Optimum Structural Design

of Masonry Shear Walls

by

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OPTIMUM STRUCTURAL DESIGN OF MASONRY SHEAR WALLS

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ABSTRACT

There are a large number of variables to be considered in the structural design of masonry buildings which complicate the traditional design process based on trial-and-error. Faced with this complexity, masonry designers tend to overestimate factors of safety, which results in uneconomic buildings. In this thesis, a pseudo-discrete model for the optimum design of masonry shear walls is developed. The model is applied to single and multistorey masonry shear walls. For the single storey shear wall, structural constraints representing stress and load limitations are derived from the Canadian Code CAN3-S304-M84, and/or elasticity principles after being manipulated mathematically to fit what is termed the "acceptable discrete form". Because of the complexity involved in manipulating the structural constraints for multistorey shear walls, simulated functions are developed to replace the original functions. Computer programs are developed to prepare the mathematical formulation of the model. The input data of the computer programs are selected by the user, the practicing engineer. Linear programming, with its integer capabilities based on the Branch and Bound technique, is used to solve the design optimization problem. Suggestions and recommendations are provided to overcome the computational difficulties associated with the modeling and solution process of such large-size optimization problems. A number of study cases are investigated to demonstrate the practicality and efficiency of the models. The results of these study cases suggest a great saving in both computation time and cost of materials. A number of recommendations for further research work in the areas of masonry structural behaviour and masonry construction are provided.

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NOMENCLATURE

Because of the large number of discrete variables introduced throughout Chapters 3, 4 and 5, it is found helpful to define the superscripts and subscripts associated with those variables in a general term before introducing the list of notations. These sub- and superscripts are defined as follows:

$e = 1, 2, \ldots, ET$	Number of spacing options for horizontal shear reinforcement for single storey problem.
$i = 1, 2, \ldots, IT$	Number of grouting conditions for both single and multi-storey problems.
$j = 1, 2, \ldots, J$	Number of stories for multi-storey problem.
$k = 1, 2, \ldots, KT$	Number of outside (or end) steel options for both single and multi-storey problems.
$m = 1, 2, \ldots, MT$	Number of inside steel options for multi-storey problem.
$p = 1, 2, \ldots, PT$	Number of inside steel options for single storey problem.
$q = 1, 2, \ldots, QT$	Number of horizontal shear reinforcement options for single storey problem.

Examples:

1. <u>For single storey problem</u>:

$(D.V.)_i^*$ A discrete variable defining the i^{th} grouting condition.

$(D.V.)^{k}$ — A discrete variable defining the k^{th} outside steel option.

$(D.V.)_{i}^{k}$ — A discrete variable defining the k^{th} outside steel option for the i^{th} grouting condition.

$(D.V.)^{p}$ — A discrete variable defining the p^{th} inside steel option.

$(D.V.)_{i}^{p}$ — A discrete variable defining the p^{th} inside steel option for the i^{th} grouting condition.

$(D.V.)_{i}^{kp}$ — A discrete variable defining the p^{th} inside steel option for the k^{th} outside steel option for the i^{th} grouting condition.

$(D.V.)^{e}$ — A discrete variable defining the e^{th} spacing option for horizontal shear reinforcement.

$(D.V.)^{q}$ — A discrete variable defining the q^{th} horizontal shear reinforcement option.

$(D.V.)_{i}^{e}$ — A discrete variable defining the e^{th} spacing option for the i^{th} grouting condition.

$(D.V.)_{i}^{eq}$ — A discrete variable defining the q^{th} horizontal shear reinforcement for the e^{th} spacing for the i^{th} grouting condition.

2. <u>For multi-storey problem</u>:

$(C.V.)^{j**}$ — A continuous variable for the j^{th} storey.

$(D.V.)_{i}^{j}$ — A discrete variable defining the i^{th} grouting condition for j^{th} storey.

$(D.V.)_{k}^{j}$ — A discrete variable defining the k^{th} outside steel option for j^{th} storey.

$(D.V.)^{j}_{ik}$ A discrete variable defining the k^{th} outside steel option for the i^{th} grouting condition for j^{th} storey.

$(D.V.)^{j}_{m}$ A discrete variable defining the m^{th} inside steel option for j^{th} storey.

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* (D.V.) Discrete variable

** (C.V.) Continuous variable

LIST OF NOTATIONS

AA	A function of (C_s, f_m, A_m, M and P_d).
AB	A function of (C_s, f_m, A_{so}, n, M and L).
AC	Cross-sectional area of one core.
ADD	A function of (ING, C_s, f_m, A_c, f_s, A_{si}, CW, h, UW and L).
ADDL	Additional applied dead load due to grouting more cores (for multi-storey problem).
ADL	Total applied (Dead + Live) load (for singel shear wall problem).
ADRC	Additional axial compressive resisting capacity due to adding compression steel and more grouted cores (for multi-storey problem).
A_g	Total gross cross-sectional area of a wall per metre.
A_{gi}	Gross cross-sectional area of the inside part of a wall.
A_{go}	Gross cross-sectional area of the outside part of a wall.
A_m	Mortar bedded area.
AN	Compressive stress surplus in case of compression violation.
A_s	Cross-sectional area of vertical steel (uniformly distributed).
A_{se}	End vertical steel cross-sectional area in case of unreinforced wall.
A_{si}	Inside vertical steel total cross-sectional area.
$A_{s\ell}$	Horizontal total shear reinforcement cross-sectional area.

A_{so}	Outside vertical steel total cross-sectional area.
BA	A function of (M and L).
BB	A function of (j, D_ℓ and ECE).
BC	A function of (j, D_ℓ and ECD).
CB	A function of (ECD, ECE, h and L).
CC	A function of (ECD, $(ECE)_u$, h and L).
C_e	Eccentricity coefficient.
CM	Masonry compressive resisting capacity.
C_s	Slenderness coefficient.
CS	Compressive steel resisting capacity.
CW	Core width.
D	Effective depth of a section under tension.
D'	Distance from center of a steel combination to the nearest end of the wall under tension.
D_ℓ	Dead load of one roof.
DL	Total dead load.
DLI	Applied (dead + live) load for the inside part of a wall.
DLIA	Extra dead load (for single shear wall problem).
DLS	Applied (dead + live) load for the outside part of a wall.
e=1,...ET	Number of lateral reinforcement spacing options.
E	Eccentricity ratio.
EC	A function of (P_d and E).
ECC	A function of (E, EC and P_d).
ECD	A function of (M and L).
ECE	A function of (C_s, f_m, A_m, L, M and EC).
E_m	Masonry modulus of elasticity.

E_s Steel modulus of elasticity.

f_{cs} Contribution of dead load to compressive stress (in the shear stress calculation).

f_m Allowable axial compressive stress for masonry.

f_{ma} Actual value of masonry compressive stress.

f_{ms} Simulated value of masonry compressive stress.

f_s Allowable tensile steel stress.

f_{sa} Actual value of tensile steel stress.

f'_s Allowable compressive steel stress.

f'_{sa} Actual value of compressive steel stress.

f_{ss} Simulated value of tensile steel stress.

f'_{ss} Simulated value of compressive steel stress.

f_v Allowable shear steel stress ($f_v = f_s$).

h Height of a single storey wall (roof to roof).

i=1,...IT Number of grouting conditions.

ING Number of inside grouted cores.

k=1,...KT Number of vertical steel standard sizes options.

K Zero-one variable associated with vertical side steel.

L Total length of a wall.

LI Inside length of a wall designed for tension or compression

LL Total live load.

L_ℓ Live load for one storey.

m=1,...MT Number of inside vertical steel options.

M In-plane bending movement.

n Modular ratio.

N	Zero-one variable associated with grouting conditions for the multi-storey problem (for DL + LL case).
NC	Extra number of inside grouted cores (or number of inside steel bars).
NS	Zero-one variable associated with inside reinforcement for the multi-storey problem (for DL + LL case).
OAD	Original applied load.
ONG	Original number of grouted cores.
ORC	Original axial compressive resisting capacity.
p=1,...PT	Number of inside vertical steel options (for single storey problem).
P	Unit weight of the wall described in sequential discrete form.
P_d	Allowable axial dead load.
P_r	Flextural compressive resisting capacity of masonry.
q=1,...QT	Number of lateral (shear) reinforcement options.
Q	Zero-one variable associated with outside steel options.
r	(Grouted cores/Ungrouted cores) ratio.
R	Zero-one variable associated with the inside vertical steel.
RCI	Axial compressive resisting capacity of the inside part of the wall.
RCIA	Additional axial compressive resisting capacity of the inside part of the wall.
RCM	Total axial compressive resisting capacity of the wall.
RCS	Axial compressive resisting capacity of the outside part

of the wall.

S Zero-one variable asociated with shear discrete variables

S_a Zero-one variable associated with lateral (shear) reinforcement options.

S_b Zero-one variable associated with spacing options for lateral (shear) reinforcement.

S_ℓ Spacing of lateral (shear) reinforcement.

SM Section modulus.

SNG Number of side grouted cores.

S_{se} Standard size of end cores vertical steel.

S_{si} Standard size of inside vertical steel.

$S_{s\ell}$ Standard size of lateral (shear) reinforcement.

S_{so} Standard size of outside vertical steel.

t Thickness of a wall.

T Tensile steel resisting capacity.

TNC Total number of cores in a wall.

TNG Total number of grouted cores in a wall.

TSS Total outside steel area.

UW Unit weight of a wall.

V Maximum shear force.

v_a Actual shear stress.

v_a' Discrete value of the actual shear stress.

v_ℓ Maximum allowable shear stress.

v_m Limited value for the allowable shear stress.

W Zero-one variable associated with grouting conditions for multi-storey problem (subset-4-DL + WL).

WL Wind load.

WS Zero-one variable associated with side steel options for multi-storey problem (subset-4-DL + WL).

X Zero-one variable associated with j^{th} storey and/or i^{th} grouting condition.

XA Axial load stress for masonry wall.

XB A function of (L and P_d).

XC A function of (M, L, C_s, f_s', and A_{so}).

XD A function of (L, C_s, f_m and A_m).

XE A function of (ECC, L, M, EC, j, D_ℓ, h and L).

XF A function of (A_m, L, A_{so}, h, L, M and EC).

XK A function of (h, C_s, f_m and A_m).

XL A function of (M, h and L).

XM A function of (AA, BB and CC).

XN A function of (AB and CB).

XO A function of (BA, BC and CC).

XP A function of (M, L, j, D_ℓ and h).

XQ A function of (C_s, f_m, A_m, L and h).

XS Zero-one variable associated with outside steel options for multi-storey problem (subset-1-, DL + WL).

Y Zero-one variable associated with grouting conditions for multi-storey problem (subset-3-, (DL + WL).

YA In-plane bending moment stress for masonry shear walls.

YC A function of M.

YS Zero-one variable associated with outside steel options for multi-storey problem (subset-3-, DL + WL).

Z Zero-one variable associated with j^{th} storey and/or i^{th} grouting conditon for multi-storey problem (subset-2-, DL + WL).

ZA Allowable masonry compressive stress modified by slenderness coefficient.

ZS Zero-one variable associated with outside steel options for multi-storey problem (subset-2-, DL + WL).

σ Reinforcement distribution factor.

ϕ (Depth to the neutral axis/effective depth of a wall) ratio.

ϕ_s Simulated value of ϕ.

CHAPTER I

INTRODUCTION

1.1 MASONRY

Masonry consists of different types of materials put together to produce a composite material. Modular units such as stones, bricks or concrete blocks are the main components, while mortar is a binding component. Grout and reinforcing steel, on the other hand, are essential only in some cases, as shown in Figure 1-1.

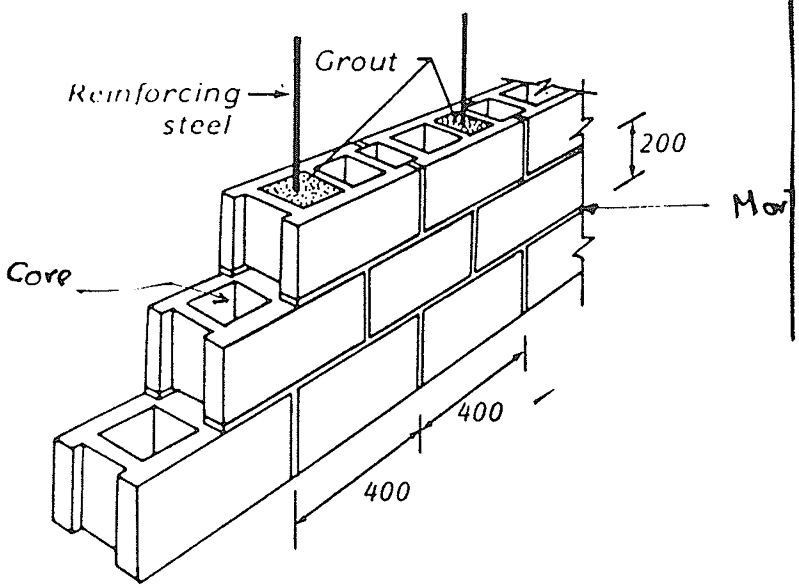


Figure 1-1. Reinforced masonry wall

Masonry is the oldest of significant building materials. Its use can be traced back thousands of years to the ancient Egyptians, Greeks and Romans. For centuries, masonry structures predominated, being used in a wide variety of structural forms. With the turn of the century and with the rapid development of concrete and steel structures, the use of masonry declined. Many factors contributed to the decline of the masonry industry. Structural steel and reinforced concrete have fewer component materials than masonry and there is, consequently, a clearer understanding of their structural analysis and design. This understanding is reflected in the corresponding codes. While steel and concrete codes are comprehensive and clear in presenting the required information, masonry codes have always been ambiguous and subject to individual interpretation. Other factors such as problems associated with the environment, resistance to flexure, manufacturing and building techniques, and the economy are among the reasons that favoured the new systems.

The above are, perhaps, the most important factors that have driven masonry from its once prominent position as a traditional building material to, in many cases, a secondary position among other competitive choices. During the last few decades, a movement has begun to strengthen the position of masonry among these engineered systems. Researchers have concentrated on areas related to the behaviour of this composite material, development of new types of masonry units, design and analysis, codes and standards. A few studies have considered the economic issue. Most of these studies[9,10,32,33] have focussed on the areas of marketing and on consumer and designer preferences. These studies are based on measuring the perceptions of those parties involved, mainly through conducting comparisons between masonry and other building materials. To face the competition of other systems, the masonry structure must be at its most economical form before conducting such comparisons.

In this thesis, a method for designing such "most economical" or "optimum" masonry structures is developed as a first step in the comparison between masonry structures and other types of structural systems.

1.2 OPTIMIZATION

The word "optimization" has been defined by many researchers, engineers and mathematicians. A.B. Templeman [21] has defined optimization in a practical way. He stated "By optimization is meant any logical process by means of which measure of the efficiency of a design or plan is improved". By analyzing such definitions, one realizes that the optimization process is not a solving process but, rather, a solution improving process. However, as is the case in most optimization processes, a solution method must accompany the solution improvement process.

The general representation of optimization problems is known as models. A model can take any of several alternative forms, one of which is a set of mathematical equations, which is the most commonly known form. In constructing a model, three steps can be identified; first, by defining the unknown variables; second, by expressing system constraints as a set of mathematical relationships; and, finally, by defining the criteria function that describes the objective of solving the problem. Different problems require different types of modelling. It can be said that both the nature of the problem and that of the constraints describing the system affect the type of model.

Having constructed the model, an appropriate solving technique is applied. Solving techniques are selected according to the nature of the model. For example, linear programming, through employing the Simplex

Method, may be applied for those models expressed only by linear relationships.

Mathematical programming was first applied to management problems. It was intended to help managers make better decisions by solving problems more efficiently. Having succeeded in this area, it has been adapted to other disciplines such as natural sciences, mathematics and engineering. Attempts at utilizing the optimization techniques in the structural design area have appeared over the last two decades. However, research related to this new field progressed slowly. Among the factors affecting this slow rate are: unfamiliarity of the practitioners with this new mathematical programming concept, the cost of an optimization process that may exceed the cost of simple analysis and the impracticality noticed in some optimization results. It appears that if practicality of a certain design and a lower cost of optimization process are achieved, then practicing engineers will become more familiar with such process and be willing to use these methods to achieve their aim, which is, mainly, a reliable, sensible and economic design.

In this thesis, the optimization models developed emphasize the practicality needed in the structural design by practicing engineers. This practicality is reflected through the interaction between the design codes and the relationships developed. Also, allowing the designer to be a major source of input data, through selecting the available materials for example, provides this practical interaction.

1.3 MASONRY OPTIMIZATION

As mentioned above, in the last few decades, through the work of engineers and researchers there is an improved knowledge and understanding of masonry. This work has helped in setting new guidelines for structural analysis and design and in more rational codes and standards. However, since economic considerations require that a "most economic" structure be selected from several competitive construction materials attention must be directed toward fulfilling this objective for masonry. This objective can be achieved by developing optimum masonry design models.

To develop such models successfully, three factors must be considered: First, the nature of the components of any masonry structure, secondly, the nature of the design relationships as introduced by the appropriate code and, third, the type of optimization technique that best suits the problem.

Any masonry structure, like any other structural system, passes through successive stages from the preliminary design stage to the final construction. Although one attempt to accommodate such stages in a single model has been made, as reported in the next chapter, the main models developed in this work consider only the structural design stage.

Three main component materials are subject to optimization in the structural design stage. These are the concrete blocks, grout and reinforcement. Concrete blocks and steel bars are available in standard sizes. On the other hand, grout is always confined by the areas of the cores in the blocks, which means that it also has discrete values. Having discrete values for such component variables requires optimization techniques that differ from those applied to continuous variable problems (e.g. optimum design of concrete beams).

These discrete optimization techniques assume prior knowledge of the variable values beforehand, while the main optimization problem will, then, be to select the most economical combination from among several choices. This selection process is performed through the application of what is known as the "Branch-and-Bound"[11] method (see Appendix B) with the help of what can be referred at as "zero-one" variables[23]. This type of approach, besides its efficiency in solving the discrete problems, is also appropriate for the design relationships used in the model as constraints. Most of these relationships, introduced by the codes(s) and elasticity principles, are non-linearly expressed and then require the type of programming known as "non-linear". However, since the discrete models are based on the fact that the discrete values of the different variables are known beforehand, part of these non-linear relationships, as will be discussed in later chapters, can be expressed in a "discrete form". This "discrete form" could be expressed, after being associated with the appropriate zero-one variables, in a linear form. For example, if X and Y are two continuous linear variables, the term XY is, then, non-linear. On the other hand, if discrete values are assigned to both X and Y, then their product, XY, is recognized as a third discrete value. This third discrete value, say Z, can be assigned a zero-one variable, say α, and the product of $Z\alpha$ can be considered a linear term.

Furthermore, when complexity is involved in expressing the design relationships in "discrete" form, simplified expressions are developed, as will be discussed in Chapters III, IV and V, that represent, as closely as possible, the original relationships. These "simulated" functions, after being tested, are then expressed in a linear "discrete" form.

In this thesis, the standard values of concrete blocks and reinforcement are selected from sets of catalogues available to practicing engineers. The design relationships, on the other hand, are adapted from the Canadian Code.[4] Linear programming through its integer capabilities is found to be the most efficient optimization technique in solving such kinds of linear discrete models. Therefore, an established optimization computer package, LINDO,[24] is utilized.

1.4 Objectives

The main objective of this study is to develop design optimization model(s) that are able to replace the traditional masonry structural design approach which is based on a trial-and-error procedure. These models, when solved, yield the optimum or the most economical masonry structural design. Although some of the applications are directed toward different types of masonry structural members (e.g. bearing walls), the main contribution in this work is toward shear walls, single and multistorey. Other objectives are as follows:

1. Developing simplified, and yet identically representative, design relationships that can be practically used in the optimum as well as traditional masonry structural design.
2. Studying the possible expansion of this work to accommodate:
    a) analysis of behaviour and physical characteristics of masonry as a composite material;
    b) a complete masonry structural system that consists of all possible structural elements.
3. Developing optimization models, that generally:
    a) are user-friendly and minimize the computation time;
    b) are transferable to practitioners.

CHAPTER II

LITERATURE SURVEY

The purpose of this survey is:

1. To review previous research work related to structural optimization in general;
2. To present and review all research work related to masonry optimization that has led to the development of the main models reported in this thesis;
3. To present studies conducted in attempts to extend such type of research to broader areas such as construction management and behaviour of masonry materials.

2.1 RESEARCH WORKS RELATED TO THE OPTIMUM DESIGN OF STEEL AND CONCRETE STRUCTURAL SYSTEMS

This survey presents the more significant work in the field of optimization for steel and concrete structures and focuses on the progressive applications of optimization techniques to steel and concrete structures from the early simple cases to the latest forms of adaptation of such techniques to structural systems. The survey is in two parts: the first presents the work done in the area of continuous variable optimization design problems. The second part presents the development of discrete models that are the most suitable to structural optimization problems.

2.1.a. Continuous Variables Optimization Problems:

F. Moses[30] has developed a process for obtaining the optimum (lightest) steel trusses and frames. The model consists of an objective function and a set of constraints that describe equilibrium and compatibility of displacements. As the relationships used are originally non-linear a trial procedure is performed to determine the initial values of the different variables. Once this initial vector is determined, a linearization process that replaces the non-linear equations by their linear first-order Taylor series is performed. Linear programming is then applied to solve the problem and the resulting optimum values of different parameters, such as areas and stresses, are found as continuous values. An optimization approach has been specially de~~veloped in~~ optimizing the cost of steel frames, members and connections.(43) The design variables considered consist of cross-sectional areas, the plastic moment capacity of the members and the moment resistance at each connection relative to the plastic moment capacity of the connected member. The objective function is formulated to express both the material and the connection cost in terms of these design variables. The solution process is iterative and based on trial-and-error. The optimum solution yields continuous values of section dimensions which are, further, approximated to their closest standard sizes.

A non-linear programming method utilizing Lagrange multipliers has been used to determine, *a priori*, which constraints will control the minimum weight design.(18) It has been concluded for the minimum weight design of a large class of structural systems subjected to single load conditions that these structures must be fully stressed. Again, the unknown variables, namely, the cross-sectional areas of the members and the stresses, are determined as continuous variables.

The problem of multiple loading conditions in the minimum weight design of plate girders has been considered and solved using the gradient projection method of non-linear programming.[13] Piecewise linearization of constraints has been achieved for use in Simplex Methods for optimum design of a pin-jointed steel framework.[26] The optimal design of a multistorey steel frames under gravity load has been developed using an analytical approach and also by employing a modified dynamic programming technique.[15] Elastic behaviour in the form of working stress limitations and ultimate capacity have been considered for the optimum design of framed structures in the form of an iterative set of linear programming problems.[27] With the help of plastic analysis, steel frames have been designed for minimum weight employing linear programming, where side-sway is accounted for by restricting the moment carrying capacity of the column.[2] The optimal elastic design of reinforced concrete continuous beams and frames has been obtained through linear programming techniques such that under any possible load combination, a certain specified minimum load factor is guaranteed against collapse of the structure and the first yield of its critical section.[7] Compatibility, limited ductility, equilibrium and serviceability criteria have been considered to arrive at the minimum amount of reinforcement.[14,31,19,20] The Simplex algorithm has been used to solve the linear programming problem,-[31,19,20] and the examples have indicated a saving in steel between 6% and 17%.[31] Adapting the flexibility method for analysis, reinforced concrete frames have been optimized for minimum cost treating width, depth and reinforcement of each member as design variables and subject to ACI Building Code regulations.[45] A study which focuses on obtaining realistic trends and guidelines for the optimization of building systems has been attempted.[28] Characteristic parameters of the mathematical model of the structure are unit weights for the floors, walls and spandrel beams and areas of steel, reinforced concrete columns and main girders. The optimization problem has been linearized using a Taylor series and solved using linear programming techniques.

2.1.b Discrete Variable Optimization Problems

Since engineers are often confronted with limited sets of standard sizes of many structural materials, e.g., reinforcing bars, steel beams and bricks, a few researchers have worked on the problems of discrete optimization. Optimum design using available sections has been worked out, for steel frames, using a search technique that has been derived from Gomary's algorithm.[46] It has been concluded that the method is inefficient because of convergence difficulties, especially for problems with a large number of variables. A multi-level decision making programming problem has been formulated to minimize the overall structural weight of rigid-plastic plane frames using discrete sections and has been solved using the Dual Simplex Method.[17] A discrete formulation of minimum cost design of framed structures has been developed to choose member sizes from a catalogue of commercially available sections.[6]

Reinschmidt[41] applied zero-one integer programming to optimize steel frames and trusses. He used limited available sections for linear structures where the nonlinear constraints were linearized using Taylor's first-order series. Further, he employed the implicit enumeration method to solve the problem. Faced with the problem of determining the global optimum solution, he suggested an initial solution or initial local solution, to generate starting points. Cella and Logcher[5] have developed a semi-automated optimization algorithm for discrete variable problems. Three phases are built into the algorithm. Phase I requires an initial lower and rather infeasible solution as input. In this phase a feasible region is generated through sequential steps parallel to the coordinate axis. In phase II, a local optimum is selected through a filtering process. In phase III a better local optimum is selected through a branch-and-bound process. The whole process is considered a multi-stage solution process where a "blockage", due to any computation problem, can cause the termination of the solution procedure. Although the algorithm has been applied to multi-member structures, such as space trusses, simplified assumptions, such as considering only one load case at a time and eliminating some important constraints, have diverted attention from the significance and capabilities of the algorithm.

Morris[29] has presented a linear programming solution to some common prestressed concrete design problems. A trial section is assumed while the unknown variables are the adequacy of the concrete section, the prestressing force and the permissible zone for the proposed force. The method has suggested a format for further design to accommodate discrete components sizes, standardized geometric arrangements and other loading conditions.

2.2 RESEARCH WORKS RELATED TO MASONRY OPTIMUM STRUCTURAL DESIGN

The structural optimization research survey has indicated that the area of masonry structural optimization has received no attention prior to 1985 when the first work was presented at The Third North American Masonry Conference.[39] Being the first attempt in this field, the model developed was simple in nature, where simplifying assumptions as well as simple application were being used. The model itself is a linear programming model that consists of a set of linear constraints derived from the Canadian Code.[4] The unknowns are the sizes of the blocks and the steel areas while the objective function is the minimization of the total cost. The model was applied to three masonry walls subjected to in- and out-of-plane bending moments as well as axial loads. The comparison between the traditional design approach and the linear programming solution showed substantial savings in both costs of material and computation time.

Mixed-integer programming models have been developed subsequently for the optimum design of masonry bearing walls.[40,41] The first has been applied to bearing walls with low eccentricities (eccentricity < thickness of wall/3). In this first paper, analysis of different design approaches for such walls was performed and it was concluded that the Coefficient Method of the Code[4] gives more economical solutions than the Unity Equation or the Load Deflection Method. Since the main expressions describing the relationships between the resisting capacity and the applied loads are non-linear, piecewise linearization was applied to linearize such expressions. The linearization was performed using the end points of any particular curve representing a non-linear relationship as grid points for the linearized relationship. This method of linearization was justified in the paper. The model developed considered axially applied vertical loads and out-of-plane bending moments due to wind pressure. It also considered the multi-choice of available blocks, namely 190, 240 and 290 mm thick blocks, and the load cases (dead + live), (dead + wind) and 0.75 (dead + live + wind). In addition, the cases of fully grouted, partially grouted and ungrouted walls with or without reinforcement were included. The objective function is the minimization of the total cost of materials, while the constraint set consists of the linearized relationships describing limitations of stresses and loads as well as code and integer requirements.

Two subsets of linear constraints have been developed, the first for the unreinforced case and the second for the reinforced case. Although the two subsets are included in the same model, artificial variables are incorporated in the first subset of constraints to differentiate between the two cases. These artificial variables have a very high cost in the objective function, a situation which prevents the binding of the constraints associated with it whenever they are not supposed to. Zero-one variables have been assigned to the discrete values of the different variables (e.g. grouted cores, block sizes, reinforcement, etc.). The model has been applied to an example problem and the optimum results, again, proved the saving achieved in both cost of material and in computation time.

The second paper[41] studied the problem of the optimum design of bearing walls with high eccentricities, (eccentricity > thickness of wall/3). Transformed Section Analysis formed the basis for the structural design relationships developed. Since tension exists in such walls, reinforcement represents an integral part of the wall. Therefore relationships between applied loads and moment resisting capacities for ranges of reinforcements, block sizes as well as several grouting conditions were first developed. Then these relationships were linearized using a Separable Programming technique. The problem was finally simplified to a linear programming problem. Each set of block size, reinforcement and a grouted condition represents a discrete solution that corresponds to a particular set of applied load and bending moment. Zero-one variables were then incorporated into the problem which was solved using the LINDO[64] optimization package.

Since masonry shear walls are, perhaps, the most important structural element in a masonry building, their optimal design forms the core of this thesis. However, prior to the development of the main, and comprehensive models presented in Chapters III and V, two attempts were made in this area. The first attempt[38] presented a simplified model for the optimum design of single storey (floor-to-roof) masonry shear walls. In developing the model, a few general as well as specific assumptions were made. It was assumed that no tension or shear violations exist in the wall, and only two load cases were investigated. Two submodels were developed, each corresponding to a governing load case. The governing load case has to be determined prior to the development of the corresponding formulation.

In case one, the (dead + live) load case governs, and two options can be identified. The first is an unreinforced wall where it is assumed that the wall resists any load lower than the maximum resisting capacity of the fully-grouted section. The model itself is a mixed-integer programming model where the constraints express the relationship between the grouted areas and the masonry compressive stress and the applied loads. These relationships, which are non-linear, were then linearized using piecewise linearization. Since the values of the grouted areas and compressive stresses yielded are continuous, a second set of constraints was developed to tie those continuous values to their corresponding discrete values. The second option within this case is for a reinforced wall where reinforcement was assumed to be uniformly distributed. The reinforcement determined from this model is treated as a continuous variable, and as such is a deficiency in the model. Whenever the (dead +

wind) load case governs, a second sub-model is applied. The relationships in this sub-model were derived from the Unity Equation and the reinforcement can be placed at each end of the wall to resist any compression violation. Again, the steel is considered to be a continuous variable. In the above work, the load cases have been separated, which simplifies handling both the formulation and the solution of the problem.

The second attempt was the development of an optimization model for the multi-storey shear wall.[13] The model, being the first attempt for such problems, was simple in nature. A few simplifying assumptions were made, such as limiting the application to one load case and one standard size of steel. The computation process was divided into two main parts. The first used a computer program specially developed for this problem which generated the required constraints for the model. This part also played a role in the optimization process by eliminating the unnecessary and unrealistic constraints as well as linearizing the non-linear relationships. In the second part, the model developed from the first step was solved using a linear programming technique. The constraint set in the model was divided into three subsets, the first was related to the case where tension did not exist, the second for cases when tension existed and the third consists of relationships used in both cases. The relationships for the first part were initially developed using the Unity Equation, and then delivered to the model storage space. Once tension exists in the wall, analysis of the section, provided that a particular steel size is available, is performed. As soon as the resulting masonry and steel stresses, from this analysis, are found to conform to their allowable limits, the program selects the most economical combination and delivers it to the model storage space. The analysis performed in this part is based on Transformed Section Analysis. The third part of the constraints consists of those general defining constraints which are necessary to complete the model. The discrete formulation for this model has proved its efficiency through an application to a seven-storey shear wall. Due to the promising results of the last two attempts, it was felt that more investigation must be done, which led to the main models presented in the following chapters.

2.3 OPTIMUM MASONRY RESEARCH WORK IN AREAS OTHER THAN STRUCTURAL DESIGN

Two attempts have been made in masonry optimization other than the optimal design of masonry structures. The first investigated the problem of optimum masonry construction while the second has considered the problem of stress behaviour of concrete masonry blocks. A goal programming approach to masonry construction was developed and applied to a hypothetical case.(1) The differences between such an approach and the common structural design optimization techniques have been discussed. While the optimum structural design problems are often concerned with a single objective function, such as minimizing the materials cost or minimizing the weight of the structure, the optimum construction problems are characterized by multi-objective nature. Such problems, although complex, are more realistic and are applied to more practical cases. In this particular approach, the joint consideration of a number of issues such as structural design, code requirements, material transportation cost, construction time and labour force management, was addressed. Since these issues are not directly comparable in a single mathematical expression, it is necessary to employ a technique which is capable of recognizing and analyzing the fundamental differences among them. The

Goal Programming approach is found suitable for this type of analysis. In this approach, a few objectives, as mentioned above, are mathematically expressed in linear relationships where a "target" or a "goal" has been set for each objective. A set of deviational variables is then been assigned to each objective statement to describe the over and under-achievement of such particular objective. The objective function, on the other hand, is the minimization of all these deviational variables which are themselves ranked according to their relative importance and assigned priority weights. The model has been applied to a hypothetical but reasonable problem, and the results have yielded a list of alternatives from which the practitioner can select the "best" one according to the corresponding practical situation. The approach, although efficient, needs more modifications and refining so that it can be applied to more realistic and complex problems.

Two investigations[36,37] have involved the behaviour of materials problems, one of which has been directed to the masonry concrete blocks.[36] The problem is to determine the optimum proportioning of fly ash concrete mixes for hollow-core concrete blocks. Since the behaviour of materials is the main concern in this type of investigation, the optimization problem is rather complex. Its complexity arose from the need to simulate some of the data and relationships that had not been developed but which were necessary to formulate the model. Therefore, the problem was formulated as a "cycling" process. It starts by gathering the available data and information through limited test results and literature review. Then, simulated data and relationships are developed which complete the formulation of the model of the first cycle. The model is then solved to get a set of "predicted" optimum results. These results are checked and evaluated through conducting experimental tests from which a second set of data and information are obtained. This "refined" set of information forms the basis for the second cycle of the process by modifying the simulated relationships from the first cycle. The process is then repeated till convergence between the experimental results and the simulated data occurs. Due to the non-linearity of most of the simulated relationships and for the purpose of attaining the closest possible results, a non-linear programming model was developed. The model is solved using the Reduced Gradient Method through a commercially available computer package.[25] The promising results suggest an extension of this process to other material problems.

2.4 CONCLUSION

The works reported in this survey cover the main and most significant developments in the optimum design of structures. Although a large number of problems have been "solved" by researchers, most of them can be classified under a limited number of categories. The survey has indicated that masonry had no share in the area of optimization till 1985. It has also revealed that although several optimization techniques are available in the literature, discrete optimization appears to be the most suitable and applicable approach to masonry optimization.

CHAPTER III

PSEUDO-DISCRETE MODELING FOR THE OPTIMUM DESIGN OF MASONRY SINGLE SHEAR WALLS

3.1 INTRODUCTION

Although this problem had been investigated in an early work,[38] it was limited in scope because of a number of simplifying assumptions. The intent of this chapter is to present a general basic pattern to solve the same problem after relaxing those simplifying assumptions. This pattern combines both the structural and the mathematical programming components into an efficiently solvable model. The structural part, although mainly based on the Code,[4] is approached with more realistic understanding of the material performance under various loading conditions, of force distribution and redistribution and finally of each design relationship employed in different cases. The mathematical part is essentially based on a "pseudo-discrete" type of modelling,[15] that is, the discrete solution is embedded in the continuous solution region. The continuous area represents the main structural constraints in the form of inequalities, while the discrete region is created by those equations describing the discrete variables. The optimum solution is selected through the use of 'zero-one' variables which are associated with the different alternatives. In presenting the general mathematical formulation for the pseudo-discrete optimization models developed in this study, the masonry discrete variables must be identified first. Since blocks, grout, vertical reinforcement and horizontal reinforcement are, generally, the components of most masonry structures, masonry discrete variables must be related to each and to every one of those components. Throughout the models developed herein, the standard size of blocks is considered constant and taken as 190x190x390 mm for all cases. In designing masonry structures grouting is usually considered first, then the reinforcing steel is determined according to the grouting level. Therefore, the grouted area and all those variables that are directly determined as functions of the grouting conditions are considered Independent Discrete Variables (INDV). Vertical reinforcement,and all those variables that are directly determined as functions of the steel options are considered Dependent Discrete Variables. Furthermore, in this study there are two types of vertical reinforcement; that is, outer vertical reinforcement used to resist tension or compression violation resulting from (Dead Load + Wind Load) application, A_{so}, and interior vertical steel used to resist compression violation resulting from (Dead Load + Live Load) application, A_{si}. In this study A_{si} is dependent on A_{so} which in turn is dependent on the grouting conditions. Therefore A_{so} and all those variables determined as functions of A_{so} options are recognized as Directly Dependent Discrete Variables (DDDV), while A_{si} and all those variables determined as functions of A_{si} options are identified as Indirectly Dependent Discrete Variables (IDDV). On the other hand, since lateral reinforcement (shear reinforcement) is mainly determined according to the grouting levels, it is considered a DDDV.

According to this classification, the general mathematical formulation for the pseudo-discrete masonry optimization model can be expressed as follows:

$$\text{Minimize} \quad \sum_{o=1}^{OT} C_o \quad \gamma_o \tag{3-1}$$

subject to

$$\sum_{b=1}^{BT} \hat{g}_{cb}{*} \ \psi_b \begin{Bmatrix} \leq \\ \geq \\ = \end{Bmatrix} 0.0 \qquad \text{for } c = 1,2,\ldots,CT \tag{3-2}$$

$$\beta S1_\ell \ + \sum_{i=1}^{IT} g_{\ell i} \ {*}\alpha_i = 0.0 \qquad \text{for } \ell = 1,2,\ldots,S1T \tag{3-3}$$

$$\beta S2_\ell \ + \sum_{i=1}^{IT} \sum_{k=1}^{KT} g^k_{\ell i} \ {*}\alpha^k_i = 0.0 \qquad \text{for } \ell = 1,2,\ldots,S2T \tag{3-4}$$

$$\beta S3_\ell \ + \sum_{i=1}^{IT} \sum_{k=1}^{KT} \sum_{p=1}^{PT} g^{kp}_{\ell i} \ {*}\alpha^{kp}_i = 0.0 \qquad \text{for } \ell = 1,2,\ldots,S3T \tag{3-5}$$

where

$$\sum_{i=1}^{IT} \alpha_i = 1.0 \tag{3-6}$$

$$\sum_{k=1}^{KT} \alpha^k_i = \alpha_i \qquad \text{for } i = 1,2,\ldots,IT \tag{3-7}$$

$$\sum_{p=1}^{PT} \alpha^{kp}_i = \alpha^k_i \qquad \text{for } i = 1,2,\ldots,IT \text{ and } k = 1,2,\ldots,KT \tag{3-8}$$

where

S1T = Number of INDV

S2T = Number of DDDV

S3T = Number of IDDV

CT = Number of continuous structural constraints representing load and stress limitations

BT = Number of continuous variables $(BT \in (S1T + S2T + S3T))$

OT = Number of discrete variables representing the components of masonry structure $(OT \in (S1T + S2T + S3T))$

IT = Number of $S1T^{th}$ options (grouting conditions)

KT = Number of $S2T^{th}$ options (outside reinforcements)

PT = Number of $S3T^{th}$ options (inside reinforcements)

γ_o = Discrete variable representing the components of masonry structures (e.g. A_{so} and A_{si}) - contained within INDV, DDDV and IDDV

C_o = Cost coefficients associated with γ_o

$\hat{g}_{cb}$ = Coefficient of b^{th} continuous variable for c^{th} continuous constraint

ψ_b = b^{th} continuous variable - contained within INDV, DDDV, and IDDV

$\beta S1_\ell$ = ℓ^{th} independent discrete variable (INDV)

$\beta S2_\ell$ = ℓ^{th} directly dependent discrete variable (DDDV)

$\beta S3_\ell$ = ℓ^{th} indirectly dependent discrete variable (IDDV)

$g_{\ell i}$ = Coefficient of ℓ^{th} INDV for the i^{th} condition (grouting condition)

$g_{\ell i}^{k}$ = Coefficient of ℓ^{th} DDDV for k^{th} option (outside steel) and i^{th} condition

$g_{\ell i}^{kp}$ = Coefficient of ℓ^{th} IDDV for p^{th} option (inside steel) ,for k^{th} option and for the i^{th} grouting condition

α_i = (0-1) variable for the i^{th} condition

α_i^k = (0-1) variable for the k^{th} option and the i^{th} condition

α_i^{kp} = (0-1) variable for the p^{th} option, k^{th} option and i^{th} condition

The model for the above general formulation is generated using a computer program (Appendix C), which is based on two types of input. First, the conditional type of input that describes the applied loads and the geometry of the structure and, second, the assumed input that provides the discrete sets of the main parameters, e.g. steel and grout. With the help of the first type of input data, the governing load case is determined; the program is then directed toward a systematic process, during which the second type of input data provides the essential information for that particular load case. Finally, when the model is completed, interaction takes place between the data set where the model is stored and LINDO[23] where the optimization problem is solved. The method of solution is based on the Branch-and-Bound technique[21] which has proven to be the most reliable method in solving such types of problems. The continuous part of the constraints directs the solution, in the beginning, to a continuous linear programming solution which, in most cases, is feasible but unacceptable because of the fractional values assigned to the discrete variables (for example, a #15 [200 mm^2] steel bar could have a cross-sectional area of 183 mm^2 in the LP Continuous Solution). Then the branch-and-bound process starts the search for the optimal integer programming (IP) solution. The period of search depends mainly on how "tight" the formulation is and on how close the integer solution is to the continuous solution. Because this process is not based on trial-and-error procedure, an initial solution is not given as input to the problem, which eliminates the possibility of fast convergence of the solution. Fortunately, this disadvantage can be ignored with a "skillfully" formulated model. The number of constraints, the number of integer variables, as well as the format of the final relationships, play a very important role in minimizing the time of the entire process of optimization, as will be discussed in later sections.

The combinatorial design process developed in this chapter is applied to the design of an individual shear wall (a floor-to-floor wall). As mentioned before, shear walls are structural elements subjected to in-plane bending moments, shear forces as well as, in most cases, axial loads. In this study, two load cases have been considered for investigation, namely, the Dead & Wind (DL + WL) and Dead & Live (DL + LL) load cases. It should be noted that, as a matter of importance and for the purpose of organizing the design process, the load case (DL + WL) is investigated first while the (DL + LL) load case is embedded within the sequential process of the first load case investigation. Therefore, assurance is always made that the optimum design for the first case is safely applicable to the second case (while the contrary is less convenient). In addition, shear stresses are tested, automatically, for each situation and a revised design does or does not take place according to whether or not a violation of shear stresses occur.

A few different cases are tested using the developed model and the results are presented diagrammatically as well as in a tabular form.

The following sections describe the development of the model. Emphases are placed on the general assumptions for each load case, the structural relationships employed, the incorporation of zero-one variables and the differences in the computation process resulting from different types of formulations. The sections are presented as follows:

3.2 ADAPTATION OF THE STRUCTURAL DESIGN RELATIONSHIPS

In this section, the structural design relationships used in the model are developed. Analysis of the adequacy of different methods of design, whether recommended by the Code or derived from the elasticity principles, is performed. Then, the adaptation process for each selected method is discussed and presented in its "discrete form".

3.2.a (DL+WL) and (DL+LL) Design Relationships

As mentioned before, the first load case to be tested is the (DL+WL) case, followed, at a particular stage by the case of (DL+LL).

The Canadian Code CAN3-S304-M84, Clause 5-7-1, permits the use of the Coefficient Method (C.M.) in designing shear walls. C.M. is mathematically expressed as follows:

$$P_d = C_e C_s \, f_m \, A_m \quad \text{for unreinforced walls} \tag{3-9}$$

and

$$P_d = C_e C_s \, [f_m \, A_m + 0.8 \, f_s' \, A_s] \quad \text{for reinforced walls} \tag{3-10}$$

where P_d = The allowable vertical dead load (N/m)

C_e = Eccentricity coefficient

C_s = Slenderness coefficient

f_m = The allowable axial compressive stress for masonry (N/mm^2)

A_m = Mortar bedded area (mm^2/m)

f_s' = The allowable compressive stress for steel (N/mm^2)

A_s = The cross-sectional area of vertical steel (mm^2/m)

Glanville[12] has indicated that the use of C_e is more applicable to out-of-plane bending, where strain change takes place across the thickness, t, of the wall, than to in-plane bending, where the change of strain is along the considerably greater length, L, of the wall. On that basis, he has recommended the use of the following relationship, derived from the Unity Equation, (U.E.):

$$\frac{P_d}{A_m} + \frac{M}{SM} \leq C_s f_m \quad \text{to check compression violation} \tag{3-11}$$

and

$$\frac{P_d}{A_m} - \frac{M}{SM} > 0.0 \quad \text{to check tension violation} \tag{3-12}$$

where M = In-plane bending moment (N-mm)

SM = Section modulus (mm^3)

and other variables as defined before.

He has further strengthened his recommendation by applying the two approaches to the same example where the first yields a safe design while the second approach shows a compression violation. In conclusion, therefore, the second approach, (hereinafter referred to as U.E.), appears more appropriate in the case of shear wall design.

Three subcases are to be examined within this load case, namely, the case when neither compression nor tension violation exists, the case when compression violation occurs and finally the case when tension violation exists. These three subcases are represented graphically in Figure 3-1.

In Equations 3-11 and 3-12 and Figure 3-1:

P_d = The applied dead load (N)

A_m = Mortar bedded area (mm^2)

M = In-plane bending moment (N-mm)

SM = Section modulus (mm^3)

In the process of checking the stress violations in the wall, subcase 3 is considered first. If it is found that no tension violation exists, then either subcase 2 or 1 can be checked. It is also assumed that whenever tension or compression violation occurs, the resisting grout and/or steel is placed in both ends of the wall. This arrangement assures that both ends of the wall are safe against changes in direction of horizontal forces. On the other hand, having tested subcase 3 first assures that no tension violation occurs when either of subcases 2 or 3 are further considered.

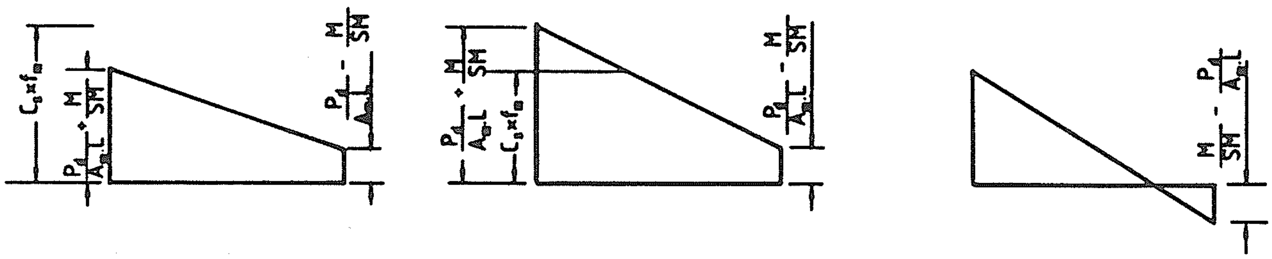
Rewriting Relationships 3-11 and 3-12, with respect to the i^{th} grouting condition:

$$\frac{(P_d)_i}{(A_m)_i} + \frac{M}{(SM)_i} \leq C_s(f_m)_i \tag{3-13}$$

$$\frac{(P_d)_i}{(A_m)_i} - \frac{M}{(SM)_i} > 0.0 \tag{3-14}$$

where

i = 1,2,.......,IT = Number of grouting conditions



SUBCASE-1-   SUBCASE-2-   SUBCASE-3-

Figure 3.1 Stress distribution due to the application of (DL + WL) for the three subcases examined for a single-storey shear wall (U.E.)

$(P_d)_i$ = Roof dead load = $(h * L * (UW)_i)$ (N)

h = Total height of the wall (roof-to-roof) (m)

L = Total length of the wall (m)

$(UW)_i$ = Unit weight of the wall for i^{th} grouting condition (N/m^2)

and $(A_m)_i$ = Mortar bedded area for i^{th} grouting conditions (mm^2)

$(SM)_i$ = Section modulus for i^{th} grouting condition (mm^3)
= $(t)_i L^2/6$ for a rectangular section

where $(t)_i$ = Equivalent wall thickness for i^{th} grouting condition (mm)

L = Total length of the wall (mm)

C_s = Slenderness Coefficient
= $1.2 - \frac{8h}{300t}$ for concentrically loaded walls

where h = The total height of the wall (mm)

and $(f_m)_i$ = The allowable axial compressive stress for masonry for i^{th} grouting condition (N/mm^2)

and other variables are as defined before.

Using discrete values for $(P_d)_i/(A_m)_i$, $M/(SM)_i$ and $C_s(f_m)_i$, Relationships 3-13 and 3-14 can be rewritten in the following discrete form:

$$(XA)_i + (YA)_i \leq (ZA)_i \quad (3\text{-}15)$$

$$(XA)_i - (YA)_i > 0.0 \quad (3\text{-}16)$$

Relationships 3-15 and 3-16 describe the stress limitations for both compression and tension, for a shear wall. However, any stress violation, due to the applied loads, requires the consideration of other design methods or the adaptation of the above method, i.e., the U.E. Therefore, for each of the three subcases shown in Figure 3-1, the design process and structural relationships adaptation is described as follows:

A. IF $(XA)_i - (YA)_i < 0.0$ (Subcase 3)

Once tension has been detected, i.e., $(XA)_i - (YA)_i < 0.0$, the Transformed Section Analysis (T.S.A.) method is applied to solve the problem. The following are the assumptions on which the process is based:

1. Steel is used in equal amounts at each end of the wall.
2. For each grouting condition, i, there are KT different steel combinations.
3. The effective depth, $(D)^k$, for k^{th} steel is taken from the centerline of the steel combination, on the tension side, to the end of the wall, on the compression side.
4. The number of grouted cores for the purpose of placing the end steel is always one core less, at each side, as the two end cores are assumed to be always grouted.

Figure 3-2 illustrates graphically the stress distribution, the applied loads and the resisting capacities for a section subjected to tension. The general procedures of analysis and relationships adaptation is as follows:

1. The following cubic equation, combining the two equilibrium relationships ($\sum$ Forces = 0 and $\sum$ Moments = 0), in one unknown, that is $(\phi)_k^i$, the ratio of the depth to the neutral axis to the effective depth $(D)^k$, is solved to determine $(\phi)_i^k$

$$(A1 * \phi^3)_i^k + (A2 * \phi^2)_i^k + (A3 * \phi)_i^k + (A4)_i^k = 0.0 \qquad (3\text{-}17)$$

where

$k = 1,2,\ldots\ldots,KT$ = Number of outside steel options

$$(A1)_i^k = ((P_d * t)_i * (D^3)^k)/3 \qquad (3\text{-}18)$$

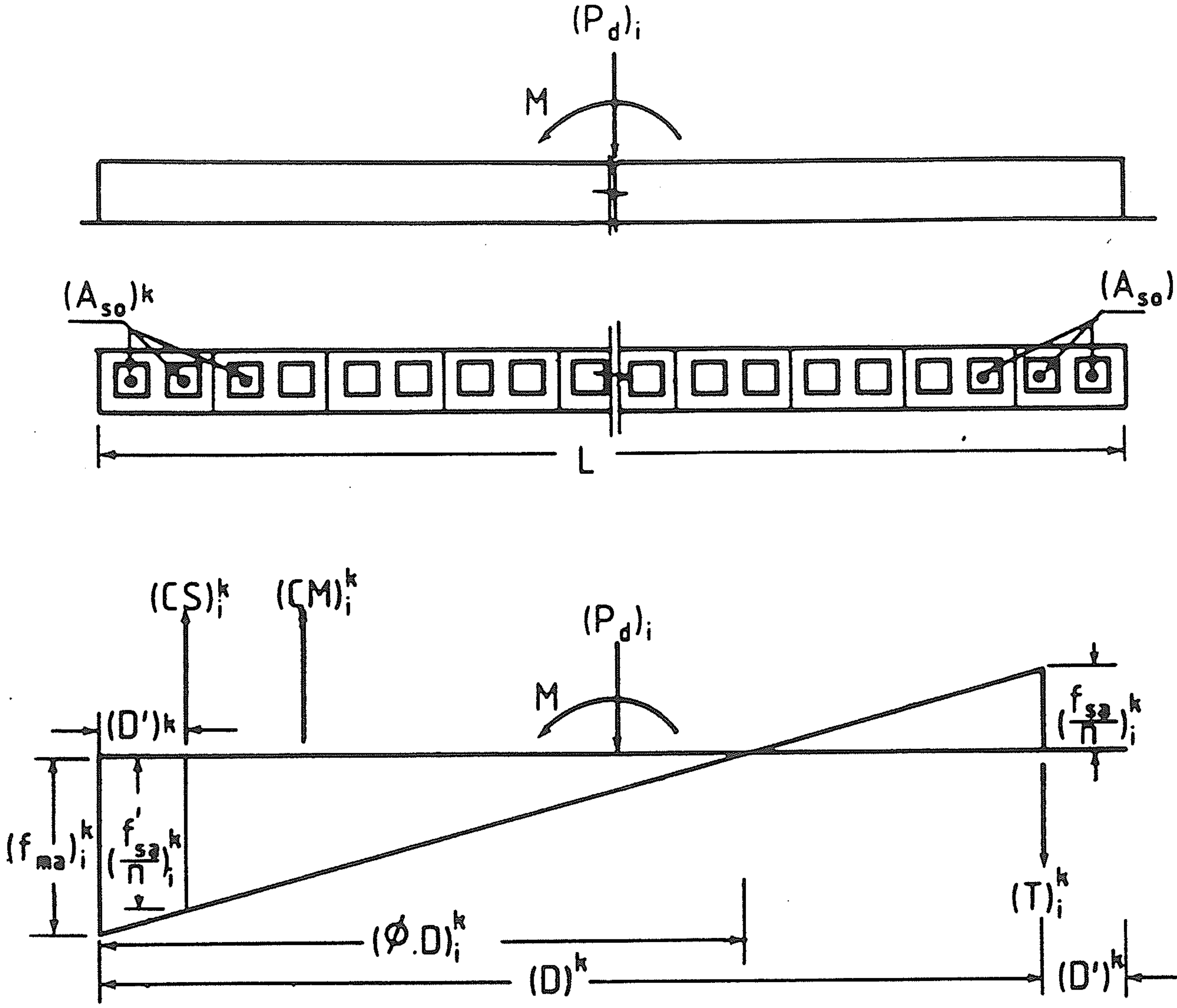


Figure 3.2 Stress distribution, applied (DL + WL) and resisting capacity for a section subjected to tension (T.S.A.)

$$(A2)_i^k = (t)_i * (D^2)^k * \{M - [(P_d)_i * L/2)]\} \tag{3-19}$$

$$(A3)_i^k = 4M * (n)_i * (D * A_{so})^k \tag{3-20}$$

$$(A4)_i^k = 2(n)_i * (A_{so})^k * (M\text{-}L - ((P_d)_i * (D\text{-}D')^k) * (L/2 - (D')^k)) \tag{3-21}$$

where $(D)^k$ = Effective depth for k^{th} steel combination (mm)

$(D')^k$ = The distance between the centerline of k^{th} combination and the nearest side of the wall (mm)

$(n)_i$ = Modular ratio for i^{th} grouting condition

$(A_{so})^k$ = The k^{th} steel combination's cross-sectional area (mm^2)

and all other variables as defined before.

2. Once $(\phi)_i^k$ is determined, an either/or situation rises. First, $(\phi)_i^k$ is greater than one, a situation that actually happens when tension is very small, and which gives an indication that tension is not a problem at all. In this case, the calculation is terminated at this point and investigation of the stress situation at the other end of the wall takes place (subcases 1 or 2).

3. In the second situation $(\phi)_i^k$ is less than one. Therefore, tension must be resisted by $(A_{so})^k$. The values of actual masonry stress $(f_{ma})_i^k$, actual compressive steel stress $(f'_{sa})_i^k$ and actual tensile steel stress $(f_{sa})_i^k$ are then calculated. These values are to be checked against their allowable values.

At this particular stage the (DL + LL) case is considered for testing. The method used is the C.M., where Equations 3-9 and 3-10, derived from the Canadian Code, with minor variations discussed below, are applied.

The wall designed for tension, as described in the above sections, for grouting condition i and side steel k, as well as the axial resisting capacity distribution is graphically presented in Figure 3-3. As the (dead + live) load is, in most cases, applied with uniform distribution, a force re-distribution most likely occurs due to the resisting capacity distribution. Accordingly, Equations 3-9 and 3-10 can be expressed in a general mathematical expression for the whole wall as follows:

$$(RCI)_i^k + 2(RCS)_i^k \geq (DLI)_i^k + 2(DLS)_i^k \tag{3-22}$$

where $(RCI)_i^k$ = the axial compressive resisting capacity of the inside portion of the wall for grouting condition i and k^{th} steel combination (N)

$(RCS)_i^k$ = the axial compressive resisting capacity of the outside portion of the wall for grouting condition i and k^{th} steel combination (N)

$(DLI)_i^k$ = applied (dead + live) load for the inside part of the wall for grouting condition i and k^{th} steel combination (N)

$(DLS)_i^k$ = applied (dead + live) load for the outside part of the wall for grouting condition i and k^{th} steel combination (N)

Expression 3-22 describes the relationship between the resisting capacity and the applied loads for two segments of the wall. Therefore, the same relationship can be further simplified by applying it to

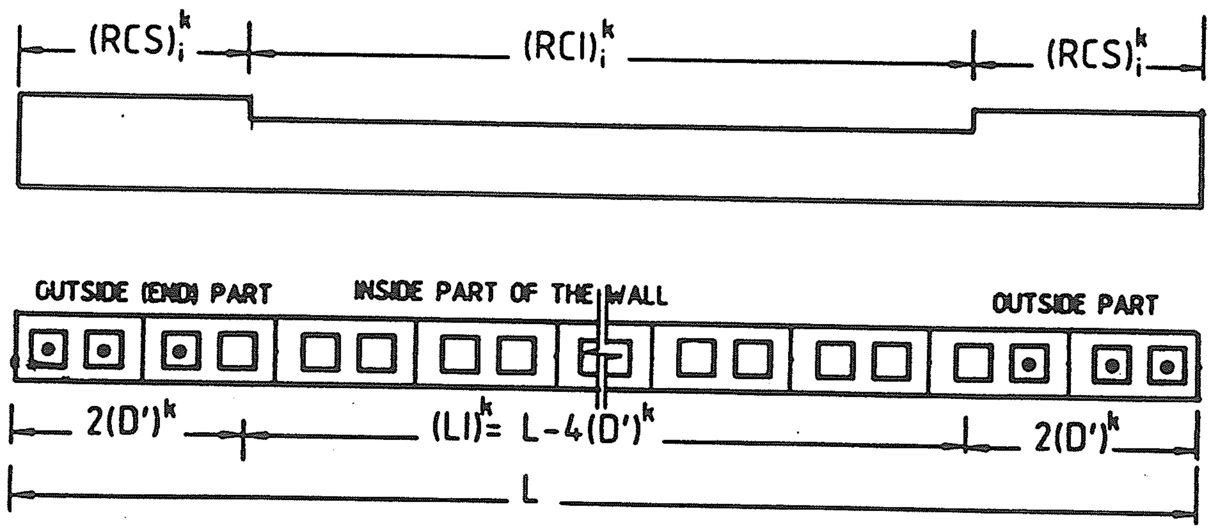


Figure 3-3. Axial compressive resisting capacity distribution for a wall designed for tension

each segment of the wall, separately, as follows:

$$(RCI)_i^k \geq (DLI)_i^k \tag{3-23}$$

$$(RCS)_i^k \geq (DLS)_i^k \tag{3-24}$$

The resisting capacities $(RCI)_i^k$ and $(RCS)_i^k$ and the applied loads $(DLI)_i^k$ and $(DLS)_i^k$ can be mathematically expressed as follows:

$$(RCI)_i^k = C_s * (f_m * A_m)_i * (LI)^k \tag{3-25}$$

$$(RCS)_i^k = C_s * [((f_m * A_m)_{IT} * 2(D)^k) + (0.8\ f_s' * (A_{so})^k)] \tag{3-26}$$

$$(DLI)_i^k = \frac{(ADL)_i}{L} * (LI)^k \tag{3-27}$$

and

$$(DLS)_i^k = \frac{(ADL)_{IT}}{L} * 2(D)^k \tag{3-28}$$

where

$(LI)^k$ = L - $4(D')^k$ (mm)

= The total inside length of the wall (as shown in Figure 3-3)

$(f_m)_{IT}$ = The allowable axial compressive stress for fully grouted wall (i.e. i = IT) (N/mm)

$(A_m)_{IT}$ = The mortar bedded area for fully grouted wall (mm^2)

f_s' = The allowable compressive steel stress (N/mm^2)

$(ADL)_i$ = The total applied (Dead + Live) load for i^{th} grouting condition (N)

$(ADL)_{IT}$ = The total applied (Dead + Live) load for fully grouted wall (N)

and other variables are as defined before.

It is evident that $(RCS)_i^k$ is always greater than $(DLS)_i^k$, which leaves only Relationship 3-23 to more further consideration. If a compression violation occurs, i.e., $(DLI)_i^k$ exceeds $(RCI)_i^k$, then the resisting capacity $(RCI)_i^k$ must be increased to cover the difference between $(DLI)_i^k$ and $(RCI)_i^k$. This increase in $(RCI)_i^k$ consists of a combination of steel and more grouted cores where the steel bars are placed. At the same time, the extra grouted cores add extra dead load on the already determined value $(DLI)_i^k$. The mathematical expression that describes this relationship is as follows:

$$(RCI)_i^k + (RCIA)_i^{kp} \geq (DLI)_i^k + (DLIA)_i^{kp} \tag{3-29}$$

where $P = 1, 2, \ldots, PT$ = Inside steel bar options

$(RCIA)_i^{kp}$ = Extra axial compressive resisting capacity (N)

$$= C_s[((f_m)_{IT} * (NC)_i^{kp} * AC) + (0.8 f_s' * (NC)_i^{kp} * (A_{si})^P)] \tag{3-30}$$

and $(DLIA)_i^{kp}$ = Extra dead load (N)

$$= (NC)_i^{kp} * CW * h * (UW)_{IT} \tag{3-31}$$

where $(NC)_i^{kp}$ = The extra number of grouted cores, or number of inside steel bars, p , for i^{th} grouting condition and k^{th} outside steel

$$(NC)_i^{kp} = \frac{(DLI)_i^k - (RCI)_i^k}{C_s\ [((f_m)_{IT} * AC) + (0.8 f_s' * (A_{si})^P)]} \tag{3-32}$$

AC = The cross-sectional area of one core (taken as 25% of the total area of a block) (mm^2)

$(A_{si})^p$ = The cross-sectional area of p^{th} inside steel bars (mm^2)

CW = The core width (mm)

and

$(UW)_{IT}$ = The unit weight of a fully grouted wall (N/mm^2)

and other variables are as defined before.

The differences between the C.M. in its original formulation as presented in the Canadian Code(4) (Relationships 3-9 and 3-10) and the mathematically manipulated new expression, represented by Relationships 3-24 and 3-29, are as follows:

1. The new adapted relationships assure that whatever the tension design results are or, more specifically, the end steel, these results must form an initial solution to the (dead + live) load case. Therefore the search for the governing load case, i.e. (DL + WL) or (DL + LL), is eliminated, which in turn saves calculation time.

2. The original form of the C.M. assumes a uniformly distributed load (forces) and, in turn, uniformly distributed values of mortar bedded area (A_m) and vertical steel area. It also either ignores the effect of the extra grouted cores for the purpose of adding compression steel on the originally applied (DL + LL), or it assumes that the added bars are placed in the already grouted cores. The first assumption leads to

inexact results and, moreover, it overestimates the resisting capacity. The second assumption is correct but not economical. The reason is, while the grout offers higher resisting capacity in compression than steel bars, its cost is less. Therefore, for a particular compression deficiency, two options are available. The first, according to the original relationships, offers a particular amount of steel placed in the already grouted cores. The second, according to the modified relationships, offers a lesser amount of steel and extra amounts of grout. Comparison shows that in almost in all cases the latter option is more economical.

The above design process for (DL + LL) can be summarized as follows:

1. For a section designed for tension, $(RCI)_i^k$, $(RCS)_i^k$, $(DLI)_i^k$ and $(DLS)_i^k$ are calculated.
2. If $(RCI)_i^k$ is greater than, or equal to, $(DLI)_i^k$, then the i^{th} grouting condition is sufficient to resist the applied loads.
3. If $(RCI)_i^k$ is less than $(DLI)_i^k$, then compression steel as well as extra grouted cores for the purpose of placing the steel bars must be considered. The increase in the resisting capacity, $(RCIA)_i^{kp}$, along with the original value of $(RCI)_i^k$ must exceed the original value of $(DLI)_i^k$, and the extra added load $(DLIA)_i^{kp}$.

B. <u>IF $(XA)_i + (YA)_i > (ZA)_i$ (Subcase 2)</u>

If no tension exists in the wall (i.e. $(XA)_i - (YA)_i > 0.0$), or if tension is negligible (i.e.$(\phi)_i^k$ >1.0), then the computation process is directed toward checking the stress situation on the other side of the wall. One possibility is the existence of compression violation, i.e., $(XA)_i + (YA)_i > (ZA)_i$. This situation is graphically illustrated in Figure 3-4.

Once a compression violation exists, additional compressive resisting capacity, represented by the additional stress $(AN)_i^k$ must be added.(12) This surplus is gained by adding compression steel. It should be noted that the compression steel is added to each end of the wall. The amount of deficient compression is over a length $(LX)_i$ of the wall and is represented by the shaded area in Figure 3-4. The value of $(AN)_i^k$ can be calculated using the following equation.(12)

$$(AN)_i^k = C_S * \frac{3.2(YA)_i * f'_s * (A_{so})^k}{L * (t)_i * [(XA)_i + (YA)_i - (ZA)_i]} \qquad (3\text{-}33)$$

where

$(AN)_i^k$ = The value of the stress that makes the difference between $(XA)_i + (YA)_i$ and $(ZA)_i$ for i^{th} grouting condition and k^{th} steel combination (N/mm^2)

$(A_{so})^k$ = The cross-sectional area of the k^{th} steel combination (It should be noted that A_{so} options are the same for compression as for tension) (mm^2)

and all other variables are as defined before.

Having determined $(AN)_i^k$, the main relationship representing this subcase can be expressed mathematically as follows:

$$(XA)_i + (YA)_i \leq (ZA)_i + (AN)_i^k \qquad (3\text{-}34)$$

After designing the wall for (DL + WL) using Relationship 3-34, the computation process automatically considers (DL + LL) case. The

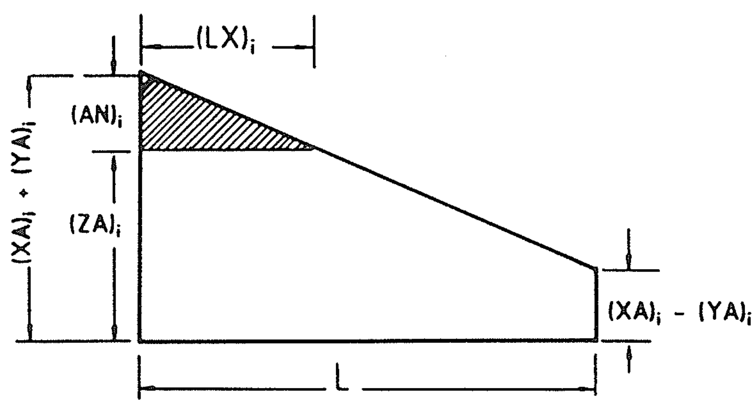


Figure 3-4. Stress distribution for shear wall subjected to compression violation

procedure is identical to that for the tension violation case.

C. <u>IF $(XA)_i + (YA)_i \leq (ZA)_i$ (Subcase 1)</u>

If neither tension nor compression violation exist in either end of the wall, i.e. $(XA)_i + (YA)_i \leq (ZA)_i$, and $(XA)_i - (YA)_i \geq 0.0$, then the wall under consideration is safe to resist (DL + WL). (Notice that Relationship 3-33 can be used in this case, with the exception that $(AN)_i^k = 0.0$). Proceeding to check the (DL + LL) case, minor modifications are required in the calculation process. As there is no force re-distribution, both initial values of the applied (dead + live) load, $(ADL)_i$, and the resisting capacity, $(RCM)_i$, for the i^{th} grouting condition are considered for evaluation. If $(RCM)_i$ is greater than or equal to $(ADL)_i$, then the wall is safe under the axial compression. However, in this particular case and as a standard engineering practice, both end cores are grouted and reinforced with steel bars #15. On the other hand, if $(RCM)_i$ is less than $(ADL)_i$, compression steel is placed in extra grouted cores. The procedure is identical to that described for subcase 3, except for determining $(NC)_i^p$ where:

$$(NC)_i^p = \frac{(ADL)_i - (RCM)_i}{C_s\ [((f_m)_{IT} * AC) + (0.8\ f_s' * (A_{si})^p)]} \qquad (3\text{-}35)$$

3.2.b <u>Shear Stress Design Relationships</u>

The shear relationships used herein are based on elasticity principles[12], which in turn are based on the requirements of the Canadian Code.[4] The relationships themselves are expressed mathematically as follows:

$$(v_a)_i \le (v_\ell)_i \tag{3-36}$$

where $(v_a)_i$ = actual shear stress for grouting condition i (N/mm)

$$= \frac{1.5\ V}{(t)_i * L} \quad \text{(for rectangular section)} \tag{3-37}$$

where V = The maximum shear force (N)

and $(v_\ell)_i$ = The maximum allowable shear stress (N/mm^2)

$$= v_m + 0.3\ (f_{cs})_i \tag{3-38}$$

where v_m = a limited value for the allowable shear stress (when no axial force is applied) (N/mm^2)

and $\{.3(f_{cs})_i\}$ = the contribution of the compression stress due to dead load for grouting condition i

$$= 0.3 * \frac{(P_d)_i}{(A_m)_i} \tag{3-39}$$

The value of v_m is derived from the Canadian Code and it varies according to whether or not the wall is reinforced and whether or not there is shear violation.

If no shear violation exists in the wall, then there is no need for shear reinforcement or to grout more cores. In case of shear stress violation, shear reinforcement must be added to the particular i^{th} grouting condition. According to some researchers,(36,44,16) it has been concluded that the horizontal steel is more effective in resisting shear as opposed to vertical web reinforcement or vertical end steel. Therefore, adding such horizonal steel requires employing the following relationships.

$$(v'_a)_i^{eq} = \frac{(A_{sl})_i^{eq} * f_v}{(S_\ell)_i^{e} * (t)_i} \tag{3-40}$$

where

$e = 1,2,\ldots\ldots.,ET$ = Number of horizontal steel spacing options

$q = 1,2,\ldots\ldots.,QT$ = Number of horizontal steel options

$(v_a')_i^{eq}$ = The actual discrete value of shear stress for i^{th} grouting condition, q^{th} horizontal steel area and e^{th} spacing (N/mm^2)

$(A_{sl})_i^{eq}$ = the q^{th} horizontal shear steel cross-sectional area, for the i^{th} grouting condition and e^{th} spacing (mm^2)

f_v = The allowable steel shear stress (taken the same as the allowable tensile steel stress) (N/mm^2)

and $(S_\ell)_i^e$ = the e^{th} spacing for the i^{th} grouting condition (mm)

It should be noted that, if a continuous solution is sought, then (v_a) must equal to (v_a'), but using the discrete values of the different variables yields a value of $(v_a')_i^{eq}$ that should be equal to or exceed $(v_a)_i$. The main expressions used in describing these relationships are as follows:

$$(v_a')_i^{eq} \leq (v_\ell)_i \tag{3-41}$$

$$(v_a')_i^{eq} \geq (v_a)_i \tag{3-42}$$

It should also be noted that as a general engineering practice, a horizontal joint reinforcement (4.1 mm diam.) is often placed at one, two, or three course spacings, according to the designer. In this study, it is assumed to be placed in every third course. The joint reinforcement at its specified spacing, is used to resist shear as well as its main function of providing a continuous tie along the wall to control shrinkage cracking and to provide a tie between two wythes.[12] In case of shear stress violation, the first option is to reduce the spacing of such lateral joint reinforcement. If that option fails, then other steel bar sizes with different spacings are tested.

The different arrangements of vertical steel do not affect the shear stress test except for the fact that the wall is being reinforced or not. Therefore the process of checking shear stress is independent of (DL + WL) and (DL + LL) case tests, as will be seen in the discrete formulation of the model.

3.3 DEVELOPMENT OF THE PSEUDO-DISCRETE MODEL

In the previous section, structural relationships, whether derived from the Canadian Code or from the elasticity principles, for different loading conditions have been adapted in a realistic fashion, in order to act as constraints in the optimization model. The various variables included in these constraints have also been evaluated on the basis of their discrete values. In this section, the general mathematical pseudo-discrete formulation of the model is described. The model consists of an objective function and a set of constraints. Three subsets of constraints are included within the constraint set. These subsets are: a continuous constraint subset, a discrete value constraint subset, and finally, the constraint subset describing the interrelationship among zero-one variables incorporated in the second type of constraints. The following sections describe the formulation of each subset as well as the development of the objective function.

3.3.1 Continuous (Structural) Constraints

The constraints within this section describe the general limitations of stress, resisting capacities and applied loads for different load cases. Also, limitations of different steel areas and grouted core areas are presented. These relationships are in the form of inequalities, so that the continuous feasible region of solution can be created.

The first constraint represents the relationship between the combined stresses for the applied (DL + WL) and their allowable value which can be expressed mathematically as follows:

$$XA + YA \leq ZA + AN \tag{3-43}$$

It should be noted that Relationship 3-43 is binding, but only in subcases two and one described earlier. On the other hand, the relationship between the actual masonry compressive stress, steel compressive stress and steel tensile stress, respectively, and their allowable value, for subcase three, tension violation, are as follows:

$$f_{ma} \leq ZA \tag{3-44}$$

$$f'_{sa} \leq f'_{s} \tag{3-45}$$

$$f_{sa} \leq f_{s} \tag{3-46}$$

Whenever tension is absent, Relationships 3-44, 3-45 and 3-46 do not appear in the model, so that no conflict between subcases 1 and 2 and subcase 3 occurs.

The continuous constraints describing the relationships between the axial resisting capacities and applied dead and live loads for two segments of the wall are expressed mathematically as follows:

For the inside part: $RCI + RCIA \geq DLI + DLIA,$ (3-47)

and for the outside part: $RCS \geq DLS$ (3-48)

Notice that whenever RCI exceeds DLI in subcases 2 or 3, both of RCIA and DLIA collapse to zero. Also, in subcase 1, where neither tension nor compression violation occurs, both of RCS and DLS disappear from the model, while both of RCI and DLI take the original values of the resisting capacity, RCM and the applied (DL + LL), ADL. Values of RCIA and DLIA then either disappear when RCM exceeds ADL or are used to determine both the extra required resistance and the additional applied dead load for the entire wall.

The limitations for shear stresses are as follows:

$$v_a \leq v_\ell \quad (3\text{-}49)$$

and $$v'_a \geq v_a \quad (3\text{-}50)$$

where Relationship 3-49 limits the value of the actual shear stress to its corresponding allowable value. Constraint 3-50 appears only for shear stress violation to assure that the discrete value of the actual shear stress is equal to or greater than its corresponding continuous original value.

The following constraint controls the total number of grouted cores in a wall. This number has an upper bound derived from the total number of cores. Therefore the constraint can be expressed as follows:

$$TNG \leq TNC \quad (3\text{-}51)$$

where TNG = Total number of grouted cores

and TNC = Total number of cores in a wall

Finally, as required by the Canadian Code, Clause 5.8.1.1, the vertical steel must exceed a certain value specified as $(0.002.Ag.\sigma)$, where

Ag = The gross cross-sectional area of the wall per metre

σ = reinforcement distribution factor, varies between 0.33-0.67 (taken in this study as 0.67)

Since vertical steel can be applied to two segments of the wall, inside and outside, the gross area corresponding to each is different and is calculated:

for inside steel:

$$\text{the gross area is} = \frac{\text{gross area of a block}}{2} * \text{number of inside grouted cores} \tag{3-52}$$

and the relationship describing the inside steel area limitation is:

$$A_{si} \geq A_{gi} * (0.002 * \sigma) \tag{3-53}$$

where A_{gi} = the gross area of the inside part of the wall.

For the outside steel:

$$\text{the gross area is} = \frac{\text{gross area of a block}}{2} * \text{number of outside grouted cores for } k^{th} \text{ side steel} \tag{3-54}$$

and the relationship describing the outside steel area limitation:

$$A_{so} \geq A_{go} * (0.002*\sigma) \tag{3-55}$$

where A_{go} = the gross area of the outside part of the wall

3.3.2 Discrete Variable Constraints

This subset of constraints serve the following purposes:

a. Defining the various variables used in the continuous relationships presented in the previous section.

b. Offering a variety of options for different masonry structural components, e.g., A_{so}.

c. Connecting the continuous constraints to zero-one variable constraints.

The values of the discrete variables are either entirely assumed as input or determined from assumed values using the structural relationships presented before. These discrete values are to be associated with zero-one variables so that the discrete region is created within the continuous region developed by the continuous structural constraints. These relationships are in the form of equalities and can be described with reference to the continuous constraints, as follows: With reference to Relationships 3-43:

$$XA = \sum_{i=1}^{IT} (XA)_i * (X)_i \tag{3-56}$$

$$YA = \sum_{i=1}^{IT} (YA)_i * (X)_i \tag{3-57}$$

$$ZA = \sum_{i=1}^{IT} (ZA)_i * (X)_i \tag{3-58}$$

$$AN = \sum_{i=1}^{IT} (AN)_i \tag{3-59}$$

where

$$(AN)_i = \sum_{k=1}^{KT} (AN)_i^k * (K)_i^k \qquad \forall\ i \tag{3-60}$$

where

$(XA)_i$ = The discrete value of masonry compressive stress due to dead load for grouting condition i

$(YA)_i$ = The discrete value of masonry compressive stress due to in-plane bending moment for grouting condition i

$(ZA)_i$ = The discrete value of the allowable masonry compressive stress for grouting condition i

$(AN)_i$ = The discrete value of additional allowable compressive stress for grouting condition i

$(AN)_i^k$ = The discrete value of additional allowable compressive stress, as calculated in Equation 3-33, for i^{th} grouting condition and k^{th} vertical side steel

$(X)_i$ = {0-1} variable associated with the i^{th} grouting condition

and $(K)_i^k$ = {0-1} variable associated with the i^{th} grouting condition and k^{th} side steel

With reference to Relationships 3-44, 3-45 and 3-46:

$$f_{ma} = \sum_{i=1}^{IT} (f_{ma})_i \tag{3-61}$$

$$\text{where } (f_{ma})_i = \sum_{k=1}^{KT} (f_{ma})_i^k * (K)_i^k \qquad \forall\ i \tag{3-62}$$

$$f'_{sa} = \sum_{i=1}^{IT} (f'_{sa})_i \tag{3-63}$$

$$\text{where } (f'_{sa})_i = \sum_{k=1}^{KT} (f'_{sa})_i^k * (K)_i^k \qquad \forall\ i \tag{3-64}$$

$$\text{and } f_{sa} = \sum_{i=1}^{IT} (f_{sa})_i \tag{3-65}$$

$$\text{where} \quad (f_{sa})_i = \sum_{k=1}^{KT} (f_{sa})_i^k * (K)_i^k \qquad \forall\ i \tag{3-66}$$

$(f_{ma})_i$, $(f'_{sa})_i$ and $(f_{sa})_i$ are the values of actual stresses for masonry, compressive steel and tensile steel, respectively, for the i^{th} grouting condition. $(f_{ma})_i^k$, $(f'_{sa})_i^k$ and $(f_{sa})_i^k$ are the discrete values of the actual masonry compressive stress, steel compressive stress and steel tensile stress, respectively, as calculated from the T.S.A., for the case of tension violation, for the i^{th} grouting condition and the k^{th} vertical side steel.

With reference to Relationship 3-47:

$$RCI = \sum_{i=1}^{IT} (RCI)_i \tag{3-67}$$

$$\text{where} \quad (RCI)_i = \sum_{k=1}^{KT} (RCI)_i^k * (K)_i^k \qquad \forall\ i \tag{3-68}$$

$$DLI = \sum_{i=1}^{IT} (DLI)_i \tag{3-69}$$

$$\text{where} \quad (DLI)_i = \sum_{k=1}^{KT} (DLI)_i^k * (K)_i^k \qquad \forall\ i \tag{3-70}$$

$$RCIA = \sum_{i=1}^{IT} (RCIA)_i \tag{3-71}$$

$$\text{where} \quad (RCIA)_i = \sum_{p=1}^{PT} (RCIA)_i^p * (NC)_i^p \qquad \forall\ i \tag{3-72}$$

$$DLIA = \sum_{i=1}^{IT} (DLIA)_i \tag{3-73}$$

$$\text{where} \quad (DLIA)_i = \sum_{p=1}^{PT} (DLIA)_i^p * (NC)_i^p \qquad \forall\ i \tag{3-74}$$

and $$(NC)_i^p = \sum_{k=1}^{KT} (NC)_i^{kp} * (R)_i^{kp} \qquad \forall\ p \qquad (3\text{-}75)$$

where

$(RCI)_i$ and $(DLI)_i$ are the values of masonry axial compressive resisting capacity and applied (DL + LL), respectively, for the inside part of the wall for the i^{th} grouting condition.

$(RCIA)_i$ and $(DLIA)_i$ are the values of the extra resisting capacity and applied dead load, respectively, for the inside part of the wall for the i^{th} grouting condition.

$(RCI)_i^k$ = The discrete value of the axial compressive resisting capacity of the inside part of the wall, as determined from Equation 3-25, for the i^{th} grouting condition and k^{th} vertical side steel.

$(DLI)_i^k$ = The discrete value of the axial applied (DL + LL), for the inside part of the wall, as determined from Equation 3-27, for the i^{th} grouting condition and k^{th} vertical side steel.

$(RCIA)_i^p$ = The discrete value of the extra axial compressive resisting capacity due to inside reinforcement p and i^{th} grouting condition.

$(DLIA)_i^p$ = The discrete value of the added dead load, in the inside part of the wall, for p^{th} inside vertical steel and i^{th} grouting condition

It should be noted that values of $(RCIA)_i^p$ and $(DLIA)_i^p$ are for one core (as determined from Equations 3-30 and 3-31).

and $(NC)_i^p$ = Number of grouted cores in the inside part of the wall for grouting condition i and p^{th} vertical inside steel

$(NC)_i^{kp}$ = The discrete number of grouted cores, in the inside part of the wall, for p^{th} inside vertical steel, k^{th} outside vertical steel and i^{th} grouting condition (as determined from Equation 3-32)

and finally,

$(R)_i^{kp}$ = {0-1} variable associated with the i^{th} grouting condition, k^{th} side vertical steel and p^{th} inside vertical steel.

Constraints 3-67 through 3-75 appear exactly the same in subcases 2 or 3 (compression or tension violation due to in-plane bending moment), whenever $(DLI)_i$ exceeds $(RCI)_i$. If $(DLI)_i$ is less than $(RCI)_i$, the values of $(NC)_i^p$ collapse to zero followed by values of $(RCIA)_i$, and $(DLIA)_i$.

On the other hand, if subcase 1 exists (neither compression or tension violation, due to in-plane bending moment, exists), the values of $(RCI)_i$ and $(DLI)_i$ are to be taken the same as the initial total values of $(RCM)_i$ and $(ADL)_i$. In this case, if compression violation due to applied (DL + LL) occurs, Constraint 3-75 will be re-written as follows:

$$(NC)_i = \sum_{p=1}^{PT} (NC)_i^p * (R)_i^p \qquad \forall\ i \qquad (3\text{-}76)$$

where

$(NC)_i^p$ = The number of the inside grouted cores for the purpose of adding p^{th} steel for the i^{th} grouting condition (as determined from Relationship 3-35).

and

$(R)_i^p$ = {0-1} variable associated with the i^{th} grouting condition and p^{th} inside vertical steel.

At the same time, Constraints 3-72 and 3-74 remain the same. Otherwise, in the situation where there is no axial compression violation in subcase 1, $(NC)_i^p$ followed by $(RCIA)_i$ and $(DLIA)_i$ collapse to zero.

With reference to Relationship 3-48:

$$RCS = \sum_{k=1}^{KT} (RCS)^k * (Q)^k \tag{3-77}$$

$$DLS = \sum_{k=1}^{KT} (DLS)^k * (Q)^k \tag{3-78}$$

where

$(RCS)^k$ = The discrete value of the outside axial compressive resisting capacity for k^{th} side steel

$(DLS)^k$ = The discrete value of the outside applied (DL + LL) for k^{th} vertical side steel

$(Q)^k$ = {0-1} variable associated with k^{th} side steel (Note that Q is independent of the i^{th} grouting condition).

With reference to Relationships 3-49 and 3-50:

$$v_a = \sum_{i=1}^{IT} (v_a)_i * (x)_i \tag{3-79}$$

$$v_\ell = \sum_{i=1}^{IT} (v_\ell) * (x)_i \tag{3-80}$$

and $$v'_a = \sum_{i=1}^{IT} (v'_a)_i \tag{3-81}$$

where

$$(v'_a)_i = \sum_{e=1}^{ET} \sum_{q=1}^{QT} (v'_a)_i^{eq} * (S)_i^{eq} \qquad \forall\ i \tag{3-82}$$

where $(v_a)_i$ = The initial actual value of shear stress for the i^{th} grouting condition as determined from Equation 3-37

$(v_\ell)_i$ = The discrete value of the allowable shear stress for the i^{th} grouting condition as determined from Equation 3-38

$(v'_a)_i$ = The discrete shear stress for the i^{th} grouting condition

$(v'_a)_i^{eq}$ = The discrete value of the actual shear stress, as determined from Equation 3-40, for the i^{th} grouting condition, e^{th} spacing and q^{th} lateral steel.

and $(S)_i^{eq}$ = {0-1} variable associated with i^{th} grouting condition, e^{th} spacing and q^{th} lateral reinforcement

It should be noted that in the case of no shear stress violation, $(v'_a)_i$ is taken the same as $(v_a)_i$.

With reference to Relationship 3-51:

$$TNG = ONG + ING + SNG \tag{3-83}$$

where

$$ONG = \sum_{i=1}^{IT} (ONG)_i * (X)_i \tag{3-84}$$

$$SNG = \sum_{k=1}^{KT} (SNG)^k * (Q)^k \tag{3-85}$$

$$ING = \sum_{i=1}^{IT} (NC)_i \tag{3-86}$$

where $$(NC)_i = \sum_{p=1}^{PT} (NC)_i^p \qquad \forall\ i \tag{3-87}$$

where:

$(ONG)_i$ = Original number of grouted cores for the i^{th} grouted condition .

$(SNG)^k$ = Number of side grouted cores for the purpose of placing k^{th} vertical outside steel combination

$(NC)_i$ = Number of inside grouted cores for i^{th} grouting condition

and $(NC)_i^p$ = The discrete value of the number of inside grouted cores for the i^{th} grouting condition and p^{th} inside vertical steel (as expressed in Relationship 3-75).

It must be noted that the value of TNC is provided as input.

With reference to Relationship 3-53:

$$A_{si} = \sum_{i=1}^{IT} (A_{si})_i \tag{3-88}$$

where $$(A_{si})_i = \sum_{p=1}^{PT} (A_{si})_i^p \qquad \forall\ i \tag{3-89}$$

and $$(A_{si})_i^p = \sum_{k=1}^{KT} (A_{si})_i^p * (NC)_i^{kp} \qquad \forall\ i\ \&\ p \tag{3-90}$$

$$A_{gi} = A_{gb} * ING \tag{3-91}$$

where $(A_{si})_i$ = Inside vertical steel area for the i^{th} grouting condition

$(A_{si})_i^p$ = The discrete value of the inside p^{th} vertical steel and i^{th} grouting condition.

$$A_{gb} = \frac{\text{gross area of a block}}{2} * 0.002 * 0.67$$

ING = Inside number of grouted cores

and other variables are as defined before

With reference to Relationship 3-55:

$$A_{so} = \sum_{k=1}^{KT} (A_{so})^k * Q^k \tag{3-92}$$

$$A_{go} = \sum_{k=1}^{KT} [A_{gb} * (SNG)^k] * (Q)^k \tag{3-93}$$

where

$(A_{so})^k$ = The discrete value of the cross-sectional area of vertical k^{th} side steel

$[A_{gb} * (SNG)^k]$ = Gross cross-sectional area of masonry blocks for the k^{th} vertical side steel.

In addition to the above defining discrete relationships, i.e. Equations 3-56 through 3-93, there are others necessary to complete the model, they are described and expressed mathematically as follows:

The total vertical steel area can be expressed in the following equation:

$$TSA = A_{si} + A_{so} + A_{se} \tag{3-94}$$

where $$A_{se} = \sum_{i=1}^{IT} (A_{se})_i * (X)_i \qquad (3\text{-}95)$$

where $(A_{se})_i$ = The end steel (vertical) cross-sectional area in case of unreinforced wall is selected, for the i^{th} grouting condition (1 # 15 is placed in the end cores)

On the other hand, the standard sizes of the vertical reinforcement, wherever they located, can be expressed mathematically as follows:

$$S_{si} = \sum_{p=1}^{PT} (S_{si})^p * (R)^p \qquad (3\text{-}96)$$

$$S_{so} = \sum_{k=1}^{KT} (S_{so})^k * (Q)^k \qquad (3\text{-}97)$$

and

$$S_{se} = \sum_{i=1}^{IT} (S_{se})_i * (X)_i \qquad (3\text{-}98)$$

where $(S_{si})^p$ = The standard size for p^{th} inside vertical steel

$(S_{so})^k$ = The standard size for k^{th} outside vertical steel

$(S_{se})_i$ = The standard size of end cores reinforcement in case of unreinforced wall, for i^{th} grouting condition.

The constraints defining the lateral steel for shear and the spacing required for such steel, are as follows:

$$A_{s\ell} = \sum_{i=1}^{IT} (A_{s\ell})_i \qquad (3\text{-}99)$$

where $$(A_{s\ell}) = \sum_{e=1}^{ET} \sum_{q=1}^{QT} (A_{s\ell})_i^{eq} * (S)_i^{eq} \quad \forall\ i \qquad (3\text{-}100)$$

and $S_\ell = S_{\ell a} + S_{\ell b}$ (3-101)

where $S_{\ell a} = \sum_{e=1}^{ET} (S_{\ell a})^e * (S_b)^e$ (3-102)

$S_{\ell b} = \sum_{e=1}^{ET} (S_{\ell b})_i * (X)_i$ (3-103)

and $S_{s\ell} = S_{s\ell a} + S_{s\ell b}$ (3-104)

where $S_{s\ell a} = \sum_{q=1}^{QT} (S_{s\ell a})^q * (S_a)^q$ (3-105)

where $S_{s\ell b} = \sum_{i=1}^{IT} (S_{s\ell b}) * (X)_i$ (3-106)

where $(A_{s\ell})_i$ = The lateral steel area (for shear) for the i^{th} grouting condition

$(As\ell)_i^{eq}$ = The discrete value of the lateral cross-sectional area (for shear) as determined from Relationship 3-40, for the ith grouting condition, e^{th} spacing and q^{th} horizontal steel

and $(S_{\ell a})^e$ = The e^{th} lateral steel spacing in case of shear violation (If no shear violation, this value, $S_{\ell a}$, does not appear in the model)

$(S_{\ell b})_i$ = The lateral joint reinforcement spacing for the i^{th} grouting condition in case of no shear violation (If shear violation exists, $S_{\ell b}$ does not appear in the model)

$(S_{s\ell a})^q$ = The standard size of the q^{th} horizontal steel in case of shear violation (otherwise it does not appear in the model)

$(S_{s\ell b})_i$ = The standard size of the lateral joint reinforcement (4.1 mm diam.), for the i^{th} grouting condition in the case of no shear violation (If no shear violation, it does not appear in the model)

and $(S_a)^q$ = {0-1} variable associated with the q^{th} standard size of lateral reinforcement

$(S_b)^e$ = {0-1} variable associated with the e^{th} spacing of lateral reinforcement

It should be noted that in case of no shear violation, Relationship 3-99, representing the value of the cross-sectional area of lateral reinforcement, will be re-written as follows:

$$A_{s\ell} = \sum_{i=1}^{IT} (A_{s\ell})_i * (X)_i \qquad (3\text{-}107)$$

where $(A_{s\ell})_i$ = Lateral joint reinforcement cross-sectional area for the i^{th} grouting condition.

Finally, values of the mortar bedded area, A_m, and the masonry allowable compressive stress, f_m, are expressed, for the discrete modelling, as follows:

$$A_m = \sum_{i=1}^{IT} (A_m)_i * (X)_i \qquad (3\text{-}108)$$

$$f_m = \sum_{i=1}^{IT} (f_m)_i * (X)_i \qquad (3\text{-}109)$$

where $(A_m)_i$ and $(f_m)_i$ are the discrete values of mortar bedded area and masonry allowable compressive stress for the i^{th} grouting condition (A complete derivation of these values is shown in Appendix A).

3.3.3 Inter-relationships Among (0-1) Variables

The following subset of constraints describes the interrelationships among zero-one variables. These relationships are developed in such a way that the total number of constraints is reduced with a corresponding significant reduction in the number of zero-one variables. As a result, the number of branches in the branch-and-bound process, and the total computation time, is decreased. The actual relationships are expressed mathematically as follows:

$$\sum_{i=1}^{IT} (X)_i = 1.0 \tag{3-110}$$

$$\sum_{k=1}^{KT} (K)_i^k = (X)_i \tag{3-111}$$

(except for subcase 1, where $\sum_{k=1}^{KT} (K)_i^k = 0$)

$$\sum_{i=1}^{IT} (K)_i^k = (Q)^k \tag{3-112}$$

$$\sum_{p=1}^{PT} (R)_i^{kp} = (K)_i^k \qquad \forall\ k\ \&\ i \tag{3-113}$$

$$\sum_{k=1}^{KT} (R)_i^{kp} = (R)_i^p \qquad \forall\ p\ \&\ i \tag{3-114}$$

(except for subcase 1, where $\sum_{k=1}^{KT} (R)_i^{kp} = 0$)

$$\sum_{i=1}^{IT} (R)_i^p = (R)^p \tag{3-115}$$

Constraints 3-111 through 3-114 are effective in subcases 2 and 3, but in subcase 1 they collapse to zero and the following relationship appears in their place:

$$\sum_{p=1}^{PT} (R)_i^p = (X)_i \qquad (3\text{-}116)$$

The following relationships are also necessary for the inclusion of shear:

$$\sum_{e=1}^{ET} \sum_{q=1}^{QT} (S)_i^{eq} = (X)_i \qquad \forall\ i \qquad (3\text{-}117)$$

(except in case when no shear violation exists, then $\sum_{e=1}^{ET} \sum_{q=1}^{QT} (S)_i^{eq} = 0$)

also $$\sum_{i=1}^{IT} \sum_{e=1}^{ET} (S)_i^{eq} = (S_a)^q \qquad \forall\ q \qquad (3\text{-}118)$$

and $$\sum_{i=1}^{IT} \sum_{q=1}^{QT} (S)_i^{eq} = (S_b)^e \qquad \forall\ e \qquad (3\text{-}119)$$

Relationships 3-110 through 3-119 can be represented in the flow diagram in Figure 3-5.

3.3.4 The Objective Function

The three constraint subsets presented above represent, in a general form, the necessary constraints for the problem of the optimum design of masonry single shear wall, regardless of the governing load case(s) or the assumed values of different masonry structural components.

The objective function, on the other hand, simply describe the total cost of the main components in the wall, namely, blocks, grout, vertical and horizontal steel.

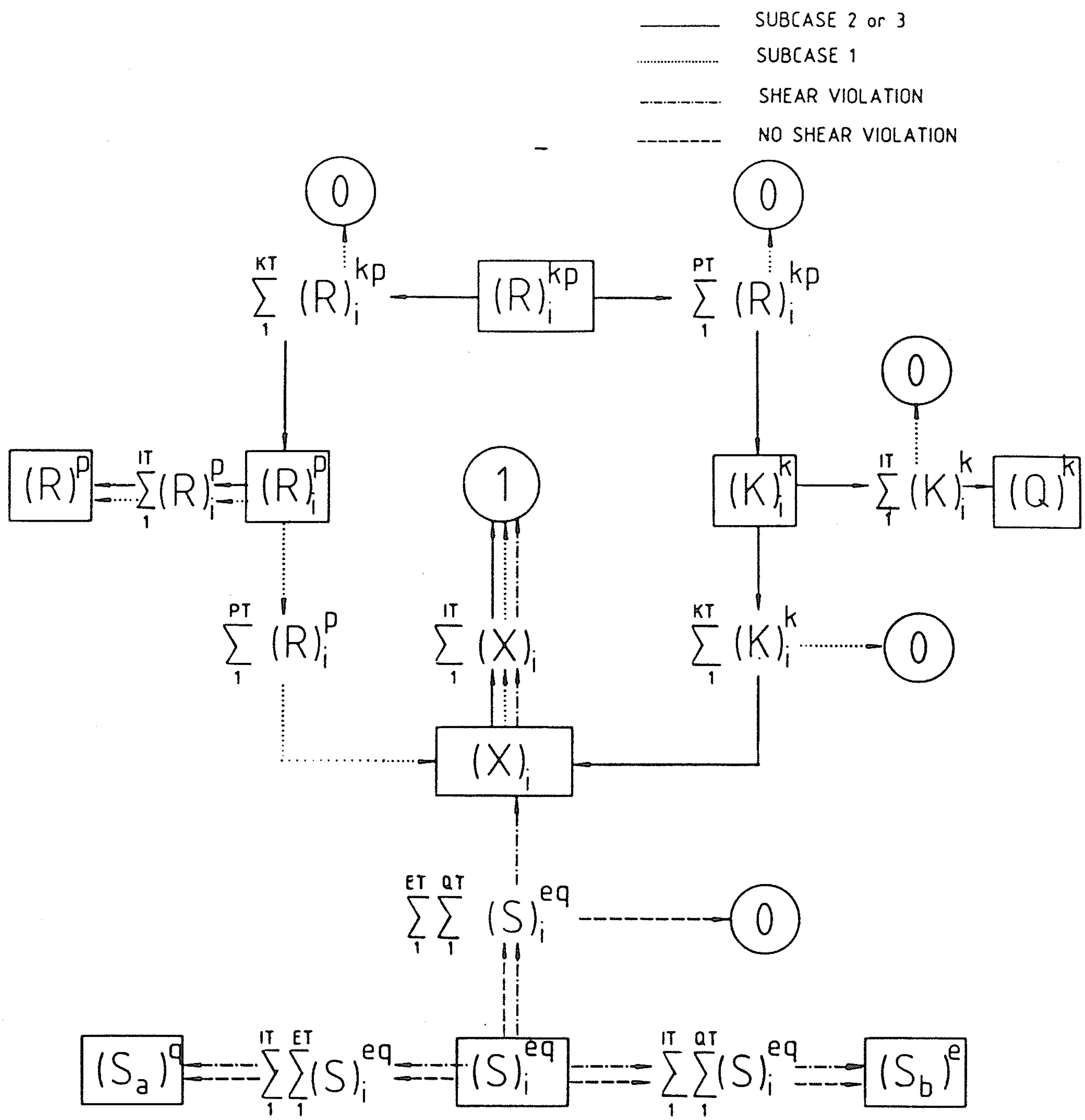


Figure 3-5. Flow diagram illustrating the interrelationships among (0/1) variables

The objective function is mathematically expressed as follows:

Minimize $TC = CB + CG + CVS + CLS$ (3-120)

where TC = Total Cost (\$)

CB = Cost of block (constant in this study) (\$)

CG = Cost of grouting

$= UCG * TNG$ (3-121)

where UCG = Cost of grouting one core for the total height of the wall (\$/core)

TNG = Total number of grouted cores

and CVS = Cost of vertical reinforcement

$= UCVS * TSA$ (3-122)

where $UCVS$ = Unit cost of vertical reinforcement's cross-sectional area for the total height of the wall (\$/mm^2)

and TSA = Total vertical steel cross-sectional area (mm^2)

CLS = Cost of lateral (shear) reinforcement

$= UCLS * A_{s\ell}$ (3-123)

where $UCLS$ = Unit cost of lateral reinforcement's cross-sectional area for the total length of the wall (\$/mm^2)

and $A_{s\ell}$ = Total lateral steel cross-sectional area (mm^2)

3.4 STUDY CASES

The following study cases are selected to demonstrate the capabilities of the model and the efficiency of the formulation. Emphasis is placed on the effect of changing the governing load cases on the optimal design results and the number of branches and pivots required as well as the execution time. Therefore, a wide range of load cases and a variety of stress and resisting capacities violations are investigated in these study cases.

The wall shown in Figure 3-6 is an individual shear wall of a masonry high-rise building. The configuration of the wall and the masonry block compressive strength (20 MPa) are assumed to be constant. Type S mortar is used and axial gravity loads are assumed to be applied. Tensile steel as well as horizontal shear steel are assumed to have an allowable stress of 165 MPa while compressive steel has 160 MPa allowable stress. Tables 3-1 through 3-5 present the assumed discrete values of the main masonry structural components used in the model namely, the grouting condition i, the vertical side steel k, the vertical inside steel p, the horizontal shear steel q and the spacing of the horizontal steel e.

The models for each case are produced using a computer program (Appendix C) which stores the generated model in data sets which in turn interact with LINDO[23] for the optimum solution process. The computer facility used is Amdahl 5870 at the University of Manitoba.

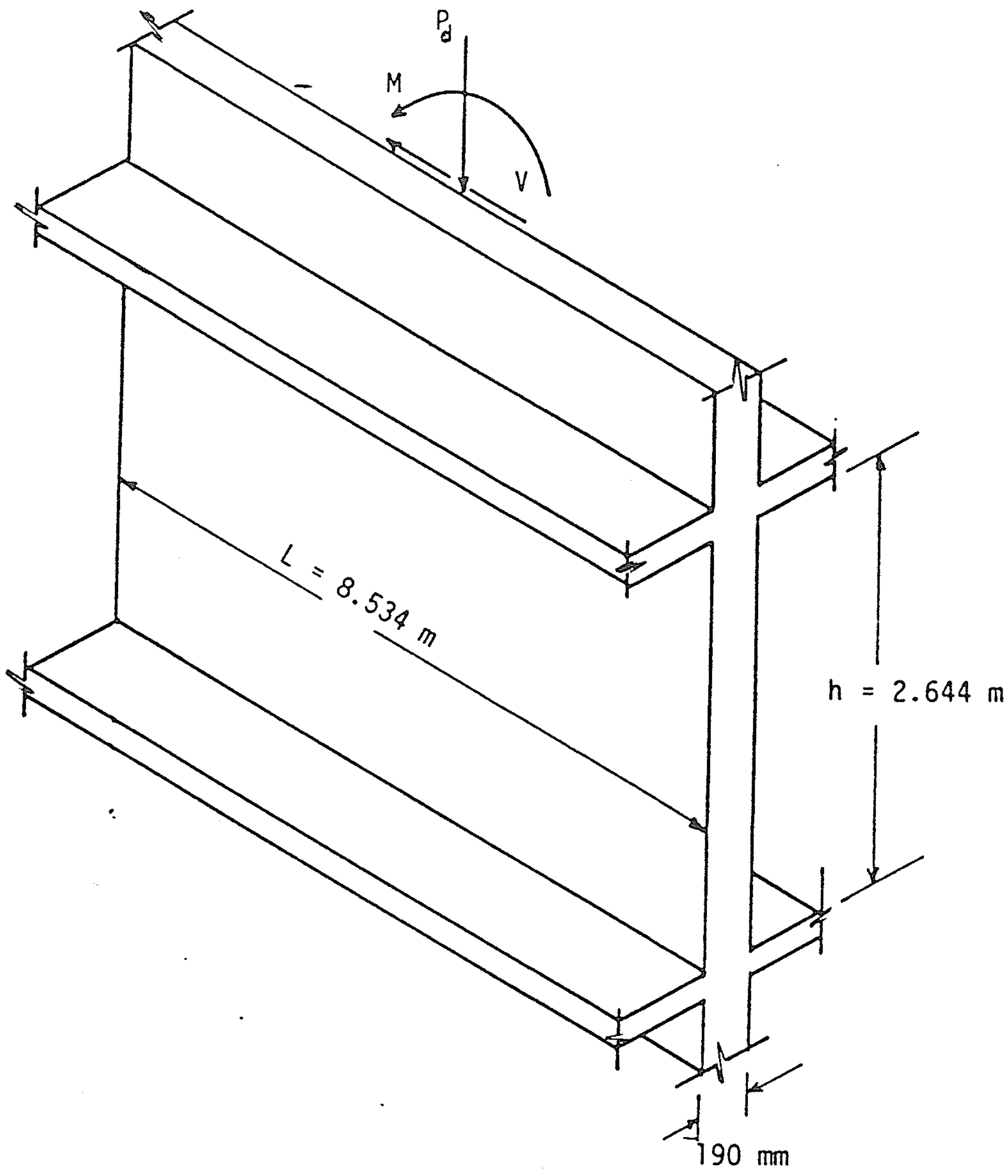


Figure 3-6. A single shear wall subjected to shear force, in-plane bending moment and applied axial load

Table 3-1. The assumed grouting conditions

i	1	2	3	4	5	6	7
Grouting Condition	Ungrouted	6th core	5th core	4th core	3rd core	2nd core	Fully Grouted

Table 3-2. The assumed vertical end steel options

k	1	2	3	4	5	6	7	8	9	10	11	12
Vertical Side Steel	1#15	1#20	2#15	1#25	2#20	3#15	4#15	3#20	2#25	4#20	3#25	4#25

Table 3-3. The assumed vertical inside steel options

p	1	2	3
Vertical Inside Steel	1#15	1#20	1#25

Table 3-4. The assumed horizontal shear steel options

q	1	2	3
Horizontal Shear Steel	2(4.1 mm)	1#10	1#15

Table 3-5. The assumed spacings for horizontal shear steel options

e	1	2	3
Spacing for lateral steel (mm)	200	400	600

Case 1: Dead load = 2000 kN Live load = 1000 kN

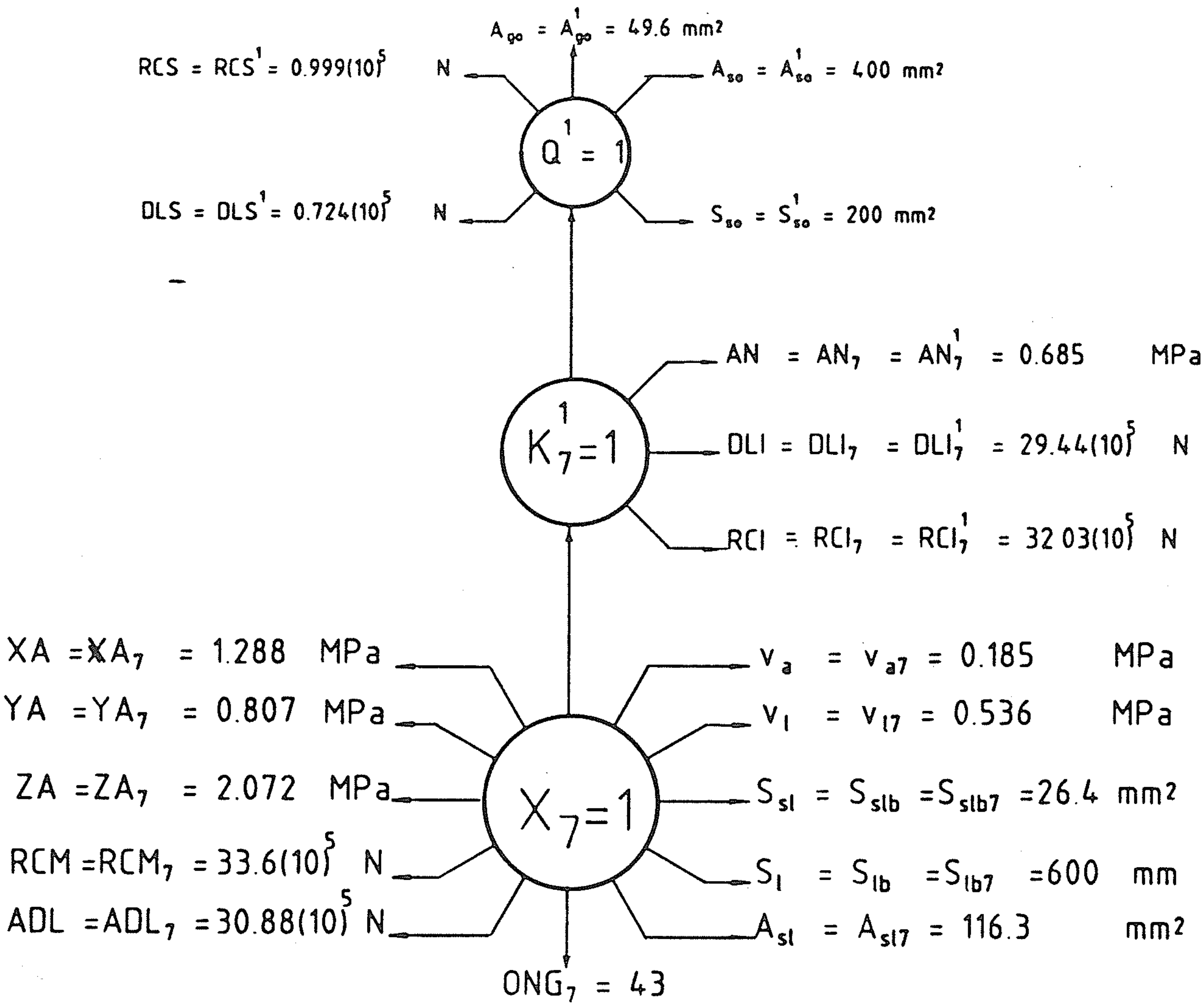
In-plane bending = 2000 kN-m Shear force = 200 kN

The stress relationship due to (DL + WL) for IT=7 grouting conditions and the violated cases due to (DL + WL), (DL + LL) or shear are presented in Table 3-6.

In Table 3-6, a compression violation occurs due to (DL + WL) in all seven grouting conditions, which means that vertical end steel is required to provide the additional strength for the compressive resisting capacity. Twelve vertical end steel combinations are considered, of which one, some or all will satisfy Relationship 3-43 ($XA + YA \leq ZA + AN$). Each steel combination creates two segments, namely the inside and outside segments, with different axial resisting capacity. As shown in Table 3.6, the resisting capacity of the outside part always exceeds the applied (DL + LL), while for the first six grouting conditions and for all end steel combinations, the internal resisting capacity is less than the applied (DL + LL). This situation suggests providing inside vertical steel and additional grouted cores. Three choices are considered for each outside steel combination, from which one, some or all will satisfy Relationship 3-47, ($RCS + RCIA \geq DLI + DLIA$). The inside resisting capacity of the seventh case is greater than the applied load which indicates that there is no axial compression violation and there is no need for providing inside steel. On the other hand, no tension or shear violation exists in the wall. Figures 3-7 and 3-8 present the optimal results in terms of the optimal tree and optimal cross-section of the wall, respectively. Table 3-7 presents the optimum cost, the optimum execution time, the L.P. (continuous) optimum cost and the number of branches and pivots required to complete the enumeration.

Table 3-6 Stress relationships and violation cases for IT = 7 grouting conditions for case study 1.

Compression due to DL + WL	Compression due to DL + LL	Tension due to DL + WL	Shear Stress	Remarks
$XA_1 + YA_1 > ZA_1$	$RCS_1 > DLS_1$ $RCI_1 < DLI_1$ (for all values of k)	$XA_1 > YA_1$	$v_{a1} < v_{\ell 1}$	-No Tension Violation
$XA_2 + YA_2 > ZA_2$	$RCS_2 > DLS_2$ $RCI_2 < DLI_2$ (for all values of k)	$XA_2 > YA_2$	$v_{a2} < v_{\ell 2}$	-Compression violation due to (DL + WL)
$XA_3 + YA_3 > ZA_3$	$RCS_3 > DLS_3$ $RCI_3 < DLI_3$ (for all values of k)	$XA_3 > YA_3$	$v_{a3} < v_{\ell 3}$	-Compression violation due to (DL + LL)
$XA_4 + YA_4 > ZA_4$	$RCS_4 > DLS_4$ $RCI_4 < DLI_4$ (for all values of k)	$XA_4 > YA_4$	$v_{a4} < v_{\ell 4}$	-No Shear Stress violation
$XA_5 + YA_5 > ZA_5$	$RCS_5 > DLS_5$ $RCI_5 < DLI_5$ (for all values of k)	$XA_5 > YA_5$	$v_{a5} < v_{\ell 5}$	
$XA_6 + YA_6 > ZA_6$	$RCS_6 > DLS_6$ $RCI_6 < DLI_6$ (for all values of k)	$XA_6 > YA_6$	$v_{a6} < v_{\ell 6}$	
$XA_7 + YA_7 > ZA_7$	$RCS_7 > DLS_7$ $RCI_7 > DLS_7$ (for all values of k)	$XA_7 > YA_7$	$v_{a7} < v_{\ell 7}$	-No Tension violation -Compression violation due to DL + WL -No compression violation due to DL + LL -No shear violation



Note: The arrows in the optimal tree point to the discrete values that have been chosen due to the selection of a particular zero-one variable (inside the circle).

Figure 3-7. The optimal tree for Case 1.

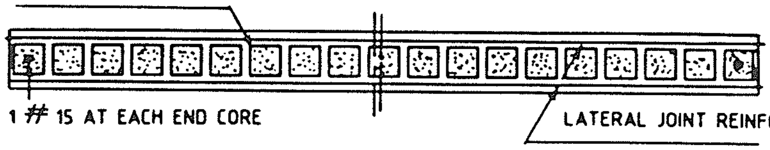


Figure 3-8. The optimal cross-section of the wall for Case 1.

Table 3-7. The optimal solution related information for Case 1.

LP** (continuous) Optimum cost $	IP (discrete) Optimum Cost, $	Central Processing Time* (sec)		No. of Branches	No. of Pivots
		Generating	Solving		
1368.03	1375.61	0.53	48.63	9	2027

* The execution time is almost 2/3 of the central processing time
** LP continuous solution is the solution just before starting branch-and-bound process

Case 2: Dead load = 1350 kN, Live load = 1000 kN

In-plane bending = 1350 kN-m, Shear force = 200 kN

Table 3-8 Shows the stress relationship due to (DL + WL) for grouting conditions, IT=7, and the violated cases due to (DL + WL), (DL + LL) or shear.

As shown in Table 3-8, no tension or shear violation exists in this case. However, the compression violation due to (DL + WL) occurs for the first five grouting conditions. Also, the compression violation due to (DL + LL) occurs only for the first five grouting cases. Figures 3-9 and 3-10 present the optimal tree and the optimal cross-section of the wall while Table 3-9 presents information related to the optimum solution.

Table 3-8. Stress relationships and violated cases with grouting condition IT = 7 for Case 2.

Compression due to DL + WL	Compression due to DL + LL	Tension due to DL + WL	Shear Stress	Remarks
$XA_1 + YA_1 > ZA_1$	$RCS_1 > DLS_1$ $RCI_1 < DLI_1$ (for all values of k)	$XA_1 > YA_1$	$v_{a1} < v_{\ell 1}$	-No Tension Violation
$XA_2 + YA_2 > ZA_2$	$RCS_2 > DLS_2$ $RCI_2 < DLI_2$ (for all values of k)	$XA_2 > YA_2$	$v_{a2} < v_{\ell 2}$	-Compression Violation due to DL + WL -Compression Violation due to DL + LL
$XA_3 + YA_3 > ZA_3$	$RCS_3 > DLS_3$ $RCI_3 < DLI_3$ (for all values of k)	$XA_3 > YA_3$	$v_{a3} < v_{\ell 3}$	-No Shear Stress Violation
$XA_4 + YA_4 > ZA_4$	$RCS_4 > DLS_4$ $RCI_4 < DLI_4$ (for all values of k)	$XA_4 > YA_4$	$v_{a4} < v_{\ell 4}$	
$XA_5 + YA_5 > ZA_5$	$RCS_5 > DLS_5$ $RCI_5 < DLI_5$ (for all values of k)	$XA_5 > YA_5$	$v_{a5} < v_{\ell 5}$	
$XA_6 + YA_6 < ZA_6$	$RCM_6 > ADL_6$	$XA_6 > YA_6$	$v_{a6} < v_{\ell 6}$	-No Tension Violation -No Compression Violation due to DL + WL
$XA_7 + YA_7 > ZA_7$	$RCM_7 > ADL_7$	$XA_7 > YA_7$	$v_{a7} < v_{\ell 7}$	-No Compression Violation due to DL + LL -No Shear Violation

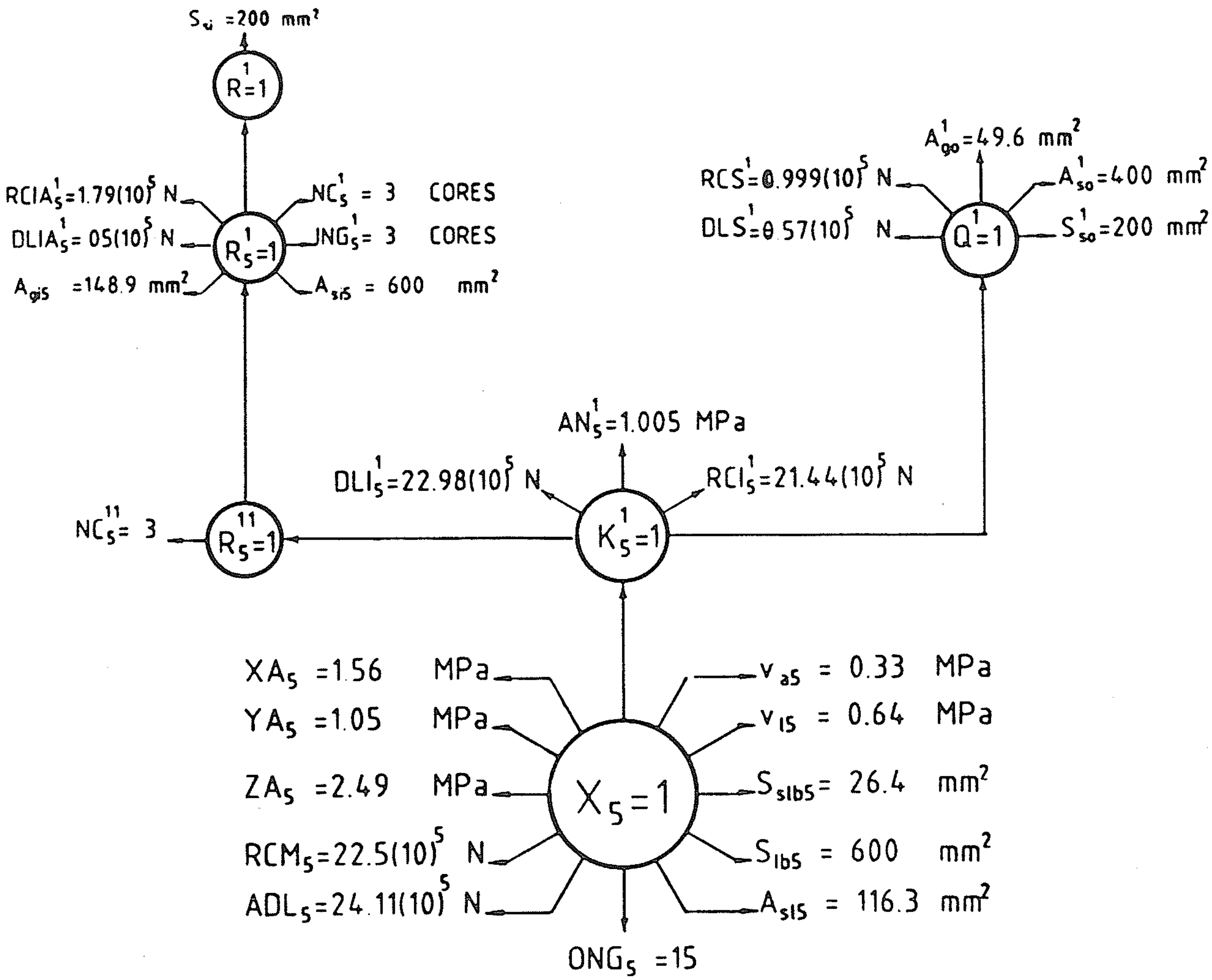


Figure 3-9. The optimal tree for Case 2.

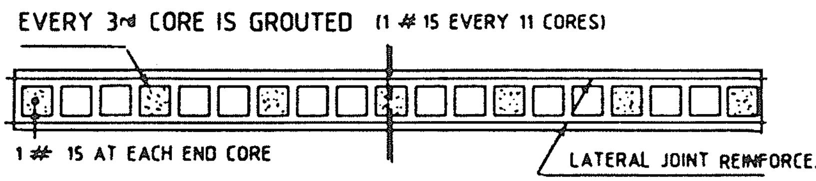


Figure 3-10. The optimal cross-section of the wall for Case 2.

Table 3-9. The optimal solution related information for Case 2.

LP (continuous) Optimum cost $	IP (discrete) Optimum Cost, $	Central Processing Time (sec)		No. of Branches	No. of Pivots
		Generating	Solving		
1301.2	1301.2	0.5	17.77	0	627

Case 3. Effect of changing (reducing) live load

Dead load = 1350 kN Live load = 200 kN

In-plane bending = 1350 kN-m Shear force = 200 kN

Table 3-10 shows the stress relationship due to (DL + WL) for grouting conditions, IT=7, and the violated cases due to (DL + LL), (DL + LL) or shear.

In this case, the compression violation due to (DL + LL) occurs only in the first grouting condition (ungrouted wall), while the stress distribution for other cases is the same as in case 2. Figures 3-11 and 3-12 present the optimal tree and the optimal cross-section of the wall, respectively, for this case. Table 3-11 presents information related to the optimum solution.

Table 3.10. Stress relationships and violated cases with grouting condition IT = 7 for case 3.

Compression due to DL + WL	Compression due to DL + LL	Tension due to DL + WL	Shear Stress	Remarks
$XA_1 + YA_1 > ZA_1$	$RCS_1 > DLS_1$ $RCI_1 < DLI_1$ (for all values of k)	$XA_1 > YA_1$	$v_{a1} < v_{\ell 1}$	-No Tension Violation -Compression Violation due to DL + WL -Compression Violation due to DL + LL -No Shear Violation
$XA_2 + YA_2 > ZA_2$	$RCS_2 > DLS_2$ $RCI_2 > DLI_2$ (for all values of k)	$XA_2 > YA_2$	$v_{a2} < v_{\ell 2}$	-No Tension Violation -Compression Violation due to DL + WL
$XA_3 + YA_3 > ZA_3$	$RCS_3 > DLS_3$ $RCI_3 > DLI_3$ (for all values of k)	$XA_3 > YA_3$	$v_{a3} < v_{\ell 3}$	-No Compression Violation due to DL + LL
$XA_4 + YA_4 > ZA_4$	$RCS_4 > DLS_4$ $RCI_4 > DLI_4$ (for all values of k)	$XA_4 > YA_4$	$v_{a4} < v_{\ell 4}$	-No Shear Violation
$XA_5 + YA_5 > ZA_5$	$RCS_5 > DLS_5$ $RCI_5 > DLI_5$ (for all values of k)	$XA_5 > YA_5$	$v_{a5} < v_{\ell 5}$	
$XA_6 + YA_6 < ZA_6$	$RCM_6 > ADL_6$	$XA_6 > YA_6$	$v_{a6} < v_{\ell 6}$	-No Tension Violation -No Compression Violation due to DL + WL -No Compression Violation due to DL + LL
$XA_7 + YA_7 < ZA_7$	$RCH_7 > ADL_7$	$XA_7 > YA_7$	$v_{a7} < v_{\ell 7}$	-No Shear Violation

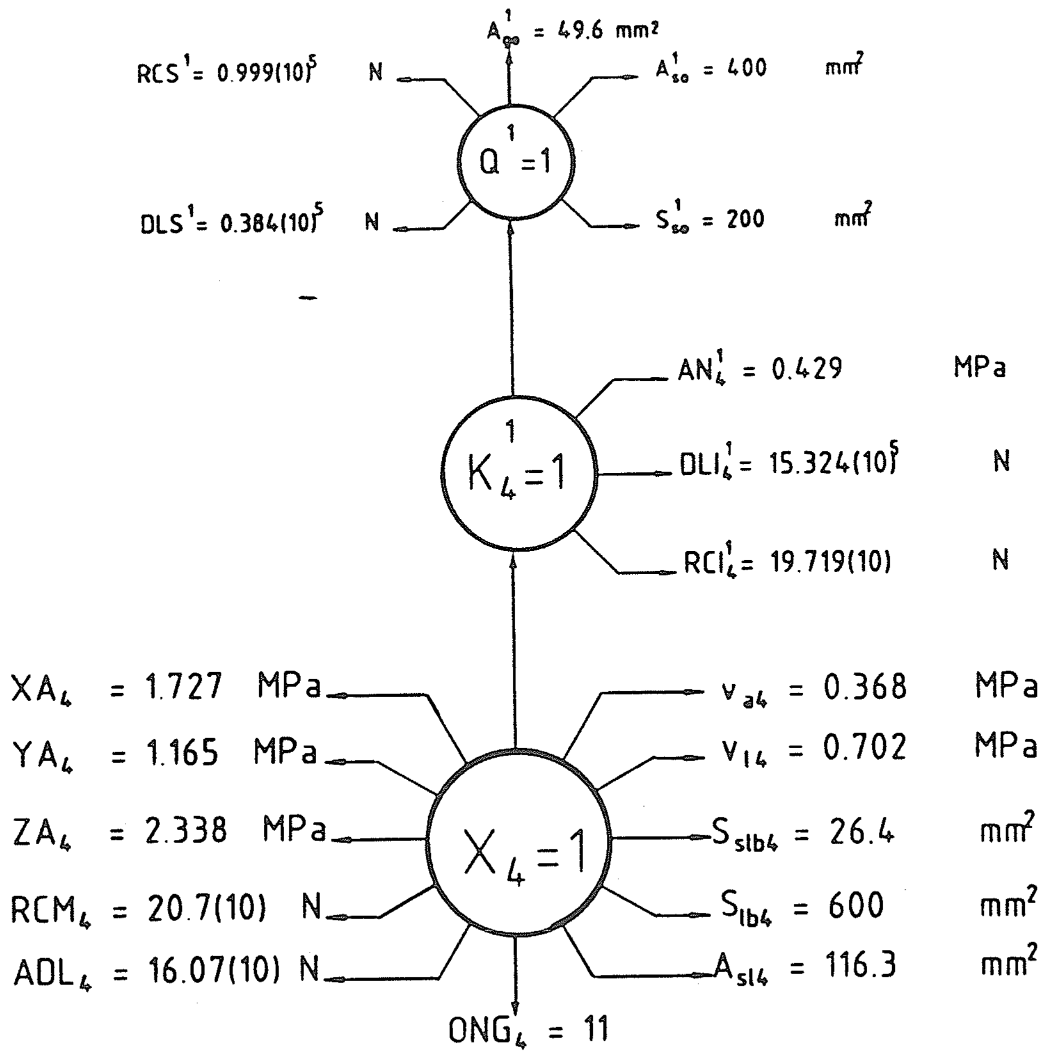


Figure 3-11. The optimal tree for case 3.

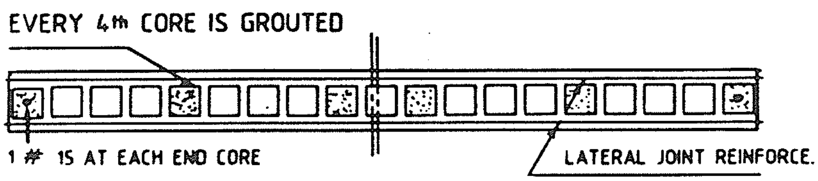


Figure 3-12. The optimal cross-section for case 3.

Table 3-11. The optimal solution related information for Case 3.

LP (continuous) Optimum cost $	IP (discrete) Optimum Cost, $	Central Processing Time (sec)		No. of Branches	No. of Pivots
		Generating	Solving		
1259.96	1266.34	0.45	34.38	8	1351

Case 4 Effect of shear violation on the optimum design of case 2.

Dead load = 1350 kN   Live load = 1000 kN

In-plane bending = 1350 kN-m   Shear force = 450 kN

Table 3-12 shows the stress relationship due to (DL + WL) for grouting conditions, IT=7, and the violated cases due to (DL + WL), (DL + LL) or shear.

This case reflects the effect of shear violation on the optimum solution of case study 2. Figures 3-13 and 3-14 present the optimal tree and the optimal cross section of the wall, while Table 3-13 shows the optimal solution related information.

Table 3-12. Stress relationships and violated cases with grouting condition IT = 7 for case 4.

Compression due to DL + WL	Compression due to DL + LL	Tension due to DL + WL	Shear Stress	Remarks
$XA_1 + YA_1 > ZA_1$	$RCS_1 > DLS_1$ $RCI_1 < DLI_1$ (for all values of k)	$XA_1 > YA_1$	$v_{a1} > v_{\ell 1}$	-No Tension Violation
$XA_2 + YA_2 > ZA_2$	$RCS_2 > DLS_2$ $RCI_2 < DLI_2$ (for all values of k)	$XA_2 > YA_2$	$v_{a2} > v_{\ell 2}$	-Compression Violation due to DL + WL
$XA_3 + YA_3 > ZA_3$	$RCS_3 > DLS_3$ $RCI_3 < DLI_3$ (for all values of k)	$XA_3 > YA_3$	$v_{a3} > v_{\ell 3}$	-Compression Violation due to DL + LL
$XA_4 + YA_4 > ZA_4$	$RCS_4 > DLS_4$ $RCI_4 < DLI_4$ (for all values of k)	$XA_4 > YA_4$	$v_{a4} > v_{\ell 4}$	-Shear Violation
$XA_5 + YA_5 > ZA_5$	$RCS_5 > DLS_5$ $RCI_5 < DLI_5$ (for all values of k)	$XA_5 > YA_5$	$v_{a5} > v_{\ell 5}$	
$XA_6 + YA_6 < ZA_6$	$RCM_6 > ADL_6$	$XA_6 > YA_6$	$v_{a6} > v_{\ell 6}$	-No Tension Violation -No Compression Violation due to DL + WL -No Compression Violation due to DL + LL -Shear Violation
$XA_7 + YA_7 < ZA_7$	$RCH_7 > ADL_7$	$XA_7 > YA_7$	$v_{a7} > v_{\ell 7}$	-No Tension Violation -No Compression Violation due to DL + WL -No Compression Violation due to DL + LL -No Shear Violation

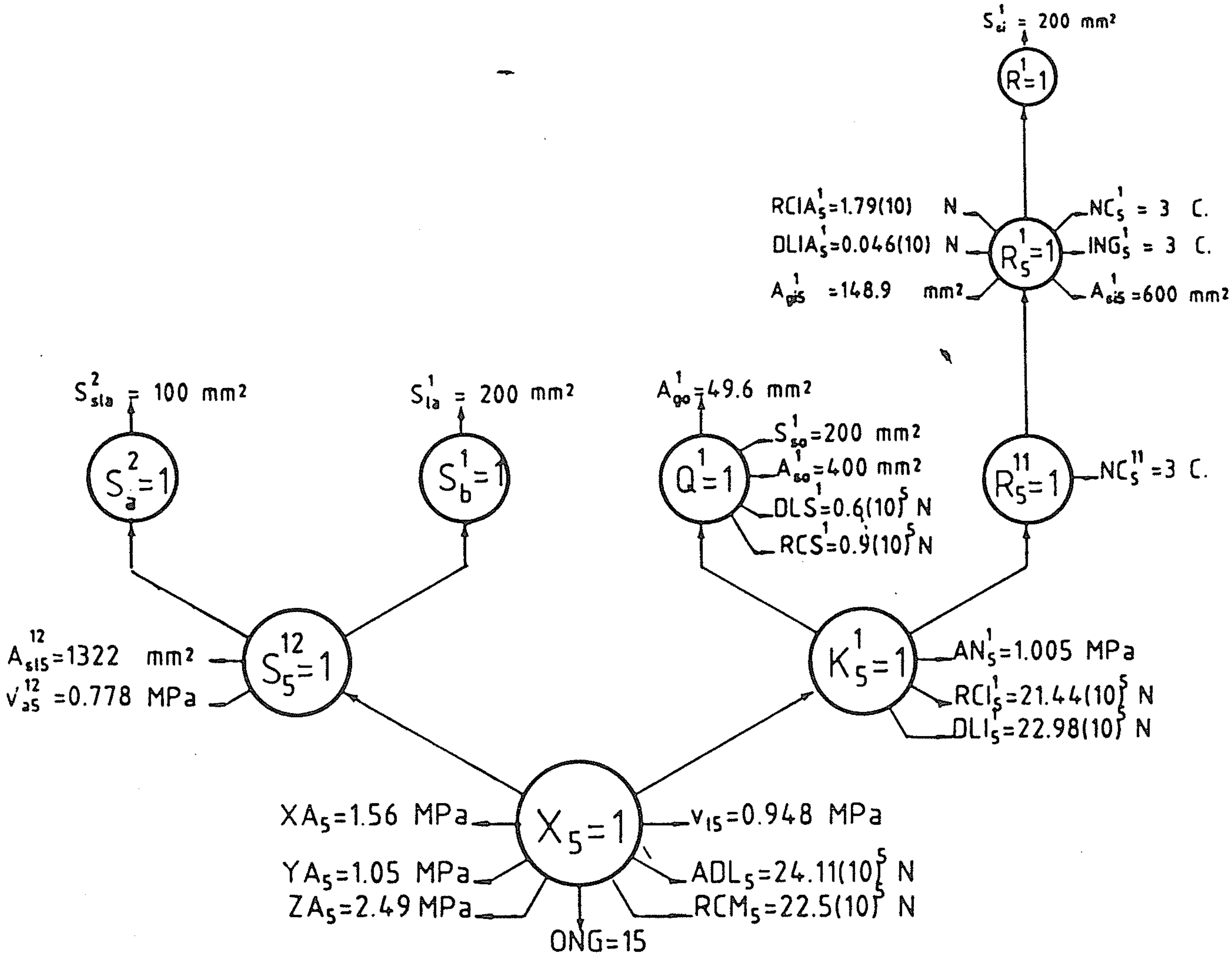


Figure 3-13. The optimal tree for case 4.

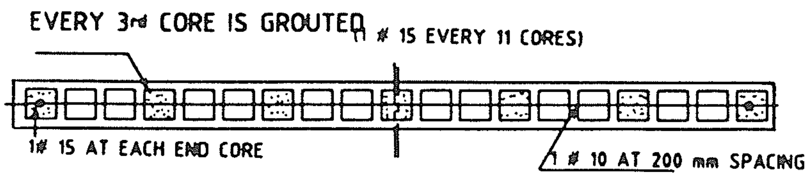


Figure 3-14. The optimal cross-section for case 4.

Table 3-13. The optimal solution related information for Case 4.

LP (continuous) Optimum cost $	IP (discrete) Optimum Cost, $	Central Processing Time (sec)		No. of Branches	No. of Pivots
		Generating	Solving		
1358.8	1372.3	0.48	8.93	16	2995

Case 5 Effect of simple cases (fewer or no violated cases) on the optimality of the computation process.

Dead load = 1000 kN Live load = 1000 kN

In-plane bending = 1000 kN-m Shear force = 200 kN

Table 3-14 shows the stress relationship due to (DL + WL) for grouting conditions, IT=7, and the violated cases due to (DL + WL), (DL + LL) or shear.

The applied loads in this case study are selected so that violated cases are fewer than in previous cases. The intent is to test the model partitioning effect on the computation time. Figures 3-15 and 3-16 present the optimal tree and the optimal cross-section of the wall respectively. Table 3-15 presents the information related to the optimum solution.

Table 3-14. Stress relationships and violated cases for grouting conditions IT = 7 for case 5

Compression due to DL + WL	Compression due to DL + LL	Tension due to DL + WL	Shear Stress	Remarks
$XA_1 + YA_1 > ZA_1$	$RCS_1 > DLS_1$ $RCI_1 < DLI_1$ (for all values of k)	$XA_1 > YA_1$	$v_{a1} < v_{11}$	-No Tension Violation -Compression Violation due to DL + WL -Compression Violation due to DL + LL -No Shear Violation
$XA_2 + YA_2 < ZA_2$	$RCM_2 < ADL_2$ (for all values of k)	$XA_2 > YA_2$	$v_{a2} < v_{\ell 2}$	-No Tension Violation -No Compression Violation due to DL + WL
$XA_3 + YA_3 < ZA_3$	$RCM_3 < ADL_3$ (for all values of k)	$XA_3 > YA_3$	$v_{a3} < v_{\ell 3}$	-No Compression Violatoon due to DL + LL -No Shear Violation
$XA_4 + YA_4 < ZA_4$	$RCM_4 > ADL_4$ (for all values of k)	$XA_4 > YA_4$	$v_{a4} < v_{\ell 4}$	-No Tension Violation
$XA_5 + YA_5 < ZA_5$	$RCM_5 > ADL_5$ (for all values of k)	$XA_5 > YA_5$	$v_{a5} < v_{\ell 5}$	-No Compression Violation due to DL + WL
$XA_6 + YA_6 < ZA_6$	$RCM_6 > ADL_6$ (for all values of k)	$XA_6 > YA_6$	$v_{a6} > v_{\ell 6}$	-No Compression Violation due to DL + LL -No Shear Violation
$XA_7 + YA_7 < ZA_7$	$RCM_7 > ADL_7$ (for all values of k)	$XA_7 > YA_7$	$v_{a7} < v_{\ell 7}$	

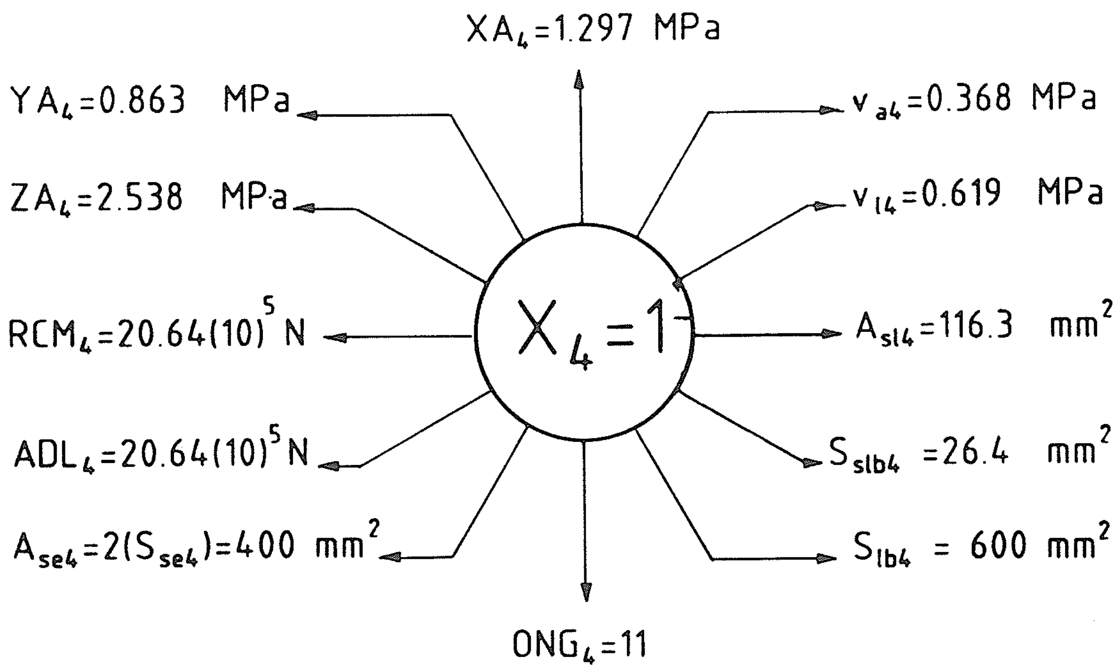


Figure 3-15. The optimum tree for case 5.

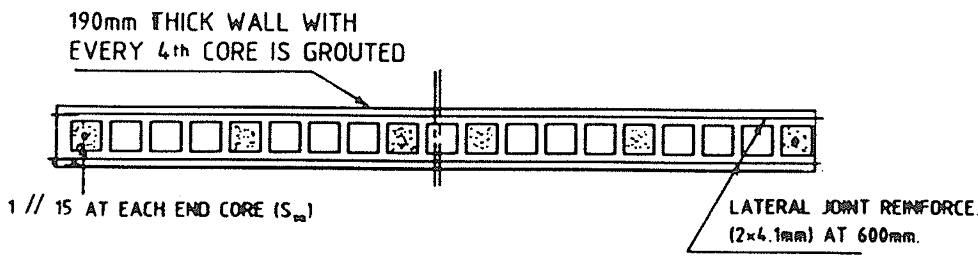


Figure 3-16. The optimum cross-section of the wall for case 5.

Table 3-15. The optimal solution related information for Case 5.

LP (continuous) Optimum cost $	IP (discrete) Optimum Cost, $	Central Processing Time (sec)		No. of Branches	No. of Pivots
		Generating	Solving		
1266.34	1266.34	0.42	6.8	0	212

Case 6 Tension Violation*

Dead Load = 500 kN Live load = 1000 kN

In-plane bending = 1500 kN-m Shear force = 200 kN

Table 3-16 presents the stress relationship due to (DL + WL) for grouting conditions IT=7 for this case study.

The applied loads in this case lead to substantial tension violation in all grouting conditions. As assumed previously, the steel selected for tension(both sides of the wall) takes care of any compression violation due to (DL + WL) that might occur. Figures 3-17 and 3-18 show the optimum tree and the optimal cross-section of the wall, respectively. Table 3-17 presents the related information to the optimal solution.

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* It is assumed that whenever tension violation occurs, the compression due to DL + WL is disregarded unless the tension is negligable as will be in case study 7.

Table 3-16. The stress relationships and violated cases for grouting IT = 7 for case 6.

Compression due to DL + WL	Compression due to DL + LL	Tension due to DL + WL	Shear Stress	Remarks
$XA_1 + YA_1 > ZA_1$	$RCS_1 > DLS_1$ $RCI_1 < DLI_1$ (for all values of k)	$XA_1 < YA_1$	$v_{a1} > v_{11}$	-Tension Violation -Compression Violation due to DL + WL -Compression Violation due to DL + LL -Shear Violation
$XA_2 + YA_2 < ZA_2$	$RCS_2 > DLS_2$ $RCI_2 > DLI_2$ (for all values of k)	$XA_2 < YA_2$	$v_{a2} < v_{\ell 2}$	-Tension Violation
$XA_3 + YA_3 < ZA_3$	$RCS_3 > DLS_3$ $RCI_3 > DLI_3$ (for all values of k)	$XA_3 < YA_3$	$v_{a3} < v_{\ell 3}$	-No Compression Violation due to DL + WL
$XA_4 + YA_4 < ZA_4$	$RCS_4 > DLS_4$ $RCI_4 > DLI_4$ (for all values of k)	$XA_4 < YA_4$	$v_{a4} < v_{\ell 4}$	-No Compression Violation due to DL + LL
$XA_5 + YA_5 < ZA_5$	$RCS_5 > DLS_5$ $RCI_5 > DLI_5$ (for all values of k)	$XA_5 < YA_5$	$v_{a5} < v_{\ell 5}$	-No Shear Violation
$XA_6 + YA_6 < ZA_6$	$RCS_6 > DLS_6$ $RCI_6 > DLI_6$ (for all values of k)	$XA_6 < YA_6$	$v_{a6} < v_{\ell 6}$	
$XA_7 + YA_7 < ZA_7$	$RCS_7 > DLS_7$ $RCI_7 > DLI_7$ (for all values of k)	$XA_7 < YA_7$	$v_{a7} < v_{\ell 7}$	

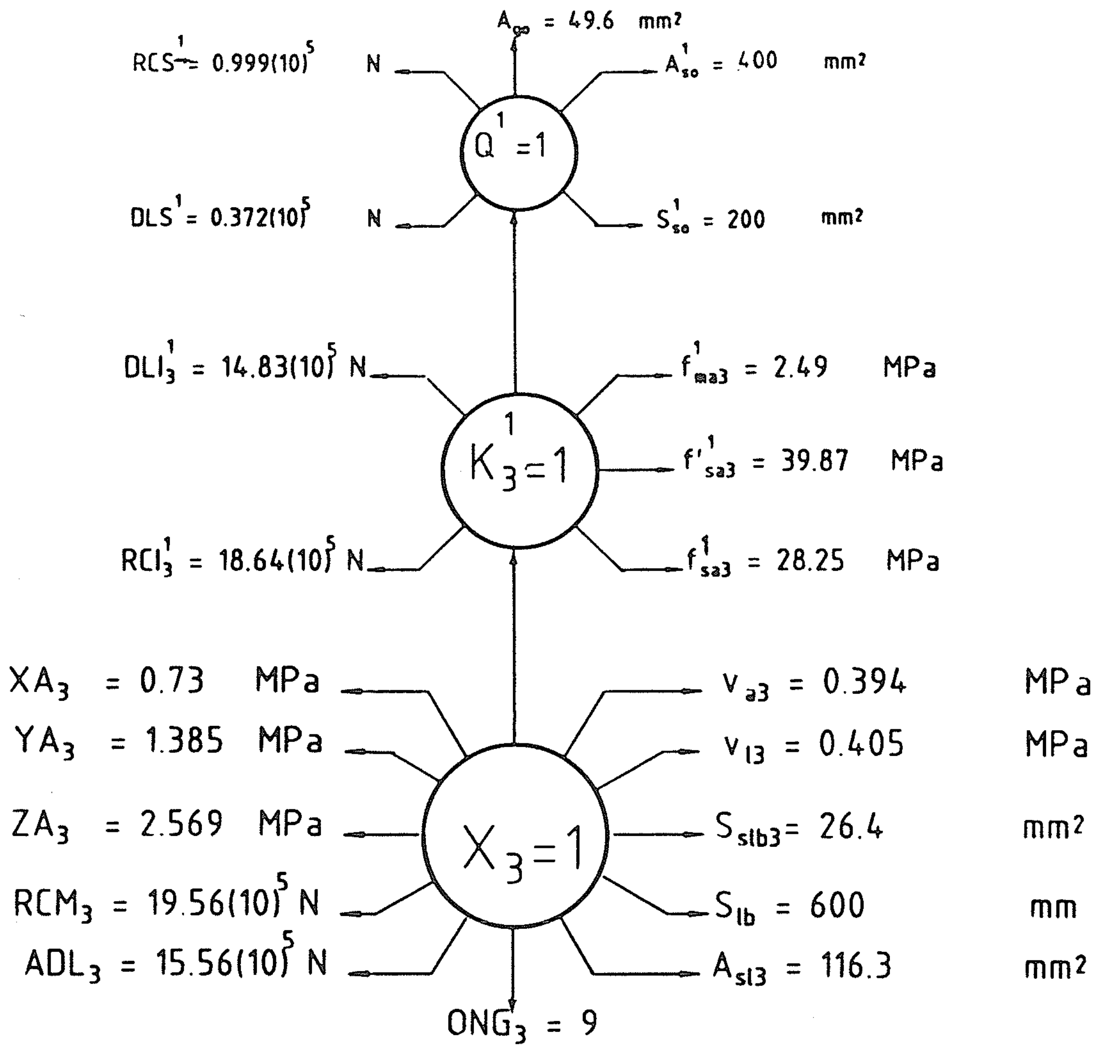

Figure 3-17. The optimum tree for case 6.

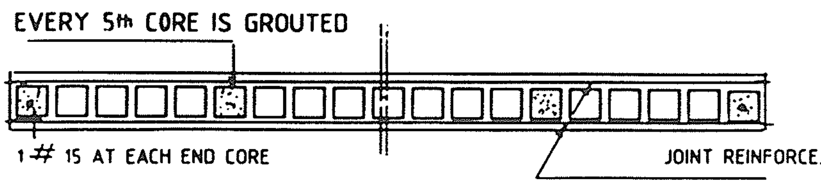

Figure 3-18. The optimum cross-section for the wall in case 6.

Table 3-17. The optimal solution related information for Case 6.

LP (continuous) Optimum cost $	IP (discrete) Optimum Cost, $	Central Processing Time (sec)		No. of Branches	No. of Pivots
		Generating	Solving		
1256.9	1259.5	1.02	23.67	4	636

Case 7 Negligible tension exists

Dead load = 1000 kN Live load = 2500 kN

In-plane bending = 1510 kN-m Shear force = 200 kN

Table 3-18 presents the stress relationship due to (DL + WL) for grouting conditions, IT=7, for case 7.

In this case study, tension violation exists. However, the tension is very small, and almost negligible, therefore the design for tension is disregarded and the design process is directed toward checking stress on the other end of the wall .

Figures 3-19 and 3-20 show the optimum tree and the optimum cross-section for the wall under study, while Table 3-19 presents the optimum solution related information.

Table 3-18. The stress relationships and violated cases with grouting condition IT = 7 for case 7.

Compression due to DL + WL	Compression due to DL + LL	Tension due to DL + WL	Shear Stress	Remarks
$XA_1 + YA_1 > ZA_1$	$RCS_1 > DLS_1$ $RCI_1 < DLI_1$ (for all volumes of k)	$XA_1 < YA_1$	$v_{a1} < v_{\ell 1}$	-Tension Violation -Compression Violation due to DL + WL -Compression Violation due to DL + LL -No Shear Violation
$XA_2 + YA_2 > ZA_2$	$RCS_2 > DLS_2$ $RCI_2 > DLI_2$ (for all values of k)	$XA_2 < YA_2$	$v_{a2} < v_{\ell 2}$	-Tension Violation -Compression Violation due to DL + WL -No Compression Violation due to DL + LL -No Shear Violation
$XA_3 + YA_3 > ZA_3$	$RCS_3 > DLS_3$ $RCI_3 > DLI_3$ (for all values of k)	$XA_3 < YA_3$	$v_{a3} < v_{\ell 3}$	
$XA_4 + YA_4 > ZA_4$	$RCS_4 > DLS_4$ $RCI_4 > DLI_4$ (for all values of k)	$XA_4 < YA_4$	$v_{a4} < v_{\ell 4}$	
$XA_5 + YA_5 < ZA_5$	$RCM_5 > ADL_5$	$XA_5 < YA_5$	$v_{a5} < v_{\ell 5}$	-Tension Violation -No Compression Violation due to DL + WL -No Compression Violation due to DL + LL -No Shear Violation
$XA_6 + YA_6 < ZA_6$	$RCM_6 > ADL_6$	$XA_6 > YA_6$	$v_{a6} < v_{\ell 6}$	-No Tension Violation -No Compression Violation due to DL + WL -No Compression Violation due to DL + LL -No Shear Violation
$XA_7 + YA_7 < ZA_7$	$RCM_7 > ADI_7$	$XA_7 > YA_7$	$v_{a7} < v_{\ell 7}$	

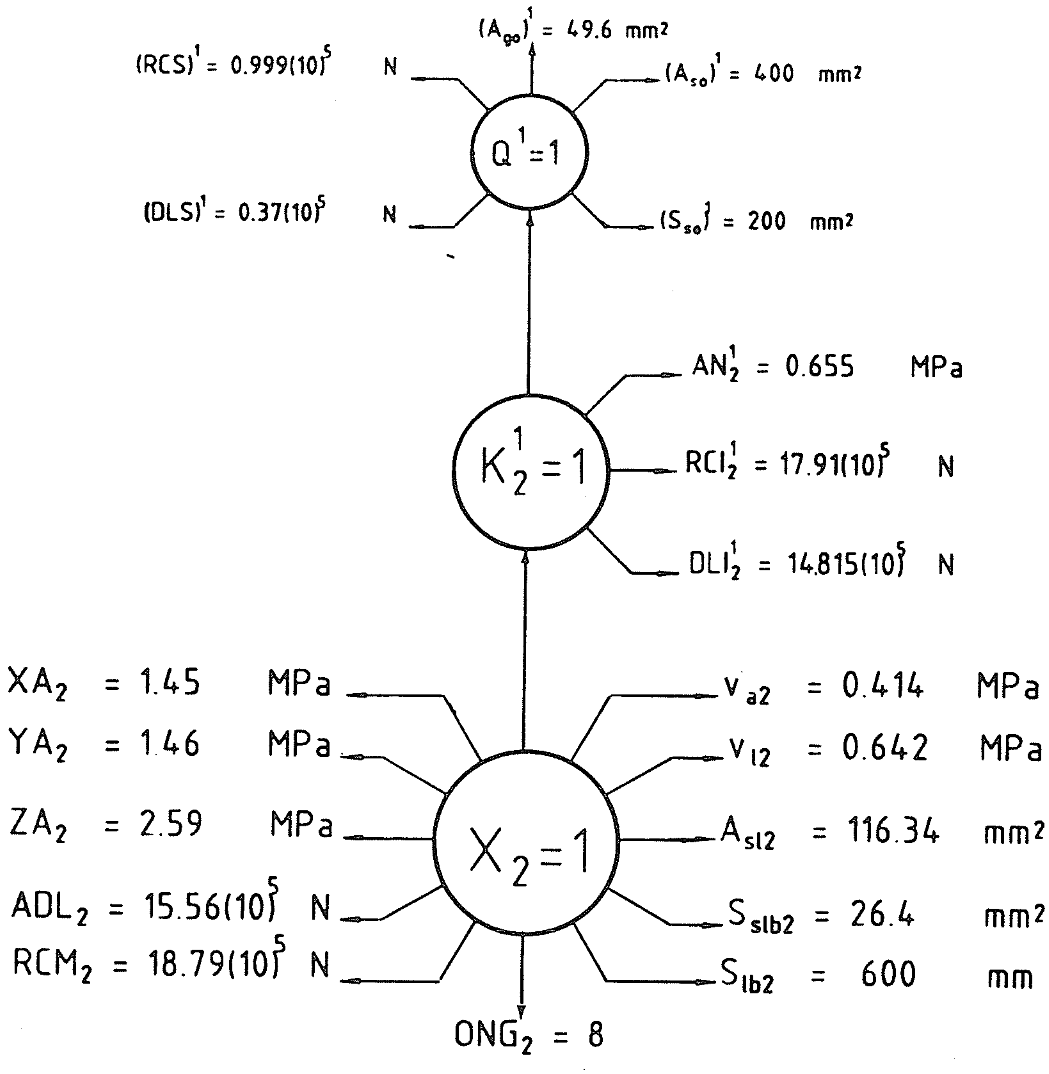

Figure 3-19. The optimal tree for case 7.

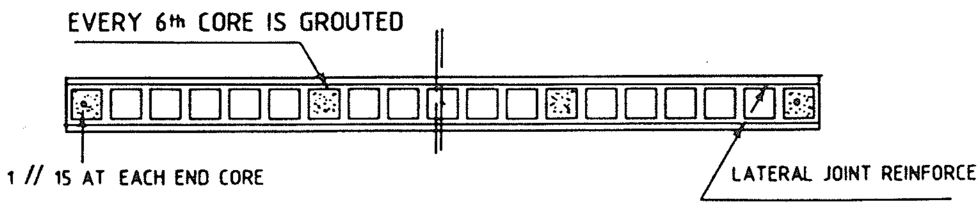

Figure 3-20. The optimum cross-section of the wall for case 7.

Table 3-19. The optimal solution related information for Case 7.

LP (continuous) Optimum cost $	IP (discrete) Optimum Cost, $	Central Processing Time (sec)		No. of Branches	No. of Pivots
		Generating	Solving		
1251.3	1256.1	0.48	21.62	4	945

3.4.1 Analysis of results of the study cases

Study cases 1 through 7 were selected so that they covered a broad range of stress and load cases. This selection demonstrates the ability of the approach to develop a variety of models that correspond to different loading conditions. These cases also support claims for the efficiency of the formulation discussed in the conclusion. The following important issues are made evident in the results of the study cases:

1. The cost of adding steel is higher than the cost of grouting more cores. At the same time, it is evident that grouting more cores increases the compressive resisting capacity of the wall, regardless of whether the conditions causing the compression are due to (DL + WL) or (DL + LL). Therefore, and as evident in the optimum results, the option of adding steel is always secondary to the option of grouting more cores.
2. The efficiencies of the "model generator" computer program and the model itself are evident from noticing their corresponding central processing time. However, although there is an obvious relationship between the time of generating the model and its size, the solution time for the model is much more complicated to analyze because of the many contributing factors affecting this time. One noticeable factor is the closeness of the LP solution to the final IP solution as shown in the two extreme cases, cases 2 and 5, where the LP solution is exactly the IP solution.

   A number of other factors affecting the computation time such as, total number of constraints, the size of individual constraints, the number of discrete variables entering the model and the number of branches and pivots required to complete the enumeration. Perhaps the most important factor, however, is the number of zero-one variables. Because of the nature of the interrelationships among zero-one variables, the model produces a constant number of zero-one variables for each case and condition. This factor does not, therefore, count when discussing the relative time differences among different cases.

3. The limitations (or the boundaries) of the discrete values of masonry structural components, and in turn the application of the model, are not discussed herein. However, these discrete values, e.g., the grouting conditions, steel options, etc. , can be easily determined by fixing one condition at a time and varying others to the extreme bounds.

3.5. SUMMARY AND CONCLUSIONS

A model for the optimum design of a masonry single shear wall is developed in this chapter. The suggested formulation of the model can be considered an efficient and practical approach to the optimum design of such walls. The several advantages can be summarized as follows:

1. The generated model is based on a realistic and practical approach to the design of individual masonry single shear walls.
2. The process does not need an initial solution. However, the assumed discrete values of the different structural variables must be selected on a rational basis.
3. The nature of the mathematical formulation, i.e., embedding the

discrete solution into a region of continuous solution, is efficient for such types of problems, as the pure discrete formulation would yield infeasible solutions in some cases unless an initial feasible solution is suggested.

4. Partitioning the problem and subsequently the model, in the way described through the development of the constraints, serves a number of purposes. First, it minimizes the computation time of generating the model. Second, it produces a model that can be easily interpreted and understood by unfamiliar users. Third, it reduces the size of each individual constraint significantly, which could in many cases affect or more specifically reduce the execution time. Fourth, it reduces the number of constraints in many cases, and subsequently the number of discrete variables entering the model which, again, affects the computation time. However, this partitioning in some cases, e.g., when compression violation occurs for all i's grouting conditions, produces a large number of constraints. As long as this number is within the tolerance limit of the available solving system, the problem will only be limited by the execution time.

The case studies presented demonstrate the efficiency and flexibility of the model through a wide variety of applied loads and resulting stress violation cases.

CHAPTER IV

SIMULATION OF THE STRUCTURAL RELATIONSHIPS FOR THE DISCRETE MODELLING OF THE MASONRY MULTISTOREY SHEAR WALL OPTIMIZATION PROBLEM

4.1 INTRODUCTION

Discrete modelling, using zero-one variables, for the optimum design of shear walls in multistorey masonry buildings requires more discussion and analysis than that required for single storey shear walls. Although the original basic design relationships used in both models are the same, more adaptations for these relationships are required for multistory shear wall problems. Since these adaptations are in some cases beyond simple mathematical manipulation, they are rather based on developing simulated functions, and the process of adaptations can be looked at as a "simulation" process.

The following sections contain:

4.2 The reason for simulation

4.3 The simulation process

4.3.1 Defining the original problem

4.3.2 The simulated functions

4.3.3 Comparisons between the simulated and original functions

4.4 Summary and conclusions

4.2 THE REASONS FOR SIMULATION

Throughout the models developed in this work, discrete variables can generally be classified under two categories. First are those discrete variables which are directly obtainable from tables containing specifications and standard sizes, such as block sizes, steel sizes and grouting conditions. Second are discrete variables that are determined with respect to the first type of variables, such as: mortar bedded area, allowable masonry compressive stress, actual steel tensile stress and axial load resisting capacity. For the single storey shear wall problems, both the grouting condition and allowable masonry compressive stress, for example, can be easily connected through the use of the same zero-one variables set. For example, when the U.E. is used for an unreinforced wall, it can be expressed as follows:

$$XA + YA \leq ZA \tag{4-1}$$

where

$$XA = \sum_{i=1}^{IT} (XA)_i * (X)_i \tag{4-2}$$

$$YA = \sum_{i=1}^{IT} (YA)_i * (X)_i \tag{4-3}$$

$$ZA = \sum_{i=1}^{IT} (ZA)_i * (X)_i \tag{4-4}$$

where

$$(XA)_i = \left(\frac{P_d}{A_m * L}\right)_i \tag{4-5}$$

$$(YA)_i = \left(\frac{M}{SM}\right)_i \tag{4-6}$$

and

$$(ZA)_i = (C_s * f_m)_i \tag{4-7}$$

Also

$$A_m = \sum_{i=1}^{IT} (A_m)_i * (X)_i \tag{4-8}$$

and

$$f_m = \sum_{i=1}^{IT} (f_m)_i * (X)_i \tag{4-9}$$

where all variables are as defined previously.

As can be seen, $(P_d)_i$ and $(A_m)_i$ are combined together to produce the discrete values of $(XA)_i$. While the above adaptation for the U.E. is adequate for the single storey shear wall problems, it is inadequate and nearly computationally impossible for multistorey problems for the following reasons. For the top and subsequent floors in a multistorey building, the U.E. can be expressed as follows:

$$(XA)^j + (YA)^j \leq (ZA)^j \qquad \text{for storey } j \tag{4-10}$$

$$(XA)^{j-1} + (YA)^{j-1} \leq (ZA)^{j-1} \qquad \text{for storey } j-1 \tag{4-11}$$

where

$$(XA)^j = \sum_{i1=1}^{IT1} (XA)^j_{i1} * (X)^j_{i1} \tag{4-12}$$

$$(YA)^j = \sum_{i1=1}^{IT1} (YA)^j_{i1} * (X)^j_{i1} \tag{4-13}$$

$$(ZA)^j = \sum_{i1=1}^{IT1} (ZA)^j_{i1} * (X)^j_{i1} \tag{4-14}$$

$$(XA)^{j-1} = \sum_{i2=1}^{IT2} (XA)^{j-1}_{i2} * (X)^{j-1}_{i2} \tag{4-15}$$

$$(YA)^{j-1} = \sum_{i2=1}^{IT2} (YA)^{j-1}_{i2} * (X)^{j-1}_{i2} \tag{4-16}$$

$$(ZA)^{j-1} = \sum_{i2=1}^{IT2} (ZA)^{j-1}_{i2} * (X)^{j-1}_{i2} \tag{4-17}$$

where

j is the top storey

$i1 = 1,2,\ldots\ldots.,IT1$= Number of grouting conditions for storey j

$i2 = 1,2,\ldots\ldots.,IT2$= Number of grouting options for storey $j-1$

The number of zero-one variables for the top storey is IT1 while the number of zero-one variables required for the subsequent storey is

$IT2 = (IT1)^2 + IT1$. The reason for this large increase in the number of zero-one variables lies in the combination $\frac{(P_d)_{i2}^{j-1} + (P_d)_{i1}^{j}}{(A_m)_{i2}^{j-1} * L}$ which is represented by $(XA)_{i2}^{j-1}$, and which upon further simplification amounts to the following mathematical expression:

$$(XA)_{i2}^{j-1} = \frac{(P_d)_{i2}^{j-1}}{(A_m)_{i2}^{j-1} * L} + \frac{(P_d)_{i1}^{j}}{(A_m)_{i2}^{j-1} * L} \tag{4-18}$$

The first term in the right-hand side of Equation 4-18 represents a combination of $(P_d)_{i2}^{j-1}$ and $(A_m)_{i2}^{j-1}$ for storey j-1 and produces a number of discrete values equal to IT2 = IT1. The second term in the same equation represents the combination of $(P_d)_{i1}^{j}$, that is, the axial dead load corresponding to grouting condition i1 in the j^{th} storey, and $(A_m)_{i2}^{j-1}$, the mortar bedded area corresponding to grouting condition i2 for $(j-1)^{th}$ storey. This combination is diagrammatically, presented in Figure 4-1.

As can be seen, this particular combination results in an excessive number of the discrete variables derived from the increased number of stories in a building.

Not only does the size of the problem, as related to the number of discrete variables and consequently zero-one variables, require reformulating the relationship employed in the model, but the computation process that precedes solving the model also requires more modifications. This complication in the computation process is more noticeable when the T.S.A. method is required to determine the steel area for walls subject

J Number of Stories

IT1 Number of Grouting Conditions for Jth Storey

IT2 Number of Grouting Conditions for J-1th Storey

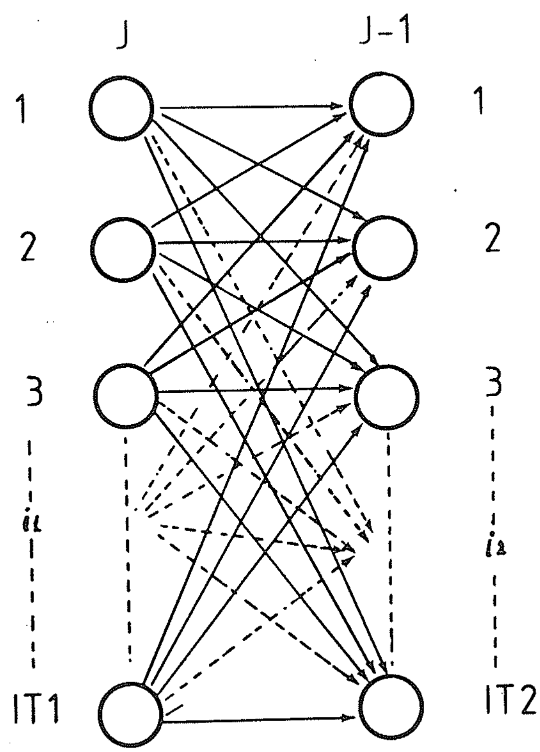

Figure 4-1. The possible grouting conditions for two subsequent stories in a high-rise masonry shear wall.

to tension violation. For a limited number of combinations, as in the case of single storey problems, the resulting third order equation in the unknown ϕ, as discussed in the previous chapter, can be solved outside the computer program used to generate the optimization model itself. The values of the unknowns yielded from the solution (ϕ and subsequently f_{ma}, f'_{sa} and f_{sa}) are then incorporated into the corresponding relationships that enter the model. With the resulting excessive number of combinations, the process of solving this cubic equation becomes time consuming and costly, contradicting the objective of optimization.

The above two reasons are the most obvious and important that require development of a "simulated" model that is closely representative of the original model. A summary of the "needs" for simulating functions is as follows:

1. The need to include all stress and load combinations and limitations resulting from applying different load cases in a single model.
2. The need to reduce the size of the model as much as possible so that it can be handled by any readily available solving technique (or package).
3. The need to express the axial compressive dead load, P_d, which is the only variable representing the connection between subsequent stories in a perfect linear discrete form and without association with any other discrete variables. This particular point can be viewed as a requirement for what can be called an "acceptable discrete form" for the relationships in the model developed for the optimum design of multistory shear walls.

4.3 THE SIMULATION PROCESS

As noted previously, all possible stress and load conditions resulting from applying different load cases must be considered in a single model. Two distinct load cases are considered in this study, namely, (Dead load + Wind load) and (Dead load + Live load). The U.E. (Unity Equation) is used to define stress conditions resulting from the (DL + WL) case, while the C.M. (Coefficient Method) is used to describe the axial compressive resisting capacity of the wall due to (DL + LL) case. While simple mathematical manipulation is sufficient to modify some of the resulting relationships, simulated functions are necessary to substitute others. Since most of the simulated functions are for (DL + WL) case, due to the complexity in expressing their original functions in "acceptable discrete forms", only this case is investigated herein. The process of simulation is carried out as indicated in Figure 4-2.

4.3.1 Defining the Original Problem

To define the original problem, the U.E. is used to identify the stress possibilities for each assumed grouting condition. Therefore, if seven grouting conditions, namely, ungrouted, every sixth, fifth, fourth, third and second cores are grouted and, finally, fully grouted, conditions, are assumed, general interaction diagrams can be presented as in Figures 4-3 through 4-9.

Figures 4-3 through 4-9 present P_d/M limitations for possible stress situations using different assumed grouting conditions. Focussing

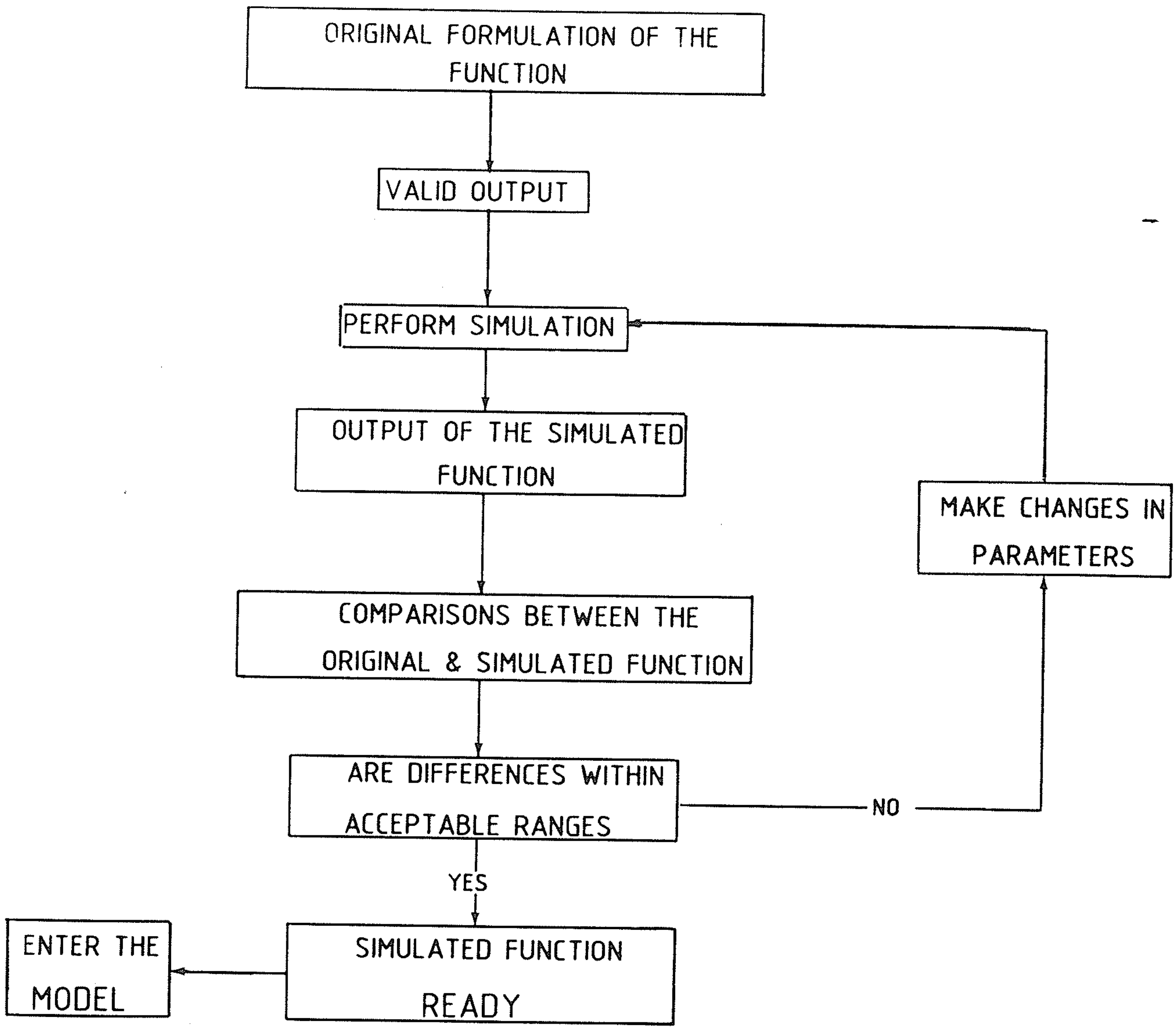

Figure 4-2 Flow diagram illustrating the simulation process.

A-NORMAL STRESS DISTRIBUTION ZONE
B-COMPRESSION VIOLATION ZONE
C-COMP. AND TENSION VIOLATION (COMP. GOVERNS)
D-COMP. AND TENSION VIOLATION (TEN. GOVERNS)
E-TENSION VIOLATION ZONE

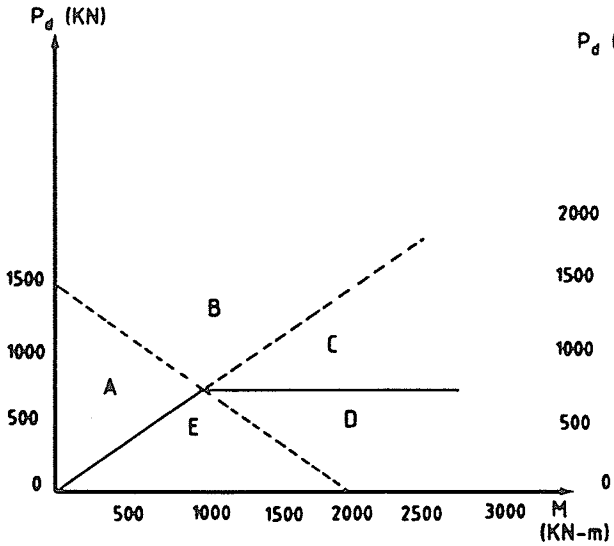

Figure 4-3 P_d/M limitations for possible stress situations for ungrouted wall

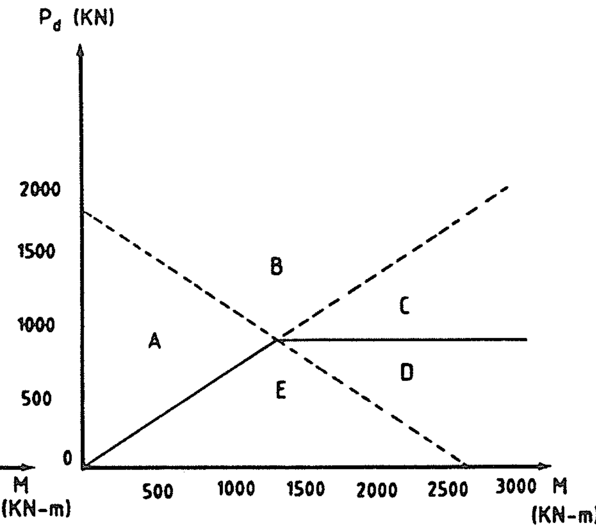

Figure 4-4 P_d/M limitations for possible stress situations for a wall with every 6th core grouted

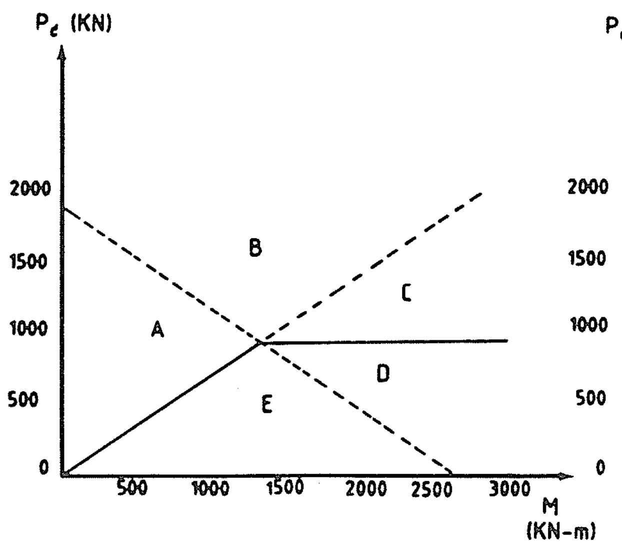

Figure 4-5 P_d/M limitations for possible stress situations for a wall with every 5th core grouted

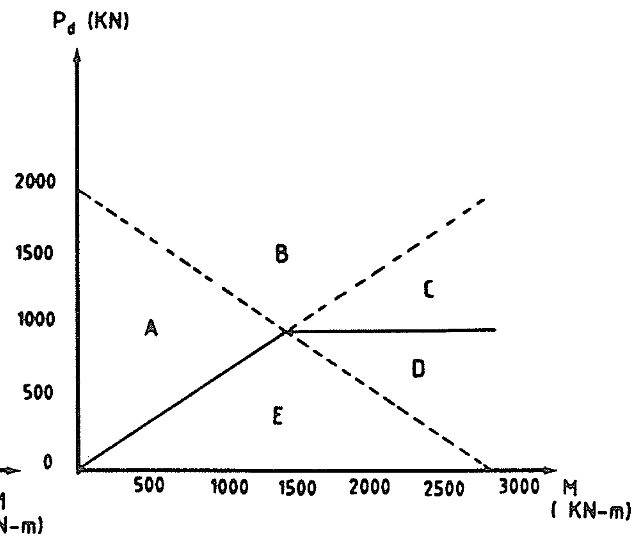

Figure 4-6 P_d/M limitations for possible stress situations for a wall with every 4th core grouted

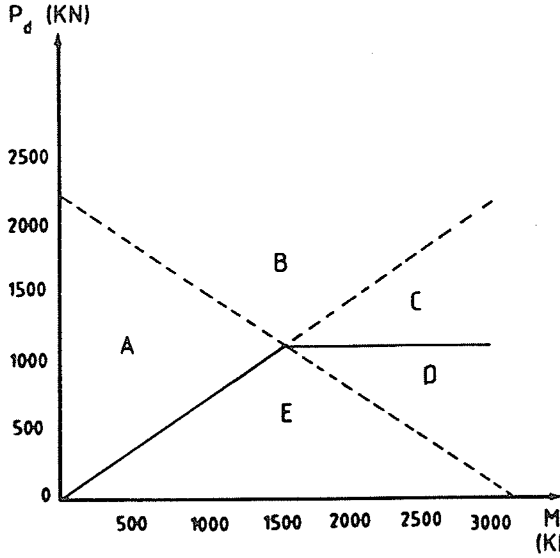

Figure 4-7 P_d/M limitations for possible stress situations for a wall with every 3rd core grouted

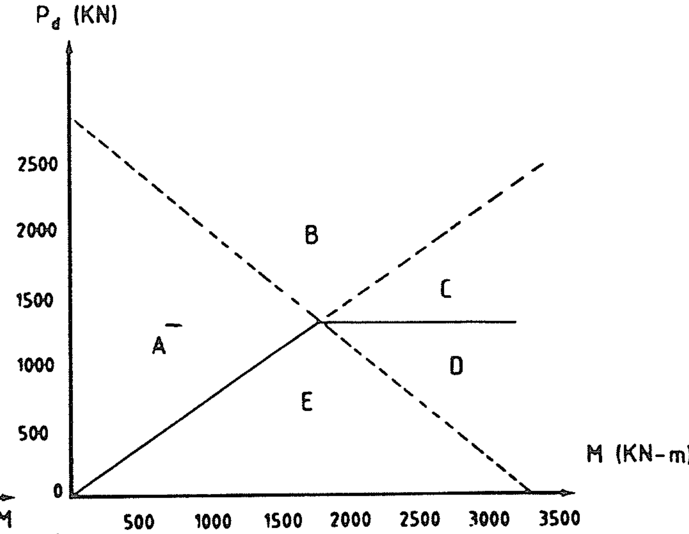

Figure 4-8 P_d/M limitations for possible stress situations for a wall with every 2nd core grouted

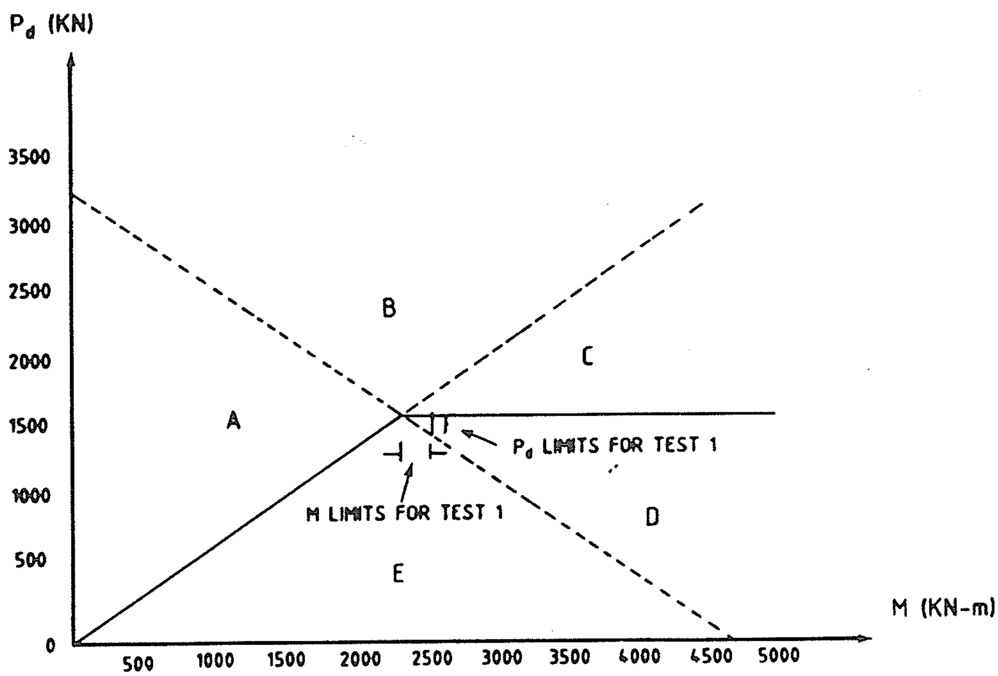

Figure 4-9 P_d/M limitations for possible stress situation for a fully grouted wall on each zone, five regions can be recognized in each of these figures. In region A, any P_d/M combination will yield a normal distribution, as discussed above. In region B, any P_d/M combination will yield compression violation on the compression side while tension is not violated on the tension side. Any P_d/M combination in region C will yield both compression and tension violation, with compression governing the design. In region D, any P_d/M combination will yield stress violation on both sides. However, since the tension violation is more severe, tension governs the design. Finally, in region E, any combination yields tension violation while the compression stress on the compression side is within the allowable limits.

Figures 4-3 through 4-9 define clearly the boundaries at each zone for each grouting condition. These boundaries, as will be seen later, are essential in developing the simulated functions.

For zones A, B and C, only simple mathematical manipulation, as will be discussed in Chapter 5, is necessary to convert the design relationships into their "acceptable discrete form". For zones, D and E, where tension governs the design and where the original design relationships, using T.S.A. method, cannot be described in "acceptable discrete form", simulated functions are developed to substitute such original functions. However, for the purpose of illustration, the simulated functions in zone D are studied in detail in this chapter, while Chapter 5 briefly discusses the development of the simulated functions for zone E, which essentially follows the same process.

4.3.2 <u>The Simulated Functions</u>

Zone D is characterized by the following stress relationships:

$$\frac{P_d}{A_m * L} + \frac{M}{SM} > C_s * f_m \tag{4-19}$$

$$\frac{M}{SM} > \frac{P_d}{A_m * L} \tag{4-20}$$

and

$$\left(\frac{M}{SM} - \frac{P_d}{A_m * L}\right) > \left(\frac{P_d}{A_m * L} + \frac{M}{SM} - C_s * f_m\right) \tag{4-21}$$

where all variables are as defined before.

For any assumed grouting condition, whenever the P_d/M combination satisfies the above three constraints, tension governs the design. The Transformed Section Analysis (T.S.A.) method is then applied to determine the required amount of steel and the actual stresses in the masonry, and in compressive and tensile steel. In solving the T.S.A. relationships it is evident that the masonry is fully utilized whenever the minimum amount of reinforcement is selected. Note that the compressive steel is the same as the tensile steel. Full utilization of masonry means that its actual compressive stress is approaching its allowable compressive stress. This important fact suggests the following assumption: Whenever Relationships 4-19, 4-20 and 4-21 are satisfied, the actual masonry compressive stress is to be taken as the allowable masonry compressive stress, $C_s * f_m$, and the depth to the neutral axis is taken from the original stress distribution resulting from applying the Unity Equation (U.E.), as shown in Chapter 5. The above assumption results in the following relationships :

$$f_{ms} = C_s * f_m \tag{4-22}$$

$$(\phi_s * D) = L * \frac{\frac{P_d}{A_m * L} \quad \frac{M}{SM}}{2 * \frac{M}{SM}} \tag{4-23}$$

$$f'_{ss} = n * C \ * f_m * \frac{(\phi_s * D) - D'}{\phi_s * D} \tag{4-24}$$

and

$$f_{ss} = n * C_s * f_m * \frac{L - (\phi * D) - D'}{\phi_s * D} \tag{4-25}$$

where the subscript s, in for example the f_{ms} term, refers to the simulated value of the variable, and other variables are as defined before.

In developing the simulated function that describes the relationship between the axial applied load and compressive resisting capacity, the following relationship is used:

$$P_d \leq P_r \tag{4-26}$$

where P_d is the actual applied compressive load (N)

and P_r is the compressive resisting capacity (N)

Using the assumed variables in Relationships 4-22 through 4-25, the following mathematical expressions can be used to define P_r:

$$P_r = CM + CS - T \tag{4-27}$$

where CM = Masonry compressive resisting capacity (N)

CS = Compressive steel resisting capacity (N)

T = Tensile steel resisting capacity (N)

where

$$CM = 0.5 * C_s * f_m * \frac{A_m}{1000} \left\{ L \left[\frac{\frac{P_d}{A_m * L} + \frac{6000\ M}{A_m * L^2}}{2(\frac{6000\ M}{A_m * L^2})} \right] \right\} \tag{4-28}$$

$$CS = A_{so} * n * C_s * f_m \left\{ \frac{L(\frac{P_d}{A_m * L} + \frac{6000\ M}{A_m * L^2}) - 2D' \frac{6000\ M}{A_m * L^2}}{L\ (\frac{P_d}{A_m * L} + \frac{6000\ M}{A_m * L^2})} \right\} \tag{4-29}$$

and

$$T = A_{so} * n * C_s * f_m \left\{ \frac{L(\frac{P_d}{A_m * L} - \frac{6000\ M}{A_m * L^2}) - 2D' \frac{6000\ M}{A_m * L^2}}{L\ (\frac{P_d}{A_m * L} + \frac{6000\ M}{A_m * L^2})} \right\} \tag{4-30}$$

It should be noted that the equivalent thickness of the wall, t, is replaced by $A_m/1000$ while the section modulus, SM, is replaced by $A_m * L^2/6000$ (for rectangular sections). Substituting Equations 4-28, 4-29 and 4-30 into Relationship 4-26, results in the following relationship:

$$P_d \left(\frac{24\ M}{L} - \frac{C_s * f_m * A_m * L}{1000} \right) \leq M \left(\frac{12\ C_s * f_m * A_m}{1000} \right) +$$

$$\left(\frac{M}{P_d * L} \right) \left(\frac{36\ C_s * f_m * A_m * M}{1000} \right) + \left(\frac{48n * A_{so} * C_s * f_m * M}{L} \right) - \left(\frac{144\ M^2}{L^2} \right) \tag{4-31}$$

There are two problems associated with Relationship 4-31. The first is that the term $M/(P_d * L)$ cannot be expressed in an "acceptable discrete form", particularly when it is associated with other discrete combinations, e.g., $36*C_s*f_m*A_m*M/1000$. The second problem is noticed in the term $(48*n*A_{so}*C_s*f_m*M/L)$. This term includes two different types of discrete combinations, $(n*f_m)$ and (A_{so}). The first combination corresponds to the grouting conditions while the second corresponds to steel options. This second problem can be solved exactly by considering all steel options with each grouting condition separately. However, this exact solution requires an excessive number of discrete values and accordingly an excessive number of zero-one variables. Therefore, an approximate solution, as discussed later, is used instead of the exact solution.

To solve the first problem, an important assumption is made; that is, Relationship 4-31 is developed to be applied to each storey whenever the conditions in Relations 4-19, 4-20 and 4-21 are met. At any storey, the in-plane bending moment, M, is considered constant while the axial compressive load, P_d, varies according to levels of grouting in the stories above and according to the grouting options at this particular storey. Based on this assumption, the relationship between (M/P_d*L) and P_d at each storey can be analyzed in an attempt to replace (M/P_d*L) by a linear term in P_d. In developing such a relationship, a wide range of reasonable values of P_d and M are used in an extensive analysis to study the effects of variation in P_d on (M/P_d*L) and vice versa. Figure 4-10 shows graphically the general resulting functions. In the figure, a_1, a_2, a_3 and a_4 represent schematically the ranges over which the axial load varies for four consecutive floors. The values of the in-plane bending moment, M, on the other hand are constant for each level. However, these values of M are increasing from one level to the next.

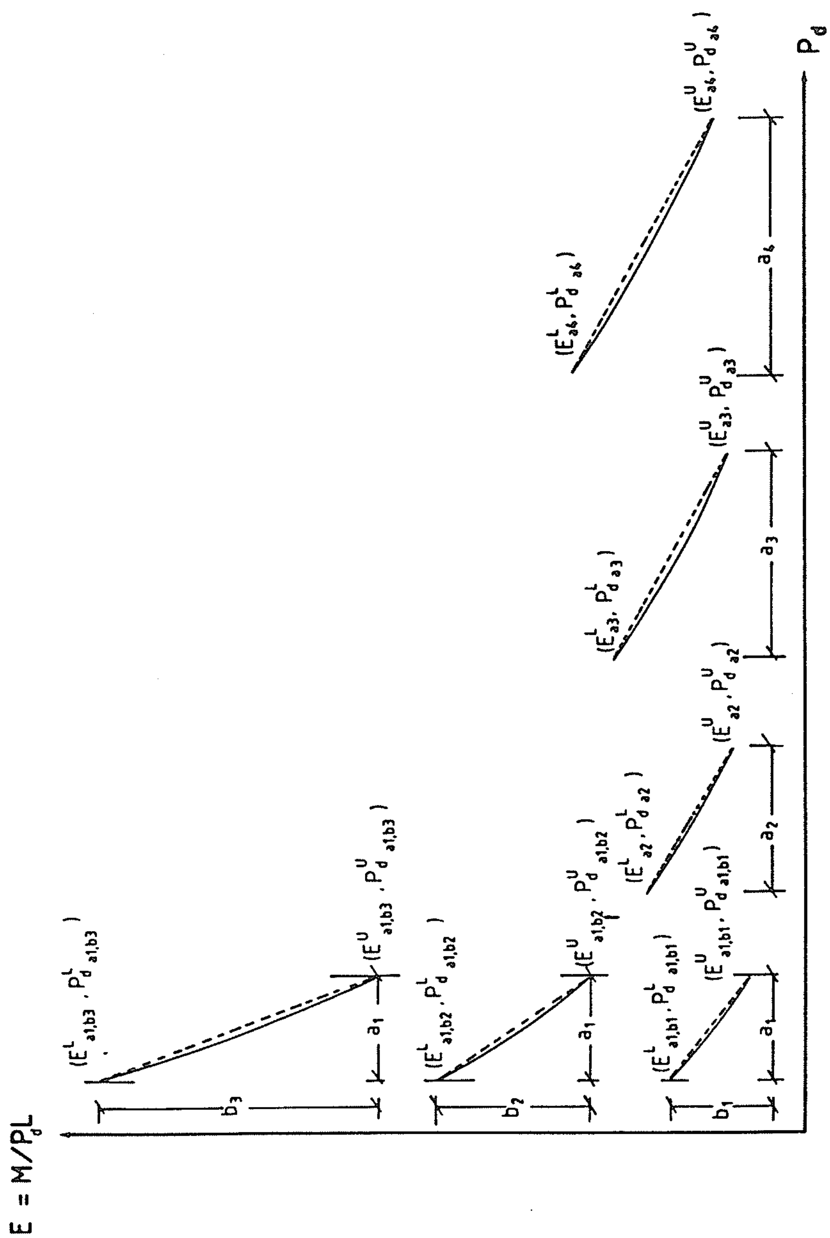

Figure 4-10 The general relationship between E = $M/(P_d{*}L)$ and P_d

Also, in the figure, b_1, b_2, and b_3 represent three different values of M, (increasing from bottom to top of the figure), applied to the same storey at which P_d varies in the range al. As mentioned before, because of the infinite number of cases that could be presented, Figure 4-10 shows the general shape of the function $f(E)$ or $f(M/P_d*L)$, rather than the specific function. However, the intent of this general illustration is to show the small curvature representing these functions, even with severe cases. This small curvature suggests linearizing these functions by considering the end points as grid points. The end points at these functions represent the lower and upper bounds of the applied axial load, P_d. The lower bound represents the case of ungrouted sections for the level under consideration as well as all of the levels above. The upper bound, on the other hand, represents the case of fully grouted sections for the level under consideration and all levels above. These two end points can be represented by (E^L, P_d^L) and (E^U, P_d^U), where $E = M/(P_d*L)$ = the eccentricity ratio. The linearized relationship between E and P_d, can then be expressed mathematically as follows:

$$E = (E)^L - (b/a)*(P_d)^L + (b/a)*(P_d) \qquad (4\text{-}32)$$

where $a = (P_d)^L - (P_d)^U$

and $b = (E)^L - (E)^U$

and all other variables as defined before.

Substituting Equation 4-32 in Relationship 4-31 results in the following mathematical expression:

$$P_d \leq \left\{ \frac{ZA * A_m * M\ (.012+.036(E)^L-.036(P_d)^L *(b/a))}{(24M/L)-\ .001\ ZA * A_m\ (L-\ 36M\ (b/a))} \right\}$$

$$+ \left\{ \frac{48 * n * A_{so} * ZA * M}{(24M/L)-\ .001\ ZA * A_m\ (L-\ 36M\ (b/a))} \right\}$$

$$- \left\{ \frac{144\ (M^2\ /L^2)}{(24M/L)-\ .001\ ZA * A_m\ (L-\ 36M\ (b/a))} \right\} \qquad (4\text{-}33)$$

where $ZA = C_s * f_m$.

To solve the second problem mentioned earlier, namely the association of A_{so} with the combination (n*ZA), a sensitivity analysis is performed to study the changes in the function containing the two combinations, that is f(n ,ZA) and A_{so}, due to changing both combinations. From this analysis it is concluded that, compared to its sensitivity towards changes in the steel options, the sensitivity of the function towards change in the grouting condition is negligible. Accordingly, it is assumed that only values of n and ZA that correspond to fully grouted sections are used in all cases of steel options but only in the term shown above, i.e., $[48*n*A_{so}*ZA*M]/[(48M/L)-0.001\ ZA*A_m(L-36M(b/a))]$. The reason for selecting the values of the fully grouted condition is that they yield, relatively, and in most cases, the lowest values of the function which in turn decreases the value of the resisting capacity P_r. This decrease in P_r, or the right hand side of Relationship 4-33, although insignificant, is considered a conservative assumption.

Relationship 4-33 is further rearranged, as will be discussed in

Chapter 5, so that it can be expressed in an "acceptable discrete form" that can be included in the discrete optimization model.

4.3.3 Comparison Between the Simulated and the Original Function

Having developed the simulated function, Relationship 4-33, does not necessarily mean that it is representative of the original function. The simulated variables, and output of the simulated functions must be compared with the original results to ensure reliability of the function. Before performing any tests, it should be recalled that in multistorey problems the in-plane bending moment is assumed to be constant at each level while the applied axial load is varying. Therefore, in performing these tests an assumed value of M along with different assumed values of P_d are considered for each set of tests. For purposes of illustration, one test is performed below for a particular grouting condition in a particular storey. Figures 4-3 through 4-9 are used to select the assumed values of P_d and M and define the limitations of both variables bounded within zone D. The tests are first performed on the assumed variables (e.g. f_{ms}, f_{ss}' and f_{ss}, ..., etc.), and then on the resisting capacity, P_r, resulting from the simulated Function 4-33.

The first set of tests is for a fully grouted wall. Referring to Figure 4-9, the assumed values of M and P_d are within the range shown.

Figures 4-11 through 4-13 illustrate graphically the differences between the variables f_{ms}, ϕ_s and f_{ss}' as calculated from Equations 4-22 through 4-24 respectively and the actual values, i.e., f_{ma}, ϕ and f_{sa}', as calculated from the T.S.A. method.

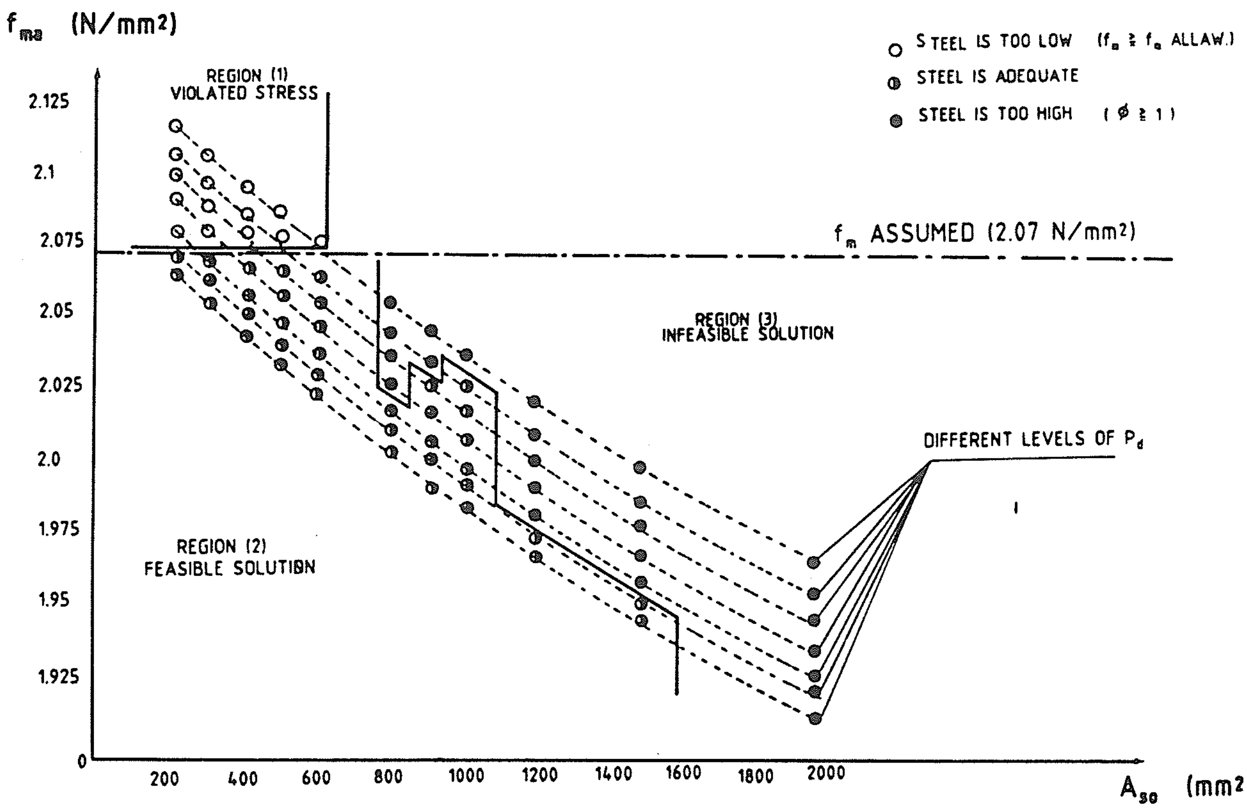

Figure 4-11 The relationship between A_{so} and both of the actual and assumed masonry compressive stress, f_{ma} and f_{ms} for assumed values of M and P_d

In Figure 4-11, the relationship between A_{so} and both of the actual and assumed masonry compressive stresses, f_{ma} and f_{ms}, for a particular assumed value of the in-plane bending, M, and different compressive loads, P_d, is shown. Three regions can be recognized on the graph. In the first region, the area of steel is not sufficient to yield an actual masonry compressive stress that is lower than or equal to its allowable limit. As a result the actual masonry compressive stress exceeds this limit, which means that a larger amount of steel is needed. In the third region, the area of steel is more than enough, which results in placing the whole section of the wall under compression. As a result of this stress redistribution, the value of (ϕ*D), the depth of the neutral axis, is found to exceed D, which in turn results in infeasible values (negative values) of f_{ma}, f'_{sa} and f_{sa}. In the second region, the steel area is adequate to resist any stress violation that could exist in the wall. The resulting values of f_{ma}, due to applying the T.S.A. method are feasible and less than the allowable values. As can be noticed from the figure, for any level of P_d, the minimum amount of steel that is adequate for the section yields an actual masonry compressive stress, f_{ma}, that approaches the allowable value of f_m. Therefore, by being close to these minimum values of steel, the assumed values of f_m, f_{ms}, which equal the allowable value as described by relationship 4-22, actually contribute to the optimization process.

Figures 4-12 and 4-13, respectively, show the comparisons between ϕ and ϕ_s, and f'_{sa} and f'_{ss}, with respect to A_{so} and for different values of D', the distance from the centroid of the steel area to the nearest end of the wall. The discrete values of ϕ and f'_{sa} show an agreement with the

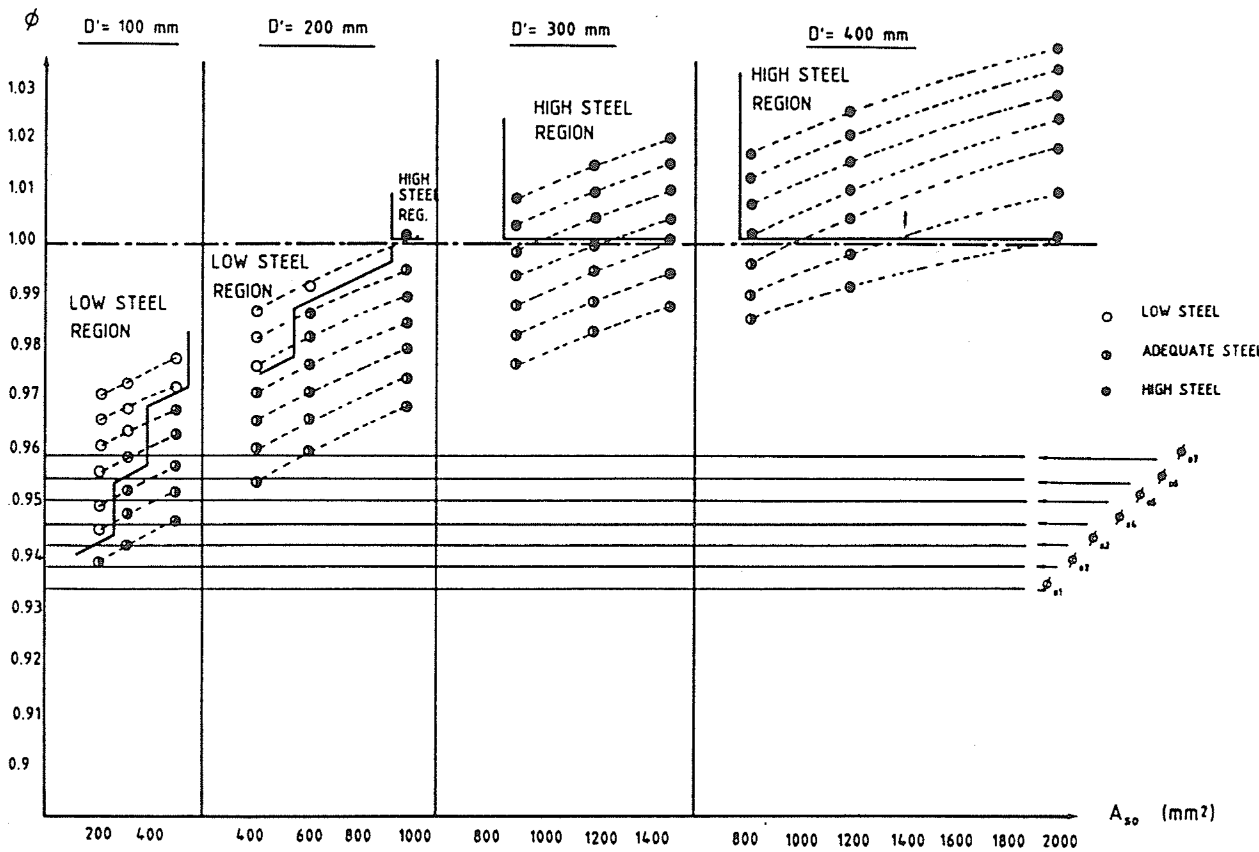

Figure 4-12 Relationship between A_{so} and both of the actual and assumed values of ϕ, for different values of D' and assumed values of M and f_d

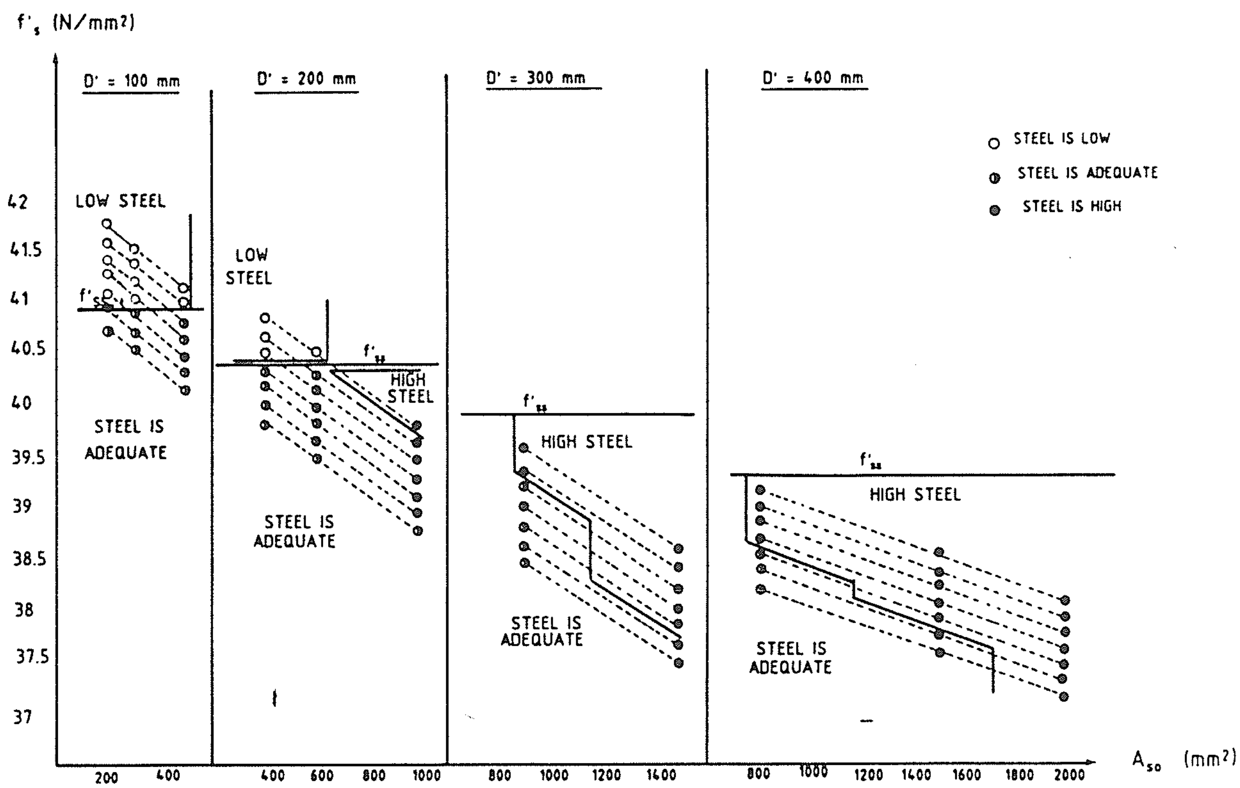

Figure 4-13 Relationship betwen A_{so} and both of the actual and assumed values of steel compressive stress for f'_{sa} and f'_{ss} for assumed values of M and f_d

discrete values of f_{ma}, in Figure 4-11, with respect to the three regions defined before. They are also in agreement with the fact that the minimum adequate steel area that yields a feasible solution is close to the assumed values of either ϕ_s or f'_{ss}. However, as can be seen from Figures 4-11 and 4-13, the values of f_{ma} and f'_{sa}, as well as f_{sa} which is not shown, are slightly lower than the assumed values, f_{ms} and f'_{ss}. On the other hand, as shown in Figure 4-12, the actual values of ϕ are slightly higher than the assumed values, ϕ_s. To study the effect of changes in these variables on the resulting assumed resisting capacity, P_r, and the actual resisting capacity (taken the same as P_d), the relationship between the two values, P_r and P_d, in terms of different discrete values of A_{so} and using both actual and assumed parameters was examined (the second set of tests).

Figure 4-14 shows this relationship graphically. In this figure, seven levels of compressive loads are shown, along with the corresponding seven levels of the resisting capacity P_r. The first level of P_d, which is in fact the actual resisting capacity, lies entirely under the assumed value of P_r, for all values of steel. Notice in the figure that the lower the steel amount is, the narrower the difference between the actual and assumed values (P_d and P_r respectively). The second horizontal line, corresponding to the actual resisting capacity of the second level of P_d, intersects the line corresponding to the assumed value of P_r, for the same applied load, at point θ. Point θ, as shown, lies on the right hand side of 200 mm^2 steel area. This intersection at point θ yields a value of P_r at 200 mm^2 steel area that is lower than the actual value of P_d which violates Constraint 4-26. As a result of this violation,

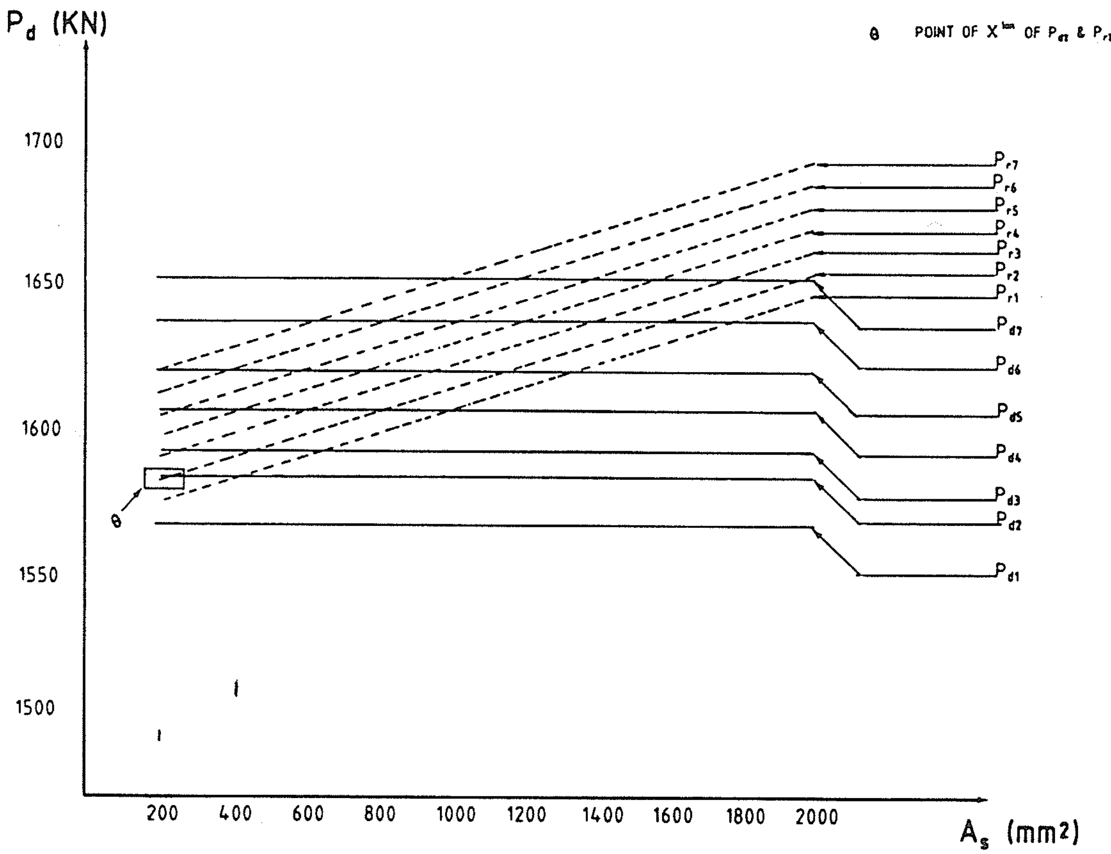

Figure 4-14 Relationship between A_{so} and both of the actual and simulated values of the axial resisting capacity, P_d and P_r for assumed values of grouting conditions

Constraint 4-26 does not recognize the 200 mm^2 steel value and proceeds to the next steel discrete value which is 300 mm^2. At 300 mm^2 steel area value, the actual resisting capacity lies below the assumed value which satisfies Constraint 4-26. Also, all subsequent values of steel yield assumed resisting capacities that are higher than the actual values. Referring to Figures 4-11 through 4-13, the first actual discrete values of f_{ma}, ϕ and f'_{sa}, due to 200 mm^2 steel area, lie in the insufficient steel area region, region 1, while the rest of the discrete values (except the last value corresponding to 2000 mm^2) are within the feasible region, region 2. Therefore an assured agreement of validity for the assumed values, f_{ms}, ϕ and f'_{ss}, and the resulting resisting capacity, P_r, can be concluded.

For further interpretation of the closeness of assumed values to actual values, reconsider point θ, in Figure 4-14, at which the steel area is insufficient. At this point, all calculated actual variables, from application of the T.S.A., are higher than the assumed corresponding values. If the value of f_{ma}, the only value that violates the allowable requirement, is reduced to f_{ms}, the values of other variables, e.g., f'_{sa}, are reduced subsequently, which in turn results in reducing the assumed value of P_r. Proceeding with the increase in the applied axial load P_d, it appears that there is always an agreement regarding the choice of the minimum amount of steel that satisfies Relationship 4-26, between the actual and the assumed values of the resisting capacity till P_d reached P_{d5}. At this level, and as indicated on Figures 4-11 through 4-13, the first three discrete values of steel are insufficient for resisting the applied loads. Figure 4-14, on the other hand, shows that the fourth value of steel also yields an actual resisting capacity that exceeds the assumed one. Therefore, to satisfy Relationship 4-26, the fifth value of steel is selected rather than the fourth value, which would actually be more than sufficient to resist the applied loads. This underestimate of the assumed resisting capacity comes as a result of higher differences between the assumed value ϕ_s and the actual value ϕ at this higher level of loading, and is clearly shown in Figure 4-12. This underestimate of P_r results, however, in selecting a higher level of steel, which is a relatively uneconomic but safe solution.

Although the high steel regions in Figures 4-11 through contain unrealistic values of f_{ma}, ϕ and f'_{sa}, respectively, the corresponding steel areas can still be applied. Although they represent uneconomic solutions, these high areas of steel do not impair the safety of the wall. Therefore, Figure 4-14 can be applied for such high levels of steel where the assumed resisting capacities exceed the actual resisting capacities.

4.4 SUMMARY AND CONCLUSIONS

Simulated functions are developed to replace the original design relationships. These functions are developed because of the difficulty of expressing the original design relationships in "acceptable discrete forms". The T.S.A. method is the basis for the original design relationships where different steel areas are tried until the actual resisting capacity becomes equal to or greater than the applied loads and, at the same time, the actual masonry and steel stresses are within their allowable limits. The simulated function is based on the reverse analogy, i.e., masonry and steel stresses are assumed first and with the trial of different steel areas, a resisting capacity, P_r, is determined which must exceed the actual (original) resisting capacity (or the applied load, P_d). The assumed variables are based on extensive and rational analysis which is evident in the comparisons between their assumed and actual values. Not only are these assumed variables close to their corresponding actual values, but they also contribute to the optimization process by being close to the minimum values of steel areas that are required to resist the applied loads. For the range of reinforcement examined, the assumed resisting capacities, P_r, are found to be in close agreement with the actual values.

CHAPTER V

PSEUDO-DISCRETE MODELLING FOR THE OPTIMUM DESIGN OF MULTISTOREY MASONRY SHEAR WALLS

5.1 INTRODUCTION

In Chapter 3, a model was developed for the optimum design of single, one storey masonry shear walls. That model was basically the result of a computational process that tested the stress distribution for each grouting condition at each end of the wall, determined the governing stress and provided the necessary relationships for the model. The computational process was performed using a computer program that was capable of testing a limited number of cases, i.e., the grouting conditions, with great efficiency in a very short time. The main constraints used in the model were derived from and represent the Unity Equation (U.E.), Transformed Section Analysis (T.S.A.), and the Coefficient Methods (C.M.), in the manner described in Chapter 3.

As the design proceeds to include more stories, however, the problem becomes more difficult, if not impossible, to handle in the same manner as that for a single storey. Although briefly discussed in Chapter 4, this difficulty is re-examined in detail before developing the model for the multistorey optimization problem.

A number of simplifying assumptions are made to enable solving the problem. Some of these assumptions include simulating expressions to substitute the original functions, as discussed in Chapter 4. General simplifying assumptions are discussed in the next section, while specific simplifying assumptions are introduced throughout the development of the model.

The model itself consists of a linear objective function and three different sets of linear constraints, the first set describing the different stress and loading limitations, the second set defining the different discrete values of the different variables used and the final set consisting of constraints organizing the relationship among zero-one variables. The first set of constraints are structural constraints developed for this model and represent a suggested simplifying method for designing such walls. The constraints in this set are in the form of inequalities and, as such, they create the initial (continuous) feasible region of solution. The second set of constraints consists of two groups of relationships. The first includes those equations that either define the discrete values of the variables included in the first set of constraints, or define those structural variables given as input to the model which are essential to complete the solution. The second group includes those constraints that describe the relationships among the so called "Transfer Variables". This second set of constraints is embedded within the continuous feasible region. As noted earlier, the final set of constraints includes the equations that define the relationships among zero-one variables. Although the technique used in modelling the problem is the same as that for single storey shear walls, i.e., pseudo-discrete modelling with the use of zero-one variables, the zero-one constraints, as will be discussed later, are constructed differently than in the previous model.

A computer program (flow chart is given in Appendix D) is developed to formulate the model. Once the model is complete, the solution process proceeds using the solving package LINDO[24] through its integer capability. The model is flexible in terms of accommodating any number of stories, grouting conditions and steel options. However, there are a few limitations regarding the size of the problem and the computation time that may require the restriction of the numbers of options within reasonable limits.

The following sections include:

5.2 THE GENERAL NATURE OF THE PROBLEM

The problem can be schematically represented as a multistage problem as shown in Figure 5-1.

The columns in Figure 5-1 represent the stories while the rows represent the grouting conditions. The figure illustrates the possible paths for any particular grouting condition from one storey to the next-knowing that the level of grouting increases downward (the lowest and highest levels are 1 and IT, respectively) while j represents the top floor and 1 the ground floor. Figure 5-1 shows that a particular grouting level in a top floor moves only to the same or higher grouting

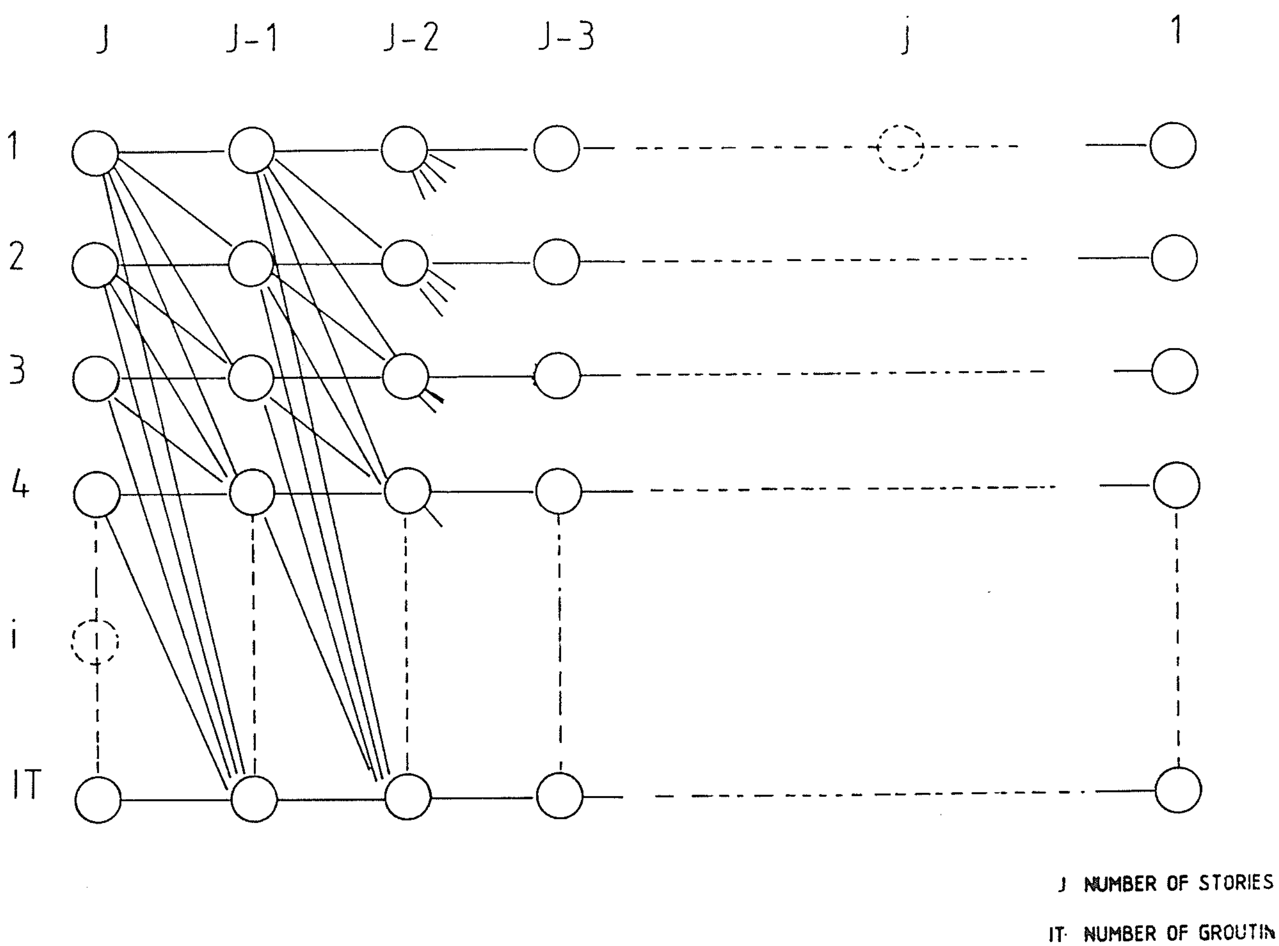

Figure 5-1 The possible routes of grouted conditions for J stories

level in a subsequent lower floor. Not only is this assumption structurally logical, but it also reduces the number of combinations to be checked. The total number of combinations, according to Figure 5-1, is too large to be tested and accommodated in one model. If the formulation adapted in Chapter 3 is employed for this problem, the number of constraints as well as the number of variables will be beyond the capacity of any existing solving package. Not only will the size of the problem increase drastically, but also the computational process preceding developing the model will require an immense number of iterations and consequently time, which contradicts the objective of optimization. The difficulties summarized above require simplifying the problem as well as developing another approach for modelling.

The following are general assumptions which help in reducing the size of the problem.

The first general assumption is based on the fact that a stiff diaphragm causes all shear walls to deflect almost the same amount[44]. Therefore, horizontal forces are distributed according to the relative stiffnesses of the walls.

Secondly, the wall is designed for the following two loading conditions:

(a) Dead load + Wind load (DL + WL)

(b) Dead load + Live load (DL + LL)

The shear load case is eliminated in this model for the following reasons:

1. As is evident from the single storey shear wall problem, the

allowable shear stress, with reasonable lateral reinforcement, is rarely violated, even with high shear forces.

2. In view of the first reason, the addition of more constraints increases the size of the problem, and, for purposes of accuracy of results and computational time, is not recommended.

However, it is assumed that joint reinforcement (4.1 mm diameter), is placed in each course.

Thirdly, the model considers one configuration of walls, namely a straight wall with no openings.

Finally, only the cost of material is considered for optimization, which means that the final decision as to the practicality of results is left to the practicing engineer. However, in developing the model, a parallel effort is made to enforce the practicality of the model in the same way as the theoretical basis is maintained.

Although the optimization model developed for this problem is based on the same concept adapted for the single storey optimization problem, i.e., pseudo-discrete modeling, the model itself is developed in a different pattern because of the large size of the problem. This pattern is described in the following sections.

5.3. DEVELOPMENT OF THE MODEL

The model developed for the optimum design of multi-storey shear walls consists of a linear objective function and three different sets of constraints.

The following sections describe the development of each type of constraint and the objective function. An illustrative example then describes the application of the model. Finally, an important part of the modelling, namely discussing problems associated with such large models and making suggestions for overcoming some of them, is presented.

5.3.1 THE CONTINUOUS CONSTRAINTS

Two subsets of constraints are included in this set, the first includes those relationships that describe stresses and loading limitations while the second subset includes the constraints that organize the choices of particular variables, e.g., reinforcement. These two subsets are developed as follows:

5.3.1a Stresses and Loading Limitation Constraints

Two load cases are considered in this study, namely, Dead load + Live load, (DL + LL), and Dead load + Wind load, (DL + WL). As was the case for a single storey shear wall, the (DL + WL) constraints are developed first followed by the constraints describing (DL + LL) case.

(DL + WL) Case Relationships. Again, the U.E. and T.S.A. methods are used as the basis for developing the relationships for this case.

The problem of not having an "IF" statement in any mathematical programming model, which in turn makes the "Either-Or" situations a difficult option to be implemented in the model, requires the knowledge of all possible conditions before developing the model. These possible conditions, or situations, in this particular design problem, can be investigated by studying the U.E. The U.E., as mathematically expressed below, is used to check the stress condition at each end of the wall, which leads to defining the possible governing case. It should be noted that subcases one, two and three described in Chapter 3 have been further subdivided in this chapter, as discussed in Chapter 4, to simplify modelling the problem of multistorey shear walls.

$$\frac{P_d}{A_m * L} + \frac{M}{SM} \leq C_s * f_m \qquad \text{for the compression side} \qquad (5\text{-}1)$$

and

$$\frac{P_d}{A_m * L} + \frac{M}{SM} \geq 0.0 \qquad \text{for the tension side} \qquad (5\text{-}2)$$

where P_d = the axial compressive dead load (N)

A_m = the mortar bedded area (mm^2/m)

L = the total length of the wall (m)

M = the in-plane bending moment (N-mm)

SM = the section modulus (mm^3)

C_s = the slenderness coefficient

f_m = the allowable compressive stress of masonry (N/mm^2)

Five different possibilities (subcases) can be recognized from checking the stress situations at each end of the wall, which leads to the development of four different types of constraints that accommodate all these possibilities that may exist in the wall. These five possibilities are as presented in Table 5-1 and shown graphically in Figures 5-2(a) through (e).

It should be noted that the possibilities, or subcases, illustrated in Table 5-1 correspond to the five regions, A through E, discussed in Chapter 4 (Figures 4-3 through 4-9). It should also be noted that subcases 1 and 2 are accommodated in one subset, as is explained below. The four subsets of constraints are developed as follows:

Table 5-1 The five stress possibilities on each side of a shear wall due to the (DL + WL) case

Subcase	Compression End	Tension End	Remarks
1	No compression violation, i.e., $\frac{P_d}{A_m*L} + \frac{M}{SM} \le C_s f_m$	No tension violation, i.e., $\frac{P_d}{A_m*L} - \frac{M}{SM} \ge 0$	
2	Compression violation, i.e., $\frac{P_d}{A_m*L} + \frac{M}{SM} > C_s f_m$	No tension violation, i.e., $\frac{P_d}{A_m*L} - \frac{M}{SM} \ge 0$	
3	Compression violation, i.e., $\frac{P_d}{A_m*L} + \frac{M}{SM} > C_s f_m$	Tension violation, i.e., $\frac{P_d}{A_m*L} - \frac{M}{SM} < 0$	and $(\frac{P_d}{A_m*L} + \frac{M}{SM} - C_s f_m) \ge (\frac{P_d}{A_m*L} - \frac{M}{SM})$
4	Compression violation, i.e., $\frac{P_d}{A_m*L} + \frac{M}{SM} > C_s f_m$	Tension violation, i.e., $\frac{P_d}{A_m*L} - \frac{M}{SM} < 0$	and $(\frac{P_d}{A_m*L} + \frac{M}{SM} - C_s f_m) < (\frac{P_d}{A_m*L} - \frac{M}{SM})$
5	No compression violation, i.e., $\frac{P_d}{A_m*L} + \frac{M}{SM} \le C_s f_m$	Tension violation, i.e., $\frac{P_d}{A_m*L} - \frac{M}{SM} < 0$	

<u>SUBSET 1</u>: This subset of constraints represents two possible stress conditions that have one common limitation, that is, no tension violation in the wall (subcases 1 & 2). In a single storey shear wall, the U.E. is applied to represent this case in the following form:

$$\frac{(P_d)^j}{(A_m)^j*L} + \frac{(M)^j}{(SM)^j} \le C_s * (f_m)^j \tag{5-3}$$

or

$$(XA)^j + (YA)^j \le (ZA)^j \tag{5-4}$$

where

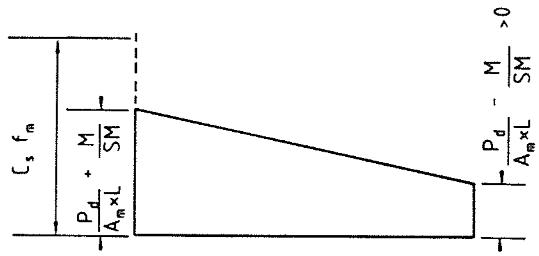

Figure 5-2-a Stress distribution for a wall subjected to (DL+WL) according to U.E. where no stress violation exists in the wall

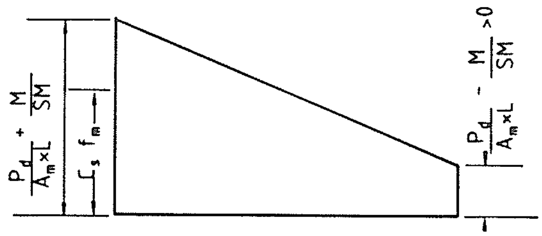

Figure 5-2-b Stress distribution for a wall subjected to (DL+WL) according to U.E. where compression violation exists in the compression end of the wall

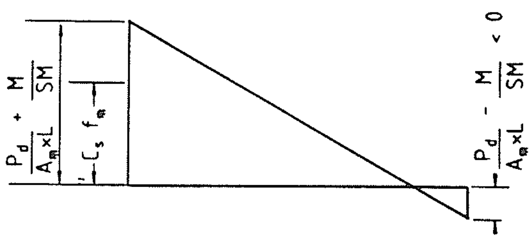

Figure 5-2-c Stress distribution for a wall subjected to (DL+WL) according to U.E. where both compression and tension violation exist and where tension is more

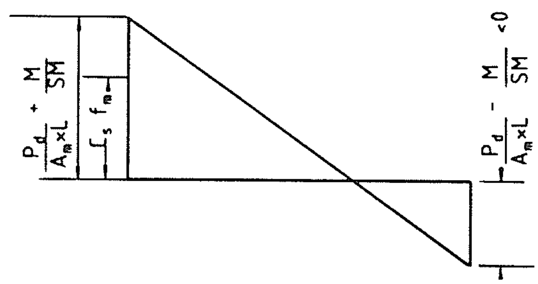

Figure 5-2-d Stress distribution for a wall subjected to (DL+WL) according to U.E. where both compression and tension violation exist and where tension is more

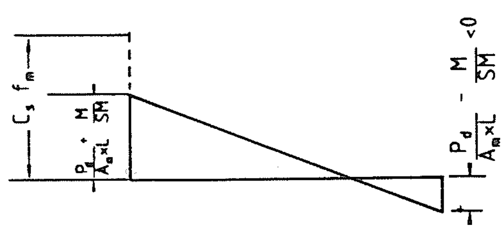

Figure 5-2-e Stress distribution for a wall subjected to (DL+WL) according to U.E. where only tension violation exists in the tension end of the wall

$$(XA)^j = \frac{(P_d)^j}{(A_m)^j * L} = \sum_{i=1}^{IT} \left(\frac{P_d}{A_m * L}\right)_i^j * (X)_i^j \tag{5-5}$$

$$(YA)^j = \frac{(M)^j}{(SM)^j} = \sum_{i=1}^{IT} \left(\frac{M}{SM}\right)_i^j * (X)_i^j \tag{5-6}$$

and

$$(ZA)^j = C_s * (f_m)^j = \sum_{i=1}^{IT} (C_s * f_m)_i^j * (X)_i^j \tag{5-7}$$

where $i = 1,\ldots,IT$ = number of the grouting condition

$j = 1,\ldots,J$ = number of storey

$(X)_i^j$ = zero-one variable associated with i^{th} grouting condition and j^{th} storey and other variables are as defined before.

The problem, so far, seems tolerable, since expressing $(P_d)^j/(A_m)^j{*}L$, $(M)^j/(SM)^j$, and $C_s{*}(f_m)^j$ by discrete values, say $(XA)_i^j$, $(YA)_i^j$ and $(ZA)_i^j$ will result in a limited number of variables (essentially zero-one variables) representing the grouting conditions for a particular j^{th} storey.

Assuming the j^{th} storey to be the top one, then moving downward to the $(j\text{-}1)^{th}$ storey requires writing Relationship 3 as follows:

$$\frac{(P_d)^j + (P_d)^{j-1}}{(A_m)^{j-1} * L} + \frac{(M)^{j-1}}{(SM)^{j-1}} \le C_s * (f_m)^{j-1} \tag{5-8}$$

or

$$\left(\frac{(P_d)^j}{(A_m)^{j-1} * L} + \frac{(P_d)^{j-1}}{(A_m)^{j-1} * L}\right) + \frac{(M)^{j-1}}{(SM)^{j-1}} \le C_s * (f_m)^{j-1} \tag{5-9}$$

If Relationship 5-9 is expressed in the same manner as Relationship 5-4, the resulting discrete values as well as the required number of zero-one variables will be so large that the increase in the size of the problem is beyond any existing solving technique capacity.

Relationship 5-3 must therefore be rearranged so that the number of discrete values and subsequently zero-one variables caused by adding more stories, is kept within a tolerable level.

The problem, as becomes obvious, lies in the axial dead load represented by P_d. Since this variable, called the "Transfer" variable, is the only one that connects stories to each other (e.g., $(P_d)^{j-1}$ is a function of $(P_d)^j$ and $(P_d)^{j-2}$ is a function of both $(P_d)^j$ and $(P_d)^{j-1}$ and so on), the re-arrangement of Relationship 5-3 must therefore consider the following important point. $(P_d)^j$ must not be associated with any other variables representing the grouting condition i. It should be noted that all other variables in Relationship 5-3 are confined to individual stories. This important point represents the basis for developing all relationships for all possible stress conditions.

Expressing Relationship 5-3 in the same general form as that shown in Chapter 3 (Relationships 3-34) gives:

$$(XA)^j + (YA)^j \leq (ZA)^j + (AN)^j \qquad (5\text{-}10)$$

where $(AN)^j$ is the discrete value of stress in N/mm^2 that makes up the difference between $[(XA)^j + (YA)^j]$ and $(ZA)^j$ in case a compressive violation exists in the wall.

Since $(SM)^j$ can be expressed as follows:

$$(SM)^j = \frac{(A_m)^j * L^2}{6000} \quad \text{for rectangular section} \tag{5-11}$$

Relationship 5-10 can be rewritten as follows, with respect to the values of XA, YA and ZA:

$$((P_d)^j * L) + 6(M)^j \leq \{(C_s * L^2) * (A_m * f_m)^j + (L^2 * (AN * A_m)^j\} \tag{5-12}$$

On the other hand, the variable $(AN)^j$ introduced in Chapter 3 can be expressed as follows:

$$(AN)^j = \sum_{i=1}^{IT} \sum_{k=1}^{KT} (AN)_{ik}^j * (X)_{ik}^j \tag{5-13}$$

$$\text{where } (AN)_{ik}^j = C_s * \frac{3.2 * f_s' * (\frac{M}{SM})^j * (A_{so})_k^j}{L * (t)_i^j * [(\frac{P_d}{A_m * L} + \frac{M}{SM})_i^j - (C_s * f_m)_i^j]} \tag{5-14}$$

where $(A_{so})_k^j$ = the k^{th} side steel for the j^{th} storey in (mm^2)

f_s' = the allowable compressive steel stress (N/mm^2)

$(t)_i^j$ = the equivalent thickness of the wall in mm

$= (A_m)_i^j/1000$

and $(X)_{ik}^j$ = zero-one variable associated with i^{th} grouting condition, k^{th} steel area and j^{th} storey, and other variables are as defined before.

Without loss of accuracy, the following assumption can be made:

$$(AN)_{ik}^j = [(\frac{P_d}{A_m * L} + \frac{M}{SM})^j - (C_s * f_m)_i^j] \tag{5-15}$$

Relationship 5-14 can now be rewritten as follows:

$$(AN)_{ik}^j = \frac{1}{j} \sqrt{\frac{19.2 * M^j * C_s * f_s' * (A_{so})_k^j}{L^3}} \tag{5-16}$$

$(A_m)_i$

where $(AN)^j_{ik}$ is in N/mm^2

Substituting Equation 5-16 into Relationship 5-12 results in the following general relationship for the j^{th} storey.

$$L + (P_d)^j + 6(M)^j \leq (L^2 * (C_s * f_m * A_m)^j) + 4.4 \sqrt{L * C_s * f_s' * (M*A_{so})^j} \qquad (5\text{-}17)$$

where
- L is in mm
- $(P_d)^j$ is in N
- $(M)^j$ is in N-mm
- $(f_m)^j$ is in N/mm^2
- $(A_m)^j$ is in $(mm^2/1000)$
- f_s' is in N/mm^2
- $(A_{so})^j$ is in mm^2

As can be noticed from Relationship 5-17, the term including $(P_d)^j$ is independent of any other variable in the relationship. Also, the term including the effect of the end steel, $(A_{so})^j$, becomes independent of the grouting conditions, i, which has the advantage of reducing the size of the model.

In a general form, Relationship 5-17 can be expressed as follows:

$$(XB)^j + (YC)^j \leq (XD)^j + (XC)^j \qquad (5\text{-}18)$$

where
$$(XB)^j = L * (P_d)^j$$
$$(YC)^j = 6 * (M)^j$$
$$(XD)^j = L^2 * C_s(f_m*A_m)^j$$
$$(XD)^j = 4.4 \sqrt{L * C_s * f_s' * (M * A_{so})^j}$$

and
$$(P_d)^j = L * h * ((UW)^{j-1} + (UW)^j)$$

where h = Height of one storey (mm)

$(UW)^j$ = The unit weight of masonry wall for j^{th} storey (N/mm^2)

As Relationship 5-18 represents the stress condition at the compression end of the wall, and as it must hold only within its subcase, (subcase 1 or 2), another constraint must be added to restrict its application to these subcases alone (when tension does not exist). This constraint is mathematically expressed as follows:

$$\frac{(P_d)^j}{(A_m)^j * L} \geq \frac{(M)^j}{(SM)^j} \tag{5-19}$$

or

$$\frac{(P_d)^j}{(A_m)^j * L} \geq \frac{6000\ (M)^j}{(A_m)\ \ * L^2} \tag{5-20}$$

from which the following relationship results

$$(P)^j \geq \frac{6(M)^j}{(h * L^2)} \tag{5-21}$$

or

$$(P)^j \geq (XL)^j \tag{5-22}$$

where $(XL)^j = \dfrac{6\ (M)^j}{h * L^2}$

and $(P)^j = (UW)^{j+1} + (UW)^j$

In summary, Subset 1 is represented by both Relationships 5-18 and 5-22.

SUBSET 2: This subset represents subcase (3), shown in Figure 5-2(c).

In this subcase, although tension exists on the tension side of the wall, the compression violation is more serious than the tension violation. Therefore, the design for compression must govern. As a result of this assumption, Relationship 5-18 must also be applied in this subcase. However, as there must be a complete separation between the two subsets, the relationship associated with this case written as follows:

$$(XB2)^j + (YC2)^j \leq (XD2)^j + (XC2)^j \qquad (5\text{-}23)$$

For purpose of organizing the model, the numbers to the right hand side of the variables in Relationship 5-23 that appear in more than one subset indicate the subset number at which this particular variable is applied. Therefore Relationships 5-18 and 5-22, for subset 1, are rewritten as:

$$(XB1)^j + (YC1)^j \leq (XD1)^j + (XC1)^j \qquad (5\text{-}24)$$

and

$$(P1)^j \geq (XL1)^j \qquad (5\text{-}25)$$

As mentioned above, tension exists in this subcase, i.e., subcase 3. However, since compression violation is more serious than tension violation, a guiding constraint must be added as a "key" constraint to Relationship 5-23. This "key" constraint is presented as:

$$\left[\left(\frac{(P_d)^j}{L * (A_m)^j} + \frac{(M)^j}{(SM)^j} \right) - (C_s*(f_m)^j) \right] \geq \left(\frac{(M)^j}{(SM)^j} - \frac{(P_d)^j}{L * (A_m)^j} \right) \qquad (5\text{-}26)$$

or

$$\frac{2(P_d)^j}{(L * (A_m)^j)} \geq C_s * (f_m)^j \tag{5-27}$$

which leads to:

$$(P)^j \geq C_s * (f_m * A_m)^j/2h \tag{5-28}$$

or

$$(P2)^j \geq (XK2)^j \tag{5-29}$$

where $(XK2)^j = C_s * (f_m*A_m)^j/2h$

In summary, subset 2 is represented by Relationships 5-23 and 5-29.

<u>SUBSET 3</u>: This subset represents subcase 4, shown in Figure 5-2(d).

In this subcase, tension violation becomes more serious than the compression violation. Therefore, design for tension rather than compression must be considered. Normally, the design for tension is done using the Transformed Section Analysis (T.S.A.), where the resulting relationships cannot be expressed in "acceptable discrete forms". Therefore simulated functions are developed, as discussed in Chapter 4, for this particular subcase, to substitute for the original functions. The simulation process discussed in Chapter 4 is based on rational assumptions where the main variables, considered as unknown variables in the T.S.A., were replaced by corresponding assumed variables, (f_{ms}, f'_{ss}, f_{ss} and ϕ_s). These assumed variables and the final simulated function, Relationship 4-33, were compared to the original variables as well as to the results yielded by the original function. The close agreement of results in most cases approves the use of Relationship

4-33 in the model for subcase 3. Relationship 4-33 can be expressed in an "acceptable discrete form" as follows:

$$(XM)^j + (XN)^j \geq (XO)^j + (P3)^j \tag{5-30}$$

where

$$(XM)^j = \frac{(AA - BB)^j}{(CC)^j} \tag{5-31}$$

$$(XN)^j = \frac{(AB)^j}{(CB)^j} \tag{5-32}$$

and $$(XO)^j = \frac{(BA + BC)^j}{(CC)^j} \tag{5-33}$$

where

$$(AA)^j = \{(12C_s*(f_m*A_m*M)^j) + (1 + (3ECC^j)\}/1000 \tag{5-34}$$

$$(BB)^j = j * (D_\ell) * (ECE)^j \tag{5-35}$$

$$(CC)^j = (ECD + ECE)^j * (h * L) \tag{5-36}$$

$$(AB)^j = (48 * C_s * ((n)^u * A_{so} * f_m*M)^j)/1000*L \tag{5-37}$$

$$(CB)^j = (ECD + (ECE)^u)^j * (h*L) \tag{5-38}$$

$$(BA)^j = 144*(M^j)^2/L^2 \tag{5-39}$$

and

$$(BC)^j = j * (D_\ell) * (ECD)^j \tag{5-40}$$

where

$$(ECC)^j = [((E)^L * (EC))^j - (P_d)^{Lj}]/(EC)^j \tag{5-41}$$

$$(EC)^j = ((Pd)^L - (P_d)^U)^j / ((E)^L - (E)^U)^j \tag{5-42}$$

$$(ECE)^j = -.001 * C_s * (f_m*A_m)^j * (L + (36 * (M/EC)^j)) \tag{5-43}$$

$$(ECD)^j = 24*(M)^j/L \tag{5-44}$$

and

$$(ECE)^{Uj} = -.001 * C_s * (f_m*A_m)^{Uj} * (L + (36*(M/EC)^j)) \tag{5-45}$$

where $(n)^{Uj}$ = the upper bound of the modular ratio

$(f_m)^{Uj}$ = the upper bound of the allowable masonry compressive stress (MPa)

h = the height of storey (mm)

L = total length of the wall (mm)

D_ℓ = the dead load of the roof for one storey (N)

$(P_d)^{Lj}$ = the lower bound of the axial load for storey j (N)

$(P_d)^{Uj}$ = the upper bound of the axial load for storey j (N)

$(E)^{Lj}$ = the lower bound of eccentricity ratio for storey j

$(E)^{Uj}$ = the upper bound of eccentricity ratio for storey j

and

$(A_m)^{Uj}$ = the upper bound of the mortar bedded area for storey j (mm^2/m)

and all other variables are as defined before.

It should be mentioned that the lower and upper bounds of the variables defined above are with respect to the lower and upper bounds of the grouting conditions.

Not only does Relationship 5-30 consider the case when tension exists in the wall, but also when the tension violation is more serious than the compression violation. Therefore, "key" constraints must be associated with Relationship 5-30 so that its application is restricted to this subcase, i.e. subcase 3 alone. These constraints are mathematically expressed as follows:

$$(XL3)^j \geq (P3)^j \tag{5-46}$$

and

$$(XK3)^j \geq (P3)^j \tag{5-47}$$

where $(XL3)^j$, $(XK3)^j$ and $(P3)^j$ are as defined before except that they are associated with subset 3.

Relationships 5-46 and 5-47 correspond to the cases when

$$\left(\frac{(M)^j}{(SM)^j} > \frac{(P_d)^j}{(A_m)^j * L}\right)$$

and $$\left\{\left(\left(\frac{(P_d)^j}{(A_m)^j * L} + \frac{(M)^j}{(SM)^j} - (C_s * (f_m)^j)\right) < \left(\frac{(M)^j}{(SMj)} - \frac{(P_d)^j}{(A_m)^j * L}\right)\right)\right\}$$

<u>SUBSET 4</u>:

This subset represents the situation where tension exists at the tension end, while the stress at the compression end falls within its allowable limit (subcase 5). The actual stress distribution according to the U.E. is as shown in Figure 5-3-a.

If the problem is solved using T.S.A., the resulting stress distribution is that shown in Figure 5-3-b. Figure 5-3-c shows the difference between the two distributions. Again, as for subset 3, an extensive study of the difference between the values of the variables produced using U.E. and those determined using T.S.A. shows that the latter are generally less than 3% higher than the former. This result suggests the following important assumption: If tension exists in the wall, i.e., $M/SM > P_d/(A_m*L)$, and if there is no compression violation, i.e., $(P_d/(A_m*L)) + M/SM \leq C_s*f_m$, then the actual values of masonry compressive stress and the depth to the neutral axis can be taken as $((P_d/(A_m*L)) + M/SM)$ and (ϕ_s*D), respectively.

It can be concluded from the above assumption and from Figures 5-3-a,b, and c that the material stresses are underestimated, which, subsequently, causes an underestimate of the total resisting capacity of the wall. Not only does this result in a conservative design, but it also serves to simplify the modeling for this case, as will be discussed later. In developing the main constraint for this case, Relationships 4-26 and 4-27 are used again and by substituting values of f_{ms}, f'_{ss} and f_{ss} as developed from similar triangles in Figure 5-3-c, the following relationship results:

$$P_d \left(\frac{L}{24M}\right) + \left[\frac{(A_m*L) + (4000n*A_{so})}{(2L * A_m)}\right] + 1.5\ E \geq 1.0 \qquad (5\text{-}48)$$

Using the piecewise linearization discussed in Chapter 4 for subset 3, to linearize the relationship between E and P_d, and substituting the equivalent value of E into Relationship 5-48, the resulting general mathematical expression, for storey j, is as follows:

$$(P4)^j + (XF)^j \geq (XE)^j \qquad (5\text{-}49)$$

where

$$XF^j = (((A_m)^j*L+4000(n)^j*(A_{so})^j)/(2(A_m)^j*L))/((h*L) + ((L/24(M)^j) + (1.5/(EC)^j)) \qquad (5\text{-}50)$$

and

$$XE^j = \{(((1.5*(ECC)^j) - 1.0)/((L/24(M)^j) + (1.5/(EC)^j))) + (j*D_\ell)\}/(h*L) \qquad (5\text{-}51)$$

and other variables are as defined before.

The "key" constraint for this case is mathematically expressed as follows:

$$(XL4)^j \geq (P4)^j \qquad (5\text{-}52)$$

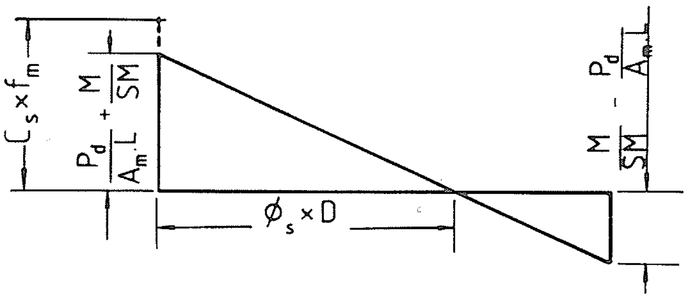

Figure 5-3-a The actual stress distribution according to the U.E. for a wall subjected to (DL+WL)

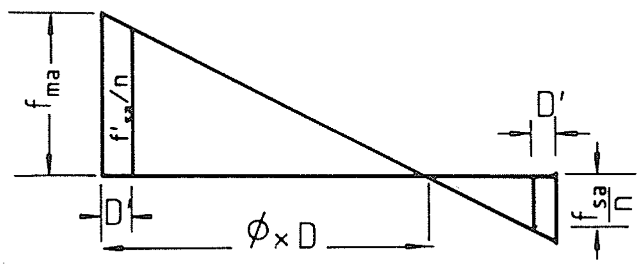

Figure 5-3-b The actual stress distribution according ot T.S.A. for a wall subjected to (DL+WL)

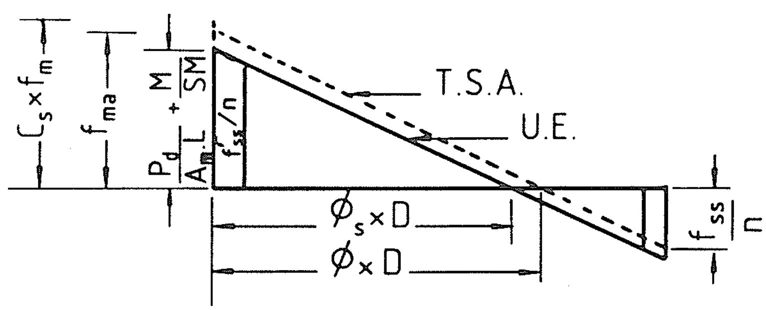

Figure 5-3-c The combined stress distribution according to both U.E. and T.S.A. for a wall subjected to (DL+WL)

Relationship 5-52 indicates the fact that tension exists in the wall. However, for complete separation between subset 3, which includes a similar "key" constraint, and subset 4, another "key" constraint must be added. This constraint reflects the fact that in subset 3, in addition to tension violation, there is compression violation i.e., $(P_d/(A_m*L)) + M/SM > C_s*f_m$, while in subset 4 the compression violation is eliminated, i.e., $(P_d/(A_m*L)) + M/SM \le C_s*f_m$. Therefore, this "key" constraint is mathematically expressed as follows:

$$(P3)^j + (XP3)^j > (XQ3)^j \qquad \text{for subset 3} \tag{5-53}$$

$$(P4)^j + (XP4)^j \le (XQ4)^j \qquad \text{for subset 4} \tag{5-54}$$

where

$$(XP3)^j \text{ or } (XP4)^j = ((6M/L)+(j*D_\ell))/(h*L) \tag{5-55}$$

and

$$(XQ3)^j \text{ or } (XQ4)^j = (C_s*L*(f_m*A_m)^j)/(1000h*L) \tag{5-56}$$

The continuous constraints subsets representing loads and stress limitations for the four cases within the (DL + WL) case are now complete. These constraints can be summarized as:

<u>Subset 1</u>

$$(P1)^j \ge (XL1)^j \qquad \text{(key constraint)} \tag{5-57}$$

$$(XB1)^j + (YC1)^j \le (XD1)^j + (XC1)^j \tag{5-58}$$

<u>Subset 2</u>

$$(P2)^j \ge (XK2)^j \qquad \text{(key constraint)} \tag{5-59}$$

$$(XB2)^j + (YC2)^j \le (XD2)^j + (XC2)^j \tag{5-60}$$

Subset 3

$$\left.\begin{aligned}(XL3)^j &\geq (P3)^j \qquad (5\text{-}61)\\ (XK3)^j &\geq (P3)^j \qquad (5\text{-}62)\\ (P3)^j + (XP3)^j &\geq (XQ3)^j \qquad (5\text{-}63)\end{aligned}\right\} \text{(key constraints)}$$

$$(XM)^j + (XN)^j \geq (XO)^j + (P3)^j \qquad (5\text{-}64)$$

Subset 4

$$\left.\begin{aligned}(XL4)^j &\geq (P4)^j \qquad (5\text{-}65)\\ (P4)^j + (XP4)^j &\leq (XQ4)^j \qquad (5\text{-}66)\end{aligned}\right\} \text{(key constraints)}$$

$$(P4)^j + (XF)^j \geq (XE)^j \qquad (5\text{-}67)$$

The above constraints can accommodate any combination that could result in any type of stress in the wall due to (DL + WL) application.

(DL + LL) CASE RELATIONSHIPS

Although the Coefficient Method is the basis for developing the main relationship in the case of (DL + LL), the expression is slightly different than that developed for a single storey because of simplifying assumptions adapted in this case as discussed below.

The process of developing the relationship is as follows:

Axial compressive resisting capacity of the wall ≥ applied (DL + LL).

The applied (DL + LL) consists of the following two components:

1. Original applied load, OAD, = $[j*(D_\ell + L_\ell) + (h*L*UW)]$ (5-68)

2. Additional applied dead load due to grouting more cores (in case compression steel is needed), ADDL, = ING $[CW*h*(UW)^U]$ (5-69)

where D_ℓ = Applied dead load (roof weight) for one storey (N)

L_ℓ = Applied live load for one storey (N)

j = Number of stories

CW = Core width (mm)

$(UW)^U$ = The upper limit of the unit weight of grout in N/mm^2

and

ING = The required number of additional inside grouted cores

and other variables are as defined before.

On the other hand, the axial compressive resisting capacity consists of two components, namely:

1. Original axial compressive resisting capacity,

$$ORC = C_s * L * f_m * A_m \quad (5\text{-}70)$$

2. Additional axial compressive resisting capacity due to adding compression steel and more grouted cores,

$$ADRC = ING * [C_s * (((f_m)^L * AC) + (0.8 * f_s' * A_{si}))] \quad (5\text{-}71)$$

where $(f_m)^L$ = The lower bound on the masonry allowable compressive stress, (i.e. for fully grouted wall) (MPa)

AC = Cross sectional area of one core (mm^2)

A_{si} = The compression steel cross sectional area (mm^2)

and other variables are as defined before.

Combining Equations 5-68, 5-69, 5-70 and 5-71 results in the following relationship:

$$(C_s*f_m*A_m/h) + \{ING*[C_s(((f_m)^L*AC) + (0.8*f_s'*A_{si})) - (CW*h*(UW)^L)]/(h*L)\} \geq (j*(D_\ell+L_\ell)/(h*L)) + UW \quad (5\text{-}72)$$

So far, one assumption has been made, that is, both the applied load and the axial compressive resisting capacity are independent of the grouting and steel combination which result at each end of the wall from the (DL + WL) application. This assumption underestimates both the applied load and the axial resisting capacity of the entire wall by almost the same negligible percentage. However, Relationship 5-72 is now applied only once at each storey instead of KT times, corresponding to KT end steel options, thereby reducing the size of the model.

A second assumption is that only one size of steel is used for compression steel which leaves the value of ING as the only continuous value. Therefore a third simplifying assumption is made, that is the use of specific assumed values for ING. These assumed values, as in the case of other discrete masonry components in the model, e.g. grouting condition, side steel, etc., are left to the designer.

After assuming these values for ING, Relationship 5-72 can be expressed in a discrete linear form, for storey j, as follows:

$$(ORC)^j + (ADD)^j \geq (OAD)^j + (P)^j \qquad (5\text{-}73)$$

where $(ORC)^j = \{C_s * (f_m * A_m)^j\}/h$

$$(ADD)^j = ING * \{C_s(((f_m)^L * AC)^j + (0.8f_s' * A_{si})^j) - (CW*h*(UW)^{Lj}\}/(h*L)$$

$$(OAD)^j = j * (D_\ell + L_\ell)/(h*L)$$

$$(P)^j = (UW)^{j+1} + (UW)^j$$

5.3.1b Continuous Organizing Constraints

Figure 5-1 shows the possible number of grouting combinations that can exist in a multistorey wall. Without controlling constraints, the number of options to be checked at each storey is almost (IT**j), and the total number for j stories is almost $\sum_{j=1}^{J}$ (IT**j) which represents a very difficult problem with the increasing number of stories. Not only is it extremely difficult to accommodate such a computational process, but the model can also lead to "impractical" design. For example, if there is no control on selecting the grouting conditions for subsequent floors, a top floor may have a higher grouting level and lesser steel than the next floor. Therefore, the following subset of constraints is essential to control the choices of such variables, e.g. original number of grouted cores and thereby produce a practical design. These constraints are expressed mathematically as:

$$(ONG)^{j} \geq (ONG)^{j+1} \tag{5-74}$$

$$(TSS)^{j} \geq (TSS)^{j+1} \tag{5-75}$$

$$(A_{si})^{j} \geq (A_{si})^{j+1} \tag{5-76}$$

$$(TNG)^{j} \leq (TNC)^{j} \tag{5-77}$$

where ONG = Original number of grouted cores

TSS = Total side steel area (mm^2)

A_{si} = Inside steel area (mm^2)

TNG = Total number of grouted cores

TNC = Total number of cores in the wall

j = Storey number

The first three Constraints 5-74, 5-75 and 5-76 ensure that both the grouting condition and the steel area, whether outside or inside, for storey j, are greater than or equal to those corresponding to storey j+1. Constraint 5-77 limits the total number of grouted cores to be within the limits of the total number of cores in the wall.

5.3.2 DEFINING CONSTRAINTS

The defining constraints consist of three types of relationships. The first defines the discrete values of those variables included in the stress and load limitation constraints (Relationships 5-57 through 5-67). The second defines the discrete values of those variables selected by the designer, and the third type defines values of the Transfer Variables, e.g. $(P)^j$.

The discrete constraints for the first type of relationships are expressed mathematically as follows:

A. Discrete Constraints Define Values of Variables Included in Subset 1 (Relationships 5-57 and 5-58)

$$(XL1)^j = (XL)^j * (X)^j \tag{5-78}$$

$$(YC1)^j = (YC)^j * (X)^j \tag{5-79}$$

$$(XB1)^j = \sum_{i=1}^{IT} (XB)_i^j * (X)_i^j \tag{5-80}$$

and

$$(XC1)^j = \sum_{k=1}^{KT} (XC)_k^j * (XS)_k^j \tag{5-81}$$

where $(XL1)^j$, $(YC1)^j$, $(XB1)^j$ and $(XC1)^j$ are the variable names for subset 1 (DL + WL) for storey j

and $(XL)^j$, $(YC)^j$, $(XB)_i^j$ and $(XC)_k^j$ are respectively the coefficients, or the discrete values of the above variables.

and $(X)^j$ = Zero-one variable associated with the j^{th} storey for subset 1

$(X)_i^j$ = Zero-one variable associated with the i^{th} grouting condition for j^{th} storey for subset 1

and $(XS)_k^j$ = Zero-one variable associated with the k^{th} side steel for j^{th} storey for subset 1

B. <u>Discrete constraints defining values of variables included in subset-2-(Relationships 5-59 and 5-60)</u>:

$$(XK2)^j = \sum_{i=1}^{IT} (XK)_i^j * (Z)_i^j \tag{5-82}$$

$$(YC2)^j = (YC)^j * (Z)^j \tag{5-83}$$

$$(XB2)^j = \sum_{i=1}^{IT} (XB)_i^j * (Z)_i^j \tag{5-84}$$

and

$$(XC2)^j = \sum_{k=1}^{KT} (XC)_k^j * (ZS)_k^j \tag{5-85}$$

where $(XK2)^j$, $(YC2)^j$, $(XB2)^j$ and $(XC2)^j$ are the variable names for subset 2 for storey j

and $(XK)_i^j$, $(YC)^j$, $(XB)_i^j$ and $(XC)_k^j$ are respectively the discrete values of the above variables

and $(Z)^j$ = zero-one variable associated with the j^{th} storey for subset 2

$(Z)_i^j$ = zero-one variable associated with the i^{th} grouting condition for j^{th} storey for subset 2

and $(ZS)_k^j$ = zero-one variable associated with the k^{th} side steel and j^{th} storey for subset-2-

C. <u>Discrete constraints defining values of variables included in Subset 3 (Relationships 5-61 through 5-64):</u>

$$(XL3)^j = (XL)^j * (Y)^j \tag{5-86}$$

$$(XK3)^j = \sum_{i=1}^{IT} (XK)_i^j * (Y)_i^j \tag{5-87}$$

$$(XP3)^j = (XP)^j * (Y)^j \tag{5-88}$$

$$(XQ3)^j = \sum_{i=1}^{IT} (XQ)_i^j * (Y)_i^j \tag{5-89}$$

$$(XM)^j = \sum_{i=1}^{IT} (XM)_i^j * (Y)_i^j \tag{5-90}$$

$$(XN)^j = \sum_{k=1}^{KT} (XN)_k^j * (Y_S)_k^j \tag{5-91}$$

$$(XO)^j = \sum_{i=1}^{IT} (XO)_i^j * (Y)_i^j \tag{5-92}$$

where $(XL3)^j$, $(XK3)^j$, $(XP3)^j$, $(XQ3)^j$, $(XM)^j$, $(XN)^j$ and $(XO)^j$ are variables names for subset 3 for storey j.

$(XL)^j$, $(XK)_i^j$, $(XP)^j$, $(XQ)_i^j$, $(XM)_i^j$, $(XN)_k^j$ and $(XO)_i^j$ are, respectively, the discrete values of the above variables.

and $(Y)^j$ = zero-one variable associated with the j^{th} storey for subset 3

$(Y)_i^j$ = zero-one variable associated with the i^{th} grouting condition and j^{th} storey for subset 3

and $(YS)_k^j$ = zero-one variable for k^{th} side steel and j^{th} storey for subset 3 and (DL + WL).

D. <u>Discrete constraints defining values of variables included in Subset 4 (Relationships 5-65, 5-66 and 5-67)</u>:

$$(XL4)^j = (XL)^j * (W)^j \tag{5-93}$$

$$(XP4)^j = (XP)^j * (W)^j \tag{5-94}$$

$$(XQ4)^j = \sum_{i=1}^{IT} (XQ)_i^j * (W)_i^j \tag{5-95}$$

$$(XF)^j = \sum_{i=1}^{IT} \sum_{k=1}^{KT} (XF)_{ik}^j * (WS)_{ik}^j \tag{5-96}$$

$$(XE)^j = (XE)^j * (W)^j \tag{5-97}$$

where $(XL4)^j$, $(XP4)^j$, $(XQ4)^j$, $(XF)^j$ and $(XE)^j$ are variables names for subset 4 for storey j.

and $(XL)^j$, $(XP)^j$, $(XQ)_i^j$, $(XF)_{ik}^j$ and $(XE)^j$ are respectively the discrete values of the above variables.

and $(W)^j$ = zero-one variable for the j^{th} storey for subset 4

$(W)_i^j$ = zero-one variable for the i^{th} grouting condition and j^{th} storey for subset 4

$(WS)^j_{ik}$ = zero-one variable for the i^{th} grouting condition, k^{th} side steel and j^{th} storey for subset 4

On the other hand, the defining constraints for the (DL + LL) relationship (Relationship 5-73) are as following:

$$(ORC)^j = \sum_{i=1}^{IT} (ORC)^j_i * (N)^j_i \tag{5-98}$$

$$(ADD)^j = \sum_{m=1}^{MT} (ADD)^j_m * (NS)^j_m \tag{5-99}$$

and

$$(OAD)^j = (OAD)^j \tag{5-100}$$

where $(ORC)^j$, $(ADD)^j$ and $(OAD)^j$ are the variable names for (DL + LL) case for storey j.

and $(ORC)^j_i$, $(ADD)^j_m$ and $(OAD)^j$, are respectively the discrete values of the above variables.

and $(N)^j_i$ = zero-one variable for the i^{th} grouting condition and j^{th} storey for (DL + LL) case.

and $(NS)^j_m$ = zero-one variable for the m^{th} number of inside steel bars and j^{th} storey for (DL + LL) case.

The second type of relationships define those variables given directly as input to the problem by the designer, and can be expressed mathematically as follows:

$$(TSS)^j = 2(A_{so})^j \tag{5-101}$$

$$(A_{so})^j = (A1_{so})^j + (A2_{so})^j + (A3_{so})^j + (A4_{so})^j \tag{5-102}$$

where

$$(A1_{so})^j = \sum_{k=1}^{KT} (A_{so})_k^j * (XS)_k^j \tag{5-103}$$

$$(A2_{so})^j = \sum_{k=1}^{KT} (A_{so})_k^j * (ZS)_k^j \tag{5-104}$$

$$(A3_{so})^j = \sum_{k=1}^{KT} (A_{so})_k^j * (YS)_k^j \tag{5-105}$$

$$(A4_{so})^j = \sum_{i=1}^{IT} \sum_{k=1}^{KT} (A_{so})_{ik}^j * (WS)_{ik}^j \tag{5-106}$$

where $(A1_{so})^j$, $(A2_{so})^j$, $(A3_{so})^j$ and $(A4_{so})^j$ are the end steel areas associated with subsets 1, 2, 3 and 4, respectively.

$(A_{so})^j$ = optimum end steel cross-sectional area for storey j (for one end of the wall)

and $(TSS)^j$ = optimum total end steel cross-sectional area for storey j (for both ends of the wall)

The number of end grouted cores constraints are as follows:

$$(SNG)^j = (SNG1)^j + (SNG2)^j + (SNG3)^j + (SNG4)^j \tag{5-107}$$

where

$$(SNG1)^j = \sum_{k=1}^{KT} (SNG)_k^j * (XS)_k^j \qquad (5\text{-}108)$$

$$(SNG2)^j = \sum_{k=1}^{KT} (SNG)_k^j * (ZS)_k^j \qquad (5\text{-}109)$$

$$(SNG3)^j = \sum_{k=1}^{KT} (SNG)_k^j * (YS)_k^j \qquad (5\text{-}110)$$

$$(SNG4)^j = \sum_{i=1}^{IT} \sum_{k=1}^{KT} (SNG)_{ik}^j * (WS)_{ik}^j \qquad (5\text{-}111)$$

where $(SNG1)^j, \ldots, (SNG4)^j$ are the number of end grouted cores for subsets 1, through 4, respectively for the (DL + WL) case.

$(SNG)^j$ = Optimum number of end grouted cores for storey j.

Also,

$$(A_m)^j = \sum_{i=1}^{IT} (A_m)_i^j * (N)_i^j \qquad (5\text{-}112)$$

$$(ONG)^j = \sum_{i=1}^{IT} (ONG)_i^j * (N)_i^j \qquad (5\text{-}113)$$

$$(ING)^j = \sum_{m=1}^{MT} (ING)_m^j * (NS)_m^j \qquad (5\text{-}114)$$

$$(A_{si})^j = \sum_{m=1}^{MT} (A_{si})_m^j * (NS)_m^j \qquad (5\text{-}115)$$

$$(TS)^j = (TSS)^j + (A_{si})^j \qquad (5\text{-}116)$$

and

$$(TNG)^j = (ONG)^j + (SNG)^j + (ING)^j \tag{5-117}$$

where $(Am)^j$, $(ONG)^j$, $(ING)^j$, $(A_{si})^j$, $(TS)^j$, $(TNG)^j$ are the mortar bedded area, original number of grouted cores, number of inside grouted cores, inside steel area, total steel area and total number of grouted cores respectively for storey j.

The final type of defining constraints are those which link the "Transfer" variables to each other according to their corresponding levels. These constraints are expressed mathematically as:

$$(P1)^j = (P)^{j+1} + \sum_{i=1}^{IT} (UW)_i^j * (X)_i^j \tag{5-118}$$

$$(P2)^j = (P)^{j+1} + \sum_{i=1}^{IT} (UW)_i^j * (Z)_i^j \tag{5-119}$$

$$(P3)^j = (P)^{j+1} + \sum_{i=1}^{IT} (UW)_i^j * (Y)_i^j \tag{5-120}$$

$$(P4)^j = (P)^{j+1} + \sum_{i=1}^{IT} (UW)_i^j * (W)_i^j \tag{5-121}$$

$$(XB1)^j = (XB)^{j+1} + \sum_{i=1}^{IT} (XB)_i^j * (X)_i^j \tag{5-122}$$

and

$$(XB2)^j = (XB)^{j+1} + \sum_{i=1}^{IT} (XB)_i^j * (Z)_i^j \tag{5-123}$$

where all variables are as defined before.

However, there is a problem associated with the formulation of Relationships 5-118 through 5-123. This problem is discussed in Section 5.3.5.

5.3.3 ZERO-ONE CONSTRAINTS:

The zero-one constraints include those equations which organize both direct and indirect relationships among zero-one variables so that they lead the computational process toward the optimum solution. These constraints are expressed mathematically for storey j as follows:

$$\sum_{k=1}^{KT} (XS)_k^j = (X)^j = \sum_{i=1}^{IT} (X)_i^j \qquad (5\text{-}124)$$

$$\sum_{k=1}^{KT} (ZS)_k^j = (Z)^j = \sum_{i=1}^{IT} (Z)_i^j \qquad (5\text{-}125)$$

$$\sum_{k=1}^{KT} (YS)_k^j = (Y)^j = \sum_{i=1}^{IT} (Y)_i^j \qquad (5\text{-}126)$$

$$\sum_{i=1}^{IT} \sum_{k=1}^{KT} (WS)_{ik}^j = (W)^j = \sum_{i=1}^{IT} (W)_i^j \qquad (5\text{-}127)$$

$$(N)_i^j = (X)_i^j + (Z)_i^j + (Y)_i^j + (W)_i^j \quad \forall\ i \qquad (5\text{-}128)$$

$$(X)^j + (Z)^j + (Y)^j + (W)^j = 1.0 \qquad (5\text{-}129)$$

and

$$\sum_{m=1}^{MT} (NS)_m^j = 1.0 \tag{5-130}$$

5.3.4 THE OBJECTIVE FUNCTION:

The objective function is presented as follows:

$$\text{Minimize} \sum_{j=1}^{J} (CO)^j \tag{5-131}$$

where $$(CO)^j = SUC * (TS)^j + GUC * (TNG)^j + BUC * (Ag)^j \tag{5-132}$$

where $(CO)^j$ = cost of material for storey j ($)

SUC = steel cross-sectional area unit cost ($/mm^2)

GUC = grouted cores unit cost ($/grouted core)

BUC = block unit cost (in terms of gross cross sectional area of the block) ($/Ag)

Ag = wall gross cross-sectional area (mm^2/m)

and $(TS)^j$ and $(TNG)^j$ are as defined in Relationships 5-116 and 5-117.

5.3.5 THE MODELING PROBLEMS:

Having developed relationships for such large problems does not necessarily mean that the resulting model is either "solvable" or a "tight" model. A structurally infeasible solution could result from a "bad" formulation and the computational process could vary in terms of computation time, for example, according to different formulations for the same problem. It is therefore obvious that the ability of solving a certain model and the efficiency of solution depend on many factors. There are two kinds of factors on which a "good" formulation depends -- general and specific factors. These factors can be thought of as requirements for production of a "good" formulation. The following sections summarize the modelling problems associated with developing "good" formulations together with those general and specific requirements.

5.3.5a. General Requirements:

It is known that in an integer programming formulation, the number of zero-one variables affect the solution time. As this number increases, the solution time increases dramatically. Therefore, efforts must be made to reduce this number as much as possible without harming the formulation. This particular requirement has been met in the model developed earlier. The interrelationship among zero-one variables associated with the (DL + LL) case, $(N)_i^j$, and zero-one variables associated with the (DL + WL) case, $(X)_i^j$, $(Z)_i^j$, $(Y)_i^j$ and $(W)_i^j$, as expressed in Equation 5-128 saved the model the addition of (3 * MT) new zero-one variables that should have been associated with variables ADD, ING and A_{si}, for each storey j.

Another important general requirement that must be considered is the location of the decisive integer variables. Due to the use of the Branch and Bound approach[22] in solving the model, (see Appendix B for illustrative example), the placement of these decisive integer variables, i.e. zero-one variables, as early as possible in the constraint set for the model, helps in reducing the number of branches required and subsequently the computation time. For a particular problem examined, when the zero-one constraints were placed in a sequential order according to the order of stories, the optimum solution required 130 branches, 23283 pivots and 8.09 minutes of computation (CPU) time. When the same problem was solved with the zero-one constraints for all stories located at the top of the model, the optimum solution required only 48 branches, 9536 pivots and 2.9 minutes computation time.

5.3.5b. Specific Requirements:

The problem of the optimum design of a multi-storey shear wall is relatively lengthy. If IT = number of grouting conditions, KT = number of end steel options and MT = number of inside steel options, then the number of zero-one variables required for J stories are J*(5IT + 4KT + M + 4). The number of constraints, on the other hand, is, approximately, 130*J, while the number of discrete variables is, approximately, 60*J. As J increases, these numbers increase until they reach the capacity of any solving method.

Not only must the size of the model be considered but also the nature of the formulation. As discussed before, the model is built in such a way that two kinds of either/or situations are embedded into it. The first decides upon the governing (DL + WL) subcase (subsets 1 through 4), while the second helps in selecting the discrete values that yield the optimum solution. Because the relationships in the model are solved simultaneously and not on an iterative basis, it is important to ensure that all relationships in the model are feasible.

Because of the above size and feasibility problems, and based upon experience with the model, the following recommendations are found to be very helpful, if not essential, in solving such kinds of problems:

A. Recommendations regarding the feasibility of the model

As mentioned above, there is a problem associated with the way the "Transfer" variables (Relationships 5-118 through 5-123) are formulated. This problem results from the fact that the variable $(P)^{j+1}$ and $(XB)^{j+1}$ always have positive values, which force the variable $(P1)^{j}$ through $(P4)^{j}$, $(XB1)^{j}$ and $(XB2)^{j}$ to have positive values, which in turn creates an infeasible region for some of the cases presented before. At the meantime $(P)^{j+1}$ and $(XB)^{j+1}$ are not to be directly associated with $(P1)^{j}$, $(P2)^{j}$, $(P3)^{j}$, $(P4)^{j}$, $(XB1)^{j}$ and $(XB2)^{j}$ because:

1. $(P)^{j+1}$ and $(XB)^{j+1}$ are already associated with zero-one variables for the $j+1^{th}$ storey. Therefore, they cannot be associated with these zero-one variables for the j^{th} storey at the same time.

2. If $(P)^{j+1}$ is to be linked to $(P1)^{j}$, i.e., developing a value of $(P1)^{j+1}$ that is linked only to $(P1)^{j}$, the intrinsically incorrect assumption of imposing a conditional situation on $(P)^{j+1}$ is made. In other words, if $(P1)^{j}$ holds, then $(P1)^{j+1}$ must hold, which should not be the case. It is therefore necessary to develop another formulation, which allows for the following situation. If $(P1)^{j}$, for example, should not hold, then $(P)^{j+1}$ corresponding to it must

collapse to zero. The formulation necessary to improve this condition is as follows:

$$(P1)^{j} = (P1)_{j}^{j+1} + \sum_{i=1}^{IT} (UW)_{i}^{j} * (X)_{i}^{j} \qquad (5\text{-}133)$$

$$(P2)^{j} = (P2)_{j}^{j+1} + \sum_{i=1}^{IT} (UW)_{i}^{j} * (Z)_{i}^{j} \qquad (5\text{-}134)$$

$$(P3)^{j} = (P3)_{j}^{j+1} + \sum_{i=1}^{IT} (UW)_{i}^{j} * (Y)_{i}^{j} \qquad (5\text{-}135)$$

$$(P4)^{j} = (P4)_{j}^{j+1} + \sum_{i=1}^{IT} (UW)_{i}^{j} * (W)_{i}^{j} \qquad (5\text{-}136)$$

$$(XB1)^{j} = (XB1)_{j}^{j+1} + \sum_{i=1}^{IT} (XB)_{i}^{j} * (X)_{i}^{j} \qquad (5\text{-}137)$$

and

$$(XB2)^{j} = (XB2)_{j}^{j+1} + \sum_{i=1}^{IT} (XB)_{i}^{j} * (Z)_{i}^{j} \qquad (5\text{-}138)$$

where

$$(P)^{j+1} = (P1)_{j}^{j+1} + (P2)_{j}^{j+1} + (P3)_{j}^{j+1} + (P4)_{j}^{j+1} \qquad (5\text{-}139)$$

$$(XB)^{j+1} = (XB1)_{j}^{j+1} + (XB2)_{j}^{j+1} \qquad (5\text{-}140)$$

$$(P)^{j} = (P1)^{j} + (P2)^{j} + (P3)^{j} + (P4)^{j} \qquad (5\text{-}141)$$

and

$$(XB)^{j} = (XB1)^{j} + (XB2)^{j} \tag{5-142}$$

In the above formulation, if $(XB)^{j+1}$ holds, then either $(XB1)_{j}^{j+1}$ or $(XB2)_{j}^{j+1}$ holds, which in turn causes either $(XB1)^{j}$ or $(XB2)^{j}$ to hold which means that either subset 1 or subset 2 must govern all the time to avoid any infeasibility. To overcome this problem, $(XB3)^{j}$ and $(XB4)^{j}$ constraints can be added to subsets 3 and 4 even though they are not required in any relationships in either subset. These two constraints are presented as follows:

$$(XB3)^{j} = (XB3)_{j}^{j+1} + \sum_{i=1}^{IT} (XB)_{i}^{j} * (Y)_{i}^{j} \tag{5-143}$$

$$(XB4)^{j} = (XB4)_{j}^{j+1} + \sum_{i=1}^{IT} (XB)_{i}^{j} * (W)_{i}^{j} \tag{5-144}$$

where Constraints 5-140 and 5-142 can be rewritten as follows:

$$(XB)^{j+1} = (XB1)_{j}^{j+1} + (XB2)_{j}^{j+1} + (XB3)_{j}^{j+1} + (XB4)_{j}^{j+1} \tag{5-145}$$

$$(XB)^{j} = (XB1)^{j} + (XB2)^{j} + (XB3)^{j} + (XB4)^{j} \tag{5-146}$$

This modified formulation for the "Transfer" variables allows $(P1)_{j}^{j+1}$, $(P2)_{j}^{j+1}$, $(P3)_{j}^{j+1}$ and/or $(P4)_{j}^{j+1}$ and $(XB1)_{j}^{j+1}$, $(XB2)_{j}^{j+1}$, $(XB3)_{j}^{j+1}$ and/or $(XB4)_{j}^{j+1}$ to have part of the full value of $(P)^{j+1}$ and $(XB)^{j+1}$, respectively, and to be just sufficient to overcome any infeasible situation. To ensure that $(P1)_{j}^{j+1}$, $(P2)_{j}^{j+1}$, $(P3)_{j}^{j+1}$, or $(P4)_{j}^{j+1}$ and $(XB1)_{j}^{j+1}$, $(XB2)_{j}^{j+1}$, $(XB3)_{j}^{j+1}$ or $(XB4)_{j}^{j+1}$ reach the full value of $(P)^{j+1}$ and $(XB)^{j+1}$, respectively, whenever the corresponding case holds, the following additional constraints are added:

$$(P1)_j^{j+1} \leq (P)^{Uj} * (X)^j \tag{5-147}$$

$$(P2)_j^{j+1} \leq (P)^{Uj} * (Z)^j \tag{5-148}$$

$$(P3)_j^{j+1} \leq (P)^{Uj} * (Y)^j \tag{5-149}$$

$$(P4)_j^{j+1} \leq (P)^{Uj} * (W)^j \tag{5-150}$$

$$(XB1)_j^{j+1} \leq (XB)^{Uj} * (X)^j \tag{5-151}$$

$$(XB2)_j^{j+1} \leq (XB)^{Uj} * (Z)^j \tag{5-152}$$

$$(XB3)_j^{j+1} \leq (XB)^{Uj} * (Y)^j \tag{5-153}$$

$$(XB4)_j^{j+1} \leq (XB)^{Uj} * (W)^j \tag{5-154}$$

where $(P)^{Uj}$ = the upper limit of $(P)^j$

$(XB)^{Uj}$ = the upper limit of $(XB)^j$

It should be noted that both $(P)^{Uj}$ and $(XB)^{Uj}$ are constants at the j^{th} storey, i.e., they are direct input data to the model. The above formulation is now adequate to define the "Transfer" variables completely.

Another feasibility problem with this model that must be considered is the negative value of some of the variables. It is evident that some loadings could yield negative values for some of the variables included in the model, e.g., XL for top stories, with certain loadings. Therefore, a value of zero should be assigned to these variables so that the non-negativity requirement of the model is met. This particular modification is done outside the model and does not change the formulation in any way.

B. <u>Recommendations regarding the size of the problem</u>

To reduce the size of the problem, the following suggestions can be considered:

1. It is evident that in most multi-storey wall problems, tension stress violation is unlikely to exist in the top storeys. Therefore, a simple test could be done in the computer program which develops the model, to identify these stories where tension does or does not exist. This separation between upper and lower stories greatly helps in reducing the size of the model. The reason for the potential reduction is that for those top stories where no tension exists, only subset 1, (DL + WL), as described before, governs the design and consequently, only the relationships associated with it will appear in the model. The other three subsets will not appear till tension starts in the lower storeys.

2. It is evident, by checking the values of $(XF)^j_{ik}$ for subset 4 in Relationship 5-96 for different loadings, that the variation in the discrete values due to variation in k is negligible. Therefore, it can be concluded that the tension in this case can be resisted with the minimum amount of steel. Following this conclusion, Relationships 5-96 and 5-106 can be rewritten as:

$$(XF)^j = \sum_{i=1}^{IT} (XF)^j_{ik} * (W)^j_i \qquad \text{for } k=1 \qquad (5\text{-}155)$$

and

$$(A4_{so})^j = (A_{so})^j_k * (W)^j \quad \text{for } k=1 \tag{5-156}$$

Since only the minimum steel area is required, the minimum grouting condition can accommodate this value. Therefore, Relationship 5-111 is removed from the model. This simplification reduces the number of zero-one variables by approximately (IT * KT) for each storey j.

3. It was mentioned previously that the inter-relationship between the zero-one variables, $(N)^j_i$, associated with (DL + LL) case and the zero-one variables, $(X)^j_i$, $(Z)^j_i$, $(Y)^j_i$ and $(W)^j_i$, associated with (DL + WL) case as expressed in Equation 5-128, saved the model the addition of (3 * MT) new zero-one variables that should have been associated with variables ADD, SNG and A_{si} for each storey. Further savings can be achieved in this particular case by linking variables $(ORC)^j$, $(ONG)^j$ and $(A_m)^j$ to the four subsets of (DL + WL). This is done by rewriting Relationships 5-98, 5-112 and 5-113 as follows:

$$(ORC1)^j = \sum_{i=1}^{IT} (ORC)^j_i * (X)^j_i \tag{5-157}$$

$$(ORC2)^j = \sum_{i=1}^{IT} (ORC)^j_i * (Z)^j_i \tag{5-158}$$

$$(ORC3)^j = \sum_{i=1}^{IT} (ORC)^j_i * (Y)^j_i \tag{5-159}$$

$$(ORC4)^j = \sum_{i=1}^{IT} (ORC)_i^j * (W)_i^j \qquad (5\text{-}160)$$

$$(ONG1)^j = \sum_{i=1}^{IT} (ONG)_i^j * (X)_i^j \qquad (5\text{-}161)$$

$$(ONG2)^j = \sum_{i=1}^{IT} (ONG)_i^j * (Z)_i^j \qquad (5\text{-}162)$$

$$(ONG3)^j = \sum_{i=1}^{IT} (ONG)_i^j * (Y)_i^j \qquad (5\text{-}163)$$

$$(ONG4)^j = \sum_{i=1}^{IT} (ONG)_i^j * (W)_i^j \qquad (5\text{-}164)$$

$$(A1_m)^j = \sum_{i=1}^{IT} (Am)_i^j * (X)_i^j \qquad (5\text{-}165)$$

$$(A2_m)^j = \sum_{i=1}^{IT} (Am)_i^j * (Z)_i^j \qquad (5\text{-}166)$$

$$(A3_m)^j = \sum_{i=1}^{IT} (Am)_i^j * (Y)_i^j \qquad (5\text{-}167)$$

$$(A4_m)^j = \sum_{i=1}^{IT} (Am)_i^j * (W)_i^j \qquad (5\text{-}168)$$

Meanwhile, Relationship 5-128 can be removed from the model for all i values.

The above modification reduces the number of zero-one variable by (IT) at each storey, while only increasing the number of constraints and the discrete variables slightly.

The above general and specific requirements and recommendations are considered helpful and may under certain circumstances be essential for solving the problem and for producing a "tight" formulation.

It should finally be mentioned that if the model gets extremely large, for example if $J > 15$, a "trial-and-error" process along the following lines could be used.

1. Whenever tension normally exists in the lower floors, remove three of the four (DL + WL) subsets presented earlier, from the model and try solving the model with the fourth subset.
2. If this yields an infeasible solution, remove this fourth subset completely from further consideration. If it yields an optimum solution, remove the fourth subset for the time being.
3. Add the other three cases, one at a time and repeat step 2.
4. Comparing the results for the four solutions and selecting the least cost solution will result in the optimum solution.

The above "trial-and-error" process can be thought of as a "tedious" process. It is, however, not only essential in some cases, but it may also reduce the computation time in many cases.

5.4 ILLUSTRATIVE EXAMPLE:

The model developed in this chapter is applied to a ten-storey shear wall in a residential building[8]. The following assumptions were made in formulating the problem.

1. Only wind pressure is considered to develop the in-plane bending.
2. 190 mm block with compressive strength 20 MPa is used.
3. Mortar bedded-area rather than net block area is considered.
4. Whenever strength requirements are satisfied for an ungrouted wall, the two end cores will each be grouted and reinforced with 1 # 15 bar.

The input data for the problem is as follows:

- D_ℓ = floor weight (64 mm concrete slab on 356 mm Hambro composite steel joists) = 107.00 kN/floor
- L_ℓ = live load = 84.7 KN/floor
- Relative stiffness = 0.047
- The in-plane-bending moment, after being modified by the relative stiffness is as shown in Table 5-2

Table 5-2 In-plane bending movement for the 10 storey shear wall

Floor	KN-m
10	84.924
9	231.945
8	437.613
7	697.762
6	1003.967
5	1347.866
4	1721.093
3	2115.235
2	2526.156
1	2945.443

The following data is assumed as direct input to the problem. Table 5-3 presents the assumed values of the grouting ratio (r) where (r) = the ratio between grouted/ungrouted cores in 1.0 m length, the unit weight (UW) for each of the grouting conditions and the modular ratio n, when n = Em/Es and Em and Es are masonry and steel modulus of elasticity respectively. The values used for A_m and f_m are also shown in Table 5-3.

Table 5.3 Values of r, UW, n, A_m, f_m as assumed.

r	0.0	1/6	1/5	1/4	1/3	1/2	1.0
UW(KN/m^2)	2.11	2.41	2.47	2.56	2.71	3.01	3.91
n	15.4	16.17	16.32	16.55	16.9	17.7	20.0
A_m (mm^2/m)	$64(10)^3$	$85(10)^3$	$89.2(10)^3$	$95.5(10)^3$	$106(10)^3$	$127(10)^3$	$190(10)^3$
f_m (MPa)	3.25	3.125	3.1	3.00	3.06	2.87	2.5

Table 5-4 presents the assumed values of end steel, A_{so} and the assumed distance from the centroid of the end steel to the end of the wall.

Table 5-4 The assumed values of A_{so} and D'.

A_{so} mm^2	200	300	400	500	600	600	800	900	1000	1200	1500	2000
D' mm	100	100	200	100	200	300	400	300	200	400	300	400

It should be noted that discrete values of n, A_m and f_m are developed as shown in Appendix (A).

Discussion of the Results:

The optimum design problem described for the example problem of a 10-storey wall above is formulated using the model developed in the previous sections. The optimum results are summarized in Table 5-5 and graphically presented in Figure 5-4. It should be noted that the values shown in Table 5-5 are for those variables representing the "binding" relationships or subsets. In other words, these variables are reflections of the governing stresses and of the specific discrete values of masonry components associated with them. Other variables that do not appear in the table are included in those relationships or subsets that do not hold and subsequently their values collapse to zero.

For example, in stories 6, 7, 8, 9 and 10, the zero-one variables named $(X)^j$ have been assigned 1.0 which means that the (DL + WL) case for subset 1, as expressed before in Relationships 5-57 and 5-58, has governed the design. Also, in the same stories, $(X)_i^j = (X)_1^j = 1.0$ indicating that the lower bounds of the grouting conditions have been selected which, in turn, means that the ungrouted wall complies with the stress requirements. The choice of $(XS)_k^j = (XS)_1^j = 1.0$, for the same stories, means that no end steel is required for the moment compressive resisting capacity. Therefore, only 1 # 15 steel bar at each end of the wall is added and this is from practical design view

Table 5-5. Optimum design results for the 10-storey shear wall.

Storey / Variable	10	9	8	7	6	5	4	3	2	1
$(X)^j$	1.0	1.0	1.0	1.0	1.0					
$(X)_i^j$	$(X)_1^{10}=1.0$	$(X)_1^9=1.0$	$(X)_1^8=1.0$	$(X)_1^7=1.0$	$(X)_1^6=1.0$					
$(XS)_k^j$	$(XS)_1^{10}=1.0$	$(XS)_1^9=1.0$	$(XS)_1^8=1.0$	$(XS)_1^7=1.0$	$(XS)_1^6=1.0$					
$(Z)^j$						1.0	1.0	1.0	1.0	1.0
$(Z)_i^j$						$(Z)_1^5=1.0$	$(Z)_2^4=1.0$	$(Z)_4^3=1.0$	$(Z)_5^2=1.0$	$(Z)_6^1=1.0$
$(ZS)_i^j$						$(ZS)_1^5=1.0$	$(ZS)_5^4=1.0$	$(ZS)_6^3=1.0$	$(ZS)_{10}^2=1.0$	$(ZS)_{11}^1=1.$
$(NS)_k^j$	$(NS)_1^{10}=1.0$	$(NS)_1^9=1.0$	$(NS)_1^8=1.0$	$(NS)_1^7=1.0$	$(NS)_1^6=1.0$	$(NS)_1^5=1.0$	$(NS)_1^4=1.0$	$(NS)_2^3=1.0$	$(NS)_3^2=1.0$	$(NS)_4^1=1.0$
$(AS_o)^j$	200	200	200	200	200	500	500	600	1000	1200
$(TSS)^j$	400	400	400	400	400	1000	1000	1200	2000	2400
$(A_{si})^j$	0	0	0	0	0	0	0	200	400	600
$(TS)^j$	400	400	400	400	400	1000	1000	1400	2400	3000
$(SNG)^j$	0	0	0	0	0	0	0	2	2	6
$(ONG)^j$	2	2	2	2	2	2	8	11	15	22
$(ING)^j$	0	0	0	0	0	0	0	1	2	3
$(TNG)^j$	2	2	2	2	2	2	8	14	19	31
$(COST)^j$	1228.75	1228.75	1228.75	1228.75	1228.75	1239.71	1260.2	1280.92	1309.19	1354.06

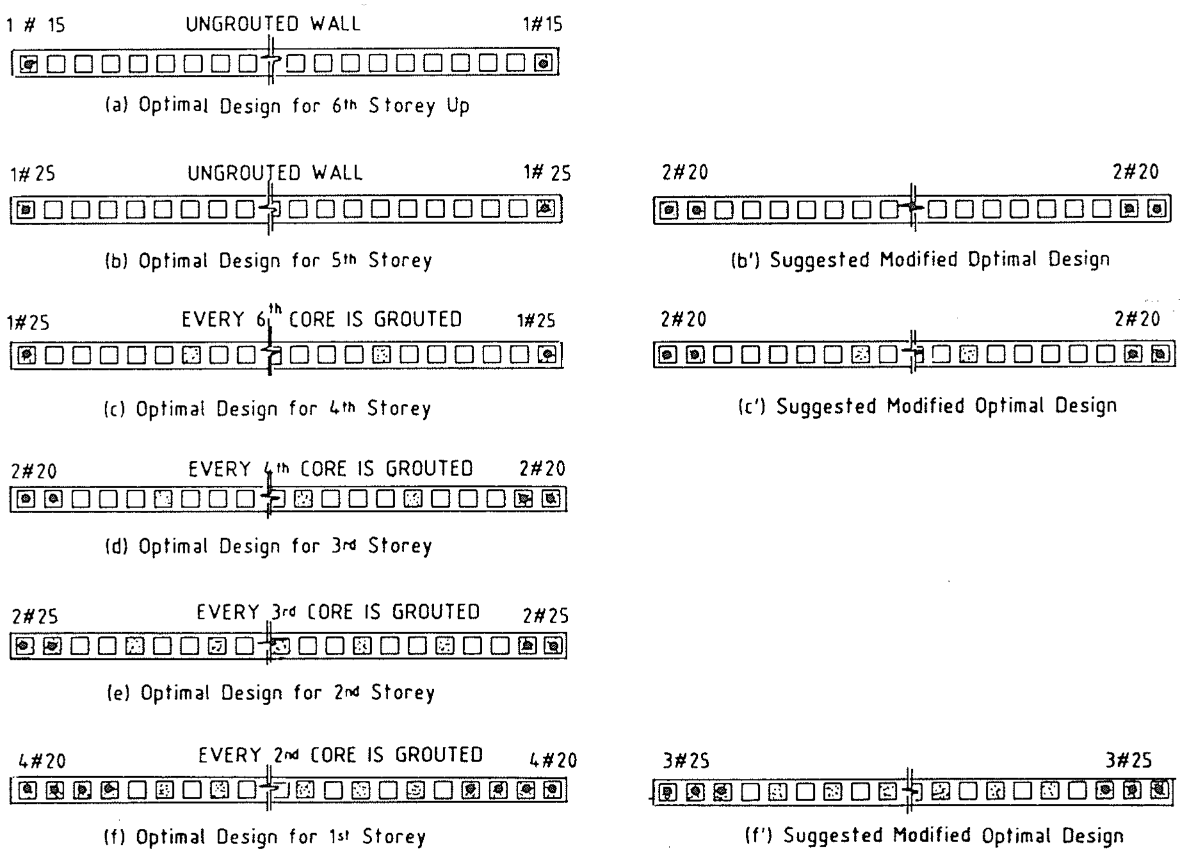

Figure 5-4 Optimal cross-sections for the different stories in a 10-storey shear wall

point. On the other hand the selection of $(NS)_m^j = (NS)_1^j = 1.0$ pointsto the fact that the axial compression resisting capacity exceeds the applied dead + live load, i.e., there is no need to add compression steel.

Moving down to the fifth floor, testing the wall for the DL + WL case has shown that subset 2 governs the design. This has been indicated by assigning the value of 1.0 to the zero-one variable designated by $(Z)^5$. Again, an ungrouted wall is found sufficient to resist the applied bending moment plus axial load combination. However, the selection of $(ZS)_k^5 = (ZS)_5^5 = 1.0$ indicates that the compression violation begins to be a serious problem which requires end steel to increase the resisting capacity. In the fourth floor, subset 2 again governs the design and $(Z)^4$ has been selected. However, the grouting condition of the wall has been increased, as required by the stress limit, from an ungrouted wall to a wall with every sixth core grouted. This condition is indicated in the choice of $(Z)_i^4 = (Z)_2^4 = 1.0$. Again end steel is found necessary to increase the resisting capacity. The optimum side steel for both stories 5 and 4 is 1 # 25 at each end $((ZS)_5^5 = (ZS)_5^4 = 1.0)$.

In the third storey, the optimum design yields a wall with every fourth core grouted, which is reflected in the choice of $(Z)_4^5 = 1.0$, while at the same time end steel has been increased to 2 # 20 at each end. At this point a call for judgement is required by the practical designer. If the continuity of steel bars with increasing sizes from top to bottom is preferred, then a modified design is required for stories five and four, so that 2 # 20 steel bars replace 1 # 25 at each end, which is suggested in Figures 5-4(b) and 5-4(c).

This suggested modification could also be applied to the first storey where the optimum design results in requirements of 4 # 20 at each end of the wall (as shown in Figure 5-4(f)), while, as shown in Figure 5-4(e), 2 # 25 at each end of the wall are required for the second storey. Therefore, the designer could replace the 4 # 20 steel bars in the first storey by 3 # 25 at each end as shown in Figure 5-4(f).

It should also be noted in Table 5-5 that the value of $(OAD)^3$ exceeds $(ORC)^3$ in the third floor which means that the original applied (DL + LL) exceeds the original resisting capacity of the section which, in turn, means that additional resisting capacity is required. This requirement is satisfied by selecting only 1 # 15 to be placed in the inside part of the wall. For the second and first storey, the required inside steel has been increased as reflected in the choices of $(NS)_3^2 = 1.0$ and $(NS)_4^1 = 1.0$.

5.5 SUMMARY AND CONCLUSIONS

In this chapter, a model has been developed for the optimum design of multi-storey masonry shear walls. The model consists of a linear objective function that calls for minimizing the total cost of blocks, grout and steel, and three types of linear constraint sets. The first type of constraints contain those relationships that describe the limitation of the loadings and stresses for two different load cases, namely, (DL + WL) and (DL + LL). It also contains the constraints that control the relationships among some of the variables in the problem. The second type includes those constraints which define the discrete values of all variables included in the model. It also defines the relationships among the "Transfer" variables. The final type of constraints organize the relationships among zero-one variables. The relationships were developed using a computer program (described Appendix D) which depends on two types of information. The first types of data are input data to be selected by the designer for some of the basic masonry structural variables in the model, e.g., selected standard sizes of steel bars. The second types of data are calculated data that are produced as a result of applying stress and load limitations relationships, e.g. $(XD)_i^j$, $(XB)_i^j$,...,etc.

The model, after being formulated, interacts with "LINDO"[24] which starts the solution process using a Branch and Bound technique[22] and continues until the optimum solution is found.

The size of the model depends, mainly, on the number of stories J, and as this number increases the size increases and, accordingly, the solution time increases. The computation time is not the only problem associated with the size of the model, but also the nature of the formulation. A "good" or "tight" formulation increases the efficiency of solution while a "bad" or "loose" formulation decreases such efficiency and may yield infeasible solutions. A few recommendations and requirements that are helpful and in some cases rather essential to produce an efficient model are presented in this chapter. However, more work is needed to produce improved models and subsequently, improved solutions, as is discussed in the recommendations in the next chapter.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

6.1 SUMMARY

The models described in this thesis are an important step toward restoring masonry, an "under-researched" material, to a competitive position among other structural materials such as concrete and steel. The primary objective of the study, namely, to develop a means whereby optimum masonry structural designs can be obtained, has been achieved.

Prior to the development of the masonry structural design optimization models described herein, a few studies were conducted on simple cases where different structural design methods, e.g., Transformed Section Analysis (T.S.A.), the Coefficient Method (C.M.), the Unity Equation (U.E.) and the Load-Deflection Method were evaluated on the basis of practical applicability and economic efficiency. Those studies also covered an evaluation of different optimization techniques, e.g., Linear Programming (L.P.), Non-linear Programming (N.P.), Goal Programming (G.P.) and Integer Programming (I.P.) on the basis of their suitability for different masonry optimization problems. Well-established guidelines derived from all those previous studies have resulted, and they provide the basis for the masonry optimization models included in this thesis. This basis is the foundation for the following aspects of masonry design, namely, the most important component of masonry structure, the most reliable structural design approaches, and the most appropriate optimization techniques for the problem of the masonry structural design optimization.

Since shear walls are an integral part of any masonry structure, being the elements which provide both resistance to lateral forces, particularly forces due to wind or earthquake, and also lateral stability for the structure, they were selected for application of the optimization models described in this study. However, these models can be easily extended to accommodate other components such as beams or columns.

Analysis of different structural design approaches provided by either the code or elasticity principles led to a number of important observations. Although the Coefficient Method yields the most economical solution in the case of bearing walls subjected to out-of-plane bending, its use is not recommended in the design of shear walls (as explained in Section 3-2-a). The Load-Deflection Method, as described in the code[4], on the other hand, ignores the contribution of compression reinforcement to the load resisting capacity and consequently has limited application. Although the Unity Equation alternative always gives the most conservative solution among methods that recognize compression steel, it is more suitable to the design of shear walls because of its conservative nature. This is particularly the case for those cases with low eccentricities, for normal stress cases, or for cases when compression governs the design. On the other hand, the Transformed Section Analysis approach gives the most economical design for high eccentricities or for cases when tension governs the design[39,40].

Evaluation of different optimization techniques resulted in recognition that linear Mixed Integer (Linear-Discrete) programming was the most appropriate optimization technique for such types of problems.

The reasons are: first, the closeness of the linearized relationships to their original non-linear forms; second, the efficiency and capabilities of linear optimization solving packages; and, third, the discrete nature of masonry elements.

Two main models have been developed in this thesis. The first is for the optimum design of an individual shear wall and the second is for the optimum design of multi-storey shear walls. Due to the limited number of grouting and steel combinations to be tested for different stress cases in the first model the Transformed Section Analysis, the Unity Equation, and the Coefficient Method relationships were applied as originally derived directly from the code requirement, and/or elasticity principles after being manipulated mathematically to fit what is termed as "Acceptable Discrete Form".

Due to the difficulties of carrying out the tension analysis (T.S.A.) for the resulting large number of steel and grouting combinations, simulated functions were developed for the second model to replace the original tension relationships. The simulated variables were derived from the Unity Equation after extensive comparisons between the simulated and original values determined from the Transformed Section Analysis approach.

A few difficulties were encountered in modeling for the highrise optimization problem, due mainly to the large size of the model. A few suggestions, modification, and recommendations are presented to assist in simplifying the problems for practical applications.

Pseudo-discrete optimization formulations were developed for both problems. The models are made up of four main parts: A linear function that describes the objective of the problem, namely, the minimization of the total cost of materials, i.e., blocks, steel, and grout; a set of continuous constraints in the form of inequalities that describe the general boundaries for different stress and load possibilities; a set of discrete constraints that define the discrete values of different variables contained in the continuous constraints. These last constraints are in the form of equalities where the discrete values of different variables are associated with different sets of zero-one (0-1) variables which correspond to the different stress and load possibilities. Finally, a set of constraints that expresses the relationships among the (0-1) variables is required. This last set of constraints was constructed in such a manner that the number of (0-1) variables is minimized.

In both problems, the optimization model was generated using computer programs which depend on input data provided by the user, the practicing engineer. The computation process that precedes the solution of the model helps in filtering the different stress and load possibilities, eliminates the unnecessary constraints, and provides the model with the necessary relationships in their "Acceptable Discrete Form" and is organized in such a way that it increases the efficiency of the solution procedure.

The optimization models were solved using a well-known commercially available optimization package, LINDO, while the computer facility used was the Amdahl 5850 at the University of Manitoba.

6.2 CONCLUSIONS

In addition to the obvious advantage that by optimizing masonry structural design, a percentage of the total cost of materials can be saved (which can be significant in some larger buildings), a number of more general conclusions have been reached. These are as follows:

1. The discrete optimization models developed allow the structural design relationships to be solved simultaneously, which eliminates the time involved in the trial-and-error process used in traditional design.
2. The confusing problem of designing a wall subjected to very small tension, a problem that has previously been solved either in incorrect or impractical ways can now be solved in a rational manner. Analysis, instead of design, of a section subjected to tensile stresses can be performed by applying different amounts of tensile steel and checking the resulting actual stresses against their allowable values. If analysis of the section, at the tension end of the wall, with the minimum amount of reinforcement, reveals a value of $(\phi)_1^k$ that is greater than one, the section is then considered safe for tension and stresses at the compression end of the wall are tested instead.
3. In designing a wall for axial applied (dead + live) loads, the axial resisting capacity re-distribution discussed in Chapter 3 achieved a two-fold purpose. First, it ensures that the design for (dead + wind) loads is maintained during the design for (dead + live) loads. Second, it contributes to the optimization process by considering the extra weight of reinforced grouted cores, a

situation which is normally ignored in traditional design.

4. From the study cases analyzed throughout this work, it appears that shear stress does not impose a serious problem in most normal masonry structures. However, horizontal joint reinforcement is always recommended.

5. It is evident that the use of the Unity Equation approach can be extended to cover cases where tension exists. The simulation analysis conducted in Chapters 4 and 5 show that in cases of tension, the U.E. can be used to determine variables needed to apply the Transformed Section Analysis approach, which permits formulating the resulting relationships in the discrete form. In the first case where both tension and compression violation exist in the wall and where tension is the more severe violation, it is concluded that with minimum steel requirement, the masonry is fully utilized and the stress distribution resulting from T.S.A. is almost identical to the U.E. stress distribution where $f_{ma} = C_s*f_m$. In the second case, where the section is subjected to tension only, the resulting stress distribution from applying T.S.A., again with the minimum required reinforcement, is found to be almost identical to the U.E. stress distribution where $f_{ma} = [P_d/(A_m*L) + M/SM]$.

It should be noted that the necessity for the simulation process arose from the fact that the direct application of T.S.A. is almost impossible, especially for highrise problems.

6.3 RECOMMENDATIONS

The research conducted in this thesis is the first of its kind in the area of masonry structures. As a new approach, the research opens doors for further work that should be directed toward investigating different aspects and problems that could assist in upgrading the understanding of this building material and subsequently strengthening its position among other building materials. The recommended further work can be summarized as follows:

First - In the area of the optimum masonry structural design:

More investigation should be directed to the optimum design of different masonry structural elements other than shear walls, e.g., columns and beams. Also, the integration of these different elements into complete system design is essential.

Second - In the area of optimization techniques adapted:

Although the general scheme of the optimization technique used in this work has been applied in the past to relatively few different optimization problems, the overall optimization process including the computational procedure prior to model development together with the inter-relationships among zero-one variables addressed in this study have not been introduced before. The large size of the models developed in this thesis necessitates the development of such inter relationships to minimize the number of constraints, zero-one variables and the computation time. Although a significant time savings has been achieved in solving most of the cases tested in this work, there remains a need and potential for developing more efficient optimization techniques. The reason for this need can be summarized as follows.

The general formulation of the pseudo-discrete model, as presented in Chapter III, can be graphically represented for two discrete variables as shown in Figure 6.1.

It should be noted that the discrete points shown on the figure present values of $\beta S1$ and $\beta S2$, only when α_i and α_i^k takes the value of 1.0. When the problem is solved using LINDO, a continuous linear programming solution representing the solution of the continuous constraints $\hat{g}_c$ $(\beta S1, \beta S2)$ $\{ \begin{smallmatrix} \leq \\ \geq \\ = \end{smallmatrix} \}$ 0, is found at the first step. The Branch-and-Bound(B&B) technique is then applied to determine the discrete values of $\beta S1$ and $\beta S2$. The branching process selects, on the basis of closeness of the variable to its feasible values of 0 or 1.0, the values of $\beta S1$ and $\beta S2$ by sequentially setting the values of α_i and α_i^k to 1.0. The process continues in a systematic manner until the optimum discrete solution is found. Further research work is recommended to develop a better search technique that selects values for $\beta S1$ and $\beta S2$ rather than α_i and α_i^k such that $\beta S1$ and $\beta S2$ are close to the vertices of the optimum feasible solution region.

Third - In the area of masonry behaviour:

The effect of additives, such as fly ash, on the strength characteristics and the cost of concrete block is an important recommended research work. The main objective of such work is to find the optimum fly ash/cement mix that minimizes the cost of producing concrete blocks while still meeting the requirements of block strength. The work should also investigate the effect of this combination on the structural behaviour of different elements (e.g. flexural and shear).

Fourth - In the area of masonry construction:

The overall masonry construction problem should also be investigated. The problem should consider the availability of different resources (e.g. materials and labour) as well as the time spans during which different construction stages (e.g. planning, designing the final building construction) are implemented with the required level of satisfaction in the most optimum periods of time.

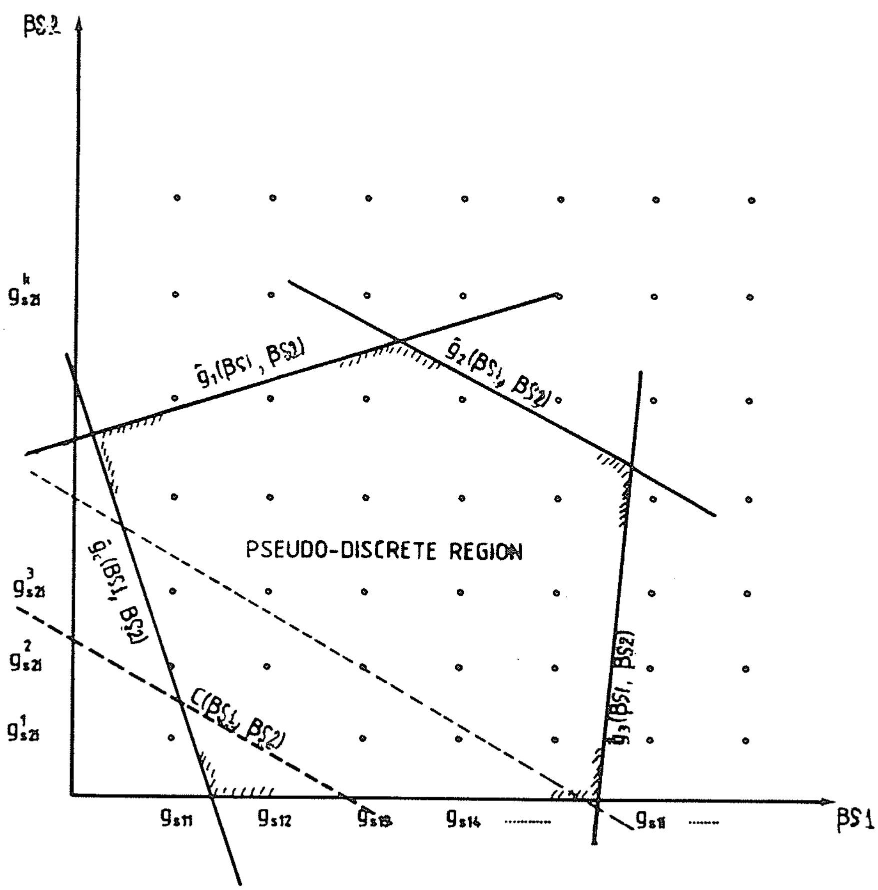

Figure 6-1. The graphical representation for the general formulation of relationships 3-1 to 3-4 (for three variables).

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Development of A_m, f_m, UW and n values:

The Canadian Code, CAN3 - 5304 M84, defines the net block area as being the net area at the mortar bed. Since, almost, one third of a block at the mortar bed area is solid, the code definition leads to the following linear relationship:

$$A_m = [0.34 + r(0.66)]\ A_g \qquad \text{(A-1)}$$

where A_m = Mortar bedded area in one metre length of the wall (mm^2/m)

r = grouted/ungrouted cores ratio (varies between 0 and 1)

A_g = the gross cross-sectional area of a wall per metre (mm^2/m)

For = 0, $A_m = A_m^{min} = 0.34\ A_g$, while for $r = 1$, $A_m = A_m^{max} = A_g$. The code has also defined two bounds for f_m associated with the two ultimate cases of grouted walls, namely, fully grouted and ungrouted. Further a linear interpolation has been permited. Therefore, if the two extreme bounds for f_m are f_m^{min} and f_m^{max}, which are associated with fully grouted and ungrouted situations respectively, the linear interpolation between A_m and f_m can be expressed as follows:

$$f_m + 0.005981\ A_m = 3.63636 \qquad \text{(A-2)}$$

where f_m: the allowable masonry compressive stress in MP_a and A_m is determined from relationship A-1, according to the variation in the ratio r.

Similarily the relationships between UW, (the unit weight of a masonry wall) or, n (the modular ratio) and A_m can be linearly interpolated and expressed mathematically in the same way as relationship A-2.

If the ratio r takes seven different values (as for the models developed in this thesis), then the discrete values of A_m, f_m, UW and n can be shown as in Table A-1.

Table A-1. Discrete Values of A_m, f_m, UW and n due to linear interpolation.

r	A_m mm^2/m	f_m MP_a	UW $KN/_{m}2$	n
1.0	$190 * 10^3$	2.5	3.91	20
1/2	$127.3 * 10^3$	2.87	3.01	17.41
1/3	$106.4 * 10^3$	3.0	2.71	16.75
1/4	$95.95 * 10^3$	3.06	2.56	16.42
1/5	$89.7 * 10^3$	3.09	2.47	16.22
1/6	$85.5 * 10^3$	3.12	2.41	16.08
0	$64 * 10^3$	3.25	2.11	15.4

Outline of Branch and Bound (B and B) method with the incorporation f zero-one variable for solving discrete optimization problems:

(N.B. The example shown herein is derived from LINDO(23) user manual)

Maximize $75\ X\ 1 + 6\ X\ 2 + 3\ X\ 3 + 33\ X\ 4$ (B-1)

subject to

$774\ X\ 1 + 76\ X\ 2 + 22\ X\ 3 + 42\ X\ 4 \leq 875$ (B-2)

$67\ X\ 1 + 27\ X\ 2 + 794\ X\ 3 + 53\ X\ 4 \leq 875$ (D-3)

where X1, X2, X3 and X4 are restricted to 0.1

The search process that a computer might follow in finding an integer optimum is illustrated in Figure B-1. First the problem is solved as an LP with the constraints X1, X2, X3, X4 $\leq$ 1. This solution is summarized in the box labeled 1. The solution has fractional values for X2 and X3 and is therefore unacceptable. At this point X2 is arbitrarily selected and the following reasoning is applied. At the integer optimum X2 must equal either 0 or 1. Therefore, replace the original problem by two new subproblems, one with X2 constrained to 1 (Box or node 2) and the other with X2 constrained to 0 (node 8). If we solve both of these new IPs, then the better solution must be the best solution to the original problem. This reasoning is the motivation for using the term "branch." Each subproblem created corresponds to a branch in an enumeration tree.

The numbers to the upper left of each node indicate the order in which the nodes (or equivalently, subproblems) are examined. The variable Z is the objective function value When the subproblem with X2 constrained to 1 (node 2) is solved as an LP, we find that X1 and X3 take fractional values. Arguing as before but now with variable X1, two new subproblems are created: one with X1 constrained to 0 (node 7) and one with X1 constrained to 1 (node 3). This process is repeated with X4 and X3 until node 5. At this point an integer solution with Z = 81 is found. We do not know that this is the optimum integer solution, however, because we must still look at subproblems 6 through 10.

Subproblem 6 need not be pursued further because there are no feasible solutions having all of X2, X1, and X4 equal to 1. Subproblem 7 need not be pursued further because it has a Z of 42 which is worse than an integer solution already in hand.

At node 9 a new and better integer solution with Z = 108 is found when X3 is set to 0. Node 10 illustrates the source for the "bound" part of "branch-and-bound." The solution is fractional; however, it is not examined further because the Z-value of 86.72 is less than the 108 associated with an integer solution already in hand. The Z-value at any node is a bound on the Z-value at any offspring node. This is true because an offspring node or subproblem is obtained by appending a constraint to the parent problem. Appending a constraint can only hurt. Interpreted in another light, this means that the Z-values cannot improve as one moves down the tree.

A Branch-and-Bound Search Tree

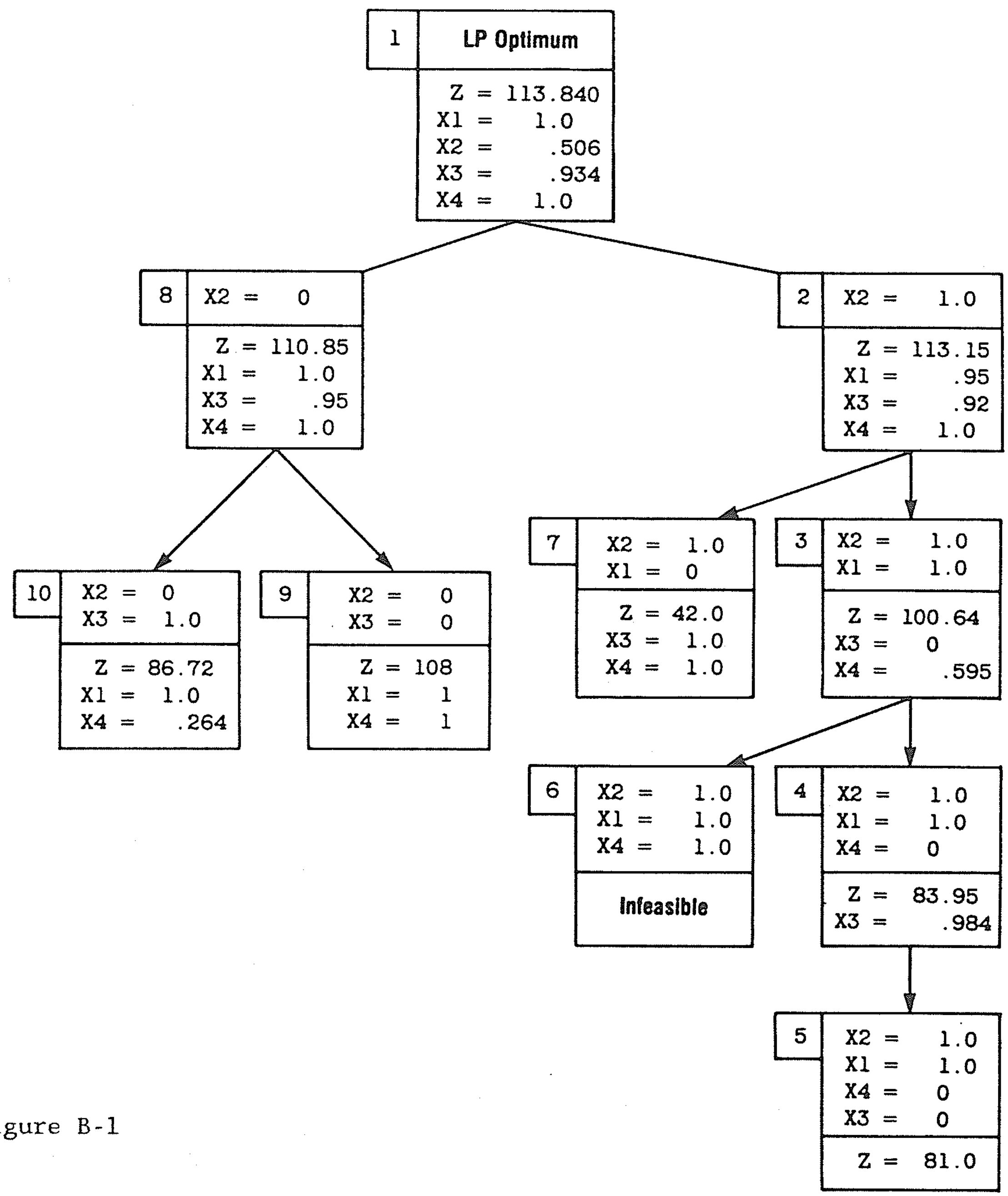


Figure B-1

The tree presented in the preceding figure was only one illustration of how the tree might be searched. Other trees could be developed for the same problem by playing with the following two degrees of freedom:

a. Choice of next node to examine, and

b. Choice of branching variable when splitting a node.

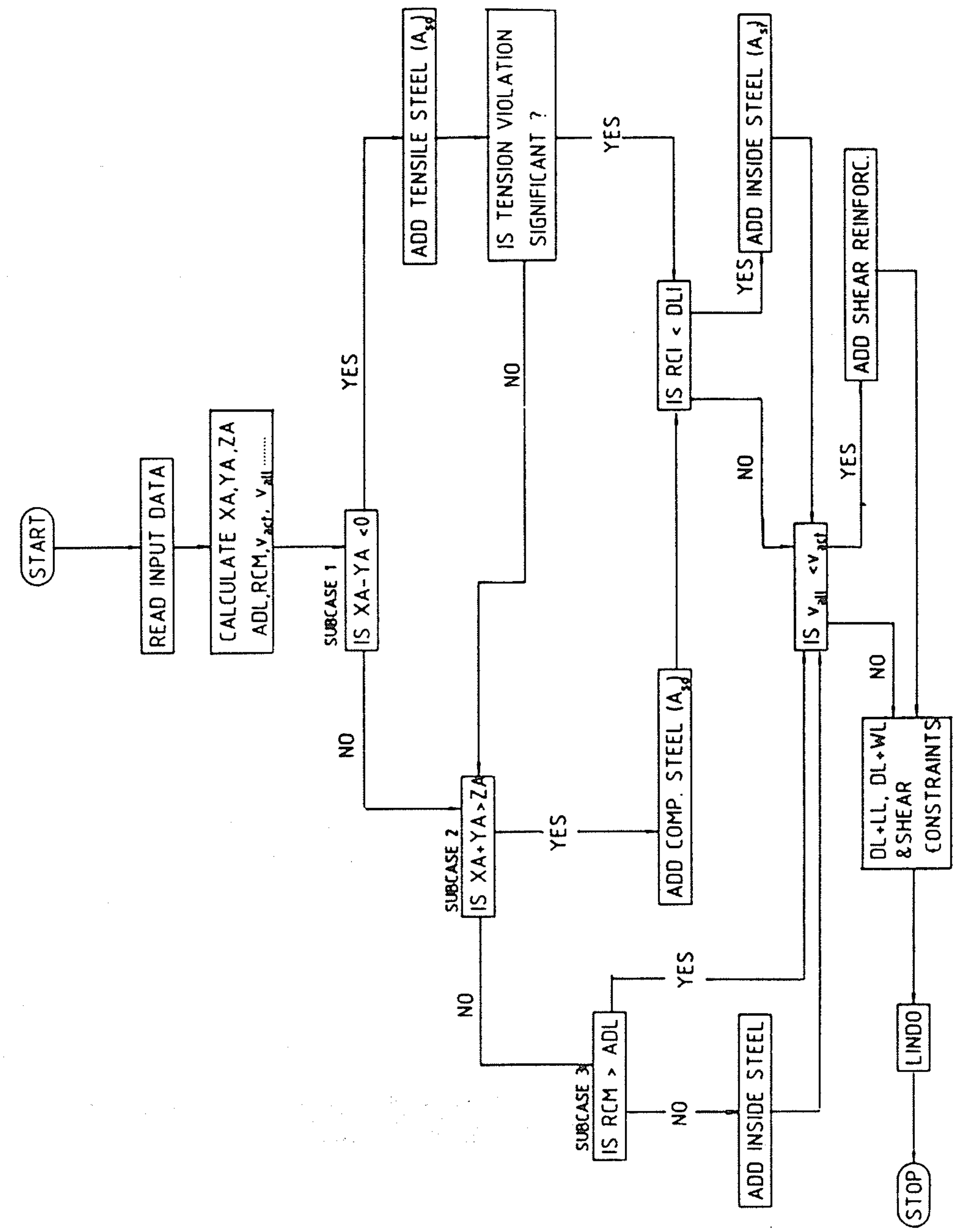
For example, if nodes 8 and then 9 were examined immediately after node 1 then the solution with Z = 108 would have been found quickly. further, nodes 4, 5, and 6 could then have been skipped because the Z-value at node 3 (100.64) is worse than a known integer solution (108) and therefore no offspring of node 3 would need examination.

In the example tree the first node is split by branching on the possible values for X2. One could have just as well chosen X3 or even X1 as the first branching variable.

The efficiency of the search is closely related to how wisely the choices are made in (a) and (b) above. In (b) one wants to choose variables which are "decisive." In general, the computer will make intelligent choices and the user need not be aware of the details of the search process. The user should, however, keep the general B & B process in mind when formulating a model. If the user has a prior knowledge that an integer variable x is decisive, then for the LINDO program it is useful to place x early in the formulation to indicate its importance. This general understanding should drive home the importance of a "tight" LP formulation. A tight LP formulation is one which, when solved, has an objective function value close to the IP optimum. The LP solutions at the subproblems are used as bounds to curtail the search. If the bounds are poor, many early nodes in the tree may be explicitly examined because their bounds look good even though in fact these nodes have no good offspring.

APPENDIX C

Flowchart for the computation process for the single-storey shear wall problem.



APPENDIX D

Flowchart of the computation process for the multi-storey masonry shear wall optimization problem: