THEORY AND ESTIMATION OF DISEQUILIBRIUM MACROECONOMICS

by

GOPALAN CHENGALATH

A thesis
presented to the University of Manitoba
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Department of Economics

Winnipeg, Manitoba

ISBN 0-315-78054-1

THEORY AND ESTIMATION OF DISEQUILIBRIUM MACROECONOMICS

BY

GOPALAN CHENGALATH

A Thesis submitted to the Faculty of Graduate Studies of the University of Manitoba in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

To Prof. Wayne Simpson and fellow Canadians

Contents

List of Figures

List of Tables

Abstract

The objective of the dissertation was to estimate a small disequilibrium macroeconomic model. A disequilibrium macroeconometric model is one in which a system of equations specifies a regime, and there are three or four such regimes. An observation of the data is assigned to the regime which has the highest probablity of occurrence. Alternatively, the model allows for switching between regimes and each regime characterized by a system of equations. The method of estimation was Full–Information Maximum Likelihood. The data is Canadian from 1972.1 to 1985.3. We were able to obtain estimates which are quite reasonable. The model also replicated, for a large part of the sample, the business cycle quite well.

Acknowledgements

In the course of the preparation of this thesis, I have been bestowed a great deal of compassion and kindness by many. It is time now to acknowledge them. The assistance of Wayne Simpson, thesis supervisor, has been inestimable. His patience and perspicacity knew no bounds, and without it I would not have survived those dark days when the model was being estimated. I would also like to thank him for making available to me the necessary computer packages. I would like to thank Norman E. Cameron for suggesting this topic to me, and leading me by the hand through the thicket of the theoretical literature. The choice of this difficult area of investigation has considerably enhanced my appreciation of Econometrics. George Chuchman deserves a special word of gratitude for his support and assistance during the course of the study. Acknowledgements are due to Martin Yeh for consenting to be one of my examiners. I would like to take this opportunity to record my appreciation for John Loxley. Without his advice, encouragement and unshakeable faith in my abilities, the thesis could not have seen the light of the day. Allow me to extend my heart–felt gratitude to Costas Nicolaou and John Gray for everything they did to see this work through to a happy conclusion. I am indebted to Sam Wong, Computer Services, University of Manitoba, for assistance in the installation of GQOPT4. I am grateful to Brent Depape for assistance with the graphs in Chapter 5.

Chapter 1

Introduction

Since Clower's (1965) pathbreaking reinterpretation of Keynes, a great deal of attention has been showered on disequilibrium macroeconomics. Clower's insights into the working of the macroeconomy was initially formalized by Barro and Grossman (1971), and subsequently elaborated upon by Malinivaud (1977) and Muellbauer and Portes (1978). By now the theory of disequilibrium macroeconomics had reached a stage of maturity. Alongside developments in macroeconomics, there occurred parallel developments in its microfoundations. The existence, uniqueness and stability of microeconomic behavior under rationing was investigated and elegant theorems ensued. The importance of rationiong schemes was highlighted. Drèze (1975) and Benassy (1975) were the pioneers in this effort. The literature on microfoundations has furthered our understanding of disequilbirium in a macroeconomy.

The stage was now set for econometric estimation. But econometric theory was lagging behind economic theoretic developments. Estimation of disequlibrium models was confined to one–market, and there were problems in the straightforward extension of the one market case to two markets. Econometricians grappled with the problem of spill–overs (see Quandt: 1978 and Amemiya: 1977) from one market to the other in the event of rationing and its specification. Also there was the problem of the derivation of the appropriate likelihood function under such an eventuality. A definite break–through in both these areas was achieved by Ito (1980) and Gourieroux, Laffont and Monfort (1980a, 1980b). Not only were spill–over effects correctly specified, and its properties fully enunciated, the appropriate likelihood function was spelled out. Applied Econometricians could no longer lament about a lack of theory. And rise they did to the occasion. The studies of Artus, Laroqe and Michel (1985), Kooiman and Kloeck (1985) and Sneessens (1981, 1983) are a testimony to these early efforts.

One of the curious developments, and fit object of study by the sociologists of knowledge, has been the predominance of Continental European Economists, notably the French, in not only pure theory, but also in Econometrics. Why have the Anglo–American economists been reluctant to investigate disequilibrium macroeconomics? Is disequilibrium only to be noticed in the economies of Continental Europe? True, the absence of articulation of sticky wages and prices has detracted many an economist, but why should all these, but for a few notable exceptions, be from the Anglo–American circles?

The dissertation focusses its attention on four aspects of disequilbrium.

Chapters II and III survey the theoretical literature both in its macroeconomic and microeconomic aspects. The choice of topics for discussion in these chapters have been inspired by the need to provide a clear understanding of the theory to facilitate estimation, since the current study is an exercise in the estimation of a macroeconomic disequilibrium model. Chapter IV deals with the theory of econometrics of disequilibrium macromodels. Chapter V, following on the footsetps of Artus et al. and Kooiman and Kloek, concerns itself with the estimation of a disequilibrium macroeconomic model for Canada for the period 1972.1 to 1985.3. The estimation technique is Full–Information Maximum Likelihood. The study terminates with some concluding remarks.

Chapter 2

Theory of Disequilibrium Macroeconomics

2.1 Introduction

Macroeconomic theory when markets do not clear is the outgrowth of disenchantment with the way economists have viewed the contribution of the General Theory to economics, just as the General Theory was a reaction to Classical orthodoxy. Keynes's theoretical contribution showed how unemployment was the outcome of deficiency of aggregate demand. More importantly, he demonstrated that unemployment once manifest can persist for an indefinite length of time, and the economic system may show no tendency to correct the malady. In other words, he seriously questioned the automatic mechanism of the economic system which was considered an article of faith by classical economists.[1] Not long after that Hicks (1937) provided an encap-

[1] And Keynes was quite alive to the theoretical departure from his predecessors, and the methodological novelty that his new perspective entailed. The emphasis on deficiency of aggregate demand as one of causes of unemployment was one which economists had not

sulated version of the substantive contribution ushered in by General Theory in the guise of IS-LM model, which has since become the bread and butter of undergraduate teaching in macroeconomics.

Differing opinions as to what constituted the essence of General Theory were not long in coming. One stream of thought contended that if the neoclassical synthesis or IS-LM model summed up the essentials of General Theory, then it differed little in substance from the economics that preceded it, and all talk about the Keynesian revolution was much ado about nothing. The first salvo at what has since become Keynesian orthodoxy was fired by Clower (1965). He argued that if the main problem that exercised Keynes's mind was unemployment, then his brand of economics was not one where all the variables are in equilibrium, but in which variables differed from their equilibrium values. If households experience excess supply in the labor market, then there cannot be equilibrium in other markets, and their behavior in other markets are constrained by their inablity to sell as much labor as they desired. To be specific, the excess supply in the labor market would force agents to be off their demand curve for goods.

Though Clower's definitive paper can legitimately be considered as pro-

given much credence until the General Theory, and Keynes can claim a rightful place in the pantheon of forerunneres who have given birth to the microfoundations of macroeconomic disequilibrium. The importance of quantity or income adjustments that deficiency of aggregate demand pointed out was in complete contrast to the price adjustment mechanics of the Classics. "As I have said above, the initial novelty lies in my maintaining that it is not the rate of interest, but the level of incomes which ensures equality between savings and investments." (Keynes: 1937) viding the initial impetus to the study of disequilibrium macroeconomics,[2] Patinkin (1956) had seen the possibility of unemployment when the demand for goods is less than output. Firms unable to sell as much goods as they wished at the prevailing prices would demand less labor. The point is that it is the demand for goods, besides wage rate, which determined the demand for labor. Since his approach was partial equilibrium, he failed to see the possibility of the causal chain which ran from unemployment to excess supply in the goods market. Clower's framework was essentially general equilibrium in conception, but he examined only the latter case. Barro and Grossman (1971) elegantly synthesized the insights of Patinkin and Clower in a general equilibrium framework with an explicit choice theoretic basis.

2.2 Effective Demand and Supply

When markets are in equilibrium, the demand and supply functions derived from the optimization problem is dependent only on relative prices. Equilibrium models by introducing the fiction of either Edgeworthian recontracting

[2]It is worthy of mention that Clower was preceded by an impressive array of modern thinkers, who in spite of bringing to bear their considerable theoretical acumen on issues concerning non-clearing markets, failed to make much headway due to the absence of a coherent and unifying framework on which to build their edifices. Prominent among these were Samuelson (1947) and Tobin and Houthakkar (1950) who studied the theory of demand under conditions of rationing. Hansen (1951) introduced the concept of active demand, a near sibling of effective demand, and quasi-equilibrium to connote a situation which required contrasting with Walrasian equilibrium. Hahn and Negeshi (1962) studied non-tatonnement processes where trading took place at situations which departed from Walrasian equilibria. Hicks (1965) recognized quantity adjustments when he conceived the concept of fix-price. See Benassy(1987).

or Walrasian tatonnement posit that excess supplies and demands adjust instantaneously and that trading takes place only at equilibrium prices. The equally realistic case of trading at false prices is not addressed, and its implication not spelled out in Walrasian models. Dissatisfaction with market clearing models have of late resulted in a spurt of theoretical activity into the implications of what are known as quantity rationing or disequilibrium models. If markets do not clear instantaneously or, in other words, have long adjustment lags, then the Walrasian or notional demands and supplies which specify that households can supply as much labor as they wished at the prevailing wage rate, or demand as much goods as they wished at prevailing relative prices, and that firms can supply as much as output as they wished, given the relative prices, or demand as much labor as they wished, given the wage rate, is no longer true. If this is the case, then notional demands and notional supplies are not descriptive of the behavioral relationships inherent in a disequilibrium system. If the households and firms are constrained in their behavior, then besides relative prices, quantity constraints enter into the behavioral relationships. For example, if there is excess supply in the labor market, the households find it unable to sell as much labor as they wished. And the existing demand would have to be rationed in some more less systematic way among the households. Similarly, when there is excess demand in the goods market, the available output has to be rationed among the households in a more or less systematic way. And rationing that agents experience in one market has non-trivial consequences on their behavior in other markets. If the households are rationed in the labor market, the demand function which is now dependent on relative prices and the labor supply constraint can no longer be expected to be the notional demand, and will in fact turn out to less than the notional demand. Thus demand and supply curves which explicitly consider quantity rationing in other markets are termed effective demand and supply curves. If no quantity restriction or rationing is experienced in other markets, the notional demand for the market under consideration coincides with the effective demand. The actual quantity traded in the market is the minimum of effective demand and supply.

In models where markets clear, individuals view prices as exogenous, while in non–market clearing models both prices and quantities become exogenous. The importance of both price and quantity rationing is highlighted in non–market clearing models. Exogeneity of quantity rationing is attributed to rigidity of wages and prices. Rigidity of wages and prices are, in turn, attributed to institutional factors, uncertanities, and information costs involved in a decentralized economy.

Following the pioneering work by Clower (1965), and based on the studies by Barro and Grossman (1971) and Malinvaud (1977), Muellbauer and Portes (1978) have classified disequilibrium situations into four regimes. In three of those regimes, households are constrained in either the labor market or goods market or both. In Repressed Inflation (excess demand in both markets) and Keynesian Unemployment (excess supply in both markets), rationing occurs in goods market and labor market respectively, while in Classical Unemployment case (excess demand in goods and excess supply in labor), there is rationing in both markets. In the Underconsumption case (excess supply in goods and excess demand in labor) households are unconstrained in both markets. And when households are constrained in the goods market, firms are unconstrained and vice versa, since rationing can take place only on the long side of the market. The Table 2.1 below classifies the various regimes from the point of view of rationing faced by the households. X indicates rationing and empty space, absence of rationing.

Table 2.1: Theoretical Regime Classification

	RI	KU	CU	UC
Rationed in goods market	X		X	
Rationed in labor market		X	X	

2.3 The Concept of Equilibrium

A novice will be quick to raise the question as why there should equilibrium in disequilibrium models. In that sense, the rubric 'disequilibrium' to connote situations when markets do not clear is unfortunate and a recipe for confusion. But some pondering over this issue should make it clear that though a market may not be in equilibrium (Walrasian or notional) in the sense that agents are not transacting quantities up to the desired amounts and therefore no equality between desired supplies and demands, there has to be some equality between purchases and sales. This solution of a quantity rationing model have been variously called in the literature as 'temporary equilibrium,' 'quasi equilibrium,' 'fix price equilibrium' and so on. To conform with the literature, we shall use the term quantity rationing and disequilibrium synonymously fully realizing that the appellation 'disequilibrium' does not really imply absence of equilibrium, but an equilibrium which is non-Walrasian.

An equilibrium with quantity rationing should comply with three conditions. Firstly, as alluded to earlier, purchases must equal sales in the market. Obviously, if a market is off the Walrasian equilibrium, purchase cannot be thought of as being dictated both by the demand curve (Walrasian) and sales by the supply curve (Walrasian). The minimum condition, Q=Min(D,S), stipulate that sales or purchase decrease to meet demand or supply; whether it is the demand or supply quantity that is traded depends on which is less.[3] Secondly, no agent must be coerced to transact more than his desired quantity. If this condition is not observed, the minimum condition is bereft of content. Workers coerced to supply more than their desired levels of labor becuase collective agreement relegates the right to determine overtime work

[3]The minimum condition, it may be noted, is not the conventional wisdom in Keynesian macroeconomics, where the quantity traded is determined by the aggregate demand, and the supply rises to meet the demand. In disequilibrium macroeconomics, the quantity traded is determined either by the aggregate demand or aggregate supply, and there will, in general, be no confluence of demand and supply.

exclusively to management, without the workers having the right to refuse the proffered overtime, run counter to the second condition. Thirdly, there cannot be both rationed buyers and sellers in the same market. In other words, only the long side of the market is rationed. However, instances are not wanting where there are unsatisfied suppliers and demanders. A case in point is the simultaneous co–existence of unfilled jobs (unsatisfied demand) and unemployment (unsatisfied supplies). This unsavory fact (from a theorist point of view) does not admit of an easy solution, and we shall ignore this possibility.[4]

It was noted above that a distinction need be observed between sales and supplies, on the one hand, and purchases and demands, on the other. It may be instructive to probe deeper into this dichotomy. The divergence of trades and their notional counterparts, it is recognized, is due to the constraints that forbid the agents to trade their desired amounts. But how do the agents perceive the constraints that are operative in the market? Surely, agents do have some information regarding the market conditions, and the rationing that is likely to be observed there, however erroneous it may be. For example, an unemployed individual knows that there is excess supply in the labor market, though he may not possess exact information regarding the extent of unemployment and the demand price of his labor. But in course of time,

[4]In deference to the reality of unsatisfied supplies and demands in the same market, macroeconometric disequilibrium models allows for the co–existence of unsatisfied supplies and demands in sub–markets, while observing discrete switching between regimes in aggregate markets. (Sneessens and Dreze: 1986 and Laroque and Salanie: 1987)

in the process of job search, perception of constraints undergoes change, through a process of learning from experience, and eventually conform to reality. Similarly, firm's perceived rations are sales, an indication of which can be had by observing the inventories. Monitoring changes in inventories, starting from an erroneous perception of constraints, firms will eventually arrive at the correct perception. In other words, the perceived constrains converge to real constraints. Convergence is crucial for equilibrium; for its absence would imply that the agent is faced with a discrepancy between trades and perceived rations which would impell him to change the trade offers subsequently. This violates equilibrium, as equilibrium implies absence of change or a state of rest. Therefore inherent in condition 1 is a sub–condition for equilibrium: perceived constraints and actual constraints have converged.[5]

2.4 The Model

2.4.1 Assumptions and Accounting

The ensuing discussion leans heavily on Muellbauer and Portes (1978). The simple economy considered here has no financial assets and is closed to international trade. Firms produce for two reasons: sales and accumulation of inventories, $Y = X + \Delta inv$, and purchases of the product of firms are

[5]The three conditions stated above will not, in general, be sufficient, for the determination of equilibrium. Complete determination of equilibrium require a full specification of the rationing scheme. The relationship between rationing scheme and the uniqueness of equilibrium, *inter alia*, constitute the subject matter of chapter III.

made by households and government. Government is never rationed and only households are rationed. Saving (S) of households is excess of income (Y) over expenditure (PC). Expenditure (PC) is financed by dividends (D) paid out by firms plus wage income (WL); $S = WL + D - PC = \Delta M_h$. Since there are no financial assets, change in savings is a change in money holdings. The profit of firms $\pi = PX - WL$ less dividend payments determine the change in the money holdings of firms; $\Delta M_f = \pi - D$. Therefore $\Delta M = \Delta M_h + \Delta M_f = P(X - C) = PG$, i.e., government expenditure is financed through the issue of money.

2.4.2 Microeconomic Basis

Behavior of Households

The model proposed is a two-period intertemporal model of the household; not only current period (period 0) consumption demand and labor supply is taken into account, but also explicit attention is paid to the future (period 1) consumption demand and labor supply. Money balances, it may be noted, perform two roles: transactions and store of value function. The transactions function of money derives from the fact that the wages are paid in money which in turn is exchanged for goods. The store of value function of money recognizes that expectation regarding period 1 transaction of goods and labor is transmitted to the current period through a change in the money holdings. If households expect to be rationed in the goods market in the next period, they would be averse to carry forward the same amount of money balances as they would if there were no rationing, and therefore reduce their stock of money holdings by increasing expenditure in the current period. On the other hand, if households expect unemployment in the next period, they would reduce consumption in the current period in order to carry forward additional money balances into the next period. It is important to note that it is money balances that link the present to the future, and it is the nexus created by money balances between the present and the future that is instrumental in the generation of disequilibrium in the current period. The 'bootstrap' character of the model cannot be overemphasized. If households expect an excess supply, because of the connection between today and tomorrow through money balances, unemployment becomes a reality today. Bootstrapping is self–fulfilling prophesy.

The utility function of the household is as follows:

$$U = U(C_0, T_0 - L_0, C_1, T_1 - L_1). \tag{2.1}$$

The $T - L$ refer to leisure and the subscript 0 and 1 refer to periods 0 and 1. We would like to derive the utility function in period 0 which explicitly takes into account the constraints that the household expects in period 1. If the household does not expect any quantity constraint in the next period either for goods or labor, the consumption for period 1 is determined by the budget constraint for that period, assuming that it does not wish to hold any money at the end of period 1,

$$M_0 + D_1 + W_1 L_1 = P_1 C_1. \tag{2.2}$$

P_1 and W_1 are the expected price and wage for period 1. C_1 and L_1 would then depend on M_0, which in turn depend on C_0, $T_0 - L_0$, P_1, W_1, $T_1 - L_1$ and exogenous D_1. If, however, rationing is expected in goods or labor or both, the budget constraint above needs to specify the rationing that is expected. With rationing expected for period 1 in the labor market, $L = \bar{L_1}$, the expected consumption in period 1 is $C_1 = (M_0 + D_1 + W_1 \bar{L_1})/P_1$. Substituting C_1 into in the utility function,

$$U = U(C_0, T_0 - L_0, \frac{M_0 + D_1 + W_1 \bar{L_1}}{P_1}, T_1 - \bar{L_1}). \tag{2.3}$$

The form of the function will be different depending on the rationing expected for the next period. Since marginal utility of money balances M_0 is conditional on expected price, wage and employment constraint, the utility function may now be simplified to,

$$U = U(C_0, T_0 - N_0, M_0, \theta), \tag{2.4}$$

where θ stands for future endowments of time, expectation regarding price, wage, and quantity constraints. The term M_0 stresses the fact that it is the current period holding of money balances that changes to accommodate future expectations. The utility function takes into account the future, and expectations regarding the future impinge on current utility.

Having incorporated the future into the current objective function, let us turn our attention to the current period decision making of the household. The period 0 budget constraint is

$$M_{-1} + D_0 + W_0 L_0 - P_0 C_0 = M_0 \tag{2.5}$$

where M_{-1} is the money stock at the end of the last period. The utility function may now be rewritten, where (5) replaces M_0 in (4)

$$U = U(C_0, T_0 - L_0, M_{-1} + D_0 + W_0 L_0 - P_0 C_0, \theta) \tag{2.6}$$

The function will assume different forms depending on the constraints expected. In the absence of perfect foresight[6] the utility function will be a weighted average of the utility function for the different regimes, the weights being the probablity of outcome of various regimes.

In Figure 2.1 is given the constant U contours in Y, L space. The effective demand (Y_d) BH is the loci of the tangencies of the iso–U curves with the vertical lines, the labor rations. Similarly, effective labor supply (L_s) CH is the loci of the tangencies of the of the iso–U curves with horizontal lines, the goods rations. The labor and goods rations are $\bar{L}$ and $\bar{Y}$ respectively. Note that Y=C+G, and since G is never rationed, employment constrained output demand is in fact the employment constrained consumption demand. At H, there is no rationing. At D, the household faces rationing in both the goods and labor markets.

[6] Neary and Stiglitz (1983) develop a disequilibrium macroeconomic model with perfect

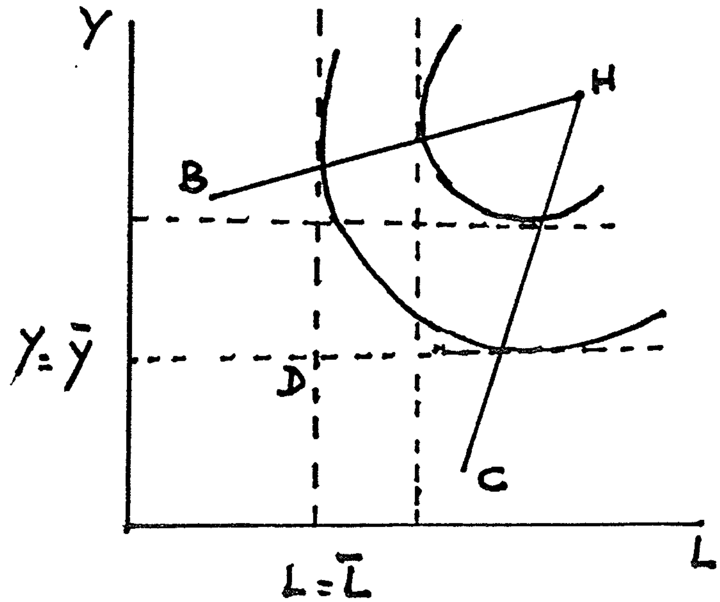


Figure 2.1: Household's Effective Demand and Supply

Behavior of Firms

Production is directed to two uses: sales and accumulation of inventories (Δinv). Assuming that the firm does not wish to hold inventories at the end of period 1, the sales constraint for period 1 is

$$inv_0 + Y(L_1) = X_1 \tag{2.7}$$

The period 1 profit level (π) is given by

$$\pi = P_1 X_1 - W_1 L_1 \tag{2.8}$$

foresight. The model exhibits perfect bootstrapping. If agents expect excess supply in the next period, excess supply is a certain outcome in this period.

If the firm is unrationed in both markets, maximization of profit subject to sales constraint yields labor demand as a function of $\frac{W_1}{P_1}$ and inv_0. If, however, the firm is constrained in sales at the level $\bar{X}_1$, the sales constraint becomes $Y(L_1) = \bar{X}_1 - inv_0$; profit maximization given the new constraint calculates L_1. Therefore depending on constraints that are relevant (sales or labor), the optimal period 1 profit can be calculated. Given the firm's expectation regarding the outcome of various regimes, a weighted average of the profit levels associated with different regimes can be calculated. The period 0 decision problem is maximize

$$\pi = \pi(X_0, inv_0, \Phi) \tag{2.9}$$

subject to

$$inv_0 = inv_{-1} + Y(L_0) - X_0 \tag{2.10}$$

The presence of inventories in the profit function enables the firm to incorporate future into the current period profit function. If it expects constraints on the sale of product, it is manifested in the current period through a reduction in the inventory holdings of firms. The term Φ stands for all the variables that help the firm to make expectations regarding the future. It is apparent by now that the role of inventories for the firm is similar to that of money balances for the household.

Figure 2.2 traces the effective goods supply (Y_s) FB and effective labor demand (L_d) FC of the firm from the loci of the iso-π curves with labor

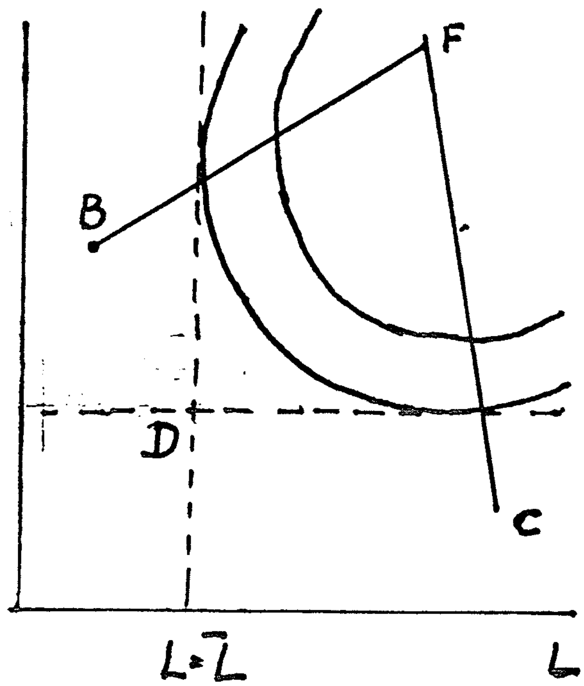


Figure 2.2: Firm's Effective Demand and Supply

and sales constraint. At F firm is demanding and supplying at its Walrasian level.

2.4.3 The Regimes

We now digress into the familiar market clearing situation where the behavior of both households and firms are determined by their notional curves. Assuming that equilibrium wage and price prevails, and households and firms expect no rationing in either market in the next period, the quantity transacted in the goods and labor market by the household and firms coincide with the Walrasian demands and supply as shown in Figures 2.3(a) and 2.3(b). Figure 2.3(a) depicts the goods and 2.3(b) the labor market. The goods

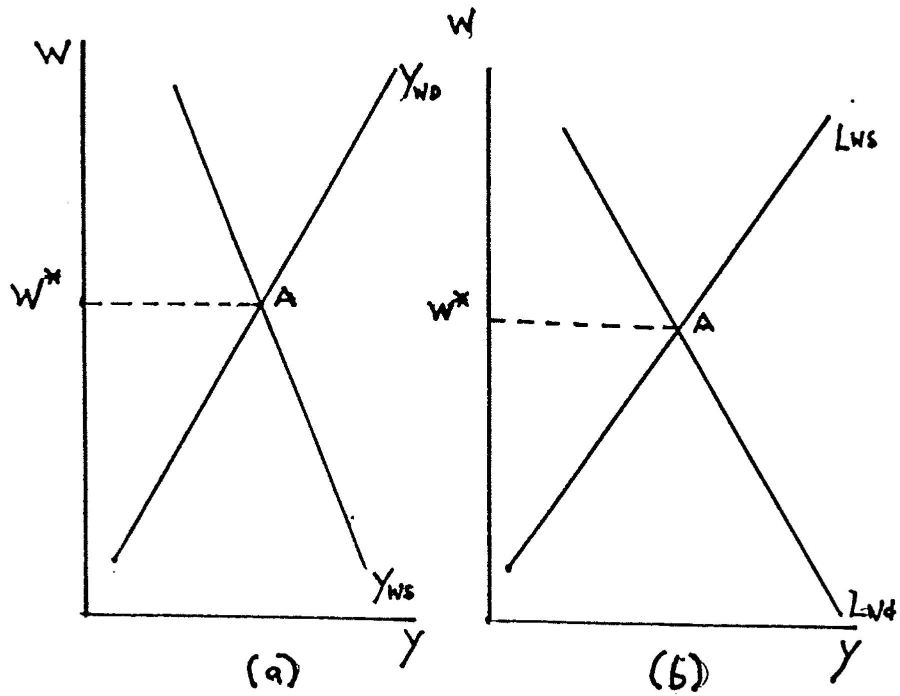


Figure 2.3: Walrasian Demand and Supply

market have real wage rate on the ordinate and quantity on the abscissa. The notional supply curve for goods is a downward sloping function and the notional demand curve for goods an upward sloping function of the real wage (Barro and Grossman: 1971). The upward sloping demand curve suggests that as real wages increase, the opportunity cost of leisure goes up, providing incentive to work more. Also, the income effect is positive. The notional labor supply and demand curves are the ones encountered in elementary text–books and bear no explanation.

The households and firms have to be aggregated into order to arrive at the market effective demand and supply. We assume that all households and firms are identical, and when rationed identical also. The implication is that all the households have not only the same exogenous initial money stock M_{-1} and dividends D, they are also awarded the same labor and goods rations in the event of their being rationed in the goods and labor markets.[7] It may be appropriate to start with a situation where households and firms are in Walrasian equilibrium, point A in Figure 2.4. The point A in Figure 2.4 correspond to the same point in Figure 2.3. The slopes of Y_s and L_s are drawn steeper than the slopes of Y_d and L_d. An explanation of this will have to wait till the regimes have been delineated.

Convergence of the intentions of the households and firms is absent in disequilibrium situations. Therefore, in order to generate disequilibrium, it is necessary to create a disharmony in the intentions of the agents. This is diagrammatically achieved when point A is no longer the point of departure for both households and firms. The Y_d and L_s of the households should intersect Y_s and L_d of the firms not at point A, but some other point such as K , for example, in Figure 2.5. Figure 2.5 depicts Keynesian Unemployment, where quantity traded is determined by the $Y_d < Y_{ws}$ and $L_d < L_{wd}$. In order for this to be possible, the firm should move in the northwesterly direction, which would be the case when government expenditure falls. As a result, the

[7] If M_{-1} and D vary across households, and suppose there is involuntary unemployment, and each household is is awarded the same labor ration. Then rich households can afford more leisure than poor and therefore not all rich households will be subjected to rationing. The effective labor supply curve will be a weighted average of the unrationed labor supply of the rich and the rationed labor supply of the poor. There does not seem to be any problem in assuming different intial endowments and dividends. For simplicity, we shall, however, assume that they are same across households.

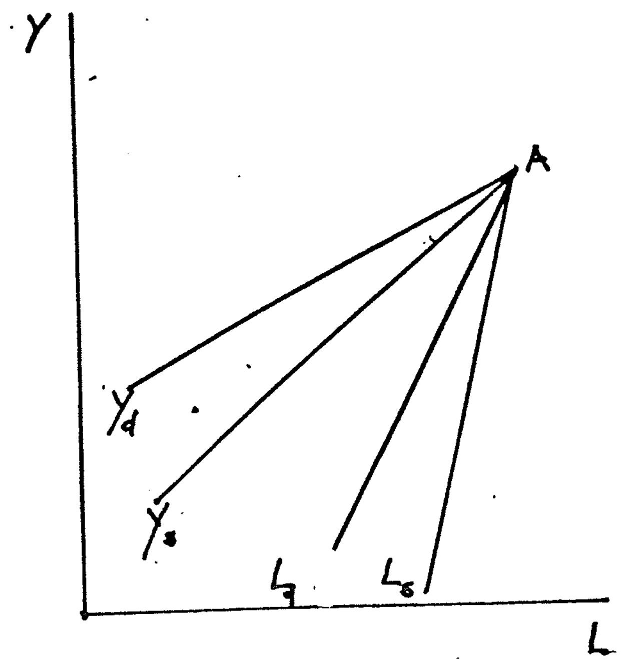


Figure 2.4: Walrasian Equilibrium

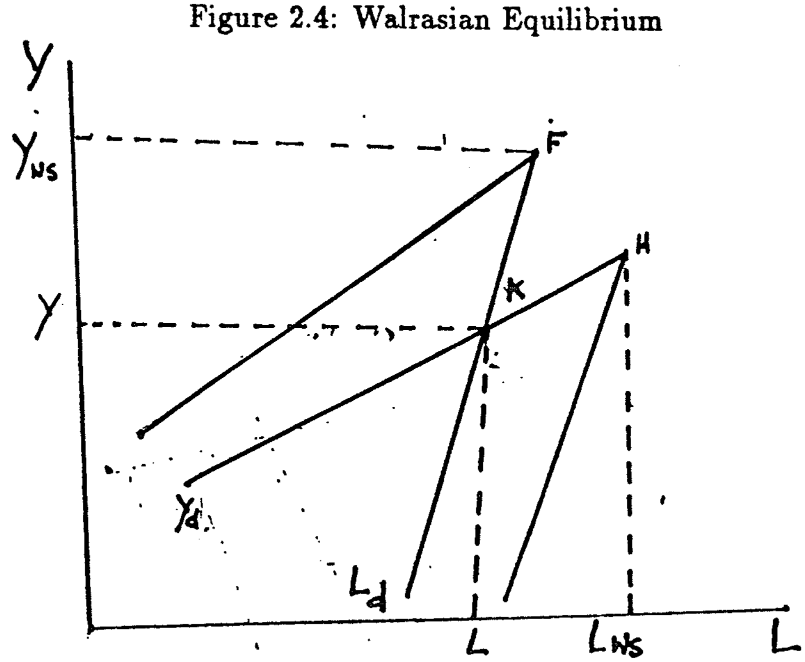


Figure 2.5: Keynesian Unemployment

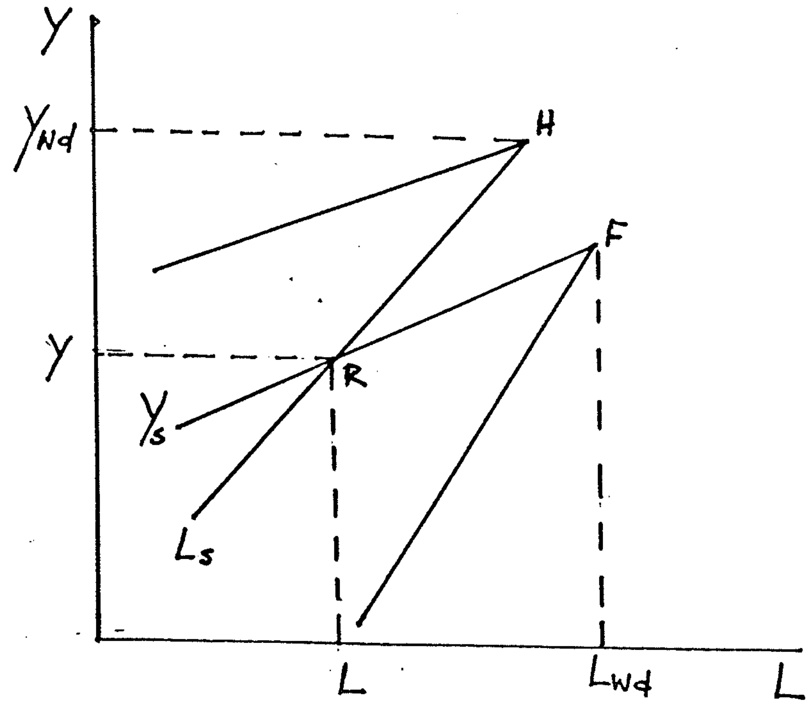


Figure 2.6: Repressed Inflation

multiplier process is set into motion. Decrease in goods demand will decrease labor demand, and decrease in labor demand decreases goods demand even further, until the process converges to K.

In Repressed Inflation, the quantity traded is determined by $Y_s < Y_{wd}$ and $L_s < L_{wd}$. Households are frustrated in the purchases of goods and firms in the purchase of labor. In response to this frustration, households withdraw labor and firms curtail production. Quantity constrained equilibrium is achieved when Y_s attains equality with L_s as in R. Such equality is obtained when the firms move from the Walrasian position of A in Figure 2.4 and travel in southeasterly direction due to some exogenous change. As output supplied falls, the labor supply falls, cutting production even further.

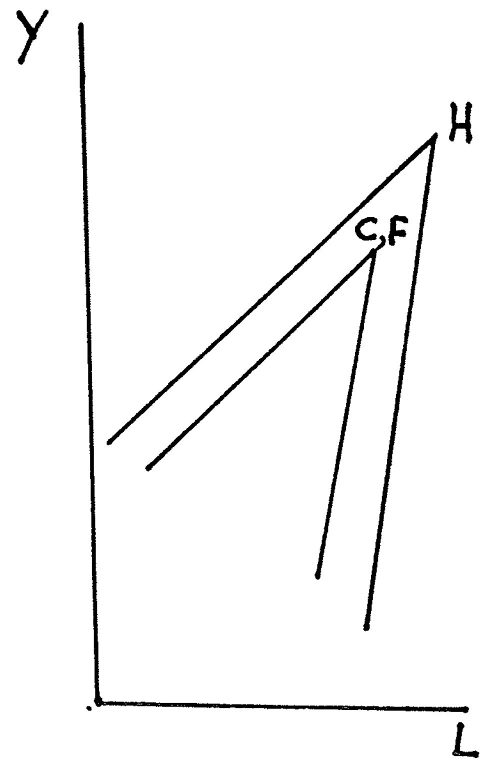


Figure 2.7: Classical Unemployment

Barro and Grossman (1971) calls it the supply multiplier.

In Classical Unemployment, since there is excess demand for goods and excess supply of labor, households are rationed in both markets. And in Repressed Inflation, there is excess demand for labor and excess supply goods; the firms are rationed in both markets. In Classical Unemployment, households are willing to buy more goods, if only firms would supply. But firms have no incentive to hire more labor and supply more goods; wage rate is too high. An increase in G is not the panacea here; it would only exacerbate the excess demand. In Underconsumption, the firms would like to hire more labor for purposes of inventory accumulation, but additional labor is not found.

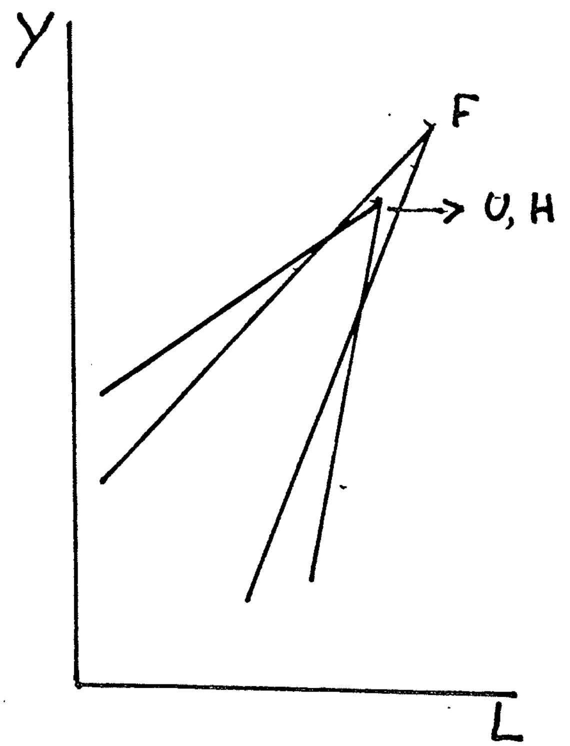


Figure 2.8: Underconsumption

Having delineated the various regimes, it is now appropriate to provide the explanation for the relatives slopes of Y_d, Y_s L_d and L_s curves in Figure 2.4. Assume that the economy is in Keynesian Unemployment. An increase in government expenditure shifts F upward and moves the point of intersection of L_d with Y_d upward; the exact movement upward can be calculated by the government expenditure multiplier. In this regime, quantity traded in the labor and goods markets are $L = L_d(Y)$ and $Y = Y_d(L_d(Y))$. Since $Y = C + G$, $Y = Y_d(L_d(C + G)$. Totally differentiating the last equation, and since a change in C is the same as change in Y,

$$dY = dC = \frac{\delta Y_d}{\delta L}\frac{\delta L_d}{\delta C}(dC + dG),$$

which can be simplified to

$$dY = dC = \frac{(\delta Y_d/\delta L)(\delta L_d/\delta C)}{1-(\delta Y_d/\delta L)(\delta L_d/\delta C)} dG. \tag{2.11}$$

In order to derive the supply multiplier associated with Repressed Inflation, recall that an increase in government expenditure decreases consumption expenditures, since government is never rationed and households will withdraw labor even further. In Figure 2.6 F shifts downward, and along with it, intersection of Y_d with L_d. Since $C = Y_s(L_s(C)) - G - inv$, the supply multiplier, is,

$$dY = dC = -\frac{1}{1-(\delta Y_s/\delta L)(\delta Ls/\delta C)} dG - dinv \tag{2.12}$$

In order for the model to possess stability, the denominators of eqs. (11) and (12) should be positive. And the denominators will be positive if and only if

$$0 \leq \delta Y_d/\delta L < \frac{1}{\delta L_d/\delta C} \tag{2.13}$$

and

$$0 \leq \delta Y_s/\delta L < \frac{1}{\delta L_s/\delta C} \tag{2.14}$$

Equation (13) suggests that the slope of the effective goods demand curve be less than the slope of the effective labor demand curve, and equation (14) suggests that the slope of the effective goods supply curve is less than effective labor supply curve. In order for this to be possible, the firm's effective trade offers should be enclosed within the household's effective trade offers.

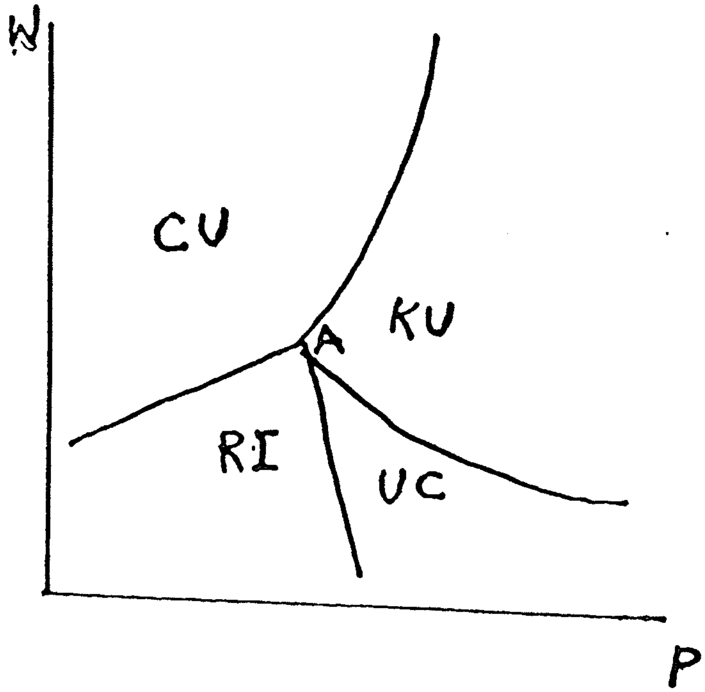


Figure 2.9: Wage–Price Constellation of Regimes

Following Malinvaud (1977), the domains of the four regimes in PW space is shown in Figure 2.9. Keyensian Unemployment occurs when the prices are too high given the money balances M and the government expenditure G. Classical Unemployment is a situation where the real wages are too high. Repressed Inflation is the situation of low wages and prices. Under Underconsumption, wages are low, households are fully employed, and not rationed in the goods market. Because excess demand for labor in the presence of excess supply for goods is a rare phenomenon, the region for this regime is small in the diagram.

2.5 Conclusion

The purpose of the above exercise was to derive the stability conditions (13) and (14) of a disequilibrium macreconomic model. In macroeconometric models, these conditions play a vital role. There, as we shall see in chapters IV and V, they appear in a different guise and are often termed spill–over coefficients. The conditions the spill–over coefficients have to satisfy are called coherency or solveability conditions, and are required for the existence of well defined reduced form (Ito: 1980 and Gourieroux, Laffont and Montfort: 1980).

Chapter 3

Microfoundations of Disequilibrium Macroeconomics

3.1 Introduction

The last chapter directed its attention to the macroeconomic manifestations of disequilibrium. It was shown there that macroeconomic Walrasian equilibrium is one of the many equilibria possible, and depending on the constellation of wages and prices, the other possible eqiuilbria are the Keynesian Unemployment, Repressed Inflation, Classical Unemployment and Underconsumption. The microeconomic underpinnings of Walrasian equilibrium has been investigated thoroughly, and the finest example of that effort is the Arrow–Debreu general equilibrium model. It does not require a great deal of sagacity to realize that if disequilibrium models have to be put on a surer footing, then its microeconomic ramifications have to be investigated. In the last decade and a half, a great deal of abstract thinking of superlative quality has done just that. This chapter is a synoptic view of their methods and results.

3.2 Partial and General Equilibrium Analysis

The search for microfoundations of disequilibrium models that is being pursued here is perhaps clearly comprehended if they are considered in the perspective of partial and general equilibrium models, both of which have hoary tradition in economic analysis. In fact, the leading exponents of general disequilibrium models insist that general equilibrium models are a special case of general disequilibrium models (Benassy: 1975). It may be instructive to view the partial, general equilibrium and general disequilibrium models as a continuum from least to most generality.

Partial equilibrium analysis investigates the price formation in one market, holding constant the behavior of all other markets. The behavior of the market under scrutiny is determined by the supply and demand pertaining to that market alone. Let d_{ih} and s_{ih} stand for the demand and supply for good h by the buyer i and seller i. There are n buyers and n sellers, indexed $i = 1, \ldots\ldots, n$. The good is exchanged for money at price p. The demand and supply of each of the agents, $d_i(p)$ and $s_i(p)$ denote that supply and demand are functions of p, the price of h. The market demand D(p) and supply S(p) of good h are

$$D(p) = \sum_{i}^{n} d_i(p)$$

$$S(p) = \sum_{i}^{n} s_i(p)$$

The equilibrium price $p*$ is determined by the equality of market demand and supply. At $p*$, the agents can demand and supply as much as they desire.

General equilibrium theory, in contrast, studies the price formation of all the markets simultaneously, and in so doing explicitly recognizes the interrelationship between markets. The individual agent's trade offers are conditional not only on the price of that commodity, but also the prices of all the commodities that are traded in the economy. Suppose there are n agents, indexed $i = 1, 2....n$ and l markets, indexed $h = 1, ..., l$. These l goods are traded for each other at the rate of exchange or relative price p_h, $h = 1, ..., l$ or the price vector p. The desired transaction of an agent i on market h, d_{ih} or s_{ih} is the function of price vector p, and the desired transaction of agent i on all the markets is $d_i(p)$ and $s_i(p)$. These vector functions for each agent is arrived at by the optimization of a decision criteria subject to the constraint,

$$\sum_{h=1}^{l} p_h d_{ih}(p) = \sum_{h=1}^{l} p_h s_{ih}(p).$$

The market demand $D_h(p)$ and supply $S_h(p)$, where p is again the price vector, is

$$D_h(p) = \sum_{i=1}^{n} d_{ih}(p)$$

$$S_h(p) = \sum_{i=1}^{n} s_{ih}(p)$$

At Walrasian equilibrium price vector $p*$,

$$D_h(p*) = S_h(p*) \quad \forall h$$

At price $p*$, the agent is able to buy or sell, depending on whether he is a buyer or a seller, all he desires of every good.

3.3 Non–Walrasian General Equilibrium

3.3.1 Institutional Setting

The economy under consideration is a monetary exchange economy. Money performs dual roles: medium of exchange and store of value. There are n agents, indexed $i = 1, ...n$ and l markets, indexed $h = 1, ...l$, where goods are exchanged for money at the monetary price p_h. The change in money holding of an agent, Δm, is the difference between sale s_{ih} and purchase d_{ih} in all the l markets.

$$\Delta m = \sum_{h=1}^{l} p_h s_{ih} - \sum_{h=1}^{l} p_h d_{ih}$$

3.3.2 Demands Versus Transactions

When markets do not clear, it is important to make the distinction between transactions and demand. In general equilibrium models, price $p*$ ensures that transactions are always equal to demand and supply. In general disequilibrium models, there will only be equality between either purchase and demand or sale and supply and not both, though the accounting identity ensures that total purchase always equals total sales. Denoting $\tilde{d}_{ih}$ and $\tilde{s}_{ih}$ as the demand and supply and $d*_{ih}$ and $s*_{ih}$ for purchases and sales by agent i on market h

$$\sum_{i=1}^{n} d*_{ih} = \sum_{i=1}^{n} s*_{ih}.$$

Note that total demand will not equal total supply in disequilibrium,

$$\tilde{D} = \sum_{i}^{n} \tilde{d}_i \neq \sum_{i}^{n} \tilde{s}_i = \tilde{S}.$$

3.3.3 Rationing Schemes

In the case when markets do not clear, there is either excess supply or excess demand since the quantity traded is less than the supply or demand respectively; the suppliers or demanders will be rationed in some systematic way. The actual rationing would obviously depend on the way exchanges are organized in these markets. Two rationing schemes to organize exchanges in a market are *proportional* and *priority* rationing schemes (Benassy: 1986). In the *proportional* rationing scheme, agents on the short side always realize their demands or supplies. On the other hand, if agents are on the long side, the available demand or supply would be rationed among them. The rationing coefficient would be a ratio of quantity traded to demand or supply depending on which is on the long side. For example, if there is excess supply, agents on the long side would be rationed; the quantity each of the agents can transact is rationing coefficient times good supplied. The rationing coefficient is calculated as the ratio of demand and supply i.e. $\frac{\tilde{D}}{\tilde{S}}$. Similarly, if there is excess demand, all agents on the long side would be rationed, and the rationing coefficient would be $\frac{\tilde{S}}{\tilde{D}}$. All the agents on the long side have the same rationing coefficient. We may now formalize the proportional rationing scheme as follows.

$$d*_i = \tilde{d}_i \times min(1, \frac{\tilde{S}}{\tilde{D}})$$
$$s*_i = \tilde{s}_i \times min(1, \frac{\tilde{D}}{\tilde{S}})$$

where

$$\tilde{D} = \sum_{j=1}^{n} \tilde{d}_j \quad \tilde{S} = \sum_{j=1}^{n} \tilde{s}_j$$

In the *priority* or *queueing* or *hierarchical system*, a queue is formed of the $n-1$ demanders arranged in the order $i = 1, ..., n-1$, and they are served in that order. Their individual demands can be expressed as $\tilde{d}_i$. The agent n is the supplier who trade offer $\tilde{s_n}$. When the turn of demander i arrives, the maximum he can obtain is the what is left after the j demanders before him have obtained, $j < i$.

$$\tilde{s}_n - \sum_{j<i} d*_j = max(0, \tilde{s}_n - \sum_{j<i} \tilde{d}_j).$$

His purchase is

$$d*_i = min[\tilde{d}_i, max(0, \tilde{s}_n - \sum_{j<i} \tilde{d}_j)].$$

The supplier sells the minimum of his supply and his total demand,

$$s*_n = min(\tilde{s}_n, \sum_{j=1}^{n-1} \tilde{d}_j).$$

It is imperative that rationing scheme satisfy some properties so that they conform to market reality and facilitate aggregation. One property is *voluntary exchange.* No agent is forced to trade more than he wants. Sellers are not coerced to sell and buyers are not coerced to buy.

$$d*_i \leq \tilde{d}_i \qquad s*_i \leq \tilde{s}_i$$

This implies that total quantity transacted by all the agents in all the markets, the aggregate purchase and sale, is less than both aggregate demand and supply.

$$D* = S* \leq min(\tilde{D}, \tilde{S})$$

The second property, *market efficiency*, pertains to the requirement that, at the quantity rationed equilibrium, all mutually advantageous trades are exhausted. If there is an unsatisfied buyer and an unsatisfied seller in a market, the *market efficiency* condition is violated, for there exist the possibility of trade between these unsatisfied agents.

The consequence of *voluntary exchange* and *market efficiency* is the short side rule. The short side rule determines that the agents on the short side always realize their transactions and it is agents on the long side who are rationed. Exchanges have to be based on free volition of the agents, according to the *voluntary exchange.* The property of *efficient* or *frictionless* market ensures that there are no unsatisfied agents on the short side. So, if there has to be *voluntary exchange* and *frictionless* market, then in the presence of rationing, it is the short side which should determine the quantity of goods traded in that market. Aggregating across all agents and markets, we then have the fundamental equation of disequilibrium economics,

$$D* = S* = min(\tilde{D}, \tilde{S}).$$

It is obvious that in the absence of *voluntary exchange*, the short side need not determine the quantity transacted, and in the absence of *frictionless* market, not all agents who are on the short side will transact their desired trades. Therefore the minimum condition require, and is in fact the direct consequence of the two properties that all rationing schemes must fulfill.[1]

Rationing schemes (or allocation procedures) can also be distinguished as *manipulable* and *nonmanipulable*. Let us suppose that an agent is on the long side of the market. In other words, he is not able to supply as many goods as he would wish to. Let us also suppose that the amount of the good he can sell depends on the amount of the good that he is willing to offer, and the offers to sell and buy of other agents. In short, the amount of the good he transacts is a function his own supply, and the supply and demand of other agents. Then, if the rationing scheme is *manipulable*, he can manipulate the market by offering to supply more than his desired quantity, thereby ensuring that the desired quantity is allocated to him by the rationing scheme. But, if on the other hand, the rationing scheme is *nonmanipulable*,

[1]The assumption that the markets are *efficient* is a *deus ex machina* to construct macroeconomic models, and is of doubtful empirical relevance. It may be relevant in highly centralized markets and small decentralized markets. In large decentralized markets, the assumption is more often than not violated.

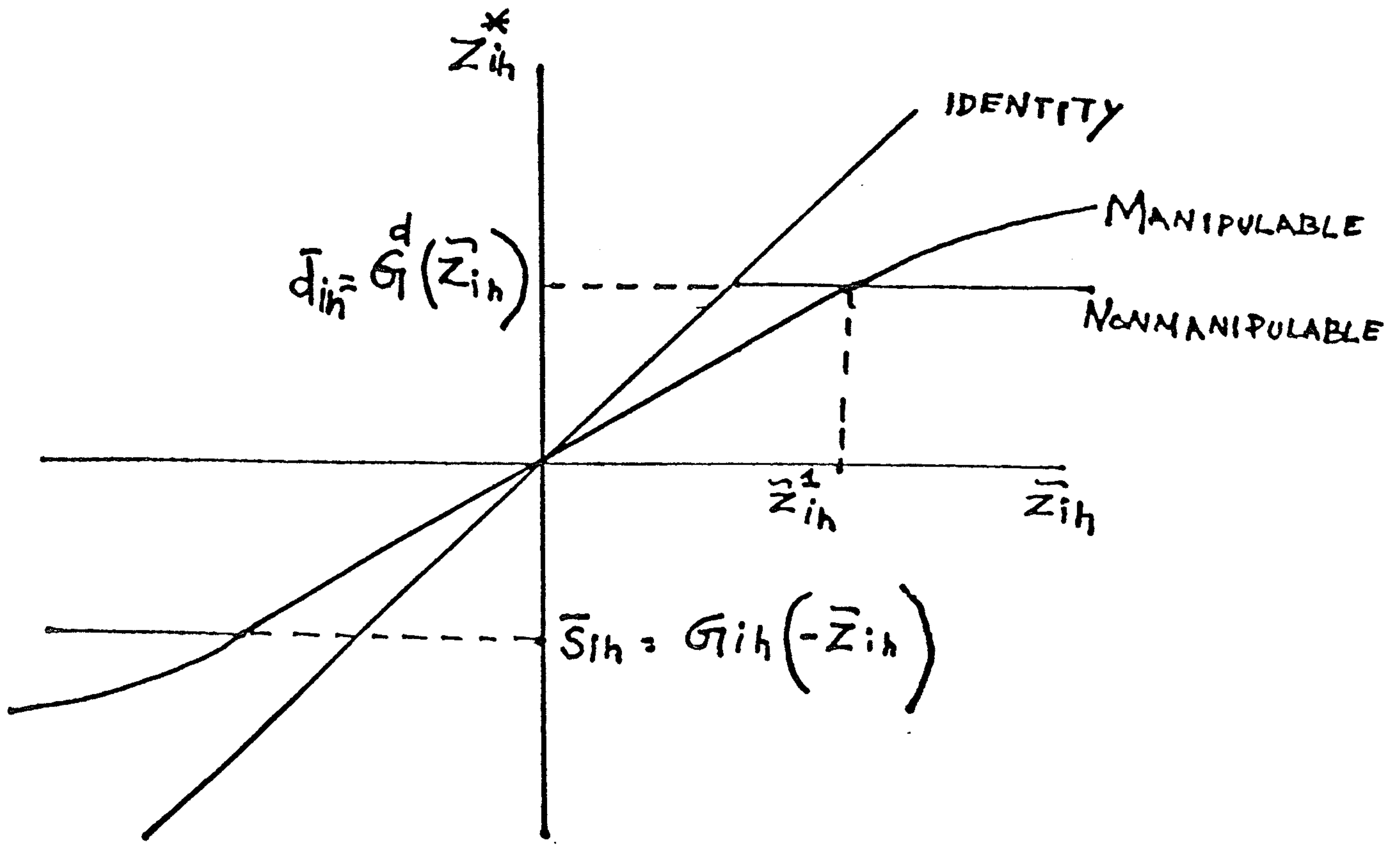


Figure 3.1: Manipulable and Nonmanipulable Rationing Schemes

if he is confronted with constraints in the markets which is dependent on the demands and supplies of other agents in the market, he is no position to overcome the barrier of constraints. For under *nonmanipulable* rationing, the quantity constraint is no longer dependent on his own supply, but the supply and demands of the others. He finds it unable to manipulate the market, and it is therefore a scheme of *nonmanipulable* rationing. Figure 3.1 below shows the difference between *manipulable* and *nonmanipulable* rationing schemes. The quantity transacted under the *nonmanipulable* rationing scheme can be expressed as follows. Denoting the constraints on the purchases and sales as

$\bar{d}$ and $\bar{s}$,

$$d*_i = min(\tilde{d}_i, \bar{d}_i)$$

$$s*_i = min(\tilde{s}_i, \bar{s}_i)$$

Let $z*_{ih}$ refers to the net transactions in the market.

$$z*_{ih} = d*_{ih} - s*_{ih}$$

Since quantity purchased should always equal to quantity sold, $\sum_{i=1}^{n} z*_{ih} \equiv 0$, $\forall$ h. Also, net demand $\tilde{z}_{ih} = \tilde{d}_{ih} - \tilde{s}_{ih}$, and there no reason for them to balance if the markets are out of equilibrium, i.e., $\sum_{i=1}^{n} \tilde{z}_{ih} \neq 0$ We remark that algebraically, the net quantity demanded $\tilde{d}_{ih} = \tilde{z}_{ih}$ and net quantity supplied supplied $\tilde{s}_{ih} = -\tilde{z}_{ih}$ (see Figure 3.1). A *nonmanipulable* rationing scheme in algebraic notation is

$$z*_{ih} = \begin{cases} min(\tilde{z}_{ih}, \bar{d}_{ih}) & \tilde{z}_{ih} \geq 0 \\ max(\tilde{z}_{ih}, -\bar{s}_{ih}) & \tilde{z}_{ih} \leq 0 \end{cases}$$

In general, a rationing scheme is *manipulable*, if an agent can increase his transactions by increasing his demand (supply) and *nonmanipulable* if the agent faced with the lower and upper bounds on his transactions cannot sell more than lower bound and buy more than the upper bound. The limits on the purchase and sale, $\bar{d}$ and $\bar{s}$, depend only on the net demands of other agents. In order to make this explicit, we may write the rationing scheme as

$$z*_{ih} = F_{ih}(\tilde{z}_{ih}, \tilde{Z}_{ih})$$

where

$$\tilde{Z}_{ih} = (\tilde{z}_{ih}, \ldots.., \tilde{z}_{i-1,h}, \tilde{z}_{i+1,h}, \ldots\ldots\ldots, \tilde{z}_{nh}).$$

Therefore $\tilde{Z}_{ih}$ is the net demands of all the agents on market h, excluding i.

Manipulable rationing schemes which are deterministic are unstable due to explosive overbidding (Benassy: 1975, 1986)[2] and therefore the exposition in section 3.3.4 assumes a *nonmanipulable rationing*. The bounds on trades $\bar{d}$ and $\bar{s}$, which are called *perceived constraints*, may be expressed as

$$\bar{d} = G^d_{ih}(\tilde{Z}_{ih}) \quad \bar{s} = G^s_{ih}(\tilde{Z}_{ih})$$

3.3.4 Walrasian, Clower–Benassy, Drèze Demands

Walrasian equilibrium and demand of the monetary economy can derived by maximizing the utility of the agents. Let $U_i(x_i, m_i)$ be utility function of each agent in the economy which depends on the consumption vector x_i and money holdings m_i. The utility function is assumed to be strictly concave in all its arguments. The Walrasian net demands are solution in z_i to the problem given below.

$$Maximize\ \ U_i(x_i, m_i) \quad s.t.$$

$$x_i = \omega_i + z_i \geq 0$$

[2]Benassy proves that under non–stochastic manipulable rationing schmes, agents have a tendency to inflate their trade offers in order to enable them to obtain their optimal transactions. But in due course, the rationing scheme itself would undergo a change transforming the same trade offers into lower and lower transactions. In response, the agents would proffer larger and larger demands, and the phenomenon will not converge to a stable situation unless additional restrictions are imposed on the demands.

$$m_i = \bar{m}_i - pz_i \geq 0.$$

$\bar{m}$ denotes the initial holding of money, ω_i is the initial endowment and $z_i = \sum_{h=1}^{l} d_{ih} - \sum_{h=1}^{l} s_{ih}$. At the Walrasian price vector $p*$, net excess demand in each of the h markets sum to zero.

$$\sum_{i=1}^{n} z_{ih}(p*) = 0 \quad \forall h$$

Drèze demands (1975) are obtained by taking into account the quantity constraints also. The transactions are limited by the constraints

$$-\bar{s}_{ih} \leq z_{ih} \leq \bar{d}_{ih}$$

The best transaction that the agent can obtain, denoted by $\zeta *_i(p, \bar{d}_i, \bar{s}_i)$, is the solution in z_i of the following maximization problem.

$$Maximize \;\; U_i(x_i, m_i) \quad s.t.$$

$$x_i = \omega_i + z_i \geq 0$$

$$m_i = \bar{m}_i - pz_i \geq 0$$

$$-\bar{s}_{ih} \leq z_{ih} \leq \bar{d}_{ih} \quad \forall h$$

The solution will be unique since U_i is strictly concave in x_i. Drèze invokes the fixed point argument to prove the existence of equilibrium, and is defined as the vector of optimal transactions $\zeta *_i(p, \bar{d}_i, \bar{s}_i)$ such that total purchases equal total sales in each market, $\sum_{i=1}^{n} z*_{ih} = 0$. Drèze does not employ a specific rationing scheme and different equilbria will ensue depending on the specificity of the rationing scheme. A Walrasian auctioneer sets the upper and lower bounds on transactions for each agent, and agents respond to their respective constraints with constrained demands. Aggregate excess demands and supplies prompt the auctioneer to change the bounds on trades. The Drèze equilibrium is a fixed point of perceived constraints or limits to trade and demands.

Benassy demands (1975), denoted by $\tilde{\zeta}_{ih}(p, \bar{d}_i, \bar{s}_i)$ are solution in z_{ih} to the following maximization problem.

$$Maximize\ \ U_i(x_i, m_i)\ \ s.t.$$

$$x_i = \omega_i + z_i \geq 0$$

$$m_i = \bar{m}_i - pz_i \geq 0$$

$$-\bar{s}_{ik} \leq z_{ik} \leq \bar{d}_{ik} \quad k \neq h$$

The demand for commodity h is the trade that maximizes utility taking into account the constraints in all other markets, excluding h. The process is repeated for all the markets sequentially, and a vector of effective demands $\tilde{\zeta}*_i(p, \bar{d}_i\bar{s}_i)$ is computed. The demands are termed effective demands or Clower–Benassy demands because of the inclusion of spill–over effects (the appearance of which in the literature dates back to Clower's (1965) *dual–decision hypothesis*) from other markets in the demand function of the agent i for commodity h. The equilibrium is a fixed point of effective demands and perceived constraints. Benassy calls it K-equilibrium. Alternatively, an equilibrium may be viewed as the set of effective demands and perceived constraints that are reproduced identically over time. At equilibrium, agents have the correct perception of the quantity constraints. The equilibrium reached is the same for both Drèze and Benassy (Green: 1980)

A problem with the Drèze demands is that the agents are prohibited from expressing their demands and supplies beyond upper and lower bounds, $\bar{d}_i$ and $\bar{s}_i$, and therefore information regarding the extent of rationing is not forthcoming. Drèze demands therefore preclude a measure of excess demand.[3] For example, unemployed are not permitted to elicit information regarding the extent of their unemployment. As opposed to Drèze, in Benassy equilibrium, demand need not equal actual trades, only that demands "reproduce" themselves. Comparison of actual quantity traded with the effective trade offer generates a measure of excess demand. However, excess demands so generated do not provide a reliable measure dissatisfaction. As the rationing scheme is *nonmanipulable*, it is constant beyond $\tilde{z}^1_{ij}$ (see Figure 3.1). So, if $d*^1_{ij}$ is the quantity traded for demand $\tilde{z}^1_{ij}$, the difference between them is not a reliable measure of excess demand. The agent could have chosen any other level of $\tilde{z}_{ij} > \tilde{z}^1_{ij}$ in order to transact $d*^1_{ij}$. Thus there is no unique and optimal demand associated with the trade $d*^1_{ij}$, and therefore a unique measure of excess demand. This problem is directly attributable

[3] Due to the lack of distinction between actual trades and demands, Drèze demands are inappropriate for estimation purposes. One of the objectives of disequilibrium macroeconometric research programme is to capture the spill–over effects, and without a measure of excess demand, such an effort is doomed to fail at the very outset.

to the *nonmanipulable* nature of the rationing scheme. Yet another problem with the specification of Benassy–Clower demand functions is its weak choice theoretic basis. Benassy–Clower trade offers are made independently and sequentially in each market without considering the outcome of these trade offers, i.e., the final transactions. Since the quantity realized in market h is less than the trade offer, the spill–over effect will bring about a series of chain reaction in $h - 1$ markets, resulting in original demand being, in general, infeasible for the agent.[4] Further, as Green (1980) has pointed out, the sequential optimization procedure of Benassy does not conform with the operation of any real markets.

3.3.5 Stochastic Rationing and Equilibrium

It was pointed out above that Clower–Benassy demands do not possess a valid measurement of excess demands, and that it could be traced to the deterministic rationing scheme of the *nonmanipulable* type. It was therefore small wonder that the next batch of invstigators directed their attention to the other possibilities lying dormant in the rationing scheme.

Some, if not all, problems associated with Drèze and Benassy–Clower can be eliminated by a using a stochastic rationing scheme (Green: 1980, Svensson: 1980, Weinrich: 1984). Under stochastic rationing, the agent is uncertain as to the trade that will be realized for his trade offer. However,

[4]For a review of literature of these and related issues, see Gradmont (1977) and Drazen (1980).

the uncertainty over transactions to his offer is mitigated to some extend by the information he has of the market in question. There obviously is some information to be had regarding the quantity traded in the market as well as his own trade offers and trades in the past periods. This information can be utilized to form a probability distribution over transactions associated with each offer. If the range of such a distribution extends from $\tilde{z}_{ih}$ to zero, then it is clear that expected realization, E($z*_{ij}$), will be less than $\tilde{z}_{ih}$ (see Figure 3.2). Therefore in order to transact $\tilde{z}_{ih}$ the agent has an incentive to overstate his trade offers. But his overbidding will not be without bounds, as there is always a probability, that the inflated bid is transacted. So some moderation will be exercised on his bids, because of the necessity to the satisfy the budget constraint under all possible states of the world. The stochastic demands, $\hat{\zeta}_{ih}(p, \bar{d}_i, \bar{s}_i)$, are the solutions in z_{ih} to the optimization problem below.

$$Maximize \quad EU_i(x_i, m_1),$$

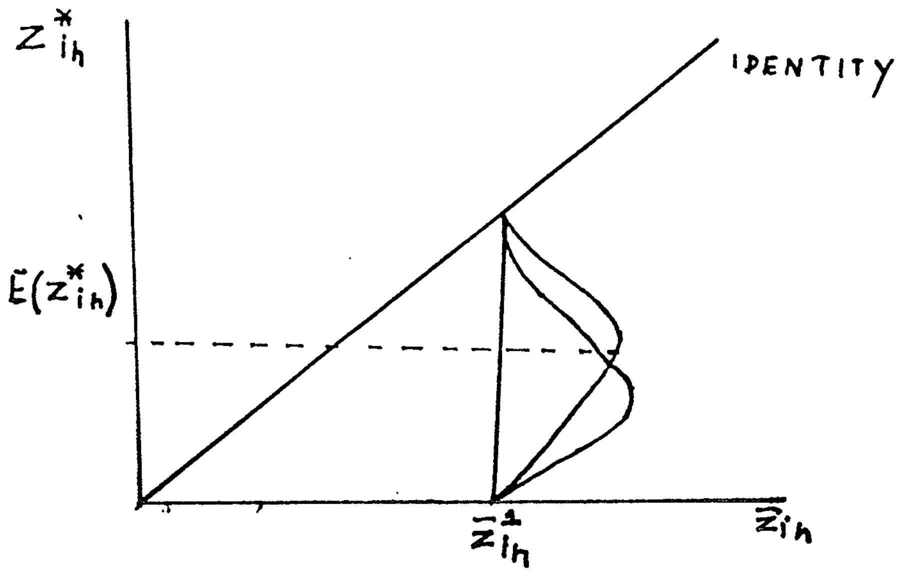
$$x_i = \omega_i + z_i \geq 0 \;\; with \;\; probability \;\; one,$$

$$m_i = \bar{m}_i - pz \geq 0 \;\; with \;\; probability \;\; one,$$

$$-\bar{s}_{ih} \leq z_{ih} \leq \bar{d}_{ih} \;\; \forall \;\; h \;\; with \;\; probablity \;\; one.$$

The above process will generate effective demands which do not have the problems associated with Drèze demands (absence of a measure of discrepancy between quantity realized and demanded) and Benassy–Clower demands (absence of a valid optimization procedure). Nevertheless, in view

Figure 3.2: Stochastic Rationing



of the fact there is an incentive to overstate demands in stochastic rationing schemes, however bounded it may be, the question of how much confidence one can place on the excess demands so derived remains unresolved.

The rationing scheme in the deterministic case was

$$z*_{ih} = F_{ih}(\tilde{z}_{ih}, \tilde{Z}_{ih}).$$

where $\tilde{z}_{ih}$ and $\tilde{Z}_{ih}$ are the net demand of agent i on market h and the net demand of all other agents. In the absence of accurate information regarding other agents' net demands, $\tilde{Z}_{ih}$, he will forced be to base his judgements regarding rationing on the available but partial information. Therefore partial information regarding aggregate demands and supplies will have to be employed to make assessments of the possible outcomes associated with a given trade offer. As this in itself is not sufficient to make such assessments, they will be supplemented by some random variables, which transform the deterministic rationing scheme into a stochastic one,

$$z*_{ih} = f_{ih}(\tilde{z}_{ih}, \tilde{Z}_{ih}(\phi)),$$

where ϕ is a random variable. The upper and lower bounds on trades $\bar{d}$ and $\bar{s}$ are also stochastic since the rationing scheme is stochastic.

$$\bar{d} = H^{d}_{ih}(\tilde{Z}_{ih}(\phi)) \quad \bar{s} = H^{s}_{ih}(\tilde{Z}_{ih}(\phi))$$

The equilibrium of the economy is a vector of effective demands $\hat{\zeta}_{ih}(p, \bar{d}_i, \bar{s}_i)$ In equilibrium, the demand and quantity constraints reproduce themselves identically over time. As Green (1980) and Weinrich (1984) have demonstrated, the rationing scheme should not only be stochastic but also manipulable.

3.4 Conclusion

The literature that was surveyed above was certainly not very voluminous, but its brevity is more than made up by the technical complexity. And there seems to be unanimous consensus that in order to obtain well defined demand functions within a disequilibrium framework, the rationing scheme should be stochastic and manipulable. However, the problem of exaggerated demands continue to persist even when the rationing scheme is stochastic and manipulable.

Chapter 4

Econometrics of Disequilibrium Macroeconomics

4.1 Introduction

A well developed theory provides a foundation on which to erect the scaffolding of econometric models. In fact, the degree of success that can be achieved, to a large measure, is predicated on how well the theoretical basis has been articulated. Viewed, therefore, from econometric lenses, gauging from the survey of the theoretical literature in the last two chapters, one would suppose that the theoretical winds that have been sowed will enable econometricians to reap a whirlwind. However, a cursory survey of the the empirical literature would reveal that nature has been very niggardly in baring her secrets to disequilibrium macroeconometricians. Highly sophisticated econometric theory has conjoined with even more sophisticated estimation techniques, to be sure, to produce results which are sensible, but intellectual and other human costs have been enormous. It would be interesting to see the verdict of econometric historians after the debits and credits have been balanced. This chapter is concerned with the econometric theory and the subsequent one with empirical implementation.

4.2 Coherency Conditions

In Chapter II, it was shown that four regimes are possible under quantity constrained equilibrium. The household's trade offers in LY space were drawn such that L_s was steeper than Y_d (Figure 2.1),

$$0 \leq \delta Y_d/\delta L < \frac{1}{\delta L_s/\delta Y}.$$

Similarly, the firm's effective trade offers were drawn such that L_d was steeper than Y_s (Figure 2.2),

$$0 \leq \delta Y_s/\delta L < \frac{1}{\delta L_d/\delta Y}.$$

Though it was not made explicit there, these conditions are the outcome of the assumption of concave utility and production functions (Laroque: 1978). Also, it was noted there that in order for the model to be stable, certain other conditions need to be satisfied. Those conditions (equations (13) and (14) of chapter II), reproduced here for convenience, are,

$$0 \leq \delta Y_d/\delta L < \frac{1}{\delta L_d/\delta C}$$

and

$$0 \leq \delta Y_s/\delta L < \frac{1}{\delta L_s/\delta C}$$

The four conditions together determine the relative slopes of firm's and household's effective trade offers (Figure 2.4)

The slopes of the effective trade offers given above are termed spill-over coefficients, and their relative magnitudes determine whether or not a disequilibrium macroeconometric model possesses a unique solution. In the terminology of macroeconometrics of disequilibrium, the conditions required for a unique solution are called solveability or coherency conditions.

For a thorough appreciation of the import of the coherency conditions in a disequilibrium macroeconometric model, it may be worthwhile to derive the firm's and household's effective trade curves from specific utility and production functions (as opposed to the general functional forms used in Chapter II). The exposition provided here closely follows Ito (1980). Let L, and Y stand for labor and goods transacted; L_s, L_d Y_s, Y_d be the effective trade offers of labor and goods; and L_{ws}, L_{wd} Y_{ws}, Y_{wd} be the notional trade offers of labor and goods. Let us suppose that the utility of a consumer is a function of consumption Y, leisure defined as difference between endowment of time and labor supply, $T-L$, and stock of money balances at the end of the previous period, M_{-1}. Further, if we impose a Cobb–Douglass specification, the decision problem of the consumer is to

$$Max\ \ U = (Y)^{\alpha}(T-L)^{\beta}(M/p)^{\gamma}, \alpha, \beta, \gamma > 0$$

subject to

$$M_{-1} + wL = pY + M.$$

The notional demands for goods, money and labor supply from the above optimization exercise is

$$Y_{wd} = \frac{\alpha}{(\alpha+\beta+\gamma)p}(M_{-1} + wT),$$

$$(M/p)_{wd} = \frac{\gamma}{(\alpha+\beta+\gamma)}(M_{-1} + wT),$$

$$L_{ws} = \frac{\alpha+\gamma}{(\alpha+\beta+\gamma)}T - \frac{\beta}{(\alpha+\beta+\gamma)}\frac{M_{-1}}{w}.$$

The effective trade offers of the household takes into account the constraints it faces in the labor and goods markets. The constraint could be either the inablitiy to sell as much labor as it desires or as many goods as desired. The effective demand for goods when it experiences constraint in the labor maket is calculated by maximizing

$$U = (Y)^{\alpha}(T-L)^{\beta}(M/p)^{\gamma}, \alpha, \beta, \gamma > 0$$

subject to

$$M_{-1} + wL = pY + M$$

and

$$L = \bar{L},$$

where $\bar{L}$ is the quantity constraint experienced in the labor market. The above optimization results in the following effective demand for goods,

$$Y_d = \frac{\alpha}{(\alpha+\gamma)p}(M_1 + w\bar{L}),$$

which may be manipulated to yield the following:

$$Y_d = Y_{wd} + \frac{\alpha}{(\alpha+\gamma)}\frac{w}{p}(L - L_{ws}).$$

If we replace $\frac{\alpha}{(\alpha+\gamma)}\frac{w}{p}$ by η_3, the above equation can be written as

$$Y_d = Y_{wd} - \eta_3(L_{ws} - L).$$

Effective demand for goods is, therefore, the difference between Walrasian demand and spill–over effect from the labor market where η_3 is the spill–over coefficient.

The effective labor supply when the household faces constraint in the goods market is calculated by maximizing

$$U = (Y)^{\alpha}(T - L)^{\beta}(M/p)^{\gamma}, \alpha, \beta, \gamma > 0$$

subject to

$$M_{-1} + wL = pY + M$$

and

$$Y = \bar{Y}.$$

The result is

$$L_s = L_{ws} + \frac{\beta}{(\beta+\gamma)}\frac{p}{w}(Y - Y_{wd}).$$

Replacing $\frac{\beta}{(\beta+\gamma)}\frac{p}{w}$ by η_4, we can rewrite the effective labor supply equation as

$$L_s = L_{ws} - \eta_4(Y_{wd} - Y).$$

As in the case of effective demand for goods, the effective labor supply is the difference between Walrasian labor supply and spill–over effect from the goods market.

In an analogous manner, we can derive the effective trade offers of the firms. Without further ado, they are

$$Y_s = Y_{ws} - \eta_2(L_{wd} - L)$$

and

$$L_d = L_{wd} - \eta_1(Y_{wd} - Y).$$

It is important to realize that spill–over effects are proportional to the dissatisfaction, or linear. The linearity of the spill–over effect is the direct consequence of the Cobb–Douglass utility function used to derive the effective trade offers. It is not clear what the resultant spill–over effect would be if a different variety of utility function was assumed.

The structural form of the two–market disequilibrium model can now be summarized as follows:

$$L_d = L_{wd} - \eta_1(Y_{ws} - Y) + u_1,$$

$$Y_s = Y_{ws} - \eta_2(L_{wd} - L) + u_2,$$

$$Y_d = Y_{wd} - \eta_3(L_{ws} - L) + u_3,$$

$$L_s = L_{ws} - \eta_4(Y_{wd} - Y) + u_4,$$

$$Y = Min(Y_d, Y_s),$$

$$L = Min(L_d, L_s.)$$

$\eta_i \;\; i = 1,..,4$ are positive, and u_1, u_2, u_3 and u_4 are error terms.

It may be apparent by now that the spill–over coefficients are nothing but the slopes of the effective trade offers of households and firms noted above:

$$\frac{1}{\delta L_d/\delta Y} = \frac{1}{\eta_1}, \quad \delta Y_s/\delta L = \eta_2,$$

$$\delta Y_d/\delta L = \eta_3, \quad \frac{1}{\delta L_s/\delta Y} = \frac{1}{\eta_4}.$$

And in order for the model to possess a unique equilibrium,

$$1 - \eta_1\eta_2 > 0, 1 - \eta_2\eta_3 > 0, 1 - \eta_3\eta_4 > 0, 1 - \eta_1\eta_4 > 0$$

If the above conditions are not observed, there will be multiplicity of equilibria. The uniqueness condition also satisfies the stability condition (Gourieroux, Laffont, Montfort: 1980a; see also Chapter I).[1] The econometric interpretation of the uniqueness and stability condition is that each observation of the data will be allocated to one and only one regime. In other words, the model has a well–defined reduced form. To sum up, in the context of a disequilibrium macroeconometric model which is also piece–wise linear, the uniqueness, stability and solveability condition and the requirements for well–defined reduced form are synonymous. The solveablity conditions were independently discovered by Ito (1980) and Gourieroux et al. (1980a),

[1]Gourieroux et al. (1980b), drawing upon the insights offered by the disequilibrium model, generalizes the result to all piece–wise linear simultaneous equation models.They include self–selectivity models and simultaneous Probit and Tobit models.

and, curiously enough, appeared as consecutive articles in the same issue of Econometrica.[2]

4.3 Estimation Methodologies

There are three approaches to the estimation of a disequilibrium macroeconometric model. They are 1) the Maximum Likelihood approach of Maddala and Nelson (1974), 2) the Mathematical Programming approach of Ginsburgh, Tishler and Zang (1980) and 3) the Pseudo–Maximum Likelihood approach of Laroque and Salanie(1989). A one–market model is examined first, and the two–market case is taken up next.

One–Market Disequilibrium Model

The most common disequilibrium model that one usually encounters in the literature has a demand equation, a supply equation and a minimum condition:

$$D_t = \alpha_1 p_t + \beta_1{}' x_{1t} + u_{1t},$$

[2]Here is yet another example of what Merton calls 'multiple discovery'. Intellectual history is replete with such multiple discoveries. One cannot find a more celebrated instance of this phenomenon than attempts by mathematicians to derive as a theorem the fifth (or parallel) postulate of Euclid from the remaining nine axioms and postulates of the *Elements*. The elusive search lasted 2000 years, and to it is credited some of the most far-reaching developments of modern mathematics. C. F. Gauss (1777-1855) of Germany, J. Bolayi (1802-1860) of Hungary and N. I. Lobachevsky (1793-1856) of Russia working in isolation reached the surprising conclusion that the parallel postulate is independent of the remaining postulates and therefore cannot be deduced from them (Eves: 1976 pp. 382-386). In Economics, the classic example is the independent discovery of the principle of marginal utility by Jevons, Menger and Walras in the mid-nineteenth century (Blaug: 1985).

$$S_t = \alpha_2 p_t + \beta_2\prime x_{2t} + u_{2t},$$

$$Q_t = Min(D_t, S_t).$$

x_{1t}, and x_{2t} are vectors of exogenous variables, u_{1t} and u_{2t} are jointly normal with mean vector zero, covariance matrix Σ and serially uncorrelated with each other. Since D_t and S_t are unobserved, the only observed random variable is Q_t.

Maddala and Nelson (1974) have proposed the appropriate likelihood function for the above model. Let $h(Q_t)$ be the probability density function for Q_t. Then

$$h(Q_t) = f(Q_t|D_t < S_t)Pr(D_t < S_t) + f(Q_t|D_t \geq S_t)Pr(D_t| \geq S_t),$$

where the conditional p.d.f.

$$f(Q_t|D_t < S_t) = \int_{Q_t}^{\infty} g(Q_t, S_t|D_t < S_t)dS_t = \frac{1}{Pr(D_t < S_t)} \int_{Q_t}^{\infty} g(Q_t, S_t)dS_t$$

and similarly for $f(Q_t|D_t \geq S_t)$. Therefore,

$$h(Q_t) = \int_{Q_t}^{\infty} f(Q_t, S_t)dS_t + \int_{Q_t}^{\infty} f(Q_t, D_t)dD_t.$$

The likelihood function is

$$\begin{aligned} L \;=\; & \left\{ \prod_{t=1}^{T} \frac{1}{\sqrt{2\Pi}\sigma_1} exp\left(-\frac{(Q_t - \alpha_1 p_t - \beta_1\prime x_{1t})^2}{2\sigma_1{}^2}\right) \times \left(1 - \Phi\left(\frac{Q_t - \alpha_2 p_t - \beta_2\prime x_{2t}}{\sigma_2}\right)\right) \right. \\ & \left. + \frac{1}{\sqrt{2\Pi}\sigma_2} exp\left(-\frac{(Q_t - \alpha_2 p_t - \beta_2\prime x_{2t})^2}{2\sigma_2{}^2}\right) \times \left(1 - \Phi\left(\frac{Q_t - \alpha_1 p_t - \beta_1\prime x_{1t}}{\sigma_1}\right)\right) \right\}, \end{aligned}$$

where Φ is the probablity distribution function. Maximization of the above likelihood function is in practice quite treacherous due to the unbounded nature of the function. There are points on the boundary of the parameter space where σ_1 and σ_2 attains the value of zero, which makes the value of L to be infinity. For an exposition of the unboundedness of the likelihood function, let

$$L = \prod_{t=1}^{T} (\alpha_t \beta_t + \gamma_t \delta_t),$$

where

$$\begin{aligned}
\alpha_t &= \frac{1}{\sqrt{2\Pi}\sigma_1} exp\left[-\frac{(Q_t - \alpha_1 p_t - \beta_1\prime x_{1t})^2}{2\sigma_1^2}\right], \\
\beta_t &= 1 - \Phi\left(\frac{Q_t - \alpha_2 p_t - \beta_2\prime x_{2t}}{\sigma_2}\right), \\
\gamma_t &= \frac{1}{\sqrt{2\Pi}\sigma_2} exp\left[-\frac{(Q_t - \alpha_2 p_t - \beta_2\prime x_{2t})^2}{2\sigma_2^2}\right], \\
\delta_t &= 1 - \Phi\left(\frac{Q_t - \alpha_1 p_t - \beta_1\prime x_{1t}}{\sigma_1}\right).
\end{aligned}$$

Consider some values α_1, β_1 such that for some observations $Q_t - \alpha_1 p_t - \beta_1 x_{it} = 0$ and for others negative. There are instances when this would be the case for a demand function with an intercept term. Let α_2, β_2, and σ_2 be any admissible value. Let us now investigate the consequences of σ_1 converging to zero. For those observations of the demand equation where the difference between the actual and predicted is zero, α_t converges to ∞ and $\lim_{\sigma_1 \to 0} \delta_t = 1/2$. For the observations for which the difference between the actual and the predicted is negative, $\lim_{\sigma_1 \to 0} \delta_t = 1$. β_t and γ_t are nonzero for all observations. Therefore, for some observations the value of the likelihood function tends to infinity, thus making the value the likelihood function for all the observation to tend to infinity.[3]

The Ginsburgh et. al approach to the estimation of the disequilibrium model has a stochastic minimum condition,

$$Q_t = Min(D_t, S_t) + \epsilon_t.$$

The implication of the above minimum condition is that demand and supply equations are deterministic:

$$\begin{aligned} D_t &= \alpha_1 p_t + \beta_1{\prime} x_{1t}, \\ S_t &= \alpha_2 p_t + \beta_2{\prime} x_{2t}. \end{aligned}$$

The sample–separation or switching between demand and supply regimes is facilitated by the use of a binary variable:

$$Q_t = \alpha_1 p_t + \beta_1{\prime} x_{1t} + q(\alpha_2 p_t + \beta_2{\prime} x_{2t} - \alpha_1 p_t + \beta_1{\prime} x_{1t}) + \epsilon_t,$$

where q(z) = max(0, z). Assuming that the ϵ_t is independently, identically and normally distributed, the parameters are estimated by maximizing the likelihood function

$$\begin{aligned} L = & \left[\prod_{T_1} \frac{1}{\sqrt{2\Pi}\sigma_\epsilon} exp \left(-\frac{(Q_t - \alpha_1 p_t - \beta_1{\prime} x_{1t})^2}{2\sigma_\epsilon} \right) \right] \times \\ & \left[\prod_{T_2} (\frac{1}{\sqrt{2\Pi}\sigma_\epsilon} exp \left(-\frac{(Q_t - \alpha_2 p_t - \beta_2{\prime} x_{2t})^2}{2\sigma_\epsilon} \right) \right], \end{aligned}$$

[3]The proclivity of the likelihood function to be unbonded is noticed only where the sample-separation is unknown, which suggests that it is lack of information for sample-separation that causes the likelihood function to behave in this whimsical manner. For a very readable survey of literature of econometrics of disequilibrium, see Quandt (1982).

where $T_1 = (t|q(z) = 0)$ and $T_2 = (t|q(z) = z)$ (Quandt: 1982). The above likelihood function is discontinuous. The problem with discontinuous function stems from the absence of the first and second derivatives for all points in the parameter space. In other words, the function is only non–continuously differentiable, and optimization routines requiring continuous first and second derivatives become ineligible for maximization of the function. Smoothing procedures enable the function to be maximized by an algorithm which requires first and second derivatives. In order to smooth the function, q(z) may replaced by $q\prime(z)$, where $q\prime(z)$ is

$$
\begin{aligned}
q\prime(z) &= 0 \quad if \quad z \leq -\kappa, \\
&= \frac{3}{16}(z/\kappa)^5 - \frac{5}{8}(z/\kappa)^3 + \frac{15}{16}z/\kappa + \frac{1}{2} \quad if \quad -\kappa \leq z \leq \kappa, \\
&= z \quad if \quad z \geq \kappa
\end{aligned}
$$

where κ is a positive parameter. When $q\prime(z)$ is embedded into the Ginsburgh et al. likelihood function, the step function becomes a twice–continuously differentiable function (Tishler and Zang: 1977). Tishler and Zang suggest that κ be reduced gradually in the iteration process. For sufficiently small values of κ, the approximating model gives the exact solution to the original model.

The Ginsburgh et al. approach is weak on several grounds. The most serious of these seems to be the absence of statistical justification for the specified equations. The demand and supply equations are exact. This would suggest that there are no measurement errors in the the endogenous variable, the specification is accurate (the econometrician steps into the shoes of the Almighty), and the agents make no errors in the optimization of their respective objective functions. The second problem is that over a large sub-set of the parameter space, they remain unidentified. Assume, for example, that in reality all observations correspond to the demand equation. Then, assuming that the approximation is reasonable or, in other words, the value of κ is sufficiently small, the parameters β_2 and α_2 remains unidentified. This, however, is not the case with Maddala and Nelson approach, where even if the observations all belong to the demand regime, the supply regime exerts a small influence on the likelihood function (Quandt: 1982).

The problem of unboundedness and numerical problems involved in the maximization of the likelihood function, and the absence of a sound statistical foundation for the Ginsburgh et al. approach motivated efforts towards innovations in estimation techniques. The Pseudo–Maximum Likelihood technique of Laroque and Salanie (1989) is a case in point. The likelihood function that is maximized is based on the first and second order moments of the endogenous variables and hence the rubric 'pseudo'. Let u_1 and u_2 be independently, normally distributed with mean zero and unit variance. The first and second order moments for the one–market model is (Laorque and Salanie: 1989)

$$E(Q_t/p_t, x_t) = -s\phi\left(\frac{D_t - S_t}{s}\right) + D_t\Phi\left(\frac{S_t - D_t}{s}\right) + S_t\Phi\left(\frac{D_t - S_t}{s}\right),$$

$$
\begin{aligned}
E(Q_t^2/p_t, x_t) &= (D_t^2+\sigma_1^2)\Phi\left(\frac{S_t-D_t}{s}\right)+(S_t^2+\sigma_2^2)\Phi\left(\frac{D_t-S_t}{s}\right) \\
&\quad -s(D_t+S_t)\phi\left(\frac{D_t-S_t}{s}\right),
\end{aligned}
$$

where $s^2=\sigma_1^2+\sigma_2^2$ and ϕ and Φ are the density and the cumulative distribution of a normal variable with mean zero and unit variance. The above expressions show that α, β and s are identifiable in the first stage, and α, β, σ in the second stage. There are three Pseudo–Maximum Likelihood estimation procedures all of which produce consistent estimates (Gourieroux, Monfort, and Trognon: 1984). Two of them are given below.

Let $\theta=(\alpha,\beta)$. The pseudo–model is

$$Q_t = EQ_t(\theta, s)+\mu_{1t},$$

where μ_{1t} is independently, normally distributed with mean 0 and variance 1. The maximum likelihood estimator, $(\hat{\theta}_1, \hat{s_1})$ is the nonlinear least squares estimator obtained by minimizing the function

$$\frac{1}{2}\sum_{t=1}^{T}(Q_t-EQ_t(\theta,s))^2.$$

$(\hat{\theta}_2, \hat{\sigma}_2)$, the maximum likelihood estimator of the pseudo–model

$$Q_t = EQ_t(\theta, s)+\mu_{2t},$$

where μ_{2t} has the same properties of μ_{1t}, is obtained by minimizing the function

$$\frac{1}{2}\sum_{t=1}^{T}\left(\frac{Q_t-E(Q_t(\theta,s))^2}{VQ_t(\theta,\sigma)}+logVQ_t(\theta,\sigma)\right),$$

and $VQ_t(\theta, \sigma)$ is the variance of Q_t.[4] While the first estimates only θ and s, the second estimates θ and σ.

Two–Market Disequilibrium Model

The obvious starting point for the discussion of the two–market model is the regime classification, deriveable from the prototype model introduced earlier.

$$\begin{aligned}
&\textit{Regime I}: \ \textit{Underconsumption}\\
L_d &= L_{wd} - \eta_1(Y_{ws} - Y) + u_1\\
Y_s &= Y_{ws} - \eta_2(L_{wd} - L) + u_2\\
Y_d &= Y_{wd} + u_3\\
L_s &= L_{ws} + u_4\\
Y &= Y_d; L = L_s
\end{aligned}$$

$$\begin{aligned}
&\textit{Regime II}: \ \textit{Classical Unemployment}\\
L_d &= L_{wd} + u_1\\
Y_s &= Y_{ws} + u_2\\
Y_d &= Y_{wd} - \eta_3(L_{ws} - L) + u_3\\
L_s &= L_{ws} - \eta_4(Y_{wd} - Y) + u_4\\
Y &= Y_s; L = L_d
\end{aligned}$$

[4] Presumably, variance is calculated from the familiar formula, $\sigma^2 = E(X^2) - [E(X)]^2$.

$$Regime\ III:\ Keynesian\ Unemployment$$

$$L_d = L_{wd} - \eta_1(Y_{ws} - Y) + u_1$$

$$Y_s = Y_{ws} + u_2$$

$$Y_d = Y_{wd} - \eta_3(L_{ws} - L) + u_3$$

$$L_s = L_{ws} + u_4$$

$$Y = Y_d; L = L_d$$

$$Regime\ IV:\ Repressed\ Inflation$$

$$L_d = L_{wd} + u_1$$

$$Y_s = Y_{ws} - \eta_2(L_{wd} - L) + u_2$$

$$Y_d = Y_{wd} + u_3$$

$$L_s = L_{ws} - \eta_4(Y_{wd} - Y) + u_4$$

$$Y = Y_s; L = L_s$$

In Underconsumption regime, despite excess supply in the goods market, firms are constrained in the labor market; production is geared towards inventory accumulation besides satisfying the current demand. This leads to the seemingly paradoxical situation of being constrained in the goods market and labor market simultaneously. In Classical Unemployment, households are constrained in both the goods and labor.markets. High wage rates induce households to supply more labor and firms to restrict demand. High wage rates also increase consumption demand. In Keynesian Unemployment, excess supply in the goods market forces firms to reduce their intake of labor, and households have to reduce their consumption demand since they are rationed in the labor market. The supplies of goods and labor are at their Walrasian levels; firms are not constrained in the availability of labor and households in the availability of goods. Quantity traded are the effective demands for goods and labor. Under Repressed inflation, excess demand in the labor market forces firms and households off their respective supply curves. Labor and goods demand, however, will be determined by their notional curves, as neither firms nor households face excess supply in the goods and labor market. Quantity traded are the effective supplies of goods and labor.

The likelihood function of the two–market disequilibrium model is an extension of the one–market model. However, as we shall see, the the likelihood function require an amendment because of the presence of the spill–over coefficients in the two–market model (Ito: 1980).[5] The likelihood function is the sum of four constituent likelihood functions, one function for each regime:

$$\begin{aligned} L &= \prod_t \Big[\int_{Y<Y_s} \int_{L<L_d} f_I(Y, Y_s, L_d, L) dY_s dL_d \\ &\quad + \int_{Y<Y_d} \int_{L<L_s} f_{II}(Y_d, Y, L, L_s) dY_d dL_s \end{aligned}$$

[5]The need for spill–over coefficients was first pointed out by Quandt (1978), though he could not arrive at such a specification. Besides, his likelihood function was incorrect. Amemiya (1977) corrected Quandt, but his specification suffered from the same problem that vitiated Quandt, the absence of explicit recognition of spill–over coefficients.

$$+\int_{Y<Y_s}\int_{L<L_s} f_{III}(Y, Y_s, L, L_s)dY_s dL_s$$
$$+\int_{Y<Y_d}\int_{L<L_d} f_{IV}(Y_d, Y, L_d, L)dY_d dL_d\Big] .$$

Though u_1, u_2, u_3 and u_4 are independently, identically distributed, it is incorrect to factor f_I into $(f^1(Y_d).f^2(Y_s).f^3(L_d).f^4(L_s))$ because of the spill–over effects. The determinant of the Jacobian of (u_1, u_2, u_3, u_4) with respect to Y_d, Y_s, L_d, L_s has to be considered unlike the one–market case where it is one.

Assume that u_1, u_2, u_3, u_4 are independently, identically and normally distributed for all t and are independent of each other:

$$u_{it} \sim N(0, \sigma_i^2) \quad i = 1, 2, 3, 4 \;\; \forall t.$$

Define

$$\varphi_j(g_{ij}) = \frac{1}{\sigma_j\sqrt{2\pi}} exp\left[-\frac{1}{2\sigma_j^2}(g_{ij})^2\right].$$

Then

$$f_i = k_i \prod_{j=1}^{4} \varphi_j(g_{ij}), \quad i = I, II, II, IV,$$

where k_i is the jacobian of transformation. The likelihood function now becomes

$$\begin{aligned} L \;=\; \prod_t \Bigg\{ & k_I \int_{Y_t}^{\infty}\int_{L_t}^{\infty} \prod_{j=1}^{4} \varphi_j(g_{Ij}|Y_d = Y_t, L_s = L_t)dL_d dY_s \\ & + k_{II} \int_{Y_t}^{\infty}\int_{L_t}^{\infty} \prod_{j=1}^{4} \varphi_j(g_{IIj}|Y_s = Y_t, L_d = L_t)dL_s dY_d \\ & + k_{III} \int_{Y_t}^{\infty}\int_{L_t}^{\infty} \prod_{j=1}^{4} \varphi_j(g_{IIIj}|Y_d = Y_t, L_d = L_t)dL_s dY_s \end{aligned}$$

$$+ k_{IV} \int_{Y_t}^{\infty} \int_{L_t}^{\infty} \prod_{j=1}^{4} \varphi_j(g_{IVj} | Y_s = Y_t, L_s = L_t) dL_d dY_d \Bigg\} .$$

Pioneers of the implementation of the the maximum likeliood technique are Artus, Laroque and Michel (1984) and Kooiman and Kloeck (1985).

The Ginsburgh et al. (1980) approach to the estimation of a macroeconomic model is a straight–forward extension of the one–market model examined above. The implementation of the Ginsburgh et al. approach was attempted by Sneessens (1981, 1983) and Vilares (1982,1986). The exposition that is followed here is an adaptation of Sneessens (1981, 1983). Firms based on the past experience form expectations regarding the current demand for goods. Given the relative prices, technology and capital stock, the expected demand is treated as the target level of production. The labor requirements for the target level of production is then obtained from the labor market. Trading is postulated to take place sequentially, with transaction occurring initially in the labor market and subsequently in the goods market. If the demand for goods is greater than the potential output, the production of goods is restricted by the potential output, the maximum possible, and households are rationed. If the labor demanded by firms, calculated on the basis of anticipated demand, is greater than labor supply, firms face rationing in the labor market. The production plans are, accordingly, revised downwards. The above conceptualization can be expressed mathematically as,

$$Y = Y_s = A.L^{\nu},$$

$$L = Min(L_d, L_s),$$

where

$$\begin{aligned} L_d &= (A^{-1}Y_d^e)^{\frac{1}{\nu}} \;\; if \;\; Y_d^e \le Y_p, \\ &= (A^{-1}Y_p)^{\frac{1}{\nu}} \;\; otherwise, \\ L_s &= e^{\eta 0}LF. \end{aligned}$$

Y_d^e is the expected demand, and Y_p is the potential output. LF is the available labor force. The first equation suggests that quantity traded in the goods market is always output supplied; $Y = Y_s = Min(Y_d, Y_s)$. Labor demanded is determined by the production function $Y = Y_s = A.L^\nu$. However, there are two possible labor demands; labor demand could either be the one corresponding to potential output, Y_p, or expected demand, Y_d^e. Labor demanded, as long it does not exceed the available labor, L_s, is the minimum of the above two,

$$L_d = A^{-\frac{1}{\nu}} Min(Y_d^{e\frac{1}{\nu}}, Y_p^{\frac{1}{\nu}}).$$

Note that labour supply is always Walrasian,

$$L_s = L_{ws}.$$

The term $e^{\eta 0}$ in the labor supply equation stands for frictional unemployment.

Since quantity traded in the goods market is always output supplied, the implication is that there is no rationing in the goods market. Hence Sneessens has a minimum condition only for the labor market. The regime classification is, therefore, accomplished by distinguishing between the various level of labor traded. As was noted earlier, there are two possible levels of labor demand, each associated with different levels output produced and traded. Obviously the output produced when the firms are producing at their potential level is different from the one produced, when demand is less than the potential output. The labor demand when firms are producing their potential output, with the proviso that $L_d < L_s$, is called Classical Unemployment. When the firms find that expected demand is less than the potential output, the corresponding labor demand is used to identify the Keynesian Unemployment. The Repressed Inflation occurs when firms are constrained in the labor market ($L_d > L_s$). In Chapter II, while discussing regime classification, it was observed that firms occasionly demand more labor than is warranted by the current output, for purposes of inventory accumulation. This would give rise to the regime of Underconsumption. Empirically, the implication is that a distinction between output produced for sales and inventory accumulation be maintained. However, if the measure of goods traded is GNP or GDP, the distinction is not preserved, and Unconsumption cannot be isolated from other regimes. Therefore, Sneessens ignores Underconsumption. Expressed mathematically,

$$Y = AL^{\nu},$$

$$L = Min(L_{ku}, L_{cu}, L_{ri}),$$

with

$$L_{ku} = (A^{-1}Y_d^e)^{\frac{1}{\nu}}, \quad L_{cu} = (A^{-1}Y_p)^{\frac{1}{\nu}}, \quad L_{ri} = e^{\eta 0}LF.$$

The estimable version of the model may be summarized as follows after transformation into logarithms:

$$\ln Y_t = \ln A + \nu \ln L + \epsilon_{1t},$$

$$\ln L_t = Min(\ln L_{ku}, \ln L_{cu}, \ln L_s) + \epsilon_{2t}.$$

From the stochastic specification, it is immediatly apparent that the estimation technique is Ginsburgh et al.(1980). The labor equations in the minimum condition are determininistic; the error term appears outside the minimum condition. Assuming that ϵ_{1t} and ϵ_{2t} are uncorrelated, the system of two equations can be estimated recursively. Sneessens estimates the production function using ordinary least squares, and the estimates of A and ν are substituted into the employment equation in the second stage, which is then estimated for the remaining parameters. The two–stage technique of Sneessens, though resulting in inefficient estimates, steers clear of the estimation problems encountered in maximum likelihood functions.

It was noted earlier that the adoption of Ginsburgh et al. approach renders the function to be maximized discontinuous. Therefore, in order to obtain a continuous function amenable for the application of optimization methods requiring first and second derivatives, a smoothing procedure becomes inevitable. A similar exercise also becomes necessary for estimating the Sneessens' employment equation; the smoothing technique of Tishler and

Zang (1979) is employed. Let the log–likelihood function be

$$L = \sum_t h_t(\ln L)$$

where $h_t(\cdot)$ is logarithm of density of $\ln L$ at time t. The three possible regimes can be represented by three binary variables:

$$\begin{aligned} r_1 &= 1 \ if \ \ L_{ku} > L_{cu} \\ &= 0 \ \ otherwise \end{aligned}$$

$$\begin{aligned} r_2 &= 1 \ if \ \ L_{ku} > L_{ri} \\ &= 0 \ \ otherwise \end{aligned}$$

$$\begin{aligned} r_3 &= 1 \ if \ \ L_{cu} > L_{ri} \\ &= 0 \ \ otherwise \end{aligned}$$

For values of $r_1 = r_2 = 0$, the outcome is Keynesian Unemployment; if $r_1 = 1$ and $r_3 = 0$, Classical Unemployment, and if $r_2 = r_3 = 1$, Repressed Inflation. Let $f(\cdot)$ stand for the logarithm of the density of the disturbance term,ϵ_{2t}. The log–density of $\ln L$ can be written as

$$\begin{aligned} h_t(\ln L) &= (1-r_1)(1-r_2)[ln|J| + f(\ln L - \ln L_{ku})] \\ &\quad + r_1(1-r_3)f(\ln L - \ln L_{cu}) + r_2 r_3 f(\ln L - \ln L_{ri}). \end{aligned}$$

where J is the jocobian of transformation from ϵ_2 to $\ln L$ in Keynesian Unemployment. If ϵ_{2t} is normally distributed, then the likelihood function can be concentrated with respect to the variance of ϵ_{2t}. If $|J| = 1$ maximizing the concentrated likelihood function and minimization of the error sum of squares are equivalent procedures.

4.4 Conclusion

Three major approaches to the estimation of a macroeconomic model were surveyed. It was found that the Ginsburgh et al. (1980) approach is unacceptable on statistical grounds. Even from an economic theoretic ground, it may not be a true representation of a general disequilibrium model. In the interest of tractability, the modeling of the goods market assumes away rationing. True, Sneessens' (1981, 1983) model behaves very well when judged against predictive accuracy. But econometric models are not judged on the basis of predictive potential alone; it must be faithful to the theoretical model it is attempting to estimate. Of the three methods surveyed here, Sneessens' estimating technique is the most accessible. Kooiman and Kloeck (1985) encountered considerable difficulty in maximizing the likelihood function, using the Full–Information Maximum Likelihood technique, which cast doubt on the efficacy of the estimating technique itself. On the contrary Artus et al. (1984) and the present study (FIML also, presented in the next chapter) go a long way in dispelling those doubts; both report none of the problems of Kooiman and Kloeck. However, the three studies agree that FIML is problematic because maximization of the likelihood function involved is no mean task. The Pseudo–Maximum Likelihood technique of Laroque and Salanie (1989) was devised to overcome the problems of both the Ginsburgh et al. and Maddala and Nelson (FIML) approaches. Laroque and Salanie report that their approach has none of the problems encountered by the Maddala and Nelson approach. Ample proof of this assertion is forthcoming not only in Laroque and Salanie (1989) where they estimate a disequilibrium macroeconomic model via three Pseudo–Maximum Likelihood techniques, but also in Laroque (1989), where the ambitious project of estimating for four countries (France, Germany, the U.K. and the U.S.A.) is successfully undertaken.

Chapter 5

Estimation of Disequilibrium Macroeconomics

5.1 Introduction

The last decade has been a witness to a flowering of interest in the estimation of disequilibrium macroeconomic models. Prominent among these are Artus, Laroque and Michel (1984) and Kooiman and Kloeck (1985) (the Maddala and Nelson approach) Laroque and Salanie (1989) and Larouqe (1989) (the Pseudo–Maximum Likeihood approach) and Sneessens (1981, 1983) and Vilares (1982, 1986) (the Ginsburgh et al. approach). The distinguishing feature of these studies is the existence of three or four mutually exclusive regimes. This approach to disequilibrium macroeconomics is in contrast to Coen and Hickman (1987). Confining attention to 2 regimes, Classical and Keynesian Unemployments, they allow for its coexistence. Yet another approach stems from the recognition that while aggregate markets may switch from regime to regime, micromarkets exhibit coexistence of regimes. Such is the case of Kooiman (1984), Sneessens and Dreze (1986), Lambert (1988), and Laroque and Salanie (1989).

The study at hand attempts an estimation of disequilibrium macro model using Canadian quarterly data from 1972.1 to 1985.3, and the technique of estimation is FIML, as it is Maddala and Nelson in spirit. Estimated model consists of two markets, aggregate goods and labor. Kinship of the model presented here to Artus et al. (1984) and Kooiman and Kloeck (1985) cannot be stressed enough, and therefore it may be instructive to make a comparative evaluation of the major features of the three.[1] Artus et al. use quarterly data of the French economy for the period 1963.2 to 1978.4 and Kooiman and Kloeck yearly data of The Netherlands from 1952 to 1979. While the French study confines its attention to the business sector and the Dutch to the manufacturing sector, our data pertain to the macroeconomy. Kooiman and Kloeck have imposed restrictions on the spill–over coefficients to satisfy the coherency conditions so that the model has a unique fix–price equilibrium. Artus et al. have a non–linear specification of the productive sector, and the model is no longer piece–wise linear. Conditions for coherency or a unique fix–price equilibrium developed by Gourieroux et al. (1980a, 1980b) and Ito (1980) are valid only for piece–wise linear model, and could not therefore be checked analytically. Simulations, however, have shown that the model in fact possess a fix–price equilibrium for a range of parameter estimates,

[1]For an insightful survey of disequilibrium macroeconometric literature, see Laffont (1985).

including the ones obtained. The present exercise has a piece–wise linear model, and we were able to obtain parameter estimates for spill–over coefficients which satisfy the uniqueness condition. Kooiman and Kloeck report considerable difficulty in maximizing the likelihood function; the likelihood function tended to stray into the unbounded region. A penalty function was introduced into the objective function to restrain it from its persistent proclivity towards unbounded regions of the function. Artus et al. and the current study have not evinced any such tendencies. We, however, were not able to obtain estimates for labor demand under Keynesian Unemployment regime which are sensible (of this more anon); nothing of this sort seem to plague either of the other studies.

5.2 The Structure of the Model

Firms maximize profits subject to a short-run production function and, therefore, the first order conditions define the following notional aggregate labor demand and output supply equations:

$$L_{wd} = L_{wd}(w) + u_1, \tag{5.1}$$

$$Y_{ws} = f(L_{wd}(w)) + u_2, \tag{5.2}$$

where w is the real wage rate and u_1 and u_2 represent the the errors made by the firms in the process of maximization of profits. The error terms are intended to capture aspects of technical and allocative inefficiency. While eqs.(5.1) and (5.2) represent the notional magnitudes of the firm, in order to arrive at their effective counterparts, the spill-over from labor and goods markets have to be appended. Accordingly, the notional aggregate labor demand and output supply functions are transformed as follows:

$$L_d = L_{wd} - \eta_1(Y_s - Y) \quad 0 \leq \eta_1 \leq 1, \tag{5.3}$$

$$Y_s = Y_{ws} - \eta_2(L_d - L) \quad 0 \leq \eta_2 \leq 1. \tag{5.4}$$

Of the components of aggregate demand, only consumption and exports are treated endogenous, leaving import content of investment goods and government expenditure, investment and government expenditure exogenous. It is assumed that excess supply in the labor market does not propagate through the system and constrain households in the demand for goods. Therefore, $\eta_3 = 0$ and notional aggregate demand is always equal to effective aggregate demand, regardless of regime.[2] Modeling of aggregate demand within the context of disequilibrium, however, demands recognition of the additional complexity of rationing of demand for goods and exports during periods

[2]The unfortunate consequence of this assumption is the inability to estimate the impact of excess supply in the labor market on goods demand, a major tenent of disequilibrium macroeconomics. Most of the empirical studies in this area suffer from the same inadequacy. The reason for this inadequacy might be sought in the difficulty to incorporate both rationing and spill–over coeffcients in the the effective aggregate demand equation. In Classical Unemployment, if one were to be faithful to the theory, one would need a spill–over coefficient since there is excess supply in the labor market, and a rationing coefficient since there excess demand in the goods market in the effective aggregate demand equation. The likelihood function would then become even more cumbersome, and the maximization of the function more difficult.

when excess demand for goods prevails. Such is the case in Classical Unemployment and Repressed Inflation regimes. This type of constrained behavior by households and exports is introduced through a rationing parameter θ. Consumption demand C_d and export demand X_d decrease by proportions θ and $(1-\theta)$ of excess demand for goods, and θ is postulated to be between 0 and 1. Denoting actual transactions by C and X,

$$C = C_d(\cdot) - \theta(Y_{wd} - Y) + u_3 \tag{5.5}$$

and

$$X = X_d(\cdot) - (1-\theta)(Y_{wd} - Y) + u_4. \tag{5.6}$$

Given the accounting identity $Y = C - M_C + X + EXO$, where M_C is the imported component of consumption goods and EXO stand for the exogenous components of effective aggregate demand ($EXO = I + G - M_I - M_G$, where M_I and M_G are the import content of investment and government expenditure respectively),

$$Y_{wd} = C_d(\cdot) - M_C + X_d(\cdot) + EXO + u_3 + u_4. \tag{5.7}$$

(5.5), (5.6) and (5.7) constitute a system of interdependent equations. Therefore, one of the above is redundant in the formulation of the likelihood function. For example, inclusion of consumption and export equations renders the accounting identity irrelevant for estimation purposes, and it is quite arbitrary as to which one of the equations in the above system is excluded. We, therefore, decided to exclude the consumption equation. The implication is that the consumption equation is estimated as part of the aggregate demand equation.

Symmetry and internal consistency demand that there be no spill over from excess demand in the goods market to the aggregate labor supply equation ($\eta_4 = 0$); recall that demand side in the goods market is never constrained due to rationing of households in labor market. Thus effective labor supply (L_s) is always identical to notional labor supply (L_{ws});

$$L_s = L_{ws}(\cdot) + u_5. \tag{5.8}$$

The likelihood function was derived under the assumption that the underconsumption regime has an insignificant role to play, and, therefore, was deleted from the likelihood function. It may be recalled that Underconsumption regime is characterized by excess supply in the goods market and excess demand in the labor market. This curious phenomenon is due to the desire of firms to accumulate inventories for the future. It is important to note that inventory accumulation is included in the aggregate demand function, as the data used for ouput demanded is value added, and therefore, it is impossible to distinguish between output produced for sales and inventory accumulation. Consequently, Underconsumption is indistinguishable from other regimes. Yet another reason for the appearence of this regime is labor hoarding by firms in anticipation of increased demand in future. Though, in theory such a possibility cannot be ruled out, in the empirical realm it may well be insignificant.[3]

Thus, taking into consideration all the amendments proposed above to the prototype model, the estimated variant of the three regimes is given below. The symbols in lower case denote that they are expressed in logarithms.

$$Classical\ Unemployment:$$

$$l = l_{wd} + u_1,$$

$$y = y_{ws} + u_2,$$

$$Y_d = Y_{wd} + u_3 + u_4,$$

$$l_s = l_{ws} + u_5.$$

$$Repressed\ Inflation:$$

$$l_d = l_{wd} + u_1,$$

$$y = y_{ws} - \eta_2(l_d - l) + u_2,$$

$$Y_d = Y_{wd} + u_3 + u_4,$$

$$l = l_{ws} + u_5.$$

$$Keynesian\ Unemployment:$$

$$l = l_{wd} - \eta_1(y_s - y) + u_1,$$

[3] Aruts et. all assigns only two observation and Kooiman and Kloeck none for this regime, while Sneessens (1981) and Vilares (1982) do not have this regime at all. If it is assumed that Underconsumption is non-existent, then we would have to impose different assumptions on the stochastic terms. Truncated error terms have to be introduced which would then impair the tractablity of the likelihood function. All that is assumed is that Underconsumption regime is absent in the data chosen.

$$y_s = y_{ws} + u_2,$$

$$Y = Y_{wd} + u_3 + u_4,$$

$$l_s = l_{ws} + u_5.$$

5.3 Estimation

The maximized likelihood function is provided in Appendix B. The period of estimation is from 1972.1 to 1985.3, and data are seasonally adjusted. Sources of data are provided in Appendix A. The likelihood function was maximized by the quadratic hill-climbing algorithm of the GQOPT4 software package.[4] Both the first and second derivatives were evaluated numerically. Computations were done on AMDHAL 5870 at the University of Manitoba.[5]

[4]A method which requires second derivatives such as Newton Raphson would perform poorly if the initial values are far from the maximum. Hence some modifications have been suggested by and Goldfeld, Quandt and Trotter (1966), and is known is as the quadratic hill-climbing method. Suppose the matrix of second derivatives, $\mathbf{G}$, at any iteration is not negative definite. This would be the case if the initial estimates are far from the optimum, and iterations would then proceed in the "wrong" direction. The quadratic hill–climbing algorithm replaces the non–negative definite $\mathbf{G}$ by $\mathbf{G} - \alpha\mathbf{I}$ where α is chosen so that $\mathbf{G} - \alpha\mathbf{I}$ is negative definite, and $\alpha > \lambda_{max}$, where λ_{max} is the largest eigenvalue of $\mathbf{G}$, when $\mathbf{G}$ is not negative definite. The rationale for this is sought in the properties of quadratic functions (Goldfeld and Quandt: 1972 and Quandt: 1983).

[5]For the large scale optimization problem of the type handled here (there are 22 parameters), it soon became apparent that it is very important to have initial estimates which are as close to the local optimum as possible. In order to obtain the initial estimates, the strategy adopted was as follows. We split the two–market model to two single markets. Since the number of parameters to be estimated for each of the two markets independently was not large, poor initial estimates did not vitiate the optimization process. Subsequently, the two markets were estimated together, this time with the inclusion of the spill–over and rationing coefficients. The estimates from the independent estimation of the single markets were used as the initial estimates, and since there are only two spill–over

Both dependent and independent variables were scaled for the purpose of estimation and the parameter estimates were rescaled to conform to its original units.[6] The optimization took between two to five minutes of CPU time depending on the initial value of the estimates. Perturbation of the initial values realized the same maximum. Though the maxima from different initial values were the same, the t-values were very volatile from run to run. The t-values were reliable only in one case, while in others they were all unreliable due to inaccuracy in the calibration of first and second derivatives. This may well be due to flatness of likelihood function in the vicinity of the optimum.[7] The results presented are for the maximum for which the t-values are reliable.

---

coefficients and one rationing coefficient, only three values of the initial estimates had to be guessed. In the next stage, we enlarged our model, to accommodate the consumption function, for until now estimation was done using an aggregate ouput demand function. In the next stage, the ouput demand equation was further disaggregated to include an equation for export demand. Therefore, at each stage we had to guess at most only the values of four initial estimates, for the remaining initial values were ones from the local optimum of the previous stage. It should be borne in mind that at every stage of the optimization a different likelihood function had to be maximized.

[6]The transformation employed for scaling variables can be written as $X = \tilde{X}d+s$, where $\tilde{X}$ is the scaled variable, $d = (\max X - \min X)\frac{1}{2}$, $s = (\max X + \min X)\frac{1}{2}$ and $\max X$ and $\min X$ are the maximum and minimum values of X. The transformation ensures that $\tilde{X}$ values are in the range $[-1, +1]$. First and second derivatives are also now scaled, and the effect on the convergence rate of the optimization is substantial. Also, since the likelihood function has cumulative distribution function $\Phi(\cdot)$ as one its arguments, the transformed function is often much better than the original function (Gill, Murray and Wright: 1981, pp.274-275).

[7]Failure of the numerical approximation to the Hessian "is most frequent in cases where the function is extremely flat. It is not clearly not an acceptable outcome in any event, but particularly in the case of maximum likelihood estimation for then the negative inverse of the Hessian is used as an estimate of the asymptotic covariance matrix".(Quandt: 1983, pp. 734-735)

Specifications of equations (5.1) and (5.2) for estimation purposes are as follows. The notional aggregate output supply behavior is explained using a Cobb-Douglas production function and notional aggregate labor demand is derived by equating marginal product of labor to the real wage rate. The parameter estimates and t–values (in parenthesis beneath the parameter estimates) are

$$\log(L_{wd}) = \underset{(22.60)}{0.121} - \underset{(8.69)}{0.477}\log(w_g) + \underset{(32.92)}{0.780}\log(K) \tag{5.9}$$

and

$$\log(Y_{ws}) = \underset{(0.085)}{-0.751} + \underset{(0.042)}{0.412}\log(L) + \underset{(52.75)}{0.835}\log(K). \tag{5.10}$$

w_g is the nominal wage rate divided by the GNP deflator, K is the capital stock and L employment. The statistical insignificance of the labor coefficient may be attributed to the absence of large variation in employment between booms and busts due to labor hoarding by firms. This speculation receives some confirmation from the estimated value of η_1 (see below). Magnitude of η_1 is indicative of labor hoarding by firms during recessions. Labor demand does not fall to extent warranted by the excess supply in the goods market. The standard error of the random terms u_1 and u_2 are

$$\sigma_1 = \underset{(5.35)}{0.187} \; and \; \sigma_2 = \underset{(8.60)}{0.048}.$$

We assume that households never experience rationing in the goods market, an empirically reasonable proposition, due to imports rising to bridge the gap, should consumers desire to demand more than domestic production. Actual transactions C, therefore, is always equal to desired transactions C_d. Desired consumption is specified to be dependent on the level of disposable income, and the outcome of the estimation is

$$C_d = \underset{(184.84)}{34.545} + D(L)\underset{(39.72)}{0.699}DI. \tag{5.11}$$

$D(L) = 0.4 + 0.3L + 0.2L^2 + 0.1L^3$. Estimate of the marginal propensity to consume is a trifle low. The oft-quoted Canadian estimates are in the neighborhood of 0.85. The standard error for the random term u_3,

$$\sigma_3 = \underset{(8.36)}{40.20.}$$

Specification of consumption as a function of disposable income is bound to lead to some statistical inconsistency. In Keynesian Unemployment regime quantity traded in the goods market is aggregate demand and, therefore, disposable income is no longer exogenous as in other regimes. The inconsistency stems from deletion of the inverse of Keynesian multiplier in the jacobian factor in the likelihood function that pertains to Keynesian Unemployment regime. This may be unavoidable in the present context, as the solution to this problem points in the direction of modeling the tax and social security system, which in our judgement is not advisable. The likelihood function becomes even more complex and may render it intractable.[8]

[8]Artus et al (1984) has wage income endogenous through its dependency on employment, and transfer income exogenous. This may not be quite satisfactory as transfer

Export demand employs a well-worn specification; a distributed lag of the ratio of the export prices and import prices and world demand determine desired exports. The lag polynomial D(L) is a priori fixed as $0.4 + 0.3L + 0.2L^2 + 0.1L^3$.

$$X_d = \underset{(77.05)}{-763.59} - \underset{(2.43)}{424.50}\Delta(D(L)\log(TOT)) + \underset{(31.28)}{435.16}\log(WD) \tag{5.12}$$

and

$$\sigma_4 = \underset{(5.59)}{11.5}.$$

Labor supply is specified along the lines of Rosen and Quandt (1978).

$$\log(L_{ws}) = \underset{(4.25)}{1.720} + \underset{(4.18)}{0.654}\log(w_c) + \underset{(4.36)}{0.418}\log(LF), \tag{5.13}$$

where w_c is the nominal wage rate deflated by the CPI and LF the labor force and

$$\sigma_5 = \underset{(5.23)}{0.366}.$$

Coherency conditions enunciated by Gourieroux et al. (1981a and 1981b) and Ito (1981) require that $\eta_1\eta_2 < 1$, and that is indeed the case;

$$\eta_1 = \underset{(8.73)}{0.845} \; and \; \eta_2 = \underset{(12.51)}{0.666}.$$

And finally the estimate for the rationing coefficient

$$\theta = \underset{(1.54)}{0.139}.$$

income is also dependent on the level of income. Treating wage income as dependent on employment and transfer income as independent of employment, the multiplier effect is correspondingly overestimated. See Kooiman and Kloeck (1985).

	CU	RI	KU
721	000	000	100*
722	000	000	100*
723	000	000	100*
724	004	000	096*
731	094*	000	006
732	096*	004	000
733	047	053*	000
734	057*	043	000
741	024	076*	000
742	001	099*	000
743	091*	009	000
744	093*	007	000
751	041	000	059*
752	061*	000	039
753	002	000	098*
754	060*	000	040
761	100*	000	000
762	100*	000	000
763	100*	000	000
764	100*	000	000
771	100*	000	000
772	100*	000	000
773	100*	000	000
774	100*	000	000
781	100*	000	000
782	100*	000	000
783	100*	000	000
784	100*	000	000
791	100*	000	000
792	100*	000	000
793	098*	002	000
794	093*	007	000
801	094*	006	000
802	027	073*	000
803	000	016	084*
804	022	050*	028
811	036	062*	002
812	045	050*	005
813	002	098*	000
814	000	047	053*
821	000	003	097*
822	000	000	100*
823	000	000	100*
824	000	000	100*
831	000	000	100*
832	000	009	091*
833	000	099*	001
834	000	085*	015
841	000	100*	000
842	000	100*	000
843	000	100*	000
844	000	100*	000
851	000	100*	000
852	000	100*	000
853	000	100*	000

Table 5.1: Regime Probabilities

The parameter estimates at the optimum value of the log-likelihood function can be used to generate probabilities for the existence of regimes for each of the observations and is given in table 3.[9] The $*$ indicates the most probable regime. Classical Unemployment is the most prevalent regime (24 observations), followed by Repressed Inflation (17 observations) and Keynesian Unemployment (14 observations). From 1975.4 to 1980.1 there is an uninterrupted dominance of Classical Unemployment regime with probabilities either one or very close to one. For the Canadian economy, 1975 marks the beginning of a rise in unemployment rates until it hits the high level of 8% in 1978. Unemployment starts to fall thereafter, and hits the low rate of 6.5% in 1981. The period was characterized by high inflation rates, caused by the second round of OPEC output restrictions, a supply shock, and an expansion of money supply due to the reduction in the demand for M1 balances, a demand shock. The period 1973-1974 alternates between Classical Unemployment and Repressed Inflation regimes with five observations assigned to the first. While the period witnessed high inflation rates as indicated by the model, it also was a period of low unemployment rates. The consequence of the oil-shock of 1974 is detected as Keyensian Unemployment in 1975.1 and 1975.3 and Classical Unemployment in 1975.2 and 1975.4. The recession caused by a restrictive monetary policy initiated in 1981 is indicated from 1981.3 to 1983.2. From the latter half of 1983 to end of sample period, Re-

[9]The conditional probabilities of each observation is equal to $\mathrm{Pr}_i = H_i/H; i \in (cu, ri, ku)$, where H is the probability density function. See Appendix for details.

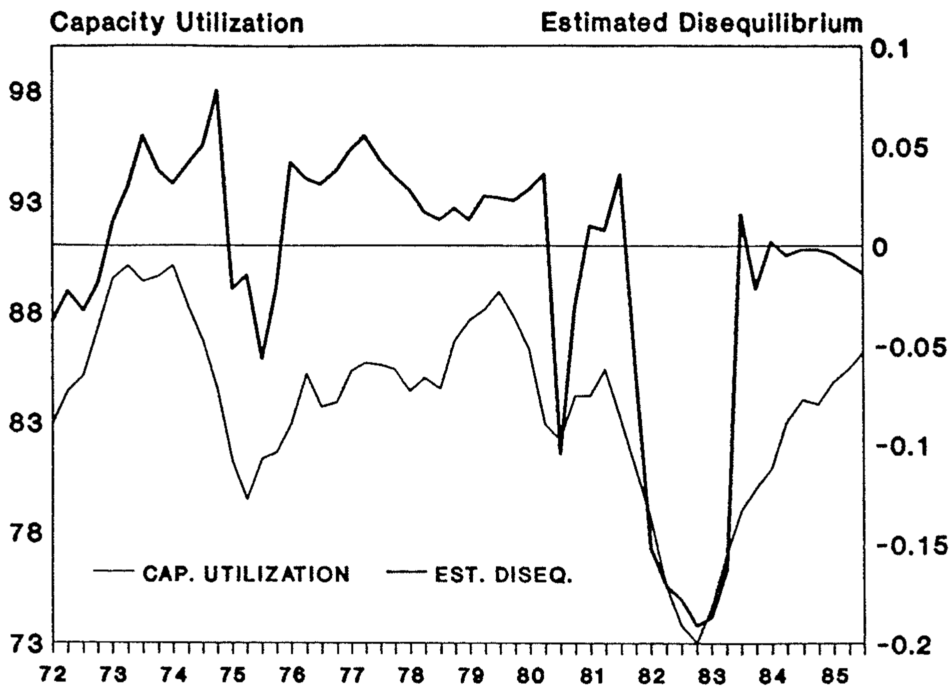


Figure 5.1: Capacity Utilization and Estimated Disequilibrium (Goods)

pressed Inflation is the most probable regime. In early 1984, the inflation rate fell to 5%, and thereater stabilized at around 4%. As regards unemployment, from a peak of 12.8% in the late 1982, there has been a steady downward trend. Therefore while excess demand for labor as indicated by the model may be quite realistic, the excess demand for goods suggested is not borne out by facts. The model is, by and large, able to classify regimes accurately, except for 1972. Keynesian Unemployment is the regime assigned to 1972. This was by no means a period of recession; unemployment rates were low and inflation rates high, a period of expansionary fiscal and monetary policy.

Yet another method available to assess the predictive potential of the

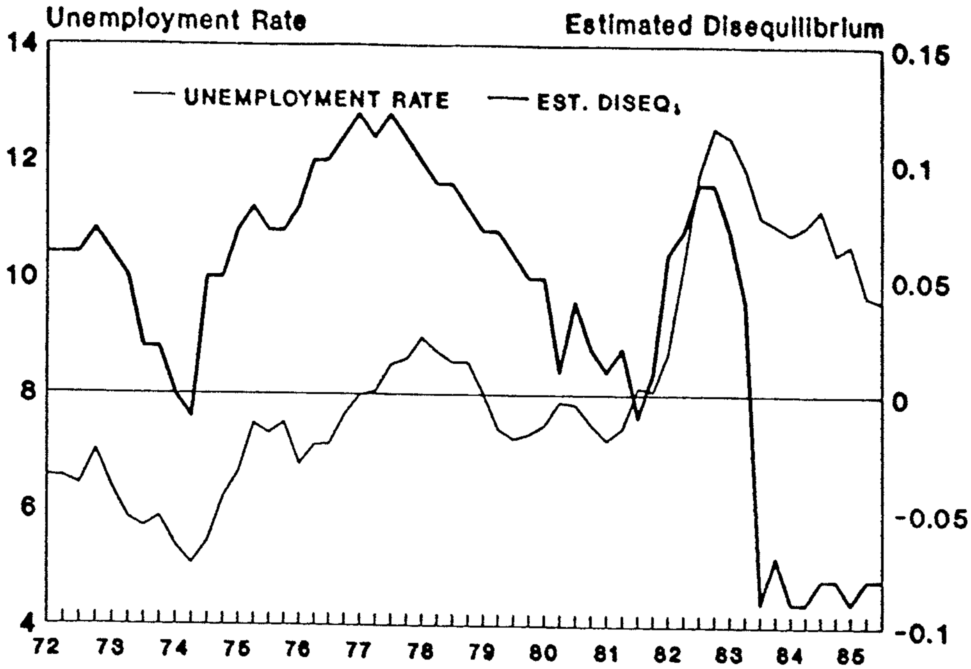


Figure 5.2: Unemployment and Estimated Disequilibrium (Labor)

model is to compare the capacity utilization of the economy with disequilibrium in the goods market and unemployment with disequilibrium in the labor market. Such a measure of disequilibrium for the goods market is obtained by calculating $(E(Y_d)-Y)\Pr(CU)+(E(Y_d)-Y)\Pr(RI)+(Y-E(Y_s))\Pr(KU)$ for each of the observations. Similarly a measure of disequilibrium in the labor market is $(E(L_s)-L)\Pr(CU)+(L-E(L_d)\Pr(RI)+(E(L_s)-L)\Pr(CU)$. A plot of the actual and predicted measures of disequilibrium in the goods and labor market is given in figures 5.1 and 5.2. The correspondence between predicted excess demand and capacity utilization rates is good, except for the periods 1979-1980 and 1983-1985. As for the labor market, the co-movements of unemployment rates and excess supply breaks down for 1983-1985. Regime classification assigns the latter period to Repressed Inflation unequivocally, and the comparison of the actual and predicted suggests that it may well be incorrect. Zero probabilites for other regimes do not reflect reality.

Minimum conditions require that excess demands and excess supplies be non-negative, and so must be their estimated counterparts. This is in fact true for all the estimated equations, except for labor demand under Keynesian Unemployment, as can be seen from figures 5.3 and 5.4. L_d for that regime is lower than L throughout the period of estimation. In order to correct for this unacceptable situation, we estimated the labor demand with cross restriction with the production function. The parameter estimates for the production function became implausible, and the regime classification absurd. We surmise that specification of labor demand require further thought and investigation.

5.4 Conclusion

A small disequilibrium macroeconomic model was estimated. It was found that the likelihood function is very sensitive to the initial estimates. Poor initial estimates may lead the iterations to a local optimum where the values of the parameter estimates are implausible. The problem seems to be the the presence of multiple optima. Specification searches are virtually ruled out; for with a given specification, it may take unreasonably large amount of real time, not to speak of CPU time, to optimize the function. Coupled

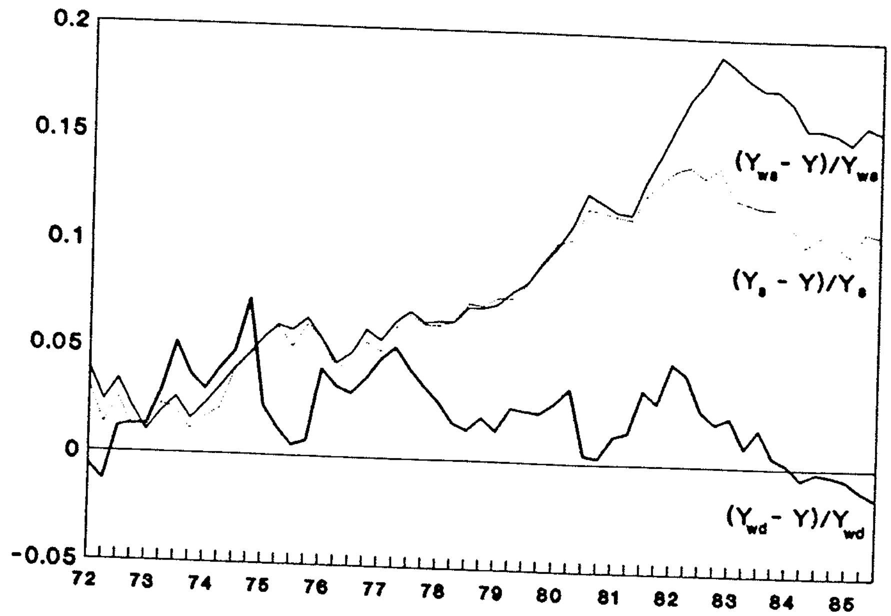


Figure 5.3: Estimated Trade Offers and Trade in the Goods Market

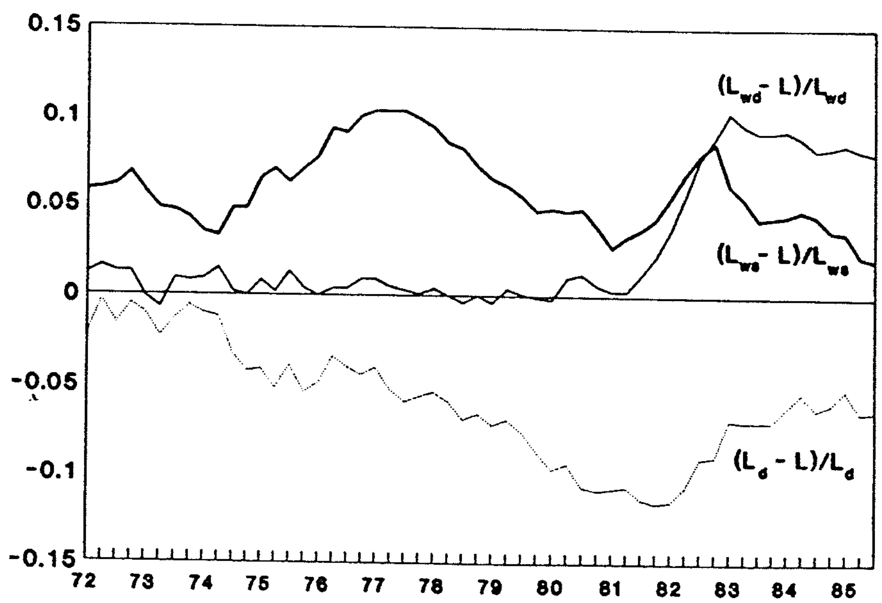


Figure 5.4: Estimated Trade Offers and Trade in the Labor Market

with that, there is the problem of the flat likelihood function. In the case of the likelihood function, this is particularly distressing, as was noted above, for the standard error of the parameter estimates are inaccurate. This has also been noticed in other studies (Kooiman and Kloeck: 1985 and Goldfeld and Quandt: 1975).

In spite of all these problems, it is gratifying to note that we were able to obtain estimates and regime classification which are quite reasonable. The magnitude of spill–over coefficients are not only as one would expect, but also statistically significant, therby providing some proof of the utility of disequilibrium theory in modeling business cycles. The model simulates the Canadian business cycle quite well for a large part of the sample period.

Appendix A: Glossary of Terms and sources of data

C: Personal expenditure on goods and services at 1981 prices, billions of dollars (Cansim: D 20032)

DI: Personal disposable income at 1981 prices, billions of dollars (Cansim: D 10111)

G: Government expenditures on goods and services at 1981 prices, billions of dollars

I: Investment expenditures at 1981 prices, billions of dollars

K: Gross Capital stock excluding housing, at 1971 prices, quarterly figures calculated from mid-year figures by smooth interpolation (Statistics Canada, 13-211)

L: Employment for both sex sexes, 15 years and over, in thousands (Cansim: D 767608)

LF: Labor Force, 15 years and over, in thousands (Cansim: D 767606)

M: Import of goods and services at 1981 prices in billions of dollars (Cansim: D 20048)

UNEMP: Unemployment rate

TOT: Price index of exports at 1981 prices (Cansim: B 1300) $\div$ Price index of imports at 1981 prices (Cansim: B 1350)

w_g: (Nominal wage rate (Statistics Canada, 11-206) $\div$ GNP deflator at 1981 prices)(Cansim: D 20337)$\times$ 100

w_c: (Nominal wage rate (Statistis Cananda, 11-206) $\div$ Consumer Price Index)(Cansim: D 20338)$\times$ 100

WD: Index of industrial production of OECD countries (Selected Economic Statistics, OECD)

X : Exports of goods and services at 1981 prices billions of dollars (Cansim: D 20044)

Y_{wd}: GDP at 1981 prices, billions of dollars (Cansim: D 20031)

Y_{ws}: GDP at 1981 prices, millions of dollars (Cansim: D 20031)

CUR: Capacity Utilization Rates (Cansim: B 6000)

Appendix B: The Likelihood Function

The likelihood function is derived on the assumption that the errors u_i, i=1, 2,....,6 are independently normally distributed with zero means and constant variances, σ_i^2. There is also no correlation between the error terms so that it is sufficient to derive the likelihood of a single observation. The endogenous variables are Y, C, X and L. Variables in lower case denote that they appear in logarithms in the maximization process. The joint density is given by H=H_{cu}+H_{ri}+H_{ku} where

$$H_{cu} = \int_{l}^{\infty}\int_{y}^{\infty} g_{cu}(l, y, X, l_{ws}, Y_{wd})dY_{wd}dl_{ws}$$

$$H_{ri} = \int_{l}^{\infty}\int_{y}^{\infty} g_{ri}(l, y, X, l_{wd}, Y_{wd})dY_{wd}dl_{wd}$$

$$H_{ku} = \int_{l}^{\infty}\int_{Y}^{\infty} g_{ku}(l, Y, X, l_{ws}, y_{ws})dy_{ws}dl_{ws}$$

$g_{cu}(\cdot)$, $g_{ri}(\cdot)$ and $g_{ku}(\cdot)$ are the joint density function of the endogenous variables. The following residuals are defined. $e_1 = l - l_{dw}(w_g)$, $e_2 = y - f(l_{dw}(w_g))$, $e_3 = Y - Y_{wd}(\cdot) - e_4$, $e_4 = X - X_d(\cdot)$ and $e_5 = l - l_{ws}(\cdot)$. Let $n(\cdot, \mu, \sigma^2)$ and $N(\cdot, \mu, \sigma^2)$ be normal density and distribution functions with mean μ and variance σ^2 and let $\phi(\cdot) = n(\cdot; 0, 1)$ and $\Phi(\cdot) = N(\cdot; 0, 1)$.

Classical Unemployment

$$l = l_{wd}(w_g) + u_1 \tag{5.14}$$

$$y = f(l_{wd}(w_g)) + u_2 \tag{5.15}$$

$$X = X_d - (1-\theta)(Y - Y_{wd}(\cdot)) - (1-\theta)u_3 + \theta u_4 \tag{5.16}$$

$$Y_{wd} = Y_{wd}(\cdot) + u_3 + u_4 \tag{5.17}$$

$$l_{ws} = l_{ws}(\cdot) + u_5 \tag{5.18}$$

The joint density function of $g_{cu}(l, y, X, l_{ws}, Y_{wd})$ can be factorized as

$$g_{cu}(\cdot) = g^1_{cu}(l)g^2_{cu}(y)g^3_{cu}(l_{ws})g^4_{cu}(X)g^5_{cu}(Y_{wd}|X).$$

The first two factors are $(\sigma_1\sigma_2)^{-1}\phi(e_1/\sigma_1)\phi(e_2/\sigma_2)$and the third factor is $n(l_{ws}; l_{ws}, \sigma_5)$. Integrating for $l_{ws} > l$ yields $1 - \Phi(e_5/\sigma_5)$. The fourth factor is obtained as $\sigma_{cu}^{-1}\phi(e_{cu}/\sigma_{cu})$ where e_{cu}= $(1-\theta)e_3 - \theta e_4$ and σ_{cu} as $(1-\theta)^2\sigma_3{}^2 + \theta^2\sigma_4^2$. $g^5_{cu}(Y_{wd}|X)$is

$$n\Big(Y_{wd}; Ywd(\cdot) + \frac{((1-\theta)\sigma_3^2 - \theta\sigma_4^2)e_{cu}}{\sigma^2_{cu}}, \frac{\sigma_3^2\sigma_4^2}{\sigma^2_{cu}}\Big).$$

Integrating for $Y_{wd} > Y$ yields

$$1 - \Phi\Big(\frac{\theta\sigma_4^2 e_3 + (1-\theta)\sigma_3^2 e_4}{\sigma_3\sigma_{cu}\sigma_4}\Big).$$

Collecting terms

$$H_{cu} = (\sigma_1\sigma_2\sigma_{cu})^{-1}\phi(e_1/\sigma_1)\phi(e_2/\sigma_2)\phi(e_{cu}/\sigma_{cu}) \times (1 - \Phi(e_5/\sigma_5)) \\ \times \left(1 - \Phi\Big(\frac{\theta\sigma_4^2 e_3 + (1-\theta)\sigma_3^2 e_4}{\sigma_3\sigma_{cu}\sigma_4}\Big)\right).$$

Repressed Inflation

$$l_{wd} = l_{wd}(w_g) + u_1 \tag{5.19}$$

$$y = f(l_{wd}(w_g)) + \eta_2(l - l_{wd}(w_g)) - \eta_2 u_1 + u_2 \tag{5.20}$$

$$X = X_d - (1-\theta_2)(Y - Y_{wd}(\cdot)) - (1-\theta)u_3 + \theta u_4 \tag{5.21}$$

$$Y_{wd} = Y_{wd}(\cdot) + u_3 + u_4 \tag{5.22}$$

$$l = l_{ws}(\cdot) + u_5 \tag{5.23}$$

Factorizing the joint density function $g_{ri}(l, y, X, Y_{wd}, L_{wd})$ as

$$g_{ri}(\cdot) = g^1_{ri}(l) g^2_{ri}(y) g^3_{ri}(X) g^4_{ri}(l_{wd}|y) g^5_{ri}(Y_{wd}|X),$$

we obtain $g^1_{ri}(l)$ as $\sigma_5^{-1}\phi(e_5/\sigma_5)$. As (20) and (21) identical to Classical Unemployment, they result in the same expression. $g^2_{ri}(y) = \sigma_{ri}^{-1}(e_{ri}/\sigma_{ri})$ where $e_{ri} = u_2 - \eta_2 e_1$ and $\sigma^2_{ri} = \eta_2^2\sigma_1^2 + \sigma_2^2$. Finally $g^4_{ri}(l_{wd}|y)$ is obtained as

$$n\Big(l_{wd}; l_{wd} - \frac{\eta_2\sigma_1^2 e_{ri}}{\sigma^2_{ri}}, \frac{\sigma_1^2\sigma_1^2}{\sigma^2_{ri}}\Big)$$

Integrating for $l_{wd} > l$, we get

$$1 - \Phi\Big(\frac{\sigma_2^2 e_1 + \eta_2\sigma_1^2 e_2}{\sigma_1\sigma_2\sigma_{ri}}\Big).$$

Collecting terms,

$$H_{ri} = (\sigma_5\sigma_{cu}\sigma ri)^{-1}\phi(e_5/\sigma_5)\phi(e_{cu}/\sigma_{cu})\phi(e_{ri}/\sigma_{ri})$$
$$\times 1 - \Phi\Big(\frac{\theta\sigma_4^2 e_3 + (1-\theta)\sigma_3^2 e_4}{\sigma_3\sigma_{cu}\sigma_4}\Big) \times 1 - \Phi\Big(\frac{\sigma_2^2 e_1 + \eta_2\sigma_1^2 e_2}{\sigma_1\sigma_2\sigma_{ri}}\Big).$$

Keynesian Unemployment

$$l = l_{wd}(w_g) + \eta_1(y - f(l_{wd}(w_g)) + u_1 - \eta_1 u_2 \tag{5.24}$$

$$y_{ws} = f(l_{wd}(w_g)) + u_2 \tag{5.25}$$

$$X = X_d(\cdot) + u_3 \tag{5.26}$$

$$Y = Y_{wd}(\cdot) + u_3 + u_4 \tag{5.27}$$

$$l_{ws} = l_{ws}(\cdot) + u_5 \tag{5.28}$$

Factorizing the joint density function $g_{ku}(l, y, X, \mathrm{Y}_{ws}, l_{ws})$ as

$$g_{ku} = g^1_{ku}(l)g^2_{ku}(Y)g^3_{ku}(X)g^4_{ku}(y_{ws}|l)g^5_{ku}(l_{ws}),$$

we obtain the second and third factors as $(\sigma_3\sigma_4)^{-1}\ \phi(e_3/\sigma_3)\phi(e_4/\sigma_4)$ and the last factor when integrated for $l_{ws} > l$ is the same as in Classical Unemployment. We obtain $g^1_{ku}(l)$ as $\sigma^{-1}_{ku}\phi(e_{ku}/\sigma_{ku})$, where $e_{ku} = e_1 - \eta_1 e_2$ and $\sigma^2_{ku} = \sigma^2_1 + \eta^2_1\sigma^2_2$. Finally, $g^5_{cu}(y_{ws}|l)$ is derived as

$$n\Big(Y_{ws}; f(l_{wd}(w_g)) + \frac{\eta_1\sigma^2_2 e_{ku}}{\sigma^2_{ku}}, \frac{\sigma^2_1\sigma^2_2}{\sigma^2_{ku}}\Big).$$

Integrating for $y_{ws} > y$, we have

$$1 - \Phi\Big(\frac{\eta_1\sigma^2_2 e_1 + \sigma^2_1 e_2}{\sigma_1\sigma_2\sigma_{ku}}\Big).$$

Therefore,

$$\begin{aligned} H_k u = {} & (\sigma_3\sigma_4\sigma_{ku})^{-1}\phi(e_3/\sigma_3)\phi(e_4/\sigma_4)\phi(e_{ku}/\sigma_{ku}) \\ & \times 1 - \Phi(e_5/\sigma_5) \times 1 - \Phi\Big(\frac{{\sigma_2}^2 e_1\eta_1 + {\sigma_1}^2 e_2}{\sigma_1\sigma_2\sigma_{ku}}\Big) \end{aligned}$$

Chapter 6

Conclusion

The objective of this dissertation was to estimate a disequilibrium macroeconomic model. The motive behind this goal was two-fold. One was to estimate a disequilibrium model in the specific context of Canadian data, and the second was to investigate the efficacy and appropriateness of the existing methodologies. At the time of the inception of the research, two methods of estimation were available to the investigator. They were the Full-Information Maximum Likelihood approach of Maddala and Nelson (1974) and the Mathematical Programming Approch of Ginsburgh, Tishler and Zang (1980). Since then another appraoch has made its appearence, the Pseudo-Maximum Likelihood approach of Laroque and Salanie (1989).

For reasons outlined in Chapter IV, the Full-Information Maximum Likelihood was found superior, and hence was adopted for estimation. This entailed the derivation of a likelihood function. Earlier studies of Kooiman and Kloeck (1984) and Artus et al.(1985) greatly fecilitated the present author in his own formulation of the likelihood function. The maximization of the likelihood function (using the numerical optimization programme GQOPT4) proved to be a very difficult task. But we are happy to report that an unique local optimum was finally obtained.

Kooiman and Kloeck (1985) and Artus et al. (1984) and the present study agree that FIML is problematic because maximization of the likelihood function involved is no mean task. The Pseudo–Maximum Likelihood technique of Laroque and Salanie (1989) was devised to overcome the problems of both the Ginsburgh et al. and Maddala and Nelson (FIML) approaches, and may very well be the methodology of the future. Laroque and Salanie report that their approach has none of the problems encountered by the Maddala and Nelson approach. Ample proof of this assertion is forthcoming not only in Laroque and Salanie (1989) where they estimate a disequilibrium macroeconomic model via three Pseudo–Maximum Likelihood techniques, but also in Laroque (1989), where the ambitious project of estimating for four countries (France, Germany, the U.K. and the U.S.A.) is successfully undertaken.

References

- Amemiya, T., "The Solveability of a Two–market Disequilibrium Model," Working Paper No. 82, IMSSS, Stanford University, 1977.

- Artus, P., G. Laroque and G. Michel, "Estimation of Quarterly Macroeconomic Model with Quantity Rationing," Econometrica 52, 1984

- Barro, R. J., and H. I. Grossman, "A General Disequilibrium Model of Income and Employment," American Economic Review 61, 1971.

- Benassy, J. P., "Neo–Keyensian Disequilibrium Theory in a Monetary Economy," Review of Economic Studies 42, 1975.

- Benassy, J. P., Macroeconomics An Introduction to the Non–Walrasian Approach (Academaic Press, 1986).

- Benassy, J. P., "Disequilibrium Analysis" in J. Eatwell, M, Wingate and P. Newman, eds., New Palgrave Dictionary of Economics, Vol.1 (1987)

- Blaug, M., Economic Theory in Retrospect (Cambridge, 1985)

- Clower, R. J., "The Keynesian Counter-revolution: A Theoretical Appraisal," in F. H. Hahn and F. Brechling, eds., The Theory of Interest Rate (Macmillan, London, 1965), Ch. 5.

- Coen, R. M., and B. G. Hickman,"Keynesian and Classical Unemployment in Four Countries," Brooking Papaers on Economic Activity, 1987
- Drazen, A., "Recent Developments in Macroeconomic Disequilibrium Theory," Econometrica 48, 1980.
- Drèze, J. H., "Existence of Exchange Equilibrium under Price Rigidities," International Economic Review 16, 1975.
- Eves, H., An Introduction to the History of Mathematics (Holt, Rinehart and Winston, 1976)
- Gill, E. P., W. Murray, and M. H. Wright, Practical Optimization (Academic Press, 1981)
- Ginsburgh, V., A. Tishler and I. Zang, "Alternative Estimation Methods for Two Regime Models: A Mathematical Programming Approach," European Economic Review 13, 1980.
- Goldfeld, S. M. and R. E. Quandt, Nonlinear Methods in Econometrics (North-Holland, 1972)
- Goldfeld, S. M. and R. E. Quandt, "Estimation of a Disequilibrium Model and the Value of Information," Journal of Econometrics 3, 1975.
- Goldfeld, S. M., R. E. Quandt and H. F. Trotter, "Maximization by Quadratic Hill Climbing," Econometrica 34, 1966

- Gourieroux, C., J-J. Laffont and A. Monfort, "Disequilibrium Econometrics in Simultaneous Equation System," Econometrica 48, 1980a

- Gourieroux, C., J-J. Laffont and A. Monfort, "Coherency Conditions in Simultaneous Equations Models with Endogenous Switching Regimes," Econometrica, 48, 1980b

- Gourieroux, C., A. Montfort, and A. Trognon, "Pseudo Maximum Likelihood Methods: Theory," Econometrica 52, 1984

- Gradmont, J-M, "The Logic of Fix Price Method," Scandinavian Journal of Economics 79, 1977

- Green, J., "On the Theory of Effective Demand," Economic Journal 90, 1980

- Greenstadt, J., "On the Relative Efficiencies of Gradient Methods," Mathematics of Computation 21, 1967

- Hahn, F. H. and T. Negeshi, "Theorem on Non-Tatonnement Stability," Econometrica 30, 1962

- Hansen, B., A Study on the Theory of Inflation (Allen and Unwin, London, 1951)

- Hendry, D. F., "Whither Disequilibrium Econometrics," Econometric Reviews 1(1), 1982.

- Hicks, J. R., "Mr. Keynes and the Classics," Econometrica, 2, 1937.
- Hicks, J. R. Capital and Growth (Oxford University Press, 1965)
- Ito, T., "Methods of Estimation for Multi-Market Disequilibrium Models," Econometrica 48, 1980.
- Keynes, J. M., "Alternative Theories on Rate of Interest," Economic Journal 47, 1937.
- Kooiman, P., "Smoothing Aggregate Fix Price Model and Use of Business Survey Data," Economic Journal 94, 1984
- Kooiman, P. and T. Kloek, "An Empirical Two Market Disequilibrium Model for Dutch Manufacturing," European Economic Review 29, 1985.
- Laffont, J-J, "Fix Price Models: A Survey of Recent Empirical Work," in Arrow and Hankapohja, eds., Frontiers in Economics (Oxford, 1985)
- Lambert, J-P., Disequilibrium Macroeconomic Models (Cambridge University Press, 1988)
- Laroque, G., "The Fix–Price Equilibria: Some Results in Local Comparative Statics," Econometrica 46, 91978.
- Laroque, G., "Comparative Estimates of a Macroeconomic Disequilibrium Model," European Economic Review 33, 1989.

- Laroque, G. and B. Salanie, "Estimation of Multi-Market Fix-Price Models: An Application of Psuedo Maximum Likelihood Methods," Econometrica 57, 1987.
- Maddala, G.S. and F. D. Nelson, "Maximum Likelihood Methods for Models of Market Disequilibrium," Econometrica 42, 1974.
- Malinvaud, E., The Theory of Unemployment Reconsidered (Blackwell, Oxford, 1977).
- Muellbauer. J., and R. Portes, "Macroeconomic Models with Quantity Rationing," The Economic Journal 88, 1978.
- Patinkin, D., Money, Interest and Prices, 2nd. ed. (New York, 1965).
- Quandt, R. E. "Maximum Likelihood Estimation of Disequilibrium Model," in T. Bagliotti and G. Franco, eds., Pioneering Economics (Edizoni Cedam, Padova, 1978)
- Quandt, R. E., "Econometric Disequilibrium Models," Econometric Reviews 1(1), 1982
- Quandt, R. E., "Computational Problems and Methods," in Z. Griliches and M. D. Intrilligator, Handbook of Econometrics (North-Holland, 1983) Chap. 12.

- Rosen, H. S. and R. E. Quandt, "Estimation of a Disequilibrium Aggregate Labor Market," The Review of Economics and Statistics, 60, 1978
- Samuelson, P. A., Foundations of Economic Analysis (Harvard University Press, 1947)
- Sneessens, H. R., Theory and Estimation of Macroeconomic Rationing Models (Springer Verlag, New York, 1981)
- Sneessens, H. R., "A Macroeconomic Rationing Model of the Belgian Economy," European Economic Review 20, 1983
- Sneessens H. R. and J. H. Dreze, "A Discussion of Belgian Unemployment, Combining Traditional Concepts and Disequilibrium Econometrics,"
- Svensson, L., "Effective Demand and Stochastic Rationing," Review of Economic Studies 47, 1980. Economica 53, 1986.
- Tishler, A. and I. Zang, "A Switching Regression Method Using Inequality Conditions," Journal of Econometrics 11, 1989
- Tobin, J. and H. S. Houthakker, "The Effect of Rationing on Demand Elasticities," Review of Economic Studies 18, 1950
- Vilares, M. J., "A Macroeconometric Model with Structural Change and Disequilibrium," Mimeo, Faculty of Economics, Porto, 1982.

- Vilares, M. J., Structural Change in Macroeconomic Models: Theory and Estimation (Martinus Nijhoff, 1986).
- Weinrich, G., "On the Theory of Effective Demand under Stochastic Rationing," Journal of Economic Theory 34, 1984