STATISTICAL ANALYSIS OF BANK EROSION OF THE BRAHMAPUTRA RIVER IN BANGLADESH

by

MD. ABU OBAIDA ANSARI KHAN

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ABSTRACT

The object of the study is predicting the amount of riverbank erosion and deposition along the Brahmaputra river between Bahadurabad and Aricha, Bangladesh. For this purpose the Transfer Function Noise Model of Box and Jenkins has been used in the space-time domain to identify a model and to estimate its parameters. The data base for this method is the satellite imagery of banklines at ninety seven locations (1525 metres apart) over ten years. The appropriate Transfer Function was found to be a first order moving average model with a noise term that may be modelled by a first order autoregressive process. The performance of the model was found to be better than that of any univariate model. For example, the one-year ahead forecast of the model, using the average parameters derived from the record can predict whether erosion or deposition occurs correctly 95% of the time. The predicted amount of bank movement has a standard error of about 270 metres. The mean west bankline was found to be shifting westward at an average rate of about 120 metres per year with no definite pattern of variation with discharge. Other parameters, including those of the noise model, do show significant relations with mean discharge.

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Chapter I

INTRODUCTION

The problem of bank erosion of the Brahmaputra river has long been a serious concern to the people and to the government of Bangladesh. Each year, a large number of people are dislocated from their homesteads and hundreds of hectares of land are lost. The country, being the world's most densely populated, has difficulties handling the social and economic pressure caused by bank erosion. Also, any developmental planning near the river is hampered by the uncertainty in the future location of the riverbank.

Of the hundreds of rivers of Bangladesh the major three, the Ganges, the Brahmaputra and the Meghna, have drawn worldwide attention because of their vastness and the magnitude of the problems which flooding and bank erosion creates. Rahman (1978) studied the erosion of the Padma. Galay (1980) found that serious bank erosion exists in all three rivers. In general, the bank erosion of the Brahmaputra river is the most serious (Petrobangla, 1983) and calls for special attention.

Erosion within the main river banks is an additional problem. It causes subchannels to widen or to reduce in size. At bifurcations near islands, channels often change direction thereby causing erosion of the semi-permanent islands, where people live and cultivate their crops. This problem sometimes becomes so serious that a whole island is wiped out.

1.1 GENERAL DESCRIPTION OF THE RIVER

Originating in the northernmost ranges of the Himalayas in Tibet, the Brahmaputra river flows through India and enters Bangladesh in the north (Figure 1.1). It joins the Ganges in Bangladesh near Goalundo and then takes the name Padma. Further downstream the river meets the Meghna and drains into the Bay of Bengal as the Lower Meghna.

The Brahmaputra receives snowmelt from the Himalayas and other mountains in the region. A large part of its discharge comes from the heavy rainfall caused by the monsoon in India and Bangladesh. The river also receives discharges from numerous tributaries. Prior to 1787, the river followed a different course located more towards the southeast. By 1830, however, one of its distributaries suddenly became the main channel and the old river course, became a mere overflow channel now known as the Old Brahmaputra. Apart from this major avulsion, the river banks have experienced many changes over the years.

Chapter II gives a physical description of the river in the Bangladesh reach including the geology of the area, the drainage basin, hydrological and morphological features and

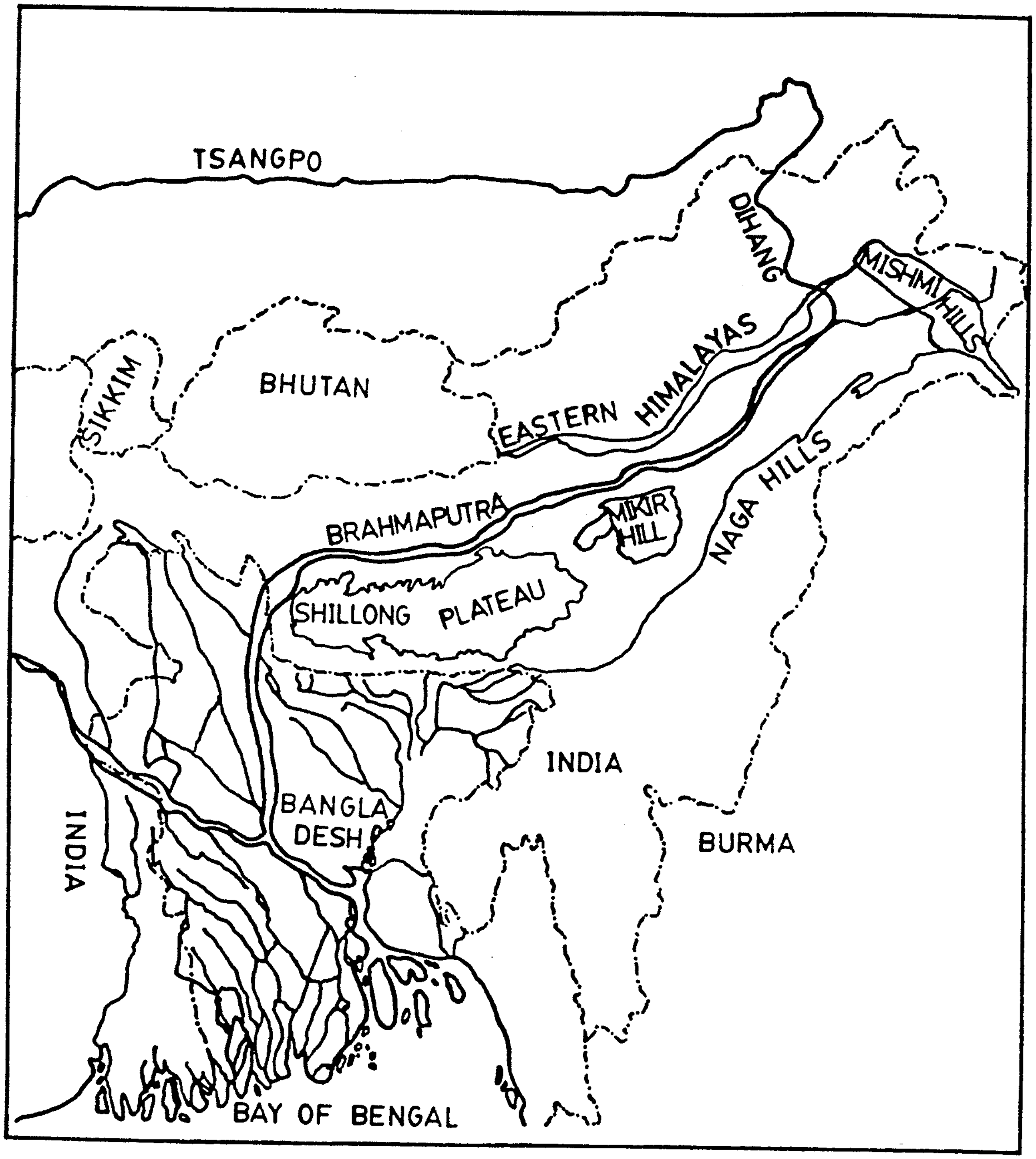


Figure 1.1: Course of the Brahmaputra River (After Murthy and Sastri,1972).

associated computational results. The history and the magnitude of the bank erosion are also presented in the same chapter.

1.2 OBJECTIVE OF THE STUDY

The enormity of the bank erosion problem along the Brahmaputra river makes any method of arresting river erosion unrealistic. The object of the study is therefore, to develop a conceptual, statistical model that is capable of predicting on a short term basis the amount of erosion or deposition one can expect along the outer bankline of the river. The results of this study can be used in connection with development planning of river bank areas and to help alleviate the socio-economic consequences resulting from loss of land and dislocation of population by river bank erosion and deposition. Channel changes within the outer banklines will not be addressed in this study.

Major changes in channel course, like the avulsion which happened by 1830, cannot be predicted statistically. The investigations do, however, include looking for any progressive changes which may in the future cause a major shift in the course of the river.

1.3 RESULTS OF EARLIER STUDIES

The Brahmaputra river is a braided stream that has experienced severe bank erosion throughout recorded history in a manner that seemed to defy analysis or prediction. Earlier studies showed that on the average the river tends to move westward but locally the movement appeared to be erratic. Also, the presence of some nodal points with little or no change has been reported (Coleman,1969). But, it remained unclear whether the nodes are a transient or a permanent feature. In all previous studies the continuity in either space or time has not been analysed because of the complexity of the problem. As a result, the reported rate of erosion or deposition has not been consistent. Also, no attempt was made in earlier studies to predict the bankline position over space and time.

The changes in location and/or cross-section of the bankfull channel as well as the branches inside are associated with the braiding process. The main aspects of the braiding process, as presently understood, is briefly described below.

1.3.1 Process of braiding

Due to its complexity there has been very little success in explaining the braiding process. On the other hand, the relatively simple meandering process has been given considerable emphasis, which has led to the development of a unified bar-bend theory (Blondeaux and Seminara, 1985) and other aspects of time dependency of meandering rivers (Parker and Andrews, 1986).

Two distinct approaches in explaining the braiding process are mentioned in the literature. The empirical approach tries to relate the observed characteristics of the braided channel among themselves and with other variables, e.g., the discharge, the slope, etc. From field and experimental data, coefficients of the relationships have been evaluated in the past. Obviously, these attempts do not answer the basic question as to why the river braids or meanders in the first place. Also, the relationships obtained are experiment or region specific. Therefore, the understanding of the overall braiding process from these attempts is only partial.

In the analytical approach, the unified treatment of meandering and braiding has resulted in theories that describe these two processes as different results of the same instability phenomenon. Three types of models have been used to investigate this problem. Reynolds (1965) used the potential flow model. Adachi (1967), Callander (1969), and Parker (1976) investigated this problem by means of a shallow water flow model, and Engelund and Skovgaard (1973) used a shear flow model. In all of these models, a basic or undisturbed flow is assumed over which a perturbation is superimposed. The factor causing the instability of erodible bed is the phase difference between the shear stress gradient (or the tractive force gradient in the case of potential flow model) and the bed form gradient. Callander (1969) used a two-dimensional model that neglects the velocity variation along verticals and includes the internal friction by a one-dimensional description. Engelund and Skovgaard (1973) considered the three dimensional character of flow, introducing an assumption of an "effect of gravity" on the transverse component of sediment transport. These stability models have succeeded in explaining a number of aspects of meandering and braiding. So far, no attempt has been made to quantify the lateral extent (or amplitude) of the braids.

These theories all lead to the conclusion that, in all cases, the rate of growth of the braids increases with the increase in number of braids which does not seem right. Hayashi and Ozaki (1980) used a different approach to obtain the dominant meander and braid length. The spatial lag distance of sediment, parameter originally proposed and dealt with by Kennedy (1963), was used to modify their shallow water flow model. This lag distance in non-uniform flow was obtained analytically from the Einstein-Brown (1950) formula modified by Tsubaki and Saito (1967). The perturbation method used was similar to that of Parker (1976). Hayashi and Ozaki's dominant wave length and initial growth formulae are given in Appendix-A. The braiding mode, in their formulae, that prevails in nature is the one corresponding to the condition when the initial growth rate is positive and maximum.

The available analytic or empirical theories do not allow quantification of erosion or deposition along the river. Some other means of modelling bankline erosion/deposition using physical and/or historical information must therefore be sought.

1.4 SELECTION OF MODELLING APPROACH

In principle, two approaches to mathematical modelling are available, one deterministic and the other stochastic. The deterministic approach of modelling channel changes concentrates on the formulation and solution of the equations based on physical laws. The stochastic approach, on the other hand, deals with the preservation of the history of the channel together with conservation of the basic physical laws. The latter approach can be used successfully where the formulation and solution of the desired physically based equations become too complex (Nordin et al., 1967).

Most of the available deterministic models of river channel changes have been developed to simulate the bed aggradation or degradation as a result of natural or man-made changes. These mathematical models have used continuity, momentum and sediment transport equations without considering the changes in the river bank. Deposition in the bed is, however, often accompanied by channel widening, while channel bed erosion is often associated with a reduction in channel width.

In a series of papers, Chang (1976,1977,1978,1980,1982, 1984) has reported on a model capable of incorporating the interrelated changes in channel bed profile, width and lateral migration in channel bends. This model employs a space-time domain and routes water assuming it to be uncoupled with the sediment process. The sediment routing is done by solving the sediment continuity equation in the longitudinal direction and computing the bed material load using a formula suitable for the physical condition and bed material composition. By observing the analogy of the minimum stream power with the minimum energy principle in thermodynamics and least work in solid mechanics, Chang (1980) defined the hypothesis of minimum stream power as :

> "For an alluvial channel, the necessary and sufficient condition for equilibrium is when the stream power is a minimum subject to given constraint."

and used this hypothesis to simulate width variation. It may be noted that the unit stream power concept of Yang (1976) will produce the same result only if the cross-sectional area remains the same (Chang, 1978). The lateral migration in a bend has been modelled by considering the transverse current, transverse sediment movement and the continuity equation of sediment in the same direction. Since the bank stability depends upon a large number of local factors, the rate of bank erosion has been multiplied by a coefficient determined from the river data to account for such factors.

The major problem of deterministic erosion modelling is that the work on processes of morphological change of a river as a whole and within channels is not well developed (Alexander, 1979). The physical process of sedimentation cannot be adequately characterized as morphological changes except for certain bed forms (Kishi and Kuroki, quoted by Krishnappan and Snyder,1979). However, the prediction of these bed forms is not accurate even though the resistance of flow offered by the bed forms can be found out once the form is known. Even if it is possible to create an adequate model of the friction flow at the channel boundaries, the resulting increments and losses in each part of the cross-section as erosion occurs, will be affected by the way the boundary is made numerically discrete (Bridge, 1976). The turbulent and three dimensional characteristics of flow as it occurs in the natural channel also have to be included in the model. Although the theory of turbulence is well advanced, eroding flows in natural channels are usually too complex for the theory to be applied without great intricacy. Tracey (1965) and Chiu et al. (1973) have presented the methods for solving three dimensional forms of continuity and momentum equations, but these methods are restricted to steady flow and downstream changes can only be modelled for simple conditions. More data are required to solve these equations compared to those for the simpler forms.

It has been suggested that deterministic mathematical modelling is essential to the understanding of the flow patterns in such complex systems as the Brahmaputra-Ganges delta in Bangladesh (Cunge et al., 1982, p. 133). But, in view of the above mentioned difficulties and for the following reasons, it has been decided to adapt a statistical approach.

The process of bankline changes in the Brahmaputra river is too complex and involves variables whose interactions are not yet completely understood. Even simplified analytical techniques that use mechanics of erosion and deposition require detailed topographic and extensive hydraulic data. The highly braided and unstable nature of this river requires these data to be closely spaced over time and space. These data are not available at the present time. Also, modelling of the complex local processes using simplified methods may not be suitable for application on the whole river reach due to the oversimplified assumptions needed by the methods and the data.

The variables pertinent to the process of erosion and deposition are stochastic in nature and hence should not be treated as deterministic. Uncertainties involved in these variables are not included in the methods based on the dynamics of water and sediment transport. Therefore, statistical methods that use the recorded bankline history must be employed for an assessment of the bank erosion and deposition of this river.

1.5 AVAILABLE DATA

As in many developing countries, the spatial and temporal distribution of available hydrologic and geomorphic data of the Bangladesh rivers are neither continuous nor abundant. Except for the preparation of a few old land survey maps, a rigorous and systematic data collection was not started until the mid-sixties. However, water level records at a few stations are available from 1949. Geomorphic data describing the locations of the banklines and the thalwegs over time, hydrologic data of water and sediment discharge (including water surface elevation), and geologic data describing the division and history of the area have been available since the early 1960's.

1.5.1 Geomorphic data

The first survey of the river was done by Major James Rennel between 1764 and 1773. At that time the river flowed through the Old Brahmaputra channel. The earliest survey of the new course of the river was made by Captain Wilcox in 1830 (Latif, 1969). Location maps from land surveys made by the former East Pakistan Water and Power Development Authority

(EPWAPDA) and the present Bangladesh Water Development Board (BWDB) are available from 1956 onwards. Multispectral Scanner (MSS) images from LANDSAT are available from 1972. Thus geomorphic data are available in two forms: (a) location maps from land surveys, and (b) LANDSAT satellite data.

1.5.2 Location maps from land survey

Bankline and thalweg location data of 1830, 1956, 1963, 1966, 1974, 1976, 1977, 1982, and 1983 are available from the maps published by BWDB (1978b). These maps are based on the measured records at 34 stations between Jalangarkuri in the upstream near the junction with the Teesta, and Nagarbari in the downstream near the Ganges-Brahmaputra confluence, a river reach of approximately 206 km (121 miles). Although some of the earlier maps were subsequently checked with aerial photographs and ground controls during the period of 1953 to 1957 and necessary corrections were made, it has been suggested not to use the pre-1952 maps due to serious errors (IECO, 1964).

The available maps at a scale of 1:120,000 are insufficient cartographically. The drawn banklines between cross-sections that average 6.4 km(4 miles) apart are approximate. Bankline location maps of 1867, 1875, 1935, 1944 and 1952 are also available (Coleman, 1969).

1.5.3 LANDSAT Satellite data

The LANDSAT satellite data are available in photographic and digital format. The digital data contained in Computer Compatible Tapes (CCT) display reflectance values of bands 4,5,6 and 7 for each ground resolution or size of 79mX79m. These data can provide valuable information on the land-water interface. The more recent LANDSAT Satellite can provide higher resolution with the Thematic Mapper (TM), which have an instantaneous field of view of 30m square (Richason, 1983). The Thematic Mapper data are available from 1982.

Cross-sections taken during the winter low flow period, are available for thirty-two sections along the river for the period 1976 to 1983 (excluding 1982).

1.5.4 Hydrologic data

The hydrologic data collection stations on the Brahmaputra river in Bangladesh are shown in Figure 1.2. Daily water discharge measurements are available at Bahadurabad, Chilmari, Sirajganj and Nagarbari. The longest available record is at Bahaduarabad, just below the diversion of the Old Brahmaputra, where daily mean discharge records from 1965 onwards are available. The stations at Chilmari, above the point of junction with the Teesta; at the Railway Ferry at Sirajganj, and at Nagarbari above the Ganges-Brahmaputra confluence have provided continuous data since 1965

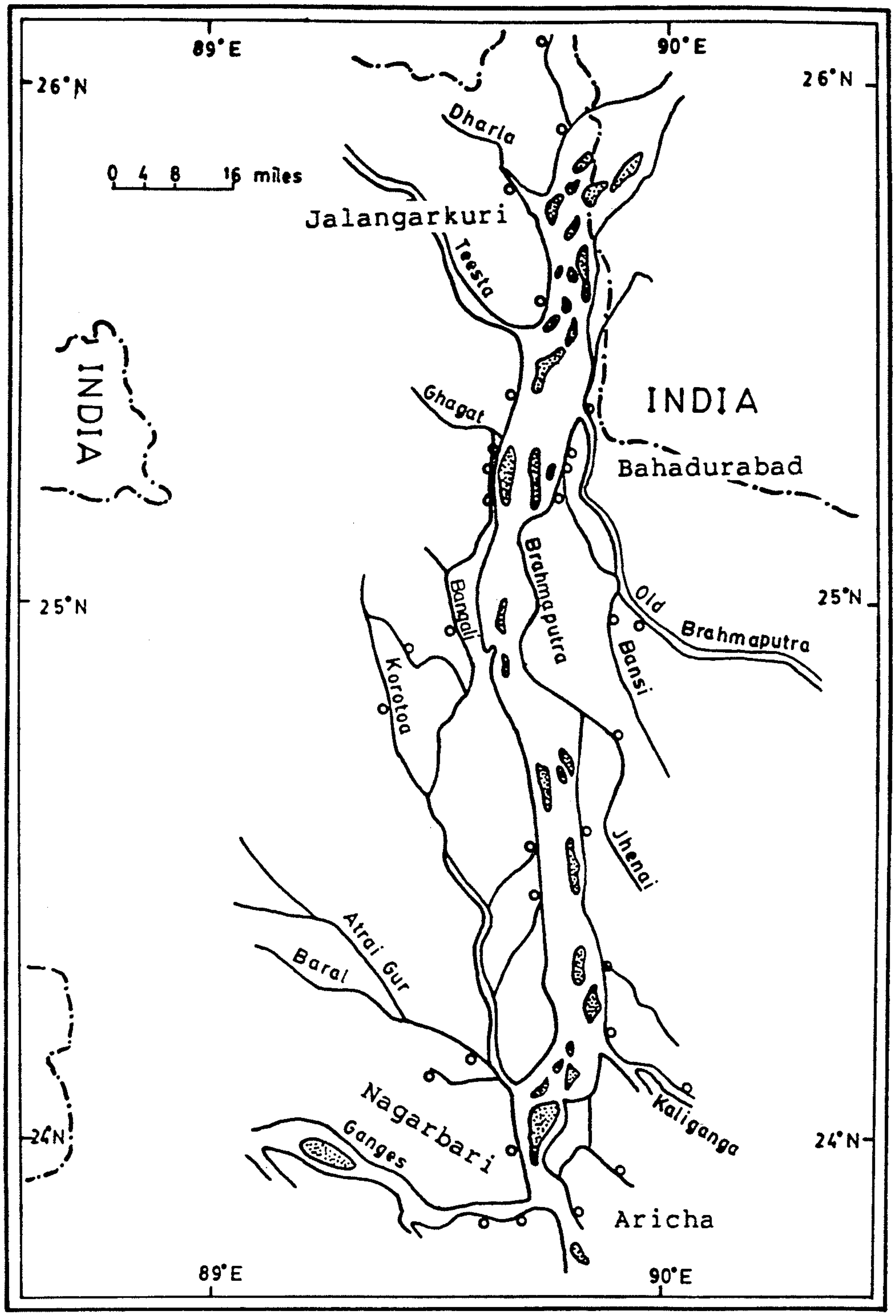


Figure 1.2: Data Collection Stations on the Brahmaputra River (stations are shown as circles)

(Latif,1969) and daily mean flow records are available since that time. Records at other stations are available with BWDB. Pre-1965 data are reported to be incomplete and are less reliable (Petrobangla, 1983).

Daily water level records are available at several measuring stations including Chilmari, Bahadurabad, Kazipur, Porabari, Sirajganj and Nagarbari. The longest record, starting from 1941 is available at Porabari. Water level measurements at other stations on the various tributary and distributary channels of the Brahmaputra are also available.

Unlike the water discharge, the sediment discharge data have not been collected continuously. Records of suspended sediment loads during flood events are available at some selected stations, like Sirajganj and Nagarbari. Size distribution of these point integrated sediment samples are also available (Latif,1969). As no bed load data are available, sediment load as discussed in the text represents only the suspended portion.

1.5.5 Geologic data

Since the location of a river depends on the geologic history and the present geologic charactersitics of the basin, it is important to look at these data. Geologic subdivision and characteristics of the Bengal basin have been reported by Morgan and McIntire (1959). Since 1959, modifications of these subdivisions and their characteristics, along with mapping of tectonic activities within the basin, have been carried out by the Geological Survey of Bangladesh.

Chapter II

PHYSICAL CHARACTERISTICS OF THE RIVER

The process of riverbank displacement is influenced by the geology, the topography and other characteristics of the river basin. This chapter will provide a brief description of the location, extent and geology of the area as well as the origin, geomorphic and hydraulic characteristics of the Brahmaputra river.

2.1 LOCATION

Situated between latitude 20^0 35' N and 26^0 75' N, and longitude 88^0 03'E and 92^0 75' E, Bangladesh is a part of the world's largest delta (Figure 1.1). Bounded by the Bhagirathi-Hoogly on the west, the Padma and the Meghna on the east, the Bay of Bengal on the south, the Bengal delta has an area of 595,000 sq. km. (230,000 sq. miles) (Bhattacharya, 1973).

2.1.1 Formation of the Bengal basin

According to the theory of plate tectonics, the component units or the plates of the earth's crust have collected together, broke-up and reformed several times during the past history of the earth. About 200 million years ago, the latest continental break-up occurred (Gordon, 1972).

Consequently, the Indian portion of the supercontinent moved northward and collided with the East Asian and Eurasian plates resulting in the rise of the Himalayas and the Arakan-Yomas, and was subducted under the East Asian plate along the line of the Himalayas. The northeastern part of this plate fractured and sank below sea level in the Oligocene period to be filled up by sediments washed down from the highlands over the next 37 million years to form the Bengal delta (Rashid, 1978). The Ganges-Brahmaputra-Meghna river system must have played the most important role in this land building process.

2.1.2 Description of Bengal basin

The Bengal basin is bounded by the Lower Jurrasic trap rocks in hills on the west, the Eocene sandstone and limestone Shillong Hills on the northeast, the Tripura and Chittagong Hills on the east, and the Bay of Bengal on the south (Coleman, 1969). Except for the four major Pliestocene alluvial terraces, the Barind, the Madhupur Tract, and the two flanking terraces in the east and the west, the remaining regions consist predominantly of Recent alluvial and deltaic sediments (Figure 2.1). The Pliestocene sediments which have been subjected to tectonic activity, are highly oxidized, reddish-brown in color, and more compacted and weathered; their boundaries with the Recent sediments being straight and distinct (Coleman, 1969). On the other hand, the loosely compacted and gray Recent

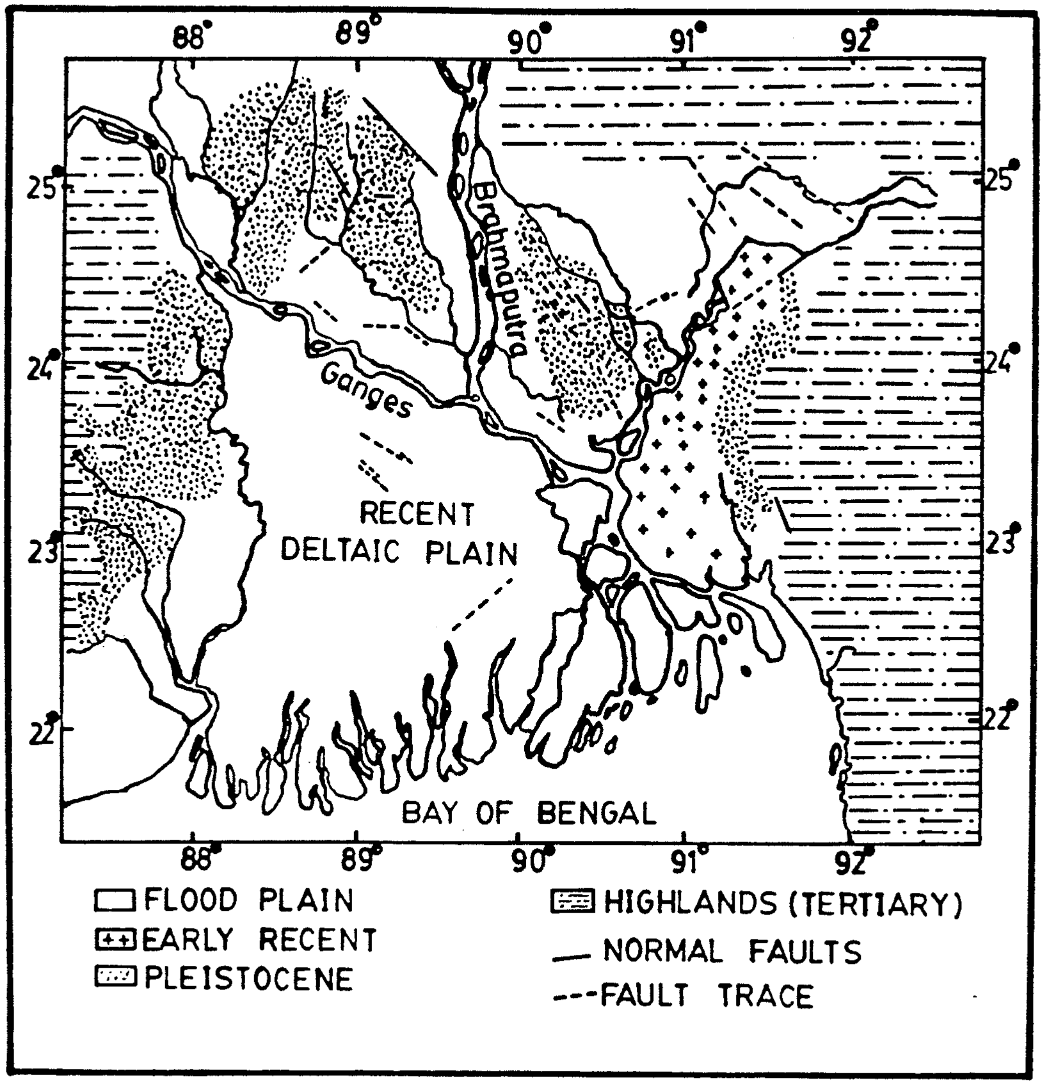


Figure 2.1: Geology of Bengal Basin (After Morgan and McIntire,1959)

sediments are composed of sand and silt in the upper valley, silt, clay and peat in the lower valley, and have a higher water content.

2.1.3 Tectonic activity in the Bengal basin

The sharp changes in the river course have been studied in relation to the subsurface structure of the region. It has been concluded that the tectonics of the area have controlled the course of the Brahmaputra river (Murthy et al., 1981). The active tectonic nature of the region is due to the subduction faults in the north and transform faults in the east. The severe earthquakes of 1762 and 1897 and more than twenty other earthquakes since 1900 are the evidence of this tectonic activity.

Figure 2.2 shows a plot of epicentres of earthquakes from historical and instrumental earthquake data up to 1971 (Chaudhury et al., 1981). The earthquakes shown are of siesmic magnitude of five or more on the Richter scale. The figure shows clearly that the Brahmaputra flows through an area which is highly tectonic in nature. Devastating earthquakes have occurred in the vicinity of the river. Within Bangladesh, earthquakes of siesmic magnitude of eight in the Richter scale have occurred.

The earthquakes of 1897 and 1950 in Assam, both of Richter magnitude 8.7, caused landslips, rockfalls on hillslopes, subsidence of ground and changes in course and

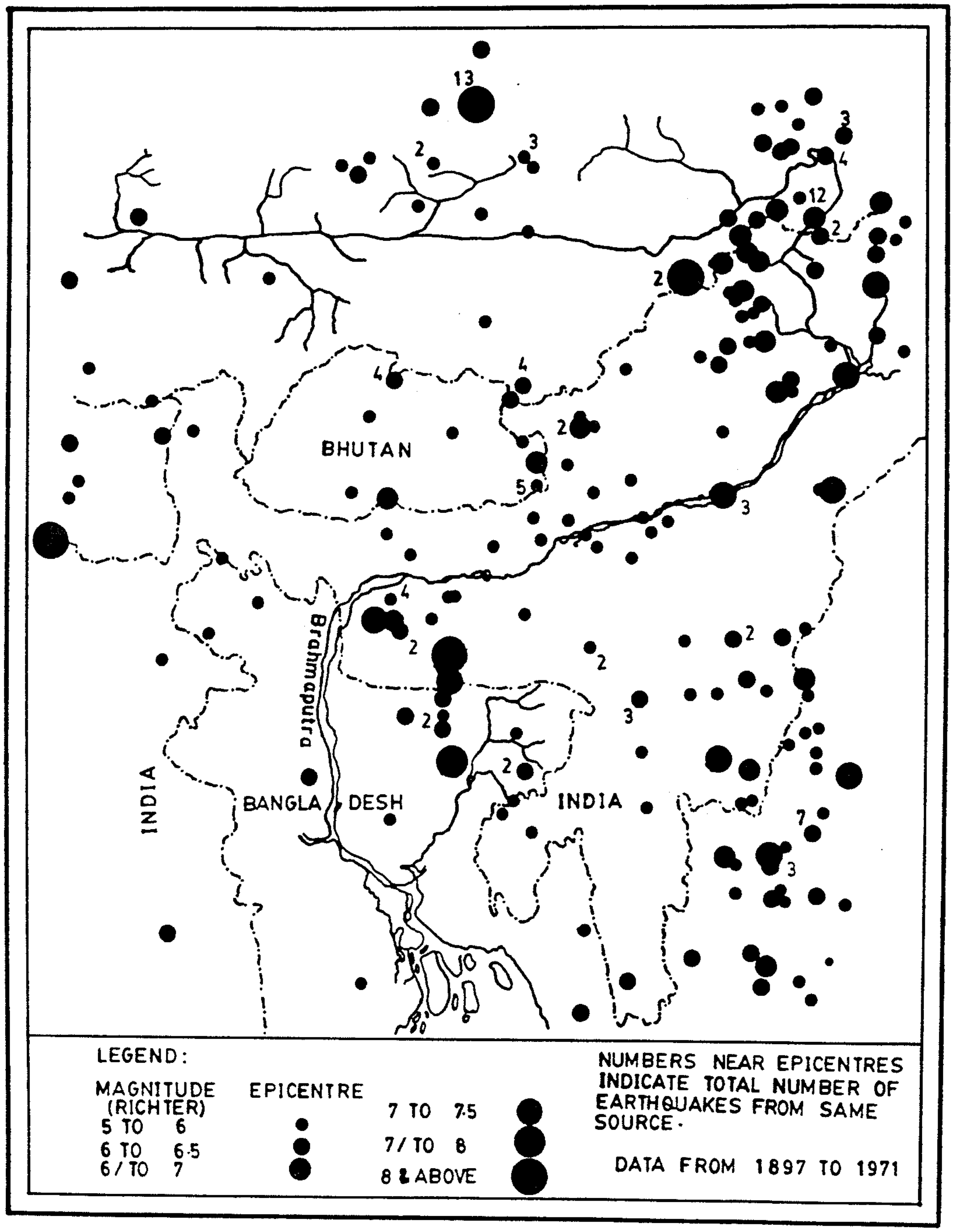


Figure 2.2: Spatial Distribution of Earthquake Epicentres (After Chaudhury et al., 1972)

morphology of some tributary channels. The courses of the Subansiri in Assam, the Dihang and Dibang in Arunachal Pradesh in India were temporarily blocked by the landslide caused by the 1950 earthquake. Subsequent release of the ponded water by sudden bursting of the dams created devastating floods downstream (Poddar,1952, quoted by Goswami, 1983) and supplied enormous amounts of sediment into the river system. A portion of that sediment, some believe, is still in the river system and is being carried to the sea.

2.2 THE BRAHMAPUTRA RIVER

The Brahmaputra river originates in the northernmost ranges of the Himalayas in Tibet as the Tsangpo river (Figure 1.1) and flows eastward for about 1125 km(700 miles). It then turns northeast, runs through many gorges and takes a southwest turn. After receiving flows from the tributaries Dihang and Luhit, it takes the name Brahmaputra and flows south into Assam and curves sharply west for about 725 km (450 miles) before entering northern Bangladesh with a sharp southern turn. Continuing along this course, it joins the Ganges near Goalundo. From its source to the Ganges-Brahmaputra confluence, the length of the river is 2900 km (1800 miles) of which 275 km (170 miles) is in Bangladesh (BWDB, 1978b).

The Brahmaputra signifies the son of Brahma, the creator of the universe. From the viewpoint of its vastness, grandeur, distinctive and erratic behaviour, it is in fact one of the greatest creations on earth and in a figurative sense might be entitled to that appellation. Downstream of the diversion at Bahadurabad, the river is locally known as the Jamuna. Some of the government agencies refer to this river as the Brahmaputra-Jamuna.

2.2.1 Change in course of the Brahmaputra

The river had a different course within Bangladesh before 1787. At that time, it followed the course now known as "Old Brahmaputra" in a south easterly direction and joined the Meghna at Bhairab Bazar. The Jenai and Konai rivers were flowing through the area of the present Brahmaputra main channel. After 1787 these two spill channels enlarged and by 1830, they formed the present course carrying nearly all the flow.

Different opinions were put forward as to the possible cause of this change. Ferguson (1863) suggested that an uplift in the Madhupur tract may have caused the diversion but was disregarded by La Touche (1919) who advocated that the beheading of the Tsangpo river of Tibet by the Dihang tributary increased the flow thereby initiating the change (Rashid, 1978). Hirst (1916) suggested a zone of subsidence between the Barind and the Madhupur Pliestocene blocks as the cause of this diversion. This theory was opposed by Hayden and Pascoe (1919) who supported the argument of La Touche. Morgan and McIntire (1959) supported Hirst's argument of the existence of a zone of subsidence and concluded that the change in the Brahmaputra river was in response to the steeper gradient and was initiated by a single flood event.

2.2.2 Drainage basin

The drainage basin of the Brahmaputra has an area of 580,000 sq. km (224,000 sq. miles) of which 293,000 sq. km (113,000 sq. miles) are in Tibet, 241,000 sq. km (93,000 sq. miles) in India, and 46,000 sq. km (18,000 sq. miles), about 8%, in Bangladesh. The river carries snowmelt from the mountains and rainwater from the drainage basin. The heavy rainfall in Assam has a major effect on the discharge of the river. The rainfall distrubution in Bangladesh is shown in Figure 2.3.

Within Bangladesh, the major tributaries to the Brahmaputra are the Dharla, the Teesta and the Korotoya-Atrai-Hurasagar, and the major distributaries from the river are the Old Brahmaputra and the Dhaleswari. A large number of small streams also join to and take off from the river. The major tributaries and distributaries are shown in Figure 1.2.

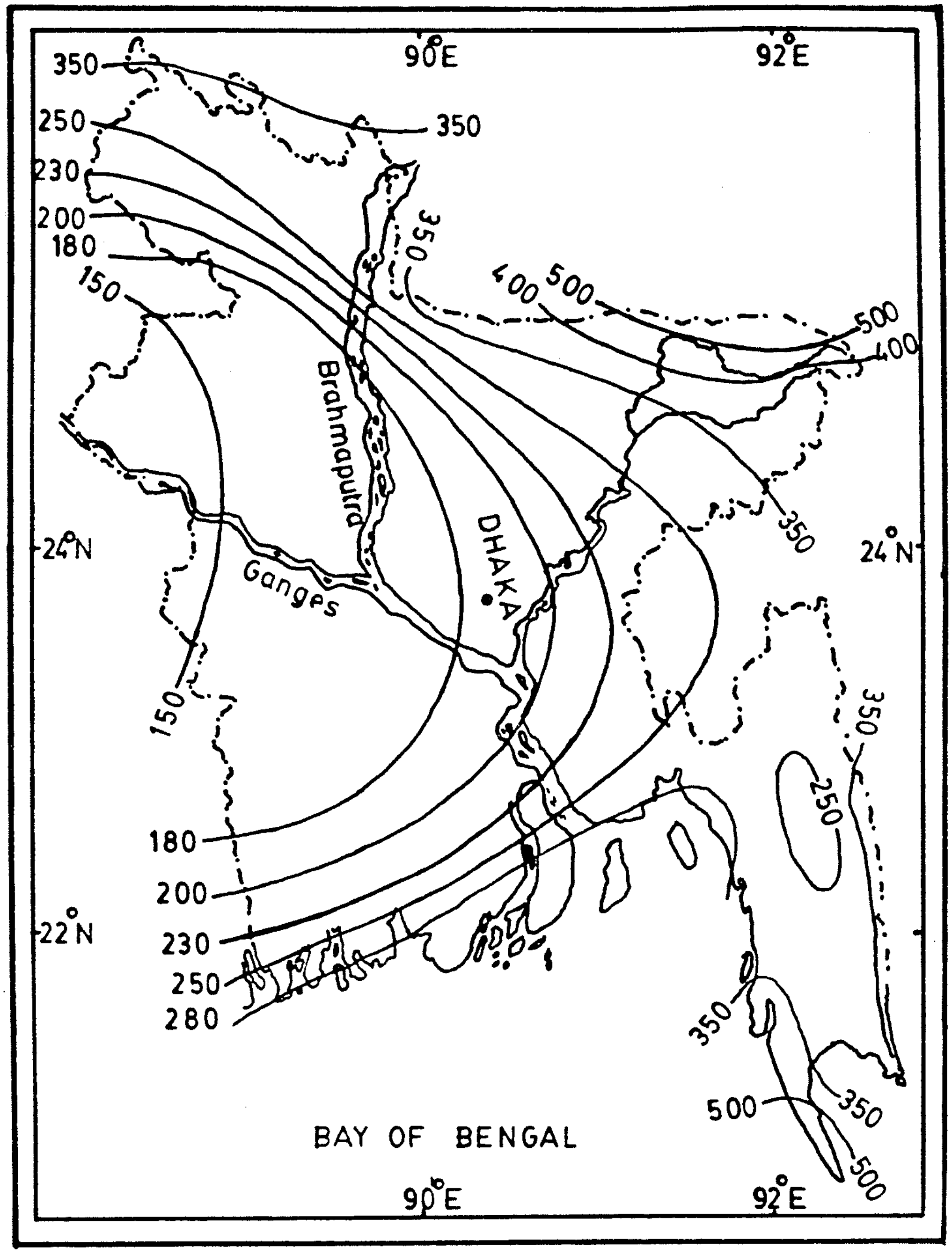


Figure 2.3: Mean Annual Rainfall in Bangladesh (After Rashid, 1977)

2.2.3 Distance elevation plot

Distance elevation data have been taken from 32 cross-sections with water levels for the months of November through April. These cross sections are approximately evenly spaced between Jalangarkuri in the upstream and Aricha near the Ganges-Brahmaputra confluence in the downstream (Figure 1.2).

It has been observed that water surface elevations in various channels of this multi-channel river are different for several cross sections even on the same date. At places this difference is as high as 0.9 m (3 ft). In addition, elevations in the same channel measured an hour later can vary as much as 0.15 m(0.5 ft). The effect of superelevation of water in a bend, the time lag of elevation measurement and the effects of wind might be the causes of water elevation variations.

Water surface elevations of the low water months for the period 1970-1984 have been plotted against the downstream distance from Jalangarkuri. The longitudinal profile (Figure 2.4) can be divided into two different slope regions. The upper reach is steeper (slope 0.000105) compared to the lower one (slope 0.000053). The average slope is 0.0000819 i.e., 0.432 ft/mile compared to the average flood water surface slope of 0.35 ft/mile (Coleman, 1969). The bank elevation plot with distance for the entire river has been taken from Goswami (1985) and is also shown in Figure 2.4.

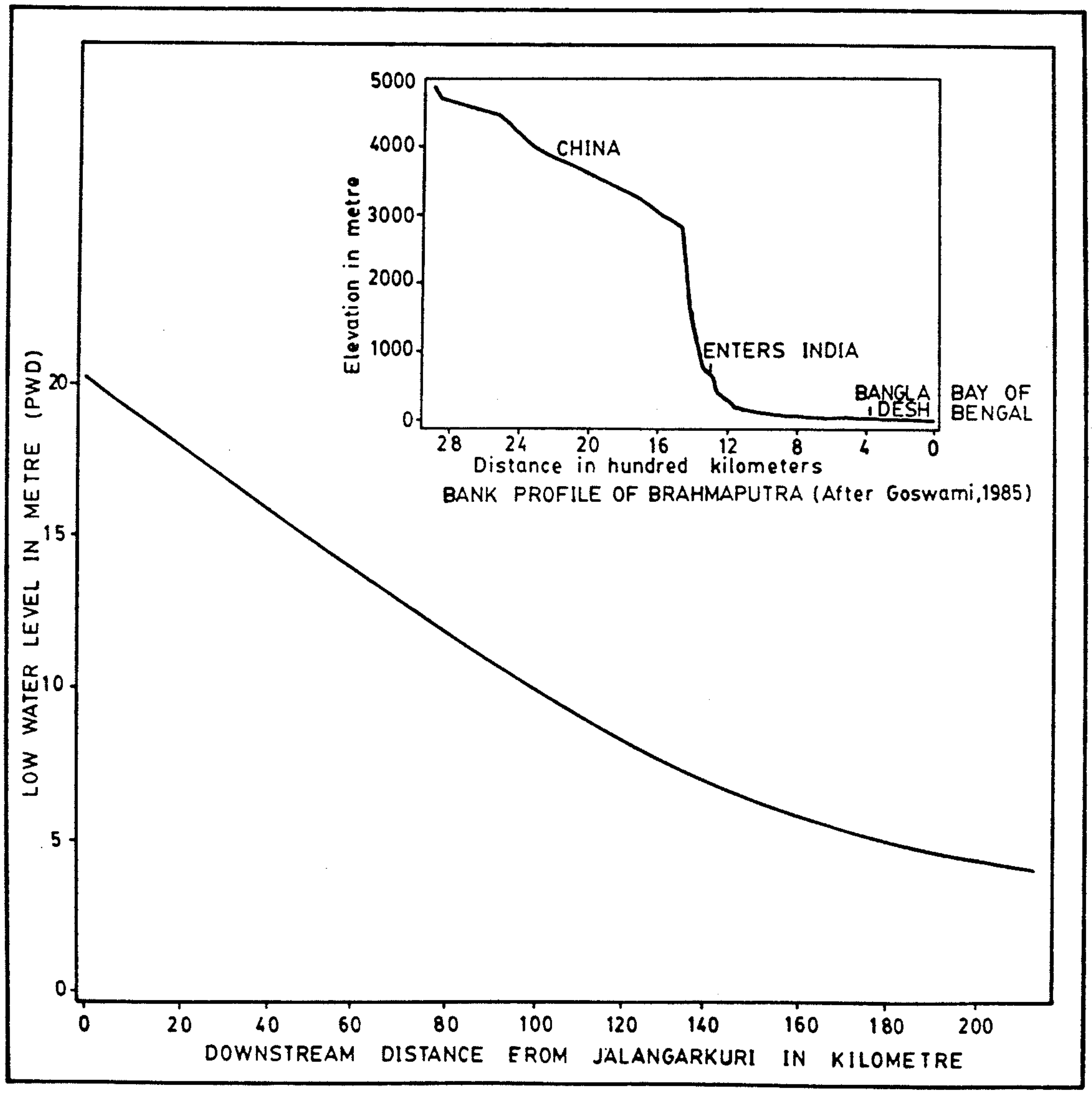


Figure 2.4: Distance Elevation Plot of the Brahmaputra River in Bangladesh

2.2.4 Composition of bed and bank material

The river bed is composed of medium and fine sands, and similar material are found up to depths of more than 50 metres (164 ft) below the river bed (Petrobangla, 1983). During flood periods, the river carries about 5 million tons of sediments per day, with the total annual sediment discharge being in the order of 600 million tons. The variation in sediment size distribution across the channel and at different locations along the channel has been studied by different organizations. There is difference between the size distributions at Bahadurabad and Sirajganj with D_{50} being within 0.25 to 0.15 mm (FAO, 1966-67, quoted by Petrobangla, 1983). The D_{65} did not vary during 1966 to 1969 across the channel but dropped from 0.3 mm in February to 0.22 mm in October. From Nagarbari to Sirajganj, D_{65} was fairly constant at 0.18 mm. At all the stations, the bed material was found to become coarser during the flood season.

The estimated Manning's roughness coefficients at different locations of the river are as follows (BWDB, 1978b) :

Nagarbari	:	0.022
Sirajganj	:	0.018
Gabargaon	:	0.021
Bahadurabad	:	0.035

2.2.5 River morphology

In the early years following the 1830 change of course, the Brahmaputra was a meandering channel, perhaps, due to the early meandering course of the Jenai and the Konai which were the original rivers in the course (Thomas, 1970). At present, however, the river is braided with multiple channels, between widely spaced high banks, separated by large sand shoals, or islands, locally known as chars. The width of the river varies between 1.5 km (0.93 mile) and 17 km (10.6 miles).

2.2.6 Water discharge

The river carries snowmelt from the mountains and rainwater from the basin with discharges as high as 91,000 cumecs (3,217,000 cusecs) and as low as 2860 cumecs (101,000 cusecs). The ratio of the maximum discharge to the minimum discharge is 31.85. More than one flood peak is present on the hydrographs as demonstrated by the flow data shown in Figure 2.5. The first peak occurs between between late May and mid June, and is the result of snowmelt. Second or more peaks occur generally in July or August and is caused by heavy rainfall in the basin.

The time variation of the day of occurrence of peak flow is shown in Figure 2.6. Maximum flow may occur as early as late June and as late as early October and is usually caused by rainwater. Generally, it occurs between July and August.

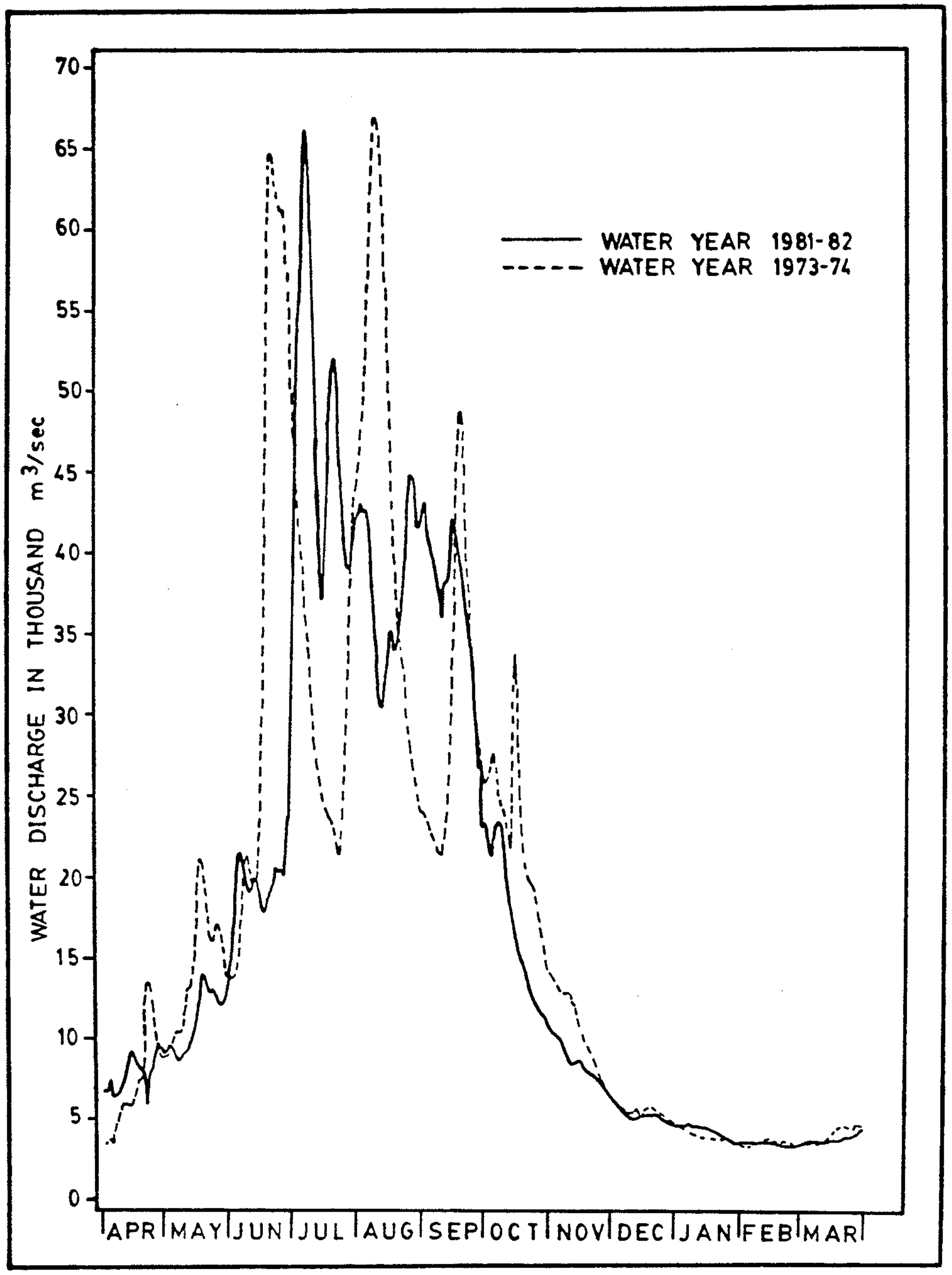


Figure 2.5: Water Discharge Hydrograph (Brahmaputra River at Bahadurabad)

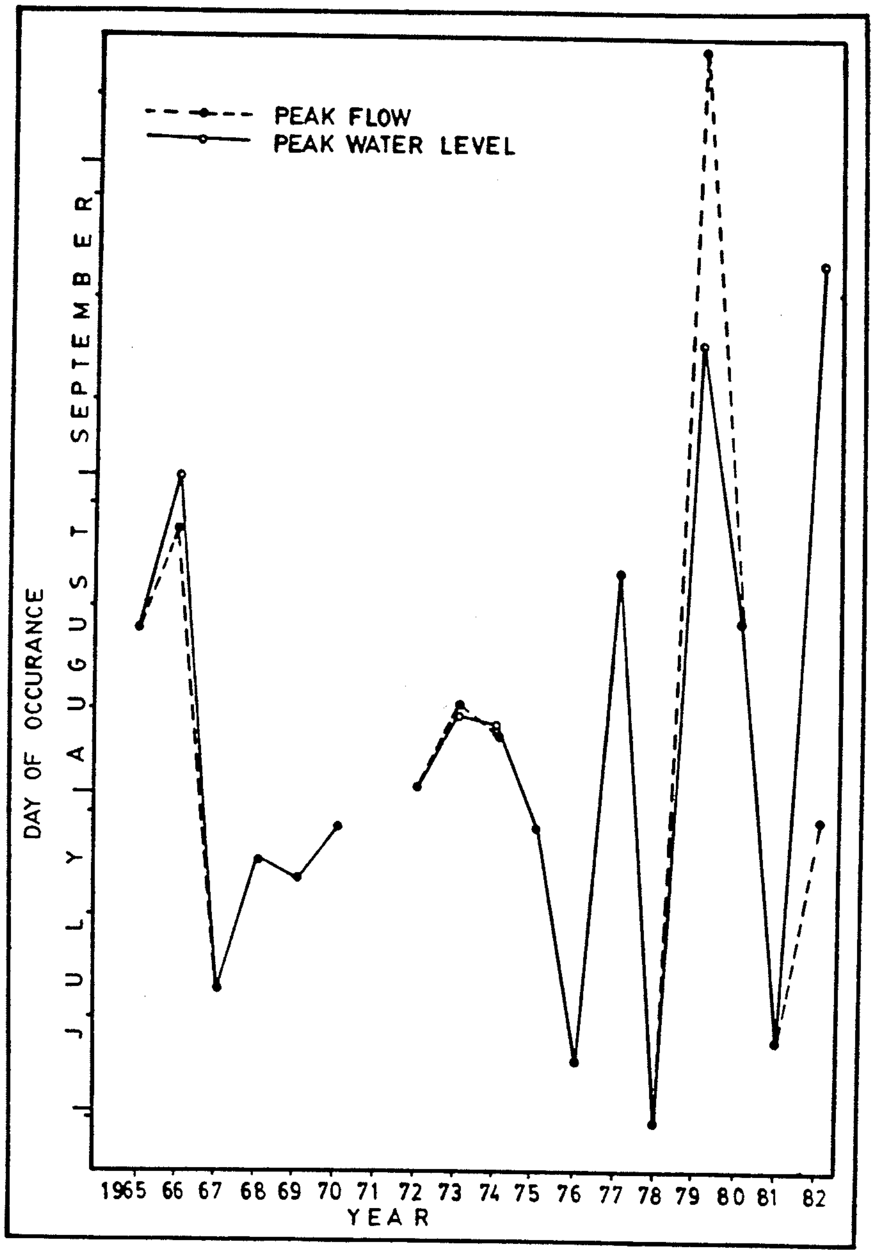


Figure 2.6: Time Variation of Day of Peak Flow, Brahmaputra River at Bahadurabad (1965-82)

Minimum flow is between January and April (Figure 2.7) with a general occurrence period between February and March. Variations of annual maximum, annual mean and annual minimum discharges with time are shown in Figure 2.8. The recorded maximum discharge varied between 50,000 cumecs to 91,000 cumecs with a mean of about 60,000 cumecs. The mean annual discharge is about 20,000 cumecs. The variation of annual range of discharge is shown in Figure 2.9. As can be seen from Figure 2.9, the range varies substantially from year to year with values from about 48,000 cumecs to 88,000 cumecs. The total annual runoff varies between 425 billion cubic metres and 735 billion cubic metres (Figure 2.10). The total annual runoff in water years 1981-82 and 1982-83 are very low compared to other recorded years.

2.2.7 Water level

The recorded maximum water level at Bahadurabad is 20 m (65.6 ft) and the minimum water level is 12.8 m (42.1 ft) above P.W.D. (Public Works Department) datum with a range of 7.20 m (23.62 ft). The values at Sirajganj are 14.24 m (46.7 ft), 6.40 m (21.0 ft), and 7.84 m (25.7 ft) respectively. The danger level at Bahadurabad set by BWDB (Bangladesh Water Development Board) is 19.36 m (63.5 ft). The annual lowest water level shows a rising trend at Bahadurabad. Since the minimum flow does not show such a trend it has been concluded that the river bed at that station is rising (BWDB, 1978b).

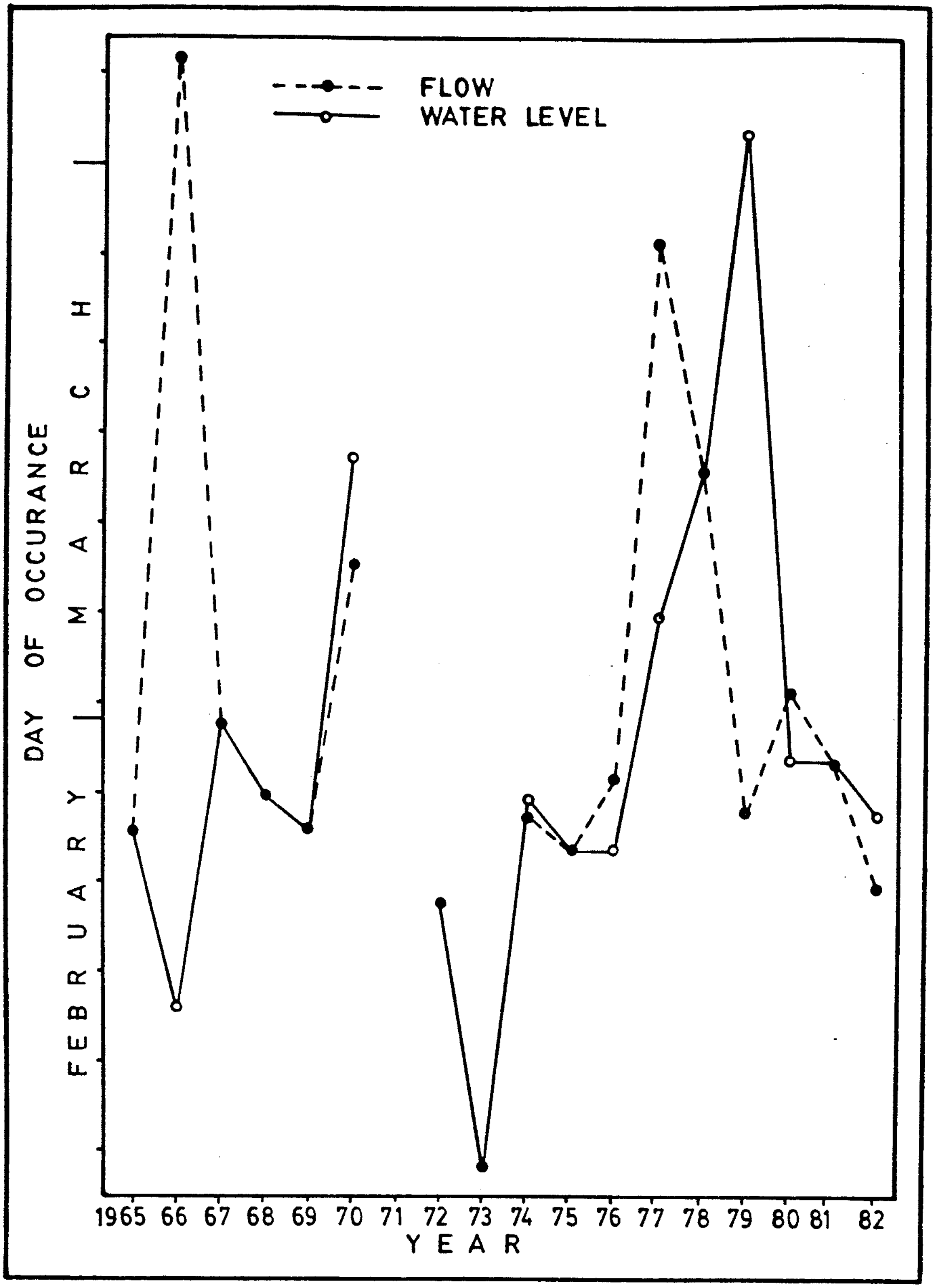


Figure 2.7: Time Variation of Day of Minimum Flow, Brahmaputra River at Bahadurabad (1965-82)

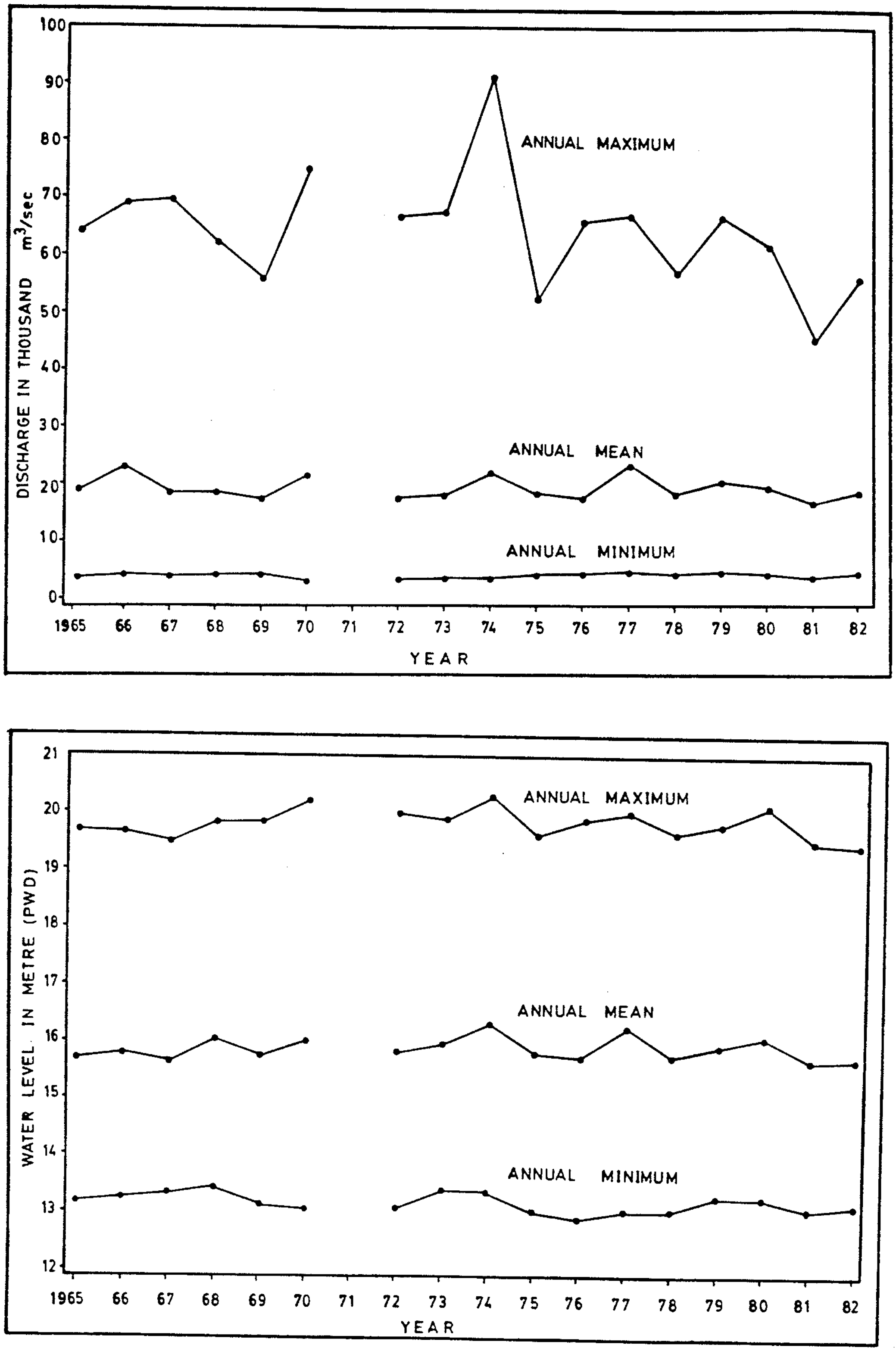


Figure 2.8: Variation of Water Discharge with time, Brahmaputra River at Bahadurabad (1965-82)

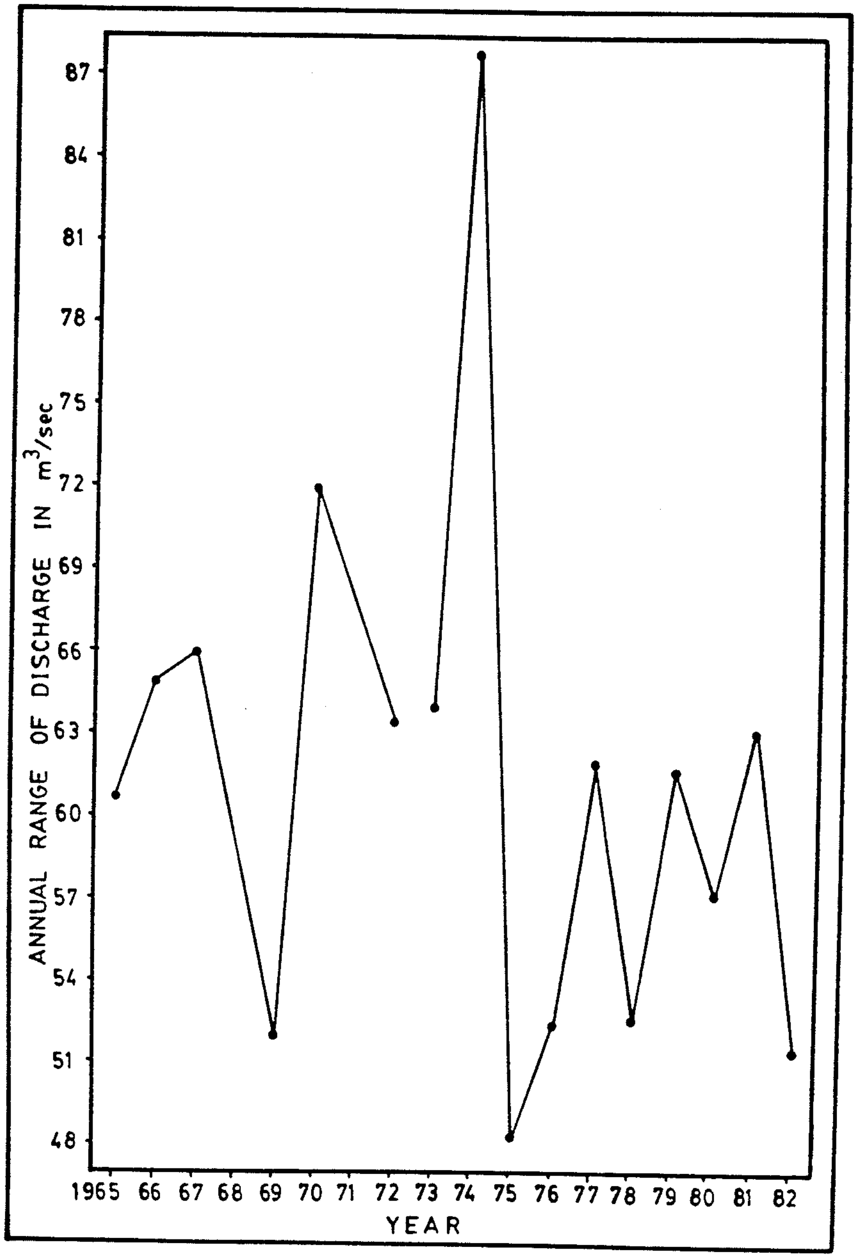


Figure 2.9: Time Variation of Annual Range of Discharge, Brahmaputra River at Bahadurabad (1965-82)

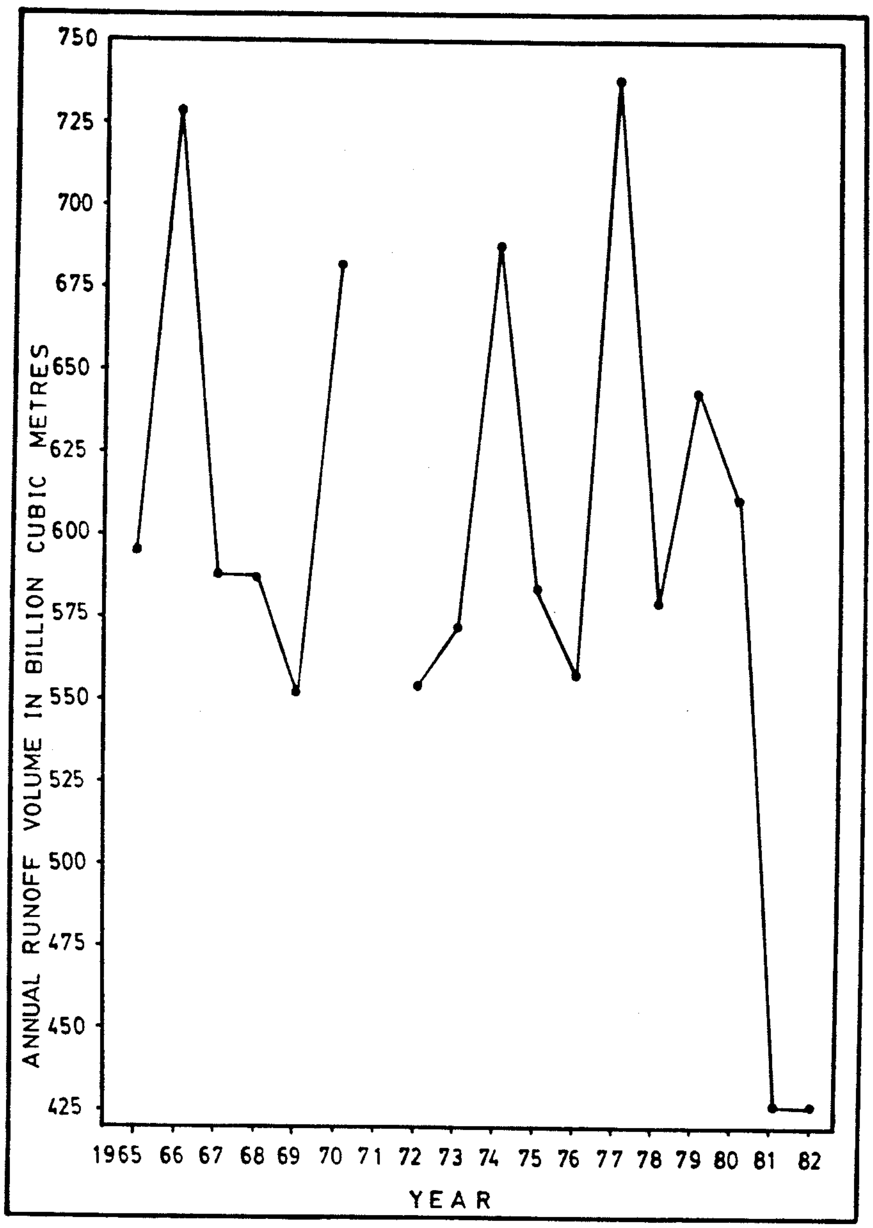


Figure 2.10: Variation of Annual Runoff with time, Brahmaputra River at Bahadurabad (1965-82)

Some of the water level hydrographs of the Brahmaputra river at Bahadurabad are shown in Figure 2.11. These hydrographs also contain more than one peak and are almost in phase with the discharge hydrograph. The inconsistency in phase relationships between water level and discharge will be discussed later. Time variation of the day of occurrence of peak water level is shown in Figure 2.6 in which the phase relationship between water level and discharge has also been demonstrated. The peak water level occured between late June and mid October with a tendency to occur generally between July and August. The minimum water level is quite out of phase with the discharge and occurs between January to April. The variation of annual maximum, mean and minimum water level with time is shown in Figure 2.8. Time variation of annual range of water level is shown in Figure 2.12.

2.2.8 <u>Stage discharge relationship</u>

The water level and discharge data of the Brahmaputra river at Bahadurabad were analysed to obtain the relationship. This recording station is in the upper steeper reach of the distance elevation plot (Figure 2.4). Seventeen years of daily discharge and water level records between 1965 and 1982 were used. No data of 1971 are available due to the Bangladesh liberation. The recorded water year starts on the first of April and ends on March 31.

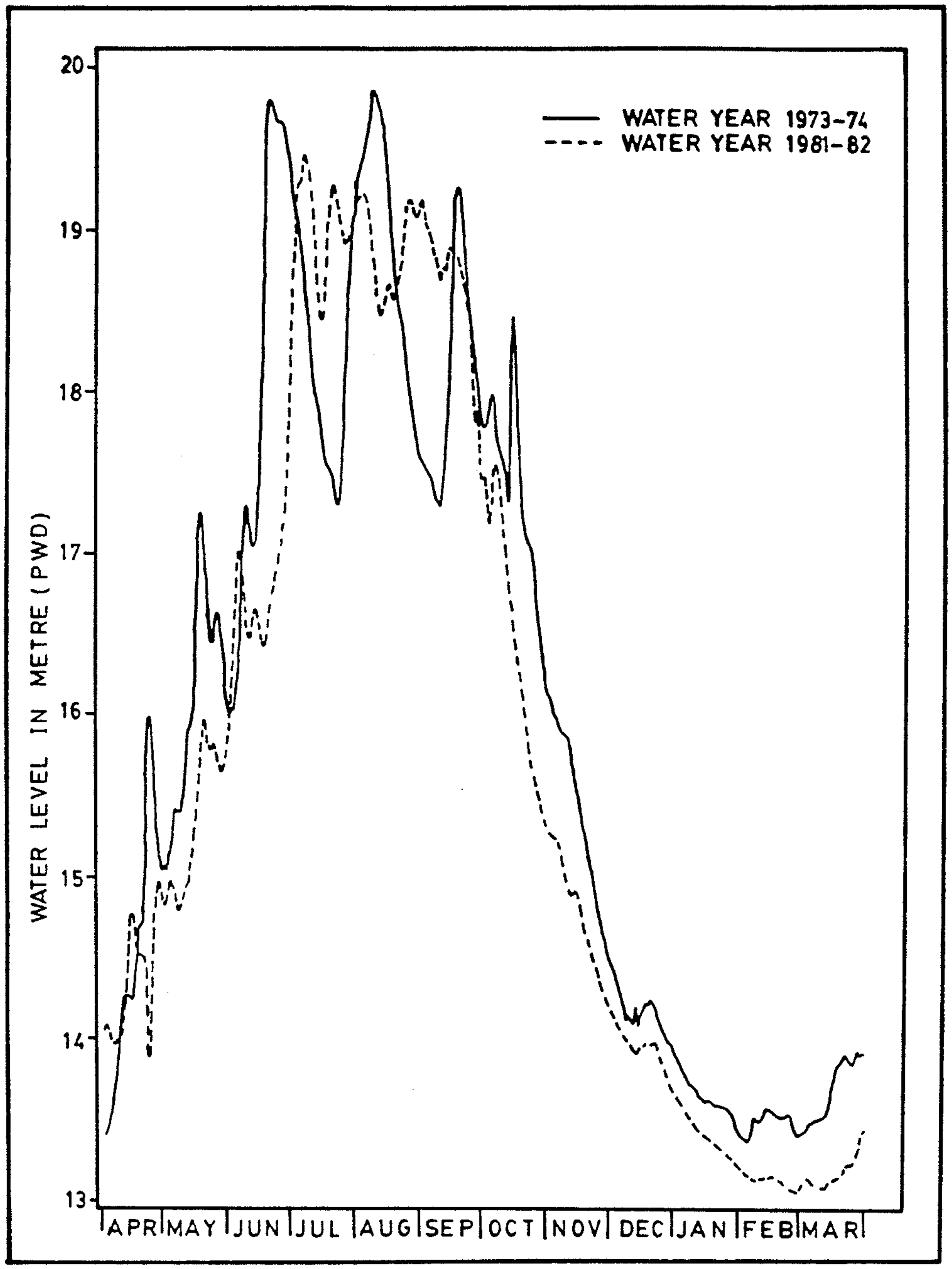


Figure 2.11: Water Level hydrograph, Brahmaputra River at Bahadurabad

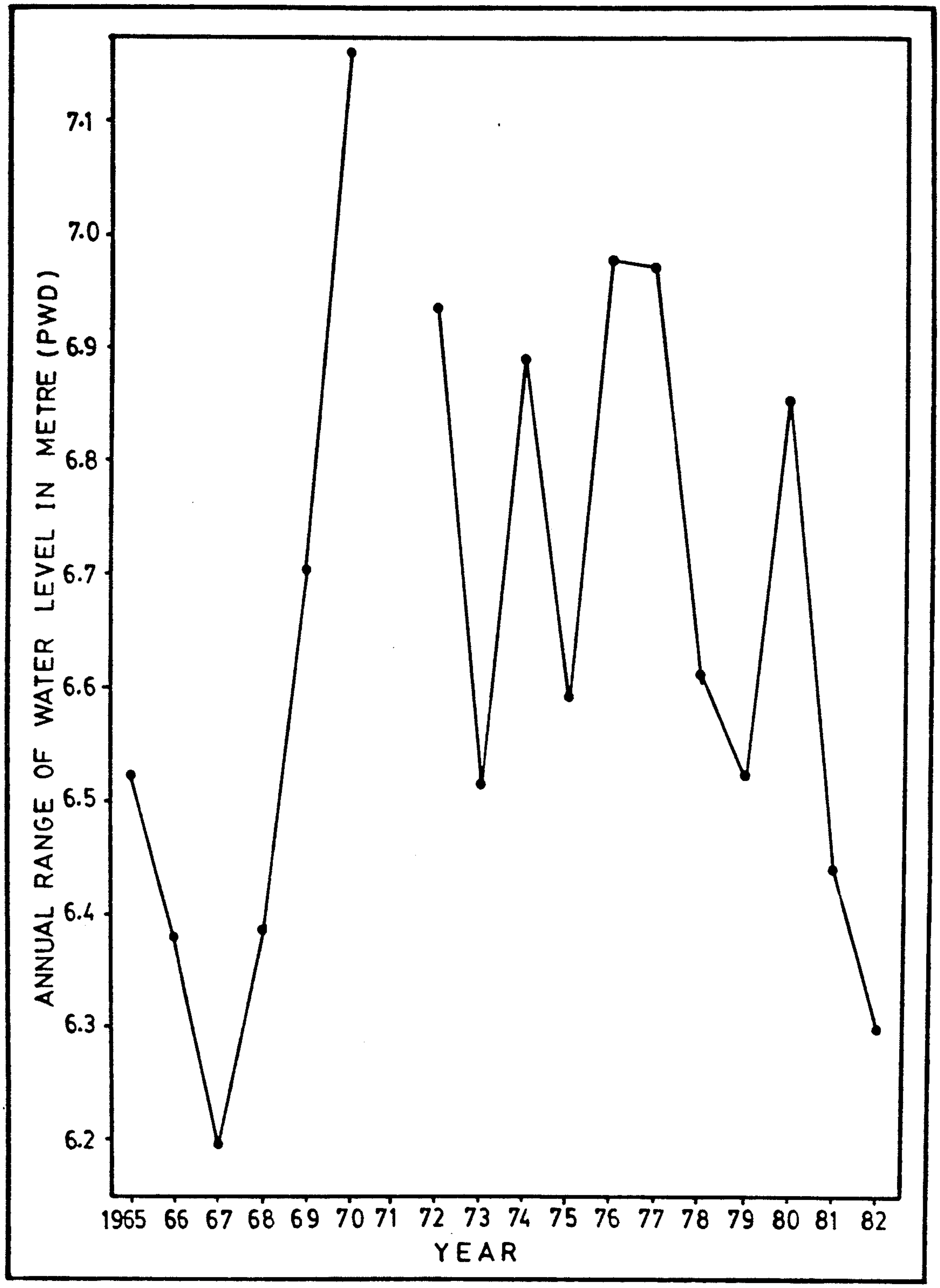


Figure 2.12: Time Variation of Annual Range of Water Level, Brahmaputra River at Bahadurabad (1965-82)

From the plot of stage versus discharge, it is found that unlike the Ganges at Hardinge Bridge (Jansen et al., 1978, p. 57), eleven out of seventeen rating curves are looped. Sample plots showing a single rating curve (1973-74) and more typical looped curve (1967-68) are shown in Figure 2.13. The loops have higher water surface elevations for the same discharge in the after peak portion of the curve. The looping patterns vary from year to year. Some are smooth with opening at the lower part (Figure 2.13) while others are zigzag with distinct bifurcation at the upper part and looping in the middle part (Figure 2.14). This complex phenomenon can be explained as follows :

In the case of uniform flow there exists a direct relationship between the discharge and the water depth at a cross-section. This relationship is given in Chezy's law and in the case of wide rectangular cross-section (Jansen et al., 1979, p. 56) :

$$Q = BCS^{1/2} h^{3/2} \qquad (2.1)$$

where Q is the discharge, B is the width, S is the slope, h is the depth and C is constant. However, this simple relationship is seldom completely valid in the case of the rivers because :

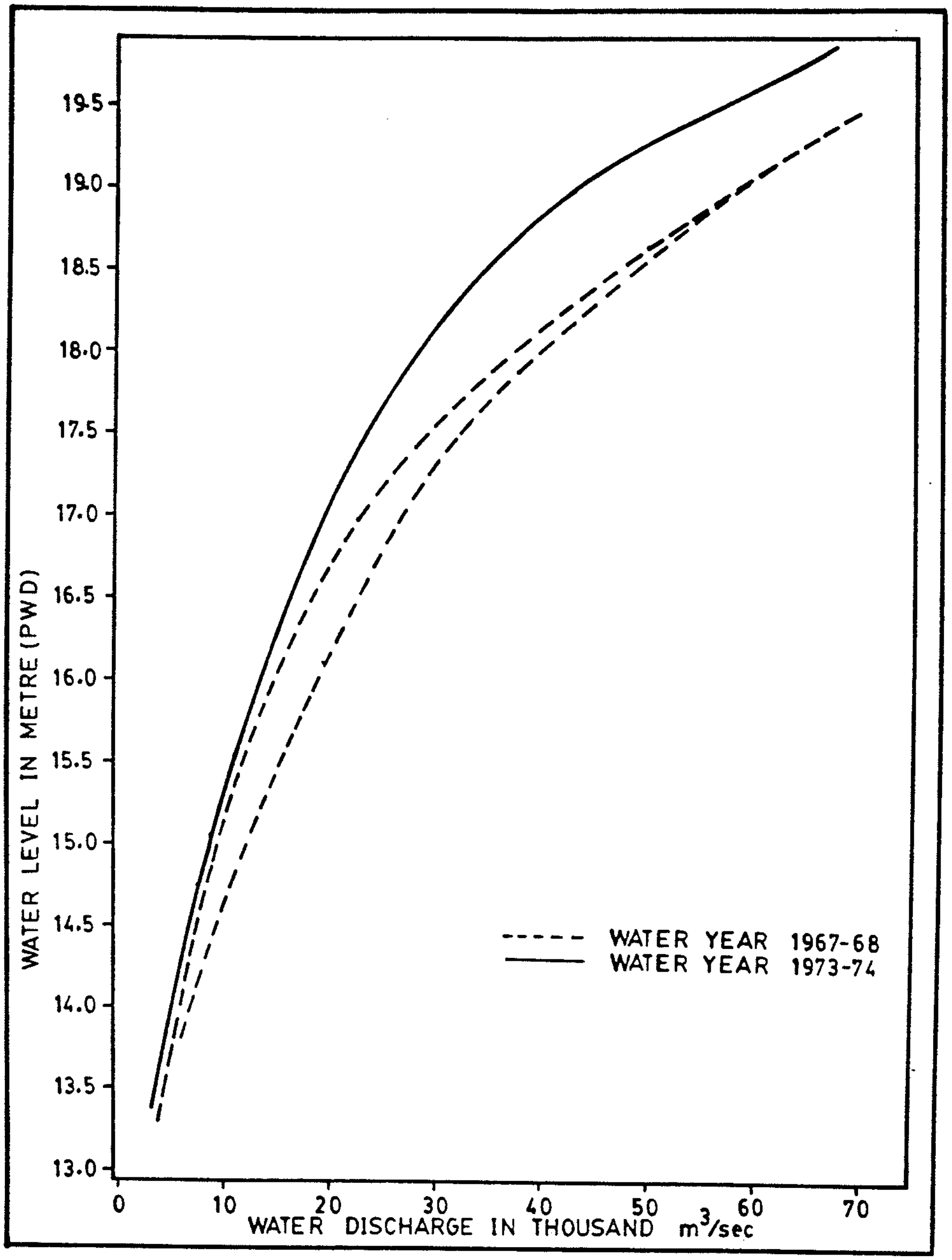


Figure 2.13: Stage Discharge Relationship, Brahmaputra River at Bahadurabad at Bahadurabad.

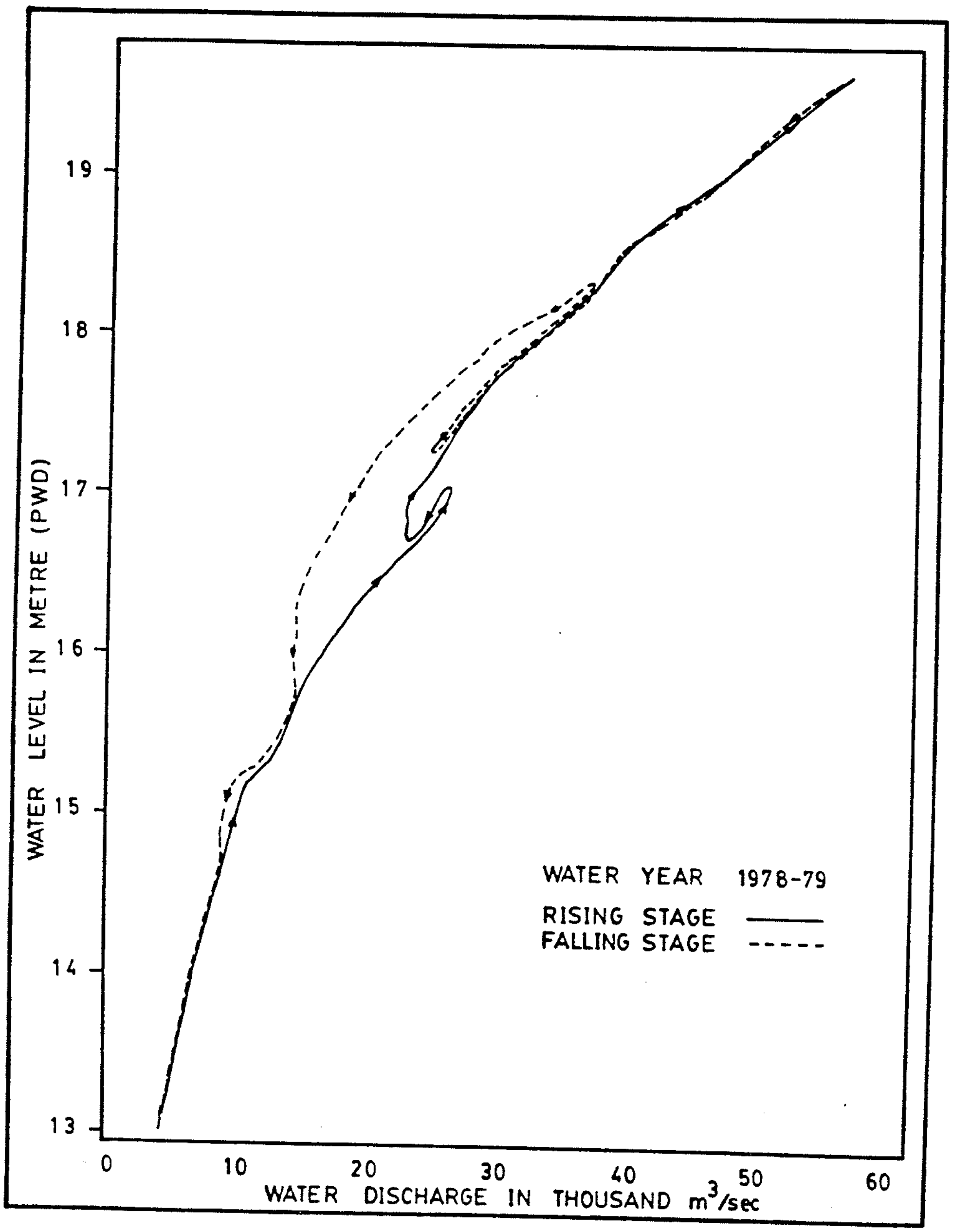


Figure 2.14: Stage Discharge Relationship, Brahmaputra River at Bahadurabad

1. The Chezy coefficient C is generally dependent on the water depth.
2. The dependence of the conveying cross-section on the water depth invalidates the simple relation (as in the flow over the flood plains).
3. Non-uniformity of flow, and
4. Unsteadiness of flow.

Factors which intervene in this direct relationship are acceleration of the flow in time and space, roughness and evolution of downstream water stages as verified in State Hydrologic Institute of Leningrad (USSR) experiments (Ivanova, 1967, quoted by Cunge et al., 1980, p. 197). If the relation of steady flow is denoted by

$$Q_S = A_S C\sqrt{RS_b} \qquad (2.2)$$

where A_S is the area of the conveying section corresponding to steady state discharge Q_S, R is the hydraulic radius and S_b is the bottom slope; then assuming hydraulically rough bottom, the discharge at the same depth during the passage of a flood wave will be

$$Q = A_S C\sqrt{[\ R(S_b - \frac{\partial h}{\partial x})\ ]} = Q_S\sqrt{(1 - \frac{\partial h}{\partial x} S_b^{-1})} \qquad (2.3)$$

where, x is downstream distance from the reference point. Again, from kinematic-wave approximation,

$$\frac{\partial h}{\partial x} = \frac{dh}{dQ}\frac{\partial Q}{\partial x} = -\frac{1}{B}\frac{dh}{dQ}\frac{\partial h}{\partial t} = -\frac{1}{c}\frac{\partial h}{\partial t} \qquad (2.4)$$

where c is kinematic wave velocity. This would be exactly correct if dh/dQ were the slope of the unknown rating curve for unsteady flow. It is only possible in practice to use the curve for steady flow, when Jone's formula (Henderson, 1963) is obtained assuming a direct proportionality between $\partial h/\partial x$ and $\partial h/\partial t$:

$$Q \simeq Q_S \left(1 + \frac{1}{S_b c} \frac{\partial h}{\partial t}\right) \qquad (2.5)$$

with which the rating curve can be constructed from a known hydrograph. The assumption can not be true near the wave maxima. This formula can be used to derive the deviation $(Q-Q_S)$ from the rating curve for steady flow by assuming the relative difference to be small as :

$$Q - Q_S = -\frac{Q_S}{2S_b} \frac{\partial h}{\partial x} \simeq \frac{Q_S}{2S_b c} \frac{\partial h}{\partial t} \qquad (2.6)$$

The quantity in the right hand side of Equation (2.6) can be positive or negative suggesting that the rating curve is transformed into a loop with the deviation positive in the rising part and negative in the falling part (i.e., from rising to falling stage, the rating curve follows a counter clockwise direction). Also, for slow natural floods the loop is more or less symmetric about the steady flow rating curve; deviations may be large but are nearly equal in absolute value during rising and falling level. For flood waves quickly varying with time, this symmetry may disappear

(Cunge et al., 1980, p. 196). The greater the bed slope the smaller the deviation from the steady state condition. In the absence of back water, loops are not observed for bed slope greater than 0.001. With the increase in roughness, the flood peak decreases but the loop opens up somewhat.

Because of the above complicated and simultaneously occurring reasons, the rating curve should be determined in practice by measurements. The formulas given may be useful as rough estimates and guidelines in extrapolation (Jansen et al., 1979, p. 56).

Besides the above described reasons, this hysterisis phenomenon suggests a major change in the form roughness of the channel during the peak flood event. The formation of the new chars (islands) from remnants of giant dunes developed during the major flood events (Coleman, 1969) supports this change. The creation of new and closure of existing channels (Latif, 1969), and the division of the channel into many branches contributes to this increase in roughness coefficient. Also, this time dependency of the stage-discharge relationship is due to the changes in bed elevation caused by scour or deposition during the passage of flood and changes in energy grade lines (Simons et al., 1973).

It should be noted that all of the rating curves except those in the period of 1970-77 are looped. From Equation

(2.6) it follows that the relative width of the loop is directly dependent on the rate of rise or fall, $\partial h/\partial t$. If the rate is large, the loop will be appreciable. However, this does not take inertia into account, which may be important for rapid change in depth. The absence of loops in the rating curves of some of the years can be partly explained by the fact that the rate of change in water level is slower compared to the other years. In other words, the flood wave was slower in those years, which can be seen from the change in water level records. It is interesting to note, that although, the maximum discharge was recorded in 1974, the influx of flood water was slow in that year. This phenomenon can be further explained by the distribution and intensity of rainfall in the drainage basin, since rainfall is the major factor in determining the maximum flood (usually the second peak) flow in any year in the river.

To obtain a stage-discharge relationship, water level was regressed on water discharge for each year of record. A similar kind of regression was performed seperately with data of the before peak flood and that of after peak flood. All of these regressions show a very strong relationship between water level and discharge. The coefficient of determination (R-square) for all of the regression equations are 0.99 or higher. The resulting equations are shown in Table 2.1. Table 2.2 and Table 2.3 give the summary of the stage and discharge data.

TABLE 2.1

Stage Discharge Relationship of Brahmaputra River at Bahadurabad

WATER YEAR	TOTAL EQUATION	RISING LIMB	FALLING LIMB	REMARKS
1965-66	$WL=4.04Q^{0.143}$	$WL=3.86Q^{0.148}$	$WL=4.09Q^{0.142}$	
1966-67	$WL=3.66Q^{0.151}$	$WL=4.24Q^{0.137}$	$WL=3.46Q^{0.156}$	Weak Loop
1967-68	$WL=4.48Q^{0.132}$	$WL=3.94Q^{0.143}$	$WL=4.50Q^{0.132}$	Strong Loop
1968-69	$WL=4.13Q^{0.143}$	$WL=3.89Q^{0.147}$	$WL=3.98Q^{0.148}$	Strong Loop
1969-70	$WL=3.54Q^{0.158}$	$WL=3.96Q^{0.146}$	$WL=3.37Q^{0.163}$	Strong Loop
1970-71	$WL=3.80Q^{0.150}$	$WL=3.34Q^{0.162}$	$WL=3.86Q^{0.148}$	
1971-72				No data available
1972-73	$WL=4.19Q^{0.143}$	$WL=3.95Q^{0.158}$	$WL=3.99Q^{0.147}$	Strong Loop
1973-74	$WL=4.31Q^{0.139}$	$WL=4.26Q^{0.140}$	$WL=4.29Q^{0.139}$	
1974-75	$WL=4.40Q^{0.137}$	$WL=4.53Q^{0.135}$	$WL=4.37Q^{0.138}$	
1975-76	$WL=3.50Q^{0.158}$	$WL=3.53Q^{0.157}$	$WL=3.49Q^{0.158}$	
1976-77	$WL=3.29Q^{0.165}$	$WL=3.42Q^{0.161}$	$WL=3.27Q^{0.165}$	
1977-78	$WL=3.13Q^{0.169}$	$WL=3.69Q^{0.153}$	$WL=3.12Q^{0.168}$	Mild Loop
1978-79	$WL=3.61Q^{0.155}$	$WL=3.84Q^{0.148}$	$WL=3.56Q^{0.156}$	Strong Loop
1979-80	$WL=3.72Q^{0.151}$	$WL=3.24Q^{0.165}$	$WL=4.65Q^{0.126}$	Strong Loop

Table 2.1 (continued)

WATER YEAR	TOTAL EQUATION	RISING LIMB	FALLING LIMB	REMARKS
1980-81	$WL=3.46Q^{0.160}$	$WL=3.34Q^{0.164}$	$WL=3.48Q^{0.160}$	Strong Loop
1981-82	$WL=3.73Q^{0.153}$	$WL=3.66Q^{0.154}$	$WL=3.72Q^{0.153}$	Strong Loop
1982-83	$WL=3.83Q$	$WL=4.22Q$	$WL=3.59Q$	Strong Loop

Note: WL = Water elevation in metre (PWD)
Q = Water discharge in cubic metre per second

TABLE 2.2

Discharge of Brahmaputra River at Bahadurabad

WATER YEAR	MAXIMUM DISCHARGE	DATE OF MAXIMUM	MINIMUM DISCHARGE	DATE OF MINIMUM	YEARLY MEAN	REMARKS
1965-66	64,200	Aug 16	3,500	Feb 22	18,867	
1966-67	68,900	Aug 26	3,950	Apr 4	23,130	
1967-68	69,600	July 12	3,620	Feb 28	18,595	
1968-69	62,300	July 25	3,880	Feb 24	18,635	
1969-70	56,000	July 22	4,020	Feb 22	17,527	

Table 2.2 (continued)

WATER YEAR	MAXIMUM DISCHARGE	DATE OF MAXIMUM	MINIMUM DISCHARGE	DATE OF MINIMUM	YEARLY MEAN	REMARKS
1970-71	75,000	July 28	2,860	Mar 9	21,660	
1971-72						No data available
1972-73	66,600	Aug 1	3,140	Feb 18	17,572	
1973-74	67,300	Aug 9	3,280	Feb 3	18,110	
1974-75	91,100	Aug 6	3,200	Feb 23	21,889	Recorded maximum
1975-76	52,200	July 28 Aug 2	4,050	Feb 21	18,475	Two same peaks
1976-77	56,500	July 5	3,990	Feb 25	17,660	
1977-78	66,600	Aug 22	4,530	Mar 27	23,436	
1978-79	56,600	June 29	4,050	Mar 14	18,367	
1979-80	66,100	Oct 12	4,330	Feb 23	20,925	
1980-81	61,200	Aug 18	4,020	Mar 2	19,419	
1981-82	66,500	July 7	3,500	Feb 26	17,189	
1982-83	55,700	July 30	4,190	Feb 19	17,826	

Mean over the period of record = 19,369 cumec

Note : Discharge is in cubic metre per second

TABLE 2.3

Water Level of Brahmaputra River at Bahadurabad

WATER YEAR	MAXIMUM ELEVATION	DATE OF MAXIMUM	MINIMUM ELEVATION	DATE OF MINIMUM	YEARLY MEAN	REMARKS
1965-66	19.690	Aug 16	13.165	Feb 22	15.718	
1966-67	19.615	Aug 31	13.235	Feb 12	15.818	
1967-68	19.495	July 12	13.300	Feb 28	15.635	
1968-69	19.795	July 25	13.410	Feb 24	16.064	
1969-70	19.840	July 22	13.135	Feb 22	15.765	
1970-71	20.195	July 28	13.030	Mar 15	16.053	
1971-72						No data available
1972-73	19.980	Aug 1	13.045	Feb 18	15.837	
1973-74	19.880	Aug 8	13.365	Feb 3	15.986	
1974-75	20.245	Aug 7	13.350	Feb 24	16.343	Recorded maximum
1975-76	19.600	July 28 Aug 2	13.010	Feb 21	15.809	Two same peaks
1976-77	19.865	July 5	12.884	Feb 21	15.753	
1977-78	19.990	Aug 22	13.015	Mar 6	16.266	
1978-79	19.630	June 29	13.015	Mar 14	15.739	

Table 2.3 (continued)

WATER YEAR	MAXIMUM ELEVATION	DATE OF MAXIMUM	MINIMUM ELEVATION	DATE OF MINIMUM	YEARLY MEAN	REMARKS
1979-80	19.630	June 29	13.015	Mar 14	15.739	
1980-81	20.102	Aug 19	13.244	Feb 26	16.091	
1981-82	19.480	July 7	13.040	Feb 26	15.680	
1982-83	19.420	Sept 21	13.120	Feb 23	15.702	

Note : Water elevations are in metre above PWD datum

The constant term of these equations varies between 3.131 to 4.496 for the total equation, between 3.124 to 4.504 for the falling limb and between 3.237 to 4.53 for the rising limb. The exponent of the discharge varies between 0.132 to 0.169 for the total equation, between 0.135 to 0.165 for the rising limb, and between 0.126 to 0.168 for the falling limb. The rising and the falling limb equations are statistically different at 5% significance level even though the rating curve does not show a prominent loop.

The plot of water level for the same discharge over time shows a slight rising trend in the water level. Such plots for different discharges (10000, 15000, 20000, 25000, 30000, 35000 and 40000 cumecs) show a common trend and the variation pattern is similar although the amount of variation is different for the different discharges (Figure 2.15). This phenomenon may be the result of a land building process by which the bed level is rising, or the result of deposition by which the area is being reduced since there is no substantial change in water surface slope over time. A close look at the cross-sections at Bahaduarabad is necessary. It should be noted here that most of the rating curves are looped, and the water level corresponding to same discharge is different for the before peak and the after peak portion of the curve. The mean value has been taken from the curve. Similar plots for different discharges taking water level in the before peak and the after peak portion of the rating curve show similar rising trend in water level. Variation patterns and magnitudes are different for the before peak and the after peak plots. Although both of these plots have similar rising trend, the water level values in the after peak portion are generally higher than those in the before peak portion. The similar rising trend, pattern and magnitude of variation of all of the abovementioned plots suggests that this variation should be taken into account in defining a bankline and hence the erosion/deposition process. Whatever might be the reasons of this rising trend in water level, this phenomenon poses a potential threat of overtopping of the banks and creating a new channel which may, in future, be the new course of the river. Unless this land building process is distributed in

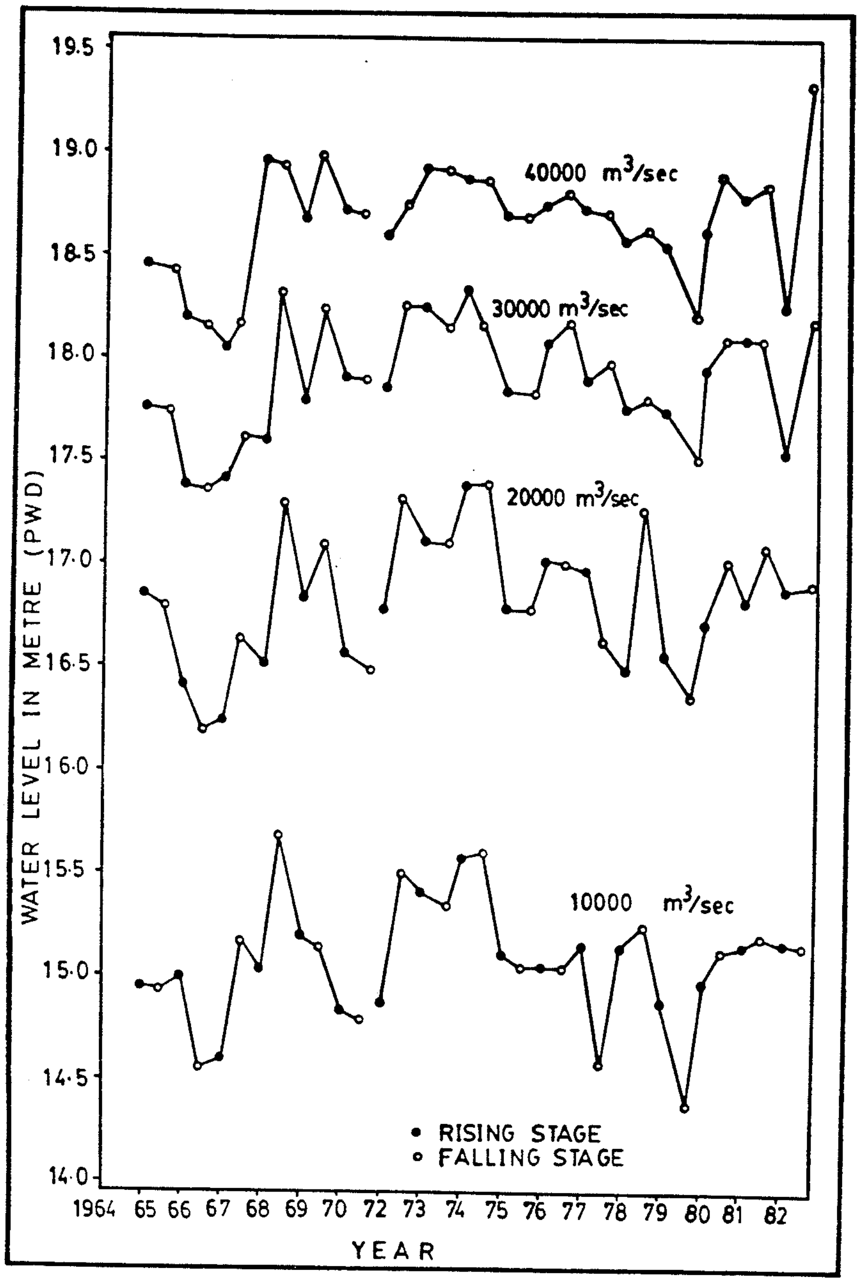


Figure 2.15: Water Level at Selected Discharges, Brahmaputra River at Bahadurabad

all of the small channels, this phenomenon will remain as a threat. Especially, the Bangali river (Figure 1.2), which is now very near to the Brahmaputra Right Flood Embankment, is a possible diversion channel. It has been noted that the distance between the Bangali river and the Brahmaputra river is decreasing over the years (BWDB, 1978b). A similar rising trend in the Brahmaputra river has been found in the Assam area upstream of Bangladesh (Goswami,1983). These observations suggest the need for a close examination of this phenomenon in future for both flood protection and bank erosion reasons.

2.2.9 Sediment discharge

Although the Brahmaputra river is the fourth largest river in the world in terms of the average water discharge at the mouth, it is the second to the Yellow river in relation to the sediment transported per unit of drainage area (Holeman, 1968; Milliman and Meade, 1983). A recent study (Goswami, 1985) on the Indian part of the Brahmaputra has placed the river in fourth place on basis of measurements of the average annual suspended load at Pandu in Assam, India. More than 95% of the annual suspended load is transported by the river during the rainy season (May through October), with the major portion of the load (about 95%) being carried by the moderate events of relatively frequent occurrence (Goswami, 1985).

The sediment discharge records are not closely spaced with respect to time and space. Some of the recorded sediment discharge hydrographs are shown in Figure 2.16. As in the water level and water discharge hydrographs, more than one peak exists in the sediment transport graph. Since these depth-integrated measurements are generally weekly (fortnightly at times), it is not unlikely that peak concentrations were not recorded in some years. Concentration of suspended sediment rapidly increases with the increase of water discharge in the river. Thus the possibility that the peak concentration is missed meaning that the estimation of the total sediment load will be quite approximate. However, recorded daily transported load varies from a high of 2.26 million tons/day to a low of 0.02 million tons/day. But, as reported by Coleman (1969), the river carries about 5.0 million tons of sediment per day, and the total amount of sediment is in the order of 600 million tons. It should be noted here that Coleman's (1969) result is based on the measurements made in the mid-sixties, but the official government records used here are of seventies.

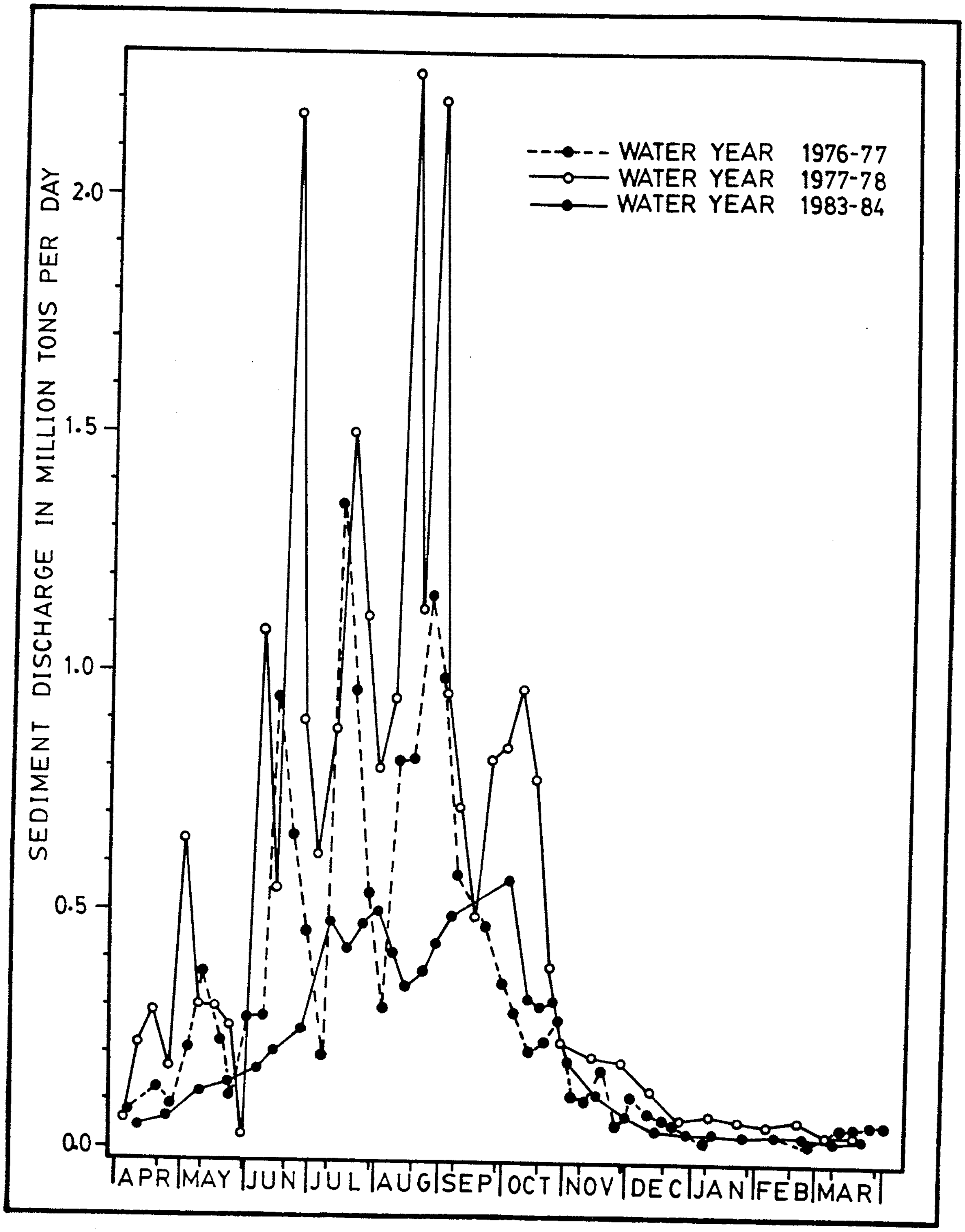


Figure 2.16: Sediment Discharge Hydrograph, Brahmaputra River at Bahadurabad

2.2.10 Water sediment discharge relationship

The logarithmic plot of sediment discharge versus water discharge is shown in Figure 2.17. This plot uses all the available records of the period of 1976-84 and shows a positive relationship between water and sediment discharge. Sediment discharge corresponding to higher flows shows a steeper trend denoting a sharp increase in concentration.

Regressions of sediment transport on water discharge for each year, and for the combined data were performed. The resulting regression equations for both before and after peak water discharge together with the number of data points are shown in Table 2.4. The constant term of the equations vary from 35.9 to 0.05 for the before peak portion, from 1.48 to 0.05 for the after peak portion and from 1.82 to 0.045 for the total equation. The exponent of the water discharge in the equation varies from 1.57 to 0.91 for the before peak portion, from 1.48 to 1.23 for the after peak portion and from 1.58 to 1.17 for the combined data of each year. All of the equations are statistically different from one another, at 5% significance level, even, those of the same year.

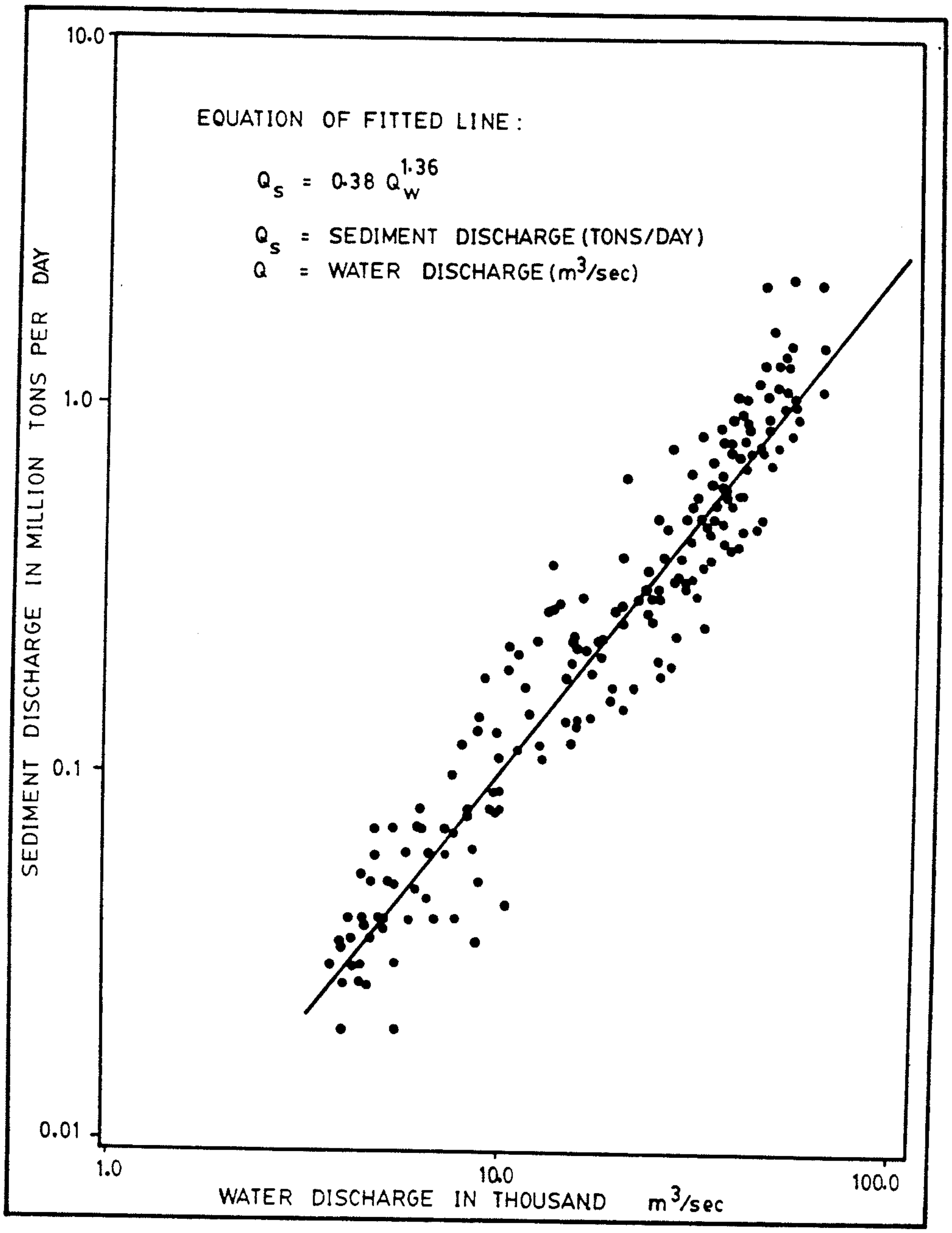


Figure 2.17: Water-Sediment Discharge Relationship, Brahmaputra River at Bahadurabad

TABLE 2.4

Sediment-Water Discharge Relationship, Brahmaputra River at Bahadurabad

WATER YEAR	TOTAL EQUATION	BEFORE PEAK	AFTER PEAK	REMARKS
1976-77	$Qs=0.49Qw^{1.342}$ R-square=0.88 (49 values)	$Qs=35.9Qw^{0.909}$ R-square=0.58 (14 values)	$Qs=0.168Qw^{1.455}$ R-square=0.96 (35 values)	
1977-78	$Qs=0.67Qw^{1.324}$ R-square=0.89 (41 values)	$Qs=0.23Qw^{1.421}$ R-square=0.68 (21 values)	$Qs=0.73Qw^{1.321}$ R-square=0.96 (20 values)	
1978-79	$Qs=0.11Qw^{1.476}$ R-square=0.95 (34 values)	----------	$Qs=0.11Qw^{1.476}$ R-square=0.95 (34 values)	No data before peak
1979-80	$Qs=0.34Qw^{1.367}$ R-square=0.96 (23 values)	$Qs=0.10Qw^{1.480}$ R-square=0.96 (18 values)	$Qs=0.45Qw^{1.352}$ R-square=0.97 (5 values)	
1980-81	$Qs=0.26Qw^{1.374}$ R-square=0.91 (15 values)	$Qs=4.76Qw^{1.10}$ R-square=0.96 (5 values)	$Qs=0.33Qw^{1.34}$ R-square=0.91 (10 values)	
1981-82	$Qs=0.57Qw^{1.322}$ R-square=0.93 (18 values)	$Qs=0.05Qw^{1.573}$ R-square=0.97 (6 values)	$Qs=1.48Qw^{1.226}$ R-square=0.93 (12 values)	
1982-83	$Qs=0.05Qw^{1.577}$ R-square=0.97 (11 values)	$Qs=0.32Qw^{1.396}$ R-square=0.90 (5 values)	$Qs=0.05Qw^{1.56}$ R-square=0.96 (6 values)	
1983-84	$Qs=1.82Qw^{1.174}$ R-square=0.98 (31 values)	$Qs=0.87Qw^{1.239}$ R-square=0.97 (16 values)	$Qs=0.75Qw^{1.28}$ R-square=0.99 (15 values)	
Equation considering all of the values is : $Qs=0.381Qw^{1.359}$				

NOTE : Qs = sediment discharge in tons per day
Qw = water discharge in cubic metres per second

2.2.11 Water surface slope

The water surface slope of different river reaches are given in Table 2.5 (BWDB,1978a). The average slopes were found to be similar across the river. A transverse slope, as mentioned earlier, was also observed at some cross-sections. The water surface slope is higher during the high water season.

2.2.12 Structures on the river

River channels in Bangladesh are almost uninterrupted by human activities with the exception of some structures constructed on small channels in the coastal areas as described by Burger (1982). The Brahmaputra river has some structures which have minor effect on the regime of flow. The major structure built along the channel is the 135 miles(227 km) Brahmaputra Right Flood Embankment with eight T-headed groins, constructed along the channel in 1968. This project was designed to protect the Rangpur, Bogra and Pabna districts from flooding and was not an excercise in bank stabilization. However, Thomas (1970) expresses his concern about tipping the balance in regime by the confinement of flow and thereby preventing the channel from changing to its original meandering form. He believes that the river will again turn to its meandering characterstics and that it has already started the process. Although this embankment was not intended to produce a change in regime, it has

TABLE 2.5

Water Surface Slopes at Different Reaches, Brahmaputra River

RIVER REACH	HIGH FLOW SEASON SLOPE	LOW FLOW SEASON SLOPE
Nagarbari to Sirajganj	0.000055 (0.29 ft/mile)	0.000053 (0.28 ft/mile)
Sirajganj to Milanpur	0.000070 (0.37 ft/mile)	0.000064 (0.34 ft/mile)
Milanpur to Golna	0.000072 (0.38 ft/mile)	0.000066 (0.35 ft/mile)
Golna to Porarchar	0.000093 (0.49 ft/mile)	0.000089 (0.47 ft/mile)

eliminated the storage on the right side of the river causing an increase in the downstream peak discharge. Also the eroding nature of the river has affected the embankment itself. The embankment was originally constructed about 1500 metres from the river bank, but the bank erosion was such that by 1978, it was threatened in ten locations and a back up embankment had to be constructed further away from the bank (BWDB, 1977).

The other important structure across the river is the East-West Interconnector, a power line crossing, at Nagarbari. This overhanging powerline is supported by 40 ft (12 m) diameter piers, and may have some effect on the downstream flow conditions.

2.3 BANK EROSION IN BRAHMAPUTRA RIVER

2.3.1 History

Historically the rivers of the Bengal basin have been changing their courses. The Brahmaputra is not an exception, but is rather the major river on which most of the changes took place. The major change in course in the early nineteenth century has been previously discussed. Besides this major change, the banks throughout its length are being continually eroded and material deposited. This fact has been documented in many old books and reports (Martin, 1838, The Imperial Gazetteer of India 1881 and 1882). Martin (1838, p.358) wrote about the rivers of Rangpur and Assam :

> Since the survey was made by Major Rennel, the rivers of this district have undergone such changes, that I find the utmost difficulty in tracing them. The soil is so light and the rivers in descending the mountains have acquired such force, that frequent and great changes are unavoidable; so that whole channels have been swept away by others, and new ones are constantly forming.

About the Brahmaputra he wrote (p. 304) :

> Below the mouth of the Chhonnokosh, again, the right bank of the Brohmoputro has been gaining, and the channels on that side have been diminishing, so that many of the chars and islands have been united with the main, but I had no opportunity of being able to trace the alterations

> in particular manner. Near Chilmari .... as I have said before, the river threatens to carry away all the vicinity of Dewangunj, and perhaps, to force its way through the Konayi into the heart of Natore.

On another perspective he wrote (p. 399) :

> It is only when sudden changes take place that great evils arise, and none such has happened since the year of Bengal era 1194, or for 20 years before this year 1809. The change which then took place in the Tista, owing to a great storm, was accompanied with a deluge, by which one half of both people and cattle were swept from the whole of the country near the new course, which the river assumed.

Although some of the historical facts and event have been misrepresented in some of these old books and reports, statements of this kind can be taken as facts since there was no political interest involved whatsoever. Therefore, it can be seen that not only the Brahmaputra but most of the rivers of this area have been changing their courses.

2.3.2 Present condition of bank erosion

The migrating river courses were mapped by Coleman (1969) from aerial photo mosaics. The map shows that the river is continuing to migrate to the west.

From a study of maps and air photos, Galay (1980) observed that three major channel shifting processes are in action . These are :

1. A gradual downstream meander migration of the major flow channel,

2. Rapid shifting of the major flow channel within the bankful channels, and
3. Sudden creation of new channels that are outside of the bankful channel and on the local floodplain.

Although the pattern of alternate point bars, obliquity of current, large and random changes in the river flow, sediment discharge and flood stages are predominant factors affecting bank migration, it was found that the bank erosion is not necessarily dependent on the direct attack by the main channel flow but is often the result of minor by-pass channels (BWDB, 1978b).

Chapter III

MODEL DEVELOPMENT

The development of a stochastic prediction model of riverbank erosion through time and space requires a close examination of available data, a selection of the pertinent variables, an investigation of the space and time dependency of the variables and interrelationships between them, and finally, the testing of the developed model.

The model development must take into account the limited and sometimes poor quality data that are available in a developing country. The model should therefore be capable of easily incorporating new information for updating as it becomes available.

3.1 IDENTIFICATION OF THE STUDY REACH

Although the whole reach of the Brahmaputra river within Bangladesh experiences erosion or deposition, the history of such changes is not available for the entire river reach. Therefore, the identification of the study reach is restricted by the availability of the information.

The channel reach between Bahadurabad in the north, upstream of the Old-Brahmaputra diversion, and Aricha in the south, above the Ganges-Brahmaputra confluence has been selected as the study reach (Figure 3.1). It is 92 miles (148km) in length, and its significant tributaries include the Bangali-Korotoya. The major distributaries are the Old Brahmaputra and the Dhaleswari.

3.2 DEFINITION OF BANKLINE

A statistical analysis of bank erosion requires in the first place a clear and unequivocal definition of the bankline that is independent of water level fluctuations. In this study, the bankline of the Brahmaputra river has been defined as the line separating the existing landmass and the stream during the low-water period of November to March or any sand deposits that appear to be transient. This definition has been adopted for the following reasons:

1. The bankline of the Brahmaputra river in the low water period is almost vertical throughout the river reach with a height range from 3m to 10m between the water surface and the top of the bank. The variation of water level within the low-water period and from one year to the next is less than this height of the vertical bank at most places. Therefore, where water surface is the dividing criterion, a small change in water level will not result in a significant change in the position of the bankline.
2. The line as defined above separates cultivated and settled land from either water or char land which is

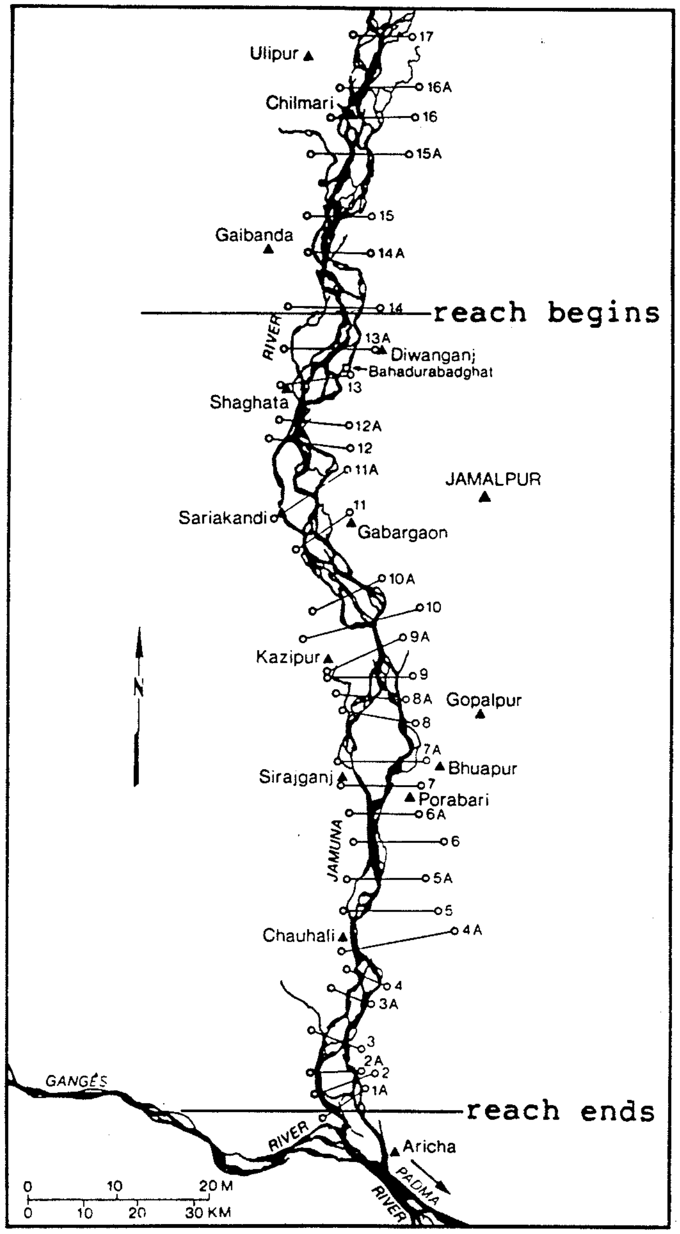


Figure 3.1: Selected Study Reach Limit (numbers are Water Board crosssection monitoring stations, Stene, 1987)

seasonally occupied during low-water periods. The char lands in between the banklines are very fertile and are effectively used for settlement and crop production during low water period. People living in this buffer area, however, are dislocated temporarily during high water periods.

3. Summer monsoon rain frequently produces floods and affects the amount of erosion or deposition. But, the position of the high water line does not give a direct measure of the total land loss and/or gain. The amount of erosion and deposition becomes evident when the flood subsides.

4. It may safely be assumed that the available survey maps were prepared during the low water period because conventional survey work is nearly impossible during the rainy season due to flooding and heavy rainfall. This assumption is supported by the work of IECO(1964), and makes the use of available maps possible.

The above definition of the bankline will cause some errors in the computation of changes due to the change in water level at different times and also possibly due to a rising trend of the bed level as suggested for the Brahmaputra in Assam (Goswami,1985). This error is small, however, compared to the total change in the bankline from year to year and hence can be neglected for the time being.

3.3 DATA COLLECTION AND PROCESSING

Available maps of the river have been collected from Bangladesh government agencies. Unfortunately, these maps are not produced at regular time and space intervals and could not be used effectively. In this study, an attempt was made to interpolate the data for the missing years by polynomial fitting (Snyder,1978) and spline interpolation using Akima's (1978a,1978b) algorithm. The theoretical detail of these interpolation functions can be found in Meinguet (1979). The interpolated values were then used for further analysis. The erosion-deposition process at a given point of the river is not uniformly progressive in time, however. For this reason interpolation between data that are seperated by substantial time intervals was found to be inappropriate and gave misleading and inconsistent results.

The only source of bankline location data which are continuous in time and space is the ERTS satellite imageries. Dry period imagery of 1973 and 1976 through 1985 were obtained from EROS Data Centre, South Dakota, USA and from Remote Sensing Division of National Research Council, Bangkok, Thailand. The year of the collected data as mentioned in this study refers to the dry period following the previous year flood or the dry period before the flood of the same year. For example, the satellite imagery of December, 1978 will be referred to as data of 1979 and that of February, 1980 as data of 1980. When several imageries of the same year were available, selection was made on the basis of quality (minimum cloud cover) and/or proximity of the recording time to the period of January-February. The collected imageries have minimal cloud cover and clearly show the bankline as defined earlier.

The collected imageries show that along the recorded reach four points exist where the river is a single channel and no significant changes have occurred over the period of record. These points have been called the "nodal points" in this study. The extent and intensity of changes, including the tendency to create new branch channels, are not the same for reaches between different pairs of consecutive nodal points.

To explain the existence of nodal points Coleman (1969) suggested that the river bed is composed of the sediments carried by numerous rivers that existed in the area, and at the nodal points, possibly, previous deposition of highly cohesive clayey material prevents the erosion process. A difference in bed material could also explain different rates of changes in the reaches between the nodal points. However, there is no evidence that the bed material is significantly different in terms of erodibility along the river. Thus Coleman's explanation lacks an objective basis. The stability theory suggests the existence of similar points at regular intervals along the river implying that if a node exists at some point, it will reoccur after some regular interval of space.

In the case of the Brahmaputra river, the distance between two successive nodal points are of the order of 25 km, except for one pair, which is about 45 km. It should be noted that this reach of 45 km length experiences more changes than the others. One might thus tentatively conclude that this reach is unstable and that a node could form here some time in the future. In fact, there existed a semi-nodal point in this reach in 1973.

3.3.1 Mapping bankline position

To obtain coordinates of the banklines in successive years, enlarged photographs of satellite imagery were oriented with respect to a satellite mosaic map of Bangladesh produced by NASA. The orientation was performed by identifying three permanent points (e.g., water bodies) which are distinctly visible on each photograph and on the map. Two arbitrary orthogonal straight lines, essentially parallel to the North-South and East-West orientation, were drawn on each photograph and were used as reference lines. These lines were then transformed to other photographs by using the same permanent points. The procedure was repeated and checked.

Starting from the upstream point of the study reach, distances of bankline positions from the vertical reference (North-South line) were measured at a 1525 metre (5000 foot) space interval along the vertical reference. The distance interval of 1525 metres was taken arbitrarily for the convenience of measurement from the available maps. Bankline measurements following the above procedure were performed for the 1973 and 1976 through 1985 satellite photographic enlargements. The 1973 bankline position was then subtracted from those of the other years which made the 1973 bankline the reference for displacement. The resulting ten displacement series of each bank (east and west) at a space interval of 1525 metres for 1976 through 1985 obtained following the above procedure have been used for analysis in this study. It is to be noted here that the positive values of the displacement series denote deposition while the negative values denote erosion with respect to 1973 bankline position.

3.4 TREATMENT OF THE DATA

The techniques for analysing any series where, the observations are at regular intervals, are referred to as time series analysis in the literature. The series that are most frequently analysed using these techniques are observed at regular intervals of time. But, as long as the observations are made at regular intervals, these techniques can be applied to any domain, space or time, as discussed by Jenkins and Watts (1968). Therefore, in this study, techniques developed for time series analysis will be used to model the bankline displacement of the Brahmaputra river at discrete length or spatial intervals along the river.

To construct a model of bank movement, it was assumed that the position of the bankline at any location is the output of a dynamic system in which the bankline of the previous year is the input. For example, the displacement series of 1977 was considered as the input to a dynamic system and that of 1978 was considered as the output from that system. Again, the displacement series of 1978 was taken as input and that of 1979 as output. Other variables such as the mean discharge in a year are lumped into the system but their effect on the model parameters can be assessed seperately. This is essentially a lumped black-box approach which is often used when the process underlying the system is not well understood and the detailed data are unavailable.

For this type of stochastic-deterministic model, the Transfer Function Noise Model of Box and Jenkins (1976) is suitable. This model has been successfully used in a wide variety of applied fields such as river flow forecasting (Fay et al.,1983), chemical process study (Box and Jenkins,1976), economic system analysis (Jenkins,1979), etc.

The Transfer Function Noise Model for the bankline displacement series is developed in the following section. The theoretical detail of this model may be found in Box and Jenkins (1976). Only one time step and ninety seven space steps are considered in this study. This approach is equivalent to saying that the information that could be obtained from earlier bankline positions at each discrete space steps is fully contained in the position of the previous year. The relevant definitions of the statistical terms used in this section and thereafter are given in Appendix-B.

3.5 TRANSFER FUNCTION NOISE MODEL

The bank displacement Y_{st}, at bank location s in year t is measured relative to the reference bankline of 1973. This displacement can be expressed as the deviation from the spatial mean displacement during year t by writing :

$$y_{st} = Y_{st} - \left(\frac{1}{n}\right) \sum_{s=1}^{n} Y_{st} \qquad (3.1)$$

where, n is the number of measurement locations. The deviation y_{st} which now has a zero mean value along the river consists of a deterministic component, D_{st}, and a random component, N_s. Thus

$$y_{st} = D_{st} + N_s$$

The random component, also called the noise term, is independent of y_{st} since all relationships are aggregated in the deterministic component. But, in general, the series of noise terms is spatially correlated with an autoregressive and a moving average part and can be written as :

$$N_s = \phi_1 N_{s-1} + \phi_2 N_{s-2} + \ldots\ldots + \phi_p N_{s-p}$$

$$+ a_s - \theta_1 a_{s-1} - \theta_2 a_{s-2} - \ldots\ldots - \theta_q a_{s-q} \qquad (3.2)$$

where, θ and ϕ are coefficients. The terms with ϕ are the autoregressive terms and those with θ are the moving average terms.

The notation can be simplified by using the backshift operator. It is defined by

$$B^K(x_S) = x_{S-K} \text{ (k is a positive integer)}$$

and the notations

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots\ldots\ldots - \phi_p B^p$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots\ldots\ldots - \theta_q B^q$$

Thus one can write :

$$\phi(B) N_S = \theta(B)\, a_S$$

$$\text{or } N_S = \phi^{-1}(B)\ \theta(B) a_S \qquad (3.3)$$

where a_S is white noise with zero mean. N_S is now the noise term with zero mean which can be any autoregressive moving average (ARMA) process.

Since only one time step is considered the term D_{St} is considered to be related to the terms $D_{S-1,t}$, $D_{S-2,t}$ etc. and to the terms $D_{S,t-1}$, $D_{S-1,t-1}$, $D_{S-2,t-1}$ etc.

To simplify the notation all displacement terms relating to year t will be designated y_S and those relating to year t-1 will be designated x_S .

One can then write :

$$y_S - N_S = \delta_1(y_{S-1} - N_{S-1}) + \delta_2(y_{S-2} - N_{S-2}) + \ldots\ldots + \delta_r(y_{S-r} - N_{S-r}) + w_0 x_S - w_1 x_{S-1} - w_2 x_{S-2} - \ldots\ldots - w_c x_{S-c} \quad (3.4)$$

Using the backshift operator this expression can be re-written as:

$$\delta(B)(y_S - N_S) = w(B)\, x_S$$

$$\text{or} \quad y_S - N_S = \delta^{-1}(B)\, w(B) x_S$$

$$\text{or} \quad y_S = \delta^{-1}(B)\, w(B)\, x_S + N_S$$

$$\text{or} \quad y_S = \delta^{-1}(B)\, w(B)\, x_S + \phi^{-1}(B)\theta(B) a_S \qquad (3.5)$$

where, $\delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \ldots\ldots - \delta_r B^r$, and

$$w(B) = 1 - w_1 B - w_2 B^2 - \ldots\ldots - w_c B^c$$

A slightly more general expression is required if there is a lag in interrelationship by a space interval of b. Then Equation (3.5) could be written as :

$$y_S = \delta^{-1}(B)\, w(B)\, x_{S-b} + \phi^{-1}(B)\theta(B) a_S \qquad (3.6)$$

where b is called the pure system delay. Equation (3.6) denotes the general Transfer Function Noise Model. In accordance with conventional notation the Transfer Function in Equation (3.6) will be designated as a TF of order (r,c,b) and the Noise term in the model described by Equation (3.6) as Noise of order (p,d,q) where d is the degree of differencing needed in case of nonstationary series.

Other forms of representation of discrete and continuous Transfer Function Models that are widely used in literature are given in Appendix-C.

The appropriate Transfer Function Noise Model must be determined from the crosscorrelation between the input and the output series of bank displacement. The crosscorrelation between two series, however, can be entirely misleading when the series are autocorrelated (Bartlett, 1935). This has been demonstrated by Jenkins (1979) and others (Hipel et al., 1985). Therefore, it is very important to remove the autocorrelation or to "prewhiten" both series so as to get white noise residuals before computing and modelling the crosscorrelation. This can be achieved by using an appropriate prewhitening filter to each series. The periodicity and trend, if present in the series, may have to be removed as well.

In order to select a proper prewhitening filter, statistical properties (univariate properties) of the individual series must be examined. This is done in the following section.

3.6 UNIVARIATE STATISTICAL PROPERTIES

In this section, the univariate statistical properties of individual series that are important in selecting a proper prewhitening filter are discussed. The computing formulae are given here as well.

The univariate properties of each displacement series of west and east banklines were computed. Only the results of 1977 and 1978 west bankline displacement series are described here in detail. Results from the other series will be summarized at the end of this chapter.

3.6.1 Plots and Moments

The series have been plotted against downstream distance from the east-west reference line and are given in Figures 3.2 and 3.3. The mean, standard deviation, skewness and other quantiles have been calculated for each series using method of moments. The computing formulas are :

$$\text{Mean, } \bar{x} = \left(\frac{1}{n}\right) \sum_{s=1}^{n} x_s \tag{3.9}$$

$$\text{Variance, } S^2 = \frac{1}{n-1} \sum_{s=1}^{n} (x_s - \bar{x})^2 \tag{3.10}$$

$$\text{Skewness coefficient, } g = \frac{n}{(n-1)(n-2)S^3} \sum_{s=1}^{n} (x_s - \bar{x})^3 \tag{3.11}$$

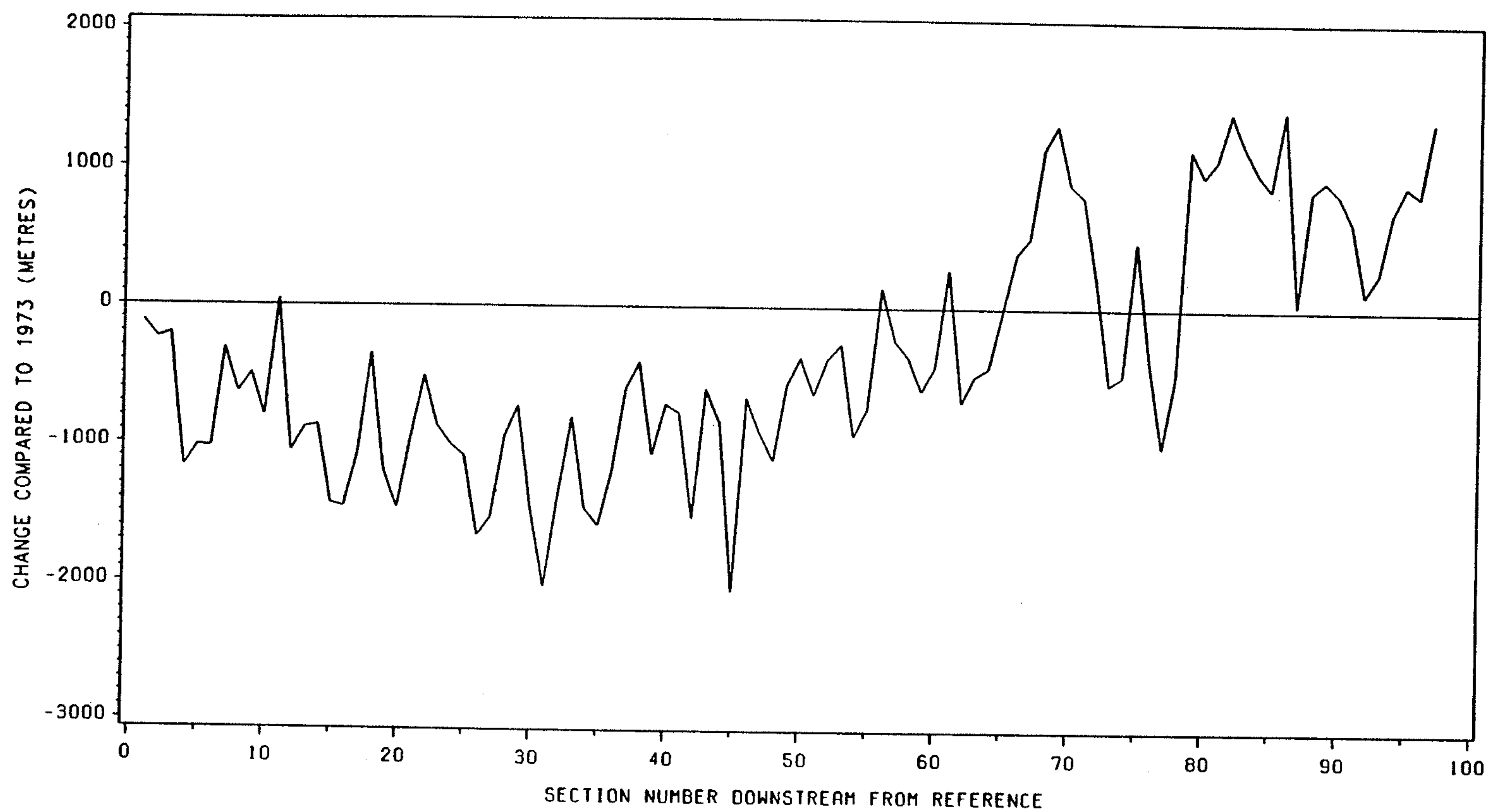


Figure 3.2: West Bank Displacement Series, 1977

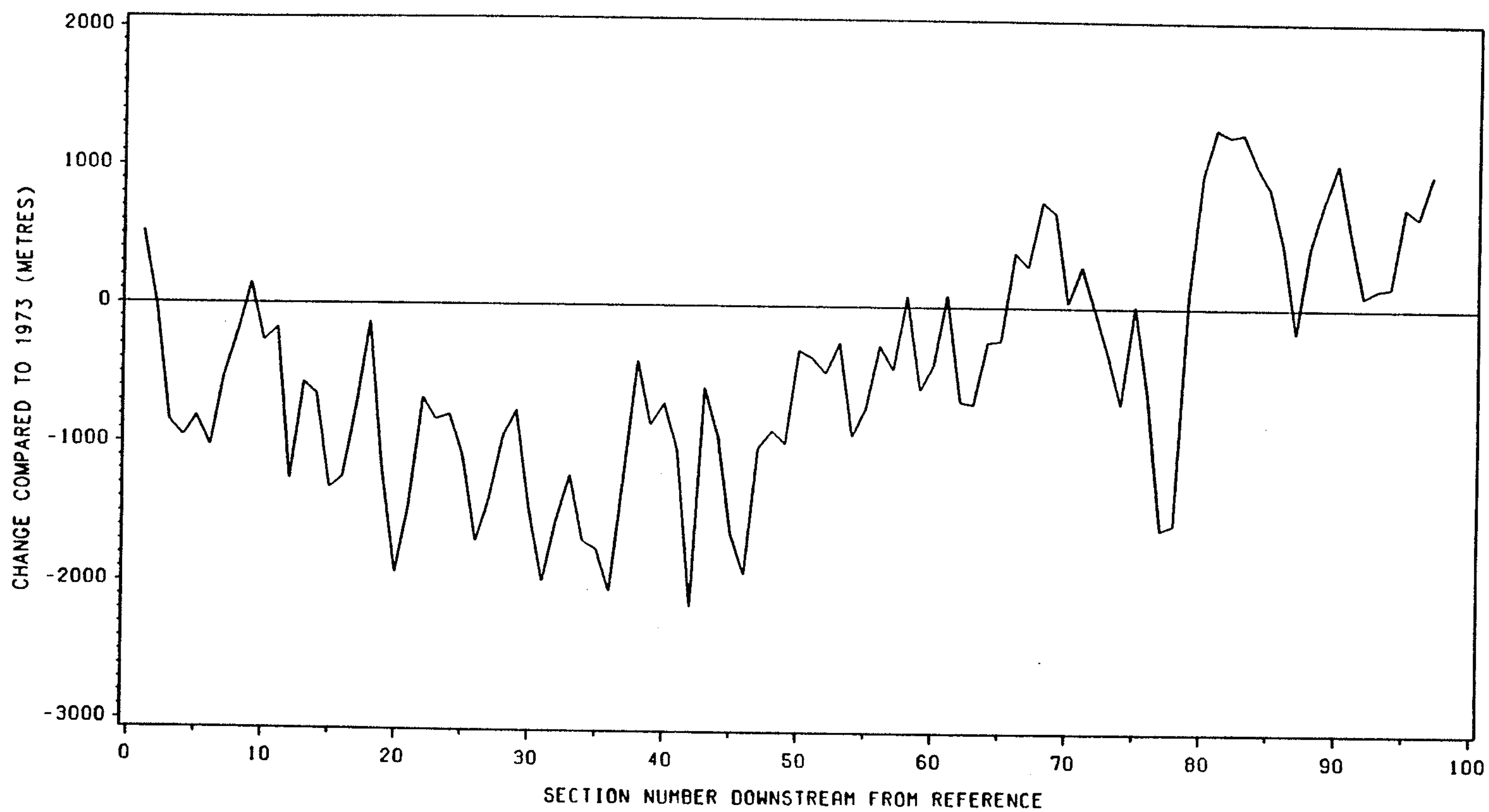


Figure 3.3: West Bank Displacement Series, 1978

where n is the number of observations. These statistics are given in Tables 3.1 and 3.2.

All of the series were tested for normality using Kolmogorov D-statistic, schematic or box plot (Tukey, 1977), skewness test for normality (Snedecor and Cochran, 1967, p.552) and normal probability plot. All tests suggest that the series can be taken as normal and that no transformation is needed.

3.6.2 Autocorrelation Structure

The autocorrelation structure is specially important for identifying a proper prewhitening filter. The Autocorrelation Function (ACF), Partial Autocorrelation Function (PACF) and the Inverse Autocorrelation Function (IACF) for different space lag were calculated and plotted (Figures 3.4 to 3.9). The computing formulas are given below:

$$\text{Autocorrelation for lag } k = \frac{\sum_{s=1}^{n} (x_S - \bar{x})(x_{S+K} - \bar{x})}{\sum_{s=1}^{n} (x_S - \bar{x})^2} \qquad (3.12)$$

The Partial Autocorrelation Functions $\emptyset_K(k)$ have been obtained recursively by using Durbin's (1960) relations:

TABLE 3.1

Univariate Properties of 1978 data

MOMENTS

N	97	SUM WGTS	97
MEAN	-464.049	SUM	-45012.7
STD DEV	843.473	VARIANCE	711447
SKEWNESS	0.156824	KURTOSIS	-0.593573
USS	89187012	CSS	68298807
CV	-181.784	STD MEAN	85.6417
T:MEAN=0	-5.41848	PROB>|T|	0.0001
SGN RANK	-1300.5	PROB>|S|	0.0001
NUM ^= 0	97		
D:NORMAL	0.0554445	PROB>D	>.15

QUANTILES(DEF=4)

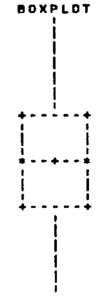
100% MAX	1299.14	99%	1299.14
75% Q3	93.0498	95%	1037.62
50% MED	-568.12	90%	768.579
25% Q1	-1019.57	10%	-1598.19
0% MIN	-2164.11	5%	-1925.85
		1%	-2164.11
RANGE	3463.25		
Q3-Q1	1112.62		
MODE	458.899		

EXTREMES

LOWEST	HIGHEST
-2164.11	1035.66
-2052.01	1055.3
-1985.63	1249.7
-1930.42	1268.32
-1925.34	1299.14

```
STEM LEAF                  #
  12 570                   3
  10 46                    2
   8 778                   3
   6 79477                 5
   4 06626                 5
   2 00                    2
   0 247990456             9
  -0 87331                 5
  -2 732878541             9
  -4 9738520               7
  -6 963200988510         12
  -8 8644395421           10
 -10 8421                  4
 -12 364320                6
 -14 985750                6
 -16 5093                  4
 -18 933                   3
 -20 65                    2
     ----+----+----+----+
 MULTIPLY STEM.LEAF BY 10**+02
```

BOXPLOT



NORMAL PROBABILITY PLOT

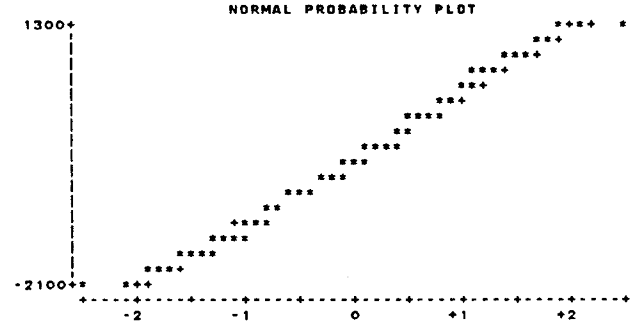


TABLE 3.2

Univariate Properties of 1977 data

MOMENTS

N	97	SUM WGTS	97
MEAN	-342.066	SUM	-33180.4
STD DEV	867.033	VARIANCE	751747
SKEWNESS	0.409335	KURTOSIS	-0.690761
USS	83517613	CSS	72167711
CV	-253.469	STD MEAN	88.0339
T:MEAN=0	-3.88562	PROB>|T|	0.00018774
SGN RANK	-987	PROB>|S|	.000505738
NUM ^= 0	97		
D:NORMAL	0.119852	PROB>D	<.01

QUANTILES(DEF=4)

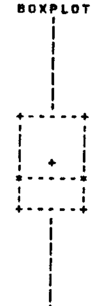
100% MAX	1432.58	99%	1432.58
75% Q3	277.117	95%	1197.68
50% MED	-495.814	90%	973.543
25% Q1	-979.805	10%	-1456.76
0% MIN	-2054.04	5%	-1534.35
		1%	-2054.04
RANGE	3486.62		
Q3-Q1	1256.72		
MODE	1148.09		

EXTREMES

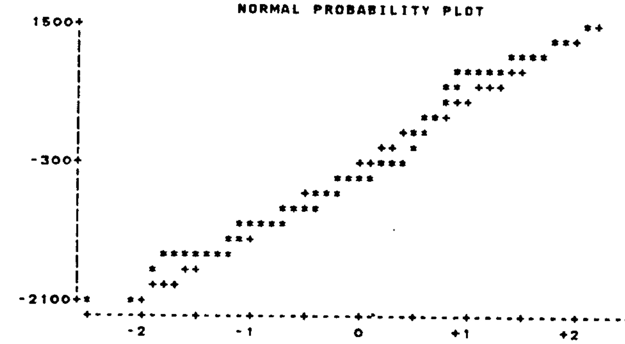
LOWEST	HIGHEST
-2054.04	1183.65
-2027.96	1323.86
-1659.83	1363.49
-1581.59	1419.03
-1529.1	1432.58

```
STEM LEAF                     #
  14 23                       2
  12 26                       2
  10 9558                     4
   8 1446701479              10
   6 41                       2
   4 081                      3
   2 088                      3
   0 4424                     4
  -0 23                       2
  -2 7665518441              10
  -4 998510998220            12
  -6 622086330                9
  -8 6431976300              10
 -10 7087652200              10
 -12 910                      3
 -14 83376663                 8
 -16 6                        1
 -18
 -20 53                       2
     ----+----+----+----+
 MULTIPLY STEM.LEAF BY 10**+02
```

BOXPLOT



NORMAL PROBABILITY PLOT



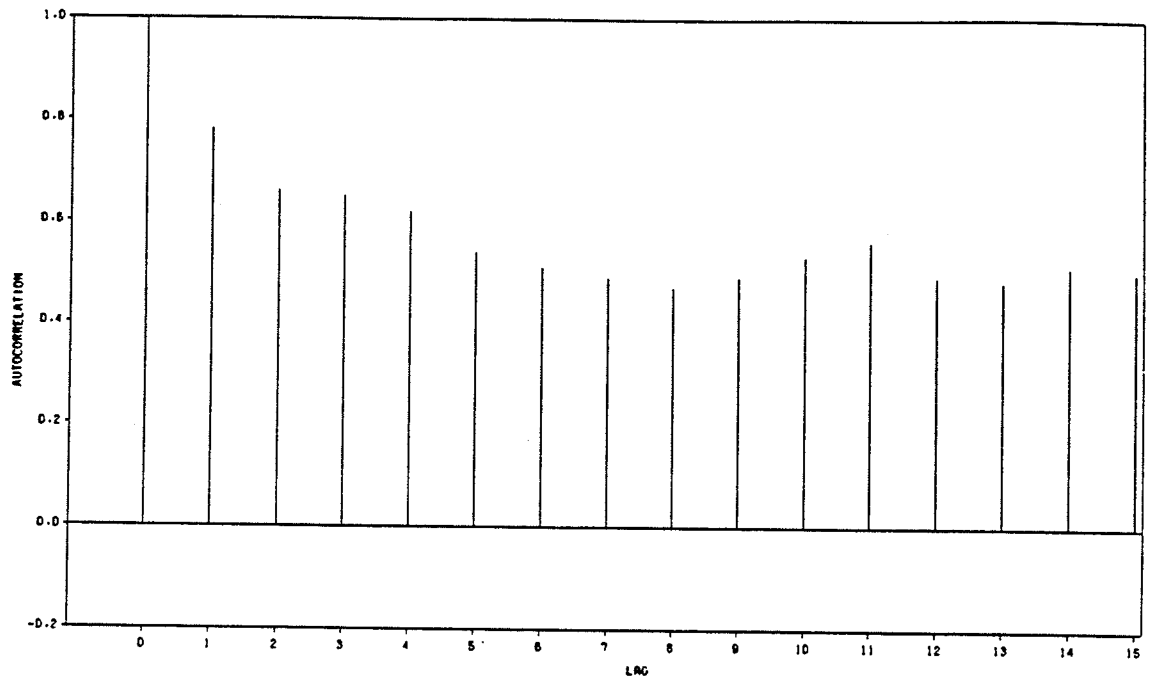


Figure 3.4: Autocorrelation Function of 1977 data

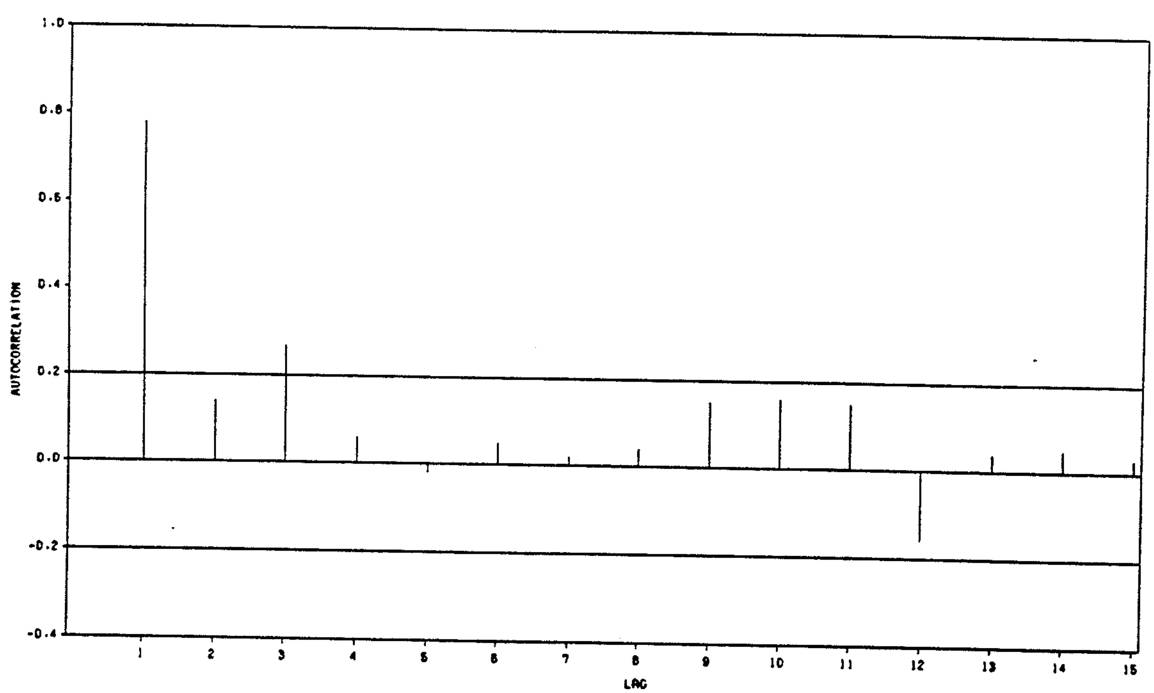


Figure 3.5: Partial Autocorrelation Function of 1977 data

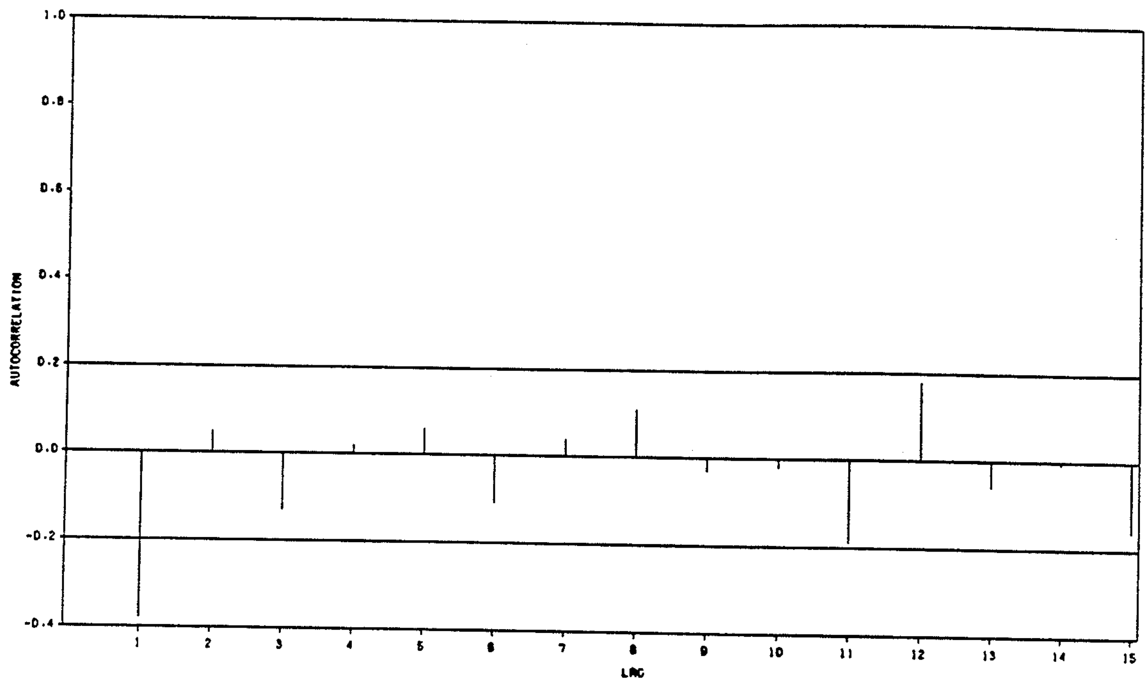


Figure 3.6: Inverse Autocorrelation Function of 1977 data

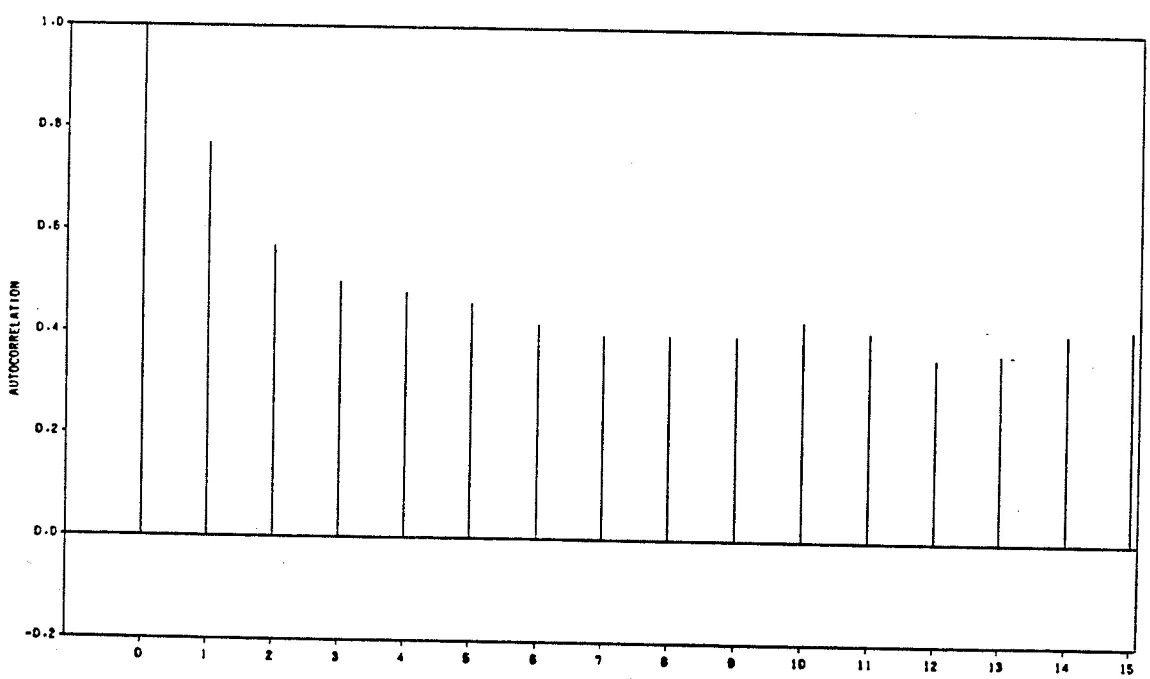


Figure 3.7: Autocorrelation Function of 1978 data

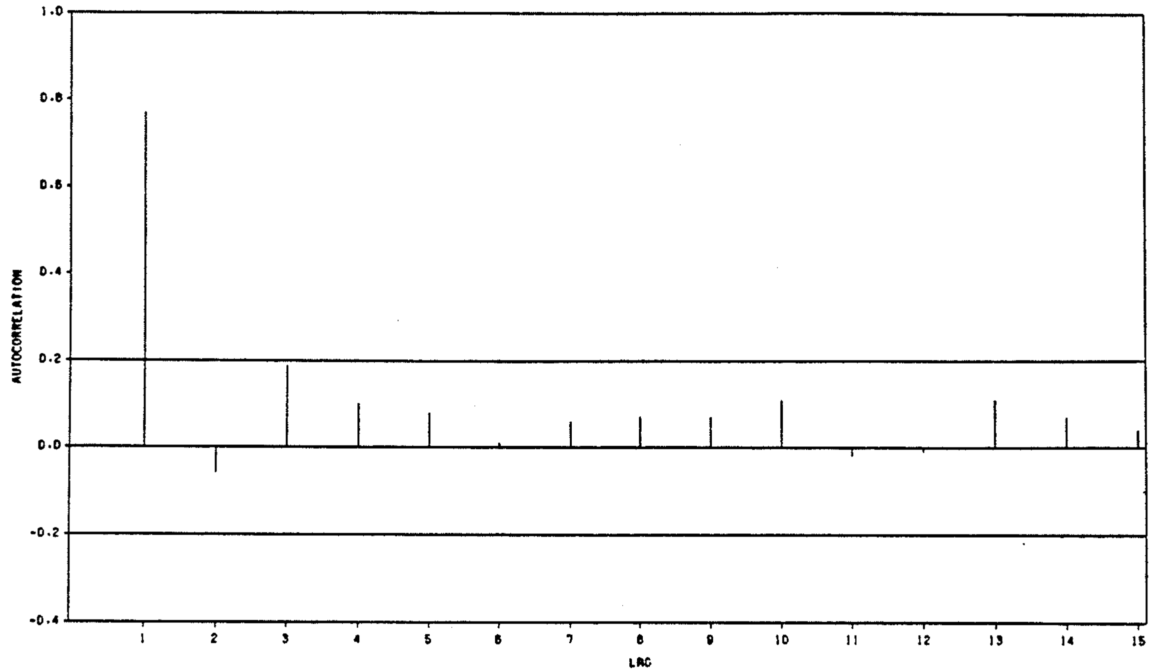


Figure 3.8: Partial Autocorrelation Function of 1978 data

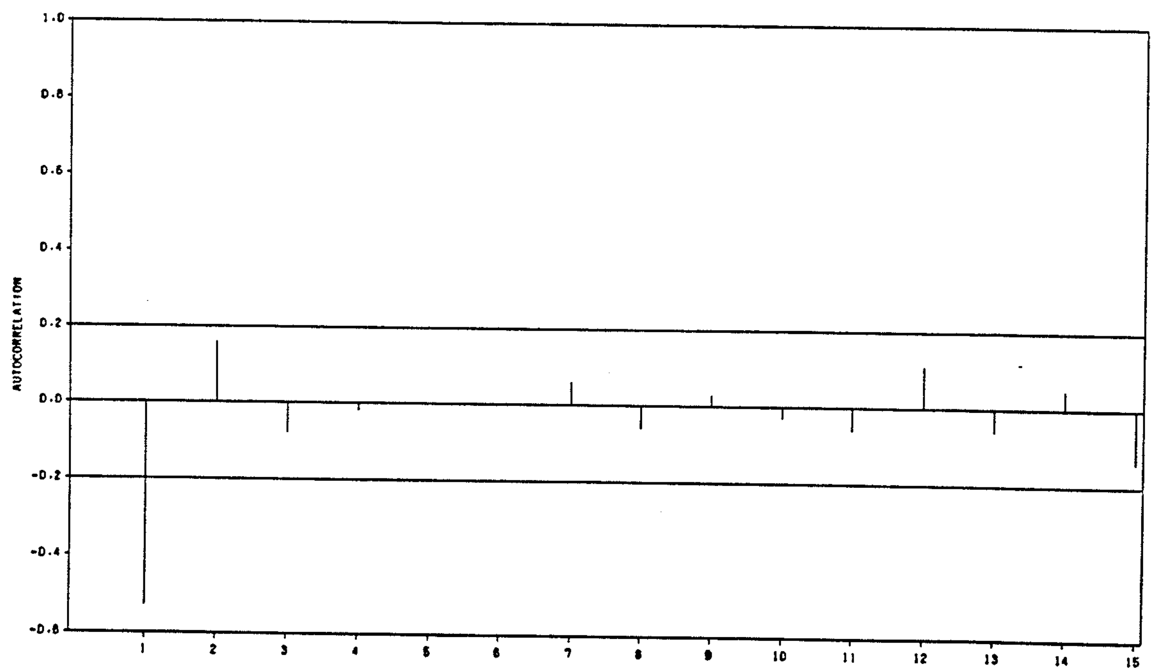


Figure 3.9: Inverse Autocorrelation Function of 1978 data

$$\phi_1(1) = r_1$$

$$\phi_2(2) = \frac{r_1\ (1 - r_2\ )}{(1 - r_1^2\ )} \qquad \phi_2(2) = \frac{r_2 - r_1^2}{(1 - r_1^{2)}} \tag{3.13}$$

$$\phi_K(k) = \frac{r_K - \sum_{j=1}^{k-1} \phi_K(k-1)\ r_{K-j}}{1 - \sum_{j=1}^{k-1} \phi_j(k-1)\ r_j}$$

$$\phi_j(k) = \phi_j(k-1) - \phi_K(k)\ \phi_{K-j}(k-1)$$

where r is the autocorrelation.

The autocorrelation function of the dual model (see Appendix-B) is called the Inverse Autocorrelation Function of the original model. Detailed discussion can be found in Priestly (1981). A high order autoregressive model was fitted to the data by Yule-Walker equations. The inverse autocorrelation function was then calculated as the autocorrelation function that corresponds to the autoregressive operator when treated as a moving average operator. That is, the autoregressive coefficients are convolved with themselves and treated as autocovariances.

The autocorrelation functions are first of all used to remove periodicity and trend if present, and secondly to select the prewhitening filter.

3.6.3 Periodicity and Trend

In order to check the presence of periodicity in the displacement series, spectral analysis of data were performed. For each series, data were decomposed into a sum of sine and cosine waves using Finite Fourier Transform of the general form :

$$x_s = (a_0/2) + \sum_{k=1}^{m} (a_K \cos w_K s + b_K \sin w_K s) \qquad (3.14)$$

where, $m = (n-1)/2$ (as n is odd)

$a_o = 2\bar{x}$

a_K = are the cosine coefficients

b_K = are the sine coefficients

$w_K = 2\pi k/n$

The periodogram is defined as

$$l_K = n(a_K^2 + b_K^2)/2 \qquad (3.15)$$

A triangular kernel or spectral window was used to smoothen the periodogram to estimate the spectral density of the series. Details of this procedure may be found in Fuller (1976), Jenkins and Watts (1968). The periodogram and spectral densities are plotted in Figures 3.10 and 3.11. Using Fisher's Kappa and Bartlett's Kolmogorov-Smirnov statistic (Fuller, 1976, p. 283), the series have been found not to be white noise. Periodicities of period 15 space units were found to be significant at the 5% significance level.

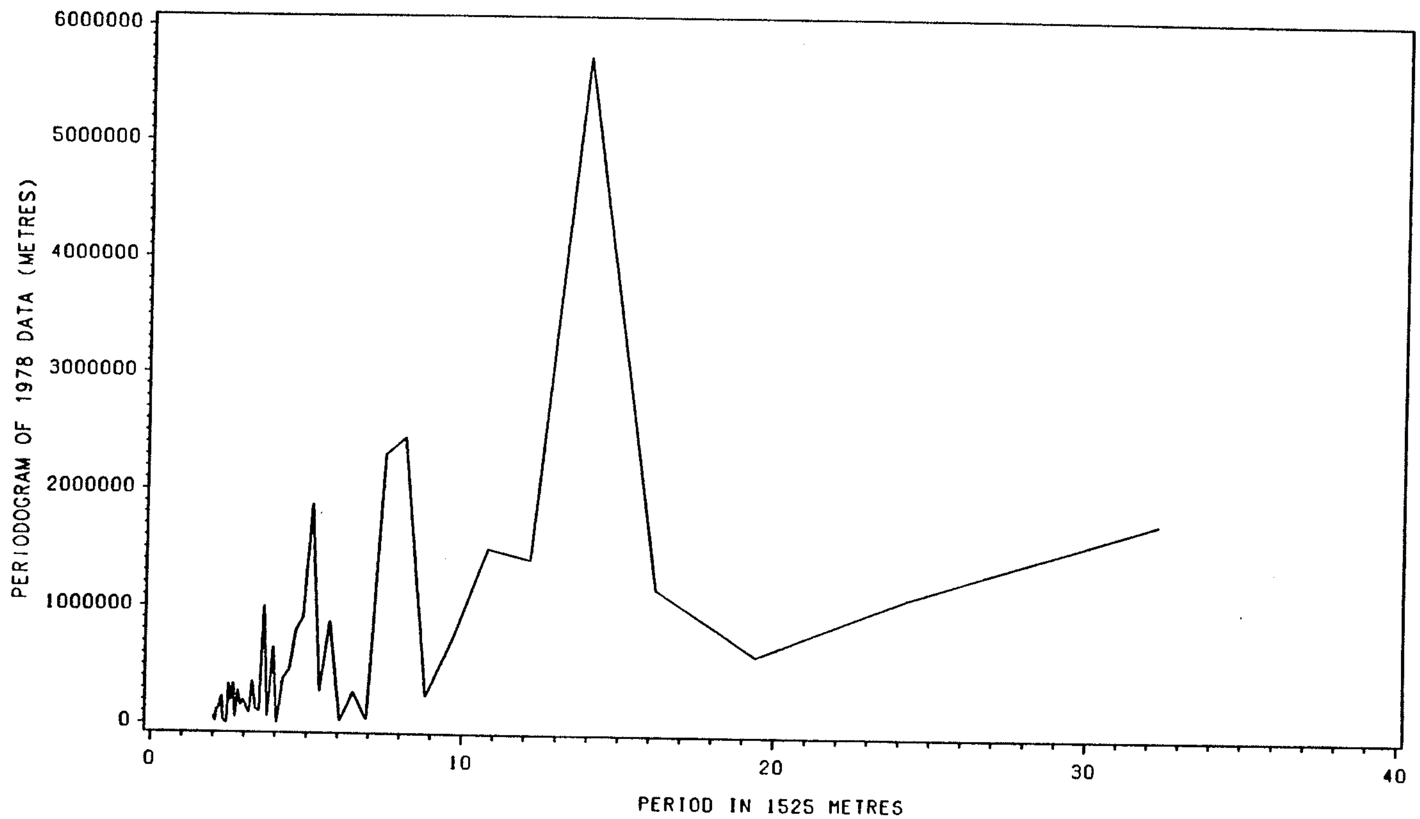


Figure 3.10: Periodogram of 1978 data

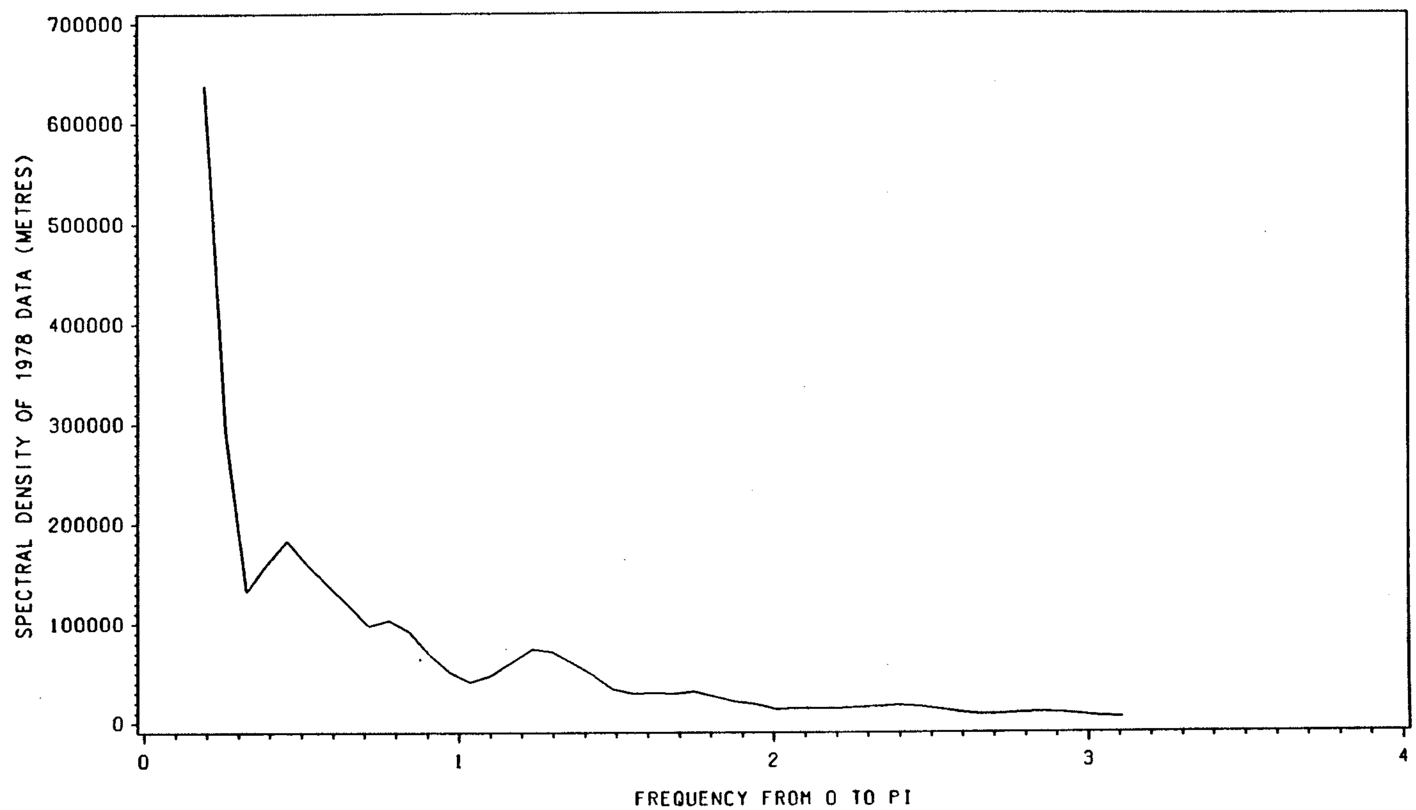


Figure 3.11: Spectral Density of 1978 data

To check the presence of trend in the series, a trend function (a polynomial in s of order two) was fitted to each displacement series. It was found that the series are nonhomogeneous and a significant trend exists in each of them.

3.7 PREWHITENING FILTER

As discussed earlier, the prewhitening filter should be such that it transforms the input and the output series into white noise residuals. In the previous section it was found that a periodicity of 15 space units, as well as a trend, is present in each displacement series. Removal of this periodicity and trend may be necessary but was not done in this study because of the paucity of data, possibility of model distortion and the need for more careful study as explained below:

1. The periodicity of 15 space units (15x1525 metres) is related to the average distance between nodes. Consideration of the periodicity in the prewhitening filter will be equivalent to taking these nodal points as permanent features of the river. But, so far, it has not been conclusively established whether they are permanent or transient. Also, there is no sound theoretical treatment available at present which is able to answer that question. The unified treatment of meandering and braiding process by the

general instability theory (see Hayashi and Ozaki, 1980) does state the existence of similar points at regular intervals along the river (so called wave length; (e.g., meander wave length) implying that if a node exists at some point, it will reoccur after some regular interval of space. It does not, however, clearly explain why a point should exist as a single channel in a river which is otherwise a braiding one. Furthermore, the bank material that exists at the nodal points is not significantly different from the material at other locations of the river reach under study. Until physical evidence about the condition of the nodal points is obtained, it has been decided not to consider the periodicity in the selection of prewhitening filter. Also, no meandering characteristic is obvious in the river that can explain this periodicity. Therefore, no differencing on the data was performed.

2. Only a limited channel reach has been considered in this study. Had there been a longer channel reach considered, perhaps, no overall trend could be demonstrated. Therefore, consideration of such a trend will be meaningless when the information on the entire river is not known. Hence, no trend function was subtracted from the data.

It is to be noted that the idea of prewhitening the input and the output series is related to easier detection of significant causal relationship between two series. Removal of periodicity becomes essential in order to infer the significance of such relationships when the crosscorrelation is insignificant at all lags (including zero) other than the lag corresponding to the period present in the series. As will be seen in section 3.8.1, there exists a highly significant crosscorrelation at lag zero in addition to that at lag +15. Therefore, it can be safely assumed that no serious error in the inference of significant relationship has been made by not removing the periodicity from the series.

3.7.1 Fitting a Proper Prewhitening Filter

In general the prewhitening filter transforms the series into a series of white noise with zero mean. The filter was selected by fitting a suitable Autoregressive Integrated Moving Average (ARIMA) model (Box and Jenkins,1976) of order (p', d', q') to each input series such that :

$$\phi'(B)\ (x_S - m_x) = \theta'(B)\ u_S \qquad (3.16)$$

where m_x = the mean of the series

B = backshift operator

$\phi'(B)$ = autoregressive operator of order p'

$= 1 - \phi'_1 B - \phi'_2 B^2 - \ldots\ldots - \phi'_{p'} B^{p'}$

$\theta'(B)$ = moving average operator of order q'

$= 1 - \theta'_1 B - \theta'_2 B^2 - \ldots\ldots - \theta'_{q'} B^{q'}$

u_s = white noise (also called innovations or disturbances) with a mean of zero and variance of 6_u^2, and

d' = degree of differencing = 0.

The subscripts x denote the input series.

In the above model, the operator $\phi'(B)$ should be such that the roots of the characterstic equation $\phi'(B)=0$ should lie outside the unit circle for stationarity. Also, the roots of the charcterstic equation $\theta'(B)=0$ should lie outside the unit circle for invertibility. The linear nature of the operators $\phi'(B)$ and $\theta'(B)$ insures that u_s and v_s are causally related in the same way as x_s and y_s , v_s being the white noise residual of the output series.

To select the appropriate prewhitening filter, sixteen models up to order three (inclusive) for both of the autoregressive part p' and the moving average part q' were tested. The model with the minimum Akaike Information Criterion (AIC) and Schwartz Bayesian Criterion (SBC) was retained. These criteria are defined as :

$$AIC = -2 \ln(L) + 2k \qquad \text{(Akaike, 1974)}$$

$$SBC = -2 \ln(L) + \ln(n)k \qquad \text{(Schwartz, 1978)}$$

where, L= the likelihood function

k = number of free parameters

n = number of residuals that can be computed for the series.

The selected prewhitening filter was found to be of order (1,0,0). This simplifies Equation (3.16) to :

$$(1 - \phi_1' B)(x_s - m_x) = u_s \qquad (3.17)$$

which is a simple first order autoregressive model. For example, the prewhitening filter of 1977 west bankline displacement series is :

$$(1 - 0.81B)(x_s - m_{77}) = u_s \qquad (3.18)$$

and that for 1978 west bankline displacement series is :

$$(1 - 0.80B)(x_s - m_{78}) = u_s \qquad (3.19)$$

where, m_{77} and m_{78} are the mean bank displacement of 1977 and 1978 respectively.

The residual series produced by the ARIMA(1,0,0) model were checked for normality, independence and lack of fit. The plots of residual autocorrelation function of 1977 and 1978 data are given in Figures 3.12 and 3.13 and found to be adequate. The χ_m^2 value used in the lack of fit test has been computed with the formula :

$$\chi_m^2 = n(n+2) \sum_{k=1}^{m} r^2 /(n-k) \qquad (3.20)$$

where, $r_K = \sum_{k=1}^{n-k} a_s a_{s+K} / \sum_{k=1}^{n} a_s^2$

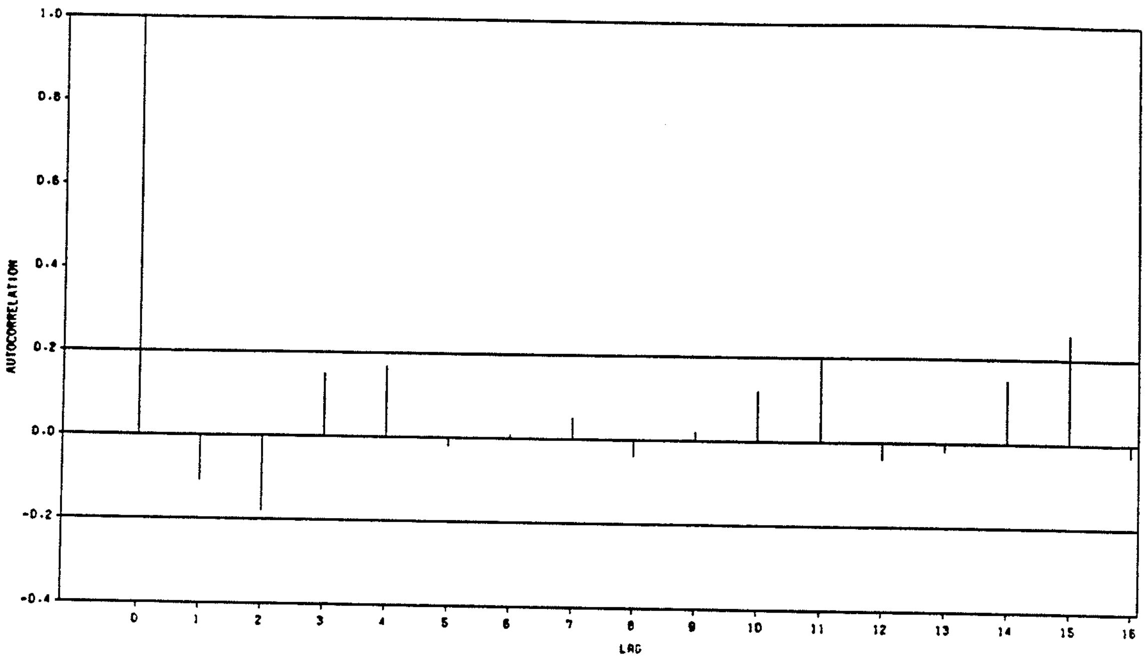


Figure 3.12: Residual Autocorrelation Function of 1977 data after Prewhitening

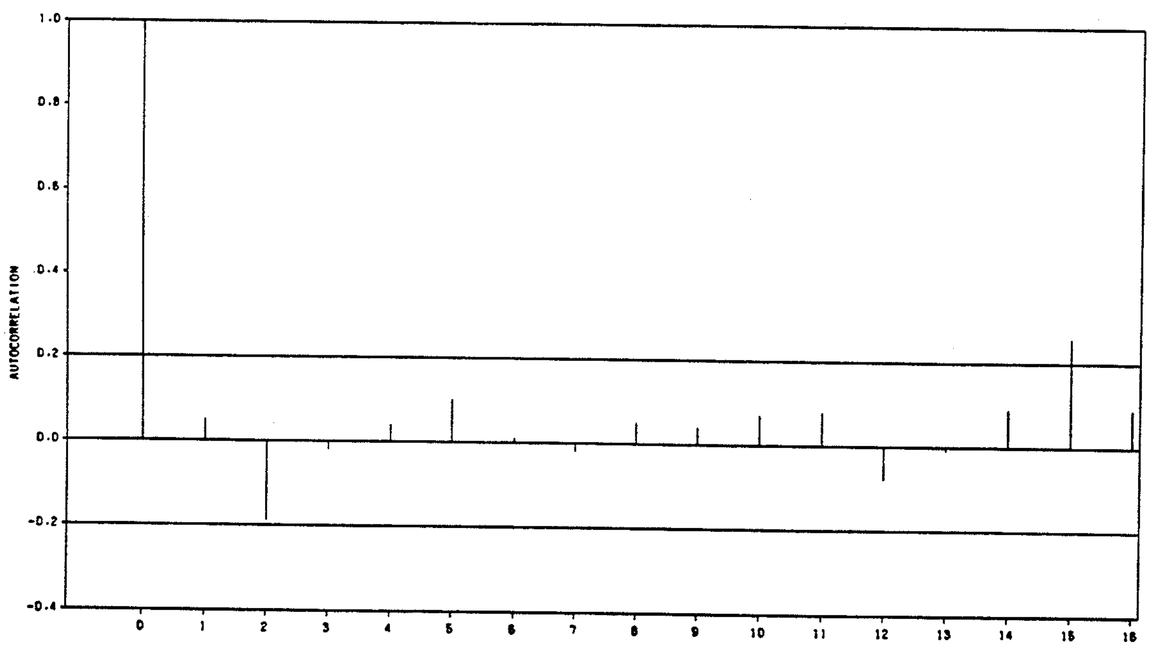


Figure 3.13: Residual Autocorrelation Function of 1978 data after Prewhitening

and a_s is the residual sequence. This formula has been suggested as yielding a better fit to the asymptotic Chi-square distribution (Ljung and Box,1978). The residual sequence passes these tests.

It has to be noted that the prewhitening filter corresponding to the input was used for both of the input and the output series as suggested by Box and Jenkins (1976). This means that when the 1977 series is taken as input and the 1978 series as the output, the prewhitening filter of the 1977 series was used for both of the 1977 and 1978 series.

3.8 RELATIONSHIP BETWEEN THE INPUT AND THE OUTPUT SERIES

From the physical point of view, the continuous and varying actions of water and sediment discharge on a bankline for a year produce the output bankline of the next year. The discharge, in turn, depends on the climatological factors, e.g. the amount, intensity and distribution of rainfall, etc. The magnitude and distribution of erosion along the bankline depends on bank material and other geological properties. Therefore, the output bankline series, in essence, is produced by multiple input including the bankline of the previous year to the system.

All of the above factors act on the existing bankline to transform it into the bankline of the next year. Therefore, the position of the bankline in the previous year is a very important factor in determining the present bankline position. Also, the bankline at a point along the river does not move miles away from the present position without any change at a point immediately upstream of it. In that respect, a continuity of bankline at a fairly reasonable scale exists. Therefore, if the space interval of the displacement series is small enough to account for such continuity, a significant relationship between the input and the output series should exist from the physical point of view.

The significance of the relationship between the input and the output series can be statistically tested and thus forms the very basis of the Transfer Function. In time series analysis literature, this relationship is referred to as causality. Granger (1969) presented a formal statistical definition of causality in terms of predictability between two series which states that : a variable x_t causes another variable y_t , with respect to a given universe or information set that includes x_t and y_t , if y_t can be better predicted by using past values of x_t than by not doing so, all other relevant information being used in either case, i.e.,

$$\sigma^2(y_t \,|\bar{A}_t) < (y_t \,|\bar{A}_t - \bar{x}_t) \qquad (3.21)$$

where, $\bar{A}_t$ is the given information set that includes at least x_t and y_t .

The types of causality (i,e, instantaneous or having feedback etc.) can be determined from the crosscorrelation structure of the prewhitened series as discussed below.

3.8.1 Crosscorrelation

After prewhitening the series with the selected filter, the Cross Correlation Function (CCF) at lag k between the white noise input residual u_s and the white noise output residual v_s can be computed using :

$$r_{uv}(k) = c_{uv}(k)/[c_u(0)\ c_v(0)] \qquad (3.22)$$

where, $$c_{uv}(k) = \frac{1}{n}\sum_{s=1}^{n} u_s v_{s+K} \qquad \text{for } k>0$$

$$= \frac{1}{n}\sum_{s=1-k}^{n} u_s v_{s+K} \qquad \text{for } k<0$$

= the estimated crosscorrelation function at lag k between the residual series

$c_u(0)$= estimated variance of u_s

$$= \frac{1}{n}\sum_{s=1}^{n} u_s^2$$

$c_v(0)$= estimated variance of v_s

$$= \frac{1}{n}\sum_{s=1}^{n} v_s^2$$

Since, unlike the autocorrelation function, the crosscorrelation function is not symmetrical about lag zero, the properties of $r_{uv}(k)$ must be examined for both positive and negative values of k. The r_{uv} is normally distributed with a mean of zero and variance 1/n for white noise. It has been a practice to use two standard deviations as the confidence limit (i,e, $\pm 2/\sqrt{n}$ in place of $1.96/\sqrt{n}$ for a significance level of 5%).

There are many possible types of causal interactions between x_s and y_s which can be categorized according to the restrictions on $\gamma_{uv}(k)$ (Pierce and Haugh,1977). From the crosscorrelation function plot (Figure 3.14) it can be seen that $r_{uv}(k) = 0$ (i.e., less than $2/\sqrt{n}$) for all $k < 0$ which suggests that there is no feedback. Also, $r_{uv}(k)$ is not equal to zero for some $k > 0$. Therefore, it can be inferred that x_s causes y_s without any feedback. It is to be noted that if x_s causes y_s only instantaneously, i.e., r_{uv} is not equal to zero for $k = 0$ only, causality exists only if there is no feedback (Price,1979, Pierce and Haugh,1979). Violation of this restriction did not happen in any of the cases. It should be noted here that the relatively large correlation at a lag of +15 is related to the average distance between nodes.

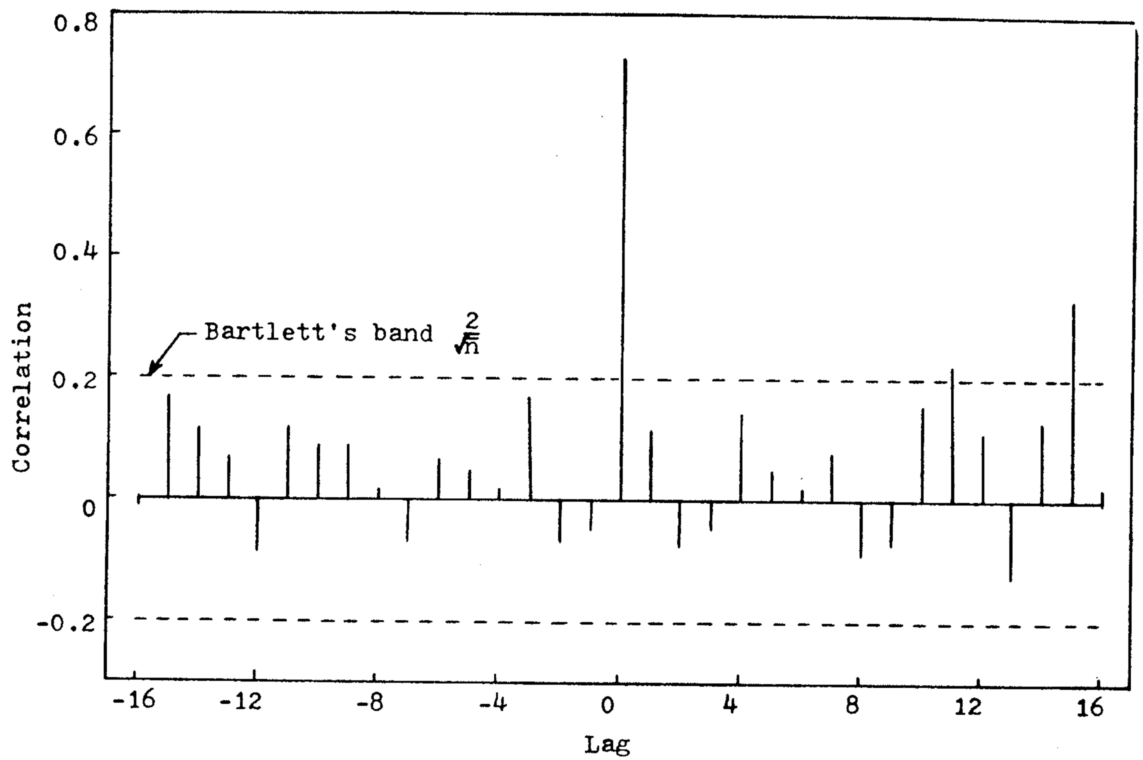


Figure 3.14: Crosscorrelation of 1978-1977 West Bank data

Therefore, it is seen that the series of 1977 is statistically related to the series of 1978. Although it is true that the output bank location is caused not only by the input bankline, but by other variables, the single input-single output model has been taken as a first step, as suggested by Jenkins (1979). The inclusion of other variables may be made to improve and elaborate the model.

3.9 MODEL IDENTIFICATION AND PARAMETER ESTIMATION

This section deals with the order identification of the Transfer Function and the Noise Model, the estimation of the parameters and the diagnostic checking of the selected model. These steps are interlinked which gives a repetitive form of model selection procedure. The parameters of the preliminary selected model are estimated and diagnostic checking is done. If the preliminary model is found to be inadequate, another model of different order is chosen and the same procedure follows. This procedure is suggested by Box and Jenkins (1976). Other forms of model identification procedures that are available (see Hipel et al.,1985) are almost the same in terms of computing efficiency, and have not been used in this study.

3.9.1 Identification of the Transfer Function Model

Identification of the Transfer Function Model involves estimation of the order r,c and b of the model (see Section 3.5). The preliminary estimation of the model order can be made from the crosscorrelation function of the prewhitened series as obtained in the previous section.

The crosscorrelation function of Figure 3.14 shows the correlation at zero lag to be highly significant. This means that there is no pure system delay implying that b=0 in Equation (3.6). Also, since the correlation at positive lags near zero are insignificant (Figure 3.14), the values of r and c are either 0 or 1 (at most 2). The relative large correlation at a lag of +15 is related to the average distance between nodes and, as discussed earlier, has not been considered in this study. Therefore, all combinations of r,c,b between 0 and 2 were studied to identify the proper order of Transfer Function model starting with the order (0,0,0). The reason for starting with a low order simple model is the lack of uniqueness of the Transfer Function model as discussed below.

If a particular system is represented by the model

$$y_S = \delta^{-1}(B)w(B)x_{S-b} + \phi^{-1}(B)\theta(B)a_S \tag{3.6}$$

then the system can also be represented by :

$$G(B)y_S = G(B)\,\delta^{-1}(B)w(B)x_{S-b} + G(B)\phi^{-1}(B)\theta(B)a_S \tag{3.23}$$

where G(B) is any arbitrary operator. Also, if G(B) is taken to be equal to $\phi(B)$, the above equation can be written as :

$$G(B)y_S = G(B)\delta^{-1}(B)w(B)x_{S-b} + \theta(B)a_S \tag{3.24}$$

There is a chance of getting towards an unnecessarily complicated model (e.g., Equation 3.23 or 3.24) and parameter redundancy in the 'trial and error' procedure of model selection. Also, it is very difficult to find out and to eliminate the common factor G(B) once such a model is estimated. This becomes difficult because there may exist factors which are not exactly the same but are close in magnitude (near redundancy), and have the same effect of instability in parameter estimation. The chance of getting into a situation like this is greatly reduced if the starting model form is kept simpler.

3.9.2 Identification of the Noise Model

The Noise Model can be identified from the autocorrelation function of the estimated noise term. The noise term can be estimated from Equation (3.6) using the estimated $\delta(B)$ and w(B) as :

$$\hat{n}_S = y_S - \hat{\delta}^{-1}(B)\,\hat{w}(B)\,x_{S-b} \qquad (3.25)$$

where, ^ denotes the estimated values. The identification procedure involves fitting an ARMA model to the estimated noise term $\hat{n}_S$ such that :

$$\phi(B)\,\hat{n}_S = \theta(B)a_S$$

where, $\phi(B)$ is the autoregressive operator, $\theta(B)$ is the moving average operator and a_S is the white noise residuals. The restrictions on the operator $\phi(B)$ and $\theta(B)$ are the same as those of the prewhitening stage.

In order to select the operator $\phi(B)$ and $\theta(B)$, the autocorrelation function of $\hat{n}_S$ should be studied. The necessary steps involved are the same as those in fitting a prewhitening filter and, hence, are not repeated here. The criterion for selecting the proper order are the same as well. The result of this identification step is dependent on the identification and estimation of the Transfer Function model and are discussed in the 'Diagnostic and Aptness' section that follows.

3.10 PARAMETER ESTIMATION

Following the identification procedure described in the previous section, the Transfer Function Noise Model

$$y_S = \delta^{-1}(B)w(B)x_{S-b} + \phi^{-1}(B)\theta(B)a_S$$

has been tentatively identified. The problem now is to estimate the parameters b, $\underline{\delta}$, $\underline{w}$, $\underline{\phi}$ and $\underline{\theta}$ of the model efficiently and simultaneously. It is to be noted that all of the parameters, with the exception of b, are vectors, and, usually have more than one coefficient.

Three methods, that have been widely used in the literature, were employed for parameter estimation in order to facilitate comparison between models. These methods are the Conditional Least Square (CLS) method, the Unconditional Least Square (ULS) method and the Maximum Likelihood Estimation (MLE) method. A very brief discussion on these methods are given here. Details may be found in the literature, such as Box and Jenkins (1976, Chapter 7). The results will be discussed in the diagnostic checking section that follows.

3.10.1 <u>The Conditional Least Square (CLS) method</u>

Given the data, for any choice of parameters $(b,\underline{\delta},\underline{w},\underline{\phi},\underline{\theta})$ and starting values $(\underline{x}_0, \underline{y}_0, \underline{a}_0)$ prior to the beginning of the series, the innovations $\underline{a}_s = a_s(b,\underline{\delta},\underline{w},\underline{\phi},\underline{\theta} \mid \underline{x}_0, \underline{y}_0, \underline{a}_0)$ can be calculated successively for s = 1,2,3 ... n . The parameters can be found by minimizing the conditional sum of square function

$$S_0(b,\underline{\delta},\underline{w},\underline{\phi},\underline{\theta}) = \sum_{s=1}^{n} a_s^2 (b,\underline{\delta},\underline{w},\underline{\phi},\underline{\theta} \mid \underline{x}_0, \underline{y}_0, \underline{a}_0) \qquad (3.26)$$

If the starting values $\underline{x}_0, \underline{y}_0, \underline{a}_0$ are unknown, the summation on the right hand side can be done over the available innovations setting the unknown a's to be zero.

3.10.2 <u>The Unconditional Least Square (ULS) method</u>

The unconditional sum of square function is given by :

$$S_0(b,\underline{\delta},\underline{w},\underline{\phi},\underline{\theta}) = \sum_{s=-\infty}^{n} [a_s \mid b,\underline{\delta},\underline{w},\underline{\phi},\underline{\theta}] \quad \text{where,} \qquad (3.27)$$

$$[a_s \mid b,\underline{\delta},\underline{w},\underline{\phi},\underline{\theta}] = E[a_s \mid b,\underline{\delta},\underline{w},\underline{\phi},\underline{\theta}]$$

and denotes the expectation of a_s conditional on $b, \underline{\delta}, \underline{w}, \underline{\phi}$ and $\underline{\theta}$. The parameters are estimated by minimizing the sum of square S_0 and are known as Least Square estimates.

3.10.3 The Maximum Likelihood Estimation (MLE)

The likelihood function has been estimated by the algorithm suggested by Ansley (1977) and maximization is done by nonlinear least square using Marquardt's method.

3.11 DIAGNOSTIC CHECKING AND APTNESS

After estimating the parameters of the tentatively selected model, the residuals are used for diagnostic checking. The fundamental assumptions in the Transfer Function Noise Model are that :

1. The residuals are independent, normally distributed and of constant variance, and that;
2. The residuals are independent of the input variable(s).

Violation of the first assumption suggests that the Noise Model is inadequate, whereas, violation of the second suggests an inadequate Transfer Function.

If a Transfer Function of order (0,0,0) and a Noise model of order (0,0) is taken, both of the above assumptions are violated suggesting that both of the models are inadequate and should be of higher order. Combination of TF(0,0,0) and

Noise(1,0) indicates inadequacy of the the Transfer Function showing a significant cross-correlation of a_S with the input x_S.

In this way sixteen combinations of Transfer Function and Noise model, both of them up to order 2 inclusive, have been tried and the best and simpler ones retained on the basis of Akaike Information Criterion and Scwartz Bayesian Criterion. The procedure resulted in a selected Transfer Function for both banks of order (0,1,0) and a Noise Model of order (1,0,0). The three methods of parameter estimation that were used gave almost identical answers.

3.11.1 Final Model

The selected model for west bank of 1978 with that of 1977 as input is :

$$(y_S - m_{78}) = (0.73+0.21B)(x_S - m_{77}) + (1/(1-0.36B))a_S \quad (3.28)$$

where m_{78} and m_{77} are the mean banklines of 1978 and 1977 respectively. Here w_0 =0.73, w_1 =-0.21 and ϕ_1 =0.36 and

$$\delta(B) = 1$$

$$w(B) = 0.73 + 0.21\ B$$

$$\phi(B) = 1 - 0.36\ B$$

$$\theta(B) = 1$$

Equation (3.28) can be written in expanded form as :

$$y_s = 0.36y_{s-1} + 0.73x_s - 0.06x_{s-1} - 0.08x_{s-2} + 0.64m_{78} - 0.59m_{77} + a_s \qquad (3.29)$$

The maximum likelihood estimation converged in only 10 iterations and the conditional least square estimation converged in only 4 iterations. This is also a good indication of the model's stability. The t-ratios of the parameters are significant at 5% level of significance. The stationarity and invertibility conditions are met since all of the parameters lie outside the unit circle i,e,

$$-1 < \text{parameter} < 1$$

The residuals were also able to be taken as normally distributed. The autocorrelation function and partial autocorrelation function, and also the Chi-square test show that the residuals are independent implying that the Noise model is sufficient. The cross correlation of the residuals with the input shows that the residuals are independent of the input, which indicates that the Transfer Function is sufficient. The plot and statistics are given in Figures 3.15 and 3.16.

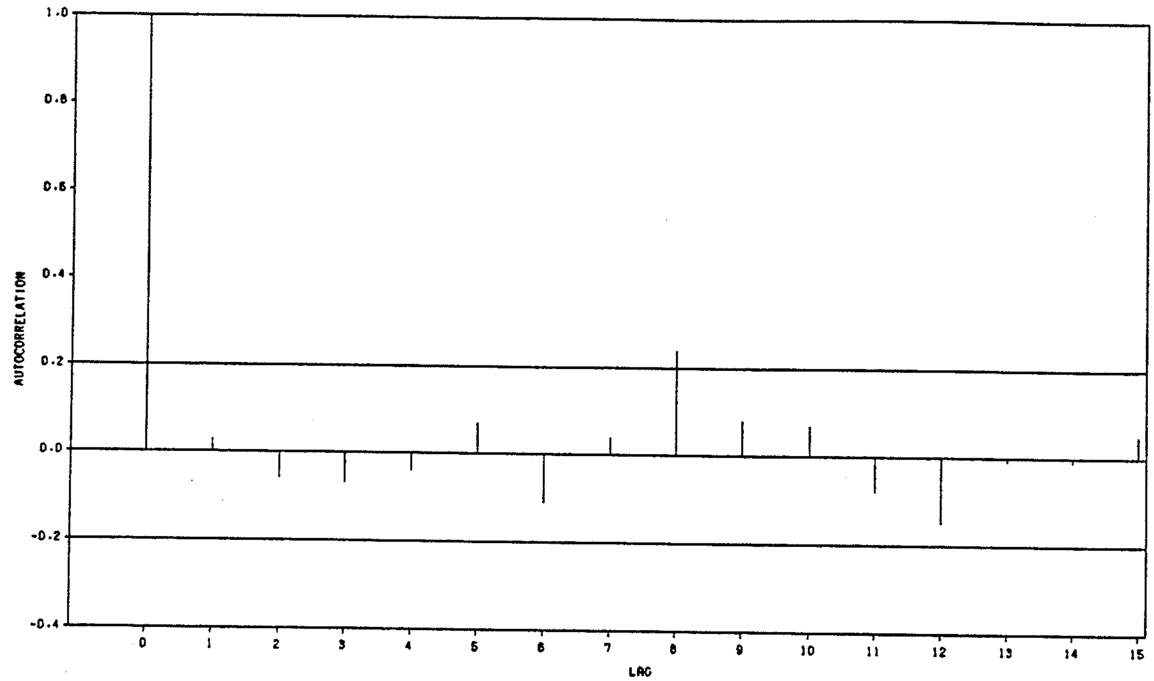


Figure 3.15: Residual Autocorrelation Function

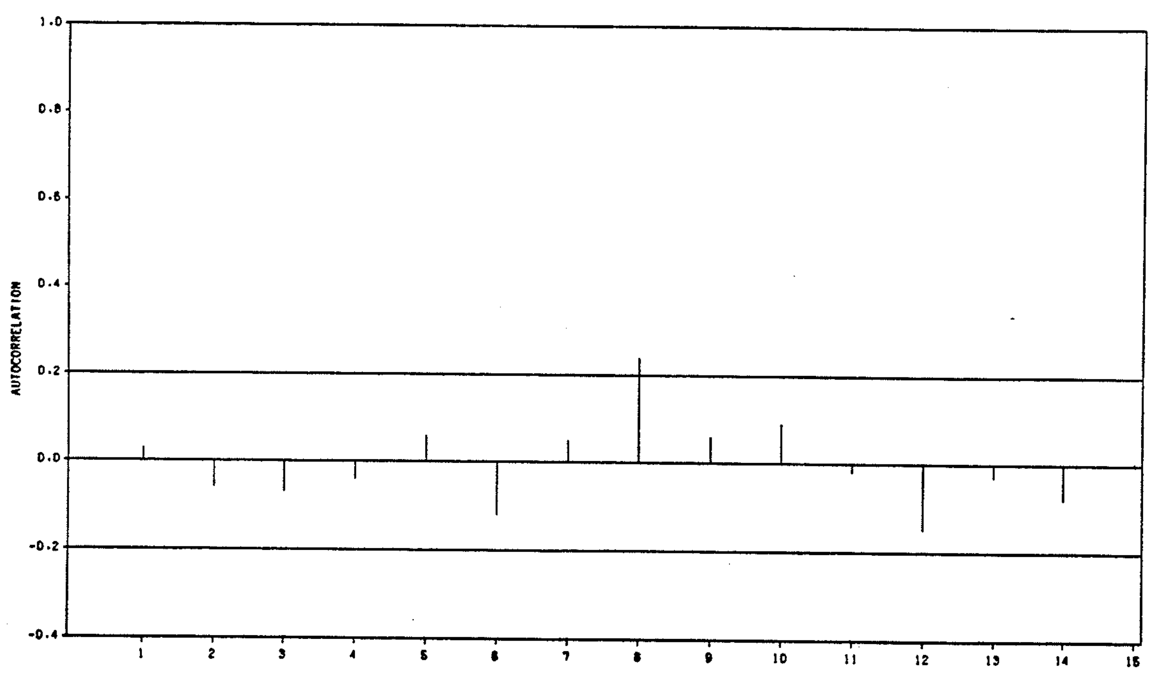


Figure 3.16: Residual Partial Autocorrelation Function

The other selected models for the west bankline of the other years, using the backshift operator, are :

$$(y_s - m_{77}) = (0.62+0.21B)(x_s - m_{76}) + (1/(1-0.71B))a_s$$

$$(y_s - m_{78}) = (0.73+0.21B)(x_s - m_{77}) + (1/(1-0.36B))a_s$$

$$(y_s - m_{79}) = (0.77+0.10B)(x_s - m_{78}) + (1/(1-0.35B))a_s$$

$$(y_s - m_{80}) = (0.59+0.22B)(x_s - m_{79}) + (1/(1-0.75B))a_s$$

$$(y_s - m_{81}) = (0.72+0.14B)(x_s - m_{80}) + (1/(1-0.47B))a_s \quad (3.30)$$

$$(y_s - m_{82}) = (0.45+0.05B)(x_s - m_{81}) + (1/(1-0.74B))a_s$$

$$(y_s - m_{83}) = (0.24+0.15B)(x_s - m_{82}) + (1/(1-0.72B))a_s$$

$$(y_s - m_{84}) = (0.64+0.10B)(x_s - m_{83}) + (1/(1-0.53B))a_s$$

$$(y_s - m_{85}) = (0.64+0.16B)(x_s - m_{84}) + (1/(1-0.78B))a_s$$

In all of the above equations, m denotes the mean bankline position and the subscript of m denotes the year of observation. It is to be noted that the 1982 satellite photograph that was used for analysis does not cover the whole study reach and therefore was excluded from determining the relationship between the model parameters and the discharge in subsequent analysis. Similar results were found for the east bankline.

3.12 PROPERTIES OF THE MODEL PARAMETERS

The selected Transfer Function Noise Model for both of the West and the East bankline displacement series has the form:

$$(y_s - m_y) = (w_0 - w_1 B)(x_s - m_x) + (1/(1 - \phi_1 B))a_s$$

where m_x and m_y respectively are the means of the input and the output displacement series, y_s is the displacement at bankline position s of year t, and x_s is the displacement at bankline position s of year (t-1). The relationships between the displacement values as defined by the parameters are shown in Figure 3.17.

The parameter w_0 shows the relationship between the bankline position at any point, y_s , and the bankline position at the same point in the previous year. The parameter w_1 shows the relationship between y_s and the bankline position one step upstream of that point of previous year, while ϕ_1 shows the relationship between y_s and the bankline position one step upstream of that point of the same year.

All of the model parameters show a strong relationship with the mean water discharge and the total volume of runoff in a year (Figures 3.18-3.20). The inverse relationships of w_1 and ϕ_1 with discharge show that with increased discharge the dependence of the bank displacement at any location on the displacement at an adjacent location decreases. On the other hand, w_0 has a positive relationship with discharge.

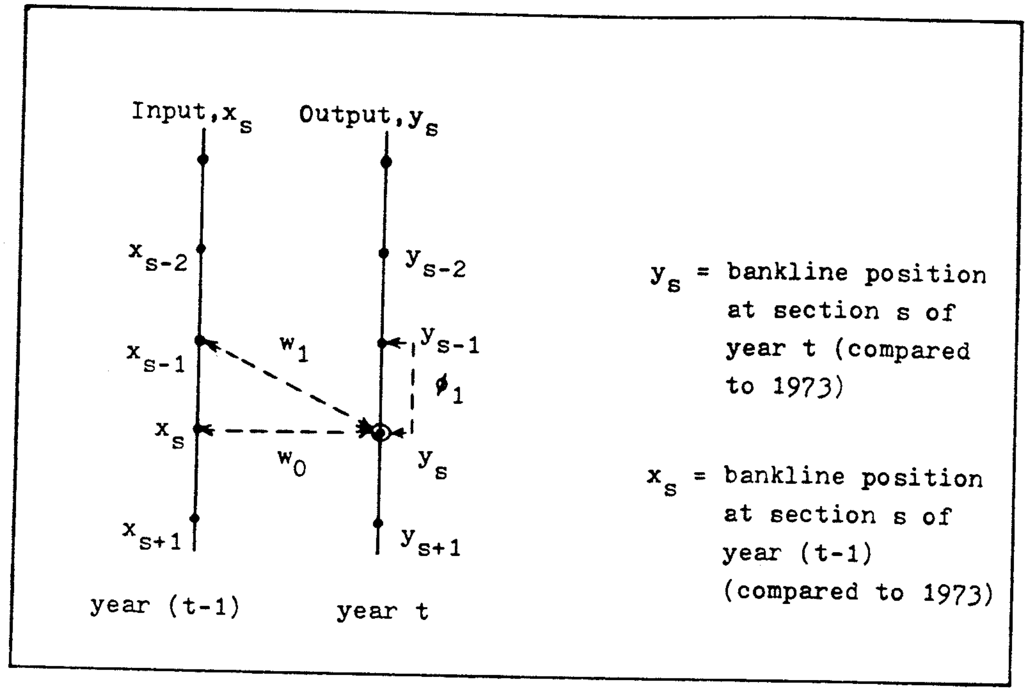


Figure 3.17: Definition Sketch Showing the Model Parameters

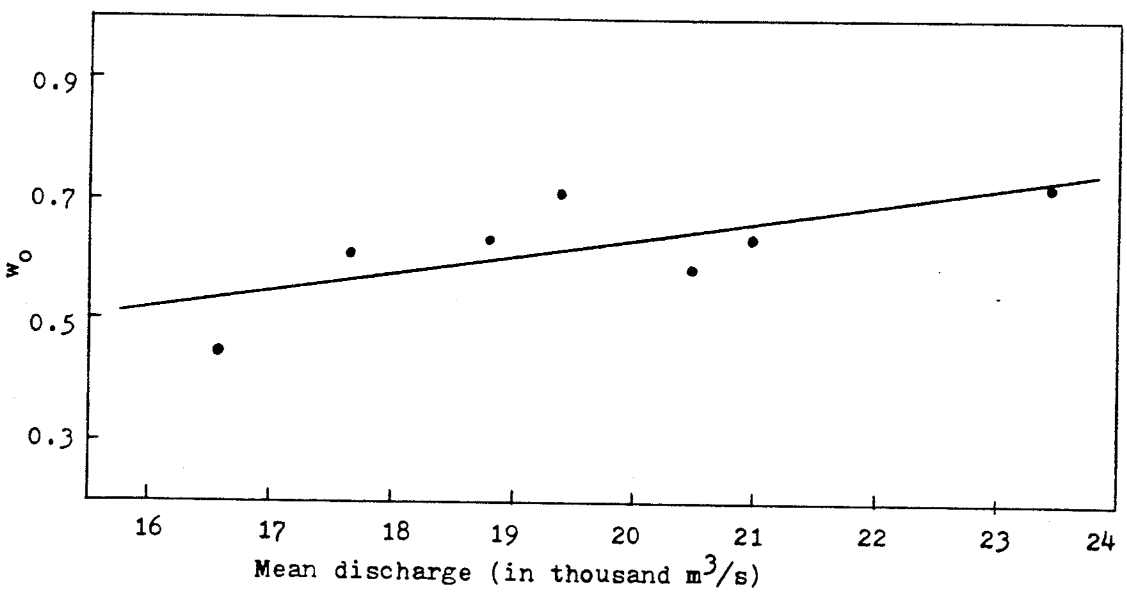


Figure 3.18: Variation of w_0 with mean discharge (West bank)

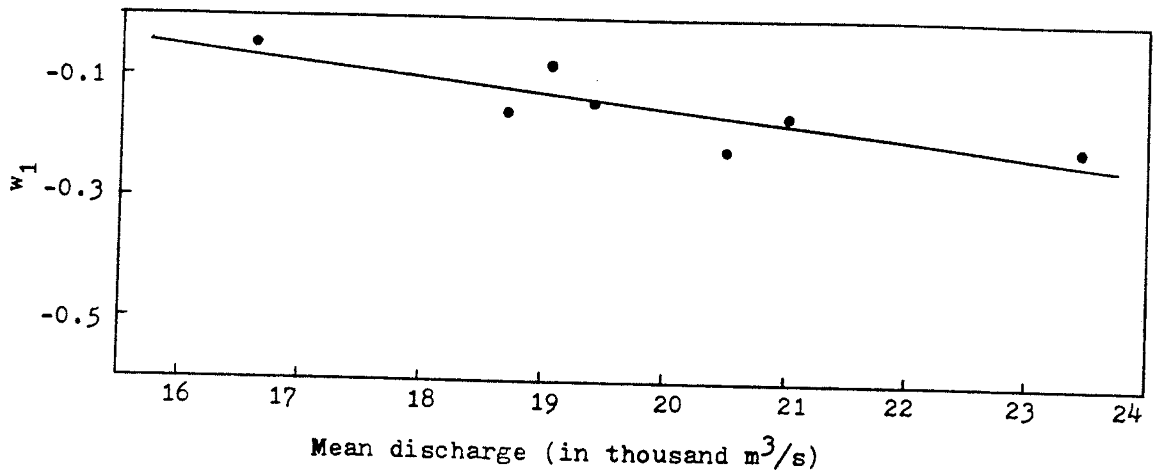


Figure 3.19: Variation of w_1 with mean discharge (West bank)

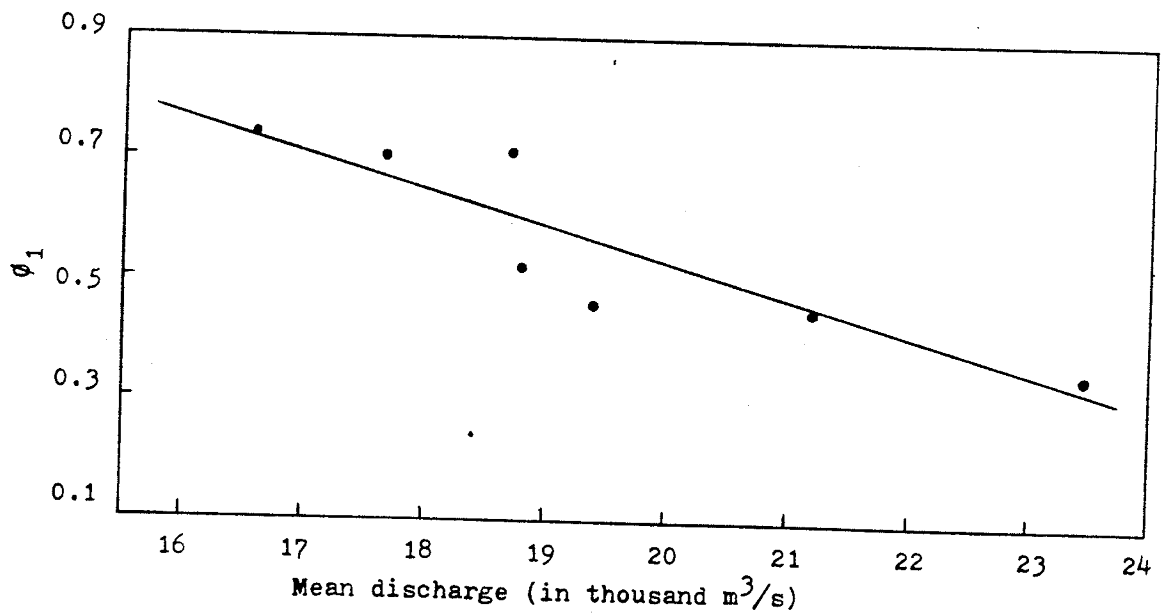


Figure 3.20: Variation of ϕ_1 with mean discharge (West bank)

This shows that higher discharges increase the dependence of the displacement at a given location with the displacement at that location at the previous time step. In other words, relative movement at any point increases with higher discharge. The relationships of the model parameters with discharge for the east bank lead to the same conclusion. They are shown on Figures 3.21, 3.22 and 3.23.

Although at any point erosion and deposition may alternate in time, the mean west bankline in the study reach was found to be shifting westward at an average rate of about 120 metres per year (Figure 3.24) with no definite pattern of variation with water discharge (Figure 3.25). This means that during the period under study the river had a tendency to shift westward in the study reach. It is to be noted that not all of the subreaches were experiencing this mean westward movement to the same extent. Also, the east bank did not show such a consistent trend in any direction over time.

3.13 FORECASTING BANKLINE MIGRATION

To forecast the bankline position at a point in any year, one must know the parameters w_0, w_1, ϕ_1, the input variables x_S, x_{S-1} and y_{S-1} and the mean bankline (see Figure 3.17). The parameters depend on the discharge. Since the discharge is not known in advance the average values of the parameters of the model over time (Equation 3.30) have been

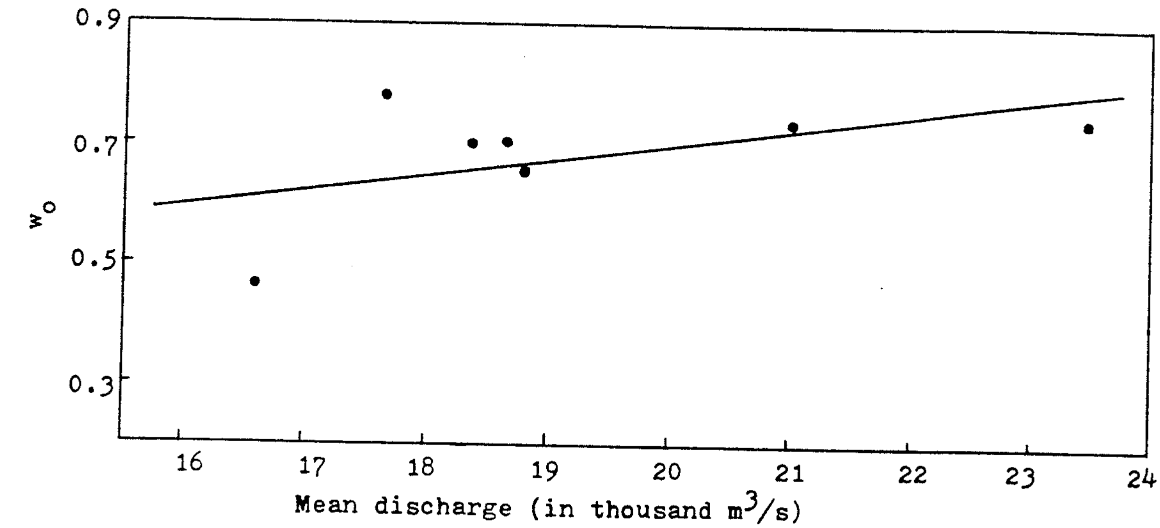


Figure 3.21: Variation of w_0 with mean discharge (East bank)

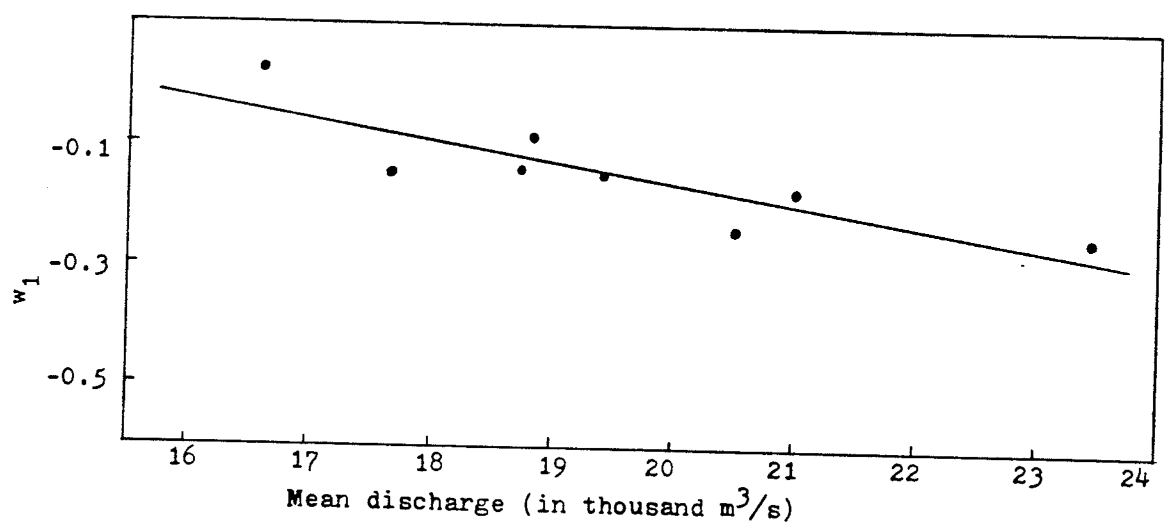


Figure 3.22: Variation of w_1 with mean discharge (East bank)

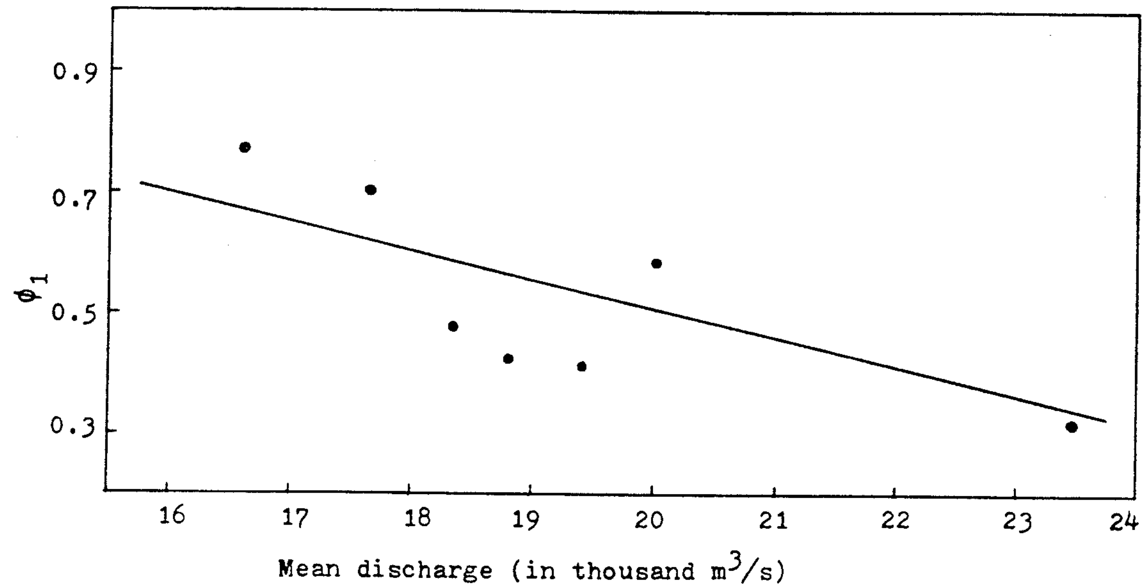


Figure 3.23: Variation of ϕ_1 with mean discharge (East bank)

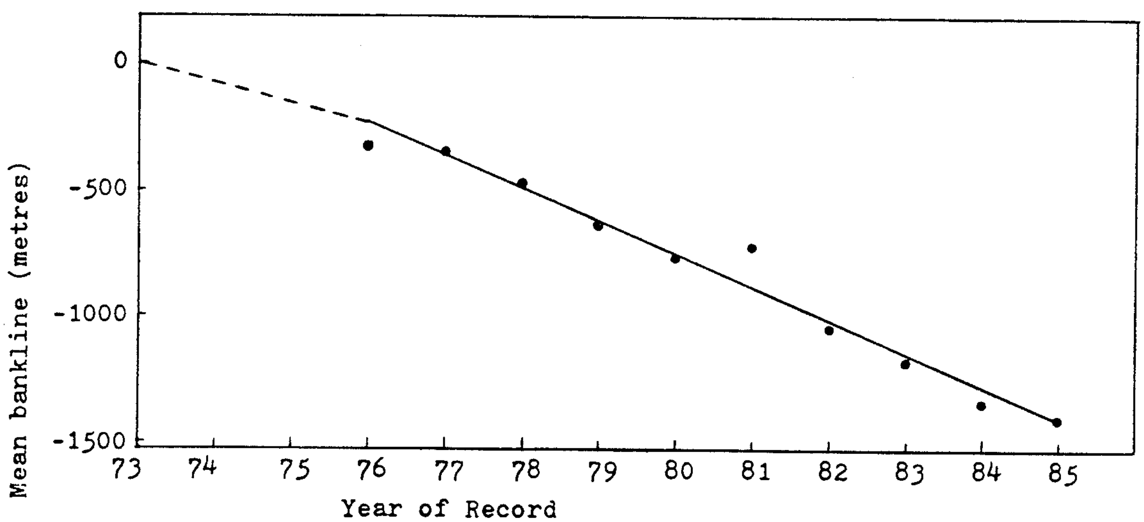


Figure 3.24: Time plot of mean west bankline series

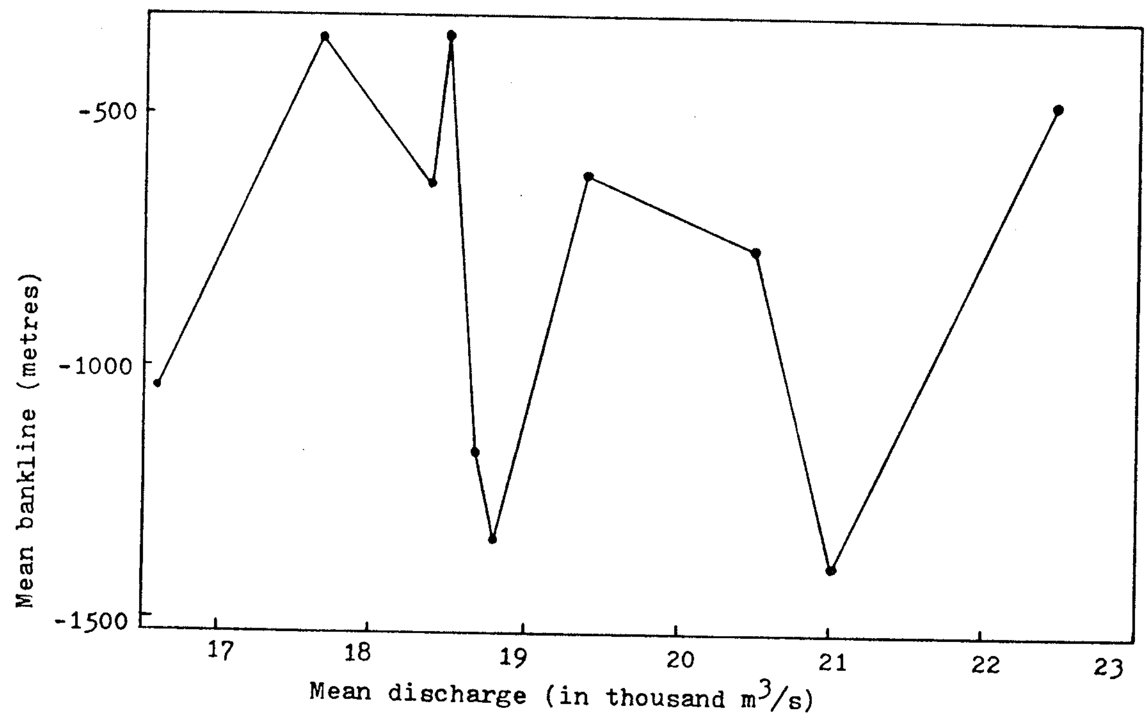


Figure 3.25: Plot of mean discharge with mean west bankline series

used for a one-year-ahead forecasting. The position of the bankline of the previous year (x_s and x_{s-1}) are of course known from the recorded data. For the input y_{s-1} the forecasted value at one step upstream of the point was taken. The mean forecasted bankline also had to be calculated. It is to be noted that the effect of the mean terms (Equation 3.29) on the forecasted values in the study period is not very high. Starting at the most upstream point of the study reach with mean bankline value as y_s and proceeding as outlined above, the bankline positions at all points along the river were forecasted.

An example is given in Figure 3.26, where the west bankline of 1978 was forecast using the known values of 1977. The standard error of forecasted values is 270 metres. The forecast predicted 95% of the time accurately whether erosion or deposition would occur. Using a univariate model, that is assuming the erosion or deposition at any point to be a simple Markov chain, the standard error of the forecasted values was found to be 530 metres. This shows that consideration of 1977 bankline as an input variable has made a substantial improvement over a univariate model, and thereby satisfies Granger's (1969) definition of causality.

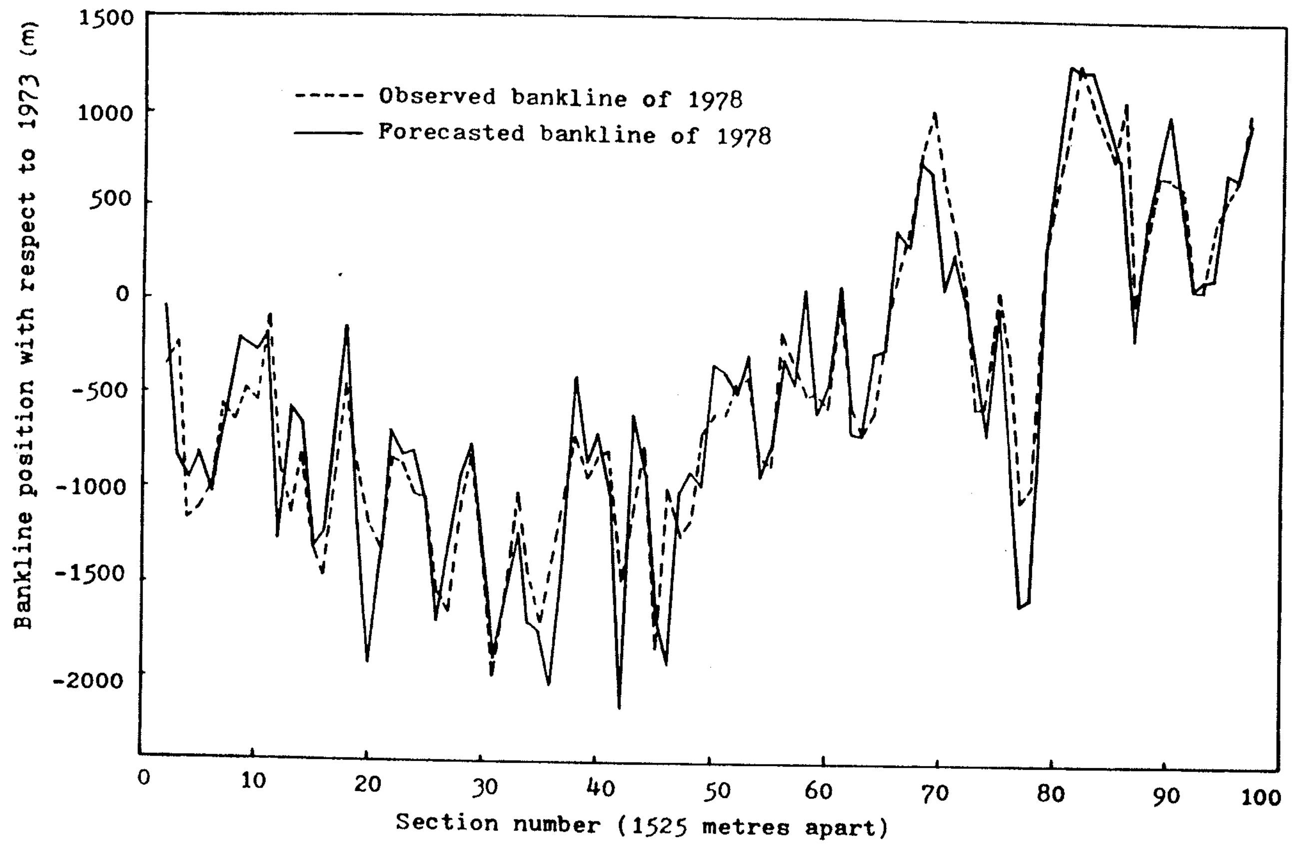


Figure 3.26: One time step ahead forecasted bankline of 1978 with 1977 bankline as input (West bank)

3.14 LIMITATIONS AND IMPROVEMENTS

The main limitation caused by the limited data base of the present model include the variability of the Transfer Function and the Noise coefficients as evidenced from Equation 3.30. There may well be a trend and/or periodicity present in the series which does not show up in the model. Cosequently, the following improvements of the present model are recommended for future work in this area.

The model parameters do not show any regular pattern of variation with time or with maximum discharge. But they do have a significant relationship with mean discharge. This relationship can be used to predict the parameter values from known values of the mean discharge for use in the model. For forecasting purposes, prediction of mean discharge from its past values and other physical variables like rainfall and temperature may be considered.

The periodicity present in the series may have been caused by the presence of the nodal points. These nodal points should be investigated and, if they are permanent, should be included as such in the model. The inclusion of the periodicity (i.e., consideration for these nodal points) can be done by following the procedure outlined by Box and Jenkins (1976).

The probability of erosion at points along the river for a longer time basis should be known in order to facilitate planning of future projects and protection of the existing ones near the river. Simulated results from the final model should be used to assign such probability values. This may be done by generating a large number (say 200) of values for year t with random components using the known values of year t-1 as input and registering the number of times it eroded past a reference line. This registered number of occurrence of erosion divided by the number of generated series will give the probability of erosion. The mean of the generated series will serve as input to the model for generating values of year t+1. Continuing as above, probability of erosion can be assigned to points along the river for a certain period of time (say 10 years).

Chapter IV

CONCLUSIONS AND RECOMMENDATIONS

4.1 CONCLUSIONS

The conclusions that can be drawn from this study are :

1. The Brahmaputra river in Bangladesh has experienced severe bank erosion and deposition throughout recorded history. Yet, there are a number of points along the river reach under study where the river experienced very little or no change over the recorded time. The average distance between two such consecutive nodes is in the order of 25 kilometres, except for one pair, which is about 45 kilometres. This reach undergoes more changes inside the outer banklines than the rest of the river under study.
2. Although, at any point, erosion and deposition may alternate in time, the mean west bankline of the Brahmaputra river in the study reach was found to be shifting westward at an average rate of 120 metres per year. The east bankline did not show such a consistent trend in either direction over time.
3. Statistical methods rather than the methods based on the dynamics of water and sediment transport should be used for an assessment of the bank erosion and

deposition of the Brahmaputra river in Bangladesh. The stochastic nature of the pertinent variables involved in the process, lack of complete understanding of the process and unavailability of extensive data needed by the present simplified deterministic modelling procedures, require the use of such methods.

4. The combined Transfer Function model of order (0,1,0) and Noise Model of (1,0,0) can be used to improve prediction of the seemingly erratic erosion and deposition along the outer banks of the Brahmaputra river in Bangladesh from one year to the next. The performance of the model in terms of the prediction of the erosion and deposition phenomenon is satisfactory. Also, the prediction of bankline movement from this model is better than any univariate model in the study period.

5. The parameters of the Transfer Function Noise Model do have a strong relationship with the annual mean water discharge and the total volume of yearly runoff through the river. This relationship suggests that the average discharge condition is important in the erosion/deposition process.

6. No significant relationship between the parameter values and the yearly maximum discharge could be found in the period of study. Lack of this kind of relationship suggests that the effect of maximum

discharge on the process of erosion and deposition was not significant in the period of study.

4.2 RECOMMENDATIONS

The issues that can be addressed in future studies are :

1. The presence of the nodal points should be investigated in order to determine whether these points are the permanent feature of the river. If so, they should be included in the model.
2. New variables, such as the mean water discharge, should be included in the model to account for the variability in the model parameters.
3. Changes inside the outer banklines (near char lands) should be modelled and the resulting model should be integrated with that of the outer bankline.
4. The model should be improved to assess the predictability on a longer time basis (e.g., 20 years). Probability of erosion and deposition at points along the river should be assessed on a longer time basis.
5. Finally, the pocess of erosion or deposition along the river should be considered as a single space-time event (on a longer time basis), and should be modelled using stochastic methods.

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Appendix A

Formulas given by Hayashi and Ozaki (1980)

The spatial distance by which local bed load transport rate lags behind local shear stress :

$$\delta = \lambda_1 d \ [\ 1+40\alpha_* \{ \frac{D_0 \ S}{(s-1)d} \}^3 \ ]$$

$$= \lambda_1 d \ [\ 1+5\alpha_* \{ \frac{D_0 \ f}{(s-1)d} \}^3 F^6 \ ]$$

where, λ_1 = the dimensionless measure for the length of single step of a particle

$\simeq$ 100

d = characterstic sediment diameter

α_* = a dimensionless constant $\simeq$ 43.5

D_0 = the depth of undisturbed flow

S = the mean channel slope

s = relative density of the sediment

f = mean friction factor of the bed

$= 2(u_*/U)^2 = 2S/F^2$

$F = U/\sqrt{gD_0}$

U = undisturbed flow velocity (mean)

CASE 1 :

When, $\left(\frac{BS}{m\lambda D_0}\right)^2 \ll 1$

Dominant wave length (L/B) = f_1 (F)

where, L = dominant wave length

B = width of the channel

$$f_1(F) = \frac{2}{m}\left[\frac{(10 + 19F^2 + 7F^4)(1 - F^2)}{-(5+8F^2+5F^4)+(25+110F^2+141F^4+44F^6+8F^8)^{1/2}}\right]$$

F = Froude number

Initial growth rate KC_I :

$$KC_I = A_s \frac{S}{F^2}\left\{5 - \frac{27}{50}\left(\frac{m\pi D_o^2}{B}\,\frac{\delta_r}{S}\,F^6\right)\right\} \text{ for } F^2 \ll 1$$

$$= A_s \frac{S}{F^2}\left\{\frac{49}{8} - \frac{9}{56}\left(\frac{m\pi D_o}{D}\right)^2 \frac{\delta_r}{S}\,\frac{1}{F^2}\right\} \text{ for } F^2 \gg 1$$

where, $A_s = \frac{T_0}{\sqrt{gD_o^3}} = [(s-1)d^3/D_o^3]^{1/2} F_f\,\phi_0$

$$\phi_o = 40\ T_{*o}^3$$

$$T_{*o} = T_* / [\ \rho(s-1)gd\ ]$$

$$F_f = \left[\frac{2}{3} + \frac{36\nu^2}{gd^3(s-1)}\right]^{1/2} - \left[\frac{36\nu^2}{gd^3(s-1)}\right]^{1/2}$$

KC_I is maximum for m=1. This formula is for meandering.

CASE 2 :

When, $\left(\dfrac{BS}{m\pi D_o} \right)^2 \gg 1$

$$\frac{L}{B} = \frac{6}{\sqrt{2\pi}} \frac{1}{m^{3/2}} (BS/D_o)^{1/2}$$

$$KC_I = A_s S m^2 \pi^2 \left(\frac{BS}{D_o} \right)^{-2} \left[1 - \frac{4\sqrt{2\pi}}{3} m \left(\frac{BS}{D_o} \right)^{-1} \right]$$

This formula is for braiding.

CASE 3 :

When, $\left(\dfrac{BS}{m\pi D_o} \right)^2 \simeq 1$

$$\frac{L}{B} = \frac{1}{m} f_2 (F)$$

$$f_2(F) = 2\sqrt{2} \left[\frac{18-10F^2+7F^4}{9+5F^2} \right]^{1/2}$$

$$KC_I = A_s S \left[\frac{9}{58} - \frac{3906}{107648} \frac{\delta_r}{S} \frac{1}{F^2} m^2 \left(\frac{\pi D_o}{B}\right)^2 \right] \text{ for } F^2 \ll 1$$

$$= A_s \frac{S}{F^2} \left[\frac{5}{2} - \frac{3713}{1568} \frac{\delta_r}{S} \frac{1}{F^2} m^2 \left(\frac{\pi D_o}{B}\right)^2 \right] \text{ for } F^2 \gg 1$$

This formula is also for meandering.

Appendix B

DEFINITIONS

The definitions in this appendix are taken from the sources given at the end of each definition. Some notations may have been changed to be consistent with the text.

B.1 STOCHASTIC PROCESS

A time-varying (or space varying) quantity, $Z(t)$, is called a stochastic process if the situation is such that, for each t, it is not possible to determine theoretically a precise value for $Z(t)$, a range of possible values with an associated probability distribution describing the relative likeliness of each possible value must be used instead. An observed record of a stochastic process is merely one record out of a whole collection of possible records which might be observed. The collection of all possible records is called the "ensemble" and each particular record is called a "realization" of the process (Priestley,1981).

B.2 WHITE NOISE

The process $Z(t)$, $t=0,\pm1,\pm2,\ldots$, is called a purely random process or white noise if it consists of a sequence of uncorrelated random variables, i.e., if for all $T\neq t$, $\mathrm{Cov}\{Z(T),Z(t)\}=0$. This is the simplest of all discrete parameter models and corresponds to the case where the process has "no memory", in the sense that the value of the process at time t is not correlated with all past values up to time (t-1) (and, in fact, with all future values of the process). Some authors (e.g., Jenkins and Watts, 1968) use the term "purely random process" to describe more particularly, a stationary sequence of independent random variables, a(t). Of course, if the a(t) are independent then they must be uncorrelated, but the converse is not necessarily true (unless a(t) is a Gaussian process). The purely random process is called "white noise", particularly in the engineering literature. This alternative description is due to the fact that a purely random process has a power spectrum which is "flat", i.e., has the same value at all frequencies. The term "white noise" thus arises from the analogy with "white light" in which all frequencies (i.e., "colors") are present in equal amounts (Priestley,1981).

B.3 STATIONARITY

The process $Z(t)$ is said to be completely stationary if, for any admissible t_1, t_2,...., t_n, and any k, the joint probability distribution of $\{Z(t_1), Z(t_2), \ldots, Z(t_n)\}$ is identical with the joint probability distribution of $\{Z(t_1+k), Z(t_2+k), \ldots, Z(t_n+k)\}$. The property described above may be summarized by saying that the probabilistic structure of a completely stationary process is invariant under a shift of the time origin.

Complete stationarity is, however, a severe requirement, and can be relaxed by introducing the notion of "stationarity up to order r" which is a weaker condition but nevertheless describes roughly the same type of physical behavior. Under this weaker condition it is not necessary to insist that, e.g., the probability distribution of $Z(t_1)$ must be identical with the probability distribution of $Z(t_1+k)$, but merely that the main features of these two distributions should be the same, i.e., that their moment up to a certain order, should be the same. Similarly, it is not necessary to insist that the joint distribution of $\{Z(t_1), Z(t_2)\}$ must be identical to the joint distribution of $\{Z(t_1+k), Z(t_2+k)\}$, but merely that, up to a certain order, their joint moments are equal, and so on. Therefore, the process $Z(t)$ is said to be stationary up to order r if, for any admissible $t_1, t_2, \ldots, t_n$, and any k, all the joint moments up to order r of $\{Z(t_1), Z(t_2), \ldots, Z(t_n)\}$ exist and equal the corresponding joint moments up to order r of $\{Z(t_1+k),\ Z(t_2+k), \ldots\ldots, Z(t_n+k)\}$. For example, if a process is stationary up to order 2, then it has the same mean value and same variance at all time points; and the covariance between the values at any two time points, T,t, depends only on (T-t), the interval between the time points, and not on the location of the points along the time axis (Priestley,1981).

B.4 LINEAR FILTER MODEL

The stochastic models employed are based on the idea that a series in which successive values are highly dependent can be usefully regarded as generated from a series of independent "shocks" a(t). These shocks are random drawings from a fixed distribution, usually assumed Normal and having mean zero and variance S^2. The white noise process a(t) is supposed to be transformed to the process Z(t) by what is called a linear filter. The linear filtering operation simply takes a weighted sum of previous observations, so that

$$\begin{aligned} Z(t) &= m + a(t) + \psi_1 a(t-1) + \psi_2 a(t-2) + \ldots \\ &= m + \psi(B)a(t) \qquad \text{(B.1)} \end{aligned}$$

where m is a parameter that determines the "level" of the process, and

$$\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \ldots\ldots$$

is the linear operator that transforms a(t) into Z(t) and is called the transfer function of the filter. B is the backshift operator.

The sequence ψ_1, ψ_2, ... formed by the weights may, theoretically, be finite or infinite. If this sequence is finite, or infinite and convergent, the filter is said to be stable. The parameter m is then the mean about which the process varies (Box and Jenkins,1976).

B.5 AUTOREGRESSIVE (AR) MODELS

A stochastic model in which the current value of the process is expressed as a finite, linear aggregate of previous values of the process and a shock a(t) is called the autoregressive model. Denote the values of a process at equally spaced times t, t-1, t-2, t-3,.... by Z(t), Z(t-1), Z(t-2), Z(t-3),..... Also let z(t), z(t-1), z(t-2),.... be deviations from the parameter m (m is the mean for a stationary process); for example, z(t)=Z(t)- m. Then

$$z(t) = \phi_1 z(t-1)+\phi_2 z(t-2)+.....+\phi_p z(t-p)+a(t) \qquad (B.2)$$

is called an autoregressive (AR) process of order p. In Equation (B.2) the variable z is regressed on previous values of itself; hence the model is autoregressive. The model contains p+2 unknown parameters which in practice have to be estimated from the data. Equation (B.2) can be re-written as:

$$\phi(B)z(t) = a(t)$$

where, $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots\ldots - \phi_p B^p$ is the autoregressive operator of order p. It can be seen that the autoregressive model is a special case of the linear filter model (Equation B.1) (Box and Jenkins,1976).

B.6 MOVING AVERAGE (MA) MODEL

A stochastic model in which the current value of the process is linearly dependent on a finite number of previous shocks is called the moving average model. Thus

$$z(t)=a(t)-\theta_1 a(t-1)-\theta_2 a(t-2)-\ldots.-\theta_q a(t-q) \qquad (B.3)$$

is called a moving average (MA) process of order q. The name "moving average" is somewhat misleading because the weights $1, -\theta_1, -\theta_2, \ldots\ldots, -\theta_q$, which multiply the a's, need not total unity nor need they be positive. However, this nomenclature is in common use. Equation (B.3) can be re-written as :

$$z(t) = \theta(B)a(t)$$

where, $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots. - \theta_q B^q$ is the moving average operator of order q. This model contains q+2 unknown parameters which in practice have to be estimated from the data (Box and Jenkins,1976).

B.7 AUTOREGRESSIVE-MOVING AVERAGE (ARMA) MODEL

To achieve greater flexibility in fitting of actual time series, it is sometimes advantageous to include both autoregressive and moving average terms in the model. This leads to the mixed autoregressive moving average model:

$$z(t)=\phi_1 z(t-1)+.....+\phi_p z(t-p)+a(t)-\theta_1 a(t-1)-....-\theta_q a(t-q)$$

or

$$\phi(B)z(t) = \theta(B)a(t) \qquad (B.4)$$

which employs p+q+2 unknown parameters that are estimated from the data (Box and Jenkins,1976).

B.8 AUTOREGRESSIVE INTEGRATED MOVING AVERAGE (ARIMA) MODEL

Many series exhibit nonstationary behavior and in particular do not vary about a fixed mean. Such series may nevertheless exhibit homogeneous behavior of a kind. In particular, although the general level about which fluctuations are occurring may be different at different times, the broad behavior of the series, when differences in level are allowed for, may be similar. Such behavior may be represented by a generalized autoregressive operator $\phi(B)$, in which one or more of the zeroes of the polynomial $\phi(B)$ (that is one or more of the roots of the equation $\phi(B)=0$) is unity. Thus the operator $\phi(B)$ can be written as $\phi(B)=\phi(B)(1-B)^d$ where $\phi(B)$ is a stationary operator. Thus a general model, which can represent homogeneous nonstationary behavior, is of the form:

$$\phi(B)z(t) = \phi(B)(1-B)^d z(t) = \theta(B)a(t)$$

that is

$$\phi(B)w(t) = \theta(B)a(t) \tag{B.5}$$

where, $w(t) = \nabla^d z(t)$ (B.6)

∇ being the backward difference operator defined as

$$\nabla z(t) = z(t) - z(t-1) = (1-B)z(t)$$

and is the inverse of the summation operator S defined as

$$Sz(t) = z(t) + z(t-1) + z(t-2)..........$$

Homogeneous nonstationary behavior can therefore be represented by a model which calls for the d'th difference of the process to be stationary.

The process defined by Equations (B.5) and (B.6) provides a powerful model for describing stationary and nonstationary time series and is called an autoregressive integrated moving average (ARIMA) process of order (p,d,q). The process is defined by

$$w(t)=\phi_1 w(t-1)+...+\phi_p w(t-p)+a(t)-\theta_1 a(t-1)-...-\theta_q a(t-q) \tag{B.7}$$

Thus, the general autoregressive integrated moving average process may be generated from white noise a(t) by means of three filtering operations, namely, the moving average filter, the stationary autoregressive filter and the nonstationary summation filter (Box and Jenkins,1976).

B.9 DUAL MODEL

Let $w(t)$ be generated by the autoregressive moving average (ARMA) process of order (p,q) as :

$$\phi(B)\, w(t) = \theta(B)\, a(t) \tag{B.8}$$

where $a(t)$ is a white noise sequence. If $\theta(B)$ is invertible (i.e., has no roots less than or equal to 1 in magnitude, considered as polynomial in B), then the model

$$\theta(B) z(t) = \phi(B) a(t) \tag{B.9}$$

is also a valid ARMA(q,p) model, and is reffered to as the dual model.

The autocorrelation function (ACF) of dual model is called the inverse autocorrelation function (IACF) of the original model. It can be noted that if the original model was a pure autoregressive one, then the inverse autocorrelation function is an autocorrelation function corresponding to a pure moving average model. Thus, it cuts off sharply when the lag is greater than p. This behavior of inverse autocorrelation function is similar to the behavior of the partial autocorrelation function (PACF).

B.10 YULE-WALKER EQUATION

The recurrence relation for the autocorrelation function of a stationary autoregressive process (see Equation B.2) is given by:

$$\rho_K = \phi_1 \rho_{K-1} + \phi_2 \rho_{K-2} + \ldots + \phi_p \rho_{K-p} \quad k>0$$

If we substitute k=1,2,...,p, we obtain a set of linear equations for $\phi_1, \phi_2, \ldots, \phi_p$ in terms of $\rho_1, \rho_2, \ldots \rho_p$, that is

$$\begin{array}{lllll} \rho_1 = \phi_1 & + \phi_2 \rho_1 & + \ldots + & \phi_p \rho_{p-1} & \\ \rho_2 = \phi_1 \rho_1 & + \phi_2 & + \ldots + & \phi_p \rho_{p-2} & \\ \vdots & \vdots & \vdots & \vdots & \quad (B.10) \\ \rho_p = \phi_1 \rho_{p-1} & + \phi_2 \rho_{p-2} & + \ldots + & \phi_p & \end{array}$$

These equations are called the Yule-Walker equations (Box and Jenkins,1976).

Appendix C

OTHER FORMS OF TRANSFER FUNCTION REPRESENTATION

The impulse response function representation of the Transfer Function in the discrete case and the Transfer Function representation in the continuous case that is used in literature are given in this Appendix.

C.1 IMPULSE RESPONSE FUNCTION

The term $\delta^{-1}(B)w(B)$ in Equation (3.6) can be expanded into an infinite polynomial in B and the equation becomes :

$$y_S = h(B)\ x_{S-b} + N_S \qquad \text{(C.1)}$$

where, h(B) is the Transfer Function, an infinite polynomial in B, defined as :

$$h(B) = 1 - h_1\ B - h_2\ B^2 - \ldots\ldots\ldots\ldots\infty$$

and the coefficients h_i are called the impulse response function. It is to be noted that in Equation (3.7), the system will be a stable one if the series h(B) converges for $|B| \leq 1$. Therefore, if the number of parameters are more than needed, it can lead to an unstable and inaccurate estimation. In using this impulse function notation, one has to stop at the proper number of parameters (Jenkins and

Watts, 1968). This problem can be avoided by the other form of representation (Equation 3.6) the parameters of which can be better estimated and, also, through which the infinite nature of the impulse response function is preserved at the same time.

It is to be noted here that by using the preliminary estimates of the impulse response weights $\hat{h}_i$ of Equation (C.1), the Transfer Function order can be estimated. It was shown that the crosscorrelation function between the prewhitened input and correspondingly transformed output is directly proportional to the impulse response function (Box and Jenkins, 1976, p.380).

C.2 CONTINUOUS TRANSFER FUNCTION

In another form of representation, the discretely coincident system of Equation (3.6) can be expressed by the convolutional integral (Robinson and Sylvia, 1981) as :

$$y = \int_0^\infty h(T)x(s-T)dT \qquad (C.2)$$

$$= h(s)*x(s)$$

$$= x(s)*h(s)$$

for any arbitrary lag of T, if the input x and output y are continuous functions in s. This form of representation is widely used in the field of electrical engineering and geophysics etc., where processing of continuous signals is done extensively.