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FORECASTING DEMAND FOR REPLACEMENT PARTS IN SUPPORT OF MILITARY AIRCRAFT ON DEPLOYMENTS

by

MICHAEL T. O'SHEA

A Thesis

Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF SCIENCE

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ABSTRACT

Operational Research and the military have an historical connection. For many years OR professionals have carried out effective studies centred on military problems in Canada and other countries. The research carried out in this thesis was motivated by spare parts forecasting problems encountered by the Canadian Air Force when planning for operational deployments during wartime conditions.

An analytic method will be proposed which can be used to predict deployment spares based on flying rates, number of aircraft and first line MTBF. A simulation model is developed to test the robustness of the analytic model. Actual deployment data gathered from the CF Hornets stationed in Qatar during the golf war is used to validate the model.

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LIST OF ABBREVIATIONS

AMMIS	Aircraft Maintenance Management Information System
CAF	Canadian Air Force
CF	Canadian Forces
CFSS	Canadian Forces Supply System
CFSD	Canadian Forces Supply Depots
DND	Department of National Defence
EOQ	Economic Order Quantity
GPSS/H	General Purpose Simulation System H
LRA	Line Replaceable Item
METRIC	Multi-Echelon Techniques for Recoverable Item Control
MTBF	Mean Time Between Failures
NORS	Non-Operationally Ready Supply
PUCK	Pack-Up Contingency Kit
ROSAM	Recursive Optimization Sparing Analysis Model
SRA	Shop Repairable Item
VMR	Variance to Mean Ratio
WAMP	Wartime Aircraft Maintenance Procedures

CHAPTER I

INTRODUCTION

A number of the aircraft fleets operated by the Department of National Defence (DND) have specific roles in support of contingency based planning. A portion of these aircraft fleets will be expected to operate from adverse locations with minimum host nation support. Furthermore, the re-supply chain back to Canada may be such that uninterrupted replenishment of replacement aircraft parts is not always possible nor practical. Hence, methods must be developed to determine the adequate level of parts that would be required to support a minimum of seven days at operational flying rates without replenishment or repair of the failed part. The determination of these parts can be classified as a "scale" or Pack-Up Contingency Kit (PUCK) to be used in support of the deployment. All parts assigned to a PUCK will come from existing stocks and/or be borrowed ("cannibalized") from other aircraft as required.

The inventory of aircraft spare parts in DND is administered by inventory item managers (located in Ottawa, Ontario) and controlled by a distributed data processing system referred to as the Canadian Forces Supply System (CFSS). At the heart of the CFSS, is an IBM mainframe on which most processing takes place, all databases are maintained and most applications reside. Each base, station, depot,..etc has limited local processing capability to control local inventories.

Canadian Air Force Inventory Terminology

The CFSS inventory can be classified as a multi-item, multi-location, and multi-repair level system. The repair and stocking echelons range from 1 to 4. Level 1 signifies "first line", level 2 "second line", and so on. First line level of repair is at the unit level (i.e. a CF 18 Tactical Fighter Squadron) and is concerned with minor, quick repairs which usually require no more than four hours to complete. Spare parts held at first line tend to be "black boxes" or other complete assemblies. The second line repair level is at the Wing[1] and provides in-depth repair. Second line spares are managed by the Wing Supply Officer on behalf of the Wing Commander. The range of parts held for an individual aircraft type at second line, can involve as many as 20,000 different line items. For the most part, third and fourth line level of repair for aircraft are synonymous. Civilian commercial facilities (i.e. Bristol Aerospace and Canadair) are contracted to provide aircraft overhaul and extensive aircraft or major assemblies rebuilding. However,

[1] A Wing is defined as the formation of a base with other Air Command Units (i.e. Air Squadrons) co-located at same location.

with some aircraft fleets, the third/fourth line repair facilities are at the manufacturer from which the aircraft or assembly was purchased. Third line aircraft parts are maintained in Canadian Forces Supply Depots (CFSD). Limited parts are also held at the civilian contractor's plant. Fourth line parts are those which can be purchased through the aerospace industry or through bi-lateral agreements with foreign governments.

Aircraft parts that can be repaired/replaced at first line are referred to as Line Replaceable Units Assemblies (LRA) and parts that can be repaired/replaced at second line are referred to Shop Replaceable Assemblies (SRA). Spare parts held at first line tend to be "black boxes" and other complete LRA assemblies. The number of indenture levels[2] as defined by Silver and Peterson [1], in a assembly[3] can go as high as 5 (i.e. transistor, circuit card, black box, navigation system, and then the avionics system itself). An aircraft can be dissected into indenture levels, and thus is referred to as a machine which uses multi-indentured assemblies.

Many of the circuit cards can quickly and easily be replaced at the line or "field level", however DND does not have portable test equipment to bench test the black box

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[2] Indenture level is the hierarchy of parts which an assembly is made up of.

[3] An assembly can be defined as part which has at least one other part as a component.

prior to re-installation into the aircraft. Thus, for all intents and purposes this thesis will only be concerned with parts that can currently be removed and installed with first line maintenance capabilities.

System replacement/spare parts are derived initially from the determination of the "mean time between failure" (MTBF) of the part divided into the forecasted flying hours during the procurement lead time. The MTBF is established by the vendor's preventative maintenance program, product technical bulletins, empirical data, and/or engineering reports. Once actual demand experience gathered, demand forecasts are then based on the previous demands (using exponential smoothing) vis-a-vis the MTBF.

After a forecast is finalized, a "scale of issue" is promulgated as authority for a unit to hold or draw the scaled stock. Within this thesis, the terms "forecast of parts" and "scale" can be regarded as synonyms as well as "spare parts" and "replacement parts".

Inventory Concepts

In the CFSS, the algorithms which determine stock levels, repair levels ...etc (also known as reprovisioning algorithms) for aircraft parts are identical to those used to determine reprovisioning data for non-aircraft parts (i.e. vehicle parts used by the Land Element of the CF).

Although aircraft such as helicopters, are used in direct support of Land and Naval Operations, the serviceability of all Canadian Forces aircraft remains the responsibility of the Commander of the Air Force.

Aircraft parts are categorized as either repairable or non-repairable. Most non-repairable parts are easier to manage due to low costs and sufficient availability. However, the repairables require extensive manual manipulation due to the idiosyncrasies associated with a multi-echelon [4], multi-indentured [5] inventory of expensive and scarce parts. Some parts can cost over $1,000,000 each (i.e. an aircraft engine) and are composed of numerous sub-components.

The authors Schonberger and Knod [2] state the ABC analysis is an important goal of material management and that it makes sense to tightly manage costly materials and to loosely manage cheap ones. They go on to outline some criteria which can be used to identify what constitutes a class A, B or C item. Their definitions are as follows:

a. Class A items may be an expensive, seldom used-ordered item or a low-cost item that is ordered often or in large quantities;

[4] A multi-echelon inventory system implies that there may be holdings for the identical part j at different inventory levels (i.e. first, second, and third line).

[5] A multi-indentured inventory comprises of bit pieces, components, "black boxes, subassemblies, and complete assemblies.

b. Class B items are of moderate costs; and

c. Class C items are low costs.

Forecasts for A items should be done using several computer models with resolution by a forecasting committee, B items by simple trend projection and C items by buyer's best guess. Bertrand and Reding [3] argue that Class A items typically comprise only 40-20% of the total inventory but 60-80% of the value (figure 1).

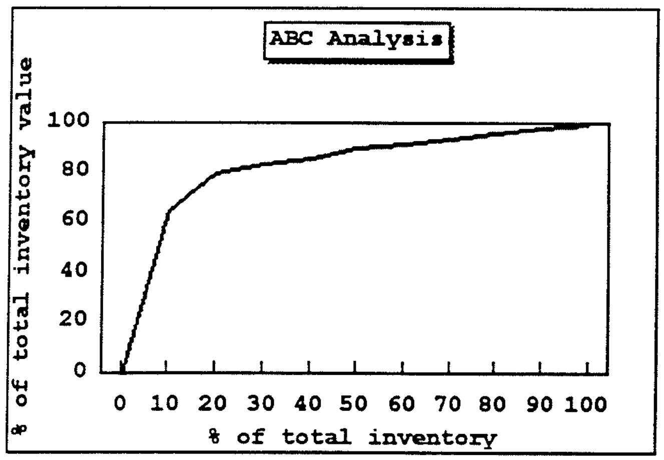

Figure 1. ABC Analysis of an Inventory System

The history behind the ABC item classification system was to maximize the use of human and computer resources to provide the forecasts which were the greatest concern of the applicable inventory holder. However, software and hardware has evolved to point where even C items can be forecasted cost effectively using computers. Furthermore, with the widespread application of "Just In Time" (JIT) inventory practices, it becomes more apparent that accurate demand forecasts are necessary to stay competitive. Stevenson [14] expands upon the ABC item classification system to include all items in the Class A management category which are of strategic importance to the company. This includes items which can directly affect profit, holding costs, and customer satisfaction. This criteria can be amended somewhat and applied to a military environment where cost does not necessarily signify strategic relevance. It can be reasonably assumed that all items or parts to be included in a PUCK are of strategic importance to the deployment and thus be managed as Class A items.

The difficulty in forecasting and subsequent provision of parts can be discussed from two distinct perspectives:

a. the selection of the type and quantity of spare (also referred to as the range and depth) parts to be held at the various echelons to meet the long term life cycle objectives of the aircraft. Consideration must taken regarding the indenture level of the part, reparability of the part and the repair capacity of the echelon; or

b. the selection of an optimal PUCK to be deployed for specific military flying operations. The range and depth of the parts must be capable of supporting the operational flying rates of the deployment for a minimum of seven days.

The first approach accounts for the steady state operation of the aircraft fleet and attempts to consider all facets of the potential utilization of the aircraft throughout its life cycle. A myriad of inventory and life cycle material managers are equipped with one of the world's largest mainframe computer to tackle this problem from the inception of the aircraft to its eventual "disposal". For non-repairable material, classical inventory methods such as EOQ formulation can be used. Repairable items generally require more complex algorithms to estimate or derive optimality. Hence, for repairables, mathematical inventory models such as Multi-Echelon Technique for Recoverable Item Control (METRIC) and MOD-METRIC have been developed. These models have been translated into computer programs and assist the inventory managers in the decision making process. METRIC was developed by Sherbrooke [4] and the RAND Corporation for the US Air Force . It was the first multi-echelon multi-item inventory model ever implemented. The MOD-METRIC is an extension of METRIC to account for the multi-indenture characteristics of most aircraft parts. Petrovic et al [5] explains that MOD-METRIC accounts for logistical relationships between an assembly and its subassemblies, as well as their different roles in the maintenance process.

The latter approach, that which deals with deployment kits, has received a lot of second guessing but very little quantitative analysis. In Chapter 2 the various methods available are discussed.

The Aim of this Thesis

The aim of this thesis is to examine spare parts forecasting problems encountered by the Canadian Air Force when planing for operational deployments. An analytical method based on several held axioms (i.e. machinery failure is an exponential process [10]) will be proposed which can be used to predict deployment spares based on flying rates, number of aircraft and the first line MTBF[6] . A simulation model will be utilized to validate the robustness of the analytic method. Actual deployment data gathered from the CF 18 Hornets stationed in Qatar during the Gulf War is used in the numerical example.

Outline of the Thesis

The second chapter of this thesis is a general discussion on Forecasting Demand as well as factors to be considered when determining deployment spares. The

[6] First line MTBF is the failure rate of the part while the aircraft is in operation or on post-flight inspection.

particular debate within the Canadian Air Force (CAF) as to the empirical source of demand data (i.e. AMMIS or CFSS), as well as the demand to flying hours relationship will be reviewed. The elements which contribute to the MTBF of parts will also be examined. This chapter contains a brief literature review of current trends particular to forecasting demand for aircraft parts. The final three sections of the chapter examine forecasting approaches in which the failure/demand rate (λ) is determined by MTBF of the part, historical demand, and a Bayesian approach.

In the third chapter, an Analytic Probability Model is introduced which determines the quantity of part j required on a deployment given certain assumptions and conditions. A numerical sample is used to compare the various demand forecasting approaches.

The fourth chapter discusses the application of simulation to validate the assumptions needed in the analytical model. An overview with the advantages and disadvantages of simulation is also presented. The chapter continues with a practical application of the simulation model to validate the analytical results. Further experimentation is conducted and the chapter concludes with a summary of the experiment results.

Finally, the fifth chapter compares the findings from chapters 3 and 4 as well as summarizing the results. The chapter concludes with suggested areas for further study.

CHAPTER II

FORECASTING OF DEMAND FOR DEPLOYMENTS

The CFSS application software is too rigid to accommodate activity based replacement parts forecasting. A Recursive Optimization Sparing Model (ROSAM) was developed by DND Director of Logistics Analysis, and can be used to assess PUCK compositions and optimize based on weight and volume as well as costs. The model uses the MTBF as the impetus behind the forecast of parts and is essentially a large non-linear programming model. Albeit, optimization can provide an adequate means to determine PUCK, more tractable methods can be utilized which focus strictly on meeting operational objectives. That is, within some probability, ensuring that no more than quantity "x" aircraft are unserviceable due to a lack of part *j*.

AMMIS vis-a-vis CFSS Debate in the CAF

Although ROSAM has proven to be a capable model, it ignores actual usage data from the CFSS and relies strictly on the data from the Aircraft Maintenance Management Information System (AMMIS). In a study by Funk [6], serious anomalies in the AMMIS data are recognized. Although the CFSS data may not be readily available in a format that can be used to forecast demands for deployments, the integrity of the data is greater than the data obtainable from AMMIS. This is primarily due to the built in audit function of the CFSS, to ensure that government owned inventory is managed and controlled according to regulations. The data captured by the AMMIS is not normally subjected to federal government audit, hence less emphasis is placed on the completeness of the input data. The CAF aerospace engineering community has a better understanding of the data from the AMMIS. Conversely, the Logistics professionals (i.e. supply officers and inventory managers) are more conversant with the data from the CFSS. Because the AMMIS and the CFSS are each autonomous computer systems, the data is often in conflict. In theory, the AMMIS can provide MTBF figures based on actual maintenance activities (i.e. replacement of parts on a particular aircraft). Furthermore, AMMIS can distinguish between parts replaced as a result of failure in-flight, pre-flight failure, post flight failure, pre-installation failure, scheduled maintenance... etc. Whereas, the demand/failure data captured CFSS is less specific, in that, the CFSS only captures the aggregated demand for specific periods of time. Unlike the AMMIS, the CFSS demand data does not distinguish between activities which created the demand. However, as stated, the integrity of the data in the CFSS is more widely accepted by the logistics community than the AMMIS data. The importance of the data quality is underscored by the lack of credibility in ROSAM sparing forecasts. The net result is that the current sparing recommendations tend to be based on out-dated and/or incomplete data. The common method which is currently relied upon to determine PUCK for deployed operations, consists of a mix of ROSAM predictions, actual demand experience, personal experience, past deployments, management intervention and luck. In general this ad hoc heuristic combination lacks the precision to be expected when determining a PUCK for an operational deployments. This thesis will present a demand forecasting method which formalizes the utilization of the best available data to derive the appropriate PUCK.

Demand to Flying Hours Relationship

Brown and Rogers [8], claim that predictions of demand based on simple linear relations between demands (or failures) and flying hours is "overly naive". Brown and Rogers do acknowledge the demand to flying hours relationship as a complex non-linear relationship. They state that only when the variance in the flying hours is large with regards to its mean, does a strong correlation exist between flying hours and demand. Brown and Rogers recommend a Bayesian approach using the Negative Binomial distribution where actual demand data is used to update the original forecast which was based on MTBF. An optimization model is used to minimize costs while maximizing aircraft availability. Other researches such as Brooks and Lu [17] assume that demands are correlated to the number of sorties or flights flown.

In a paper by Gross and Craig [18] a Standard Bayes forecasting scheme is presented where the conjugate prior distribution is assumed to exponential with a known mean. The posterior distribution is gamma with parameters derived from combining the actual demand data for part j, with the prior mean namely, the MTBF of part j. Hence, the forecast is based on flying hours and actual demand.

Reliability Analysis - MTBF Factors

Factors which can influence the MTBF are operation time, aircraft utilization, environmental conditions, and level of maintenance. Switzer [9] recommends that the mission criticality of the part should be considered when forecasting requirements. For the purpose of aircraft operations all parts are critical, in that, the failure of the part could result in the destruction of the aircraft either through hostile action or aircraft failure. Thus in this thesis, no distinction between mission criticality of a part will be made.

Operating Time

In DND, MTBF implies the number of operating hours a part *j* is expected to remain serviceable once it is installed on an aircraft (end-product). Hours in which the aircraft is in stand-by are not included in the MTBF calculations, nor are the hours that part *j* spends in inventory or in transit. Petrovic et al [5} describe a method in which the aggregate of three constant failure rates in electronic parts is used as the MBTF. The failure rates are specified for each part *j* under the following three different scenarios:

a. a part *j* is installed on an aircraft and the aircraft is idle;

b. a part *j* is install on an aircraft and the aircraft is in operation (flying); and

c. a part *j* is serviceable but is held in stock or is in transit.

It is felt that MTBF determined according to the above criteria would provide a realistic view of part failure rates and highlight areas to improve reliability through different storage and handling methods. This would be very useful to DND, however there is no practical or inexpensive means to capture the required data.

Aircraft Utilization

In addition to considering the operating time of a aircraft, one must also weigh the utilization of the aircraft. An aircraft can be utilized in three basic ways; training role, exercise support role, and combat missions. The combat mission is the area that is of greatest concern. It was an accepted practice to assume that the MTBF of parts in the combat role would be much higher than the two other roles. However, during the Gulf War this assumption was proven to be unfounded, and the average MTBF of part *j* increased. This paradox can be explained using two arguments. The first being that Wartime Aircraft Maintenance Procedures (WAMP) were invoked; this resulted in extensions to periodic maintenance schedules and the relaxation of some performance standards. The second cause for the increased MTBF, was the nature of the type of mission flown. Many of the missions flown were in "close air support" (CAP) for other aircraft. These types of missions are relatively tolerant on the aircraft thus resulting in less failures. The paradox implies that for planning purposes it is best to use the "everyday MBTF" unless specific mission types are known.

Environmental Factors

The impact of the operating climate on the actual parts is difficult to measure and as proven in the Gulf, may not be a significant factor on the MTBF. However, it should not be discarded but further analysis is required before being included in forecasting models.

Level of Maintenance

There is considerable variation in the MTBF of part j on an aircraft in operation and that on an aircraft which is undergoing second or third line maintenance. Indeed, the failure rate of the part j during maintenance is much higher than its failure when functioning on an operating aircraft. Thus, when the information is available, it would be preferential to use actual operating MTBF.

For the purposes of this thesis, the MTBF will be modelled according to Ross [10] who maintains that the memoryless property of the failure rate function suggests that MTBF should be modelled using an exponential distribution. This implies that the MTBF for any particular part j is independent of the amount of time that the part has already operated.

Brown and Rogers [8] argue that classical forecasting techniques may not be appropriate when forecasting aircraft spare parts. They claim that statistical estimation procedures are more suitable when there is an absence of deterministic predictors. In the literature, the statistical estimation of demand is often formulated in an optimization model to determine the quantity of part j required to meet objectives such as cost, weight, and aircraft availability.

For situations where the average demand over the lead time is at least 10 units, Silver and Peterson [1] recommend using a forecast method which assumes a demand pattern following the normal distribution. However, if the average is below 10 units or a Class A item, then a discrete processes such as the Poisson or combined Poisson process should be used. They recommend that when the variance to mean ratio (VMR) for the demand is less than or equal to one, use a Poisson distribution and when this is not the case, somewhat more elaborate distributions of the Compound Poisson family should be examined. They also recommended that approximation techniques for class A items should not be used and that the benefits of using a more accurate representation are higher.

The Compound Poisson process is defined by

Petrovic [5] as "a generalization of Poisson processes and describes a series of customers who arrive following a simple Poisson process, each of whom demands an amount of spares that is independently and identically distributed according to some compound distribution". The compound process gives a greater flexibility to demand description, which improves the fitting of actual demands to a theoretical distribution. Brown and Rogers [8] show that one member of the Compound Poisson family, the Negative Binomial distribution is a valid distribution when modelling a demand forecast based on two different demand processes.

According to Laws and Kelton [11], the deterministic demand rate from MTBF calculations can be modelled as a Poisson process, while the stochastic demands from corrective maintenance can be modelled using a Negative Binomial process. Sherbrooke [4] recommends that an exponential smoothing method is the best method to model the demand for aircraft replacement parts. He argues that to model a Poisson process, the assumption that the variance equal the mean, must be made. Still, empirical studies on aircraft demand rates indicate that this assumption to be inconsistent with actual data and therefore another demand distribution process should be used to model demand rates. Sherbrooke's method requires the demand VMR, for each part be greater to one. Outliers are to be identified by using a statistical test and subsequently discarded from the demand history. However, White [12] contends that there is insufficient evidence to reject the Poisson assumption when referring to the demand rate for aircraft replacement parts. White's paper presents an optimization model to predict aircraft replacement parts based on the Poisson assumption of part *j* failure/demand rates.

A paper by Landsdown and Morey [7] dealt with the problems associated with deriving an optimal PUCK for a large self contained unit such as a ship, however their approach (optimization) has a limited application to a self-sustaining air operation.

Two of the forecasting demand schemes reviewed in a paper by Gross and Craig [18] follow a Bayesian approach. In both the Standard and Adaptive Bayes schemes, it is necessary to assume a prior distribution on the part *j* failure/demand rate. With the Standard Bayes, the prior distribution is assumed to exponential with a known mean. The posterior distribution is gamma with parameters derived from combining the actual demand data with the prior mean namely, the MTBF. To determine the part *j* demand/failure rate, the Adaptive Bayes uses the marginal distribution of demand for the *n*th period given the sum of the past *n*-1 periods of demand. The resulting cumulative distribution is negative binomial.

Brown and Rogers [8] have formulated an optimization model to determine the optimal inventory levels for aircraft spare parts. A negative binomial part j demand/failure distribution is used to derive the inventory levels. The model may be used to not only determine inventory requirements, but also to evaluate budgetary and operational ramifications of support policies.

MTBF Approach

As stated previously, the initial MTBF is typically a subjective estimate, however as actual failure data is gathered, the MTBF can be adjusted as required to better reflect the true failure rate. The MTBF is usually expressed in hours, that is the number of operating/flying hours between failures. Assuming that the time between failure is exponential, implies that future failures of part j are independent of previous operating/flying hours. This is commonly referred to as the memoryless property of the exponential distribution. Thus, the failure rate for a particular part is expressed as quotient of estimated flying hours per replenishment period over MTBF and is given by:

$$\frac{\text{Total flying hours per replenishment period}}{MTBF_j} = \theta_j \qquad (2.1)$$

where

$MTBF_j$ = MTBF for part j

θ_j = mean demand/failure rate of part j per replenishment period

Historical Demand Approach

Silver and Peterson [1] argue that forecasting demand based on historical demand/failure data is the most widely accepted and employed method for short-term forecasting. Historical data can be used for forecasting either using the Maximum Likelihood or Exponential Smoothing.

Maximum Likelihood estimator for the mean of a Poisson random variable is the arithmetic mean of the past observations and is given by:

$$\frac{\sum_{i=1}^{n} t_{ij}/\alpha}{n} = \tau_j \qquad (2.2)$$

where t_{ij} = sum of demands for part j during the observation periods

n = number of equal length observation periods

α = number of aircraft for which the demand was experienced

τ_j = average demand for part j per aircraft per observation period

It may be necessary to adjust τ_j to account for difference in the length of the deployment forecast if the duration varies from the length of the observation periods.

Exponential Smoothing is a forecasting technique that averages historical demand with weights that decrease exponentially; thus, stressing more recent data. The Exponential Smoothing routine used by Gross and Craig [18] is given by:

$$\lambda_{n,j} = ax_{n-1}+(1-a)\lambda_{n-1,j} \tag{2.3}$$

where

$\lambda_{n,j}$ = the estimated mean for part j in period n

a = smoothing constant $(0 \le a \le 1)$

x_{n-1} = the demand observed for part j in period $n-1$

To initiate the technique, the initial condition is set to:

$$\lambda_2 = (x_2 + x_1)/2$$

thus (3) is valid for $n \ge 3$

Bayesian Approach

The Bayesian approach generally involves an estimated parameter which is assumed to have a *prior distribution* reflecting initial beliefs in the failure rate of a part. Actual demand data is gathered, summarized by the likelihood, and is combined with the prior distribution to provide a *posterior distribution* for the parameter.

In the Adaptive Bayes scheme examined by Gross et al [18] and demonstrated by Brown et al [8], the total number of expected demands for a part *j* over a specified time period (the parameter of the Poisson demand distribution) was taken to be a random variable λ_j with a Gamma *prior distribution* used to estimate its behaviour. Bayesian methods then can be used to revise λ_j as actual demand information is obtained. This revised version of the prior is referred to as the posterior distribution.

Assuming that demand is Poisson, the conditional distribution of X_j given λ_j is:

$$P(X_j=x/\lambda_j)=\frac{(\lambda_j)^x.e^{\lambda_j}}{x!}$$

If we let $F(\lambda_j)$ be the prior distribution of λ_j then the probability of having x demands during the deployment period is:

$$
\begin{aligned}
P(X_j=x) &= \int_0^\infty P(X_j=x/\lambda_j)\ dF(\lambda_j) \\
&= \int_0^\infty \frac{(\lambda_j)^x e^{-\lambda_j}}{x!}\ dF(\lambda_j) \qquad \textbf{(2.4)} \\
&= \int_0^\infty \frac{(\lambda_j)^x e^{-\lambda_j}}{x!}\ f(\lambda_j)\, d\lambda_j
\end{aligned}
$$

where $$f(\lambda_j) = \frac{dF(\lambda_j)}{d\lambda_j} \equiv \textit{prior density function of } \lambda_j$$

Brown and Rogers use the criteria as established by Raiffa and Schlaifer [20], when selecting the prior distribution $F(\lambda_j)$. They consider the distribution to be appropriate if it is suitable and analytically tractable. The authors maintain that the two parameter Gamma distribution meets all the criteria, with a probability density of:

$$f(\lambda_j) = \frac{(\beta_j)^{\alpha_j}}{\Gamma(\alpha_j)} (\lambda_j)^{\alpha_j - 1} e^{-\beta_j \lambda_j} \quad , \quad \lambda_j,\ \beta_j,\ \alpha_j\ > 0$$

Substituting this into equation (2.4), the probability of having x demands during the deployment was derived by Brown and Rogers to be:

$$P(X_j=x) = \binom{x+\alpha_j-1}{\alpha_j-1} \left(\frac{\beta_j}{\beta_j+1}\right)^{\alpha_j} \left(\frac{1}{\beta_j+1}\right)^{x}, \quad \alpha_j=1,2,3,\ldots,$$

which is the Negative Binomial distribution with parameters α_j and β_j/β_j+1. As actual demand data for part *j* is obtained, it can be used to revise the estimated distribution of λ_j to a closer approximation of the true demand distribution for part j. That is the prior mean of λ is α/β and α is updated with actual demand data. The drawback with the Adaptive Bayes approach is the problem of establishing the initial values for β_j. Marty and Waller [21] interpret the slope parameter α as being the "pseudo number of failures in a prior life test of duration β pseudo time units". The MTBF established for each part *j* does not provide the requisite information to derive both α_j and β_j.

The Standard Bayes approach as described by Gross and Craig [18] is a combination of MTBF and historical demand. With Bayesian methods it is necessary to assume a prior distribution of the demand/failure rate (λ), in that, under Bayesian analysis, λ is a random variable. Thus, the prior distribution of λ is assumed to be exponential with a known mean of θ. The estimate for λ_n is the expected value of the posterior distribution for λ given the sum of the past n-1 observations which is denoted by t_{n-1}. Gross and Criag [18] prove that given t_{n-1}, the posterior distribution is gamma with parameters n-1+1/θ and t_{n-1}+1. Thus, the expected value of λ_n is given by:

$$\lambda_j=(t_{n-1}+1)/(n-1+1/\theta) \qquad (2.5)$$

where

λ_j = the estimated mean for part j during the replenishment period

τ_j = observed demand for part j over period n-1 from equation (2.2)

θ_j = is the prior mean for part j from equation (2.1)

Conclusions

Forecasting schemes such as the Maximum Likelihood and Exponential Smoothing are not tied to any assumptions concerning the distribution of demand (i.e. Poisson). Nevertheless, these methods ignore one other source of usage data (AMMIS) when forecasting the quantity (s_j) of part j to take on aircraft deployments. Thus, when there is conflicting dogma as to the sourcing of forecasting data, it is prudent to respect all the credible available sources, and attempt to reach a consolidated solution. As stated, this tends to be the case in the CAF with regards to AMMIS and CFSS data, consequently a Bayesian approach is strongly recommended. However, if the integrity of the data from one source is beyond reproach compared to the other, then every effort should be made to unite opinion as to the preferred source of forecasting data.

The Standard Bayes is more straight forward and thus more likely to be understood by the "users", than the Adaptive Bayes. It also provides for the inclusion of MTBF estimates and actual demand data. Granted the Adaptive Bayes is more robust, the problems encountered when estimating the Gamma parameters reduces the appeal of this approach. In the paper by Gross and Craig [18] they argue that when there is sufficient demand data, the difference between the Standard and Adaptive Bayes schemes is negligible. Thus, to reduce the complexity of the model, the Standard Bayes approach is the method followed in this thesis.

CHAPTER III

ANALYTIC PROBABILITY MODEL

Analytic models are particularly useful when a quick or inexpensive solution is required [19]. However, analytical models can break down when too many of the simplifying assumptions are relaxed. In real life situations, many complications can arise that render some analytic models intractable, thus making simulation a more practical tool in these circumstances.

Within the context of this thesis, an Analytic Probability Model can be formulated given a demand rate (λ) for part *j* and an exponential assumption regarding the time between failures of part *j*; to derive the quantity (s_j) of part *j* required so that 98% of the time there are sufficient quantities of part *j* for 95% of the aircraft on the deployment. Hence, the model will determine the quantity of part *j* so that the probability of not running out of a particular part *j* must be greater than or equal to a critical probability as determined by senior management.

Optimization Approach

In papers by Brown and Rogers [8], Switzer [9] and, Lansdowne and Morey [7], optimization models are used to derive PUCKs for military equipment on deployments. Brown and Rogers, as well as Lansdowne and Morey, demonstrate models that forecast demand for aircraft on deployments. Whereas, Switzer's model illustrates a method to determine PUCKs for Army equipment on deployments. The optimization models all have constraints which to restrict storage space, weight, holding costs and/or purchase costs. The following optimization model (where the Negative Binomial distribution is assumed, to determine the demand/failure rate of part j) illustrates the use of optimization to select a PUCK, where size, weight, and cost are limiting factors.

$$MIN \sum_{j \in N} B(S_j)$$

subject to

$$\sum_{j \in N} V_j S_j \leq V \quad (volume\ constraint)$$

$$\sum_{j \in N} W_j S_j \leq W \quad (weight\ constraint)$$

$$\sum_{j \in N} C_j S_j \leq C \quad (costs\ constraint)$$

$$V_j\ ,\ W_j\ ,\ C_j\ ,\ S_j\ \in +I \text{ for all } j\ \in N$$

where

$$B(S_j) = \sum_{x=s+1}^{\infty} (x-s)\, P(x_i = x)$$

$$P(X_j=x) = \binom{x+\alpha_j-1}{\alpha_j-1} \left(\frac{\beta_j}{\beta_j+1}\right)^{\alpha_j} \left(\frac{1}{\beta_j+1}\right)^x , \ \alpha_j=1,2,3,\ldots,$$

However, in CAF deployments, there is minimal requirement for the PUCK to be mobile, in that, a static location is established to serve as an operating base in or near the theatre of operations. Thus, eliminating (within reason) weight and volume constraints. When the Government of Canada issues orders for the CAF to deploy aircraft in response to a crisis situation, the budgetary concerns become tertiary when determining the composition of a PUCK to support the deployed aircraft. Hence, given the fact that external constraints are not applicable to the determination of s_j of part j in a PUCK, a straight forward analytical model can be used more effectively than an optimization model.

Assumptions

The following assumptions are considered reasonable within the scope of the research previously conducted and thus, were built into the model:

1. Items to be regarded for inclusion in the PUCK are equally essential for all missions and failure of the item will render an aircraft

unserviceable;

2. The quantity demanded for part *j* in a fixed period is assumed to follow a stationary probability distribution (Poisson);
3. All the aircraft are 100% serviceable (i.e. operationally ready) at the outset of the deployment;
4. The failure rate of part *j* is independent of the other parts;
5. Repair is limited to first line (i.e. no part repair capability exists);
6. Given that appropriate parts are available, unserviceable aircraft can be repaired within 4 hours or by the start of the next fly day;
7. Each aircraft in the deployment is configured with only 1 of each of part *j* from the PUCK; and
8. The ability to cannibalize unserviceable aircraft for parts to fill holes on other unserviceable aircraft is not a feature of this model.

Determination of Failure Rates

As discussed in Chapter II, the demand forecasting methods deemed more suitable to short-term requirements such as deployments, are the MTBF Approach, Historical Demand, or the Bayesian Approach. Table 1 is the data selected from the actual PUCK used by the Canadian CF-18 Task Group in Qatar during the Gulf War.

Table 1. Input Data

PART j	NSN	NAME	MTBF (hrs)	DEMAND	FAILURE RATES (λ)		
					MTBF	HISTORICAL	BAYESIAN
1	1270-01-161-4141	Computer_Armament	394	3	.053	.013	.0411
2	5865-00-177-3418	Dispenser_Module	3189	7	.007	.030	.0083
3	5985-01-150-6891	Antenna	289	3	.073	.013	.0500
4	6605-01-271-4573	Inertial_Nav_Unit	76	33	.276	.143	.1833
5	6610-01-226-8582	Computer_Air	586	5	.036	.022	.0327

In Table 1, are the actual demand quantities and the MTBFs for the selected part j. The failure rates for the three schemes (MTBF, Historical and, Bayesian) were computed according to the methods given in Chapter II. The MTBF was calculated given a daily flying rate of 3 hours per aircraft or 21 hours per aircraft per replenishment period (7 days in this particular case). The Historical Demand was gathered during a 60 day period for 27 aircraft. The demand/failure rate (λ) was calculated using a 7 day replenishment period and 27 aircraft in the deployment.

Model

We have

n aircraft

k parts inventory

The objective is to have at least r aircraft flying with a critical probability P_c . We want the minimum k that will accomplish this.

$1/\lambda$ = MTBF per part per aircraft

X_i = number of breakdowns of part j (for a particular aircraft i) in a replenishment period.

A discrete random variable X has a Poisson distribution with parameter λ if, for x = 0,1,2,...

$$P(X_i=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

where (x = 0,1,2,...) (3.1)

λ = expected demand/failure rate

Y = number of parts that break down in a replenishment period (all aircraft)

$Y = X_1 + X_2 + \ldots + X_n$

If the uncertain quantities of X_i are probabilistically independent then Y is also Poisson with parameter $n\lambda$. The optimal number of parts k in inventory is the smallest non-positive integer k such that:

$$\text{Prob}(r \le n+k-Y) \ge P_c$$

or

$$\text{Prob}(Y \le n+k-r) \ge P_c$$

or

$$\sum_{j=0}^{n+k-r} \frac{e^{-n.\lambda}(n.\lambda)^j}{j!} \ge P_c \qquad (3.2)$$

where n = number of aircraft in the deployment

k = quantity of parts *t* in the PUCK

r = minimum number of operational aircraft

j = 0...(n+k-r)

λ = the demand/failure rate of part *t*

Equation 3.2 can be computed using a spreadsheet, Mathcad, or using a third generation computing language. The value of the variable "r" is a policy variable set by senior management. For the purposes of this thesis, "r" is set to equal 95% of the total aircraft in the deployment (rounded down to nearest integer).

Numerical Example

The results from the various methods (MTBF, Historical and, Bayesian) used to determine demand/failure rate (λ) of part j items from Table 1, are used in the Analytical Model to derive the results as depicted in Table 2.

Table 2. Analytical Results

PART j	NSN NAME	PUCK Recommended Quantity		
		MTBF	HISTORICAL	BAYESIAN
1	1270-01-161-4141 Computer_Armament	3	1	3
2	5865-00-177-3418 Dispenser_Module	0	2	1
3	5985-01-150-6891 Antenna	4	1	3
4	6605-01-271-4573 Inertial_Nav_Unit	13	7	9
5	6610-01-226-8582 Computer_Air	2	2	2

Figure 2 provides a graphical representation of the results from Table 2.

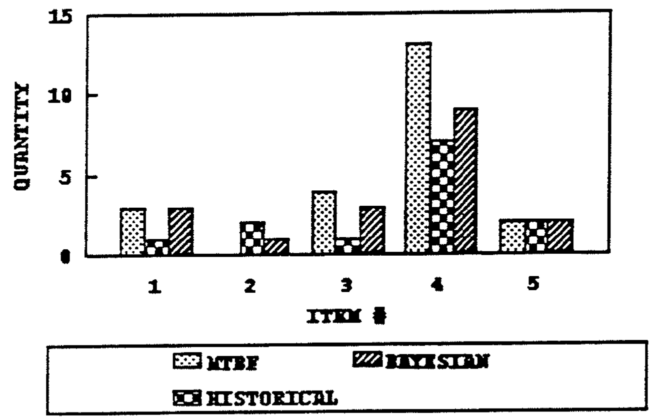

Figure 2. **Analytical Results**

Alternative Model

Another approach to solving a problem similar to the one presented in this thesis, is to determine how many items k_t do we need for a particular part t so that the probability of not running out of some part must be at least P_c. If there are N parts that are essential for an aircraft, we could find the minimum number of items k_t such that:

$$\sum_{j=0}^{n+k-r} \frac{e^{-n.\lambda_t}(n.\lambda_t)^j}{j!} \geq P_c^{\frac{1}{N}}$$

for each part t where there are N total number of different parts.

Policy Variable P_c

The critical probability or P_c provides some pre-specified level of protection against stockouts over a given horizon. According to Lansdowne and Morey [7], the U.S. Navy may use a P_c of 90%. However, within the CAF a P_c of no less than 98% is preferred. Moreover, P_c is a policy variable subject to change by senior management.

The effect of changing the value of P_c can greatly influence the PUCK composition. To illustrate this effect, the value of P_c in the analytic model was modified, giving the results in Table 3.

Table 3. **Experiments to P_c**

PART j	NSN NAME	λ	PUCK Recommended Quantity			
			P_c=.95	P_c=.98	P_c=.99	P_c=.999
1	1270-01-161-4141 Computer_Armament	.0411	2	3	3	4
2	5865-00-177-3418 Dispenser_Module	.0083	0	1	1	2
3	5985-01-150-6891 Antenna	.0500	2	3	4	5
4	6605-01-271-4573 Inertial_Nav_Unit	.1833	8	9	10	12
5	6610-01-226-8582 Computer_Air	.0327	2	2	3	4

Conclusion

As expected, the Bayesian method of determining failure/demand rate provides a compromise between using strictly the MTBF or the Historical Approaches. This illustrates a practical solution to ascertain PUCK requirements when different data sources (i.e. CFSS and AMMIS) are in conflict with one another. Furthermore, seemingly minor changes in policy variable can have a significant effect on the quantity of part *j* held in a PUCK.

CHAPTER IV

SIMULATION MODEL

Simulation allows us to use computer models to duplicate the behaviour of the "real system". Analytic mathematical methods or non-simulation specialized software packages (i.e. SPSS or LINDO) should be considered first before writing a simulation program written in a specialized simulation language (i.e. GPSS/H). However, if the degree of complexity of the real system is such that quick analytical models are incapable of capturing all the important aspects of the system, then simulation can be used. In simulation, a computer reacting to a series of instructions through the simulation software is used to evaluate a model numerically. However, simulation is only an estimate of the true system. Chew et al [15] claim that "simulations are by definition incomplete representations of reality". The strength of simulations is that changes can be made with relative ease to facilitate experimentation and sensitivity analysis.

Systems, Models, and Simulation

Schmidt and Taylor [16] define a system as "a collection of entities that act and interact toward the accomplishment of some logical end". In practice, this definition does not adequately describe the inherent flexibility found in most systems. For instance, the set of entities which comprise one system in one study, may be only be a subset of the overall system of another study. Systems tend to be dynamic, that is their status is changing over time. Therefore, it is more helpful to capture information about the system as it changes from state to state. Winston [13] defines a state of a system as the collection of variables necessary to describe the status of the system at any given time. Thus to describe the changes in status over time in a system, a set of variables called state variables are used. The objects of interest in a system are called entities and the properties associated with an entity is called an attribute.

Systems can either be discrete or continuous events. In a discrete system, state variables change only during countable points in time. Whereas, in a continuous system, state variables change continuously over time.

Simulation models can be classified as static or dynamic, as well as deterministic or stochastic. Stochastic models contain one or more random variables (i.e. MTBF), while deterministic models contain no random variables. Simulations are also either terminating or steady-state. In terminating simulations, one observation per replication is captured; the focus is to determine the state of a system over a specific period of time. In steady-state simulation the behaviour of the system once in steady-state is of interest.

The forecasting model in this paper can be classified as a discrete event terminating model. Law and Kelton [11] describe a discrete event simulation as a model of a system which evolves over time in which the state variables change only at specific (countable) points of time.

Advantages and Disadvantages of Simulation

Albeit, most simulation methods are easier to apply than analytical methods, the quality of the analysis depends on the quality of the simulation model. One should carefully weight the strengths and weakness of both before making a decision to proceed solely with simulation.

Advantages

Simulation is a widely used and increasing popular method for studying complex systems. Some of the advantages of simulation are as follows:

1. many complex real-world systems, with stochastic elements cannot be accurately described by a mathematical model;
2. simulation allows one to study a system over a

long period of time;

3. in simulation, the control of variables in the experiments is much easier to manage than would be possible if experimenting with the real system;
4. simulation can be used to check the validity of the assumptions made in an analytic model; and
5. overall, simulation is considerably more flexible than analytic models and is better suited for experimentation.

Disadvantages

Simulation is not without its drawbacks. Some of the more common disadvantages are as follows:

1. simulation model building is an art, thus the model is only as good as the artist;
2. it is often difficult to determine the extent to which an observation made during a simulation run is due to a significant underlying relationship in the system being modelled or due to the built-in randomness of the run; and
3. simulation can be time consuming and an expensive process;

Statement of the Problem

The purpose of this simulation model is to check the robustness of the assumptions applied to the analytical model in Chapter II and to provide a model which will recommend inventory requirements under less restrictive conditions than the analytic model of Chapter III. The model must be representative of the demand/failure rate of part j installed on CAF aircraft on deployments where resupply and repair is restricted.

Assumptions

The following assumptions are essentially a repeat of the assumptions needed for the analytical model:

1. The failure of part j will render an aircraft unserviceable;
2. The quantity demanded for part j in a fixed period is assumed to follow a stationary probability distribution (Poisson);
3. All the aircraft are 100% serviceable (i.e. operationally ready) at the outset of the deployment;
4. The failure rate of an part j is independent of the other parts;

5. Repair is limited to first line (i.e. no part repair capability exists);
6. Given that appropriate parts are available, unserviceable aircraft can be repaired within four hours or by the start of the next fly day;
7. Each aircraft in the deployment is configured with only one of each part *j*; and
8. The ability to cannibalize unserviceable aircraft for parts to fill holes on other unserviceable aircraft is not a feature of this model.

Description of the Base Model

The model simulates the demand for an aircraft part *j* based on the MTBF and the number of hours flown. A representative sample of five A Class CF 18 parts (out of a population of 137) was selected from the parts held in the PUCK during the Gulf War. The MTBF was calculated based on first and second line failure data as determined through the AMMIS. The model simulates seven days of operating without resupply to determine if the quantity of part *j* as determined, by the analytic model, meets the established performance objectives. In the base model, the resupply ("service") time is constant because resupply (including procurement) or part repair will not occur before seven days have expired. The failure/demand rate of each part *j* assumes an exponential distribution. Petrovic et al [5] affirm that it is a reasonable assumption to model forecasts based on reliability analysis as exponential failure rates (resulting in demands) for repairable parts. The simulation model overlooks the aircraft repair process and assumes that all aircraft unserviceable for parts on a given day, will be repaired by the beginning of the next flying day, granted that the required part is available.

In the model an ASCII file containing the part j data will be read record by record (each part is a record). The model will determine if the stock levels produced by the analytical model are sufficient, via statistical hypothesis testing. A simulation reports are produced for each experiment. A sample report is submitted at Appendix 1.

Model Structure

Referring to figure 3, the simulation model structure can best be explained by separating the model into four main sections. The first section of the model (Model Segments 1, 2, 3, and 4) consist of the input of the data to the variables through reading the data from an ASCII file and interactively via the keyboard. In the second section (Model Segment 5), the individual failure/demand rate of part j is compared to the number of spares available. The third section (Model Segment 6) controls the running of the

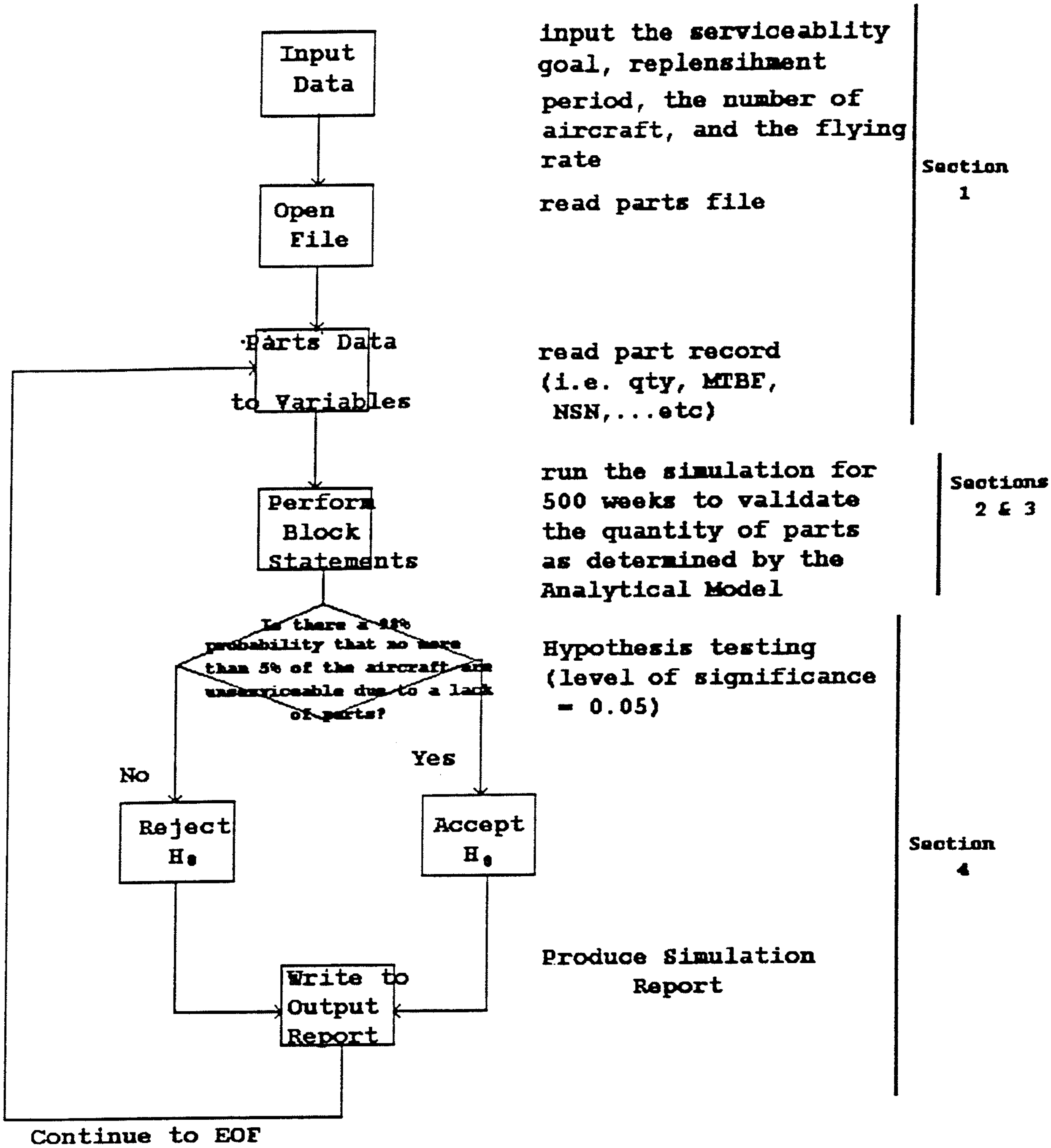

Figure 3. Overview Simulation Model

simulation. Every seven days (replenishment period in the base run), a control transaction is produced which causes the model to evaluate the contents of a queue and establish if there were sufficient quantities of part *j* in the PUCK to meet the failure/demand rate of part *j* installed on the aircraft. In the base model, the simulation is run for 500 weeks to reduce the impact of initial bias. Section four of the model conducts the hypothesis testing and produces the simulation reports. The Block Sections (sections 2 and 3) are illustrated in figures 4 and 5.

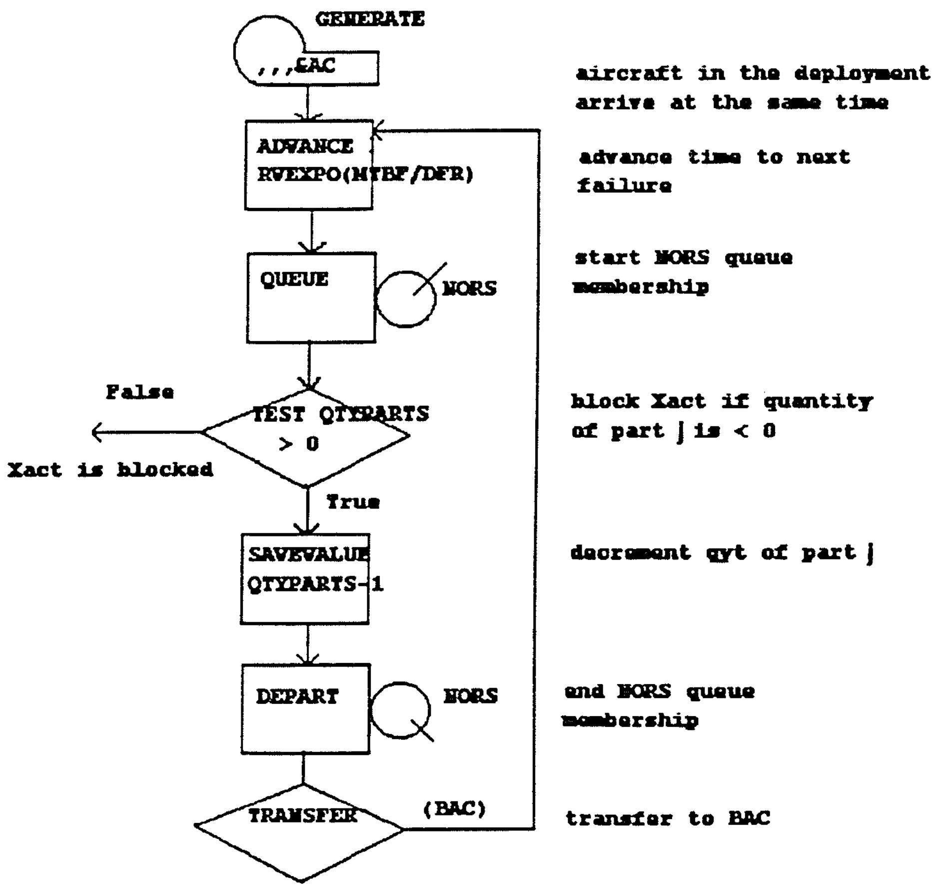

Figure 4. Model Segment 5

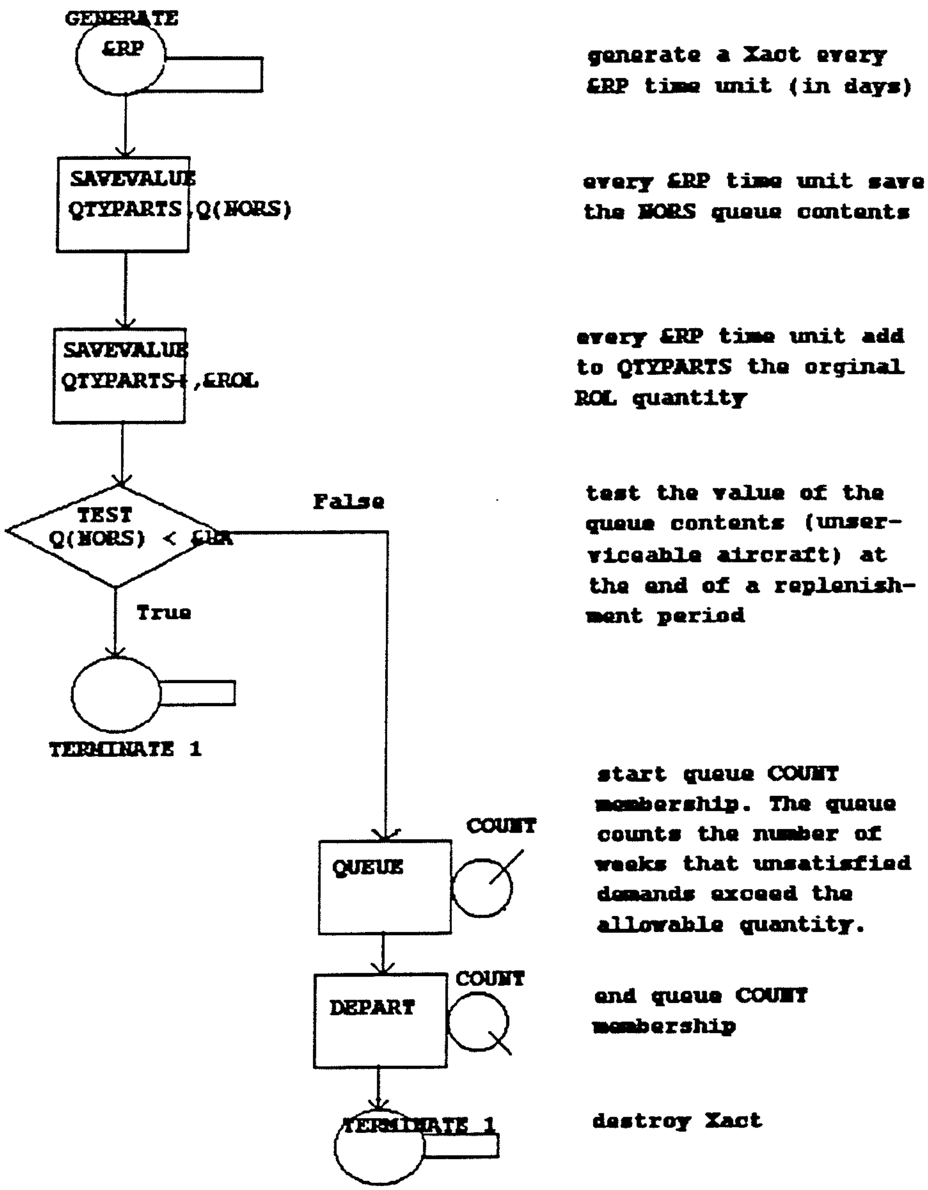

Figure 5. Model Segment 6

The modified source echo of the GPSS/H model is submitted at Appendix 2. The program was written to accommodate simple changes to the objective serviceability rate, the length of the resupply process, the number of aircraft on the deployment, and the daily flying rate per aircraft. The user will be prompted by the program to insert the above variables. As well, changes can easily be made to experiment with the resupply process.

Demand Process

The manner in which the simulation program handles the demand process warrants further description. For each part j installed on an aircraft, a random exponential time between failures is generated and a daily failure/demand rate is calculated.

Definitions

The Table of definitions for the GPSS/H model is shown in Table 4.

Table 4. Table of Definitions

Base Time Unit: 1 DAY (24 hours)

GPSS/H Entity	Interpretation
Transactions	
Model Segment 5	Aircraft
Model Segment 6	A control-Xact
Queues	
NORS	The outstanding demand for a part j
COUNT	Keeping track of unsatisfied demands

Validation of the Model

As mentioned, a sample data file, using actual CF 18 parts data gathered during the Gulf War, will be applied to the analytical and simulation model. For each part j, a record (see Table 1) is maintained with the following information:

a. Nato Stock Number (NSN);

b. Description;

c. MTBF; and

d. usage during the Gulf War.

Given the input variables, the results of the simulation model, as shown in Table 5, validate the assumptions used in the formulation of the analytical model. The simulation model was ran under the assumptions of the analytic model. A "Pass" indicates that the quantity of spares as derived from the analytical model were statistically sufficient to meet the serviceability goals with a 98% or greater probability. Thus the results of the simulation are totally consistent with the theoretical results predicted by the analytic model.

Table 5. Base Run[1]

PART j	NSN	NAME	QTY Derived by Analytic Model[2]	Z Test Simulation Model[3]
1	1270-01-161-4141	COMPUTER_ARMAMENT	3	Pass
2	5865-00-177-3418	DISPENSER_MODULE	0	Pass
3	5985-01-150-6891	ANTENNA	4	Pass
4	6605-01-271-4573	INERTIAL_NAV_UNIT	13	Pass
5	6610-01-226-8582	COMPUTER_AIR	2	Pass

Table 5 Notes

1. The input data used in the analytic and simulation models is as follows:
   Serviceability Goal: 95%
   Resupply Period Mean: 7 days
   Total Number of Aircraft: 27
   Daily Flying Rate per Aircraft: 3 hours

2. The analytic model is given by:

$$f(j|\lambda) = \sum_{j=0}^{n+k-r} \frac{e^{-(n.\lambda)} \cdot (n.\lambda)^j}{j!} \geq P_c$$

where P_c = a given probability of 98%
   $j = 0 \ldots (n+k-r)$
   n = total aircraft
   k = number of parts
   λ = part failure rate per aircraft part per replenishment period
   r = the performance objective (number of serviceable aircraft required)

3. A Normal Approximation to the Binomial Distribution can be used to test the results of the simulation model. A one tailed test (level of significance α = .05) was used, and quantity of spares which produced probabilities greater than 98% were acceptable.
Where $Z = x \sim N(np, npq)$ and is given by:

$$= \frac{a - .98}{\sqrt{\frac{(.98)(.02)}{n}}}$$

where a = number of successes in the sample n
   n = number of weeks simulated (sample size)

Experimentation Runs

Experimentation on the model can focus on several distinct areas including analyzing a proposed PUCK, different distributions for the MTBF and resupply periods, as well as variations in the flying rate and so forth. The specific experiments were chosen to parallel plausible real time variations in the model variables. The experiments were measured as to the success of statistical hypothesis testing given a quantity (s_j) of part j (as generated by the analytical model once with a daily flying rate of 3 hrs and again using 6 hrs). Table 6 summarizing the experiments parameters can be found preceding a brief description of each experiment.

Experiment 1

In experiment 1, the distribution of the resupply period was changed to triangular with a min of 6 days, mode of 7 days, and a max of 8 days. All the other variables are as per the Base Run.

Experiment 2

In experiment 2, the distribution of the resupply period was changed to triangular with a min of 6 days, mode of 7 days, and a max of 9 days. All the other variables are as per the Base Run.

Experiment 3

In experiment 1, the distribution of the resupply period was changed to triangular with a min of 4 days, mode of 7 days, and a max of 10 days. All the other variables are as per the Base Run.

Experiment 4

In experiment 4, the distribution of the resupply period was changed to triangular with a min of 6 days, mode of 7 days, and a max of 8 days. The daily flying was changed to 6 hours per day. All the other variables are as per the Base Run.

Experiment 5

In experiment 5, the distribution of the resupply period was changed to triangular with a min of 6 days, mode of 7 days, and a max of 9 days. The daily flying was changed to 6 hours per day. All the other variables are as per the Base Run.

Experiment 6

In experiment 6, the distribution of the resupply period was changed to triangular with a min of 4 days, mode of 7 days, and a max of 10 days. The daily flying was changed to 6 hours per day. All the other variables are as per the Base Run.

Experiment 7

In experiment 7, the distribution of the resupply period was changed to triangular with a min of 6 days, mode of 7 days, and a max of 10 days. The daily flying was changed to 6 hours per day. All the other variables are as per the Base Run.

Table 6. Summary of Experiment Inputs

Variables	EXPERIMENTS						
	1	2	3	4	5	6	7
Serviceability Goal	95%	95%	95%	95%	95%	95%	95%
Resupply Period	6,7,8 DAYS	6,7,9 DAYS	4,7,10 DAYS	6,7,8 DAYS	6,7,9 DAYS	4,7,10 DAYS	6,7,10 DAYS
Total Number of Aircraft	27	27	27	27	27	27	27
Daily Flying Rate per Aircraft	3 HRS	3 HRS	3 HRS	6 HRS	6 HRS	6 HRS	6 HRS

Experiment Results

The results from the experiments (Table 7) illustrate inherent ability of the analytical model to originate quantities of part *j* which can accommodate the variability of real time conditions. Only when there is a substantial increase to the daily flying rate and considerable variation in the resupply period does the analytical model fails to meet the objective.

Table 7. Experiment Results

PART j	NSN NAME	EXPERIMENTS[4]						
		1	2	3	4	5	6	7
1	1270-01-161-4141 COMPUTER ARMAMENT	Pass	Pass	Pass	Pass	Pass	Pass	Pass
2	5865-00-177-3418 DISPENSER MODULE	Pass	Pass	Pass	Pass	Pass	Pass	Pass
3	5985-01-150-6891 ANTENNA	Pass	Pass	Pass	Pass	Pass	Pass	Pass
4	6605-01-271-4573 INERTIAL NAV UNIT	Pass	Pass	Pass	Pass	Pass	Fail	Fail
5	6610-01-226-8582 COMPUTER AIR	Pass	Pass	Pass	Pass	Pass	Pass	Fail

Table 7 Notes

4. A Normal Approximation to the Binomial Distribution can be used to test the results of the simulation model. A one tailed test (level of significance α = .05) was used, in that results which produced probabilities greater than 98% were acceptable.
Where $Z = x \sim N(np, npq)$ and is given by:

$$= \frac{a-.98}{\sqrt{\frac{(.98)(.02)}{n}}}$$

where a= number of successes in the sample n
n= number of weeks simulated (sample size)
Reject H_0 when $Z \leq -1.645$

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The selection of critical spares for Canadian Air Force aircraft deployments can be accomplished by the use of a simulation model assuming a Poisson process. The focus of this thesis was to find ways to improve the initial process of deciding what quantity of spares should be allowed in a PUCK for deployments. The actual process of establishing the PUCK quantity depends totally upon the following factors: number of aircraft, daily flying rate during the replenishment period, failure/demand rate of part j, and the number of days between replenishment periods.

When there is dichotomy as to the sourcing of forecasting data, it is prudent to respect all the credible available sources, and attempt to reach a consolidated solution. As stated, this tends to be the case in the CAF with regards to AMMIS and CFSS data, consequently a Bayesian approach is strongly recommended. However, if the integrity of the data from one source is beyond reproach compared to the other, then every effort should be made to unite opinion as to the preferred source of forecasting data.

Indeed, a Bayesian method of determining failure/demand rate provides a compromise between using strictly the MTBF or the Historical Approaches. This illustrates a practical solution to ascertain PUCK requirements when different data sources (i.e. CFSS and AMMIS) are in conflict with one another.

The Standard Bayes approach is more straight forward and thus more likely to be understood by the "users", than the Adaptive Bayes. It also provides for the inclusion of MTBF estimates and actual demand data. Granted the Adaptive Bayes is more robust, the problems encountered when estimating the Gamma parameters reduces the appeal of this approach.

An analytical model can be used to accurately predict the quantity of part j required to support aircraft on operational deployments. Moreover, the analytic method reflects the real life variability expected during a deployment. Only when there is a substantial increase to the daily flying rate and considerable variation in the resupply period does the analytical model fails to meet the objective.

Field experience plays a dominant role in determining the composition of a PUCK when such experience is available. Nothing can replace the insight of experience equipment managers during PUCK deliberations. However, when experience is limited, the model developed in this thesis can provide practical and needed information.

Recommendations for Further Research

The analytic model presented in this thesis does not account for additional parts that may become available from cannibalization of other aircraft. This type of action is common especially on deployments, and should be factored in when determining the contents of a PUCK. Further study is also required to develop forecasting models which can predict parts required to support a number of specific types of aircraft missions of a predetermined length of time. Military Commanders prefer to know that sufficient spares are available to perform a desired number of missions, as opposed to knowing that there are "x" number of days of spares available.

APPENDIX 1

SIMULATION REPORT - BASE RUN
REPLENISHMENT PERIOD CONSTANT AT 7 DAYS
DAILY FLYING 3 HOURS

Hypothesis test #1
Ho: p >= .98
H1: p < .98

NSN: 1270-01-161-4141 NAME: COMPUTER_ARMAMENT MTBF: 394 QTY: 6

Percentage of Demands Satisfied 99.40%
Number of Weeks in the simulation run 500

Accept Ho: Z value 2.236
Conclude that, given the quantity of spares as determined by the analytical model, the probability of maintaining a maximum of 5% of the aircraft unserviceable due to parts during the replenishment period is equal to or greater than 98%.

Hypothesis test #2
Ho: p >= .98
H1: p < .98

NSN: 5865-00-177-3418 NAME: DISPENSER_MODULE MTBF: 3189 QTY: 1

Percentage of Demands Satisfied 97.80%
Number of Weeks in the simulation run 500

Accept Ho: Z value -0.319
Conclude that, given the quantity of spares as determined by the analytical model, the probability of maintaining a maximum of 5% of the aircraft unserviceable due to parts during the replenishment period is equal to or greater than 98%.

APPENDIX 1

SIMULATION REPORT - BASE RUN
REPLENISHMENT PERIOD CONSTANT AT 7 DAYS
DAILY FLYING 3 HOURS

Hypothesis test #3
Ho: p >= .98
H1: p < .98

NSN: 5985-01-150-6891 NAME: ANTENNA MTBF: 298 QTY: 7

Percentage of Demands Satisfied   98.20%
Number of Weeks in the simulation run 500

Accept Ho: Z value 0.319
Conclude that, given the quantity of spares as determined by the analytical model, the probability of maintaining a maximum of 5% of the aircraft unserviceable due to parts during the replenishment period is equal to or greater than 98%.

Hypothesis test #4
Ho: p >= .98
H1: p < .98

NSN: 6605-01-271-4573 NAME: INERTIAL_NAV_UNIT MTBF: 76 QTY: 22

Percentage of Demands Satisfied   99.40%
Number of Weeks in the simulation run 500

Accept Ho: Z value 2.236
Conclude that, given the quantity of spares as determined by the analytical model, the probability of maintaining a maximum of 5% of the aircraft unserviceable due to parts during the replenishment period is equal to or greater than 98%.

APPENDIX 1

SIMULATION REPORT - BASE RUN
REPLENISHMENT PERIOD CONSTANT AT 7 DAYS
DAILY FLYING 3 HOURS

Hypothesis test #5
Ho: p >= .98
H1: p < .98

NSN: 6610-01-226-8582 **NAME:** COMPUTER_AIR **MTBF:** 586 **QTY:** 4

Percentage of Demands Satisfied 98.80%
Number of Weeks in the simulation run 500

Accept Ho: Z value 1.278
Conclude that, given the quantity of spares as determined by the analytical model, the probability of maintaining a maximum of 5% of the aircraft unserviceable due to parts during the replenishment period is equal to or greater than 98%.

APPENDIX 2

GPSS/H MODEIFIED SOURCE ECHO

This appendix contains a glossary of the variables in the GPSS/H program and the modified source echo of the program.

APPENDIX 2

GPSS/H MODEIFIED SOURCE ECHO

GPSS/H Variables

&AC	number of aircraft
&COUNT	a queue to track the number of stockouts
&DFR	daily flying rate
&MTBF	mean time between failures
&NAME	part description
NORS	when a demand for parts can not be satisfied from existing stock, the demand waits in the NORS queue.
&NSN	Nato Stock Number (i.e. part number)
&QTY	quantity held in the PUCK
&PR	unit price of a part
&ROL	reorder quantity
&RP	replenishment period (i.e. 7 days)
&RQTY	the adjusted qty to be held in the PUCK
RVEXPO	a random number generator which follows a exponential distribution.
&S	number of successes
&SG	serviceability goal (i.e.70%)
&UA	unservicable aircraft
&WEEKS	number of weeks in the simulation run
&X	number of stockouts
&Z	Z test result

APPENDIX 2

GPSS/H MODEIFIED SOURCE ECHO

```
PERSONAL GPSS/H RELEASE 2.0 (AY130)      08 Jun 1994    15:07:17     FILE: 6THE.gps

LINE# STMT#  IF DO  BLOCK#  *LOC OPERATION      A,B,C,D,E,F,G  COMMENTS

                                 SIMULATE    2
                            ********************************************************************
                            *                         SIMULATION MODEL                           *
                            *                         THESIS                                     *
                            *                         BASE TIME UNIT: 1 DAY                      *
                            ********************************************************************
                            *                                                                    *
                            *                          Compiler Directives                       *
                            *              (REAL;INTEGER;VCHAR;OPERCOL;UNLIST;REALLOCATE)        *
                            ********************************************************************
                            *
                                 REALLOCATE COM,64000       increase memory requirements
                                 OPERCOL    35              search for operands up to col 35
                                 UNLIST     CSECHO          do not echo Control Statements
                                 NOXREF
                            *
                            ********************************************************************
                            *      Define Ampervariables
                            ********************************************************************
                            *
                                 INTEGER    &I                  index for GETLIST loop
                                 INTEGER    &QTY                quantity in stock
                                 INTEGER    &RP                 replenishment period
                                 INTEGER    &ROL                reorder qty
                                 INTEGER    &AC                 number of aircraft
                                 INTEGER    &MTBF               mean time between failures
                                 INTEGER    &RMTBF              mean time between failures
                                 INTEGER    &UA                 unserviceable aircraft
                            *
                                 REAL       &DFR                daily flying rate
                                 REAL       &S                  % of successes
                                 REAL       &SG                 serviceability goal
                                 REAL       &WEEKS              number of weeks
                                 REAL       &X                  number of stock outs
                                 REAL       &Z                  Z test result
                            *
                                 VCHAR*30   &NAME               name of part
                                 VCHAR*20   &NSN                nato stock number of part
                            *
                            ********************************************************************
                            *                   Interactive Input-Model Segment 1
                            *   Input serviceability goal,  resupply period, total aircraft
                            *   in the deployment, and the daily flying rate per aircraft
                            ********************************************************************
                            *
                               PUTSTRING  ('                                  ')
                               PUTSTRING  ('                                  ')
                               PUTSTRING  ('ENTER THE SERVICEABILITY GOAL')
                               GETLIST  &SG                  serviceability rate
                               PUTSTRING  ('                                  ')
                               PUTSTRING  ('ENTER THE RESUPPLY PERIOD IN DAYS')
                               GETLIST  &RP                  replenishment period
                               PUTSTRING  ('                                  ')
                               PUTSTRING  ('ENTER THE TOTAL NUMBER OF AIRCRAFT?')
                               GETLIST  &AC                  number of aircraft
                               PUTSTRING  ('                                  ')
                               PUTSTRING  ('ENTER DAILY FLYING RATE/ AIRCRAFT IN HOURS?')
                               GETLIST  &DFR                 daily flying rate per ac
                            *

                            ********************************************************************
                            *          Control Statements (LET)
                            ********************************************************************
```

APPENDIX 2

GPSS/H MODEIFIED SOURCE ECHO

```
                *
                      LET &SG=(&SG/100)*&AC          calculate the min
                *                                    number of unserviceable
                *                                    aircraft
                      LET &UA=(&AC-&SG)+1
                *
                *****************************************************************
                *                Read Parts File Data-Model Segment 2
                *****************************************************************
                *
                 DATA  FILEDEF    '6PARTS.DAT'       link physical input file to
                *                                    a logical file
                *

                *****************************************************************
                * Perform Simulation for each Part in Data File-Model Segment 3
                *****************************************************************
                *
  1                   DO &I=1,5,1                    read each record
  1             *
  1             *****************************************************************
  1             *Input Data-Model Segment 4
  1             *****************************************************************
  1             *
  1                    GETLIST  FILE=DATA,END=TERM,(&NSN,_
  1                                  &NAME,&MTBF,&QTY)
  1             *                                     read the record elements
  1             *****************************************************************
  1             *          Control Statement (LET)
  1             *****************************************************************
  1             *
  1                   LET &RMTBF=&MTBF               capture MTBF
  1                   LET &ROL=&QTY                  set ROL to original qty
  1                   LET &MTBF=(&MTBF/&DFR)         determine daily MTBF
  1             *
  1                   LET &WEEKS=500                 number of weeks
  1             *
  1             *****************************************************************
  1             *          GPSS/H Block Section-Model Segment 5
  1             *****************************************************************
  1             *
  1                   INITIAL    X$QTYPARTS,&ROL     initialize stock qty
  1        1          GENERATE   ,,,&AC              generate transactions
  1        2    BAC   ADVANCE    RVEXPO(3,&MTBF)     part failure rate
  1        3          QUEUE      NORS                start NORS queue membership
  1        4          TEST G     X$QTYPARTS,0        continue if parts available
  1        5          SAVEVALUE  QTYPARTS-,1         decrement the qty of parts
  1        6          DEPART     NORS                depart queue
  1        7          TRANSFER   ,BAC                continue
  1             *
  1             *****************************************************************
  1             *        GPSS/H Control Statements-Model Segment 6
  1             *****************************************************************
  1             *
  1        8          GENERATE   &RP                 simulate for resupply
  1             *                                    period
  1        9          SAVEVALUE  QTYPARTS,Q(NORS)    save queue current contents
  1       10          SAVEVALUE  QTYPARTS+,&ROL      backfill unsat demands
  1             *
  1       11          TEST L     Q(NORS),&UA,JMP1    test for unsat demands
  1       12          TERMINATE  1                   terminate transaction
  1             *
  1       13    JMP1  QUEUE COUNT                    use a queue as an unsat
  1             *                                    demand counter
  1       14          DEPART COUNT                   depart queue
  1       15          TERMINATE 1                    terminate transaction
  1             *
  1                   START      &WEEKS,NP           lenght of simulation
  1             *
```

APPENDIX 2

GPSS/H MODEIFIED SOURCE ECHO

```
*****************************************************************
*
*  Run-Control Statements  and Customized Reporting Statements
*
*****************************************************************
*
*************************Print Table Headings********************
*
          PUTPIC  LINES=9,FILE=SYSPRINT,(&NSN,&NAME,&MTBF,&QTY)
                         SIMULATION REPORT

Hypothesis test
Ho: p >= .98
H1: p <  .98


   NSN: *  NAME: *  MTBF: *  QTY: *
   ----------------------------------------------------------
*
********Perform Calculations, Print and Save Results************
*
*
          PUTPIC LINES=1                         terminal message
WORKING...PLEASE STAND-BY...
*

          LET &X=QC(COUNT)/&WEEKS               percentage of
*                                                stock
*                                                outs
*
          LET &S=(1-&X)                          % of successes
*
          LET &Z=(&S-.98)/(SQRT((.98*.02)_       Z test
                                 /&WEEKS))
          LET &S=&S*100                          save this observation
*
          PUTPIC LINES=3,FILE=SYSPRINT,&S,&WEEKS
   Percentage of Demands Satisfied   ***.**%
   Number of Weeks in the simulation run *
   ----------------------------------------------------------
*
* Hypothesis Testing
*
          IF (&Z<=-1.645)
            PUTPIC LINES=9,FILE=SYSPRINT,&Z

Reject Ho: Z value **.***
Conclude that, given the quantity of spares as determined by the
analytical model, the probability of maintaining a maximum of 5%
of the aircraft unserviceable due to parts during the
replenishment period is less than 98%.



          GOTO JMP2
          ENDIF
*
*Accept Ho
*
          PUTPIC LINES=9,FILE=SYSPRINT,&Z

Accept Ho: Z value **.***
Conclude that, given the quantity of spares as determined by the
analytical model, the probability of maintaining a maximum of 5%
of the aircraft unserviceable due to parts during the
replenishment period is equal to or greater than 98%.



JMP2    HERE
        CLEAR
```

APPENDIX 2

GPSS/H MODEIFIED SOURCE ECHO

```
        ENDDO                         read the next part record
*
***********************************************************************
*  End of Simulation-Model Segment 9
***********************************************************************
*
 TERM   HERE                           end of simulation
        PUTSTRING ('SIMULATION COMPLETED')
        END
```

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