PARENTS' PERCEPTIONS OF MATHEMATICS EDUCATION: ALIGNING METHODS OF INSTRUCTION

By

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A Thesis
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I dedicate this work to my loving parents. To my late father, Wilfred William Watt, whose wisdom and patience continue to be a part of my journey; and to my mother, Reine, always loving, caring and encouraging.

Abstract

This thesis explored the nature of the learning that occurred as parents attuned their perceptions of mathematics instruction and learning for themselves and their children and aligned their practices with that of the current methodology espoused in the Manitoba mathematics curriculum and by the National Council of Teachers of Mathematics.

Parents' learning was interpreted and analyzed through phenomenographic research methods. A four-part intervention program became the phenomenon that provided the stimulus for learning. Data was collected by field notes, observation, participant notes and through interviews.

The limited number of "qualitatively different and logically interrelated ways" (Marton, 1997, p. 97) that participants understand a phenomenon was classified according to categories of description. Participants perceived the program on a personal level, as learners who have taken on procedures for learning mathematics and working with their children, and as learners who have taken on conceptual knowledge of mathematics education. The degrees of awareness represented by the collective bank of data suggested that participants took on and internalized their learning with varying depth. Sometimes ideas from the program were valued but when voiced by the participants were lacking a sense of ownership. Other perceptions were stated with a sense of ownership, but varied little from the original presentation. Another level suggested that participants were able to transform concepts from the program into unique and personal ideas. Individuals' learning was characterized by Piaget's theories regarding assimilation and accommodation.

This research study has shown that parents' perceptions of math education and their methods of assistance to their children can be brought into alignment with current mathematics education methodology. Through experiencing the processes of communicating, connecting, reasoning and problem solving, participants have become proponents of these practices.

Parents' Perceptions of Math Education: Aligning Methods of Instruction

Chapter 1

Home – School Misalignment: Expressing the Need

Amidst the busy schedule of any middle school is a desire to offer parents a greater role in the education of their children. These noble intentions often become clouded in a myriad of other responsibilities of the classroom teacher and the school. The role that parents play in the education of their children is profound; yet many lack the resources to assist their children particularly in their mathematical studies. Assistance with homework assignments is often grounded in a traditional methodological view of instruction. This may leave a misalignment between the intent of the current mathematics classroom and the intent of the parent's assistance. Lehrer and Shumow (1997) describe a possible remedy in *Aligning the Construction Zones of Parents and Teachers for Mathematics Reform.*

> Providing parents with an opportunity to acquire knowledge about their child's mathematical thinking would serve to align the construction zones of parents and teachers… (a perspective that) suggests that with greater knowledge of children's thinking, parents could assist children in a manner more like that of teachers, in part because knowing about their children's thinking would enable them to provide less direct forms of assistance (p. 66).

As mathematics teachers, we learn about how children think through listening to their reasoning. We also gather data about learning through repeated experiences, as well as by reading research and participating in teacher inservices. Parents are not privy to these resources. Their view of learning and thus teaching will be different from that of the classroom teacher and the education field in general. As such they may vocalize "reservations about the amount of talk during mathematics, the lack of focus on teaching algorithms, the dubious value of spatial-reasoning for primary-grade children, and the continued availability of resources such as unifix cubes to assist the problem solving of second-grade children" (Lehrer and Shumow, 1997 p. 47). If parents are given the opportunity to see the value embedded in these practices, they may choose to align their support with current mathematics instruction.

Faced with the desire to assist and support their children in their studies but unable to align with current practices, parents may question the validity of teachers' practices. Lehrer and Shumow (1997) reported, "parents found particularly controversial the practices of collaborative interaction and discourse that we and others (...) believe essential to learning mathematics. This implies that parents as a whole might be less likely than teachers to think about mathematics as a meaning-making venture" (p. 56). When engaging in the reform of mathematics education, mathematics educators require the support of these key stakeholders to carry the momentum forward.

My desire is to assist parents in understanding current mathematics education methods both philosophically and practically as they endeavour to guide their children. I arrived at this topic after considering many fields in mathematics education such as the nature of problem solving, the constitution of complex activities, the success of mathematics teachers trained in other fields and the application of generative thinking in teacher planning. I also questioned which of the many different stake-holding groups would be my participant group. Unsure of whether I wanted to carry out research with my school-aged or university students, I turned to a new venue, the parents. This is a group that I believe has been largely ignored in our mathematics reform attempt, and one that would allow me to re-examine my other interests in a different light.

My varying roles as an educator have led me to this thesis topic. As an instructor of mathematics methods courses at the University of Manitoba and the University of

Winnipeg in recent years I've had the opportunity to observe the nature of adults venturing into the world of mathematics teaching. These groups of inexperienced teachers are always open to sharing their beliefs about mathematics instruction, both their experiences and their intent with their own students. A common trend is apparent each year: approximately forty percent share an unease or to some extent, phobia, towards mathematics; half are indifferent to the study of mathematics but are confident that they can handle teaching the subject; and a mere ten percent enjoy the study of mathematics and are eager to teach the subject. It is common for these students to express that success in math is linked to natural ability. Pedagogically, they most often begin with the idea that mathematics teaching consists of: explaining the topic, giving examples, assigning the work, and then correcting. Those with little math success are the most fearful here as they believe that with little mathematical understanding, they are lost at step one. With these observations garnered informally from university students, I began to wonder if these ideas are not also commonly held by the parents of the children we teach.

It was my intention to engage future teachers in experiences that young children might have in the classroom. On the surface, teaching adults to learn with methods we employ with children seems to be simplistic. Surely if a child is expected to do a particular question, then a fifth year university student would be bored with this expectation. On the contrary, the complexity that challenges, intrigues and draws in children to their sense of power as a learner is equally evident in the pre-service teachers with which I have worked. These adults manipulate mathematically representative materials, are encouraged to engage in reasoning and discourse about strategies, develop the connections among mathematical concepts, and solve problems in a community of learners, and all the while sitting back and analyzing the effectiveness of these methods. It is not uncommon to hear an adult pre-service student comment 'I wish I had learned math this way. I finally get it'. I began to wonder if parents couldn't also benefit from a program that would enable them to live the practices of their children and in turn serve to enhance the mathematical understanding that their children are developing.

As a middle years mathematics teacher for the past 17 years, I myself have undergone the transformation from traditional practices of instruction where first I taught the skill, then they practiced and finally I marked, to one where learning to think through reasoning, carefully selected activities and construction of conceptual ideas and discourse became the norm. Grounding my work with students in the tenets of the National Council of Teachers of Mathematics (NCTM) became a driving force. Our curriculum calls for fifty percent of the students' time to be occupied with problem solving resulting in more engaged flexible thinkers. As students draw upon their own mathematical knowledge and discuss problems with peers, they begin to realize the applicability of mathematics in real situations; they feel the triumph of discovery and the power of thought. In truth, they work as a mathematician might. Yet while my students' engagement in thinking, understanding, and enjoyment of mathematics has climbed steadily over the years, I still hear the troubled comments of parents who were taught in a traditional program.

Parents' traditional instructional backgrounds influence their perception of mathematics instruction. Two examples serve to illuminate the need for further work with parents. In order for students to engage in thoughtful work, they must have a stimulus that ignites their thought process. Having given a particular problem to my senior one students, a parent questioned, 'My pharmacist friend couldn't get the answer. How can my son get the answer if you haven't taught him how to solve the problem?' It appeared that teaching 'how to do it' is the frame of reference for good mathematics teaching. A similar case can be made for 'repetition'. This is a most difficult perception for a mathematics teacher to respond to. A parent's suggestion that "My child would learn his math if you would just repeat your instruction over and over, maybe after school, like the teacher last year did", is not easily handled in a short interview session. Parents are in need of a forum which allows them to not only ask questions, but to experience the learning of mathematical thinking beyond the act of simply 'doing' the arithmetic.

My experience as a member of the parent community also has an impact upon my desire to work with parents in my study. The interaction of community members often revolves around issues in their schools. On the edges of the soccer field, the arena, the ballet class, conversations about mathematics instruction are carried out. As a mathematics educator, I'm encouraged to hear that many of my colleagues are persevering with involving children in problem solving situations in their mathematics classes. What is less encouraging is overhearing the parents' reactions: "Did you get last night's problem?", " I phoned_____ for the answer" and "Why are they doing these stupid problems?" or "Kids should still be learning to multiply and divide. Apparently it's not important any more." Ignorance of the underlying principles of current instructional practices cause people to fall back on what they know; that is, what they themselves experienced as a student.

I hold a strong belief that our current trend in math education as espoused by the NCTM Standards and reflected in the Manitoba mathematics curriculum is supportive of developing powerful mathematical thinking in many students, not just a select top percentage. Mathematics should and can be interesting and engaging for all.

Parents' roles in the success of mathematics reform should not be underestimated. One has only to look to current trends in California (Wilson, 2003) where the Back to

Basics agenda is gaining momentum to realize the strength of this backward shift in mathematics education. A political group in Iowa, running on the platform 'Target the Basics', claims that "academics were being short-changed, that children's need to learn basic facts was being compromised, and that portfolio assessment did not hold students to universal standards and therefore compromised their future life chances" (Lehrer and Shumow, 1997, p. 47). With parental groups advocating for oppositional changes to how mathematics is taught, largely based on traditional practices, the need for knowledge of current practices has never been greater.

One of the most pointed causes of the demise of mathematics education reform is the "mismatch between the aspirations of reformers and the expectations of the community in which reform takes place" (Mirel, 1994, in Lehrer and Shumow, 1997, p. 42). Often parents, not unlike my curriculum and instruction students, see mathematics education as a 'telling-retrieving of information' venture. Lehrer and Shumow (1997) described the parents' instructional practices as product driven and based in "telling, lecturing and controlling" (p. 42). This creates great dissonance with the reformer's vision of mathematics learning as one of constructing knowledge.

Lehrer and Shumow (1997) provide a significant example for this study. The goals of my intervention are very similar to their goals, that is, aligning the mathematics learning conceptions and methods of parents with that of current reform efforts. Lehrer and Shumow claim that "models of parental involvement should consider the potentially important role played by shared knowledge of student thinking. We anticipate that collectively held knowledge about children's (mathematical) thinking can serve to nurture and sustain reform" (p. 72). The goal of their study was to promote deeper mathematical conceptual understanding in children and for parents to realize that this result is superior to simply learning algorithmic knowledge.

There exists a need to pursue an alignment between parents' understanding of math instruction and how children learn with the pedagogy currently used in schools. With this purpose, I decided to design a program to offer parents an opportunity to realize the potential that exists in mathematics education by attuning their traditionally based perceptions with that of understanding-based practices. This program aimed to align parents' instructional methods with that of current mathematics curriculum pedagogy by learning to carefully listen to their children's thinking and by offering less indirect assistance. Lehrer and Shumow (1997) found "Congruence, or lack thereof, between practices in school and at home has been credited with impacting school performance of children, the quality of home-school relations, and community support of school policies" (p. 43).

Substantial rationale exists for the necessity to develop a parent program for mathematics education. The goals of such a program are to cause learning for the participants regarding the rationale which drives our curriculum, their own learning of mathematics, how math is learned, and ways to increase their effectiveness in helping their own children in mathematics. My research intention was to study the nature and range of this learning among participants in a parent education program. With this intent, my research question was formulated:

> **What is the nature of the learning that occurs as parents attune their perceptions of mathematics instruction and learning for themselves and their children and align their practices with that of the current methodology espoused in the Manitoba mathematics curriculum and by the National Council of Teachers of Mathematics?**

Chapter 2
Examining Math Education Reform & Parents' Perceptions

This chapter will establish the tenets of mathematics education reform as expressed in the academic and professional literature. It will examine the significant influence parents have on their children's learning by looking at how they impact on their children's attitudes toward and achievements in mathematics. It will describe how an understanding of teachers' approaches to mathematics teaching contributes to parents' abilities to help their children. It will describe the factors that may influence a parent's approach to assisting their child with mathematics. The final element looks at what parents want from parent education programs.

Reform of Mathematics Teaching and Learning

Studies of society's economic demands recognize the changing needs of a society as we move from an "economy of mass production" to a need for "people who are sufficiently independent in their thinking to be adaptive, innovative, and inventive" (Lappan, 1999, p. 134). To this end, Lynn Steen, professor of mathematics and former president of the Mathematics Association of America, feels we must set goals in school mathematics courses that are "appropriate for the demands of a global economy in an age of information" (Steen, 1989, p. 19). Our provincial (Manitoba) response to this need began by constructing a curriculum that was built on the groundwork of the National Council of Teachers of Mathematics (NCTM) publication of Curriculum and Evaluation Standards for School Mathematics (the Standards) in 1989. NCTM began their push for mathematics instructional methods that would encourage mathematics students to value mathematics, to gain confidence in their ability to do math, to be problem solvers, to communicate mathematically and to learn to reason mathematically and in turn become mathematically literate individuals (NCTM, 1989). As well, teachers were expected to pose "worthwhile and engaging mathematical tasks, manage the intellectual activity in the room, including the discourse; help students to understand mathematical ideas and to monitor their own understanding" (Silver & Smith, 1996, p. 20).

Contrary to traditional approaches to mathematics education, the Standards call for both the construction of cognitive development as well as attention to affective factors. Hart and Walker (1993) suggest that researchers "are beginning to recognize that cognitive and affective aspects of learning are present when students construct mathematical understandings, that focusing on one without the other limit what teachers and researchers know about the teaching and learning of mathematics. When a student works on a mathematical task, he or she brings a whole set of beliefs and attitudes about mathematics and what it means to do mathematics" (p. 23). In effect, we as teachers or parents must concern ourselves with the students' "interest, frustrations, excitement or anxiety" (Hart and Walker, 1993, p. 23) as they engage in a mathematical task. There is a direct correlation between the student's affective response and the ability to construct knowledge.

Current reform efforts "commonly characterize teaching as guidance of the students' knowledge construction" where classrooms are characterized as "activity settings in which the mutual construction of knowledge arises through collaborative interaction, shared meaning, and assistance. Teaching suggests the assistance of performance and learning involves guided participation" (Lehrer & Shumow, 1997, p. 43). Lappan (1999) characterizes this classroom as one where students have "opportunities to muck about with ideas; to struggle to make sense of new mathematics situations; to try out ideas; and with the help of classmates and the teacher, raise those ideas to powerful mathematics strategies and ways of thinking" (p. 132). The intent is for students to construct understanding and thus for students to become more mathematically literate, because "understanding mathematics is more generative and adaptive than simply recalling past procedures and practices" (Lehrer & Shumow, 1997, p. 65). These practices are often described as enacting *constructivist* theory.

Constructivism is "essentially a theory about limits of human knowledge, a belief that all knowledge is necessarily a product of our own cognitive acts" (Confrey, 1990, p. 108). As a theory of learning, its application is open to varied interpretations in a classroom. However, reform in mathematics education is aligned with the precepts of constructivism. Constructivism requires a stimulus, such as a mathematical problem, to initiate the learning process. Instruction must be tailored that allows the learners to mesh their thinking with that of expert knowledge (Baroody and Ginsburg, 1990) while allowing the child to 'play around' with ideas and engage in mathematical conversations. Palinscar (1998) comments on the role teachers play in student discourse. "Teachers play an important role in mediating classroom discourse by seeding the conversation with new ideas and alternatives to be considered that push the students' thinking and discussion and prepare them for conversation" (p. 365). Piaget's theory of intellectual organization and adaptation (Wadsworth, 1975) has also contributed to constructivist theory as students work to reorganize and situate mathematical concepts within their own minds. The teacher's goal is to create cognitive conflict that requires a student to question assumptions and conjectures. As mathematicians approach their work in a thoughtful and reflective manner, so should our students. Students will work as mathematicians when they are encouraged to conjecture, reason, problem solve, validate and connect. These processes are the ground rules for discourse with others. Constructivist learning is conceived as a social venture. One must interact with good questions, intellectual conflict, and the ideas of others in order to sort through that which they believe to be true. Guided instruction, inquiry, activity based instruction (with and without manipulatives), and cooperative groupings can all play a part in assisting children to constructively learn mathematics. To this end, Wood and Sellers (1997) have described mathematics teaching reform.

> Current reforms suggest a change in the nature of math taught in the classroom as well as a different view of what it means to do math – that is, a constructivist view of learning. From a psychological point of view, the contention is that students learn mathematics most effectively if they construct meanings for themselves, rather than simply being told. In support of this view, a great deal of evidence has shown that young children develop an intuitive and informal sense of mathematical concepts and procedures long before they enter school (p. 164).

The NCTM Standards recommend that learning about number and number relationships, algebra, statistics, probability, geometry and measurement is accomplished by the actions of reasoning, problem solving, communicating and connecting. The intent of each of these actions is described in the Curriculum and Evaluation Standards (NCTM, 1989) for grades 5 to 8 (Fig. 2.1).

Mathematics as Problem Solving:
In grades 5-8, the mathematics curriculum should include numerous and varied experiences with problem solving as a method of inquiry and application so that students can –

- Use problem solving approaches to investigate and understand mathematical content;
- Formulate problems from situations within and outside mathematics;
- Develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;
- Verify and interpret results with respect to the original problem situation;
- Generalize solutions and strategies to new problem situations;
- Acquire confidence in using mathematics meaningfully (NCTM, p. 75)

Mathematics as Communication:
In grades 5-8, the study of mathematics should include opportunities to communicate so that students can-

- Model situations using oral, written, concrete, pictorial, graphical, and algebraic methods;
- Reflect on and clarify their own thinking about mathematical ideas and situations;
- Develop common understandings of mathematical ideas, including the role of definitions;
- Use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas;
- Discuss mathematical ideas and make conjectures and convincing arguments;
- Appreciate the value of mathematical notation and its role in the development of mathematical ideas (NCTM, p. 78)

Mathematics as Reasoning
In grades 5-8, reasoning shall permeate the mathematics curriculum so that students can-
Recognize and apply deductive and inductive reasoning;

- Understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs;
- Make and evaluate mathematical conjectures and arguments;
- Validate their own thinking;
- Appreciate the pervasive use and power of reasoning as a part of mathematics.

(NCTM, p. 81)

Mathematics as Connections:
In grades 5-8, the mathematics curriculum should include the investigation of mathematical connections so that students can-

- See mathematics as an integrated whole;
- Explore problems and describe results using graphical, numerical, physical, algebraic, and verbal mathematical models or representations;
- Use a mathematical idea to further their understanding of other mathematical ideas;
- Apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business;
- Value the role of mathematics in our culture and society. (NCTM, p. 84)

Figure 2.1 Mathematical Processes.

Parental Impact on Learning

Research shows that parental involvement in their children's education affects both academic achievement as well as their attitudes toward the study of mathematics. In

1969 the U.S. Department of Education collected data from over 20,000 students, and found "parent and community involvement had a substantial, significant effect on the math achievement and educational aspirations of students even when controlling for ability and family educational background" (McDill, Rigsby, and Meyers, 1969, in University of Minnesota, 2003, Ch.1, para.8).

Academic success. A child's desire to be successful and the ability of the child to act on this desire is affected by the parent's involvement. Two ideals of parent involvement have a significant effect on a child's academic achievement. Parents must have realistic, high expectations for their children's performance at school (Amato and Ochiltree, 1986) and thus be able to set realistic goals for their children, and they must have knowledge of their children's work. These ideals become interrelated, as high expectations must be balanced with goals, which depend on parents' knowledge of their children's work (Kellaghan, Sloans, Alvarez & Bloom, 1993, in University of Minnesota, 2003, Ch. 1, para.7). Other studies have also shown a positive correlation between student success and parent involvement in homework. "Russell (1996) found that students whose parents were directly involved in their homework were more successful academically than students who received instruction primarily at school" (Hradnansky, 1999, p. 24).

Attitudes. Directly related to reform efforts in the classroom is the need to affect children's attitudes toward math. The literature supports a need to assist parents in realizing a positive attitude towards the learning of mathematics, as the transference to their children is evident. All too often, parents will report that they too 'couldn't do math neither… so that's why my child can't'. Such negative and inaccurate beliefs affect a child's attitude towards mathematics. Stanic (1989) found that parents' influences were significant as well and that "Children tended to adopt their parents' beliefs about achievement" (Hradnansky, 1999, p. 19).

A parent education program should work to establish or enhance a positive attitude within parents so that the subsequent impact on their children's attitude may be realized. Research has shown that the child's disposition toward mathematics can be positively impacted when parents and teachers work cooperatively together (Hartog, Diamantis and Brosnan, 1998, in Hradnansky, 1999, p. 24):

> A child's attitude toward mathematics was influenced by parental attitudes towards the subject. Parents who were enthused and excited about math in everyday situations were likely to have children who developed an enthusiasm for math (Hartog and Brosnan, 1994, in Hradnansky, 1999, p. 19).

Parental Misalignment in Instructional Methodology

One can speculate on the reasons why a misalignment may occur between a parent's instructional strategies and teachers' methods in reform-based classrooms. Parents' practices are largely influenced by their own experiences in mathematics instruction and by media and 'common beliefs' in the public at large.

Parents' personal histories. Often parents' perspectives of what students should be taught mirror their own experiences as a student. In Hradnansky's (1999) work with parents of primary children, comments written on children's papers by their parents would make reference to a need and call for "students learning more rote, memorization, and traditional worksheet type of homework" (p. 14). At parent conferences these same parents made reference to when they were in school and how memorization and pages of examples were the norm. They would also relate their child's difficulty in the fact that they themselves were not good at mathematics, putting their children at a disadvantage (Hradnansky, 1999).

Parents' experiences in mathematics classrooms were most likely the result of a 'traditional' methodology where mathematics was delivered to the students without question. The intent was to provide the algorithms and skills of a set of rules via direct instruction. The idea was that "mathematics is received and is based on computational speed and accuracy" (Lehrer and Shumow, 1997, p. 74.) The role of the teacher in traditional mathematics instruction has been largely described as "essentially to transmit knowledge to, and validate answers for, students, who are expected to learn alone and in silence" (Silver & Smith, 1996, p. 20). Given that this method of instruction is familiar for parents, it is understandable that they would rely on that which they know to offer instruction to their own children. It is also realistic that they might expect this of their child's school. They might not be ready to conceptualize that other methods lend themselves to deeper understanding and can assist in developing a genuine affinity for doing mathematics.

What must be called into question are parents' desires to have their education replicated for their children. Figure 2.2 relates examples of memories extracted from parents in a parent information group conducted by Morse and Wagner (1998). These vignettes are not unlike the messages conveyed to me by preservice students in my classes- messages of dislike, discomfort, embarrassment and misunderstanding of mathematical concepts.

Painful, misunderstanding, miserable:
Third grade – copying my best friend's math (fractions) homework, not having a clue what they meant – Painful sessions with my father over word problems, usually ending in tears (mine) – all memories of elementary school were miserable (ie. Humiliation and anxiety) – The only positive experience was geometry.

Authoritarian, Memorizing:
Math class consisted of each kid in turn standing up and reciting the addition tables and multiplication tables. For each wrong answer or missed step, the teacher would step up and smack the palm of your hand with a ruler...I remember that despite the way math was presented, it could not entirely kill the interest and wonder of the child. I remember noticing for example, the patterns in numbers, the "thrill" of realizing there were patterns. I remember having a glimmering in my brain that it all meant more than I could see. But having no means to articulate my thoughts, I memorized... and I held out my hand for the ruler.

Tolerated, Dull –repetition, Waste of numerical understanding:
I merely "tolerated" about 80 percent of the mathematical coursework I performed as a student in elementary school and in college. Now... I wonder how I would have approached college calculus and statistics if I had been taught differently in elementary and high school. I doubt I would have settled for the dull repetition of 'algorithms'. If I had utilized manipulatives, graphs, and more hands-on materials, would I have been as mystified by calculus?...I wonder, why didn't I probe more, insist on understanding, and spend more time simply thinking about what I was doing? I mourn the gigantic waste of numerical understanding that I suffered along with many of my friends. I see that I frequently "shut down" when I saw math problems. Did I carry this approach to learning into other subject areas I found difficult?

Figure 2.2 Comments from parents regarding their own mathematics class experiences. (Morse and Wagner, 1998, p. 362)

Although not generalizable to all parents' experiences, these excerpts serve to illuminate the need for teaching methods that differ from the methods that many of today's parents may have experienced. In the last excerpt, the impact of reform methodology surfaces with very conflicting and positive thoughts for the learner.

Influence of media. Much of parents' knowledge of current mathematics instructional methodology is not only fuelled by their own school mathematics experiences but also by media. An article published in a major Canadian newspaper, the *Globe and Mail*, May 2001, entitled *Go Figure, Ashley Can't* serves to illuminate this position. The author, Margaret Wente, described reform attempts as "a pedagogical fad."

She described reform proponents as those who work at universities and education ministries while reform critics were the "leading mathematicians, top learning experts, angry parents, and quite a few math teachers." She then claimed that this new math "discourages the teaching of standard algorithms [that is, the methods used to solve problems]. In fact, it discourages all direct instruction, repetition and memory work. Times-tables drills (and, in some schools, long division and fractions) are out the window. These are thought to deaden interest in learning." She went on to quote an angry parent who claimed, "One plus one equals two! It's a fact! There's nothing to explore! In the elementary grades, it is not necessary to understand why!" A web site for "extensive discussion" (www.mathematicallycorrect.com) is given for those who wish to explore this topic further. A look at this site provides parents with support for believing that this 'new' math is "counterproductive and downright dangerous" and destroying the minds of our youth. Such biased interpretations make it difficult for parents to rationally examine the validity of mathematics reform.

The mass media abounds with countless lists of *math myths*, all of which tend to reflect many of the same ideas. Although refuted many times, many of these myths have emerged as truths in commonly held public perceptions regarding the learning of mathematics. Founded on the many studies of public perceptions of mathematics learning, these myths have found their entry into the common knowledge of our culture. Figure 2.3 outlines myths that exemplify these ubiquitous perceptions (Kogelman and Warren, 1979, pp. 30-43).

1. Men are better at math than women
2. Math requires logic, not intuition
3. Math is not creative
4. You must always know how you got the answer
5. There is a best way to do math problems
6. It's always important to get the answer exactly right
7. It's bad to count on your fingers
8. Mathematicians do problems quickly, in their heads
9. Math requires a good memory
10. Math is done by working intensely until the problem is solved
11. Some people have a 'math mind' and some don't
12. There is a magic key to doing math

Figure 2.3 Commonly held math myths.

Parents whose beliefs are represented by these myths are not likely to understand reform-based mathematics teaching. For example, if parents believed that in order to be 'mathematical' students must be able to do all work quickly in their heads (Myth #8), they would be apt to ignore the thought process required to construct a deep connected understanding of, for example, volume. They might tend to want to put undue emphasis on memorization of algorithms and underemphasize (or ignore) understanding and searching for meaning and application. Similarly if parents believed that there is a magic key to doing math (Myth #12), then they might expect the teacher to simply drill this 'key' until the student gets it. In terms of parent-child interaction, this may unfold as a 'kill and drill' exercise, resulting in short term memory of the concept rather than long term understanding with ability to apply and use the concept. Frustration on the part of the child who 'just doesn't get it' may also be detrimental to the child-parent relationship.

Critics of Standards-based mathematics learning are quick to spread their ill-conceived notions with the public. In *On My Mind: Dispelling Myths about Reform in School Mathematics*, Frances Curcio, a teacher at New York University and former president of NCTM, points to five areas she believes math reform critics are grossly misinformed. These include the criticisms that "basic computation is ignored... answers that are close to correct are good enough... only one right way exists to teach mathematics... textbooks identified as 'standards-based' support reform efforts and no research is available to support reform efforts" (Curcio, 1999, pp. 282-283). Having been engaged in this reform practice for 14 years, I have confidence that each of these criticisms is indeed inaccurate.

Research conducted by David Bradley focused on dispelling common myths he has encountered in his position as a curriculum specialist in K-12 mathematics. Bradley provides readers with empirical evidence that public concerns are unfounded, including, "Mathematics achievement was much better in the Seventies; NCTM Standards have not been proven to work; if children are allowed to use calculators, they won't learn the basic math facts; and, anyone can teach mathematics" (Bradley, 1999, p. 1).

With ill-informed ideologies and experiences embedded in traditional practices, parents are assailed with reasons not to support mathematics reform. Parents should be provided the opportunity to examine the intent of mathematics reform through first hand experiences and discussions and thus become better equipped to make decisions regarding their child's learning.

Enablers for School-Home Alignment

The intent of the parent education program (to be described later in this proposal as the research intervention) is to enable parents to work more productively with their children by aligning their current perceptions with that of mathematics curriculum reform. This alignment will be mobilized if parents can adopt the enablers identified by

Ford, Follmer & Litz (1998). These examples for home/school alignment enablers, shown in figure 2.4, are a result of an increase in parents' knowledge of reform practices and an active involvement in these practices.

1. As parents become more confident in their mathematics-related interactions with their children, they discover that wrong answers may tell them more about their children's understanding than will a focus on just the correct answer. Wrong answers become a parent's lens for looking further, asking probing questions, and seeing what a child does and does not understand.

2. As parents understand how and why mathematical thinking empowers their children to become numerically literate, they become more supportive of a Standards-based curriculum. This support not only positively influences their children's successes in, and attitudes toward, mathematics but also extends to other parents.

3. When parents and teachers form home-school partnerships, children are more likely to see a unified front between parents and teacher. Rather than struggle with two seemingly different opinions and methods of doing mathematics, children are able to share the same enjoyable and meaningful mathematics activities with their parents at home as they do with their teacher in school.

4. Opening the lines of communication at school also sends an important message to children that the mathematics they are experiencing in school is worthy and important.

Figure 2.4 Examples of home/school alignment enablers (Ford, Follmer & Litz, 1998, p. 311)

As a result of my intervention, I hoped that parents would come to work more easily with their child on non-traditional mathematics activities. I hoped that this work would be enhanced by a focus on the child's understandings, more support of a Standards-based program, an increased enjoyment through aligned methodologies, and send the message that math is important.

I hoped also that parents would benefit from the intervention on a personal level by rethinking and relearning mathematics for themselves. Simmt found that parents could enjoy their role as fellow learner with their child as "a person whose thinking stimulates the child's and whose thinking is stimulated by the child's" (Simmt, 1996, p. 109).

Intellectual stimulation can occur as much for the parent as for the child. This realization occurs when parents are engaged in activities with the child that go beyond "simply helping them memorize basic number facts or doing long division" (Simmt, p. 109). When a parent realizes the impact that constructivist methods have on them as learners, they can more readily accept this methodology for their children.

What Parents Want in an Education Program

Alongside the benefits demonstrated by parent education programs is the evidence as presented in the literature that parents desire such programs. Having information on parents' needs and concerns helped me to establish a balance between the subject matter of my intervention-curriculum and the parents' desires in such a program. To this mix, I also included the desires of my own participants.

As was the case with my curriculum and instruction classes, students tended to fall into three categories: fear of mathematics, comfortable but needy in terms of teaching mathematics, and confident. Simmt (1996) found a similar range of parent attitudes. She found that some parents admit to a fear of mathematics that perpetuates a dislike for the subject. In turn they believe that they cannot help their child with mathematics. Others, comfortable with mathematics themselves, would like advice on helping their child. Some saw a parent program as a way to spend more quality time with their children.

Parents value information about their child's mathematics education. Parents want accurate information on grade specific outcomes; they want enhanced communication about upcoming work; and they want to better understand the content and processes of mathematics and their child's progress; and, they want to know how to help their child at home (Levine, 1998). It was not my intention in the intervention for this project to provide child-specific instruction. However, this intervention intended to assist parents to get a better understanding of the content and processes of mathematics and how to help their child at home.

On a provincial level, Manitoba Education and Training published *Math Matters*, an information booklet for parents, which was distributed in all schools in 1998. (See Appendix A.) This booklet encapsulates the mathematics reform agenda as it touches on needs of society, student learning outcomes, standards testing, supporting children at home, the strands of the curriculum, problem solving, communicating, math classrooms, technology, positive attitudes and the ubiquity of math. Currently, the province does not have any parent information initiatives in effect. This study may inform future efforts to pursue parent-school alignment in mathematics learning.

From the academic and professional literature, I found myself armed with a vision of parents' desires and needs in terms of mathematics instruction and their children's investments. My experiences as a mathematics educator also helped to prepare me to engage parents in the intent of NCTM's math reform practices. As the study began, I looked forward to listening carefully and analyzing the learning of my parent participants as they engaged in an informative and interactive mathematics education program that spoke to learning to be mathematical rather than doing rote exercises.

Chapter 3
Purpose into Action

This chapter will provide an overview of the research design: both its intervention method and research method. A general overview of curriculum development and phenomenography will be presented. Then, the particular elements of how the study progressed will be outlined.

My intentions in this study were twofold. As a researcher my intention was to examine learning that occurs in participants in the study. The other intention was to design a program, the intervention, which would cause this learning to occur. Being the stimulus for learning, the structure of the intervention was carefully constructed to maximize the learning of the participants. With this importance in mind, curriculum models were reviewed to realize a model for the intervention program.

The research was modelled on a phenomenographic design such that it was an "empirical study of the differing ways in which people experience, perceive, apprehend, understand, and conceptualize various phenomena in and aspects of the world around us," (Marton, 1997, p. 96). Data is collected in a phenomenographic study to encapsulate the conceptions of its participants and thus allow the researcher to examine these conceptions and draw out understandings of their existence and search for relationships.

Curriculum Development

Traditionally, curriculum design is characterized by many differing schools of thought. Three perspectives tend to emerge (Marsh & Willis, 1999). One can proceed with a procedural means-ends orientation such as that proposed by Tyler; a naturalistic or descriptive approach such as that of Walker, whereby the actions and decisions of the curriculum participants are at the forefront; or, a critical view such as that of Eisner that places social equity and personal freedom as the crucial impetus in curriculum development (Marsh & Willis, 1999). The goals and content of my intervention were largely decided prior to the construction of this 'parent education curriculum'. As a consequence, organizing the structure of this content was my focus here. I used Giles' model (Ornstein & Hunkins, 1988), a model very similar in its elements to Tyler's, as the framework for organizing my intervention. Giles' model (Figure 3.1), although consistent with Tyler's in question posing, suggests that the elements of curriculum design are interactive. That is, as one proposes, develops and implements each of the elements in the design, they are constantly being altered based on how each of the elements interacts.

Alterations will occur as participants' questions and concerns arise out of their interaction with the subject matter of the intervention. In this study, a question posed by Rob (discussed in Chapter 6, p. 107), caused the focus of the session to centre in on examples of questioning children to determine their initial thinking and thus establish a starting point for a cognitive partnership and scaffolding. The ability to alter elements of the program when the opportunity arises naturally out of discussion enables a greater sense of connectedness for the participants.

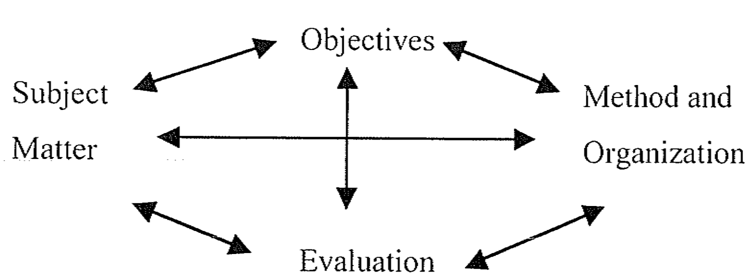


Figure 3.1 Giles' Model (Giles, 1942, in Ornstein & Hunkins, 1988, p. 166)

Later in this chapter, the elements of this model will be described as I illustrate how the specific content of the intervention program is organized under these categories.

Phenomenography

Phenomenography was conceived as a form of educational research because it sought to answer questions about thinking and learning (Marton, 1986). It is a study of how people experience a certain phenomenon in the world around them (Orgill, 2003). People experience phenomena in their own way, which results in many varying perceptions. Phenomenography seeks to describe these perceptions and expose the range that exists. By characterizing the logical relationships that exist between individuals experiencing the same phenomenon and within individuals' perceptions, the phenomenographer can establish 'categories of description' (Marton, 1997) which in turn can be analyzed for 'awareness' and hierarchal order.

To understand these labels, we might look to a mathematical example. Young children may experience a situation where it's necessary to find the sum of three and fourteen. This situation would be considered to be the phenomenon that the children encounter. A study of their conceptions of this task would reveal many varying methods for determining the sum. Some may choose to model the sum by making a pile of three and a pile of fourteen objects, put them together, then count the total number of objects; another student may start at 3 then parallel count to 17; another may choose to start at 14 and parallel count to 17, while another may be able to mentally construct a group of ten plus seven. Each of these methods could be determined to be a 'category of description'. Upon analysis, one might begin to establish a hierarchy based on efficiency of the method.

Phenomenography seeks to collect data that describes the participants' awareness, "their total experience of the world at a given point in time" (Marton, 1997, p. 98). Awareness, as conceived by the phenomenographer, does not suggest a dichotomy of 'to be aware or not', but refers to varying degrees of awareness. One might see aspects of a phenomenon that are obviously explicit, but behind these aspects are other more implicit levels of awareness, followed by yet other ways of being aware. Awareness is viewed as a "continuous variation" (Marton, 1997, p. 98). For example, when we view participants' awareness of a mathematical concept such as 7, we may note their awareness of its cardinal value and the use of the symbol on an explicit level. We may note as well that some participants see seven in other ways- its relation to other numbers, what 7 on a number line represents, seven as the absolute value of –7, 7 as the summation of a part-part configuration, 7 as a fraction or decimal, or 7 as a spatial representation.

In phenomenographic research, data should be collected which encourages participants' reflections on their awareness. Data collection usually includes interviews, whereby the interviewer seeks to cause the interviewee to reflect on situations arising from the phenomenon. These interviews should not involve preconceived sets of questions but should follow from what the subject has to say (Marton, 1997).

Analysis of the data requires that the researcher set aside his/her perceptions of the phenomenon and focus on the perceptions of the participants. Data should be analyzed from two perspectives:

1. The connection between participants and their perceptions will be separated, as the researcher looks for similarities and differences amongst the data. This causes certain understandings to surface across the participant group; and,

2. The researcher will also align the individuals with their perceptions, as examining the total package of each participant's perceptions offers different meanings.

An analysis of the ways in which participants are 'aware' may lead to an establishment of hierarchical order.

Reliability in a phenomenographic study suggests that another researcher will be able "to recognize instances of the different ways of experiencing the phenomenon in question" (Marton, 1997, p. 100) but would not be required to apply repeated measures to get the same result. Another researcher would be able to judge which categories of description are applicable to the perceptions found in the data. Phenomenography is not a measure, but a revealing of discovery. The discovery does not need to be replicable.

Marton (1997) suggests that the application of phenomenography lends itself to using the different ways that understanding is learned to develop educational programs that can establish the desired knowledge and skill.

Research Method: Participants

The participants in this study were parents of children in grade three to eight who attended a kindergarten to Senior 1 school in South Winnipeg. The community was approximately 20 years old where the limits of new home construction were only recently occurring. Most children in this community lived in single-family homes with the family unit intact and where both parents worked outside of the home.

The children of these participants attended a school where they were exposed to the current Mathematics curriculum in varying degrees. Since the initiation of the current curriculum in 1994 to the date of the study (2004), the children continued to experience instruction that could vary from traditional methodology to a solid immersion in NCTM espoused methodology.

Parents were recruited by an introductory letter inviting them to participate in the study. This letter, sent home with each kindergarten to grade eight student in the school, outlined the participant's part in the research and the data gathering processes. This study was designed to accept up to twenty participants as any larger a group would have affected the quality of the open discussion format intended. Minimally, I would have liked to have at least six participants in the study.

More in-depth data sources were selected from this participant group based on their desire to be interviewed or by my request to have them become more actively involved in the research. It was my intention to select at least four participants from this group to serve as more intensive data sources. Written consent was obtained from the participants who agreed to partake in informal interviews. Identity of the participants was concealed through the use of pseudonyms and any quotes suggesting identity were removed.

Research Intervention Development: The Phenomenon

My intervention for parents was premised on the NCTM Standards and the learning processes described in the Manitoba 5-8 mathematics curriculum. Using these tenets as the subject matter, I analyzed parents' learning as they responded to their engagement in the reality of current mathematics instructional methodology. This intervention was formulated by implementing elements of Giles curriculum design: objectives, subject matter, method and organization, and evaluation.

Objectives. This intervention served as the stimulus that caused learning in the participants of this study. Objectives of the parent education intervention program were separate from the research goals of this study. Generally, the objectives of the intervention program were:

1. To assist parents in understanding the rationale that drives our curriculum
2. To enable parents to rethink how math is taught through experiencing methods other than those from their own experience
3. To enable parents to align the difference in teaching methods between home and school
4. To allow for in-depth dialogue with and among parents
5. To continue the promotion of the principles espoused by the NCTM

Development of subject matter. Subject matter refers to those elements that were critical in causing learning and alignment to occur for parents as they endeavoured to learn more about the mathematics curriculum, mathematics learning itself, as well as how to align their instructional strategies with that of the current mathematics instruction methodology. These elements formed the backbone of this intervention, as success of this program was predicated on the parents learning through these elements. Subject matter in this model was determined based on input from various sources. Tyler (1949) suggests that contemporary life outside the school should influence objectives, thus a literature review that includes both media and studies on parent perceptions had been undertaken. I also drew upon input from specialists in the field; for example, the philosophical positions of both the NCTM Standards and the Manitoba 5-8 Mathematics Framework for Implementation. My own experiences as a curriculum committee member, experienced teacher and university instructor also affected these decisions. From these sources, a tentative set of elements was determined with the intention that alterations would occur as a result of my interactions with the participants and through the analytical stage of the research process.

Subject matter in this intervention program included the mathematical processes of reasoning, communicating, connecting and problem solving, as outlined in the literature review.

Method and organization. The intervention consisted of 120-minute sessions with the participants on four Wednesday evenings in January. The intervention, based on the four processes of mathematical learning, also included an introduction to these processes, opportunity for open discussion, and opportunity to actively participate in the reality of the chosen process through activities that modeled the essence of the process. The structure of these sessions reflected practices described earlier regarding my interaction with fifth year education students. Similarly, I had my participants engaging in mathematical activities that a child might do to learn a particular concept. Parent participants, as was the case with my education students, were encouraged to view the activity in terms of the sense making that occurs for them mathematically. I anticipated that parents would analyze the activities differently than education students. Where education students might analyze these activities by sifting through the nature of many types of learners and by sorting through classroom management issues, parent participants were encouraged to analyze the activity by connecting to their own child as a learner and to themselves as learners.

A general overview of the original organization of the program follows. Issues regarding data collection will not be expressed in this section on curriculum development.

Session One – Introduction. The goals of this session are twofold. Participants must be introduced to the nature and processes of being mathematical but must also have an opportunity to be actively involved. I anticipate the latter will be a difficult step for the adults in my program as they will be coming together for the first time and I don't want to be put them on the spot intellectually. With these thoughts in mind, my plan is to begin with a small opening activity in which everyone can be successful. I'll ask them to mentally solve a two-digit by two-digit multiplication question. This will open the door for examining the multitude of methods that people employ to solve arithmetic problems as well as the learning that occurs when we discuss algorithms as a community of learners. I'll follow this entry with a presentation of the Big Ideas of our Manitoba curriculum and an introduction to the four processes of mathematical learning. This will include a brief explanation of each.

Cognizant of the need to keep the participants actively involved, we'll move on to look at examples of how these processes unfold in the classroom. Participants will look at an algebraic connecting activity that demonstrates the power of connecting through language-pictorial- symbolic modes of thought. We'll also look at the power of communication through a geometric construction activity.

The session will close with a short video "Mathematics: What are You Teaching My Child?" (Corporate Source, 1994). This video is designed to help parents understand why mathematics teaching has changed, how it will benefit their children, and what they (parents) can do to help. It will also provide examples of children working in a contemporary mathematics classroom.

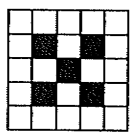
Session Two – Building the Processes. This session will focus on a complex activity which will model the of development of a concept (volume) through reasoning and scaffolded assistance. Participants will look at the conceptual development of volume as a child might who has no pre-conceived notion of 'formula'. We'll build this concept from the ground up as we look at non-standard ideas of 'fill', connecting this to a need for standard measure, onward through the filling of containers, to developing rudimentary formulas using cross-sectional area, to a measuring of containers and finally connecting with the formula for volume of rectangular prism. This will also lead to a reasoning of formulas for other regular shapes. Connections within math will also be discussed throughout this session as we expose the various mathematical concepts that arise in this activity. The value of increased communication between mathematical learners and the problem solving involved in the activity will also be addressed.

Session Three – Algebra: I've chosen to look at the development of algebraic ideas with parents for a number of reasons. First, this seems to me to be an area where parents assist their children by jumping to the final stage. Rather than work with their child on developing algebraic thinking and inductive reasoning, it becomes a lesson in 'just do it like this'. Algebra also offers me the opportunity to help parents realize that the 'patterns' that their children have been 'playing' with for many years, do in fact have purpose. I feel that seeing the developmental connections within algebra (patterns, graphing, equations and problem solving) is particularly important for parents; as is the effective use of manipulative materials. This work in algebra will also serve as an introduction to the idea of routine problem solving.

My intention is to begin the session with a geometric pattern, then look at the idea of pattern growth and relationship between patterns. We'll look at how this representation can be connected to a graphical representation and an algebraic equation. Participants will use algebra tiles to model the construction of equation balancing skills. Participants will also see the application of algebra through using equations to solve problems. This will lead to discussion on the idea of routine problem solving.

Session Four – Non-routine Problem Solving: The final session will focus on the intellectual and affective power of problem solving. Participants will solve two problems in a communicative setting and will receive guidance on how to assist 'solvers' engaged in this process. I intend to use two problems for this session.

One is entitled "What's Left', a geometric cube problem, whereby a 5 x 5 x 5 cube built of 125 smaller cubes is altered by removing sections of the cube. Each face of the cube, as below, shows where the 'pushes' are made through the large cube.



The other problem is a numerical problem entitled 'The Calendar'.

A friend asks you to select a 2 by 2 block on a calendar. When you give him the total of the 4 dates, he then gives you the first date. How did he do it?

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Each of these problems will clearly demonstrate that many solutions are equally eloquent, and that one singular 'right' way to solve the problem does not exist; and that the affective power of problem solving comes, as Polya (1957) stated, from the triumph of discovery.

Evaluating the effectiveness of learning experiences. As with any curriculum construction, its effectiveness must be evaluated. In a general sense, the effectiveness of the program will be evaluated through participant exit comments expressed through an exit log interview. See Appendix D for sample questions.

On a more in-depth level, the interpretation and analysis of the data collected throughout the intervention will speak to the success of the program. The intensity or quality of learning that occurs for the participants as viewed over the duration of the intervention will cast light on the effectiveness of each of the elements of the intervention as well as the methodology used to enable an understanding of these elements.

Phenomenographic Research Design

Participants in this study interacted with a parent education program for mathematics education. This intervention program served as the phenomenon that parents would experience. The conceptions parents formed while interacting with this program were the source of study. I collected data that illuminated the range of understandings parents developed as they interacted with the content of the program. The following section will describe the data collection methods used in this study as well as methods of interpretation and analysis.

Log-interviews. Parent participants began this program with beliefs about mathematics instruction and learning established through interaction with other phenomena, be it their own schooling, media, or their experiences with their children's schooling. In order to examine my participants' learning throughout the intervention, I established a point of entry for their beliefs. This allowed for a point of departure on their journey into current mathematics pedagogy. This data was gathered through a log-interview (Taylor & Bogdan, 1998) prior to the intervention. Sometimes participants responded to questions regarding their perceptions of mathematics instruction and learning. (See Appendix B.) Log-interviews were also used throughout the intervention so that participants had an immediate option for responding to various elements of the intervention.

Participants were invited to share their thoughts in writing via the log when they felt that something in the intervention was affecting their beliefs or connecting with their understanding of mathematics or their work with their own child. They were invited to write at any time during the sessions but time was also scheduled at the end of each session specifically for this purpose. Participants engaged in writing any thoughts on the topic or responded to cuing questions. (See Appendix C.) I wanted the participants to feel comfortable with a method for writing that suited their style. For example, some parents chose to respond to the cues in point form. These logs provided the researcher with information for further programming, acted as stimuli for informal interviews, and added to the bank of data from which categories of description were drawn.

Informal interviews. Informal interviews are such that a natural flow of conversation will occur between researcher and participant therefore a formal list of questions was not generated for the interview. Appendix E provided some guiding questions. After the second session I realized that I was going to have to interview as many participants as possible because their conversations in the sessions were limited. The purpose of the interviews was to extract reflection from the participants about their learning. A search for degrees of awareness, as outlined earlier in the general section on phenomenography, was the focus of conversation. I met with seven of the participants on days separate from the sessions for thirty to forty-five minutes each. Interviews were recorded and transcribed. Interviewing of the participants is viewed as the most important data source in a phenomenographic study as researchers seek to interpret meaning and relational connections in reactions to the phenomenon.

Notes. Researcher notes were written throughout the intervention. These notes were recorded from two sources. A colleague offered to assist in the construction of field notes as the sessions of the intervention took place. These field notes were to be a chronological account of the sessions with particular attention to comments and actions made by participants regarding their learning. The field note writer was to participate in the events of the sessions as a participant observer allowing for a close connection with the content and participants. I kept a journal throughout the intervention to record my observations of the participants' learning.

Analysis and Interpretation The recurring principle of a phenomenographic design is that "whatever phenomenon or situation people encounter, we can identify a limited number of qualitatively different and logically interrelated ways in which the phenomenon or the situation is experienced or understood" (Marton, 1997, p. 97). Interpretations focused on organizing the participants' understandings as categories of description based on the similarities and differences evident in the way in which the intervention program appears to the participants. Categories of description were drawn and redrawn until the categories were "sufficiently descriptive and indicative of the data" (Orgill, 2003, para 11). Analysis generated meaning for the categories that composed the outcome space by attuning to the degree of awareness present in the participants in relation to the elements of the intervention. A search for hierarchy was conducted as I examined how the categories related to each other as well as to the degree of awareness present within the categories.

Chapter 4
Data Descriptors

Participants

This group of parents brought together to learn about math education came with varying purposes, be it as parents of toddlers, or teenagers. They came with varying degrees of confidence with mathematics. They came with varying levels of education and work experiences. The data gathered from these people also varied in its depth. Some participants quite willingly agreed to be interviewed; others shared their thoughts only through writing, and some said very little. The vast majority of the participants came to all four sessions, but others came for as few as one evening.

Recruiting participants for this study proved to be revealing in itself. Conversations with people interested in the program often shifted to 'I can't do math' or 'Norma would like to join but is too afraid to do math' or ' My husband can do math. I'll send him'. In fact this was how Rob and Richard became group members. Time was another issue for parents, as they attempted to fit the program between their children's hockey and dance lessons and winter holidays.

I asked each participant on the first day to respond to a rating scale for their mathematical confidence level. The results showed that three were confident with math and enjoyed it at school. Four stated that math was okay; they could take it or leave it. Seven of the group were hoping that they would never have to journey down the math path ever again.

My participant group consisted of sixteen individuals, ten of whom were consistent in their attendance, fit the requirements for participation in my study (ie. they were parents in the community) and were the main sources of data in this phenomenographic study. The six individuals outside of this group were a mother from a neighboring community who came with a friend to one session; two teachers from Korea interested in learning more about our mathematics education methods; a teacher from the school, who participated in order to be of any assistance to me; and Eric, who attended the first two sessions, and his wife, Lydia, who attended the first session but then declined further sessions. Although this last couple have provided me with little information about themselves, I will make mention of them in my data, albeit with limited knowledge, as their involvement appears to be an important outlier in the participation group.

I've grouped the ten complete data sources according to their rating of themselves on their own competencies in mathematics. Donna, Rob and Terry were the three participants who enjoyed math as students (especially opportunities to problem solve), and were employed in positions that requires competency in this area.

Donna was married with two children. Her elder child had attended the community school in the study but was at the time, in high school. The younger, Riley, was in grade three. Of the participants in my group, Donna appeared to have had school math experiences that were mostly positive. She fondly remembered teachers who were progressive and challenging in their approach to math and classmates that were eager to be successful in their work in math. She was also the only participant that spoke of using any form of manipulative materials in her early math classes. She described them as rods, likely Cuissenaire rods. Donna went on to study mathematics as a part of her course work at university and has also tutored students. She worked as an administrative assistant. Donna joined this group so that she could develop more skills in dealing with helping

Riley with his schoolwork and to offer her support to my master's program. She also knew and said she trusted the facilitator.

Although I was not able to interview Rob, he was probably one of the most talkative in the group and offered insights into his schoolwork assistance with his children. Because January is a very demanding time of year in the financial industry the commitment without interviews was already taxing Rob's time. Rob was also very busy raising his two young sons, Chris who was in grade four and Patrick in grade two. Rob joined the group as his wife volunteered him. Rob entered the program wanting his child's math class to provide him with the basic skills of addition, subtraction, multiplication and division, without the use of a calculator. He also recognized the need to apply these skills to problem solving. He commented on the affective needs of his children: appreciation and enjoyment of mathematics and developing a desire to improve in mathematics. Rob joined the group hoping to learn ways to alleviate some of the frustrations he had in working with his sons on their homework and to learn why math is being taught differently than when he went to school. As a confident student of mathematics, Rob recalled that his greatest enjoyment of the subject came from the challenge of problem solving, an activity that was usually reserved for those who had completed their regular assignment.

Terry was very enthusiastic about the program. He enjoyed math as a child, something he attributes to his mother, who always challenged him with 'puzzles'. His schooling was indicative of traditional programming whereby he recalled memorizing tables. He remembered his own initiative as well, in looking for shortcuts in his work. Terry entered the program realizing that math instruction should offer children a method of teaching that allows them to connect; that different styles should be incorporated so that the children understand the principle. Terry had two boys, one in grade nine, the other in grade four, whom he referred to as his super heroes. Terry's enthusiasm for the program resulted in many after session discussions; he was always the last to leave. He joined the program with the simply stated reason that he wished to "understand and learn."

Those that found math okay as students, where they could take it or leave it, were Julie and Janice. Eric and Lydia were also in this group. Julie's enjoyment of her mathematical studies was marred by an encounter with a grade eleven teacher who, as an authoritarian, would resort to throwing chalk and slamming books if the students came up with incorrect answers. This resulted in Julie dropping the course and moving down to a lower academic math course. After completing high school, she was able to upgrade her math for university entrance. Subsequent to her grade eleven encounter, Julie enjoyed her study of mathematics. Julie has a son in kindergarten and a younger child at home. Julie entered the program confident in her desire to be a resource for her children while empowering them to learn for themselves.

Janice remembered math as being very disciplined with lots of memorizing and that children were grouped according to ability with indicative names such as rabbits and turtles. Grade nine was the year when math became too difficult for her and, as a result she ended up in the lower academic stream in high school math. Janice stated at the first session that she joined this program not so much as a parent, but to advance her understanding of how children learn math and how she, as an instructional assistant at a local school, can learn how to initiate the learning process in her students. Janice's daughters were in grade six and high school, and as Janice herself stated, her participation in such a program as this, would have been more practical for her if it had occurred years before. Both of her daughters were quite independent in their work and their math was moving beyond the scope of both Janice and her husband.

Eric and Lydia were a couple who were both born and raised in a foreign country and were also much older than the other participants in the group. They had a grown daughter, not much younger than the other parents in the group, and a daughter in grade five at the school. They were visibly tired and appeared bored throughout the first session, something attributed to jet lag from a recent trip. Lydia did not return to another session, citing a lack of babysitter and cold weather as the reason. Eric did join in for the second session, politely requesting that we take a break in the middle of the evening's session. Interestingly, I offered the break but no one stopped his or her work or discussions, except Eric, who grabbed a coffee. In describing their thoughts on their own schooling, Eric's experience was one of "very hard, have to memorize all multiplications by heart. If you make a mistake, be prepared for a good punishment" and Lydia remembered "a lot of memorizing and plenty of math exercises." When asked what they believed should be happening in their child's math class, both offered concerns about problem solving. Lydia wanted her daughter to be interested in math, like it, and have the confidence to solve problems. Eric wanted the teacher to "explain in relatively simple terms." He believed that "some children are not smart to understand what teacher is trying to say and solve the problem." Eric also identified a desire to learn Canadian methods of math education and "to follow your thoughts and learning techniques." Eric returned for another session on his own but offered little in terms of response to the evening in his departing notes other than an apology for their lack of enthusiasm in the last session due to their jet lag. He also informed me that he wouldn't be attending the next week as he had a meeting. The only other information I had about Eric's participation came from Julie who worked in his group the second evening. As she eagerly discussed her participation in the activity at hand, trying to consider how her son might see the situation, she was somewhat dismayed at Eric's apparent conclusion that the task was simple for him. Julie referred to him as the really smart engineer who said, I use this everyday, and I know what it is just by looking at it. She felt that he may have seen her as simple-minded as she chose not to look at answering the problem itself but to look at it, as was suggested to the group, from a child's perspective. Although Eric may have served as a revealing data source, his departure from the group terminated such opportunity.

Those that responded to the math confidence rating scale with a three were identifying with a group that had hoped that math was behind them and that they would never have to venture into this study again. Little did they realize that having children would once again bring those fears to the surface. I was fortunate to have seven people in this category as from initial conversations with people in the community I felt this group would be underrepresented, despite being the ones most likely in need of such a program. Two of these participants were not major data sources for this study. One, Tan, was a teacher from Korea, who was working as an instructional assistant in my classroom and was not a parent. The other was Ed, the grade six teacher who offered his assistance to my study. The remaining five of this group require further introduction.

Rena was a single mom with a daughter in high school and a son in grade three. Math was a misery for Rena as a child as she feared having to display her inability to understand in front of the class as she was often called to the chalkboard to present her solutions. Ten years prior to this program, Rena had returned to school to upgrade her grade twelve, a courageous move but one that once again left her feeling vulnerable as a learner. Her presence in a group of young people was one issue; the other was once again the call to display her thoughts in front of others. Rena did see at this time though, that learning math was easier as an adult who has had varied experiences in life. Rena's frustrations working with her son and a desire to learn different strategies and techniques were the motivation that drew her to the program. Rena worked as a receptionist.

Richard wrote that he was not interested in math as a student and that he found it difficult. He attempted 300 Math (academic grade twelve) in high school but then dropped down to 301 (non-academic), a course he found to be more practical. Richard's comments revealed that the practical, applicable learning of math was desirable to that of memorizing information. He wished for his son in grade one that math be fun and practical. Although Richard came to the group as a parent with concerns for his son's education, he was also motivated by his own interest in returning to school. He felt that perhaps this program might show him a better way to learn. Richard worked in the farm implement manufacturing industry.

Ron and Marsha were parents of a boy in grade three and a girl in kindergarten and friends of Janice. Without Janice's persuasion, they would not have joined this program. Ron described his math experiences as "frustrating, always feeling one step behind, not fun and a chore." He missed a few months of school in grade three as his family moved. He felt he had difficulties in math after this time. In the summer before his grade ten year, Ron received a call from the high school telling him they were moving him into math 101 (non-academic grade ten) because they didn't think he was strong enough for the 100 level (academic grade ten). Even so, Ron graduated from university as an economics major. He was able to do some upgrading at the university to allow him entry into the calculus courses. At the time, he was employed as a drug company representative. Ron did not want his experience replicated for his kids. He didn't want to pass his negative attitude about math on to his children. Thus he stepped forward to try to overcome his negative attitude and to learn strategies that would prepare him to be a knowledgeable resource for his children.

Marsha's qualms about math were more pronounced than that of her husband. Her constant struggle with mathematics in school parlayed into a fear of basic math that she carried with her. Marsha joined with much reservation and under pressure from her friend and husband. Her motivation also came from wanting math to be enjoyable for her children and to become more able to assist them. As well, Marsha wanted to learn more about what she could expect with her children's math education. Marsha only participated in two of the sessions. She missed on the evening that Ron was out of town and did not attempt the evening on problem solving.

Bev, married with two sons (grades one and six), had been studying to be a primary teacher. Her math experience as a student was one dominated by a lot of memorization, whereby it was okay if you didn't understand; you were to simply follow the formula. Her motivation for joining was threefold. She was interested in helping out with my research. She wanted to understand and be more able to talk to her own children about math and she wanted to do a better job of teaching math in the early years.

Information relevant to the study is summarized in Table 4.1.

Table 4.1 Participant Information.

Math Comfort Level	Participant	Children	Occupation
1	Donna	Andrea- high school Riley - grade 4	Administrative assistant
1	Rob	Chris – grade 4 Patrick – grade 2	Financial planner
1	Terry	William – S1 Brian – grade 4	Coster for Hydro
2	Julie	Carter – Kindergarten Matthew – age 3	Project manager analyst
2	Janice	Lauren – high school Pam – grade 6	Instructional assistant
3	Rena	Erin – high school Ryan – grade 3	Receptionist
3	Richard	John – grade 1	Farm implement manufacturing
3	Ron	Shawn – grade 3 Paige – Kindergarten	Drug company representative
3	Marsha	Shawn – grade 3 Paige – Kindergarten	Unknown
3	Bev	Trevor – grade 6 Jake – grade 1	Student teacher

Data Descriptors

The group as a whole were eager, polite and cooperative. They were always very focussed on the activities we explored, offering little discussion outside of the task at hand. This made it difficult for me to gather data about their thoughts and concerns via field notes. I also found that the writing at the end of the evening was useful but slim. I realized after the second session that I would have to conduct as many interviews as was possible. This was where the bulk of my data was generated. Each method of data collection, however minimal, contributed to the descriptions of the variation in how the phenomenon was experienced by the participants. Following are descriptions of each data collection method as it pertained to this particular study. Quotes are recorded verbatim. Any grammatical errors present in speech have remained unaltered from their original presentation.

Log-interview. This term, coined by Taylor & Bogdan (1998), was essentially a journal where participants were invited to respond to my questions or write any notes they wanted as the session progressed. Initially I had hoped that the log would provide a rich collection of their inner thoughts on the process. I soon realized that after a long day at work and two hours of math education, participants were not really up to composing lengthy records of their thinking. I also realized that they were uneasy with putting their thoughts down, for fear what they had to say wasn't what I was looking for or that it would be substandard in writing quality. I believe that as our relationship formed, participants became more at ease with both of these concerns. They started to realize that I was interested in how the program was affecting them, that were no wrong answers and that I wasn't grading their writing form. Each of the participants' entries was photocopied and the originals were returned to them for their use.

I began the log interview process by asking parents to respond to some entry questions that would give me a look at their thoughts prior to any intervention. I was interested in learning about their math experiences, what they wanted for their children in math education and why they had joined the program. Julie's response to these questions provides an example of the process.

What was math like for you as a child?
Enjoyable up to grade 11 when I had a teacher who was very authoritarian (threw chalk at students, slammed books on his desk) when we had wrong answers. As a result, I transferred from 300 Math to 301. The lower level math was easy for me, however I did have to upgrade as an adult for University entrance.

What do you believe should be happening in your child's math class?
-variety of learning styles considered in activity/exploratory based learning opportunities.
-building on our child's strengths to develop confidence. Offering alternate ways to reach the answer/solve problems.

Why are you here?
-want to support my child(ren) – Carter is a Kinder, and Conner (4 yrs) will be in Kindergarten next year.
-want to be a resource to my children while empowering them to learn for themselves.
- want to learn and understand how and what (approach) to be effective supports and resources for my children.

To encourage participants that they were indeed on the right track with their writing and to enhance the quality of the learner-teacher relationship, I chose to personally respond to each participant weekly. Participants appeared pleasantly surprised when they arrived to find a response to their own writing. My responses to Julie's initial entry above and to Richard are noted below.

Julie,
Your enthusiasm for your children's learning will be rewarded. When you talk about the counting of trees and adding games, you are fostering a view of the world through math. Children with these experiences develop a stronger number sense, which enables them to see the big 'net'.

From the few notes I've read, sounds like we're on the same page – 'empowering them to learn for themselves' – I really believe this is a key to their success. Without realizing it, wanting their children to be successful, many parents actually 'steal' learning away from their children.

I'm looking forward to hearing your reactions to this program as we move along. I'd really like an opportunity to speak with you in more detail.
Deb

Richard,
Appreciated your comments about positive experiences in math education. Interesting to me that the point you made about application is what we are trying to create throughout a child's math experiences. Unfortunately for you, this experience didn't occur until high school.

Hoping this program will have an effect on your learning of math – perhaps your searching for connections and reasoning will help. Seeking practical applications for your son as you assist him through the years will cause him to understand more deeply as well. When kids have difficulty understanding an algorithm, I often go to a practical situation. Last week I had a grade 7 boy trying to do something like 11.8 divided by 0.5. He wasn't sure what to do with the decimals, so we started by thinking about what the question might mean and estimating. Eg. If the teacher has about $12 and wants to give some students 0.5, or 0.50 or more meaningfully 50 cents, how many people could get 50 cents? He could easily reason about 24 people. So when we went back to moving the decimals over, he could see where they move. The meaningful story also helps him to understand the decimal answer – no we won't get exactly 50 cents or 0.50 but a little less. Why and how we can just move over the decimals is another story. The point is, any situation can be better understood by connecting with a situation that makes sense.

Deb

The second evening began by having them reflect on the first session and write about what stood out for them or stuck with them over the week. Rena wrote:

> I thought it was interesting about the different ways that are accepted to get the same answer for example 144 the question that you had asked us what the answer was which was 168. I came to the answer by the "traditional" way, but there are so many other ways that are acceptable. I feel bad now, that when my kids required help I was getting upset with them for not showing there work, I actually thought they were being lazy trying to get there work done as quickly as possible.

The second and third sessions ended with their writing reflections on the evening. I did put up a list of guiding questions, as shown below, with an invitation to follow the list only if they desired. My attempt was to get them to write that which was important to them, not just what I asked for, while still providing direction for those feeling uneasy about selecting their own topic choice. Eric was the only participant who chose not to respond at all. Some, such as Terry, chose to follow the list of questions:

1. Has your understanding of math learning been altered today? How?
Yes, it is exciting to see the changes being incorporated. I feel it is a far more realistic approach to today's work environment.

2. Has your learning of mathematics been affected?
Yes. It has expanded my thought process to look at alternate ways of arriving at a solution.

3. Have any ideas made you excited or angry?
Yes. In terms of discussing problem solving with the "Super Heroes" (my boys) I am excited.

4. Was something particularly helpful? Why?
I think that the building off of information previously learned and applying it in a number of different ways is reflective of how people function in everyday life.

Others chose to free write, such as Richard's response to the second session and Rena's response to the third session.

Richard (January 21, 2004):
- I am amazed how you brought in so many topics just by doing a volume class
- It shows the web that math really is
-The triangle volume made sense at the end
- If you have any questions *****@mts.net

Rena (January 28, 2004):

I was surprised by how much of the material that seemed to come back to me as we were working on the problems. Working with the tiles really helps to pull it all together. The visual demonstration of the tiles makes the overall concept much easier to grasp and understand.

Entries in some logs, such as that of Julie and Bev, merged a record of what they perceived to be important ideas from the session with their personal responses to these ideas. An excerpt from Julie's notes on the last session exemplifies this process. Julie records a process that people might go through as they problem solve (my thoughts) then draws a parallel with a previous comment I had made about kids playing video games and her own experience as a counsellor.

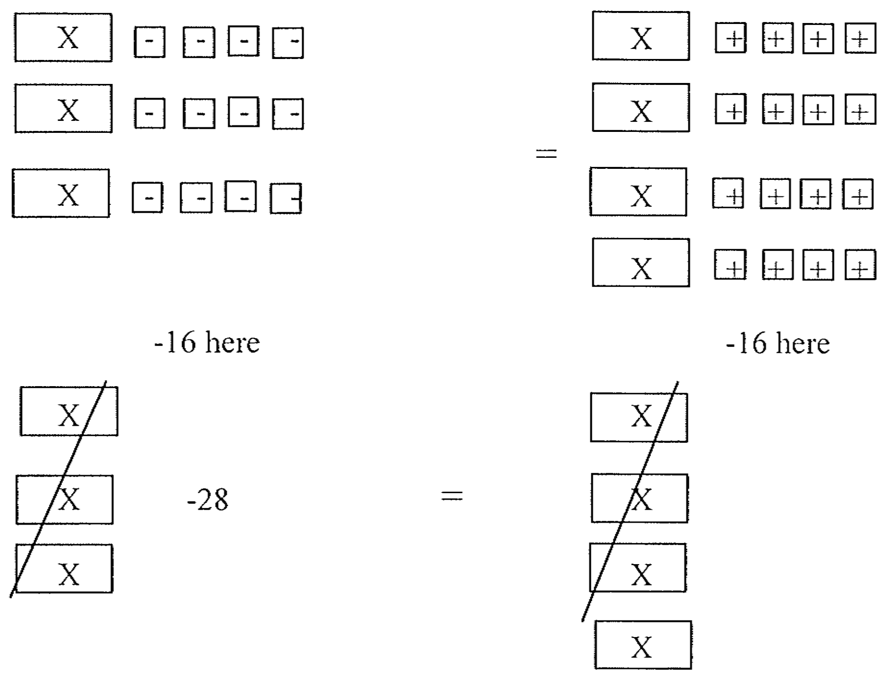
Process of Problem Solving:
1. trying to understand the problem
2. feelings (frustration, stress could also be anxiety)
3. different routes/strategies
   eg. drew picture, looked for patterns, algebraic equation, guess & check, verification
4. reread / relook at problem
5. take break if stuck
6. collaborative effort
7. persistence, keep trying
8. moment of clarity
9. check out again
10. triumph / relief / elation

parallels drawn - Nintendo
- counselling (my profession)

* amazing parallel to counselling profession problem solving therapy techniques used with clients

Bev showed examples of how she used the algebra tiles and reflected on her learning at the same time. The actual example $(3(x - 4) = 4(4 + x))$ had been used in a different context earlier in the session. Bev brought the example back and used that which she had learned from the simple examples with tiles to solve the more difficult equation.

> The algebraic symbols seemed so easy, with the initial pictorial representations. What a wonderful way to learn! I loved algebra as a student in high school and today you brought a lot of it back – seeing it with tiles was so easy to solve.
>
> $$3(x-4) = 4(4+x)$$
>
> I could remember how to get my x to one side & with the tiles it seemed so easy. I did it a little different but it worked.



-28 = X

I wish I had tiles to work with in high school. The equation doesn't seem so scary anymore
My e-mail ****@mts.net

Some entries were very minimal such as Marsha's entry after the second session.

It's not such a blur. I still have a lot of holes in my net, but seeing things again after a long time has helped me understand a bit more tonight. Thanks.

Electronic Mail. Because participants were justifiably tired by the end of the evening, I invited them to correspond by e-mail at some point during the week. Donna accepted this invitation within the first week, passing on some reflections she had had about the first session. She continued to interact both in her class writing and by e-mail. Richard responded to an e-mail I had sent him after he offered his e-mail address to me. This data collection method allowed my participants to take the time to reflect about their learning in the program. The following is demonstrated through my communication with Richard.

----- Original Message -----

From: Deb Woloshyn

To:

Sent: Thursday, January 22, 2004 10:48 AM

Subject: math program

Richard,

Thanks for the e-mail address. I do in fact have some questions.

I'm intrigued by your using this information to 'play with math' with your children. This is the 'setting the stage' idea that will affect your children's number and math sense. You mentioned that you like the 'backpack' idea. Can you tell me more about your thoughts on this idea? How do you see this transpiring in your family?

In your initial notes a part of your reason for joining was to affect your own learning of math. Working at all yet?

Deb

It has only been two sessions with you and I am already "seeing the light" on the how children learn and what can we as parents do to help foster the seed of mathematics that I am starting to believe is in each one of them.

For example, this weekend (it might sound silly) I was cooking bacon. There were 13 pieces and three people. I saw an opportunity to slip the concept of fractions into breakfast. I asked my son John how many pieces of beacon will each of us have. He thought for a second and said "You and Mom will have 4 each and I will get 5" (he really like bacon). So I asked him how could we divide up the 13 pieces so that we all had the same amount. He said to cut the last one in 3 pieces. So I said I told him that we would each get 4 and 1/3 pieces of bacon. I might have lost him on the wording but I know that he understand the concept.

I am going to keep looking for situations that I can use to challenge John mathematically

As for me the challenges that I am going to be facing will math. This program is showing me how many holes are in my net of understanding.

My problem is probably the same as many adults. You can teach me almost anything and I will understand it and be able to do it at that time but, once I am finished the class give me a week or so and it is like I never learn t it.

I hope this is helpful information.

Richard

Field notes. There were two kinds of field notes; those taken by a colleague during the sessions and the notes that I wrote at the end of the evenings. I had one colleague who attended all of the sessions with the intent of keeping a chronological account of the sessions while paying particular attention to comments and actions made by participants regarding their learning. The resulting notes turned out to be less useful than I had anticipated. Although he did collect a chronological account, I had hoped he would gather and record participant-based data that connected to particular events of the evening. This occurred in a very abbreviated form as demonstrated in the example that follows. One problem that my colleague and I found was that participant discussions were minimal as they concentrated on the mathematics at hand. I also believe I did not clearly indicate to my colleague the particulars that I was looking for as data. I wasn't sure of this myself in the beginning and failed to make my emerging requirements for data explicit to him. He dismissed as unimportant, conversations about how their child might look at an event or how this looked like their work. Or quite possibly, these conversations didn't occur when he sat at a table. Regardless, it was worthwhile to have Ed join the group as both a parent and teacher at the school. He felt comfortable to interject with ideas of his own on a number of appropriate occasions. The following are Ed's recorded notes from the second session:

Goal to talk about big ideas
-thinking and reasoning

-volume of rectangular prism (grade 6 outcome)
-look at model to break it apart

Connect - modes of thinking

Speak hear move it build it

Pictorial $\rightarrow$ Concrete

1	3
2	3

-regular polyhedra – 3d shape
-polygon – 2d shape
-What is a prism? – 3rd group because it's a 3D rectangle
-It's the math terms √ + throw
- prism like the light
- 3 rows of 4
- 6 rows of 2
- its terms √ terms √+ bug me
- zone of proximal learning – optimal learning
-Math is a universal language 2 $\rightarrow$ L W

3D $\rightarrow$ L W H

- Bev prime # only 2 factors
Composite - more factors
- L X W X H ÷ 2 =

- If you work off what you know
- quoting jobs / had to relearn to figure it out
- L x W^2
-William uses language at home grade 9
- enter into your child's path of learning
- Ask Marty -

My field notes, recorded after the sessions, collected some of my observations of the evening but were also less productive than I had intended. I had anticipated coming home and not being able to type fast enough to record the gist of my participants' conversations. This wasn't the case; as mentioned earlier, each participant was more drawn up in the mathematics than freely chatting about their children or other matters important to them. I also found it difficult to clearly detail the conversations I did have at the end of the evening. I contemplated trying to tape record these conversations and even went so far as to set up a machine and tape, ready to turn on at the end of the evening. I found the idea of turning on the tape after the session to record informal conversations to be intrusive and destructive of the moment. My field notes did however help me to organize my thoughts about direction in my data collection and enabled further organization in terms of communication with my participants. An example of these notes is included as Appendix F.

My reflections through field notes helped me to realize that I was going to have to interview as many participants as possible. I increased the solicitation of interviewee candidates and found by the third session that many previously interview-shy participants were warming up to the idea. By the end of the intervention program I had conducted seven interviews. Each interview was based on the questions outlined in Appendix E, but strayed from the outline as participants' ideas led the direction. My intent was to attempt a more conversation-like atmosphere rather than interview format. This changed some as I moved through the interviews because I questioned if I was leading their response somewhat with my ideas and wanted to lessen this influence. I believe my later interviews contained less of my thoughts than the first two. All interviews were transcribed in order to ease the interpretation of the data.

Chapter 5
Categories of Description

What is the nature of the learning that occurs as parents attune their perceptions of mathematics instruction and learning for themselves and their children and align their practices with that of the current methodology espoused in the Manitoba mathematics curriculum and by the National Council of Teachers of Mathematics?

A phenomenographic study should expose the range of participants' perceived experiences of the phenomenon and seek to analyze the learning of these participants. The data collected should and will reveal only so many ways that a phenomenon is experienced. I have organized the experiences of my participants into categories that gather their experiences by commonalities. As my participants discussed their learning, three main categories of description emerged:

1. Parents often talked about how the information of the phenomenon *fit with their lives*; how it spoke to their own careers as well as how it addressed their own children or affected them emotionally.
2. Parents also provided evidence that they participated in the act of learning by taking on procedures in their roles as parents or teachers and as learners of mathematics themselves. I have called this category their *procedural learning.*
3. Parents also gained *conceptual knowledge* of mathematics education throughout the program as they reflected upon their memories of their own mathematics learning history and also that of the current mathematics curriculum.

This section serves to present the data as organized by these themes. Unless stated otherwise, all quotes are recorded verbatim from the participants' interviews. Please note that Terry and Rena will be discussed in greater detail in Chapter 6. To avoid duplication and to provide clarity for Chapter 6, often quotes associated with these two participants are found in Chapter 6. When mention of an incident with Terry or Rena occurs in Chapter 5 without a direct quote, a referenced page number is given for the latter chapter.

The Personal Impact

Parents' discussions resulting from their involvement in this mathematics education program often pointed to aspects of their own lives. How the elements of the program were reflective of their own work and how ideas were lived out in their own homes surfaced repeatedly. The emotional impact resulting from their participation in the group also speaks to the degree that the program has affected their personal being. Examples indicative of the personal impact this program has had on the participants have been categorized according to work, children and the affective element. Each of these aspects of their lives has been drawn upon as participants worked to make sense of the program.

Work

Julie presented herself as an active learner who was also willing to share that which she was learning with me. On more than one occasion Julie connected with her work in the health field. Her first realization that what we were doing in this program was being played out daily in her work life came in an interview that occurred half way through our sessions. Julie had recently accepted a new position as a project manager analyst. Her work involved pulling together the project charters, plans and milestone charts and then implementing them with the project teams that she supervised. She described a moment at work whereby she consciously realized that mathematics played a role in her work.

> It was the first time I'm thinking, I'm using it, I'm using math to keep myself organized in these graphs that I have to make that I funnel up to my teams and just the project plans where it's 1.0 or 1.01 and you kind of go down and you're actually using it to keep yourself organized and to communicate with other people. Just from last week, a light bulb went on 'math is a language' and I've never thought of it that way until you started speaking about it like that.

Julie was also surprised on another occasion when the process by which we solve mathematical problems was indicative of that which she has used in a previous position as a social worker. She described the parallel to problem solving therapy techniques in the counselling profession as amazing. These techniques ask clients to, for example, understand the problem, manage frustration, and look for different routes. (A more detailed list from Julie's notes appeared in the data descriptors section. See Chapter 4, p. 55)

Ron's connection to his work in sales was also significant. Ron drew parallels between the changes in math instruction from his student years to current methods on the one hand, and the evolution that had occurred in the work force on the other. Ron's math education consisted mainly of memorizing steps and formulas. Similarly his work in the early days of his career consisted of being:

> ...told how you do a sales call. Memorize it and do it, where it's impossible, everything's different. You have to be able to take a look at a problem, come up with a solution and learn from that solution if it's wrong. And that's what life is about and that's more of what I got out of these sessions. That's how you as a teacher and the curriculum is building towards the math. I think it's great.

He saw that math education was replicating the methods implemented and desired in his work. Ron also saw the communicative nature of constructivist learning in his sales meetings where they share openly about their work and when they present information to clients in an interactive setting. This has changed from the days when information was disseminated in an instructive lecture from a knowing body:

> That's the way we meet for our sales meetings and it's not a didactic lecture anymore, it's all workshops, we're going to break out into groups. You guys do this question, you guys do this question, and then we're going to present that. (Me: So it used to be lecturing?) Oh yeah. We used to go into sales meetings and somebody would stand up there and talk for 20 minutes and then you'd have a break and you'd come back and talk for another 20 minutes. And when we did stuff to present to the clients, it would be okay we're going to find a specialist who'll get up in front. He'll speak for 45 minutes, we'll do 15 minutes of questions and answers and that will be the... That's not how people work and that's not how people learn. So now it's all workshops.

Terry was constantly drawn to connecting his involvement in the sessions to his work during the week. In the beginning Terry talked about the amount of math he used in his work as he does costing and determines and converts measures. As we worked into more problem solving, Terry recognized that his job was problem solving. (Donna also made this statement about her work as an administrative assistant.) He expressed the belief that any career requires that one be an able problem solver. (See p. 131.) He also spoke to the need for high levels of interaction amongst the employees in his organization as they determine construction plans, expenses, time allotments and customer needs. During the sessions Terry's interpretations of instruction methods also parlayed into an assessment of needs in the work force. He described math learning in his day as creating a plethora of people who are afraid to fail. (See p. 132.) Today's methods lend themselves to children taking risks as they explore, discuss and try to bring understanding to mathematical ideas. Work in the manufacturing industry requires that one can take risks and manage those risks as they learn from the errors (in Terry's example inaccurate pricing). Taking risks may also, in Terry's estimation, lead someone to the attainment of that desired contract.

Breaking into groups and discussing ideas was also prevalent in Terry's work environment as adults must be able to work together, listening to and valuing each other's opinions. Following the sessions, Terry shared that this program had actually influenced the way in which he does his work. He said that he had actually applied a couple of the principles and approached problems differently than he had in the past. Although it was difficult for me to draw out and understand exactly what this difference was, it appeared that he now approached his work in a more holistic fashion where he often asked himself 'what does this mean', rather than accept numbers and figures on the surface. This "whole different approach to cost estimating" was allowing him to determine a more precise cost for his product. (Further details appear in Chapter 6, p. 131.)

Janice discussed how ideas from this program affected her work as an instructional assistant. More often than not, her thoughts were connected to enhancing her already developing ideas about learning math, such as how motivation is an enabling factor and how success in problem solving can breed this motivation. She recognized how a teacher's perspective on learning, as either that of understanding or as merely content attainment, can affect a student's desire to learn. Janice discussed an incident in her log whereby a teacher she worked with told his class, "'I know you hate this and you think you'll never use it so why do we need to learn it'." Janice saw that "with the right approach and a positive attitude kids can be taught that the challenges of math can be fun" in its own right. She believed my message to them was that "students should challenge yourself, try to put it together, figure it out and feel the pride when they solve it"; in effect, a teacher's excitement about math is a part of the success. Janice's questions throughout the sessions pertained to how one initiates the learning process. She determined in the end that she needed to transfer from "move over, I'll show you how" to a method that required her to ask questions about the student's current understanding which allow him to realize that he can, in fact, think for himself. Once she has established where his thinking is at, she'll join him as a cognitive partner and work from there to build understanding.

Although I hadn't anticipated that their work would play a role in this program, this aspect of their lives surfaced regularly in the sessions and within their writing and interviews. This served to reinforce my belief that learning is situated within who we are and what we do, what constructivists might describe as "assimilating knowledge into the learner's existing mental framework" (Hanley, 1994).

Children

As expected, participants connected their learning in this program to the context of their lives with their children. Data in this regard was vast and varied. Some participants were spurred on by the program to reflect on situations with their children where they hadn't previously seen the depth of their children's mathematical thoughts. Others expressed stories about where they now see mathematical learning opportunities that they hadn't recognized before.

Recognizing math in past experiences with their children. After the second session, Rob relayed a story in his notes (and elaborated on it afterwards in conversation) that reminded him of the difference between being mathematical and following mathematical procedure. While driving his son to an activity one day, Rob's son told him "Integers are like looking in the rear view mirror. The positives are like looking into the mirror and the negatives are what you see on the other side." Rob remembered being impressed at the time but hadn't initially realized the power of Chris' analogy as a way of learning math that differed from his own. Chris wasn't working from an algorithm to solve integer operation questions but from an understanding that made sense to him.

Ron reflected on a situation regarding his nine-year-old son's portfolio presentation. As Shawn proudly demonstrated his understanding of three-digit addition with Dienes blocks, Ron sat quietly beside him having no idea what his son was attempting to show him. After working with algebra tiles in this program, Ron realized that the blocks his son was using actually represented varying amounts. He saw that his son could do the demonstration but could not explain it to him.

> Ron: It would have been nice to do more stuff from the younger age groups. Just see what they're doing. I know that they're working with, they have blocks. Shawn did a problem for me, we added up a hundred and twelve or something plus whatever and he used the blocks, and one was tens and one was hundreds, and I really didn't understand that until we started doing some stuff in here where you showed, this is what it is. He could do it, but he couldn't explain it to me.
>
> Me: But from what we did, you connected the … yourself? (Yeah) That exercise made sense to you, even though you didn't do it, because of what we did here?
>
> Ron: Yeah, because of what we did. The one day you were showing us this block is x squared. I went okay they're assigning a block to a value, and okay, he was saying these five singles make a, these ten make up these one shape, and okay now I understand what he was doing and how they got to where it was. He understood it; he just couldn't verbalize it to me. But now that made sense because, when I left I was, okay, I'm trying to explain it to Marsha because she couldn't go to that parent teacher interview. I said he's doing something with blocks and he added something up and I didn't understand it. But now I understand what he was getting at so.

Ron was able to transfer his learning with tiles in algebra to an understanding of his son's previous work. Ron also talked about previous frustrations with his son's homework, as questions would often come home with no explanation on how to do the problem. "Yeah and I think that's going to be good, especially for my son. My daughter is only in Kindergarten, but Shawn is in grade three. There's been some issues with …

hopefully this is going to be able to help us to help him. Instead of...you know...I know sometimes he gets frustrated with me over doing his homework. 'Dad that's not how it's supposed to be done'." His understanding of these events has now been altered as he realizes the purpose of the activity in a more global sense.

Terry's stories about his boys were representative of past events as well as new perspectives on current experiences. Shopping with the boys has regularly been an opportunity for his sons to put their math skills to the test as Terry often asks the boys to do some price comparisons and find the best deal. (See p. 136.) This is likely a math experience that many families face at some point and one that would be easily identified as proof that everyone does mathematics. Terry's other stories however were deeper, in the sense that they got at elements of math that were beyond arithmetic. He talked about his older son being able to establish what the question was that needed to be addressed in a situation that the younger son had initiated.

> You know what? Now I gotta think, 'cause Brian had a question regarding something and I had an answer for him. One of the things I try and do is, not necessarily give the boys the answer, but give them the tools to find the answer, and so we talked about it and Will just, he just came up with the formula. And it caught me so off guard, and I had to think about it actually, for a second, and go yeah, that's absolutely correct. What he did was, what was neat was, he's taken what he's learned and he's applying it to real life, and he's determining the question first. And that's what struck me, it wasn't that he had the answer or any of that stuff, it was that he would, he had figured out the question. Because once you figure out the question, then the answer can be easy. And that's neat about the curriculum at this school, 'cause most people don't ask for the question; they just do what they need to do. And they're answering questions and they don't know what the question is.

Although Terry couldn't communicate the details or even the context of the situation, he felt that this act on William's part was somehow significant in his son's ability to be mathematically creative rather than just follow what he'd been told to do via an algorithm.

Terry also spoke of a situation with Brian's homework where he pulled out tiles to represent the fraction work. But Terry didn't leave it at fractions, they went on to talk about squares and square roots: "I mentioned that Brian had that one fraction problem, he was kind of struggling with and him and I went through it. We probably went further than we needed to but (laugh). But we, we used the tiles, the white tiles, and black tiles, and … and bringing numbers down to their root. That's what we did. And that was the extra step that we took that we didn't have to." This appears to be Terry's attempt to, as was discussed in the program, set the stage for later learning.

Recognizing math in new experiences with their children. Julie and Richard, parents of primary-aged children, both talked of new experiences with their children. Having little homework with which to assist their children, both saw their role as one whereby they should help orchestrate mathematical moments that are implicit in nature and occur in day-to-day functions.

While waiting at the doctor's office with her five-year-old son, Julie pulled her calculator from her purse and made a game out of using the calculator. They would ask each other simple addition questions then check it on the calculator and they'd talk about "Let's read it together Carter. I'm using my finger like I do when I'm helping with his reading. I'm starting to see a lot of potential for us as parents to really encourage our children. All the time. It doesn't have to be structured, let's do math now." Her son was also matching words as he pressed the buttons ("two plus two equals") and looking at repeating patterns on the display. Prior to the program Julie told me that she was "brought up to believe that the calculator was cheating", and had assumed that her children would learn their operations long before they would be allowed to use a calculator. Julie had begun to see the value of setting the stage for later learning at school by playing with mathematical ideas informally at home.

After the session on developing volume concepts, Julie went home with the intent of playing a new game with her children. All of the family spent an evening filling different boxes with Lego blocks, while estimating how many would fit, talking about aligning the blocks to maximize the number, finding other boxes that would fit the same amount and also looking at patterns with the blocks. Julie was excited with the thought that the opportunity for mathematical experiences with her children was endless. Prior to the program she felt that their social needs and getting in touch with their feelings were paramount to their development. She stated, "I think I'm just starting to notice that it's just a part of everyday life. More so than, I guess I was taking it for granted. Being in the health field, I would always be concerned about my children's self esteem, mental health, feelings. I do a lot of stuff on feelings and empathy with them. Didn't do a lot on math because I thought, it wasn't such a great experience for me; they'll figure that out."

I did not interview Richard but did share discussions with him through his writing in class and e-mails. Richard had made a personal commitment to look for math learning opportunities with his son. He wrote, after the second session, that he "planned to 'slip' math into their family life as 'fun' or a game/trivia.". In a subsequent e-mail, he relays a story about their Sunday morning breakfast.

> There were 13 pieces of bacon and three people. I saw an opportunity to slip the concept of fractions into breakfast. I asked my son John how many pieces of bacon will each of us have. He thought for a second and said 'You and mom will have 4 each and I will get 5' (he really likes bacon). So I asked him how we could divide up the 13 pieces so that we all had the same amount. He said to cut the last one in 3 pieces. So I told him that we would each get 4 and 1/3 pieces of bacon. I might have lost him on the wording but I know he understood the concept. I am going to keep looking for situations that I can use to challenge John mathematically.

This example demonstrates Richard's ability to connect the idea of providing math experiences for children before the concept is formalized.

In her interview, Rena talked about a new experience with her son while he worked on a mathematics homework assignment.

> Rena: Um, sort of, but just minor stuff in terms of how… there's rows of blocks and a certain number of ones times a certain number of blocks and you …. So I think I dealt with it differently. Not that it was very difficult but you could feel him responding to me. ……(Interviewer: So what did you try to do?) Well, I just (pause-thinking) okay, I just counted. I started to count backwards, so count these for me, and this is the number and this is ones and then count across and then do the number across and then the number down. … whereas just looking at the picture, you have no clue.
>
> Interviewer: It sounds like you helped him break it apart some?
>
> Rena: Yeah. (to solve it?) Whereas before, I wouldn't have known where to start. …..I didn't know what they're trying to get at.
>
> Interviewer: Do you feel you have a better grip on that?
>
> Rena: I think so, yeah. I feel that so far I've gotten stuff that I needed.

Even though she perceived the task to be minor, the effect was grand. As her son worked to understand the area model for multiplication, Rena helped him break down the concept by having him count the tiles on each side. She felt that she had helped him deal with the work differently than she had in the past as she could feel the difference in the response he had towards her input. In the past, she said she wouldn't have had a clue as to where to start with him, as she hadn't known what the task was trying to get at. Rena's attempts to apply aspects of the program to her life with her son are apparent.

These stories could very well belong to any parent as they try to provide and reveal moments of mathematical learning in their children. Each of these stories suggests that participants saw that their role as their child's teacher isn't just about teaching how to do a mathematical procedure, but to see mathematics in our lives. They also saw that the primary goal of working with children should be about their understanding. The stories say much about how parents were internalizing their learning from the program, something that will be discussed later in the analysis.

The Affective Element

> *" I'm more accepting about math because I understand and I see, I feel that I understand it better now than I ever have, just from those four sessions. So I'm really happy with that. It's not something that I necessarily dread but it's more, 'how are we going to do it?' Which I think is going to be good for the kids because, for me, if I'm, like I said before, if I'm not excited about it or if I've got a negative attitude towards it, that's going to filter down to them. So I'm pretty happy with it. It's a positive not a negative."* *Ron*

When my participants revealed how the program related to them on a personal level, talk of their work and their children emerged naturally. There was another element that lay below the surface: how the program affected them on an emotional level. In some cases, evidence suggested that their sense of identity was being affected and for others it was a responsive emotion expressed through a few descriptive words.

Affecting Identity. It was Rena's comments about herself on an emotional level that drew my search for categories of description in this direction. Rena often made reference to her lack of confidence with mathematics, not trusting herself, and feeling inadequate when helping her son. After telling me about the occasion when she helped her son with the 'tiles' assignment, she said, "I also get scared, because he's .... grade three. Not knowing how to help him. And I feel I'm inadequate. It's kind of ...

inadequacy is a very uncomfortable feeling for me. So I feel like I don't know what I'm doing." I asked her if she felt that this was changing for her. Her response was "Yeah, I'm glad to do that and so, so far so good." She said she's working to reduce tense moments by avoiding reactions and taking the time to work together as a team. Rena, as a student, found mathematics very difficult and stressful. As we moved through the program, Rena's confidence in learning math grew. She attributed this to the method of instruction. She found herself very comfortable working in groups of three or four and sharing her ideas. (See p. 126.) Confidence grew as her ideas became accepted and validated by others. She said, "It's not uncomfortable feeling, you're part of a group, and knowing that what you're doing is valid." Being able to contribute to the solving of a problem became empowering for Rena, just as I would hope it would be for a child in the classroom. Learning some strategies for working with her son contributed to a sense of acceptance and brought about less negative reactions.

Terry and Bev confirmed their sense of identity as they claimed that many ideas in the program reinforced what they believed about how children learn. Terry held a belief that in order to help someone learn math, they "have to be able to understand" and that you have to meet them where they are at on a conceptual level. (See p. 136.) Bev, who was studying to be a teacher, wrote, "My philosophy seems to fit with your beliefs. I was very aware of the curriculum and the importance of developing a deep understanding for each new concept. Thank you for the reinforcement of my beliefs." Julie spoke of her desire to share what she was learning with her husband and mother. She wanted to explain "this whole thing" to her mother because she sat with the boys when Julie's husband was out of town. She was excited that her boys would enter into a school system where they could "explore" and "blossom." Julie also shared that the program allowed her to "challenge" herself and to "learn."

A wonderful way to learn. Positive comments about the program revealed the impact the program had on them on an emotional level. From a number of people I read, 'I wish I had learned math this way'. Donna described their group work as "empowering and just a really valuable learning experience." Rena conveyed this message as well. Ron used numerous adjectives to describe his experience: "enjoyable...fun and worthwhile...neat... the learning method was great...amazing ...and mathematics is becoming clear."

Janice used terms such as "wonderful" to describe the possibility that a program such as this could occur for other parents as well. She felt that the program could actually "take away their fear." And she used 'wonderful' again along with "incredible" to describe what a child may gain from hands on exploratory activities that will later on become more formal mathematically.

Bev remarked that "this is a wonderful way to learn" and that " you have excited me about math." She also spoke for the group in her writing when she stated, " you have empowered us and make us feel good about math." I was also intrigued by Bev's comment that "problem solving is exciting, a little tense, but it really seems to get my adrenaline going." Bev's comments were influenced not only by her participation in this parent program, but also by her work for the past two years in a math education program at the university. Recall that Bev labelled herself as a math avoider stemming from her childhood experiences. She had also stated in her writing that she was devastated when she realized that she was going to have to take a math education course at university and wondered why she should have to, considering that she was going to teach the early years.

Such a change in perspective may be attributed to the power of the math curriculum and NCTM Standards.

Terry labelled his sharing of his evening sessions with his boys as a "terrific experience" and spoke of his son's ability to formulate the right questions from a situation as "powerful." Terry's summation of the group seems fitting to end this section:

> *I think people were excited because there is, you could see the light go on. That was kind of neat. You could see the light go on and the realization and the frustrations in some cases were, I spent all those years, not knowing something that's very elemental, that I could have learned, and saved this time.*

Procedural Learning

> *In a short span that so far things have been great ....I try to do more things with him...trying to be more accepting...he's doing things and he's not fighting me or not running away from me. He's actually putting his head on my arm and doing things and that's okay. I need to keep this up....*
>
> Rena

My intention in this program was to provide a learning situation that could meet all learners; not one where everyone would learn the same content, but one where each could extract much that would fit his/her own needs and situation. From this, parents drew out ideas that affected how they work with their own children and how they themselves approach the learning of mathematics. This section will discuss ways that parenting practices regarding work with their children have been altered and aligned with NCTM ideology. Examples of these alignment practices will be given. Participants have also gained new procedures for learning math for themselves. Examples from the data will illustrate this point.

Of Parenting

If parents are to learn about helping their own children, they must learn what works for their child. A technical 'how to' program on math instruction may not meet the unique needs of the parents and their children. A philosophically sound and idea rich program may allow them to extract their own 'how to', precisely tuned to their own child. As with any learner, one must assess importance based on their own judgement of their own needs and situation. This 'how to work with my child' theme emerged as a valuable collection of procedural ideas, very unlike a recipe for success that each should follow. I used an analogy with the parents of raising a moral child. There is no set of rules, no tried and true recipe to follow, but a set of guiding principles that we each comprehend and adapt to our own situations. My goal then was to present a set of 'principles' about math education and have parents draw out what applies to the context of their own parenting situation. These principles centered around an underlying philosophical stance, as presented at the first session, and the goal of helping children to become more mathematical by engaging in the processes of reasoning, communicating, connecting, and problem solving. The discussion that follows represents the ideas that parents valued in altering or improving their parenting role.

Learning should be social. Some parents recognized the value of the social aspect of learning. Donna, Julie, Rena and Janice were participants who commented on a need to direct their children to call peers to discuss mathematical ideas, even though their situations were very different. After the second session, Donna wrote "I have struggled to try to relearn the concepts independently (using a dictionary and or the textbook) before providing help/teach." Her new plan, as she wrote on the last day, was to "try to be more patient and provide an opportunity for him to 'lead' as we problem solve together. I will pose more questions to him. I will encourage my child to consult with friends with homework." In the past, Rena said she saw peer interaction in mathematics as a form of cheating as she could only imagine that children would just be giving each other answers. (See p. 123.) After completing the program, she herself felt the difference, as interactive learners support and spur ideas in each other. Julie, the mother of young children, saw that she could encourage her sons to interact in mathematical ideas through play with their peers. Janice, whose girls were twelve and eighteen, realized that her girls' mathematical learning had surpassed that of herself and her husband, and that calling on their peers for assistance was a valuable learning strategy. "Like right now if my kids come home with math, it's like, forget it, I can't help you. Phone a friend. My husband is the same."

Although some, as identified earlier, saw the power for themselves as learners working in an interactive setting, some also transferred this idea to how they should work with their own children. Ron, Rena, Rob, and Terry all talked about working together with their children as intellectual partners rather than in a tutor-student perspective. Most conveyed stories of assisting their children with homework that resulted in a frustrating venture for both child and parent. They recognized the need to work together, as they had in their work groups in the program, rather than in a purely instructive mode. Ron, for example, talked about the frustrations of a past situation with homework. (See p. 68) After completing the program he talked about some of his acquired cognitive partnership strategies. "Yeah you tell me Shawn and I'll try to hint you along.....Now I'll say... before it was this is how it's done. If I got that worksheet now, it would be okay 'what do you think it is Shawn? And how would you look at doing this?' .... instead of me saying …'cause he's not learning anything from me if I say here's what you need to do......It's frustrating for both of us. Now it's more we're working together." My reading here, as was told to parents in the group, is that the frustrations felt may have been caused by two people working on parallel paths. In this case the cognitive partnership would never occur, as they would metaphorically never meet, even though the parent may be offering accurate instruction. If an adult doesn't meet a child where the child is at cognitively or procedurally, the child may not understand the parent's point and thus may not be able to follow. These parents identified with this dissonance. Terry used the term "collective reasoning" to describe the power of social interaction in problem solving. He said "Yeah, that's neat. Collective reasoning is so powerful. It's something that you are going to use throughout your life. I mean most people are going to go, what do you think about… ? What do you think about…?" He shared stories of his work with his boys that exemplified the power of this idea. For example, Terry said he enjoyed having problems of the week sent home so that he could engage in the process with his boys: "It's … problem of the week (laugh). And they like to do them. It's a neat thing to do together as well."

Affecting their children's being. There was also a shift in desire to help develop changes in their children's perspectives. Donna noted the need to lessen her son's dependence on her and elevate his level of responsibility as a learner. In an e-mail after the third session she wrote: "With Riley I am feeling more confident that my role is to provide a positive learning environment, encourage him but ensure that he takes responsibility for completing/attempting his homework."

Janice and Ron were drawn to the idea of developing self-esteem and confidence through mathematical learning. Ron saw that drawing out his son's ideas about a problem first and then working with Shawn's ideas rather than having his son follow his thinking, would increase his son's self esteem as a mathematical learner: "...work on the positives, like okay, that's a good idea, keep going with that, what if you tried this, how many other ways do you think you could do that, kind of things, instead of 'no, this is what you do'. [Me: Do you think that will affect other aspects of their learning?] I hope. I hope so. Yeah. Their self esteem. Absolutely."

For Janice, imparting the feeling to her daughter that she is capable of thinking with her (as an adult) on an equal level, affected her daughter's confidence as a learner. "They can think equally. Like I'm often amazed at what Pam will think as opposed to, I didn't even see that. You know different, not necessarily with math but with different life situations. She'll come across with a totally different insight to something. Just because I'm a grown up doesn't mean I'm always right."

Encouraging the mathematical. Although not stated with consistency by many participants, Rob and Terry both realized the need to encourage 'being mathematical' and 'thinking mathematically' with their children. I believe this was the greatest change for Rob, whose early questions and comments during the sessions dealt with doing homework, completing it and sending it back to school correct. Rob questioned just how much assistance he should offer Chris, as he didn't want to do his son's homework for him but also didn't want his son to be embarrassed at school by having the wrong answers to his work. Rob's focus changed from 'correct answer' mode to learning and being mathematical. He commented, regarding problem solving, that math "focuses on finding a path to the solution, rather than the solution itself" and that he will allow his son "to get frustrated" and that he'll "allow him to go back to school with the wrong answer as long as he has worked on the problem." I'm not sure Terry underwent a major change on this idea, as much as a reinforcing of what he already knew. Terry recognized that his son's math problems were a great opportunity for them to interact, that math was about them and that math was a thinking tool rather than just a subject at school. What changed for Terry was his confidence in pursuing the 'mathematical' with his sons. (Further detail in Chapter 6.)

Aligning instruction methods. Key 'how to' ideas expressed by many participants circled around four main ideas:

- Rather than tell their children what to think, they would enhance communication by asking more questions to ascertain their children's thoughts on the topic.
- They would seek to meet their children at their understanding of the concept, and then proceed from this starting point.
- They would set the stage for future mathematical learning by playing games, looking for mathematical moments in a natural setting, and by playing with concepts on an informal level.
- They would deal with frustration as a natural by-product of problem solving.

Ron's perspective on working with his son has moved from directives such as 'this is what you do' to questions that will require his son to think and allow Ron to get a glimpse of his son's understandings before they begin their work together. Ron's new plan was to ask Shawn: "What do you think? How would you do this?"; and to "work on the positives"; and to respond with "that's a good idea, keep going"; and hint him through his work with "What if you tried this?"; and "How many other ways do you think you could do that?". Donna talked about beginning to realize that she didn't need to know the entire grade three curriculum. She said she was "becoming more comfortable with Riley's ability to teach me and explain to me what he has learned." Janice also spoke of a transition from telling kids how to do something to getting them "to vocalize what they are thinking." She recognized that hearing "their thought process is important" when working with children. This topic was also one of discussion within our group. It seemed so obvious to many that getting their children to vocalize their thinking would alter the way that they work with their children. Many were open to admit that their method of assistance was 'here's what you do'. This method is most likely a reflection of having been a product of traditional methods of math instruction; their actions formulated from ways in which they themselves were taught. Just like students who have within them the ideas they need to problem solve, parents needed assistance to see teaching/helping in another way.

Parents saw that, as adults, we could better assist children by meeting them at their point of understanding. Expecting children to join our thought processes may prove to be too challenging. We discussed the idea of looking for their 'zone of proximal development' (Vygotsky, 1978), then attempt to scaffold to an understanding of the concept with adult intervention. Janice, Terry and Julie found this idea to be realistic and functional. Janice referred to this idea as "going back and bringing to maybe where you are or to where you see, the baby steps, because some stuff we know, but how did we learn that or why did we know that." Terry saw that in order for someone to understand, it's necessary to come exactly to where they're at and bring them to where you are. Likewise, Julie talked about "moving at a child's space and pace" as well as to "discover or find out where the kids are at and then link to their level."

Parents also spoke of the value of play and informal mathematical learning, something I have referred to as a part of 'setting the stage'. Some were surprised that, as a parent myself, my children have never been the recipients of extra math worksheets. There seemed to be a belief that one would get better at math by practice sheets and that surely a math teacher would provide this 'opportunity' for her own children. Julie told me of her sister, a teacher, who used to have her kids doing worksheets at the cabin in the summer before they could go and play. After discussing ways that kids can begin to feel and see math around them, my participants began to talk about 'setting the stage'. For Julie and her preschool children, the idea was exciting. She began to imagine counting and sorting activities they could do at the lake, games they could play with blocks and ways they could talk about number without formalizing the concept. Richard, as I spoke about earlier (p. 71), was eager to look for ways that he and his son could put on their math-coloured glasses. Cooking breakfast became an informal opportunity to talk and see math. Terry's venture into squares and square roots with his ten-year-old son was not intended as a learning outcome but only as a glimpse into the future. He'll learn the concept with more depth and formality later.

A sense of clarity also seemed to follow as parents recognized their own sense of frustration while solving problems. If one has no frustration, they likely aren't really challenged or are not really problem solving. Frustration, as a natural side effect of problem solving, is a normal function of the task. Feeling this emotion themselves has helped parents to see that it's okay for their children to get frustrated while working. It also offered them a sense of relief. The problem of the week (POW) used to be a source of tension in Donna's home. Knowing that the point of the problems is not to just have the correct answer, but that the process itself is the value of the task, ("I remember early on when the POW came, just the pressure to solve it, now I'm more relaxed, no that's not the important thing"), allowed Donna to help Riley in seeing that being frustrated was in fact, part of that process. Rob's focus from getting the answer to developing process, even with the accompanying frustration, was also evident in his writing and discussion. He wrote "I will allow him to get frustrated, as long as he doesn't quit. I will allow him to go back to school with the wrong answer, as long as he has worked on the problem." Rena's decision to be more accepting and work to ease her son's frustrations appear to be changing their working relationship. She said, "As far as now with the math and stuff, I'll help him to be less frustrated and stuff."

The richness of the ideas that parents intend to put into practice may indeed make a difference in their interactions and the relationships that develop with their children. Hearing how they plan to implement these ideas speaks to the depth of their learning as they reach out to make the ideas come alive for themselves and their families.

Of Math for Themselves

"I look around me and it's everywhere but I didn't really... you know...."

Julie

"If I had learned math like this...."

Rena, Ron, Terry, Donna

The other aspect of procedure that affected my participants had to do with how their own procedures for doing mathematics have been altered. Parents came away from these sessions feeling that their own understanding of mathematics or their own learning was somehow affected. As I looked at the many concepts that they talked about in terms of their own learning of math, I realized these concepts really fell back into the four categories that directed the planning of the sessions, those being the mathematical standards of communicating, problem solving, connecting and reasoning. As parents learned about these processes of mathematics, their own learning was reflective of the same processes. This section will provide examples to illustrate the procedures that participants have realized are important in enabling their own abilities to learn math.

Communication. Julie and Janice learned about communication of math in two different ways. Julie spoke of realizing that math itself is a language. It's a way of talking about our world and communicating ideas to others. She began to see the math not only in her children's play but also in her work. It seemed to appear for her where she didn't see it before (p. 64). Janice used the term 'language' when she referred to math terms (rectangular prism for example) she didn't know or couldn't recall. She felt that work with others in mathematics is hindered if we don't share the same terms.

Some participants realized that learning could be enhanced by discussing mathematical ideas with others.Terry's slant on communication paralleled that of a social constructivist view. Learning to communicate with each other as a method of instruction reflected his work world and was seen as essential for children to learn. He also stated that math "really is a form of communication. In a lot of ways. In a lot of different ways." Rena also voiced this opinion. "Well, when you are discussing it with other people, you have different outlooks, …their response, there is not only just one way of doing it. There are different ways that you can …something. If you are doing it by yourself, you are not going to ... So you can see different insights." The strength of the group in which she worked was impacted upon by her confidence as a learner. She compared learning in groups with her previous experiences where she was often left to struggle on her own and then present her lack of understanding in front of the class. "We had to go to the board and I didn't like that 'cause, 'cause I didn't know whether I was right or not. There were all these kids there…." Being able to see different insights into a situation became a revealing moment for Rena as a learner. Donna enjoyed the learning that occurred through her interaction with others. Memories were spurred or ideas formulated by discussing problems with other people. She said, "I felt I could not have solved that problem on my own but with that group it was empowering and just a really valuable learning experience."

Problem solving. Although not all participants wrote about their newly realised or awakened sense of enjoyment of problem solving, most were visibly empowered by the process. Whether a confident problem solver in the beginning or a nervous participant to start, there was a sense of triumph and enjoyment in the room. There were also a couple of exceptions. Richard was noticeably withdrawn from the process, being paired with a couple of confident problem solvers. Janice also spoke of sitting back and listening. She quoted the adage "it's better to be thought a fool than to open your mouth and remove all doubt. So I tend to be quieter in group situations like that because I don't want to remove all that doubt." Most were vocal in sharing their positive experience. While Bev spoke of getting the adrenaline going, Terry was ever enthusiastic about his involvement, his ability to problem solve being rejuvenated. Ron, with the self-professed negative attitude towards math, stated point blank that he "liked the problem solving."

For some it was the feeling of worth gained in the process, that sense of 'I was a part of the solution'. For others it was realizing that there are many ways to get the correct answer. Rena commented on this idea the first day when we looked at how they solved a simple two-digit multiplication question. Rena mentally calculated the multiplication as she had been taught, thinking that was the only way it can or should be done. She was pleased to realize other methods were equally effective and acceptable. This seemed to free her from the 'do as I say' philosophy of math instruction. This same activity had a similar impact on Ron who also took the question to ten or twelve friends to see their solving process. The fact that he shared this idea with so many people must speak to the value he placed on it.

Ron and Donna commented on the learning of strategies to solve problems. For Ron, this was a rather novel idea. He expressed gratitude for "The strategies. Nobody had ever done that before. I think back and it's okay, I used to do that, I used to do that. But here are some strategies and I think that helps out." Although he recognized some of the strategies as ones that he had used before, he found that their being made overt was very helpful. He also voiced that it wasn't the problems themselves that were important to his learning but the process or strategies that affected his learning. He found it amazing that "if you just look at a problem in a different way"; say by setting the data into a table, then "the whole picture came together." Donna also spoke of this same experience. Attempting to solve the calendar problem, she found her group was floundering with their random approach. "I thought oh, that step at organizing it, the visual with the different problem solving strategies. Yeah, that's the one I was doing. So I thought, oh, once you're in the middle of that problem solving you can reflect back and see what are those strategies. I wasted some time on that randomness. If I had…." A glance at the list of strategies on the overhead got them organizing their data in a table then finding clarity in the problem.

From my observations, it seemed that the idea of applying strategies to solve a problem was a covert operation for most of the participants in my group, as was identified by Ron and Donna. Making the process more important than the problem appeared to be a reversal for some. Parents also found the thought of doing problem solving more appealing, even those who dreaded this aspect of math in their formative years. Perhaps it allowed for more risk taking and validated their mathematical ideas, as was the case for Rena (stated earlier in the section on affective elements).

Connections. Connections within mathematics learning apply at many different levels. One can connect math learning with that of actions in our world or connect one math concept with another or connect varying modes of understanding such as the symbolic, pictorial, concrete and linguistic. All three of these definitions were represented within this group.

To Our World. If we talk of one's learning of math as being able to recognize the role math plays in their lives, we can look at the examples provided by Janice, Terry and Julie. Janice said we need to "look at the math in every situation" and look beyond thinking that "you are never going to use this" or that "this will never apply to your life." She saw that the process and thinking developed through mathematical study is reaching farther than the content of a problem. Terry saw that math even lies in "areas where you don't think you use math." He said, "Math belongs to everything that we do." Julie was also swept up in her realization that math was everywhere. "I didn't see it in the environment. Now I look around me and it's everywhere but I didn't really… you know…the gas station. Ding, ding, ding, ding. [The bells went off.]"

To Other Math Concepts. Connections may also refer to how one math concept fits with another. Richard noted his surprise at seeing how many concepts can be used or learned within one activity. He said "I was amazed how you brought in so many topics just by doing a volume class." He was referring to the activity we did to build conceptions of volume through reasoning but also seeing factors, multiples, area and other concepts connected to the developing concept. Ron made reference to the net analogy where one mathematical idea is conceptually linked to another. He realized that through visiting a few mathematical concepts over the course of these sessions that he was "beginning to fill in some of the holes in his net."

Through Various Modes. Connecting through symbolic, pictorial, concrete and linguistic means held significant influence for Rena. Rena's learning was enhanced by the use of visuals. She recognized that "some people learn better by visuals, and it makes more sense, you can make sense out of it, instead of trying to imagine it." She liked to work through a problem while literally seeing what she was doing. Terry commented as well that his learning was enhanced by the use of visual aids as he could relate to the concept better. (See p. 136.) Ron's understanding of algebra concepts was relearned through the use of algebra tiles. He felt that the tiles really helped to pull it all together and make the concept much easier to grasp and understand. A connection with his son's use of tiles in a place value activity was also established. Donna also said, "The balance scale and tiles helped me to make sense of balancing an equation …(and) understanding the concept of x squared." Bev echoed this sentiment as she felt that "the algebraic symbols seemed so easy to understand with the pictorial representations." These various modes of learning have affected the participants' understandings of mathematics for them on a personal level.

Reasoning. If one is to reason they must see and expect that mathematics makes sense (NCTM, 1989). This seemed to be readily accepted by my participants as they began to understand rather than memorize mathematics. Janice felt that anyone could memorize, but to be able to remember and use the skill is the true meaning of learning. "Because everybody can memorize … you can memorize things but it's to actually remember them and use them… to know them. Because cramming before a test, and the next day it's gone. You write your test, you might do fine, but then what have you really learned?". Rena saw that learning algebra with tiles helped her to make sense, understanding "why she was solving for 'y'" and thinking about "what 'y' represents" was very different for her from her days of 'do it like this'. (See p. 124.) She felt that by reasoning through a concept that it would stay with her longer. Julie recognized that learning algebra by breaking down the steps through reasoning made it much easier to understand. "I learned it as two, bracket, three, times, equals, and it was just all formula, it didn't break it down the way you did it." Donna told me that learning by reasoning was very powerful for her. She pointed to the idea of seeing the surface area of a cylinder as a label on a soup can, which allowed her to see and understand. "If I had been taught to understand I could have solved for volume or area without relying on the memory."

Affecting Long Term Memory. Some categories didn't fit into the four processes but I feel are important to mention here. Some participants' learning was about pulling some forgotten concepts out of long-term memory. Donna appreciated this opportunity to re-establish her understanding of some algebra processes. Ron impressed himself by seeing how much he had actually remembered but also remarked, "seeing it in a different way, it now made more sense." Terry also commented "ideas I had forgotten were refreshed such as the order of operations."

Terry gave his assessment of parents' learning throughout this program:

> "I think people were excited because there is, you could see the light go on. That was kind of neat. You could see the light go on and the realization and the frustrations in some cases were, I spent all those years, not knowing something that's very elemental. That I could have learned, and saved this time."

I question whether or not these participants felt a sense of exoneration, in that it wasn't them who couldn't learn math, but in that it was the methods they were taught by that had caused their inability to learn well. Rena herself said that if she had learned this way, things might have been different for her. Their learning of math by methods that varied from that of their own experience has paved the way for changing the assistance they will provide for their children.

Conceptual Knowledge of Mathematics Education

Parents' knowledge of mathematics education likely represents the most significant alteration to their existing schemata. This section will look at parents' recall of their own math education experiences as well as their newfound knowledge regarding current mathematics education experiences.

Math Education Remembered

Parents' recollections fell into a few distinctive categories. The methods of instruction as well as their methods of learning seemed to be quite similar for all participants. Instruction mainly consisted of 'do it my way' and drills through textbook work were the norm. Students were expected to work individually and memorize steps and procedures.

Julie described instruction as "delivery of information" where the expectation was that she get "just the answer." Speaking of a representative teacher, she said, "he didn't care if you didn't get it and we didn't focus on how people were thinking." Learning was characterized by lots of memorizing. She said, "You memorize. I was really good at the memorization and formulas and you apply it and you walk away." She had little knowledge of how this fit in her world, only that it was something that was done in math class.

While discussing her friend's reluctance to join this program, Janice said "she too has learned by the old school learning and she is also very much, do it my way." Learning consisted of seeing an example, then following the example with different numbers. They "learned by drills and papers, never hands on and never in groups. It was all individual and individual accomplishment." Students were inspired to be competitive as the 'correct' people were handed accolades and the others were neglected. She referred to her math education as a "stigmatism" that one must overcome. Janice was also haunted by "the segregation of boys and girls." In her day she felt boys were considered to be naturally better at mathematics while "girls weren't allowed to be smarter." She also felt that the message given was that learning math was for people who endeavoured to be "an astronaut or an engineer or whatever, in order to deal with mathematical things."

Rena's recollections were marred by fear and lack of confidence with mathematics. Students were expected to work alone, in fact talking with anyone, was perceived as cheating. "When they work in groups they can get so much out of it. They can [inaudible] their notes and I find that, I guess I couldn't understand that concept before. I just figured, I associated that with cheating. If you were copying from someone, you're not working your own thing and you're not getting your own answers." Questions were to be directed to the teacher: "If you had a problem, you asked the teacher, and you didn't discuss it with somebody else." Much of Rena's fear was associated with being called upon in front of the class to share her lack of mathematical knowledge. "We had to go to the board and I didn't like that 'cause, 'cause I didn't know whether I was right or not. There were all these kids there." Her willingness to take a risk was affected by this fear of being wrong. She said there was only one way to work out a mathematical solution and you were required to show the identical steps. She felt that the teacher's role was to watch the class. Memorizing formulas was once again the expectation and that after the test most content was forgotten. "The old way, after you have written a test or exam, unless you use what you were tested on, more than likely you aren't going to remember what you were doing without reviewing it again, and that is if it is reviewed within a reasonable time frame."

Terry felt that the teacher's role was to present the rules and that students were to fit inside the box. Students weren't to ask questions, they were to simply do what they needed to do. The method of instruction and learning was very structured. Terry's thoughts on this topic are detailed in Chapter 6.

Donna's recollection of her math education was the most positive on an affective level, marked by interested teachers and equally enthused classmates; yet Donna's experience with memorizing rather than understanding was also evident. She recalled learning about measurement from examples in a textbook, "The old textbook, the examples from the old textbook, the problems, yeah, there was no milk carton." They never pulled out a can or a box or measured an actual object. In terms of learning, calling a friend to discuss a problem would have been viewed as cheating as the purpose would have been likely only to get the correct answer. She said, "Even times, oh no, you don't call your friends for help. It's just sort of like, that's cheating. You've got to do your math on your own."

Math Education Today: Teaching

Participation in the program allowed participants to change their views of mathematics education. Ron described mathematics education today as "a whole concept" or "a new attitude"; most recognized that something very different was happening. Rena saw that math education today was "a change in perspective." Not one participant talked about memorizing or following one method or having a negative feeling. In fact, much of their conceptual knowledge was characterized by polar opposites to their past experiences. The richness of their views on current math education may indeed speak to the necessity of such a program being readily available for all parents. Some of the ideas in this section may duplicate some of the previously mentioned data as their global conceptual understanding of math education may be reflective of and intertwined with representations of their own personal and procedural learning.

To these participants, mathematics education was about living a mathematical experience with others, about making sense of mathematics, about enjoying learning, and about life. Teaching and learning mathematics looks and feels quite different from that of their own past experiences.

Teacher's role. Participants' views of mathematics teaching were definitely altered. The teacher's role was described as that of facilitator. Donna saw the teacher's role as a "facilitator assessing where they are at, and providing steps to assist them with the process." She said that she "wouldn't see it as the teacher's role to teach the step by step how to solve it." Rena saw the role change from what she perceived to be 'teachers watching the class' to being readily available to deal with students' questions.

More than one participant expressed awareness of the teacher's responsibility for creating a safe place to learn. Julie saw this role as one who creates "a safe environment.

It's a creative environment" in which children can explore and take risks. She talked about telling her husband about the program: "Oh dear it's so exciting and I was, I was really on a high thinking how wonderful for our children that they are going to learn in an environment that allows them to reach for the sky and try new things and it's trial and error and it's resilience for life too. It's transferable, not just in math, but in every day life. You keep trying, you keep trying. You'll get there." Terry saw the math classroom as an environment where risk taking is expected, as the students engage in thinking and reasoning through problem solving activities. (See p. 132.) Rena also suggested a risk-free learning approach when she spoke about the difference between having to present her ideas at the board versus being able to discuss comfortably with others in a group. (See p. 126.)

Teaching tools. The materials used in instruction were not expected by the participants but were readily received and acknowledged as valuable teaching tools. The use of manipulatives was the most talked about teaching strategy. Every participant praised their use as it helped them to visualize math concepts and thus connect with the concept on a different level than that offered solely by textbook instruction. Parents recognized that the use of manipulatives appealed to their different learning styles. Richard was impressed by the hands-on nature of mathematics; something he had said on the first day was imperative for his learning style. His past experience with a technical course helped him to recognize this need. He wrote in his notes after the third session: "Using the tiles as visual tools to simplify the problem is a great tool for the kids (and adults)." Donna linked the use of manipulatives to developing understanding when she said "Balancing an equation made sense with a balance scale…the tiles really helped to understand x squared." Rena, who referred to manipulatives as apparatus, commented on the visual support as she moved through developing a concept. (See p. 123.) Ron suggested that the visuals helped the whole picture come together. Terry, who said he often drew diagrams to explain ideas, already knew the value of a visual representation, and was excited to have this as an additional tool for his children as they work to understand mathematical concepts. Julie also commented on "the fresh look" that manipulatives brought to problem solving and algebra.

Rena also made mention of the lack of a single textbook in a course. Previous to the program, Rena wondered why her children didn't each have a textbook. "They're always talking about lack of funds for schooling. That was my thinking. I thought that that was just the one thing that they were cutting out …the necessity. But now I have an understanding of why they do that way." She then commented on understanding how the use of many sources and group interaction is superior to the methods with which she was familiar as she saw and felt the difference herself. (See p. 125.)

Setting the stage. Parents also saw that the curriculum supports the need to open up thinking on particular concepts before they become formulized in the classroom. The model presented for building the concept of volume over the years helped them to see the value of this approach. Rather than being introduced to volume of a rectangular prism as a formula with examples in a textbook, children spend many years internalizing the concept first. Parents liked the idea and saw how they can play a role in this 'setting of the stage'. Ron identified that children are learning concepts at an earlier age. "I recognized everything that was being done. It just seems to me that they're doing it at a younger age, not necessarily in … like here's the quadratic equation or whatever, but it's starting the process early. I think it's a big advantage. 'Cause you walk in one day to algebra and 'okay today we're going to do the quadratic formula. Here it is. This is how you do it.'"

He saw this as a big advantage, as his kids will walk into a class someday when the formality of the concept is being taught, and they will recognize it, based on their experience with informal development, and ideally, connect the formula with their pre-constructed knowledge. Janice talked about kids building knowledge and vocabulary before they may even realize what they are learning; again, her way of realizing the idea of building mathematical knowledge. "What I'm getting from you and what I was hearing is, that it starts already in K, and using the proper language so that the kids know, oh, that's a variable. Before they even realize that's what they're knowing. It stays with them so that when they get to grade 7, they know that's a variable, but as a parent, I have to go, what was the variable?" She felt that children will eventually put the hands-on conceptual development into perspective. Julie spoke excitedly about encouraging play with her boys that had its basis in mathematical conceptual development. She saw the role that she can play in helping her boys to be mathematical.

The idea of 'setting the stage' for further learning helped parents to see that even though the purpose in an activity may not be apparent at the time, there is validity for learning further down the line. And it made them aware of looking for purpose other than 'get it done because it's homework', as was the case with Rob. It gave parents a different direction in helping their children think rather than just search for the correct answer (perhaps with a call to other parents for the answer). It also held them back from using their adult knowledge about concepts and telling their kids just do it this way and you don't need to do all those extra steps. For example, if an adult's understanding lies with a memorized formula, they may be inclined to just use this formula rather than help their child fill boxes to find the volume.

Math Education Today: Learning

Learning is about life. Most often, parents saw that math instruction by today's vision helped their children to build skills that they will use everyday as an adult. Their comments weren't directed to the utilitarian nature of mathematical operations but toward either a way of thinking, or toward how people interact with each other on a daily basis, especially in the work force. Terry said, "Now it's more, more of a world environment. It's more like a realistic approach to problem solving. Math really is problem solving in a primary element."

Terry was likely the most vocal in this regard, his desire to connect on this level the most apparent. He saw today's math class as encouraging children to take risks and thus become effective risk managers. In his business, risk equated to profit and loss. (See p. 132.) He saw the power in interactive learning, something he referred to as collective reasoning, a highly desired skill in the work world. He saw the power that his son held as he has learned to apply the mathematics that he has learned to problems that he encounters outside of school. He commented on the fact that it wasn't that William had the answer but that he could figure out the question. Terry also saw that his boys were learning to be responsible decision makers through their mathematical study. (See p. 136.) Rather than be told how to do something, they were to think, make plans, and ultimately make decisions based on their thinking. He also saw that his boys were learning strategies, tools as Terry called them, to solve problems on their own and they were learning when and where to use those tools. Terry said, "Today's child is being taught with methods that equate to an understanding of the environment" in that "the curriculum mirrors real life situations and provides strategies required to succeed, through the use of many problem solving techniques versus one formula."

Ron talked about our method of working with children when he turned "this is how you do it" into "how do you think you should do it and build from there." He said that this was what life was about. He went on to use examples at his work that reflected these understandings when he spoke of how sales information meetings had changed. He saw that his colleagues are now more apt to analyze problems and come up with solutions, similar to what teachers are doing in the mathematics classroom (p. 64). Ron also saw that "children working together towards solutions is the way that life is going."

Rena saw that current teaching methods help children to develop skills that reach beyond mathematics. She identified that children are learning "to question", which she saw as a valuable life skill. She also felt that children were learning to trust themselves mathematically, a key to their future success.

Janice saw that children were learning that math is about everyday. It's more than a subject at school. She said you don't need to be in a mathematically based profession "in order to deal with mathematical things." She saw "life is problem solving" and that children are learning strategies that allow them to deal with life's problems as well. Janice commented, "Well if it's a problem with friends, it's okay, and she'll come, and it's okay 'well let's look at it. Let's look at the pieces. Let's look at where things went wrong and let's see what could have been done better'. You have to pull it apart and lots of things. I think that's an important part of what we're teaching them. We're not just teaching just a mathematical skill." She saw that there was carry over into life or the 'bigger picture'. She saw that her daughters were becoming better equipped to help their own children as they lack the fear of mathematics and the stigma that girls can't do math. She seemed to have equated these instructional methods with confidence as a learner.

Julie spoke of life skills as she referred to telling her husband about the program. She felt today's methods would teach her children to keep trying as it was okay to be wrong. Mistakes tell us a lot about what we know and how we think. She felt this skill would transfer to other aspects of her children's lives (p. 94). Julie was excited by the determination that mathematical problem solving might instil in her children.

Learning is a social construction. The way that children are learning math today is something that parents see as a valid and superior alternative to their past experience with mathematical learning. The meeting of minds that comes from learners interacting with each other to construct understanding and knowledge was seen as learning that will stay with the learner for a longer period of time and with greater depth. Each participant was intrigued by the power that group interaction had on his or her own learning. Many of these examples have been documented earlier in this data review. Janice recognized that group work was "good, because I think it spurs when you come to a stagnated part, working in a group, kind of spurs your, one thing somebody may say or one idea, even if it's wrong or if it's not quite in the right direction, at least it twigs more of your own. [I think that empowers people, to have that little tweaking.] Yeah, when they kind of run up against a wall, with a problem or something, and somebody else says 'what if', and then, oh, and then that makes you think of something totally different and all of a sudden a light comes on." Having your ideas noted and validated by others gives one confidence, as was noted by Rena. They saw that there was power in collective reasoning, the term coined by Terry. They saw that talking about math with others was in fact, mind stretching, rather than cheating, as was the case with Donna and Rena. Rena e-mailed to me: " If you are working on the question by yourself, you may not think of all the different ways of coming up with the answers. I found that having the discussions, you can see different ways, and if you can't see that way, or it doesn't seem to make sense to you, you are able to discuss it with the other parties in your group." They saw that learning and thinking together was reflective of expectations put on professionals in the work force. And they saw and felt that this was the underlying force of today's mathematics learning,

Learning is an affective endeavour. Parents personally felt the affective element of mathematics learning and confidently wished this same effect for their own children. They learned that mathematics was about enjoying seeing patterns and solving problems. It was about sense making and that felt good. They saw that current instructional methods built motivation. Terry felt "a program where children interact and think and see the math, would keep them interested and thus motivated far beyond that of paperwork and repetition." Julie saw that math should allow children to blossom and enter into school without fear. She felt that an exploratory setting where the children's thinking is acknowledged will help them to enjoy their work.

Learning is Understanding. Parents also realized that learning math was about understanding the concepts rather than memorizing formulas, as was the case for most of my participants. Each of the participants who attended three or four of the sessions expressed this idea in some way. It was the case for Terry when he talked about his work in the 'old days' when people were educated to do a job whereby "you went out, this is what you do, and this is how you do it. Don't ask any questions. Carry on." Today he said children will be able to "understand the concrete, the logical, and the abstract" when they reach university. He felt that "not a lot of people today are able to do that." Terry also commented on what he felt was the gist of the program: "We want them to understand what the question is asking and that's exactly what we're doing in this class, and it makes sense."

Rena's thoughts on this topic were not entirely clear on the tape. She referred to "they work more on, not memorization but more… that will stay with you longer… They learn to question." These few ideas garnered from the tape pointed towards her view that understanding stays with one longer than memorizing steps and formulas. It was Julie as she made note in her log of the value of reasoning. She noted "reasoning is a process to get to the answer. (It's the) how (or) explanation. (It's) not just the right or wrong answer. (It's) understanding why we do things." Donna commented after the second session that she saw that "understanding as being superior to memory was clearly demonstrated." Following this session I asked her what had made her come to this conclusion. In a later e-mail, she told me that an example I had used about finding surface area of a cylinder by using a soup can and taking off the label to reveal a rectangle whose base is the circumference of the circle, made her realize the power of using understanding over memorizing formulas. She wrote, "Your example last week with the label and the soup can. Seeing/understanding allowed me to pull those old formulae out of memory. If I had been taught to understand I could have solved the volume or memory without relying on the memory."

On the last day I asked participants what they had learned about the current mathematics curriculum. Rob wrote "There is a logical reason for it. It focuses on finding a path to the solutions, rather than the solution itself. It allows many ways to solve a problem rather than the one 'right' answer." Bev commented that she was "very aware of the importance of developing a deep understanding for each new concept" and that " (the algebra tiles) are a great way to learn in order to encourage understanding."

Janice also talked about the process of understanding when she tried to explain the program to her friend. She said "And that's the message I was trying to get cross to her about this new math and new thinking with math, is that it's not necessarily the answer but it's the how you came to that answer and what thinking you've had going on while you were coming to the answer, what things were you looking for to solve this problem and how did you go about doing it. And that's what I got from this program."

When discussing the idea of problem solving becoming clear for him, Ron speculated about what this must be like for children. "Yeah, and you know if the kids can get that, it's okay, I understand this, here's where it comes from. Then I think it's... instead of saying okay, that's just part of a formula." Ron actually said that he came to the sessions expecting some kind of new formula-like process for doing math, but then said that it was "a new attitude." I believe he was referring to the idea of learning for understanding rather than memorization.

Parents, as identified in the section on their own learning of problem solving and in the section on mathematical instruction reflecting life, recognized that problem solving is an imperative method of learning in today's math curriculum. Whether they were learning through problem solving, as was the case with the volume exercise, or learning about problem solving as they did at the last session, the main understanding was that the process of problem solving is empowering to learners. Donna said that she " had a relatively strong understanding of the curriculum, but through the program, the emphasis on problem solving and communication and group work really became obvious. Especially when we as adults were participating in those problem solving exercises. I didn't realize, I never visualized the kids in the classroom, being so social, and team building and problem solving. That was exciting, I didn't realize that it was at that level."

Once again I'll give Terry the closing word as he commented on instructional methods, learning and life:

> *"Well a huge part of the world today is risk management and I think some of that is being incorporated into the classrooms as well. In terms of, number one, the kids are, the kids are allowed to take risks with having a safety net. A great example is when I was really, really young. Mom and dad have pictures when I started to walk and stand and stuff like that. I have a picture of me standing in my dad's hand with just one leg, and standing there. A huge amount of trust goes with that and a huge amount of risk as well, right? I see the kids have the opportunity to do that, here, in the curriculum, which is really neat. I mean that's what I always loved about this school, is that they encouraged taking risks and I've seen it in the reports of the kids and that's actually a word that's incorporated into their development or factored into their development. People as a whole don't have that skill. And that's neat, that is really neat!"*

Chapter 6
Degrees of Awareness

The expressed experiences of the participants in this program formed a body of data that has been categorized into three emergent themes; a) participants experienced the phenomenon of the series of four sessions on a personal level, b) they represented it as enabling them to develop procedures for working with children and with mathematics, and c) they described developing knowledge of mathematics teaching and learning. These categories represent the collective experiences of individuals experiencing the same phenomenon. Analysis of these descriptive categories will focus on the phenomenographic concept of 'awareness'. Awareness (Marton, 1986) is described as a certain way of understanding something. It is not a dichotomy of aware or unaware, but is an exploration of the differing ways that we are aware of a certain phenomenon. Phenomenographers look for differences in the structure of awareness. The experiences of the participants in this study will be analyzed in two ways, as suggested by phenomenographic practices:

1. The underlying organization within the *collective* experiences, articulates the degree of awareness represented within the participant group. As such, all data will be viewed as a whole, as to reveal varying ways that a participant group internalizes the phenomenon.

2. A phenomenographic study must also look at the data from an *individual's* perspective, revealing the awareness that exists within each individual's perceptions. This analysis will explore the learning of Terry and Rena, exemplars of the range that exists amongst individuals in the program as they took on constructs that enriched or changed their views on math education.

Awareness within the Collective Group

Participants in this program experienced learning in varying ways. The data suggests that individuals take in and use information differently. How they express their intake and understanding of the information, demonstrates degrees of awareness. Participants discussed elements of the program that held importance in their schemata of math education, both in teaching and learning. The word schema is used here in reference to Piaget's theory of cognitive development. Piaget found that people organize their environment according to their own internal cognitive structures or schemata (Wadsworth, 1975). In this case, participants entered this program having their own cognitive structures regarding mathematics learning and teaching practices.

The depth of their understandings may be classified according to how the information was internalized, owned and reported. Sometimes, participants would offer their understandings of an idea by reflecting back ideas that I had presented. For example, if I told participants that children needed to work with manipulatives to develop deeper understandings, they would later report this same idea in virtually the same manner in which it was originally presented, yet relay it in a way that suggested that it was their own idea. This is not to be construed as a negative value; the idea is merely being presented as one that they now own, the original source having been detached. At other times, participants would take the information and make it their own, as demonstrated by an ability to express the idea transformed into the context of their own lives. In the above example, this might mean that parents would internalize the idea of manipulatives then go home and create a mathematically rich game for their children to play with blocks. A third, more implicit category also emerged. There were times when participants were simply connecting to ideas, the suggestive aspect of ownership not evident. In the example here, participants would relay to me that they liked when I used the algebra tiles or that the manipulatives did in fact help them to learn.

As the internal perceptions of the participants may not have been revealed, I seek only to express an examination of that to which I was privy. I also do not presume that an individual's learning is representative of a particular level of learning, as described here, only that this level exists within the participant group and that this is one such example. Each participant's learning fluctuated within the range of awareness at varying times, and only a subset of his or her learning was expressed.

1. Reflected Ideas: With Ownership

Parents took on ideas that have become a part of their internal schemata, suggesting that they have put a level of importance on the construct. As such, these ideas represented an aspect of their learning. This level describes a degree of awareness whereby participants have gathered a 'knowing of ideas', garnered from their participation in the phenomenon. I intend to portray the incidents from the program that support their gathering of these ideas and suggest that the ideas have been internalized in a form similar to their original presentation.

Teacher-student: A cognitive partnership. Working with children in mathematics is not about telling children what to do and how to think. Mathematics teaching has more to do with helping students to construct understandings that make sense to them. It's about providing experiences to set the stage for further learning and helping students to find the understanding within them. Rob opened up a discussion in the second session regarding helping his son with a math problem that Chris had received for homework. Rob was concerned about his son's work being wrong and he questioned whether or not he should work with him until the answer was correct. This created much frustration between father and son but Rob didn't want his son to be embarrassed by returning to school with the wrong answers. Here's the infamous problem of the week situation presented in the initial chapters of this report; the parent takes on the responsibility to teach the solution to the child. This question initiated some discussion on how parents might work with their child. I shared my take on working with a student. Foremost, my instruction isn't about my ability to tell someone how to do a mathematical problem; my job is to enter into the child's path of learning. I need to find out just what they have already put together by asking them questions such as, 'What have you tried already? Do you know what this means? Do you remember the....? Was there something from...that might look like...?' The point is to get the student thinking about the situation and to teach me about where they are at in terms of understanding the problem. Then I can attempt to pick up from where they are at and move onward from there. This is where I also talked to parents about Vygotsky's (1978) 'zone of proximal development'. We need to identify the child's zone and then attempt to scaffold to our 'expert' adult understanding of the problem or concept. At the time, this paralleled nicely with the intent of the volume exercise done on this day where we looked at the development of the concept of volume. Parents could see and feel that learning a concept isn't being told about the final stage. Knowing is building sensible constructs that connect, and that thinking for yourself builds a sense of mathematical confidence.

I also talked to the parents about my daughter's piano instruction where she had experienced two opposing forms of instruction. Her former teacher would look at learning as black and white; 'you've got it' or 'that's wrong' (usually the latter). Her current teacher works from the level where Kas is currently playing and adds to and builds from there. Kas leaves the session feeling that she has accomplished a great deal and is open to moving on from there. Playing the same piece for months feels quite different, as one methodology is saying 'keep doing it, it's still wrong'; and the other is about constructing knowledge upon that which has already been accomplished. The message was that learning is about moving along a continuum, not right or wrong.

My participants picked up on these ideas and in their interviews passed the ideas back to me with a sense of internalized ownership. Janice determined that one initiated the learning process by getting them to 'vocalize what they are thinking' and to hear 'their thought processes'. She also reiterated ideas from the session when she talked about going back to where they are at and bringing them forward with baby steps (p. 82). Ron also talked about drawing out the child's ideas first and transferring from telling his son what to do, to asking questions (quite similar to my examples) that would initiate his son's thinking and responsibility for the task. He also spoke of working on the positives to encourage the milestones in learning, very similar to Kas' learning of piano (pp. 69 and 81). Donna talked about the teacher's role as one of facilitator (p. 94) rather than one who should give the step-by-step and that she was becoming more comfortable with Riley's ability to teach and explain his learning to her (p. 82). Terry talked about the only way that someone can learn from your instruction is when the instructor mentally moves to where the learner is at and then brings the learner to his understanding (p. 82). Julie also reported this idea when she spoke about moving to a child's 'space and pace' (p. 82). Both Ron and Janice commented on the confidence and self esteem that comes from children feeling that they have the math within them and that they can in fact do mathematics (p. 79). In all these instances, parents have reflected my thoughts on the 'teacher'- student partnership and have communicated these ideas with a sense of ownership.

Group interaction: Another cognitive partnership. After my participants' first round of group work to discuss and develop mathematical constructs, I shared my understanding of the value of group work. A group of people working together to solve a problem or construct mathematical ideas has the advantage of many minds working together as one. When someone has no idea where to go, another participant's thoughts might tweak his or her thinking. The shared knowledge may be, and often is, enough to reach a solution. This leaves all members of the group with the feeling that they were a part of the solution, thus allowing them to have the triumph of having succeeded with a difficult problem. The teacher's role is to offer suggestions, as would any group member, thus ensuring that success will follow. We compared this strategy to one where children would sit alone with a problem they perhaps can't understand and may not have all the mathematical skill required to solve. We talked about the advantage of group work for this child who can now feel some ownership for the solution. Janice, as recorded earlier (p. 100), but repeated here to offer the reader an example of the similarity, discusses the value of group work. "Good, because I think it spurs when you come to a stagnated part, working in a group, kind of spurs your, one thing somebody may say or one idea, even if it's wrong or if it's not quite in the right direction, at least it twigs more of your own. … when they kind of run up against a wall, with a problem or something, and somebody else says what if, and then, oh, and then that makes you think of something totally different and all of a sudden a light comes on."

There were also less global ideas that participants demonstrated that they owned, reflected to me in phrases very similar to my original comments. For example, I tied the value of group interaction to having their children continue this cognitive partnership outside of school. Rather than the parent help with homework, they should have the child phone friends who have spent the day together trying to work out mathematical ideas. The child's reconstruction of the thinking already invested in the class would allow him/her to further his/her understanding, with the intent of developing a solution to the problem. Donna reiterated this idea when she wrote that she would encourage her son to call a friend rather than rely on her for the answers (p. 77).

Value of understanding. The intent of the volume exercise was made explicit to the participants in the group. We want children to develop a sense of why a particular formula works for finding a measure rather than to memorize the formula. This is accomplished by having students reason by building on that which they already know and, by providing experiences that will both fill in any gaps in experience and also set the stage for later development. I began this session with a simple example regarding the finding of equivalent fractions. One could tell children the rule for doing this calculation or one could have children fold papers into varying sections and then discuss similarities and differences. From this activity children can construct a rule for finding equivalent fractions. We then proceeded to the development of volume, beginning with kindergarten children who fill containers with sand and marbles and acorns, to using a standard measure, then filling containers with cm^3, to developing a rudimentary sense of a formula, to measuring boxes with rulers, to developing a formula, to finally generalizing and abstracting to transfer this knowledge to the volume of other geometric shapes. I asked participants before beginning this exercise to "suspend your knowledge of volume.

Consider the stages of development here. What do you think a child will think? Practice listening to how your partner thinks through the situation and links ideas. To do this you are going to have to think out loud. Remember, our intent (although this may have been the main focus when you went to school) is not just to get the answer, we want, as parents, to analyze the thinking." Following the development of volume through reasoning and constructing, I demonstrated the learning of this concept from a traditional perspective whereby I showed them examples of a textbook where the left page says 'do it like this' and the right page gives the practice. We also talked about how a child might be able to complete the exercise with little to no understanding of the concept.

I also passed on to the participants that learning from an understanding viewpoint rather than memorizing a list of formulas would stay with the learner longer. For example, I talked about memorizing the formula for surface area of a cylinder versus imagining the cylinder as a soup can where one can visibly see that it is composed of two circles and the label, a rectangle whose base is the circumference of the circle and whose height is the height of the can. Participants were encouraged to consider other mathematical concepts in this way, where the learner constructs meaning as they connect with that which they know, and that concept development is a path not an endpoint.

I found the idea of valuing understanding and process being valued over that of memorizing and 'getting the answer correct' being reiterated by the participants in the study. Rob wrote on the last day that he would be focussing on the path with his son when they worked together rather than just the correct solution (p. 80). Donna also communicated this idea when she spoke about the process in a POW (problem of the week) being 'the important thing' (p. 83). Janice connected with the value of understanding when she spoke of being able to remember and use the skill (p. 89). Rena said that reasoning through a concept will stay with you longer (p. 81). Donna's comments about her own learning, where she felt that learning through understanding was favourable to relying on memory (p. 90), were also indicative of the message presented at that time. Ron's conclusions about children saying that they understand, they know where the idea comes from, rather than saying that's just part of a formula (p. 103), also reflected my direct sentiments from the session.

I'll refer to one last example to make my point here. In this case, Terry had adopted some of the actual vocabulary used in the sessions (p. 98). He referred to his sons having a body of tools (his metaphor) within their backpack or net. Terry's vocabulary here referred to two metaphors I had used in the sessions. The 'backpack' is a collection of problem solving strategies that everyone carries with them. The idea is that each person reaches into a personal backpack for a strategy when confronted with a problem and that learners add to their backpack when they solve problems in conversation with others, thus making the strategy their own and ready for future use. The net referred to my analogy of mathematical concepts being connected in the form of a net. Each concept learned represents a new knot in the net, the net being woven by interconnecting concepts. As learners move through the grades, their nets get larger and new knots are tied. When a new concept is 'thrown' at the learner, a full net would allow for an easy 'catch'. Likewise when a net has many or large holes from misunderstood concepts, the learner has little with which to 'catch' a new or future concept.

I do not intend to suggest that participants here were merely repeating my words and claiming ownership. Participants also experienced these ideas, thus their internalizations may very well be the result of having 'lived' the cognitive construct. The point here is that the construct changed very little from its original verbal form to the moment in time at which it was voiced. This resulted in what I have described as a reflecting back (in the physics sense of the word) of the ideas as they were presented.

2. Adopted and Transformed Ideas: With Ownership

Participants' awareness can also be expressed in their abilities to transform ideas presented in the sessions into that which has particular meaning in their lives. This involves a cognitive processing of an idea that is particularly meaningful for them by weaving the idea into their own lives so that it appears in a form different than that expressed in the session. Figure 6.1 outlines a number of transformations present in the participant group. I will be discussing two significant ways of transforming with examples from this collection. I will also be suggesting the event in the phenomenon that I believe was the stimulus for the transformations and thus linking their transformed ideas with the phenomenon.

Idea Presented in Sessions	Transformed Idea as Presented by Participant	Participant	Page in Ch. 4/5
Problem solving strategies	Counselling techniques	Julie	64/55
Children need to problem solve	The need in the workforce	Terry	65
Instructional methods	Looks like the workforce	Ron	64/99
Problem solving strategies	Looks like life (Daughters solving disagreements)	Janice	99
Instructional methods	Looks like life	Julie	100
Problem solving/risk taking	Looks like the workforce	Terry	65/98
Building understanding/reasoning	The need in the workforce	Terry	65/101
Communicative process	Looks like the workforce	Terry	65
Communicative process	Looks like the workforce	Ron	64
Being mathematical	Example with son (mirror)	Rob	67
Being mathematical	Example with son (William's developing the question)	Terry	69
Manipulative representation with Algebra tiles	Manipulative representation with Denes blocks	Ron	68
Students helping each other with Homework	Pre-kindergarten playtime	Julie	78
Math is a way we talk in our world	Math is everywhere – gas station/ work	Julie	85/88
Setting the stage	Laying the beginnings for squares and square roots	Terry	69
Setting the stage	Calculator and block play	Julie	70
Setting the stage	Cooking breakfast with son	Richard	71
Setting the stage	Laying the beginnings for quadratic equations	Ron	70

Figure 6.1 Examples of Transformations.

These transformations appeared in two distinct fashions. In some cases participants transformed the presented idea into a personal and unique idea. Other transformations occurred as an action, not merely an idea. Not only did the participants create their transformations of the idea, they also acted upon the transformation. The example that follows highlights these two ways of internalizing and using the information. Because the concept of 'setting the stage' transformed into both an idea and action, I'll use this concept as an example to depict these two varying transformations.

My version of setting the stage as presented to the participants revolved around two premises. First, as teachers and parents we must provide experiences for children such that they can connect with 'formal mathematical' ideas in the future. Second, beginning or opening up a thought process on a concept has educational value. It is not always necessary to formalize the concept. I pointed to a dice game that I played with my son when he was four. Drew would instinctively keep track of the points gained in his turn thereby requiring that he add hundreds and thousands in his head. He had no idea what this looked like on paper, yet when he reached this point at school, he could easily connect with the idea as he had internalized that concept in varying ways in an informal sense. I also told a story of my daughter attempting to convince me to buy the large bag of oatmeal, as it would fit into the two ice cream pails that we had at home. It wasn't necessary for me to talk about volume of cylinders at this point, only that estimating volume and then testing this estimate would begin to establish the idea of volume measurement. I also talked about reading to children when they are young or taking them to visit many places. This establishes a vast view of the world upon which later learning may be built. Mathematical ideas can be established in much the same way as that of vocabulary and linguistic experiences. In the sessions, the volume activity also served to show the participants how this setting of the stage in the early years would evolve into a formal concept later on. It appeared obvious to participants at the end of the program that learning volume formulas without having something to connect to in their lives was a rather isolated construction with little possibility for application in the future.

Transformed into an action. Julie, Richard and Terry each acted upon this idea of setting the stage by creating a situation that fit with their interactions with their children. Julie, a parent of primary children, turned to 'designing' play activities whereby her children would be establishing mathematical ideas. She turned to the calculator as a tool of play as she interacted with Carter while waiting to see the doctor (p. 70). In addition she had 'family fun night' when the four of them played with boxes and blocks, with a focus on measuring and comparing. This also evolved into an activity on patterning. (See p. 70.) Recall Richard's experience with his son John while they were making breakfast. Richard had his seven-year-old son determine how to divide thirteen pieces of bacon, working with the idea of division and fractions in a real setting but void of the pressures that formal development of these concepts might evoke. Richard also said that he would look for other moments when he can see mathematics in their everyday play and interactions (p. 71). Terry's action on this idea was slightly different than that of Julie and Richard, as Terry was actually engaged in a homework assignment with Brian, rather than a casual event. Brian's homework involved the construction of equivalent fractions. Terry immediately was drawn to the idea of using tiles to help Brian see the situation. However, Terry didn't stop at the equivalent fractions. He went on to open the door on the idea of square numbers and square roots, something that Terry knew would be formalized at a later date (p. 69). Each of these participants had sincerely expressed these ideas as important ways in which they have internalized and acted upon events of the phenomenon they encountered.

Transformed into an idea. Ron also transformed the idea of setting the stage, but did so with an idea rather than an action. Ron spoke about how his son might learn about aspects of the quadratic equation before he actually sees it in a formal way in the classroom. Although Ron does not express any idea of how this might transpire, he uses this example, transformed from the original, to depict his understanding of the idea of setting the stage (p. 96).

3. Connecting Ideas: No Ownership

While searching through the data, it became apparent that there was another category of awareness present in the learner's use of the information. As described above, participants took on information and reported these ideas with a sense of ownership. Participants also signified that the ideas were not necessarily their own, but still held significance for them. Sometimes participants, such as Julie, made overt statements that the idea was garnered from the program. Julie prefaced her comments about seeing math in her work with: "Just from last week, a light bulb went on, math is a language" or "I'm starting to see what you're saying"; when she spoke of the interconnectedness of math (p. 64). Janice would also make no claim to the ownership of the idea when she said, "What I'm getting from you and what I'm hearing is..." (p. 97).

Sometimes the statements were less explicit about ownership, but the suggestion remained that the idea was important to the learner but perhaps not as internalized as those spoken with a sense of ownership. These might have appeared with statements such as "(This idea) helped me to...", suggesting that the idea still belongs to an outside source. The difference may be in considering the statements, 'I see that...' as opposed to the sense of ownership in 'I believe that ...'. Figure 6.2 presents incidents in the data that portray an agreement with ideas that were presented in the program but at the point in time when they were conveyed had not been internalized with a sense of personal ownership.

Julie's comment about algebra (in boldface in the table) provides a good example. She said that when she had learned algebra it was very much 'put the x here' and 'then do this'. She said that it was never broken down like I had done with them. With "the way you did", Julie suggested that the process of breaking down components of algebra (patterning, skills building, tile representations, graphical representations, tables, problem solving) was one she could appreciate but not one that she could do. The ownership of the idea still lay with me, as the facilitator; Julie's understanding of the idea was only one of observation.

Significant Idea from the program that affects the participant's understanding	**Participant**	**Page in Chapter 5**
I see the math in my work	Julie	63
Communication while doing math isn't cheating	Rena	123
Collective reasoning was so powerful	Terry	79
Problem solving enjoyment rejuvenated	Terry	86
There are many ways to get the correct answer	Rena	86
Strategies aid in solving problems	Ron	87
Strategies aid in solving problems	Donna	87
There really are many concepts within one mathematical problem	**Richard**	**88**
The tiles helped me to...	Donna	95
The ...symbols seemed so easy with the pictorial representation	Bev	89
Likes the visual support	Rena	89 / 95
Visuals helped the whole picture come together	Ron	89
Tiles as visual tools is a great tool	Richard	95
It didn't break it down the way you did.	**Julie**	**90**
If I relearn the algebra, this will help my son when he gets there	Donna	90
Seeing it in a different way, it makes more sense now	Ron	90
Teachers are readily available to answer questions	Rena	94
Child's learning environment will be wonderful	Julie	94 / 101
I like hands on learning	Richard	95
Method helps children to understand their environment	Terry	98
They learn to question	Rena	99
They learn to question	Terry	101
These methods build motivation	Terry	101
What I'm hearing from you is...setting the stage	Janice	97

Figure 6.2 Examples of Connecting Ideas: No Ownership.

Likewise, when Richard talked about the many concepts embedded within an activity (in boldface in the table) it was in reference to the list that I had put on the overhead after doing the volume exercise. Richard voiced an appreciation for the multitude of concepts, but offered no suggestion that he would have arrived at this conclusion. His comment, which voiced a reaction to the idea, contained the preface (similar to those noted earlier about ownership) that "I was amazed how you brought so many...." Richard leaves the ownership of the idea of multiple concepts within an activity, once again, with the facilitator.

Individual Awareness

I struggled to find a 'measure' of learning, a way to understand degrees of awareness of each individual. How could I say that one of my participants had learned more or more deeply than another? Through my investigation of each participant's learning, I was constantly drawn to Rena and Terry. As individual learners, they appeared to me as contrasting ends of the group. Rena exemplified those who, admittedly, knew very little about doing mathematics and described her ability to help her son with mathematics as "inadequate." Her increase in confidence throughout the program was noticeable both visibly and through her notes and interview. Terry, on the other hand was confident, both as a parent and as a doer of mathematics, when he entered the program. His learning curve appeared less dramatic than that of Rena. Entering into this program, a teacher may have viewed Rena as a student who struggled, and Terry as a highly skilled, capable student. They were the extremes of my group. However, to say that Rena's learning was significant because she came to understand a math learning philosophy that she hadn't imagined and Terry's learning was only moderate for he already had a self-established understanding of learning math, would be an inaccurate assessment of the depth of each individual's learning. To say that Terry's level of understanding was better than Rena's because he was more capable with mathematics and more confident in helping his children, would also not evaluate the learning that occurred. They both seemed to have learned a lot and appreciated the degree to which they were learning. However, I felt they had learned in different ways and indeed they had.

I intend to show that Rena's learning is well characterized as *accommodation.* Accommodation, as theorized by Piaget, occurs when incoming data does not fit with pre-existing schemata. In order to adopt the information, the learner will accommodate by doing one of two things. "(He) can create a new schema into which he can place the stimulus, or he can modify an existing schema so that the stimulus will fit into it; both are forms of accommodation" (Wadsworth, 1975). Rena had preconceived notions of how to help her son that were mostly based on her math experiences. She also had ideas about math instruction that were at odds with current methodology. These are the schemata with which Rena entered the program. I will describe Rena's initial schemata in these two areas and provide evidence that supports Rena's engagement in the cognitive act of accommodation.

Conversely, Terry's learning in the program was most often characterized as that of *assimilation* of information. "Assimilation is the cognitive process by which the person integrates new perceptual matter or stimulus events into existing schemata or patterns of behavior" (Wadsworth, 1975). As is the case with Rena, I intend to present some of the schemata from which Terry was operating, and show evidence of how information from the program was assimilated into these schemata.

Rena's Learning

Although assimilation and accommodation are interconnected processes, which means that Rena was obviously assimilating information as well, the data suggest that Rena was most often engaging in accommodation as she was constantly presented with ideas that didn't fit into her pre-existing schemata about math instruction.

Rena's schemata related to math instruction prior to the program presented a rigid view of what children and teachers do in the classroom. She portrayed an image of children working individually at their own desks (lest they be thought to be cheating) with their own textbooks. There was only one way to do each mathematical concept: the way that the teacher told the class in the initial instruction. The teacher's role, following instruction, was to 'watch the class'. Rena's affective view of math education was one of great stress.

Some passages from the interview have appeared earlier in this document. They are repeated here to provide a cohesive view of Rena's previous schemata and her emerging schemata through the process of accommodation. Also note that when (…) is used in the quotation, it signifies a section of the recording that was inaudible.

Communication. Rena's image of a class of students working quietly on their own in a math classroom was soon altered. Her emerging view of an interactive group engaging in the construction of mathematical ideas created a new schema for Rena about mathematics instruction. Often Rena would tell me what she used to think and how that perception has changed because of her involvement in this program. The following excerpts from her interview (Fig. 6.3) demonstrate the accommodation process in regard to Rena's new understanding of group work and the role of communication and discourse in mathematics learning as opposed to her old view that talking equated to cheating.

Rena: I really like the idea that they work in groups. When they work in groups they can get so much out of it. They can … their notes and I find that, I guess I couldn't understand that concept before. I just figured, I associated that with cheating. If you were copying from someone, you're not working your own thing and you're not getting your own answers. If you had a problem, you asked the teacher, and you didn't discuss it with somebody else. ….

Me: So your perspective must have changed a bit.

Rena: Totally did. ……

Rena: Well, when you are discussing it with other people, you have different outlooks, …their response, there is not only just one way of doing it. There are different ways that you can …something. If you are doing it by yourself, you are not going to …So you can see different insights.

Rena: You're teaching them here, in the schools to … You have to work as a team, you can't just work as an individual and you're not going to get anywhere, you work as a team and you work together. …. You learn to interact with others whether you have differences or not. It's just ….. I think people should get some training in that. But, yeah, I don't see it where I am, I'm on the phone. I'm a receptionist …… It's encouraging for kids. ….

Figure 6.3 Excerpts from interview: Rena accommodates her new understanding of communication in mathematics learning

Manipulative materials. Rena's sense of the use of manipulatives for developing mathematical constructs was also one that developed as a result of her participation in the program. Note that she had an idea that 'apparatus' was used in mathematics teaching but she needed to establish an accommodation to define its use as a mathematical tool. She expressed this point in her interview: "I get to see blocks and stuff in the classes but I never really knew what they were used for." Then later, Rena talked about how her learning was being affected by the use of manipulatives. "Just the apparatus that you used, and you can work through the problems, and actually see what you're doing with them." As a result of her ability to comprehend mathematics concepts more easily through visual materials, Rena commented "some people learn better by visuals. And it makes more sense, you can make sense out of it, instead of trying to imagine it."

Teacher-student interaction. Her view of the teacher-student interaction in the classroom also underwent an accommodating process. Where once Rena had a vision of a non-communicative atmosphere, she soon came to realize a different perspective. She said, "Yeah and you're not, teachers aren't just watching the class... Students are more, I would think, free to approach you if they have problems. Then they can ask...."

From memorization to sense-making. Learning mathematics also took on a shift in perspective, from the expectation of memorization to the value of sense making. Rena realized that this sense making approach would have made a difference for her, as a learner of math, had she had this opportunity when she was younger. Rena responded to the work we did with algebra.

> It gave more of a (pause) ... of explaining it. It broke it down in such a way that ... if I had learned it like this back then, it would have made more sense. Solving for y, why are you solving for y, and what y represents. The way you explain it is, you make sense. .... I know that kids learn today differently. ...... They'll learn it because they know they have to but at least... They need it they have it. Like you were saying, they work more on, not memorization, but more.. that will stay with you more. .... They learn to question. I think that's a lot of their ... They have a job to do....and then you don't use it for a while and to do it you have to take out a period of time that you shouldn't have to take and do it. ....Not just review but..(relearning). Yeah.

The work we did in algebra has aided Rena in making the accommodation to see mathematical learning as working to understand rather than memorizing.

Rena also made a shift from believing that learning mathematics was about the one way to do it right to valuing the sense making process. "'Cause it makes sense. It's not the only way. Or you came up with the same thing, nobody else ... ." In regard to her pre-existing schema she said, "There was one way and that was the only way. And show your steps." Rena was able to make the accommodation for a sense making process as she herself had felt success through engaging in the process.

From textbook use to interactive learning. Rena brought up the issue of textbook use more than once. An e-mail half way through the program stated that she was beginning to see why the textbook wasn't such a necessity.

> As far as the paper versus the textbook, with being able to figure things out in that way that you do now, as opposed to in the past, the paper form really does give you all that you need. You also work through things differently now in the way that you work with other items to work things through so you can see how you come to the answers. In this format you get to know what you are doing and things that make sense to you, instead of memorizing things and making sure that you know the formulas for what you are doing.

During her interview, Rena commented on the lack of textbook use in the mathematics classroom.

> Rena: I don't feel like ..... school just seems to be cutting back....
> Me: Did you think we weren't using them (textbooks) because the schools didn't have the money for them?)
> Rena: Yeah, basically yeah. But my, because they're always talking about lack of funds for schooling. That was my thinking. I thought that that was just the one thing that they were cutting out. ..the necessity.

This textbook issue was a surprise for me as a teacher who uses a textbook very little. In retrospect though, I can understand Rena's position, as the textbook preconception was representative of her schema regarding the math classroom. If children were to learn math by doing their textbook exercises and Rena hadn't imagined a different view of math instruction, then her comments about the lack of funds make sense. So this example served to demonstrate the extent of Rena's accommodation as she made way for a new schema involving interactive learning. She went on to say that learning without the consistent use of a textbook was "sort of justified" and that "now I have an understanding of why they do it that way ...."

Affective perception of mathematics. Rena's affective perception of mathematics learning was another area where the data showed that a great deal of accommodation had occurred. Rena viewed math learning for herself as a stressful endeavour in which she felt incompetent. For Rena to begin to view mathematics with an entirely different affective sensitivity suggested that she was more highly engaged in accommodation than assimilation. Rena painted a picture of her entering schema regarding learning math and each time followed her comment with the change she had undergone, suggesting a new affective schema was emerging (see Fig. 6.4).

Entering Schema: ....but still they didn't really work in groups like they do now, it was grade 11/12. We had to go to the board and I didn't like that 'cause, 'cause I didn't know whether I was right or not. There were all these kids there, (put you in an uncomfortable position.) Yeah. **Emerging Schema:** So I'd rather be .....Yeah, but it's just not they see my thinking. I'm dealing with 3 maybe 4 people, and then there're saying, oh I never saw it that way, I never thought of it that way. You know, you feel good. (It validates your reasoning.)
Entering Schema: I never found math easy at all. I found school stressful and math was one of the contributors to that. ...... **Emerging Schema:** If it had been more like this, it may have worked out completely different for me.
Entering Schema: Yeah. It is because, for me personally I don't like being asked any questions, no matter what it was in. Math being a really big one for me. I didn't want to be wrong but I didn't want to ... either. **Emerging Schema:** So I'd rather be in my area, comfort zone. And it's not uncomfortable feeling you're part of a group, and knowing that what you're doing is valid....
Entering Schema: And I feel I'm inadequate. It's kind of ....inadequacy is a very uncomfortable feeling for me. So I feel like I don't know what I'm doing ..... **Emerging Schema:** Me: are you telling me that this is changing for you? Yeah, I'm glad to do that and so, so far so good. (Also on p. 73.)

Figure 6.4 Excepts from Interview: Examples of Rena's entering and emerging schemata regarding her affective view of mathematics learning.

Working with children. Rena did not present an elaborate view of her schema regarding her assistance with her children's math homework. One can see from her comments about what was happening in her home however, that as a result of this program, she had once again developed an emerging schema that differed from her entering point of view. Rena presented a schema for working with her children characterized by 'do it like this', frustration, and not knowing what to do. The following excerpts from her interview (Fig. 6.5) point to an emerging view of assisting her children with their homework.

Rena: And that was another thing too I thought, working with my kids prior to this. About showing their work. Like, you can't just put the answer down. 'Cause how are they supposed to know where you got it. But they are getting it somehow. ......

And I was almost getting very angry because, especially times with the oldest one. The first one, you don't use .... There was a war between the two of us.

Me: Do you think this will change how you work with (your son)?

Rena: I think so. As far as now with the math and stuff, I'll help him to be less frustrated and stuff. Whereas before, I wouldn't have known where to start. .....I didn't know what they're trying to get at.

Me: Do you feel you have a better grip on that?

Rena: I think so, yeah. I feel that so far I've gotten stuff that I needed

Me: The work that they do at home should be able to help you develop that teamwork; the thinking it through together, like you did when you sat with other people.

Rena: Yeah, I need that .... instead of having a reaction.....

Me: Yeah, enjoy it.

Rena: We took the time.....

Figure 6.5 Excerpts from Interview: Examples of Rena's entering and emerging schemata regarding her view of assisting her children.

Rena's comments regarding her entering schema for working with her children are fraught with emotional reactions such as frustration or confusion. Rena accommodates by creating new schemata for mathematics learning methods and new ways to work with her children. The courage Rena found through a sense-making constructivist approach to mathematics allowed her to abandon her preconceptions and adopt alternate perceptions.

Terry's Learning

The data will show that Terry most often engaged in the cognitive process of assimilating information into firmly established schemata regarding math learning, his view of his work and his involvement with his sons as he assisted them with their learning. I will present evidence of Terry's integration of information from the program into these existing schemata.

Math education. Terry entered the program with a sense that mathematics instruction is and should be different from his own experiences as a mathematics student. He wrote in his entry notes to the program, "In terms of style, I was taught in the traditional sense of memorizing tables. What I found myself doing was looking for shortcuts in problem solving. This has continued into my adult life. As a side note [name deleted] has some interesting theories on math which I have enjoyed." Terry suggested that even as a child in a traditional classroom, he was attempting to invent his own mathematical methods. In response to being asked what he believed should be happening in his child's math class, he wrote, "I believe that it is fundamental to teach children in a method to which they can connect. By this I mean that different styles must be incorporated so that the child can understand the principle. If they don't understand one way, try a different method (approach). I also believe in balance. There are fundamentals which need to be learned like multiplication tables (but done creatively)." These initial comments demonstrated Terry's concern for learning by understanding rather than memorization.

Prior to the program Terry had already recognized that math education was different from his schooling and had accepted these changes as a positive direction. He stated, "It's a consensus, I would think, 'cause anyone who has sat down with their kids and done homework, sees that it's different. Even the questions are different, the wording is more complex, and that's a positive thing." Terry described his mathematics education as "here are the rules, you know, it's got to fit into this little box and away you go." The following excerpt from his interview provides further insight into Terry's schema for math education, which included the sense that mathematics teaching had changed and that he required some answers.

> Me: And throughout the program was that difference (from 'fit in the box' math) very apparent to you?
>
> Terry: Oh, very much so.
>
> Me: Did you expect that before you came?
>
> Terry: Nope. (laughs)
>
> Me: What did you expect?
>
> Terry: I didn't know really what to expect. I didn't realize that math had really changed that much. In some regards, yeah, the teaching of it. I mean when you first open up a math book, it kind of looks the same. But of course there were some questions that I wasn't sure why they were (laugh) doing these questions, or looking for these types of answers. I think the problem that we sometimes, sometimes those things would drive you crazy because you weren't sure what the purpose was, you didn't know yourself what the question was, right.

Raising his children: Terry also entered the program with a philosophical position on what was important for his boys to have cognitively and he had a method by which to make this happen with his sons. On a number of informal discussions Terry would reconfirm, through anecdotes, his desire and ability to work closely with his boys. Decision-making formed a foundation for his work with his sons.

> Yeah, and that's something I've always tried to do with the boys; is try and give them the base to work off of. That's all you can really do. I can't spend my life telling them what to do, (laugh) because I don't want to be 80 and still telling them what to do. At some point in time they need to know how they are going to do it. So, tell them what to do and hold them accountable for what they do. And that's pretty cool. It's like you can do what ever you want, I can't really do anything about that, but this is pretty much what you are looking at. And from there, you have to make a decision. 'Cause decision-making is a lacking skill out there. And I wouldn't say I have the best skill but people have a lot of trouble making decisions.

Work. Terry's schemata regarding his employment focussed on problem solving, risk taking, decision making and mathematical calculation. As I discuss how elements of the program were assimilated into his work view, a clearer picture of this schema will arise.

Assimilation

Terry provided little indication that he was accommodating information from the program However, there were numerous occasions whereby he vocalized his assimilation of ideas into his employment schema and his perception of working with his sons. His view of mathematics as demonstrated above was being added to throughout the program as well, but the assimilation of particular ideas was difficult to detect in our conversation. I attribute this to the fact that many ideas fit so smoothly with his existing schema that the changes could not be readily detected.

Problem solving. Throughout the program, the focus of mathematics education was presented as a problem solving venture. From the first session to the last, participants engaged in mathematics by problem solving or learning about how to problem solve. Activities, such as looking at the development of volume and algebra concepts, required that the participants reason and connect to establish an understanding of these concepts.

Our last session saw parents learning about problem solving by learning various strategies and examining reasons why their children need to engage in problem solving and what should be valued in the process.

As a result of this program, Terry saw that problem solving was the root of our attempts to improve math education. He commented on the change from his day then proceeded to assimilate these ideas into what he knew about the work world:

> It (mathematics education) was very structured when I grew up. Now it's more, more of a world environment, it's more like a realistic approach to problem solving. Math really is problem solving in a primary element. And then taking it, 'cause any problem that can be broken to its key elements can be solved. And that's I guess in my line of work, is really relevant because that's what I do, I problem solve. So I spend a lot of time, pretty much every career that you are going to have, it's going to be there.

The view he had about mathematics education taking on more of a 'world view' because of the problem solving was clearly assimilated into his view of the work world as one of problem solving. This viewpoint was not likely, as described earlier, one that he held about math education but had been smoothly assimilated into his math education schema; a small and natural step from his seeing math as puzzles given by his mother and finding his own methods to solve problems.

Learning for understanding. Previous to this program, Terry had an established method by which he figured the costing of goods. As a result of taking on new procedures in problem solving from his engagement in the program, Terry said he had actually taken on "a whole different approach to cost estimating" in his work. Terry had assimilated his new knowledge with math learning into his view of cost estimating. From the interview, Terry said:

> Yeah, I see, I have actually applied a couple of different principles and approached problems maybe a little differently than I would have in the past.

> I am trying some things differently, maybe on how we view the product as a whole, mathematically. Okay. Do we look at this thing as just a, traditionally what you would do is you would, I, it's a very obscure thing that I do, in a lot of cases, so what I actually end up doing is taking the whole thing apart. Now is there a better, faster way in looking at things as a whole and what does it mean. In this class, we talked a lot about, okay what does that mean, when we use a formula, an algebra formula, or when we're trying to figure out the answer to an algebra formula. What does that mean? And I heard a lot of talking about, okay, when you are growing up, younger, from K on, take a block, what is a block, so I've been taking that same principle and saying, okay, what is this part, is it a block, is it a shell, is it a box?

Although Terry is still doing many aspects of his job in the same manner, he has introduced another viewpoint as he looks more deeply at understanding. He has improved his method of costing by bringing more clarity or efficiency to the process, something, according to his story about searching for improvement by asking more questions (p.134); he strives to do on a daily basis.

Taking risks. Terry's take on children learning about and being able to take risks in the classroom also fit into his view of that which is required in the work world; risk management. Terry saw that when children work in groups to solve problems, not only do they have many decisions to make (nobody is telling them the steps to solve the problem), but they must also learn to speak up and offer their own ideas about finding a solution.

> Well and a huge part of the world today is risk management and I think some of that is being incorporated into the classrooms as well, in terms of, number one, the kids are, the kids are allowed to take risks with having a safety net.
>
> See in math, it's always been known as black and white, right? You succeed or you fail. And I think that's why people now a days go, well I can't do math. 'Cause they're scared to fail. In our organization, okay, you learn, oh, we didn't price that one really well. (laugh) Okay, why didn't we price it well? So it becomes risk management and your risk becomes loss.

Math learning processes to life experiences schemata. Terry presented a number of examples whereby he was constantly interconnecting his learning from the program with that of his work and his children. I will discuss my analysis of one of these situations by explaining an aspect of the program that may have been the stimulus for his comments and how I see that Terry was assimilating into existing schemata. Other examples from his interview (Fig.6.6) will further demonstrate Terry's assimilation into these various schemata.

The session on the construction of volume concepts once again played a role in the formation of ideas about learning. Terry assimilated ideas about ' being mathematical' into existing schemata. One element I discussed with the participants following the construction activity was that real-life problems don't present themselves with a set of 'do it like this' instructions as a traditional textbook might. We questioned whether or not children are actually problem solving if they are told how to begin and how to proceed. In the case of volume of a rectangular prism, a student might just in fact nudge his friend after the instruction and ask, 'What do we do here?' to which the response might be ' Just multiply the three numbers'. It wouldn't be necessary for the student to even consider what the question was asking, or for what the 'multiplication' was being used, in order to get a correct answer to the problem. I also shared with participants an example of a perimeter test item whereby students are given a six-sided polygon with attached measurements and asked to find the perimeter. We discussed the fact that a child could very well get the answer correct and have little to no understanding of perimeter. Given that the six numbers aren't likely to be subtracted multiplied or divided, the logical choice would be to add them together. A question that requires the learner to construct a six-sided polygon with a perimeter of thirty four will require him to consider the question more deeply and come up with a path of action.

Another idea that may have contributed to Terry's thoughts about math learning occurred in the last session. I began the problem solving examples with a quote "One can learn without getting an answer. One can get an answer without learning" (Van de Walle, 1994). The point was to get participants thinking about that which has value within the problem solving process. With this idea parents began to solve problems while also thinking about the process they were using. One of the first elements of the process they encountered was to determine the question, 'Just what is it that we are being asked to do?' With no guidance on how to proceed with the problem, participants relied on themselves and each other to make decisions that would lead them to a solution. As was the case for participants in this group, we also ask children to determine the question and come up with a path that may lead them to the solution.

Armed with these ideas, Terry began the process of 'seeing' them in schema that he already has regarding his work and his children. He discussed an experience with his older son.

> And it caught me so off guard, and I had to think about it actually, for a second, and go yeah, that's absolutely correct. What he did was, what was neat was, he's taken what he's learned and he's applying it to real life, and he's determining the question first. (That's powerful.) Oh, it's huge. And that's what struck me, it wasn't that he had the answer or any of that stuff, it was that he would, he had figured out the question. Because once you figure out the question, then the answer can be easy. And that's neat about the curriculum at this school, 'cause most people don't ask for the question, they just do what they need to do. And they're answering questions and they don't know what the question is (laugh).

His comprehension of learning to be mathematical by questioning and decision making also assimilated into what he saw in his work, from his early years in the business to current methods of business interaction.

> Yeah, yeah, in the old days you did your job. Right? You went out, this is what to do, and this is how you do it. Don't ask any questions. Carry on. And that's how manufacturing was, years ago.
>
> The one thing that I found; I worked on a very complex project just recently and I had a lot of questions for our supplier because we didn't have all of the information. And by the time it was done, I ended up going to the UK for some of the answers to

the questions. I really had to search, because they didn't have the answers either. So it's kind of a collaborative effort. At the time it was all said and done, the other day we submitted our quote, and then sat back and I said, so what happened with the other suppliers? I mean, they must have had some questions. And they said, no, none of them questioned anything. I said, well then how did they come up with the answer? And he goes, I have no idea. They guessed. And they couldn't have. And the customer realized that as well. 'Cause they didn't have enough information. That's a frightening world (laugh).

Terry saw that as William worked to deal with a situation and he worked at his job to ask the questions and use this to make decisions, so should math education support this process. To Terry, the information assimilated easily into the perspectives that he held. New ideas about math education have deepened his understanding and enriched his schemata.

Program Concept: Use of visual Aids **Assimilated into:** a)How he learns:	**Quote from Interview:** Me: What about, you mentioned the visual aids, I imagine you didn't expect that. a) Terry : No, and you know what, it was very good. You see, for myself, I'm very visually orientated. You know, draw a picture or something. When I explain something, a lot of times I'll draw a picture. 'Cause, not well, but I will draw it. Because that's how I explain; if I have a pen in my hands, I could do it now. That sort of thing. So visual is a big part of my world so I can relate to visual aids that you have. Whereas somebody else might be more auditory.
b) How he works with his children:	b) You go grocery shopping and you have, you see it, you have the visual aids, you have the pricing structures. You know what I really like with the shopping thing, is the comparison; like this is what it really costs you. And I use that for the boys, and I say, okay go get cereal. Find a good deal on cereal. It's on sale this week. And they'll go, oh a great big box. It's got to be the best price, and I'll go let's check it out. Or a two for, let's check it out. Really it's not a good deal. As a matter of fact, you haven't really saved much of anything. You dropped it like 2 cents a hundred grams.
Program Concept: Children learning to make decisions/ developing problem solving strategies/ instructional method regarding learning **Assimilated into:** What Terry believes in terms of learning	**Quote from Interview:** Me: ... or working together to make decisions. Terry: There's not a lot of things that would happen, if it weren't for the last minute. (laugh) As a matter of fact, I don't know if anything would happen if it wasn't for the last minute. People procrastinate and keep muddling over the same problem and it becomes an issue. But I know that within this system right now with the boys, is that if you have... And they do have to learn different decision-making techniques. And one of them is being able to have a body of tools within their backpack or net, to do that. 'Cause the more resources you have, you know, how is this problem being tackled, then it comes back to problem solving (and how do you get the resources?) You have to be able to understand. Until you can understand, until, the only way you can teach somebody is to come to exactly where they're at and bring them to where you're at. So until you've done that, they're out there. Me: And that wasn't a new idea to you? Terry: No, no. That's something I've always believed. It just reinforced what I believe.
Program Concept: The value of learning through group interaction **Assimilated into:** What Terry sees in his work	**Quote from Interview:** Me: Any other things you felt were reinforced by doing this program? (Pause) Well you know what, just breaking into groups and working together. That was kind of cool and ... 'cause in a lot of cases you have that in a work environment as well. Sometimes you're going to get that with your brother or sister, and you're going to play, you're going to have different environments, you're going to have different opinions, you basically have ... You have to work together and have a different opinion. (And it's okay to have a different opinion.) Well, and it's, you know what, yeah absolutely, and it was kind of neat to see even the dynamics because, there are some people in the group that don't want to say anything and there's some people in the group that just take charge, right? And you have those dynamics, but by having everybody rotate and having to explain, meant that they had to understand. And that was really cool, because that doesn't always happen in the real world, and it should.

Figure 6.6 Terry assimilates ideas from the program into his existing schemata.

Chapter 7
Voice

This chapter will offer the reader conclusions regarding the phenomenographic research process. This study has met the criteria of a phenomenographic examination as the differing ways in which people experience a phenomenon have been realized, categorized and examined as to expose the range of experiences and the degrees and hierarchy of awareness. Phenomenographic research should also speak to the enhancement of the phenomena under study. Therefore I intend to draw conclusions about the necessary elements of such a program. This study also has much to say about the possibility of establishing alignment of math learning methods between home and school by addressing the needs of an important but often neglected link in the process of change within our schools, the parents. This chapter will suggest that parents can indeed be brought into synchrony with the intent and practices espoused by the Manitoba mathematics curriculum and the NCTM Standards.

Research Intentions

This study began by establishing descriptive categories of the participants' expressed experiences. These categories were then analyzed such that varying degrees of awareness were revealed. These elements of the study will be recapitulated and further conclusions regarding hierarchy will be drawn.

Categories of Description

It was found that participants' expressed experiences could be categorized according to (1) the connections they drew with their lives, (2) the procedures they gained from their participation in the program and (3) the knowledge they gained about mathematics education.

Connecting with their life experiences. Although I had anticipated and expected parents to draw connections with their experiences with their children, I was somewhat surprised by the additional linkages that appeared regarding their work world. In hindsight, and as a result of examining the data for categories of description, the connection is a logical one. If people are learning, they are connecting with their own experiences. One might consider that the defining of our personal experiences is in fact through our families and our work.

Procedural learning. It was also apparent that participants were changing their procedures for working with their children as well as their own procedures for doing mathematics. Participants said that asking their children questions and attempting to meet them at a cognitive point of understanding was obviously different from their past procedures and also more realistic for them to enact. Most of the participants revealed that their methods of assistance had been 'do it like this'; an expected procedure from one educated in traditional methodology of mathematics education. The other significant change in procedure, their own mathematics learning, suggests a favourable reaction to current methodology. Participants were most appreciative of learning with manipulative materials that allowed them to 'see' what was happening. They felt a sense of safety as concepts were built and linked in a developmental manner. Participants learned math by connecting new concepts to those that they already understood. A sense of enjoyment for problem solving and learning in groups was reinforced for some participants and experienced for the first time by others.

Knowledge learning. The final category of description was knowledge based. Participants gained conceptual knowledge of the philosophical stance of the NCTM and the provincial mathematics curriculum methodology. By recalling their own math learning experiences as children and reflecting upon their experiences in this program, parents were able to draw comparisons that suggest mathematics education reform is of value. Parents conveyed ideas about math education that they deemed to be of importance. Views regarding the role of the teacher suggest an understanding of the need to be facilitative rather than authoritative. Perceptions of the math learner are of an interactive, sense making, reasoning, risk taking, questioning, problem solving individual.

Variation of Awareness within Categories

Analyzing these interpretive categories led me to realize that the variation of awareness within the categories was in how the information was internalized and reported. The data was viewed from the two perspectives suggested by phenomenographic studies. First, the group was viewed as a whole for variations in learning and then the individuals' perspectives were analyzed. Within the large group, I found that the participants' degrees of ownership and internalization of the information varied. A hierarchy of awareness can be determined across the whole group. By looking at each participant individually, two methods by which they were internalizing data became apparent. As both modes of internalization are significant in value and suitable for each learner, I shall not extend any suggestion of hierarchy on an individual basis.

Group awareness and hierarchy. As participants talked about their learning in the program, they did so with a variation in ownership of the ideas. There were times when participants would respond to an element of the program with a sense of appreciation for the idea, but with no suggestion that they could replicate the idea or that the idea was even their own. Then there were incidents when participants would talk about what they believe is important and true about math learning. This category transpired in two different ways. There was the sense that the idea was that of the participant, one now adopted as his/her own. There were also cases whereby the participant had taken on an idea as his/her own then transformed the idea into a situation in his/her own life, be it with children or work.

These three degrees of awareness do suggest a hierarchical order. For participants to comment on ideas suggests that they value the idea but are perhaps not yet able to use the information or maybe haven't gathered enough elements of the idea to confidently make it their own. I would see this as a beginning step to taking on new constructs. The act of adopting ideas without transformation can be viewed as a further step in learning about changes in mathematics education. The implicit statement of ownership suggests that the learner may keep this idea with them for transformation at a later, and maybe more opportune time. For one to be able to adopt an idea and then transform the idea into a personal situation represents a greater depth of internalization. Once someone has viewed information from an alternate and personal perspective, the opportunity for retention and use is more likely. I believe this to be the basis for and representation of, depth of learning.

One must keep in mind that the data collected in this study was a snapshot in time. As participants continue to reflect on the program and continue to assist their children with their homework, one might expect that a move to ownership and transformation of ideas will occur.

Individual awareness and hierarchy. Grouping the data as representations of individual learners, Rena and Terry surfaced as the contrasting ends of the group. As reported earlier, the processing of data was analyzed in terms of Piaget's theory of intellectual organization and adaptation (Wadsworth, 1975). Although Piaget's theory plays a part in the development of constructivist theory and in turn a role in mathematics reform, I had no intention in this study of analyzing my data according to these frameworks. The awareness of individuals spoke rather overtly to this theory. As participants talked about their learning, as quoted in the analysis, they often revealed within one paragraph the new idea they were adopting and then suggested whether or not they had a schema into which they could place the idea. Piaget's theory of assimilation and accommodation does not place a hierarchy on either of these cognitive acts for both are needed in learning. In fact it is theorized that one will seek equilibrium, a balance between assimilation and accommodation. When a person "experiences a new event, disequilibrium sets in until he is able to assimilate and accommodate the new information and thus attain equilibrium" (Ginn, 2004). This study supports this claim, as the value of Terry or Rena's learning appears to match each learner's unique need and purposes appropriately.

Phenomenographic Study to Future Programs

The results of a phenomenographic study should be able to speak to the facilitation of a program that can establish the desired knowledge and skill of the phenomenon. Therefore, it is the recommendation of this study that a program of this nature be designed such that participants are provided the opportunity to actually experience learning that is indicative of the knowledge and practices we wish them to gain. The government's publication of written material offering methods by which parents can assist their children (Appendix A) may be useful but cannot take the place of actual experiences. Participants must be encouraged to connect with past experiences of their own mathematics learning and with their current lived experiences. The strength of this program lies in the participants' abilities to reach depth in their learning by internalizing and transforming ideas into their own lives. As participants will enter into a program with varying levels of mathematics ability and confidence as well as varying methods of working with their children, a program must allow for these participants to engage in both the assimilation and accommodation of ideas. As is the case with any group of learners, the program must not overwhelm the students who are accommodating to a large extent but also must have a balance so as to not underwhelm those who are mostly assimilating information. Again, I believe this can be achieved by encouraging each participant to connect with his/her own circumstances. This program has demonstrated that learners' multiple distinct points of entry can be accommodated so that maximum learning occurs for each participant.

This study is not without its limitations. Throughout the program, the group was positive and accepting of change in mathematics learning methodology. I did not encounter any opposition to these changes. Could this be attributed to their familiarity with the presenter? Could this be attributed to the strength and viability of mathematics reform? This study was not organized to address these questions.

I also have come away with a feeling that in some regard I have deceived my participants. When I speak of mathematics reform and what is happening in the mathematics classroom, do they believe that this reform is happening in all classes in this same way? Have they distinguished between that which is a factor of my personal teaching style and that of basic fundamental changes within each classroom? I cannot begin to measure the influence that my personal style has had on the effectiveness of this program. However, I can conclude that their methods of working with their children have been positively affected and is likely to continue to be in the future. Their knowledge of math education may however, over the course of their children's education, conflict with that which they may experience through the individual teacher's interpretation and implementation of the mathematics curriculum.

Implementing Change

Parents came into this program with a view of what math education should be, formulated by both their own experiences in math education but also by what they see from their children's homework assignments. Recognizing aspects of their own education (students still do algebra and operations on numbers, for example) parents maintained a belief that their own experiences with learning math is the manner in which their children should learn as well. But they were also confused by the teacher's processes. They wondered, in many cases, why the teacher wouldn't just tell them what to do. Why, for example, are they bothering with this roundabout way to get to the solution? Why are they using these blocks? How can they learn without consistent use of a textbook? Parents were also experiencing bouts of frustration as their children would recognize that 'we didn't do it like that at school'. It seems that there were times that the parent's explanations and the child's understanding didn't meet. These findings were consistent with my preliminary assessment of the school / home misalignment outlined in the introductory chapters. It is my belief that parents can and should have the opportunity to align their views of math assistance to their children with that of curriculum-based practices. Participants identified two areas that were significant in causing the alignment to occur: (1) learning about an alternative approach to helping someone learn math and (2) learning for themselves through experiences that knowing math is about understanding processes not simply performing methods by rote.

Aligning Instructional Methods

Parents' methods for helping their children, as was the method they experienced in their traditional programming, consisted for the most part of 'just do it like this'. Parents didn't recognize that they were teaching from the endpoint of their own understandings and that their children were further back on the learning continuum. Nor did they recognize that there was in fact a continuum until they experienced this path themselves through the volume activity. From their experiences, having been told that volume was a particular formula, seemed to be the way to do it with their children. Parents in the program began to recognize that asking questions of their children to ascertain their current understandings and then working with their children in a cognitive partnership from that point was far more conducive to reaching understanding. They also recognized, through experiences in the program, the value of the cognitive partnership in learning. Thus they began to realize why the textbook isn't always necessary in the math learning process. Work with other students isn't cheating but is a viable learning resource that should be encouraged. In the end, parents were very open to encouraging their children to call classmates when they encountered difficulty in their homework, and thus continuing the interaction that began at school during the day.

Understanding Mathematics Education

Parents came to accept that learning mathematics is about understanding and connecting not just memorizing formulas and performing rote exercises. This acceptance was realized as the participants linked experiences from the program with facets of their own work experiences and their interactions with their children. Although most parents came into the program content with their expectations that math is learned by repeating rote exercises, they left the program accepting that these practices are not a reality in today's world. Some participants were able to point to the changes they've seen in the work force over the years. The parallel between math education methods experienced in this program and the reality of the work force (be it problem solving strategies, decision making, risk taking, question posing, application, communication, or teamwork) was the confirmation for participants that learning math for understanding is far superior to their past experiences and a necessary requirement in today's world. Parents were also convinced that learning for understanding is a valid necessity as many found themselves commenting 'I wish I had learned math this way'. They came to realize that solving problems is about becoming a better problem solver by gaining more strategies to use in the future, not just getting the answer correct. Having experienced the ease at which understanding is built by linking to previous experiences, the enjoyment that comes from 'getting it now', seeing why we do a certain mathematical function, and experiencing the triumph of having solved a problem, participants were assured that the changes being encouraged in mathematics education are not only viable but necessary.

As parents engaged in learning about mathematics education by methods that are implemented in the classroom, they came to realize the actions necessary for alignment between home and school. They expressed an excitement and optimism toward their future journeys with their children in mathematics learning. I am fortunate to have had the opportunity to initiate this journey and to be a part of their enthusiasm for change.

Closing Remarks

This study has reinforced my belief that learning mathematics can be empowering and enjoyable for all. My experiences with teenagers, pre-service teachers, teachers and now parents confirm that our current attempts at mathematics reform can be and should be realized. Breaking down the stronghold of traditional practices is a challenge that can be overcome by offering people the experience of a viable alternative. I see that I am empowered by being a part of the vehicle that enacts change. I see and believe that traditional practices of rote learning are damaging to the confidence of a learner and to his/her ability to learn mathematics. This study has reinforced my belief that:

- learners need experiences with which to connect new ideas,
- math concepts are linked to each other and to the world around us,
- being able to think through problems and processes with others is superior to learning on one's own,
- reasoning and connecting embed the learner in thought which in itself is far more rewarding than performing repetitive rote mathematical algorithms
- learning should be an enjoyable process.

I have learned that parents are not kept 'in the know' well enough. Misconceptions, such as 'working with others on math is viewed as cheating', may cause parents to question the teacher's methods and thus weaken their support for change in mathematics education. Parents can be instrumental in improving the quality of instruction in mathematics education, as their support is necessary in the wheels of change. Parents, through math education programs, will be better equipped to dispel the myths that surround mathematics learning. With knowledge, they won't be asking the teacher to 'just repeat it many times' or 'just tell him how to do it'. They will be actively engaged in promoting mathematics learning through understanding and connecting.

I appreciate the journey that I have taken. My new relationships with parents in the community will be lasting. The satisfaction that comes from believing that my research has had a positive impact upon their relationships with their children is rewarding.

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Appendix A: Math Matters Booklet

Appendix B: Initial Log Interview Questions

What was math class like for you as a child?

What do you believe should be happening in your child's math class?

Why are you here?

Appendix C: Log Interview Cuing Questions

Have you learned anything today?

Have any ideas made you excited or angry?

Was something particularly helpful? Why?

Was something a waste of your time?

Will any ideas discussed today affect your work with your child? In what way?

Do you have questions about today's session that you would like to pursue further?

Would you like to arrange a time to discuss any of these ideas?

Appendix D: Log Interview Exit Questions

What have you learned about the current mathematics curriculum?

What idea/activity stands out the most for you?

Do you foresee any changes in how you will work with your child in mathematics?

Has your own learning of math been affected?

Some of the group members have questioned their own mathematical abilities. There are others who chose not to attend these sessions because of their feelings of inadequacy in mathematics. Did this ever surface as a concern for you as we worked through these sessions?

If this program were to be run again, what would you like to see changed?

Do you have any comments you'd like to make in general?

Appendix E: Informal Interviews – Guiding Questions

Tell how your perceptions of math instruction are changing? (Curriculum and process)

Has your own learning of mathematics been affected?

Is your acceptance of current teaching methods favourable? Why or why not?

What is one element in this program that has resulted in learning that can be applied to a particular aspect of your life?

Have your methods of helping your children changed as a result of this program? Please explain.

What makes this program valuable? What elements were most supportive for your learning?

Have you spoken to anyone about this program? What did you talk about?

Appendix F: Researcher Generated Field Notes

January 14, 2004
My initial recruitment letter for this program brought back 6 responses. This number started to slowly grow as I received calls asking about the program. My discussions with parents in the community started to reveal some of the problems. Talking to some parents the time commitment is difficult; every one is running with evening practices and such, some are going on holidays. But the most common reason for not participating in the program was their fear of their own mathematical ability. Some woman told me they were not good at mathematics but perhaps they would volunteer their husbands. Others wouldn't touch a math course. My attempts to alleviate this fear resulted in some signups but not all. I suspect a large number in the community felt the program is really only for those who can 'do math'.

My thoughts : I struggled with one of 2 directions for the first evening with parents. On one hand I wanted to establish a philosophical stance the first evening, one that would get them ' hooked' wondering how will we accomplish or reach this 'stance'. I wanted them to want to come back next week to find out. In order to 'set the bait', I felt I needed to inundate them with info, release something that each of my learners would grasp on to. So, I needed to do a lot of the talking, a lecture, if you will. There is a risk in starting a parent session like this. What is they find the evening too long, just sitting and listening, and decide they are not going to spend 3 more evenings listening to someone else talk. They do have an immediate desire to find ways to help their child. Will they wonder if I can help them meet that desire when I'm waxing philosophically about learning and mathematics? This is also a negative approach in terms of data gathering. If I'm doing 95% of the talking, what data will I gather from them other than their initial notes? This may also negatively impact my desire for discussion in a community of learners. What if they come to depend on me to be 'the talker', having already set the stage for this direction on day 1?

My other choice was to start with some activities and build slowly to the philosophical stance. Although the participants would be more actively engaged from the onset, I worried that the stance would not become apparent. Without an adoption of the stance, I worry that mathematics may become little more than fun activities, and that parents would 'miss the point' in guiding their children with their homework.

So I took the risk of losing some participants in favor of the stance and thus went with the lecture. As I expressed to my parents, what they need to establish is a guiding direction (stance) by which they will work with their own children in their mathematics. There is no 'how to' of math instruction. If there was, it would look ironically like a traditional approach to math instruction – do this and follow me. It would take us years to work though the curriculum grade by grade with the parents. Every child is different in their learning as well, we'd have to break down their child's style and then ultimately create a different path for each of their children. I suggested to the parents that this guiding focus might be similar to guiding your child to be a moral person. There is no ' how to'. We do things with our children to guide them in morality. As we will, to aid them in becoming mathematical (as opposed to being able to do some mathematics).
Will I write about my reasons for each section of my lecture? The number 123 thing

I left for the evening with some questions about my programming. I wondered if I might have pushed the traditional approach with too much negativity. I don't want to lead the parents in their thinking; I want them to 'own' their beliefs. There are problems with traditional approaches that people seem to want to follow no matter what. You know, math should be taught this way because that's how it's done. I wanted them to recognize that approaches from their day have not been entirely successful for a large % of the population without insulting those that did find success. Even though I began the evening having them write about their experiences, I did not see what they had written. I went on the assumption, based on my past experiences with many groups of university students, that their math experiences were mostly consumed with only one right answer, lots of frustrations, lots of arithmetic, straight rows, no talking. I used this assumption to created a cognitive conflict. On the other side I asked participants to consider what it is like for them when they are really learning. Then I put a list on the overhead created by a different group of adults and had them compare their thoughts. The list is hardly disputable. These ideas of learning are surely the direction we want for our children and as was necessary for them to see, in direct opposition to their memories of learning in a traditional classroom.

I Know I'm Learning When ......

* I have an inner feeling of accomplishment
* I feel involved
* My thoughts are sparked
* I can apply a concept to a new situation
* My answer was not important to me, it was how the answer came about
* I am able to teach / explain to someone else
* I can make personal meaning / relate to things in my real life
* I am doing
* I can remember it 2 months down the road / retain
* I feel confident
* I am enjoying what I am doing
* I go home and think about what we have discussed
* Something once frustrating now makes sense
* I can discover things I never knew understood before
* I feel challenged
* I feel compelled to extend / expand
* My interest is stronger than my struggle to catch on
* I can picture the steps
* I am actively involved / engaged
* I can understand why / the process
* I have internalized ideas, they become a part of how I think and who I am
* I can make associations
* I lose track of time
* I can get creative with the material
* A concept or idea clicks / makes sense
* I feel motivated, inquisitive, intrigued
* I trust my teacher
* I concentrate 100 %

I brought this issue of traditional programming up with some parents who lingered at the end of the session. Rob(2), someone I knew to work in a math related field, may certainly have felt that he wanted the same traditional experience for his children as it likely worked for him. This wasn't necessarily the case. I believe from our conversation that he recognized the need for changes in approach and was looking forward to seeing how this will transpire. I'll need to carry this conversation further.

Terry (12) was energized by the session. I learned in our conversation that he too, uses a great deal of mathematics in his field. He connected with the estimating/ problem solving that he uses at work on a daily basis as well as his need to be exact in his calculations. His job requires that he think mathematically not just perform arithmetic calculations. He too seemed very supportive of the direction our mathematics instruction is taking and appeared to have internalized aspects of the philosophical stance I was attempting to establish. He talked about how aspects of this stance relate to each of his 2 boys. With his youngest son, he talked about establishing understanding of what it is his son is doing. He understood why his son had been taught a 'different method' for long division other than just the standard algorithm he learned in school. He also has had discussions with his son regarding why it is being done this particular way. Terry was supportive about developing understanding in learning rather than simply memorizing. Another point Terry connected with was that of helping children to take on responsibility for their own learning. His oldest son had little of this in grade 7 but, by grade 9, has taken on full responsibility, much to

Terry's relief. Terry, in part, seems to working with his boys as we would like to work with children in a mathematics classroom.

I was encouraged by the response of these two fathers to what had they had heard from the evening. Even though these are 2 of my 'comfortable with math' people, they recognize that a traditional approach isn't the best for their children. I'm confident that those that identified themselves as uneasy with mathematics, will be 'sold' on a change of instructional methods for their children as they are not too keen to have their experience replicated for their children. Thus, my focus on the negative aspects of a traditional program may have provided them some commiserating comfort in their unease.

I also spoke with Julie(4) who talked about the mathematical outlook of her 5 year old when he counts the number of trees it takes to arrive at a destination or how he likes to play adding games. The evening left her 'excited for my child' in that he would learn mathematics in an exploratory, thinking environment.

After reading their accounts of their own math education after the session was over, I'm confident in the direction I took.

1 – positive experience, easy, enjoyed meeting the challenge of p.s., studied math at U., economist

2 – math was easy most of the time, liked the challenge

3 – lack of understanding of math, upgraded as adult and found easier

4 – enjoyable up to 11 then encountered authoritarian teacher who through chalk, slammed books, upgraded as adult for u. entrance

5 –

6 – not interested in math, difficult,

7 – memorizing, plenty of math exercises

8 – very hard, memorizing, punishment for mistake

9 – frustrating, always feeling one step behind, not fun, a chore

10 – a struggle, even now a fear of math

11 – very disciplined, a lot of memorizing, ability grouping with indicative names (turtles, rabbits) grade 9 became too difficult

12 – enjoyed math, logical, problem solving, mom encouraged lots of 'puzzles'. At school taught to memorize.

13 – (educated in Korea) simple work (operations) were easy, when there was lots of reading, then very difficult, secondary- enjoyed math because of the teacher.

14 – a lot of memorizing – this is my way to do it – so just do it and it's okay if you don't understand why just follow the formula.

16 – simply put a "3"

Analyzing the above, I would set the number ratings at:

1- confident with mathematics, enjoyed in school (3)
2. math okay – can take or leave (5)
3. Was hoping I wouldn't have to journey down the math path ever again – (6)

Should say something to their honesty and my reading of the situation.

I was also encouraged to hear that many aspects of the stance that I discussed came up as important to my participants in their initial writing about what they wanted for their children. This would certainly help to put us on the same page and thus have them 'buy into' this math education program as well as the instructional methods employed with their children.

1 – support to develop p.s. skills, creativity encouraged, be challenged, enjoy math, learning to work with others, to learn from and teach others

2 - learn basic skills without calc, appreciation of math skills in life, enjoy math, gain desire to improve in math, gain ability in p.s.

3 – interesting and fun, concern about his difficulty in p.s.

4 – variety of learning styles, activity- exploratory based, building on strengths, developing confidence, looking at alternative strategies in p.s.

5 -

6 – turn kids on to math, should be fun and practical

7 – don't know, to be interested in math, have confidence to solve problems

8 – teacher to try to explain in relatively simple terms as possible, (some children are not smart to understand what teacher is trying to say and solve the problems)

9 – give students a foundation whereby they will have the skills needed to see him through his life

10 – positive and fun while learning to solve problems

11 – a place where she can explore new ideas and concepts

12 – teach so children can connect, different styles must be incorporated so the child can understand the principle, balance – fundamentals (mult facts) need to be learned.

13 – no children but if did would want them to feel comfortable and enjoy

14 – to understand what they are doing, more automaticity of facts in early years

15 –

16- the language to approach p.s. develop grade appropriate strategies.

Comments from class:

Unsure of my focus on the problems with traditional based instruction, I asked my group if they had in fact **positive experiences** from they mathematics education that come to mind. Only Richard responded. He felt that his course at Winnipeg Tech, one where he finally got to see what the math he was learning was used for and then again when he took an astronomy course were 2 occasions where his mathematics learning was positive. No others responded to this question. Interestingly, the application of mathematics and its explicit connections to the outside world are foci of our current intent in math education. Nice to see that that which parents desire as a learner are those very things that we are trying to provide.

When addressing elements that I feel should be nurtured in children, the first presented idea was getting children to **accept mathematics as a challenge to be conquered**. This opened some conversation as to 'how do we do this' and, 'my youngest will accept the challenge but not my oldest'. We talked about paralleling our observations of our children in situations where they do take on the challenge and showing children how this aspect of their character can be used elsewhere, like in problem solving. (I've been thinking about these comments. The transference from one activity to another may also have something to do with how we approach kids with this idea. If one were to say why don't you use that energy in your math, their response may be, well I like Nintendo. Maybe the approach has to be more of a coaching, guiding nature. I saw the determination you had when playing that game. You have what it takes to solve math problems. Use it here. Lots of kids won't take on the challenge because they believe at the onset that they are not capable of finding success.)

Here's my group:

1 Donna – parent of 2 - high school and Riley gd 3
- Works as an

2 Rob – father of 3 boys grade 5,3 and a 4 year old

3. Rena – single mother of 2 - Grades 10 and 4

4. Julie – mother of 2 boys K and 4 year old
- works as project manager

5.

6. Richard – father of boy grade 2
- work –farm implements

7. Lydia – mother of daughter grade 5 as well as a daughter in her 30's

8. – Lydia's husband

9. Ron – son grade 3 and daughter K

10. Marsha – Ron's wife Ron and Marsha were persuaded to join by Janice

11. Janice – mother of 2 girls – high school and grade 6
- works as an IA

12. Terry – single father of 2 boys – grade 9 and 4
- works for Hydro

13. Tan – my IA – no children – from foreign country- learning about Canadian schooling

14. Bev – mother of 2 boys – grades 6 and 1
- studying to be a teacher

16. Ed – teacher at the school – helps with field notes – father of 2

Donna,
When I look at your list of what you believe should be happening in your son's math class, I can see the elements of the NCTM Standards that we'll talk about today: reasoning, communication, problem solving and connections.

Appreciated your questions regarding the diversity of children as they come to accept a challenge. How true it is; we see it in the classroom every day. Do you think birth order may play a part here? Degree may be another issue. I certainly saw the difference in Michael's 'taking the challenge' over the 2 years I spent with him.

You had many positive experiences in mathematics as did I. You also have many ideas on what today's classroom should do. Often people who feel that their experiences were positive want this traditional math classroom replicated for their children; thus they may not want to hear about reforms in math – we hear about 'back to the basics' more often from this group. I was concerned that my negative position on traditional methods might be considered unfounded to someone with your positive mathematical background. Comments?

Thanks for your support.
Deb

Rob,
Interesting that many of the things that you believe should be happening in your son's math class, actually came up later such as enjoyment, gaining desire, and ability to apply concepts. You also brought up the calculator, I'd like to get to this topic at some time during the next 3 weeks. A calculator can actually teach us about fundamental ideas with operations.

The basic skills of the operations are fundamental in math but even more so, I believe number sense derived from the learning of algorithms is more important. It doesn't really bother me if students don't get 80-100 % on a list of operation questions. I'm more confident when I can listen to an accurate process and know that they understand why they can do the steps they can do. If I see this, I won't drill and test for higher scores. I'll have to tell you about my experiences with operations with grade 7 students.

Thanks for helping out a friend.
Deb

Rena,

I often tell kids that if you had difficulty with something when you were younger, it doesn't mean you can't do it now. Fractions are often a stumbling block for students, they get the idea that they just can't do it. Your comments provide support for this idea.

I wonder if your difficulty understanding in grade school was brought on by frustration. As teachers/parents it seems we must attune ourselves to children's feelings as much as the content. I'll get into this idea more when we look at problem solving.

I also think that when children's difficulties hit the wall at about grade 7 or 8, it has to do with how they are internalizing math. Is it 3500 different ideas to be memorized or is it a connected

whole with a few main channels (like the net idea). We need to help children see the connections. I'll be talking about this today.
Deb

Julie,
Your enthusiasm for your children's learning will be rewarded. When you talk about the counting of trees and adding games, you are fostering a view of the world through math. Children with these experiences develop a stronger number sense, which enables them to see the big 'net'.

From the few notes I've read, sounds like we're on the same page – 'empowering them to learn for themselves' – I really believe this is a key to their success. Without realizing it, wanting their children to be successful, many parents actually 'steal' learning away from their children.

I'm looking forward to hearing your reactions to this program as we move along. I'd really like an opportunity to speak with you in more detail.
Deb

Richard,
Appreciated your comments about positive experiences in math education. Interesting to me that the point you made about application is what we are trying to create throughout a child's math experiences. Unfortunately for you, this experience didn't occur until high school.

Hoping this program will have an effect on your learning of math – perhaps your searching for connections and reasoning will help. Seeking practical applications for your son as you assist him through the years will cause him to understand more deeply as well. When kids have difficulty understanding an algorithm, I often go to a practical situation. Last week I had a grade 7 boy trying to do something like 11.8 divided by 0.5. He wasn't sure what to do with the decimals, so we started by thinking about what the question might mean and estimating. Eg. If the teacher has about $12 and wants to give some students 0.5, or 0.50 or more meaningfully 50 cents, how many people could get 50 cents. He could easily reason about 24 people. So when we went back to moving the decimals over, he could see where they move. The meaningful story also helps him to understand the decimal answer – no we won't get exactly 50 cents or 0.50 but a little less. Why and how we can just move over the decimals is another story. The point is, any situation can be better understood by connecting with a situation that makes sense.
Deb

Lydia,
It sounds to me that you experienced a traditional math experience with lots of memorizing. I imagine this included not only the facts, but also the formulas and procedures. Did you have much application of these skills?

You are very right when you say you want your child to have 'confidence' in problem solving. This is my number one concern with my grade 8 classes. If I can help them to build confidence as a problem solver, I will have met a very important goal. I have worked on and altered my teaching plan many times over the years to meet this goal. More about this in session 4.

How was the first session for you? We didn't get a chance to talk after.
Deb

Eric,
I chuckled at your term 'good punishment', something like a 'good' snowstorm I imagine. Eric, do you feel that this instructional method had a positive or negative effect on your learning. I know that sometimes we do things for children, discipline wise, that are for their own good. I question though that learning math (or not) should receive the same approach as bad behavior, for example.

I hope that my techniques will make a difference for you in your work with Kelsey. Sometimes adults try to use the final stage to start teaching rather than the beginning. Easy to do, as often it's the last stage we remember. (Or in traditional practices, the only one we learned.) I try to hook on to the stage they're at and move forward from there. More on this today.

Thanks for joining in; Lydia wasn't sure you were going to be able to make it.
Deb

Ron,
Knowing you'll be away today, I'll try to fill you in on some key ideas. We will have looked at developing the concept of volume from K to grade 8. Julie will fill you in on the process we followed. The purpose of this is to have parents see that the curriculum builds conceptual understanding. One could easily have started at grade 6 with this topic and told the children that Volume of a box is length x width x height and they could add this to their memorized bag of information. Too bad for the kid that starts to mix up all this info because they don't 'own' the idea of volume. Was this you Ron?

Another purpose in this activity is to have parents see the value of communication as children work together to reason an understanding of volume. Another is for parents to see how any concept can break down into many parts. Our job as teachers/parents, is to listen to what children have put together and then help them 'scaffold' to a new or deeper understanding. We should enter into their learning process rather than have them follow our plan (aka. Here's how to do it!). Learning is about linking with what we already know. This helps to explain why we sometimes appear to be taking the long way around.

And another point is that of learning math as a connected whole rather than a whole pile of isolated concepts. Kids who learn math concepts in isolation often fall apart about grade 7, as they no longer can manage so many separate ideas. If we teach connectedly, although more complex, they see math as connected (which it is). That means while we worked through volume, we also visited factors, decimals, linear measurement, estimating, etc.

And lastly, what we have done today is also problem solving for children. When the curriculum suggests that 50% of what children do is p.s., it doesn't necessarily mean that we're doing word problems.

See you next week.
Deb

Julie,
I'm impressed that you made the step to join in to this program. After we look at building the concept of volume with reasoning and understanding, I hope your fears will be lessened. I believe that we must enter into a child's learning, trying to find out just how far they've put something

together and then carry on from there. Too often, with traditional methods, if you missed a point in instruction, you might have missed the whole concept. No one asked you what you were thinking and helped you 'scaffold' to further understanding. In traditional programming, the content order was also so tied to closed units. For example, if fractions were studied in October, they would not appear again until next year (when many kids would have forgotten). With problem solving situations, we revisit topics throughout the year. This not only allows children to continue working on understanding the topic but also to revisit and refresh their memory.

I agree with your statement that your child's math class should be positive. The affective side of learning cannot be ignored.
Deb

Janice,
It's hard to believe that ability grouping was ever perceived as a viable instructional method. Were we only concerned with educating the 'top' students?
The NCTM (National Council of Teachers of Mathematics) strongly advises that we consider a child's affective and cognitive development in math education.

This can be a difficult task when you see 120 students in a day and their affective needs vary. Certainly, labelling them as turtles seems like a really bad start. But encouraging them to realize just how much they actually do know, might be a good start. Building confidence in a child who perceives themselves as weak in math can go a long way. Luckily for you as a parent, this isn't an issue.

Have you had any conversations about last week. I'd love to hear them.
Deb

Terry,
I enjoyed our conversation last week. The direction you take with your boys in terms of valuing understanding is positive. I wondered after reading your initial entry if your mom's influence (puzzles) was stronger than your own math education influences.

You already seem onboard with a need to change instructional styles to incorporate an understanding based program. Hoping these next few sessions will give you some different ideas. I believe one important concept that I've learned as a teacher is to apply a 'breakdown' structure to all concepts that children in my grades study. So as I breakdown volume today, look towards what this looks like in long division, etc. Sounds like you're working in this direction already. Perhaps this program will expand your push in this direction.

Deb

Tan,
Although you are not a parent, I know these sessions will bring us closer together philosophically in terms of direction with our students. The model I use today to build volume, I use quite consistently with all conceptual development. I go back and break it down so that students can enter in at their point and move forward. When I work individually with the students I try to listen for what they have already internalized and them move forward from there.

When you write, why don't you use our students as the children with which you are connecting these ideas. Good to have you here.
Deb

Bev,
Your presence here is truly appreciated. I believe it's valuable for parents to hear the connections with what you do in class (eg. the discussion of solutions) and it gave me a breather last week.

The structure of the group will change this week, should be more open discussion. Any points on these conversations would help with my notes later.

The ownership and responsibility of one's learning is certainly life long. If children learn this from the onset, I believe success will follow them. A funny society we live in though, seems like people think they're bad parents if they don't do, do, do for their kids. Take science projects for example. Isn't there an underlying belief that a parent's involvement as a partner in the project is expected? Sometimes parents lose sight of their role.

Thanks again,
Deb

Ed,
Language is an interesting topic. This may greatly impact on how a teacher will set up the structure of a class. One teacher might believe that children need the language and then commence with a definition-based program. Another might recognize that to develop the language, one needs to speak the words not just hear them and define them. Consider the parent as well who is so used to a 'teacher as teller' traditional based program, that they do all the talking as they instruct their child and thus miss out on the opportunity not only to help their child develop the language but also the opportunity to hear what they have already internalized and go from there. More on the 'path of learning' idea today.

Your last point is valid: How can we as parents support our children if the answer is not obvious to us. As a teacher I would never give students a problem that I myself have not worked through yet. But… parents are put in this position. What happens before and after the homework assignment is crucial here. And as teachers we should be filling parents in on our expectations with problem solving.

As teachers, you and I, we could discuss a structure for this at a later time. I've analyzed and revamped my ideas on this often throughout the years. I'm somewhat pleased with my approach with the 8's and 9's now.
Deb