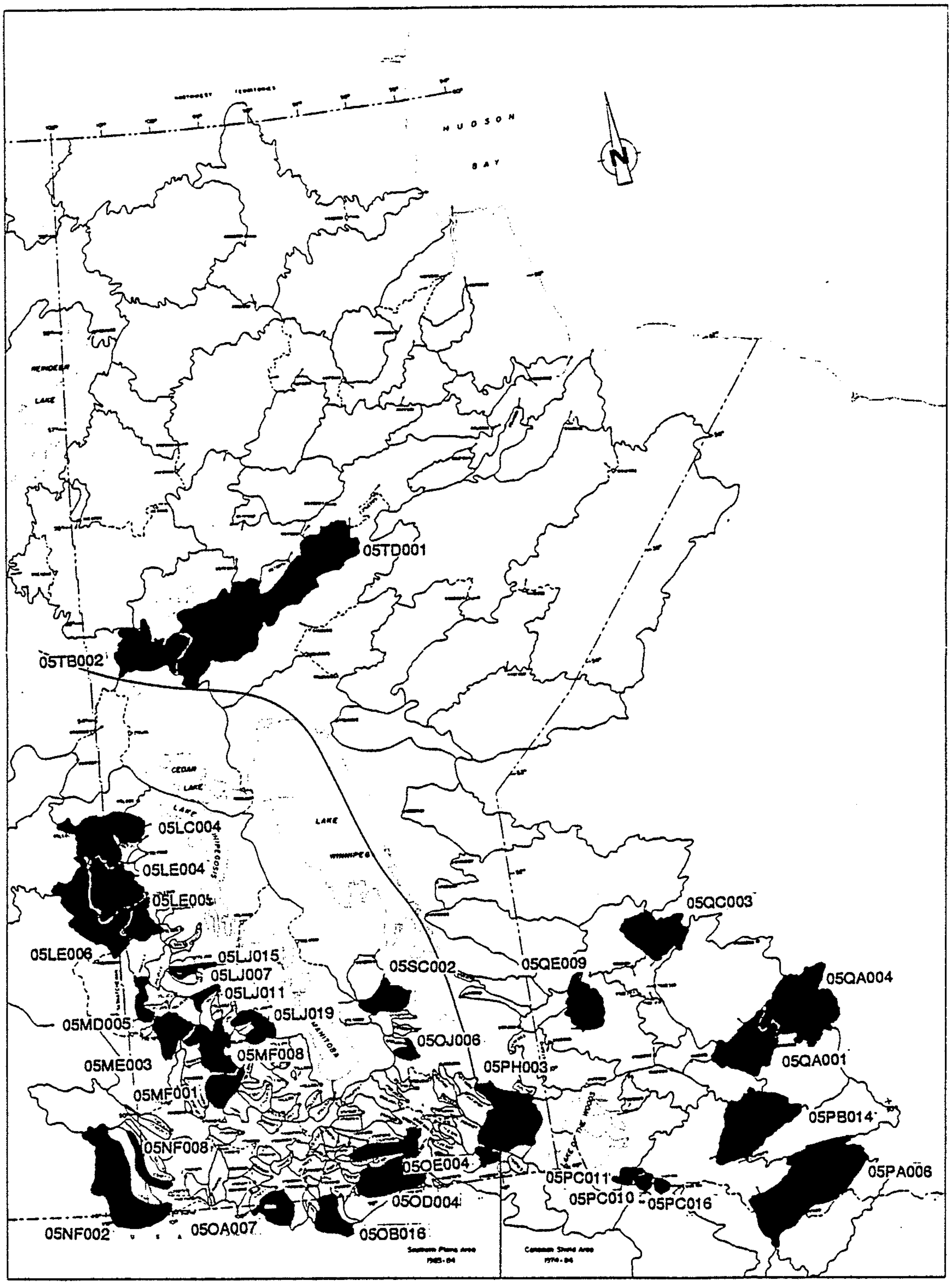
In this chapter, a hierarchical approach to the estimation of parameters of the distribution function is applied to the region of influence approach, and refer to as the hierarchical region of influence approach, in order to improve the estimation of parameters for a desired regional distribution function. The approach is demonstrated with an application through a Monte Carlo simulation. Results and discussion are presented at the end of the chapter.

5.1. DESCRIPTION OF THE APPLICATION

The hierarchical approach, combined with the ROI technique, is applied to a set of parent distribution parameters derived from 32 catchments on naturally flowing rivers in Manitoba and Northern Ontario, as shown in Figure 5.1. This area can be divided into two distinct groups, based on a combination of physiographic characteristics and climatic regions. The first group is southern Manitoba consisting of the Southern Plains area, where a total of 20 stations are located. The land in this area is mainly used for agricultural purposes with drainage from this area to Lake Winnipeg mainly through the Saskatchewan and Assiniboine Rivers. The second group comes from the Canadian Shield area where a total of 12 stations are located, two in northern Manitoba and 10 in northern Ontario. The Canadian Shield is characterized by rugged rocks, heavy woodlands, and many lakes, rivers, marshes and waterfalls. Water from the two northern Manitoba catchments drains directly into the Nelson River, while water from the rest of the catchments considered drain into the Winnipeg River system. The range for years of record at the stations is from 21 to 68 years with a mean value of 35.6 years.

Basically, the area considered could be divided into two or three regions with fixed boundaries according to physiographic, climatic and administrative criteria. However, regions obtained in this fashion would preclude any information exchange between stations belonging to other regions but having similar extreme flow characteristics. As well, the number of stations in northern Manitoba is limited to only two, which makes regional analysis very inefficient. Therefore, the ROI approach will again be adopted for the analysis.

Figure 5.1 Location of gauged sites and their watersheds for the application



5.2. APPLICATION OF THE HIERARCHICAL APPROACH

In order to perform a regional flood frequency analysis, the appropriate regional distribution function must again be selected. The selection was made by calculating the Z statistics (Hosking, 1991) as described earlier. A summary of the Z statistics for common distribution functions is presented in Table 5.1. It can be noticed that the acceptable alternative is the Generalized Extreme Value (GEV) distribution function. The visual information from the L-moment ratio plot, shown in Figure 5.2, ensures that the GEV distribution would be an appropriate parent distribution function. The GEV distribution function, with the probability weighted moment (PWM) parameter estimators, are described in section 4.2. The selection of the distribution function is made for the entire set of stations, which does not consider the possibility that for some of the ROIs, other distribution functions may provide a better fit. The main idea is to have the same parent distribution function for each region in order to be able to evaluate the power of the hierarchical approach for the estimation of distribution parameters. The evaluation of the hierarchical approach when more than one parent distribution function is employed becomes more complicated due to an increased number of factors involved in the evaluation, (i.e., various distribution functions with various parameter estimators have differing efficiencies).

Table 5.1 Output of the goodness of fit program by Hosking (1991)

***** GOODNESS-OF-FIT MEASURES *****	
GENERALIZED LOGISTIC	Z VALUE= 3.28
GENERALIZED EXTREME VALUE	Z VALUE= 0.76 *
PEARSON TYPE III	Z VALUE= -2.57
GENERALIZED PARETO	Z VALUE= -5.50

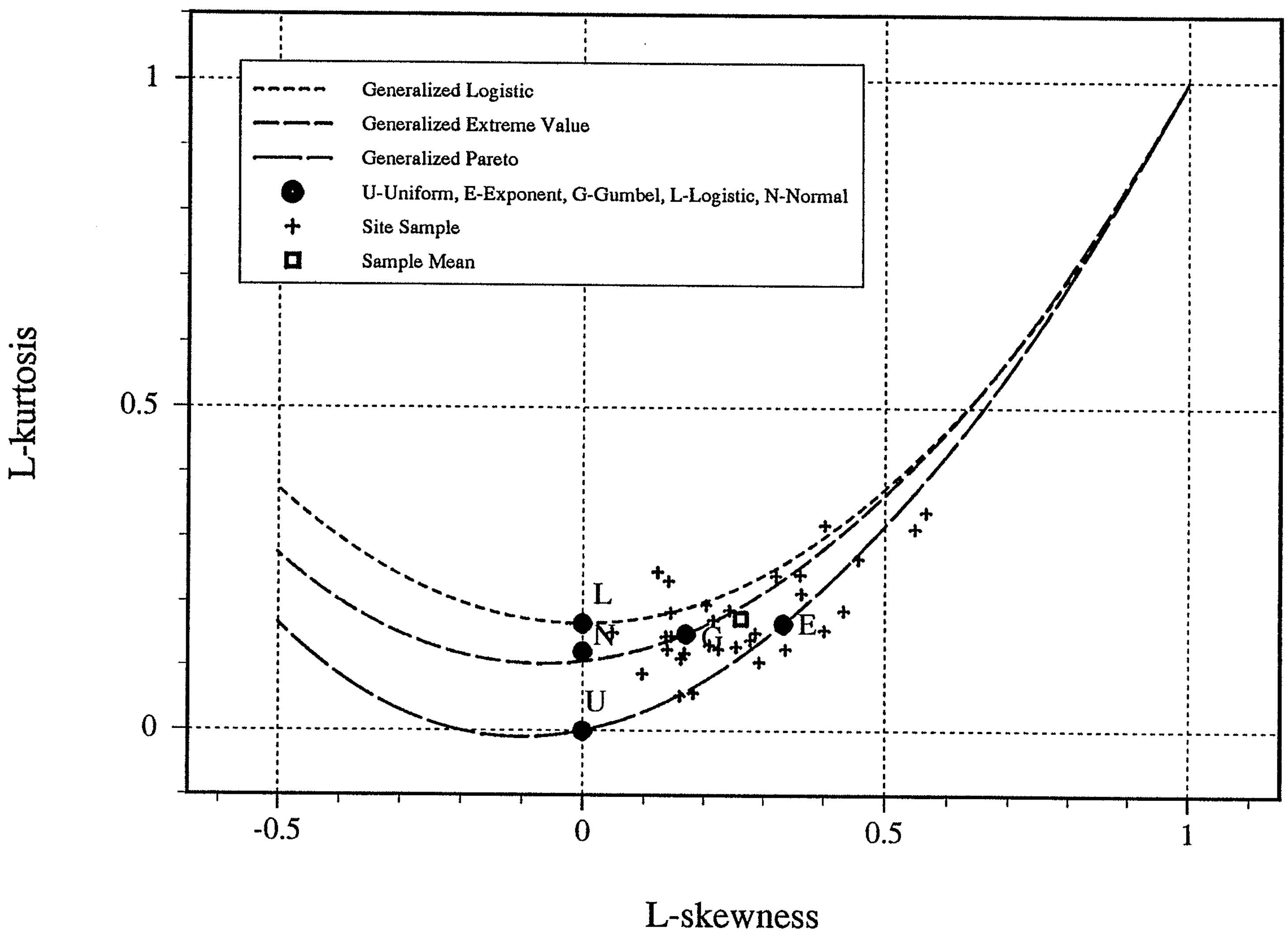


Figure 5.2 L-moment ratio diagram for the application data

The selection of sites for the site specific ROI follows the procedure described in Chapter 3. All of the attributes considered for use in the calculation of the proximity measure must be readily available for all stations. An additional criterion is that the attributes must be closely related to the extreme flow response at the station. Because of the limited information describing physical features of the catchment areas (i.e. size of the drainage basin), attributes were extracted from the series of annual maximum daily flow measurements in order to calculate the station proximity. The attributes used are a plotting position estimate of the standardized 10 year flow event (Q10) interpolated from the available annual flood series, a variation of the Pearson skewness (PS), and the coefficient of variation (CV) of the annual flood series. Similarity in these statistical measures would indicate that stations have similar extreme flow characteristics such that a common flood frequency relationship can successfully represent all of the stations. It is to be expected that these attributes will also reflect the flood generating mechanism which is a function of a variety of drainage basin factors such as the slope, the soil type, and the infiltration potential. The summary of the attributes for each site, along with the available data record length, are presented in Table 5.2.

Using the selected set of attributes, the station similarity is calculated for each station pair. The rank of the stations, according to the similarity to the selected site, is presented in Table 5.3. This table of station ranks differs, however, from the station order in Table 4.3 of section 4.2. Here, for each site, the closest station to the site is the site itself and therefore the next closest station is labelled as the second closest station, and so on.

In the formation of the site specific ROI, the homogeneity test described in Section 3.4.1 was applied. Since the selected attributes are assumed to closely describe the properties of the annual flood series, the ROIs were created without applying the skipping feature described in section 3.3. In order to have ROIs with a different level of hydrologic homogeneity, different significance levels were applied in the homogeneity test. The number of stations included in the ROI for various significance levels can be determined from Table 5.4.

Table 5.2 Summary of flow characteristics used in the calculation of station similarity

#	Gauged Station Name	ID	CV	Q10	PS	Rec.L
1	Red Deer River near the mouth	05LC004	1.675	2.024	0.020	28
2	Woody River Near Bowsman	05LE004	1.189	2.195	0.226	37
3	Roaring River near Minitonas	05LE005	0.989	2.213	0.261	32
4	Swan River near Minitonas	05LE006	1.397	2.183	0.145	30
5	Turtle River near Laurier	05LJ007	1.143	1.687	0.247	42
6	Wilson River near Dauphin	05LJ011	1.188	1.953	0.299	34
7	Fishing River near Fork River	05LJ015	0.648	2.702	0.322	21
8	Mink Creak near Ethelbert	05LJ019	1.007	2.290	0.241	37
9	Shell River near Inglis	05MD005	1.442	1.955	0.257	40
10	Birdtail Creek near Birtle	05ME003	1.303	2.475	0.341	38
11	Little Saskatchewan River near Minnedosa	05MF001	1.336	2.158	0.191	32
12	Rolling River near Ericson	05MF008	1.265	2.178	0.372	30
13	Antler River near Melita	05NF002	0.806	2.840	0.427	47
14	Braham Creek near Melita	05NF008	0.607	2.782	0.401	37
15	Badger Creek near Cartwright	05OA007	0.675	3.960	0.392	32
16	Snowflake Creek near Snowflake	05OB016	0.874	2.808	0.453	29
17	Rosseau River at Gardenton	05OD004	2.146	1.706	0.147	29
18	Rat River near Sundown	05OE004	1.775	1.680	0.033	30
19	Cooks Creek at Cooks Creek	05OJ006	1.387	2.324	0.073	31
20	Whitemouth River near Whitemouth	05PH003	1.696	1.708	0.102	49
21	Icelandic River near Riverton	05SC002	1.151	2.349	0.277	33
22	Grass River at Wekusko Falls	05TB002	2.237	1.599	0.060	33
23	Grass River above Standing Stone Falls	05TD001	3.211	1.446	0.054	25
24	Namakan River at outlet of Lac La Croix	05PA006	2.497	1.450	-0.089	68
25	Turtle River near Mine Centre	05PB014	2.234	1.581	0.065	58
26	Sturgeon River near Barwick	05PC010	1.996	1.710	0.067	35
27	Pinewood River near Pinewood	05PC011	1.682	1.772	0.099	39
28	La Vallee River near Devlin	05PC016	2.176	1.616	-0.176	27
29	English River near Sioux Lookout	05QA001	2.053	1.670	0.266	60
30	Sturgeon River at McDougall Mills	05QA004	2.066	1.892	0.288	29
31	Troutlake River above Big Falls	05QC003	2.230	1.426	0.053	21
32	Sturgeon River at outlet of Salvesen Lake	05QE009	2.049	1.565	0.202	26

Table 5.3 Rank of stations in flow characteristic space

	Station Order																														
#	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**
1	9	19	27	4	11	20	18	12	2	30	6	26	10	25	5	21	29	17	3	8	22	28	32	31	24	7	16	13	14	23	15
2	21	11	3	8	4	12	19	6	10	9	1	5	27	7	16	20	13	18	30	25	26	14	29	17	22	32	28	31	24	15	23
3	8	2	21	6	12	11	4	19	10	5	9	1	7	13	16	27	20	18	14	30	25	26	29	17	32	22	28	31	24	15	23
4	19	11	2	12	9	21	1	10	6	8	3	27	5	20	18	30	7	26	16	25	13	29	17	22	28	32	14	31	24	15	23
5	6	9	3	11	12	2	18	27	20	4	19	1	8	21	10	25	26	29	30	32	17	22	28	7	31	16	13	24	14	23	15
6	12	9	11	5	2	3	4	19	8	21	1	10	27	20	18	25	30	26	29	7	16	17	13	32	22	28	31	14	24	15	23
7	13	16	14	8	21	10	3	2	12	4	19	11	6	15	9	1	5	27	20	18	30	25	26	29	17	22	28	32	31	24	23
8	21	3	2	12	10	11	4	6	19	7	9	16	13	5	1	14	27	20	18	30	25	26	29	17	32	22	28	15	31	24	23
9	11	6	12	1	4	19	27	2	5	20	18	3	21	10	8	30	25	26	29	17	32	22	28	31	7	16	13	24	14	23	15
10	21	19	12	2	4	11	8	3	9	6	1	16	7	13	5	27	20	30	14	18	25	26	29	17	22	32	28	31	15	24	23
11	4	19	12	2	9	6	21	10	1	3	8	5	27	20	18	30	25	26	7	29	16	13	17	22	32	28	31	14	24	15	23
12	11	2	6	9	21	4	19	10	3	8	1	5	27	20	18	7	16	30	13	25	26	29	17	32	22	14	28	31	24	15	23
13	16	7	14	8	21	10	3	2	12	4	19	11	15	6	9	1	5	27	20	18	30	25	26	29	17	22	32	28	31	24	23
14	13	7	16	15	8	21	10	3	2	12	4	19	11	6	9	1	5	27	20	18	30	25	26	29	17	22	28	32	31	24	23
15	14	16	13	7	10	21	8	2	3	19	4	12	11	6	9	1	5	27	30	20	18	25	26	29	17	22	28	32	31	24	23
16	13	7	14	21	8	10	3	2	12	4	19	11	6	15	9	1	5	27	20	18	30	25	26	29	17	22	32	28	31	24	23
17	29	25	22	26	30	32	31	28	20	27	18	24	1	9	11	19	4	6	5	12	23	2	10	21	3	8	16	7	13	14	15
18	20	27	26	25	29	1	32	9	30	17	22	5	28	31	6	11	4	19	12	2	24	3	10	21	8	7	16	23	13	14	15
19	4	11	2	1	12	9	10	21	6	8	3	27	5	20	18	30	26	7	25	16	13	29	17	22	28	32	14	31	24	15	23
20	27	18	26	25	29	1	30	9	17	32	22	5	28	11	6	31	19	4	12	2	24	3	10	21	8	7	16	13	23	14	15
21	8	2	10	3	12	4	11	19	6	9	7	16	13	1	5	27	14	20	18	30	25	26	29	17	22	32	28	31	15	24	23
22	31	17	28	26	32	29	25	24	30	20	18	27	1	9	23	11	19	4	5	6	12	2	10	21	3	8	7	16	13	14	15
23	24	31	22	28	17	32	29	26	25	30	20	18	27	1	9	19	11	4	5	6	12	10	2	21	3	8	16	7	13	14	15
24	31	22	28	32	17	26	29	25	30	23	18	20	27	1	9	19	11	4	5	6	12	2	10	21	3	8	7	16	13	14	15
25	29	17	30	26	32	20	22	27	18	31	28	1	9	24	11	6	19	5	4	12	2	10	21	3	8	23	16	7	13	14	15
26	17	29	22	25	30	20	28	32	27	18	31	1	24	9	11	19	4	5	6	12	2	10	21	3	8	23	7	16	13	14	15
27	20	18	1	26	25	9	30	29	17	32	11	22	6	5	19	4	28	12	31	2	3	10	21	24	8	7	16	13	23	14	15
28	22	31	26	24	17	32	29	25	30	20	18	27	1	9	19	23	4	11	5	6	12	2	10	21	3	8	7	16	13	14	15
29	25	17	26	30	32	22	20	31	27	18	28	24	1	9	11	19	6	5	4	12	2	10	21	23	3	8	16	7	13	14	15
30	25	17	29	26	27	22	20	18	32	1	28	31	9	24	11	19	4	12	6	5	10	2	21	3	8	23	16	7	13	14	15
31	22	32	24	28	17	26	29	25	30	18	20	27	1	23	9	5	11	19	4	6	12	2	10	3	21	8	7	16	13	14	15
32	31	22	29	17	25	26	28	18	24	20	30	27	1	9	5	6	11	19	4	23	12	2	10	3	21	8	7	16	13	14	15

Table 5.4 Size of ROI with various homogeneity level

	Significance level α					
#	**0.005**	**0.01**	**0.025**	**0.05**	**0.10**	**0.25**
1	17	17	17	16	14	13
2	17	17	17	16	16	15
3	17	16	16	15	14	14
4	18	18	17	17	17	16
5	18	17	17	16	16	16
6	18	17	17	16	16	1
7	16	16	9	9	8	6
8	17	16	16	15	13	13
9	18	17	17	17	17	16
10	17	16	16	14	14	14
11	18	17	17	17	17	16
12	18	18	18	17	17	16
13	16	16	9	9	8	6
14	16	16	10	10	9	7
15	16	16	15	10	5	5
16	16	16	9	9	8	6
17	17	17	16	15	15	14
18	20	18	17	16	15	15
19	18	18	18	17	17	16
20	20	18	17	15	15	14
21	17	16	16	14	13	13
22	15	15	15	15	15	14
23	15	14	13	12	1	1
24	15	14	13	12	11	10
25	16	16	16	15	15	14
26	17	17	16	15	15	14
27	20	16	15	14	13	13
28	16	16	16	16	15	14
29	17	17	16	15	15	14
30	17	17	16	15	15	14
31	15	14	14	14	14	14
32	16	16	16	15	15	14

The composition of the ROI for a selected site includes the site itself, since flow measurements are available in the context of gauged analysis. Table 5.4 shows that the largest number of stations included in an ROI is 20 (for stations 18, 20 and 27) out of the total possible stations of 32. At these stations, station 2 cannot be included in the ROI even when the lowest significance level is applied. The construction of the ROI at other sites terminates earlier, when the station selected on the basis of station proximity in attribute space no longer results in a homogeneous region.

It can be seen that stations 6, 7, 13, 14, 15 and 23 are at the end of the rank for most of the listed stations. In addition, for these stations, with a high significance level for the homogeneity test, only a limited number of stations can be included in the corresponding ROI. This is especially true for stations 6 and 23 for which with a high significance level, no station can be added while maintaining the homogeneity of the formed region. The ROI therefore consists only of the station itself. This phenomenon indicates that these stations are substantively different from the rest of the stations. Stations 6, 7, 13, 14 and 15 are geographically positioned in the south west corner of Manitoba (see Figure 5.1). In addition, these stations (with the exception of station 15) have a very pronounced and extended longitudinal dimension relative to the width of the catchment. Therefore, the flood generating process for these catchments may well be different from the rest of the catchments involved in the case study.

Although these latter stations showed different extreme flow characteristics according to the homogeneity test, they are retained in the application in order to evaluate the impacts of a situation where outlier stations are not detected at the beginning of the process of flood frequency analysis. For the rest of the listed stations, the size of ROIs do not change greatly with the different significance levels used in the homogeneity test. This implies that the expansion of the size of ROI is prohibited by some stations that will introduce heterogeneity into the region of influence. The cut off point for the expansion of the ROIs occurred at approximately the same ROI size, regardless of the significance level applied.

In addition to the homogeneity test used above, a homogeneity test suggested by Hosking and Wallis (1991), and described in section 3.4.2, was also employed in the composition of ROIs. There are several reasons for the application of the second homogeneity test. Firstly, this provides a second, independent analysis of the homogeneity. In addition, the homogeneity test by Hosking and Wallis (1993) is based on the variability of different order sample L-moments calculated from the at-site measurements. Various order L-moments have similar properties to the corresponding distribution parameters, namely to vary less in space as the order of L-moment increases. This homogeneity test is based on the variability of three different measures and thus adopts a similar approach to the hierarchical regionalization. Finally, since this test is not distribution specific, it demonstrates the potential utility of the techniques developed herein when the GEV distribution is not an appropriate parent distribution.

With the application of this homogeneity test, it is possible to create ROIs which are homogeneous in the space of L-CV, L-CV and L-skewness, and L-skewness and L-kurtosis. Each homogeneity criterion incorporates the variability of the considered order of L-moments, which results in different sizes of ROI. The summary of ROI sizes, corresponding to various orders of homogeneity, are presented in Table 5.5. Table 5.5 summarizes both the strictly homogeneous and the probably homogeneous homogeneity criteria, which correspond to the H value as $H<1$ and $1 \leq H<2$ respectively.

Table 5.5 Summary of ROI sizes for various homogeneity criteria

#	Strictly Hom.			Probably Hom.		
	L-Ku	**L-Sk**	**L-CV**	**L-Ku**	**L-Sk**	**L-CV**
1	25	15	11	28	18	12
2	19	15	14	22	17	14
3	18	14	13	20	16	14
4	21	17	12	24	18	16
5	24	13	10	28	18	16
6	23	16	14	28	19	16
7	14	5	4	18	6	5
8	17	13	10	19	16	12
9	26	16	15	28	19	16
10	19	14	12	22	14	13
11	22	16	14	28	19	16
12	19	16	14	26	18	16
13	13	5	4	19	6	5
14	7	6	5	19	7	6
15	11	5	5	18	7	5
16	13	5	4	19	6	4
17	25	18	13	29	20	14
18	23	17	8	27	21	12
19	21	16	14	27	18	16
20	24	18	8	28	21	12
21	17	13	11	19	14	12
22	25	18	13	29	21	14
23	25	18	12	29	20	14
24	25	18	12	29	20	14
25	24	16	13	29	19	14
26	24	18	13	29	20	15
27	24	15	9	28	21	12
28	25	18	13	29	21	14
29	25	17	13	28	19	14
30	25	17	13	29	20	14
31	25	16	13	29	20	14
32	25	15	13	29	19	14

It can be seen that with the application of the homogeneity criteria suggested by Hosking and Wallis (1993), more stations can be added into the ROI before the regionalization process terminates. ROIs with only one station do not occur even for the case of a strictly homogeneous criteria. This can be viewed as an indication that this homogeneity test is not as rigorous as the homogeneity test suggested by Chowdhury et al. (1991). Thus, as was stated earlier by Hosking and Wallis (1993), the H criteria depends on the size of the formed region. With a small number of stations, the homogeneity test can give a false indication of whether or not the region is homogeneous. It would appear that regions are being accepted as homogeneous when in fact they probably are not.

5.3 BASIS FOR COMPARISON OF RESULTS

The characteristics of the 32 catchments are used to create realistic parents which are then used to generate the extreme flows. The at-site extreme flow quantiles were estimated by the employment of higher order distribution functions, with more than three parameters. The kappa distribution function, with four parameters, and the Wakeby distribution function, with five parameters, satisfied this criterion. These distribution functions are more flexible due to an additional parameter(s) and can be applied to a wider characteristic range of the random variable than the three parameter GEV distribution function. The kappa distribution function can be defined as:

$$x(F) = \xi + \frac{\alpha}{\kappa}\left[1 - \left(\frac{1-F^{h}}{h}\right)^{\kappa}\right] \tag{5.1}$$

where ξ, α are the location and scale parameters respectively while κ and h are the two shape parameters. For the estimation of these parameters, a Newton-Raphson iteration is used to solve the equations which express τ_3 and τ_4 as functions of κ and h. α and ξ are calculated as functions of λ_1, λ_2, κ and h. To ensure a one to one relationship between parameters and L-moments, the parameter space is restricted according to Hosking (1988) as follows:

$$\kappa > -1; \tag{5.2}$$

$$\text{if } h < 0 \quad \text{then} \quad h*k > -1; \tag{5.3}$$

$$h > -1; \text{ and} \tag{5.4}$$

$$\kappa + 0.725h > -1 \tag{5.5}$$

If the parameters do not satisfy the constraints defined above, then the kappa distribution is not uniquely defined and a Wakeby distribution is used as the parent distribution function for the purpose of generating at-site flows. The Wakeby distribution function is defined as:

$$x(F) = \xi + \frac{\alpha}{\beta}(1-(1-F)^{\beta}) - \frac{\gamma}{\delta}(1-(1-F)^{-\delta}) \tag{5.6}$$

where ξ, α are the location and scale parameters respectively while β, γ and δ are shape parameters. The basic method for the estimation of parameters of the Wakeby distribution is the method described by Landwehr et al. (1979b), but the calculations are expressed in terms of L-moments rather than in terms of PWMs. The parameter estimation requires the calculation of the first five L-moments (Hosking, 1986). If no solution is found, ξ is set equal to zero and a solution process continues to find the other four parameters as a function of the first four L-moments. Fortran routines are available for parameter estimation of the kappa and the Wakeby distribution functions (Hosking, 1991).

A summary of the estimation of parent flows for various return periods along with an indication in the last column as to which distribution function represents the parent at the particular site, is presented in Table 5.6.

Table 5.6 Summary of the estimated flow quantiles from the site record

	Return Period [years]						
#	**T=5**	**T=10**	**T=20**	**T=50**	**T=100**	**T=200**	**Distribution**
1	137.41	171.54	201.36	235.72	258.54	279.01	kappa
2	90.68	124.85	164.29	227.54	286.11	356.16	kappa
3	45.2	64.19	89.61	137.28	188.18	256.87	Wakeby
4	147.34	187.08	216.59	244.17	258.82	269.63	kappa
5	70.3	97.75	129.42	179.75	225.74	280.06	kappa
6	67.27	90.28	111.65	137.77	156.09	173.29	kappa
7	12.89	20.48	29.41	43.66	56.62	71.81	kappa
8	11.23	16.11	20.46	25.39	28.56	31.32	kappa
9	32.45	43.03	53.35	66.69	76.59	86.33	kappa
10	27.54	37.07	45.24	54.15	59.66	64.29	kappa
11	39.79	53.23	66.01	82.05	93.6	104.68	kappa
12	23.83	33.1	43.28	58.42	71.32	85.65	kappa
13	37.29	59.74	81.85	109.59	129.21	147.62	kappa
14	4.74	8.59	13.43	21.57	29.32	38.72	kappa
15	48.17	84.93	129.37	200.5	264.79	339.44	kappa
16	10.95	16.85	22.31	28.65	32.8	36.44	kappa
17	78.11	92.06	103.15	114.66	121.53	127.15	kappa
18	20.6	25.59	30.06	35.35	38.97	42.3	kappa
19	42.43	55.71	69.64	89.61	106.18	124.19	kappa
20	112.21	140.8	171.2	215.97	254.28	297.13	kappa
21	82.22	116.22	150.03	194.3	227.45	260.29	kappa
22	28.13	33.6	38.75	45.28	50.08	54.8	kappa
23	153.6	177.72	204.45	244.25	278.16	315.73	Wakeby
24	415.61	478.27	535.39	606.13	657.16	706.49	kappa
25	162.36	200.77	236.06	278.86	308.72	336.62	kappa
26	28.51	33.84	40.17	50.62	60.51	72.53	Wakeby
27	55.61	67.88	76.83	85.05	89.35	92.49	kappa
28	44.42	53.1	60.9	70.22	76.66	82.66	kappa
29	384.88	473.46	563.32	687.28	786.21	890.26	kappa
30	148.95	183.54	215.78	255.73	284.25	311.48	kappa
31	53.14	62.97	74.94	94.86	113.81	136.91	Wakeby
32	50.12	60.28	69.86	82.06	91.06	99.9	kappa

For the estimation of the "true" flows, parameters were estimated via L-moments. L-moments were calculated from the series of at-site annual maximum daily flows. Once the parameters for the distribution function were obtained, the extreme flow with a desired return period can be obtained, which will be used as the reference "true" extreme flow quantile.

5.4 MONTE CARLO SIMULATION

A Monte Carlo experiment was employed to investigate the effectiveness of the hierarchical approach as applied in conjunction with the ROI approach. The analysis proceeded in the following steps:

(1) For the application data set with 32 sites, "true" extreme flow quantiles were estimated using the kappa, or the Wakeby distribution. Extreme flow quantiles were calculated for return periods of 5, 10, 20, 50, 100 and 200 years.

(2) In order to simulate different spatial variability of shape and scale parameters, various ROI sizes were used. Different sizes of ROIs were obtained by the usage of various significance levels in the homogeneity test in the regionalization phase, as presented in Table 5.4 and Table 5.5. When parameters were estimated based on an ROI with the same degree of homogeneity, the extreme flow quantile is identical to the estimation of parameters with a fixed size ROI.

(3) For each site, the same number of flow values were generated as the original number of years of available record for the particular site, using the kappa or Wakeby distribution as defined in step (1). Then, using the generated flow values, the hierarchical ROI approach was applied in order to obtain an extreme flow estimation.

(4) Once the extreme flow quantiles were obtained, an evaluation of these extreme flow quantiles proceeded. Extreme flow evaluation is made by calculating the contribution to the

mean squared error (MSE) by Equation (4.10) and the BIAS by Equation (4.11).

(5) Steps (3) and (4) are repeated 10000 times and then the MSE and BIAS for the entire set of realizations are calculated. The calculated MSE and BIAS values are presented in Table 5.7.1 and in Table 5.7.2 respectively. These tables summarize different quantile estimation options when the location parameter was estimated based on data available at-site and the estimation of the scale parameter uses data from an ROI which passes the homogeneity test with a significance level of 0.25.

Table 5.7.1 Summary of MSE

ROI criteria for Shape (κ) parameter					
0.005	**0.01**	**0.025**	**0.05**	**0.10**	**0.25**
0.047490	0.047676	0.048403	0.048812	0.049561	0.050205

Table 5.7.2 Summary of BIAS

ROI criteria for Shape (κ) parameter					
0.005	**0.01**	**0.025**	**0.05**	**0.10**	**0.25**
0.010315	0.010291	0.010432	0.010556	0.010146	0.010204

It can be seen that the MSE is decreasing as the size of the ROI for the estimation of the shape parameter increases. The changes in BIAS have no obvious pattern and the variation can likely be attributed to noise in the simulation process. The positive value of the BIAS indicates that the extreme flows are overestimated relative to the corresponding "true" flows. However, the values of BIAS, which for practical purposes can be considered to be essentially constant, in combination with the decreased MSE value, indicates that as the size of the ROI is increasing for the estimation of the shape parameter, the corresponding extreme flows are providing better estimates of the "true" extreme flow quantiles.

This result justifies the earlier assumption that the shape parameter has less variability than the scale parameter. For example, if for the estimation of the shape and scale parameters the size of the ROI which corresponds to the largest considered significance level (0.25) is used, the calculated MSE over 32 stations and 6 different return periods is larger than in cases when the shape parameter was estimated from one of the ROIs which were formed with a lower significance level. In other words, if the ROI for the estimation of the shape parameter is formed with a lower homogeneity level, which means more stations in the ROI, the parameters of the GEV distribution can be estimated more precisely. This improvement in the parameter estimation can be demonstrated through the improved quantile estimation. With an enlarged ROI for the estimation of the shape parameter, the calculated MSE from the estimated flow quantiles is smaller than the MSE value for a case when the shape and the scale parameters were calculated from the same fixed size ROI.

Table 5.8.1 and Table 5.8.2 show a summary of MSE and BIAS values respectively, for the case when the ROIs are formed with the Hosking and Wallis (1993) homogeneity test employed in the regionalization process. Both tables consist of two parts, namely the case where the strictly homogeneous criterion is applied ($H<1$) and the case where the probably homogeneous criterion is used ($1 \leq H<2$).

Table 5.8.1 Summary of MSE

Strictly Homogeneous			Probably Homogeneous		
L-Ku	**L-Sk**	**L-CV**	**L-Ku**	**L-Sk**	**L-CV**
0.048007	0.050974	0.052743	0.046652	0.049781	0.051320

Table 5.8.2 Summary of BIAS

Strictly Homogeneous			Probably Homogeneous		
L-Ku	**L-Sk**	**L-CV**	**L-Ku**	**L-Sk**	**L-CV**
0.010204	0.010157	0.010133	0.010390	0.010318	0.010176

MSE and BIAS values are practically the same for the case when ROIs were composed with an application of the strict homogeneity criterion and the case when ROIs were composed with an application of the probable homogeneity criterion. In addition, MSE and BIAS values for the cases when the ROIs are formed with the Hosking and Wallis (1993) homogeneity test applied in the regionalization process are very similar to the MSE and BIAS values obtained in the previous case when the ROIs were formed using the homogeneity test suggested by Chowdhury et al. (1991).

Therefore, the common conclusion from the results is that MSE values are decreasing as the ROIs for the estimation of the shape parameter are expanded from the ROI used for the estimation of the scale parameter. Therefore, regardless of the homogeneity test applied in the regionalization process for the creation of ROIs, there is an improvement in the quantile estimation using a hierarchical ROI approach to the parameter estimation.

Table 5.9.1 and Table 5.9.2 summarize MSE and BIAS respectively for each station for a case when ROIs are obtained by the application of the homogeneity test suggested by Chowdhury et al. (1991).

Table 5.10.1 and Table 5.10.2 summarize MSE and BIAS respectively for each station for the case when the Hosking and Wallis homogeneity test was applied. Values of MSE and BIAS are the average values from the six different return periods calculated. The tables include both the strict homogeneity and the probably homogeneous criteria.

Table 5.9.1 Summary of MSE for each site

#	**0.005**	**0.010**	**0.025**	**0.050**	**0.100**	**0.250**
			Significance level			
1	0.0068	0.0133	0.0136	0.0138	0.0142	0.0146
2	0.0139	0.0147	0.0146	0.0151	0.0151	0.0155
3	0.0694	0.0659	0.0649	0.0660	0.0674	0.0678
4	0.0819	0.0939	0.0945	0.0939	0.0938	0.0940
5	0.0131	0.0100	0.0101	0.0106	0.0107	0.0106
6	0.0329	0.0365	0.0363	0.0375	0.0371	0.0373
7	0.0776	0.0314	0.0390	0.0396	0.0415	0.0479
8	0.1357	0.1686	0.1689	0.1684	0.1703	0.1709
9	0.0136	0.0245	0.0246	0.0244	0.0246	0.0246
10	0.1338	0.1108	0.1103	0.1109	0.1115	0.1107
11	0.0317	0.0330	0.0334	0.0333	0.0330	0.0339
12	0.0155	0.0134	0.0133	0.0138	0.0137	0.0142
13	0.2034	0.2401	0.2465	0.2482	0.2505	0.2536
14	0.0826	0.0617	0.0674	0.0669	0.0691	0.0728
15	0.0710	0.0794	0.0790	0.0839	0.0957	0.0958
16	0.2747	0.2722	0.2794	0.2800	0.2786	0.2814
17	0.0108	0.0100	0.0099	0.0099	0.0099	0.0100
18	0.0133	0.0094	0.0094	0.0094	0.0096	0.0097
19	0.0054	0.0053	0.0052	0.0054	0.0054	0.0057
20	0.0062	0.0082	0.0081	0.0088	0.0085	0.0089
21	0.0432	0.0546	0.0546	0.0545	0.0561	0.0552
22	0.0022	0.0019	0.0019	0.0019	0.0019	0.0020
23	0.0341	0.0162	0.0145	0.0138	0.0161	0.0162
24	0.0030	0.0022	0.0023	0.0024	0.0024	0.0025
25	0.0040	0.0141	0.0139	0.0138	0.0139	0.0146
26	0.0254	0.0269	0.0271	0.0274	0.0273	0.0274
27	0.0612	0.0571	0.0570	0.0577	0.0574	0.0575
28	0.0037	0.0034	0.0035	0.0034	0.0035	0.0036
29	0.0019	0.0029	0.0030	0.0031	0.0032	0.0033
30	0.0068	0.0085	0.0085	0.0086	0.0087	0.0089
31	0.0524	0.0396	0.0394	0.0394	0.0394	0.0393
32	0.0086	0.0020	0.0020	0.0021	0.0021	0.0022

Table 5.9.2 Summary of BIAS for each site

#	Significance level 0.005	0.010	0.025	0.050	0.100	0.250
1	0.0217	0.0312	0.0321	0.0322	0.0320	0.0326
2	-0.0145	-0.0181	-0.0180	-0.0179	-0.0183	-0.0181
3	-0.0342	-0.0268	-0.0258	-0.0261	-0.0257	-0.0262
4	0.0877	0.0904	0.0907	0.0903	0.0903	0.0902
5	-0.0253	-0.0156	-0.0154	-0.0160	-0.0158	-0.0158
6	0.0318	0.0351	0.0347	0.0357	0.0353	0.0355
7	-0.0470	-0.0270	-0.0282	-0.0274	-0.0275	-0.0278
8	0.0550	0.0628	0.0629	0.0626	0.0628	0.0630
9	0.0166	0.0242	0.0242	0.0239	0.0242	0.0239
10	0.0802	0.0689	0.0685	0.0684	0.0687	0.0682
11	0.0294	0.0302	0.0306	0.0304	0.0302	0.0311
12	0.0014	-0.0046	-0.0044	-0.0043	-0.0045	-0.0040
13	0.0221	0.0267	0.0265	0.0270	0.0269	0.0265
14	-0.0562	-0.0735	-0.0739	-0.0742	-0.0742	-0.0744
15	-0.0623	-0.0635	-0.0644	-0.0642	-0.0657	-0.0655
16	0.0634	0.0533	0.0537	0.0539	0.0529	0.0522
17	0.0413	0.0382	0.0380	0.0375	0.0378	0.0379
18	0.0363	0.0259	0.0255	0.0252	0.0255	0.0257
19	0.0024	-0.0048	-0.0050	-0.0046	-0.0052	-0.0044
20	-0.0085	-0.0132	-0.0125	-0.0137	-0.0126	-0.0133
21	0.0159	0.0234	0.0236	0.0227	0.0235	0.0228
22	0.0014	0.0052	0.0051	0.0048	0.0051	0.0050
23	-0.0420	-0.0340	-0.0260	-0.0216	-0.0339	-0.0338
24	-0.0084	-0.0049	-0.0047	-0.0049	-0.0046	-0.0050
25	0.0202	0.0308	0.0301	0.0298	0.0300	0.0308
26	-0.0183	-0.0222	-0.0226	-0.0225	-0.0226	-0.0219
27	0.0844	0.0805	0.0804	0.0808	0.0804	0.0804
28	0.0175	0.0169	0.0173	0.0168	0.0170	0.0173
29	-0.0030	0.0019	0.0019	0.0019	0.0015	0.0016
30	0.0216	0.0231	0.0225	0.0226	0.0229	0.0229
31	-0.0338	-0.0371	-0.0370	-0.0367	-0.0369	-0.0366
32	-0.0170	0.0058	0.0054	0.0055	0.0054	0.0058

Table 5.10.1 Summary of MSE for each site

#	Strictly Homogeneous L-Ku	Strictly Homogeneous L-Sk	Strictly Homogeneous L-CV	Probably Homogeneous L-Ku	Probably Homogeneous L-Sk	Probably Homogeneous L-CV
1	0.0130	0.0138	0.0152	0.0127	0.0130	0.0150
2	0.0138	0.0162	0.0164	0.0126	0.0142	0.0162
3	0.0626	0.0673	0.0704	0.0615	0.0641	0.0687
4	0.0937	0.0939	0.0971	0.0919	0.0934	0.0942
5	0.0088	0.0134	0.0159	0.0073	0.0094	0.0106
6	0.0355	0.0374	0.0379	0.0352	0.0363	0.0369
7	0.0346	0.0538	0.0597	0.0309	0.0476	0.0533
8	0.1695	0.1708	0.1736	0.1681	0.1679	0.1726
9	0.0233	0.0249	0.0249	0.0228	0.0236	0.0244
10	0.1096	0.1115	0.1119	0.1088	0.1118	0.1117
11	0.0322	0.0339	0.0344	0.0315	0.0326	0.0337
12	0.0135	0.0143	0.0155	0.0116	0.0136	0.0142
13	0.2454	0.2581	0.2687	0.2406	0.2523	0.2572
14	0.0740	0.0774	0.0819	0.0612	0.0732	0.0767
15	0.0826	0.0953	0.0961	0.0784	0.0883	0.0965
16	0.2752	0.2907	0.2973	0.2755	0.2869	0.2981
17	0.0097	0.0099	0.0100	0.0096	0.0096	0.0102
18	0.0093	0.0100	0.0114	0.0087	0.0089	0.0101
19	0.0048	0.0059	0.0065	0.0039	0.0050	0.0056
20	0.0080	0.0088	0.0119	0.0073	0.0076	0.0096
21	0.0544	0.0563	0.0575	0.0532	0.0554	0.0563
22	0.0014	0.0017	0.0021	0.0013	0.0013	0.0020
23	0.0158	0.0161	0.0166	0.0155	0.0157	0.0161
24	0.0019	0.0020	0.0023	0.0017	0.0018	0.0021
25	0.0132	0.0138	0.0145	0.0132	0.0133	0.0143
26	0.0263	0.0269	0.0278	0.0255	0.0262	0.0273
27	0.0571	0.0570	0.0578	0.0564	0.0570	0.0576
28	0.0032	0.0034	0.0037	0.0031	0.0031	0.0036
29	0.0023	0.0029	0.0034	0.0020	0.0025	0.0033
30	0.0081	0.0085	0.0094	0.0079	0.0083	0.0090
31	0.0380	0.0391	0.0397	0.0377	0.0384	0.0392
32	0.0016	0.0021	0.0023	0.0014	0.0016	0.0022

Table 5.10.2 Summary of BIAS for each site

#	Strictly Homogeneous L-Ku	Strictly Homogeneous L-Sk	Strictly Homogeneous L-CV	Probably Homogeneous L-Ku	Probably Homogeneous L-Sk	Probably Homogeneous L-CV
1	0.0317	0.0315	0.0323	0.0317	0.0317	0.0327
2	-0.0177	-0.0185	-0.0186	-0.0175	-0.0175	-0.0180
3	-0.0257	-0.0257	-0.0274	-0.0259	-0.0254	-0.0271
4	0.0905	0.0900	0.0911	0.0897	0.0901	0.0906
5	-0.0150	-0.0155	-0.0154	-0.0154	-0.0154	-0.0156
6	0.0348	0.0353	0.0351	0.0353	0.0354	0.0351
7	-0.0270	-0.0280	-0.0286	-0.0271	-0.0281	-0.0280
8	0.0633	0.0626	0.0628	0.0632	0.0628	0.0634
9	0.0240	0.0241	0.0238	0.0238	0.0237	0.0236
10	0.0686	0.0686	0.0682	0.0684	0.0688	0.0683
11	0.0301	0.0309	0.0305	0.0304	0.0305	0.0306
12	-0.0040	-0.0042	-0.0043	-0.0038	-0.0034	-0.0039
13	0.0270	0.0263	0.0274	0.0268	0.0260	0.0263
14	-0.0746	-0.0748	-0.0754	-0.0732	-0.0746	-0.0748
15	-0.0643	-0.0657	-0.0658	-0.0637	-0.0653	-0.0651
16	0.0531	0.0534	0.0536	0.0546	0.0535	0.0540
17	0.0381	0.0379	0.0375	0.0381	0.0381	0.0384
18	0.0255	0.0260	0.0249	0.0252	0.0256	0.0255
19	-0.0050	-0.0055	-0.0040	-0.0044	-0.0045	-0.0052
20	-0.0132	-0.0124	-0.0129	-0.0134	-0.0134	-0.0131
21	0.0235	0.0235	0.0236	0.0232	0.0237	0.0229
22	0.0048	0.0050	0.0049	0.0051	0.0049	0.0049
23	-0.0345	-0.0342	-0.0343	-0.0340	-0.0339	-0.0335
24	-0.0047	-0.0045	-0.0043	-0.0043	-0.0046	-0.0049
25	0.0300	0.0300	0.0302	0.0305	0.0300	0.0303
26	-0.0224	-0.0223	-0.0213	-0.0224	-0.0224	-0.0228
27	0.0809	0.0801	0.0799	0.0806	0.0810	0.0804
28	0.0176	0.0176	0.0168	0.0175	0.0174	0.0174
29	0.0014	0.0016	0.0016	0.0015	0.0016	0.0011
30	0.0231	0.0230	0.0235	0.0232	0.0234	0.0232
31	-0.0369	-0.0369	-0.0369	-0.0370	-0.0372	-0.0365
32	0.0062	0.0058	0.0056	0.0058	0.0054	0.0055

Through the application of either of the mentioned homogeneity tests, the relative contribution of each site to the overall MSE is very much the same. Major contributions are from sites 8, 10, 13 and 16. These stations are located in south west Manitoba and as was mentioned earlier, the corresponding catchments have a very different extreme flow response. For some sites, the difference in the extreme flow characteristics are detected early in the regionalization process. With a high homogeneity criterion, those stations can form an ROI of only a limited size. Examples are stations 6, 7, 14, 15 and 23. For these stations, the error contribution is just above average.

At each site, there is a tendency for a decrease in the error as the size of the ROI increases for the estimation of the shape parameter. For some sites, however, the minimum error does not occur with the application of the ROI with the largest size for the estimation of the shape parameter, but rather for the second largest and sometimes even for the third largest. This indicates that there is a limit to the expansion of the ROI for the estimation of the shape parameter. The limit is governed by the variability of the L-kurtosis in an ROI. This is clearly illustrated in Table 5.10.1 when ROIs are formed with the application of the Hosking and Wallis homogeneity test. For these results, the error constantly decreases as the size of the ROIs increase. This is true for both the strictly and the probably homogeneous criteria. In the detection of the appropriate size of the ROI to be used for the estimation of the shape parameter, the Hosking and Wallis (1993) homogeneity test showed superior results to those obtained using the homogeneity test suggested by Chowdhury et al.(1991).

Analyzing the BIAS tables, it can be noticed that no substantial BIAS value can be detected at any sites. In most cases either over or underestimation occurs. At sites with greater than average error contribution, an overestimation always occurred. From the practical point of view, this means that design engineers have a conservative extreme flow estimate. When underestimation occurs, the error is generally not as severe as in the case of overestimation.

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

The first major contribution of this study to the estimation of extreme flow event magnitudes is the incorporation of a homogeneity test into the regionalization process. This ensures the construction of a region of influence which is homogeneous and can therefore serve as an ideal basis for the estimation of extreme flow events at the site of interest. This added feature to the regionalization technique ensures hydrological homogeneity as opposed to most previous regionalization approaches which assume that hydrologic homogeneity will result.

The approach developed herein for forming regions of influence can be applied to the analysis of gauged sites or for the case of ungauged sites. The selection of stations that are considered to be similar to the site of interest, in terms of extreme flow characteristics, is based on: (i) a weighted combination of catchment geophysical attributes; (ii) statistical characteristics of the measured extreme flows; or (iii) a combination of these two groups of attributes. In the case when an ROI is formed for a gauged site, the flow measurements at the site of interest are explicitly considered in the test of homogeneity. This, however, is not possible for an ungauged site since flow measurements at the site of interest are not available. Therefore, flow measurements from all sites in the ROI except the site of interest, are used in the homogeneity test for this case.

The second major contribution of this study is the introduction of multiple ROIs for a site of interest. ROIs formed with various homogeneity levels provide an ideal basis for the hierarchical approach to parameter estimation for the selected distribution function. For the estimation of the shape parameter of the distribution function, a ROI with a lower homogeneity level is used in comparison with the homogeneity level of the ROI that is used for the estimation of the scale parameter. Results show that expanding the amount of information through increasing the size of the shape parameter ROI leads to improved estimation of extreme flows.

Uncertainty associated with the estimation of the so called "true" extreme flows which serve as reference points in the evaluation of the estimation of extreme flows is always high. Usually, the return period of interest exceeds the duration of the recorded data set, which necessitates the use of extrapolation in order to estimate the extreme flow quantile. The usual procedure in such situations is to select a parent distribution function that has a greater flexibility in reproducing the characteristics of the sample data. However, in a situation when the length of the at-site data record is short, the application of a distribution function that has additional parameter(s) will not necessarily lead to a better data description. In such cases, a regional estimation of the "true" extreme flow represents a potential solution to this problem. The ROI approach to the estimation of an extreme flow quantile can serve as a good alternative in this situation, as was demonstrated in the case study in Chapter 4.

To summarize, regional flood frequency analysis with an ROI approach can lead to an improved extreme flow estimation for both ungauged (Chapter 4) and gauged (Chapter 5) analysis. The attractive features of the ROI approach can be beneficial to design engineers in conducting various extreme flow related designs.

6.2 RECOMMENDATIONS

The major error in the extreme flow quantile estimation at an ungauged site is often primarily due to estimating the at-site index flood as opposed to estimating the regional growth factor. The index flood is generally estimated through regression analysis. Improvements in the estimation of regression parameters can generally be obtained using weighted least squares (WLS), or generalized least squares (GLS) versus the ordinary least squares (OLS) parameter estimators. The use of WLS or GLS will, however, provide improvements from a statistical point of view, but not necessarily result in a better estimation of the index flood. The main problem is that the attributes used in the regression as independent variables are not always highly correlated with the dependent variable, the index flood.

An estimate of the index flood from a non-regression based approach may provide an improvement in the estimation of flows at the ungauged sites. Such an approach can be based on a weighted combination of the mean annual floods of stations incorporated in the ROI.

An explicit method for the determination of appropriate weights to apply to the selected attributes in the calculation of the station proximity in the selected attribute space is also needed. Some attributes used in the regional flood frequency analysis have a higher correlation with the flood generating mechanism at some sites, while others have an important role in the flood generating mechanism at other stations. Therefore, it is difficult to weight the contribution of each attribute that is applied. In addition, an efficient method for selecting attributes is also needed.

A good data base of basin characteristics is important for accurate regional analysis. In order to estimate an extreme flow at an ungauged site, the preferred basin attributes have to be available at each site involved in the regional analysis. The application of a Geographic Information System (GIS) may be a good direction to improve this aspect of the methodology.

REFERENCES

Abrahams, A.D. (1972), Factor analysis of drainage basin properties: Evidence for stream abstraction accompanying the degradation of relief, Water Resources Research, 8(3), 624-533.

Acreman, M.C. (1987), Regional flood frequency analysis in the UK: Recent research - new ideas, Institute of Hydrology, Wallingford, UK.

Acreman, M.C. and C.D. Sinclair (1986), Classification of drainage basins according to their physical characteristics; an application for flood frequency analysis in Scotland, Journal of Hydrology, 84, 365-380.

Acreman, M.C. and S.E. Wiltshire (1987), Identification of regions for regional flood frequency analysis, EOS68(44), 1262. (Abstract)

Burn, D.H. (1988), Delineation of groups for regional flood frequency analysis, Journal of Hydrology, 104, 345-361.

Burn, D.H. (1989), Cluster analysis as applied to regional flood frequency, Journal of Water Resources Planing and Management, 115(5), 576-582.

Burn, D.H. (1990a), An appraisal of the "region of influence" approach to flood frequency analysis, Hydrological Sciences-Journal, 35(2), 149-165.

Burn, D.H. (1990b), Evaluation of regional flood frequency analysis with a region of influence approach, Water Resources Research, 26(10), 2257-2265.

Chow, K.C.A. and W.E. Watt (1992), Use of Akaike information criteria for selection of flood frequency distribution, Canadian Journal of Civil Engineering, 19, 616-626.

Chowdhury, J.U., J.R. Stedinger, and L.H. Lu (1991), Goodness of fit test for regional generalized extreme value flood distributions, Water Resources Research, 27(7), 1765-1776.

Cunnane, C. (1985), Factors affecting choice of distribution for flood series, Hydrological Sciences Journal, 30(1), 25-36.

Filliben, J.J. (1975), The probability plot correlation coefficient test for normality, Technometrics, 17(1), 111-117.

Gabriele, S. and N. Arnell (1991), A hierarchical approach to regional flood analysis, Water Resources Research, 27(6), 1281-1289.

Gingras, D. and K. Adamowski (1992), Coupling of nonparametric frequency and L-moment analysis for mixed distribution identification, Water Resources Bulletin, 28(2), 263-272.

Greenwood, J.A., J.M. Landwehr, N.C. Matalas, and J.R. Wallis (1979), Probability weighted moments: definition and relation to parameters of several distributions expressible in inverse form, Water Resources Research, 15(5), 1049-1054.

Greis, N.P. and E.F. Wood (1981), Regional flood frequency estimation and network design, Water Resources Research, 17(4), 1167-1177.

Guetzkow, L.C. (1977), Techniques for estimating magnitude and frequency of floods in Minnesota. U.S. Geological Survey Water Resources Investigations 77-31, 33pp.

Hogg, R.V. and A.T. Craig (1969), Introduction to mathematical statistics, Second edition, The Macmillan Company, New York.

Hosking, J.R.M. (1986), The theory of probability weighted moments, IBM research report RC12210, IBM, Yorktown Heights, New York.

Hosking, J.R.M. (1988), The 4-parameter kappa distribution, IBM research report RC13412, IBM, Yorktown Heights, New York.

Hosking, J.R.M. (1990), L-moments: Analysis and estimation of distributions using linear combinations of order statistics, Journal of the Royal Statistical Society, Series B, 52(1), 105-124.

Hosking, J.R.M. (1991), Fortran routines for use with the method of L-moments, Version-2, IBM research report RC17097, IBM, Yorktown Heights, New York.

Hosking J.R.M. and J.R. Wallis (1993), Some statistics useful in regional flood frequency analysis, Water Resources Research, 29(2), 271-281.

Hosking, J.R.M., J.R. Wallis, and E.F. Wood (1985a), An appraisal of the regional flood frequency procedure in the UK "Flood Studies Report", Hydrological Sciences Journal, 30(1), 85-109.

Hosking, J.R.M., J.R. Wallis, and E.F. Wood (1985b), Estimation of the generalized extreme value distribution by the method of probability-weighted moments, Technometrics, 27(3), 251-261.

Institution of Engineers. Australia, (IEA), (1977), Australian rainfall and runoff: flood analysis and design, IEA, Canberra, 149 pp.

Jin, M. and J.R. Stedinger (1989), Flood frequency analysis with regional and historical information, Water Resources Research, 25(5), 925-936.

Kirby, W. (1974), Algebraic boundedness of sample statistics, Water Resources Research, 10(2), 220-222.

Landwehr, J.M., N.C. Matalas, and J.R. Wallis (1979a), Probability weighted moments compared with some traditional techniques in estimating Gumbel parameters and quantiles, Water Resources Research, 15(5), 1055-1064.

Landwehr, J.M., N.C. Matalas, and J.R. Wallis (1979b), Estimation of parameters and quantiles of Wakeby distributions, Water Resources Research, 15(6), 1361-1379. Correction: Water Resources Research, 15(6), 1672.

Lettenmaier, D.P., J.R. Wallis, and E.F. Wood (1987), Effect of regional heterogeneity on flood frequency estimation, Water Resources Research, 23(2), 313-323.

Lu, L.H. and J.R. Stedinger (1992), Sampling variance of normalized GEV/PWM quantile estimators and a regional homogeneity test, Journal of Hydrology, 138, 223-245.

Nathan, R.J. and T.A. McMahon (1990), Identification of homogeneous regions for the purpose of regionalization, Journal of Hydrology, 121, 217-238.

Natural Environment Research Council, (NERC), (1975), Flood Studies Report, Vols. 1-5, NERC, London, 1100 pp.

Panu, U.S. and D.A. Smith (1988), Estimating flood flows at ungauged sites in Newfoundland, Water for World Development Proceedings of the VIth IWRA World Congress on the Water Resources, Volume II, 207-221.

Parr, W.C. (1983), A note on the jackknife, the bootstrap and the delta method estimators of bias and variance, Biometrika, 70(3), 719-722.

Pilon, P. and K. Adamowski (1992), The value of regional information to flood frequency analysis using the method of L-moments, Canadian Journal of Civil Engineering, 19, 137-147.

Potter, K.W. (1987), Research on flood frequency analysis: 1983-1986. Reviews of Geophysics, 25(2), 113-118.

Ribeiro-Correa, J. and J. Rousselle (1993), A hierarchical and empirical Bayes approach for the regional Pearson type III distribution, Water Resources Research, 29(2), 435-444.

Stedinger, J.R. and G.D. Tasker (1985), Regional hydrologic analysis, 1. ordinary, weighted, and generalized least squares compared, Water Resources Research, 21(9), 1421-1432.

Stedinger, J.R. and G.D. Tasker (1986), Regional hydrology analysis, 2, model-error estimators, estimations of sigma and log-Pearson type 3 distributions, Water Resources Research, 22(10), 1487-1499.

Tasker, G.D. (1982), Comparing methods of hydrologic regionalization, Water Resources Bulletin, 18(6), 965-970.

Tasker, G.D., J.H. Eychaner, and J.R. Stedinger, (1986), Application of generalized least squares in regional hydrologic regression analysis, U.S. Geological Survey, Water Supply Paper, 2310, 107-115.

Tasker, G.D. and J.R. Stedinger (1989), An operational GLS model for hydrologic regression, Journal of Hydrology, 111, 361-375.

Vogel, R.M. and C.N. Kroll (1989), Low-frequency analysis using probability-plot correlation coefficients, Journal of Water Resources Planning and Management, 115(3), 338-357.

Vogel, R.M. and D.E. McMartin (1991), Probability plot goodness-of-fit and skewness estimation procedures for the log-Pearson type 3 distribution, Water Resources Research, 27(12), 3149-3158.

Wallis, J.R., N.C. Matalas, and J.R. Slack (1974), Just a moment!, Water Resources Research, 10, 211-219.

Wandle, S.W. (1977), Estimating the magnitude and frequency of flood on natural streams in Massachusetts. U.S. Geological Survey Water Resources Investigations 77-39, 27pp.

White, E.L. (1975), Factor analysis of drainage basin properties: Classification of flood behaviour in terms of basin geomorphology, Water Resources Bulletin, 11(4), 676-687.

Wiltshire, S.E. (1985), Grouping basins for regional flood frequency analysis, Hydrological Sciences Journal 30(1), 151-159.

Wiltshire, S.E. (1986a), Regional flood frequency analysis I: Homogeneity statistics, Hydrological Sciences Journal 31(3), 321-333.

Wiltshire, S.E. (1986b), Regional flood frequency analysis II: Multivariate classification of drainage basins in Britain, Hydrological Sciences Journal, 31(3), 335-346.

Wiltshire, S.E. (1986c), Identification of homogeneous regions for flood frequency analysis, Journal of Hydrology, 84, 287-302.

Zrinji, Z. and D.H. Burn (1994), Flood frequency analysis for ungauged sites using a region of influence approach, Journal of Hydrology, 153(1-4), 1-21.

Zrinji, Z. and D.H. Burn (1993), Hydrologic regionalization using a homogeneity test, proceedings of the International ASCE conference held in San Francisco, 641-646.

APPENDIX A

A set of common distribution functions used in the flood frequency analysis are presented here along with the corresponding estimations of parameters via L-moments, adapted from Hosking (1990).

In the first column, distribution functions are presented in the quantile function form, x(F), except for the Log-normal and Gamma functions. These distribution functions are presented in the form of cumulative distribution function (F(x)), because x(F) for these distributions has no explicit analytical form. The second column represents the parameter estimators using L-moments. The sample L-moments are presented in a notation of l_i, where i indicates the order of L-moment and t_i is noted for the L-moment ratio, where for i=2 the $t_2=l_2/l_1$ represents the L-coefficient of variation (L-CV), and for i=3 the $t_3=l_3/l_2$ represents the L-skewness (L-Sk).

DISTRIBUTION	**ESTIMATORS**
Gumbel	
$x=\xi-\alpha \log(-\log F)$	$\alpha=l_2/\log 2$; $\xi=l_1-\gamma\alpha$
Logistic	
$x=\xi+\alpha \log[F/(1-F)]$	$\alpha=l_2$; $\xi=l_1$
Generalized Pareto	
$x=\xi+\alpha[1-(1-F)^{\kappa}]/\kappa$	(ξ known), $\kappa=l_1/l_2-2$; $\alpha=(1+\kappa)l_1$
Generalized Extreme Value	
$x=\xi+\alpha[1-(-\log F)^{\kappa}]/\kappa$	$c=2/(3+t_3)-\log 2/\log 3$; $\kappa\approx 7.8590c+2.9554c^2$
	$\alpha=l_2\kappa/(1-2^{-\kappa})\Gamma(1+\kappa)$; $\xi=l_1\alpha[\Gamma(1+\kappa)-1]/\kappa$

Generalized Logistic

$x=\xi+\alpha\{1-[(1-F)/F]^{\kappa}\}/\kappa$ $\kappa=-t_3$, $\alpha=l_2/\Gamma(1+\kappa)\Gamma(1-\kappa)$, $\xi=l_1+(l_2-\alpha)/\kappa$

Log Normal

$F=\Phi\{[\log(x-\xi)-\mu]/\sigma\}$ $c=\sqrt{(8/3)}\Phi^{-1}\{(1+t_3)/2\}$, $\sigma=0.999281c-0.006118c^3+0.000127c^5$,

$\mu=\log\{l_2/\mathrm{erf}(\sigma/2)\}-\sigma^2/2$, $\xi=l_1-\exp(\mu+\sigma^2/2)$

Gamma

$F=\beta^{-\alpha}\int_0^x t^{\alpha-1}\exp(-t/\beta)dt/\Gamma(\alpha)$ $t=l_2/l_1$

if $0<t<½$ then $c=\pi t^2$ and

$\alpha\approx(1-0.308c)/(c-0.05812c^2+0.01765c^3)$

if $½<t<1$ then $c=1-t$ and

$\alpha\approx(0.7213c-0.5947c^2)/(1-2.1817c+1.2113c^2)$, $\beta=l_1/\alpha$

Notes: Φ is the standard normal distribution function, Φ^{-1} is the inverse standard normal distribution function and Φ can be expressed in the form as:

$$\phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\varphi(t)\,dt \qquad (A.1)$$

and φ can be expressed in the form of:

$$\varphi(x)=\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}} \qquad (A.2)$$

erf() is the error function which is a special case of the incomplete Gamma function given as:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{A.3}$$

Constants in this section have the following values:

γ=0.5772 the Euler's constant, and

e=2.7182 the base of the natural logarithm.